









TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

EDINBURGH.

VOL. XXV.

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EDINBURGH:

PUBLISHED BY ROBERT GRANT & SON, 54 PRINCES STREET.  
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TRANSACTIONS

ROYAL SOCIETY

EDINBURGH

VOL. XXV



EDINBURGH

PUBLISHED BY WILKIE GIBBS & CO. 21, PRINCE STREET,

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1857

L A W S

OF THE

ROYAL SOCIETY OF EDINBURGH,

AS REVISED 31<sup>ST</sup> OCTOBER 1869.



# L A W S.

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[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

## I.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Title.  
Honorary Fellows.

## II.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.\*

The fees of Ordinary Fellows residing in Scotland.

## III.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from farther payment.

Payment to cease after 25 years.

## IV.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any farther payment. In the case of any Resident Fellow ceasing to reside in Scot-

Fees of Non-Resident Ordinary Fellows.

Case of Fellows becoming Non-Resident.

\* At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s., to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

A modification of this rule, in certain cases, was agreed to 3d January 1831.

land, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

## V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

## VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

## VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

## VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

## IX.

Mode of Recommending Ordinary Fellows.

No person shall be proposed as an Ordinary Fellow without a recommendation subscribed by *One* Ordinary Fellow, to the purport below.\* This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of the election, and shall lie upon the table during that time.

## X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall

\* "A. B., a gentleman well skilled in several branches of Science (*or Polite Literature, as the case may be*), being to my knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, I hereby recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

This recommendation to be accompanied by a request of admission signed by the Candidate.

consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

## XI.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X. Royal Personages.

## XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below\*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election. Recommendation of Honorary Fellows.  
Mode of Election.

## XIII.

The election of Ordinary Fellows shall take place at the Ordinary Meetings of the Society. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote. Election of Ordinary Fellows.

## XIV.

The Ordinary Meetings shall be held on the first and third Mondays of every month from November to June inclusive. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make. Ordinary Meetings.

## XV.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall The Transactions.

\* We hereby recommend \_\_\_\_\_  
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

## XVI.

How Published. The *Transactions* shall be published in Parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

The Council. There shall be elected annually, for conducting the publications and regulating the private business of the Society, a Council, consisting of a President; Six Vice-Presidents, two at least of whom shall be resident; Twelve Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

## XVII.

Retiring Councillors. Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

## XVIII.

Election of Office-Bearers. An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

## XIX.

Special Meetings; how called. Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

## XX.

Treasurer's Duties. The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

## XXI.

Auditor. At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

## XXII.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes, he shall, when necessary, employ a clerk, to be paid by the Society.

General Secretary's  
Duties.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the donors' names, to the Curator of the Library and Museum: they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

Secretaries to  
Ordinary Meetings.

## XXIII.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

Curator of Museum  
and Library.

## XXIV.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint.

Use of Museum  
and Library.

## XXV.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

Register Book.



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## ERRATA.

In Professor TAIT's Paper,

Page 281, line 6, omit the sign of integration.

In Dr DICKSON's Paper,

Page 641, note, line 1, omit "only."

Page 646, line 3, *for* "extremities" *read* "extremity."

Plate XXIX. fig. 14, *for* "pt" *read* "pl."



# TRANSACTIONS.

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## I.—*On Polyzomal Curves, otherwise the Curves*

$$\sqrt{U} + \sqrt{V} + \&c. = 0.$$

By Professor CAYLEY. Communicated by Professor TAIT.

(Read 16th December 1867.)

IF  $U, V, \&c.$ , are rational and integral functions  $(*)(x, y, z)^r$ , all of the same degree  $r$ , in regard to the co-ordinates  $(x, y, z)$ , then  $\sqrt{U} + \sqrt{V} + \&c.$  is a polyzome, and the curve  $\sqrt{U} + \sqrt{V} + \&c. = 0$  a polyzomal curve. Each of the curves  $\sqrt{U} = 0, \sqrt{V} = 0, \&c.$  (or say the curves  $U = 0, V = 0, \&c.$ ) is, on account of its relation of circumscription to the curve  $\sqrt{U} + \sqrt{V} + \&c. = 0$ , considered as a girdle thereto ( $\zeta\hat{\omega}\mu\alpha$ ), and we have thence the term “zome” and the derived expressions “polyzome,” “zomal,” &c. If the number of the zomes  $\sqrt{U}, \sqrt{V}, \&c.$  be  $= \nu$ , then we have a  $\nu$ -zome, and corresponding thereto a  $\nu$ -zomal curve; the curves  $U = 0, V = 0, \&c.$ , are the zomal curves or zomals thereof. The cases  $\nu = 1, \nu = 2$ , are not, for their own sake, worthy of consideration; it is in general assumed that  $\nu$  is  $= 3$  at least. It is sometimes convenient to write the general equation in the form  $\sqrt{lU} + \&c. = 0$ , where  $l, \&c.$  are constants. The Memoir contains researches in regard to the general  $\nu$ -zomal curve; the branches thereof, the order of the curve, its singularities, class, &c.; also in regard to the  $\nu$ -zomal curve  $\sqrt{l(\Theta + L\Phi)} + \&c. = 0$ , where the zomal curves  $\Theta + L\Phi = 0$ , all pass through the points of intersection of the same two curves  $\Theta = 0, \Phi = 0$  of the orders  $r$  and  $r-s$  respectively; included herein we have the theory of the depression of order as arising from the ideal factor or factors of a branch or branches. A general theorem is given of “the decomposition of a tetrazomal curve,” viz., if the equation of the curve be  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} + \sqrt{pT} = 0$ ; then if  $U, V, W, T$  are in involution, that is, connected by an identical equation  $aU + bV + cW + dT = 0$ , and if  $l, m, n, p$ , satisfy the condition  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$ , the tetrazomal curve breaks up into two trizomal curves, each expressible by means of any three of the four functions  $U, V, W, T$ ; for

example, in the form  $\sqrt{l} \bar{U} + \sqrt{m} \bar{V} + \sqrt{p} \bar{T} = 0$ . If, in this theorem, we take  $p = 0$ , then the original curve is the trizomal  $\sqrt{l} \bar{U} + \sqrt{m} \bar{V} + \sqrt{n} \bar{W} = 0$ ,  $T$  is any function  $= -\frac{1}{d}(aU + bV + cW)$ , where, considering  $l, m, n$  as given,  $a, b, c$  are quantities subject only to the condition  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ , and we have the theorem of "the variable zomal of a trizomal curve," viz., the equation of the trizomal  $\sqrt{l} \bar{U} + \sqrt{m} \bar{V} + \sqrt{n} \bar{W} = 0$ , may be expressed by means of any two of the three functions  $U, V, W$ , and of a function  $T$  determined as above, for example in the form  $\sqrt{l} \bar{U} + \sqrt{m} \bar{V} + \sqrt{n} \bar{T} = 0$ ; whence also it may be expressed in terms of three new functions  $T$ , determined as above. This theorem, which occupies a prominent position in the whole theory, was suggested to me by Mr Casey's theorem, presently referred to, for the construction of a bicircular quartic as the envelope of a variable circle.

In the  $\nu$ -zomal curve  $\sqrt{l(\Theta + L\Phi)} + \&c. = 0$ , if  $\Theta = 0$  be a conic,  $\Phi = 0$  a line, the zomals  $\Theta + L\Phi = 0$ ,  $\&c.$  are conics passing through the same two points  $\Theta = 0, \Phi = 0$ , and there is no real loss of generality in taking these to be the circular points at infinity—that is, in taking the conics to be circles. Doing this, and using a special notation  $\mathbf{A}^\circ = 0$  for the equation of a circle having its centre at a given point  $A$ , and similarly  $\mathbf{A} = 0$  for the equation of an evanescent circle, or say of the point  $A$ , we have the  $\nu$ -zomal curve  $\sqrt{l\mathbf{A}^\circ} + \&c. = 0$ , and the more special form  $\sqrt{l\mathbf{A}} + \&c. = 0$ . As regards the last-mentioned curve,  $\sqrt{l\mathbf{A}} + \&c. = 0$ , the point  $A$  to which the equation  $\mathbf{A} = 0$  belongs, is a focus of the curve, viz., in the case  $\nu = 3$ , it is an ordinary focus, and in the case  $\nu > 3$ , it is a special kind of focus, which, if the term were required, might be called a foco-focus; the Memoir contains an explanation of the general theory of the foci of plane curves. For  $\nu = 3$ , the equation  $\sqrt{l\mathbf{A}} + \sqrt{m\mathbf{B}} + \sqrt{n\mathbf{C}} = 0$  is really equivalent to the apparently more general form  $\sqrt{l\mathbf{A}^\circ} + \sqrt{m\mathbf{B}^\circ} + \sqrt{n\mathbf{C}^\circ} = 0$ . In fact, this last is in general a bicircular quartic, and, in regard to it, the before-mentioned theorem of the variable zomal becomes Mr Casey's theorem, that "the bicircular quartic (and, as a particular case thereof, the circular cubic) is the envelope of a variable circle having its centre on a given conic and cutting at right angles a given circle." This theorem is a sufficient basis for the complete theory of the trizomal curve  $\sqrt{l\mathbf{A}^\circ} + \sqrt{m\mathbf{B}^\circ} + \sqrt{n\mathbf{C}^\circ} = 0$ ; and it is thereby very easily seen that the curve  $\sqrt{l\mathbf{A}^\circ} + \sqrt{m\mathbf{B}^\circ} + \sqrt{n\mathbf{C}^\circ} = 0$  can be represented by an equation  $\sqrt{l\mathbf{A}'} + \sqrt{m'\mathbf{B}'} + \sqrt{n'\mathbf{C}'} = 0$ . But for  $\nu > 3$  this is not so, and the curve  $\sqrt{l\mathbf{A}} + \&c. = 0$  is only a particular form of the curve  $\sqrt{l\mathbf{A}^\circ} + \&c. = 0$ ; and the discussion of this general form is scarcely more difficult than that of the special form  $\sqrt{l\mathbf{A}} + \&c. = 0$ , included therein. The investigations in relation to the theory of foci, and in particular to that of the foci of the circular cubic and bicircular quartic, precede in the Memoir the theories of the trizomal curve

$\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} = 0$ , and the tetrazomal curve  $\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} + \sqrt{pD^\circ} = 0$ , to which the concluding portions relate. I have accordingly divided the Memoir into four parts, viz., these are—Part I, On Polyzomal Curves in general; Part II., Subsidiary Investigations; Part III., On the Theory of Foci; and Part IV., On the Trizomal and Tetrazomal Curves where the zomals are circles. There is, however, some necessary intermixture of the theories treated of, and the arrangement will appear more in detail from the headings of the several articles. The paragraphs are numbered continuously through the Memoir. There are four Annexes, relating to questions which it seemed to me more convenient to treat of thus separately.

It is right that I should explain the very great extent to which, in the composition of the present Memoir, I am indebted to Mr Casey's researches. His Paper "On the Equations and Properties (1.) of the System of Circles touching three circles in a plane; (2.) of the System of Spheres touching four spheres in space; (3.) of the System of Circles touching three circles on a sphere; (4.) on the System of Conics inscribed in a conic and touching three inscribed conics in a plane," was read to the Royal Irish Academy, April 9, 1866, and is published in their "Proceedings." The fundamental theorem for the equation of the pairs of circles touching three given circles was, previous to the publication of the paper, mentioned to me by Dr Salmon, and I communicated it to Professor Cremona, suggesting to him the problem solved in his letter of March 3, 1866, as mentioned in my paper, "Investigations in connection with Casey's Equation," "Quarterly Math. Journal," t. viii. 1867, pp. 334–341, and as also appears, Annex No. IV. of the present Memoir.

In connection with this theorem, I communicated to Mr Casey, in March or April 1867, the theorem No. 164 of the present Memoir, that for any three given circles, centres  $A, B, C$ , the equation  $\overline{BC} \sqrt{A^\circ} + \overline{CA} \sqrt{B^\circ} + \overline{AB} \sqrt{C^\circ} = 0$  (where  $\overline{BC}, \overline{CA}, \overline{AB}$ , denote the mutual distances of the points  $A, B, C$ ) belongs to a Cartesian. Mr Casey, in a letter to me dated 30th April 1867, informed me of his own mode of viewing the question as follows:—"The general equation of the second order  $(a, b, c, f, g, h) (a, \beta, \gamma)^2 = 0$ , where  $a, \beta, \gamma$  are circles, is a bicircular quartic. If we take the equation  $(a, b, c, f, g, h) (\lambda, \mu, \nu)^2 = 0$  in tangential co-ordinates (that is, when  $\lambda, \mu, \nu$  are perpendiculars let fall from the centres of  $a, \beta, \gamma$  on any line), it denotes a conic; denoting this conic by  $F$ , and the circle which cuts  $a, \beta, \gamma$  orthogonally by  $J$ , I proved that, if a variable circle moves with its centre on  $F$ , and if it cuts  $J$  orthogonally, its envelope will be the bicircular quartic whose equation is that written down above;" and among other consequences, he mentions that the foci of  $F$  are the double foci of the quartic, and the points in which  $J$  cuts  $F$  single foci of the quartic, and also the theorem which I had sent him as to the Cartesian, and he refers to his Memoir on Bicircular Quartics as then nearly finished. An Abstract of the Memoir as read before the Royal Irish

Academy, 10th February 1867, and published in their "Proceedings," pp. 44, 45, contains the theorems mentioned in the letter of 30th April, and some other theorems. It is not necessary that I should particularly explain in what manner the present Memoir has been, in the course of writing it, added to or altered in consequence of the information which I have thus had of Mr Casey's researches; it is enough to say that I have freely availed myself of such information, and that there is no question as to Mr Casey's priority in anything which there may be in common in his memoir on Bicircular Quartics and in the present Memoir.

PART I. (Nos. 1 to 55).—ON POLYZOMAL CURVES IN GENERAL.

*Definition and Preliminary Remarks*—Art. Nos. 1 to 4.

1. As already mentioned,  $U, V, \&c.$  denote rational and integral functions (\*)  $(x, y, z)^r$ , all of the same degree  $r$  in the co-ordinates  $(x, y, z)$ , and the equation

$$\sqrt{U} + \sqrt{V} + \&c. = 0$$

then belongs to a polyzomal curve, viz., if the number of the zomes  $\sqrt{U}, \sqrt{V}, \&c.$ , is  $= \nu$ , then we have a  $\nu$ -zomal curve. The radicals, or any of them, may contain rational factors, or be of the form  $P\sqrt{Q}$ ; but in speaking of the curve as a  $\nu$ -zomal, it is assumed that any two terms, such as  $P\sqrt{Q} + P'\sqrt{Q}$ , involving the same radical  $\sqrt{Q}$ , are united into a single term, so that the number of distinct radicals is always  $= \nu$ ; in particular ( $r$  being even), it is assumed that there is only one rational term  $P$ . But the ordinary case, and that which is almost exclusively attended to, is that in which the radicals  $\sqrt{U}, \sqrt{V}, \&c.$  are distinct irreducible radicals without rational factors.

2. The curves  $U = V = 0, \&c.$  are said to be the zomal curves, or simply the zomals of the polyzomal curve  $\sqrt{U} + \sqrt{V} + \&c. = 0$ ; more strictly, the term zomal would be applied to the functions  $U, V, \&c.$  It is to be noticed, that although the form  $\sqrt{U} + \sqrt{V} + \&c. = 0$  is equally general with the form  $\sqrt{lU} + \sqrt{mV} + \&c. = 0$  (in fact, in the former case, the functions  $U, V, \&c.$ , are considered as implicitly containing the constant factors  $l, m, \&c.$ , which are expressed in the latter case), yet it is frequently convenient to express these factors, and thus write the equation in the form  $\sqrt{lU} + \sqrt{mV} + \&c.$  For instance, in speaking of any given curves  $U = 0, V = 0, \&c.$ , we are apt, disregarding the constant factors which they may involve, to consider  $U, V, \&c.$  as given functions; but in this case the general equation of the polyzomal with the zomals  $U = 0, V = 0, \&c.$ , is of course  $\sqrt{lU} + \sqrt{mV} + \&c. = 0$ .

3. Anticipating in regard to the cases  $\nu = 1, \nu = 2$ , the remark which will be presently made in regard to the  $\nu$ -zomal, that  $\sqrt{U} + \sqrt{V} + \&c. = 0$  is the curve represented by the rationalised form of this equation, the monozomal curve  $\sqrt{U} = 0$  is merely the curve  $U = 0$ , viz., this is any curve whatever  $U = 0$  of the

order  $r$ ; and similarly, the bizomal curve  $\sqrt{U} + \sqrt{V} = 0$  is merely the curve  $U - V = 0$ , viz. this is any curve whatever  $\Omega = 0$ , of the order  $r$ ; the zomal curves  $U = 0$ ,  $V = 0$ , taken separately, are not curves standing in any special relation to the curve in question  $\Omega = 0$ , but  $U = 0$  may be any curve whatever of the order  $r$ , and then  $V = 0$  is a curve of the same order  $r$ , in involution with the two curves  $\Omega = 0$ ,  $U = 0$ ; we may, in fact, write the equation  $\Omega = 0$  under the bizomal form  $\sqrt{U} + \sqrt{\Omega + U} = 0$ . In the case  $r$  even, we may, however, notice the bizomal curve  $P + \sqrt{U} = 0$  ( $P$  a rational function of the degree  $\frac{1}{2}r$ ); the rational equation is here  $\Omega = U - P^2 = 0$ , that is  $U = \Omega + P^2$ , viz.,  $P$  is any curve whatever of the order  $\frac{1}{2}r$ , and  $U = 0$  is a curve of the order  $r$ , touching the given curve  $\Omega = 0$  at each of its  $\frac{1}{2}r^2$  intersections with the curve  $P = 0$ . I further remark that the order of the  $\nu$ -zomal curve  $\sqrt{V} + \&c. = 0$  is  $= 2^{\nu-2}r$ ; this is right in the case of the bizomal curve  $\sqrt{U} + \sqrt{V} = 0$ , the order being  $= r$ , but it fails for the monozomal curve  $\sqrt{U} = 0$ , the order being in this case  $r$ , instead of  $\frac{1}{2}r$ , as given by the formula. The two unimportant and somewhat exceptional cases  $\nu = 1$ ,  $\nu = 2$ , are thus disposed of, and in all that follows (except in so far as this is in fact applicable to the cases just referred to),  $\nu$  may be taken to be  $= 3$  at least.

4. It is to be throughout understood that by the curve  $\sqrt{U} + \sqrt{V} + \&c. = 0$  is meant the curve represented by the rationalised equation—

$$\text{Norm} (\sqrt{U} + \sqrt{V} + \&c.) = 0$$

viz. the Norm is obtained by attributing to all but one of the zomes  $\sqrt{U}$ ,  $\sqrt{V}$ , &c., each of the two signs  $+$ ,  $-$ , and multiplying together the several resulting values of the polyzome; in the case of a  $\nu$ -zomal curve, the number of factors is thus  $= 2^{\nu-1} r$  (whence, as each factor is of the degree  $\frac{1}{2}r$ , the order of the curve is  $2^{\nu-1} \cdot \frac{1}{2}r = 2^{\nu-2} r$ , as mentioned above). I expressly mention that, as regards the polyzomal curve, we are not in any wise concerned with the signs of the radicals, which signs are and remain essentially indeterminate; the equation  $\sqrt{U} + \sqrt{V} + \&c. = 0$ , is a mere symbol for the rationalised equation, Norm  $(\sqrt{U} + \sqrt{V} + \&c.) = 0$ .

*The Branches of a Polyzomal Curve—Art. Nos. 5 to 12.*

5. But we may in a different point of view attend to the signs of the radicals; if for all values of the co-ordinates we take the symbol  $\sqrt{\quad}$ , and consider  $\sqrt{U}$ ,  $\sqrt{V}$ , &c. as signifying determinately, say the *positive* values of  $\sqrt{U}$ ,  $\sqrt{V}$ , &c.; then each of the several equations  $\pm \sqrt{U} \pm \sqrt{V} + \&c. = 0$ , or, fixing at pleasure one of the signs, suppose that prefixed to  $\sqrt{U}$ , then each of the several equations  $\sqrt{U} \pm \sqrt{V} \pm \&c. = 0$ , will belong to a *branch* of the polyzomal curve: a  $\nu$ -zomal curve has thus  $2^{\nu-1}$  branches corresponding to the  $2^{\nu-1}$  values respec-

tively of the polyzome. The separation of the branches depends on the precise fixation of the significations of  $\sqrt{U}$ ,  $\sqrt{V}$ , &c., and in regard hereto some further explanation is necessary.

6. When  $U$  is real and positive,\*  $\sqrt{U}$  may be taken to be, in the ordinary sense, the positive value of  $\sqrt{U}$ , and so when  $U$  is real and negative,  $\sqrt{U}$  may be taken to be  $= i$  into the positive value of  $\sqrt{-U}$ ; and the like as regards  $\sqrt{V}$ , &c. The functions  $U$ ,  $V$ , &c., are assumed to be real functions of the co-ordinates; hence, for any real values of the co-ordinates,  $U$ ,  $V$ , &c. are real positive or negative quantities, and the significations of  $\sqrt{U}$ ,  $\sqrt{V}$ , &c. are completely determined.

7. But the co-ordinates may be imaginary. In this case the functions  $U$ ,  $V$ , &c. will for any given values of the co-ordinates acquire each of them a determinate, in general imaginary, value. If for all real values whatever of  $a$ ,  $\beta$ , we select once for all one of the two opposite values of  $\sqrt{a + \beta i}$ , calling it the *positive* value, and representing it by  $\sqrt{a + \beta i}$ , then, for any particular values of the co-ordinates,  $U$  being  $= a + \beta i$ , the value of  $\sqrt{U}$  may be taken to be  $= \sqrt{a + \beta i}$ ; and the like as regards  $\sqrt{V}$ , &c.  $\sqrt{U}$ ,  $\sqrt{V}$ , &c. have thus each of them a determinate signification for any values whatever, real or imaginary, of the co-ordinates. The co-ordinates of a given point on the curve  $\sqrt{U} + \sqrt{V} + \text{\&c.} = 0$ , will in general satisfy only one of the equations  $\sqrt{U} \pm \sqrt{V} \pm \text{\&c.} = 0$ ; that is, the point will belong to one (but in general only one) of the  $2^{v-1}$  branches of the curve; the entire series of points the co-ordinates of which satisfy any one of the  $2^{v-1}$  equations, will constitute the branch corresponding to that equation.

8. The signification to be attached to the expression  $\sqrt{a + \beta i}$  should agree with that previously attached to the like symbol in the case of a positive or negative real quantity; and it should, as far as possible, be subject to the condition of continuity, viz., as  $a + \beta i$  passes continuously to  $a' + \beta' i$ , so  $\sqrt{a + \beta i}$  should pass continuously to  $\sqrt{a' + \beta' i}$ ; but (as is known) it is not possible to satisfy universally this condition of continuity; viz., if for facility of explanation we consider  $(a, \beta)$  as the co-ordinates of a point in a plane, and imagine this point to describe a closed curve surrounding the origin or point  $(0, 0)$ , then it is not possible so to define  $\sqrt{a + \beta i}$  that this quantity, varying continuously as the point moves along the curve, shall, when the point has made a complete circuit, resume its original value. The signification to be attached to  $\sqrt{a + \beta i}$  is thus in some measure arbitrary, and it would appear that the division of the curve into branches is affected by a corresponding arbitrariness, but this arbitrariness relates only to the imaginary branches of the curve: the notion of a real branch is perfectly definite.

9. It would seem that a branch may be impossible for any series whatever of points real or imaginary. Thus, in the bizomal curve  $\sqrt{U} + \sqrt{V} = 0$ , the branch  $\sqrt{U} + \sqrt{V} = 0$  is impossible. In fact, for any point whatever, real or

imaginary, of the curve, we have  $U = V$ , and therefore  $\sqrt{U} = \sqrt{V}$ ; the point thus belongs to the other branch  $\sqrt{U} - \sqrt{V} = 0$ , not to the branch  $\sqrt{U} + \sqrt{V} = 0$ ; the only points belonging to the last-mentioned branch are the isolated points for which simultaneously  $\sqrt{U} = 0$ ,  $\sqrt{V} = 0$ ; viz., the points of intersection of the two curves  $U = 0$ ,  $V = 0$ .

10. It is not clear to me whether the case is the same in regard to the branch  $\sqrt{U} + \sqrt{V} + \sqrt{W} = 0$  of a trizomal curve. In fact, for each point of the curve  $\sqrt{U} + \sqrt{V} + \sqrt{W} = 0$  we have  $(U - V - W)^2 = 4 VW$ , and therefore,  $U - V - W = \pm 2 \sqrt{V} \sqrt{W}$ ; there may very well be points for which the sign is +; that is, points for which  $U = V + W + 2 \sqrt{V} \sqrt{W}$ , and for these points we have  $\pm \sqrt{U} = \sqrt{V} + \sqrt{W}$ ; for real values of the co-ordinates the sign on the left hand must be + (for otherwise the two sides would have opposite signs), but there is no apparent reason, or at least no obviously apparent reason, why this should be so for imaginary values of the co-ordinates, and if the sign be in fact —, then the point will belong to the branch  $\sqrt{U} + \sqrt{V} + \sqrt{W} = 0$ .

11. But the branch in question is clearly impossible for any series of real points; so that, leaving it an open question whether the epithet “impossible” is to be understood to mean impossible for any series of real points (that is, as a mere synonym of imaginary), or whether it is to mean impossible for any series of points, real or imaginary, whatever, I say that in a  $\nu$ -zomal curve some of the branches are or may be impossible, and that there is at least one impossible branch, viz. the branch  $\sqrt{U} + \sqrt{V} + \&c. = 0$ .

12. For the purpose of referring to any branch of a polyzomal curve it will be convenient to consider  $\sqrt{U}$  as signifying determinately  $+\sqrt{U}$ , or else  $-\sqrt{U}$ ; and the like as regards  $\sqrt{V}$ , &c., but without any identity or relation between the signs prefixed to the  $\sqrt{U}$ ,  $\sqrt{V}$ , &c., respectively; the equation  $\sqrt{U} + \sqrt{V} + \&c. = 0$ , so understood, will denote determinately some one (that is, any one at pleasure) of the equations  $\sqrt{U} \pm \sqrt{V} \pm \&c. = 0$ , and it will thus be the equation of some one (that is, any one at pleasure) of the branches of the polyzomal curve — all risk of ambiguity which might otherwise exist will be removed if we speak either of the *curve*  $\sqrt{U} + \sqrt{V}, \&c. = 0$ , or else of the *branch*  $\sqrt{U} + \sqrt{V} + \&c. = 0$ . Observe that by the foregoing convention, when only one branch is considered, we avoid the necessity of any employment of the sign  $\pm$ , or of the sign —; but when two or more branches are considered in connection with each other, it is necessary to employ the sign — with one or more of the radicals  $\sqrt{U}$ ,  $\sqrt{V}$ , &c.; thus in the trizomal curve  $\sqrt{U} + \sqrt{V} + \sqrt{W} = 0$ , we may have to consider the branches  $\sqrt{U} + \sqrt{V} + \sqrt{W} = 0$ ,  $\sqrt{U} + \sqrt{V} - \sqrt{W} = 0$ ; viz., either of these equations apart from the other denotes any one branch at pleasure of the curve, but when the branch represented by the one equation is fixed, then the branch represented by the other equation is also fixed.

*The Points common to Two Branches of a Polyzomal Curve—Art. Nos. 13 to 17.*

13. I consider the points which are situate simultaneously on two branches of the  $\nu$ -zomal curve  $\sqrt{U} + \sqrt{V} + \&c. = 0$ . The equations of the two branches may be taken to be

$$\begin{aligned}\sqrt{U} + \&c. + (\sqrt{W} + \&c.) &= 0, \\ \sqrt{U} + \&c. - (\sqrt{W} + \&c.) &= 0,\end{aligned}$$

viz., fixing the significations of  $\sqrt{U}$ ,  $\sqrt{V}$ ,  $\sqrt{W}$ , &c. in such wise that in the equation of one branch these shall each of them have the sign +, we may take  $\sqrt{U}$ , &c. to be those radicals which, in the equation of the other branch, have the sign +, and  $\sqrt{W}$ , &c. to be those radicals which have the sign —. The foregoing equations break up into the more simple equations

$$\sqrt{U} + \&c. = 0, \quad \sqrt{W} + \&c. = 0,$$

which are the equations of certain *branches* of the *curves*  $\sqrt{U} + \&c. = 0$ , and  $\sqrt{W} + \&c. = 0$ , respectively, and conversely each of the intersections of these two curves is a point situate simultaneously on some two branches of the original  $\nu$ -zomal curve  $\sqrt{U} + \sqrt{V} + \&c. = 0$ . Hence, partitioning in any manner the  $\nu$ -zome  $\sqrt{U} + \sqrt{V} + \&c.$  into an  $\alpha$ -zome,  $\sqrt{U} + \&c.$  and a  $\beta$ -zome  $\sqrt{W} + \&c.$  ( $\alpha + \beta = \nu$ ), and writing down the equations

$$\sqrt{U} + \&c. = 0, \quad \sqrt{W} + \&c. = 0$$

of an  $\alpha$ -zomal curve and a  $\beta$ -zomal curve respectively, each of the intersections of these two curves is a point situate simultaneously on two branches of the  $\nu$ -zomal curve; and the points situate simultaneously on two branches of the  $\nu$ -zomal curve are the points of intersection of the several pairs of an  $\alpha$ -zomal curve and a  $\beta$ -zomal curve, which can be formed by any bipartition of the  $\nu$ -zome.

14. There are two cases to be considered:—First, when the parts are 1,  $\nu - 1$  ( $\nu - 1$  is  $> 1$ , except in the case  $\nu = 2$ , which may be excluded from consideration), or say when the  $\nu$ -zome is partitioned into a *zome* and *antizome*. Secondly, when the parts  $\alpha$ ,  $\beta$ , are each  $> 1$  (this implies  $\nu = 4$  at least), or say when the  $\nu$ -zome is partitioned into a pair of *complementary parazomes*.

15. To fix the ideas, take the tetrazomal curve  $\sqrt{U} + \sqrt{V} + \sqrt{W} + \sqrt{T} = 0$ , and consider first a point for which  $\sqrt{U} = 0$ ,  $\sqrt{V} + \sqrt{W} + \sqrt{T} = 0$ . The Norm is the product of ( $2^3 =$ ) 8 factors; selecting hereout the factors

$$\begin{aligned}\sqrt{U} + \sqrt{V} + \sqrt{W} + \sqrt{T}, \\ \sqrt{U} - \sqrt{V} - \sqrt{W} - \sqrt{T},\end{aligned}$$

let the product of these

$$= U - (\sqrt{V} + \sqrt{W} + \sqrt{T})^2$$

be called  $F$ , and the product of the remaining six factors be called  $G$ ; the

rationalised equation of the curve is therefore  $FG = 0$ . The derived equation is  $GdF + FdG = 0$ ; at the point in question  $\sqrt{U} = 0, \sqrt{V} + \sqrt{W} + \sqrt{T} = 0$ ;  $G$  and  $dG$  are each of them finite (that is, they neither vanish nor become infinite), but we have

$$F = 0, dF = dU - (\sqrt{V} + \sqrt{W} + \sqrt{T})(dV \div \sqrt{V} + dW \div \sqrt{W} + dT \div \sqrt{T}), = dU,$$

and the derived equation is thus  $GdU = 0$ , or simply  $dU = 0$ . It thus appears that the point in question is an ordinary point on the tetrazomal curve; and, further, that the tetrazomal curve is at this point touched by the zomal curve  $U = 0$ . And similarly, each of the points of intersection of the two curves  $\sqrt{U} = 0, \sqrt{V} + \sqrt{W} + \sqrt{T} = 0$ , is an ordinary point on the tetrazomal curve; and the tetrazomal curve is at each of these points touched by the zomal curve  $U = 0$ .

16. Consider, secondly, a point for which  $\sqrt{U} + \sqrt{V} = 0, \sqrt{W} + \sqrt{T} = 0$ ; to form the Norm, taking in this case the two factors

$$\begin{aligned} &\sqrt{U} + \sqrt{V} + \sqrt{W} + \sqrt{T}, \\ &\sqrt{U} + \sqrt{V} - \sqrt{W} - \sqrt{T}, \end{aligned}$$

let their product

$$= (\sqrt{U} + \sqrt{V})^2 - (\sqrt{W} + \sqrt{T})^2$$

be called  $F$ , and the product of the remaining six factors be called  $G$ ; the rationalised equation is  $FG = 0$ , and the derived equation is  $FdG + GdF = 0$ . At the point in question  $G$  and  $dG$  are each of them finite (that is, they neither vanish nor become infinite), but we have

$$F = 0, dF = (\sqrt{U} + \sqrt{V})(dU \div \sqrt{U} + dV \div \sqrt{V}) - (\sqrt{W} + \sqrt{T})(dW \div \sqrt{W} + dT \div \sqrt{T}), = 0,$$

that is, the derived equation becomes identically  $0 = 0$ ; the point in question is thus a singular point, and it is easy to see that it is in fact a node, or ordinary double point, on the tetrazomal curve. And similarly, each of the points of intersection of the two curves  $\sqrt{U} + \sqrt{V} = 0, \sqrt{W} + \sqrt{T} = 0$  is a node on the tetrazomal curve.

17. The proofs in the foregoing two examples respectively are quite general, and we may, in regard to a  $\nu$ -zomal curve, enunciate the results as follows, viz., in a  $\nu$ -zomal curve, the points situate simultaneously on two branches are either the intersections of a zomal curve and its antizomal curve, or else they are the intersections of a pair of complementary parazomal curves. In the former case, the points in question are ordinary points on the  $\nu$ -zomal, but they are points of contact of the  $\nu$ -zomal with the zomal; it may be added, that the intersections of the zomal and antizomal, each reckoned twice, are *all* the intersections of the  $\nu$ -zomal and zomal. In the latter case, the points in question are nodes of the  $\nu$ -zomal; it may be added, that the  $\nu$ -zomal has not, *in general*, any nodes other than the points which are thus the intersections of a pair of complementary parazomals, and that it has not *in general* any cusps.

*Singularities of a  $\nu$ -zomal Curve—Art. Nos. 18 to 21.*

18. It has been already shown that the order of the  $\nu$ -zomal curve is  $= 2^{\nu-2}r$ . Considering the case where  $\nu$  is  $= 3$  at least, the curve, as we have just seen, has contacts with each of the zomal curves, and it has also nodes. I proceed to determine the number of these contacts and nodes respectively.

19. Consider first the zomal curve  $U = 0$ , and its antizomal  $\sqrt{V} + \sqrt{W} + \&c. = 0$ , these are curves of the orders  $r$  and  $2^{\nu-3}r$  respectively, and they intersect therefore in  $2^{\nu-3}r^2$  points. Hence the  $\nu$ -zomal touches the zomal in  $2^{\nu-3}r^2$  points, and reckoning each of these twice, the number of intersections is  $= 2^{\nu-2}r^2$ , viz., these are all the intersections of the  $\nu$ -zomal with the zomal  $U = 0$ . The number of contacts of the  $\nu$ -zomal with the several zomals  $U = 0$ ,  $V = 0$ , &c., is of course  $= 2^{\nu-3}r^2\nu$ .

20. Considering next a pair of complementary parazomal curves, an  $\alpha$ -zomal and a  $\beta$ -zomal respectively ( $\alpha + \beta = \nu$ ), these are of the orders  $2^{\alpha-2}r$  and  $2^{\beta-2}r$  respectively, and they intersect therefore in  $2^{\alpha+\beta-4}r^2 = 2^{\nu-4}r^2$  points, nodes of the  $\nu$ -zomal. This number is independent of the particular partition ( $\alpha, \beta$ ), and the  $\nu$ -zomal has thus this same number,  $2^{\nu-4}r^2$ , of nodes in respect of each pair of complementary parazomals; hence the total number of nodes is  $= 2^{\nu-4}r^2$  into the number of pairs of complementary parazomals. For the partition ( $\alpha, \beta$ ) the number of pairs is  $= [\nu]^\nu \div [\alpha]^\alpha [\beta]^\beta$ , or when  $\alpha = \beta$ , which of course implies  $\nu$  even, it is one-half of this; extending the summation from  $\alpha = 2$  to  $\alpha = \nu - 2$ , each pair is obtained twice, and the number of pairs is thus  $= \frac{1}{2} \sum \{ [\nu]^\nu \div [\alpha]^\alpha [\beta]^\beta \}$ ; the sum extended from  $\alpha = 0$  to  $\alpha = \nu$  is  $(1 + 1)^\nu = 2^\nu$ , but we thus include the terms 1,  $\nu$ ,  $\nu$ , 1, which are together  $= 2\nu + 2$ , hence the correct value of the sum is  $= 2^\nu - 2\nu - 2$ , and the number of pairs is the half of this  $= 2^{\nu-1} - \nu - 1$ . Hence the number of nodes of the  $\nu$ -zomal curve is  $= (2^{\nu-1} - \nu - 1) 2^{\nu-4}r^2$ .

21. The  $\nu$ -zomal is thus a curve of the order  $2^{\nu-2}r$ , with  $(2^{\nu-1} - \nu - 1) 2^{\nu-4}r^2$  nodes, but without cusps; the class is therefore

$$= 2^{\nu-3}r [(\nu + 1)r - 2],$$

and the deficiency is

$$= 2^{\nu-4}r [(\nu + 1)r - 6] + 1.$$

These are the general expressions, but even when the zomal curves  $U = 0$ ,  $V = 0$ , &c., are given, then writing the equation of the  $\nu$ -zomal under the form  $\sqrt{lU} + \sqrt{mV} + \&c. = 0$ , the constants  $l : m : \&c.$ , may be so determined as to give rise to nodes or cusps which do not occur in the general case; the formulæ will also undergo modification in the particular cases next referred to.

*Special Case where all the Zomals have a Common Point or Points—Art. Nos. 22 to 27.*

22. Consider the case where the zomals  $U = 0$ ,  $V = 0$  have all of them any number, say  $k$ , of common intersections—these may be referred to simply as the common points. Each common point is a  $2^{\nu-2}$ -tuple point on the  $\nu$ -zomal curve; it is on each zomal an ordinary point, and on each antizomal a  $2^{\nu-3}$ -tuple point, and on any  $\alpha$ -zomal parazomal a  $2^{\alpha-2}$ -tuple point. Hence, considering first the intersections of any zomal with its antizomal, the common point reckons as  $2^{\nu-3}$  intersections, and the  $k$  common points reckon as  $2^{\nu-3} k$  intersections; the number of the remaining intersections is therefore  $= 2^{\nu-3} (r^2 - k)$ , and the zomal touches the  $\nu$ -zomal in each of these points. The intersections of the zomal with the  $\nu$ -zomal are the  $k$  common points, each of them a  $2^{\nu-2}$ -tuple point on the  $\nu$ -zomal, and therefore reckoning together as  $2^{\nu-2} k$  intersections; and the  $2^{\nu-3} (r^2 - k)$  points of contact, each reckoning twice, and therefore together as  $2^{\nu-2} (r^2 - k)$  intersections ( $2^{\nu-2} k + 2^{\nu-2} (r^2 - k) = 2^{\nu-2} r^2 = r \cdot 2^{\nu-2} r$ ); the total number of contacts with the zomals  $U = 0$ ,  $V = 0$ , &c., is thus  $= 2^{\nu-3} (r^2 - k) \nu$ .

23. Secondly, considering any pair of complementary parazomals, an  $\alpha$ -zomal and a  $\beta$ -zomal, each of the common points, being a  $2^{\alpha-2}$ -tuple point and a  $2^{\beta-2}$ -tuple point on the two curves respectively, counts as  $2^{\alpha+\beta-4} = 2^{\nu-4}$  intersections, and the  $k$  common points count as  $2^{\nu-4} k$  intersections; the number of the remaining intersections is therefore  $= 2^{\nu-4} (r^2 - k)$ , each of which is a node on the  $\nu$ -zomal curve; and we have thus in all  $2^{\nu-4} (2^{\nu-1} - \nu - 1) (r^2 - k)$  nodes.

24. There are, besides, the  $k$  common points, each of them a  $2^{\nu-2}$ -tuple point on the  $\nu$ -zomal, and therefore each reckoning as  $\frac{1}{2} 2^{\nu-2} (2^{\nu-2} - 1) = 2^{2\nu-5} - 2^{\nu-3}$  double points, or together as  $(2^{2\nu-5} - 2^{\nu-3}) k$  double points. Reserving the term *node* for the above-mentioned nodes or proper double points, and considering, therefore, the double points (dps.) as made up of the nodes and of the  $2^{\nu-2}$ -tuple points, the total number of dps. is thus

$$\begin{aligned} & 2^{\nu-4} (2^{\nu-1} - \nu - 1) (r^2 - k) + (2^{2\nu-5} - 2^{\nu-3}) k, \\ & = 2^{\nu-4} (2^{\nu-1} - \nu - 1) r^2 + (\nu + 1) 2^{\nu-4} - 2^{\nu-3} k; \end{aligned}$$

or finally this is

$$= 2^{\nu-4} \{ (2^{\nu-1} - \nu - 1) r^2 + (\nu - 1) \};$$

so that there is a gain  $= 2^{\nu-4} (\nu - 1) k$  in the number of dps. arising from the  $k$  common points. There is, of course, in the class a diminution equal to twice this number, or  $2^{\nu-3} (\nu - 1) k$ ; and in the deficiency a diminution equal to this number, or  $2^{\nu-4} (\nu - 1) k$ .

25. The zomal curves  $U = 0$ ,  $V = 0$ , &c., may all of them pass through the same  $\nu^2$  points; we have then  $k = r^2$ , and the expression for the number of dps.

is  $= (2^{2\nu-5} - 2^{\nu-3})r^2$ , viz., this is  $= \frac{1}{2} 2^{\nu-2} (2^{\nu-2} - 1)r^2$ . But in this case the dps. are nothing else than the  $r^2$  common points, each of them a  $2^{\nu-2}$ -tuple point, the  $\nu$ -zomal curve in fact breaking up into a system of  $2^{\nu-2}$  curves of the order  $r$ , each passing through the  $r^2$  common points. This is easily verified, for if  $\Theta = 0$ ,  $\Phi = 0$  are some two curves of the order  $r$ , then, in the present case, the zomal curves are curves in involution with these curves; that is, they are curves of the form  $l\Theta + l'\Phi = 0$ ,  $m\Theta + m'\Phi = 0$ , &c., and the equation of the  $\nu$ -zomal curve is

$$\sqrt{l\Theta + l'\Phi} + \sqrt{m\Theta + m'\Phi} + \&c. = 0.$$

The rationalised equation is obviously an equation of the degree  $2^{\nu-2}$  in  $\Theta$ ,  $\Phi$ , giving therefore a constant value for the ratio  $\Theta : \Phi$ ; calling this  $q$ , or writing  $\Theta = q\Phi$ , we have

$$\sqrt{lq + l'} + \sqrt{mq + m'} + \&c. = 0,$$

viz., the rationalised equation is an equation of the degree  $2^{\nu-2}$  in  $q$ , and gives therefore  $2^{\nu-2}$  values of  $q$ . And the  $\nu$ -zomal curve thus breaks up into a system of  $2^{\nu-2}$  curves each of the form  $\Theta - q\Phi = 0$ , that is, each of them in involution with the curves  $\Theta = 0$ ,  $\Phi = 0$ . The equation in  $q$  may have a multiple root or roots, and the system of curves so contain repetitions of the same curve or curves; an instance of this (in relation to the trizomal curve) will present itself in the sequel; but I do not at present stop to consider the question.

26. A more important case is when the zomal curves are each of them in involution with the same two given curves, one of them of the order  $r$ , the other of an inferior order. Let  $\Theta = 0$  be a curve of the order  $r$ ,  $\Phi = 0$  a curve of an inferior order  $r - s$ ;  $L = 0$ ,  $M = 0$ , &c., curves of the order  $s$ ; then the case in question is when the zomal curves are of the form  $\Theta + L\Phi = 0$ ,  $\Theta + M\Phi = 0$ , &c., the equation of the  $\nu$ -zomal is

$$\sqrt{l(\Theta + L\Phi)} + \sqrt{m(\Theta + M\Phi)} + \&c. = 0,$$

where  $l$ ,  $m$ , &c., are constants. This is the most convenient form for the equation, and by considering the functions  $L$ ,  $M$ , &c. as containing implicitly the factors  $l^{-1}$ ,  $m^{-1}$ , &c. respectively, we may take it to include the form  $\sqrt{l\Theta + L\Phi} + \sqrt{m\Theta + M\Phi} + \&c. = 0$ , which last has the advantage of being immediately applicable to the case where any one or more of the constants  $l$ ,  $m$ , &c. may be  $= 0$ .

27. In the case now under consideration we have the  $r(r - s)$  points of intersection of the curves  $\Theta = 0$ ,  $\Phi = 0$  as common points of all the zomals. Hence, putting in the foregoing formula  $k = r(r - s)$ , we have a  $\nu$ -zomal curve of the order  $2^{\nu-2}r$ , having with each zomal  $2^{\nu-2}rs$  contacts, or with all the zomals  $2^{\nu-2}rs\nu$  contacts, having a node at each of the  $2^{\nu-4}rs$  intersections (not being common points  $\Theta = 0$ ,  $\Phi = 0$ ) of each pair of complementary parazomals; that is, together  $2^{\nu-4}(2^{\nu-1} - \nu - 1)rs$  nodes, and having, besides, at each of the

$r(r - s)$  common points, a  $2^{\nu-2}$ -tuple point, counting as  $2^{2\nu-5} - 2^{\nu-3}$  dps., together as  $(2^{2\nu-5} - 2^{\nu-3}) r(r - s)$  dps.; whence, taking account of the nodes, the total number of dps. is  $= 2^{\nu-4} r[(2^{\nu-1} - 2)r - (\nu-1)s]$ .

*Depression of Order of the  $\nu$ -zomal Curve from the Ideal Factor of a Branch or Branches—*  
 Art. Nos. 28 to 37.

28. In the case of the  $r(r - s)$  common points as thus far considered, the order of the  $\nu$ -zomal curve has remained throughout  $= 2^{\nu-2}r$ , but the order admits of depression, viz., the constants  $l, m, \&c.$ , and those of the functions  $L, M, \&c.$ , may be such that the norm contains the factor  $\Phi^\omega$ ; the  $\nu$ -zomal curve then contains as part of itself ( $\Phi^\omega = 0$ ) the curve  $\Phi = 0$  taken  $\omega$  times, and this being so, if we discard the factor in question, and consider the residual curve as being the  $\nu$ -zomal, the order of the  $\nu$ -zomal will be  $= 2^{\nu-2}r - \omega(r - s)$ .

29. To explain how such a factor  $\Phi^\omega$  presents itself, consider the polyzome  $\sqrt{l(\Theta + L\Phi)} + \&c.$ , or, what is the same thing,  $\sqrt{l} \sqrt{\Theta + L\Phi} + \&c.$ , belonging to any particular branch of the curve, we may, it is clear, take  $\sqrt{\Theta + L\Phi}, \&c.$  each in a fixed signification as equivalent to  $\sqrt{\Theta + L\Phi}, \&c.$ , respectively, and the particular branch will then be determined by means of the significations attached to  $\sqrt{l}, \sqrt{m}, \&c.$  Expanding the several radicals, the polyzome is

$$\sqrt{l} \left\{ \sqrt{\Theta} + \frac{1}{2} L \frac{\Phi}{\sqrt{\Theta}} - \frac{1}{8} L^2 \frac{\Phi^2}{\Theta \sqrt{\Theta}} + \&c., \right\} + \&c.;$$

or, what is the same thing, it is

$$\sqrt{\Theta} \left( \sqrt{l} + \&c. \right) + \frac{1}{2} \frac{\Phi}{\sqrt{\Theta}} \left( L \sqrt{l} + \&c. \right) - \frac{1}{8} \frac{\Phi^2}{\Theta \sqrt{\Theta}} \left( L^2 \sqrt{l} + \&c. \right) + \&c.$$

which expansion may contain the factor  $\Phi$ , or a higher power of  $\Phi$ . For instance, if we have  $\sqrt{l} + \&c. = 0$ , the expansion will then contain the factor  $\Phi$ ; and if we also have  $L\sqrt{l} + \&c. = 0$  (observe this implies as many equations as there are aszygetic terms in the whole series of functions  $L, M, \&c.$ ; thus, if  $L, M, \&c.$ , are each of them of the form  $aP + bQ + cR$ , with the same values of  $P, Q, R$ , but with different values of the co-efficients  $a, b, c$ , then it implies the three equations  $a\sqrt{l} + \&c. = 0, b\sqrt{l} + \&c. = 0, c\sqrt{l} + \&c. = 0$ ; and so in other cases), if I say  $L\sqrt{l} + \&c.$  be also  $= 0$ , then the expansion will contain the factor  $\Phi^2$ , and so on; the most general supposition being, that the expansion contains as factor a certain power  $\Phi^\alpha$  of  $\Phi$ . Imagine each of the polyzomes expanded in this manner, and let certain of the expansions contain the factors  $\Phi^\alpha, \Phi^\beta, \&c.$ , respectively. The produce of the expansions is identically equal to the product of the unexpanded polyzomes—that is, it is equal to the Norm; hence, if  $\alpha + \beta + \&c. = \omega$ , the Norm will contain the factor  $\Phi^\omega$ .

30. It has been mentioned that the form  $\sqrt{l(\Theta + L\Phi)}$  is considered as including the form  $\sqrt{l\Theta + L\Phi}$ , that is, when  $l = 0$ , the form  $\sqrt{L\Phi}$ . If in the equation of the  $\nu$ -zomal curve there is any such term—for instance, if the equation be  $\sqrt{L\Phi} + \sqrt{m(\Theta + M\Phi)} + \&c. = 0$ —the radical  $\sqrt{L\Phi}$  contains the factor  $\Phi^{\frac{1}{2}}$ ; but if  $L$  contains as factor an odd or an even power of  $\Phi$ , then  $\sqrt{L\Phi}$  will contain the factor  $\Phi^{\alpha}$  where  $\alpha$  is either an integer, or an integer  $+ \frac{1}{2}$ . Consider the polyzome  $\sqrt{L\Phi} + \sqrt{m(\Theta + M\Phi)} + \&c.$ , belonging to any particular branch of the curve; the radical  $\sqrt{L\Phi}$  contains, as just mentioned, the factor  $\Phi^{\alpha}$ , and if the remaining terms  $\sqrt{m(\Theta + M\Phi)} + \&c.$ , are such that the expansion contains as factor the same or any higher power of  $\Phi$ , then the expansion of the polyzome  $\sqrt{L\Phi} + \sqrt{m(\Theta + M\Phi)} + \&c.$ , belonging to the particular branch will contain the factor  $\Phi^{\alpha}$ ; and similarly we may have branches containing the factors  $\Phi^{\alpha}, \Phi^{\beta}, \&c.$ , whence, as before, if  $\omega = \alpha + \beta + \&c.$ , the Norm will contain the factor  $\Phi^{\omega}$ ; the only difference is, that now  $\alpha, \beta, \&c.$ , instead of being of necessity all integers, are each of them an integer, or an integer  $+ \frac{1}{2}$ ; of course, in the latter case the integer may be zero, or the index be  $= \frac{1}{2}$ . It is clear that  $\omega$  must be an integer, and it is, in fact, easy to see that the fractional indices occur in pairs; for observe that  $\alpha$  being fractional, the expansion of  $\sqrt{m(\Theta + M\Phi)} + \&c.$ , will contain not  $\Phi^{\alpha}$ , but a higher power,  $\Phi^{\alpha+q}$ , where  $\alpha + q$  is an integer; whence *each* of the polyzomes  $\sqrt{L\Phi} \pm (\sqrt{m(\Theta + M\Phi)} + \&c.)$  will contain the factor  $\Phi^{\alpha}$ .

31. Observe that in every case the factor  $\Phi^{\alpha}$  presents itself as a factor of the expansion of the polyzome corresponding to a particular branch of the curve; the polyzome itself does not contain the factor  $\Phi^{\alpha}$ , and we cannot in anywise say that the corresponding branch contains as factor the curve  $\Phi^{\alpha} = 0$ ; but we may, with great propriety of expression, say that *the branch ideally contains the curve*  $\Phi^{\alpha} = 0$ ; and this being so, the general theorem is, that if we have branches ideally containing the curves  $\Phi^{\alpha} = 0, \Phi^{\beta} = 0, \&c.$  respectively, then the  $\nu$ -zomal curve contains not ideally but actually the factor  $\Phi^{\omega} = 0$  ( $\omega = \alpha + \beta + \&c.$ ), the order of the  $\nu$ -zomal being thus reduced from  $2^{\nu-2}r$  to  $2^{\nu-2}r - \omega(r - s)$ ; and conversely, that any such reduction in the order of the  $\nu$ -zomal arises from factors  $\Phi^{\alpha} = 0, \Phi^{\beta} = 0, \&c.$ , ideally contained in the several branches of the  $\nu$ -zomal.

32. It is worth while to explain the notion of an ideal factor somewhat more generally; an irrational function, taking the irrationalities thereof in a determinate manner, may be such that, as well the function itself as all its differential co-efficients up to the order  $\alpha - 1$ , vanish when a certain parameter  $\Phi$  contained in the function is put  $= 0$ ; this is only saying, in other words, that the function expanded in ascending powers of  $\Phi$  contains no power lower than  $\Phi^{\alpha}$ ; and, in this case, we say that the irrational function contains *ideally* the factor  $\Phi^{\alpha}$ . The rationalised expression, or Norm, in virtue of the irrational function (taken determinately as above) thus ideally containing  $\Phi^{\alpha}$ , will actually contain the factor

$\Phi^\alpha$ ; and if any other values of the irrational function contain respectively  $\Phi^\beta$ , &c., then the Norm will contain the factor  $\Phi^{\alpha + \beta + \&c.}$

33. A branch ideally containing  $\Phi^\alpha = 0$  may for shortness be called integral or fractional, according as the index  $\alpha$  is an integer or a fraction; by what precedes the fractional branches present themselves in pairs. If for a moment we consider integral branches only, then if the  $\nu$ -zomal contain  $\Phi = 0$ , this can happen in one way only, there must be some one branch ideally containing  $\Phi = 0$ ; but if the  $\nu$ -zomal contain  $\Phi^2 = 0$ , then this may happen in two ways,—either there is a single branch ideally containing  $\Phi^2 = 0$ , or else there are two branches, each of them ideally containing  $\Phi = 0$ . And generally, if the  $\nu$ -zomal contain  $\Phi^\omega = 0$ , then forming any partition  $\omega = \alpha + \beta + \&c.$  (the parts being integral), this may arise from there being branches ideally containing  $\Phi^\alpha = 0$ ,  $\Phi^\beta = 0$ , &c. respectively. The like remarks apply to the case where we attend also to fractional branches,—thus, if the  $\nu$ -zomal contain  $\Phi = 0$ , this may arise (not only, as above mentioned, from a branch ideally containing  $\Phi = 0$ , but also) from a pair of branches, each ideally containing  $\Phi^{\frac{1}{2}} = 0$ . And so in general, if the  $\nu$ -zomal contain  $\Phi^\omega = 0$ , the partition  $\omega = \alpha + \beta + \&c.$  is to be made with the parts integral or fractional ( $= \frac{1}{2}$  or integer  $+ \frac{1}{2}$  as above), but with the fractional terms in pairs; and then the factor  $\Phi^\omega = 0$  may arise from branches ideally containing  $\Phi^\alpha = 0$ ,  $\Phi^\beta = 0$ , &c. respectively.

34. Any zomal, antizomal, or parazomal of a  $\nu$ -zomal curve,  $\sqrt{l(\Theta + L\Phi)} + \&c. = 0$ , is a polyzomal curve (including in the term a monozomal curve) of the same form as the  $\nu$ -zomal; and may in like manner contain  $\Phi = 0$ , or more generally,  $\Phi^\omega = 0$ , viz., if  $\omega = \alpha + \beta + \&c.$  be any partition of  $\omega$  as above, this will be the case if the zomal, antizomal, or parazomal has branches ideally containing  $\Phi^\alpha = 0$ ,  $\Phi^\beta = 0$ , &c. respectively. It is to be observed that if a zomal, antizomal, or parazomal contain  $\Phi = 0$ , or any higher power  $\Phi^\omega = 0$ , this does not in anywise imply that the zomal contains even  $\Phi = 0$ . But if (attending only to the most simple case) a zomal and its antizomal, or a pair of complementary parazomals, each contain  $\Phi = 0$  inseparably (that is, through a single branch ideally containing  $\Phi = 0$ ), then the  $\nu$ -zomal will have two branches, each ideally containing  $\Phi = 0$ , and it will thus contain  $\Phi^2 = 0$ . In fact, if in the zomal and antizomal, or in the complementary parazomals, the branches which ideally contain  $\Phi = 0$  are

$$\sqrt{l(\Theta + L\Phi)} + \&c. = 0, \quad \sqrt{n(\Theta + N\Phi)} + \&c. = 0$$

respectively (for a zomal, the  $+ \&c.$  should be omitted, and the first equation be written  $\sqrt{l(\Theta + L\Phi)} = 0$ ), then in the  $\nu$ -zomal there will be the two branches

$$(\sqrt{l(\Theta + L\Phi)} + \&c.) \pm (\sqrt{n(\Theta + N\Phi)} + \&c.) = 0,$$

each ideally containing  $\Phi = 0$ .

Conversely, if a  $\nu$ -zomal contain  $\Phi^2 = 0$  by reason that it has two branches each ideally containing  $\Phi = 0$ , then either a zomal and its antizomal will each of them, or else a pair of complementary parazomals will each of them, inseparably contain  $\Phi = 0$ .

35. Reverting to the case of the  $\nu$ -zomal curve

$$\sqrt{l(\Theta + L\Phi)} + \sqrt{m(\Theta + M\Phi)} + \&c. = 0,$$

which does not contain  $\Phi = 0$ , each of the  $r(r - s)$  common points  $\Theta = 0$ ,  $\Phi = 0$  is a  $2^{\nu-2}$ -tuple point on the  $\nu$ -zomal; each of these counts therefore for  $2^{\nu-2}$  intersections of the  $\nu$ -zomal with the curve  $\Phi = 0$ , and we have thus the complete number  $2^{\nu-2} r(r - s)$  of intersections of the two curves, viz., the curve  $\Phi = 0$  meets the  $\nu$ -zomal in the  $r(r - s)$  common points, each of them a  $2^{\nu-2}$ -tuple point on the  $\nu$ -zomal, and in no other point.

36. But if the  $\nu$ -zomal contains  $\Phi^\omega = 0$ , then each of the  $r(r - s)$  common points is still a  $2^{\nu-2}$ -tuple point on the aggregate curve; the aggregate curve therefore passes  $2^{\nu-2}$  times through each common point; but among these passages are included  $\omega$  passages of the curve  $\Phi = 0$  through the common point. The residual curve—say the  $\nu$ -zomal—passes therefore only  $2^{\nu-2} - \omega$  times through the common point; that is, each of the  $r(r - s)$  common points is a  $(2^{\nu-2} - \omega)$  tuple point on the  $\nu$ -zomal. The curve  $\Phi = 0$  meets the  $\nu$ -zomal in  $\{2^{\nu-2} r - \omega(r - s)\}$   $(r - s)$  points, viz., these include the  $r(r - s)$  common points, each of them a  $(2^{\nu-2} - \omega)$  tuple point on the  $\nu$ -zomal, and therefore counting together as  $(2^{\nu-2} - \omega) r(r - s)$  intersections; there remain consequently  $\omega s(r - s)$  other intersections of the curve  $\Theta = 0$  with the  $\nu$ -zomal.

37. In the case where the  $\nu$ -zomal contains the factor  $\Phi^\omega = 0$ , then throughout excluding from consideration the  $r(r - s)$  common points  $\Theta = 0$ ,  $\Phi = 0$ , the *remaining* intersections of any zomal with its antizomal are points of contact of the zomal with the  $\nu$ -zomal, and the *remaining* intersections of each pair of complementary parazomals are nodes of the  $\nu$ -zomal, it being understood that if any zomal, antizomal, or parazomal contain a power of  $\Phi = 0$ , such powers of  $\Phi = 0$  are to be discarded, and only the residual curves attended to. The number of contacts and of nodes may in any particular case be investigated without difficulty, and some instances will present themselves in the sequel, but on account of the different ways in which the factor  $\Phi^\omega = 0$  may present itself, ideally in a single branch, or in several branches, and the consequent occurrence in the latter case of powers of  $\Phi = 0$  in certain of the zomals, antizomals, or parazomals, the cases to be considered would be very numerous, and there is no reason to believe that the results could be presented in any moderately concise form; I therefore abstain from entering on the question.

*On the Trizomal Curve and the Tetrizomal Curve—Art. Nos. 38 and 39.*

38. The trizomal curve

$$\sqrt{U} + \sqrt{V} + \sqrt{W} = 0$$

has for its rationalised form of equation

$$U^2 + V^2 + W^2 - 2VW - 2WU - 2UV = 0;$$

or as this may also be written,

$$(1, 1, 1, -1, -1, -1)(U, V, W)^2 = 0;$$

and we may from this rational equation verify the general results applicable to the case in hand, viz., that the trizomal is a curve of the order  $2r$ , and that

$$\begin{array}{llll} U = 0, & \text{at each of its } r^2 \text{ intersections with } V - W = 0 \\ V = 0, & \text{,,} & \text{,,} & W - U = 0 \\ W = 0, & \text{,,} & \text{,,} & U - V = 0 \end{array}$$

respectively touch the trizomal. There are not, in general, any nodes or cusps, and the order being  $= 2r$ , the class is  $= 2r(2r - 1)$ .

39. The tetrizomal curve

$$\sqrt{U} + \sqrt{V} + \sqrt{W} + \sqrt{T} = 0$$

has for its rationalised form of equation

$$(U^2 + V^2 + W^2 + T^2 - 2UV - 2UW - 2UT - 2VW - 2VT - 2WT)^2 - 64UVWT = 0,$$

and we may hereby verify the fundamental properties, viz., that the tetrizomal is a curve of the order  $4r$ , touched by each of the zomals  $U = 0, V = 0, W = 0, T = 0$  in  $2r^2$  points, viz. by  $U = 0$  at its intersections with  $\sqrt{U} + \sqrt{W} + \sqrt{T} = 0$ , that is,  $V^2 + W^2 + T^2 - 2VW - 2VT - 2WT = 0$ ; and the like as regards the other zomals), and having  $3r^2$  nodes, viz., these are the intersections of  $(\sqrt{U} + \sqrt{V} = 0, \sqrt{W} + \sqrt{T} = 0)$ ,  $(\sqrt{U} + \sqrt{W} = 0, \sqrt{V} + \sqrt{T} = 0)$ ,  $(\sqrt{U} + \sqrt{T} = 0, \sqrt{V} + \sqrt{W} = 0)$ , or, what is the same thing, the intersections of  $(U - V = 0, W - T = 0)$ ,  $(U - W = 0, V - T = 0)$ ,  $(U - T = 0, V - W = 0)$ . There are not in general any cusps, and the class is thus  $= 4r(4r - 1) - 6r^2, = 10r^2 - 4r$ .

*On the Intersection of two  $\nu$ -Zomals having the same Zomal Curves—Art. Nos. 40 and 41.*

40. Without going into any detail, I may notice the question of the intersection of two  $\nu$ -zomals which have the same zomal curves—say the two trizomals  $\sqrt{U} + \sqrt{V} + \sqrt{W} = 0, \sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ , or two similarly related

tetrazomals. For the trizomals, writing the equations under the form

$$\sqrt{U} + \sqrt{V} + \sqrt{W} = 0, \quad \sqrt{l} \sqrt{U} + \sqrt{m} \sqrt{V} + \sqrt{n} \sqrt{W} = 0,$$

then, when these equations are considered as existing simultaneously, we may, without loss of generality, attribute to the radicals  $\sqrt{U}$ ,  $\sqrt{V}$ ,  $\sqrt{W}$ , the same values in the two equations respectively; but doing so, we must in the second equation successively attribute to all but one of the radicals  $\sqrt{l}$ ,  $\sqrt{m}$ ,  $\sqrt{n}$ , each of its two opposite values. For the intersections of the two curves we have thus

$$\sqrt{U} : \sqrt{V} : \sqrt{W} = \sqrt{m} - \sqrt{n} : \sqrt{n} - \sqrt{l} : \sqrt{l} - \sqrt{m},$$

viz., this is one of a system of four equations, obtained from it by changes of sign, say in the radicals  $\sqrt{m}$  and  $\sqrt{n}$ . Each of the four equations gives a set of  $r^2$  points; we have thus the complete number,  $= 4r^2$ , of the points of intersection of the two curves.

41. But take, in like manner, two tetrazomal curves; writing their equations in the form

$$\begin{aligned} \sqrt{U} + \sqrt{V} + \sqrt{W} + \sqrt{T} &= 0, \\ \sqrt{l} \sqrt{U} + \sqrt{m} \sqrt{V} + \sqrt{n} \sqrt{W} + \sqrt{p} \sqrt{T} &= 0, \end{aligned}$$

then  $\sqrt{U}$ ,  $\sqrt{V}$ ,  $\sqrt{W}$ ,  $\sqrt{T}$  may be considered as having the same values in the two equations respectively, but we must in the second equation attribute successively, say to  $\sqrt{m}$ ,  $\sqrt{n}$ ,  $\sqrt{p}$ , each of their two opposite values. For the intersections of the two curves we have

$$\begin{aligned} (\sqrt{m} - \sqrt{l}) \sqrt{V} + (\sqrt{n} - \sqrt{l}) \sqrt{W} + (\sqrt{p} - \sqrt{l}) \sqrt{T} &= 0 \\ (\sqrt{l} - \sqrt{m}) \sqrt{U} + (\sqrt{n} - \sqrt{m}) \sqrt{W} + (\sqrt{p} - \sqrt{m}) \sqrt{T} &= 0 \end{aligned}$$

viz., this is one of a system of eight similar pairs of equations, obtained therefrom by changes of sign of the radicals  $\sqrt{m}$ ,  $\sqrt{n}$ ,  $\sqrt{p}$ . The equations represent each of them a trizomal curve, of the order  $2r$ ; the two curves intersect therefore in  $4r^2$  points, and if each of these was a point of intersection of the two tetrazomals, we should have in all  $8 \times 4r^2 = 32r^2$  intersections. But the tetrazomals are each of them a curve of the order  $4r$ , and they intersect therefore in only  $16r^2$  points. The explanation is, that not all the  $4r^2$  points, but only  $2r^2$  of them are intersections of the tetrazomals. In fact, to find *all* the intersections of the two trizomals, it is necessary in their two equations to attribute opposite signs to one of the radicals  $\sqrt{W}$ ,  $\sqrt{T}$ ; we obtain  $2r^2$  intersections from the equations as they stand, the remaining  $2r^2$  intersections from the two equations after we have in the second equation reversed the sign, say of  $\sqrt{T}$ . Now, from the two equations as they stand we can pass back to the two tetrazomal equations, and the first mentioned  $2r^2$  points are thus points of intersection of the two tetrazomal curves — from the two equations after such reversal of the sign of  $\sqrt{T}$ , we cannot pass back to the two tetrazomal equations, and the last-mentioned  $2r^2$  points are thus

not points of intersection of the two tetrazomal curves. The number of intersections of the two curves is thus  $8 \times 2r^2, = 16r^2$ , as it should be.

*The Theorem of the Decomposition of a Tetrazomal Curve—Art. Nos. 42 to 45.*

42. I consider the tetrazomal curve

$$\sqrt{lU} + \sqrt{mV} + \sqrt{nW} + \sqrt{pT} = 0,$$

where the zomal curves are in involution,—that is, where we have an identical relation,

$$aU + bV + cW + dT = 0;$$

and I proceed to show that if  $l, m, n, p$ , satisfy the relation

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0,$$

the curve breaks up into two trizomals. In fact, writing the equation under the form

$$(\sqrt{lU} + \sqrt{mV} + \sqrt{nW})^2 - pT = 0,$$

and substituting for  $T$  its value, in terms of  $U, V, W$ , this is

$$(ld + pa)U + (md + pb)V + (nd + pc)W + 2\sqrt{mnd} \sqrt{VW} + 2\sqrt{nld} \sqrt{WU} + 2\sqrt{lmd} \sqrt{UV} = 0;$$

or, considering the left-hand side as a quadric function of  $(\sqrt{U}, \sqrt{V}, \sqrt{W})$ , the condition for its breaking up into factors is

$$\begin{vmatrix} ld + pa, & d\sqrt{lm}, & d\sqrt{ln} \\ d\sqrt{ml}, & md + pb, & d\sqrt{mn} \\ d\sqrt{nl}, & d\sqrt{nm}, & nd + pc \end{vmatrix} = 0,$$

that is

$$p^2(lbcd + meda + ndab + pabc) = 0,$$

or finally, the condition is

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0.$$

43. Multiplying by  $ld + pa$ , and observing that in virtue of the relation we have

$$\begin{aligned} (ld + pa)(md + pb) &= lmd^2 - \frac{abd}{c}pn, \\ (ld + pa)(nd + pc) &= lnd^2 - \frac{acd}{b}pm, \end{aligned}$$

the equation becomes

$$\left( (ld + pa)\sqrt{U} + d\sqrt{lm}\sqrt{V} + d\sqrt{ln}\sqrt{W} \right)^2 = \frac{ad}{bc}p \left( b\sqrt{n}\sqrt{V} - c\sqrt{m}\sqrt{W} \right)^2.$$

or as this is more conveniently written

$$\left( \left( \sqrt{l} + \frac{ap}{d\sqrt{l}} \right) \sqrt{U} + \sqrt{m}V + \sqrt{n}W \right)^2 = \frac{a}{bcd} \frac{p}{l} \left( b\sqrt{n}V - c\sqrt{m}W \right)^2,$$

an equation breaking up into two equations, which may be represented by

$$\sqrt{l_1}U + \sqrt{m_1}V + \sqrt{n_1}W = 0, \quad \sqrt{l_2}U + \sqrt{m_2}V + \sqrt{n_2}W = 0,$$

where

$$\begin{aligned} \sqrt{l_1} &= \sqrt{l} + \frac{a}{d} \frac{p}{\sqrt{l}} & , & \quad \sqrt{l_2} = \sqrt{l} + \frac{a}{d} \frac{p}{\sqrt{l}} \\ \sqrt{m_1} &= \sqrt{m} - \sqrt{\frac{a}{bcd} \frac{p}{l}} b \sqrt{n} & , & \quad \sqrt{m_2} = \sqrt{m} + \sqrt{\frac{a}{bcd} \frac{p}{l}} b \sqrt{n} \\ \sqrt{n_1} &= \sqrt{n} + \sqrt{\frac{a}{bcd} \frac{p}{l}} c \sqrt{m} & , & \quad \sqrt{n_2} = \sqrt{n} - \sqrt{\frac{a}{bcd} \frac{p}{l}} c \sqrt{m}, \end{aligned}$$

where, in the expressions for  $\sqrt{l}$ , &c., the signs of the radicals

$$\sqrt{l}, \sqrt{m}, \sqrt{n}, \sqrt{\frac{a}{bcd} \frac{p}{l}},$$

may be taken determinately in any way whatever at pleasure; the only effect of an alteration of sign would in some cases be to interchange the values of  $(\sqrt{l_1}, \sqrt{m_1}, \sqrt{n_1})$  with those of  $(\sqrt{l_2}, \sqrt{m_2}, \sqrt{n_2})$ . The tetrazomal curve thus breaks up into two trizomals.

44. It is to be noticed that we have

$$\begin{aligned} \frac{l_1}{a} + \frac{m_1}{b} + \frac{n_1}{c} &= \frac{l}{a} + \frac{ap^2}{d^2l} + 2\frac{p}{d} \\ &+ \frac{m}{b} + \frac{a}{cd} \frac{np}{l} \\ &+ \frac{n}{c} + \frac{a}{bd} \frac{mp}{l} \\ &= \left( l + \frac{ap}{dl} \right) \left( \frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} \right); \end{aligned}$$

that is

$$\frac{l_1}{a} + \frac{m_1}{b} + \frac{n_1}{c} = 0.$$

And similarly we have

$$\frac{l_2}{a} + \frac{m_2}{b} + \frac{n_2}{c} = 0.$$

The meaning is, that, taking the trizomal curve  $\sqrt{l_1}U + \sqrt{m_1}V + \sqrt{n_1}W = 0$ , this regarded as a tetrazomal curve,  $\sqrt{l_1}U + \sqrt{m_1}V + \sqrt{n_1}W + \sqrt{0}T = 0$ , satisfies the condition  $\frac{l_1}{a} + \frac{m_1}{b} + \frac{n_1}{c} + \frac{0}{d} = 0$ ; and the like as to the trizomal curve  $\sqrt{l_2}U + \sqrt{m_2}V + \sqrt{n_2}W = 0$ .

45. The equation by which the decomposition was effected is, it is clear, one of twelve equivalent equations; four of these are

$$\begin{aligned} & \left( \sqrt{l} + \frac{ap}{d\sqrt{l}}, \sqrt{m}, \sqrt{n}, 0 \right) \left( \sqrt{U}, \sqrt{V}, \sqrt{W}, \sqrt{T} \right)^2 = \\ & \qquad \qquad \qquad \frac{a}{bcd} \frac{p}{l} \left( b\sqrt{nW} - c\sqrt{mW} \right)^2 \\ & \left( 0, \sqrt{m} + \frac{bl}{a\sqrt{m}}, \sqrt{n}, \sqrt{p} \right) \left( \begin{array}{c} \text{,,} \\ \text{,,} \end{array} \right)^2 = \\ & \qquad \qquad \qquad \frac{a}{cda} \frac{l}{m} \left( c\sqrt{pW} - d\sqrt{nT} \right)^2 \\ & \left( \sqrt{l}, 0, \sqrt{n} + \frac{cm}{b\sqrt{n}}, \sqrt{p} \right) \left( \begin{array}{c} \text{,,} \\ \text{,,} \end{array} \right)^2 = \\ & \qquad \qquad \qquad \frac{c}{dab} \frac{m}{n} \left( d\sqrt{lT} - a\sqrt{pU} \right)^2 \\ & \left( \sqrt{l}, \sqrt{m}, 0, \sqrt{p} + \frac{dn}{c\sqrt{p}} \right) \left( \begin{array}{c} \text{,,} \\ \text{,,} \end{array} \right)^2 = \\ & \qquad \qquad \qquad \frac{d}{abc} \frac{n}{p} \left( a\sqrt{mU} - b\sqrt{lV} \right)^2, \end{aligned}$$

and the others may be deduced from these by a cyclical permutation of  $(U, V, W)$ ,  $(a, b, c)$ ,  $(l, m, n)$ , leaving  $T, d, p$  unaltered.

*Application to the Trizomal; the Theorem of the Variable Zomal—Art. Nos. 46 to 51.*

46. I take the last equation written under the form

$$(a\sqrt{mU} - b\sqrt{lV})^2 = \frac{abc}{dn} \left( \sqrt{lpU} + \sqrt{mpV} + \left( p + \frac{dn}{c} \right) \sqrt{T} \right)^2,$$

which, putting therein  $p = 0$ , is

$$(a\sqrt{mU} - b\sqrt{lV})^2 = \frac{abd}{c} nT,$$

which is in fact the trizomal curve,

$$a\sqrt{mU} - b\sqrt{lV} + \sqrt{\frac{abd}{c} nT} = 0,$$

viz., the trizomal curve  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ ,—if  $a, b, c$  be any quantities connected by the equation

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0,$$

(the ratios  $a, b, c$  thus involving a single arbitrary parameter); and if we take  $T$  a function such that  $aU + bV + cW + dT = 0$ ; that is,  $T = 0$ , any one of the

series of curves  $aU + bV + cW = 0$ , in involution with the given curves  $U = 0$ ,  $V = 0$ ,  $W = 0$ ,—has its equation expressible in the form

$$a\sqrt{mU} - b\sqrt{lV} + \sqrt{\frac{abd}{c}}nT = 0;$$

that is, we have the curve  $T = 0$  (the equation whereof contains a variable parameter) as a zomal of the given trizomal curve  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ ; and we have thus from the theorem of the decomposition of a tetrazomal deduced the theorem of the variable zomal of a trizomal. The analytical investigation is somewhat simplified by assuming  $p = 0$  *ab initio*, and it may be as well to repeat it in this form.

47. Starting, then, with the trizomal curve

$$\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0,$$

and writing

$$aU + bV + cW + dT = 0$$

as the definition of  $T$ , the coefficients being connected by

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0,$$

the equation gives

$$lU + mV + 2\sqrt{lmUV} - nW = 0;$$

or substituting in this equation for  $W$  its value in terms of  $U, V, T$ , we have

$$(an + cl)U + (bn + cm)V + 2c\sqrt{lmUV} + dnT = 0,$$

which by the given relation between  $a, b, c$ , is converted into

$$-\frac{ac}{b}mU - \frac{bc}{a}lV + 2c\sqrt{lmUV} + dnT = 0;$$

that is

$$a^2mU + b^2lV - 2ab\sqrt{lmUV} = \frac{abd}{c}nT,$$

viz., this is

$$(a\sqrt{mU} - b\sqrt{lV})^2 = \frac{abd}{c}nT,$$

or finally

$$a\sqrt{mU} - b\sqrt{lV} + \sqrt{\frac{abd}{c}}nT = 0.$$

48. The result just obtained of course implies that when as above

$$aU + bV + cW + dT = 0, \frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0,$$

the trizomal curve  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$  can be expressed by means of any three of the four zomals  $U, V, W, T$ , and we may at once write down the four forms

$$\left( \begin{array}{cccc} \cdot & \sqrt{\frac{n}{c^2}}, & -\sqrt{\frac{m}{b^2}}, & -\sqrt{\frac{ld}{abc}} \\ -\sqrt{\frac{n}{c^2}}, & \cdot & \sqrt{\frac{l}{a^2}}, & -\sqrt{\frac{md}{abc}} \\ \sqrt{\frac{m}{b^2}}, & -\sqrt{\frac{l}{a^2}}, & \cdot & -\sqrt{\frac{nd}{abc}} \\ \sqrt{\frac{ld}{abc}}, & \sqrt{\frac{md}{abc}}, & \sqrt{\frac{nd}{abc}}, & \cdot \end{array} \right) (\sqrt{U}, \sqrt{V}, \sqrt{W}, \sqrt{T}) = 0,$$

the last of which is the original equation  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ . It may be added that if the first equation be represented by  $\sqrt{m_1V} + \sqrt{n_1W} + \sqrt{p_1T} = 0$ ,—that is, if we have

$$\sqrt{m_1} = \sqrt{\frac{n}{c^2}}, \quad \sqrt{n_1} = -\sqrt{\frac{m}{b^2}}, \quad \sqrt{p_1} = \sqrt{\frac{ld}{abc}},$$

and therefore,

$$\frac{m_1}{b} + \frac{n_1}{c} + \frac{p_1}{d} = \frac{l}{bc} \left( \frac{l}{a} + \frac{m}{b} + \frac{n}{c} \right), = 0;$$

or if the second equation be represented by  $\sqrt{l_2U} + \sqrt{n_2W} + \sqrt{p_2T} = 0$ ,—that is, if we have

$$\sqrt{l_2} = -\sqrt{\frac{n}{c^2}}, \quad \sqrt{n_2} = \sqrt{\frac{l}{a^2}}, \quad \sqrt{p_2} = \sqrt{\frac{md}{abc}},$$

and therefore

$$\frac{l_2}{a} + \frac{n_2}{c} + \frac{p_2}{d} = 0;$$

or if the third equation be represented by  $\sqrt{l_3U} + \sqrt{m_3V} + \sqrt{p_3T} = 0$ ,—that is, if we have

$$\sqrt{l_3} = \sqrt{\frac{m}{b^2}}, \quad \sqrt{m_3} = -\sqrt{\frac{l}{a^2}}, \quad \sqrt{p_3} = \sqrt{\frac{md}{abc}},$$

and therefore

$$\frac{l_3}{a} + \frac{m_3}{b} + \frac{p_3}{d} = 0,$$

then the equation of the trizomal may also be expressed in the forms—

$$\left( \begin{array}{cccc} \cdot & \sqrt{m_1} & \sqrt{n_1} & \sqrt{p_1} \\ -\sqrt{m_1} & \cdot & \sqrt{\frac{p_1 bc}{ad}} & -\sqrt{\frac{n_1 bd}{ac}} \\ -\sqrt{n_1} & -\sqrt{\frac{p_1 bc}{ad}} & \cdot & \sqrt{\frac{m_1 cd}{ab}} \\ -\sqrt{p_1} & \sqrt{\frac{n_1 bc}{ad}} & -\sqrt{\frac{m_1 cd}{ab}} & \cdot \end{array} \right) (\sqrt{U}, \sqrt{V}, \sqrt{W}, \sqrt{T}) = 0$$

$$\left( \begin{array}{cccc} : & , & -\sqrt{l_2} & , & -\sqrt{\frac{p_2 ac}{bd}} & , & \sqrt{\frac{n_2 ad}{bc}} \\ \sqrt{l_2} & , & . & , & \sqrt{n_2} & , & \sqrt{p_2} \\ \sqrt{\frac{p_2 ac}{bc}} & , & -\sqrt{n_2} & , & . & , & -\sqrt{\frac{l_2 cd}{ab}} \\ -\sqrt{\frac{n_2 ad}{bc}} & , & -\sqrt{p_2} & , & -\sqrt{\frac{l_2 cd}{ab}} & , & . \end{array} \right) (\sqrt{U}, \sqrt{V}, \sqrt{W}, \sqrt{T}) = 0$$

and

$$\left( \begin{array}{cccc} . & , & \sqrt{\frac{p_3 ab}{cd}} & , & -\sqrt{l_3} & , & -\sqrt{\frac{m_3 ad}{bc}} \\ -\sqrt{\frac{p_3 ab}{cd}} & , & . & , & -\sqrt{m_3} & , & -\sqrt{\frac{l_3 bd}{ac}} \\ \sqrt{l_3} & , & \sqrt{m_3} & , & . & , & \sqrt{p_3} \\ \sqrt{\frac{m_3 ad}{bc}} & , & -\sqrt{\frac{l_3 bp}{ac}} & , & -\sqrt{p_3} & , & . \end{array} \right) (\sqrt{U}, \sqrt{V}, \sqrt{W}, \sqrt{T}) = 0$$

49. These equations may, however, be expressed in a much more elegant form. Write

$$a' = \frac{a}{(\beta\gamma\delta)}, \quad b' = -\frac{b}{(\gamma\delta\alpha)}, \quad c' = \frac{c}{(\delta\alpha\beta)}, \quad d' = \frac{-d}{(\alpha\beta\gamma)},$$

where, for shortness,  $(\beta\gamma\delta) = (\beta - \gamma)(\gamma - \delta)(\delta - \beta)$ , &c.;  $(\alpha, \beta, \gamma)$  being arbitrary quantities: or, what is the same thing,

$$a : b : c : d = a'(\beta\gamma\delta) : -b'(\gamma\delta\alpha) : c'(\delta\alpha\beta) : -d'(\alpha\beta\gamma).$$

Assume

$$l : m : n = \rho a'(\beta - \gamma)^2 : \sigma b'(\gamma - \alpha)^2 : \tau c'(\alpha - \beta)^2;$$

then the equation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$  takes the form

$$\rho(\beta - \gamma)(\alpha - \delta) + \sigma(\gamma - \alpha)(\beta - \delta) + \tau(\alpha - \beta)(\gamma - \delta),$$

and the four forms of the equation are found to be

$$\left( \begin{array}{cccc} . & , & \sqrt{\tau}(\delta - \gamma) & , & \sqrt{\sigma}(\beta - \delta) & , & \sqrt{\rho}(\gamma - \beta) \\ \sqrt{\tau}(\gamma - \delta) & , & . & , & \sqrt{\rho}(\delta - \alpha) & , & \sqrt{\sigma}(\alpha - \gamma) \\ \sqrt{\sigma}(\delta - \beta) & , & \sqrt{\rho}(\alpha - \delta) & , & . & , & \sqrt{\tau}(\beta - \alpha) \\ \sqrt{\rho}(\beta - \gamma) & , & \sqrt{\sigma}(\gamma - \alpha) & , & \sqrt{\tau}(\alpha - \beta) & , & . \end{array} \right) (\sqrt{a'U}, \sqrt{b'V}, \sqrt{c'W}, \sqrt{dT}) = 0$$

viz., these are the equivalent forms of the original equation assumed to be  $(\beta - \gamma) \sqrt{\rho a'U} + (\gamma - \alpha) \sqrt{\sigma b'V} + (\alpha - \beta) \sqrt{\tau c'W} = 0$ .

50. I remark that the theorem of the variable zomal may be obtained as a transformation theorem—viz., comparing the equation  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$  with the equation  $\sqrt{lx} + \sqrt{my} + \sqrt{nz} = 0$ ; this last belongs to a conic touched by the three lines  $x = 0, y = 0, z = 0$ ; the equation of the same conic must, it is clear, be expressible in a similar form by means of any other three tangents

thereof, but the equation of any tangent of the conic is  $ax + by + cz = 0$ , where  $a, b, c$  are any quantities satisfying the condition  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ ; whence, writing  $ax + by + cz + dw = 0$ , we may introduce  $w = 0$  along with any two of the original zomals  $x = 0, y = 0, z = 0$ , or, instead of them, any three functions of the form  $w$ ; and then the mere change of  $x, y, z, w$  into  $U, V, W, T$  gives the theorem. But it is as easy to conduct the analysis with  $(U, V, W, T)$  as with  $(x, y, z, w)$ , and, so conducted, it is really the same analysis as that whereby the theorem is established *ante*, No. 47.

51. It is worth while to exhibit the equation of the curve

$$\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0,$$

in a form containing three new zomals. Observe that the equation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$  is satisfied by  $a = l\phi\chi, b = m\chi\theta, c = n\theta\phi$ , if only  $\theta + \phi + \chi = 0$ ; or say, if  $\theta = a' - a'', \phi = a'' - a, \chi = a - a''$ . The equation

$$\begin{aligned} & \lambda \sqrt{(a-a')(a-a'')lU + (a'-a'')(a'-a)mV + (a''-a)(a''-a')nW} \\ & + \mu \sqrt{(b-b')(b-b'')lU + (b'-b'')(b'-b)mV + (b''-b)(b''-b')nW} \\ & + \nu \sqrt{(c-c')(c-c'')lU + (c'-c'')(c'-c)mV + (c''-c)(c''-c')nW} = 0 \end{aligned}$$

is consequently an equation involving three zomals of the proper form; and we can determine  $\lambda, \mu, \nu$  in suchwise as to identify this with the original equation  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW}$ , viz., writing successively  $U = 0, V = 0, W = 0$ , we find

$$\begin{aligned} (a'-a'') \lambda + (b'-b'') \mu + (c'-c'') \nu &= 0, \\ (a''-a) \lambda + (b''-b) \mu + (c''-c) \nu &= 0, \\ (a-a') \lambda + (b-b') \mu + (c-c') \nu &= 0, \end{aligned}$$

equations which are, as they should be, equivalent to two equations only, and which give

$$\lambda : \mu : \nu = \begin{vmatrix} 1, 1, 1 \\ b, b', b'' \\ c, c', c'' \end{vmatrix} : \begin{vmatrix} 1, 1, 1 \\ c, c', c'' \\ a, a', a'' \end{vmatrix} : \begin{vmatrix} 1, 1, 1 \\ a, a', a'' \\ b, b', b'' \end{vmatrix},$$

and the equation, with these values of  $\lambda, \mu, \nu$  substituted therein, is in fact the equation of the trizomal curve  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$  in terms of three new zomals. It is easy to return to the forms involving one new zomal and any two of the original three zomals.

*Remark as to the Tetrazomal Curve—Art. No. 52.*

52. I return for a moment to the case of the tetrazomal curve, in order to show that there is not, in regard to it in general, any theorem such as that of the variable zomal. Considering the form  $\sqrt{lx} + \sqrt{my} + \sqrt{nz} + \sqrt{pw} = 0$  (the co-

ordinates  $x, y, z, w$  are of course connected by a linear equation, but nothing turns upon this), the curve is here a quartic touched twice by each of the lines  $x = 0, y = 0, z = 0, w = 0$  (viz., each of these is a double tangent of the curve), and having besides the three nodes  $(x = y, z = w), (x = z, y = w), (x = w, y = z)$ . But a quartic curve with three nodes, or trinodal quartic, has only four double tangents—that is, besides the lines  $x = 0, y = 0, z = 0, w = 0$ , there is no line  $\alpha x + \beta y + \gamma z + \delta w = 0$  which is a double tangent of the curve; and writing  $U, V, W, T$  in place of  $x, y, z, w$ , then if  $U, V, W, T$  are connected by a linear equation (and, *à fortiori*, if they are not so connected), there is not any curve  $\alpha U + \beta V + \gamma W + \delta T = 0$  which is related to the curve in the same way with the lines  $U = 0, V = 0, W = 0, T = 0$ ; or say there is not (besides the curves  $U = 0, V = 0, W = 0, T = 0$ ), any other zomal  $\alpha U + \beta V + \gamma W + \delta T = 0$ , of the tetrazomal curve. The proof does not show that for special forms of  $U, V, W, T$  there may not be zomals, not of the above form  $\alpha U + \beta V + \gamma W + \delta T = 0$ , but belonging to a separate system. An instance of this will be mentioned in the sequel.

*The Theorem of the Variable Zomal of a Trizomal Curve resumed—Art. Nos. 53 to 56.*

53. I resume the theorem of the variable zomal of the trizomal curve  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ . The variable zomal  $T = 0$  is the curve  $aU + bV + cW = 0$ , where  $a, b, c$  are connected by the equation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ ; that is, it belongs to a single series of curves selected in a certain manner out of the double series  $aU + bV + cW = 0$  (a double series, as containing the *two* variable parameters  $a : b : c$ ). These are the whole series of curves in involution with the given curves  $U = 0, V = 0, W = 0$ , or being such that the Jacobian of any three of them is identical with the Jacobian of the three given curves; in particular, the Jacobian of any one of the curves  $aU + bV + cW = 0$ , and of two of the three given curves, is identical with the Jacobian of the three given curves. I call to mind that, by the Jacobian of the curves  $U = 0, V = 0, W = 0$ , is meant the curve

$$J(U, V, W) = \frac{d(U, V, W)}{d(x, y, z)} = \begin{vmatrix} d_x U, d_y U, d_z U \\ d_x V, d_y V, d_z V \\ d_x W, d_y W, d_z W \end{vmatrix} = 0$$

viz., the curve obtained by equating to zero the Jacobian or functional determinant of the functions  $U, V, W$ . Some properties of the Jacobian, which are material as to what follows, are mentioned in the Annex No. I.

For the complete statement of the theorem of the variable zomal, it would be necessary to interpret geometrically the condition  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ , thereby

showing how the single series of the variable zomal is selected out of the double series of the curves  $aU + bV + cW = 0$  in involution with the given curves. Such a geometrical interpretation of the condition may be sought for as follows. but it is only in a particular case, as afterwards mentioned, that a convenient geometrical interpretation is thereby obtained.

54. Consider the fixed line  $\Omega = px + qy + rz = 0$ , and let it be proposed to find the locus of the  $(r-1)^2$  poles of the line  $\Omega = 0$  in regard to the series of curves  $aU + bV + cW = 0$ , where  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ . Take  $(x, y, z)$  as the co-ordinates of any one of the poles in question, then in order that  $(x, y, z)$  may belong to one of the  $(r-1)^2$  poles of the line  $\Omega = px + qy + rz = 0$  in regard to the curve  $aU + bV + cW = 0$ , we must have

$$d_x(aU + bV + cW) : d_y(aU + bV + cW) : d_z(aU + bV + cW) = p : q : r ;$$

or, what is the same thing—

$$= d_x\Omega : d_y\Omega : d_z\Omega ;$$

and these equations give without difficulty

$$a : b : c = J(V, W, \Omega) : J(W, U, \Omega) : J(U, V, \Omega) ,$$

whence, substituting in the equation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ , we have

$$\frac{l}{J(V, W, \Omega)} + \frac{m}{J(W, U, \Omega)} + \frac{n}{J(U, V, \Omega)} = 0$$

as the locus of the  $(r-1)^2$  poles in question. Each of the Jacobians is a function of the order  $2r-2$ , and the order of the locus is thus  $= 4r-4$ . As the given curves  $U = 0$ ,  $V = 0$ ,  $W = 0$  belong to the single series of curves, it is clear that the locus passes through the  $3(r-1)^2$  points which are the  $(r-1)^2$  poles of the fixed line in regard to the curves  $U = 0$ ,  $V = 0$ ,  $W = 0$  respectively.

55. In the case where the given trizomal is

$$\sqrt{l(\Theta + L\Phi)} + \sqrt{m(\Theta + M\Phi)} + \sqrt{n(\Theta + N\Phi)} = 0 ,$$

$s = r-1$ , that is, where the zomals  $\Theta + L\Phi = 0$ ,  $\Theta + M\Phi = 0$ ,  $\Theta + N\Phi = 0$  are each of them curves of the order  $r$ , passing through the  $r$  intersections of the line  $\Phi = 0$  with the curve  $\Theta = 0$ , then, taking this line  $\Phi = 0$  for the fixed line  $\Omega = 0$ , we have

$$J(V, W, \Omega) = J(\Theta + M\Phi, \Theta + N\Phi, \Phi) = \Phi\{M, N\} ,$$

if, for shortness,  $\{M, N\} = J(M-N, \Theta, \Phi) + \Phi J(M, N, \Phi)$ , and the like as to the other two Jacobians, so that, attaching the analogous significations to  $\{N, L\}$  and  $\{L, M\}$ , the equation of the locus is

$$\frac{l}{\{M, N\}} + \frac{m}{\{N, L\}} + \frac{n}{\{L, M\}} = 0 ,$$

where observe that each of the curves  $\{M, N\} = 0$ ,  $\{N, L\} = 0$ ,  $\{L, M\} = 0$  is

a curve of the order  $2r - 3$ ; the order of the locus is thus  $= 4r - 6$ , and (as before) this locus passes through the  $3(r - 1)^2$  points which are the  $(r - 1)^2$  poles of the line  $\Phi = 0$  in regard to the curves  $\Theta + L\Phi = 0$ ,  $\Theta + M\Phi = 0$ ,  $\Theta + N\Phi = 0$  respectively.

56. In the case  $r = 2$ , the trizomal is

$$\sqrt{l(\Theta + L\Phi)} + \sqrt{m(\Theta + M\Phi)} + \sqrt{n(\Theta + N\Phi)} = 0,$$

where the zomals are the conics  $\Theta + L\Phi = 0$ ,  $\Theta + M\Phi = 0$ ,  $\Theta + N\Phi = 0$ , each passing through the same two points  $\Theta = 0$ ,  $\Phi = 0$ ; the locus of the pole of the line  $\Phi = 0$ , in regard to the variable zomal, is the conic

$$\frac{l}{\{M, N\}} + \frac{m}{\{N, L\}} + \frac{n}{\{L, M\}} = 0,$$

viz.,  $\{M, N\} = 0$ ,  $\{N, L\} = 0$ ,  $\{L, M\} = 0$ , are here the lines passing through the poles of the line  $\Phi = 0$  in regard to the second and third, the third and first, and the first and second of the given conics respectively: treating  $l$ ,  $m$ ,  $n$  as arbitrary, the locus is clearly *any* conic through the poles of the line  $\Phi = 0$  in regard to the three conics respectively. The Jacobian of the three given conics is a conic related in a special manner to the three given conics, and which might be called the Jacobian conic thereof, and it would be easy to give a complete enunciation of the theorem for the case in hand. (See as to this, Annex No. I, above referred to.) But if, in accordance with the plan adopted in the remainder of the memoir, we at once assume that the points  $\Theta = 0$ ,  $\Phi = 0$  are the circular points at infinity, then the theorem can be enunciated under a more simple form—viz., if  $\mathbf{A}^0 = 0$ ,  $\mathbf{B}^0 = 0$ ,  $\mathbf{C}^0 = 0$  are the equations of any three circles, then in the trizomal

$$\sqrt{l\mathbf{A}^0} + \sqrt{m\mathbf{B}^0} + \sqrt{n\mathbf{C}^0} = 0,$$

the variable zomal is any circle whatever of the series of circles cutting at right angles the orthotomic circle of the three given circles, and having its centre on a certain conic which passes through the centres of the given circles. Moreover, if the co-efficients  $l$ ,  $m$ ,  $n$  are not given in the first instance, but are regarded as arbitrary, then the last-mentioned conic is any conic whatever through the three centres, and there belongs to such conic and the series of zomals derived therefrom as above, a trizomal curve  $\sqrt{l\mathbf{A}^0} + \sqrt{m\mathbf{B}^0} + \sqrt{n\mathbf{C}^0} = 0$ . This is obviously the theorem, that if a variable circle has its centre on a given conic, and cuts at right angles a given circle, then the envelope of the variable circle is a trizomal curve  $\sqrt{l\mathbf{A}^0} + \sqrt{m\mathbf{B}^0} + \sqrt{n\mathbf{C}^0}$ , where  $\mathbf{A}^0 = 0$ ,  $\mathbf{B}^0 = 0$ ,  $\mathbf{C}^0 = 0$  are any three circles, positions of the variable circle, and  $l$ ,  $m$ ,  $n$  are constant quantities depending on the selected three circles.

PART II. (Nos. 57 to 104).—SUBSIDIARY INVESTIGATIONS.

*Preliminary Remarks*—Art. Nos. 57 and 58.

57. We have just been led to consider the conics which pass through two given points. There is no real loss of generality in taking these to be the circular points at infinity, or say the points  $I, J$ —viz., every theorem which in anywise explicitly or implicitly relates to these two points, may, without the necessity of any change in the statement thereof, be understood as a theorem relating instead to any two points  $P, Q$ . I call to mind that a circle is a conic passing through the two points  $I, J$ , and that lines at right angles to each other are lines harmonically related to the pair of lines from their intersection to the points  $I, J$  respectively, so that when  $(I, J)$  are replaced by any two given points whatever, the expression a circle must be understood to mean a conic passing through the two given points; and in speaking of lines at right angles to each other, it must be understood that we mean lines harmonically related to the pair of lines from their intersection to the two given points respectively. For instance, the theorem that the Jacobian of any three circles is their orthotomic circle, will mean that the Jacobian of any three conics which each of them passes through the two given points is the orthotomic conic through the same two points, that is, the conic such that at each of its intersections with any one of the three conics, the two tangents are harmonically related to the pair of lines from this intersection to the two given points respectively. Such extended interpretation of any theorem is applicable even to the theorems which involve distances or angles—viz., the terms “distance” and “angle” have a determinate signification when interpreted in reference (not to the circular points at infinity, but instead thereof) to any two given points whatever (see as to this my “Sixth Memoir on Quantics,” Nos. 220, *et seq.*\*) And this being so, the theorem can, without change in the statement thereof, be understood as referring to the two given points.

58. I say then that any theorem (referring explicitly or implicitly) to the circular points at infinity  $I, J$ , may be understood as a theorem referring instead to any two given points. We might of course give the theorems in the first instance in terms explicitly referring to the two given points—(viz., instead of a circle, speak of a conic through the two given points, and so in other instances); but, as just explained, this is not really more general, and the theorems would be given in a less concise and familiar form. It would not, on the face of the investigations, be apparent that in treating of the polyzomal curves

$$\sqrt{L(\Theta + L\Phi)} + \sqrt{m(\Theta + M\Phi)} + \&c. = 0,$$

( $\Theta = 0$  a conic,  $\Phi = 0$  a line, as above), that we were really treating of the

\* Phil. Transactions, vol. cxlix, (1859), pp. 61–90. See p. 86.

curves the zomals whereof are circles, and therein of the theories of foci and focofoci as about to be explained. And for these reasons I shall consider the two points  $\Theta = 0$ ,  $\Phi = 0$ , to be the circular points at infinity  $I, J$ , and in the investigations, &c., make use of the terms circle, right angles, &c., which, in their ordinary significations, have implicit reference to these two points.

The present Part does not explicitly relate to the theory of polyzomal curves, but contains a series of researches, partly analytical and partly geometrical, which will be made use of in the following Parts III. and IV. of the Memoir.

*The Circular Points at Infinity; Rectangular and Circular Co-ordinates*—Art. Nos. 59 to 62.

59. The co-ordinates made use of (except in the cases where the general trilinear co-ordinates  $(x, y, z)$ , or any other co-ordinates, are explicitly referred to), will be either the ordinary rectangular co-ordinates  $x, y$ , or else, as we may term them, the circular co-ordinates  $\xi, \eta$  ( $= x + iy, x - iy$  respectively,  $i = \sqrt{-1}$  as usual), but in either case I shall introduce for homogeneity the co-ordinate  $z$ , it being understood that this co-ordinate is in fact  $= 1$ , and that it may be retained or replaced by this its value, in different investigations or stages of the same investigation, as may for the time being be most convenient. In more concise terms, we may say that the co-ordinates are either the rectangular co-ordinates  $x, y$ , and  $z (= 1)$ , or else the circular co-ordinates  $\xi, \eta$ , and  $z (= 1)$ . The equation of the line infinity is  $z = 0$ ; the points  $I, J$  are given by the equations  $(x + iy = 0, z = 0)$  and  $(x - iy = 0, z = 0)$ , or, what is the same thing, by the equations  $(\xi = 0, z = 0)$  and  $(\eta = 0, z = 0)$  respectively; or in the rectangular co-ordinates the co-ordinates of these points are  $(-i, 1, 0)$  and  $(i, 1, 0)$  respectively, and in the circular co-ordinates they are  $(1, 0, 0)$  and  $(0, 1, 0)$  respectively. It is of course, only for points at infinity that the co-ordinate  $z$  is  $= 0$  (and observe that for any such point the  $x$  and  $y$  or  $\xi$  and  $\eta$  co-ordinates may be regarded as finite); for every point whatever not at infinity the co-ordinate  $z$  is, as stated above,  $= 1$ .

60. Consider a point  $A$ , whose co-ordinates (rectangular) are  $(a, a', 1)$  and (circular)  $(a, a', 1)$ , viz.,  $a = a + a'i, a' = a - a'i$ ; then the equations of the lines through  $A$  to the points  $I, J$ , are

$$x - az + i(y - a'z) = 0, \quad x - az - i(y - a'z) = 0$$

respectively, or they are

$$\xi - az = 0 \quad , \quad \eta - a'z = 0$$

respectively. These equations, if  $(a, a')$  or  $(a, a')$  are arbitrary, will, it is clear, be the equations of any two lines through the points  $I, J$ , respectively.

61. We have from either of the equations in  $(x, y, z)$

$$(x - az)^2 + (y - a'z)^2 = 0,$$

that is, the distance from each other of any two points  $(x, y, 1)$ , and  $(a, a', 1)$  in a

line through  $I$  or  $J$  is  $= 0$ . And in particular, if  $z = 0$ , then  $x^2 + y^2 = 0$ ; that is, the distance of the point  $(\alpha, \alpha', 1)$  from  $I$  or  $J$  is in each case  $= 0$ .

62. Consider for a moment any three points  $P, Q, A$ ; the perpendicular distance of  $P$  from  $QA$  is  $= 2$  triangle  $PQA \div$  distance  $QA$ ; if  $Q$  be any point on the line through  $A$  to either of the points  $I, J$ , and in particular if  $Q$  be either of the points  $I, J$ , then the triangle  $PQA$  is finite, but the distance  $QA$  is  $= 0$ : that is, the perpendicular distance of  $P$  from the line through  $A$  to either of the points  $I, J$ , that is, from any line through either of these points, is  $= \infty$ . But, as just stated, the triangle  $PQA$  is finite, or say the triangles  $PIA, PJA$  are each finite; viz., the co-ordinates (rectangular) of  $P, A$  being  $(x, y, z = 1), (\alpha, \alpha', 1)$  or (circular)  $(\xi, \eta, z = 1), (\alpha, \alpha', 1)$ , the expressions for the doubles of these triangles respectively are

$$\begin{vmatrix} x, y, z \\ -i, 1, 0 \\ \alpha, \alpha', 1 \end{vmatrix}, \quad \begin{vmatrix} x, y, z \\ i, 1, 0 \\ \alpha, \alpha', 1 \end{vmatrix}$$

that is, they are (rectangular co-ordinates)  $x - \alpha z + i(y - \alpha' z), x - \alpha z - i(y - \alpha' z)$ , or (circular co-ordinates)  $\xi - \alpha z, \eta - \alpha' z$ .

Representing the double areas by  $PIA, PJA$ , respectively, and the squared distance of the points  $A, P$  by  $A$ , we have—

$$\begin{aligned} A &= (x - \alpha z)^2 + (y - \alpha' z)^2 \\ &= (\xi - \alpha z)(\eta - \alpha' z), = PIA, PJA. \end{aligned}$$

*Antipoints; Definition and Fundamental Properties—Art. No. 63.*

63. Two pairs of points  $(A, B)$  and  $(A_1, B_1)$  which are such that the lines  $AB, A_1B_1$  bisect each other at right angles in a point  $O$  in such wise that  $OA = OB = i OA_1 = i OB_1$ , are said to be antipoints, each of the other. In rectangular co-ordinates, taking the co-ordinates of  $(A, B)$  to be  $(\alpha, 0, 1)$  and  $(-\alpha, 0, 1)$ , those of  $(A_1, B_1)$  will be  $(0, \alpha i, 1)$  and  $(0, -\alpha i, 1)$  respectively, whence joining the points  $(A, B)$  with the points  $(I, J)$ , the points  $A_1, B_1$  are given as the intersections of the lines  $AI$  and  $BJ$ , and of the lines  $AJ$  and  $BI$  respectively. Or, what is the same thing, in any quadrilateral wherein  $I, J$  are opposite angles, the remaining pairs  $(A, B)$  and  $(A_1, B_1)$  are antipoints each of the other.

64. In circular co-ordinates, if the co-ordinates of  $A$  are  $(\alpha, \alpha', 1)$ , and those of  $B$  are  $(\beta, \beta', 1)$ , then the equations of

$$\begin{aligned} AI, AJ \text{ are } \xi - \alpha z = 0, \quad \eta - \alpha' z = 0 \\ BI, BJ \text{ ,, } \xi - \beta z = 0, \quad \eta - \beta' z = 0 \end{aligned}$$

whence the equations of

$$\begin{aligned} A_1 I, A_1 J \text{ are } \xi - \alpha z = 0, \quad \eta - \beta' z = 0 \\ B_1 I, B_1 J \text{ ,, } \xi - \beta z = 0, \quad \eta - \alpha' z = 0. \end{aligned}$$

65. Considering any point  $P$  the co-ordinates of which are  $\xi, \eta, z$  ( $= 1$ ), let  $A, B, A_1, B_1$  be its squared distances from the points  $A, B, A_1, B_1$  respectively; then by what precedes—

$$\begin{aligned} A &= (\xi - \alpha z) (\eta - \alpha' z), \\ B &= (\xi - \beta z) (\eta - \beta' z), \\ A_1 &= (\xi - \alpha z) (\eta - \beta' z), \\ B_1 &= (\xi - \beta z) (\eta - \alpha' z), \end{aligned}$$

and thence

$$A \cdot B = A_1 \cdot B_1 ;$$

that is, the product of the squared distances of a point  $P$  from any two points  $A, B$ , is equal to the product of the squared distances of the same point  $P$  from the two antipoints  $A_1, B_1$ . This theorem, which was, I believe, first given by me in the *Educational Times* (see reprint, vol. vi. 1866, p. 81), is an important one in the theory of foci. It is to be further noticed that we have

$$\begin{aligned} A + B - A_1 - B_1 &= (\alpha - \beta) (\alpha' - \beta') z^2 \\ &= K z^2 = K, \end{aligned}$$

if  $K, = (\alpha - \alpha') (\beta - \beta')$ , be the squared distance of the points  $A, B, = -$  squared distance of points  $A_1, B_1$ .

*Antipoints of a Circle—Art. No. 66.*

66. A similar notion to that of two pairs of antipoints is as follows, viz., if from the centre of a circle perpendicular to its plane and in opposite senses, we measure off two distances each  $= i$  into the radius, the extremities of these distances are antipoints of the circle. It is clear that the antipoints of the circle and the extremities of any diameter thereof are (in the plane of these four points) pairs of antipoints. It is to be added that each antipoint is the centre of a sphere radius zero, or say of a cone sphere, passing through the circle: the circle is thus the intersection of the two cone spheres having their centres at the two antipoints respectively.

*Antipoints in relation to a Pair of Orthotomic Circles—Art. No. 67.*

67. It is a well-known property that if any circle pass through the points  $(A, B)$ , and any other circle through the antipoints  $(A_1, B_1)$ , then these two circles cut at right angles. Conversely if a circle pass through the points  $A, B$ , then all the orthotomic circles which have their centres on the line  $AB$  pass through the antipoints  $A_1, B_1$ . In particular, if on  $AB$  as diameter we describe a circle and on  $A_1 B_1$  as diameter a circle, then these two circles—being, it is clear, concentric circles with their radii in the ratio  $1 : i$ , and as concentric circles touching each other at the points  $(I, J)$ —cut each other at right angles; or say they are concentric orthotomic circles.

*Forms of the Equation of a Circle—Art. Nos. 68 to 71.*

68. In rectangular co-ordinates the equation of a circle, co-ordinates of centre  $(a, a', 1)$  and radius =  $a''$ , is

$$A^\circ = (x-az)^2 + (y-a'z)^2 - a''^2 z^2 = 0;$$

and in circular co-ordinates, the co-ordinates of the centre being  $(a, a', 1)$ , and radius =  $a''$  as before, the equation is

$$A^\circ = (\xi - az)(\eta - a'z) - a''^2 z^2 = 0.$$

69. I observe in passing, that the origin being at the centre and the radius being = 1, then writing also  $z = 1$ , the equation of the circle is  $\xi\eta = 1$ , that is the circular co-ordinates of any point of the circle, expressed by means of a variable parameter  $\theta$ , are  $(\theta, \frac{1}{\theta}, 1)$ .

70. Consider a current point  $P$ , the co-ordinates of which (rectangular) are  $x, y, z (= 1)$ , and (circular) are  $\xi, \eta, z (= 1)$ , then the foregoing expression

$$\begin{aligned} A^\circ &= (x-az)^2 + (y-a'z)^2 - a''^2 z^2 \\ &= (\xi - az)(\eta - a'z) - a''^2 z^2 \end{aligned}$$

denotes, it is clear, the square of the tangential distance of the point  $P$  from the circle  $A^\circ = 0$ .

71. But there is another interpretation of this same function  $A^\circ$ , viz., writing therein  $z = 1$ , and then

$$A^\circ = (x-a)^2 + (y-a')^2 + (a''i)^2,$$

we see that  $A^\circ$  is the squared distance of  $P$  from either of the anti-points of the circle (points lying, it will be recollected, out of the plane of the circle), and we have thus the theorem that the square of the tangential distance of any point  $P$  from the circle is equal to the square of its distance from either anti-point of the circle.

*On a System of Sixteen Points—Art. Nos. 72 to 77.*

72. Take  $(A, B, C, D)$  any four concyclic points, and let the anti-points of

$$\begin{aligned} (B, C), (A, D) &\text{ be } (B_1, C_1), (A_1, D_1), \\ (C, A), (B, D) &\text{ ,, } (C_2, A_2), (B_2, D_2), \\ (A, B), (C, D) &\text{ ,, } (A_3, B_3), (C_3, D_3), \end{aligned}$$

then each of the three new sets  $(A_1, B_1, C_1, D_1)$ ,  $(A_2, B_2, C_2, D_2)$ ,  $(A_3, B_3, C_3, D_3)$  will be a set of four concyclic points.

73. Let  $O$  be the centre of the circle through  $(A, B, C, D)$ , say of the circle  $O$ , and then, the lines  $BC, AD$  meeting in  $R$ , the lines  $CA, BD$  in  $S$ , and the lines  $AB, CD$  in  $T$ , let each of these points be made the centre of a circle orthotomic to  $O$ , viz., let these new circles be called the circles  $R, S, T$  respectively.

As regards the circle  $R$ , since its centre lies in  $BC$ , the circle passes through  $(B_1, C_1)$ ; and since the centre lies in  $AD$ , the circle passes through  $(A_1, D_1)$ , that is, the four points  $(A_1, B_1, C_1, D_1)$  lie in the circle  $R$ . Similarly  $(A_2, B_2, C_2, D_2)$  lie in the circle  $S$ , and  $(A_3, B_3, C_3, D_3)$  in the circle  $T$ .

74. The points  $R, S, T$  are conjugate points in relation to the circle  $O$ ; that is,  $ST, TR, RS$  are the polars of  $R, S, T$  respectively in regard to this circle; and they are, consequently, at right angles to the lines  $OR, OS, OT$  respectively; viz., the four centres  $O, R, S, T$  are such that the line joining any two of them cuts at right angles the line joining the other two of them, and we see that the relation between the four sets is in fact a symmetrical one; this is most easily seen by consideration of the circular points at infinity  $I, J$ , the four sets of points may be arranged thus:—

$$\begin{array}{c} A, A_3, A_2, A_1, \\ B_3, B, B_1, B_2, \\ C_2, C_1, C, C_3, \\ D_1, D_2, D_3, D, \end{array}$$

in such wise that any four of them in the same vertical line pass through  $I$ , and any four in the same horizontal line pass through  $J$ ; and this being so, starting for instance with  $(A_3, B_3, C_3, D_3)$  we have anti-points

$$\begin{array}{l} \text{of } (B_3, C_3), (A_3, D_3) \text{ are } (B_2, C_2), (A_2, D_2), \\ \text{,, } (C_3, A_3), (B_3, D_3) \text{ ,, } (C_1, A_1), (B_1, D_1), \\ \text{,, } (A_3, B_3), (C_3, D_3) \text{ ,, } (A, B), (C, D), \end{array}$$

and similarly if we start from  $(A_1, B_1, C_1, D_1)$  or  $(A_2, B_2, C_2, D_2)$ .

75. I return for a moment to the construction of  $(A_1, B_1, C_1, D_1)$ ; these are points on the circle  $R$ , and  $(B_1, C_1)$  are the anti-points of  $(B, C)$ ; that is, they are the intersections of the circle  $R$  by the line at right angles to  $BC$  from its middle point, or, what is the same thing, by the perpendicular on  $BC$  from  $O$ . Similarly  $(A_1, D_1)$  are the anti-points of  $(A, D)$ ; that is, they are the intersections of the circle  $R$  by the perpendicular on  $AD$  from  $O$ . And the like as to  $(A_2, B_2, C_2, D_2)$  and  $(A_3, B_3, C_3, D_3)$  respectively.

76. Hence, starting with the points  $A, B, C, D$  on the circle  $O$ , and constructing as above the circles  $P, Q, R$ , and constructing also the perpendiculars from  $O$  on the six chords  $AB, AC, \&c.$ ,

$$\begin{array}{l} \text{the perpendiculars on } BC, AD \text{ meet circle } R \text{ in } (B_1, C_1), (A_1, D_1), \\ \text{,, } CA, BD \text{ ,, ,, } S \text{ ,, } (C_2, A_2), (B_2, D_2), \\ \text{,, } AB, CD \text{ ,, ,, } T \text{ ,, } (A_3, B_3), (C_3, D_3), \end{array}$$

so that the whole system is given by means of the circles  $P, Q, R$ , and the six perpendiculars.

77. If to fix the ideas  $(A, B, C, D)$  are real points taken in order on the real circle  $O$ , then the points  $R, S, T$  are each of them real; but  $R$  and  $T$  lie outside,  $S$  inside the circle  $O$ . The circles  $R$  and  $T$  are consequently real, but the circle  $S$  imaginary, viz., its radius is  $= i$  into a real quantity; the imaginary points  $(A_1, B_1, C_1, D_1)$  are thus given as the intersections of a real circle by a pair of real lines, and the like as to the imaginary points  $(A_3, B_3, C_3, D_3)$ ; but the

imaginary points  $(A_2, B_2, C_2, D_2)$  are only given as the intersections of an imaginary circle (centre real and radius a pure imaginary) by a pair of real lines. The points  $(C_2, A_2)$  *quâ* anti-points of  $(C, A)$  are easily constructed as the intersections of a real circle by a real line, and the like as to the points  $(B_2, D_2)$  *quâ* anti-points of  $(B, D)$ , but the construction for the two pairs of points cannot be effected by means of the same real circle.

*Property in regard to Four Confocal Conics—Art. Nos. 78 to 80.*

78. All the conics which pass through the four concyclic points  $A, B, C, D$ , have their axes in fixed directions; but three such conics are the line-pairs  $(BC, AD)$ ,  $(CA, BD)$ , and  $(AB, CD)$ , whence the directions of the axes are those of the bisectors of the angles formed by any one of these pairs of lines; hence, in particular, considering either axis of a conic through the four points, the lines  $AB$  and  $CD$  are equally inclined on opposite sides to this axis, and this leads to the theorem that the anti-points  $(A_3, B_3)$   $(C_3, D_3)$  are in a conic confocal to the given conic through  $(A, B, C, D)$ ; whence, also, considering any given conic whatever through  $(A, B, C, D)$ , the points  $(A_1, B_1, C_1, D_1)$ ,  $(A_2, B_2, C_2, D_2)$   $(A_3, B_3, C_3, D_3)$  lie severally in three conics, each of them confocal with the given conic.

79. To prove this, consider any two confocal conics, say an ellipse and a hyperbola, and let  $F$  be one of their four intersections; join  $F$  with the common centre  $O$ , and let  $OT, ON$  be parallel to the tangent and normal respectively of the ellipse at the point  $F$ .  $OF, OT$  are in direction conjugate axes of the ellipse, and  $OF, ON$  are in direction conjugate axes of the hyperbola; and if they are also the axes in magnitude, that is, if the points  $T, N$  are the intersections of  $OT$  with the ellipse and of  $ON$  with the hyperbola respectively, then it is easy to show that  $\overline{OT}^2 + \overline{ON}^2 = 0$ . And this being so, imagine on the ellipse any two points  $A, B$  such that the chord  $AB$  is parallel to  $OT$ , that is conjugate to  $OF$ ;  $AB$  is bisected by  $OF$ , say in a point  $K$ , or we have parallel to  $OT$  the semichords or ordinates  $KA = KB$ ; and we may, perpendicularly to this or parallel to  $ON$ , draw through  $K$  in the hyperbola a chord  $A_3B_3$ , which chord will be bisected in  $K$ , or we shall have  $KA_3 = KB_3$ . Hence  $KA, KA_3$  are in the ellipse and the hyperbola respectively ordinates conjugate to the same diameter  $OF$ , and the semi-diameters conjugate to  $OF$  being  $OT, ON$  respectively, we have  $KA^2 (= KB^2) : \overline{KA}_3^2 (= \overline{KB}_3^2) = \overline{OT}^2 : \overline{ON}^2$ , that is,  $\overline{KA}^2 = \overline{KB}^2 = -\overline{KA}_3^2 = -\overline{KB}_3^2$ ; or  $(A_3, B_3)$  will be the anti-points of  $(A, B)$ .

80. Conversely, if in the ellipse we have the two points  $(A, B)$ , then drawing the diameter  $OF$  conjugate to  $AB$ , and through its extremity  $F$ , the confocal hyperbola, then the anti-points  $(A_3, B_3)$  will lie on the hyperbola. And similarly, if on the ellipse we have the two points  $(C, D)$ , then drawing the diameter  $OG$  conjugate to  $CD$ , and through its extremity  $G$  a confocal hyperbola, the

anti-points  $(C_3, D_3)$  will lie on the hyperbola. Suppose  $(A, B, C, D)$  are concyclic, then, as noticed,  $AB$  and  $CD$  will be equally inclined on opposite sides to the transverse axis of the ellipse—the conjugate diameters  $OF, OG$  will therefore be equally inclined on opposite sides of the transverse axis—and the points  $F$  and  $G$  will therefore be situate symmetrically on opposite sides of the transverse axis, that is, the points  $F$  and  $G$  will respectively determine the same confocal hyperbola, and we have thus the required theorem, viz., if  $(A, B, C, D)$  are any four concyclic points on an ellipse, or say on a conic, and if  $(A_3, B_3)$  are the anti-points of  $(A, B)$ , and  $(C_3, D_3)$  the anti-points of  $(C, D)$ , then  $(A_3, B_3, C_3, D_3)$  will lie on a conic confocal with the given conic.

*System of the Sixteen Points, the Axial Case—Art. Nos. 81 to 85.*

81. The theorems hold good when the four points  $A, B, C, D$  are in a line; the anti-points  $(B_1, C_1)$  of  $(B, C)$ , &c., are in this case situate symmetrically on opposite sides of the line, so that it is evident at sight that we have  $(A_1, B_1, C_1, D_1)$ ,  $(A_2, B_2, C_2, D_2)$ ,  $(A_3, B_3, C_3, D_3)$ , each set in a circle; and that the centres  $R, S, T$  of these circles lie in the line. The construction for the general case becomes, however, indeterminate, and must therefore be varied. If in the general case we take any circle through  $(B, C)$ , and any circle through  $(A, D)$ , then the circle  $R$  cuts at right angles these two circles, and has, consequently, its centre  $R$  in the radical axis of the two circles; whence, when the four points are in a line, taking any circle through  $(B, C)$ , or in particular the circle on  $BC$  as diameter, and any circle through  $(A, D)$ , or in particular the circle on  $AD$  as diameter,—the radical axis of these two circles intersects the line in the required centre  $R$ , and the circle  $R$  is the circle with this centre cutting at right angles the two circles respectively; the circles  $S$  and  $T$  are, of course, obtained by the like construction in regard to the combinations  $(C, A; B, D)$  and  $(A, B; C, D)$ , respectively. It may be added, that we have

$$\left. \begin{array}{l} R \\ S \\ T \end{array} \right\} \text{centre and} \left\{ \begin{array}{l} \text{extremities } R \\ \text{of diameter } S \\ \text{of circles } T \end{array} \right\} \text{sibconjugate points of involutions} \left\{ \begin{array}{l} B, C; A, D, \\ C, A, B, D, \\ A, B, C, D, \end{array} \right.$$

and that (as in the general case) the circles  $R, S, T$  intersect each pair of them at right angles; and they are evidently each intersected at right angles by the line  $ABCD$  (or axis of the figure), which replaces the circle  $O$  in the general case.

82. If the points  $A, B, C, D$  are taken in order on the line, then the points  $R, S, T$  are all real, viz., the point  $R$  is situate, on one side or the other, outside  $AD$ , but the points  $S$  and  $T$  are each of them situate between  $B$  and  $C$ ; the circles  $R$  and  $T$  are real, but the circle  $S$  has its radius a pure imaginary quantity.

83. If one of the four points, suppose  $D$ , is at infinity on the line, then the anti-points of  $(A, D)$ , of  $(B, D)$ , and of  $(C, D)$  are each of them the two points

(*I, J*). It would at first sight appear that the only conditions for the circles *R, S, T* were the conditions of passing through the anti-points of (*B, C*), of (*C, A*), and of (*A, B*) respectively, and that these circles thus became indeterminate; but in fact the definition of the circles is then as follows, viz., *R* has its centre at *A*, and passes through the anti-points of (*B, C*): (whence squared radius = *AB . AC*). And similarly, *S* has its centre at *B*, and passes through anti-points of (*C, A*), (squared radius = *BA . BC*); and *T* has its centre at *C*, and passes through anti-points of (*A, B*), (squared radius = *CA . CB*); these three circles cut each other at right angles. As before, *A, B, C* being in order on the line, the circles *R, T* are real, but the circle *S* has its radius a pure imaginary quantity.

84. That the circles are as just mentioned appears as follows: taking the line as axis of *x*, and *a, b, c, d* for the *x* co-ordinates of the four points respectively, then the co-ordinates of *A*<sub>1</sub>, *D*<sub>1</sub> are

$$\frac{1}{2}(a + d), \pm \frac{1}{2}i(a - d);$$

whence, *m* being arbitrary, the general equation of a circle through *A*<sub>1</sub>, *D*<sub>1</sub> is

$$x^2 + y^2 - 2mzx + [m(a + d) - ad]z^2 = 0,$$

writing herein

$$m = a - \frac{k^2}{d}$$

this becomes

$$x^2 + y^2 - 2\left(a - \frac{k^2}{d}\right)zx + \left(a^2 - k^2 - \frac{ak^2}{d}\right)z^2 = 0,$$

viz., for *d* = ∞ it is

$$(x - az)^2 + y^2 - k^2z^2 = 0,$$

which is a circle having *A* for its centre, and its radius an arbitrary quantity *k*. If the circle passes through the anti-points of *B, C*, the co-ordinates of these are

$$\frac{1}{2}(b + c), \pm \frac{1}{2}i(b - c),$$

and we find

$$k^2 = \left[\frac{1}{2}(b + c) - a\right]^2 - \frac{1}{4}(b - c)^2 = (a - b)(a - c).$$

85. Reverting to the general case of four points *A, B, C, D* on a line, the theorem as to the confocal conics holds good under the form that, drawing any conic whatever through (*A*<sub>1</sub>, *B*<sub>1</sub>, *C*<sub>1</sub>, *D*<sub>1</sub>) the points (*A*<sub>2</sub>, *B*<sub>2</sub>, *C*<sub>2</sub>, *D*<sub>2</sub>), and (*A*<sub>3</sub>, *B*<sub>3</sub>, *C*<sub>3</sub>, *D*<sub>3</sub>) lie in confocal conics, these conics have their centre on the line, and axes in the direction of and perpendicular to the line. When *D* is at infinity, the confocal conics become any three concentric circles through (*B*<sub>1</sub>, *C*<sub>1</sub>), (*C*<sub>2</sub>, *A*<sub>2</sub>), and (*A*<sub>3</sub>, *B*<sub>3</sub>) respectively.

*The Involution of Four Circles.*—Art. Nos. 86 to 91.

86. Consider any four points *A, B, C, D*, the centres of circles denoted by these same letters, and let *A*<sup>o</sup>, *B*<sup>o</sup>, *C*<sup>o</sup>, *D*<sup>o</sup> signify as usual, viz., if (in orthogonal

co-ordinates)  $(a, a', 1)$  are the co-ordinates of the centre, and  $a''$  the radius of the circle  $A$ , then  $A^\circ$  stands for  $(x - az)^2 + (y - a'z)^2 - a''^2z^2$ , and the like for  $B^\circ, C^\circ, D^\circ$ . Write also

$$a : b : c : d = BCD : -CDA : DAB : -ABC,$$

where  $BCD$ , &c., are the triangles formed by the points  $(B, C, D)$ , &c.; the analytical expressions are

$$a : b : c : d = \begin{vmatrix} b & b' & 1 \\ c & c' & 1 \\ d & d' & 1 \end{vmatrix} : - \begin{vmatrix} c & c' & 1 \\ d & d' & 1 \\ a & a' & 1 \end{vmatrix} : \begin{vmatrix} d & d' & 1 \\ a & a' & 1 \\ b & b' & 1 \end{vmatrix} : - \begin{vmatrix} a & a' & 1 \\ b & b' & 1 \\ c & c' & 1 \end{vmatrix}$$

so that

$$\begin{aligned} a + b + c + d &= 0, \\ aa + bb + cc + dd &= 0, \\ aa' + bb' + cc' + dd' &= 0; \end{aligned}$$

this being so, it is clear that we have

$$\begin{aligned} aA^\circ + bB^\circ + cC^\circ + dD^\circ &= \\ z^2[a(a^2 + a'^2 - a''^2) + b(b^2 + b'^2 - b''^2) + c(c^2 + c'^2 - c''^2) + d(d^2 + d'^2 - d''^2)] &= Kz^2 = K, \end{aligned}$$

a constant.

87. I am not aware that in the general case there is any convenient expression for this constant  $K$ ; it is  $= 0$  when the four circles have the same orthotomic circle; in fact, taking as origin the centre of the orthotomic circle, and its radius to be  $= 1$ , we have

$$a^2 + a'^2 - a''^2 = 1, \text{ \&c. ,}$$

whence

$$K = a + b + c + d = 0;$$

that is, if the circles  $A, B, C, D$  have the same orthotomic circle, then  $A^\circ, B^\circ, C^\circ, D^\circ$ ,  $a, b, c, d$ , signifying as above, we have

$$aA^\circ + bB^\circ + cC^\circ + dD^\circ = 0,$$

and, in particular, if the circles reduce themselves to the points  $A, B, C, D$  respectively, then (writing as usual  $A, B, C, D$  in place of  $A^\circ, B^\circ, C^\circ, D^\circ$ ) if the four points  $A, B, C, D$  are on a circle, we have

$$aA + bB + cC + dD = 0.$$

88. This last theorem may be regarded as a particular case of the theorem

$$aA + bB + cC + dD = Kz^2 = K,$$

viz., the four circles reducing themselves to the points  $A, B, C, D$ , we can find for the constant  $K$  an expression which will of course vanish when the points are on a circle. For this purpose, let the lines  $BC, AD$  meet in  $R$ , the lines

$CA, BD$  in  $S$ , and the lines  $AB, CD$  in  $T$ ; we may, to fix the ideas, consider  $ABCD$  as forming a convex quadrilateral,  $R$  and  $T$  will then be the exterior centres,  $S$  the interior centre;  $a, b, c, d$ , may be taken equal to  $BCD, -CDA, DAB, -ABC$ , where the areas  $BCD, \&c.$ , are each taken positively. The expression  $aA + bB + cC + dD$  has the same value, whatever is the position of the point  $P (x, y, z = 1)$ ; taking this point at  $R$ , and writing for a moment

$$RA = \alpha, RB = \beta, RC = \gamma, RD = \delta,$$

then

$$BCD = (RCD - RBD) = \frac{1}{2}RD (RC - RB) \sin R = (\gamma - \beta)\delta \sin R,$$

with similar expressions for the other triangles; and we thus have

$$aA + bB + cC + dD = \frac{1}{2}z^2 \cdot \sin R \begin{pmatrix} \alpha^2(\gamma - \beta)\delta \\ -\beta^2(\delta - \alpha)\gamma \\ +\gamma^2(\delta - \alpha)\beta \\ -\delta^3(\gamma - \beta)\alpha \end{pmatrix} = \frac{1}{2}z^2 \sin R (\beta\gamma - \alpha\delta)(\gamma - \beta)(\delta - \alpha),$$

that is, replacing  $\alpha, \beta, \gamma, \delta$ , by their values, and writing also  $z = 1$ , we have

$$aA + bB + cC + dD = \frac{1}{2} \sin R \cdot (RB \cdot RC - RA \cdot RD)BC \cdot AD,$$

where  $\frac{1}{2} \sin R \cdot BC \cdot AD$  is in fact the area of the quadrilateral  $ABCD$ ; we have thus

$$\begin{aligned} aA + bB + cC + dD &= (RB \cdot RC - RA \cdot RD) \square \\ &= (SC \cdot SA - SB \cdot SD) \square \\ &= (TA \cdot TB - TC \cdot TD) \square \end{aligned}$$

where it is to be observed that  $SA, SC$  being measured in opposite directions from  $S$ , must be considered, one as positive, the other as negative, and the like as regards  $SB, SD$ . This expression for the value of the constant is due to Mr Crofton. In the particular case where  $A, B, C, D$ , are on a circle, we have as before

$$aA + bB + cC + dD = 0.$$

89. If the four points  $A, B, C, D$ , are on a circle, then, taking as origin the centre of this circle and its radius as unity, the circular co-ordinates of the four points will be

$$\left(\alpha, \frac{1}{\alpha}, 1\right), \left(\beta, \frac{1}{\beta}, 1\right), \left(\gamma, \frac{1}{\gamma}, 1\right), \left(\delta, \frac{1}{\delta}, 1\right),$$

the corresponding forms of  $A^\circ, \&c.$ , being

$$A^\circ = \left(\xi - \alpha z\right) \left(\eta - \frac{1}{\alpha} z\right) - \alpha'^2 z^2, \&c.$$

the expressions for  $a, b, c, d$ , observing that we have

$$\begin{vmatrix} \beta, \beta^{-1}, 1 \\ \gamma, \gamma^{-1}, 1 \\ \delta, \delta^{-1}, 1 \end{vmatrix} = \frac{1}{\beta\gamma\delta} \begin{vmatrix} 1, \beta, \beta^2 \\ 1, \gamma, \gamma^2 \\ 1, \delta, \delta^2 \end{vmatrix} = \frac{1}{\beta\gamma\delta} (\beta\gamma\delta), \&c.$$

if  $(\beta\gamma\delta)$ , &c., denote  $(\beta - \gamma)(\gamma - \delta)(\delta - \beta)$ , &c., become

$$a : b : c : d = \alpha(\beta\gamma\delta) : -\beta(\gamma\delta\alpha) : \gamma(\delta\alpha\beta) : -\delta(\alpha\beta\gamma),$$

which are convenient formulæ for the case in question.

90. If the points  $A, B, C, D$ , are on a line, then taking this line for the axis of  $x$ , we may write  $\mathbf{A}^{\circ} = (x - az)^2 + y^2 - a''^2 z^2$ , &c. It is to be remarked here that we can, without any relation whatever between the radii of the circles, satisfy the equation

$$a\mathbf{A}^{\circ} + b\mathbf{B}^{\circ} + c\mathbf{C}^{\circ} + d\mathbf{D}^{\circ} = 0;$$

in fact this will be the case if we have

$$\begin{aligned} a &+ b &+ c &+ d &= 0, \\ aa &+ bb &+ cc &+ dd &= 0, \\ a(a^2 - a''^2) &+ b(b^2 - b''^2) &+ c(c^2 - c''^2) &+ d(d^2 - d''^2) &= 0, \end{aligned}$$

equations which determine the ratios  $a : b : c : d$ . In the case where the circles reduce themselves to the points  $A, B, C, D$ , these equations become

$$\begin{aligned} a &+ b &+ c &+ d &= 0, \\ aa &+ bb &+ cc &+ dd &= 0, \\ aa^2 &+ bb^2 &+ cc^2 &+ dd^2 &= 0, \end{aligned}$$

giving

$$a : b : c : d = (bcd) : - (cda) : (dab) : - (abc);$$

if for shortness  $(bcd)$ , &c. stand for  $(b - c)(c - d)(d - b)$ , &c.; and for these values, we have

$$a\mathbf{A} + b\mathbf{B} + c\mathbf{C} + d\mathbf{D} = 0.$$

91. A very noticeable case is when the four circles are such that the foregoing values of  $(a, b, c, d)$  also satisfy the equation

$$a\mathbf{A}^{\circ} + b\mathbf{B}^{\circ} + c\mathbf{C}^{\circ} + d\mathbf{D}^{\circ} = 0;$$

the condition for this is obviously

$$aa''^2 + bb''^2 + cc''^2 + dd''^2 = 0;$$

or, as it may also be written,

$$\frac{a''^2}{(a-b)(a-c)(a-d)} + \frac{b''^2}{(b-c)(b-d)(b-a)} + \frac{c''^2}{(c-d)(c-a)(c-b)} + \frac{d''^2}{(d-a)(d-b)(d-c)} = 0.$$

*On a Locus connected with the foregoing Properties.*—Art. No. 92.

92. If, as above,  $A, B, C, D$  are any four points, and  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$  are the squared distances of a current point  $P$  from the four points respectively, then the locus of the foci of the conics which pass through the four points is the tetrazomal curve

$$a\sqrt{\mathbf{A}} + b\sqrt{\mathbf{B}} + c\sqrt{\mathbf{C}} + d\sqrt{\mathbf{D}} = 0.$$

In fact the sum  $a\mathbf{A} + b\mathbf{B} + c\mathbf{C} + d\mathbf{D}$  has, it has been seen, a constant value for

all positions of the point  $P$ ; taking  $P$  to be the other focus, its squared distances are  $(k - \sqrt{A})^2$ , &c., whence for the first-mentioned focus we have

$$aA + bB + cC + dD = a(k - \sqrt{A})^2 + b(k - \sqrt{B})^2 + c(k - \sqrt{C})^2 + d(k - \sqrt{D})^2;$$

or recollecting that  $a + b + c + d = 0$ , we have for the locus in question  $a\sqrt{A} + b\sqrt{B} + c\sqrt{C} + d\sqrt{D} = 0$ ; this locus will be discussed in the sequel. I remark here, that in the case where the four points are on a circle, then (as mentioned above), the axes of the several conics are in the same fixed directions; there are thus two sets of foci, those on the axis in one direction, and those on the axis in the other direction; it might therefore be anticipated, and it will appear, that in this case the tetrazomal breaks up into two trizomal curves.

*Formulae as to the two Sets (A, B, C, D), and (A<sub>1</sub>, B<sub>1</sub>, C<sub>1</sub>, D<sub>1</sub>), each of four Concyelic Points—Art. Nos. 93 to 98.*

93. Consider the four points  $A, B, C, D$  on a circle, then taking, as before, their circular co-ordinates to be  $(\alpha, \alpha', 1), (\beta, \beta', 1), (\gamma, \gamma', 1), (\delta, \delta', 1)$ , the condition that the points may be on a circle is

$$\begin{vmatrix} 1, \alpha, \alpha', \alpha\alpha' \\ 1, \beta, \beta', \beta\beta' \\ 1, \gamma, \gamma', \gamma\gamma' \\ 1, \delta, \delta', \delta\delta' \end{vmatrix} = 0,$$

viz., this equation may be written

$$\begin{aligned} &(\beta - \gamma)(\alpha - \delta) : (\gamma - \alpha)(\beta - \delta) : (\alpha - \beta)(\gamma - \delta) \\ &= (\beta' - \gamma')(\alpha' - \delta') : (\gamma' - \alpha')(\beta' - \delta') : (\alpha' - \beta')(\gamma' - \delta'); \end{aligned}$$

or, if for shortness, we take

$$\begin{aligned} a &= \beta - \gamma, & f &= \alpha - \delta, & a' &= \beta' - \gamma', & f' &= \alpha' - \delta', \\ b &= \gamma - \alpha, & g &= \beta - \delta, & b' &= \gamma' - \alpha', & g' &= \beta' - \delta', \\ c &= \alpha - \beta, & h &= \gamma - \delta, & c' &= \alpha' - \beta', & h' &= \gamma' - \delta', \end{aligned}$$

and consequently

$$\begin{aligned} af + bg + ch &= 0, & a'f' + b'g' + c'h' &= 0, \\ a &= g - h, & a' &= g' - h', \\ b &= h - f, & b' &= h' - f', \\ c &= f - g, & c' &= f' - g', \\ a + b + c &= 0, & a' + b' + c' &= 0, \end{aligned}$$

then the equation is

$$af : bg : ch = a'f' : b'g' : c'h'.$$

94. Let  $a, b, c, d$ , denote as before ( $a : b : c : d = BCD : -CDA : DAB : -ABC$ ), then we have

$$a : b : c : d = \begin{vmatrix} \beta, \beta', 1 \\ \gamma, \gamma', 1 \\ \delta, \delta', 1 \end{vmatrix} : - \begin{vmatrix} \gamma, \gamma', 1 \\ \delta, \delta', 1 \\ \alpha, \alpha', 1 \end{vmatrix} : \begin{vmatrix} \delta, \delta', 1 \\ \alpha, \alpha', 1 \\ \beta, \beta', 1 \end{vmatrix} : - \begin{vmatrix} \alpha, \alpha', 1 \\ \beta, \beta', 1 \\ \gamma, \gamma', 1 \end{vmatrix}$$

and we may write

$$\begin{aligned} a &= \quad \quad \quad ah' - a'h, \quad ag' - a'g, \quad gh' - g'h, \\ b &= bh' - b'h, \quad \quad \quad , \quad bf' - b'f, \quad hf' - h'f, \\ c &= cg' - c'g, \quad cf' - c'f, \quad \quad \quad , \quad fg' - f'g, \\ d &= cb' - c'b, \quad ac' - a'c, \quad ba' - b'a, \quad \quad \quad . \end{aligned}$$

viz., the expressions in the same horizontal line are equal, and a, b, c, d are proportional to the expressions in the four lines respectively.

95. I say that we have

$$\frac{c'f'}{ah}a = \frac{c'g'}{bh}b = \frac{a'f'}{af}c = \frac{f'g'}{ab}d,$$

viz., this will be the case if

$$\begin{aligned} bc'a &= hg'd, \\ ac'b &= hf'd, \\ a'bc &= fg'd, \end{aligned}$$

and selecting the convenient expressions for a, b, c, d, these equations become

$$\begin{aligned} bc'(gh' - g'h) &= g'h(cb' - c'b), \\ ac'(hf' - h'f) &= f'h(ac' - a'c), \\ a'b(fg' - f'g) &= fg'(ba' - b'a), \end{aligned}$$

viz., these equations are respectively  $bgc'h' = b'g'ch$ ,  $cha'f' = c'h'af$ ,  $afb'g' = a'f'bg$ , and are consequently satisfied. It thus appears that the equation

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$$

is transformable into

$$\frac{c'f'}{ah}l + \frac{c'g'}{bh}m + \frac{a'f'}{af}n + \frac{f'g'}{ab}p = 0.$$

which is of course one of a system of similar forms.

96. Take  $(A_1, D_1)$  the anti-points of  $A, D$ ;  $(B_1, C_1)$  the anti-points of  $(B, C)$ ; or say that the circular co-ordinates of  $A_1, B_1, C_1, D_1$  are  $(\alpha, \delta', 1)$ ,  $(\beta, \gamma', 1)$ ,  $(\gamma, \beta', 1)$ ,  $(\delta, \alpha', 1)$  respectively; the points  $A_1, B_1, C_1, D_1$  are, as above mentioned, on a circle, the condition that this may be so being in fact

$$\begin{vmatrix} 1, \alpha, \delta', \alpha\delta' \\ 1, \beta, \gamma', \beta\gamma' \\ 1, \gamma, \beta', \gamma\beta' \\ 1, \delta, \alpha', \delta\alpha' \end{vmatrix} = 0,$$

equivalent to

$$af : bg : ch = a'f' : b'g' : c'h'.$$

97. Let  $(a_1, b_1, c_1, d_1)$  be the corresponding quantities to  $(a, b, c, d)$ , viz.  $a_1 : b_1 : c_1 : d_1 = B_1C_1D_1 : -C_1D_1A_1 : D_1A_1B_1 : -A_1B_1C_1$ ; we have

$$a_1 : b_1 : c_1 : d_1 = \begin{vmatrix} \beta, \gamma', 1 \\ \gamma, \beta', 1 \\ \delta, \alpha', 1 \end{vmatrix} : - \begin{vmatrix} \gamma, \beta', 1 \\ \delta, \alpha', 1 \\ \alpha, \delta', 1 \end{vmatrix} : \begin{vmatrix} \delta, \alpha', 1 \\ \alpha, \delta', 1 \\ \beta, \gamma', 1 \end{vmatrix} : - \begin{vmatrix} \alpha, \delta', 1 \\ \beta, \gamma', 1 \\ \gamma, \beta', 1 \end{vmatrix}$$

giving rise to a similar set of forms

$$\begin{aligned} a_1 &= \dots, -ac' + ha', & a'g + b'a, & -c'g - b'h, \\ b_1 &= -c'b - g'h, & \dots, & -f'b - g'f, -f'h + e'f, \\ c_1 &= b'e + h'g, -f'e + h'f, & \dots, & f'g + g'f, \\ d_1 &= g'c + h'b, -h'a + a'e, & -a'b - g'a, & \dots \end{aligned}$$

and leading to

$$\frac{cf}{a'e'} a_1 = -\frac{cg}{c'g'} b_1 = \frac{af}{a'f'} c_1 = -\frac{fg}{a'g'} d_1,$$

so that the equation  $\frac{l_1}{a_1} + \frac{m_1}{b_1} + \frac{n_1}{c_1} + \frac{p_1}{d_1} = 0$ , is transformable into

$$\frac{cf}{a'e'} l_1 - \frac{cg}{c'g'} m_1 + \frac{af}{a'f'} n_1 - \frac{fg}{a'g'} p_1 = 0.$$

98. Let  $A, B, C, D$ , be, as above, points on a circle;  $(A_1, D_1)$  and  $(B_1, C_1)$  the anti-points of  $(A, B), (B, C)$  respectively. Write

$$\begin{aligned} A &= (\xi - \alpha z)(\eta - \alpha'z), & A_1 &= (\xi - \alpha z)(\eta - \delta'z), \\ B &= (\xi - \beta z)(\eta - \beta'z), & B_1 &= (\xi - \beta z)(\eta - \gamma'z), \\ C &= (\xi - \gamma z)(\eta - \gamma'z), & C_1 &= (\xi - \gamma z)(\eta - \beta'z), \\ D &= (\xi - \delta z)(\eta - \delta'z), & D_1 &= (\xi - \delta z)(\eta - \delta'z); \end{aligned}$$

then we have identically

$$\begin{aligned} (\delta - \alpha)(\delta' - \alpha') B &= (\beta - \delta)(\beta' - \delta') A + (\beta - \alpha)(\beta' - \alpha') D - (\beta - \delta)(\beta' - \alpha') A_1 - (\beta - \alpha)(\beta' - \delta') D_1, \\ (\delta - \alpha)(\delta' - \alpha') C &= (\gamma - \delta)(\gamma' - \delta') A + (\gamma - \alpha)(\gamma' - \alpha') D - (\gamma - \delta)(\gamma' - \alpha') A_1 - (\gamma - \alpha)(\gamma' - \delta') D_1, \\ (\delta - \alpha)(\delta' - \alpha') B_1 &= (\beta - \delta)(\gamma' - \delta') A + (\beta - \alpha)(\gamma' - \alpha') D - (\beta - \delta)(\gamma' - \alpha') A_1 - (\beta - \alpha)(\gamma' - \delta') D_1, \\ (\delta - \alpha)(\delta' - \alpha') C_1 &= (\gamma - \delta)(\beta' - \delta') A + (\gamma - \alpha)(\beta' - \alpha') D = (\gamma - \delta)(\beta' - \alpha') A_1 - (\gamma - \alpha)(\beta' - \delta') D_1, \end{aligned}$$

or in the foregoing notation

$$\begin{aligned} ff' B &= gg' A + cc' D + gc' A_1 + cg' D_1, \\ ff' C &= hh' A + bb' D - hb' A_1 - bh' D_1, \\ ff' B_1 &= gh' A - cb' D - gb' A_1 + ch' D_1, \\ ff' C_1 &= g'h A - bc' D + hc' A_1 - bg' D_1. \end{aligned}$$

*Further Properties in relation to the same Sets (A, B, C, D) and (A<sub>1</sub>, B<sub>1</sub>, C<sub>1</sub>, D<sub>1</sub>)—*

Art. Nos. 99 to 104.

99. It is be shown that in virtue of these equations, and if moreover  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$ , then it is possible to find  $l_1, m_1, n_1, p_1$ , such that we have identically

$$-lA + mB + nC - pD + l_1A_1 - m_1B_1 - n_1C_1 + p_1D_1 = 0.$$

This equation will in fact be identically true if only

$$\begin{array}{rcl} -ff'l + gg'm + hh'n & . & -gh'm_1 - gh'n_1 = 0, \\ ce'm + bb'n - ff'p & . & +cb'm_1 + bc'n_1 = 0, \\ gc'm - hb'n & +ff'l_1 & +gb'm_1 - hc'n_1 = 0, \\ cg'm - bh'n & . & +ch'm_1 + bg'n_1 + ff'p_1 = 0. \end{array}$$

From the first and second equations eliminating  $m_1$  or  $n_1$ , the other of these quantities disappears of itself, and we thus obtain two equations which must be equivalent to a single one, viz., we have

$$\begin{array}{l} bc'ff'l + c'g'afm + bh'a'f'n + g'h'ff'p = 0, \\ b'c'ff'l + cg'a'f'm + b'h'a'f'n + g'h'ff'p = 0; \end{array}$$

which equations may also be written

$$\begin{array}{l} \frac{c'f'}{ah}l + \frac{c'g'}{bh}m + \frac{a'f'}{af}n + \frac{f'g'}{ab}p = 0, \\ \frac{cf}{a'h}l + \frac{cg}{b'h}m + \frac{af}{a'f'}n + \frac{fg}{a'b}p = 0; \end{array}$$

and it thus appears that the equations are equivalent to each other, and to the assumed relation

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0.$$

100. Similarly, from the third and fourth equations eliminating  $m$  or  $n$ , the other of these quantities disappears of itself, and we find

$$\begin{array}{l} cg'ff'l_1 - cg'a'f'm_1 + afc'g'n_1 - c'gff'p_1 = 0, \\ bh'ff'l_1 - afb'h'm_1 + bh'a'f'n_1 - bh'ff'p_1 = 0, \end{array}$$

equations which may be written

$$\begin{array}{l} \frac{cf}{a'c}l - \frac{cg}{c'g}m + \frac{af}{a'f'}n - \frac{fg}{g'a}p = 0, \\ \frac{f'h'}{ah}l - \frac{b'h'}{bg}m + \frac{a'f'}{af}n - \frac{b'g'}{ab}p = 0, \end{array}$$

where we see that the two equations are equivalent to each other and to the equation

$$\frac{l_1}{a_1} + \frac{m_1}{b_1} + \frac{n_1}{c_1} + \frac{p_1}{d_1} = 0.$$

It thus appears that the quantities  $l_1, m_1, n_1, p_1$ , must satisfy this last equation. It is to be observed that the first and second equations being, as we have seen, equivalent to a single equation, either of the quantities  $m_1, n_1$ , may be assumed at pleasure, but the other is then determined; the third and fourth equations then give  $l_1, p_1$ ; and the quantities  $l_1, m_1, n_1, p_1$ , so obtained, satisfy *identically* the equation  $\frac{l_1}{a_1} + \frac{m_1}{b_1} + \frac{n_1}{c_1} + \frac{p_1}{d_1} = 0$ .

101. Now writing

$$\begin{aligned} ff'l_1 &= -g(c'm + b'm_1) + h(b'n + c'n_1), \\ ff'p_1 &= -c(g'm - h'm_1) + b(h'n - g'n_1), \end{aligned}$$

and

$$\begin{aligned} ff'p &= c(c'm + b'm_1) + b(b'n + c'n_1), \\ ff'l &= g(g'm - h'm_1) + h(h'n - g'n_1), \end{aligned}$$

we find

$$\begin{aligned} f^2 f'^2 (l_1 p_1 - lp) &= -(bg + ch)[(c'm + b'm_1)(h'n - g'n_1) + (g'm - h'm_1)(b'n + c'n_1)] \\ &= (bg + ch)(b'g' + c'h')(m_1 n_1 - mn) \\ &= aa'ff'(m_1 n_1 - mn) \end{aligned}$$

that is,

$$ff'(l_1 p_1 - lp) = aa'(m_1 n_1 - mn)$$

viz., this equation is satisfied identically by the values of  $l_1, m_1, n_1, p_1$  determined as above.

102. Hence if  $m_1 n_1 = mn$ , we have also  $l_1 p_1 = lp$ , and we can determine  $m_1, n_1$ , so that  $m_1 n_1$  shall =  $mn$ , viz., in the first or second of the four equations (these two being equivalent to each other, as already mentioned), writing  $m_1 = \theta n$ , and therefore  $n_1 = \frac{1}{\theta} m$ , we have

$$\begin{aligned} -ff'l + gg'm + hh'n - gh'n\theta - g'hm\frac{1}{\theta} &= 0, \\ cc'm + bb'n - ff'p + cb'n\theta + bc'm\frac{1}{\theta} &= 0, \end{aligned}$$

which are, in fact, the same quadric equation in  $\theta$ , viz., we have

$$\frac{-ff'l + gg'm + hh'n}{cc'm + bb'n - ff'p} = -\frac{gh'}{cb'} = -\frac{gk'}{bc}.$$

The final result is that there are two sets of values of  $l_1, m_1, n_1, p_1$ , each satisfying the identity

$$-lA + mB + nC - pD + l_1 A_1 - m_1 B_1 - n_1 C_1 + p_1 D_1 = 0,$$

and for each of which we have

$$\frac{l_1}{a_1} + \frac{m_1}{b_1} + \frac{n_1}{c} + \frac{p_1}{d_1} = 0, \quad l_1 p_1 = lp, \quad m_1 n_1 = mn.$$

103. Consider, in particular, the case where  $p = 0$ ; the relation

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0,$$

here becomes

$$l = -\frac{ag'}{bf'}m - \frac{a'h}{cf}n.$$

The equation in  $\theta$  is

$$(c'm + bb'n)\theta + cb'n\theta^2 + bc'm = 0,$$

viz., this is

$$(c\theta + c'm)(b'n\theta + b) = 0,$$

giving

$$\theta = -\frac{b}{c}, \quad m_1 = -\frac{bn}{c}, \quad n_1 = -\frac{cm}{b},$$

\* or else

$$\theta = -\frac{c'm}{b'n}, \quad m_1 = -\frac{c'm}{b'}, \quad n_1 = -\frac{b'n}{c'}.$$

Since in the present case  $l_1 p_1 = 0$ , we have either  $l_1 = 0$ , or else  $p_1 = 0$ , and as might be anticipated, the two values of  $\theta$  correspond to these two cases respectively, viz., proceeding to find the values of  $l_1, p_1$ , the completed systems are

$$\theta = -\frac{b}{c}, \quad l_1 = \frac{a}{bcf'} (cc'm - bb'n), \quad m_1 = -\frac{bn}{c}, \quad n_1 = -\frac{cm}{b}, \quad p_1 = 0.$$

$$\theta = -\frac{c'm}{b'n}, \quad l_1 = 0, \quad m_1 = -\frac{c'm}{b'}, \quad n_1 = -\frac{b'n}{c'}, \quad p_1 = \frac{a'}{b'c'f} (cc'm - bb'n),$$

so that for the first system we have

$$\frac{l_1}{a_1} + \frac{m_1}{b_1} + \frac{n_1}{c_1} = 0, \quad m_1 n_1 = mn, \quad -lA + mB + nC = -l_1 A_1 + m_1 B_1 + n_1 C_1,$$

and for the second system

$$\frac{m_1}{b_1} + \frac{n_1}{c_1} + \frac{p_1}{d_1} = 0, \quad m_1 n_1 = mn, \quad -lA + mB + nC = -p_1 D_1 + m_1 B_1 + n_1 C_1.$$

104. The whole of the foregoing investigation would have assumed a more simple form if the circular co-ordinates had been taken with reference to the centre of the circle  $ABCD$  as origin, and the radius of this circle been put = 1; we should then have  $a' = \frac{1}{\alpha}$ , &c., and consequently

$$a' = -\frac{1}{\beta\gamma} a, \quad b' = -\frac{1}{\gamma\alpha} b, \quad c' = -\frac{1}{\alpha\beta} c, \quad f' = -\frac{1}{\alpha\delta} f, \quad g' = -\frac{1}{\beta\delta} g, \quad h' = -\frac{1}{\gamma\delta} h;$$

but the symmetrical relation of the circles  $ABCD$  and  $A_1 B_1 C_1 D_1$  would not have been so clearly shown.

I will however give the investigation in this simplified form, for the identity  $-lA + mB + nC = -l_1 A_1 + m_1 B_1 + n_1 C_1$ ; viz., in this case we have

$$\frac{l}{\alpha} = -\frac{m(\beta - \gamma)(\beta - \delta)}{\beta(\gamma - \alpha)(\alpha - \delta)} - \frac{n(\beta - \gamma)(\gamma - \delta)}{\gamma(\alpha - \beta)(\alpha - \gamma)},$$

and the identity to be satisfied is

$$\begin{aligned} & - l(\xi - \alpha z) \left( \eta - \frac{1}{\alpha} z \right) = - l_1(\xi - \alpha z) \left( \eta - \frac{1}{\delta} z \right) \\ & + m(\xi - \beta z) \left( \eta - \frac{1}{\beta} z \right) + m_1(\xi - \beta z) \left( \eta - \frac{1}{\gamma} z \right) \\ & + n(\xi - \gamma z) \left( \eta - \frac{1}{\gamma} z \right) + n_1(\xi - \gamma z) \left( \eta - \frac{1}{\beta} z \right); \end{aligned}$$

writing  $\xi = \alpha z$ ,  $\eta = \frac{1}{\beta} z$ , we find  $m_1$ , and writing  $\xi = \alpha z$ ,  $\eta = \frac{1}{\gamma} z$ , we find  $n_1$ , and it is then easy to obtain the value of  $l_1$ , viz., the results are

$$\frac{l_1}{\delta} = \frac{m(\alpha - \beta)(\beta - \gamma)}{\beta(\gamma - \alpha)(\alpha - \delta)} + \frac{n(\beta - \gamma)(\gamma - \alpha)}{\gamma(\alpha - \beta)(\alpha - \delta)}, \quad m_1 = -n \frac{\gamma - \alpha}{\alpha - \beta}, \quad n_1 = -m \frac{\alpha - \beta}{\gamma - \alpha},$$

and therefore  $m_1 n_1 = mn$ ; it may be added that we have

$$- \frac{l_1}{\delta} = \frac{\beta - \gamma}{\alpha - \delta} \left( \frac{m_1}{\gamma} + \frac{n_1}{\delta} \right),$$

viz., this is the form assumed by the equation  $\frac{l_1}{a_1} + \frac{m_1}{b_1} + \frac{n_1}{c_1} = 0$ .

PART III. (Nos. 105 to 157.)—ON THE THEORY OF FOCI.

*Explanation of the General Theory*—Art. Nos. 105 to 110.

105. If from a focus of a conic we draw two tangents to the curve, these pass respectively through the two circular points at infinity, and we have thence the generalised definition of a focus as established by PLÜCKER, viz., in any curve a focus is a point such that the lines joining it with the two circular points at infinity are respectively tangents to the curve; or, what is the same thing, if from each of the circular points at infinity, say from the points  $I, J$ , tangents are drawn to the curve, the intersections of each tangent from the one point with each tangent from the other point are the foci of the curve. A curve of the class  $n$  has thus in general  $n^2$  foci. It is to be added that, as in the conic the line joining the points of contact of the two tangents from a focus is the directrix corresponding to that focus, so in general the line joining the points of contact of the tangents from the focus through the points  $I, J$  respectively is the directrix corresponding to the focus in question.

106. A circular point at infinity  $I$  or  $J$ , may be an ordinary or a singular point on the curve, and the tangent at this point then counts, or, in the case of a multiple point, the tangents at this point count a certain number of times, say  $q$  times, among the tangents which can be drawn to the curve from the point; the number of the remaining tangents is thus  $= n - q$ . In particular, if the circular point at infinity be an ordinary point, then the tangent counts twice, or

we have  $q = 2$ ; if it be a node, each of the tangents count twice, or  $q = 4$ ; if it be a cusp, the tangent counts three times, or  $q = 3$ . Similarly, if the other circular point at infinity be an ordinary or a singular point on the curve, the tangent or tangents there count a certain number of times, say  $q'$  times, among the tangents to the curve from this point; the number of the remaining tangents is thus  $= n - q'$ . And if as usual we disregard the tangents at the two points  $I, J$  respectively, and attend only to the remaining tangents, the number of the foci is  $= (n - q) (n - q')$ .

107. Among the tangents from the point  $I$  or  $J$  there may be a tangent which, either from its being a multiple tangent (that is, a tangent having ordinary contact at two or more distinct points), or from being an osculating tangent at one or more points, counts a certain number of times, say  $r$ , among the tangents from the point in question. Similarly, if among the tangents from the other point  $J$  or  $I$ , there is a tangent which counts  $r'$  times, then the foci are made up as follows, viz. we have—

Intersections of the two singular tangents counting as	$r'r$ foci.
Intersections of the first singular tangent with each of the ordinary tangents from the other circular point at infinity, as . . . . .	$(n - q' - r')r$ „
Do. for second singular tangent, . . . . .	$(n - q - r)r'$ „
Intersections of the ordinary tangents, . . . . .	$(n - q - r) (n - q' - r')$ „
Giving together the . . . . .	<hr style="width: 100%; border: 0.5px solid black;"/> $(n - q) (n - q')$ foci :

and the like observation applies to the more general case where the tangents from each of the points  $I, J$  include more than one singular tangent.

108. There is yet another case to be considered ; the line infinity may be an ordinary or a singular tangent to the curve: assuming that it counts  $s$  times among the tangents from either of the circular points at infinity, the numbers of the remaining tangents are  $n - q - s, n - q' - s$  from the two points  $I, J$  respectively, and the number of foci is  $= (n - q - s) (n - q' - s)$ .

109. In the case of a real curve the two points  $I, J$  are related in the same manner to the curve, and we have therefore  $q = q'$ ; the singular tangents (if any) from the two points respectively being the same as well in character as in number. Writing  $n - q - s = n - q' - s, = p$ , and not for the present attending to the case of singular tangents, I shall assume that the number of tangents to the curve from each of the two points is  $= p$ ; the number of foci is thus  $= p^2$ ; and to each focus there corresponds a directrix, viz., this is the line through the points of contact of the tangents from the focus to the two points  $I, J$  respectively.

110. Consider any two foci  $A, B$  not *in lineâ* with either of the points  $I, J$ , then joining these with the points  $I, J$ , and taking  $A_1, B_1$  the intersections of  $AI, BJ$  and of  $AJ, BI$  ( $A_1, B_1$  being therefore by a foregoing definition the anti-

points of  $(A, B)$ , then  $A_1, B_1$  are, it is clear, foci of the curve. We may out of the  $p^2$  foci select, and that in  $1 \cdot 2 \dots p$  different ways, a system of  $p$  foci such that no two of them lie *in lineâ* with either of the points  $I, J$ ; and this being so, taking the anti-points of each of the  $\frac{1}{2} p(p - 1)$  pairs out of the  $p$  foci, we have, inclusively of the  $p$  foci, in all  $p + 2 \cdot \frac{1}{2} p(p - 1)$ , that is  $p^2$  foci, the entire system of foci.

*On the Foci of Conics—Art. Nos. 111 to 117.*

111. A conic is a curve of the class 2, and the number of foci is thus = 4. Taking as foci any two points  $A, B$ , the remaining two foci will be the anti-points  $A_1, B_1$ . In order that a given point  $A$  may be a focus, the conic must touch the lines  $AI, AJ$ ; similarly, in order that a given point  $B$  may be a focus, the conic must touch the lines  $BI, BJ$ ; the equation of a conic having the given points  $A, B$  for foci contains therefore a single arbitrary parameter.

112. In the case, however, of the parabola the curve touches the line infinity; there is consequently from each of the points  $I, J$  only a single tangent to the curve, and consequently only one focus: the parabola having a given point  $A$  for its focus is a conic touching the line infinity and the lines  $AI, AJ$ , or say the three sides of the triangle  $AIJ$ ; its equation contains therefore two arbitrary parameters.

113. Returning to the general conic, there are certain trizomal forms of the focal equation, not of any great interest, but which may be mentioned. Using circular co-ordinates, and taking  $(\alpha, \alpha', 1)$  and  $(\beta, \beta', 1)$  for the co-ordinates of the given foci  $A, B$  respectively, the conic touches the lines  $\xi - \alpha z = 0, \eta - \alpha' z = 0, \xi - \beta z = 0, \eta - \beta' z = 0$ ; the equation of a conic touching the first three lines is

$$\sqrt{l(\xi - \alpha z)} + \sqrt{m(\xi - \beta z)} + \sqrt{n(\eta - \alpha' z)} = 0,$$

where  $l, m, n$  are arbitrary, and it is easy to obtain, in order that the conic may touch the fourth line  $\eta - \beta' z = 0$ , the condition

$$n = -\frac{\beta - \alpha}{\beta' - \alpha'}(m - l).$$

114. In fact,  $n$  having this value, the equation gives

$$l(\xi - \alpha z) + m(\xi - \beta z) + 2\sqrt{lm(\xi - \alpha z)(\xi - \beta z)} = -\frac{\beta - \alpha}{\beta' - \alpha'}(m - l)(\eta - \beta' z + (\beta' - \alpha')z),$$

and taking over the term  $\frac{\beta - \alpha}{\beta' - \alpha'}(m - l)(\beta' - \alpha')z, = (\beta - \alpha)(m - l)z,$

this gives

$$l(\xi - \beta z) + m(\xi - \alpha z) + 2\sqrt{lm(\xi - \alpha z)(\xi - \beta z)} = -\frac{\beta - \alpha}{\beta' - \alpha'}(m - l)(\eta - \beta' z),$$

which puts in evidence the tangent  $\eta - \beta'z$ . It is easy to see that the equation may be written in any one of the four forms

$$\sqrt{l(\xi - \alpha z)} + \sqrt{m(\xi - \beta z)} + \sqrt{-\frac{\beta - \alpha}{\beta' - \alpha'}(m - l)(\eta - \alpha'z)} = 0,$$

$$\sqrt{m(\xi - \alpha z)} + \sqrt{l(\xi - \beta z)} + \sqrt{-\frac{\beta - \alpha}{\beta' - \alpha'}(m - l)(\eta - \beta z)} = 0,$$

$$\sqrt{l(\eta - \alpha'z)} + \sqrt{m(\eta - \beta'z)} + \sqrt{-\frac{\beta' - \alpha'}{\beta - \alpha}(m - l)(\xi - \alpha z)} = 0,$$

$$\sqrt{m(\eta - \alpha'z)} + \sqrt{l(\eta - \beta'z)} + \sqrt{-\frac{\beta' - \alpha'}{\beta - \alpha}(m - l)(\xi - \beta z)} = 0.$$

viz., in forms containing any three of the four radicals  $\sqrt{\xi - \alpha z}$ ,  $\sqrt{\xi - \beta z}$ ,  $\sqrt{\eta - \alpha'z}$ ,  $\sqrt{\eta - \beta'z}$ . The conic is thus expressed as a trizomal curve, the zomals being each a line, viz., they are any three out of the four focal tangents; the order of the curve, as deduced from the general expression  $2^{\nu-2}r$ , is = 2; so that there is here no depression of order.

115. But the ordinary form of the focal equation is a more interesting one: viz., **A**, **B** being as usual the squared distances of the current point from the two given foci respectively, say

$$\mathbf{A} = (\xi - \alpha z) \eta - \alpha'z),$$

$$\mathbf{B} = (\xi - \beta z) \eta - \beta'z),$$

then  $2a$  being an arbitrary parameter, the equation is

$$2az + \sqrt{\mathbf{A}} + \sqrt{\mathbf{B}} = 0,$$

viz., the equation is here that of a trizomal curve, the zomals being curves of the second order, that is, the zomals are ( $z^2 = 0$ ) the line infinity twice, and the line-pairs  $AI$ ,  $AJ$  and  $BI$ ,  $BJ$  respectively: the general expression  $2^{\nu-2}r$  gives therefore the order = 4; but in the present case there are two branches, viz., the branches

$$2az + \sqrt{\mathbf{A}} - \sqrt{\mathbf{B}} = 0, \quad 2az - \sqrt{\mathbf{A}} + \sqrt{\mathbf{B}} = 0,$$

each ideally containing ( $z = 0$ ) the line infinity; the curve contains therefore ( $z^2 = 0$ ) the line infinity twice, and omitting this factor the order is = 2, as it should be.

116. To express the equation by means of the other two foci  $A_1$ ,  $B_1$ , writing the equation under the form

$$\mathbf{A} + \mathbf{B} + 2\sqrt{\mathbf{A}\mathbf{B}} - 4a^2z^2 = 0,$$

and then if  $\mathbf{A}_1$ ,  $\mathbf{B}_1$  are the squared distances of the current point from  $A_1$ ,  $B_1$  respectively, we have (*ante*, No. 65).

$$\mathbf{A}\mathbf{B} = \mathbf{A}_1\mathbf{B}_1,$$

$$\mathbf{A} + \mathbf{B} - \mathbf{A}_1 - \mathbf{B}_1 = kz^2,$$

where  $k$  is the squared distance of the foci  $A, B, = 4a^2e^2$  suppose: whence putting  $a^2(1 - e^2) = b^2$ , the equation becomes

$$A_1 + B_1 + 2\sqrt{A_1B_1} - 4b^2z^2 = 0,$$

that is

$$\sqrt{A_1} + \sqrt{B_1} + 2bz = 0,$$

which is the required new form. It is hardly necessary to remark that the equation  $2az + \sqrt{A} + \sqrt{B} = 0$ , putting therein  $z = 1$ , and expressing  $A, B$  in rectangular co-ordinates measured along the axes, is the ordinary focal equation  $2a = \sqrt{(x - ae)^2 + y^2} + \sqrt{(x + ae)^2 + y^2}$ .

117. I remark that the equation  $2az + \sqrt{A} + \sqrt{B} = 0$ , gives rise to  $4a^2z^2 + A - B + 4az\sqrt{A} = 0$ , but here  $A - B = -4aexz$ , so that the equation contains  $z = 0$ , and omitting this it becomes  $(az - ex) + \sqrt{A} = 0$ , a bizomal form, being a curve of the order = 2, as it should be; this is in fact the ordinary equation in regard to a focus and its directrix.

*Theorem of the Variable Zomal as applied to a Conic—Art. Nos. 118 to 123.*

118. The equation  $2kz + \sqrt{A^\circ} + \sqrt{B^\circ} = 0$  is in like manner that of a conic; in fact, this would be a curve of the order = 4, but there are as before the two branches  $2kz + \sqrt{A^\circ} - \sqrt{B^\circ} = 0, 2kz - \sqrt{A^\circ} + \sqrt{B^\circ} = 0$ , each ideally containing ( $z = 0$ ) the line infinity, and the order is thus reduced to be = 2. Each of the circles  $A^\circ = 0, B^\circ = 0$  is a circle having double contact with the conic (this of course implies that the centre of the circle is on an axis of the conic). We may if we please start from the form  $2kz + \sqrt{A} + \sqrt{B} = 0$ , and then by means of the theorem of the variable zomal introduce into the equation one, two, or three such circles.

119. It is in this point of view that I will consider the question, viz., adapting the formula to the case of the ellipse, and starting from the form

$$2az + \sqrt{(x - aez)^2 + y^2} + \sqrt{(x + aez)^2 + y^2} = 0,$$

the equation of the variable zomal or circle of double contact may be taken to be

$$\frac{4a^2z^2}{-2} + \frac{(x - aez)^2 + y^2}{1 - q} + \frac{(x + aez)^2 + y^2}{1 + q} = 0,$$

where  $q$  is an arbitrary parameter; writing for greater simplicity  $z = 1$ , and reducing, the equation is

$$(x - qae)^2 + y^2 = b^2(1 - q^2).$$

120. If  $q < 1$ , then writing  $q = \sin \theta$ , we obtain the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

as the envelope of the variable circle

$$(x - ae \sin \theta)^2 + y^2 = b^2 \cos^2 \theta ,$$

viz., of a circle having its centre on the major axis at a distance =  $ae \sin \theta$  from the centre, and its radius =  $b \cos \theta$ . (I notice, in passing, that this gives in practice a very convenient graphical construction of the ellipse.) It may be remarked that for  $\theta = \pm \sin^{-1} e$ , the circle becomes

$$\left(x \pm \left(a - \frac{b^2}{a}\right)\right)^2 + y^2 = \frac{b^4}{a^2} ,$$

viz., this is the circle of curvature at one or other of the extremities of the major axis; as  $\theta$  passes from 0 to  $\pm \sin^{-1} e$  we have a series of real circles, which, by their continued intersection, generate the ellipse; as  $\theta$  increases from  $\theta = \pm \sin^{-1} e$  to  $\pm 90^\circ$ , the circles continue real, but the consecutive circles no longer intersect in any real point,—and ultimately for  $\theta = \pm 90^\circ$ , the circles become evanescent at the two foci respectively.

121. In the case  $q > 1$ , we have a real representation of

$$(x - qae)^2 + y^2 + b^2(q^2 - 1) ,$$

as the squared distance of the point  $(x, y)$  from a point  $(X, 0, Z)$  out of the plane of the figure, viz., putting this

$$= (x - X)^2 + y^2 + Z^2 ,$$

we have

$$qae = X, \quad Z^2 = b^2(q^2 - 1) ,$$

whence

$$Z^2 = b^2 \left( \frac{X^2}{a^2 c^2} - 1 \right) ,$$

or what is the same thing,

$$\frac{X^2}{a^2 - b^2} - \frac{Z^2}{b^2} = 1 ;$$

that is, the locus is the focal hyperbola, viz., a hyperbola in the plane of  $zx$ , having its vertices at the foci, and its foci at the vertices of the ellipse.

122. If instead of the form first considered, we start from the trizomal form

$$2bz + \sqrt{x^2 + (y - aeiz)^2} + \sqrt{x^2 + (y + aeiz)^2} = 0 ,$$

then we have the zomal or circle of double contact under the form

$$x^2 + (y - qaei)^2 = a^2(1 - q^2) ;$$

or putting herein  $q = -i \tan \phi$ , this is,

$$x^2 + (y - a \tan \phi)^2 = a^2 \sec^2 \phi ;$$

so that we have the ellipse as the envelope of a variable circle having its centre on the minor axis of the ellipse, distance from the centre =  $a \tan \phi$ , and radius

$= a \sec \phi$ . This is, in fact, Gergonne's theorem, according to which the ellipse is the secondary caustic or orthogonal trajectory of rays issuing from a point and refracted at a right line into a rarer medium. It is to be remarked that for  $\tan \phi = \pm \frac{ae}{b}$ , the equation of the circle is

$$x^2 + \left( y \pm \left( b - \frac{a^2}{b} \right) \right)^2 = \frac{a^4}{b^2},$$

viz., this is the circle of curvature at one or other extremity of the minor axis; from  $\phi = 0$  to  $\phi = \pm \tan^{-1} \frac{ae}{b}$ , the intersections of the consecutive circles are real, and give the entire real ellipse; from  $\phi = \pm \tan^{-1} \frac{ae}{b}$ , to  $\phi = \pm 90^\circ$ , the circles are still real, but the intersections of consecutive circles are imaginary.

123. If in the equation of the generating circle we interchange  $x, y, a, b$ , the equation becomes

$$(x - ae i \tan \phi)^2 + y^2 = b^2 \sec^2 \phi,$$

which is (as it should be) equivalent to the former equation

$$(x - ae \sin \theta)^2 + y^2 = b^2 \cos^2 \theta,$$

the identity being established by means of the equation

$$\cos \theta = \frac{1}{\cos \phi}, \text{ and } \therefore \sin \theta = i \tan \phi, \tan \theta = i \sin \phi,$$

which is Jacobi's imaginary transformation in the theory of Elliptic Functions.

*Foci of the Circular Cubic and the Bicircular Quartic*—Art. Nos. 124 to 126.

124. For a cubic curve, the class is in general = 6, and the number of the foci is = 36. But a specially interesting case is that of a circular cubic, viz., a cubic passing through each of the circular points at infinity. Here, at each of the circular points at infinity, the tangent at this point reckons twice among the tangents to the curve from the point; the number of the remaining tangents is thus = 4, and the number of the foci is = 16. If from any two points whatever on the curve tangents be drawn to the curve, then the two pencils of tangents are, and that in four different ways, homologous to each other, viz., if the tangents of the first pencil are (1, 2, 3, 4), and those of the second pencil, taken in a proper order, are (1', 2', 3', 4'), then we have (1, 2, 3, 4) homologous with each of the arrangements (1', 2', 3', 4'), (2', 1', 4', 3'), (3', 4', 1', 2'),

(4', 3', 2', 1'). And in each case the intersections of the four corresponding tangents lie on a conic passing through the two given points on the curve.\*

125. Hence taking the points on the curve to be the circular points at infinity, we have the sixteen foci lying in fours upon four different circles—that is, we have four tetrads of concyclic foci. Let any one of these tetrads be  $A, B, C, D$ , then if

$$\begin{aligned} \text{Anti-points of } (B, C) (A, D) & \text{ are } (B_1, C_1), (A_1, D_1), \\ \text{,, } (C, A) (B, D) & \text{ ,, } (C_2, A_2), (B_2, D_2), \\ \text{,, } (A, B) (C, D) & \text{ ,, } (A_3, B_3), (C_3, D_3), \end{aligned}$$

the four tetrads of concyclic foci are

$$\begin{aligned} A, B, C, D ; \\ A_1, B_1, C_1, D_1 ; \\ A_2, B_2, C_2, D_2 ; \\ A_3, B_3, C_3, D_3 . \end{aligned}$$

It is to be observed that if  $A, B, C, D$  are any four points on a circle, then if, as above, we pair these in any manner, and take the anti-points of each pair, the four anti-points lie on a circle, and thus the original system  $A, B, C, D$ , of four points on a circle, leads to the remaining three systems of four points on a circle. The theory is in fact that already discussed *ante*, No. 72 *et seq.*

126. The preceding theory applies without alteration to the bicircular quartic, viz., the quartic curve which has a node at each of the circular points at infinity. The class is here = 8, but among the tangents from a node each of the two tangents at the node is to be reckoned twice, and the number of the remaining tangents is = 4: the number of foci is = 16. And, by the general theorem that in a binodal quartic the pencils of tangents from the two nodes respectively are homologous, the sixteen foci are related to each other precisely in the manner of the foci of the circular cubic. The latter is in fact a particular case of the former, viz., the bicircular quartic may break up into the line infinity, and a circular cubic.

\* It may be remarked that if the equation of the first pencil of lines be

$$(x - ay)(x - by)(x - cy)(x - dy) = 0,$$

and that of the second pencil

$$(z - aw)(z - bw)(z - cw)(z - dw) = 0,$$

then the equations of four conics are

$$\begin{aligned} xw - yz &= 0, \\ (a + d - b - c) xz + (bc - ad)(xw + yz) + (ad(b + c) - bc(a + d))yw &= 0, \\ (b + d - c - a) xz + (ca - bd)(xw + yz) + (bd(c + a) - ca(b + d))yw &= 0, \\ (c + d - a - b) xz + (ab - cd)(xw + yz) + (cd(a + b) - ab(c + d))yw &= 0. \end{aligned}$$

*Centre of the Circular Cubic, and Nodo-Foci, &c. of the Bicircular Quartic*—Art. Nos. 127 to 129.

127. The tangents at  $I, J$  have not been recognised as tangents from  $I, J$ , giving by their intersection a focus, but it is necessary in the theory to pay attention to the tangents in question. It is clear that these tangents are in fact asymptotes—viz., in the case of the circular cubic they are the two imaginary asymptotes of the curve, and in the case of a bicircular quartic, the two pairs of imaginary parallel asymptotes; but it is convenient to speak of them as the tangents at  $I, J$ .

128. In the case of a circular cubic, the tangents at  $I$  and  $J$  meet in a point which I call the centre of the curve, viz., this is the intersection of the two imaginary asymptotes.

129. In the case of a bicircular quartic, the two tangents at  $I$  and the two tangents at  $J$  meet in four points, which (although not recognising them as foci) I call the nodo-foci; these lie in pairs on two lines, diagonals of the quadrilateral formed by the four tangents (the third diagonal is of course the line  $IJ$ ), which diagonals I call the “nodal axes;” and the point of intersection of the two nodal axes is the “centre” of the curve. The nodo-foci are four points, two of them real, the other two imaginary, viz., they are two pairs of anti-points, the lines through the two pairs respectively being, of course, the nodal axes; these are consequently real lines bisecting each other at right angles in the centre (with the relation  $1 : i$  between the distances). The centre may also be defined as the intersection of the harmonic of  $IJ$  in regard to the tangents at  $I$ , and the harmonic of this same line in regard to the tangents at  $J$ . Speaking of the tangents as asymptotes, the nodo-foci are the angles of the rhombus formed by the two pairs of parallel asymptotes; the nodal axes are the diagonals of this rhombus, and the centre is the point of intersection of the two diagonals; as such it is also the intersection of the two lines drawn parallel to and midway between the lines forming each pair of parallel asymptotes.

*Circular Cubic and Bicircular Quartic; the Axial or Symmetrical Case*—Art. No. 130.

130. In a circular cubic or bicircular quartic, the pencil of the tangents from  $I$  and that of the tangents through  $J$ , considered as corresponding to each other in some one of the four arrangements, may be such that the line  $IJ$  considered as belonging to the two pencils respectively shall correspond to itself, and when this is so, the four foci,  $A, B, C, D$ , which are the intersections of the corresponding tangents in question, will lie in a line (viz., the conic which exists in the general case will break up into a line-pair consisting of the line  $IJ$  and another line). The line in question may be called the focal axis; it will presently be shown that in the case of the circular cubic it passes through the centre, and that in the case of the bicircular quartic it not only passes through the centre, but

coincides with one or other of the nodal axes, viz., with that passing through the real or the imaginary nodo-foci; that is, the curve may have on the focal axis two real or else two imaginary nodo-foci. The focal axis contains, as has been mentioned, four foci—the remaining twelve foci are situate symmetrically, six on each side of the focal axis, the arrangement of the sixteen foci being as mentioned *ante*, No. 81 *et seq.*; the focal axis is in fact an axis of symmetry of the curve, and if preferred it may be named the axis of symmetry, transverse axis, or simply the axis. And the curve (circular cubic, or bicircular quartic) is in this case a “symmetrical” or “axial” curve.

*Circular Cubic and Bicircular Quartic: Singular Forms—Art. Nos. 131 to 140.*

131. The circular cubic may have a node or a cusp. If this were at one of the points  $I, J$  the curve would be imaginary, and I do not attend to the case; and for the same reason, for the bicircular quartic I do not attend to the case where *one* of the points  $I, J$  is a cusp. There remain then for the circular cubic and for the bicircular quartic the cases where there is a node or a cusp at a real point of the curve; and for the bicircular quartic the case where each of the points  $I, J$  is a cusp—in general the curve has no other node or cusp, but it may besides have a node or cusp at a real point thereof.

132. I consider first the case of the bicircular quartic where each of the points  $I, J$  is a cusp. The curve is in this case of necessity symmetrical\*—it is in fact a Cartesian; viz., the Cartesian may be taken by definition to be a quartic curve having a cusp at each of the circular points at infinity. But in this case, as distinguished from the general case of the bicircular quartic, there is an essential degeneration of all the focal properties, and it is necessary to explain what these become. The centre is evidently the intersection of the cuspidal tangents; the nodo-foci (so far as they can be said to exist) coalesce with the centre, and they do not in so coalescing determine any definite directions for the nodal axes; that is, there are no nodal axes, and the only theorem in regard to the focal axis or axis of symmetry is, that it passes through the centre. Of the four tangents through the point  $I$ , one has come to coincide with the line  $IJ$ ; and similarly, of the four tangents through the point  $J$  one has come to coincide with the line  $JI$ : there remain only three tangents through  $I$  and three tangents through  $J$ , and these by their intersections determine nine foci—viz., three foci  $A, B, C$  on the axis, and besides  $(B_1, C_1)$  the anti-points of  $(B, C)$ :  $(C_2, A_2)$  the anti-points of  $(C, A)$  and  $(A_3, B_3)$  the anti-points of  $(A, B)$ .

\* It will appear, *post* Nos. 161–164, that if starting with three given points as the foci of a bicircular quartic, we impose the condition that the nodes at  $I, J$  shall be each of them a cusp, then either the quartic will be the circle through the three points taken twice, in which case the assumed focal property of the given three points disappears altogether, or else the three points must be *in lineâ*, or the curve be symmetrical, that is, a Cartesian.

133. The remaining seven foci have disappeared, viz., we may consider that one of them has gone off to infinity on the focal axis, and that three pairs of foci have come to coincide with the points  $I, J$  respectively. The circle  $O$  (as in the general case of a symmetrical quartic) has become a line, the focal axis; the circles  $R, S, T$  (contrary to what might at first sight appear) continue to be determinate circles, viz., these have their centres at  $A, B, C$  respectively, and pass through the points  $(B_1, C_1), (C_2, A_2),$  and  $(A_3, B_3)$  respectively, see *ante*, No. 83. But on each of these circles we have not more than two proper foci, and it is only on the axis as representing the circle  $O$  that we have three proper foci, the axial foci  $A, B, C$ : in regard hereto it is to be remarked that the equation of the curve can be expressed not only by means of these three foci in the form  $\sqrt{lA} + \sqrt{mB} + \sqrt{nC} = 0$ ; but by means of any two of them in the form  $\sqrt{lA} + \sqrt{mB} + K = 0$ , where  $K$  is a constant, or, what is the same thing ( $z$  being introduced for homogeneity in the expressions of  $A$  and  $B$  respectively), in the form  $\sqrt{lA} + \sqrt{mB} + Kz^2 = 0$ .

134. Using for the moment the expression “twisted” as opposed to symmetrical—(viz., the curve is twisted when there is not any axis of symmetry but the foci lie only on circles)—then the classification is

Circular Cubics, twisted,	
”          ”          symmetrical,	
Bicircular Quartics, twisted,	
”          ”          symmetrical,	{ Ordinary,
	Bicuspidal = Cartesian,

and each of these kinds may be general, nodal, or cuspidal—viz., for the two last mentioned kinds there may be a node or a cusp at a real point of the curve.

135. In the case of a node, say the point  $N$ ; first if the curve (circular cubic or bicircular quartic) be twisted—then of the four foci  $A, B, C, D$  we have two, suppose  $B$  and  $C$ , coinciding with  $N$ ; and the sixteen foci are as follows, viz.

$B, C, A, D$	are	$N, N, A, D$ ;
$B_1, C_1, A_1, D_1$	”	$N, N,$ Anti-pts. of $(A, D)$ ;
$C_2, A_2, B_2, D_2$	”	Anti-pts. of $(N, A),$ Anti-pts. of $(N, D)$ ;
$A_2, B_3, C_3, D_3$	”	Do. do.

viz., we have the points  $(A, D)$  each once, the node  $N$  four times, the anti-points of  $(A, D)$  once, and the anti-points of  $(N, A)$  and of  $(N, D)$ , each pair twice. But properly there are only four foci, viz., the points  $A, D$  and their anti-points. The circle  $O$  subsists as in the general case, and so does the circle  $R$  ( $BC, AD$ ), viz., this has for centre the intersection of the line  $AD$  by the tangent at  $N$  to the circle  $O$ , and it passes through the point  $N$ , of course cutting the circle  $O$  at right angles: the circles  $S$  and  $T$  each reduce themselves each to the point  $N$  considered as an evanescent circle, or what is the same thing to the line-pair  $NI, NJ$ .

136. The case is nearly the same if the curve be symmetrical, but in the case of the bicircular quartic excluding the Cartesian: viz., we have on the axis the foci  $B, C$  coinciding at  $N$ , and the other two foci  $A, D$ ; the sixteen foci are as above—and the circle  $R$  is determined by the proper construction as applied to the case in hand, viz., the centre  $R$  is the intersection of the axis by the radical axis of the point  $N$  (considered as an evanescent circle) and the circle on  $AD$  as diameter; that is  $\overline{RN}^2 = RA \cdot RD$ . And the circles  $S$  and  $T$  reduce themselves each to the point  $N$  considered as an evanescent circle.

137. Next if we have a cusp, say the point  $K$ : first if the curve (circular cubic or bicircular quartic) be twisted—then of the four foci  $A, B, C, D$ , three, suppose  $A, B, C$ , coincide with  $K$ ; and the sixteen foci are as follows, viz.,

$B, C, A, D$	are	$K, K, K, D$ ,
$B_1, C_1, A_1, D_1$	„	$K, K$ , Anti-points of $(K, D)$ ,
$C_2, A_2, B_2, D_2$	„	Do. do.
$A_3, B_3, C_3, D_3$	„	Do. do.

viz., we have the point  $D$  once, the point  $K$  nine times, and the anti-points of  $K, D$  three times. But properly the point  $D$  is the only focus. The circle  $O$  is, it would appear, *any* circle through  $K, D$ , but possibly the particular circle which touches the cuspidal tangent may be a better representative of the circle  $O$  of the general case—the circles  $R, S, T$  reduce themselves each to the point  $K$  considered as an evanescent point.

138. The like is the case if the curve be symmetrical, but in the case of the bicircular quartic excluding the Cartesian; the circle  $O$  is here the axis, which is in fact the cuspidal tangent.

139. For the Cartesian, if there is a node  $N$ ; then of the three foci  $A, B, C$ , two, suppose  $B$  and  $C$ , coincide with  $N$ ; the nine foci are  $A$  once,  $N$  four times, and the anti-points of  $N, A$  twice: but properly the point  $A$  is the only focus. And if there be a cusp  $K$ ; then all the three foci  $A, B, C$  coincide with  $K$ ; and the nine foci are  $K$  nine times; but in fact there is no proper focus.

140. A circular cubic cannot have two nodes unless it break up into a line and circle; and similarly a bicircular quartic cannot have two nodes (exclusive of course of the points  $I, J$ ) unless it break up into two circles; the last-mentioned case will be considered in the sequel in reference to the problem of tactions.

*As to the Analytical Theory for the Circular Cubic and the Bicircular Quartic respectively—*

Art. No. 141.

141. It may be remarked in regard to the analytical theory about to be given, that although the investigation is very similar for the circular cubic and for the bicircular quartic, yet the former cannot be deduced from the latter case. In fact if for the bicircular quartic, using a form somewhat more general than that

which is ultimately adopted, we suppose that for the two nodes respectively  $(\xi = 0, z = 0)$  and  $(\eta = 0, z = 0)$ , then if  $l\xi + mz = 0, l'\xi + m'z = 0, n\eta + pz = 0, n'\eta + p'z = 0$  are the tangents at the two nodes respectively, the equation will be

$$(l\xi + mz)(l'\xi + m'z)(n\eta + pz)(n'\eta + p'z) + ez^2\xi\eta + z^3(a\xi + b\eta) + cz^4 = 0,$$

and if (in order to make this equation divisible by  $z$ , and the curve so to break up into the line  $z = 0$  and a cubic) we write  $l = 0$  or  $n = 0$ , then the curve will indeed break up as required, but we shall have, not the general cubic through the two points  $(\xi = 0, z = 0), (\eta = 0, z = 0)$ , but in each case a nodal cubic, viz., if  $l = 0$  there will be a node at the point  $(\eta = 0, z = 0)$ , and if  $n = 0$  a node at the point  $(\xi = 0, z = 0)$ .

*Analytical Theory for the Circular Cubic—Art. Nos. 142 to 144.*

142. I consider then the two cases separately; and first the circular cubic. The equation may be taken to be

$$\xi\eta(p\xi + q\eta) + ez\xi\eta + z^2(a\xi + b\eta + cz^3) = 0,$$

or what is the same thing

$$\xi\eta(p\xi + q\eta + ez) + z^2(a\xi + b\eta + cz) = 0,$$

viz.  $(\xi, \eta, z)$  being any co-ordinates whatever, this is the general equation of a cubic passing through the points  $(\xi = 0, z = 0), (\eta = 0, z = 0)$ , and at these points touched by the lines  $\xi = 0, \eta = 0$  respectively. And if  $(\xi, \eta, z = 1)$  be circular co-ordinates, then we have the general equation of a circular cubic having the lines  $\xi = 0, \eta = 0$  for its asymptotes, or say the point  $\xi = 0, \eta = 0$  for its centre; the equation of the remaining asymptote is evidently  $p\xi + q\eta + ez = 0$ ; to make the curve real we must have  $(p, q)$  and  $(a, b)$  conjugate imaginaries,  $e$  and  $c$  real.

143. Taking in any case the points  $I, J$  to be the points  $\xi = 0, z = 0$  and  $\eta = 0, z = 0$  respectively, for the equation of a tangent from  $I$  write  $p\xi = \theta z$ ; then we have

$$\theta\eta(\theta z + q\eta + ez) + z(a\theta z + bp\eta + cpz) = 0,$$

that is

$$z^2(a\theta + cp) + \eta z(\theta^2 + e\theta + bp) + \eta^2 \cdot q\theta = 0,$$

and the line will be a tangent if only

$$(\theta^2 + e\theta + bp)^2 - 4q\theta(a\theta + cp) = 0,$$

that is, the four tangents from  $I$  are the lines  $p\xi = \theta z$ , where  $\theta$  is any root of this equation. Similarly the four tangents from  $J$  are the lines  $q\eta = \phi z$ , where  $\phi$  is any root of the equation

$$(\phi^2 + e\phi + aq)^2 - 4p\phi(b\phi + cq) = 0,$$

writing the two equations under the forms

$$\left\{ \begin{array}{l} 6, \\ 3e, \\ c^2 + 2bp - 4aq, \\ 3cbp \quad - 6cpq, \\ 6b^2p^2, \end{array} \right\} (\theta, 1)^4 = 0, \quad \left\{ \begin{array}{l} 6, \\ 3e, \\ c^2 + 2aq - 4bp, \\ 3caq \quad - 3cpq, \\ 6a^2q^2, \end{array} \right\} (\varphi, 1)^4 = 0,$$

the equations have the same invariants; viz., for the first equation the invariants are easily found to be

$$I = 3(c^2 - 4bp - 4aq)^2 + 72 ce - 2ab) pq,$$

$$J = -(c^2 - 4bp - 4aq)^3 - 36 ce - 2ab) pq(c^2 - 4bp - 4aq) - 216 c^2 p^2 q^2,$$

and then by symmetry the other equation has the same invariants. The absolute invariant  $I^3 \div J^2$  has therefore the same value in the two equations; that is, the equations are linearly transformable the one into the other, which is the before-mentioned theorem that the two pencils are homographic.

144. The two equations will be satisfied by  $\theta = \phi$ , if only  $bp = aq$ ; that is, if  $p = \frac{a}{k}$ ,  $q = \frac{b}{k}$ ; putting for convenience  $\frac{e}{k}$  in place of  $e$ , the equation of the curve is then

$$\xi\eta(a\xi + b\eta + cz) + kz^2(a\xi + b\eta + cz) = 0.$$

In this case the pencils of tangents are  $a\xi = k\theta z$ ,  $b\eta = k\theta z$ , where  $\theta$  is determined by a quartic equation, or taking the corresponding lines (which by their intersections determine the foci  $A, B, C, D$ ) to be ( $a\xi = k\theta_1 z$ ,  $b\eta = k\theta_1 z$ ), &c., these four points lie in the line  $a\xi - b\eta = 0$ , which is a line through the centre of the curve, or point  $\xi = 0, \eta = 0$ : the formulæ just obtained belong therefore to the symmetrical case of the circular cubic. Passing to rectangular co-ordinates, writing  $z = 1$ , and taking  $y = 0$  for the equation of the axis, it is easy to see that the equation may be written

$$(x^2 + y^2)(x - a) + k(x - b) = 0;$$

or, changing the origin and constants,

$$xy^2 + (x - a)(x - b)(x - c) = 0.$$

*Analytical Theory for the Bicircular Quartic—Art. Nos. 145 to 149.*

145. The equation for the bicircular quartic may be taken to be

$$k(\xi^2 - \alpha^2 z^2)(\eta^2 - \beta^2 z^2) + ez^2 \xi\eta + z^3(a\xi + b\eta) + cz^4 = 0,$$

viz.  $(\xi, \eta, z)$  being any co-ordinates whatever, this is the equation of a quartic curve having a node at each of the points  $(\xi = 0, z = 0)$  and  $(\eta = 0, z = 0)$ : the equations of the two tangents at the one node are  $\xi - \alpha z = 0$ ,  $\xi + \alpha z = 0$ ; and those of the two tangents at the other node are  $\eta - \beta z = 0$ ,  $\eta + \beta z = 0$ ;  $\xi = 0$  is thus the harmonic of the line  $z = 0$  in regard to the tangents at  $(\xi = 0, z = 0)$ ,

and  $\eta = 0$  is the harmonic of the same line  $z = 0$  in regard to the tangents at  $(\eta = 0, z = 0)$ . If  $(\xi, \eta, z = 1)$  be circular co-ordinates, then we have the general equation of the bicircular quartic having the lines  $\xi + \alpha z = 0, \xi - \alpha z = 0$  for one pair, and the lines  $\eta - \beta z = 0, \eta + \beta z = 0$  for the other pair of parallel asymptotes; and therefore the point  $\xi = 0, \eta = 0$  for centre, and the lines  $\beta\xi - \alpha\eta = 0, \beta\xi + \alpha\eta = 0$  for nodal axes. In order that the curve may be real we must have  $(\alpha, \beta), (a, b)$  conjugate imaginaries,  $k, e, c$  real. The points  $(\xi = 0, z = 0)$  and  $(\eta = 0, z = 0)$  are as before the points  $I, J$ . If  $\alpha = 0$ , the node at  $I$  becomes a cusp, and so if  $\beta = 0$ , the node at  $J$  becomes a cusp; the form thus includes the case of a bicuspidal or Cartesian curve.

146. To find the tangents from  $I$ , writing in the equation of the curve  $\xi = \theta\alpha z$ , we have

$$k\alpha^2(\theta^2 - 1)(\eta^2 - \beta^2 z^2) + e\alpha\theta\eta z + z(\alpha a\theta z + b\eta) + cz^2 = 0;$$

that is

$$\begin{aligned} &\eta^2 \cdot k\alpha^2(\theta^2 - 1), \\ &+ \eta z \cdot e\alpha\theta + b, \\ &+ z^2 \cdot -k\alpha^2\beta^2(\theta^2 - 1) + \alpha a\theta + c = 0, \end{aligned}$$

and the condition of tangency is

$$4k(\theta^2 - 1)\{k\alpha^2\beta^2(\theta^2 - 1) - \alpha a\theta - c\} + \left(e\theta + \frac{b}{\alpha}\right)^2 = 0;$$

viz., the tangents from  $I$  are  $\xi = \theta\alpha z$ , where  $\theta$  is any root of this equation. Similarly, if we have

$$4k(\phi^2 - 1)\{k\alpha^2\beta^2(\phi - 1) - b\beta\phi - c\} + \left(e\phi + \frac{a}{\beta}\right)^2 = 0,$$

the tangents from  $J$  are  $\eta = \phi\beta z$ , where  $\phi$  is any root of this equation.

147. The two equations may be written

$$\left\{ \begin{array}{l} 24k^2\alpha^2\beta^2, \\ - 6ka\alpha, \\ - 8k^2\alpha^3\beta^2 - 4kc + e^2, \\ 6a\alpha \quad + 3e\frac{b}{\alpha}, \\ 24k^2\alpha^2\beta^2 + 24kc + 6\frac{b^2}{\alpha^2} \end{array} \right\} (\theta, 1)^4 = 0, \left\{ \begin{array}{l} 24k^2\alpha^2\beta^2, \\ - 6kb\beta, \\ - 8k^2\alpha^2\beta^2 - 4kc + e^2, \\ 6kb\beta \quad + 3e\frac{\alpha}{\beta}, \\ 24k^2\alpha^2\beta^2 + 24kc + 6\frac{\alpha^2}{\beta^2} \end{array} \right\} (\phi, 1)^4 = 0,$$

which equations have the same invariants; in fact for the first equation the invariants are found to be as follows, viz., if for shortness

$$C = - 8k^2\alpha^2\beta^2 - 4kc + e^2,$$

then

$$I = 576k^4\alpha^4\beta^4 + 576k^3c\alpha^2\beta^2 + 144k^2(a^2\alpha^2 + b^2\beta^2) + 72kab + 3C^2,$$

$$\begin{aligned} J = C\{ &576k^4\alpha^4\beta^4 + 576k^3c\alpha^2\beta^2 + 144k^2(a^2\alpha^2 + b^2\beta^2) + 36ke\alpha\beta - C^2\} \\ &- 864k^3eab\alpha^2\beta^2 - 216k^2e^2(a^2\alpha^2 + b^2\beta^2) - 216k^2a^2b^2, \end{aligned}$$

and then by symmetry the other equation has the same invariants. The

absolute invariant  $I^3 \div J^2$  has thus the same value in the two equations, that is, the equations are linearly transformable the one into the other, which is the before-mentioned theorem that the pencils are homographic.

148. The equations will be satisfied by  $\theta = \phi$  if only  $a\alpha = b\beta$ , that is, if  $a, b = m\beta, m\alpha$ ; or by  $\theta = -\phi$  if only  $a\alpha = -b\beta$ , that is, if  $a, b = m\beta, -m\alpha$ : the equation of the curve is these two cases respectively—

$$\begin{aligned} k(\xi^2 - \alpha^2 z^2)(\eta^2 - \beta^2 z^2) + cz^2 \xi \eta + mz^3(\beta \xi + \alpha \eta) + cz^4 &= 0, \\ k(\xi^2 - \alpha^2 z^2)(\eta^2 - \beta^2 z^2) + cz^2 \xi \eta + mz^3(\beta \xi - \alpha \eta) + cz^4 &= 0. \end{aligned}$$

If to fix the ideas we attend to the first case, then the equation in  $\theta$  is

$$\left\{ \begin{array}{l} 24k^2\alpha^2\beta^2, \\ -6kma\beta, \\ -8k^2\alpha^2\beta^2 - 4kc + c^2, \\ 6kma\beta \quad \quad \quad + 3me, \\ 24k^2\alpha^2\beta^2 + 24kc + 6m^2 \end{array} \right\} (\theta, 1)^4 = 0;$$

and we may take as corresponding tangents through the two nodes respectively  $\xi = \theta az, \eta = \theta \beta z$ ; the foci  $A, B, C, D$ , which are the intersections of the pairs of lines ( $\xi = \theta_1 az, \eta = \theta_1 \beta z$ ), &c., lie, it is clear, in the line  $\beta \xi - \alpha \eta = 0$ , which is one of the nodal axes of the curve. Similarly, in the second case, if  $\theta$  be determined by the foregoing equation, we may take as corresponding tangents through the two nodes respectively  $\xi = \theta az, \eta = -\theta \beta z$ ; the foci ( $A, B, C, D$ ), which are the intersections of the pairs of lines ( $\xi = \theta_1 az, \eta = -\theta_1 \beta z$ ), &c., lie in the line  $\beta \xi + \alpha \eta = 0$ , which is the other of the nodal axes of the curve. In either case the foci  $A, B, C, D$  lie in a line, that is, we have the curve symmetrical; and, as we have just seen, the focal axis, or axis of symmetry, is one or other of the nodal axes.

149. In the case of the Cartesian, or when  $a=0, \beta=0$ , viz., the equation  $a\alpha = b\beta$  is satisfied identically, and this seems to show that the Cartesian is symmetrical; it is to be observed, however, that for  $a=0, \beta=0$  the foregoing formulæ fail, and it is proper to repeat the investigation for the special case in question. Writing  $a=0, \beta=0$  the equation of the curve is

$$k\xi^2\eta^2 + cz^2\xi\eta + z^3(\alpha\xi + b\eta) + cz^4 = 0,$$

and then, taking  $\xi = \theta bz$  for the equation of the tangent from  $I$ , we have

$$\begin{aligned} \eta^2 \cdot kb^2\theta^2 \\ + \eta z \cdot b(e\theta + 1) \\ + z^2 \cdot ab\theta + c &= 0. \end{aligned}$$

and the condition of tangency is

$$4k\theta^2(ab\theta + c) - (e\theta + 1)^2 = 0;$$

viz, we have here a cubic equation. Similarly, if we have  $\eta = \theta az$  for the equation of a tangent from  $J$ , then

$$4k\theta^2(ab\theta + c) - (e\theta + 1)^2 = 0.$$

Hence  $\theta$  being determined by the cubic equation as above, we may take  $\phi = \theta$ , and consequently the equations of the corresponding tangents will be  $\xi = \theta bz$ ,  $\eta = \theta az$ , viz., the foci  $A, B, C$  will be given as the intersections of the pairs of lines ( $\xi = \theta_1 bz$ ,  $\eta = \theta_1 az$ ), &c. The foci lie therefore in the line  $a\xi - b\eta = 0$ ; or the curve is symmetrical, the focal axis, or axis of symmetry, passing through the centre.

*On the Property that the Points of Contact of the Tangents from a Pair of Concyelic Foci lie in a Circle—Art. Nos. 150 to 158.*

150. We have seen that the foci form four concyclic sets  $(A, B, C, D)$ ,  $(A_1, B_1, C_1, D_1)$ ,  $(A_2, B_2, C_2, D_2)$ ,  $(A_3, B_3, C_3, D_3)$ , that is,  $A, B, C, D$  are in a circle. We may, if we please, say that any one focus is concyclic—viz., it lies in a circle with three other foci; but any two foci taken at random are not concyclic; it is only a pair such as  $(A, B)$  taken out of a set of four concyclic foci which are concyclic, viz., there exist two other foci lying with them in a circle. The number of such pairs is, it is clear = 24. Let  $A, B$  be any two concyclic foci, I say that the points of contact of the tangents  $AI, AJ, BI, BJ$ , lie in a circle.

151. Consider the case of the bi-circular quartic, and take as before ( $\xi = 0$ ,  $z = 0$ ), and ( $\eta = 0$ ,  $z = 0$ ) for the co-ordinates of the points  $I, J$  respectively. Let the two tangents from the focus  $A$  be  $\xi - az = 0$ ,  $\eta - a'z = 0$ , say for shortness  $p = 0$ ,  $p' = 0$ , then the equation of the curve is expressible in the form  $pp'U = V^2*$ , where  $U = 0$ ,  $V = 0$  are each of them circles, viz.,  $U$  and  $V$  are each of them quadric functions containing the terms  $z^2$ ,  $z\eta$ ,  $z\xi$ , and  $\xi\eta$ . Taking an indeterminate coefficient  $\lambda$ , the equation may be written

$$pp'(U + 2\lambda V + \lambda^2 pp') = (V + \lambda pp')^2,$$

and then  $\lambda$  may be so determined that  $U + 2\lambda V + \lambda^2 pp' = 0$ , shall be a 0-circle, or pair of lines through  $I$  and  $J$ . It is easy to see that we have thus for  $\lambda$  a cubic equation, that is, there are three values of  $\lambda$ , for each of which the function  $U + 2\lambda V + \lambda^2 pp'$  assumes the form  $(\xi - \beta z)(\eta - \beta' z) = qq'$  suppose: taking any one of these, and changing the value of  $V$  so as that we may have  $V$  in place of  $V + \lambda pp'$ , the equation is  $pp'qq' + V^2$ , where  $V = 0$  is as before a circle, the equation shows that the points of contact of the tangents  $p = 0$ ,  $p' = 0$ ,  $q = 0$ ,  $q' = 0$  lie in this circle  $V = 0$ . The circumstance that  $\lambda$  is determined by a cubic equation would suggest that the focus  $q = 0$ ,  $q' = 0$  is one of the three foci  $B, C, D$  concyclic with  $A$ ; but this is the very thing which we wish to prove, and the investigation, though somewhat long, is an interesting one.

152. Starting from the form  $pp'qq' = V^2$ , then introducing as before an arbitrary coefficient  $\lambda$ , the equation may be written

$$pp'(qq' + 2\lambda V + \lambda^2 pp') = (V + \lambda pp')^2,$$

\* This investigation is similar to that in Salmon's Higher Plane Curves, p. 196, in regard to the double tangents of a quartic curve.

and we may determine  $\lambda$  so that  $qq' + 2\lambda V + \lambda^2 pp' = 0$  shall be a pair of lines. Writing  $V = H\xi\eta - L\eta z - L'\xi z + Mz^2$ , and substituting for  $pp'$  and  $qq'$  their values  $(\xi - \alpha z)(\eta - \alpha'z)$  and  $(\xi - \beta z)(\eta - \beta'z)$ , the equation in question is

$$(1 + 2\lambda H + \lambda^2) \xi\eta - (\beta + 2\lambda L + \lambda^2\alpha) \eta z - (\beta' + 2\lambda L' + \lambda^2\alpha') \xi z + (\beta\beta' + 2\lambda M + \lambda^2\alpha\alpha') z^2 = 0,$$

and the required condition is

$$(1 + 2\lambda H + \lambda^2) (\beta\beta' + 2\lambda M + \lambda^2\alpha\alpha') = (\beta + 2\lambda L + \lambda^2\alpha) (\beta' + 2\lambda L' + \lambda^2\alpha');$$

or reducing, this is

$$\begin{aligned} & (2M + 2H\beta\beta' - 2L'\beta - 2L\beta) \\ & + \lambda (\alpha - \beta) (\alpha' - \beta') + 4HM - 4LL' \\ & + \lambda^2 (2M + 2H\alpha\alpha' - 2L'\alpha - 2L\alpha') = 0, \end{aligned}$$

viz.,  $\lambda$  is determined by a quadric equation. Calling its roots  $\lambda_1$ , and  $\lambda_2$ , the foregoing equation, substituting therein successively these values, becomes  $(\xi - \gamma z)(\eta - \gamma'z) = 0$ , and  $(\xi - \delta z)(\eta - \delta'z) = 0$  respectively, say  $rr' = 0$  and  $ss' = 0$ .

153. We have to show that the four foci ( $p = 0, p' = 0$ ), ( $q = 0, q' = 0$ ), ( $r = 0, r' = 0$ ), ( $s = 0, s' = 0$ ) are a set of concyclic foci; that is, that the lines  $p = 0, q = 0, r = 0, s = 0$  correspond homographically to the lines  $p' = 0, q' = 0, r' = 0, s' = 0$ ; or, what is the same thing, that we have

$$\begin{vmatrix} 1, \alpha, \alpha', \alpha\alpha' \\ 1, \beta, \beta', \beta\beta' \\ 1, \gamma, \gamma', \gamma\gamma' \\ 1, \delta, \delta', \delta\delta' \end{vmatrix} = 0,$$

or, as it will be convenient to write this equation,

$$\frac{\alpha - \beta}{\alpha' - \beta'} \frac{\gamma - \delta}{\gamma' - \delta'} = \frac{\alpha - \delta}{\alpha' - \delta'} \frac{\beta - \gamma}{\beta' - \gamma'}.$$

154. We have

$$\begin{aligned} \gamma &= \frac{\beta + 2\lambda_1 L + \lambda_1^2 \alpha}{1 + 2H\lambda_2 + \lambda_1^2}, & \gamma' &= \frac{\beta' + 2\lambda_1 L' + \lambda_1^2 \alpha'}{1 + 2H\lambda_1 + \lambda_1^2} \\ \delta &= \frac{\beta + 2\lambda_2 L + \lambda_2^2 \alpha}{1 + 2H\lambda_2 + \lambda_2^2}, & \delta' &= \frac{\beta' + 2\lambda_2 L' + \lambda_2^2 \alpha'}{1 + 2H\lambda_2 + \lambda_2^2} \end{aligned}$$

The expressions of  $\alpha - \delta$ , &c., are severally fractions, the denominators of which disappear from the equation; the numerators are

$$\begin{aligned} \text{for } \alpha - \delta, &= \alpha(1 + 2\lambda_2 H + \lambda_2^2) - (\beta + 2\lambda_2 L + \alpha\lambda_2^2), \\ &= \alpha - \beta + 2\lambda_2 (\alpha H - L); \\ \text{for } \beta - \gamma, &= \beta(1 + 2\lambda_1 H + \lambda_1^2) - (\beta + 2\lambda_1 L + \alpha\lambda_1^2), \\ &= \lambda_1 \{2(\beta H - L) (\alpha - \beta)\}; \\ \text{for } \gamma - \delta, &= (\beta + 2L\lambda_1 + \alpha\lambda_1^2) (1 + 2H\lambda_2 + \lambda_2^2) \\ &\quad - (\beta + 2L\lambda_2 + \alpha\lambda_2^2) (1 + 2H\lambda_1 + \lambda_1^2), \\ &= (\alpha' - \beta') \{2H^2\alpha\beta - 2HL(\alpha + \beta) + 2L^2 + \frac{1}{2}(\alpha - \beta)^2\} \end{aligned}$$

and it hence easily appears that the equation to be verified is

$$\frac{2H^2\alpha\beta - 2HL(\alpha + \beta) + 2L^2 + \frac{1}{2}(\alpha - \beta)^2}{2H^2\alpha'\beta' - 2HL'(\alpha' + \beta') + 2L'^2 + \frac{1}{2}(\alpha' - \beta')^2} = \frac{\alpha - \beta + 2(\alpha H - L)\lambda_2}{\alpha' - \beta' + 2(\alpha' H - L')\lambda_2} \cdot \frac{2(\beta H - L) - (\alpha - \beta)\lambda_1}{2(\beta' H - L') - (\alpha' - \beta')\lambda_2}$$

155. This is

$$\frac{B - C}{B' - C'} = \frac{A + B\lambda_1 + C\lambda_2 + D\lambda_1\lambda_2}{A' + B'\lambda_1 + C'\lambda_2 + D'\lambda_1\lambda_2},$$

if for shortness

$$\begin{aligned} A &= 2(\alpha - \beta)(\beta H - L), & A' &= 2(\alpha' - \beta')(\beta' H - L'), \\ B &= -(\alpha - \beta)^2, & B' &= -(\alpha' - \beta')^2, \\ C &= 4(\alpha H - L)(\beta - L), & C' &= 4(\alpha' H - L')(\alpha' H - L'), \\ D &= -2(\alpha - \beta)(\alpha H - L), & D' &= -2(\alpha' - \beta')(\alpha' H - L'), \end{aligned}$$

and the equation then is

$$AB' - A'B + CA' - C'A - (\lambda_1 + \lambda_2)(B'C - B'C) + \lambda_1\lambda_2(CD' - C'D - (BD' - B'D)).$$

156. Calculating  $AB' - A'B, CA' - C'A, CD' - C'D, BD' - B'D$ , these are at once seen to divide by  $\{(\alpha\beta' - \alpha'\beta)H + L(\alpha' - \beta') - L'(\alpha - \beta)\}$ ; we have, moreover,

$$\begin{aligned} BC' - B'C &= -4(\alpha - \beta)^2(\alpha' H - L')(\beta' H - L') + 4(\alpha' - \beta')^2(\alpha H - L)(\beta H - L) \\ &= -\{(\alpha\alpha' - \beta\beta')H - L(\alpha' - \beta') - L'(\alpha - \beta)\} \{(\alpha\beta' - \alpha'\beta)H + L(\alpha' - \beta') - L'(\alpha - \beta)\}, \end{aligned}$$

viz., this also contains the same factor; and omitting it, the equation is found to be

$$\begin{aligned} &\{(\alpha - \beta)(\alpha' - \beta') - 4(\beta H - L)(\beta' H - L')\} \\ &- 2\{(\alpha\alpha' - \beta\beta')H - L(\alpha' - \beta') - L'(\alpha - \beta)\}(\lambda_1 + \lambda_2) \\ &+ \{- (\alpha - \beta)(\alpha' - \beta') + 4(\alpha H - L)(\alpha' H - L')\} \lambda_1\lambda_2 = 0; \end{aligned}$$

viz., substituting for  $\lambda_1 + \lambda_2$  and  $\lambda_1\lambda_2$  their values, this is

$$\begin{aligned} &\{(\alpha - \beta)(\alpha' - \beta') - 4(\beta H - L)(\beta' H - L')\} (M + H\alpha\alpha' - L\alpha' - L'\alpha) \\ &- \{(\alpha\alpha' - \beta\beta')H - L(\alpha' - \beta')\} \{(\alpha - \beta)(\alpha' - \beta') + 4HM - 4LL'\} \\ &+ \{- (\alpha - \beta)(\alpha' - \beta') + 4(\alpha H - L)(\alpha' H - L')\} \{M + H\beta\beta' - L\beta' - L'\beta\} = 0, \end{aligned}$$

which should be identically true. Multiplying by  $H$ , and writing in the form

$$\begin{aligned} &\{(\alpha - \beta)(\alpha' - \beta') - 4(\beta H - L)(\beta' H - L')\} (HM - LL' + (\alpha H - L)(\alpha' H - L')) \\ &- \{(\alpha H - L)(\alpha' H - L') - (\beta H - L)(\beta' H - L')\} \{(\alpha - \beta)(\alpha' - \beta') + 4(HM - LL')\} \\ &+ \{- (\alpha - \beta)(\alpha' - \beta') + 4(\alpha H - L)(\alpha' H - L')\} (HM - LL' + (\beta H - L)(\beta' H - L')) = 0, \end{aligned}$$

we at once see that this is so, and the theorem is thus proved, viz., that the equation being  $pp'qq' = V^2$ , the foci ( $p = 0, p' = 0$ ) and ( $q = 0, q' = 0$ ) are concyclic.

157. By what precedes,  $\lambda$  being a root of the foregoing quadric equation, we may write

$$qq' + 2\lambda V + \lambda^2 pp' = K^2 rr',$$

where the focus  $r = 0$ ,  $r' = 0$  is concyclic with the other two foci; but from the equation of the curve  $V = \sqrt{pp'qq'}$ , that is we have

$$qq' + 2\lambda \sqrt{pp'qq'} + \lambda^2 pp' = Krr',$$

or, what is the same thing,

$$\lambda \sqrt{pp'} + \sqrt{qq'} + K \sqrt{rr'} = 0,$$

viz., this is a form of the equation of the curve; substituting for  $p, p', q, q', r, r'$  their values, writing also

$$A = (\xi - \alpha z)(\eta - \alpha' z),$$

$$B = (\xi - \beta z)(\eta - \beta' z),$$

$$C = (\xi - \gamma z)(\eta - \gamma' z),$$

and changing the constants  $\lambda, K$  (viz.  $\lambda : 1 : K = \sqrt{l} : \sqrt{m} : \sqrt{n}$ ) the equation is

$$\sqrt{lA} + \sqrt{mB} + \sqrt{nC} = 0,$$

viz., we have the theorem that for a bicircular quartic if  $(\xi - \alpha z = 0, \eta - \alpha' z = 0)$ ,  $(\xi - \beta z = 0, \eta - \beta' z = 0)$ ,  $(\xi - \gamma z = 0, \eta - \gamma' z = 0)$  be any three concyclic foci, then the equation is as just mentioned; that is, the curve is a trizomal curve, the zomals being the three given foci regarded as 0-circles. The same theorem holds in regard to the circular cubic, and a similar demonstration would apply to this case.

158. It may be noticed that we might, without proving as above that the two foci  $(p = 0, p' = 0)$ ,  $(q = 0, q' = 0)$  were concyclic, have passed at once from the form  $pp'qq' = V^2$ , to the form  $\sqrt{\lambda pp'} + \sqrt{qq'} + K \sqrt{rr'} = 0$  (or  $\sqrt{lA} = \sqrt{mB} = \sqrt{nC} = 0$ ), and then by the application of the theorem of the variable zomal (thereby establishing the existence of a fourth focus concyclic with the three) have shown that the original two foci were concyclic. But it seemed the more orderly course to effect the demonstration without the aid furnished by the reduction of the equation to the trizomal form.

PART IV. (Nos. 159 to 206).—ON TRIZOMAL AND TETRAZOMAL CURVES WHERE THE ZOMALS ARE CIRCLES.

*The Trizomal Curve—The Tangents at I, J, &c.*—Art. Nos. 159 to 165.

159. I consider the trizomal

$$\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} = 0,$$

where  $A, B, C$  being the centres of three given circles,  $A^\circ$ , &c. denote as before, viz., in rectangular and in circular co-ordinates respectively, we have

$$A^\circ = (x - \alpha z)^2 + (y - \alpha' z)^2 - \alpha''^2 z^2, = (\xi - \alpha z)(\eta - \alpha' z) - \alpha''^2 z^2,$$

$$B^\circ = (x - \beta z)^2 + (y - \beta' z)^2 - \beta''^2 z^2, = (\xi - \beta z)(\eta - \beta' z) - \beta''^2 z^2,$$

$$C^\circ = (x - \gamma z)^2 + (y - \gamma' z)^2 - \gamma''^2 z^2, = (\xi - \gamma z)(\eta - \gamma' z) - \gamma''^2 z^2.$$

By what precedes, the curve is of the order = 4, touching each of the given circles twice, and having a double point, or node, at each of the points  $I, J$ ; that is, it is a bicircular quartic: but if for any determinate values of the radicals  $\sqrt{l}, \sqrt{m}, \sqrt{n}$ , we have

$$\sqrt{l} + \sqrt{m} + \sqrt{n} = 0,$$

then there is a branch

$$\sqrt{lA^{\circ}} + \sqrt{mB^{\circ}} + \sqrt{nC^{\circ}} = 0,$$

containing ( $z = 0$ ) the line infinity; and the order is here = 3: viz., the curve here passes through each of the points  $I, J$  and through another point at infinity (that is, there is an asymptote), and is thus a circular cubic.

160. I commence by investigating the equations of the nodal tangents at the points  $I, J$  respectively; using for this purpose the circular co-ordinates ( $\xi, \eta, z = 1$ ), it is to be observed that, in the rationalised equation, for finding the tangents at ( $\xi = 0, z = 0$ ) we have only to attend to the terms of the second order in ( $\xi, z$ ), and similarly for finding the tangents at ( $\eta = 0, z = 0$ ) we have only to attend to the terms of the second order in ( $\eta, z$ ). But it is easy to see that on any term involving  $a'', b'',$  or  $c''$  will be of the third order at least in ( $\xi, z$ ), and similarly of the third order at least in ( $\eta, z$ ); hence for finding the tangents we may reject the terms in question, or, what is the same thing, we may write  $a'', b'', c''$  each = 0, thus reducing the three circles to their respective centres. The equation thus becomes

$$\sqrt{l(\xi - \alpha z)(\eta - \alpha'z)} + \sqrt{m(\xi - \beta z)(\eta - \beta'z)} + \sqrt{n(\xi - \gamma z)(\eta - \gamma'z)} = 0.$$

For finding the tangents at ( $\xi = 0, z = 0$ ) we have in the rationalised equation to attend only to the terms of the second order in ( $\xi, z$ ); and it is easy to see that any term involving  $\alpha', \beta', \gamma'$  will be of the third order at least in ( $\xi, z$ ), that is, we may reduce  $\alpha', \beta', \gamma'$  each to zero; the irrational equation then becomes divisible by  $\sqrt{\eta}$ , and throwing out this factor, it is

$$\sqrt{l(\xi - \alpha z)} + \sqrt{m(\xi - \beta z)} + \sqrt{n(\xi - \gamma z)} = 0,$$

viz., this equation which evidently belongs to a pair of lines through the point ( $\xi = 0, z = 0$ ) gives the tangents at the point in question; and similarly the tangents at the point ( $\eta = 0, z = 0$ ) are given by the equation

$$\sqrt{l(\eta - \alpha'z)} + \sqrt{m(\eta - \beta'z)} + \sqrt{n(\eta - \gamma'z)} = 0.$$

161. To complete the solution, attending to the tangents at ( $\xi = 0, z = 0$ ), and putting for shortness

$$\begin{aligned} \lambda &= l - m - n, \\ \mu &= -l + m - n, \\ \nu &= -l - m + n, \\ \Delta &= l^2 + m^2 + n^2 - 2mn - 2nl - 2lm, \end{aligned}$$

the rationalised equation is easily found to be

$$\begin{aligned} & \xi^2 \cdot \Delta \\ & - 2\xi z(l\lambda\alpha + m\mu\beta + n\nu\gamma) \\ & + z^2(l^2\alpha^2 + m^2\beta^2 + n^2\gamma^2 - 2mn\beta\gamma - 2nl\gamma\alpha - 2lm\alpha\beta) = 0. \end{aligned}$$

And it is to be noticed that in the case of the circular cubic or when  $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$ , then  $\Delta = 0$ , so that the equation contains the factor  $z$ , and throwing this out, the equation gives a single line, which is in fact the tangent of the circular cubic.

162. Returning to the bicircular quartic, we may seek for the condition in order that the node may be a cusp: the required condition is obviously

$$\Delta(l^2\alpha^2 + m^2\beta^2 + n^2\gamma^2 - 2mn\beta\gamma - 2nl\gamma\alpha - 2lm\alpha\beta) - (l\lambda\alpha + m\mu\beta + n\nu\gamma)^2 = 0,$$

or observing that

$$\begin{aligned} \Delta - \lambda^2 &= -4mn, \text{ \&c.} \\ \Delta + \mu\nu &= -2l\lambda, \text{ \&c.} \end{aligned}$$

this is

$$l\alpha^2 + m\beta^2 + n\gamma^2 + \lambda\beta\gamma + \mu\gamma\alpha + \nu\alpha\beta = 0,$$

or substituting for  $\lambda, \mu, \nu$ , their values, it is

$$l(\alpha - \beta)(\alpha - \gamma) + m(\beta - \gamma)(\beta - \alpha) + n(\gamma - \alpha)(\gamma - \beta) = 0,$$

or as it is more simply written

$$\frac{l}{\beta - \gamma} + \frac{m}{\gamma - \alpha} + \frac{n}{\alpha - \beta} = 0.$$

163. If the node at  $(\eta = 0, z = 0)$  be also a cusp, then we have in like manner

$$\frac{l}{\beta' - \gamma'} + \frac{m}{\gamma' - \alpha'} + \frac{n}{\alpha' - \beta'} = 0.$$

Now observing that

$$\begin{aligned} (\gamma - \alpha)(\alpha' - \beta') - (\gamma' - \alpha')(\alpha - \beta) &= \begin{vmatrix} \alpha, \alpha', 1 \\ \beta, \beta', 1 \\ \gamma, \gamma', 1 \end{vmatrix} \\ &= (\alpha - \beta)(\beta' - \gamma') - (\alpha' - \beta')(\beta - \gamma), \\ &= (\beta - \gamma)(\gamma' - \alpha') - (\beta' - \gamma')(\gamma - \alpha), \end{aligned}$$

=  $\Omega$  suppose: the two equations give

$$l : m : n = \Omega(\beta - \gamma)(\beta' - \gamma') : \Omega(\gamma - \alpha)(\gamma' - \alpha') : \Omega(\alpha - \beta)(\alpha' - \beta');$$

or if  $\Omega$  is not = 0, then

$$l : m : n = (\beta - \gamma)(\beta' - \gamma') : (\gamma - \alpha)(\gamma' - \alpha') : (\alpha - \beta)(\alpha' - \beta').$$

164. If

$$\Omega = \begin{vmatrix} \alpha, \alpha', 1 \\ \beta, \beta', 1 \\ \gamma, \gamma', 1 \end{vmatrix}, = 0,$$

or, what is the same thing, if

$$\begin{vmatrix} a, a', 1 \\ b, b', 1 \\ c, c', 1 \end{vmatrix} = 0,$$

the centres  $A, B, C$  are in a line; taking it as the axis of  $x$ , we have  $a = a' = a, \beta = \beta' = b, \gamma = \gamma' = c$ ; and the conditions for the cusps at  $I, J$  respectively reduce themselves to the single condition

$$\frac{l}{b-c} + \frac{m}{c-a} + \frac{n}{a-b} = 0,$$

so that this condition being satisfied, the curve

$$\sqrt{l\{(x-az)^2 + y^2 - a''^2z^2\}} + \sqrt{m\{(x-bz)^2 + y^2 - b''^2z^2\}} + \sqrt{n\{(x-cz)^2 + y^2 - c''^2z^2\}} = 0$$

is a Cartesian; viz., given any three circles with their centres on a line, there are a singly infinite series of Cartesians, each touched by the three circles respectively; the line of centres is the axis of the curve, but the centres  $A, B, C$  are not the foci, except in the case  $a'' = 0, b'' = 0, c'' = 0$ , where the circles vanish. The condition for  $l, m, n$  is satisfied if  $l : m : n = (b-c)^2 : (c-a)^2 : (a-b)^2$ ; these values writing  $\sqrt{l} : \sqrt{m} : \sqrt{n} = b-c : c-a : a-b$ , give not only  $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$ , but also  $a\sqrt{l} + b\sqrt{m} + c\sqrt{n} = 0$ ; these are the conditions for a branch containing ( $z^2 = 0$ ) the line infinity twice; the equation

$$(b-c)\sqrt{(x-az)^2 + y^2 - a''^2z^2} + (c-a)\sqrt{(x-bz)^2 + y^2 - b''^2z^2} + (a-b)\sqrt{(x-cz)^2 + y^2 - c''^2z^2} = 0,$$

is thus that of a conic, and if  $a'' = 0, b'' = 0, c'' = 0$ , then the curve reduces itself to  $y^2 = 0$ , the axis twice.

165. If  $\Omega$  is not  $= 0$ , then we have

$$l : m : n = (\beta - \gamma)(\beta' - \gamma') : (\gamma - \alpha)(\gamma' - \alpha') : (\alpha - \beta)(\alpha' - \beta'),$$

viz.,  $l, m, n$  are as the squared distances  $\overline{BC^2}, \overline{CA^2}, \overline{AB^2}$ , say as  $f^2 : g^2 : h^2$ ; or when the centres of the given circles  $A, B, C$  are not in a line, then  $f, g, h$  being the distances  $BC, CA, AB$  of these centres from each other, we have, touching each of the given circles twice, the *single* Cartesian

$$f\sqrt{A^o} + g\sqrt{B^o} + h\sqrt{C^o} = 0,$$

which, in the particular case where the radii  $a'', b'', c''$  are each  $= 0$ , becomes

$$f\sqrt{A} + g\sqrt{B} + h\sqrt{C} = 0,$$

viz., this is the circle through the points  $A, B, C$ , say the circle  $ABC$ , twice.

*Investigation of the Foci of a Conic represented by an Equation in Areal Co-ordinates—*  
Art. Nos. 166 to 169.

166. I premise as follows: Let  $A, B, C$  be any given points, and in regard to the triangle  $ABC$  let the areal co-ordinates of a current point  $P$  be  $u, v, w$ : that is, writing  $PBC$ , &c., for the areas of these triangles, take the co-ordinates to be

$$u : v : w = PBC : PCA : PAB ,$$

or, what is the same thing in the rectangular co-ordinates ( $x, y, z = 1$ ), if

$$(a, a', 1), (b, b', 1), (c, c', 1) ,$$

be the co-ordinates of  $A, B, C$  respectively, take

$$u : v : w = \begin{vmatrix} x, y, z \\ b, b', 1 \\ c, c', 1 \end{vmatrix} : \begin{vmatrix} x, y, z \\ c, c', 1 \\ a, a', 1 \end{vmatrix} : \begin{vmatrix} x, y, z \\ a, a', 1 \\ b, b', 1 \end{vmatrix}$$

or in the circular co-ordinates ( $\xi, \eta, z = 1$ ), if  $(a, a', 1), (\beta, \beta', 1), (\gamma, \gamma', 1)$  be the co-ordinates of the three points respectively, then

$$u : v : w = \begin{vmatrix} \xi, \eta, z \\ \beta, \beta', 1 \\ \gamma, \gamma', 1 \end{vmatrix} : \begin{vmatrix} \xi, \eta, z \\ \gamma, \gamma', 1 \\ a, a', 1 \end{vmatrix} : \begin{vmatrix} \xi, \eta, z \\ a, a', 1 \\ \beta, \beta', 1 \end{vmatrix}$$

167. For the point  $I$  we have  $(\xi, \eta, z) = (0, 1, 0)$ , and hence if its areal co-ordinates be  $(u_0, v_0, w_0)$ , we have

$$u_0 : v_0 : w_0 = \beta - \gamma : \gamma - a : a - \beta ,$$

and hence also,  $(u, v, w)$  referring to the current point  $P$ , we find

$$v_0 w - w_0 v = (\gamma - a)[(a' - \beta)(\xi - az) - (a - \beta)(\eta - a'z)] \\ - (a - \beta)[(\gamma' - a')(\xi - az) - (\gamma - a)(\eta - a'z)] = \Omega(\xi - az) ,$$

if 
$$\Omega = (\gamma - a)(a' - \beta) - (a - \beta)(\gamma' - a') = \begin{vmatrix} a, a', 1 \\ \beta, \beta', 1 \\ \gamma, \gamma', 1 \end{vmatrix} ;$$

whence

$$v_0 w - w_0 v : w_0 u - w u_0 : u_0 v - w v_0 = \xi - az : \xi - \beta z : \xi - \gamma z ,$$

and in precisely the same manner, if  $u'_0, v'_0, w'_0$  refer to the point  $J$ , then

$$u'_0 : v'_0 : w'_0 = \beta' - \gamma' : \gamma' - a' : a' - \beta' ,$$

and

$$v'_0 w - w'_0 v : w'_0 u - w u'_0 : u'_0 v - w v'_0 = \eta - a'z : \eta - \beta'z : \eta - \gamma'z .$$

168. Consider the conic

$$(a, b, c, f, g, h)(u, v, w)^2 = 0 ,$$

where  $u, v, w$  are any trilinear co-ordinates whatever; and take the inverse co-efficients to be  $(A, B, C, F, G, H)$  ( $A = bc - f^2$ , &c.), then for any given point the co-ordinates of which are  $(u_0, v_0, w_0)$ , the equation of the tangents from this point to the conic is, as is well known,

$$(A, B, C, F, G, H)(v_0 w - w_0 v, w_0 u - u_0 w, u_0 v - v_0 u)^2 = 0;$$

consequently for the conic

$$(a, b, c, f, g, h)(u, v, w)^2 = 0,$$

where  $(u, v, w)$  are areal co-ordinates referring, as above, to any three given points  $A, B, C$ , the equation of the pair of tangents from the point  $I$  to the conic is

$$(A, B, C, F, G, H)(\xi - \alpha z, \xi - \beta z, \xi - \gamma z)^2 = 0,$$

and that of the pair of tangents from  $J$  is

$$(A, B, C, F, G, H)(\eta - \alpha' z, \eta - \beta' z, \eta - \gamma' z)^2 = 0,$$

these two line-pairs intersecting, of course, in the foci of the conic.

169. In particular, if the conic is a conic passing through the points  $A, B, C$ , then taking its equation to be

$$l v w + m w u + n u v = 0,$$

the inverse co-efficients are as  $(l^2, m^2, n^2, -2mn, -2nl, -2lm)$ , and we have for the equations of the two line-pairs

$$\begin{aligned} \sqrt{l(\xi - \alpha z)} + \sqrt{m(\xi - \beta z)} + \sqrt{n(\xi - \gamma z)} &= 0, \\ \sqrt{l(\eta - \alpha' z)} + \sqrt{m(\eta - \beta' z)} + \sqrt{n(\eta - \gamma' z)} &= 0. \end{aligned}$$

*The Theorem of the Variable Zomal—Art. No. 170.*

170. Consider the four circles

$$A^\circ = 0, B^\circ = 0, C^\circ = 0, D^\circ = 0 \quad (A^\circ = (x - az)^2 + (y - a'z)^2 - a''z^2, \text{ \&c.}),$$

which have a common orthotomic circle; so that as before

$$aA^\circ + bB^\circ + cC^\circ + dD^\circ = 0,$$

where

$$a : b : c : d = BCD : -CDA : DAB : -ABC.$$

I consider the first three circles as given, and the fourth circle as a variable circle cutting at right angles the orthotomic circle of the three given circles; this being so, attending only to the ratios  $a : b : c$ , we may write

$$a : b : c = DBC : DCA : DAB,$$

that is,  $(a, b, c)$  are proportional to the areal co-ordinates of the centre of the variable circle in regard to the triangle  $ABC$ .

171. Suppose that the centre of the variable circle is situate on a given conic, then expressing the equation of this conic in areal co-ordinates in regard to the triangle  $ABC$ , we have between  $(a, b, c)$  the equation obtained by substituting these values for the co-ordinates in the equation of the conic; that is, the equation of the variable circle is

$$aA^\circ + bB^\circ + cC^\circ = 0.$$

where  $(a, b, c)$  are connected by an equation,

$$(a, b, c, f, g, h)(a, b, c)^2 = 0.$$

Hence  $(A, B, C, F, G, H)$  being the inverse co-efficients, the equation of the envelope of the variable circle is

$$(A, B, C, F, G, H)(A^\circ, B^\circ, C^\circ)^2 = 0,$$

and, in particular, if the conic be a conic passing through the points  $A, B, C$ , and such that its equation in the areal co-ordinates  $(u, v, w)$  in regard to the triangle  $ABC$  is

$$lvw + mvu + nuv = 0,$$

then the equation of the envelope is

$$(l^2, m^2, n^2, -mn, -nl, -lm)(A^\circ, B^\circ, C^\circ)^2 = 0;$$

that is, it is

$$(1, 1, 1, -1, -1, -1)(lA^\circ, mB^\circ, nC^\circ)^2 = 0,$$

or, what is the same thing, it is

$$\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} = 0.$$

172. It has been seen that the equations of the nodal tangents at the points  $I, J$  respectively are respectively

$$\begin{aligned} \sqrt{l(\xi - \alpha z)} + \sqrt{m(\xi - \beta z)} + \sqrt{n(\xi - \gamma z)} &= 0, \\ \sqrt{l(\eta - \alpha' z)} + \sqrt{m(\eta - \beta' z)} + \sqrt{n(\eta - \gamma' z)} &= 0, \end{aligned}$$

and that these are the equations of the tangents to the conic  $lvw + mvu + nuv = 0$  from the points  $I, J$  respectively. We have thus Casey's theorem for the generation of the bi-circular quartic as follows:—The envelope of a variable circle which cuts at right angles the orthotomic circle of three given circles  $A^\circ = 0, B^\circ = 0, C^\circ = 0$ , and has its centre on the conic  $lvw + mvu + nuv = 0$  which passes through the centres of the three given circles is the bicircular quartic, or trizomal

$$\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} = 0,$$

which has its nodo-foci coincident with the foci of the conic.

173. To complete the analytical theory, it is proper to express the equation of

the orthotomic circle by means of the areal co-ordinates  $(u, v, w)$ . Writing for shortness  $a^2 + a'^2 - a''^2 = \tilde{a}$ , &c., and therefore

$$A^\circ = x^2 + y^2 - 2axz - 2a'yz - a'z^2, \text{ \&c.},$$

then if as before

$$u : v : w = \begin{vmatrix} x, y, z \\ b, b', 1 \\ c, c', 1 \end{vmatrix} : \begin{vmatrix} x, y, z \\ c, c', 1 \\ a, a', 1 \end{vmatrix} : \begin{vmatrix} x, y, z \\ a, a', 1 \\ b, b', 1 \end{vmatrix},$$

and therefore

$$x : y : z = au + bv + cw : a'u + b'v + c'w : u + v + w,$$

the equation of the orthotomic circle is

$$\begin{vmatrix} x - az, y - a'z, ax + a'y - a'z \\ x - bz, y - b'z, bx + b'y - b'z \\ x - cz, x - c'z, cx + c'y - c'z \end{vmatrix} = 0,$$

viz., throwing out the factor  $z$ , this is

$$u(ax + a'y - a'z) + v(bx + b'y - b'z) + w(cx + c'y - c'z) = 0,$$

or what is the same thing, it is

$$(au + bv + cw)x + (a'u + b'v + c'w)y - (a'u + b'v + c'w)z = 0,$$

viz., it is

$$(au + bv + cw)^2 + (a'u + b'v + c'w)^2 - (a'u + b'v + c'w)(u + v + w) = 0,$$

that is, substituting for  $a', b', c'$  their values, it is

$$\begin{aligned} & a''^2 u^2 + b''^2 v^2 + c''^2 w^2 \\ & + (b''^2 + c''^2 - (b - c)^2 - (b' - c')^2) vw \\ & + (c''^2 + a''^2 - (c - a)^2 - (c' - a')^2) wu \\ & + (a''^2 + b''^2 - (a - b)^2 - (a' - b')^2) uv = 0, \end{aligned}$$

and it may be observed that using for a moment  $\alpha, \beta, \gamma$  to denote the angles at which the three circles taken in pairs respectively intersect, then we have  $2b''c'' \cos \alpha = b''^2 + c''^2 - (b - c)^2 - (b' - c')^2$ , &c., and the equation of the orthotomic circle thus is

$$(1, 1, 1, \cos \alpha, \cos \beta, \cos \gamma) (a''u, b''v, c''w)^2 = 0.$$

174. We have in the foregoing enunciation of the theorem made use of the three given circles  $A, B, C$ , but it is clear that these are in fact *any* three circles in the series of the variable circle, and that the theorem may be otherwise stated thus:—

The envelope of a variable circle which has its centre in a given conic, and cuts at right angles a given circle, is a bi-circular quartic, such that its nodofoci are the foci of the conic.

*Properties depending on the relation between the Conic and Circle—*

Art. Nos. 175 to 177.

175. I refer to the conic of the theorem simply as the conic, and to the fixed circle simply as the circle, or when any ambiguity might otherwise arise, then as the orthotomic circle. This being so, I consider the effect in regard to the trizomal curve, of the various special relations which may exist between the circle and the conic.

If the conic touch the circle, the curve has a node at the point of contact.

If the conic has with the circle a contact of the second order, the curve has a cusp at the point of contact.

If the centre of the circle lie on an axis of the conic, then the four intersections lie in pairs symmetrically in regard to this axis, or the curve has this axis as an axis of symmetry.

If the conic has double contact with the circle (this implies that the centre of the circle is situate on an axis of the conic) the curve has a node at each of the points of contact, viz., it breaks up into two circles intersecting in these two points. The centres of the two circles respectively are the two foci of the conic, which foci lie on the axis in question. Observe that in the general case there are at each of the circular points at infinity two tangents, without any correspondence of the tangents of the one pair singly to those of the other pair, and there are thus four intersections, the four foci of the conic; in the present case, where the curve is a pair of circles, the two tangents to the same circle correspond to each other, and intersect in the two foci on the axis in question. The other two foci, or anti-points of these, are each of them the intersection of a tangent of the one circle by a tangent of the other circle.

If the conic has with the circle a contact of the third order (this implies that the circle is a circle of maximum or minimum curvature, at the extremity of an axis of the conic), then the curve has at this point a tacnode, viz., it breaks up into two circles touching each other and the conic at the point in question, and having their centres at the two foci situate on that axis of the conic respectively.

176. If the conic is a parabola, then the curve is a circular cubic having the four intersections of the parabola and circle for a set of concyclic foci, and having the focus of the parabola for *centre*. The like particular cases arise, viz.,

If the circle touch the parabola, the curve has a node at the point of contact.

If the circle has, with the parabola, a contact of the second order, the curve has a cusp at the point of contact.

If the centre of the circle is situate on the axis of the parabola, then the four intersections are situate in pairs symmetrically in regard to this axis, and the curve has this axis for an axis of symmetry.

If the circle has double contact with the parabola (which, of course, implies

that the centre lies on the axis), then the curve has a node at each of the points of contact, viz., the curve breaks up into a line and circle intersecting at the two points of contact, and the circle has its centre at the focus of the parabola.

If the circle has with the parabola a contact of the third order (this implies that the circle is the circle of maximum curvature, touching the parabola at its vertex), then the curve has a tacnode, viz., it breaks up into a line and circle touching each other and the parabola at the vertex, that is, the line is the tangent to the parabola at its vertex, and the circle is the circle having the focus of the parabola for its centre, and passing through the vertex, or what is the same thing, having its radius =  $\frac{1}{2}$  of the semi-latus rectum of the parabola.

177. If the conic be a circle, then the curve is a bi-circular quartic such that its four nodo-foci coincide together at the centre of the circle; viz., the curve is a cartesian having the centre of the conic for its cuspo-focus, that is, for the intersection of the cuspidal tangents of the cartesian. The intersections of the conic with the other circle, or say with the orthotomic circle, are a pair of non-axial foci of the cartesian; viz., the anti-points of these are two of the axial foci. The third axial focus is the centre of the orthotomic circle.

*Case of Double Contact, Casey's Equation in the Problem of Tactions—Art. No. 178.*

178. In the case where the conic has double contact with the orthotomic circle, then (as we have seen) the envelope of the variable circle is a pair of circles, each touching the variable circle; or, if we start with three given circles and a conic through their centres, then the envelope is a pair of circles, each of them touching each of the three given circles; that is, we have a solution of the problem of tactions. Multiplying by 2, the equation found *ante*, No. 173, for the variable circle, and then for the moment representing it by  $(a, b, c, f, g, h)(u, v, w)^2 = 0$ ; then attributing any signs at pleasure to the radicals  $\sqrt{a}, \sqrt{b}, \sqrt{c}$ , the equation of a conic through the centres of the given circles, and having double contact with the orthotomic circle, will be

$$(a, b, c, f, g, h)(u, v, w)^2 - (u\sqrt{a} + v\sqrt{b} + w\sqrt{c})^2 = 0,$$

viz., representing this equation as before by

$$lvw + mwu + nuw = 0,$$

we have

$$l : m : n = f - \sqrt{bc} : g - \sqrt{ca} : h - \sqrt{ab},$$

that is, substituting for  $a, b, c, f, g, h$  their values, and taking, for instance,  $a, b, c = a''\sqrt{2}, b''\sqrt{2}, c''\sqrt{2}$ , we find

$$\begin{aligned} l : m : n &= (b'' - c'')^2 - (b - c)^2 - (b' - c')^2 \\ &: (c'' - a'')^2 - (c - a)^2 - (c' - a')^2 \\ &: (a'' - b'')^2 - (a - b)^2 - (a' - b')^2, \end{aligned}$$

that is,  $l, m, n$  are as the squares of the tangential distances (direct) of the three circles taken in pairs, and this being so, the equation of a pair of circles touching each of the three given circles is  $\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} = 0$ . It is clear that, instead of taking the three direct tangential distances, we may take one direct tangential distance and two inverse tangential distances, viz., the tangential distances corresponding to any three centres of similitude which lie in a line; we have thus in all the equations of four pairs of circles, viz., of the eight circles which touch the three given circles. This is Casey's theorem in the problem of tactions.

*The Intersections of the Conic and Orthotomic Circle are a set of four Concyelic Foci—*

Art. No. 179.

179. The conic of centres intersects the orthotomic circle in four points, and for each of these the radius of the variable circle is  $= 0$ , that is, the points in question are a set of four concyclic foci ( $A, B, C, D$ ) of the curve. Regarding the foci as given, the circle which contains them is of course the orthotomic circle; and there are a singly infinite series of curves, viz., these correspond to the singly infinite series of conics which can be drawn through the given foci. As for a given curve there are four sets of concyclic foci, there are four different constructions for the curve, viz., the orthotomic circle may be any one of the four circles  $O, R, S, T$ , which contain the four sets of concyclic foci respectively; and the conic of centres is a conic through the corresponding set of four concyclic foci. We have thus four conics, but the foci of each of them coincide with the nodofoci of the curve, that is, the conics are confocal; that such confocal conics exist has been shown, *ante*, Nos. 78 to 80.

*Remark as to the Construction of the Symmetrical Curve—Art. Nos. 180 and 181.*

180. It is to be observed that in applying as above the theorem of the variable zomal to the construction of a symmetrical curve, the orthotomic circle made use of was one of the circles  $R, S, T$ , not the circle  $O$ , which is in this case the axis; in fact, we should then have the conic and the orthotomic circle each of them coinciding with the axis. And the variable circle, *quà* circle having its centre on the axis, cuts the axis at right angles whatever the radius may be; that is, the variable circle is no longer sufficiently determined by the theorem. The curve may nevertheless be constructed as the envelope of a variable circle having its centre on the axis; viz., writing  $A^\circ = (x - az)^2 + y^2 - a''^2 z^2$ , &c., and starting with the form

$$\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} = 0,$$

then recurring to the demonstration of the theorem (*ante*, No. 47), the equation of the variable circle is  $aA^\circ + bB^\circ + cC^\circ = 0$ , where  $a, b, c$  are any quantities

satisfying  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ , or, what is the same thing, taking  $q$  an arbitrary parameter, and writing  $\frac{l}{a} = 1 + q$ ,  $\frac{m}{b} = 1 - q$ ,  $\frac{n}{c} = -2$ , the equation of the variable circle is

$$\frac{1}{1+q} lA^\circ + \frac{1}{1-q} mB^\circ - \frac{1}{2} nC^\circ = 0.$$

Compare Nos. 118–123 for the like mode of construction of a conic; but it is proper to consider this in a somewhat different form.

181. Assume that the equation of the variable circle is

$$D^\circ = (x - dz)^2 + y^2 - d''^2 z^2 = 0;$$

we have therefore identically

$$aA^\circ + bB^\circ + cC^\circ + dD^\circ = 0,$$

viz., this gives

$$\begin{aligned} a + b + c &= -d, \\ aa + bb + cc &= -dd, \\ a(a^2 - d''^2) + b(b^2 - b''^2) + c(c^2 - c''^2) &= -d(d^2 - d''^2), \end{aligned}$$

and from these equations we obtain  $a, b, c$  equal respectively to given multiples of  $d$ ; substituting these values in the equation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ ,  $d$  divides out, and we have an equation involving the parameters of the given circles, and also  $d, d''$ , the parameters of the variable circle; viz., an equation determining  $d''$ , the radius of the variable circle, in terms of  $d$ , the co-ordinate of its centre. I consider in particular the case where the given circles are points; that is, where the given equation is

$$\sqrt{lA} + \sqrt{mB} + \sqrt{nC} = 0.$$

The equations here are

$$\begin{aligned} a + b + c &= -d \\ aa + bb + cc &= -dd \\ aa^2 + bb^2 + cc^2 &= -d(d^2 - d''^2), \end{aligned}$$

and from these we obtain

$$\begin{aligned} a(a-b)(a-c) &= -d((d-b)(d-c) - d''^2) \\ b(b-c)(b-a) &= -d((d-c)(d-a) - d''^2) \\ c(c-a)(c-b) &= -d((d-a)(d-b) - d''^2), \end{aligned}$$

so that the equation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$  becomes

$$\frac{l(a-b)(a-c)}{(d-b)(d-c) - d''^2} + \frac{m(b-c)(b-a)}{(d-c)(d-a) - d''^2} + \frac{n(c-a)(c-b)}{(d-a)(d-b) - d''^2} = 0,$$

or, as this is more conveniently written

$$\frac{l}{b-c} \frac{1}{(d-b)(d-c)-d'^2} + \frac{m}{c-a} \frac{1}{(d-c)(d-a)-d'^2} + \frac{n}{a-b} \frac{1}{(d-a)(d-b)-d'^2} = 0,$$

viz., considering  $d, d''$  as the abscissa and ordinate of a point on a curve, and representing them by  $x, y$  respectively, the equation of this curve is

$$\frac{l}{b-c} \frac{1}{(x-b)(x-c)-y^2} + \frac{m}{c-a} \frac{1}{(x-c)(x-a)-y^2} + \frac{n}{a-b} \frac{1}{(x-a)(x-b)-y^2} = 0,$$

which is a certain quartic curve; and we have the original curve

$$\sqrt{lA} + \sqrt{mB} + \sqrt{nC} = 0,$$

as the envelope of a variable circle having for its diameter the double ordinate of this quartic curve.

Write for shortness  $\frac{l}{b-c}, \frac{m}{c-a}, \frac{n}{a-b} = L, M, N$  respectively, then the equation of the quartic curve may be written

$$\Sigma L [(x-a)^2 (x-b)(x-c) - y^2(x-a)(2x-b-c) + y^4] = 0,$$

viz., this is

$$\begin{aligned} \Sigma L [ & x(x-a)(x-b)(x-c) \\ & - y^2(2x^2 - (a+b+c)x + (ab+ac+bc)) + y^4 \\ & - a(x-a)(x-b)(x-c) + y^2(ax+bc) ] = 0, \end{aligned}$$

or what is the same thing, the equation is

$$\begin{aligned} (L+M+N) [ & x(x-a)(x-b)(x-c) - y^2(2x^2 - (a+b+c)x + ab+ac+bc) + y^4 ] \\ & - (La+Mb+Nc)(x-a)(x-b)(x-c) \\ & + y^2 \{ La+Mb+Nc \} x + Lbc + Mca + Nab \} = 0. \end{aligned}$$

In the particular case where  $L + M + N = 0$ , that is, where

$$\frac{l}{b-c} + \frac{m}{c-a} + \frac{n}{a-b} = 0,$$

the quartic curve becomes a cubic, viz., putting for shortness

$$- \delta = \frac{Lbc + Mca + Nab}{La + Mb + Nc},$$

the equation of the cubic is

$$y^2 = \frac{(x-a)(x-b)(x-c)}{x-\delta},$$

viz., this is a cubic curve having three real asymptotes, and a diameter at right angles to one of the asymptotes, and at the inclinations  $+45^\circ, -45^\circ$  to the other two asymptotes respectively—say that it is a “rectangular” cubic. The relation

$\frac{l}{b-c} + \frac{m}{c-a} + \frac{n}{a-b} = 0$  implies that the curve  $\sqrt{lA} + \sqrt{mB} + \sqrt{nC} = 0$  is a cartesian, and we have thus the theorem that the envelope of a variable circle having for diameter the double ordinate of a rectangular cubic is a cartesian.

I remark that using a particular origin, and writing the equation of the rectangular cubic in the form  $y^2 = x^2 - 2mx + a + \frac{2A}{x}$ , the equation of the variable circle is

$$(x - d)^2 + y^2 = d^2 - 2md + a + \frac{2A}{d},$$

that is

$$x^2 + y^2 - a - 2d(x - m) - \frac{2A}{d} = 0,$$

where  $d$  is the variable parameter. Forming the derived equation in regard to  $d$ , we have

$$x - m = \frac{A}{d^2},$$

and thence

$$x^2 + y^2 - a = \frac{4A}{d},$$

$$(x^2 + y^2 - a)^2 = \frac{16A^2}{d^2} = 16A(x - m),$$

that is, the equation of the envelope is  $(x^2 + y^2 - a)^2 = 16A(x - m) = 0$ , which is a known form of the equation of a Cartesian.

*Focal Formulæ for the General Curve—Art. Nos. 182 and 183.*

182. Considering any three circles centres  $A, B, C$ , and taking  $A^\circ$ , &c., to denote as usual, let the equation of the curve be

$$\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} = 0;$$

then considering a fourth circle, centre  $D$ , a position of the variable circle, and having therefore the same orthotomic circle with the given circles, so that as before

$$aA^\circ + bB^\circ + cC^\circ + dD^\circ = 0,$$

the formulæ No. 47 (changing only  $U, V, W, T$  into  $A^\circ, B^\circ, C^\circ, D^\circ$ ) are at once applicable to express the equation of the curve in terms of any three of the four circles  $A, B, C, D$ .

In particular, the circles may reduce themselves to the four points  $A, B, C, D$ , a set of concyclic foci, and here, the equation being originally given in the form

$$\sqrt{lA} + \sqrt{mB} + \sqrt{nC} = 0,$$

the same formulæ are applicable to express the equation in terms of any three of the four foci.

183. It is to be observed that in this case if the positions of the four foci are given by means of the circular co-ordinates  $\left(a, \frac{1}{a}\right)$  &c., which refer to the centre of the circle  $ABCD$  as origin, and with the radius of this circle taken as unity, then the values of  $a, b, c, d$  (*ante*, No. 90), are given in the form adapted to the formulæ of No. 49, viz., we have

$$a : b : c : d = \alpha(\beta\gamma\delta) : -\beta\gamma\delta\alpha : \gamma(\delta\alpha\beta) : -\delta(\alpha\beta\gamma),$$

where  $(\beta\gamma\delta) = (\beta - \gamma)(\gamma - \delta)(\delta - \beta)$ , &c. The relation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ , putting therein  $l : m : n = \rho\alpha(\beta - \gamma)^2 : \sigma\beta(\gamma - \alpha)^2 : \tau\gamma(\alpha - \beta)^2$ , (or, what is the same thing, taking the equation of the curve to be given in the form  $(\beta - \gamma)\sqrt{\rho\alpha\mathbf{A}} + (\gamma - \alpha)\sqrt{\sigma\beta\mathbf{B}} + (\alpha - \beta)\sqrt{\tau\gamma\mathbf{C}} = 0$ ), becomes

$$\rho(\beta - \gamma)(\alpha - \delta) + \sigma(\gamma - \alpha)(\beta - \delta) + \tau(\alpha - \beta)(\gamma - \delta) = 0,$$

viz., this equation, considering  $\rho, \sigma, \tau, \alpha, \beta, \gamma$  as given, determines the position of the fourth focus  $D$ , or when  $A, B, C, D$  are given, it is the relation which must exist between  $\rho, \sigma, \tau$ ; and the four forms of the equation are

$$\left( \begin{array}{cccc} \sqrt{\tau}(\delta - \gamma), & \sqrt{\sigma}(\beta - \delta), & \sqrt{\rho}(\gamma - \beta), & \sqrt{\delta\mathbf{D}} \\ \sqrt{\tau}(\gamma - \delta), & \sqrt{\rho}(\delta - \alpha), & \sqrt{\sigma}(\alpha - \gamma), & \sqrt{\delta\mathbf{D}} \\ \sqrt{\sigma}(\delta - \beta), & \sqrt{\rho}(\alpha - \delta), & \sqrt{\tau}(\beta - \alpha), & \sqrt{\delta\mathbf{D}} \\ \sqrt{\rho}(\beta - \gamma), & \sqrt{\sigma}(\gamma - \alpha), & \sqrt{\tau}(\alpha - \beta), & \sqrt{\delta\mathbf{D}} \end{array} \right) \left( \sqrt{a\mathbf{A}}, \sqrt{b\mathbf{B}}, \sqrt{c\mathbf{C}}, \sqrt{d\mathbf{D}} \right) = 0,$$

viz., the curve is represented by means of any one of these four equations involving each of them three out of the four given foci  $A, B, C, D$ .

*Case of the Circular Cubic—Art. Nos. 184 and 185.*

184. In the case of a circular cubic, we must have

$$\begin{aligned} \rho(\beta - \gamma)(\alpha - \delta) + \sigma(\gamma - \alpha)(\beta - \delta) + \tau(\alpha - \beta)(\gamma - \delta) &= 0, \\ \sqrt{\rho}(\beta - \gamma) + \sqrt{\sigma}(\gamma - \alpha) + \sqrt{\tau}(\alpha - \beta) &= 0, \end{aligned}$$

which, when the foci  $A, B, C, D$  are given, determine the values of  $\rho : \sigma : \tau$  in order that the curve may be a circular cubic. We see at once that there are two sets of values, and consequently two circular cubics having each of them the given points  $A, B, C, D$  for a set of concyclic foci. The two systems may be written

$$\sqrt{\rho} : \sqrt{\sigma} : \sqrt{\tau} = \sqrt{a\delta} - \sqrt{\beta\gamma} : \sqrt{\beta\delta} - \sqrt{\gamma\alpha} : \sqrt{\gamma\delta} - \sqrt{\alpha\beta},$$

viz., it being understood that  $\sqrt{a\delta}$  means  $\sqrt{a}\sqrt{\delta}$ , &c., then, according as  $\sqrt{\delta}$  has one or other of its two opposite values, we have one or other of the two

systems of values of  $\rho : \sigma : \tau$ . To verify this, observe that writing the equation under the form

$$\sqrt{a\rho} : \sqrt{\beta\sigma} : \sqrt{\gamma\tau} = a\sqrt{\delta} - \sqrt{a\beta\gamma} : \beta\sqrt{\delta} - \sqrt{a\beta\gamma} : \gamma\sqrt{\delta} - \sqrt{a\beta\gamma},$$

the second equation is verified; and that writing them under the form

$$\rho : \sigma : \tau = -(\beta + \gamma)(a + \delta) + M : -(\gamma + \alpha)(\beta + \delta) + M : -(a + \beta)(\gamma + \delta) + M,$$

where

$$M = \beta\gamma + \alpha\delta + \gamma\alpha + \beta\delta + \alpha\beta + \gamma\delta - 2\sqrt{a\beta\gamma\delta},$$

the second equation is also verified.

185. If we assume for a moment  $a = \cos a + i \sin a = e^{ia}$ , &c., viz., if  $a, b, c, d$  be the inclinations to any fixed line of the radii through  $A, B, C, D$  respectively, then we have

$$\begin{aligned} \sqrt{a\delta} \pm \sqrt{\beta\gamma} &= e^{\frac{1}{2}(a+b+c+d)i} \left\{ e^{\frac{1}{2}(a+d-b-c)i} \pm e^{-\frac{1}{2}(a+d-b-c)i} \right\}, \\ \sqrt{a}(\beta - \gamma) &= e^{\frac{1}{2}(a+b+c)i} \left\{ e^{\frac{1}{2}(b-c)i} - e^{-\frac{1}{2}(b-c)i} \right\}, \end{aligned}$$

and thence

$$\begin{aligned} \sqrt{a\rho}(\beta - \gamma) : \sqrt{\beta\sigma}(\gamma - \alpha) : \sqrt{\gamma\tau}(\alpha - \beta) &= \cos \frac{1}{4}(a + d - b - c) \sin \frac{1}{2}(b - c) \\ &: \cos \frac{1}{4}(b + d - c - a) \sin \frac{1}{2}(c - a) \\ &: \cos \frac{1}{4}(a + d - a - b) \sin \frac{1}{2}(a - b); \end{aligned}$$

or else

$$\begin{aligned} &= \sin \frac{1}{4}(a + d - b - c) \sin \frac{1}{2}(b - c) \\ &: \sin \frac{1}{4}(b + d - c - a) \sin \frac{1}{2}(c - a) \\ &: \sin \frac{1}{4}(c + d - a - b) \sin \frac{1}{2}(a - b). \end{aligned}$$

Putting in these formulæ,

$$\begin{aligned} \frac{1}{4}(a - b - c) &= A, & \text{then we have} & & B - C &= \frac{1}{2}(b - c), \\ \frac{1}{4}(b - c - a) &= B, & & & C - A &= \frac{1}{2}(c - a), \\ \frac{1}{4}(c - a - b) &= C, & & & A - B &= \frac{1}{2}(a - b), \end{aligned}$$

and for either set of values the verification of the relation

$$\sqrt{a\rho}(\beta - \gamma) + \sqrt{\beta\sigma}(\gamma - \alpha) + \sqrt{\gamma\tau}(\alpha - \beta) = 0,$$

will depend on the two identical equations

$$\begin{aligned} \sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B) &= 0, \\ \cos A \sin(B - C) + \cos B \sin(C - A) + \cos C \sin(A - B) &= 0: \end{aligned}$$

although the foregoing solution for the case of a circular cubic is the most elegant one, I will presently return to the question and give the solution in a different form.

*Focal Formulæ for the Symmetrical Curve—Art. No. 186.*

186. In the symmetrical case, where the foci  $A, B, C, D$  are on a line, then if, as usual,  $a, b, c, d$  denote the distances from a fixed point, we have the ex-

pressions of (a, b, c, d) in a form adapted to the formulæ of No. 49, viz.,

$$a : b : c : d = (b-c)(c-d)(d-b) : -(c-d)(d-a)(a-c) : (d-a)(a-b)(b-d) : -(a-b)(b-c)(c-a),$$

so that, assuming

$$l : m : n = \rho(b-c)^2 : \sigma(c-a)^2 : \tau(a-b)^2,$$

the equation

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0,$$

becomes

$$\rho(b-c)(a-d) + \sigma(c-a)(b-d) + \tau(a-b)(c-d) = 0,$$

and the equation of the curve may be presented under any one of the four forms

$$\left( \begin{array}{cccc} \sqrt{\tau}(d-c), & \sqrt{\sigma}(b-d), & \sqrt{\rho}(c-b), & \\ \sqrt{\tau}(c-d), & \sqrt{\sigma}(d-a), & \sqrt{\rho}(a-c), & \\ \sqrt{\sigma}(d-b), & \sqrt{\rho}(a-d), & \sqrt{\tau}(b-a), & \\ \sqrt{\rho}(b-c), & \sqrt{\sigma}(c-a), & \sqrt{\tau}(a-b), & \end{array} \right) (\sqrt{A}, \sqrt{B}, \sqrt{C}, \sqrt{D}) = 0.$$

*Case of the Symmetrical Circular Cubic—Art. No. 187.*

187. For a circular cubic we must have

$$\begin{aligned} \rho(b-c)(a-d) + \sigma(c-a)(b-d) + \tau(a-b)(c-d) &= 0 \\ \sqrt{\rho}(b-c) + \sqrt{\sigma}(c-a) + \sqrt{\tau}(a-b) &= 0. \end{aligned}$$

These equations give  $\sqrt{\rho} : \sqrt{\sigma} : \sqrt{\tau} = 1 : 1 : 1$  (values which obviously satisfy the two equations), or else

$$\sqrt{\rho} : \sqrt{\sigma} : \sqrt{\tau} = a + d - b - c : b + d - c - a : c + d - a - b.$$

In fact, these values obviously satisfy the second equation; and to see that they satisfy the first equation, we have only to write them under the form

$$\rho : \sigma : \tau = M - 4(b+c)(a+d) : M - 4(c+a)(b+d) : M - 4(a+b)(c+d),$$

where  $M = (a+b+c+d)^2$ . The first set gives for the curve

$$(b-c)\sqrt{A} + (c-a)\sqrt{B} + (a-b)\sqrt{C} = 0,$$

but this contains the line  $z = 0$  not once only, but twice; it in fact is ( $y^2 = 0$ ), the axis taken twice; the only proper cubic with the foci  $A, B, C, D$  in *lineâ* is therefore

$$(b-c)(a+d-b-c)\sqrt{A} + (c-a)(b+d-c-a)\sqrt{B} + (a-b)(c+d-a-b)\sqrt{C} = 0,$$

the equation of which is, of course, expressible in each of the other three forms.

*Case of the General Circular Cubic—Art. Nos. 188 to 192.*

188. Returning to the general case of the circular cubic, the lines  $BC, AD$

meet in  $R$ , and if we denote by  $a_1, b_1, c_1, d_1$ , the distances from  $R$  of the four points respectively, so that  $b_1 c_1 = a_1 d_1 = \text{rad.}^2 R$ , then observing that  $a, b, c, d$  are proportional to the triangles  $BCD, CDA, DAB, ABC$ , with signs such that  $a + b + c + d = 0$ , we find

$$a : b : c : d = -d_1(b_1 - c_1) : c_1(a_1 - d_1) : -b_1(a_1 - d_1) : a_1(b_1 - c_1);$$

and this being so, the equations  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ ,  $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$ , give two systems of values of  $\sqrt{l} : \sqrt{m} : \sqrt{n}$ , viz., these are

$$\sqrt{l} : \sqrt{m} : \sqrt{n} = b_1 - c_1 : c_1 - a_1 : a_1 - b_1,$$

and

$$= b_1 - c_1 : c_1 + a_1 : -a_1 - b_1.$$

(To verify this, observe that for the first set we have

$$\begin{aligned} \frac{l}{a} + \frac{m}{b} + \frac{n}{c} &= \frac{(b_1 - c_1)^2}{-d_1(b_1 - c_1)} + \frac{(c_1 - a_1)^2}{c_1(a_1 - d_1)} + \frac{(a_1 - b_1)^2}{-b_1(a_1 - d_1)} \\ &= \frac{b_1 - c_1}{-d_1} + \frac{1}{a_1 - d_1} \left( c_1 + \frac{a_1^2}{c_1} - b_1 - \frac{a_1^2}{b_1} \right) \\ &= \frac{b_1 - c_1}{-d_1} + \frac{b_1 - c_1}{a_1 - d_1} \left( \frac{a_1^2}{b_1 c_1} - 1 \right) \\ &= -\frac{b_1 - c_1}{d_1} + \frac{b_1 - c_1}{a_1 - d_1} \left( \frac{a_1}{d_1} - 1 \right) = 0; \end{aligned}$$

and the like as regards the second set).

189. These values of  $\sqrt{l} : \sqrt{m} : \sqrt{n}$  give the equations of the two circular cubics with the foci  $(A, B, C, D)$ , the equation of each of them under a fourfold form, viz., we have

$$\begin{pmatrix} \cdot & , & d_1 - c_1 & , & b_1 - d_1 & , & c_1 - b_1 \\ c_1 - d_1 & , & \cdot & , & d_1 - a_1 & , & a_1 - c_1 \\ d_1 - b_1 & , & a_1 - d_1 & , & b_1 - a_1 & , & b_1 - a_1 \\ b_1 - c_1 & , & c_1 - a_1 & , & a_1 - b_1 & , & \cdot \end{pmatrix} (\sqrt{A}, \sqrt{B}, \sqrt{C}, \sqrt{D}) = 0$$

(first curve),

and

$$\begin{pmatrix} \cdot & , & -c_1 - d_1 & , & d_1 + b_1 & , & -b_1 + c_1 \\ d_1 + c_1 & , & \cdot & , & a_1 - d_1 & , & c_1 - a_1 \\ -b_1 - d_1 & , & d_1 - a_1 & , & \cdot & , & a_1 + b_1 \\ b_1 - c_1 & , & c_1 + a_1 & , & -a_1 - b_1 & , & \cdot \end{pmatrix} (\sqrt{A}, \sqrt{B}, \sqrt{C}, \sqrt{D}) = 0$$

(second curve).

190. Similarly  $CA$  and  $BD$  meet in  $S$ , and if we denote by  $a_2, b_2, c_2, d_2$  the distances from  $S$  of the four points respectively, so that  $c_2 a_2 = b_2 d_2 = \text{rad.}^2 S$  (observe that if as usual  $A, B, C, D$  are taken in order on the circle  $O$ , then  $A, C$  are on opposite sides of  $S$ , and similarly  $B, D$  are on opposite sides of  $S$ , so that

taking  $a_2, b_2$  positive  $c_2, d_2$  will be negative) we have

$$a : b : c : d = c_2(b_2 - d_2) : d_2(c_2 - a_2) : -a_2(b_2 - d_2) : -b_2(c_2 - a_2),$$

and then the equations  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ ,  $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$ , are satisfied by the two sets of values

$$\begin{aligned} \sqrt{l} \sqrt{m} : \sqrt{n} &= b_2 - c_2 : c_2 - a_2 : a_2 - b_2, \\ &= -b_2 - c_2 : c_2 - a_2 : a_2 + b_2, \end{aligned}$$

and

and we have the equations of the same two cubic curves, each equation under a fourfold form, viz., these are

$$\left( \begin{array}{cccc} \cdot, & -c_2 + d_2, & -d_2 + b_2, & -b_2 + c_2 \\ c_2 - d_2, & \cdot, & d_2 - a_2, & -c_2 + a_2 \\ -b_2 + d_2, & a_2 - d_2, & \cdot, & -a_2 + b_2 \\ b_2 - c_2, & c_2 - a_2, & a_2 - b_2, & \cdot \end{array} \right) (\sqrt{A}, \sqrt{B}, \sqrt{C}, \sqrt{D}) = 0$$

(first curve),

and

$$\left( \begin{array}{cccc} \cdot, & c_2 + d_2, & -d_2 + b_2, & -b_2 - c_2 \\ -d_2 + c_2, & \cdot, & a_2 + d_2, & c_2 - a_2 \\ -b_2 + d_2, & -d_2 - a_2, & \cdot, & a_2 + b_2 \\ b_2 + c_2, & -c_2 + a_2, & -a_2 - b_2, & \cdot \end{array} \right) (\sqrt{A}, \sqrt{B}, \sqrt{C}, \sqrt{D}) = 0.$$

(second curve).

191. And again  $AB$  and  $CD$  meet in  $T$ , and denoting by  $a_3, b_3, c_3, d_3$  the distances from  $T$  of the four points respectively, so that  $a_3 b_3 = c_3 d_3 = \text{rad.}^2 T$ , we have

$$a : b : c : d = b_3(c_3 - d_3) : -a_3(c_3 - d_3) : -d_3(a_3 - b_3) : c_3(a_3 - b_3).$$

The equations  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ ,  $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$ , then give for  $\sqrt{l}, \sqrt{m}, \sqrt{n}$  two sets of values, viz., these are

$$\begin{aligned} \sqrt{l} : \sqrt{m} : \sqrt{n} &= b_3 - c_3 : c_3 - a_3 : a_3 - b_3, \\ &= b_3 + c_3 : -c_3 - a_3 : a_3 - b_3; \end{aligned}$$

and

and we again obtain the equations of the two cubics, each equation under a fourfold form, viz., these are

$$\left( \begin{array}{cccc} \cdot, & -c_3 + d_3, & -d_3 + b_3, & c_3 - b_3 \\ -d_3 + c_3, & \cdot, & -a_3 + d_3, & a_3 - c_3 \\ -b_3 + d_3, & -d_3 + a_3, & \cdot, & b_3 - a_3 \\ b_3 - c_3, & c_3 - a_3, & a_3 - b_3, & \cdot \end{array} \right) (\sqrt{A}, \sqrt{B}, \sqrt{C}, \sqrt{D}) = 0,$$

and

$$\left( \begin{array}{cccc} \cdot, & c_3 - d_3, & d_3 + b_3, & -c_3 - b_3 \\ d_3 - c_3, & \cdot, & -a_3 - d_3, & a_3 + c_3 \\ b_3 - d_3, & d_3 + a_3, & \cdot, & b_3 - a_3 \\ b_3 + c_3, & -c_3 - a_3, & a_3 - b_3, & \cdot \end{array} \right) (\sqrt{A}, \sqrt{B}, \sqrt{C}, \sqrt{D}) = 0.$$

192. The three systems have been obtained independently, but they may of course be derived each from any other of them: to show how this is, recollecting that we have

$$\begin{aligned} RA, RB, RC, RD &= a_1, b_1, c_1, d_1, \\ SA, SB, SC, SD &= a_2, b_2, -c_2, -d_2, \\ TA, TB, TC, TD &= a_3, b_3, c_3, d_3; \end{aligned}$$

then to compare

$$(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2);$$

similar triangles

$$\begin{aligned} SBC \text{ give } b_1 - c_1 &: -c_2 : b_2, \\ SAD &= a_1 - d_1 : -d_2 : a_2, \end{aligned}$$

and similar triangles

$$\begin{aligned} RAC \text{ give } a_2 - c_2 &: c_1 : a_1, \\ RBD &= b_2 - d_2 : d_1 : b_1; \end{aligned}$$

using these equations to determine the ratios of  $a_2, b_2, c_2, d_2$  we have

$$\frac{a_2 - c_2}{b_2 - d_2} = \frac{c_1}{d_1}, \text{ or } d_1 a_2 - d_1 c_2 - c_1 b_2 + c_1 d_2 = 0;$$

that is

$$b_2 \left\{ -c_1 + d_1 \frac{a_1 - d_1}{b_1 - c_1} \right\} + c_2 \left\{ -d_1 + c_1 \frac{a_1 - d_1}{b_1 - c_1} \right\} = 0;$$

and hence

$$b_2(-b_1 c_1 + c_1^2 + a_1 d_1 - d_1^2) + c_2(-b_1 d_1 + c_1 d_1 + a_1 c_1 - c_1 d_1) = 0,$$

that is

$$b_2(c_1^2 - d_1^2) + c_2(a_1 c_1 - b_1 d_1) = 0,$$

but

$$a_1 c_1 - b_1 d_1 = \frac{b_1}{d_1}(c_1^2 - d_1^2),$$

or the equation gives  $b_2 + \frac{b_1}{d_1} c_2 = 0$ , or say  $b_2 : c_2 = b_1 : -d_1$ , and this with

$\frac{b_1 - c_1}{a_1 - d_1} = \frac{c_2}{d_2} = \frac{b_2}{a_2}$ , gives all the ratios, or we have

$$a_2 : b_2 : c_2 : d_2 = b_1(a_1 - d_1) : b_1(b_1 - c_1) : -d_1(a_1 - d_1) : -d_1(b_1 - c_1).$$

We have then for example

$$b_2 - c_2 : c_2 - a_2 : a_2 - b_2 = b_1 - c_1 : c_1 - a_1 : a_1 - b_1; \text{ \&c.,}$$

showing the identity of the forms in  $(a_1, b_1, c_1, d_1)$  and  $(a_2, b_2, c_2, d_2)$ .

*Transformation to a New Set of Concyelic Foci.*—Art. No. 193.

193. Consider the equation

$$\sqrt{lA} + \sqrt{mB} + \sqrt{nC} = 0,$$

which refers to the foci  $A, B, C$ , and taking  $D$  the fourth concyclic focus, let

$(A_1, D_1)$  be the antipoints of  $(A, D)$  and  $(B_1, C_1)$  the antipoints of  $(B, C)$ ; so that  $(A_1, B_1, C_1, D_1)$  are another set of concyclic foci. We have  $B_1 \cdot C_1 = B \cdot C$ , and it appears, *ante* No. 104, that we can find  $l_1, m_1, n_1$ , such that identically

$$-lA + mB + nC = -l_1A_1 + m_1B_1 + n_1C_1$$

and that  $m_1 n_1 = mn$ . The equation of the curve gives

$$-lA + mB + nC + 2\sqrt{mnBC} = 0,$$

we have therefore

$$-l_1A_1 + m_1B_1 + n_1C_1 + 2\sqrt{m_1n_1B_1C_1} = 0,$$

that is,

$$\sqrt{l_1A_1} + \sqrt{m_1B_1} + \sqrt{n_1C_1} = 0,$$

viz., this is the equation of the curve expressed in terms of the concyclic foci  $A_1, B_1, C_1$ .

*The Tetrizomal Curve, Decomposable or Indecomposable.*—Art. No. 194.

194. I consider the tetrizomal curve

$$\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} + \sqrt{pD^\circ} = 0,$$

where the zomals are circles described about any given points  $A, B, C, D$  as centres.

There is not, in general, any identical equation  $aA^\circ + bB^\circ + cC^\circ + dD^\circ = 0$ , but when such relation exists, and when we have also  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$ , then the curve breaks up into two trizomals. When the conditions in question do not subsist, the curve is indecomposable. But there may exist between  $l, m, n, p$  relations in virtue of which a branch or branches ideally contain ( $z^a = 0$ ) the line infinity a certain number of times, and which thus cause a depression in the order of the curve. The several cases are as follows:—

*Cases of the Indecomposable Curve.*—Art. No. 195.

195. I. The general case;  $l, m, n, p$  not subjected to any condition. The curve is here of the order = 8; it has a quadruple point at each of the points  $I, J$  (and there is consequently no other point at infinity); it is touched four times by each of the circles  $A, B, C, D$ ; and it has six nodes, viz., these are the intersections of the pairs of circles

$$\begin{aligned} \sqrt{mB^\circ} + \sqrt{nC^\circ} &= 0, & \sqrt{lA^\circ} + \sqrt{pD^\circ} &= 0, \\ \sqrt{nC^\circ} + \sqrt{lA^\circ} &= 0, & \sqrt{mB^\circ} + \sqrt{pD^\circ} &= 0, \\ \sqrt{lA^\circ} + \sqrt{mB^\circ} &= 0, & \sqrt{nC^\circ} + \sqrt{pD^\circ} &= 0; \end{aligned}$$

the number of dps. is  $6 + 2 \cdot 6 = 18$ , and there are no cusps, hence the class is = 20, and the deficiency is = 3.

II. We may have

$$\sqrt{l} + \sqrt{m} + \sqrt{n} + \sqrt{p} = 0 ;$$

there is in this case a single branch ideally containing ( $z = 0$ ) the line infinity ; the order is = 7. Each of the points  $I, J$  is a triple point, there is consequently one other point at infinity ; viz., this is a real point, or the curve has a real asymptote. There are 6 nodes as before ; dps. are  $6 + 2 \cdot 3, = 12$  ; class = 18, deficiency = 3.

III. We may have

$$\sqrt{l} + \sqrt{m} = 0, \sqrt{n} + \sqrt{p} = 0 ;$$

there are then two branches each ideally containing ( $z = 0$ ) the line infinity ; the order is = 6. Each of the points  $I, J$  is a double point, and there are therefore two more points at infinity. These may be real or imaginary ; viz., the curve may have (besides the asymptotes at  $I, J$ ) two real or imaginary asymptotes. The circles  $\sqrt{lA} + \sqrt{mB} = 0, \sqrt{nC} + \sqrt{pD} = 0$ , each contain ( $z = 0$ ) the line infinity, or they reduce themselves to two lines, so that in place of two nodes we have a single node at the intersection of these lines ; number of nodes is = 5. Hence dps. are  $5 + 2 \cdot 1, = 7$ . Class is = 16, deficiency = 3.

IV. We may have

$$\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p} = a : b : c : d$$

there is here a single branch containing ( $z^2 = 0$ ) the line infinity twice ; the order is = 6. Each of the points  $I, J$  is a double point, and there are therefore two more points at infinity, that is (besides the asymptotes at  $I, J$ ), there are two (real or imaginary) asymptotes. The number of nodes, as in the general case, is = 6. Hence dps. are  $6 + 2 \cdot 1, = 8$  ; class is = 14 ; deficiency = 2.

I notice the included particular case where the circles reduce themselves to their centres ; viz., we have here the curve

$$a\sqrt{A} + b\sqrt{B} + c\sqrt{C} + d\sqrt{D} = 0,$$

which (see *ante* No. 93) is in fact the curve which is the locus of the foci of the conics which pass through the four points  $A, B, C, D$ . It is at present assumed that the four points are not a circle ; this case will be considered *post* No. 199. If we have  $BC, AD$  meeting in  $R$  ;  $CA, BD$  in  $S$ , and  $AB, CD$  in  $T$ , then these points  $R, S, T$  are three of the six nodes. In fact, writing down the equations of the two circles

$$b\sqrt{B} + c\sqrt{C} = 0, a\sqrt{A} + d\sqrt{D} = 0,$$

and observing that when the current point is taken at  $R$ , we have  $B : C = \overline{RB}^2 : \overline{RC}^2 = (BAD)^2 : (CAD)^2 = c^2 : b^2$ , and similarly  $A : D = \overline{RA}^2 : \overline{RD}^2 = (ABC)^2 : (DBC)^2 = d^2 : a^2$ , we see that each of the two circles passes through

the point  $R$ , or this point is a node. Similarly, the points  $S$  and  $T$  are each of them a node.

V. If 
$$\sqrt{l} = \sqrt{m} = \sqrt{n} = \sqrt{p},$$

there are here three branches, each ideally containing ( $z = 0$ ) the line infinity; the order is thus = 5. Each of the points  $I, J$  is an ordinary point on the curve; there are besides at infinity three points, all real, or one real and two imaginary; that is (besides the asymptotes at  $I, J$ ) there are three asymptotes, all real, or one real and two imaginary. Each of the circles  $\sqrt{A} + \sqrt{B} = 0$ , &c., contains the line infinity, and is thus reduced to a line; the number of nodes is therefore = 3. Hence also, dps. = 3; class = 14; deficiency = 3.

*Cases of the Indecomposable Curve, the Centres being in a Line.*—Art. No. 196.

196. There are some peculiarities in the case where the centres  $A, B, C, D$  are on a line; taking as usual ( $a, b, c, d$ ) for the  $x$ -co-ordinates or distances of the four centres from a fixed point on the line, I enumerate the cases as follows:—

- I. No relation between  $l, m, n, p$ ; corresponds to I. *supra*.
- II.  $\sqrt{l} + \sqrt{m} + \sqrt{n} + \sqrt{p} = 0$ ; corresponds to II. *supra*.
- III.  $\sqrt{l} + \sqrt{m} = 0, \sqrt{n} + \sqrt{p} = 0$ ; corresponds to III. *supra*.
- IV.  $\sqrt{l} + \sqrt{m} + \sqrt{n} + \sqrt{p} = 0, a\sqrt{l} + b\sqrt{m} + c\sqrt{n} + d\sqrt{p} = 0$ ; corresponds to IV. *supra*, viz., there is a branch ideally containing ( $z^2 = 0$ ) the line infinity twice. But, observe that whereas in IV. *supra*, in order that this might be so, it was necessary to impose on  $l, m, n, p$  three conditions giving the definite systems of values  $\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p} = a : b : c : d$ , in the present case only two conditions are imposed, so that a single arbitrary parameter is left.

V.  $\sqrt{l} = \sqrt{m} = \sqrt{n} = \sqrt{p}$ ; corresponds to V. *supra*.

VI.  $\sqrt{l} + \sqrt{m} = 0, \sqrt{n} + \sqrt{p} = 0, a\sqrt{l} + b\sqrt{m} + c\sqrt{n} + d\sqrt{p} = 0$ , or what is the same thing,  $\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p} = c - d : d - c : b - a : a - b$ ; the equation is thus  $(c - d)(\sqrt{A^\circ} - \sqrt{B^\circ}) - (a - b)(\sqrt{A^\circ} - \sqrt{B^\circ}) = 0$ . There is here one branch ideally containing ( $z^2 = 0$ ) the line infinity twice, and another branch ideally containing ( $z = 0$ ) the line infinity once; order is = 5. Each of the points  $I, J$  is an ordinary point on the curve, the remaining points at infinity are a node ( $A^\circ = B^\circ, C^\circ = D^\circ$ ), as presently mentioned, counting as three points, viz., one branch has for its tangent the line infinity, and the other branch has for its tangent a line perpendicular to the axis; or what is the same thing, there is a hyperbolic branch having an asymptote perpendicular to the axis, and a parabolic branch ultimately perpendicular to the axis. The number of nodes is = 5, viz., there is the node  $A^\circ = B^\circ, C^\circ = D^\circ$  just referred to; and the two pairs of nodes  $((c - d)\sqrt{A^\circ} - (a - b)\sqrt{C^\circ} = 0, -(c - d)\sqrt{B^\circ} + (a - b)\sqrt{D^\circ} = 0)$  and  $(c - d)\sqrt{A^\circ} + (a - b)\sqrt{D^\circ} = 0, (c - d)\sqrt{B^\circ} + (a - b)\sqrt{C^\circ} = 0)$ , each

pair symmetrically situate in regard to the axis. Hence also dps. = 5; class = 10; deficiency = 1.

And there is apparently a seventh case, which, however, I exclude from the present investigation, viz., this would be if we had

$$\left( \begin{array}{cccc} 1 & , & 1 & , & 1 & , & 1 \\ a & , & b & , & c & , & d \\ a^2 & , & b^2 & , & c^2 & , & d^2 \\ a''^2 & , & b''^2 & , & c''^2 & , & d''^2 \end{array} \right) (\sqrt{l}, \sqrt{m}, \sqrt{n}, \sqrt{p}) = 0,$$

that is, a, b, c, d denoting as before, if we had

$$\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p} = a : b : c : d, \quad \text{and} \quad aa''^2 + bb''^2 + cc''^2 + dd''^2 = 0.$$

For observe that in this case we have

$$aA^\circ + bB^\circ + cC^\circ + dD^\circ = 0, \quad \text{and} \quad \frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0;$$

that is, the supposition in question belongs to the decomposable case.

*The Decomposable Curve—Art. No. 197.*

197. We have next to consider the decomposable case, viz., when we have

$$aA^\circ + bB^\circ + cC^\circ + dD^\circ = 0;$$

see *ante*, Nos. 87 *et seq.*—it there appears that (unless the centres *A, B, C, D* are in a line) the condition signifies that the four circles have a common ortho-  
tomic circle; and when we have also

$$\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0.$$

The formulæ for the decomposition are given *ante*, Nos. 42 *et seq.* Writing therein  $A^\circ, B^\circ, C^\circ, D^\circ$  in place of  $U, V, W, T$  respectively, it thereby appears that the tetrazomal curve  $\sqrt{lA^\circ} + \sqrt{mB^\circ} + \sqrt{nC^\circ} + \sqrt{pD^\circ} = 0$ , breaks up into the two trizomal curves

$$\sqrt{l_1A^\circ} + \sqrt{m_1B^\circ} + \sqrt{n_1C^\circ} = 0, \quad \sqrt{l_2A^\circ} + \sqrt{m_2B^\circ} + \sqrt{n_2C^\circ} = 0,$$

where

$$\sqrt{l_1} = \sqrt{l} + \frac{a}{d} \frac{p}{\sqrt{l}}, \quad \sqrt{l_2} = \sqrt{l} + \frac{a}{d} \frac{p}{\sqrt{l}},$$

$$\sqrt{m_1} = \sqrt{m} - \sqrt{\frac{a}{bcd} \frac{p}{l}} b \sqrt{n}, \quad \sqrt{m_2} = \sqrt{m} + \sqrt{\frac{a}{bcd} \frac{p}{l}} b \sqrt{n},$$

$$\sqrt{n_1} = \sqrt{n} + \sqrt{\frac{a}{bcd} \frac{p}{l}} c \sqrt{m}, \quad \sqrt{n_2} = \sqrt{n} - \sqrt{\frac{a}{bcd} \frac{p}{l}} c \sqrt{m},$$

and where we have

$$\frac{l_1}{a} + \frac{m_1}{b} + \frac{n_1}{c} = 0, \quad \frac{l_2}{a} + \frac{m_2}{b} + \frac{n_2}{c} = 0.$$

*Cases of the Decomposable Curve, Centres not in a line*—Art. Nos. 198 to 203.

198. I assume, in the first instance, that the centres of the circles are not in a line; we have the following cases:—

I. No further relation between  $l, m, n, p$ ; the order of the tetrazomal is = 8 the order of each of the trizomals is = 4, that is each of them is a bicircular quartic.

II.  $\sqrt{l} + \sqrt{m} + \sqrt{n} + \sqrt{p} = 0$ ; the order of the tetrazomal is = 7, that of one of the trizomals must be = 3.

To verify this, observe that we have

$$\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1} = \sqrt{l} + \sqrt{m} + \sqrt{n} + \frac{ap}{d\sqrt{l}} + \frac{\sqrt{p}}{\sqrt{l}} \sqrt{\frac{a}{bcd}} (c\sqrt{m} - b\sqrt{n}),$$

or substituting for  $\sqrt{l} + \sqrt{m} + \sqrt{n}$  the value  $-\sqrt{p}$ , this is

$$= \frac{\sqrt{p}}{d\sqrt{l}} \left\{ a\sqrt{p} - d\sqrt{l} + \sqrt{\frac{ad}{bc}} (c\sqrt{m} - b\sqrt{n}) \right\},$$

and similarly for  $\sqrt{l_2} + \sqrt{m_2} + \sqrt{n_2}$ , the only change being in the sign of the radical  $\sqrt{\frac{ad}{bc}}$ . But from the two conditions satisfied by  $l, m, n, p$  it is easy to deduce

$$(a\sqrt{p} - d\sqrt{l})^2 - \frac{ad}{bc} (c\sqrt{m} - b\sqrt{n})^2 = 0,$$

and hence one or other of the two functions

$$\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1}, \quad \sqrt{l_2} + \sqrt{m_2} + \sqrt{n_2} \text{ is } = 0;$$

that is, one of the trizomal curves is a cubic.

III.  $\sqrt{l} + \sqrt{p} = 0, \sqrt{m} + \sqrt{n} = 0$ ; order of the tetrazomal is = 6; and hence order of each of the trizomals is = 3. To verify this, observe that here

$$l\left(\frac{1}{a} + \frac{1}{d}\right) + m\left(\frac{1}{b} + \frac{1}{c}\right) = 0,$$

which since  $a + b + c + d = 0$ , gives  $\frac{l}{m} = \frac{ad}{bc}$ ; so that properly fixing the sign

of the radical, we may write  $\sqrt{l} + \sqrt{\frac{ad}{bc}} \sqrt{m} = 0$ . We have then

$$\sqrt{l_1} = \frac{a+d}{d} \sqrt{l}, \quad \sqrt{m_1} + \sqrt{n_1} = \sqrt{\frac{a}{abc}} (b+c) \sqrt{m};$$

which last equation, using  $\sqrt{\frac{ad}{bc}}$  to denote as above, but properly selecting the signification of  $\pm$ , may be written

$$\sqrt{m_1} + \sqrt{n_1} = \pm \frac{b+c}{d} \sqrt{\frac{ad}{bc}} \sqrt{m} .$$

Hence

$$\begin{aligned} \sqrt{l_1} \mp (\sqrt{m_1} + \sqrt{n_1}) &= \frac{1}{a} \left\{ (a+d) \sqrt{l} + (b+c) \sqrt{\frac{ad}{bc}} \sqrt{m} \right\} \\ &= \frac{a+d}{d} \left\{ \sqrt{l} + \sqrt{\frac{ad}{bc}} \sqrt{m} \right\} , = 0 , \end{aligned}$$

viz.,  $\sqrt{l_1} \mp (\sqrt{m_1} + \sqrt{n_1})$  with a properly selected signification of the sign  $\mp$  is  $= 0$ ; and similarly  $\sqrt{l_2} \mp (\sqrt{m_2} + \sqrt{n_2})$  with a properly selected signification of the sign  $\mp$  is  $= 0$ ; that is, each of the trizomals is a cubic.

199. IV.  $\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p} = a : b : c : d$  (values which, be it observed, satisfy of themselves the above assumed equation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$ ); the order of the tetrazomal is  $= 6$ ; and the order of each of the trizomals is here again  $= 3$ . We in fact have  $\sqrt{l_1} = a + d$ ,  $\sqrt{m_1} + \sqrt{n_1} = b + c$ , and therefore  $\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1} = 0$ ; and similarly  $\sqrt{l_2} + \sqrt{m_2} + \sqrt{n_2} = 0$ ; that is, each of the trizomals is a cubic.

I attend, in particular, to the case where the four circles reduces themselves to the points  $A, B, C, D$ ; these four points are then in a circle; and the curve under consideration is

$$a\sqrt{A} + b\sqrt{B} + c\sqrt{C} + d\sqrt{D} = 0 ;$$

in the general case where the points  $A, B, C, D$  are not on a circle, this is, as has been seen, a sextic curve, the locus of the foci of the conics which pass through the four given points; in the case where the points are in a circle then the sextic breaks up into two cubics (viz., observing that the curve under consideration is  $\sqrt{lA} + \sqrt{mB} + \sqrt{nC} + \sqrt{pD} = 0$ , where  $\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p} = a : b : c : d$ , these values do of themselves satisfy the condition of decomposability  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$ ), that is, the locus of the foci of the conics which pass through four points on a circle is composed of two circular cubics, each of them having the four points for a set of concyclic foci. It is easy to see why the sextic, thus defined as a locus of foci, must break up into two cubics; in fact, as we have seen, the conics which pass through the four concyclic points  $A, B, C, D$  have their axes in two fixed directions; there is consequently a locus of the foci situate on the axes which are in one of the fixed directions, and a separate locus of the foci

situate on the axes which lie in the other of the fixed directions; viz., each of these loci is a circular cubic.

200. Adopting the notation of No. 188, or writing

$$RA = a_1, RB = b_1, RC = c_1, RD = d_1,$$

(and therefore  $b_1c_1 = a_1d_1$ ) we have

$$a : b : c : d = -d_1(b_1 - c_1) : c_1(a_1 - d_1) : -b_1(a_1 - d_1) : a_1(b_1 - c_1).$$

Moreover

$$\begin{aligned} \sqrt{l_1} &= a + d & , & \quad \sqrt{l_2} = a + d, \\ \sqrt{m_1} &= b + \sqrt{\frac{bcd}{a}} & , & \quad \sqrt{m_2} = b - \sqrt{\frac{bcd}{a}}, \\ \sqrt{n_1} &= c - \sqrt{\frac{bcd}{a}} & , & \quad \sqrt{n_2} = c + \sqrt{\frac{bcd}{a}}, \end{aligned}$$

and we have

$$\frac{bcd}{a} = (a_1 - d_1)^2 \frac{a_1 b_1 c_1}{d_1} = a_1^2 (a_1 - d_1)^2, \quad \sqrt{\frac{bcd}{a}} = -a_1(a_1 - d_1) \quad \text{suppose};$$

and thence

$$\begin{aligned} \sqrt{l_1} &= (a_1 - d_1)(b_1 - c_1), & \sqrt{l_2} &= (a_1 - d_1)(b_1 - c_1) \\ \sqrt{m_1} &= (a_1 - d_1)(c_1 - a_1), & \sqrt{m_2} &= (a_1 - d_1)(c_1 + a_1) \\ \sqrt{n_1} &= (a_1 - d_1)(a_1 - b_1), & \sqrt{n_2} &= (a_1 - d_1)(-a_1 - b_1), \end{aligned}$$

that is

$$\begin{aligned} \sqrt{l_1} : \sqrt{m_1} : \sqrt{n_1} &= b_1 - c_1 : c_1 - a_1 : a_1 - b_1, \\ \sqrt{l_2} : \sqrt{m_2} : \sqrt{n_2} &= b_1 - c_1 : c_1 + a_1 : -a_1 - b_1, \end{aligned}$$

agreeing with the formulæ No. 188.

The tetrazomal curve

$$-d_1(b_1 - c_1) \sqrt{A} + c_1(a_1 - d_1) \sqrt{B} - b_1(a_1 - d_1) \sqrt{C} + a_1(b_1 - c_1) \sqrt{D} = 0$$

is thus decomposed into the two trizomals

$$\begin{aligned} (b_1 - c_1) \sqrt{A} + (c_1 - a_1) \sqrt{B} + (a_1 - b_1) \sqrt{C} &= 0, \\ (b_1 - c_1) \sqrt{A} + (c_1 + a_1) \sqrt{B} - (a_1 + b_1) \sqrt{C} &= 0. \end{aligned}$$

201. Observe that the tetrazomal equation is a consequence of either of the trizomal equations; taking for instance the first trizomal equation, this gives the tetrazomal equation, and consequently any combination of the trizomal equation and the tetrazomal equation is satisfied if only the trizomal equation is satisfied.

Multiply the trizomal equation by  $-a_1 + d_1$  and add it to the tetrazomal equation; the resulting equation contains the factor  $a_1$ , and omitting this, it is

$$(b_1 - c_1)(-\sqrt{A} + \sqrt{D}) + (a_1 - d_1)(\sqrt{B} - \sqrt{C}) = 0,$$

where observe that  $b_1 - c_1$  is the distance  $BC$ , and  $a_1 - d_1$  the distance  $AD$ . But in like manner multiplying the second trizomal equation by  $-a_1 + d_1$ , and adding it to the original tetrazomal equation, the resulting equation, omitting the factor  $a_1$ , is

$$(b_1 - c_1)(-\sqrt{A} + \sqrt{D}) - (a_1 - d_1)(\sqrt{B} - \sqrt{C}) = 0;$$

viz., it is in fact the same tetrazomal equation as was obtained by means of the first trizomal equation.

201. The new tetrazomal equation, say

$$(b_1 - c_1)(-\sqrt{A} + \sqrt{D}) + (a_1 - d_1)(\sqrt{B} - \sqrt{C}) = 0,$$

is thus equivalent to the original tetrazomal equation; observe that it is an equation of the form  $\sqrt{lA} + \sqrt{mB} + \sqrt{nC} + \sqrt{pD} = 0$ , where

$$\sqrt{l} = -(b_1 - c_1), \quad \sqrt{m} = a_1 - d_1, \quad \sqrt{n} = (a_1 - d_1), \quad \sqrt{p} = b_1 - c_1,$$

and where consequently  $\sqrt{l} + \sqrt{p} = 0$ ,  $\sqrt{m} + \sqrt{n} = 0$ , that is an equation of the form (198) III., decomposable, as it should be, into the equations of two circular cubics. Writing

$$\frac{-\sqrt{A} + \sqrt{D}}{a_1 - d_1} = \theta, \quad \frac{\sqrt{B} - \sqrt{C}}{b_1 - c_1} = \theta;$$

where  $\theta$  is an arbitrary parameter, the curve is obtained as the locus of the intersections of two similar conics having respectively the foci  $(A, D)$  and the foci  $(B, C)$ ; (see SALMON, *Higher Plane Curves*, p. 174): whence we have the theorem, that if  $A, B, C, D$  are any four points on a circle, the two circular cubics which are the locus of the foci of the conics which pass through the four points  $A, B, C, D$ , are also the locus of the intersections of the similar conics, which have for their foci  $(A, D)$  and  $(B, C)$  respectively; and of the similar conics with the foci  $(B, D)$  and  $(C, A)$  respectively; and of the similar conics with the foci  $(C, D)$  and  $(A, B)$  respectively.

202. V.  $\sqrt{l} = \sqrt{m} = \sqrt{n} = \sqrt{p}$ . The order of the tetrazomal is = 5, whence those of the trizomals should be = 3 and = 2 respectively. To verify this observe that the equation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$  gives  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 0$ , and combining with  $a + b + c + d = 0$ , these are only satisfied by one of the systems

( $a + b = 0, c + d = 0$ ), ( $a + c = 0, b + d = 0$ ), ( $a + d = 0, b + c = 0$ ). Selecting to fix the ideas the first of these, or writing

$$(a, b, c, d) = (a, -a, c, -c),$$

so that we have identically

$$a(A^\circ - B^\circ) + c(C^\circ - D^\circ) = 0,$$

an equation which signifies that the radical axis of the circles  $A, B$  is also the radical axis of the circles  $C, D$ ; then, writing as we may do,  $\sqrt{\frac{a}{bcd} \frac{p}{l}} (= \sqrt{\frac{1}{c^2}}) = \frac{1}{c}$ , we have

$$\begin{aligned} \sqrt{l_1} &= 1 - \frac{a}{c}, & \sqrt{l_2} &= 1 - \frac{a}{c}, \\ \sqrt{m_1} &= 1 + \frac{a}{c}, & \sqrt{m_2} &= 1 - \frac{a}{c}, \\ \sqrt{n_1} &= 1 + 1, = 2, & \sqrt{n_2} &= 1 - 1, = 0. \end{aligned}$$

Here  $\sqrt{l_1} + \sqrt{m_1} - \sqrt{n_1} = 0$ , which gives one of the trizomals a cubic, viz., this is the trizomal

$$\left(1 - \frac{a}{c}\right) \sqrt{A^\circ} + \left(1 + \frac{a}{c}\right) \sqrt{B^\circ} + 2\sqrt{C^\circ} = 0.$$

The other trizomal reduces itself to the bizomal  $\sqrt{A^\circ} + \sqrt{B^\circ} = 0$ , which regarded as a trizomal, or written under the form  $(\sqrt{A^\circ} + \sqrt{B^\circ})^2 = 0$ , is the line  $A^\circ - B^\circ = 0$  twice, viz., this is the radical axis of the circles  $A, B$  twice; and the order is thus = 2. By what precedes, the line in question is in fact the common radical axis of the circles  $A, B$  and of the circles  $C, D$ .

*Cases of the Decomposable Curve, the Centres in a Line—Art. Nos. 203 to 206.*

203. We have yet to consider the decomposable case when the centres  $A, B, C, D$  are on a line; the equation  $aA^\circ + bB^\circ + cC^\circ + dD^\circ = 0$  here subsists universally, whatever be the radii  $a'', b'', c'', d''$ . We establish as before the relation  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$ . The cases are as follows:—

I. No further relation between  $l, m, n, p$ , order of tetrazomal = 8, of trizomals 4 and 4.

II.  $\sqrt{l} + \sqrt{m} + \sqrt{n} + \sqrt{p} = 0$ ; order of tetrazomal = 7; of trizomals = 4 and 3; same as II. *supra*.

III.  $\sqrt{l} + \sqrt{p} = 0, \sqrt{m} + \sqrt{n} = 0$ ; order of tetrazomal = 6; of trizomals 3 and 3; same as III. *supra*.

204. IV.  $\sqrt{l} + \sqrt{m} + \sqrt{n} + \sqrt{p} = 0$ ,  $a\sqrt{l} + b\sqrt{m} + c\sqrt{n} + d\sqrt{p} = 0$ ; order of tetrazomal = 6; this is a remarkable case, the orders of the trizomals are either 3, 3 or else 4, 2.

To explain how this is, it is to be noticed that in the absence of any special relation between the radii, the above conditions combined with  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$  give  $\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p} = a : b : c : d^*$ ; when  $l, m, n, p$  have these values, the case is the same as IV. *supra*, and the orders of the trizomals are 3, 3. But if the radii of the circles satisfy the condition

$$\begin{vmatrix} 1 & , & 1 & , & 1 & , & 1 \\ a & , & b & , & c & , & d \\ a^2 & , & b^2 & , & c^2 & , & d^2 \\ a''^2 & , & b''^2 & , & c''^2 & , & d''^2 \end{vmatrix} = 0,$$

then the two conditions satisfy of themselves the remaining condition  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$ , and the ratios  $\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p}$  instead of being determinate as above, depend on an arbitrary parameter.

We have

$$\sqrt{l_1} = \sqrt{l} + \frac{a}{d} \frac{p}{\sqrt{l}}, \quad \sqrt{m_1} = \sqrt{m} - \sqrt{\frac{a}{bcd} \frac{p}{l}} b \sqrt{n}, \quad \sqrt{n_1} = \sqrt{n} + \sqrt{\frac{a}{bcd} \frac{p}{l}} c \sqrt{m},$$

and between  $l, m, n, p$  only the relations

$$\sqrt{l} + \sqrt{m} + \sqrt{n} + \sqrt{p} = 0, \quad a\sqrt{m} + b\sqrt{n} + c\sqrt{p} = 0.$$

We find first

$$\begin{aligned} \sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1} &= \sqrt{l} + \sqrt{m} + \sqrt{n} \\ &+ \frac{\sqrt{p}}{\sqrt{l}} \left\{ \frac{a}{d} \sqrt{p} - \sqrt{\frac{a}{bcd}} (b\sqrt{n} - c\sqrt{m}) \right\} \\ &= - \frac{\sqrt{p}}{\sqrt{l}} \left\{ \frac{1}{d} (d\sqrt{l} - a\sqrt{p}) - \sqrt{\frac{a}{bc}} (b\sqrt{n} - c\sqrt{m}) \right\}, \end{aligned}$$

\* Writing  $x^2, y^2, z^2, w^2$  in place of  $\sqrt{l}, \sqrt{m}, \sqrt{n}, \sqrt{p}$ , we have to find  $x, y, z, w$  from the conditions

$$\begin{aligned} x + y + z + w &= 0, \\ ax + by + cz + dw &= 0, \\ \frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} + \frac{w^2}{d} &= 0, \end{aligned}$$

where the constants are connected by the relation

$$aa + bb + cc + dd = 0.$$

It readily appears that the line represented by the first two equations *touches* the quadric surface in the point  $x : y : z : w = a : b : c : d$ , so that these are in general the only values of  $\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p}$ . In the case next referred to in the text the line lies in the surface, and the values are not determined.

and then

$$\begin{aligned}(d-a)\sqrt{l} &= (b-d)\sqrt{m} + (c-d)\sqrt{n}, \\ (d-a)\sqrt{p} &= (a-b)\sqrt{m} + (a-c)\sqrt{n},\end{aligned}$$

whence

$$d\sqrt{l} - a\sqrt{p} = \frac{b-c}{d-a}(b\sqrt{n} - c\sqrt{m}),$$

and we have thus

$$\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1} = \frac{\sqrt{p}}{d\sqrt{l}} \left( \frac{b-c}{d-a} - \sqrt{\frac{ad}{bc}} \right) (b\sqrt{n} - c\sqrt{m}).$$

And similarly

$$\sqrt{l_2} + \sqrt{m_2} + \sqrt{n_2} = \frac{\sqrt{p}}{d\sqrt{l}} \left( \frac{b-c}{d-a} + \sqrt{\frac{ad}{bc}} \right) (b\sqrt{n} - c\sqrt{m});$$

(observe that in the case not under consideration  $b\sqrt{n} - c\sqrt{m} = 0$ , and therefore  $\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1} = 0$ ,  $\sqrt{l_2} + \sqrt{m_2} + \sqrt{n_2} = 0$ ). In the present case we have

$$a:b:c:d = (b-c)(c-d)(d-b) : -(c-d)(d-a)(a-b) : -(d-a)(a-b)(b-d) : -(a-b)(b-c)(c-a),$$

and thence

$$\frac{ad}{bc} = \frac{(b-c)^2}{(d-a)^2},$$

so that only one of the two sums  $\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1}$ ,  $\sqrt{l_2} + \sqrt{m_2} + \sqrt{n_2}$  is 0, viz., assuming

$$\sqrt{\frac{ad}{bc}} = \frac{b-c}{d-a},$$

we have

$$\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1} = 0.$$

And then also

$$\begin{aligned}a\sqrt{l_1} + b\sqrt{m_1} + c\sqrt{n_1} &= a\sqrt{l} + b\sqrt{m} + c\sqrt{n} \\ &+ \frac{\sqrt{p}}{\sqrt{l}} \left\{ \frac{aa\sqrt{p}}{d} - \sqrt{\frac{a}{bcd}} (bb\sqrt{n} - cc\sqrt{m}) \right\} \\ &= - \frac{\sqrt{p}}{\sqrt{l}} \left\{ \frac{1}{d}(dd\sqrt{l} - aa\sqrt{p}) - \sqrt{\frac{a}{bcd}} (bb\sqrt{n} - cc\sqrt{m}) \right\};\end{aligned}$$

but we find

$$dd\sqrt{l} - aa\sqrt{p} = \frac{b-c}{d-a}(bb\sqrt{n} - cc\sqrt{m}),$$

and thence

$$a\sqrt{l_1} + b\sqrt{m_1} + c\sqrt{n_1} = \frac{\sqrt{p}}{d\sqrt{l}} \left( \frac{b-c}{d-a} - \sqrt{\frac{ad}{bc}} \right) (bb\sqrt{n} - cc\sqrt{m}), = 0,$$

in virtue of  $\sqrt{\frac{ad}{bc}} = \frac{b-c}{d-a}$ . Hence  $\sqrt{l_1} : \sqrt{m_1} : \sqrt{n_1} = b-c : c-a : a-b$ , or the corresponding trizomal is a conic, but the other trizomal is a quartic.

205. V.  $\sqrt{l} = \sqrt{m} = \sqrt{n} = \sqrt{p}$ ; order of tetrazomal is = 5; orders of trizomals = 3, 2; same as V. *supra*.

VI.  $\sqrt{l} + \sqrt{p} = 0$ ,  $\sqrt{m} + \sqrt{n} = 0$ ,  $a\sqrt{l} + b\sqrt{m} + c\sqrt{n} + d\sqrt{p} = 0$ ; order of tetrazomal = 5; orders of trizomals are 3, 2.

We have here

$$\begin{aligned}\sqrt{l_1} &= \frac{a+d}{d} \sqrt{l}, \\ \sqrt{m_1} &= \sqrt{m} + \sqrt{\frac{a}{bcd}} b \sqrt{m}, \\ \sqrt{n_1} &= \sqrt{m} + \sqrt{\frac{a}{bcd}} c \sqrt{m},\end{aligned}$$

or writing the values of  $\sqrt{m_1}$ ,  $\sqrt{n_1}$  in the form

$$\begin{aligned}\sqrt{m_1} &= \sqrt{m} + \sqrt{\frac{ad}{bc}} \frac{b}{d} \sqrt{m}, \\ \sqrt{n_1} &= -\sqrt{m} + \sqrt{\frac{ad}{bc}} \frac{c}{d} \sqrt{m},\end{aligned}$$

then observing that as before  $l = \frac{ad}{bc} m$ , if to fix the ideas we assume

$\sqrt{l} = \sqrt{\frac{ad}{bc}} \sqrt{m}$ , the equations are

$$\begin{aligned}\sqrt{l_1} &= \frac{a+d}{d} \sqrt{l} & \text{and similarly } \sqrt{l_2} &= \frac{a+d}{d} \sqrt{l} \\ \sqrt{m_1} &= \sqrt{m} + \frac{b}{d} \sqrt{l}, & \sqrt{m_2} &= \sqrt{m} - \frac{b}{d} \sqrt{l}, \\ \sqrt{n_1} &= -\sqrt{m} + \frac{c}{d} \sqrt{l}, & \sqrt{n_2} &= \sqrt{m} - \frac{c}{d} \sqrt{l},\end{aligned}$$

whence

$$\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1} = 0, \quad \sqrt{l_2} - \sqrt{m_2} - \sqrt{n_2} = 0.$$

We have moreover

$$\begin{aligned}a\sqrt{l_1} &= \frac{aa+ad}{d} \sqrt{l}, \\ b\sqrt{m_1} + c\sqrt{n_1} &= (b-c)\sqrt{m} + \frac{bb+cc}{d} \sqrt{l},\end{aligned}$$

and thence

$$a\sqrt{l_1} + b\sqrt{m_1} + c\sqrt{n_1} = (a-d)\sqrt{l} + (b-c)\sqrt{m} = 0,$$

so that

$$\sqrt{l_1} : \sqrt{m_1} : \sqrt{n_1} = b-c : c-a : a-b;$$

the corresponding trizomal is thus a conic, and it has been seen that the other trizomal is a cubic.

$$\text{VII. If we have } \begin{vmatrix} 1, & 1, & 1, & 1 \\ a, & b, & c, & d \\ a^2, & b^2, & c^2, & d^2 \\ a''^2, & b''^2, & c''^2, & d''^2 \end{vmatrix} = 0, \text{ and } \begin{pmatrix} 1, & 1, & 1, & 1 \end{pmatrix} \begin{pmatrix} \sqrt{l}, \sqrt{m}, \sqrt{n}, \sqrt{p} \end{pmatrix} = 0.$$

the tetrazomal has a branch ideally containing ( $z^3 = 0$ ) the line infinity 3 times: order is = 5; orders of the trizomals are 3, 2. We have here

$$\sqrt{l} : \sqrt{m} : \sqrt{n} : \sqrt{p} = a : b : c : d,$$

and thence

$$\begin{aligned} \sqrt{l_1} &= a + d, & \sqrt{l_2} &= a + d \\ \sqrt{m_1} &= b - \sqrt{\frac{bcd}{a}}, & \sqrt{m_2} &= b + \sqrt{\frac{bcd}{a}} \\ \sqrt{n_1} &= c + \sqrt{\frac{bcd}{a}}, & \sqrt{n_2} &= c - \sqrt{\frac{bcd}{a}}, \end{aligned}$$

which give

$$\sqrt{l_1} + \sqrt{m_1} + \sqrt{n_1} = 0, \quad \sqrt{l_2} + \sqrt{m_2} + \sqrt{n_2} = 0.$$

Moreover

$$\begin{aligned} a\sqrt{l_1} + b\sqrt{m_1} + c\sqrt{n_1} &= a(a+d) + bb + cc \\ &\quad - (b-c)\sqrt{\frac{bcd}{a}} \\ &= (a-d)d - (b-c)\sqrt{\frac{bcd}{a}} \\ &= d \left\{ (a-d) - (b-c)\sqrt{\frac{bc}{ad}} \right\}, \end{aligned}$$

and similarly

$$a\sqrt{l_2} + b\sqrt{m_2} + c\sqrt{n_2} = d \left\{ (a-d) + (b-c)\sqrt{\frac{bc}{ad}} \right\};$$

whence in virtue of

$$\frac{ad}{bc} = \frac{(b-c)^2}{(d-a)^2},$$

one of the two expressions is = 0; and the trizomals are thus a conic and a cubic.

*The Decomposable Curve; Transformation to a different set of Concyelic Foci—Art. No. 206.*

206. Consider the decomposable case of

$$\sqrt{lA} + \sqrt{mB} + \sqrt{nC} + \sqrt{pD} = 0;$$

viz., the points  $A, B, C, D$  lie here in a circle, and we have  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} + \frac{p}{d} = 0$ .

Taking  $(A_1, D_1)$  the anti-points of  $(A, D)$ ;  $(B_1, C_1)$  the anti-points of  $(B, C)$ ; then  $A_1 D_1 = AD$ ,  $B_1 C_1 = BC$  (No. 65) and referring to the formulæ, *ante*, Nos. 100 *et seq.*, it appears that we can find  $l_1, m_1, n_1, p_1$  such that identically

$$-lA + mB + nC - pD = -l_1A_1 + m_1B_1 + n_1C_1 - p_1D_1,$$

and moreover that  $lp = l_1p_1, mn = m_1n_1$ .

The equation of the curve gives

$$-lA + mB + nC - pD - 2\sqrt{lpAD} + 2\sqrt{mnBC} = 0,$$

which may consequently be written

$$-l_1A_1 + m_1B_1 + n_1C_1 - p_1D_1 - 2\sqrt{l_1p_1A_1D_1} + 2\sqrt{m_1n_1B_1C_1} = 0;$$

viz., this is

$$\sqrt{l_1A_1} + \sqrt{m_1B_1} + \sqrt{n_1C_1} + \sqrt{p_1D_1} = 0;$$

that is, the two trizomals expressed by the original tetrazomal equation involving the set of concyclic foci  $(A, B, C, D)$  are thus expressed by a new tetrazomal equation involving the different set of concyclic foci  $(A_1, B_1, C_1, D_1)$ ; and we might of course in like manner express the equation in terms of the other two sets of concyclic foci  $(A_2, B_2, C_2, D_2)$  and  $(A_3, B_3, C_3, D_3)$  respectively. It might have been anticipated that such a transformation existed, for we could as regards each of the component trizomals separately pass from the original set to a different set of concyclic foci, and the two trizomal equations thus obtained would, it might be presumed, be capable of composition into a single tetrazomal equation; but the direct transformation of the tetrazomal equation is not on this account less interesting.

ANNEX I.—*On the Theory of the Jacobian.*

Consider any three curves  $U = 0, V = 0, W = 0$ , of the same order  $r$ , then writing

$$J(U, V, W) = \frac{d(U, V, W)}{d(x, y, z)} = \begin{vmatrix} d_x U, d_x V, d_x W \\ d_y U, d_y V, d_y W \\ d_z U, d_z V, d_z W \end{vmatrix},$$

we have the Jacobian curve  $J(U, V, W) = 0$ , of the order  $3r - 3$ .

A fundamental property is that if the curves  $U = 0, V = 0, W = 0$  have any common point, this is a point on the Jacobian, and not only so, but it is a node, or double point, that is, for the point in question we have  $J = 0$ , and also  $d_x J = 0, d_y J = 0, d_z J = 0$ .

It follows that for the three curves  $l\Theta + L\Phi = 0, m\Theta + M\Phi = 0, n\Theta + N\Phi = 0$  ( $\Theta = 0$  of the order  $r - s', \Phi = 0$  of the order  $r - s, l = 0, m = 0, n = 0$  each of

the order  $s'$ ,  $L = 0$ ,  $M = 0$ ,  $N = 0$  each of the order  $s$ ) which have in common the  $(r - s')(r - s)$  points of intersection of the curves  $\Theta = 0$ ,  $\Phi = 0$ , each of these points is a node on the Jacobian, and hence that the Jacobian must be of the form

$$J(l\Theta + L\Phi, m\Theta + M\Phi, n\Theta + N\Phi) = A\Theta^2 + 2B\Theta\Phi + C\Phi^2 = 0,$$

where obviously the degrees of  $A$ ,  $B$ ,  $C$  must be  $r + 2s' - 3$ ,  $r + s + s' - 3$ ,  $r + 2s - 3$  respectively. In the particular case where  $s' = 0$ , that is where  $l$ ,  $m$ ,  $n$  are constants, we have  $A = 0$ ; the Jacobian curve then contains as a factor  $(\Phi = 0)$ , and throwing this out, the curve is  $B\Theta + C\Phi = 0$ , viz., this is a curve of the order  $2r + s - 3$  passing through each of  $r(r - s)$  points of intersection of the curves  $\Theta = 0$ ,  $\Phi = 0$ .

In particular, if  $r = 2$ ,  $s = 1$ , that is, if the curves are the conics  $\Theta + L\Phi = 0$ ,  $\Theta + M\Phi = 0$ ,  $\Theta + N\Phi = 0$ , passing through the two points of intersection of the conic  $\Theta = 0$  by the line  $\Phi = 0$ , then the Jacobian is a conic passing through these same two points, viz., its equation is of the form  $\Theta + \Omega\Phi = 0$ . This intersects any one of the given conics, say  $\Theta + L\Phi = 0$  in the points  $\Theta = 0$ ,  $\Phi = 0$ , and in two other points  $\Theta + \Omega\Phi = 0$ ,  $\Omega - L = 0$ ; at each of the last-mentioned points, the tangents to the two curves, and the lines drawn to the two points  $\Theta = 0$ ,  $\Phi = 0$ , form a harmonic pencil.

Although this is, in fact, the known theorem that the Jacobian of three circles is their orthotomic circle, yet it is, I think, worth while to give a demonstration of the theorem as above stated in reference to the conics through two given points.

Taking  $(z = 0, x = 0)$ ,  $(z = 0, y = 0)$  for the two given points  $\Theta = 0$ ,  $\Phi = 0$ , the general equation of a conic through the two points is a quadric equation containing terms in  $z^2$ ,  $zx$ ,  $zy$ ,  $xy$ ; taking any two such conics

$$\begin{aligned} cz^2 + 2fyz + 2gzx + 2hxy &= 0, \\ Cz^2 + 2Fyz + 2Gzx + 2Hxy &= 0, \end{aligned}$$

these intersect in the two points  $(x = 0, z = 0)$ ,  $(y = 0, z = 0)$  and in two other points; let  $(x, y, z)$  be co-ordinates of either of the last-mentioned points, and take  $(X, Y, Z)$  as current co-ordinates, the equations of the lines to the fixed points and of the two tangents are

$$\begin{aligned} Xz - Zx &= 0, \quad Yz - Zy = 0, \\ (hy + gz)(Xz - Zx) + (hx + fz)(Yz - Zy) &= 0, \\ (Hy + Gz)(Xz - Zx) + (Hx + Fz)(Yz - Zy) &= 0, \end{aligned}$$

whence the condition for the harmonic relation is

$$(hy + gz)(Hx + Fz) + (hx + fz)(Hy + Gz) = 0,$$

that is

$$(fG + gF)z^2 + (hF + fH)yz + (gH + hG)zx + 2hHxy = 0,$$

but from the equations of the two conics multiplying by  $\frac{1}{2}H$ ,  $\frac{1}{2}h$  and adding, we have

$$\frac{1}{2}(cH + hC)z^2 + (hF + fH)yz + (gH + hG)zx + 2hHxy = 0 ;$$

viz., the condition is thus reduced to

$$cH + hC - 2(fG + gF) = 0 ,$$

so that being satisfied for one of the points in question, it will be satisfied for the other of them. Now for the three conics

$$\begin{aligned} cz^2 + 2fyz + 2gzx + 2hxy &= 0 , \\ c'z^2 + 2f'yz + 2g'zx + 2h'xy &= 0 , \\ c''z^2 + 2f''yz + 2g''zx + 2h''xy &= 0 , \end{aligned}$$

forming the Jacobian, and throwing out the factor  $z$ , we may write the equation in the form

$$Cz^2 + 2Fyz + 2Gzx + 2Hxy = 0 ,$$

where the values are

$$\begin{aligned} C &= g(f'c'' - f''c') + g'(f''c - fc'') + g''(fc' - f'c) , \\ H &= g(h'f'' - h''f') + g'(h''f - hf'') + g''(hf' - h'f) , \\ 2F &= h(f'c'' - f''c') + h'(f''c - fc'') + h''(fc' - f'c) , \\ 2G &= h(c'g'' - c''g') + h'(c''g - cg'') + h''(cg' - c'g) ; \end{aligned}$$

and we thence obtain

$$\begin{aligned} cH + hC &= -(fg' - f'g)(c'h - ch') + (f''g - fg'')(ch' - c'h) \\ &= 2(fG + gF) , \end{aligned}$$

viz., the condition is satisfied in regard to the Jacobian and the first of the three conics; and it is therefore also satisfied in regard to the Jacobian and the other two conics respectively.

I do not know any general theorem in regard to the Jacobian which gives the foregoing theorem of the orthotomic circle. It may be remarked that the use in the Memoir of the theorem of the orthotomic circle is not so great as would at first sight appear: it fixes the ideas to speak of the orthotomic circle of three given circles rather than of their Jacobian, but we are concerned with the orthotomic circle less as the circle which cuts at right angles the given circles than as a circle standing in a known relation to the given circles.

ANNEX II.—On CASEY'S Theorem for the Circle which touches three given Circles.

The following two problems are identical:—

1. To find a circle touching three given circles.
2. To find a cone-sphere (sphere the radius of which is = 0) passing through three given points in space.

In fact, in the first problem if we use  $z$  to denote a given constant (which may be  $= 0$ ), then taking  $a, a'$  and  $i(z - a'')$  for the co-ordinates of the centre and for the radius of one of the given circles; and similarly  $b, b', i(z - b'')$ ;  $c, c', i(z - c'')$  for the other two given circles; and  $S, S', i(z - S'')$  for the required circle; the equations of the given circles will be

$$\begin{aligned}(x - a)^2 + (y - a')^2 + (z - a'')^2 &= 0, \\(x - b)^2 + (y - b')^2 + (z - b'')^2 &= 0, \\(x - c)^2 + (y - c')^2 + (z - c'')^2 &= 0,\end{aligned}$$

and that of the required circle will be

$$(x - S)^2 + (y - S')^2 + (z - S'')^2 = 0.$$

In order that this may touch the given circles, the distances of its centre from the centres of the given circles must be  $i(S'' - a'')$ ,  $i(S'' - b'')$ ,  $i(S'' - c'')$  respectively; the conditions of contact then are

$$\begin{aligned}(S - a)^2 + (S' - a')^2 + (S'' - a'')^2 &= 0, \\(S - b)^2 + (S' - b')^2 + (S'' - b'')^2 &= 0, \\(S - c)^2 + (S' - c')^2 + (S'' - c'')^2 &= 0,\end{aligned}$$

or we have from these equations to determine  $S, S', S''$ . But taking  $(a, a', a'')$ ,  $(b, b', b'')$ ,  $(c, c', c'')$  for the co-ordinates of three given points in space, and  $(S, S', S'')$  for the co-ordinates of the centre of the cone-sphere through these points, we have the very same equations for the determination of  $(S, S', S'')$ , and the identity of the two problems thus appears.

I will presently give the direct analytical solution of this system of equations. But to obtain a solution in the form required, I remark that the equation of the cone-sphere in question is nothing else than the relation that exists between the co-ordinates of any four points on a cone-sphere; to find this, consider any five points in space, 1, 2, 3, 4, 5; and let  $\bar{1}2$ , &c. denote the distances between the points 1 and 2, &c.; then we have between the distances of the five points the relation

$$\begin{vmatrix} 0, & 1, & 1, & 1, & 1, & 1 \\ 1, & 0, & \bar{1}2^2, & \bar{1}3^2, & \bar{1}4^2, & \bar{1}5^2 \\ 1, & \bar{2}1^2, & 0, & \bar{2}3^2, & \bar{2}4^2, & \bar{2}5^2 \\ 1, & \bar{3}1^2, & \bar{3}2^2, & 0, & \bar{3}4^2, & \bar{3}5^2 \\ 1, & \bar{4}1^2, & \bar{4}2^2, & \bar{4}3^2, & 0, & \bar{4}5^2 \\ 1, & \bar{5}1^2, & \bar{5}2^2, & \bar{5}3^2, & \bar{5}4^2, & 0 \end{vmatrix} = 0;$$

whence taking 5 to be the centre of the cone-sphere through the points 1, 2, 3, 4,

we have  $\overline{15} = \overline{25} = \overline{35} = \overline{45} = 0$ ; and the equation becomes

$$\begin{vmatrix} 0, & \overline{12}^2, & \overline{13}^2, & \overline{14}^2 \\ \overline{21}^2, & 0, & \overline{23}^2, & \overline{24}^2 \\ \overline{31}^2, & \overline{32}^2, & 0, & \overline{34}^2 \\ \overline{41}^2, & \overline{42}^2, & \overline{43}^2, & 0 \end{vmatrix} = 0,$$

which is the relation between the distances of any four points on a cone-sphere; this equation may be written under the irrational form

$$\overline{23} \cdot \overline{14} + \overline{31} \cdot \overline{24} + \overline{12} \cdot \overline{34} = 0.$$

Taking  $(a, a', a''), (b, b', b''), (c, c', c''), (x, y, z)$  for the co-ordinates of the four points respectively, we have

$$\begin{aligned} \overline{23} &= \sqrt{(b-c)^2 + (b'-c')^2 + (b''-c'')^2}, & \overline{14} &= \sqrt{(x-a)^2 + (y-a')^2 + (z-a'')^2}, \\ \overline{31} &= \sqrt{(c-a)^2 + (c'-a')^2 + (c''-a'')^2}, & \overline{24} &= \sqrt{(x-b)^2 + (y-b')^2 + (z-b'')^2}, \\ \overline{12} &= \sqrt{(a-b)^2 + (a'-b')^2 + (a''-b'')^2}, & \overline{34} &= \sqrt{(x-c)^2 + (y-c')^2 + (z-c'')^2}, \end{aligned}$$

or the symbols having these significations, we have

$$\overline{23} \cdot \overline{14} + \overline{31} \cdot \overline{24} + \overline{12} \cdot \overline{34} = 0$$

for the equation of the cone-sphere through the three points; or rather (since the rational equation is of the order 4 in the co-ordinates  $(x, y, z)$ ) this is the equation of the pair of cone-spheres through the three given points; and similarly it is in the first problem the equation of a pair of circles each touching the three given circles respectively.

In the first problem the radii of the given circles were  $i(z-a'')$ ,  $i(z-b'')$ ,  $i(z-c'')$  respectively; denoting these radii by  $\alpha, \beta, \gamma$ , or taking the equations of the given circles to be

$$\begin{aligned} (x-a)^2 + (y-a')^2 - \alpha^2 &= 0, \\ (x-b)^2 + (y-b')^2 - \beta^2 &= 0, \\ (x-c)^2 + (y-c')^2 - \gamma^2 &= 0, \end{aligned}$$

the symbols then are

$$\begin{aligned} \overline{23} &= \sqrt{(b-c)^2 + (b'-c')^2 - (\beta-\gamma)^2}, & \overline{14} &= \sqrt{(x-a)^2 + (y-a')^2 - \alpha^2}, \\ \overline{31} &= \sqrt{(c-a)^2 + (c'-a')^2 - (\gamma-\alpha)^2}, & \overline{24} &= \sqrt{(x-b)^2 + (y-b')^2 - \beta^2}, \\ \overline{12} &= \sqrt{(a-b)^2 + (a'-b')^2 - (\alpha-\beta)^2}, & \overline{34} &= \sqrt{(x-c)^2 + (y-c')^2 - \gamma^2}, \end{aligned}$$

and the equation of the pair of circles is as before

$$\overline{23} \cdot \overline{14} + \overline{31} \cdot \overline{24} + \overline{12} \cdot \overline{34} = 0;$$

where it is to be noticed that  $\overline{23}$ ,  $\overline{31}$ ,  $\overline{12}$  are the tangential distances of the circles 2 and 3, 3 and 1, 1 and 2 respectively; viz., if  $\alpha, \beta, \gamma$  are the radii taken positively, then these are the direct tangential distances. By taking the radii positively or negatively at pleasure, we obtain in all four equations—the tangential distances being all direct as above, or else any one is direct, and the other two are inverse; we have thus the four pairs of tangent circles.

The cone-spheres which pass through a given circle are the two spheres which have their centres in the two anti-points of the given circle; and it is easy to see that the foregoing investigation gives the following (imaginary) construction of the tangent circles; viz., given any three circles  $A, B, C$  in the same plane, to draw the tangent circles. Taking the anti-points of the three circles, then selecting any three anti-points (one for each circle) so as to form a triad, we have in all four complementary pairs of triads. Through a triad, and through the complementary triad draw two circles, these are situate symmetrically on opposite sides of the plane; and combining each anti-point of the first circle with the symmetrically situated anti-point of the second circle, we have two pairs of points, the points of each pair being symmetrically situate in regard to the plane, and having therefore an anti-circle in this plane; these two anti-circles are a pair of tangent circles; and the four pairs of complementary triads give in this manner the four pairs of tangent circles.

I return to the equations

$$\begin{aligned}(x - S)^2 + (y - S')^2 + (z - S'')^2 &= 0, \\(x - S)^2 + (a' - S)^2 + (a'' - S'')^2 &= 0, \\(b - S)^2 + (b' - S)^2 + (b'' - S'')^2 &= 0, \\(c - S)^2 + (c' - S)^2 + (c'' - S'')^2 &= 0;\end{aligned}$$

by eliminating  $(S, S', S'')$  from these equations we shall obtain the equation of the pair of cone-spheres through the points  $(a, a', a''), (b, b', b''), (c, c', c'')$ . Write  $x - S, y - S', z - S'' = X, Y, Z$ , then we have

$$X^2 + Y^2 + Z^2 = 0,$$

and if, for shortness, we put

$$\begin{aligned}\mathbf{A} &= (a - x)^2 + (a' - y)^2 + (a'' - z)^2, \\ \mathbf{B} &= (b - x)^2 + (b' - y)^2 + (b'' - z)^2, \\ \mathbf{C} &= (c - x)^2 + (c' - y)^2 + (c'' - z)^2,\end{aligned}$$

then by means of the equation just obtained the other three equations become

$$\begin{aligned}\mathbf{A} + 2[(a - x)X + (a' - y)Y + (a'' - z)Z] &= 0, \\ \mathbf{B} + 2[(b - x)X + (b' - y)Y + (b'' - z)Z] &= 0, \\ \mathbf{C} + 2[(c - x)X + (c' - y)Y + (c'' - z)Z] &= 0.\end{aligned}$$

These last equations give

$$\begin{aligned} X : Y : Z &= \lambda A + \mu B + \nu C \\ &: \lambda' A + \mu' B + \nu' C \\ &: \lambda'' A + \mu'' B + \nu'' C, \end{aligned}$$

where

$$\begin{aligned} \lambda &= b'c'' - b''c' + (c' - b')z - (c'' - b'')y, \\ \mu &= c'a'' - c''a' + (a' - c')z - (a'' - c'')y, \\ \nu &= a'b'' - a''b' + (b' - a')z - (b'' - a'')y, \\ \lambda' &= b''c - bc'' + (c'' - b'')x - (c - b)z, \\ \mu' &= c'a - ca'' + (a'' - c'')x - (a - c)z, \\ \nu' &= a''b - ab'' + (b'' - a'')x - (b - a)z, \\ \lambda'' &= bc' - b'c + (c - b)y - (c' - b')x, \\ \mu'' &= ca' - c'a + (a - c)y - (a' - c')x, \\ \nu'' &= ab' - a'b + (b - a)y - (b' - a')x; \end{aligned}$$

and the result of the elimination then is

$$\begin{aligned} &(\lambda A + \mu B + \nu C)^2 \\ &+ (\lambda' A + \mu' B + \nu' C)^2 \\ &+ (\lambda'' A + \mu'' B + \nu'' C)^2 = 0. \end{aligned}$$

But substituting for **A**, **B**, **C** their values, and writing, for shortness,

$$\begin{aligned} -i &= b'c'' - b''c' + c'a'' - c''a' + a'b'' - a''b, \\ -j &= b''c - bc'' + c'a - ca'' + a''b - ab'', \\ -k &= bc' - b'c + ca' - c'a + ab' - a'b, \\ \Delta &= a(b'c'' - b''c) + a'(b''c - bc'') + a''(bc' - b'c), \\ -p &= (b'c'' - b''c')(a^2 + a'^2 + a''^2) + (c'a'' - c''a')(b^2 + b'^2 + b''^2) + (a'b'' - a''b')(c^2 + c'^2 + c''^2), \\ -q &= (b''c - bc'')(a^2 + a'^2 + a''^2) + (c''a - ca'')(b^2 + b'^2 + b''^2) + (a''b - ab'')(c^2 + c'^2 + c''^2), \\ -r &= (bc' - b'c')(a^2 + a'^2 + a''^2) + (ca' - c'a')(b^2 + b'^2 + b''^2) + (ab' - a'b')(c^2 + c'^2 + c''^2), \\ -l &= (c - b')(a^2 + a'^2 + a''^2) + (a - c')(b^2 + b'^2 + b''^2) + (b - a')(c^2 + c'^2 + c''^2), \\ -m &= (c' - b'')(a^2 + a'^2 + a''^2) + (a' - c'')(b^2 + b'^2 + b''^2) + (b' - a'')(c^2 + c'^2 + c''^2), \\ -n &= (c'' - b'')(a^2 + a'^2 + a''^2) + (a'' - c'')(b^2 + b'^2 + b''^2) + (b'' - a'')(c^2 + c'^2 + c''^2), \end{aligned}$$

we find

$$\begin{aligned} &\lambda A + \mu B + \nu C \\ &= -i(x^2 + y^2 + z^2) \\ &\quad + 2i(x^2 + y^2 + z^2) - 2x(ix + jy + kz) - 2\Delta x + ny - mz - p, \end{aligned}$$

with similar expressions for  $\lambda'A + \mu'B + \nu'C$ ,  $\lambda''A + \mu''B + \nu''C$ , and the result is

$$\begin{aligned} &\{i(x^2 + y^2 + z^2) - 2x(ix + jy + kz) - 2\Delta x + ny - mz - p\}^2 \\ &+ \{j(x^2 + y^2 + z^2) - 2y(ix + jy + kz) - nx - 2\Delta y + lz - q\}^2 \\ &+ \{k(x^2 + y^2 + z^2) - 2z(ix + jy + kz) + mx - ly - 2\Delta z - r\}^2 = 0, \end{aligned}$$

viz., this is

$$\begin{aligned} & (x^2 + y^2 + z^2)^2 (i^2 + j^2 + k^2) \\ & + (x^2 + y^2 + z^2) \{4\Delta(ix + jy + kz + 2(i(ny - mz) + j(lz - nx) + k(mx - ly))) \\ & \quad + 4\Delta^2 - 2(ip + jq + kr) + (l^2 + m^2 + n^2)\} \\ & - (lx + my + nz)^2 + 4(ix + jy + kz)(px + qy + rz) \\ & + 4\Delta(px + qy + rz) - 2(p(ny - mz) + q(lz - nx) + r(mx - ly)) \\ & + p^2 + q^2 + r^2 = 0. \end{aligned}$$

viz., this is in the rational form the equation of the pair of cone-spheres. The function on the left hand side must, it is clear, be save to a numerical factor the norm of

$$\begin{aligned} & \sqrt{(b-c)^2 + (b'-c')^2 + (b''-c'')^2} \cdot \sqrt{(x-a)^2 + (y-a')^2 + (z-a'')^2} \\ & + \sqrt{(c-a)^2 + (c'-a')^2 + (c''-a'')^2} \cdot \sqrt{(x-b)^2 + (y-b')^2 + (z-b'')^2} \\ & + \sqrt{(a-b)^2 + (a'-b')^2 + (a''-b'')^2} \cdot \sqrt{(x-c)^2 + (y-c')^2 + (z-c'')^2}, \end{aligned}$$

the numerical factor of the expression in question is in fact  $= -4$ , that is, the norm is

$$= -4(x^2 + y^2 + z^2)^2 (i^2 + j^2 + k^2) + \&c. ;$$

so that attending only to the highest powers in  $(x, y, z)$  we ought to have

$$\begin{aligned} \text{Norm } \{ & \sqrt{(b-c)^2 + (b'-c')^2 + (b''-c'')^2} + \sqrt{(c-a)^2 + (c'-a')^2 + (c''-a'')^2} + \sqrt{(a-b)^2 + (a'-b')^2 + (a''-b'')^2} \} \\ & = -4(i^2 + j^2 + k^2). \end{aligned}$$

It is easy to see that the norm is in fact composed of the terms

$$\begin{aligned} & 2(b'-c')^2 \{ (b-c)^2 - (c-a)^2 - (a-b)^2 \}, \\ & + 2(c'-a')^2 \{ -(b-c)^2 + (c-a)^2 - (a-b)^2 \}, \\ & + 2(a'-b')^2 \{ -(b-c)^2 - (c-a)^2 + (a-b)^2 \}, \end{aligned}$$

and of the similar terms  $(a, b, c)$ ,  $(a'', b'', c'')$  and in  $(a', b', c')$ ,  $(a'', b'', c'')$ ; the above written terms are  $= -4$  into

$$\begin{aligned} & (b'-c')^2 (a-b)(a-c) \\ & + (c'-a')^2 (b-c)(b-a) \\ & + (a'-b')^2 (c-a)(a-b), \end{aligned}$$

which is

$$\begin{aligned} & = a'^2 (b-c)^2 + b'^2 (c-a) + c'^2 (a-b)^2 \\ & \quad + 2b'c' (a-b)(c-a) + 2c'a' (b-c)(a-b) + 2a'b' (c-a)(b-c) \\ & = \{a'(b-c) + b'(c-a) + c'(a-b)\}^2 \\ & = k^2 ; \end{aligned}$$

and the value of the norm is thus  $= -4(i^2 + j^2 + k^2)$ , as it should be.

ANNEX III.—On the Norm of  $(b-c)\sqrt{A^\circ} + (c-a)\sqrt{B^\circ} + (a-b)\sqrt{C^\circ}$ , when the Centres are in a Line.

The norm of  $\sqrt{U} + \sqrt{V} + \sqrt{W}$  is

$$= (1, 1, 1, -1, -1, -1)(U, V, W)^2,$$

whence that of  $\sqrt{U+U'} + \sqrt{V+V'} + \sqrt{W+W'}$  is

$$\begin{aligned} &= (1, 1, 1, -1, -1, -1)(U, V, W)^2 \\ &+ (1, 1, 1, -1, -1, -1)(U', V', W')^2 \\ &+ 2(1, 1, 1, -1, -1, -1)(U, V, W)(U', V', W'), \end{aligned}$$

where the last term is = 2 into

$$\begin{aligned} &U'(\sqrt{U} - \sqrt{V} - \sqrt{W}) \\ &+ V'(-\sqrt{U} + \sqrt{V} - \sqrt{W}) \\ &+ W'(-\sqrt{U} - \sqrt{V} + \sqrt{W}). \end{aligned}$$

And the norm of  $\sqrt{U+U'+U''} + \sqrt{V+V'+V''} + \sqrt{W+W'+W''}$  is obviously composed in a similar manner.

Now, applying the formula to obtain the norm of

$$(b-c)\sqrt{a^2 + \theta + \alpha} + (c-a)\sqrt{b^2 + \theta + \beta} + (a-b)\sqrt{c^2 + \theta + \gamma},$$

the expression contains six terms, two of which are at once seen to vanish; and writing for shortness (,,) in place of (1, 1, 1, -1, -1, -1) the remaining terms are

$$\begin{aligned} &(,,) \left\{ (b-c)^2 \alpha, (c-a)^2 \beta, (a-b)^2 \gamma \right\}^2 \\ &+ 2(,,) \left\{ (b-c)^2 \alpha, (c-a)^2 \beta, (a-b)^2 \gamma \right\} \left\{ (b-c)^2 \alpha^2, (c-a)^2 \beta^2, (a-b)^2 \gamma^2 \right\} \\ &+ 2\theta(,,) \left\{ (b-c)^2 \alpha, (c-a)^2 \beta, (a-b)^2 \gamma \right\} \left\{ (b-c)^2, (c-a)^2, (a-b)^2 \right\} \\ &+ 2\theta(,,) \left\{ (b-c)^2 \alpha^2, (c-a)^2 \beta^2, (a-b)^2 \gamma^2 \right\} \left\{ (b-c)^2, (c-a)^2, (a-b)^2 \right\}; \end{aligned}$$

the first of these terms requires no reduction; the second, omitting the factor 2, is

$$\begin{aligned} &(b-c)^2 \alpha \left[ (b-c)^2 \alpha^2 - (c-a)^2 \beta^2 - (a-b)^2 \gamma^2 \right] \\ &+ (c-a)^2 \beta \left[ -(b-c)^2 \alpha^2 + (c-a)^2 \beta^2 - (a-b)^2 \gamma^2 \right] \\ &+ (a-b)^2 \gamma \left[ -(b-c)^2 \alpha^2 - (c-a)^2 \beta^2 + (a-b)^2 \gamma^2 \right]; \end{aligned}$$

which is

$$= 2(a-b)(b-c)(c-a) \left[ bc(b-c)\alpha + ca(c-a)\beta + ab(a-b)\gamma \right].$$

Similarly the third term, omitting the factor 2 $\theta$ , is

$$\begin{aligned} &(b-c)^2 \alpha \left[ (b-c)^2 - (c-a)^2 - (a-b)^2 \right] \\ &+ (c-a)^2 \beta \left[ -(b-c)^2 + (c-a)^2 - (a-b)^2 \right] \\ &+ (a-b)^2 \gamma \left[ -(b-c)^2 - (c-a)^2 + (a-b)^2 \right], \end{aligned}$$

which is

$$= 2(a-b)(b-c)(c-a) [(b-c)\alpha + (c-a)\beta + (a-b)\gamma],$$

and for the last term, omitting the factor  $2\theta$ , this may be deduced therefrom by writing  $(a^2, b^2, c^2)$  in place of  $(\alpha, \beta, \gamma)$ , viz., it is

$$= -2(a-b)^2(b-c)^2(c-a)^2.$$

Hence, restoring the omitted factors, and collecting, we find

$$\begin{aligned} & \text{Norm} \{ (b-c)\sqrt{a^2 + \theta + \alpha} + (c-a)\sqrt{b^2 + \theta + \beta} + (a-b)\sqrt{c^2 + \theta + \gamma} \} \\ &= (b-c)^4\alpha^2 + (c-a)^4\beta^2 + (a-b)^4\gamma^2 - 2(c-a)^2(a-b)^2\beta\gamma - 2(a-b)^2(b-c)^2\gamma\alpha - 2(b-c)^2(c-a)^2\alpha\beta \\ & \quad + 4\theta(a-b)(b-c)(c-a) [(b-c)\alpha + (c-a)\beta + (a-b)\gamma] \\ & \quad + 4(a-b)(b-c)(c-a) [bc(b-c)\alpha + ca(c-a)\beta + ab(a-b)\gamma] \\ & \quad - 4\theta(a-b)^2(b-c)^2(c-a)^2. \end{aligned}$$

Hence, first writing  $a-x, b-x, c-x$  in place of  $a, b, c$ ; then  $y^2$  for  $\theta$ , and  $(-a'', -b'', -c'')$  for  $(\alpha, \beta, \gamma)$ ; and finally introducing  $z$  for homogeneity, we find

$$\begin{aligned} & \text{Norm} \{ (b-c)\sqrt{(x-az)^2 + y^2 - a''^2z^2} + (c-a)\sqrt{\phantom{(x-az)^2 + y^2 - a''^2z^2}} + (a-b)\sqrt{\phantom{(x-az)^2 + y^2 - a''^2z^2}} \} = z^2 \text{ into} \\ & \quad z^2((b-c)^4a''^4 + (c-a)^4b''^4 + (a-b)^4c''^4 \\ & \quad \quad - 2(c-a)^2(a-b)^2b''^2c''^2 - 2(a-b)^2(b-c)^2c''^2a''^2 - 2(b-c)^2(c-a)^2a''^2b''^2) \\ & \quad - 4y^2(b-c)(c-a)(a-b) [ (b-c)a''^2 + (c-a)b''^2 + (a-b)c''^2 ] \\ & \quad - 4(b-c)(c-a)(a-b) \{ (b-c)a''^2 (z^2bc - zx(b+c) + x^2) \\ & \quad \quad \quad + (c-a)b''^2 (z^2ca - zx(c+a) + x^2) \\ & \quad \quad \quad + (a-b)c''^2 (z^2ab - zx(a+b) + x^2) \} \\ & \quad - 4y(b-c)^2(c-a)(a-b)^2. \end{aligned}$$

so that the equation  $(b-c)\sqrt{A^{\circ}} + (c-a)\sqrt{B^{\circ}} + (a-b)\sqrt{C^{\circ}} = 0$ , in its rationalised form, contains  $(z^2 = 0)$  the line infinity twice, and the curve is thus a conic. If  $a''^2 = b''^2 = c''^2 = k''^2$ , then the expression of the norm is

$$= z^2 \text{ into } -4(a-b)^2(b-c)^2(c-a)^2(y - k''^2z^2),$$

viz., when the three circles have each of them the same radius  $k''$ , the curve is the pair of parallel lines  $y^2 - k''^2z^2 = 0$ ; and in particular when  $k'' = 0$ , or the circles reduce themselves each to a point, then the curve is  $y^2 = 0$ , the axis twice.

ANNEX IV.—On the Trizomal Curves  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ , which have a Cusp, or two Nodes.

The trizomal curve  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ , has not in general any nodes or cusps: in the particular case where the zomal curves are circles, we have

however seen how the ratios  $l : m : n$  may be determined so that the curve shall acquire a node, two nodes, or a cusp; viz., regarding  $a, b, c$  as current areal co-ordinates, we have here a conic  $\frac{l}{a} + \frac{m}{b} + \frac{n}{c} = 0$ , the locus of the centres of the variable circle, and the solution depends on establishing a relation between this conic and the orthotomic circle or Jacobian of the three given circles. I have in my paper "Investigations in connection with CASEY'S Equation," Quart. Math. Jour. vol. viii. (1867), pp. 334-342, given, after Professor CREMONA, a solution of the general question to find the number of the curves  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ , which have a cusp, or which have two nodes, and I will here reproduce the leading points of the investigation. I remark, that although one of the loci involved in it is the same as that occurring in the case of the three circles (viz., we have in each case the Jacobian of the given curves), the other two loci  $\Sigma$  and  $\Delta$ , which present themselves, seem to have no relation to the conic of centres which is made use of in the particular case.

We have the curves  $U = 0, V = 0, W = 0$ , each of the same order  $r$ ; and considering a point the co-ordinates whereof are  $(l, m, n)$ , we regard as corresponding to this point the curve  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ , say for shortness, the curve  $\Omega$ , being as above a curve of the order  $2r$ , having  $r^2$  contacts with each of the given curves  $U = 0, V = 0, W = 0$ . As long as the point  $(l, m, n)$  is arbitrary, the curve  $\Omega$  has not any node, and in order that this curve may have a node, it is necessary that the point  $(l, m, n)$  shall lie on a certain curve  $\Delta$ ; this being so, the node will, it is easy to see, lie on the curve  $J$ , the Jacobian of the three given curves; and the curves  $J$  and  $\Delta$  will correspond to each other point to point, viz., taking for  $(l, m, n)$  any point whatever on the curve  $\Delta$ , the curve  $\Omega$  will have a node at some one point of  $J$ ; and conversely, in order that the curve  $\Omega$  may be a curve having a node at a given point of  $J$ , the point  $(l, m, n)$  must be at some one point of the curve  $\Delta$ . The curve  $\Delta$  has, however, nodes and cusps; each node of  $\Delta$  corresponds to two points of  $J$ , viz., for  $(l, m, n)$  at a node of  $\Delta$ , the curve  $\Omega$  is a binodal curve having a node at each of the corresponding points of  $J$ ; each cusp of  $\Delta$  corresponds to two coincident points of  $J$ , viz. for  $(l, m, n)$  at a cusp of  $\Delta$ , the curve  $\Omega$  has a node at the corresponding point of  $J$ . The number of the binodal curves  $\Omega$  is thus equal to the number of the nodes of  $\Delta$ , and the number of the cuspidal curves  $\Omega$  is equal to the number of the cusps of  $\Delta$ ; and the question is to find the Plückerian numbers of the curve  $\Delta$ . This Professor CREMONA accomplished in a very ingenious manner, by bringing the curve  $\Delta$  into connexion with another curve  $\Sigma$  (viz.,  $\Sigma$  is the locus of the nodes of those curves

$lU + mV + nW = 0$  which have a node), and the result arrived at is that for the curve  $\Delta$

Order	=	$3(r-1)(3r-2)$ ,
Class	=	$6(r-1)^2$ ,
Nodes	=	$\frac{3}{2}(r-1)(27r^3 - 63r^2 + 22r + 16)$ ,
Cusps	=	$3(r-1)(7r-8)$ ,
Double tangents	=	$\frac{3}{2}(r-1)(12r^3 - 36r^2 + 19r + 16)$ ,
Inflexions	=	$12(r-1)(r-2)$ ;

so that, finally, the number of the cuspidal curves  $\sqrt{lU} + \sqrt{mV} + \sqrt{nW} = 0$ , is  $= 3(r-1)(7r-8)$ , and the number of the binodal curves of the same form is  $= \frac{3}{2}(r-1)(27r^3 - 63r^2 + 22r + 16)$ . When the given curves are conics, or for  $r = 2$ , these numbers are  $= 18$  and  $36$  respectively; but the formulæ are not applicable to the case where the conics have a point or points of intersection in common; nor, consequently, to the case of the three circles.



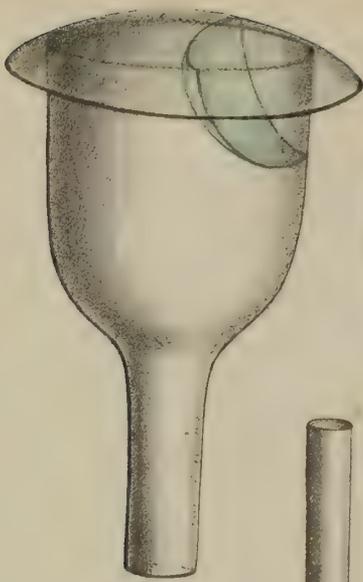


FIG. 1.

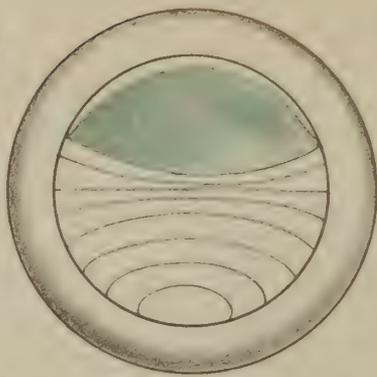


FIG. 2.

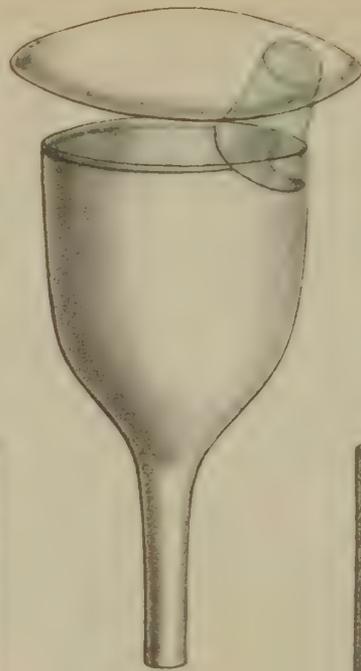


FIG. 3.



FIG. 4.



FIG. 5.



FIG. 6.



FIG. 7.

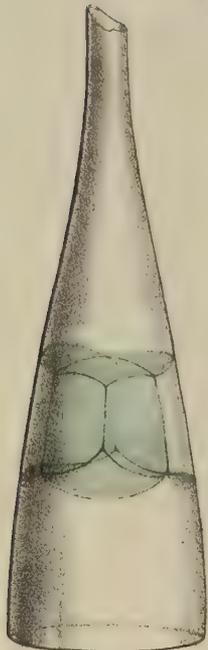


FIG. 8.

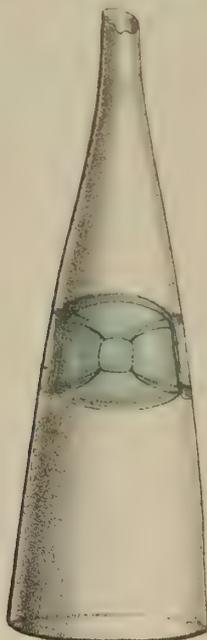


FIG. 9.

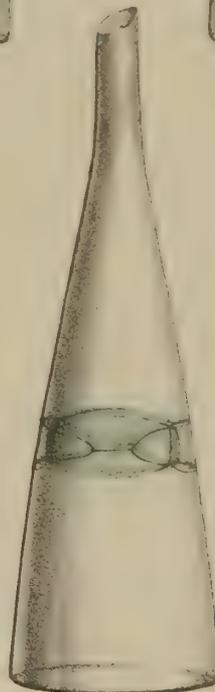


FIG. 10.



FIG.



FIG. 12.

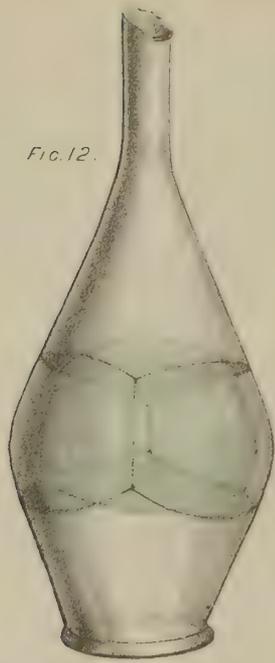


FIG. 13.



FIG. 14.



FIG. 15.

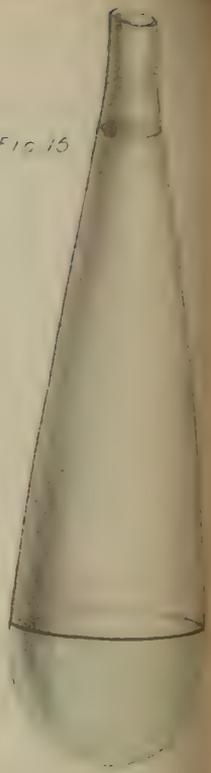


FIG. 16.

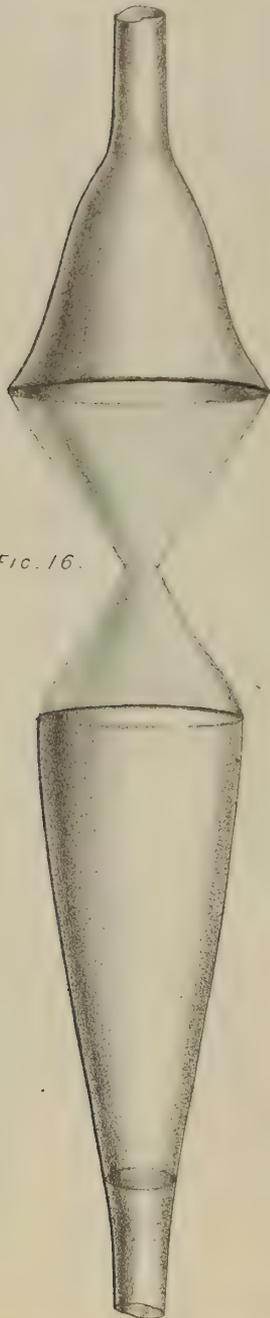


FIG. 17.



FIG. 18.

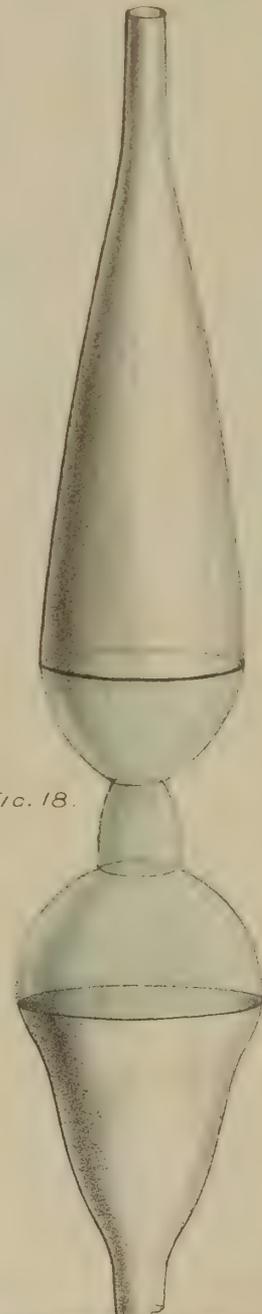
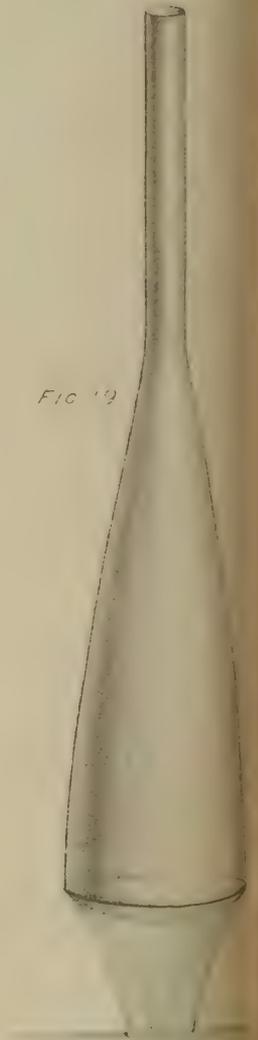


FIG. 19.



II.—*On the Motion, Equilibrium, and Forms of Liquid Films.* By the late Sir DAVID BREWSTER, K.H., D.C.L., &c. (Plates I. and II.) Communicated by FRANCIS DEAS, Esq., LL.B.

(Read 6th April 1868.)

[This paper was transmitted to the Council by Sir DAVID BREWSTER, on the 8th February 1868, with the following remarks:—“ I have tried in vain to finish the two most important of my papers on Liquid Films, but the most beautiful drawings of all the phenomena which its purpose was to describe have been finished. I think, therefore, that my friend Mr DEAS will, by means of these drawings, produce an interesting paper. The drawings are numerous and large, but many of them may be reduced in size. As this is the last of my papers, I hope the Council will not grudge the expense of having them well lithographed.”]

In another letter, Sir DAVID BREWSTER expresses a wish, that in the event of the paper being printed in the “Transactions,” notice should be taken of the fact that the drawings were executed by his friend Miss DICKENSON.]

I. *On some Transformations in Films when brought in contact with Surfaces of Glass.\**

(1.) Let a film be formed on the rim of a cylindrical wine-glass, at or very near its margin; cover it immediately with a watch-glass, and holding the latter firmly in its place, invert the whole, so that the film is placed in a vertical position. The film will now attach itself to the watch-glass at the lowest point where it is in contact with the margin of the wine-glass, and will run up the concave surface of the former. At the same time, the film will leave the margin of the wine-glass at its upper edge, and retreat into the glass, running down its inner surface. A film of the form of the segment of a sphere will thus be produced, which, with the upper portions of the inner surfaces of the watch-glass and wine-glass, will form a hollow filled with air, as shown in fig. 1. This state of matters will remain the same, in whatever position we now place the wine-glass, the figure which has thus been produced being one of equilibrium. The phenomenon produced arises from the fact, that when the original film is first taken up on the margin of the wine-glass, a drop of liquid always remains in the bottom of the glass, and when the glass is inverted, so as to bring the film into the vertical

\* The experiments under Head 1, are best performed by using a watch-glass of considerable concavity, but they will all succeed more or less perfectly by using a piece of perfectly flat glass, or even by employing the convex instead of the concave surface of the watch-glass, provided we take care that the surface of the film on the wine-glass does not project in any part above its rim.

position, this drop runs down the inner surface of the glass till, reaching its lowest point, it brings the film into close contact with the watch-glass; the film now attaches itself to and spreads itself over the surface of the latter in a similar way to what takes place when a bubble blown from a pipe is brought into contact with any smooth surface, whilst the original system of equilibrium being now disturbed, the upper part of the film is put in motion, and a new system of equilibrium is formed.

In making this experiment, the rise of the film upon the surface of the watch-glass is generally so rapid, that it is difficult to watch its progress, but by careful inspection it may be observed to commence first at a single point; *the edge of attachment* of the film to the watch-glass then becomes first elliptical, then circularly concave, then a straight line, then convex, as seen in fig. 2, where the lines represent the edge of the film on the watch-glass in its different stages of progress. The changes which take place in the curvature of the concave spherical film within the wine-glass during these stages are also very curious. The concave film may sometimes be produced without inverting the glass, by holding the watch-glass firmly in position with the thumb, and briskly shaking the glass. The drop of liquid is thus thrown upon the margin of the film.

(2.) The concave spherical film being thus formed, we can reduce matters to their original state (*i. e.* reproduce the single original film on the margin of the wine-glass) by slowly and carefully removing the watch-glass. The experiment is best made by lifting the watch-glass from its connection with the wine-glass at its upper margin, keeping the two in close contact at their lower margin. The upper edge of the concave film will thus again rise in the wine-glass, while its lower edge will descend along the watch-glass; and when it has reached the point where the two glasses are in contact, the watch-glass may be removed, and the original film left on the wine-glass. The edge of the film, as it descends the watch-glass, exhibits the same series of curves as it did in ascending in the last experiment; but, of course, in a reversed order. The nature of these curves may be much more satisfactorily observed in this experiment than in the last, as their progress is much less rapid. Before entirely removing the watch-glass, we can cause the film to ascend or descend at pleasure, with any degree of rapidity, by approximating or separating the two glasses.

In making the first experiment, the drop of fluid which causes the film to attach itself to the wine-glass generally entirely escapes; consequently, if we repeat this experiment upon the film as re-formed in the second experiment, it will seldom succeed, there being no superfluous fluid to produce the necessary contact. If, however, we place a drop of fluid, either on the surface of the reformed film or on the inner surface of the watch-glass, the experiment will succeed, and may be repeated indefinitely, till the film bursts—a fact which clearly proves the drop of fluid to be the agent in producing the phenomenon.

These experiments succeed best with freshly-formed films, but they may be performed even after the colouring matter has formed on the film, though with less certainty. In re-forming the original film, as above described, the wine-glass may be held either horizontally, vertically, or even inverted entirely with the watch-glass held beneath it.

(3.) If, after forming the concave film, as in the first experiment, we raise the watch-glass from the wine-glass, maintaining the contact between the two only at one point, and then cause the watch-glass to rotate by this point round the wine-glass, the concave film will likewise move round the wine-glass; and by continuing the movement, the concave film may be gradually enlarged, till it passes into the original single film, adherent to the wine-glass only as in the last experiment, the curvature of its margin passing through the forms already observed.

(4.) If, having formed the concave film as in the first experiment, we lift the watch-glass perpendicularly from the margin of the wine-glass, the edges of the concave film will remain adherent to the wine-glass and watch-glass respectively, the film being stretched out, while the fluid between the glasses at their previous line of contact will be drawn into a second film, which unites with the concave film to form a cylindrical bag attached above to the watch-glass, and below to the wine-glass.\* By continuing to raise the watch-glass, this bag will sometimes become detached from the watch-glass, and return into the form of the concave film; at other times it will leave the wine-glass entirely, and take the form of a lens upon the watch-glass. Fig. 3 exhibits the cylindrical bag thus formed, still attached to both wine-glass and watch-glass.

As seen from the figure, the remainder of the wine-glass is now covered by a separate film, upon which the cylindrical bag partly rests. This may be regarded as that part of the original film which, in the first experiment, ran up and attached itself to the concave surface of the watch-glass, and which, in the process of perpendicularly lifting the watch-glass, has become restored to its original position, and which, for the sake of distinctness, we may call the complementary film. Whether the cylindrical bag will adhere to the watch-glass or return into the concave film, seems to depend on the relation of the radius of curvature of the concave film to that of the film covering the remainder of the glass, and to that of the watch-glass itself. The smaller the concave film, and the more convex the complementary film, the more readily will the bag leave the latter, and attach itself to the watch-glass in the form of a lens, while the less concave the watch-glass, the more difficult it is to produce this result, till, when we use a perfectly flat glass, it will rarely take place in any case. When, however, the complementary film is either accidentally or purposely broken, so that only the concave film remains, the latter will invariably pass into the form

\* The experiment succeeds best, by first raising the portion of the watch-glass most distant from the concave film, and then lifting the whole watch-glass vertically.

of the lens. The curious bag shape of the film in this latter form of the experiment, while thus passing from the wine-glass to the watch-glass, is shown in fig. 4.

(5.) The lens, when obtained upon the watch-glass, may be restored to the wine-glass in the form of the concave film, by merely replacing the watch-glass so that the lens shall touch the edge of the wine-glass, and it may be taken up and replaced several times, and that whether the complementary film has remained on the wine-glass or not.

(6.) If, when the complementary film has remained on the wine-glass after removing the lens upon the watch-glass, we replace the lens so as to form the concave film, we can, by lifting the watch-glass, or causing it to rotate, as in the previous experiments, reproduce the original single film, which in this case consists of the same film which was taken up as a lens upon the watch-glass.

## II. On the Motions and Figures of Equilibriums of Films within Single Hollow Cones.

In the following experiments, a glass cone of the form in the annexed diagram was used—the perpendicular height of the cone being 3 inches, the diameter of the base being 1 inch. The aperture at the apex can be opened or closed at pleasure, by inserting a plug at  $MN$ . When this vessel is dipped in the soap solution ( $MN$  being open), it gives, when raised, a plane film at  $CD$ . This film (as shown in the previous paper on “Liquid Films,” vol. xxiv. p. 503, of the “Transactions”) does not remain at  $CD$ , but at once commences to ascend towards  $AB$ , with a velocity increasing—*firstly*, with the angle of the cone itself; and, *secondly*, with the angle of inclination at which the cone is held. Arrived at  $AB$ , where the cone ends in the cylindrical tube, the film becomes stationary.



The motions and disposition of films within the cone may be well studied, by blowing small bubbles from a quill or small tube, and inserting them within the vessel. Thus, if a bubble of moderate size be placed on the side of the cone near its base, it will gradually ascend till its convex surface comes in contact with the opposite side of the cone, when it will separate into two plane films, the lower of which will remain stationary at  $AB$ , the upper a little way up the tube. This movement of the bubble is represented in fig. 5.

Again, having obtained a plane film, a little way up the cone, insert a small bubble against the side of the cone, touching the under surface of the plane film, the upper surface of the bubble will unite with the adjacent portion of the plane film to form a single film, and produce the system shown in fig. 6. Insert now, on the opposite side, a second bubble of the same size with the first, and we obtain the system shown in fig. 7, the upper surface of each of the two hollow figures thus produced being convex, the lower concave, their surface of contact being a

plane film. The same result is produced, though no plane film is first formed within the cone, by simply inserting two bubbles of equal size, so as to come in contact with one another. In a similar manner, three bubbles may be inserted in place of two, the result is the system shown in fig. 8, where the three bubbles are united to one another by three plane films meeting in a central vertical straight line. The system produced by inserting four bubbles in a similar manner to the last, is a very curious and beautiful one, peculiarly interesting on account of its analogy to the system already described in the previous paper, as formed within the wire cube (*vide* "Transactions," vol. xxiv. p. 505). This system, shown in fig. 9, consists of four similar figures, the curvatures of whose sides are extremely curious, united in their centre, just as in the case of the system within the wire cube, by a plane quadrilateral film bounded by convex lines. In the case of the wire cube, it was seen that this plane film could be transposed from the vertical to the horizontal position, by blowing upon its margin through a small tube. It is difficult, if possible, to repeat the experiment in this form upon the similar system within the cone, but the same result may be produced, *i.e.* the system may be obtained with the plane film, in the horizontal position, by means of the following process.

It has been seen that in the case of the systems formed of two and three bubbles respectively, it is immaterial whether we proceed by first obtaining a plane film upon which the bubbles are inserted, or by simply inserting the bubbles upon the sides of the cone without the previous existence of the plane film. But in the case of the system now under consideration, if we proceed by first inserting a plane film as a basis upon which to place the four bubbles, and take care that these are not too large, the result is the system shown in fig. 10, in which it will be seen the plane quadrilateral film connecting the four bubbles, instead of being vertical, is horizontal, consisting in fact of the central portion of the original plane film which has remained *in situ*, the remainder of it having amalgamated with the films of the bubbles. A beautiful variety of this experiment may be made in the following manner:—Having obtained the system with the plane film in the horizontal position, withdraw the plug from *MN*, so as to allow the system to rise in the cone (a result which can always be insured by gently sucking out the air by the mouth from *MN*); the system being thus contracted, the sides of the bubbles are brought in contact with one another, the horizontal film is first contracted to a point, and then (the system being now unstable) passes into the vertical position. By again blowing air in at *MN*, the system may be made again to descend, and the plane film to pass once more into the horizontal position.

It will be remembered that, in the case of the wire cube, a beautiful system was produced by forming a small cubical film in the centre of the polyhedron (*vide* "Transactions," vol. xxiv. Plate xxxiv. Fig. 4). This, it was seen, could be produced either by dipping the cube a second time in the solution, or by blow-

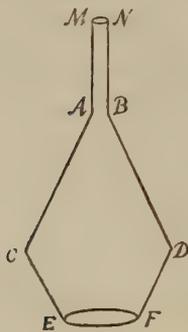
ing a small bubble in the centre of the polyhedron. A precisely similar result may be obtained with the similar system within the cone, and that by either of the same methods. The result of causing this system to ascend the cone is strikingly beautiful; the cube becomes narrowed above, assuming the form of a truncated pyramid; and if caused still further to ascend, the horizontal film at its apex entirely disappears, and a perfect pyramid is produced, resembling that shown in Plate xxxiv. Fig. 10, of vol. xxiv. of "The Transactions."\*

At the same instant that this takes place, the four original bubbles become united above the pyramid in a vertical straight line, but this system being unstable, the line quickly passes into a vertical plane film.

As the system continues still further to ascend the cone, it generally settles into one or other of the forms already described, *i.e.*, two, three, or four similar hollow figures united by one or more plane vertical films. At any time before the pyramid disappears, we can again reduce the system to its original form by blowing air in at  $MN$ . This experiment is best made with a cone whose vertical angle is more obtuse than that described.

### III. On the Motions and Figures of Equilibrium of Films within Double Hollow Cones.

In these experiments a glass vessel was used of the form of that in the annexed diagram, consisting of two cones united by their bases. The vessel, as in the last case, can be closed or opened by a plug fitting at  $MN$ . When  $MN$  is open, and the vessel is dipped into the soap solution, it gives, when raised, a single *plane* film at  $EF$ . When the vessel is dipped so that the liquid rises in it a little below  $CD$ , and  $MN$  is then entirely or nearly closed, we obtain, on raising the vessel so that the liquid runs out, a convex film below  $CD$ , and a concave one at  $EF$ .



If we make the same experiment with the vessel dipped so deep that the liquid rises above  $CD$ , we obtain a variety of regular or irregular systems of films, which, when destroyed, often leave a fine concave film above  $CD$ . On opening  $MN$ , this concave film rises to  $AB$ , the rings or bands of colour rising to higher orders by the thickening of the film. This concave film is sometimes formed along with and above the regular or irregular systems of films, and separate from them.

When a film is formed exactly at  $CD$ , it is fixed, and moves neither towards  $AB$  nor  $EF$ ; but if it be made slightly to advance towards  $AB$ , it will rise to  $AB$ , or if made to advance in the direction of  $EF$ , it will fall to  $EF$ , and there become stationary. Regular binary, ternary, or quaternary systems, similar to those already described as formed within the single cone (the quaternary system

\* The so-called cube is of course more or less a truncated pyramid from the first, owing to the conical form of the vessel used.

having the plane film in its centre, sometimes vertical, sometimes horizontal), may be produced by simply dipping the vessel in the solution and raising it with  $MN$  closed; but these systems can all be obtained with much greater ease and certainty by blowing small bubbles within the vessel in the same way as was done with the single cone. Figs. 11 to 14 exhibit these systems respectively. The system formed by thus inserting two bubbles of equal size is shown in fig. 11; that by inserting three such, in fig. 12; that by inserting four, in fig. 13. This last system is, like that formed within the single cone, united in its centre by a plane quadrilateral film, which, as in the experiment with the single cone, may be obtained in the horizontal instead of in the vertical position, by introducing a plane film at or near  $CD$ , before inserting the bubbles, *vide* fig. 14. All the preceding experiments with single and double cones may be performed with cones of any angle, the forms of the curvatures of the figures produced being modified accordingly.

These experiments may likewise be further varified by using cylindrical tubes instead of cones. If a system of three bubbles be adopted, a small bubble inserted in their centre will take the form of a triangular prism;\* with four, a cube; with five, a pentagonal prism; with six, a parallelopiped, and so on; or two cubes, two parallelopipeds, &c., may be inserted one above the other, forming systems analogous to that in Plate xxxiv. Fig. 5, of vol. xxiv. of "The Transactions." If two cubes be thus inserted into the system formed in the single cone, and the system be then allowed to rise in the cone, the lower cube will retain more or less its quadrilateral form, while the upper passes into the pyramid—a state of things which may be compared to that previously described with the wire pyramid, and shown in Plate xxxiv. Fig. 11, of vol. xxiv. of "The Transactions."

When a cylinder is used, the vertical lines of the prism, cube, &c. thus introduced are straight lines, their horizontal lines being outwardly convex.

#### IV. *On some Miscellaneous Experiments on Paraboloidal, Conical, and Cylindrical Films.*

If the cone previously described be dipped in the solution, so that it is filled to  $AB$ , and  $MN$  be then closed, we obtain, on raising the cone, a paraboloidal film at  $CD$ , and a plane film about half way up the cone (fig. 15). The paraboloid, as the excess of fluid escapes from its apex, becomes first a hemisphere, then the segment of a sphere, and then bursts.

If two very convex films be placed in contact with one another, they will become united by a more or less flattened film, which, as already shown in the previous paper on this subject (Vol. xxiv. p. 503, of the "Transactions"), will be plane, convex, or concave, according to the relative convexity of the bubbles

\* This figure is best obtained by inserting the small bubble before inserting the *third* of the large bubbles.

themselves. If we now draw the bubbles gently apart, they will form two truncated cones united by a small circular film at their apices, which may sometimes be reduced to a mere point before bursting, thus forming two perfect cones, as shown in fig. 16.

If, instead of bringing the two convex films into direct connection with one another, we unite them by means of a small bubble placed between them, this intermediate small bubble will assume the barrel shape shown in figs. 17 and 18. If we now decrease the distance between the large convex films, the small bubble will assume the flattened form of fig. 17. If we increase the distance between the large films by drawing them apart, the small bubble will become elongated, as in fig. 18. The larger end of the barrel will always be that in connection with the film of least convexity; in other words, with the film whose radius of curvature is the greater, and when the system is drawn apart so as to break the chain of connection, it is to this film (that, namely, of greater convexity) that the small bubble will always adhere.

The form of the barrel is always that of a more or less perfect cylinder, all of whose bounding lines are convexly curved.

When the large convex films have their radii exactly equal, the barrel-shaped cylinder will be perfectly symmetrical, *i.e.*, the circles at its upper and lower ends will be equal. When this is the case, or nearly so, the system can be drawn apart without suffering disruption till the ends of the cylindrical bubble become almost mere points.

To make the two large films of *exactly* equal convexity is of course impossible practically, but theoretically it would appear that in such a case the small central bubble ought to part connection with each of the large bubbles simultaneously, and fall, resuming its original form of a perfect sphere.

If a cylindrical or conical tube, such as that used in the previous experiments, be slightly dipped into the solution, and then gradually raised vertically, the film formed on its lower aperture will remain attached to the surface of the liquid. The curvatures of the film as we gradually raise the tube are very curious. At first, while the surface of attachment is large, the film is convexly curved; its sides then become straight lines; then concave, then, just before parting with the surface of the liquid, they become slightly convex above, concave below. In fig. 19 these changes of curvature are shown.

If the liquid into which we dip the tube be shallow and small in quantity (that, for example, contained in a watch-glass), it will be elevated bodily when we raise the tube, its whole surface becoming more and more convex as the film becomes more concave. If we insert a film half way up the tube before making this experiment, it will sink in the tube whilst the lower film is convex, and again rise when the lower film begins to become concave.

III.—*On the Temperature of the Common Fowl* (*Gallus domesticus*). By  
JOHN DAVY, M.D., F.R.SS. Lond. and Edin.

Read 17th February 1868.

During the last three years I have made a large number of observations on the temperature of the common fowl under different circumstances, the results of which I now beg leave to submit to the Society, with the hope that they may be considered not altogether uninteresting to the physiologist.

The fowls tried were chiefly of the pure Dorking breed. At the time they appeared to be healthy, and all in good condition. They had all the run of a field adjoining the poultry yard.

In all the trials the same thermometer was used,—each degree of which, that of Fahrenheit, was divided into ten parts, and had been warranted correct by the makers after comparison with a standard. The quantity of mercury in the bulb of the instrument was so small that in a minute or two, when introduced into the rectum of the bird, it reached the maximum; and in every instance the rectum was the part of which the temperature was ascertained, presuming it to be there the same as that of the interior of the body generally.

Though the temperature of the air was mostly noted down, as well as other circumstances likely to affect the results, I do not think it necessary to enter into minute details respecting them, partly for the sake of brevity, and partly from their not appearing to influence materially the results. I may, however, remark that the observations were mostly made, whatever the season of the year, between 10 and 11 A.M., and that in the majority of instances the birds had been kept in confinement during the night and early morning, and had not been fed since the day preceding.

1. *Of the Average Temperature.*

The total number of observations made during the whole period on birds of different sexes and different ages, varying in age from five weeks to five years, the majority from six months to two years, amounted to 163; the average temperature deducible from them was  $107^{\circ}81$ .

2. *Of the Temperature of the Male and Female.*

The number of males tried was 68, of females 95. The average temperature from the former was  $108^{\circ}39$ ; from the latter,  $107^{\circ}36$ . The highest temperature observed in any one instance of the males was  $110^{\circ}$ —this in August, in sultry weather, when the thermometer in the shade was  $81^{\circ}$ ; the lowest was  $106^{\circ}5$ , whilst the highest noticed in the other sex was  $109^{\circ}25$ , the lowest  $105^{\circ}$ —this in a hen on the sixteenth day of her incubation.

3. *Of the Temperature of the Sexes previous to Maturity.*

The term maturity is used as implying the stage at which the female begins to lay and the male to exercise the generative faculty—in the instance of the former about the sixth or seventh month, in that of the latter a month or two earlier.

The number of both sexes tried was 31,—of males, 16 ; of females, 15,—yielding conjointly an average temperature of  $108^{\circ}\cdot5$ , and separately, in the instance of the males,  $108^{\circ}\cdot4$  ; in that of the females,  $108^{\circ}\cdot66$ . The highest male temperature was  $109^{\circ}\cdot25$ , the lowest  $107^{\circ}\cdot5$  ; whilst the highest female temperature was  $109^{\circ}\cdot5$ , the lowest  $108^{\circ}$ .

4. *Of the Temperature of the Mature Male.*

The temperature of the same male was taken during one year monthly, with the omission of one month. At the beginning the bird was two years old ; at the end, when three years old, it weighed nine pounds and a half. During the whole time it seemed in vigorous health. The results were the following :—

In October, . . . . .	$109^{\circ}\cdot5$	In May, . . . . .	$108^{\circ}\cdot5$
November, . . . . .	$109^{\circ}\cdot5$	June, . . . . .	$109^{\circ}\cdot5$
December, . . . . .	$108^{\circ}\cdot5$	July, . . . . .	$108^{\circ}\cdot75$
January, . . . . .	$108^{\circ}\cdot25$	August, . . . . .	$110^{\circ}\cdot00^*$
February, . . . . .	$107^{\circ}\cdot75$	September, . . . . .	$108^{\circ}\cdot77^{\dagger}$
April, . . . . .	$108^{\circ}\cdot25$		

affording an average temperature of  $108^{\circ}\cdot77$ .

5. *Of the Temperature of the Female whilst Laying.*

The number of females tried was 12, varying in age from six or seven months to four or five years. The average temperature reducible from them was  $107^{\circ}\cdot5$  ; the lowest temperature noticed in any one was  $105^{\circ}\cdot5$ —this in a hen five years old ; the highest,  $108^{\circ}\cdot5$ .

6. *Of the Temperature during Incubation.*

Of 14 sitting hens tried, the average temperature was found to be  $107^{\circ}$  ; the lowest temperature observed was  $105^{\circ}$  ; the highest,  $109^{\circ}\cdot5$ .

The average weight of the fowls at the beginning of sitting was 5 lbs. 13 oz. ; the average loss at the end of the process was 1 lb. 7 oz. ; the smallest loss in any one instance was 4 oz.—this in that of the fowl the temperature of which at the end was  $109^{\circ}\cdot5$  ; the greatest, and the same in two instances, was 1 lb. 8 oz., and of both the temperature was  $107^{\circ}$ .

\* Thermometer in shade,  $81^{\circ}$ .

† Moulting.

7. *Of the Temperature during Moulting.*

Of 10 hens undergoing this change the average temperature was  $108^{\circ}44$ ; the highest in any one instance was  $109^{\circ}5$ ; the lowest,  $107^{\circ}$ . It is noteworthy that the highest degree was observed in the middle stage of the process, when the surface of the abdomen was nearly destitute of feathers.

*Conclusions.*

The results, I would beg to remark, are offered merely as approximations as such they seem to show—

1. That the temperature of the common fowl is  $107^{\circ}81$ .
2. That the temperature of the sexes before maturity is comparatively high, being  $108^{\circ}5$ , whilst that of the two sexes at this stage varies very little.
3. That the temperature of the male, on the whole, irrespective of any particular age, is higher than that of the female, being as  $108^{\circ}39$  to  $107^{\circ}3$ .
4. That the temperature of the fully matured male (a single instance) is  $108^{\circ}77$ .
5. That the temperature of the laying hen is  $107^{\circ}4$ .
6. That during incubation the temperature falls, and is as low as  $107^{\circ}$ .
7. And that during moulting it rises to  $108^{\circ}44$ .

The variations of temperature, from the highest to the lowest, noticed in the several instances, are less perhaps than might have been expected. No doubt, they were connected with peculiarities of circumstance, which very careful observation might possibly have detected,—circumstances of weather, not only as to temperature, but also as to moisture of air, degree of exposure, and varying strength of wind, not to insist on other conditions, such as the quantity of food taken, the period of fast, amount of exercise, the precise state of health. The very few observations I have made, tending at all to illustrate modifying circumstances in relation to temperature, are the following:—

A hen, seven months old, after having been confined in a basket thirty-six hours without food, the thermometer part of the time below the freezing point, was of the temperature  $106^{\circ}5$ .

Another of the same age, confined without food twenty-six hours, the thermometer during the time between  $40^{\circ}$  and  $50^{\circ}$ , was  $107^{\circ}$ .

An old hen, the leg of which had been broken two or three days previous to trial, and had ate little since the preceding evening, was  $105^{\circ}$ .

A fowl labouring under difficulty of breathing of some hours, was  $100^{\circ}$ .

A male bird, in full vigour, of the temperature  $108^{\circ}75$ , was let loose, and driven till it stopped, after four minutes,—now its temperature was found to be  $109^{\circ}$ .

On the principal results, as given in the summary, I shall comment but little. Some of them, especially the near equality of the temperature of the sexes before maturity, and the lowering of the temperature of the female at the period of laying, may be viewed as physiological problems; others, as the loss of weight during incubation, with a fall of temperature, and the contrary as to temperature during the period of moulting, seem in harmony with what might be expected according to the theory of animal heat. In the one case, there being a loss of substance of the individual in connection with diminished nutriment, without apparently a febrile disturbance of health; whilst in the other case, it would seem there was such a disturbance, with an elevation of temperature, such as is witnessed in febrile diseases.

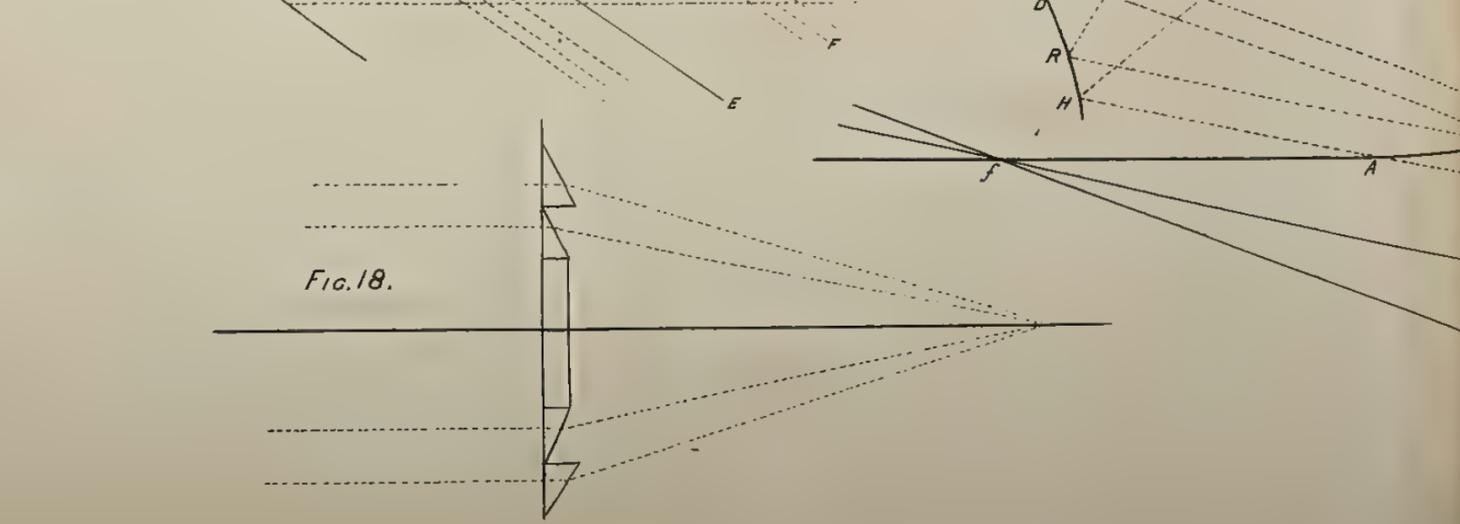
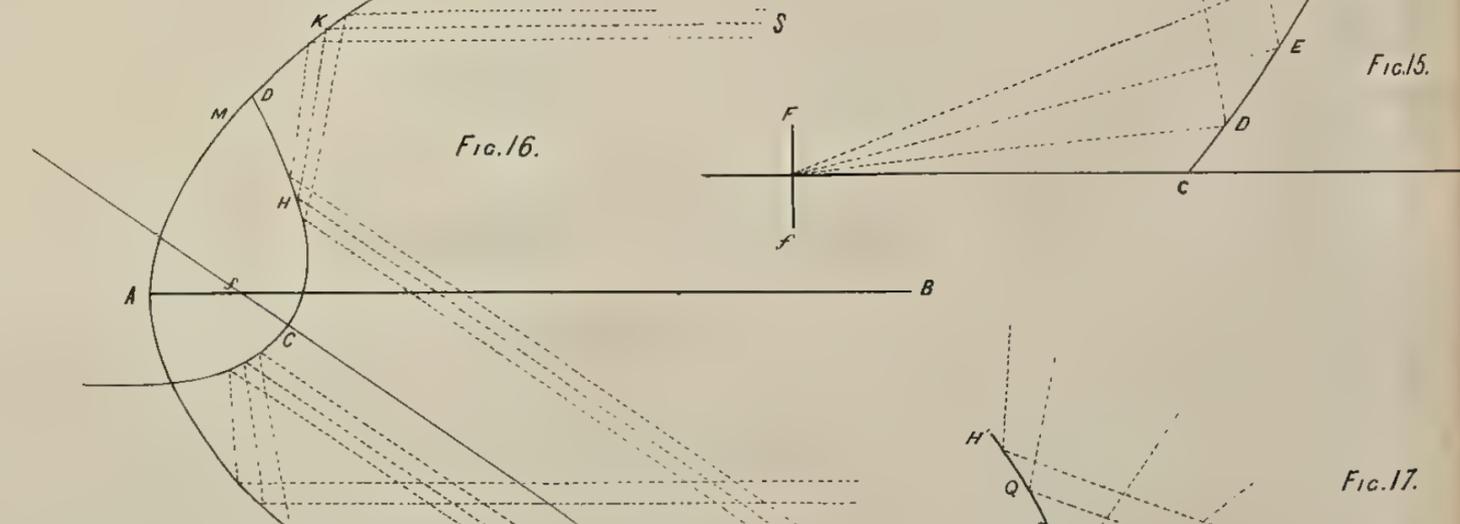
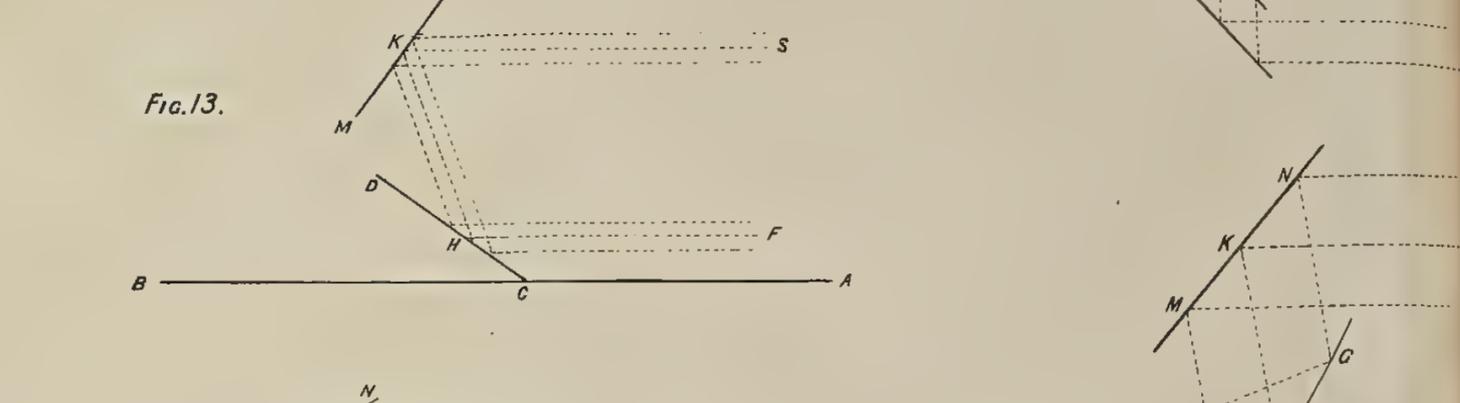
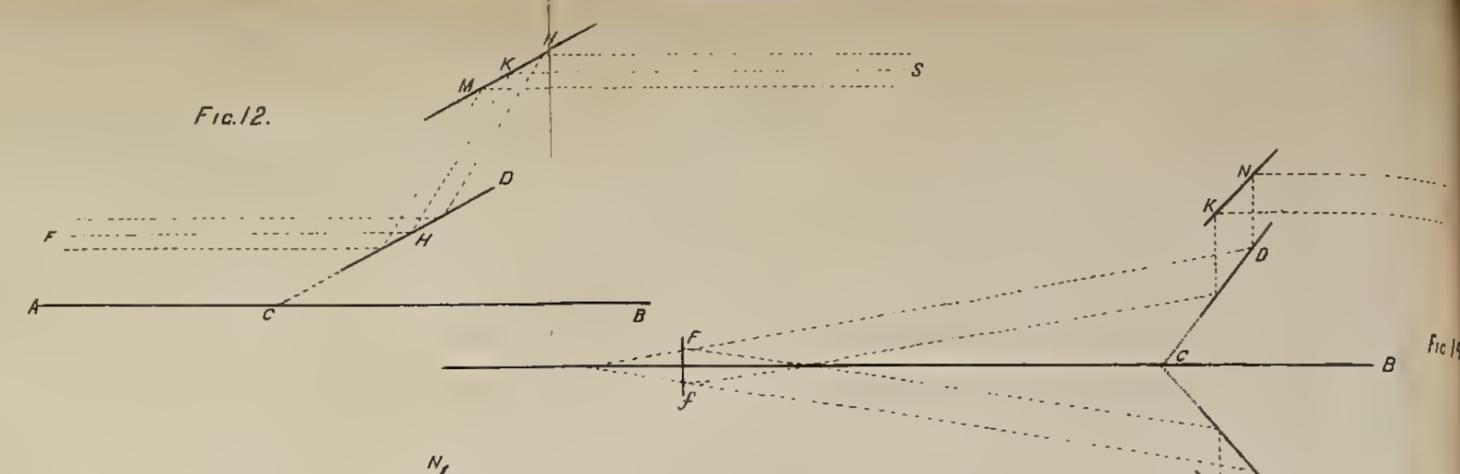
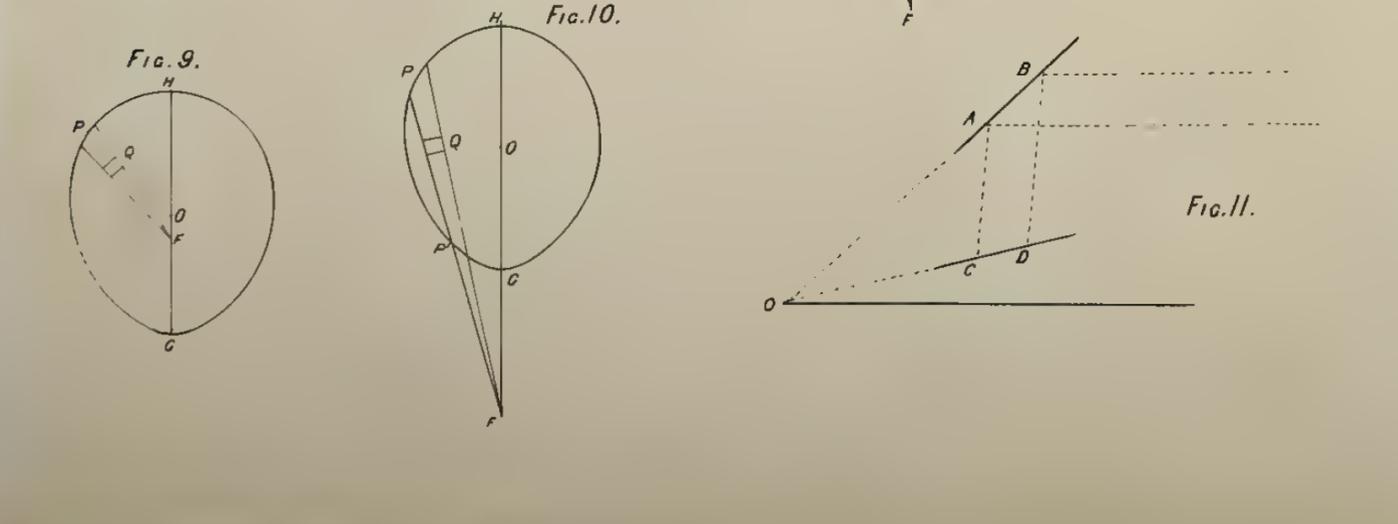
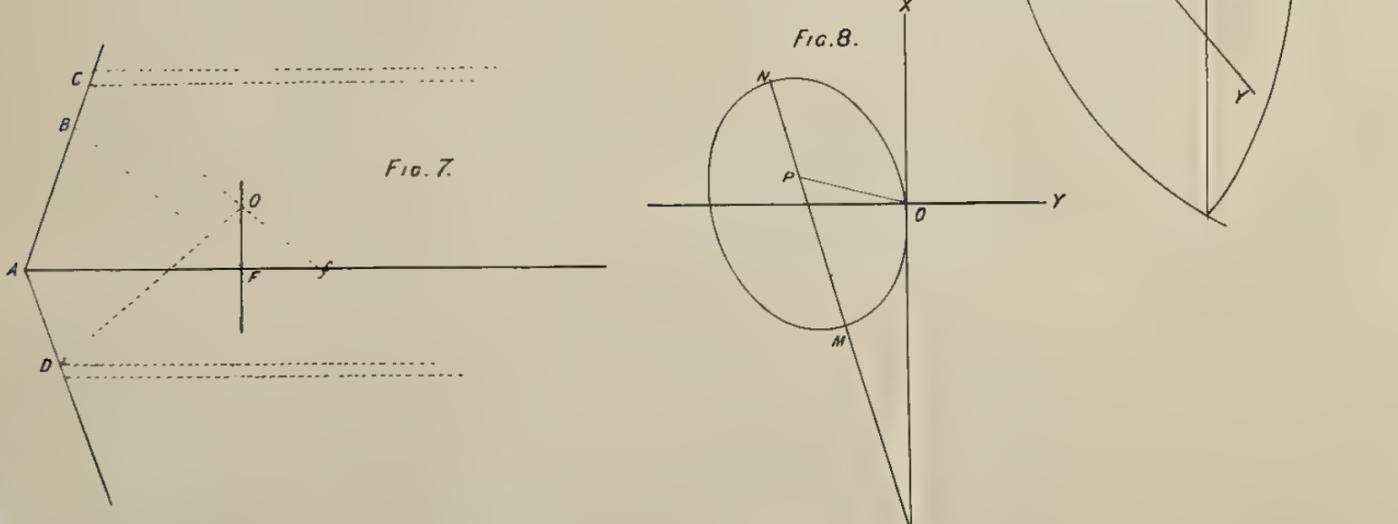
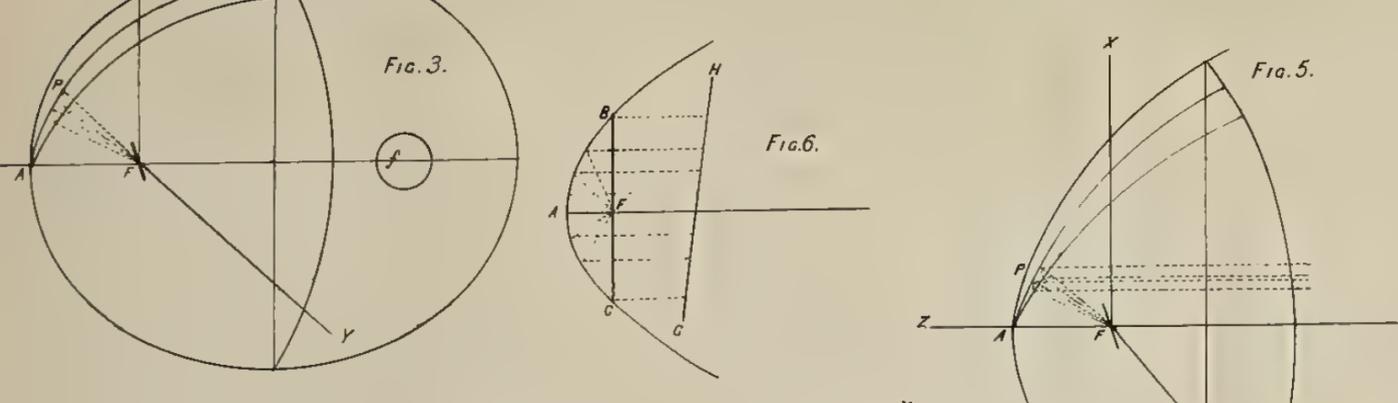
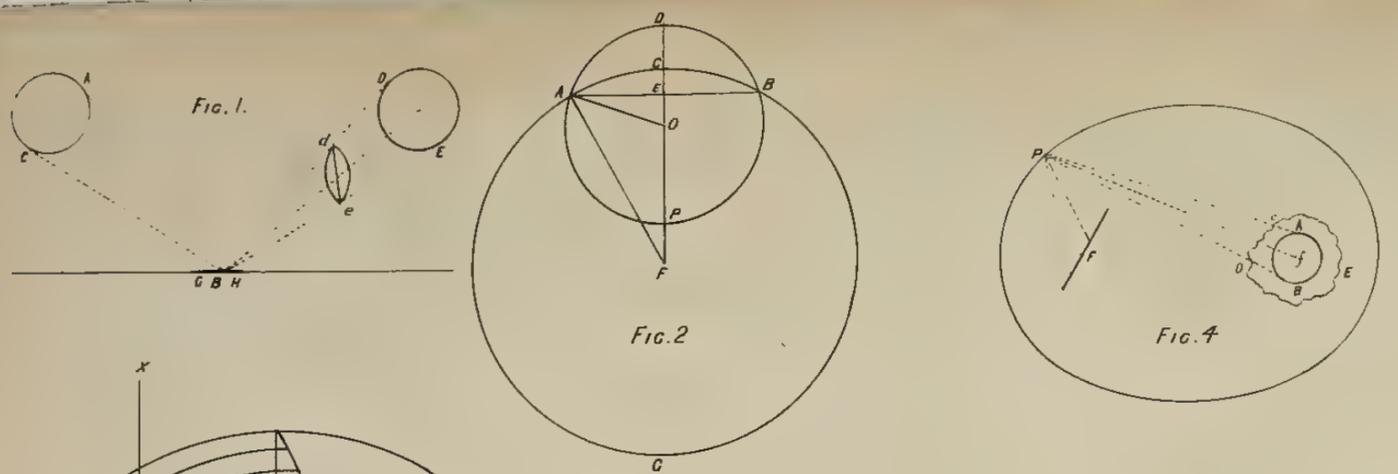
The change of temperature that takes place in the chick on quitting the egg is remarkable, and, as it appears to me, strongly in support of that view of animal heat, in which respiration and the formation of carbonic acid by the union of oxygen with carbon, is considered its principal source. In the egg, just before hatching, the chick is of a temperature rarely exceeding  $100^{\circ}$ , and that derived more from the incubating mother than from the organic changes in progress; but no sooner is the hatching completed, and the young bird freely respire, than there is a sudden elevation of temperature. In one instance, in which I had an opportunity of watching a gosling in the hatching act, the temperature actually rose, and that suddenly, from about  $100^{\circ}$  to  $106^{\circ}$ , after the manner of a hibernating animal, such as the dormouse, in which in passing from its torpid to a state of activity without taking food, there has been, as I have noticed, a rise of temperature from  $56^{\circ}$  to  $99^{\circ}5$ .\*

It is also remarkable how soon the young fowl, after becoming tolerably fledged and capable of securing adequate food, which would seem to be simultaneous with the consumption of the internal provisional yolk, its earliest nourishment, it attains a comparatively high temperature. Thus of 11—7 of them thirty-one days old, 4 thirty-five—the average temperature in the middle of October was  $108^{\circ}73$ ; the lowest in any one instance  $108^{\circ}$ , the highest  $109\cdot5$ : and I have found the temperature of nestlings also comparatively high,—a young swallow, fully fledged, just after being taken from the nest on the 28th July, was  $108^{\circ}$ .

Dr William Edwards, in his very interesting work, "On the Influence of the Physical Agents on Life," has come to the conclusion that "the power of producing heat in warm-blooded animals is at its maximum at birth, and increases successively until adult age,"—a conclusion which seems to me questionable, and requiring, if admitted at all, to be received with many restrictions. Some of the results I have obtained are opposed to it, and others might be mentioned tending to invalidate it.

\* See *Physiological Researches* (1863), p. 85.





IV.—*On the Burning Mirrors of Archimedes, with some Propositions relating to the concentration of Light produced by Reflectors of different forms.* By JOHN SCOTT, Esq., Tain. (Plate III.)

(Read 6th January 1868).

As the reputed fact of ARCHIMEDES having burned the Roman ships engaged in the siege of Syracuse, by concentrating on them the solar rays, has not only been doubted but disbelieved by some of the most eminent scientific men, I shall briefly give the evidence on both sides.

The burning of the ships of MARCELLUS is mentioned by most of the ancient writers who refer to the machines which ARCHIMEDES employed in the defence of his native city, and their statements have been repeated by succeeding authors, without any doubts having been expressed until comparatively recent times. Our earliest authorities on the subject are DIODORUS SICULUS, LUCIAN, GALEN, DION, and PAPPUS. At a later period the architect ANTHEMIUS, of Tralles, in a fragment entitled *περι παραδοξων μηχανηματων* (Wonderful Machines), not only dwells particularly on the burning mirror of ARCHIMEDES, but adds, besides, that it was universally admitted in his time that ARCHIMEDES had destroyed the Roman fleet by means of burning mirrors. It is also mentioned by HERO, a writer on military engines—about the middle of the seventh century—and by EUSTATHIUS, ZONARES, and TZETZES, who flourished in the twelfth. The last two have transmitted to us passages extracted from the work of PAPPUS on the siege of Syracuse, which was then extant, but has since disappeared. We give that by TZETZES as the more definite and circumstantial:—“When MARCELLUS had placed the ships a bow-shot off, the old man (ARCHIMEDES) contrived a hexagonal mirror. He placed at proper distances from the mirror smaller mirrors of the same kind, and which were moved by means of their hinges and certain square plates of metal. He placed it in the midst of the solar rays at noon both in summer and winter. The rays of the sun being reflected by this, a dreadful fire was excited on the ships, which reduced them to ashes at the distance of a bow-shot.”

In the sixteenth century mirrors similar to that of ARCHIMEDES seem to have engaged the attention of LEONHARD DIGGES, and of Baron NAPIER of Merchiston. At a subsequent period Father KERSHER took up the same subject, prosecuting it with such assiduity that he travelled to Sicily to examine the coast in the vicinity of Syracuse, and came to the conclusion that ARCHIMEDES might have opportunities of placing his mirror within 30 yards of the ships. He also mentions in his “*Magica Catoptrica*,” as the result of experiments which he had performed, that the superimposed rays reflected from five plane mirrors at the distance of

more than 100 feet, produced a heat which could scarcely be endured. Apparently convinced of the practicability of the achievement by means of plane mirrors, he entreats future mathematicians to prosecute the subject. BUFFON, following in his steps, completely established the fact that combustible materials can be set on fire at distances corresponding to the accounts we have of the mirror of ARCHIMEDES. This he effected by means of a combination of plane reflectors, consisting of ordinary looking-glasses, 8 inches by 6, attached to a single frame, each glass, as well as the supporting frame, being capable of motion in every direction. With forty of these glasses he set on fire tarred beech at a distance of 66 feet. A plank, smeared with tar and brimstone, was ignited at 126 feet by 98 glasses. A combination of 128, with a clear sun, inflamed very suddenly a plank of tarred fir at 150 feet, the conflagration springing up at once over a space of 16 inches in diameter—the whole reflected image of the sun at that distance. In addition to these experiments made about the beginning of April, others were exhibited with the summer sun, by which wood was kindled at 200 and 210 feet, and silver and other metals were melted at distances varying from 25 to 40 feet.

Let us now consider the evidence on the opposite side of the question. DESCARTES and others have treated the whole affair as fabulous, from the belief that the burning glass must have consisted of a single spherical or parabolic reflector. But since no mention is made of the kind of specula employed by ARCHIMEDES, such objections, after the successful experiments of BUFFON, necessarily become irrelevant. Another and quite different ground of doubt has arisen from the circumstance, that POLYBIUS, LIVY, and PLUTARCH make no mention of the destruction of the Roman fleet by means of burning glasses, although they describe somewhat in detail the ballistæ and other military engines constructed by ARCHIMEDES to resist the assailants. How far the silence of the forementioned writers should be taken as negative evidence, it is not easy to determine. It seems to indicate, at least, that the damage effected by the burning mirror was confined to comparatively narrow limits; for, if prominent in the defence, we naturally expect that it would have received special notice. But allowing that the fleet sustained no serious injury from this source, some fact must be left sufficient to account for the belief prevalent among ancient authors in favour of the achievement. Nothing less, I conceive, will suffice than to admit that ARCHIMEDES made an attempt to destroy the Roman fleet in the manner described. Such an admission, however, implies that he must have previously tried his combination of reflectors in private, and was able to ignite combustible substances at considerable distances. The mere attempt to bring an artillery so singular and subtle to bear on the fleet is in itself a conclusive proof that an experiment, similar to those exhibited by BUFFON in the Jardin-du-Roi at Paris, had been successfully performed 2000 years before within the walls of Syracuse. We can no more suppose the contrary than believe that any of the nations of modern Europe would send

into the field of actual warfare a novel piece of ordnance without subjecting it to a previous trial. Though on this hypothesis the element of success as an engine of war is questionable, the invention of the mirror as a fact in the history of science remains entire. Neither should it be forgotten, as perhaps a reason for the silence of POLYBIUS, LIVY, and PLUTARCH, that with the fall of Syracuse and the death of the illustrious inventor, all definite information relating to the scientific principles of the mirror seems to have perished—a result not improbable, when we consider that its application to the art of war would induce the original possessors to retain its construction, as far as possible, a secret in their own hands. Finally, it is an admitted axiom in estimating historical evidence, that the silence of one author respecting an event is never considered sufficient to invalidate a plain and consistent statement of that event made by another. That our conclusions should be formed in strict accordance with the principle enunciated, may be made apparent by striking and well-ascertained facts, some of which have been inaccurately recorded, and others altogether omitted by the most reliable contemporary historians. As an instance of the former, modern authorities maintain that the account given by LIVY of the route by which HANNIBAL conducted his army across the Alps cannot be reconciled with that by POLYBIUS, and an extract from Sir CHARLES LYELL'S "Principles of Geology" will show an historical omission equally inexplicable. Speaking of the first eruption of Vesuvius, he says, "The younger PLINY, although giving a substantial detail of so many physical facts, and describing the eruption and earthquake and the shower of ashes which fell at Stabiae, makes no allusion to the sudden overwhelming of two large and populous cities, Herculaneum and Pompeii. In explanation of this omission, it has been suggested that his chief object was simply to give TACITUS a full account of the particulars of his uncle's death. It is worthy of remark, however, that had the buried cities never been discovered, the accounts transmitted to us of their tragical end might well have been discredited by the majority, so vague and general are the narratives, or so long subsequent to the event. TACITUS, the friend and contemporary of PLINY, when adverting in general terms to the convulsions, says merely cities were consumed or buried. . . . Suetonius, although he alludes to the eruption incidentally, is silent as to the cities. They are mentioned by MARTIAL in an epigram as buried in cinders; but the first historian who alludes to them by name is DION CASSIUS, who flourished about a century and a half after PLINY."

Returning to BUFFON'S combination of reflectors—when the focus had to be changed, a numerous staff of assistants required about half an hour to re-adjust the mirrors. After all, the superposition of the reflected light would be imperfect, each operator being liable to mistake the deviation of the image reflected by some other glass for that of which he had charge. PEYRARD has appended to his translation of the works of ARCHIMEDES a memoir of his own, in which he calls atten-

tion to these defects, and to remedy them proposes that a telescope and somewhat complex apparatus be attached to each reflector, that with fewer hands the adjustment might be rendered more accurate and speedy.

It is obvious that the improvement suggested by PEYRARD is only partial and its success doubtful. I conceive the Archimedean mirror to have been a compound reflecting apparatus, free from these defects, capable of being directed by one eye and guided by a single hand. From TZETZES we learn that the mirror was hexagonal, that like BUFFON'S it consisted of a combination of reflectors, and that at proper distances from the outer mirror were placed other smaller ones of the same kind. This last peculiarity of the Archimedean mirror has no parallel in that of BUFFON; and yet the arrangement of the smaller mirrors, at proper distances from the larger, indicates that the relation of the two kinds to each other formed an essential feature of the combination. Although the above passage conveys no information respecting the nature of the specula, there is such a description of the connection of the parts as an intelligent observer might carry away, and yet be unacquainted with the scientific principles involved in the construction.

In the sequel (Arts 13 and 14) we show how larger and smaller specula, all of the same kind, can be so connected as to form a single compound reflector capable of concentrating on a single spot the reflected rays, and of darting them instantaneously in any direction, when they will produce the effects ascribed to the mirror of ARCHIMEDES. The results thus being the same, and the construction of the combination coinciding with the description given by TZETZES, we therefore infer that the real principle of the Archimedean mirror has been attained, and that the accounts which have come down to us respecting it are in the main authentic. This will be brought out more fully after the following general propositions have been considered.

For the historical facts contained in the preceding, I am chiefly indebted to PEYRARD'S edition of the "Works of Archimedes," and to the article "Burning Glasses," in the Encyclopædia Britannica.

ARTICLE I.—Prop. *When the Light emanating from a Luminous Sphere of small angular diameter falls on a very small Plane Mirror; to find the Intensity of the reflected Light at any distance from the Mirror.*

Let GH (fig. 1, Plate III.) be the mirror, B the point of reflection, AC the small luminous sphere, and DE the plane on which the reflected light falls. It follows, as a consequence of the equality of the angles of incidence and reflection, that the angle which DE subtends at B is equal to that which AC subtends, DE being supposed perpendicular to the cone of rays reflected from B.

Whatever may be the form of the infinitesimal mirror at B, the perpendicular section of the reflected rays, at all finite distances from B, is a circle.

Let  $\alpha$  = the angle ABC = DBE.

$r$  = radius of the sphere AC.

$c$  = the distance from B, measured along the slant side of the cone of rays, at which the diameter of the circle of reflected light is equal to unity.

$I$  = the intensity of the light radiating from the sphere at its surface.

$I'$  = intensity of the light at the mirror, on a plane perpendicular to the axis of the cone of rays falling on B.

$\therefore kI'$  = intensity of the reflected light at the mirror,  $k$  being a constant less than unity.

$d$  = distance of the centre of the sphere from B.

$d'$  = distance of DE from B, measured along the slant side of the cone of reflected rays.

$\delta A$  = sectional area of the light incident on small mirror at B.

$$\text{Area of the circle DE} = \pi d'^2 \sin^2 \frac{\alpha}{2} \quad (1),$$

$$\text{,,} = \frac{\pi d'^2}{4c^2} \quad (2),$$

because  $c : d' :: \frac{1}{2} : \text{radius of the circle DE}$ .

Since the intensity of the light, emanating from the surface of a luminous sphere and falling on a concentric spherical surface, is inversely as the square of the distance from its centre,

$$r^2 : d^2 :: I' : I \quad \therefore I = \frac{d^2 I'}{r^2} \quad (3).$$

But  $\sin \frac{\alpha}{2} = \frac{r}{d} = \frac{1}{2c} \quad \therefore I = \frac{I'}{\sin^2 \frac{\alpha}{2}} \quad (4)$

$$= 4c^2 I' \quad (5).$$

$$\text{Intensity of the reflected light at DE} = \frac{kI' \delta A}{\text{area of circle DE}} \quad (6),$$

$$\text{,,} = \frac{kI' \delta A}{\pi d'^2 \sin^2 \frac{\alpha}{2}} \quad (7),$$

$$\text{,,} = \frac{4c^2 kI' \delta A}{\pi d'^2} \quad (8).$$

When the plane on which the reflected light falls is not perpendicular to the

axis of the cone of rays, as  $de$  (fig. 1), the intensity of the light at  $e$  will exceed the intensity at  $d$  from the greater obliquity of the rays at the latter point. But if the angle  $DBE$  be small, and the plane  $de$  cut the axis of the cone at a considerable angle, the intensity will be nearly uniform over the whole ellipse  $de$ ; and the centre of the ellipse may be viewed as situated in the axis of the cone, because the elongated cone approaches nearly to a cylinder.

$$\therefore \text{intensity on } de = \frac{kI \delta A}{\text{area of ellipse DE}} \quad (9).$$

In the case of the sun's light the above formulæ will give pretty accurate approximations, since  $a = 32'$  and  $c = 107.4$  nearly . . . . . (10).

ARTICLE 2.—PROP. *When a Cylindrical Beam of Solar Light is reflected from a Plane Mirror; to find the Intensity on a Plane Surface perpendicular to the direction of the reflected Beam, and at any distance from the Mirror.*

Since the rays which emanate from any single point in the sun's disc may be considered as perfectly parallel, however large the mirror, it follows that those from the centre of the disc will, after reflection from the mirror, form a perfectly cylindrical beam of parallel rays, and will cast on the given plane a circle of light of uniform intensity.

Let  $ABG$  (fig. 2) represent this circle, each point in its area is the centre of a circular image of the sun's disc reflected from a corresponding point, or infinitesimal area, of the mirror.

If  $O$  be one of these points, and the circle  $ADBP$  the image of the sun's disc at the given distance, or, in other words, the base of a cone of rays whose apex is at the mirror, as shown in fig. 1, Art. 1, the illumination or intensity of the light at  $O$  will be that due to the superposition of all the images of the sun's disc whose centres fall within the area  $ACBPA$ ; for none of the images of the sun's disc, whose centres fall without the above area, can extend so far as  $O$ . The point  $O$  is therefore illuminated by a portion of the incident beam equal in area to  $ACBPA$ , every increment of which, after reflection, gives rise to a conical pencil of rays, a part of whose base overspreads the point  $O$ .

Let  $I'$  = intensity of the solar light on a plane perpendicular to the direction of the incident beam.

Let  $AO = r$ ,  $FA = \rho$ ,  $FO = z$ ,  $AFO = \theta$ ,  $AOC = \phi$ , and  $u_1 u_2 u_3 u_4 u_5$ , &c., the sectional areas, at the mirror, of the respective pencils whose light overspreads the point  $O$ . Whatever may be the form of these small increments, the base of the cone of light to which they give rise will be a circle at all finite distances from the mirror, as shown Art. 1.



The preceding limiting cases are also evident from the geometrical consideration of the problem taken in connection with equation 2; and the same principles can be applied to find the intensity, whatever the shape of the perpendicular section of the incident beam.

ARTICLE 3.—When there are  $n$  mirrors similar to the preceding, the light from each making an angle of incidence  $i$ , with the perpendicular to the plane on which it is thrown, the intensity of the central spot in each of the preceding cases becomes—

$$\text{In 1st case, } nkI' \cos i, \quad . \quad . \quad . \quad . \quad (1).$$

$$\text{In 2d case, } \frac{nkI'g^2 \cos i}{r^2}, \quad . \quad . \quad . \quad . \quad (2).$$

ARTICLE 4.—Prop. *A Small Luminous Sphere has its centre in one of the foci of a Prolate Elliptic Mirror, to find the Intensity on any Small Plane surface situated in the other Focus.*

Let  $f$  (fig. 3) represent the luminous sphere.

$\beta$  = the angle which the small plane at F makes with FX: the axis of  $z$  coinciding with the axis of the mirror, and the plane of  $xz$  being perpendicular to the small plane passing through F; the axis of  $y$ , which is at right angles to the plane of  $xz$ , will therefore coincide with the small plane which passes through F.

$r$  = FP.

$\theta$  = the angle PFZ.

$\phi$  = the angle which the projection of  $r$  on the plane of  $xy$  make with FX.

$a$  = radius of the luminous sphere at  $f$ .

$V$  = the angle which  $r$  makes with the normal to the small plane at F.

$I$  = the intensity of the light at the surface of the small sphere.

Since the well-known differential of a volume  $r^2 \sin \theta d\theta d\phi dr$  has for its perpendicular section, at the surface of the spheroid,  $r^2 \sin \theta d\theta d\phi$ ; we may suppose the whole surface of the elliptic mirror to be divided into small areas, each of which receives from the sphere at  $f$ , and reflects to F, a pencil of light, whose perpendicular section, at any point P of the mirror, is  $r^2 \sin \theta d\theta d\phi$ .

Moreover, the intensity at P perpendicular to  $Pf = \frac{\alpha^2 I}{Pf^2}$  (by Art. 1, Equa. 3), and after reflection it becomes  $\frac{\alpha^2 k I}{Pf^2}$ .

The cone of rays reflected from the increment of surface at P, in the direction PF, will have expanded at F into a circle (perpendicular to the radius vector), whose radius by similar triangles is  $\frac{ar}{Pf}$  and area  $\frac{\pi a^2 r^2}{Pf^2}$ .

By Art. 1. Equa. 6, the intensity of the light on this circle at F =  $r^2 \sin \theta d\theta d\phi \times \frac{\alpha^2 k I}{Pf^2} \div \frac{\pi a^2 r^2}{Pf^2} = \frac{k I}{\pi} \sin \theta d\theta d\phi$ .

and  $\frac{kI}{\pi} \sin \theta \, d\theta \, d\phi \cos V =$  its intensity at F on the small plane, whose equation is  $z \cos \beta - x \sin \beta = 0,$

$$(1),$$

$$\cos V = \frac{z}{r} \cos \beta - \frac{x}{r} \sin \beta = \cos \beta \cos \theta - \sin \beta \sin \theta \cos \phi.$$

Substituting for  $\cos V$  its value we obtain for the whole intensity at F, on the small plane,

$$\frac{kI}{\pi} \iint (\cos \beta \cos \theta - \sin \beta \sin \theta \cos \phi) \sin \theta \, d\theta \, d\phi, \quad (2).$$

Denoting this integral by  $u,$

$$u = \frac{kI}{2\pi} \int (\cos \beta \sin^2 \theta + \sin \beta \sin \theta \cos \theta \cos \phi - \theta \sin \beta \cos \phi) \, d\phi + C.$$

Now  $\cot \theta = \tan \beta \cos \phi,$  when  $V = 90^\circ,$  that is, when PF coincides with the plane denoted by Equation 1,

$$\therefore \theta = \cot^{-1}(\tan \beta \cos \phi)$$

$$\sin \theta = \frac{1}{(1 + \tan^2 \beta \cos^2 \phi)^{\frac{1}{2}}}$$

$$\cos \theta = \frac{\tan \beta \cos \phi}{(1 + \tan^2 \beta \cos^2 \phi)^{\frac{1}{2}}}$$

Taking the integral between the limits  $\theta = 0,$  and  $\theta = \cot^{-1}(\tan \beta \cos \phi),$

$$\begin{aligned} u &= \frac{kI}{2\pi} \int \left\{ \frac{\cos \beta (1 + \tan^2 \beta \cos^2 \phi)}{1 + \tan^2 \beta \cos^2 \phi} - \cot^{-1}(\tan \beta \cos \phi) \sin \beta \cos \phi \right\} d\phi. \\ &= \frac{kI}{2\pi} \left\{ \int \cos \beta \, d\phi - \sin \beta \sin \phi \cot^{-1}(\tan \beta \cos \phi) + \int \frac{\sin \beta \tan \beta \sin^2 \phi \, d\phi}{1 + \tan^2 \beta \cos^2 \phi} \right\} \\ &= \frac{kI}{2\pi} \left\{ \int \cos \beta \, d\phi - \sin \beta \sin \phi \cot^{-1}(\tan \beta \cos \phi) - \int \cos \beta \, d\phi + \int \frac{\sec \beta \cdot d(\tan \phi)}{\sec^2 \beta + \tan^2 \phi} \right\} \\ &= \frac{kI}{2\pi} \left\{ \tan^{-1}(\cos \beta \tan \phi) - \sin \beta \sin \phi \cot^{-1}(\tan \beta \cos \phi) \right\} + C. \end{aligned}$$

and between the limits  $\phi = 0$  and  $\phi = 90^\circ,$

$$u = \frac{kI}{2\pi} \left( \frac{\pi}{2} - \frac{\pi}{2} \sin \beta \right),$$

$$= \frac{kI}{4} (1 - \sin \beta), \text{ which is the intensity at F of the light reflected from}$$

that portion of the spheroid bounded by the planes passing through the co-ordinate axes of  $+x$  and  $+z,$   $+y$  and  $+z,$  and the small plane produced.

Similarly  $\frac{kI}{4}(1 + \sin \beta)$  gives the intensity of the light, from that portion enclosed by the planes passing through the axes  $-x$  and  $+z,$   $+y$  and  $+z,$  and the small plane at F produced. Hence the intensity of all the light which can fall on the side of the small plane towards A (fig. 3).

$$\begin{aligned} &= 2 \left\{ \frac{kI}{4} (1 - \sin \beta) + \frac{kI}{4} (1 + \sin \beta) \right\}, \\ &= kI, \end{aligned} \quad (3).$$

Also  $\frac{kI}{2} \sin \beta$  is the intensity at F due to the light reflected from the part of the mirror intercepted between the small plane produced, and the co-ordinate plane of  $xy$ .

In the same manner it can be shown, that the intensity of the light concentrated at F, on the opposite side of the small surface, and reflected from the remaining portion of the spheroid, is also equal to  $kI$ .

These results are independent of  $a$ , the radius of the luminous sphere, and are equally true for all spheroids which have F for one of their foci, wherever the other may be situated.

It appears then, in conclusion, that the light emanating from a small luminous sphere, with its centre in one of the foci of a prolate elliptic mirror, produces at the other focus a nucleus of radiant light and heat, equal in intensity to the radiation at the luminous surface diminished by the quantity lost by reflection.

Again, putting  $\beta = 0$  (in Equation 2), we obtain intensity

$$= \frac{kI}{\pi} \iint \sin \theta \cos \theta \, d\theta \, d\phi, \quad . \quad . \quad . \quad (4).$$

Integrating between

$$\phi = 0, \text{ and } 2\pi;$$

intensity

$$= 2\pi \times \frac{kI}{\pi} \int \sin \theta \cos \theta \, d\theta,$$

$$= 2kI \int \sin \theta \cos \theta \, d\theta;$$

and between  $\theta^\circ$ , and zero,

$$= kI \sin^2 \theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

This expresses the intensity at the focus of the light reflected from a segment of the spheroid intercepted between the vertex and a plane perpendicular to the axis; and the intensity produced by a zone intercepted between two planes, perpendicular to the axis of revolution, is

$$kI (\sin^2 \theta - \sin^2 \theta') \quad . \quad . \quad . \quad . \quad (6).$$

ARTICLE 5.—The preceding proposition is true, independently of the size and form of the luminous body in the focus  $f$  (figs. 3 and 4).

Since radiant light and heat diminish in the inverse ratio of the square of the distance, it follows that the quantities received from circular areas of equal angular magnitudes are equal, whatever their absolute magnitudes, when the intensities at the radiating surfaces are equal. Taking this principle in connection with the fact, that a luminous surface appears equally bright when viewed at any angle, the light emanating from CD, part of the surface of DCE, will therefore have at F the same intensity as if it had proceeded from the small sphere AB (fig. 4). But the light reflected from P to F can only emanate from some part of the surface DC, which lies within the cone described by PA, revolving about Pf. Hence the

intensity at F of the light radiating from DC, and reflected at P, is equal to that which would have resulted from the sphere AB; and the same is true for every increment of the elliptic mirror. Therefore, the total illumination at F, from the luminous surface DCE, is equivalent to that from the small sphere AB.

From this and the preceding article, we infer that a zone or segment of an elliptic reflector may be used as a pyrometer. For if such a zone, contained between two planes perpendicular to the axis, be placed before an opening in a furnace, the place of the focus  $f$  falling within the heated body, the heat reflected to F may be reduced, by diminishing the breadth of the zone, until it can be measured by a Fahrenheit thermometer; and I, the intensity of the total radiation from any point  $f$  within the furnace, can be determined in degrees of Fahrenheit by Art. 4, Equa. 6.

ARTICLE 6.—Prop. *When a Parabolic Reflector has its axis directed to the centre of the Sun, to find the intensity of the converging Rays which fall on a small Plane Disc at the Focus.*

Let  $a$  = angular diameter of the sun, which is about  $32'$ ,

$c$  = the distance at which the reflected image of the sun expands into a circle equal to unity in diameter, being about  $107\cdot4$ ,

$r$  = PF (fig. 5),

$I'$  = intensity of the sun's rays at the earth's surface,

$I$  = intensity at the surface of the sun,

$u, k, \beta, \theta, \phi$ , and  $V$  = the same as in proposition (Art. 4).

Then  $\frac{\pi r^2}{4c^2}$  = area of the circle, which the light reflected at P occupies at F perpendicular to PF.

The intensity of the light reflected from the increment of surface at P on this circle by Art. 1, Equa. 6.

$$= r^2 \sin \theta \, d\theta \, d\phi \times kI' \div \frac{\pi r^2}{4c^2},$$

$$= \frac{4c^2 kI'}{\pi} \sin \theta \, d\theta \, d\phi;$$

and on the small disc at F, =  $\frac{4c^2 kI'}{\pi} \sin \theta \, d\theta \, d\phi \cos V$ .

$$\cos V = \cos \beta \cos \theta - \sin \beta \sin \theta \cos \phi,$$

$$\therefore u = \frac{4c^2 kI'}{\pi} \iint (\cos \beta \cos \theta - \sin \beta \sin \theta \cos \phi) \sin \theta \, d\theta \, d\phi, \quad (1).$$

Integrating as in Article 4, we obtain for the intensity of the light reflected from the corresponding sections of the parabolic mirror

$$c^2 kI' (1 - \sin \beta), \quad . \quad . \quad . \quad . \quad (2).$$

and

$$c^2 kI' (1 + \sin \beta), \quad . \quad . \quad . \quad . \quad (3).$$

Hence the total intensity at F (fig. 5) on the side of the plane disc towards

A, resulting from the light reflected from the segment of the paraboloid cut off by the plane of the disc produced,

$$= 2\{c^2kI'(1 - \sin \beta) + c^2kI'(1 + \sin \beta)\} = 4c^2kI' = kI \quad (4),$$

I being the intensity at surface of the sun.

But  $c = 107$  nearly, therefore the numerical value of this equation

$$= 45796kI'$$

nearly, which is a degree of concentration several times that of the most powerful burning glass ever constructed.

Again putting  $\beta = 0$  (in Equation 1),

$$u = \frac{4c^2kI'}{\pi} \iint \sin \theta \cos \theta d\theta d\phi;$$

and integrating as in Article 4, Equation 4,

$$u = 4c^2kI' \sin^2 \theta; \quad (5),$$

which gives the intensity at the focus of the light reflected from a segment of a paraboloid, intercepted between the vertex and a plane perpendicular to the axis; and the intensity produced by a zone, intercepted between two planes perpendicular to the axis, is

$$4c^2kI' (\sin^2 \theta - \sin^2 \theta') = kI (\sin^2 \theta - \sin^2 \theta') \quad (6).$$

Equation 5 shows that the concentration at the focus varies as  $\sin^2 \theta$ : it is a maximum when  $\theta = 90^\circ$ , and is independent of the parameter of the parabola. It may therefore be inferred that a reflector employed to detect the heat of the lunar rays should be as large a segment of a paraboloid as possible; and the same condition is essential in improving to its utmost limit the space-penetrating power of the reflecting telescope.

Again, suppose the parabolic mirror to extend to infinity, it can also be shown that the light concentrated at the focus on the other side of the small disc is equal to  $4c^2kI' = kI$ . What has been proved respecting the intensity at the focus is approximately true for every point on the plane of the small disc not farther from F than  $\frac{p}{8c}$ , the quantity  $\frac{p}{8c}$  being the radius of the sun's image reflected from the vertex of the paraboloid, and  $p$  the parameter of the generating parabola. Thus, in every position in space, when the axis of a parabolic mirror, whose extent of surface is not less than that cut off by a plane passing through the focus, is directed to the sun, a circle of radiant light and heat is formed equal in intensity to the radiation at the solar surface minus the quantity lost by reflection.

ARTICLE 7.—The intensity at the focus of a parabolic reflector is independent both of the form and distance of the luminous body.

Let BAC (fig. 6) represent a section of the parabolic reflector, and GH that of a luminous surface of uniform intensity; it can be shown, as in the case of the spheroid (Art. 5), that the concentration at F produced by the only rays which can fall on it, namely, those emanating from GH parallel to the axis AF, is equal to the intensity at the luminous surface GH, minus the quantity lost by reflection. It is evident that the section of the luminous body must not be less than CB.

ARTICLE 8.—Prop. *When the Axis of a Mirror in the form of a Right Cone is directed to the centre of the Sun, to find the Intensity of the reflected Light on any point in a Plane placed perpendicular to its Axis.*

Let CAD (fig. 7) represent a section of the mirror,

O the point on which we wish to determine the intensity of the reflected light.

Every small increment of the mirror gives rise to a cone of rays which casts an ellipse of light on the plane at F, the major axis of which passes through the point F. The light of all these ellipses, whose centres fall within a certain distance of the point O, will overspread it and increase the intensity at that point.

If P (fig. 8) be the centre of one of these ellipses NOM, considerably magnified, whose circumference passes through O, then P is a point in the curve within which must fall the centres of all the ellipses whose light can overspread O.

To find the equation to this curve,

let  $FP = \rho$   
 $FO = z$

$\alpha$  and  $\beta$  = the co-ordinates of P, referred to rectangular axes whose origin is at O,

$\theta$  = the angle PFO.

Now  $a^2y^2 + b^2x^2 = a^2b^2$  is the equation to the ellipse NOM, the centre being the origin.

When referred to the axes OX and OY; by substituting,

$$y = (y' - \beta) \cos \theta - (x' - \alpha) \sin \theta,$$

$$x = (y' - \beta) \sin \theta + (x' - \alpha) \cos \theta,$$

$$\tan \theta = \frac{\beta}{z + \alpha}, \text{ we obtain}$$

$$(a^2 \cos^2 \theta + b^2 \sin^2 \theta)(y' - \beta)^2 + (a^2 \sin^2 \theta + b^2 \cos^2 \theta)(x' - \alpha)^2 - 2(a^2 - b^2)(y' - \beta)(x' - \alpha) \sin \theta \cos \theta = a^2 b^2.$$

Putting  $x' = 0$ ,  $y' = 0$ , there results the equation to the required curve, which is

the locus of the centres of all the preceding ellipses whose transverse axes intersect in F and their circumferences in O,

$$(a^2 \cos^2 \theta + b^2 \sin^2 \theta) \beta^2 + (a^2 \sin^2 \theta + b^2 \cos^2 \theta) \alpha^2 - 2(a^2 - b^2) \alpha \beta \sin \theta \cos \theta = a^2 b^2 \tag{1},$$

$$\therefore (a^2 + b^2 \tan^2 \theta) \beta^2 + (a^2 \tan^2 \theta + b^2) \alpha^2 - 2(a^2 - b^2) \alpha \beta \tan \theta = a^2 b^2 (1 + \tan^2 \theta).$$

Substituting for  $\tan \theta$  its value, we obtain

$$\{a^2(z + \alpha)^2 + b^2 \beta^2\} \beta^2 + \{a^2 \beta^2 + b^2(z + \alpha)^2\} \alpha^2 - 2(a^2 - b^2)(z + \alpha) \alpha \beta^2 = a^2 b^2 \{(z + \alpha)^2 + \beta^2\},$$

$$\therefore a^2 z^2 \beta^2 + b^2 \{\beta^2 + \alpha(z + \alpha)\}^2 = a^2 b^2 \{(z + \alpha)^2 + \beta^2\} \tag{2}.$$

By substituting in this  $z + \alpha = \rho \cos \theta$ , and  $\beta = \rho \sin \theta$ , we have the polar equation to the curve, F being the origin,

$$a^2 z^2 \rho^2 \sin^2 \theta + b^2 (\rho^2 \sin^2 \theta + \rho^2 \cos^2 \theta - z \rho \cos \theta)^2 = a^2 b^2 \rho^2,$$

$$\therefore a^2 z^2 \rho^2 \sin^2 \theta + b^2 \rho^2 (\rho - z \cos \theta)^2 = a^2 b^2 \rho^2,$$

and 
$$\rho = z \cos \theta \pm a(1 - \frac{z^2}{b^2} \sin^2 \theta)^{\frac{1}{2}} \tag{3}.$$

It is evident that the form of the curve represented by the Equations 2 and 3 will vary with the relative values of the constants  $a$ ,  $b$ , and  $z$ ; but in every case it is symmetrical with respect to the axis FX (fig. 8).

When  $z=0$ , the point O is situated on the axis of the mirror, and the curve becomes the circle  $\rho = a$ .

And if  $z=b$ , the equation breaks up into two circles whose centres lie in the line FO, and which touch one another externally at the point F, their diameters being  $a + b$  and  $a - b$  respectively.

When the point F falls without the curve, the radius vector becomes a tangent for the value  $\sin \theta = \frac{b}{z}$ .

Putting  $\theta=0$  in Equation (1), we get for a plane mirror

$$a^2 \beta^2 + b^2 \alpha^2 = a^2 b^2,$$

an ellipse with O for its centre, and equivalent to

$$a^2 y^2 + b^2 x^2 = a^2 b^2.$$

Now, if PHG (figs. 9 and 10) represent the curve, considerably magnified, expressed by equation 3, in deducing the intensity at O the proposition divides itself into cases depending on the relative positions of the points F and O (as in figs. 9 and 10).

*Case 1.* When the distance FO from the axis (fig. 7) is so small compared with FB that the distance of any point within the curve similar to PHG from CB, that part of the mirror where the light which overspreads O is reflected, may be considered equal to FB.

Let  $i$  = angle of incidence on the mirror,  
 $R$  = perpendicular distance of B from the axis,  
 $r$  = perpendicular distance of any point in the curve similar to HPG (fig. 9),  
 as represented by FQ,  
 $v$  = FA = FB, because BFA is an isosceles triangle (fig. 7),  
 $u$  = intensity at O,  
 $I'$  = intensity of the sun's light at the earth's surface ;  
 then  $2i$  = angle BFA,  
 $90^\circ - i$  = angle BAF,  
 $a = b \sec 2i$ ,  
 $\pi ab = \pi b^2 \sec 2i$  = area of ellipse, semi-axes  $a$  and  $b$ ,  
 $z = FO$ , in this case less than  $b$ .

Since the rays reflected from C and D (fig. 7) fall upon the same point O, the circumference of the circle described by O in its revolution about the axis is illuminated by the light reflected from the two annuli described by C and D. Besides the point O is situated so near the axis, that the perpendicular distances of C and D from the axis may be considered as equal to one another. Thus, to find the concentration which results from the converging of the rays to the axis, we have,

$$2\pi r : 4\pi R :: I' : \text{intensity at distance } r \text{ from F, } \left( = \frac{2RI'}{r} \right),$$

the intensities being estimated on planes perpendicular to the rays. Wherefore the intensity on the plane at F of the reflected light

$$= k \times \frac{2RI'}{r} \times \cos 2i = \frac{2kRI' \cos 2i}{r},$$

which would give the intensity on the increment at Q (figs. 9 and 10), if the sun's rays were perfectly parallel. But instead of this light being confined to the increment at Q, it is spread over an ellipse whose area =  $\pi b^2 \sec 2i$ , and hence the intensity at O due to this increment

$$= \frac{2kRI' \cos 2i}{r} \times \frac{r dr d\theta}{\pi b^2 \sec 2i} = \frac{2kRI' \cos^2 2i}{\pi b^2} dr d\theta;$$

and the same is true for a corresponding increment on the other side of the axis GH (figs. 9 and 10),

$$\begin{aligned} \therefore u &= \iint 2 \times \frac{2kRI' \cos 2i}{r} \times \frac{r dr d\theta}{\pi b^2 \sec 2i} \\ &= \frac{4kRI' \cos^2 2i}{\pi b^2} \iint dr d\theta \quad (7). \end{aligned}$$

This, taken between the limits  $r = \rho$ , and  $r = 0$ ; and  $\theta = \pi$ , and  $\theta = 0$ , gives the total intensity

$$u = \frac{4k RI' \cos^2 2i}{\pi b^2} \int_0^\pi \rho^2 d\theta .$$

Now, from Equa. 3,  $\rho = z \cos \theta + a \left( 1 - \frac{z^2}{b^2} \sin^2 \theta \right)^{\frac{1}{2}}$ ,  $z$  being less than  $b$ ,

$$\therefore u = \frac{4k RI' \cos^2 2i}{\pi b^2} \int_0^\pi \left\{ a \left( 1 - \frac{z^2}{b^2} \sin^2 \theta \right)^{\frac{1}{2}} + z \cos \theta \right\} d\theta .$$

Expressed in terms of elliptic functions,

$$\begin{aligned} u &= \frac{4k RI' \cos^2 2i}{\pi b^2} \left[ a E_{\frac{z}{b}}(\theta) + z \sin \theta \right] + C . \\ &= \frac{4k RI' \cos^2 2i}{\pi b^2} a E_{\frac{z}{b}}(\pi) \end{aligned} \quad (8).$$

But by Art. 1,  $R \operatorname{cosec} 2i : c :: b : \frac{1}{2}$ , ( $R \operatorname{cosec} 2i$  being = FB, fig. 7),

$$\begin{aligned} \therefore b &= \frac{R \operatorname{cosec} 2i}{2c} , \\ &= \frac{R}{2c \sin 2i} , \end{aligned}$$

and

$$a = b \sec 2i = \frac{b}{\cos 2i} .$$

Substituting in Equation 8, we get

$$\begin{aligned} u &= \frac{8ck I' \sin 2i \cos 2i}{\pi} E_{\frac{z}{b}}(\pi) , \\ &= \frac{4ck I' \sin 4i}{\pi} E_{\frac{z}{b}}(\pi) \end{aligned} \quad (9).$$

An expression which may be put in the following form:—If a circle be described with F as a centre (figs. 7, 8, or 9) and  $2b$  as a diameter, and an ellipse with O for one of its foci and the same diameter as a major axis; the circumference of the circle will be to the circumference of the ellipse as the intensity at the axis to the intensity at O.

When  $z = 0$ , then  $E_{\frac{z}{b}}(\pi) = \pi$

and

$$u = \frac{4ck I' \sin 4i}{\pi} \times \pi = 4ck I' \sin 4i , \quad (10),$$

which is the expression for the intensity at the axis, and shows that in the same conical mirror it is constant at every point in the axis; whereas, in conical mirrors

of different inclinations, there are two which produce a maximum effect at the axis, viz., when  $4i = 90^\circ$  and  $270^\circ$  (in Equa. 10), that is, when BAF (fig. 7) =  $67\frac{1}{2}^\circ$  or  $22\frac{1}{2}^\circ$ .

When

$$z = b \text{ (Equa. 9), } E_z(\pi) = 2,$$

and

$$u = \frac{8ckI' \sin 4i}{\pi} \tag{11}.$$

If  $k = \frac{1}{2}$ , the maximum intensity at the axis expressed numerically is

$$u = 4ckI' \sin 90 = 4 \times 107 \times \frac{1}{2} I' = 214 I'.$$

When  $k = \frac{1}{2}$  and  $4i = 2^\circ 36'$ , it can be shown by Equa. 10 that  $u = 10I'$  nearly, a heat sufficient to ignite wood and other combustibles. This can be effected at a distance of 130 feet with a segment of the reflector 18 inches broad, and having a mean diameter of 6 feet.

*Case 2.* When the point O is situated at a considerable distance from the axis,  $z$  being much greater than  $b$ , the distances CO and DO will now differ perceptibly from one another and from  $FA = FB = v$  (fig. 7).

Let R and R' represent the distances of the points C and D respectively from the axis, then

$$R = fC \sin 2i = (v + z \cot 2i) \sin 2i,$$

and

$$CO = fC - fO = FA + Ff - fO = v + z \cot 2i - z \operatorname{cosec} 2i = v - z \tan i.$$

Similarly for the point D,

$$R' = (v - z \cot 2i) \sin 2i,$$

and  $DO = v + z \tan i$ .

The ellipses of light which overspread O reflected from each increment of the space around C have for their minor and major axes respectively,

$$b = \frac{v - z \tan i}{2c}, \quad a = \frac{v - z \tan i}{2c \cos 2i};$$

and the corresponding quantities with respect to the space around D are

$$b' = \frac{v + z \tan i}{2c}, \quad a' = \frac{v + z \tan i}{2c \cos 2i}.$$

It can be shown, as in *Case 1*, Equa. 7, that the concentration at O, due to the light reflected from the space around C, is

$$\begin{aligned} &= \frac{2kRI' \cos^2 2i}{\pi b^2} \iint dr d\theta, \text{ and taken between } \xi_1 \text{ and } \xi_2, \\ &= \frac{2kRI' \cos^2 2i}{\pi b^2} \int (\xi_1 - \xi_2) d\theta, \\ &= \frac{4kRI'a \cos^2 2i}{\pi b^2} \int \left(1 - \frac{z^2}{b^2} \sin^2 \theta\right)^{\frac{1}{2}} d\theta, \end{aligned}$$

because (fig. 10)

$$\xi_1 = FP = z \cos \theta + a \left(1 - \frac{z^2}{b^2} \sin^2 \theta\right)^{\frac{1}{2}},$$

$$\xi_2 = FP' = z \cos \theta - a \left(1 - \frac{z^2}{b^2} \sin^2 \theta\right)^{\frac{1}{2}},$$

$$\xi_1 - \xi_2 = 2 \left(1 - \frac{z^2}{b^2} \sin^2 \theta\right)^{\frac{1}{2}}.$$

Taking the integral between the required limits  $\sin \theta = \frac{b}{z}$ , and  $\theta = 0$  we obtain

$$\frac{4kRI'a \cos^2 2i}{\pi b^2} (A - H)_{\frac{z}{b}} \quad (12),$$

$(A - H)_{\frac{z}{b}}$  denoting the difference between the asymptote and the infinite hyperbolic arc whose major axis is unity, and eccentricity  $\frac{z}{b}$ , a finite quantity, though A and H are severally infinite. But, if the distance from the centre to the focus be equal to unity, the transverse axis is  $\frac{z}{b}$  and  $(A - H)_{\frac{z}{b}} = \frac{z}{b} (A - H)$ , which expressed in complete elliptic functions of the second order gives

$$(A - H)_{\frac{z}{b}} = \frac{z+b}{b} E_{e_1} - \frac{z}{b} E_e,$$

$e$  being put for  $\frac{b}{z}$ , the relation between the moduli  $e$  and  $e_1$ , being

$$e_1 = \frac{2\sqrt{e}}{1+e} = \frac{2\sqrt{bz}}{z+b},$$

and  $e'$  and  $e'_1$  denoting the corresponding quantities when  $b'$  is put for  $b$ .

Substituting in Equation 12, we get for the light reflected from the space around C,

$$\frac{4kRI'a \cos^2 2i}{\pi b^2} \left\{ \frac{z+b}{b} E_{e_1} - \frac{z}{b} E_e \right\};$$

and similarly the light reflected from the space around D,

$$= \frac{4kR'I'a \cos^2 2i}{\pi b'^2} \left\{ \frac{z+b'}{b'} E_{e_1} - \frac{z}{b'} E_{e'} \right\}.$$

Hence the total intensity at O is

$$u = \frac{4kR'I'a \cos^2 2i}{\pi b^2} \left\{ \frac{z+b}{b} E_{e_1} - \frac{z}{b} E_e \right\} + \frac{4kR'I'a' \cos^2 2i}{\pi b'^2} \left\{ \frac{z+b'}{b'} E_{e_1} - \frac{z}{b'} E_{e'} \right\},$$

and substituting for  $a, b, a', b', R,$  and  $R'$

$$u = \frac{4c(v+z \cot 2i)kI' \sin 4i}{\pi(v-z \tan i)} \left\{ \frac{v+(2c-\tan i)z}{v-z \tan i} E_{e_1} - \frac{2cz}{v-z \tan i} E_e \right\} + \frac{4c(v-z \cot 2i)kI' \sin 4i}{\pi(v+z \tan i)} \left\{ \frac{v+(2c+\tan i)z}{v+z \tan i} E_{e_1} - \frac{2cz}{v+z \tan i} E_{e'} \right\}, \quad (13),$$

the values of the moduli in terms of  $c, v, z,$  and  $i$  being

$$e_1 = \frac{2\sqrt{2cz(v-z \tan i)}}{v+(2c-\tan i)z}, \quad e = \frac{v-z \tan i}{2cz}, \quad e'_1 = \frac{2\sqrt{2cz(v+z \tan i)}}{v+(2c+\tan i)z} \text{ and } e' = \frac{v+z \tan i}{2cz}.$$

Since the value of  $u$  will not be altered by substituting for  $v$  and  $z$  any two quantities having the same ratio, it follows that the intensity of the reflected light is uniform along the line which joins A and O (fig. 7).

ARTICLE 9.—Corollary. The value of  $k$ , the fraction which expresses the relation between the intensities of the reflected and incident rays, may be found by means of a conical reflector, thus:—

Let  $R$  = distance from the axis of a small zone described by AB (fig. 11).

$r$  = distance from the axis at which the reflected light or heat becomes, by convergence to its axis, equal in intensity to the incident.

Then  $kI' : I' :: r : R,$

$$\therefore kRI' = rI', \text{ and } k = \frac{r}{R}.$$

This result, calculated on the assumption that all the rays emanating from the sun are parallel, will not deviate perceptibly from the truth, except when  $r$  is small compared with  $R$ .

The value of  $k$  may also be found by using the combination of  $n$  plane mirrors. By Article 3, Equation 2, intensity

$$= \frac{nkI_s^2 \cos i}{r^2},$$

but if the direct rays of the sun fall on the same plane with the light reflected by the combination, the intensity will be

$$\frac{nkI'g^2 \cos i}{r^2} + I'.$$

Bringing the combination nearer the plane on which the light is thrown, the intensity of the  $n$  mirrors can be made equal to the above equation by diminishing the value of  $r$ .

Hence

$$\frac{nkI'g^2 \cos i}{r'^2} = \frac{nkI'g^2 \cos i}{r^2} + I',$$

$$\therefore k = \frac{r^2 r'^2}{ng^2(r^2 - r'^2) \cos i}.$$

As these results are independent of the absolute value of  $I'$ , the equality of temperature may be detected by a Fahrenheit thermometer, or any more delicate means of indicating equal temperature.

When  $k$  is accurately known, this combination, or the conical reflector, may obviously be used to ascertain the intensity of the solar beams at different hours of the day and different periods of the year, and will thereby furnish data for estimating accurately the heat or light absorbed by the atmosphere. The light lost by the solar rays in penetrating the atmosphere being known, the intensity of the radiation at different parts of the solar disc may be found by (Article 6, Equation 6), if a zone or segment of a parabolic reflector can be constructed having a focal length of 70 or 80 feet.

ARTICLE 10.—Prop. *When two Conical Mirrors have a common Axis, their Surfaces being either perpendicular or parallel, if the rays incident on the exterior Reflector parallel to the Axis meet after reflection the interior one, they will be again reflected parallel to the Axis in a beam of increased intensity.*

Let AB (figs. 12 and 13) be the common axis of two conical reflectors described by the revolution of the lines MN and CD about the axis AB, CD being either perpendicular or parallel to MN.

When AB (fig. 12) is directed to the centre of the sun, the rays which fall on the surface described by MN will make with it the angle

$$SKN = MKH = KHD = FHC = DCB,$$

because DCB = SKN, therefore FH is parallel to AB (Euclid, 1-27). The intensity of the finally reflected beam at H is to that incident at K as the perpendicular distance of K from AB is to the perpendicular distance of H from

AB. Since every cylindrical annulus of rays incident on the exterior mirror forms after reflection from the interior a cylindrical annulus of equal thickness, the intensities must be inversely as the mean radii, the loss from reflection being neglected, or, if taken into account, equal to  $k^2$  times the preceding intensity nearly.

ARTICLE 11.—The preceding annulus of rays may be thrown upon a circular area whose diameter is equal to the breadth of the zone, which forms a section of the annulus. To effect this, we have only so to increase the angle DCB, that the rays may meet the axis at the required distance, as shown in fig. 14.

In like manner, the parallel rays, from any extent of reflecting surface may be thrown upon the area whose section is Ff (fig. 15) by constructing one or both of the conical mirrors of frustums having the required extent of curved surface and the requisite inclination, the same axis being common to all. But when the breadths of the annuli are small compared with the distance of the focus Ff from the reflectors, the diameter of the circular area mentioned must be increased by the diameter of the sun's image for that distance.

*Cor.* When the number of lines CD, DE, EG, &c. (fig. 15), is indefinitely increased, their lengths being diminished, CDEG becomes part of a parabola.

ARTICLE 12.—Prop. *If two Parabolic Reflectors have a common Focus, the Solar Rays which are made to converge by reflection from the exterior Mirror will again form a beam of parallel Rays by reflection from the surface of the interior one.*

Let MKN and DHC (fig. 16) be sections of two confocal parabolic reflectors of which AB is the axis of the exterior and CE that of the interior,  $f$  being their common focus.

When AB is directed to a point in the sun's disc, the rays which fall on the exterior mirror parallel to the axis AB, in converging to  $f$ , will meet the surface of the interior mirror, and be reflected parallel to its axis  $f$ CE, as indicated by the course of the rays SKHF. Thus the solar beam of light which falls on the exterior and larger mirror is again reflected into a beam of parallel rays, and the intensity of the final beam will be greater than that of the incident, as explained in the preceding article. Moreover, since the axis  $f$ E of the interior mirror may make any angle with AB the axis of the exterior, the final beam may be thrown in any direction.

ARTICLE 13.—Prop. *The Rays which converge to the focus of the exterior Parabolic Mirror may be thrown by a second reflection on a given circular space, by constructing the interior reflector in the following manner:—*

Let DH and H'D (fig. 17) be parts of parabolas, whose common focus is  $f$ , their axes being respectively  $fC$  parallel to  $RG$ , and  $fC'$  to  $QG$ .

By causing these to revolve about the line  $fA$ , a surface will be described, such that the rays converging to  $f$  and falling on DH, will be reflected parallel to  $fC$ , and fall on the plane  $FF'$ .

In like manner, the rays converging to  $f$  and falling on H'D, will be reflected parallel to  $fC'$ , and will intersect the axis of revolution at  $G$ , and fall upon the plane  $FF'$ .

If this surface be substituted for the inner reflector DHC (fig. 16), the rays reflected from the outer mirror whose axis is directed to the sun, when converging to  $f$ , will meet the inner reflector described by H'QDH, and be reflected (as indicated in fig. 17) so as to intersect its axis of revolution at  $G$ , and fall upon the plane  $FF'$ . And this is true, whatever angle  $fG$  makes with the axis of the exterior reflector.

*Cor.* When the number of parts in H'QDRH are indefinitely increased, and their lengths diminished, it evidently becomes the arc of a hyperbola whose foci are  $f$  and  $G$ .

ARTICLE 14. The convergence of the solar rays upon a given area can also be effected by combining a number of exterior reflectors, each with its corresponding interior, as indicated in fig. 16, the axes of all the exterior parabolic reflectors being directed to the centre of the sun's disc, while the axes of the interior are directed to the centre of the given spot, on which the light has to be cast. Neither is it necessary that the respective reflectors should be complete symmetrical paraboloids: the exterior may consist of a series of large plates, each forming a part of a paraboloid of revolution, with a corresponding plate cut from a less paraboloid for its inner reflector. If the axes of all the exterior plates be in the same straight line, such a combination may have a common focus, each interior having that diameter of its generating parabola, which passes through the centre of the plate directed to the spot on which the light is required to fall. The practicability of such a combination is evident, from fig. 16, where  $K$  may be viewed as the centre of one of the exterior plates, and  $H$  that of its corresponding interior, having the diameter of its generating parabola, which passes through  $H$ , directed to the plane on which the light is concentrated. The exterior plates may be joined together to move as one piece, and in like manner the interior. This combination is capable of casting the finally reflected beam in a direction

making any angle with the axis of the exterior plates; and by a readjustment of the inner plates, the distance at which the rays finally meet may be varied at pleasure.

ARTICLE 15. It is stated that the Archimedean burning mirror was hexagonal. Let us consider if the combinations we have been illustrating can be made to conform to that figure. The term hexagonal may have reference either to the appearance of the mirror as a whole, or to the form of each individual reflector. Figs. 14 and 15 will correspond to the former; for if the external and internal conical frustums be each divided into six equal segments, with sufficient space between the segments to admit of free motion, the combination, viewed at a distance, will resemble a hexagonal polygon. Assuming the other meaning to be the correct one, we have only to suppose the form of the parabolic plates, which constitute the exterior and interior reflectors previously explained, to be hexagonal. From this would result two advantages:—they could be so formed that the different six-sided figures would unite together without leaving any interval, and the section of the beam cast by each on the required spot, approximating to a circle, would approach more nearly to the maximum effect with a given section of solar light.

ARTICLE 16.—Prop. *As the Expansion of the Sun's image is in proportion to the distance from the Point of Reflection, no greater accuracy is required for the construction of curved surfaces, capable of producing Combustion at distances of 150, 200, and 300 feet, than for those of a focal length of only a few inches.*

The expansion being about 1 foot in diameter for every 108 feet of focal distance, it follows that a reflector is sufficiently accurate for a burning glass, if it can concentrate the rays which fall on each part of its surface from the centre of the sun's disc, within a circular area, whose diameter is the same multiple or part of 1 foot which its focal length is of 108 feet.

The same principle may be exhibited in another and more definite form. Burning-glasses, which produce at the focus an intensity equal to parabolic ones, may be constructed of plane reflectors arranged as tangent planes to a paraboloid of revolution.

Since the increment of intensity is  $\frac{kI\delta A}{\text{area of circle DE}}$  (Art. 1, Equa. 6, fig. 1), if we take a portion of the surface of the paraboloid subtending an angle at the focus not greater than the sun's disc, the denominator of the foregoing fraction may be considered constant; and the intensity at the focus reflected from such an extent will be

$$\frac{kI' \text{ area of circle DE}}{\text{area of circle DE}} = kI'.$$

But a plane mirror which is a tangent to the paraboloid at the same spot, and of just sufficient area to reflect the circular beam of light whose section is equal to the circle DE, will also produce at the focus an intensity equal to  $kI'$  (Art. 2, Equa. 5); that is, the concentration at the focus is the same, whether a circular beam of the section mentioned be reflected from the surface of the paraboloid or from its tangent plane; and the same will evidently apply to any polygonal beam capable of being inscribed in the circular.

Hence a burning mirror, scarcely inferior in its effects to a parabolic one, may be formed of plane hexagonal reflectors, their sizes, of course, depending on the distance of the focus. For example, as the sun's image overspreads an area of 1 inch in diameter at a distance of 9 feet, a burning mirror of that focal length may be formed of plane hexagonal pieces, each side about half an inch; whereas at 108 feet distance, the sides of the plane hexagonal plates need not be less than half a foot, and so on in proportion.

Plates of the latter size being greater than those with which BUFFON performed his experiments, we infer that his combination, at distances exceeding 100 feet, would be little inferior in power to a parabolic segment of equal focal length, and capable of reflecting exactly the same sectional area of the solar beams.

Again, what has been proved true of plane mirrors, tangents to a paraboloid of revolution, must be equally true of a series of tangential circumscribing conical frustums. In all these cases, however, it is probable that the advantage in practice will remain with the parabolic figure, from the light at its focus having a greater area of maximum intensity.

From these results, as well as from independent calculations, we conclude that refracting burning-glasses may be constructed, by placing at some distance from an axis a series of acute-angled conical zones, or wedge-shaped pieces of glass (fig. 18), built up like the compound lens of BREWSTER, which will produce combustion at as great distances as BUFFON'S combination of reflectors.

ARTICLE 17. That the practicability of the Archimedean mirror may be made still more apparent, we shall now apply Equations 1 and 2, Article 3, to find the numerical intensity of the light in the focus of BUFFON'S combination. This was attempted by PEYRARD, but his conclusions are vitiated by the false premises from which he set out. He assumed that the intensity is uniform at every part of the luminous image reflected by a plane mirror,—a supposition proved incorrect by Art. 2. In our calculation we shall suppose that each of BUFFON'S mirrors, which were 8 inches by 6, produced an effect equivalent to a circular beam of 6 inches diameter, when it leaves the mirror.

Taking  $k = \frac{1}{2}$ , and  $\cos i = 1$ , we obtain by substitution, in the equations of Art. 3, the following results :—

On 23d March, heat produced by 40 glasses at 66 ft. French	=	$\frac{40 \times \frac{1}{2} \times 6^2}{\left(\frac{66}{9}\right)^2}$	I' = 13.3 I'
Do. do. 98	„	126 ft.	= 9 I'
3d April, 4 P.M., do. 112	„	138 ft.	= 8.57 I'
10th April, after 12 noon, do. 128	„	150 ft.	= 8.29 I'
10th April, 2.30 P.M., do. 148	„	150 ft.	= 9.95 I'
11th April, 2.30 P.M., do. 21	„	20 ft. = $21 \times \frac{1}{2}$ I'	= 10.5 I' (by Art. 3, Eq. 1)
Do. 2.30 P.M., do. 12	„	20 ft. = $12 \times \frac{1}{2}$ I'	= 6 I'

In the first experiment, on the 23d March at noon, tarred beech was ignited with the 40 glasses; but the mirror not being mounted on a stand, acted at a disadvantage. On the same day, when 98 glasses ignited a plank smeared with tar and brimstone, the mirror is said to have been still more disadvantageously placed. The experiment on the 3d April was at 4 o'clock P.M., with the mirror mounted, and placed on its stand. The sun being weak, a slight inflammation was produced on a plank covered with threads of wool.

On the 10th April, with a clear sun, the 128 glasses very suddenly kindled a plank of tarred fir. At half-past 2 o'clock on the same day, the combination of 148 glasses was tried on a plank of beech tarred in part, and covered in some places with shreads of wool. The inflammation, which was very sudden, commenced on those parts of the wood which were uncovered. Beech previously charred was the material ignited with 21 glasses, and little combustible materials were the substances set on fire by 12 glasses, on the 11th April.

An inspection of the results in the preceding table shows that if  $k = \frac{1}{2}$  be correct, wood done over in the manner mentioned can be ignited by a heat varying from eight to nine times that of the direct mid-day rays of the sun at Paris in April, and finely divided combustible substances by a heat considerably less, as proved by the experiment with 12 mirrors. But if we assume  $k = \frac{2}{3}$ , which is probably nearer the truth, the heat required to produce the same effect will vary between ten and eleven times the sun's mid-day heat. If the number in the right-hand column of the preceding table be multiplied by  $\frac{1}{k}$ , it gives the minimum number of plane mirrors capable in each case of producing ignition—that is, the number of mirrors which come under Equa. 1, Art. 3.

Supposing  $k = \frac{1}{2}$ , we find the numbers, for the first five experiments adduced, to be respectively 26.6, 18, 16.58, 17.14, and 19.18. The sizes of these mirrors will, of course, depend on the distance of the focus, and the angle at which they receive the incident light. When the distance is about 108 feet, each of them should have an extent sufficient to reflect a beam of solar light, not less than one foot in diameter, and their dimensions vary in the same ratio for other focal lengths.

As these minimum combinations have been calculated on the assumption

that the centre of the luminous circle reflected by each of the mirrors can be directed with precision to a given point, which in practice is not attainable, their number, or the size of each, must be somewhat increased, to compensate for defective adjustment. After making such an allowance, it will appear that at distances not exceeding 150 feet, between 16 and 20 plane mirrors, each  $2\frac{1}{2}$  by  $1\frac{1}{2}$  feet, may be substituted for the numerous combinations of BUFFON, the adjustment of which required so much time and trouble.

What has been shown respecting the power of the solar rays to produce combustion after one reflection, can easily be extended to the case in which the light undergoes two reflections, the intensity of the final beam being then reduced to about one-fourth that of the direct solar rays. To compensate for this diminution, the exterior reflectors must have about twice the area of BUFFON'S combination. Taking, for example, the 128 plane mirrors which kindled combustibles at 150 feet, the reflecting surface of the combination is  $\frac{8 \times 6 \times 128}{144} = 42\frac{2}{3}$  square feet, and the sectional area of solar light, which we supposed it to reflect, was  $\frac{\pi}{4} \times 6^2 \times 128 \div 144 = 25$  square feet nearly. It seems then, that about 80 feet of reflecting surface, or an extent capable of reflecting 50 square feet of solar light, will be more than sufficient to inflame such a material as tarred wood, &c., at the distance of 150 feet, after having undergone two reflections,—an extent of surface not too great to be united in one compound mirror, constructed in the manner explained in the foregoing articles.

Considering the scepticism which has prevailed respecting the Archimedean achievement in the most favourable circumstances, we are the less surprised to find that some recent authors, in quoting the passage from TZETZES, omit the statement which refers to the burning of the Roman ships in *winter*. Instead of ignoring these winter attacks, let us examine them in the light which BUFFON'S experiments supply. That performed with 112 mirrors, at the distance of 138 feet, was at 4 o'clock on the afternoon of the 3d April, at which time the altitude of the sun in the sky of Paris would nearly correspond to his meridian altitude at mid-winter in the more southern latitude of Syracuse; and as the difference between the meridian altitude of the sun at the summer and winter solstice amounts to above  $46^\circ$ , it must be admitted that this additional fact corroborates in a striking manner the evidence already adduced.

Having now shown how compound burning mirrors can be constructed corresponding in every respect to the description which TZETZES gives of the one invented by ARCHIMEDES, and that every statement in the passage is in accordance with well-established facts, we conclude that his narrative is no fiction, but, on the contrary, a true account of a real mirror, capable of producing all the effects ascribed to it. While this ancient discovery can be tried, after the lapse

of two thousand years, by the light of modern science, and pass with credit through the ordeal, the pretended discoveries of comparatively modern times, when subjected to the same test, fall to pieces.

An instance may be given without digressing from the subject of our paper THOMAS DIGGES, who republished in 1591 a work by his father, LEONHARD DIGGES, entitled "Pantometria," would make us believe, in the preface to this edition, that he had seen his father at sundry times fire gunpowder and discharge ordnance at a distance of half-a-mile or more, by means of the sun's beams. Had he been aware that to accomplish such a feat would require at least four thousand square feet of reflecting surface, we may venture to affirm that he would not have overstepped so far the Archimedean range.

We may observe, in conclusion, that the experiments of BUFFON, taken in connection with the preceding deductions, are calculated to produce a strong conviction that, in clear and comparatively warm climates, the sun's rays may be made, at a small expense, to supersede in some respects the fires employed in culinary operations. Further, when it is considered with what ease a combination of plane mirrors, or a series of conical reflecting zones, can be constructed, capable of producing a heat exceeding that of the most intense furnace (Art. 16), we infer that the solar beams may also be turned to account by the chemist and metallurgist. For these purposes, one reflection only is required, as the reflected light can be made to fall always on the same spot, by directing the axis of the reflector to the centre of the sun's disc, and causing it to follow the sun's motion in the heavens, by revolving round a fixed axis parallel to that of the earth.



V.—*On the Connection between Chemical Constitution and Physiological Action.*  
 Part. I.—*On the Physiological Action of the Salts of the Ammonium Bases, derived from Strychnia, Brucia, Thebaia, Codeia, Morphia, and Nicotia.* By  
 Dr A. CRUM BROWN and Dr THOMAS R. FRASER.

(Read 6th January 1868, under the title “On the Changes produced by direct Chemical Addition on the Physiological Action of certain Poisons.”)

There can be no reasonable doubt that a relation exists between the physiological action of a substance and its chemical composition and constitution, understanding by the latter term the mutual relations of the atoms in the substance. There are numerous indications of such a relation, and attempts have been made to express it formally in certain cases. Thus it has been long observed, that the salts of the same base have a common physiological action, and it has been pointed out by Mr BLAKE\* that, with some exceptions, the salts of isomorphous bases have a similar action. A corresponding likeness in physiological action may be traced in salts having the same acid, but beyond these generalisations we are not aware that any approach has been made to the statement of a law connecting the physiological action of a substance with its chemical constitution.

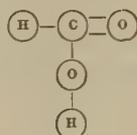
Some observers have endeavoured to connect physiological action with composition, looking for the cause of the peculiar action of substances in the presence or proportion of particular elements. It is a sufficient answer to this to point to isomeric or polymeric bodies—bodies having identically the same composition—which differ totally in action, such as acetic acid ( $C_2H_4O_2$ ), and sugar ( $C_6H_{12}O_6$ ); glycocoll ( $C_2H_5NO_2$ ), and nitrite of ethyl ( $C_2H_5NO_2$ ); or to instance kakodylic acid, which is inert, although perfectly soluble, and containing more than 54 per cent. of metallic arsenic.

Examples such as these clearly show that composition alone is quite insufficient to explain physiological action, and that constitution must also be taken into account in every attempt to connect the chemistry of substances with their action on the animal body.

The most direct way of making such an attempt would obviously be to compare physiological action and chemical constitution in a sufficiently large number of cases, and by classifying the results to deduce a law; but, unfortunately, the data which we possess are quite insufficient for this. We know, indeed, the “structure” of a considerable number of substances; that is, we know the *order*

\* Proceedings of the Royal Society of London, vol. iv. Jan. 28, 1841, p. 285.

in which the atoms of these substances are related to each other, but something more than this is implied in the term *constitution*, as we have used it above. For this involves not only the "structure," or the arrangement of the equivalents in atoms and in mutually united pairs, but also what we may call the *potential* of each pair of united equivalents.\* For instance, the structural formula of formic acid is



which indicates—1st, That the four carbon equivalents form one atom, the four oxygen equivalents two atoms, and the two hydrogen equivalents two atoms; 2d, that these equivalents are united in pairs, thus—*co, co, co, ch, ho*, but it does not in any way indicate (and we do not know) what is the *potential* of each of these pairs—that is, how much energy would be required to separate the equivalents from each other. We know that this potential depends upon the structure, and we can to a certain extent trace the nature of this dependence, but we cannot as yet express the potential numerically, or give a rule for finding its value from the structure, and till we can do this we do not fully know the constitution.

But even the structure of the majority of substances is not at all, or only very imperfectly known, and this is especially the case with those whose physiological action has been most fully investigated, such as the natural alkaloids.

Seeing, then, that we could not follow the direct road of induction, it occurred to us that a by-path might be found, by making use of a method resembling in its main features a mathematical *calculus of finite variations*. This method consists in performing upon a substance a chemical operation which shall introduce a known change into its constitution, and then examining and comparing the physiological action of the substance before and after the change. We may express this in mathematical language thus:—Let  $C$  represent the constitution of the original substance and  $\Phi$  its physiological action. After the operation,  $C$  becomes  $C + \Delta C$  and  $\Phi$ ,  $\Phi + \Delta\Phi$ . Here  $\Delta C$ ,  $\Phi$ , and  $\Phi + \Delta\Phi$  are known, and by applying the method to a sufficient number of substances, and by varying  $\Delta C$ , we might hope to determine what function  $\Phi$  is of  $C$ . The only reason why this method is not a strictly mathematical one is, that we cannot express our known terms  $\Delta C$ ,  $\Phi$ , and  $\Phi + \Delta\Phi$  with sufficient definiteness to make them the subjects of calculation. But although, on this account, we cannot obtain an accurate mathematical definition of  $f$  in the equation  $\Phi = fC$ , we may be able, in an approximate manner, to discover the nature of the relation.

In applying this method, we must select a chemical operation which satisfies

\* More correctly, "the exhaustion of the potential energy" of each pair of united equivalents. See THOMSON and TAIT'S *Treatise in Natural Philosophy*, § 547.

the following conditions:—1st, That it is unambiguous; that is, that the change of structure produced by it is susceptible of only one interpretation. 2d, That the change of *structure* produced by the operation is, in all cases investigated, the same, and the change of *constitution* ( $\Delta C$ )—that is, the change of structure and *potential*—as nearly as possible the same. 3d, That the operation is completely under our control, so that it cannot be either performed or reversed spontaneously, in ordinary circumstances, within the animal body. 4th, That the substance is equally suitable for absorption into the system before and after the change (that is, that  $\Phi$  and  $\Phi + \Delta\Phi$  are observed under similar conditions); and 5th, That a decided change of physiological action is, in some cases at least, produced (that is, that  $\Delta\Phi$  is not always = 0).

Chemical operations may be divided into two classes—1st, operations of substitution; and 2d, operations of addition or subtraction. In the first, an atom or group of atoms is replaced by an *equivalent* atom or group of atoms, without any change taking place in the active atomicity of any atom or radical in the substance.

In the case of addition (and by subtraction we mean to express merely the inverse operation to addition), the active atomicity of one or more atoms or radicals in the compound is increased, and the bonds thus set free, or rendered active, are saturated by atoms or radicals (the sum of whose active atomicity is of course an even number), which are thus *added* to the substance. We shall apply the name *condensation* to capability of being added to in whatever way the addition takes place, and distinguish two kinds of condensation, *intra-atomic* and *inter-atomic*; in the first of which it is an atom, and in the second a compound radical, the active atomicity of which is increased. Thus, carbonic oxide, sulphide of methyl, and protochloride of tin, are examples of *intra-atomic* condensation; olefiant gas, the dibasic anhydrous acids, and allylic alcohol, of *inter-atomic* condensation; while hydrocyanic acid (if we assume for it the formula  $\textcircled{\text{H}}-\textcircled{\text{N}}=\textcircled{\text{C}}$ ) shows both.

Many operations of addition and also of substitution satisfy the 1st, 2d, 3d, and 4th of the five conditions mentioned above; but when we examine them in reference to the 5th condition, we find a marked difference. Operations of substitution (satisfying the 1st, 2d, 3d, and 4th conditions) do not appear greatly to change the physiological activity of a substance, except, 1st, where the activity depends on direct local action; or 2d, where the operation removes or introduces an atom or radical, the compounds of which are as a rule active. As examples of the first exception, we may take sulphuric acid ( $\text{H}_2\text{SO}_4$ ) and caustic soda ( $\text{HNaO}$ ), both poisonous; while sulphate of soda ( $\text{Na}_2\text{SO}_4$ ) and water ( $\text{H}_2\text{O}$ ) are not: as examples of the second, acetate of lead and cyanide of sodium, both poisonous, acetate of potash and chloride of sodium not. Besides the exceptions which can be reduced to the two classes just mentioned, there are several isolated

cases of change of activity produced by replacement, such as the singular inertness of ferrocyanide of potassium and of the analogous double cyanides, as compared with the activity of cyanide of potassium and its analogues.

On the other hand, operations of addition, particularly where the condensation diminished by the addition is intra-atomic, seem, in many cases, to produce very decided change both in the kind and in the degree of the physiological activity of the substance acted on. The following examples will illustrate this statement.

Some are cases of direct and some of indirect addition, and in all of them the change of structure produced is known, and there is in none of them much risk of fallacy arising from the change taking place spontaneously in the animal system. The first column contains the names and formulæ of the substances before addition, the second the atoms or groups added, and the third the names and formulæ of the substances produced.

I.	II.	III.
Carbonic oxide, CO	O	Carbonic acid, CO <sub>2</sub>
Hydrocyanic acid, HCN	2H <sub>2</sub> + HCl	Hydrochlorate of methylamine, CNH <sub>6</sub> Cl.
Arsenious acid, As <sub>2</sub> O <sub>3</sub> , [HAsO <sub>2</sub> ]	(CH <sub>3</sub> ) <sub>2</sub>	Kakodylic acid, AsC <sub>2</sub> H <sub>7</sub> O <sub>2</sub> *
Strychnia, C <sub>21</sub> H <sub>22</sub> N <sub>2</sub> O <sub>2</sub>	(CH <sub>3</sub> (HO))	Methyl-strychnia (hydrate), C <sub>22</sub> H <sub>26</sub> N <sub>2</sub> O <sub>3</sub> †
Brucia, C <sub>23</sub> H <sub>26</sub> N <sub>2</sub> O <sub>4</sub>	(CH <sub>3</sub> (HO))	Methyl-brucia (hydrate), C <sub>24</sub> H <sub>30</sub> N <sub>2</sub> O <sub>5</sub> ‡

It will be observed that all the substances in the first column are highly poisonous, while those in the third column are either quite inert, or possess an action entirely different in kind from that of the bodies from which they are derived, and very much less in degree.

A consideration of the hitherto isolated facts collected in the above table leads not unnaturally to a suspicion that *condensation* (and in particular *intra-atomic condensation*) is in some way connected with physiological activity, as the first is, and the second appears to be, diminished or removed by chemical addition. This suspicion is strengthened when we observe that in a very large proportion of the cases as yet investigated saturated bodies (that is, bodies whose condensation is 0) are inert, or nearly so.

Kakodylic acid, as already mentioned, is a remarkable example of this, and the salts of tetrethyl-arsonium § seem to be equally inert. Similarly, the salts of tetramethyl-stibonium || are not emetic. So that, as far as experiment goes, it would seem that the stable compounds of pentatomic arsenic and antimony have a very different and much less strongly marked action than the compounds in which these elements are contained as triads, or than those (such as arsenic acid)

\* BUNSEN, *Annalen der Chemie and Pharmacie*, vol. xlvi. p. 10 (1843).

† STAHLSCHMIDT, *Poggendorff's Annalen*, vol. cviii. p. 523 (1859).

‡ *Ibid.* p. 541.

§ LANDOLT, *Annalen der Chemie und Pharmacie*, vol. lxxxix. p. 331 (1854).

|| *Ibid.* vol. lxxxiv. p. 49 (1852).

in which, although present as pentads, they are easily reduced by subtraction to the state of triads.

In reference to this, we cannot avoid referring to a very remarkable passage in BUNSEN'S admirable paper on kakodylic acid. After describing the experiments by which he proved the inert character of this acid, he says, "Gehen wir auf den Grund dieser unerwarteten Erscheinung zurück, so bietet sich dafür nur in der Annahme eine Erklärung dar, dass die Verbindungsweise des Arseniks im Kakodyl eine andere ist, als in seinen unorganischen Verbindungen. Indem es darin aufgehört hat, für sich einen Angriffspunkt der Verwandtschaft zu bilden, hat es zugleich seine Reaction auf den Organismus verloren." (Annalen, vol. xlvi. 1843, p. 11.) While it is plain that BUNSEN does not here refer to the different degree of saturation of the arsenic in arsenious and kakodylic acids, both because the whole theory of saturation is of a much later date, and because he makes no distinction between the mode of combination of the arsenic in those compounds in which kakodyl is monad and arsenic triad, and those in which kakodyl is triad and arsenic pentad, he points out in an exceedingly clear manner the striking coincidence of peculiar chemical constitution and peculiar physiological action in the case of kakodylic acid.

While, however, the cases mentioned incline us to believe that physiological activity is related to condensation, the occurrence of saturated substances, such as alcohol, corrosive sublimate, and oxalic acid, having a well marked poisonous action, and of condensed substances, such as benzoic acid and salicine, which are comparatively inert, shows that condensation is not the only condition of physiological activity. There can, at the same time, be little doubt that if the effect of condensation were discovered and eliminated, the other conditions might be much more hopefully sought for.

Under these circumstances, we turned our attention, in the first place, to the effect of chemical addition in altering the physiological action of the natural alkaloids. We were led to do so, partly by a consideration of the ease with which, by means of iodide of methyl, the nitrogen of nitrile bases can be rendered stably pentatomic, and partly by the hope, grounded on the observations of STAHLSCHEMIDT in reference to the salts methyl-strychnium and methyl-brucium, that we should obtain marked changes of physiological action.

The great majority of natural alkaloids belong to the class of *nitrile bases*, that is, they contain one or more atoms of triatomic nitrogen directly united to carbon by three bonds. This nitrogen atom (or, in the case of poly-acid bases, atoms) can become pentatomic, as in the formation of salts; thus in the formation of hydrochlorate of morphia the nitrogen takes up H and Cl, thus becoming pentatomic, united by three bonds to carbon, by one to hydrogen, and by one to chlorine. But by this change it is not rendered permanently or stably

pentatomic; it easily loses the hydrogen and chlorine it has acquired, and returns to the triatomic state. The action of alkalies, or, in many cases, even of alkaline carbonates, is sufficient to effect this, and reprecipitate the alkaloid. It is obvious, therefore, that the chemical addition of an acid does not satisfy the third condition mentioned above, for it is certain that the addition can be performed in the stomach, which is acid, and very probable that it may be reversed in the blood and other alkaline fluids of the body. But if, instead of an acid, we make use of such a substance as iodide of methyl, we find that while the triatomic nitrogen takes up  $\text{CH}_3$  and I, and becomes pentatomic (just as in the former case it took up H and Cl), it does not lose these newly-acquired atoms when the substance is treated with alkalies, but remains pentatomic even when subjected to attacks more violent than any to which it can be exposed in the animal system. This operation, the addition of iodide of methyl to nitrile bases, satisfies the first condition, for we know precisely what change of structure is produced. It satisfies the second, for the change of structure is the same in all nitrile bases; and the change of *potential*, as far as can be judged from a very rough estimate of the heat produced by the change, and from the general character of the substances produced, is not very different in different cases. It satisfies the third, as we have seen above; and as the iodides of the compound ammoniums thus formed from the alkaloids are all tolerably soluble in warm water, and can easily be transformed into other salts very readily soluble, it satisfies the fourth condition; and the observations of STAHLSCHMIDT show, and the sequel of this paper will further prove, that it satisfies the fifth.

It deserves to be noted that this operation only removes the condensation of the typical nitrogen (that is, of one atom of nitrogen for each molecule of a mono-basic acid that the alkaloid can saturate), and leaves any other condensation which may exist in the substance unaffected; so that even if physiological action should depend upon condensation, it would be unreasonable to expect  $\Phi + \Delta \Phi$  to be in all cases zero, that is, that the new bodies should be quite inert.

In the present paper we communicate the results of the application of the method described to strychnia, brucia, thebaia, codeia, morphia, and nicotia. In each case we shall first describe the action of the alkaloid itself, then give the method of preparing the derived substances, and describe their physical characters, and, with some detail, their physiological action. Our investigation of the physiological action of these substances has been chiefly directed to the determination of their poisonous activity, and of the most prominent differences between the nature of their action and that of the alkaloids from which they are derived.

## STRYCHNIA.

It is well known that strychnia acts on the living economy in a distinctly defined and characteristic manner, and that it is one of the most active of poisons. When administered subcutaneously, doses varying from one-twentieth to one-fiftieth of a grain rapidly produce in rabbits the most violent tetanic convulsions, and in a few minutes kill the animal. Few poisons have been more carefully studied, and it is now almost undoubtedly established that the phenomena produced by strychnia are due to a localisation of its action on the spinal cord.

*Iodide of methyl-strychnium.*—Strychnia ( $C_{21}H_{22}N_2O_2$ ) is a mono-acid nitrile base, that is, it contains one atom of nitrogen united by three bonds to carbon; the structure of the radical or radicals ( $C_{21}H_{22}NO_2$ )<sup>'''</sup> is unknown. How first demonstrated that strychnia is a nitrile base by subjecting it to the action of iodide of ethyl, and described, in a paper read before this Society,\* the ethyl-strychnium and amyl-strychnium compounds. STAHLSCHEMIDT subsequently prepared and described the compounds of methyl-strychnium.† We prepared the iodide of methyl-strychnium by STAHLSCHEMIDT'S method. Strychnia, in fine powder, was treated, in a flask, with excess of pure iodide of methyl;‡ the flask was allowed to stand in the cold for some hours, then heated in the water-bath, the excess of iodide of methyl distilled off, and the iodide of methyl-strychnium dissolved in boiling water, filtered, and recrystallised.

Iodide of methyl-strychnium ( $C_{21}H_{22}N_2O_2CH_3I$ ) crystallises in brilliant white scales, tastes distinctly bitter, though not so strongly or persistently so as strychnia, and when treated with strong sulphuric acid and peroxide of manganese, or bichromate of potash, it gives the colour reaction of strychnia, somewhat obscured by the presence of free iodine. It dissolves in 133 parts of water at 37° C., and in 385 parts of water at 9° C.

STAHLSCHEMIDT has published a statement to the effect that the methyl-strychnium compounds are inert. As the sequel will show, we do not confirm this assertion; but it is proper to admit that our investigation arose principally from it.

We first examined the effects of this substance by subcutaneous administration. For this purpose, it was reduced to the form of very fine powder, suspended and dissolved in warm distilled water, and injected into a previously formed

\* Transactions, vol. xxi. p. 32 (1854).

† POGGENDORFF'S Annalen, vol. cviii. p. 513 (1859).

‡ As iodide of methyl prepared directly from pyroxylic spirit is apt to become acid, it is advisable, if such impure iodide of methyl be used, to add a small quantity of an alkali (such as carbonate of potash), in order to prevent any of the strychnia being converted into a salt, and thus remaining unacted on by the iodide of methyl.

cavity in the subcutaneous cellular tissue. In this way, by a series of progressively increasing doses, it was found that as much as twelve grains could be given to a rabbit, weighing three pounds and four ounces, without any effect whatever. Fifteen grains, however, produced serious symptoms, though followed by recovery, and death was caused by the exhibition of twenty grains. Short abstracts of the majority of the experiments will be found in the table at the end of this paper; we shall, however, give some details of several experiments, in order to illustrate the mode of action.

EXPERIMENT VII.—Two very small incisions were made through the skin, one in either flank, of a rabbit, weighing three pounds and eight ounces; and by inserting an aneurism needle into these incisions, two cavities were formed in the cellular tissue. Into each of these we injected seven and a-half grains of iodide of methyl-strychnium (in all fifteen grains), suspended and dissolved in warm distilled water. No effect was caused until forty-five minutes, when the rabbit moved about uneasily, the limbs gradually yielded, and it soon lay on its chin and abdomen. When placed on the side, it remained quiet, without any efforts to recover a normal posture. Irritation did not cause any spasm nor give the slightest evidence of any increase in the reflex excitability. In one hour, when lifted by the ears, it hung in a perfectly flaccid and unresisting condition; the respirations were sixty-four per minute; and there were no voluntary movements. In one hour and thirteen minutes, a few spontaneous movements occurred in the limbs, but these, apparently, were merely feeble efforts to change its position. The external temperature appeared to be somewhat elevated, and the respirations were sixty-five per minute. In an hour and twenty-two minutes, a few twitches of the body, and especially of the abdominal muscles, occurred during the respiratory movements, which were now at the rate of sixty-six per minute; the eyelids did not contract when the conjunctiva or cornea was touched; but the animal was still conscious. In two hours, the condition was nearly the same as at last note, except that faint twitches of the eyelids could be excited by gentle irritation of their edges. In two hours and fifteen minutes, a number of very feeble spasmodic-like movements of the limbs occurred along with the twitches of the body, and these could also be excited by irritation. In two hours and thirty-five minutes, the condition of the rabbit had greatly improved. Efforts to rise were frequently made, in the intervals between which it lay perfectly quiet and flaccid, and the sensibility of the conjunctiva and cornea appeared to be normal.

The observations were now stopped until the following morning, when the rabbit was found jumping actively about, and apparently in a perfectly normal condition.

EXPERIMENT VIII.—We injected ten grains of iodide of methyl-strychnium, suspended and dissolved in warm distilled water, into each of two subcutaneous

cavities (twenty grains in all) of a rabbit, weighing three pounds and two and a-half ounces. Fifty minutes afterwards, the animal was lying flaccid, and exhibited the continuance of life only by slow and laboured respiratory movements. In one hour, tremulous movements of the body and limbs accompanied the respirations; and it was extremely difficult to excite even a feeble reflex movement by pretty strong stimulation. In one hour and ten minutes, the rabbit was dead.

The autopsy was immediately made: the heart was contracting with regularity and considerable force, at the rate of 160 beats per minute; the intestinal peristalsis seemed normal; galvanic stimulation of the exposed muscles caused energetic contractions, and continued to do so until more than thirty minutes after death; and similar stimulation of the exposed sciatic nerves caused contractions of the posterior extremities at four minutes after death, but ceased to do so in other five minutes.

These experiments are sufficient to illustrate the physiological effects that are produced when iodide of methyl-strychnium is administered to rabbits by subcutaneous injection. We have made similar experiments, with exactly analogous results, on dogs and cats, the more important details of which are mentioned in the table at the end of this paper.

The effects of internal administration were examined by passing a gum-elastic catheter down the œsophagus of a rabbit, and so injecting iodide of methyl-strychnium, suspended and dissolved in warm distilled water. It is unnecessary to give any description of these experiments, at this place, as no effect was produced by this method of exhibition, although as much as thirty grains was given at one time, and it was inconvenient, as well as unnecessary, to give larger doses. It is well known that to produce symptoms with a poison in a rabbit, a much larger quantity is required when it is administered by the stomach than when it is injected subcutaneously. The contrast between the action of iodide of methyl-strychnium and strychnia itself was, however, well shown in the rabbit to which thirty grains of the former had been given without any effect; for one-tenth of a grain of strychnia, also administered by the stomach, quickly produced violent tetanic convulsions, and, in a few minutes, killed the animal.

As iodide of methyl-strychnium is a sparingly soluble substance, it appeared proper, in conformity with our fourth condition, and in order to compare the actions of strychnia and of methyl-strychnium, that the properties of the sulphate of the latter, which is extremely soluble, should be examined.

*Sulphate of methyl-strychnium* ( $(C_{21}H_{22}N_2O_2CH_3)_2SO_4$ ) was prepared by precipitating a hot aqueous solution of the iodide by a hot solution of sulphate of silver, the slight excess of the latter was precipitated by chloride of sodium, the filtrate evaporated to dryness, and the sulphate of methyl-strychnium extracted by means of alcohol. It crystallises in delicate white needles, is very soluble in cold

water, tastes like the iodide, and gives the usual strychnia-reaction with oxidising agents.

As had been anticipated, it is much more active than the iodide. One grain, dissolved in water, and injected under the skin of a small rabbit, caused its death in eighteen minutes. Half-a-grain, however, produced no effect. When eight-tenths of a grain was similarly administered, the following symptoms were produced, but death did not result.

EXPERIMENT XXIII.—Eight-tenths of a grain of sulphate of methyl-strychnium, dissolved in a few minims of distilled water, was injected into the subcutaneous tissue over the abdomen of a rabbit, weighing three pounds and three and a-half ounces. It caused no immediate uneasiness, and the animal was unaffected for about twenty-five minutes, after which, however, it became restless. In twenty-eight minutes, movements of the limbs were made with obvious difficulty, and the rabbit occasionally stumbled. In twenty-nine minutes, the limbs could no longer support the body, and a position was assumed in which the rabbit lay on the abdomen with the chin resting on the table. It was now perfectly flaccid, and remained on the side when so placed. There was no evidence of exaggeration in the reflex motor function; indeed, an extremely violent stimulus was required to produce even a faint reflex movement. In thirty-two minutes, slight quiverings occurred, and the respirations were laboured, and at the rate of sixty-eight per minute. This condition continued until one hour after the administration, and during all this time consciousness seemed unaffected, and sensibility was not lost, as was shown by stimulation of the conjunctiva or cornea causing movements of the eyelids. Repeated efforts were, however, now made to recover a normal posture, and the frequency of the respirations increased. In one hour and eleven minutes, the head was raised from the table; and in eleven minutes afterwards, the rabbit succeeded in rising on its feet and maintained itself thus, though at first somewhat unsteadily. In one hour and twenty-two minutes, all the symptoms had disappeared. The rabbit was perfectly well on the following morning.

The sequence of symptoms to a fatal termination, and the *post mortem* appearances, are well shown in the experiment where one grain was exhibited (Experiment XXV.).

EXPERIMENT XXV.—We dissolved one grain of sulphate of methyl-strychnium in fifteen minims of distilled water, and injected this solution into the subcutaneous tissue of a rabbit, weighing two pounds and fourteen ounces. In eleven minutes, the first symptom, unsteadiness, appeared. In twelve minutes, the rabbit was lying on the abdomen and chest, with the lower jaw resting on the table. There were no voluntary movements; strong irritation caused feeble reflex movements only, and the respirations were shallow and laboured, and at the rate of sixty per minute. In sixteen minutes, quivering movements of the chest and abdominal muscles occurred, from which it was nearly impossible to distinguish the

respiratory movements; and the sensibility of the eyeball was greatly impaired. In seventeen minutes, there were no movements, except occasional faint twitches of the muscles of the body, while irritation of the skin or of the eyeball did not cause any reflex movements. The rabbit was quite dead in eighteen minutes.

Four minutes after death, the heart was contracting in proper rhythm and with regularity, at the rate of 164 beats per minute, and the intestinal peristalsis was well marked; the heart had however ceased to contract in other twenty-four minutes, but the intestinal peristalsis continued for some time after this. Six minutes after death, the gluteal muscles were exposed, and exposure caused them to twitch. The sciatic nerves were at the same time stimulated with galvanism and mechanical irritation, but no contractions were produced. *Rigor mortis* commenced about two hours and forty minutes after death.

When sulphate of methyl-strychnium is administered to rabbits by the stomach, twenty-five grains appears to be about the minimum fatal dose. The symptoms and mode of death are the same as those that result from subcutaneous injection.

These experiments clearly prove that the methyl derivatives of strychnia possess a very different action from strychnia itself. In none of our experiments, not even in the fatal cases, were the symptoms those of strychnia-poisoning; no starts nor spasms occurred, nor did stimulation give evidence of the slightest increase of reflex excitability. In fact, a condition exactly the reverse of that produced by strychnia was produced by these compounds. In place of violent spasmodic contractions and muscular rigidity, the appearances were those of paralysis, with a perfectly flaccid condition of the muscles. The limbs of the animal first yielded, its head gradually sank until it rested on the table, by-and-by, it lay in a perfectly relaxed condition, and when death occurred, it was due to stoppage of the respiratory movements. In the autopsy, further evidence was obtained to distinguish the effects of the methyl-strychnium compounds from those of strychnia. The heart was found acting with nearly its normal rapidity; the spinal motor nerves were either paralysed or nearly so; and, in place of the almost immediate occurrence of *rigor mortis* that follows the action of strychnia, the muscles continued flaccid, contractile, and alkaline for many hours.

These symptoms are sufficient to suggest a close resemblance between the action of the methyl derivatives of strychnia and that of curare (*vourali*), a well known and elaborately studied poison. In a recent publication, Professor SCHROFF, of Vienna, has indicated a resemblance of this kind between the nitrate of methyl-strychnium and curare.\* Both substances undoubtedly produce a condition of

\* Wochenblatt der Zeitschrift der k. k. Gesellschaft der Aertze in Wien; vi. Band, 1866, pp. 157-162.

general paralysis; but the special characteristic of curare-poisoning is, that this paralysis is the result of an impairment or destruction of the function of the peripheral terminations (end-organs) of the motor nerves. It is impossible to demonstrate such an action without undertaking experiments of a special character. We, accordingly, extended our research for the purpose of examining this question.

EXPERIMENT XXVIII.—The sciatic artery and vein were tied at the knee of a frog, and one-tenth of a grain of sulphate of methyl-strychnium, dissolved in distilled water, was injected under the skin of the back. Eight minutes afterwards, the frog was lying in a perfectly flaccid state, and, in ten minutes, irritation of any portion of the skin produced energetic movements of the tied limb, *below the points of ligature*, but nowhere else. The sciatic nerve of the untied limb was now exposed, and on stimulating it with a weak, interrupted galvanic current, movements occurred in the tied limb only; not the slightest effect occurred in any part to which the poison had access. At the same time, the muscles were everywhere active, and freely contracted when directly stimulated. The sciatic nerve was then exposed in the tied limb, *above the points of ligature*, and on stimulating it, energetic movements occurred below the knee of that limb, and there only. The heart was, at this time, acting at the rate of fifty per minute.

This experiment was repeated with one grain of iodide of methyl-strychnium, and the same general results were obtained. The evidence that was thus acquired in favour of an action on the peripheral terminations of the motor nerves was strengthened by a modification of this method of experiment.

EXPERIMENT XXIX.—The right gastrocnemius muscle of a frog was carefully dissected from its connections, excepting that its origin and insertion, and the nerve-fibres entering it, were untouched, and that all its blood-vessels were ligatured. One-tenth of a grain of sulphate of methyl-strychnium, dissolved in five minims of distilled water, was then injected under the skin of the back. Twenty minutes afterwards, the animal being in a perfectly relaxed and motionless condition, the two sciatic nerves were exposed. Galvanism of the left produced no movement in the left limb, while galvanism of the right produced energetic movements of the right limb, which were seen to be due solely to contractions of the right gastrocnemius muscle, the other muscle remaining motionless. At the same time, direct stimulation by galvanism caused contractions as freely in the poisoned muscles as in the non-poisoned right gastrocnemius.

In an experiment, in which iodide of methyl-strychnium was substituted for sulphate, the effects were the same. We have, therefore, demonstrated that the methyl-strychnium derivatives produce paralysis and death by destroying the function of the motor nerve end-organs, and that their mode of action is, therefore, identical with that of curare. This conclusion is an extremely curious and

interesting one. It is difficult to imagine a more decided modification in the action of any substance than has been produced by the addition of iodide or sulphate of methyl to strychnia. The striking characteristic of strychnia-action is the great and uncontrollable activity of the muscular system; that of curare, of iodide, and sulphate of methyl-strychnium, and, as we shall presently see, of several other similarly modified poisons, is the flaccid and motionless condition caused by the impossibility of exciting muscular action through the nervous system. So opposite are their effects that physiologists look upon curare as a powerful counteragent to strychnia, while physicians have employed it with success in the treatment of strychnia-poisoning and of tetanus. It is remarkable that by so simple a chemical process so thorough a change should be produced in physiological action.

The experiments we have already described have also shown that this change in chemical constitution has greatly reduced the poisonous activity of strychnia. This effect is still more clearly exhibited in the following table:—

No.*	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.
VII.	Iodide of methyl-strychnium.	Rabbit, 3 lbs. 8 oz.	Subcutaneously.	15 grs. (containing 10·5 grs. of strychnia).	Paralysis in 50 minutes, continuing for more than 2 hours, and followed by recovery.
XIII.	Strychnia (suspended in distilled water).	Do. (same rabbit as in Expt. VII.)	Subcutaneously.	0·05 gr.	Tetanus in 15 minutes; death in 30 minutes.
XVII.	Iodide of methyl-strychnium.	Do., 3 lbs. 13 oz.	By stomach.	30 grs. (containing 21·1 grs. of strychnia).	No effect.
XIX.	Strychnia (as hydrochlorate).	Do., (same rabbit as in Ex. XVII.)	By stomach.	0·1 gr.	Tetanus in 22 minutes; death in 31 minutes.
XXIII.	Sulphate of methyl-strychnium.	Do., 3 lbs. 3½ oz.	Subcutaneously.	0·8 gr. (containing 0·67 gr. of strychnia).	Paralysis in 29 minutes, continuing for 53 minutes, and followed by recovery.
XXXIII.	Sulphate of methyl-strychnium.	Do., 3 lbs. 5¼ oz.	By stomach.	20 grs. (containing 16·8 grs. of strychnia).	No effect.

We have made experiments with nitrate of methyl-strychnium and hydrochlorate of ethyl-strychnium, and have found that their action is identical with that of the iodide or sulphate of methyl-strychnium.

\* The numbers in this, and in the other short tables that are appended to the description of the physiological action of the derivatives of each alkaloid, have reference, in common with the numbers in the text, to the arrangement in the complete table at the end of the paper.

## BRUCIA.

Brucia is a poisonous alkaloid derived from some plants belonging to the genus *Strychnos*. It possesses a physiological action exactly similar in character to that of strychnia, but less in degree.

*Iodide of methyl-brucium* ( $C_{23}H_{26}N_2O_4CH_3I + 8H_2O$ ).—Brucia ( $C_{23}H_{26}N_2O_4 + 4H_2O$ ) is, like strychnia, a mono-acid nitrile base: here also the structure of the group ( $C_{23}H_{26}NO_4$ )''' is unknown, but the action of nitric acid on brucia renders it

probable that it contains the radical— $\text{O} - \text{C} - \text{N}$ . The ethyl-brucium compounds



were discovered and described by GUNNING,\* and the methyl-brucium compounds by STAHLSCHMIDT.† We prepared the iodide of methyl-brucium by adding excess of iodide of methyl to a saturated solution of brucia in rectified spirit, allowing the mixture to stand for some hours, evaporating, and recrystallising from hot water.

It forms thin white scales, and dissolves in 79 parts of water at 37°C, and in 225 parts of water at 9°C. Its taste resembles that of the corresponding strychnia compound.

When administered by subcutaneous injection, iodide of methyl-brucium was reduced to the form of a very fine powder, and suspended and dissolved in warm distilled water. In a series of experiments, it was found that as much as twelve grains could be thus given to a rabbit without any effect, that fifteen grains produced marked symptoms, and that eighteen grains was about the minimum fatal dose. Its method of action is well shown in the following experiment.

EXPERIMENT XL.—We injected seven and a-half grains of iodide of methyl-brucium, suspended and dissolved in warm distilled water, into each of two cavities (fifteen grains in all) previously formed in the subcutaneous cellular tissue over the abdomen of a rabbit, weighing four pounds. This did not produce the slightest effect until two hours and forty-three minutes after the administration, when the rabbit's movements became sluggish. Shortly after, a difficulty was observed in standing, and this posture soon become impossible on account of the increasing feebleness of the limbs. In three hours and three minutes, the rabbit subsided on the abdomen and chest, with the lower jaw resting on the table. The condition was one of perfect quietness, there being no twitches; and, though frequently tested, the reflex excitability appeared normal.

\* Journal für praktische Chemie, vol. lxxvii. p. 46.

† POGGENDORFF'S Annalen, vol. cviii. p. 535 (1859).

It remained on the side when so placed, but unsuccessful resistance was made to this change of position. In three hours and thirty-eight minutes, the flaccid state was even more marked, the position was changed without any resistance on the part of the rabbit, severe pinching only occasionally excited a reflex movement, but the respiratory movements were at the rate of sixty-eight per minute. These symptoms continued for other twenty minutes, when some voluntary movements were made, and soon after, the flaccid condition had nearly disappeared. On the following morning, the animal appeared to be perfectly well.

In the experiment we next give, a fatal dose was administered.

EXPERIMENT XLI.—We injected, in all, eighteen grains of iodide of methyl-brucium, suspended and dissolved in warm distilled water, into two subcutaneous cavities formed over the abdomen of a rabbit, weighing three pounds and twelve ounces. No result was observed until twenty-seven minutes, when uneasiness was manifested by restless movements, and slight quivers were seen in the muscles of the neck. In thirty minutes, there was great difficulty in supporting the head, which shook tremulously, and frequently fell on the table, where it eventually remained at thirty-two minutes. The body was still supported on the limbs, though by no means steadily. In thirty-seven minutes, it lay on the table and remained on the side, unresisting and flaccid. The respirations were, at this time, at the rate of forty-eight per minute, and were occasionally interrupted by faint quivering movements, but these had no spasmodic character. In forty-five minutes, the respirations were thirty-six per minute, and the heart's contractions 160 per minute. In one hour, the respirations were twenty-five per minute; and irritation of the conjunctiva did not now cause any movements of the eyelids. In one hour and seven minutes, the respiratory movements were irregular and shallow, only about sixteen occurring in the minute, while the heart was contracting at the rate of 120 per minute. The limbs were perfectly flaccid and motionless. The respiratory movements gradually became less apparent, a series of feeble quivers occurred in the muscles of the face, and death immediately afterwards occurred, one hour and thirteen minutes after the administration.

In the autopsy, the cardiac action was found to be regular and rhythmical, though only at the rate of seventy-four per minute. In three minutes after death, galvanism of the sciatic, phrenic, and other nerves, did not produce any muscular contraction; while it was found by direct galvanism that the muscles retained their contractibility for many minutes afterwards. *Rigor mortis* did not occur until more than one hour after death.

For the purpose of contrasting these symptoms with those that are caused by brucia itself, we shall describe, very briefly, an experiment in which the rabbit, that recovered after the administration of fifteen grains of iodide of methyl-brucium, was rapidly killed by a somewhat large dose of brucia.

EXPERIMENT XLIII.—One-fifth of a grain of brucia was dissolved in ten minims of very dilute hydrochloric acid, and injected, with Wood's syringe, into the subcutaneous tissue of the rabbit that had, some days previously, been subjected to an experiment with fifteen grains of iodide of methyl-brucium. In seven minutes, a constrained position was assumed by the rabbit, and the slightest touch caused a sudden spasmodic contraction of the four limbs by which the body was swiftly elevated. In eight minutes, the rabbit sprang to a considerable height, and fell in a well-marked tetanic convulsion, which lasted about fifteen seconds. After this, a series of violent tetanic convulsions, of a distinctly opisthotonic character, followed each other in rapid succession; and at the termination of one of these, eighteen minutes and thirty seconds after the injection of the poison, the rabbit died. There was distinct *rigor mortis* thirty minutes after death.

For internal administration, the iodide of methyl-brucium was also reduced to a very fine powder, and suspended and dissolved in warm distilled water. It was then introduced into the stomach, by means of a gum-elastic catheter. In this way, we performed several experiments, but never succeeded in producing any effect, although as large a dose as thirty grains was at one time administered. It is well known that there is considerable difficulty in affecting a rabbit by a poison introduced into the stomach. That this difficulty was not due, in the present instance, to any recognised cause peculiar to the stomach of the rabbit, was shown by an experiment in which we produced tetanic symptoms and death by introducing two grains of brucia into the stomach of the rabbit that had previously received thirty grains of iodide of methyl-brucium without any effect whatever.

*Sulphate of methyl-brucium* ( $(C_{23}H_{20}N_2O_4CH_3)_2SO_4$ , dried at  $100^\circ C$ ) was prepared by precipitating a hot solution of the iodide by means of sulphate of silver. It forms a white crystalline mass, readily soluble in water, and, as well as the iodide, gives the ordinary brucia reaction with nitric acid. It is freely soluble in cold water.

We examined the effects of this substance by subcutaneous injection and by introduction into the stomach. For the former purpose, it was dissolved in a few minims of distilled water, and injected under the skin with a Wood's syringe. In a rabbit, one grain could be thus given without any effect, two grains caused marked effects, which were not, however, fatal; while two grains and a-half soon killed the animal. The symptoms were the same as those of the iodide, and, therefore, very different from the exaggerated reflex action, convulsions, and tetanus, which are caused by brucia itself. They are illustrated in the following experiments.

EXPERIMENT LIII.—We injected two grains of sulphate of methyl-brucium, dissolved in fifteen minims of distilled water, under the skin of a rabbit, weighing two

pounds and thirteen ounces and three-quarters. In ten minutes, the animal had obviously some difficulty in moving about, and it could not stand steadily. The limbs soon after yielded, and it lay down on the abdomen, chest, and lower jaw ; while occasional quivering movements occurred in the muscles of the body. In thirty-four minutes, it lay unresisting and quiet on the side, and the respirations were at the rate of seventy-four per minute. In forty minutes, the respirations were at the rate of fifty-four per minute. It lay in a perfectly relaxed and quiet condition, and when the skin was severely irritated, only extremely feeble reflex movements followed. In one hour and two minutes, the respirations were at the rate of forty-eight per minute ; and though irritation of the cornea or conjunctiva did not cause any movement of the eyelids, reflex movements could be excited by severe pinching of the skin. This condition of helpless prostration continued for about thirty minutes, during which some faint twitches of the body and jerking movements of the limbs occasionally occurred. Soon after this, however, a marked improvement was observed : the respirations became fuller and more frequent ; irritation of the eyeball was followed by contractions of the eyelids ; and, at last, well-directed efforts were made to recover a normal position, and these ultimately proved successful at about two hours after the poison had been injected. The rabbit recovered perfectly.

EXPERIMENT LIV.—Two and a-half-grains of sulphate of methyl-brucium was dissolved in fifteen minims of distilled water, and administered by subcutaneous injection to a rabbit, weighing three pounds and fourteen ounces and a-half. In twenty-two minutes, the animal was lying on the abdomen and chest, but the head was still supported by the muscles of the neck ; there was distinct congestion of the ears and conjunctiva. In thirty-five minutes, the head had fallen on the table, and the rabbit was perfectly flaccid, and apparently unable to make any voluntary movements. The respirations were at the rate of eighty-two per minute. In fifty-three minutes, the number of the respirations had diminished to twenty-four per minute, while their character was extremely feeble and shallow. In one hour and two minutes, the respiratory movements occurred at long intervals, and were accompanied with a faint tremor of the body and limbs ; and it was ascertained that the cardiac contractions were occurring regularly, at the rate of 160 beats per minute. In one hour and ten minutes, the respirations altogether ceased, and death occurred. During the progress of the symptoms, the reflex excitability was frequently tested, with the result that not the slightest increase was ever observed.

The autopsy was immediately made : the heart was found contracting at the rate of 120 per minute ; the vermicular action of the intestines was well marked ; the conductivity of the sciatic nerves was lost three minutes after death ; and idiomuscular irritability persisted for more than twenty minutes afterwards. *Rigor mortis* had not commenced forty minutes after death.

For administration by the stomach, we dissolved this substance in warm distilled water, and introduced the solution through a gum-elastic catheter. We found that as much as twenty grains could be thus given without any effect, and it was not considered advisable to increase this dose. Its magnitude is apparent when we recollect that it contains about seventeen grains of brucia; and we have already seen that when two grains of this alkaloid is introduced into the stomach of a rabbit, the most violent tetanic convulsions are quickly produced, and death soon follows.

The short account we have given of a few of our experiments with iodide and sulphate of methyl-brucium is sufficient to show that these substances have an action that is very different from that of brucia itself. Brucia is a violent convulsant poison, and it causes death by either exhaustion or asphyxia; its methyl derivatives never produce convulsions, nor do they even increase the reflex activity; and although they cause death by asphyxia, this asphyxia, in place of being the result of prolonged and continuous muscular action, due to abnormal nerve activity, is the result of muscular paralysis, due to partial or complete absence of normal nerve activity. We have demonstrated the latter effect by the following experiments, which further show that the influence of the methyl derivatives of brucia is exercised on the terminations of the motor nerves.

EXPERIMENT LVI.—The left iliac artery of a frog, weighing 608 grains, was tied, after exposing it by removing a portion of the sacrum, and one-fifth of a grain of sulphate of methyl-brucium, dissolved in ten minims of distilled water, was then injected into the abdomen. In four minutes, every portion of the frog except the left leg was paralysed. In five minutes and thirty seconds, weak interrupted galvanism, applied to any portion of the skin, caused violent movements of the left leg, and of it alone, every other part of the body remaining motionless. The heart, as ascertained by its impulse, was contracting thirty times per minute. In seven minutes, the right sciatic nerve was exposed—the incisions necessary for which excited energetic reflex movements of the left limb—and on galvanising it, strong contractions of the left limb occurred, but no movement occurred in the right limb. The muscles were everywhere in a normal state, and freely responded to direct galvanic stimulation; and the heart still contracted at the rate of thirty beats per minute.

In a similar experiment, with half a grain of iodide of methyl-brucium, the same effects were observed. It is, therefore, apparent that these substances do not directly influence the action of the heart, of the muscles, of the spinal cord, or of the sensory (afferent) nerves, but that the paralysis, which they so prominently cause, is the result of an action on the motor nerves. In the above experiment, the whole course of the sciatic nerve, from the pelvis to the extremity of the left posterior limb, was protected from the influence of the poison. The experiment does not, therefore, show if the methyl-brucium compounds have

an elective action for any special portion of the nerve. In the next experiment, a much more limited portion of the nerve was protected from the poisonous action.

EXPERIMENT LVII.—In a frog, weighing 542 grains, the right gastrocnemius muscle was exposed; the muscle was separated from all its connections, excepting its origin and insertion and the nerve-fibres that entered it. One-sixth of a grain of sulphate of methyl-brucium, dissolved in ten minims of distilled water, was then injected into the abdomen. In twenty minutes, a condition of complete paralysis was present everywhere except in the right leg. The two sciatic nerves were exposed, and on galvanising the left nerve, feeble movements occurred in the right leg, and there only. When the right nerve was galvanised, movements occurred in the right leg, which were observed to be solely due to contractions in the right gastrocnemius muscle.

In this experiment, the terminations of the sciatic nerve in the right gastrocnemius muscle were alone protected from the direct influence of sulphate of methyl-brucium. This substance had access to all the other terminations of the right sciatic nerve, to the trunk of this nerve, and to all the other nerves of the body. No manifestation of vitality was obtained anywhere, except in the right limb, and it was restricted to contractions of one muscle of that limb. As these contractions could be produced by a stimulus originated in and conducted along the nerve trunk, it is obvious that the vitality of this portion of the nerve was not lost. And as the stimulus produced no effect on the terminations of the nerves to which sulphate of methyl-brucium had access, while it produced an effect on those that were protected from its direct influence, it is evident that this poison acts on the peripheral terminations of the motor nerves.

The physiological action of brucia is, therefore, completely changed by the addition of iodide or sulphate of methyl. It is also apparent that its activity as a poison is greatly lessened; and the following table, which contains a succinct statement of some of the previously-mentioned facts, will clearly illustrate this:—

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.
XL.	Iodide of methyl-brucium.	Rabbit, 4 lbs.	Subcutaneously.	15 grs. (containing 8·7 grs. of dry brucia).	Paralysis in 3 hours and 3 minutes, continuing for more than 28 minutes, and followed by recovery.
XLIII.	Brucia (crystallised).	Do. (same rabbit as in Experiment XL.)	Subcutaneously.	0·2 gr. (containing 0·17 gr. of dry brucia).	Tetanus in 8 minutes; death in 18 minutes 30 seconds.
XLVI.	Iodide of methyl-brucium.	Do., 4 lbs. 2 oz.	By stomach.	30 grs. (containing 17·4 grs. of dry brucia).	No effect.
LI.	Brucia (crystallised).	Do. (same rabbit as in Experiment XLVI.)	By stomach.	2 grs. (containing 1·7 gr. of dry brucia).	Tetanus in 44 minutes; death after 3 hours.
LIII.	Sulphate of methyl-brucium, dried.	Do., 2 lbs. 13 $\frac{3}{4}$ oz.	Subcutaneously.	2 grs. (containing 1·7 gr. of dry brucia).	Paralysis in 20 minutes, continuing for about 1 hour and 40 minutes, and followed by recovery.
LVIII.	Sulphate of methyl-brucium, dried.	Do., 4 lbs. 2 oz.	By stomach.	20 grs. (containing 17·2 grs. of dry brucia).	No effect.

## THEBAIA.

One of the active principles of opium possesses an action in all respects the same in character as that of strychnia or brucia. We principally owe our knowledge of the method in which thebaia acts to the admirable researches of CLAUDE BERNARD. This distinguished physiologist has further demonstrated that thebaia does not possess any soporific property, that it is the most active toxic principle in opium, and that it ranks first among the alkaloids of this drug that have a convulsant action.\* From our experience of its properties, we should assign to it a lower rank than brucia as a toxic and convulsant substance.

*Iodide of methyl-thebium.*—The close analogy in physiological action that exists between thebaia ( $C_{19}H_{21}NO_3$ ) on the one hand, and strychnia and brucia on the other, led us to subject this alkaloid to the action of iodide of methyl. The method adopted was the same as that described for the preparation of iodide of methyl-brucium, and the reaction takes place as readily. The product crystallises from alcohol in hard, shining, transparent crystals, which, when air-dried, have the composition ( $C_{19}H_{21}NO_3CH_3I$ ). They dissolve in 16·5 parts of water at 37°C., and in 63·5 parts of water at 9°C.† When a hot, saturated, aqueous solution is allowed to cool, it gelatinises, and the jelly, when left to itself, in some hours, and, when stirred, in a few minutes, is converted into a mass of minute silky needles, which when dried in the air, have the same composition as the crystals obtained from the alcoholic solution.

\* Comptes Rendus, vol. lix. 1864, p. 413.

† The methyl derivatives of thebaia have not been described. We shall take some other opportunity of giving details of their chemical relations.

It is much more soluble in water than the iodides of methyl-strychnium and methyl-brucium, and, on this account, we commenced its administration in relatively small quantities. We found that doses of one, five, and six grains, nearly completely dissolved in very dilute spirit, and administered to rabbits by subcutaneous injection, produced absolutely no effect. When, however, the dose was increased to ten grains, partial and then complete paralysis was caused, and death quickly occurred; while serious symptoms were caused by eight grains, but they did not terminate fatally. We shall give some details of these two experiments.

EXPERIMENT LXII.—We dissolved eight grains of iodide of methyl-thebium in very dilute alcohol, and injected the solution, with Wood's syringe, into the subcutaneous cellular tissue of a rabbit, weighing two pounds and twelve ounces. Symptoms of uneasiness occurred in thirty minutes, and were soon followed by quivering movements of the head and ears, and, to a slight extent, of the rest of the body. It was soon apparent that the neck muscles were scarcely able to support the head, for it frequently fell on the table, but the rabbit did not permit it to remain there until forty-five minutes after the administration. At this time, the respirations were at the rate of seventy-eight per minute, and, although the head was resting on the table, the body of the animal was supported, in a comparatively normal posture, on the limbs. There were occasional tremulous movements of the body, but no exaggeration of the reflex function could be discovered. The rabbit remained in this state for about thirty minutes; but soon after this, the tremulous movements disappeared, the head was raised and supported normally, and a perfectly natural posture was assumed. Every symptom had disappeared within two hours after the administration.

EXPERIMENT LXIII.—Ten grains of iodide of methyl-thebium, reduced to a very fine powder, was partially dissolved and partially suspended in very dilute alcohol, and injected under the skin of a rabbit, weighing two pounds and eleven ounces. There was no obvious effect until ten minutes, when it was observed that the animal moved with difficulty. Tremulous movements then occurred, the limbs occasionally yielded, and the head frequently fell. In twelve minutes, the rabbit lay on the abdomen and chest, with the lower jaw resting on the table; and the tremulous movements only occurred at intervals. It could now be lifted without any struggles. In nineteen minutes, the condition was one of complete flaccidity, the only movements were an occasional gasping respiration, but common sensibility was still retained. It continued thus, on the very verge of death, for about four minutes, when a few quivering contractions occurred in the muscles of the face and neck, and the respirations altogether ceased. During the course of the symptoms, there was never the slightest trace of any exaggeration in the reflex activity, nor of spasmodic or convulsive movements.

The autopsy was immediately made. The heart was found contracting, in normal rhythm, at the rate of eighty-five per minute, and its spontaneous contractions did not cease until eight minutes; and the intestinal peristalsis was active. The sciatic nerves were exposed four minutes after death, and stimulated with weak and strong currents of interrupted galvanism, but no muscular contractions were thereby caused. The muscles themselves readily contracted when the poles were applied directly to their surface, and continued to do so for more than fifteen minutes after death. There was no appearance of *rigor mortis* one hour and five minutes after death, and the muscles were, at this time, alkaline in reaction.

We administered to the rabbit, which had survived the administration of eight grains of iodide of methyl-thebium (Experiment LXII.), a fatal dose of the thebaia from some of which the methyl compound had been prepared. The striking contrast in the symptoms that were produced will be seen from the following account of the Experiment.

EXPERIMENT LXVI.—We injected one-fifth of a grain of thebaia, dissolved in very dilute hydrochloric acid, into the subcutaneous cellular tissue of the rabbit, which had been subjected to an experiment, some days previously, with eight grains of iodide of methyl-thebium. The injection did not appear to cause much annoyance, as the animal jumped about naturally for forty minutes after it. Soon after, however, its movements became more constrained and cautious, and occasional twitches occurred in the muscles of the back. These gradually became more marked and powerful, and in forty-eight minutes, they assumed the character of spasmodic starts. In forty-nine minutes, a touch, even when very gentle, of any portion of the skin excited a violent spasmodic jump, and in fifty-two minutes, a spontaneous violent opisthotonic convulsion took place, and continued for forty-five seconds. The rabbit now lay on its side; every respiratory movement provoked a short fit of tetanus, while, occasionally, a violent and prolonged fit occurred. This condition lasted for two minutes, when, at the termination of one of the more violent of these fits, death occurred,—fifty-four minutes after the administration of the poison.

It was found, in the autopsy, that the sciatic nerves retained their motor conductivity for at least fifteen minutes after death. A certain degree of muscular rigidity was observed at twenty-eight minutes, and *rigor mortis* was perfectly established at forty minutes, when all the muscles were acid in reaction, although the temperature of the abdominal cavity was as high as 95° F.

The internal administration of iodide of methyl-thebium was effected in the same way as we have described for the corresponding strychnia and brucia compounds. It was found that, with this substance also, so large a dose as thirty grains could be introduced into the stomach of a rabbit without any effect. Well-marked symptoms were produced in the same animal by three, and, on another

occasion, by three and a-half grains of thebaia similarly administered, but it recovered after both doses. Four grains was, however, a fatal dose, as will be seen from the following Experiment.

EXPERIMENT LXXIV.—Four grains of thebaia, almost completely dissolved in very dilute hydrochloric acid, was introduced, by a gum-elastic catheter, into the stomach of the rabbit that had received thirty grains of iodide of methyl-thebium (Experiment LXXI.) In six minutes, a violent tetanic convulsion occurred; after this, the rabbit remained on the side, and convulsion succeeded convulsion until its death, nineteen minutes after the administration of thebaia. *Rigor mortis*, with an acid reaction of the muscles, was completely established at thirty-seven minutes after death.

*Sulphate of methyl-thebium* ( $(C_{19}H_{21}NO_3CH_3)_2SO_4$ , dried at  $100^\circ C.$ ), was prepared by precipitating an aqueous solution of the iodide by means of sulphate of silver. It forms a white, indistinctly crystalline mass. It dissolves readily in water, and gives, with sulphuric acid, the reaction of thebaia.

We found it to be a less active substance than the corresponding derivative of either strychnia or brucia, as doses of four and of four-and-a-half grains were not fatal, though they produced symptoms, when injected into the subcutaneous cellular tissue of rabbits. Five grains appears to be about the smallest quantity that can produce death when administered to rabbits in this manner. The experiments in which four and a-half and five grains were given are sufficient to illustrate the general physiological effects of this substance.

EXPERIMENT LXXVII.—We dissolved four and a-half grains of sulphate of methyl-thebium in fifteen minims of distilled water, and injected this solution into the subcutaneous tissue at the flank of a rabbit, weighing three pounds and eleven ounces and a-half. In seventeen minutes, the rabbit had some difficulty in jumping about, for it occasionally stumbled, and rested for a few seconds on the chest. In twenty-one minutes, it was lying on the abdomen, with the lower jaw resting on the table; and, occasionally, a series of shivering tremors took place in the muscles of the back. In thirty minutes, it remained on the side, when so placed, and was perfectly flaccid. The respirations were at the rate of sixty per minute. In forty-one minutes, the respirations had diminished in frequency to forty per minute, and during inspiration the abdominal muscles contracted in a tremulous manner. In fifty-five minutes, the respirations had increased in number to seventy-one per minute, and in one hour and thirty minutes, they appeared to have regained their normal rapidity; but it was impossible to ascertain this definitely, on account of frequent interruptions by tremulous movements of the abdominal muscles. The rabbit was still lying on the side in a perfectly flaccid state. In one hour and thirty-two minutes, however, it suddenly raised the head, rose, and assumed a normal posture; but the trembling continued. This trembling, very faint and not at

all spasmodic, was the last symptom to disappear, which it did about two hours and thirty minutes after the injection of the poison. We frequently tested the condition of the reflex activity, and did not find it increased at any period during the experiment.

EXPERIMENT LXXVIII.—Five grains of sulphate of methyl-thebium was dissolved in thirty minims of distilled water, and injected under the skin of a rabbit, weighing four pounds and half an ounce. Its effects began to be seen in thirteen minutes, when, after a few restless movements, the rabbit subsided on the abdomen and chest. Complete flaccidity soon after occurred; and the respirations became shallow and gasping, and they diminished in frequency until, at twenty-five minutes after the injection, they were only at the rate of twenty-three per minute. Occasional, very weak, tremulous movements occurred at this time. In thirty-five minutes, severe pinching of the skin caused only a feeble reflex movement, while the contraction of the eyelids, after irritation of the eyeball, was almost imperceptible. The rabbit appeared still to retain consciousness. In fifty minutes, no movement followed severe pinching of the skin, or irritation of the eyeball, and the respirations were gasping and infrequent. In fifty minutes, a few twitches occurred in the muscles of the face, and either immediately before or during these the rabbit expired.

In the autopsy, which was immediately performed, the heart was seen contracting at the rate of seventy-eight per minute, and the intestinal peristalsis seemed normal. Four and a-half minutes after death, neither a weak nor a powerful galvanic current could excite any muscular contraction when applied to the trunk of a sciatic nerve; but idio-muscular irritability was not lost for many minutes after this. At two hours and thirty minutes after death, the rabbit was still perfectly flaccid, and there was not the slightest appearance of muscular rigidity.

We have not observed any symptoms follow the internal administration of this substance, as no effect was produced when we introduced twenty grains, dissolved in warm water, into the stomach of a rabbit. It has been shown by Experiment LXXIV. that four grains of thebaia is a fatal dose when thus exhibited.

The experiments we have narrated contain the most satisfactory proof that the chemical addition of iodide and sulphate of methyl has produced a complete change in the physiological action of thebaia. The nature of the change appears to be identical with that we have described as occurring under similar circumstances in strychnia and brucia. Thebaia acts in the same way as these alkaloids; for it causes increase of the reflex activity, convulsions, and tetanus by an action on the spinal cord. The action of iodide and sulphate of methyl-thebium is strikingly different; for they diminish reflex excitability, and produce a condition of paralysis in which death occurs by asphyxia. This paralysis, as we have seen, is dependent on an effect on the spinal nerve system.

We will now describe an experiment in which we endeavoured to determine what portion of this system is affected.

EXPERIMENT LXXIX.—The sciatic artery and the two principal veins were tied in the right thigh of a frog, weighing 420 grains, and one-fifth of a grain of sulphate of methyl-thebium, dissolved in seven minims of distilled water, was injected into the abdominal cavity. In six minutes, the animal was flaccid and motionless, and in other four minutes the respiratory movements of the chest and abdomen had ceased, while those of the throat continued, and did so for several minutes longer. In sixteen minutes, galvanic stimulation by an interrupted current, applied to any portion of the skin, caused movements of the right leg *below the points of ligature*, but nowhere else. In twenty-one minutes, the left sciatic nerve was exposed, and on galvanising it, energetic movements occurred in the right leg, while the left leg and every other part of the body remained motionless. The heart was now contracting at the rate of thirty-six beats in the minute. The muscles that had been laid bare in the left leg, by the dissection necessary for the exposure of the left sciatic nerve, were stimulated by the direct application of an interrupted galvanic current, and they contracted powerfully. This condition continued during other two days; on the second day, even a feeble stimulus applied to the left sciatic nerve was followed by well-marked contractions of the right leg, below the points of ligature; while it caused no movements in those parts of the frog that had been directly acted upon by the poison, although the muscles everywhere contracted when directly stimulated.

We learn from this experiment that sulphate of methyl-thebium produces paralysis by destroying the conductivity of the motor nerves, and not by interfering with the function of the spinal cord, or of the sensory (afferent) nerves. The next experiment was made with the view to determine what portion of the motor nerve is paralysed by this substance.

EXPERIMENT LXXX.—The left gastrocnemius muscle was exposed in the leg of a frog, weighing 604 grains. The blood-vessels that entered it were ligatured or twisted, and it was carefully separated from all its connections, excepting that its origin and insertion were untouched, and that the nerve fibres that entered it were not divided. Immediately after this somewhat tedious preparation, one-fifth of a grain of sulphate of methyl-thebium, dissolved in ten minims of distilled water, was injected in the abdomen. Omitting the details of the effects that ensued, it is sufficient to mention that, at thirty minutes after this injection, the sciatic nerve was exposed in each thigh and galvanised, with the result that in the case of the right nerve movements followed in the left leg alone, and in the case of the left nerve movements followed in the left leg, and there only. It was seen that these movements in the left leg were entirely caused by contractions of the left gastrocnemius muscle, that is, of the muscle which had been protected from the direct influence of the poison.

We obtained the same results on repeating these experiments with half-grain doses of iodide of methyl-thebium.

These experiments demonstrate clearly that the methyl derivatives of thebaia produce their principal physiological effects by impairing and destroying the function of the peripheral termination of the motor nerves—an action that is very different from that of thebaia itself. They also differ from thebaia in being considerably less potent as poisons. Several of these characters are summarised in the appended table.

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.
LXII.	Iodide of methyl-thebium.	Rabbit, 2 lbs. 12 oz.	Subcutaneously.	8 grs. (containing 5·5 grs. of thebaia).	Paralysis in 45 minutes, continuing for about 30 minutes, and followed by recovery.
LXVI.	Thebaia.	Do. (same rabbit as in Experiment LXII.)	Subcutaneously.	0·2 gr.	Tetanus in 52 minutes, and death in 54 minutes.
LXXI.	Iodide of methyl-thebium.	Do., 4 lbs. 6 oz.	By stomach.	30 grs. (containing 20·6 grs. of thebaia).	No effect.
LXXIV.	Thebaia.	Do. (same rabbit as in Exp. LXXI.)	By stomach.	4 grs.	Tetanus in 6 minutes, and death in 19 minutes.
LXXVII.	Sulphate of methyl-thebium.	Do., 3 lbs. 11½ oz.	Subcutaneously.	4·5 grs. (containing 3·7 grs. of thebaia).	Paralysis in 21 minutes, continuing for 2 hours and 9 minutes, and followed by recovery.
LXXXI.	Sulphate of methyl-thebium.	Do., 4 lbs. 4 oz.	By stomach.	20 grs. (containing 16·6 grs. of thebaia).	No effect.

#### CODEIA ( $C_{18}H_{21}NO_3 + H_2O$ ).

We have examined the effect of the addition of iodide and sulphate of methyl to codeia—an opium alkaloid, which, according to CLAUDE BERNARD, is the second in toxic activity, and possesses distinct convulsant but feeble soporific properties.\*

*Iodide of methyl-codeium.*—How † obtained by the action of iodide of ethyl on codeia, iodide of ethyl-codeium, and from it a number of ethyl-codeium compounds, and proved that codeia is a nitrile base. As was to be expected, iodide of methyl acts even more readily on codeia. ‡ It is only necessary to heat codeia with a little alcohol and an excess of iodide of methyl to 100° C. for an hour, in a sealed tube, to complete the reaction. The excess of iodide of methyl is distilled off, the alcohol evaporated, and the product crystallised from hot water. It

\* Comptes Rendus, vol. lix. (1864) p. 413.

† Chemical Society's Quarterly Journal, vol. vi. (1853) p. 134.

‡ We shall give details of the chemical relations of the methyl derivatives of codeia on some other occasion.

forms large transparent prisms, soluble in 14.5 parts of water at 37°C., and in 49 parts of water at 9°C. Its solution is not precipitated by caustic potash, and in all respects, except in the appearance of its crystals, agrees with iodide of ethyl-codeium.

As iodide of methyl-codeium is tolerably soluble in warm water, we could administer it by subcutaneous injection in the form of solution. It was found, in rabbits, that a dose of five grains was quite inert, that one of fifteen grains caused prolonged and serious symptoms which were recovered from, and that one of twenty grains produced death in a short time. The following details include the principal symptoms that appeared when fifteen and twenty grains were thus administered.

EXPERIMENT LXXXIV.—Fifteen grains of iodide of methyl-codeium was dissolved in some warm distilled water, to which a few drops of rectified spirit had been added, and the solution was injected into the subcutaneous cellular tissue of a rabbit, weighing two pounds and fourteen ounces. The rabbit remained sitting quietly until twenty-two minutes afterwards, but in a few seconds more it had some difficulty in retaining a sitting posture, and, on standing, the fore-limbs occasionally yielded, until, at twenty-five minutes, it subsided on the abdomen, chest, and lower jaw. In thirty minutes, it remained on the side without struggling; and now, after considerable intervals, faint twitches occurred in the body and limbs, which, however, had no convulsive character. In thirty-seven minutes, irritation of the cornea or conjunctiva did not cause any movement in the eyelids, but the respirations, though weak, shallow, and somewhat jerking, were at the rate of sixty-seven in the minute. In forty-five minutes, the frequency of the respirations had diminished to sixty in the minute, and there were now no twitches. The rabbit continued to lie in this flaccid state for about two hours longer; at the end of which time, twitches reappeared, at first extremely faint, but, by-and-by, of considerable strength, and involving the muscles of the abdomen, chest, neck, and limbs. In four hours and twenty minutes, the rabbit was again in a perfectly quiet state, the twitches had disappeared, and the common sensibility was in a normally active condition. Frequent attempts were made, soon after, to recover a natural position, and success was at length attained, four hours and twenty-five minutes after the injection of the poison. There were no further symptoms.

EXPERIMENT LXXXV.—We injected twenty grains of iodide of methyl-codeium, dissolved as in the preceding experiment, into the subcutaneous cellular tissue of a rabbit, weighing two pounds and twelve ounces and a-half. The animal began to tremble in thirteen minutes, and the head, after being unsteadily supported for a short time, fell on the table. In fifteen minutes, the rabbit remained on the side; the respirations were weak and irregular, and slight starts occurred occasionally. Severe irritation of the skin was now required to cause

even an extremely feeble reflex movement. In twenty-four minutes, no movement followed irritation of either the skin or eyeball, and the respirations were mere gasping jerks. In thirty-two minutes, a series of feeble twitches occurred in the face-muscles, and then the respirations entirely ceased.

We immediately exposed the sciatic nerves, and examined their condition: when they were stimulated with galvanism, slight movements followed in the hind limbs at one minute after death; but no movement could be excited at one minute and thirty seconds. The heart was found to be contracting in regular rhythm, at the rate of eighty-two in the minute. Forty-five minutes after death, the body was perfectly flaccid, and there was not the slightest appearance of muscular rigidity.

We may best display the marked differences between these physiological effects and those that are caused by codeia, by describing an experiment in which the rabbit that survived the administration of fifteen grains of iodide of methyl-codeium, was quickly killed by the subcutaneous injection of one grain of codeia.

EXPERIMENT LXXXIX.—We dissolved one grain of codeia in some warm distilled water, to which a few drops of rectified spirit had been added, and injected the solution into the subcutaneous tissue of the rabbit, which was some days previously the subject of Experiment LXXXIV. In fifteen minutes, faint twitches occurred in some of the muscles of the back; and, soon after, a slight touch excited a violent start. Spontaneous spasmodic starts now followed each other, until one hour and eleven minutes, when a violent tetanic convulsion of an opisthotonic character occurred. For some time before this, it was observed that the hind limbs trailed slightly when movements were attempted, indicating, apparently, a slight degree of motor paralysis. The first tetanic convulsion was followed by trismus, which lasted for a few seconds, and by a succession of slight spasms; and soon after its occurrence, unsuccessful efforts were made to recover a normal position. In one hour and thirty minutes, a second violent tetanic convulsion took place, and this presented the character of emprostotonos rather than of opisthotonos. Such convulsions now recurred after intervals of a few minutes, and at the termination of one of them, one hour and forty-five minutes after the administration of the poison, the rabbit died. In fifteen minutes after death, strong *rigor mortis* was present.

We introduced iodide of methyl-codeium into the stomach of rabbits on two occasions. In one of these, fifteen grains were thus administered, and in the other, thirty grains; but no effect was produced by either dose. Codeia itself, however, is by no means a violent poison when given to rabbits in this manner. We made a considerable number of experiments, but did not succeed in causing death even with fifteen grains. In the following experiment we employed ten grains.

EXPERIMENT XCIV.—By means of a gum-elastic catheter, we injected ten grains of codeia, dissolved in warm distilled water to which a few drops of

dilute hydrochloric acid had been added, into the stomach of a rabbit, weighing three pounds and thirteen ounces. In twenty-four minutes, some symptoms of sleepiness were observed, which chiefly manifested themselves by nodding movements of the head. In thirty-nine minutes, the reflex excitability seemed increased, as a slight touch caused a sudden, somewhat spasmodic start. In one hour, the sleepy condition had so far increased, that the head rested on the table, and the eyelids were semi-closed. In one hour and twenty minutes, the rabbit could be placed in almost any position, provided physical rest were allowed; and it would remain sleeping in these attitudes until roused by sounds or by pretty violent irritations. It continued in this condition for more than two hours; but in three hours, the sleepiness was less marked, and on the following morning the rabbit was in a perfectly natural state.

*Sulphate of methyl-codeium* was prepared from the iodide, by precipitating it by means of sulphate of silver. It forms a white crystalline mass, readily soluble in cold water.

It is a rather more active poison than the iodide, for we found that ten grains, exhibited subcutaneously, was sufficient to kill a rabbit. We observed only slight symptoms with eight grains.

EXPERIMENT XCVI.—Eight grains of sulphate of methyl-codeium was dissolved in twenty minims of distilled water, and injected under the skin of a rabbit, weighing four pounds. No distinct effect was observed until thirty minutes, when some uneasiness was shown by restless movements of the limbs; and, soon after, a little trembling occurred. Weakness of the limbs was then exhibited by occasional stumbles, and, in thirty-three minutes, the rabbit fell, and remained resting on the abdomen, with the lower jaw on the table. There were no starts nor spasms, and even the trembling had now ceased; while severe irritation of the skin caused merely slight reflex movements. After remaining in this state for twenty minutes, the symptoms gradually improved, and the rabbit appeared to be quite well two hours after it had received the poison.

EXPERIMENT XCVII.—Ten grains of the sulphate of methyl-codeium was dissolved in distilled water, and injected under the skin of a rabbit, weighing four pounds and four ounces. In twenty-three minutes, the head and portions of the body shook in a quivering manner; and, gradually, the head sank until it rested on the table. In twenty-five minutes, the legs gave way, and the animal fell; faint twitches occurred over the body, but otherwise the condition was one of complete flaccidity. In thirty-five minutes, it remained on the side, without any resistance. In thirty-eight minutes, the respirations were laboured, and at the rate of thirty-six per minute; and in other four minutes, they had fallen to twenty per minute. In forty-one minutes, these movements were extremely shallow and irregular; and in forty-two minutes, they altogether ceased. In the course of this experiment, no convulsive symptoms occurred, and no hypnotism was observed.

In the autopsy, the heart was seen acting, one minute after death, at the rate of 160 beats per minute, and the intestinal peristalsis was found to be normal. The motor conductivity of the sciatic nerves was retained at three minutes after death, but it had disappeared in other four minutes; while the idio-muscular irritability was not lost until more than sixty minutes after death.

For internal administration, we followed the plan already described. No symptom whatever was observed when the large dose of twenty grains was introduced into the stomach of a rabbit. We did not, accordingly, consider it advisable to continue this method of administration any further.

As we have already stated, and as the experiments we have narrated clearly show, the principal effects that are caused by codeia are convulsions and hypnotism. In our experiments with rabbits, the latter effect was manifested only when large doses were introduced into the stomach. It was not seen when this alkaloid was administered by subcutaneous injection, probably because sleep was then prevented by the spasmodic starts and convulsions that were so prominently caused. We learn from our experiments that the iodide and sulphate of methyl-codeium have a very different action from codeia. We have never observed any hypnotic effect follow their administration, and, in place of convulsions, we have seen that they produce paralysis. This, indeed, is the only marked symptom that follows their administration, and it is apparent that it does not depend on an effect on the muscles, nor on the cerebral lobes. We endeavoured to determine the exact cause of this paralysis by experiments with localised poisoning on frogs.

EXPERIMENT XCVIII.—Having tied the right sciatic artery and vein of a frog, weighing 722 grains, one grain of sulphate of methyl-codeium, dissolved in distilled water, was injected into the abdominal cavity. In fifteen minutes, voluntary movements had disappeared, and the frog was lying on the abdomen, in a flaccid state. In thirty minutes, pinching of the skin with a pair of forceps excited movements in all the limbs, but these were most energetic in the right posterior extremity. In one hour and thirty minutes, similar stimulation excited no movement except in the right posterior extremity (where the vessels had been tied). The application of an interrupted galvanic current to the exposed trunk of the left sciatic nerve was now followed by active movements of the right leg, but of no other part; while, at the same time, the muscles in the poisoned regions freely responded to galvanic stimulation directly applied to them. In two hours and forty minutes, the condition was the same, and, judging from the cardiac impulse, the heart was contracting at the rate of thirty-five per minute.

We need not again enter into the reasons for concluding from such an experiment that the paralysis caused by sulphate of methyl-codeium is due to an action on the motor nerves. As has been already done with the corresponding substances treated of in the previous portion of this paper, we, in the next place, determined what portion of the motor nerve—trunk or periphery—is acted on.

EXPERIMENT XCIX.—In a frog, weighing 694 grains, the left gastrocnemius muscle was prepared in the manner described in Experiments XXIX., LVII., and LXXX., and one grain of sulphate of methyl-codeium, dissolved in distilled water, was injected into the abdomen by means of a Wood's syringe. In one hour after this, a condition of flaccidity being present, the trunks of the two sciatic nerves were exposed, and stimulated with an interrupted galvanic current. When the right nerve was thus treated, some contractions followed in the left leg, and nowhere else; and when the left nerve was thus treated, vigorous contractions followed in the left leg; and it was observed that the movements of the left leg were caused by contractions restricted to its gastrocnemius muscle, that is, the muscle to which the poison had no direct access. At this time, the muscles in all parts of the body contracted freely when the poles of the battery were applied to their surfaces, and continued to do so for many hours longer.

We repeated these last experiments with iodide of methyl-codeium, and obtained the same general results.

We have, therefore, demonstrated that iodide and sulphate of methyl-codeium produce paralysis, by destroying the function of the peripheral terminations (end-organs) of the motor nerves—a mode of action that distinguishes them, as physiological agents, in a most striking manner from codeia. It will also be seen from the following table, that the poisonous (toxic) activity of the codeia in these methyl-compounds is considerably diminished.

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.
LXXXIV.	Iodide of methyl-codeium.	Rabbit, 2 lbs. 14 oz.	Subcutaneously.	15 grs. (containing 10·2 grs. of dry codeia).	Paralysis in 25 minutes, continuing for about 3 hours, and followed by recovery.
LXXXIX.	Codeia (crystallised).	Do. (same rabbit as in Ex. LXXXIV.)	Subcutaneously.	1 gr. (containing 0·94 gr. of dry codeia).	Spontaneous twitches in 15 minutes, tetanus in 1 hour and 11 minutes, and death in 1 hour and 45 minutes.
XCI.	Iodide of methyl-codeium.	Do., 2 lbs. 13 oz.	By stomach.	30 grs. (containing 20·3 grs. of dry codeia).	No effect.
XCIV.	Codeia (crystallised).	Do. (same rabbit as in Ex. XCI.)	By stomach.	10 grs. (containing 9·4 grs. of dry codeia).	Sleepiness in 24 minutes, increase of reflex excitability in 39 minutes, and followed by recovery in more than 3 hours.
XCVI.	Sulphate of methyl-codeium.	Do., 4 lbs.	Subcutaneously.	8 grs. (containing 6·6 grs. of dry codeia).	Paralysis in 33 minutes, continuing for more than 20 minutes, and followed by recovery.
C.	Sulphate of methyl-codeium.	Do., 4 lbs. 13 oz.	By stomach.	20 grs. (containing 16·5 grs. of dry codeia).	No effect.

## MORPHIA.

The most recent and trustworthy investigations show that, among the opium alkaloids, morphia ( $C_{17}H_{19}NO_3 + H_2O$ ) is next in activity as a soporific to narceia, that it possesses a less convulsant action than codeia, and that its fatal dose is one of the largest of those of the active principles of opium.\*

*Iodide of methyl-morphium* ( $C_{17}H_{19}NO_3CH_3I$ ).—How subjected morphia to the action of iodide of ethyl and of iodide of methyl, prepared and described a number of the ethyl-morphium and methyl-morphium compounds, and proved that morphia is a nitrile base.† We prepared the iodide of methyl-morphium by How's method, viz., by treating morphia with alcohol and an excess of iodide of methyl in a sealed tube, at  $100^\circ C.$ , for an hour, distilling off the excess of iodide of methyl, and recrystallising from hot water.

It forms long, transparent, prismatic needles; and dissolves in 34 parts of water at  $37^\circ C.$ , and in 88.5 parts of water at  $9^\circ C.$

As it is well known that comparatively large doses of morphia are required to produce any symptom in such animals as rabbits, we at once commenced the administration of iodide of methyl-morphium in very large doses. We were unable to produce any effect whatever when so large a dose as twenty grains was injected under the skin of a small rabbit; and, as this could only be administered as a fine powder, suspended in warm distilled water, it was extremely inconvenient to give any larger quantity in a form necessarily so bulky. Eight grains of morphia was afterwards exhibited, in the same way, to this rabbit, and it caused the usual symptoms and death. It may be interesting and satisfactory to give some details of these two experiments.

EXPERIMENT CI.—Twenty grains of iodide of methyl-morphium was reduced to a fine powder, mixed with two drachms of warm distilled water, and injected into two previously formed subcutaneous cavities at the flanks of a rabbit, weighing two pounds and fourteen ounces. The rabbit was carefully observed for four hours, but no symptom occurred during this time. It was perfectly well on the following morning.

EXPERIMENT CV.—Eight grains of morphia, suspended in warm distilled water, was introduced into the subcutaneous cellular tissue of the rabbit that had been employed, two days previously, in Experiment CI. In one hour and four minutes, an inclination to sleep was observed, the eyelids closed, and the head sank on the table, but a slight sound immediately roused the rabbit. In two hours, the soporific effect was more marked; and the animal remained in almost any position in which it could be placed, provided the change was made gradually and gently;

\* CLAUDE BERNARD, *Comptes Rendus*, vol. lix. 1864, p. 413.

† *Chemical Society's Quarterly Journal*, vol. vi. (1853) p. 126.

and, however unnatural the position might seem to be, if it were consistent with rest, sleep immediately occurred. In three hours, there was some difficulty in rousing it, and when this was done, it remained awake for a few seconds only. In six hours, the respirations had fallen to the slow rate of twenty-six in the minute. This condition lasted, altogether, for about forty-eight hours, when spasms made their appearance, which, by-and-by, assumed all the characters of epileptiform convulsions. These epileptic fits frequently recurred, and could be excited, at any time, by pinching the skin. They consisted of tonic spasms of the limbs and of the abdominal muscles, followed by twisting of the head to the right, grinding movements of the lower jaw, and violent opisthotonos. The rabbit was found dead on the morning of the third day after the administration.

The two subcutaneous cavities into which the morphia had been introduced were laid open, and a small quantity of unabsorbed morphia was found in both. The cavities into which iodide of methyl-morphium had been introduced were also laid open, but none of this substance was found.

We were unsuccessful in producing any symptoms by the internal administration of iodide of methyl-morphium. Thirty grains was found to be perfectly inert when exhibited by the stomach, while the same rabbit was decidedly narcotised with five grains of morphia similarly exhibited. It is interesting, for the purpose of comparison, to give a short account of these two Experiments.

EXPERIMENT CVI.—We suspended thirty grains of finely-powdered iodide of methyl-morphium in distilled water, and injected the mixture into the stomach of a rabbit, weighing three pounds and twelve ounces. It was observed for more than two hours, but no symptoms could be detected.

EXPERIMENT CVII.—We suspended five grains of finely-powdered morphia in distilled water, and injected the mixture into the stomach of the rabbit that was used, two days previously, in Experiment CVI. In one hour and six minutes, the rabbit was observed to be sleepy, and it soon after laid its head on the table. This sleepy condition became gradually more marked: in one hour and twenty-five minutes, the rabbit could be placed in almost any position, and slept thus; while about the same time, a condition resembling that of catalepsy was present, for when we placed the rabbit on the back and raised the fore legs perpendicularly upwards, it remained in this extraordinary attitude for several minutes. In two hours and forty-one minutes, it was observed that the pupils, which were small, did not contract on the approach of a bright light, nor did this stimulus excite any movement of the body; but the common sensibility was not lost. The condition of cataleptic-like hypnotism lasted, altogether, about three hours and twenty minutes. Soon after this, some voluntary movements were made, and the rabbit gradually recovered to a perfectly normal state.

Any conclusion drawn from experiments on such animals as rabbits, with a substance whose predominating action is a soporific one, are always liable to

objection. For this reason, we were induced to try the effect of iodide of methyl-morphium on man. One of us,\* who is perfectly susceptible to the action of morphia, took on one occasion, half a grain of iodide of methyl-morphium, in the form of powder; but this produced no effect. On another occasion, one grain was taken, also as a powder; but not the slightest soporific or other action was caused. The latter dose contained about three-fourths of a grain of morphia, and this is certainly much above the usual narcotic dose of this substance.

It is important to mention, that although we have failed in causing any symptoms in warm-blooded animals with this substance, we have found that it acts with considerable energy on frogs. The nature of this action will be explained in the description of the effects of sulphate of methyl-morphium.

*Sulphate of methyl-morphium* ( $(C_{17}H_{19}NO_3CH_3)_2SO_4$ ), was prepared by precipitating a solution of the iodide by means of sulphate of silver. It forms a white crystalline mass, very soluble in water. It gives the ordinary blue colour-reaction of morphia with persalts of iron.

This salt of methyl-morphium is much more active than the iodide. By subcutaneous injection, doses of two, three, four, five and eight grains caused marked symptoms; while a dose of ten grains was sufficient to kill a large rabbit. The effects of eight and of ten grains are described in the two following Experiments.

EXPERIMENT CXII.—Eight grains of sulphate of methyl-morphium, dissolved in distilled water, was injected under the skin, over the two flanks of a rabbit, weighing three pounds and one ounce. In twelve minutes, it appeared to be rather sleepy, and disinclined to move. In fourteen minutes, the head fell on the table, and the animal remained in this position, without any movements, except those that were necessary for respiration. In twenty-five minutes, the hypnotism was extremely well-marked; it was possible to place the animal in any position, and if this were compatible with stability, sound sleep occurred. A considerable stimulus was now required before the rabbit could be roused from sleep. In two hours and twenty minutes, this condition still continued, but the observations were now discontinued. On the following morning, the rabbit appeared to be perfectly well. No convulsive symptoms nor exaggeration of reflex activity was observed in this Experiment.

EXPERIMENT CXIII.—We dissolved ten grains of sulphate of methyl-morphium in 200 minims of distilled water, and injected the solution under the skin of a rabbit, weighing three pounds and eight ounces. In seven minutes, difficulty in moving about was observed; and, in rapid succession, some stumbles occurred, the limbs yielded, and the animal lay in a state of flaccidity, on the abdomen, chest, and lower jaw. It could now be placed without any resistance in almost

\* Dr FRASER.

any position. In twenty-four minutes, the respirations were very feeble and shallow, and at the rate of twenty-four in the minute; the rabbit was perfectly quiet and flaccid; and severe pinching of the skin excited only feeble reflex movements. There was not the slightest appearance of muscular rigidity, nor of starts, spasms, or even quivering movements. In forty-seven minutes, the respirations were extremely weak and jerky, and at the rate of ten per minute, while the sensibility of the conjunctiva and cornea had greatly diminished. In fifty-six minutes, the respirations occurred only eight times in the minute, and no movement of the eyelids could be excited by irritating the conjunctiva or cornea. Exophthalmos was now markedly present. Death occurred in one hour and two minutes after the administration of the poison.

In the autopsy, the heart was found to be distended, and acting irregularly and slowly. There was no appearance of *rigor* two hours after death.

When administered by the stomach, twenty grains of sulphate of methylmorphium produced no effect on a rabbit.

Our experiments with morphia confirmed the observations made by others, which show that this alkaloid has two prominent actions on rabbits—a convulsant and a hypnotic one. We shall now consider how far each of these is modified by the addition of sulphate of methyl to morphia. The addition of iodide of methyl appears, no doubt, to have produced a very important change, but as this is rather in the direction of diminishing, or, as our experiments indicate, altogether destroying, the physiological activity of morphia, the iodide of methylmorphium may, in the mean time, be removed from consideration.

It has been proved, in a most satisfactory manner, that sulphate of methylmorphium possesses no convulsant action; for neither in the experiments we have described in detail, nor in any of the others we performed with this substance, was there any trace of spasmodic action or of exaggeration of the reflex function. It, however, undoubtedly causes hypnotic symptoms. In small non-fatal doses, hypnotism was chiefly manifested, and this rendered it somewhat difficult to judge whether paralysis were present or not. In large non-fatal doses, and in fatal doses, on the other hand, paralysis appeared to be the chief effect, though hypnotism was also present. It would, therefore, seem that sulphate of methylmorphium agrees with morphia in possessing a hypnotic action, but differs from it in producing paralysis, and in being free from all convulsant action. It is obvious that an objection might be urged against the latter part of this statement; for both the absence of convulsions and the production of paralysis might be merely the effects of hypnotism. Though we were ourselves convinced, from our experiments on rabbits, that such is not the case, we made some experiments on frogs to determine this more clearly.

EXPERIMENTS CXV. and CXX.—The blood-vessels were tied in one limb near the knee of two frogs, selected because of their resemblance to each other in weight

and in activity. One grain of sulphate of methyl-morphium, in solution, was injected into the abdominal cavity of one of these frogs (*a*), and three-fourths of a grain of morphia, dissolved in very dilute sulphuric acid, into the abdominal cavity of the other (*b*).

(*a*). *Frog with Sulphate of Methyl-morphium.*

In eight minutes, the limbs yielded, and the frog subsided on the abdomen and chest.

In twenty minutes, it was perfectly flaccid, and the respirations had entirely ceased. Pinching of any portion of the skin excited energetic movements of the leg whose vessels were tied, and feeble movements in various other parts.

In thirty minutes, the two sciatic nerves were exposed; galvanism applied to their trunks caused contractions of the *tied limb, below the ligatures*, and nowhere else. The heart was now acting at the rate of forty-two in the minute, and the idio-muscular irritability was normal everywhere.

In twenty-four hours, the frog was still perfectly flaccid, the heart was contracting at the rate of thirty per minute, and the muscles of the poisoned and non-poisoned regions contracted when directly galvanised. Galvanism of the sciatic nerve of the poisoned leg, however, produced no movement; but galvanism of the sciatic nerve of the non-poisoned leg, even when applied to a part where the poison had access, still caused vigorous movements below the ligatures.

(*b*). *Frog with Sulphate of Morphia.*

In sixteen minutes, some slight sprawling occurred, before which the frog was jumping about vigorously.

In fifty minutes, pinching of the skin occasioned a series of clonic spasms, in which both posterior extremities were forcibly and slowly extended and then withdrawn, somewhat regularly, during three or four minutes, about four times in the minute. The movements then ceased, but they could be again excited.

In one hour, there was marked increase of the reflex excitability, a slight touch causing a spasmodic start.

In one hour and thirty-eight minutes, a slight touch of the skin excited a short tetanic convulsion.

In two hours, the same condition existed, and a tetanic convulsion could be at any time excited by a slight touch. During these convulsions, the muscles in the non-poisoned limb were contracted as forcibly as those in the poisoned regions.

In twenty-four hours, the frog was found dead, with all its muscles rigid.

These experiments prove distinctly that sulphate of methyl-morphium does not possess, in any degree, the convulsant action of morphia, but that it causes paralysis in place of convulsions. They also prove that this paralysis is due to an effect on the motor nerves. We have further determined, by the same method of experiment as has been already frequently described, that the peripheral terminations are the parts of the motor nerves which are primarily affected.

Iodide of methyl-morphium produces the same effects on frogs as sulphate, only a larger dose is required.

The poisonous activity of sulphate of methyl-morphium does not appear to be very different from that of a salt of morphia; for we have seen that for rabbits ten grains is about the minimum fatal dose of the former by subcutaneous injection, and this contains about eight grains of morphia, which is little above the fatal dose when subcutaneously exhibited. We have placed these and several other results, in a form convenient for comparison, in the following table.

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.
CI.	Iodide of methyl-morphium.	Rabbit, 2 lbs. 14 oz.	Subcutaneously.	20 grs. (containing 13·3 grs. of dry morphia).	No effect.
CV.	Morphia.	Do. (same rabbit as in Experiment CI.)	Subcutaneously.	8 grs. (containing 7·5 grs. of dry morphia).	Sleep in 1 hour and 4 minutes, epileptic convulsions in about 48 hours, and death some hours afterwards.
CVI.	Iodide of methyl-morphium.	Do., 3 lbs. 12 oz.	By stomach.	30 grs. (containing 20 grs. of dry morphia).	No effect.
CVII.	Morphia.	Do. (same rabbit as in Experiment CVI.)	By stomach.	5 grs. (containing 4·7 grs. of dry morphia).	Sleep in 1 hour and 6 minutes, and catalepsy in 1 hour and 25 minutes; these symptoms lasted for nearly 3 hours and 30 minutes, and were followed by recovery.
CXII.	Sulphate of methyl-morphium.	Do., 3 lbs. 1 oz.	Subcutaneously.	8 grs. (containing 6·6 grs. of dry morphia).	Sleep and partial paralysis in 14 minutes, continuing for more than 2 hours and 16 minutes, and followed by recovery.
CXIII.	Sulphate of methyl-morphium.	Do., 3 lbs. 8 oz.	Subcutaneously.	10 grs. (containing 8·2 grs. of dry morphia).	Paralysis in 8 minutes, and doubtful sleepiness in 10 minutes; the paralysis became gradually more complete, and terminated in death, at 1 hour and 2 minutes after the administration of the poison.
CXXI.	Sulphate of methyl-morphium.	Do., 4 lbs. 3½ oz.	By stomach.	20 grs. (containing 16·4 grs. of dry morphia).	No effect.

## NICOTIA.

The last substance in which we have now to describe the modifications produced by chemical addition is nicotia. This is a liquid alkaloid of great poisonous energy, derived from tobacco. It is a di-acid nitrile base, and has the formula ( $C_{10}H_{14}N_2$ ).

*Iodide of methyl-nicotium.*—VON PLANTA and KEKULÉ\* investigated the action of iodide of ethyl on nicotia, and described a number of the ethyl-nicotium salts. The compounds of methyl-nicotium were investigated and described by STAHL-SCHMIDT.† When excess of iodide of methyl is added to nicotia, a considerable amount of heat is developed, and it is advisable to immerse the flask in which the mixture is made in cold water, in order to moderate the action; by this means the product (iodide of methyl-nicotium ( $C_{10}H_{14}N_2(CH_3I)_2$ )) is obtained nearly colourless, and crystallises almost as soon as it is cold. The crystalline

\* *Annalen der Chemie und Pharmacie*, vol. lxxxvii, p. 1 (1853).

† *Ibid.*, vol. xc, p. 222 (1854).

powder is washed with a little cold alcohol, and crystallised from hot rectified spirit. Thus obtained, it forms tolerably large prismatic crystals, perfectly transparent and colourless, and free from the peculiar odour of nicotia. It is extremely soluble in water, so that for our purpose it was scarcely necessary to prepare the sulphate. More for the sake of symmetry, however, than because we expected to find any difference in action, we did so.

A dose of five grains of iodide of methyl-nicotium, exhibited by subcutaneous injection, produced no effect on a rabbit. Ten grains caused trembling and slight impairment of motility; and the same symptoms occurred, in a somewhat exaggerated form, after the administration of fifteen grains: but recovery took place after both doses. The subcutaneous injection of twenty grains was followed, after several hours, by death. In the following account of the experiments in which fifteen and twenty grains were exhibited, it will be seen that no convulsive movements occurred during the progress of the symptoms.

EXPERIMENT CXXVII.—We injected fifteen grains of iodide of methyl-nicotium, dissolved in ninety minims of distilled water, into the subcutaneous cellular tissue of a rabbit, weighing three pounds. In eleven minutes, some trembling occurred, which, however, did not continue long; but it recurred in twenty-three minutes. In thirty minutes, it was observed that the head was supported with great difficulty, and shortly after it fell on the table, and the rabbit assumed a crouching attitude. There was no trembling so long as it was not disturbed; but whenever this was done, and when attempts were spontaneously made to assume some different position, the trembling recommenced. It continued in this condition for about an hour; soon afterwards the head was raised, and the trembling ceased. The rabbit was jumping about in a perfectly normal state two hours and three minutes after the administration.

EXPERIMENT CXXVIII.—We injected twenty grains of iodide of methyl-nicotium, dissolved in ninety minims of distilled water, into the subcutaneous cellular tissue of a rabbit, weighing about three pounds. In eight minutes, some trembling of the fore-legs was observed, which, however, soon ceased, and the rabbit sat down and remained quiet. In twenty minutes, the head fell upon the table, the neck muscle being apparently unable to support it; and in twenty-eight minutes, the paralysis had so far extended to the body that the rabbit, being unable to maintain even a crouching attitude, fell on the side. In one hour, it was in the flaccid condition of the last note, but the respiratory movements were few and feeble. In two hours and ten minutes, the respirations consisted of occasional gasps merely, and death appeared imminent. The observations were unfortunately now (4 P.M.) interrupted until the following morning, when (10·15 A.M.) the rabbit was dead, and in *rigor mortis*.

In accordance with the plan followed in this investigation, we shall now

describe the effects that are produced by nicotia itself,—and in order to obtain as exactly comparable data as possible, a portion of the nicotia used in the preparation of the iodide of methyl-nicotium employed in Experiments CXXVII. and CXXVIII. was administered to the rabbit which recovered from fifteen grains of the latter substance.

EXPERIMENT CXXXII.—One half-minim of nicotia (about 0.5 grain) was dissolved in fifteen minims of very dilute sulphuric acid, and the solution was injected into the subcutaneous cellular tissue of the rabbit employed, a week previously, in Experiment CXXVII. Symptoms were rapidly produced. In two minutes, spasmodic contractions occurred in the four limbs, which became extended, and raised the body in a convulsive manner. In three minutes, violent tremors occurred, and the whole body was convulsively agitated. In a few seconds afterwards, the limbs altogether yielded; the rabbit lay on the abdomen; and strong twitches occurred in the muscles of the neck, by which the head was jerked upwards, and in the limbs, by which the body was partially raised. This condition continued until ten minutes, when the spasmodic twitches ceased, and the rabbit fell on the side. It was now perfectly flaccid, with only twenty-five laboured respirations in the minute. In fourteen minutes, the respiratory movements were so feeble as to be scarcely visible; and, in fifteen minutes, they altogether ceased.

In the autopsy, the heart was found contracting, five minutes after death, at the rate of 160 per minute, but its contractions were feeble. The vermicular movements of the intestines appeared to be normal. The trunk of a sciatic nerve was irritated, ten minutes after death, and energetic movements followed in the limb to which the nerve was distributed.

Having found, in the case of iodide of methyl-nicotium, that so large doses of an extremely soluble substance were necessary to affect a rabbit by subcutaneous injection, we did not consider it advisable to determine how much was required to produce symptoms when it is exhibited by the stomach. For it may be almost positively asserted that, in the latter case, a much larger dose would be necessary; and while the administration of this would be inconvenient, because of its bulkiness, and of the difficulty of obtaining a large quantity in a perfectly pure form, the data obtained by subcutaneous injection are sufficient to prove the principal change that the addition of iodide of methyl produces in the physiological action of nicotia—namely, a great diminution in its poisonous activity.

*Sulphate of methyl-nicotium* ( $C_{10}H_{14}N_2(CH_3)_2SO_4$ ) was prepared by precipitating a solution of the iodide by means of sulphate of silver. It forms a white, crystalline mass, extremely soluble in water.

On account of the readiness with which iodide of methyl-nicotium dissolves in water, it was not to be expected that any change in poisonous activity would be caused by its conversion into a sulphate; and the following experiment con-

firms this surmise, by showing that the activity of the sulphate is apparently no greater than that of the iodide.

EXPERIMENT CXXXIII.—Ten grains of sulphate of methyl-nicotium, dissolved in ninety minims of distilled water, was injected into the subcutaneous cellular tissue of a rabbit, weighing four pounds and three ounces. In ten minutes, some trembling occurred, accompanied with partial paralysis of the fore-legs. In twenty minutes, the head fell on the table, and, at intervals, series of tremors shook the whole body. It continued in this condition, the body being still supported by the legs, until fifty minutes, when ineffectual attempts were made to raise the head. These attempts were frequently repeated, and were finally successful at one hour and ten minutes; but the trembling, though now very slight, did not altogether cease until one hour and twenty minutes. After this, the rabbit seemed perfectly well.

In the absence of any very trustworthy or complete investigation into the mode in which nicotia acts, we cannot ascertain exactly how far its physiological properties are modified by chemical addition. It would appear, however, that the convulsive movements which are described as always occurring during nicotia poisoning, and which were well marked in Experiment CXXXII., are not among the symptoms produced by either iodide or sulphate of methyl-nicotium. The action of these substances is characterised by paralysis, accompanied with tremors, but unattended with spasms or convulsions. We performed the following experiments on frogs, in order to determine if this change were due not only to the disappearance of convulsive action, but also to the appearance of a paralysing action on motor nerves, similar to that so prominently possessed by the methyl derivatives of the other alkaloids examined in this paper.

EXPERIMENT CXXX.—The blood-vessels were tied in the left thigh of a frog, weighing 430 grains, and one grain of iodide of methyl-nicotium, dissolved in fifteen minims of distilled water, was then injected into the abdomen. In ten minutes, the anterior extremities had become so weak that they could not altogether support the thorax, but still the frog jumped about with considerable activity. In twenty-five minutes, the movements were sluggish, and the jumps were by no means so active as formerly, while some trailing of the posterior extremities was observed. The heart was acting at the rate of forty-two in the minute. In thirty-five minutes, irritation of any portion of the skin was followed by contractions of all the limbs, but these appeared to be rather more energetic in the left posterior (non-poisoned) limb than in the others. In forty minutes, the respirations were feeble, but the frog was sufficiently powerful to turn itself when placed on the back. In fifty-five minutes, severe pinching caused only slight reflex movements, of nearly equal strength, in both posterior extremities. In fifty-seven minutes, it was unable to turn when placed on the back, and the heart's contractions were at the rate of thirty-seven per minute. In one hour and

thirty-nine minutes, irritation of any portion of the skin was followed by feeble, but nearly equal, movements of the four limbs. The observations were now interrupted. On the following morning the frog was jumping about normally.

EXPERIMENT CXXXI.—The blood-vessels were tied at the right knee of a frog, weighing 630 grains, and three grains of iodide of methyl-nicotium, dissolved in twenty minims of distilled water, was injected into the abdomen. In twenty-six minutes, the frog was lying, flaccid, on the abdomen and chest; and when the skin was irritated, reflex movements of equal strength were caused in the four limbs. In one hour and sixteen minutes, the flaccid state had become more marked, and, now, a somewhat stronger irritation was requisite in order to cause reflex movements, while these appeared to be of greatest strength in the right posterior (non-poisoned) limb. In two hours and forty-six minutes, the condition was exactly the same as last noted. The observations were now interrupted; and on the following morning the frog was found dead, and in *rigor*.

We obtained similar results with the sulphate.

It would, therefore, appear that though the convulsant effects of nicotia are not produced by its methyl derivatives, these derivatives do not possess any paralyzing action on motor nerves. The change that is produced in the physiological action of nicotia is not the same as that which we have described in strychnia, brucia, thebaia, codeia, and morphia. We are inclined to believe, on account of this difference, that the convulsions of nicotia are not due to the same cause as in the other alkaloids we have examined.

A great diminution in physiological activity has, however, been produced by this chemical addition, and this will be at once recognised by referring to the following table:—

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.
CXXVII.	Iodide of methyl-nicotium.	Rabbit, 3 lbs.	Subcutaneously.	15 grs. (containing 5·4 grs. of nicotia).	Trembling in 11 minutes, and partial paralysis in 30 minutes; these continued for about 49 and 50 minutes respectively, and a perfect recovery afterwards occurred.
CXXXII.	Nicotia (as sulphate).	Do. (same rabbit as in Experiment CXXVII.)	Subcutaneously.	0·5 min. (0·5 gr. nearly).	Convulsions in 3 minutes, and partial paralysis in less than 4 minutes; followed by death, 15 minutes after administration.
CXXXIII.	Sulphate of methyl-nicotium.	Do., 4 lbs. 3 oz.	Subcutaneously.	10 grs. (containing 5·6 grs. of nicotia).	Trembling in 10 minutes, slight paralysis in 20 minutes; perfect recovery in 1 hour and 20 minutes after administration.

Some experiments were made to determine, for our satisfaction, the physiological effects of iodide of methyl. The only bearing of these on the present investigation is, that no evidence was obtained in support of the extremely improbable hypothesis, that some of the changes produced in the action of the substances we have described might have been due to addition of the physiological action of the methyl compounds.

We have thus shown that chemical addition produces some important modifications in the action of those poisons which have been treated of in this communication. The action of strychnia, brucia, thebaia, codeia, morphia, and nicotia is evidently greatly diminished in degree, and, at the same time, strikingly changed in character.

The former effect is shown with all these alkaloids, especially when their action is compared with that of the iodides of their methyl derivatives. As all these iodides are much less soluble than the salts of the alkaloids themselves, it might be supposed that the diminution in activity could be explained by this difference in solubility. Some support is given to this supposition, by examining the relations between various of the substances included in this investigation. Thus, it has been demonstrated, on the one hand, that, for rabbits, the fatal dose of iodide of methyl-strychnium administered subcutaneously, is about twenty grains, and that of iodide of methyl-thebium is about ten grains; while the former is soluble in 133 parts of distilled water, at a temperature of 37° C., and the latter in 16.5 parts at the same temperature. On the other hand, the fatal dose for rabbits, of sulphate of methyl-strychnium, is about four-fifths of a grain, and that of sulphate of methyl-thebium is about five grains; while both substances are freely soluble, and with nearly equal readiness, in cold water. In these examples, the greater activity of strychnia over thebaia is manifested when a soluble salt of the methyl derivative of strychnia is employed; but when an extremely insoluble salt—the iodide—is employed, its activity is nearly the same as that of a corresponding preparation of thebaia; although the latter alkaloid is itself considerably less energetic than strychnia. It is, therefore, apparent that poisonous activity may be modified by the degree of solubility,—a well-recognised principle in toxicological physiology. But while the diminished activity of the iodides of many of these methyl derivatives may be greatly due to the difficulty of dissolving them, this explanation is inapplicable to iodide of methyl-nicotium,—an extremely soluble substance,—and it is insufficient to account for the differences of activity between the majority of the sulphates of the methyl derivatives and the salts of the alkaloids themselves. Our investigation has not furnished us with any explanation of the change in these sulphates. There are several possible explanations, but we shall not specially allude to them, as their discussion can only be properly undertaken after experimental examination of a laborious and difficult nature, and but indi-

rectly connected with our present subject. When, however, we compare the activity of the sulphates of the majority of the ammonium bases considered in this paper with that of the corresponding iodides, we observe striking differences, which cannot be explained by differences of solubility alone, but which, we believe, must be also due to the remarkable stability possessed by these iodides. Strychnia is a much less soluble substance than iodide of methyl-strychnium, and yet a rabbit that survived the administration of fifteen grains of iodide of methyl-strychnium, was killed in a few minutes by the administration of one-twentieth of a grain of strychnia. Before absorption, the strychnia may have been converted into a more soluble form, and this change may have facilitated its absorption, and permitted it to be carried by the blood-stream to the tissues it affects; but the great *stability* of the iodide of methyl-strychnium prevents its conversion into a more soluble form, and so impedes greatly the absorption. Just as in the more familiar case of the salts of lead, the sulphate is inert while the carbonate is poisonous, although they are both insoluble; and this difference of physiological action is undoubtedly due to the fact, that the carbonate, on account of its instability, is readily converted in the stomach into a soluble salt, while no such change takes place in the case of the sulphate. Stability may also influence the physiological activity of these iodides, even after their absorption, by preventing those chemical actions on the tissues by which many of the effects of poisons are probably caused.

The change in the *character* of the physiological action is remarkably illustrated by strychnia, brucia, and thebaia, whose purely spinal-stimulant action is converted into a paralysing action on the periphery (end-organs) of motor nerves; it is apparent in codeia and morphia, whose convulsant action is also converted into a paralysing action on motor nerve end-organs, and whose hypnotic action is apparently altogether destroyed in the case of codeia, and certainly greatly diminished in that of morphia; and it is obviously, though less so than with the others, in the case of nicotia, whose convulsant action is diminished if not altogether removed. We may conclude from these facts, that when a nitrile base possesses a strychnia-like action, the salts of the corresponding ammonium bases have an action identical with that of curare.

It is well known that curare and strychnia are derived from plants belonging to the same *genus*, and it is, therefore, interesting to observe such a relationship. It may not, however, be altogether superfluous to add, that strychnia, brucia, and the other spinal-stimulant alkaloids examined in this paper, have not been converted by chemical addition into curarina,—the active principle of curare. The action of the methyl derivatives of these bases is of precisely the same character as that of curare, and they possess the same peculiarity of slow absorption by the mucous membrane of the digestive system, but the degrees of their activity are very different. If we confine our

attention to the salts of the methyl derivatives of strychnia, brucia, and thebaia, where the action is uncomplicated, we observe that they form a series in which the fatal dose varies for each, while this dose in the case of the most active of the three is considerably above that of curare, and greatly above that of curarina. Besides, curarina has a characteristic colour reaction that belongs to none of these bodies; and the latter further prove this dissimilarity by each of them possessing special colour reactions by which they may be distinguished from each other.

It is not only of great interest, but probably of some practical value, that five new compounds should be found having the physiological action of curare. The great difficulty of obtaining this substance has hitherto proved a serious barrier to its therapeutical employment. Although none of the compounds that we have shown to act as curare does are so energetic as that substance, three of them—sulphate of methyl-strychnium, sulphate of methyl-brucium, and sulphate of methyl-thebium—are sufficiently so to fulfil all possible therapeutical requirements, and even to rank as powerful poisons. Moreover, they may be readily obtained in a state of perfect purity, and, therefore, of constant strength; and, in this respect, they possess a great advantage over curare.

The six alkaloids we have examined may be divided into two classes, according to the readiness with which they combine with iodide of methyl. The one class includes strychnia, brucia, thebaia, and nicotia; and the other, codeia and morphia; and the combination is much more easy with the former than with the latter class. Without attaching any general significance to the occurrence, it may not be altogether unworthy of being pointed out that in our experience, therefore, the more active poisons are the more readily acted upon by iodide of methyl.

It is curious, though not unexpected, that the ordinary colour reactions of the alkaloids are retained by their methyl derivatives. This may possibly prove of some importance to the medical jurist; and as these compounds are not precipitated by alkalies, nor by the carbonates of the alkalies, some difficulty may be met with in discovering their presence in cases of poisoning.

TABULAR SUMMARY OF EXPERIMENTS.

No of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of symptoms.	Notes.
I.	Iodide of methyl-strychnium.	Rabbit, 3 lbs.	Subcutaneously.	5 grs.	None.			
II.	Do.	3 lbs. 14 oz.	Do.	6 grs.	None.			
* III.	Do.	3 lbs. 12 oz.	Do.	8 grs.	None.			
IV.	Do.	3 lbs. 4 oz.	Do.	8 grs.	None.			
V.	Do.	3 lbs. 4 oz.	Do.	10 grs.	None.			
VI.	Do.	3 lbs. 4 oz.	Do.	12 grs.	None.			
VII.	Do.	3 lbs. 8 oz.	Do.	15 grs.	Paralysis followed by recovery.	45 minutes.	More than 2 hours.	{ Same rabbit, on three consecutive days.
VIII.	Do.	3 lbs. 2½ oz.	Do.	20 grs.	Death, preceded by paralysis.	20 minutes.	50 minutes.	
IX.	Do.	Cat (full grown).	Do.	1 gr.	None.			
X.	Do.	Do.	Do.	5 grs.	None.			
XI.	Do.	Frog, 541 grs.	Do.	1 gr.	Paralysis.	11 minutes.	Not observed.	
* XII.	Do.	Do., 410 grs.	Do.	0.5 gr.	Paralysis.	12 minutes.	Not observed.	
XIII.	Strychnia (suspended in distilled water).	Rabbit, 3 lbs. 8 oz.	Do.	0.05 gr.	Death, preceded by spasms and numerous tetanic convulsions.	8 minutes.	22 minutes.	Same rabbit as survived 15 grs. of iodide of methyl-strychnium (Experiment VII.)
XIV.	Iodide of methyl-strychnium.	Rabbit, 2 lbs. 10 oz.	By stomach.	12 grs.	None.			{ Same rabbit. Experiment XIV. was made 48 hours after Experiment XV.
XV.	Do.	2 lbs. 10 oz.	Do.	15 grs.	None.			
XVI.	Do.	3 lbs. 2 oz.	Do.	20 grs.	None.			
XVII.	Do.	3 lbs. 13 oz.	Do.	30 grs.	None.			
XVIII.	Strychnia (as hydrochlorate).	Do. 3 lbs. 2 oz.	Do.	0.075 gr.	Death, preceded by numerous spasms and tetanic convulsions.	Exaggeration of reflex function in 24 minutes.	37 minutes.	Same rabbit as survived 20 grs. of iodide of methyl-strychnium (Experiment XVI.)

\* Several of the experiments included in this Table were performed after the reading of the paper. They are distinguished by an asterisk; and it will be observed that the details of a few of them have been incorporated in the text.

TABULAR SUMMARY OF EXPERIMENTS—continued.

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of symptoms.	Notes.
XIX.	Strychnia (as hydrochlorate).	Rabbit, 3 lbs. 13 oz.	By stomach.	0·1 gr.	Death, preceded by numerous spasms and tetanic convulsions.	17 minutes.	24 minutes.	Same rabbit as survived 30 grs. of iodide of methyl - strychnium (Experiment XVII.)
XX.	Sulphate of methyl strychnium.	Do., 2 lbs. 5 oz.	Subcutaneously.	0·1 gr.	None.			
XXI.	Do.	Do., 3 lbs. 3½ oz.	Do.	0·2 gr.	None.			
XXII.	Do.	Do., 2 lbs. 5 oz.	Do.	0·5 gr.	None.			
XXIII.	Do.	Do., 3 lbs. 3½ oz.	Do.	0·8 gr.	Paralysis, followed by recovery.	28 minutes.	5½ minutes.	
XXIV.	Do.	Do., 3 lbs. 1 oz.	Do.	0·8 gr.	Death, preceded by paralysis.	Paralysis in 9 minutes.	21 minutes.	Some doubt of the absolute purity of the sulphate used in this Experiment.
XXV.	Do.	Do., 2 lbs. 14 oz.	Do.	1 gr.	Do.	11 minutes.	7 minutes.	
XXVI.	Do.	Do., 3 lbs. 10 oz.	Do.	5 grs.	Do.	13 minutes.	4 minutes.	
XXVII.	Do.	Dog, 40 lbs. (?).	Do.	12 grs.	Do.	Partial paralysis in 4 minutes.	15 minutes.	
XXVIII.	Do.	Frog, 820 grs.	Do.	0·1 gr.	Paralysis.	4 minutes.	Not observed.	
* XXIX.	Do.	Do., 615 grs.	Do.	0·1 gr.	Paralysis.	3 minutes.	Not observed.	
* XXX.	Do.	Rabbit, 3 lbs. 3 oz.	By stomach	2 grs.	None.	...	...	Same rabbit. Experiments XXX. and XXXI. were performed after an interval of 48 hours, and Experiments XXXI. and XXXII. after one of 24 hours.
* XXXI.	Do.	Do., 3 lbs. 3 oz.	Do.	5 grs.	None.	...	...	
* XXXII.	Do.	Do., 3 lbs. 3 oz.	Do.	10 grs.	None.	...	...	
XXXIII.	Do.	Do., 3 lbs. 5½ oz.	Do.	20 grs.	None.	55 minutes.	More than 2 hours.	
* XXXIV.	Do.	Do., 3 lbs. 3 oz.	Do.	25 grs.	Death, preceded by tremors and paralysis.			

TABULAR SUMMARY OF EXPERIMENTS—continued.

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of symptoms.	Notes.
XXXV.	Nitrate of methyl-strychnium.	Rabbit, 4 lbs. 1 oz.	Subcutaneously.	10 grs.	Death, preceded by tremors and paralysis.	6 minutes.	4½ minutes.	
XXXVI.	Hydrochlorate of ethyl-strychnium.	Frog, 240 grs.	Placed on skin.	0.5 gr. (?).	Paralysis, followed by recovery.	2 minutes and 30 sec.	More than 4 hours.	
XXXVII.	Do.	Do., 400 grs.	Subcutaneously.	0.5 gr.	Death, preceded by paralysis.	2 minutes.	About 15 ho.	
XXXVIII.	Iodide of methyl-brucium.	Rabbit, 3 lbs. 11 oz.	Do.	8 grs.	None.			
XXXIX.	Do.	Do., 3 lbs. 11 oz.	Do.	12 grs.	None.			
XL.	Do.	Do., 4 lbs.	Do.	15 grs.	Paralysis, followed by recovery.	2 hours and 43 minutes.	More than 1 hour and 15 minutes.	
XLI.	Do.	Do., 3 lbs. 12 oz.	Do.	18 grs.	Death, preceded by paralysis.	27 minutes.	46 minutes.	
XLII.	Brucia (as hydrochlorate).	Do., 4 lbs.	Do.	0.1 gr.	None.			
XLIII.	Do.	Do., 4 lbs.	Do.	0.2 gr.	Death, preceded by numerous spasms and tetanic convulsions.	7 minutes.	11 minutes and 30 sec.	Same rabbit as survived 15 grs. of iodide of methyl-brucium (Experiments XL.)
XLIV.	Iodide of methyl-brucium.	Frog, 380 grs.	Do.	0.5 gr.	Death, preceded by paralysis.	16 minutes.	Not observed.	
XLV.	Do.	Rabbit, 3 lbs.	By stomach.	20 grs.	None.			
XLVI.	Do.	Do., 4 lbs. 2 oz.	Do.	30 grs.	None.			
* XLVII.	Brucia (as hydrochlorate).	Do., 3 lbs.	Do.	0.2 gr.	None.			
* XLVIII.	Do.	Do., 2 lbs. 1 oz.	Do.	0.3 gr.	None.			
* XLIX.	Do.	Do., 3 lbs. 11 oz.	Do.	0.5 gr.	None.			
* L.	Do.	Do., 2 lbs. 13 oz.	Do.	1 gr.	None.			
LI.	Do.	Do., 1 lbs. 2 oz.	Do.	2 grs.	Death, preceded by spasms and tetanic convulsions.	40 minutes.	More than 2 hours and 10 minutes.	Same rabbit as survived 30 grs. of iodide of methyl-brucium (Experiment XLVI.)

TABULAR SUMMARY OF EXPERIMENTS—*continued*.

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of symptoms.	Notes.
LII.	Sulphate of methyl-brucium.	Rabbit, 2 lbs. 4 oz.	Subcutaneously.	1 gr.	None.			
LIII.	Do.	Do., 2 lbs. 13 $\frac{3}{4}$ oz.	Do.	2 grs.	Paralysis, followed by recovery.	10 minutes.	About 1 hour and 50 min.	
LIV.	Do.	Do., 3 lbs. 14 $\frac{1}{2}$ oz.	Do.	2.5 grs.	Death, preceded by paralysis.	20 minutes.	50 minutes.	
* LV.	Do.	Do., 3 lbs. 5 oz.	Do.	3 grs.	Do.	12 minutes.	16 min. and 15 seconds.	
LVI.	Sulphate of methyl-brucium.	Frog, 608 grs.	Injected into abdomen.	0.2 gr.	Do.	3 to 4 minutes.	Not observed.	
* LVII.	Do.	Do., 542 grs.	Do.	0.16 gr.	Do.	6 minutes.	Not observed.	
LVIII.	Do.	Rabbit, 4 lbs. 2 oz.	By stomach.	20 grs.	None.	...	...	
LIX.	Iodide of methyl-thebium.	Do., 2 lbs. 12 oz.	Subcutaneously.	1 gr.	None.	...	...	
LX.	Do.	Do., 2 lbs. 12 oz.	Do.	5 grs.	None.	...	...	
LXI.	Do.	Do., 2 lbs. 12 oz.	Do.	6 grs.	None.	...	...	
LXII.	Do.	Do., 2 lbs. 12 oz.	Do.	8 grs.	Paralysis, followed by recovery.	30 minutes.	1 hour and 30 minutes.	
LXIII.	Do.	Do., 2 lbs. 11 oz.	Do.	10 grs.	Death, preceded by paralysis.	10 minutes.	13 minutes and 30 sec.	
* LXIV.	Do.	Do., 2 lbs. 15 oz.	Do.	12 grs.	Do.	Less than 11 minutes.	10 minutes nearly.	
LXV.	Thebaia (as hydrochlorate).	Do., 2 lbs. 12 oz.	Do.	0.1 gr.	Slight exaggeration of the reflex function.	About 50 minutes.	About 1 hour.	
LXVI.	Do.	Do., 2 lbs. 12 oz.	Do.	0.2 gr.	Death, preceded by spasms and tetanic convulsions.	40 minutes.	14 minutes.	
LXVII.	Do.	Do., 2 lbs. 14 oz.	Do.	0.2 gr.	Do.	33 minutes.	1 hour, 12 minutes and 44 seconds.	
LXVIII.	Iodide of methyl-thebium.	Frog, 460 grs.	Do.	0.5 gr.	Death, preceded by paralysis.	12 minutes.	Not observed.	
* LXIX.	Do.	Do., 540 grs.	Do.	0.5 gr.	Do.	16 minutes.	Not observed.	

Same rabbit; intervals of at least 24 hours elapsed between each experiment.

Same rabbit as survived 8 grs. of iodide of methyl-thebium (Experiment LXII.)

TABULAR SUMMARY OF EXPERIMENTS—continued.

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of Symptoms.	Notes.
* LXX.	Iodide of methyl-thebainum.	Rabbit, 3 lbs.	By stomach.	10 grs.	None.			
LXXI.	Do.	Do., 4 lbs. 6 <sup>1</sup> / <sub>2</sub> oz.	Do.	30 grs.	None.	16 minutes.	More than 1 hour.	} Same rabbit as survived 30 grs. of iodide of methyl-thebainum (Experiment LXXI.) The experiments were made at intervals of at least 48 hours.
LXXII.	Thebaia (as hydrochlorate).	Do., 4 lbs. 6 oz.	Do.	3 grs.	Spasms, followed by recovery.	14 minutes.	More than 2 hours.	
LXXIII.	Do.	Do., 4 lbs. 6 oz.	Do.	3·5 grs.	Spasmodic convulsions, followed by recovery.	4 minutes.	15 minutes.	
LXXIV.	Do.	Do., 4 lbs. 6 oz.	Do.	4 grs.	Death, preceded by numerous tetanic convulsions.			
LXXV.	Sulphate of methyl-thebainum.	Do., 2 lbs. 14 oz.	Subcutaneously.	2 grs.	None.			
LXXVI.	Do.	Do., 3 lbs. 15 oz.	Do.	4 grs.	Slight increase of reflex activity, followed by recovery.	22 minutes.	15 minutes.	Faintly coloured preparation.
* LXXVII.	Do.	Do., 3 lbs. 11 <sup>1</sup> / <sub>2</sub> oz.	Do.	4·5 grs.	Paralysis, followed by recovery.	15 minutes.	About 2 hours and 15 min.	Pure preparation.
LXXVIII.	Do.	Do., 4 lbs. 0 <sup>1</sup> / <sub>2</sub> oz.	Do.	5 grs.	Death, preceded by paralysis.	13 minutes.	37 minutes.	
* LXXIX.	Do.	Frog, 420 grs.	Injected into abdomen.	0·2 gr.	Do. do.	5 minutes.	Not observed.	
* LXXX.	Do.	Do., 604 grs.	Do.	0·2 gr.	Do. do.	7 minutes.	Not observed.	
* LXXXI.	Do.	Rabbit, 4 lbs. 4 oz.	By stomach.	20 grs.	None.			
LXXXII.	Iodide of methyl-codeium.	Do., 3 lbs. 1 oz.	Subcutaneously.	5 grs.	None.			
LXXXIII.	Do.	Do., 3 lbs.	Do.	10 grs.	Paralysis, followed by recovery.	27 minutes.	More than 2 hours.	
LXXXIV.	Do.	Do., 2 lbs. 14 oz.	Do.	15 grs.	Do. do.	22 min. and 20 seconds.	4 hours and 3 minutes.	
* LXXXV.	Do.	Do., 2 lbs. 12 <sup>1</sup> / <sub>2</sub> oz.	Do.	20 grs.	Death, preceded by paralysis.	13 minutes.	19 minutes.	

TABULAR SUMMARY OF EXPERIMENTS—*continued.*

No. of Experiments.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of Symptoms.	Notes.
LXXXXVI.	Iodide of methyl-codeium.	Frog, 701 grs.	Subcutaneously.	2 grs.	Death, preceded by paralysis.	21 minutes.	Not observed.	
*LXXXXVII.	Do.	Do., 610 grs.	Do.	2.5 grs.	Do.	11 minutes.	Not observed.	
LXXXXVIII.	Codeia.	Rabbit, 3 lbs.	Do.	0.5 gr.	Abnormal frequency of respiration, increase of reflex activity, and slight hypnotism, followed by recovery.	14 min. and 30 seconds.	About 4 hours.	
LXXXXIX.	Do.	Do., 2 lbs. 14 oz.	Do.	1 gr.	Death, preceded by spontaneous twitches, and numerous tetanic convulsions.	15 minutes.	1 hour and 30 minutes.	Same rabbit as survived 15 grs. of iodide of methyl-codeium (Experiment LXXXXIV.)
XC.	Iodide of methyl-codeium.	Do., 3 lbs. 12 oz.	By stomach.	15 grs.	None.			
XCI.	Do.	Do., 2 lbs. 13 oz.	Do.	30 grs.	None.			
XCII.	Codeia (as hydrochlorate).	Do., 2 lbs. 5 oz.	Do.	5 grs.	None.			
*XCIII.	Do.	Do., 2 lbs. 5 oz.	Do.	7 grs.	Hypnotism, and increase of reflex cavity, followed by recovery.	43 minutes.	More than 2 hours and 30 seconds.	
*XCIV.	Do.	Do., 3 lbs. 13 oz.	Do.	10 grs.	Do.	24 minutes.	More than 2 hours and 36 minutes.	Same rabbit as received into the stomach, without any effect, 30 grs. of iodide of methyl-codeium (Experiment XCI.)
XC.V.	Sulphate of methyl-codeium.	Do., 3 lbs. 2 oz.	Subcutaneously.	4 grs.	None.			
*XCVI.	Do.	Do., 4 lbs.	Do.	8 grs.	Paralysis, followed by recovery.	30 minutes.	About 1 hour and 20 min.	No hypnotism was observed in either of these experiments.
XCVII.	Do.	Do., 4 lbs. 4 oz.	Do.	10 grs.	Death, preceded by paralysis.	23 minutes.	19 minutes.	

TABULAR SUMMARY OF EXPERIMENTS—*continued.*

No. of Experiments.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of Symptoms.	Notes.
XCVIII.	Sulphate of methyl-codeium.	Frog, 722 grs.	Injected into abdomen.	1 gr.	Death preceded by paralysis.	14 minutes.	Not observed.	
* XCIX.	Do.	Do., 694 grs.	Do.	1 gr.	Do. do.	11 minutes.	Not observed.	
C.	Do.	Rabbit, 4 lbs. 13 oz.	By stomach.	20 grs.	None.			
CI.	Iodide of methyl-morphium.	Do., 2 lbs. 14 oz.	Subcutaneously.	20 grs.	None.			
CII.	Do.	Do., 2 lbs.	Do.	20 grs.	None.			
* CIII.	Do.	Frog, 501 grs.	Do.	1 gr.	Paralysis, followed by recovery.	1 hour and 25 minutes.	More than 24 hours.	
* CIV.	Do.	Do., 410 grs.	Do.	2 grs.	Death, preceded by paralysis.	50 minutes.	Not observed.	
CV.	Morphia.	Rabbit, 2 lbs. 14 oz.	Do.	8 grs.	Death, preceded by hypnotism and epileptic convulsions.	1 hour and 4 minutes.	More than 48 hours.	Same rabbit as received 20 grs. of iodide of methyl-morphium (Experiment CI.)
CVI.	Iodide of methyl-morphium.	Do., 3 lbs. 12 oz.	By stomach.	30 grs.	None.			
CVII.	Morphia.	Do., 3 lbs. 12 oz.	Do.	5 grs.	Hypnotism and catalepsy, followed by recovery.	1 hour and 6 minutes.	About 3 hours and 30 min.	Same rabbit as received 30 grs. of iodide of methyl-morphium (Experiment CVI.)
CVIII.	Sulphate of methyl-morphium.	Do., 3 lbs.	Subcutaneously.	2 grs.	Hypnotism (and paralysis?), followed by recovery.	54 minutes.	More than 3 hours.	
* CIX.	Do.	Do., 3 lbs. 5 oz.	Do.	3 grs.	Do. do.	15 minutes.	More than 2 hours.	
* CX.	Do.	Do., 3 lbs. 1 oz.	Do.	4 grs.	Do. do.	18 minutes.	More than 3 hours.	
* CXI.	Do.	Do., 3 lbs. 8 oz.	Do.	5 grs.	Do. do.	15 minutes.	More than 3 hours.	
CXII.	Do.	Do., 3 lbs. 1 oz.	Do.	8 grs.	Hypnotism and paralysis, followed by recovery.	12 minutes.	More than 2 hours.	

TABULAR SUMMARY OF EXPERIMENTS—continued.

No. of Experiments.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of Symptoms.	Notes.
CXXIII.	Sulphate of methyl-morphium.	Rabbit, 3 lbs. 8 oz.	Subcutaneously.	10 grs.	Death, preceded by paralysis.	7 minutes.	55 minutes.	
* CXXIV.	Do.	Do., 2 lbs. 13 oz.	Do.	15 grs.	Do. do.	5 minutes.	7 minutes.	
CXXV.	Do.	Frog, 480 grs.	Injected into abdominal cavity.	1 gr.	Do. do.	8 minutes.	Not observed.	
* CXXVI.	Do.	Do., 444 grs.	Do.	1 gr.	Do. do.	4 minutes.	Not observed.	
CXXVII.	Morphia (as sulphate).	Rabbit, 3 lbs. 8½ oz.	Subcutaneously.	2 grs.	Hypnotism, followed by recovery.	20 minutes.	More than 3 and less than 24 hours.	
CXXVIII.	Do.	Do., 3 lbs. 2 oz.	Do.	4 grs.	Do. do.	7 minutes.	More than 48 and less than 72 hours.	
CXXIX.	Do.	Do., 3 lbs. 7 oz.	Do.	5 grs.	Do. do.	7 minutes.	Not observed.	
CXXX.	Do.	Frog, 510 grs.	Injected into abdominal cavity.	0.75 gr.	Death, preceded by increase of the reflex function, spasms, tetanic convulsions, and partial paralysis.	16 minutes.	More than 1 hour and 44 min. and less than 24 hours.	
CXXXI.	Sulphate of methyl-morphium.	Rabbit, 4 lbs. 3½ oz.	By stomach.	20 grs.	None.			
CXXXII.	Nitrate of methyl-morphium.	Do., 3 lbs. 2 oz.	Subcutaneously.	20 grs.	Death, preceded by tremors and paralysis.	3 minutes and 30 seconds.	10 minutes.	
CXXXIII.	Iodide of methyl-nicotium.	Do., 3 lbs. 7 oz.	Do.	5 grs.	None.			
* CXXXIV.	Do.	Do., 2 lbs. 3 oz.	Do.	5 grs.	None.			
CXXXV.	Do.	Do., 3 lbs. 4 oz.	Do.	10 grs.	Tremors and partial paralysis, followed by recovery.	10 minutes.	1 hour and 18 minutes.	Not an absolutely pure preparation; the colour being yellowish brown.
CXXXVI.	Do.	Do. 2 lbs. 10 oz.	Do.	10 grs.	Slight tremors, followed by recovery.	13 minutes.	About 20 min.	Pure, nearly colourless and odourless preparation.

TABULAR SUMMARY OF EXPERIMENTS—continued.

No. of Experiment.	Substance employed.	Animal and its weight.	Method of exhibition.	Dose.	Effect.	Interval between administration and commencement of symptoms.	Duration of symptoms.	Notes.
CXXXVII.	Iodide of methyl-nicotium.	Rabbit, 3 lbs.	Subcutaneously.	15 grs.	Tremors and partial paralysis, followed by recovery.	11 minutes.	49 minutes.	
CXXXVIII.	Do.	Do., 3 lbs.	Do.	20 grs.	Death, preceded by tremors and paralysis.	8 minutes.	About 2 hours.	
* CXXXIX.	Do.	Do., 3 lbs. 2 oz.	Do.	25 grs.	Do.	6 minutes.	1 hour and 38 minutes.	
CXXX.	Do.	Frog, 430 grs.	Injected into abdominal cavity.	1 gr.	Partial paralysis, followed by recovery.	15 minutes.	More than 1 and 30 min. and less than 15 hours.	
* CXXXI.	Do.	Do., 630 grs.	Do.	3 grs.	Death, preceded by paralysis.	10 minutes.	More than 2 and 26 min. and less than 20 hours.	
CXXXII.	Nicotia (as sulphate).	Rabbit, 3 lbs.	Subcutaneously.	6.5 minim (= about 0.5 gr.).	Death, preceded by tremors, violent convulsions, and partial paralysis.	2 minutes.	13 minutes.	Same rabbit as survived 15 grs. of iodide of methyl-nicotium (Experiment CXXXVII.)
CXXXIII.	Sulphate of methyl-nicotium.	Do., 4 lbs. 3 oz.	Do.	10 grs.	Slight tremors, and partial paralysis, followed by recovery.	10 minutes.	1 hour and 10 minutes.	
CXXXIV.	Do.	Frog, 683 grs.	Injected into abdominal cavity.	3 grs.	Death, preceded by paralysis.	32 minutes.	More than 2 and less than 48 hours.	



VI.—*On the Products of the Destructive Distillation of Animal Substances.*

Part V. By THOMAS ANDERSON, M.D., Professor of Chemistry in the University of Glasgow.

(Read 20th April 1868.—Sent for publication, November 1868).

In the fourth part of these researches, I described a new base produced by the action of sodium upon picoline, to which I gave the name of Parapicoline, because it has the same composition as picoline, although the circumstances of its formation appeared to show that it had been produced by the combination of two molecules of that substance, so that its true formula would be  $C_{12}H_{14}N_2$ . Unfortunately, its high boiling point, and tendency to decompose when distilled, made it impossible to determine its vapour density, which afforded the only means of ascertaining whether this hypothesis was correct; and it was only assumed, because similar cases of polymerisation had been established beyond a doubt in the case of other classes of organic compounds. In the hope of obtaining a similar base of lower boiling point, and therefore better adapted to the necessary experiments, I have submitted pyridine to the action of sodium, and the results of the inquiry are contained in the following pages.

My earlier experiments were conducted in precisely the same manner as those with picoline. Dry pyridine was heated to its boiling point along with small pieces of sodium, amounting to about one-fifth of its weight, in a flask furnished with a long cohobating tube. As the temperature rose, the pieces of sodium became covered with a brown coating; purple streaks appeared in the fluid, which, however, soon disappeared again; and after some hours the whole fluid was converted into a dark-brown or black mass, which was viscous when hot, and on cooling solidified into a hard brittle resin. In this a few white powdery nodules are disseminated, which explode violently when brought in contact with water. A large portion of the sodium employed remains unacted on, and if the operation has been well performed, is generally found collected into one or two large pieces, which can be easily separated from the resinous mass. After the sodium has been removed as thoroughly as possible, the crude product is thrown into water in small successive portions, so as to avoid the risk of explosions from any particles of sodium which may have remained disseminated through it. The water soon becomes alkaline, owing to the presence of caustic soda; unchanged pyridine makes itself manifest by its powerful smell, and the resin is slowly converted into a thick, viscid oil of dark-brown colour, and nearly, or altogether, insoluble in water, which collects at the bottom of the vessel. This oil is washed several times with water, dried over calcic-chloride, and distilled. The distillation is best effected in a current of hydrogen, and at a temperature

below the boiling point of the oil. A little unchanged pyridine distils at first, accompanied by a small quantity of a light oil, insoluble in water, having a pungent smell similar to, and yet appreciably different from, that of pyridine; and which, as we shall afterwards see, appears to be a mixture of several bases. As the temperature rises, a thick, heavy, and yellowish oil, having a peculiar smell, in no degree pungent, but dull, heavy, and somewhat resembling that of soot, passes over. As the distillation proceeds, crystals make their appearance in the neck of the retort. At a certain stage of the process the product becomes nearly solid, and on cooling, crystals are deposited from the fluid distillate. Towards the close of the distillation some ammonia and very volatile bases are evolved, obviously produced by the decomposition of the oil passing over; and a dark resinous mass remains in the retort, which can be forced over by raising the temperature, in doing which a large part of it is decomposed, and a residue of charcoal is left in the retort.

The products of the action being obviously complicated, the whole was cautiously redistilled, and the portion which solidified in the neck of the retort collected apart, while the fluid portions having been introduced into a freezing mixture of snow and salt, soon gave an abundant crop of crystals. These were purified by pressure between folds of filtering paper, and crystallisation from water or alcohol, in both of which they are soluble, until they have lost the smell of the oil by which they are accompanied.

After having proceeded some way in the investigation, I found that the same substances could be obtained with greater certainty by a modification of the process just described. It is by no means necessary to heat the sodium and pyridine together, for the action takes place in the cold; but in this case it is slower, and the phenomena are somewhat different. The brown appearance on the surface of the sodium and the purple streaks appear in the fluid at the beginning of the action, but the pyridine does not become brown, it retains its colour, and the sodium is covered with a black crust, which, after two or three days, exceeds it in bulk, is quite brittle, and sometimes shows a tendency to separate into layers. The pyridine acquires a yellowish tint, and then contains in solution an oil insoluble in water. When the action is judged to have gone sufficiently far, the sodium with its crust is removed from the fluid and washed with a small quantity of pure pyridine, so as to get rid of any of the oily base which may remain attached to it. The crust is then detached as thoroughly as possible from the sodium and thrown into water, any sodium still adhering to it burns, and a dark gray, almost black, powder falls to the bottom of the glass. This is washed first by decantation, and afterwards on a cloth filter until it is free from soda, and on being opened out and exposed for some time to the air, it is entirely converted into a snow-white mass of interlaced acicular crystals identical with those obtained by the first process.

## DIPYRIDINE.

The substance so obtained is a base to which, for reasons that will be immediately apparent, I give the name of dipyridine. It forms white crystals fusing at  $108^{\circ}$  Cent., and solidifies on cooling into a crystalline mass. It volatilises slowly at  $100^{\circ}$ , and sublimes unchanged at a high temperature, giving long needle-shaped crystals. It is rather sparingly soluble in cold but readily in boiling water, and the fluid, on cooling, becomes filled with a mass of interlaced needles. It is readily soluble in alcohol and ether, and the boiling solutions give acicular crystals on cooling. It likewise dissolves in pyridine and in volatile oils. From the oily base along with which it distils in the first process of preparation, it crystallises in short, thick, four-sided prisms, which are transparent so long as they remain in the fluid, but soon become opaque when they are removed from it. When well purified they are inodorous, but in general they have a faint smell, due, apparently, to a trace of the fluid base adhering to them. Dipyridine is a very stable compound. It is not decomposed by hydrochloric, sulphuric, or nitric acids. Potash and ammonia precipitate it from its solutions in acids as a mass of minute crystals. Its aqueous solution gives no precipitates with solutions of sulphate of magnesia, zinc, nickel, acetate of lead, or perchloride of iron. With sulphate of copper it gives a pale bluish-white precipitate, with corrosive sublimate a white amorphous powder insoluble in boiling water, and with nitrate of silver a white precipitate insoluble in cold and sparingly in boiling water, from which the compound is obtained in crystals on cooling. Its most characteristic reactions, however, are those it gives with the ferro- and ferri-cyanides of potassium. If a few drops of the ferrocyanide of potassium be added to a not too dilute solution of the dipyridine hydrochlorate, a pale precipitate makes its appearance, which rapidly changes to a dirty indigo colour, increasing at the same time in quantity. If the proper concentration is hit, the precipitate consists entirely of very minute needle-shaped crystals having a dark indigo colour. They dissolve in boiling water, forming a very deep and rather dull purple solution, and are again deposited on cooling; but if the boiling be continued for some time, the compound appears to undergo some change, for the fluid retains its red colour at ordinary temperatures, though a great part of the substance is still deposited in crystals. A saturated cold solution of dipyridine in water gives no precipitate with ferrocyanide of potassium, but on the addition to the mixture of a drop or two of hydrochloric acid the dark precipitate instantly makes its appearance, and is deposited in small crystals. The precipitate is readily soluble in excess of hydrochloric acid. When ferricyanide of potassium is added to dipyridine hydrochlorate no immediate effect is observed, but on standing, the interior of the test-tube becomes lined with minute prisms of sulphur yellow colour and high lustre. If the solution be boiled, it acquires a dark colour, and partial decomposition takes place.

Dipyridine carefully dried in the water bath was found on analysis to give these results:—

I.	{	0·2802	gramme	dipyridine	gave
		0·7782	...	carbonic acid,	and
		0·1497	...	water.	
II.	{	0·3000	gramme	dipyridine	gave
		0·8367	...	carbonic acid,	and
		0·1575	...	water.	

	Experiment.		Calculation.		
	I.	II.			
Carbon, . . .	75·74	76·07	75·94	C <sub>5</sub>	60
Hydrogen, . . .	5·94	5·83	6·33	H <sub>5</sub>	5
Nitrogen, . . .	...	...	17·73	N	14
			100·00		79

These numbers lead to the formula C<sub>5</sub>H<sub>5</sub>N, which is that of pyridine itself.

The platinum compound of the base, which is thrown down as a yellow crystalline powder, gave the following results:—

I.	{	0·3345	gramme	gave
		0·1175	...	platinum.
II.	{	0·2007	gramme	gave
		0·0685	...	platinum.
III.	{	0·4335	gramme	gave
		0·1473	...	platinum.
IV.	{	0·4052	gramme	gave
		0·3087	...	carbonic acid, and
		0·0905	...	water.

	Experiment.			Calculation.		
	I.	II.	III.			
Carbon, . . .	...	...	20·78	21·02	C <sub>10</sub>	120
Hydrogen, . . .	...	...	2·48	2·10	H <sub>12</sub>	12
Nitrogen, . . .	...	...	...	4·90	N <sub>2</sub>	28
Chlorine, . . .	...	...	...	37·30	Cl <sub>6</sub>	213
Platinum, . . .	34·11	34·12	34·03	34·68	Pt	197·4
				100·00		570·4

This agrees with the formula (C<sub>5</sub>H<sub>5</sub>NHC1)<sub>2</sub>PtCl<sub>4</sub>, which is identical with that of the pyridine salt. In order to fix the true constitution of the base, it was necessary to determine its vapour density, and as its boiling point was beyond the range of the mercurial thermometer, it was necessary to use a bath of metallic lead and an air thermometer. The air thermometer was a bulb of the same size as that used for containing the vapour, and the details of the experiment were as follows:—

Weight of vapour bulb empty, . . . . .	22.2530
... .. full of vapour, . . . . .	22.4806
Temperature at weighing, . . . . .	13°5
Barometer, . . . . .	753 mm.
Volume of bulb, . . . . .	127.1 C.C.
Residual air, . . . . .	0.2 C.C.

Hence

Weight of bulb and vapour, . . . . .	22.4806
... .. air displaced, . . . . .	0.1552
<hr/>	
Weight of bulb and vapour in vacuo, . . . . .	22.6358
... .. bulb, . . . . .	22.5230
<hr/>	
Residual air, . . . . .	0.3828
<hr/>	
Residual air, . . . . .	0.0003
<hr/>	
Weight of vapour, . . . . .	0.3825
Weight of air thermometer, . . . . .	264 grains.
... .. partly full of mercury, . . . . .	14391
Barometer 753 — 67.5 . . . . .	685.5 mm.
Temperature, . . . . .	17°5 C.
Air thermometer full of mercury, . . . . .	26930

Hence

Weight of mercury partially filling bulb, . . . . .	14127
... .. completely, . . . . .	26666

and

$$26666 - 14127 = 12539.$$

Now, we have here in the formula

$$W(1 + kt) = w \cdot \frac{h}{H} \times \frac{1 + at}{1 + at'}$$

$$\begin{aligned} W &= 26666 \\ w &= 12539 \\ H &= 753 \\ h &= 685.5 \\ a &= 0.00366 \\ k &= 0.00003 \\ t' &= 17^\circ.5 \end{aligned}$$

whence

$$t = 414^\circ.4 \text{ Cent.}$$

Now

$$127.1 - 0.5 = 126.5,*$$

and

$$\frac{126.5}{1 + 414^\circ.4 \times 0.00366} \times \frac{753}{760} \times \frac{1.29366}{1000} = 0.0646,$$

and

$$\frac{0.3825}{0.0646} = 5.92.$$

The formula  $C_{10}H_{10}N_2$  requires 5.46. This result is as close as could be expected under the circumstances, and proves that the base must be formed by the com-

\* 0.5 is the volume of the residual air at 414°.

bination of two molecules of pyridine, and hence the name of dipyridine which I have applied to it.

*Salts of Dipyridine.*—Though dipyridine is not a very powerful base, it gives a number of salts, most of which crystallise well, though some of them are not easily obtained of definite composition.

*Hydrochlorate of Dipyridine.*—This salt is best obtained by adding a slight excess of hydrochloric acid to the aqueous solution of the base and evaporating to crystallisation; the crystals, after being pressed and recrystallised from water, are sufficiently pure for analysis. They are flat needles readily soluble in water, especially when hot—insoluble in ether. The salt is very apt to retain hydrochloric acid, and it is advisable to heat it to 130° for analysis.

{ 0·6104 gramme of the hydrochlorate gave  
0·7633 ... iodide of silver.

				Experiment.	Calculation.		
Carbon,	.	.	.	...	51·94	C <sub>10</sub>	120
Hydrogen,	.	.	.	...	5·19	H <sub>12</sub>	12
Nitrogen,	.	.	.	...	12·13	N <sub>2</sub>	28
Chlorine,	.	.	.	30·93	30·74	Cl <sub>2</sub>	71
					100·00		231

*Sulphate of Dipyridine.*—Dipyridine is added in slight excess to dilute sulphuric acid, and this fluid is evaporated nearly to dryness; on cooling, crystals of the sulphate are deposited; they are washed with alcohol, in which they are scarcely soluble, and again crystallised from water. It is thus obtained in needle-shaped crystals, which deliquesce in moist air. One determination of sulphuric acid was made of a specimen of this salt dried in vacuo over sulphuric acid, which gave 26·85 per cent. of SO<sub>3</sub>. This would correspond with a salt containing two molecules of water of crystallisation with the formula C<sub>10</sub>H<sub>10</sub>N<sub>2</sub>H<sub>2</sub>SO<sub>4</sub> + 2H<sub>2</sub>O, which requires 27·39 per cent.

*Nitrate of Dipyridine* is obtained by adding a slight excess of nitric acid to solution of dipyridine, evaporating on the water-bath and recrystallising. It forms pale yellow needles of moderate solubility in water. When exposed for some time to a temperature of 100°, it acquires an orange colour, but is not sensibly decomposed. A combustion of this salt gave

I. { 0·5162 gramme of the nitrate gave  
0·7915 ... of carbonic acid, and  
0·1820 ... of water.

				Experiment.	Calculation.		
Carbon,	.	.	.	41·81	42·25	C <sub>10</sub>	120
Hydrogen,	.	.	.	3·92	3·92	H <sub>12</sub>	12
Nitrogen,	.	.	.	...	20·38	N <sub>4</sub>	56
Oxygen,	.	.	.	...	33·45	O <sub>6</sub>	96
					100·00		284

*Double Salt with Chloride of Zinc.*—This substance is prepared by adding to dipyridine a mixture of hydrochloric acid and zinc chloride, and allowing it to stand for some time, when small prismatic crystals are deposited. Their formation is materially assisted by the addition of alcohol and ether. The compound may even be obtained from the crude product, in which the dipyridine is mixed with the oily base distilling along with it. For this purpose zinc chloride, along with hydrochloric acid and a considerable excess of alcohol and ether, must be added to the crude product, when, especially on stirring, the zinc salt deposits, and can be purified by crystallisation from water. This process can even be employed for separating the two bases. The salt is in long white needles, soluble in eight times their weight of water, less so in alcohol, and quite insoluble in ether. On the addition of potassic hydrate in excess it gives the pure base in minute crystals. A chlorine determination gave

{ 0.3660 gramme zinc salt gave  
 { 0.5755 ... dichloride of zinc.

		Experiment.	Calculation.		
Carbon,	. . .	...	32.70	C <sub>10</sub>	120
Hydrogen,	. . .	...	3.27	H <sub>12</sub>	12
Nitrogen,	. . .	...	7.62	N <sub>2</sub>	28
Chlorine,	. . .	38.90	38.68	Cl <sub>4</sub>	142
Zinc,	. . .	...	17.73	Zn	65
			100.00		367

*Double Salt with Nitrate of Silver.*—This salt is best obtained by mixing hot solutions of dipyridine hydrochlorate, and silver nitrate, the latter being in excess, and at once filtering off the precipitated silver chloride. On cooling, the salt is deposited in brilliant needles, of sparing solubility in water. This compound could not be obtained of constant composition, but one specimen gave 35.09 per cent. of silver, while the formula C<sub>10</sub>H<sub>10</sub>N<sub>2</sub>(HNO<sub>3</sub>)<sub>2</sub>(AgNO<sub>3</sub>)<sub>2</sub> requires 34.61.

*Platino-chloride of Dipyridine.*—The analysis of this salt has been already given. It is obtained as a crystalline yellow powder of very sparing solubility.

*Palladio-chloride of Dipyridine* is obtained as an orange precipitate on mixing the hydrochlorates.

#### DERIVATIVES OF DIPYRIDINE.

*Diethylo-Dipyridine.*—The compounds of this base were obtained in the usual manner. The ethyl-iodide is easily prepared by exposing dry dipyridine with iodide of ethyl to the temperature of 100° in hermetically sealed tubes. The action is complete in half an hour. It is obtained in acicular crystals, which are brilliant and perfectly colourless if they have not been exposed to the air. They are very soluble in water, much less so in alcohol and in ether.

- I. { 0·3997 gramme ethyl-iodide gave  
 0·5195 ... carbonic acid, and  
 0·1530 ... water.
- II. { 0·3512 gramme gave  
 0·3513 ... iodide of silver.

	Experiment.	Calculation.	
Carbon, . . .	35·44	35·74	C <sub>14</sub> 168
Hydrogen, . . .	4·25	4·26	H <sub>20</sub> 20
Nitrogen, . . .	...	5·96	N <sub>2</sub> 28
Iodine, . . .	54·05	54·04	I <sub>2</sub> 254
		100·00	470

Corresponding with the formula C<sub>10</sub>H<sub>20</sub>N<sub>2</sub>(C<sub>2</sub>H<sub>5</sub>I)<sub>2</sub>. Heated with silver chloride and water this salt was converted into the chloride, which, on the addition of platinic-chloride gave the platino-chloride in very sparingly soluble small red needles.

- { 0·2780 gramme platino-chloride gave  
 0·0685 ... platinum.

This corresponds with 31·12 per cent., and the formula C<sub>10</sub>H<sub>10</sub>N<sub>2</sub>(C<sub>2</sub>H<sub>5</sub>Cl)<sub>2</sub>PtCl<sub>4</sub> requires 30·44.

The base itself, when separated from the iodide by silver oxide, forms a highly alkaline solution, having generally a red or purple colour, which, on evaporation, leaves a dark-coloured uncrystallised residue. It obviously belongs to the class of ammonium bases, but I have not pursued its investigation further.

*Dibromo-Dipyridine.*—This base is thrown down when bromine is added to a solution of dipyridine hydrochlorate, or hydrobromate, as a white powder, insoluble in water, sparingly soluble in cold, more so in hot alcohol, from which it is deposited in flattened needles on cooling. If too much bromine has been used in its preparation, these crystals are pink. Its basic properties are extremely feeble, and it is a somewhat unstable compound; for, on boiling with water, or with hydrochloric acid, the original base appears to be more or less completely regenerated. An analysis, in which the hydrogen was lost, gave

- I. { 0·3560 gramme dibromo-dipyridine gave  
 0·4902 ... carbonic acid.
- II. { 0·3420 gramme gave  
 0·4078 .. silver iodide.

	Experiment.	Calculation.	
Carbon, . . .	37·56	37·97	C <sub>10</sub> 120
Hydrogen, . . .	...	2·53	H <sub>8</sub> 8
Nitrogen, . . .	...	8·87	N <sub>2</sub> 28
Bromine, . . .	50·74	50·87	Br <sub>2</sub> 160
		100·00	316

Its platinum compound could not be obtained in a state fitted for analysis. On boiling with hydrochloric acid, and adding platinic-chloride, a yellow precipitate was obtained, which, in one experiment, gave 32.39 per cent. of platinum; and in another, in which the boiling was continued longer, 33.53 per cent. was obtained. Dibromo-dipyridine requires 30.94, and dipyridine itself 34.68; so that there can be little doubt that the latter has been regenerated.

## OILY BASE.

The oily base from which the dipyridine was deposited in crystals, has been as yet but imperfectly examined. It was purified by redistillation and cooling, by which it yielded a small additional quantity of dipyridine, and this was repeated as long as it gave crystals. The base so obtained is a rather thick, pale-yellow oil, heavier than water, having a peculiar heavy smell, quite distinct from that of pyridine. It is insoluble in water, but dissolves with great ease in alcohol and ether. It boils at a high temperature, and if distilled rapidly, it undergoes partial decomposition, yielding a small quantity of what appears to be a mixture of several bases with pungent smell, and sparingly soluble in water. If, however, the distillation be carried on very cautiously at a temperature below its boiling point, it passes over unchanged. It dissolves in acids and forms salts, most of which, however, are uncrystallisable, and dry up into gummy masses. It was prepared for analysis by drying over calcic-chloride, distilling and separating the first part of the distillate which might retain moisture.

I.	{	0.3420 gramme of the base gave
	{	0.9605 ... carbonic acid.
	{	0.2135 ... water.
II.	{	0.3662 gramme of the base gave
	{	1.0140 ... carbonic acid.
	{	0.2300 ... water.

	Experiment.		Calculation.		
	I.	II.			
Carbon, . . .	76.59	75.51	75.94	C <sub>5</sub>	60
Hydrogen, . .	6.94	6.98	6.33	H <sub>5</sub>	5
Nitrogen, . .	...	...	17.73	N <sub>5</sub>	14
			100.00		79

These results, it will be seen, correspond with those given by pyridine and dipyridine, and they are confirmed by the analysis of a platinum salt, prepared in the usual way.

{	0.2991 gramme of the platinum salt gave
{	0.1015 ... platinum.

This corresponds to 33.94, and calculation for the formula (C<sub>5</sub>H<sub>5</sub>NHCl)<sub>2</sub>PtCl<sub>4</sub> requires 34.68. It is obvious, therefore, that this is another polymer of the

original pyridine; but, unfortunately, there is no means by which its molecular constitution can be determined. It is impossible to determine its vapour density, because it undergoes partial decomposition at its boiling point; and as its salts do not crystallise, and probably, like the platinum compound, all correspond with those of the original pyridine, there is no prospect of satisfactory conclusions being drawn from them. In the absence of experimental evidence, any assumption may be made regarding the constitution of this base, and at first sight the most reasonable view of the matter is to suppose it to be the product of a further polymerisation, and to be formed by the combination of three or four molecules of the original pyridine. Its boiling point, which is certainly lower than that of dipyridine, however, appears to militate against this view; and taking its properties and those of its compounds into consideration, I am inclined to believe it to be another dipyridine, and an example of those cases of physical isomerism of which so many are now known. As there was no means of ascertaining the constitution of this base, and the properties of its compounds were not encouraging, I have not pursued their investigation further.

#### LIGHT BASES.

It has been stated at the commencement of this paper that when dipyridine was prepared by the first of the processes there described, that a light basic oil was obtained at the beginning of the rectification of the crude product. This oil, which is insoluble in water, was collected, dried, and rectified when it was found to consist of several bases. The distillate was collected in several fractions, which were analysed, but the quantity was far too small to admit of any systematic attempt to separate them. The results, both of the combustion of the bases themselves and of the platinum determinations in their platinum compounds, seem to show that they are a class of bases isologous with the pyridine series. I give here the results of these analyses:—

I.	{	0·3025	gramme of base boiling between 225° and 240° F. gave
		0·8375	... .. carbonic acid, and
		0·2302	... .. water.
II.	{	0·4120	gramme of base boiling between 270° and 290° F. gave
		1·1385	... .. carbonic acid, and
		0·3230	... .. water.
III.	{	0·2711	gramme of base boiling about 291° F. gave
		0·7572	... .. carbonic acid, and
		0·1984	... .. water.
IV.	{	0·2708	gramme of base boiling between 287° and 291° F. gave
		0·7371	... .. carbonic acid, and
		0·2092	... .. water.
V.	{	0·2355	gramme of base boiling between 291° and 360° F. gave
		0·6690	... .. carbonic acid,
		0·1795	... .. water.

	I.	II.	III.	IV.	V.
Carbon, . . .	75.51	75.36	76.18	74.23	77.47
Hydrogen, . . .	8.46	8.71	8.13	8.58	8.47
Nitrogen, . . .	16.03	15.93	15.69	17.19	14.06
	<hr/> 100.00				

It must be distinctly understood that all the substances analysed were obviously mixtures, and the degrees above given do not denote true boiling points, but only that the fractions they represent were collected at these points. It will be observed that all these analyses are characterised by yielding a percentage of hydrogen far above that contained in pyridine, or any of its homologues. To render this more obvious, I place here the calculated numbers for pyridine and picoline, along with those required by bases containing two atoms of hydrogen more than these compounds:—

	$C_5H_5N$	$C_6H_7N$	$C_5H_7N$	$C_6H_9N$
Carbon, . . .	75.94	77.42	74.04	75.75
Hydrogen, . . .	6.33	7.53	8.64	9.47
Nitrogen, . . .	17.73	15.05	17.32	14.78
	<hr/> 100.00	<hr/> 100.00	<hr/> 100.00	<hr/> 100.00

The experimental results above given are manifestly incompatible with the first two of these formula, but would agree tolerably with a mixture of substances containing more hydrogen, and this is further confirmed by several platinum determinations in platinum compounds obtained from these substances. These results appear to indicate the existence of a series of bases having the general formula  $C_nH_{2n-3}N$ . The further investigation of these substances would be of interest, but as they are obtained only in minute quantity, and are clearly secondary products of the action of sodium on pyridine, it is scarcely possible to obtain them in sufficient quantity for this purpose.

At the beginning of this paper, mention has been made of a black or dark-gray substance, obtained in the second process for preparing dipyrindine. This compound was only obtained towards the close of the investigation, and I have not had time to examine its properties and relations minutely. It is a black amorphous powder, quite insoluble in water. When exposed to the air, it is rapidly converted into a mass of crystals of pure dipyrindine. I was at first disposed to consider this substance to be a sodium compound of dipyrindine, but I soon found that this was not the case, and that its properties more nearly corresponded with a hydrogen compound of that base; and it seems probable that its formula will turn out to be  $C_{10}H_{12}N_2$ , in which case it would be related to

dipyridine in the same manner as ammonium is to ammonia. Should this view be correct, it seems probable that, on the addition of hydrochloric acid, it should, like a metal, evolve hydrogen. An experiment was made to ascertain whether this occurred by introducing a quantity of the compound into a jar over mercury, and bringing hydrochloric acid in contact with it; but the anticipated result was not obtained,—no hydrogen was evolved, but a brownish precipitate appeared in the fluid, and the gray powder at once disappeared. I have been unable to pursue this subject further, but propose to return to it on a future occasion.

Numerous experiments have been made, in the hope of throwing light on the nature of the chemical changes occurring during the first process of preparing dipyridine; but it is obviously of a very complex kind, and some of the products must be the result of secondary decompositions. My impression is, that sodio-dipyridine must be first formed, probably  $C_{10}H_8Na_2N_2$ . In this case hydrogen must be given off during the action, and this is actually the case, as was established by direct experiment; but the quantity evolved is trifling compared with that of the sodium consumed, so that if the action takes place in this way, a large part of the hydrogen must be converted into some other compounds within the mixture itself. The light bases already mentioned might account for this, if it were not that they are produced in very small quantity. Altogether I am inclined to think that, in the first process, a number of secondary reactions take place, which greatly complicate matters, and that it is through some modification of the second by which an explanation will most probably be obtained. I am still engaged with the subject, and have already nearly perfected a process by which some of the products can be obtained with greater certainty and in larger quantity than by either of those described in this paper, and which I hope will enable me to subject the constitution and relations of these curious compounds to a more minute examination.

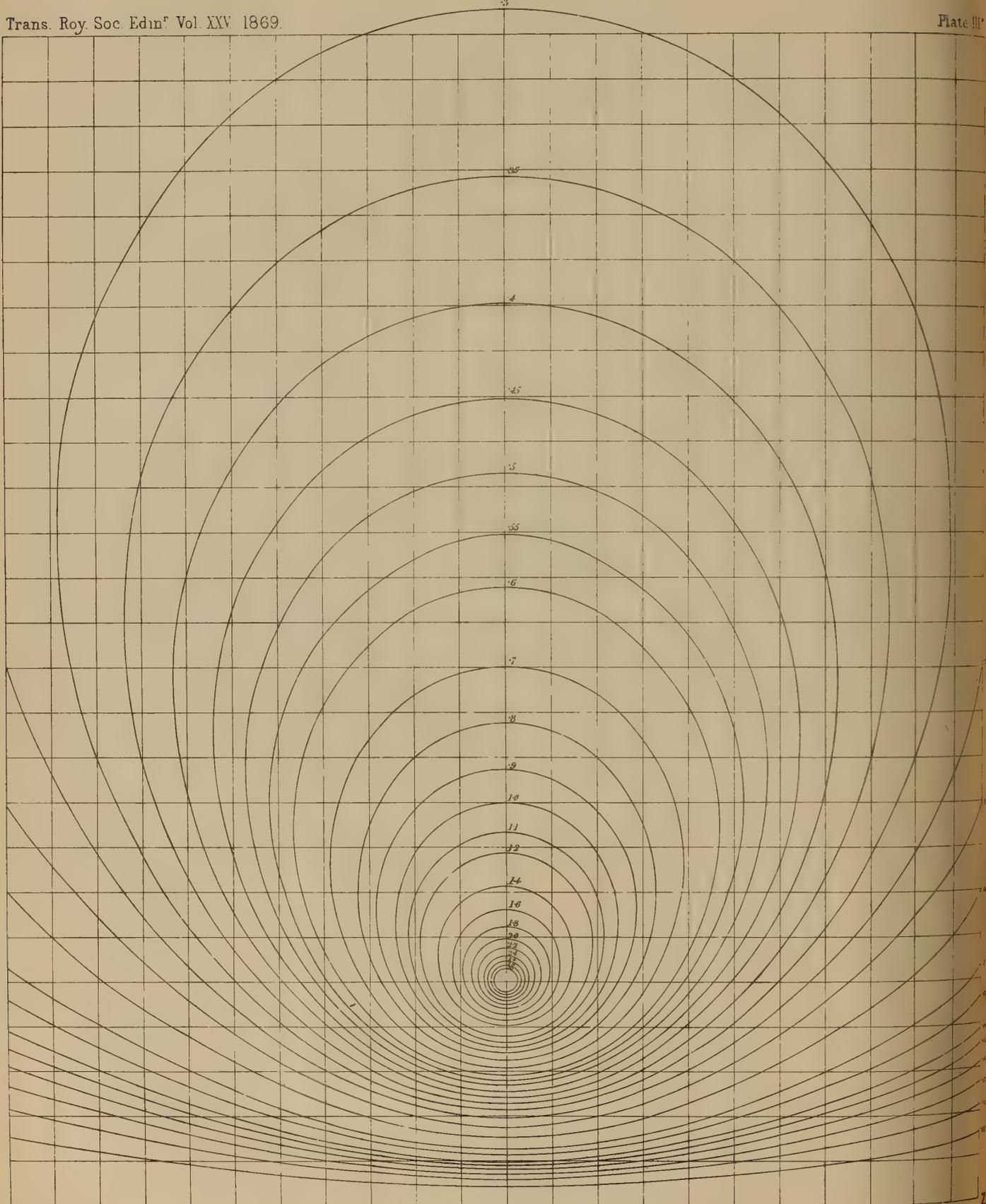


Diagram showing stream lines traversing circular ring.

X

Trans. Roy. Soc. Edin<sup>r</sup>. Vol. XXV. 1869.

Plate III



VI.—*On Vortex Motion.* By Sir W. THOMSON.

(Read 29th April 1867.)

( §§ 1-59 recast and augmented 28th August to 12th November 1868.)

1. The mathematical work of the present paper has been performed to illustrate the hypothesis, that space is continuously occupied by an incompressible frictionless liquid acted on by no force, and that material phenomena of every kind depend solely on motions created in this liquid. But I take, in the first place, as subject of investigation, a finite mass of incompressible frictionless\* fluid completely enclosed in a rigid fixed boundary.

2. The containing vessel may be either *simply* or *multiply continuous*.† And I shall frequently consider solids surrounded by the liquid, which also may be either simply or multiply continuous. It will not be necessary to exclude the supposition that any such solid may touch the outer boundary over some finite area, in which case it is *not* surrounded by the liquid; but each such solid, whether surrounded by the liquid or not, and whether moveable or fixed, must be considered as a part of the whole boundary of the liquid.

3. Let the whole fluid be given at rest, and let no force, except pressure from the containing vessel, or from the surfaces of solids immersed in it, ever act on any part of it. Let there be any number of solids, perfectly incompressible, and of the same density as the fluid; but either perfectly rigid, or more or less flexible, with perfect or imperfect elasticity. Some of these may at times be supposed to lose rigidity, and become perfectly liquid; and portions of the liquid may be supposed to acquire rigidity, and thus to constitute solids. Let the solids act on one another with any forces, pressures, frictions, or mutual distant actions, subject only to the law of “action and reaction.” Let motions originate among them and in the liquid, either by the natural mutual actions of the solids or by the arbitrary application of forces to them during some limited time. It is of no consequence to us whether these forces have reactions on matter outside the containing vessel, so that they might be called “natural forces” in the present state of science (which admits action and reaction at a distance); or are applied arbitrarily by supernatural action without reaction. To avoid circumlocution,

\* A frictionless fluid is defined as a mass continuously occupying space, whose contiguous portions press on one another everywhere exactly in the direction perpendicular to the surface separating them.

† HELMHOLTZ—*Ueber Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen*: Crelle (1858); translated by TAIT in *Phil. Mag.* 1867, i. RIEMANN—*Lehrsätze aus der Analysis situs, &c.* Crelle (1857). See also § 58, below.

and, at the same time, to conform to a common usage, we shall call them *impressed forces*.

4. From the homogeneousness as to density of the contents of the fixed bounding vessel, it follows that the centre of inertia of the whole system of liquid and solids immersed in it remains at rest; in other words, the integral momentum of the motion is zero. Hence (THOMSON and TAIT'S "Natural Philosophy," § 297) the time integral of the sum of the components of *pressure on the containing vessel*, parallel to any fixed line, is equal to the time-integral of the sum of the components of *impressed forces* parallel to the same line. This equality exists, of course, at each instant during the action of the impressed forces, and continues to exist for the constant values of their time integrals, after they have ceased. Thus, in the subsequent motion of the solids, and of the fluids compelled to yield to them, whatever pressure may come to act on the containing vessel, whether from the fluid or from some of the solids coming in contact with it, the components of this pressure, parallel to any fixed line, summed for every element of the inner surface of the vessel, must vanish for every interval of time during which no impressed forces act. If, for example, one of the solids strikes the containing vessel, there will be an impulsive pressure of the fluid over all the rest of the fixed containing surface, having the sum of its components parallel to any line, equal and contrary\* to the corresponding component of the impulsive pressure of the solid on the part of this surface which it strikes [see § 8, and consider oblique impulse of an inner moving solid, on the fixed solid spherical boundary]. *But, after the impressed forces cease to act, and as long as the containing vessel is not touched by any of the solids, the integral amount of the component of fluid pressure on it, parallel to any line, vanishes.*

5. If now forces be applied to stop the whole motion of fluid and solids [as (§ 62) is done, if the solids are brought to rest by forces applied to themselves only], the time integrals of the sums of the components of these forces, parallel to any stated lines, *may or may not in general be equal and contrary* to the time integrals of the corresponding sums of components of the initiating impressed forces (§ 3). But we shall see (§§ 19, 21), that *if the containing vessel be infinitely large, and all of the moving solids be infinitely distant from it during the whole motion*, there must be not merely the equality in question between the time integrals of the components in contrary directions of the initiating and stopping impressed forces, but there must be (§ 21) *completely equilibrating opposition between the two systems.*

6. To avoid circumlocution, henceforth I shall use the unqualified term *impulse* to signify a system of impulsive forces, to be dealt with as if acting on a rigid body. Thus the most general impulse may be reduced to an impulsive force, and couple

\* I shall use the word *contrary* to designate merely directional opposition; and reserve the unqualified word *opposite*, to signify *contrary and in one line*.

in plane perpendicular to it, according to POINSOT; or to two impulsive forces in lines not meeting, according to his predecessors. Further, I shall designate by *the impulse of the motion at any instant*, in our present subject, the system of impulsive forces on the moveable solids which would generate it from rest; or any other system which would be equivalent to that one if the solids were all rigid and rigidly connected with one another, as, for instance, the POINSOT resultant impulsive force and minimum couple. The line of this resultant impulsive force will be called the *resultant axis of the motion*, and the moment of the minimum couple (whose plane is perpendicular to this line) will be called the *rotational moment of the motion*.

7. But, having thus defined the terms I intend to use, I must, to warn against errors that might be fallen into, remark that the momentum of the whole motions of solids and liquid is *not* equal to what I have defined as *the impulse*, but (§ 4) is equal to zero; being the force-resultant of “the impulse” and the impulsive pressure exerted on the liquid by the containing vessel during the generation of the motion: and that the moment of momentum of the whole motion round the centre of inertia of the contents of the vessel is *not* equal to the *rotational moment*, as I have defined it, but is equal to the moment of the couple constituted by “the impulse” and the impulsive pressure of the containing vessel on the liquid. It must be borne in mind that however large, and however distant all round from the moveable solids, the containing vessel may be, it exercises a finite influence on the momentum and moment of momentum of the whole motion within it. But if it is infinitely large, and infinitely distant all round from the solids, it does so by infinitely slow motion through an infinitely large mass of fluid, and exercises no finite influence on the finite motion of the solids or of the neighbouring fluid. This will be readily understood, if for an instant we suppose the rigid containing vessel to be not fixed, but quite free to move as a rigid body without mass. The momentum of the whole motion will then be not zero, but exactly equal to the force-resultant of the impulse on the solids; and the moment of momentum of the whole motion round the centre of inertia will be precisely equal to the resultant impulsive couple found by transposing the constituent impulsive forces to this point after the manner of POINSOT. But the finite motion of the immersed solids, and of the fluid in their neighbourhood which we shall call the *field of motion*, will not be altered by any finite difference, whether the containing vessel be held fixed or left free, provided it be infinitely distant from them all round. It is, therefore, essentially indifferent whether we keep it fixed or let it be free. The former supposition is more convenient in some respects, the latter in others; but it would be inconvenient to leave any ambiguity, and I shall adhere (§ 1) to the former in all that follows.

8. To further illustrate the impulse of the motion, and its resultant impulsive force and couple, according to the previous definitions, as distinguished from

the momentum, and the moment of momentum, of the whole contents of the vessel, let the vessel be spherical. Its impulsive pressure on the liquid will always be reducible to a single resultant in a line through its centre, which (§ 4) will be equal and contrary to the force-resultant of "the impulse;" and, therefore, with it will constitute in general a couple. The resultant, of this couple and the couple-resultant of the impulse, will be equal to the moment of momentum of the whole motion round the centre of the sphere (which is the centre of inertia). But if the vessel be infinitely large, and infinitely distant all round from the moveable solids, the moment of momentum of the whole motion is irrelevant; and what is essentially important, is the impulse and its force and couple-resultants, as defined above.

9. The following way of stating (§§ 10, 12), and proving (§§ 11—15), a fundamental proposition in fluid motion will be useful to us for the theory of the impulse, whether of the moveable solids we have hitherto considered or of vortices.

10. The moment of momentum of every spherical portion of a liquid mass in motion, relatively to the centre of the sphere, is always zero, if it is so at any one instant for every spherical portion of the same mass.

11. To prove this, it is first to be remarked, that the moment of momentum of that part of the liquid which at any instant occupies a certain fixed spherical space can experience no change, at that instant (or its *rate* of change vanishes at that instant), because the fluid pressure on it (§ 1), being perpendicular to its surface, is everywhere precisely towards its centre. Hence, if the moment of momentum of the matter in the fixed spherical space varies, it must be by the moment of momentum of the matter which enters it not balancing exactly that of the matter which leaves it. We shall see later (§§ 20, 17, 18) that this balancing is vitiated by the entry of either a moving solid, or of some of the liquid, if any there is, of which spherical portions possess moment of momentum, into the fixed spherical space; but it is perfect under the condition of § 10, as will be proved in § 15.

12. First, I shall prove the following purely mathematical lemmas; using the ordinary notation  $u, v, w$  for the components of fluid velocity at any point  $(x, y, z)$ .

Lemma (1.) The condition (last clause) of § 10 requires that  $u dx + v dy + w dz$  be a complete differential,\* at whatever instant and through whatever part of the fluid the condition holds.

Lemma (2.) If  $u dx + v dy + w dz$  be a complete differential of a single valued function of  $x, y, z$ , through any finite space of the fluid, at any instant, the condition of § 10 holds through that space at that instant.

\* This proposition was, I believe, first proved by STOKES in his paper "On the Friction of Fluids in Motion, and the Equilibrium and Motion of Elastic Solids."—"Cambridge Philosophical Transactions," 14th April 1845.

13. The following is STOKES' proof of Lemma (1):—First, for any motion whatever, whether subject to the condition of § 10 or not, let L be the component moment of momentum round OX of an infinitesimal sphere with its centre at O. Denoting by  $\iiint$  integration through this space, we have

$$L = \iiint (wy - vz) dx dy dz \quad (1).$$

Now let  $\left(\frac{dw}{dx}\right)_0$ ,  $\left(\frac{dw}{dy}\right)_0$ , &c. denote the values at O of the differential coefficients.

We have, by MACLAURIN'S theorem,

$$w = x \left(\frac{dw}{dx}\right)_0 + y \left(\frac{dw}{dy}\right)_0 + z \left(\frac{dw}{dz}\right)_0,$$

and so for  $v$ . Hence, remembering that  $\left(\frac{dw}{dx}\right)_0$ , &c. are constants for the space through which the integration is performed, we have

$$\iiint dx dy dz wy = \left(\frac{dw}{dx}\right)_0 \iiint xy dx dy dz + \left(\frac{dw}{dy}\right)_0 \iiint y^2 dx dy dz + \left(\frac{dw}{dz}\right)_0 \iiint zy dx dy dz.$$

The first and third of the triple integrals vanish, because every diameter of a homogeneous sphere is a principal axis; and if A denote moment of momentum of the spherical volume round its centre, we have for the second

$$\iiint y^2 dx dy dz = \frac{1}{2} A.$$

Dealing similarly with  $vz$  in the expression for L, we find

$$L = \frac{1}{2} A \left[ \left(\frac{dw}{dy}\right)_0 - \left(\frac{dv}{dz}\right)_0 \right] \quad (2).$$

But L must be zero according to the condition of § 10; and, therefore, as the centre of the infinitesimal sphere now considered may be taken at any point of space through which this condition holds at any instant, we must have, throughout that space,

$$\left. \begin{aligned} \frac{dw}{dy} - \frac{dv}{dz} &= 0 \\ \frac{du}{dz} - \frac{dw}{dx} &= 0 \\ \frac{dv}{dx} - \frac{du}{dy} &= 0 \end{aligned} \right\} \quad (3);$$

and similarly

which proves Lemma (1.)

14. To prove Lemma (2.), let

$$u = \frac{d\phi}{dx}, \quad v = \frac{d\phi}{dy}, \quad w = \frac{d\psi}{dz} \quad (4);$$

and let L denote the component moment of momentum round OX, through any spherical space with O in centre. We have [(1) of § 13],

$$L = \iiint dx dy dz (wy - vz) . . . . . (5),$$

$\iiint$  denoting integration throughout this space (not now infinitesimal). But by (4)

$$yw - vz = \left( y \frac{d}{dz} - z \frac{d}{dy} \right) \phi = \frac{d\phi}{d\psi} . . . . . (6);$$

if  $\frac{d}{d\psi}$  denote differentiation with reference to  $\psi$ , in the system of co-ordinate  $x, \rho, \psi$ , such that

$$y = \rho \cos \psi, z = \rho \sin \psi . . . . . (7).$$

Hence, transforming (5) to this system of co-ordinates, we have

$$L = \iiint dx d\rho \rho d\psi \frac{d\phi}{d\psi} . . . . . (8).$$

Now, as the whole space is spherical, with the origin of co-ordinates in its centre, we may divide it into infinitesimal circular rings with OX for axis, having each for normal section an infinitesimal rectangle with  $dx$  and  $d\rho$  for sides. Integrating first through one of these rings, we have

$$dx d\rho \rho \int_0^{2\pi} \frac{d\phi}{d\psi} d\psi,$$

which vanishes, because  $\phi$  is a single-valued function of the co-ordinates. Hence  $L = 0$ , which proves Lemma (2.).

15. Returning now to the dynamical proposition, stated at the conclusion of § 11; for the promised proof, let R denote the radial component velocity of the fluid across any element,  $d\sigma$ , of the spherical surface, situated at  $(x, y, z)$ ; and let  $u, v, w$  be the three components of the resultant velocity at this point; so that

$$R = u \frac{x}{r} + v \frac{y}{r} + w \frac{z}{r} . . . . . (9).$$

The volume of fluid leaving the hollow spherical space across  $d\sigma$  in an infinitesimal time,  $dt$  is  $Rd\sigma \cdot dt$ , and the moment of momentum of this moving mass round the centre has, for component round OX,

$$(wy - vz) Rd\sigma dt.$$

Hence, if L denote the component of the moment of momentum of the whole, mass within the spherical surface at any instant,  $t$ , we have (§ 11),

$$\frac{dL}{dt} = \iint (wy - vz) R d\sigma, . . . . . (10).$$

Now, using Lemma (1.) of § 12, and the notation of § 14, we have

$$wy - vz = \frac{d\phi}{d\psi},$$

and, by (9),

$$R = \frac{d\phi}{dr}$$

where  $\frac{d}{dr}$  denotes rate of variation per unit length perpendicular to the spherical surface, that is differentiation with reference to  $r$ , the other two co-ordinates being directional relatively to the centre. Hence, using ordinary polar co-ordinates,  $r$ ,  $\theta$ ,  $\psi$ , we have

$$\frac{dL}{dt} = r^2 \iint \frac{d\phi}{dr} \frac{d\phi}{d\psi} \sin \theta \, d\theta d\psi \quad \dots \quad (11).$$

But the "equation of continuity" for an incompressible liquid (being

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0),$$

gives\*  $\nabla^2 \phi = 0$ , for every point within the spherical space; and therefore [THOMSON & TAIT, App. B]

$$\phi = S_0 + S_1 r + S_2 r^2 + \&c. \quad \dots \quad (12).$$

a converging series, where  $S_0$  denotes a constant, and  $S_1, S_2, \&c.$ , surface harmonics of the orders indicated.

Hence

$$R = \frac{d\phi}{dr} = S_1 + 2r S_2 + 3 r^2 S_3 + \&c. \quad \dots \quad (13).$$

And it is clear from the synthesis of the most general surface harmonic, by zonal, sectional, and tesseral harmonics [THOMSON & TAIT, §781], that  $\frac{dS_i}{d\psi}$  is a surface harmonic of the same order as  $S_i$ : † from which [THOMSON & TAIT, App. B (16)], it follows that,

\* By  $\nabla^2$  we shall always understand  $\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$ .

† This follows, of course, from the known analytical theorem that the operations  $\nabla^2$  and  $(y \frac{d}{dz} - z \frac{d}{dy})$  are commutative, which is proved thus:—

By differentiation we have

$$\frac{d^2}{dy^2} \left( y \frac{d\phi}{dz} \right) = y \frac{d^2}{dy^2} \frac{d\phi}{dz} + 2 \frac{d}{dy} \frac{d\phi}{dz};$$

and therefore, since  $\frac{d}{dy} \frac{d\phi}{dz} = \frac{d}{dz} \frac{d\phi}{dy}$ ,

$$\nabla^2 \left( y \frac{d\phi}{dz} - z \frac{d\phi}{dy} \right) = y \nabla^2 \left( \frac{d\phi}{dz} \right) - z \nabla^2 \left( \frac{d\phi}{dy} \right) = \left( y \frac{d}{dz} - z \frac{d}{dy} \right) \nabla^2 \phi$$

or

$$\nabla^2 \left( y \frac{d}{dz} - z \frac{d}{dy} \right) \phi = \left( y \frac{d}{dz} - z \frac{d}{dy} \right) \nabla^2 \phi,$$

$\phi$  being any function whatever. Hence, if  $\nabla^2 \phi = 0$  we have

$$\nabla^2 \left( y \frac{d\phi}{dz} - z \frac{d\phi}{dy} \right) = 0.$$

$$\iint S_i \frac{dS_i}{d\psi} \sin \theta d\theta d\psi = 0,$$

except when  $i' = i$ . But this is true also when  $i' = i$  because

$$S_i \frac{dS_i}{d\psi} = \frac{1}{2} d \frac{(S_i^2)}{d\psi},$$

and therefore, as in § 14, the integration for  $\psi$ , from  $\psi = 0$  to  $\psi = 2\pi$  gives zero. Hence (11) gives

$$\frac{dL}{dt} = 0,$$

This and § 11 establish § 10.

16. Lemma (1) of § 11, and § 10 now proved, show that in any motion whatever of an incompressible liquid, whether with solids immersed in it or not,  $udx + vdy + wdz$  is always a complete differential through any portion of the fluid, for which it is a complete differential at any instant, to whatever shape and position of space this portion may be brought in the course of the motion. This is the ordinary statement of the fundamental proposition of fluid motion referred to in § 9, which was first discovered by LAGRANGE. (For another proof see § 60.) I have given the preceding demonstration, not so much because it is useful to look at mathematical structures from many different points of view, but (§ 19) because the dynamical considerations and the formulæ I have used are immediately available for establishing the theory of the impulse (§§ 3 . . . 8), of which a fundamental proposition was stated above (§ 5). To prove this proposition (in § 19) I now proceed.

17. Imagine any spherical surfaces to be described round a moveable solid or solids immersed in a liquid. The surrounding fluid can only press (§ 1) perpendicularly; and therefore when any motion is (§ 3) generated by impulsive forces applied to the solids, the moment round any diameter of the momentum of the matter within the spherical surface at the first instant, must be exactly equal to the moment of those impulsive forces round this line. And the moment round this line, of the momentum of the matter in the space between any two concentric spherical surfaces is zero, provided neither cuts any solid, and provided that, if there are any solids in this space, no impulse acts on them.

18. Hence, considering what we have defined as "the impulse of the motion," (§ 6), we see that its moment round any line is equal to the moment of momentum round the same line, of all the motion within any spherical surface having its centre in this line, and enclosing all the matter to which any constituent of the impulse is applied. This will still hold, though there are other solids not in the neighbourhood, and impulses are applied to them: provided the moments of momentum of those only which are within S are taken into account, and provided none of them is cut by S.

19. The statements of § 11, regarding fluid occupying at any instant a fixed spherical surface, are applicable without change to the fluids and solids occupying

the space bounded by S, because of our present condition, that no solid is cut by S. Hence every statement and formula of § 15, as far as equation (11), may be now applied to the matter within S; but instead of (12) we now have [THOMSON & TAIT, § 736], if we denote by  $T_1, T_2, \&c.$ , another set of surface spherical harmonics,

$$\left. \begin{aligned} \varphi = S_0 + S_1 r + S_2 r^2 + \&c. \\ + T_1 r^{-2} + T_2 r^{-3} + \&c \end{aligned} \right\} \quad (14).^*$$

for all space between the greatest and smallest spherical surface concentric with S, and having no solids in it, because through all this space, § 16, and the equation of continuity prove that  $\nabla^2 \phi = 0$ . Hence, instead of (13), we now have

$$\left. \begin{aligned} R = \frac{d\varphi}{dr} = S_1 + 2r S_2 + 3r^2 S_3, \&c. \\ - \frac{2}{r^3} T_1 - \frac{3}{r^4} T_2 - \frac{4}{r^5} T_3 + \&c. \end{aligned} \right\} \quad (15).$$

Hence finally

$$\frac{dL}{dt} = \sum_{i=0}^{i=\infty} \iint \left[ i S_i \frac{dT_i}{d\psi} - (i+1) T_i \frac{dS_i}{d\psi} \right] \sin \theta d\theta d\psi \quad (16).$$

Now if, as assumed in § 5, neither any moveable solids, nor any part of the boundary exist within any finite distance of S all round;  $S_1, S_2, \&c.$ , must each be infinitely small: and therefore (16) gives  $\frac{dL}{dt} = 0$ . This proves the proposition asserted in § 5: because a system of forces cannot have zero moment round every line drawn through any finite portion of space, without having force-resultant and couple-resultant each equal to zero

20. As the rigidity of the solids has not been taken into account, all or any of them may be liquefied (§ 3) without violating the demonstration of § 19. To save circumlocutions, I now define a *vortex* as a portion of fluid having any motion that it could not acquire by fluid pressure transmitted through itself from its boundary. Often, merely for brevity, I shall use the expression *a body* to denote either a solid or a vortex, or a group of solids or vortices.

21. The proposition thus proved may be now stated in terms of the definitions of § 6, which were not used in § 5, and so becomes simply this:—*The impulse of the motion of a solid or group of solids or vortices and the surrounding liquid remains constant as long as no disturbance is suffered from the influence of other solids or vortices, or of the containing vessel.*

This implies, of course (§ 6), that the magnitudes of the force-resultant and the rotational moment of the impulse remain constant, and the position of its axis invariable.

\* There is no term  $\frac{T_0}{r}$ , because this would give, in the integral of flow across the whole spherical surface, a finite amount of flow out of or into the space within, implying a generation or destruction of matter.

22. In POINSON'S system of the statics of a rigid body we may pass from the resultant force and couple along and round the central axis to an equal resultant force along the parallel line through any point, and a greater couple the resultant of the former (or minimum) couple, and a couple in the plane of the two parallels, having its moment equal to the product of their distance into the resultant force. So we may pass from the force-resultant and rotational moment of the impulse along and round its axis, to an equal force-resultant and greater moment of impulse, by transferring the former to any point,  $Q$ , not in the axis (§ 6) of the motion. This greater moment is (§ 18) equal to the moment of momentum round the point  $Q$ , of the motion within any spherical surface described from  $Q$  as centre, which encloses all the vortices or moving solids.

23. Hence a group of solids or vortices which always keep within a spherical surface of finite radius, or a single body, moving in an infinite liquid, can have no permanent average motion of translation in any direction oblique to the direction of the force-resultant of the impulse, if there is a finite force-resultant. For the matter within a finite spherical surface enclosing the moving bodies or body, cannot have moment of momentum round the centre increasing to infinity.

24. But there may be motion of translation when the force-resultant of the impulse vanishes; and there will be, for example, in the case of a solid, shaped like the screw-propeller of a steamer, immersed in an infinite homogeneous liquid, and set in motion by a couple in a plane perpendicular to the axis of the screw.

25. And when the force-resultant of the impulse does *not* vanish, there may be no motion of translation, or there may even be translation in the direction opposite to it. Thus, for example, a rigid ring, with cyclic motion, established (§ 63) through it, will, if left at rest, remain at rest. And if at any time urged by an impulse in either direction in the line of the force-resultant of the impulse of the cyclic motion, it will commence and continue moving with an average motion of translation in that direction; a motion which will be uniform, and the same as if there were no cyclic motion, when the ring is symmetrical. If the translatory impulse is contrary to the cyclic impulse, but less in magnitude, the translation will be contrary to the whole force-resultant impulse.

If the translatory impulse is equal and opposite to the cyclic impulse, there will be translation with zero force-resultant impulse—another example of what is asserted in § 24. In this case, if the ring is plane and symmetrical, or of any other shape such that the cyclic motion (which, to fix ideas, we have supposed given first, with the ring at rest,) must have had only a force-resultant, and no rotational moment, we have a solid moving with a uniform motion of translation through a fluid, and both force and couple resultant of the whole impulse zero.

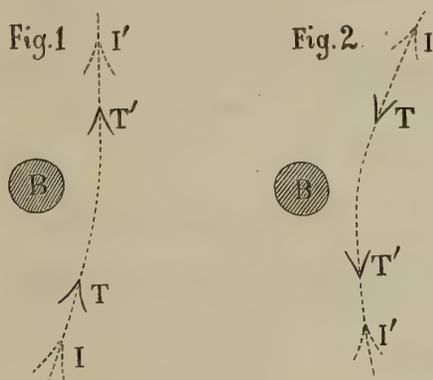
26. From §§ 21 and 4, we see that, however long the time of application of the impressed forces may be—provided only that, during the whole of it, the





system of impulsive attractions towards the latter, everywhere in the direction of the normal, and amounting to  $\frac{1}{2} \rho q^2 dt$  per unit of area. But it must not be forgotten that the term  $\phi$  in the expression [§ 31 (5)] for  $p$  produces, as shown in § 30 (1), an influence during the collision, the integral effect of which only disappears from the expression [§ 32 (7)] for the impulse after the collision is completed; that is (§ 29) after the moving system has passed away so far as to leave no sensible fluid motion in the neighbourhood of the fixed body.

33. Hence, and from § 23, we see that when there is no impact of moving solid against the fixed body, and when the moving solid or group of solids passes altogether on one side of the fixed body, the direction of the translation will be deflected, as if there were, on the whole, an attraction towards the fixed body, or a repulsion from it, according as (§ 25) the translation is in the direction of the impulse or opposite to it. For, in each case, the impulse is altered by the introduction of an impulse towards the fixed body upon the moving body or bodies as they pass it; and (§ 23) the translation before and after the collision is always along the line of the impulse, and is altered in direction accordingly. This will be easily understood from the diagrams, where, in each case B represents the fixed body, the dotted line ITT', and arrow-heads II', the directions of the force-resultant of the impulse at successive times, and the full arrow-heads TT', the directions of the translation.



All ordinary cases belong to the class illustrated by fig. 1. The case of a rigid ring, with cyclic motion (§ 25) established round it as core, belongs to the class illustrated by fig. 2, if the ring be projected through the fluid in the direction perpendicular to its own plane, and contrary to the cyclic motion through its centre.

34. When (§ 66) we substitute vortices for the moving solids, we shall see (§ 67) that the translation is probably always in the direction *with* the impulse. Hence, as illustrated by fig. 1, there is always the deflection, as if by attraction, when a group of vortices pass all on one side of a fixed body. This is easily observed, for a simple Helmholtz ring, by sending smoke rings on a large scale, according to

Professor TAIT's plan, in such directions as to pass very near a convex fixed surface. An ordinary 12-inch globe, taken off its bearings and hung by a thin cord, answers very well for the fixed body.

35. The investigation of §§ 30, 31, 32, is clearly applicable to a vortex or a moving body, or of a group of vortices or moving bodies, which keep always near one another (§ 23), passing near a projecting part of the fixed boundary, and being, before and after this collision (§ 29), at a very great distance from every part of the fixed boundary. Thus, a Helmholtz ring projected so as to pass near a projecting angle of two walls, shows a deflection of its course, as if caused by attraction towards the corner.

36. In every case the force-resultant of the impulse is, as we shall presently see (§ 37), determinate when the flow of the liquid across every element of any surface completely enclosing the solids or vortices is given; but not so, from such data, either the axis (§ 6) or the rotational moment, as we see at once by considering the case of a solid sphere (which may afterwards be supposed liquefied) set in motion by a force in any line not through the centre, and a couple in a plane perpendicular to it. For this line will be the "axis," and the impulsive couple will be the rotational moment of the whole motion of the solid and liquid. But the liquid, on all sides, will move exactly as it would if the impulse were merely an impulsive force of equal amount in a parallel line through the centre of the sphere, with therefore this second line for "axis" and zero for rotational moment. For illustration of rotational moment remaining latent in a liquid (with or without solids) until made manifest by actions, tending to alter its axis, or showing effects of centrifugal force due to it; see § 66, and others later.

37. The component impulse in any direction is equal to the corresponding component momentum of the mass enclosed within the surface  $S$ , containing all the places of application of the impulse, together with that of the impulsive pressure outwards on this surface. But as the matter enclosed by  $S$  (whether all liquid or partly liquid and partly solid) is of uniform density, its momentum will be equal to its mass multiplied into the velocity of the centre of gravity of the space within the surface  $S$  supposed to vary so as to enclose always the same matter, and will therefore depend solely on the normal motion of  $S$ ; that is to say, on the component of the fluid velocity in the direction of the normal at every point of  $S$ . And the impulsive fluid pressure, corresponding to the generation of the actual motion from rest, being the time integral of the pressure during the instantaneous generation of the motion, is (§§ 31, 32) equal to  $-\phi$ , the velocity potential; which (§ 61) is determinate for every point of  $S$ , and of the exterior space when the normal component of the fluid motion is given for every point of  $S$ . Hence the proposition asserted in § 36. Denoting by  $d\sigma$  any element of  $S$ ;  $N$  the normal component of the fluid velocity;  $\alpha$  the inclination to  $OX$ , of the normal drawn *outwards* through  $d\sigma$ ; and  $X$  the  $x$ -component of the impulse; we

have for the two parts of this quantity considered above, and its whole value, the following expressions; of which the first is taken in anticipation from § 42—

$$\left. \begin{aligned} x\text{-momentum of matter, within } S, &= \iint Nx \, d\sigma \quad (8) \text{ of } \S 42 \\ x\text{-component of impulsive pressure on } S, \text{ outwards,} &= -\iint \phi \cos \alpha \, d\sigma \end{aligned} \right\} (1).$$

$$X = \iint (Nx - \phi \cos \alpha) \, d\sigma \quad (2).$$

It is worthy of remark that this expression holds for the impulse of all the solids or vortices within  $S$ , even if there be others in the immediate neighbourhood outside: and that therefore its value must be zero if there be no solids or vortices within  $S$ , and  $N$  and  $\phi$  are due solely to those outside.

38. If  $\phi$  be the potential of a magnet or group of magnets, some within  $S$  and others outside it, and  $N$  the normal component magnetic force, at any point of  $S$ , the preceding expression (2) is equal to the  $x$ -component of the magnetic moment of all the magnets within  $S$ , multiplied by  $4\pi$ . For let  $\rho$  be the density of any continuous distribution of positive and negative matter, having for potential, and normal component force,  $\phi$  and  $N$  respectively, at every point of  $S$ . We have [THOMSON & TAIT, § 491 (c)]  $\epsilon = -\frac{1}{4\pi} \nabla^2 \phi$ , and therefore

$$\iiint \epsilon x \, dx \, dy \, dz = -\frac{1}{4\pi} \iiint x \left( \frac{d^2 \phi}{dx^2} + \frac{d^2 \phi}{dy^2} + \frac{d^2 \phi}{dz^2} \right) dx \, dy \, dz \quad (3).$$

Now, integrating by parts,\* as usual with such expressions, we have

$$\iiint x \frac{d^2 \phi}{dx^2} dx \, dy \, dz = \iint x \frac{d\phi}{dx} dy \, dz - \iiint \frac{d\phi}{dx} dx \, dy \, dz = \iint \left( x \frac{d\phi}{dx} - \phi \right) dy \, dz.$$

Hence, integrating each of the other two terms of (3) once simply, and reducing as usual [THOMSON & TAIT, App. A (a)] to a surface integral, we have

$$\iiint \epsilon x \, dx \, dy \, dz = -\frac{1}{4\pi} \iint (Nx - \phi \cos \alpha) \, d\sigma \quad (4);$$

which proves the proposition, and also, of course, that if there be no matter within  $S$ , the value of the second member is zero.

39. Hence, considering the magnetic and hydrokinetic analogous systems with the sole condition that at every point of some particular closed surface, the magnetic potential is equal to the velocity potential, we conclude that  $4\pi$  times the magnetic moment of all the magnetism within any surface, in the magnetic system, is equal to the force-resultant of the impulse of the solids or vortices within the corresponding surface in the hydrokinetic system; and that the directions of the magnetic axis and of the force-resultant of the impulse are the same. For the theory of magnetism, it is interesting to remark that indeterminate distributions of magnetism within the solids, or portions of fluid to which initiating

\* The process here described leads merely to the equation obtained by taking the last two equal members of App. A (1) (THOMSON & TAIT) for the case  $\alpha = 1$ ,  $U = \phi$ ,  $U' = x$ .



at different parts of the boundary of this area, in the infinitely small time  $dt$ , contribute no increments or decrements to  $\iint [x^2 dy dz]$ , as we see most easily by first supposing  $S$  to be a surface everywhere convex outwards. Hence

$$\frac{d}{dt} \iint [x^2 dy dz] = \iint \left[ \frac{d(x^2)}{dt} dy dz \right] = 2 \iint \left[ x \frac{dx}{dt} dy dz \right] \quad (6).$$

But if  $N$  denote the velocity with which the surface moves in the direction of its outward normal at  $(x, y, z)$ , we have, in the preceding expression

$$\frac{dx}{dt} = N \sec \alpha \quad (7),$$

if  $\alpha$  be the inclination of the outward normal to  $OX$ . Hence

$$\frac{d(\mathbf{V}\bar{x})}{dt} = \iint [xN \sec \alpha dy dz].$$

But the condition as to limits indicated by [ ] are clearly satisfied, if,  $d\sigma$  denoting an element of the surface, such that

$$dy dz = \cos \alpha d\sigma,$$

we simply take  $\iint d\sigma$  over the whole surface. Thus we have

$$\frac{d(\mathbf{V}\bar{x})}{dt} = \iint xN d\sigma \quad (7);$$

42. In any case in which  $V$  is constant, this becomes

$$V \frac{d\bar{x}}{dt} = \iint x N d\sigma \quad (8).$$

If now the varying surface,  $S$ , is the boundary of a portion of the matter—fluid or solid—of uniform density unity, with whose motions we are occupied, the  $x$ -component momentum of this portion is  $V \frac{dx}{dt}$ ; and, therefore, equation (8) is the required (§ 40) expression.

43. The same formulæ (7) and (8) are proved more shortly of course by the regular analytical process given by POISSON\* and GREEN† in dealing with such subjects; thus, in short. Let  $u, v, w$  be the components of velocity, of any matter, compressible or incompressible, at any point  $(x, y, z)$  within  $S$ ; and let  $c$  denote the value at this point of  $\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}$ , so that

$$\frac{du}{dx} = c - \left( \frac{dv}{dy} + \frac{dw}{dz} \right), \quad (9).$$

We have, for the component momentum of the whole matter within  $S$ , if of unit density at the instant considered,

$$\iiint u dx dy dz = \iint ux dy dz - \iiint x \frac{du}{dx} dx dy dz \quad (10).$$

\* Théorie de la Chaleur, § 60.

† Essay on Electricity and Magnetism.

But by (9)

$$\iiint x \frac{du}{dx} dx dy dz = \iiint cx dx dy dz - \iiint x \left( \frac{dv}{dy} + \frac{dw}{dz} \right) dx dy dz$$

and by simple integrations,

$$\iiint x \left( \frac{dv}{dy} + \frac{dw}{dz} \right) dx dy dz = \iint x (v dx dz + w dx dy).$$

Using these in (10), and altering the expression to a surface integral, as in THOMSON & TAIT, App. A (a), we have

$$\begin{aligned} \iiint u dx dy dz &= \iint x (u dy dz + v dz dx + w dx dy) - \iint cx dx dy dz \\ &= \iint x N d\sigma - \iint cx dx dy dz \end{aligned} \quad (11),$$

which clearly agrees with (7).

When this mass is incompressible, we have  $c = 0$  by the formula so ill named the equation "of continuity" (THOMSON & TAIT, § 191), and we fall upon (8.)

The proper analytical interpretation of the differential coefficients  $\frac{du}{dx}$ , &c., and of the equation of continuity, when, as at the surfaces of separation of fluid and solids,  $u, v, w$  are discontinuous functions, having abruptly varying values, presents no difficulty.

44. In the theory of the impulse applied to the collision (§ 29) of solids or vortices moving through a liquid, the force-resultant of the impulse corresponds, as we have seen, precisely to the resultant momentum of a solid in the ordinary theory of impact. Some difficulty may be felt in understanding how the zero-momentum (§ 4) of the whole mass is composed; there being clearly positive momentum of solids and fluids in the direction of the impulse in some localities near the place of its application, and negative in others. [Consider, for example, the simple case of a solid of revolution struck by a single impulse in the line of its axis. The fluid moves in the direction of the impulse, before and behind the body, but in the contrary direction in the space round its middle.] Three modes of dividing the whole moving mass present themselves as illustrative of the distribution of momentum through it; and the following propositions (§ 45) with reference to them are readily proved (§§ 46, 47, 48).

45. I. Imagine any cylinder of finite periphery, not necessarily circular, completely surrounding the vortices (or moving solids), and any other surrounding none, and consider the infinitely long prisms of variously moving matter at any instant surrounded by these two cylinders. The component momentum parallel to the length of the first is equal to the component of the impulse parallel to the same direction; and that of the second is zero.

II. Imagine any two finite spherical surfaces, one enclosing all the vortices

or moving solids, and the other none. The resultant-momentum of the whole matter enclosed by the first is in the direction of the impulse, and is equal to  $\frac{2}{3}$  of its value. The resultant-momentum of the whole fluid enclosed by the second is the same as if it all moved with the same velocity, and in the same direction, as at its centre.

III. Imagine any two infinite planes at a finite distance from one another and from the field of motion, but neither cutting any solid or vortex. The component perpendicular to them of the momentum of the matter occupying at any instant the space between them (whether this includes none, some, or all of the vortices or moving solids) is zero.

46. To prove these propositions:—

I. Consider in either case a finite length of the prism extending to a very great distance in each direction from the field of motion, and terminated by plane or curved ends. Then, the motion being, as we may suppose (§ 61) started from rest by impulsive pressures on the solids [or (§ 66) on the portions of fluid constituting the vortices]; the impulsive fluid pressure on the cylindrical surface can generate no momentum parallel to the length; and to generate momentum in this direction there will be, in case 1, the impressed impulsive forces on the solids, and the impulsive fluid pressures on the ends; but in case 2 there will be only the impulsive fluid pressure on the ends. Now, the impulsive fluid pressures on the ends diminish [§ 50 (15)] according to the inverse square of the distance from the field of motion, when the prism is prolonged in each direction, and are therefore infinitely small when the prisms are infinitely long each way. Whence the proposition I.

47. By using the harmonic expansions § 19, (14), (15), in the several expressions (1), (2), of § 37, (1), (2); and the fundamental theorem

$$\iint \dot{S}_i \dot{S}_v d\sigma = 0,$$

of the harmonic analysis [THOMSON & TAIT, App. B. (16)]; and putting  $S_i = 0$  for one case, and  $T_i = 0$  for the other; we prove the two parts of Prop. II., § 45 immediately.

48. To prove Prop. II., § 45, the well-known theory of electric images in a plane conductor\* may be conveniently referred to. It shows that if  $N_1$  denotes the normal component force at any point of an infinite plane due to any distribution,  $\mu$ , of matter in the space lying on one side of the plane, a distribution of matter over the plane having  $\frac{1}{2\pi} N_1$  for surface density at each point exerts the same force as  $\mu$  through all the space on the other side of the plane, and therefore that the whole quantity of matter in that surface distribution is equal to the

\* THOMSON, Camb. and Dub. Math. Journal, 1849; LIOUVILLE'S Journal, 1845 and 1847; or Reprints of Electrical Papers, (Macmillan, 1869.)



the equality to zero asserted in this proposition is proved in § 48 to be approximated to when the planes are extended to distances all round, which, though infinitely short of the distances to the containing vessel, are very great in comparison with their perpendicular distances from the most distant parts of the field of motion.

50. The convergencies concerned in § 46, I., III. may be analysed thus. Perpendicular to the resultant impulse draw any two planes on the two sides of the field of motion, with all the moving solids and vortices between them, and divide a portion of the space between them into finite prismatic portions by cylindrical (or plane) surfaces perpendicular to them. Suppose now one of these prismatic portions to include all the moving solids and vortices, and without altering the prismatic boundary, let the parallel planes be removed in opposite directions to distances each infinite (or very great) in comparison with distance of the most distant of the moving solids or vortices. By § 46, I., the momentum of the motion within this prismatic space is (approximately) equal to the force-resultant,  $I$ , of the impulse, and that of the motion within any one of the others is (approximately) zero.

But the sum of these (approximately) zero values must, on account of § 46, III., be equal to  $-I$ , if the portions of the planes containing the ends of the prismatic spaces be extended to distances very great in comparison with the distance between the planes. To understand this, we have only to remark that if  $\phi$  denotes the velocity potential at a point distant  $D$  from the middle of the field, and  $x$  from a plane through the middle perpendicular to the impulse, we have (§ 53) approximately,

$$\phi = - \frac{I x}{4\pi D^3} \quad \dots \quad (15),$$

provided  $D$  be great in comparison with the radius of the smallest sphere enclosing all the moving solids or vortices. Hence, putting  $x = \pm a$  for the two planes under consideration, denoting by  $A$  the area of either end of one of the prismatic portions, and calling  $D$  the proper mean distance for this area, we have (§ 46) for the momentum of the fluid motion within this prismatic space, provided it contains no moving solids or vortices,

$$- 2 \frac{I a}{4\pi D^3} A \quad \dots \quad (16).$$

This vanishes when  $\frac{A}{D^2}$  is an infinitely small fraction (as  $\frac{a}{D}$  is at most unity); but it is finite if  $\frac{A}{D^2}$  is finite, provided  $\frac{a}{D}$  be not infinitely small. And its integral value (compare § 48, footnote) converges to  $-I$ , when the portion of area included in the integration is extended till  $\frac{a}{D}$  is infinitely small for all points of its boundary.

51. Both as regards the mathematical theory of the convergence of definite integrals, and as illustrating the distribution of momentum in a fluid, it is interesting to remark that,  $u$  denoting component velocity parallel to  $x$ , at any point  $(x, y, z)$ , the integral  $\iiint u \, dx \, dy \, dz$ , expressing momentum, may, as is readily proved, have any value from  $-\infty$  to  $+\infty$  according to the portions of space through which it is taken.

52. As a last illustration of the distribution of momentum, let the containing vessel be spherical of finite radius  $a$ .

We have, as in § 19,

$$\left. \begin{aligned} \varphi &= S_0 + S_1 r + S_2 r^2 + \&c., \\ &+ T_1 r^{-2} + T_2 r^{-3} + \&c., \end{aligned} \right\} \dots \dots \dots (14),$$

each series converging, provided  $r$  is less than  $a$ , and greater than the radius of the smallest concentric spherical surface enclosing all the solids or vortices. Now, by the condition that there be no flow across the fixed containing surface, we must have

$$\frac{d\varphi}{dr} = 0, \text{ when } r = a \dots \dots \dots (15),$$

which gives

$$S_i = \frac{i + 1}{i} \frac{T_i}{a^{2i+1}} \dots \dots \dots (16);$$

and (14) becomes

$$\varphi = \frac{T_1}{r^2} \left( 1 + 2 \frac{r^3}{a^3} \right) + \frac{T_2}{r^3} \left( 1 + \frac{3}{2} \frac{r^5}{a^5} \right) + \&c. \dots \dots \dots (17).$$

But [§ 37 (1) ] if the whole amount of the  $x$ -component of impulsive pressure exerted by the fluid within the spherical surface of radius  $r$ , upon the fluid round it be denoted by  $F$ , we have

$$F = -\iint \varphi \cos \theta \, d\sigma \dots \dots \dots (18),$$

$\theta$  being the inclination to  $OX$  of the radius through  $d\sigma$ . Now  $\cos \theta$  is a surface harmonic of the first order, and therefore all the terms of the harmonic expansion, except the first, disappear in the integral, which consequently becomes

$$F = -\left( 1 + 2 \frac{r^3}{a^3} \right) \iint T_1 \cos \theta \frac{d\sigma}{r^2} \dots \dots \dots (19).$$

Now let

$$T_1 = -\frac{Ax + By + Cz}{r} \dots \dots \dots (20),$$

this being [THOMSON & TAIT, App. B, §§ i, j] the most general expression for a surface harmonic of the first order. We have  $\cos \theta = \frac{a}{r}$ ; and therefore (by spherical harmonics, or by the elementary analysis of moments of inertia of a uniform spherical surface),

$$-\iint T_1 \cos \theta \frac{d\sigma}{r^2} = \frac{A}{r^4} \iint x^2 d\sigma = \frac{4\pi A}{3} \quad (21);$$

and (19) becomes

$$F = \left(1 + 2 \frac{r^3}{a^3}\right) \cdot \frac{4\pi A}{3} \quad (22);$$

Whence, if X denote the *x*-momentum of the fluid at any instant in the space between concentric spherical surfaces of radius *r* and *r'*,

$$X = -\frac{2}{3} \frac{r^3 - r'^3}{a^3} 4\pi A \quad (23).$$

If *r* and *r'* be each infinitely small in comparison with *a*, this expression vanishes, as it ought to do, in accordance with § 45, II. But if

$$\left. \begin{aligned} \frac{r}{a} = 0, \quad \& \quad r = a, \\ X = -\frac{2}{3} \cdot 4\pi A \end{aligned} \right\} \quad (24),$$

it becomes

fulfilling § 4, by showing in the fluid outside the spherical surface of radius *r'* a momentum equal and opposite to that (§ 45, II.) of the whole matter, whether fluid or solid, within that surface.

53. Comparing § 47 and § 52, we see that if X, Y, Z be rectangular components of the force-resultant of the impulse, the term  $T_1 r^{-2}$  of the harmonic expansion (14) is as follows:—

$$T_1 r^{-2} = \frac{Xx + Yy + Zz}{4\pi r^3} \quad (25),$$

provided all the solids and vortices taken into account are within a spherical surface whose radius is very small in comparison with the distances of all other vortices or moving solids, and with the shortest distance to the fixed bounding surface.

54. HELMHOLTZ, in his splendid paper on Vortex Motion, has made the very important remark, that a certain fundamental theorem of GREEN'S, which has been used to demonstrate the determinateness of solutions in hydrokinetics, is subject to exception when the functions involved have multiple values. This calls for a serious correction and extension of elementary hydrokinetic theory, to which I now proceed.

55. In the general theorem (1) of THOMSON & TAIT, App. A let  $\alpha = 1$ . It becomes

$$\begin{aligned} \iiint \left( \frac{d\phi}{dx} \frac{d\phi'}{dx} + \frac{d\phi}{dy} \frac{d\phi'}{dy} + \frac{d\phi}{dz} \frac{d\phi'}{dz} \right) dx dy dz &= \iint d\sigma \phi \nabla \phi' - \iiint dx dy dz \phi \nabla^2 \phi' \\ &= \iint d\sigma \phi' \nabla \phi - \iiint dx dy dz \phi' \nabla^2 \phi \quad (1), \end{aligned}$$

which is true without exception if  $\phi$  and  $\phi'$  denote any two *single-valued* functions of *x, y, z*;  $\iiint dx dy dz$  integration through the space enclosed by any finite closed



if in its other terms the value of  $\tan^{-1}\frac{y}{x}$  is reckoned continuously round from one side of the plane ZOY to the other : or

$$- 2\pi \iint dy dz \left( \frac{d\phi}{dx} \right)_{x=0},$$

if the continuity be from one side of ZOY to the other ; to render it really equal to the first member of (1). Thus, taking for example the first form of the added term, we now have for the corrected double equation (1) for the case of  $\phi' = \tan^{-1}\frac{y}{x}$ ,  $\phi$  any single valued function, and S the surface, composed of the two co-axial cylinders and two parallel planes specified above :

$$\iiint^x \frac{\frac{d\phi}{dz} - y \frac{d\phi}{dx}}{x^2 + y^2} dx dy dz = 0 = 2\pi \iint dx dz \left( \frac{d\phi}{dy} \right)_{y=0} + \iint d\sigma \tan^{-1}\frac{y}{x} \nabla\phi - \iiint dx dy dz \tan^{-1}\frac{y}{x} \nabla^2\phi \quad (6).$$

But if we annex to S any barrier stopping circulation round the inner cylindrical core, all ambiguity becomes impossible, and the double equation (1) holds. For instance, if the barrier be the portion of the plane ZOY, intercepted between the co-axial cylinders and parallel planes constituting the S of § 55, so that  $\iint d\sigma$  must now include integration over each side of this rectangular area ; (6) becomes simply the strict application of (1) to the case in question.

57. The difficulty of the exceptional interpretation of GREEN'S theorem for the class of cases exemplified in §§ 55 and 56, depends on the fact that  $\int Fds$  may have different values when reckoned along the lengths of different curves, drawn within the space bounded by S, from a point P to a point Q ;  $ds$  being an infinitesimal element of the curve, and F the rate of variation of  $\phi$  per unit of length along it. Let PCQ, PC'Q be two curves for which the  $\int Fds$  has different values ; and let both lie wholly within S. If we draw any curve from P to Q ; make it first coincide with PCQ, and then vary it gradually until it coincides with PC'Q ; it must in some of its intermediate forms cut the bounding surface S : for we have

$$Fds = \frac{d\phi}{dx} dx + \frac{d\phi}{dy} dy + \frac{d\phi}{dz} dz$$

throughout the space contained within S, and  $\frac{d\phi}{dx}, \frac{d\phi}{dy}, \frac{d\phi}{dz}$ , are each of them unambiguous by hypothesis ; which implies that  $\int Fds$  has equal values for all

gradual variations of one curve between P and Q, each lying wholly within S. Now, in a simply continuous space, a curve joining the points P and Q may be gradually varied from any curve PCQ to any other PC'Q, and therefore if the space contained within S be simply continuous, the difficulty depending on the multiplicity of value of  $\phi$  or  $\phi'$  cannot exist. And however multiply continuous (§ 58) the space may be, the difficulty may be evaded if we annex to S a surface or surfaces stopping every aperture or passage on the openness of which its multiple continuity depends; for these annexed surfaces, as each of them occupies no space, do not disturb the triple integrations (1), and will, therefore, not alter the values of its first member; but by removing the multiplicity of continuity, they free each of the integrations by parts, by which its second or third members are obtained, from all ambiguity. To avoid circumlocution, we shall call  $\beta$  the addition thus made to S; and further, when the space within S is (§ 58) not merely doubly but triply, or quadruply, or more multiply, continuous, we shall designate by  $\beta_1, \beta_2$ ; or  $\beta_1, \beta_2, \beta_3$ ; and so on; the several parts of  $\beta$  required in any case to stop all multiple continuity of the space. These parts of  $\beta$  may be quite detached from one another, as when the multiple continuity is that due to detached rings, or separate single tunnels in a solid. But one part  $\beta_1$  may cut through part of another,  $\beta_2$ , as when two rings (§ 58, diagram) linked into one another without touching constitute part of the boundary of the space considered. And we shall denote by  $\iint d\sigma$ , integration over the surface  $\beta$ , or over any one of its parts,  $\beta_1, \beta_2$ , &c. Let now P and Q be each infinitely near a point B, of  $\beta$ , but on the two sides of this surface. Let  $\kappa$  denote the value of  $\int F ds$  along any curve lying wholly in the space bounded by S, and joining PQ without cutting the barrier; this value being the same for all such curves, and for all positions of B to which it may be brought without leaving  $\beta$ , and without making either P or Q pass through any part of  $\beta$ . That is to say,  $\kappa$  is a single constant when the space is not more than doubly continuous; but it denotes one or other of  $n$  constants  $\kappa_1, \kappa_2, \dots \kappa_n$ , which may be all different from one another, when the space is  $n$ -ply continuous. Lastly, let  $\kappa'$  denote the same element, relatively to  $\phi'$ , as  $\kappa$  relatively to  $\phi$ . We find that the first steps of the integrations by parts now introduce, without ambiguity, the additions

$$\sum \kappa \iint d\sigma \mathbf{d}\phi', \text{ and } \sum \kappa' \iint d\sigma \mathbf{d}\phi \quad \dots \dots \dots (6),$$

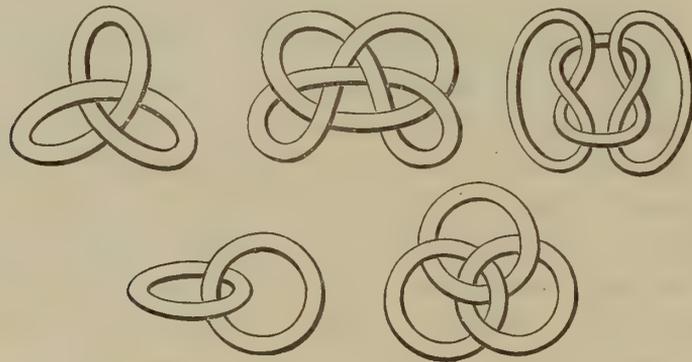
to the second and third numbers of (1):  $\Sigma$  denoting summation of the integrations for the different constituents  $\beta_1, \beta_2, \dots$  of  $\beta$ ; but only a single term when the space is (§ 58) not more than doubly continuous. GREEN'S theorem thus corrected becomes

$$\begin{aligned} \iiint \left( \frac{d\phi}{dx} \frac{d\phi'}{dx} + \frac{d\phi}{dy} \frac{d\phi'}{dy} + \frac{d\phi}{dz} \frac{d\phi'}{dz} \right) dx dy dz &= \iint d\sigma \phi \mathbf{d}\phi' + \sum \kappa \iint d\sigma \mathbf{d}\phi' - \iiint \phi \nabla^2 \phi' dx dy dz \\ &= \iint d\sigma \phi' \mathbf{d}\phi + \sum \kappa' \iint d\sigma \mathbf{d}\phi - \iiint \phi' \nabla^2 \phi dx dy dz \quad \dots \quad (7). \end{aligned}$$

58. Adopting the terminology of RIEMANN, as known to me through HELMHOLTZ, I shall call a finite position of space  $n$ -ply continuous when its bounding surface is such that there are  $n$  irreconcilable paths between any two points in it. To prevent any misunderstanding, I add (1), that by a portion of space I mean such a portion that any point of it may be travelled to from any other point of it, without cutting the bounding surface; (2), that the "paths" spoken of all lie within the portion of space referred to; and (3), that by irreconcilable paths between two points P and Q; I mean paths such, that a line drawn first along one of them cannot be gradually changed till it coincides with the other, being always kept passing through P and Q, and always wholly within the portion of space considered. Thus, when all the paths between any two points are reconcilable, the space is simply continuous. When there are just two sets of paths, so that each of one set is irreconcilable with any one of the other set, the space is doubly continuous; when there are three such sets it is triply continuous, and so on. To avoid circumlocutions, we shall suppose S to be the boundary of a hollow space in the interior of a solid mass, so thick that no operations which we shall consider shall ever make an opening to the space outside it. A tunnel through this solid opening at each end into the interior space constitutes the whole space doubly continuous; and if more tunnels be made, every new one adds one to the degree of multiple continuity. When one such tunnel has been made, the surface of the tunnel is continuous with the whole bounding surface of the space considered; and in reckoning degrees of continuity, it is of no consequence whether the ends of any fresh tunnel be in one part or another of this whole surface. Thus, if two tunnels be made side by side, a hole anywhere opening from one of them into the other adds one to the degree of multiple continuity. Any solid detached from the outer bounding solid, and left, whether fixed or movable in the interior space, adds to the bounding surface an isolated portion, but does not interfere with the reckoning of multiple continuity. Thus, if we begin with a simply continuous space bounded outside by the inner surface of the supposed external solid, and internally by the boundary of the detached solid in its interior, and if we drill a hole in this solid we produce double continuity. Two holes, or two solids in the interior each with one hole (such as two ordinary solid rings), constitute triple continuity, and so on. A sponge-like solid whose pores communicate with one another, illustrates a high degree of multiple continuity, and it is of no consequence whether it is attached to the external bounding solid or is an isolated solid in the interior. Another type of multiple continuity, that presented by two rings linked in one another, was referred to in § 57.

When many rings are linked into one another in various combinations, there are complicated mutual intersections of the several partial barriers  $\beta_1, \beta_2, \dots$  required to stop all multiple continuity. But without having any portion of the

bounding solid detached, as in that case in which one at least of the two rings is loose, we have varieties of multiple continuity curiously different from that illustrated by a single ordinary straight or bent tunnel, illustrated sufficiently by the simplest types, which are obtained by boring a tunnel along a line agreeing in form with the axis of a cord or wire on which a simple knot is tied; and by fixing the two ends of wire with a knot on it to the bounding solid, so that the surface of the wire shall become part of the bounding surface of the space considered, the knot not being pulled tight, and the wire being arranged not to touch itself in any point; or by placing a knotted wire, with its ends united, in the interior of the space. No amount of knotting or knitting, however complex, in the cord whose axis indicates the line of tunnel, complicates in any way the continuity of the space considered, or alters the simplicity of the barrier surface required to stop the circulation. But it is otherwise when a knotted or knitted wire forms part of the bounding solid. A single simple knot, though giving only double continuity, requires a curiously self-cutting surface for stopping barrier: which, in its form of minimum area, is beautifully shown by the liquid film adhering to an endless wire, like the first figure, dipped in a soap solution and removed. But no complication of these types, or of combinations of them with one another, eludes the statements and formulæ of § 57.



59. I shall now give a dynamical lemma, for the immediate object of preparing to apply GREEN'S corrected theorem (§ 57) to the motion of a liquid through a multiply continuous space. But later we shall be led by it to very simple demonstrations of HELMHOLTZ'S fundamental theorems of vortex motion; and shall see that it may be used as a substitute for the common equations of hydrokinetics.

(Lemma). An endless finite tube\* of infinitesimal normal section, being given full of liquid (whether circulating round through it, or at rest) is altered in shape,

\* A finite length of tube with its ends done away by uniting them together.

*Instalment, received Nov.—Dec. 1869* [§ 59 – § 64 (5)].

length, and normal section, in any way, and with any speed. The average value of the component velocity of the fluid along the tube, reckoned all round the circuit (irrespective of the normal section), varies inversely as the length of the circuit.

59. (a). To prove this, consider first a single particle of unit mass, acted on by any force, and moving along a smooth guiding curve, which is moved and bent about quite arbitrarily. Let  $\rho$  be the radius of curvature, and  $\xi, \eta$  the component velocities of the guiding curve, towards the centre of curvature, and perpendicular to the plane of curvature, at the point P, through which the moving particle is passing at any instant. Let  $\zeta$  be the component velocity of the particle itself, along the instantaneous direction of the tangent through P. Thus  $\xi, \eta, \zeta$  are three rectangular components of the velocity of the particle itself. Let  $\mathcal{Z}$  be the component in the direction of  $\zeta$ , of the whole force on P. We have, by elementary kinetics,

$$\frac{d\zeta}{dt} = \mathcal{Z} + \frac{\zeta\xi}{\rho} + \xi \frac{d\xi}{ds} + \eta \frac{d\eta}{ds} \quad \dots \quad (1),*$$

\* This theorem (not hitherto published?) will be given in the second volume of THOMSON and TAIT's "Natural Philosophy." It may be proved analytically from the general equations of the motion of a particle along a varying guide-curve (WALTON, "Cambridge Mathematical Journal," 1842, February); or more synthetically, thus—Let  $l, m, n$  be the direction cosines of PT, the tangent to the guide at the point through which the particle is passing at any instant;  $(x, y, z)$  the co-ordinates of this point, and  $(\dot{x}, \dot{y}, \dot{z})$  its component velocities parallel to fixed rectangular axes. We have

$$\zeta = l\dot{x} + m\dot{y} + n\dot{z}; \text{ and } \mathcal{Z} = l\ddot{x} + m\ddot{y} + n\ddot{z},$$

and from this

$$\frac{d\zeta}{dt} = l\ddot{x} + m\ddot{y} + n\ddot{z} + \dot{l}\dot{x} + \dot{m}\dot{y} + \dot{n}\dot{z} = \mathcal{Z} + \dot{l}\dot{x} + \dot{m}\dot{y} + \dot{n}\dot{z}.$$

But it is readily proved (THOMSON and TAIT's "Natural Philosophy," § 9, to be made more explicit on this point in a second edition) that the angular velocity with which PT changes direction is equal to  $\sqrt{\dot{l}^2 + \dot{m}^2 + \dot{n}^2}$ , and, if this be denoted by  $\omega$ , that

$$\frac{l}{\omega}, \frac{m}{\omega}, \frac{n}{\omega}$$

are the direction cosines of the line PK, perpendicular to PT in the plane in which PT changes direction, and on the side towards which it turns. Hence,

$$\frac{d\zeta}{dt} = \mathcal{Z} + \kappa\omega$$

if  $\kappa$  denote the component velocity of P along PK. Now, if the curve were fixed we should have  $\omega = \frac{\zeta}{\rho}$ , by the kinematic definition of curvature (THOMSON and TAIT, § 5); and the plane in which PT changes direction would be the plane of curvature. But in the case actually supposed, there is also in this plane an additional angular velocity equal to  $\frac{d\xi}{ds}$ , and a component angular velocity in the plane of PT and  $\eta$ , equal to  $\frac{d\eta}{ds}$ ; due to the normal motion of the varying curve. Hence the whole angular velocity  $\omega$  is the resultant of two components,

$$\frac{\zeta}{\rho} + \frac{d\xi}{ds} \text{ in the plane of } \xi,$$

where  $\rho$  denotes the radius of curvature, and  $\frac{d\xi}{ds}, \frac{d\eta}{ds}$  rates of variation of  $\xi$  and  $\eta$  from point to point along the curve at one time.

59. (b). Now, instead of a single particle of unit mass, let an infinitesimal portion,  $\mu$ , of a liquid, filling the supposed endless tube, be considered. Let  $\omega$  be the area of the normal section of the tube in the place where  $\mu$  is, and  $\delta s$  the length along the tube of the space occupied by it, at any instant; so that (as the density of the fluid is called unity),

$$\mu = \omega \delta s.$$

Further, let  $\frac{dp}{ds}$  denote the rate of variation of the fluid pressure along the tube, so that

$$Z = -\omega \frac{dp}{ds} \delta s.$$

Thus we have, by (1),

$$\frac{d\zeta}{dt} = \frac{\zeta\xi}{\rho} + \xi \frac{d\xi}{ds} + \eta \frac{d\eta}{ds} - \frac{dp}{ds} \quad \dots \quad (2).$$

(c). Now, because the two ends of the arc  $\delta s$  move with the fluid, we have, by the kinematics of a varying curve,

$$\frac{d\delta s}{dt} = \frac{d\zeta}{ds} \delta s - \frac{\xi}{\rho} \delta s \quad \dots \quad (3);$$

and, therefore,

$$\frac{d(\zeta\delta s)}{dt} = \frac{d\zeta}{dt} \delta s + \zeta \left( \frac{d\zeta}{ds} \delta s - \frac{\xi}{\rho} \delta s \right) \quad \dots \quad (4).$$

Substituting in this for  $\frac{d\zeta}{dt}$  its value by (2), we have

$$\frac{d(\zeta\delta s)}{dt} = \left( \xi \frac{d\xi}{ds} + \eta \frac{d\eta}{ds} - \frac{dp}{ds} + \zeta \frac{d\zeta}{ds} \right) \delta s,$$

or

$$\frac{d(\zeta\delta s)}{dt} = \delta(\frac{1}{2}q^2 - p) \quad \dots \quad (5),$$

if  $q$  denote the resultant fluid velocity; and  $\delta$ , differences for the two ends of the arc  $\delta s$ . Integrating this through the length of any finite arc  $P_1P_2$  of the fluid, its ends  $P_1, P_2$ , moving with the fluid, we have

$$\frac{d\Sigma_1(\zeta\delta s)}{dt} = (\frac{1}{2}q^2 - p)_2 - (\frac{1}{2}q^2 - p)_1 \quad \dots \quad (6),$$

the suffixes denoting the values of the bracketed function, at the points  $P_2$  and

and

$$\frac{d\eta}{ds} \text{ in the plane of } \eta.$$

Hence

$$\xi \left( \frac{\zeta}{\rho} + \frac{d\xi}{ds} \right) + \eta \frac{d\eta}{ds} = \kappa\omega,$$

and the formula (1) of the text is proved.

$P_1$ , respectively; and  $\Sigma_1^2$  denoting integration along the arc from  $P_1$  to  $P_2$ . Let now  $P_2$  be moved forward, or  $P_1$  backward, till these points coincide, and the arc  $P_1P_2$  becomes the complete circuit; and let  $\Sigma$  denote integration round the whole closed circuit. (6) becomes

$$\frac{d\Sigma(\zeta\delta s)}{dt} = 0 \quad (7);$$

and we conclude that  $\Sigma\zeta\delta s$  remains constant, however the tube be varied. This is the proposition to be proved, as the "average velocity" referred to is found by dividing  $\Sigma(\zeta\delta s)$  by the length of the tube.

59. (d). The tube, imagined in the preceding, has had no other effect than exerting, by its inner surface, normal pressure on the contained ring of fluid. Hence the proposition\* at the beginning of § 59 is applicable to any closed ring of fluid forming part of an incompressible fluid mass extending in all directions through any finite or infinite space, and moving in any possible way; and the formulæ (5) and (6) are applicable to any infinitesimal or infinite arc of it with two ends not met. Thus in words—

PROP. (1.) *The line-integral of the tangential component velocity round any closed curve of a moving fluid remains constant through all time.*

And, PROP. (2), The rate of augmentation, per unit of time, of the space integral of the velocity along any terminated arc of the fluid is equal to the

\* Equation (6), from which, as we have seen, that proposition follows immediately, may be proved with greater ease, and not merely for an incompressible fluid, but for any fluid in which the density is a function of the pressure, by the method of rectilinear rectangular co-ordinates from the ordinary hydrokinetic equations. These equations are—

$$\frac{Du}{Dt} = -\frac{d\varpi}{dx}, \quad \frac{Dw}{Dt} = -\frac{d\varpi}{dy}, \quad \frac{Dw}{Dt} = -\frac{d\varpi}{dz},$$

if  $\frac{D}{Dt}$  denote rate of variation per unit of time, of any function depending on a point or points *moving with the fluid*; and  $\varpi = \int \frac{dp}{\rho}$ ,  $\rho$  denoting density. In terms of rectangular rectilinear co-ordinates we have

$$\zeta\delta s = u\delta x + v\delta y + w\delta z.$$

Hence

$$\frac{D(\zeta\delta s)}{Dt} = \frac{Du}{Dt}\delta x + u\frac{D\delta x}{Dt} + \&c.$$

Now

$$\frac{D\delta x}{Dt} = \delta u, \quad \frac{D\delta y}{Dt} = \delta v, \quad \text{and} \quad \frac{D\delta z}{Dt} = \delta w.$$

These and the kinetic equations reduce the preceding to

$$\frac{D(\zeta\delta s)}{Dt} = u\delta u + v\delta v + w\delta w - \frac{d\varpi}{dx}\delta x - \frac{d\varpi}{dy}\delta y - \frac{d\varpi}{dz}\delta z = \delta\left[\frac{1}{2}(u^2 + v^2 + w^2) - \varpi\right] \quad (8);$$

whence, by  $\Sigma$  integration, equation (6) generalised to apply to compressible fluids.

excess of the value of  $\frac{1}{2}q^2 - p$ , at the end towards which tangential velocity is reckoned as positive, above its value at the other end.

59. (e). The condition that  $u dx + v dy + w dz$  is a complete differential [proved above (§ 13) to be the criterion of irrotational motion] means simply

*That the flow [defined § 60 (a)] is the same in all different mutually reconcilable lines from one to another of any two points in the fluid; or, which is the same thing,*

*That the circulation [§ 60 (a)] is zero round every closed curve capable of being contracted to a point without passing out of a portion of the fluid through which the criterion holds.*

From Proposition (1), just proved, we see that this condition holds through all time for any portion of a moving fluid for which it holds at any instant; and thus we have another proof of LAGRANGE'S celebrated theorem (§ 16), giving us a new view of its dynamical significance, which [see for example § 60 (g)] we shall find of much importance in the theory of vortex motion.

(f). But it is only in a closed curve, *capable of being contracted to a point without passing out of space occupied by irrotationally moving fluid*, that the circulation is necessarily zero, in irrotational motion. In § 57 we saw that a continuous fluid mass, occupying doubly or multiply continuous space, may move altogether irrotationally, yet so as to have finite circulation in a closed curve  $PP'QQ'P$ , provided  $PP'Q$  and  $PQ'Q$  are "irreconcilable paths" between P and Q. *That the circulation must be the same in all mutually reconcilable closed curves* (compare § 57), is an immediate consequence from the now proved [§ 59 (Prop. 2)] equality of the flows [§ 60 (a)] in all mutually reconcilable conterminous arcs. For by leaving one part of a closed curve unchanged, and varying the remaining arc continuously, no change is produced in the flow, in this part; and, by repetitions of the process, a closed curve may be changed to any other reconcilable with it.

60. *Definitions and elementary propositions (a)*. The line-integral of the tangential component velocity along any finite line, straight or curved, in a moving fluid, is called the flow in that line. If the line is endless (that is, if it forms a closed curve or polygon), the *flow* is called *circulation*. The use of these terms abbreviates the statements of Propositions (2) and (1) of § 59 to the following:—

[§ 59, Prop. (2)]. The rate of augmentation, per unit of time, of the flow in any terminated line which moves with the fluid, is equal to the excess of the value of  $\frac{1}{2}q^2 - p$  at the end from which, above its value at the end towards which, positive flow is reckoned.

[§ 59, Prop. (1)]. The circulation in any closed line moving with the fluid, remains constant through all time.

(b). If any open finite surface, lying altogether within a fluid, be cut into

parts by lines drawn across it, the circulation in the boundary of the whole is equal to the sum of the circulations in the boundaries of the parts. This is obvious, as the latter sum consists of an equal positive and negative flow in each portion of boundary common to two parts, added to the sum of the flows in all the parts into which the single boundary of the whole is divided.

60. (c). Hence the circulation round the boundaries of infinitesimal areas, infinitely near one another in one plane, are simply proportional to these areas.

(d). *Proposition.* Let any part of the fluid rotate as a solid (that is, without changing shape); or consider simply the rotation of a solid. The "circulation" in the boundary of any plane figure moving with it is equal to twice the area enclosed, multiplied by the component angular velocity in that plane (or round an axis perpendicular to that plane). For, taking  $r, \theta$  to denote polar co-ordinates of any point in the boundary,  $A$  the enclosed area, and  $\omega$  the component angular velocity in the plane, and continuing the notation of § 59, we have

$$\zeta = r\omega \frac{rd\theta}{ds},$$

and therefore

$$\Sigma \zeta \delta s = \omega \Sigma r^2 \frac{d\theta}{ds} \delta s = \omega \Sigma r^2 \delta \theta = \omega \times 2A.$$

(e). *Definition.* (For a fluid moving in any manner), the circulation round the boundary of an infinitesimal plane area, divided by double the area, is called the *component rotation* in that plane (or round an axis perpendicular to that plane) of the neighbouring fluid.

In this statement, the single word "rotation" is used for *angular velocity of rotation*: and the definition is justified by (c) and (d); also by § 13 (2) above, applied to (p) below. It agrees, in virtue of (p), with the definition of rotation in fluid motion given first of all, I believe, by STOKES, and used by HELMHOLTZ in his memorable "Vortex Motion," also in THOMSON and TAIT'S "Natural Philosophy," §§ 182 and 190 (j).

(f). *Proposition.* If  $\xi, \eta, \zeta$  be the components of rotation at any point, P, of a fluid, round three axes at right angles to one another, and  $\omega$  the component round an axis, making with them angles whose cosines are  $l, m, n$ ,

$$\omega = \xi l + \eta m + \zeta n.$$

To prove this, let a plane perpendicular to the last-mentioned axis cut the other three in A, B, C. The circulation in the periphery of the triangle ABC is, by (b), equal to the sum of the circulations in the peripheries PBC, PCA, and PAB. Hence, calling  $\Delta$  and  $\alpha, \beta, \gamma$  the areas of these four triangles, we have, by (e),

$$\omega \Delta = \xi \alpha + \eta \beta + \zeta \gamma.$$

But  $\alpha, \beta, \gamma$  are the projections of  $\Delta$  on the planes of the pairs of the rectangular axes; and so the proposition is proved.

It follows, of course, that the composition of rotations in a fluid fulfils the law of the compositions of angular velocities of a solid, of linear velocities, of forces, &c.

60. (g). Hence, in any infinitesimal part of the fluid, the circulation is zero in the periphery of every plane area passing through a certain line;—the resultant axis of rotation of that part of the fluid. But (a) the circulation remains zero in every closed line moving with the fluid, for which it is zero at any time. Hence

(h). The axial lines [defined (i)] move with the fluid.

(i). *Definition.* An axial line through a fluid moving rotationally, is a line (straight or curved) whose direction at every point coincides with the resultant axis of rotation through that point.

(j). *Proposition.* The resultant rotation of any part of the fluid varies in simple proportion to the length of an infinitesimal arc of the axial line through it, terminated by points moving with the fluid. To prove this, consider any infinitesimal plane area,  $A$ , moving with the fluid. Let  $\omega$  be the resultant rotation, and  $\theta$  the angle between its axis and the perpendicular to the plane of  $A$ . This makes  $\omega \cos \theta$  the component rotation in the plane of  $A$ ; and therefore  $A\omega \cos \theta$  remains constant. Now, draw axial lines through all points of the boundary of  $A$ , forming a tube whose area of normal section is  $A \cos \theta$ . The resultant rotation must vary inversely as this area, and therefore (in consequence of the incompressibility of the fluid) directly as the length of an infinitesimal line along the axis.

(k). Form a surface by axial lines drawn through all points of any curve in the fluid. The circulation is zero round the boundary of any infinitesimal area of this surface; and therefore (b) it is zero round the boundary of any finite area of it.

(l). Let the curve of (k) be closed, and therefore the surface tubular. On this surface let  $ABCA, A'B'C'A'$  be any two curves closed round the tube, and  $ADA'$  any arc from  $A$  to  $A'$ . The circulation in the closed path,  $ADA'B'C'A'DACBA$ , is zero by (h). Hence the circulation in  $ABCA$  is equal to the circulation in  $A'B'C'A'$ —that is to say,

The circulations are equal in all circuits of a vortex tube.

(m). *Definitions.* An *axial surface* is a surface made up of axial lines. A *vortex tube* is an axial surface through every point of which a finite endless path, cutting every axial line it meets, can be drawn. Any such path, passing just once round, is called a *circuit*, or *the circuit* of the tube. The *rotation of a vortex tube* is the circulation in its circuit. A *vortex sheet* is (a portion as it were of a collapsed vortex tube) a surface on the two sides of which the fluid moves with different tangential component velocities.

60. (n.) Draw any surface cutting a vortex tube, and bounded by it. The surface integral of the component rotation round the normal has the same value for all such surfaces; and this common value is what we now call the rotation of the tube.

(o). In an unbounded infinite fluid, an axial tube must be either finite and endless or infinitely long in each direction.\* In an infinite fluid with a boundary (for instance, the surface of an enclosed solid), an axial tube may have two ends, each in the boundary surface; or it may have one end in the boundary surface, and no other; or it may be infinitely long in each direction, or it may be finite and endless. In a finite fluid mass, an axial tube may be endless, or may have one end, but, if so, must have another, both in the boundary surface.

(p). *Proposition.* Applying the notation of (f), to axes parallel to those of co-ordinates  $x, y, z$ , and denoting, as formerly, by  $u, v, w$ , the components of the fluid velocity at  $(x, y, z)$ , we have—

$$\xi = \frac{1}{2} \left( \frac{dw}{dy} - \frac{dv}{dz} \right), \quad \eta = \frac{1}{2} \left( \frac{du}{dz} - \frac{dw}{dx} \right), \quad \zeta = \frac{1}{2} \left( \frac{dv}{dx} - \frac{du}{dy} \right).$$

The proof is obvious, according to the plan of notation, &c., followed in § 13 above.

(q). Hence by (f), (e), and (b)—

$$\iint dS \left\{ l \left( \frac{dw}{dy} - \frac{dv}{dz} \right) + m \left( \frac{du}{dz} - \frac{dw}{dx} \right) + n \left( \frac{dv}{dx} - \frac{du}{dy} \right) \right\} = \int (u dx + v dy + w dz).$$

where  $\iint dS$  denotes integration over any portion of surface bounded by a closed curve;  $\int (u dx + \&c.)$  integration round the whole of this curve; and  $(l, m, n)$  the direction cosines of any point  $(x, y, z)$  in the surface. It is worthy of remark that the equation of continuity for an incompressible fluid does not enter into the demonstration of this proposition, and therefore  $u, v, w$  may be any functions whatever of  $x, y, z$ . In a purely analytical light, the result has an important bearing on the theory of the integration of complete or incomplete differentials. It was first given, with the indication of a more analytical proof than the preceding, in THOMSON and TAIT'S "Natural Philosophy," § 190 (j).

(r). Propositions (h) (j) (n) (o) of the present section (§ 60) are due to HELMHOLTZ; and with his integration for associated rotational and cyclic irrotational motion in an unbounded fluid, to be given below, constitute his general theory of vortex motion. (n) and (o) are purely kinematical; (h) and (j) are dynamical.

(s). Henceforth I shall call a *circuit* any closed curve not continuously reducible to a point, in a multiply continuous space. I shall call *different circuits*, any

\* Vortex tubes apparently ending in the fluid, for instance, a portion of fluid bounded by a figure of revolution, revolving round its axis as a solid, constitute no exception. Each infinitesimal vortex tube in this case is completed by a strip of vortex sheet and so is endless.

two such closed curves if mutually irreconcilable (§ 58); but different mutually reconcilable closed curves will not be called different circuits.

60. (*t*). Thus,  $(n+1)$ ply continuous space, is a space for which there are  $n$ , and only  $n$ , different circuits. This is merely the definition of § 58, abbreviated by the definite use of the word circuit, which I now propose. The general terminology regarding simply and multiply continuous spaces is, as I have found since § 58 was written, altogether due to HELMHOLTZ; RIEMANN'S suggestion, to which he refers, having been confined to two-dimensional space. I have deviated somewhat from the form of definition originally given by HELMHOLTZ, involving, as it does, the difficult conception of a stopping barrier;\* and substituted for it the definition by reconcilable and irreconcilable paths. It is not easy to conceive the stopping barrier of any one of the first three diagrams of § 58, or to understand its singleness; but it is easy to see that in each of those three cases, any two closed curves drawn round the solid wire represented in the diagrams are reconcilable, according to the definition of this term given in § 58, and therefore, that the presence of any such solid adds only one to the degree of continuity of the space in which it is placed.

(*u*). If we call a *partition*, a surface which separates a closed space into two parts, and, as hitherto, a *barrier*, any surface edged by the boundary of the space, HELMHOLTZ'S definition of multiple continuity may be stated shortly thus:—

*A space is  $(n+1)$ ply continuous if  $n$  barriers can be drawn across it, none of which is a partition.*

(*v*). HELMHOLTZ has pointed out the importance in hydrokinetics of many-valued functions, such as  $\tan^{-1}\frac{y}{x}$ , which have no place in the theories of gravitation, electricity, or magnetism, but are required to express electro-magnetic potentials, and the velocity potentials for the part of the fluid which moves irrotationally in vortex motion. It is, therefore, convenient, before going farther, that we should fix upon a terminology, with reference to functions of that kind, which may save us circumlocutions hereafter.

(*w*). A function  $\phi(x, y, z)$  will be called *cyclic* if it experiences a constant augmentation every time a point P, of which  $x, y, z$  are rectangular rectilinear co-ordinates, is carried from any position round a certain circuit to the same position again, without passing through any position for which either  $\frac{d\phi}{dx}$ ,  $\frac{d\phi}{dy}$ , or  $\frac{d\phi}{dz}$  becomes infinite. The value of this augmentation will be called the cyclic

\* But without this conception we can make no use of the theory of multiple continuity in hydrokinetics (see §§ 61–63), and HELMHOLTZ'S definition is, therefore, perhaps preferable after all to that which I have substituted for it. Mr CLERK MAXWELL tells me that J. B. LISTING has more recently treated the subject of multiple continuity in a very complete manner in an article entitled "Der Census räumlicher Complexe."—*Königl. Ges. Göttingen*, 1861. See also Prof. CAYLEY "On the Partition of a Close."—*Phil. Mag.* 1861.

constant for that particular circuit. The cyclic constant must clearly have the same value for all circuits mutually reconcilable (§ 58), in space throughout which the three differential coefficients remain all finite.

60. (*x*). When the function is cyclic with reference to several different mutually irreconcilable circuits, it is called polycyclic. When it is cyclic for only one set of circuits, it is called monocyclic.

EXAMPLE.—The apparent area of a circle as seen from a point  $(x, y, z)$  anywhere in space, is a monocyclic function of  $x, y, z$ , of which the cyclic constant is  $4\pi$ .

The apparent area of a plane curve of the  $(2n)$ th degree, consisting of  $n$  detached closed (that is finite endless) branches (some of which might be enclosed within others) is an  $n$ -cyclic function, of which the  $n$  cyclic constants are essentially equal, being each  $4\pi$ .

Algebraic equations among three variables  $(x, y, z)$ , may easily be found to represent tortuous curves, constituting one or more finite, isolated, endless branches (which may be knotted, as shown in the first three diagrams of § 58, or linked into one another, as in the fourth and fifth). The integral expressing what, for brevity, we shall call the *apparent area* of such a curve, is a cyclic function, which, if polycyclic, has essentially equal values for all its cyclic constants. By the *apparent area of a finite endless curve* (tortuous or plane), I mean the *sum of the apparent areas of all barriers edged by it, which we can draw without making a partition*.

It is worthy of notice that every polycyclic function may be reduced to a sum of monocyclic functions.

(*y*). Fluid motion is called *cyclic* unless the circulation is zero in every closed path through the fluid, when it is called *acyclic*. Rotational motion is (*e*) essentially cyclic.

(*z*). Irrotational motion may [ § 59 (*f*)] be either acyclic or cyclic. If cyclic it is *monocyclic* if there is only one distinct circuit, or *polycyclic* if there are several distinct circuits, in which there is circulation. It is *purely cyclic* if the boundary of the space occupied by irrotationally moving fluid is at rest. If the boundary moves and the motion of the fluid is cyclic, it is *acyclic compounded with cyclic*.

61. (*a*). We are now prepared to investigate the most general possible irrotational motion of a single continuous fluid mass, occupying either simply or multiply continuous space, with for every point of the boundary a normal component velocity given arbitrarily, subject only to the condition that the whole volume remains unaltered.

(*b*). \**Genesis of acyclic motion*. Commencing, as in § 3, with a fluid mass at rest throughout, let all multiplicity of the continuity of the space occupied by it be done away with by temporary barrier surfaces,  $\beta_1, \beta_2 \dots$  stopping the circuits, as described in § 57. The bounding surface of the fluid, which ordinarily consists

of the inner surface of the containing vessel, will thus be temporarily extended to include each side of each of these barriers. Let now, as in § 3, any possible motion be arbitrarily given to the bounding surface. The liquid is consequently set in motion, purely through fluid pressure; and the motion is [§§ 10–15, or 60, 59] throughout irrotational. Hence irrotational motion fulfilling the prescribed surface conditions is possible, and the actual motion is, of course (as the solution of every real problem is), unambiguous. But from this bare physical principle we could not even suspect, what the following simple application of GREEN'S equation proves, that the surface normal velocity at any instant determines the interior motion irrespectively of the previous history of the motion from rest.

61. (c). *Determinacy of irrotational motion in simply continuous space.* In § 57 (1), which is immediately applicable, as the volume is now simply continuous, make  $\varphi' = \varphi$ , and put  $\nabla^2\varphi = 0$ , so that  $\varphi$  may be the velocity potential of an incompressible fluid. That double equation becomes the following single equation—

$$\iiint \left( \frac{d\varphi^2}{dx^2} + \frac{d\varphi^2}{dy^2} + \frac{d\varphi^2}{dz^2} \right) dx dy dz = \iint d\sigma \varphi \nu \varphi,$$

where the surface integration  $\iint d\sigma$  must now include each side of each of the barrier surfaces  $\beta_1, \beta_2, \dots$ . Hence, if  $\nu\varphi = 0$  for every point of the bounding surface, we must have

$$\iiint \left( \frac{d\varphi^2}{dx^2} + \frac{d\varphi^2}{dy^2} + \frac{d\varphi^2}{dz^2} \right) dx dy dz = 0,$$

which requires that

$$\frac{d\varphi}{dx} = 0, \quad \frac{d\varphi}{dy} = 0, \quad \frac{d\varphi}{dz} = 0:$$

that is to say, if there is no motion of the boundary surface in the direction of the normal, there can be no motion of the irrotational species in the interior; whence it follows that there cannot be two different internal irrotational motions with the same surface normal component velocities. Thus, as a particular case, beginning with a fluid at rest, let its boundary be set in motion; and brought again to rest at any instant, after having been changed in shape to any extent, through any series of motions. The whole liquid comes to rest at that instant.

A demonstration of this important theorem, which differs essentially from the preceding, and includes what the preceding does not include, a purely analytical proof of the possibility of irrotational motion throughout the fluid, fulfilling the arbitrary surface-condition specified above, was first published in THOMSON and TAIT'S "Natural Philosophy," § 317 (3), and is to be given below, with some variation and extension. In the meantime, however, we satisfy ourselves as to the *possibility* of irrotational motions fulfilling the various surface-conditions with which we are concerned, because the surface motions are possible and require the fluid to move, and [§§ 10–15, or § 59] because the fluid cannot acquire

rotational motion through fluid pressure from the motion of its boundary; and we go on, by aid of GREEN'S extended formula [§ 57 (7)], to prove the determinateness of the interior motion under conditions now to be specified for multiply continuous space, as we have done by his unaltered formula [§ 57 (1)] for simply continuous space.

62. *Genesis of Cyclic Irrotational Motion.*—In the case of motion considered in § 61, the value of the normal component velocity is not independently arbitrary over the whole boundary, but has equal arbitrary values, positive and negative, on the two sides of each of the barriers  $\beta_1, \beta_2, \&c.$  We must now introduce a fresh restriction in order that, when the barriers are liquefied, the motion of the fluid may be irrotational throughout the space thus re-opened into multiple continuity. For although we have secured that the normal component velocity is equal everywhere on the two sides of each barrier, we have hitherto left the tangential velocity unheeded. If they are not equal on the two sides, and in the same direction, there will be a finite slipping of fluid on fluid across the surface left by the dissolution of the infinitely thin barrier membrane; constituting [§ 60 (*m*) above], as HELMHOLTZ has shown, a "vortex sheet." The analytical expression of the condition of equality between the tangential velocities is that the variation of the velocity potential in tangential directions shall be equal on the two sides of each barrier. Hence, by integration, we see that the difference between the values of the velocity potential on the two sides must be the same over the whole of each barrier. This condition requires that the initiating pressure be equal over the whole membrane. For, at any time during the instituting of the motion, let  $p_1, p_2$  be the pressures at two points  $P_1, P_2$  of the fluid, and moving with the fluid, infinitely near one another on the two sides of one of the membranes, so that the pressure  $\varpi$ , which must be applied to the membrane to produce this difference of fluid pressure on the two sides, is equal to  $p_1 - p_2$  in the direction opposed to  $p_1$ . And let  $\varphi_1, \varphi_2$  be the velocity potentials at  $P_1$  and  $P_2$ , so that if  $\int ds$  denote integration from  $P_1$  to  $P_2$ , along any path  $P_1PP_2$  whatever from  $P_1$  to  $P_2$ , altogether through the fluid (and therefore cutting none of the membranes), and  $\zeta$  the component of fluid velocity along the tangent at any point of this curve, we have

$$\int \zeta ds = \varphi_2 - \varphi_1 \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Hence, by (6) of § 59,

$$\frac{d(\varphi_2 - \varphi_1)}{dt} = \varpi - \frac{1}{2}(q_1^2 - q_2^2) \quad . \quad . \quad . \quad . \quad (2),$$

where  $q_1, q_2$  denote the resultant fluid velocities at  $P_1$  and  $P_2$ . Now, the normal component velocities at  $P_1$  and  $P_2$  are necessarily equal; and therefore, if the components parallel to the tangent plane of the intervening membrane are also equal, we have

$$q_1 = q_2$$

and the preceding becomes

$$\frac{d(\varphi_2 - \varphi_1)}{dt} = \varpi \quad \dots \quad (3).$$

But if the tangential component velocities at  $P_1$  and  $P_2$  are not only equal, but in the same direction,  $\varphi_2 - \varphi_1$  must, as we have seen, be constant over the membrane, and therefore  $\varpi$  must also be constant.

Suppose now that after pressure has been applied for any time in the manner described, of uniform value all over the membrane at each instant, it is applied no longer, and the membrane (having no longer any influence) is done away with. The fluid mass is left for ever after in a state of motion, which is irrotational throughout, but cyclic. The "circulation" [ $\S$  60 (*a*)], or the cyclic constant being equal to  $\varphi_2 - \varphi_1$ , for every circuit reconcilable with  $P_1 P P_2 P_1$  is given by the equation

$$\varphi_2 - \varphi_1 = - \int \varpi dt \quad \dots \quad (4),$$

$\int dt$  denoting a time-integral extended through the whole period during which  $\varpi$  had any finite value.

The same kind of operation may be performed, on each of the  $n$  barriers temporarily introduced in  $\S$  61 to reduce the  $(n+1)$ fold continuity of the space occupied by the fluid, to simple continuity.

The velocity potential at any point of the fluid will then be a polycyclic function [ $\S$  60 (*x*)] equal to the sum of the separate values corresponding to the pressure separately applied to the several barriers. Thus we see how a state of irrotational motion, cyclic with reference to every one of the different circuits of a multiply continuous space, and having arbitrary values for the corresponding cyclic constants, or circulations, may be generated. But the proof of the possibility of fluid motion fulfilling such conditions, founded on this planning out of a genesis of it, leaves us to imagine that it might be different according to the infinitely varied choice we may make of surfaces for the initial forms of the barriers, or according to the order and the duration of the applications of pressure to them in virtue of which these figures may be changed more or less, and in various ways, before the initiating pressures all cease; and hitherto we have seen no reason even to suspect the following proposition to the contrary.

63. (PROP.) The motion of a liquid moving irrotationally within an  $(n+1)$ ply continuous space is determinate when the normal velocity at every point of the boundary, and the values of the circulations in the  $n$  circuits, are given.

This is proved by an application of GREEN'S extended formula (7) of  $\S$  57, showing, as the simple formula (1) of the same section showed us in  $\S$  61 for simply continuous space, that the difference of the velocity potentials of two motions, each fulfilling this condition, is necessarily zero throughout the whole

fluid. Let  $\phi, \phi'$  be the velocity potentials of two motions fulfilling the prescribed conditions, and let

$$\psi = \phi - \phi'.$$

At every point of the boundary (the barriers not included) the prescribed conditions require that  $\nabla\phi = \nabla\phi'$ , and therefore  $\nabla\psi = 0$ . Again, the cyclic constants for  $\phi'$  are equal to those for  $\phi$ ; those for  $\psi$ , being their differences, must therefore vanish. Hence, if the  $\phi$  and  $\phi'$  of § 57 (7) be made equal to one another and to avoid confusion with our present notation we substitute  $\psi$  for each, the second members of that double equation vanish, and it becomes simply

$$\iiint \left( \frac{d\psi^2}{dx^2} + \frac{d\psi^2}{dy^2} + \frac{d\psi^2}{dz^2} \right) dx dy dz = 0;$$

which, as before (§ 61), proves that  $\psi = 0$ , and therefore  $\phi' = \phi$ ; and so establishes our present proposition.

EXAMPLE (1). The solution  $\phi = \tan^{-1} \frac{y}{x}$  considered in § 56, fulfils LAPLACE'S equation,  $\nabla^2\phi = 0$ ; and obviously satisfies the surface condition, not merely for the annular space with rectangular meridional section there considered, but for the hollow space bounded by the figure of revolution obtained by carrying a closed curve of any shape round any axis (OZ) not cutting the curve; which, for brevity, we shall in future call a *hollow circular ring*. Hence the irrotational motion possible within a fixed hollow circular ring is such that the velocity potential is proportional to the angle between the meridian plane through any point, and a fixed meridian.

EXAMPLE (2). The solid angle,  $\alpha$ , subtended at any point  $(x, y, z)$ , by an infinitesimal plane area,  $A$ , in any fixed position, fulfils LAPLACE'S equation  $\nabla^2\alpha = 0$ . This well-known proposition may be proved by taking  $A$  at the origin, and perpendicular to OX, when we have

$$\alpha = \frac{Ax}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} = A \frac{d}{dz} \frac{-1}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} \quad (5),$$

for which  $\nabla^2\alpha = 0$  is verified.

The solid angle subtended at  $(x, y, z)$  by any single closed circuit is the sum of those subtended at the same point by all parts into which we may divide any limited surface having this curve for its bounding edge. [Consider particularly curves such as those represented by the first three diagrams of § 58.] Hence if we call  $\phi$  the solid angle subtended at  $(x, y, z)$  by this surface, LAPLACE'S equation  $\nabla^2\phi$  is fulfilled. Hence  $\phi$  represents the velocity potential of the irrotational motion possible for a liquid contained in an infinite fixed closed vessel, within which is fixed, at an infinite distance from the outer bounding surface, an infinitely thin wire bent into the form of the closed curve in question.

The particular case of this example for which the curve is a circle, presents us with the simplest specimen of cyclic irrotational motion not confined [as that of Example (1) is] to a set of parallel planes. The velocity potential being the apparent area of a circular disc (or the area of a spherical ellipse) is readily found, and shown to be expressible readily in terms of a complete elliptic integral of the third class, and therefore in terms of incomplete elliptic functions of the first and second classes. The equi-potential surfaces are therefore traceable by aid of LEGENDRE'S tables. But it is to HELMHOLTZ that we owe the remarkable and useful discovery, that the equations of the *stream lines* (or lines perpendicular to the equi-potential surfaces) are expressible in terms of complete integrals of the first and second classes. They are therefore easily traceable by aid of LEGENDRE'S tables. The annexed diagram, of which we shall make much use later, show these curves as calculated and drawn by Mr MACFARLANE from HELMHOLTZ'S formula, expressed in terms of rectangular co-ordinates. An improved method of tracing them is described in a note by Mr CLERK MAXWELL, which he has kindly allowed me to append to this paper.

EXAMPLE 3. The motion described in Example 2 will remain unchanged outside any solid ring formed by solidifying and reducing to rest a portion of the fluid bounded by stream lines surrounding the infinitely thin wire. Thus we have a solid thick endless wire or bar forming a ring, or an endless knot as illustrated in the first three diagrams of § 59, of peculiar sectional figure depending on the stream lines round the arbitrary curve of Example 2; and the cyclic irrotational motion which, if placed in an infinite liquid it permits, is that whose velocity potential is proportional to the solid angle defined geometrically in the general solution given under Example 2.

64. *Kinetic energy of compounded acyclic and polycyclic irrotational motion—kinetico-statics.* The work done in the operation described in § 62 is calculated directly by summing the products of the pressure into an infinitesimal area of the surface, into the space through which the fluid contiguous with this area moves in the direction of the normal, for all parts of the surface, whether boundary or internal barrier, where the genetic pressure is applied, and for all infinitesimal divisions of the whole time from the commencement of the motion.

(a). Let  $w$  denote the work done, and  $\int dt$  time-integration, from the beginning of motion up to any instant. At any previous instant let  $p$  be the pressure,  $q$  the velocity, and  $\phi$  the velocity potential, of the fluid contiguous to any element  $d\sigma$  of the bounding surface,  $k$  the difference of fluid pressures on the two sides of any element,  $ds$ , of one of the internal barriers, and  $N$  the normal component of the fluid velocity contiguous to either  $d\sigma$  or  $ds$ . The preceding statement expressed in symbols is

$$W = \int dt [ -\iint p N d\sigma + \sum \iint k N ds ] \quad . \quad . \quad . \quad . \quad (6),$$

$\Sigma$  denoting summation for the several barriers if there are more than one. According to the general hydrokinetic theorem for irrotational motion [§ 59 (6) compare with § 31 (5)], with  $\phi$  expressed in terms of the co-ordinates of a point moving with the fluid, we have

$$p = -\frac{d\phi}{dt} + \frac{1}{2}q^2 \quad \dots \quad (7).$$

Now, let us suppose the pressure to be impulsive, so that there is infinitely little change of shape either of the bounding surface or of the barriers during the time  $\int dt$ .

This will also imply that  $\frac{d\phi}{dt}$  is infinitely great in comparison with  $\frac{1}{2}q^2$ ; so that

$$p = -\frac{d\phi}{dt} \quad \dots \quad (8).$$

And according to the notation of § 57 we have

$$N = \mathfrak{D}\phi \quad \dots \quad (9).$$

Also  $k$  is constant over each barrier surface.

Hence (6) becomes

$$W = \int dt \left[ \iint \frac{d\phi}{dt} \mathfrak{D}\phi d\sigma + \Sigma k \iint \mathfrak{D}\phi ds \right] \quad \dots \quad (10).$$

64. (b). The initiating motion of the bounding surface and the pressures on the barriers may be varied quite arbitrarily from the beginning to the end of the impulse; so that the history within that period of the acquisition of the prescribed final velocity may be altogether different, and not even simultaneous, in different parts of the bounding surface. Thus  $k_1$  and  $k_2$  may be quite different functions of  $t$ ; provided only  $\int k_1 dt$  and  $\int k_2 dt$  have the prescribed values, which we shall denote by  $\mathfrak{k}_1$  and  $\mathfrak{k}_2$  respectively.

(c). But, for one example, we may suppose  $\phi$  to have at each instant of  $\int dt$  everywhere one and the same proportion of its final value; so that if the latter denoted by  $\Phi$ , and if we put

$$\frac{\phi}{\Phi} = m \quad \dots \quad (11),$$

$m$  is independent of co-ordinates of position, but may of course be any arbitrary function of the time. Hence, observing that

$$\int dt m \frac{dm}{dt} = \frac{1}{2},$$

as the final value of  $m$  is 1, (10) becomes

$$W = \frac{1}{2} [\iint \Phi \mathfrak{D}\Phi d\sigma + \Sigma \mathfrak{k} \iint \mathfrak{D}\Phi ds] \quad \dots \quad (12).$$

(d). The second member of this equation doubled agrees with the two equal

second members of (7) § 57 with  $\phi$  and  $\phi'$  each made equal to  $\Phi$ . And the first member of that equation becomes twice the kinetic energy of the whole motion. Hence, when  $\phi' = \phi$ , and  $\nabla^2\phi = 0$ , (7) of § 57 expresses the equation of energy for the impulsive generation, of the fluid motion corresponding to velocity potential  $\phi$ , by pressures varying throughout according to the same function of the time; the first member being twice the kinetic energy of the motion generated, and the second twice the work done in the process.

64. (e). As another example, let us suppose the initiating pressures to be so applied as first to generate a motion corresponding to velocity potential  $\phi$ , and after that to change the velocity potential from  $\phi$  to  $\phi + \phi'$ , denoting by  $\phi$  and  $\phi'$  any two functions, such that  $\phi + \phi' = \Phi'$ , and each fulfilling LAPLACE'S equation: and let the augmentation from zero to  $\phi$ , and again from  $\phi$  to  $\phi + \phi'$  be uniform through the whole fluid. The work done in the first process, found as above (12),

$$\frac{1}{2}[\iint\phi\mathfrak{D}\phi\,d\sigma + \Sigma\kappa\iint\mathfrak{D}\phi\,ds] \quad . \quad . \quad . \quad (13),$$

if  $\kappa_1, \kappa_2, \&c.$ , denote the cyclic constants relative to  $\phi$ , as  $\kappa_1, \kappa_2, \&c.$ , relatively to  $\Phi$ , and the additional work done in the second process, similarly found, is

$$\frac{1}{2}[\iint\phi'(2\mathfrak{D}\phi + \mathfrak{D}\phi')\,d\sigma + \Sigma\kappa'\iint(2\mathfrak{D}\phi + \mathfrak{D}\phi')\,ds] \quad . \quad . \quad . \quad (14).$$

(f). Now, as we have seen (§ 63) that the actual fluid motion depends at each instant wholly on the normal velocity at each point of the bounding surface and the values of the cyclic constants, it follows that the work done in generating it ought to be independent of the order and law, of the acquisition of velocity at the bounding surface, and of the attainment of the values of the several cyclic constants. Hence, the the sum of (13) and (14) ought to be equal to (12). But if, for  $\Phi$  in (12) we substitute  $\phi + \phi'$ , the difference between its value and that of the sum of (13) and (14) is found to be

$$\frac{1}{2}[\iint(\phi\mathfrak{D}\phi' - \phi'\mathfrak{D}\phi)\,d\sigma + \Sigma(\kappa\iint\mathfrak{D}\phi'\,ds - \kappa'\iint\mathfrak{D}\phi\,ds)] \quad . \quad . \quad . \quad (15);$$

which, being the half the difference between the two equal second members of (7) § 57 for the case of

$$\nabla^2\phi = 0 \quad \text{and} \quad \nabla^2\phi' = 0,$$

is equal to zero. Hence, the equality of the second members of (7) § 57, constitutes the analytical reconciliation of the equations of energy for different modes of generation of the same fluid motion.

VIII.—*On the Rotation of a Rigid Body about a Fixed Point.* By Professor TAIT.

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Although it is very improbable that there remains to be discovered any new, and at the same time simple, fact connected with a question which has been elaborately treated by many of the greatest mathematicians of this and the preceding century, the employment of a new mathematical method may enable us to present some of their results in a more intelligible form, and with far less expenditure of analytical power than has hitherto been deemed necessary; and it may give us such an insight into the question, that we shall be able easily to discover the mutual relations among the various processes which have been already employed; so far, at least, as these differ in principle, and not merely in the peculiar co-ordinates assumed for the purpose of simplifying the equations. Such a method is that of *Quaternions*, which seems to be expressly fitted for the symmetrical evolution of truths which are usually obtained by the ordinary Cartesian methods only after great labour of calculation, and by modes of attack so indirect, and at first sight so purposeless, as to bewilder all but a very small class of readers. Quaternions afford so clear a view of the nature of the question they are applied to, that even the student, if he have some little knowledge of them, can often see *why* a transformation is made, whose object he would have been unable to discover had the problem been masked in the unnecessarily artificial difficulties of Cartesian geometry, or the outrageously repulsive formulæ of spherical trigonometry.

By far the most elegant and most easily intelligible representations of the motion of a solid body yet discovered, are due to POINSON. With the following extract from his splendid work, *Théorie Nouvelle de la Rotation des Corps* (*Liouville's Journal*, 1851), I most cordially agree,—though it appears to me that, when he does condescend to use analytical methods, he is by no means so happy as others have been, who, trusting to mathematical analysis alone, had not the benefit of his beautiful geometrical representations. But in perusing the extract, let the reader bear in mind that a *quaternion* equation is quite as suggestively intelligible, to those who understand it, as any geometrical diagram can possibly be. In fact, I might almost say, that it is *more* readily intelligible than diagrams usually

are; for, in reading a work illustrated by figures, we have generally to go through a laborious explanation of what the figure is intended to represent before we can make use of it for further developments. On the other hand, a purely quaternion formula draws, as it were, its own figure in the reader's mind, and saves him at least the trouble just mentioned. In this way every one has his figures drawn so as best to suit himself, and is not perplexed by having to pick up the principles on which they have been drawn for him by another, very probably of a different mode of thought. Still, such words as the following, when properly applied, *not* to quaternions but, to ordinary so-called analysis, must always convey a much-needed warning:—"Gardons-nous de croire qu'une science soit faite quand on l'a réduite à des formules analytiques. Rien ne nous dispense d'étudier les choses en elles-mêmes, et de nous bien rendre compte des idées qui font l'objet de nos spéculations. N'oublions point que les résultats de nos calculs ont presque toujours besoin d'être vérifiés, d'un autre côté, par quelque raisonnement simple, ou par l'expérience. Que si le calcul seul peut quelquefois nous offrir une vérité nouvelle, il ne faut pas croire que, sur ce point même, l'esprit n'ait plus rien à faire: mais, au contraire, il faut songer que, cette vérité étant indépendante des méthodes ou des artifices qui ont pu nous y conduire, il existe certainement quelque démonstration simple qui pourrait la porter à l'évidence: ce qui doit être le grand objet et le dernier résultat de la science mathématique."

. . . . . "Ce n'est qu'une apparente fécondité de cette méthode de pur calcul qu'on appelle assez improprement l'*analyse*. Car si les théorèmes sont déjà connus on découvre bien vite les transformations à faire pour que les équations y répondent; mais quand on n'a aucune idée de ces théorèmes, on ne transforme guère qu'au hasard, et le plus souvent on n'arrive à rien. La vraie analyse est dans l'examen attentif du problème à résoudre, et dans ces premiers raisonnements qu'on fait pour le mettre en équations. Transformer ensuite ces équations, c'est-à-dire les combiner ensemble, ou en poser d'autres évidentes que l'on combine avec elles, n'est au fond que de la synthèse; à moins que l'idée de chaque transformation ne nous soit donnée par quelque vue nouvelle de l'esprit, ou quelque nouveau raisonnement, ce qui nous fait rentrer dans la véritable analyse. Hors de cette voie lumineuse, il n'y a donc plus d'analyse, mais une obscure *synthèse* de formules algébriques que l'on *pose*, pour ainsi dire, l'une sur l'autre, et sans trop prévoir ce que pourra donner cette combinaison. Voilà les idées nettes qu'il faut attacher aux mots: et c'est au fond ce que tout le monde paraît sentir, puisqu'on dit très-bien une *heureuse* transformation, et qu'on ne dit point un *heureux* raisonnement, ni une *heureuse* analyse."

I was led to the following investigations by a desire to simplify, if possible, by a symmetrical process, the usual modes of treating the rotation of a rigid body. The methods ordinarily employed are essentially unsymmetrical, *e.g.* the determination, by means of three angles, of the position of the body at a given time, when

its angular velocities about its principal axes are given, or can be found. It was not till after my investigations were nearly completed, and the chief fundamental equations had been communicated to the British Association at Norwich, that I became aware of the existence of Professor CAYLEY'S\* admirable *Second Report on Theoretical Dynamics*, which contains an immense amount of valuable information, especially bearing on the present subject. From this I found that the notion of attaining symmetry, by seeking the single rotation which would bring the body from some initial position to its actual position at a given time, which had been suggested to me by HAMILTON'S† beautiful results, is due to EULER; and I also found that, by the help of certain formulæ due to RODRIGUES, CAYLEY has completely solved the question in the "Cambridge Mathematical Journal," vol. iii. (1843).‡ Comparative symmetry, however, is only attained by means of a brilliant display of analytical power at a great expense of time and bewilderment to the ordinary reader. In the "Philosophical Magazine," 1848, ii., CAYLEY has translated some of his formulæ into quaternions, and has thus arrived, though by a very circuitous route, at the fundamental kinematical equation of the present paper (§ 7 below). He does not give it in its simplest form, and he remarks that he has "not ascertained whether it leads to any results of importance." Under these circumstances, I have had no hesitation in laying this paper before the Society; for although many of its more important results have been otherwise obtained, few, with the exception of those due to HAMILTON (which will be given in their turn), have hitherto been arrived at so easily or in such simple forms.

As symmetry has been the particular object which I have had in view, by far the greater part of the investigation bears upon the determination of the quaternion, by which the transition can at one step be effected from any initial position to the actual position of the body at a given time; and a good many results have been retained, which are of more interest as properties of quaternions, than as regards their connection with the physical question. In the kinematical part of the paper, to which I proceed as a necessary preliminary, I have exhibited, for facility of comparison with other works on the subject, the values of this quaternion in terms of the various sets of co-ordinates usually employed. This, I need hardly say, does *not* lead to very simple or elegant results; but the fault is due, not to quaternions, but to the *unnaturalness* and want of symmetry of these common methods of attacking the problem. On the other hand, nothing can be neater than the set of formulæ which are suggested directly by quaternions.

\* Report on the Progress of the Solution of certain Special Problems of Dynamics.—*Brit. Ass. Report*, 1862.

† Proc. R. I. A., 1846. See also §§ 1 and 4 below.

‡ See also *Cambridge and Dublin Math. Journal*, vol i. (1846).



4. If  $q$  be any quaternion, the operator

$$q ( \quad ) q^{-1}$$

turns the vector, quaternion, or system, to which it is applied, about the axis of  $q$  through double the angle of  $q$ .

This was one of HAMILTON'S early\* discoveries in his new calculus, but it was independently obtained by CAYLEY (only a month or two later)† by the help of the formulæ of RODRIGUES already referred to. Conversely, when its truth has been established by an independent process, these formulæ may be at once derived from it: not only far more simply, but even in a somewhat improved form.

The quaternion  $q$  may obviously be considered as a mere versor, since its tensor does not appear in the operator  $q ( \quad ) q^{-1}$ , and a glance at the annexed

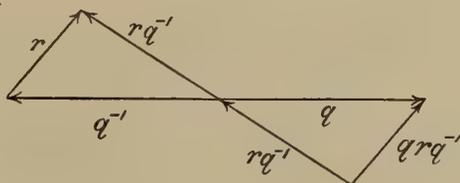


figure proves, by the multiplication of versor arcs, the theorem above stated. (See TAIT'S *Quaternions*, § 353, or HAMILTON'S *Lectures*, § 282, and *Elements*, § 308 (9).)

5. In quaternions we have, of course, whatever be  $q$  and  $r$ ,

$$(qr)^{-1} = r^{-1}q^{-1}.$$

Hence

$$q \cdot r ( \quad ) r^{-1} \cdot q^{-1} = qr ( \quad ) (qr)^{-1},$$

which shows how to combine any two rotations into a single one.

6. Given the initial and final positions of any two vectors of a rigid system, drawn from the fixed point; to find the quaternion operator by which the rotation can be effected. Let them be  $\alpha, \beta, \alpha_1, \beta_1$ , and let  $q$  be the required quaternion, then

$$q\alpha q^{-1} = \alpha_1, \quad q\beta q^{-1} = \beta_1,$$

or

$$q\alpha = \alpha_1 q, \quad q\beta = \beta_1 q \quad \dots \quad (3).$$

Hence

$$S(\alpha - \alpha_1)q = 0, \quad S(\beta - \beta_1)q = 0,$$

or

$$Vq \parallel V(\alpha - \alpha_1)(\beta - \beta_1)$$

\* *Proc. R. I. A.* November 11, 1844.

† *Phil. Mag.* Feb. 1845.

as we might at once have seen by the geometry of the question.  
Hence

$$q = x + yV(\alpha - \alpha_1)(\beta - \beta_1).$$

By the help of this, the first of equations (3) becomes

$$0 = x(\alpha - \alpha_1) + y \{V(\alpha - \alpha_1)(\beta - \beta_1) \cdot \alpha - \alpha_1 V(\alpha - \alpha_1)(\beta - \beta_1)\}$$

or

$$0 = x + yS(\alpha + \alpha_1)(\beta - \beta_1).$$

[The second of equations (3) merely gives us a condition which is equivalent to this, because

$$S(\alpha + \alpha_1)(\beta - \beta_1) = -S(\alpha - \alpha_1)(\beta + \beta_1)$$

or

$$S\alpha\beta = S\alpha_1\beta_1.]$$

Thus, finally,

$$\begin{aligned} q &= y \{-S(\alpha + \alpha_1)(\beta - \beta_1) + V(\alpha - \alpha_1)(\beta - \beta_1)\} \\ &= -y[(\beta - \beta_1)\alpha + \alpha_1(\beta - \beta_1)] \end{aligned}$$

where, as was to be expected, the tensor is left indeterminate.

7. *Given the instantaneous axis in terms of the time, it is required to find the single rotation which will bring the body from any initial position to its position at a given time.*

If  $\alpha$  be the initial vector of a point of the body,  $\varpi$  the value of the same at time  $t$ , and  $q$  the required quaternion, we have

$$\varpi = qaq^{-1} \quad \dots \quad (4).$$

Differentiating with respect to  $t$ , this gives

$$\begin{aligned} \dot{\varpi} &= \dot{q}aq^{-1} - qa\dot{q}q^{-1}, \\ &= \dot{q}q^{-1} \cdot qaq^{-1} - qa\dot{q}q^{-1}, \\ &= 2V \cdot (V\dot{q}q^{-1} \cdot qaq^{-1}). \end{aligned}$$

But

$$\dot{\varpi} = V\varepsilon\varpi = V \cdot \varepsilon qaq^{-1}.$$

Hence, as  $qaq^{-1}$  may be any vector whatever in the displaced body, we must have

$$\varepsilon = 2V\dot{q}q^{-1} \quad \dots \quad (5).$$

This is the fundamental kinematical relation already referred to. CAYLEY'S\* quaternion form of it (which will be understood by the help of § 13 below) is

$$\kappa(ip + jq + kr) = 2 \frac{d\Delta}{dt} \Delta + \frac{d\kappa}{dt},$$

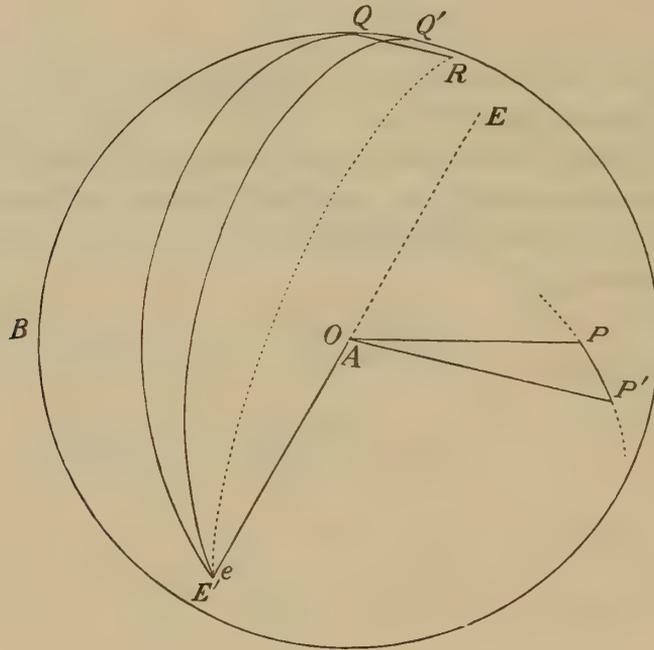
\* *Phil. Mag.*, Sept. 1848.



$$\begin{aligned}
 V \cdot \epsilon q a q^{-1} &= 2q V (V q^{-1} \dot{q} \cdot a) q^{-1} \\
 &= q (V q^{-1} \dot{q} \cdot a - a V q^{-1} \dot{q}) q^{-1} \\
 &= q (V q^{-1} \dot{q}) q^{-1} \cdot q a q^{-1} - q a q^{-1} \cdot q (V q^{-1} \dot{q}) q^{-1} \\
 &= V \dot{q} q^{-1} \cdot q a q^{-1} - q a q^{-1} \cdot V \dot{q} q^{-1} \\
 &= 2V (V \dot{q} q^{-1} \cdot q a q^{-1})
 \end{aligned}$$

the same equation as in § 7.

9\*. [*Inserted Dec. 19th, 1868.*] A geometrical investigation may also easily be given, if for no other purpose than to serve as an instance of the justice of my introductory remarks on diagrams as compared with quaternion equations.



Let  $Q, Q'$  be the poles, on the unit-sphere, of the versor angles  $BQE', BQ'E'$ , whose bounding arcs intersect in  $E'$ ; and let  $P, P'$  be the poles of these bounding arcs,  $A$  the pole of  $QQ'B$  [ $A$  coincides with the projection of  $O$ , the centre of the sphere]. Then evidently  $\widehat{AP} (=q)$  and  $\widehat{AP'} (=q')$  are the versor arcs, corresponding to the above versor angles. Obviously the point  $E'$  is deduced from a point  $e$  on the other side of the sphere [whose projection coincides with that of  $E'$ ], by a rotation about  $Q$  through double of  $BQE'$ , or about  $Q'$  through double of  $BQ'E'$ . Hence we have obviously

$$\overline{OE'} = q \overline{Oe} q^{-1} = q' \overline{Oe} q'^{-1}.$$

Thus a rigid body may pass from the position  $q ( ) q^{-1}$  to the position  $q' ( ) q'^{-1}$ , whatever be  $q$  and  $q'$ , by a rotation about  $OE'$ . Also, by  $q ( ) q^{-1}$ ,  $Q$  remains fixed; but by  $q' ( ) q'^{-1}$  it moves to  $R$ , where  $\angle QE'R = 2 \angle QE'Q' = 2 \angle POP'$ .

Hence if  $\overline{OE}' = -U_\epsilon = (U_\epsilon)^{-1}$ , the versor arc  $PP'$  may be expressed by either of the equal quantities

$$(U_\epsilon)^{\frac{2PP'}{\pi}} = q'q^{-1}.$$

But the actual rotation about  $\epsilon$  is  $2PP'$ , because  $Q$  moves to  $R$ . Hence if we put

$$q' = q + \dot{q}\delta t + \&c.,$$

we have

$$T_\epsilon \delta t = 2PP',$$

and thus

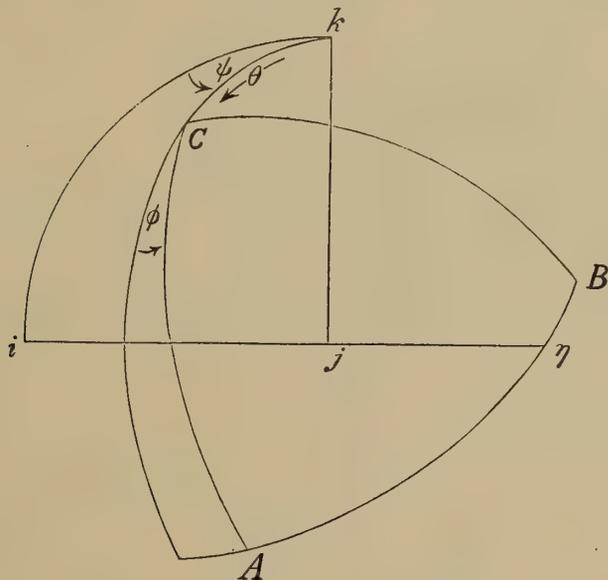
$$\begin{aligned} 1 + \dot{q}q^{-1}\delta t + \&c. &= (U_\epsilon)^{\frac{T_\epsilon \delta t}{\pi}} = \cos \frac{\delta t T_\epsilon}{2} + U_\epsilon \sin \frac{\delta t T_\epsilon}{2} \\ &= 1 + \frac{\epsilon}{2} \delta t + \&c. \end{aligned}$$

Hence, as in (6), when  $\delta t$  is indefinitely small

$$2\dot{q}q^{-1} = \epsilon.$$

**10.** To express  $q$  in terms of the usual angles  $\psi, \theta, \phi$ .

Here the vectors  $i, j, k$  in the original position of the body correspond to



$\overline{OA}, \overline{OB}, \overline{OC}$ , respectively, at time  $t$ . The transposition is effected by—*first*, a rotation  $\psi$  about  $k$ ; *second*, a rotation  $\theta$  about the new position of the line

originally coinciding with  $j$ ; *third*, a rotation  $\phi$  about the final position of the line at first coinciding with  $k$ .

Let  $i, j, k$  be taken as the initial directions of the three vectors which at time  $t$  terminate at A, B, C respectively.

The rotation  $\psi$  about  $k$  has the operator

$$k^{\frac{\psi}{\pi}} ( \quad ) k^{-\frac{\psi}{\pi}}.$$

This converts  $j$  into  $\eta$ , where

$$\eta = k^{\frac{\psi}{\pi}} j k^{-\frac{\psi}{\pi}} = j \cos \psi - i \sin \psi.$$

The body next rotates about  $\eta$  through an angle  $\theta$ . This has the operator

$$\eta^{\frac{\theta}{\pi}} ( \quad ) \eta^{-\frac{\theta}{\pi}}.$$

It converts  $k$  into

$$\begin{aligned} \overline{OC} = \zeta &= \eta^{\frac{\theta}{\pi}} k \eta^{-\frac{\theta}{\pi}} = \left( \cos \frac{\theta}{2} + \eta \sin \frac{\theta}{2} \right) k \left( \cos \frac{\theta}{2} - \eta \sin \frac{\theta}{2} \right) \\ &= k \cos \theta + \sin \theta (i \cos \psi + j \sin \psi). \end{aligned}$$

The body now turns through the angle  $\phi$  about  $\zeta$ , the operator being

$$\zeta^{\frac{\phi}{\pi}} ( \quad ) \zeta^{-\frac{\phi}{\pi}}.$$

Hence

$$\begin{aligned} q &= \zeta^{\frac{\phi}{\pi}} \eta^{\frac{\theta}{\pi}} k^{\frac{\psi}{\pi}} \\ &= \left( \cos \frac{\phi}{2} + \zeta \sin \frac{\phi}{2} \right) \left( \cos \frac{\theta}{2} + \eta \sin \frac{\theta}{2} \right) \left( \cos \frac{\psi}{2} + k \sin \frac{\psi}{2} \right) \\ &= \left( \cos \frac{\phi}{2} + \zeta \sin \frac{\phi}{2} \right) \left[ \cos \frac{\theta}{2} \cos \frac{\psi}{2} + k \cos \frac{\theta}{2} \sin \frac{\psi}{2} + \sin \frac{\theta}{2} \cos \frac{\psi}{2} (j \cos \psi - i \sin \psi) + \sin \frac{\theta}{2} \sin \frac{\psi}{2} (i \cos \psi + j \sin \psi) \right] \\ &= \left( \cos \frac{\phi}{2} + \zeta \sin \frac{\phi}{2} \right) \left[ \cos \frac{\theta}{2} \cos \frac{\psi}{2} - i \sin \frac{\theta}{2} \sin \frac{\psi}{2} + j \sin \frac{\theta}{2} \cos \frac{\psi}{2} + k \cos \frac{\theta}{2} \sin \frac{\psi}{2} \right] \\ &= \cos \frac{\phi}{2} \cos \frac{\theta}{2} \cos \frac{\psi}{2} + \sin \frac{\phi}{2} \sin \frac{\theta}{2} \sin \frac{\psi}{2} \sin \theta \cos \psi - \sin \frac{\phi}{2} \sin \frac{\theta}{2} \cos \frac{\psi}{2} \sin \theta \sin \psi - \sin \frac{\phi}{2} \cos \frac{\theta}{2} \sin \frac{\psi}{2} \cos \theta \\ &\quad + i \left( -\cos \frac{\phi}{2} \sin \frac{\theta}{2} \sin \frac{\psi}{2} + \sin \frac{\phi}{2} \cos \frac{\theta}{2} \cos \frac{\psi}{2} \sin \theta \cos \psi - \sin \frac{\phi}{2} \sin \frac{\theta}{2} \cos \frac{\psi}{2} \cos \theta + \sin \frac{\phi}{2} \cos \frac{\theta}{2} \sin \frac{\psi}{2} \sin \theta \sin \psi \right) \\ &\quad + j \left( \cos \frac{\phi}{2} \sin \frac{\theta}{2} \cos \frac{\psi}{2} + \sin \frac{\phi}{2} \cos \frac{\theta}{2} \cos \frac{\psi}{2} \sin \theta \sin \psi - \sin \frac{\phi}{2} \sin \frac{\theta}{2} \sin \frac{\psi}{2} \cos \theta - \sin \frac{\phi}{2} \cos \frac{\theta}{2} \sin \frac{\psi}{2} \sin \theta \cos \psi \right) \\ &\quad + k \left( \cos \frac{\phi}{2} \cos \frac{\theta}{2} \sin \frac{\psi}{2} + \sin \frac{\phi}{2} \cos \frac{\theta}{2} \cos \frac{\psi}{2} \cos \theta + \sin \frac{\phi}{2} \sin \frac{\theta}{2} \sin \frac{\psi}{2} \sin \theta \sin \psi + \sin \frac{\phi}{2} \sin \frac{\theta}{2} \cos \frac{\psi}{2} \sin \theta \cos \psi \right) \\ &= \cos \frac{\phi + \psi}{2} \cos \frac{\theta}{2} + i \sin \frac{\phi - \psi}{2} \sin \frac{\theta}{2} + j \cos \frac{\phi - \psi}{2} \sin \frac{\theta}{2} + k \sin \frac{\phi + \psi}{2} \cos \frac{\theta}{2} \end{aligned}$$

which is, of course, essentially unsymmetrical.

11. To find the usual equations connecting  $\psi, \theta, \phi$  with the angular velocities about three rectangular axes fixed in the body.

Having the value of  $q$  in last section in terms of the three angles, it may be useful to employ it, in conjunction with equation (6) of § 8, partly as a verification of that equation. Of course, this is an exceedingly roundabout process, and does not in the least resemble the simple one which is immediately suggested by quaternions.

We have

$$2\dot{q} = \varepsilon q = \{\omega_1 \overline{OA} + \omega_2 \overline{OB} + \omega_3 \overline{OC}\} q,$$

whence

$$2q^{-1}\dot{q} = q^{-1}\{\omega_1 \overline{OA} + \omega_2 \overline{OB} + \omega_3 \overline{OC}\}q,$$

or

$$2\dot{q} = q(i\omega_1 + j\omega_2 + k\omega_3).$$

This breaks up into the four (equivalent to three independent) equations

$$2 \frac{d}{dt} \left( \cos \frac{\phi + \psi}{2} \cos \frac{\theta}{2} \right) = -\omega_1 \sin \frac{\phi - \psi}{2} \sin \frac{\theta}{2} - \omega_2 \cos \frac{\phi - \psi}{2} \sin \frac{\theta}{2} - \omega_3 \sin \frac{\phi + \psi}{2} \cos \frac{\theta}{2}$$

$$2 \frac{d}{dt} \left( \sin \frac{\phi - \psi}{2} \sin \frac{\theta}{2} \right) = \omega_1 \cos \frac{\phi + \psi}{2} \cos \frac{\theta}{2} - \omega_2 \sin \frac{\phi + \psi}{2} \cos \frac{\theta}{2} + \omega_3 \cos \frac{\phi - \psi}{2} \sin \frac{\theta}{2}$$

$$2 \frac{d}{dt} \left( \cos \frac{\phi - \psi}{2} \sin \frac{\theta}{2} \right) = \omega_1 \sin \frac{\phi + \psi}{2} \cos \frac{\theta}{2} + \omega_2 \cos \frac{\phi + \psi}{2} \cos \frac{\theta}{2} - \omega_3 \sin \frac{\phi - \psi}{2} \sin \frac{\theta}{2}$$

$$2 \frac{d}{dt} \left( \sin \frac{\phi + \psi}{2} \cos \frac{\theta}{2} \right) = -\omega_1 \cos \frac{\phi - \psi}{2} \sin \frac{\theta}{2} + \omega_2 \sin \frac{\phi - \psi}{2} \sin \frac{\theta}{2} + \omega_3 \cos \frac{\phi + \psi}{2} \cos \frac{\theta}{2}.$$

From the second and third eliminate  $\dot{\phi} - \dot{\psi}$ , and we get by inspection

$$\cos \frac{\theta}{2} \cdot \dot{\theta} = (\omega_1 \sin \phi + \omega_2 \cos \phi) \cos \frac{\theta}{2},$$

or

$$\dot{\theta} = \omega_1 \sin \phi + \omega_2 \cos \phi \quad \dots \quad (8).$$

Similarly, by eliminating  $\dot{\theta}$  between the same two equations,

$$\sin \frac{\theta}{2} (\dot{\phi} - \dot{\psi}) = \omega_3 \sin \frac{\theta}{2} + \omega_1 \cos \phi \cos \frac{\theta}{2} - \omega_2 \sin \phi \cos \frac{\theta}{2}.$$

And from the first and last of the group of four

$$\cos \frac{\theta}{2} (\dot{\phi} + \dot{\psi}) = \omega_3 \cos \frac{\theta}{2} - \omega_1 \cos \phi \sin \frac{\theta}{2} + \omega_2 \sin \phi \sin \frac{\theta}{2}.$$

These last two equations give

$$\dot{\phi} + \dot{\psi} \cos \theta = \omega_3 \quad \dots \quad (9).$$

$$\dot{\phi} \cos \theta + \dot{\psi} = (-\omega_1 \cos \phi + \omega_2 \sin \phi) \sin \theta + \omega_3 \cos \theta.$$

From the last two we have

$$\dot{\psi} \sin \theta = -\omega_1 \cos \phi + \omega_2 \sin \phi \quad \dots \quad (10).$$

(8), (9), (10) are the forms in which the equations are usually given.



Substituting the above values of  $\zeta$  and  $\eta$ , multiplying out and arranging, we find finally

$$\begin{aligned} q &= \cos \frac{\phi}{2} \cos \frac{\theta}{2} \cos \frac{\psi}{2} - \sin \frac{\phi}{2} \sin \frac{\theta}{2} \sin \frac{\psi}{2} \\ &+ i \left( \cos \frac{\phi}{2} \cos \frac{\theta}{2} \sin \frac{\psi}{2} + \sin \frac{\phi}{2} \sin \frac{\theta}{2} \cos \frac{\psi}{2} \right) \\ &+ j \left( \cos \frac{\phi}{2} \sin \frac{\theta}{2} \cos \frac{\psi}{2} - \sin \frac{\phi}{2} \cos \frac{\theta}{2} \sin \frac{\psi}{2} \right) \\ &+ k \left( \cos \frac{\phi}{2} \sin \frac{\theta}{2} \sin \frac{\psi}{2} + \sin \frac{\phi}{2} \cos \frac{\theta}{2} \cos \frac{\psi}{2} \right). \end{aligned}$$

The expressions for  $\omega_1, \omega_2, \omega_3$  in terms of  $\phi, \theta, \psi$  and their differential coefficients are not very simple, and can scarcely be of any use.

We see by the equation of § 11 that

$$- \omega_1 = 2S \cdot iq^{-1}\dot{q}.$$

If we put

$$q = w + ix + jy + kz$$

this gives

$$- \omega_1 = 2(x\dot{w} - w\dot{x} + y\dot{z} - zy\dot{y})$$

from which the required expression may be obtained.

I have not examined the question, but I fancy that to deduce the constituents of the above value of  $q$  by means of spherical trigonometry would not be very easy.

**13.** *To deduce expressions for the direction-cosines of a set of rectangular axes in any position in terms of rational functions of three quantities only.*

Let  $\alpha, \beta, \gamma$  be unit-vectors in the directions of these axes. Let  $q$  be, as in § 7, the requisite quaternion operator for turning the co-ordinate axes into the position of this rectangular system. Then

$$q = w + xi + yj + zk$$

where, as in § 8, we may write

$$1 = w^2 + x^2 + y^2 + z^2.$$

Then we have

$$q^{-1} = w - xi - yj - zk,$$

and therefore

$$\begin{aligned} \alpha &= qi q^{-1} = (wi - x - yk + zj)(w - xi - yj - zk) \\ &= (w^2 + x^2 - y^2 - z^2) i + 2(wz + xy)j + 2(xz - wy)k, \end{aligned}$$

where the coefficients of  $i, j, k$  are the direction-cosines of  $\alpha$  as required. A similar process gives by inspection those of  $\beta$  and  $\gamma$ .

As given by CAYLEY, after RODRIGUES, they have a slightly different and

somewhat less simple form—to which, however, they are easily reduced by putting

$$w = \frac{x}{\lambda} = \frac{y}{\mu} = \frac{z}{\nu} = \frac{1}{\alpha^{\frac{1}{2}}}.$$

The geometrical interpretation of either set is obvious from the nature of quaternions. For (taking CAYLEY'S notation) if  $\theta$  be the angle of rotation:  $\cos f$ ,  $\cos g$ ,  $\cos h$ , the direction-cosines of the axis, we have

$$q = w + xi + yj + zk = \cos \frac{\theta}{2} + \sin \frac{\theta}{2} (i \cos f + j \cos g + k \cos h),$$

so that

$$w = \cos \frac{\theta}{2}$$

$$x = \sin \frac{\theta}{2} \cos f$$

$$y = \sin \frac{\theta}{2} \cos g$$

$$z = \sin \frac{\theta}{2} \cos h.$$

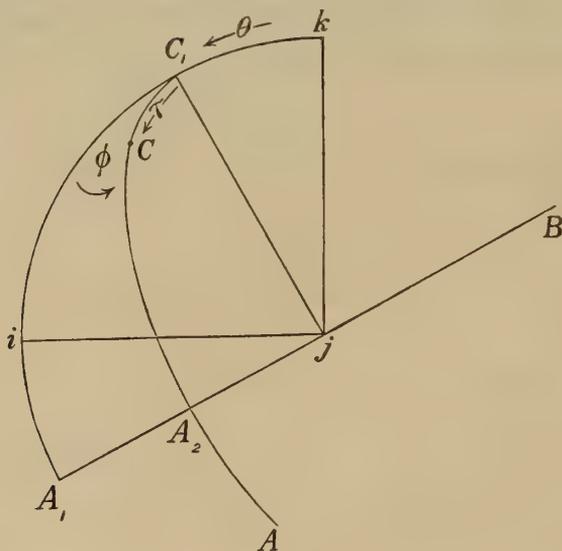
From these we pass at once to RODRIGUES' subsidiary formulæ,

$$\alpha = \frac{1}{w^2} = \sec^2 \frac{\theta}{2}$$

$$\lambda = \frac{x}{w} = \tan \frac{\theta}{2} \cos f$$

$$\&c. = \&c.$$

14. *In the system of three angles, corresponding to that usually employed in astronomy—viz.,  $\theta$  the longitude of node,  $\phi$  the inclination of orbit,  $\tau$  the angle from node in plane of orbit—to find the quaternion operator.*



Here we relapse into the essential asymmetry of the method of § 10. *First,*

a rotation  $\theta$  about  $j$ ; *second*, a rotation  $\phi$  about the new position of  $k$ ; *third*, a rotation  $\tau$  about the final position of what was originally  $j$ . The connection of this process with that of § 10 is sufficiently obvious.

Here  $j^{\frac{\theta}{\pi}} ( ) j^{-\frac{\theta}{\pi}}$  is the operator for  $\theta$ , and converts  $k$  into

$$\begin{aligned} \overline{OC}_1 = \eta &= \left( \cos \frac{\theta}{2} + j \sin \frac{\theta}{2} \right) k \left( \cos \frac{\theta}{2} - j \sin \frac{\theta}{2} \right) \\ &= i \sin \theta + k \cos \theta. \end{aligned}$$

Next, the operator for  $\phi$  is

$$\eta^{\frac{\phi}{\pi}} ( ) \eta^{-\frac{\phi}{\pi}},$$

and converts  $j$  into

$$\begin{aligned} \overline{OB} = \zeta &= \left( \cos \frac{\phi}{2} + \sin \frac{\phi}{2} (i \sin \theta + k \cos \theta) \right) j \left( \cos \frac{\phi}{2} - \sin \frac{\phi}{2} (i \sin \theta + k \cos \theta) \right) \\ &= -i \sin \phi \cos \theta + j \cos \phi + k \sin \phi \sin \theta. \end{aligned}$$

Hence we have

$$\begin{aligned} q &= \zeta^{\frac{\tau}{\pi}} \eta^{\frac{\phi}{\pi}} j^{\frac{\theta}{\pi}} \\ &= \left[ \cos \frac{\tau}{2} + \sin \frac{\tau}{2} (-i \sin \phi \cos \theta + j \cos \phi + k \sin \phi \sin \theta) \right] \left( \cos \frac{\phi}{2} + \sin \frac{\phi}{2} (i \sin \theta + k \cos \theta) \right) \left( \cos \frac{\theta}{2} + j \sin \frac{\theta}{2} \right) \\ &= \left[ \cos \frac{\tau}{2} + \sin \frac{\tau}{2} (-i \sin \phi \cos \theta + j \cos \phi + k \sin \phi \sin \theta) \right] \left( \cos \frac{\phi}{2} \cos \frac{\theta}{2} + i \sin \frac{\phi}{2} \sin \frac{\theta}{2} + j \cos \frac{\phi}{2} \sin \frac{\theta}{2} + k \sin \frac{\phi}{2} \cos \frac{\theta}{2} \right) \\ &= \cos \frac{\tau + \theta}{2} \cos \frac{\phi}{2} + i \sin \frac{\theta - \tau}{2} \sin \frac{\phi}{2} + j \sin \frac{\tau + \theta}{2} \cos \frac{\phi}{2} + k \cos \frac{\theta - \tau}{2} \sin \frac{\phi}{2}. \end{aligned}$$

As a verification, we have by § 11

$$\begin{aligned} \overline{OA} &= qi q^{-1} \\ &= (w^2 + x^2 - y^2 - z^2) i + 2(wz + xy) j + 2(ax - wy) k \\ &= \left[ \cos(\theta + \tau) \cos^2 \frac{\phi}{2} - \cos(\theta - \tau) \sin^2 \frac{\phi}{2} \right] i + \cos \tau \sin \phi j + \left[ \sin(\theta - \tau) \sin^2 \frac{\phi}{2} - \sin(\theta + \tau) \cos^2 \frac{\phi}{2} \right] k \\ &= (\cos \theta \cos \tau \cos \phi - \sin \theta \sin \tau) i + \cos \tau \sin \phi j + (-\sin \theta \cos \tau \cos \phi - \cos \theta \sin \tau) k. \end{aligned}$$

The coefficients of  $i, j, k$ , in this are the usual expressions for three of the direction-cosines. The other six may be obtained by the same process.

To express the angular velocities about  $\overline{OA}, \overline{OB}, \overline{OC}$  in terms of the three angles  $\theta, \phi, \tau$ , we have at once

$$\begin{aligned} -\omega_1 &= 2S \cdot i q^{-1} \dot{q} \\ &= 2(x\dot{w} - w\dot{x} + y\dot{z} - z\dot{y}) \\ &= -\dot{\theta} \cos \tau \sin \phi - \dot{\phi} \sin \tau. \end{aligned}$$

And the others can be found in a similar manner.



as given by HAMILTON; and we shall omit for the present the consideration of whether  $\gamma$  is a constant vector or not.

19. Let  $a$  be the initial position of  $\varpi$ ,  $q$  the quaternion by which the body can be at one step transferred from its initial position to its position at time  $t$ . Then

$$\varpi = qaq^{-1}$$

and HAMILTON's equation (14) becomes

$$\Sigma . mqaq^{-1} \nabla . \varepsilon qaq^{-1} = \gamma,$$

or

$$\Sigma . mqa \{ aS . \alpha q^{-1} \varepsilon q - q^{-1} \varepsilon q \alpha^2 \} q^{-1} = \gamma.$$

Let

$$\phi \varepsilon = \Sigma . m(aS \alpha \varepsilon - \alpha^2 \varepsilon) \dots \dots \dots (16),$$

where  $\phi$  is a self-conjugate linear and vector function, whose constituent vectors are fixed in the body in its initial position. Then the previous equation may be written

$$q\phi(q^{-1}\varepsilon q)q^{-1} = \gamma,$$

or

$$\phi(q^{-1}\varepsilon q) = q^{-1}\gamma q.$$

For simplicity let us write

$$\left. \begin{aligned} q^{-1}\varepsilon q &= \eta \\ q^{-1}\gamma q &= \zeta \end{aligned} \right\} \dots \dots \dots (17).$$

Then HAMILTON's dynamical equation becomes simply

$$\phi \eta = \zeta \dots \dots \dots (18).$$

20. It is easy to see what the new vectors  $\eta$  and  $\zeta$  represent. For we may write (17) in the form

$$\left. \begin{aligned} \varepsilon &= q\eta q^{-1} \\ \gamma &= q\zeta q^{-1} \end{aligned} \right\} \dots \dots \dots (17)',$$

from which it is obvious that  $\eta$  is that vector in the initial position of the body which, at time  $t$ , becomes the instantaneous axis in the moving body. When no forces act,  $\gamma$  is constant, and  $\zeta$  is the initial position of the vector which, at time  $t$ , is perpendicular to the invariable plane.

21. The complete solution of the problem is contained in equations (7), (17), (18).\* Writing them again we have, attending to (17), while introducing  $\eta$  instead of  $\varepsilon$  into (7),

\* To these it is unnecessary to add

$$Tq = \text{constant},$$

as this constancy of  $Tq$  is proved by the form of (7). For, had  $Tq$  been variable, there must have been a quaternion in place of the vector  $\eta$ . In fact,  $\frac{d}{dt}(Tq)^2 = 2S \cdot \dot{q}Kq = (Tq)^2 S\eta = 0$ .



square roots of the moments of inertia about them,\* so that, if  $i, j, k$  be taken as unit vectors in the directions of its axes respectively, we have

$$\left. \begin{aligned} S i \phi i &= -A, \\ S j \phi j &= -B, \\ S k \phi k &= -C, \end{aligned} \right\} \dots \dots \dots (22),$$

A, B, C, being the principal moments of inertia. Consequently

$$\phi \rho = -\{A i S i \rho + B j S j \rho + C k S k \rho\} \dots \dots \dots (23).$$

Thus the equation (21) for  $\eta$  breaks up, if we put

$$\eta = i \omega_1 + j \omega_2 + k \omega_3$$

into the three following scalar equations

$$\left. \begin{aligned} A \dot{\omega}_1 + (C - B) \omega_2 \omega_3 &= 0, \\ B \dot{\omega}_2 + (A - C) \omega_3 \omega_1 &= 0, \\ C \dot{\omega}_3 + (B - A) \omega_1 \omega_2 &= 0, \end{aligned} \right\}$$

which are the same as those of EULER. Only, it is to be understood that the equations just written are not primarily to be considered as equations of rotation. They rather express, with reference to fixed axes in the initial position of the body, the motion of the extremity,  $\omega_1, \omega_2, \omega_3$ , of the vector corresponding to the instantaneous axis in the moving body. If, however, we consider  $\omega_1, \omega_2, \omega_3$  as standing for their values in terms of  $w, x, y, z$  (§ 27 below), or any other coordinates employed to refer the body to fixed axes, they *are* the equations of motion.

Similar remarks apply to the equation which determines  $\zeta$ , for if we put

$$\zeta = i \varpi_1 + j \varpi_2 + k \varpi_3,$$

(20) may be reduced to three scalar equations of the form

$$\dot{\varpi}_1 + \left( \frac{1}{C} - \frac{1}{B} \right) \varpi_2 \varpi_3 = 0.$$

24. EULER'S equations in their usual form are easily deduced from what precedes. For, let

$$\phi \rho = q \phi (q^{-1} \rho q) q^{-1}$$

whatever be  $\rho$ ; that is, let  $\phi$  represent with reference to the moving principal axes what  $\phi$  represents with reference to the principal axes in the initial position of the body, and we have

$$\begin{aligned} \phi \dot{\rho} &= q \phi (q^{-1} \dot{\rho} q) q^{-1} &= q \phi (\dot{\rho}) q^{-1} \\ &= q \dot{\zeta} q^{-1} &= q V(\zeta \phi^{-1} \zeta) q^{-1} \end{aligned}$$

\* For further information about this equation, see HAMILTON, *Proc. R. I. A.* 1847, and *Elements of Quaternions*, p. 755. Also TAIT, *Quaternions*, § 367.

$$\begin{aligned}
&= -qV(\eta\phi\eta)q^{-1} \\
&= -V.q\eta\phi(\eta)q^{-1} \\
&= -V.q\eta\eta^{-1}q\phi(q^{-1}\varepsilon q)q^{-1} \\
&= -V.\varepsilon\phi\varepsilon,
\end{aligned}$$

which is the required expression.

But perhaps the simplest mode of obtaining this equation is to start with HAMILTON'S unintegrated equation (11), which for the case of no forces is simply

$$\sum .mV_{\omega}\ddot{\omega} = 0.$$

But from

$$\dot{\omega} = V_{\varepsilon}\omega$$

we deduce

$$\begin{aligned}
\ddot{\omega} &= V_{\varepsilon}\dot{\omega} + V_{\dot{\varepsilon}}\omega \\
&= \omega\varepsilon^2 - \varepsilon S_{\varepsilon}\omega + V_{\dot{\varepsilon}}\omega,
\end{aligned}$$

so that

$$\sum .m(V_{\varepsilon}\omega S_{\varepsilon}\omega - \dot{\varepsilon}\omega^2 + \omega S_{\dot{\varepsilon}}\omega) = 0.$$

If we look at equation (16), and remember that  $\phi$  differs from  $\phi$  simply in having  $\omega$  substituted for  $\alpha$ , we see that this may be written

$$V_{\varepsilon}\phi\varepsilon + \phi\dot{\varepsilon} = 0,$$

the equation before obtained. The first mode of arriving at it has been given because it leads to an interesting set of transformations, for which reason we append other two.

By (17)

$$\gamma = q\zeta q^{-1},$$

therefore

$$0 = \dot{q}q^{-1}.q\zeta q^{-1} + q\dot{\zeta}q^{-1} - q\zeta q^{-1}\dot{q}q^{-1},$$

or

$$\begin{aligned}
q\dot{\zeta}q^{-1} &= 2V.\gamma V\dot{q}q^{-1} \\
&= V\gamma\dot{\varepsilon}.
\end{aligned}$$

But, by the beginning of this section, and by (14), this is again the equation lately proved.

Perhaps, however, the following is neater.\*

By (14)

$$\phi\varepsilon = \gamma.$$

Hence

$$\begin{aligned}
\phi\dot{\varepsilon} &= -\dot{\phi}\varepsilon = -\sum .m(\dot{\omega}V_{\varepsilon}\omega + \omega V_{\dot{\varepsilon}}\omega) \\
&= -\sum .m\dot{\omega}S_{\varepsilon}\omega \\
&= -V.\varepsilon\sum .m\omega S_{\varepsilon}\omega \\
&= -V_{\varepsilon}\phi\varepsilon.
\end{aligned}$$

\* [Inserted Dec. 19, 1868.] I have lately found that HAMILTON, in his *Elements of Quaternions* (1866), has obtained this equation in a manner almost identical with that last given.

25. However they are obtained, such equations as those of § 23 were shown long ago by EULER to be integrable as follows.

Putting

$$2\int\omega_1\omega_2\omega_3 dt = s,$$

we have

$$A\omega_1^2 = A\Omega_1^2 + (B - C)s$$

with other two equations of the same form. Hence

$$2dt = \int \frac{ds}{\left(\Omega_1^2 + \frac{B-C}{A}s\right)^{\frac{1}{2}} \left(\Omega_2^2 + \frac{C-A}{B}s\right)^{\frac{1}{2}} \left(\Omega_3^2 + \frac{A-B}{C}s\right)^{\frac{1}{2}}},$$

so that  $t$  is known in terms of  $s$  by an elliptic integral. Thus, finally,  $\eta$  or  $\zeta$  may be expressed in terms of  $t$ ; and in some of the succeeding investigations for  $q$  we shall suppose this to have been done. It is with this integration, or an equivalent one, that most writers on the farther development of the subject have commenced their investigations.

26. By § 16,  $\gamma$  is evidently the vector moment of momentum of the rigid body; and the kinetic energy is, as in § 17,

$$-\frac{1}{2}\sum m\dot{\mathbf{x}}^2 = -\frac{1}{2}S\varepsilon\gamma.$$

But

$$S\varepsilon\gamma = S \cdot q^{-1}\varepsilon\eta q^{-1}\gamma q = S\eta\zeta,$$

so that when no forces act

$$S\zeta\phi^{-1}\zeta = S\eta\phi\eta = -h^2.$$

But, by (17), we have also

$$T\zeta = T\gamma, \text{ or } T\phi\eta = T\gamma,$$

so that we have, for the equations of the cones described in the initial position of the body by  $\eta$  and  $\zeta$ , that is, for the cones described in the moving body by the instantaneous axis and by the perpendicular to the invariable plane,

$$h^2\zeta^2 + \gamma^2 S\zeta\phi^{-1}\zeta = 0,$$

$$h^2(\phi\eta)^2 + \gamma^2 S\eta\phi\eta = 0.$$

This is on the supposition that  $\gamma$  and  $h$  are constants. If forces act, these quantities are functions of  $t$ , and the equations of the cones then described in the body must be found by eliminating  $t$  between the respective equations. The final results to which such a process will lead must, of course, depend entirely upon the way in which  $t$  is involved in these equations, and therefore no general statement on the subject can be made.

27. Recurring to our equations for the determination of  $q$ , and taking first the case of no forces, we see that, if we assume  $\eta$  to have been found (as in § 25) by means of elliptic integrals, we have to solve the equation

$$q\eta = 2\dot{q},^*$$

that is, we have to integrate a system of four other differential equations harder than the first.

Putting, as in § 23,  $\eta = i\omega_1 + j\omega_2 + k\omega_3,$

where  $\omega_1, \omega_2, \omega_3$  are supposed to be known functions of  $t$ , and

$$q = w + ix + jy + kz,$$

this system is

$$\frac{1}{2}dt = \frac{dw}{W} = \frac{dx}{X} = \frac{dy}{Y} = \frac{dz}{Z},$$

where

$$W = -\omega_1x - \omega_2y - \omega_3z,$$

$$X = \omega_1w + \omega_3y - \omega_2z,$$

$$Y = \omega_2w + \omega_1z - \omega_3x,$$

$$Z = \omega_3w + \omega_2x - \omega_1y.$$

or, as suggested by CAYLEY to bring out the skew symmetry,

$$X = \omega_3y - \omega_2z + \omega_1w,$$

$$Y = -\omega_3x + \omega_1z + \omega_2w,$$

$$Z = \omega_2x - \omega_1y + \omega_3w,$$

$$W = -\omega_1x - \omega_2y - \omega_3z.$$

Here, of course, one integral is

$$w^2 + x^2 + y^2 + z^2 = \text{constant}.$$

\* To get an idea of the nature of this equation, let us integrate it on the supposition that  $\eta$  is a constant vector. By differentiation and substitution, we get

$$2\ddot{q} = \dot{q}\eta = \frac{1}{2}\eta^2q.$$

Hence

$$q = Q_1 \cos \frac{T\eta}{2}t + Q_2 \sin \frac{T\eta}{2}t.$$

Substituting in the given equation we have

$$T\eta \left( -Q_1 \sin \frac{T\eta}{2}t + Q_2 \cos \frac{T\eta}{2}t \right) = \left( Q_1 \cos \frac{T\eta}{2}t + Q_2 \sin \frac{T\eta}{2}t \right) \eta.$$

Hence

$$T\eta \cdot Q_2 = Q_1\eta,$$

$$-T\eta \cdot Q_1 = Q_2\eta$$

which are virtually the same equation—and thus

$$\begin{aligned} q &= Q_1 \left( \cos \frac{T\eta}{2}t + U\eta \sin \frac{T\eta}{2}t \right) \\ &= Q_1 (U\eta)^{\frac{tT\eta}{\pi}}. \end{aligned}$$

And the interpretation of  $q(\quad)q^{-1}$  will obviously then be a rotation about  $\eta$  through the angle  $tT\eta$ , together with any other arbitrary rotation whatever. Thus any position whatever may be taken as the initial one of the body—and  $Q_1(\quad)Q_1^{-1}$  brings it to its required position at time  $t=0$ .

It may suffice thus to have alluded to a possible mode of solution, which, except for very simple values of  $\eta$ , involves very great difficulties. The quaternion solution, when  $\eta$  is of constant length and revolves uniformly in a right cone, will be given later.

28. If, on the other hand, we eliminate  $\eta$ , we have to integrate

$$q\phi^{-1}(q^{-1}\gamma q) = 2\dot{q},$$

so that one integration theoretically suffices. But, in consequence of the present imperfect development of the quaternion calculus, the only known method of effecting this is to reduce the quaternion equation to a set of four ordinary differential equations of the first order. It may be interesting to form these equations.

Put

$$q = w + ix + jy + kz,$$

and

$$\gamma = ia + jb + kc,$$

then, by ordinary quaternion multiplication, we easily reduce the given equation to the following set:—

$$\frac{dt}{2} = \frac{dw}{W} = \frac{dx}{X} = \frac{dy}{Y} = \frac{dz}{Z} \quad \dots \quad (24),$$

where

$$\begin{aligned} W &= -x\mathfrak{A} - y\mathfrak{B} - z\mathfrak{C} & \text{or} & & X &= & y\mathfrak{C} - z\mathfrak{B} + w\mathfrak{A} \\ X &= w\mathfrak{A} + y\mathfrak{C} - z\mathfrak{B} & & & Y &= -x\mathfrak{C} & + z\mathfrak{A} + w\mathfrak{B} \\ Y &= w\mathfrak{B} + z\mathfrak{A} - x\mathfrak{C} & & & Z &= x\mathfrak{B} - y\mathfrak{A} & + w\mathfrak{C} \\ Z &= w\mathfrak{C} + x\mathfrak{B} - y\mathfrak{A} & & & W &= -x\mathfrak{A} - y\mathfrak{B} - z\mathfrak{C} \end{aligned}$$

and

$$\begin{aligned} \mathfrak{A} &= \frac{1}{A} \left[ a(w^2 - x^2 - y^2 - z^2) + 2x(ax + by + cz) + 2w(bz - cy) \right] \\ \mathfrak{B} &= \frac{1}{B} \left[ b(w^2 - x^2 - y^2 - z^2) + 2y(ax + by + cz) + 2w(cx - az) \right] \\ \mathfrak{C} &= \frac{1}{C} \left[ c(w^2 - x^2 - y^2 - z^2) + 2z(ax + by + cz) + 2w(ay - bx) \right] \end{aligned}$$

W, X, Y, Z are thus *homogeneous* functions of  $w, x, y, z$  of the third degree.

Perhaps the simplest way of obtaining these equations is to translate the group of § 21 into  $w, x, y, z$  at once—instead of using the equation from which  $\zeta$  and  $\eta$  are eliminated.

We thus see that

$$\eta = i\mathfrak{A} + j\mathfrak{B} + k\mathfrak{C}.$$

One obvious integral of these equations ought to be

$$w^2 + x^2 + y^2 + z^2 = \text{constant},$$

which has been assumed all along. In fact, we see at once that

$$wW + xX + yY + zZ = 0$$

identically, which leads to the above integral.

These equations appear to be worthy of attention, partly because of the homogeneity of the denominators  $W, X, Y, Z$ , but particularly as they afford (what does not appear to have been sought) the means of solving this celebrated problem *at one step*, that is, without the previous integration of EULER'S equations (§ 23).

A set of equations identical with these, but not in a homogeneous form (being expressed, in fact, in terms of  $\kappa, \lambda, \mu, \nu$  of § 13, instead of  $w, x, y, z$ ), is given by CAYLEY (*Camb. and Dub. Math. Journal*, vol. i. 1846), and completely integrated (in the sense of being reduced to quadratures) by assuming EULER'S equations to have been previously integrated. (Compare § 27.)

CAYLEY'S method may be even more easily applied to the above equations than to his own; and I therefore leave this part of the development to the reader, who will at once see (as in § 27) that  $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}$  correspond to  $\omega_1, \omega_2, \omega_3$  of the  $\eta$  type § 23.

29. It may be well to notice, in connection with the formulæ for direction cosines in § 13 above, that we may write

$$\begin{aligned}\mathfrak{A} &= \frac{1}{A} \left[ a(w^2 + x^2 - y^2 - z^2) + 2b(xy + wz) + 2c(xz - wy) \right], \\ \mathfrak{B} &= \frac{1}{B} \left[ 2a(xy - wz) + b(w^2 - x^2 + y^2 - z^2) + 2c(yz + wx) \right], \\ \mathfrak{C} &= \frac{1}{C} \left[ 2a(xz + wy) + 2b(yz - wx) + c(w^2 - x^2 - y^2 + z^2) \right].\end{aligned}$$

These expressions may be considerably simplified by the usual assumption, that one of the fixed unit-vectors (*i* suppose) is perpendicular to the invariable plane, which amounts to assigning definitely the initial position of one line in the body; and which gives the relations

$$b = 0, c = 0.$$

30. When forces act,  $\gamma$  is variable, and the quantities  $a, b, c$  will in general involve all the variables  $w, x, y, z, t$ , so that the equations of last section become much more complicated. The type, however, remains the same if  $\gamma$  involves  $t$  only; if it involve  $q$  we must differentiate the equation, put in the form

$$\gamma = 2q \phi(q^{-1}\dot{q})q^{-1},$$

and we thus easily obtain the differential equation of the second order

$$\psi = 4V \cdot \dot{q} \phi(q^{-1}\dot{q})q^{-1} + 2q \phi(V \cdot q^{-1}\dot{q})q^{-1};$$

if we recollect that, because  $q^{-1}\dot{q}$  is a vector, we have

$$S \cdot q^{-1}\ddot{q} = (q^{-1}\dot{q})^2.$$





This may easily be put in the simpler form

$$2\delta = V\delta\eta - V \cdot (\gamma + \zeta)V \cdot (\gamma - \zeta)^{-1}\eta\delta \quad (34).$$

Reduced to scalars, this gives three *linear* differential equations of the first order, the coefficients being functions of  $t$ . These can, of course, be reduced to depend upon one linear differential equation of the third order with coefficients functions of  $t$ .

36. As a verification of the preceding work, we may try whether the result is consistent, as it ought to be, with the condition (assumed throughout).

$$\text{Constant} = (Tq)^2 = 2\gamma^2\delta^2 + 2S \cdot \gamma\delta\zeta\delta.$$

This expression gives, by differentiation,

$$0 = -\delta^2 S\gamma\dot{\zeta} + 2(\gamma^2 - S\gamma\zeta)S\delta\dot{\delta} + 4S\gamma\delta S\gamma\dot{\delta}.$$

Substituting for  $\dot{\delta}$  its value from (34), we have

$$\begin{aligned} 0 &= -\delta^2 S\gamma\dot{\zeta} + S \cdot \delta\gamma\zeta S\delta\eta + 2S\gamma\delta(S \cdot \gamma\delta\eta - \frac{1}{2}S \cdot (\gamma + \zeta)\delta\eta) \\ &= -\delta^2 S\gamma\dot{\zeta} + S \cdot \delta\gamma\zeta S\delta\eta + S\gamma\delta S \cdot \gamma\delta\eta - S\gamma\delta S \cdot \zeta\delta\eta \\ &= -\delta^2 S\gamma\dot{\zeta} + S \cdot \delta\{\eta S \cdot \gamma\zeta\delta + \zeta S \cdot \eta\gamma\delta + \gamma S \cdot \zeta\eta\delta\} \\ &= -\delta^2 S\gamma\dot{\zeta} + S \cdot \delta(\delta S \cdot \gamma\zeta\eta) \end{aligned}$$

which is true, because by (20)

$$\zeta = V\zeta\eta.$$

37. Another mode of attacking the problem, at first sight entirely different from that in § 19, but in reality identical with it, is to seek the linear and vector function which expresses the *Homogeneous Strain* which the body must undergo to pass from its initial position to its position at time  $t$ .

Let

$$\varpi = \chi\alpha$$

$\alpha$  being (as in § 19) the initial position of a vector of the body,  $\varpi$  its position at time  $t$ . In this case  $\chi$  is a linear and vector function. (TAIT'S *Quaternions*, § 355.)

Then, obviously, we have,  $\varpi_1$  being the vector of some other point, which had initially the value  $\alpha_1$ ,

$$S\varpi\varpi_1 = S \cdot \chi\alpha\chi\alpha_1 = S\alpha\alpha_1$$

(a particular case of which is

$$T\varpi = T\chi\alpha = T\alpha)$$

and

$$V\varpi\varpi_1 = V \cdot \chi\alpha\chi\alpha_1 = \chi V\alpha\alpha_1.$$

These are necessary properties of the strain-function  $\chi$ , depending on the fact that in the present application the system is rigid.

38. The kinematical equation

$$\dot{\alpha} = V_{\varepsilon\alpha}$$

becomes

$$\dot{\chi}\alpha = V \cdot \varepsilon\chi\alpha,$$

(the function  $\dot{\chi}$  being formed from  $\chi$  by the differentiation of its constituents with respect to  $t$ ).

HAMILTON'S kinetic equation

$$\Sigma \cdot m_{\alpha} V_{\varepsilon\alpha} = \gamma,$$

becomes

$$\Sigma \cdot m\chi\alpha V \cdot \varepsilon\chi\alpha = \gamma.$$

This may be written

$$\Sigma \cdot m(\chi\alpha S \cdot \varepsilon\chi\alpha - \varepsilon\alpha^2) = \gamma,$$

or

$$\Sigma \cdot m(\alpha S \cdot \alpha\chi'_{\varepsilon} - \chi^{-1}_{\varepsilon} \cdot \alpha^2) = \chi^{-1}\gamma,$$

where  $\chi'$  is the conjugate of  $\chi$ .

But, because

$$S \cdot \chi\alpha\chi\alpha_1 = S\alpha\alpha_1,$$

we have

$$S\alpha\alpha_1 = S \cdot \alpha\chi'\chi\alpha_1,$$

whatever be  $\alpha$  and  $\alpha_1$ , so that

$$\chi' = \chi^{-1}.$$

Hence

$$\Sigma \cdot m(\alpha S \cdot \alpha\chi^{-1}_{\varepsilon} - \chi^{-1}_{\varepsilon} \cdot \alpha^2) = \chi^{-1}\gamma,$$

or, by § 19,

$$\phi\chi^{-1}_{\varepsilon} = \chi^{-1}\gamma.$$

39. Thus we have, as the analogues of (17), (17'), the equations

$$\chi^{-1}_{\varepsilon} = \eta,$$

$$\chi^{-1}\gamma = \zeta,$$

and the former result

$$\dot{\chi}\alpha = V \cdot \varepsilon\chi\alpha$$

becomes

$$\dot{\chi}\alpha = V \cdot \chi\eta\chi\alpha = \chi V\eta\alpha.$$

This is our equation to determine  $\chi$ ,  $\eta$  being supposed known. To find  $\eta$  we may remark that

$$\phi\eta = \zeta$$

and

$$\zeta = \widehat{\chi^{-1}\gamma}.$$

But

$$\chi\chi^{-1}\alpha = \alpha,$$

so that

$$\dot{\chi}\chi^{-1}\alpha + \chi\widehat{\chi^{-1}\alpha} = 0.$$

Hence

$$\begin{aligned} \dot{\zeta} &= -\chi^{-1}\dot{\chi}\chi^{-1}\gamma \\ &= -V \cdot \eta\chi^{-1}\gamma = V\zeta\eta = V \cdot \zeta\phi^{-1}\zeta, \end{aligned}$$

or

$$\phi\dot{\eta} = -V\eta\phi\eta.$$

These are the equations we obtained before. Having found  $\eta$  from the last we have to find  $\chi$  from the condition

$$\chi^{-1}\dot{\chi}\alpha = V\eta\alpha.$$

**40.** We might, however, have eliminated  $\eta$  so as to obtain an equation containing  $\chi$  alone, and corresponding to that of § 21. For this purpose we have

$$\eta = \phi^{-1}\zeta = \phi^{-1}\chi^{-1}\gamma,$$

so that, finally,

$$\chi^{-1}\dot{\chi}\alpha = V \cdot \phi^{-1}\chi^{-1}\gamma\alpha,$$

or

$$\widehat{\chi^{-1}\alpha} = V \cdot \chi^{-1}\alpha\phi^{-1}\chi^{-1}\gamma,$$

which may easily be formed from the preceding equation by putting  $\chi^{-1}\alpha$  for  $\alpha$ , and attending to the value of  $\widehat{\chi^{-1}}$  given in last section.

**41.** We have given this process, though really a disguised form of that in §§ 19, 21, and though the final equations to which it leads are not quite so easily attacked in the way of integration as those there arrived at, mainly to show how free a use we can make of symbolic functional operators in quaternions without risk of error. It would be very interesting, however, to have the problem worked out afresh from this point of view by the help of the old analytical methods: as several new forms of long-known equations, and some useful transformations, would certainly be obtained.

**42.** As a verification, let us now try to pass from the final equation, in  $\chi$  alone, of § 40 to that of § 21 in  $q$  alone.

We have, obviously,

$$x = q\alpha q^{-1} = \chi\alpha$$

which gives the relation between  $q$  and  $\chi$ .

[It shows, for instance, that, as

$$S . \beta \chi \alpha = S . \alpha \chi' \beta$$

while

$$S . \beta \chi \alpha = S . \beta q \alpha q^{-1} = S . \alpha q^{-1} \beta q ,$$

we have

$$\chi' \beta = q^{-1} \beta q ,$$

and therefore that

$$\chi \chi' \beta = q(q^{-1} \beta q)q^{-1} = \beta ,$$

or

$$\chi' = \chi^{-1} , \text{ as above.}]$$

Differentiating, we have

$$\dot{q} \alpha q^{-1} - q \alpha q^{-1} \dot{q} q^{-1} = \dot{\chi} \alpha .$$

Hence

$$\chi^{-1} \dot{\chi} \alpha = q^{-1} \dot{q} \alpha - \alpha q^{-1} \dot{q}$$

$$= 2V . V(q^{-1} \dot{q}) \alpha .$$

Also

$$\phi^{-1} \chi^{-1} \gamma = \phi^{-1}(q^{-1} \gamma q) ,$$

so that the equation of § 40 becomes

$$2V . V(q^{-1} \dot{q}) \alpha = V . \phi^{-1}(q^{-1} \gamma q) \alpha ,$$

or, as  $\alpha$  may have any value whatever,

$$2V . q^{-1} \dot{q} = \phi^{-1}(q^{-1} \gamma q) ,$$

which, if we put

$$Tq = \text{constant}$$

as was originally assumed, may be written

$$2\dot{q} = q\phi^{-1}(q^{-1}\gamma q)$$

as in § 21.

**43.** Let  $\rho$  be the vector joining the extremity of  $\varepsilon$  to the intersection of  $\gamma$  with the invariable plane. Then

$$\rho + x\gamma = \varepsilon .$$

Operating by  $S . \gamma$ , and remembering the condition

$$S\varepsilon\gamma = -h^2 ,$$

we have

$$x\gamma^2 = -h^2 ;$$

so that

$$\rho = \varepsilon + \frac{h^2}{\gamma^2} \gamma .$$

In the initial position of the body this vector, considered as being drawn from the fixed point, was

$$\begin{aligned} \sigma &= q^{-1} \varepsilon q + \frac{h^2}{\gamma^2} q^{-1} \gamma q \\ &= \eta + \frac{h^2}{\gamma^2} \zeta , \\ &= \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right) \zeta . \end{aligned}$$

In the initial position of the body, therefore, this vector passes through the intersection of the ellipsoid

$$S \cdot \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right)^{-1} \sigma \phi^{-1} \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right)^{-1} \sigma = S\zeta\eta = -h^2,$$

with a second ellipsoid

$$T \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right)^{-1} \sigma = T\zeta = T\gamma.$$

It therefore lies on the cone

$$\gamma^2 S \cdot \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right)^{-1} \sigma \phi^{-1} \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right)^{-1} \sigma + h^2 S \cdot \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right)^{-1} \sigma \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right)^{-1} \sigma = 0,$$

or

$$S \cdot \sigma \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right)^{-1} \sigma = 0.$$

[We might have saved the last seven lines by noticing that

$$S\gamma\eta = 0$$

in the present position of the body, involves

$$S\zeta\sigma = 0$$

in the initial state, which, with the value of  $\zeta$  in terms of  $\sigma$  above, gives the result at once.]

44. This cone is seen at once to be normal to the  $\zeta$ -cone in the initial body, viz., by § 26,

$$S \cdot \zeta \phi^{-1} \zeta = -\frac{h^2}{\gamma^2} \zeta^2,$$

or

$$S \cdot \zeta \left( \phi^{-1} + \frac{h^2}{\gamma^2} \right) \zeta = 0.*$$

The vector  $\sigma$  constantly changes so as to be perpendicular to  $\zeta$ . Hence in the

\* In fact any equation such as

$$S_g \psi_g = 0,$$

where  $\psi$  is a constant self-conjugate linear and vector function, gives

$$S \psi_g d_g = 0,$$

whence

$$\nu = \psi_g$$

where  $\nu$  represents the normal-vector. For its locus, we have

$$g = \psi^{-1} \nu,$$

and by substitution for  $g$  and  $\psi_g$  in the given equation, we have

$$S \nu \psi^{-1} \nu = 0.$$

moving body, the vector  $\rho$ , which is always in the plane through the fixed point and perpendicular to  $\gamma$ , belongs to a cone of which  $\gamma$  is a normal, and which therefore *rolls* on that plane. But the cone also *slides*, because the vector  $\rho$  which is in contact with the plane is not the instantaneous axis of the body. This construction for the illustration of the motion is also due to POINSON, and the complete analytical solution of the problem has been given, from this point of view, by RUEB and JACOBI.\* It is easy to see that the angular velocity of the sliding motion is the *constant* resolved angular velocity of the body about the fixed line  $\gamma$ , which has the value

$$-S \cdot \epsilon U \gamma = \frac{h^2}{T \gamma}.$$

45. When two of the moments of inertia of the rigid body are equal, *i.e.*, when the symbolical cubic in  $\phi$  or  $\varphi$  has two equal roots, all the previous dynamical work becomes immensely simplified. In fact, if we now take  $\alpha, \beta, \gamma$  as unit-vectors coinciding with the principal axes of the moving body, we have by (23)

$$\varphi \xi = -A \alpha S \alpha \xi - B \beta S \beta \xi - C \gamma S \gamma \xi.$$

But

$$\xi = -\alpha S \alpha \xi - \beta S \beta \xi - \gamma S \gamma \xi,$$

so that

$$\varphi \xi = B \xi - (A - B) \alpha S \alpha \xi \dots \dots \dots (35),$$

and thus depends upon the position of the *one* vector  $\alpha$ . We may attempt to determine the motion without at first introducing the consideration of the quaternion which has been our principal object of study in this paper.

46. The general equation of § 24

$$\varphi \dot{\epsilon} = -V \epsilon \varphi \epsilon$$

becomes, by substituting for  $\varphi$  from (35),

$$B \dot{\epsilon} - (A - B) \alpha S \alpha \dot{\epsilon} = -(A - B) V \alpha \epsilon S \alpha \epsilon \dots \dots \dots (36).$$

Operating by  $S \cdot \alpha$ , we have

$$S \alpha \dot{\epsilon} = 0 \dots \dots \dots (37).$$

Omitting, therefore, this term from (36) and operating by  $S \cdot \epsilon$ , we have

$$S \epsilon \dot{\epsilon} = 0,$$

whose integral is

$$\epsilon^2 = \text{constant} = -\Omega^2, \text{ suppose, } \dots \dots \dots (38).$$

But we have always by § 1

$$\dot{\alpha} = V \epsilon \alpha$$

because  $\alpha$  is fixed in the body.

From this we see that

$$S \epsilon \dot{\alpha} = 0.$$

\* See CAYLEY, *B. A. Report*, 1862.

This, taken in conjunction with (37), gives

$$S\alpha\dot{\varepsilon} + S\varepsilon\dot{\alpha} = 0,$$

whose integral is

$$S\alpha\varepsilon = \text{constant} = -\Omega \cos \beta, \text{ suppose,} \quad . \quad . \quad . \quad (39).$$

Equation (36) may now be written

$$B\dot{\varepsilon} = -(A-B)\Omega\dot{\alpha} \cos \beta,$$

or

$$B\varepsilon = -(A-B)\Omega\alpha \cos \beta + \text{constant vector.}$$

But we have always, by (14), (see § 24)

$$\varphi\varepsilon = \gamma,$$

or by, (35), (36), (39),

$$B\varepsilon + (A-B)\alpha\Omega \cos \beta = \gamma \quad . \quad . \quad . \quad (40).$$

So that the constant vector is  $\gamma$ .

Thus we see that  $\alpha$  and  $\varepsilon$  are always coplanar with  $\gamma$ , and that each remains constantly at the same inclination to it.

47. Operating on (40) by  $S \cdot \varepsilon$ ,  $S \cdot \alpha$ ,  $S \cdot \gamma$ , respectively, we have

$$\begin{aligned} -B\Omega^2 - (A-B)\Omega^2 \cos^2 \beta &= -h^2, \\ -B\Omega \cos \beta - (A-B)\Omega \cos \beta &= S\alpha\gamma, \\ -Bh^2 + (A-B)S\alpha\gamma \Omega \cos \beta &= \gamma^2, \end{aligned}$$

and these give, in order,

$$\begin{aligned} (A \cos^2 \beta + B \sin^2 \beta)\Omega^2 &= h^2, \\ -A\Omega \cos \beta &= S\alpha\gamma, \\ -(A^2 \cos^2 \beta + B^2 \sin^2 \beta)\Omega^2 &= \gamma^2. \end{aligned}$$

The first and third determine  $\beta$  and  $\Omega$  in terms of the given constants  $h$  and  $T\gamma$ , and the second gives the value of the constant inclination of  $\alpha$  to the fixed line  $\gamma$ .

Introducing  $-\alpha^2$ , which is unity, as a multiplier of  $\gamma^2$  in the third equation, and adding to its members the squares of the corresponding members of the second, we have

$$-B^2\Omega^2 \sin^2 \beta = V^2 \alpha\gamma.$$

48. We get equations immediately derivable from these by seeking at once the equations of the fixed and rolling cones, by which the motion may be exhibited. Thus the locus of  $\varepsilon$  in the body, *i.e.*, the rolling cone, has by (14) and (38) the equation

$$\Omega T\varphi\varepsilon = T\gamma T\varepsilon,$$

which may be transformed as follows—

$$\Omega^2 \{B^2 \varepsilon^2 - 2B(A-B)S^2 \alpha \varepsilon - (A-B)^2 S^2 \alpha \varepsilon\} = -\gamma^2 \varepsilon^2,$$

$$\Omega^2 (B^2 \varepsilon^2 - (A^2 - B^2)S^2 \alpha \varepsilon) = -\gamma^2 \varepsilon^2,$$

$$(B^2 \Omega^2 + \gamma^2) \varepsilon^2 - (A^2 - B^2) \Omega^2 S^2 \alpha \varepsilon = 0,$$

and finally

$$\varepsilon^2 \cos^2 \beta + S^2 \alpha \varepsilon = 0.$$

This might have been written down at once by inspection of (38) and (39).

The locus of  $\varepsilon$  in space, *i.e.*, the fixed cone, has the equation

$$S^2 \gamma \varepsilon + \frac{h^4}{\Omega^2} \varepsilon^2 = 0.$$

**49.** In the preceding solution we began with the very simple equation for  $\varepsilon$ , which immediately presented itself. Let us now apply to the same problem the general equation of § 21, *viz.*,

$$2\dot{q} = q\phi^{-1}(q^{-1}\gamma q).$$

Here, of course, we have

$$\begin{aligned} \phi^{-1}\varepsilon &= -\frac{1}{A} iS i\varepsilon - \frac{1}{B} jS j\varepsilon - \frac{1}{B} kS k\varepsilon, \\ &= \left(\frac{1}{B} - \frac{1}{A}\right) iS i\varepsilon + \frac{\varepsilon}{B}. \end{aligned}$$

Hence

$$2\dot{q} = q \left\{ \left(\frac{1}{B} - \frac{1}{A}\right) iS \cdot i q^{-1} \gamma q + \frac{1}{B} q^{-1} \gamma q \right\}$$

which, because

$$\alpha = q i q^{-1},$$

becomes

$$\varepsilon = 2\dot{q}q^{-1} = \left(\frac{1}{B} - \frac{1}{A}\right) \alpha S \alpha \gamma + \frac{1}{B} \gamma$$

which is (40) of § 46, as we see by substituting for  $S\alpha\gamma$  from § 47.

**50.** Employing this value of  $\varepsilon$  in the kinetic equation

$$\dot{\alpha} = V\varepsilon\alpha,$$

we have

$$\dot{\alpha} = -\frac{1}{B} V\gamma\alpha.$$

Hence

$$\begin{aligned} \ddot{\alpha} &= -\frac{1}{B} V\gamma\dot{\alpha} = \frac{1}{B^2} V \cdot \gamma V\gamma\alpha \\ &= \frac{\gamma^2}{B^2} \alpha - \frac{\gamma}{B^2} S\alpha\gamma, \end{aligned}$$

of which the integral is obviously

$$\alpha = \gamma^{-1} S\alpha\gamma + x \cos \frac{T\gamma}{B} t + \lambda \sin \frac{T\gamma}{B} t,$$

where  $\kappa$  and  $\lambda$  are vector constants of integration.

The two last terms must be, together, equal to

$$\gamma^{-1} V\gamma\alpha,$$

and, as they vanish alternately, the tensors of  $\kappa$  and  $\lambda$  must be equal. Also unless

$$Sx\lambda = 0$$

the tensor of this part of  $\alpha$  will vary. Hence

$$\alpha = -U\gamma S\alpha U\gamma + TV\alpha U\gamma \cdot \left( Ux \cos \frac{T\gamma}{B} t + U\lambda \sin \frac{T\gamma}{B} t \right).$$

Let us, for simplicity, take the usual  $i, j, k$  of quaternions as coinciding with  $U\gamma, U\kappa, U\lambda$ , and let

$$-S\alpha U\gamma = \cos \beta.$$

Then

$$TV\alpha U\gamma = \sin \beta.$$

Also let

$$\frac{T\gamma}{B} = n.$$

Thus we have

$$\alpha = i \cos \beta + (j \cos nt + k \sin nt) \sin \beta$$

whence

$$2\dot{q}q^{-1} = -\left(\frac{1}{B} - \frac{1}{A}\right)nB \cos \beta [i \cos \beta + (j \cos nt + k \sin nt) \sin \beta] + ni$$

$$= 2ai + 2b(j \cos nt + k \sin nt),$$

where

$$2b = -nB \left(\frac{1}{B} - \frac{1}{A}\right) \cos \beta \sin \beta$$

$$2a = -nB \left(\frac{1}{B} - \frac{1}{A}\right) \cos^2 \beta + n = n \left( \sin^2 \beta + \frac{B}{A} \cos^2 \beta \right).$$

51. For the complete solution of the problem, it remains that we integrate the equation above, which we may write as

$$\begin{aligned} \dot{q} &= [ai + b(j \cos nt + k \sin nt)] q \\ &= (ai + b\varpi) q \quad \dots \dots \dots (41), \end{aligned}$$

if we put

$$\varpi = j \cos nt + k \sin nt.$$

This gives at once the following results, which are necessary in the elimination of  $\varpi$  by differentiation,

$$\begin{aligned} \varpi^2 &= -1, & \dot{\varpi} &= ni\varpi, \\ \varpi\dot{\varpi} &= ni, & i\dot{\varpi} &= -n\varpi, \\ \ddot{\varpi} &= -n^2\varpi. \end{aligned}$$

Also, because

$$Si\varpi = 0,$$

we have

$$(ai + b\varpi)^2 = -(a^2 + b^2).$$

Differentiating (41), and simplifying at every step by the above auxiliary equations, we have

$$\begin{aligned} \dot{q} &= (ai + b\varpi)q \\ \ddot{q} &= -(a^2 + b^2)q + b\dot{\varpi}q \\ \dot{\ddot{q}} &= -(a^2 + b^2)\dot{q} - bn^2\varpi q + bn(a\varpi - bi)q \\ \ddot{\ddot{q}} &= -(a^2 + b^2)\ddot{q} - (bn^2 - bna)\dot{\varpi}q + (bn^2 - bna)\left(\frac{a}{n}\dot{\varpi} + b\right)q - b^2n\left(-a + \frac{b}{n}\dot{\varpi}\right)q \\ &= -(a^2 + b^2)\ddot{q} - (bn^2 - 2bna + ba^2 + b^3)\dot{\varpi}q + b^2n^2q. \end{aligned}$$

Eliminating  $\dot{\varpi}q$  from the last equation by means of the second, we have for the determination of  $q$  the linear equation of the fourth order with constant coefficients

$$\ddot{\ddot{q}} + [2(a^2 + b^2) + n^2 - 2na]\ddot{q} + [(a^2 + b^2)^2 + (a^2 + b^2)(n^2 - 2na) - b^2n^2]q = 0 \quad (42).$$

Assume, as a particular integral,

$$q = Q\epsilon^{mt},$$

where  $Q$  is an arbitrary, but constant, quaternion, and  $\epsilon$  is the base of NAPIER'S Logarithms. Then we find for  $m$  the equation

$$m^4 + [2(a^2 + b^2) + n^2 - 2na]m^2 + (a^2 + b^2 - na)^2 = 0,$$

or

$$m^2 + a^2 + b^2 - na = \pm \sqrt{-m^2n^2}.$$

Hence  $m$  is imaginary, so we may write

$$m = \mu\sqrt{-1},$$

and our equation gives

$$\mu^2 \pm \mu n = a^2 + b^2 - na,$$

whence

$$\mu = \pm \frac{n}{2} \pm \sqrt{\left(a - \frac{n}{2}\right)^2 + b^2}.$$

By § 50 this may be written

$$\mu = \pm \frac{T\gamma}{2B} \left\{ 1 \pm \left(1 - \frac{B}{A}\right) \cos \beta \right\} \dots \dots \dots (43).$$

These values may be called  $\pm \mu_1$ ,  $\pm \mu_2$ , and we have

$$\mu_1 + \mu_2 = n.$$

52. The complete solution of the equation (42) is therefore

$$q = Q_1 \cos \mu_1 t + Q_2 \sin \mu_1 t + Q_3 \cos \mu_2 t + Q_4 \sin \mu_2 t.$$

This, however, is far too general for the solution of the original problem, for it involves *sixteen* arbitrary constants instead of *four*. But it is a mere piece of ordinary analysis to find twelve of these in terms of the other four.

Thus, let us write

$$Q_1 = H_1 + I_1 i + J_1 j + K_1 k,$$

$$Q_2 = H_2 + I_2 i + J_2 j + K_2 k,$$

$$Q_3 = H_3 + I_3 i + J_3 j + K_3 k,$$

$$Q_4 = H_4 + I_4 i + J_4 j + K_4 k.$$

If these values be substituted in the above expression for  $q$ , and the resulting value of  $q$  be used in the equation

$$\dot{q} = [ai + b(j \cos nt + k \sin nt)]q,$$

we find, on replacing *products* of sines and cosines of multiples of  $t$  by *sums* of sines or cosines, two sets of terms. One of these is of the type

$$\cos (n - \mu_1)t,$$

which, being equal to

$$\cos \mu_2 t,$$

may be allowed to remain in the equation. The other set is of the type

$$\cos (n + \mu_1)t,$$

and the terms introducing it must vanish identically.

This consideration gives us the following relations among the sixteen constants above

$$I_2 = H_1, H_2 = -I_1, J_2 = -K_1, K_2 = J_1,$$

$$I_4 = H_3, H_4 = -I_3, J_4 = -K_3, K_4 = J_3;$$

so that the values of eight are assigned in terms of the remainder.

Next, by equating coefficients of each such distinct term as

$$i \cos \mu_1 t, k \sin \mu_2 t, \text{ \&c.},$$

we obtain sixteen additional equations, of which, however, eight are mere repetitions of the other eight. Rejecting them, we find the remainder to be

$$\begin{aligned}
 bH_3 &= (a - \mu_1) K_1 & bK_1 &= - (a - \mu_2) H_3 \\
 bI_3 &= (a - \mu_1) J_1 & bJ_1 &= - (a - \mu_2) I_3 \\
 bJ_3 &= - (a - \mu_1) I_1 & bI_1 &= (a - \mu_2) J_3 \\
 bK_3 &= - (a - \mu_1) H_1 & bH_1 &= (a - \mu_2) K_3
 \end{aligned}$$

These are, again, identical in pairs; for each pair containing the same two constants agrees with the others in giving

$$\frac{a - \mu_1}{-b} = \frac{b}{a - \mu_2},$$

or

$$a^2 + b^2 - (\mu_1 + \mu_2)a + \mu_1\mu_2 = 0.$$

But, by (43), we have

$$\begin{aligned}
 \mu_1 + \mu_2 &= n \\
 \mu_1\mu_2 &= \frac{n^2}{4} - \left(a - \frac{n}{2}\right)^2 - b^2
 \end{aligned}$$

and the condition is satisfied identically.

The final value of the quaternion in the case of the uniform rolling of one right cone on another is therefore

$$\begin{aligned}
 q &= (H_1 + I_1 i + J_1 j + K_1 k) \cos \mu_1 t \\
 &\quad - (I_1 - H_1 i + K_1 j - J_1 k) \sin \mu_1 t \\
 &\quad + \frac{a - \mu_1}{b} (K_1 + J_1 i - I_1 j - H_1 k) \cos \mu_2 t \\
 &\quad - \frac{a - \mu_1}{b} (J_1 - K_1 i - H_1 j + I_1 k) \sin \mu_2 t. *
 \end{aligned}$$

Putting

$$q = w + ix + jy + kz,$$

the ordinary differential equations, corresponding to that just solved, are

\* The tensor of  $q$  has been assumed constant. Accordingly we find by this formula

$$\begin{aligned}
 & \left[ H_1 \cos \mu_1 t - I_1 \sin \mu_1 t + \frac{a - \mu_1}{b} (K_1 \cos \mu_2 t - J_1 \sin \mu_2 t) \right]^2 + \left[ I_1 \cos \mu_1 t + H_1 \sin \mu_1 t + \frac{a - \mu_1}{b} (J_1 \cos \mu_2 t + K_1 \sin \mu_2 t) \right]^2 \\
 & + \left[ J_1 \cos \mu_1 t - K_1 \sin \mu_1 t - \frac{a - \mu_1}{b} (I_1 \cos \mu_2 t - H_1 \sin \mu_2 t) \right]^2 + \left[ K_1 \cos \mu_1 t + J_1 \sin \mu_1 t - \frac{a - \mu_1}{b} (H_1 \cos \mu_2 t + I_1 \sin \mu_2 t) \right]^2 \\
 & = (H_1^2 + I_1^2 + J_1^2 + K_1^2) \left[ 1 + \left( \frac{a - \mu_1}{b} \right)^2 \right] \\
 & = (H_1^2 + I_1^2 + J_1^2 + K_1^2) \left( 1 - \frac{a - \mu_1}{a - \mu_2} \right).
 \end{aligned}$$

$$\begin{aligned} \dot{w} &= -ax - by \cos nt - bz \sin nt, \\ \dot{x} &= aw + bz \cos nt - by \sin nt, \\ \dot{y} &= bw \cos nt + bx \sin nt - az, \\ \dot{z} &= bw \sin nt + ay - bx \cos nt. \end{aligned}$$

By substitution in these the above result may be verified.

**53.** Consider, as an example of applied forces, a homogeneous solid of revolution moving about a fixed point in its axis, which is not its centre of gravity. To determine the motion.

If  $\alpha$ , a unit-vector, represent at time  $t$  the position of the axis of the solid, we may choose the tensor of  $\gamma$ , a vertical vector, so that the couple due to gravity is  $V\alpha\gamma$ . Hence the equation of motion is §§ 24, 22,

$$\varphi\dot{\varepsilon} + V\varepsilon\varphi\varepsilon = V\alpha\gamma.$$

But

$$\varphi\varepsilon = B\varepsilon - (A - B)\alpha S\alpha\varepsilon,$$

so that

$$B\dot{\varepsilon} - (A - B)\alpha S\alpha\dot{\varepsilon} - (A - B)V\varepsilon\alpha S\alpha\varepsilon = V\alpha\gamma \quad . \quad . \quad . \quad (44).$$

This, with the kinematical relation

$$\dot{\alpha} = V\varepsilon\alpha \quad . \quad (1),$$

contains the complete solution of the problem.

**54.** Operating on (44) by  $S \cdot \alpha$ , we have

$$S\alpha\dot{\varepsilon} = 0.$$

But, by (1), we have

$$S\dot{\alpha}\varepsilon = 0.$$

Hence

$$S\alpha\varepsilon = \text{constant} = \Omega \quad . \quad (45)$$

(that is, the angular velocity about the axis of revolution of the solid is constant) and (44) is reduced to the form

$$B\dot{\varepsilon} - (A - B)\Omega\dot{\alpha} = V\alpha\gamma \quad . \quad (46).$$

But, by (45) and (1),

$$\varepsilon\alpha = \Omega + \dot{\alpha},$$

or

$$\varepsilon = -\Omega\alpha + \alpha\dot{\alpha} \quad . \quad (47).$$

Since  $\alpha\dot{\alpha}$  is a vector, we have (as in § 30)

$$S\alpha\ddot{\alpha} = -\dot{\alpha}^2 \quad . \quad (48),$$



57. From (49) we have at once, by operating by  $S \cdot \gamma$  and integrating,

$$B S \cdot \gamma \dot{\alpha} \dot{\alpha} = A \Omega S \gamma \alpha + C' \quad . \quad . \quad . \quad (52).$$

Also, operating by  $S \cdot V \gamma \alpha$ ,

$$B S \cdot \gamma \alpha V \alpha \ddot{\alpha} = A \Omega S \cdot \gamma \alpha \dot{\alpha} - (V \alpha \gamma)^2 \quad . \quad . \quad . \quad (53),$$

or

$$\begin{aligned} B (-S \gamma \ddot{\alpha} - S \gamma \alpha S \alpha \ddot{\alpha}) &= A \Omega S \cdot \gamma \alpha \dot{\alpha} + \alpha^2 \gamma^2 - S^2 \alpha \gamma \\ &= \frac{A^2 \Omega^2}{B} S \gamma \alpha + \frac{A \Omega C'}{B} - \gamma^2 - S^2 \alpha \gamma, \end{aligned}$$

by (52).

This may be written

$$B \left[ -S \gamma \ddot{\alpha} - S \gamma \alpha \left( -\Omega^2 - \frac{2S \gamma \alpha + C}{B} \right) \right] = \frac{A^2 \Omega^2}{B} S \gamma \alpha + \frac{A \Omega C'}{B} - \gamma^2 - S^2 \alpha \gamma,$$

which leads, by integration, to the ordinary expression for  $S \gamma \alpha$  in terms of an elliptic function. It is to be observed, however, that this quantity is not one which the quaternion calculus directly points out as an object of research; the propriety of seeking  $\alpha$  in the first place being clearly indicated.

58. From the above equations all the ordinary results connected with this problem may be at once deduced by any one who has a little skill in quaternion analysis: but the determination of the quaternion which gives the position of the body at any time does not appear, so far as I have yet examined the question, to lead to any very simple expressions.

If we could, generally, integrate equation (49),  $\epsilon$  would be at once given by (47), and the determination of the motion would be reduced to comparative simplicity. The equation for the direct determination of  $\epsilon$  may be formed as follows, but it is not so simple as that for  $\alpha$ .

From the equation

$$B \dot{\epsilon} - (A - B) \Omega V \epsilon \alpha = V \alpha \gamma,$$

we have, by operating by  $V \cdot \epsilon$ , the result

$$B V \epsilon \dot{\epsilon} - (A - B) \Omega (\alpha \epsilon^2 - \epsilon \Omega) = \Omega \gamma - \alpha S \gamma \epsilon,$$

which gives

$$\alpha = \frac{B V \epsilon \dot{\epsilon} + (A - B) \Omega^2 \epsilon - \Omega \gamma}{(A - B) \Omega \epsilon^2 - S \gamma \epsilon}.$$

The condition

$$\dot{\alpha} = V \epsilon \alpha$$

gives, by substituting this value of  $\alpha$ ,

$$\begin{aligned} B V \epsilon \ddot{\epsilon} + (A - B) \Omega^2 \dot{\epsilon} - \frac{B V \epsilon \dot{\epsilon} + (A - B) \Omega^2 \epsilon - \Omega \gamma}{(A - B) \Omega \epsilon^2 - S \gamma \epsilon} (2(A - B) \Omega S \epsilon \dot{\epsilon} - S \gamma \dot{\epsilon}) \\ = B (\dot{\epsilon} \epsilon^2 - \epsilon S \dot{\epsilon} \epsilon) - \Omega V \epsilon \gamma. \end{aligned}$$

59. Processes very similar to these may be applied to the motions of the Gyroscope and to Precession and Nutation. I confine myself at present to the formation of the equation for the latter question, reserving for another communication the *details* of the solutions of these three problems; as they involve some curious and delicate points of quaternion analysis.

60. *To form the equation for Precession and Nutation.* Let  $a$  be the vector, from the centre of inertia of the earth, to a particle  $m$  of its mass: and let  $\rho$  be the vector of the disturbing body, whose mass is  $M$ . The vector-couple produced is evidently

$$\begin{aligned} & M\Sigma . mV . a \frac{U(\rho - a)}{T^2(\rho - a)} \\ &= M\Sigma . m \frac{V\alpha\rho}{T^3(\rho - a)} \\ &= M\Sigma . \frac{mV\alpha\rho}{T^3\rho} \frac{1}{\left(1 + \frac{2S\alpha\rho}{T^2\rho} + \frac{T^2\alpha}{T^2\rho}\right)^{\frac{3}{2}}} \\ &= M\Sigma . \frac{mV\alpha\rho}{T^3\rho} \left(1 - \frac{3S\alpha\rho}{T^2\rho} + \&c.\right), \end{aligned}$$

no farther terms being necessary, since  $\frac{T\alpha}{T\rho}$  is always small in the actual cases presented in nature. But, because  $a$  is measured from the centre of inertia,

$$\Sigma . ma = 0 .$$

Also, as in § 19,

$$\phi\rho = \Sigma . m(\alpha S\alpha\rho - a^2\rho) .$$

Thus the vector-couple required is

$$\frac{3M}{T^5\rho} V . \rho\phi\rho .$$

Referred to co-ordinates moving with the body,  $\phi$  becomes  $\varphi$  as in § 24, and § 24 gives

$$\varphi\varepsilon = \gamma = 3M \int \frac{V . \rho\varphi\rho}{T^5\rho} dt .$$

Introducing the value of  $\varphi$  from § 53—*i.e.*, assuming that the earth has two principal axes of equal moment of inertia, we have

$$B\varepsilon - (A - B)\alpha S\alpha\varepsilon = 3M(A - B) \int \frac{V\alpha\rho S\alpha\rho}{T^5\rho} dt .$$

This gives, as in § 54,

$$S\alpha\varepsilon = \text{const.} = \Omega ,$$

whence

$$\varepsilon = -\Omega\alpha + \alpha\dot{\alpha} ,$$

so that, finally,

$$BV_{\alpha\ddot{\alpha}} - A\Omega\dot{\alpha} = \frac{3M}{T^2g} (A - B)S_{\alpha g}V_{\alpha g}.$$

The most striking peculiarity of this equation is that the *form* of the solution is entirely changed, not modified as in ordinary cases of disturbed motion, according to the nature of the value of  $\rho$ .

Thus, when the right hand side vanishes, we have the equation (49) with the restriction that the body moves about its centre of inertia (easily seen to be identical with that at the beginning of § 50); which, in the case of the earth, would represent the rolling of a cone fixed in the earth on one fixed in space, the angles of *both* being exceedingly small.

If  $\rho$  be finite, but constant, we have a case nearly the same as that of the top in §§ 53, 54, the axis on the whole revolving conically about  $\rho$ .

But if we assume the expression

$$\rho = r(j \cos mt + k \sin mt)$$

(which represents a circular orbit described with uniform velocity)  $\alpha$  revolves on the whole conically about the vector  $i$ , perpendicular to the plane in which  $\rho$  lies.

I hope, on a future occasion, to give detailed solutions of these problems, to a sufficient degree of approximation.



IX.—*On the Structure of the British Nemerteans, and some New British Annelids.*

By W. CARMICHAEL M'INTOSH, M.D., F.L.S., Murthly, Perthshire. Communicated by Professor TURNER. (Plates IV.-XVI.)

(Read 20th April 1868.)

The anatomy of the soft worms variously arranged under the Nemertean Order has, even in recent times, not been carried out with that completeness necessary for their thorough elucidation, a state of matters partly due to the confounding of the structure of one family with another, and predicating of the series what investigation has but proved in one group. Few British comparative anatomists have paid much attention to these animals; indeed, Dr GEORGE JOHNSTON,\* Mr HARRY GOODSIR,† and Dr THOMAS WILLIAMS,‡ are the only three who have left researches of any moment on the subject. The observations of the first-mentioned naturalist were made many years ago, with the aid of inferior instruments, and, though conscientious enough, are very meagre and unsatisfactory; and those of Dr WILLIAMS, while also showing the defects just noted, bear evident traces of imagination. Mr H. GOODSIR'S interpretation of structures was, from his limited observations, likewise very erroneous. On the Continent, again, the investigators have been more numerous, and a long list of distinguished names attest the interest which the subject has received at their hands. I do not deem it necessary on the present occasion to enumerate the older writers at full length, since this has already been accomplished very satisfactorily by MM. DE QUATREFAGES§ and KEFERSTEIN,|| but shall refer to such of their views under the respective heads as may be required for the complete elucidation of the subject. Of those, however, who led the way to a more correct appreciation of the structure of these animals, I may particularise MM. DUGÈS,¶ BLANCHARD,\*\* and DE QUATREFAGES,§ in France; EHRENBERG,†† RATHKE,‡‡ MAX SCHULTZE,§§ and KEFER-

\* Mag. Zool. and Bot. vol. i. 1837; and Catalogue of Worms, 1865.

† Annals N. Hist. xv. 1845.

‡ Report Brit. Assoc. 1851.

§ Annales des Soc. Nat. 3<sup>me</sup> ser. vi. 1846; and Voyage en Sicilie, vol. ii. par MM. EDWARDS, DE QUATREFAGES, and BLANCHARD.

|| Zeitschrift für wiss. Zool. xii. 1863.

¶ Annales des Sc. nat. tom. xxi. 1830.

\*\* Annales des Sc. nat. 3<sup>me</sup> ser. xii. 1849.

†† Symbolæ Physicæ, 1831.

‡‡ Neueste Schrift. der Naturforsch. in Danzig, 1842.

§§ Beiträge zur Naturg. der Turbellarien, 1851; and Zeitsch. für wiss. Zool. iv. 1853, &c.

STEIN,\* in Germany; ÆRSTED,† in Denmark; VAN BENEDEN,‡ in Belgium; CLAPARÈDE,§ in Switzerland; and DELLE CHIAJE,|| in Italy.

The confusion in regard to the structural characteristics of the order is well illustrated in the descriptions given in the lately published "Catalogue of the British Museum," and in the first chapter of Dr COBBOLD's "Entozoa," where little else than an array of doubts is produced as a solution of this question. In France, again, the valuable Lectures of M. MILNE EDWARDS,¶ for instance, are chiefly of interest on the subject of the Nemertean as stimulants for further investigation. An examination of the discrepancies existing between the comparatively recent and excellent researches of MM. DE QUATREFAGES,\*\* MAX SCHULTZE, CLAPARÈDE, VAN BENEDEN, and KEFERSTEIN, demonstrate the same necessity for further elucidation. MM. DE QUATREFAGES, VAN BENEDEN, and KEFERSTEIN have, perhaps, gone more minutely than the others into the question, but all have confounded the structure, or certain parts of the structure, of the Ommatopleans with the Borlasians, whether one or both groups have been examined. M. DE QUATREFAGES investigated the Ommatoplean group more extensively than the Borlasian; while Prof. KEFERSTEIN paid more attention to the latter; but he has not entered so minutely into structural detail as the former, though his observations are, on the whole, more exact. VAN BENEDEN likewise predicated of one group what he had found in the other, and hence sometimes gave an erroneous interpretation of the parts. While thus reviewing the labours of these distinguished naturalists, it must not be understood that I in the least degree undervalue their investigations; but rather, that from a more continued series of observations, structures—about which they were in doubt—have been more clearly determined, and many additional facts brought to light. Indeed, no one who is acquainted with the patience and experience necessary for unravelling the anatomy of these delicate creatures, will wonder at the occurrence of errors of omission or commission, either in the labours of others or his own. Ever restless when alive, prone to rapid dissolution when dead or too much pressed, and comparatively few of the requisite transparency for examination, it is only by a happy combination of circumstances that the structure of these animals can be successfully demonstrated.

One of the main objects of this paper is to show the essential differences between the Ommatopleans and the Borlasians, from the skin even to the micro-

\* Zeitschrift für wiss. Zool. xii. 1863.

† Entwurf einer Syst., &c., der Plattwürmer, 1844.

‡ Memoires des Sc. des Acad. Roy. de Belgique, tom. xxxii. 1861.

§ Recherches Anatomiques, &c., dans les Hebrides, 1861.

¶ Mem. sulle Storia, &c., vol. ii. Naples, 1825.

¶¶ Leçons sur la Physiol. et l'Anat. Comparée, tom. 5<sup>me</sup>, pp. 460-65.

\*\* In his "General Outline of the Animal Kingdom," 3d edit. 1861, Professor RYMER JONES strictly follows this author.

scopic structure of the proboscis, as well as to advance our knowledge of the minute anatomy of these animals and their immediate allies.

I use the terms Ommatopleans and Borlasians provisionally in the meantime, because the majority of these soft animals group themselves round two centres, represented respectively by the common *Ommatoplea alba* and *Borlasia olivacea*. The terms, indeed, are nearly equivalent to MAX SCHULTZE'S *Enopla* and *Anopla*, and to Prof. KEFERSTEIN'S *Tremacephalidæ* and *Rhynchmocephalidæ*. I do not think it advisable to call by the name of Borlasia, as the last-mentioned author has done, a family whose structure is quite different from that of the animal originally so termed, and hence I have preferred EHRENBERG'S name, *Ommatoplea*, on the one hand, and substituted *Borlasia* for KEFERSTEIN'S *Nemertes*, on the other, both because it (*Borlasia*) has the priority, was applied to an animal similar in structure, and because there are strong claims to perpetuate the name of the early English zoologist. So comprehensive are the above terms, that almost in every minute particular all the known British forms, with the exception of *Cephalothrix* and another, resolve themselves at once under their respective heads.

*Ommatoplea alba* (and variety *rosea*) may, as above mentioned, be conveniently taken as the type of the Ommatoplean group, both from its size and abundance, and accordingly a systematic examination of its anatomy shall first engage our attention, the additional observations made on its immediate allies being appended and contrasted therewith. It is also fair to state, that I could not have pursued the following inquiries if a liberal and ever-ready supply of living animals from the St Andrews' rocks had not been perseveringly forwarded by a relative, to whom I owe the deepest obligations in this and other departments of zoology.

*Dermal Tissues*.—The body of the animal, like that of each in the Order, is universally covered with cilia, some longer ones being present at the proboscidian aperture and mouth, and others at the tip of the tail. The ciliary motion is most active at the openings of the cephalic pits. In *Tetrastemma variegatum*, it is interesting to watch the cilia at the anterior end, especially around the aperture of the proboscis, as the long cilia bend outwards and inwards with a less rapid motion than the shorter. Those at the posterior end cause a complete vortex, the longer cilia often remaining quiescent. The granules in the surrounding water are directed by the cilia of the sides of the tail towards the tip, where, after coming in contact, the two opposing currents dash outwards, frequently again to curve round, and cause their granules to come under the action of the lateral cilia. The whole appearances very much resemble the currents of water in a vessel after the application of heat. This action would be of little service to an animal whose posterior end was quite closed. The cilia, as long known, perform a respiratory function; at least there exist no other special organs for the purpose.

The skin is somewhat opaque, and presents a cellular or cellulo-granular appearance. In a small living specimen it is represented as a transparent object in Plate IV. fig. 8, the entire field being definitely covered with glandular cells, and the reddish pigment grouped here and there in varying and irregular granular masses. On snipping a portion of skin from an adult living specimen, and placing it under moderate pressure (Plate IV. fig. 3), it presents the aspect of a series of ovate or spathulate cells, which contain soft and minutely granular contents, interspersed with large clear masses of mucus (like oil) of a somewhat similar figure, the latter becoming more numerous as the pressure increases. There are also numerous pigment and other granules scattered over the field. Changes, however, rapidly ensue in this delicate texture, as noted by M. DE QUATREFAGES, both in this group and in Planaria, and the masses of mucus pass rapidly to the nearest free border and there accumulate, the granular contents of the cells following a similar course, but not coalescing. Some of these free globules are shown in Plate IV. fig. 7, *a* being the granular masses, and *b* a group of mucous globules like oil. The former structures, though very mobile, are less so than the latter. A transparent gelatinous basis-substance, often of a reticulated aspect, remains after the extrusion of the foregoing elements from the skin.

When a transverse section is made of an animal hardened in spirit and mounted in chloride of calcium, the appearance of the dermal textures (Plate IV. fig. 2) is as follows:—In rapidly prepared and newly mounted specimens, a structureless film is sometimes observed to separate from the exterior of the skin, as indicated by the double line at the edge of the figure. Chloride of calcium would seem to destroy this delicate structure, as after a time it becomes indistinct, and I have not seen it in those hardened in chromic acid. The cellular cutis (*a*) is found to have undergone an alteration, being streaked perpendicularly, an appearance due to the collapsed state of the areolæ and cells, whose contents to a greater or less degree have escaped, and thus given greater prominence to the hyaline intercellular substance. It is granular throughout, and rather more so towards the outer and inner edges. In most of the transverse sections, the pressure of the cover has caused flattening of the skin, so that the increased cellular appearance of the outer edge is partly due to the fact that the texture is seen from the surface, and not laterally. Towards the inner edge, the skin in this state sometimes assumes a crenate aspect, and adjoins a pale and structureless basis-layer (*b*), which separates it from the subjacent muscular walls of the body. In longitudinal sections of the textures, especially in those much hardened or slightly exposed to air, spurious annulations are caused by the folding inwards or wrinkling of the skin, but such crenations do not affect the muscular layers, and have no connection with the segmentation of the digestive chamber, or true annuli. A thin longitudinal section from the surface of the skin shows a series of meshes with crenated edges, the size of the spaces being variable. In *Omma-*

*topea purpurea* and *O. gracilis* the cells of the skin are much smaller than in *O. alba*. In *O. gracilis*, indeed, the skin resembles microscopic mosaic work, from the granules and plaits in each space or cell.

The function of this elaborate glandular arrangement is doubtless the secretion of the abundant mucus so characteristic of these animals, and which is often of a most tenacious description. I have seen a specimen rapidly form a sandy investment by this means, when placed in a vessel containing a little sand; and whether the sand particles simply adhered to the gelatinous mucus by accident or not, the animal took full advantage of the protection. The same habit is extensively followed by the Ommatopleans of our southern shores, apparently to protect themselves from the increased danger of desiccation. On placing a living specimen on a glass slip, and causing it to emit some mucus, the secretion proved to be a minutely granular fluid, intermingled with a few larger corpuscles. The silky sheaths formed by *Tetrastemma variegatum* and others are well-known examples of this cutaneous secretion. The tube constructed by *Polia involuta*, VAN BEN.,\* is the densest yet seen, and it has an areolar aspect, from the granules or globules being set in a hyaline matrix, sometimes at considerable intervals from each other. Moreover, when viewed in profile, these globules are found to be elevated above the external surface, like a series of low pale warts. M. BENEDEN says it is simply tessellated. The tube is attached to the hairs of the abdominal feet of female crabs (*C. mœnas*) bearing ova, and is evidently of intrinsic importance to the species, both as a protection against injury and desiccation. That some of the characters of this group of worms are due to the thick and soft cutaneous layers is demonstrated by the appearance which they present when such are removed, as by improper preservation. Two specimens of *O. pulchra*, dredged off the Hebrides by Mr JEFFREYS, were in this condition; and as the proboscis had been thrown off in the one first examined, it appeared like a new type of non-bristled worms, characterised by the simple arrangement of its digestive system, and its glistening and elastic investment, so different from the dull, whitish, and non-elastic covering of an ordinary preparation.† Another interesting feature in regard to the skin of the Ommatopleans (in common with the Borlasians), is the reaction which ensues on testing with litmus-paper. In this group an acid reaction occurs in *O. alba*, *O. melanocephala*, and *O. gracilis*; while, on the other hand, a reaction not less distinctly alkaline characterises *O. purpurea* and *O. pulchra*.

M. DE QUATREFAGES' description of the tegumentary structures differs materially from that just given, a discrepancy arising partly from his confounding the

\* *Nemertes carcinophilus*, Kölliker.

† The comparison of the external tissues of certain remarkable processes, occurring on a new Annelid from the Gulf of Suez, to the Nemertean skin, as described by M. LE DR LÉON VAILLANT in the "Ann. des Sc. nat" for 1865, is certainly far fetched and unlikely. The processes referred to are considered buds, but they seem to me to be no more buds or parasites, than the processes on the long tentacles of our British *Mœa mirabilis*.

structure of *Ommatoplea* (his *Polia*) with *Borlasia*, and partly from incorrect observations. He refers to the cells or areolæ of the integument as "simples vacuoles ovoïdes ou arrondies," which refract light, takes no note of their contents, and apparently considers them empty. His separation of the skin into two layers, the exterior composed of smaller, the interior of larger cells, is not evident in *Ommatoplea*. Smaller cells sometimes do occur towards the ciliated surface, but the entire integument-proper is continuous as a single layer. The only representative of his "fibrous" layer, which is described as lying within the former, is our structureless basement-layer. Dr SCHULTZE\* figures a small portion of the skin of his *Tetrastemma obscurum*, showing a series of large cells under the epidermis, with a few granular bodies interspersed, but the view is diagrammatic. Prof. KEFERSTEIN'S observations on the cutaneous and muscular structures apply almost entirely to our *Borlasians*.

*Muscular Layers of the Body.*—A very distinct belt of circular muscular fibres (Plate IV. fig. 2, *c*) occurs next the basement-layer of the cutis. They (the fibres) are compact throughout, and less bulky than the next coat, with which their filaments do not mix. The succeeding layer (*d*) forms a powerful wall of longitudinal muscular fibres, which, in transverse sections, is generally somewhat crenated on its inner border, and fasciculated throughout. The interfascicular substance is transparent and structureless, and evidently as mobile and contractile as the fibres themselves. Numerous fibrous bands stretch from the inner surface in connection with the various contents of the body. The muscular coats in *Tetrastemma* are formed on the same plan as the foregoing. The appearances of these muscles in transverse section resemble those recently given by Professor KÖLLIKER of the muscles in crabs.† Thus there are only two distinct muscular coats of non-striated fibres around the body of the *Ommatopleans*, making an essential difference in this respect between them and the *Borlasians*, to which (latter) previous observers have for the most part confined their investigations.

M. DE QUATREFAGES describes the muscular coats both in *Borlasia* and *Nemertes* (specially instancing *Nemertes balmea*, our *Ommatoplea gracilis*), as consisting of "external longitudinal and internal transverse" fibres. In *Ommatoplea*, as just described, it is exactly the reverse, the circular fibres being external, and the longitudinal internal. He also speaks of another layer, within the internal, as forming an aponeurosis, apparently referring to the fibrous prolongations from the internal or longitudinal coat.‡ Thus Sig. DELLE CHIAJE, instead of being in error, as averred by M. DE QUATREFAGES, is correct in stating that the external coat is circular, and the internal longitudinal. Physiologically, it is certainly a

\* Beiträge zur Naturges. der Turbellarien, tab. vi. fig. 4.

† Zeitsch. für wiss. Zool. bd. xvi. 1866, p. 375

‡ VAN DER HOEVEN, apparently from following M. DE QUATREFAGES, makes the same errors—*Handbuch der Zoologie*, vol. i. p. 212.

better arrangement for such an animal, which has only two muscular coats, to have the longitudinal fibres internal, for, on the occurrence of rupture, they, as well as the other tissues, are constricted by the circular; whereas, in the supposed arrangement of M. DE QUATREFAGES, the longitudinal are beyond the reach of the constricting belt. Other organs also in the same animal, such as the proboscidian sheath and long posterior gland, have their circular fibres exterior to the longitudinal. The actions of this muscular system are very varied, and include swimming or floating on the surface of the water, an action performed, as in the Nudi-branchiate mollusca, by aid of the mucous exudation, and not, as stated by M. DE QUATREFAGES, chiefly by the cilia.

Anteriorly the body-wall terminates in a rounded snout—of the usual cutaneous textures, presenting in transverse section an areolar and granular appearance, the soft contents of the areolæ having for the most part escaped. The aperture for the proboscis lies at the ventral border of such a section. Somewhat behind this, but yet in front of the ganglia, a remarkable interlacement of fibres (Plate IV. fig. 1), occupying almost the entire cephalic region, occurs. Powerful bands of fibres (1) pass below both the buccal cavity and the tube for the proboscis, meet, and cross each other in an oblique manner, forming afterwards, by their divergence, extensive lateral connections; indeed, it will be observed, that towards the inner muscular layer the fibres just mentioned form a broad fan-shaped arrangement. Some of the fibres (2) pass upwards by the side of the central canal, and mingle with those descending from this region; while others (3) curve downwards to the ventral wall. The fibres (4) that meet above the central canal cross each other obliquely in the middle line, so as to form a firm arch; and, besides, there are some transverse fibres (5) that cross over the canal, and spread out on each side. Other bands of fibres (6) slant downwards and inwards on each side of the cavity, and meet inferiorly. The arrangement of these bands and fibres is so intricate, that each seems to blend with the other, and form a continuous anastomosis of contractile meshes. In addition to these oblique and radiating fibres, there is a powerful series of longitudinal fibres interwoven with them in an intricate manner, besides the denser grouping (*e*) at the margin (which indicates the inner muscular coat of the body), and the glandular masses in the centre. It will be observed that the bands which pass beneath the central canal are the most powerful, and afford a much greater resistance to the bulging of the proboscis and its sheath than the superior fibres, so that in extrusion the organ is mainly directed upwards. This will be understood by referring to Plate IV. fig. 5, which represents a section of an animal which had protruded a small portion of its proboscis after chloroforming and immersion in spirit. The inferior commissure of the ganglion is thus somewhat protected by the arrangement of the fibres in front of it. The blood-vessel (Plate IV. fig. 1, *l*) lies on each side in a sheltered position, in an angle between two series of fibres; and its calibre

would not seem to be much interfered with except in extreme protrusion of the proboscis. All the oblique or transverse fibres are connected with the body-wall and the inner muscular layer, as are also the longitudinal at the tip of the snout.

This elaborate interlacement provides in the best possible manner for the varied changes which this region undergoes during protrusion and retraction of the proboscis, and the ordinary motions of a tactile and mobile, yet not too yielding snout. The arrangement of the oblique and circular fibres around the longitudinal layer of the central canal also must act the part of a constrictor, and adapt the cavity to its ever-varying calibre. On the whole, the stroma in this group, from the greater predominance of granular elements, is less dense than in *Borlasia*, and the interlacing of the fibres, though not more complex, is more beautiful, because possessing greater distinctness and regularity.

The posterior end of the body has no such intricate arrangement, but the muscular fibres blend together at the tip and close in the cavity, with the exception of the small and sometimes indistinct opening of the great longitudinal digestive chamber. The modes of fracture of these muscular coats in some of the Ommatopleans in a sick and perishing condition are interesting, the body being separated into a number of beads from the constriction and rupture of the body-wall at somewhat regular intervals.

My observations would lead me to follow a different arrangement in the description of the cavities within the body-wall, from that pursued by MM. DE QUATREFAGES, KEFERSTEIN, and VAN BENEDEN, since there exist some differences as regards interpretation of structures. Instead of speaking of a "general cavity of the body," I shall first refer to that chamber in which the proboscis lies, and which may be termed the cavity of the proboscidian sheath.

*Cavity of Proboscidian Sheath.*—In *Ommatoplea alba* as well as in *Tetrastemma*, this chamber commences just in front of the ganglionic commissures, and continues without interruption nearly to the posterior end of the worm. It is recognised in the living animal under the lens, or even with the naked eye, as that forming a pale dorsal streak, and containing a transparent fluid. The commencement of the chamber is shown in Plate VI. fig. 1, where a fold (*a*) from the tube of the proboscis becomes attached to the parenchyma of the head, or where, instead of a canal (*ab*) simply hollowed out in the tissues of the head, free and distinct walls to the proboscis become apparent. This reflection is the anterior boundary of the proboscidian sheath under ordinary circumstances, and it is against this obstruction that the wave of proboscidian fluid first impinges in the evolution of the proboscis. The cavity gradually increases in diameter, and again diminishes towards the posterior end, where it terminates in a distinct *cul-de-sac*, a short distance in front of the tail. Its general appearance, when viewed from above, as a transparent object, is seen in Plate VI. figs. 3 and 8, but it varies much according to the position, degree of extension or contraction of the

animal, sometimes almost clasping the elongated proboscis, at others being attenuated over the doubled organ.

The various transverse sections of the worms also render the relations of the cavity more apparent. Like the proboscis, its anterior end passes through the ring formed by the arching of the superior commissure, the inferior commissure, and the sides of the ganglia. The nervous matter must thus occasionally undergo very great stretching, or else the proboscis is rarely launched out. This will be more particularly noticed in the description of the ganglia, and a reference to Plate IV. fig. 5, will suffice in the present instance. The inferior commissure separates it entirely from the chamber of the great ciliated œsophagus. The relation of the parts in the ganglionic region is represented in Plate V. fig. 1, *o* being the wall of the proboscidian sheath somewhat compressed, so as to show both longitudinal and circular fibres; for it may be mentioned, that the structure of the chamber wall is powerfully muscular, as evinced by its ever-varying condition. At this point, however, the fibres have not attained a great degree of development. In a section made further back (as in Plate V. fig. 2, *o*), and in the other transverse sections, this muscularity is more distinctly exhibited, though, of course, the spirit has shrivelled all the parts, especially the muscular. Externally the wall of the chamber is furnished with a layer of circular, and internally with a series of longitudinal fibres, both becoming thinner posteriorly. The comparatively large size of the cavity during life has doubtless caused several observers to err, by confounding it with the supposed general cavity of the body. The presence of ova or sperm-sacs has a considerable influence in modifying the size of the chamber, which in the ripe animal is pressed upwards and towards the median line, while in the spawned worm it expands freely in all directions. It is a mistake, however, to suppose, with M. DE QUATREFAGES, that no cavity exists posteriorly in the ripe animal, for this chamber holds the same anatomical relations from the ganglia to the tail as at other seasons, only its calibre is encroached on posteriorly, and the consequent distention by the proboscis and fluid makes it more conspicuous in front. The chamber is absent in the aberrant form *Polia involuta*, VAN BENEDEN.

In the foregoing cavity the proboscis floats in a clear fluid, rich in large flattened discs, which have a minutely granular appearance. In the living animal, these generally have a fusiform outline, from a slight thickening in the middle (Plate IV. fig. 9, *b*). They are accompanied by certain granules and globules, which are also represented in this figure. The discs vary in size, and adhere together in a dying animal very easily, from the highly coagulable nature of the transparent fluid in which they float; and occasionally fibrinous shreds may be observed attached to them under the same circumstances. The fluid, indeed, is highly organised, and very different from sea-water, to which Dr T. WILLIAMS compares it. When the proboscis has been gently protruded under chloroform,

the discs in the interspace may by-and-by be seen grouping together, so as to form stellate bodies, resembling miniature solasters, spiked bodies like thorn apples, flattened structures with pectinate ends, and various other forms. In *O. melan-cephala* the discs are comparatively small, some being clear, spindle-shaped bodies, others granular and rounded. The enormous increase of cells and granular masses in the proboscidian fluid, after the discarding of a proboscis, is well seen in this species. In *Tetrastemma* the discs (Plate IV. fig. 14), though similar in shape to those of *O. alba*, are comparatively large; and in a variety of *T. varicolor*, which I am at present inclined to regard as the *Polia sanguirubra* of M. DE QUATREFAGES, they are tinged pinkish or reddish by transmitted light (Plate IV. fig. 11). They are not all similarly tinted, some being pale, others yellowish, while many are bright red—the colour in all cases being in the nuclei. Circular bodies and granules are present, as in Ommatoplea. The skin of this specimen contained many minute reddish pigment specks, so that to the naked eye it had a delicate salmon-pink appearance. Reddish granular masses occasionally occur in the proboscidian chamber of *O. alba*, and in other species of *Tetrastemma*, generally associated with reddish specks in the skin; and it is curious that a cast-off proboscis in *T. algæ*, and other species, assumes the same hue by transmitted light. With the foregoing exceptions, the only changes noticed in the colour of the discs were those caused by refraction of the rays of light. After extrusion into the water, their shape soon alters, and they adhere together, and become translucent.

M. CÆRSTED\* gives a small figure of a transverse section of his *Notospermus flaccidus*, and characterises the proboscidian cavity as “canalis in quo penis est,” indicating by a blank beneath what might have been the digestive tract. His interpretation of structures, however, is more distinct in his section explanatory of the Family Amphiporina,† in which the digestive cavity is correctly alluded to.

The reflection of the walls of the proboscis before-mentioned, in front of the ganglionic commissures, is the only barrier (and a very effectual one) I have observed separating the proboscidian chamber from the tissues of the head. In no species examined has such a cephalic diaphragm as described by M. DE QUATREFAGES been found; but the peculiar ciliated chamber or œsophagus, to be described hereafter, takes its place, and leads one to infer that the distinguished naturalist has misinterpreted the structure. Besides, the head is not a hollow organ, requiring such definition from the other parts of the body. This author, while explaining a transverse section through *Nemertes Borlasii*‡ (vel *Borlasia Angliæ*), shows a canal surrounding the proboscis; but in his description he confounds it with the general cavity of the body, and figures (fig. 5 same plate) the proboscis as

\* Entwurf einer syst. Eintheilung, &c. der Plattwürmer, tab. iii. fig. 51.

† Entwurf &c., p. 94, fig. 18 (woodcut).

‡ Recherches Anat. and Zool. vol. ii. pl. xviii. fig. 4.

occupying the centre of the general cavity posteriorly. This description, no doubt, refers to a Borlasian; but he states that the same arrangement occurs in the Ommatopleans, and represents in *Polia*\* a series of transverse fibres as forming a platform (*plancher*) at the anterior and upper portion of the general cavity of the body, indicating its presence in his figures by a dark shading. No such arrangement of transverse fibres has been seen by me, but the characteristic ciliated œsophagus occupies this situation, and has probably misled the observer. The somewhat erroneous views he entertained with respect to the relations of the corpusculated fluid of the proboscidian chamber may be seen by a glance at one of his figures,† which depicts in *Polia sanguirubra* the proboscidian bodies as floating in what he calls the genital cavity, and in which the genital cæca are supposed to lie. I cannot corroborate his statement that these discs become much more numerous at the epoch of reproductive activity. The diminished size of the chamber may cause a slight crowding anteriorly, but this is not an increase. He did not recognise the complete muscular sheath for the proboscis and the proboscidian fluid. Dr JOHNSTON likewise confounded the cavity-proper of the proboscis with the general cavity of the body; and Dr WILLIAMS,‡ who styled the canal the œsophageal intestine, stated that it opened externally on the side of the body, not far from the head, after the manner of the Sipunculidæ. M. VAN BENEDEN,§ however, alludes to the sheath for the proboscis in *Polia obscura*, and compares the fluid and discs therein to pale blood. Professor KEFERSTEIN,|| again, follows the majority of his predecessors, in so far as he also describes the proboscidian discs as floating in the general cavity of the body, in which, moreover, he locates the proboscis (Rüssel); thus ignoring the special and complete muscular sheath just described.

The structure of the proboscidian discs, and the highly organised condition of the transparent liquid in which they float, point them out as being, in all probability, concerned in nutrition, as first mentioned by M. DE QUATREFAGES, though he likewise associated generation therewith. Some very interesting questions, however, are raised by their entire absence in the curious *Polia involuta*, VAN BEN., especially to those who, like the late Dr WILLIAMS, consider the fluid analogous to the peritoneal or perivisceral fluid in the true Annelids—a fluid, we may remark, which Professor HUXLEY¶ considers as the true blood, while he thinks the red fluid in the branching vessels analogous to the water vascular system in the Annuloida. If in *Polia involuta* the proboscidian fluid had been more important in nutrition than that in the vessels, it certainly would not have given way to the latter. It is to be remembered, too, that this absence coincides with the

\* *Op. cit.* fig. 1, pl. xviii., and fig. 1, pl. xix.

† Report, Brit. Assoc. 1851.

‡ *Zeitsch. für wiss. Zool.* xii. pp. 69 and 71.

¶ Notes of Lectures at the Roy. Coll. Surgeons, *Med. Times and Gaz.*, March 7, 1868.

† *Op. cit.* pl. xxii. fig. 1.

§ *Op. cit.* p. 26.

atrophied condition of the proboscis itself and all its apparatus. It cannot be affirmed, also, of the Nemerteans, that the fluid in the so-called blood-vessels is devoid of corpuscles, for they occur in several species. Again, I think there can be no doubt the fluid and discs exercise a very important influence on the reproduction of the proboscis, a process hereafter to be described, as well as promote the absorption of the debris of the discarded organ when it happens to be included in the chamber. But while thus affirming the fluid has a certain influence on, and bears a certain relation to, the development of the proboscis, it cannot be said to be indispensable for the appearance of the latter, since there is a small proboscis in *P. involuta*, where the fluid is altogether absent. The views of Dr THOMAS WILLIAMS in regard to this corpusculated liquid, which he termed the "chylaqueous fluid," are so much at variance with accuracy, that I cannot pass them over in silence. He says—"In the case of the Borlasiadæ, Planariadæ, and Liniadæ, the chylaqueous fluid is contained in the digestive cæca and diverticula. In some of the Planariadæ, however, I have proved that a space does actually exist between the digestive diverticula and the solid structure of the body, *which is lined by a vibratile epithelium*, and into which probably the external water is in some way admitted. By this water, thus situated, the contents of the digestive cæca are aërated. The fluid oscillating in these cæcal appendages of the stomach is thickly charged with corpuscles, which, from their regular character, prove this fluid to have already reached a high standard of organisation. They occur as elliptical cells in the *Borlasia* from which the illustration (fig. 25) was taken; the fluid abounded also in small orbicular points, constituting the 'molecular basis' of the digestive product. In this worm, it is this fluid, and not the true blood, that is aërated; the latter system is too little developed."\* The above clearly shows that he was quite unaware that the so-called "elliptical cells" are always confined within the cavity of the proboscidian sheath, as well as points out the erroneous notion he entertained of the true digestive tract, which in all cases can readily admit salt water (by mouth or anus), if such is required, but certainly not for the purpose of converting it into "a *vital* organised fluid." The proboscidian fluid and discs, as I have previously shown, are very far removed from sea-water.

In the Ommatopleans, the aperture for the extrusion of the proboscis is situated towards the ventral edge of the tip of the snout, and under favourable circumstances in the living animal, may be seen as a terminal pore, surrounded by a closely set series of radiating lines; as, for instance, when the snout is bent upwards towards the tube of the microscope (Plate IV. fig. 13). It is furnished with longer cilia even in the young animal; and in the adult these (cilia) form, when the lips are slightly pouting, a very pretty arrangement (Plate VI. fig. 1, *a c*), similar to the analogous opening in *Borlasia* (Plate X. fig. 1). The striated ring

\* Phil. Trans. Part ii. 1852, p. 627, pl. xxxii. fig. 25.

surrounding this orifice in transverse sections of the tip of the snout indicates the special muscular coat pertaining thereto. The canal proceeds in a straight line backwards from this aperture to a point in front of the commissures of the ganglia, where it meets the differentiated walls of the proboscis, as shown in Plate VI. fig. 1, *a b*; and the cilia can be traced backwards to this region, but no further. This canal is simply hollowed out in the tissues of the head, and is quite independent of the motions of the proboscis. It is furnished with a series of longitudinal muscular fibres beneath the ciliated mucous surface, and the strong oblique and circular bands (Plate IV. fig. 1) form a very efficient constricting investment. When the proboscis is about to be ejected, it commences to fold over like the turning of the finger of a glove inside out, at the point (Plate VI. fig. 1, *a*) in front of the ganglionic commissures, and not at the tip of the snout, a fact which has escaped previous observers. In withdrawal also, it may be noticed that, towards the conclusion of the process, the last wrinkle of the proboscis glides within the terminal aperture, and is seen slowly passing backwards till this point is reached, when the wrinkle ceases, and the organ is once more in its ordinary condition, any change that afterwards ensues being due to the stretching of the shortened organ backwards—a process of simple elongation. Thus the anterior portion structurally and functionally differs from the succeeding, the walls of the proboscis always intervening between it and the proboscidian fluid.

The attenuated coats of the proboscis curve outwards all round, and become fixed to the walls of the foregoing canal and other cephalic tissues just in front of the ganglia; and so the reflection constitutes the *point d'appui* against which the wave of proboscidian fluid impinges, when the organ is about to be extruded. The thin anterior walls of the proboscis unroll, the terminal canal is distended by a pouch of fluid, and then the organ is rapidly launched forth. To judge from the description and drawings of M. DE QUATREFAGES, the entire force of this liquid would dash against the posterior part of his nerve-ganglia, and the straitened border of his hypothetical “diaphragm” would not pass further forwards. In my specimens, the waves of the proboscidian fluid debouch readily into the yielding anterior canal in front of the commissures, and then externally into the loop of the extruded proboscis. I have never seen the very pretty lozenge-shaped arrangement of muscular bands in the snout, as figured\* by M. DE QUATREFAGES, and whose function, he says, is to dilate the “oral” orifice, and carry the “gullet” forwards; but the elaborate stroma, shown in Plate IV. fig. 1, would amply suffice for this. During the motions of the proboscis, the reflection in front of the ganglia assumes various postures, and it frequently does stretch obliquely forwards and outwards from the tube, especially when that is drawn backwards. On the other hand, when the tube is thrust forwards, the fibres slope forwards and inwards.

Dr JOHNSTON, M. DE QUATREFAGES, and Dr WILLIAMS agreed in considering

\* *Op. cit.* pl. xix. fig. 1.

the terminal aperture the mouth, and indeed it could not be otherwise, since the proboscis was regarded by them as the true alimentary organ. My observations, while leading me to differ from M. VAN BENEDEN and Professor KEFERSTEIN, who aver that the Ommatoplean mouth is situated on the under surface *behind* the ganglia, as in the Borlasians, coincide with the three former only in so far as this anterior opening lies close to the real mouth (communicating with the ciliated sac or œsophagus). Dr MAX SCHULTZE, almost alone amongst foreign authors, seems to have noticed the true position of the mouth in his *Tetrastemma obscurum*. The aperture for the proboscis lies just at the ventral border of the snout, while the mouth forms a slit on the ventral surface immediately behind the former. In this respect, therefore, there is a marked distinction between the Ommatoplea and its allies on the one hand, and Borlasia and Cephalothrix on the other, the mouth in the first group opening quite in front of the ganglia, while in the other it is situated considerably behind the ganglia. Analogy gives no grounds for supposing the proboscis to be the alimentary organ.

I shall divide, for convenience of description, the Ommatoplean proboscis into three regions, viz., the anterior, middle, and posterior. The first (Plate VI. fig. 3, A) comprehends that somewhat cylindrical portion between the reflection in front of the ganglionic commissures and the commencement of the stylet-region—the *trompe* of M. DE QUATREFAGES; the second (B) includes the stylet-region proper and the well-marked swelling of the great muscular sac—the œsophagus of M. DE QUATREFAGES; and the third (C) is represented by the long posterior gland—the *intestin* of M. DE QUATREFAGES.

*Anterior Region of Proboscis.*—From the point of reflection backwards, the proboscis (*trompe*, Rüssel) gradually increases in diameter until its full size is attained. The entire organ is proportionally on a larger scale than in Borlasia, and its anatomy more apparent; though I doubt, even in this group, if we can assign it the ideal office of a vertebral column. The general appearance of the commencement of the organ in *O. alba* is seen in Plate VI. fig. 3, and in *Tetrastemma algæ*, in Plate VIII. fig. 3. At the point of reflection there is sometimes seen a kind of *os*, from the slight turning over of the lips of the organ in the early stage of ejection (Plate VI. fig. 1, *a*). This figure also represents the longitudinal fibres of the proboscis as most conspicuous in this region. Sometimes the organ assumes a twisted position under examination, so as to give the fibres a spiral appearance, and in such a state the structure might fancifully be likened to the spiral arrangement of the muscular fibres in the œsophagus of the higher animals, but the condition is purely accidental. I fear, however, it has led M. DE QUATREFAGES into an erroneous interpretation of the anatomy of the organ in *Polia glauca*,\* which (organ) is described and figured as having regular spiral belts at its commencement.

\* *Op. cit.* plate xx. fig. 3.

The anterior fibres of the proboscis, as further shown in the various transverse sections, and in the ruptured organ when extruded, are chiefly longitudinal, and while the thinness of the coats renders the exact structure of this region in transverse section less distinct, a very definite arrangement is observable as soon as the tube has attained larger proportions. Dr JOHNSTON, indeed, considered the organ to be homogeneous;\* and M. DE QUATREFAGES describes its commencement in *Polia mutabilis* as consisting of two longitudinal muscular coats, separated from each other by a cellular layer, which, he explains, is a provision for enabling these muscular coats to act independently. He also observes, that no circular fibres were seen in this species, in *P. filum*, and some others. In very small specimens of the British examples the transparency of the tissues renders definition of the coats somewhat obscure, especially after mounting in chloride of calcium, but, so far as I have observed, the structure is as follows:—Externally, there is a layer of what appears to be elastic tissue (Plate IV. fig. 4, *g*, Plate V. fig. 4, *g*, &c.). It is more distinctly striated in transverse than in longitudinal sections of the organ, hence it may be inferred that its fibres are chiefly circular in direction, as seen on comparing the last-mentioned figures. Towards its free border, also, certain obscure granular markings observed in the longitudinal section (Plate IV. fig. 4), show that the direction of the external fibres is different from the others; indeed, in some views, the appearance is such as to raise a suspicion of the presence of the cut ends of a few fine circular muscular fibres, the rest being nearly homogeneous. Within this is a somewhat narrow belt of longitudinal muscular fibres (*f*, same figures), which may be termed the *external longitudinal muscular coat*. It consists of pale, unstriped muscular fibres, whose cut ends are seen in Plate V. fig. 4. Intervening between this coat and the other longitudinal layer is a remarkable stratum, the *reticulated* or *beaded layer* (*e*), in the same figures, which in transverse sections (Plate V. fig. 4) assumes a regularly moniliform appearance, from an increase of its constituent substance at certain points. In longitudinal sections, I was for a time puzzled by the appearance of the cut ends of fibres in this layer, as if it had been composed of circular fibres; and a more minute examination showed that such was due to certain intermediate bands which passed between the thicker or beaded portions. If a thin longitudinal slice from the organ in *O. pulchra* is hardened and mounted in chloride of calcium, numerous well-marked homogeneous longitudinal belts are seen at regular intervals, from one end of the anterior region of the proboscis to the other, and between them are many connecting transverse fibres, which pass from each edge of the belt. The cut ends of the fibres in the longitudinal sections have therefore been caused by the knife severing the transverse meshes between two longitudinal belts. Thus the tube is surrounded by a complete investment of this

\* Catalogue Brit. Museum, p. 285.

homogeneous though complex layer, which, doubtless, has its physiological use in the varied movements of the organ. The next layer (*d*, same figures) consists of a strong coat of longitudinal fibres, fully twice as thick as the external longitudinal layer, and which may be termed the *inner longitudinal muscular coat*. In essential structure it resembles the exterior, differing only in bulk. In sections prepared by hardening in alcohol, these fibres, in common with others in this organ, present a much coarser appearance in transverse section than after hardening in chromic acid. It may be mentioned also, that there is a considerable histological difference between these muscular fibres and those in the higher animals, such as absence of nuclei and greater homogeneousness. The fifth layer from without inwards is a strong band of circular fibres (*e*, same figures), the *circular muscular coat*, which forms a counterpoise to the preceding. Lying on the inner side of these fibres is a *basement-layer* of pale translucent texture, best observed in the longitudinal sections (Plate IV. fig. 4), where it is marked *h*. In transverse sections this coat is apt to be confounded with the inner layer of circular fibres, but the distinction between the two is sufficiently apparent in longitudinal sections. It has, on the whole, a cheesy or cartilaginous aspect. Upon this layer rest the peculiar glandular papillæ, which arise from a distinct margin on its inner edge, as indicated at *b* in the last-mentioned figure, where some of the basal streaks of the papillæ are represented. A glance at the other figures will show the relations and proportions of these organs. In the ordinary transverse sections of the proboscis they form *en masse* a somewhat foliated or frilled arrangement, often more strictly symmetrical than the view here given (Plate V. fig. 4). In some contracted specimens they block up the entire cavity, or else a transparent mucous film which has exuded from them does so. The form of the glands in the fresh specimen under pressure is seen in *O. alba* in Plate V. fig. 7, and in *Tetrastemma* in Plate V. figs. 6 and 11. The largest glands are situated some distance in front of the stylets, for towards this region they become smaller, and finally the fundus is clothed only by minute papillæ. In typical examples of *Tetrastemma variegatum* the glandular papillæ are leaf-shaped, and somewhat crenated at the free border, where there is a regularly streaked appearance from the arrangement of the globules. Under pressure they are granular in the interior, and furnished with numerous globular or wedge-shaped mucous masses, that refract the light like oil. Sometimes in *O. alba* they present a coarsely fringed appearance, with large granules in their interior; and when the tube has been turned inside out, they have a villous aspect, the tough mucosity adverted to above projecting in strings from the papillæ under the slightest pressure. I have generally observed also, towards the first portion of the protruded organ, fine motionless processes like cilia projecting from the apices of the glands, and they are probably homologous with the minute spikes which occur on the glands of the posterior region after rupture from pressure.

The foregoing description of the structure of this region differs much from that given by M. DE QUATREFAGES, almost the only author who has entered into the minute anatomy of the Ommatoplean proboscis. He states, like Mr H. GOODSIR,\* that externally the tube is furnished with a series of transverse muscular bridles, which maintain it in position within the body of the worm, and he gives a section of the parts in *Nemertes balmea*, which bears out his description very well; but he did not observe that if such bridles exist, they would have to pass through the muscular sheath in which the proboscis glides, before reaching the body-wall of the animal. Apparently he has not made out the two diverse structures. His minute anatomy of the proboscis is chiefly taken from the examination of *Borlasia Angliæ*, and hence cannot apply in any degree to the Ommatopleans, though he considered it the type of both. He makes out only two muscular layers in the wall of this organ, and though in his section from *B. Angliæ* he indicates "traces de fibres transversales," by a few lines crossing these longitudinal coats, he distinctly observes that they are not apparent in the smaller species. These longitudinal coats are separated, says he, by a transparent homogeneous tissue, which forms a great number of bridles of very elastic fleshy columns, making, in other words, an elastic cellular layer; and he figures this in the before-mentioned section, adding that this lax cellular coat will give the two longitudinal muscular coats that independence of action necessary for the proper performance of their functions. No such cellular layer has been seen in the British species, but between the two longitudinal coats there is found the remarkable reticulated layer. He mentions a transparent homogeneous coat within his longitudinal muscular layer, corresponding to the mucous coat of the higher animals, and adds that the papillæ of the latter are all covered with vibratile cilia. M. DE QUATREFAGES thus describes only four coats, viz., mucous, internal longitudinal, elastic cellular, and external longitudinal; and if the stays or bridles which he notes as connecting the tube to the body-wall be taken into account, it may be surmised that the muscular sheath for the proboscis is included in his reckoning. No cilia are present in this organ. Professor KEFERSTEIN does not enter into the structure of this region in Ommatoplea.

*Middle Region.*—The elongated chamber just described terminates posteriorly in a sort of *cul-de-sac*, into which three small apertures converge—one at each side from the lateral stylet-sacs, and a central one in the pit of the cavity connected with the peculiar reservoir which succeeds.

The walls of the proboscis undergo a considerable change in this region, especially in regard to the deeper layers. Externally there is the investing coat continued from the anterior region on to the commencement of the reservoir (Plate IX. fig. 11), and which has a crenated border in the contracted state of the

\* Ann. Nat. Hist. xv. 1845.

parts, with transverse markings or rugæ; but such an appearance does not of necessity mean that it is composed of circular fibres, for the contraction of the longitudinal layer underneath would cause even a very feebly elastic coat to assume similar markings. The thin subjacent layer of longitudinal fibres is likewise continued to a similar extent on the reservoir-region, and assists in connecting the divisions. These two layers lie exterior to the stylet-sacs.

The structure of the pit or termination of the anterior chamber ( $\eta$ , Plate IX. fig. 3) merits special notice, since it has certain important functions to perform. The large glands of the inner wall gradually diminish in size until the floor is covered only by small, densely arranged, and minutely granular processes, so that the whole forms a somewhat sharply defined border, which in the ordinary state of the parts knuckles backwards all round the central stylet in the manner shown in the figure, becomes firmly bound together so as to constitute a sphincter for the aperture, and gently bending outwards and backwards, is lost in the obscurity of the parts, caused by the external circlet of glands—somewhat behind the anterior termination of the wedge-shaped investment of the sac at the base of the stylet. This floor of the chamber is composed of a series of muscular fibres, whose direction, in the ordinary state of the parts, is outwards and backwards, as shown in the drawing, but which assume various aspects during the motions of the organ. Thus the floor passes from the conical form with the apex directed backwards to that of a transverse platform; and in the everted condition forms a cone whose apex is directed forwards (Plate VI. fig. 2). In the latter position the secure binding of the fibres which knuckle round the central aperture just permits the stylet to project, but no more. The whole arrangement constitutes a large muscular pit with very powerful and mobile walls, capable of many and varied alterations of form. In firm contraction of the region the floor or pit of the cavity is pouted forwards (Plate XII. fig. 9), causing a radiated or slanting appearance of the fibres. A firm constriction of the tube just in front of the stylet-region often takes place, separating the pit of the organ from the more glandular region in front, and causing a double swelling of the parts. Just in front of the stylet-sacs lie some coarse granular glands, which, however, are less conspicuous than in *O. gracilis* and others. Professor KEFERSTEIN\* speaks of this region as having only a longitudinal muscular coat (though the crenated border of the anterior chamber is continued thereon in his figure), and as possessing much pigmentary and granular matter. The latter is not well marked in *Ommatoplea alba* or *Tetrastemma*, as the entire apparatus is either translucent or white; but in certain species, as will hereafter be shown, an increase in the granular matter occurs. The longitudinal fibres of the last-mentioned author end at the posterior border of the stylet-region.

The *Lateral Stylet-Sacs*—poches styligènes, QUATREF., Taschen, KEF., &c.

\* *Op. cit.*

(*v*, Plate IX. fig. 3)—occupy the exterior portion (covered only by the elastic coat and external longitudinal fibres) of the somewhat solid wall of the section immediately succeeding the foregoing cavity, and in some views cause a distinct bulging. They are conspicuous by their aqueous translucency, as well as by the nail-shaped stylets in their interior, though the exact position of their long axes is rather difficult to determine. In ordinary views, when the animal is examined as a transparent object under pressure, their long diameter is antero-posterior, or slightly oblique; but when the worm has been killed and hardened in alcohol, their long diameter is often found to be transverse (Plate V. fig. 5). Each sac is somewhat ovoid in outline, has a thin, transparent, contractile investment (sufficiently tough to prevent the points of the stylets piercing it during the motion of the worm), which lies immediately under the superficial layers of the section, and a duct passing from its central region to communicate with the pit of the anterior chamber of the proboscis. The direction of this duct under ordinary circumstances (*i.e.* when the animal is viewed from above as a transparent object) is forwards and inwards, but, like other structures pertaining to this mobile organ, it is liable to many alterations, and is occasionally much stretched and attenuated. It is also slightly narrowed on approaching the sac, and has at its junction therewith a series of protecting fibres (Plate VI. fig. 9, *a*). MM. DE QUATREFAGES and MAX SCHULTZE do not notice the duct at all, and M. CLAPARÈDE's figure\* shows it distorted from pressure in *Tetrastemma*, but M. KEFERSTEIN's representation is more accurate. Each sac contains a variable number of the characteristic nail-shaped stylets ( $\beta$ ), from three to five, more or less—in different stages of development, as well as certain clear fluid vesicles ( $\xi$ ), globules and granules, and is quite filled by a transparent fluid. The relations of the sac and its contents are shown in the various figures. In *Tetrastemma algæ* I have seen, besides the ordinary stylets, a group of minute crystalline spinets, which had no connection with the clear vesicle of the sac. The stylets very much resemble a lath-nail, and are formed of a translucent calcareous secretion; indeed, they appear like spikes of the purest glass. The head is bulged, rounded at the edges, and somewhat flattened on the top, from which an elongated conical spike proceeds to a sharp apex. The perfect spike or spikes in these sacs are usually about the size of the central stylet, and there are often three or four that can scarcely be distinguished from each other. Besides the perfect spikes, there are some with heads not fully developed, but complete in other respects; others again present the form of simple spikes of various lengths devoid of any head. In some instances the centre both of the head and point of the stylet is granular, while the superficial portion is of the usual homogeneous aspect. These stylets are secreted by the sac, yet I do not think they are *always* developed originally in one of the contained globules, as Dr SCHULTZE says; and this would not signify much, since the entire cavity must

\* Recherches Anat. sur les Annélides, &c. plate v. fig. 6.

act as a secreting chamber, else the large ones could receive no increase after they had outgrown the capacities of the globules. They seem to be formed by gradual increase of layer upon layer of the calcareous glassy secretion, as is well shown in some specimens mounted in chloride of calcium, where they have assumed a stratified or laminated appearance. Sometimes a process (Plate IV. fig. 10), probably a remnant of the globule, passes from the head down the shaft of the spike for a short distance, as indicated by Dr SCHULTZE in *Tetrastemma*,\* though seldom to such an extent in the adult stylet. The knob on the head figured by this author must be rare, and probably represents a casual globule. The stylets are dissolved in weak acetic acid, as first noted by M. DE QUATREFAGES, and are roughened or corroded by strong liquor potassæ.

In a large animal an interesting arrangement of the stylet-sacs occurred on one side, for there were two of nearly equal size, which communicated with each other at one end, so that an interchange of fluid and granular contents took place. Only one had a duct of communication with the anterior chamber of the proboscis. The opposite side had a single sac of the usual formation, containing two large and perfect stylets, and a shorter without a head. On the abnormal side the outer sac (in this view) had two fully formed stylets, a larger and a smaller clear globule, besides some other minute globules and granules; the inner, which possessed the duct of communication, had one stylet as large as the preceding, and fully formed; another somewhat less, but also having a head; a third slender spike of greater length than the latter, but headless; and a fourth, rather more than half the length of the last mentioned. No globule existed in the inner sac. It is interesting to notice the different degrees of perfection of these spikes in relation to what Dr SCHULTZE avers as to their development, viz., that they are the products of the smaller contained vesicles. In the one there were two large globules, and two perfect stylets, yet no trace of a developing spike; in the other there were three completely formed stylets, yet each varied in length; while the long spike without a head was fully as long as the largest in that sac—head included. The stylets in the outer sac were quite as large as the central stylet. Thus at present, though I have often seen spikes inside, and connected with the fluid vesicles, I cannot support Dr SCHULTZE'S notion that the spikes *must* be developed therein. M. CLAPARÈDE says he has never seen the spikes inside those vesicles,† but I observe, in a more recent publication,‡ he figures a developing stylet in a globule in *Prosorhochmus Claparèdii*.

In a specimen that had often been under the microscope, I found on one occasion a pair of stylets, apparently from the lateral sac of one side, advanced nearly to the ganglionic portion of the proboscis. One lateral pouch, as it

\* Beiträge zur Naturges der Turb. tab. vi. fig. 10, a.

† Recherches Anat., &c. p. 79.

‡ Beobachtungen über Anat. und Entwicklung, &c., 1863.

happened, was thus emptied, while the other retained its three stylets. The loose stylets were very slowly moved forwards, scarcely any progress being made during an hour's observation. At this time the sac from which they had been liberated contained numerous granules, but no circular or ovoid vesicle. Twenty-four hours after, the stylets had disappeared. The sac is now observed to be much less than its fellow of the opposite side, and somewhat shrivelled and undefined; but it contains a small ovoid vesicle, which is traversed by a minute slender spike, whose long diameter exceeds that of the globule, and therefore it cannot be supposed to be within it. In addition, there is a free spike about a third the length of the larger one. The former has assumed the shape of a stylet without a head; the latter is as yet nearly cylindrical (Plate VI. fig. 4). Whatever the function of these stylets in the lateral sacs may be, there can be no doubt they have nothing to do with the supply of the central apparatus, for that furnishes its own stylet.

The middle or stylet-region is likewise the seat of other structures of importance, viz., the *central stylet and its basal sac*, the *ejaculatory duct* or canal of communication with the reservoir, and the *circlet of granular glands*. It is of the same vitreous translucency posteriorly as the succeeding region, while both the anterior chamber and the posterior region are of an opaque white in the fresh specimen. Externally there is the investing layer (Plate IX. fig. 3, *g*), continued from the anterior chamber, and which passes backwards to the next region. Beneath this lies a series of very powerful and conspicuous longitudinal muscular fibres (*f*, same plate), apparently to some extent continuous with the more bulky longitudinal layer of the preceding region, but few of which pass on to the next. Internally oblique and radiating fibres occur, the former slanting forwards and outwards from the setting of the central stylet, and forming a kind of muscular sling, well marked in *O. melanocephala* (Plate VI. fig. 7). This layer is distinctly separated at its posterior border from the succeeding region or reservoir by a pale boundary-line under pressure, so that the parts have a somewhat jointed appearance. In transverse section, the complicated structure of this part is well observed (Plate V. fig. 5). The longitudinal fibres form a thick belt exteriorly, and send gradually diminishing bundles inwards towards the central point. This peculiar appearance in transverse section must be due to some difference in the arrangement of the ultimate fibres, as such sections of other muscles usually show a much coarser, more fasciculated, and less granular aspect. There can be no mistake as to the true structure and arrangement of these fibres, since I have cut them both obliquely and transversely in the same specimen. The last-mentioned transverse section also shows a complicated arrangement round the central stylet-apparatus; exteriorly there is a firm setting, next a layer which seems to be closely united with the coat of the ejaculatory duct in front, and other two more immediately connected with the granular sac itself. Some of

these appearances may have been due to the action of the chemicals in mounting, but they were very distinct. The ejaculatory duct has a single ring or coat surrounding it. The exact arrangement of the fibres of this region is difficult to unravel, but some evidently curve across the region, while those at the sides bend backwards, the latter in some views simulating the walls of a cavity. In *Tetrastemma vermiculus* (as a living transparent object) the region has its deep mass formed of fibres which curve outwards and forwards from the central setting (Plate IX. fig. 12). Through this region the ejaculatory duct ( $\mu$ ) passes to the point where it opens into the muscular space behind the constrictor of the central aperture in the floor of the anterior chamber. The aperture of the duct ( $\mu'$ ) is generally obscured by the central stylet-apparatus, unless the observer sees it at the moment of contraction of the powerful muscular walls of the reservoir, when the mucous or villous lining is driven forward so as to render the channel more apparent, and a vigorous jet of the minutely granular fluid is propelled into the muscular sac, and then through the stylet-aperture into the floor of the anterior chamber. Closer observation, even when such convulsive contractions are absent, occasionally shows the minutely granular fluid passing onwards to the anterior chamber; and when the ejaculatory duct is not obscured by the glands, the dancing granules of this peculiar fluid are seen therein. Moreover, when the large compound cells (Plate V. fig. 3) have been detached under pressure, and squeezed forwards into the reservoir and along the duct, the calibre of the opening into the muscular sac may be ascertained with tolerable accuracy, and, so far as I could see, is such that only a single file of cells at a time can be transmitted. The duct has a bent-conical form, a shape that avoids interference with the basal sac of the stylet, which occupies the centre of the region; and its posterior end (that opening into the reservoir) is capable of a certain amount of constriction, as indicated in one of M. CLAPARÈDE'S figures, but I have rarely met with the organ in this position. In the latter state the inner or convex side of the duct is glandular, while the outer or concave side is not. A layer of longitudinal fibres, continued from the reservoir posteriorly, constitutes the proper wall of the tube, and is represented in transverse section in Plate V. fig. 3. Internally the tube has a mucous lining, which anteriorly is for the most part quite free from glandular papillæ; a few small glands, however, are generally observed towards its posterior end. Its wall is not very dilatable, the cavity becoming elongated, but not much increased in diameter, even under violent expansive force. It can be firmly closed by the contraction of the region surrounding it, so as to be marked by a mere central streak (Plate XII. fig. 9,  $\mu$ ). The villous lining of the reservoir is often pushed forwards along the duct during violent contractions. The whole structure of the channel, and its relations to surrounding parts, show that it is formed, not for transmitting fluids from before backwards, but entirely in the opposite direction. The mobile

muscular space ( $\epsilon$ , various figures) into which this duct opens, forms a kind of sac that is occasionally distended with the cells and granules, before they reach, through the central pore, the pit of the anterior chamber.

The cavity or *reservoir* ( $\rho$ , Plate IX. fig. 3, and other figures), from which the duct proceeds, is a somewhat globular or ovoid chamber, with its long diameter for the most part directed transversely; or it may be compared to the bowl of a short and wide wine-glass, the stem being formed by the peculiar channel of communication with the long posterior chamber. It is liable to much variation in shape, from the contractility of its inner wall, independently of the action of the massive exterior muscular investment. Extreme contraction of the region transforms the globular cavity into a mere transverse slit. Its inner surface is provided with a series of glands, the larger and more distinct having minutely granular contents (Plate IX. fig. 3,  $\sigma$ ), and easily distinguished from those of the anterior chamber or long posterior gland. Towards the opening of the ejaculatory duct the glands are smaller than in the swollen part of the reservoir, and they again decrease in size before the organ narrows to its posterior channel of communication. In this comparatively large chamber the dancing granules, hereafter to be described, have free scope for the display of their movements, and not only do they move themselves, but they cause such large bodies as the compound gland-cells from the posterior chamber, when they happen to be present, to revolve and jerk also, a state of matters that has probably helped to mislead M. DE QUATRE-FAGES as to the ciliation of the organ. Such, however, is very distinct from ciliary motion. The reservoir diminishes posteriorly, so as to form in the contracted state of the parts a very narrow duct ( $\phi$ ), which by-and-by expands, and becomes continuous with the long glandular posterior chamber, the whole forming an hour-glass contraction, as represented in the various figures.

Before, however, proceeding with the description of the posterior chamber, it may be as well to complete the narration of the structure of the two translucent regions in which the foregoing duct and cavity lie.

In addition to the ejaculatory duct of the reservoir, the anterior division possesses also the central stylet and its peculiar arrangements, with the external circlet of granular glands. The former projects straight forward in the usual state of the parts, and is generally about the same size as the largest stylet in the lateral pouches, with which it likewise agrees in structure and composition (Plate IX. fig. 3, &c.). Its point under examination seems generally to project into the pit of the anterior chamber, though the thick muscular floor occasionally closes round it. The base of the stylet is fixed to the granular sac ( $\lambda$ ); the arrangement being not inaptly likened by Dr JOHNSTON to an awl, the anterior or smaller end of the sac sending its investing substance over the head of the stylet, and grasping part of the spike. The basal sac (or awl-handle) is narrowed anteriorly, gradually widens backwards, is then marked by a constriction, and

again is terminated by a wider portion, which may represent the butt of the awl. This structure is shorter in proportion to the stylet, and has its constriction placed further backwards than in *Tetrastemma algæ*. The entire sac is opaque white, and coarsely granular from an early age, the granules disappearing with effervescence under the action of weak acetic acid, and rendered paler (in some cases dissolved) by liquor potassæ. These granules would not seem to be simply inclosed in the structure, as if in an ordinary sac, but they adhere together and form a consistent whole, as proved, amongst other things, by their not falling out of the fragment when the anterior part is cast off with the stylet, as will be hereafter described. I have also seen the stylet and its granular basal sac thrown off together in a discarded proboscis in the proboscidian chamber of *O. melanocephala* and other species. This peculiar body or sac is set in a firm wedge of translucent yet compact muscular substance (marked  $\theta$  in the various figures) which often has its posterior border curved in a saddle-shaped manner, projecting backwards in the middle, and with a curve on each side directed forwards. The anterior part of this wedge proceeds about as far forwards as the shoulder of the first swelling of the awl-handle, and there becomes lost on the coat of the latter. Though this generally appears like a wedge of translucent and structureless cartilage, the addition of liquor potassæ and acetic acid shows distinct striæ, chiefly of a transverse character when viewed under pressure, and therefore of a radiating nature with regard to the central granular sac. In front of the wedge-shaped division lies the muscular cavity ( $\epsilon$ , Plate IX. fig. 3), into which the ejaculatory duct opens (at  $\mu'$ ). This cavity is formed by the knuckling outwards of the floor of the anterior chamber all round, and it is furnished with a distinct inner muscular coat. The walls are thus very mobile, and I have seen them form an hour-glass contraction in the middle, quite distinct from the narrowing between the sac (whose greatest diameter is in front) and the firm wedge behind. Its anterior border can be projected to the tip of the central stylet; while in the extruded state of the parts ( $\epsilon$ , Plate VI. fig. 2) it forms, when seen from above, a compressed process at each side of the basal sac of the central stylet; more correctly, however, and if viewed from the front, it has the shape of a muscular umbrella, which slopes all round the anterior portion of the basal sac. M. CLAPARÈDE does not mention this arrangement at all, and M. DE QUATREFAGES seems to have mistaken it for a pair of glands, which, he explains, probably secrete poison for cankering the wounds inflicted by the stylet, a supposition unsupported by any anatomical basis as regards this spot. Prof. KEFERSTEIN'S anatomy of the region also requires correction, since he does not distinguish the separation between this cavity and the floor of the anterior chamber; thus in his representation\* of the extruded proboscis, the central stylet projects smoothly into the water, and the ejaculatory duct opens directly into the latter at a short distance from the stylet.

\* *Op. cit.* tab. v. fig. 3.

A very interesting condition was found in two specimens of *Tetrastemma varicolor*, which directly bears on the physiology of this region. In each a fragment of the granular sac, with the central stylet attached, lay towards the anterior end of the first region of the proboscis; and since injury would scarcely have caused a result so systematic, it is evident the stylet had been thrown off by the animal. In both instances the central stylet-apparatus was complete, only in one the anterior part of the basal sac appeared paler, and there was a slight irregularity in its outline, similar to that in fig. 14, Plate V. In each, the lateral stylet-sacs had their full complement of stylets, one or two of which equalled the central stylet in size. There appears to be only one explanation of this state of matters, viz., the fact that the central stylet can be thrown off, and somewhat rapidly regenerated; for it is unlikely that in each case it found its way there from without, and it is still less likely to have been driven in by an enemy. Former experience in regard to the stylets from the lateral sacs shows that such bodies take some time to gain the exterior of the worm, and hence our surprise is lessened at the perfection of the new structures while the old have not yet escaped from the proboscis. Besides, the structure of the parts in *O. pulchra* will by-and-by throw still farther light on this subject.

Lastly, across this region passes the belt of granular glands ( $\pi$ , various figures), which have the form of lobules, with their long axes parallel to that of the proboscis, and are situated beneath the two external layers of the part. The granules are proportionally larger in *Tetrastemma*. I have not found any structural guide to their function, though they are invariably present in the Ommatopleans. A curious appearance was noticed in a small specimen of *Tetrastemma varicolor*, which had its stylet-region in front of the granular glands covered by an external coating of large cells, with a nucleus and faintly granular contents; such, however, may have been due to an abnormality.

The structure of the next division—that of the great Reservoir—has now to be examined ( $\rho$ , Plate IX. fig. 3). On reaching the point previously mentioned ( $a$ , Plate IX. fig. 11), the elastic coat and the external longitudinal muscular fibres of the proboscis for the most part cease. Before this occurs, however, the muscular fibres ( $\tau$ ) peculiar to the region arise, sweep backwards in a beautiful fan-like manner over the reservoir, loop round and meet those from the opposite side, and leave only a small space in the centre posteriorly, through which the channel of communication with the third region passes. When viewed as a transparent object under pressure, or in longitudinal section, the direction of these fibres is backwards and inwards. This great muscular mass does not receive accessions from the outer wall, but the whole of the loops come from the front. By the varied crossings of these fibres, a felted aspect is produced under examination in some species, such as *O. purpurea* and *O. alba* (Plate IX. fig. 11), and is doubtless present in all. In addition, there are circular and longitudinal fibres

within the former, and to whose presence the independent wrinkles of the inner structures are due. The longitudinal layer ( $\tau_0$ ) is innermost, and forms a kind of spindle-shaped arrangement; the anterior fibres—commencing with the ejaculatory duct (of which they form the special wall)—soon spread out to cover the dilated cavity of the reservoir, then become narrowed as they surround the channel of communication, and proceeding backwards, merge into the longitudinal coat of the posterior chamber. In some positions, these fibres assume a crossed or spiral aspect in the channel of communication; but, as in the case of the ganglionic region of the proboscis, this is purely accidental. The margins of the reservoir and the channel of communication are marked under pressure by the ends of muscular fasciculi, especially posteriorly; an appearance due to the doubling of the looped fibres, but also partly to the presence of the thin circular coat, which lies without the longitudinal. By the contraction of these various fibres, the chamber of the reservoir is squeezed with great force in every direction, like a thick caoutchouc ball or globular syringe in the hand. Its transverse diameter is lessened, and still more, its antero-posterior, while a jet of the minutely granular fluid is squirted into the anterior chamber; and, in spasmodic efforts, even a prolapsus of its glandular lining occurs. In contraction, the entire region is much shortened, and the mass of the looping muscle increased posteriorly. Not only does the peculiar looping of the fibres cause most powerful squeezing of the cavity, but the posterior aperture has a tendency to be closed, and slightly carried forwards, the anterior being less subject to interference. The closing of the posterior aperture (channel of communication) is also greatly assisted by the circular fibres which are situated outside the longitudinal. The varying conditions of the reservoir may be understood by comparing Plate IX. fig. 3 with Plate XII. fig. 9, the former showing the organ in its ordinary state, the latter in a somewhat contracted condition.

The peculiar looping of the fibres of the reservoir causes a transverse section through its posterior part (Plate IX. fig. 10) to assume a finely radiated spiral arrangement, the whole reminding one strongly of Dr PETTIGREW'S beautiful diagrams of the arrangement of the muscular fibres of the heart; and in this case no better structure could have been devised for the complete and forcible evacuation of the chamber. Professor KEFERSTEIN describes only oblique and longitudinal muscular fibres in this region.

*Posterior Region.*—Behind the translucent region just described, the opaque white long posterior chamber (C) (*intestin*, QUATREF., *Drusentheils* of the Germans) occurs. It communicates with the reservoir in front, as previously mentioned, but its posterior end is caecal. The contractile nature of the parts renders comparison uncertain, but it is generally not much shorter than the anterior chamber in the perfect animal. Sometimes, indeed, it exceeds the latter chamber in length, the simpler structure of its walls giving greater extensibility. In young

specimens and in regenerating organs, again, it assumes a nearly globular form in contraction. Externally, it is covered by a very delicate investing layer. Within this lies a series of powerful circular muscular fibres, which towards the tapering posterior end become indistinct, and finally disappear altogether, after the cæcal tip is reached (Plate VII. fig. 4). The next coat is formed of an equally strong series of longitudinal fibres, the anterior or primary ones being continuous with the longitudinal layer of the reservoir, as previously mentioned. These run throughout the entire length of the posterior chamber, becoming proportionally more developed as the central cavity diminishes towards the cæcal end, and finally merging into the muscular ribands which terminate the organ. The mucous layer with its glands lies within the latter, though in several views, both in the living animal and in transverse sections, I fancied some sub-mucous circular fibres were present; they are at any rate insignificant, and the two chief layers explain all the motions which ensue in this division. This mucous layer in contraction of the organ forms many rounded folds, which are especially distinct in *O. gracilis* (Plate IX. fig. 16). A transverse section of the chamber is represented in Plate IX. fig. 14, and the great increase of the glandular mucous layer in contraction is conspicuous. The two muscular coats are about equal in thickness. From the commencement of the region behind the translucent reservoir almost, but not quite, to its cæcal tip, its entire inner surface is covered with a series of glandular papillæ, which differ materially in structure from those of the previous regions. Viewed as a transparent object under moderate pressure (Plate V. fig. 9), the field is found to be covered with globular glands containing clear rounded vesicles in their interior. In contraction, and when the wall is less compressed, the glands have an enlarged and coarse appearance, only the external wall of each being visible. When the pressure has been increased, these glands, especially towards the posterior end (where, from their lessened numbers, a clearer view can be obtained), alter their shape apparently by bursting (fig. 10, same plate), and seem like a double ring of a minutely hirsute aspect, while the contained globules are scattered over the membrane. If the organ has been ruptured and partly inverted, the free edge of the laceration and the shrivelled glands have the appearance shown in fig. 8, same plate. The globules from the glandular papillæ (fig. 3) and glands whose contents have been evacuated (and which are minutely hirsute) readily pass forwards to the reservoir, and roll through the ejaculatory duct under pressure. The function of the vast array of glands in this chamber would seem to be the formation and elaboration of the remarkable fluid with the dancing granules previously alluded to. This secretion is produced in considerable quantities, and towards the posterior portion frequently distends the organ into a translucent pouch (Plate VII. fig. 4, *a*), wherein the moving granules are in full action, and even the experienced are apt to err in regard to the nature of the movements, so like are they to those caused by ciliary currents.

Under a high power (700–1000 diam.), the moving bodies appear as mere specks or points, and they retain this remarkable motion for upwards of twenty-four hours after extrusion from the cavity into the surrounding salt water. There is thus a peculiar fluid rich in these granules secreted by the posterior chamber or gland; and continued observation, and the whole anatomy of the parts, show that this fluid passes forwards into the reservoir, where it is probably mixed with a small quantity of another secretion from the glandular walls of the latter, and then propelled with force through the ejaculatory duct into the anterior chamber. What its peculiar function in the anterior chamber, or when discharged into the surrounding medium in the extruded state of the parts, may be, can only be conjectured at present; but from the elaborate structure of the parts concerned in its economy its action would seem to be important. I have no observations in support of the view that this granular fluid is poisonous. It cannot pass into a wound at any rate until the stylet is withdrawn; and if it really acts as a poison to animals when introduced into their tissues, it might reasonably be supposed to affect them injuriously when discharged into the water around them. Whether the fluid has any influence on the secretion of the stylets in the lateral sacs, or in the central apparatus, I am unable to say; but, as already mentioned, a minutely granular fluid has been seen in the former, and a large though imperfect stylet in the posterior chamber of *O. pulchra*. MM. DE QUATREFAGES, VAN BENEDEN, and others, state that the proboscis and the foregoing apparatus are used in attacking prey; but, we may ask, Do the Borlasians use their feeble and unarmed structure for the same purpose? So far as I have seen, the proboscis is a somewhat precarious aggressive weapon, since it frequently adheres to the attacking body, and is thrown off. It is true we may assign, with an air of probability, an aggressive function to the central stylet; but we cannot do so with the very same organs in the lateral sacs; for, being developed in a free condition within almost closed cavities, they are quite useless as offensive weapons.

In extrusion of the proboscis (Plate VI. fig. 2), the entire spike of the stylet projects, the floor of the anterior chamber forms all round a thick and powerful umbrella-shaped cushion (whose independent structure has escaped Prof. KEFERSTEIN), the lateral stylet-sacs are under cover, and the region of the reservoir is shortened and widened. The position of the muscular chamber ( $\epsilon$ ), which forms a second small umbrella round the apex of the basal sac of the central stylet, has already been mentioned. The separation between the longitudinal fibres of the stylet-region proper ( $\nu$ ) and the looping fibres ( $\tau$ ) of the reservoir is well marked in this condition. It will also be observed that the stylet-region is widened by the forcible wedging forwards of the reservoir.

The walls of the posterior chamber, after forming the *cul-de-sac*, are continued backwards in the form of one or two long translucent muscular ribands of extreme flexibility and contractility ( $\psi$ , fig. 4, Plate VIII.), and which are attached to the

walls of the proboscidian sheath, rather behind the middle of the animal, the fibres spreading out in a fan-shaped manner, and mingling with those of the tube. The motions of these muscular bands is most interesting, now jerking into numerous graceful folds or coils, by a sudden contraction, like the stalk of a *Vorticella*, now shortening more gradually—the curves being thickened here and there by the bulging of the fibrillæ. They are simply muscular fasciculi, with very fine longitudinal lines—the marks of the fibrillæ, and seem to restrain the irregular protrusion of the proboscis and assist in its retraction. This muscular arrangement is also the *ultimum moriens*, showing contractions when all other signs of life have fled. In a young *Tetrastemma variegatum*, in which the riband had been ruptured from its attachment, the fibres (Plate VI. fig. 6,  $\psi$ ) had assumed a clavate aspect from contraction, and only very faint longitudinal markings were visible.

Before reviewing the statements of previous investigators with regard to the general structure of the foregoing parts, a description of the peculiarities of the regions in other species of *Ommatoplea* will be narrated.

In *Ommatoplea melanocephala* (JOHNST.), the proboscis is somewhat larger in proportion than in *O. alba*; and, while the type of structure is adhered to, there are several important differences in detail. The stylet-region (Plate VI. fig. 7) is peculiar in having the lateral stylet-sacs carried considerably forwards, so that they lie quite in front of the central apparatus, and the floor of the anterior region has consequently to form a deep pit to reach the spike of the stylet. In this figure the organ is shown comparatively free from pressure, and the encroachments of the lateral sacs on the cavity may thus be correctly estimated. The basal sac of the central stylet is proportionally large, while its wedge-shaped setting is comparatively meagre. The powerful series of oblique or radiating fibres which pass outwards and forwards from the latter, in the usual position of the organ under pressure, are very distinctly shown, and, as it appears, sling the apparatus. The points of the stylets (central and lateral) are rather blunt, and their shape, on the whole, resembles that found in *Tetrastemma algæ*. Some of the looped muscular fibres of the reservoir seemed to pass inwards beyond the exterior ring in front, so that a continuous series of fibres would thus be formed, as in certain viscera\* (bladder, &c.) of the higher animals, and the chamber environed with the exception of the anterior and posterior openings. The circlet of granular glands is much developed in this species, and often renders the subjacent parts obscure.

The remarks and figures of M. DE QUATREFAGES† relating to this species (his *Polia coronata*) require amendment. He mentions that it is the only exception

\* Vide the admirable Researches of Dr PETTIGREW, *Philos. Trans.* part ii. 1867.

† *Recherches Anat. &c.* p. 166.

he has met with to the uniform arrangement of the stylet-apparatus, as, in addition to the forward position of the lateral stylet-sacs, the central stylet and its surroundings are placed in his second œsophageal cavity—that is, in our reservoir; and his figure\* bears out his description, representing, moreover, the organ as placed at the commencement of the posterior channel. The species is easily identified by the position of the lateral stylet-sacs and other peculiarities, and there is certainly no such abnormality of the central apparatus or alteration of type as noted and figured by this naturalist.

In a very pretty new species—dredged in Lochmaddy—of a salmon hue, striped down the back with two brown and a white central streak, having also a transverse brown bar at the posterior part of the head, and only two eyes,† the stylets were similar in shape to those of *O. melanocephala*, but decidedly smaller.

This shows that while distinctions in size and shape are valuable specifically, they should not be too much relied on.

The anterior chamber in *O. gracilis* (Plate VII. fig. 1) is very short in proportion to the great elongation of the animal, the stylet-region being found only a short distance behind the ganglia; indeed, in this respect, it is not far removed from *Polia involuta*, VAN BENEDEN. The floor of the anterior chamber has generally a bilobed aspect under examination, and hence differs considerably from that of *O. alba*. On each side of the floor in front of the stylet-sacs the end of the proboscis has not the massive muscular structure usually found in this position, but internally has a somewhat opaque mobile lobulated glandular arrangement, which, when the organ is everted, projects as two semi-opaque whitish papillæ (one on each side), the stylet-sacs being sometimes prolapsed into their interior. The central stylet and its apparatus do not easily project in this condition. The stylet-region proper, consisting of that part from the floor of the anterior chamber to the border of the reservoir, is somewhat opaque, on account of the glandular nature of the walls anteriorly, and the layer of granular glands posteriorly. The latter are placed far back, and in developing specimens form an opaque granular mass on each side of the ejaculatory duct, sometimes entirely filling up the angle (at *a*, same fig.), and consist of a dense grouping of minute clear granules, and occasionally coarser particles in lobulated glands, which are apparently homologous with the granular glands of other species. The lateral stylet-sacs have very long ducts, and each encloses from seven to ten stylets of a characteristic shape, besides other contents. The central stylet is appended to a basal sac of great length, the sac indeed resembling the outline of some long bone, such as the radius, the stylet being articulated to the head, while the distal extremity of the bone is represented by the swollen posterior end of the sac. The latter has

\* *Op. cit.* pl. xiii. fig. 8.

† The *Nareda superba* of STIMPSON has likewise two eyes, but has no longitudinal stripes.—*Synopsis Mar. Invert.* of G. MANAN, N. Brunswick, p. 28, fig. 17, 1853.

the usual granular contents, but the exterior firm setting, so characteristically wedge-shaped in other species, does not proceed half-way forwards, the slender anterior portion having only a thin covering for its support, as indicated in the figure. While in ordinary views the stylet and sac seem straight, both have a decided curve when seen laterally (Plate VI. fig. 12). Just in front of the point where the clear setting of the sac becomes indistinct, the ejaculatory duct opens into the peculiarly elongated muscular cavity ( $\epsilon$ ), which extends forwards to the circular opening in the floor of the anterior chamber. This channel shows a distinct inner layer of longitudinal fibres, which, however, seem to act only in company with the external oblique fibres surrounding them. The presence of this special inner coat demonstrates that it is not the mere doubling of the floor of the anterior chamber that forms this cavity, as indeed certain appearances, previously observed, had led me to suspect. The central and lateral stylets have the same shape, and the majority agree in size. In its usual position the stylet has the form of a spear-head (Plate VI. fig. 13), being sharp-pointed, then dilating gradually till near the posterior end, where a slight diminution occurs, and then a marked constriction, just in front of the somewhat small head. If minutely examined, both central and lateral stylets show a small secondary swelling or ring above the head (Plate VII. fig. 9). The ejaculatory duct is comparatively large and boldly marked, comprising at its posterior end almost the entire region of the reservoir, a slight demarcation, however, marking off the dilated posterior end into a portion pertaining to the reservoir, and another to the duct. The widened posterior end is covered with small glands, which are continued along the tube to its opening into the long muscular chamber behind the floor. One peculiarity in the elongated reservoir is the comparative thinness of the looped fibres towards the anterior end, and the thickness of the longitudinal layer, which seems to afford compensation for the diminished strength of the exterior coat. This deviation from the usual structure is doubtless in connection with the enlarged posterior end of the ejaculatory duct, and the gradual continuation of the cavity of the reservoir into it. The bulk of the looped fibres is grouped posteriorly, and in action would seem to compress the reservoir, so as to throw its contents forward to the gaping aperture of the duct. On this account also the posterior channel of communication is long. The external layer, continued from the preceding division, passes about half-way backwards over the reservoir. Another peculiarity is the presence of numerous clear cells and granules amongst the looped fibres, most distinctly seen at the posterior part of the chamber. Some of the cells contain nuclei, and others do not. The glandular papillæ in the interior of the reservoir are large and prominent. The very great length of the posterior chamber as compared with the anterior is remarkable.

M. DE QUATREFAGES seems to have devoted considerable attention to the ana-

tomy of the foregoing species (his *Nemertes balmea*), and his deviations from accuracy, therefore, surprise us. He represents\* the stylet-region as having the lateral sacs placed rather behind the long central granular sac, each of the former having a carunculated gland attached to its posterior end, while the latter has two longer structures of the same description. None of these carunculated appendages have been seen by me, since it can scarcely be supposed he refers to the opaque granular condition of the angle (at *a*, fig. 1, Plate VII.), previously described. His description of the contents of the lateral stylet-sacs is erroneous; for though the position of the stylets is of no moment, the assertion † (and corresponding figure) that each has a developing sac attached to its extremity is very wide of the correct account. The outline of the stylets given by this author is inaccurate, since no constriction is represented in front of the head, and no mention is made of their curvature. The other objections to his views are noticed elsewhere.

The proboscis in the long purple species, *O. purpurea*, while approaching that of *O. gracilis* in slenderness and in tenuity of the posterior region, is yet more closely allied to *O. alba* in the structure of its comparatively short stylet-region proper. The floor of the anterior chamber in this species is furnished with very minute glands. Notwithstanding the great length of the worm, there is no corresponding elongation of the stylets, and the granular basal sac of the central apparatus is likewise short ( $\lambda$ , fig. 2, Plate VII.) The lateral stylet-sacs are small, and somewhat rounded, and their ducts are sometimes spindle-shaped, from marked constrictions situated respectively at the sac and opening into the floor of the anterior chamber. The stylets are at once distinguished by their short, stout form and peculiar longitudinal markings, which resemble the longitudinal streaks in polished mahogany (Plate V. fig. 12), and are due to irregularities in the outline. The granular sac of the central stylet (Plate V. fig. 13,  $\lambda$ ) has only a slight constriction in the middle, so that the lateral line, from the apex of the spike to the base of the sac, is nearly straight. The opening of the ejaculatory duct into the cavity behind the floor of the anterior chamber is wide. The reservoir is much elongated, and it may be observed that its fibres, as pressed between glasses, are not seen in a looping series down the sides of the cavity, but form a densely felted arrangement on each side. When freed from pressure these fibres are observed to cover the reservoir with most elaborate crossings, from the diverse directions which they pursue. In the same region the longitudinal fibres are much developed anteriorly, though they are only well seen on stretching the parts, otherwise the felted arrangement of the looping fibres obscures them. The glands of the reservoir are smaller and less distinct than in *O. gracilis*, especially anteriorly. The channel

\* *Op. cit.* pl. x. fig. 8.

† "Quelquefois, surtout chez le *Némerte balmée*, on aperçoit même un commencement de la tige du stylet."—*Op. cit.* p. 166.

of communication with the posterior chamber is somewhat short and wide, and in marked contrast with the same part in the latter form. The long posterior chamber has its inner surface thrown into more prominent rugæ than in most species, so that they sometimes appear like large papillæ covered with the glands-proper of the cavity. These plaits are not mere wrinkles and folds caused by the contraction of the elongated organ, but are present under severe pressure; indeed, they are characteristic and original processes of the chamber (Plate IX. fig. 16). The granules of the peculiar fluid therein are also very distinct. It may be mentioned here, that after prolonged confinement the integrity of the proboscis in this and other species is affected, the stylets degenerating, and even disappearing altogether, both from the central and lateral structures. Not only is this the case in the adults themselves, but under the same circumstances the more advanced young in the interior of *Prosorhochmus Claparèdii* undergo a like degeneration. In a specimen of the former species where this had occurred, the wave of granular fluid driven forward by the contraction of the reservoir distended the muscular cavity in front of the granular basal sac of the central apparatus (which in this instance was devoid of a stylet), and as the aperture into the anterior chamber permitted only a limited discharge at a time, the fluid rushed into the centre of the granular sac, and distended it and its wedge-shaped setting with every impulse. The absence of proper nutriment and free aeration—for the salt water was but rarely changed during the year—are sufficient causes for the above-mentioned degeneration.

In *O. pulchra* (JOHNST.) the anterior region of the proboscis has a decidedly pinkish hue, and numerous small clear globules at its commencement, as well as over the reservoir. The large glandular papillæ in the anterior chamber have their marginal globules less distinctly marked than in *O. alba* or *Tetrastemma*, and hence the structure has a smoother or finer appearance. The lateral stylet-sacs (Plate VII. fig. 3,  $\nu$ ) are very large, and each contains, in well-developed specimens, from five to nine stylets, a large circular globule, and a granular orange pigment-mass, besides a fluid rich in moving granules, similar to the secretion from the long posterior chamber. It is, however, in the apparatus of the central stylet that the greatest deviation from the typical structure occurs. The basal sac of the stylet (Plate VI. fig. 11,  $\lambda$ ) is small, elliptical rather than ovoid, and its granules are very minute. In addition to the ordinary stylet (*a*) fixed to its anterior end, another stylet (*b*) projects into its posterior portion, enclosed in a kind of sheath, and whose point extends forwards almost to the butt of the anterior stylet. This reserve-stylet is not in all cases fully formed, but apparently awaits the rejection of its progenitor for complete development. The head of the reserve-stylet projects into a large cavity formed by a peculiar disposition of the fibres composing the setting of the basal sac and the region behind. Instead of the usual wedge-shaped structure, radiating fibres pass outwards from the sides

of the sac, curve backwards, and arch over a large cavity (Plate VII. fig. 3, *o v*) filled with a clear fluid, part of the floor being formed by the anterior fibres of the reservoir. In certain states of contraction the central (reserve) stylet may be seen pressed backwards, so that its butt rests on the latter—a position quite easily attained on account of the yielding nature of the cavity and tissues which lie immediately behind and around it. Some granular streaks, probably due to the granular glands, are also observed passing from the central sac along the arch of the fibres. The granular glands themselves are distinct enough if the specimen is not too much pressed. The peculiar cavity behind the central granular sac might be supposed to assist in the rapid formation of the reserve-stylet, yet it cannot be absolutely necessary for its development, since the stylet is as readily replaced in front of the sac in *O. alba*, and others, where no such space exists. Physiologically the cavity may also act as an elastic buffer or cushion when the stylet is driven into any structure, if such ever occurs. The ejaculatory duct is large, and being surrounded by a yielding region, is more mobile than in the typical forms. The clear globules interspersed amongst the looped fibres of the reservoir are numerous, so that under pressure the cavity seems covered with them; and if pressure is severe, they escape into the reservoir, and pass forwards into the ejaculatory duct. Posteriorly these looped fibres have a laminated appearance. During examination the walls of the reservoir were frequently contracted in the manner shown in the drawing (Plate VII. fig. 3), thus indicating very clearly the presence of circular fibres. The entire region had more translucent walls and greater mobility than in *O. alba*, and the coats were somewhat diminished in total bulk posteriorly, so that the channel of communication was short. The glands are large transparent structures, with clear globules in their interior, and in general aspect differ from any hitherto observed. Those of the posterior chamber of the organ were longer than in *O. alba* and *Tetrastemma*. In one specimen several stylets lay in the *cul-de-sac* of the latter chamber, showing that they had passed along the ejaculatory duct, or else had been formed in the cavity.

The muscular and other structures of the anterior region of the proboscis of *O. pulchra* present, in transverse section, a slight variation from the common type, as seen in *O. alba*. The beaded layer (Plate VII. fig. 10, *e*) is very distinctly marked, and the external angle of the somewhat lozenge-shaped enlargements (longitudinal bands) is connected with the outer layer (*g*), while a process from the opposite angle passes inwards towards the circular coat (*c*), so as to cut the great longitudinal layer (*d*) into a number of separate fascicles, which, in the specimen represented, amount to fourteen. The changes which ensue in the various layers, when the organ is completely everted, are portrayed in the figure; and the characteristic appearance of the beaded layer (*e*) is to be noted, as well as the swollen segments of the usually thin external longitudinal layer (*f*).

Dr JOHNSTON\* observes of this species, that "the structure of the stomach" (proboscis) "is like that of its congeners, excepting in there being five or six spines on each side of it, instead of three, which is the usual number." He does not refer at all to the remarkable arrangement of the central stylets, though an incomplete woodcut in one of his early papers† shows that it had not entirely escaped the notice of his accomplished artist.

The general arrangement of the proboscis in *Tetrastemma algæ* agrees with that in *O. alba*, though there are some minor differences in the details of the stylet-region. If under examination the ejaculatory duct is placed on the left of the central stylet-apparatus, an explanation is obtained of the mistake into which M. CLAPARÈDE‡ had fallen in his description of the region in *Tetrastemma varicolor*, CÆRST. (the figure, however, appears to me to be very like that of *T. algæ*). The central stylet and its sac have been slightly pressed backwards so that the radiating fibres which sling them have been brought out distinctly, and sometimes a faint line of demarcation is seen on the right side (in such a position) simulating the presence of a separation; but numerous fibres are prolonged past this, and, moreover, a slight contraction or change of position obliterates this line, while the curved or radiating fibres are rendered more distinct. On the left side the only boundary line to the supposed distinct coat around the wedge-shaped setting is the wall of the ejaculatory duct. The basal sac of the central stylet in *T. algæ* (to continue the description) has rather more shape than in *O. melanocephala*, and is proportionally more elongated. I thought I could detect a slight difference between this species and *T. variegatum*, for the stylet in *T. algæ* is generally shorter in proportion to the length of the sac than in *T. variegatum*. Considerable variations exist in the size of the several stylets in *T. algæ*, independently of the size of the animal, a fact, perhaps, the less to be wondered at when the reproduction of the tube is remembered; but the greater size is generally diagnostic when compared with other species. In a developing or recently repaired central apparatus (Plate V. fig. 14) the basal sac is thinned off anteriorly from contraction of the parietes, and the difference in size between this central stylet and one from the lateral stylet-sac (Plate V. fig. 15) of the same animal is marked. In *Tetrastemma variegatum* the structure of the stylet-region, while agreeing generally with *O. alba*, is yet more particularly allied to *T. algæ*. The stylets are on the whole more slender than in the latter, and the central longer in proportion to its basal sac. In *T. vermiculus* the structure is similar to the two former (Plate IX. fig. 12). The shape of the basal sac of the central apparatus in *T. varicolor* is characteristic (Plate VI. fig. 5), the stylet being more slender than in the other two species, larger in proportion to the sac, and the lateral lines of the latter nearly straight. The proportionally large size of the glands in the

\* Catalogue of Worms, &c. p. 292.

† Mag. Zool. and Bot. vol. i. p. 531, fig. 4.

‡ Recherches Anat. sur les Annel. Turb., &c. p. 81, plate v. fig. 6.

reservoir in *Tetrastemma* is well illustrated in this species, where they form very prominent structures with granular contents, and more nearly allied to those in the posterior chamber than in *O. alba*. In transverse section the microscopic structure of the organ in the foregoing species agrees with that in *Ommatoplea*.

In *Polia involuta*, VAN BENEDEN, the proboscis and its apparatus are reduced to a minimum. The anterior region (Plate VII. fig. 5, A) is very short, and has in general a somewhat conical outline, the base of the cone being formed by the floor of the chamber. Its walls are proportionally thick and muscular, and internally have a minutely granular aspect, a condition probably due to indications of papillæ. Posteriorly it terminates in the usual floor, into which, however, only one aperture leads, viz., that of the central stylet. The next, or stylet-region proper, while still retaining the *Ommatoplean* type, differs much from that of any other British species. Instead of the usual well-defined arrangement of longitudinal and radiating fibres, the entire muscular structure is obscured by numerous granular or cellulo-granular bodies ( $\gamma$ ), which give a characteristic appearance to the somewhat conical region. There is no trace of lateral stylet-sacs. The central stylet is minute, and furnished with an elongated and faintly granular basal sac, which is fixed in the usual transparent muscular setting, the mobile muscular chamber into which the ejaculatory duct opens being situated immediately in front. Though the whole apparatus is very minute, I have seen the stylet thrust forwards by the contraction of the fibres of its basal setting, so that its point projected into the floor of the anterior chamber of the proboscis. The ejaculatory duct is large, and, owing to its central position in ordinary examinations, causes the stylet-region proper to appear bifid posteriorly; but this is due only to the greater translucency of the duct, which, for the time being, renders the denser granular masses at the sides more conspicuous. The region of the reservoir is fairly developed, the walls being striated with transparent muscular fibres in the usual manner, and the granular glands lining the inner surface. The walls might be seen now and then contracting with force, and driving the contents forwards into the ejaculatory duct and muscular chamber behind the floor of the first region. The posterior channel of the reservoir led into a posterior chamber of comparatively small dimensions, but having thicker walls than usually found in this region, and terminating in a *cul-de-sac* and rounded end, a short distance behind the œsophageal apparatus. This chamber had a cellulo-granular lining internally, and in some specimens the posterior end was observed under pressure to be distended with a transparent fluid containing a few compound cells of similar aspect to those found in other species. This posterior region is kept in position by fibres from the strong bands at the posterior part of the œsophageal apparatus.

All that M. VAN BENEDEN says with regard to the structure of this organ is

that it is very short, and bears an "isolated" stylet, while his enlarged drawing\* is incomplete.

M. DE QUATREFAGES considered the *posterior chamber* of the Ommatoplean proboscis the intestine-proper, but there is no support for this view; and, indeed, his minute anatomy of the organ is somewhat inaccurate. I have not observed that the dilatations and contractions of the channels of the reservoir (his oesophagus) vary in the manner he refers to in different species. He describes two bulgings of this "oesophagus," a large lozenge-shaped one at its commencement, and another corresponding to our reservoir, these dilatations being connected by a straight channel. The former may refer to the mobile muscular chamber behind the stylet-aperture in the floor of the anterior region, but his descriptions and drawings are indistinct. He aptly likens the two central divisions (*stylet-region*) to crystal; but he says he required the action of hydrochloric and acetic acids to distinguish fibres, which, he observes, have a transverse direction, and he especially notes that he could not see any longitudinal fibres. I have always been able to see these fibres in the fresh and living animals without any addition to the sea-water in which they happened to float; and, moreover, the presence of longitudinal, looped, and other fibres previously described show how much more complex the structure is than this author imagined. He correctly reports the absence of vibratile cilia from this region; but he again errs by affirming that they occur in the posterior chamber. His figures of the stylets are different from any seen by me, since they exhibit a bulging and then a contraction in front of the head. The basal sac is termed the "body" of the central stylet, and he narrates how in *Nemertes balmea* (our *O. gracilis*) this body has an exterior coat composed of the same structure as the point. Nothing more than the usual firm muscular setting is really present (see p. 335). Again, the statement that the "body" acquires greater solidity is not borne out in fact, for the granular contents of the sac are homogeneous throughout. He speaks of a pouch containing a granular glandular substance in which the stylet and its "body" are placed in this species, and thinks it probably secretes the latter (body); and, though he has not seen it in *Polia*, he considers its existence likely. The author has evidently fallen into confusion here, for the granular sac (or so-called "body") is fixed in a clear setting of the firm muscular substance. He next describes and figures other two cavities, which are said to exist at the borders of the "stylet-pouch," semi-opaque and glandular in *N. balmea*, very transparent in *Polia*; and he considers that these two glandular organs secrete a poisonous fluid for use in offence and defence, which fluid is poured into the pit in front of the stylet-region. Entomostraca, moreover, were killed instantaneously by wounds of the stylet, an effect which could not be due to mechanical injury only, but to the presence of an

\* Mémoires l'Acad. Roy. &c. pl. iii. fig 6.

active poison. It is true he was not able to distinguish these glands or their cavities in many species, so that if they existed they must have been confounded with the neighbouring tissues by reason of their transparency. Such glands have never occurred in any of the British species, and the opaque granular substance really present in *O. gracilis* (*N. balmea*, QUATREF.) totally differs in structure and function from his representations. The folding downwards of the floor of the anterior chamber and the presence of the muscular space behind this have probably caused the error—an opinion shared by Prof. KEFERSTEIN; and, indeed, it may be remarked, that the time and opportunities necessary for a correct appreciation of these complex structures make those best acquainted with them least surprised at such mistakes. The two muscular bands, also, which M. DE QUATREFAGES figures and describes as for the probable purpose of carrying forward the stylet-apparatus, and compressing his hypothetical poison-glands, have not been seen, and the explanation of the parts already given renders such useless. With regard to the observation, that the lateral stylet-sacs are free in *N. balmea*, but placed in the thick walls of the œsophagus in *Polia*, I can only state that the type of structure is the same in all, and that they occupy corresponding positions in the species referred to. It is probable also that the finding of only a single lateral stylet-sac in *Polia quadrioculata* and *P. humilima* was accidental, and not by any means a characteristic of such species (*Tetrastemma*). I have also very little doubt that the presence of the toothed cartilaginous plate, which he describes as occupying the usual place of the central stylet in *Cerebratulus spectabilis*, has been due to some mistake or confusion in his notes. Indeed, the author himself does not speak with certainty on the subject, since he states that he regrets he had mislaid his drawing of the actual relations of this organ to the other parts. The remark, that in *Polia vermiculus* one sac was placed on the dorsal and the other on the ventral surface, is of no consequence when the ever-changing condition of this very mobile organ is remembered. This author further describes the “intestin” (our posterior chamber) as having the same coats entering into its composition as the anterior region, though, he adds, the muscular layers are proportionally thinner. As already stated, the structure of the walls of the two regions is essentially different, just as their functions disagree. He is correct in averring that the cavity ends in a *cul-de-sac*; but wrong in saying it is ciliated, and that the terminal ribands are attached “à la paroi abdominale.” His distinguished countryman, M. MILNE EDWARDS,\* is also in error in regard to both of these points. Lastly, M. DE QUATREFAGES is only certain of the muscularity of these ribands in *Polia coronata* (*O. melanocephala*), and he gives a curious figure (which cannot be verified in the British examples) of their termination in this species—as a series of arborescent fibres.

\* Leçons sur la Physiol. et l'Anat. Comp. tome v. p. 464, 1859.

Dr JOHNSTON'S\* description of the stylet-region is as follows:—"First, we perceive on each side a small circular spot or cavity, in each of which are three spines with their sharp points directed outwards; beneath these there is a cup-shaped organ encircled above with a faintly plaited membrane, and armed in the centre with a strong spine, which can be compared to nothing more aptly than a cobbler's awl in miniature, the part representing the handle being very dark, and the point transparent and crystalline. This apparatus is placed within the intestine, is visible only when this is compressed, and is, as I believe, stomachial, having some distant analogy with the proper digestive organs of *Laplysia* and *Bulla*." His anatomy is thus imperfect, and he, moreover, considered that the "intestine," as he termed the organ, proceeded to the tip of the body and terminated in a distinct anus.

Dr WILLIAMS† observes with regard to the proboscis (his alimentary organ), "The extremity of this organ is armed with several styleted jaws, which, from their construction, seem only designed to fix the suctorial end by perforating the alimentary object. When the proboscis is withdrawn into the interior of the body, fitting admirably into a short œsophagus, these sharp instruments are packed and folded upon themselves," the sides of the tubes closing round them. The correct examination of a single extruded organ would have at once dispelled such notions. His supposition—that the glands in the interior of this structure furnished an important secretion for the digestive process, which secretion was exuded into the "œsophagus" (apparently, judging from his figure,‡ the proboscidian sheath), and thence into the great alimentary organ—rests upon no facts. He is also wrong in stating that the outlet of this organ is situated not far from the cephalic end of the body; but his remark, that there is no open communication between the œsophageal tube (proboscidian sheath) and the "alimentary cæcum" is correct.

Dr MAX S. SCHULTZE, in his account of *Tetrastemma obscurum*,§ gives no definite description of the ending of the proboscis, and figures the central stylet as projecting freely into the cavity. He indicates the presence of the muscular space behind this, but confounds its structure with the wedge-shaped setting of the basal sac, the whole forming, he says, a quadrangular mass. He falls into the same error as M. DE QUATREFAGES and others, in describing the terminal ribands of the organ as attached to the wall of the body. His figure|| of the exerted stylet-region is incomplete in detail, for he omitted to notice the ducts of the lateral stylet-sacs, though he regarded the latter as the producers of the

\* Mag. of Zool. and Bot. vol. i. p. 530, 1837, copied into "Catalogue" Brit. Mus. pp. 285-6 1865.

† Report Brit. Assoc. 1851.

‡ *Op. cit.* pl. xi. fig. 64.

§ Beiträge zur Naturges der Turb. 1851, p. 62, tab. vi. figs. 2-10.

|| *Op. cit.* tab. vi. fig. 3.

stylets for the central organ. He first indicated, however, the connection between the developing spikes and the clear globules in the lateral sacs, showing that they are sometimes seen in their interior. Finally, he has not discriminated the structure of the reservoir-region, and its relation to the neighbouring parts; and, indeed, his anatomy of the animal, from the limited nature of his observations, is somewhat imperfect.

M. CLAPARÈDE,\* in his remarks on *Tetrastemma varicolor*, describes the sac of the central stylet as set in a pale space of a triangular form, and he leaves the stylet-apparatus to hang therein, apparently by its anterior end. He has evidently mistaken the translucent wedge-shaped setting of the sac for a cavity, and the triangular muscular structure shown exterior to this has no existence as figured (*vide* p. 339). He has correctly observed the presence of a duct to the lateral sac, though his figure is somewhat distorted from pressure, and represents the duct by far too wide. He is, moreover, of the opinion that these chambers are not for the sake of furnishing new stylets for the central organ, as Dr SCHULTZE avers, but for the lodgment of those discarded from the latter; a view quite as erroneous as the other. Each supplies its own stylets. He did not observe any connection between the clear globule in the lateral sacs and the developing spikes. His representation of the muscular fibres of the stylet-region is faulty. In mentioning the cavity of our reservoir, he properly describes the presence of a liquid containing minute granules in suspension (but not in motion), and that it (reservoir) communicates with the "trompe" by means of an efferent canal: but he fell into the error of regarding the long posterior chamber as a "muscle retracteur." His figure is inaccurate in other respects, such as in the mode of opening of the ejaculatory duct, and in the absence of the muscular space behind the stylet-aperture in the floor of the anterior chamber. He regards the reservoir as a poison-gland, which squirts its contents along the ejaculatory duct into the wounds inflicted by the stylet. This author is scarcely correct in saying that M. DE QUATREFAGES had in reality figured this poison-gland without the efferent canal in *Polia mandilla*; for the French naturalist figures and describes the part as one of the bulgings of his œsophagus, and which, therefore, communicated both with the "trompe" and "intestin." In a still more recent publication† M. CLAPARÈDE exhibits the structure of this region in KEFERSTEIN'S *Prosorhochmus Claparèdi*, a viviparous species, but he gives no details of muscular structure. The central stylet and its sac are placed in the middle of a continuous and apparently homogeneous oblong body, the wedge-shaped enclosure of the basal sac and the muscular cavity in front being confounded. The opening of the ejaculatory duct of his poison-gland (reservoir) has the same position as in his previous figure, viz., at some distance from the stylet, and passing directly

\* Recherches Anat. sur les Annélides, Turb. &c. 1861, p. 81, pl. v. fig. 6.

† Beobach. über Anat. und Entwicklung. wirb. Thiere, &c. 1863, p. 23, tab. iv. fig. 10-12.

into the floor of the anterior chamber. He now refers to the posterior chamber, which, he says, occupies the centre of *the muscle of the organ*, a modified but scarcely satisfactory description. The external granular glands show certain peculiarities when contrasted with other species, viz., complete separation, large number and minute size of the divisions or lobules—modifications that I have not been able to verify.

M. VAN BENEDEN'S brief remark on the proboscis in *Polia involuta* has already been adverted to. It may also be stated, however, that, in addition to the incompleteness of his figure, he represents certain lines,\* which indicate a sheath (one of his *culs-de-sac*) around the proboscis—a state that has not been seen in our examples. The structure of the stylet-region, as observed by him in *Polia obscura* (*Tetrastemma varicolor*?), is erroneous. He represents no ducts to the lateral stylet-sacs; no ejaculatory duct. The division of the reservoir has a cavity in the centre, but is likewise furnished with two hypothetical oval vesicles or cavities, and the muscular structure, the floor or ending of the anterior chamber, and other important points, are absent. The statement, that the lateral stylet-sacs contained stylets of a smaller size than the central, and of a different form at the base, shows the learned author did not possess good opportunities for examining these creatures. He follows Dr SCHULTZE in calling the lateral sacs pouches of replacement, and therefore is not aware of the true physiology of the parts. While he states that the proboscis is enclosed in a separate sheath, he distinctly adds, that its muscular retractor is attached to the skin of the animal posteriorly; and that there may be no misunderstanding on the question, he again repeats the statement when drawing up his conclusions, by erroneously averring that the internal surface of the proboscis is ciliated, and that it is fixed to the bottom of the digestive tube by a retractor muscle, as in the stomach of the *Bryozoa*.†

Prof. KEFERSTEIN'S‡ remarks, so far as they go, upon this region in *Polia mandilla*, are decidedly in advance of his predecessors. He, however, does not mention the *minute* glands on the floor of the anterior chamber, and shows the central aperture for the stylet in the same by far too large, so that in extrusion the muscular space ( $\epsilon$  in our figures) becomes obliterated. The muscular setting of the granular sac is also continued too far forwards in his figure. He indicates no oblique fibres from the pit of the anterior region (as shown in Plate IX. fig. 3), and the thick coat of the reservoir is described as composed of longitudinal fibres. The external granular glands are not distinctly described; and the disproportion between the central and lateral stylets is so great, that I fear some

\* Mém. de l'Acad. Roy. des Sc. de Belgique, tom. xxxii. pl. iii. fig. 7.

† *Op. cit.* p. 44. Unfortunately this author has not lettered his plates, so that I have often been at a loss as to his interpretation of structures of which no mention is made in the text.

‡ Zeitsch. für wiss. Zool. Bd. xii. p. 72, taf. v. fig. 4.

slip has occurred in their delineation. Lastly, his crenated border (external elastic coat) does not pass the constriction between the stylet-region and the reservoir-region, whereas, as already shown, both this and the longitudinal coat are continued some distance on the latter division.

*Reproduction of Proboscis.*—So far as I am aware, no author has alluded to the reproduction of this organ. The process was first observed in *Ommatoplea melanocephala*, but it has since been seen in *O. gracilis*, *Tetrastemma algæ*, and others. In a specimen of the former (*O. melanocephala*), from which three days before the proboscis had been removed, there existed a pale conical papilla, which projected a short distance behind the ganglionic commissures. Two days after considerable progress had been made, and the organ proceeded backwards as a slender rod tapered posteriorly (Plate VIII. fig. 1, *a*). There was a distinct exterior coat from one end to the other, and an inner terminating at the commencement of the posterior narrow portion. The former had a crenated edge in contraction. The organ gradually increases in size and complexity, but continues quite free posteriorly for a considerable time, until, indeed, the stylets are well developed. At a further stage of growth (Plate VIII. fig. 2), the walls are defined almost as in the complete structure, but of course are much more delicate and plastic; and the extreme contractility and elasticity of the entire organ are most interesting, and raise a doubt as to the identity of its muscular fibres with those of the higher invertebrates, since it so much surpasses them in mobility. The floor of the anterior chamber ends in the usual pit, which is swollen on account of the shortening of the organ. The walls of the muscular cavity behind the floor of the anterior region are not well defined, though the space itself is large, and contains a granular fluid. There is no central stylet, and the basal sac is represented by a somewhat triangular group of the usual granules, round which the radiating fibres are placed. The wedge-shaped setting within the latter (fibres) is mobile and translucent. A somewhat indistinct streak (*f*) in the central line indicates the canal for the central stylet, and now and then this became bulged by projected fluid. The lateral stylet-sacs, from the bulging of the chamber in this instance, seem pressed backwards, but in reality they have their distinctive position. Each contained a stylet or two, a few granules, and a clear globule.

The reservoir at this stage had assumed its characteristic shape, though its glands were barely visible. The shortening and bulging of the anterior and posterior chambers have annihilated the usual prominent appearance of this part, and the last has encroached very much on the cavity posteriorly. The glands were formed in the posterior chamber, though their contents were not elaborated, and the cavity terminated in the usual *cul-de-sac*. A few rounded papillæ at the posterior end indicated the early condition of the muscular retractor or riband. It is clear that at some time or other the latter becomes attached to the wall of

the proboscidian sheath, and that, too, in a definite manner, since no great deviation in a series of specimens is met with.

In the developing organ of *O. gracilis*, a very good analysis of the somewhat complicated structure is obtained, so that doubtful anatomical points are cleared up satisfactorily. The sac at the base of the central stylet is sometimes seen to be composed of granules in rounded masses; and they are all grouped posteriorly at an early stage, and thus present a similar form to that seen in other species which have no such elongated sac in the complete state. It is curious to witness the accuracy with which the stylets are reproduced in this and other species. There is never any confusion, but each invariably produces them of their respective sizes and curves as infallibly as if they had been struck out of the same mould. Yet these bodies are not in any way organically connected with the tissues of the proboscis, but only spring from a secretion poured into the lateral sacs, or from the central apparatus. In the concentric arrangement of their constituent substance, and some other particulars, these spicula are analogous to those of the sponges, whose microscopic anatomy has been so excellently investigated by Dr BOWERBANK.\* Indeed, the morphology of the stylets of the Ommatopleans offers elements for deeper reflection than even the hooks and bristles of the higher annelids, which are often so diagnostic of genus and species.

Besides the developing organ, the proboscidian chamber contains (unless in cases where the organ has been violently expelled) the cast-off proboscis; and it is a curious sight to observe a fully-developed organ floating freely in the chamber, and still endowed with contractile power, while the new proboscis has advanced to the stage of the advent of stylets. The discarded organ soon becomes opaque, appearing reddish by transmitted light, and the stylets leave their positions. As there is no mode of exit after the new proboscis has begun to develop, the aborted one can only (not to speak of rupture) be removed by disintegration and absorption; and hence in the proboscidian chambers of such animals there is a vast increase of cells, granules, and granular debris.

*Digestive System.*—Though no such transverse muscular plate, as described by M. DE QUATREFAGES, occurs at the anterior part of the body of the worm, yet there exists a very distinct and comparatively large ciliated œsophageal chamber or sac, as first described by Sig. DELLE CHIAJE, apparently in a Borlasian.† The figures of the supposed transverse plate given by the former, indeed, show some degree of doubt, since in the large figure † both wavy and longitudinal fibres are represented, while in the small figure there are only transverse fibres. I fear the wavy longitudinal lines owe their presence to those actually existing in the œsophageal sac. Dr JOHNSTON'S figure§ of *O. melanocephala* indicates this structure, to which he thus

\* Monograph of the British Spongiadæ, Ray Society, vol. i. p. 5, *et seq.*

† Mémoire sulla, &c. vol. ii. 1835.

‡ *Op. cit.* pl. xix. fig. 1, *m.*

*Op. cit.* pl. ii. a fig. 5.\*

refers under the head of *O. pulchra*:—"Immediately under the hearts" (ganglia) "we observe a large, somewhat muscular, viscus, apparently hollow, and lying in the course of the intestine, but seemingly unconnected with it. Of its office and nature I can form no opinion; but I may remark, that in all the species a greater duskiness in its site shows that a similar organ exists in all." Prof. KEFERSTEIN'S notice\* of the organ in *Ærstedtia pallida* is very brief; and he has abstained from figuring its relations, though affirming that its opening (constituting the mouth) is on the ventral surface behind the ganglia, as in the Borlasians. M. VAN BENEDEN,† while indicating an outline of the structure in *Polia capitata*, makes no reference thereto in his descriptions. The same omission is made by M. CLAPARÈDE with regard to his figure of *Prosorhochmus Claparèdii*, KEF. †

In every specimen of *Ommatoplea* and *Tetrastemma* the great œsophageal organ above-mentioned has been easily observed (Plate VIII. fig. 3, *j*) as an elongated sac, slightly narrowed posteriorly, and usually thrown into various longitudinal wrinkles. In ordinary views from above, it is seen to narrow somewhat abruptly behind the ganglionic commissure, and to pass forwards beneath the inferior one, to open at the tip of the snout just at its ventral border, as a short longitudinal slit. I have seen the sac turned inside out here, and projecting beyond the head in an animal which had been subjected to chloroform. Both apertures may frequently be observed at once,—that for the proboscis being circular, while the mouth forms a short longitudinal slit beneath the former. The observations on this point have been often repeated, out of deference to the distinguished foreign authors who hold different views, but I have never seen any other aperture in the British Ommatopleans, and it were hard for such to exist in the free portion of the œsophageal tube behind the ganglia. Moreover, as shown in Plate IV. fig. 1, the narrow anterior part of the glandular œsophagus lies close to the chamber for the proboscis, when the latter is in this region. The two organs, proboscis and œsophagus, become more evidently separated from each other in most sections, just in front of the ganglia, and the interposition of the broad inferior commissure soon renders the distinction more evident; thereafter they have the tunnel of the proboscis as a partywall, together with that portion of the fibrous stroma of the extra-proboscidian region in which the median blood-vessel is situated. The œsophagus, moreover, occupies a special chamber, bounded by a series of well-marked fibres (Plate V. fig. 2, *k*), which pass downwards from the upper wall by the side of the proboscidian sheath, and unite in the median line below it. The anterior narrow portion is generally translucent; and just behind the commissure a pursed arrangement is often seen, which is followed by the more opaque portion with its longitudinal rugæ. The pursed arrangement is very similar to that which is caused by tying the mouth of a

\* *Op. cit.*, p. 70.

† *Mém. l'Acad. Belgique*, pl. iv. fig. 13.

‡ *Beobachtungen über*, &c. pl. v. fig. 10. ph.

leathern bottle, and is due to the narrowing of the sac in front. The pale portion immediately behind the ganglia shows cilia in active motion very distinctly, but I have never seen anything like an aperture; indeed, the great and peculiar stretching of this pale portion, as it is dragged backwards from the region in front of the ganglia during the motions of the animal under pressure, at once demonstrates the fallacy of supposing it connected with any post-ganglionic aperture, as in *Borlasia*. The wall of the sac evidently contains some contractile fibres, which cause it to dimple inwards here and there during its motions; and in anterior transverse sections the cut ends of longitudinal muscular fibres are shown very distinctly, though they are finer than those of the proboscis. Posteriorly, the organ opens into the digestive cavity; but the communication is not actually seen in ordinary views, from the folding together of the walls, and I have not been so fortunate as to observe the animals feeding. In *Polia involuta*, VAN BENEDEN, the œsophagus is short and nearly globular under moderate pressure, being also conspicuously tied posteriorly by strong transverse bands. In this species the posterior aperture is very apparent.

The relations of the œsophagus to surrounding organs may be observed in the sections (Plate IV. fig. 5, and Plate V. fig. 2, at *j*). The walls increase in thickness after passing the narrowed portion in front, form considerable parietes, and again slightly diminish posteriorly. In transverse sections of specimens hardened in spirit, and mounted in the usual manner, the structure has a streaked and fibrillated aspect, or marked by a series of vertical striæ, and minutely granular, an appearance due to the position of the glandular follicles with respect to the inner surface, and the change caused by the preparation. It will also be observed that in these sections the organ is thrown into numerous characteristic longitudinal folds. In life considerable differences in appearance are observed, according to the degree of pressure—as, for instance, between the flattened follicles of the organ in a small *Tetrastemma*, and the thicker structure in a good-sized *O. alba* (Plate VII. fig. 7). In the latter, the inner edge (*a*) of the glandular tube has a somewhat translucent and well-defined border, garnished with moderately long and most vigorous cilia, whose activity is in strong contrast with the motion of the same organs on the epidermis, and which seem to play an important part in the economy of the tube. Under the microscope the fresh specimen is always thrown into numerous wrinkles, and is crossed by pale streaks—the ciliated edges of the folds (*b*). The entire organ is studded internally with a series of granular glands or follicles, and numerous brownish pigment-granules. The glands taper towards the free ciliated edge of the rugæ.

In *O. melanocephala* the organ is curiously narrowed posteriorly; and in *O. pulchra* the granular glands are distinct and large. In *T. varicolor* the glandular appearance in a small specimen under pressure is somewhat finer and more translucent, but the structure is essentially the same as in *Ommatoplea*.

This ciliated glandular structure is physiologically and homologically an organ of great interest. It is entirely Ommatoplean in the condition just described, since what is shown here in the complete form is only indicated in *Borlasia* by the turning inwards of the margins at the junction of the two regions of the alimentary canal. The granular glands and cells which coat the latter in *Ommatoplea* arise (in the case of the cells, at least) on the sides considerably in front of the posterior end of the œsophageal region—in some cases, indeed, almost touching the ganglia (Plate VIII. fig. 3), and besides, the first region has been demonstrated to occupy a special pouch in which it rolls. The rich ciliation of this œsophageal region, and the somewhat indistinct ciliary movements seen in the posterior division of the alimentary chamber, are points of importance when contrasted with the arrangement in *Borlasia*, and show that from structure to structure the essential differences between the groups meet the inquirer at every step. In *Vortex*, again, the homologue of this region is seen in the “Schlund” of the German authors.

*The Digestive Cavity-Propriety*—The detailed description of the general cavity of the worm (all within the muscles) given by M. DE QUATREFAGES, shows that he had no clear conception of this structure, for, after explaining the hypothetical transverse diaphragm, to which we have already alluded, he goes on to say,\*—“Le reste de la cavité générale occupe tout le corps proprement dit; mais les cloisons verticales auxquelles sont suspendus les organes générateurs le partagent en trois chambres distinctes, l'une médiane, qui renferme le tube digestif dans une portion de son étendue; les deux autres latérales, dans lesquelles flottent les ovaires ou les testicules, et qui à l'époque de la reproduction se remplissent d'œufs ou de zoospermes.” In his figures† the scalloped shaded portion, which he terms “ovaires ou testicules,” is, as Prof. KEFERSTEIN has pointed out, the glandular wall of the digestive cavity. I am at a loss to understand how M. DE QUATREFAGES did not correct his error on contrasting his figures of the male and female elements in his *Nemertes balmea* (*O. gracilis*), for the very same organ is made in the one case ovary, and its gland-cells developing ova, and in the other respectively testicle and sperm-cells. Dr JOHNSTON‡ recognised the structure as “a close series of vesicles or cells, formed, in the true *Nemertes*, apparently by the folds of a membrane.” The cæca, he adds, are always full of some opaque matter, which varies “in intensity at least according to the nature of the animal's food.” He thought the structure was connected with the digestive system, though not in communication with the proboscis (his alimentary organ). Dr WILLIAMS§ had also an inexact idea of this cavity, for he speaks of it as a great spongy mass, or “great alimentary cæcum,” which commences anteriorly

\* *Op. cit.* p. 152.

† *Mag. Zool. and Bot.* vol. i. p. 532.

‡ *Op. cit. e.g.* pl. xviii. fig. 1, and pl. xix. fig. 1.

§ Report Brit. Assoc. 1851, pp. 244–5.

immediately behind the hearts (ganglia), under the character of a cæcal end, and as "a perfectly closed sac, containing a milky fluid." The walls of this cavity, he says, act upon the exuded food, after its passage through the walls of the "œsophagus." He is correct in denying the ovarian character of the organ, and in showing that the so-called ova consisted only of oil-globules. He has also some reason for considering the transverse segmentation of the organ as an indication of annuli.\* Dr MAX SCHULTZE† described it as a straight canal in *Tetrastemma obscurum*, ciliated on its inner surface, and opening anteriorly and posteriorly, and figures‡ the cells in its walls as altered by extrusion into the water. M. CLAPARÈDE, in the before-mentioned figure of *Prosorhochmus*, shades the region, but makes no mention of it in his description.

The digestive cavity is a somewhat moniliform or lamellated canal, in so far as its surface is increased by the numerous diverticula. Its appearance under pressure is well seen in *Tetrastemma* (Plate VIII. fig. 3) as a lobulated glandular organ, usually of a pale flesh or slightly pinkish hue, extending from a short distance behind the ganglia to the tip of the tail, and forming (in the individual in which the reproductive elements are not developed) a lining to the body-wall, except where interrupted by the proboscidian sheath. In the ripe animal, however, the gradual enlargement of the ova or sperm-sacs pushes in the yielding organ, so that it occupies a more median position, and has its ventral portion increased in bulk. It is also well to bear in mind that the body of the adult worm is only rounded in contraction, and partly so when the ova or spermatozoa are mature, but at other times it is flattened, and very mobile; thus, what is space in the transverse section is often filled up in the living animal by the collapsing and contraction of the yielding tissues in the neighbourhood. Anteriorly the only opening leading into this chamber is that of the posterior end of the rugose œsophagus; posteriorly it terminates in an anal pore, less easily seen than the similar structure in *Borlasia*, from the absence of the strongly ciliated internal line. In intimate structure the walls of this cavity resemble the anterior or œsophageal portion, only the gland-cells are larger and more numerous, and the fatty elements in greater abundance, so that although the type of structure remains, there are considerable differences in microscopic appearances. I was for a long time in doubt about the ciliation of this chamber in *Ommatoplea*, since I have seldom been able to see cilia satisfactorily in the uninjured *O. alba*, though in the latter, *O. purpurea*, *Tetrastemma*, and especially in *Polia involuta*, VAN BENEDEN, peculiar motions of the cells were apparent. When a specimen is kept for some time under pressure, a few moving granules are observed at some particular point; these continue to increase in number, and sometimes a few cells accompany them, the group gradually enlarging and

\* Philos. Transact. 1858.

† *Op. cit.* p. 64.‡ *Op. cit.* taf. i. fig. 35.

revolving with great velocity. Such motions are doubtless due to the ciliation of the chamber. On making a transverse section of the living animal (*O. alba*), I have seen the inner margin of the digestive cavity cause motion in the surrounding particles, but the cilia were indistinct, and the appearances very different from the richly ciliated tube of *Borlasia*, or its own œsophageal portion anteriorly. It is thus much more feebly ciliated than the others.

In the walls of this complex cavity are a vast series of gland-cells, which, with M. VAN BENEDEEN, I consider as having some analogy with the liver of the higher forms, notwithstanding the adverse opinion of Prof. KEFERSTEIN, who, however, probably refers more particularly to the Porlasians. Microscopically the cells consist of a delicate membrane containing a number of fatty globules (Plate X. fig. 6), the average size of the cell being  $\frac{1}{800}$ th of an inch. Under pressure, and when highly magnified (700 diam.), it is seen to consist of a number of granular fatty bodies (Plate X. fig. 7). After extrusion from a living specimen into salt water, a remarkable motion occasionally ensues in the contents of the cell before breaking up, a condition which causes the observer to fancy the entire organ ciliated. The contained bodies jerk about within the cell, and soon a number of very minute granules appear, having burst from the larger bodies, in which their presence is indicated by obscure markings. The peculiar motions would seem to be due to the action of the water, and ultimately the minute contained bodies are all set free. The various appearances of the bodies from the cells are shown in Plate X. fig. 8, some being granular, others presenting faint concentric lines like starch-globules (though probably fatty), while three oil-globules are indicated on the right. The deep port-wine oil-globule is somewhat sparingly scattered throughout the wall of the tract, the yellowish red being abundant, and the pale globule still more plentiful. These cells have a similar structure in *Tetrastemma*, and often escape under pressure posteriorly. The quantity of deep yellow oil in this organ in *T. algæ* is unusually great. The foregoing glandular structure undergoes partial absorption at the period of reproductive activity, so that after spawning the animal is much flattened; but by-and-by it regains its plumpness, and often becomes of a greyish hue, apparently from the increased development of this tissue, which is exuded as a pale, salmon-coloured, semi-fluid substance on rupture of the body-wall. In *O. gracilis* the posterior division of the digestive system has a somewhat regularly ramified arrangement, when viewed from the ventral surface, and this is especially evident some time after spawning, when the animal has regained its condition. The colour of the region is of a deep green by transmitted light, whereas the œsophageal division is brownish. The lamellæ of this region in *O. pulchra* form simple tapering papillæ under pressure. In *Polia involuta*, V. BEN., the cavity is greatly developed, both as regards the rest of the body and its individual structures; and it also presents a firmer and more consistent aspect than usual on transverse section.

The absence of the proboscidian sheath and its contents leaves the central space almost entirely at its disposal.

In *O. alba* and *rosea* Mr E. RAY LANKESTER\* found many Gregarinæ, but they were rare in the specimens from St Andrews. In *Tetrastemma varicolor* a few gregariniform parasites (Plate IV. fig. 12) occurred in the digestive cavity towards the tail.

Another parasitic structure was found in January in a large male specimen of *Ommatoplea alba* in the form of an ovum enveloped in a granular lobulated mass—lying close behind the ganglion of one side (Plate XIV. fig. 9, *y*), to the exterior of the proboscidian sheath, and altogether unconnected with the œsophagus. Externally there was a distinct hyaline capsule or cyst, to which certain fragments of the fibro-granular lobulated covering adhered. The embryo was furnished with a very conspicuous opaque granular mass, and two discs; while the general stroma was cellulo-granular, here and there closely streaked by minute lines, apparently from its external investment. No motion of the included animal was observable, except an alteration of the size and aspect of the pores and discs after a period of eight or nine hours. There was no doubt as to this being a Trematode-larva in its capsule, and by rupturing the latter a complete view of the embryo was obtained (Plate XIV. fig. 10). The oral sucker (*c*) was considerably smaller than the ventral (*b*), and this formed a marked feature in the general aspect of the animal. The œsophageal bulb (*d*) appears as a distinct swelling close behind the margin of the oral disc, and from the tube behind the former the alimentary cæca (*e, e*) branched off and became lost in the cellular tissues posteriorly. The opaque mass of cells and granules (at *a*) corresponded to those observed in the Trematode-larva of the *Carcinus mænas*,† though, from the immature condition of this example, these and other structures were much less definite. There were also two large circular granular bodies (generative organs) (*f* and *g*); but only a trace of the excretory tubes existed in front near the oral sucker.

Microscopically, the alimentary organ has scarcely the regular and firm glandular appearance of the same structure in *Borlasia*, but is more friable and cellular. Its analogy with that of the higher annelids is also borne out; for, although the biliary matter is not arranged as a distinct organ exterior to the alimentary, it is incorporated therewith, and probably has a similar function. The fluid, however, which bathes the liver in the higher forms (if we suppose that inside the sheath for the proboscis to be the homologue of the former), is here separated by the muscular walls of its special tube. The large size of the proboscis in the *Ommatopleans* renders this system very obscure from the dorsal aspect, and it is only when the ventral surface is upturned that a correct know-

\* Jour. Micros. Sc. 1865.

† Jour. Micros. Sc. vol. v. N. S., pl. viii. fig. 5, *k*.

ledge of its relations is obtained. I have not been able to see *O. alba* feed in captivity, and have not found any food in the alimentary cavities of those examined. It is a curious fact, that in this group the digestive system lies quite beneath the nervous system anteriorly, whereas the proboscis passes through the nervous collar.

*Circulatory System.*—The circulatory system is composed of three great longitudinal trunks—one central and two lateral—besides the cephalic arch and anastomotic vessels. Commencing with the great central trunk posteriorly (Plate VI. fig. 8, *p*) in *Ommatoplea*, it is found that the vessel, which in this region is about twice the diameter of the lateral, arises from the point of junction of the two last-mentioned, just within the posterior border of the worm. It travels forward beneath the proboscidian chamber in a very undulated manner—as usually seen—to the region behind the ganglionic commissures, where it bifurcates (Plate VI. fig. 3, *q*), a branch passing to either side to join the lateral trunk (*r*), which bends inwards to meet it. From this point of junction also a single vascular arch (cephalic) proceeds forwards into the tissues of the snout (*l*, same figure, and in Plate IV. fig. 6, the latter showing the vessels in transverse section), the pillars of the arch thus meeting the lateral and the anastomotic vessels of each side. From the same point of union each lateral trunk passes backwards under the nerve-cord of its side to the tail, where it meets its fellow of the opposite side, and gives origin to the single central vessel with which the circuit commenced. The lateral vessels appear to diminish slightly posteriorly. The median vessel does not actually touch the wall of the proboscidian sheath, though transverse sections usually show a close apposition, but is situated in a layer of transparent elastic tissue which intervenes between this organ and the digestive tract. At the ganglionic region the vessels which go to form the cephalic arch pass below the commissures, and unite in front beneath the channel of the snout. In *O. purpurea* there are three main longitudinal trunks as in *O. alba*; but it can be observed that the lateral communicate with the central, as in *Borlasia*, by transverse branches, which, however, are proportionally smaller. Whether such anastomoses occur in the pale *Ommatopleans* is thus an open question; but they are distinct enough in this species. Two lateral trunks only could be discovered in *Polia involuta*, VAN BENEDEN (Plate VIII. fig. 5, *r*), which trunks unite by a very short loop just in front of the commissures. This loop (*l*) is distinguished from the ordinary arrangement by its not extending forwards into the tissues of the snout. The lateral vessels are not so clear or well defined as in *O. alba* and *Tetrastemma*, and are observed to have internal transverse bands or partial septa in front; while the contained fluid has a few clear granules, as in *O. purpurea* and others. The contractions in the lateral vessels are very vigorous, and even a minute central vessel could not have been passed over if a trace of such had existed.

The course of the circulation, so far as I can see, is as follows:—Posteriorly a gentle contraction from behind forwards drives the contained fluid along the great central vessel to the front, where it is forced through the anastomotic into the lateral vessels and the cephalic arch. The lateral trunk may be seen to swell with the wave, and the fluid then passes to the posterior end to enter the median as before-mentioned. In addition to the stream poured into the lateral trunks, another passes into the cephalic arch by the vessel on each side, and the counter-currents must meet and commingle, returning again during the diastole of the central vessel. I have not made out any branches in the British species except in *O. purpurea*; but this is a somewhat difficult task, on account of the transparency of the circulating medium and channels.

In many species the fluid contained in these vessels is transparent and homogeneous. M. DE QUATREFAGES, however, found corpuscles in his *Polia bembix*, Prof. KEFERSTEIN small oval discs in the reddish blood of his *Borlasia splendida*,\* and I have seen in *Ommatoplea purpurea* minute granular corpuscles, but both they and the fluid are colourless. Minute colourless globules also occur in the blood of *O. pulchra*.

Such, in the Ommatopleans, is a brief outline of the circulation, which, although resembling that of M. DE QUATREFAGES, in so far as each describes three main trunks, differs considerably in detail. The first point to be noticed in the descriptions of this author is the statement that the lateral trunks pass through the cephalic diaphragm—a structure which has not been seen. He is slightly in error also when he states that the median vessel lies immediately under the subcutaneous muscles. The arrangement shown in his two sections of *Borlasia anglia* cannot apply to this group. I have not been able to verify the elaborate curves which this author gives† each anastomotic division of the central vessel anteriorly, and which may be described as first forming a loop behind the ganglion, with its curve directed outwards, and a second inversely curved round its anterior border—in its passage outwards to join the lateral, which is scarcely bent inwards at all, but occupies a space where no vessel occurs in the British forms. The mere shortening of the anastomotic will not retrieve this anatomical error. The cephalic arch is also placed otherwise than “immédiatement au-dessous des couches sous-cutanées,” as already described (Plate IV. fig. 1). He mentions the presence of distinct walls to these vessels, which, however, he learned from *Borlasia anglia*, and in this I concur (Plate IV. figs. 1 and 6). The walls are highly contractile, and in the latter figure the vessels have been cut across just before they complete the cephalic arch; they are observed to be surrounded by a ring of finely granular texture. M. DE QUATREFAGES likewise

\* This species has since been found in the Channel Islands. It is the *Cerebratulus spectabilis* of M. DE QUATREFAGES.

† *Op. cit.* pl. xviii. fig. 1. and pl. xix. fig. 1; also in his recent *Hist. Nat. des Annelés*, pl. iv. figs. 2 and 3.

states, that though fixed in front the vessels are elsewhere free, and only connected here and there to the body-wall by ligamentous bridles; and in one of his plates\* figures the ova between the lateral vessels and the wall of the body. All our transverse sections show that such could hardly occur, for the vessels occupy a secure position beneath the nerve-trunks; and while the ovaries or sperm-sacs sometimes press the vessels downwards towards the ventral surface, and increase the distance between them and the nerve-trunks, they never actually intervene between the latter and the body-wall in the perfect worm.

Many of the older authors confounded the ganglia with hearts, such as EHRENBERG, HUSCHKE, DELLE CHIAJE, DUGÉS, ØRSTED, and more recently our countrymen, Drs WILLIAMS and JOHNSTON. The latter mentions that the only blood-vessel he has seen is one "winding down the middle, along the surface of the alimentary canal," but he can neither trace its origin nor termination. Dr MAX SCHULTZE† seems to have mistaken the edge of the proboscidian sheath under pressure for the blood-system, which he figures as two long straight trunks on each side of the digestive tract. The true blood-vessels he describes as the water-vascular system, but shows neither beginning nor ending, though numerous large branches are represented as issuing from them throughout their course. Prof. KEFERSTEIN‡ does not distinguish with sufficient clearness the different blood-systems of the Ommatopleans and the Borlasians; and, indeed, applies the definition of the former to the latter; but so far as they go his descriptions and representations of the arrangement in this group are good. He, moreover, shows an elaborate series of minute transverse anastomosing vessels in his *Borlasia splendida*, whose structure therefore differs from that usually exhibited by the British Ommatopleans. M. CLAPARÈDE,§ though his publication is more recent, is less correct than the latter author, for he figures the dorsal vessel as passing above the ganglionic commissure before giving off the anastomotic to join the lateral, and thus a somewhat stiff square is formed in the cephalic region, while the lateral vessels have to pass to the outside and front of the ganglia before meeting the anastomotic. The vessel appears also to be placed on the dorsum of the proboscis.

*Nervous System.*—In the living animal two carmine, pinkish, or reddish colorations are observed on the snout some distance behind the tip: these mark the position of the cephalic ganglia or nervous centres. As previously mentioned not a few authors, misled by their colour, pronounced them to be hearts. The aspect of the ganglia under pressure is indicated in Plate VI. figs. 1 and 3, *h*; and in large specimens they are pear-shaped under a lens. Each ganglion consists of two divisions—a superior, shaped somewhat like an almond, and an inferior, continuous with the great nerve-trunks. The first-mentioned portion is chiefly

\* *Op. cit.* pl. xxi. fig. 3. *Polia sanguirubra*.

† *Zeitsch. für wiss. Zool.* pp. 85–87, taf. v. & vi.]

‡ *Op. cit.* p. 64, pl. vi. fig. 2.

§ *Beobach. über, &c.*, taf. v. fig. 10.

cellular, being composed of minutely granular nerve-cells, and is connected with its fellow of the opposite side by the *long* or *superior commissure* (Plate V. fig. 1, *f*), which passes over the proboscis. In ordinary circumstances, this commissure is less than half as broad as the inferior, but it is considerably longer. It is a simple ribbon of transverse fibres. As observed in the living animal, these fibres pass on to the superior lobe, where they diverge, some turning slightly forwards, but the majority passing obliquely backwards to the pale central part of the lobe. The only remark made by M. DE QUATREFAGES with regard to the physiology of this band is, that it removes the somewhat surprising state of matters of having a brain composed of two lateral masses, and only one ("sub-œsophageal") commissure. To me, however, this band seems of more importance, since, during the enormous distention which takes place in the extrusion of the proboscis, it is the superior commissure which is stretched to an extreme degree of tenuity. The proboscis, as mentioned, passes through a complete ring of nervous texture, and, during extrusion, forces this outwards in all directions, but chiefly superiorly, the inferior commissure, indeed, being little altered. Nearly half the circumference of the proboscis projects above the level of the ganglion (Plate IV. fig. 5), and the superior commissure must be correspondingly elongated; hence, if this is purely a nervous band, we have a very interesting example of the elasticity of such texture. It may possess elastic as well as nervous fibres, but such are not distinguishable. The *inferior commissure* consists of a thick mass of nerve-fibres, the majority of which sweep backwards to form the lateral nerve-trunks; thus it becomes an isthmus between these cords. A few of the anterior fibres are connected with the central region of the former division of the ganglion.

In long species, such as *O. gracilis* and *O. purpurea*, the ganglia are not correspondingly lengthened, but are rather rounded. In *Tetrastemma* the arrangement of these organs is very similar to that in *O. alba*, so that a special description need not at present be given, further than by referring to Plate VIII. fig. 7, which represents the ganglia in a small specimen of *T. varicolor*, where the inferior commissure is shorter and broader, and the lobes more elongated. This is also the case in *Prosorhochmus*. In the aberrant form, *Polia involuta*, VAN BENED., the ganglia are strictly Ommatoplean in shape, and the lateral nerves, which are not shown by the discoverer of the species, comparatively large. M. BENEDEN'S figure of the anterior branches of the ganglia is erroneous. The lateral nerves lie quite within the longitudinal muscular coat.

Carefully made transverse sections show how incomplete is the impression conveyed by the examination of the parts in a compressed, though living animal. Instead of forming a flattened organ, whose greatest transverse diameter is across the plane of the body, each ganglion has its longest (transverse) diameter nearly perpendicular to the latter (Plate IV. fig. 5, and Plate V. fig. 1). The nerve-cells do not appear to be confined to the superior portion, but occur in the inferior also

(Plate VI. fig. 1), where they are seen on each side of the origin of the great nerve-trunks. In the fresh specimen the sheath of the ganglion is moderately resistant; for under pressure the nerve-cells from the softer interior do not pass through this, but escape by travelling along a portion of the great lateral trunk, and rushing out at its torn end, or pass along other branches, such as the superior and inferior commissures, and the anterior nerves, or through accidental punctures. The nerve-cells are of a yellowish tinge, and minutely granular (Plate VII. fig. 11), and rapidly alter their appearance after escape into the water. Many contain a larger reddish granule or granules, to which the colour of the organ is partly due; but I cannot say I saw all the numerous larger pigment-granules so located, although they might have been. In the fresh as well as in the prepared condition (Plate IV. fig. 5), the entire ganglion is dotted with minute pigment-specks and granules, which are also continued along the great nerve-trunk for a considerable distance. The superior commissure is faintly tinged with colouring matter, but the inferior more so; both are paler than the masses of the ganglia. The colour of the ganglion is not destroyed by sulphuric ether, but is rendered paler by acetic acid.

M. DE QUATREFAGES mentions that in a large *Borlasia (anglicæ?)* he found the cephalic ganglia surrounded by a sheath forming a sort of dura mater, but he could see none in the smaller species. In the Ommatopleans, the muscular and other structures of the head form a somewhat condensed capsule round the ganglia, independently of the delicate sheath-proper of the nervous matter. The longitudinal fibres of the former, indeed, form powerful bands between the ganglia and the inner muscular layer of the body-wall. M. DE QUATREFAGES mentions the occurrence of ventricles in the interior of these organs (ganglia), and figures them in *Polia berea*; such have never appeared in any British form, though, under pressure, collections of oil closely resemble the drawing given by this author. I have also never been able to see so many branches proceeding from the ganglia (as he shows)\* to the eyes, cephalic fossæ, "mouth," and other tissues from the anterior borders, in addition to the great trunks and other twigs posteriorly. The arrangement in the British Ommatopleans is represented in Plate VI. fig. 1, and consists of the following, viz., three very distinct branches on each side of the superior lobe anteriorly; two about equal in size; and a third much smaller, to the outer side. Traces of a fourth branch are also present. The outline of the ganglion throughout the rest of its extent is quite smooth. Various branches from these trunks proceed in the direction of the eyes; but the nature of the cephalic tissues renders it very difficult to trace such an object as a pale nerve-branch with certainty. Dr M. SCHULTZE† gives a tolerably correct view of the ganglia and nerve-trunks in *Tetrastemma obscurum*; no branches, however, occur on the trunks in his figure. This author, in a later publication,‡ founded

\* *Op. cit. e.g.* pl. xv. fig. 14; pl. xviii. fig. 1; pl. xix. fig. 1; and the whole of pl. xxiv.

† *Beiträge zur Naturges. Turb.*, 1851.

‡ *Zeitsch. für wiss. Zool.* iv. 1852.

one of the chief distinctions of his *Enopla* and *Anopla* (Tremacephalidæ and Rhochmocephalidæ) on the structure of the ganglia. Prof. KEFERSTEIN figures only two branches, proceeding from the anterior part of each superior lobe to the eyes in his *Borlasia splendida*, but he represents a kind of mesh-work, formed by three or four trunks between the side of the lobe and the cephalic sac, and a pair of nerves from the inferior commissure. No equivalent arrangement to the two latter series has been seen in our species. M. CLAPARÈDE\* figures the proboscis as passing beneath the great or inferior nervous commissure in *Prosorhochmus Claparedii*, and the central blood-vessel as placed above both.

The *great nerve-trunks* (*n*, in the various transverse sections), springing from the inferior lobes of the cephalic ganglia, pass backwards in this group within the inner (longitudinal) muscular layer of the body-wall to the posterior end of the worm, where they terminate near the tip. They are surrounded by a coat of the usual delicate fibroid stroma of the parts. The branches given off by these trunks are generally pale and indistinct, but by the use of dilute acetic acid in *O. alba*, and in others without such aid, they can be satisfactorily observed. They are easily seen, for instance, in *O. pulchra*, the reddish hue which tinges them at their commencement shining through the translucent integuments. An elaborate plexus of branches from the lateral trunks has also been noticed in the same species. In this form also there remains, even after continued pressure, a peculiar narrowing of the great trunks immediately behind the ganglia, which, if not an original condition, may be due either to comparative immunity from pressure, or a tougher investment. The same constriction is seen in *O. purpurea*. In transverse section the nerves present a delicately granular appearance from the ends of the cut fibres. No one who has seized on such specimens as *O. gracilis* in semi-contraction (though unwrinkled), and drawn them out to treble the length and upwards, can doubt the peculiar elasticity that must pertain to the lateral nerves in these animals. †

The nerve-trunks were said by M. DE QUATREFAGES to lie "between the external longitudinal and internal transverse muscular fibres" of the body-wall; a description which may in some respects apply to the Borlasians, but is inapplicable to the present group. FREY and LEUCKART ‡ mention that the lateral trunks lie to the inside of the muscular coats; but while indicating the different arrange-

\* Beobachtungen, &c. pl. v. figs. 10 and 12.

† The arrangement of the nervous system in the curious foreign Turbellarian, described under the names of *Bipalium*, STIMPSON and GRUBE, *Sphyrocephalus*, SCHMARDA, and *Dunlopea*, PERCEVAL WRIGHT, presents a considerable variation from the foregoing, just as the external form of the head and the digestive system do. SCHMARDA represents the cephalic ganglia as quite separated from each other, except by connecting cords, and the great nerve-trunks placed close together in the median line, with an intervening ganglion at regular distances.

‡ Beiträge zur Kenntniss Wirb. Thiere, p. 72.

ments of the "brain" in *Tetrastemma* and *Borlasia*, they do not explain the distinction in regard to the position of the nerve-trunks in these species. Prof. KEFERSTEIN likewise did not observe this essential distinction between the two groups, but considered *Cerebratulus* the type of the whole. He describes an otolite or two in the middle of the ganglion in a young *Ersteddia pallida*, but I fear such are only pigment-granules and cells, or collections of oil. E. GRAEFFE,\* again, in some brief remarks on a *Tetrastemma* from Nice, states that he found a small cluster of otolite-capsules between the eyes, each capsule containing a crowd of minute otolites. If such were not pigment-cells or structures pertaining to the cephalic sacs, the Mediterranean form shows a most interesting advance on the British in this respect, as well as in having lenses to its eyes. Unfortunately, the author has not figured the structures.

The only British Ommatoplean, so far as I have seen, which shows a special structure in its eye-specks, is *O. pulchra*. In this species the pigment is grouped within a distinct capsule (Plate VII. fig. 8, from a dead, and therefore slightly injured specimen). The eyes in the living animal have a clear patch in the centre, from the projection of the lens-like capsule. In *O. gracilis* and others, a few of the eye-specks are frequently connected together by bridges of the pigmentary substance. Though a pale portion is sometimes seen in the specks of the former, I have not satisfactorily made out a lenticular structure. In *Tetrastemma vermiculus*, which has frequently been sent me from St Andrews, the eyes of each side are connected by a longitudinal patch of dark pigment, so that in contraction the animal seems only to have two large crescentic eyes, of a very characteristic appearance.

*Cephalic Sacs and Furrows.*—Midway between the tip of the snout and the anterior border of the ganglion in *O. alba*, a furrow runs inwards and slightly forwards on the dorsum, ceasing, however, before the middle line is reached; and on the ventral surface a similar though shorter furrow exists, the two meeting in a dimple, furnished with longer cilia, on the side (where the cilia are more active and powerful than usual), which depression leads into the cephalic sac. A short distance behind the ganglia two other superficial furrows occur, each slanting backwards and inwards to meet its fellow of the opposite side in the middle line. These furrows are also continued inferiorly, but with a slightly different direction, so that they meet under the ganglia. The two sets of furrows are very distinctly marked in a flattened head as lateral notches. From the dimple mentioned in connection with the anterior furrows, a thick-walled ciliated duct on each side leads into a considerable ovoid, pyriform, or almond-shaped glandular mass, which lies in front of and rather exterior to the ganglion of the side (Plate VI. fig. 1, *m*); and from what is seen in translucent species, such as *Tetrastemma vermiculus*, it would appear to end in a *cul-de-sac*; the walls, moreover, under pressure are marked

\* Beobach. über Rad. und Würmer in Nizza, Zürich, 1858.

with transverse rugæ. Towards its first part the duct is surrounded by a minutely granular glandular structure, which usually has a somewhat triangular figure. Several glandular masses lie behind, one to the outer, and another to the inner side in this position. The glandular substance around and behind the posterior part of the ciliated external duct contains numerous granules and finely granular circular cells. From the posterior end of the outer mass in such a view, a structure that appears to be a pale duct passes obliquely towards the superior lobe of the ganglion, crossing this for some distance in a direction inwards and backwards. Traces of a cavity are apparent at its commencement, and, besides, it is distinguished from the adjoining nerve-trunks under pressure by not being continuous with the ganglion at its edge. In transverse sections of the snout, each sac is seen to occupy a position to the outside of the cephalic blood-vessel, and somewhat above it (Plate IV. fig. 1, *m*), and to have a special space in the muscular stroma of the head. In large specimens the sacs contain many reddish pigment-granules, and occasionally a large cell filled with coarse granules. Behind the foregoing glandular apparatus lie the coiled ciliated ducts (*m'*), which are sometimes pale and irregularly bulged from included fluid, or else collapsed and minutely granular in aspect. In some specimens of *O. alba* the commencement of the duct is tinged of a faint reddish hue. There seems to be no ground for the supposition that the sacs are connected with other organs. In *O. melanocephala* they are less dilated than in *O. alba*. The coils of the ciliated duct in *O. gracilis* are most elaborate, and can be traced for a long distance backwards by the side of the nerve-trunk. In *O. purpurea*\* the external apertures are not so evident as in *O. alba* and *Tetrastemma*, because the furrows are less distinct when viewed as transparent objects. They are best seen when the ventral surface is upturned, and occur in the angle of the furrow some distance from the margin of the head in this position. The ciliated pit leading inwards is short. Like other parts of the animal, there is a considerable variation in *O. pulchra* from the typical form in the shape and position of these sacs as well as in regard to the furrows. The latter species has numerous short longitudinal or accessory furrows on the front of the ventral grooves, and in this respect is allied to the *Borlasia splendida* of Prof. KEFERSTEIN. Instead of lying in front of the ganglia (in the ordinary position under examination), the sacs are situated laterally and posteriorly, forming somewhat elongated pyriform organs, which adapt themselves to the curves of the ganglia. Each sac is filled with rounded granular cells, reddish pigment and other granules, has a ciliated duct anteriorly, which opens at the constriction or lateral dimple of the head just in front of the ganglia, and posteriorly ends in a ciliated tube which by-and-by bifurcates and extends for a

\* I have a strong suspicion that this is the same species as the *Borlasia camillea* of M. DE QUATREFAGES, which he places next *B. anglicæ*, an association, if I am correct, founded on erroneous principles.

considerable distance backwards by the side of the lateral nerve-trunk. Besides these sacs there is in the snout of this worm a series of well-marked glandular organs in front of the ganglia, one of which lies on each side of the blood-vessel, and is connected with a large lobulated mass in the middle line. In structure these glands are allied to the foregoing, having in their interior rounded granular cells, pigment, and other granules. What in some views appeared to be a duct passed from the posterior end of the external lobule towards the cephalic sacs. Traces of similar glandular masses were seen in other species (*e.g.* *O. alba*) near the middle line of the snout, behind the cephalic sacs, and elsewhere. In *Tetrastemma* the sacs agree essentially in structure with those of *O. alba*, and in such translucent specimens as *T. varicolor* the ciliated posterior ducts are easily traced.

The slight furrows just described on the head in this group have been noticed by few investigators, and only Prof. KEFERSTEIN\* and M. CLAPARÈDE† mention the occurrence of the sacs; the former using the term *Seitenorgane* for their signification, but his notice is very brief. He figures and describes his *B. splendida* as furnished with sacs at the side of the ganglia, but without the ciliated ducts posteriorly; while in *B. mandilla* the latter reach no further back than the ganglia. The former species has a curious series of oblique furrows on the side just behind the snout, which are evidently homologous with those described in *O. pulchra*. M. CLAPARÈDE again figures on each side of the eyes in the young of *Prosorhochmus Claparèdii* a blind sac, apparently unconnected with the ciliated pits above-mentioned; moreover, in the drawing of the adult animal (fig. 10) there is on each side a ciliated duct, but no sac. M. DE QUATREFAGES only noticed traces of these structures in the Ommatopleans; for he describes bridles or bands as passing outwards to the "fossettes céphaliques." In his *Polia bembix* he represents a large nerve passing from the anterior part of each lateral nerve-column, not far behind the ganglion, and which, after a course directed obliquely forwards, ends in a swollen granular manner at the cephalic fossa. A similar arrangement occurred in *P. humilis*; but in this instance the nerve arose from the superior lobe of the ganglion, passed obliquely forwards and outwards, and ended in several branches at the fossa. In *Cerebratulus crassus* and *Nemertes peronea*, again, he figures the nerve as springing from the posterior part of the superior lobe. He does not seem surprised that the nerve-trunks to these fossæ should spring from sites so diverse as the front and back of the superior lobe and the lateral trunk. The disposition of an important nerve-branch in species of the same genus, or even in allied genera, is seldom so varied. The structure appears to have been misinterpreted in Ommatoplea, the sac having been overlooked, and the process or duct, which sometimes crosses to the origin of the great nerve-trunk and ganglion of its side, assumed to be a nerve-branch. M.

\* Zeitsch. für wiss. Zool. xii. 1863, pp. 81 and 82.

† Beobach. über, &c. pl. v. fig. 12.

VAN BENEDEN, though he noticed the sac in *Borlasia*, does not mention more than "fossettes céphaliques" in this group.

*Organs of Reproduction.*—The sexes are separate, and the generative products developed between the inner muscular layer of each lateral region of the body and the glandular digestive chamber, and enclosed in special cavities (Plate XI. fig. 2) formed by transparent membranous sacs (*e*), which are connected with the inner muscular layer of the body-wall. In the matured specimen the ova are observed to extend from the œsophagus almost to the tip of the tail, each ovary containing from one to seven ova, which, when fully developed, are seen with the naked eye through the attenuated parietes of the body. They attain a comparatively large size before leaving the body of the parent; and it is curious that they are not much less in bulk in small specimens, though few in number. The female in the ripe state has a greyish-white appearance, with the dorsal tube for the proboscis extending nearly from end to end, though its diameter is lessened posteriorly from the encroachments of the ovaries. The sperm-sacs in the male generally have a pyriform or flask-shaped aspect, especially in the early condition, being attached to the body-wall by a narrow tubular neck, which at the proper period doubtless gives transit to the contents of the sac. In the early condition the latter is finely granular, then cellulo-granular; and in the mature state it has a finely fibrous or streaked appearance from the spermatozoa. Sometimes both granules and spermatozoa occur in the same sac, and then the former are often observed to be somewhat regularly arranged (Plate VII. fig. 12). The spermatozoa in *O. alba* (Plate VIII. fig. 13) have a slight curve of the body, which gently widens from the tip and ends in a perceptibly larger rounded knob, from which the long tail proceeds. The mature males are easily distinguished from the females by their whitish or pinkish aspect, and their bodies are less bulged. The spermatozoa of *O. gracilis* (Plate IX. fig. 8) are most active wriggling structures, of a more slender shape than in *O. alba* or *Tetrastemma* (Plate VIII. fig. 14), appearing under a power of 1000 diameters as simple rods, slightly larger towards the end from which the elongated and very fine tail proceeds. The sperm-sacs are very numerous in *Polia involuta*; but the tenuity of the spermatozoa (Plate IX. fig. 9) renders their exact structure somewhat obscure. The body of the spermatozoon is elongated, gently curved, and slightly thickened at the end from which the tail proceeds. It is very common, moreover, to observe one or more minute clear globules attached to the body of the spermatozoon, so that the structure seems to have a tail at both ends, or a large flattened head. These appearances have misled even so experienced an observer as M. VAN BENEDEN, who figures\* these organs as possessed of a somewhat globular body, with a tail at each pole. But, independently of the strange exception which such a condition would make in Nemertean physiology, the frequent occurrence

\* Recherches, &c. Mém. l'Acad. Belg. t. xxxii. pl. iii. fig. 11.

of more than one globule on these thread-like organisms, and the comparative steadiness of the body of the spermatozoon, contrasted with the lashing of the tail, might have raised a doubt in the mind of the distinguished foreign author. The spermatozoa in *Tetrastemma vermiculus* (Plate VIII. fig. 12), though minute, are amongst the most active of the group. These structures are slender at one end, and slightly dilate towards the opposite, which is furnished with a very long tail. Just in front of the posterior end there is in certain views a somewhat abrupt swelling of the body, as if from an adhering globule, but none were observed without the enlargement. The ova and spermatozoa in *O. alba* would seem to attain full development in February, March, and April; but the breeding-season of other Ommatopleans ranges from the latter month to November. When fully developed, the mode of depositing the ova and spermatozoa may be illustrated by the following account:—Two specimens, male and female, of *O. gracilis* were taken from a deep vessel, and subjected to examination in a large glass cell. In a very few minutes after the male had been placed on the bottom of the cell tiny jets or jet-like wreaths of sperm-fluid were observed to issue from the sides of the body, rather past the middle, and gradually increased in number, both in front and behind. The body of the animal was soon enveloped in a wavy cloud of the milky substance, whose borders were slowly commingling with the surrounding water, while the numerous coiling jets, like so many miniature wreaths of white smoke from the sides of the worm, were constantly adding to the central mass. This operation lasted only a few minutes, and thereafter the animal crawled about the vessel. The female specimen was now observed to protrude her snout from the mass of sand and mucus in which she was coiled, and crawling to the side of the vessel, deposited in a few minutes a group of ova, about three inches distant from the white edges of the sperm-cloud, and she retired again under the mass of sand and mucus. The change of water probably caused the male to eject his matured spermatozoa, and some sympathetic influence, it may be the diffusion of the latter, induced the female at once to evacuate her generative organs, so as to afford the ova the benefit of the male element. A very few ova were found on examination to remain in the body of the female, and they differed in no respect from those deposited in the vessel. The apertures by which the respective elements passed out in these specimens were readily observed as pale specks, each furnished with a central opening, round which ciliation for the time being was well marked. These openings, as in *Borlasia*, occur a little above the lateral nerve-trunk on each side, and even in specimens of *O. alba* not fully ripened, pressure forces the contents of the generative sacs in the same direction, although no aperture is visible.

Specimens of *O. alba*, which had been in confinement for seven months, deposited their ova about the middle of February; and that this is not later than in the free examples, the receipt of many mature specimens from St Andrews at

the end of March proves. The actual number of ova was not counted; but in one instance the ova of a single specimen covered a circular space of more than half an inch in diameter. Occasionally, in a crowded vessel, they are found above the water-line, adhering to the glass in an irregular mass; but they are not connected together by other than accidental mucus, and easily fall asunder. There is, therefore, a marked difference in regard to the deposition of the ova between this group and *Borlasia*; for in the latter they have a totally different shape, and a special investment of tough mucus. The only exception, so far as I have yet found, in regard to the deposition of the ova in a free condition, occurs in the aberrant form *Polia involuta*, VAN BEN. The bulk of the worm considerably diminishes after spawning, and the body assumes a flattened form, especially marked in large examples. That impregnation of the ova (in *O. alba*) takes place only after deposition, is proved by segregating a female ready to spawn, for then it is found that no further change ensues in the egg. Hence the large size of the male organs, as in fishes and other animals that shed their secretion into the surrounding water.

It is a mistake to describe, as Dr JOHNSTON, M. DE QUATREFAGES, and Drs FREY and LEUCKART have done, the ova as occurring in a free condition between the body-wall, and the *Darm* or digestive cavity. They are always contained in ovisacs. M. DE QUATREFAGES observes that he found at the reproductive season a milky liquid, containing corpuscles of conglomerated globules, in the generative cæca; and the succeeding descriptions and illustrations make it clear, as already stated, that he refers to the walls of the digestive cavity, and the special elements contained therein. Thus it is no wonder he had some difficulty in distinguishing the sexes in the early condition of the generative products, since the cells would be identical in every specimen. He indeed gives a tolerable figure of a cell from the wall of the digestive cavity, as one of the true stages in the growth of the spermatozoa;\* and again refers (Plate XXII. fig. 2) to the glandular wall of the said cavity as representing generative cæca. The spermatozoa, therefore, which he shows, had either been discharged externally, or procured from a specimen in such a condition as to leave no room for doubt. His figure† of the spermatozoa of *N. balmea* is incorrect, for the body is too short and thick. He considered that it was only after the granular corpuscles fell out of the cæca into the lateral cavities that they assumed their special characteristics as sperm-cells. He thus failed to make out the correct anatomy of the parts and the physiology of the process. Dr WILLIAMS‡ states that the "segmental organs" in Lineus, *Borlasia*, and Nemertes correspond in number with the transverse divisions of his great "alimentary cæcum" (digestive cavity), and that there is only one British species (*Polia quadrioculata*) in which it is possible to demonstrate the segmental organs *in situ*

\* *Op. cit.* pl. xxi. fig. 4, and still more plainly in pl. xxii. fig. 2.

† *Op. cit.* pl. xix. fig. 6.

‡ *Philos. Transact.* 1858, p. 131.

as transparent objects. It is almost unnecessary to contradict the last statement, since small specimens of most of the species are more or less translucent. This author also maintains that the group agrees in the structure of its generative organs with the type of the lateral ovarian pouches of the Hirudineï, differing from the latter, however, in having the sexes separate. MM. VAN BENEDEN and KEFERSTEIN give a correct account of the position of the ova and sperm-sacs in the body of the species examined by them; but the term "biliary cæca" used by the former is objectionable, as tending to confound the generative and digestive systems.

M. DE QUATREFAGES makes no mention as to how the ova are extruded, though he points out that CERSTED and DUGÈS were wrong in averring that they escaped through the walls of the body. CERSTED's observation, however, is correct, as subsequently proved by MM. BENEDEN and KEFERSTEIN. FREY and LEUCKART erroneously conjectured that the ripe ova were shed from the posterior end of the body, "as in *Arenicola*."

The unimpregnated ova in *O. alba* (Plate VIII. fig. 8) are pure white, and measure about  $\frac{1}{7\frac{1}{2}}$  of an inch in diameter, the pale spot just before deposition being about  $\frac{1}{2\frac{1}{5}}$ th of an inch. The ovum has two coats—an external hyaline investment (*a*), which becomes considerably firmer after extrusion, and an inner membranous sheath (*b*) of greater delicacy enveloping the vitellus (*c*). With the exception of the pale spot the ovum is uniformly granular, the granules on gaining freedom showing very active molecular motion in the surrounding water. At a particular point there is a very distinct process (micropyle?) (*d*), as if from the remains of a tube that led through the outer coat. In a few hours after deposition and impregnation the pale spot disappears, the yolk divides into two masses, and shortly afterwards into four. On the second day they are almost all in the mulberry-stage. In seven or eight days the contained embryo is observed to revolve within the capsule by aid of its cilia, and the majority are extruded from the 12th to the 14th day. The young animal is furnished with two eyes before bursting the egg (Plate VIII. fig. 11), and the coarse granular matter and globules of the digestive tract are apparent. In such a condition the wall of the ovum is readily ruptured, and in several instances the posterior end of the animal emerged first. No sooner did the young get over their labours of extrusion than they glided rapidly off, head first, in a manner that showed no training was necessary to enable them to progress. Probably the action of the cilia may have some influence in determining their course. In these young animals, which are just visible to the naked eye as minute specks, the proboscis is marked by a paler space (Plate IX. fig. 1), that has on each side of it a dense mass of the granules of the digestive canal. To the outside of the latter are two pale stripes, broader in front, caused by the nervous ganglia and trunks. Two longer cilia mark the posterior end. A further stage of development (after an interval of about eight

days) is shown in Plate IX. fig. 2, under somewhat less pressure. It will now be observed that there are four eyes, the anterior pair of which are largest, and correspond to the first pair. Occasionally a few have an additional pigment-speck or two on one side of the posterior pair. The anterior pair are nearer each other than the posterior, differing in this respect from those of the young *Tetrastemma*, whose eyes are equidistant in both pairs.\* The two ganglia (*h*) are large, pale, distinctly outlined, connected by the two commissures, and give off the lateral nerves (*n*), which approach each other very closely at their posterior termination. The oesophageal sac (*j*) behind the ganglia is well defined; and two pale streaks mark the cephalic sacs (*m*). The proboscis has its anterior opening, and the first region (*a*) its glands, the posterior border being marked by a transverse line (*b*), after which follows an indistinct stylet and reservoir-region. No stylets are visible until much crushed, and then in one specimen two slender spikes, probably from the lateral sacs, were seen. The posterior region of the proboscis bends forwards, and becomes lost at *c*. Shortly after this the lateral stylet-pouches become very evident in some, opening by a short and wide tube into the floor of the anterior chamber, and either containing granules or small stylets, while the central apparatus has no stylet (Plate VII. fig. 6). The specimen had really only granules in its sacs; but to save multiplication of figures one of them was deleted, and filled in with correct drawings of stylets from another example. There is no trace of a central stylet, but the central sac is filled with coarse granules, and they moved with the muscular setting around them, for at this time the latter showed distinct contractions. The muscular space ( $\epsilon$ ) behind the floor of the anterior chamber shows traces of an inner and special lining, which forms a transverse boundary in front. The basal sac is irregular in outline at present, and the shape less defined than in the adult, but, as development advances, the form of the "awl-handle" becomes more characteristic. The lateral stylet-sacs in a few days afterwards were generally furnished with stylets, but these organs were not so sharp and smoothly finished as in the older examples. When the central stylet appears, the granules of the basal sac have a more definite shape than represented in the figure. An outline of the two kinds of stylets is shown in fig. 6, Plate VIII., from the same specimen, and the disproportion between them is evident, thus confirming the previous statement, that each apparatus furnishes its own stylets. The central stylet (*a*) is generally more slender and acute, as well as longer than the lateral (*b*), which have a more globular head than in the adult. As the specimen increases in age, the disproportion between the two sets of stylets lessens—one or more of the lateral being equal to the central in size. The long posterior chamber of the proboscis now contains the peculiar fluid with moving granules, and the reservoir sometimes con-

\* It is curious that in the young of *Planaria* also four eyes should be a common arrangement: indeed, they are present in some species before the embryo leaves the egg.

tracted with force, so as to propel the granules, and even the glandular lining of the cavity itself, forwards to the front of the basal sac. The superficial granular glands of the stylet-region are also well developed.

Some weeks afterwards (and there was no difficulty in preserving them for this period, even without a change of water) four eyes were observed in the majority. The head of the worm is distinctly marked in crawling, and the cuticle richly ciliated, a few longer cilia occurring at the snout and tail. Ciliation is also very active in the cephalic pits, whose openings are circular; and there is, moreover, a slight constriction at this point between the two pairs of eyes. The dermal tissues are well seen, and the ganglia are still relatively large. Every structure pertaining to the proboscis now shows considerable advancement; and it may be noted that the posterior glandular organ is wider and shorter in proportion than in the adult. In each lateral stylet-sac (Plate IX. fig. 13) there are at least three well-developed stylets, whose heads still appear somewhat more globular than in the perfect animal, besides a headless fragment or two, and one or two clear globules. The normal position of these organs in the lateral pouches seems to be transverse. The stylet on the central apparatus is completely formed, and likewise has a somewhat globular head. The muscular cavity ( $\epsilon$ ) is kept in constant jerking contractions under pressure, while the posterior part ( $\theta$ ) is quite still. The other structures, such as the cells of the digestive cavity, had made corresponding advancement, but no blood-vessels were apparent. It may be mentioned, in passing, that the cuticular tissues of these domesticated examples become less transparent than in the wild forms brought from the rocks, and the examination of the internal organs is consequently interfered with. In these young animals also (under pressure) the proboscis generally escaped by rupture at the posterior end, as in *Tetrastemma variegatum*, probably by passing through the anus. In the adult protrusion rarely occurs posteriorly, but almost invariably anteriorly.

The ova of *O. gracilis* (Plate VIII. fig. 9) are much smaller than those of *O. alba*, and when first deposited adhere together slightly, so that they may be pushed *en masse*, but they afterwards lie flatly on the bottom of the vessel. Each likewise possesses two coats. The vitellus is of a dull yellow hue. Though there is no doubt the spermatozoa in this, as in other species, rapidly diffuse themselves throughout a large bulk of water, yet they were applied directly to the ova by means of a pipette. In about four hours many were adhering to the exterior of the hyaline coat, others were within this, while a few seemed to have penetrated both capsules (Plate VIII. fig. 10). In six hours cleavage had proceeded much farther, so as to cause many to have the usual mulberry-aspect. In *O. pulchra* the contents of the ovaries are of a beautiful rose-red colour, with a clear spot in the centre. Each ovisac in the middle of the body contains from twelve to twenty ova, therefore it is unlikely that this is a viviparous species, unless only a single ovum happened to be detained in an ovisac here and there, impregnated and developed.

Numerous specimens of *Polia involuta*, VAN BEN., were sent from St Andrews in April, loaded with ova, and their development could easily be followed out. The newly deposited eggs (Plate XIV. fig. 1), are somewhat ovoid, about  $\frac{1}{250}$ th of an inch in their long and  $\frac{1}{320}$ th to  $\frac{1}{350}$ th in their short diameter, and appear to possess only a single investment. They are not simply enclosed in a sheath, as M. VAN BENEDEN says, but the animal, during deposition, envelopes them and its body in a tough hyaline mucus, afterwards withdrawing itself therefrom, as in *Borlasia*, so that the whole forms a tunnel of mucus, with the ova in its walls. The spiral condition of some of the masses was due to the coiled condition of the animal during deposition. After extrusion the ova pass through the usual stages, and the embryo in each is sometimes ciliated on the tenth day (Plate XIV. fig. 2), although entire dependence cannot be placed on this date, since development occurs within as well as without the body of the parent. In a short time the young are extruded either with a pair of eye-specks, or without them, and furnished with a very long anterior, and a shorter posterior ciliary tuft or whip (Plate XIV. fig. 6). Moreover, numerous adult specimens are found towards the end of April to contain ova with ciliated young, showing that impregnation, as may easily be understood, can take place through the genital pores. In many of the ova the embryo had two reddish eyes, and some were extruded from the body of the parent in a free state, so that they sailed about actively through the water as ciliated pyriform bodies. The ciliation of the œsophageal region in those with the eyes was very distinct; indeed, after the other and apparently more delicate tissues of the animal had become disintegrated, this region was left in active ciliation—dissected out, as it were, by rapid decay. This somewhat globular œsophageal region has probably been mistaken by M. VAN BENEDEN for a mouth. The same author fell into the error of supposing that a form having a smooth outline was developed within its progenitor with the long ciliary tuft, the former representing the *scolex*, and the latter the *proglottis*; in short, as he says, a case of digenesis, and not a metamorphosis. But his figure\* represents the so-called *proglottis* as furnished with two eyes exactly in the same manner as the *scolex*, yet he neither mentions having seen the one form inside the other, nor figures this interesting condition. No such mode of development has ever been seen by me, either in the case of those ova deposited in the unimpregnated condition, or in those developed within the body of the parent; but the same gradual changes ensue in the young of this animal as in *Tetras-temma*, and, as will afterwards be seen, also in *Cephalothrix*.

Many of the parent-specimens having developing young in their interior are feeble, and almost in a decaying condition inside their sheaths, so that their inert bodies seem but the *nidi* for the growth of their progeny, each of which, pro-

\* *Op. cit.* pl. iii. fig. 28.

vided with two boldly marked eyes, and other differentiated tissues, revolves rapidly within its capsule. This evolution of the ova in these decaying adults is a feature analogous to the elaboration of the respective generative products in the headless fragments of male and female specimens of *Lineus longissimus* and others—the last efforts of the parental tissues being devoted to the reproduction of the species.

In *Tetrastemma variegatum* the ova are found in the body of the adult in June and August, and are deposited freely in the vessel. The same changes ensue in the egg as in the other forms, and the young are found in swarms beside the adults in the beginning of July and September. These young forms (Plate IX. fig. 15) are so mobile, that one scarce sees the body of the same shape for two consecutive seconds. The surface is coated with long cilia, by whose aid they are piloted through the water like infusorial animalcules; while, in addition, they are furnished with a single long tuft anteriorly, as described by M. VAN BENEDEN, in the young of his *Polia involuta*. The cutaneous textures are not distinguishable as separate layers, and the entire body has a cellular appearance, probably from the individual elements of the digestive cavity and the cuticular areolæ. No eyes are visible. About a week afterwards considerable progress had been made in size, but the cilia had become shorter in proportion to the bulk of the animal; and though the anterior and posterior ends showed a few conspicuous cilia, the long tuft was absent. There are now four eyes. In another week the stylet-region of the proboscis is nearly complete, the lateral often appearing before the central stylets. The usual mode by which the proboscis escapes under pressure is by rupture per anum. Thus there is a slight divergence in the development of this species, whose young move freely as eyeless organisms, each provided with a long ciliary tuft; while in *O. alba* two well-marked eyes appear in the young in ovo.

Dr SCHULTZE\* first observed that his *Tetrastemma obscurum* was viviparous. He likewise stated that, in the development of the proboscis, the lateral stylets appear before the central, and as the animal grows older, he figures it with two loose stylets lying in the pit of the proboscis—an arrangement, as he supposes, for the supply of the central apparatus. I have also seen a loose stylet or two lying in the anterior chamber of the proboscis, but this occurred both when there was, and when there was not, a stylet on the central apparatus. The physiology of the region, as previously explained, demonstrates that there is no connection between the lateral and central stylets, save perhaps in the composition of the fluid with which both are bathed. Prof. KEFERSTEIN† again details the development of *Prosorhochmus Claparedii*—a species in which the young animals attain considerable advancement before extrusion, for they are found with four eyes, a well-developed

\* *Op. cit.* p. 65, tab. v. figs. 7, 8, and 9.

† *Op. cit.* pp. 89 and 90, taf. vi. figs. 2 and 3.

proboscis, and other organs, before they leave the body of the parent, and on being set free have the same general form as the latter. The larger examples are often doubled within the body of the parent, and apparently invested by the stretched covering of the ovisac, or in large cavities produced by the coalescing of many ovisacs; at any rate, it is clear that to describe them (as Prof. KEFERSTEIN and M. CLAPARÈDE\* have done) as simply within the body-cavity of the worm, is wanting in structural accuracy. It is certainly a curious sight to see these large young animals moving within the body of the adult, apparently without causing the latter any inconvenience. Such, then, appears to be a further stage of the type of development seen in certain species (*e.g.*, *Polia involuta*, VAN BENEDEN), in which, after deposition of the majority, a few ova are left in the body of the parent for subsequent evolution. It remains, however, to be proved whether all the ova in *Prosorhochmus* are so developed (in which case they must be very few), or whether part are deposited at one or different periods or stages, and the rest evolved in the body of the parent. By the examination of this species, I have been enabled to confirm many of the excellent observations of Prof. KEFERSTEIN and M. CLAPARÈDE; but, on the other hand, the determination of the actual position of the mouth in the same animal shows that it does not deviate from the typical Ommatopleans, and that the organ is situated not behind the ganglia, as asserted, but, like the others previously described, quite in front of the commissures. The mouth, moreover, is the most distinct of any I have examined.

It appears to me that such viviparous species do not form a group *sui generis*, but are connected by insensible gradations with the true oviparous forms. Doubtless, in the majority, some of the ova only are retained in the ovisacs, impregnated by the ubiquitous spermatozoa through the genital pores, developed in the sacs, and space afforded for the growth of the young animals by the stretching or rupturing of the membranous walls of the latter. It is a very interesting fact in connection with this subject, that Prof. KEFERSTEIN† has lately discovered a Hermaphrodite Nemertean (*Borlasia hermaphroditica*) at St Malo, in which the anterior sacs were found full of mature spermatozoa, and the posterior distended with developing ova. This can only be explained in one of two ways—either that the species is truly a hermaphrodite one, or that the spermatozoa are passed from the body of a male (in apposition) into certain sacs of the female through the genital pores, there to remain until the other contents of the female generative organs are evacuated.

#### BORLASIA.

*Cuticular Tissues.*—The skin in this group, for which *Borlasia olivacea* may be taken as the type, is allied in structure to that of Ommatoplea, though in the living animal its condition is frequently rendered obscure by the much

\* Beobachtungen über Anat., &c., p. 23.

† Ann. Nat. Hist., 4th ser. vol. i. 1868, p. 229; and Archiv für Naturges. 1868.

greater development of pigment. The body is everywhere covered with cilia, which are most active in the lateral fissures, but longest on the papillæ of the snout. They may be seen in active motion under a lens in good light. Sometimes the motion of the cilia in the lateral fissures is suspended, and again set agoing, without evident cause. Dr WILLIAMS first asserts that the cilia are confined to the dorsal half of the body,\* and then seems to contradict himself by saying farther on that the whole body is ciliated.† The latter, as above-mentioned, is the correct view.

In the living animal the cutis has a cellular aspect (Plate IX. fig. 4), the cells or areolæ measuring  $\frac{1}{1000}$ th of an inch or more, and most distinctly seen towards the tip of the tail in the adult. Sometimes a number of minute clear granules are observed overlying the larger cells, as shown at the lower third of the figure. The pigment-cells and granules reach their greatest development anteriorly, and some of the former contain very dark brownish black pigment in circumscribed masses. The dorsal pigment has in general a longitudinally streaked appearance (Plate IX. fig. 5), a state probably due to the peculiar arrangement of the fibres of the external muscular layer hereafter to be described. In some pale red specimens the coloration is observed to be due to a uniform impregnation of the cutis, and the tint is much deeper than that of the ganglia, which are thus rendered conspicuous by their pallor. Occasionally one or two pigment-cells of exceptionally large size are present anteriorly (Plate IX. fig. 6), and there were three clear granules in the larger of the two figured. The cuticular cells are finer in *Borlasia lactea*, MONT. MS., and the body is not clouded by the granular pigmentary matter. The superficial arrangement in *Meckelia annulata* (Plate IX. fig. 7) is similar, though the cells or areolæ are smaller, and the pigment-granules do not form themselves into streaks.

There are three tactile papillæ on the snout, one of which, from its situation, falls to be described with the opening of the canal for the proboscis. The other two are placed on each side of the central (Plate X. fig. 1), but are not always so prominent. Each is furnished with a series of cilia of greater length than those on the general surface, and which extend from the erected papilla in a radiating or fan-shaped manner. They are probably of great tactile service to the worm. Prof. KEFERSTEIN refers to a "transverse" tactile papilla on the snout of his *Cephalothrix longissima*, which differs from those usually seen in *Borlasia*, and resembles a slight pouting of the lining membrane of the canal for the proboscis.

Under pressure granular masses and globules of mucus resembling oil are extruded from the skin, as in *Ommatoplea*, and often congregate round the borders of fresh transverse sections. But, while in *Ommatoplea* there are only the ciliated and structureless epidermis, a single layer of cutis-cells and the basement-layer, before

\* Report Brit. Assoc. 1851, p. 171.

† *Op. cit.* p. 243.

the circular (external) muscular fibres are reached, in *Borlasia* the structure of the dermal layer is more complicated. Fine transverse sections of *B. olivacea* demonstrate that underneath the ciliated epidermis (*c*, Plate XI. fig. 8), a somewhat thick layer (*d*) composed of granular cells and globules in areolæ, occurs. From the facility with which these contents escape, the drawings show the parts in a slightly altered condition. Beneath this lies a pale structureless basement-layer (*d'*), the presence of which in *Cerebratulus* had misled Prof. KEFERSTEIN into the idea that it was a layer of circular muscular fibres; but an attentive examination of that genus, as well as the present, demonstrates that, while one may be deceived if only transverse sections are made, no doubt can exist in longitudinal sections. This point may readily be settled without reference to the more explicit, because larger, condition of the parts in the great *Lineus longissimus*. A thick compound layer is next encountered in *B. olivacea*, consisting externally of pigment-granules and cuticular globules (*d''*), and internally of a series of powerful longitudinal muscular fibres (*e*). Under a low power, indeed, this compound layer in transverse section appears as one, the pigment and other cells, and the cut ends of the muscular fibres, presenting a similar aspect. The amount of pigment varies of course in different specimens, and is always much more developed dorsally than ventrally. Towards the anterior end of the animal this layer of the cutis (*d''*) becomes thicker, and its reticulations more distinctly marked. Fine longitudinal sections of the snout from above downwards show superficially a series of very beautiful reticulations of a somewhat regular aspect, the chief interstitial bands having a longitudinal direction. Towards the tip of the snout the texture becomes denser in transverse section (Plate X. fig. 4), and the pigmentary matter increases, especially just within the pale external layer of the cutis. A section still further back (Plate XII. fig. 2) exhibits a less dense arrangement, and the pigment is now for the most part grouped into a dorsal and ventral band. The general stroma consists of radiating and longitudinal fibres, the cut ends and granular matter being often situated in the axils of the radiating series. The pigment anteriorly attains its greatest density immediately beneath the pale external layer of the cutis, diminishing in quantity from this point inwards. The snouts of these mobile animals resemble in structure the elaborate arrangements which are sometimes met with in certain organs (such as the tongue) in the higher animals, where extensive and delicate motions are combined with great tactile power.

In *Cerebratulus bilineatus*,\* the arrangement of the two white median dorsal stripes is characteristic, for the pigment is strictly confined to the region corresponding to *d''* and *e* in *Borlasia*; and in transverse section they appear as two patches with an intervening pale space, bounded anteriorly by the basement-

\* *Gordius tænia*, DALYELL, Pow. Creat. vol. ii.

layer of the pale exterior coat, and internally by the circular muscular fibres. In transverse section the cutis of *Meckelia annulata* contains rather small cells (Plate XIV. fig. 11), which retain much of their ordinary shape after mounting. The characteristic opaque white dorsal and lateral pigment-stripes pass throughout the entire thickness of this tissue, while the white touches on the sides that apparently correspond with the openings of some of the ovaries or sperm-sacs do not traverse the entire thickness, but lie towards its inner border.

The skin in many of the Borlasians, e.g., *Lineus longissimus*, *Borlasia olivacea*, *B. octoculata*, *B. lactea*, *Micrura (Stylus) purpurea* and *M. fasciolata*, gives a marked acid reaction when tested with litmus-paper.

*Muscular Coats.*—The longitudinal muscular coat (*e*), which is incorporated with the former cutaneous layer at its commencement, is thick and powerful, and has a well-marked fasciculated aspect in transverse section. At the sides of the mouth, where this coat attains great development, and forms a strong lateral support, there is a very pretty radiated or somewhat arborescent arrangement of the interfascicular substance on transverse section (Plate XI. fig. 1, 2). Such a condition would permit great stretching in all directions without actual separation of the muscular bundles, and is thus eminently adapted for the functions of the parts. The intimate connection of the outer fibres of this layer with the adjoining coat is well brought out in some superficial longitudinal sections of the body, which show the outer bundles of fibres quite separated from each other by rows of pigment and other cells and granules,—the whole having a curiously streaked appearance. Anteriorly this longitudinal layer becomes lost in the tissues of the snout. The next coat (*e'*) consists of a series of circular muscular fibres of considerable thickness, and it is between this and the former that the nerve-trunks are situated. It passes by the sides of the ganglia, and appears to merge into the wall of the passage for the proboscis in front of these organs. In *Cerebratulus bilineatus* this coat is decidedly thicker than usual, a condition which may be connected with the somewhat rounder form of the body generally in the species. Within the last-mentioned coat is a layer (*e''*) of longitudinal muscular fibres, similar in structure to the corresponding stratum in Ommatoplea. Like the former the fibres pass the ganglia to become connected with the muscular channel for the proboscis in the snout.

Certain peculiarities are observable in the dermal tissues of the large *Lineus longissimus* (*Borlasia angliæ*, QUATREF.), and since this species has been taken as the type of the Nemerteans by M. DE QUATREFAGES and others, it is necessary to enter somewhat minutely into the anatomy of the parts, as shown in the transverse and longitudinal sections (Plate XI. figs. 6 and 7). The external cuticular layer (*d*) is proportionally thinner than in the common species. The pigmentary layer (*d'*, *d''*) is divided by a definite black band (*z*), and is distinctly separated from the first or external longitudinal muscular layer by a curious translucent

stratum (3, 3), which in transverse section (fig. 6) has a transversely barred arrangement with linear interruptions, which divide it into numerous and somewhat regular elongated spaces. In longitudinal section, again (fig. 7), this stratum has a wavy aspect, or, if much contracted, presents a series of moniliform streaks. That this layer, however elastic, is not muscular, a glance at the position of the parts in fig. 7 at once demonstrates. It belongs entirely to the cuticular elements, and with the interior pigmentary layer corresponds to the region *d''* in *B. olivacea*, which, in the larger species, attains much greater perfection, and becomes distinctly separated from the longitudinal muscular fibres. The only peculiarities in the muscular coats consist in the very evident transverse streaking of the external longitudinal layer (fig. 7, *e*), and in the presence of certain parasitic (?) cellular masses in it and the next outer layer. These masses lie in definite spaces, and consist of groups of rounded cells filled with granules. In the contracted state of the animal, as after preservation in spirit, the fibres of the circular coat in longitudinal sections are grouped in a wavy manner (*e'*, fig. 7), apparently from the extreme shortening of the parts.

In the arrangement of the muscular system of the body-wall the curious specimen from Balta is distinguished from all other British forms yet encountered. Externally (Plate X. fig. 2, *d'*), beneath the basement-layer of the cutis (which in the fragmentary specimen was almost absent), there is a layer of circular fibres (*e'*). Within the latter is a very powerful layer of longitudinal fibres (*e*), which (layer), however, is not continuous, as in *Ommatoplea* and *Borlasia*, but has at least one very distinct point of separation. Upon approaching the middle line of the dorsum in transverse section, this longitudinal coat becomes thinned off, so as to end on each side of the centre in a blunt point. In addition, there is a somewhat triangular portion (*ea*) cut off by interfascicular substance and fibres. The dorsal curve of the proboscidian sheath is closely applied to this central point of separation, apparently receiving therefrom a few fibres, which retain it in position, while other fibres pass downwards to join the circular layer (*ja*), which here encloses the space for the digestive tract. The separation of the great longitudinal layer of the body-wall is marked externally by a distinct median line, which is rendered more conspicuous by the occurrence of the transverse striæ of the dorsum on each side of it. There is also a slightly marked fissure of this muscular coat inferiorly. This arrangement therefore conforms to the Meckelian type, as seen in *M. annulata*, in which there are two muscular coats, with intermediate lateral nerve-trunks. The deviations from the ordinary aspect in the Zetlandic specimen may prove to be accidental.

The elaborate system of muscles in the body-wall of these worms enables them to perform the most varied and complex motions, so that they have not inaptly been compared to a piece of living caoutchouc. When irritated, the larger species, such as *Borlasia lactea*, MONT., and the true *B. octoculata*, suddenly contract in a

spiral manner like a cork-screw or the stalk of a *Vorticella*, or twist their bodies into a rope of various strands. The great *Lineus longissimus* may now and then be observed in its native pools extended between the Fuci of opposite sides in numerous loops, each several yards in length, and so intricately arranged, that they can scarcely be unravelled by other than the animal itself. The extreme stretching which the body undergoes before it snaps—as in attempting to secure a specimen in an intricate and inaccessible pool—and the extraordinary shortening on immersion in spirit, are only well-marked conditions into which the animal throws its yielding textures at will. A *Micrura*, again, from the deep water of St Andrews' Bay, swims freely on its edge like a fresh-water *Nephele*, or its own ally *O. pulchra*, lashing the water with alternate strokes of its muscular and flattened posterior extremity. Sir J. G. DALYELL likewise noticed this edge-motion in his great "*Gordius*" *fragilis*, but he was not sure whether it was a natural condition, or caused by the confined vessel. *Meckelia annulata* forms in captivity a beautiful silky sheath by its cutaneous secretions, within which it lies in comparative security, until, tempted perhaps by love of change, it searches for a fresh site, whereon to manufacture a new chamber for its protection. In unhealthy and slowly dying animals the skin becomes raised into pale bullæ, not only from corrugation, but from degeneration of the cutaneous textures.

The posterior end of the body in *Micrura* (*Stylus*) requires special mention, since there is superadded a peculiar elongated and contractile style. This appendage seems to be formed by a prolongation of the cutaneous and part of the muscular (longitudinal and circular) textures of the body-wall of the animal. The entire organ in contraction has a granular appearance, the coarsest granules, and occasionally a few circular masses of brownish pigment, being at the tip. Within these coats is a central chamber, which undergoes various alterations in size, and contains a transparent fluid. This cavity is not connected with the digestive tract, which opens by a terminal pore at the base of the process, nor can proboscidian discs be seen therein. I have not as yet ascertained with what system it communicates, but its connection with the circulatory appears most probable. The style is richly ciliated externally, and undergoes many and varied motions, now forming a verrucose knob, now stretched to an extreme degree of tenuity, and apparently assisted in the latter action by the fixing of the tip, whose warty formations seem to perform the functions of suckers, for the animal may be observed crawling about with a loose style, then the tip of the latter suddenly becomes fixed upon the clean and smooth glass, and the whole organ is elongated accordingly. The fixed portion at the tip is usually more dilated than the succeeding part of the style.

In *Cephalothrix*, ERST. (including *Astemma*), the dermal tissues, and indeed the entire body-wall, deviate from the ordinary structure in *Ommatoplea*, *Borlasia*, and *Meckelia*; and while the minute anatomy of this genus bears out the

distinctions—based on external characters, and the form of the nerve-ganglia—given by Prof. KEFERSTEIN, its independent position can be more satisfactorily demonstrated. Externally (Plate X. fig. 3) there is the usual ciliated coating, whose action is most vigorous in the cephalic region. The cutaneous textures are exceedingly transparent, the pigment, if present, being only developed at the snout in front of the ganglia as a rose-pink or reddish shading within the superficial cuticular layer of the parts. The cutis (*d*), composed of the usual granular cells and gelatinous matter in areolæ, has along its inner margin a trace of a translucent homogeneous basement-layer. A very thin layer of circular fibres (*e'*) comes next, the exact structure of which is best demonstrated in the fresh animals, after the addition of a little dilute acetic acid. The fibres are also evident in fine longitudinal sections, but are not satisfactorily seen in transverse sections on account of their tenuity. Beneath this lies a very powerful longitudinal muscular coat (*e''*), the cut ends of the fibres having the usual fasciculated appearance, the inner being somewhat coarser than the outer. At each side a distinct increase occurs at the region of the nerve, where the coat is separated into two portions by a septum of fibres from the circular coat, the nerve lying in the line of demarcation. This arrangement is quite characteristic, and the position of the nerve-trunk probably points to the compound nature of the great longitudinal layer, viz., as analogous to the two longitudinal layers in *Borlasia*, the circular muscular coat cutting off only the lateral portions (*e*), instead of dividing it completely. This genus shows the mobility of the race even in a greater degree than the others. In crawling about the long yielding snout is used as an exploratory or boring organ, which it stretches hither and thither with ceaseless energy, and by its aid is able to push aside its own mobile body in any direction; while through any narrow loop of mucus the latter is drawn like a thread of semi-fluid, yet coherent substance. These animals also progress readily on the surface of the water. When tested with blue litmus-paper the skin of *Cephalothrix* gives a most vivid red stain.

DELLE CHIAJE'S\* description of the structure of the body-wall, if applied to the Ommatopleans, is correct enough, viz., that there is an external layer of circular fibres and an internal longitudinal coat; hence the criticism of M. DE QUATREFAGES requires qualification. The *Polia siphunculus*, D. CH., however, seems to have been a Borlasian, judging from the large triangular slit which lies at a considerable distance behind the snout. H. RATHKE† gives *Borlasia striata* two coats,—an epidermis, and a corium,—combining under the latter both the pale and the pigmentary layers of the skin. He has omitted to notice the external longitudinal muscular layer, and mentions only an outer circular and an inner longitudinal muscular coat. It is somewhat difficult to comprehend the views held by M. DE QUATREFAGES with regard to the same structures, since his descriptions and

\* Memorie sulla storia, &c., vol. ii. 1825.

† Neueste Schriften der Naturforschenden, &c. p. 95, 1842.

figures do not seem to coincide with each other. He divides the skin into three coats, viz., the ciliated epidermis, cutis, and the fibrous coat. Moreover, the cutis has two layers—an outer, formed of a homogeneous transparent substance, presenting in its mass a number of cells or simple rounded vacuoles refracting the light, and an inner, of large elongated cells in a double row; but in his figure\* the muscular elements occupy a bulk so insignificant that some error appears to have been committed, especially as the third layer of the skin is stated to be a transverse fibrous one. It is at all events difficult to see how the enlarged transverse section just noted agrees with his figures iv. and v., pl. 18. Two muscular coats only are described by this author—an external longitudinal and an internal circular—the internal longitudinal being omitted, or rather considered as an aponeurotic layer. He also commits a serious error in affirming that the structure of the dermal tissues in Ommatoplea corresponds with that in *Borlasia anglicæ*. FREY and LEUCKART likewise describe only two muscular coats—an outer longitudinal and an internal circular. Prof. KEFERSTEIN,† while representing the cutaneous textures of *Cerebratulus* (a Borlasian) with greater accuracy, also falls into the mistake of applying what he found in this animal to all the Nemer-teans. He describes the skin as composed of two coats,—a cuticula covered with cilia, and an inner thick, finely granular coat which contains the pigment,—a definition which is scarcely comprehensive enough for the nature of the parts in such as *Lineus longissimus*. He mentions the occurrence of crystals of the form of arragonite in the pigmentary layer of *Cephalothrix ocellata*, but such have not been seen in the British forms, except under the action of chemicals, or after the evaporation of the salt water. His statement, that in *Cerebratulus marginatus* there are four muscular coats—an external circular under the pigment-layer of the cutis, a longitudinal, a circular, and lastly an internal longitudinal—has already been noticed. No more than three muscular coats are present in the Borlasians. Lastly, Dr ANTON SCHNEIDER, in his remarks on the muscles of worms, and their importance in the system,‡ states that in *Nemertes* the following layers occur:—Circular, longitudinal, and circular, besides radiating muscles—a description that is unsatisfactory as regards the British species.

*Cavity of the Proboscidian Sheath.*—This forms a shut sac, as in Ommatoplea, from the bridge of the ganglionic commissure to the posterior end of the worm. The long proboscis glides smoothly in this chamber, whose walls are united with it and other tissues just in front of the commissure. The other contents are the clear proboscidian fluid and its discs. The latter are circular granular bodies, similar to, though smaller than, those of Ommatoplea, and when seen on the edge present a fusiform outline, having a swollen middle and two tapering ends. There are also a few small granules and granular cells. The muscular wall of

\* *Op. cit.* pl. xxiii, fig. 1.

† *Op. cit.* pp. 66–68.

‡ Müller's Archiv für Anat. 1864, p. 595.

this chamber and other points agree so closely, both structurally and functionally, with the same parts in *Ommatoplea*, that it is unnecessary to describe farther than refer to the aspect of the parts in the living animal (Plate X. fig. 1, *o*); and to the various transverse sections, in which the wall of the chamber is lettered *o*, and the cavity *ao*. Sometimes near its diminished posterior end the latter shows a series of moniliform spaces, from internal bridles, and often does not quite reach the tip of the tail either in this group or in *Cephalothrix*. In *Meckelia annulata* the proboscidian sheath is not continued to the tip of the tail either, and it is an interesting fact that this absence coincides, as in the last-mentioned genus, with greatly enlarged lateral vessels. In *Cephalothrix* the chamber presents certain peculiarities, being subdivided by transverse bands of contractile tissue throughout its entire length, so that during the motions of the worm the anterior region is occasionally thrown into a series of moniliform spaces. These contractile septa (though imperfect in the middle), doubtless prove of much service during rupture—an occurrence so liable in this lengthened animal. Moreover, the wall of the chamber is thin, and the circular muscular fibres of the body not much developed; hence the advantages afforded by these safeguards against the inconvenient bulging of the chamber during the motions of the worm. The transparent liquid of the cavity in this genus (*Cephalothrix*) contains flask-shaped bodies and minute clear corpuscles.

Prof. KEFERSTEIN\* seems to have had no definite idea of this chamber as a cavity with special muscular walls, but speaks of the peculiar discs as floating in the body-cavity (*Leibeshöhle*)—an error of some importance. In his two transverse sections of *Cerebratulus marginatus*, he appears to have confounded the wall of the tunnel with that of the proboscis. He is thus less correct than his predecessors FREY and LEUCKART,† who noticed the sheath of the proboscis and its contents.

*Terminal Aperture in the Snout for the Proboscis.*—A channel, ciliated for some distance, leads inwards from the terminal pore to the reflection of the proboscis just in front of the commissures. This channel, shortly after its commencement (Plate X. fig. 4, *a*), is surrounded by an elaborate series of muscular loops (indicated at 2), which, while keeping it closed under ordinary circumstances, permit of rapid and easy dilatation. Immediately within these is a series of longitudinal muscular fibres, which attain a more distinct development somewhat posterior to this point (*a*, Plate XII. fig. 2). A very beautiful group of circular and diverging fibres lies to the outside of the first-mentioned series (2, in the last-mentioned figure), crossing each other in a striking manner superiorly and inferiorly, as well as less distinctly at intermediate points, and forming with the longitudinal and other fibres the intricate stoma of the snout. The terminal

\* *Op cit.* pp. 68 and 69.

† *Beiträge zur Kenntniss, &c.* p. 70.

pore is furnished with a prominent papilla, covered with a fan-shaped brush of cilia, the whole being only occasionally extruded, and no doubt assisting the papillæ previously mentioned in the tactile functions of the snout. This central papilla is sometimes bilobed, and each of the divisions supplied with cilia. In spirit-preparations of large examples of *Lineus longissimus* the proboscidian aperture is distinguished by a slight slit on the inferior surface immediately behind the tip of the snout.

*Proboscis*.—The proboscis (Plate X. fig. 1, *a*) commences as a somewhat slender tube just in front of the commissures, gradually enlarges, continues for a considerable distance of nearly equal calibre, and then, diminishing, terminates posteriorly in a long muscular ribbon ( $\psi$ , sometimes bifid), which, curving forwards in the ordinary state of the parts, becomes attached to the wall of the proboscidian tunnel. Its cavity is continued in front into the canal of the snout, and posteriorly terminates in a *cul-de-sac* at the commencement of the muscular ribbon. It differs from the Ommatoplean organ in certain respects, such as the absence of the stylets, its more slender proportions, and the shape of the glandular papillæ on its internal surface. Experience, indeed, generally enables the observer to distinguish by external characters the proboscis of a Borlasian from that of an Ommatoplean in spirit-preparations, by the abrupt diminution of the calibre at the posterior portion in the latter, caused by the presence of the stylet-region and swollen reservoir; but even where the organ is incomplete, a transverse section at once puts the question beyond doubt. This was illustrated in a well-preserved though shrunken fragmentary specimen brought by Mr GWYN JEFFREYS, the distinguished conchologist, from North Unst, Shetland. At first sight it looked like a Borlasian organ, on account of the absence of the stylet and posterior regions, and from its large size I thought it would demonstrate the structure in that family favourably, but a transverse section gave a true Ommatoplean anatomy, with the characteristic beaded and other layers; and an examination of the animal itself at once confirmed its relationship. In the living animal the organ is proportionally longer than in Ommatoplea, and when cast off becomes thrown into numerous screw-like coils. Thus do the two great groups of soft worms differ in essential characters; and we are taught how unsafe is that classification, *e.g.*, such as SCHMARDA'S,\* which proceeds on other than anatomical grounds.

A transverse section of the proboscis of a Borlasian (*Micrura*) from St Andrews is represented in Plate XII. fig. 1. Externally there is a coat (*a*) similar to that in Ommatoplea, apparently composed of homogeneous elastic tissue, yet showing some granular markings towards its outer border. This coat is tougher than any of the others, and often retains its integrity after they have ruptured. A powerful longitudinal muscular layer (*b*) lies within the former, its cut fibres in transverse

\* Neue Turbel. Rotat. und Anneliden, vol. i. pt. 1, 1859.

section having the same histological characters as in Ommatoplea. At opposite or nearly opposite poles of the circle, however, a remarkable interposition severs the continuity of the layer (as seen at *g*, *g'*). At one pole, two symmetrical bundles of fibres spring from the succeeding circular layer, and, slanting outwards, cross each other in such a manner as to disconnect the longitudinal coat just mentioned, and for a portion of its circumference wedge it between two bands of circular fibres. The outer or oblique bands of circular fibres become lost in the external coat of the organ. The longitudinal layer (*b*) is thus diminished to a blunt point on each side of the crossing of these peculiar fibres, and a region is formed externally which is occupied by a special and somewhat lozenge-shaped group of longitudinal fibres, through which the dotted line *g* passes. The longitudinal layer, especially near the wedge-shaped ends (where the fibres are often grouped in a thicker mass in these preparations, is marked in the centre by a faint linear streak, as if composed of two layers, but this does not continue all round, and is not apparent in every specimen, nor in *B. olivacea*. At the other pole there is a variation in this arrangement, for it is found that an elongated portion (*g'*) is cut off without apparent crossing, the ends of the great longitudinal coat (*b*) being widely apart. It generally happens that towards this side the bulging of the contracted organ occurs, and, it may be, such forces the edges of the longitudinal coat apart, and aids in causing the above appearances; but it would not account for them all. In contraction this coat is sometimes thrown into a silky belt of regularly waved fibres. Within the longitudinal layer is an equally powerful belt of circular fibres (*c*) which, at opposite poles in the transverse sections, gives off the peculiar oblique bands previously mentioned. A basement-layer (*d*), better marked in this species than in the common form (*B. olivacea*), is situated on the inner surface of the latter. There is also present in this species an incomplete belt of longitudinal fibres (*e*) within the basement-layer, and which is not evident in the species just mentioned. Attached to the inner surface of the basement-layer, or in the latter case partly to the incomplete longitudinal layer, is the glandular mucous coat (*f*), which, from lengthened preservation, has in this case become somewhat altered. The glandular bodies are scattered chiefly towards its inner or free surface. In fresh preparations, *i. e.*, in those made from the organ immediately after extrusion from the living animal, a very pretty radiated arrangement of this coat is constantly observed, as if a series of explosions had occurred in the mucous substance so as to scatter the globules and gelatinous bands in a fan-shaped manner. Indeed, the aspect resembles thick and graceful tufts of grass with large spikes, for the granular glands are mostly at the tips of the streaks of mucus, a state doubtless due to their passage outwards under compression. Prof. KEFERSTEIN\* figures this in

\* *Op. cit.* taf. v. fig. 16.

*Borlasia splendida*, but he does not refer thereto in his descriptions. In the fresh specimen it is found that the glandular papillæ are much smaller than in *Ommatoplea*, and widely different in shape (Plate VI. fig. 10, and Plate X. fig. 5), the former representing them in the extruded proboscis, the latter as viewed from without. Under ordinary circumstances they appear to have an ovoid shape, and to vary from  $\frac{1}{1500}$ th to  $\frac{1}{2000}$ th of an inch in size. Under pressure they become either flattened circular bodies or assume an elongated and slightly barred aspect; and, after escape into the surrounding water, the contents are club-shaped or rounded (Plate XIII. fig. 9).

The usual crossing occurs at one of the poles of the circular section of the proboscis in *Lineus longissimus* (Plate XIV. fig. 8), but the separated piece at the opposite pole is somewhat larger than in *B. olivacea*. Like the latter, it also has no inner longitudinal fibres grouped exterior to the mucous layer. In the remarkable form\* dredged in 50 fathoms off Balta by Mr JEFFREYS—and the structure of whose body-wall coincided with the Meckelian type rather than the Borlasian—the proboscis proceeded backwards from the tip of the snout in the usual manner, but instead of the posterior end diminishing insensibly into the long muscular ribbon, the organ divided into two nearly equal trunks (Plate XIV. fig. 12), each about as large as the entire portion, and terminated in a somewhat abrupt and swollen end, from which the long muscular ribbon proceeded. The wall of this peculiar proboscis, so far as I could make out from the single and rather unfavourable example, had the following structure:—Externally there was a circular layer which showed a few granules on the outer margin in transverse section; within this lay a powerful and apparently continuous longitudinal muscular coat, from whose inner surface the granular papillary mucous lining projected. The inner or free margin of the latter was comparatively smooth, a result probably due to the minuteness of the papillæ. Each of the forked portions had the same structure as the anterior region, and the thick longitudinal coat, after bending inwards at the posterior end of the swollen termination, became continuous with the muscular ribbon. The proboscis thus differed from the ordinary Meckelian form in the bifurcation, and in having no distinct circular coat within the longitudinal. It had no closer analogy with the Borlasian or other type.

In *Micrura (Stylus)*, a true Borlasian, the organ is furnished with somewhat slender papillæ, which, under pressure, became lanceolate and pedicled, fusiform, or rounded with granular contents. When viewed laterally, the rounded or flattened papillæ that formerly seemed granular appear to be composed of a series of minute rods set closely together. In some of the elongated structures, however, under pressure, the striæ are longitudinal. When extruded from the organ into the water the elongated bodies in the papillæ cling together in some instances like fibrillæ, and their appearance in the prepared specimens is quite charac-

\* See p. 375.

teristic, the inner or free surface of the coat being covered with a vast number of these elongated glandular structures. These are the bacillary bodies described by Dr MAX. MÜLLER,\* but I have never observed in the British species any of the urticating organs mentioned by this author. The minute structure of the wall of the proboscis agrees with that in *Borlasia*, only the lozenge-shaped portion (*g*, Plate XII. fig. 1) in some specimens was longer than in *B. olivacea*, from the more gradual slanting of the fibres to the exterior.

In *Cephalothrix* the papillæ of the proboscis are acicular, and they are longest towards the anterior part of the organ (Plate XI. fig. 9). In transverse section the walls present a simpler structure than in *Borlasia*; and, though in the living animal an external circular and internal longitudinal muscular coat are apparent, the tissues become so confused after mounting, that I have not yet satisfactorily unravelled them.

Under the action of powerful irritants, such as alcohol, the animal detaches, in its spasms, both the anterior and posterior connections of the proboscis at once, so that the extruded organ remains in its ordinary condition when expelled, and is not turned inside out. In *Cephalothrix*, again, it sometimes ruptures near the ganglia, and is drawn backwards by the ribbon of attachment and its own elasticity; and the animal seems to be unaffected by the injury, which regeneration soon repairs. I have never seen the worm use the proboscis for any purpose; and though M. VAN BENEDEN has observed it extruded in his *Cerebratulus Ærstedii* (which is only DALYELL'S *Gordius taenia*), and threatening its prey, I fear it could not do much harm. The life-like vermicular motions of this muscular tube, both *in situ* and when cast off, have misled Mr BEATTIE† and others, so that they have described the organ as a young animal, and the possessor as viviparous, or else have considered the expelled portion a parasite. This is at once apparent on examining Mr BEATTIE'S specimen of the supposed young animal in the British Museum.‡ The proboscis is reproduced in the same manner as in *Ommatoplea*; and the discarded organ, if not ejected, may be seen floating in the proboscidian cavity amidst much granular debris. Sir J. DALYELL§ states that the usual colour of the proboscis in *Lineus longissimus* is vivid red; our specimens have generally had white or faintly pinkish organs.

M. VAN BENEDEN|| does not mention the tissues to which the muscular retractor of the proboscis is attached in his *Nemertes communis*, and speaks of it as suspended freely in the cavity of the body, like the digestive tube of the Bryozoa. A further remark with regard to the organ in *Cerebratulus Ærstedii* (*G. taenia*).

\* Observat. Anat. de Vermibus quibusdam Maritimis, Berolini, 1852.

† Ann. Nat. Hist., 1859.

‡ Dr BAIRD, in describing *Serpentaria Berryi*, n. sp., also alludes to the very common practice of ejecting the proboscis (not the alimentary canal) after immersion in spirit. It is a habit common to all the Nemerteans.—*Proceed. Zool. Soc.* Feb. 12, 1866.

§ Pow. Creat. vol. ii.

|| *Op. cit.* p. 10.

DALYELL) makes his error still more apparent, for he says, "Toute la trompe se meut librement dans la cavité intestinale."\* Prof. KEFERSTEIN gives a small figure† of a transverse section of the organ in *Cerebratulus marginatus* turned inside out; but, though he indicates the lozenge-shaped space formed by the crossing of the fibres, it is misplaced on one side, and the entire figure is too indistinct for reference.

*Digestive System.*—The mouth in *Borlasia olivacea* is a longitudinal fissure on the ventral surface, situated a short distance behind the ganglia, and varying in size according to the motions of the animal, and the degree of contraction or relaxation. Its ordinary appearance under examination is represented in Plate X. fig. 1. *w*. Certain broad pale lines radiate from the lips of the fissure (which lines in dark specimens are generally pale), an arrangement which led Dr G. JOHNSTON into the error of considering it a nerve-ganglion and branches. These radiating lines or folds are due to the same structural cause as those in the ciliated œsophageal region of *Ommatoplea*—viz., prominent longitudinal rugæ of the thick glandular texture of the organ, which, in this case, permit great dilatation of the parts during ingestion. The number of these rugæ varies, as may be observed by a comparison of the figures. In *Borlasia lactea*, MONT. MS., the mouth is situated very far back, leaving a long space between it and the ganglia. In *Cerebratulus*, again, the aperture is a longitudinal slit, somewhat less marked than in *Borlasia*. The mouth leads into a great ciliated œsophageal chamber (*j*), which commences anteriorly as a *cul-de-sac* behind the ganglia and cephalic sacs, and nearly closing in by its anterior wall the vascular lacunæ there, while it may be said to terminate posteriorly at a distinct incurving of its wall, by becoming continuous with the digestive cavity-proper. In the transverse section (Plate XI. fig. 1), the anterior part of this chamber is seen under favourable circumstances, as a thickly folded glandular mass (*j*), with the ventral slit (*w*) leading quite freely into it. The cavity has not yet attained its full size, and the mouth is severed at its anterior border. Superiorly, a large space is occupied by the proboscidian sheath (*a*), and the great lacunæ (*s, s*), and indications of some other vascular meshes are seen at the sides. The lips of the mouth (*w*) curve inwards, and gradually merge into the ciliated glandular texture of the cavity. A little further back the glandular substance becomes confined to the inner surface of the body-wall (though actually not closely applied thereto), leaving a large central space. In full perfection the chamber and glandular texture are seen in Plate XIII. fig. 6. The minute structure of the wall of this portion of the digestive cavity is similar to that of the ciliated œsophageal region in *Ommatoplea*, being composed of a thick layer of granular gland-cells and basement-substance, raised here and there into prominent rugæ, and richly ciliated on the

\* *Op. cit.* p. 17.

† *Op. cit.* taf. vii. fig. 5.

inner surface. The turning in of the borders of the region is an interesting circumstance, and demonstrates the distinction between it and the succeeding region, even from the earliest condition of the worm, without for the moment regarding the other cardinal facts relating to the peculiar arrangement of the circulating channels on the walls, the thicker texture of the latter, and the total absence of the gregariniform parasites. Moreover, it is only in this region that the ciliated character of the digestive cavity is apparent, probably because the greater firmness of the walls keeps the chamber somewhat distended. In certain lateral views of the animal, the distinction between the œsophageal and the succeeding region is very evident.

Though in the various drawings of transverse sections of *Borlasia* this chamber (œsophageal) is seen in its normal condition, it is well to remember that it undergoes very marked alterations in size, according to the condition of the proboscidian cavity in its vicinity, for the proboscis most readily distends the latter in this region, and bulges it so much that the walls of the former are pressed flatly together at the ventral surface. In the contracted condition of the worm, as after immersion in spirit, the communication between the œsophageal and the succeeding portion of the digestive system is almost obliterated by firm closure.

The second or great division of the alimentary tube extends from the point of inflection previously mentioned to the posterior end of the worm, as a ciliated chamber with glandular and sacculated walls; but the cilia, with the exception of a streak near the tip of the tail, are only favourably seen on making a transverse section of the living animal, though they are actually longer and more active than those on the cuticular surface. In pale species, such as *Borlasia lactea*, MONT. MS., the digestive canal is very distinctly divided, for the posterior region is not only more opaque than the œsophageal, on account of the greater development of its glandular elements, but its borders are crenate from the sacculations. The posterior aperture or anus is situated slightly in front of the tip of the tail, and is well guarded by the muscular structures surrounding it, as may be observed before granular matter escapes, for it requires the impulse of numerous waves of fluid before yielding under pressure. In some favourable specimens masses of cells and debris may be seen revolving within the dilated anus before extrusion. In various examples a distinct anal papilla (Plate XII. fig. 7), furnished with a tuft of longer cilia, is seen projecting posteriorly.

In transverse section (Plate XII. fig. 3), the encroachment made on the cavity by the ovaries, during the period of their activity, is well shown, and also the gregariniform parasites, which often occur so abundantly in these worms. The parasites were first alluded to and figured by Dr G. JOHNSTON,\* afterwards

\* Magaz. Zool. and Botany, vol. i. p. 534, pl. xviii. fig. 1 \*\*.

by FREY and LEUCKART,\* KÖLLIKER,† MAX SCHULTZE,‡ VAN BENEDEN,§ KEFERSTEIN,|| and lately they and certain ova in this species by the author;¶ so that the subject need not be further alluded to here, save to observe that they are strictly confined to the region behind the streaked œsophageal division of the digestive tract, that they hang freely into the cavity, and that the ova mentioned in the last paper probably may not be connected with this particular species of parasite. The occurrence of these ova, however, in specimens so widely different in habitat as St Andrews and South Devon, shows that there is some constancy in their presence. The parasites occur in young specimens scarcely a quarter of an inch in length, and vary in size. When the animal has regained its condition in its native haunts after spawning, the granular cells of the digestive chamber become largely developed, so that in transverse section the body is rounder, and the entire central region filled up by the mass, with the exception of an irregular fissure in the centre; whereas considerable atrophy of these elements occurs during long confinement, or the exigencies of reproduction. Towards the posterior end of the worm, the tract becomes considerably diminished in size, and, in the living animal, more evidently ciliated when viewed from above. The minute structure of the wall of the cavity (Plate XII. fig. 10) has a considerable resemblance under pressure to that of the ciliated œsophageal region in *Ommatoplea*, having a basement-substance, in which are imbedded a vast array of granular glands, and with the inner surface richly ciliated. The contents of the glands (Plate XIII. fig. 7) consist of granular cells and globules, which readily escape from the free border of the organ, and are often ejected per anum.

In *Cephalothrix* the lips of the oral aperture are frequently pouted outwards in the form of a short funnel, so that the animal resembles an elongated *Distoma*, and the ciliation of the entire canal is more apparent than in *Borlasia*. Some circular fibres around the mouth are evident in this genus, and probably exist also in *Borlasia*. The general arrangement in transverse section is seen in fig. 3, Plate X., and the same gregariniform parasites before mentioned, as well as an *Opalina*, likewise occur. In minute structure, the first or œsophageal portion has a much more lax and cellular aspect than the succeeding densely granular region; and from the translucency of the animal, the distinctions in this respect are more exaggerated than in *Borlasia*. In one specimen sent from St Andrews in April, the digestive chamber was coloured of a fine pea-green instead of the usual pale pinkish hue—a state due to the uniform tinting of the cellular elements.

It may now be proper to refer to the presence of another parasitic animal which was found in several specimens of *Borlasia olivacea* from St Andrews in

\* Beiträge zur Kenntniss, &c. † Zeitsch. f. wiss. Zool. bd. i. pp. 1 and 2, taf. i. fig. 4.

‡ Beiträge zur Naturges. Turb., &c. § *Op. cit.* || *Op. cit.* p. 70.

¶ Quart. Jour. Micros. Sc. &c., April 1867.

November. The animals infested by this parasite present a remarkable aspect, the posterior half of the dorsum appearing under the lens to be honey-combed and tracked by pale channels in every direction, as if a microscopic *Tomiscus typographus* had been at work in their bodies. Under the microscope the vast net-work of pale channels have a minutely granular appearance, and numerous small, opaque, ovoid granular bodies likewise occurred. Upon rupturing the body of the worm, a large number of the peculiar structures (Plate XII. fig. 4) slid out of their investments, and sailed about in the surrounding water, generally, though not always, with the upper end in the figure first. They differed totally from the gregarinæ above-mentioned, many of which, however, were present in the same hosts. Externally, they are coated with long cilia, whose activity in the free state is of somewhat short duration, for after a time the animals remain quiet, and they drop off. The body is distinctly segmented, and tapers slightly towards the posterior end; while the surface is marked by very fine longitudinal lines, as in *Opalina*, though in a much more minute degree. Anteriorly, there is a conical portion (*a*), composed of three rather indistinctly-marked segments. Two well-marked annuli (*b*) succeed, the posterior part of the last being narrowed, so as to cause an evident constriction of the body-wall in many positions. Behind these are six nearly equal divisions (*c*), each of which often appears double, that is, has a broad anterior and a narrow posterior belt, as indicated in the figure. The posterior region (*d*) consisted of three indistinct segments. The body was minutely granular throughout, and an internal cavity was apparent from the fourth segment to the last; commencing in the former by a rounded end, and terminating just within the border of the latter. No aperture was observed at either end. The opaque ovoid granular bodies (Plate XII. fig. 6), scattered profusely throughout the infected portions of the Borlasian, were evidently young stages in the development of this species, and they too were ciliated. Upon subjecting them to gentle pressure (Plate XII. fig. 5), transverse segmentation was apparent, the number of segments varying according to the degree of advancement. The parasites were very delicate structures; and in the free state soon broke up into cells and granules, after discarding their cilia as above-mentioned. Transverse section of the affected animals showed that they occurred both in the skin and in walls of the digestive tract; their ravages in the pigmentary layer of the former tissue causing the curious appearances which led to their detection. It is a somewhat difficult point to determine whether the skin, muscles of the body-wall, and the digestive canal, constitute the common area of this creature's depredations; or if it was piercing the former on its way to the surface, or again passing towards the alimentary cavity to be voided per anum. The differently segmented condition of the full-grown specimens, and their internal structure, exhibit a higher type of organisation than the ordinary *Opalina* and *Pachydermon*, which again are more elevated than the *Gregarinæ*. The ease with which

so soft and delicate an organism bores through and tunnels the tissues of its host is wonderful.\*

The *Borlasia* readily feed upon fragments of mussel (as first noticed by Sir J. G. DALYELL). When a specimen has come in contact with a suitable portion, the mouth is enormously dilated, and the bolus, even though of considerable size, rapidly swallowed. The snout of the animal during this process is curved backwards, doubtless to afford assistance by its tactile properties, but there is no extrusion of the proboscis. They also feed on dead specimens of *Nereis pelagica*, ejecting the bristles and indigestible portions afterwards per anum. A specimen measuring about three inches in length boldly seized the head of a large Nephthys, upwards of four inches long, and partially ingulfed its prey. The danger of putting rare specimens, such as *Micruræ*, together in a vessel is great, as the larger generally makes a meal of the smaller. While thus predatory and voracious, they are in turn tolerant of much injury; for instance, one specimen had its head and anterior portion seized and held in the stomach of a *Sagartia troglodytes* for ten minutes, yet the worm subsequently got free, and crawled about as if nothing had happened. After being put in spirit, they occasionally turn their bodies inside out, and expose the inner surface of the digestive cavity. In *Cephalothrix* the contents of the latter are easily observed, and often consist of fragments of its fellows of the same species.

EHRENBERG and DE QUATREFAGES considered the mouth to be the genital orifice, the former observing that a large quantity of mucus was discharged therefrom. Mr H. GOODSIR† thought the canal common to the respiratory, digestive, and generative systems. "In *Serpentaria*," says he, "it acts almost as an organ of digestion, while in *Nemertes* there is a trumpet-shaped exsertile proboscis, which, contrary to the opinion of RATHKE and other naturalists, and according to the opinion already expressed by EHRENBERG, is the intestinal canal." He agreed with EHRENBERG in supposing that the ova escaped into this chamber. His views were rather erroneous, such as supposing that the first region of these worms was composed of a single annulus; but the succeeding or terminal of many, each about an  $\frac{1}{8}$ th of an inch in length; moreover, that each of the separated annuli contained all the elements of the perfect or original animal, viz., a male and female generative apparatus, the cavity common to the generative, digestive, and respiratory functions, and a small dorsal vessel analogous to the intestinal canal of *Nemertes*. *Serpentaria*, therefore, he explains, "is a com-

\* Since the foregoing was communicated to the Society, I find that Prof. KEFERSTEIN, in a recent paper, gives a drawing of a parasite very similar to the above, but he does not say more about it than simply mention, under the explanation of the plate, that it is an enigmatical body from the stomach of a *Leptoplana tremellaris*. Beiträge zur Anat. u. Entwicklungsgeschichte Seeplanarien von St. Malo (Der K. Gesellsch. der Wissensch. vorgel. am 4. Januar 1868), p. 37, taf. ii. fig. 8. It is probable that the same parasite, as in the case of the Gregarinæ, may have a wide distribution.

† Annals Nat. Hist. xv. 1845.

posite animal, each perfect individual consisting of numerous and apparently still unformed or imperfectly formed individuals." Modern researches do not support any of these suppositions. Of the other British zoologists who have examined these animals, Dr WILLIAMS,\* while admitting the digestive nature of this chamber, misinterpreted its true relations. He considered the organ as a closed sac filled with a milky fluid, and having many diverticula, into which the nutritive matter passed by exudation from the proboscis. He appears thus to have drawn up his description from an Ommatoplean, which possessed no large slit leading into the chamber. He denied the existence of the proper anus. While thus deviating from the true structure of the parts, he was correct at least in viewing the chamber as digestive, and quite independent of the generative system placed to its exterior. Sir J. G. DALYELL,† whose untiring scrutiny of the habits of such animals is worthy of all praise, saw a Borlasian (his *Gordius gesserensis*) feeding by the ventral slit, which he therefore correctly termed the mouth. Dr JOHNSTON, in his Catalogue, observes—"There is another and much larger aperture in front, behind and underneath the head. Long mistaken for the mouth, this has been usually described of late as genital, but the orifice is doubtful." M. VAN BENEDEN does not demonstrate that the so-called biliary elements are simply constituents of the wall of the digestive cavity, and not special cæca attached to the sides of the canal. In *Cerebratulus taenia* (his *C. (Erstedii)*) he states that the digestive canal is divided into three compartments—the first short, and corresponding to the œsophagus; the second twice or thrice the length of the former, and representing the stomach; the third extending to the posterior extremity of the worm and constricted at regular intervals, and corresponding to the intestine. I have not as yet noticed this in the British examples, which agree with the typical Borlasian form in the structure of the chamber, although the external aperture or mouth is somewhat smaller. Prof. KEFERSTEIN'S‡ description of the cavity as applied to Borlasia, though brief, is good, and his criticism of VAN BENEDEN'S view, in regard to the "liver" in the same group, fair.

*Nervous System.*—The cerebral ganglia or central organs form two large and conspicuous pale red masses situated a short distance behind the snout of the worm (Plate X. fig. 1). They differ in shape, as seen under slight pressure, from the same organs in Ommatoplea, each half being narrower and more elongated, so as to cause the entire arrangement to have the appearance of a horse-shoe magnet. In some specimens, instead of being more deeply tinted than the rest of the cephalic tissues, they are paler, on account of the deep red coloration of the latter; while, in others, they can scarcely be distinguished under the dense blackish-green coating of cutaneous pigment. They are surrounded by the usual

\* Rept. Brit. Assoc. 1851.

† Powers of the Creator, vol. ii. p. 73.

‡ Zeitsch. f. wiss. Zool. xii. p. 70.

fibres of the cephalic region, besides the sheath-proper of the ganglia. The inferior commissure, often of a deep red hue, is well marked, and placed quite at the front. The anterior curves of the ganglia do not bulge so much forwards on each side as in *Ommatoplea*, and thus the anterior margin of the system forms a nearly uniform transverse line. The superior commissure is smaller and less distinct; indeed, it is with difficulty seen in the living animal as a transparent preparation. Each ganglion is composed of a superior and an inferior lobe; and in minute structure of the nervous matter agrees with that in *Ommatoplea*. On making a transverse section through the ganglionic mass just behind the commissure, the superior lobe is found to be more rounded than the inferior, and to communicate with its fellow of the opposite side by the superior commissure. The inferior is somewhat ovoid, and the great commissure joins it with its fellow; while posteriorly each gives off the great nerve-trunk. In front the two lobes are soldered together, but towards the posterior part a section is now and then found, which shows the posterior end of the upper lobe separated from the inferior. This severing of the end of the upper lobe is not to be confounded with the free rounded sac which lies close behind, as demonstrated in a section in which the knife has cut the left ganglion somewhat further back than the right, and so indicated this separation on that side. The presence of the trumpet-shaped mouths of the ducts of the cephalic sacs in such a section shows that these bodies are posterior and not yet reached by the instrument. Longitudinal sections of the head of the worm exhibit the positions of the ganglia and the cephalic sacs with great clearness, each of the former often presenting different appearances on the respective sides from obliquity of section, but the posterior borders are always distinctly separated from the sacs.

In all the sections of the ganglia a peculiar change occurs after mounting in chloride of calcium, the oily matter of the tissue collecting in curious streaks and circles, and apparently at some parts resisting the penetration of the fluid.

Considerable difficulty is experienced in making out the anterior branches of the ganglia, from the opacity of the snout; but three or four trunks of note are occasionally apparent—two large branches superiorly, and one or two smaller beneath. Some twigs seemed to proceed in the direction of the eye-specks, but their ultimate distribution could not be traced.

The great nerve-trunks (Plate X. fig. 1, *n*) leave the posterior end of the inferior lobe as in *Ommatoplea*, proceed along each side of the body, and terminate a little within the tip of the tail. Their calibre slightly diminishes as they course backwards; and their position is nearer the ventral than the dorsal surface. Branches no doubt exist, but only faint traces of such are seen in the longitudinal sections, for the opacity of the textures in the living animal prevents their being satisfactorily made out. The trunks are imbedded in a fibro-granular matrix of the same reddish hue, and have, in addition, the proper sheath of the

nerve. In some pale species they are marked externally as two pinkish dorsal streaks. These trunks, as already indicated, have a very different position from the Ommatoplean nerves, being situated outside the circular muscular coat, and between it and the great longitudinal. Two muscular coats (circular and internal longitudinal) thus intervene between the nerves and the body-cavity and its contents, whereas in Ommatoplea the nerves are within all the muscular layers. In *Meckelia annulata*, the nerve-trunks are not placed as in *Cerebratulus taenia*, which conforms to the Borlasian type, but lie between the external circular and internal longitudinal muscular coats. This arrangement is characteristic of the Meckelian type.

In *Cephalothrix*, the peculiarity of the ganglia (as first pointed out by Prof. KEFERSTEIN) is the advance of the almond-shaped upper lobes, so that the superior commissure is quite in front of the inferior (Plate XIII. fig. 1). The lateral nerves are placed between an isolated longitudinal fasciculus and the great longitudinal muscular coat of the worm.

In regard to the innervation of the body by the lateral trunks, it is interesting to observe the very long time during which detached fragments of the body survive in several of the long Borlasians, such as *Cerebratulus taenia*, DALYELL, and the great *Lineus longissimus*. A specimen of the latter, for instance, sent from St Andrews in September, broke into pieces on the journey; yet six months afterwards most of the fragments were alive, although the sea-water had not been changed more than once. The head and anterior portion of the worm, which scarcely measured two inches at first, had now grown a body and tail that when crawling measured at least seven inches, and of course capable of much greater extension, so that it looked like an independent animal; and this was accomplished without the aid of any food, except perhaps what it might have acquired from the fragments of its own body in the neighbourhood. Some of the latter measured about a foot in length, and all lay coiled in various ways, with the ends puckered, and in most cases fixed by a whitish cicatrix, which was firmer at one end than the other, and occasionally tapered. A similar power of regeneration was observed in the anterior end of *Borlasia*, *Cerebratulus*, *Micrura*, and *Cephalothrix*, when only a fragment of the body was left behind the mouth; and in *Borlasia octoculata*, a very fragile species, reproduction of a complete head upon each of the fragments ensues, if not with rapidity, at least with certainty.\* One of the most remarkable features, to continue the case of *L. longissimus* as a type, was the gradual development and elaboration of the products of the generative organs (in this case the male elements) in the headless fragments, so that when in February they were placed in clean sea-water, some gave exit to milky clouds of perfect spermatozoa. This would seem in these animals to be the main aim of such a provision, since their very length and softness, if not fragility, apparently court

\* *Proced. Linn. Soc.*, June 1868.

disseverance. The formation of a complete individual, and the prolonged retention of certain functions by the headless fragments, under circumstances so adverse as the above, may give us some idea of the powers of regeneration and vitality possessed by these worms in their native haunts.

Mr H. GOODSIR criticises M. DE QUATREFAGES' description of the nervous system in *Serpentaria* and *Nemertes*, and denies its existence altogether, averring that microscopically the so-called nerve-trunks showed no nervous elements at all, but were the testicles of the worms. I fear, however, this worthy naturalist depended rather upon analogy than actual observation in this case. He accounts for the nervous fibres seen by RATHKE\* (the first who correctly described the Borlasian ganglia) passing out from the cerebral ganglia to the narrow furrows on each side of the head, by supposing them to be seminal tubes on their way to the furrows (his seminal apertures). M. DE QUATREFAGES confined his examinations chiefly to Ommatoplean ganglia. FREY and LEUCKART,† again, confound the cephalic sacs with the posterior part of the ganglia. M. VAN BENEDEN‡ makes a curious remark in regard to his *Nemertes Quatrefagii*—viz., that the “collier œsophagien” is peculiar for its red colour, which hue, he says, is less marked in the other species of *Nemertes*. This colour, he explains, is not due, as believed for a long time, to the nerve-ganglia, but to the vessels which surround them, and it can easily be understood how the ganglia were confounded with the nerve-trunks. Nothing akin to this has ever come under my observation, and the minute anatomy of the region is adverse to the view. M. GRUBE§ had previously made the same remark in describing *Nemertes purpurea*, JOHNST., a species which (judging from the descriptions) seems to differ very materially from *Ommatoplea purpurea*, and is apparently a Borlasian form, but I have not as yet seen any British representative. Prof. KEFERSTEIN is scarcely accurate in affirming that the ganglia in this group are larger than those of the Ommatopleans. In his figure of the parts viewed from the dorsum (Taf. vii. fig. 1), the cephalic sacs are not discriminated.

*Lateral Fissures.*—On each side of the head in *Borlasia* is situated an extensive fissure (Plate X. fig. 1, and Plate XII. fig. 2, *b*), which commences as a shallow groove at the anterior border of the snout, and terminates, as a reddish pit, somewhat abruptly, just beyond the entrance to the cephalic sac. A distinct narrowing of the anterior region occurs behind the fissures in *B. olivacea*, thus marking off the cephalic boundary. There is nothing special in the anatomy of these fissures, for they are formed by a simple extension of the cutaneous elements superiorly and inferiorly, as represented in the transverse section (Plate XII. fig. 2). Their entire surface is covered with very active cilia, which, as before mentioned, I have often seen cease abruptly, and again begin to play vigorously.

\* Neueste Schriften, &c.

† *Op. cit.* p. 16.

‡ Beiträge zur Kenntniss wirb. Thiere, p. 73, taf. i. fig. 15.

§ Archiv für Naturges. 1855, p. 150.

The vapour of chloroform, if applied in sufficient quantity, causes them to cease entirely, but they again commence vibration on the partial recovery of the animal. Mr H. GOODSIR thought that the fissures were the apertures of the male generative system, a supposition, as mentioned, scarcely requiring refutation. Prof. KEFERSTEIN gives a very good summary of the views of previous observers, but, while agreeing with none, he advances no new interpretation of these structures. He concludes by criticising M. VAN BENEDEN'S statements, with which he disagrees, but he has scarcely reviewed them at sufficient length. M. VAN BENEDEN observes that the cephalic fissures are furnished posteriorly with a pit leading into a ciliated funnel, and that the lateral vessels when they approach the ganglia swell out into vesicles ("ils se renflent là en vesicules"), which simulate the ganglia, and which lead their contents to the exterior by the ciliated funnel just mentioned.\* He considers that the central point of this apparatus lies immediately beneath the ganglia on each side; and he has seen, under compression, the pit of the lateral slit adjoin a large canal, which terminated exteriorly by a sort of funnel, and this led into a pouch behind the nerve-ganglia. He did not see any vibratile movement within the vesicle; and states his conviction that this apparatus is similar to that in the Trematoda and Cestoidea. Thus, as Prof. KEFERSTEIN says, he has nearly retrograded to the time of HUSCHKE, who regarded these fissures as connected with the lateral nerves, which he took for canals. In his enlarged figure,† however, he represents the position of the cephalic sacs fairly, but he has a large blood-vessel running to the exterior of the nerves, and extending to the tip of the snout; this, of course, is quite at variance with a true interpretation of the structures in Borlasia.

The cephalic fissures, as characteristic of the Borlasians, are absent in *Meckelia annulata*, their places being supplied by two pale curved grooves on the dorsum and two continuous transverse furrows on the ventral surface of the snout. The furrows are richly ciliated. In the remarkable form from Balta, the snout is surmounted by two curious frilled processes (Plate XIV. fig. 12, *b*), which terminate posteriorly in a long filament. Whether the latter, however, is a structure *sui generis*, or only some normal constituent of the body (such as a nerve) in a peculiar position, the state of the specimen forbids our determining.

*Cephalic Sacs*.—At the posterior end of each lateral fissure, a funnel-shaped tube (*m'*, Plate X. fig. 1) leads into a large globular structure (*m*), often of a pinkish or reddish hue, and the apparent homologue of the cephalic sac in Ommatoplea. This globular sac lies over the origin of the great nerve-trunk on each side, and abuts so closely on the posterior prominence of the upper lobe of the ganglion, as to have led some observers into the error of supposing it only a continuation of the ganglionic texture. Very carefully made preparations and examinations of the adult animal, as well as observations on the young at various

\* Mém. de l'Acad. Roy. des Sc. de Belgique.

† *Op. cit.* pl. i. fig. 5.

stages, remove all doubt on this subject, and show that these globular bodies belong neither to the nervous nor the circulatory system. The funnel-shaped duct (*m'*) is richly ciliated, and the cilia may be traced to the sac, wherein they are continued as a linear streak along its exterior border, but its general mass is not ciliated. The ciliated curve along the external border is well seen in young specimens, but the exact superficial extent of the ciliation is difficult to determine. In favourable examples the walls are observed to be furnished with finely granular cells, which have a clear and distinct nucleus. These cells are most evident on the inner and posterior curves, the outer curve being pale. The sacs project posteriorly into two large cavities (Plate XI. fig. 1, *s, s*) on each side of the proboscidian tunnel, and are thus laved by the circulating fluid, which rushes forwards from the walls of the digestive cavity; but there is nothing to support M. VAN BENEDEN'S views\* as to their continuity with the circulatory system. Their relations to the ganglia have been adverted to previously, and are well shown in some horizontal sections, where one sac has been severed considerably lower than the other. Just in front of the external border of the curved dorsal groove on the snout of *Meckelia annulata* is an ovoid body apparently homologous with the foregoing; but I have not yet been able to trace its anatomy, on account of the opacity of the cutaneous tissues in this animal.

The functions of these bodies would seem to be excretory. Their gradual advance in position and proportional diminution in size in the developing animal would seem to indicate that their function is more important in the young than in the adult. They are quite absent in *Cephalothrix*.

Prof. KEFERSTEIN does not enter into structural detail with regard to these organs in this group, but states they lie at the posterior end of the lateral fissures.

*Eyes.*—These are simply masses of black pigment, arranged on the sides of the snout with greater or less regularity, and without any special optical structure. The textures of the head and nerve-fibres themselves are so unfavourable for observation that I have had difficulty in making out nerve-branches thereto. A more definite structure is observed in the Ommatopleans, both as regards nervous elements and complexity of organisation. Some Borlasians have no eyes (a remark, however, which does not apply to *Lineus longissimus*), or have them only temporarily in their young state, like the developing oysters and *Terebratulæ*; while all the Ommatopleans possess them. It is a curious fact that in transverse sections of the snout (such as Plate X. fig. 4) considerable pigment-specks are seen towards the ventral surface.

\* *Op. cit.* p. 12.—“ En avant, ces vaisseaux aboutissent au-dessous des ganglions cérébraux, et, si nous ne nous trompons, ils se renflent là en vésicules qui semblent appartenir aux ganglions mêmes, et qui conduisent leur contenu à l'extérieur par un court canal excréteur aboutissant au fond de la fossette latérale.”

*Circulatory System.*—The circulation in *Borlasia* diverges considerably from that in *Ommatoplea*, the vessels differing in definition, size, coiling, and contents. The main vessels indeed somewhat resemble long cavities, with contractile walls, within which floats a transparent fluid with corpuscles. I have referred to this system as the circulatory, but the current is driven by the contraction of the vessels now backwards, now forwards, so that it is rather a kind of oscillation.

There are three great longitudinal trunks—confining the description at present to the region behind the œsophageal division of the digestive tract—a dorsal (*p*) and two ventral, *r, r* in the various transverse sections, and in Plate XIII. fig. 2. These three vessels in *Borlasia* were first mentioned by RATHKE.\* The dorsal is a large trunk situated immediately to the outside and to the ventral surface of the proboscidian sheath; while the ventral, also considerable trunks, lie on a lower plane, and nearer the middle line than the nerves. Indeed, when the three trunks are distended in *B. olivacea* and *B. octoculata*, they occupy nearly the entire breadth of the worm under gentle pressure. These vessels are frequently swollen in various ways, sometimes being irregularly moniliform from dilatations, crenate, or simply distended as long pale spaces. The three trunks are intimately connected by an array of simple and rather large transverse anastomosing branches (*y*, Plate XIII. fig. 2), some of which are forked. These transverse vessels have special contractile walls, and are not mere random channels, as may be seen in the longitudinal sections of the worms (Plate XI. fig. 7, 4). They are subject to the various changes of form noted in the larger trunks. The great longitudinal trunks are further connected by meeting at the tip of the tail (Plate XIII. fig. 2). The dorsal vessel generally contracts from behind forwards, and this causes the corpuscular fluid, not only to rush to the front, but also to flow through the transverse branches into the lateral trunks. The latter propel their contents in both directions.

At the posterior end of the œsophageal division of the alimentary canal the three great vessels, for the most part, lose their individuality, and, so far as I have observed, form an elaborate meshwork of vascular spaces (*u, u*, Plate X. fig. 1) around this organ, again meeting in the lacunæ (*s, s*) in front of the cavity, and bathing the bulbs of the cephalic sacs which lie therein. These lacunæ or channels pass forwards to unite at the ganglionic commissures, and the granules of the contained fluid may be seen rushing forwards in the one and backwards in the other. In addition to the smaller meshes surrounding the œsophageal region, two larger spaces are seen on each side of the proboscidian sheath in transverse section, which may be held as the continuations of the dorsal vessel. The reticulations formed by this system are seen under favourable conditions in the living animal (*e.g.*, as represented in Plate X. fig. 1), as well as in numerous transverse

\* *Neueste Schriften, &c.* Danzig, 1842.

sections. I have not been able to see any blood-vessel in the tissues of the head in *Borlasia*. A distended pale portion may often be noticed in the central line between the snout and the ganglionic commissures, as if the animal had gulped water by the aperture for the proboscis, so as to distend the channel, but this has no connection with the circulatory system. Transverse section demonstrates that there is no other channel in the snout in front of the ganglia than that just referred to.

In long pale species, such as *Lineus lactea*,\* MONT. MS., the intervention of an elongated region between the posterior end of the ganglia and the anterior border of the œsophageal region renders a special modification of the circulatory channels necessary. Accordingly, it is found that after the fluid collects in the spaces in front of the alimentary organ, it is conveyed by two long channels forwards to the ganglia, where the same ending occurs as in the other species. These channels seem to be simple elongations of the ordinary lacunæ, and are represented in transverse section in Plate XII. fig. 8; thus forming an intermediate link between *Borlasia olivacea* and the still more elongated post-ganglionic region in *Cephalothrix*.

In *Meckelia annulata* there are two great longitudinal vascular trunks (Plate XIV. fig. 11, *r*), which lie within the inner or longitudinal muscular coat opposite the nerve-trunks, and they are peculiar on account of their large size and the granular nature of their contained fluid. They form a coarse network in the œsophageal region as in *Borlasia*, and are continued forwards just within the border of the snout to meet in a vascular arch.

Whatever special function the œsophageal region may perform in regard to digestion, it is clear that the circulatory fluid bathing its outer wall is placed in a favourable condition for oxygenation, as the mouth now and then must give entrance and exit to sea-water, under the influence of the powerful ciliary currents caused by the entire surface of this division. Besides, it is evident that during the varied actions of the oral aperture (*e.g.*, during feeding) the circulation would sometimes be much interfered with if such a *rete mirabile* did not exist.

In *Cephalothrix* I can only make out two great longitudinal vessels, whose positions are seen in the transverse section (Plate X. fig. 3, *r*), viz., nearly opposite the nerve-trunks (*n*), from which they are separated by the chief longitudinal muscular coat. There is thus in this system also a deviation from the ordinary Borlasian type. The size of the vessels is proportionally larger than in the latter, and their transparent fluid contains a number of minute corpuscles. In the living animal each lateral vessel may be observed to contract regularly and swiftly from before backwards, sending a wave of fluid towards its posterior

\* I am indebted to Mr PARFITT for living specimens of this species from Devonshire.

end, at which the contraction ceases. A reversed movement by-and-by takes place, the contents being propelled towards the snout. Anteriorly the two vessels course forwards by the side of the œsophageal portion of the alimentary canal without sub-division, pass along the sides of the proboscidian sheath in special cavities (*v*), as in *Lineus lactea*, in front of the former, and reach the ganglia, where they communicate. I have not actually seen a junction posteriorly, but analogy would lead us to suppose such to exist. There appeared to be little regularity or rhythm in the movement of the fluid in these vessels, both of which were occasionally seen contracting from before backwards at the same time. Generally, however, the contractions were alternate.

In the fragmentary specimen from Balta, transverse section of the anterior region (Plate X. fig. 2) showed a large ovoid and probably vascular tube (*r*) placed at the inner border of the great longitudinal muscular coat on each side, while the nerve-trunk (*n*) lay outside the latter. The cavity was partly filled in the preparation with minute granular cells. This agrees with the arrangement in *Meckelia*.

Both Dr G. JOHNSTON and Dr WILLIAMS mistook the ganglia for hearts, and the inferior commissure for a connecting vascular trunk. The blood, says the latter author, derived from the cutaneous system of capillaries, is poured by a dorsal vessel into one of the chambers of the heart (the dorsal). From the latter it is sent into the ventral cavity, and thence distributed over the integumentary and intestinal systems. He, moreover, says the blood is red, and always devoid of corpuscles. Such remarks are not based on correct observations. E. BLANCHARD,\* in his examination of *Cerebratulus liguricus*, describes the nervous centres as lodged in a cavity into which the vascular trunks open, and this can only refer to the post-ganglionic lacunæ, though such do not by any means surround the ganglia. I have not seen any vascular space surrounding the "trompe" in front of the commissures, as described and figured by this author; and the fluid of the proboscidian cavity could only have been seen there during the ejection of the proboscis. He shows several longitudinal vessels in *Nemertes*, which are not present in the British forms. I cannot agree with M. VAN BENEDEEN'S† views of the circulation in *Borlasia*, for he describes the lateral vessels as swelling out into vesicles when they approach the ganglia, and their contents conducted to the exterior by a ciliated funnel. The erroneous nature of this supposition has already been noticed under 'Cephalic sacs.' He also mentions that each lateral trunk posteriorly communicates only with that of the opposite side, and concludes doubtfully thus:—"Le long des parois du tube digestif, on voit en outre plusieurs vaisseaux, mais dont les aboutissants sont difficiles à découvrir." Another deviation from accuracy is apparent from his remark (under *Cerebratulus œrstedii*) that "En arrière un gros vaisseau très-

\* Ann. des Sc. Nat. 3<sup>me</sup> ser. tom viii. pl. ix. fig. 5.

† *Op. cit.* p. 12, &c.

large, à parois très-contractiles, qui parait et disparaît par intervalles, occupe la ligne médiane et semble s'ouvrir au bout de la queue." A reference to his figure\* and its explanation at once makes it apparent that he has mistaken the proboscidian sheath for a blood-vessel. Prof. KEFERSTEIN again does not enter into detail with regard to the circulation in *Borlasia*, and his figures and descriptions apply to *Ommatoplea*, with two exceptions, † which represent transverse sections of *Cerebratulus marginatus*. In that through the anterior part of the body five circular vessels at least are transversely cut in the meshes round the œsophageal region, and, moreover, they are connected together by a pink band in the figure, as if from a connecting trunk. I fear the author has been misled by the carmine used in the preparation, for in the British examples of *Cerebratulus* a true Borlasian arrangement is found.

*Generation and Development.*—The sexes are separate, as in *Ommatoplea*, and the ova and spermatozoa developed in their respective sacs between the inner muscular layer of the body and the digestive cavity. The glandular elements in the walls of the latter indeed undergo a certain amount of atrophy during the period of reproductive perfection, as observed in the transverse section through a specimen just before spawning (Plate XII. fig. 3).

In *Borlasia olivacea* the spermatozoa (Plate X. fig. 9) have the aspect of slender rods, with a scarcely perceptible enlargement at the end from which the filiform tail proceeds. When a mass is taken from a living animal, they often adhere to a point by one end, and, spreading around this in a radiating manner, lash the surrounding water with their tails. The spermatozoa of *B. octoculata* (Plate XI. fig. 5) are more minute than the former, and somewhat resemble an awl-handle in shape, with the filament projecting from the butt, which is thus frequently agitated, while the tapered end is comparatively still. In *Lineus longissimus* the outline of the body of the spermatozoon (Plate XI. fig. 4) is less regular than in *Borlasia*, and it seems slightly crenated or moniliform. A very long filament proceeds from the body at the larger end. In *Micrura fasciolata* there is likewise a slight constriction in the middle of the spermatozoon, and the tail proceeds from the larger extremity.

The ova are few and large in *B. olivacea*, smaller and more numerous in *B. octoculata*. Both ova and spermatozoa escape by pores on each side a little above the nerve-trunks, these apertures being often indicated by pale specks along the sides of the worm, and occasionally, as in *Meckelia annulata*, they are boldly marked by white spots. In this species also the rudimentary condition of the generative organs may be seen in transverse section as a series of small globular or pyriform sacs, filled with granules and globules, and situated above the lateral vessel on each side of the body. Thus far there is a certain

\* *Op. cit.* pl. iii. fig. 4.

† *Zeitsch. f. w. Zool.* xii taf. vii. figs. 3 and 4.

resemblance between Ommatoplea and Borlasia (to take, for example, *B. olivacea*), but the moment the ova pass from the animal, and the condition in which they do so, a decided divergence occurs. Instead of being deposited as free circular bodies, the products are here placed within a flask-shaped membrane, with one end narrowed to a fine point, and the whole enclosed in a tough covering of gelatinous mucus, which is fixed either to stone or glass, in the form of a bulky cord, as noticed by ØRSTED.\* When a female specimen is about to deposit ova, she seeks the water-line, or a space above it, and quietly settles along the vessel. By-and-by a copious exudation of tough translucent mucus takes place, which envelopes the entire animal. In this mucus, which when fresh is crowded with small ovoid granular corpuscles from the cutis, the ova are deposited in the flask-shaped capsules, each of the latter corresponding to an ovary, and containing all its ova, viz., from one to seven. Hence, by the nature of the parts, the ova are arranged in a somewhat irregular double row along each side, the extremities of the cord—corresponding on the one hand to the head and œsophageal portion of the digestive tract, and on the other to the extreme tip of the tail—being free from ova. In some instances, the posterior end of the animal was curiously frilled and grooved on the ventral surface during deposition. When newly deposited the mucus is softer and less tenacious than it afterwards becomes, and the same may be said of the membranous flasks. The solidifying of the mucus is analogous to what takes place, under similar circumstances, in the egg-capsules of certain mollusca, e.g., *Buccinum undatum* and others. If one end of the animal be disturbed from its original site on the glass before the ova are all deposited, four rows will be found there instead of two, for sufficiently obvious reasons. The ova of *B. olivacea* are of two shades, viz., white and pale-brownish; and though the dark-greenish examples often lay white eggs, they do not seem to do so always. Each ovum measures from  $\frac{1}{70}$ th to  $\frac{1}{80}$ th of an inch in diameter. The deposition takes place in January and February in those long confined; but some specimens sent from the St Andrews rocks towards the end of April likewise deposited ova, so that some latitude in regard to date is necessary. The American examples deposited their ova in January, and those from Cuxhaven in March; but the *Nemertes communis* of M. VAN BENEDEN only did so in September. It is often observed that impurity of the water causes recently captured animals to lay their ova rapidly, as if from a kind of abortion.

The development of the ova in *Borlasia obscura*—a species apparently identical with our *B. olivacea*—has been described by E. DESOR† up to the period of the extrusion of the young from the capsules; and MAX SCHULTZE‡ and KROHN§ have also investigated the subject, especially the former, so that I shall only dwell on such points as have not been elucidated. Our British forms seem to

\* Entwurf einer Syst., &c., p. 25.

† Zeitch. für wiss. Zool. bd. iv. 1853.

‡ Boston Jour. Nat. Hist. vol. vi. No. 1, 1850.

§ Archiv für Anat. 1853.

offer great facilities for such investigations, and I have had no difficulty in rearing the *Borlasia* at a long distance from the sea.

The ova on deposition in the flask-shaped capsules are uniformly granular and opaque; and when broken up, are seen to be composed of a granular oily matter, which forms streaks and rounded masses, and is not cellular, as described by E. DESOR. The clear, semi-transparent spot mentioned by the latter as occurring in the ova after deposition is seldom visible, though the germinal vesicle (*a*) and dot (*b*) are apparent enough in the centre of a pale oleaginous space, while they are yet in the body of the female (Plate XIII. fig. 8). The cleavage of the vitellus generally commences on the second day, when in some it is found divided into two and in others into four parts. As first pointed out by MAX SCHULTZE, DESOR committed an error when he stated that the irregularity of the divisions of the vitellus distinguished this species from other animals. The divisions proceed regularly and somewhat rapidly; for ova which presented four lobes at 9 A.M. were found at 1 P.M. broken up into a number of rounded masses, so that the ovum had a nodulated or mulberry-aspect. No clear spot was observed in the centre of these secondary masses. During the next four or five days the changes which ensue in the ova consist chiefly of sub-divisions of the vitellus, which daily become finer. There is now a pale spot in the ovum, and a few free granules and cells in the flask, as noticed by DESOR. The ova gradually become smoother in the outline from sub-division of the vitellus, and then only a few nodules appear here and there on the otherwise even circumferences. E. DESOR found the ova ciliated on the twelfth and fourteenth days, MAX SCHULTZE on the eleventh and twelfth, and I have struck the average amongst the British examples on the latter date. The ova, again, which had been left entirely above the water-line did not develop so quickly. At first the ciliation does not cause the mass to revolve, but subsequently this motion takes place with vigour. They continue in this condition for about a month, and then a further change ensues in the contents of the flasks (Plate XIII. fig. 4); and the latter drawing will explain E. DESOR's discovery, as well as enable us to correct a slight inaccuracy into which he has fallen. The opaque ciliated mass previously noticed by-and-by shows a double outline under pressure, caused by the development of the young *Borlasian* within the ciliated coating; indeed, at an advanced stage, as in the middle of the flask represented in Plate XIII. fig. 4, the embryo seems as if shrouded in a layer of fatty cells and oil-globules (*b*), within which it distinctly moves. In such a condition the animal readily escapes from its investment, and at the upper part of the same flask a free example (*a*) is seen. E. DESOR commits an error in his excellent description, when he states that the cells in the interior of the embryo are the "residue of the vitellus destined for the support of the animal;" they are nothing else than the cells in the developing wall of the alimentary canal. The large dark ciliated mass (*c*) at the lower part of the

flask, and the scattered cells and granules, are portions of the discarded external covering of the embryo; and it is to be observed that the cilia on this texture are, if anything, longer than those on the free young animal, though their motion is less vigorous. The "cells" of which this rejected covering is made up are entirely of a fatty nature (Plate XI. fig. 10)—in short, an aggregation of fatty granules, with an oil-globule or two, and capable of changing form accordingly. It is a fact that this debris after a time quite disappears from the flask, and therefore it probably acts as nourishment for the young (being swallowed by the mouth, as in the case of the embryo of *Purpura lapillus*) just as the yolk-sac, by a different mode, does in other animals. In escaping from the flask, the young animals, in many cases, seem to have thrust themselves along the narrow apex, dilating it and bursting through. For a considerable time afterwards they crawl about in swarms amongst the gelatinous mucus, so that the latter has a curious aspect, being filled, in addition, with the transparent flasks from which they have escaped, and a few undeveloped ova. Moreover, it is a common practice for the adult animals to crawl through these masses, and several are generally coiled in proximity. The number of undeveloped ova is extremely small, showing how easy it is to rear these animals, even with very limited supplies of fresh sea-water.

The foregoing development is thus much less complicated than the remarkable evolution of the Nemertean worm, called *Alardus caudatus*, BUSCH., from *Pylidium gyrans*, as described by J. MÜLLER.\* This form would seem to be allied to Sir J. DALYELL'S *Stylus (Micrura)*, since it is furnished with a process posteriorly; and the author states that most examples are eyeless. LEUCKART and PAGENSTECHE† have also recorded another species of *Pylidium*, and the development of the Borlasian worm therein; and they remark that the mouth of the worm is in connection with that of the *Pylidium*—indeed the organ in the latter opens into it—a statement verified in the same volume of the "Archiv" by KROHN.

The young Borlasians, at the stage previously mentioned, are visible to the naked eye as small elongated worms, somewhat tapered at both ends, pale, or rather translucent in front, and opaque-whitish posteriorly (Plate XIII. fig. 5), and in structure now closely approach the adult. The whole surface of the body is richly ciliated, the cilia being especially active in the cephalic fissures, and still more so at the openings of the cephalic sacs. The ganglia are indicated by a paler space (*h*) on each side, but their actual outline is indistinct. There are in all cases at least two well-marked eyes. The cephalic sacs (*m*) are large and well defined, indeed very much larger proportionally than they are in the adult; and from their present position with respect to the ganglia, demonstrate the true

\* Archiv für Anat. &c. 1854, p. 75, taf. 4.

† Archiv für Anat. 1858.

form of the latter, as well as the error into which those authors have fallen who have confounded the sacs in the adult animals with posterior ganglionic enlargements. The sacs open by their ducts at the posterior part of the cephalic fissures (*b*), and the ciliary action can be traced inwards from these points. The œsophageal division (*j*) of the digestive canal is distinguished by its pallor, more evident ciliation, and the well-defined border of the succeeding opaque region (*j'*). The proboscis (*a*) is marked by a central streak of papillæ, and, after tapering posteriorly, it curves forwards, and disappears. The proboscidian sheath (*o*) is observed to be banded here and there anteriorly by transverse bridles; and a clear line is occasionally visible on each side of the opaque alimentary tube, as if from circulatory undulation. An anal papilla (Plate XII. fig. 7), with a ciliated line connecting it with the digestive cavity, is also apparent.

Shortly after reaching the degree of advancement shown in Plate XIII. fig. 5, the young *Borlasians* leave the gelatinous masses, and congregate at the water-line. Hundreds now perish from want of sufficient food, which in their native haunts is doubtless both abundant and suitable, while in the artificial circumstances and confined vessel it is denied them. Two and a half months afterwards the young animals are found still of the same whitish hue, and possessing only two eyes, rarely with an additional pigmentary fragment. The proboscis has much increased in size; indeed, at this time it has attained a comparatively larger development than the digestive cavity, which is in active use, since the young animal is entirely dependent on its own exertions for a supply of food. The œsophageal region is very distinctly marked, though its dimensions are proportionally small when contrasted with the length of the head; at present it is not a quarter the length of the latter, whereas in the adult it is several times longer. Its space is also considerably encroached on by the large cephalic sacs.

At a further stage of development the animal is much elongated, yet still possesses only two eyes. In this condition it has, doubtless, been mistaken for the representative of a different genus, and is probably that referred to by Dr JOHNSTON, under the name of *Cephalothrix* (*Vermiculus lineatus*, DALYELL).

M. DE QUATREFAGES observes that the reproductive organs are digitate in *Borlasia anglie*, and figures them after this manner;\* but such is scarcely a correct definition; neither have any cilia been detected in connection with these structures. Indeed, he has probably mistaken the digestive canal and its sacculations for the reproductive system, as he mentions that out of season the cæca are filled with a fluid more or less opaline. M. VAN BENEDEN remarks that the ovisacs contain from one to a hundred ova in his *Nemertes communis*; but although deposited in a membranous sheath in September, no change had ensued

\* *Op. cit.* p. 182, pl. xx. fig. 8.

in November. His figure of the spermatozoa of this species\* is not correct, as no tails are present, and he describes them as simple rods. He makes the interesting statement, † that in the same animal he found the embryos in some ova covered with vibratile cilia while yet in the body of the parent, while others were only fecundated during or after deposition. The development of the curious form described by Mr ALEX. AGASSIZ, ‡ which, commencing with an oral and anal circlet of cilia, gradually loses these and two short antennæ which subsequently appear, and assumes the form of Nareda (GIRARD) with two eyes, shows that the type of growth is different from that of any British species yet observed. The opening of the mouth (to all appearance) behind the ganglia points to some affinity with the Borlasians; but the absence of so important an organ as the proboscis, which very soon becomes conspicuous in all the young British forms, again leaves us in doubt as to its actual position. The young Nemertean described by Dr BUSCH, § under the name *Alardus caudatus*, would seem to have some relation to *Stylus* (*Micrura*), since it possesses a very distinct tail. The apparent segmentation of the latter, however, is characteristic.

In *Cephalothrix* (*Astemma*) the ova and spermatozoa are developed in a dense series of sacs (that give the animal a transversely barred aspect), which commence a short distance behind the mouth and continue nearly to the tip of the tail. The males are distinguished by their somewhat paler aspect when their reproductive organs are fully developed, viz., towards the end of January and during the subsequent spring months. The spermatozoa (Plate XI. fig. 3) consist of short flattened spindles with rounded instead of pointed ends, that to which the tail is attached being somewhat smaller than the other. In swimming about the two ends appear as clear dots. Though the animal is extremely elongated, the bodies or "heads" of the spermatozoa are comparatively short. The body of the female, with matured ova, presents a dusky or slightly fawn-coloured aspect, the ova, under slight pressure in the living animal, being arranged in dense transverse rows in each ovary. The total number of ova produced by a single female must be very great. In transverse sections they are seen to occupy a large ovoid space on each side of the alimentary canal, upwards of twenty ova—very prettily arranged in a concentric manner—occurring in a single thin slice. The space of the digestive canal in these preparations had thus assumed the form of the letter *x*, the walls approaching each other in the middle, but diverging superiorly and inferiorly; while a wedge-shaped fold from the dorsum below the proboscis, and another from the ventral surface, completed the resemblance. This was the more marked, if the

\* *Op. cit.* pl. i. fig. 13.

† *Op. cit.* p. 13.—"La vésicule germinative ayant disparu, le vitellus s'organise, et, avant la ponte, nous avons trouvé des embryons couverts de cils vibratiles."

‡ *Ann. Nat. Hist.*, 3d Ser. vol. xix. 1867, pp. 208-214, pl. v. figs. 3-17.

§ *Beobacht. über Anat. u. Entwickelung einiger Wirbellos.* Seeth. Berlin, 1851, p. 111, taf. xi. fig. 8.

proboscis had been ejected. The ova are deposited from the beginning of February till June; sometimes adhering together in irregular masses by their edges or a little accidental mucus, at others scattered about the vessel in detached groups. In several instances, however, they were deposited in a translucent sheath of mucus. On deposition they have the aspect shown in Plate XIII. fig. 3, being of a granular structure throughout, with a clear spot and globule, and measuring about  $\frac{1}{80}$ th of an inch in diameter. The ova pass rapidly through the usual stages, and on the 11th of February the embryos were revolving rapidly in the egg by aid of their cilia, and in some cases hatched. The extruded animal (Plate XIV. fig. 3), under moderate pressure, has a globular form, but assumes various shapes when freed—the ordinary one being that of an apple—the long ciliary process representing the stalk, while the body slightly tapers towards the posterior end. It revolves rapidly between the glasses. The body is opaque and granular, with the exception of the margin, which is somewhat paler, from the slight differentiation of the cutaneous textures. Externally, it is coated with long cilia, by aid of which it executes rapid motions, and a tuft anteriorly had the form of a long whip-like process, as during the progress of the animal it appeared like a single mobile thread. The body is sometimes pitted at the origin of the latter, while a slight papilla projects at the posterior end. When fixed between the glasses the cilia were soon pitched off, and the animal resolved itself into a number of cells and granules (Plate XII. fig. 11). In two days the animal is found somewhat elongated (Plate XIV. fig. 4), and the mouth (*a*) becomes more evident as a strongly ciliated slit placed nearly in the centre of the body, which, with the above-mentioned exception, is still uniformly granular. A longer tuft of cilia at the anus is now more distinctly seen. Two days later considerable increase has occurred in the length of the body (Plate XIV. fig. 5), and from the more anterior position of the mouth, it is apparent that the chief increment has taken place in the posterior region. The outline is now pear-shaped, the snout being much less tapered than the tail. The cutaneous textures are more distinctly marked, and the cells, with their refracting contents, very apparent; there is also a corresponding advance in the growth of the granules of the alimentary canal, its ciliation, and the posterior sacculations. The whip-like tuft of cilia on the snout is somewhat shorter, and there now exist a few longer cilia on the side of the head, the posterior group of which (*c*) are evidently the precursors of the long ciliary tuft, which by-and-by appears. There is as yet no trace of eye-specks. A few cylindrical papillæ are observed on the snout and tail, and one or two along the sides, which processes do not seem to result from pressure. In a day or two afterwards some are furnished with one and others with two eye-specks. Moreover, the tuft of cilia on the snout is gradually diminishing in length, while the lateral cilia (*c*) before-mentioned are becoming longer. During a period stretching from March to the beginning of June, the various vessels swarmed with

successive broods of young (from different individuals), which as minute white specks darted about most actively. They did not crawl along the bottom, but, like the young of *Phyllodoce* and other Annelids, swam freely throughout the water after the manner of Infusoriæ, or danced to and fro like Ephemerae in the air. Externally at this further stage of advancement they have still a coating of very long cilia (Plate XIV. fig. 7), which serve as natatory organs, the tuft (*c*) on each side being about thrice as long as the rest, while the long anterior whip has disappeared. There are two large well-defined black eyes, no doubt provided by nature for the exigencies of the youthful state, just as the young of certain molluscs and Balani are similarly furnished. The mouth (*a*), the œsophageal, and succeeding region of the digestive cavity are all richly ciliated. The whole animal is soft and delicate, and none of my specimens survived this stage.

We have thus in *Cephalothrix* a certain resemblance to the development of M. VAN BENEDEN'S *Polia involuta*, already described (see p. 369), and the phases of the growth of the present species likewise corroborate everything that has been advanced in contradistinction to the interpretations of the Belgian author. His views in regard to the *scolex* and *proglottis* receive no support from the foregoing observations, for all the changes that occur are only the gradual and very perceptible shedding of certain cilia, and the general advance of organisation as shown by the differentiation of tissues and the appearance of pigment in the eye-specks. The shedding of the long anterior tuft of cilia by the young *Cephalothrix* has its analogue in the loss of the ciliated ring by the young *Phyllodoce* and others, in the casting of the temporary bristles noticed by BUSCH and LEUCKART\* in the young of a *Nerine*, and by M. DE QUATREFAGES in the young stages of *Hermella*.† I think there can be no doubt that the remarkable tuft of cilia which occurs in the young *Cephalothrix* on each side of the snout, and which attains its full development after the long anterior tuft has ceased to be conspicuous, is connected homologically with the entrance to the cephalic sacs in the Ommatopleans and the fissures in the Borlasians, as well as with the ciliated ring of *Phyllodoce* above-mentioned. It is an embryonic type of a structure which disappears entirely in the adult form. The delicacy of the young at the period of the full development of the eye-specks is an interesting feature; but it prevented my observing their growth into perfect animals.

Thus, so far as development goes, *Cephalothrix* is nearly allied to the Ommatopleans, especially to *Tetrastemma variegatum*, *Polia involuta*, and probably to others of the group not yet investigated; while, in the structure of its digestive system, circulatory apparatus, and the unarmed proboscis with its bridled sheath, it leans rather towards the Borlasians. Prof. KEFERSTEIN in his proposed classi-

\* Ann. Nat. Hist. 2d ser. vol. xvi. p. 259, pl. vii.

† Annales des Sc. Nat. 3d ser. tom. x. 1848.

fication of the Order\* rightly places the genus in a special Family, called *Gymnocephalidæ*, whose chief characteristics as described by him are:—Absence of cephalic fissures; brain like that of Polia, but the superior ganglion covers the inferior much less, and is advanced in front of it. He bases his statement of the relationship to the Ommatopleans, as it appears to me, on somewhat questionable grounds, for the ganglia are by no means closely allied in form and structure to those of that group.

#### ANNELIDA.

In the following part of the paper I purpose making a few remarks on the structure of some recent additions to the Annelidan fauna of Britain, as well as of a few species believed to be new to science. Many of them have been known to me for years, and, indeed, were figured and described in my MS. long before the appearance of M. DE QUATREFAGES'S "Annelés" and Dr A. J. MALMGREN'S "Catalogue of Northern Annelids;" but the publication of these and other recent works on the subject has occasionally anticipated me in nomenclature—a kind of loss, however, which I esteem rather lightly, since so much yet remains to be done in the minute structure of the entire class.

*Amphinome vagans*, LEACH(?)—Two genera have hitherto represented the British Amphinomea, viz., *Euphrosyne* and *Spinther*, and this species introduces with certainty a third. Two very minute specimens ( $\frac{1}{4}$ th of an inch in length), from St Magnus Bay, occurred in an extensive collection made last year (1867) by Mr GWYN JEFFREYS, while dredging in the Shetland seas. The segments numbered in the one twenty-three, and in the other twenty-seven. The head agrees with that of *Hipponœ*, AUD. and ED.,† with which genus I at first thought it most closely allied, but the feet are biramous. In these specimens also no caruncle can be observed, the head forming a smooth rounded eminence, from which a subulate antenna projects. No eyes are present. There are two antennæ in front of the median at the anterior border of the snout, and two others at a distance behind. The bristles (Plate XV. fig. 1) of the superior and inferior lobes of the feet agree in structure, and consist (1) of a somewhat stout kind (*b, c*), which has serrations on one side, and thus not observable in all positions; and (2) of various modifications of a peculiar bifid bristle, some of which (*a*), especially towards the posterior end of the body, show a swollen part below the bifurcation, with a short and simple limb, and a longer process serrated on one side, while others have the serrated limb extremely elongated and tapered to a fine point, and with little or no swelling at the bifurcation. The inferior cirrus is very small. A large specimen from the Channel Islands seems to belong to the same or a closely allied species, but there are sixty-seven

\* Zeitsch. für wiss. Zool. xii. 1863.

† Hist. Nat. du litt., &c. tom. ii. p. 128, pl. ii B. fig. 10.

segments and four distinct eyes. The head in the latter is pale, somewhat horse-shoe-shaped, with two short conical tentacles in front, and two longer ones a little behind—opposite the swollen part of the snout. A curved line separates the anterior from the posterior region of the head, the former being flattened, the latter more elevated, and furnished with four reddish eyes, the anterior pair of which are about twice the size of the posterior. A little behind the anterior pair a filiform tentacle projects upwards in the middle line, and close behind this a wrinkled ridge (caruncle) extends to the anterior border of the third bristled segment. The sulci between the first three bristled segments are somewhat less marked, and the slope of the bristles more oblique, but the rest are very distinctly separated; indeed, the body has a somewhat moniliform aspect. The branchial tuft springs from a point behind, and rather below the dorsal fascicle, and consists of about four pale finger-like processes, which arise from a common basis; they commence on the second segment, and continue almost to the tip of the tail. In this example, the swelling below the tip of the bristles, corresponding to fig. 1, *a*, was not very evident, and the serrations of the extremely elongated distal portion widely separated; and, indeed, I was at one time disposed to regard the animal as specifically different. The bristles of these animals are extremely fragile, and the majority are broken during the efforts to decipher their structure. The crop commences at the posterior third of the fourth bristled segment, and extends to the posterior border of the sixth; it is truncated anteriorly and posteriorly, and swollen in the middle. The commencement is marked by two brownish specks. The published descriptions of the species of *Amphinome* make it somewhat difficult to determine them with accuracy, and I am by no means certain at present that SAVIGNY refers to this form under the above-mentioned name. I had provisionally termed the two minute eyeless specimens from the Shetlands *Hipponöe jeffreysii*,\* but I think they may more correctly be grouped with the example last described. The *Eurythöe borealis* of Sars† is a very closely allied form.

*Lætmonice filicornis*, KINBERG.‡—Three British species of the family Aphroditaceæ are recorded in the Catalogue of the British Museum, and one since the publication of the latter by Dr BAIRD; but I agree with Dr MALMGREN in considering *A. borealis*, JOHNSTON, only the young of *A. aculeata*, and the *Lætmonice kinbergi*, described by Dr BAIRD,§ as *L. filicornis* of KINBERG,|| a species which abounds on our north-western and northern shores, just as *Hermione hystrix* does on our southern coasts. KINBERG does not show the recurved fang towards the extremity of the ventral bristles—an error probably due to the inaccuracy of his artist. The dorsal bristles are very large and powerful, and

\* Ann. Nat. Hist. Oct. 1868.

† Kongliga svenska Fregatten Eugen., &c., 1851–1853, p. 7, taf. iii. fig. 7.

§ Dr BAIRD is now of the same opinion.

‡ Christ. vid. Selsk. Forh. 1861, p. 56.

|| Proc. Linnean Soc. vol. viii. p. 180.

taper towards both ends, especially the terminal. The recurved fangs of the latter are not always readily seen until the bristle is turned round.

*Lepidonotus pellucidus*, EHLERS.\*—Amongst the Lepidonoti dredged in Lochmaddy, North Uist, in 1865, this peculiar species occurred. The head has two rounded and prominent lobes in front, that do not form the acute angles seen in the common species. The eyes of each side are placed close together, while the pairs are widely separated, and situated far back. EHLERS's description and figures of the bristles need improvement. These structures throughout are pale and translucent, the superior fascicle of the foot having a series of slightly curved bristles (Plate XV. fig. 2, *a*), whose rows of secondary spines (about eight in number) are very wide apart inferiorly, while the tip of the bristle is notched, as if from a minute claw. Those of the ventral bundle are equally peculiar (*b*, same figure), having a short but well-marked claw at the tip, with a small spike adjoining. The terminal portion is somewhat flattened, and marked by oblique rows of secondary spikes, while it gradually widens inferiorly, and terminates in an abruptly dilated shoulder, furnished with a projecting series of secondary processes. The latter appeared to be similar to the spikes of the dorsal bristles, and the intervening angle was filled with debris. Dr EHLERS does not discriminate the bifid nature of the inferior bristles.†

*Polynoe longisetis*, GRUBE,‡ a species described as British by Mr E. RAY LANKESTER,§ under the name of *Harmothoe malmgreni*, though unfortunately, owing to the engraver, its bristles have not been figured with anything like recognisable accuracy, has been found after storms at St Andrews. It is distinguished at once from *L. cirratus* (*Harmothoe imbricata*) by the paler and more resplendent bristles which flank its sides, by the structure and greater pellucidity of its scales, and by the structure of its dorsal cirri. The dorsal bristles are almost identical, except in length, with those of *H. imbricata*; while the ventral, though formed on the same plan, characteristically exceed those of the latter in the length of the terminal spiked portion (Plate XV. fig. 3). The dorsal cirri (Plate XV. fig. 3, *a*) present scarcely any swelling below the tip, are pale throughout, and have only a few pale warts, so that the entire organ is much smoother than in the common species. *P. longisetis* exhibits a very close affinity with *Lœnilla glabra*, MALMGREN.||

*Halosydna gelatinosa*, SARS,¶ a species first found on the shores of Norway by this celebrated naturalist, and afterwards by KINBERG\*\* and LOVEN,†† is abundant

\* Die Borstenwürmer, &c. p. 105, taf. iii. fig. 5, 7–13, and taf. iv. fig. 1–3.

† M. CLAPARÈDE probably refers to this species (in his recent work "Les Annélides Chétopodes du Golfe de Naples"), under the name of *Hermadion fragile*.

§ Archiv für Naturges. xxix. 1863.

‡ Linnean Trans. vol. xxv. p. 375, tab. 51, fig. 28.

|| Nordiska Hafs-Annulater, 1865, p. 73, tab. 9, fig. 5.

¶ Beskrivelser og Jattagelser, &c. p. 62, pl. ix. fig. 25.

\*\* Kongliga svenska Fregatten Eugénies, &c. p. 19, taf. v. fig. 26.

†† Cited by MALMGREN, *op. cit.* p. 82.

in the stomachs of cod captured off St Andrews Bay, and a few specimens also occur at low water under stones. In the scale of the living animal a series of radiating lines are observed to stretch outwards from the point of attachment. The dorsal tuft of bristles is not conspicuous, and consists of a series of delicate translucent bristles, with faint serrations at the tip. The bristles of the ventral bundle are characteristic (Plate XV. fig. 6, 6 *a*, 6 *b*), being pale, elongated, and flattened out at the tip in varying degrees. The claw at the extremity of the broad examples is short and strong, while the inferior division is slender. The oblique transverse lines from the rows of spines are also very distinctly marked.

*Sthenelais dendrolepis*, CLAP.\* was dredged in 90 fathoms, off North Unst, Shetland, by Mr JEFFREYS.—It has rather the aspect of *S. boa*, JOHNSTON, than *Sigalion mathildæ*, AUD. and ED., but it can at once be observed that its bristles are more elongated than in either of these species. The form of the anterior scales also approaches that in *S. boa*, being somewhat quadrate, with one end rounded; but instead of having the simple papillæ which characterise the margin of the scales in the latter, the new species has peculiar pinnate processes (Plate XII. fig. 12); the whole having a tree-like figure, while the shape of the pinnæ and the contour of the process in general readily distinguish it again from the pinnate appendages on the scales of *S. mathildæ*. The process in the latter has a less robust form, its pinnæ are hyaline cylindrical processes; whereas in the present species they are lanceolate and granular lamellæ, with a narrowed papillary tip. The specific differences are likewise very apparent in the form of the feet and their appendages, the superior lobe being somewhat leaf-shaped or ovate, with a simple terminal process superiorly, and shorter than in *S. mathildæ*; the inferior lobe again has the spine-papilla much more prominent than in the last-mentioned species. While the bristles of *S. mathildæ* are proportionally more slender than in *S. boa*, here they exceed both in length, especially as regards the terminal process. There is a general resemblance in all the three species as regards the superior fascicle, but the inferior groups differ very characteristically. In the new species the superior bristles of the series with the jointed tips (which adjoin the short tapering-spiked forms) have the terminal portion of the shaft covered with whorls of somewhat sparse spikes (Plate XV. fig. 5), which (spikes) are much more numerous than in either of the others before mentioned; while the stouter series next these (Plate XV. fig. 4) have the same portion of the shaft closely and transversely rowed with minute spikes. The spikes on the terminal portion of the shaft of the inferior bristles are likewise more distinct, and the terminal clawed portion longer.

\* Les Annélides Chétopodes du Golfe de Naples, p. 99, pl. iv. fig. 4, and pl. v. fig. 1. I had described this new species under the name of *S. buskii*, but the unavoidable delay in the publication of the present paper gives M. CLAPARÈDE's title the priority, if, as I am inclined to think, it refers to the same species.

*Sthenelais limicola*, EHLERS.\*—Another species of *Sthenelais*, brought in numbers by Mr JEFFREYS from the Shetland seas, seems to be identical with Dr EHLERS'S species from Quarnero, in the Adriatic. The anterior scales are furnished, towards the outer margin, with peculiar processes, which, so far as regards our examples, are uncharacteristically represented by the German naturalist. The processes are irregular, either simple, bifid, or divided into several pieces, and the margin of the scale is generally folded back under examination, so as to render them indistinct. In the first scale the processes are papillary and undivided. The dorsal lobe of the foot has four or five elongated papillary processes superiorly, and a peculiar broad and curved lobule projects upwards from the inferior lobe. The inferior bristles have their terminal clawed portions shorter than in *S. mathildæ*, and those corresponding to figs. 4 and 5, Plate XV. (*S. dendrolepis*), have only two or three spines at the terminal portion of the shaft. Dr EHLERS'S figures of the bristles are not good, whether as applied to this or any other species of *Sthenelais*—no compound claw, for instance, appearing on the terminal process. The animal also possesses four eyes, instead of the two mentioned by the foregoing author, the anterior pair being hidden from ordinary observation in two sulci under the squamous processes at the base of the median tentacle. This may be the *Aphrodita arcta* of Sir J. DALYELL,† a species likewise brought from Shetland.

*Notophyllum polynoides*, ERSTED.—A specimen was procured from the deep-sea fishing, off St Andrews Bay. The feet are described by Dr MALMGREN,‡ as having the dorsal lamellæ of an elliptico-subrectangular or unequally reniform shape; and in this the new or regenerated plates were somewhat reniform, especially posteriorly, while the older inclined to an elliptico-subrectangular form. The new lobes are even at the edges, but the older are slightly frilled or waved—an appearance intensified by the coloured border of rich blackish-brown, which glistens in the play of light with a purplish-red iridescence. They are also characteristically marked with small groups of white grains. The structure of the bristles is represented in (Plate XV. fig. 9), and consists of a long smooth shaft, which terminates in the swollen end and jointed tip, seen laterally in 9 *a*, and in profile in 9 *b*. The terminal portion is finely serrated, and on each side of its base the shaft of the bristle sends off a series of short spikes, which are inclined towards the serrated edge of the terminal division.

*Ophiodromus vittatus*, SARS.§—Dredged rather abundantly on a bottom of tenacious grey clay and mud in Lochmaddy, in from four to eight fathoms, and rarely met with there under immersed stones at extreme low-water. Length,  $2\frac{1}{4}$  inches; head small, distinct, furnished with five tentacles—two lateral on each side, and a median; the inferior or external lateral being furnished with a thick

\* Die Borstenwürmer, &c. 1864, p. 120, taf. iv. fig. 4–7, and taf. v.

† Pow. Creat. vol. ii. p. 170, pl. xxiv. fig. 14.

‡ Nord. Hafs-Annulater, p. 93.

§ Forhandlinger i Videnskabs-Selskabet, 1861, pp. 87, 88.

basal joint, and a more slender distal portion. Eyes four, the anterior pair being larger as well as more distant from each other than the posterior pair. The colour need not be referred to further here, than by simply mentioning that the dorsum is of various shades of lustrous brown, banded at intervals with belts of pale iridescent blue; while the under surface is of a deep, dark madder-brown. The body dilates behind the head, attains its maximum about the anterior third, and then tapers towards the tail. It is proportionally thicker than in its allies (such as *Castalia* and *Psamathe*), and garnished at each side with long resplendent bristle-tufts, that glance with the varied hues of the rainbow, the effect being heightened by the two long hair-like cirri that stretch beyond them. The tail terminates in two long slender styles, which are shorter, however, than the cirri of the fourth foot from behind. Through the mouth is protruded a large proboscis, which is unfurnished with jaws or tentacular processes; and this assumes various forms after immersion of the living animal in spirit, or when killed by the salt water being impure in any degree,—sometimes being cylindrical, or presenting a constriction between the swollen base and distal rim.

The first four segments after the head bear modified limbs, each consisting of two long cirri. As soon as the foot attains its perfect condition, it is found to be distinctly biramous, thus at once demonstrating its distinction from all the *Hesionea* except SCHMARDA'S *Cirrosyllis* (*Pseudosyllis*, QUATREF.) and ERSTED'S *Castalia*. The superior lobe, as observed in a fine spirit-preparation, consists of the long superior cirrus, which has a soft articulation at its base; an inferior cirriform branch, from the upper and basal part of which spring a series of elongated, slender, and tapering bristles, simple throughout. After attaining some thickness, the shaft (Plate XV. fig. 8) is observed to be striated longitudinally, and to have minute transverse touches, which, however, attain a larger development in the next series. The inferior branch of the foot also consists of two portions, a ventral cirrus, and a bristle-bearing process, from the posterior surface of which the somewhat stiff fan of jointed bristles emerges. In such forms the bristles of the anterior feet have shorter tips, while those of the posterior feet have more elongated terminal processes. Besides, in each foot in this species the terminal pieces vary in length, the shorter occurring superiorly and inferiorly, or at the edges of the fan. When highly magnified (Plate XV. fig. 7), the claw at the tip of the terminal piece is seen to be somewhat faintly marked, from the blocking of its curvature by a process beneath, and the serratures of the edge of the process very fine, indeed scarcely distinguishable near the end. The shaft of the bristle is obliquely striated towards the articulation, longitudinally throughout the rest of its extent, except as usual at the pale diminished base (where the striæ become indistinct), and marked by a close series of transverse specks or touches. The tip of this division of the foot ends in a cirriform prolongation. The jointed bristles of the inferior branch of the foot differ, as we

might expect, from any other allied British form, such as *Psamathe fusca*, JOHST., and *Castalia punctata*, MÜLL., each of which possesses similarly jointed bristles, and has the serrated terminal portion peculiarly clawed. In *Psamathe* the larger size, the structure of the shaft, and the coarser serratures of the terminal portion (Plate XVI. fig. 2), distinguish it from *Castalia*; while the latter again (Plate XVI. fig. 1) has a much broader and proportionally longer terminal process than the present form. The distinction in this respect between the *Periboea* and *Podarke* of Dr EHLERS\* and the latter is very apparent. Dr GRUBE's genus *Oxydromus*,† with which the foregoing has certain affinities, has also an unarmed proboscis, but the feet are uniramous.

I may also remark that two very distinct species, or rather genera, have been included—on the one hand, by Dr JOHNSTON in Britain, and on the other, by several continental authors—under the name of *Psamathe punctata*. Some of the most recent foreign publications—such as the work of Dr EHLERS and that of Dr MALMGREN—do not sufficiently recognise the distinctions between the two. M. DE QUATREFAGES,‡ however, correctly separates them into genera, yet he places the synonym *Castalia punctata*, “ÆRST.” under both. Dr MALMGREN,§ while correctly including the *Halimede venusta* of RATHKE || under *Castalia punctata*, MÜLL., falls into the error of comprising Dr JOHNSTON's species under the same head—a slip which would not have happened if this excellent observer had seen a specimen. Dr JOHNSTON's species, for which, notwithstanding Dr EHLERS's views, the original name of *Psamathe fusca*¶ may be retained, has a uniramous foot, with the terminal portion of the bristles characteristically marked shortly after its commencement by a series of larger serrations, which gradually rise to a maximum, and similarly diminish, before arriving at the middle of the process, into fine serrations that disappear before the clawed tip is reached (Plate XVI. fig. 2). All the bristles of the foot are not so boldly marked as this example, but in each there is a tendency to have a shorter terminal piece, with coarser serrations, than in those of its immediate allies, and the clawed portion at the tip is very distinctly seen, so that the bristle can be distinguished specifically at a glance. The shaft has also coarser transverse markings, and its distal end is somewhat less clavate than in *C. punctata*. The *Psamathe cirrata* of Prof. KEFERSTEIN,\*\* also described by M. CLAPARÈDE,†† seems to me to be allied in the closest manner to *P. fusca*, if, indeed, it is not identical therewith. M. DE QUATREFAGES, however,‡‡ considers the annelids

\* Die Borstenwürmer, &c. pp. 190 and 199, taf. viii.

† TROSCHEL's Archiv für Naturges. 1855, p. 98.

‡ Annelés, vol. ii. 1865, pp. 100–102 and 106.

§ Annulata Polychæta Spetsbergiæ, &c. 1867, p. 31.

|| Beiträge zur Fauna Norwegens, &c. (Nov. Act. Acad. C. L. C. Nat. Cur. &c.), p. 168, tab. vii. fig. 1–4.

¶ Loud. Mag. Nat. Hist. vol. ix. p. 15, 1836.

\*\* Zeitsch. für wiss. Zool. bd. xii. p. 107, taf. ix. figs. 32–36.

†† Beobach. über Anat. &c. p. 55, taf. xiv. figs. 1–7.

‡‡ *Op. cit.* vol. ii. p. 41.

described by the two foregoing authors as distinct species (which I hardly think is the case), and classes them under the genus *Kefersteinia*. They also appear to me to be in all respects much more nearly allied to the *Hesionea* than the *Syllidea*. The *Castalia punctata*, MÜLL., again, has a biramous foot, whose compound bristles (Plate XVI. fig. 1) have on the whole a longer terminal portion, with finer serrations than in *P. fusca*. I have found it on various parts of the British coast.

*Autolytus pictus*, EHLERS.\*—I agree with Dr R. GREEFF,† who, in his remarks on *Autolytus prolifer*, observes that the above species (the *Proceræa picta* of EHLERS) is, in truth, an *Autolytus*. It was first found in Britain under a stone in a rock-pool at Paible, North Uist, and again, in greater numbers, at St Andrews. Its length is about an inch. The dorsum is very prettily marked by a pale central band, with numerous and rather regular transverse branches, which, uniting with a pale lateral belt on each side, cut the sepia-brown pigment-masses into oblong spaces. The latter are minutely striated under the lens by fine pale lines, and the intervals dotted by almost microscopic pale grains. The first twelve or thirteen segments are darker in hue dorsally, and the intersecting lines paler; and in some the oblongs are decidedly paler in this region. Below the pale lateral belt, and just above the feet, a dark-brown band runs from end to end, intersected only here and there opposite the pale transverse belts by narrow pale lines. A dark patch of brown is placed behind the median tentacle, and from the latter two characteristic diverging pale lines proceed backwards. The under surface is of a pale whitish or flesh colour. These markings were well seen in specimens preserved for upwards of a year in spirit. The head is rather small, and appears at first sight to be supplied with two eyes only, which are situated laterally, and somewhat in front of the great median tentacle, but a careful examination shows two clear lens-like structures on each side, the larger towards the front of the pigment-mass, and the smaller behind. There is thus some difference between our description and that of Dr EHLERS, since he shows a posterior pair of eyes considerably behind the median tentacle, and quite separated from the compound group in front. This ocular region is richly ciliated, and so is the dark pigmentary portion on the sides immediately behind. The median tentacle had its place supplied in a few instances by two of equal length, but this is simply an abnormality. The segments (upwards of 100 in number) behind the three or four anterior rings are furnished with a rather short dorsal cirrus, a few simple spines, and a fascicle of bristles (Plate XV. fig. 11), which possess a short terminal piece, with a bidentate apex. I have not seen any with a tridentate terminal portion, as shown by Dr EHLERS. Towards the tail there is only a single conspicuous spine in each bundle of bristles, and, finally, a larger and smaller spine form the sole appendages to the feet. Here,

\* Die Borstenwürmer, &c. p. 256, taf. xi.

† Archiv für Naturges. 1866, and Annals N. Hist. March, 1868.

also, as in many of the Syllidea, the terminal joint of the bristles undergoes various changes throughout the course of the body, being very short anteriorly, then lengthening, and again diminishing in size towards the tail. The latter is terminated by two short curved styles. Dr EHLERS found his specimens at Martinsica. M. DE QUATREFAGES groups this species under his Myrianida, as *M. picta*.\*

*Pionosyllis malmgreni*, n. s.—This species, dredged in the Minch, off Lochmaddy, and also procured at the latter under a littoral stone, seems to belong to Dr MALMGREN'S genus *Pionosyllis*,† but is distinct from the species described by him. The elongated terminal portion of the bristles (Plate XVI. fig. 10) is peculiar, from the somewhat rapid widening below the bidentate apex. Faint serrations are observed on the terminal or articulating end of the shaft. The present is distinguished from MALMGREN'S species, *P. compacta*, by the following particulars:—A shorter terminal portion to the bristles; the absence of the elongated simple bristles in the non-budding animal; the greater length of the palpi; and in the much more elongated condition of the tentacula and cirri, which, moreover, are distinctly moniliform. In a specimen having a two-eyed bud posteriorly, the latter had, besides the ordinary kind, a tuft of slender simple bristles, which did not reach beyond the others.

Under the title *Syllis armillaris*, Dr JOHNSTON seems to have included two very distinct species, the *S. armillaris*, MÜLL.—a form occurring very abundantly between tide-marks, and having a single claw to the tip of the terminal piece of the bristle, and another annelid equally common in the laminarian region and deep water, whose membranous tubes occur in hundreds on the blades of *Laminaria saccharina*, tossed on shore by storms. The latter is probably the species referred to by Mr GOSSE‡ under the name of *Syllis tubifex*, though various characteristics, such as the single tooth of the proboscis, and the exact structure of the bristles, are omitted. The palpi are of considerable length, joined at the base in front of the snout, and richly ciliated, besides having in front some motionless microscopic spinules. The processes of the head and the two next segments are most distinctly moniliform, as well as longest, and the succeeding cirri show the crenations in a diminishing degree. All have the microscopic spinules. The proboscis has a denticulated edge, though a third of the circumference is only minutely crenated, and it is furnished with a single pyramidal tooth. This region is usually thrown into prominent wrinkles. Several elongated papillæ are present in front of the anterior edge of the proboscis—some apparently directed forwards, others backwards. The proventriculus is studded with minutely granular glands. Segments about fifty-six in number. The bristles, which are similar to those represented in Plate XV. fig. 21, have a short,

\* Annelés, vol. ii. p. 63.

† Annulata Polychæta, &c. p. 39.

‡ Ann. Nat. Hist. 2d ser. vol. xvi. p. 31.

bidentate apical portion. The colours of this species are very beautiful, and it is brilliantly phosphorescent. It appears to fall under Dr MALMGREN'S genus *Eusyllis*, and to be most nearly allied to, though not identical with, his *E. monilicornis*. Another new British species, characterised by indistinctly articulated tentacles and cirri, four very large and unusually distinct eyes, very short bidentate apical portions to the compound bristles, and the presence of long simple hairs, seems to be the *E. blomstrandii* of the same author. It was dredged in the Minch in 1865.

*Syllis krohnii*, EHLERS,\* var.?—Found under a stone in a rock-pool at Paible, North Uist, in a tube of sand. In this animal every alternate dorsal cirrus is a third larger, more opaque, speckled with white dots, and, instead of passing transversely outwards like the others, curves upwards in a very graceful fashion, and is often coiled at the tip. The others are smaller, paler, also speckled with white dots, and longer than the diameter of the body. The ventral cirrus is very small. The bristles (Plate XVI. fig. 14) have a stout terminal portion, with an entire claw at the apex, and the edge is serrated. The curves of the terminal portion of the shaft are peculiar, and, in this respect, allied to MALMGREN'S *Syllis borealis*,† from which, however, the animal is readily distinguished by the characters of the dorsal cirri, and the more elongated condition of the cephalic lobes. Unless we are to mistrust the descriptions and figures of the dorsal cirri given by Dr EHLERS, the British form varies very considerably from the typical one. In no state were the alternate cirri club-shaped, and those of the third and fourth segments were small and nearly equal; whereas he shows them furnished with a clavated pair, and all much more distinctly annulated than in the British example.

*Syllis cornuta*, RATHKE.‡—A *Syllis*, dredged off the Hebrides by Mr JEFFREYS, presents certain characteristics which point to its identity with the above-mentioned species of H. RATHKE; and since it is doubtful (from the description at least) whether Dr JOHNSTON'S remarks§ apply to this animal or not, I shall briefly allude to its structure. The body, composed of fully 100 segments, is about an inch in length, and of a highly iridescent aspect, from the close plaiting of the fine muscular fibres. All the tentacles and cirri are moniliform. Each foot has a dorsal cirrus, divided usually into twelve segments, a bristle-papilla, and a short lingulate inferior lobe. The bristle-bundle is chiefly composed of the form *b* (Plate XVI. fig. 15), which at first sight resemble simple bristles, as their articulating processes are usually hidden amongst the others. They have, however, a most minute bidentate tip. Some (*a*) again have an extremely elongated terminal process. Dr MALMGREN'S figure|| represents the dorsal cirrus as furnished

\* Die Borstenwürmer, p. 234, taf. x. † Annulat. Polychæta Spetzbergiæ, &c. 1867.

‡ Beiträge zur Fauna Norwegens, p. 164, taf. vii. fig. 12.

§ Catalogue, &c. p. 192. || Annulat. Polychæta, &c. p. 43, taf. vii. fig. 45 c.

with at least double the number of annulations described above, and the bristles are not characterised by the minute bidentate apex; moreover, only a linear or profile view of the elongated kind is exhibited, so that the characters required some further elucidation.

A species allied to the *Syllis macrocera*, GRUBE,\* was found under a littoral stone at Lochmaddy. It had about the same number of segments as the foregoing, smooth cirri, and a very short apical piece to the bristles. It was of a dull orange-yellow colour, with the head about as long as broad, the central tentacle longer than the lateral, and all extending beyond the lobes. The bristles (Plate XV. fig. 12) of the several fascicles do not vary to the same degree as in such as *S. armillaris*, MÜLL., and each has a blunt claw at the apex, with a rough edge, for the notches are irregular. The articular portion of the shaft ends bluntly.

*Sphaerosyllis hystrix*, CLAPARÈDE.†—Two forms of this species were found at North Uist in 1865, the one in the littoral region at Lochmaddy, and the other in the Minch. The littoral form (apparently that described by M. CLAPARÈDE) was marked down the centre of its pale body by a moniliform yellow band (intestine). The body tapered anteriorly, and ended in a small snout formed by the united palpi. Eyes four, placed close together in pairs, the anterior only furnished with lenses. Segments thirty-two. The tuberculated dorsal cirri with their swollen bases were well marked. At the eleventh segment a series of flask-shaped bodies (buds)—two in each segment—commenced, and continued almost to the tail. These bodies were of a pale rose-pink hue, with a reddish spot in the centre, where the oil-globules were massed. They were nearly equal in size throughout, had the usual processes at the ends, and were all thrown off when the animal was placed in spirit. The tail terminated in two swollen cirri. The bristle-bearing papillæ were distinctly tuberculated, and furnished throughout with compound bristles (Plate XV. fig. 10, *b*), which had a delicate and rather elongated apical portion with a simple claw at the tip, and a stout simple bristle (fig. 10, *a*) slightly bent towards the attenuated extremity. In addition, from the ninth segment backwards nearly to the tail, each foot was provided with a tuft of long filiform bristles, which stretched far beyond the others. It seemed an inactive animal, and lay rolling on the bottom of the vessel; and the numerous parasitic organisms on the bristles would likewise indicate a sluggish habit. In the other form (from the Minch) there were none of the last-mentioned filiform bristles, and the compound series, moreover, had a more elongated apical piece (Plate XVI. fig. 9). The eyes also were in one specimen six, two larger ones posteriorly on each side, quite separated from each other, and two small round

\* QUATREF. Annelés, vol. ii. p. 28.

† Beobach. &c. p. 45, taf. xiii. figs. 36, 37, and Glanures Zootomiques, &c. p. 86, pl. vi. fig. 1.

specks in front. In the second specimen the two anterior eyes were absent. Segments about thirty

*Staurocephalus kefersteini*, n. s.—On both the eastern and western shores of North Uist a species of Prof. GRUBE'S genus *Staurocephalus*\* occurred under stones near low-water mark. Body of an orange hue, paler towards head and tail; length about an inch. Eyes two, black, situated near the posterior border of the head. The latter conical, the snout forming a somewhat blunt apex. Tentacles four, the anterior, arising from the infero-lateral region of the head, by much the largest, and having a short jointed process at the tip; the posterior pair, springing from the outer side of each eye, are annulated and much less. The large anterior pair can be coiled and twisted very prettily. The feet, instead of being furnished with a dorsal and ventral cirrus, as in most of the species, have only a small ovate dorsal and ventral process (Plate XVI. fig. 11, *f* and *g*) as their representatives, and they are scarcely more prominent than the bristle-papillæ; thus it approaches *S. eruceiformis*, MGRN.; from which, however, it differs in the structure of the bristles and other respects. The superior fascicle of bristles consists of two series, a stout bifid kind (Plate XVI. fig. 11 *b*) with the long limb of the fork flattened and slightly clawed at the tip, the shorter truncate and rounded. The second series (fig. 11 *a*) are more slender, elongated, finely tapered, and definitely curved, with a limited number of slight serrations on the distal and convex side of the curve. The bristles of the inferior fascicle again are all compound and of one kind, the terminal portion being somewhat elongated, clawed at the tip, and without evident serrations on its edge. In regard to the length of the terminal piece, these bristles present a gradational arrangement, the longest terminal portions being superior, the shortest inferior. The extreme bristles of a single foot are shown in figs. 11 *c* and 11 *d*; and it will be observed that the swollen terminal portion of the shaft has a few serrations. The tail is terminated by two styles of moderate length, which, like the processes of the feet, are much shorter than in *S. ciliatus*. MALMGREN,† alludes to a drawing of a species of "Prionognathus," apparently different from the latter, which had been sent him by A. BOECK from Norway; but he gives no description.

*Notocirrus scoticus*, n. s.—At least three species of the Family *Lumbrinereidæ* have been hitherto described as frequenting the British shores, viz., *Lysidice ninetta*, AUD. and ED., *Lumbrinereis tricolor*, MONT., and *L. latreillii*, AUD. and ED. The two latter, however, have in all probability been sometimes confounded with the *L. fragilis* of MÜLLER, a species abounding on our northern and southern coasts. A fourth and very well-marked form, which I have designated by the above name, was dredged amongst tenacious grey clay in 6 to 9 fathoms in Lochmaddy, and subsequently in several parts of the Hebridean seas by Mr GWYN JEFFREYS. The

\* *Prionognathus*, KEFERSTEIN, Zeitsch. für wiss. Zool. vol. xii. p. 99, taf. viii. figs. 13–19.

† Annulat. Polychæt. &c. 1867, p. 62.

head is of an acutely conical form, with two distinct eye-specks at its posterior border, close to the first transverse sulcus. Its body is much more slender than that of *L. fragilis*, and at once attracts notice by its characteristically marked segments, which, with the exception of a few anteriorly, assume quite a moniliform appearance. In the structure of its feet it differs from all the foregoing species. Each foot is furnished with a small branchial lobe (Plate XVI. fig. 17 *a*) in which a single vascular loop is observed; and thus it would appear to fall under the genus *Notocirrus*, SCHMARDA,\* though the possession of the eyes is exceptional. The tip of a stout spine or two (*b*) projects beyond the foot amongst the bristles. The latter (*c*) have simple shafts with a broad spear-tip, which tapers to a fine point, and is faintly serrated along part of the edge.

*Hyalinæcia sicula*, QUATREF. (?)—This is a small representative of the *Onuphididæ*, dredged in 90 fathoms off North Unst, Shetland, by Mr GWYN JEFFREYS, F.R.S. It is characterised in spirit by two parallel bands of brown which course along the lustrous dorsum from a transverse belt of the same colour immediately behind the head, and by a brown spot between each foot from the fifth backwards. There are three elongated tentacula (a median and two lateral), and two shorter in front, as in *H. tubicola*, MÜLL. The small black eyes are situated at the outer side of the base of the long lateral. All the tentacles have a crenated base. The antennæ are similar to those of *H. tubicola*, or perhaps slightly longer. In the structure of the bristles of the anterior feet, however, a very diagnostic feature occurs; for instead of the large unjointed winged hooks, which are found in the latter and in *Nothria conchilega*, SARS, there are peculiar jointed structures (Plate XVI. fig. 3); and the bristles (fig. 3 *c*) are slender, and furnished with a very narrow wing, whereas in both of the other species they are shaped like a Valentin's knife. Posteriorly the jointed hooks are supplanted by two simple ones (fig. 3 *b*), which are stouter and slightly curved. Some of these occasionally present no wing at the tip. The bristles in this region are also shorter, and some are characteristically curved at the point. None of the peculiar brush-shaped bristles common in the two species above-mentioned occurred in this animal. No tube accompanied it; but I have since found that this species inhabits a tube composed of gravel and shell-fragments, and thus differs very considerably in its habitation from *H. tubicola*, while the length and form of the tube also distinguish it at once from that of *Nothria conchilega*. The foregoing animal has certain close affinities with the *Onuphis sicula* of M. DE QUATREFAGES,† but differs from the description of that author in so far as the bases of the tentacles do not occupy the whole surface of the head, which in the Sicilian species is very small. The body is rounded in the latter, flattened in the British; and the bristles of the former are said by M. DE QUATREFAGES to present a great resemblance to those of

\* Neue wirb. Thiere, &c. tom. i. ii. p. 114.

† Annelés, vol. i. p. 352.

*O. tubicola*, a statement at variance with the characteristics of the present species. The persistent brown stripes and spots also had not been seen by M. DE QUATREFAGES.

*Eumenia jeffreysii*, n. s.—This curious form, which I have been unable to identify with any known species, occurred amongst the annelids dredged by Mr JEFFREYS off the Hebrides in 1866, and again amongst those from Shetland in 1867. The length is about  $1\frac{1}{2}$  inch, and the outline of the body somewhat fusiform, the greatest diameter being at the anterior third. The head is small, furnished with two short thick tentacles, which give it a bilobed aspect, and is generally retracted within the papillose anterior region in the preparations. The mouth opens on the ventral surface just behind the snout. The structure of the skin and the arrangement of the rugose annulations resemble the same parts in *Travisia*, *Scalibregma*, *Eumenia*, and their allies; but the animal essentially differs from each of the foregoing in having no trace of branchial filament or appendage. The tail has several elongated processes around the anus. The ventral surface is in some cases marked by an elevated median line. There are about thirty segments, each of which has three rings. A double row of isolated papillæ runs along each side from the snout to the tail, the summit of each process giving exit to a fascicle of bristles composed of two kinds, viz., numerous long, simple, hair-like bristles, tapering to a very fine point, and a shorter forked series (Plate XVI. fig. 5). The only other case in which I have up to this time met with such bristles, is in a remarkable fragment of the posterior end of a small yellow annelid from Lochmaddy, which may have some relation to MONTAGU'S *Nereis pinnigera*. The foot had an elongated unjointed dorsal, and a shorter ventral lobe, and possessed two fascicles of bristles, each of which consists of long simple bristles, and a few of the forked kind mentioned above.

There is much in the foregoing description that agrees with *Eumenia crassa*, ERST., but the absence of the branchial filaments is diagnostic. Dr BAIRD had received this species from the same source, and kindly sent it, with other rare and doubtful specimens, for my examination. He likewise recognised the absence of the branchiæ, and his preparation was labelled "*E. ebranchiata* (?)." The *Vermiculus crassus* of DALYELL\* had no bristles, and cannot easily be recognised from the description or figure.

*Chloræmidæ*.—Two examples of this family have been recorded as British, viz., *Trophonia plumosa* and *Siphonostoma uncinata*, both of which abound in Scotland. Another species of *Trophonia*, dredged by Mr JEFFREYS in the Hebridean and Zetlandic seas, is recognised specifically by the absence of hooks in the inferior rows, and the substitution of the jointed bristles. It agrees with the *T. glauca* of MALMGREN;† but this author does not specially point out the essen-

\* *Op. cit.* p. 83, pl. x. fig. 11.

† *Annulat. Polychæt. &c.* p. 82.

tial change in regard to the inferior appendages of the feet. As contrasted with the common species, the joints or transverse markings of the bristles are much more boldly indicated in this form, especially in those from the inferior fascicles (Plate XV. fig. 13 *b*). The latter bristles (inferior) are shorter than the superior, and both, as usual, have larger joints than the anterior series. A second representative of the family (*Siphonostoma buskii*, n. s.), from the Minch, off Lochmaddy, North Uist, is remarkable for its deep-red colour throughout, a hue so unusual in the group. The two long tentacles or palpi are pale, but the branchial filaments are deep red. The surface of the body is furnished with minute papillæ, which have the enlarged terminal portion furnished with a knob at the tip. The hooks (Plate XVI. fig. 4) differ very much from those of *S. uncinata*, in so far as the shaft is much longer and less robust, and the terminal claw more elongated, and abruptly curved. When the latter breaks off, it separates obliquely at *a*, a little above the articulation, leaving the short spike through which the dotted line passes. A bristle is shown in fig. 4 *a*, and a fragment more highly magnified in Plate XV. fig. 13 *a*.

*Maldanidæ*.—Two species of this family (*Clymene*, QUATREF.) are mentioned as British by the authors of the Catalogue, both of which are of doubtful identity, and apparently referable to the common *Clymene lumbricalis*, FABR.\* (*Nicomache lumbricalis*, MGRN.), though this is by no means certain. Mr E. R. LANKESTER, in his list of the Annelids collected at Guernsey† in 1865, notices a third species—viz., the *Clymene amphistoma* of SAVIGNY. The explorations of the coast line in the Hebrides, and dredging in the surrounding seas by Mr JEFFREYS and myself, as well as the cruise to the Shetland Islands last summer by the former experienced investigator of our seas, have considerably augmented the number of the British representatives. One of the most remarkable species is the *Rhodine Loveni*, MALMGREN,‡ which combines an entire anal funnel, with a pointed snout, and has its characteristic hooks (Plate XV. fig. 16) in a double instead of a single row, thus materially differing from the others pertaining to the family. The outlines of the hooks of the British species differ in some details from those represented by Dr MALMGREN—a discrepancy in all probably due to the inaccuracy of his artist. The *Axiiothea catenata*, MGRN.,§ was dredged recently by Mr JEFFREYS, off St Magnus Bay, Shetland, in 80 to 100 fathoms. Besides having an infundibuliform anal funnel, with alternate longer and shorter filaments, the base of the cup is marked exteriorly on the ventral surface by a continuation of the median line. There are about forty processes on the margin of the funnel, a smaller one, and sometimes two, occurring between the longer filiform divisions. The base of the funnel is surrounded

\* Faun. Grœnland. p. 374.

+ Annals Nat. Hist. May, 1866.

‡ Nordiska Hafs-Annulater, &c. 1865, p. 189; and Annulat. Polychæta, &c. p. 99, tab. x. fig. 61

§ Annulat. Polychæt. p. 99, tab. x. fig. 59.

by a distinct cup or fold exteriorly. The anal nipple, moreover, is roughened by small papillæ. The bristles are slightly winged below the tip, and under a power of 700 diameters show minute serrations at the margins of the wings. Instead of hooks, the first bristle-bearing segment has three very stout spines gently curved at the tip, and the second and third four of the same character. They have a distinct shoulder, and the chitinous substance is strongly striated longitudinally. Only a small portion of the tip is usually seen beyond the skin. The hooks in the segments which immediately follow have the processes above the great tooth somewhat fewer (four to five), but the rest have six; and in those of the last row, in front of the anal funnel, the denticles are even more numerous towards the crown. The great tooth comes off somewhat stiffly at the base, and its upper curve is not sinuous. Dr MALMGREN does not notice the peculiar spines anteriorly, but simply mentions that the hooks are fewer in those segments, and omits several characteristics described above. There are no hooks on the soft lobulated processes which succeed the last bristle-bearing segment, with its conspicuous transverse pad. The frontal lobes form two very prominent laminae.

Another species, the *Praxilla prætermissa*, MALMGREN,\* is not uncommon on our western and northern coasts, inhabiting sandy mud at a depth varying from four to eight fathoms. In a large specimen the teeth of the anal funnel are 27 in number. The hooks are characteristic, having about six teeth above the large fang, and a well-marked interval between the latter and the origin of the spinous tufts. The first three segments have simple and strong spines with the apex slightly curved. There are also a few shallow crenations on the margins of the cephalic lobes. A somewhat rarer species is *Praxilla gracilis*, SARS,† two specimens of which appeared in the rich collection brought by Mr JEFFREYS from the Shetlands. The hooks of the first three bristled segments differ from the others, and are spines with the apices more curved than in *Axiothea*, so as to resemble a hook furnished with the large fang only. A third species of *Praxilla* from the same region (North Unst, St Magnus Bay, and the Outer Haaf), while agreeing in several particulars with *P. prætermissa*, has its funnel-teeth much more filiform and distinct—in one instance 14 in number. The hooks (Plate XVI. fig. 13) have the large fang short and powerful, with the spinous filaments arising close underneath, and a numerous array (seven to eight) of diminishing teeth superiorly, the whole forming a very elevated crown, indeed it is the most elevated of the series in this respect. The curves of the hook, especially the posterior, are characteristic. The bristles are also peculiar, for instead of the usual winged margin, the whole shaft is flattened out towards the translucent tip, very minutely serrated at the edges, and tapered to a delicate point. The shaft below the flattened portion is, as usual, finely striated longitudinally.

\* Nordiska Hafs-Annulater, p. 191.

† Fauna litt. Norveg. ii. p. 15, tab. 2, figs. 18–22.

This may be MALMGREN's *P. artica*,\* but as he only says as to its characteristics that it is similar to *P. prætermissa*, with the exception of having six teeth on the crown of the hook, we are left quite in doubt as to his species.

The anterior portion of a specimen of *Clymene ebiensis*, AUD. and ED.,† also came from Shetland. It is recognised by the pointed snout, the somewhat swollen anterior segments, and the absence of the usual frontal flattening. The shape of the hooks (Plate XVI. fig. 12) is peculiar, the chief fang being short, and the crown somewhat flattened. There are five or six teeth above the former. The curves of the organ and its coarse striæ are also characteristic. I could not find in this specimen either spines or hooks in the first three segments. The figure of the hooks given in the "Règne Animal" is quite unfit for identification. The species is also allied to Prof. GRUBE's *Clymene leiopygos*,‡ from Cherso, though his drawing of the hooks is widely different.

The *Ammochares ottonis*, GRUBE,§ has been found abundantly, at St Andrews, in the stomachs of cod, at Lochmaddy under stones near low water, and dredged by Mr JEFFREYS in Shetland and the Minch. The bristles are rendered hirsute by microscopic spines, as shown by Dr MALMGREN;|| but the hooks of the rasp-like belts have a much more characteristic shape than represented by this author's artist, since they are figured without any shoulder, and with the curve at the back of the beak too prominent. Their exact condition is shown in Plate XV. fig. 14. There are three tufts of longer and more delicate bristles in the British specimens on the first region, instead of two, as shown by Drs GRUBE and MALMGREN; but one may have been overlooked from its minuteness. I am inclined to believe, judging from MALMGREN's paper, that the *A. assimilis* of SARS is the same species as the above. Dr CARRINGTON of Eccles describes this species¶ under the name of *Ops digitata*.

Of the family of the *Ampharetea*, MALMGREN, several representatives new to Britain have occurred. One species, the *Amphicteis gunneri*, SARS, though unnoticed in the recent Catalogue of the British Museum, had been found by Mr GOSSE at Ilfracombe, and described by him under the name of *Crossostoma midas*.\*\* Dr MALMGREN†† mentions another form, the *A. sundevalli*, which is characterised by having nineteen hook-bearing processes posteriorly, whereas the former has but fifteen; the bristles also have the winged portion striate, and the upper part of each hook widest, while in *A. gunneri* the corresponding region of the bristle is smooth, and the hook widest in the middle. Our common Hebridean and Zetlandic *Amphicteis* has certain of the characters ascribed to each of these species,

\* Annulat. Poly. Spetz. &c. p. 100.

† Cuv. Reg. An. iii. pl. xxii. fig. 4.

‡ Archiv für Naturges. 1860, p. 91, taf. iv. fig. 3.

§ Archiv für Naturges. 1846, p. 163, taf. v. fig. 2 a, b, c.

|| Annulat. Polychæta, &c. tab. xi. fig. 65 D.

¶ Proc. Lit. & Phil. Soc. Manchester, 1865.

\*\* Ann. Nat. Hist. vol. xvi. 1855, p. 310, pl. viii. figs. 7-12.

†† Nordiska Hafs-Annul. taf. xix. fig. 46 D.

for the bristles agree with those of *A. sundevalli* in having the winged portion striate, while the hooks are widest in the middle (Plate XIV. fig. 14), and there are but fifteen hook-bearing processes posteriorly. A boreal form, not uncommon in the Scotch seas, is *Ampharete artica*, MALMGREN, the hooks in this species being furnished with a large number of teeth (Plate XIV. fig. 13). The former examples possess frontal bristles, but two species in Mr JEFFREYS' Hebridean and Zetlandic collections have none. The first is the *Sabellides sexcirrata*, SARS,\* wherein the hooks have for the most part five teeth, though some of the larger have six (Plate XVI. fig. 16 *a* and 16 *b*). Occasionally one occurs in the centre of the row with only four large teeth. The other species was in a very imperfect state, but seems to be an *Amage*, MGRN., having about fourteen bristle-bundles on each side, somewhat club-shaped smooth tentacles, and the ventral bars very distinctly marked. The hooks (Plate XIII. fig. 10 and 10 *a*) have four or five teeth, and differ so much from *A. auricula*, MGRN., that in all probability the animal is distinct.

The descriptions of the British *Terebellæ* given in the Catalogue of the British Museum stand very much in need of revision, it being difficult, indeed, in some cases to understand what species is meant. Thus *T. conchilega* could not be identified from the characteristics there noted. The *T. nebulosa* of Dr JOHNSTON is not that of MONTAGU, but a very different form, with 24 pairs of bristle-bundles (he says 23), and well-marked hooks, with the chief fang very long and several smaller processes above it. It may be remarked in passing, that in such a profile view all the small hooks on the crown are not seen, and hence the armature is greater than at first sight appears. This species attains a very large size on our western shores. Dr MALMGREN† proposes for it the name of *Amphitrite Johnstoni*, but Sir J. DALYELL had long previously called it *T. figulus*.‡ The true *T. nebulosa* is described in the Catalogue under *T. tuberculata*, DALYELL, and MONTAGU'S name, at any rate, must stand instead of MALMGREN'S recent title, *T. debilis*. The hook of this species has two very distinct fangs and a greatly elongated base.

In addition to the twelve species mentioned in the Catalogue no less than eight new British forms require notice. In *Terebella* (*Nicolæa*) *zostericola*, ÆRST., a very abundant species, the hooks (Plate XV. fig. 15) are furnished with a single fang above the large one, and in some cases with a trace of a second. *Pistacristata*, MÜLLER, a species with a single pair of whorled branchiæ, was first got at Lochmaddy, and since at various parts of the coast; its hooks are characterised by the singular form represented in Plate XV. fig. 20, with three or four prongs above the chief fang, and a powerful process for the ligament at the posterior end

\* Fauna litt. Norveg. ii. p. 23.

† Nord. Hafs-Ann. p. 377.

‡ Pow. Creat. vol. ii. p. 191, pl. xxvii. figs. 1 and 2.

of the enlarged base. This animal is quite different from the *T. maculata* of Sir J. DALYELL, which may be a species having a speckled aspect in spirit, a single pair of branchiæ, and hooks of the form shown in Plate XIV. fig. 15. The *Grymœa bairdi*, MGRN., a form nearly allied to *Thelepus circinnatus*, FABR. (*Venusia punctata*, JOHNST.), was dredged in 90 fathoms off St Magnus Bay, Shetland, by Mr JEFFREYS. It is at once distinguished from the latter by the much greater prominence of the bristle-papillæ, and the greater length and lustre of the bristles themselves throughout the entire body. The hooks resemble those of the common species (*T. circinnatus*) very much, but the process for the ligament is not so near the tip of the upper curve as in the latter, and the organs are proportionally smaller. The tube is composed of fine grains of muddy sand, instead of the coarser and stronger structure of *T. circinnatus*.

Amongst the Polycirridea from the same region is a very interesting form, called by Dr MALMGREN *Lysilla loveni*, and distinguished by the largely dilated cephalic lobe, furnished with numerous clavated grooved tentacles along its margin, and a cluster of tangled filiform processes inferiorly at each side. The whole of the anterior dorsal region is densely tuberculated with papillæ, which, from the intervening lines, assume a transverse arrangement. On the ventral surface, which is thrown in contraction into two prominent longitudinal folds with a central depression, the swollen portions are covered with somewhat larger tubercles than the dorsum, but the depressed central region forms a nearly smooth line of demarcation. There are six pairs of foot-papillæ in front, each having a short tuft of simple slender bristles, whose tips in the preparation are entirely within the summit. From the same source as the latter there is also the anterior fragment of another curious and new example of the same sub-family, *Polycirrus tribullata*, n. s., which has neither bristles nor hooks. The head and tip have the usual tentacles. The body has no ventral plates, but only a raised central line. There are three pairs of well-marked circular truncated papillæ (on the sixth, seventh, and eighth segments), each consisting of a raised ring externally, with an elevation in the centre. Two minute papillæ were visible in front of the first flattened process, but only a trace of an elevation occurred on the lateral region of the succeeding segments, which were two-ringed. The cuticle has a minutely granular aspect. The remarkable lateral processes may act as suckers. Two species, which come under Dr MALMGREN'S recently constituted genus *Ereutho*, are not uncommon in Britain. They are distinguished from other Polycirridea by having thirteen pairs of bristle-bundles. The first, which seems closely allied to *E. smitti*, MGRN., has hooks (Plate XV. fig. 17), which possess only two fangs, and a very much produced and characteristically striated basal process. The hooks of the other species (Plate XV. figs. 18 and 19) are much smaller than the foregoing, and so exactly resemble the figure by MALMGREN from a specimen of *P. aurantiacus*, GRUBE—forwarded by Prof. GRUBE

himself—that one may be allowed to have some doubt as to the correctness of previous descriptions with regard to the number of the bristle-bundles. The last of the group is *Trichobranchus glacialis*, a species which Dr MALMGREN has only described from a spirit-preparation.\* This form was dredged in six to eight fathoms in Lochmaddy, in 1865, as well as got under a stone amongst sandy mud at low water. Length about  $\frac{3}{4}$ ths of an inch when moderately extended. Of a general blood-red hue, or dark-red anteriorly, paler posteriorly. In shape the body is irregularly fusiform, ending anteriorly in rich red lips, with a translucent projecting collar at each side, leaving the dorsal and ventral edges free. From the dorsum, slightly posterior to the fissure thus left, spring a tangled series of tentacles, which are easily differentiated into three groups, even in the spirit-preparation. The most conspicuous, long, thick cylindrical processes, varying from four to six in number, arise distinctly behind the others, from the dorsal edge posterior to the cephalic frill, and are distinguished by a bright-red central vessel, as well as by the frequency with which they are thrown into spiral curves. They are capable of great extension, and seem more especially homologous with the branchiæ of the Terebellæ. In front of the latter series is a dense mass of short, pale-pink, thread-like tentacles, while a number of larger, clavated, red-streaked ones, arising from the border of the lip, are in the centre of these. The latter become grooved in contraction. In fine specimens, the varying habit of these three groups of tentacles is very marked. Four annulations occurred on the ventral, and three on the dorsal aspect (the first not being visible after immersion in spirit), before the bristles appeared. These are ranged on fifteen prominent papillæ, and during life are frequently directed forwards. The arrangement of the bristles in the fascicles is peculiar, for they are grouped in pairs—a large and small one alternately—to the number of six (twelve bristles). The latter (Plate XVI. fig. 8) are proportionally strong, and taper from a little above the base to a slightly bent apex. For about a third of the distal portion, there is a very narrow wing or border at each side, which has minute striæ directed forwards and outwards. A row of hooks runs in a transverse manner on the ventral surface from each bristle-papilla, the anterior rows being closer to the papilla than the posterior. These hooks (Plate XVI. figs. 6 and 7) have an elongated and slightly-curved form like those of Terebellides, the head possessing a strong beak, behind which are a series of small processes or fangs. There is a distinct narrowing or neck below the head, and the hook gradually tapers from the succeeding shoulder backwards. This form of hook is confined to the somewhat prominent pads of the bristle-bearing segments. A series of elevated mamillæ succeed the latter, each being furnished with a row of short hooks, which differ entirely from the foregoing (Plate XVI. fig. 7 a). Each has a short and wide basal process, a

\* Nord. Hafs-Annulat. p. 395, tab. xxiv. fig. 65.

characteristic notch between this and the large beak, and numerous curved fangs of smaller size above the latter. The fangs above the larger beak are not simply arranged in a linear manner, but, as it were, form a spined knob, with the points curved obliquely downwards. The ventral surface of the annelid is marked by a central blood-vessel, and in spirit thrown into prominent transverse rugæ. In my specimens the posterior part of the body tapered to a blunt tail, terminated by two soft papillæ; but these represented the ordinary processes, and probably the tail was absent. The peritoneal bodies are of a pale-red colour, and, as usual in such animals, very large. Dr MALMGREN describes the posterior hooks as bidentate, but does not figure them. If this remark is accurate, then the foregoing differs specifically.

#### EXPLANATION OF THE PLATES.

*The following letters have been employed as far as possible in designating similar organs in Ommatoplea and allies.*

<i>a.</i> Proboscis	A. First region of proboscis.
<i>ac.</i> Reflection of proboscis in front of ganglia.	B. Second do. do.
<i>b.</i> Epidermis.	C. Third do. do.
<i>ab.</i> Channel in snout for proboscis.	Ϟ. Globule in lateral stylet-sac.
<i>c.</i> Cutis.	β. Stylets in do.
<i>d.</i> Circular muscular coat.	δ. Duct of lateral stylet-sac.
<i>e.</i> Longitudinal muscular coat.	ε. Muscular chamber behind the floor of the anterior region of proboscis.
<i>f.</i> Superior commissure of ganglia.	η. Floor of anterior chamber of proboscis.
<i>g.</i> Inferior commissure of ganglia.	θ. Muscular setting of granular basal apparatus.
<i>h.</i> Superior lobe of ganglion.	λ. Granular basal sac.
<i>i.</i> Inferior lobe of do.	μ. Ejaculatory duct.
<i>j.</i> Œsophageal apparatus.	μ'. Aperture of ejaculatory duct into chamber ε.
<i>j'</i> . Digestive canal-proper.	π. External granular glands.
<i>k.</i> General stroma of snout.	ρ. Reservoir.
<i>l.</i> Cephalic vessel.	σ. Glands of reservoir.
<i>m.</i> Cephalic sac.	τ. Looping muscular fibres of the walls of reservoir.
<i>m'</i> . Duct of do.	τo. Longitudinal muscular fibres of the walls of reservoir.
<i>n.</i> Great lateral nerve-trunk.	φ. Duct of communication with the posterior chamber.
<i>o.</i> Proboscidian sheath.	χ. Wall of posterior chamber.
<i>p.</i> Dorsal blood-vessel.	ψ. Muscular ribbons.
<i>q.</i> Anastomotic branch.	
<i>r.</i> Lateral blood-vessel.	
<i>ov.</i> Ova <i>in situ</i> .	
<i>v.</i> Lateral stylet-sacs.	
<i>z.</i> Anus.	

*Letters used to designate similar parts in Borlasia and Cephalothrix.*

<i>a.</i> Proboscis.	<i>d'</i> . Basement-layer.
<i>ao.</i> Tube for proboscis in snout.	<i>d''</i> . Pigment-layer in <i>B. olivacea</i> .
<i>b.</i> Cephalic fissures.	<i>e.</i> External (longitudinal) layer.
<i>c.</i> Ciliated epidermis.	<i>e'</i> . Circular muscular layer.
<i>d.</i> External layer of cutis.	<i>e''</i> . Inner (longitudinal) muscular layer.

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|--|--|
| <i>f.</i> Superior ganglionic commissure.    | <i>r.</i> Lateral blood-vessel.  |
| <i>g.</i> Inferior do. do.                   | <i>s.</i> Lacunæ behind ganglia.   |
| <i>h.</i> Ganglia.                           | <i>u.</i> Vascular meshes around the œsophageal region.  |
| <i>h'</i> . Superior lobe of ganglion.       | <i>v.</i> Larger vascular cavity at each side of the sheath for the proboscis in front.                |
| <i>h''.</i> Inferior lobe of do.             | <i>w.</i> Mouth.   |
| <i>j.</i> Œsophageal region.                 | <i>y.</i> Constriction marking the junction of the œsophageal region with the digestive cavity-proper. |
| <i>j'</i> . Alimentary cavity-proper.        | <i>z.</i> Anus.  |
| <i>k.</i> General stroma of cephalic region. | <i>ov.</i> Ovaries and their remains.  |
| <i>m.</i> Cephalic sacs.                     | <i>ψ.</i> Muscular ribbons of proboscis.   |
| <i>m'</i> . Ducts of cephalic sacs.          |  |
| <i>n.</i> Great lateral nerves.              |  |
| <i>o.</i> Proboscidian sheath.               |  |
| <i>p.</i> Dorsal blood-vessel.               |  |

## PLATE IV.

- Fig. 1. Transverse section, a short distance behind the tip of the snout of *Ommatoplea alba*, in front of the ganglia, somewhat flattened from pressure. 1, 2, 3, 4, 5, 6, the various bands of fibres described in the text; *e*, longitudinal muscular fibres; *l*, section of cephalic blood-vessel; *m*, section of cephalic sac. × 210 diameters.
- ... 2. Transverse section of the body-wall of *O. alba*, after hardening in spirit and mounting in chloride of calcium; *a*, cutis, with its cells and areolæ, somewhat compressed; *b*, structureless basement-layer; *c*, circular muscular coat; *d*, longitudinal muscular coat; *e*, delicate fibres proceeding from the latter to the viscera. × 700 diameters.
- ... 3. View of a portion of skin snipped from a living specimen, and submitted to moderate pressure. × 350 diameters.
- ... 4. Longitudinal section of the anterior region of the proboscis of *O. alba*. The same letters are used as in fig. 4, Pl. V. × 90 diameters.
- ... 5. Transverse section through the anterior part of the cephalic ganglia, in a specimen which had been chloroformed and then immersed in strong alcohol, so as to protrude a small portion of the proboscis. The inferior commissure (*g*) is not much stretched, but the superior (*f*) is almost imperceptible; *j*, œsophagus. × 55 diameters. In this and other drawings, accuracy has been preferred to symmetry.
- ... 6. Section of the snout in front of fig. 1, showing the channel for the proboscis (*a*), and the cephalic blood-vessels (*l*), just before they complete the arch. × 210 diameters.
- ... 7. Elements as they escape from the fresh skin of the same animal; *a*, granular cells; *b*, mucous or gelatinous masses, having the appearance of oil-globules. × 350 diameters.
- ... 8. Skin of *O. alba*, as seen near the tail of a small living specimen, under slight compression. × 350 diameters.
- ... 9. Corpuscles of the proboscidian fluid; *a*, minute nucleated cells and granules; *b*, spindle-shaped corpuscles. × 500 diameters.
- ... 10. Stylet, from a lateral sac of the same species, showing a "wing" at base (from remains of globule), and an abnormal point. × 210 diameters.
- ... 11. Discs of proboscidian fluid, from a specimen of *Tetrastemma varicolor*. × 350 diameters.
- ... 12. Small gregariniform parasite, from the digestive cavity of *Tetrastemma variegatum*. × 210 diameters.
- ... 13. Proboscidian aperture in snout of *O. alba*. × 210 diameters.
- ... 14. Discs of proboscidian fluid from *Tetrastemma variegatum*. × 350 diameters.

## PLATE V.

- Fig. 1. Transverse section through the cephalic ganglia of *O. alba*, in the line of the commissures, the superior of which, from the flattening of the preparation, is shown very plainly; *a*, proboscis; *d*, circular muscular fibres of the body-wall; *k*, muscular and glandular stroma of the region. × 90 diameters.

- Fig. 2. Transverse section through the body of the same animal some distance behind the ganglia. The sheath for the proboscis now separates the latter from the œsophagus, which has attained a considerable size. The lateral nerve-trunks have nearly reached their proper position, viz., to the inner side of the internal muscular layer of the body-wall; *s*, granular masses at the sides of the œsophagus.  $\times 55$  diameters.
- ... 3. Isolated gland-cells from the posterior chamber of the proboscis.  $\times 350$  diameters.
- ... 4. Transverse section through the anterior region of the proboscis in a large *O. alba*, after hardening in spirit and mounting in chloride of calcium; *a*, central cavity; *b*, the papillary glandular layer; *c*, internal circular muscular coat; *d*, inner longitudinal layer; *e*, peculiar reticulated or beaded layer; *f*, external longitudinal muscular layer; *g*, external layer; *h*, basement-layer.  $\times 90$  diameters.
- ... 5. Transverse section through the stylet-region of the proboscis of the same species, in the line of the lateral sacs.  $\times 350$  diameters.
- ... 6. Glandular papillæ from the proboscis of *Tetrastemma vermiculus*, seen on the free edge of the everted organ.  $\times 700$  diameters.
- ... 7. Glandular papillæ in the anterior region of the proboscis of *O. alba*, seen in the ordinary condition of the organ under pressure.  $\times 210$  diameters.
- ... 8. Portion of the everted inner surface of the posterior chamber of the proboscis of the same species. The glands have for the most part burst and become minutely hirsute.  $\times 350$  diameters.
- ... 9. Portion of the glandular surface of the posterior chamber of the proboscis in its normal condition.  $\times 350$  diameters.
- ... 10. Portion of the inner surface of the same chamber, viewed *in situ* under pressure. The papillæ are hirsute, and their contents scattered over the surface of the organ.  $\times 350$  diameters.
- ... 11. Lanceolate and pedicled papillæ from the anterior part of the proboscis of *T. vermiculus*.  $\times 300$  diameters.
- ... 12. Central stylet of *Ommatoplea purpurea*.  $\times 700$  diameters.
- ... 13. Central stylet and basal apparatus of the same species.  $\times 350$  diameters.
- ... 14. Developing or recently repaired central stylet-apparatus in *T. algæ*.  $\times 700$  diameters.
- ... 15. Stylet from a lateral sac of the same animal.  $\times 700$  diameters.

## PLATE VI.

- Fig. 1. Head of *O. alba*.  $\times 210$  diameters.
- ... 2. Proboscis of the same species, gently but completely extruded under chloroform, so as to render the central stylet prominent.  $\times 55$  diameters.
- ... 3. View of the nervous and circulatory systems in the anterior end of *O. alba*.
- ... 4. Abnormal stylet-region in the same species; *a*, perfect stylet-sac of the left side; *b*, shrivelled sac of the right side.  $\times 210$  diameters.
- ... 5. Stylet-region of the proboscis of *T. varicolor*, with the reservoir somewhat contracted.  $\times 210$  diameters.
- ... 6. Extremity of the posterior chamber of the proboscis in *T. variegatum*, apparently after rupture of the muscular ribands from the sheath of the organ.  $\times 350$  diameters.
- ... 7. Stylet-region in *Ommatoplea melanocephala*.  $\times 90$  diameters.
- ... 8. Circulation, &c., in the posterior end of *O. alba*; a fragment of the same drawing from which fig. 3 was cut.
- ... 9. Isolated lateral stylet-sac of *O. alba*; *a*, a few fibres which probably act as constrictors of the aperture of the duct. The laminated arrangement of the calcareous layers of the stylets is indicated in this figure.  $\times 350$  diameters.
- ... 10. Tip of the snout of *Borlasia olivacea*, with proboscis partly protruded.  $\times 210$  diameters.
- ... 11. Central stylet-apparatus in *Ommatoplea pulchra*; *a*, central stylet; *b*, reserve-stylet *in situ*.  $\times 210$  diameters.
- ... 12. Central stylet and its basal granular apparatus in *O. gracilis*, turned round so as to demonstrate the curvature of both.  $\times 100$  diameters.
- ... 13. Isolated central stylet of the foregoing.  $\times 420$  diameters.

## PLATE VII.

- Fig. 1. Stylet-region of the proboscis of *O. gracilis*.  $\times 210$  diameters.  
 ... 2. Stylet-region of the proboscis of *O. purpurea*.  $\times 210$  diameters.  
 ... 3. Stylet-region of the proboscis of *O. pulchra*.  $\times 90$  diameters.  
 ... 4. Extremity of the posterior region of the proboscis of *O. alba* distended with fluid; *a*, a group of the peculiar dancing granules.  $\times 90$  diameters.  
 ... 5. Stylet-region of the proboscis of *Polia involuta*, VAN BENEDEEN.  $\times 700$  diameters.  
 ... 6. Stylet-region of the proboscis of a young *O. alba*, illustrating the first appearance of the stylets, and the development of the parts. The organ is drawn as it bulged from a wound in the body-wall of the animal.  $\times 700$  diameters.  
 ... 7. Fragment of the œsophagus from a living animal; *a*, inner edge of ciliated fold; *b*, sulcus between two folds.  $\times 350$  diameters.  
 ... 8. Eye of *Ommatoplea pulchra*.  $\times 210$  diameters.  
 ... 9. Central stylet and portion of basal apparatus in a large *O. gracilis*.  $\times 350$  diameters.  
 ... 10. Transverse section of an everted proboscis in a small specimen of *O. pulchra*. The papillose mucous surface has been injured in the manipulation.  $\times 90$  diameters.  
 ... 11. Nerve-cells from a cephalic ganglion of *O. alba*.  $\times 400$  diameters.  
 ... 12. Portion of a sperm-sac in *Tetrastemma varicolor*, showing a streaked and granular aspect, from the varying nature of the contents.  $\times 350$  diameters.

## PLATE VIII.

- Fig. 1. Aspect of the developing proboscis in *O. melanocephala*, about the fifth day after the removal of the original organ.  $\times 55$  diameters.  
 ... 2. Stylet-region of a developing proboscis in the same species; *f*, canal, which by-and-by is occupied by the central stylet. The organ is contracted.  $\times 350$  diameters.  
 ... 3. Anterior region of *Tetrastemma algæ*, showing the arrangement of the digestive system. Enlarged.  
 ... 4. Termination of the posterior chamber of the proboscis (C) in *O. alba*, with muscular ribands.  $\times 210$  diameters.  
 ... 5. Head and anterior portion of *Polia involuta*, V. BEN.; *f*, powerful transverse band of fibres which retains the posterior part of the œsophagus *in situ*.  $\times 180$  diameters.  
 ... 6. The central (*a*) and lateral stylets (*b*) from a young *O. alba*, on the first appearance of the former.  $\times 700$  diameters.  
 ... 7. Cephalic ganglia of *Tetrastemma varicolor*.  $\times 210$  diameters.  
 ... 8. Unimpregnated ovum of *O. alba*; *a*, outer coat; *b*, inner coat; *c*, vitellus; *d*, "micropyle," or cicatrix-like arrangement.  $\times 90$  diameters.  
 ... 9. Ovum of *O. gracilis* after impregnation; *a*, outer coat; *b*, inner coat; *c*, vitellus.  $\times 90$  diameters.  
 ... 10. The inner coat and vitellus of an ovum (of *O. gracilis*) at the same stage of development, with the relations of the spermatozoa.  $\times 210$  diameters.  
 ... 11. Ovum of *O. alba*, just before the extrusion of the embryo.  $\times 90$  diameters.  
 ... 12. Spermatozoa of *Tetrastemma vermiculus*.  $\times 1000$  diameters.  
 ... 13. Spermatozoa of *O. alba*.  $\times 800$  diameters.  
 ... 14. Spermatozoa of *T. variegatum*.  $\times 400$  diameters.

## PLATE IX.

- Fig. 1. Young *O. alba*, on extrusion from the egg, somewhat compressed.  $\times 55$  diameters.  
 ... 2. Young *O. alba* eight days older than the preceding; *b*, stylet-region; *c*, point where the posterior chamber of the proboscis becomes lost, after curving forwards.  $\times 90$  diameters.  
 ... 3. Structure of the stylet- and reservoir-regions in *O. alba*. Considerably magnified.  
 ... 4. View of the cutis in a living specimen of *Borlasia olivacea* as a transparent object.  $\times 210$  diameters.  
 ... 5. Streaked arrangement of the cutis of *B. olivacea*, from the dorsum.  $\times 210$  diameters.  
 ... 6. Pigment-cells from the anterior part of the dorsum of the same species.  $\times 350$  diameters.  
 ... 7. View of the skin of a living *Meckelia annulata*.  $\times 350$  diameters.  
 ... 8. Spermatozoa of *Ommatoplea gracilis*.  $\times 700$  diameters.  
 ... 9. Spermatozoa of *Polia involuta*.  $\times 950$  diameters.

- Fig. 10. Transverse section through the contracted reservoir-region of *O. alba*, showing the complex spiral arrangement of the fibres.  $\times 55$  diameters.
- ... 11. Superficial structure of the reservoir- and stylet-regions in the same species.
- ... 12. Central stylet and basal apparatus with radiating fibres in *Tetrastemma vermiculus*.  $\times 350$  diameters.
- ... 13. Stylet-region of a young *O. alba*, some weeks older than that represented in fig. 2.  $\times 350$  diameters.
- ... 14. Transverse section through the posterior chamber of the proboscis in a large example of *O. alba*. The circular and longitudinal muscular and the mucous coats are well shown.  $\times 90$  diameters.
- ... 15. Young *Tetrastemma variegatum*, shortly after extrusion from the egg, and somewhat compressed, so as to show its cellulo-granular structure.  $\times 350$  diameters.
- ... 16. Portion of the long posterior chamber of the proboscis of *O. purpurea*, showing the characteristic plaits of the mucous surface.  $\times 90$  diameters.

## PLATE X.

- Fig. 1. Enlarged view of the anterior end of *B. olivacea* as a transparent object.
- ... 2. Transverse section through the curious example (probably a variety of *Meekelia*) from Balta; *d*, external layer of cutis; *d'*, basement-layer; *e*, longitudinal muscular layer; *ea*, dorsal sub-divisions of the latter coat in the central line; *e'*, circular muscular coat; *j*, section of the œsophageal region of the digestive tract; *ja*, distinct band of muscular fibres enclosing the latter; *n*, lateral nerve; *o*, sheath for proboscis; *r*, vascular spaces.  $\times 55$  diameters.
- ... 3. Transverse section through the body of *Cephalothrix filiformis*. The proboscis is coiled in its sheath.  $\times 90$  diameters.
- ... 4. Transverse section just behind the tip of the snout of *Borlasia olivacea*. The grouping of the pigment (3) readily enables the observer to distinguish the dorsal from the ventral surface; 2, powerful series of fibres arching over the channel leading to proboscis, and which radiate into the surrounding stroma (*k*).  $\times 55$  diameters.
- ... 5. Portions of the inner surface of the proboscis of the same species, showing the glandular papillæ, slightly compressed.  $\times 700$  diameters.
- ... 6. Gland-cells from the wall of the digestive cavity of *Ommatoplea alba*.  $\times 400$  diameters.
- ... 7. One of the same slightly compressed glands.  $\times 700$  diameters.
- ... 8. Contents of the same gland-cells, with oil-globules.  $\times 700$  diameters.
- ... 9. Spermatozoa of *Borlasia olivacea*.  $\times 700$  diameters.

## PLATE XI.

- Fig. 1. Transverse section through *Borlasia olivacea*, just at the commencement of the œsophageal region; 2, radiated or slightly arborescent arrangement of the external longitudinal muscular coat at the sides of the mouth. The thick folds of the œsophagus are seen almost at the termination of the anterior cul-de-sac.  $\times 90$  diameters.
- ... 2. Arrangement of the ova in the ovisacs of *Tetrastemma vermiculus*; *a*, proboscis; *o*, proboscidian sheath. Only a fragment of the body is represented.  $\times 24$  diameters.
- ... 3. Spermatozoa of *Cephalothrix filiformis*.  $\times 900$  diameters.
- ... 4. Spermatozoa of *Lineus longissimus*.  $\times 900$  diameters.
- ... 5. Spermatozoa of *Borlasia octoculata*.  $\times 800$  diameters.
- ... 6. Transverse section through the body-wall of *Lineus longissimus* at a somewhat narrow portion; *d*, external cuticular layer; *d''*, pigmentary layer divided into two strata by a definite black band (2); 3, curious translucent stratum, cut into somewhat regular spaces. Other letters as usual.  $\times 210$  diameters.
- ... 7. Longitudinal section of the tissues of the body-wall in the same species; 4, 4, sections of the transverse connecting trunks between the lateral and dorsal vessels; 5, granular stroma within the inner longitudinal muscular coat, supporting the former and various other tissues.  $\times 90$  diameters.
- ... 8. Transverse section of the body-wall of *Borlasia olivacea*.  $\times 350$  diameters.
- ... 9. Proboscis of *Cephalothrix filiformis*, slightly everted, so as to exhibit the acicular papillæ.  $\times 350$  diameters.

Fig. 10. Aggregations of fatty granules from the discarded coating of the embryo of *B. olivacea*.  
× 210 diameters.

## PLATE XII.

- Fig. 1. Transverse section through the proboscis of a Borlasian (*Micrura*) from St Andrews; *a*, external coat; *b*, great longitudinal muscular layer; *c*, belt of circular muscular fibres; *d*, basement-layer; *e*, incomplete series of longitudinal fibres which do not occur in the common species; *f*, glandular mucous coat; *g*, peculiar lozenge-shaped portion of longitudinal fibres, formed by the splitting and crossing of two bands from the circular muscular coat; *g*, separated segment at the other pole of the circle. × 90 diameters.
- ... 2. Transverse section of the snout of *Borlasia olivacea*, somewhat behind that shown in fig. 4. Pl. X., and through the anterior part of the cephalic fissures. The channel for the proboscis has become more central in position. The superior pigmentary belt (3) is somewhat narrower, and an inferior (4) has now appeared. The central channel has a layer of longitudinal muscular fibres internally, and a powerful series of oblique and circular fibres (2, 2) form a very efficient exterior investment. × 55 diameters.
- ... 3. Transverse section of a specimen of *B. olivacea*, in which the ova are well developed. The shrunken condition of the walls of the digestive cavity (*j'*), with the numerous array of gregariniform parasites, is in strong contrast with the state of the animal after spawning. The specimen had been in spirit for a considerable time before dissection. × 55 diameters.
- ... 4. Parasitic ciliated animal from the tissues of the same species. The letters, *a*, *b*, *c*, and *d*, correspond with the groups of segments described in the text. × 350 diameters.
- ... 5. The foregoing parasite in an earlier state of development. × 350 diameters.
- ... 6. The last-mentioned specimen subjected to slight pressure, so as to exhibit the segments. × 350 diameters.
- ... 7. Posterior end of a young *B. olivacea*, showing the anal papilla. × 210 diameters.
- ... 8. Transverse section through the post-ganglionic region of *Lineus lactea*, MONT. MS., showing the long vascular lacunæ (*s*, *s*) in front of the œsophageal region. The slice of the proboscis has fallen out of its sheath (*o*). × 90 diameters.
- ... 9. Stylet-region in *Tetrastemma variegatum*, somewhat contracted, and with the floor of the anterior chamber pouted forwards. The latter condition is more easily seen in *Tetrastemma* than in *O. alba*. × 210 diameters.
- ... 10. Fragment of the wall of the digestive chamber-proper, from the living *Borlasia olivacea*. The cilia mark the inner surface. × 350 diameters.
- ... 11. Cells from the digestive cavity of a young *Cephalothrix filiformis*. × 700 diameters.
- ... 12. One of the pinnate processes of the scale of *Sthenelais dendrolepis*, CLAP. × 90 diameters.

## PLATE XIII.

- Fig. 1. Highly magnified view of the anterior end of *Cephalothrix filiformis* (Astemma); *b*, *b*, bridles of sheath for proboscis.
- ... 2. Arrangement of the vessels at the posterior extremity of *Borlasia olivacea*. Magnified.
- ... 3. Ovum of *Cephalothrix filiformis* immediately after deposition. × 350 diameters.
- ... 4. Flask from the mucous cord of *B. olivacea*, with two young animals somewhat compressed; *a*, embryo forced from its ciliated cellulo-granular fatty coating, the bulk of which lies at *c*; *b*, embryo still within the ciliated coating. × 55 diameters.
- ... 5. Young *B. olivacea* immediately after leaving the flask-shaped capsule; *b*, opening of the cephalic sac of the right side. The other letters as in the adult. × 90 diameters.
- ... 6. Transverse section through the middle of *B. olivacea* after the second or great region of the digestive cavity has attained its full size. The difference between such a view and the indistinct mass formed by the Ommatoplean digestive cavity, after section, is characteristic. × 55 diameters.
- ... 7. Cellular elements of the wall of the digestive chamber of the same species. × 700 diameters.
- ... 8. Pale oily region with germinal vesicle (*a*), and germinal dot (*b*), in an ovum removed from the body of the female *B. olivacea*. × 350 diameters.
- ... 9. Elements of the glandular papillæ of the proboscis of *B. olivacea*, after their escape into the surrounding water. × 700 diameters.

Fig. 10. } Hooks of *Amage*. × 700 diameters.  
 ... 10 a. }

## PLATE XIV.

- Fig. 1. Ovum of *Polia involuta*, VAN BEN., immediately after deposition. × 350 diameters.  
 ... 2. Ovum of the same species about the 10th day, showing the ciliated embryo revolving therein. × 350 diameters.  
 ... 3. Young of *Cephalothrix filiformis* shortly after extrusion from the egg. × 350 diameters.  
 ... 4. A young specimen of *Cephalothrix*, two days older than that shown in fig. 3; *a*, mouth; *b*, granules of digestive cavity. × 210 diameters.  
 ... 5. A specimen about three days older than the foregoing (fig. 4). × 210 diameters.  
 ... 6. Young *Polia involuta*, extruded from the body of the adult under pressure. It has the same appearance when originating in a free ovum. × 350 diameters.  
 ... 7. Young *C. filiformis*, after shedding the long anterior whip of cilia, but having the lateral tufts (*c*) and eyes; *a*, mouth; *b*, granules of digestive cavity. × 210 diameters.  
 ... 8. Transverse section through the proboscis of *Lineus longissimus*. × 55 diameters.  
 ... 9. Magnified view of the ganglionic region of a large *Ommatoplea alba*, in which a parasitic ovum (*y*) lay imbedded in a granular lobulated mass (*y'*).  
 ... 10. Parasite extruded from capsule; *a*, opaque cellular and granular mass; *b*, ventral disc; *c*, oral disc; *d*, œsophageal bulb; *e*, alimentary cæca; *f* and *g*, large circular granular bodies.  
 ... 11. Transverse section through the body of a large *Meckelia annulata*. × 55 diameters.  
 ... 12. Head and proboscis (*a*) of a remarkable variety of *Meckelia*, brought from Shetland (Balta) by Mr Gwyn Jeffreys; *b*, curiously frilled arrangement of the enlarged homologue of the superior lip of the cephalic fissure; *w*, prolapsus of textures from mouth. Magnified under a lens.  
 ... 13. Hook of *Ampharete artica*, MGRN. × 700 diameters.  
 ... 14. Hook of *Amphicteis gunneri*. × 700 diameters.  
 ... 15. Hook of *Terebella*, from the Hebrides. × 700 diameters.

## PLATE XV.

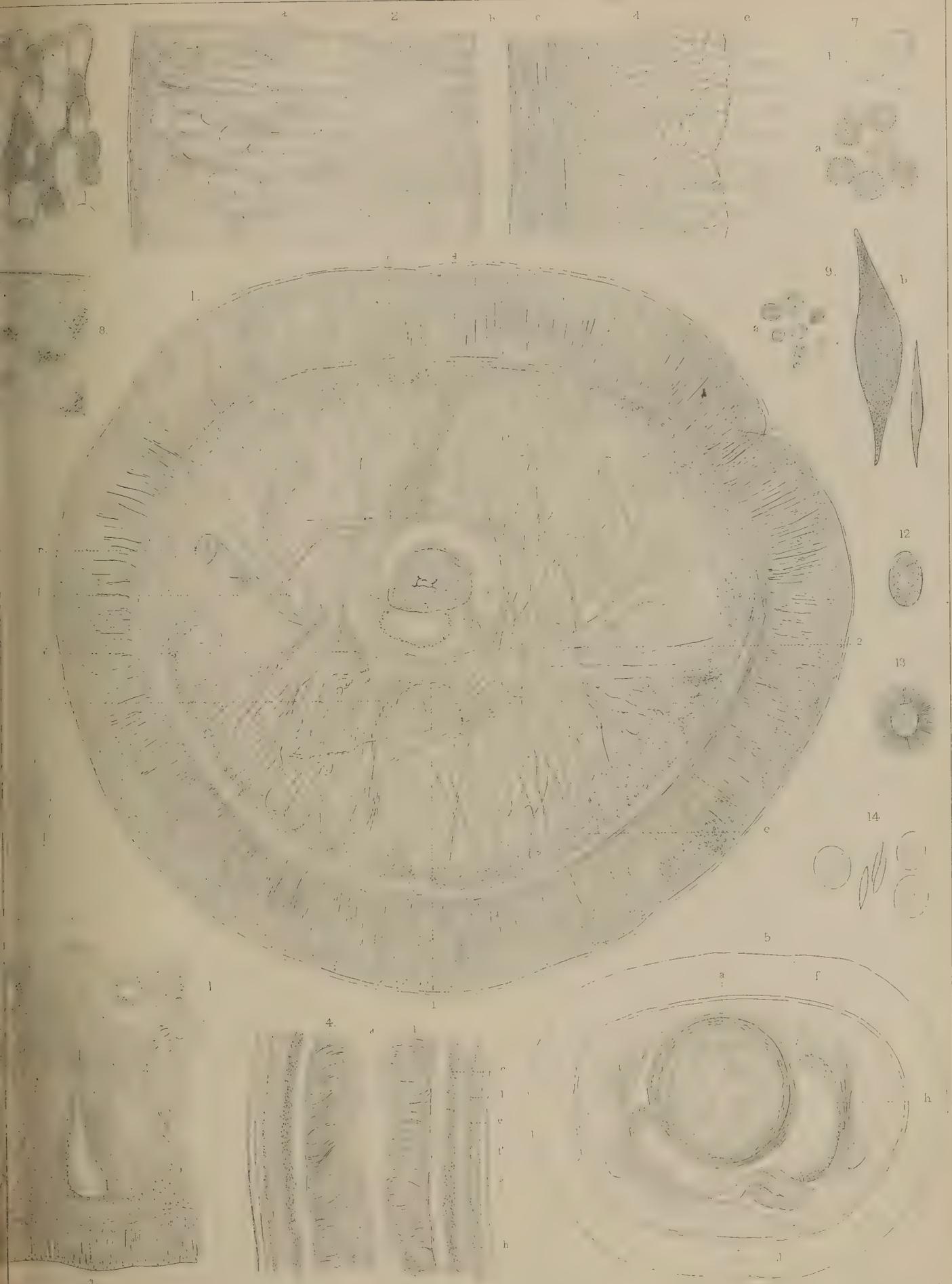
- Fig. 1. Bristles of *Amphinome vagans*; *a*, bristle from the inferior lobe of foot; *b*, *c*, bristles of the superior lobe. × 700 diameters.  
 ... 2 *a*. Dorsal bristle of *Lepidonotus pellucidus*, EHLERS. × 700 diameters.  
 ... 2 *b*. Ventral bristle of the same species. × 700 diameters.  
 ... 3. Ventral bristle of *Polynœ longisetis*, GRUBE. × 350 diameters.  
 ... 3 *a*. Tip of the dorsal cirrus of the same species. × 55 diameters.  
 ... 4. } Two of the characteristic bristles (with jointed tips) of *Sthenelais dendrolepis*, CLAP.  
 ... 5. } × 350 diameters.  
 ... 6. }  
 ... 6 *a*. } Ventral bristles of *Halosydna gelatinosa*, SARS. × 180 diameters.  
 ... 6 *b*. }  
 ... 7. Ventral bristle of *Ophiodromus vittatus*, SARS., with short terminal process. × 700 diameters.  
 ... 8. Fragment of a bristle from the dorsal lobe of the same animal. × 700 diameters.  
 ... 9. Bristle of *Notophyllum polynoides*, ÆRST. × 420 diameters.  
 ... 9 *a*. Lateral view of the end of the shaft and its processes in the same bristle. × 700 diameters.  
 ... 9 *b*. Profile view of the same. × 700 diameters.  
 ... 10. Bristles of the littoral form of *Sphaerosyllis hystrix*, CLAPARÈDE; *a*, simple spine; *b*, jointed bristle. × 700 diameters.  
 ... 11. Jointed bristles of *Autolytus pictus*, EHLERS. × 700 diameters.  
 ... 12. Bristle of *Syllis*, resembling *S. macrocera*, GRUBE. × 700 diameters.  
 ... 13 *a*. Fragment of the frontal bristle (of *Siphonostoma buskii*) represented in fig. 4 *a*, Pl. XVI. × 350 diameters.  
 ... 13 *b*. Piece of a corresponding bristle from *Trophonia glauca*, MALMGREN. × 350 diameters.  
 ... 14. Hook from the rasp-like surface of *Ammochares ottonis*, GRUBE. × 900 diameters.  
 ... 15. Hook of *Terebella zostericola*, ÆRST. × 700 diameters.

- Fig. 16. Hook of *Rhodine loveni*, MGRN.  $\times 700$  diameters.  
 ... 17. Hook of a species allied to *Ereutho smitti*, MGRN.  $\times 900$  diameters.  
 ... 18 } Hooks of a form closely resembling *Polycirrus aurantiacus*, GRUBE.  $\times 900$  diameters.  
 ... 19 }  
 ... 20. Hook of *Pista cristata*, MÜLLER.  $\times 700$  diameters.  
 ... 21. Bristles of *Syllis tubifex* (?), GOSSE; *a*, *a*, from middle of body; *b*, spine; *c*, bristle from the third foot.  $\times 280$  diameters.

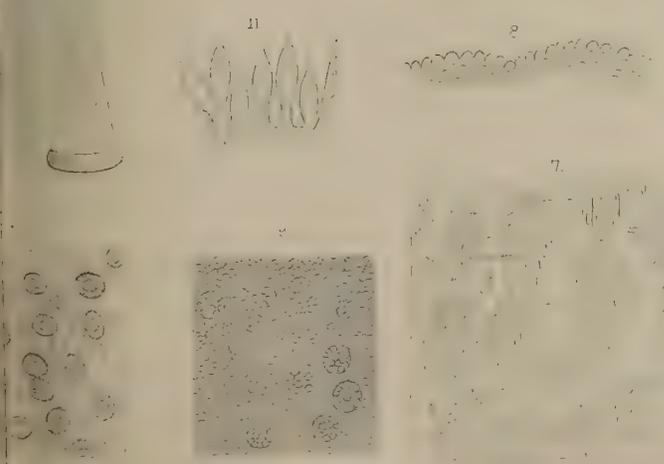
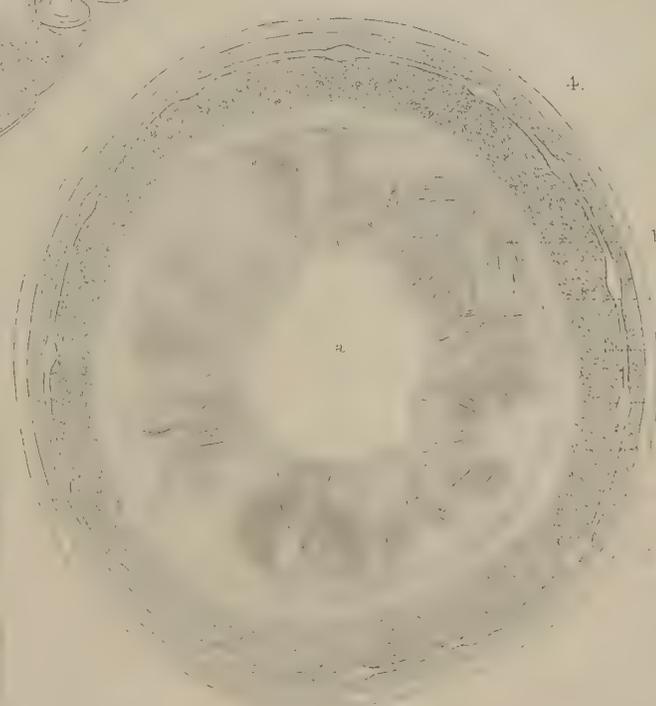
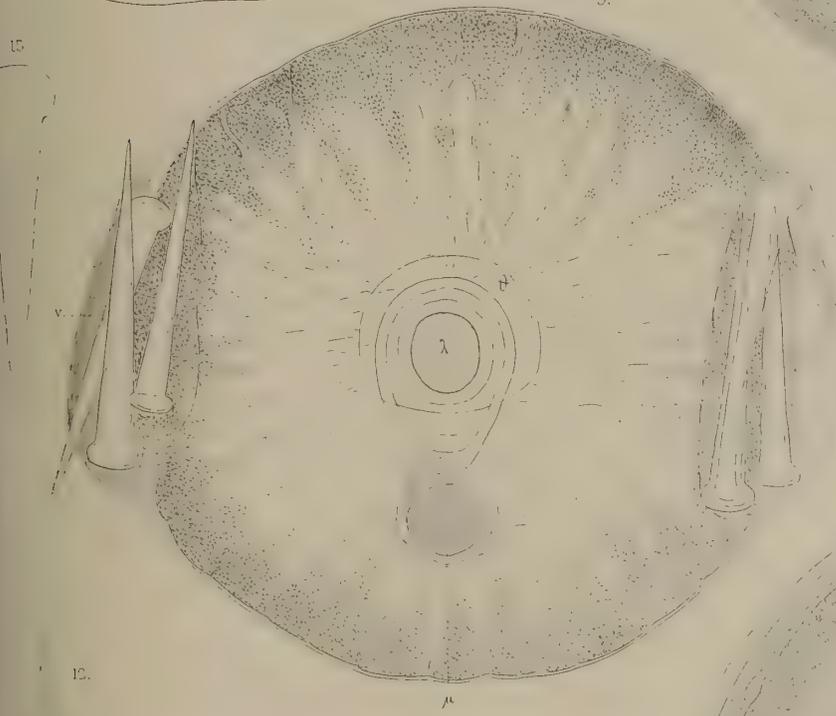
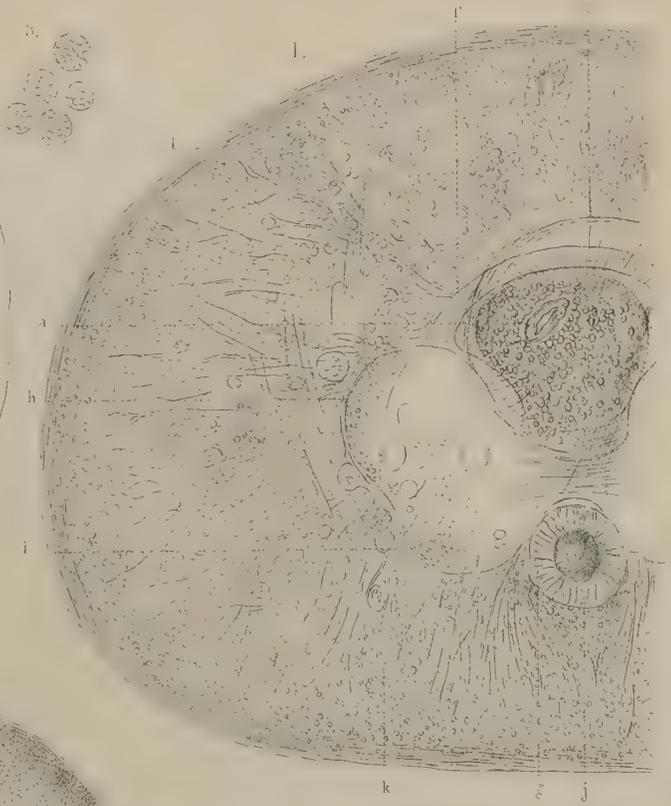
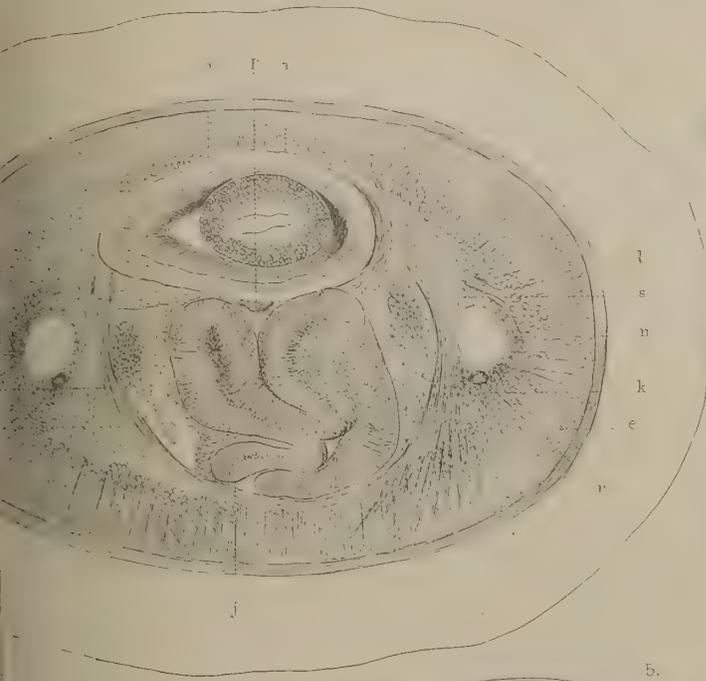
## PLATE XVI.

- Fig. 1. Ventral bristle of *Castalia punctata*, MÜLL.  $\times 700$  diameters.  
 ... 2. Bristle of *Psamathe fusca*, JOHNST.  $\times 700$  diameters.  
 ... 3. Jointed hook from the anterior segments of *Hyalinæcia sicula*, QUATREF.  $\times 700$  diameters.  
 ... 3 *b*. Simple hook from the posterior region of the same.  $\times 700$  diameters.  
 ... 3 *c*. Bristle of the foregoing species.  $\times 700$  diameters.  
 ... 4. Hook of *Siphonostoma buskii*, n. sp.  $\times 350$  diameters.  
 ... 4 *a*. Bristle from the frontal series of the same species.  $\times 90$  diameters.  
 ... 5. Forked bristle of *Eumenia jeffreysi*, n. sp.  $\times 700$  diameters.  
 ... 6. Hook from the bristle-bearing segments of *Trichobanchus glacialis*, MGRN.  $\times 90$  diameters.  
 ... 7. The same.  $\times 700$  diameters.  
 ... 7 *a*. Hooks from the posterior segments of the same annelid.  $\times 700$  diameters.  
 ... 8. Bristle of *T. glacialis*.  $\times 350$  diameters.  
 ... 9. Bristle of *Spherosyllis* from the Minch.  $\times 700$  diameters.  
 ... 10. Bristle of *Pionosyllis malmgreni*, n. sp.  $\times 700$  diameters.  
 ... 11. Foot of *Staurocephalus kefersteini*, n. sp.; *f*, superior cirrus; *g*, inferior cirrus.  $\times 210$  diameters.  
 ... 11 *a*. }  
 ... 11 *b*. } The varieties of the bristles in the same species, as described in the text.  $\times 700$   
 ... 11 *c*. } diameters.  
 ... 11 *d*. }  
 ... 12. Hook of *Clymene ebiensis*, AUD. & ED.  $\times 350$  diameters.  
 ... 13. Hook of *Praxilla (artica?)* MGRN.  $\times 350$  diameters.  
 ... 14. Bristle of *Syllis krohni*, EHLERS.  $\times 700$  diameters.  
 ... 15 *a*. } Bristles of *Syllis cornuta*, RATHKE.  $\times 700$  diameters.  
 ... 15 *b*. }  
 ... 16 *a*. } Hooks of *Sabellides sexcirrata*, SARS.  $\times 700$  diameters.  
 ... 16 *b*. }  
 ... 17. Foot of *Notocirrus scoticus*, n. sp.; *a*, branchial lobe; *b*, spine; *c*, bristles.  $\times 350$  diameters.

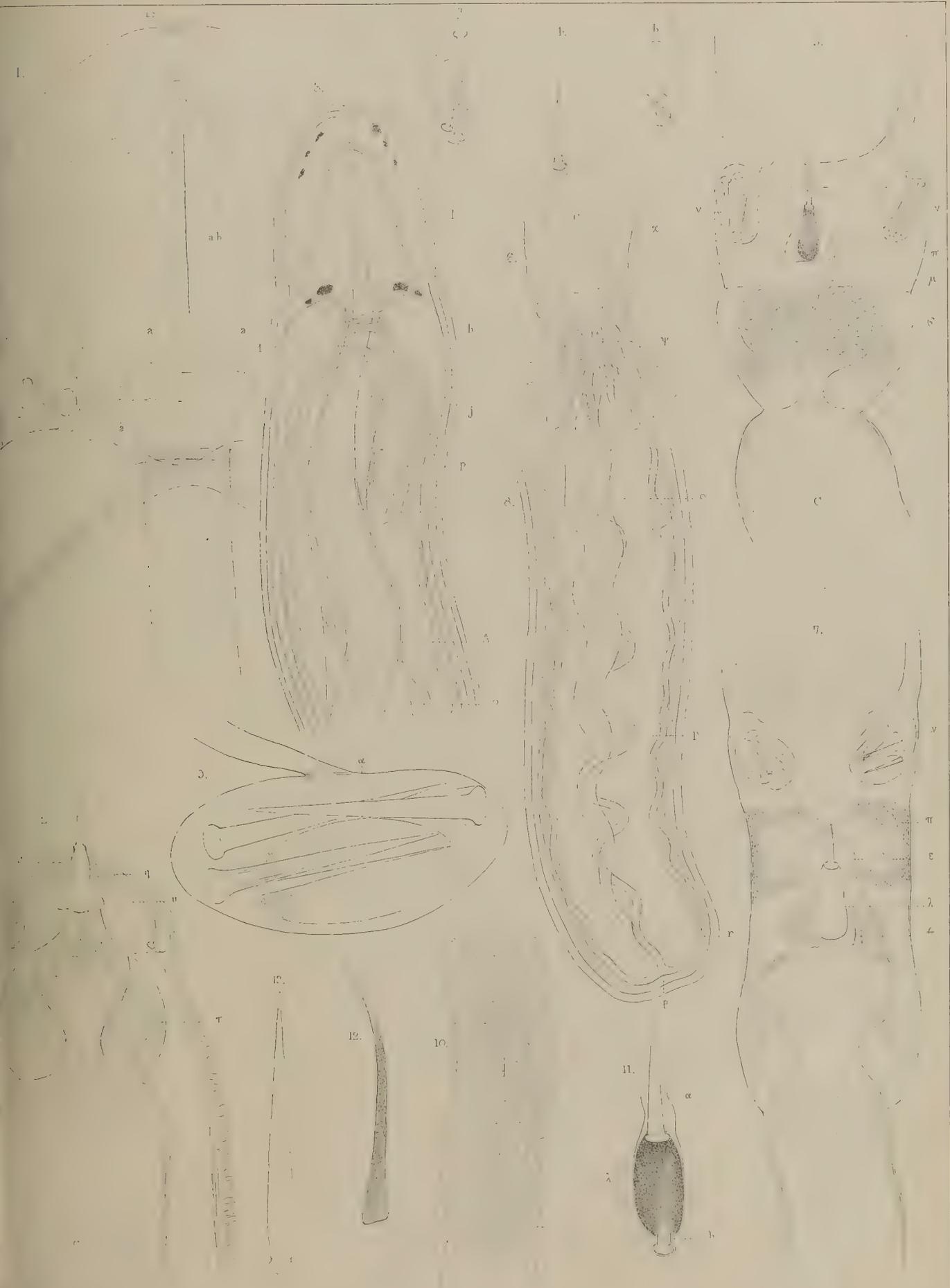




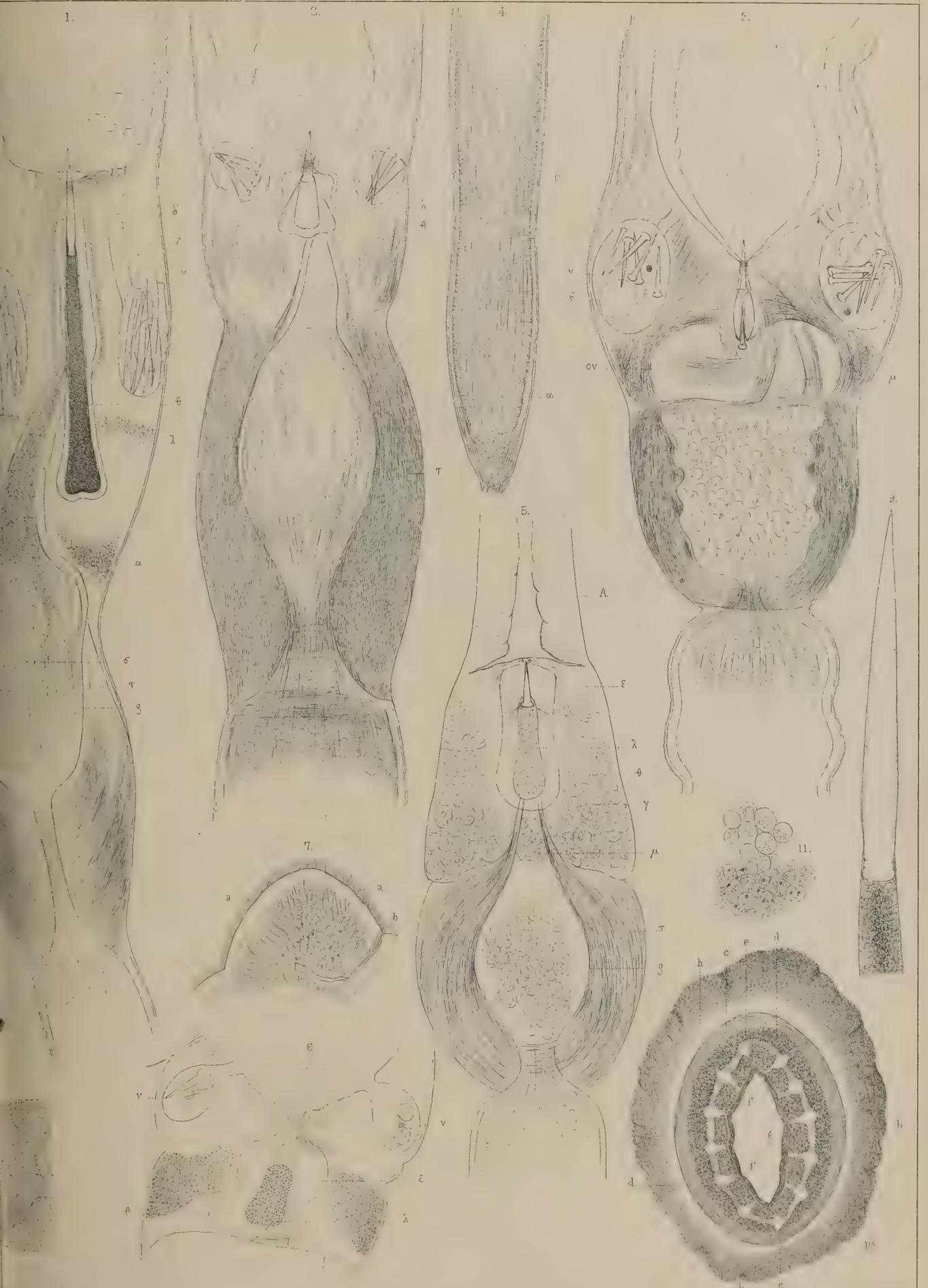




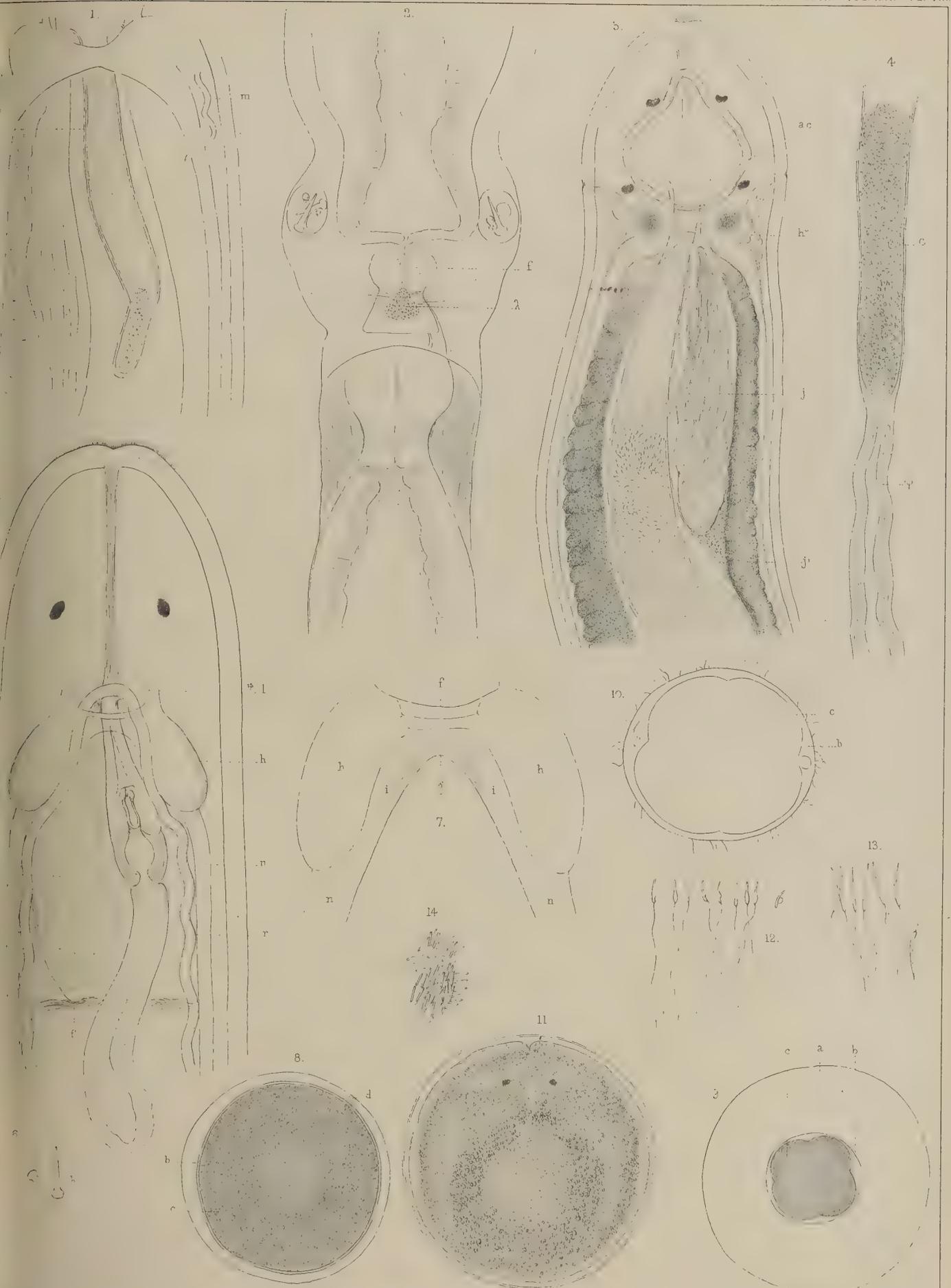




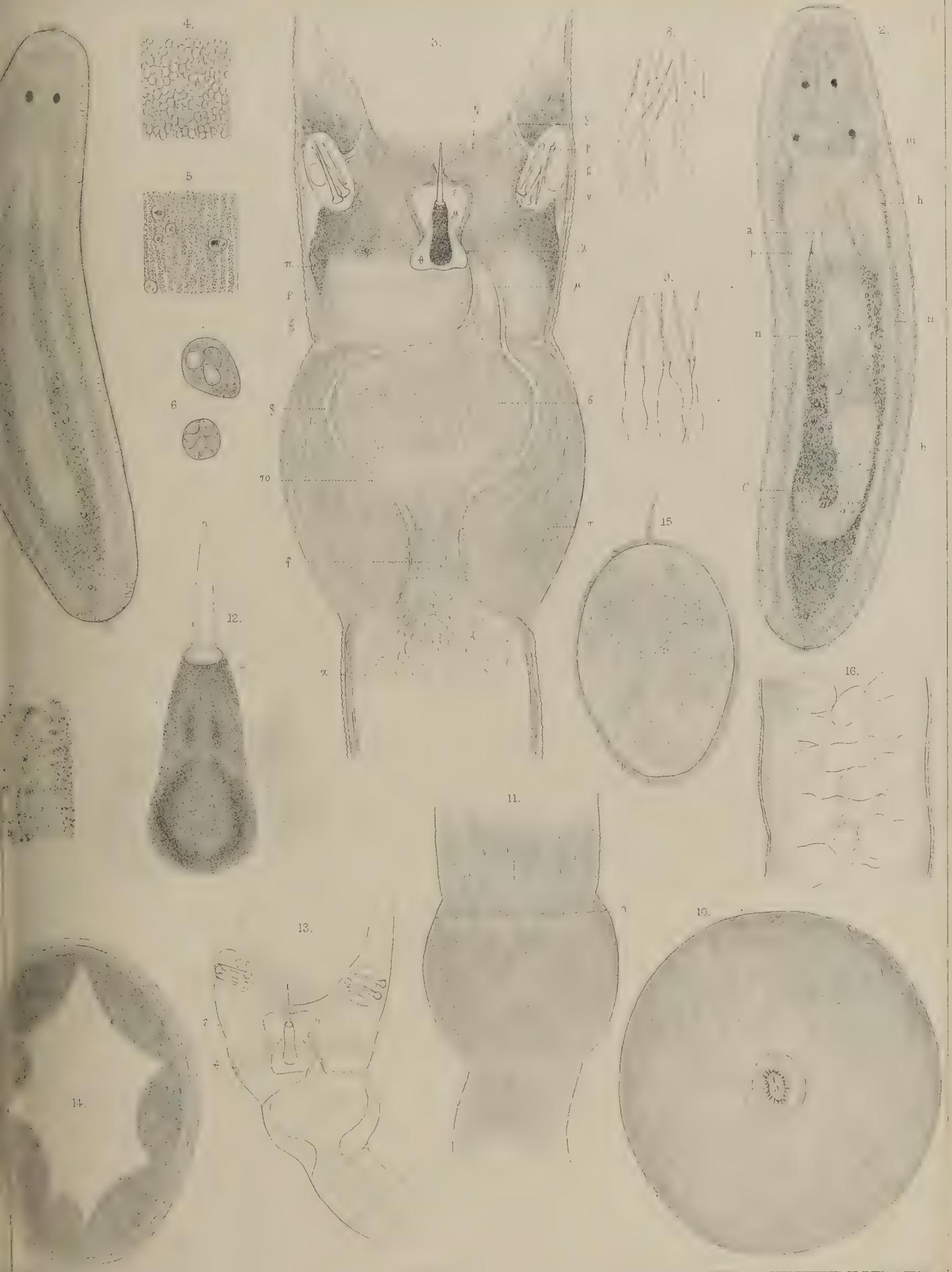




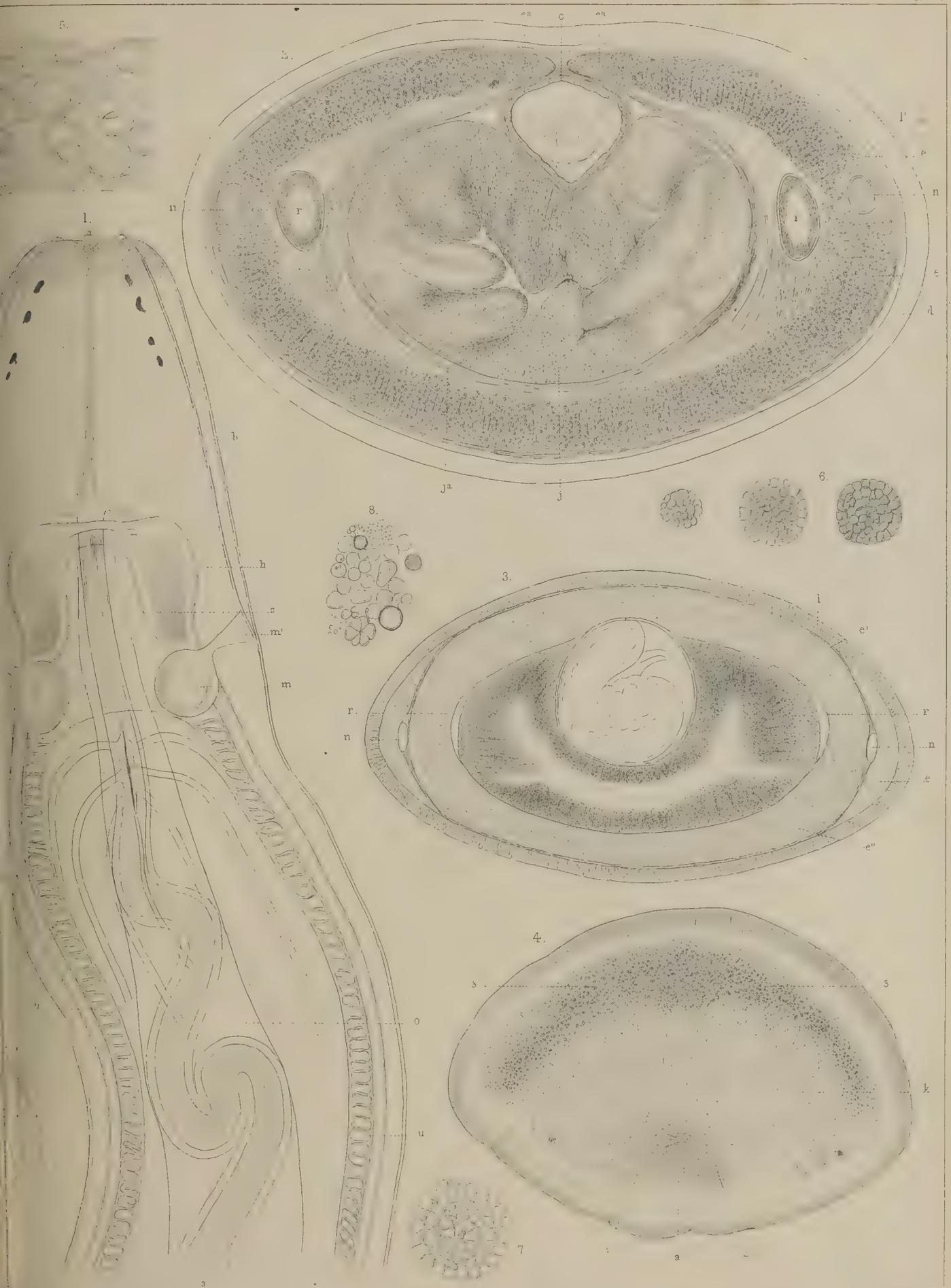














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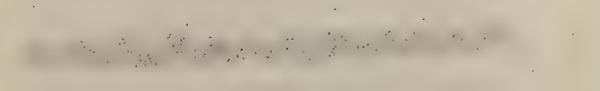
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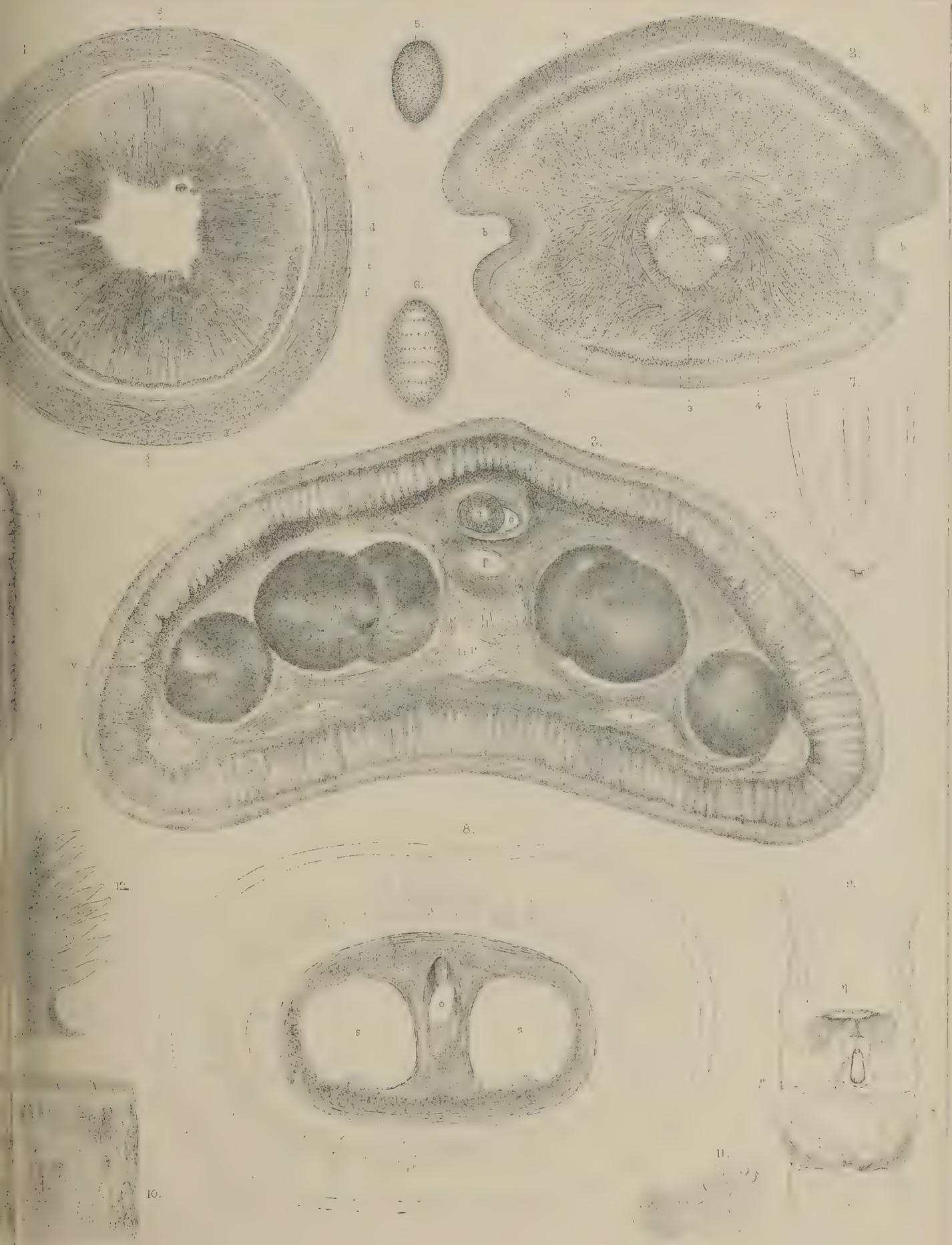
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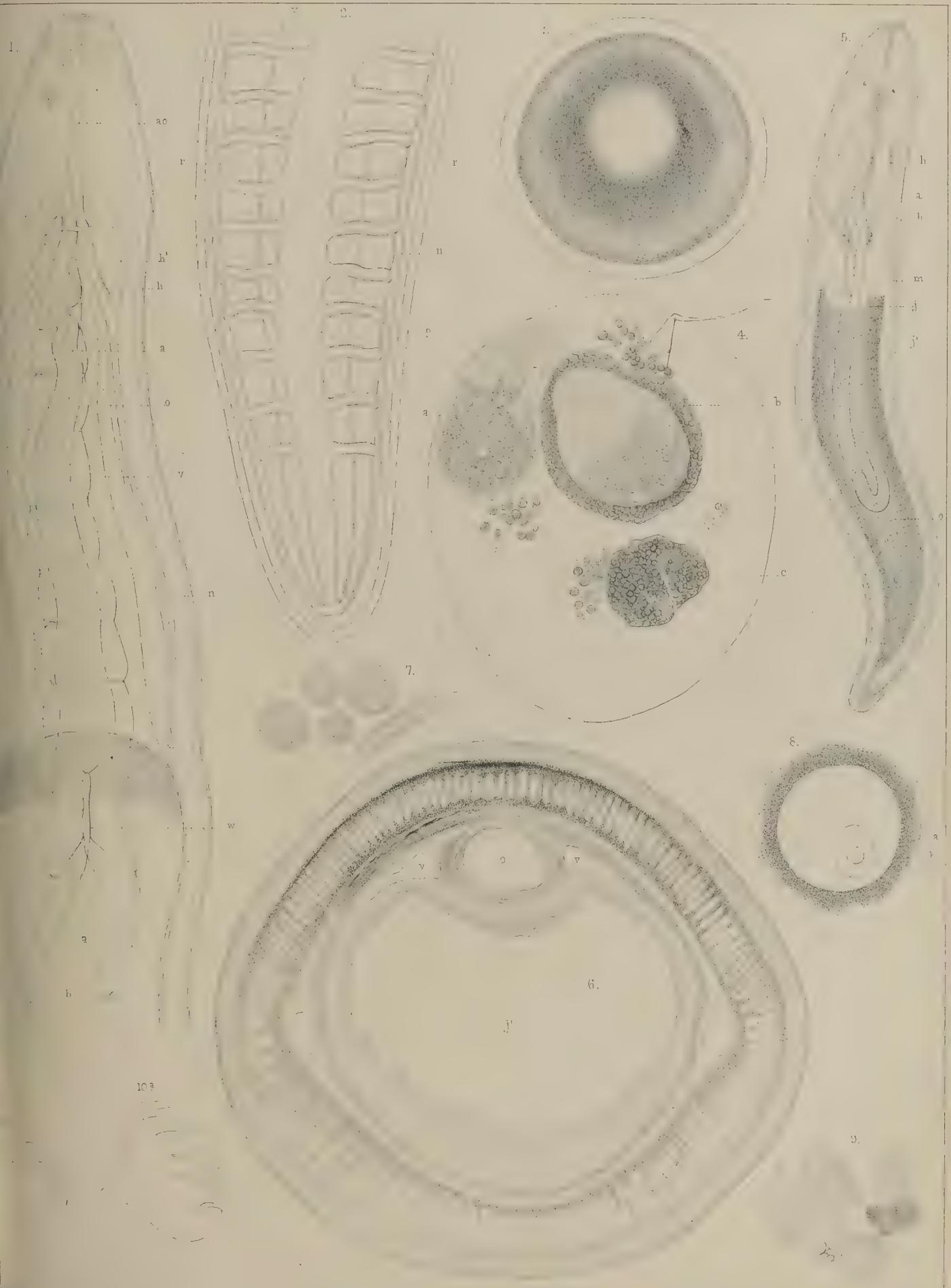
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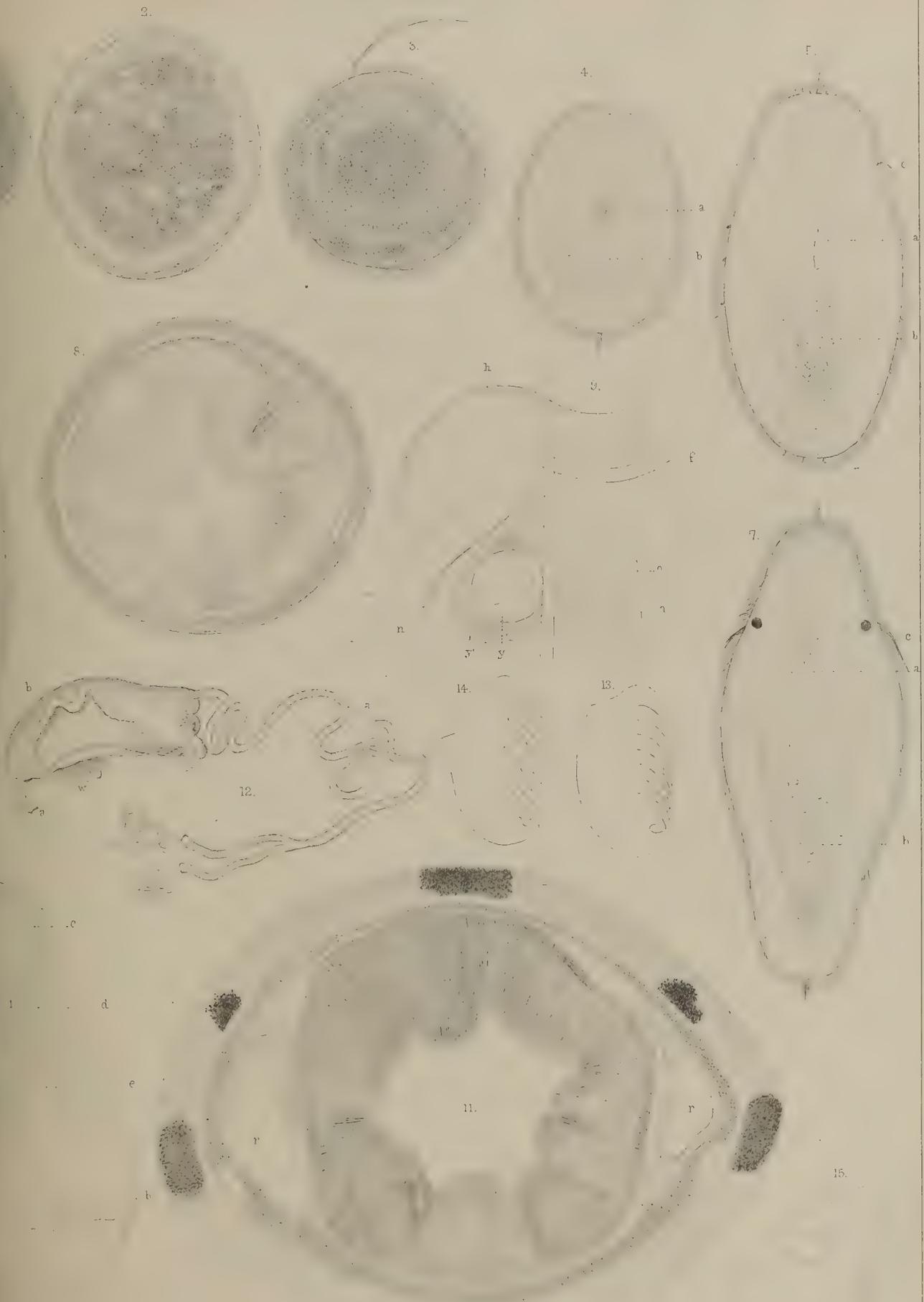




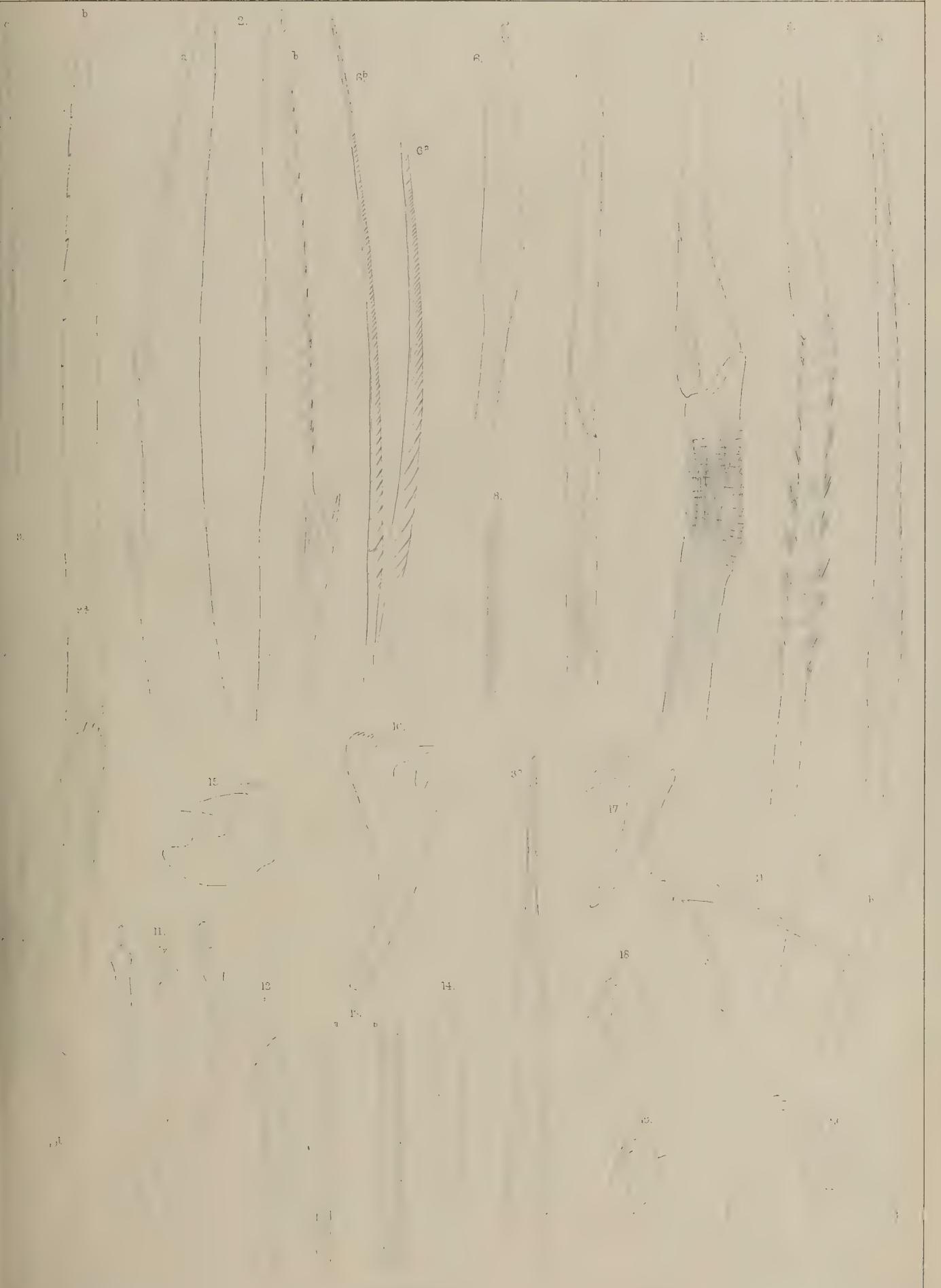


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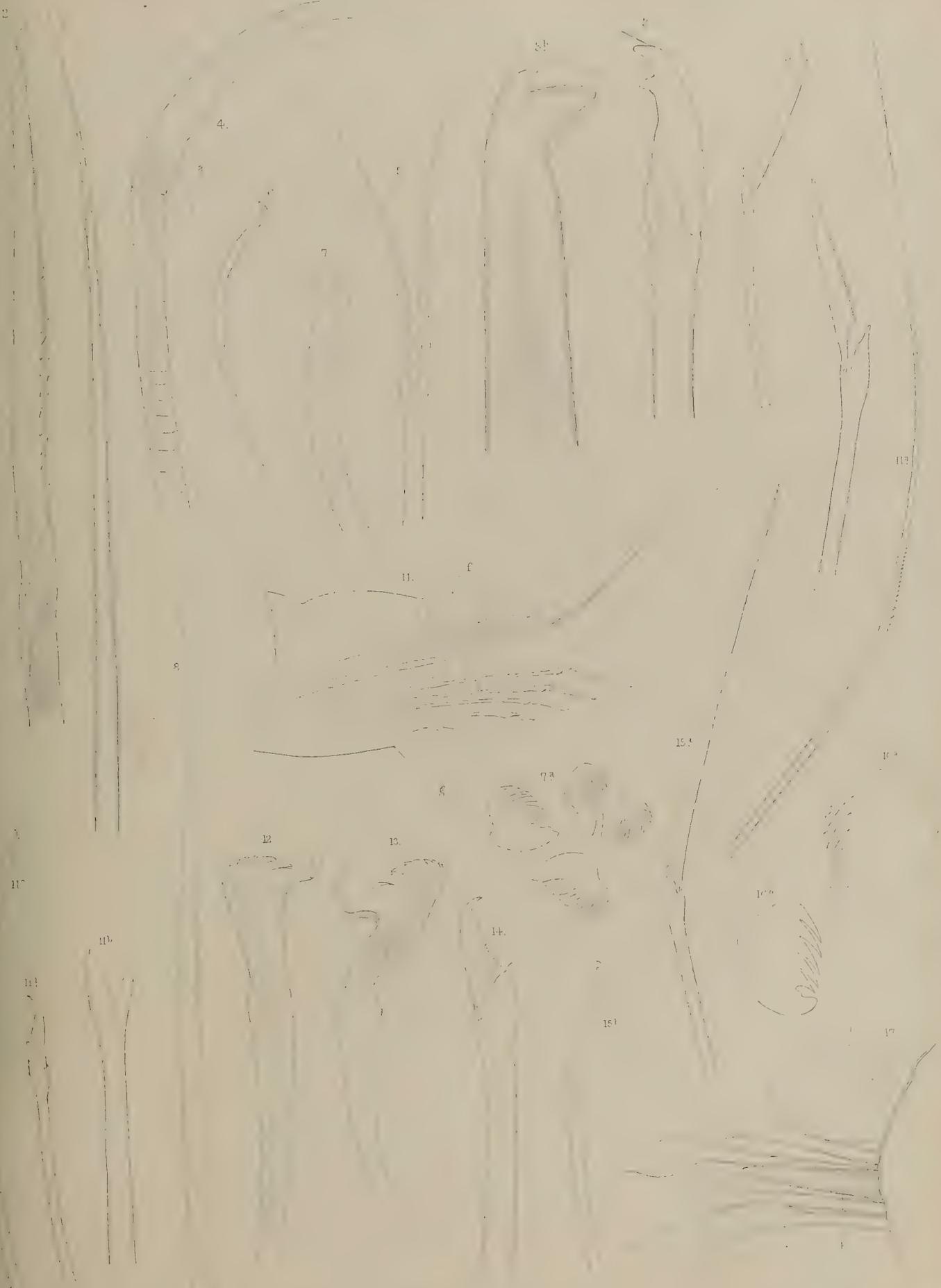














X.—*Observations on the Temperature of Newly-Born Children.* By T. J. MACLAGAN, M.D., Dundee. Communicated by Dr J. MATTHEWS DUNCAN.

(Read 5th April 1869.)

The observations which form the basis of this paper were made on newly-born children with the object of determining whether their temperature differed from that of the adult, and if so, how and to what extent. They were made during a recent residence in the Edinburgh Maternity Hospital, with the full sanction of the attending physician, Dr CHARLES BELL. The thermometers\* used were CASELLA'S straight self-registering. The rectum was the part chosen for the insertion of the instrument. The utmost expedition was used in ligaturing the cord, and separating the child from its mother. This being effected, the bulb of the thermometer was at once introduced into the rectum, and the child was wrapped in flannel, and committed to the charge of a nurse, who held the instrument steadily *in situ*. In five minutes it was removed, and the temperature noted. The observations were repeated every fifteen minutes during the first hour, every thirty minutes during the second and third hours, and then every hour up till the sixth hour after birth; after that at wider intervals up to twenty-four hours; and then only twice a day between ten and eleven in the morning, and between six and seven in the evening.

By the adoption of the above method, the first observation was made whilst the child yet retained the temperature imparted by the mother, whilst any subsequent change could not fail to be noted in consequence of the frequency with which the observations were repeated.

In order that the peculiarities of the child's temperature may be made as clear as possible, the facts observed shall be treated of under three different heads. We will consider (*a*) the temperature at the time of birth; (*b*), the range noted during the first twenty-four hours of extra-uterine life; (*c*), that of the next five days.

(*a*). The temperature of the child at birth is the same as that of the mother. If hers be high during the second stage of labour, that of the infant at birth will show a corresponding elevation; if normal, so also will the child's be. The mean range of the maternal temperature at the time of delivery was found to be

\* The instruments were verified by the maker by a standard which is in perfect accordance with that of the Royal Kew Observatory.

99°·154; that of the child at birth, 99°·872. The highest was 103°·5 for the mother, and 104° for the child; the lowest 97° and 98°·1. As the former case was altogether abnormal, it ought perhaps to be excluded in striking the average. By doing so, we get a mean range for the mother of 98°·6, and for the child of 99°·3—a difference of °·7 in favour of the child. This slightly higher range on the part of the child is fully accounted for by the fact that the infant's temperature was taken in the rectum, whilst the mother's was taken in the axilla. From observations made on the adult with the object of determining the point in question, I found the temperature of the former locality to be from half a degree to a degree above that of the latter. Making allowance for this, the range of the mother and child may be regarded as the same. The case with the high range, in virtue of its abnormality, serves well to illustrate the close connection which exists between the two. In it the mother's temperature towards the end of labour rose five degrees above the normal standard; the child's at birth was found to have undergone a similar elevation.

(b.) The range of the first few hours after birth is altogether peculiar. When the child is separated from the parent, and commences its new mode of existence, a marked change takes place. The temperature acquired from the mother is no longer sustained. The thermometer introduced into the rectum shows a fall which varies in different cases both in rapidity and extent, but which is never altogether wanting.

In children born at the full time, the average period after birth at which the temperature reached its lowest point was two hours. The average extent of the fall was five degrees below the normal standard of the adult; the greatest was to 90°·8, the least to 96°—the average being to 93°·4. The mean time which elapsed before the temperature again rose to what might be regarded as its normal range was 22·25 hours after birth; the shortest was two hours; the longest forty-four. In one sickly child it was four days before the depression was recovered from. In a seven months' child the temperature fell to 90°·2, more than eight degrees below the adult standard of health; and during the thirteen days on which it was under observation the highest point reached was 94°·6, the mean range being 92°·3.

(c.) So much for the first twenty-four hours. The observations made after that time were made only morning and evening. I have selected sixteen cases, in which no disturbing element intervened at all likely to affect the normal range, and find that the mean range for the first five days immediately succeeding the time at which the normal standard was attained was in the morning 97°·43, and in the evening 98°·06, the average being 97°·74—that is, more than half a degree below the normal standard of the adult. As the temperature of these cases was invariably taken in the rectum (in which we have seen that the range is higher than in the axilla), it may be inferred that the child's tempera-

ture during the first few days of its existence is a degree lower than that of the adult. It must also be mentioned, however, that all these observations were made during the winter, and that possibly the external atmosphere may have a greater effect on the temperature of the child than on that of the adult. I have never had the opportunity to repeat them during the warm weather of summer.

How are these peculiarities to be accounted for? Why should the child's temperature fall so rapidly, and to such an extent, immediately after birth? And why should it, on recovering from this temporary depression, still be lower than that of the adult?

(a.) That the child at birth should have the same temperature as the mother is what would naturally be expected. Considering the close connection which exists between them, and the manner in which the child is nourished by the mother, it could not well be otherwise. The circumstance calls for no explanation whatever.

(b.) With regard to the peculiar range of the first few hours of the child's separate existence, it is quite different; and we have now to inquire why it is that the child's temperature should fall so rapidly, and to such an extent, immediately after birth. It is probable that the sudden change from the high temperature of the womb to the low temperature of the external air exercises to some extent a chilling influence on the child—an influence which it can the less resist, and with the more difficulty recover from, in consequence of the peculiarities of its circulatory system allowing of the passage of so much venous blood into the arterial circulation. That, however, is not of itself sufficient to account for a fall so sudden, so great, and of so short duration; for assuredly, if that were the sole cause, recovery from a very low range so produced would be a much slower process than it is. There must be some other and more powerful agent at work; and this we have in the first necessary act of the child's independent existence—respiration. I believe that the passage of air into the lungs has at first a refrigerating influence, and is the chief, if not the sole, agent in producing the great and sudden fall which takes place immediately after birth. This explanation is to a certain extent borne out by what was observed in one case in which the child was apparently still-born, and in which considerable difficulty was experienced in inducing the respiratory act. The temperature in the rectum half an hour after birth (immediately after respiration was established) was  $98^{\circ}3$ ; in the next half-hour it fell to  $92^{\circ}6$ . The state of the child at birth was such that attention was directed solely to the respiration, and until that was right the temperature was not taken. Just before delivery, however, the mother's stood at  $100^{\circ}$ , so that the child's may with propriety be supposed to have been  $100^{\circ}7$  in the rectum. In ordinary cases, in which breathing commenced at once, the mean fall during the first half hour was in full-grown healthy children  $5^{\circ}2$ , and during the second half hour  $1^{\circ}4$ . In this case it was during the first half

hour  $2^{\circ}4$ , and during the second half hour (after respiration was established)  $5^{\circ}7$ . In all other cases in which a comparison could be made the fall was much greater during the first half hour than during the second; in this one the reverse was the case; and the only explanation of this circumstance is to be found in the tardy establishment of the respiratory act. So far as one case can do so, this one shows that it is not until the child breathes that the temperature falls to any great extent; though the diminished range may also be partly explained by the cooling influence of the external air on the blood in the very active cutaneous circulation.

But the question naturally arises, Why should the respiratory act, which in the adult has a heat-producing effect, have an opposite result in the infant? The answer involves a brief consideration of the whole question of the production of animal heat. To the various theories which have at different times been advanced to account for this I shall not allude further than to say, that all have given place to that which ascribes it to chemical action—to the changes which are constantly going on in the blood in all parts of the capillary system—general and pulmonic. As these changes take place in organs and parts which are dependent for the proper performance of their functions on the integrity of the nervous system, it follows that the amount of heat produced is apt to be modified by the operation of that system. Medicine abounds in illustrative cases in which a part of the body, a limb for instance, in consequence of being deprived of its nervous supply by disease or accident, has a lower temperature than it had when that supply remained intact.

Sir B. C. BRODIE (I quote from KIRKES' "Physiology") "found that if artificial respiration was kept up in animals killed by decapitation, division of the medulla oblongata, destruction of the brain, or poisoning with worara poison, the action of the heart continued, and the blood underwent the usual changes in the lungs, as shown by the analysis of the air respired, but that the heat of the body was not maintained; on the contrary, being cooled by the air forced into the lungs, it became cold more rapidly than the body of an animal in which artificial respiration was not kept up."

Absence of the due nervous influence is, I believe, the true explanation of the rapid and transient lowering of the child's temperature during the first few hours of extra-uterine life. It is, indeed, unlikely that the child should have its connection with the mother severed, and commence its new and independent existence with patent foramen ovale, unclosed ductus arteriosus, and lungs hitherto untried, and from the first maintain the temperature imparted to it by the parent; but, as already explained, the existence of this peculiar state of the organs of circulation is inadequate to account for a fall so very rapid and of so short duration. Closure of the foramen ovale and ductus arteriosus cannot explain the speedy return to the normal range, for these passages are not

obliterated for some considerable time after birth; respiration goes on as from the beginning, and why should it after a few hours cease to exert the same lowering influence which it exercised at first? It seems to me that the only feasible explanation is to suppose that the hitherto unexercised influence of the nervous system over the respiratory function is not at once called into vigorous and efficient action—that though the influence required for inducing the muscles of respiration to act is in full force from the beginning, there is still wanting, or only partially supplied, that more delicate and less easily explained agency without which, even though the blood may undergo the usual changes, the due amount of heat is not generated; and that coincidently with the establishment of this influence does the temperature of the child rise to its normal range. This explanation is quite in harmony with the fact that in delicate and premature children the fall is greater than in vigorous ones and those born at the full time, in whom also the normal standard is more rapidly reached, in consequence probably of the more speedy establishment of the due nervous influence.

(c). During the five days immediately succeeding that on which the temperature rose to a height which might be regarded as normal, we have seen that the range was one degree below that of the adult, after due allowance had been made for the difference resulting from the manner in which the thermometer was applied.

This lower range of the early period of extra-uterine life admits, I think, of a very ready explanation.

As a general rule, the degree of heat produced bears a definite relation to the activity of the respiration: in birds, in which respiration is very active, the temperature is high; in reptiles, with a sluggish respiration, there is a low temperature.

Anything which interferes with the proper oxygenation of the venous blood, or with the due supply of the purified fluid to the tissues, has a lowering effect on the temperature. Such agencies are constantly at work in the newly-born child. The patent condition of the foramen ovale and ductus arteriosus permit of so free a commingling of the venous and arterial blood, that a lower temperature than exists after the closure of these passages must result; for, in the first place, less blood goes to the lungs at each contraction of the right ventricle, and so less heat is produced there; and, in the second place, the blood which goes to the tissues is less pure, and consequently less of an interchange takes place in the capillaries of the systemic circulation.

How long the child's temperature continues lower than that of the adult I am not prepared to say, as my observations were limited to the first week of extra-uterine life, but should think it probable that the adult standard of health is not maintained till the foramen ovale and ductus arteriosus are closed, or nearly so. This, however, is a mere hypothesis, and must remain so, as the time of closure of

these orifices cannot be accurately ascertained. Their obliteration is extremely rare before the completion of the first week of separate existence; but cases are on record in which the foramen ovale, and others in which the ductus arteriosus has been found obliterated before birth; whilst there are recorded several in which the foramen ovale was patulous for many years.

The natural vigour of the child seems to exercise some influence on the extent and duration of the fall which takes place after birth.

I made some calculations with the object of finding out whether the weight of the child at birth or the weight of the placenta bore any fixed relationship to the child's range of temperature, but failed to establish any necessary connection. The only conclusion to which I came was, that those children which gave the most decided indications of being in a vigorous healthy state were also those in whom the temperature fell least, and in whom it soonest again rose to the normal standard. In a feeble seven months' child the range fell to  $90^{\circ}\cdot 2$ , and only once rose as high as  $94^{\circ}\cdot 6$ , during the thirteen days that it was under observation. In another case, a vigorous healthy child, born at the full time, the lowest point was  $96^{\circ}$ , and in two hours after birth the standard of health was reached.

It is very probable that the vigour of a healthy child, and the higher range which it shows, are both merely evidences of a better developed state of the organs generally, and especially of the nervous and circulatory systems, on the integrity of which the production of a due amount of heat is dependent.



Fig. 3. Frame 1.  
Scale 1/2 inch to 10 feet.

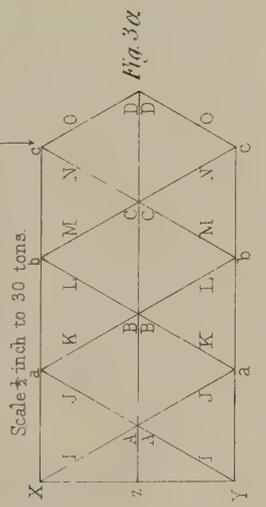
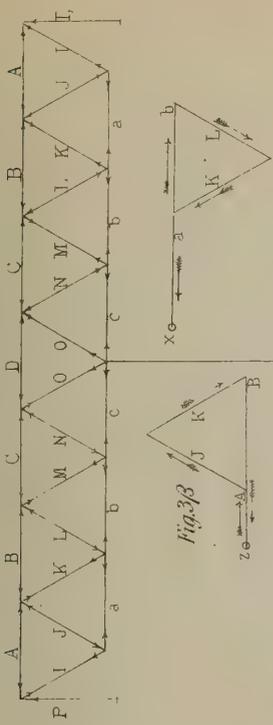


Fig. 4. Frame 1.  
Scale 1/2 inch to 10 feet.

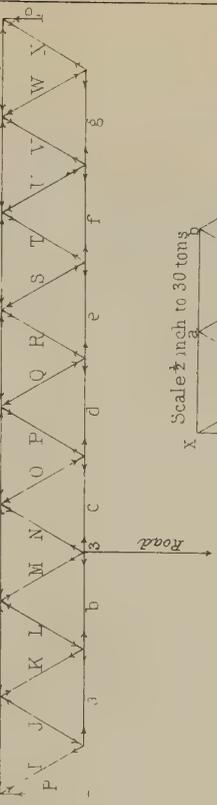


Fig. 4α



Fig. 4β

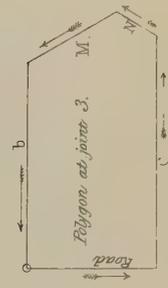


Fig. 1. Frame 1.  
Scale 1/2 inch - 10 feet.

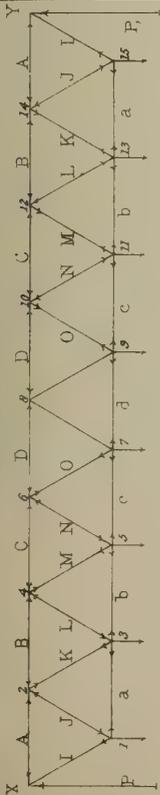


Fig. 1α. Frame 1.  
Scale 1/2 inch - 30 tons.

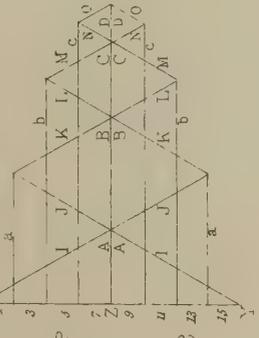


Fig. 1β



Fig. 2. Frame 1.  
Scale 1/2 inch - 10 feet.

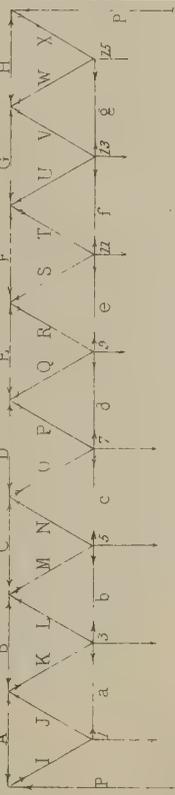


Fig. 2α

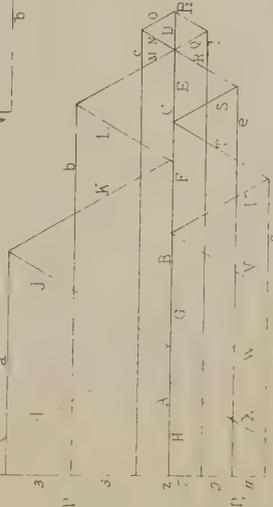


Fig. 2β





UNIFORMLY LOADED GIRDER

Scale 20 feet =  $\frac{1}{2}$  inch

Fig. 5. Frame II.

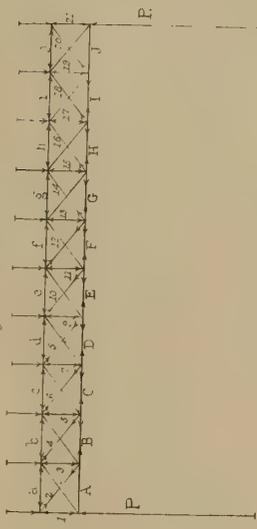


Fig. 5  $\alpha$ .

Scale 30 tons =  $\frac{1}{2}$  inch

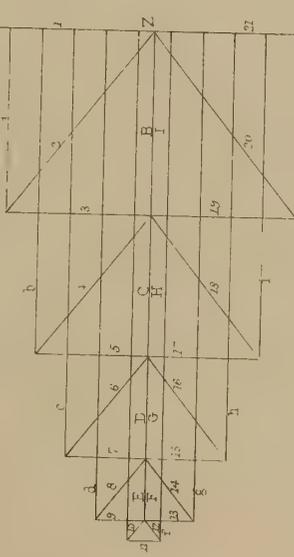
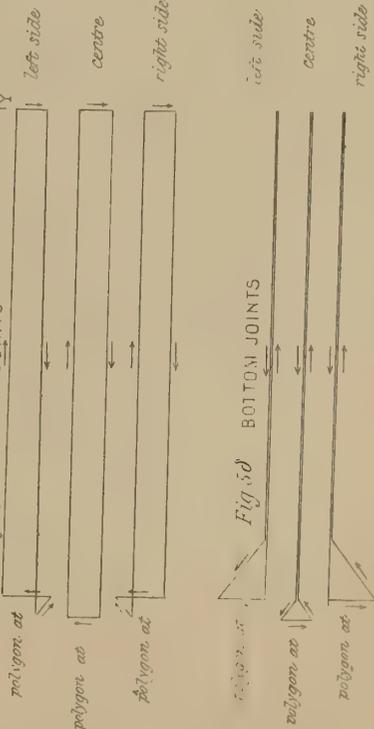


Fig. 5  $\beta$ .



PARTIALLY LOADED GIRDER

Scale 20 feet =  $\frac{1}{2}$  inch

Fig. 6. Frame II.

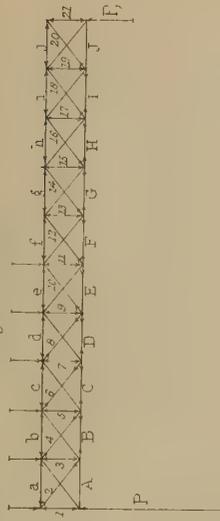
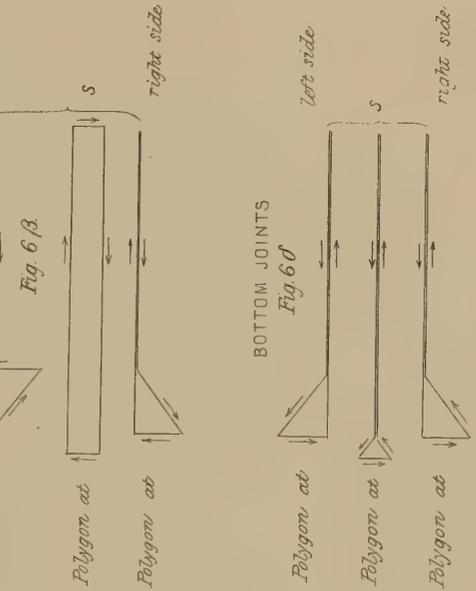


Fig. 6  $\alpha$ .

Scale 30 tons =  $\frac{1}{2}$  inch



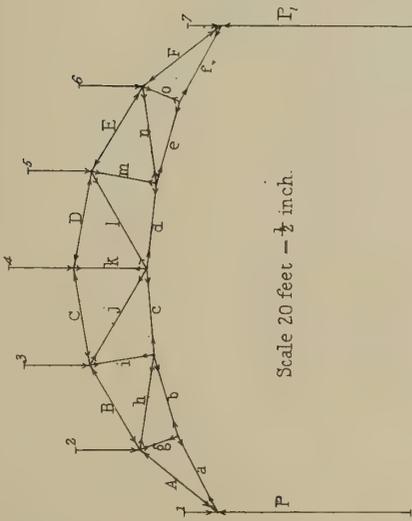
Fig. 6  $\beta$ .





UNIFORMLY LOADED ROOF  
WITH TIES

Fig. 7 Frame III.



Scale 20 feet =  $\frac{1}{2}$  inch.

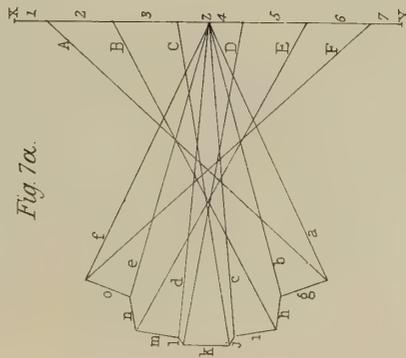
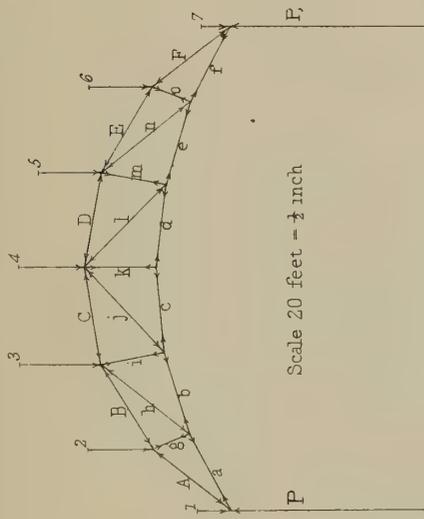


Fig. 7a.

Scale 15 cwt. =  $\frac{1}{2}$  inch.

UNIFORMLY LOADED ROOF  
WITH STRUTS

Fig. 8 Frame IV.



Scale 20 feet =  $\frac{1}{2}$  inch.

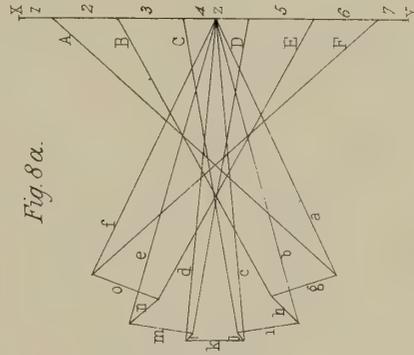


Fig. 8a.

Scale 15 cwt. =  $\frac{1}{2}$  inch.



ROOF TRUSS  
UNIFORM LOAD

Fig. 10.

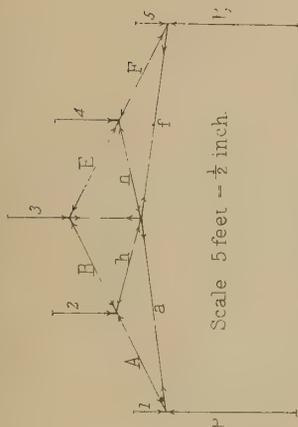


Fig. 10a

Scale  $\frac{1}{2}$  inch = 30 cwt.

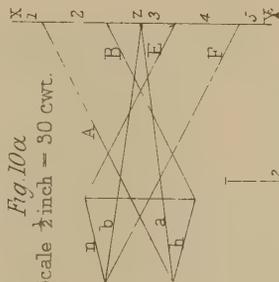


Fig. 11.

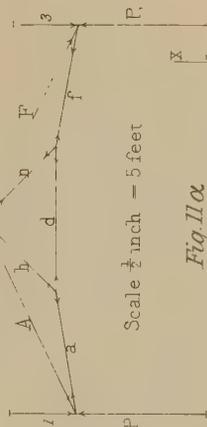
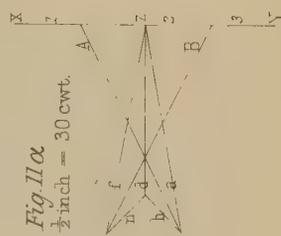


Fig. 11a

Scale  $\frac{1}{2}$  inch = 30 cwt.



ROOF WITH EXTRA LOAD ON JOINTS 1 & 2.

Fig. 9 Frame IV

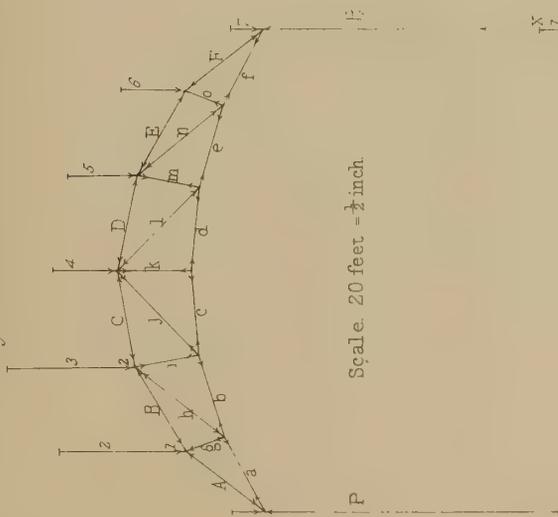
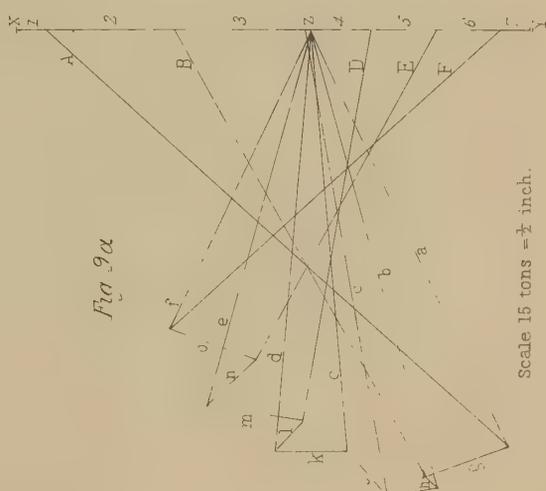


Fig. 9a





ROOF  
Fig. 13

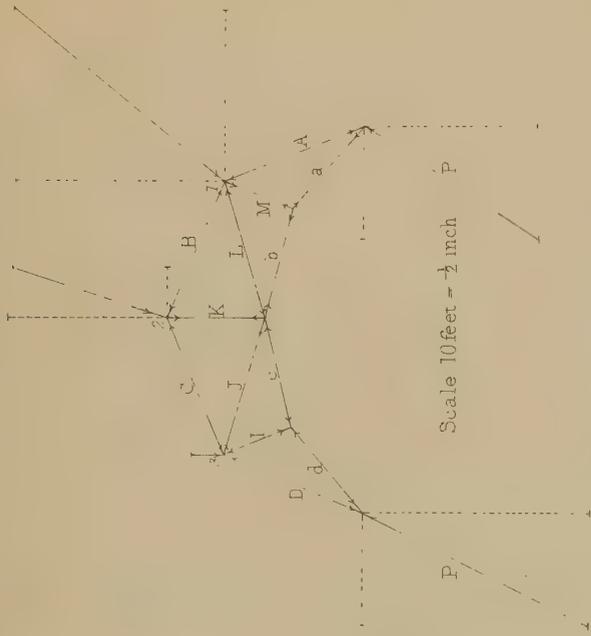
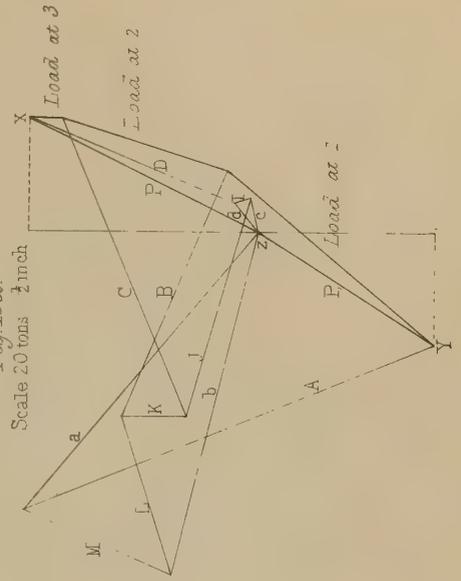


Fig. 13a.



ROOF  
Fig. 12

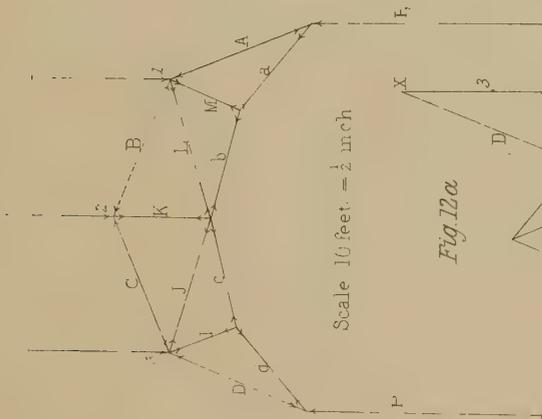


Fig. 12a

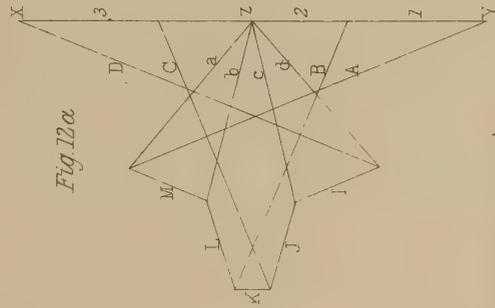
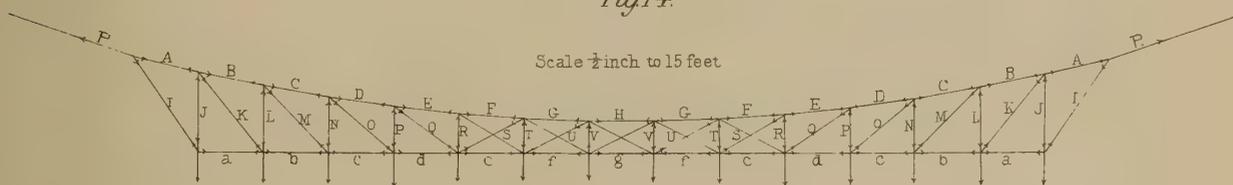




Fig. 14.

Scale  $\frac{1}{2}$  inch to 15 feet



Scale  $\frac{1}{2}$  inch to 30 tons

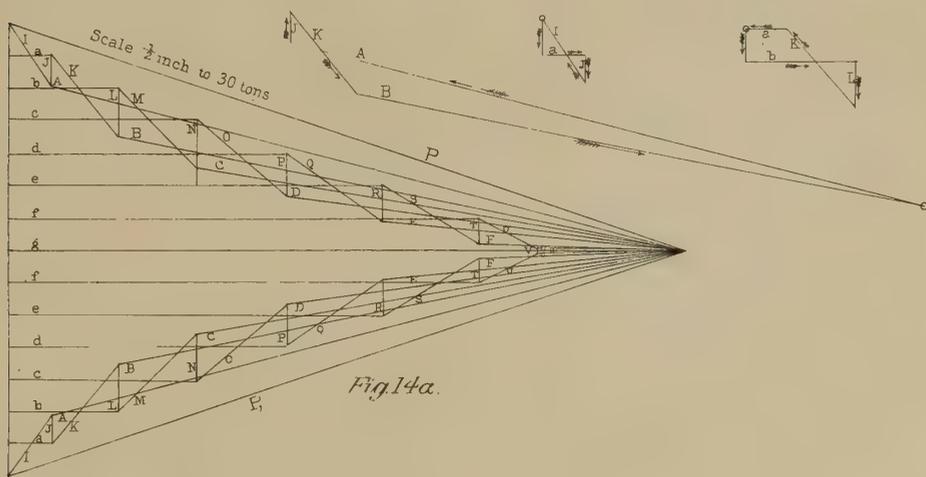


Fig. 14a.



XI.—*On the Practical Application of Reciprocal Figures to the Calculation of Strains on Framework.* By PROFESSOR FLEEMING JENKIN. (Plates XVII. to XXII.)

(Read 15th March 1869.)

The theory of reciprocal figures used as diagrams of forces was first completely stated by Professor T. CLERK MAXWELL, in a paper published in the "Philosophical Magazine," April 1864. The following definition of reciprocal plane figures, and their application to statics, are there given as follows:—

"Two plane figures are reciprocal when they consist of an equal number of lines, so that corresponding lines in the two figures are parallel, and corresponding lines which converge to a point in one figure form a closed polygon in the other."

"If forces represented in magnitude by two lines of a figure be made to act between the extremities of the corresponding lines of the reciprocal figure, then the points of the reciprocal figure will all be in equilibrium under the action of these forces."

The demonstration of this statement is given. The conditions under which stresses are determinate, and some examples of reciprocal figures, are also given in the paper, which leaves nothing to be desired by the mathematician.

Few engineers would, however, suspect that the two paragraphs quoted put at their disposal a remarkably simple and accurate method of calculating the stresses in framework; and the author's attention was drawn to the method chiefly by the circumstance that it was independently discovered by a practical draughtsman, Mr TAYLOR, working in the office of the well-known contractor, Mr J. B. COCHRANE. The object of the present paper is to explain how the principles above enunciated are to be applied to the calculation of the stresses in roofs and bridges of the usual forms.

The construction of a reciprocal figure for any frame requires the exercise of a little discrimination, and the method employed can be best explained by examples; those frames only being considered which are so braced as to be stiff, but have not more members than is sufficient for this purpose.

The simplest example is that of a triangle loaded in the middle, and supported at the two ends.

At each point three forces are acting. Thus we have at point 1 the upward

reaction of the support, the push due to the compressed member A, and the pull due to the extended member C. The directions of the three triplets of forces are shown by arrows.

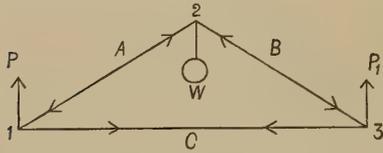


Fig. 1.

In the reciprocal figure lines parallel to each triplet of forces must make a triangle, and the whole figure must consist of six lines. Begin

with the joint 1, and draw one triangle  $mnq$  with the sides P, A, C, such that they are parallel to P, A, C in fig. 1; while the line P is equal on any convenient scale to the known force due to the upward reaction at the point of support. Then PAC, fig. 2, is the simple polygon of forces acting at 1.

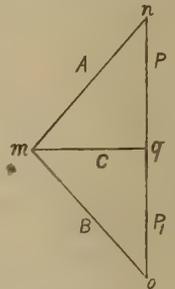


Fig. 2.

Next beginning at point  $m$ , and using the line A to express the upward push at 2 (fig. 1) draw the triangle  $mno$ , which represents the polygon of forces at joint 2. We next find that  $mgo$  represents the polygon of forces at joint 3, and that the fig. 2 is the reciprocal of fig. 1.

The reciprocal figure is thus built up of a series of the well-known polygons of forces acting at each joint, but so arranged that the line representing the elastic force exerted by each member does not require to be drawn twice, but forms part of two polygons, in which, however, it represents forces acting in opposite directions.

Thus the line  $mq$  in fig. 2 forms part of the triangle  $qnm$ , in which it represents a pull from  $m$  to  $q$ , acting at point 1, and it also forms part of the triangle  $mgo$ , in which it represents a pull from  $q$  to  $m$ , acting at joint 3. It is in choosing the form of the component polygons, and in their arrangement, that a certain discrimination is required.

To aid in the construction of the figures, we may observe that the lines representing the external forces acting on a rigid frame in equilibrium must in the reciprocal figure form a closed polygon, and when these lines are parallel, as when weights only are applied, this polygon becomes infinitely thin, and is represented by a single straight line, subdivided into parts proportional to the forces. Thus, in figs. 1 and 2, the three vertical forces are represented by the lines  $no$ ,  $oq$ , and  $qn$ , which represent respectively the solid weight, acting from  $n$  to  $o$ , the upward reaction P acting from  $o$  to  $q$  and P from  $q$  to  $n$ . This line ( $on$ ), subdivided in the ratio of the loads, may be conveniently termed the line of loads, and reappears in all reciprocal figures of framework under parallel forces.

The reciprocal figures corresponding to the ordinary Warren girder will now be described in Plate XVII. fig. 1. The load is supposed to be applied at the bottom joints, and will just be assumed as equal to 10 tons at each of the eight joints. These weights are represented by short vertical lines in fig. 1.

Fig. 1  $\alpha$  is the reciprocal figure of Frame I., thus uniformly loaded and supported at X and Y. The line  $xy$  is the vertical line of loads, equal to 80 tons in all, and equally subdivided, because the load at each joint is equal. From each of these subdivisions horizontal lines are ruled, and the lines IJKLMNO in the reciprocal figure are drawn parallel to the lines similarly lettered in the frame. The lengths of each of the lines in the reciprocal figure measure the stresses on the members in the frame. The figure can be drawn in five minutes; whereas the algebraic computation of the stresses, though offering no mathematical difficulty, is singularly apt, from mere complexity of notation, to result in error.

The figure and the direction of each stress will be easily understood when decomposed into its component polygons. The triangle PIA corresponds to the polygon of forces at X, in which the direction of all the forces is that in which the pen moves, starting from Z towards  $x$ . The polygon of forces acting on joint 1, beginning with the forces determined by the previous polygon, and proceeding in the direction in which the forces act on the joint 1, is I, 1,  $a$ , J.

The polygon at joint 2 is shown separately at fig. 1  $\beta$ , being AIJB; the polygon at joint 3 is also shown separately, the directions of the forces being indicated by arrows. The complete fig. 1  $\alpha$  is built up of separate polygons similar to these two; the origin or starting-point on each being indicated by a small circle in fig. 1  $\beta$ .

Each line in fig. 1  $\alpha$  serves as a part of two component polygons, but it would be passed over in opposite directions in the two polygons by a pencil following the directions of the forces in the two polygons. This fact is of assistance in drawing the reciprocal figures, making it easy to find the starting-point or origin of each new polygon, since the lines representing forces already known must be traversed in the opposite direction to those forces; thus the polygon at joint 4 will include the force due to B and L. These have already formed part of polygons 2 and 3; but in these the direction of the forces was from the joint 4, and hence in the new polygon the direction will be to the joint 4, and the polygon will begin at Z, running BLMC.

It must be observed that the lines ABCD all begin at Z, ending at the intersection of I and J, K and L, M and N respectively. The stress on the two centre diagonals is nil, and with the uniform load the second half of the reciprocal figure is exactly symmetrical with the first half. The stress on  $d$  is equal to that on D.

When the load is not uniform (fig. 2), the weights supported on the two piers are not equal; in other words, the forces P and  $P_1$  are not equal, and the line of loads must be subdivided at Z into two portions, P and  $P_1$  (fig. 2  $\alpha$ ), equal to the loads borne by the piers. The divisions 1, 3, 5, 7, 9, 11, 13, 15, are made equal

to the various loads on the several joints, these unequal loads being, as before, indicated by vertical lines in fig. 2. The two halves of the reciprocal figures are now no longer symmetrical, but it is as easily drawn as the simpler case. Two of the component polygons are shown, as in the previous case. The direction of the stress on each member is found by going round each separate polygon, beginning with some strain the direction of which is known. Thus at joint 7 the polygon is  $c, 7, d, P_2, O$ , and knowing that the direction of the weight 7 is down, we find at once that  $P_2$  and  $O$  must both be pulled upwards. Care must be taken as before to measure ABCDEFGH from the origin  $Z$  in each case. As soon as the values of  $P$  and  $P_1$  have been determined, the most complicated arrangement of loads presents no more difficulty than the very simplest, the typical form being identical in all cases, and easily remembered.

In figs. 3 and 3a the reciprocal figure for the same frame with a single load at the centre is shown. The strains on  $a, b, c$  are represented by the lines  $xa, xb, xc$ ; the strains on  $A, B, C$  by the lines  $zA, zB, zC$ . This will be clear from an inspection of the component polygons.

In figs. 4 and 4a the reciprocal figure for a single weight hung at any joint is shown, and will readily be understood from the explanations already given.

If the frame were inverted, and the loads applied at the top, the strains would remain the same in amount, but be altered in direction in the diagonals; the reciprocal figures would be identical in form with those already given, but would lie on the other side of the line of loads, as if simply turned over through  $180^\circ$  on that line as a hinge.



Fig. 3.

If the loads in Frame I. were applied at the top joints instead of along the bottom, the strains and the reciprocal figure would be modified, the component polygons for the bottom joints being of the annexed type, fig. 3, and the component polygons for the top joint of the annexed type, fig. 4.  $P_1$  is also placed above  $P$ , so that the upper and lower halves of the figure change places.

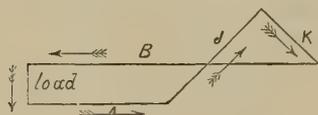


Fig. 4.

It is not, however, necessary to recollect these changes of arrangement, since the known reaction at one pier and the first polygon of forces determine at once the general arrangement of the figure.

In the example just given the members of the frame are simply sufficient in number to make the frame stiff; such a frame is incapable of being self-strained, that is to say, any member might be lengthened or shortened without throwing a strain on the other members. When this condition is fulfilled the stresses on a frame under the action of known external pressure determinate; but when more members are used than suffice to render the frame stiff, the stresses are indeter-

minate, and the frame may be self-strained. In these cases, therefore, the reciprocal figure is useless to determine the stresses.

Such a frame as this shown in Plate XVIII. fig. 5; in which, if the diagonals and verticals were all adapted to resist tension and compression, the stresses could not be determined by the use of reciprocal figures or any ordinary method of computation. If, however, the verticals be alone suited to resist the compression, the diagonals being fit to sustain tension only, the stresses become determinate, half the diagonals being with any given load wholly inoperative. The reciprocal figure can be used to discover which are the active members, as they may be called, and what are the strains upon them.

Fig. 5*a* shows the reciprocal figure for a uniform load.

The inactive members in Frame II. are not numbered. The component polygons of fig. 5*a* are shown in figs. 5*β*, 5*δ*. These figures require no explanation beyond that already given for frame I.

Fig. 6 shows the active and inactive members of Frame II. partially loaded. The active members have arrows on them, showing the direction of the stresses.

Fig. 6*a* is the reciprocal of fig. 6, and figs. 6*a* and 6*β* show the component polygons as before.

It must be remembered that in fig. 6*a*, as in fig. 5*a*, that the lines representing the stresses on B, C, D, E, F, G, H, I, all start from Z.

In fig. 7 we have a third frame not unfrequently used in roof work. Fig. 7*a* is the reciprocal of fig. 7, and is thus constructed. XY is the line of loads subdivided at Z in the ratio of the loads borne by the two piers. It is further subdivided into the parts 1, 2, 3, 4, 5, 6, 7, the loads directly borne by each joint. The lines A, B, C, D and F are in fig. 7*a* all drawn parallel to the top members of the same name in fig. 7, and start from points in the line of loads determined by the subdivision into partial loads. The lines *a, b, c, d, e, f* all radiate from Z, and these two sets of lines are joined by the zigzag line *g, h, i, j, k, l, m, n, o*, each of course parallel to the corresponding member in fig. 7. The figure, although a little complex at first sight, is extremely easily and rapidly constructed. In building it up out of successive polygons we should as usual begin with the reaction of one pier; starting at Z we draw the line ZX, return along load 1 directly borne by the pier, and complete the first polygon by drawing A*a*. The second polygon is A, 2, *b, H, g*, and the remaining component polygons corresponding to each joint can easily be traced in like manner. It will be observed that all the members except A, B, C, D, E, F are in tension.

Fig. 8 and fig. 8*a* (Plate XIX.) show a slightly different frame with the corresponding reciprocal figure when uniformly loaded. *h, j, l, n* are in compression instead of in tension, as in Frame III.

Figs. 9 and 9*a* show Frame IV. and its reciprocal figure when not uniformly loaded.

The reciprocal figure now begins to appear very complicated, but it is drawn on precisely the same plan as fig. 8*a*; but the lines of loads being no longer equally subdivided, the reciprocal figure no longer presents two symmetrical parts.

Figs. 10 and 10*a* with 11 and 11*a* show frames commonly used as roofs, with reciprocal figures. They are only simplified cases of the roof already described.

Fig. 11*a* may be compared with fig. 76 in RANKINE'S "Applied Mechanics." The series of figures 75 in the same work are true elementary reciprocal figures.

Figs. 12 and 12*a* show a simple roof uniformly loaded, which is drawn in order to render more intelligible the comparatively complex case in figs. 13 and 13*a*. Fig. 13 shows the roof under a series of external forces which are no longer parallel, but represented by the inclined lines 1, 2, 3, which have been somewhat arbitrarily chosen as corresponding to a possible distribution of stresses produced by the lateral and vertical pressure of wind. These external forces are met by the two reactions P and P<sub>1</sub> at the piers, calculated on the hypothesis that each pier or wall takes half of the horizontal strain.

Fig. 13*a* is the curiously distorted reciprocal figure which results from these assumptions. It is drawn by precisely the same rule as the comparatively simple figs. 12*a* and 7*a*. In each the lines *a*, *b*, *c*, *d* radiate from a centre Z, which divides the lines P and P<sub>1</sub> representing the reactions on the piers. In each the members A, B, C, D diverge from points separating the successive loads on the joints 1, 2, 3; but in fig. 13*a* the line of loads 1, 2, 3 with the lines P and P<sub>1</sub> representing the reactions at the piers, build up a polygon enclosing a space, whereas in figs. 7*a* and 12*a* this polygon was represented by two straight lines superimposed.

Again, if the zig-zag line corresponding to the diagonals be traced, it will be found to run in an essentially similar manner in figs. 12*a* and 13*a*; thus I joins *d* and *c* in both, the end of the line *d* having been determined by its intersection with D, both starting from Z and X. Looked at by the light of fig. 12*a*, fig. 13*a* becomes readily intelligible, and serves to show how the theory of reciprocal figures can be applied to the most complex conditions of stress which are conceivable, without any greater essential complication than occurs in the simplest examples.

As a final example, the reciprocal figure is given of a braced suspension bridge or arch uniformly loaded, figs. 14 and 14*a*. The strains are drawn on the hypothesis that the direction and magnitude of the resultant thrust are known. This thrust can be determined by Professor J. CLERK MAXWELL'S method for calculating the equilibrium of Frames, published at the same time as his account of reciprocal figures.

In conclusion, a few words may be said of the advantage of the diagrams of

forces, now explained as reciprocal figures, over the ordinary methods of calculation used by engineers.

The graphic method of calculation hitherto employed has been to draw a separate polygon of forces for each joint. To do this it was necessary at each joint to start afresh, setting out the known forces, and from them determining the unknown forces. In thus continually measuring and setting out new lines considerably greater accumulations of error, and more frequent errors, are probable, than when each line when determined by an intersection is used where it lies, and if the successive polygons are drawn to any considerable scale, they lap over one another on the drawing in an awkward and complex manner. Moreover, it is impossible to complete the diagram of the reciprocal figure without taking every line into account; and the closing of the diagram by the final line is an almost perfect check on the accuracy of the delineation.

When compared with algebraic methods, the simplicity and rapidity of execution of the graphic method is very striking; and algebraic methods applied to frames such as the Warren girders, in which there are numerous similar pieces, are found to result in frequent clerical errors, owing to the cumbrous notation which is necessary, and especially owing to the necessary distinction between odd and even diagonals. If this is the case when the loads are uniform or symmetrical, the advantage is much more strikingly in favour of the graphic method when the loads are not symmetrical, and when they are inclined, as in fig. 13, or in such cases as the framed arch and suspension bridge. In fine, the diagram once drawn acts as a sort of graphic formula for the strain on every part of the bridge or roof, and it is a formula which can hardly be misapplied.

In conclusion, the author begs to acknowledge with thanks the assistance of his students, Mr T. H. CUNNINGHAM and Mr A. M'CULLOCH, in preparing the diagrams; and takes the opportunity of repeating, that the merit of discovering the method is entirely due to Professor MAXWELL and Mr TAYLOR, the object of the present paper being to put the theory in such a form as should be intelligible to the engineer and mechanician.



XII.—*An Investigation into some previously undescribed Tetanic Symptoms produced by Atropia in Cold-Blooded Animals, with a Comparison of the Action of Atropia on Cold-Blooded Animals and on Mammals.* By THOMAS R. FRASER, M.D.

(Read 21st December 1868.)

Authorities on the action of medicinal substances agree in including convulsions among the effects on man of belladonna, and of its active principle, atropia.\* Similar effects are described as occurring when large doses of these substances are administered to dogs, rabbits, and other mammals, and to various birds. The recent remarkable progress of our knowledge of the exact and ultimate physiological action of many medicinal substances is greatly due to investigations that have been made on animals of a lower type of organisation; and, accordingly, numerous observers have instituted experiments with atropia on such animals, and especially on frogs. Hitherto, however, convulsions and tetanus have not been described among the effects of atropia-poisoning in cold-blooded animals.†

While making a series of experiments, in April 1868, to determine the minimum fatal dose of atropia for frogs, I was somewhat surprised to find that symptoms of greatly increased reflex excitability occasionally occurred at a certain stage in the poisoning. Believing that a careful examination of these symptoms might probably serve to throw some light on the causation of several of the complicated effects of a substance that has long occupied an important position as a therapeutic agent, I have made a number of experiments (A), to determine accurately the character of these convulsive effects; (B), to ascertain the dose necessary for their production; (C), to differentiate, as far as possible,

\* CHRISTISON, "A Treatise on Poisons," 1845, p. 836; TROUSSEAU and PIDOUX, "Traité de Thérapeutique et de Matière Médicale," tome ii. 1862, p. 55; PEREIRA, "The Elements of Materia Medica and Therapeutics," vol. ii. part i. 1855, p. 549; STILLÉ, "Therapeutics and Materia Medica," vol. i. 1868, p. 770; GUBLER, "Commentaires Thérapeutiques du Codex Medicamentarius," 1868, p. 602; TH. and A. HUSEMANN, "Handbuch der Toxikologie," Erste Hälfte, 1862, p. 465; TARDIEU, "Étude Médico-Légale et Clinique sur l'Empoisonnement," 1867, p. 750; TAYLOR, "The Principles and Practice of Medical Jurisprudence," 1865, p. 358; SCHROFF, "Lehrbuch der Pharmacologie," 1868, p. 508.

† Since this was written, I have communicated with Dr JOHN HARLEY, of London (the author of several important papers on the physiological action and therapeutical employment of belladonna), and have had the satisfaction of learning that he also has observed tetanus, and other symptoms of abnormal reflex activity, in frogs during protracted atropia-poisoning.

2d March 1869.—I quote the following reference to these symptoms from a work which Dr HARLEY has published since this paper was communicated:—"The action of atropia leaves the frog in an excessively nervous state; the least disturbance causes great agitation, with increase of the respiratory movements, and a touch often throws the animal into a tetanic convulsion."—*The Old Vegetable Neurotics*, 1869, p. 240.

the structures on whose affection they depend; and (D), to harmonise these effects with analogous ones in warm-blooded animals, and explain their appearance in certain special circumstances only, in both frogs and mammals. This investigation is limited to the consideration of these objects. Only those effects of atropia that are directly connected with the convulsive symptoms will, therefore, be considered.

#### SECTION A.

Soon after a small fatal dose, or one rather less than fatal, of a salt of atropia is administered to a frog, a slight degree of weakness occurs in the anterior extremities; the respiratory movements of the chest cease, those of the throat continuing; and the motor power becomes gradually more and more impaired, until at length no voluntary or respiratory movements occur, and the animal lies on the abdomen and chest in a perfectly flaccid state. If the condition of the heart be now examined, it will be observed that the cardiac impulse is scarcely perceptible, and that the contractions are reduced to a very few in the minute. At this time, the application of various stimuli shows that the functions of the afferent and efferent nerves and of the spinal cord are retained, though in a greatly impaired condition.

Several hours afterwards—it may be not until the following day—the action of the poison is still further advanced; for the afferent and efferent nerves are completely paralysed, while but an occasional and scarcely perceptible cardiac impulse can be discovered, the only signs of vitality being this imperfect cardiac action, and the retained irritability of the striped muscles. This condition may last for many hours, or for several days. Previous observers have apparently mistaken it for one of death, and have therefore failed to observe the symptoms that subsequently appear, and to which, more particularly, I wish to draw attention.

The first of these symptoms is usually caused by a change that occurs in the flaccid condition of the animal; the anterior extremities becoming gradually more and more flexed, until they assume a state of rigid and continuous contraction, with the webs pressed either against each other, or against the opposite elbows—tonic spasm of the muscles of the chest helping to keep the anterior extremities in this position. At this time, a touch of any portion of the skin increases the spasm of the anterior extremities and of the chest muscles, and causes some slight spasmodic movements in the posterior extremities. After a varying interval, the respiratory movements reappear, the cardiac action improves greatly in strength and in frequency, and the posterior extremities assume an extended position, with the webs more or less stretched. If the skin be now touched, a violent attack of tetanus occurs (at this time usually opisthotonic), which may last for from two to ten seconds, and which is succeeded by a

series of clonic spasms. During the first attacks of tetanus the posterior extremities are often more or less abducted, and immediately after each attack they become flaccid; but the anterior extremities almost always remain rigidly flexed. At a somewhat later period tetanus of a still more violent character, and of longer duration, may be excited, and the attacks are now almost invariably emprosthotonic. During them, the posterior extremities are rigidly extended; while at their conclusion, not only do the anterior extremities remain flexed, but the head continues bent downwards by tonic spasm of the muscles of the abdomen, chest, and neck.

A series of such attacks may be produced by repeated touches of the skin; but when a number are excited in quick succession, the convulsions become shorter, and rather less violent, though they reacquire all their former violence after a period of rest.

During the convulsive stage, and especially at its latter portion, the animal may execute various movements; but from the difficulty with which these are performed, even when they do not themselves excite spasms and convulsions, it is apparent that the power of voluntary movement is still considerably impaired.

The period during which this tetanic condition remains was found to vary greatly in different experiments; and, as might have been anticipated, the larger, within certain limits, the dose of atropia administered, the longer the continuance of this condition. It has been observed to continue in some experiments for only a few hours; in others, for several days; and, in one experiment, for even so long as seventeen days.

This great protraction of the stage of tetanus occurred in an experiment in which a small fatal dose of the sulphate was administered, and this experiment will now be described, as it admirably illustrates the usual sequence of the phenomena.

*Experiment XIX.\**—A solution of 0.45 grain of sulphate of atropia, in eight minims of distilled water, was injected, by means of a Wood's hypodermic syringe, † into the abdominal cavity of a healthy male frog, weighing 455 grains. For some minutes afterwards, the frog jumped about very actively; but in about eight minutes its movements were slow and sluggish, and some weakness occurred in the anterior extremities, and in ten minutes it was unable to jump with normal activity, and when undisturbed lay quietly on the abdomen and chest. A few minutes later, the respiratory movements of the abdominal and chest muscles ceased, those of the throat muscles, however, continuing, and the head rested on the lower jaw. In twenty-one minutes, the frog was placed on the back, and it then made some feeble voluntary movements of the limbs, which

\* The numbers of the experiments of which detailed descriptions are given in this section have reference to the arrangement in Table I. at the end of the section.

† This instrument was employed in all the experiments in this investigation.

were insufficient to change its condition, though apparently designed to do so. There were now no respiratory movements whatever, and the heart's action, as ascertained by its impulse, was reduced to twenty-four feeble beats in the minute. In thirty-one minutes, the frog was in a perfectly flaccid condition; it was obviously unable to perform any voluntary movements, and merely feeble reflex twitches could be excited by strong stimulation. In two hours, reflex movements could still be excited, but the rate of the cardiac contractions had diminished to nineteen per minute. In six hours, however, the nerve-paralysis was more complete; stimulation did not excite any reflex movement; and even direct galvanic excitation of an exposed sciatic nerve failed to produce any muscular contraction, although the muscles themselves readily contracted when the poles were applied to their surfaces. The colour of the frog's skin was now much darker than before the exhibition of atropia.

At the earlier portion of the following day—eighteen hours after the administration of the poison—the frog was in the same state as last noted, except that the heart's action was still more feeble, the contractions being distinctly vermicular, and at the rate of only fifteen beats per minute. Twenty-four hours after the administration, however, an extremely faint twitch of the foot could be excited by galvanism of a sciatic nerve; though a strong current passed through the cord caused no movement beyond that resulting from direct stimulation of several of the muscles of the back, and it was impossible to excite any reflex contraction.

On the third day—forty-four hours after the administration—the frog was lying in the same flaccid condition. The heart's impulse was extremely feeble, and the beats occurred only ten times in the minute.

On the fourth day—sixty-eight hours after the administration—a change of position had occurred, for the anterior extremities were flexed, and formed an arch-like prop on which the raised head and thorax were supported. Reflex movements were now more easily excited, though still very sluggish and feeble; and when such movements occurred the muscles continued in a contracted state for several seconds before they again slowly relaxed. This peculiarity in the contractions was most marked in the muscles of the thorax, anterior extremities, and head. A respiratory movement of the throat occasionally took place; and the rate of the heart's beats was increased to twelve in the minute.

On the fifth day—ninety-two hours after the administration—the frog was lying on the side with the anterior extremities strongly flexed, the webs being in close contact with the opposite elbows, while the posterior extremities were normally flexed. At frequent intervals, the contraction of the muscles of the anterior extremities and of the front of the thorax relaxed somewhat, and, apparently taking advantage of the intervals, the frog made slight voluntary movements, which always excited short attacks of emprosthotonic tetanus. These

attacks ceased first in the posterior extremities, continuing for several seconds longer in the anterior extremities and thorax. During the intervals the anterior extremities were constantly flexed by tonic spasm in their muscles, and in those passing to them from the thorax (especially in the pectoralis major). If the skin were touched, or if the frog were otherwise gently stimulated, an attack of emprosthotonic tetanus followed, during which the flexion of the anterior extremities was rendered more rigid, and the posterior extremities were extended and considerably abducted. These attacks of tetanus lasted for about five seconds, when the posterior extremities became flaccid; but the increased spasm of the anterior extremities and of the chest muscles continued for about sixty seconds longer. When the frog was not suffering from general tetanus there were occasional respiratory movements of the throat and chest. It was now impossible to observe any cardiac impulse, because of the constant spasm of the chest muscles.

On the sixth day—one hundred and sixteen hours after the administration—the frog was lying on the back with the anterior extremities rigidly retained in the position already described; but the head was bent forwards (downwards) by spasm of the anterior abdominal and chest muscles, and the posterior extremities were loosely extended, with the webs slightly stretched. The respiratory movements of the throat were frequent, but those of the thorax but rarely occurred. If a posterior web were now touched, a pretty powerful attack of tetanus followed, during which the body was curved in the form of an arch, with the head bent forwards, while the anterior extremities were strongly clasped against the chest, and the posterior rigidly extended in a straight line. This was general for about six seconds, when the posterior extremities became flaccid; but the increased spasm of the muscles of the anterior extremities and of the anterior surfaces of the abdomen, chest, and throat continued for twelve seconds longer. Still more powerful and prolonged tetanus could be excited by stimulating the skin of the head; and these attacks lasted for eight seconds in the posterior extremities, and for two minutes in the anterior extremities and in the muscles of the anterior surface of the chest. So powerful was the tetanus at this stage that it was possible to lift the frog by the feet and hold it horizontally for eight seconds, with either the back or front of the animal uppermost. The faintest voluntary movements almost invariably excited a tetanic convulsion, and, indeed, such attacks could be produced even by excitations through the organs of vision, as by the sudden approach of any object.

The frog remained in this remarkable condition, without any notable change, until the fourteenth day.

At this time—three hundred and ten hours after the administration—it still continued in the position last described, but, although violent tetanus could still be excited, this did not occur so invariably as before. Frequently, indeed, excitation produced only slow and stiff movements of the posterior extremities,

with violent convulsive spasms of the anterior extremities and of the trunk. Moreover, to excite general tetanus, stimuli of a rather more severe, though still slight, character were now required.

With this modification in the constancy and readiness with which tetanus could be excited, and with a slight diminution in the period during which an attack lasted, the condition of the frog remained unchanged until the twentieth day.

At this time—four hundred and fifty hours after the administration—the frog had assumed a most extraordinary and ungainly attitude. It lay on the right side, with the head bent downwards, by strong tonic spasm of the muscles of the front of the abdomen, chest, and neck; with the anterior extremities rigidly clasped against the thorax (the webs being, as before, pressed against the opposite elbows); and with the right posterior extremity extended, and the left drawn forwards and slightly flexed. The disagreeable appearance resulting from this attitude was greatly increased by the emaciation of the frog, which had gradually increased for several days, until it had so far advanced that the frog now weighed only 385 grains—its weight before the sulphate of atropia was given having been 455 grains. At this time, excitation usually produced merely stiff and slow movements of the posterior extremities, and increase of the tonic spasm of the muscles in the other regions. Occasionally, however, a short attack of emprosthotonic tetanus could still be excited.

For other two days the frog remained in this condition; but on the twenty-third day—five hundred and twenty-six hours after the administration—general tetanus could not be excited by any stimulus, however strong. Stimulation only slightly increased the tonic spasm of the muscles of the throat, anterior extremities, chest, and abdomen, and caused slow and feeble movements of the posterior extremities.

On the following day—the twenty-fourth of the experiment—the frog was found dead and in rigor; the emprosthotonic curve of the body and the rigid flexion of the anterior extremities being retained in death.

In this experiment the tetanic stage lasted for seventeen days, which in this investigation is the longest period during which it has been observed to continue. During eight days the attacks were extremely violent and prolonged; during six days neither could they be invariably excited, nor did they continue for quite so long a period as before; and during the three days that immediately preceded the death of the animal they could but rarely be produced.

Loss of weight is by no means an invariable occurrence after a long continuance of the tetanic stage. Indeed, I am inclined to think that an opposite effect, namely, augmentation of weight, more commonly occurs. The latter is caused by general anasarca, and is sometimes very considerable.

Allusion has been made in the general description of these remarkable phenomena to the stage of complete paralysis having continued for several days. In the following experiment its duration was five days.

*Experiment XXXIII.*—I injected into the abdomen of an active male frog, weighing 251 grains, a solution of 0.3 grain of sulphate of atropia, in five minims of distilled water. Flaccidity and motionlessness occurred rather more rapidly than in the previous experiment; and eight hours after the injection it was found that the conductivity of the sciatic nerves was completely suspended—the muscles, however, freely contracting when directly galvanised—and that the heart's action was extremely feeble, and at the rate of only nineteen beats in the minute.

On the following day—twenty-two hours after the administration—the heart's impulse was even less apparent, and contractions occurred only seven times in the minute; while galvanism of the sciatic and brachial nerves was not followed by any muscular contraction, although idio-muscular contractility was apparently unaffected.

The frog remained in this state of complete nerve-paralysis for other four days. The cardiac action, however, improved during the latter portion of this period; and on the sixth and seventh days the contractions occurred sixteen and nineteen times respectively in the minute.

On the seventh day—at about one hundred and forty-six hours after the administration—a change occurred. The frog still lay on the abdomen and chest, with the posterior extremities flaccidly extended; but the anterior extremities were now slightly arched, there were infrequent respiratory movements of the throat, and a slight touch of the skin excited a feeble, momentary, and sudden movement of the whole body.

On the eighth day—one hundred and sixty-eight hours after the administration—excitation produced a violent attack of tetanus, which was slightly opisthotonic in character, and was succeeded, after lasting for eight seconds, by a series of quivering movements of the posterior extremities. It was not necessary, however, to apply excitation, in order to produce tetanic convulsions, for they also frequently occurred when voluntary movements were attempted. At one hundred and seventy hours, the tetanus was emprosthotonic.

On the ninth day—one hundred and ninety hours after the administration—the tetanic condition was exactly the same as at the latter part of the previous day, and the frog remained extended horizontally for five seconds when lifted in that position by the ankles. The heart was now contracting at the rate of twenty-two beats in the minute.

On the tenth day—two hundred and sixteen hours after the administration—the frog was lying on the lower jaw, chest, and abdomen, the anterior extremities being extended at right angles to the body, while the posterior were stretched backwards. Excitation now caused a feebler and stiffer movement of the limbs; and however powerful the excitation, it was impossible to cause tetanic convulsions.

On the eleventh day—two hundred and thirty-eight hours after the adminis-

tration—only feeble movements could be excited, and there was now a slight degree of continuous general stiffness. The heart's action was at the rate of twenty-two beats in the minute, but the respiratory movements were feeble and very infrequent.

On the twelfth day—two hundred and sixty hours after the administration—the stiffness was more marked, no reflex movements whatever could be excited, and it was found that the sciatic nerves were completely paralysed, and that the muscles responded but faintly to direct galvanic stimulation. The heart's action was at the rate of only eleven beats in the minute.

On the thirteenth day, the frog was dead, and in rigor.

The dose of atropia administered in this experiment was exceptionally large, when compared with the weight of the frog; but the frog was a small one, and had been kept in the laboratory for many months—conditions which appear to favour a certain amount of tolerance. With a frog recently obtained from its natural *habitat* so large a dose, however, would most probably have proved fatal before tetanus occurred. The stage of complete paralysis of motor nerves lasted altogether about five days and ten hours.

Each of these two experiments has been distinguished by an exceptional circumstance: Experiment XIX. by the long continuance of the stage of tetanus, and Experiment XXXIII. by the long continuance of the stage of complete paralysis of the motor nerves. In the experiment which will now be described the duration of the phenomena was such as more frequently occurred.

*Experiment XXIII.*—A solution of 0.4 grain of sulphate of atropia, in four minims of distilled water, was injected under the skin at the left flank of a frog, weighing 386 grains. As usual, after such a dose, in the course of an hour the frog was flaccid, and unable to perform any voluntary movements.

On the following day—eighteen hours after the administration—the frog was lying motionless on the abdomen and lower jaw. It was ascertained by galvanic stimulation that the conductivity of the sciatic nerves was suspended, while the contractility of the voluntary muscles was apparently unaffected. At twenty-two hours after the administration, however, a weak stimulus produced feeble reflex movements. The heart's impulse was now barely perceptible, and contractions occurred but eight times in the minute.

On the third day—fifty hours after the administration—the frog was still lying on the abdomen, but the chest and head were slightly raised by continuous flexion of the anterior extremities. The reflex function was in a more active state, for a slight stimulus applied to the skin of the head caused an increase in the flexion of the anterior extremities, by which the head was still further raised, and a sudden extreme abduction of the two posterior extremities. Irregular respiratory movements of the throat were now observed.

On the fourth day—seventy-three hours after the administration—a faint

touch of the skin of the head was followed by an attack of opisthotonic tetanus, lasting for four seconds; and during it the anterior extremities were rigidly arched, while the posterior were extended straight backwards. When the stimulus was applied to any other region, the only effect was an increase in the tonic spasm of the anterior extremities, and a sudden somewhat spasmodic flexion of the posterior.

On the fifth day—ninety-five hours after the administration—the frog was lying on the back with the anterior extremities rigidly flexed, the webs being pressed against each other, and with the posterior extremities stiffly extended. A slight touch of the skin of any region was immediately followed by a sudden and violent attack of emprosthotonic tetanus. These convulsions were usually general for ten seconds; but the tetanic spasm continued in the anterior extremities for several seconds longer than elsewhere. The respiratory movements had now become more frequent and regular.

During the two following days the frog remained in this condition.

On the eighth day—one hundred and sixty-four hours after the administration—it was more difficult to excite general tetanus, somewhat irregular convulsions most commonly occurring. When the skin of an ankle was touched, tetanus occurred in that limb and in the two anterior extremities for five seconds; but merely spasms, without extension, occurred in the opposite posterior extremity. General tetanus could be excited only when the irritation was applied to the head. The cardiac impulse had now greatly improved in character, while the rate of contraction had increased to twenty-two beats in the minute.

After this a daily improvement was apparent. On the twelfth day the frog had resumed a normal sitting posture, the anterior extremities being, however, still slightly arched; and on the sixteenth day the tonic spasm of the chest muscles and of the anterior extremities had completely disappeared, while slow, voluntary movements could be cautiously performed: but during all this time it was possible to excite a short attack of general tetanus, though severe or frequently repeated stimulation had to be employed.

On the seventeenth day—three hundred and eighty-two hours after the administration—stimulation, even when severe, excited mere stiff reflex movements of the two posterior extremities, and comparatively slight and short tetanus of the two anterior.

The complete disappearance, however, of the exaggerated activity of the reflex function was but slowly effected, and did not occur until about the twenty-fourth day, or five hundred and fifty hours after the administration. For several days after this, the frog was in a somewhat torpid state, moving about very sluggishly, and obviously preferring to remain quiet; but ultimately it recovered perfectly.

The total duration of the symptoms was considerably shorter in a few of the experiments, of which the following is an example:—

*Experiment XVII.*—Three-twentieths of a grain of sulphate of atropia was dissolved in four minims of distilled water, and injected under the skin at the left flank of a male frog, weighing 156 grains; but during some vigorous movements of the frog, which succeeded this injection, a small quantity of the solution escaped from the subcutaneous tissue. In a few minutes, the frog was lying on the abdomen; chest, and lower jaw in a flaccid state; but even in three hours the conductivity of the motor nerves was found to be retained. The observations were now interrupted until the following day.

At this time—twenty-four hours after the administration—the frog was lying as last described; but the conductivity of the motor nerves was found to be completely suspended, while idio-muscular contractility was apparently unaffected. The heart's contractions were very feeble, and occurred twenty-six times in the minute.

On the third day—fifty hours after the administration—the state of flaccid paralysis had disappeared, and the frog was sitting in a nearly normal posture, except that the anterior extremities were unnaturally and somewhat rigidly flexed. On touching any part of the body, a violent attack of opisthotonic tetanus occurred, during which the animal was turned on the back. Such attacks could be excited at any time, at short intervals, during the next three hours, at the end of which period the observations were interrupted. They were general for five seconds; but the tetanic contraction continued in the anterior extremities for five seconds longer than elsewhere. In the intervals between them the frog turned itself from the back, and executed feeble and slow voluntary movements. The heart's contractions were of fair strength, and at the rate of forty-six in the minute; and the respiratory movements of the chest and throat were of nearly normal frequency.

On the fourth day—seventy-four hours after the administration—the frog seemed to have perfectly recovered: it moved and jumped about freely, and no trace of exaggerated activity of the reflex function could be discovered.

The description that has been given, and the illustrative experiments that have been narrated, are sufficient to indicate the usual characters and sequence of the phenomena with such a dose of atropia as produces tetanus. Experiments have, however, been made in which the functions of the cerebro-spinal nervous system were not observed to be completely suspended in the stage of the poisoning, antecedent to the appearance of tetanus. Only impairment of these functions was observed; but, as the state of flaccidity often lasts for several days, it is obviously impossible to make observations so continuously during its existence, as to authorise the assertion that total suspension did not occur. At the same time, there is no reason for supposing that complete paralysis is a necessary antecedent to tetanus.

It has also happened that in one or two experiments symptoms of exaggerated activity of the reflex function occurred, without being observed to assume the violence of tetanus. The following is an example of such an experiment:—

*Experiment X.*—I injected under the skin, at the left flank of a male frog, weighing 231 grains, one-fifth of a grain of sulphate of atropia, dissolved in four minims of distilled water. In ten minutes, the frog was resting flaccidly on the abdomen and chest. In two hours, all voluntary and respiratory movements had ceased, but stimulation still excited feeble reflex contractions; and the heart was contracting at the rate of twenty beats in the minute.

On the second day—twenty hours after the administration—the frog had resumed a natural position, the thorax and head being supported by the anterior extremities, while the posterior extremities were normally flexed; and the throat and chest respirations were frequent. When the skin was touched, or when any object was rapidly approached to the eyes, a sudden, spasmodic, and momentary contraction occurred simultaneously in the four extremities; but it was impossible to excite a tetanic convulsion even by severe stimulation. These spasmodic starts—for they were only such—were often preceded by a “croak,” and when suddenly and unexpectedly excited, were sufficiently strong to raise the body upwards for about a second. During the following day, these symptoms continued; but on the fourth day the only symptom was a slight degree of stiffness when the frog jumped.

On the fifth day the frog was perfectly well.

It is almost superfluous to allude to the resemblance in frogs between the tetanic symptoms of atropia and those of strychnia. There are, however, certain peculiarities connected with the tetanus caused by atropia—altogether apart from the remarkable circumstance of this tetanus being preceded by more or less complete paralysis—that distinguish it from the tetanus caused by strychnia. After poisoning by atropia, the symptoms of exaggerated excitability of the reflex function, as has been shown, are extremely slight on their first appearance, and they acquire their greatest violence only after some considerable time. When these convulsant effects have become fully developed, the state of the animal is one of nearly constant tonic spasm—this tonic spasm being rarely general, but almost always restricted to certain regions,—so that the attacks of tetanus are of the nature rather of exacerbations of existing spasm than of successive and independent convulsions. Strychnia tetanus, on the other hand, becomes fully developed with great rapidity; and, during the stage of remission, the animal is usually in a perfectly flaccid state.\*

\* Although this is “usually” the case, continuous tonic spasm of the anterior extremities may be produced by strychnia also, if an extremely small dose be given. I have found that a dose equivalent to about the  $\frac{1}{450000}$ th of the weight of a male frog (or of a female in whom the abdomen is not greatly enlarged by distended oviducts) will almost invariably cause continuous spasm and arch-like flexion of the anterior extremities; and TARDIEU (*op. cit.* p. 983) describes the same effect in an experiment with a minute dose of strychnia.

Further, in atropia-poisoning, the attacks of tetanus can seldom be excited by the very slight stimuli that are sufficient to do so in strychnia-poisoning.

The tetanus of atropia, also, is characterised by various irregularities. Some of these have already been described; such as the continuous spasm of the muscles of the throat, of the front of the thorax and abdomen, and of the anterior extremities, either only accompanying a condition in which tetanus in the posterior extremities may be excited, or also persisting after this condition has ceased: and of the others, it is sufficient to mention the occurrence of tonic spasm of one group of muscles in one limb, and of another group in another; of contractions of unequal force in the muscles at the opposite sides of the thorax and neck, causing lateral curvature during a tetanic convulsion; and of tetanus in the posterior extremities, with only slight increase of reflex excitability in the anterior.

The last of these irregularities was observed in the following among other experiments:—

*Experiment XXIX.*—A solution of four-tenths of a grain of sulphate of atropia, in four minims of distilled water, was injected under the skin, at the right flank of a female frog, weighing 361 grains, whose oviducts were greatly distended.

On the following day, the frog was lying on the abdomen, chest, and lower jaw, with the posterior extremities flexed, and the anterior extended at right angles to the body. When the skin was stimulated, some feeble movements followed in the toes of both posterior extremities; but no reflex contraction could elsewhere be excited.

On the third day—fifty-two hours after the administration—the posture of the frog was the same as on the previous day; but the paralysis was now more complete, for no reflex movement whatever could be excited, and it was found, on examination, that the motor conductivity of the sciatic nerves had disappeared.

On the fourth day—seventy-two hours after the administration—the condition of flaccidity was no longer present. The anterior extremities were now slightly flexed, so as to raise the head and chest; and now and then a feeble voluntary movement, and a barely perceptible respiration, occurred. These movements generally excited an attack of violent tetanus in the posterior extremities, and a comparatively feeble spasmodic extension of the anterior, during which the latter assumed a more perpendicular direction than before. Feeble irritation caused similar attacks. They usually lasted for eight seconds in the posterior extremities, and for only three in the anterior.

During the fifth and sixth days, the frog continued to suffer from tetanus, and the character of the symptoms was exactly the same as on the fourth day. Neither abnormal flexion nor tonic spasm occurred in the anterior extremities, and such convulsive movements as appeared in them were always much less violent, and of shorter duration, than in the posterior extremities.

On the seventh day, the posture of the frog was quite normal; voluntary movements took place with considerable activity; and although some exaggeration of the reflex function was still present, it was exhibited principally by stiff spasmodic movements of the posterior extremities, and general tetanus could not be excited.

The frog recovered completely in a few days.

The absence of rigid and continuous flexion of the anterior extremities, which is illustrated by this experiment, has been an invariable occurrence in frogs with distended oviducts. With this exception alone, so far as my experience has shown, rigid and continuous flexion of the anterior extremities is a constant, prominent, and early symptom of the motor-stimulant action of atropia.

When the dose of atropia exhibited is not a fatal one, the animal usually recovers completely and rapidly. In one experiment, however, this was not the case. The symmetrical tonic spasm of the two anterior extremities passed into unsymmetrical tonic spasm, which persisted for several months after the disappearance of every other symptom. This sequela will be best described by a short narration of the experiment.

*Experiment XXVIII.*—I injected three-tenths of a grain of sulphate of atropia, dissolved in four minims of distilled water, into the abdominal cavity of an active and perfectly healthy male frog, weighing 275 grains. The usual paralytic effects followed. During the second and third days, the motor-nerve conductivity was completely suspended; but on the fourth day it reappeared, though in an extremely imperfect form, galvanism of a nerve trunk producing only faint twitches.

On the fifth day, the frog was lying on the abdomen with the head and chest raised on the anterior extremities, which had become symmetrically flexed, the webs being rigidly and continuously pressed against the opposite elbows. Slight irritation now excited a short attack of opisthotonic tetanus.

On the sixth day, the attacks of tetanus were somewhat more violent, but they were still opisthotonic.

On the seventh, eighth, ninth, and tenth days, the attacks of tetanus were very violent, and emprosthotonic in character.

On the eleventh day, the violence and duration of the tetanic convulsions had somewhat diminished.

On the twelfth, thirteenth, and fourteenth days, the frog still retained the strongly flexed symmetrical posture of the anterior extremities; but irritation now excited merely a sudden momentary extension of the posterior extremities, and an increase of the tonic spasm of the anterior.

On the fifteenth day, the frog had assumed a nearly normal sitting posture. The anterior extremities were, however, still rigidly flexed, and the frog could move about only by a vigorous use of the posterior extremities. Irritation now

caused merely a slight increase of the flexion of the anterior extremities, and perfectly normal movements in the posterior.

On the sixteenth day, it was observed that the posture of the anterior extremities was somewhat unsymmetrical; the right being rather more flexed than before, while the left was extended backwards so as to be nearly parallel with the left side of the body.

For several days afterwards, the only change was an increase in the rigidity and tonic spasm of the anterior extremities; until, on the twenty-first day, they had assumed the following positions:—The right anterior extremity was in extreme flexion, the upper-arm being at a right angle to the body, with the fore-arm below it, while the hand was everted at the wrist, and had its dorsal surface closely pressed against the anterior surface of the shoulder; the left anterior extremity was extended backwards (towards the posterior extremities), the arm being in close contact with the left side of the thorax, with the fore-arm slightly flexed, inverted, and pressed against the abdomen, while the web was firmly applied, by its palmar surface, against the lower part of the anterior surface of the abdomen. Both anterior extremities were rigidly maintained in these postures, and the frog had no control over them. Changes of position were effected by the use of the posterior extremities alone; and, while irritation produced normal reflex contractions in the posterior extremities, it produced only extremely faint movements in the anterior without changing their unsymmetrical postures. When the frog was placed in water, vigorous swimming movements of the posterior extremities occurred, but the anterior remained motionless.

This sequela first appeared seventy-four days before this description was written; and at this time, there is neither the slightest abatement in the rigidity, nor any other change in the character of the distortion.

Although the physiological action of atropia has been frequently and elaborately studied, these very striking and remarkable convulsive phenomena have hitherto escaped attention. I have, therefore, thought it necessary to enter with considerable detail into the description of these effects, so as to indicate with accuracy their usual character, and to point out the principal irregularities that have been observed. I have likewise shown their relations to some of the other effects that are produced by this substance.

The following Table contains a succinct account of a number of experiments on frogs with sulphate of atropia, in which the progress of the symptoms was not interfered with, either by ligature of blood-vessels before the administration or by division of nerve-structures before or after the atropia-effects were initiated. The latter classes of experiments will be described in a subsequent portion of this paper.

TABLE I.—SUMMARY OF EXPERIMENTS ON FROGS WITH SULPHATE OF ATROPIA, IN WHICH THE EFFECTS WERE NOT MATERIALLY INTERFERED WITH BY OPERATIONS OR OTHERWISE.

Number of Experiment.	In Grains.		Relation of Dose to Weight of Frog.	Region of Injection.	Effect.
	Weight of Frog.	Dose.			
I.	413	0.05	$\frac{1}{8260}$	Under skin of left flank.	No obvious effect.
II.	275	0.05	$\frac{1}{5500}$	Into abdominal cavity.	None observed in 6 hours.
III.	333	0.1	$\frac{1}{3330}$	Under skin of left thigh.	No obvious effect.
IV.	294	0.1	$\frac{1}{2940}$	Do.	Slight incomplete paralysis 1st day; and perfect recovery 2d day.
V.	298	0.2	$\frac{1}{1490}$	Do.	Incomplete paralysis 1st day; exaggerated reflex movements 2d day; <i>tetanus</i> 3d day; and recovery 4th day.
VI.*	275	0.2	$\frac{1}{1375}$	Into abdominal cavity.	Incomplete paralysis 1st day; <i>tetanus</i> 2d to 5th days; stiff spasmodic reflex movements 6th to 8th days; and perfect recovery 10th day.
VII.	273	0.2	$\frac{1}{1365}$	Do.	Incomplete paralysis 1st day; exaggerated reflex movements 2d to 5th days; and recovery 6th day.
VIII.	376	0.3	$\frac{1}{1253}$	Do.	Incomplete paralysis 1st to 3d days; tonic spasm and <i>tetanus</i> of anterior extremities and chest muscles, with only slightly exaggerated reflex movements of posterior extremities 4th to 10th days; and perfect recovery 12th day.
IX.	250	0.2	$\frac{1}{1250}$	Do.	Incomplete paralysis 1st day; <i>tetanus</i> 2d to 6th days; stiff spasmodic reflex movements 7th and 8th days; and perfect recovery 9th day.
X.	231	0.2	$\frac{1}{1155}$	Under skin of left flank.	Incomplete paralysis 1st day; exaggerated reflex movements 2d and 3d days; and perfect recovery 5th day.
XI.	346	0.3	$\frac{1}{1153}$	Into abdominal cavity.	Incomplete paralysis 1st to 8th days; and death, with commencing rigor, 9th day.
XII.	230	0.2	$\frac{1}{1150}$	Do.	Incomplete paralysis 1st and 2d days; slight <i>tetanus</i> 3d day; stiff spasmodic reflex movements 4th to 6th days; and recovery 7th day.
XIII.	340	0.3	$\frac{1}{1133}$	Under skin of right flank.	Incomplete paralysis 1st day; <i>tetanus</i> 2d day; stiff spasmodic reflex movements 3d day; and perfect recovery 5th day.
XIV.	447	0.4	$\frac{1}{1117}$	Do.	Incomplete paralysis 1st day; spasms 2d day; incomplete paralysis 3d day; <i>tetanus</i> 4th to 6th days; stiff reflex movements 7th day; and recovery 8th day.

\* The same frog was used in this experiment as in Experiment II. The second dose (0.2 gr.) was administered 21 hours after the first, and when the frog appeared to be in normal health. The influence of the first dose might, however, have been still partially present; and the exceptional severity of the symptoms described as being caused by a dose equivalent to only the  $\frac{1}{1375}$ th of the weight of the frog may thus be accounted for. If this view be correct, the dose of Experiment VI. should be about 0.25 grain, or the  $\frac{1}{1100}$ th of the weight of the frog.

TABLE I.—SUMMARY OF EXPERIMENTS ON FROGS—*continued.*

Number of Experiment.	In Grains.		Relation of Dose to Weight of Frog.	Region of Injection.	Effect.
	Weight of Frog.	Dose.			
XV.	432	0.4	$\frac{1}{1080}$	Under skin of right flank.	Complete paralysis 1st and 2d days; <i>tetanus</i> 3d to 8th days; stiff spasmodic movements 9th day; and perfect recovery 10th day.
XVI.	372	0.35	$\frac{1}{1063}$	Do.	Complete paralysis 1st and 2d days; <i>tetanus</i> 3d to 6th days; stiff spasmodic movements 7th to 9th days; and recovery 10th day.
XVII.	156	0.15	$\frac{1}{1040}$	Under skin of left flank.	Incomplete paralysis 1st day; complete paralysis 2d day; <i>tetanus</i> 3d day; and perfect recovery 4th day.
XVIII.	508	0.5	$\frac{1}{1016}$	Into abdominal cavity.	Complete paralysis 1st to 4th days; incomplete paralysis 5th day; <i>tetanus</i> 6th to 8th days; stiff reflex movements 9th to 11th days; and perfect recovery 12th day.
XIX.	455	0.45	$\frac{1}{1011}$	Do.	Complete paralysis in 6 hours and at earlier part of 2d day; incomplete paralysis, 2d to 4th days; <i>tetanus</i> 5th to 22d days; stiff spasmodic movements 23d day; and death, with commencing rigor, 24th day.
XX.	404	0.4	$\frac{1}{1010}$	Under skin of left flank.	Complete paralysis, 1st and 2d days; <i>tetanus</i> 3d to 9th days; stiff spasmodic movements 10th and 11th days; and recovery 12th day.
XXI.	300	0.3	$\frac{1}{1000}$	Do.	Incomplete paralysis 1st day; slightly exaggerated reflex movements 2d day; and death 3d day.
XXII.	400	0.4	$\frac{1}{1000}$	Do.	Incomplete paralysis 1st day; complete paralysis 2d day; <i>tetanus</i> 3d to 6th days; irregular <i>tetanus</i> , with twisting of body to left, 7th and 8th days; stiff spasmodic movements, also with twisting of body to left, 9th to 13th days; dead, with rigor, 14th day.
XXIII.	386	0.4	$\frac{1}{968}$	Do	Complete paralysis 1st and earlier part of 2d day; exaggerated reflex movements 3d day; <i>tetanus</i> 4th to 16th days; stiff spasmodic movements 17th to 21st day; stiff movements 22d to 24th days; and perfect recovery 27th day.
XXIV.	482	0.5	$\frac{1}{964}$	Under skin of both flanks.	Incomplete paralysis 1st and 2d days; complete paralysis 3d day; exaggerated reflex movements 4th day; <i>tetanus</i> 5th to 7th days; stiff spasmodic movements 8th and 9th days; and death, with rigor, 10th day.
XXV.	337	0.35	$\frac{1}{963}$	Under skin of left flank.	Incomplete paralysis 1st to 5th days; and death, with commencing rigor, 6th day.
XXVI.	234	0.25	$\frac{1}{936}$	Do.	Incomplete paralysis 1st to 3d days; <i>tetanus</i> 4th to 12th days; stiff spasmodic movements 13th and 14th days; and perfect recovery 15th day.
XXVII.	465	0.5	$\frac{1}{930}$	Do.	Incomplete paralysis 1st day; <i>tetanus</i> 2d to 4th days; and death, with commencing rigor, 5th day.

TABLE I.—SUMMARY OF EXPERIMENTS ON FROGS—*continued.*

Number of Experiment.	In Grains.		Relation of Dose to Weight of Frog.	Region of Injection.	Effect.
	Weight of Frog.	Dose.			
XXVIII.	275	0.3	$\frac{1}{9 \frac{1}{7}}$	Into abdominal cavity.	Incomplete paralysis 1st day; complete paralysis 2d and 3d days; incomplete paralysis again on 4th day; <i>tetanus</i> 5th to 11th days; stiff spasmodic reflex movements 12th to 14th days; stiff reflex movements 15th day; and unsymmetrical continuous rigidity of the anterior extremities 16th day, remaining for more than 74 days afterwards.
XXIX.	361	0.4	$\frac{1}{9 \frac{1}{2}}$	Under skin of right flank.	Incomplete paralysis 1st and 2d days; complete paralysis 3d day; <i>tetanus</i> 4th to 6th days; stiff spasmodic movements 7th and 8th days; and perfect recovery 10th day.
XXX.	270	0.3	$\frac{1}{9 \frac{1}{10}}$	Into abdominal cavity.	Incomplete paralysis 1st to 5th days; and death, with commencing rigor, 6th day.
XXXI.	355	0.4	$\frac{1}{8 \frac{1}{3}}$	Under skin of right flank.	Incomplete paralysis 1st day; slightly exaggerated reflex movements 2d day; and death 3d day.
XXXII.	340	0.4	$\frac{1}{8 \frac{1}{5}}$	Under skin of both flanks.	Incomplete paralysis 1st day; exaggerated reflex movements 2d day; <i>tetanus</i> 3d and 4th days; stiff spasmodic movements 5th day; and perfect recovery 7th day.
XXXIII.	251	0.3	$\frac{1}{8 \frac{1}{3}}$	Into abdominal cavity.	Complete paralysis 1st to 6th days; slightly exaggerated reflex movements 7th day; <i>tetanus</i> 8th and 9th days; stiff reflex movements 10th and 11th days; complete paralysis 12th day; and death, with commencing rigor, 13th day.
XXXIV.	410	0.5	$\frac{1}{8 \frac{1}{2}}$	Under skin of both flanks.	Complete paralysis 1st day; and death, with rigor, 2d day.
XXXV.	403	0.5	$\frac{1}{8 \frac{1}{10}}$	Do.	Do.
XXXVI.	638	0.8	$\frac{1}{7 \frac{1}{3}}$	Under skin of left flank.	Incomplete paralysis 1st day and earlier part of 2d; exaggerated reflex movement latter part of 2d day; <i>tetanus</i> 3d day; stiff reflex movements 4th day; and death 5th day.
XXXVII.	396	0.5	$\frac{1}{7 \frac{1}{2}}$	Under skin of left thigh.	Complete paralysis 1st and 2d days; incomplete paralysis 3d day; <i>tetanus</i> 4th to 6th days; exaggerated reflex movements 7th day; and death 8th day.
XXXVIII.	147	0.2	$\frac{1}{7 \frac{1}{3}}$	Into abdominal cavity.	Incomplete paralysis 1st day; exaggerated reflex movements 2d day; <i>tetanus</i> 3d to 10th days; stiff spasmodic movements 11th to 13th days; and perfect recovery, 15th day.
XXXIX.	214	0.3	$\frac{1}{7 \frac{1}{3}}$	Do.	Complete paralysis, 1st and 2d days; and death, with commencing rigor, 3d day.
XL.	322	0.5	$\frac{1}{6 \frac{1}{4}}$	Under skin of both flanks.	Complete paralysis 1st day; and death, with rigor, 2d day.
XLI.	254	0.4	$\frac{1}{6 \frac{1}{3}}$	Into abdominal cavity.	Complete paralysis 1st to 3d days; incomplete paralysis 4th and 5th days; and death, with commencing rigor, 6th day.
XLII.	259	0.5	$\frac{1}{3 \frac{1}{8}}$	Under skin of left flank.	Complete paralysis 1st day; and death, with commencing rigor, 2d day.
XLIII.	340	1.	$\frac{1}{3 \frac{1}{10}}$	Do.	Do. do.

Some experiments were made with acetate of atropia also. I have not considered it necessary to describe these in detail, or to include them in the above Table, as the symptoms were the same as those observed with corresponding doses of the sulphate.

It appeared of interest to examine if analogous symptoms were produced in other cold-blooded animals, and, with this view, a number of experiments were made with *Triton cristatus*—a species of water-newt, which abounds in many of the lochs in the neighbourhood of Edinburgh.

It was found that sulphate of atropia produces in this animal the same general paralytic and convulsant effects as in the frog. After the subcutaneous administration of doses that were somewhat smaller than the minimum fatal, a condition of partial but marked paralysis was, in the first place, produced; and this was succeeded, in less than twenty-four hours, by a condition of slight impairment of the power of voluntary movement with decided increase of reflex excitability, which condition persisted, in many of the experiments, for more than fourteen days. The reflex excitability manifested itself by sudden starts, when the skin was gently touched; and by tetanic spasm, lasting for from fifteen to forty seconds, when the irritation was more prolonged and powerful, as when produced by a series of taps with the handle of a scalpel. The appearance of this tetanus was somewhat peculiar, and its characters varied considerably. Frequently, the trunk of the body was curved laterally, with the tail curled in three or four coils, and the head twisted round to such an extent that the snout was in contact with the outside coil of the tail; at other times, the body was curved in an opisthotonic spasm, with the tail elevated either in a straight oblique line or in coils, and with the head raised; while, not unfrequently, the trunk of the body was irregularly contorted, with the head and tail in one or other of the above positions.

#### SECTION B.

The experiments that have been made are sufficiently numerous to show what dose is required to produce these extraordinary convulsive phenomena. Tetanus, or, at least, a state of greatly exaggerated reflex excitability, may be looked for with great confidence, when a dose of the sulphate or acetate of atropia, equivalent to about the  $\frac{1}{1000}$ th of the weight of the frog, is administered by injection, either under the skin or into the abdominal cavity. If the latter region be selected, it is necessary to puncture the abdominal parietes at a point as far removed from the heart as possible, in order to prevent a powerful local action on that organ; at the same time, taking care to avoid injuring the lungs. It is also important to dissolve the atropia-salt in only a few minims of water—from four to eight is quite a sufficient quantity. Not only is the danger of affecting the heart by local contact thereby diminished when the exhibition is by the

abdominal cavity, but the small bulk of such a solution is also an advantage when atropia is exhibited by injection under the skin. In the latter case, the energetic movements that nearly invariably occur when the frog is set free, are very apt to press some of the solution out of the subcutaneous tissue; and even with this precaution I have found it difficult to prevent all loss.

It appears from the experiments contained in Table I. that tetanus is also pretty constantly produced by doses somewhat greater or less than the  $\frac{1}{1000}$ th of the weight of the frog; indeed, by the majority of the doses included between the  $\frac{1}{735}$ th and the  $\frac{1}{1250}$ th. The larger doses usually produce the most violent tetanic symptoms, and they may be given with considerable confidence to very small animals, and to such as have been kept in a laboratory for several months. The smaller doses seem best adapted for large frogs, and for such as have been recently obtained from their natural *habitats*. If a dose be employed smaller than those above indicated, impairment of the functions of the cerebro-spinal nervous system and of the heart may be caused, but general tetanus will seldom follow, although spasms, generally restricted to certain regions, may occasionally appear. The tetanic state resulting from the largest doses usually terminates in death, that from the smallest in recovery.

In the above Table, the smallest dose of sulphate of atropia that produced tetanus is equivalent to the  $\frac{1}{1490}$ th of the weight of a frog of 298 grains (Experiment v.), the largest to the  $\frac{1}{735}$ th of the weight of a frog of 147 grains (Experiment xxxviii.)

Without attaching undue weight to an experience which is insufficient to justify generalisation on such a subject, it may be mentioned that tetanic symptoms were usually produced most readily, and continued for longest periods, in the experiments that were made in winter, when the temperature of the laboratory was low.

### SECTION C.

It is by no means an easy matter to ascertain what structures are concerned in the production of the convulsant effects of atropia, for the protracted intervals that often elapse between the administration of the poison and the appearance of tetanus, and the difference in the duration and severity of the tetanic symptoms that follow even the most carefully calculated doses, render the inquiry an unusually difficult one, and frequently necessitate a patient repetition of the experiments. It is obvious, at the outset, that experimental investigation is required. It cannot be maintained that these remarkable convulsant and tetanic effects are merely secondary results of certain degrees of the primary paralysing action of atropia on the cerebro-spinal system and on the heart. Against such a supposition it were easy to bring a mass of opposing evidence, derived from the physiological effects of other active substances. It is well known that after the administration

of curara (wourali) frogs may remain for lengthened periods in a state of absolute motor paralysis, with the cardiac action greatly impaired, and nevertheless perfectly recover without the occurrence of the slightest degree of abnormal activity of the reflex function. Professor VULPIAN has recently shown that iodide of phosphethylamine may likewise produce in frogs complete temporary paralysis, yet this is not succeeded by any spasmodic symptoms.\* In many experiments, also, which I have made with physostigma—one of which is described in a paper communicated to this Society†—complete motor paralysis and great diminution of the cardiac action were produced, yet the animal gradually recovered therefrom, without any symptom of exaggeration in the reflex activity having been observed. Similar evidence may be accumulated from many other sources; but it is sufficient to mention an interesting experiment by VULPIAN, which has a direct bearing on the question. That eminent physiologist ligatured the aorta at its origin from the heart of a frog, so as completely to stop the arterial circulation. In the course of a few hours, the excitability of the spinal cord was suspended, and, soon after, the conductivity of the motor nerves was considerably impaired. The frog presented all the phenomena of death; for although the heart still continued to contract, it was unable to propel any blood. The ligature was then removed, and the circulation became re-established. By-and-by, respiratory movements reappeared; in one or two hours, reflex contractions could be readily excited, while voluntary movements were freely executed; and, soon after, the animal regained all its suspended functions. Yet, although the recovery was established by gradual stages, no symptoms of exaggerated activity of the reflex function were observed. ‡

It is unnecessary to discuss this hypothesis further, especially as sufficient evidence will be adduced, in this and the following Section, to demonstrate that the convulsant effects of atropia are caused by a direct action on the nervous system.

In the attempt to differentiate the structures on whose affection these effects depend, I have considered the possibilities of their being dependent on the muscles, on the efferent (motor) nerves, on the afferent (sensory) nerves, on the spinal cord, or on the encephalon.

In the first series of experiments, the atropia was prevented from reaching certain limited regions of the body, while it had access to all other regions.

*Experiment XLIV.*—The blood-vessels were ligatured at the upper third of the right thigh of an active male frog, weighing 272 grains; and immediately afterwards a solution containing one-fourth of a grain of sulphate of atropia, in four minims of distilled water, was injected under the skin of the left flank. In the course of two hours voluntary movements had ceased, and a state of motionless flaccidity was produced.

\* Archives de Physiologie Normale et Pathologique, 1868, p. 472.

† Transactions of the Royal Society of Edinburgh, vol. xxiv. part iii. p. 743.

‡ Leçons sur la Physiologie Générale et Comparée du Système Nerveux. Paris, 1866, p. 457.

On the following day—twenty-two hours after the administration—this state of flaccidity was present everywhere, except in the right (non-poisoned) leg, which was extended somewhat stiffly. The skin of this leg, below its upper third, was also paler than that of any other part of the body, and, occasionally, spasmodic contractions occurred spontaneously in the right foot, the rest of the body remaining motionless. When the skin of any region—poisoned or non-poisoned—was touched, a sudden and violent tetanic convulsion occurred in the right (non-poisoned) leg, continuing in it sometimes for four seconds, and at others for from six to eight seconds; while at the same time merely feeble twitches occurred in the left leg and in the two anterior extremities. After several such attacks had been excited in rapid succession, a repetition of the excitation still caused well-marked tetanus of the right leg, but it did not cause any movement whatever in the poisoned regions. The heart was now contracting only twelve times in the minute, and the respirations were very infrequent.

On the third day—fifty hours after the administration—the anterior extremities had become flexed, so as slightly to raise the head; there was some improvement in the character of the respiratory movements; and tetanic convulsions, which frequently lasted for ten seconds, could be excited in the right (non-poisoned) leg, while merely clonic spasms appeared in the other extremities.

On the fourth day—seventy-two hours after the administration—a slight irritation of the skin was followed by a general tetanic convulsion, during which, however, the right (non-poisoned) leg was very slightly affected. A weak interrupted galvanic current applied to the right sciatic nerve, below the ligatures, excited only some faint movements in that limb, while it excited a violent attack of tetanus in the rest of the body (poisoned regions).

For several days afterwards, tetanic convulsions could still be excited; but now the right posterior extremity took no part whatever in these—the stoppage of the circulation having obviously destroyed the vitality of its structures.

*Experiment XLV.*—Immediately after ligaturing the right sciatic artery and veins of a frog, weighing 322 grains, I injected a solution of three-tenths of a grain of sulphate of atropia, in four minims of distilled water, under the skin of the left flank.

On the second day—at forty-five hours after the administration—faint reflex movements could everywhere be excited by gentle stimulation of the skin; but these movements were most marked in the right (non-poisoned) posterior extremity.

On the third day—at forty-five hours after the administration—the reflex movements rarely occurred anywhere but in the right posterior extremity. On the same day, at fifty-one hours, the right (non-poisoned) posterior extremity became extended in violent tetanus when stimulation was applied to any portion of the skin, while everywhere else only feeble reflex movements occurred.

On the fourth day—seventy-three hours after the administration—stimulation

excited severe and prolonged general tetanus, which affected both posterior extremities equally.

On the fifth day, the right posterior extremity was continuously rigid; but for other three days tetanic convulsions could be excited, in which the whole of the body except the right (non-poisoned) posterior extremity was involved.

*Experiment XLVI.*—The right sciatic artery and veins were ligatured in a frog, weighing 315 grains, and immediately afterwards three-tenths of a grain of sulphate of atropia, dissolved in four minims of distilled water, was injected under the skin of the left flank.

On the third day, rigid incurvation of the anterior extremities, and stiff extension of the right posterior extremity, had occurred. A faint touch of the skin of any region caused a violent attack of tetanus; in which the right (non-poisoned) posterior extremity was rigidly extended with the web stretched, for five seconds, while the left was almost unaffected, merely becoming momentarily extended at the commencement of the attack.

On the fourth day, some rigor had appeared in the right posterior extremity; but general tetanus could be excited in all the other regions of the body.

*Experiment XLVII.*—I tied the left sciatic artery and veins of a frog, weighing 211 grains, and then injected a solution of one-fifth of a grain of sulphate of atropia, in six minims of distilled water, under the skin of the right flank.

On the second day—twenty-two hours after the administration—general tetanic convulsions could readily be excited by touching the skin in any region; and both posterior extremities—poisoned as well as non-poisoned—were equally involved in the convulsions.

On the third day—forty-seven hours after the administration—the left (non-poisoned) posterior extremity was somewhat rigid, and took no part in the convulsions.

In the next experiment both the posterior extremities were protected from the direct influence of the sulphate of atropia.

*Experiment XLVIII.*—By excising the sacrum, I exposed the sacral nerves and blood-vessels within the abdomen of a male frog, weighing 210 grains, and then by passing a strong thread below these nerves, I firmly ligatured the abdomen, including all its blood-vessels, but excluding the sacral nerves. After this operative procedure, the frog retained an apparently normal control over the movements of the posterior extremities. Three minutes afterwards, I injected eleven-hundredths of a grain of sulphate of atropia, dissolved in four minims of distilled water, under the skin at the left side of the thorax.

On the following day—at twenty-one hours after the administration—the frog was lying on the abdomen, with the anterior extremities flexed inwards and supporting the head and chest, and with the posterior extremities normally flexed. A slight touch of any portion of the skin was followed by ordinary reflex move-

ments of all the extremities; but a somewhat severe excitation of the head, as by a smart tap, was followed by pretty strong spasm of the anterior extremities, and of the chest muscles, and by violent tetanus of the two posterior extremities, the latter lasting for four seconds.

On the same day—at twenty-five hours after the administration—stimulation caused increase of the tonic spasm of the muscles anterior to the ligature, and, simultaneously, sudden and rigid contraction of the muscles posterior to it—in fact, a general tetanic convulsion—the latter lasting for seven seconds. These convulsions could readily be excited by moderate stimulation of the skin of any region—below the ligature (non-poisoned regions) as well as above it.

On the third day—at forty-four hours after the administration—it was impossible to excite tetanic spasm in the non-poisoned regions. At forty-six hours, the posterior extremities were slightly rigid, and soon afterwards stimulation of the sacral nerves did not produce in them any movement whatever; but violent spasm could still be excited in the anterior extremities, and in the other poisoned regions.\*

The evidence contained in these experiments is sufficient to exclude the muscles and the afferent and the efferent nerves from being held to be directly concerned in the production of the spasmodic and convulsive symptoms of atropia. In each experiment, certain regions of considerable extent were protected from the direct influence of the poison, and yet freely participated in the spasmodic and tetanic effects, thereby proving that these effects were not caused by a direct action of atropia on either muscles or efferent nerves. The evidence that excludes the afferent nerves is quite as satisfactory; for in each experiment an excitation of the skin of the non-poisoned region readily produced spasms and general tetanus, thereby proving that a direct action on the afferent nerves is not required for the production of these symptoms.

We are now obliged to look for the cause of these effects to a direct action of atropia on the central nerve-organs. The predominance of cerebral symptoms during atropia-poisoning in animals of a higher development, suggested the possibility of the tetanic symptoms being caused in frogs by an influence originating in the cerebral lobes, or, more probably, in the ganglia at the summit of the medulla. Accordingly, on several occasions, the spinal cord of a frog in the stage of tetanus was divided immediately below the brachial enlargement, with results such as are described in the following experiment.

*Experiment XLIX.*—A solution of fifteen-hundredths of a grain of sulphate of atropia, in four minims of distilled water, was injected under the skin at the left flank of a frog, weighing 152 grains. The stage of paralysis continued until

\* The experiments in this series were all performed in winter, when the low temperature of the laboratory was favourable to a long-continued retention of nerve-irritability in parts cut off from the circulation.

the end of the third day; and on the fourth day the frog had entered the tetanic stage—a slight touch of the skin being followed by opisthotonic tetanus lasting for four seconds.

On the sixth day, at 1.50 P.M., a touch of the left ankle caused a violent attack of emprosthotonic tetanus, during which both posterior extremities were rigidly extended for three seconds. At 1.51, a touch of the right ankle caused a similar tetanic convulsion, which lasted in both posterior extremities for three seconds.

On the same day (sixth), at 2.10 P.M., the spinal cord was divided immediately below the brachial enlargement; and, it is important to note, extremely little blood was lost by this operation (about a drop only). At 2.21 P.M., a touch of the right ankle caused rigid tetanic extension of both posterior extremities for two seconds. At 2.22, a touch of the left ankle caused similar extension for three seconds. The anterior portion of the body took no part whatever in these convulsions; but when the skin at the head or anywhere anterior to the section of the cord was touched, the regions supplied by the anterior segment of the divided cord were at once thrown into a state of tetanus. These latter attacks lasted for about four seconds; and during them the regions supplied by the posterior segment of the divided cord were unaffected.

On the seventh day, a touch of either ankle caused violent tetanus of both posterior extremities and of the muscles at the lower part of the flanks, lasting for five seconds; and immediately after the tetanic contraction had ceased, a series of clonic spasms occurred in these regions for other fifteen seconds. A touch of the skin anterior to the position of the section of the cord was followed by violent tetanus of the anterior portion of the frog, lasting for about eight seconds.

This condition of independently excitable tetanus of the anterior and posterior segments continued, with but little change in the character of the excited attacks, until the nineteenth day of the experiment. There were, however, in this prolonged period, some differences in the duration of the tetanic convulsions: for, on the eighth day, after slight stimulation, the posterior extremities were rigidly extended for six seconds, and were then affected with clonic spasms for seventy seconds; on the eleventh day, the rigid extension lasted for four seconds, and the succeeding clonic spasms for twelve; on the fourteenth day, the rigid extension lasted for four seconds, and the clonic spasms for only two; and on the eighteenth day, the rigid extension lasted for four seconds, but in place of a series of clonic spasms, it was succeeded by merely two or three faint twitches.

On the nineteenth day, tetanus of the posterior extremities was caused only when the stimulation was severe; and there was now no evidence of increased reflex excitability in the anterior part of the frog.

The observations were now stopped.

In performing this experiment, care must be taken to avoid injuring important blood-vessels, as any considerable loss of blood would completely vitiate the results. This is at once apparent, if we compare the experiments contained in Table II. with those in Table III.

TABLE II.—SUMMARY OF EXPERIMENTS IN WHICH THE SPINAL CORD WAS DIVIDED DURING ATROPIA-TETANUS, *without any material loss of blood.*

Number of Experiment.	In Grains.		Relation of Dose to Weight of Frog.	Effects before the Division of the Cord.	Time of Division.	Effects after the Division of the Cord.
	Weight of Frog.	Dose.				
XLIX.	152	0.15	$\frac{1}{1018}$	Incomplete paralysis 1st to 3d days; tetanus 4th to 6th days.	6th day.	Tetanus in both segments 6th to 17th days, and in posterior segment alone 17th to 19th days.
L.	294	0.3	$\frac{1}{880}$	Incomplete paralysis 1st and 2d days; tetanus 3d and 4th days.	4th day.	Tetanus in both segments 4th to 7th days. Frog was killed on 7th day.
LI.	358	0.4	$\frac{1}{898}$	Complete paralysis 1st and 2d days; incomplete paralysis 3d day; tetanus 4th and 5th days.	5th day.	Tetanus in both segments 5th and 6th days. Frog was killed on 6th day.

TABLE III.—SUMMARY OF EXPERIMENTS IN WHICH THE SPINAL CORD WAS DIVIDED DURING ATROPIA-TETANUS, *with considerable loss of blood.*

Number of Experiment.	In Grains.		Relation of Dose to Weight of Frog.	Effects before the Division of the Cord.	Time of Division.	Effects after the Division of the Cord, and loss of Blood.
	Weight of Frog.	Dose.				
LII.	272	0.25	$\frac{1}{1088}$	Incomplete paralysis 1st to 3d days; tetanus 4th to 7th days.	7th day.	Tetanus ceased in 15 minutes, and rigor mortis was present on 8th day.
LIII.	288	0.3	$\frac{1}{960}$	Complete paralysis 1st and 2d days; incomplete paralysis 3d day; tetanus 4th and 5th days.	5th day.	Tetanus of posterior segment ceased in 10 min., but tonic spasm continued in the anterior segment for 24 hours longer.
LIV.	310	0.35	$\frac{1}{888}$	Incomplete paralysis 1st to 4th days; tetanus 4th to 6th days.	6th day.	Tetanus ceased in both segments in 5 minutes.*

\* In this experiment a large quantity of blood was rapidly lost, as the apex of the heart was excised immediately after the cord had been divided.

I performed the following experiment to ascertain how far the mere division of the cord affects reflex excitability.

*Experiment LV.*—In a healthy male frog the reflex excitability was tested in various ways, and found to be normal. The spinal cord was then divided immediately below the brachial enlargement, with the loss of only one or two drops of blood. Except that some general quivering occurred at the time of the division, and for a few minutes afterwards, the frog remained quietly in a normal posture, and showed no symptoms of exaggerated reflex activity until the second day.

On the second day—at twenty-one hours after the division of the cord—slight stimulation of an ankle was followed by a series of feeble twitches of various muscles in both legs, lasting for three seconds. These twitches were so slight that they caused no movements of either posterior extremity, and during their occurrence both posterior extremities remained normally flexed. Similar series of twitches were produced other three times by stimulating each ankle alternately at intervals of a minute; but when the same stimulation was repeated for the fifth time, no effect whatever followed.

On the third day, the posture of the frog was still perfectly normal; but stimulation of an ankle excited merely a feeble twitch in both posterior extremities.

On the fourth and fifth days, stimulation of a more energetic character was required to excite similar feeble twitches in the posterior extremities; but such series of twitches as appeared on the second day could not be caused by even powerful excitation.

The experiment was now terminated.

It is therefore apparent that, in Experiments XLIX., L., and LI., the tetanic symptoms that were present in the posterior extremities after division of the spinal cord were not caused by the division.

We have thus obtained most satisfactory evidence in favour of the conclusion that these convulsive symptoms are due to a direct action of atropia on the spinal cord.

Moreover, the results of some further experiments made to test this conclusion are entirely confirmatory thereof. Two of these may be briefly described.

*Experiment LVI.*—Immediately after dividing the left sciatic nerve in the thigh of a frog, weighing 360 grains, I injected a solution of seven-twentieths of a grain of sulphate of atropia, in four minims of distilled water, under the skin of the right flank.

On the third day, the frog was lying on the abdomen, with the anterior extremities rigidly arched, with the right posterior extremity stiffly extended, and with the left posterior extremity flexed and somewhat flaccid. A touch anywhere, except in the left posterior extremity below the middle of the thigh, excited a violent tetanic convulsion, in which the left leg took no part. Galvanic stimulation, when applied to the cut end of the distal portion of the left sciatic

nerve, caused normal contractions of the left leg ; but when applied to the cut end of its central portion, excited a violent tetanic paroxysm in which the left leg took no part.

*Experiment LVII.* differed from *Experiment LVI.* mainly in the nerve division having been postponed until the tetanic stage was entered into. Before the right sciatic nerve was divided, excitation caused a violent attack of tetanus, during which the right posterior extremity was rigidly extended for eight seconds. After the right sciatic nerve was divided, the right leg took no part in the tetanic convulsions. Galvanic stimulation also of the cut end of the distal portion of the nerve caused merely normal movements of the right leg.

*The symptoms of increased reflex excitability that occur so prominently in frogs after the exhibition of certain pretty well-defined doses of atropia, are therefore caused by a direct action of this poison on the spinal cord.*

Having reached this stage in the investigation, we are naturally tempted to proceed a step farther, and to inquire what is the nature of the action on the spinal cord, by which atropia produces convulsive and tetanic symptoms? At first sight, the solution of this question might appear to be an easy one. Investigation has shown that atropia is a powerful agent in influencing the condition of at least certain portions of the vascular system, although there is a difference of opinion among investigators as to the nature of the influence. The probabilities are, however, in favour of the view of MEURIOT, that when large doses are given, atropia first diminishes the calibre, and increases the vermicular contraction, of the blood-vessels; and, subsequently, increases their calibre by paralyzing the contractile walls.\* It may be supposed that the latter effect—dilatation of blood-vessels—is the cause of the tetanic symptoms; for such dilatation might operate either by permitting the augmented blood-supply that many suppose to be essential for abnormal activity, or by causing irritation of the cord directly, by congestion, or even rupture of its blood-vessels.†

The plausibility of this view is strengthened by the opinion of so eminent a physiologist as BROWN-SÉQUARD, who maintains that vascular dilatation is one of the primary causes of the tetanic effects of strychnia;‡ and by the *post mortem* appearances of engorgement of the vessels of the spinal cord, after poisoning by atropia, which several observers have drawn attention to in mammals,§ and which I have frequently observed in frogs also.

There are, however, several grave objections to the adoption of a theory of the

\* De la Méthode Physiologique en Thérapeutique et de ses applications a l'étude de la Belladone. Paris, 1868, p. 51, &c.

† MEURIOT, *op. cit.* p. 98.

‡ Lectures on the Diagnosis and Treatment of the Principal Forms of Paralysis of the Lower Extremities. Philadelphia, 1861, pp. 51 and 112.

§ ROSENBERGER, quoted by TARDIEU, *op. cit.* p. 752; SCHROFF, Lehrbuch der Pharmacologie. Wien, 1868, p. 508.

vascular causation of atropia-tetanus, founded on the above arguments. It is true that the demonstration appears to be perfect of great dilatation of the blood-vessels of the skin, muscles, abdominal and thoracic viscera, and several other structures occurring at an advanced stage of atropia poisoning—probably, indeed, this dilatation is contemporaneous with the tetanus—but we have yet to wait for the demonstration of a dilatation of the blood-vessels of the spinal cord during the life of the animal. It is even difficult to believe that the analogous tetanic symptoms of strychnia are due to vascular engorgement, for a frog may be bled as perfectly as possible, and still the subsequent direct application of strychnia to the spinal cord will cause tetanus.\* Farther, the discovery of vascular engorgement after death from tetanus is insufficient to prove that the production of the tetanus is in any way dependent on that engorgement. It might be urged, with equal reason, that the tetanus is the cause of the spinal engorgement, the mechanical effect of the muscular contractions tending to force the blood into those regions where this effect cannot operate. Besides, there are no good grounds for assuming that an engorged state of the vessels of the cord will necessarily increase reflex excitability or originate tetanic convulsions.

It is obvious that this question can be solved by direct proof only. An apparent approach to such a solution might appear to be contained in the results of the Experiments in Table III. Tetanus and convulsions rapidly disappeared after copious bleeding. If, however, the blood be freely and abundantly abstracted from a frog in a normal condition, the reflex excitability will be quickly impaired, and, very soon afterwards, it will altogether disappear.

While, however, we cannot at present accept the view that the tetanic effects of atropia are produced by dilatation of the blood-vessels of the spinal cord, such a method of production is not disproved by any known fact. The question of the exact nature of the causation of atropia-tetanus—in common with similar questions in relation, probably, to every active substance—is, therefore, still open for future research. Meanwhile, by restricting actions to certain organs and structures, we gain an essential advance towards the solution of such problems.

#### SECTION D.

In this section an attempt will be made to show that the convulsive and tetanic symptoms that have been described in frogs are represented among the symptoms of atropia-poisoning in rabbits, dogs, and other mammals; and that, in both cases, the causation and special characters of these symptoms, as well as the peculiarities of their occurrence, are the results of exactly the same actions.

\* MM. MARTIN-MAGRON et BUISSON, "Action Comparée de l'Extrait de Noix Vomique et du Carare," *Journal de la Physiologie de l'Homme et des Animaux*, 1859, p. 487; Dr A. J. SPENCE, "On the Mode of Action of Strychnia," *Edinburgh Medical Journal*, July 1866, p. 50.

There can be little doubt that, in many cases, the convulsions that appear during poisoning by atropia in man, dogs, rabbits, and other mammals, are due chiefly to asphyxia, caused by impairment of the functions of the cerebro-spinal nervous system. These convulsions are, however, due also to a special and primary stimulant action of atropia on the spinal cord. The latter method of production has been recognised by observers who were fully alive to the possibility of such symptoms being produced by asphyxia alone.\* Several experiments in dogs have satisfied me—so far as evidence short of direct demonstration can do so—that this is the case; for, after the administration of doses that were about the minimum fatal, I have, on several occasions, observed a condition of combined unconsciousness, partial paralysis, and exaggerated reflex activity continue for more than twenty-four hours, while, during a considerable portion of this time, the respirations were of fair character.

The remarkable position that the convulsive symptoms occupy in frogs—occurring subsequently to either a partial and short, or a complete and protracted paralysis of the cerebro-spinal nervous system—at first sight appears to lend but little support to the assertions that atropia has a primary spinal-stimulant action in mammals, and that atropia-convulsions are caused by the same action in both frogs and mammals. It is, however, necessary to remember, that *in atropia the amount of spinal-stimulant is in all animals less than the amount of paralysing action, and that paralysis, compared with spinal-stimulation, is more rapidly produced by atropia in frogs than in mammals.*

The first of these propositions—that the amount of spinal-stimulant is in all animals less than the amount of paralysing action—is founded on the fact, that the principal symptoms produced by an aggregate of various doses are those of paralysis. Thus, in frogs, the smallest doses that affect motricity cause slight paralysis without any obvious symptom of spinal-stimulation (Experiment IV. Table I.); somewhat larger doses cause more decided paralysis, with slight symptoms of spinal-stimulation (Experiments V. and VII. Table I.); still larger doses cause complete paralysis, and violent symptoms of spinal-stimulation (Experiments XV. XVI. XVII. XVIII. XIX. XX. XXI., &c. Table I.); and doses so large as to produce death rapidly, cause complete paralysis without any manifestation of a spinal-stimulant action (Experiments XLII. XLIII., &c. Table I.) In mammals, the symptoms are in like manner confirmatory of the proposition. We may safely refer to almost every investigation in which different doses of atropia have been administered to animals of the same species; but the following short account of two experiments, which are described with minute detail in an inge-

\* MEURIOT, *loc. cit.* p. 98, &c. BROWN-SÉQUARD, "Lectures on the Diagnosis and Treatment of Functional Nervous Affections," 1868, p. 66. Both authors account for the increased excitability of the spinal cord by dilatation of blood-vessels—a method of causation which, I believe, cannot be established by any evidence that we at present possess.

nious and elaborate paper by Dr LEMATTRE, contains ample evidence in support of the proposition. Both experiments were performed on dogs, and in both a solution of sulphate of atropia was injected into a jugular vein. In the first experiment, the dose was one decigramme (= 1.54 grain). In six minutes, paralytic symptoms occurred, which gradually became well-marked and severe; and in two hours and forty minutes, “une convulsion réflexe” followed pinching of the skin. *This is the only symptom of a spinal-stimulant action that is mentioned*, although the details of the experiment are described with great minuteness. This dog recovered.\* In the second experiment, the dose was five decigrammes (= 7.7 grains). In about five minutes, paralytic symptoms were observed; and in about one hour and ten minutes, some spasms occurred. The paralytic symptoms became very obvious soon after their first appearance, while the spinal-stimulant reached more gradually such an intensity as to cause *frequent tetanic convulsions*. This dog died six hours after the administration.†

The second proposition—that the paralysis, compared with the spinal-stimulation, is produced in frogs more rapidly than it is in mammals—may likewise be established by an appeal to observation. The experiments described in Table I. show that in frogs complete paralysis (and, therefore, absolute suspension of reflex activity) may be caused by doses of atropia considerably below the minimum fatal. On the other hand, it is well known that in mammals even fatal doses do not completely suspend reflex activity before death. Indeed, it is not to be expected that they should do so, for an amount of paralysis considerably short of complete suspension of reflex activity would undoubtedly cause such an embarrassment of respiration as to produce death by asphyxia.‡ Hence, it is necessary to employ artificial respiration, in order to produce complete paralysis of motor nerves with even so powerful a paralyzing agent as curara (wourali).§ It has been amply demonstrated in Section C. that large doses of atropia completely suspend the conductivity of motor nerves.|| This one method, among seve-

\* “Recherches Expérimentales et Cliniques sur les Alcaloïdes de la famille des Solanées.” Archives Générales de Médecine, 1865, vol. ii. p. 175.

† *Op. cit.* p. 177. I have in my possession notes of many experiments supporting this proposition, but have preferred to quote evidence obtained from an investigation in which this marked difference between the effects of different doses is not specially alluded to.

‡ That this difficulty in causing complete paralysis does not occur with frogs, is due to their endowment with the function of cutaneous respiration. In this animal, reflex activity may be so far impaired by the action of a poison, that pulmonary respiration is rendered impossible, and yet asphyxia may not take place to such an extent as to bring the circulation to a stand-still, and the poison may thus be allowed sufficient time to produce on the living nerve-structures its complete physiological effects.

§ VULPIAN, *op. cit.* p. 196.

|| This action has already been demonstrated by BOTKIN, VIRCHOW'S Archiv, Bd. 24, 1862, p. 84; by VON BEZOLD and BLOEBAUM, Untersuchungen aus dem Physiologischen Laboratorium in Würzburg, 1<sup>tes</sup> Heft, 1867, p. 13; and by MEURIOT, *op. cit.* p. 90. The last author attempted to prove that it is the result of a local action on the nerves by imbibition, and not of poisoning through the blood; but his arguments seem insufficient to establish this view. I hope to refer more fully to this objection on some future opportunity.

ral, by which it produces paralysis, is, therefore, sufficient to account for the greater readiness with which complete paralysis is produced in frogs than in mammals.

It is thus seen why atropia produces paralysis so much more rapidly and completely in frogs than in mammals, and also why in both frogs and mammals spinal-stimulant effects are obviously manifested only when atropia is administered in doses that are near the minimum fatal—that is, in doses containing the largest amount of spinal-stimulant action consistent with the production of a prolonged duration of symptoms.

When a dose of atropia near the minimum fatal is given to a frog, paralysis is caused with such rapidity and completeness, that the spinal-stimulant action is at first prevented from exhibiting itself; but when a similar dose is given to a mammal, paralysis is caused so slowly and incompletely that a sufficient amount of reflex activity remains to allow the spinal-stimulation to manifest itself by exaggerated reflex movements and convulsive spasms. In the frog, the spinal-stimulation is, in the first stage, concealed by the impossibility of its effects being manifested, and the first symptoms are, therefore, those of paralysis; but, as this paralysis is being recovered from, the spinal-stimulation becomes apparent. In the mammal, the spinal-stimulation is merely impaired by the partial paralysis; and during the whole course of the poisoning, the symptoms are, therefore, those of a paralyzing combined with a spinal-stimulant action, the former merely lessening the violence, without concealing the effects of the latter.

This combined action, and the variety produced by it on the symptoms in frogs and mammals, may be graphically illustrated by two curves, one of which represents the paralyzing, and the other the spinal-stimulant action. The forms of these curves are to a great extent arbitrary, and they must of necessity be so until we possess some exact method of estimating degrees of action, and thereby obtaining ordinates that may have some pretension to accuracy. Thus, in the curve  $op_1p_2$  &c., of Diagram 1, the motor nerve paralysis is complete, so far as our methods of examination can show, at  $pc$ ; but between  $pc$  and  $p_2$  there is a considerable interval, during which the degree of action may or may not have been constant. What is termed complete paralysis does not represent the maximum of action, for we know that the paralysis may go on to permanent suspension of motility, or death, as well as return to normal activity. As, therefore, the ordinates are but roughly determined, these curves are in no sense accurate delineations of the paralytic and spinal-stimulant actions. They may, however, serve the useful purpose of exhibiting clearly the relations between the effects of these two actions. For the sake of simplicity, the best marked paralytic action of large doses of atropia—that on the motor nerves—will alone be considered.

Diagram 1 is a delineation, on this plan, of Experiment XVI. of Table I. In

this experiment, seven-twentieths of a grain of sulphate of atropia, administered to a frog, weighing 372 grains, produced complete paralysis of the motor nerves on the first and second days; tetanus on the third (slight), fourth, fifth, and sixth days; and stiff spasmodic movements on the seventh and eighth days. Complete recovery had taken place on the tenth day.

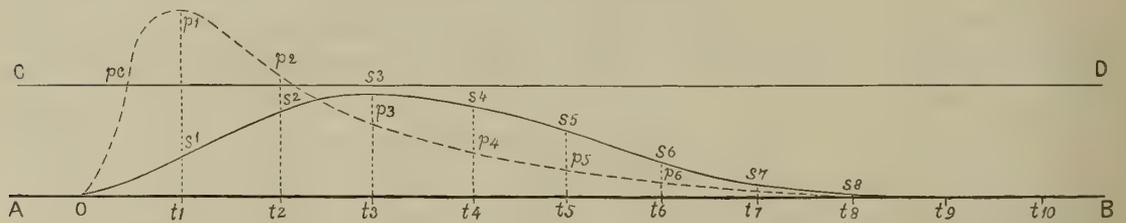


Diagram 1.\*

In the diagram, the curve of paralysis,  $op_1p_2p_3$ , &c., rises abruptly from the line of normality, AB. The symptoms are those of paralysis only, until the curve of spinal-stimulation,  $os_1s_2s_3$ , &c., cuts that of paralysis between  $t_2$  and  $t_3$ , after the descent of the latter from the line of complete paralysis, CD. Tetanus then becomes the predominant symptom. At first, its violence is considerably checked by the coexisting paralysis; but as the ordinates of the paralytic curve diminish in length, while those of the tetanic curve increase, the tetanic symptoms gradually acquire greater prominence and force, until they reach their maximum intensity between  $t_3$  and  $t_5$ . They then, in their turn, also diminish.

This diagram further shows how a paralytic and a spinal-stimulant (tetanic) action may be coexistent, while the effects of only one of these are apparent. Between  $o$  and  $t_2$ , the symptoms are those of paralysis alone, because the spinal-stimulant action is altogether masked by the complete paralysis; between  $t_2$  and  $t_3$ , tetanic symptoms appear, because the paralysis is incomplete, and reflex movements are therefore permitted to occur; and between  $t_3$  and  $t_5$ , the spinal-stimulant action, being but slightly checked by paralysis, manifests itself by violent tetanic convulsions.

The effects of this combined action on a mammal are graphically represented in Diagram 2. The experiment (Experiment LVIII.) I have selected is one in which eight grains of sulphate of atropia in solution was injected under the skin of a dog, weighing fifteen pounds. Slight paralysis was observed at twenty minutes, and feeble spasms occurred at forty minutes; they together reached their maximum intensities at about seventy minutes; and the

\* AB, line of normality, each division of which,  $ot_1$ ,  $t_1t_2$ ,  $t_2t_3$ , &c., represents a period of twenty-four hours; CD, line of complete paralysis;  $op_1p_2p_3$ , &c., curve of paralysis;  $os_1s_2s_3$ , &c., curve of spinal-stimulation (tetanus, &c.);  $t_1p_1$ ,  $t_2p_2$ , &c., ordinates whose length roughly represents the amount of the paralytic action;  $t_1s_1$ ,  $t_2s_2$ , &c., ordinates whose length roughly represents the amount of spinal-stimulant action.

spasms ceased at one hundred minutes, while the paralysis continued until about one hundred and twenty minutes. The dog recovered perfectly.

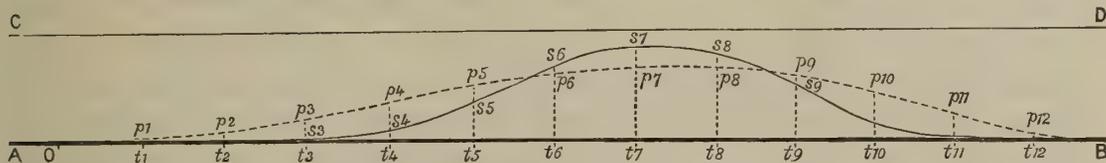


Diagram 2.\*

In this diagram, the curve of paralysis,  $op_1p_2$ , &c., leaves the line of normality, AB, before the curve of spinal-stimulation, the first symptoms being those of paralysis. As the curve  $op_1p_2$ , &c., never attains the level of the line of complete paralysis, CD, the paralytic action is never sufficiently great to prevent the manifestation of the considerable spinal-stimulant action present. Although the ordinates  $t_6s_6$ ,  $t_7s_7$ , and  $t_8s_8$ , are considerably longer than the ordinates  $t_5s_5$  and  $t_9s_9$ , the effects of spinal-stimulation are not proportionally greater at  $t_6$ ,  $t_7$ , and  $t_8$  than at  $t_5$  and  $t_9$ , for the ordinates of the paralytic curves also are longer at the former than at the latter times, reaching their highest point at about the same time as those of the spinal-stimulant curve, and the spinal-stimulant action is, accordingly, more masked about the time of its greatest intensity than at times somewhat anterior and subsequent thereto. These curves, therefore, admirably represent the effects that were observed, the convulsive symptoms having been of nearly uniform intensity throughout the whole time of their occurrence.

It has been taken for granted that the paralytic and spinal-stimulant actions coexist in frogs after the administration of large doses of atropia. Some evidence in support of this view may with propriety be given at this place.

*Experiment LIX.*—In a frog, weighing 270 grains, the abdominal aorta was ligatured immediately above its bifurcation into the two iliacs, and one-fifth of a grain of sulphate of atropia, dissolved in four minims of distilled water, was then injected under the skin at the right side of the thorax. Great impairment of motility and other symptoms of atropia action had occurred in one hour, when the observations were interrupted.

On the following day—twenty-three hours after the administration—the frog was lying on the abdomen, chest, and lower jaw, the anterior extremities being perfectly flaccid, while the posterior were extended with the webs stretched. A slight touch of the head caused a sudden attack of tetanus in the two posterior extremities, which lasted for three seconds; but it was impossible to

\* AB, line of normality, each division of which,  $ot_1$ ,  $t_1t_2$ ,  $t_2t_3$ , &c., represents a period of ten minutes; CD, line of complete paralysis;  $op_1p_2p_3$ , &c., curve of paralysis;  $os_1s_2s_3$ , &c., curve of spinal-stimulation (tetanus, &c.);  $t_3p_3$ ,  $t_4p_4$ , &c., ordinates whose length roughly represents the amount of the paralytic action;  $t_4s_4$ ,  $t_5s_5$ , &c., ordinates whose length roughly represents the amount of the spinal-stimulant action.

excite any movement whatever in the anterior extremities, or in any other part of the poisoned region. At twenty-eight hours, the position of the frog was the same as that last described, there being perfect flaccidity and complete motionlessness in the poisoned region, while the posterior extremities (non-poisoned region) were rigidly extended. A touch of the skin anywhere now excited violent tetanus of the posterior extremities, lasting for five seconds; but no movement occurred elsewhere, and the anterior extremities were perfectly flaccid. To test the condition of motor conductivity in the poisoned region, the right brachial nerve was exposed, and subjected to galvanic stimulation; no movement of the right anterior extremity was thereby produced, but tetanus of the two posterior extremities invariably followed each stimulation.

On the third day, the frog was lying on the abdomen, but the chest and head were now raised by the anterior extremities, which had become rigidly flexed. On stimulating the skin, an attack of general opisthotonic tetanus occurred, involving the poisoned as well as the non-poisoned regions.

In this experiment, the spinal-stimulant action would have been completely masked by the paralytic, if the posterior extremities had not been protected from the direct influence of the poison. Yet even when this is done, evidence of the spinal-stimulant action will only exceptionally be obtained at so early a stage. Atropia causes paralysis by an action not only on the motor nerves, but also on the sensory (afferent) and on some portion of the reflex apparatus in the spinal cord. In this experiment, two of these causes of paralysis (suspension of the function of the sensory nerves, and suspension of that of some portion of the reflex apparatus in the spinal cord) ceased before the third (suspension of the function of motor nerves); for the conductivity of the poisoned motor nerves was still completely suspended when the poisoned spinal cord and sensory nerves had regained their functions. Usually the return to normality occurs much more simultaneously in these different structures. It is still more difficult to obtain evidence in frogs of a spinal-stimulant action occurring soon after the administration of large fatal doses. Complete paralysis is so rapidly produced that no opportunity is given to the spinal-stimulant action to manifest itself. The evidence in support of an early stimulation of the cord is, however, readily obtained in mammals; for the paralytic effects are never so great as to prevent the manifestation of the spinal-stimulant action.

The two already mentioned propositions—namely, that in atropia the amount of paralyzing is, in all mammals, greater than the amount of spinal-stimulant action, and that atropia-paralysis is more readily produced in frogs than in mammals—are also sufficient to explain why different effects are produced in frogs and mammals by different doses of atropia.

When a large fatal dose of atropia is administered to a frog, the predominating

paralysis is so rapidly and completely produced that no spinal-stimulant symptom can be exhibited. Death results either from an extreme degree of the paralytic action, or, possibly, from some other effect of atropia. In neither case, however, does the paralytic action diminish sufficiently (if it diminish at all) to permit any effects of spinal-stimulation to appear, for death occurs during a high intensity of the paralytic action. Diagram 3 represents an experiment (XLII. of Table I.) in which a large fatal dose of sulphate of atropia was administered to a frog. The reflex activity was destroyed in a few minutes, and the conductivity of the motor nerves was completely suspended in one hour and forty minutes. Death occurred during the complete paralysis.

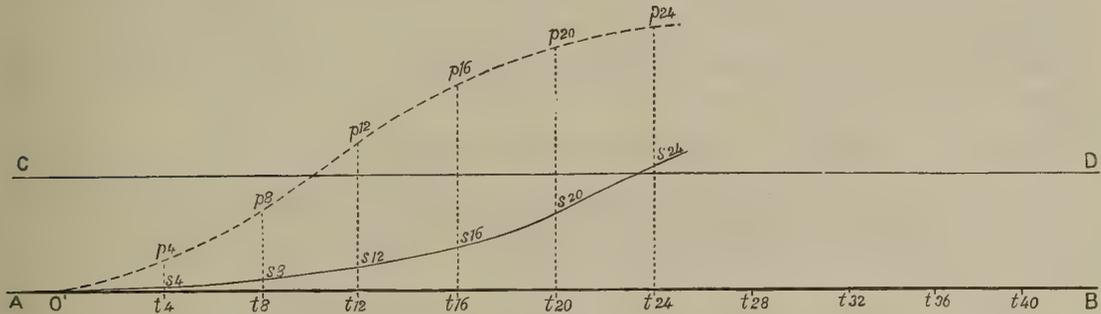


Diagram 3.\*

As the curve of paralysis,  $op_4p_8$ , &c., rises rapidly to the line of complete paralysis, CD, and crossing, terminates above it (at the time of the occurrence of death), while the curve of spinal-stimulation,  $os_4s_8$ , &c., rises with comparative slowness, it is obvious that no spinal-stimulant effect can possibly be manifested. In frogs, the only symptoms of a fatal dose of atropia are, accordingly, those of paralysis, notwithstanding that such a dose exerts a large amount of spinal-stimulant action, which is represented in the diagram by the curve  $os_4s_8s_{12}$ , &c.

In mammals, fatal doses of atropia invariably produce spasms and convulsions. We at once see why this should be so, if we bear in mind that mammals are less susceptible than frogs to a paralytic action. I have delineated in Diagram 4 the symptoms that were observed in an experiment (Experiment LX.) in which a solution, containing fifteen grains of sulphate of atropia, was injected under the skin of a dog, weighing nine pounds. Partial, but distinct, paralysis was first observed in eleven minutes, and spasms, with increased reflex excitability, in sixteen minutes. They both gradually increased in severity—the paralytic action causing inability to stand in twenty-two minutes, and the spinal-stimulant producing the first of a series of frequently recurring tetanic convulsions in nineteen minutes; and death took place one hour and eighteen minutes after the administration.

\* AB, line of normality, each division of which,  $ot_4, t_4t_8, t_8t_{12}$ , &c., represents a period of forty minutes; CD, line of complete paralysis;  $op_4p_8p_{12}$ , &c., curve of paralysis;  $os_4s_8s_{12}$ , &c., curve of spinal-stimulation (tetanus, &c.)

In this diagram (Diagram 4), the curve of paralysis,  $op_1p_2p_3$ , &c., does not at any time rise to the level of the line of complete paralysis, CD; whereas in the

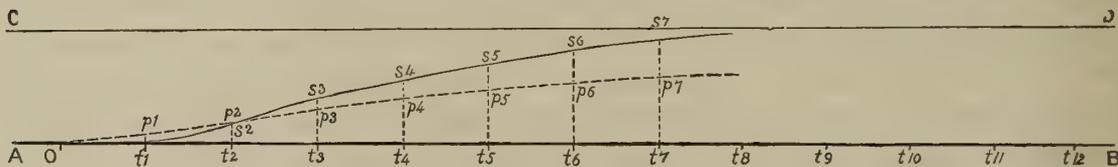


Diagram 4.\*

diagram representing the effects of a fatal dose in a frog (Diagram 3), the curve of paralysis very quickly reaches this line.† Accordingly, in this experiment, the spinal-stimulant action, which was considerable, was not prevented from manifesting itself, and spasms and tetanus coexisted with paralysis up to the time at which death occurred.

When a dose of atropia considerably below the minimum fatal, and just sufficiently large to produce obvious effects on motility, is administered to a frog, the effects are such as have been roughly delineated in the next diagram.

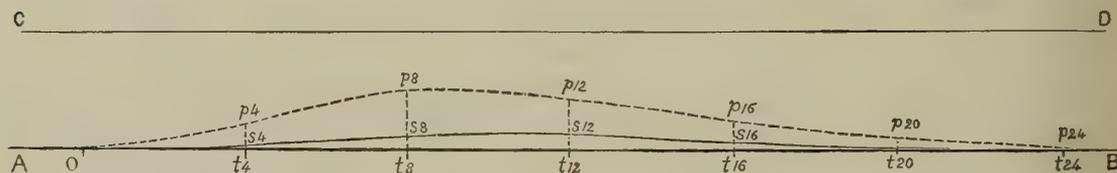


Diagram 5.‡

In a mammal the effect of such a dose would be represented by a diagram, in which the differences between the ordinates of the paralytic and spinal-stimulant curves are less than in the above.

The symptoms being but slight in both cases, the ordinates of the curves are very short; and as the amount of paralytic action in atropia is greater than the amount of spinal-stimulant, the area enclosed by the curve  $op_4p_8$ , &c., and the

\* AB, line of normality, each division of which,  $ot_1, t_1t_2, t_2t_3$ , &c., represents a period of ten minutes; CD, line of complete paralysis;  $op_1p_2p_3$ , &c., curve of paralysis;  $os_1s_2s_3$ , &c., curve of spinal-stimulation (tetanus, &c.);  $t_1p_1, t_2p_2$ , &c., ordinates whose length roughly represents the amount of the paralytic action;  $t_2s_2, t_3s_3$ , &c., ordinates whose length roughly represents the amount of the spinal-stimulant action.

† It is probable that a stage of tetanus occurring subsequently to a stage of paralysis has never been observed in mammals, after the administration of atropia, because a sufficiently large dose cannot be administered without causing death while the paralytic effects are being developed. It is, however, possible that separate paralytic and tetanic stages might be produced in mammals, if artificial respiration were employed after the administration of a very large dose.

‡ AB, line of normality, each division of which,  $ot_4, t_4t_8, t_8t_{12}$ , &c., represents a period of four hours; CD, line of complete paralysis;  $op_4p_8p_{12}$ , &c., curve of paralysis;  $s_4s_8s_{12}$ , &c., curve of spinal-stimulation (tetanus, &c.);  $t_4p_4, t_8p_8$ , &c., ordinates whose length roughly represents the amount of the paralytic action;  $t_4s_4, t_8s_8$ , &c., ordinates whose length roughly represents the amount of the spinal-stimulant action.

abscissa  $ot_{24}$  is greater than the area enclosed by the curve  $s_4s_8$ , &c., and the abscissa  $t_4t_{20}$ . The symptoms are, accordingly, those of paralysis; the spinal-stimulant action being so slight that its effects are not perceived. With small doses of atropia, spinal-stimulant effects are more likely to be observed in mammals than in frogs. If the minimum dose that produces paralytic effects be determined, and then a series of doses gradually increasing from this be administered, it will be found that in frogs a considerable increase may be effected before any spinal-stimulant symptom is produced; but that in mammals an extremely slight increase will cause spasmodic symptoms to make their appearance. The explanation of this also is to be found in the propositions.

*It has thus been shown that the tetanic symptoms produced by atropia in frogs are represented, though in a somewhat different form, in animals of a higher development. Atropia, therefore, forms no exception to the general law that poisons affect the same structures in the same way, in whatever animals these structures occur.*

*It has also been shown that the differences in the symptoms that are produced by different doses in animals of the same species may be explained by the paralysing action of atropia being greater than the spinal-stimulant.*

Paralysis, combined with spinal-stimulation, forms, therefore, the leading characteristic of the action of large doses of atropia on the cerebro-spinal nervous system, and unless this combination be taken into account, which it has not hitherto been, the symptoms that are produced by such doses cannot be rationally explained. In the antecedent portion of this paper, and especially in section C, this combined action on the nervous system has been demonstrated by a process of physiological *analysis*. I now propose to add to this some further proof, derived from what may be termed a process of physiological *synthesis*.

So long as we are unable to separate from one another those elements or groups of elements in atropia that produce its different effects—allowing that it is legitimate to suppose that such elements or groups of elements exist—a strict synthetical method cannot be applied to the investigation of its effects; but an imperfect synthetical method may be applied, in which we imitate these effects by combining various substances of clearly defined action. For this purpose I have selected strychnia, as the best known and most typical of the spinal-stimulants, and sulphate of methyl-strychnium, as one of the simplest and, for such purposes, certainly one of the most convenient of the paralyzers of motor nerves.\*

It was found that a dose of strychnia, in the form of a salt, equivalent to

\* This action of sulphate of methyl-strychnium has been demonstrated by Dr A. CRUM BROWN and the author in a paper read before this Society, and published in the Transactions, vol. xxv. part i. pp. 151–203. I prefer this substance to curara, because of its strength being constant; and on this ground I would recommend it to physiologists and physicians.

about the  $\frac{1}{450000}$ th of the weight of the frog, was sufficient to produce in it violent tetanic symptoms, which lasted for many hours, and terminated in recovery. When such a dose is given to a male frog, or to a female whose abdomen is not greatly enlarged by distended oviducts, it is of interest to note that the anterior extremities become incurved at the commencement of the tetanic symptoms, and continue so until these disappear; thus imitating a symptom produced by atropia that has been fully described in this paper (Section A.)

It was also found that doses of sulphate of methyl-strychnium, varying from the  $\frac{1}{8000}$ th to the  $\frac{1}{15000}$ th of the weight of the frog, were sufficient to produce complete and prolonged paralysis of the motor nerves, without causing death.

Guided by these results, I then administered to frogs combined doses of strychnia and sulphate of methyl-strychnium. After making several experiments, I at length discovered that the remarkable combination of paralytic and tetanic symptoms that has been described in Section A. of this paper could be exactly imitated by administering to frogs a mixture of strychnia with sulphate of methyl-strychnium in a certain proportion.

The following experiment is sufficient to confirm this statement:—

*Experiment LXI.*—One minim of a mixture of two minims of liquor strychniæ (B.P.) in eighteen minims of distilled water (equivalent to  $\frac{8}{100000}$ ths of a grain of strychnia), was added to three minims of a solution of one-tenth of a grain of sulphate of methyl-strychnium, in ten minims of distilled water (equivalent to three-hundredths of a grain of sulphate of methyl-strychnium), and the four minims of solution thus obtained was injected under the skin at the right flank of a male frog, weighing 355 grains. In two minutes, some sprawling was observed; in three minutes, the frog was unable to jump, and the respiratory movements of the chest had ceased; in twelve minutes, only extremely feeble and sluggish reflex movements could be excited by pinching the skin; in fourteen minutes, the lower jaw rested on the table, the respiratory movements of the throat ceased, and the frog was perfectly flaccid; and in thirty-five minutes, it was impossible to cause any reflex movement whatever even by severe excitation of the skin. During all this time, there was not the faintest strychnic symptom. The reflex activity was frequently tested. At the commencement of the experiment, it was perfectly normal; and as the symptoms advanced, the only change observed was a gradually increasing feebleness. At forty minutes, it was found, by exposing a sciatic nerve, and subjecting it to galvanic stimulation, that motor-nerve conductivity was completely suspended. At this time, the cardiac impulse was good, and the heart's contractions occurred twenty-eight times in the minute.

On the second day—twenty hours after the administration—the frog was lying on the abdomen with the thorax raised and supported by the anterior extremities, *which were rigidly incurved*, and there were infrequent respiratory

movements. A slight touch of the skin caused a violent attack of emprosthotonic tetanus, which lasted for seven seconds, and was succeeded by a series of clonic spasms in the posterior extremities and the abdominal muscles. During the tetanus, the posterior extremities were stiffly extended, with their webs stretched, while the anterior were rigidly incurved. Tetanic convulsions could be excited at any time by repetitions of the excitation, and they sometimes occurred spontaneously. In the intervals between them, the anterior extremities continued rigidly flexed.

During the third, fourth, and fifth days, the frog remained in this state, except that, on the fifth day, the convulsions were less powerful and prolonged.

On the sixth, seventh, and eighth days, excitation caused merely sudden spasmodic movements; but the anterior extremities were still rigidly flexed inwards.

On the ninth and tenth days, the frog was in a normal position; voluntary movements were freely executed; but there was still a slight increase in the activity of the reflex function.

On the twelfth day, the symptoms had completely disappeared.

This experiment, therefore, proves that combined doses of strychnia and sulphate of methyl-strychnium may produce symptoms that in every detail imitate the most obvious of the effects of atropia on the cerebro-spinal nervous system of frogs. By comparing the diagrammatic representation of this experiment with that of Experiment XVI. (p. 480), it will be seen how close are the resemblances.

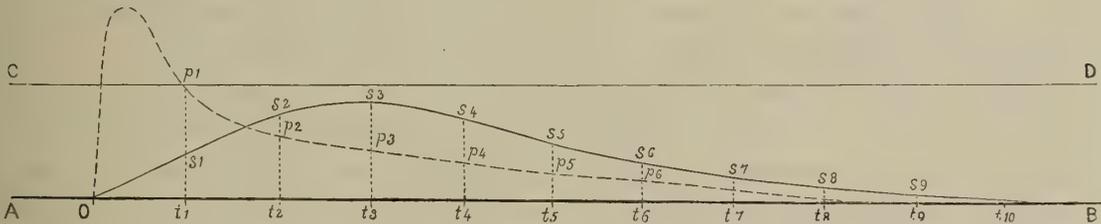


Diagram 6.\*

It is therefore possible, by combining a paralyzing with a convulsant substance, to produce in frogs paralytic and tetanic effects, which in their relative and general characters are undistinguishable from the paralytic and tetanic effects of atropia.

The next step was to administer these substances simultaneously to a mammal.

It has been shown in a paper communicated to this Society by Dr A. CRUM

\* AB, line of normality, each division of which,  $ot_1, t_1t_2, t_2t_3$ , &c., represents a period of twenty-four hours; CD, line of complete paralysis;  $op_1p_2p_3$ , &c., curve of paralysis;  $os_1s_2s_3$ , &c., curve of spinal-stimulation;  $t_1p_1, t_2p_2$ , &c., ordinates whose length roughly represents the amount of the paralytic action;  $t_1s_1, t_2s_2$ , &c., ordinates whose length roughly represents the amount of the spinal-stimulant action.

BROWN and myself, that four-fifths of a grain of sulphate of methyl-strychnium is about the minimum fatal dose for a full-grown rabbit.\* The amount of strychnia that should be combined with this dose, in conformity with the ratio of the last experiment, is about two-hundredths of a grain (or 2·66 minims of *liquor strychniæ*).

*Experiment LXII.*—2·66 minims of *liquor strychniæ* (containing 0·02 grain of strychnia) was mixed with a solution of four-fifths of a grain of sulphate of methyl-strychnia, in twenty-five minims of distilled water, and this solution was injected under the skin at the right flank of a rabbit, weighing three pounds and four ounces. The first symptoms occurred in seven minutes, and consisted of a slight degree of exaggeration in the starts that were caused by irritating the nostrils of the animal. In nine minutes, a series of spontaneous spasms occurred in the anterior extremities; and it was now obvious that the motor power of the posterior extremities was slightly diminished. In nine minutes and thirty seconds, a further series of spontaneous spasms occurred, but the spasms now involved the whole body. In ten minutes, the rabbit lay down on the abdomen and chest; and after remaining quietly in this position for a minute, it was again affected with spasms, during which it fell over on the side. In twelve minutes and ten seconds, there was an attack of opisthotonic tetanus, which lasted for only ten seconds, and was immediately succeeded by a second attack, and, on its termination, by a third, both also of short continuance. The rabbit was still lying on the side, and appeared unable to change its position. At frequent intervals, a series of feeble spasms now succeeded each other; and at the termination of one of these death occurred, fourteen minutes after the administration.

The symptoms of paralysis and of spinal-stimulation observed in this experiment do not, in their relation to each other, exactly resemble those of atropia. Indeed, it was not anticipated that they would do so; but it was anticipated rather that the paralytic phenomena would be less marked, and that the spinal-stimulant would, consequently, acquire a greater prominence than with atropia. In the mixture of strychnia and sulphate of methyl-strychnium, the paralysis is produced by an action on the motor nerves alone, which action affects frogs much more rapidly and powerfully than mammals; whereas in atropia, it is produced not only by an action on the motor nerves, but also by actions on the sensory nerves, and, probably, on a portion of the spinal cord, and the additional actions seem to affect frogs and mammals nearly equally. Therefore, while frogs are more readily and completely paralysed than mammals by both atropia and sulphate of methyl-strychnium, mammals are less readily paralysed by the latter than by the former substance. Accordingly, the effects of the combination of sulphate of methyl-strychnium and of strychnia more closely resemble those of atropia in frogs than in mammals.

\* *Loc. cit.* pp. 160 and 196.

At the same time, the last experiment is in all respects a satisfactory one, for it clearly demonstrates that such a combination of a paralysing with a spinal-stimulant substance as produces in frogs paralysis followed by convulsions, will produce in mammals paralysis coexisting with convulsions, and impeding their manifestation. So that by a process of what may be termed physiological synthesis, further evidence has been obtained in support of the conclusions, *that the effects of large doses of atropia on the cerebro-spinal nervous system (mental phenomena excluded) are due to combined spinal-stimulant and paralysing actions of that substance, and that the differences in the relations of these effects to each other, which are seen in different species of animals, may be explained by this combination acting on special varieties of organisation.*

It is generally admitted that atropia produces both paralytic and convulsive symptoms in mammals, but no satisfactory attempt has hitherto been made to define the relations of these symptoms to each other. This investigation has shown in what manner the paralysing is related to the convulsant action both in mammals and in frogs; and it has also accounted for the differences in the manifestation of these actions after different doses of atropia. It may, without presumption, be asserted, that it throws a new light on the causation of some of the symptoms of atropia, and also of many other substances, whose action, like that of atropia, produces a combination of paralytic and convulsive symptoms.

The principal results that have been obtained may be thus summarised:—

1. Atropia produces in frogs well-marked convulsive and tetanic symptoms, which, when present in an extreme degree, form a separate stage in the poisoning, succeeding that of paralysis.
2. Tetanic symptoms follow the subcutaneous administration of a dose of sulphate of atropia, equivalent to the  $\frac{1}{1000}$ th of the weight of the frog, and of doses a little greater or less than this.
3. These symptoms are due to a direct action of atropia on the *medulla (oblongata and spinalis)*.
4. The differences between the paralytic and convulsive symptoms that occur in frogs and those that occur in mammals may be explained by the greater susceptibility of the former to the action of a paralysing agent, and by the amount of paralysing being greater in atropia than the amount of convulsant action.
5. The different symptoms that are produced by different doses of atropia in animals of the same species may be explained by its paralysing being greater than its convulsant action.
6. The paralysing and convulsant actions of atropia can be imitated in both frogs and mammals by a combination of a paralysing with a convulsant substance.



XIII.—*Hegel and the Metaphysics of the Fluxional Calculus.* By W. ROBERTSON SMITH, M.A., Assistant to the Professor of Natural Philosophy in the University of Edinburgh. Communicated by Professor TAIT.

(Read 17th May 1869.)

It is now many years since Dr WHEWELL drew the attention of the Cambridge Philosophical Society to the courageous, if somewhat Quixotic, attempts of HEGEL to cast discredit on NEWTON'S law of gravitation, and on the mathematical demonstrations of KEPLER'S laws given in the "Principia." At the time when WHEWELL wrote, it would probably have been difficult to find in Britain any one ready to maintain the cause of HEGEL in this matter, or even to hint that the astounding arguments of the Naturphilosophie flowed from any deeper source than self-complacent ignorance.

The present state of matters is different. The philosophy of HEGEL is now for the first time beginning to have a direct and powerful influence on British speculation. Men are beginning to study HEGEL; and an author whose works confessedly demand the labour of years, if they are to be fully understood, can hardly be studied at all except by devoted disciples. A man whose determination to master HEGEL'S philosophy survives the repelling impression which the obscurity and arrogance of the philosopher are sure to produce at first, is very likely to be carried away by the calm assumption of omniscience which runs through HEGEL'S writings. It is not, therefore, surprising that Dr STIRLING extends his admiration to HEGEL'S physical positions; and if he does not venture to say that HEGEL'S proof of KEPLER'S laws is right, at least feels sure that it would repay the attention of mathematicians.

It would not, perhaps, be impossible to rob Dr STIRLING of even this sorry consolation; but there is the less occasion for retracing any part of the ground gone over by WHEWELL, in so much as "The Secret of Hegel" calls attention to another point, in which HEGEL criticises NEWTON, and in which Dr STIRLING has no hesitation in pronouncing HEGEL'S findings "perfectly safe from assault," and NEWTON guilty of an obvious mathematical blunder.

Such a statement, proceeding from the most powerful of our living metaphysicians, and recently reiterated in the newspaper press, as a sort of challenge to mathematicians, seems to call for some remark from a mathematical point of view. It is true that a confirmed Hegelian is not likely to be influenced by any reasoning that we can offer. "The judgment of a pure mathematician," we are

told, "has really been so peculiarly trained that, perhaps, any such will never prove decisive as regards any Hegelian element." We are told, too, that HEGEL'S "most important note" on the mathematical infinite "has remained hitherto absolutely sealed," for C. FRANTZ, who does take up the subject "as in opposition to, is to be assumed ignorant of, the views of HEGEL, which plainly, so far as they go, are inexpugnable" (!)

Now I do not profess to be able to treat this question from the stand-point of HEGEL'S own philosophy. I have no desire to criticise HEGEL'S doctrine of the Infinite, in so far as it forms an integral part of his system. But the note to which Dr STIRLING calls attention is itself a critical note, in which HEGEL proposes "to consider in detail the most remarkable attempts to justify the use of the mathematical notion of the Infinite, and get rid of the difficulties by which the method feels itself burdened" (HEGEL'S *Werke*, iii. 286).<sup>\*</sup> What HEGEL seeks to show is, "that the mathematical Infinite is at bottom the true Infinite" (p. 283); imperfectly conceived, however, by the mathematicians, who have therefore never been able to put the higher calculus on a basis thoroughly free from confusion, or even error. Thus, not to speak of FERMAT, LEIBNITZ, EULER, and others, whose views HEGEL takes up more or less fully, we are told that NEWTON himself, although his fundamental thought was quite in harmony with HEGEL'S views, was not so far master of his own thought as to be able fairly to deduce the practical rules of his method. In the actual application of the new instrument, NEWTON clung "to the formal and superficial principle of omission because of relative smallness." He thus fell into real errors; and even so fundamental a problem as the determination of the fluxion of a product was solved in a manner analytically unsound. Now these, I maintain, are assertions that can fairly be examined by one who does not profess to have mastered HEGEL'S system. They even afford a fair test whether that system is really so complete in all its parts, and so light-giving in its applications, as we are told to believe. If NEWTON is really confused and in error, it must be possible to make this clear by an argument based on NEWTON'S own principles. For if to the mathematician NEWTON'S method is perfectly clear and self-contained, and if its errors can only be observed from an entirely different point of view, we have not one truth, but two truths, mutually destructive. And this surely Dr STIRLING will not assert.

It is possible, however, to go further than this. To the subject of the calculus HEGEL devotes two notes. The first of these alone is taken up by Dr STIRLING. And in this note HEGEL adds to the destructive criticism of which we have been speaking only a very general account of the principles on which he would base the calculus. These general principles are, as HEGEL says, "abstract" (we would rather say vague), "and therefore in themselves also easy" (p. 327). The real

<sup>\*</sup> Here and elsewhere I adopt, as far as possible, the language of Dr STIRLING'S own translations from HEGEL, which may be viewed as authoritative.

difficulty lies "in the concrete side," in the deduction from these generalities of the practical rules of the method. To this subject HEGEL devotes his second note, professing to point out a purely analytical method whereby, without any application of the doctrine of limits, everything necessary for practice can be deduced. If we can demonstrate that the analytical method is radically unsound, producing results mathematically false, it will surely be vain to appeal in defence to any "deficiency in the judgment of a pure mathematician."

The plan that suggests itself is therefore the following:—*First*, to consider the real character of NEWTON'S method, and to show what may, I think, be made quite clear to an unprejudiced mind, that that great man really did know what he was doing; and, in the *second* place, to show that HEGEL having refused to be instructed by NEWTON'S real knowledge, but having acutely enough caught sight of something like the ghost of an idea, which he could not for want of solid knowledge really make his own, was first ensnared by the plausible but fallacious method of LAGRANGE, and then, in attempting to improve that method, lost any glimpse of the truth that he had before, and was swamped in hopeless absurdity.

The ingenuity of a great deal that HEGEL has said on this subject I do not wish to dispute. No doubt he,

——— "with as delicate a hand,  
Could twist as tough a rope of sand"

as any man that ever lived. But the question is, after all, one of plain truth and error; and however much we may admire the chivalry with which HEGEL rushes into an unequal encounter with so gigantic an antagonist as NEWTON, it will never do to

"Coin a formal lie on't  
To make the knight o'ercome the giant."

We must begin, then, by examining the principles on which NEWTON based his doctrine of Fluxions. In doing this, it is not necessary to inquire how far NEWTON'S own views varied during his life. HEGEL knows NEWTON'S method from the Principia only, and a quotation from the second Lemma of the Second Book (*Werke*, iii. 305) shows that it was the current text of the Principia (that of the second edition) which he had before him. In fact, HEGEL'S acquaintance with NEWTON'S writings was clearly of the most superficial character, embracing apparently little if anything beyond the section on Prime and Ultimate Ratios, and the Lemma just referred to. These facts make all merely bibliographical inquiries superfluous in dealing with HEGEL'S objections. I may refer, however, to a paper by Professor DE MORGAN, in the "Philosophical Magazine" for 1852, on the "Early History of Infinitesimals in England," in which it is shown "that NEWTON never varied in his meaning of  $\dot{x}$ ;" or, in other words, that NEWTON "held to the conception of the velocity or fluxion," although he at first "used the infinitely small increment" (only of the first order, however), "as a means of

determining it." What follows will, I hope, serve to show that these facts imply that NEWTON had all along a firm grasp of the principle of his method, and that his frequent employment of abbreviated practical processes was really based on a consciousness of the strength of his method, according to the general principle of mathematicians, who never hesitate to apply the boldest symbolical methods in detail, when they feel confident of the starting-point in the use of these symbols. This, in fact, is a point that metaphysicians have never properly attended to. One is disposed to cap Dr STIRLING'S wish that some great analyst would study HEGEL, by expressing a hope that some metaphysician of real ability may pay sufficient attention to what are technically called the "Symbolical Methods" of mathematics, to enable him to appreciate BOOLE'S profound preface to his treatise on "Differential Equations." This exercise would at least make it clear that metaphysical criticism on mathematics is still—I speak without any desire to be disrespectful—in the circle-squaring stage, *i.e.*, still treats as the real questions for discussion points that mathematicians have long seen to be merely special cases of general principles, and therefore to be no longer possessed of independent interest.

To return from this digression. NEWTON saw that there were two ways in which quantities might be conceived as generated. The first of these is that which the usual processes of arithmetic have made familiar to everybody, *viz.*, the addition of discrete units. The theory of numbers thus viewed is contained in the arithmetic of integers, to which may be added the doctrine of arithmetical fractions as an extension of the method, reached by supposing the unit itself to change in value. NEWTON was especially attentive to the importance of the doctrine of decimal fractions, in which the change of unit is so regulated as to give the greatest possible increase of power that the *arithmetical* conception of quantity admits of; and the opening pages of his "Geometria Analytica" are expressly directed to show that these advantages may be made available in literal as well as in numerical calculations. [See also the treatise "De Analysi per Equationes Numero Terminorum Infinitas."]

NEWTON saw, however, that arithmetic in its most perfect form could give full mastery over quantity, only on the supposition that quantity, as it comes before us in the universe, is always produced by the synthesis of ultimate units, or, in other words, of indivisibles. And this, says NEWTON, is contrary to what EUCLID has proved concerning incommensurables in the tenth book of the Elements (Princ. lib. i. sec. i. schol.)

Instead, therefore, of endeavouring to eke out this view of quantity by arbitrary assumptions, NEWTON resolved to turn to Nature herself, and inquire how quantity is really generated in the objective universe. "Lineae," he writes "describuntur ac describendo generantur non per appositiones partium sed per motum continuum punctorum; superficies per motum linearum; solida per

motum superficierum; anguli per rotationem laterum; tempora per fluxum continuum et sic in cæteris. *Hæ Geneses in rerum natura locum vere habent et in motu corporum quotidie cernuntur.*" (Introd. ad Quad. Curv.)

In a word, NEWTON'S fundamental position is, that the arithmetical conception of quantity is not that with which nature herself presents us, and is not, therefore, universally applicable. On the other hand, every quantity that has objective reality [*i.e.*, is an object of real intuition] is generated by continuous motion, with definite (constant or variable) velocity within definite limits of time. The metaphysical nature of time and motion NEWTON has nothing to do with. It is enough for him that mathematical time, conceived as an independent variable flowing uniformly, is clearly the *true* time made known to us in nature (Principia; Schol. to the Defs.), and that the existence of a definite velocity at each point of a motion is in like manner an undoubted physical fact.

By means of these profound yet simple considerations, NEWTON is at once able to revolutionise the whole theory of quantity, and to substitute for the relation of unit and sum that of velocity and quantity generated, or, in NEWTON'S own language, of fluxion and fluent. It must be remembered that we have said nothing of *space*, so that fluent is not limited to extensive quantity, while velocity, or as we should rather say *rate*, has a correspondingly wide application. Thus, any fluxion may itself be treated as a fluent quantity, and its fluxion sought, the only independent variable being time, which is thus a fluent which has no variable fluxion.

This conception of time, as the one absolute and independent variable, is undoubtedly one of the most splendid and fruitful in the history of human thought, and well deserves the attention of metaphysicians. Only let it be said that no criticism of NEWTON'S *time*, which starts from the arithmetical view of quantity, and urges the old objections about infinite divisibility, and so forth, is competent; for the arithmetical theory is a product of abstract reflection, and so stands on a lower platform than the pure objective notion of NEWTON.

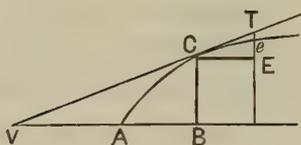
There is no difficulty in comprehending the mathematical power which the conception of fluxions at once puts in NEWTON'S hands, if we remember that it is not in any sense an extension of the theory of numbers that he is seeking. It is true that the calculus has revolutionised algebra as well as geometry; but it has done so by transforming algebra from the abstract science of numbers to a *physical* science—the science of pure time. In NEWTON'S own mind, however, this conception was probably not explicitly present. What he did see was, that all difficulties in geometry (and to NEWTON, as to the old geometers, geometrical magnitude is the type and exponent of all magnitude whatsoever, when viewed with respect to its generation) are reducible to the general form:—"Given the fluent as a function of time to determine the fluxion and *vice versâ*."

The one class of problems that can be thoroughly treated without explicit

reference to a generation by flux, is that which has for its geometrical type systems of straight lines; and thus geometers were tempted to introduce the fiction of indivisibles, in order to reduce higher problems to this type.

But these higher problems are not simply complicated cases of the rectilinear type; on the contrary, that type is produced by one of the two essentially distinct elements (generated and generating quantity), which usually appear side by side, ceasing to be explicitly manifest.

Take, for example, NEWTON'S own instance at the beginning of the "De Quadratura." Suppose the abscissa AB of a curve to flow uniformly, in which case it may be taken as the graphic representation of the independent variable, *i.e.*, of time, while the ordinate BC is of course a function



of the abscissa. Then NEWTON shows that the reason why the determination of the tangent at C is a difficult problem, is that the ratio of the ordinate BC to the sub-tangent VB is the graphical representation of the fluxion of the ordinate. In fact, the

*meaning* of the tangent is, that it is the direction in which the curve is flowing at the point C; and all attempts to give it another explanation without reference to motion simply ignore the real gist of the problem, and of course end in difficulties that can be escaped only by violent assumptions. It is only in the straight line where the fluxion of the ordinate is constant, or the tangent sinks into the curve, that the conception of *rate* can be dispensed with.

Before we go farther, it is proper to remark that in criticising NEWTON, HEGEL coolly ignores the whole foundation of the doctrine of fluxions as here developed. "The thought," says he (*Werke*, iii. 302; STIRLING, ii. 354), "cannot be more correctly determined than NEWTON has given it; that is, the conceptions of movement and velocity (whence fluxion) being withdrawn as burdening the thought with inessential forms and interfering with due abstraction"—*i.e.*, because HEGEL thought that the calculus should be based, after the manner of LAGRANGE, on purely analytical considerations, it never enters his head that if NEWTON thought otherwise there might be some deeper ground for this course than a want of insight into his own method. On the contrary, HEGEL comments in the most edifying manner on the "early still naïve period of the calculus" in which "mathematicians sought to express, in words and propositions, results of the newly invented calculus, and to present them in geometrical delineations," assigning to the "definitions and propositions so presented a real sense *per se*," in which sense they were "applied in proof of the main positions concerned." If there is any meaning at all in these statements, which are the gist of a somewhat lengthy discussion (*Werke*, iii. 324; STIRLING, ii. 375), that meaning must be that NEWTON and others first differentiated a function, then sought a geometrical construction to suit, and finally invented a physical proposition to correspond.

Purely analytical considerations without any physical basis were held, HEGEL thinks, to furnish in this way physical laws. In support of this view, HEGEL triumphantly refers to "the Newtonian proof of his fundamental proposition in the theory of gravitation compared with SCHUBERT'S 'Astronomy,' where it is admitted that . . . in the point, which is the nerve of the proof, the truth is not as NEWTON assumes it" [!] And so upheld by the dictum of this forgotten astronomer, HEGEL goes on to inveigh against the mere jugglery by which NEWTON, already knowing KEPLER'S results, avails himself of the "mist of the infinitely little" to bring out apparent mathematical proofs of these results. One does not know whether the singular perversity of this accusation against NEWTON'S moral character, or the incredible ignorance of the argument by which it is supported, is most to be wondered at; for, not only do the reasonings of the "Principia" rest throughout on the experimental laws of motion on which NEWTON'S first proposition is expressly based, but the proof itself depends not on the interpretation of an analytical process, but on the essentially physical or, more definitely, kinematical considerations above developed. Nay, so little is it the case, that the "mist of the infinitely little" is needed to give a show of plausibility to NEWTON'S process, that the whole gist of the proof lies in the one conception of quantity generated at a definite though variable rate, and that thus, without any change in the spirit of the proof, by simply introducing explicitly a theorem about moments of velocity which the demonstration in the "Principia" implies, the law of equal areas can be deduced without even that apparent use of the infinitely little which, as NEWTON himself warns his readers, is always merely apparent (THOMSON and TAIT'S "Natural Philosophy," § 234). In one word, NEWTON'S proofs are always physical throughout, and really belong to the essence of the thing to be proved; while HEGEL first shuts his eyes to the real import of the fluxional method, insisting that it must be made purely analytical, and then rails at NEWTON for using the method to do work for which, if it had been purely algebraic, it would not have been fit. A Hegelian calculus, as we shall see, would certainly have been of little service to physics; but the doctrine of fluxions is itself a part of physics, and absolutely indispensable in some form or other to the right understanding of physical problems.

We have still, however, to see how it is that NEWTON'S system comes to have anything at all to do with the infinitely little which, as he himself says (Introd. ad Quad. Curv. § 11), it is the peculiar merit of that system to render unessential. The reason is simply, as we are told in the scholium at the end of the first section of the "Principia," that he was anxious to provide for ease of conception, and also to introduce all legitimate abbreviations in his arguments. When NEWTON is called upon to justify his method, he always refers to the simple fact that a velocity definite, yet never for the shortest space of time uniform, is a notion really furnished by nature, and that the true measure of that velocity is to be

got by allowing the motion at any point to become uniform for a unit of time. But if one wishes, as HEGEL would say, to substitute for this *notion* a convenient "Vorstellung" to assist the imagination, NEWTON is ready, by means of the doctrine of prime and ultimate ratios, to point out a way in which we may avail ourselves of the method of indivisibles, always remembering that this method shall have merely a symbolic value, and so must be used with caution.

If two quantities have the same fluxion at any moment, they begin at that moment to increase at the same rate. It does not follow from this that the two quantities shall receive equal increments in any space of time however small, unless during that time the rates of flow remain constant. But NEWTON shows that in a very large class of cases, which he takes up one by one in the first section of the "Principia," not only may we, by taking the time of flow small enough, make the difference of the increments generated in that time as small as we please, but if we enlarge both increments on the same scale up to any given size, we may make the differences of the increased increments as small as we please, while the time of flow has still a definite value. Since, then, the ratio of the increments is always nearer to unity the less the time of flow, and may be brought as near to unity as we please by taking the time short enough, but still finite, the ratio must ultimately be unity—*i.e.*, that quantity which, varying according to a definite rule, always represents at any given time the ratio of the increments, may still be constructed when the time is made zero, and is now equal to unity, or is equal to the ratio at which the increments *start*, which NEWTON calls their prime or ultimate ratio.

The practical application of this reasoning is, of course, that in virtue of it, we may in certain cases with strict accuracy treat the increments of two variables (of a curve, for example, and its tangent) as equal, if, before closing our reasoning, we proceed to *take the limit*. Thus, if any one finds that it assists his imagination to deal with magnitudes as if they were composed of indivisibles, instead of confining himself to fluxions, NEWTON provides in the method of prime ratios a criterion by which the applicability of the process may be judged. The details by which it is shown in the "Principia," that the limit of the ratio of the increments is equal to the ratio of the fluxions whenever the fluents may be geometrically represented as curves of continuous curvature, involve no new principle in geometry. Everything is as plainly and undeniably reduced to ordinary geometrical intuition as anything in EUCLID, when we once bring with us the fundamental kinematical ideas of velocity and acceleration. It is obvious, moreover, that to NEWTON the fraction  $\frac{0}{0}$ , as above explained, means simply the ratio of the rates at which two quantities are flowing at the moment at which they pass together through the point from which we have agreed to reckon their magnitude backwards and forwards. Except where such rates can be assigned

possessing a definite ratio, NEWTON does not pretend to recognise  $\frac{0}{0}$  as a mathematical reality.

This outline of NEWTON'S principles is, of course, very meagre. It will probably, however, suffice to enable us to estimate the real value of HEGEL'S criticisms.

HEGEL highly approves of NEWTON'S statement of what he means by prime and ultimate ratios, viz., that he always deals not with indivisible but with vanishing divisibles. This is very satisfactory so far, but the next paragraph makes one doubt whether HEGEL knew what he was approving.

"NEWTON," we are told, "only explained what he means by his terms, without showing that such a notion has internal truth."\*

This is an accusation constantly recurring in various forms. Its source is, of course, that determination which we have already noticed in HEGEL to pay no regard to considerations of velocity and motion. Now it is quite true that NEWTON does not condescend to offer any explanation of his "notion" to the man who has failed to familiarise himself by actual intuition with the nature of velocity, and acceleration, and the genesis of quantities by flux. But these notions are just as truly capable of being constructed by pure intuition as those of ordinary geometry, and so NEWTON'S definitions enjoy fully the advantage which KANT ascribes to mathematical definitions in general. They cannot err, because they simply unfold a construction by means of which the notion is actually produced.

If HEGEL, however, shut his eyes to NEWTON'S notion, he has got one of his own, which he is sure is just what NEWTON wanted. I do not intend to attempt to take up anything but the concrete applications of this notion; but perhaps it may be well to give here part of HEGEL'S abstract statement of what he conceives to be the mathematical infinite. "Das unendliche Quantum . . . ist nicht mehr irgend ein endliches Quantum, nicht eine Grössebestimmtheit, die ein Daseyn als Quantum hätte sondern es ist einfach, und daher nur als Moment; es ist eine Grössebestimmtheit in qualitativer Form; seine Unendlichkeit ist als eine qualitative Bestimmtheit zu seyn" (iii. 289; STIRLING, ii. 341). Now, says HEGEL, this is clearly what NEWTON needs. His vanishing magnitudes have ceased to exist as quanta, and exist only as sides of a relation; but farther, the relation itself, in so far as it is a quantum, vanishes. "The limit of a quantitative relation is that in which it both is and is not, or, more accurately, that in which the quantum has disappeared, and there remains the relation only

\* Dr STIRLING (ii. 355) seems to have read "Nach dem damaligen Stande der wissenschaftlichen Methode wurde *nun* erklärt." In the collected edition of the "*Werke*," iii. 303, I read "wurde *nur* erklärt," which seems to give a more intelligible sense.

as qualitative relation of quantity." This sentence must mean that in the equation

$$\text{Lt } \frac{\delta y}{\delta x} = \frac{\dot{y}}{\dot{x}}$$

the left hand side vanishes as quantum in the same sense in which  $\delta x$  and  $\delta y$  vanish, or, as HEGEL often puts it,  $\frac{dy}{dx}$  is "infinite," just as truly as  $\delta y$  and  $\delta x$ . Now, we are told again and again that the "infinity" of the  $\delta x$  and  $\delta y$  does not lie in their being infinitely small, but in their having ceased to be any determinate magnitude, and only representing the qualitative principle of a magnitude. To this statement NEWTON would probably not have objected, as his whole use of infinitely small quantities is, as we have seen, merely to help the imagination, and scientific strictness is given to his method from another side. But certainly he would never have dreamed of admitting that  $\frac{\dot{y}}{\dot{x}}$  is also indeterminate; for both numerator and denominator of this fraction are in their nature definite quantities. That the fraction can be expressed as  $\frac{0}{0}$  is to NEWTON by no means the essential point. On the contrary, he argues distinctly that  $\frac{0}{0}$  must have a definite value, just because this is the form in which certain processes present to us a quantity which, from kinematical grounds, we know to be definite. To HEGEL, however, the fascinating element is just this  $\frac{0}{0}$ , which for his ends would be quite spoiled by being evaluated. That would reduce it to a mere quantum; but, in the meantime, it is "a qualitative relation of quantity," which is a far finer thing. Not unnaturally, however, HEGEL has now to ask himself, what is to be the practical use of this  $\text{Lt } \frac{\delta y}{\delta x}$ , which certainly "expresses a certain value which lies in the function of variable magnitude." In asking this question, he still supposes himself to be criticising NEWTON and the mathematicians, and accordingly proceeds, with much severity of manner, to knock down the indeterminate  $\frac{dy}{dx}$  which he has just set up (p. 318). To apply the conception of limit in the concrete we must determine the limit. This is done by TAYLOR'S theorem, from which if  $y = f(x)$  we get

$$\frac{\delta y}{\delta x} = p + q \delta x + \text{\&c.},$$

and then letting  $\delta x$  and  $\delta y$  vanish  $\text{Lt } \frac{\delta y}{\delta x} = p$ ;—not as it should have been  $= \frac{0}{0}$ . This, of course, is sadly inconsistent; for instead of our fine qualitative determination, here is a stubborn quantum turning up. Now, says HEGEL, the

mathematicians try to get over this by saying that  $p$  is not really  $= \frac{0}{0}$ , but is only a definite value, to which  $\frac{0}{0}$  comes as near as you please. Of course, if this is so, it is as evident as anything can be that the difference between  $p$  and  $\frac{0}{0}$  is not a quantitative one. But, adds the philosopher, naïvely enough, that doesn't help one over  $\frac{dy}{dx} = \frac{0}{0}$ . Suppose now that we were to say  $\frac{dy}{dx}$  really  $= p$  (a definite quantity), as, in fact, mathematicians *do* say, then it is obvious that  $\delta x$  couldn't have been  $= 0$ . Or if, finally, it is conceded that  $\frac{\delta y}{\delta x} = 0$  (which HEGEL seems to think most likely, since  $\delta y$  and  $\delta x$  vanish together), then what can  $p$  be?

Now, can any one say that the man who devised this argument knew what he was doing? When did any mathematician suppose that after evaluation  $\frac{0}{0}$  is indeterminate? Or had HEGEL never read NEWTON'S first lemma, with its "fiunt ultimo æquales"? Or, again, if HEGEL allows that there is no quantitative difference between  $p$  and  $\frac{0}{0}$ , why does he assume a qualitative one? Or, above all, why try to explain NEWTON'S doctrine without ever deigning more than a contemptuous glance at the one central point of the whole? HEGEL boasts that half an hour would suffice to learn the calculus. Certainly he might have employed a good many hours in unlearning his false conceptions of it.

HEGEL has next something to say about the way in which mathematicians have developed the details of the calculus. Since none of them had a clear notion of the matter in hand, their proofs, we are told, are very weak. They always fall back into methods merely approximate, subjecting infinitely small quantities to the laws of finite quanta, and yet rejecting them as relatively unimportant, in despite of these laws. Of course, adds HEGEL, we need not look for the rigour of demonstration of the old geometry, for the analysis of the infinite is of a nature essentially higher than that geometry. However, mathematicians have sought this rigour, and they have all failed.—Of course, it would be easy for any one to point out numerous mathematicians who have failed; but let us simply ask whether NEWTON has done so. HEGEL unhesitatingly affirms that he has, and Dr STIRLING is jubilant at the discovery.

The error is supposed to lie in the deduction in Prin. ii. Lem. 2, of the fluxion of a product. The statement of NEWTON is as follows:—If A, B be two quantities increasing continuously, and their moments or rates of change  $a$  and  $b$ , the moment or change of the rectangle AB is  $aB + bA$ . By moment NEWTON does not mean the increment actually received in any time, however short, but the

nascent principle of the fluent quantity—a notion, of course, made clear by the previous discussion of prime and ultimate ratios. The moments, in fact, are any quantities proportional to the rates at which A and B are flowing—the products of the fluxions of A and B by an arbitrary increment of time. If moments, then, are called increments, the meaning is increments which would be received if the rate of flow remained constant, and the ratio of two moments is simply the ratio of the fluxions, and therefore equal to the limit of the ratio of the actual increments, while it is quite independent of the magnitude of the separate moments. Now, says NEWTON, when A and B are diminished by half their moments, the rectangle is  $AB - \frac{1}{2} aB - \frac{1}{2} bA + \frac{1}{4} ab$ ; and when A and B are increased by half their moments, it is  $AB + \frac{1}{2} aB + \frac{1}{2} bA + \frac{1}{4} ab$ ; and so to the increments  $a$  and  $b$  in the sides corresponds an increment  $aB + bA$  in the rectangle. This demonstration is certainly very curt, and intended only for those who have mastered NEWTON's fundamental notions, and may therefore be saved the tedium of a long *reductio ad absurdum*. More at length, the proof would be of this kind. The fluxion of the rectangle must, since the flow is continuous, be a definite quantity, depending only on the magnitudes and fluxions of the sides at each moment. Thus the fluxion of AB will be unchanged, if we suppose that from the values  $A - \frac{1}{2} a$ ,  $B - \frac{1}{2} b$  the sides flow with uniform velocity, equal to  $\dot{A}$  and  $\dot{B}$ , until they attain the values  $A + \frac{1}{2} a$ ,  $B + \frac{1}{2} b$ . In this case the increments  $a$  and  $b$  will represent exactly upon the same scale the fluxions  $\dot{A}$  and  $\dot{B}$ . Meantime, the rectangle has been flowing with a constantly increasing velocity, which at the moment when the value AB was reached, was the velocity NEWTON is seeking to determine. The whole increment of the rectangle is  $aB + bA$ , which therefore represents the average velocity of the rectangle on the same scale as  $a, b$  represent the uniform velocities of the sides. Clearly the average velocity with which the increment is described is greater than the velocity at the beginning of the motion, and less than that at the end, and therefore, since the velocity is continuous, is strictly the velocity at some intermediate point. But this point can be none other than that at which the rectangle = AB, for were it any other point, we could take  $a$  and  $b$  small enough to throw this point out, and there would still be another point at which the fluxion of the rectangle must =  $aB + bA$ . But this is contrary to the intuitive fact that the velocity is continuously increasing. To the mathematician, however, this round-about process is unnecessary. He sees at once that if the average velocity is independent of the duration of flow, and depends solely on a certain point being included within the flow considered, the velocity at that point must be strictly the average velocity, for *in the limit* the two coincide.

Now, HEGEL, of course, did not see this, because he would not admit the kinematical reality of fluxions. He, therefore, supposes that NEWTON wants to find the *differential* of AB—a way of stating the problem which NEWTON would

have rejected as misleading. The differential can be nothing else than  $(A + dA)(B + dB) - AB$ . But NEWTON writes instead of this  $(A + \frac{1}{2} dA)(B + \frac{1}{2} dB) - (B - \frac{1}{2} dA)(B - \frac{1}{2} dB)$ , thereby making an error in so elementary a process as the multiplication of two binomials!—But where is HEGEL's justification for saying that what NEWTON is seeking is  $(A + dA)(B + dB) - AB$ ? NEWTON says nothing about differentials at all; his  $a$  is, as we have seen, not the infinitely small increment of  $A$ , but an arbitrary multiple of the fluxion of  $A$ , which need not be infinitely small. NEWTON'S  $\frac{aB + bA}{a}$  is, if you please,

$$= \text{Lt} \frac{(A + dA)(B + dB) - AB}{dA};$$

but even this, which is very different from what HEGEL writes, is simply a different, by no means a more fundamental, view of the problem than NEWTON'S.

Dr STIRLING tells us that HEGEL'S expression is what NEWTON'S says his is, "the excess of the increase by a whole  $dA$  and  $dB$ ." But what NEWTON says is only that when the sides are increased from  $A - \frac{1}{2} a$  and  $B - \frac{1}{2} b$ , through increments  $a$  and  $b$  the rectangle increases by  $aB + bA$ . That this is true surely cannot be denied. In fact  $(A + a)(B + b) - AB$  would have represented not the velocity at value  $AB$ , but the average velocity of the rectangle during the interval between values  $AB$  and  $(A + a)(B + b)$ , and therefore the real velocity at a point between these limits which NEWTON was not wanting. We know, in fact, that it would have been the velocity when the sides are  $A + \frac{a}{2}$  and  $B + \frac{b}{2}$ .

Instead, therefore, of NEWTON rejecting a quantity on the ground of relative smallness, we find that HEGEL has gratuitously introduced such a quantity.

Of course, the Hegelian will reply to all this, that our method is "rendered impure by the concrete adjunct of motion." And here, of course, we can say nothing, except that the fluxional calculus is essentially kinematical, and that to construct it apart from motion is as likely a task as to make a geometry without lines. To make bricks without straw is a light task compared with that which HEGEL has set himself.

Happily unconscious of these difficulties, HEGEL goes on to moralise with much satisfaction upon NEWTON'S melancholy self-deception, in palming on himself such a proof.

After this specimen of HEGEL'S analytical subtilty, it is perhaps sufficient to confront the assertion which immediately follows (*Werke*, iii. 313; STIRLING, ii. 364), that NEWTON, in finding fluxions by the method of expansions, uses a process analogous to his method of solving approximately numerical equations, constantly "neglecting what is relatively unimportant," with the explicit words of the *De Quadratura* (Introd. § 5)—"Errores quam minimi in rebus mathematicis non sunt contemnendi." The terms omitted are, of course, always terms which we

know to become not relatively but absolutely zero in proceeding to the limit. The motive for using such expressions as “*minuatur quantitas o in infinitum*,” instead of simply saying, let  $o = \text{zero}$ , is merely to show that  $o$  becomes zero not by a discontinuous process, as subtraction, but by a continuous flow. Nay, cries HEGEL, for in the 3d Problem of Book ii. of the “*Principia*,” NEWTON fell into an error, by “throwing out, as LAGRANGE has shown, the very term which—for the problem in hand—was wanted. NEWTON had erred from adhering to the formal and superficial principle of omission from relative smallness.” This error, by the way, is only in the first edition of the “*Principia*,” which HEGEL, one may safely affirm, had never seen. The whole statement here is taken from LAGRANGE, and applies much better to LAGRANGE’S analytical way of putting NEWTON’S argument, than to that argument in its geometrical form.

NEWTON, in fact, investigating the law of resistance, that a body under gravity may describe a given path, seeks a geometrical expression for the *moment* of the sagitta—a small quantity of the third order. It is clear, therefore, that no such expression can be exact unless account is taken of every small quantity of an order not higher than the third in the geometrical construction involved, for such quantities will not vanish in the limit, or are not “relatively small,” in a mathematical sense. The principle of the problem, then, presents no difficulty on NEWTON’S method; and the true account of the error is, that by a mere slip in the details of a complicated process, NEWTON failed to see that he was omitting a term (or better, a line) *not* small relatively to the moment of the sagitta. HEGEL, however, conceives that so far as this goes NEWTON was all right. The error, according to him, lies in neglecting a term which, though “relatively small,” “possessed the qualitative value sought.” “In mechanic, a particular import is attached to the terms of the series in which the function of a motion is developed, so that the first term, or the first function, relates to the moment of velocity, the second to the accelerating force, and the third to the resistance of forces.” The terms are thus to be regarded as “qualitative moments of a whole of the notion;” and, of course, in a problem about resistances NEWTON needed the third term.—Now here we have, *firstly*, a laxness in the use of terms so gross, as to make it hardly possible to criticise our author fairly. Luckily, we can see that HEGEL is leaning entirely on LAGRANGE, and that “the series in which the function of a motion is developed,” must therefore mean the series which expresses space in ascending powers of time. And this enables us to ask, *secondly*, What reason HEGEL has for supposing that it is in this series that we are to find the basis for a truly philosophical view of kinetics? It was HEGEL’S misfortune to live at a time when, among other fruits of the “*Aufklärung*,” LAGRANGE’S “formal and superficial” method of treating physics was in great repute; and surely it was a cruel fate that the great enemy of the *Aufklärung* should, through a defective mathematical education, be made a willing captive to a mathematical

“Aufklärung,” which has, from its intrinsic weakness, fallen as fast as it rose. In details, it is true, HEGEL is keen enough in detecting the unsatisfactory character of LAGRANGE’S stand-point [see, for example, a note at this very point]; but that the whole method was artificial he could not see, not for want of mental power, but because, having never studied the subject, he knew nothing whatever about it—had not even mastered its technicalities. Then, again, if it is true that successive differential coefficients have a qualitative difference, how can that be brought out except in virtue of the relations established in mathematics between quantity and quality, relations which are not reached by pure analysis, but only in NEWTON’S way, *i.e.*, by intuition? And would not these relations be violated, and all mathematics rendered absurd, if the term that is qualitatively important *could* be quantitatively negligible? And, last of all, let me challenge HEGEL to bring forward any proof on his own principles, that the third term relates to the resistance of forces; or for that matter, to show that this statement has any real meaning whatever.

But most men, I imagine, have now had enough of HEGEL’S criticisms—criticisms which simply show that the “half hour” which he had devoted to the calculus had not sufficed to give him any just idea of that great method. It is certainly much to be regretted that so able a man did not study mathematics thoroughly, for such a course might have proved useful to the theory of mathematics, and could not have failed to be profitable to himself. As it is, he has only given us criticisms such as we have seen, and an attempt to which we now proceed to establish the calculus on a new and very inadequate basis.

The point which we have always found HEGEL urging is, that mathematical functions, when they become quantitatively indefinite or “infinite,” may still have a real qualitative value. Passing over the fact that this is not the technical sense of infinite in mathematics, we may grant that there is a kind of meaning, however vague, that may be attached to the view. Thus an incommensurable is infinite in the Hegelian sense, not because it can be expressed arithmetically only by an infinite series, but because it is essentially *not* a sum of units, but, as HEGEL vaguely says, a “relation.” For relation we should say *function*, and then we should be able to read in HEGEL’S words some meaning like this. Algebraic and geometrical functions are qualitatively different from mere arithmetical functions. They imply an entirely different way of looking at quantity, expressing, in fact, *steps* in time or space [or in kinematics, both in time and space]. So, again, the differential coefficient which takes the form  $\frac{0}{0}$  ceases to be intelligible on the mere arithmetical view, but gives us a real result of a different quality, when we understand it as equivalent to a proposition about the rates of the vanishing quantities. But then HEGEL does not seem to have seen that  $\frac{0}{0}$  has a real *quantitative* value, expressing accurately a definite quantity of a different

quality. And further, there was in HEGEL a rigid determination *not* to see the real qualitative difference between the continuous quantity of the higher analysis and of actual nature, and the discrete quantity of arithmetical abstraction.\* He thus fell into the delusion, that a writer like LAGRANGE who, from the extreme nominalistic stand-point of the eighteenth century, seeks to make analysis a merely formal instrument, in no way expressing the essence of things, and who, for example, boasts that in his *Mécanique Analytique* one will find no such unnecessary incumbrances as figures—HEGEL, I say, imagined that such a writer had really reached a higher generality than NEWTON, when he had only reached an untenable extremity of one-sided abstraction, and hence, without a moment's hesitation, resolved that by simply treating the successive differential coefficients as the successive derived functions obtained by expanding  $y$  in terms of  $x$ , we shall be quit “of the formal categories of the infinite, and of infinite approximation, and of the equally empty category of continuous magnitude” (iii. 320).

The differential calculus, then, is a special branch of mathematics which has to deal (by purely arithmetical methods) with qualitative forms of quantity, *i e.*, says HEGEL, with relations of powers. A power, it should be said, means with HEGEL a quantity raised to a higher power than the first, and the link between the clauses of the foregoing sentence is as follows:—“In the equation  $\frac{y}{x} = a$  the relation of  $y$  to  $x$  is an ordinary quantity, and  $\frac{y}{x}$  a common fraction, just like  $\frac{a}{b}$ , so that the function is only formally one of variable magnitudes. On the contrary, if  $\frac{y^2}{x} = p$ ,  $\frac{y}{x}$  has no determinate quotient, and, in fact,  $x$  has no ratio to  $y$ , but only to  $y^2$ . Now the relation of a magnitude to a power is not a quantum, but qualitative.” It is needless to say that the man who could make “no constant ratio” identical with “no ratio,” and who did not see that  $\sqrt{px}$  has a definite value for each value of  $x$ , or who did not see that  $p$  is a quantum, though not of the same dimensions as  $y^2$  (which probably was what confused HEGEL), is hardly fit to construct a new theory of the calculus. But let us pass on.

The subject matter of the calculus is then, we are to believe, equations in which one variable appears as a function of a second, one of these at least occurring in a power higher than the first. In such a case the variation of the variables is qualitatively determined, *and therefore* continuous. It would be vain to ask *why*; but since we are told that in the equation  $s = ct$  there is no scope for differentiation,  $\frac{s}{t}$  not being qualitative, we may at least conclude that HEGEL does not regard uniform motion as continuous!

So far as the principle goes it is quite sufficient, continues HEGEL, to consider

\* HEGEL absolutely identifies *analysis* with arithmetical process—“Auf analytische d. i. ganz arithmetische Weise” (iii. 328). Had HEGEL ever studied the treatment of incommensurables in ordinary algebra? If algebra is “ganz arithmetisch,” the whole doctrine of indices is false.

the equation  $x = y^n$ ; the advance to more complicated functions is quite mechanical. Now both  $y$  and  $x$  are really numbers, and so may be expressed as sums. [This, of course, is a very bold assumption, as HEGEL says nothing of the possible case of incommensurables.] The simple and yet comprehensive way of representing  $x$  as a sum is to write it as binomial. Now expand  $x^n$  as a binomial function, and we have a series of terms which are "wholly functions of the potentiation and the power." The differential calculus seeks the relation between these terms and the original components of  $x$ . As we are not concerned with the sum, but merely with the relation of the terms of the expansion, it would be best simply to expand  $(x + i)^n$ , and to define the particular "Potenzenbestimmung" got by considering the second term of this series as the first derived potence-function of  $y$ . In short, the true mathematical commencement in this part of analysis is no more than the discovery of the functions determined by the expansion of a power.

We see at once that this is simply an excessively clumsy adaptation of the method of LAGRANGE, which is based on the proposition that  $f(x + i)$  can always be expanded in a series of ascending integral powers of  $i$ , and then defines the successive fluxions [or derived functions] of  $fx$  with reference to the series. HEGEL adds to LAGRANGE nothing but confusion, and a degree of vagueness which is quite pitiable; and, of course, his method has the same fundamental fallacy as that of LAGRANGE, in so much as  $f(x + i)$  cannot always be expanded as LAGRANGE proposes, or what comes to the same thing, the details of the calculus cannot be deduced by processes purely arithmetical from the definition (for it is no more)  $\frac{dx^n}{dx} = n x^{n-1}$ . I do not, therefore, think it needful to go into details on this part of HEGEL's method. The really important point is the use to be made of these magical "Potenzenbestimmungen," which, according to HEGEL, depends on the discovery of concrete relations which can be referred to these abstract analytical forms. HEGEL proceeds as follows:—

There is always a fall of one dimension in passing to the first derived function. Hence the calculus is useful in cases where we have a similar fall in the powers. We are also to remember that, by differentiating an *equation*, we get not an equation but a *relation*. Whenever, then, we wish to investigate relations connected with any equation, but of a lower dimension, we have room for the calculus. A case in point is the investigation of the relations between the tangent, subtangent, and ordinate, for example, in a curve of the second degree. These relations are linear, while the equation contains squares. They depend, therefore, on the first derived function (pp. 341, 342, 344).

That such a statement is mere guess work is clear, if we observe that by a linear relation HEGEL means indifferently the ratio of two straight lines, or a ratio involving only first powers of  $x$  and  $y$ . Or, again, since the value of the radius of curvature is also on HEGEL's principles linear, why does

it involve the *second* derived function? Let us, however, follow our philosopher further. "Suppose we have  $2ax - x^2 = y^2$ , and take the derived function, we get a ratio  $a - x : y$ ,—a linear ratio representing the proportion of two lines. The real point is to show that these two lines are the ordinate and subtangent." This is very plausible, no doubt; but let us try a cubic equation, say  $2ax - x^2 = y^3$ . Now the resulting ratio, to put it in HEGEL'S way, is  $2(a - x) : 3y^2$ . Is this a linear ratio? Yet it still represents the ratio of the ordinate and subtangent. Clearly HEGEL does not know that when  $x$  and  $y$  become definite co-ordinates of a point on the curve the ratio  $\frac{y}{a-x}$  ceases to be a linear function of variables in any proper sense, and is simply a determinate fraction. This mistake augurs ill for the validity of HEGEL'S proof, that the two lines, whose ratio is the ratio of the derived functions, are really ordinate and subtangent. But he has LAGRANGE luckily to help him, who, he says, has entered on the truly scientific way. We get, therefore, a wordy and loose description, which would be utterly unintelligible to any one who did not know the thing before, of the way in which LAGRANGE proves that the line  $q = fx - xf'x + pf'x$  lies nearer to the curve  $y = fx$  in the neighbourhood of the point  $(x, y)$  than any other straight line through that point. HEGEL'S confusion is not diminished by the fact, that LAGRANGE deduces this proposition from a general theorem about the contact of curves, and originally writes the straight line as  $q = Fp$ . This piece of tactics so puzzles the philosopher that, after all his invective against the differentiation of linear functions, he allows LAGRANGE, without rebuke, to write  $f'x = F'x$ .

In other respects, however, we have great improvements on LAGRANGE. It is absurd to write  $q = a + bp$ \* as the equation of the line to be compared with the tangent,  $q = pb$  being quite general. That the line  $q = bp$  would not necessarily pass through the given point of the curve at all is, of course, a trifling consideration!

A still greater improvement regards the process by which LAGRANGE shows that we can always find a point (with abscissa  $x + i$ ), at which  $q = fx - xf'x + pf'x$  shall be nearer the curve than any other assigned straight line. At this point HEGEL begins to dread (not unjustly) that the conception of limit, or rather "das berüchtigte Increment," is to be employed. However "this apparently only relative smallness contains absolutely nothing empirical, *i.e.*, dependent on the quantum as such; it is qualitatively determined through the nature of the formula, when the difference of the moment on which the magnitude to be compared depends, is a difference of powers. Since this difference depends on  $i$  and  $i^2$ , and  $i$ , as a proper fraction, is necessarily greater than  $i^2$ , it is really not in place to say anything about taking  $i$  of any size we please, and any such statement is quite

\* HEGEL uses  $p = aq + b$ , but I keep LAGRANGE'S own letters throughout.

superfluous" (p. 347). One word in explanation of these. LAGRANGE takes an abscissa  $(x + i)$ , and gets

$$f(x+i) = fx + if'x + \frac{i^2}{2} f''(x+j),$$

and

$$F(x+i) = Fx + iF'x + \frac{i^2}{2} F''(x+j),$$

or for the straight line given above,

$$= fx + if'x.$$

Thus the difference of the ordinates of the curve and straight line with abscissa  $x + i$  is  $\frac{i^2}{2} f''(x+j)$ . For any other straight line the difference may be written  $mi$ .

Now, the ratio of these increments is  $i \frac{f''(x+j)}{m}$ , which may always be made less than unity by taking  $i < \frac{m}{f''(x+j)}$ . HEGEL, however, asserts that  $\frac{i^2}{2} f''(x+j) < mi$ , whenever  $i$  is a proper fraction, which is an obvious analytical absurdity, and, in fact, is equivalent to saying that it is impossible to draw a chord to a curve, the difference of the abscissæ of whose points of section is less than unity, since for the chord through  $(x, y)$  cutting the curve again at  $(x+i)$ ,  $mi = 0$ . In the face of this absurdity, it is scarcely necessary to add, that HEGEL having resolved to simplify matters, as we saw, by getting his derived functions from the expansion of  $(x+i)^n$ , has no right even to form for every curve the expansions on which LAGRANGE's proof depends.

I shall, in passing from the subject of geometry, merely enunciate a simple deduction from HEGEL's result in an intelligible form. "At any point of a curve there are an infinite number of tangents, which may be got by uniting that point with any other point on the curve whose abscissa is not different by a quantity greater than unity." I present this proposition, which is *entirely* due to HEGEL, and in the development of which my share has been "purely mechanical," for the admiration of all Hegelians whatsoever.

HEGEL's account of the application of the calculus to mechanic is much briefer, and presents less interest after what WHEWELL has written on a connected point. I cull only one or two illustrative points. For the purposes of the calculus, HEGEL classes motion as uniform, uniformly accelerated, and motion returning into itself, alternately uniformly accelerated and retarded. Variable acceleration, which in the form of harmonic motion is by far the most common in nature, is quite ignored.

Again, criticising the assertion that  $\frac{ds}{dt}$  represents the velocity at any point of a course, he tells us that it is "schiefe Metaphysik" to speak of the velocity at the end of a part of time. "This end must still be a part of time; if it were not,

there would be rest, and no motion; velocity can be measured only by the space passed through in a definite time" (p. 352).—An appeal to ATTWOOD'S machine would probably be too "empirical" for our philosopher, but the law of energy might surely convince HEGEL of the reality of a variable velocity dependent on potential energy lost or gained. It is clear, at least, that HEGEL lacked the first elements of physical notions, and these were not likely to be supplied by the method of LAGRANGE to which he adheres, beginning with  $s=ft$ , and deducing every other consideration by differentiation.

The following criticism on a remark of LAGRANGE is splendid:—"We find, says LAGRANGE, the motion represented by  $s=at^2$  in the actual fall of bodies. The next simplest motion would be  $s=ct^3$ , but nature shows no such motion, and we do not know what  $c$  could mean." [The ground of this is, of course, to be found in the law of the conservation of energy.] "If so, we have at least a motion whose equation is  $s^3=at^2$ ,—KEPLER'S law of planetary motion; and here the investigation of the first derived function  $\frac{2at}{3s^2}$ , &c., the direct treatment of this equation by differentiation, the development of the laws of that absolute movement from this starting point, must certainly be a most interesting task, in which analysis would appear in the brightest splendour" [!]. That  $t$  and  $s$  in KEPLER'S law are not variables, but constants determined for each planet; that the equation has no analogy whatsoever with the equation of motion; that its differentiation would be meaningless unless space were filled with planets; and that then it would have nothing to do with "the determinations of that absolute motion," are considerations that never entered HEGEL'S head.

It is rather hard that, from a metaphysical stand-point, a man is still allowed to write about things he has not studied; and more than this, that men so able as Dr STIRLING should be found imploring great mathematicians to come and read such utter nonsense as naturally results from the attempt. Certainly HEGEL'S fame is not likely to rise higher the more his notes on the calculus are studied; for these notes show quite clearly—*first*, substantial ignorance of the subject in hand, bolstered up by some hasty glances at the "literature of the subject;" *secondly*, great disingenuousness in criticising NEWTON, without having ever given his views a careful study; *thirdly*, almost incredible confusion of mind, in so far as he seems to have thought that he knew his own meaning when he really had no meaning at all; and *lastly*, to add nothing more, such a degree of self-complacent arrogance as led him to fancy the results of his "half-hour" more valuable than the fruit of the whole life of men like NEWTON.

This paper has already grown to such a length that it seems better to say nothing of HEGEL'S remarks on integration in the closing pages of his second note on the calculus, or of the third note, in which he treats "some other forms connected with the qualitative determination of quantity." The subject, in fact, has

a purely adventitious interest, and no one will care to linger longer over such a mass of confusion, both as to language and thought, than is absolutely necessary in self-defence. And the preceding pages may perhaps suffice to show that he who would exchange NEWTON'S clear ideas, based on nature's own showings, and alike removed from shallow empiricism and self-conceited dogmatism, for the vague pomposities of a HEGEL, exchanges

*χρύσεια χαλκείων, ἑκατόμβοι' ἑνεαβοίων.*



XIV.—*Observations on New Lichenicolous Micro-Fungi.* By W. LAUDER LINDSAY,  
M.D., F.L.S., &c. (Plates XXIII.—XXIV.)

(Read 19th April 1869.)

In the course of my studies on the Microscopic Anatomy of Lichens, during the last fifteen years, I have frequently met with various more or less minute *Parasites*—mostly black and punctiform or papillæform—sometimes disciform or maculæform—affecting either the thallus or apothecia of lichens, or both thallus and apothecia. They grow equally on foliaceous and crustaceous lichens, especially of the following genera:—*Parmelia*; *Physcia*; *Umbilicaria*; *Solorina*; *Peltidea*; *Nephromium*; *Sticta*; *Stereocaulon*; *Usnea*; *Neuropogon*; *Cladonia*; *Bæomyces*; *Squamaria*; *Placodium*; *Lecanora*; *Pertusaria*; *Thelotrema*; *Lecidea*; *Graphis*; *Endocarpon*; *Verrucaria*.

Most of these *Parasites* occur on lichens in my own Herbarium, collected by myself in 1856; or on lichens sent me for examination and determination by various British lichenologists between 1856 and 1858. They were examined, and described with figures in my Herbarium Note-books, between 1856 and 1859; in most cases without the assignation of *names*. I have not hitherto published their descriptions, or assigned names, for a variety of reasons, and especially on account of the difficulties which appear to me\* to surround the determination of what, to any single observer, seem to be (so-called) “new species.” Hence the parasites in question have been accumulating in my Herbarium, and their descriptions in my Note-books, for twelve years or upwards; and they now form a large and interesting, though obscure and puzzling, group of microscopic plants. I cannot, however, quote them in a Memoir I have in preparation on the Spermogonia and Pycnidia of the lower lichens (with which Spermogonia and Pycnidia the said parasites are frequently apt to be confounded) without placing their description—and, in certain cases at least, their names—on record in a form convenient for future reference. Nor can I otherwise contrast with them various groups of Lichenicolous Micro-Lichens,† Micro-Fungi, or Micro-Algæ, which have been described by other authors, or having been observed are yet to be described by myself. Inasmuch, moreover, as the said descriptions and names have not been, so far as I am aware, published by other authors—whether fungologists or

\* *Vide* Author’s “Contributions to New Zealand Botany,” 1868, p. 22; “Otago Lichens and Fungi,” Transactions of Royal Society of Edinburgh, vol. xxiv. p. 407; “Parasitic (lichenicolous) Micro-Lichens,” Quart. Jour. of Micro. Science, January 1869; “Polymorphism in Fructification of Lichens,” Quart. Jour. of Micro. Science, January 1868.

† *E.g.* “Enumeration of Micro-Lichens parasitic on other Lichens,” Quart. Jour. of Micro. Science, January 1869.

lichenologists—I can no longer hesitate in contributing to botanical science the following observations on the structure and place in classification of the Micro-Parasites referred to.

In the earlier years of my lichenological studies, I examined microscopically, with the greatest minuteness, large numbers of lichens for different collectors, from various parts of Britain and Ireland. I gave much more attention to the contents of other Herbaria than to those of my own; and to this circumstance, along with my reluctance to describe and name “new species”—a hesitancy to “rush into print” with accounts of mere novelties, real or supposed—I owe the fact that many of my own gatherings in different parts of the world—many of the original observations recorded in my MSS.—have been published to science, with the stamp and *éclat* of novelty, by other—mostly continental—lichenologists. This, however, I do not regard as subject for regret. Much more important than the mere discovery and nomenclature of so-called “new species”—only a small proportion of which has any claim to *permanent* rank as *species*—is, I think, the proper classification of existing material, so as to render additions to our knowledge capable at once of estimation at their proper value, and of absorption and assimilation in their proper place. So far as regards descriptive or systematic lichenology, my own aim has always been and still is to arrange on a *simple* plan of classification the data already accumulated, so that they may be readily accessible and intelligible to the student. My own studies in lichenology are and have been preferentially biological; regarding as I do questions affecting (*e.g.*) the physiology and anatomy, affinities and uses, of lichens as of higher interest than the *mere collection and nomenclature of varieties or species!*

The lichenicolous parasites above-mentioned are partly of the character of true *lichens*, partly of true *fungi*; while many partake of, or possess, the characters both of lichens and fungi, and can be appropriately referred only to the intermediate group of *fungo-lichens*.\* In the present communication I confine myself to the two last-named groups—to Parasites which are either true *fungi* or *fungo-lichens*. All of them require for proper examination the microscope, and most of them are distinguishable only under the lens. Very few, such as *Coniothecium* sometimes, are sufficiently large to be visible to the naked eye. All are rendered more conspicuous by moisture, which frequently converts punctiform into papillæform perithecia, and flat surfaces into convex ones.

In determining the *genera* under which to arrange the parasites hereinafter to be described, I have availed myself of the opinion, kindly accorded, of two of the most competent British Fungologists, who have at various times examined certain of the said parasites at my request, viz., Rev. M. J. BERKELEY, F.L.S., and FRED. CURREY, F.R.S. While agreeing with these distinguished fungologists

\* *Vide* Author's “Otago Lichens and Fungi,” p. 434, and *Arthonia melasperma*, Journal of Linnean Society (Botany), vol. ix. p. 269.

in many, I do not profess to agree with them in all, respects. When they claim parasites as indubitable *fungi*, I have no hesitation in accepting their determinations, as I have done (*e.g.*) in the case of various organisms referred by Mr CURREY to *Torula*, *Coniothecium*, and *Sphaeria*. While agreeing with Mr BERKELEY as to the close alliance between the fungi and lichens,\* I cannot subscribe to his views of the place in classification to be assigned to various members of the group of *fungo-lichens*.†

Many of the lichenicolous parasites hereinafter described belong to the *Torulacei*—to the genera *Torula* and *Coniothecium*. The majority of the remainder, which are confessedly most heterogeneous, I arrange provisionally under the genus *Microthelia*, adopting this genus only in the sense elsewhere and already explained.‡ The parasites in question are, in a manner at least, hereinafter systematically described in detail, their variations especially being made the subject of exposition. But *here* it is desirable to make certain preliminary *general* observations regarding the more prominent of their features

### 1. GENUS *Torula*. §

What I hereinafter describe as *T. lichenicola* varies considerably in its internal characters. In particular the spores are not always simple. Nevertheless all the forms described appear to me to be referable to a single type or species. Externally, *T. lichenicola* shows little diversity of form. It is black, punctiform, and superficial, resembling in this respect, and apt to be confounded with,

- (a). *Spermogonia* and } of many lichens, especially when intermixed
- (b). *Pycnidia* } therewith.
- (c). Many minute parasitic *lichens* belonging to such genera as *Verrucaria* and *Endococcus*.||
- (d). Many minute parasitic *fungo-lichens* belonging to the provisional genus *Microthelia*.

\* In various letters Mr BERKELEY has expressed himself as follows:—"So convinced am I of the near relation of lichens and fungi that in the portion of my 'Introduction to Cryptogamic Botany,' which is printed, I make one division, *Mycetales*, to include *Fungales* and *Lichenales*" (July 1856). . . . "One or two *Verrucaria* are so near *Sphaeria* that it is almost impossible to draw the line" (Dec. 1856). . . . "It is quite impossible to distinguish some *lichens* from *fungi*, and I consider the whole series as a division of *fungals*" (Feb. 1869). I hold quite as decided an opinion as to the *impossibility of distinguishing many lichens from many fungi*; or, in other words, of referring members of the group of *fungo-lichens* to the group of *fungi* rather than to the *lichens*! But I regard any classification, which arranges *lichens* as a co-division with *fungi* of a group of *fungals*, as imperfect, artificial, and arbitrary, excluding as it does the equally close alliance that subsists between lichens and *Algæ*.

† His views and my objections are fully given in a subsequent part of the present memoir (pp. 528-580).

‡ "Otago Lich. and Fungi," p. 436; *Arthonia melaspermella*, p. 279.

§ As determined by Mr CURREY, who wrote me in February 1866 as to "a curious species of *Torula*" (contained in my Herbarium) "which I do not recognise as having seen before. It is ramose, with bluish or greenish-black joints, the cells of which measure from 0.0003 to 0.0005 inch."

|| *E.g.* those described in KÖRBER'S "Parerga," p. 452, *et seq.*

- (e) Many minute parasitic *fungi*, belonging especially to the genus *Sphæria*.
- (f). Certain members of the pseudo-genus (of lichens) *Pyrenotheca*.
- (g). Granules of coal dust, or other inorganic foreign bodies.

From all of these the *Torula* is readily distinguished on microscopical examination, more especially by the presence of its peculiar spore-chains or filaments. There is no complete perithecium; but the basal cellular tissue, from which spring the spore-filaments, and which is generally sub-immersed in the host, is indistinguishable from that which constitutes the envelope or perithecium in many lichens, fungi,\* or fungo-lichens, and their spermogonia or pycnidia. The cellular tissue in question is most frequently of an indigo colour, or bluish or bluish-black, though sometimes also it is brownish; not varying, however, in colour to so great an extent as do the spores. The free surface, which is granular or powdery, consists of spores separated from their filaments, and of the apices of the spore-filaments, which are closely aggregated, just as are the sterigmata or basidia in lichen or fungus-spermogonia, and pycnidia. In the young state, these filaments are simple hyaline tubes, resembling the simple paraphyses of many lichens, broader or thicker at the distal or free end, tapering into a thread-like pedicle at the lower, basal or proximal extremity. Gradually, however, articulations appear, beginning first at the distal end; and colour is added, the filament increasing in volume. The filaments then resemble the articulated paraphyses of many lichens, *e.g.*, *Lecidea lenticularis*, Fr. Usually in maturity four or six articulations are formed, and gradually thrown off one after another from the distal end as free spores. Sometimes only one or two articulations are developed. The spore-filaments necessarily vary considerably in length, but less so in breadth. Some filaments appear to be abortive, and maintain throughout the simple or non-articulated, colourless character; not even increasing in length. These sterile filaments are intermixed with the fertile ones—just as sterile sterigmata—in this case generally elongated and ramose, and exceptionally articulated or pseudo-articulated, frequently accompany the fertile ones in the spermogonia of many lichens.† Occasionally there is atrophy of certain articulations, which then assume the character of threads connecting the normal spores. The distal half of the spore-filament is generally coloured, though the colour is sometimes faint or exceptionally absent. Where colour exists, it is always deepest at the distal or free end—in the terminal articulation. This colour is most frequently bluish (indigo or with a blackish shade); but sometimes it consists of various shades of olive or brown. The colour of the spore-filament is that also of the articulations of which it is composed. The colour of the spores is much more variable than their

\* *E.g.* *Dichæna rugosa*, Fr.

† *Vide* author's "Memoir on Spermogones and Pycnides," Trans. Royal Society of Edinburgh, vol. xxii, plates iv. v. vi. vii. viii. xi. xii.

dimensions or form. The proximal articulation is frequently colourless, while the terminal one is deeply coloured. In size the articulations always vary on the same filament—the oldest or terminal one being the largest—the younger being at least narrower in proportion to their youth. Thus the terminal spore is frequently twice as large as the proximal one, the increase in dimension relating to breadth rather than length. In maturity, and when free—thrown off from their filaments—there is much less difference in the form and size of the spores. In form they are generally oblong, with flattened ends, unless in the terminal spore, which has its free or upper end rounded, even while attached to the filament. Occasionally the corners of all the free articulations or spores are similarly rounded. Sometimes the spores are oval or ellipsoid. Their length is generally about  $\cdot 00025''$ . In structure they are usually simple, with or without double contour; sometimes, though rarely, granular; occasionally also having a central septum, or faint indications of the existence of one or more septa; more frequently containing one or more (two to three) spherical nuclei. Where there are two nuclei, they are generally arranged near the poles or extremities of the spores, to which they sometimes then give a sub-physcioid aspect, that which occupies the distal end of a spore being always the larger. This bi-nuclear character may attach to all the spores in a given specimen; and then, as well as in other cases, the spores in question resemble many lichen-sporidia. Sometimes chains or groups of spores of equal size and uniform character occur in numbers of four to eight, apparently the result of agglutination after maturity, and gaining the free state. In some cases the concatenate condition might be supposed to arise from simple absorption or disappearance of the pedicle of the filament; but in such a case the constituent articulations would probably retain, even in age, their differences in size. The site of *Torula lichenicola* is the thallus or apothecia (or both) of various lichens, mostly crustaceous, and belonging to the *Lecanora* or *Lecideæ*. It is much more common in *Lecanora subfusca* than in any other species in my own experience; and it is so markedly more common on *Irish* specimens of that *Lecanora*—mostly from the vicinity of Cork—as to give rise to the supposition that there may be some connection between the greater frequency of the parasite in Ireland and the (alleged) greater moistness of the climate of that country. There is, moreover, a frequent connection between the growth of the parasite and degeneration of the thallus or apothecia of the host; sometimes at least, obviously as productive of degeneration, *e.g.*, when the *Torula* overspreads the disk of *L. subfusca*, rendering it as black as that of *L. atra*. On the thallus of lichens it may be scattered generally over the surface; or only over particular parts thereof, *e.g.*, the periphery, or it may occupy only the areolæ or verrucæ. On the apothecia it may occur only on the disk, or both on the exciple and disk. The apothecia affected by the parasite are frequently degenerate or deformed; the disk has sometimes disappeared, and the whole apothecium has acquired an irregularly

verrucaform character. But in this case the degeneration appears to be quite unconnected with the growth of the parasite, which equally affects the thallus and apothecia, whether healthy or diseased, normal or deformed. The parasite may be scattered; or closely aggregated, becoming confluent and maculaform; or it may be copiously studded over the apothecia, and sparingly on the thallus of the same species, or *vice versa*, though the former arrangement is the more common. Generally there is a marked contrast of colour between the parasite and the whitish or greyish thallus, brownish or reddish disk of the apothecia, which it so frequently affects. Necessarily the *Torula* is most conspicuous by reason of this contrast, where the thallus and disk of the host are pale—whitish in the one case, and brownish in the other. Its structure is essentially the same on whatever lichen it be parasitic. In one case I found it occupying the cavity of spermogonia (in *Lecanora varia*).\*

## 2. GENUS *Coniothecium*.

There are various points of resemblance between *Coniothecium lichenicolum* and *Torula lichenicola*. In both cases the parasite is black, and is conspicuous from contrast of its colour to that of the pale (or whitish) thallus on which it so frequently occurs. In the *Coniothecium* the basal cellular tissue is the same. There is no complete perithecium; the granular or powdery surface consists of the free spores, which possess deep and dirty colours, mostly brown, though sometimes blackish or olive. In the young state only is the *Coniothecium* papillæform or verrucarioid, in which condition it may be confounded externally with *Torula*, or with the various organisms with which the *Torula* may itself be confounded. But there is a greater number of points of difference between these two common lichenicolous parasites. While *Torula* mostly affects *corticolous* lichens in the *fertile* state, *Coniothecium* affects only *saxicolous Lecanoræ* in the *sterile*, and frequently isidioid or other degenerate or hypertrophic, condition. In maturity, moreover, *Coniothecium* is much larger, and more conspicuous—visible for the most part to the naked eye. It is largish and flattish, discoid or lecioid, resembling some forms of the pseudo-genus (of lichens) *Spiloma*, as well as the parasitic Spilomatic fungi—*Spilomium Graphideorum*, and *Gassicurtia silacea*. It varies considerably in size, surface, and outline; in the old state frequently resembling soot-spots. It is apt to be confounded with the apothecia, especially when they are sub-degenerate, of various saxicolous *Lecideæ*; and the character of the spores is sometimes such as to assist in this confusion. These spores are typically, in the young state, spherical and single; but they gradually acquire a sub-cubical form, and are associated in groups—sometimes most irregular in outline—of two, three, or four, the form of the constituent spores then undergoing

\* *Vide* p. 520, and foot note †.

change from mutual pressure. Their colour is usually in maturity and age a deep blackish-brown; but in the young state they may be pale, or even colourless; while in older conditions they may be olive, with a blackish tinge. When aggregated in groups of two, if the form of the constituent spores remains comparatively regular, they may be indistinguishable from some figure-8-shaped lichen or fungus-sporidia. Hence, in one case, among the parasites hereinafter described (the Mangerton plant),\* it is difficult to determine whether it is *Coniothecium lichenicolum* or a separate parasite. Sometimes the spores are concatenate, as in *Torula*; but they are at once distinguished from those of *Torula* by their breadth being greater than their length, as well as by their sub-cubical form. When in aggregates of four, the spore-groups resemble wool-pack-like or *Sarcina*-like cubes. When in threes, as well as sometimes in twos or fours, they are very unlike spores, and are apt to be mistaken for fragments of cellular tissue, such as that which constitutes the perithecium of many of the lower lichens and fungi.† Whether simple or aggregate, the spores always exhibit double contour, presenting the aspect of being thick-walled. The *Coniothecium* may be scattered, as it generally is, or grouped; very rarely it is closely aggregated or even confluent. The thallus, which it affects, is frequently so altered—apart, however, from the growth of the parasite—that, in the absence of apothecia, it is impossible to determine the *species* to which it is referable. It appears generally, if not always, to belong to *Lecanora*, and, at least frequently, to the species *tartarea*, *parella*, and *glaucoma*.

### 3. GENUS *Microthelia*.

The parasites, which I have provisionally grouped in this pseudo-genus,‡ are confessedly most diverse in character, though they possess certain characters in common. For the reasons elsewhere set forth,§ I think there is an advantage in considering them *as a group* until their characters are more fully known and understood. Their common or general characters are the following:—

Most of them are microscopic, like *Torula*; black, papillæform or punctiform. The papillæform or verrucarioid condition is always rendered more distinct by moisture. Sometimes they are flattened and discoid, lecidiod or arthonioid (*e.g.*, the parasites on *Lichen dactylinus*, *Lecidea pachycarpa*, and *L. albo-atra*). Sometimes they are maculæform *ab initio* (*e.g.*, the parasites affecting *Squamaria crassa* and *S. saxicola*); at other times the maculæ are produced by the confluence or aggregation of minute papillæ (as in the parasite affecting *Parmelia perlata*).

\* (b) P. 540.

† I have seen true *Lichen-sporidia* by cohesion acquiring characters closely resembling those concatenate and woolpack-like forms of the spores of *Coniothecium lichenicolum* (pl. xxiii. fig. 28), *e.g.*, in *Lecidea dubia*, T. and B., Leight. Exs. No. 88. In several other cases, I have met with *Lichen-sporidia* cohering in such manner as to resemble cellular tissue, *e.g.*, in *Verrucaria subalbicans*, Leight. Exs. No. 200.

‡ *Vide* p. 515, and foot note †.

§ "Otago Lich. and Fungi," p. 436.

Their size varies in the same species, but mostly with age. As to site, they affect the thallus or apothecia, or both; sometimes the under as well as upper surface of the apothecia. They are, like *Torula*, more frequent on fertile than sterile lichens. As in *Torula* and *Coniothecium*, they are conspicuous where the colour of their host is pale. As in these parasites also, they may be few or numerous, scattered or aggregated, discrete or confluent; in the latter case becoming maculæform. Externally, they frequently resemble *Torula* and *Coniothecium*, or the organisms with which they are apt to be confounded; as well as certain young lichen-apothecia, belonging to the *Lecideæ*, e.g., *Abrothallus Smithii* and *oxysporus*. Generally only the base is immersed in the host, but sometimes the body of the perithecium is immersed, only the apex or ostiole projecting above, or being visible on, the surface of the host. The envelope or perithecium is in all cases the same, consisting of brown cellular tissue; frequently, if not generally, the cells being sub-hexagonal. The *Microtheliæ* are sometimes associated with, if not productive of, deformities or degenerations of the thallus or apothecia of the lichens on which they grow (e.g., *M. Stereocaulicola*, *M. Bæomycearia*, and the parasite which affects *Squamaria saxicola* as *Torula* does *Lecanora subfusca*).

Their internal structure varies considerably. Some of them are verrucarioid, in so far as they possess sporidiiferous asci, with or without paraphyses. Where paraphyses exist, they are either very delicate, filiform, more or less indistinct, without thickened or coloured tips; or they appear as a mere striated jelly. The asci are frequently saccate, as in *Arthonia*; short and broad, not tapering below into a pedicle.

In another group, no asci, paraphyses, sterigmata, nor basidia, were visible, so that it was impossible to determine whether the contained reproductive corpuscles are to be considered sporidia, spores, stylospores, or conidio-spores. Probably in the majority of cases they are really sporidia contained in asci.

In a third series, the perithecia are quite sterile, containing no reproductive structure. Some of these parasites may prove to be mere pycnidia analogous to *Phoma*, *Septoria*, *Diplodia*, and *Sphæropsis*. At least one of the parasites grouped under *Microthelia* possesses *pycnidia* in addition to sporidiiferous perithecia, viz., that affecting *Thelotrema lepadinum*.

In certain exceptional cases, the same perithecium contained not only sporidiiferous asci, but stylospores and basidia; and in one instance ramose filaments, resembling the hypertrophied sterigmata of many lichen-spermogonia\* (e.g. the parasites accompanying *Verrucaria epidermidis* v. *analepta*, and *Lecanora pyracea*). Parallel phenomena are the occurrence of sporidia and spermatia in the same perithecia in *Verrucaria atomaria* and *Sphæria Lindsayana*, as seen by myself, and in a certain section of the *Verrucariæ* as described

\* *Vide* p. 516, and foot note †.

by GIBELLI.\* This observation is one obviously of much interest in regard to the physiology of reproduction, and of the reproductive organs, in lichens. I doubt not that careful observation will yet multiply the number of instances in which *different forms of reproductive corpuscles exist in the same perithecium* or organ.

It may be provisionally convenient to classify the parasites grouped under *Microthelia*, according to the character of their contained reproductive corpuscles, as follows (omitting any specific mention of those whose reproductive structure is imperfect, and which are, therefore, for the present indeterminable):—

Sporidia, or reproductive corpuscles—

1. *Simple*. For the most part *spherical* and *brown*.

Microthelia atricola.		Parasites on (c.) Graphis scripta.
Spilonium Graphideorum.		(d.) Pertusaria.
Gassicurtia silacea.		Parasites accompanying
Parasites on		(a.) Opegrapha atra.
(a.) Lecidea rupestris.		(b.) Abrothallus Smithii.
(b.) L. sanguinaria v. affinis.		(c.) Pyrenotheca verrucosa.

2. *1-septate* (= bilocular).

A. *Brown*, in maturity; frequently or generally *soleiform*.

Microthelia Stereocaulicola.		Parasites on
rugulosaria.		(c.) Squamaria crassa.
Stictaria.		(d.) Endocarpon microsticticum.
parietinaria.		(e.) Usnea barbata v. florida.
Parasites on		(f.) Lecanora pyracea.
(a.) Lecidea pachycarpa.		Parasite accompanying
(b.) Thelotrema lepadinum.		Verrucaria fusiformis.

B. *Colourless* or *yellowish*; sometimes *simple*.

Microthelia Cookei.		Parasites on
Parasites on		(c.) Lecidea Hookeri; colourless.
(a.) Lecanora cenisia; brownish-yellow.		(d.) Verrucaria Garovaglii; colourless.
(b.) L. polytropa v. intricata; sometimes simple; colourless or yellowish.		Parasites accompanying
		Verrucaria epidermidis v. analepta; colourless.

\* So long ago as July 1856, Mr BERKELEY wrote me—"You are aware, probably, that in a species of *Tympanis*, Mr BROOME and myself have seen *on the same hymenium* the spores of a *Diplodia* and true *asci*. TULASNE doubts Mr BROOME and myself having seen *stylospores* on the *paraphyses* of a *lichen*. Nothing, however, was more clear and free from illusion. Almost in the same breath TULASNE calls in question the correctness of HOOKER and BABINGTON's observation. He should not do this. I sent TULASNE the very section we had seen the *stylospores* of the lichen in, but he could see nothing. Unfortunately, there were but two or three scattered apothecia on the roots of *Ammophila*, sent for a fungus by GARDINER. I have in vain tried to get more from the same locality." (*Vide* also my "Monogr. *Abrothallus*," p. 55; NYLANDER'S "Prodromus," p. 55; BERKELEY'S "Brit. Fungology," p. 27, plate i. fig. 13.)

In December of the same year Mr BERKELEY again wrote me (in regard probably to *Abrothallus Smithii*)—"In American specimens of your plant I sometimes find *asci*, sometimes *naked spores*, which have the same relation to the *asci* that the *stylospores* of *Diplodia* to the *asci* of the *Sphæria* to which they belong. Whether the production in question is a *lichen* or *fungus* is a knotty point. It grows on *living bark*, and therefore should be a *lichen*!"

3. 1 to 3-septate, mostly 3-septate and *brown*; sometime colourless.

Microthelia vesicularia; 1-3 septate. Umbilicariæ; 1- sometimes 2-septate. Nephromiaria; colourless. Bæomycearia.	Parasites on (a.) Physcia ciliaris. (b.) Lecidea lurida. Parasites accompanying (a.) Lecidea ferruginea v. sinapisperma. (b.) Verrucaria Taylora.
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Any such classification, however, is obviously artificial and defective; for not only in the same species, but in the same individual, the sporidia frequently vary much in character—in size, form, colour, and structure. In particular, they are frequently both simple and compound, colourless or coloured, of regular or irregular outline—according to the stage of growth. The “character” selected as the basis of classification must, therefore, be that which is presumed to prevail in maturity, and in normal conditions of growth. But what is prevalent or normal in one district or set of circumstances is not so in another, especially if the district or circumstances in or under which one systematist works are very diverse from those of others. There is, therefore, in such cases no precise or permanent basis of classification; whence it follows that the classification itself must be faulty. These remarks apply to too many modern “classifications” of lichens, based on the characters of the sporidia alone, or on any single “characters” or combination thereof!

In the group of parasites hereinafter described under *Microthelia*, the *Iodine-reaction*, which by fungologists is considered *lichenoid*, denoting the presence of *lichenine* in the lichen-tissues, is generally absent. In the parasite accompanying *Verrucaria epidermidis* v. *analepta*, however, the asci give a *blue* reaction with iodine; in *Microthelia Stictaria* they become deep *violet*; in *M. Umbilicariæ* the asci and hymenial gelatine assume various shades of violet; while in *M. Nephromiaria* the hymenial gelatine becomes violet. These exceptional reactions, however, neither prove nor disprove in themselves that the parasites, in which they occur, are *lichens* or *fungi*; for, as I have elsewhere\* sufficiently shown, this so-called lichenoid reaction occurs in indubitable *fungi*; while there are many true *lichens* destitute of any colour-reaction—indicative of the presence of starch, or its varieties or allies—with iodine.

*Iodine-reaction* is a subject of so great (supposed) importance in relation to the differential diagnosis between lichens and fungi: and as a “character” it bears so intimately on the place in classification to be assigned to the group of fungo-lichens, and to the members of the provisional genus *Microthelia*, as adopted or established by me, that it requires here some additional consideration. As a ground for regarding it as a diagnostic “character,” it appears to me necessary,

\* *Arthonia melaspermella*, p. 283; “Otago Lich. and Fungi,” p. 423; “Parasitic Micro-lichens.”

in the first place, that we should possess some trustworthy information as to the *chemistry* of the reaction, and as to the nature of those substances which, in lichens and fungi respectively, yield colour-reactions with iodine. As regards the lichens, I have carefully studied the most recent and approved standard works in chemistry; and the result is, that I find a discrepancy and confusion of assertion and opinion among chemists, nearly as great as that which exists between lichenologists and fungologists in regard to the application of the test, or of its colour-reaction, as a botanical "character."

So far as I have been able to ascertain, the substances occurring in lichens, which give colour-reactions with iodine, of the class which is now under review, are the following:—

I. *Starch* or its modifications.

A. *Lichenine*, Syn. Lichen starch, Lichenic acid. *Formulae*,  $C_6H_{10}O_5$ ,  $C_{12}H_{10}O_{10}$  (GREGORY).\* Has been examined by chemists as it occurs in *Cetraria Islandica* and *aculeata*, *Sticta pulmonaria*, *Ramalina fraxinea*, *Usnea barbata*, *Physcia parietina*.

*Chemical characters*.—Isomeric with starch. In *C. Islandica* does not occur in granules; but is uniformly distributed through the tissues in a soluble condition. *Pure* lichenine is merely coloured *yellow* by iodine; but a green or blue is often produced *from admixture of starch* (WATT).† A colourless jelly "sometimes assumes a *blue*, and sometimes a greenish tint," with iodine (GORUP BESANEZ).‡ Gives with iodine a greenish-brown colour (KANE).§ "Its solution is *not coloured* by iodine; but the jelly is rendered blue by that test" (GREGORY). Other authors describe the reaction with iodine as *blue*, and this is the reaction (generally) assumed by lichenologists as the basis of their iodine-testings.

B. *Inuline*, Syn. Dahline, Alantine, Menyanthine, Datiscline|| (GREGORY). *Formulae*,  $C_{12}H_{10}O_{10}$  (GREGORY),  $C_{24}H_{21}O_{21}$  (PARNELL). Has been examined as it occurs in *Cetraria Islandica* in association with lichenine.

*Chemical Characters*.—Also isomeric with starch. Occurs in white, crystalline grains. Sparingly soluble in cold, very soluble in hot, water. Iodine colours it slightly *brown* (GREGORY). Insoluble in alcohol. Not blue, but *yellow*, with iodine (MILLER).

C. *Starch*. *Formula*,  $C_{12}H_{10}O_{10}$ . Has been examined as it occurs in *Ramalina fastigiata* (in large quantity); and *Cladonia macilenta*, *digitata*, and *uncialis* (WATT). It does not quite clearly appear whether this is ordinary starch in its

\* "Handbook of Organic Chemistry," 4th edition (1856).

† "Dictionary of Chemistry," 5 vols. (1860-68).

‡ Quoted in "Chambers's Encyclopædia," 10 vols. (1860-68).

§ "Elements of Chemistry," 2d. ed. (1849).

|| MILLER ("Elements of Chemistry," 2d ed., 1862, p. 597) gives Datiscline ( $=C_{42}H_{22}O_{24}$ ), as the *colorific* principle of *Datisca cannabina*—not as a starch!

ordinary form. There *is* a substance in lichens that gives a beautiful and deep *blue* reaction with iodine; and if chemists are correct in asserting that such a reaction is indicative of the presence of free starch in its ordinary form, while lichenine and inuline give yellow or brown colours to the reagent, we must admit that *common starch* not only occurs in the lichen-tissues, but that it is sometimes associated with, and at other times substituted for, lichenine and inuline.

## II. *Gum*, or its modifications.

Gum has been examined as it exists in *Lecanora parella*. Gives a greenish-*blue* with iodine (SCHUNCK). Ordinary gum (= Arabin) is *not* altered in colour by iodine; but the modification thereof known as *Bassorin* gives *blue* and *red* reactions (MILLER, p. 109).

Were we to accept as a trustworthy basis for our conclusions the foregoing assertions of chemists, we would deduce that in lichens occur several forms or modifications of starch and gum that give reactions with iodine, variously blue, red, or brown, or admixtures of these shades, especially green. But it is impossible to accept as proper bases, on which to found *diagnostic* characters, statements so contradictory. The conclusion to be drawn is rather that chemists are yet ignorant in great measure of the composition and character of the mucilaginous and other components of lichens; and that at present they probably confound substances of somewhat dissimilar character. Thus the character of the iodine-reaction leads to the suspicion that what SCHUNCK describes as a *gum*, may be in reality a *starch*!

It by no means follows that the same reagent should produce the same colour-reaction in the same species of lichen, whether it is applied by the chemist in the laboratory to the separated amylaceous or mucilaginous principles, or by the lichenologist in his library to microscopical sections or preparations of the hymenium or other tissues. On the contrary, what we know of other colour-developments in lichens would lead us here to expect a certain difference in result; and, in point of fact, there *is* such a difference. And, further, differences of result in the same species, when iodine is applied as a test in microscopico-botanical diagnosis, arise in the hands of different experimenters from circumstances sometimes apparently most trivial, *e.g.*, the strength or character of the iodine solution, the age or other conditions of development of the specimen operated on. I need not, however, further pursue or illustrate the subject here, having pointed out in detail elsewhere the sources of fallacy and the causes of difference in the colour-reactions of lichens as supposed botanical characters.\*

The substances or tissues in lichens, which yield colour-reactions with iodine, are chiefly—(1.) The hymenial gelatine or mucilage, which has hitherto been

\* "On Chemical Reaction as a Specific Character in Lichens," Journ. of Linn. Soc. vol. xi. (Botany), p. 36; and "Experiments on Colour-reaction as a Specific Character in Lichens," Trans. Botanical Society of Edinburgh, vol. x.

generally assumed to be, but on insufficient grounds, lichenine; (2.) The asci; (3.) The sporidia; (4.) The medullary or other tissues of the thallus. Exceptionally colour-reaction may occur in other tissues. I have, for instance, met with a blue reaction from iodine in the interior structure of the *spermogonia* of *Abrothallus oxysporus* (Birnam, June 1856). The commonest iodine-reaction, that with which lichenologists have to do as a botanical character, is that of the *hymenial gelatine*,\* in which are imbedded the asci and paraphyses; and of the *asci* themselves, on which the reaction is generally the most intense. *Typically* this reaction is a beautiful *Prussian-blue*; that is to say, it has been generally regarded by continental lichenologists as what NYLANDER calls a “*Nota lichenosa*,”—a diagnostic “character” of *lichens* as contra-distinguished from *fungi*. But this blue is not always exhibited in different specimens of the same species, nor even in the same specimen at different times; it may, moreover, be faint† or fugacious. In a large number of lichens, instead of blue, the colour-reaction with iodine is *violet, red, brown, or yellow*; while in another large group there is *no colour-reaction!* Thus, in the genus *Verrucaria*, as defined by NYLANDER (in his “*Lich. Scand.*” p. 266), iodine develops in the hymenial gelatine of

- (a.) One section—the supposed typical or lichenic reaction.
- (b.) In another section—a wine red.
- (c.) In a third section—a faint bluish or reddish tinge.
- (d.) In a fourth section—no reaction.‡

To which it may be here added, that some *Verrucariæ* have no paraphyses, while in others they are distinct; but are always (where they exist) more or less graceful, delicate, and filiform. Further, different tissue-constituents of the same lichen, or different parts of the same organ, give frequently different colour-results with the same reagent.

These irregularities in colour-reaction may be conveniently and sufficiently illustrated by the following selection of quotations from the record of my microscopical examination of the lichens contained in the published Fasciculi of SCHÆRER (Switzerland), NYLANDER (France), and LEIGHTON (England). The advantage of using published Fasciculi is, that a standard of comparison is secured accessible

\* The term, “gelatine” or “mucilage,” is here used, and by lichenologists generally, in a popular, not in a strictly chemical, sense; for it has already been shown that the so-called “gelatine” may really be a form of starch or gum, or a mixture of forms of either or both! Compare *Arthonia melaspermella*, p. 283.

† It is faint or obscure in the following, and in many other, true lichens:—

*Collema turgidum*, Schær. Exs. 433. Asci.

*Stereocaulon condensatum*, Schær. Exs. 509. Asci.

*Calicium stigonellum*, Leight. Exs. 226. Asci.

*Lecidea Wahlenbergiana* v. *truncigena*, Ach., Leight. Exs. 123. Hymenium, mere trace.

‡ In his “*Prodromus*” he describes some species as possessing a yellow reaction, e.g. *V. xyliua* (p. 191).

to other lichenologists; and I doubt not that, if they take the trouble to make similar microscopico-botanical testings, they will meet with many marked instances in which their own results will differ both from mine and from those recorded by authors, like NYLANDER, who have given prominence in their works to chemical reaction as a botanical character.\* This arises from the circumstance, already mentioned, that the same species, under different circumstances, yields different colour-results with the same reagent. The following list could have been largely extended had I introduced quotations from similar records relating to the contents of my own Herbarium or of that at Kew. But such plenitude of illustration is, for present purposes at least, quite unnecessary. Omitting all instances of *blue*-reaction with iodine, whether distinct or faint, the other results of iodine-testing may be roughly classified as follows:—

### I. *No reaction.*

- Thelopsis rubella*, Nyl. Exs. 96. According to NYLANDER (Prod. 196), its hymenial gelatine becomes red.
- Nephromium cellulosum*, Ach. Hermite Island, Cape Horn. According to NYLANDER (Syn. 318), hymenial gelatine becomes blue.
- Urceolaria actinostoma*, Schær. Exs. 573.
- Strigula Babingtonii*, Leight. Exs. 35.
- Lecidea sphaeroides*, Smrf., accompanying *Opegrapha atra* in my copy of LEIGHTON'S Exs. 245. According to NYLANDER (Scand. 204), hymenial gelatine gives in different forms of the plant various shades of violet, or wine-red.
- L. foveolaris*, Schær. Exs. 293.
- L. Lightfootii*, Ach. v. *commutata*, Schær. Exs. 581. Apothecia here degenerate.
- Calicium turbinatum*, Schær. Exs. 6.
- Verrucaria chlorotica*, Ach., Nyl. Exs. 96.
- V. elæina*, Schær. Exs. 590.
- V. biformis*, Schær. Exs. 109 (= *V. chlorotica*, Ach.)
- V. levata*, Leight. Exs. 198.
- V. rupestris*, accompanying *V. pyrenophora* in my copy of LEIGHTON'S Exs. 245.
- Melaspilea arthonioides*, Nyl. Exs. According to NYLANDER (Prod. 159), hymenial gelatine becomes wine-red or bluish.

### II. *Reaction violet, red, or brown.*

- Lecidea luteola*, Leight. Exs. 150. Hymenium deep violet, with reddish tinge.
- L. Wahlenbergiana*, Ach., Leight. Exs. 123. Hymenium very faint purple.
- L. cupularis*, Ach., Leight. Exs. 122. Hymenial gelatine and asci deep brownish-red; hypothecium (only) blue.
- L. abietina*, Ach., Leight. Exs. 124. Same reactions as in *L. cupularis*.
- L. lurida*, Nyl. Exs. 131. Some tissues rose-red, others purple. Schær. Exs. 157, asci pale blue or wine-red; Hepp Exs. 121, asci wine-red.
- L. exilis*, Hepp Exs. 473. Asci violet.
- L. premnea*, Ach. (saxicolous), Leight. Exs. 185. Asci lilac or lake-coloured.
- L. atro-alba* v. *concentrica*, Leight. Exs. 17. Hymenial gelatine violet. According to NYLANDER (Scand. 233), it becomes deep blue.
- L. expansa*, Nyl., Leight. Exs. 186. Hymenium indistinct blue, with a lilac tinge.
- L. coarctata*, Leight. Exs. 177. Asci faint blue; contained protoplasm orange-red.

\* The majority of lichenological systematists give no attention to chemico-botanical characters, e.g., MASSALONGO, KÖRBER, TH. M. FRIES and MUDD; while NYLANDER, on the contrary, gives them decided prominence, e.g., in his "Lich. Scand."

- Lecanora atra* v. *verrucoso-areolata*, Schær. Exs. 538. Hymenium pale rose-coloured. Asci give no blue. According to NYLANDER (Scand. 192), the hymenial gelatine of *Verrucaria verrucoso-areolata* becomes yellowish-red.
- Physcia stellaris* v. *ambigua*, Schær. Exs. 351. Asci-tips pale brown.
- Squamaria crassa*, Ach., Leight. Exs. 5. Hymenium dirty palish blue or violet.
- Opegrapha dendritica*, Schær. Exs. 585. Hymenial gelatine pale lilac.
- O. vulgata*, Ach. v. *vulgata*, Leight. Exs. 194. Asci violet.
- Arthonia cinnabarina*, Wallr., accompanying *Opegrapha atra* in my copy of LEIGHTON'S Exs. 245. Some asci very pale purple.
- Verrucaria rimosicola*, Leight. Exs. 253. Hymenial gelatine violet; asci not blue.
- V. subalbicans*, Leight. Exs. 200. Hymenium violet or lilac; hypothecium pale blue.
- V. epigæa*, Ach. (apparently), accompanying *Squamaria saxicola*, in my copy of LEIGHTON'S Exs. 145. Hymenial gelatine palish purple or violet; asci not blue. According to NYLANDER (Scand. 276), hymenial gelatine becomes blue.
- V. gemmifera*, Tayl. Glenfarg, April 1858. Hymenium pale rose-red; no blue tinge.

On the other hand, I found a very marked *lichenic* reaction—a beautiful and more or less deep blue—with iodine in certain plants, generally regarded by fungologists as *fungi*; but now classed by lichenologists as lichens, on the sole ground apparently of this supposed diagnostic reaction. In *Xylographa parallela* v. *pallens*, Nyl. Exs., the asci and hymenial gelatine gave a beautiful blue. In *X. flexella*, Nyl. Exs., they gave a deep blue; and in *Agyrium rufum*, Nyl. Exs., the hymenium became blue.\* I have elsewhere † cited instances of what are still regarded, alike by lichenologists and fungologists, as *fungi*, giving so-called *lichenic* reactions with iodine. *Sphaeria ventosaria*, which Mr CURREY considers “a true *Sphaeria*,” gave me in its hymenium a violet or carmine with iodine; while in a plant lately submitted to Mr BERKELEY, and by him regarded as a “*Peziza* of the tribe *Patellea*,” the asci sometimes give a blue, sometimes no reaction with iodine.

This *Peziza* (which appears to be new, and for which, if it is so, I propose the specific name *lichenoides*), is associated with *Lecidea parasema* and *disciformis* on the bark of fir trees, Morchone, Braemar, collected by myself in August 1856. The apothecia are apparently sub-stipitate; this appearance being produced by the disintegration of the fibres of the bark on which they are seated. They vary greatly in form and size, being variously angular or oblong, or irregularly subspherical. The margins are involute to various degrees. They are always black; frequently wavy both in surface and outline, and generally thin. The paraphyses are very delicate, filiform, wavy, without coloured tips. The asci are long and sublinear or clavate, springing in groups or tufts from the hypothecium. The sporidia are innumerable in each ascus; atomic, subellipsoid or subspherical. The protoplasm, which is gradually developed into sporidia, closely occupies the cavity of the asci, separated only by a very narrow margin or double contour. Externally the *Peziza* has much of the character of a *Patellaria*, e.g., *P. atrata* (as described in

\* In BERKELEY'S “British Fungology” (1860) p. 375, both *Agyrium* and *Xylographa* figure among *fungi*, the latter having rank as a subgenus under *Stictis*. *A. rufum* and *X. parallela* are mentioned; but not *X. flexella* (unless it be as *Peziza flexella*, Fr., p. 371), which, however, appears associated with *X. parallela* in NYLANDER'S “Prodromus,” p. 148, as a *lichen*.

† “Parasitic Micro-lichens;” *Arthonia melaspermella*, p. 284.

my "Otago Lich. and Fungi," p. 427). It has also certain resemblances to a *Hysterium*, e.g., *H. pulicare*, P. The asci and sporidia are similar to those of what, now regarded as a lichen, was by FRIES the elder considered a *Peziza*, viz., *Lecidea (Peziza) resinæ*, Fr. (= *Biatorella*, Mudd, p. 191).\* From that *Lecidea* or *Peziza*, *P. lichenoides* differs only in the colour of its apothecium.

The asci of *Abrothallus Smithii*, which NYLANDER regards as a *fungus*, and BERKELEY † as a lichen, though they generally give no colour-reaction with iodine, ‡ have, in one instance at least, yielded in my hands the *lichenic blue*, less distinct, however, than in *A. oxysporus*. In the latter species, on the other hand, which is equally by fungologists and lichenologists admitted to be a lichen, while iodine generally develops a vivid and beautiful blue in the asci, this reaction is sometimes either obscure or absent; and the same remark applies to many true lichens, which generally exhibit the typical iodine-reaction.

*Sphæria (Stereocaulicola)*, Th. Fries, and *Leptosphæria (Lopadiicola)*, Th. Fries (Lich. Spitsb. p. 34), give in their hymenium a *yellow* iodine-reaction. But, according to the same careful observer, FRIES the younger (in his "Lich. Spitsbergenses," where he has recorded the iodine-reactions of most or many of the lichens therein described), the same reaction § is exhibited by the following true lichens:—

<i>Leptogium</i> lacerum and tenuissimum;	}	<i>Lecothecium</i> asperellum; protoplasm of asci.
<i>Collema</i> pulposum;		<i>Pannaria</i> arctophila; sporidia.
hymenium, except the sporidia;	}	<i>Lecidea</i> pezizoidea; sporidia and paraphyses.
<i>L. scotinum</i> ; sporidia.		<i>Endocarpon</i> pulvinatum; protoplasm of asci.
<i>Spherophoron</i> fragile; medullary tissue (of thallus.)		<i>Microglæna</i> sphinetrinoides; sporidia.
<i>Gyrophora</i> cylindrica; sporidia.		<i>Staurothele</i> clopima; sporidia and protoplasm of asci.
<i>Lecanora</i> flavida; sporidia and protoplasm of the asci.		<i>Thelidium</i> pyrenophorum; sporidia.
<i>L. mastrucata</i> ; hymenium.		<i>Verrucaria</i> extrema; do.
<i>L. calcarea</i> ; do.		<i>Arthopyrenia</i> conspurcans; protoplasm of asci.

Some of the parasites, which I have grouped meanwhile under *Microthelia*, were lately submitted to Mr BERKELEY, in the hope that he might claim a portion at least as fungi proper. But only one of them, what I have described as *M. Nephromiaria*, he refers to *Sphæria* and the *fungi*. The remainder he considers lichens belonging to the genera *Verrucaria*, *Celidium*, and *Abrothallus*. To *Verrucaria* he refers *M. Umbilicariæ* and *M. Bæomycearia*; to *Abrothallus* the parasite

\* Quoad the asci and sporidia it also resembles *Lecanora cervina*, Pers., *Lecidea morio*, Sch., *L. fossarum*, Duf. of NYLANDER'S Exs., and *L. pruinosa*, Sm.

† "I think," writes Mr BERKELEY, in Feb. 1869, "TULASNE is quite right in making *Phacopsis*, *Abrothallus*, *Celidium*, and *Scutula* lichens."

‡ Vide my "Monograph of *Abrothallus*," Quart. Journ. of Micro. Science, Jany. 1857. NYLANDER (Prod. 55) remarks, "Nullam enim mihi obtulit notam lichenosam." But what constitutes a "Nota lichenosa?" What *diagnostic* is there characteristic of a lichen as contradistinguished from a fungus? For my own part, I know of none!

§ I have never myself met with in lichens a *yellow* iodine-reaction, which I did or could not regard as the natural (unchanged) colour of the reagent itself.

which affects *Lecanora polytropa* v. *intricata*; to *Celidium*, *M. Stereocaulicola* and the parasites on *Squamaria crassa* and *S. saxicola*.

Some of the parasites formerly grouped by me under *Microthelia* and the *fungo-lichens* have been transferred to the *fungi* proper, e.g., *Sphaeria ventosaria*, by Mr CURREY. And it is most likely that other members of that provisional genus or group will from time to time be claimed as true *fungi*. But I find it impossible to perceive the validity of the claim, or the grounds of distinction. To me it appears that *Sphaeria* is quite as nondescript or heterogeneous a genus as *Microthelia*, and that it passes into *Verrucaria* by connecting links that defy differential definitions! In the "Treasury of Botany," Mr BERKELEY says that the only distinction between *Verrucaria* and *Sphaeria* consists in the presence of thalline *gonidia*; but, as I have elsewhere\* shown, no *gonidia* can be present in the large group of parasitic athalline lichens, in which the apothecia—with or without spermogonia or pycnidia—constitute the plant. In the same work he describes *Endothia* as distinguished from *Verrucaria* by its "naked spores." He also apparently regards it as consisting merely of the pycnidia or spermogonia of different lichens (Treasury, p. 1211); but he elsewhere describes "asci" (Brit. Fungology, p. 384)! I am utterly at a loss to reconcile or understand these diverse and puzzling statements. Moreover, if we may judge from NYLANDER'S description of *S. homostegia* (Prod. 56), which is flat and maculæform, the *Sphaeriæ* are not necessarily papillæform or verrucarioid. Some of them are thus arthonioid; and there are a few parasitic athalline *Arthoniæ* which have quite the facies of *Sphaeria homostegia*, e.g., *A. varians*, Dav. (NYL. Scand. 260).

Mr BERKELEY regards *M. Umbilicariæ* and *M. Bæomycearia* (in my specimens) as having a "distinct crust;" or, in other words, a proper thallus. This I quite fail to discover, on repeated examination; the perithecia appearing to me to be indubitably seated directly on an alien (lichen) thallus. No doubt, *M. Umbilicariæ* and *M. Bæomycearia* may be referred to the *Verrucariæ*; but only in the same sense in which the whole group of the *Microtheliæ* may be so transferred, constituting, with *Endococcus* or other pseudo-genera, an athalline (parasitic) section. We have already seen that the botanical "characters" of *Verrucaria* are in great measure negative; while there can be no doubt, as a genus, it is already much too (confusingly) large and heterogeneous.

None of my *Microtheliæ* have the essential characters of TULASNE'S genus *Celidium* (as defined in his Mémoire, p. 120, pl. xiv. figs. 9–13, or in my "Otago Lich. and Fungi," p. 448). According to TULASNE, the perithecia are aggregated so as to form maculæ, in the centre of which are seated spermogonia, the spermatia being linear and very slender. In the typical species *C. Stictarum*, Tul. (Mém. p. 122), the iodine-reaction is lichenic; the hymenium becoming bluish, reddish,

\* *Arthonia melaspermella*, p. 282.

brownish, or yellowish; but there is no such reaction in the *Microtheliæ*, which Mr BERKELEY refers to *Celidium*. Nor can I conceive any proper ground for assigning the parasite of *Lecanora polytropa* v. *intricata* to *Abrothallus*, which is itself a provisional and pseudo-genus.

Indeed, I am utterly at a loss to comprehend the principles of Mr BERKELEY'S classification in the parasites above referred to; and I am led to regard his opinion as another of the many illustrations that may be cited of the diversity between lichenologists and fungologists regarding the nature and affinities of a large and important group of parasites, which have been fully studied by neither class of observers, and are yet, therefore, most imperfectly known. And further, his (quite recent) opinion I accept, as strongly confirmatory of the propriety of establishing a provisional group of *Fungo-lichens*, and of resting contented with placing therein such doubtful organisms as the *Microtheliæ*, which I have hereinafter and elsewhere described,\* instead of engaging in fruitless and interminable discussion as to whether they are *fungi* or *lichens*.

When they become more generally studied and more thoroughly known, it may prove that some of the *Microtheliæ* in question are not parasitic or athalline, really possessing a proper thallus; or they may occur—as not a few true lichens do—both in the thalline and athalline state.

Only in certain cases, in describing the parasites, which form the subject of the present communication, have I ventured to assign *names*, viz., in the cases of those which may be considered typical or representative. In other cases—by reason of their resemblances to certain types or to each other, of the imperfections of their structure, or for other causes—I have deemed it preferable for the present not to assign names, either generic or specific, though all these doubtful parasites are grouped provisionally, for convenience in future study and reference, under the pseudo-genus *Microthelia*. I have little doubt that when the parasites in question, as well as the parasitic Micro-lichens and Micro-fungi that have been described by other authors, are more thoroughly studied and known, the *same* type or species will be held to include *several* of those which at present appear distinct. There will be not only a certain reduction, but abolition, of genera and species, of which there is at present a most confusing redundancy.†

### *Description of Illustrative Specimens.*

#### I. *Torula lichenicola*.

A. Parasitic on thallus or apothecia (or both) of *Lecanora subfusca*, Ach.

1. *Scotch* forms.

(a) Corticolous: Craig Choinich, Braemar, Aug. 1856, W. L. L.—Parasitic both on thallus and apothecia in one specimen of ordinary form of the *Lecanora*. The

\* "Otago Lich. and Fungi," pp. 436-442.

† An excellent illustration is to be found in the group of "Parasitic *Micro-lichens*," (*antea citat.*)

disk of the apothecium is, however, chiefly affected. Here the parasite is so abundant and so closely aggregated, that it renders the usually smooth disk quite black, and rough-granular or warted; the apothecia then resembling, especially in colour, those of *L. atra*. The spores are sometimes 1-septate or 3-nuclear, resembling many lichen-sporidia.

- (b) Corticolous: on birch bark, Corramulzie Linn, Braemar, Aug. 1856, W. L. L.—Parasitic on both thallus and apothecia. As in the last case, the disk of the apothecia is frequently quite black and roughened from the growth of the *Torula*.
- (c) Corticolous: Morchone, Braemar, Aug. 1856, W. L. L.—On thallus only.
- (d) Corticolous: on firs, base of Ben Lawers, on banks of Loch Tay, June 1856, W. L. L.—Equally on thallus and apothecia. Sometimes, as in cases *a* and *b*, the disk is blackened with the parasite, and resembles that of *L. atra*. Spores bluish.
- (e) Corticolous: on ash, roadside, Loch Tay, June 1856, W. L. L.—Copiously and generally studded over thallus; much more abundant than the spermogonia of the *Lecanora*; very distinct, black, punctiform bodies; spores generally 2-nuclear, brown.
- (f) Both corticolous and saxicolous: Kyles of Bute, Aug. 1852, W. L. L.—On thallus; spores indigo-blue; simple (no contained nuclei); narrow, and frequently longer than usual.
- (g) Corticolous: on firs and other trees, Caerlaverock road, Dumfries, Aug. 1856, W. L. L.—Scattered on thallus of var. *albella*, Pers. Spores large and more numerous than usual; frequently exhibit 2 polar nuclei; that which occupies the upper and broader end of each spore in the spore-chain being generally the larger; or there is only one nucleus at the superior or distal end of the spore.
- (h) Corticolous: near Dunglass, Cockburnspath, June 1856, Dr Murray Lindsay.—Variety of the *Lecanora*. *Torula* intermixed on thallus with spermogonia, which have not the usual characters of those of *L. subfusca*. Spores bluish. Terminal articulation of the spore-chain, as usual, darkest in colour, with a rounded apex. The other spores, when separated, have squarish or truncated ends, and an oblong form. Occasionally the mature spore contains one or two nuclei. Sometimes the spore-filament develops only one (terminal) articulation; at other times there is no articulation at all—the filament being sterile or abortive—then resembling the paraphysis of a lichen.
- (i) Corticolous: woods of Blackhall, Strichen, Aberdeenshire, July 1865, Layton.—Copiously scattered over the warts or areolæ of the subverrucose and areolate thallus, but sparingly studding the apothecia.
- (k) Corticolous: on alder; Pease Dean, Berwickshire, 1856, James Hardy.—Thallus sub-tartareous; many apothecia degenerate; disks eroded. Parasite abundant, both on disk and exciple of apothecia, and on thalline areolæ. Spores with polar nuclei, somewhat resembling certain physcioid sporidia in lichens.
- (l) Corticolous: Penmanshiel, Berwickshire, February 1857, Hardy.

## 2. Irish forms.

- (a) Corticolous: near Cork, March 1858, Isaac Carroll. Associated with *Physcia candelaria*, Ach.—Thallus subtartareous, made up of numerous closely-aggregated verrucæform areolæ; apothecia mostly degenerate; disk has disappeared, and the apothecia have assumed the appearance of irregular warts. Parasite is copiously studded over both thallus and apothecia, which have alike a black-punctate character. The *Torula* has its usual black, punctiform character. The basal cellular tissue is bluish, bluish-black, or brownish, resembling in this respect the varying colour of the spores. Each spore-filament usually develops four or five articulations, which are oblong and simple, broader above than below. The terminal ones, the largest, are about  $\cdot 00025''$  long, and  $\cdot 000111''$  broad.
- (b) Corticolous: on old beeches, Castle Bernard, Cork, Carroll. Associated with *Stigmatidium crassum*, Dub.—On both thallus and apothecia (disk and exciple

- alike). Parasite very minute, punctiform, black; basal cellular tissue deep indigo-blue. Spore-filaments about  $\cdot 001''$  long, and  $\cdot 000111''$  broad. Spores pale olive to bluish-black—about  $\cdot 00025''$  long, and  $\cdot 000111''$  broad.
- (c) Corticolous: Rathconnac, Co. Cork, Mar. 1858, Carroll.—On thalline areolæ and apothecia. Parasite resembles granules of a black powder dusted over thallus, its blackness contrasting strongly with the brown disk of the apothecium and white thallus of the *Lecanora*. Basal cellular tissue, like the spores, bluish-black or indigo colour. Spore-filament, before separation of the spores,  $\cdot 00066''$  long. It frequently throws off only two articulations from its tip; the lower or basal—that is, the longer—portion showing no division. Sometimes there are three articulations or spores; rarely more. The mature spores are  $\cdot 00025''$  long, and  $\cdot 00014''$  broad.
- (d) Corticolous: Great Island, Cork, Mar. 1858, Carroll.—On periphery of thallus. Spores pale olive or brown; oval, ellipsoid, or oblong;  $\cdot 00016''$  long, and  $\cdot 000090''$  broad.
- (e) Corticolous: Carrigaloe, Cork Harbour, Mar. 1858, Carroll. Associated with *Physcia pulverulenta*, Schreb.—Both on apothecia and thallus, on the latter somewhat inconspicuous; apothecia degenerate; margin eroded; disk black-punctate with the parasite, which is very numerous and crowded. The *Torula* is here larger than is usual. Basal cellular tissue indigo-blue. Spores brown;  $\cdot 00025''$  to  $\cdot 00033''$  long, and  $\cdot 00016''$  broad—usually simple. Associated with the sporiferous filaments are numerous sterile or non-articulated—probably abortive—colourless, very delicate and linear filaments, which resemble the paraphyses of many lichens. Sometimes they exhibit a faint appearance of septa. A parallel to these sterile filaments is to be found in the sterile hypertrophied sterigmata in the spermogonia of many lichens.\*
- (f) Corticolous: Upper Lakes, Killarney, Mar. 1858, Carroll.—Disk of apothecia destroyed, and the apothecia converted into an uniform dark purple, sterile, degenerate mass.
- (g) Corticolous: Ardrum, Carroll.—Basal cellular tissue deep indigo. Spore-filaments  $\cdot 00133''$  long,  $\cdot 000111''$  broad; spores  $\cdot 00025''$  long,  $\cdot 000111''$  broad; olive or bluish-brown; oblong, with rounded ends; simple, or frequently with two polar spherical nuclei; sometimes with double contour.
- (h) Kerry: Taylor in Herb. Mackay.—On apothecia; disks of which become sub-convex and deformed, and frequently as black as those of *L. atra*, from growth of the parasite.

### 3. *English* or other forms.

Betton, 1805, in Herb. Kew. All disks of the *Lecanora* entirely blackened by growth of the parasite, so that the lichen is apt to be mistaken for *L. atra*.

KÖRBER ("Parerga," p. 470) describes his *Pharacidia congesta* as very frequently parasitic on the apothecia of *L. subfusca* and *L. intumescens*, Rebert., in Germany. It possesses 8-spored asci; the sporidia clavate-oblong, sub-baccillar, 1-3-septate, hyaline. *Lecidea parasitica*, Flk., and *Arthonia varians*, Dav., are also occasionally parasitic either on the thallus or apothecia (or both) of *L. subfusca*.†

#### B. Parasitic on thallus of *Lecidea canescens*, Ach.

Corticolous: Aghada, Cork Harbour, Carroll.—On portions of thallus free of apothecia. Basal cellular tissue bluish-black. Spores pale indigo-blue or olive; terminal or larger ones  $\cdot 00041''$  long,  $\cdot 00016''$  broad; others  $\cdot 00025''$  long, and  $\cdot 00011''$  to  $\cdot 00016''$  broad.

#### C. Parasitic on thallus of *Lecidea parasema*, Ach.

On a specimen in Herb. Kew, from Ireland (sub nom. *Opegrapha gemmata*, Ach.) Apothecia of the *Lecidea* confluent and somewhat irregular in form and surface; colour bluish or greenish; sporidia normal. The parasite presents very numerous individuals, closely aggregated, occupying the site of, and otherwise externally resembling, spermogonia; very minute (microscopic), black, punctiform, superficial on, or more or less immersed in, thallus of the *Lecidea*. Spore-filaments about  $\cdot 0025''$  long, and  $\cdot 00016''$  broad,

\* *Vide* "Mem. Spermog." Plates IV. V. VI. VIII. XI. XII.

† *Vide* Paper on "Parasitic Micro-Lichens."

varying, however, both in length and breadth. Spores also variable in dimension; simple; oblong, with rounded ends when mature; colourless in the young state, gradually acquiring an olive tint with maturation; terminal articulation, as usual, deepest in colour. Spore-filaments consist frequently of six articulations—some of which, however, are sometimes atrophied, assuming the aspect of linear threads or ribbons connecting the normal spores.

D. Parasitic on the apothecia of *Lecidea ferruginea*, Huds.

(a) Corticolous: Craig Rossie, Dunning, Perthshire, April 1858, W. L. L.

(b) Corticolous: on silver fir: Ardrum, near Cork, Mar. 1858, Carroll.

E. Parasitic on thallus of *Lecidea anomala*, Ach.

(a) On ash and other trees, associated with *Lecanora subfusca* and *Pertusaria communis*; near Dunglass, Cockburnspath, Berwickshire, Dr Murray Lindsay, June 1856. The parasite here is associated with, and apt to be mistaken for, spermogonia. Terminal spores blackish; rounded at upper end; sometimes containing enclosed nuclei.

(b) Dunglass; thallus of the *Lecidea* white and granulate. *Torula* intermixed with spermogonia.

F. Parasitic on thallus of *Lecanora varia*, Ach.

Sub *Parmelia* in LEIGHTON'S Exs. No. 176; on fir bark, Twyford Churchyard, Shropshire.

Intermixed with spermogonia, with which the parasitic perithecia are apt to be confounded. In the same perithecia, moreover, the spore-filaments of the *Torula* are associated with the sterigmata of the *Lecanora*, taking here the place of the sterile hypertrophied sterigmata of lichen-spermogonia.\*

G. Closely associated with, but not parasitic on, various corticolous Lichens.

(a) On or with *Arthonia melaspermella*, Nyl.; corticolous; Weybridge, Surrey, Currey. In the "Journal of the Linnean Society" (Botany, vol. ix. pp. 271 and 286, tab. 6, figs. 2 c, 3 c, and 6), I have erroneously described and figured the parasite as the *Pycnides* of the *Arthonia*. Here the articulations of the spore-filament are bluish. It is impossible to determine whether or not there is a proper thallus of the *Arthonia*, on which the *Torula* occurs.

(b) With *Opegrapha atra*, Pers.; in SCHÆRER'S Exs. No. 634; right hand specimen in my copy. Spores brown; smaller than is usual.

(c) With *Verrucaria epidermidis*, Ach., Malham, Yorkshire, Oct. 1857, Dr Carrington. —Spore-filaments of two or three articulations only. Terminal spores .00025" long, very narrow and granular.

In the foregoing cases (B to G) the parasite has the same essential characters that it exhibits when frequenting its much commoner host—especially in Ireland—*Lecanora subfusca*.

It is probable that the *Torula*, which is described as destroying the apothecia of *Biatorina fraudans*, Helb., in Spitzbergen (TH. FRIES in "Lichenes Spitsbergenses," p. 35), as well as that referred to in NYLANDER'S "Synopsis" (p. 58), is *Torula lichenicola*. But of neither FRIES' nor NYLANDER'S plants have I seen specimens. *T. lichenicola* has a suspiciously close resemblance to what NYLANDER describes in his "Prodromus" (p. 86), as the *Pycnides* of *Sphaeria epicymatia*—a parasite, which, he says, is common on *Lecanora subfusca*, but which I have never found. In the same work (and same page) he mentions *Torula monilioides*, Bon.; but I have no means of knowing whether this is *T. monilioides*, Cd., or *Bispora monilioides*, Cd., of BERKELEY'S "British Fungology" (pp. 326 and 327); or *T. lichenicola*; or some plant—fungus or lichen—different from either.

\* Vide p. 532 and foot note.\*

II. *Coniothecium lichenicolum*.A. Parasitic on thallus of *Lecanora tartarea*, L.

- (a) Morchone, Braemar, Aug. 1856, W. L. L.—On isidioid, sterile states of the thallus of the *Lecanora*. Spores deep brown; sometimes spherical and simple; frequently 8-shaped (didymous); frequently also in moniliform chains, as in *Torula lichenicola*, or in groups of four or three.

B. Parasitic on thallus of *L. parella*, Ach.

- (a) Blackcairn Hill, near Newburgh, Fifeshire, May 1858, W. L. L.—On sterile forms of thallus of the *Lecanora*. Parasite more irregular in surface and outline, more crowded, and more frequently confluent than is usual. Occurs in groups on different parts of the thallus of the host. Spores sometimes 8-shaped (didymous), .00025" broad, .0005" long; more generally single or simple and spherical, about .00016" in diameter.
- (b) Morchone, Braemar, Aug. 1856, W. L. L.—On sterile states of thallus, sparingly scattered, sometimes only about periphery of thallus. Parasite has sometimes an apothecioid aspect, its black mass being girt with an obscure thalline ring. Occasionally it appears as if seated in thalline verrucæ, and is so small as to resemble spermogonia. At other times it resembles the smaller urceolate apothecia of *Lecanora cinerea*; and if the thallus of *L. parella* were more generally covered with the parasite, it might, at first sight, be confounded with a form of *L. cinerea*.
- (c) Glen Dee, Braemar, Aug. 1856, W. L. L.—On isidioid states of thallus. Parasite mostly large and flattish, with very ragged outline in the old state, resembling spots of soot; in young state, is regularly papillar. Has a close resemblance to *Spilomium Graphideorum*, Nyl., in size and irregularity of outline and surface.

C. Parasitic on *Isidium corallinum*, Ach.

- (a) Old Wall, Craigie Hill, Perth, May 1856, W. L. L.—The surface of the thallus shows little distinction between the constituent isidia, which are so closely aggregated as to form a general white subcretaceous mass, sometimes obscurely divided into areolæ. Parasite occupies generally the centre of these areolæ where they occur, or is studded generally over surface of thallus, as black, round, convex masses, varying in size. Spores in all cases show double contour; in age acquire an irregular or corrugate outline; colour generally brown, or blackish-brown, graduating into olive; sometimes very pale or almost colourless in young state.

It appears to be the same parasite which occurs on *Isidium corallinum* in MOUGEOT and NESTLER'S EXSIC. No. 74; which *Isidium* is there probably referable to *Lecanora parella*. The parasite resembles black apothecia, which have been doubtless by lichenologists of the pre-microscope era mistaken for the "fruit" of the *Isidium* or *Lecanora*.

The columns of *Isidium corallinum* have apices that are frequently coloured more darkly than the body of the isidia; which coloured apices were often mistaken by the earlier lichenologists for apothecia or "fruit" of different kinds, and which often have a close resemblance to some forms of spermogonia. The plant consists, when typically developed, of a series of minute, round, perpendicular columns, which become by close appression sometimes subangulose, and may even lose their individuality, coalescing into a general subcretaceous mass,  $\frac{1}{8}$ " to  $\frac{1}{4}$ " thick. Where such coalescence does not take place, there is seen on cross section, either natural or artificial—a honeycomb-like arrangement of columns—similar to the basaltic columns of Staffa or the Giant's Causeway on a microscopic scale. Where the apices are not cut off by natural cross section or erosion, they are discrete and papillæform, resembling, on a small scale, the young ramuscles of *Sphaerophoron*, as these are figured in my "Observations on New Zealand Lichens" (Pl. lxii. fig. 5).\* The colour of the isidia varies considerably; sometimes, especially in specimens long preserved in the

\* Transactions of Linnean Society of London, vol. xxv.

Herbarium, they are flesh-coloured; more generally they are grey. The colour is always deepest at the apex, gradually disappearing below, where the bodies of the columns coalesce into a chalky mass. The tips of the papillæ, which form the upper or free extremities of the columns, are generally brown; but this tint varies in depth or distinctness, being frequently very obscure.

*D.* Parasitic on thallus of *Lecanora atra*, Ach.

Saxicolous: Pentland Hills, Edinburgh, Aug. 1855, W. L. L.—No part of this parasite gives a blue reaction with iodine.

*E.* Parasitic on thallus of *Diplotomma calcareum*, Weis.

Clapham, Yorkshire, Dr Carrington, Oct. 1857.—What appears to be the *Coniothecium* is seated on the thalline areolæ, where they are at all distinct. It is black, generally round, sometimes irregular in form, *e.g.*, becoming sub-arthonioid; generally flat; seldom, and only in young state, verrucarioid or papillæform; sometimes confluent; superficial, the base only immersed. The parasite is sometimes indistinguishable from the apothecia of its host, save as to the inferior size of the *Coniothecium*; its size, however, is variable; it generally wants the thalline margin—which is, however, sometimes comparatively distinct, girding the apothecia. Here the *Coniothecium* exhibits no reproductive structure.

The same *Diplotomma* is in England the seat of the parasitic *Microthelia rimosicola*, Leight. (Mudd, "Brit. Lich.," p. 308, plate v. fig. 129), which has 8-spored asci, and oblong, 3-septate, brown sporidia. My note-book records, on the Yorkshire plant, the presence of *Pycnidia*, containing stylospores.

*F.* Parasitic on thallus of *Lecanora cinerea*, L.

Kerry, Taylor (in Fl. Hibern., sub-nom. *Spiloma sphaerale*).—Thallus of *Lecanora* sterile. The parasite is scattered about the periphery of the alien thallus, much more abundantly than the spermogonia of the *Lecanora* usually are. In the young state the *Coniothecium* is immersed, and then frequently resembles closely some forms of the spermogonia of *L. cinerea*; but gradually it becomes emergent and epithalline, resembling, according to its size and form—whether flattish or sub-globose—the black apothecial disks of a sessile *Calicium* or a *Lecidea*. It contains no reproductive structure, exhibiting under the microscope only its deep-brown, basal, cellular tissue.

*G.* Parasitic on sterile conditions (which are variously isidioid or variolarioid) of the thallus of several *Lecanora*.

It is in general impossible to determine in these cases what is the *species* of *Lecanora*. Sometimes only it appears to be *L. glaucoma*, *L. parella*, or *L. tartarea*. In no case is the isidioid condition so marked as to bring the thallus under the category (*C*) of *Isidium corallinum*.\*

(*a*) Scur-na-gillean, Skye, Aug. 1856, W. L. L.—Black papillæ of parasite very variable in size; generally very distinct. Spores of a sooty-brown colour.

(*b*) Saxicolous: Moors east of Reykjavik, Iceland, June 1860, W. L. L.—Here, again, no distinct reproductive structure is visible. The parasite is large, black, and conspicuous by contrast of colour on the whitish or grey thallus, having somewhat the characters of the apothecia of a *Lecidea* of the *parasema* or *contigua* group; much more irregular in form, however, and variable in size; consisting, moreover, apparently of aggregations, or glomeruli, of irregular papillæ; semi-immersed in the thalline areolæ, but projecting by an irregular, rough, granular surface above their level.

(*c*) Morchone, Braemar, Aug. 1856, W. L. L.—Abundant and in fine condition.

(*d*) North Wales, Rev. H. Davies (sub-nom. *Isidium microsticticum*, E. B. and Lich. Brit.) in Herb. Kew.—Parasite seated on, and partly in, small thalline papillæ; in which latter case it possesses a pale thalline border; small, black,

\* I have recently (August 1869) found *Coniothecium* frequent on the sterile saxicolous thalli of *Lecanora*, both in the northern and southern Highlands, *e.g.*, Helmsdale, Sutherlandshire, and St Mary's Loch district, Selkirkshire. In these districts the thalli in question are most probably referable to *Lecanora parella* or *glaucoma*, or both. On similar sterile thalli the parasite is common throughout Scotland, and probably throughout Britain.

lecidoid, convex, scabrous—studded over the thallus of the host. Spores brown; very irregular; concatenate, or Sarcinate, in groups of 4-woolpack-like masses, similar to those of *Sarcina ventriculi*, Goodsir.

(e) Kinnoull Hill, Perth, May 1856, W. L. L.—Associated with *Squamaria gelida*, L. Parasite occurs on an areolate, white, crustaceous thallus, sterile of apothecia, referable, doubtless, to some *Lecanora*. The *Coniothecium* has quite the aspect of some species of *Microthelia*, e.g., *M. rimosicola*, Leight. Spores chestnut-brown; have the appearance of portions of cellular tissue,\* composed of irregular, subcubical cellules,

(f) Barmouth, N. Wales, June 1836, Leighton (sub-nom. *Spiloma*).—Referred by Mr COOKE to "*Sporidesmium* sp." in my Herbarium.

H. Not parasitic on, though closely associated with, various saxicolous lichens.

Roadside between Sligachan and Portree, Skye, Aug. 1856, W. L. L.

Some at least of the parasites which I have referred to *Coniothecium lichenicolum* have apparently been mentioned, if not also described, by various of the earlier lichenologists, under most diverse names. Specimen (d) has the characters of *Isidium microsticticum* of "English Botany" and the "Lichenographia Britannica;" (b) has quite the appearance externally of *Spiloma nigrum*, Leight. and "English Botany," † and of *Spilomium Graphideorum*, Nyl. ‡ A plant from Clapham, Yorkshire, Dr Carrington, is apparently—partly at least—*Sclerococcum sphaerale*, Fr.; § and *Cyphelium* (or *Acolium*) *corallinum*, HEPP Exs. No. 531, and KÖRBER'S "Parerga," pp. 299 and 465. *Coniothecium lichenicolum* agrees also with *Variolaria conspurcata*, Engl. Bot., tab. 1993 (at least) with the characters of the plate.

*Coniothecium lichenicolum* has a close resemblance to some forms of *Sporidesmium*; and it is quite likely that there is a lichenicolous *Sporidesmium*, hitherto undescribed, for which the specific name *lichenicolum* would be appropriate. I have not, however, at present sufficient data for determining this. What appears to me to be *Con. lichenicolum*, occurring on the white, crustaceous, sterile thallus of a *Lecanora* from Barmouth, N. Wales, Leighton, June 1856—in my Herbarium—was (as already stated) labelled by Mr COOKE "*Sporidesmium* sp." Two of the lichens of the earlier lichenologists have been transferred by fungologists to the genus *Sporidesmium*, viz, 1. *Lepraria nigra*, Engl. Bot., t. 2409, of 1st edition, which is the *Coniothecium effusum*, Cd., and the *Sporidesmium*

\* *Vide* p. 519 and foot note.\*

† P. 45, tab. 1984, 2d ed., 1843.

‡ The specimen of *S. Graphideorum*, Nyl., contained in my copy of his "Herb. Lich. Paris," No. 72 (from Fontainebleau, on a white, mealy thallus—of some *Graphis*—coating a very rugged bark, and associated with a *Hysterium*), has the external characters, on a large scale, of a *Spiloma*. *Spiloma nigrum*, var. *variolosum*, Turn. & Borr. in LEIGHTON'S Exsic. No. 259, closely resembles it, though LEIGHTON'S plant is more crowded and more irregular in outline. The French *Spilomium* is quite visible to the naked eye; variable in size; very black; irregular in outline, though generally round; sometimes confluent; surface usually more or less convex and rough, as in *Coniothecium*, from the projecting powdery or granular spore-masses. The spores are spherical or oval; generally with double contour; simple; deep brown; about .00025" in diameter; sometimes slightly irregular in outline; cohering frequently in rouleaux like blood-corpuscles.

§ = *Spiloma sphaerale*, Ach., but not *Buellia saxatilis*, Schær. (NYLANDER, "Prod." p. 140), according to TH. FRIES ("L. Arct." p. 116).

*Lepraria*, Berk. and Broome, of BERKELEY'S "Brit. Fungology," p. 327; and 2. *Spiloma melanopum*, Engl. Bot. t. 2358 of 1st edition, which is the *Sporidesmium melanopum*, Berk. and Broome, of BERKELEY'S "Brit. Fungol." p. 327.

III. *Sphaeria ventosaria*, Linds. "Obs. on Otago Lichens and Fungi," p. 439;\* "Obs. on Greenland Lichens."†

Parasitic on the thallus of what appears to be *Lecidea grossa*, Ach., Ingleby, Cleveland, Yorkshire, Mudd; in my Herbarium. Occurs as minute, black papillæ, intermixed with the apothecia of the *Lecidea*. Externally the parasite has the character, as it has the position, of certain verrucarioid spermogonia, e.g., in the genus *Lecidea*; and it has also the facies of *Endococcus* or *Microthelia*. No distinct asci are visible; but the hymenial gelatine showed a beautiful *carmine or violet colour with iodine*, just as many true lichens do. Sporidia minute, pale brown, oblong, with rounded ends, with or without double contour, according to age; 1-septate—septum indistinct in young state. Mr CURREY regards it as "a true *Sphaeria*. . . . There are certainly thecæ filled with very numerous, brown, 1-septate sporidia, 0·0003". . . . The perithecia are so minute and so scattered, that it is extremely difficult to detach them for examination." He refers the parasite to *S. ventosaria*; but the perithecia are very different from those common in the same parasite on *Lecanora ventosa*.

#### IV. *Microthelia*.

1. *M. Cookei*. Parasitic on thallus of *Lecanora crenulata*, Dicks., Barrack wall, Chichester, W. C. Cooke, March 1866. The parasite occurs on the thicker, whiter portions of the thallus, as black, papillæform, scattered conceptacles. The hymenium gives no blue with iodine; its constituents are very indistinct, unless under the action of iodine or other colouring matter. Hypothecium is colourless. Paraphyses appear rather like a mass of striated jelly than as distinct, filiform threads—resembling in this respect those of *Pertusaria*. Apices not coloured. Asci polysporous, saccate, bulging irregularly, 0018" long, 00045" broad. Sporidia very small, 00022" long, 00009" broad; colourless; oblong-ellipsoid; 1-septate, sometimes simple.

*M. Cookei* externally resembles *M. rimosicola* of MUDD'S Exsic. No. 301; but the asci in the latter are 8-spored. It differs from all the *Microtheliæ* mentioned in MUDD'S "Manual of British Lichens," in having colourless sporidia; but in other points it resembles *M. pygmæa*, Körb., which is also polysporous.

2. *M. Stereocaulicola*. Parasitic on thallus of *Stereocaulon paschale*, L., Glen Derrie, Braemar, Croall, July 1854.

The free ends of some of the podetia of the *Stereocaulon* are the seat of bullose-looking deformities or expansions, which are compound solid verrucæ, made up of an aggregation of minute wartlets, of the same colour as the normal

\* Transactions of the Royal Society of Edinburgh, vol. xxiv.

† Transactions of the Linnean Society, vol. xxvii. (1869).

podetium and its squamules. These thalline deformities closely resemble those which are occupied by spermogonia in the same *Stereocaulon* and in *S. denudatum*, Flk., as they are figured in my "Memoir on Spermogones" (plate v. figs. 33 and 36). Moreover, the parasite closely resembles the spermogonia in question in its conceptacles forming brown or black verrucarioid papillæ; sometimes visible as mere points, the body immersed in the wartlets of the thalline deformities of the *Stereocaulon*—one or two in each wartlet—the apex alone projecting. Asci not blue with iodine, .0023" long, .00066" broad. No distinct paraphyses. Sporidia, .0005" long, .00033" broad; obovate; colourless when young, becoming olive or brown with age; 1-septate.

TH. FRIES\* has described, without naming, a *Sphæria* as parasitic on the thallus of *S. alpinum*, Laur., in Spitzbergen; whose sporidia, however, distinguish it from the *Microthelia*. The hymenium of the *Sphæria*—which may appropriately bear the specific name *Stereocaulicola*—becomes yellow with iodine. The sporidia are blackish, four in each ascus, oblong-elongate, 3-5-septate, becoming submuri-form by longitudinal division of the loculi. KÖRBER (in his "Parerga," p. 455), describes *Scutula Stereocaulorum*, Anzi, as parasitic on *Stereocaulon alpinum* and *S. fastigiatum* in the Alps of Northern Italy. But its sporidia are smallish, narrowly ellipsoid, subcymbiform, 1-septate, and colourless. Also his *Polycoccum Sauteri* ("Parerga," p. 470) as occurring on the protothallus and thallus of *S. condensatum*, Hffm. Sporidia small, dacryoid, 1-septate, brownish.

One or more species of *Sirosiphon* (an *alga* belonging to the family *Sirosiphoniaceæ*, of the *Palmellaceæ* or *Protophyta*, according to RABENHORST (Fl. Europ. Algar. Aq. dulcis, &c. p. 289), are parasitic on some species of *Stereocaulon*. Thus *Sirosiphon saxicola*, Næg. (perhaps the *S. crustacea*, Ag., of RABENHORST, *op. cit.* p. 289), is parasitic on *Stereocaulon denudatum* in Scandinavia (NYL. "Scand." p. 65); while a *Sirosiphon*, not named, and which may be also *S. saxicola*, grows on the podetia of *Stereocaulon vulcani*, Bor., and was mistaken for cephalodia by FRIES (NYL. "Lich. Exotic." p. 252). The parasite occurs as blackish pulvinuli, which are quite different in anatomical constitution from cephalodia; the latter always exhibiting on section the structure of—1. The cortical; 2. Gonimic; and 3. Medullary, tissues of the lichen-thallus.

3. *M. Umbilicariæ*. Parasitic on the sterile thallus of *Umbilicaria pustulata*, Hffm.; collected in Norway as commercial "Pustulatus moss." The parasite is copiously studded over both the bullæ and interspaces of the thallus, as black, papillæform conceptacles, varying somewhat in size and form; semi-immersed; comparatively conspicuous on the gray, cracked thallus of the host. Paraphyses very delicate and indistinct, as in *Verrucaria*. Asci very faint violet with iodine; while the hymenial gelatine shows more distinctly the same tint with the same

\* "Lichenes Spitsbergenses," p. 36. *Vide* also author's "Observations on Greenland Lichens."

reagent. Large quantity of oil globules intermixed with the hymenial constituents. Contents of young asci colourless, gradually assuming a brown tint as the protoplasm becomes distinctly partitioned into sporidia. Latter are, in maturity and when free, deep brown, generally 1- sometimes 2-septate; oval or broadly ellipsoid. It is possible the parasite on *U. pustulata* may prove referable to what KÖRBER describes in his "Parerga" (pp. 40 and 469) as *Tichothecium grossum*, which affects the thallus of *U. arctica*, Ach. The latter species of *Umbilicaria* is the seat also of *Dothidea lichenum*, Smrf., which TH. FRIES (Lich. Arct. p. 166) suggests may be KÖRBER'S *T. grossum*. I have not met with any description of SOMMERFELT'S plant. Its name, *D. lichenum*, is apt to be confounded with that of MASSALONGO'S *D. lichenicola* ("Recherche," p. 45, fig. 81), which affects the apothecia of *Pachyspora viridescens*, Mass.; has 1-septate sporidia, that, however, are colourless, elliptic-oblong, and slightly curved; and is apparently, therefore, a different plant.

4. *M. Nephromiaria*. Parasitic on thallus, and (back or under side of old) apothecia of *Nephromium cellulolum*, Ach.; Hermite Island, Cape Horn, Dr Hooker, Antarctic Expedition, 1839-43.

Parasite occurs as very minute, black, punctiform or papillæform, semi-immersed conceptacles, dotted over the thalline rugæ, or on the back of the old apothecia. It is sparingly scattered about the centre of the thallus, more plentifully on the thalline underside of the apothecia. The black apex is the only part that is superficial, the body being immersed. According to their size, and the form of the ostioles, the perithecia resemble those of many lichen-spermogonia, or of some of the smaller *Verrucariæ*. They vary in size, and are sometimes confluent, though generally scattered. The asci are sac-shaped like those of *Arthonia*, bulging broadly, not blue with iodine. Hymenial gelatine violet under iodine. Paraphyses indistinct, and as in *Verrucaria*. Sporidia 3-septate, colourless, fusiform; eight in each ascus.

The asci and sporidia agree with those of what I described in my "Memoir on Spermogones" (p. 135) as *Lecidea Alectoriæ*; and as the plants otherwise appear essentially the same, I merge the two in a single type, and abolish both the generic and specific\* designations as inappropriate. *M. Nephromiaria* also resembles externally *M. Cargilliana*;† but the simple, spherical, brown sporidia of the latter sufficiently distinguish it.

Also having 3-septate sporidia in sac-shaped asci is a parasite which covers copiously, with its punctiform perithecia, some of the laciniae of *Physcia ciliaris*, L., in SCHLÆRER'S Exs. No. 388 (sub-nom. *Parmelia ciliaris*: the lower of two specimens in my copy = ed. alt. immut., 1840). The body of the perithecium is

\* What was originally designated *Alectoria*, is now known as *Neuropogon, Taylori* (NYL. "Syn." p. 273).

† "Otago Lichens and Fungi," p. 441, pl xxx. figs. 31-34.

immersed in the thallus of the host, only the black apex being visible on the surface. The whole length of some laciniaë is dotted over with the parasite, while others are unaffected. My notes, made in 1857, do not record, unfortunately, the colour, size, or form of the sporidia.

5. *M. rugulosaria*. Parasitic on apothecia of *Placodium rugulosum*, Nyl. ("Chil." p. 193;) Tasmania, Stuart, in Herb. Kew; saxicolous.

The apothecia, which are of a deep orange-red, are abundantly studded over with the very minute, black, punctiform, or papillæform conceptacles of the parasite. The latter are prominent under the lens, especially when the apothecia are moistened, from the striking contrast of colour. They are semi-immersed in the epithecium of the *Placodium*. Paraphyses very delicate and filiform. Asci as long as the paraphyses; .0016" long, .0005" broad; 8-spored. Sporidia .0004" long, .00016" broad; 1-septate; soleaform;\* brown. Both asci and sporidia resemble those of the microspermous and microsporous forms of *Abrothallus Smithii*.

(a) Having similar sporidia—brown, soleaform, .0005" long, .00016" broad—is a similar parasite (which may prove referable to *M. rugulosaria*, or to the same type to which it may hereafter itself be referred) that affects the thallus of *Thelotrema lepadinum*, Ach.; on Holly, Ireland, Carroll. Its perithecia are black and sub-verrucarioid, resembling those of *Verrucaria fusiformis* or *epidermidis*. These sporidiiferous perithecia are accompanied by others containing stylospores, like some of those of *Lecidea abietina*; narrowly ellipsoid; .00033" long, .000111" broad; pale yellow. Both forms of perithecia are probably referable to the same parasite.

The thallus of *T. lepadinum* is also affected by *Nesolechia Nitschkii*, Körb. (Par. p. 462), which has minute, oblong-sub-bacillar, simple, hyaline sporidia; and by *Stenocbye eusporum*, Nyl.

(b) Another Irish specimen, sent me as a *Sphæria*, by CARROLL, in August 1856, from Mangerton, County Kerry, on a tartareous, white, much areolate, sterile thallus (which cannot be referred to its proper species), has figure-8-shaped, 1-septate, deep brown or olive sporidia or spores; .00033" long, and .00025" broad. The perithecia are largish and verrucarioid; very black; varying greatly in size and form; in the young state papillæform, in age flattened and lecioid; they are scattered on the thalline areolæ, and are very conspicuous on the whitish or cream-coloured thallus.

The plant has externally the facies of *Coniothecium lichenicolum*, to which it may really belong.† Mr COOKE, who examined it, describes the contained reproductive corpuscles, which may be either sporidia or spores, as "Toruloid spores."

\* Frequently erroneously written *soleaform*. *Soleaform* sporidia are necessarily 1-septate. *Vide* definition of the term in "Otago Lich. and Fungi," foot note, p. 447.

† *Vide* p. 519.

Both thallus (of the host) and disks of the parasite closely resemble those of Barmouth (North Wales) specimens of *C. lichenicolum*.

(c) *Lecanora cenisia*, Ach. (which = var. *atrynea*, Ach., of *Lecanora subfusca*, Ach. according to NYLANDER) from Ayton, Cleveland, Yorkshire, Mudd, in my Herbarium, bears on its apothecia a minute punctiform parasite, containing soleaform sporidia; here, however, of a pale, brownish-yellow hue.

(d) On thallus of *Lecidea pachycarpa*, Duf., Ireland, Admiral Jones, June 1858. Parasite is small, black, and Lecidioid, resembling certain lichen-spermogonia of the flat, discoid type. Hymenium gives no blue with iodine. Paraphyses filiform, indistinct, wavy, not coloured at tips; asci  $\cdot 0040''$  long,  $\cdot 00083''$  broad; sporidia colourless when young, gradually acquiring an olive or brown tint with age; soleaform;  $\cdot 00066''$  to  $\cdot 0010''$  long,  $\cdot 00033''$  broad.

It is impossible to confound the perithecia or sporidia of the parasite with the apothecia or sporidia of the *Lecidea*. The apothecia in question are very large and conspicuous; while the sporidia of the *Lecidea* are also very large— $\cdot 0040''$  long, and  $\cdot 00133''$  broad; 7-septate, colourless, oval-oblong. The asci of the *Lecidea* are, moreover, 1-spored,  $\cdot 0050''$  long,  $\cdot 0014''$  broad, becoming pale blue with iodine. Its paraphyses are indistinct; obscured about their irregular tips by much granular, greenish pigment-matter, and throughout the length of their bodies by the same colouring matter (in quantity), and by oil globules.

6. *M. Stictaria*. Parasitic on thallus of *Sticta Freycinetii*, Dél., Campbell's Island, Dr Hooker, Antarctic Expedition (sub-nom. *S. scrobiculata*). The conceptacles of the parasite are small, black, and superficial, easily detached. Asci 8-spored, small, subsaccate, *deep violet with iodine*. This is, at least, an unusual reaction if the plant is a *fungus*; while it does *not* appear to possess other characters of a *lichen*! Sporidia brown, soleaform (1-septate),\* resembling those of *M. rugulosaria*, but much smaller.

7. *M. parietinaria*. Parasitic on thallus of *Physcia parietina*, L., Cottishall, in Herb. Kew; on a single fragment of the *Physcia*. Parasite occurs as minute, black perithecia, variously punctiform or papillæform according very much to size; partly immersed; much crowded on the thallus of the host; variously resembling lichen-spermogonia (*e.g.* of some *Lecideæ*), or the smaller *Verrucariæ*. Asci not seen; sporidia brown, 1-septate,  $\cdot 0005''$  long,  $\cdot 00016''$  broad; soleaform as in *M. Stictaria* and *M. rugulosaria*; nearly of same size as those of latter, but larger considerably than those of former. *M. parietinaria* must not be confounded with *Phacopsis varia*, Tul. (Mém., p. 125, tab. 14, figs. 1-3; *Celidium*, Körb., Parerga, p. 456), which has 3-septate, ellipsoid, colourless sporidia, according to TULASNE; but oblong and becoming brown, according to KÖRBER.

8. *M. Bæomycearia*. Parasitic on sterile thallus of *Bæomyces rufus*, DC., Bal-

\* Frequently erroneously written *soleaform*. *Soleaform* sporidia are necessarily 1-septate. *Vide* definition of the term in "Otago Lich. and Fungi," foot note, p. 447.

thayock Woods, Perth, June 1856, W. L. L. Thallus consists of a series of minute, irregular pulvinuli, of a buff colour, on which the black perithecia of the parasite are conspicuous by contrast. This contrast is rendered greater, however, by the circumstance that the patches of thallus occupied by the parasite are lighter in colour than the rest of its surface. The parasitic perithecia are extremely minute and punctiform, so closely scattered as to give the thallus the appearance of being covered with granules of coal-dust. Sometimes they are so numerous and so closely aggregated as to become confluent in very irregular patches. Under moisture, the single perithecia assume a papillæform character. The *Microthelia* cannot be confounded with the young apothecia of the *Baeomyces*, which are brown, and much larger in almost all stages of growth. Sporidia of the *Microthelia* dark brown, oval; 3-septate; frequently or generally constricted at or opposite the septa. The thallus of the same *Baeomyces*, as well as that of *B. roseus*, Pers., is affected by *Nesolechia ericetorum*, Fw. (KÖRB., Parerga, p. 461), whose sporidia are minute, ellipsoid, sub-bacillar, simple, and hyaline. On *B. rufus* also occur *Lecidea parasitica*, Flk., *L. scabrosa*, Ach., *L. inquinans*, Tul., and *L. arenicola*, Nyl., as well as *Thelocarpon epithalinum*, Leight.\*

9. *M. atricola*. Parasitic on thallus of *Lecanora atra*, Ach., on red sandstone, Derriquin, County Kerry, Taylor in Herb. Moore, Dublin; associated with *Lecanora periclea*, Ach. (= var. of *L. sophodes*, Ach.). The parasite has the facies of a *Verrucaria* or *Endococcus*; its perithecia being minute, distinct, black cones, with sometimes a flattish or depressed apex; becoming occasionally irregular in form; seated *on*, scarcely *in*, the thallus of the host. Asci '0020" long, '00066" broad; crowded with innumerable sporidia. Sporidia spherical, simple, deep brown, about '000083" in diameter; resembling those of many *Calicia*.

*L. atra*, on the Continent, † is occasionally the seat of another parasitic fungus, *Gassicurtia silacea*, Fée (NYLANDER, Prod. p. 91; Lich. Parisienses, No. 150), which either affects the thallus or apothecia, sometimes occupying the place of the latter. The parasite consists of black filaments, forming in the aggregate brush-like masses, similar to the apothecium of *Sphærophoron* in some of its old stages of growth; it has a *Spilomatic* or glomeruliform facies. In the only authentic specimen I have examined (in NYLANDER'S Herb. Lich. Paris., No. 150; on stones in Forest of Fontainebleau), the thallus is sterile, consisting of a series of cushion-like areolæ, more or less scattered, seldom closely aggregated. Some of these white tartareous verrucæ are occupied by the parasite, which is very black, irregular in form, and easily distinguishable under the lens from the apothecia of the *Lecanora*; surface generally more or less subgranular. The spores are deep brown;

\* Vide Paper on "Parasitic Micro-Lichens" (*antea citat.*).

† And in New Zealand; LINDSAY, "Obs. on N. Z. Lichens," p. 540.

spherical, '00033" in diameter, generally with double contour—rough or granulate externally—resembling, in some respects, those of *Sphærophoron*; than which, however, they are much larger. Sometimes in age they become oval and unequally figure-8-shaped, as if in process of fission. What appear to be the nuclei of the spores also occur abundantly, as much smaller spherical corpuscles, '00016" to '00020" in diameter, pale yellow, gradually becoming olive and brown, exhibiting like the spores themselves double contour. NYLANDER describes (Prod. 91) its spores as black and spherical, and thinks the plant should be referred to the genus *Spilomium*\* (*Uredineæ*).

10. *M. vesicularia*. Parasitic on thallus of *Lecidea vesicularis*, Ach., Switzerland; in Herb. Kew. Occurs as small but distinct black papillæ, closely aggregated; superficial; scarcely immersed; externally resembling those of *M. pygmæa*. It resembles that species further in its asci being polysporous; but the sporidia are 1-3-septate, according to age; most usually the latter in maturity. The smaller ones, when 1-septate, resemble those of *M. pygmæa*. Hymenium gives no blue with iodine. Asci '0027" long, and '00083" broad. Sporidia '00033" to '00050" long, '00016" broad, but variable in size; fusiform or oval; brown.

Also having brown, minute sporidia, which are here, however, oval or ellipsoid, and are sometimes concatenate, is a parasite that affects the sterile thallus of what appears to be a *Pertusaria*, in Balthayock Woods, Perth, June 1856, W. L. L. No part of the hymenium gives blue reaction with iodine. The thallus of *Pertusaria communis* is the seat of a parasitic fungus, *Spilomium Pertusariicolum*, Nyl. (Enum. Génér. p. 91, and Synopsis, p. 144), which is conidiosporous, the spores being oblong and blackish. The same thallus is affected by *Lecidea parasitica*, Flk., *Sphinctrina turbinata*, Pers., and var. *microcephala*, Nyl., *Trachylia stigonella*, Ach., *Pseudographis elatina*, Ach., and *Opegrapha anomæa*, Nyl.†

My Herbarium contains a number of other lichenicolous parasites, having (more or less) characters resembling those of the *Microthelieæ* above described. But I cannot at present venture to assign names, or a specific place in classification, on account of the imperfections of their reproductive structure, the doubtful nature of their habitats, or other difficulties as regards their determination or description. The following are illustrations of this heterogeneous group of parasites:—

1. Associated with *Verrucaria epidermidis*, Ach., var. *analepta*, Ach.; banks of Crinan Canal, Argyleshire; on birch; Aug. 1856, W. L. L.—It has quite the thallus and aspect of a *Verrucaria* (e.g. *gemma*); and it is impossible to determine whether the thallus is that of *V. epidermidis* or other *Verrucaria*, or belongs to the plant now to be described. Intermixed with the apothecia of

\* A genus not mentioned in BERKELEY'S "British Fungology" (1860).

† Vide Paper on "Parasitic Micro-Lichens" (*antea citat.*).

*V. epidermidis* are certain other perithecia—scattered, round, minute, punctiform—somewhat prominent. Instead of paraphyses of ordinary character, the parasite possesses long, delicate, branching filaments, like those of many lichen-spermatogonia. Asci are frequently grouped and ribbon-shaped; giving a faint *blue reaction with iodine*; 8-spored. Sporidia ellipsoid, colourless, 1-septate, exactly like those of *Verrucaria gemmata*. The same hymenium, which contains sporiiferous asci and ramose paraphyses, contains also stylospores, oval or ellipsoid,  $\cdot 00066''$  long; borne on long filiform basidia, resembling paraphyses; sometimes 1-septate; occasionally exhibiting 3 nuclei, central largest.

2. Associated with *Physcia obscura*, Fr., var. *leprosa*, Hepp; Morchone, Braemar; corticolous; Aug. 1856, W. L. L. (Mem. Spermog. p. 247.)—Black and punctiform, but exhibiting no reproductive structure.

3. Associated with *Lecidea ferruginea*, Huds., var. *sinapisperma*, DC.; on dead mosses, grasses, twigs of shrubs, &c.; Hepp Exsic. No. 200 (sub-nom. *Placodium sinapispermum*, DC.)—Scattered over the decayed vegetation on which the apothecia of the *Lecidea* occur, and apparently partly intermixed with them, are very minute, black specks, which are perithecia, containing brown, 3-septate, ellipsoid, largish sporidia or spores.

4. On thallus of *Lecanora polytropa*, Ehrh., var. *intricata*, Schrad., Penmanshiel, Berwickshire; Hardy, Novem. 1856; saxicolous.—Parasite occurs on thalline areolæ as punctiform and black conceptacles, very minute, sometimes papillæ-form and Verrucarioid, varying in size; full of corpuscles, which may be either sporidia or stylospores (for neither asci nor basidia were observed), these reproductive corpuscles being very variable in size and shape—spherical to figure-8-shaped, simple to 1-septate, and colourless. The parasite is certainly not the *Thelidium epipolytropum* of MUDD (Brit. Lich. p. 298). I have also met with what appears to be the latter, externally resembling Verrucarioid spermatogonia, and containing ellipsoid, 1-septate sporidia, with pale yellow loculi, but having no distinct paraphyses; while MUDD describes the paraphyses as distinct in his plant.

5. On apothecia (disk) of *Physcia chrysophthalma*, L., var. *Dickieana*, Linds. (Nyl. and Mudd, Brit. Lich. p. 112; sub-nom. var. of *Physcia villosa*, Dub., in Linds. Mem. Spermog., plate xiii. fig. 14); Belfast, Prof. Dickie.—Parasite consists of small, round, brown, quite superficial papillæ or points, easily removable. Its envelope is composed of dark brown or bluish-brown cellular tissue, but the conceptacle contains no sporidia, stylospores, nor spermatia.

6. On thallus of *Lecidea albo-atra*, Fr.; shore of Great Island, Cork, Carroll, Sept. 1858.—Parasite is studded over areolate thallus as black papillæ, generally crowded; varying in size; frequently flattened and irregular in form; semi-immersed; sometimes confluent, and then very difform.

7. With *Verrucaria fusiformis*, Leight.; Douglas, near Cork; on ash; Carroll,

Mar. 1858.—Perithecia are black and punctiform, containing in great numbers corpuscles that are '00025" long, and '000066" broad; simple, or sometimes faintly 1-septate, brown, linear or ellipsoid-oblong, frequently somewhat constricted centrally. Neither asci nor basidia were visible, and the corpuscles above described may therefore be either sporidia or stylospores.

8. The horizontal squamules (and, to a less extent, the scales of the podetia from base to apex) of a specimen of *Cladonia bellidiflora*, Ach., collected on Kelly's Green, Ireland, by Dr MOORE, Aug. 1853, in Herb. Carroll, (Linds. Mem. Spermog. p. 163), bear, copiously scattered, a parasite, which has certain of the characters of NYLANDER'S *Lecidea Cladoniaria*\* (Enum. Génér., Suppl. p. 339). His description, however, is imperfect, e.g., as regards the sporidia, which, he hints, may sometimes be normally brown. In the Irish plant, the sporidia are eight in each ascus, arranged in one series; ellipsoid, simple, and colourless, '00033" long, '000111" broad; asci elongated, '00166" long, '00033" broad; paraphyses with discrete tips, but colourless, and not thickened. With apothecia, having externally the characters partly of those of *Abrothallus Smithii*, partly of *A. oxysporus*, are associated *Pycnidia*, containing stylospores precisely of the characters of the sporidia as respects size, form, colour, and structure, '00033" long, '00014" broad. Externally, however, these pycnidia are always brown. In my "Memoir on Spermogones and Pycnides," I have described them as *spermogones*; but their contained corpuscles have rather the characters of *stylospores*.† The apothecia have a convex surface in maturity; seldom sessile, and equally seldom altogether immersed; the body or bulk being generally immersed, and the surface nearly on the same level as the thallus of the host. They are discoid; black throughout; and their section resembles that of a double convex lens. In the young state they appear as minute, black papillæ, emergent from the thallus; in which condition they are apt to be confounded with the pycnidia.

In my "Memoir on Spermogones," I have mentioned this parasite under the head of NYLANDER'S *Lecidea Cladoniaria*; to which I have also provisionally referred a commoner parasite on *Cladonia uncialis*, Hoffm. (p. 285, plate vii. figs. 14–16). But the stylospores of the latter parasite are not the same as those of the parasite on *C. bellidiflora*; and, indeed, the two parasites seem distinct in several essential respects. Nor does NYLANDER mention either spermogonia or pycnidia as possessed by his plant. While, then, it is possible that one or other of the parasites in question is referable to NYLANDER'S plant, it is equally likely they are hitherto undescribed. Should this prove to be the case, I propose for that which affects

\* It may also be compared with his *Lecidea oxysporella* (Prod. 145), which grows on the thallus of *C. digitata* on the Splügen; and with *Lecidea Cetraricola*, Linds. ("Lichenicolous Micro-Lichens," Quart. Journal of Microscopical Science, Jan. 1869).

† I have pointed out the anatomical or morphological distinction between *stylospores* and *spermatia* in my paper on "Polymorphism in the Fructification of Lichens" (*antea citat.*).

*C. bellidiflora*—as an appropriate name—*Abrothallus Moorei*,\* in honour of the distinguished Director of the Botanic Garden of Glasnevin, Dublin, who has made so many important contributions to the Irish flora—cryptogamic as well as phænogamic.

The protothallus of various *Cladoniæ* is affected by *Nesolechia punctum*, Mass. (KÖRBER, *Parerga*, p. 461), the sporidia of which are minute, linear-fusiform, simple, and hyaline.

9. On thallus of *Squamaria crassa*, Huds.; Crosshaven, Cork Harbour, Sullivan. Parasite occurs as deep bluish-black round maculæ, surrounded frequently by a black ring; both conspicuous on the buff-coloured thallus of the host; seated on a sort of thalline papillæ; body immersed. Paraphyses Verrucarioid—very delicate, wavy, filiform, indistinct—not knobbed nor coloured at tips; asci '0028" long, and '00066" broad. Sporidia brown, soleiform, '00050" to '00066" long, '00025" broad; while in the asci always have the broadest and shortest end upwards.

This parasite is obviously different from the *Sphæria squamarioides* and *S. gelidaria* of MUDD. (*Brit. Lich.* p. 130), which affect the thallus of *Squamaria gelida*, L.

Nor does it appear to be any of these parasites which copiously affects the apothecia of *S. saxicola*, Poll., in a specimen which I collected near Jerkin, Norway (4600 feet), in August 1857. I have not in this case, however, been able to detect reproductive structure. In its young state the parasite appears as black spots on the epithecium. These gradually increase in number, and at length coalesce till they cover the whole disk; which covering leads apparently to the degeneration and consequent shrivelling of the whole apothecium. Its outline becomes most irregular; both exciple and disk acquire a very black granular surface, while the whole apothecium decreases in thickness. At a later stage it appears as a very black shapeless granular mass, frequently crowded or confluent, conspicuous on the pale stramineous thallus.

NYLANDER (*Scand.* p. 133), describes a parasitic *Sphæria* as affecting, in some parts of Scandinavia, *Squamaria saxicola* and *S. chrysoleuca*, Sm. It is black, punctiform, immersed in the thallus; spores fusiform, colourless; possessed of spermatogonia, which are also black and punctiform, containing minute straight spermatia.

KÖRBER (*Parerga*, p. 458) describes *Conida clemens*, Tul. (*Mém.* p. 124, sub *Phacopsis*) as parasitic on the apothecia of *Squamaria chrysoleuca* and *saxicola*; the sporidia being small, irregularly oblong, 1-septate, and hyaline. Also *Cercidospora Ulothii* (*Parerga*, p. 466), as affecting the thallus of *S. saxicola*; sporidia fusiform or cymbiform, 1-septate, and hyaline. *S. saxicola* is also affected with *Lecidea micraspis*, Smrf., and *Thelidium epipolytropum*, Mudd.†

\* In my MS. Notes on Moore's Irish Lichens—made in 1858—I named this parasite provisionally *Abroth. Cladoniarum*, but any such specific designation is apt to lead to confusion with NYLANDER'S *Lecidea Cladoniarum*.

† *Vide* Paper on "Parasitic Micro-Lichens" (*antea citat.*).

10. On or with *Lecanora varia*, Ach., var. *symmicta*, Ach.; on rotten stumps of *Pinus sylvestris*, Blaeberry Hill, Perth, April 1858, W. L. L. Conceptacles, externally resembling spermogonia, contain deep brown, oval or ellipsoid, apparently simple, sporidia,  $\cdot 0005''$  long,  $\cdot 00033''$  broad.

11. On *Lecidea rupestris*, Scop. (sub-nom. *Biatora rupestris*, var. *calva*, Dicks.), on limestone rocks, in HEPP'S Exsic. No. 134. Parasite consists of very small, black, punctiform perithecia, scattered among the apothecia of the *Lecidea*, externally resembling spermogonia, but containing very deep brown, oval sporidia; simple, or 1-septate, or both; the colour rendering it impossible to determine their structure. NYLANDER arranges *L. rupestris* as a variety under *Lecanora cerina*; a classification to which I cannot subscribe.

12. Accompanying *Pyrenotheca verrucosa*; on old oak, Castle Bernard, near Cork, Carroll. Parasite is seated on some of the thalline verrucæ, associated and apt to be confounded with spermogonia; sporidia spherical, brown,  $\cdot 00025''$  in diameter.

13. On thallus of *Endocarpon microsticticum*, Leight. (which appears to be only a var. of *Lecanora cervina*, Pers.; having quite the aspect of the common var. *smaragdula*, Whlnb.); Barmouth, North Wales, Leighton, 1856. Hymenium gives no blue with iodine. Asci sublinear,  $\cdot 00020''$  to  $\cdot 000233''$  long,  $\cdot 00033''$  to  $\cdot 0005''$  broad. Sporidia deep brown or olive according to age, 1-septate, oval,  $\cdot 0005''$  long,  $\cdot 00025''$  broad, arranged either in a single row, or in a double series, in each ascus.

*Endocarpon rufescens*, Ach. is the seat of *Sphæria Hookeri*, Nyl. (Prod. p. 139 and 175; LINDS. Otago Lich. and Fungi, p. 438), which has broadly fusiform, 3-septate sporidia (NYL. Prod. 139), becoming sometimes 5-septate or polyseptate and muriform (MUDD, Brit. Lich. p. 271, plate v. fig. 112), in all cases brown—sometimes constricted centrally or opposite each septum.

*Lecidea Endocarpicola*. On the thallus of *Endocarpon hepaticum*, Ach.—on walls, Lower Glanmire Road, Cork, Carroll—there is a parasitic *Lecidea* associated with the apothecia and spermogonia of the *Endocarpon*, having many of the characters of *L. aromatica*, Turn. (which, however, is not known to occur in the athalline condition). The paraphyses have deep brown or bluish-black apices, which are irregularly knobbed; their bodies constitute, however, a mere striated indistinct mass. Hymenial gelatine and asci become deep indigo-blue under iodine; latter are  $\cdot 00233''$  long, and  $\cdot 00050''$  broad. Sporidia very variable in size, length from  $\cdot 00033''$  to  $\cdot 00083''$ , and in breadth from  $\cdot 000083''$  to  $\cdot 000133''$ ; ellipsoid-oblong or linear-oblong; simple in young state, normally 3-septate in maturity.

14. On thallus of *Usnea barbata*, Fr., var. *florida*, L.; Rio Janeiro, Henry Paul, 1851. Parasite is seated on some of the thalline tubercles usually occupied by the spermogonia. Sporidia brown, 1-septate; very different from the simple, colour-

less, oval sporidia of the *Usnea*. No hymenial tissue, asci, nor paraphyses can be made out, even under iodine; which, however, produces in some places only a *blue reaction in the medullary tissue* of the *Usnea*. The structure of the spermogones of the *Usnea* is described and figured in my "Memoir on Spermogones" (p. 122, plate iv. figs. 2-5). This parasite has certain points of resemblance, especially as regards the sporidia, to *Phymatopsis dubia*, Linds., and *Abrothallus Usneæ*, Rabenh., as I have described and figured them in my "Otago Lichens and Fungi" (p. 442, *et seq.*, plate xxx. figs. 36-41).

15. On sterile thallus of *Parmelia perlata*, L.; India, M. C. Cooke, 1866; ordinary a-ciliate form of the *Parmelia* as it occurs, from Britain, Norway, New Zealand, and the Canary Islands, in my Herbarium. In what appears to be its highest stage of growth, the parasite occurs as sub-rotund or sub-diform, raised, sub-convex, black maculæ, with a granulate irregular surface. In the young state it is developed as punctiform or papillæform bodies, sometimes girt with a ragged thalline margin. In the young state the body is wholly immersed in the thallus of the host or nearly so, while in age it emerges and becomes epithalline. In none of its stages of growth does the parasite show sporidia or other reproductive structure.

16. Associated with *Verrucaria Taylori*, Carr., and *Opegrapha vulgata*, Ach.; corticolous; Dunscombes Wood, Cork, Carroll. Perithecia contain sporidia that are brown, 3-septate, bulging opposite each septum; .00083" long, and .00033" broad.

17. On thallus of *Lecidea Hookeri*, Schær. (sub-nom. *L. spherica*, Schær.) in his Exsic. No. 526. The thalline squamules are dotted over with small, black, prominent papillæ, externally resembling spermogonia, but containing sac-shaped asci, and 1-septate, colourless, ellipsoid sporidia, somewhat resembling those of some forms of *Verrucaria epidermidis*. The same thallus bears the parasitic *Sphæria Hookeri*, Nyl. (Prod. 175 and 139; LINDS. "Otago Lich. and Fungi," p. 438)\* with verrucarioid perithecia, and deep brown, 3-septate sporidia, .001"

\* Two specimens of *S. Hookeri* (sub-nom. *Verrucaria*), which I examined in the Kew Herbarium, had the following characters:—

1. Summit of Ben Lawers. Thallus Parmelioid, pale yellowish-white. Perithecia are quit those of a *Verrucaria*; seldom, however, forming regular cones or papillæ; more usually flattened and irregular as to form and size. None of the hymenial elements give blue with iodine. Sporidia broadly ellipsoid, tapering suddenly at the tips; 3-septate; becoming by longitudinal sub-division of the loculi sub-muriform; deep brown; .001" long, .0005" broad.

2. Gemmi, Switzerland. Perithecia much larger and ostioles more distinct; immersed or semi-immersed; bursting through the cortical layer of thallus, with—at least usually at first, in their young state—stellate fissuring. Thallus here again Parmelioid and simple; usually buff-coloured, sometimes pale green. The plant has an Endocarpoid facies.

In both cases the perithecia occur by themselves on a thallus, which appears to belong to them. It seems to me that it is the same plant that occurs sometimes with a proper thallus (*Verrucaria*), and at other times as an athalline parasite (*Sphæria*); that it has equal claims to rank as a *Sphæria* or *Verrucaria*; and that it matters little whether it is classed among the *Sphæriæ* or *Verrucariæ-fungi* or *lichens*—provided only fungologists and lichenologists would come to some common understanding regarding it!

long, and '0004" broad. The *Lecidea* itself (in my copy, original edition, 1847) has simple, ellipsoid, colourless sporidia; while those of the true *L. Hookeri*, according to NYLANDER (Prod. p. 139), are brown and 1-septate.

18. On thallus of *Lecidea sanguinaria*, Ach., var. *affinis*, Schær. (Exsic. No. 629, on left hand specimen in my copy, orig. ed. 1852). Intermixed with the spermogonia, and indistinguishable therefrom; but the parasitic perithecia contain round, brown spores.

19. On thalline scales of *Lecanora coarctata*, Ach., var. *involuta*, Tayl. (sub-nom. *L. involuta*, Tayl., Fl. Hibern., p. 134); Dunkerron, Taylor in Herb. Moore.; on grey sandstone; associated with *L. varia*. Parasite occurs as minute black cones, externally resembling spermogonia, but exhibiting no reproductive structure.

20. On thallus of an isidioid form of *Lecanora parella*, Ach. (sub-nom. *Lichen dactylinus*, Ach.), collected by Dr SCOTT, 1802; in Herb. Kew, where it was examined by DAWSON TURNER. Associated with spermogonia, and externally resembling them. Parasite is black and discoid; immersed, and bursting through cortical layer of thallus of host.

21. On thallus of *Verrucaria Garovaglii*, Mont. (sub-nom. *Thelotrema Schæreri*) Hepp, Exsic. No. 100; which I regard as a mere form of *V. pallida*, Ach. Parasite may be externally confounded either with the sporidiiferous perithecia or spermogonia of the *Verrucaria*; but its sporidia are oblong-ovoid, colourless, and 1-septate.

22. On thallus of *Graphis scripta*, Ach., var. *horizontalis*, Leight. Exsic. No. 244 (sub-nom. *G. serpentina*, var.); Abdon, Shropshire. Intermixed with apothecia and pycnidia, and externally resembling the latter; occurring here and there as minute black cones, full of minute brown spherical sporidia.

23. Associated with *Opegrapha atra*, Pers., var., and *Lecidea canescens*. Aghada; corticolous; Carroll. Possesses no distinct paraphyses. Sporidia simple, pale brown, '00066" long, '00033" broad—contained in asci.

24. Associated with *Lecanora pyracea*, Ach. (sub-nom. *Biatora rupestris*, var. *irrubata*) in Leighton's Exsic. No. 213, are conceptacles externally resembling pycnidia or spermogonia, which contain not only stylospores but sporidiiferous asci; in which, further, the stylospores and sporidia have the same characters. The perithecia occur as small, brown, punctiform bodies seated on the thalline scales of the *Lecanora*. Stylospores are oblong-ellipsoid—normally 1-septate; granular or occupied by two or more nuclear globules or cellules in the young and older states; borne on long, filiform basidia. Sporidia also 1-septate, and having otherwise precisely the characters of the stylospores; asci 8-spored. There is no possibility of confounding the internal structure of what appears to be a *fungus* with that of the ordinary spermogonia of the *Lecanora*, which possess arthrosterigmata, and very short rod-shaped spermatia. The existence in this fungus of stylospores and sporidia within the same perithecium—springing from the same

hymenium—is most interesting in both an anatomical and physiological point of view. I have met with the same phenomenon in several other fungi, *e.g.*, that described in this paper as associated with *Verrucaria epidermidis*. Parallel phenomena are the co-existence of sporidia and spermatia or stylospores in the same perithecia in *Sphaeria Lindsayana*, Curr. (LINDS. “Otago Lich. and Fungi,” p. 425, plate xxx. fig. 7); of sporidia and spermatia in *Verrucaria atomaria*, Ach. (LINDS. “On Polymorphism in the Fructification of Lichens,” in *Quart. Journal of Microscopical Science*, Jan. 1868); and of spermatia in the ordinary sporidiferous perithecia of *Verrucaria*, by GIBELLI (*Annals of Nat. History*, April 1866, p. 270).

25. Associated with *Lecidea lurida*, Ach., in *Herb. Kew*; “sea rocks near Bangor, July 1802.” Thallus exhibits a number of spermogonia scattered about the margins of its lobes as deep brown points, the body of the conceptacle being immersed in the thallus. Intermixed are the externally similar, but more conoid, perithecia of the parasite, whose hymenium gives no blue with iodine. Asci apparently polysporous; .0020" long, .00066" broad. Sporidia fusiform or ellipsoid, dark bottle-green or brown, irregularly 3-septate in maturity; .0005" to .00066" long, .00014" broad.

On the same sheet, and associated with *L. lurida*, are fastened specimens of what appears to be the same lichen, labelled “No. 19, on rocks by the sea, Miss Hutchins,” from Ireland doubtless. In both, the apothecia are distinctly Lecanorine in the young state, possessing a thalline margin, and thus differing altogether from the *Lecidea*. But their apothecia resemble those of *L. lurida* in the old state, when the disk becomes sub-convex, and the thalline border disappears, or is covered by the swollen disk. The disk in the Lecanorine apothecia is usually of a lighter red than in *L. lurida*. In Miss HUTCHINS' plant the thallus is much paler than in Bangor specimens. The colour of the thallus obviously varies, just as it does in *Physcia aquila*, with its degree of exposure to light; being palest when the plant grows in the shaded crevices of rocks. In the Irish plant the paraphyses are subdiscrete, with brown tips; the asci 8-spored, .00166" long, .0005" broad; the sporidia ellipsoid, colourless, apparently 1-septate, .00033" long, .00014" broad. Probably the lichen in both the Bangor and Irish specimens is *Lecidea sublurida*, Nyl. (MUDD. *Brit. Lich.* p. 172), which MUDD places in the genus *Thalloidima*, Mass.

26. Several parasitic fungi, or fungoid growths of the most diverse character, affect the apothecia of *Abrothallus Smithii*, Tul., or are associated with its pycnidia. But their apparent frequency in that lichen probably depends simply on the greater amount of attention I gave to the examination of the pseudo-genus *Abrothallus* while preparing my “Monograph” thereof\* in 1856.

\* *Quarterly Journal of Microscopical Science*, vol. v. 1857.

(a) Ben Lawers. Intermixed with the pycnidia of the *Abrothallus*, and externally indistinguishable therefrom, is a minute papillæform parasite, which consists of an envelope of dark-brown, hexagonal, cellular tissue, enclosing myriads of dark-brown spherical spores, which are frequently irregular or jagged in outline, like blood-corpuscles in a condition of shrivelling from exosmosis.

(b) Var. *Welwitzschii*, Tul.; Amulree Road, Dunkeld. Black perithecia—externally resembling pycnidia, with which they are associated—contain nothing but spherical oil globules, or corpuscles closely resembling them.

(c) Craigie Hill, Perth. As in (a) and (b), intermixed with pycnidia, and like them papillæform or punctiform. They contain—

1. Largish, spherical corpuscles, with pale brownish-yellow subgranular protoplasm, resembling the sporidia of certain lichens, *e.g.* some forms of *Lecanora cinerea*. Sometimes the protoplasm becomes distinctly circumscribed and separated from the cell-wall by a varying hyaline interspace. This protoplasm gradually acquires a nuclear character and a central position, and then divides into two or four (sometimes three) equal subspherical segments, after the manner of some of the larger forms of gonidia. In age, both cell-wall and outline of nucleus, or its segments, become irregular, as if from shrivelling.

2. Corpuscles resembling shrivelled sporidia; most irregular in form, colourless, generally with double contour, and containing one or more largish, distinct subspherical nuclei, and frequently also fine granular protoplasm. These corpuscles are often found attached to each other in groups of two or more.

3. Most irregular, ribbon-like tubules, marked by subspherical nuclei, which are sometimes of an iodine colour. In some cases these would appear to be mere chains of degenerate sporidia. Sometimes only two or three constitute the pseudo-tubule, whose septa (the walls of the sporidia) have disappeared. But at other times the outline of the sporidia remains; there is a pedicle formed by the base of the shrivelled ascus; the nuclei are polar and distinct, sometimes yellowish; or they are connected by a central canal, as in the sporidia of *Physcia parietina*.

A solitary black conceptacle, externally resembling an apothecium of the *Abrothallus*, picked off the bluish, curled squamules of the host (*Parmelia saxatilis*), consists of an envelope of dark-brown, honeycomb-like cellular tissue; rootlets being sent downwards into the tissues of the thallus of the *Parmelia*, penetrating through its cortical and gonidic layers to the medullary tissue. It contains—(1.) A parenchyma of colourless hexagonal cells, associated with mycelioid tubes—also hyaline, but short and thickish, and intermixed with much oily matter in the form of globules; (2.) Largish, spherical, colourless, sporoid corpuscles, full of a nucleiform, cellular, or granular protoplasm.

On one of the true apothecia of *A. Smithii*, from Craigie, I found a large, dark-brown, 3-septate sporidium, with bulgings opposite the loculi. Its size and form

are so different from those of the sporidia of the *Abrothallus* (as figured in my "Monograph," pl. iv.) that it is more probably referable to some *Sphæria* or fungus—not necessarily immediately associated with the *Abrothallus*, for in the course of microscopic studies on lichens, I have frequently met with *alien* sporidia—sporidia belonging to other and topographically distant lichens or fungi. Thus, in a specimen of var. *Welwitzschii* in LEIGHTON'S Exsic. No. 191, I found a number of dark-brown figure-8-shaped sporidia on and among the thalline rhizinæ of *Parmelia saxatilis*. They had not the usual soleaform character of the sporidia of *A. Smithii* and its varieties; nevertheless they probably belonged, in this case, to the parasitic *Abrothallus*.

(d) Var. *Welwitzschii* (LEIGHTON'S Exsic. No. 191). A specimen of the deformed thallus of *P. saxatilis*, without apothecia of the *Abrothallus*, bears bodies externally similar—like some degenerate forms of the apothecia of *A. Smithii*. Their envelope is of hexagonal cellular tissue, containing bodies like some forms of gonidia in process of segmentation—large spherical cells with delicate hyaline wall, enclosing centrally four bluish corpuscles, evidently resulting from segmentation into four of a central spherical nucleus.

(e) A specimen of *A. Smithii* (from Glen Dee, Braemar; on boulders, August 1856, W. L. L.), bears a Sphærioid parasite on its *apothecia*. In another specimen of the same *Abrothallus*, from Glenbeg, between Spittal of Glenshee and Braemar (on a roadside wall), August 1856, a similar parasite, occurring on the thallus of *P. saxatilis*, resembles externally the apothecia of the *Abrothallus*, and is apt to be confounded therewith. It contains a mass of minute globular brown spores, intermixed with a few partially disintegrated sporidia of the *Abrothallus*.

(f) Associated with *A. oxysporus*, Tul., and with the pycnidia of *A. Smithii* (on an old wall, top of Craig-y-Barns, Dunkeld, June 1856, W. L. L.), on thallus of *P. saxatilis*. Parasite punctiform, black, containing masses of hyaline Toruloid spore-filaments, with myriads of very minute, also colourless, globular cells, generally aggregated in irregular masses; associated with a few sporidia, partially degenerate, both of *A. oxysporus* and *A. Smithii*.

Those parasitic fungi, accompanying *Abrothallus Smithii*, that are Verrucarioid externally, are apt to be confounded not only with the *pycnidia* of the *Abrothallus*, but with young states of the *apothecia* of both *A. Smithii* and *A. oxysporus*.

27. In my copy of SCHÆRER'S Exsic. No 503, *Calicium disseminatum*, Fr. *patellæforme*, Sch., has not the sporidia of the *Calicium*, which are, according to NYLANDER (Syn. 146), blackish, oblong, and sometimes subspherical; but ellipsoid, colourless ones, .00033" long, 00013" broad, contained in asci .00133" long, .00033" broad. There are no distinct paraphyses, and the plant may be regarded as either *Verrucaria* or *Microthelia*—lichen or fungus.

## EXPLANATION OF PLATES XXIII., XXIV.

## PLATE XXIII.

Figs. 1 to 18. *Torula lichenicola*.1 to 12. On *Lecanora subfusca*.

1. Craig Choinich, Braemar.

(a) Portion of thallus, with apothecia, of the *Lecanora*.

(b) Sections of said thallus and apothecia—magnified.

(c) One of its apothecia—magnified.

(d) Spore-filaments and spores of *Torula*.\*2. Ben Lawers. Spore-filaments and spores of *Torula*.

3. Loch Tay. Do.

4. Kyles of Bute. Do.

5. Caerlaverock road, Dumfries. Spores.

6. Dunglass, Berwickshire. Spore-filaments and spores.

7. Pease Dean, Berwickshire. Spores.

8. Near Cork.

(a) Portion of thallus of the *Lecanora* with apothecia.

(b) Section of said thallus and apothecia.

(c) Spore-filaments and spores of *Torula*.

9. Castle Bernard, Co. Cork. Spore-filaments and spores.

10. Rathconnac, Co. Cork.

(a) Portion of thallus, with apothecia, of the *Lecanora*.

(b) Section of said thallus and an apothecium.

11. Great Island, Cork.

(a) Portion of thallus of the *Lecanora* with apothecia.

(b) Spores.

12. Carrigaloe, Cork.

(a) Portion of thallus of *Lecanora*.

(b) Section of do.

13. On *Lecideia canescens*. Aghada, Cork.(a) Portion of thallus of the *Lecideia* with apothecia.

(b c) Sections of said thallus and an apothecium.

14. On *Lecideia parasema*. Ireland. Spore-filaments and spores of the *Torula*.15. Accompanying *Opegrapha atra*, Schærer's Exsic. No. 634. Spore-filaments and spores.16. Accompanying *Verrucaria epidermidis*, var.; Malham, Yorkshire. Spore-filaments and spores.17. In spermogonia of *Lecanora varia*, Leighton's Exsic. No. 176.

(a) Portion of thalline scales bearing spermogonia.

(b) Section of said spermogonia.

(c) Normal spermatia and sterigmata of the *Lecanora*.(d) Associated *Torula*.

(e) Spore-filaments and spores of do.

18. *Torula lichenicola* on *Lecanora subfusca*. Ardrum, Co. Cork. Spore-filaments and spores.Fig. 19. *Coniothecium lichenicolum*.On *Lecanora parella*. Morchone, Braemar.(a) Portion of thallus of *Lecanora*, showing the parasite on its areolæ.(b) The *Coniothecium*, magnified and sectioned.Fig. 20. *Isidium corallinum*. Craigie, Perth.

(a) Section, logitudinal.

(b) Portions (terminal) of some of the Isidia, variously magnified.

(c) Portion of the plant viewed from above, showing the brown-tipped apices of the Isidia.

Fig. 21. Parasite on *I. corallinum* (sub-nom. *Lichen dactylinus*, Ach.), in Herb. Kew.

(a) Section of disk.

(b) Sporidia.

Figs. 22 to 28. *Coniothecium lichenicolum*.22. On *I. corallinum*. Craigie, Perth.(a) Portion of thallus of the *Isidium*.

(b) Do. magnified.

(c) Sections of parasite.

23. *Diplotomma calcareum*. Clapham, Yorkshire.(a) Portion of thallus of the *Diplotomma*.

(b) Do. magnified.

(c) Sections of the parasite.

24. *Coniothecium lichenicolum*. Kinnoull, Perth. Spores.

25. Do. Seuir-na-gillean, Skye. Spores.

26. Do. Sligachan, Skye. Spores.

27. Do. Mangerton, Co. Kerry.

(a) Portion of thallus of host.

(b) Sections of the parasite, variously magnified.

(c) Spores.

28. Do. North Wales; Davies, in Herb. Kew.

(a) Sections of the parasite.

(b) Spores.

\* The magnifying power is that which I have uniformly adopted in my drawings of microscopical structure in Lichens, viz., 425 diameters linear (under Objective No. 3, and Eye-piece No. 3) of a Nacet's microscope made for me in 1851 (vide "Otago Lich. and Fungi," foot note, p. 410).

- Fig. 29. *Microthelia Cookei*, on *Lecanora crenulata*. Chichester.  
 (a) Section of hymenium, showing asci and paraphyses.  
 (b) Sporidia.
- Fig. 30. *M. Stereocaulicola*, on *Stereocaulon paschale*. Glen Derrie, Braemar.  
 (a) Portions of the podetia of *Stereocaulon paschale*, showing the deformities occupied by the parasite; variously magnified.  
 (b) One of the conglomerate wartlets isolated.  
 (c) Sections of the parasite.  
 (d) Ascus, with young sporidia.  
 (e) Mature sporidium.
- Fig. 31. *M. Umbilicariae*, on *Umbilicaria pustulata*. Norway.  
 (a) Portion of thallus of the *Umbilicaria*, magnified.  
 (b) Sections of the parasite.  
 (Vide also Plate xxiv. fig. 3.)
- Fig. 32. *M. rugulosaria*, on *Placodium rugulosum*. Tasmania. Sporidia.
- Fig. 33. *M. parietinaria*, on *Physcia parietina*. Cot-tishall. Sporidia.
- Fig. 34. Parasite on *Thelotrema lepadinum*. Ireland.  
 (a) Sporidium.  
 (b) Stylospores.
- Fig. 35. Parasite on *Lecanora cenisea*. Ayton, Cleveland. Sporidia.

## PLATE XXIV.

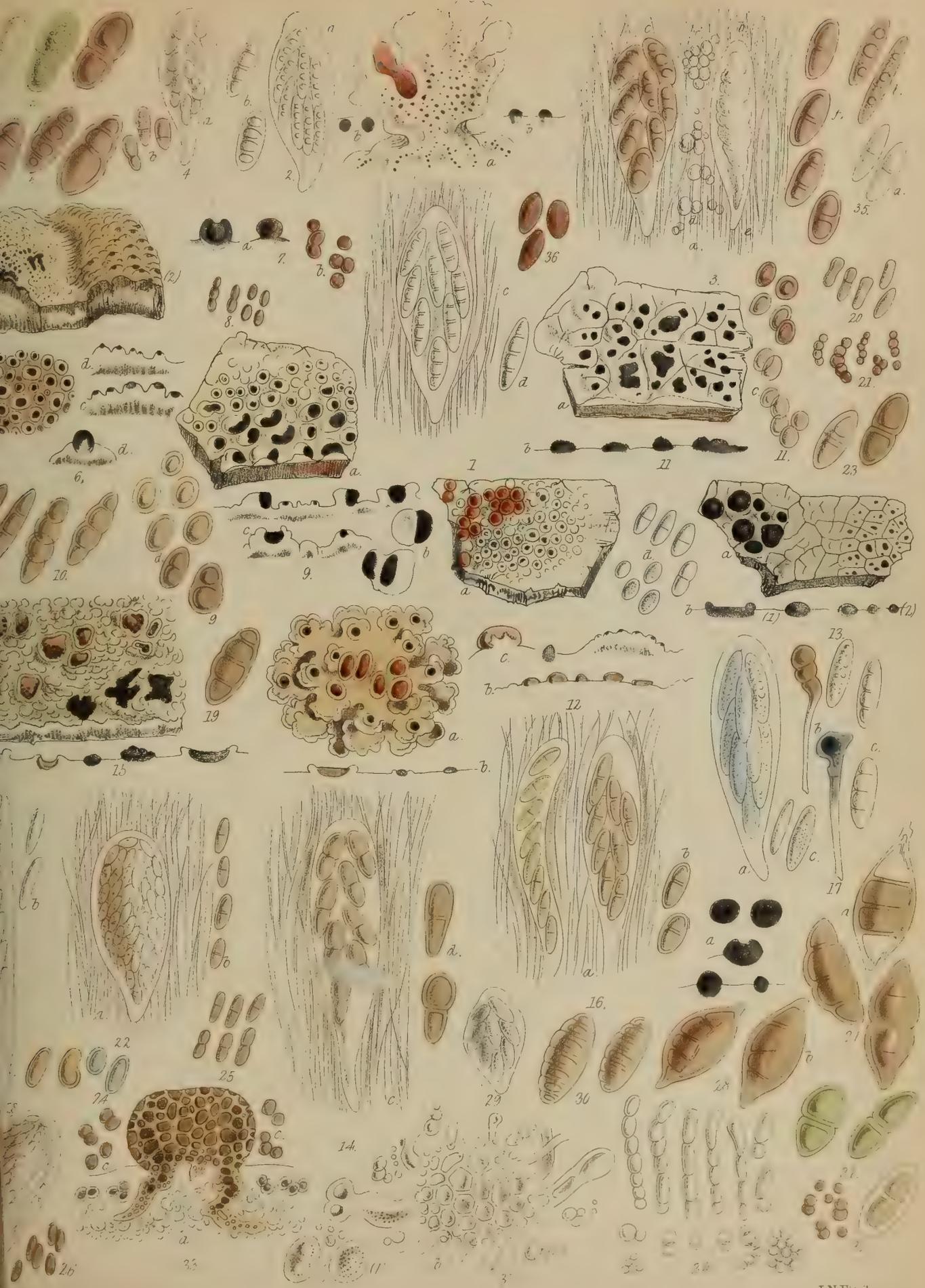
- Fig. 1. *Microthelia Nephromiaria*, on *Nephromium cellulolum*. Hermite Island.  
 (a) Old apothecium of the *Nephromium*; under side—bearing the parasite.  
 (b) Sections of the parasite.  
 (c) Section of hymenium, showing an ascus and paraphyses.  
 (d) Sporidium, mature.
- Fig. 2. *M. Nephromiaria*, on *Neuropogon Taylori*. Kerguelen's Land.  
 (a) Ascus.  
 (b) Mature sporidia.
- Fig. 3. *M. Umbilicariae*. Norway.  
 (a) Section of hymenium, showing asci and paraphyses.  
 (b) Young, and (c) Mature, ascus.  
 (d) Oil globules.  
 (e) Reaction of iodine on hymenial gelatine.  
 (f) Mature sporidia.  
 (Vide Plate xxiii. fig. 31.)
- Fig. 4. *M. Stictaria*, on *Sticta Freycinetii*. Campbell's Island.  
 (a) Ascus, showing reaction with iodine.  
 (b) Sporidia, mature and young.
- Fig. 5. Parasite on *Lecidea pachycarpa*. Ireland. Sporidia, young and mature.
- Fig. 6. *Microthelia Bæomycearia*, on *Bæomyces rufus*. Balthayock.  
 (a) Portion of thallus of the *Bæomyces*, showing—(1) its own young apothecia; and (2) the parasite.  
 (b) Portion of same thallus, further magnified, showing young apothecia.  
 (c) Sections of said apothecia.  
 (d) Sections of parasite.  
 (e) Sporidia.
- Fig. 7. *M. atricola*, on *Lecanora atra*. Derriquin, Co. Kerry.  
 (a) Perithecia, magnified; one sectioned.  
 (b) Sporidia.
- Fig. 8. Parasite on *Pertusaria*. Balthayock, Perth. Sporidia.
- Fig. 9. *Gassicurtia silacea*. Fontainebleau, Nyl. Exsic. No. 150.  
 (a) Portion of thallus of *Lecanora atra*, showing the parasite on its areolæ.  
 (b) Two of said areolæ, further magnified.  
 (c) Sections of the said areolæ and of their parasite.  
 (d) Spores.
- Fig. 10. *Microthelia vesicularia*, on *Lecidea vesicularis*. Switzerland. Sporidia.
- Fig. 11. *Spilomium Graphideorum*. Fontainebleau, Nyl. Exsic. No. 72.  
 (a) Portion of thallus of *Graphis*, bearing the parasite.  
 (b) Sections of the latter.  
 (c) Spores.
- Fig. 12. Parasite on *Lecanora polytropa*, var. *intricata*. Penmanshiel.  
 (a) Portion of thallus of the *Lecanora*, with apothecia and the parasite.  
 (b) Sections of young apothecia.  
 (c) Sections of mature apothecium and of the parasite.  
 (d) Sporidia or stylospores.
- Fig. 13. Parasite on *Lecidea albo-atra*. Great Island, Cork.  
 (a) Portion of the thallus of the *Lecidea* with apothecia and the parasite.  
 (b) Sections of (1) said apothecia and (2) parasite.
- Fig. 14. Parasite on *Squamaria crassa*. Crosshaven, Cork.  
 (a) Thallus bearing apothecia and the parasite.  
 (b) Section of an apothecium and of the parasite.  
 (c) Section of hymenium, showing ascus and paraphyses.  
 (d) Mature sporidia.
- Fig. 15. Parasite on *Squamaria saxicola*. Jerkin, Norway.  
 (a) Portion of thallus bearing apothecia and the parasite.  
 (b) Sections of said apothecia and parasite.
- Fig. 16. Parasite on *Endocarpon microsticticum*. Barmouth, N. Wales.  
 (a) Section of hymenium, showing asci and paraphyses.  
 (b) Mature sporidia.
- Fig. 17. *Lecidea Endocarpicola*, on *Endocarpon hepaticum*. Glanmire Road, Cork.  
 (a) Ascus under iodine, with young sporidia.  
 (b) Paraphyses, isolated.  
 (c) Sporidia, mature and young.
- Fig. 18. *Brothallus Moorei*, on *Cladonia bellidiflora*. Ach. Kelly's Green, Ireland.  
 (a) Ascus.  
 (b) Mature sporidia.  
 (c) Stylospores.
- Fig. 19. Associated with *Verrucaria Taylori*. Dunscombes Wood, Cork. Sporidium.

- Fig. 20. Associated with *Verrucaria fusiformis*. Douglas, Cork. Sporidia.
- Fig. 21. Associated with *Graphis scripta*. Leighton's Exsic. No. 244. Sporidia.
- Fig. 22. *Microthelia Collemaria*, on *Collema muscicolum*, Ach. On walls, Ingleby, Cleveland, Yorkshire, Mudd, 1857. (Compare Parasite (*Sphaeria* or *Microthelia*) on *C. melænum*, Ach., in LINDS. "Otago Lich. and Fungi," p. 442; and "Mem. Spermog." p. 272.)  
 (a) Section of hymenium, showing an ascus and paraphyses; with the reaction of iodine on the hymenial gelatine.  
 (b) Mature sporidia.
- Fig. 23. Parasite on *Usnea barbata*, var. *florida*. Rio Janeiro. Sporidia.
- Fig. 24. Parasite on *Urceolaria*. Glenarm, Co. Antrim. Spores or stylospores.
- Fig. 25. Associated with *Opegrapha atra*, Leighton's Exsic. No. 245. Sporidia.
- Fig. 26. Associated with *Opegrapha atra*, Schærer's Exsic. No. 634. Ascus and sporidia.
- Fig. 27. *Sphaeria Hookeri*, on *Lecidea sphaerica*. Schær. (Exsic. No. 526). Sporidia;  
 (a) One of them preparing to germinate.
- Fig. 28. *S. Hookeri* (sub-nom. *Verrucaria*), in Herb. Kew.  
 (a) Specimen from the summit of Ben Lawers; perithecia magnified and sectioned.  
 (b) Specimen from the Gemmi, Switzerland; Sporidia.
- Fig. 29. Accompanying *Endocarpon rufescens*, Ach. Schærer's Exsic. No. 465. Ascus with young sporidia. (Perhaps *Endocarpon cinereum*, Pers.?)
- Fig. 30. Associated with *Physcia astroidea*, Fr., var. *Clementi*, Turn. Sporidia. (Perithecia verrucarioid.)
- Fig. 31. Associated with *Umbilicaria hyperborea*, Hffm., Schærer's Exsic. No. 151. Sporidia.
- Fig. 32. Parasite on *Lecidea sanguinaria*, var. *affinis*. Schær. Exsic. No. 629.  
 (a) Sporidium.  
 (b) Spores.
- Fig. 33. Associated with *Abrothallus Smithii*, Tul. Craigie Hill, Perth.  
 (a) Perithecium of dark-brown hexagonienchyma; parasitic on thallus of *Parmelia saxatilis*.  
 (b) Mycelium; (1) spores, and (2) oil globules.  
 (c) Spores.
- Fig. 34. Associated with *Abrothallus oxysporus*, Tul. Craig-y-Barns, Dunkeld. Toruloid spore-filaments and spores.
- Fig. 35. Associated with *Lecidea lurida*, Ach. In Herb. Kew. Sporidia.  
 (a) On seaside rocks, Ireland, Miss Hutchins.  
 (b) Seaside rocks, Bangor, July 1802.











XV.—*On the Thermal Energy of Molecular Vortices.* By W. J. MACQUORN RANKINE, C.E., LL.D., F.R.SS. L. & E., &c.

(Read 31st May 1869.)

§ 1. *Object of this Paper.*—In a paper on the Mechanical Action of Heat, which I sent to the Royal Society of Edinburgh in December 1849, and which was read in February 1850, it was shown, that if sensible or thermometric heat consists in the motion of molecular vortices supposed to be arranged in a particular way, and combined in a particular way with oscillatory movements, the principles of thermodynamics, and various relations between heat and elasticity, are arrived at by applying the laws of dynamics to that hypothesis.\* The object of the present paper is to show how the general equation of thermodynamics, and other propositions, are deduced from the hypothesis of molecular vortices, when freed from all special suppositions as to the figure and arrangement of the vortices, and the properties of the matter that moves in them, and reduced to the following form:—*That thermometric heat consists in a motion of the particles of bodies in circulating streams, with a velocity either constant or fluctuating periodically.* This, of course, implies that the forces acting amongst those particles are capable of transmitting that motion.

§ 2. *Steady and Periodical Component Motions.*—A vortex, in the most general sense of the word, is a stream or current which circulates within a limited space. Conceive a closed vessel of any figure and volume to be filled with vortices, or circulating streams, the mean velocity of circulation in each such stream being the same; and let the velocities of the moving particles be either constant or periodic. How complex soever those motions may be, they may be resolved into the following component motions;—a motion of steady circulation with the uniform velocity already mentioned as the mean velocity; and a motion consisting in periodical fluctuations of velocity. Those two component motions may be called respectively the *steady circulation* and the *disturbance*.

§ 3. *Mean Pressure due to Centrifugal Force.*—Let an elementary circulating stream—that is, a circulating stream of indefinitely small sectional area—be supposed to flow round and round in an endless tube with the uniform velocity  $w$ ; let  $\rho$  denote the density of the stream;  $d\sigma$  the sectional area. Consider two cross sections of the stream at which the directions of motion of the particles are

\* Transactions of the Royal Society of Edinburgh, 1850, vol. xx.

contrary; and consider what resultant forces are exerted by the stream on the two parts into which those two cross sections divide the tube. The mass of matter which flows through each cross section of the tube in an unit of time is

$$\rho w d\sigma ;$$

and in each unit of time a mass of matter of that amount has its velocity reversed. The force required in order to produce that reversal of velocity is of the following amount in absolute units,

$$2\rho w^2 d\sigma ;$$

and such is the amount of each of the pair of inward pressures which the tube exerts on the stream, and of each of the pair of equal and opposite outward pressures exerted by the stream on the tube, tending to pull it to pieces. It may be called the *centrifugal tension* of an elementary stream.

The velocity of the particles flowing in the stream may undergo periodical fluctuations, positive and negative alternately; these will cause periodical variations in the centrifugal tension; but the mean value of that tension will continue to be that given by the formula.

The *mean intensity of the centrifugal tension*, in a direction tangential to the stream, is found by dividing the amount given in the preceding expression by the collective area,  $2d\sigma$ , of the two cross sections, giving the following result,

$$\rho w^2 .$$

Suppose now that the stream is cut by an oblique sectional plane, making the angle  $\theta$  with a transverse section. Then the area of that oblique section is greater than that of a transverse section in the ratio of  $1 : \cos \theta$ ; and the amount of the component tension in a direction normal to the oblique section is less than that of the total centrifugal tension in the ratio of  $\cos \theta : 1$ ; whence it follows, that the mean intensity of the component centrifugal tension in a direction making an angle  $\theta$  with a tangent to the stream is

$$\rho w^2 \cos^2 \theta .$$

Next, suppose a vessel of any invariable volume and figure to be filled with vortices or circulating streams, the velocity of steady circulation being  $w$ , and the mean density  $\rho$ . The centrifugal force will cause a pressure to be exerted in all directions against the inside of the vessel. To determine the mean intensity of that pressure, irrespectively of periodical variations, conceive the contents of the vessel to be divided into two parts by an imaginary plane, and consider what will be the mean intensity of the force with which the circulating streams tend to drive asunder the portions of matter at the two sides of that plane. The

plane will cut the streams that flow across it, some normally, others obliquely; and the tangents to those streams will have all possible directions relatively to a normal to the plane, subject to the condition, in the case of isotropic action, that the mean value of  $\cos^2\theta$  must be the same for all positions of the plane. But the sum of the mean values of  $\cos^2\theta$  for three planes at right angles to each other must be = 1; therefore the mean value of  $\cos^2\theta$  is =  $\frac{1}{3}$ ;\* and finally, the mean intensity of the centrifugal pressure is given in absolute units per unit of area, by the equation,

$$p = \frac{\rho w^2}{3} \quad \dots \quad (1).$$

§ 4. *Energy of Steady Circulation compared with Centrifugal Pressure.*—The actual energy† of the steady circulation in an unit of volume, is expressed in absolute units of work, as follows:—

$$\frac{\rho w^2}{2} \quad \dots \quad (2);$$

which, being compared with equation (1), gives the following result:—

$$p = \frac{\rho w^2}{3} = \frac{2}{3} \cdot \frac{\rho w^2}{2} \quad \dots \quad (3);$$

that is to say, *the intensity of the centrifugal pressure on the unit of area is two-thirds of the energy of the steady circulation in an unit of volume.* This is one of the propositions of the paper of 1849-50, p. 151, eq. v.; but it is now shown to be true, not merely, as in the former paper, for molecular vortices arranged in a particular way, but for molecular vortices arranged in any way whatsoever, provided their action is isotropic, and their mean velocity uniform.

A similar proposition has been proved by WATERSTON, CLAUSIUS, CLERK MAXWELL, and others, for the pressure produced by the impulse of small particles flying about in all directions within a closed vessel, and rebounding from its sides.

§ 5. *Vortices with Heterotropic Action.*—It is conceivable that in solid bodies, molecular vortices may be so arranged as to produce centrifugal pressures of different intensities in different directions. In such cases, it is to be recollected that the sum of the mean values of  $\cos^2\theta$  for the obliquities of any set of lines to any three planes at right angles to each other is = 1; whence it follows, that if  $p'$ ,  $p''$ , and  $p'''$  be the mean intensities of the centrifugal pressures in any three orthogonal directions, we have

$$p' + p'' + p''' = \rho w^2 \quad \dots \quad (4);$$

\* There is a well-known integration by which it is easily proved, that for a number of directions equally distributed round a point, the mean value of  $\cos^2\theta$  is  $\frac{1}{3}$ .

† Called by THOMSON and TAIT the "Kinetic Energy."

that is to say, *the sum of the mean intensities of the three centrifugal pressures in any three orthogonal directions is equal to twice the energy of the steady circulation in an unit of volume.* This proposition was not in the paper of 1849-50, which was confined to an isotropic arrangement of vortices.

§ 6. *Energy of the Periodical Disturbances.*—In the paper of 1849-50, p. 152, equation x., the energy of the periodical disturbances was taken into account by multiplying the energy of the steady circulation by a factor  $k$  greater than unity; thus giving for the total energy in an unit of volume the following expression,

$$\frac{\rho v^2}{2} = \frac{k\rho w^2}{2};$$

in which  $v^2$  denotes the mean of the squares of the resultant velocities of the particles with their combined motions. The values of the factor  $k$ , being the ratio which the total energy of the molecular motions bears to the energy of the steady circulation, are to be deduced in each case from the results of experiments on specific heat.

Thus the energy of the disturbances in an unit of volume is expressed by

$$(k-1) \frac{\rho w^2}{2} = \frac{3}{2}(k-1) p \quad . \quad . \quad . \quad . \quad (5).$$

It may now be observed, in addition, that the energy of the disturbances may, and indeed must, be at times partly potential as well as actual; in other words, partly due to displacement as well as to fluctuation of velocity.

Let  $\pm u$  be the greatest fluctuation of velocity; then a particle of the mass unity has the energy  $\frac{u^2}{2}$  due to that fluctuation, in addition to the energy due to the steady circulation. It is only at the instants of greatest disturbance of velocity that the energy is all actual: at every other instant the energy is partly potential. Hence  $v^2 = kw^2$  may be taken to denote, not the square of an actual velocity common to all the particles, but the value to which the square of the velocity of the particles would rise, if all the energy of the disturbances, actual and potential, were expended in increasing the velocity of steady circulation.

§ 7. *Total Energy of Thermal Motions.*—The total energy of the motion, compounded of steady circulation and periodical disturbances, in an unit of volume, is expressed, as in the paper of 1849-50, by the following equation, which also shows its relation to the centrifugal pressure,

$$\frac{k\rho w^2}{2} = \frac{3k}{2} p \quad . \quad . \quad . \quad . \quad . \quad . \quad (6);$$

in which (to recapitulate the notation)  $\rho$  is the mean density;  $w$  the velocity of steady circulation; the centrifugal pressure  $p$  is expressed in absolute units of

force on the unit of area; and the proportion  $k$ , in which the *total energy of thermal motions* exceeds the energy of steady circulation, is a quantity whose values and laws are left to be deduced from the results of experiment.

§ 8. *Determination of Centrifugal Pressures.*—The external pressure exerted by any substance, as we find it in nature, is a complex quantity, being compounded of the centrifugal pressure already mentioned, and of forces which may be classed together under the name of *cohesion*. To enable us to distinguish those components of the total pressure from each other, we have the principle, that the centrifugal pressure varies as the density simply; whereas pressure or tension, or *stress* (to use a general term), arising from cohesive forces, must vary as some function of the density of a higher order than the first power.

The *perfectly gaseous state* is an ideal state in which the substance exerts no external pressure except that which varies as the density simply; that is, centrifugal pressure. It is impossible to obtain a substance absolutely in the state of perfect gas; but the cohesive stress diminishes with increase of temperature and diminution of density in such a manner, that it is possible, as is well known, to obtain substances approaching very nearly to the perfectly gaseous state, such as atmospheric air and various other gases; and the actual pressures of such nearly perfect gases may be used, either as approximate values of the pressures in the ideal state of perfect gas, or as data for calculating the latter kind of pressures by the method of limits. We thus have the means of determining, to a close approximation, the centrifugal pressure of a given substance at a given temperature and density; the well-known formula being

$$\frac{p}{\rho} = \frac{p_0}{\rho_0} \cdot \frac{\tau}{\tau_0} \quad \dots \quad (7);$$

in which  $\tau_0$  is the absolute temperature of melting ice;  $\tau$  the actual absolute temperature; and  $\frac{p_0}{\rho_0}$  the value of the quotient  $\frac{p}{\rho}$  at the temperature of melting ice, for the particular substance in question.

§ 9. *Temperature and Specific Heat.*—It is shown in the paper of 1849–50, that temperature, according to the hypothesis of molecular vortices, is a function of the quotient found by dividing the *energy of the steady circulation in an unit of mass* by a constant depending on the nature of the substance; which constant may be defined, as the value which the energy of steady circulation in an unit of mass of the given substance assumes at a standard temperature, such as that of melting ice. The energy of the steady circulation in an unit of mass is

$$\frac{w^2}{2} = \frac{3}{2} \cdot \frac{p}{\rho};$$

whence it appears, that the principle stated as to absolute temperature is

expressed by equation (7), already given in § 8. The *total energy of the thermal motions* in an unit of mass is expressed by dividing equation (6) of § 7 by the density  $\rho$ ; hence that quantity of energy (denoted for shortness by  $Q$ ) is given in terms of the absolute temperature by the following equation,

$$Q = \frac{kv^2}{2} = \frac{3k}{2} \cdot \frac{p}{\rho} = \frac{3k}{2} \cdot \frac{p_0}{\rho_0} \cdot \frac{\tau}{\tau_0} \quad (8).$$

The real specific heat of a substance, as defined in the previous paper, when expressed in units of work per degree, is

$$Jc = \frac{dQ}{d\tau} = \frac{3kp_0}{2\rho_0\tau_0} + \frac{3p_0\tau}{2\rho_0\tau_0} \cdot \frac{dk}{d\tau} \quad (9).$$

in which  $c$  is the real specific heat, in terms of the minimum specific heat of liquid water, and  $J$ , JOULE'S equivalent, or the dynamical value of the ordinary thermal unit.

There is one part of the specific heat which is necessarily constant for a given substance in all conditions; and that is the part which expresses the rate of increase with the temperature, of the energy of the steady circulation alone in an unit of mass, viz.,

$$\frac{d}{d\tau} \left( \frac{Q}{k} \right) = \frac{v^2}{2g\tau} = \frac{3p}{2\rho\tau} = \frac{3p_0}{2\rho_0\tau_0} \quad (10).$$

The part of the specific heat which depends on periodical disturbances is expressed as follows:—

$$\frac{d}{d\tau} \left\{ \frac{(k-1)Q}{k} \right\} = \frac{3(k-1)p_0}{2\rho_0\tau_0} + \frac{3p_0\tau}{2\rho_0\tau_0} \cdot \frac{dk}{d\tau} \quad (11).$$

It is only by experiment that it can be ascertained whether this part of the specific heat is constant or variable. Experiment has proved that it is constant for the perfectly gaseous state, and nearly, if not exactly constant, for other conditions; but that its values for the same substance in the solid, liquid, and gaseous conditions are often different.\*

The apparent specific heat contains other terms, depending on the expenditure of energy in performing external and internal work, according to principles of thermodynamics which are now well known.

§ 10. *Examples of the Proportion in which the Total Energy of the Thermal Motions exceeds the Energy of the Steady Circulation.*—In the perfectly gaseous

\* According to the nomenclature used by CLAUDEUS, the phrase "real specific heat" is applied to that part only of the specific heat which is necessarily constant for a given substance in all conditions. Hence, if that nomenclature were adapted to the hypothesis of molecular vortices, the term real specific heat would be applied to the coefficient given in equation (10) only, and that given in equation (11) would be considered as part of the apparent specific heat.

state, the coefficient given in equation (9) is *the specific heat at constant volume*; and as that quantity is known to be constant at all temperatures, the second term of the right hand side of the equation disappears, and it is reduced simply to the following—

$$Jc = \frac{3k\rho_0}{2\rho_0\tau_0} \dots \dots \dots (12).$$

The specific heat, in dynamical units per degree, of a perfect gas *under constant pressure*, is expressed as follows:—

$$Jc' = Jc + \frac{p_0}{\rho_0\tau_0} = \frac{p_0}{\rho_0\tau_0} \left( \frac{3k}{2} + 1 \right) \dots \dots \dots (13);$$

and the ratio in which the latter coefficient is greater than the former is, therefore,

$$\frac{c'}{c} = 1 + \frac{2}{3k} \dots \dots \dots (14);$$

whence we have the following formulæ for deducing the proportion *k*, borne by the total energy of the thermal motions to the energy of the steady circulation, from the ratio  $\frac{c'}{c}$  as determined by experiment,

$$k = \frac{2}{3\left(\frac{c'}{c} - 1\right)} \dots \dots \dots (15).$$

This method is applicable only to substances that are nearly in the perfectly gaseous state.

There is another method, applicable to the same class of substances, which is expressed as follows:—

$$k = \frac{2\rho_0\tau_0 Jc}{3p_0} \dots \dots \dots (16).$$

This second method may be applied to liquids and solids also, under the following conditions; the quantity  $\frac{p_0}{\rho_0\tau_0}$  is to be calculated as for the perfectly gaseous state; and the specific heat *c* must be nearly constant.

The ratio which the energy of periodical disturbances in an unit of volume bears to the centrifugal pressure may be interesting in connection with hypothetical views of the constitution of matter. It is expressed as follows:—

$$\frac{3(k-1)}{2} \dots \dots \dots (17).$$

The following are some examples of the results of calculations by formulæ (15) and (17):—

Substance,	$\frac{e'}{c}$	$k$	$\frac{3}{2}(k-1)$
Atmospheric air, . . . . .	1·408	1·634	0·951
Nitrogen, . . . . .	1·409	1·630	0·945
Oxygen, . . . . .	1·400	1·667	1·000
Hydrogen, . . . . .	1·413	1·614	0·921
Steam-gas, . . . . .	1·297	2·242	1·863

§ 11. *General Equation of Thermodynamics*.—In the paper of 1849-50, pp. 158 to 164, the general equation of thermodynamics (equation 6 of that paper, p. 161) is deduced from the hypothesis of molecular vortices, on the supposition of a special form and arrangement of the vortices. In a subsequent paper, “On the Centrifugal Theory of Elasticity,” read to the Royal Society of Edinburgh in December 1851 (“Transactions,” vol. xx. pp. 433 to 436), the same general equation (being equation 25 of the latter paper, p. 436) is deduced from the hypothesis of molecular vortices, without any special supposition as to the form and arrangement of the vortices, but with certain assumptions as to the laws of the elasticity of the matter which moves in them. In a paper read to the British Association in 1865, and published in the “Philosophical Magazine” for October of that year, a further generalisation is effected; and it is shown that the general equation of thermodynamics follows from the supposition, that sensible heat consists in any kind of steady molecular motion within limited spaces, without any assumption either as to the figures of vortices, or as to the special properties of the matter that moves in them. The object of this section of the present paper is to show how the same general equation is deducible from the hypothesis of molecular vortices, as stated at the commencement of the paper; that is, freed from all special suppositions except that of a steady circulation, combined with periodical disturbances of speed, whose energy may bear any proportion, constant or variable, to that of the steady circulation.

The forces by which an elementary circulating stream, whether flowing with a steady or with a fluctuating speed, is kept in a given state of motion, and of a definite figure and dimensions, are equivalent in their action to a tension exerted at each cross-section of the stream, of an amount which, at a given cross-section, and at a given instant, is expressed in absolute units of force by the product of the mass which flows along the stream in a second into the velocity of flow at that cross-section and instant. The mean value of the tension is the product of the same mass into the mean velocity; that is, into the velocity of steady circulation. Hence the *mean centrifugal tension*, as this force may be called, is proportional to the square of the velocity of steady circulation, and therefore to the *absolute temperature*; and the work done by the forces to which the virtual



dynamical value of the specific heat of the gas at constant volume; and consequently,  $\psi(\tau) = Jc \text{ hyp. log. } \tau$ ; and the same is the value for any substance which, at the temperature  $\tau$ , is capable of approaching indefinitely near to the perfectly gaseous condition. There is some reason for believing that all substances may have that property;\* but to provide for the possibility, pointed out by CLAUSIUS ("POGGENDORFF'S Annalen," vol. xcvi. p. 73), of the existence of substances which at certain temperatures are incapable of approaching indefinitely near to the perfectly gaseous condition, we may make (as that author does),

$$\psi(\tau) = Jc \text{ hyp. log. } \tau - \chi(\tau);$$

where  $\chi(\tau)$  is a function of the temperature, which becomes = 0 at all temperatures at which an indefinitely close approximation to the perfectly gaseous state is possible; thus giving, for the complete value of the thermodynamic function,

$$\phi = Jc \text{ hyp. log. } \tau + \chi(\tau) + \int \frac{dX}{d\tau} dx + \int \frac{dY}{d\tau} dy + \&c. \quad (19).$$

That expression may be abbreviated as follows:—Let  $U$  be the potential energy of the elastic stress of unity of mass of the body at constant temperature; then

$$\phi = Jc \text{ hyp. log. } \tau + \chi(\tau) + \frac{dU}{d\tau} \quad (20);$$

and the corresponding form of the general equation of thermodynamics is as follows:—

$$dQ = \{Jc + \tau\chi'(\tau)\} d\tau + \tau d \cdot \frac{dU}{d\tau} \quad (21).$$

§ 12. *Conclusion.*—In conclusion, then, it appears that the special suppositions as to matters of detail, introduced into the hypothesis of molecular vortices in the paper of 1849–50, are not essential to the deduction from that hypothesis of the principles of thermodynamics, but that such matters of detail may be left open to be determined by future investigations.

\* See Phil. Mag. December 1865.

XVI.—*On the Alkaloids contained in the Wood of the Bebeeru or Greenheart Tree (Nectandra Rodicei, Schomb.)*. By DOUGLAS MACLAGAN, M.D., F.R.S.E., Professor of Medical Jurisprudence in the University of Edinburgh, and ARTHUR GAMGEE, M.D., F.R.S.E., Lecturer on Physiology in Surgeon's Hall, Edinburgh.

(Read 3d May 1869.)

In a paper read before the Royal Society of Edinburgh in April 1843,\* Dr MACLAGAN described the general properties of the alkaloid, whose presence had been indicated in the bark of the bebeeru or greenheart tree, by Dr RODIE of Demerara, and described the mode of preparation of its sulphate for medicinal use. The fact that bebeerine appeared to possess marked antiperiodic properties,† rendered its careful chemical study desirable, and accordingly the alkaloid, purified as far as possible, was subjected to analysis by Drs MACLAGAN and TILLEY.‡ It resulted from this research that bebeerine is an uncrystallisable base, very soluble in alcohol, less so in ether, and very sparingly so in water. It forms with acids salts which are all uncrystallisable. With perchloride of gold, mercury, copper, and platinum, it gives precipitates which are soluble to a certain extent in water and alcohol, but which are deposited in a non-crystalline form when the solution cools. To this base the author assigned the formula  $C_{35}H_{20}N_2O_6$  (C=6). VON PLANTA\* subsequently attempts to purify further the alkaloid, and assigned to it the formula  $C_{36}H_{21}O_6N$  (C=6) or  $C_{18}H_{21}O_3N$  (C=12.)

In consequence of the apparent impossibility to obtain bebeerine in a crystalline form, it is impossible to state whether the substances examined by MACLAGAN and TILLEY, or by VON PLANTA, were absolutely pure; and there is no evidence to show that the product obtained by the latter chemist was purer than that examined by the former investigators. Since the time when these papers were published, sulphate of bebeerine has found its way into medical practice, and the experience of many appears to show that it is possessed of no insignificant tonic and antiperiodic properties. The sulphate of bebeerine, as it occurs in the market, has been, we believe, almost entirely manufactured by Messrs MACFARLANE & Co. of Edinburgh.

Experimenting with various portions of the bebeeru tree, one of the members

\* Transactions of the Royal Society of Edinburgh, vol. xv. part. iii.

† MACLAGAN, Edinburgh Medical and Surgical Journal, April 1845.

‡ London and Edinburgh Philosophical Magazine, series iii. vol. xxvii. p. 253.

of that firm discovered that, on subjecting the wood to a process similar to that which had been used in the separation of bebeerine from the bark, a product was obtained which did not apparently differ from bebeerine in its physical properties. He requested us to undertake for ourselves the examination of the product which he had obtained from the wood.

The substance handed to us for examination had been prepared by subjecting the wood of the bebeeru tree to a process substantially identical with that recommended in the "British Pharmacopœia" for the extraction of the sulphate of bebeerine from the bark. The product did not differ in appearance from the latter substance as it occurs in commerce, *i.e.*, it was in the form of shining yellowish-brown scales, soluble in water, and possessed of an intensely bitter taste, not differing perceptibly from that of sulphate of bebeerine.

In the first place, a portion of this substance was dried in the water-bath, and then the amount of sulphuric acid determined.

(1.) 2.001 grms. of substance yielded 0.5300 grms. of sulphate of barium.

(2.) 2.001 grms. of substance yielded 0.483 grms. of sulphate of barium.

The mean of these two results gives to amount of sulphuric acid (calculated as  $H_2SO_4$ ) as 10.69 per cent. This would indicate that the substance examined consisted of sulphate of bebeerine, mixed with other substances; or that it was composed of the sulphate of one or more alkaloids, having a higher molecular weight than bebeerine.

One hundred grammes of the powdered but undried sulphate were dissolved in two litres and a half of distilled water. An insignificant quantity of a brownish powder was left undissolved. The fluid was filtered through calico, and precipitated carefully with solution of ammonia. The bulky precipitate was collected on calico, carefully washed, and dried on the water-bath. When dry, it was boiled for some time with chloroform. The latter fluid soon acquired a deep brownish-yellow colour. The residue was treated three successive times with chloroform. At the end of that time, the chloroform appeared to exert no action upon the tolerably abundant residue. When dried, the chloroform extract weighed 60.55 grammes; it had a brownish-yellow colour, and broke with a resinoid fracture. When powdered, it possessed a very pretty yellow colour.

We shall, in the first place, state the result of our examination of this substance before proceeding to that of the bodies which were left undissolved by the chloroform.

#### I. *Examination of Nectandria, a new Base soluble in Chloroform.*

The chloroformic extract, to which we have referred, left no ash when ignited on platinum. It was very freely soluble in rectified spirit; less soluble in absolute alcohol. It was not perceptibly dissolved by cold distilled water. When boiled with water, it very readily fused at a temperature below  $100^\circ C.$ ; and the boiling

solution, when filtered, deposited a small quantity of yellow powder, which was found to be amorphous when examined under the microscope. 49 grammes of the solution in boiling water yielded 0.07 grammes of dry residue; or 100 parts yielded 0.142 parts of solid residue. The powder was found to be entirely soluble in dilute acid; the solution possessed a yellow colour, and an intensely bitter taste. The residue, dissolved in water, and treated with solution of ammonia, or of any of the fixed alkalis, yielded a bulky yellowish precipitate. When evaporated to dryness, and redissolved in water, a perfectly neutral solution was obtained. This was abundantly precipitated by tetrachloride of platinum, the precipitate being quite amorphous, and not fusible when heated to 100° C. 0.709 grammes of this hydrochlorate yielded 0.268 grammes of chloride of silver; 100 parts, therefore, contained 9.361 per cent. of chlorine. Oil of vitriol added to the base did not blacken it, but merely caused it to assume a faint rose tint. When binoxide of manganese was added to the acid solution thus obtained, a most splendid rich green colour was developed, which, on exposure to the air, passed into a violet of great beauty, scarcely distinguishable from that procured when strychnia is similarly treated. This reaction is one of very great delicacy. On adding sulphuric acid and binoxide of manganese to a fragment of the alkaloid placed in a tube, and afterwards diluting the fluid sufficiently by means of oil of vitriol, we observed its effects on the spectrum. In the case of the green fluid first spoken of, the violet end of the spectrum was cut off, and when a sufficiently thick stratum was examined none but the red rays passed. No definite absorption band was, however, present.

After assuming the violet tint, besides a cutting off of the violet end of the spectrum, a very well-marked absorption band, situated between C and D, is noticed, as is shown in the annexed diagram.



The reaction above described was possessed equally by all compounds of the alkaloid under examination.

On heating the alkaloid on a platinum spoon, it first melts, and then burns with the evolution of fumes which are both pungent and fragrant. These are identical with the fumes evolved under the same circumstances by bebeerine.

A comparison of the properties which we have described with those of pure bebeerine, established in the clearest manner the difference between the two. The chief of these differences are—

1st, The ready fusibility in hot water of the base from the wood.

2d, The beautiful and most delicate reaction with binocide of manganese and sulphuric acid, which is not possessed by bebeerine.

3d, The much smaller solubility of the new base in ether. With regard to this point it may be stated, that, in a paper previously quoted, one of us had stated the solubility of bebeerine in ether to be 1 in 13. The ether used had a specific gravity of 730.

On repeating, however, our observations with perfectly pure ether, of density 0·715, and pure bebeerine, which had been prepared from the bark exhibited by the firm of MACFARLANE & Co. in the Great Exhibition, we ascertained the solubility to be smaller.

100 parts of this ether dissolved, at 14° C., 0·96 parts of pure bebeerine.

Under the same circumstances, 100 parts of this ether dissolved 0·201 parts of the base from the wood. After being boiled in contact with the base for ten minutes, being filtered and evaporated, 100 parts of ether was found to have dissolved 0·188 parts of our new base. These numbers are, however, higher than the truth. After the two latter determinations, it was found that the substance which had been used retained chloroform with great tenacity, and could only be freed from it by very prolonged heating and exhaustion *in vacuo*. The base, when purified by a process to be mentioned below, and thoroughly dried, was again treated with ether.

1. After standing for many days in contact with it, ether of density 715 had dissolved only 0·04 parts of alkaloid.

2. After standing for twenty-four hours only, in contact with the same sample of base, 100 parts of the same ether had dissolved only 0·021 parts of the base.

We have mentioned that the base which we obtained from the wood possessed a fine yellow colour. This colour is not, however, essential to it.

On treating the yellow solution of the hydrochlorate of the base with animal charcoal, the solution is almost completely decolorised; and when treated with solution of ammonia a precipitate is obtained, which, after being drained and allowed to dry (without the application of heat), either in the air or *in vacuo*, occurs in the form of an almost purely white powder. When heated in the water-bath, it soon acquires the fawn colour which it possessed before the treatment with charcoal.

On dissolving the white powder in alcohol, and evaporating the solution, a translucent residue of yellowish tint is obtained. All attempts to obtain this base in a crystalline form have failed. Obtained by evaporation from alcohol, chloroform, or ether, like bebeerine, it never presents the slightest crystalline form.

We have ascertained the composition of this base, to which we now assign the name of Nectandria, by the analysis of two different samples, one of which

was quite white, the other possessed of a light fawn colour. We must remark, that this alkaloid is excessively difficult to burn. In the two first analyses quoted, the substance was burned in the manner now usual with oxide of copper and oxygen gas, taking great care to have a sufficient quantity of copper turnings in the anterior part of the tube. In the third analysis, we used the method proposed by GINTL, one of ROCHLEDER'S pupils, and which consists in mixing the substance intimately with fused and powdered bichromate of potash and powdered oxide of copper, and filling up the tube as usual with granulated oxide of copper. Operating with the greatest precaution, the combination was, however, not completed until a very considerable quantity of oxygen gas had been passed through the tube.

I. 0.225 grms. of substance gave 0.139 grms. of water and 0.583 grms. of carbonic acid.

II. 0.317 grms. of substance gave 0.194 grms. of water and 0.816 grms of carbonic acid.

III. 0.272 grms. of substance gave 0.158 grms. of water and 0.6945 grms. of carbonic acid.

Nitrogen was determined by WILL and VARRENPTRAPP'S method. The ammonia evolved was collected in standard sulphuric acid.

0.4015 grms. of substance yielded ammonia, which required for neutralisation 1.3 cubic cents. of normal sulphuric acid (1 C.C. corresponded to 0.0177 ammonia, or 0.014 grms. of nitrogen).

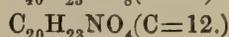
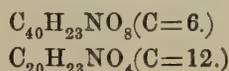
These numbers lead to the following per-centage composition:—

	I.	II.	III.	Mean.
Carbon,	70.26	70.19	69.63	70.02
Hydrogen,	6.86	6.81	6.43	6.73
Nitrogen,	4.53	4.53	4.53	4.53
Oxygen,	18.35	18.47	19.31	18.72

The double compound, with tetrachloride of platinum, was employed in the determination of the atomic weight of the base.

(1.) 0.2985 grms. of the salt gave 0.053 grms. of platinum	=	17.72	per cent. of platinum.
(2.) 0.259       "       "       "       "       "       "	=	17.7	"       "
(3.) 0.1735     "       "       "       "       "       "	=	17.0	"       "
(4.) 0.288     "       "       "       "       "       "	=	18.05	"       "
(5.) 0.329     "       "       "       "       "       "	=	18.23	"       "

The mean of these five determinations gives the per-centage composition of platinum in the double salt as 17.72, from which we deduce 348.08 as the atomic weight of the alkaloid. From the above numbers, we deduce for our new alkaloid, Nectandria, the formula—



This gives by calculation—

		Found.
Carbon, . . . . .	70.38	70.02
Hydrogen, . . . . .	6.74	6.70
Nitrogen, . . . . .	4.10	4.53
Oxygen, . . . . .	18.79	18.71
	<hr/>	<hr/>
	100.00	100.00

According to this formula, the compound with tetrachloride of platinum should contain . . . . . 18.07 per cent. of platinum,  
 Mean of five determinations, . . . . . 17.72   "   "

According to this formula, the hydrochlorate of the base should contain . . . . . 9.60 per cent of chlorine.  
 Actually found, . . . . . 9.361   "   "

Below the formula of bebeerine, as ascertained by VON PLANTA, is placed side by side with that of nectandria, as ascertained by ourselves—

Bebeerine, . . . . .	$C_{18}H_{21}O_3N.$
Nectandria, . . . . .	$C_{20}H_{23}O_4N.$

## II. Examination of a new Base insoluble in Chloroform.

In a previous part of this paper it has been stated, that the precipitate produced by ammonia in a solution of the mixed sulphates obtained from the wood of the bebeeru tree, was by treatment with chloroform subdivided into two portions, of which one, Nectandria, has been already examined.

After the treatment with chloroform, 16.7 grammes of a greyish solid matter remained. This matter was nitrogenous, soluble in dilute acids; its alcoholic solution had a very marked alkaline reaction; its solution in hydrochloric acid was abundantly precipitated by tetrachloride of platinum. On boiling the base with water, the latter soon acquired a rich yellow colour, and possessed a very bitter taste; the solution had a strongly alkaline reaction. On allowing the hot solution to cool, a yellow powder subsided. When examined with a power of 300 diameters, this powder was seen to be uniformly composed of nodules, and clustres of nodules. On repeatedly dissolving the powder in boiling water, and examining the deposit which subsided on cooling, the same forms were visible. From the portion of residue insoluble in chloroform, we obtained 5.65 grammes of this yellow substance, which our observations prove to be a powerful base.

It was entirely soluble in water.

1. 100 grammes of the boiling solution, on being evaporated, gave 2.11 grammes of solid residue.

2. 100 grammes of the solution which had cooled, and had deposited the yellow nodules, yielded 1.77 grammes of solid residue.

From these determinations, it would appear that one part of this yellow base

is soluble in 56·81 parts of cold water, and in 47·3 parts of boiling water. The aqueous solution possessed a powerfully alkaline reaction. When boiled with a solution of chloride of ammonium, ammonia was evolved abundantly; when treated with solution of nitrate of silver, a white precipitate fell, which became black on boiling.

Sulphuric acid dissolved the yellow base with the production of a dirty brownish-yellow colour. When binoxide of manganese was added to the acid solution, a magnificent green colour was produced, which, on exposure to air, changed to a rich purple, and ultimately assumed a dirty red tint. This reaction was compared with, and found to be identical with that manifested by nectandria, although, from the very remarkable differences in the action of solvents upon them, the one alkaloid could not be contaminated with the other. The yellow base is rich in nitrogen; and when heated on platinum it melts, and evolves the same fumes as bebeerine and nectandria.

When dissolved in hydrochloric acid, and treated with tetrachloride of platinum, a yellow precipitate falls, which on the fluid being heated to boiling point fuses, and is then perceptibly dissolved. A portion resists solution, and is converted into a brown substance. On cooling, the solution deposits a yellow powder, which is not amorphous, but displays, under a power of 300 diameters, clusters of nodules similar to those of the alkaloid itself, as it is deposited from its aqueous solution. The platinum was determined.

I. 0·551 grms. of the double salt gave 0·111 grms. of platinum = 20·1 of platinum per cent.

II. 0·345 grms. of the double salt gave 0·071 grms. of platinum = 20·57 of platinum per cent.

We reserve the further examination of this base for a future memoir; as far as our examination goes, it however appears to show, that this is best characterised and most clearly separated of all the products obtained from the bebeeru tree.

### III. *Examination of the Substance Insoluble in Chloroform and in Boiling Water.*

After treatment with chloroform and with boiling water, there still remains a considerable residue. It appears, from the observations which we have hitherto made upon it, that this residue possesses all the characters of a non-crystalline vegetable base, or of a mixture of such bases. It is soluble in alcohol, and almost completely insoluble in ether and chloroform. Its alcoholic solution possesses an alkaline reaction. The substance is capable of neutralising acids, and forms with platinum tetrachloride, a compound which is fusible in boiling water.

It is our intention to prosecute further the chemical characters of the alkaloids of the wood of the bebeeru tree, and more particularly to direct our attention to their therapeutical properties.

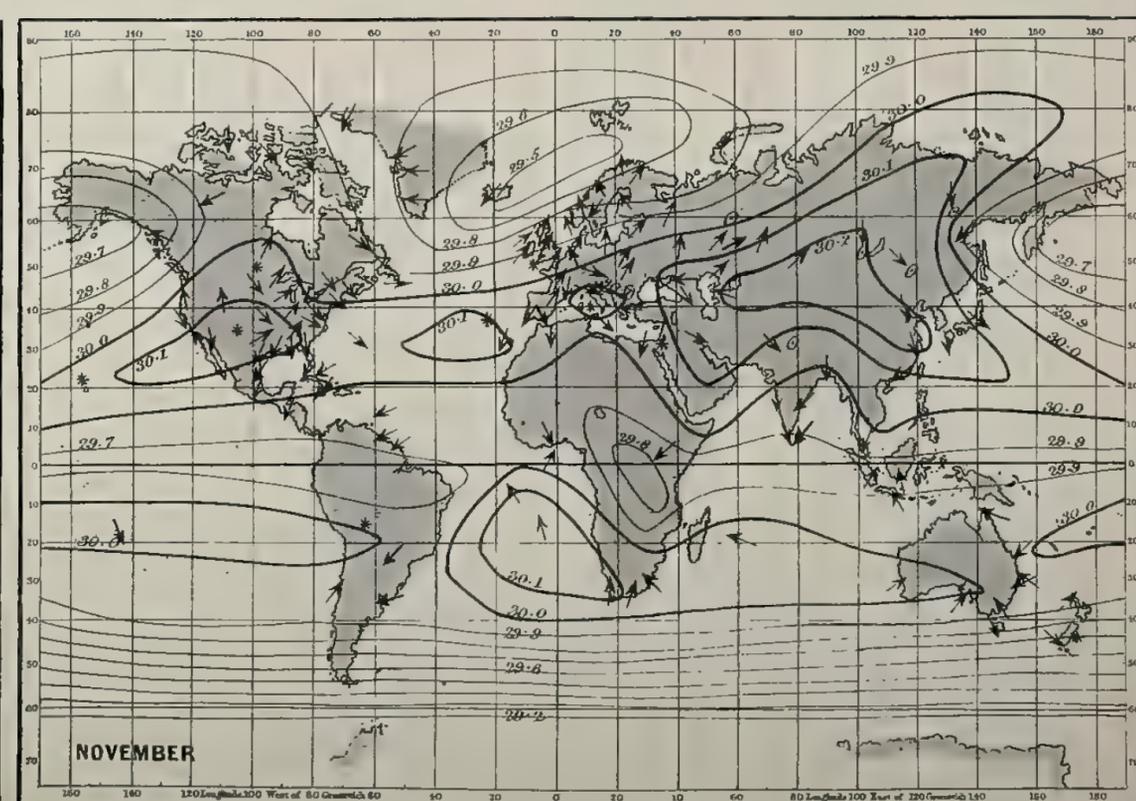
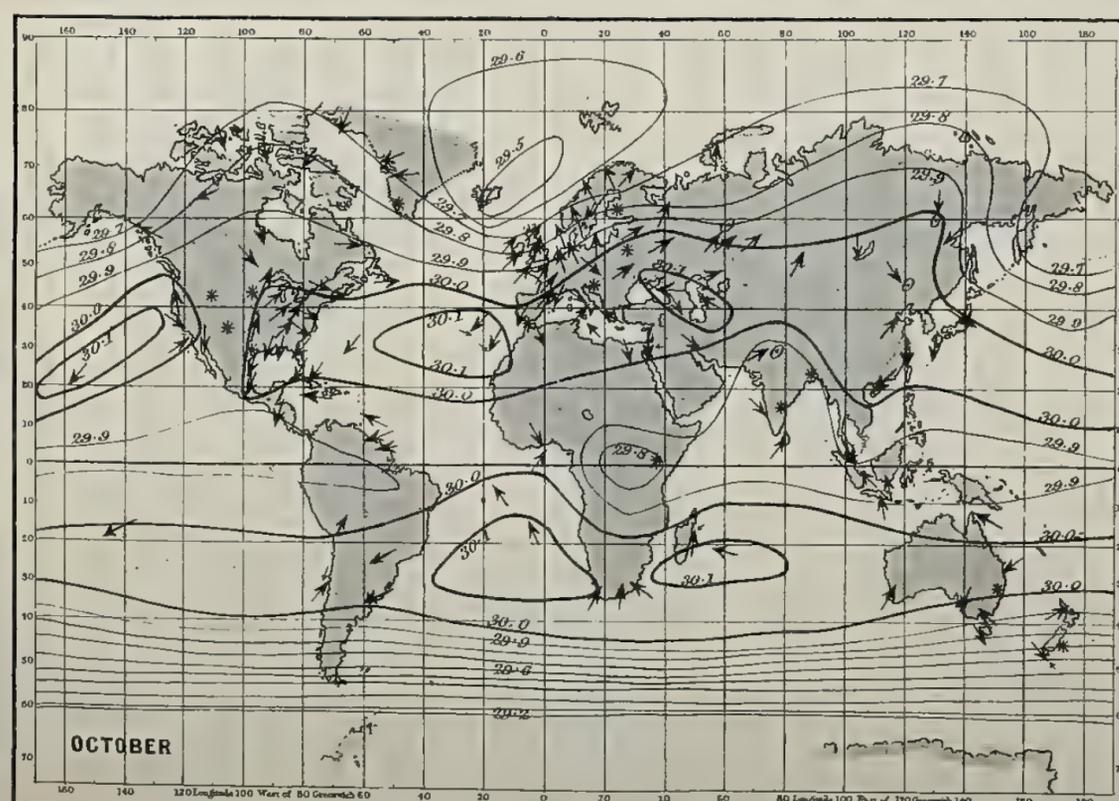
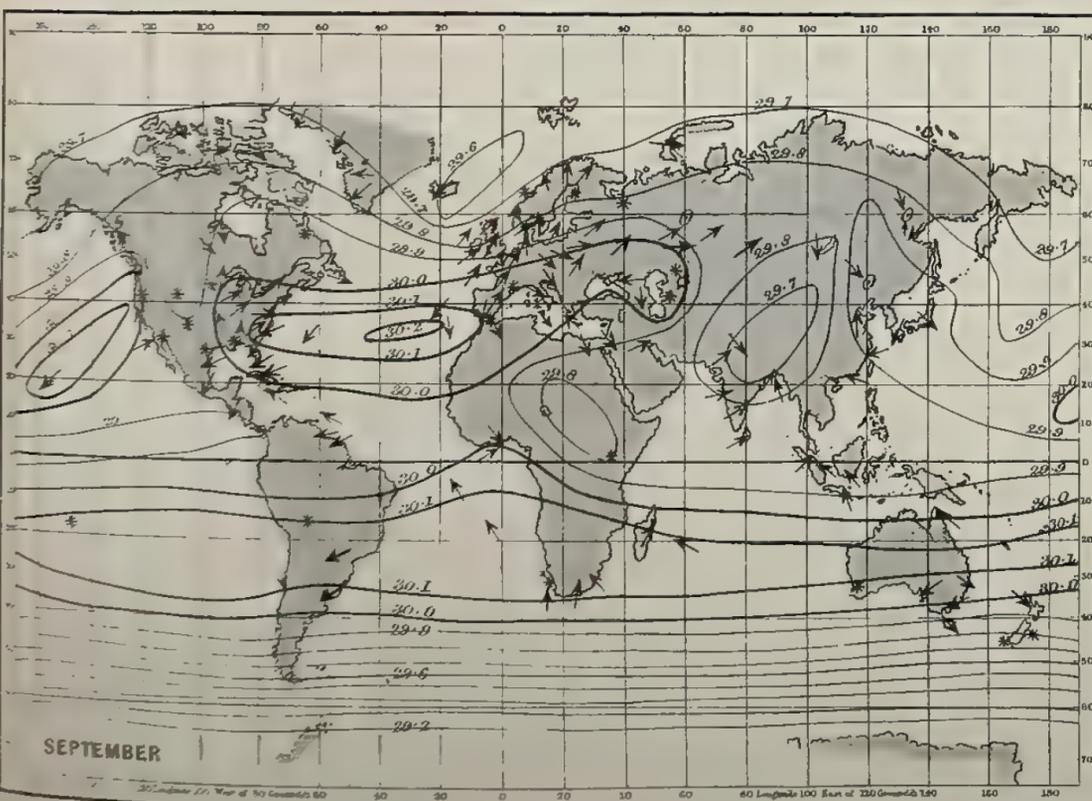
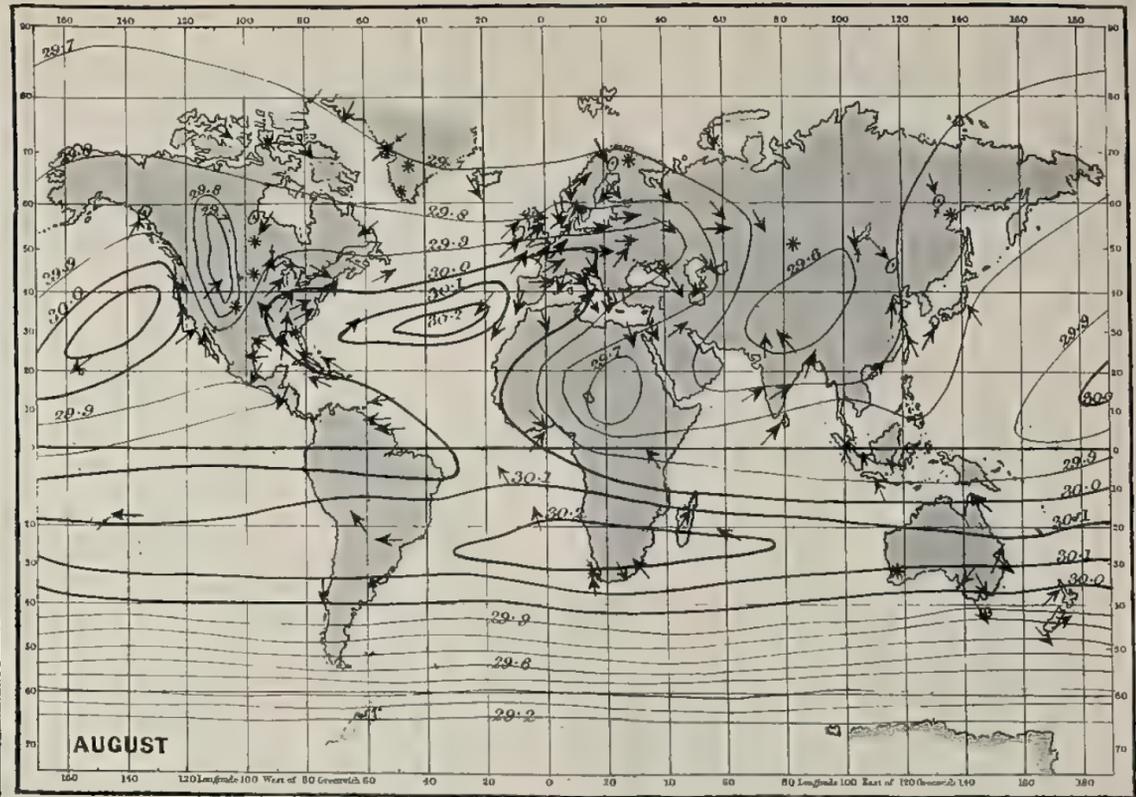
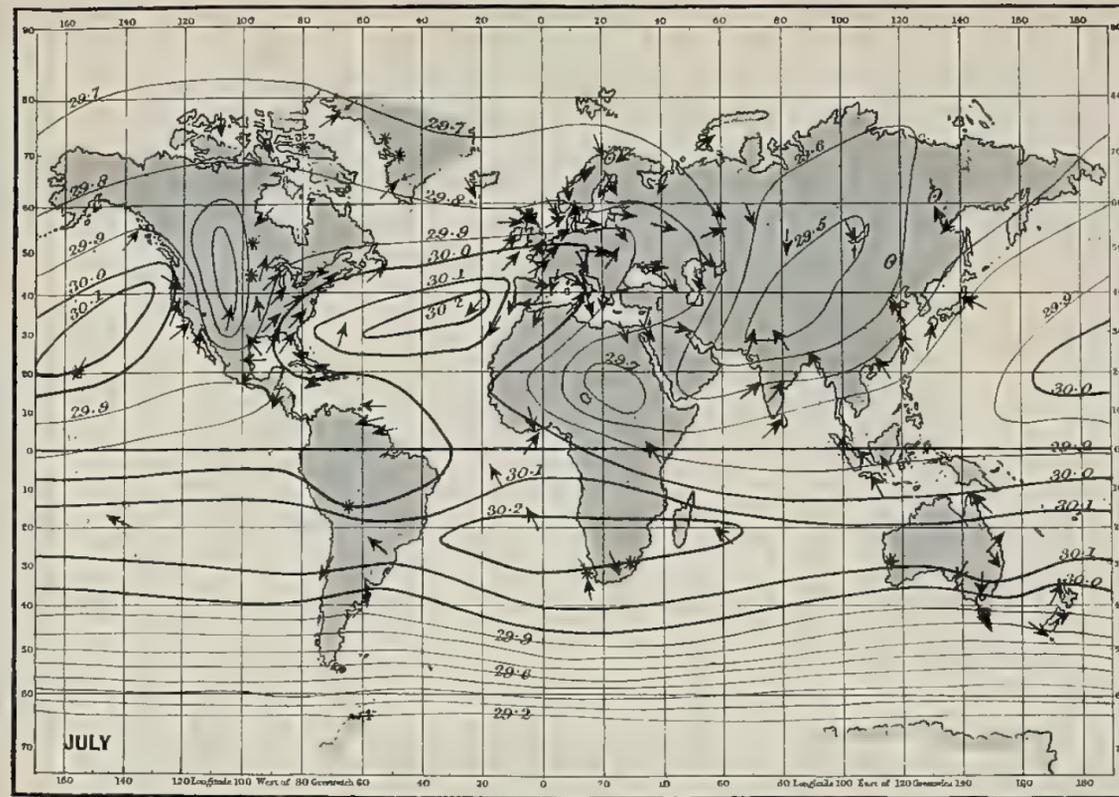
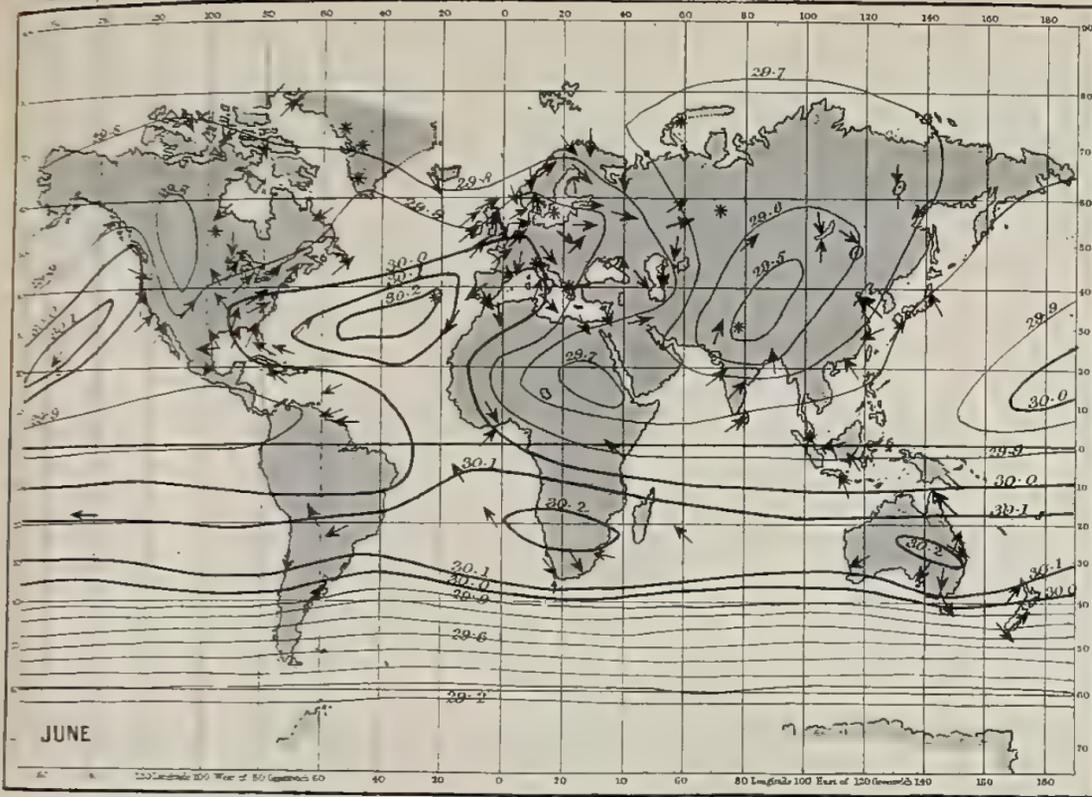






CHARTS SHEWING (1) BY ISOBARIC LINES THE MEAN PRESSURE OF THE ATMOSPHERE AND (2) BY ARROWS THE PREVAILING WINDS OVER THE GLOBE EACH MONTH FROM JUNE TO NOVEMBER.

*Calm's marked thus 0 Variable Winds thus \**



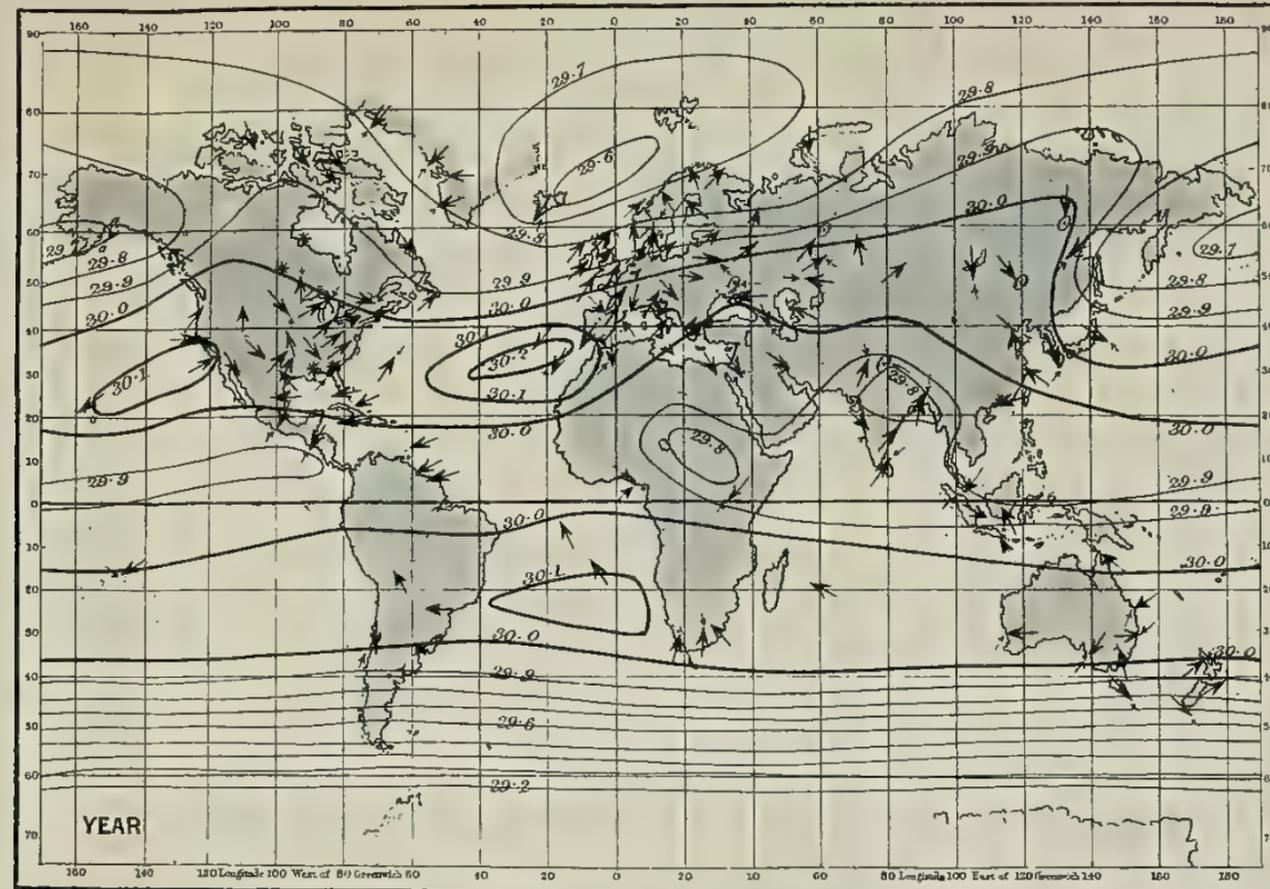


# CHART SHEWING (1) BY ISOBARIC LINES THE MEAN PRESSURE OF THE ATMOSPHERE AND (2) BY ARROWS THE PREVAILING WINDS FOR THE YEAR.

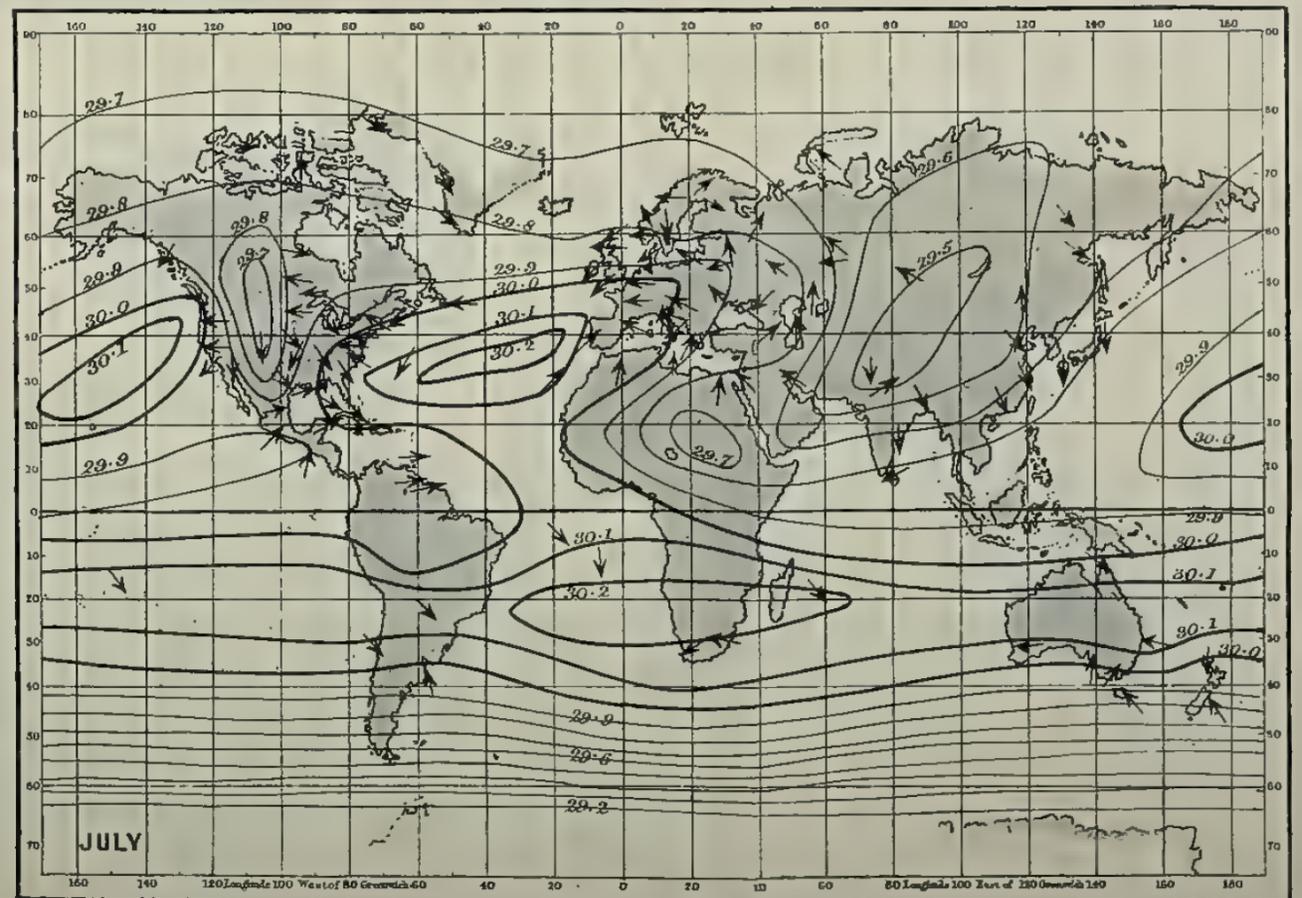
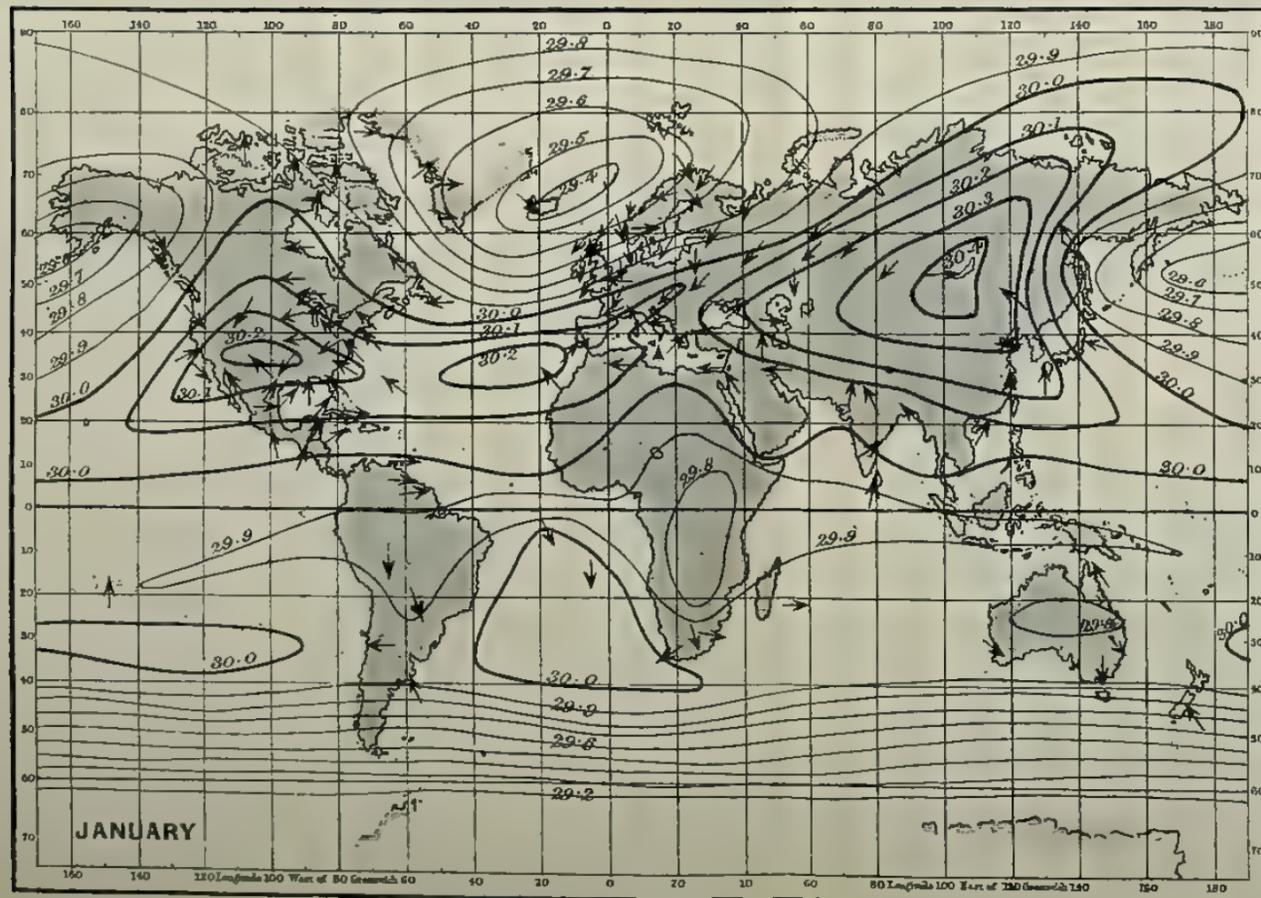
Trans. Roy. Soc. Edin. Vol. XXV.

*(calms marked thus 0 Variable Winds thus \*)*

PLATE XXVII



## CHARTS SHEWING BY ARROWS THE LEAST FREQUENT WINDS WHICH PREVAIL IN JANUARY AND JULY. THE ISOBARIC LINES ARE ALSO GIVEN FOR THESE MONTHS.





XVII.—*The Mean Pressure of the Atmosphere and the Prevailing Winds over the Globe, for the Months and for the Year.* PART II. By ALEXANDER BUCHAN, M.A., Secretary of the Scottish Meteorological Society. (Plates XXV. to XXVII.)

(Read 19th April 1869.)

CHARTS, showing by *Isobaric Lines* the mean pressure of the atmosphere over the globe during the months of the year, may be justly regarded as furnishing the key to all questions of meteorological inquiry; for without the information conveyed by such charts it is impossible to discuss satisfactorily those questions which relate to prevailing winds, the varying temperature, and the rainfall throughout the year in the different countries of the world. It is to meet this desideratum that the Charts of Mean Atmospheric Pressure of the globe which are given with this paper are offered as the first approximate solution of this great physical problem.

Since Part I.\* was read in March 1868, valuable additional information has been obtained from Australia, New Zealand, Tasmania, Africa, South America, the west coast of North America, Iceland, Norway, and Sweden, and from several isolated stations in different parts of Europe and Asia. The period for the British Islands and a large portion of Europe has been extended so as to include the eleven years from 1857 to 1867.

In this Part the complete set of Charts for the twelve months and for the year are given, together with the data from which the Charts have been constructed.

As regards PRESSURE, the stations were selected with the view of representing as well as possible the geographical distribution of the pressure. The first place was assigned to those stations at which the barometric observations were known to be, or presumably were, of the best quality; and in drawing the isobaric curves, the greatest weight was given to means deduced from these observations. Since it is the mean pressure at sea-level which is here inquired into, and since the manner of the geographical distribution of the pressure doubtless varies at different heights, stations at low elevations were preferred to those at greater heights. The pressures at a few elevated places, such as Great St Bernard and Dodabetta, are given in the Tables; but they were not made use of in drawing the curves,—their value consisting in the light they tend to throw on the movements of the upper currents of the atmosphere.

In an inquiry into the comparative distribution of atmospheric pressure, it is

\* Proceedings of the Society, vol. vi. p. 303.

evident that the first requisite, as regards time, is, that the means be deduced from observations made in the same years. In the tropics, where there is great regularity in the mean pressure of the same month from year to year, observations in the same years are of less importance; but in extra-tropical regions, where the mean pressure of the same month varies considerably from year to year, it is of the utmost importance to obtain observations for the same years. This has been the guiding principle in selecting the years from which the means of the different places in Table I. have been calculated. Thus, in the British Islands, the means are uniformly given for the eleven years from 1857 to 1867, and the means of many European stations are given for the same years; and in the United States of America the means are for the six years from 1854 to 1859.

It will be observed, that the means for two or more series of years are given for several places, such as Christiania, Upsal, Toronto, Hobart Town, Algiers, &c. At Christiania, the average for 1861-68 is that for most other Norwegian stations, and that for 1857-67 is the average adopted for the British Islands. For Upsala, the average for 1857-67 is given together with the average for 1859-66, the average of the other stations in Sweden. By the comparisons which may be instituted between these means, a closer approximation to the course of the isobaric curves over this portion of north-western Europe is obtained. Similar comparisons may be made from the data given in Table I. for different regions of the world, and thus the disadvantages, arising from the necessary use of averages of different terms of years, for different places in the same or in neighbouring regions, may, to some extent, be obviated.

In addition to these two classes of averages, the averages deduced from long series of years are given for many places, such as London, 89 years; Turin, 74 years; Bologna, 45 years; Brussels, 38 years; Christiania, 31 years; Toronto, 27 years; Stykkisholm, 23 years; Hobart Town, 28 years, &c. A comparison of these with the other averages will give some indication of the true mean pressure of the atmosphere for different regions of the globe. But for a general survey of the geographical distribution of the mass of the earth's atmosphere through the months of the year, the data, from which the isobaric lines of the charts have been drawn, may be regarded as sufficient. The closer approximations to the true mean pressure of the atmosphere, to be obtained from the accumulated observations of future years, will give the data for more detailed representations of the pressure of the atmosphere over different regions of the earth. If the isobaric curves could be drawn true for every 0.025 inch, the disturbing influence of the Mediterranean, Black and Caspian Seas, and American Lakes; and of the Pyrenees, Alps, Dovrefeld, Himalayas, and other mountain ranges, would be more apparent.

In every case, where possible, the means in the Table are the arithmetic means of the observations, reduced to  $-32^{\circ}$  Fahr. only,—no corrections being

applied for daily range or for height. For places, for which the means were obtained corrected for daily range, "red." (meaning reduced to mean daily pressure), is entered in Table I. in the column of *Hours of Observation*. Those stations for which the means are reduced to sea-level are printed in *italics*.

The next step was to apply to the figures in Table I. corrections (1) for daily range and (2) for height. So far as possible, that hour, or those hours, of observation were selected when the pressure of the atmosphere is nearly the mean of the day. For places for which this could not be done, a collection of *Mean Hourly Variations of the Barometer* was made from a considerable number of stations in different parts of the world. From these, approximate corrections for daily range were deduced, and applied to the monthly means of the stations.

For reducing to sea-level, a table was prepared from the Formula and Table XVI, given in GUYOT'S Meteorological and Physical Tables, D, p. 89. This table, calculated for each 5° Fahr. of the temperature of the air, from — 40° to 90°, was used in all cases where the height did not exceed 800 feet. For higher situations, the reduction was made by means of DIPPE'S method, as detailed in GUYOT'S Tables, D, p. 60.

The means,\* so corrected, were then entered on large polar projections of the northern hemisphere, from which the *isobars* were drawn for every tenth of an English inch of pressure. The isobars for the southern hemisphere were drawn on charts of MERCATOR'S Projection. The whole was ultimately transferred to charts of the projection on Plates XXV. to XXVII. The isobars, indicating a pressure of 30 inches, which is nearly the average pressure, and upwards, are represented on the charts by heavy lines, and lower pressures are represented by light lines.

For many of the means I have been indebted to the labours and writings of DOVE, BUYS BALLOT, SECCHI, CARL JELINCK, MOHN, JAMES, QUETELET, and KUPFFER. For the means of single stations and groups of stations, I have received most valuable assistance from Meteorologists in all parts of the world, for which I beg to return them my grateful thanks.

#### DISTRIBUTION OF ATMOSPHERIC PRESSURE, *in December, January, and February.*

—In these months, the highest pressures are grouped over the land portions of the northern hemisphere, and the larger the extent of the land the greater is the pressure. The area of high barometer (30 inches and upwards) embraces nearly all Asia; all Europe, south of the North and Baltic Seas; the North Atlantic, between 15° and 45° lat.; the West Indies; North America, except the north and north-west; and the North Pacific, between 8° and 24° lat. There are also two regions of high pressure of comparatively small extent—the one in the South Atlantic, and the other in the South Pacific.

\* The original observations are given in Table I. in preference to the corrected means deduced from them.

The regions of low pressure are the northern portions of the North Atlantic and of the North Pacific, including portions of the continents adjoining; the belt of low pressure in the equatorial regions, towards which the trade winds blow; and the remarkable depression in the Antarctic regions, which probably is subject to little variation throughout the year.

*In March*, pressure diminishes over Asia, the middle and south of Europe, and the United States of America. Everywhere else, except in the tropics, it is rising. This rise of pressure is most apparent in the temperate regions of the southern hemisphere. In the north of the Atlantic it is rapidly rising, the average pressure in Iceland now being 29·609 inches, thus showing an increase of 0·34 inch as compared with January.

*In April*, the heavy lines showing a pressure above the average have now all but left Asia, Europe, and the United States, and the isobars of 30 inches bound a belt of high pressure which completely encircles the globe in the south temperate zone. Pressure continues to rise in the north of the Atlantic, and to the north of North America, and it is probable that a space of high pressure (at least 30 inches) surrounds the North Pole. In this month pressure is more equally distributed over the globe than in any other month; for, excepting the Antarctic Ocean, it scarcely rises anywhere above 30·1 inches, or falls below 29·8 inches.

*In May*, in the north of Europe, in Greenland, and in the north of America, atmospheric pressure attains the maximum of the year. Pressure continues to increase over the south temperate zone, and the *isobar* of 30·1 inches now nearly extends round the globe. At this time the highest pressure in the southern hemisphere occurs in the south-east of Australia, where, at Deniliquin, it is 30·185 inches. Pressure is rapidly falling over Asia and the United States.

*In June, July, and August*, pressure falls in the central regions of Asia to about 29·5 inches. In this season this great diminution of pressure, which may be regarded as absolutely determining the summer climates of Asia, reaches its lowest point. Pressure falls also in the interior of North America, where at Utah, Great Salt Lake, it is only about 29·7 inches. The annual maximum of the south temperate zone is attained in these months. The isobar of 30·1 inches goes completely round the globe, and a still higher pressure prevails over the south of Africa, and over those parts of the ocean immediately to the west and east of it. In these months the arrangement of the isobars may be regarded as being, generally speaking, reversed from that of December, January, and February, and on this account a comparison of these two groups of months is very instructive.

From this period, pressures increase over the continents of the northern hemisphere, and diminish over the south temperate zone, till the distribution of pressure is regained, which has been already shown to prevail during the winter months. *In September and October*, an interesting feature of these lines is a very

rapid diminution of the pressure indicated as taking place in the north of the Atlantic and adjoining regions. This is the season of the year when the first great decrease of temperature takes place, which is accompanied by heavy rains and furious storms. The increase of pressure in Sweden in October, taken in connection with the simultaneous decrease in Greenland, Iceland, north of Norway, and the British Islands, is interesting, as bearing on the transference of masses of the atmosphere from one region to another.

*In November*, pressure rises considerably over the continents of the northern hemisphere, and falls in the south temperate zone; and the belt of low pressure in the equatorial regions may be regarded as now passing completely round the globe. This belt, towards which the trades on each side of the equator blow, does not occur in the summer months in the Indian Ocean; but, on the contrary, there is a continuous diminution of pressure northwards, from Australia and Mauritius to the interior of Asia. It will be seen that in November, as compared with October, the isobars have advanced a little northwards from the British Isles to Iceland, and eastwards from Baffin's Bay to Iceland, thus indicating a general increase of pressure over the north of the Atlantic and regions adjoining. Coincident with this increase of pressure, there occurs a diminution of pressure to the south-east of it, including Austria, Italy, and countries adjoining the Mediterranean; and in the Atlantic to the south of it, from about latitude  $45^{\circ}$  to  $15^{\circ}$  N. Probably these extensive oscillations of the pressure are parts of one general movement of the atmosphere, which in one of its manifestations has been long known to meteorologists under the name of the great November wave, but of which no very satisfactory account has yet been given.

In addition to these changes in the monthly distribution of the pressure, it is probable that a system of low pressures traverses the continent of Africa, following the sun's course; but since the grounds of this supposition have been recently laid before the Society, in a paper on "The Determination of Heights, chiefly in the Interior of Continents, by Observations of Atmospheric Pressure,"\* it is not necessary to reproduce them here. The probable pressure for the months is shown on the separate charts.

PREVAILING WINDS.—It will be seen that every one of the charts shows considerable disturbance of the equilibrium of the atmosphere at the surface of the earth. If the pressure was equal in all parts of the globe, we should have the physical conditions of a stagnant atmosphere. But such is not the case. From the different pressures which the charts show in different regions, it might be expected, from the laws of aerial fluids, that movements of the atmosphere would set in, giving rise to the prevailing winds of these regions.

\* Proceedings of the Roy. Soc. Edin. vol. vi. p. 465.

To ascertain what relation may subsist between mean atmospheric pressure and prevailing winds, Table II. has been prepared, which shows the mean number of days in each month winds from N., N.E., E., S.E., S., S.W., W., and N.W. have been observed to blow. In tropical and subtropical regions, a period of one or two years is sufficient to indicate the average direction, or the prevailing winds of the locality, owing to the steadiness with which the wind blows in these regions; but in temperate and polar regions a considerable number of years is indispensable. The direction of the winds has been generally obtained, or calculated, for the same years as the atmospheric pressure; but where such observations could not be obtained, care was taken to include in the list only those places for which a sufficient number of years was available, so as to give a good average. As regards the stations in British North America, the shortness of the time during which, in each case, the observations were made, is to a great extent compensated for by the number of places at which observations have been made, and the comparative steadiness of the winds in these high latitudes of America. Valuable assistance was obtained from Professor COFFIN'S elaborate "Treatise on the Winds of the Northern Hemisphere," though many averages given in this work could not be adopted, being based on an insufficient number of years,—a remark which applies extensively to averages of observations of the wind hitherto published.

In selecting stations, a preference was given to those which are situated in comparatively level localities, with the view of obtaining as close an approximation as possible to the true direction of the wind. To this there are, however, several exceptions, such as the stations in Norway and in Greenland, these places being given to illustrate the effect of mountain ranges in changing the mean direction of the wind. Stations at no great elevation above the sea were selected in different regions, it being evident that winds observed at great elevations are not suited to an inquiry into the movements of the atmosphere in relation to sea-level pressures.

It will be observed that the time, or the duration of the prevalence of each wind (N., N.E., E., &c.), is the only element taken into account in this inquiry. The element of force has, for several reasons, been neglected:—(1.) The force of the wind has been less generally observed than the direction; and at very many places where the force has been observed, the observations, from the manner in which they have been made, do not give the materials for arriving at absolute results. (2.) It is well known that the velocity of the wind is retarded by the land as it passes across it; thus, for example, an anemometer on the west coast of the British Isles registers considerably more wind than one erected at an inland or eastern situation. Also, more wind is registered in rising above the surface of the earth. The effect of local situation on the velocity of the different winds is very great. Hence, whilst the amount of these disturbing influences are

unknown, to attempt to determine the velocity of any general movement of the atmosphere from the observed velocity of the wind at Observatories could lead to no satisfactory result as regards the present inquiry. In the broad results aimed at in this comparison of atmospheric pressure and prevailing winds, it cannot affect the conclusions arrived at to assume, as is here done, that the mean velocities of winds from different directions are equal to each other.

From the figures given in Table II. the mean direction of the wind has been calculated in the usual way. This direction is represented in the charts by arrows flying in the direction of the wind. In cases where the winds do not preponderate from one quarter, but are nearly equally distributed over different points of the compass, an asterisk (\*) is entered on the charts, which thus represents variable winds; when calms preponderate, a circle with a dot in the centre (⊙) is used. When two maximum directions are strongly indicated, or when the smaller one is very decidedly marked, instead of resolving the two into one intermediate, which would in many cases represent a wind which scarcely ever occurs at the time, the greater of the two is represented by the ordinary arrow, and the smaller in the more marked cases by a less arrow placed beside it. By this means an important feature in climate is represented.

Thus two distinct sets of facts are exhibited on the charts, viz., lines showing the mean pressure of the atmosphere, and arrows showing the prevailing winds at the earth's surface, each being independently arrived at by the summing and averaging of observed facts. What relation is there between these two classes of facts?

I. WINDS *within, or near, a space of Low Pressure.*—Of this class, the best example is the low pressure which prevails in the north of the Atlantic and adjoining regions in the winter months. This region of low pressure is bounded to the S.W. by the high pressure of North America; to the S. by the high pressure in the Atlantic, about 30° lat. N.; to the S.E. by the high pressure in the interior of Asia. In January, the difference between the average pressure of Iceland and the interior of Asia is fully an inch.

It is seen from the charts, that in Baffin's Bay and east of the Rocky Mountains, as far south as 40° lat., the winds are N.N.W., N.W., and W.N.W. Crossing the Atlantic, winds in the British Islands, in France, and the north of Germany, are from W.S.W. to S.W.; in Denmark, S.S.W.; near Bergen, in Norway, S.; and at Christiansund and Hammerfest, S.S.E. The relation of these winds to the isobaric lines is the same as that which is illustrated by the winds in storms, in their relation to the isobaric lines of these storms. This has been already stated in a Paper by the author, published in the "Transactions of the Society," vol. xxiv. Part i. p. 201, in the following words:—"The wind in storms neither blows round the centre of least pressure in circles, or as tangents to the concentric isobaric curves, nor does it blow directly towards that centre. It

takes a direction intermediate, approaching, however, more nearly to the direction and course of the circular curves than of the radii to the centre." Or, according to Dr BUYS BALLOT, the angle is not a right angle, but from about  $60^{\circ}$  to  $80^{\circ}$ . This relation is usually called "BUYS BALLOT'S LAW OF THE WINDS."

Another well-marked depression is the low summer pressure in the interior of Asia; with reference to which, it is seen from the charts that the winds of Eastern Europe and Western Asia are from N.W. to W.N.W. and W.; at Ceylon, S.W.; at Shanghai, S.E.; and on the Sea of Okotsk, N.E.; whilst in the interior, calms generally prevail.

The behaviour of the winds, as regards the low pressure of North America, is exactly similar to that of the winds in Asia at this season. In all these cases the wind appears to flow round and in upon the space where pressures are low. Even in those instances where the depression over a limited space is comparatively small, such as in Australia during the summer months, the winds observe the same course with respect to it.

A well-known and remarkable diminution of pressure is that of the Antarctic regions; and though, except in Tasmania and the south of New Zealand, observations are wanting at particular points for a sufficiently long time to give good averages, yet the concurrent testimony of sailors and the inhabitants of these regions all go to show that, at least on the outskirts of the region, winds are chiefly N.W. or W.N.W.—that is, they appear to flow in upon the space of low pressure. The low pressure in the equatorial regions, towards which the trades blow, is an illustration of the same principle.

WINDS *within, or near, a space of High Pressure.*—The most prominent illustration of this is the high pressure in the interior of Asia in winter. It is seen from a single glance at the charts that the winds flow *out of* this space in every direction. The same *outflow* is seen with respect to the less strongly marked, but still very distinct space of high pressure in North America; owing to the large number of stations available here, this principle is amply illustrated.

The next most noteworthy area of high pressure occurs in summer between Africa and North America, out of which also the charts show the winds blowing in all directions towards and round upon the surrounding low pressures.

The following mean pressures, in inches, at  $32^{\circ}$  and sea-level, occur in Australia in June:—At Brisbane, Queensland, 30·062; Sydney, 30·116; Melbourne, 30·178; Adelaide, 30·132; Freemantle, 30·121; and at Deniliquin, in the interior, on a branch of the Murray River, 30·217. Hence a higher pressure occurs at this season (winter) in the interior, and it may be inferred that it is greatest in the southern portion of the interior. The prevailing winds are these:—At Brisbane, S.S.W.; Sydney, W. by N.W.; Melbourne, N.; Adelaide, N.E. by N.; Freemantle, N.E. by E.; in other words, the winds blow out from this space of high pressure.

This behaviour of the winds with respect to spaces of high pressure differs in

no respect from what occurs on particular days on which the isobaric lines present the same conditions of pressure. Mr FRANCIS GALTON first drew attention to this peculiarity, under the name of *Anticyclones*, by which name he intended to convey the idea that in cases of high pressure occurring over a limited area, the course of the winds is exactly the reverse of what is seen to prevail in cyclones in which the winds blow round and in upon a space of low pressure.

The *outflow* of the air from a region of high pressure, and the *inflow* upon a region of low pressure, appears to be reducible to a single principle, viz., the principle of gravitation. Given as observed facts the differences of pressure, it might almost be predicted, before calculating the averages, what the prevailing winds are. Indeed, so predominating is the influence of gravitation that it may be regarded as the sole force immediately concerned in determining the movements of the atmosphere. If there be any other force or forces which set the winds in motion, their influence must be altogether insignificant as compared with gravitation.

The effect of a mountain range interposed in the course of one of these great atmospheric currents is interesting. Of this, the best example is furnished by the mountain range of the Scandinavian peninsula, in its effect on the prevailing winds in winter. It will be observed that this mountain range lies between the low pressure about Iceland, and the high pressure in the interior of Asia.

The following are the mean directions of the wind at different places in Norway in January, deduced from Table II., to which are added the winds at other points, courteously sent by Professor MOHN:—Christiania, N.E.; Sandösund, N.N.E.; Lindesnes, N.E.; Mandal, N.E. by E.; Lister, E.; Skudesnes, S.S.E.; Udsire and Bergen, S.; Christiansund, S.S.E.; Villa, S.E.; Hammerfest, S.E. by S.; and Vardö, S.W. Thus at Christiania, Sandösund, and Lindesnes, which lie on the east side of the south spur of the mountain range, the prevailing winds are N.E. or N.N.E.; at Mandal, at the extreme south point of Norway, the wind is N.E. by E., and calms also largely prevail; and at Lister, a little to the west, the wind is E.; along the whole west coast from Skudesnes to Hammerfest, near the North Cape, winds are chiefly S., S.S.E., or S.E.; while at Vardö, to the east of the north spur of the mountain range, the prevailing winds are S.W. These directions are very much the directions water should take in flowing past and round a rock lying in the bed of the current; the Scandinavian mountains being in this case the obstacle which diverts the winds from what may be called their normal course in flowing towards and round the low pressure in the north of the Atlantic.

On the other hand, in July it is seen from the Chart (Plate XXVI.) that the lowest pressures occur in the interior of Asia, towards which there is an extensive aërial current from W.S.W., W., and N.W. over Europe and Western Asia. Here also the influence of the mountain system of Norway is very perceptible. The following are the prevailing winds in July:—Christiansund, N.W. by N.; Bergen, N.; Skudesnes, N.W.; Lindesnes, W.; Mandal, W. by S.W.; Sandösund, S.W.;

Christiania, S. Thus, this part of the great atmospheric current flows round the southern region of Norway, being N. at Bergen, thence in succession N.W. and W., and on rounding the coast becomes diverted into a SW. and S. wind. The extraordinary deflection of the isothermal lines in the different months, as they cross Norway and Sweden, is doubtless to no inconsiderable extent occasioned by the opposite prevailing winds, which arise from the obstruction presented by the mountain range to the prevailing atmospheric currents of the seasons.

The prevailing winds at Upernivik, Jacobshavn, and Godthaab, in the west of Greenland, appear to point to another principle. It will be seen from the Charts and the Table that the prevailing winds in the winter months at these stations are N.E. and E. instead of N. and N.N.W., which, from the analogy of the winds at other places, they might have been supposed to be. They are thus diverted a few points from their proper course in the direction of E., or, roughly speaking, into a direction which is perpendicular to the line of the coast; in other words, they follow the course of the ravines. The daily observations at the Greenland stations have been published by the Danish Academy of Sciences, from which the averages have been calculated. A large proportion of these winds are very light, being frequently marked 0 by the observers; that is, they were too light to be represented by the scale for wind force in use (0 to 4.) Since the west coast of Greenland is bounded immediately to the east by a steep high mountain range covered with snow, it is probable that the direction of these winds is still further modified by the same causes which give rise to the well-known class of breezes peculiar to mountain districts, of which the *Vent du Mont Blanc* is an example. These breezes are caused by the cooling of the air in immediate contact with the high ground, which, thus acquiring greater density, flows down their slopes, and thence diffuses itself over the low ground as a surface wind of inconsiderable depth. The Greenland stations are in those very situations which expose them to this wind. It is in favour of this supposition that these easterly winds occur oftenest, and blow with greatest force in the afternoon, it being at this time of the day that the difference is greatest between the temperature of the low grounds and that of the snow-covered mountains; just as sea and land breezes are strongest at those hours of the day, when the difference of temperature is greatest between the sea and land.

It will be observed that at St Helena the mean direction of the wind varies little from month to month,—being almost uniformly from S.E. or S.; and it will also be observed that the relative distribution of the pressure in neighbouring regions varies little from month to month. The result is one mean annual direction of nearly S.E. by S. At Mauritius there occurs a little variation from month to month. Thus, whilst in June, July, and August, the mean direction is about S.E. by E., in December, January, and February, it is nearly due E.; in other words, during the summer season the wind shifts a few points from S.E. by E. in

the direction of N. This northing of the winds at Mauritius is exactly what should be expected to result from the proximity to the low pressures which prevail in South Africa at this season. This change in the mean direction of the wind being small, the mean annual direction may be regarded as E.S.E.

Such slight variation, however, is limited to very few regions, for on examining the number of days each wind (N., N.E., E., &c.) has on the *mean of the year* prevailed at the different stations in Table II., it will be observed that in almost every instance there are two maximum directions, the one being considerably greater than the other. These maximum directions may arise in two ways—*1st*, At places such as Colombo, Ceylon, where the wind during summer is S.W., and during winter chiefly N.E., the two annual maximum directions are S.W. and N.E.; in like manner the maximum directions at all places in monsoon regions are occasioned. *2d*, At Greenwich two maximum directions, from about S.W. and N.E., appear in the means of every month, from which it is evident that the wind at this place blows oftener, and remains longer, in these two directions than in any other. In cases where the less maximum arises from the prevalence of winds from that direction during a few months of the year, the isobaric charts of the separate months give a ready explanation of both maxima in the annual means. But at places where both maxima appear in the same months, it is evident that the isobaric charts can only furnish data towards the explanation of the greater maximum direction; and it may be assumed as equally evident that the smaller maximum, of which the east wind of the British Islands is an illustration, can alone be legitimately discussed by daily synoptic charts of the weather. Towards the discussion of this and other questions of meteorology, the value of the Daily Synoptic Charts prepared and issued under the superintendence of M. LEVERRIER cannot be overrated.\*

There are 115 stations in Table II. situated in the north temperate zone. I have tabulated the two maximum directions at these stations where they occur according to sixteen points of the compass—viz., N., N.N.E., N.E., E.N.E., E., &c. of which the following Table shows the maximum directions most frequently observed:—

Maximum directions of wind are—			Greater Max.	Smaller Max.	at	15	stations.
			S.W.	N.E.			
"	"	"	N.	S.	"	8	"
"	"	"	N.E.	S.W.	"	7	"
"	"	"	N.W.	S.E.	"	6	"
"	"	"	S.	N.	"	5	"
"	"	"	W.	N.E.	"	5	"
"	"	"	N.W.	S.	"	4	"
"	"	"	W.	E.	"	3	"
"	"	"	E.	W.	"	3	"
"	"	"	W.S.W.	E.	"	3	"
"	"	"	W.S.W.	S.E.	"	3	"

\* Atlas des Mouvements Généraux de l'Atmosphère pour 1864-5.

Thus the number of stations, at which the greater maximum direction of the wind is S.W., and the smaller, N.E., is 15. Further, if the whole 115 stations be examined, and those picked out at which the greater maximum direction is from any point between S.S.W. and W., and the smaller maximum from any point between N.N.E. and E., the number is found to amount only to 34, or less than 30 per cent. of the whole. Now, since these are the directions in which truly equatorial and truly polar atmospheric currents should blow, it is evident that these two currents, as often represented, are not the two prevailing winds generally observed over the north temperate zone. For if the two great currents of the atmosphere were, one flowing from subtropical regions towards the poles, and the other flowing from the poles towards the tropics, it is plain that a much larger percentage of the stations than 30 would follow the course of these currents in the north temperate zone. If to these 34 stations, at which the maximum directions are in the course of the equatorial and polar currents respectively, we add 17 stations, at which the greater maximum direction is from N.N.E. to E., and the smaller from S.S.W. to W., the number of stations at which the two prevailing winds follow the course of these two currents is only 51, or about 44 per cent. of the whole—a proportion, it need scarcely be said, which could not obtain, if it be the case that there is a general flow of the atmosphere at the surface of the earth in the northern hemisphere from the tropics towards the north pole, and from the north pole towards the tropics.

Further, if the two maximum directions be separately examined, it is seen that the *greater* maximum direction being from any point of the compass from

S.S.W. to W.	occurs at 47 stations.
W.N.W. to N.	„ 33 „
N.N.E. to E.	„ 19 „
E.S.E. to S.	„ 16 „

and the *lesser* maximum direction being from any point from

S.S.W. to W.	occurs at 20 stations.
W.N.W. to W.	„ 22 „
N.N.E. to E.	„ 38 „
E.S.E. to S.	„ 32 „

Thus the chief prevailing winds in the north temperate zone blow from some point from S.S.W. to W. at 41 per cent. of the stations, leaving 59 per cent of the stations at which the prevailing winds are from other points of the compass; and the secondary prevailing winds come from some point from N.N.E. to E. at 34 per cent. of the stations, or only a third of the whole. Hence, as in the former case, while the largest percentages of prevailing winds are in the directions in which truly equatorial and polar currents should blow, the percentages from other directions are so large as to preclude the supposition of a general flow of

the surface winds of north temperate regions towards and from the polar regions.

An examination of the isobaric and wind charts for the months shows, as has been already pointed out, that where there is a mean low pressure, such as occurs in the north of the Atlantic in the winter months, and in the centre of Asia in the summer months, thitherward the winds tend in all directions in an inmoving spiral course; and where there occurs a mean high pressure, as in the centre of Asia in winter, and in the Atlantic between Africa and the United States in summer, out of this space the winds flow in all directions, or they appear to be thrown out from the space of high pressure in a manner exactly the reverse from that by which they are drawn inward upon a space of low pressure. These spaces of low and high pressures may therefore be regarded as the true poles of the winds, which blow at the surface of the earth, towards which, and from which, the great movements of the atmosphere proceed. From the unequal distribution of land and water, it results that the poles of the pressure and movements of the atmosphere are, as in the case of the poles of temperature, very far from being coincident with the north pole.

The causes which bring about an unequal distribution of the mass of the earth's atmosphere may be considered to be chiefly two, viz., the temperature primarily; and, secondarily, the moisture of the atmosphere, in their relations to the geographical distribution of land and water. From the relations of land and water to temperature, the summer temperature of continents greatly exceeds that of the ocean in the same latitudes. Hence the abnormally high temperatures which prevail in Asia, Africa, and North America during summer, in consequence of which the air becomes specifically lighter, and ascends, as from a furnace, in vast columns thousands of miles in diameter. In this way the summer pressure of continents is diminished, the amount of the decrease being greatest in Asia, the largest continent, and least in Australia, the smallest. At Barnaul, in Asia, the pressure in July is 0.418 inch below the annual average;\* whereas at Deniliquin, in Australia, the pressure in January is only 0.154 inch below the annual average: at Great Salt Lake, in North America, it is intermediate, being 0.333 inch.

In the remarks which follow on the vapour of the atmosphere, the principles laid down in the two following extracts are assumed:—1. "Air charged with vapour, or vaporised air, is specifically lighter than when without the vapour; or, in other words, the more vapour any given quantity of atmospheric air has in it, the less is its specific gravity."† 2. "It appears, therefore, that the explanation suggested by Dr JOULE is correct; and that the condensation of vapour in ascend-

\* Some part of this diminished pressure in Asia is doubtless due to the condensation of the vapour of the south-west monsoon.

† DALTON'S Meteorological Observations and Essays, 2d ed. Manchester, 1834, p. 100.

ing air is the chief cause of the cooling effect being so much less than that which would be experienced by the dry air.”\*

The influence of vapour in lowering the pressure is well illustrated by the low pressure in the tropics towards which the trades blow, this belt being characterised by a highly saturated atmosphere and heavy rains.

Again, much more vapour is observed in the air at places near the shores of the north of the Atlantic in winter than at places in the same latitudes in the interior of continents. In Great Britain, as compared with the interior of Asia, the excess is great; and in the former case the skies are generally cloudy, and in the latter clear. Also, over the same region, the atmosphere of which abounds in vapour, the mean winter temperature is much higher than it is on the continent; and from the conclusion arrived at by Sir WILLIAM THOMSON regarding the temperature of an ascending column of saturated air, the relatively higher temperature over such regions must continue to prevail up to very great heights. Hence, owing to the presence of a larger amount of vapour, and to a higher temperature, the air resting on the north of the Atlantic and regions adjoining is specifically lighter than in the continents which surround it; consequently the charts show an enormous diminution of pressure over this region, as compared with the continents. Similar depressions from like causes occur in the north of the Pacific and in the Antarctic regions.

Since dry and cold air is, on the other hand, specifically heavy, we should expect that in the interior of continents, where temperatures are low and the air is dry in winter, that pressures would be high; and observations show (see the Charts) that the highest mean pressures occur in Asia and North America at this season. For the same reason, pressures are also highest in Australia, South Africa, and the south of South America in the winter months.

There is another source from which atmospheric pressure is increased. It has been shown from the Charts that the tendency of the prevailing winds on the surface of the earth is to blow round and in upon the space where pressures are low, and out of the space where pressures are high. Now, since in this way vast volumes of air are poured into the space where pressure is low without increasing that pressure, and vast volumes flow out of the space of high pressure without diminishing that pressure, it follows that the air poured in is not allowed to accumulate over this space, but must escape into other regions; and that the air which flows out from the place of high pressure must have its place supplied by fresh accessions from above. The exchange indicated here is probably brought about in this way:—Since in winter, over the north of the Atlantic, the atmosphere is specifically lighter than in surrounding regions, there are here the conditions of an ascending current; and it may be inferred that the ascent will continue until

\* Sir WILLIAM THOMSON in Mem. Lit. and Phil. Soc. Manchester, vol. ii. 3d series, p. 131.

a height is attained at which pressures *at that level are equal*; thence the air will flow over, as an upper current, towards those regions which offer the least resistance to its course,—in other words, where the tension at that height is least. Over what part of the earth's surface is the pressure of the air least at great heights? Evidently, that region over which the air is coldest and driest near the surface of the earth; because, being thereby densest, the great mass of the air is condensed or gathered together in the lower beds of the atmosphere, thus leaving less air, or a diminished pressure, in the upper regions. Thus the extraordinarily high pressure in Asia in winter will be due both to the low temperature and great dryness of the atmosphere, and to proximity to the regions of low pressure in the north of the Atlantic, the north of the Pacific, and in the equatorial regions to the south; from which it may be inferred that upper currents flow towards the centre of Asia, and that these upper currents compensate for the drain arising from the surface currents, which flow out of this space in all directions. In corroboration of this view, it is seen that while in winter the winds in India at low levels blow from some northerly point, at Dodabetta, on the Neilgherry Hills, 8640 feet high, the mean direction of the wind during winter is from about E.S.E.; and, on the other hand, while in summer winds blow from some southerly point at low levels, at Dodabetta they are almost wholly N.W.

But by far the most striking illustration of this principle, is the high pressure in summer which prevails in the Atlantic, between Africa and North America. If the principle here suggested, as regulating the movements of the atmosphere, be correct, the following will be the explanation of this singularly high pressure:—Since, at this season, the temperature of the air resting on this part of the ocean is much lower than that of the land, it follows that the ascending currents which rise from the heated plains of Africa, South America, North America, and Europe, and from the tropical belt of calms to the south, will, on reaching the upper regions of the atmosphere, flow over upon this part of the Atlantic, because the temperature being comparatively low in the lower beds, the air is condensed there, thus leaving less pressure in the upper regions. It may also be added, that since the surface winds of this region are constantly drawing away the air poured down upon it by the upper currents, extreme saturation of the atmosphere cannot take place; and hence the atmosphere is relatively cool and dry. The high pressure maintained in the South Atlantic, between Africa and South America in the summer of the southern hemisphere, corroborates this view.

From these considerations, it may be concluded that the winds on the surface of the earth are approximately known from the isobaric lines,—the direction being from regions of high towards those of low pressure, subject to the changes in the direction of the currents produced by the earth's rotation; and that the upper currents of the atmosphere may be inferred from the isobaric lines

taken reversely, together with the isothermal lines taken directly. In other words, the regions of lowest pressure, by giving the ascending currents, point out the sources or fountains whence the upper currents flow; and the isothermals, by showing where, on account of the low temperature, the greater portion of the air is condensed in the lower beds, and so diminishing the pressure in the upper beds, point out the regions towards and over which the upper currents diffuse themselves.

1. To travellers in the interior of continents and regions at a great distance from places where Meteorological Observations are made, the Charts will be of use in showing the approximate sea-level pressures for each month of the year. They show at a glance the *zero points* from which the heights of places may be calculated, at which observations of the pressure of the atmosphere have been made.\*

2. To sailors, the Charts will be useful as showing the prevailing winds at many places in each of the twelve months, and still further as suggesting, from the connection which is here pointed out between mean atmospheric pressure and prevailing winds, the winds which are most likely to be met with in regions where little is known of the general course of the winds from actual observations. The charts of least prevailing winds in January and July (Plate XXVII.), will also be useful in this respect.

The following illustration will show the method of using the Charts in applying BUYS BALLOT'S LAW OF THE WINDS. This law has been stated at the foot of page 581, but it may be more popularly expressed thus,—Stand with your back to the wind; and the low barometer will be to your left in the northern hemisphere; † or, reversing it, stand with the high barometer to your right, and the low barometer to your left, and the wind will blow on your back. Suppose, during the summer months, a person at Lisbon to stand so, with reference to the high pressure in the Atlantic, and the low pressure in Africa, he should have a N.N.E. wind; and as he proceeded southward along the coast of Africa, the wind would wear more to eastward. On the north coast of South America, being between the high pressure of the Atlantic and the low pressure of South America, the winds should be about easterly; and on the north coast of Central America, the low pressure in the Pacific being now to his left, the winds should be about N.N.E. On passing through the West Indies towards Florida and the south-eastern States, as the influence of the low pressure in North America in its relations to the high pressure in the Atlantic comes into play, the prevailing winds should gradually become E., E.S.E., S., S.S.W., and S.W., and from this region to England to about W.S.W. These are, it need scarcely be said, the prevailing winds of these regions.

\* Note of the Determination of Heights, chiefly in the interior of Continents, from Observations of Atmospheric Pressure.—“Proceedings of the Society,” vol. vi. p. 465.

† In the southern hemisphere the low barometer will be to the right.

Further, the Table of Winds shows a total absence of westerly winds on the north coast of South America, the winds there being almost always from N.E. to E. or occasionally S.E.; whereas, at Bermuda, the winds, while mostly S.W., are more distributed over the other points of the compass. The daily pressures charted in LEVERRIER'S "Atlas des Mouvements Généraux de l'Atmosphère," give a ready explanation of the winds of these two regions—pressures in the one case being comparatively steady, whilst in the other they are fluctuating.

3. Since winds bring with them the temperature and vapour of the regions they have traversed, it follows that the data mapped on the Charts may be considered as furnishing the key to the climates of the different parts of the globe, since the approximate temperature and rainfall of the different seasons may thereby be known. Between the monthly isobars and the rainy seasons of portions of Asia, Africa, America, and Australia, there is an obvious connection. The distribution of the pressure also explains the greater rainfall which occurs in Russia and other places in the interior of Europe in summer, as compared with the other seasons. For, if the winds of July in Table II. be compared with those in January at British, French, German, Russian, and other European stations, they will be found uniformly to show a shifting of the prevailing winds farther to the west and north,—a change, doubtless, arising from the low pressures in Asia in summer. The effect of this is, to draw over these parts of Europe, during the summer months, air-currents more directly from the ocean than in the other seasons, from which result a larger rainfall and greater fertility to these regions.

The political importance even of such information will be seen when it is considered, that if there had been two or more years' Meteorological Observations, especially of atmospheric pressure and winds, at Aden, Massuah, and Suez, at the beginning of the Abyssinian war, the time of the commencement of the rainy season in Abyssinia could have been stated.

The Charts of monthly isobars, the monthly isothermals, and the information tabulated in Table II., furnish materials from which more exact information regarding the climate of a particular place may be obtained. Thus, suppose it were required to know something of the climate of Shanghai, China. The mean temperature in January is about 40°, being nearly that of the west coast of Scotland; the isobaric lines show an increase of pressure from Shanghai in the direction of the interior of the continent; the winds for January are these:—

Days each wind has prevailed (January) at Shanghai,	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
	8	2	2	3	2	2	3	10	0

Thus the mean direction of the wind is nearly N.N.W.; and, since this wind comes from the continent, it may be concluded that it is dry, and, consequently, that very low temperatures are of certain occurrence. Again, since in nine days winds blow from N.E., E., S.E., and S., or from the ocean, and these winds, especially

from the S.E. and S., may be expected to be warm and moist, it may further be inferred, that the winter climate, so far as regards the two important elements of heat and moisture, is subject to great fluctuation. On the other hand, since the July mean temperature is about  $80^{\circ}$ , and in the same month the wind blows ten days from S.E. and fourteen from S., it may be concluded that the summer climate will be hot, stifling, and relaxing. These are shown, by observation, to be the peculiar characteristics of the climate of Shanghai in winter and in summer.

4. An inquiry of still greater interest is suggested by the isobaric lines of the Charts. Their position appears to be altogether determined by the geographical distribution of land and water on the surface of the earth; and since the isobaric lines determine the prevailing winds, and these in their turn the peculiar distribution of temperature and rainfall,—in other words, the climates of the globe,—it is evident that we have here a principle applicable not merely to the present state of the earth, but also to *different distributions of land and water in past time*. In other words, there is here a principle which the geologist will require to apply in attempting to account for glacial and warm epochs, through which the climates of great Britain and other countries have passed. In this way it is possible to arrive at an approximate numerical statement, as regards temperature and rainfall, of Sir CHARLES LYELL's idea of the changes of climate brought about through the displacements of continents.

The following instances will serve to illustrate the effect of the partial displacements of continents in changing climate. On examining the chart for July (Plate XXVI.), it is seen that the fine summer climates of Western Europe, and of the Eastern States of America, are caused by south-westerly prevailing winds, which, having their origin in the region of high pressure in the Atlantic, possess, in admirable proportions, the genial qualities of warmth and moisture. Since these winds depend on the high pressure in the Atlantic between Africa and the United States, whatever would alter this arrangement of the pressure may be expected to change the character of the climates. Suppose, then, a displacement of the continents, either of Africa or South America, so that land would occupy the place of the part of the ocean lying between Africa and the United States. With this new disposition of the land, it is plain that the high pressure in the Atlantic would disappear, and the spaces of low pressure in Asia, Africa, and North America would unite into one region of low pressure, stretching from the west of North America to the east of Asia. Simultaneously with this change in the pressure, the winds of the United States and Western Europe, including Great Britain, would become northerly, and, as a consequence, the summer climates of large portions of these regions would be so seriously deteriorated that the cultivation of cereals could not be attempted.

Observations show that the lowest pressures which accompany the storms which traverse Europe, or the centres of these storms, pass eastward for the most

part, in a course lying somewhere between Iceland and Farö; as a consequence of this, and of the mean low pressure in the north of the Atlantic in the winter months, the prevailing winds in Great Britain at this season are south-westerly, and even in stormy weather the wind seldom veers further towards the north than N.W., and continues only for a short time in this quarter. To these considerations we owe the mildness and equableness of the winter climate of Great Britain. At Stykkisholm, in the north-west of Iceland, which lies on the north side of the storms' path, the great preponderance of winds in the six stormy months, from October to March, are N.E. and E., as will appear from the following Table, which gives the number of days on an average of the three years, 1866-69, winds from the different points have blown during these six months:—

Number of days the wind has blown at Stykkisholm, from October to March, on an average of three years, viz., 1866-69,	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
	4	36	44	26	23	22	10	4	13

Suppose a change in the distribution of land and water took place in this part of the globe; on the one hand, land taking the place of sea to the west of a line drawn through Spitzbergen, the north of Norway, Farö, and the east of Newfoundland; and, on the other hand, sea taking the place of land over part of the north of Africa, and over the comparatively low plains of Europe and Siberia, the following changes would take place in the distribution of atmospheric pressure in winter:—The high pressure over Asia would be reduced and contracted; the high pressure in North America would be increased and extended, so as to include Greenland; and the low pressure round Iceland would be transferred to the south-east, so that the central space of least mean pressure would probably stretch from the north of France to the Gulf of Finland. Under these new conditions, mean pressure would increase greatly from the south of Great Britain towards the north-west, and thus northerly and easterly winds would become the prevailing winds in winter; and as the mean central track of storms would lie in a line from the north of France to St Petersburg, the winds accompanying storms, particularly those in the rear of the storms, would be dry and intensely cold. Further, suppose the Gulf Stream, or any oceanic current from equatorial regions, to flow past Great Britain on its way to the Arctic Ocean, through the Baltic and White Seas, from the lower mean temperature which would be brought about by the now prevailing northerly winds, the vapour brought by the Gulf Stream would be precipitated no longer in the form of rain but of snow, and frost would be of frequent occurrence. Since the heat of summer would be insufficient to melt this snow, it would accumulate from year to year; and thus the Gulf Stream, instead of ameliorating the climate, as at present, would only the sooner and more effectually, by accumulations of snow and ice, bring back to the British Islands the climate of the glacial epoch.

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT OF THE BAROMETER

Note.—Under column of "Hours of Observation" "*red.*" signifies that a correction has been applied for Daily Range, the P.M. after it. A Minus Sign before Latitudes signifies Latitude South, and before Longitudes, it

Places.	Country.	Authorities, see page 635.	Number of Years.	Years Specified.	Hours of Observation.	Latitude.	Longitude.	Height in Eng. Feet.
Stykkisholm, . . . . .	Iceland	1	23	1846-68	noon	65 4	-22 43	37
Do., . . . . .	do.	1	11	1857-67	noon	65 4	-22 43	37
Reykjavik, . . . . .	do.	2	13	1823-35	8 or 9:	64 40	-22 0	36
Do., . . . . .	do.	3	3	1866-69	9:	64 40	-22 0	10
Eyafjord, . . . . .	do.	4	2	1811-13	?	66 0	-18 20	?
Thorshavn, . . . . .	Farø	3	3	1866-69	9: 9	62 2	-6 43	12
Armagh, . . . . .	Ireland	5	11	1857-67	10: 10	54 21	-6 49	210
Belfast, . . . . .	do.	6	11	do.	9:	54 36	-5 56	66
Dublin, . . . . .	do.	7	22	1831-52	noon	53 22	-6 21	162
Do., . . . . .	do.	8	11	1857-67	9½: 3½	53 22	-6 21	159
Monkstown, . . . . .	do.	9	8	1859-66	8½: 8½	53 18	-6 8	110
Cork, . . . . .	do.	10	11	1857-67	9: 3	51 53	-8 28	25
Sandwick, . . . . .	Scotland	3	11	do.	9: 9	59 2	-3 18	94
Stornoway, . . . . .	do.	3	11	do.	do.	58 12	-6 21	70
Tongue, . . . . .	do.	3	11	do.	do.	58 30	-4 25	40
Culloden, . . . . .	do.	3	11	do.	do.	57 30	-4 7	104
Elgin, . . . . .	do.	3	11	do.	do.	57 38	-3 19	40
Aberdeen, . . . . .	do.	3	11	do.	do.	57 9	-2 7	110
Braemar, . . . . .	do.	3	11	do.	do.	57 0	-3 24	1114
Kettins, . . . . .	do.	3	11	do.	do.	56 32	-3 16	228
Barry, . . . . .	do.	3	11	do.	do.	56 31	-2 44	38
Callton Mor, . . . . .	do.	3	11	do.	do.	56 8	-5 30	65
Glasgow, . . . . .	do.	3	11	do.	do.	55 53	-4 18	180
Nookton, . . . . .	do.	3	11	do.	do.	56 11	-3 3	80
Smeaton, . . . . .	do.	3	11	do.	do.	56 0	-2 40	100
Thirlestane Castle, . . . . .	do.	3	11	do.	do.	55 43	-2 45	558
Milne-Graden, . . . . .	do.	3	11	do.	do.	55 0	-2 12	103
Durham, . . . . .	England	11	11	do.	10: 10	54 46	-1 35	352
Silloth, . . . . .	do.	12	11	do.	9: 9	54 52	-3 23	28
Stonyhurst, . . . . .	do.	13	21	1848-68	red.	53 51	-2 28	381
Do., . . . . .	do.	14	11	1857-67	7: 1, 9	53 51	-2 28	381
York, . . . . .	do.	15	11	do.	red.	53 58	-1 5	50
Manchester, . . . . .	do.	15	11	do.	8: 11	53 39	-2 14	123
Liverpool, . . . . .	do.	16	11	do.	:1	53 25	-2 59	37
Derby, . . . . .	do.	17	11	do.	9: 3	52 56	-1 28	174
Holkham, . . . . .	do.	18	11	do.	9: 3	52 57	0 48	39
Norwich, . . . . .	do.	19	11	do.	10: 3	52 38	1 18	50
Cardington, . . . . .	do.	20	11	do.	9: 3	52 7	-0 2	109
Oxford, . . . . .	do.	21, 15	11	do.	biho.	51 46	-1 16	210
Greenwich, . . . . .	do.	22, 15	11	do.	red.	51 28	0 0	159
London, . . . . .	do.	23	89	{ 1774-80 } { 1787-1868 }	do.	various	various	various
Clifton, . . . . .	do.	15	11	1857-67	do.	51 28	-2 36	228
Worthing, . . . . .	do.	15	11	do.	do.	50 49	-0 22	34
Helston, . . . . .	do.	24	20	1849-68	9: 3, 9	50 7	-5 16	106
Do., . . . . .	do.	24	11	1857-67	9: 3, 9	50 7	-5 16	106
Guernsey, . . . . .	Channel Isles	25, 15	11	do.	9: 3	49 28	-2 32	204
Hammerfest, . . . . .	Norway	26	13	1848-60	8: 2, 8	70 40	23 46	21

REDUCED TO 32° FAHR., IN ENGLISH INCHES, AT DIFFERENT PLACES OVER THE GLOBE.

in all other cases no such correction has been applied; the Hours of the A.M. Observations are placed before the Colon [:], signifies Longitude West. The Observations are reduced to sea-level at all places which are printed in *Italics*.

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
29-303	29-457	29-634	29-747	29-806	29-730	29-700	29-688	29-615	29-554	29-600	29-388	29-602
29-308	29-400	29-648	29-723	29-845	29-747	29-701	29-669	29-549	29-550	29-628	29-360	29-593
29-467	29-307	29-449	29-663	29-724	29-717	29-687	29-660	29-514	29-457	29-482	29-274	29-534
29-431	29-228	29-719	29-608	29-877	29-836	29-792	29-723	29-718	29-395	29-772	29-328	29-619
29-433	29-246	29-602	29-904	29-948	29-717	29-753	29-699	29-566	29-468	29-619	29-797	29-646
29-571	29-394	29-746	29-691	29-913	29-896	29-872	29-782	29-757	29-485	29-827	29-485	29-702
29-541	29-623	29-539	29-673	29-691	29-692	29-690	29-639	29-623	29-581	29-650	29-650	29-633
29-807	29-901	29-787	29-927	29-949	29-932	29-922	29-887	29-865	29-834	29-870	29-900	29-882
29-684	29-693	29-752	29-747	29-796	29-732	29-772	29-751	29-754	29-688	29-600	29-747	29-726
29-678	29-759	29-640	29-777	29-789	29-796	29-795	29-740	29-734	29-681	29-757	29-775	29-743
29-761	29-873	29-730	29-955	29-915	29-899	29-944	29-857	29-847	29-798	29-814	29-882	29-856
29-813	29-934	29-823	29-894	29-932	29-998	29-959	29-929	29-909	29-833	29-881	29-933	29-903
29-535	29-661	29-574	29-774	29-820	29-786	29-753	29-697	29-658	29-642	29-716	29-620	29-686
29-523	29-660	29-580	29-743	29-788	29-776	29-748	29-688	29-634	29-628	29-700	29-628	29-676
29-586	29-690	29-639	29-809	29-840	29-820	29-808	29-739	29-698	29-697	29-798	29-704	29-736
29-534	29-665	29-564	29-758	29-788	29-764	29-732	29-673	29-631	29-617	29-705	29-636	29-672
29-627	29-752	29-652	29-836	29-867	29-840	29-803	29-750	29-732	29-697	29-779	29-718	29-754
29-598	29-730	29-614	29-793	29-830	29-802	29-759	29-715	29-692	29-663	29-747	29-704	29-721
28-494	28-607	28-495	28-678	28-709	28-703	28-684	28-628	28-611	28-589	28-647	28-592	28-620
29-495	29-610	29-494	29-668	29-688	29-665	29-632	29-584	29-571	29-548	29-635	29-600	29-599
29-700	29-828	29-708	29-883	29-905	29-885	29-839	29-795	29-776	29-763	29-847	29-832	29-813
29-617	29-730	29-624	29-786	29-803	29-777	29-778	29-722	29-704	29-676	29-771	29-721	29-726
29-534	29-641	29-515	29-694	29-712	29-694	29-677	29-622	29-606	29-581	29-668	29-615	29-630
29-682	29-803	29-674	29-853	29-876	29-853	29-825	29-780	29-770	29-749	29-820	29-788	29-789
29-651	29-769	29-646	29-835	29-850	29-826	29-803	29-750	29-744	29-717	29-786	29-765	29-762
29-171	29-288	29-157	29-328	29-365	29-341	29-314	29-261	29-249	29-228	29-293	29-269	29-274
29-675	29-793	29-650	29-823	29-846	29-811	29-793	29-747	29-736	29-712	29-778	29-768	29-761
29-415	29-530	29-379	29-560	29-555	29-559	29-510	29-500	29-500	29-460	29-523	29-524	29-501
29-739	29-860	29-714	29-880	29-895	29-896	29-868	29-818	29-823	29-772	29-850	29-858	29-831
29-457	29-499	29-438	29-486	29-513	29-520	29-510	29-478	29-523	29-408	29-481	29-453	29-481
29-411	29-497	29-341	29-491	29-516	29-514	29-503	29-466	29-459	29-403	29-487	29-496	29-465
29-770	29-865	29-701	29-866	29-868	29-867	29-842	29-806	29-830	29-773	29-847	29-886	29-827
29-736	29-832	29-671	29-822	29-836	29-830	29-813	29-781	29-781	29-733	29-805	29-839	29-790
29-834	29-922	29-776	29-927	29-939	29-938	29-931	29-881	29-889	29-827	29-867	29-932	29-889
29-688	29-769	29-589	29-750	29-758	29-770	29-756	29-716	29-723	29-662	29-734	29-788	29-725
29-870	29-950	29-763	29-908	29-922	29-914	29-905	29-870	29-882	29-840	29-914	29-984	29-894
29-906	29-975	29-807	29-979	29-973	29-971	29-958	29-904	29-950	29-894	29-955	30-008	29-940
29-806	29-880	29-705	29-852	29-860	29-863	29-858	29-824	29-844	29-777	29-858	29-905	29-836
29-699	29-779	29-598	29-745	29-749	29-761	29-753	29-721	29-733	29-666	29-737	29-800	29-728
29-760	29-829	29-650	29-792	29-793	29-810	29-814	29-778	29-794	29-721	29-790	29-853	29-782
29-947	29-962	29-959	29-941	29-965	30-001	29-967	29-982	29-975	29-909	29-904	29-943	29-955
29-668	29-744	29-572	29-709	29-716	29-730	29-731	29-696	29-704	29-628	29-705	29-764	29-697
29-926	29-967	29-797	29-933	29-934	29-947	29-965	29-925	29-931	29-869	29-937	29-988	29-926
29-810	29-986	29-857	29-863	29-866	29-929	29-924	29-900	29-903	29-796	29-856	29-893	29-882
29-849	29-919	29-772	29-886	29-888	29-930	29-932	29-899	29-893	29-796	29-867	29-947	29-882
29-777	29-789	29-651	29-766	29-759	29-811	29-819	29-788	29-780	29-695	29-737	29-836	29-767
29-515	29-366	29-614	29-726	29-798	29-728	29-726	29-682	29-651	29-568	29-594	29-478	29-620

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 635.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.	Longitude.	Height in Eng. Feet.
Alten (at 32° ?)	Norway	27	12	1837-48	9: 3, 9	69 58	23 2	?
Christiansund,	do.	28	8	1861-68	8: 2, 8	63 7	7 45	65
Alesund,	do.	28	8	do.	do.	62 29	6 9	32
Bergen,	do.	28	8	do.	do.	60 24	5 20	50
Skudesnes,	do.	28	8	do.	do.	59 9	5 16	37
Mandal,	do.	28	8	do.	do.	58 2	7 27	54
Sandöund,	do.	28	8	do.	do.	59 5	10 27	41
Christiania,	do.	29	31	1837-67	7,9,2,4,10	59 55	10 44	74
Do.,	do.	29	11	1857-67	do.	do.	do.	74
Do.,	do.	28	8	1861-68	do.	do.	do.	74
Haparanda,	Sweden	30	7½	1859-66	8:	65 50	24 11	0
Umea,	do.	30	8	do.	do.	63 50	20 17	0
Hernösund,	do.	30	8	do.	do.	62 38	17 57	0
Göteborg,	do.	30	7½	do.	do.	57 42	11 58	0
Wisby,	do.	30	7½	do.	do.	57 39	18 19	39
Jönköping,	do.	30	8	do.	do.	57 47	14 11	292
Kalmar,	do.	30	8	do.	do.	56 40	16 21	0
Carlshamm,	do.	30	8	do.	do.	56 10	14 52	0
Örebro,	do.	30	8	do.	do.	59 16	15 13	97
Upsala,	do.	31, 30	11	1857-67	7 or 8:	55 52	17 38	77
Do.	do.	31, 30	8	1859-66	8	55 52	17 32	77
Copenhagen,	Denmark	32	11	1857-67	noon	55 41	12 35	12
Kiel,	do.	33	11	do.	6:	54 19	10 20	7
Groningen,	Netherlands	33	11	do.	8: 2	53 13	6 34	49
Leeuwarden,	do.	33	25	1843-67	8: 2, 8	53 12	5 47	24
Do.,	do.	33	11	1857-67	8: 2	53 12	5 47	24
Utrecht,	do.	33	20	1849-68	8:	52 5	5 7	44
Do.,	do.	33	11	1857-67	8: 2	52 5	5 7	44
Flushing,	do.	33	11	do.	do.	51 26	3 35	0
Luxemburg,	do.	33	11	do.	do.	49 37	6 8	1020
Maestricht,	do.	33	11	do.	do.	50 52	5 37	174
Brussels,	Belgium	34	35	1833-67	noon	50 51	4 22	186
Do.,	do.	34	11	1857-67	do.	50 51	4 22	186
Liege,	do.	34	20	1847-66	do.	50 41	5 23	199
Namur,	do.	34	13	1849-63	do.	50 28	4 51	491
Metz,	France	35	22	1825-46	do.	49 7	6 10	595
Paris,	do.	35	30	1816-45	do.	48 50	2 20	216
Do.,	do.	36, 37	11	1857-67	do.	48 50	2 20	216
Strasburg,	do.	4	15	?	?	48 36	7 42	460
Dijon,	do.	38	23	1845-67	noon	47 19	5 2	806
Do.,	do.	38	11	1857-67	do.	do.	do.	806
Ahun,	do.	39	38	1828-65	?	46 6	2 0	1471
Lyon,	do.	39	6	1861-66	9:	45 46	4 49	636
Toulouse,	do.	40	22	1839-60	9.12:3,6,9	43 37	1 28	650
St Rambert,	do.	41	6	1838-43	7: 1, 7	45 37	5 26	1017
Alais,	do.	35	35	1802-36	noon	44 7	4 4	?
Orange,	do.	35	36	1813-48	9: 3	44 8	4 48	149
Montpellier,	do.	42	7	1857-63	noon	43 36	3 54	193
Bordeaux,	do.	93	10	1847-56	: 2	44 50	- 0 35	75
Oviedo,	Spain & Portugal	40	11	1852-62	9, 12: 3, 9	43 24	- 10 29	718

AND THE PREVAILING WINDS OVER THE GLOBE.

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	Mar.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
29.712	29.653	29.754	29.856	29.892	29.802	29.781	29.808	29.771	29.698	29.668	29.662	29.755
29.544	29.595	29.619	29.764	29.843	29.788	29.710	29.686	29.701	29.666	29.630	29.583	29.677
29.630	29.686	29.694	29.843	29.902	29.855	29.780	29.749	29.768	29.737	29.713	29.678	29.753
29.634	29.693	29.674	29.823	29.859	29.832	29.752	29.721	29.749	29.737	29.705	29.701	29.740
29.717	29.788	29.745	29.894	29.934	29.890	29.792	29.772	29.800	29.796	29.776	29.788	29.808
29.733	29.788	29.725	29.855	29.898	29.843	29.764	29.756	29.795	29.812	29.764	29.800	29.794
29.741	29.772	29.745	29.856	29.897	29.827	29.730	29.745	29.800	29.831	29.772	29.799	29.793
29.753	29.729	29.770	29.813	29.831	29.734	29.691	29.726	29.797	29.736	29.756	29.789	29.760
29.732	29.790	29.676	29.796	29.835	29.777	29.696	29.718	29.765	29.772	29.806	29.803	29.764
29.693	29.682	29.697	29.788	29.812	29.756	29.670	29.686	29.753	29.776	29.717	29.745	29.731
29.663	29.811	29.774	29.850	29.838	29.832	29.694	29.740	29.778	29.792	29.771	29.725	29.772
29.650	29.775	29.702	29.828	29.850	29.822	29.693	29.700	29.748	29.778	29.776	29.740	29.756
29.675	29.758	29.678	29.836	29.850	29.813	29.697	29.680	29.749	29.795	29.745	29.725	29.750
29.787	29.836	29.733	29.938	29.892	29.848	29.781	29.756	29.842	29.894	29.841	29.886	29.836
29.801	29.818	29.740	29.925	29.872	29.863	29.772	29.750	29.870	29.906	29.865	29.886	29.839
29.497	29.508	29.410	29.589	29.613	29.560	29.511	29.470	29.564	29.599	29.548	29.594	29.539
29.863	29.835	29.738	29.912	29.922	29.869	29.810	29.780	29.865	29.921	29.870	29.917	29.858
29.900	29.882	29.732	29.948	29.966	29.903	29.872	29.844	29.874	29.961	29.907	29.943	29.894
29.716	29.741	29.670	29.834	29.848	29.791	29.704	29.681	29.767	29.818	29.793	29.797	29.763
29.713	29.766	29.676	29.792	29.855	29.816	29.692	29.734	29.792	29.805	29.795	29.787	29.769
29.680	29.716	29.652	29.833	29.842	29.790	29.695	29.684	29.772	29.817	29.786	29.785	29.754
29.875	29.936	29.776	29.909	29.947	29.914	29.857	29.873	29.937	29.921	29.918	29.947	29.901
29.888	29.954	29.761	29.922	29.959	29.927	29.848	29.906	29.949	29.919	29.937	29.907	29.906
29.882	29.926	29.747	29.910	29.914	29.914	29.894	29.878	29.918	29.871	29.910	29.973	29.894
29.887	29.909	29.886	29.909	29.933	29.940	29.932	29.913	29.979	29.867	29.911	29.969	29.919
29.894	29.963	29.780	29.934	29.941	29.941	29.922	29.898	29.938	29.894	29.930	29.985	29.918
29.896	29.971	29.886	29.916	29.915	29.951	29.934	29.925	29.970	29.886	29.928	29.981	29.928
29.918	29.969	29.780	29.930	29.934	29.945	29.934	29.914	29.949	29.894	29.938	29.989	29.925
29.977	30.032	29.843	29.985	29.993	30.020	30.009	29.981	30.005	29.941	29.989	30.052	29.985
28.926	28.953	28.768	28.894	28.886	28.938	28.953	28.928	28.965	28.902	28.910	28.989	28.918
29.819	29.863	29.666	29.812	29.796	29.819	29.815	29.800	29.843	29.776	29.815	29.897	29.810
29.766	29.778	29.746	29.743	29.758	29.786	29.795	29.779	29.794	29.728	29.741	29.835	29.771
29.773	29.835	29.641	29.770	29.763	29.791	29.794	29.767	29.802	29.739	29.781	29.860	29.776
29.717	29.764	29.690	29.685	29.697	29.733	29.745	29.725	29.761	29.686	29.705	29.776	29.724
29.567	29.650	29.575	29.520	29.532	29.540	29.575	29.567	29.603	29.524	29.560	29.611	29.567
29.400	29.370	29.367	29.286	29.310	29.366	29.376	29.359	29.352	29.383	29.315	29.409	29.357
29.808	29.762	29.762	29.708	29.723	29.780	29.771	29.767	29.754	29.757	29.721	29.804	29.759
29.800	29.840	29.650	29.753	29.735	29.786	29.796	29.768	29.800	29.728	29.775	29.869	29.775
29.587	29.615	29.567	29.526	29.533	29.612	29.590	29.606	29.631	29.573	29.563	29.548	29.580
29.179	29.194	29.115	29.100	29.108	29.172	29.192	29.179	29.201	29.157	29.156	29.227	29.165
29.201	29.221	29.025	29.135	29.113	29.173	29.192	29.164	29.221	29.158	29.146	29.218	29.164
28.430	28.436	28.416	28.388	28.435	28.522	28.557	28.535	28.509	28.464	28.414	28.494	28.467
29.413	29.362	29.158	29.362	29.316	29.355	29.355	29.366	29.395	29.316	29.339	29.366	29.342
29.402	29.359	29.341	29.278	29.290	29.366	29.395	29.381	29.355	29.329	29.321	29.421	29.353
28.983	28.881	28.968	28.882	28.904	28.976	29.006	29.022	28.965	28.949	28.875	29.085	28.958
29.673	29.623	29.619	29.553	29.548	29.601	29.602	29.602	29.621	29.544	29.611	29.597	29.600
29.867	29.825	29.823	29.762	29.788	29.840	29.854	29.870	29.829	29.862	29.827	29.884	29.836
29.878	29.878	29.756	29.741	29.735	29.787	29.811	29.792	29.812	29.807	79.772	29.886	29.805
29.937	30.007	29.956	29.870	29.901	29.980	30.003	29.993	29.973	29.919	29.978	30.024	29.970
29.229	29.217	29.215	29.185	29.175	29.261	29.322	29.287	29.281	29.208	29.153	29.239	29.231

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 635.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.		Longitude.		Height in Eng. Feet.
						°	'	°	'	
<i>Barcelona</i> , . . . . .	Spain & Portugal	33	4	1864-67	7 or 8 :	41	22	2	9	?
<i>Oporto</i> , . . . . .	do.	43	5	1863-67	9 : 3	41	9	-8	27	278
<i>Madrid</i> , . . . . .	do.	40	9	1853-62	?	40	24	-3	52	2149
<i>Lisbon</i> , . . . . .	do.	43	11	1857-67	9, 12: 3, 9	38	43	-9	8	336
<i>Alicante</i> , . . . . .	do.	33	4	1864-67	7 or 8 :	38	21	-0	25	?
<i>San Fernando</i> , . . . . .	do.	33	9	1859-67	do.	36	27	-6	13	?
<i>Gibraltar</i> , . . . . .	do.	44	6	1853-59	9½: 3½	36	6	-5	21	46
<i>Gibraltar</i> , . . . . .	do.	45	3	1864-66	9 : 3	36	6	-5	21	50
<i>Zurich</i> , . . . . .	Switzerland	46	10	1837-46	9, 12: 3, 9	47	22	8	32	1432
<i>Geneva</i> , . . . . .	do.	40	25	1836-60	biho.	46	12	6	9	1335
Do., . . . . .	do.	47	11	1857-67	noon	46	12	6	9	1335
<i>Great St Bernard</i> , . . . . .	do.	47	11	do.	do.	45	51	7	11	8174
<i>Trient</i> , . . . . .	Italy	48	11	1856-66	various	46	4	11	4	622
<i>Udine</i> , . . . . .	do.	40	40	1803-42	9½:	46	4	13	14	393
<i>Milan</i> , . . . . .	do.	48	16	1848-53	{ 6, 12: 6, 12, 6, 12: 3 }	45	28	9	9	482
<i>Verona</i> , . . . . .	do.	40	7	1854-60	8: 2, 8	45	27	10	59	186
<i>Venice</i> , . . . . .	do.	48	10½	1853-63	{ sr: 2, 9 or 6: 2, 10 }	45	26	12	17	66
<i>Turin</i> , . . . . .	do.	40	74	1787-1860	?	45	4	7	41	915
<i>Brescia</i> , . . . . .	do.	40	27	1818-44	sr: 12: ss	45	32	10	13	492
<i>Bologna</i> , . . . . .	do.	40	45	1814-58	?	44	30	11	21	244
<i>Genoa</i> , . . . . .	do.	4	10	?	?	44	25	8	55	157
Do., . . . . .	do.	40	2	1860-61	9, 12: 3, 9	do.	do.	do.	do.	157
<i>Rome</i> , . . . . .	do.	40	10	1852-61	: ½	41	54	12	28	163
<i>Rome</i> , . . . . .	do.	40	15	1852-66	: ½	do.	do.	do.	do.	163
<i>Naples</i> , . . . . .	do.	40	28	1833-60	9½:	40	52	14	15	482
<i>Palermo</i> , . . . . .	do.	55	78	1791-1868	various	38	7	13	21	237
Do., . . . . .	do.	55	11	1857-67	do.	do.	do.	do.	do.	237
<i>Malta</i> , . . . . .	do.	44	6	{ 1853-55. 1858-59 }	9½: 3½	35	54	14	31	232
<i>Malta</i> , . . . . .	do.	45	2	1865-66	9 : 3	35	54	14	30	111
<i>Bodenbach</i> , . . . . .	Austria	48	19	1848-66	6: 2, 10	50	46	14	10	466
<i>Prague</i> , . . . . .	do.	48	19	do.	do.	50	5	14	23	660
Do., . . . . .	do.	48	11	1857-67	do.	do.	do.	do.	do.	660
<i>Krakau</i> , . . . . .	do.	48	19	1848-66	do.	50	4	19	55	708
Do., . . . . .	do.	48	11	1857-67	do.	do.	do.	do.	do.	708
<i>Troppau</i> , . . . . .	do.	48	7½	1858-65	do.	49	56	17	52	847
<i>Lemberg</i> , . . . . .	do.	48	17	1850-66	{ 6or7: 2, 9 or 10 }	49	50	24	0	928
Do., . . . . .	do.	48	11	1857-67	{ 6or7: 2, 9 or 10 }	do.	do.	do.	do.	928
<i>Brunn</i> , . . . . .	do.	48	19	1848-66	6: 2, 10	49	11	16	35	697
<i>Vienna</i> , . . . . .	do.	48	19	do.	do.	48	12	16	20	638
Do., . . . . .	do.	48	11	1857-67	do.	do.	do.	do.	do.	638
<i>Kremsmünster</i> , . . . . .	do.	48	19	1848-66	do.	48	3	14	6	1258
<i>Debreczin</i> , . . . . .	do.	48	11	1857-67	{ 6: 2, 10 or 9 }	47	32	21	39	417
<i>Ofen (Buda)</i> , . . . . .	do.	48	10	1856-66	6: 2, 10	47	31	19	1	420
<i>Klagenfurt</i> , . . . . .	do.	48	19	1848-66	7: 2, 9	46	37	14	16	1438
<i>Szegedin</i> , . . . . .	do.	48	12	1853-66	6: 2, 10	46	15	20	6	276

AND THE PREVAILING WINDS OVER THE GLOBE.

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	Mar.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
30.024	30.056	29.792	30.008	29.985	30.016	30.004	30.004	30.071	29.941	30.024	30.158	30.015
29.748	29.825	29.586	29.684	29.634	29.747	29.754	29.733	29.762	29.622	29.741	29.842	29.723
27.835	27.836	27.783	27.751	27.739	27.816	27.830	27.814	27.741	27.825	27.752	27.912	27.834
29.828	29.773	29.691	29.688	29.665	29.735	29.747	29.712	29.745	29.672	29.678	29.832	29.731
30.138	30.134	29.906	30.040	30.020	30.091	30.064	30.060	30.123	30.012	30.127	30.213	30.077
30.170	30.111	30.048	30.009	30.000	30.091	30.044	30.017	30.056	30.013	30.079	30.182	30.068
30.082	30.018	30.069	30.062	30.017	30.075	30.050	30.002	30.080	30.076	29.995	30.150	30.056
30.138	30.002	29.905	29.965	29.965	30.015	29.998	29.977	30.042	29.928	30.047	30.132	30.010
28.504	28.434	28.469	28.410	28.430	28.520	28.536	28.536	28.516	28.504	28.434	28.567	28.489
28.630	28.584	28.574	28.499	28.538	28.626	28.650	28.647	28.634	28.605	28.570	28.659	28.601
28.660	28.670	28.489	28.577	28.565	28.630	28.658	28.643	28.676	28.613	28.618	28.743	28.629
22.062	22.112	22.000	22.160	22.227	22.337	22.394	22.383	22.377	22.254	22.146	22.166	22.218
29.434	29.407	29.238	29.297	29.289	29.311	29.322	29.323	29.387	29.356	29.362	29.467	29.350
29.629	29.655	29.583	29.539	29.569	29.590	29.576	29.596	29.630	29.631	29.624	29.624	29.604
29.523	29.504	29.412	29.374	29.392	29.438	29.445	29.452	29.496	29.481	29.446	29.539	29.458
29.756	29.760	29.772	29.650	29.634	29.749	29.741	29.729	29.815	29.733	29.686	29.733	29.729
29.975	29.970	29.854	29.908	29.872	29.890	29.915	29.916	29.972	29.944	29.928	29.995	29.928
29.077	29.068	29.051	29.042	29.112	29.166	29.176	29.193	29.176	29.117	29.068	29.068	29.109
29.487	29.441	29.414	29.371	29.406	29.424	29.415	29.441	29.450	29.450	29.441	29.512	29.435
29.776	29.764	29.697	29.650	29.666	29.705	29.701	29.709	29.760	29.756	29.756	29.804	29.729
29.863	29.813	29.781	29.697	29.774	29.804	29.779	29.783	29.786	29.798	29.753	29.869	29.800
29.857	29.787	29.820	29.736	29.796	29.800	29.751	29.835	29.823	29.933	29.774	29.754	29.806
29.849	29.798	29.826	29.760	29.760	29.817	29.813	29.795	29.850	29.845	29.766	29.790	29.808
30.040	29.989	29.947	29.977	29.961	29.993	29.989	29.969	30.032	30.016	29.949	30.004	29.989
29.545	29.503	29.491	29.451	29.519	29.582	29.572	29.569	29.589	29.567	29.520	29.558	29.539
29.720	29.718	29.660	29.664	29.693	29.729	29.714	29.718	29.743	29.728	29.713	29.708	29.709
29.735	29.734	29.592	29.677	29.680	29.705	29.718	29.695	29.753	29.715	29.695	29.733	29.703
30.096	29.915	30.034	30.026	29.973	30.016	30.031	30.000	30.060	30.045	29.996	30.034	30.019
29.883	29.825	29.692	29.925	29.854	29.846	29.808	29.822	29.903	29.839	29.896	29.965	29.856
29.515	29.472	29.409	29.431	29.428	29.449	29.471	29.459	29.526	29.479	29.489	29.565	29.474
29.327	29.288	29.206	29.225	29.221	29.253	29.267	29.264	29.332	29.290	29.286	29.379	29.278
29.325	29.355	29.129	29.242	29.240	29.265	29.258	29.261	29.337	29.305	29.323	29.378	29.285
29.282	29.208	29.143	29.188	29.189	29.211	29.220	29.233	29.300	29.282	29.246	29.320	29.235
29.295	29.297	29.102	29.199	29.120	29.219	29.219	29.235	29.322	29.307	29.298	29.338	29.246
29.166	29.120	28.939	29.067	29.053	29.053	29.061	29.063	29.094	29.133	29.111	29.131	29.083
29.014	28.952	28.881	28.916	28.917	28.943	28.936	28.960	29.043	29.040	29.023	29.039	28.972
29.011	29.027	28.846	28.917	28.939	28.932	28.937	28.954	29.047	29.054	29.036	29.046	28.979
29.298	29.235	29.156	29.166	29.170	29.202	29.211	29.224	29.287	29.251	29.240	29.330	29.231
29.378	29.320	29.239	29.244	29.241	29.281	29.297	29.303	29.360	29.330	29.320	29.418	29.311
29.384	29.388	29.182	29.270	29.265	29.290	29.297	29.301	29.376	29.349	29.364	29.413	29.325
28.702	28.664	28.585	28.599	28.602	28.661	28.683	28.682	28.715	28.667	28.652	28.747	28.663
29.602	29.592	29.390	29.430	29.430	29.415	29.427	29.442	29.558	29.544	29.568	29.623	29.502
29.647	29.604	29.440	29.497	29.476	29.491	29.508	29.507	29.570	29.609	29.606	29.707	29.555
28.491	28.427	28.345	28.307	28.371	28.424	28.440	28.447	28.496	28.456	28.431	28.537	28.431
29.757	29.709	29.614	29.628	29.593	29.644	29.625	29.646	29.745	29.780	29.735	29.763	29.686

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Authorities, see page 685.	Number of Years.	Years Specified.	Hours of Observation.	Latitude.	Longitude.	Height in Eng. Feet.
Agram,	Austria	48	8	1857-9, 62-66	{6: 2, 10 7: 2, 9}	45 49	15 53	448
Hermanstadt,	do.	48	16	1851-66	6: 2, 10	45 47	24 7	1354
Do.,	do.	48	11	1857-67	do.	do.	do.	1354
Trieste,	do.	48	19	1848-66	7: 2, 9	45 39	13 44	79
Do.,	do.	48	11	1857-67	do.	do.	do.	79
Pancsova,	do.	48	7	1860-66	{6 or 7: 1 or 2. 8 or 9}	44 50	20 35	224
Lesina,	do.	48	9	1858-67	{6 or 7: 2, 9 or 10}	43 11	16 25	63
Munich,	Bavaria	49	10	1857-66	hourly	48 9	11 34	1676
Memel,	Prussia, &c.	50	7	1861-67	6: 2, 10	55 44	21 6	?
Königsberg,	do.	50	10	1858-67	do.	54 43	20 29	72
Do.,	do.	50, 33	7	1861-67	do.	do.	do.	72
Dantzig,	do.	4	32	?	do.	54 21	18 41	30
Do.,	do.	50	7	1861-67	do.	do.	do.	30
Putbus,	do.	50, 33	11	1857-67	do.	54 22	13 35	173
Coslin,	do.	50	7	1861-67	do.	54 12	16 15	128
Stettin,	do.	50	7	do.	do.	53 25	12 30	49
Bromberg,	do.	50	7	do.	do.	53 8	18 0	146
Berlin,	do.	50, 33	11	1857-67	do.	52 30	13 3	153
Do.,	do.	50	7	1861-67	do.	do.	do.	153
Posen,	do.	50	7	do.	do.	52 25	17 5	287
Halle,	do.	50	7	do.	do.	51 30	11 57	372
Leipzig,	do.	51	33	1835-67	do.	51 20	12 21	386
Mülhausen,	do.	50, 33	10 $\frac{1}{3}$	1857-67	do.	51 13	10 27	686
Breslau,	do.	50, 33	10 $\frac{2}{3}$	1857-67	do.	51 7	17 2	484
Erfurt,	do.	50	7	1861-67	do.	50 59	11 4	682
Bucharest,	Turkey & Greece	52	6	1863-68	do.	44 26	26 8	700
Janina,	do.	3	5	1864-69	various	39 47	20 55	1570
Corfu,	do.	44	6	1853-59	9 $\frac{1}{2}$ : 3 $\frac{1}{2}$	39 39	19 55	74
Constantinople,	do.	52	11	1858-68	9:	41 0	28 59	?
Athens,	do.	52	11	do.	8:	37 58	23 43	264
Archangel,	Russia	4	18	?	?	64 33	40 33	?
Helsingfors,	do.	26	10	1852-62	hourly	60 14	24 57	50
St Petersburg,	do.	26	19	1846-64	do.	59 56	30 18	10
Do.,	do.	26, 33	11	1857-67	8:	do.	do.	10
Baltischport,	do.	26	10	1855-64	8, 12: 3, 10	59 21	24 3	0
Dorpat,	do.	33, 53	11	1857-67	A.M.	58 17	26 47	150
Kostroma,	do.	26	7	1850-56	7: 2, 2 x 9	57 46	40 56	640
Mittau,	do.	26	12	1852-63	6: 2, 10	56 35	23 43	13
Riga,	do.	26	10	{1847-48, 1852-53, 1855-58, 1863-64}	do.	56 57	24 6	20?
Moscow,	do.	33	10	1858-67	A.M.	55 42	37 39	400
Zlalouste,	do.	26	28	1837-64	various	55 10	59 40	1444
Gorki,	do.	26	4	1851-54	6: 2, 10	54 15	30 35	690
Wilna,	do.	26	9	{1817- 1825}	?	54 41	25 17	388
Kaluga,	do.	26	13	1851-63	7, 2: 2 x 9	54 30	36 15	576

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
29-556	29-495	29-295	29-423	29-299	29-414	29-428	29-400	29-540	29-453	29-476	29-653	29-452
28-597	28-518	28-437	28-470	28-477	28-500	28-503	28-533	28-620	28-621	28-565	28-621	28-538
28-602	28-592	28-410	28-480	28-493	28-493	28-500	28-535	28-620	28-617	28-602	28-621	28-547
29-939	29-902	29-825	29-804	29-840	29-870	29-861	29-863	29-922	29-898	29-850	29-945	29-876
29-956	29-970	29-774	29-871	29-860	29-870	29-868	29-859	29-944	29-917	29-902	29-944	29-895
29-898	29-835	29-589	29-766	29-733	29-702	29-724	29-739	29-809	29-819	29-811	29-887	29-776
29-968	29-901	29-772	29-884	29-864	29-859	29-856	29-866	29-937	29-930	29-870	29-907	29-885
28-220	28-201	28-040	28-151	28-148	28-203	28-228	28-211	28-249	28-190	28-171	28-267	28-189
29-844	29-904	29-768	29-890	29-904	29-871	29-790	29-811	29-924	29-958	29-775	29-928	29-864
29-913	29-912	29-749	29-857	29-900	29-882	29-817	29-847	29-848	29-954	29-916	29-937	29-878
29-875	29-922	29-783	29-907	29-922	29-882	29-830	29-845	29-953	29-978	29-890	29-914	29-892
29-980	29-949	29-917	29-912	29-952	29-927	29-892	29-908	29-967	29-961	29-912	29-919	29-933
29-894	29-943	29-792	29-893	29-943	29-908	29-846	29-877	29-981	29-992	29-910	29-977	29-913
29-706	29-723	29-576	29-696	29-744	29-744	29-686	29-683	29-768	29-714	29-747	29-766	29-710
29-829	29-880	29-714	29-873	29-910	29-848	29-822	29-837	29-905	29-900	29-848	29-930	29-858
29-862	29-883	29-703	29-885	29-873	29-837	29-798	29-830	29-894	29-915	29-879	29-974	29-861
29-789	29-830	29-650	29-801	29-819	29-799	29-756	29-789	29-859	29-863	29-790	29-889	29-803
29-818	29-867	29-679	29-786	29-813	29-812	29-786	29-814	29-875	29-843	29-816	29-866	29-815
29-774	29-846	29-641	29-800	29-820	29-795	29-770	29-795	29-870	29-835	29-800	29-898	29-804
29-758	29-774	29-586	29-769	29-763	29-739	29-664	29-741	29-824	29-806	29-763	29-825	29-751
29-668	29-707	29-492	29-705	29-676	29-674	29-656	29-675	29-736	29-695	29-682	29-806	29-681
29-610	29-577	29-600	29-535	29-568	29-602	29-624	29-613	29-623	29-597	29-560	29-626	29-596
29-283	29-305	29-198	29-257	29-253	29-286	29-277	29-270	29-316	29-270	29-291	29-328	29-278
29-550	29-544	29-359	29-478	29-464	29-467	29-436	29-457	29-553	29-550	29-521	29-592	29-598
29-303	29-325	29-114	29-318	29-290	29-308	29-306	29-309	29-358	29-309	29-298	29-410	29-304
29-308	29-235	29-092	29-174	29-269	29-174	29-146	29-172	29-314	29-334	29-296	29-396	29-242
28-323	28-371	28-135	28-326	28-361	28-330	28-283	28-311	28-414	28-365	28-333	28-364	28-326
30-036	29-981	29-981	29-955	29-954	29-972	29-940	29-953	30-040	30-080	30-002	30-040	29-995
30-081	30-058	29-898	29-924	29-878	29-877	29-839	29-877	29-979	30-055	30-038	30-055	29-963
29-730	29-734	29-605	29-669	29-651	29-607	29-571	29-611	29-673	29-750	29-728	29-728	29-671
29-743	29-765	29-703	29-782	29-798	29-734	29-698	29-685	29-798	29-725	29-696	29-694	29-735
29-820	29-829	29-775	29-786	29-860	29-820	29-764	29-777	29-814	29-844	29-852	29-728	29-806
29-915	29-801	29-841	29-877	29-898	29-827	29-798	29-836	29-891	29-904	29-916	29-833	29-861
29-850	29-943	29-819	29-834	29-896	29-847	29-780	29-806	29-897	29-926	29-888	29-885	29-864
29-857	29-891	29-754	29-843	29-882	29-881	29-801	29-808	29-880	29-888	29-942	29-800	29-852
29-690	29-776	29-658	29-713	29-729	29-741	29-619	29-642	29-729	29-772	29-745	29-725	29-712
29-336	29-157	29-230	29-273	29-293	29-208	29-148	29-235	29-284	29-328	29-351	29-126	29-247
29-910	29-905	29-854	29-882	29-901	29-880	29-845	29-870	29-914	29-949	30-104	29-909	29-910
29-901	29-968	29-807	29-846	29-859	29-864	29-794	29-842	29-893	29-908	29-980	29-761	29-869
29-483	29-495	29-482	29-474	29-473	29-457	29-447	29-455	29-483	29-504	29-499	29-498	29-479
28-634	28-556	28-547	28-525	28-475	28-344	28-341	28-411	28-527	28-559	28-615	28-550	28-507
29-305	29-023	29-188	29-143	29-142	29-078	29-093	29-187	29-198	29-224	29-205	29-092	29-158
29-473	29-493	29-450	29-492	29-584	29-534	29-519	29-599	29-673	29-649	29-522	29-522	29-542
29-470	29-397	29-378	29-354	29-375	29-314	29-284	29-346	29-410	29-499	29-447	29-401	29-390

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 635.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.	Longitude.	Height in Eng. Feet.
Tambov, . . . . .	Russia	26	13	{ 1847-56, 1858-60 }	8: 8 or 10	52 41	41 30	580
Warsaw, . . . . .	do.	33	10	1858-67	6:	52 14	21 7	450
Orenburg, . . . . .	do.	26	21	1844-64	{ 10:2, 10 } chiefly	51 49	35 6	280
Astracan (at 32°?), . . . . .	do.	4	1	?	?	46 15	48 4	40
Kursk, . . . . .	do.	26	27	1833-59	9, 12: 3, 9	51 44	36 14	700
Morshansk, . . . . .	do.	26	4	{ 1855-56, 1858-59 }	8: 8	52 27	41 53	520
Woltehansk, . . . . .	do.	26	13	1852-64	6: 2, 10	50 17	36 56	370
Lugan, . . . . .	do.	26	23	1842-64	various	48 35	39 20	330
Nicholaieff, . . . . .	do.	26	6	1859-64	10: 10	46 58	31 58	85
Odessa, . . . . .	do.	26	9	{ 1843-50, 1860 }	9: 9	46 28	30 43	147
Kiew, . . . . .	do.	52	10½	1858-68	6:	50 27	30 34	578
Alagir, . . . . .	do.	26	16	1848-63	7: 2, 9	43 2	43 53	2060
Derbent, . . . . .	do.	26	3½	1852-55	do.	42 12	48 15	-15
Jakutsk, . . . . .	Asiatic Russia	4	1½	?	?	62 2	129 14	285
Bogoslovsk, . . . . .	do.	26	26	1839-64	various	59 45	60 2	593
Tobolsk, . . . . .	do.	26	11	1852-62	{ 6: 2, 10 } and { 7: 3, 11 }	58 12	68 16	355
Nijni-Tagilsk, . . . . .	do.	54	21	1845-65	8: 3, 8	57 59	60 19	730?
Catherinenburg, . . . . .	do.	26	19	1846-64	hourly	56 49	60 35	997
Tomsk, . . . . .	do.	26	2	1852-53	8: 8	56 30	85 10	300
Ajansk, . . . . .	do.	26	2	1847-49	7: 2, 9	56 27	138 26	?
Krasnoyarsk, . . . . .	do.	26	10	1838-47	9: or 10:	56 1	92 54	?
Udskoi, . . . . .	do.	4	1	?	?	54 30	134 28	?
Barnaul, . . . . .	do.	26	19	1846-64	hourly	53 20	83 57	400
Peterpaulshavn, . . . . .	do.	4	1	?	?	53 10	158 32	?
Irkutsh, . . . . .	do.	26	15	1830-44	7: 2, 9	52 17	122 11	1253
Nertchinsk, . . . . .	do.	26	18	1847-64	hourly	51 19	119 36	2130
Fort No. 1, . . . . .	do.	26	1	1865	6: 2, 10	45 45	64 27	170
Novo Petrovsk, . . . . .	do.	26	6	1852-57	do.	44 27	50 8	100
Kutais, . . . . .	do.	26	3	1850-52	7: 2, 9	42 31	42 27	470
Redut Kale, . . . . .	do.	26	8	1847-54	do.	42 16	41 36	20
Tiflis, . . . . .	do.	26	14	1850-63	hourly	41 42	44 50	1500
Alexandropol, . . . . .	do.	26	12	1854-65	7: 2, 9	40 48	43 49	5010
Baku, . . . . .	do.	26	17	1848-64	do.	40 22	49 50	-53
Aralikh, . . . . .	do.	26	3	1851-53	{ 7: 12 or } { 1, 8 or 9 }	39 53	44 33	2600
Lencoran, . . . . .	do.	26	5	1852-56	7: 2, 9	38 44	48 52	-65
Wernoie (132 obs.), . . . . .	do.	26	1	1859	noon	43 16	77 0	2430
Chusan (at 32°?), . . . . .	China	4	1	?	?	30 30	122 6	?
Pekin, . . . . .	do.	26	14	1842-55	{ 5, 7, 9 } &c., to 9 and hourly	39 54	116 26	?
Tien Tsin, . . . . .	do.	105	1	1860-61	9: 3	39 9	117 16	29
Shanghai, . . . . .	do.	56	2	1867-68	various	30 4	85 33	0
Canton, . . . . .	do.	4	10	?	?	23 12	113 17	?

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
29-587	29-419	29-426	29-397	29-397	29-299	29-265	29-381	29-413	29-432	29-483	29-455	29-413
29-587	29-548	29-426	29-497	29-520	29-489	29-445	29-504	29-611	29-603	29-556	29-670	29-538
29-870	29-820	29-827	29-742	29-618	29-481	29-439	29-534	29-688	29-761	29-908	29-844	29-711
30-206	30-402	30-099	30-000	29-899	29-600	29-600	29-899	29-899	30-201	30-300	30-300	30-034
29-303	29-263	29-198	29-236	29-230	29-163	29-148	29-224	29-309	29-343	29-352	29-277	29-254
29-462	29-435	29-404	29-418	29-456	29-389	29-322	29-379	29-465	29-441	29-442	29-515	29-427
29-672	29-672	29-611	29-538	29-549	29-468	29-437	29-499	29-620	29-729	29-754	29-603	29-596
29-835	29-736	29-686	29-622	29-626	29-523	29-480	29-595	29-690	29-800	29-864	29-785	29-685
30-010	29-990	29-840	29-825	29-788	29-732	29-740	29-822	29-884	29-963	30-047	29-907	29-879
29-876	29-714	29-806	29-732	29-720	29-677	29-667	29-691	29-768	29-838	29-937	29-709	29-761
29-432	29-430	29-343	29-337	29-378	29-314	29-291	29-348	29-425	29-498	29-479	29-461	29-395
27-888	27-873	27-877	27-827	27-844	27-801	27-781	27-831	27-912	28-018	28-013	27-941	27-884
30-187	30-076	30-059	30-020	30-001	29-886	29-834	29-936	30-016	30-162	30-134	30-153	30-039
29-895	29-957	29-748	29-620	29-472	29-366	29-383	29-435	29-711	29-670	29-829	30-060	29-679
29-280	29-245	29-255	29-216	29-185	29-016	29-055	29-104	29-188	29-205	29-261	29-253	29-191
30-002	29-926	29-917	29-802	29-681	29-504	29-462	29-498	29-571	29-719	29-816	29-803	29-725
29-386	29-347	29-372	29-303	29-241	29-107	29-103	29-165	29-282	29-300	29-384	29-317	29-276
29-127	28-990	29-002	28-999	28-922	28-777	28-778	28-843	28-963	29-000	29-058	28-957	28-951
30-092	30-215	29-934	29-822	29-600	29-483	29-350	29-561	29-601	29-819	29-852	29-835	29-763
29-786	29-874	29-910	29-806	29-751	29-688	29-633	29-754	29-766	29-855	29-743	29-752	29-776
30-078	30-043	29-952	29-813	29-662	29-550	29-536	29-672	29-818	29-907	30-041	30-112	29-850
30-003	29-866	29-766	29-766	29-590	29-550	29-463	29-412	29-568	29-655	29-788	29-860	29-691
29-807	29-739	29-689	29-521	29-375	29-186	29-104	29-228	29-409	29-556	29-667	29-703	29-499
29-409	29-547	29-700	29-921	29-805	29-732	29-685	29-714	29-709	29-626	29-600	29-720	29-679
28-777	28-719	28-624	28-590	28-353	28-261	28-192	28-264	28-443	28-588	28-657	28-677	28-512
27-963	27-922	27-876	27-662	27-604	27-566	27-566	27-649	27-752	27-834	27-856	27-848	27-758
29-986	29-938	29-986	29-804	29-736	29-635	29-512	29-624	29-756	29-945	30-120	29-986	29-836
30-080	30-017	29-982	29-901	29-866	29-772	29-724	29-794	29-923	30-120	30-137	30-034	29-946
29-624	29-510	29-490	29-439	29-458	29-388	29-328	29-365	29-484	29-555	29-572	29-598	29-484
30-094	29-977	29-994	29-911	29-936	29-866	29-819	29-829	29-945	30-037	30-055	30-076	29-962
28-547	28-502	28-460	28-397	28-398	28-322	28-291	28-362	28-473	28-589	28-608	28-561	28-459
24-938	24-895	24-905	24-874	24-925	24-908	24-871	24-912	24-982	25-060	25-049	24-976	24-941
30-215	30-179	30-132	30-023	29-988	29-884	29-815	29-907	30-025	30-183	30-219	30-166	30-061
27-458	27-294	27-200	27-164	27-202	27-080	27-022	27-061	27-214	27-339	27-376	27-364	27-232
30-292	30-171	30-152	30-099	30-065	29-958	29-904	29-962	30-095	30-246	30-262	30-243	30-121
27-597	27-409	27-579	27-419	27-294	27-222	27-182	27-270	27-425	27-578	27-665	27-700	27-445
30-412	30-426	...	...	...	...	...	...	30-027	30-176	30-193	30-344	...
30-244	30-170	30-009	29-826	29-668	29-517	29-470	29-586	29-813	30-008	30-154	30-217	29-890
30-313	30-336	30-132	29-950	29-867	29-642	29-571	29-645	29-927	30-076	30-121	30-245	29-985
30-254	30-186	30-108	29-944	29-848	29-707	29-719	29-729	29-903	30-062	30-211	30-205	29-990
30-175	30-099	30-018	29-849	29-761	29-731	29-656	29-659	29-685	29-912	30-071	30-123	29-895

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 635.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.	Longitude.	Height in Eng. Feet.
Macao, . . . . .	China	4	1	?	?	22 15	113 36	?
Hong Kong, . . . . .	do.	44	6	1853-59	9½: 3½	22 16	114 10	35
Nafa, . . . . .	Pelew Islands	4	1	?	?	26 14	127 46	?
Chacodate, . . . . .	Japan	26	4½	1859-63	7, 9: 2, 9	41 48	140 47	150
Decima (Nangasaki), . . . . .	do.	33	7	1845-48, 1852-55	6, 9: 3½, 10	33 44	109 42	26
Erzroum, . . . . .	Turkey in Asia	3	1	1836-38	{ thrice daily }	39 57	41 13	?
Scutari, . . . . .	do.	45	2	1865-66	9: 3	41 0	29 3	60
Larnaka, Cyprus, . . . . .	Syria	3	3	1866-69	9: 9	34 55	33 39	25
Beyrout, . . . . .	do.	4	1	?	?	33 54	35 29	?
Do., . . . . .	do.	3	1	1868-69	9: 10	33 54	35 29	160
Jerusalem, . . . . .	do.	3	8	1861-68	9:	31 47	35 13	2500
Djedda, . . . . .	do.	57	1 6	1831 and 1833	9: 3½	21 28	39 13	25
Tor, . . . . .	do.	57	1½	1831	8½: 3½	28 13	33 39	18
Aden, . . . . .	do.	4	2	?	?	12 46	45 5	199
Mooltan, . . . . .	Hindustan	58	6	1862-67	10: 4	31 11	71 33	450
Roorkee, . . . . .	do.	59	4	1865-68	do.	29 52	77 57	880
Nynee Tal, . . . . .	do.	59	4	do.	do.	29 23	79 31	6433
Agra, . . . . .	do.	59	4	do.	do.	27 10	78 5	551
Nazirabad, . . . . .	do.	4	4	?	?	26 18	74 45	1585
Benares, . . . . .	do.	59	3	1865-68	10: 4	25 2	83 5	260
Calcutta, . . . . .	do.	60	12	1856-67	hourly	22 33	88 21	19
Kurrachee, . . . . .	do.	45	1	1864	9: 3	24 51	67 2	?
Bombay, . . . . .	do.	61	14	1847-60	hourly	18 54	72 48	35
Poonah, . . . . .	do.	98	1	1830	{ 9, 10: 4-5, 10-11 }	18 31	74 6	1823
Secunderabad, . . . . .	do.	62	1	1864	4, 10: 4, 10	17 25	78 40	1700
Dodabetta, . . . . .	do.	63	5	1851-55	9: 3	11 32	76 50	8640
Madras, . . . . .	do.	64	5	1846-50	hourly	13 4	80 19	27
Do., . . . . .	do.	64	22	1822-43	red.	do.	do.	27
Merkeria, . . . . .	do.	27	3	1838-40	9: 5	12 46	75 44	4500
Trivandrum, . . . . .	do.	4	8½	?	?	8 31	77 0	130
Colombo, . . . . .	do.	44	6	1853-59	9½: 3½	6 56	79 50	18
Gangarooowa, . . . . .	do.	60	1½	1863-64	do.	7 17	80 37	1560
Ava, . . . . .	do.	4	1	?	?	21 50	96 5	?
Saigon, . . . . .	Cochin China	51	1½	1867-68	6, 10: 4, 10	10 33	106 33	?
Honolulu, . . . . .	{ East India Islands and Pacific }	4	1	?	?	21 16	- 157 59	?
Manilla, . . . . .	do.	4	1	?	?	14 36	129 0	?
Sincapore, . . . . .	do.	66	1	1866	9:	1 17	103 51	50
Raffles Lighthouse (Sin- capore, . . . . . }	do.	66	2	1866-67	noon	1 9	103 44	65
Padang, . . . . .	do.	33	3½	1850-53	6, 9: 3, 10	- 0 56	100 2	?
Batavia, . . . . .	do.	4	1	?	?	- 6 9	106 53	?
Buitenzorg, . . . . .	do.	33	12	1841-54	{ 6, 9: 3, 10 }	- 6 37	106 49	889
Samarang, . . . . .	do.	4	1	?	?	- 6 50	110 33	?

AND THE PREVAILING WINDS OVER THE GLOBE.

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OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
30-232	30-197	30-161	29-997	29-987	29-815	29-840	29-840	30-009	30-055	30-165	30-263	30-047
30-179	30-126	30-014	29-951	29-864	29-764	29-713	29-702	29-790	29-981	30-104	30-143	29-944
30-087	30-080	30-063	29-991	29-996	29-802	29-780	29-677	29-783	29-918	30-065	30-115	29-935
29-743	29-769	29-870	29-799	29-721	29-610	29-603	29-628	29-732	29-840	29-838	29-715	29-739
30-173	30-123	30-099	29-983	29-886	29-783	29-790	29-748	29-871	30-033	30-143	30-170	29-984
23-974	23-898	23-953	23-869	23-954	24-004	24-037	24-049	24-126	24-052	24-027	23-978	23-993
29-975	29-852	29-793	30-032	29-942	29-868	29-791	29-800	29-966	29-972	30-002	30-124	29-926
30-084	30-068	29-885	29-932	29-879	29-890	...	...	...	30-004	30-040	30-002	...
29-897	29-836	29-737	29-772	29-689	29-604	29-535	29-563	29-697	29-723	29-792	29-818	29-722
29-890	29-932	29-721	29-830	29-749	29-704	29-639	29-638	29-714	...	...	29-922	...
27-432	27-432	27-374	27-341	27-376	27-330	27-263	27-278	27-361	27-443	27-446	27-431	27-376
...	...	...	...	...	...	29-714	29-733	...	...	...	...	...
...	...	...	...	29-891	...	...	...	...	...	...	...	...
29-823	29-844	29-776	29-701	29-610	29-528	29-482	29-512	29-632	29-778	29-876	29-890	29-705
29-653	29-566	29-492	29-342	29-179	29-013	29-017	29-106	29-273	29-437	29-604	29-669	29-361
29-153	29-069	29-018	28-876	28-762	28-625	28-638	28-696	28-779	28-969	29-109	29-168	28-905
23-873	23-825	23-882	23-823	23-745	23-697	23-703	23-726	23-784	23-884	23-904	23-889	23-811
29-496	29-409	29-345	29-210	29-061	28-940	28-954	28-983	29-099	29-301	29-429	29-509	29-223
28-475	28-387	28-317	28-224	28-109	27-997	27-973	28-025	28-137	28-305	28-431	28-451	28-236
29-882	29-782	29-697	29-557	29-415	29-289	29-299	29-365	29-446	29-670	29-808	29-888	29-592
30-022	29-944	29-859	29-754	29-645	29-542	29-538	29-592	29-676	29-827	29-968	30-029	29-783
30-092	30-058	30-004	29-914	29-827	29-634	29-550	29-672	29-826	29-993	30-094	30-131	29-900
29-935	29-908	29-856	29-794	29-745	29-648	29-644	69-718	29-772	29-829	29-897	29-935	29-807
28-087	28-002	27-952	27-908	27-846	27-768	27-767	27-840	27-925	27-924	28-018	28-068	27-925
28-267	28-226	28-193	28-113	28-097	27-986	27-993	28-053	28-105	28-192	28-230	28-247	28-142
22-176	22-183	22-187	22-171	22-146	22-088	22-064	22-092	22-128	22-149	22-155	22-171	22-142
29-986	29-971	29-909	29-816	29-730	29-693	29-714	29-746	29-763	29-827	29-918	29-957	29-836
30-019	30-008	29-923	29-848	29-740	29-698	29-721	29-748	29-772	29-846	29-932	29-995	29-863
26-130	26-154	26-105	26-080	26-065	26-086	26-019	25-998	26-030	26-045	26-098	26-120	26-077
29-739	29-721	29-688	29-653	29-631	29-650	29-680	29-678	29-694	29-707	29-708	29-730	29-691
29-909	29-906	29-885	29-841	29-839	29-835	29-846	29-854	29-880	29-881	29-886	29-909	29-873
28-359	28-334	28-335	28-280	28-287	28-275	28-278	28-287	28-304	28-308	28-328	28-350	28-310
29-801	29-688	29-624	29-546	29-468	29-417	29-396	29-447	29-472	29-583	29-684	29-774	29-573
29-985	29-992	29-989	29-961	29-953	29-910	29-938	29-922	29-922	29-922	30-040	30-064	29-966
30-027	30-018	30-095	30-122	30-130	30-078	30-105	30-077	30-084	30-105	30-058	30-105	30-083
29-992	29-936	29-896	29-854	29-797	29-764	29-763	29-750	29-759	29-820	29-888	30-005	29-852
30-017	29-980	30-017	29-970	29-934	29-972	29-972	30-000	30-000	29-974	29-967	30-000	29-984
29-951	29-943	29-940	29-875	29-916	29-945	29-923	29-930	29-922	29-922	29-934	29-904	29-934
29-686	29-696	29-696	29-662	29-646	29-662	29-670	29-689	29-698	29-704	29-666	29-691	29-680
29-747	29-746	29-739	29-707	29-730	29-769	29-757	29-765	29-769	29-782	29-757	29-730	29-750
28-983	28-985	28-975	28-969	28-964	28-973	28-971	28-980	28-983	28-993	28-964	28-973	28-976
29-953	29-978	29-973	29-983	30-004	30-014	30-042	30-045	30-041	30-041	30-010	29-911	30-000

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 685.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.	Longitude.	Height in Eng. Feet.
<i>Tahiti</i> (said to be red. to sea-level),	East India Islands and Pacific	67	5	1855-60	{4 daily obs.}	-17 32	-149 34	?
Port de France, N. Cal- donia, . . . . .	do.	65	2	1863-64	{6,10:1, 4,10}	-22 16	166 26	22
Suez, . . . . .	Egypt	51	2	1866-68	{6,9,12: 3,6,9}	29 57	32 32	20
Ismäilia, . . . . .	do.	51	2	do.	do.	30 38	32 13	25
Port Saïd, . . . . .	do.	51	2	do.	do.	31 18	32 18	10
Alexandria, . . . . .	do.	68	2	1858-60	7: 2	31 11	29 50	50
Cairo, . . . . .	do.	4	1	?	?	30 6	31 26	?
Gondar, . . . . .	Abyssinia	57	$\frac{7}{2}$	1832-33	9: 3	15 50	37 32	7422
Massuah, . . . . .	do.	57	$\frac{7}{2}$	1831-32	9: 3 $\frac{1}{2}$	15 36	39 21	5
Tripolis, . . . . .	Tripoli	4	...	?	?	32 54	13 19	?
La Calle, . . . . .	Algeria	69	$3\frac{1}{2}$	1865-68	?	36 52	8 23	30
Dellys, . . . . .	do.	69	$3\frac{1}{2}$	do.	?	36 49	3 50	75
Algiers, . . . . .	do.	69	11	1857-67	?	36 43	3 3	66
Do., . . . . .	do.	69	$3\frac{1}{2}$	1865-68	?	do.	do.	66
Djidjilly, . . . . .	do.	69	$3\frac{1}{2}$	do.	?	36 17	5 42	49
Medeah, . . . . .	do.	69	$3\frac{1}{2}$	do.	?	36 13	2 43	3150
Aumale, . . . . .	do.	69	$3\frac{1}{2}$	do.	?	36 7	3 40	2933
Orleansville, . . . . .	do.	69	$3\frac{1}{2}$	do.	?	36 6	1 19	459
Oran, . . . . .	do.	35	13	1841-53	10: 4	35 40	-0 38	164
Do., . . . . .	do.	69	$3\frac{1}{2}$	1866-68	?	do.	do.	164
Laghouat, . . . . .	do.	69	$3\frac{1}{2}$	do.	?	33 47	2 54	2461
Casa Blanca, . . . . .	Morocco	70	1	1867-68	various	30 0	-7 30	?
St George d'Elmina, . . . . .	Ashantee	33	3	1860-62	6: 2, 9	5 5	-1 20	75
Christiansburg, . . . . .	do.	33	$7\frac{1}{2}$	{1829-40, 1833-34}	various	5 24	0 10	60
Gondokoro, . . . . .	Benin	4	1	?	?	4 30	31 40	1800
Lagos, . . . . .	Slave Coast	71	$\frac{7}{2}$	1863	?	6 12	3 25	0
Cape Town, . . . . .	Cape Colony	72	14	1842-55	{hourly; 5, 9: 1, 5, 9}	-33 56.	18 27	37
Do., . . . . .	do.	73	4	1862-65	5, 9: 1, 5, 9	do.	do.	37
Worcester, . . . . .	do.	73	4	do.	9: 1, 5	-33 38	19 23	776
Simon's Town, . . . . .	do.	73	4	do.	do.	-34 12	18 24	50
Mossel Bay, . . . . .	do.	73	$1\frac{5}{6}$	1862-63	do.	-34 12	22 5	429
Somerset West, . . . . .	do.	73	3	1861-64	do.	-34 2	18 46	124
<i>Graham's Town</i> , . . . . .	do.	44	$4\frac{1}{2}$	1864-69	$9\frac{1}{2}$ : $3\frac{1}{2}$	-33 13	26 32	1750
Graff Reinet, . . . . .	do.	73	3	1863-65	9: 1, 5	-32 18	24 51	2517
Pieter Maritzburg, . . . . .	Natal	74	8	1858-65	9: 3	-29 30	30 2	2096
Zambesi Delta, . . . . .	Mozambique	71	$\frac{1}{2}$	1858	mean	-18 24	35 30	0
Tamatave, . . . . .	Madagascar	70	$\frac{1}{4}$	1863	9: 4	-18 3	49 11	0
St Louis, . . . . .	Mauritius	75	13	1853-65	{ $3\frac{1}{2}$ , $9\frac{1}{2}$ : $3\frac{1}{2}$ , $9\frac{1}{2}$ }	-20 10	57 30	30
St Denis, . . . . .	Bourbon	4	2	?	?	-20 51	55 30	142
Socotra, . . . . .	...	4	$\frac{5}{2}$	?	?	-12 30	54 10	?
Somerset, Cape York, . . . . .	Queensland	103	$2\frac{1}{4}$	1865-67	9: 3	-10 44	142 36	70
Brisbane, . . . . .	do.	76	3	1859-61	9: 3	-27 5	153 0	70
<i>Brisbane</i> , . . . . .	do.	103	$2\frac{1}{2}$	1867-69	9: 3	-27 28	153 6	140

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
29-868	29-846	29-856	29-864	29-910	29-921	29-943	29-966	29-968	29-946	29-913	29-887	29-907
29-913	29-946	30-008	30-008	30-107	30-119	30-131	30-079	30-119	30-032	30-008	29-934	30-034
30-095	30-127	29-938	29-939	29-902	29-865	29-786	29-812	29-894	29-999	30-079	30-071	29-959
30-062	30-079	29-906	29-920	29-896	29-821	29-731	29-750	29-847	29-963	30-042	30-024	29-920
30-080	30-103	29-934	29-950	29-906	29-853	29-772	29-798	29-896	29-985	30-052	30-046	29-947
29-998	29-929	29-993	29-919	29-827	29-792	29-738	29-732	29-840	29-966	29-978	29-977	29-891
30-000	30-036	29-900	29-821	29-842	29-834	29-730	29-756	29-923	29-938	29-998	30-080	29-904
23-338	23-302	23-268	23-267	...	...	...	...	...	23-312	23-312	23-315	...
30-097	30-010	29-955	29-926	...	...	...	...	29-859	29-956	30-028	30-055	...
30-138	29-993	29-928	29-956	29-946	30-044	30-019	30-035	30-034	30-013	30-000	30-068	30-014
29-977	30-024	29-831	29-985	29-953	29-944	29-958	29-931	29-971	29-930	30-025	30-036	29-960
29-953	30-004	29-867	29-902	29-902	29-878	29-858	29-839	29-879	29-840	29-875	29-985	29-900
30-073	30-061	29-934	29-944	29-930	29-961	29-993	29-958	29-996	29-960	29-981	30-097	29-986
29-985	30-083	29-863	29-934	29-914	29-906	29-948	29-914	29-973	29-934	30-073	30-073	29-967
29-982	30-028	29-775	29-886	29-867	29-875	29-878	29-846	29-904	29-871	29-963	29-991	29-906
26-977	27-000	26-812	26-890	26-886	26-936	26-981	26-966	26-945	26-908	26-956	26-974	26-936
26-956	27-067	26-890	26-981	26-986	27-016	27-060	27-028	27-099	27-004	27-083	27-063	27-020
29-686	29-784	29-520	29-611	29-585	29-603	29-699	29-620	29-579	29-548	29-693	29-750	29-640
29-984	29-956	29-924	29-858	29-836	29-870	29-861	29-845	29-861	29-857	29-935	30-002	29-895
29-941	30-024	29-819	29-886	29-874	29-914	29-827	29-804	29-855	29-823	29-934	29-952	29-888
27-512	27-665	27-391	27-441	27-500	27-469	27-452	27-465	27-472	27-461	27-469	27-477	27-481
30-268	30-335	30-001	30-154	30-048	30-060	30-134	30-095	30-123	30-119	30-103	30-099	30-128
29-877	29-857	29-844	29-849	29-882	29-942	29-985	29-981	29-953	29-901	29-870	29-887	29-901
29-862	29-838	29-829	29-837	29-874	29-939	29-971	29-958	29-920	29-882	29-862	29-849	29-885
28-356	28-300	28-317	28-349	28-425	28-474	28-469	28-439	28-433	28-413	28-392	28-394	28-397
...	...	...	...	...	29-974	30-011	30-047	30-001	29-950	29-925	29-955	...
29-940	29-966	29-968	29-984	30-087	30-085	30-199	30-112	30-072	30-080	29-975	29-974	30-037
29-937	29-911	29-954	30-002	30-065	30-110	30-134	30-129	30-090	30-006	30-019	29-932	30-023
29-162	29-146	29-184	29-232	29-298	29-330	29-347	29-376	29-309	29-227	29-234	29-152	29-250
29-947	29-911	29-961	30-000	30-066	30-087	30-116	30-118	30-098	30-011	30-023	29-942	30-023
29-576	29-557	29-576	29-598	29-680	29-642	29-699	29-713	29-681	29-615	29-583	29-531	29-621
29-845	29-821	29-878	29-915	29-982	29-984	30-053	30-055	30-000	29-928	29-915	29-835	29-934
29-919	29-955	29-966	30-050	30-102	30-078	30-148	30-085	30-075	30-037	29-984	29-938	30-028
27-395	27-416	27-414	27-480	27-555	27-616	27-602	27-572	27-524	27-422	27-409	27-353	27-480
27-786	27-802	27-844	27-914	27-937	27-994	28-001	27-981	27-905	27-864	27-822	27-795	27-887
...	...	...	...	...	...	30-202	...	...	...	...	...	...
...	...	...	...	...	...	...	30-077	30-016	30-024	29-977	...	...
29-915	29-854	29-925	29-984	30-048	30-126	30-167	30-171	30-158	30-111	30-031	29-963	30-038
29-695	29-687	29-679	29-757	29-793	29-901	29-948	29-935	29-894	29-859	29-811	29-740	29-808
29-420	29-399	29-380	...	...	29-086	29-084	...	...	...	...	...	...
29-788	29-785	29-847	29-795	29-852	29-914	29-916	29-933	29-907	29-898	29-868	29-780	29-857
29-807	29-837	29-916	29-944	30-058	29-987	30-056	30-109	30-033	30-001	29-918	29-850	29-959
29-872	29-875	30-015	30-026	30-097	30-093	30-164	30-117	30-020	29-991	29-966	29-888	30-010

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 635.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.	Longitude.	Height in Eng. Feet.
Casino & Richmond River,	Queensland	76	5	1858-62	9: 3	-28 50	153 0	139
Armidale, . . . . .	N. South Wales	76	3½	1860-63	9:	-30 33	151 46	3195
Newcastle, . . . . .	do.	76	9	1858-66	do.	-32 57	151 47	18
Windsor, . . . . .	do.	77	4	1863-66	9: 3	-33 36	150 50	53
Paramatta, . . . . .	do.	76	3½	1858-61	do.	-33 49	151 1	60
Bathurst, . . . . .	do.	76	5	1859-63	9:	-33 24	149 37	2333
Goulburn, . . . . .	do.	76	8	1858-65	9: 3, 9, & 9:	-34 45	149 45	2129
Sydney, . . . . .	do.	76	11	1858-68	do.	-33 52	151 11	155
Do., . . . . .	do.	76	9	1858-66	9: 3, & 9:	do.	do.	155
Deniliquin, . . . . .	do.	76	9	do.	9: 3, 9, & 9:	-35 32	145 2	410
Albury, . . . . .	do.	76	8½	do.	do.	-36 6	147 0	572
Cooma, . . . . .	do.	76	7	1858-64	9: 3, & 9:	-36 13	149 9	2637
Sandhurst, . . . . .	Victoria	78	5	1863-67	9: 3, 9	-36 43	144 21	778
Ararat, . . . . .	do.	78	5	do.	9: 3	-37 18	142 58	1072
Ballarat, . . . . .	do.	78	5	do.	9½: 3½, 9½	-37 34	143 53	1438
Melbourne, . . . . .	do.	78	10	1858-67	6, 9: 3, 9	-37 50	144 59	121
Do., . . . . .	do.	78	5	1863-67	do.	do.	do.	121
Portland, . . . . .	do.	78	5	do.	do.	-38 21	141 32	37
Cape Otway, . . . . .	do.	78	5	do.	do.	-38 54	143 37	300
Kupunda, . . . . .	South Australia	79	4	1861-64	9: 6	-34 20	139 0	730
Adelaide, . . . . .	do.	79	7	1861-65 1867-68	do.	-34 53	138 39	140
Strathalbyn, . . . . .	do.	79	4	1861-64	do.	-35 8	138 57	220
Guichen Bay, . . . . .	do.	79	4	do.	do.	-37 3	139 42	19
Mount Gambier, . . . . .	do.	79	4	do.	do.	-37 51	140 53	133
Freemantle, . . . . .	West Australia	44	3	1853-55	9½: 3½	-33 2	115 45	16
Kent's Group, . . . . .	Tasmania	80	5	1861-66	6, 12: 6	-39 29	147 35	280
Swan Island, . . . . .	do.	80	2½	1864-66	do.	-40 45	148 10	14
Swansea, . . . . .	do.	80	3	do.	do.	-42 8	148 5	18
Hobart Town, . . . . .	do.	80	28	1841-68	do.	-42 52	147 21	37
Do., . . . . .	do.	80	5	1861-66	do.	do.	do.	37
Port Arthur, . . . . .	do.	80	5	1861-66	do.	-43 9	147 54	55
Auckland, . . . . .	New Zealand	{ 44, 45, 80, 81 }	11	{ 1853-58, 1863-68 }	9½: 3½	-36 50	174 51	140
Taranaki, . . . . .	do.	81	5	1864-68	10: 4	-39 4	174 5	70
Wellington, . . . . .	do.	81	5	do.	do.	-41 16	174 47	90
Nelson, . . . . .	do.	81	5	do.	9: 3	-41 16	173 19	18
Hokitika, . . . . .	do.	81	3	1866-68	10: 4	-42 42	170 59	8
Christchurch, . . . . .	do.	81	5	1864-68	9½: 3½, 9½	-42 33	172 39	21
Do., . . . . .	do.	81	3	1866-68	do.	do.	do.	21
Dunedin, . . . . .	do.	81	5½	1862-68	9½: 4½	-45 52	170 31	550
Southland, . . . . .	do.	81	10	1859-68	9: 3, 9	-46 17	168 20	79
Upernivik, . . . . .	Greenland	2	5	1833-38	noon	72 48	-55 53	15
Jacobshavn, . . . . .	do.	2	9½	1842-51	do.	69 12	-51 0	10
Godthaab, . . . . .	do.	2	5	1841-46	do.	64 10	-51 53	15
Baffin Bay, . . . . .	Arctic	82	1	1857-58	various	mean 72½° lat.	various	0
Van Rensselaer, . . . . .	do.	82	2	1853-4-5	do.	78 37	-73 0	0
Port Foulke, . . . . .	do.	82	1	1860-61	do.	78 18	-73 0	6
Port Kennedy . . . . .	do.	82	1	1858-59	do.	72 1	-94 0	0
Boothia Felix, . . . . .	do.	4	2	...	...	70 3	-95 0	0

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
29-776	29-780	29-849	29-911	29-961	29-909	29-962	30-032	29-958	29-905	29-838	29-746	29-886
26-631	26-617	26-733	26-752	26-741	26-717	26-671	26-716	26-690	26-681	26-639	26-590	26-682
29-912	29-936	30-037	30-100	30-103	30-117	30-087	30-112	30-025	29-994	29-970	29-902	30-024
29-847	29-869	30-012	30-061	30-057	30-086	29-980	30-045	29-916	29-854	29-862	29-845	29-949
29-846	29-888	29-972	30-022	29-999	30-029	30-084	30-047	29-956	29-972	29-907	29-864	29-966
27-655	27-668	27-766	27-832	27-822	27-816	27-825	27-868	27-807	27-747	27-715	27-681	27-767
27-766	27-796	27-911	27-934	27-896	27-889	27-884	27-885	27-796	27-818	27-796	27-737	27-842
29-773	29-792	29-906	29-948	29-959	29-949	29-916	29-950	29-858	29-828	29-814	29-745	29-874
29-774	29-795	29-903	29-950	29-939	29-949	29-907	29-951	29-866	29-852	29-809	29-753	29-871
29-506	29-554	29-660	29-728	29-741	29-767	29-716	29-735	29-670	29-577	29-586	29-506	29-646
29-238	29-318	29-449	29-478	29-482	29-540	29-468	29-518	29-473	29-365	29-344	29-273	29-412
27-241	27-290	27-379	27-393	27-342	27-340	27-292	27-323	27-275	27-280	27-251	27-219	27-303
29-109	29-143	29-257	29-308	29-310	29-357	29-223	29-304	29-168	29-074	29-143	29-083	29-207
28-783	28-826	28-902	28-957	28-952	28-977	28-880	28-936	28-812	28-760	28-822	28-797	28-867
28-464	28-484	28-589	28-620	28-593	28-630	28-509	28-576	28-458	28-394	28-466	28-418	28-517
29-811	29-839	29-940	29-987	29-958	30-000	29-950	29-985	29-852	29-833	29-851	29-777	29-899
29-827	29-843	29-966	30-000	29-994	30-045	29-911	29-988	29-837	29-768	29-843	29-781	29-900
29-931	29-942	30-050	30-075	30-049	30-121	29-971	30-033	29-893	29-851	29-930	29-884	29-977
29-701	29-708	29-819	29-840	29-757	29-848	29-719	29-795	29-657	29-594	29-689	29-641	29-722
29-062	29-090	29-187	29-240	29-146	29-210	29-203	29-262	29-188	29-130	29-127	29-057	29-159
29-805	29-825	29-915	29-975	30-008	29-980	29-942	30-013	29-866	29-844	29-868	29-782	29-902
29-676	29-705	29-787	29-843	29-825	29-817	29-751	29-859	29-761	29-741	29-721	29-653	29-761
29-878	29-914	30-010	30-063	30-018	30-032	29-951	30-056	29-975	29-928	29-927	29-884	29-970
29-792	29-816	29-922	29-960	29-903	29-921	29-814	29-926	29-854	29-812	29-826	29-776	29-860
29-936	29-930	29-987	30-072	30-122	30-121	30-010	30-066	30-080	30-023	30-015	29-962	30-027
29-620	29-650	29-766	29-764	29-686	29-776	29-588	29-652	29-600	29-610	29-598	29-564	29-656
29-933	30-023	30-090	30-060	30-010	30-060	29-955	30-030	29-875	30-020	29-945	29-820	29-987
29-677	29-748	29-854	29-855	29-781	29-827	29-734	29-704	29-665	29-681	29-644	29-633	29-734
29-744	29-837	29-867	29-903	29-871	29-892	29-856	29-822	29-758	29-772	29-722	29-746	29-816
29-807	29-849	29-973	29-996	29-927	30-002	29-855	29-912	29-789	29-824	29-788	29-756	29-873
29-791	29-829	29-865	29-835	29-747	29-823	29-654	29-700	29-647	29-727	29-687	29-719	29-752
29-968	29-999	30-067	30-098	29-990	29-954	29-968	29-941	29-930	29-950	29-918	29-974	29-980
29-906	29-946	30-040	30-058	29-974	29-930	29-898	29-839	29-814	29-832	29-852	29-886	29-915
29-838	29-876	29-963	30-007	29-895	29-902	29-886	29-822	29-866	29-781	29-778	29-837	29-871
29-874	29-923	29-974	30-058	29-959	29-984	30-000	29-903	29-898	29-670	29-790	29-918	29-906
29-903	29-861	30-007	30-026	30-082	29-971	29-975	29-885	29-910	29-778	29-810	29-850	29-922
29-783	29-840	29-947	29-953	29-910	29-892	29-880	29-835	29-848	29-716	29-700	29-790	29-841
29-812	29-808	29-957	29-926	30-040	29-952	29-870	29-838	29-902	29-682	29-684	29-741	29-851
29-859	29-855	30-011	29-968	30-023	30-023	29-912	29-882	29-871	29-773	29-774	29-823	29-898
29-789	29-843	29-867	29-897	29-889	29-866	29-826	29-770	29-788	29-665	29-692	29-744	29-803
29-580	29-458	29-699	29-774	29-803	29-703	29-694	29-671	29-620	29-576	29-517	29-592	29-640
29-634	29-681	29-786	29-830	29-867	29-815	29-766	29-744	29-777	29-722	29-754	29-612	29-749
29-582	29-713	29-824	29-833	29-963	29-897	29-869	29-763	29-824	29-818	29-793	29-655	29-786
29-532	29-649	29-893	29-940	30-014	29-817	29-753	29-736	29-735	29-756	29-665	29-570	29-755
29-778	29-848	29-750	29-903	29-942	29-719	29-741	29-694	29-658	29-755	29-758	29-753	29-775
29-834	29-747	29-816	30-085	29-985	29-678	29-691	29-662	29-684	29-618	30-087	30-032	29-824
29-979	29-933	30-173	30-179	30-010	29-913	29-704	29-741	29-899	29-798	30-052	29-872	29-938
29-823	29-975	29-962	29-993	30-141	30-023	29-891	29-857	29-826	29-957	29-945	29-920	29-943

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 635.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.	Longitude.	Height in Eng. Feet.
Melville Island, . . .	Arctic	4	1	1819-20	?	75 40	-112 3	0
Port Bowen, . . .	do.	4	1	1824-25	?	73 13	-88 54	0
E. of Greenland, . . .	do.	83	7	1807-12, 1822	noon	66 to 80 34	about -7 0	0
Sitka, . . . . .	Russian America	26	17	1848-64	hourly & 4, 6, & c.	56 50	-135 0	20
New Westminster, . . .	British Columbia	84	2	1860-61	9½: 3½	49 13	-122 53	54
Esquimault Harbour, . . .	Vancouver	85	1	do.	?	48 25	-123 27	0
Astoria, . . . . .	Oregon	86	2½	1859-60 1866	?	46 8	-123 48	?
Sacramento, . . . . .	California	86	9	1858-66	7: 2, 9	38 35	-121 28	81
San Francisco, . . . . .	do.	86	9	do.	do.	37 48	-122 23	86
Great Salt Lake City, . . .	Utah	87	2	1858-59	6, 9: 3, 9	40 45	-111 26	4260
St John's, . . . . .	Newfoundland	44	6	1853-59	9½: 3½	47 35	-52 42	130
Halifax, . . . . .	Nova Scotia	44	4	1853-56, 1858	do.	44 39	-63 37	8
Do., . . . . .	do.	45	2	1864-65	9: 3	44 39	-63 36	137
Albion Mines, . . . . .	do.	27	10	1843-52	noon	45 34	-62 42	128
Quebec, . . . . .	Canada East	44, 45	3½	1853-55, 1858, 1866	9½: 3½	46 48	-71 12	230
St Martin's, . . . . .	do.	87	6	1854-59	6: 2, 10	45 32	-73 36	118
Kingston, . . . . .	Canada West	44	4½	do.	9½: 3½	44 14	-76 31	294
Toronto, . . . . .	do.	88, 71	28	1840-67	6: 2, 10	43 39	-79 2	342
Do., . . . . .	do.	88	11	1857-67	do.	do.	do.	342
Do., . . . . .	do.	88	6	1854-59	do.	do.	do.	342
Hamilton, . . . . .	do.	87, 99	11	1849-59	9: 9	43 15	-79 57	290
Gardiner, . . . . .	Maine, U.S.	87	5	1855-59	7: 2, 9	44 11	-69 46	90
Steuben, . . . . .	do.	87	6	1854-59	do.	44 28	-67 50	50
Amherst, . . . . .	Massachusetts	87	6	do.	do.	42 22	-72 34	267
New Bedford, . . . . .	do.	87	6	do.	do.	41 39	-70 56	90
Nantucket, . . . . .	do.	87	6	do.	do.	41 16	-70 6	30
Burlington, . . . . .	Vermont	87	5	1854-56, 1858-59	do.	44 29	-73 11	346
Providence, . . . . .	Rhode Is.	89	29	1832-60	sr: 2, 10	41 50	-71 23	170
Do., . . . . .	do.	89	6	1854-59	do.	do.	do.	170
Rochester, . . . . .	New York	87	4	1856-59	7: 2, 9	43 8	-77 51	516
Bedford, . . . . .	Pennsylvania	87	4	1854-56, 1858	do.	40 1	-78 30	900
Harrisburg, . . . . .	do.	87	6	1854-59	do.	40 16	-76 50	280
Pittsburg, . . . . .	do.	87	6	do.	various	40 30	-80 00	960
Lambertville, . . . . .	New Jersey	87	6	do.	7: 2, 9	40 23	-74 56	?
Washington, . . . . .	{ District of Columbia }	87	6	do.	do.	38 56	-76 58	40
Do., . . . . .	do.	90	5	1862-66	{ 0, 3, 6, } { 9, 12: } { 3, 6, 9 }	38 56	-76 58	103
Portsmouth, . . . . .	Virginia	87	3	1857-59	7: 2, 9	36 50	-76 19	34
Chapel Hill, . . . . .	N. Carolina	87	6	1854-59	do.	35 54	-79 17	500
All Saints, . . . . .	S. Carolina	87	5	1855-59	do.	33 40	-79 17	20
Athens, . . . . .	Georgia	87	2	1857-59	do.	33 58	-83 30	730
Savannah, (at sea level?)	do.	87	6	1854-59	do.	32 5	-81 7	42

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
30-077	29-769	29-803	29-979	30-109	29-823	29-668	29-726	29-748	29-811	29-945	29-865	29-860
29-717	29-886	30-107	30-068	30-051	29-888	29-817	29-683	29-689	29-962	29-899	29-869	29-886
...	...	...	29-852	29-870	29-852	29-870	...	...	...	...	...	...
29-551	29-644	29-648	29-711	29-836	29-814	29-877	29-850	29-766	29-623	29-615	29-630	29-714
30-074	30-042	30-022	30-000	29-984	29-962	30-032	30-012	30-029	30-008	29-937	29-928	30-002
30-110	30-030	30-090	30-030	30-030	30-060	30-100	30-040	30-080	30-010	30-100	29-960	30-053
29-985	30-138	29-957	30-094	29-996	30-067	30-067	30-023	30-032	30-021	29-992	29-958	30-028
30-106	30-074	30-060	30-011	29-938	29-888	29-869	29-867	29-880	29-946	30-049	30-080	29-981
30-070	30-043	30-040	30-004	29-938	29-897	29-891	29-890	29-894	29-943	30-024	30-041	29-973
25-880	25-780	25-670	25-685	25-640	25-655	25-645	25-675	25-727	25-760	25-705	(25-800)	25-717
29-924	29-781	29-690	29-942	29-943	29-934	29-993	29-964	29-971	29-986	29-908	29-842	29-906
29-985	29-797	29-685	29-931	29-959	29-976	29-974	29-950	30-024	29-962	29-954	29-869	29-922
29-744	29-681	29-748	29-842	29-780	29-823	29-778	29-810	29-848	29-674	29-836	29-737	29-776
29-696	29-687	29-672	29-702	29-746	29-674	29-710	29-770	29-767	29-766	29-687	29-688	29-714
30-104	30-055	29-901	30-010	29-864	29-934	29-955	29-947	30-022	30-045	29-936	29-982	29-980
29-842	29-753	29-639	29-693	29-770	29-719	29-796	29-773	29-820	29-846	29-800	29-852	29-775
30-062	30-014	29-941	29-970	29-965	29-917	29-999	29-977	30-054	30-078	29-946	30-044	29-973
29-647	29-624	29-596	29-597	29-574	29-572	29-599	29-622	29-662	29-645	29-612	29-656	29-617
29-650	29-654	29-577	29-583	29-564	29-577	29-593	29-606	29-671	29-645	29-595	29-676	29-617
29-670	29-643	29-538	29-572	29-593	29-542	29-613	29-608	29-675	29-655	29-600	29-672	29-615
29-693	29-648	29-608	29-606	29-663	29-650	29-707	29-700	29-745	29-684	29-634	29-670	29-666
29-874	29-782	29-702	29-750	29-824	29-746	29-792	29-786	29-860	29-810	29-850	29-854	29-802
29-988	29-895	29-773	29-848	29-918	29-862	29-923	29-928	29-985	29-928	29-920	29-932	29-908
29-813	29-730	29-788	29-648	29-687	29-717	29-673	29-723	29-720	29-788	29-737	29-788	29-730
29-972	29-915	29-788	29-827	29-878	29-853	29-925	29-925	29-985	29-928	29-900	29-925	29-902
30-043	29-970	29-873	29-900	29-963	29-928	29-973	29-985	30-053	29-988	29-960	30-008	29-970
29-668	29-572	29-488	29-558	29-612	29-555	29-618	29-625	29-682	29-612	29-602	29-652	29-604
29-807	29-779	29-743	29-740	29-722	29-725	29-722	29-788	29-823	29-808	29-783	29-784	29-779
29-857	29-773	29-674	29-718	29-773	29-722	29-797	29-789	29-856	29-808	29-765	29-805	29-778
29-508	29-420	29-358	29-348	29-385	29-338	29-405	29-350	29-418	29-432	29-365	29-435	29-397
29-145	29-045	29-005	29-110	29-048	29-060	29-112	29-105	29-160	29-180	29-102	29-100	29-097
29-820	29-765	29-678	29-675	29-695	29-670	29-733	29-728	29-815	29-785	29-770	29-803	29-745
29-043	28-952	28-923	28-914	28-912	28-910	28-968	28-953	29-035	28-995	28-980	28-983	28-964
30-058	29-982	29-933	29-920	29-953	29-945	30-025	30-010	30-100	30-023	30-017	30-028	30-000
30-145	30-047	29-963	29-952	29-967	29-938	29-990	29-988	30-055	30-047	30-028	30-080	30-017
30-031	30-030	29-921	29-956	29-844	29-912	29-914	29-932	30-000	29-967	30-004	30-035	29-962
30-140	30-083	29-967	29-887	29-963	29-927	29-977	29-980	30-043	30-010	30-047	30-073	30-006
29-602	29-525	29-468	29-448	29-453	29-463	29-478	29-483	29-552	29-540	29-550	29-570	29-511
30-090	30-006	29-962	29-942	29-920	29-960	29-968	29-946	30-006	29-998	30-042	30-074	29-993
29-410	29-275	29-270	29-210	29-233	29-293	29-273	29-285	29-315	29-348	29-410	29-340	29-297
30-135	30-087	30-038	30-015	29-985	30-010	30-037	30-018	30-053	30-058	30-085	30-118	30-053

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 635.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.	Longitude.	Height in Eng. Feet.
Jacksonville, . . . . .	Florida	87	6	1854-59	7: 2, 9	30 30	-82 0	13
Warrington, . . . . .	do.	87	4 $\frac{5}{12}$	1854-57 1859	do.	30 20	-87 16	12
Auburn, . . . . .	Alabama	87	3	1855-57	do.	32 37	-85 34	?
Columbus, . . . . .	Mississippi	87	4	1856-59	do.	33 30	-88 29	227
Washington, . . . . .	Texas	87	2	1858-59	do.	30 26	-96 15	360
Sisterdale, . . . . .	do.	87	1	1859	do.	29 54	-98 35	1320
Goliad, . . . . .	do.	87	1	1858	do.	28 40	-97 35	50
Memphis, . . . . .	Tennessee	87	2 $\frac{1}{3}$	1857-59	do.	35 8	-90 00	262
Glenwood, . . . . .	do.	87	6	1854-59	do.	36 28	-87 13	481
Springdale, . . . . .	Kentucky	87	6	do.	various	38 7	-85 24	570
Cincinnati, . . . . .	Ohio	87	4	1856-59	7: 2, 9	39 6	-84 28	500
Marquette, . . . . .	Michigan	87	2 $\frac{1}{2}$	1857-59	do.	46 32	-87 41	630
Ottawa Point, . . . . .	do.	87	1 $\frac{1}{3}$	1858-59	6, 9: 3, 9	44 16	-83 25	600
New Harmony, . . . . .	Indiana	87	6	1854-59	7: 2, 9	38 8	-87 50	320
Wheaton, . . . . .	Illinois	87	2	1858-59	do.	41 49	-88 6	682
St Louis, . . . . .	Missouri	87	6	1854-59	do.	38 37	-90 16	482
Beloit, . . . . .	Wisconsin	87	6	do.	do.	42 30	-89 4	780
Dubuque, . . . . .	Iowa	87	6	do.	do.	42 30	-90 52	680
Beaver Bay, . . . . .	Minnesota	87	1 $\frac{1}{6}$	1858-59	do.	47 12	-91 19	650
Lawrence, . . . . .	Kansas	87	2	1857-59	do.	38 58	-95 12	800
Leavenworth City, . . . . .	do.	87	1 $\frac{1}{4}$	1858-59	do.	39 20	-94 33	1342
Cordova, . . . . .	Mexico	87	2	do.	9: 9	19 26	-97 51	2820
Vera Cruz, . . . . .	do.	87	2 $\frac{3}{8}$	1857-59	{6 to 9: } {9 to 11 }	19 10	-96 8	26
Guatemala, . . . . .	Central America	40	2	1860-61	7: 2, 9	14 17	-90 26	4856
Port of Anapala, . . . . .	do.	100	$\frac{1}{4}$	1857	7: 2, 9	13 8	-87 34	0
Belize, . . . . .	Brit. Honduras	71	$\frac{1}{2}$	1862-63	?	17 32	-88 9	?
Bermuda, . . . . .	West Indies	44	3 $\frac{1}{2}$	1855-59	9 $\frac{1}{2}$ : 3 $\frac{1}{2}$	32 23	-64 40	123
Nassau, Bahama, . . . . .	do.	44	6	1853-59	do.	25 4	-77 22	13
Havanna, Cuba, . . . . .	do.	40	3	1859-61	8, 12: 4, 8	23 8	-82 22	66
Up Park Camp, Jamaica, . . . . .	do.	44	6	1853-59	9 $\frac{1}{2}$ : 3 $\frac{1}{2}$	18 0	-76 56	225
Barbadoes, . . . . .	do.	44	6	do.	do.	13 5	-59 4	5
Do., . . . . .	do.	45	2	1865-66	9: 3	do.	do.	15
Port of Spain, Trinidad, . . . . .	do.	87	$\frac{5}{12}$	1856-57	9: 3	10 39	-61 34	16
Caledonia Bay, . . . . .	New Granada	67	$\frac{1}{6}$	1854	3, 9: 3, 9	8 54	-77 45	0
Cartagena, . . . . .	do.	67	$\frac{1}{4}$	do.	o.	10 22	-75 32	0
Bogota, . . . . .	do.	35	2	1848-50	9: 3	4 35	-74 14	8727
Caraccas, . . . . .	Venezuela	92	1	1860	?	10 31	-66 55	2924
George Town, . . . . .	British Guiana	91	11	1846-56	{8, 9, 10: } {2, 3, 4 }	6 50	-58 8	10
Cayenne, . . . . .	French do.	35	6	1845-52	9, 12: 3, 9	4 56	-55 39	7
Catherina Sophia, . . . . .	Dutch do.	87	2	1858-59	6: 2, 6	5 48	-56 47	?
Ceara, . . . . .	Brazil	37	1	1860	?	-3 43	-38 35	?
Pernambuco, . . . . .	do.	4	$\frac{5}{12}$	?	?	-8 0	-35 53	?
Rio de Janeiro, . . . . .	do.	4	6	1851-56	?	-22 57	-43 7	224
Asuncion (and vicinity), . . . . .	Paraguay	101	1 $\frac{1}{2}$	1854-55	{0, 3, 6, } {9, &c. }	-25 16	-57 43	307
Buenos Ayres (& vicinity), . . . . .	La Plata	101	2 $\frac{1}{3}$	1853-56	{0, 3, 6, } {9, &c. }	-34 36	-58 23	60
Monte Video, . . . . .	Uruguay	102	10	1843-52	sr: 2, ss	-34 54	-58 33	26
Santiago de Chile, . . . . .	Chili	4	3	1850-52	?	-33 22	-70 38	1900

AND THE PREVAILING WINDS OVER THE GLOBE.

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
30-220	30-143	30-100	30-080	30-023	30-058	30-087	30-060	30-083	30-088	30-127	30-178	30-104
30-136	30-092	30-088	30-037	29-912	29-972	30-004	29-992	29-996	30-034	30-054	30-102	30-027
29-443	29-377	29-353	29-343	29-260	29-300	29-320	29-357	29-380	29-383	29-307	29-420	29-361
29-990	29-865	29-812	29-768	29-752	29-780	29-800	29-768	29-855	29-868	29-880	29-920	29-838
29-750	29-685	29-540	29-575	29-620	29-625	29-660	29-595	29-630	29-675	29-705	29-750	29-650
28-850	28-650	28-560	28-610	28-570	28-670	28-680	28-580	28-660	28-730	28-720	28-760	28-670
30-060	30-040	29-940	29-870	29-900	29-940	29-970	29-950	30-020	29-920	30-080	30-065	29-980
29-890	29-775	29-685	29-650	29-680	29-745	29-755	29-720	29-800	29-817	29-790	29-830	29-761
29-683	29-587	29-545	29-507	29-477	29-513	29-552	29-543	29-593	29-598	29-558	29-630	29-566
29-500	29-415	29-387	29-365	29-320	29-377	29-398	29-390	29-448	29-453	29-418	29-470	29-412
29-600	29-513	29-488	29-435	29-375	29-455	29-430	29-453	29-522	29-533	29-500	29-570	29-490
29-325	29-330	29-160	29-220	29-330	29-290	29-370	29-330	29-330	29-370	29-277	29-320	29-304
29-440	29-340	29-150	29-260	29-380	29-400	29-410	29-370	29-445	29-405	29-385	29-395	29-365
29-787	29-702	29-665	29-612	29-585	29-598	29-643	29-635	29-688	29-702	29-667	29-723	29-667
29-270	29-215	29-090	29-070	29-165	29-210	29-240	29-215	29-235	29-240	29-200	29-240	29-199
29-632	29-540	29-498	29-437	29-428	29-448	29-502	29-512	29-547	29-543	29-517	29-578	29-515
29-250	29-188	29-085	29-085	29-117	29-105	29-188	29-193	29-215	29-220	29-128	29-205	29-167
29-397	29-338	29-293	29-238	29-242	29-243	29-310	29-327	29-340	29-345	29-300	29-352	29-310
29-220	29-220	29-020	29-160	29-190	29-160	29-220	29-160	29-190	29-220	29-205	29-200	29-180
29-120	29-080	28-950	28-945	29-000	29-050	29-077	29-095	29-130	29-095	29-010	29-115	29-055
28-745	28-750	28-660	28-560	29-580	28-550	28-570	28-640	28-640	28-540	28-735	28-720	28-641
27-200	27-125	27-090	27-070	27-065	27-095	27-165	27-135	27-120	27-140	27-185	27-180	27-131
30-100	29-995	29-930	29-920	29-860	29-863	29-957	29-980	30-020	30-020	30-105	30-085	29-986
25-269	25-264	25-252	25-233	25-222	25-225	25-247	25-235	25-208	25-208	25-254	25-276	25-241
...	...	...	...	29-887	29-860	29-893	...	...	...	...	...	...
30-100	...	...	...	...	...	...	29-970	29-950	29-980	30-080	30-060	...
30-114	30-069	30-009	30-049	30-065	30-120	30-156	30-093	30-075	30-015	30-061	30-155	30-082
30-146	30-125	30-104	30-079	30-018	30-066	30-080	30-057	30-024	29-999	30-033	30-096	30-069
30-063	30-026	29-998	29-952	29-896	29-943	29-981	29-937	29-915	29-885	29-982	30-027	29-964
30-060	30-031	30-017	29-998	29-958	29-987	30-006	29-984	29-960	29-950	29-963	30-015	29-990
29-955	29-950	29-952	29-935	29-937	29-949	29-950	29-934	29-927	29-904	29-892	29-914	29-933
30-009	30-010	30-004	29-985	29-970	30-018	30-005	29-950	29-954	29-925	29-924	29-942	29-975
29-940	29-890	...	...	...	...	...	...	...	29-870	29-850	29-890	...
...	29-922	29-858	...	...	...	...	...	...	...	...	...	...
...	...	...	29-848	29-856	29-843	...	...	...	...	...	...	...
22-048	22-060	22-061	22-079	22-060	22-060	22-058	22-062	22-076	22-068	22-049	22-034	22-060
26-960	26-971	26-954	26-964	26-984	26-975	26-980	26-964	26-956	26-946	26-938	26-959	26-963
29-943	29-965	29-957	29-944	29-933	29-962	29-966	29-954	29-938	29-913	29-877	29-910	29-939
29-903	29-932	29-924	29-925	29-916	29-946	29-957	29-961	29-944	29-917	29-880	29-889	29-924
29-890	29-900	29-880	29-880	29-870	29-895	29-915	29-890	29-890	29-855	29-870	29-870	29-884
29-823	29-863	29-855	29-831	29-851	29-875	29-898	29-875	29-918	29-871	29-823	29-819	29-859
...	...	...	...	...	...	...	30-181	30-154	30-081	30-042	30-042	...
29-745	29-765	29-796	29-822	29-912	29-970	29-979	29-917	29-904	29-815	29-754	29-744	29-844
29-910	...	...	...	30-070	30-130	30-172	30-094	30-050	30-037	29-860	29-884	...
29-824	29-865	29-962	29-990	30-068	29-954	29-950	29-988	30-015	29-925	29-849	29-860	29-938
29-841	29-876	29-924	29-965	29-959	29-990	29-974	30-049	30-012	29-940	29-900	29-860	29-938
28-025	28-021	28-043	28-076	28-074	28-114	28-095	28-151	28-117	28-109	28-046	28-044	28-077

TABLE I.—SHOWING THE MEAN MONTHLY AND ANNUAL HEIGHT

Places.	Country.	Autho- rities, see page 635.	Number of Years.	Years Specified.	Hours of Observa- tion.	Latitude.	Longitude.	Height in Eng. Feet.
Valparaiso, . . . .	Chili	67	$\frac{1}{3}$	1853	$9\frac{1}{2} : 3\frac{1}{2}$	-33 25	-71 40	0
Port Famine, . . . .	Patagonia	4	$\frac{3}{4}$		?	-53 38	-70 52	?
Port Louis, . . . .	E. Falkland Isl.	104	$\frac{1}{2}$	1842	?	-51 40	-59 0	0
St Michael (at 32°?), . .	Azores	27	10	1840-49	?	37 35	-25 30	?
Ponta Delgada, . . . .	do.	94	1	1868-69	hourly	37 40	-25 32	?
Funchal, . . . . .	Madeira	95	2	1826,	10: 10 or	32 38	-16 56	95
Orotava, . . . . .	Canaries	67	1	1852 3	7: 2, 7			
	North Atlantic	96		1856-57	about 9:	28 27	-16 38	70
	do.	96				40 to 35		0
	do.	96				35 to 30		0
	do.	96				30 to 25		0
	do.	96				25 to 20		0
	do.	96				20 to 15		0
	do.	96				15 to 10		0
	do.	96				10 to 5		0
	do.	96				-5 to 0		0
	South Atlantic	96				-0 to 5		0
	do.	96				-5 to 10		0
	do.	96				-10 to 15		0
	do.	96				-15 to 20		0
	do.	96				-20 to 25		0
	do.	96				-25 to 30		0
	do.	96				-30 to 35		0
	do.	96				-35 to 40		0
	do.	96				-40 to 45		0
	do.	96				-45 to 50		0
	do.	96				-50 to 55		0
	do.	96				-55 to 60		0
Ascension, . . . . .	do.	67	2	1854-55	$9\frac{1}{2} : 3\frac{1}{2}$	-8 8	-14 28	0
St Helena, . . . . .	do.	44	5	1854-59	do.	-15 55	-5 42	40
Do., . . . . .	do.	97	$3\frac{1}{2}$	1844-47	biho.	-15 55	-5 43	1763

ADDENDUM.

The Atmospheric Pressure of the Atlantic Ocean.—The following Pressures are the means of observations made in July and August by Captain TOYNBEE, during five voyages to India, at different points on the Outward Route:—

	Inches.		Inches.		Inches.
35° N. Lat.	30.252	10° N. Lat.	30.017	10° S. Lat.	30.082
30° "	30.166	5° "	30.025	15° "	30.142
25° "	30.092	Equator	30.042	20° "	30.245
20° "	30.022	5° S. Lat.	30.050	25° "	30.236
15° "	29.996				

The results were published in the "Proceedings of the Royal Society," June 15, 1865, but not corrected for temperature.\* The figures given above Captain TOYNBEE has kindly corrected for temperature and height.

\* The uncorrected means were used in constructing the charts exhibited in reading Part I. of this paper.

OF THE BAROMETER AT DIFFERENT PLACES OVER THE GLOBE—*continued.*

January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Year.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
...	...	...	...	30-090	30-079	30-056	30-098	...	...	...	...	...
...	29-405	29-646	29-573	29-304	29-279	29-571	29-286	...	29-293	29-539	...	...
...	...	...	29-428	29-304	29-396	29-665	29-654	29-576	...	...	29-349	...
30-214	30-166	30-255	30-247	30-237	30-253	30-311	30-238	30-210	30-142	30-115	30-201	30-216
30-020	30-268	30-343	30-056	29-961	30-180	30-192	30-243	30-052	30-241	29-989	30-071	30-135
30-012	30-092	30-016	29-929	29-978	30-068	29-992	29-997	30-025	29-941	29-898	30-048	29-998
30-290	30-116	30-177	30-110	30-100	30-140	30-094	30-095	30-125	30-121	30-119	30-180	30-144
30-210	30-110	30-150	30-060	30-070	30-200	30-190	30-160	30-110	30-090	30-020	30-060	30-119
30-260	30-180	30-200	30-160	30-120	30-200	30-240	30-190	30-150	30-180	30-030	30-180	30-174
30-250	30-160	30-130	30-170	30-220	30-230	30-190	30-160	30-100	30-140	30-060	30-180	30-166
30-100	30-110	30-100	30-100	30-140	30-160	30-090	30-070	30-030	30-070	30-010	30-000	30-082
30-010	30-060	30-040	30-000	30-060	30-030	30-000	30-000	29-990	29-990	29-980	30-010	30-014
29-950	29-970	29-990	29-970	29-990	29-960	29-970	29-930	29-930	29-950	29-960	29-960	29-961
29-900	29-940	29-910	29-920	29-940	20-940	29-980	29-970	29-950	29-930	29-940	29-910	29-936
29-880	29-910	29-890	29-900	29-920	29-930	29-980	29-970	29-980	29-950	29-920	29-910	29-928
29-890	29-910	29-900	29-920	29-940	29-940	29-990	30-000	30-010	29-960	29-940	29-930	29-944
29-950	29-940	29-940	29-940	29-990	30-010	30-020	30-030	30-030	30-020	29-990	29-960	29-985
29-970	29-980	29-960	29-990	30-040	30-050	30-050	30-060	30-090	30-070	30-040	30-000	30-025
30-020	30-010	30-010	30-030	30-090	30-090	30-090	30-130	30-110	30-100	30-050	30-050	30-065
30-060	30-050	30-070	30-050	30-060	30-140	30-110	30-160	30-170	30-180	30-080	30-080	30-101
30-070	30-050	30-060	30-030	30-140	30-090	30-130	30-180	30-130	30-160	30-080	30-080	30-100
30-050	30-050	30-040	30-030	30-100	30-040	30-120	30-100	30-080	30-080	30-090	30-000	30-065
29-980	30-040	30-020	29-980	29-900	29-900	30-040	29-940	29-950	30-050	30-050	29-970	29-985
29-920	29-950	29-990	29-950	29-880	29-890	29-950	29-930	29-960	30-020	29-940	29-950	29-944
29-710	29-780	29-770	29-760	29-720	29-650	29-820	29-830	29-870	29-770	29-700	29-670	29-754
29-410	29-440	29-490	29-440	29-410	29-480	29-530	29-560	29-570	29-480	29-310	29-430	29-462
29-250	29-230	29-250	29-200	29-260	29-280	29-250	29-280	29-290	29-100	29-100	29-210	29-225
30-018	30-049	30-037	30-036	30-011	30-085	30-092	30-086	30-080	30-058	30-091	29-977	30-052
30-048	30-043	30-030	30-039	30-079	30-126	30-163	30-152	30-137	30-107	30-079	30-085	30-091
28-241	28-238	28-228	28-249	28-279	28-328	28-351	28-349	28-305	28-286	28-262	28-247	28-280

The Pressures reduced to 32° and sea-level, on the Homeward Passage from the Cape, being the means of five voyages in 1861-62-63-64-65, in the latter part of February, in March, and beginning of April, are as follow:—

	Inches.		Inches.		Inches.
35° N. Lat.	30-034	10° N. Lat.	29-986	15° S. Lat.	30-018
30° "	30-098	5° "	29-944	20° "	30-040
25° "	30-202	Equator	29-950	25° "	30-111
20° "	30-122	5° S. Lat.	29-954	30° "	30-076
15° "	30-062	10° "	29-978		

It may be noted, that if Captain TOYNBEE'S Chart (Proc. Roy. Soc. vol. xiv. Plate VII.), and the Isobaric Charts for March and July be compared with reference to BUYS BALLOT'S LAW OF THE WIND, it will be seen that the Prevailing Wind will blow aft over nearly the whole Outward Tract and the Homeward Tract to India or the Cape.

TABLE II.—SHOWING THE AVERAGE NUMBER OF DAYS EACH MONTH THE WIND HAS PREVAILED FROM NORTH, NORTH-EAST, EAST, &C., AT DIFFERENT PLACES OVER THE GLOBE.

MONTH.	Reykjavik, Iceland. 10 Years, 1828-32. Hour A.M.										Dublin, Ireland. 11 Years, 1857-67. Hour 9.30:										Cork, Ireland. 11 Years, 1857-67. Hour 9:									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	4	5	5	3	3	5	1	1	4	1	1	2	3	3	9	8	2	2	2	2	1	5	3	8	2	8	...			
Feb.	4	5	5	3	1	6	0	1	3	2	1	3	3	2	6	8	1	2	1	1	3	2	5	1	7	3	6	...		
March	5	4	5	4	2	6	1	2	2	3	2	4	2	2	6	8	2	2	1	4	2	5	1	5	4	9	...			
April	6	5	4	4	2	5	0	3	1	2	3	6	2	2	6	6	3	1	1	4	3	5	2	6	3	6	...			
May	5	4	5	5	2	3	1	2	4	2	2	7	3	2	7	4	3	1	2	2	4	7	2	6	3	5	...			
June	5	2	3	3	3	3	2	5	4	2	2	5	2	2	6	6	4	1	1	2	1	5	3	6	4	8	...			
July	6	1	3	3	3	3	2	5	5	2	1	2	2	2	7	9	4	2	1	1	1	3	2	7	6	10	...			
Aug.	5	2	6	3	3	3	2	4	3	2	1	2	2	2	8	9	3	2	1	1	1	4	4	7	5	8	...			
Sept.	6	3	7	3	2	3	1	3	2	1	1	3	2	3	9	7	2	2	1	1	2	4	3	8	4	7	...			
Oct.	8	5	6	2	2	4	1	1	2	2	2	4	3	2	7	6	2	3	1	3	2	6	2	7	2	8	...			
Nov.	8	3	7	2	2	4	1	1	2	1	1	4	3	2	7	6	3	3	1	3	3	5	3	6	1	8	...			
Dec.	5	5	6	2	2	7	1	1	2	1	1	2	3	2	10	7	2	3	1	2	1	5	3	9	3	7	...			
Year	67	44	62	37	27	52	13	29	34	21	18	44	30	26	88	84	31	24	14	28	23	59	29	82	40	90	...			

MONTH.	Sandwick, Scotland. 11 Years, 1857-67. Hour 9: 9.										Stornoway, Scotland. 11 Years, 1857-67. Hours 9: 9.										Aberdeen, Scotland. 11 Years, 1857-67. Hours 9: 9.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	2	2	2	6	4	6	5	2	2	2	2	2	1	4	10	6	3	1	1	1	1	2	2	13	4	5	2			
Feb.	2	1	1	6	4	4	5	3	2	2	1	2	3	4	6	6	4	0	2	1	1	2	4	10	3	4	1			
March	3	2	3	6	3	4	5	3	2	3	2	6	2	2	6	5	4	1	2	2	2	3	3	7	3	7	2			
April	3	2	3	7	3	4	4	3	1	2	3	4	3	4	7	3	3	1	3	3	1	4	3	7	2	5	2			
May	3	3	3	8	1	3	5	3	2	1	4	5	4	3	6	4	3	1	3	5	2	4	4	6	1	4	2			
June	1	2	3	7	1	4	6	4	2	1	3	6	3	2	6	6	3	0	3	3	2	6	5	5	1	3	2			
July	2	2	1	4	2	4	6	8	2	2	3	4	1	2	6	7	4	2	4	3	2	3	3	7	2	5	2			
Aug.	2	1	1	8	2	5	5	4	3	4	2	2	2	5	6	6	3	1	3	2	1	3	4	9	2	5	2			
Sept.	1	1	2	6	4	5	6	3	2	2	2	2	1	5	8	6	3	1	1	1	1	3	4	12	2	5	1			
Oct.	2	2	2	7	3	5	4	4	2	3	2	3	3	4	9	4	3	0	2	1	1	4	4	10	3	5	1			
Nov.	3	2	1	6	4	3	4	4	3	4	2	2	2	3	7	5	4	1	2	1	1	3	2	10	4	6	1			
Dec.	2	1	1	7	5	5	6	3	1	2	2	1	2	4	10	6	3	1	2	0	0	3	3	15	3	4	1			
Year	26	21	23	78	36	52	61	44	24	28	28	39	27	42	87	64	40	10	28	23	15	40	41	111	30	58	19			

MONTH.	Scotland, 55 Stations. 11 Years, 1857-67. Hours 9: 9.										Clifton, England. 10 Years, 1853-62. Hour ?										Greenwich, England. 20 Years, 1841-60. Hourly.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	2	2	2	3	3	8	6	3	2	2	4	2	4	3	5	5	3	3	3	3	1	2	4	10	3	2	3			
Feb.	2	2	2	3	3	6	5	3	2	2	6	3	2	3	4	4	3	1	3	4	2	1	3	8	3	2	2			
March	3	3	4	3	2	5	6	4	1	2	5	3	3	2	4	6	5	1	4	4	3	2	2	8	3	3	2			
April	2	3	4	3	3	5	5	3	2	2	6	3	3	2	3	5	4	2	4	6	3	2	3	6	3	2	1			
May	2	3	5	4	3	5	5	2	2	3	6	4	2	2	5	4	3	2	4	7	3	2	3	7	2	1	2			
June	2	2	4	3	3	6	5	3	2	2	3	2	2	2	6	7	5	1	3	4	2	2	2	10	4	2	1			
July	2	2	3	2	3	6	7	4	2	2	2	1	2	3	5	10	5	1	3	4	1	1	3	10	4	2	3			
Aug.	2	2	2	3	3	7	6	3	3	3	2	2	2	2	6	8	5	1	3	3	1	1	3	11	4	2	3			
Sept.	1	1	2	2	3	8	7	3	3	2	4	3	3	2	5	6	4	1	4	5	2	2	2	7	2	2	4			
Oct.	2	2	3	3	3	6	6	3	3	2	4	3	3	3	4	7	3	2	3	3	1	2	3	9	4	2	4			
Nov.	3	2	3	3	3	5	5	3	3	3	5	5	3	2	3	5	2	2	4	4	2	2	3	8	2	2	3			
Dec.	2	1	2	3	3	8	7	3	2	2	4	1	4	3	5	5	2	2	3	2	2	2	3	9	4	2	4			
Year	25	25	36	35	35	75	70	37	27	27	51	32	33	29	54	70	50	19	41	49	23	21	34	103	38	24	32			

TABLE II.—continued.

MONTH.	Helston, England. 11 Years, 1857-67. Hours 9: 3, 9.										Vardö, Norway. 8 Years, 1856-63. Hours 8, 12: 6, 8.										Hammerfest, Norway. 14 Years, 1848-61. Hours 8: 3, 8.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	3	2	4	3	3	6	6	4	...	2	3	2	3	2	11	2	3	1	2	1	2	11	8	2	2	2	1			
Feb.	3	2	4	2	2	6	5	4	...	1	2	0	3	1	13	3	4	1	2	1	3	7	9	2	1	2	1			
March	4	2	5	1	2	6	6	5	...	3	3	1	3	1	11	3	5	1	3	1	3	8	8	2	2	2	2			
April	4	1	7	1	2	6	5	4	...	4	4	1	3	2	6	3	4	3	3	1	3	6	6	1	4	3	3			
May	4	2	6	2	3	7	3	4	...	3	4	3	3	1	4	3	8	2	3	2	5	3	5	2	3	4	4			
June	3	1	5	1	3	7	5	5	...	3	2	2	6	3	1	1	8	4	3	2	4	3	4	1	3	4	6			
July	3	1	3	1	2	9	5	7	...	4	6	3	7	2	1	0	5	3	2	2	4	2	4	2	3	4	8			
Aug.	2	1	3	1	3	8	6	7	...	3	4	1	8	2	2	1	7	3	2	1	4	3	4	1	3	4	9			
Sept.	3	1	4	1	3	9	5	4	...	1	2	1	6	4	6	2	7	1	2	1	2	3	8	2	4	4	4			
Oct.	3	2	7	2	3	5	5	4	...	2	3	1	5	2	10	2	5	1	3	2	3	7	7	2	3	3	1			
Nov.	3	4	6	3	2	4	5	3	...	1	5	2	1	1	13	2	4	1	3	1	3	9	6	2	2	2	2			
Dec.	3	3	4	2	3	6	6	4	...	1	3	2	3	1	15	2	3	1	2	1	3	9	7	3	3	2	1			
Year	38	22	58	20	31	79	62	55	...	28	41	19	51	22	95	24	63	22	30	16	39	71	76	22	33	36	42			

MONTH.	Christiansund, Norway. 8 Years, 1861-68. Hours 8: 2, 8.										Skudesnes, Norway. 8 Years, 1861-68. Hours 8: 2, 8.										Mandal, Norway. 8 Years, 1861-68. Hours 8: 2, 8.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	1	1	4	8	4	5	4	1	3	2	3	4	5	6	5	2	2	2	1	7	4	1	1	4	3	1	9			
Feb.	1	1	2	6	3	6	4	2	3	3	2	3	3	5	3	4	3	2	1	6	3	1	1	3	5	7	7			
March	1	2	3	8	4	5	3	2	3	3	2	4	5	6	2	2	3	4	1	9	5	2	1	2	2	1	8			
April	2	4	2	4	2	3	6	3	4	6	2	2	2	5	3	3	4	3	0	5	4	1	1	4	6	2	7			
May	3	7	2	2	1	4	5	3	4	7	1	2	2	6	4	3	4	2	1	4	4	1	1	5	6	1	8			
June	4	5	1	1	1	4	5	3	6	8	1	2	1	5	3	3	5	2	0	3	4	1	1	7	8	1	5			
July	5	5	1	1	1	3	8	4	3	8	1	1	1	4	3	4	7	2	0	2	3	2	2	6	10	2	4			
Aug.	3	6	2	3	1	3	5	3	5	6	1	1	2	6	5	3	5	2	0	3	4	1	3	6	8	2	4			
Sept.	2	2	3	5	2	4	5	2	5	5	2	2	3	6	4	3	3	2	1	4	3	2	2	4	6	2	6			
Oct.	1	1	3	7	2	5	5	3	4	4	2	2	5	6	3	3	3	3	1	5	5	2	1	4	5	1	7			
Nov.	2	2	2	7	3	6	3	2	3	4	2	4	5	5	2	3	3	2	1	6	5	2	1	3	5	2	5			
Dec.	1	1	4	8	3	5	5	2	2	2	2	4	6	6	4	4	2	1	1	6	5	1	1	4	5	2	6			
Year	26	37	29	60	27	53	58	30	45	58	21	31	40	66	41	37	44	27	8	60	49	17	16	52	69	18	76			

MONTH.	Sandösund, Norway. 8 Years, 1861-68. Hours 8: 2, 8.										Christiania, Norway. 11 Years, 1857-67. Hours 9: 2, 4.										Haparanda, Sweden. 8 Years, 1859-66. Hour 8:									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	5	7	1	2	3	6	2	2	3	5	8	4	2	4	1	2	2	3	4	4	2	4	7	4	1	1	4			
Feb.	4	5	1	2	3	7	2	2	2	4	6	3	3	4	2	1	2	3	6	3	2	4	5	3	1	2	2			
March	5	10	2	1	2	5	1	1	4	5	7	4	2	4	2	1	2	4	8	3	2	4	8	3	1	1	1			
April	3	7	1	1	2	8	2	2	4	5	5	2	3	6	3	2	2	2	6	2	2	3	5	4	2	2	4			
May	3	7	1	1	2	12	2	1	2	3	4	2	4	8	4	2	2	2	6	6	2	4	6	4	0	1	2			
June	2	5	1	2	3	11	3	1	2	2	3	2	5	8	5	2	2	1	4	4	3	4	7	3	1	2	2			
July	3	4	1	1	3	13	2	2	2	2	3	2	4	9	5	2	2	2	7	4	4	3	7	3	1	1	1			
Aug.	2	5	1	2	4	12	2	1	2	3	3	3	4	7	5	2	2	2	6	7	4	3	4	2	2	1	2			
Sept.	1	5	2	3	2	10	3	1	3	3	4	2	5	7	4	1	2	2	6	5	3	3	4	3	3	2	1			
Oct.	3	6	2	2	2	9	3	2	2	4	6	3	3	5	2	1	2	5	6	4	3	2	5	4	2	3	2			
Nov.	3	7	2	2	2	6	3	2	3	6	7	3	3	2	1	2	2	4	6	4	2	4	5	4	2	2	1			
Dec.	4	5	1	2	3	8	3	3	2	6	7	3	2	4	2	1	2	4	5	3	1	4	5	5	3	2	3			
Year	38	73	16	21	31	107	28	20	31	48	63	33	40	68	36	19	24	34	70	49	30	42	68	42	19	20	25			

TABLE II.—continued.

MONTH.	Upsala, Sweden. 8 Years, 1855-62. Hours 7: 2, 9.									Wisby, Sweden. 8 Years, 1859-66. Hour 8:									Smidstrup, Denmark. 7 Years, 1861-67. Hours 8: 2, 10.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	
Jan.	7	4	2	1	4	6	3	4	...	2	2	3	5	3	4	6	4	2	2	2	6	2	6	2	9	2	2	...
Feb.	6	4	1	1	4	6	3	3	...	3	2	3	3	3	3	4	5	2	2	2	7	1	5	3	6	3	1	...
March	8	5	2	2	5	4	2	3	...	2	3	7	5	4	3	3	2	2	2	2	10	3	4	3	6	2	1	...
April	9	6	2	2	3	4	2	2	...	3	5	3	4	2	4	5	3	1	3	3	4	3	3	2	7	5	3	...
May	10	7	3	2	3	3	2	1	...	3	5	2	3	2	5	6	3	2	3	3	5	3	3	2	5	7	3	...
June	7	6	3	2	4	4	2	2	...	2	4	3	5	2	5	4	2	3	2	2	3	2	3	2	9	7	2	...
July	7	4	4	2	4	4	3	3	...	4	3	2	2	3	6	6	4	1	3	3	1	1	3	3	7	10	3	...
Aug.	9	5	3	2	3	3	3	3	...	2	3	2	3	4	5	6	5	1	2	2	1	1	4	3	11	6	3	...
Sept.	8	4	3	1	4	4	3	3	...	2	2	2	4	5	4	6	3	2	2	2	2	2	9	4	6	4	1	...
Oct.	7	4	2	1	6	4	4	3	...	3	2	4	5	4	3	5	4	1	2	2	4	3	8	2	7	3	2	...
Nov.	8	5	2	1	3	3	4	4	...	2	2	4	5	5	4	4	4	0	2	2	4	1	7	5	7	2	2	...
Dec.	8	3	2	2	5	4	3	4	...	4	2	3	3	4	5	5	4	1	2	2	4	2	7	3	7	2	4	...
Year	94	57	29	19	48	49	34	35	...	32	35	38	47	41	51	60	43	18	27	51	24	62	34	87	53	27	...	

MONTH.	Tarum, Denmark. 7 Years, 1861-67. Hours 8: 2, 10.									Bremen, Germany. 10 Years, 1857-66. Hours 8: 3, 11.									Leeuwarden, Netherlands. 35 Years, 1843-67. Hours A.M., Noon, P.M.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	1	4	2	6	5	8	4	1	...	1	1	3	6	1	10	7	2	...	1	3	4	5	6	8	2	2	...
Feb.	2	4	2	4	3	8	4	1	...	0	2	4	5	1	8	5	3	...	1	4	3	3	3	8	3	3	...
March	2	5	4	6	3	5	3	3	...	1	3	3	6	0	9	5	4	...	2	4	4	3	3	7	4	4	...
April	2	3	2	4	3	5	5	6	...	0	4	3	5	0	5	4	9	...	3	5	3	2	2	6	3	6	...
May	2	4	3	4	2	5	6	5	...	0	5	4	5	0	6	3	8	...	4	6	3	2	2	7	2	5	...
June	2	2	3	3	3	5	7	5	...	0	2	2	5	1	8	4	8	...	2	4	2	2	2	9	4	5	...
July	1	1	1	3	4	5	10	6	...	1	2	1	3	0	8	7	9	...	2	3	1	2	3	9	5	6	...
Aug.	1	1	1	4	4	7	8	5	...	0	2	2	3	1	8	7	8	...	2	3	2	3	4	9	3	5	...
Sept.	1	1	2	7	6	6	4	3	...	0	2	3	4	1	10	6	4	...	2	3	2	4	5	7	3	4	...
Oct.	1	4	3	7	4	4	4	4	...	0	2	5	7	1	9	4	3	...	1	4	4	5	5	7	2	3	...
Nov.	2	3	2	5	5	6	4	3	...	0	3	4	8	1	9	3	2	...	1	4	5	4	5	6	2	3	...
Dec.	1	3	2	6	4	7	5	3	...	0	2	3	7	1	10	5	3	...	1	3	3	4	6	9	3	2	...
Year	18	35	27	59	46	71	64	45	...	3	30	37	64	8	100	60	63	...	22	46	36	39	46	92	36	48	...

MONTH.	Brussels, Belgium. 10 Years, 1853-62. Hourly, and on alternate hours.									St Hippolyte-de-Caton, France. 13 Years, 1837-49. Hour?									Paris, France. 30 Years, 1816-45. Hour?								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	1	2	2	2	5	12	5	2	...	13	7	0	0	4	1	1	5	...	4	3	2	3	5	6	4	3	1
Feb.	1	3	4	2	3	7	5	3	...	10	7	0	1	6	0	1	3	...	3	3	2	2	5	4	5	3	1
March	2	3	4	2	3	8	6	3	...	11	6	1	1	6	0	1	5	...	4	4	2	2	4	5	6	3	1
April	3	5	4	1	2	6	5	4	...	12	7	0	1	7	0	0	3	...	5	5	2	2	4	4	4	2	1
May	3	4	4	2	2	7	5	4	...	10	5	0	2	10	1	0	3	...	4	4	3	2	4	5	5	3	1
June	2	2	1	1	2	11	8	3	...	14	5	1	1	7	1	0	1	...	4	3	2	1	3	6	7	3	1
July	2	2	1	1	2	11	8	4	...	13	3	0	1	8	0	1	5	...	3	3	1	1	3	7	8	4	1
Aug.	3	2	2	2	3	9	7	3	...	11	5	0	1	7	1	1	5	...	3	3	2	1	3	7	8	3	1
Sept.	2	3	2	2	4	10	5	2	...	8	6	1	2	9	2	0	2	...	2	4	2	3	5	6	5	3	0
Oct.	0	2	4	3	6	11	4	1	...	10	7	1	1	6	1	1	4	...	2	3	2	3	6	6	5	3	1
Nov.	1	3	5	3	5	8	4	1	...	9	5	1	1	7	1	2	4	...	2	2	2	3	6	7	5	2	1
Dec.	1	2	4	2	5	11	5	1	...	11	7	0	1	5	1	1	5	...	2	4	2	3	5	7	5	3	0
Year	21	33	37	23	42	111	67	31	...	132	70	5	13	82	9	9	45	...	38	41	24	26	53	70	67	36	10

TABLE II.—*continued.*

MONTH.	Ahun, France. 38 Years, 1828-65. Hour?										Bordeaux, France. 10 Years, 1837-46. Hours?										Toulouse, France. 22 Years, 1839-60. Hours 9: 12: 3, 6, 9.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.	
Jan.	3	4	3	2	4	9	4	2	...	3	1	6	3	4	4	5	5	...	1	0	0	8	2	3	9	8	...			
Feb.	3	5	3	1	4	6	3	3	...	3	1	6	2	3	4	5	4	...	1	1	0	9	2	2	7	6	...			
March	4	7	2	1	3	6	5	3	...	2	2	6	2	3	4	7	5	...	2	1	0	9	2	2	6	9	...			
April	4	5	3	1	3	6	5	3	...	1	1	6	2	1	3	10	6	...	1	1	1	11	1	1	7	7	...			
May	5	6	1	1	3	6	5	4	...	1	1	4	2	1	4	11	7	...	2	2	1	9	2	2	6	7	...			
June	4	5	2	1	2	7	5	4	...	0	1	5	1	1	4	12	6	...	2	1	1	9	2	1	5	9	...			
July	4	6	1	1	1	7	6	5	...	1	1	3	0	1	4	16	5	...	4	1	0	5	1	1	7	12	...			
Aug.	4	5	1	1	2	7	7	4	...	1	1	4	1	1	4	13	6	...	4	0	1	6	1	2	6	11	...			
Sept.	3	6	3	1	3	8	3	3	...	1	1	6	2	3	5	8	4	...	2	1	0	7	3	3	5	9	...			
Oct.	1	5	2	2	5	10	4	2	...	1	1	7	2	2	4	6	8	...	2	1	1	9	3	2	6	7	...			
Nov.	3	5	3	2	4	8	3	2	...	1	1	6	5	5	5	4	3	...	1	1	1	10	2	3	6	6	...			
Dec.	3	5	4	3	4	7	3	2	...	2	2	7	5	2	4	2	7	...	1	0	1	11	3	3	6	6	...			
Year	41	64	28	17	38	87	53	37	...	17	14	66	27	26	50	99	66	...	23	10	7	103	24	25	76	97	...			

MONTH.	St Rambert-en-Bugey, France. 6 Years, 1838-43. Hours 7: 1, 7.										Oviedo, Spain. 11 Years, 1852-62. Hours 9, 12: 3, 9.										Madrid, Spain. 10 Years, 1853-62. Hour?									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.	
Jan.	7	2	1	0	3	8	4	6	...	2	7	0	1	3	4	4	8	...	3	11	4	2	2	5	2	2	...			
Feb.	9	1	1	0	6	3	3	5	...	6	6	1	0	3	3	3	7	...	6	6	3	2	3	4	2	2	...			
March	10	2	2	0	5	4	3	5	...	5	11	2	2	1	1	1	7	...	6	6	2	2	4	5	3	3	...			
April	7	4	3	3	5	3	2	3	...	5	11	1	2	1	1	1	7	...	3	5	4	3	4	6	3	2	...			
May	6	3	2	2	7	4	3	4	...	7	12	1	0	2	2	2	7	...	2	5	2	2	4	8	4	4	...			
June	7	1	2	1	5	3	7	4	...	5	13	0	0	1	1	1	6	...	3	6	1	1	2	9	4	4	...			
July	7	2	1	0	3	7	6	5	...	8	5	0	0	1	0	2	7	...	3	5	3	3	2	7	4	4	...			
Aug.	10	2	1	0	5	6	4	3	...	5	15	0	0	0	0	0	5	...	3	7	3	2	3	8	3	2	...			
Sept.	6	2	2	2	5	6	5	2	...	5	10	0	0	0	1	1	6	...	4	5	3	2	3	7	3	3	...			
Oct.	10	2	0	0	6	4	5	4	...	3	10	1	1	2	3	1	14	...	2	6	4	3	5	6	2	3	...			
Nov.	9	1	1	0	5	7	4	3	...	3	8	0	1	3	2	1	9	...	4	5	2	2	4	6	3	4	...			
Dec.	12	3	1	0	4	5	2	4	...	3	6	1	0	3	3	3	7	...	3	11	2	1	3	6	3	2	...			
Year	100	25	17	8	59	60	48	48	...	57	114	7	7	20	21	20	80	...	42	78	33	25	39	77	36	35	...			

MONTH.	Lisbon, Portugal. 9 Years, irreg., 1857-65. Hour A.M.										Gibraltar, Spain. 6 Years, 1853-59. Hours 9.30: 3.30.										Geneva, Switzerland. 35 Years, 1826-60. Hours, every twoHours.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calin.	
Jan.	8	11	3	0	2	2	3	1	1	1	2	7	4	2	3	5	7	...	5	4	2	2	3	13	1	1	...			
Feb.	7	7	3	0	3	2	4	1	1	1	0	6	2	1	5	4	9	...	7	6	1	1	3	8	1	1	...			
March	11	8	2	1	2	2	3	2	0	1	1	6	2	2	8	3	8	...	9	7	1	0	2	10	1	1	...			
April	8	5	5	1	1	2	5	3	0	0	1	3	4	1	6	3	12	...	10	6	0	1	1	10	1	1	...			
May	7	7	5	0	2	1	5	2	2	0	0	5	4	1	8	3	10	...	10	7	1	0	1	10	1	1	...			
June	10	8	2	0	0	3	4	2	1	0	1	8	5	1	8	3	4	...	9	8	0	0	1	10	1	1	...			
July	15	9	1	0	2	0	1	1	2	0	1	11	4	2	3	5	5	...	10	5	1	0	1	11	1	2	...			
Aug.	14	7	2	1	0	1	3	1	2	1	1	10	4	1	4	4	6	...	10	5	0	1	2	10	1	2	...			
Sept.	11	4	1	1	1	1	6	3	2	1	1	9	3	0	3	5	8	...	8	6	0	1	1	12	1	1	...			
Oct.	9	9	2	0	2	1	4	2	2	1	2	9	3	1	6	3	6	...	10	7	0	1	1	10	1	1	...			
Nov.	6	7	5	1	2	3	3	1	2	1	2	9	3	1	3	4	7	...	5	6	1	2	2	12	1	1	...			
Dec.	4	13	4	1	1	1	3	2	2	1	2	6	3	2	3	1	13	...	8	6	2	2	3	8	1	1	...			
Year	110	95	35	6	18	19	44	21	17	8	14	89	41	15	60	43	95	...	101	73	9	11	21	124	12	14	...			

TABLE II.—continued.

MONTH.	Bologna, Italy. 45 Years, 1814-58. Hour?									Rome, Italy. 11 Years, 1850-60. Hours 7: 0.30, 3, 9.									Naples, Italy. 28 Years, 1833-60. Hours 9, 12: 9.								
	N.	E.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	1	1	1	0	1	2	19	6	...	10	8	5	1	4	1	2	0	...	7	8	1	2	4	5	1	3	...
Feb.	1	1	2	1	1	2	14	6	...	10	4	4	1	4	2	2	1	...	5	6	1	2	3	6	2	3	...
March	2	2	4	2	1	3	11	6	...	8	3	4	1	6	4	4	1	...	5	6	1	2	4	8	2	3	...
April	2	2	7	2	2	3	7	5	...	5	2	4	1	8	4	5	1	...	3	5	1	2	5	10	2	2	...
May	3	3	6	2	3	3	7	4	...	4	2	3	1	9	5	6	1	...	3	4	1	2	6	11	2	2	...
June	2	3	6	2	2	4	7	4	...	3	3	3	1	7	5	7	1	...	2	3	1	3	5	11	2	3	...
July	3	2	7	3	1	2	8	5	...	4	3	2	1	7	7	6	1	...	1	3	1	3	6	11	3	3	...
Aug.	2	2	7	3	2	2	8	5	...	6	3	1	1	8	5	6	1	...	2	4	1	2	5	11	3	3	...
Sept.	2	2	6	2	2	3	9	4	...	7	3	3	1	8	3	4	1	...	4	5	1	2	5	8	2	3	...
Oct.	2	1	4	2	2	3	12	5	...	7	3	3	1	8	4	4	1	...	5	5	1	2	5	8	2	3	...
Nov.	2	1	2	1	1	2	16	5	...	9	6	4	1	5	2	2	1	...	7	6	1	2	4	6	2	2	...
Dec.	1	1	2	1	1	0	20	5	...	9	9	4	1	4	2	1	1	...	8	8	1	1	3	4	2	4	...
Year	23	21	54	21	19	29	138	60	...	82	49	40	12	78	44	49	11	...	52	63	12	25	55	99	25	34	...

MONTH.	Malta, Italy. 3 to 4 Years, 1853-59. Hours 9, 30: 3, 30.									Lemberg, Austria. 5 Years, 1854-58. Hours 6: 2, 10.									Vienna, Austria. 6 Years, 1856-61. Hours 6: 2, 10.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	2	8	5	4	1	3	3	5	...	2	1	2	2	4	6	12	2	0	4	0	1	6	5	2	3	10	...
Feb.	1	3	1	2	3	9	3	6	...	1	0	1	4	6	2	10	4	0	4	1	0	6	3	1	4	9	...
March	2	4	2	3	2	3	4	11	...	1	2	1	6	6	4	6	5	0	3	1	0	5	5	2	3	12	...
April	2	3	1	2	1	4	2	10	...	3	1	3	3	4	1	10	5	0	6	1	1	3	4	2	3	10	...
May	1	5	3	5	1	2	3	11	...	3	2	3	2	4	3	10	4	0	5	1	1	4	5	1	2	12	...
June	2	7	2	2	1	4	4	8	...	4	2	3	2	3	2	8	6	0	6	1	1	4	2	1	3	12	...
July	6	5	2	3	1	3	1	10	...	2	2	2	1	3	2	15	4	0	4	0	1	2	3	1	3	17	...
Aug.	2	8	2	3	1	3	2	10	...	1	1	2	3	6	4	11	3	0	4	1	1	3	3	2	4	13	...
Sept.	3	6	2	2	1	3	1	12	...	2	1	1	3	5	3	11	4	0	4	1	1	5	4	1	3	11	...
Oct.	2	3	6	5	4	3	3	5	...	2	2	1	4	6	4	9	2	1	4	1	1	8	5	1	3	8	...
Nov.	1	2	3	4	2	6	2	10	...	1	1	1	5	2	4	11	4	1	5	1	0	6	5	2	3	8	...
Dec.	2	3	1	3	2	8	4	8	...	2	1	1	4	5	4	10	4	0	4	0	1	4	5	2	4	11	...
Year	25	57	29	38	20	51	32	106	...	24	16	21	39	54	39	123	47	2	53	9	9	56	49	18	38	133	...

MONTH.	Debreczin, Austria. 5 Years, 1854-58. Hours 6: 2, 10.									Corfu, Italy. 5 Years, 1854-59. Hour 9, 30:									East of Nova Zembla, Lat. N. 70° 37'. 4½ Years, 1832-35 (irreg.) Hour 8:								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	11	1	2	1	9	1	0	1	5	5	8	7	4	1	1	1	4	0	7	5	1	3	5	2	3	2	3
Feb.	8	1	3	1	7	2	1	2	3	5	4	8	4	1	0	2	3	1	3	2	6	2	4	2	5	1	3
March	12	2	4	1	6	2	0	1	3	5	2	9	5	1	2	3	3	1	8	3	4	1	3	2	3	2	5
April	13	1	0	1	5	2	2	3	3	4	4	5	6	2	2	2	4	1	9	8	2	1	2	2	2	1	3
May	13	1	3	2	5	1	1	2	3	6	4	4	9	1	1	1	4	1	8	3	2	3	1	4	4	2	4
June	16	1	2	2	3	0	1	3	2	7	3	3	7	1	1	4	3	1	6	3	3	3	3	4	4	2	2
July	12	2	2	1	4	2	4	2	2	8	3	4	3	1	1	5	5	1	4	4	2	1	3	7	5	3	2
Aug.	10	2	2	2	4	3	3	3	2	7	3	6	4	2	0	4	5	0	6	3	1	2	3	3	6	3	4
Sept.	14	2	3	1	5	1	0	2	2	7	2	5	7	3	2	2	2	0	4	1	5	1	2	2	7	4	4
Oct.	11	2	2	1	9	0	1	1	4	6	3	7	7	2	1	2	3	0	4	2	3	5	6	4	4	2	1
Nov.	12	1	2	1	10	0	1	1	2	4	5	6	6	2	1	3	3	0	6	3	7	1	3	1	6	2	1
Dec.	14	1	2	1	8	1	0	1	3	4	4	7	5	2	2	2	5	0	4	2	7	3	6	4	1	1	3
Year	146	17	27	15	75	15	14	22	34	68	45	71	67	19	14	31	44	6	69	39	43	26	41	37	50	25	35

TABLE II.—continued.

MONTH.	Archangel, Russia. 4 Years, 1855, 1858-60. Hours 6: 2, 10.										St Petersburg, Russia. 10 Years, 1855-64. Hourly.										Gorki, Russia. 11 Years, 1844-54. Hour 10:									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	1	1	5	6	4	7	3	4	0	2	1	3	3	3	5	6	2	6	2	4	5	4	3	4	5	4	...			
Feb.	0	1	4	6	5	5	2	3	2	0	2	1	3	3	5	5	2	7	1	2	3	6	3	5	4	4	...			
March	1	1	6	7	6	6	1	2	1	2	2	2	5	3	5	5	1	6	2	2	5	5	4	2	6	5	...			
April	2	2	4	5	3	5	3	2	4	3	3	2	4	3	3	5	2	5	3	4	2	4	3	4	4	6	...			
May	4	2	3	3	4	3	2	6	4	2	4	3	3	3	3	6	3	4	2	3	5	5	2	4	4	6	...			
June	4	4	3	2	4	2	2	5	4	3	4	2	2	1	2	9	3	4	2	2	4	2	2	6	6	6	...			
July	4	5	4	2	3	3	2	5	3	3	3	2	2	3	3	8	3	4	3	3	2	4	3	5	3	8	...			
Aug.	5	4	4	4	3	3	1	5	2	2	2	2	2	3	4	8	3	5	2	3	3	3	4	6	4	6	...			
Sept.	2	3	7	2	5	4	2	3	2	2	3	2	3	3	4	6	2	5	2	5	3	3	2	5	6	4	...			
Oct.	1	2	4	3	7	6	2	2	4	2	2	1	3	4	6	6	3	4	2	3	4	4	3	5	4	6	...			
Nov.	1	3	3	3	6	6	3	2	3	2	2	2	4	4	5	5	3	3	1	2	5	5	4	6	5	2	...			
Dec.	1	2	5	4	5	7	3	2	2	1	1	2	3	3	6	6	2	7	3	4	3	3	3	4	6	5	...			
Year	26	30	52	47	55	57	26	41	31	24	29	24	37	36	51	75	29	60	25	37	44	48	36	56	57	61	...			

MONTH.	Kostroma, Russia. 10 Years, 1850-59. Hours 7: 2, 9.										Orenburg, Russia. 10 Years, 1855-64. Hour 10:										Lugan, Russia. 17 Years, 1840-56. Hourly.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	2	1	3	5	8	4	4	2	2	2	3	9	2	4	4	4	2	1	1	4	5	2	1	3	3	1	11			
Feb.	2	0	3	7	4	2	5	3	2	3	3	6	3	5	3	3	1	1	1	2	4	2	2	3	4	1	9			
March	2	1	2	6	7	3	5	3	2	4	3	11	1	5	3	3	0	1	2	3	5	1	3	3	5	2	7			
April	1	1	3	6	6	2	6	3	2	3	3	12	2	3	1	4	1	1	2	3	5	2	2	3	5	1	7			
May	2	3	2	4	5	3	6	5	1	5	3	8	1	4	1	6	2	1	1	3	6	2	2	3	4	2	8			
June	5	1	2	2	3	3	7	5	2	6	3	6	1	3	1	6	3	1	2	2	3	1	1	3	6	2	10			
July	6	2	2	2	3	3	7	5	1	7	3	4	1	3	1	7	3	2	2	2	3	1	2	2	5	3	11			
Aug.	4	2	2	2	3	4	6	6	2	6	3	5	1	3	2	7	3	1	2	4	5	1	1	1	4	2	11			
Sept.	4	1	2	2	4	5	6	4	2	5	2	6	1	4	2	7	2	1	2	2	5	2	1	2	3	2	11			
Oct.	2	1	2	5	5	4	7	4	1	3	2	5	1	5	4	8	2	1	1	2	5	1	2	2	4	1	13			
Nov.	1	1	4	4	6	3	7	2	2	4	1	5	1	5	3	8	2	1	1	3	6	1	2	2	3	1	11			
Dec.	2	0	2	6	7	4	6	3	1	3	3	9	3	4	3	4	1	1	1	3	4	1	2	3	5	1	11			
Year	33	14	29	51	61	40	72	45	20	51	32	86	18	48	28	67	22	13	18	33	56	17	21	30	51	19	120			

MONTH.	Taganrog, Russia. 16 Years, 1817-32. Hours 7: 2, 10.										Jakutsk, Siberia. 15 Years, 1829-44. Hour 7:										Bogoslovsk, Siberia. 10 Years, 1855-64. Hours various.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	3	3	11	2	3	1	3	2	3	9	1	0	0	2	0	1	1	17	1	1	0	1	2	4	4	2	16			
Feb.	3	3	10	2	3	1	2	2	2	6	1	1	0	2	0	1	1	16	2	2	1	1	2	4	3	1	12			
March	2	4	11	2	3	2	2	1	4	5	1	1	0	3	0	2	2	17	2	1	1	2	3	5	3	2	12			
April	1	2	9	3	4	2	4	2	3	6	1	1	1	3	1	3	2	12	2	3	2	2	3	5	3	3	7			
May	1	1	8	4	5	3	4	2	3	5	1	3	1	3	1	4	2	11	3	2	2	2	3	5	4	4	6			
June	2	1	6	2	4	3	6	3	3	3	1	4	2	3	1	3	1	12	3	3	2	2	3	4	5	3	5			
July	2	1	5	2	3	3	9	3	3	3	1	3	2	5	1	3	1	12	4	3	2	2	2	3	3	4	8			
Aug.	2	2	8	3	3	3	4	2	4	4	1	3	1	3	1	3	2	13	5	2	1	2	2	3	3	4	9			
Sept.	3	2	10	2	3	1	5	2	2	4	1	2	1	3	1	3	2	13	2	2	1	2	2	5	4	3	9			
Oct.	2	2	12	2	3	1	4	1	4	5	1	1	1	3	1	3	2	14	2	1	1	2	3	6	4	3	9			
Nov.	2	3	9	2	3	1	3	2	5	8	1	1	0	1	0	1	1	17	2	1	1	1	3	6	3	2	11			
Dec.	2	2	12	3	3	1	2	1	5	9	1	0	0	2	0	1	1	17	1	1	0	1	2	4	4	1	17			
Year	25	26	111	29	40	22	48	23	41	67	12	20	9	33	7	28	18	171	29	22	14	20	30	54	43	32	121			

TABLE II.—continued.

MONTH.	Catherinenburg, Siberia. 10 Years, 1855-64. Hourly.									Ajansk, Siberia. 2 Years, Aug. 1847—Sept. 1849. Hours 7: 2, 9.									Barnaul, Siberia. 10 Years, 1855-64. Hour 9:								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	1	1	1	5	2	5	7	1	8	3	3	1	3	5	8	1	1	6	1	1	0	2	4	14	4	2	3
Feb.	1	1	2	4	2	4	7	1	6	3	5	2	1	2	3	2	1	9	1	2	1	1	5	12	3	1	2
March	1	1	1	6	3	6	6	1	6	2	11	1	1	3	3	0	1	9	1	3	0	2	4	13	3	3	2
April	1	2	2	5	3	4	6	1	6	2	9	1	1	5	4	0	1	7	4	5	0	1	3	8	4	3	2
May	2	3	2	3	2	3	6	3	7	3	10	1	0	3	5	1	0	8	4	5	2	2	2	6	4	4	2
June	3	2	2	2	1	2	7	4	7	2	10	1	0	2	8	2	0	5	2	4	1	4	3	8	3	4	1
July	3	2	1	2	1	3	7	3	9	1	10	3	0	1	7	1	1	7	4	5	1	3	3	7	2	4	2
Aug.	2	2	2	3	1	4	6	3	8	2	8	2	0	2	8	1	0	8	4	6	1	3	3	7	2	3	2
Sept.	1	1	2	3	1	5	7	4	6	0	13	2	0	2	5	1	0	7	2	3	1	3	3	11	2	3	2
Oct.	2	1	1	2	2	6	9	2	6	3	6	1	1	2	5	1	2	10	2	2	0	1	4	15	3	1	3
Nov.	1	1	1	3	2	6	10	1	5	3	6	1	2	3	4	3	2	6	1	1	0	1	3	17	3	2	2
Dec.	1	1	1	5	2	5	7	1	8	2	8	1	2	3	8	1	2	4	1	1	0	3	4	16	3	2	1
Year	19	18	18	43	22	53	85	25	82	26	99	17	11	33	68	14	11	86	27	38	7	26	41	134	36	32	24

MONTH.	Irkutsk, Siberia. 13 Years, 1832-44. Hours 7: 2, 10.									Nertschinsk, Siberia. 10 Years, 1855-64. Hours 10: 4.									Novo Petrovsk, Turkestan. 8 Years, 1849-56. Hours 6: 2, 10.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	8	0	3	0	8	0	0	0	12	1	0	0	0	0	0	1	4	25	4	4	5	11	2	1	1	2	1
Feb.	8	0	1	0	8	0	0	0	11	1	1	0	0	0	1	2	5	18	4	2	7	9	1	0	2	2	1
March	10	0	0	0	12	0	0	1	8	2	2	1	1	1	2	3	6	13	6	3	5	10	1	1	2	3	0
April	13	0	0	1	8	0	0	2	6	3	2	2	1	2	4	4	8	4	4	3	7	7	2	1	3	3	0
May	12	0	0	1	10	0	0	3	5	2	3	2	2	1	4	4	10	3	5	4	5	6	2	2	2	4	1
June	10	0	0	2	10	0	0	4	4	3	4	3	2	1	2	3	6	6	5	4	4	4	2	2	4	4	1
July	9	0	0	1	11	1	0	3	6	2	3	4	2	1	2	2	5	10	6	4	3	5	1	2	4	5	1
Aug.	11	0	0	0	9	0	0	3	8	1	3	1	2	1	3	3	6	11	5	3	5	6	3	2	3	3	1
Sept.	13	0	0	0	8	0	0	1	8	3	1	1	1	2	3	4	7	8	3	3	5	8	2	1	2	5	1
Oct.	13	0	0	0	6	0	0	1	11	2	1	0	1	1	3	4	8	11	3	2	4	11	3	1	2	4	1
Nov.	13	0	1	0	6	0	0	1	9	1	0	0	0	1	2	4	5	17	3	4	5	12	1	0	1	3	1
Dec.	12	0	2	0	6	0	0	0	11	1	0	0	0	0	1	1	3	25	4	4	5	10	1	0	2	4	1
Year	132	0	7	5	102	1	0	19	99	22	20	14	12	11	27	35	73	151	52	40	60	99	21	13	28	42	10

MONTH.	Tiflis, Transcaucasia. 7 Years, 1858-64. Hourly.									Tobolsk, Siberia. 10 Years, 1852-61. Hour 7:									Pekin, China. 6 Years, 1850-55. Hours 9: 3.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	1	1	4	2	0	0	1	17	5	1	1	1	11	5	3	3	4	2	4	1	1	1	2	3	2	10	7
Feb.	2	0	5	5	1	0	0	10	5	1	1	2	10	5	2	2	3	2	2	2	1	2	5	4	2	6	4
March	2	1	5	8	1	0	1	9	4	1	1	1	8	6	4	2	4	4	3	2	1	4	8	4	1	5	3
April	2	1	3	6	1	0	1	12	4	1	1	2	7	7	5	2	4	1	2	1	1	4	9	5	1	5	2
May	3	1	2	7	1	0	1	12	4	3	2	3	4	3	3	4	7	2	2	2	1	5	11	4	1	4	1
June	3	0	1	6	1	0	1	15	3	4	2	3	4	2	5	3	6	1	3	2	2	6	8	3	1	2	3
July	3	1	1	6	1	0	1	15	3	4	3	2	5	4	3	2	6	2	3	3	2	5	7	2	1	1	7
Aug.	2	1	1	7	2	1	0	12	5	4	2	1	4	4	4	4	6	2	3	3	3	3	6	3	0	2	8
Sept.	2	0	1	8	1	0	1	12	5	1	2	1	4	4	6	5	5	2	3	2	2	2	7	3	1	5	5
Oct.	2	0	2	7	1	0	1	12	6	2	0	2	4	5	7	5	4	2	2	2	2	2	7	4	2	6	4
Nov.	2	1	2	5	0	0	1	12	7	1	1	1	5	5	7	5	3	2	4	1	1	2	5	3	2	7	5
Dec.	1	1	4	4	0	0	1	14	6	1	1	1	8	7	4	3	3	3	5	2	0	2	4	3	2	7	6
Year	25	8	31	71	10	1	10	152	57	24	17	20	74	57	53	40	55	25	36	23	17	38	79	41	16	60	55

TABLE II.—continued.

MONTH.	Shanghai, China. 2 Years, 1867-69. Hour A.M. and P.M.										Hong-Kong, China. 5 Years, 1853-59. Hours, 9.30: 3.30.										Chacodate, Japan. 3 Years, 1840-42. Hours 6, 9: 3, 10.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	8	2	2	3	2	2	2	10	0	3	7	14	2	0	0	2	1	2	2	0	1	2	0	0	8	17	1			
Feb.	7	7	5	2	0	0	2	5	0	1	9	10	1	1	1	3	0	2	0	2	0	0	0	9	10	3				
March	7	6	3	6	4	1	2	2	0	1	8	8	7	1	3	0	2	1	2	0	4	4	1	1	7	7	5			
April	3	4	6	4	5	1	3	4	0	0	8	6	10	1	2	1	0	2	1	0	3	8	2	3	6	4	3			
May	4	4	6	9	5	0	1	1	1	0	6	7	10	0	3	1	2	2	1	0	4	11	4	5	4	1	1			
June	4	6	6	3	4	5	1	1	0	1	3	5	8	1	7	2	2	1	0	0	3	11	4	4	4	2	2			
July	0	1	3	10	14	2	0	1	0	1	2	7	8	2	7	1	2	1	1	0	5	11	3	5	4	1	1			
Aug.	3	2	3	9	11	1	1	1	0	1	4	5	4	1	8	4	1	3	1	0	2	12	3	4	5	2	2			
Sept.	4	9	6	7	1	1	1	1	0	2	4	7	6	1	3	1	3	3	2	0	2	7	1	1	7	8	2			
Oct.	5	7	4	3	1	1	1	9	0	2	11	10	2	1	0	2	2	1	2	0	3	4	2	1	9	10	0			
Nov.	4	2	0	0	1	2	11	9	1	3	10	7	3	0	1	1	3	2	2	0	2	2	1	1	10	11	1			
Dec.	6	4	4	2	2	2	5	6	0	3	9	11	2	0	0	1	2	3	2	1	1	1	1	11	12	1				
Year	55	54	48	58	50	18	30	50	2	18	81	97	63	9	35	19	20	23	18	1	32	75	22	26	84	85	22			

MONTH.	Jerusalem, Syria. 5½ Years, 1863-69. Hour 9:										Mooltan, India. 1½ Year, 1866-67. Hours 10: 4.										Roorkee, India. 4 Years, 1865-68. Hours 10: 4.									
	N.	N.E.	E.	S.E.	S.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.				
Jan.	1	5	7	1	2	7	5	3	...	14	2	2	3	0	2	2	6	0	0	0	1	3	0	3	1	11	12			
Feb.	1	3	3	1	2	6	6	6	...	11	2	1	1	5	2	0	2	4	1	1	2	2	1	1	1	8	11			
March	2	3	6	2	2	4	6	6	...	6	8	1	4	3	4	1	4	0	1	2	0	2	1	2	2	8	13			
April	3	2	2	5	2	5	3	8	...	8	6	2	3	3	4	0	3	1	1	2	1	4	1	2	3	5	11			
May	4	4	4	3	1	3	4	8	...	7	2	3	2	3	7	2	3	2	0	1	1	10	2	1	3	5	8			
June	4	1	2	2	0	3	6	12	...	2	4	1	3	7	11	1	0	1	1	2	2	7	1	2	2	5	8			
July	2	1	0	0	1	1	6	20	...	3	1	0	3	6	14	0	0	4	0	2	2	14	1	1	1	2	8			
Aug.	2	1	0	0	1	2	5	20	...	4	3	0	2	14	3	0	3	2	0	1	2	14	1	1	2	2	8			
Sept.	8	1	1	1	1	1	6	11	...	2	1	1	2	11	11	1	1	0	0	1	1	4	1	2	2	8	11			
Oct.	4	3	7	3	0	2	2	10	...	6	2	0	0	6	8	4	4	1	0	1	1	7	1	2	1	3	15			
Nov.	1	4	8	2	0	5	6	4	...	5	3	1	2	3	2	9	5	0	0	1	0	5	1	1	1	3	18			
Dec.	1	4	5	2	1	9	4	5	...	7	4	2	3	4	1	2	4	4	0	1	1	4	0	2	2	5	16			
Year	33	32	45	22	13	48	59	113	...	75	38	14	28	65	64	22	35	19	4	15	14	76	11	20	21	65	139			

MONTH.	Agra, India. 4 Years, 1865-68. Hours 10: 4.										Calcutta, India. 6 Years, 1861-65. Hour 8:										Madras, India. 4 Years, 1847-50. Hourly.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	SW.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	5	2	3	1	1	1	10	2	6	11	1	2	1	3	2	3	2	...	6	19	4	0	1	1	0	0	...			
Feb.	4	3	1	1	1	1	12	2	3	4	2	2	1	6	2	3	4	...	2	13	4	4	5	0	0	0	...			
March	4	2	2	2	2	2	12	1	4	4	1	1	1	8	6	3	2	...	0	3	2	5	19	1	1	0	...			
April	2	3	3	2	3	7	4	3	0	0	2	3	15	4	1	1	...	0	1	0	6	19	2	1	1	...				
May	3	2	4	2	2	1	11	2	4	0	1	5	5	12	3	0	0	...	0	0	0	4	16	8	2	1	...			
June	2	4	4	1	1	2	10	4	2	1	2	3	3	10	5	2	1	...	0	0	0	0	2	18	9	1	...			
July	2	4	7	2	1	2	4	3	6	1	1	4	6	11	4	1	0	...	0	0	0	0	5	16	10	0	...			
Aug.	1	2	7	4	4	4	2	1	6	2	1	4	5	8	3	2	1	...	0	0	0	1	5	15	8	2	...			
Sept.	2	3	3	1	1	1	7	4	8	2	3	4	4	8	2	3	2	...	0	0	0	1	4	10	12	3	...			
Oct.	2	1	1	2	2	3	6	2	12	4	2	3	2	4	3	4	2	...	7	4	1	1	8	4	3	3	...			
Nov.	1	0	2	1	3	4	5	1	13	15	2	2	1	0	1	2	3	...	13	12	1	0	0	0	1	3	...			
Dec.	2	2	2	2	2	2	8	2	9	12	3	1	0	2	1	3	3	...	11	16	2	0	1	1	0	0	...			
Year	30	28	39	22	22	26	94	28	76	56	19	33	32	87	36	27	21	...	39	68	14	22	85	76	47	14	...			

TABLE II.—continued.

MONTH.	Colombo, Ceylon. 6 Years, 1853-59. Hour 9.30:										Dodabetta, India. 5 Years, 1851-55. Hourly.										Tahiti, Pacific. 3 Years, 1858-60. Hours, 4 times daily.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	5	14	1	1	0	1	0	3	1	5	6	6	10	3	1	0	0	...	4	9	6	1	0	0	6	3	2			
Feb.	1	11	1	1	1	1	1	1	6	3	6	10	7	2	0	0	0	...	1	7	10	1	0	0	4	5	0			
March	0	1	2	4	1	3	2	1	10	2	6	14	7	1	1	0	0	...	0	6	8	2	1	2	6	6	0			
April	0	0	1	6	3	5	4	1	6	2	10	13	5	0	0	0	0	...	1	5	9	0	0	2	8	4	1			
May	0	0	0	2	1	17	3	1	4	3	8	11	4	1	0	1	3	...	3	7	6	1	0	0	3	8	3			
June	0	0	0	0	1	18	6	0	1	1	1	1	0	0	1	7	19	...	1	5	7	5	2	2	3	4	2			
July	0	0	0	0	1	16	9	1	0	2	0	0	0	0	1	6	22	...	1	4	8	5	3	5	2	1	2			
Aug.	0	0	0	1	0	15	10	0	1	5	2	1	1	0	0	3	19	...	0	2	8	2	5	8	3	2	1			
Sept.	0	0	0	0	1	16	9	0	1	7	3	1	1	1	1	3	13	...	1	3	7	3	7	6	2	1	0			
Oct.	0	0	1	2	2	11	8	2	2	6	6	7	4	1	2	2	3	...	2	8	8	4	1	4	1	2	1			
Nov.	4	7	3	2	1	2	2	2	2	2	6	10	9	2	0	0	1	...	5	4	6	2	0	0	5	8	0			
Dec.	6	13	1	2	0	1	0	0	2	2	6	10	9	3	0	0	1	...	2	7	4	0	0	1	4	12	1			
Year	16	46	10	21	12	106	54	12	36	40	60	84	57	14	7	22	81	...	21	67	87	26	19	30	46	56	13			

MONTH.	Somerset, Cape York, Queensland. 2½ Years, 1865-67. Hours 9: 3, 9.										Sydney, N.S. Wales. 6 Years, 1860-3, 1867-8. Hour 9:										Melbourne, Victoria. 8 Years. Hourly.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	4	1	2	1	0	1	3	17	2	1	9	6	4	7	2	1	1	0	2	1	1	6	11	6	2	2	0			
Feb.	2	2	4	2	1	2	3	8	4	1	7	5	4	6	2	1	1	1	2	1	2	5	7	6	2	3	0			
March	1	1	10	7	1	1	1	6	3	1	7	6	4	5	2	3	2	1	4	1	3	4	7	6	2	4	0			
April	1	1	11	14	1	1	1	1	0	1	3	4	3	4	4	6	4	1	5	2	3	4	4	3	1	7	1			
May	0	0	13	17	1	0	0	0	0	1	2	1	2	2	3	12	7	1	8	4	6	3	2	2	1	5	0			
June	0	0	10	17	2	1	0	0	0	2	2	1	1	2	4	11	7	0	9	4	5	2	2	1	1	6	0			
July	0	0	6	22	2	1	0	0	0	2	1	1	1	2	2	11	10	1	8	4	4	3	2	2	1	6	1			
Aug.	0	0	7	22	2	0	0	0	0	1	2	1	2	3	3	11	7	1	7	4	4	4	3	1	1	7	0			
Sept.	0	0	11	16	1	1	0	1	0	2	5	3	3	2	2	7	5	1	6	4	7	4	3	1	1	4	0			
Oct.	1	0	12	16	1	0	1	0	0	1	6	4	3	4	3	5	4	1	4	3	5	6	6	2	1	3	1			
Nov.	1	3	17	7	0	0	0	2	0	1	7	6	4	6	2	2	1	1	2	1	4	6	8	4	2	3	0			
Dec.	1	0	8	3	1	3	2	9	4	1	8	7	3	6	2	2	2	0	2	1	4	7	9	4	2	2	0			
Year	11	8	110	144	13	11	11	44	13	15	59	45	34	49	31	72	51	9	59	30	48	54	64	38	17	52	3			

MONTH.	Adelaide, S. Australia. 3 Years, 1861-63. Hour 9:										Freemantle, W. Australia. 2 Years, 1854-55. Hour 9.30:										Kent's Group, Tasmania. 5 Years, 1861-66. Hours 6, 12: 6.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	4	3	2	5	4	7	2	4	...	0	5	5	7	5	8	0	1	...	2	5	4	1	1	7	10	1	...			
Feb.	4	2	2	6	4	6	1	3	...	0	3	5	5	2	9	3	1	...	2	4	5	1	0	6	9	1	...			
March	5	7	2	4	4	5	3	1	...	0	6	3	8	2	8	2	2	...	3	5	5	1	1	6	8	2	...			
April	5	10	3	4	2	3	2	1	...	0	4	5	8	4	4	2	3	...	4	4	3	2	1	4	9	3	...			
May	8	12	1	1	2	3	2	2	...	1	6	9	8	1	4	0	2	...	4	3	2	1	1	5	10	5	...			
June	8	14	2	1	2	1	1	1	...	1	13	6	4	2	1	1	2	...	2	4	4	2	1	4	9	4	...			
July	10	8	1	1	2	3	2	4	...	1	8	2	2	4	5	2	7	...	3	2	2	3	2	4	11	4	...			
Aug.	6	12	2	1	1	3	1	5	...	1	8	3	2	1	5	4	7	...	3	4	2	2	1	6	9	4	...			
Sept.	7	10	1	1	1	4	3	3	...	1	3	7	2	3	4	8	2	...	3	3	2	0	1	4	14	3	...			
Oct.	4	8	2	2	3	4	4	4	...	1	4	3	3	7	4	6	3	...	3	3	4	2	1	4	12	2	...			
Nov.	3	7	1	2	3	9	2	3	...	0	1	7	1	4	10	4	3	...	3	3	3	1	0	5	13	2	...			
Dec.	3	4	1	3	4	10	3	3	...	0	1	5	2	4	12	5	2	...	2	4	3	1	1	5	13	2	...			
Year	67	97	20	31	32	58	26	34	...	6	62	60	52	39	74	37	35	...	34	44	39	17	11	60	127	33	...			

TABLE II.—*continued.*

MONTH.	Hobart Town, Tasmania. 5½ Years, 1861-67. Hours 6, 12: 6.									Port Arthur, Tasmania. 5 Years, 1861-66. Hours 6, 12: 6.									Auckland, New Zealand. 8 Years, 1853-59, 66-67. Hours 9.30 : 3.30.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	4	2	2	7	2	4	3	7	...	1	6	1	8	2	6	3	4	...	2	7	2	2	4	9	3	2	...
Feb.	3	2	1	8	2	3	2	7	...	1	4	2	7	2	5	3	4	...	3	6	1	2	3	8	2	3	...
March	4	1	2	7	3	2	2	10	...	1	5	1	7	2	5	2	8	...	3	6	3	4	3	7	2	3	...
April	3	2	1	5	2	4	2	11	...	1	4	0	5	2	8	2	8	...	2	4	2	4	3	10	2	3	...
May	4	2	1	2	2	4	2	14	...	2	2	0	3	2	7	3	12	...	1	3	1	3	4	10	4	5	...
June	4	1	1	1	1	3	4	15	...	2	3	1	1	2	6	6	9	...	1	4	3	4	4	7	4	3	...
July	5	1	1	2	2	3	3	14	...	0	1	1	2	1	6	6	14	...	2	5	3	4	5	7	2	3	...
Aug.	4	2	1	3	2	4	2	13	...	4	2	1	2	2	8	5	7	...	1	5	3	4	3	9	2	4	...
Sept.	4	2	1	3	2	3	3	12	...	4	3	0	3	3	6	5	6	...	2	6	2	2	2	5	4	7	...
Oct.	4	2	1	6	2	4	2	10	...	2	5	1	6	3	6	4	4	...	2	4	1	1	3	10	6	4	...
Nov.	3	3	1	6	1	3	3	10	...	2	3	1	5	3	5	5	6	...	2	3	2	0	3	10	6	4	...
Dec.	3	2	3	9	2	2	2	8	...	1	6	2	10	2	5	2	3	...	5	6	1	1	4	7	4	3	...
Year	45	22	16	59	23	39	30	131	...	21	44	11	59	26	73	46	85	...	26	59	24	31	41	99	41	44	...

MONTH.	Christchurch, New Zealand. 4 Years, 1864-67. Hours 10 : 4.									Dunedin, New Zealand. 4½ Years, 1862-64, 66-67. Hours 9.30:3.30: or 4.30.									Southland, New Zealand. 8 Years, 1858-6, 1866-67. Hours 9 : 3, 9.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	1	11	5	0	1	7	1	4	1	2	7	1	2	3	4	5	2	5	1	0	3	9	0	1	8	9	...
Feb.	1	11	4	1	1	7	0	2	1	1	6	2	2	2	4	5	1	5	1	0	2	8	0	0	10	7	...
March	1	7	5	1	2	10	1	2	2	2	5	1	1	2	6	7	1	6	1	0	2	6	0	1	11	10	...
April	0	10	5	1	1	7	1	2	3	1	4	1	1	1	5	8	2	7	1	0	2	4	1	0	11	11	...
May	1	7	3	2	1	10	1	1	5	1	3	1	0	1	4	10	3	8	1	0	4	2	0	0	9	15	...
June	2	5	4	2	0	12	2	1	2	1	5	0	0	1	6	11	1	5	2	0	5	2	0	1	8	12	...
July	1	3	5	2	1	12	4	1	2	2	5	1	0	1	5	6	2	9	2	1	7	4	0	0	5	12	...
Aug.	0	7	3	1	1	12	2	2	3	2	5	1	1	1	5	8	2	6	1	0	4	3	0	1	10	12	...
Sept.	1	8	7	1	1	8	2	1	1	3	7	1	2	1	4	5	1	6	2	0	6	6	1	1	6	8	...
Oct.	1	9	6	1	1	8	1	3	1	2	7	1	3	2	6	4	2	4	1	0	2	9	0	1	9	9	...
Nov.	0	6	7	2	1	7	1	6	0	2	7	1	2	2	6	4	2	4	1	0	3	8	1	1	8	8	...
Dec.	1	10	8	1	1	6	1	2	1	2	9	2	2	3	4	3	1	5	1	0	3	9	1	1	9	7	...
Year	10	94	62	15	12	106	17	27	22	21	70	13	16	20	59	76	20	70	15	1	43	70	4	8	104	120	...

MONTH.	Cape Town, Cape Colony. 4 Years, 1862-65. Hours 5, 9: 1, 5, 9.									Graham's Town, Cape Colony. 4½ Years, 1854-59. Hours 9.30 : 3.30.									Graff Reinet, Cape Colony. 3 Years, 1863-65. Hours 9 : 1, 5.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	1	0	0	2	21	1	2	4	...	0	3	4	9	3	9	2	1	...	1	0	1	1	19	5	3	1	...
Feb.	1	0	0	2	19	1	2	4	...	1	2	2	8	3	7	2	3	...	2	1	0	2	15	6	1	1	...
March	1	0	1	2	17	1	3	6	...	1	3	2	8	3	8	1	5	...	2	1	1	2	14	5	2	4	...
April	2	0	0	3	14	2	3	6	...	1	2	2	4	2	9	3	7	...	4	1	1	2	7	5	5	5	...
May	3	0	0	2	13	1	3	9	...	1	1	1	2	1	8	3	14	...	8	1	0	2	4	3	5	8	...
June	5	0	0	1	9	3	4	8	...	1	1	0	1	1	6	5	15	...	10	0	1	2	3	1	2	11	...
July	5	0	0	1	12	2	4	7	...	1	1	0	1	1	7	6	14	...	11	1	1	3	6	2	0	7	...
Aug.	3	0	0	2	11	2	5	8	...	0	2	2	2	1	9	5	10	...	10	1	0	2	5	3	2	8	...
Sept.	2	0	0	2	12	2	5	7	...	1	3	3	3	2	9	4	5	...	5	0	0	3	12	3	2	5	...
Oct.	2	0	0	1	14	2	6	6	...	1	3	3	5	5	9	2	3	...	4	0	0	3	16	2	1	5	...
Nov.	2	0	0	2	17	1	3	5	...	0	2	4	8	4	7	3	2	...	1	0	0	2	16	6	2	3	...
Dec.	1	0	0	3	20	1	3	3	...	1	2	3	8	5	9	1	2	...	3	1	0	3	17	4	2	1	...
Year	28	0	1	23	178	19	43	73	...	9	25	26	59	31	97	37	81	...	61	7	5	27	134	45	27	59	...

TABLE II.—continued.

MONTH.	Pieter Maritzburg, Natal. 2 Years, 1858-59. Hours 9: 3.									Tamatave, Madagascar. ¼ Year. Hour A.M.									Mauritius. 6 Years, 1853-59. Hours 9.30: 3.30.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	2	2	9	10	3	1	1	3	0	...	...	...	...	...	...	...	...	...	2	6	12	6	1	1	1	2	0
Feb.	2	2	8	9	2	2	1	2	0	...	...	...	...	...	...	...	...	...	1	3	7	9	0	1	2	5	0
March	1	1	10	6	4	2	3	4	0	...	...	...	...	...	...	...	...	...	1	4	11	9	1	1	1	3	0
April	2	1	11	6	5	2	2	1	0	...	...	...	...	...	...	...	...	...	1	4	12	9	0	1	1	2	0
May	2	4	10	6	1	3	2	3	0	...	...	...	...	...	...	...	...	...	1	2	9	13	2	1	1	2	0
June	4	2	8	5	2	2	4	3	0	...	...	...	...	...	...	...	...	...	1	1	9	12	3	1	1	1	1
July	2	3	6	4	3	4	2	6	1	...	...	...	...	...	...	...	...	...	1	0	9	16	2	0	1	1	1
Aug.	3	1	6	8	3	8	2	6	0	2	0	1	3	15	10	0	0	...	1	1	10	15	1	0	1	2	0
Sept.	1	1	7	12	2	2	2	3	0	2	2	0	2	10	8	2	4	...	1	2	9	12	2	0	2	2	0
Oct.	2	3	9	10	1	2	1	3	0	3	9	1	6	5	6	0	0	...	1	2	13	11	1	0	1	2	0
Nov.	3	3	7	10	2	1	1	3	0	...	...	...	...	...	...	...	...	...	2	4	11	5	1	1	2	4	0
Dec.	1	2	9	9	4	2	1	3	0	...	...	...	...	...	...	...	...	...	2	5	13	4	1	0	2	3	1
Year	25	25	100	95	32	25	22	40	1	...	...	...	...	...	...	...	...	...	15	34	125	121	15	7	16	29	3

MONTH.	Alexandria, Egypt. 3 Years, 1858-61. Hours 7: 2.									Oran, Algeria. 12 Years, 1841-53. Hour: 2.									Upernivik, Greenland. 8 Years, 1847-54. Hour 9:								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	2	3	2	2	3	8	6	5	...	4	2	1	2	4	9	2	7	...	9	4	10	1	1	4	1	0	1
Feb.	4	2	1	1	1	7	6	6	...	6	6	1	1	2	6	2	4	...	4	2	12	1	0	7	1	0	1
March	4	3	2	5	2	4	3	8	...	6	8	1	1	1	7	1	6	...	11	3	7	2	1	6	0	0	1
April	3	3	2	3	3	2	4	10	...	8	7	1	1	1	6	1	7	...	10	4	5	2	1	6	1	0	1
May	5	3	2	3	2	2	3	11	...	9	7	0	1	1	4	1	8	...	9	5	6	2	0	6	1	1	1
June	6	1	1	1	2	1	4	14	...	9	8	0	1	1	2	0	9	...	10	2	2	2	1	9	1	1	2
July	7	1	1	0	0	0	5	17	...	10	7	0	0	1	0	1	12	...	8	2	3	2	1	11	2	1	1
Aug.	7	1	0	1	0	0	3	19	...	10	8	0	1	0	1	0	11	...	7	2	5	1	2	12	1	1	0
Sept.	11	2	0	0	0	1	1	15	...	8	7	0	1	0	1	1	12	...	7	4	8	2	1	6	0	1	1
Oct.	8	6	0	2	0	1	3	11	...	6	7	0	1	2	4	1	10	...	6	3	10	2	1	8	1	0	0
Nov.	8	4	1	2	3	4	3	5	...	4	7	0	2	2	7	2	6	...	5	4	13	1	1	6	1	1	0
Dec.	2	1	0	3	2	11	6	6	...	3	5	0	3	3	10	1	6	...	5	7	13	1	0	3	1	0	1
Year	67	30	12	23	18	41	47	127	...	83	79	4	15	18	55	13	98	...	91	42	94	19	10	82	11	6	10

MONTH.	Jacobshavn, Greenland. 11 Years, 1840-51. Hour 7:									Godthaab, Greenland. 5½ Years, 1841-45. Hour 10:									Port Foulke, Greenland. 1 Year, 1860-61. Hour, various.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	2	1	16	1	2	3	0	0	6	2	9	9	2	0	5	1	2	1	0	14	0	3	0	3	0	1	10
Feb.	3	1	12	1	2	4	0	0	5	1	9	6	4	1	4	1	1	1	3	14	0	0	0	3	0	0	8
March	6	1	11	1	3	3	0	1	5	3	11	5	4	1	4	0	1	2	1	13	0	3	0	4	0	0	10
April	7	1	10	1	3	4	0	0	4	4	9	6	2	1	5	0	1	2	0	14	0	2	0	8	0	0	6
May	8	1	8	1	2	5	0	1	5	1	8	6	2	1	8	1	1	3	0	18	0	0	0	4	1	0	8
June	8	0	5	1	3	6	1	1	5	4	5	4	0	0	9	3	2	3	0	9	0	0	0	13	0	0	8
July	4	1	4	2	3	6	1	1	9	1	6	4	1	0	10	3	2	4	0	7	1	1	0	11	0	1	10
Aug.	6	0	9	1	2	7	0	0	6	1	8	4	0	0	11	3	0	4	1	14	1	1	0	6	0	0	8
Sept.	5	1	14	1	2	4	0	0	3	2	7	7	1	0	7	2	1	3	2	21	0	1	0	1	0	1	4
Oct.	4	1	15	2	2	3	0	0	4	0	5	8	2	0	9	2	2	3	0	14	0	0	1	5	2	0	9
Nov.	3	1	20	1	2	2	0	0	1	1	6	11	5	0	4	0	1	2	0	18	0	0	0	3	0	0	9
Dec.	3	1	19	1	2	4	0	0	1	4	6	7	4	2	3	1	2	2	0	16	0	0	0	4	0	0	11
Year	59	10	143	14	28	51	2	4	54	24	89	77	27	6	79	17	30	...	7	172	2	11	1	65	3	3	101

TABLE II.—*continued.*

MONTH.	Port Kennedy, British America. 1 Year, 1858-59. Hours 8 : 8.										Melville Island, Arctic. 1 Year, 1819-20. Twice a day.										Igloodik, Arctic. 1 Year, 1822-23. Twice a day.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	2	4	1	0	0	0	2	18	4	15	1	3	2	1	1	3	5	0	10	1	3	2	2	1	2	10	0			
Feb.	0	3	1	1	0	0	6	14	3	14	0	1	1	1	0	3	5	3	7	2	1	0	0	0	4	14	0			
March	0	11	1	0	0	0	2	8	9	20	0	1	1	1	0	2	5	1	8	1	0	0	0	2	7	13	0			
April	5	10	4	0	0	0	5	1	5	12	1	4	3	1	0	0	5	4	5	0	0	0	0	2	6	14	0			
May	6	2	2	0	0	0	12	3	6	12	1	2	1	4	1	0	7	3	8	4	1	4	5	2	5	5	0			
June	3	6	4	1	0	0	4	11	1	10	0	1	4	4	3	4	4	0	12	1	1	1	2	3	4	6	0			
July	2	6	3	0	1	0	6	9	4	11	2	1	2	6	3	2	3	1	5	2	1	12	2	0	2	4	3			
Aug.	2	3	8	1	1	1	8	3	4	2	3	2	3	1	1	10	3	6	4	3	4	4	0	1	2	12	1			
Sept.	3	4	0	3	2	4	9	4	1	11	4	0	0	0	5	3	6	1	2	2	3	5	1	0	6	11	0			
Oct.	0	10	1	2	1	2	2	12	1	14	0	1	1	1	4	5	5	0	3	7	5	4	3	1	0	8	0			
Nov.	1	7	2	0	0	0	2	16	2	20	0	0	0	1	2	2	4	1	4	0	0	2	1	2	8	12	1			
Dec.	0	3	2	1	0	1	2	17	5	7	0	8	2	3	1	2	7	1	5	2	1	0	1	1	11	10	0			
Year	24	69	29	9	5	8	60	116	45	148	12	24	20	24	21	36	59	21	73	25	20	34	17	15	57	119	5			

MONTH.	Winter Island. 1 Year, 1821-22. Twice a day.										Nain, Labrador. Twice a day.										Fort Confidence, Gt Bear Lake, B. Amer. 7 Months, 1848-9. Hours 18 times daily									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	7	2	2	0	0	0	6	14	0	17	1	0	0	0	0	8	5	0	2	4	8	2	1	3	5	3	3			
Feb.	9	0	0	0	0	1	1	17	0	8	5	0	0	0	1	9	5	0	1	3	7	1	1	2	6	3	4			
March	8	1	0	2	0	2	5	14	0	18	4	2	0	0	0	2	5	0	1	4	11	4	0	1	6	0	4			
April	5	3	4	2	1	3	4	6	2	6	5	0	0	1	0	2	16	0	1	2	9	5	0	1	10	2	0			
May	6	5	0	2	1	2	5	10	0	3	10	3	2	0	1	3	9	0	...	...	...	...	...	...	...	...	...			
June	4	2	4	5	1	3	3	7	1	4	11	9	0	0	1	2	2	1	...	...	...	...	...	...	...	...	...			
July	7	4	1	3	5	0	2	8	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...			
Aug.	4	3	1	1	5	3	7	7	0	3	1	3	0	1	1	14	7	0	...	...	...	...	...	...	...	...	...			
Sept.	3	3	4	4	6	1	3	5	1	4	2	12	0	0	1	8	3	0	...	...	...	...	...	...	...	...	...			
Oct.	15	3	3	4	0	2	1	3	0	6	1	3	1	1	2	11	6	0	2	11	9	6	0	0	1	0	2			
Nov.	10	4	2	1	1	2	4	6	0	4	2	6	0	1	0	14	3	0	2	9	10	5	0	0	2	1	1			
Dec.	8	0	0	5	0	1	1	16	0	6	0	0	0	0	0	15	10	0	1	6	16	2	1	1	1	0	3			
Year	86	30	21	28	20	20	42	113	5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...			

MONTH.	Sitka, N.-W. America. 8 Years, 1855-64. Hours 9 : 9.										San Francisco, California. 3 Years, 1857-59. Hours 7 : 2, 9.										Sacramento, California. 6 Years, 1854-59. Hours 7 : 2, 9.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	1	4	7	5	2	2	2	1	7	5	2	0	1	5	1	1	2	14	6	3	2	6	3	1	1	9	...			
Feb.	1	5	7	2	1	2	2	1	7	3	1	0	1	5	2	3	1	12	5	1	2	7	4	2	1	6	...			
March	1	5	7	2	1	2	2	1	10	5	1	0	0	3	2	6	2	12	5	1	2	5	4	5	2	7	...			
April	2	5	6	3	2	2	1	1	8	2	0	0	0	1	2	12	2	11	4	1	1	4	6	6	3	5	...			
May	1	1	5	3	2	3	4	4	8	1	0	0	0	1	2	16	1	10	2	1	0	6	9	7	1	5	...			
June	1	1	3	2	3	5	4	4	7	0	0	0	0	1	0	18	1	10	2	0	1	4	11	6	2	4	...			
July	1	1	3	2	2	6	4	3	9	0	0	0	0	2	2	19	0	8	2	0	0	5	12	7	3	2	...			
Aug.	1	1	3	3	2	5	3	3	10	0	0	0	0	1	3	17	0	10	2	1	0	8	10	6	2	2	...			
Sept.	1	1	5	4	2	3	1	1	12	1	0	0	0	0	2	14	0	13	5	1	1	6	6	6	2	3	...			
Oct.	1	1	8	9	2	2	1	1	6	2	0	0	0	1	2	10	1	15	7	1	1	6	4	3	2	7	...			
Nov.	1	3	7	6	1	2	1	1	8	4	1	0	0	5	2	3	1	14	8	2	3	5	2	2	1	7	...			
Dec.	1	4	8	5	2	1	1	1	8	9	1	0	0	4	2	0	0	15	8	3	3	7	1	1	0	8	...			
Year	13	32	69	46	22	35	26	22	100	32	6	0	2	29	22	119	11	144	56	15	16	69	72	52	20	65	...			

TABLE II.—continued.

MONTH.	San Diego, California. 5 Years, 1850-54. Hours, sunrise, 9: 3, 9.										Steilacoom. 5 Years, 1850-54. Hours, sunrise, 9: 3, 9.										St John's, Newfoundland. 6 Years, 1853-59. Hour 9.30:									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	
Jan.	1	7	6	3	2	5	5	2	...	2	3	6	6	2	9	2	1	0	...	2	2	1	2	2	6	7	9	...		
Feb.	2	3	5	3	4	6	4	1	...	1	2	3	4	3	8	4	3	0	...	3	3	0	3	1	7	5	6	...		
March	1	3	4	4	3	7	7	2	...	3	2	3	5	2	10	3	3	0	...	2	2	1	4	2	9	6	5	...		
April	1	2	4	4	3	8	5	3	...	1	2	3	3	2	8	4	6	1	...	4	2	1	4	2	8	5	4	...		
May	1	1	2	4	5	11	5	2	...	3	2	1	2	3	8	3	8	1	...	2	4	1	4	2	7	7	4	...		
June	1	1	2	2	5	11	6	2	...	3	1	1	2	3	6	6	8	0	...	1	3	1	4	3	9	5	4	...		
July	1	2	2	1	3	12	8	2	...	5	1	1	1	2	6	7	8	0	...	2	4	1	2	2	10	6	4	...		
Aug.	1	2	3	3	4	10	7	1	...	4	3	1	2	2	4	7	7	1	...	2	2	2	4	3	8	6	4	...		
Sept.	1	2	4	2	2	11	7	1	...	3	3	3	4	3	4	2	5	3	...	2	4	1	3	3	6	4	7	...		
Oct.	2	2	5	3	3	8	6	2	...	4	1	2	5	5	6	2	4	2	...	3	4	1	5	1	8	4	5	...		
Nov.	2	4	5	4	4	4	6	1	...	1	2	3	4	7	6	3	3	1	...	3	2	1	2	1	8	5	8	...		
Dec.	1	4	7	4	4	5	5	1	...	1	4	4	6	8	5	2	1	0	...	3	2	1	2	2	7	6	8	...		
Year	15	33	49	37	42	98	71	20	...	31	26	31	44	42	80	45	57	9	...	29	34	12	39	24	93	66	68	...		

MONTH.	Montreal, Canada. 4½ Years, 1854-59. Hours 7: 2, 9.										Kingstown, Canada. 4 Years, 1854-59. Hour 9.30:										York Factory, Hudson B., Brit. America. 6 Years, 1843-48. Hours 9: 3, 9.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	
Jan.	5	6	0	1	1	6	6	5	1	4	4	3	1	5	3	8	3	...	6	1	2	1	6	5	2	3	5			
Feb.	3	3	0	1	2	6	6	6	1	5	3	1	2	2	5	5	5	...	6	1	2	1	4	1	2	4	7			
March	3	4	0	1	2	6	8	7	0	4	3	2	1	2	7	5	7	...	10	2	1	1	4	2	1	2	8			
April	3	7	0	2	3	4	5	6	0	2	6	2	2	4	8	2	4	...	6	4	3	1	4	1	1	1	9			
May	4	6	1	1	4	3	5	5	2	2	5	1	4	4	9	2	4	...	7	6	2	1	3	0	0	2	10			
June	1	3	1	1	3	8	5	4	4	1	4	3	3	3	13	2	1	...	3	6	4	1	4	0	1	1	10			
July	3	4	0	1	3	7	4	5	4	1	5	2	1	5	12	3	2	...	3	6	6	1	4	0	0	1	10			
Aug.	2	4	1	1	3	5	5	5	5	3	4	1	3	6	8	3	3	...	3	5	4	1	3	1	1	1	12			
Sept.	4	3	1	1	3	7	4	5	2	4	3	3	1	6	9	2	2	...	4	2	2	1	5	0	2	3	11			
Oct.	2	4	0	1	3	5	6	8	2	3	5	2	1	5	7	5	3	...	6	1	3	1	6	1	2	4	7			
Nov.	2	6	1	1	3	6	3	6	2	3	3	2	2	3	6	5	6	...	4	1	3	1	8	3	4	3	3			
Dec.	4	6	0	1	2	4	5	8	1	5	5	2	1	3	5	5	5	...	3	1	4	1	9	4	5	2	2			
Year	36	56	5	13	32	67	62	70	24	37	50	24	22	48	92	47	45	...	61	36	36	12	60	18	21	27	94			

MONTH.	Red River Settlement, British America. 4 Years, 1855-59. Hours 7: 2, 9.										Norway House, British America. 7 Years, 1841-47. Hour?										Brunswick, Maine, U.S. 50 Years, 1809-59. A.M., Noon, P.M.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	
Jan.	6	1	1	0	10	4	3	3	3	3	4	1	3	4	3	3	7	3	2	7	1	1	1	6	2	11	...			
Feb.	4	0	2	0	5	2	4	5	6	5	5	1	2	5	2	1	4	3	2	5	1	1	0	6	2	11	...			
March	8	0	1	1	11	1	1	3	5	8	4	1	2	6	2	1	4	3	1	4	1	2	1	8	2	12	...			
April	8	2	1	2	8	1	1	3	4	4	7	1	2	6	2	0	5	3	1	4	1	3	1	9	1	10	...			
May	7	1	1	1	11	3	1	2	4	4	6	1	1	9	2	0	3	5	1	4	2	4	1	11	1	7	...			
June	7	2	1	1	7	2	2	3	5	4	4	1	1	8	3	0	2	7	1	2	1	4	1	12	1	8	...			
July	5	1	2	1	10	3	3	2	4	5	3	1	1	8	2	1	5	5	1	2	1	2	2	14	2	7	...			
Aug.	5	1	2	1	7	3	4	3	5	3	2	1	1	7	3	2	7	5	1	2	1	3	1	14	2	7	...			
Sept.	8	0	1	3	8	3	4	2	1	5	2	0	3	5	2	3	6	4	1	3	1	2	1	12	1	9	...			
Oct.	6	1	1	1	11	3	4	3	1	7	3	2	2	4	2	1	7	3	1	4	1	2	1	9	2	11	...			
Nov.	4	1	1	1	8	4	3	5	3	7	3	2	2	6	1	2	4	3	2	3	1	2	1	7	2	12	...			
Dec.	6	1	1	1	9	3	3	2	5	4	4	1	2	7	2	2	5	4	2	7	1	1	0	6	2	12	...			
Year	74	11	15	13	105	32	33	36	46	59	47	13	22	75	26	16	59	48	16	47	13	27	11	114	20	117	...			

TABLE II.—continued.

MONTH.	New Bedford, Mass., U.S. 16 Years, 1818-33. Hour?									Oswego, New York, U.S. 6 Years, 1854-59. Hours 7: 2, 9.									New York, U.S. 4½ Years, 1854-58. Hours 7: 2, 9.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	2	3	3	2	2	5	2	12	...	3	2	2	4	7	3	6	3	1	1	6	1	2	1	4	5	11	...
Feb.	2	2	2	2	2	5	3	10	...	2	2	3	2	7	2	7	2	1	1	4	1	1	0	4	6	11	...
March	2	3	3	3	2	6	3	9	...	3	1	2	4	5	3	10	3	0	1	4	0	3	0	4	7	12	...
April	1	4	4	3	2	7	3	6	...	2	4	3	3	4	1	10	1	2	1	6	1	5	1	5	4	7	...
May	1	3	2	4	3	9	4	5	...	0	5	2	3	5	3	10	1	2	2	5	2	6	1	3	6	6	...
June	1	2	2	3	3	10	5	4	...	1	2	1	3	5	3	13	0	2	0	3	2	6	3	5	8	3	...
July	1	2	2	3	3	11	5	4	...	1	3	1	2	6	3	9	1	5	0	7	1	5	2	7	6	3	...
Aug.	1	3	3	3	3	9	5	4	...	1	3	2	3	7	3	9	2	1	1	4	2	5	2	6	6	5	...
Sept.	1	3	3	3	3	8	4	5	...	2	3	2	3	8	2	8	1	1	1	5	1	4	2	6	6	5	...
Oct.	2	3	3	2	2	8	4	7	...	1	4	2	3	5	3	10	2	1	2	4	1	3	2	4	8	7	...
Nov.	2	2	3	2	2	6	3	10	...	1	2	3	5	5	3	10	1	0	1	3	1	3	1	4	6	11	...
Dec.	2	3	2	2	2	6	3	11	...	1	2	4	5	6	2	8	2	1	1	5	1	2	0	4	7	11	...
Year	18	33	32	32	29	90	44	87	...	18	33	27	40	70	31	110	19	17	12	56	14	45	15	56	75	92	...

MONTH.	Washington, D. of C., U.S. 5 Years, 1862-66. Hours 0, 3, 6, 9, 12: 3, 6, 9.									Chapel Hill, N.C., U.S. 6 Years, 1855-59. Hours 7: 2, 9.									Charlestown, S.C., U.S. 5 Years, 1855-59. Hours 7: 2, 9.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	2	6	2	2	4	1	3	11	...	5	4	1	1	2	6	7	5	...	3	9	1	2	1	5	2	8	0
Feb.	3	3	1	3	4	2	4	8	...	4	4	1	0	3	5	7	4	...	2	9	2	3	0	7	1	4	0
March	3	4	2	2	5	2	4	9	...	4	3	1	1	3	6	8	5	...	2	7	1	4	1	10	1	5	0
April	2	4	3	3	5	2	4	7	...	4	4	1	2	3	6	7	3	...	1	5	2	4	1	12	0	5	0
May	3	3	2	3	6	3	4	7	...	4	6	3	1	3	4	8	2	...	1	7	3	7	1	8	1	3	0
June	3	3	2	4	6	4	3	5	...	3	2	1	1	5	7	9	2	...	1	4	1	7	2	9	1	5	0
July	4	3	1	3	6	4	4	6	...	4	5	2	1	4	5	9	1	...	1	6	1	7	2	9	2	3	0
Aug.	4	2	2	2	6	4	4	7	...	6	5	2	1	2	4	9	2	...	1	5	2	4	2	14	1	2	0
Sept.	3	3	2	3	6	3	3	7	...	6	5	2	1	2	3	8	3	...	3	11	3	5	1	4	1	2	0
Oct.	4	3	2	3	4	1	5	9	...	5	5	1	1	2	3	9	5	...	3	12	1	3	1	4	1	4	2
Nov.	3	2	1	2	6	2	4	10	...	4	4	1	1	3	5	7	5	...	2	12	1	2	1	5	1	6	0
Dec.	3	3	2	2	4	3	3	11	...	4	5	1	1	4	5	6	5	...	0	10	2	4	1	7	2	5	0
Year	37	39	22	32	62	31	45	97	...	53	52	17	12	36	59	94	42	...	20	97	20	52	14	94	14	52	2

MONTH.	Jacksonville, Flor., U.S. 6 Years, 1854-59. Hours 7: 2, 9.									Key West, Flor., U.S. 6 Years, 1854-59. Hours 7: 2, 9.									Warrington, Flor., U.S. 4½ Years, 1854-59. Hours 7: 2, 9.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	3	8	0	2	2	5	1	10	0	9	9	4	4	1	1	0	3	0	3	10	1	3	1	3	1	8	1
Feb.	2	7	1	2	2	7	2	5	0	7	5	4	5	2	1	1	3	0	3	8	2	2	1	4	2	6	0
March	2	7	1	2	1	10	2	6	0	5	4	4	7	4	1	1	4	1	2	6	2	4	1	7	2	6	1
April	1	7	1	3	3	9	1	4	1	4	5	7	5	2	1	2	4	0	2	3	1	7	3	8	1	4	1
May	1	12	1	4	1	8	1	3	0	4	5	5	8	1	1	2	4	1	1	4	1	7	3	12	0	3	0
June	1	8	0	7	2	10	1	1	0	1	3	9	11	2	1	1	2	0	1	4	1	5	2	13	1	3	0
July	0	5	2	6	1	11	3	2	1	1	2	10	11	3	2	1	1	0	1	2	1	7	5	10	1	3	1
Aug.	1	6	2	5	2	11	1	2	1	2	3	7	11	4	2	1	1	0	1	3	2	6	4	10	1	3	1
Sept.	2	14	1	3	1	5	0	4	0	3	10	9	3	2	1	1	1	0	1	6	3	6	3	7	1	2	1
Oct.	3	10	1	2	1	5	1	8	0	8	11	5	3	1	0	1	2	0	2	10	3	5	1	4	1	5	0
Nov.	2	7	1	2	1	7	1	8	1	9	8	5	3	2	0	1	2	0	2	9	2	3	2	4	1	6	1
Dec.	4	7	0	2	2	6	2	7	1	7	8	6	4	2	1	1	2	0	3	9	1	4	0	4	1	9	0
Year	22	98	11	40	19	94	16	60	5	60	73	75	75	26	12	13	29	2	22	74	20	59	26	86	13	58	7

TABLE II.—continued.

MONTH.	New Orleans, Lous., U.S. 2½ Years, 1854-56, 1857. Hours 9, 12: 3.									Austin, Texas, U.S. 3 Years, 1857-59. Hours 7: 2, 9.									Goliad, Texas, U.S. 1 Year, 1858. Hours 7: 2, 9.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	4	10	4	3	2	1	2	5	0	11	1	1	2	6	3	2	4	1	12	5	3	5	2	0	1	3	...
Feb.	5	5	4	4	4	2	2	2	0	7	2	1	4	5	4	1	3	1	14	2	1	7	3	0	1	0	...
March	5	5	4	4	4	3	3	2	1	8	2	0	6	7	4	1	2	1	6	0	0	21	1	0	0	3	...
April	3	4	6	5	6	1	2	3	0	5	2	1	7	8	4	1	2	0	4	2	1	17	1	0	1	4	...
May	2	2	7	6	4	4	3	2	1	3	4	1	7	7	4	1	3	1	3	2	1	23	1	0	0	1	...
June	3	4	6	6	6	1	2	1	1	3	2	1	8	11	3	1	1	0	4	2	4	18	2	0	0	0	...
July	4	2	3	7	7	2	2	2	2	2	3	1	9	9	5	1	1	0	1	0	1	22	5	1	1	0	...
Aug.	3	3	2	5	8	2	2	3	3	2	3	2	8	11	3	1	1	0	5	4	5	10	3	0	3	1	...
Sept.	1	6	6	7	2	1	3	1	3	5	4	3	5	9	2	1	1	0	12	4	4	6	3	0	1	0	...
Oct.	9	7	8	2	1	2	1	1	0	9	3	2	4	6	3	2	2	0	10	0	4	12	4	0	1	0	...
Nov.	7	5	3	4	5	1	2	2	1	8	2	1	2	6	4	3	3	1	15	5	1	7	2	0	0	0	...
Dec.	6	7	4	3	3	2	2	4	0	8	1	2	2	5	4	3	5	1	18	3	0	8	0	0	1	1	...
Year	52	60	57	56	52	22	26	28	12	71	29	16	64	90	43	18	28	6	104	29	25	156	27	1	10	13	...

MONTH.	Fort Towson, Indian Territory, U.S. 2 Years, 1833-42. Hours sunrise, 9: 3, 9.									Glenwood, Tenn., U.S. 6 Years, 1854-59. Hours 7: 2, 9.									Marietta, Ohio, U.S. 22 Years, 1829-50. Mean of day observations.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	2	2	3	5	5	6	3	5	...	5	3	1	5	4	4	1	5	3	6	1	1	2	5	7	6	3	...
Feb.	4	3	1	3	4	6	4	3	...	3	2	1	4	4	3	3	6	2	6	1	1	2	3	6	5	4	...
March	4	2	2	4	7	5	5	2	...	5	4	2	4	4	3	2	5	2	7	1	1	2	4	7	5	4	...
April	3	2	3	4	7	6	4	1	...	2	4	1	4	7	3	1	4	4	7	1	2	2	5	6	4	3	...
May	1	1	2	6	10	6	2	3	...	2	3	2	5	5	3	2	2	7	7	1	1	3	6	6	4	3	...
June	2	1	2	4	12	6	2	1	...	1	3	1	3	6	4	2	2	8	6	1	1	2	6	8	4	2	...
July	1	2	1	5	13	5	3	1	...	1	3	2	4	4	4	3	1	9	7	1	2	2	6	8	3	2	...
Aug.	2	2	2	4	10	7	1	3	...	2	4	2	3	4	4	2	2	8	8	1	2	5	7	5	2	1	...
Sept.	4	3	2	5	8	3	2	3	...	3	3	1	4	4	3	2	2	8	7	1	2	3	7	5	3	2	...
Oct.	4	4	1	3	6	7	2	4	...	2	3	2	5	4	3	1	3	8	8	1	2	2	6	6	4	2	...
Nov.	2	2	1	6	3	8	4	4	...	2	3	3	4	4	4	2	4	4	4	1	2	2	4	8	6	3	...
Dec.	2	3	2	6	4	6	5	3	...	3	3	1	5	4	4	2	6	3	5	1	2	2	5	6	6	4	...
Year	31	27	22	55	89	71	37	33	...	31	38	19	50	54	42	23	42	66	78	12	19	29	64	78	52	33	...

MONTH.	Detroit, Mich., U.S. 5 Years, 1854-56, 1858-59. Hours 7: 2, 9.									Greenbay, Wisc., U.S. 9 Years, 1822-30. Hours sunrise, 9: 3, 9.									Hazelwood, Minn., U.S. 5 Years, 1855-59. Hours 7: 2, 9.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	1	3	2	1	3	13	5	3	...	2	5	0	0	4	14	3	3	...	2	1	1	1	6	3	5	12	0
Feb.	1	5	2	1	2	9	4	4	...	4	5	0	0	4	11	3	1	...	2	2	2	2	4	2	3	11	0
March	2	4	2	2	1	12	5	3	...	2	10	1	1	4	9	3	1	...	2	2	1	3	6	2	3	12	0
April	1	8	2	2	1	11	3	2	...	4	9	0	1	3	9	3	1	...	2	2	1	3	5	3	2	12	0
May	3	8	6	1	2	7	3	1	...	2	11	1	1	5	9	1	1	...	1	2	2	6	7	3	1	9	0
June	2	3	2	2	3	10	6	2	...	2	9	1	1	3	10	2	2	...	1	0	1	3	8	3	2	12	0
July	1	3	3	5	5	9	4	1	...	1	10	1	1	4	12	1	1	...	1	2	0	5	10	2	1	10	0
Aug.	1	3	3	4	5	10	3	2	...	2	9	1	2	3	10	2	2	...	1	0	1	3	9	3	2	12	0
Sept.	1	6	3	3	4	6	5	2	...	3	6	2	1	4	9	3	2	...	1	1	1	4	10	1	2	10	0
Oct.	1	3	3	3	3	12	4	2	...	2	6	1	2	5	10	3	2	...	1	1	1	3	9	3	2	10	1
Nov.	2	2	3	3	2	9	6	3	...	3	6	1	1	4	10	3	2	...	3	1	1	4	5	1	2	12	1
Dec.	1	4	3	5	2	11	4	1	...	2	3	1	1	5	12	3	4	...	3	1	1	2	5	1	4	14	0
Year	17	52	34	32	33	119	52	26	...	29	89	10	12	48	125	30	22	...	20	15	13	39	84	27	29	136	2

TABLE II.—continued.

MONTH.	Fort Leavenworth, Kansas, U.S. 10 Years, 1845-54. Hour, sunrise, 9: 3, 9.										Cantonment Loring. $\frac{3}{4}$ Year, 1849-50. Hours, sunrise, 9: 3, 9.										Fort Johnston, N.C., U.S. 10 Years? Hours, sunrise, 9: 3, 9.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	
Jan.	5	3	1	4	4	4	2	8	0	6	1	2	1	16	1	2	1	...	8	2	2	0	6	3	3	7	...			
Feb.	3	3	1	3	3	5	2	8	0	9	1	3	0	12	0	1	1	...	8	1	2	0	6	1	5	5	...			
March	3	3	1	3	3	5	2	10	1	6	0	4	0	16	0	4	1	...	10	2	1	1	8	2	3	4	...			
April	3	4	2	4	4	5	2	6	0	5	0	1	0	15	1	5	0	...	0	2	1	1	12	1	3	1	...			
May	2	3	2	5	6	6	2	5	0	...	...	...	...	...	...	...	...	...	8	1	3	0	12	2	4	1	...			
June	2	3	3	5	7	5	1	3	1	...	...	...	...	...	...	...	...	...	6	0	1	1	14	2	5	1	...			
July	2	3	2	5	10	5	1	2	1	...	...	...	...	...	...	...	...	...	6	0	1	2	10	4	6	2	...			
Aug.	2	3	3	6	8	3	1	3	2	3	0	2	1	5	6	12	1	...	9	0	2	0	9	4	5	2	...			
Sept.	2	3	2	5	8	4	1	4	1	6	1	2	0	10	2	8	0	...	8	2	2	1	7	1	5	4	...			
Oct.	2	4	2	4	6	4	2	7	0	5	1	1	0	12	1	9	0	...	12	1	2	0	6	1	4	5	...			
Nov.	4	3	2	4	3	4	2	8	0	4	1	2	2	18	1	2	0	...	9	2	2	1	7	1	4	4	...			
Dec.	5	2	2	3	5	4	3	7	0	4	1	1	0	14	0	1	0	...	9	2	1	1	9	1	3	5	...			
Year	35	37	23	51	67	54	21	71	6	...	...	...	...	...	...	...	...	...	102	15	20	8	105	23	50	42	...			

MONTH.	Fort Yuma. 3 Years, 1852-54. Hours, sunrise, 9: 3, 9.										Santa Fe, N. Mexico, U.S. 5 Years, 1850-54. Hours, sunrise, 9: 3, 9.										Matamoras, Mexico. $1\frac{1}{2}$ Year, 1847-48. Hours, sunrise, 9: 3, 9.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	
Jan.	10	2	1	1	2	1	3	11	...	4	3	2	2	2	4	5	6	3	10	3	3	2	9	2	1	1	...			
Feb.	6	3	1	2	2	2	4	8	...	4	3	1	3	2	3	2	8	2	6	5	6	4	3	2	1	1	...			
March	5	3	1	2	5	3	7	5	...	2	4	2	4	2	4	4	5	4	6	1	7	5	9	1	1	1	...			
April	3	3	1	2	3	4	7	7	...	2	7	3	2	2	5	3	4	2	3	2	13	6	3	1	1	1	...			
May	4	1	1	5	6	2	6	6	...	3	4	2	4	2	4	3	5	4	1	2	13	3	10	1	1	0	...			
June	1	1	1	6	5	5	6	5	...	2	4	3	4	2	5	3	2	5	0	3	26	1	0	0	0	0	...			
July	2	1	3	7	9	4	4	1	...	2	3	2	6	2	5	3	3	5	0	1	30	0	0	0	0	0	...			
Aug.	2	2	5	5	9	2	5	1	...	2	4	4	5	3	4	4	3	2	0	0	31	0	0	0	0	0	...			
Sept.	6	3	4	3	6	3	4	1	...	3	5	4	2	2	4	3	4	3	6	5	19	0	0	0	0	0	...			
Oct.	8	5	3	2	2	2	4	5	...	5	4	3	3	2	4	4	3	3	8	6	15	1	1	0	0	0	...			
Nov.	10	4	1	1	2	2	4	6	...	5	4	2	2	2	2	5	3	5	14	3	8	0	4	0	1	0	...			
Dec.	11	4	1	1	1	0	3	10	...	6	3	2	2	1	3	5	5	4	16	0	4	1	10	0	0	0	...			
Year	68	32	23	37	52	30	57	66	...	40	48	30	39	24	47	44	51	42	70	31	175	23	49	7	6	4	...			

MONTH.	Cordova, Mexico. 2 Years, 1858-59. Hours various.										Bermuda. $3\frac{1}{2}$ Years, 1855-59. Hours 9.30:										Nassau, Bahama. 5 Years, 1853-59. Hour 9.30:									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.		N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	
Jan.	6	9	2	2	1	5	2	4	0	3	6	3	2	2	8	2	5	...	4	8	4	4	2	0	1	3	1			
Feb.	3	9	2	1	3	5	1	4	0	3	4	3	1	2	6	4	5	...	2	7	4	2	3	3	2	3	1			
March	5	9	3	2	1	5	3	3	0	5	2	1	1	3	9	5	5	...	3	6	3	6	3	2	1	2	1			
April	3	9	2	1	2	7	3	1	2	3	2	3	1	5	7	3	6	...	3	7	4	4	3	1	1	1	2			
May	2	8	1	2	1	4	6	5	2	2	3	4	3	4	9	3	3	...	1	5	6	6	3	3	1	1	1			
June	3	12	4	2	1	2	2	4	0	1	4	2	3	5	7	4	4	...	1	4	6	10	3	1	0	0	2			
July	3	12	2	3	2	3	2	3	1	1	2	2	5	7	8	3	3	...	1	2	11	8	4	2	0	0	1			
Aug.	2	11	3	3	1	5	1	4	1	1	3	2	2	5	8	7	3	...	0	3	9	9	3	1	0	0	2			
Sept.	6	14	1	1	1	2	2	3	0	3	9	3	3	4	4	1	3	...	2	6	7	4	1	2	1	1	2			
Oct.	5	11	2	2	1	2	3	4	1	5	7	5	3	2	5	1	3	...	3	11	6	2	1	2	1	2	1			
Nov.	4	10	1	2	2	4	2	5	0	5	3	2	4	1	6	4	5	...	4	7	7	1	2	1	1	2	1			
Dec.	5	12	1	2	2	4	2	3	0	5	4	2	1	3	8	2	6	...	4	5	7	4	3	1	1	1	1			
Year	47	126	24	23	18	48	29	43	7	37	49	32	29	43	85	39	51	...	28	71	74	60	31	19	10	16	16			

TABLE II.—continued.

MONTH.	Havanna, Cuba. 3 Years, 1859-61. Hours, 8, 12: 4, 8.									Up Park Camp, Jamaica. 5 Years, 1853-59. Hour 9.30:									Barbadoes. 6 Years, 1853-59. Hour 9.30:								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	4	6	8	1	1	0	0	0	...	7	13	2	8	0	0	0	1	...	0	12	17	2	0	0	0	0	0
Feb.	5	6	9	3	2	0	0	0	...	6	7	1	10	0	1	0	3	...	0	8	16	4	0	0	0	0	0
March	3	9	8	5	2	0	0	1	...	4	5	1	14	1	1	1	4	...	0	7	20	4	0	0	0	0	0
April	1	8	7	4	1	1	0	0	...	2	4	2	17	0	2	1	2	...	0	3	17	9	0	0	0	1	0
May	2	6	8	5	2	2	0	1	...	2	8	2	17	0	0	0	2	...	0	2	19	10	0	0	0	0	0
June	2	8	8	3	0	0	0	1	...	3	6	1	15	1	1	0	3	...	0	4	20	6	0	0	0	0	0
July	2	7	12	4	0	0	0	0	...	4	10	2	11	0	1	1	2	...	0	6	20	5	0	0	0	0	0
Aug.	3	4	13	3	2	1	0	0	...	5	6	2	15	0	1	1	1	...	0	10	15	5	0	1	0	0	0
Sept.	2	5	11	2	1	1	1	0	...	2	9	3	13	0	1	1	1	...	0	3	18	8	1	0	0	0	0
Oct.	2	6	12	5	0	1	0	0	...	4	10	2	12	0	0	0	3	...	1	4	18	8	0	0	0	0	0
Nov.	3	5	12	6	0	0	0	0	...	6	14	2	5	1	0	0	2	...	0	8	16	5	1	0	0	0	0
Dec.	3	6	15	2	1	0	0	1	...	7	17	2	4	0	0	0	1	...	1	12	17	1	0	0	0	0	0
Year	32	76	123	43	12	6	1	4	...	52	109	22	141	3	8	5	25	...	2	79	213	67	2	1	0	1	0

MONTH.	Georgetown, British Guiana. 5 Years, 1850-51, 1854-56. Hour?									Catherina Sophia, Dutch Guiana. 4 Years, 1856-59. Hour 7: 2, 9.									Cayenne, French Guiana. 7 Years, 1846-52. Hours, generally mean of day.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	0	11	19	0	0	0	0	0	1	2	16	7	5	0	0	1	0	...	1	23	5	0	0	0	0	0	2
Feb.	0	15	13	0	0	0	0	0	0	1	18	6	3	0	0	0	0	...	1	23	2	0	0	0	0	0	2
March	0	17	14	0	0	0	0	0	0	2	21	3	4	0	1	0	0	...	2	24	3	0	0	0	0	9	2
April	0	11	18	1	0	0	0	0	0	2	17	6	4	1	0	0	0	...	1	19	5	2	0	0	0	0	3
May	0	9	18	4	0	0	0	0	0	1	12	7	7	2	1	0	1	...	1	11	13	1	0	0	0	0	5
June	0	6	22	2	0	0	0	0	0	1	10	6	9	2	1	0	1	...	0	8	18	2	0	0	0	0	2
July	0	8	19	4	0	0	0	0	0	2	9	5	8	5	1	0	1	...	0	4	21	3	0	0	0	0	3
Aug.	0	7	21	0	0	0	1	0	2	1	10	6	9	2	2	0	1	...	0	2	24	4	0	0	0	0	1
Sept.	0	9	20	1	0	0	0	0	0	1	11	5	8	2	1	0	2	...	0	4	25	1	0	0	0	0	0
Oct.	0	11	19	1	0	0	0	0	0	1	13	6	7	3	0	0	1	...	1	7	21	1	0	0	0	0	1
Nov.	0	9	20	1	0	0	0	0	0	1	12	5	7	4	1	0	0	...	1	10	17	1	0	0	0	0	1
Dec.	0	11	19	1	0	0	0	0	0	1	12	7	9	2	0	0	0	...	0	20	9	0	0	0	0	0	2
Year	0	124	222	15	0	0	1	0	3	16	161	69	80	23	8	1	7	0	8	155	162	15	0	0	0	0	24

MONTH.	Cochahamba, Bolivia. 8 Months, 1852. Hours, 9: 3.									Asuncion, Paraguay. 11 Months, 1854. Hours various.									Buenos Ayres, La Plata. 1½ Year, 1853-4-5-6. Hours, 0, 3, 6, &c.								
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Jan.	0	1	0	15	0	9	0	2	4	2	4	7	7	5	1	0	1	4	8	7	3	2	1	5	1	4	0
Feb.	0	3	1	9	0	7	0	0	9	...	...	...	...	...	...	...	...	...	8	6	3	3	0	5	2	1	0
March	0	6	0	8	0	4	1	2	10	5	7	3	8	4	1	1	2	0	6	6	7	3	4	2	0	3	0
April	0	6	0	5	0	11	0	2	6	5	5	3	6	6	2	1	2	0	2	5	5	6	4	1	2	2	3
May	...	...	...	...	...	...	...	...	...	6	5	9	7	3	1	0	0	0	10	9	1	2	3	6	0	0	0
June	2	5	0	10	4	0	0	7	2	4	4	15	3	2	0	1	1	0	4	0	1	7	2	8	6	1	0
July	0	2	0	11	0	3	0	13	2	2	4	12	5	5	1	2	0	0	6	3	6	2	0	7	3	4	0
Aug.	1	1	0	11	0	1	1	4	2	3	5	6	7	3	1	3	0	2	6	3	7	5	4	3	1	2	0
Sept.	6	4	1	7	2	1	0	2	3	2	6	10	2	6	2	0	0	2	2	7	8	5	2	4	1	1	0
Oct.	...	...	...	...	...	...	...	...	...	4	8	7	4	3	4	0	0	1	4	5	11	4	2	4	0	1	0
Nov.	...	...	...	...	...	...	...	...	...	7	6	5	4	4	0	1	2	0	5	5	7	6	0	4	2	1	0
Dec.	...	...	...	...	...	...	...	...	...	4	8	3	5	4	1	1	2	3	8	9	4	4	0	4	1	1	0
Year	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	69	65	64	49	22	53	19	21	3

TABLE II.—continued.

MONTH.	Valparaiso, Chile. 3 Years, 1853-55. Hours 9.30: 3.30.										St Michael, Azores. 10 Years, 1860-69. Hour?										Funchal, Madeira. 3 Years, 1826-28. Hour?									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	4	1	0	0	6	9	3	3	5	1	8	1	4	3	6	1	6	0	3	9	2	4	2	2	8	1	...			
Feb.	3	2	0	1	7	3	2	1	9	1	6	1	4	1	7	1	7	0	3	10	2	1	0	0	9	3	...			
March	5	2	0	2	5	3	2	2	10	1	7	1	3	2	9	2	6	0	6	7	6	2	0	0	6	4	...			
April	8	2	1	1	4	4	1	3	6	1	9	2	2	1	4	2	8	0	2	11	4	1	1	2	7	2	...			
May	9	3	1	2	3	2	1	1	9	2	10	1	3	1	4	2	8	0	3	15	2	0	0	1	6	4	...			
June	7	3	1	1	2	2	1	2	11	1	10	1	3	1	4	2	8	0	2	14	3	0	0	0	6	5	...			
July	7	3	1	3	4	3	1	1	8	2	13	1	3	0	5	2	5	0	2	20	2	0	0	0	6	1	...			
Aug.	10	3	1	2	4	3	0	3	5	0	15	0	5	0	3	2	5	1	0	29	1	1	0	0	0	0	...			
Sept.	6	2	3	1	3	3	2	4	6	1	12	1	5	0	3	1	6	1	2	9	3	2	0	0	14	0	...			
Oct.	6	4	2	1	7	7	1	2	1	2	10	1	4	2	4	1	5	1	2	14	5	2	0	2	3	3	...			
Nov.	3	2	2	1	10	6	1	2	3	3	7	0	4	2	7	1	6	0	3	13	2	1	0	0	6	5	...			
Dec.	5	2	1	0	8	7	1	2	5	3	8	2	4	2	5	2	7	0	14	10	2	1	0	0	3	1	...			
Year	73	29	13	15	63	52	16	26	78	18	115	12	44	14	62	18	77	5	42	161	34	15	3	7	74	29	...			
ADDENDUM.																														
MONTH.	Ascension. 2 Years, 1854-55. Hours 9.30: 3.30.										St Helena. 5 Years, 1855-59. Hours 9: 30.										Alexandropol, Transcaucasia. 8 Years, 1858-65. Hours 7: 2, 9.									
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.			
Jan.	0	0	2	9	17	1	0	0	2	0	0	1	15	10	1	0	0	0	0	2	0	0	0	1	0	1	27			
Feb.	0	1	1	11	13	1	1	0	0	1	0	1	14	7	1	1	0	0	1	2	0	0	0	1	0	0	24			
March	0	0	3	11	16	0	0	1	0	0	1	0	13	10	2	1	1	0	1	3	1	0	0	1	0	0	25			
April	0	0	2	11	16	0	0	0	1	1	0	0	14	8	2	0	0	0	1	7	0	0	0	4	0	1	17			
May	0	0	5	15	10	0	0	0	1	1	1	0	13	8	2	0	1	2	0	7	1	0	0	4	1	1	17			
June	0	0	3	15	11	0	0	0	1	0	3	0	12	8	2	0	1	1	1	11	0	0	0	2	0	1	15			
July	0	0	4	17	10	0	0	0	0	0	0	0	13	11	3	0	0	0	1	17	1	0	0	1	0	0	11			
Aug.	0	0	3	18	10	0	0	0	0	1	1	1	14	8	2	1	1	1	0	18	1	0	0	2	0	0	10			
Sept.	0	0	3	17	10	0	0	0	0	0	0	0	13	12	1	0	1	1	0	12	0	0	0	2	0	1	15			
Oct.	0	1	2	22	6	0	0	0	0	0	0	0	15	9	3	0	0	1	1	6	0	0	0	2	0	0	22			
Nov.	0	0	2	12	16	0	0	0	0	0	1	0	14	9	3	0	1	0	0	3	0	0	0	1	0	0	26			
Dec.	0	0	1	9	20	0	0	0	1	1	1	0	13	10	2	0	0	0	1	2	0	0	0	1	0	0	27			
Year	0	2	31	167	155	2	1	1	6	5	8	3	163	110	24	3	6	6	7	90	4	0	0	22	1	5	236			

TABLE III.—(Supplementary to Table II.)—SHOWING THE PREVAILING WINDS EACH MONTH OF THE YEAR AT SIXTEEN PLACES.

Places.	No. of Years.	Years Specified.	Hours of Observation.	Latitude.	Longitude.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Waioli, (days of NE wind, Sand Is. { " var. "	1	1845-46	?	22 15	-160 0	{ 3 28	10 18	18 13	20 10	27 4	26 4	30 1	29 2	27 3	17 14	4 26	7 24
Decima, Japan, . . .	7	1845-48, 1852-55	{ 6, 9, 3, 10	32 44	129 42	W 88°	W 81°	W 78°	W 35°	W 57°	S 53°	S 17°	S 31°	N 23°	N 3°	W 86°	W 86°
Bombay, India, . . .	3	1858-60	hourly	18 53	72 48	W 80°	W 66°	W 46°	W 29°	W 10°	S 63°	S 65°	S 75°	W 1°	W 71°	N 3°	W 76°
Raffles' Lighthouse, Singapore, . . .	2	1866-67	A.M.	1 9	103 44	NE	NE	NE	NNE	SSW	S	SSW	SSW	SW	WSW	N	NE
Banjermassing, Borneo, . . .	9	1850-58	9: 3	- 3 35	114 30	S 70°	S 72°	S 76°	E 48°	E 52°	E 59°	E 62°	E 64°	E 60°	E 87°	E 88°	S 59°
Banjoewangie, Java, . . .	8	1850-57	6, 9, 3, 10	- 8 16	114 27	E 88°	E 45°	E 29°	E 51°	E 55°	E 70°	E 76°	E 72°	E 72°	E 81°	E 72°	E 82°
Palembang, Sumatra, . . .	7	1850-56	9: 3	- 2 50	104 53	W 7°	W 20°	W 30°	N 28°	N 79°	N 85°	E 6°	E 21°	E 18°	E 25°	S 30°	W 4°
Brisbane, Queensland, . . .	3	1859-61	9: 3, 9	- 27 5	153 0	N 80°	N 76°	N 71°	E 45°	S 43°	S 29°	S 38°	S 0°	N 69°	N 44°	N 52°	N 84°
Christiansborg, Gold Coast	5	1829-34	6:	- 8 16	114 27	W 50°	W 45°	W 44°	W 43°	W 42°	W 23°	W 21°	S 79°	W 15°	W 38°	W 43°	W 52°
Do., . . .	5	do.	: 4	- 8 16	114 27	S 38°	S 44°	S 46°	S 46°	S 44°	S 45°	S 43°	S 45°	S 45°	S 45°	S 43°	S 44°
Central Africa, Speke's obs.	1	1861-62	?	1 37	32 20	NE	NE	E by N	var.	E by S	SE	SE	SE	var.	var.	NE	NE
Guatemala, days of NNE, . . .	2	1860-61	7: 2, 9	14 41	- 90 35	{ 21 2 1 1	18 5 4 0	17 4 7 2	11 6 10 2	12 14 4 4	11 12 5 5	17 5 5 3	14 7 6 3	15 4 8 0	17 9 5 2	26 1 3 1	27 0 3 0
Andenes, Norway, . . .	6	1863-68	8: 2, 8	69 19	16 S	S	S	S	S	NE	NE	W	NE	SW	S	S	S
Villa, do., . . .	3-4	1865-68	do.	64 33	10 42	SE	SE	SE	SE	SW	SW	SW	SW	SE	SE	W of SE	SE
Bergen, do., . . .	8	1861-68	do.	60 24	5 20	S	S	S	S of N	N of S	N	N	N of S	S	S	S	S
Lister, do., . . .	6	1863-68	do.	58 6	6 34	E	W of E	E	NW	NW	NW	NW	NW	NW of E	E	NW	E
Lindesnes, do., . . .	6	1863-68	do.	58 0	7 2	NE	W	NE	NE of W	W of NE	W	W	W	W	NE	NE	W

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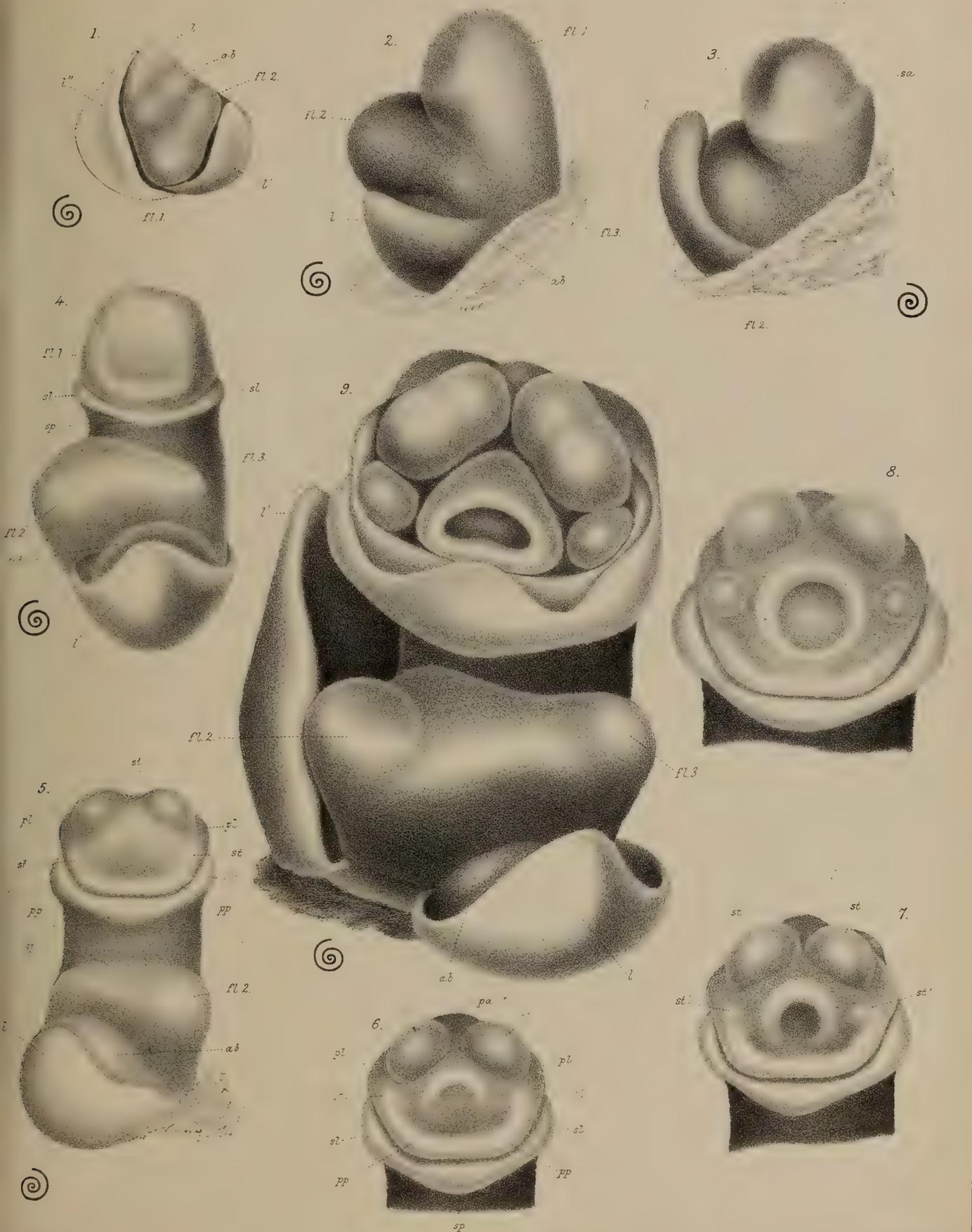
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## B, FOR WINDS (TABLES II. AND III.)

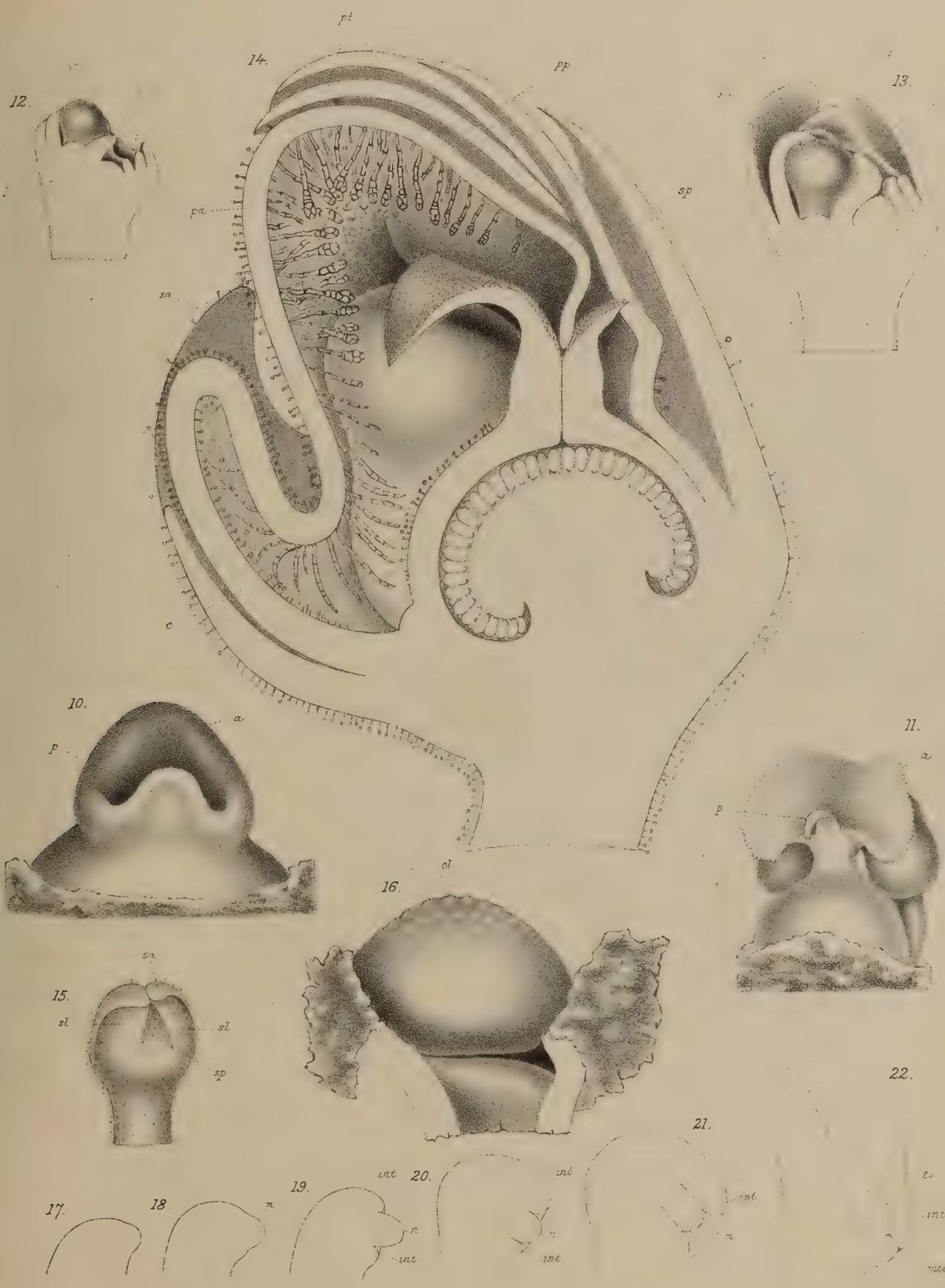
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XVIII.—*On the Development of the Flower of Pinguicula vulgaris, L.; with Remarks on the Embryos of P. vulgaris, P. grandiflora, P. lusitanica, P. caudata, and Utricularia minor.* By ALEXANDER DICKSON, M.D. Edin. & Dublin.; Regius Professor of Botany in the University of Glasgow. (Plates XXVIII.—XXX.)

(Read 19th April 1869.)

The order Lentibulariaceæ is usually described in systematic works as exhibiting affinities, on the one hand with Scrophulariaceæ, which it resembles in the bilabiate corolla, partial suppression of the andrœcium, bilabiate stigma, and two-valved capsule; and, on the other, with Primulaceæ and its allies, with which it agrees in having a truly free central placenta.

LINDLEY places the order in his alliance of Bignoniales, along with Scrophulariaceæ, apparently following Mr BENTHAM, whom he quotes in support of the supposed affinity between the families.\* Others, again, more impressed with the importance of the placental character, place the family near Primulaceæ, as has been done by PAYER.†

In the hope that the study of the development of the flower in Lentibulariaceæ might throw some light on the question of the affinities of the order, I have, from time to time during several years past, taken up the investigation of the organogeny of the flower of *Pinguicula vulgaris*, according as opportunities occurred for collecting suitable material; and I now venture to lay my results, imperfect as they still are, before this Society.

If a plant of *Pinguicula vulgaris* be examined during the flowering season, it is found to exhibit a short axis, on which are crowded a variable number of leaves, spreading out in a rosette-like manner upon the surface of the soil or turf on which the plant is found. This short axis is terminated by a contracted indefinite inflorescence, consisting of a variable number of ebracteate flowers with long pedicels—an unstalked umbel, in fact, analogous to that in the ordinary form of *Primula vulgaris*. Immediately below the inflorescence, a leaf-bud is found in the axil of the last leaf. As the fruit ripens, the leaves of the main axis gradually wither off, and the main axis itself decays; the original rosette becoming replaced in the autumn by a similar one, resulting from the development of the axillary bud of its last leaf. On the approach of winter, the

\* Vegetable Kingdom, p. 686.

† Leçons sur les Fam. Nat. des Plantes, p. 14.

expanded outer leaves of the autumn rosette disappear; the central portion remaining as a firm, bulb-like, winter-resting bud, the outer leaves of which are developed as somewhat fleshy scales. This bulb-like bud remains during the winter sunk in the soil, or among the surrounding moss; and, on the return of warm weather the next season, expands into the summer-rosette, terminated by the inflorescence as above described.

*Rudiment of the Inflorescence in Winter-Bud.*

On removing the leaves from the winter-resting bud, the following structures appear:—1st, A cushion-like mass in the axil of the last leaf, the rudiment of the bud which develops the autumn-rosette of the next season, and becomes the flowering plant of the summer thereafter. This cushion usually (always?) appears somewhat depressed, in a direction corresponding to the middle line of the supporting leaf, as I have indicated in Plate XXVIII. fig. 1; but as to the significance of this median furrow, I am unable to offer any suggestion. 2d, The termination of the main axis, which appears as an unequally three-sided cushion, nearly flat on the top, and with rounded angles, the largest and thickest of which represents the rudiment of the first flower, the angle next in size representing that of the second flower, and the remaining angle (often very obscure) that of the third flower.

These floral rudiments continue, as to position, the spiral succession of the leaves upon the main axis. If a number of plants be examined, the spiral will be found running sometimes to the right, and sometimes to the left, in about equal proportions (see figs. 1, 2, 3, 4, 5, and 9). The fraction expressing the leaf-arrangement appears to be  $\frac{5}{13}$  approximately; and the spiral succession of leaves developed upon the axillary shoot of the last leaf is homodromous with that of the main axis.

*Early Obliquity of the Floral Axis.*

Almost as soon as the young flower has begun distinctly to project from the axis of inflorescence, and before there is any appearance of sepals or other floral parts, it is seen to be more developed on the anterior aspect (that furthest from the axis of inflorescence) than on the posterior. At this stage the young flower appears as a short cylindrical body, the free extremity of which is flattened in a direction from above anteriorly, downwards posteriorly (see Plate XXVIII. fig. 2). This very early indication of irregularity is noteworthy, from the circumstance that, as a rule, irregularity commences to show itself only with, or shortly after, the appearance of the appendicular organs.

*Calyx.*

The sepals make their appearance a little below the obliquely flattened

extremity of the floral axis. The two anterior are developed first (Plate XXVIII. fig. 3). Of the lateral sepals and the posterior one, I have not been able satisfactorily to determine the relative time of appearance; but there can be little doubt that the lateral precede the posterior. The sepals soon become connate with each other; but unequally so, the two anterior with each other, and the posterior with the lateral, respectively forming an anterior lip with two lobes, and a posterior with three. These lips are almost free from each other, the antero-lateral connation being very slight. When the sepals are sufficiently developed to cover in the young flower-bud, they are found, in the great majority of cases, so arranged, that the posterior sepal is overlapped by the lateral ones, which are in turn overlapped by the anterior (Plate XXIX. fig. 15). The anterior sepals, as a rule, have not their surfaces in contact.\*

#### *Corolla.*

The examination of the earliest appearance of the corolla has been the most unsatisfactory part of my research. Its parts very soon become connate, if, indeed, they are not "congenitally" so. I am inclined to think that, as in the calyx, its anterior portion is developed first; the anterior petal appearing to me to be a more salient projection than the others in the early condition. In Plate XXVIII. fig. 4, I have represented a young flower, where the corolla is seen as a rim-like, faintly angular edging to the receptacle, just within or above the calyx, its angles alternating with the sepals. Here the stamens have not yet made their appearance, unless the very slight furrow in the middle line anteriorly be held as indicating, indirectly, the presence of the anterior stamens, one on either side of it. At this stage the centre of the receptacle is seen to exhibit a slight concavity, chiefly in the antero-posterior direction, a concavity which becomes still more marked in the subsequent stages represented in Plate XXVIII. figs. 5 and 6, and which I shall have further occasion to refer to in connection with the development of the pistil. The growth of the corolla appears to continue uninterruptedly until its full development, not exhibiting the pause which occurs so frequently in its course in other plants. As the calyx does not at all keep pace with the corolla, the latter soon forces its way from between the sepals, which at an early period are folded over it; and, in consequence of this, it is only in comparatively young flower-buds that the æstivation of the sepals can be observed. A little before the sepals are thus pushed aside, the spur of the corolla begins to appear, as a small dilatation from within of the tube of the corolla at its base, in the middle line anteriorly, indicated externally by a rounded knob-

\* Exceptions are sometimes met with. I have seen the posterior sepal overlapping only one of the lateral; or one, or both of the lateral sepals wholly external. A hasty observation of such an exception as the last, probably led PAYER (*Leçons*, p. 14) to describe the æstivation of the calyx as quincuncial, which I can hardly believe it ever is.

like projection. The process of dilatation or expansion commenced in this portion of the corolla-tube progresses gradually until the period of flowering, by which time the characteristic spur is fully developed. As in the calyx, the connation of the parts of the corolla is unequal in extent, the anterior and lateral petals forming an anterior lip, and the two posterior a posterior one. The æstivation of the corolla is similar to that of the calyx—that is to say, the odd part (here, of course, anterior) is overlapped by the lateral, which are overlapped by the other two parts.

*Andrœcium.*

In the adult condition, the andrœcium of *Pinguicula* consists of two stamens placed anteriorly. The examination of the flower in its earlier stages, however, reveals the interesting fact of the presence of two lateral rudiments or staminodes. The two fertile stamens appear first—at least they may be seen as distinctly present when the staminodes are as yet very indistinct, if not quite inappreciable. Their appearance seems to follow that of the corolla in quick succession, from the great difficulty I have experienced in finding flowers having the corolla distinctly visible, with at the same time no trace of the stamens. Indeed, even in the stage represented in Plate XXVIII. fig. 4, although the stamens can scarcely be said to be visible, yet, as I have already said, the slight indentation in the middle line anteriorly may possibly be held as indicating, indirectly, the presence of a staminal elevation on either side of it.

The stamens originate as rather large protuberances, which very soon exhibit an oblong figure, being wider from side to side than deep from without inwards. They alternate with the petals, being superposed to the two anterior sepals. In their further development there is nothing very special to be noted. As usual, the anther is formed first, becoming raised upon the subsequently developed filament. The connective forms the great bulk of the young anther, and broadens upwards in such a way that the four anther-cells lie upon its upper surface, what correspond to lateral furrows forming a single transverse one across the top of the anther. Ultimately the anther becomes one-celled, by the occurrence of absorption in the substance of the connective and consequent fusion of the anther-cells. Dehiscence takes place at the transverse furrow just mentioned.

The staminodes originate as mammillæ of small size, compared with the staminal rudiments, and are superposed to the lateral sepals. They are represented in different stages in Plate XXVIII. figs. 5-9. As a rule, they do not proceed beyond the stage represented in fig. 7, and usually become wholly obliterated by the disproportionate development of the neighbouring parts. Sometimes, however, they are developed as shorter or longer styloid processes; and I have met with a good many instances where one or both presented a terminal knob, or were even distinctly antheriferous; in the best developed cases

being scarcely distinguishable from the normal stamens. In Plate XXX. fig. 31, I have represented the essential organs of a flower where a moderate degree of this condition is to be seen, accompanied by an interesting reversion to regularity in the stigma, to which I shall afterwards refer.

*Pistil.*

The pistil appears very quickly after the development of the andrœcium; it being a matter of some difficulty to find a flower with the staminodes visible that does not, at the same time, exhibit some vestige of the pistil. It makes its first appearance as a semilunar elevation placed anteriorly just within, or (from the downward slope of the receptacle) below the two fertile stamens, with which it alternates. The extremities of this semilunar elevation gradually extend themselves around the organic centre of the receptacle, till they meet in the middle line posteriorly. The ovarian wall, thus completed, grows up as a short tube, which very soon exhibits a tendency to bilabiation, the result of preponderating growth, anteriorly and posteriorly (Plate XXVIII. fig. 9). The orifice of the short tube constituting the young ovarian wall, at first nearly circular, very soon becomes narrowed in the antero-posterior direction. This narrowing, apparently, is mainly caused by the inclination of the anterior and posterior walls towards each other, in consequence of the antero-posterior concavity of the receptacle, to which I have above alluded.\* The antero-posterior inclination towards each other of the ovarian walls, is well seen in the sections represented in Plate XXIX. figs. 12 and 13. The anterior and posterior walls thus inclined towards each other, at last come in contact, whereby the cavity of the ovary is closed in above. From this point of contact the lips of the ovarian margin, in their further development, curve away from each other; the one posteriorly as a narrow strap-like body; the other anteriorly as a broadly expanded lamina, which rests upon and ultimately wholly conceals the anthers of the two fertile stamens (Plate XXIX. fig. 11). These lips become covered on their upper surface by papillæ, and together constitute an unequally bilabiate stigma. The part where the ovarian walls are in contact becomes somewhat elongated (apparently to a variable extent), and constitutes the short style. The basal portion of the pistil becomes dilated, forming the ovary proper. It is to be noted that the ovary is to a certain extent inferior posteriorly—that is to say, its cavity posteriorly extends distinctly below the level of the insertion of the calyx and corolla.

*Placenta and Ovules.*

In the earlier stages of the development of the flower, and up to the time when

\* The slight bilabiation of the ovarian orifice seen in Plate XXVIII. fig. 9, though real, is doubtless in appearance considerably exaggerated by this antero-posterior narrowing.

the ovarian wall is completed posteriorly, by the coalescence of the extremities of the original semilunar elevation, the organic centre of the receptacle is somewhat depressed. Almost as soon, however, as the ovarian wall is complete, the receptacular centre enclosed by it begins to be developed as a more or less hemispherical protuberance—the young placenta. At no period of its development has it any connection with the ovarian wall: it is as truly “free-central” as that in *Primulaceæ*. The ovules make their appearance first on the top of this hemispherical placenta, and continue to appear in succession from above downwards, until the surface is covered by them (Plate XXIX. fig. 16). This placenta does not exhibit the slightest trace of the barren apex, which is so characteristically present in that of *Primulaceæ*—not even a bare spot,—but is uniformly and densely crowded with ovules over its whole surface. The ovules originate as small mammillæ, which become invested with a single integument, and undergo the anatropal curvature, as represented in the series given in Plate XXIX. figs. 17–22. They are placed so that the raphe is superior where the ovules project horizontally, internal where they have an upward direction, and external where they have a downward one.

*Abnormalities.*

In the course of the examination of numerous flowers, for the purposes of the foregoing investigation, I have met with a considerable number of cases of abnormality or monstrosity, some of which I think worthy of being recorded.

In Plate XXX. figs. 23 and 24, are represented two cases of remarkable modification in the symmetry. In fig. 23, the flower is dimerous and regular, with two sepals, two petals, and two stamens, in decussate succession. The ovary here is as yet only faintly indicated.\* In fig. 24, there are six sepals, of which one is anterior, one is posterior, and four are lateral, these last being conveniently distinguishable as antero-lateral and postero-lateral. Alternating with the sepals are six petals. There are five parts of the andrœcium, viz., two fertile stamens superposed to the antero-lateral sepals, and three staminodes, of which two are superposed to the postero-lateral sepals, and the third is placed between the two fertile stamens, and thus superposed to the anterior sepal.

The other abnormalities I have figured are some very interesting ones affecting the pistil. In fig. 25, the posterior wall of the ovary is deficient, the placenta and ovules being exposed; the result, doubtless, of imperfect coalescence of the extremities of the primitive semilunar elevation, a defect of development analogous to *spina bifida*, cleft-palate, hypospadias, &c., in the animal subject. In fig. 26, the posterior (small) lip of the stigma is seen to be bipartite. In this, as in the last abnormality, we have impressed upon us

\* This flower was unfortunately detached before I had ascertained whether the sepals were antero-posterior or lateral.

the fact that the posterior middle line of the ovarian wall is a line of suture, and in consequence that the small posterior lip of the stigma is potentially a double organ.\* In fig. 27, the posterior lip is normal, but the anterior (large) lip is tripartite. Fig. 28 represents a left † antero-lateral view of the same pistil, showing a slightly marked lobule (*lbl*) at the base of the antero-medial lobe; the right antero-lateral fissure, however, was found to be uncomplicated by any such lobule. Fig. 29 exhibits a nearly anterior view of an abnormality very similar to the last; but where a lobule occurs on either side of the base of the middle anterior lobe, that on the left side (to the right hand in the figure) being developed to about the same extent as the corresponding lobule in the last abnormality, while the lobule on the right side is considerably more distinct. ‡ I have represented in fig. 30 a pistil with the posterior lip somewhat broader than usual, though undivided, and the anterior lip cleft down the left side, thus exhibiting one antero-lateral fissure. § A very small notch is seen on the right side, which possibly may be held as representing a right antero-lateral fissure. In fig. 31 is seen an abnormality of quite another character. Here the stigma is altogether undivided and almost quite regular, resembling a funnel the walls of which are to a great extent turned inside out from reflection of the margin. As I previously mentioned, when treating of the androecium, the two staminodes here are well developed, with distinct filaments and anther-like terminal knobs.

*Morphological Constitution of the Ovary.*

In connection with the monstrous pistils just described, and of course always keeping in view the normal course of development, I would here make a few observations as to the probable morphological constitution of the ovary. The ordinary view has hitherto been, that the ovary in *Lentibulariaceæ* is bicarpellary, a view supported by the bilabiate stigma, bivalved capsule, and last, not least, by the fact that of the somewhat numerous vascular bundles entering its walls, the two strongest are in the mesial plane, one anteriorly the other posteriorly. This view, however, must be set aside in the face of developmental facts, which show the posterior middle line to be a line of suture. If it be objected that the presence of a strong vascular bundle in the posterior middle line constitutes a difficulty, I need only point to the interpetiolar stipules in *Cinchona*, where we have a well-marked vascular bundle occupying the middle line of the stipule, although that middle line is the line of a suture, and not of a true midrib. There are, it seems

\* I have met with three instances of this bipartite condition of the posterior lip.

† To the left of an observer supposed to stand in the axis of inflorescence.

‡ I have in my possession a third example of an ovary with tripartite anterior lip, but as I have been unwilling to remove the stamens from the specimen, I cannot say what appearance is presented on an anterior view; its posterior aspect, however, is almost identical with that given in Plate XXX. fig. 27.

§ This antero-lateral fissure is uncomplicated by any lobule.

to me, only two suppositions possessing any elements of probability and compatible with the history of development: either the ovary consists of one carpel, embracing the extremities of the receptacle; or it consists of five connate carpels, as in Primulaceæ.

With regard to the first supposition it will, I think, be admitted that it is, *à priori*, improbable that a corollifloral plant, like *Pinguicula*, should have only one carpel; all the orders with which it might possibly be compared having compound ovaries. On this ground alone I should be inclined to dismiss the idea.

On the other hand, the 5-carpellary hypothesis has the support of the monstrosities just referred to. In some we have the posterior lip of the stigma bipartite, in others the anterior lip tripartite.\* Now, if we combine these monstrosities, we obtain five parts, and these placed in the proper position—*superposed to the petals*. Were we to take the ovary of *Primula*, which originates as an entire annulus, and so modify its development that its anterior part should appear first (just as the anterior part of the calyx in *Pinguicula* appears first), we should have a structure originating in semilunar form exactly as in the young ovary of *Pinguicula*. That five connate carpels should go to form a bilabiate stigma, is just what might be expected in a family where the tendency to bilabiation is so strongly marked. To take an extreme case, I may refer to *Utricularia minor*, where the corolla, with two vascular bundles going to its upper and three to its lower part, is bilabiate with two perfectly entire lips.

#### *General Conclusions.*

A few words may be said with regard to the probable affinities of the order Lentibulariaceæ. In the first place, I shall allude to the opinion of Mr BENTHAM, as quoted by LINDLEY (*Veget. Kingd.* p. 686), to the effect that they are very closely related to Scrophulariaceæ, in “having the same calyx, corolla, stamens, and bivalve capsule, but distinguished solely by their really unilocular fruit, with a free central placenta, and the minuteness of their embryo. In respect of the former character, they come very near to *Limosella*, *Lindernia*, and other *Gratiolææ*, with parallel dissepiments and entire valves; for in these plants the dissepiment is very thin, and usually detaches itself from the valves before maturity, so that being concealed by the seeds, which fill nearly the whole capsule, it often escapes observation, and many of these genera have frequently been described as having a unilocular fruit.”

Having, as I think, satisfactorily set aside the idea that the ovary of Lentibulariaceæ is bicarpellary, it is, perhaps, unnecessary on my part to refer to Mr BENTHAM's view, that the premature detachment from the valves of the thin

\* The variable and inconstant lobules at the base of the middle anterior lobe in this form of monstrosity I am, I think, justified in considering of secondary importance.

dissepiments in the Gratiolæ is an indication of an approach to the structure of an ovary with free central placenta; I would only suggest that this is an idea of the same character, and quite as fallacious, as the popular one that the peculiar splitting of the fruit in *Platystemon* indicates an approach in that plant to the apocarpous Ranunculaceæ. If, then, any affinity with Scrophulariaceæ is to be found it must be in the floral envelopes and stamens. In Lentibulariaceæ we have, no doubt, irregular bilabiate floral envelopes and partial suppression of the andrœcium with a tendency to the didynamous structure; but the value of this combination of bilabiation with didynamy as determining the true affinities of a given plant is seriously open to question. It must, I think, be evident to any one reflecting on the subject, that such a combination of characters occurs in several very different types, by what may be called a parallelism of development or modification. Thus,

- 1st, In Scrophulariaceæ, with 2-celled ovary and axile placentation; a modification of the Solanaceous type.
- 2d, In Gesneraceæ and Orobanchaceæ, with 1-celled ovary and parietal placentation; a modification of the Hydrophyllaceous (?) type.
- 3d, In Labiatae, with gynobasic style and spuriously multiplied loculi; a modification (in spite of the difference in the position of the raphe) of the Boraginaceous type.
- 4th, In *Morina* (belonging to the order Dipsacaceæ), where we have a bilabiate corolla of five petals, and four stamens, two large and two small.

On the whole, it seems to me that we have as little right to associate Lentibulariaceæ with Scrophulariaceæ on account of bilabiate floral envelopes and more or less didynamous stamens, as a zoologist would have to associate the Echidna with Hedgehogs or with Porcupines, on account of the remarkable correspondence in their prickly defence.

With regard to the supposed affinity with Primulaceæ, we have a correspondence in what may perhaps be viewed as the most remarkable structure in the Lentibulariaceous flower, viz., the free central placenta; and I have shown at least some plausible grounds for believing the Lentibulariaceous ovary to be composed of five carpels, like that of Primulaceæ. The important differences between the orders may thus be reduced to the position of the stamens and the albuminous or exalbuminous character of the seeds.

PAYER, in his *Leçons sur les Fam. Nat. des Plantes*, places the order Salvadoraceæ (consisting of the single genus *Salvadora*) in juxta-position with Lentibulariaceæ. Both agree in the superposition of the stamens to the sepals, in having a unilocular ovary with free central or basilar placentation, and in the exalbuminous character of the seed. The question very naturally suggests itself,

have we not in *Salvadora*, with oppositi-sepalous stamens and solitary exalbuminous seed,\* a plant bearing the same relation to Lentibulariaceæ, with numerous exalbuminous seeds, as Plumbaginaceæ, with oppositi-petalous stamens and solitary albuminous seed, bears to Primulaceæ, with numerous albuminous seeds? I believe that in Salvadoraceæ with Lentibulariaceæ, on the one hand, and Plumbaginaceæ with Primulaceæ, on the other, we have two parallel nearly allied series. I shall not, however, pursue this subject further, as my personal knowledge of *Salvadora* is very limited.

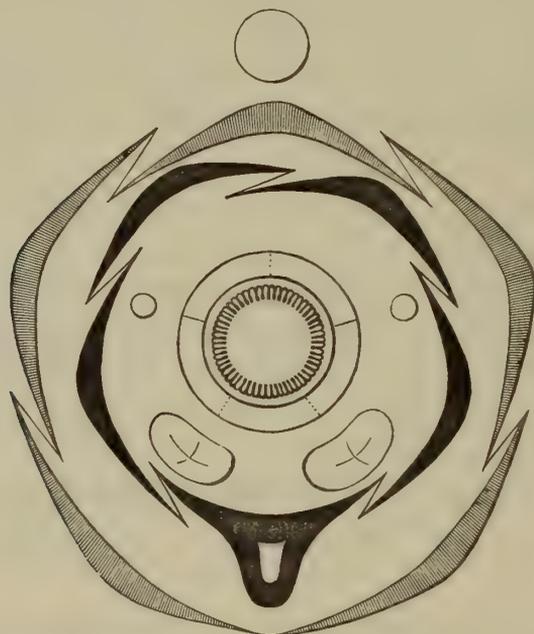


Diagram of the flower of *Pinguicula vulgaris*, L., showing the aestivation of calyx and corolla, the stamens and staminodes superposed to the anterior and lateral sepals, and the one-celled ovary with free central placenta. The wall of the ovary is represented as divided into five parts by two plain and three dotted lines, the two plain lines representing the division of the stigma into two lips or of the capsule into two valves, the three dotted lines representing the abnormal fissures in the above mentioned monstrosities.

\* WIGHT (*Icones pl. Ind. Orient.* t. 1621), ENDLICHER (*Genera*, p. 349), LINDLEY (*Veget. Kingd.* p. 652), and PAYER (*Leçons*, p. 14) agree in describing *Salvadora* as having a unilocular ovary with solitary erect ovule. Professor OLIVER has kindly examined for me flowers of *S. persica*, L., and *S. Wightiana*, Pl., from the Kew Herbarium, of which he reports in a letter as follows:—“In each of these I find a 1-celled ovary with a solitary basal ovule.” My own somewhat limited examination of the flowers of *S. persica* has led me to the same conclusion. On the other hand, PLANCHON (*Sur les Salvadoracées*, *Ann. des Sc. Nat.* 3<sup>me</sup> serie x. p. 190), and more recently MM. MAOUT and DECAISNE (*Traité de Botanique*, p. 453) describe the ovary here (PLANCHON in the genus *Salvadora*, MAOUT and DECAISNE in the order Salvadoraceæ) as bilocular, with two collateral ascending ovules in each cell. The only explanation I can suggest for the statement in the “*Traité de Botanique*,” is that the authors have probably followed PLANCHON, for M. DECAISNE had formerly described *S. oleoides* as having “ovarium . . . uniloculare, loculo uniovulato” (*Jacquemont Voyage*, p. 140, t. 144); while M. PLANCHON’s description is so opposed to the results of other botanists, and so unlike anything I myself have been able to see, that I am constrained to believe that it was some other plant, and not *Salvadora*, that he examined. I should mention, however, that DECAISNE (*Jacquemont Voy.* t. 144) gives a figure of a fruit of *S. Madurensis* containing three seeds.

*Remarks on the Embryos of Pinguicula vulgaris, P. grandiflora, P. lusitanica, P. caudata,\* and Utricularia minor.*

The remarkable diversity in the structure of the embryo in the Lentibulariaceæ is, perhaps, one of the most extraordinary circumstances connected with the order. A. DE ST HILAIRE pointed to the occurrence of a dicotyledonous embryo in *P. lusitanica*, a monocotyledonous one in *P. vulgaris*, and an acotyledonous one in *Utricularia vulgaris*, as an instance of how the most important characters may vary, even within the limits of a single order.†

TREVIRANUS, in 1838,‡ was the first to show that the embryo of *P. vulgaris* has only one cotyledon. In 1848, he published his researches on its germination, which were called forth by a statement of KLOTZSCH'S, that this embryo germinates with two cotyledons, of which one is much smaller than the other. Here, he showed that KLOTZSCH'S smaller cotyledon does not appear until germination is considerably advanced, thus proving that it does not legitimately fall under the definition of a cotyledon at all.§

*P. vulgaris*, L. (Plate XXX. figs. 33-40).

The embryo of *Pinguicula vulgaris*, taken as a whole, is of a cylindrical form, with rounded extremities, and measures about  $\frac{1}{3}$ d of an inch in length. The single cotyledon constitutes about one-half of the entire length of the embryo, and is folded upon itself in a conduplicate manner, its margins being approximate and parallel to each other, except towards the base, where they diverge rather suddenly, leaving a considerable interval, where the termination of the embryonic axis (rudimentary plumule) is to be seen (fig. 33). The apex of the cotyledon is almost constantly entire, or, at least, not sufficiently emarginate to appear distinctly so in a back view, such as is represented in fig. 34. In two, or at most three instances, however, out of the large number of embryos that I have examined, the tip of the cotyledon was somewhat bifid, as is seen in the back view in fig. 35. When sections made in the mesial plane (fig. 38) and at right angles to it (fig. 37) are compared, the rudimentary plumule is seen to be compressed laterally, having a strong convex curvature from side to side, while there is only the slightest possible convexity from before backwards. That there

\* The observations on the embryo of this species were made after the paper had been submitted to the Society.

† Morphologie, pp. 755-6.

‡ In a communication to a meeting of naturalists, at Freyburg in Br., of which I have seen no report, but which is referred to by TREVIRANUS in his subsequent paper in the Bot. Zeitung, 1848.

§ Botanische Zeitung, 1848, p. 444.

is no trace of a second cotyledon is quite evident from examination of the mesial sections.\*

*P. grandiflora*, Lam. (Plate XXX. figs. 41-42).

After examining the embryo of *P. vulgaris*, I was curious to ascertain whether there was any difference between it and that of this species, which is so nearly allied to *P. vulgaris* that some botanists are disposed to combine them together; and I was gratified to find embryonic characters by which they may readily be distinguished from each other. In front view (fig. 41), the embryo of *P. grandiflora* (which is about the same size as the last) exhibits a single cotyledon having about the same relative length to the whole as that of *P. vulgaris*. The base of the cotyledon, however, is found almost completely to surround the extremity of the embryonic axis, so that hardly a vestige of the plumule is to be seen from the outside; and on back view (fig. 42), the tip of the cotyledon is seen to be constantly and deeply bifid.† The first peculiarity is, so far as I have seen, absolutely distinctive between this embryo and that of *P. vulgaris*; while as to the second one, it is, as I have just mentioned, only in very rare cases that the cotyledon of *P. vulgaris* is bifid at its extremity. These embryonic characters, combined with some other remarkable differences (such as the number of adventitious buds produced at the bases of the outer leaves of the autumn-rosette—in *P. vulgaris*, usually only one in the middle line of each leaf; in *P. grandiflora*, a considerable number in a single transverse row), go far, in my opinion, to establish the validity of the claim of *P. grandiflora* to be ranked as a species.‡

*P. lusitanica*.

With regard to the very minute embryo of this species (about  $\frac{1}{57}$ th of an inch in length), I need not say much, beyond confirming the statements of ST HILAIRE as to there being two cotyledons. These are relatively considerably shorter than the single one of *P. vulgaris* or *P. grandiflora*. I have to note the presence of a trace of albumen in the seed here.

\* TREVIRANUS' figure of the embryo from the seed is somewhat faulty, from the cotyledon being represented as considerably too short in proportion to the radicle, and from the absence of any indication of the rudimentary plumule. There is also no indication of the plumule in his figures of the earlier stages of germination, the result, doubtless, of imperfect observation (*loc. cit.* t. iv.). He also makes a curious blunder in describing the apex of the embryo as pointed towards the hilum of the seed (*loc. cit.* p. 442), the fact being that in this, as in all anatropal seeds, the apex of the embryo points away from the hilum, the radicle being directed towards it. This mistake is probably due to the circumstance that there is often a projecting portion of the testa at the chalazal extremity, which is apt to be mistaken for the somewhat similar projection at the hilum.

† I think it not improbable that back views of this embryo may have had something to do with the statement found in most of the books, that there are two "cotyledones brevissimæ" in *Pinguicula*.

‡ I should mention that a very brief statement, by me, of the differences between the embryos of *P. vulgaris* and *P. grandiflora*, has already appeared in the report of a meeting of the Dublin Microscopical Club ("Quarterly Journal of Microscopical Science," viii. pp. 121-2). I now take this opportunity of describing them in greater detail, and with figures.

*P. caudata* (Plate XXX. figs. 43-44).

Since bringing this paper before the Society, I have succeeded in extracting an embryo, almost entire, from one of a very few seeds of this Mexican species obtained from the University Herbarium in Dublin; and I find that there are two cotyledons, whose length is about one-half of that of the embryo, which measures about  $\frac{1}{30}$ th of an inch. The embryo here, like the seed containing it, is very narrow and considerably elongated. I have given two views of this specimen, so as to show the division between the cotyledons on either side; from which the fact that there *are* two cotyledons is abundantly manifest. In the specimen figured, one cotyledon is a little shorter than the other; this, however, is accidental, as the cotyledons were of equal length in another embryo which I extracted in a somewhat mutilated condition.

*Utricularia minor*, L. (Plate XXX. fig. 45.)

The embryo here is somewhat globular, about  $\frac{1}{57}$ th of an inch in diameter, and at first sight appears to have a smooth undivided surface; on careful inspection, however, a remarkable conformation is to be observed of that end of the embryo which is remote from the hilum of the seed, viz., a minute, slightly convex *punctum vegetationis* surrounded by four slight elevations placed so as to form the somewhat incurved sides of a square. I am not exactly prepared to call these elevations cotyledons; but the whole structure is interesting, as showing this embryo to be a little in advance of a mere "embryonal globule," as are most of the embryos described as "undivided" or "acotyledonous."

### Explanation of Plates XXVIII., XXIX., XXX.

#### PLATE XXVIII.

##### *Pinguicula vulgaris.*

- Fig. 1. Extremity of winter-resting bud, showing rudiment of the inflorescence, and of the axillary bud of the last leaf. *l'*, 3d last leaf cut across; *l'*, 2d last leaf; *l*, last leaf; *ab*, axillary bud of last leaf; *f*<sup>1</sup>, indication of 1st flower; *f*<sup>2</sup>, that of 2d flower. Leaf-spiral from right to left of observer supposed to occupy the axis.  $\times 77$ .
- Fig. 2. Young inflorescence further advanced. The first flower (*f*<sup>1</sup>) distinctly projects, and exhibits irregularity, being flattened from above anteriorly, downwards posteriorly, although not even the calyx has appeared. Leaf-spiral from right to left.  $\times 77$ .
- Fig. 3. Young inflorescence, in which the anterior sepals of the 1st flower are beginning to appear (*sa*). Leaf-spiral from left to right.  $\times 77$ .
- Fig. 4. Young inflorescence. Here the calyx of the 1st flower is now complete, and the corolla is visible. *sl*, lateral sepal; *sp*, posterior sepal. Leaf-spiral from right to left.  $\times 77$ .
- Fig. 5. Young inflorescence. Fertile stamens (*st*) distinctly present, and staminodes (*st'*) faintly so in the 1st flower. *pp*, posterior, and *pl*, lateral petals. Leaf-spiral from left to right.  $\times 77$ .

- Fig. 6. Young flower. Ovary beginning to appear to the anterior side of receptacular centre as a semilunar elevation alternate with the anterior (fertile) stamens.  $\times 77$ .
- Fig. 7. Young flower. The extremities of the semilunar ovarian wall are now extending themselves round the receptacular centre.  $\times 77$ .
- Fig. 8. Young flower. The ovarian wall is now completed by union of the extremities of the semilunar elevation in the middle line posteriorly. The receptacular centre, hitherto depressed, is becoming slightly elevated; forming the rudiment of the free central placenta.  $\times 77$ .
- Fig. 9. Young inflorescence. In the first flower the ovarian wall is completed, and begins to show a tendency to bilabiation. The fertile anthers now show themselves to be 4-celled. Leaf-spiral from right to left.  $\times 77$ .

## PLATE XXIX.

*Pinguicula vulgaris.*

- Fig. 10. Young pistil, showing larger or anterior (*a*) and smaller or posterior (*p*) lip of the stigma. The disproportion between the lips is not yet very great.  $\times 85$ .
- Fig. 11.\* Young pistil, considerably further advanced, exhibiting nearly its adult form. Anterior lip of stigma broadly expanded, the posterior narrow and strap-shaped.  $\times 15$ .
- Fig. 12. Longitudinal section of young flower at about the stage represented in fig. 9, *pst*, pistil. The placental elevation (*pc*) is now commencing to appear.  $\times 30$ .
- Fig. 13. Longitudinal section of young flower at a further advanced stage. The ovarian cavity is becoming somewhat "inferior" posteriorly. As yet no ovules.  $\times 30$ .
- Fig. 14. Longitudinal section of half-mature flower-bud. The corolla now extends beyond the sepals, and its spur (*e*) is of considerable length. The ovarian cavity is now nearly half-inferior posteriorly.  $\times 30$ .
- Fig. 15. Young flower-bud, showing the æstivation of the sepals.  $\times 15$ .
- Fig. 16. Young placenta, showing the basipetal succession of the ovules (*ol*), which have as yet appeared only on its upper part.  $\times 100$ .
- Fig. 17-22. Outline-sections (partly optical) of ovules at different stages of development. Nucleus (*n*); integument (*int*). In fig. 22 the embryo-sac (*es*) appears to have wholly replaced the nucleus.

## PLATE XXX.

*Pinguicula vulgaris.*

- Fig. 23. Abnormality. Young flower with dimerous symmetry and regular; 2 sepals (*s*), 2 petals (*p*), 2 stamens (*st*). The ovary is faintly indicated.  $\times 77$ .
- Fig. 24. Abnormality. Young flower with hexamerous symmetry. Sepals—1 anterior (*sa*), 2 antero-lateral (*sal*), 2 postero-lateral (*spl*), and 1 posterior (*sp*). Petals—2 anterior (*pa*), 2 lateral (*pl*), and 2 posterior (*pp*). Two stamens (*st*), here antero-lateral; and three staminodes (*st'*), 1 anterior and 2 postero-lateral.  $\times 77$ .
- Fig. 25. Abnormal young pistil. Ovarian wall deficient posteriorly, exposing the placenta and ovules. *x*, lappet of doubtful significance.  $\times 15$ .
- Fig. 26. Abnormal young pistil, with bipartite posterior lip of the stigma. *a*, anterior lip of stigma; *p' p'*, the halves of the posterior lip.  $\times 85$ .
- Fig. 27. Abnormal young pistil. Anterior lip of stigma tripartite, being divided into an antero-median lobe (*am*), and two antero-lateral lobes, one right (*ral*), the other left (*lal*).  $\times 85$ .
- Fig. 28. Left antero-lateral view of the same pistil, showing a slightly-marked "lobule" (*ll*) at the left side of the base of the antero-median lobe.  $\times 85$ .

\* In this figure, as also in fig. 31, the capitate hairs scattered over the surface of the ovary are not represented.

- Fig. 29. Nearly anterior view of a monstrous pistil resembling the last; but where there is a "lobule" (*lbl*) on each side of the base of the antero-median lobe, that on the right side (to left hand in the fig.) being considerably the larger.  $\times 85$ .
- Fig. 30. Abnormal pistil. Posterior lip of the stigma (*p*) somewhat broader than usual, but undivided. Anterior lip with a fissure on the left side, separating off a left antero-lateral lobe (*lal*).  $\times 85$ .
- Fig. 31. Abnormal pistil. Stigma funnel-shaped, and nearly regular. The staminodes (*st'*) here are greatly developed, showing distinct filaments terminated by anther-like knobs.  $\times 15$ .
- Fig. 32. Monstrous pitcher-like leaf. The dotted line indicates where the cavity of the leaf terminates below. Natural size.
- Fig. 33. Embryo. Front view. Solitary cotyledon (*c*); radicle (*r*); rudimentary plumule or *punctum vegetationis* (*pv*).  $\times 41$ .
- Fig. 34. Embryo. Back view.  $\times 41$ .
- Fig. 35. Embryo. Back view, exhibiting an unusual bifid condition of the extremity of the cotyledon.  $\times 41$ .
- Fig. 36. Embryo. Side view.  $\times 41$ .
- Fig. 37. Embryo. Longitudinal section at right angles to the mesial plane.  $\times 41$ .
- Fig. 38. Embryo. Longitudinal section in the mesial plane.  $\times 41$ .
- Fig. 39. Embryo. Remarkably curved.  $\times 41$ .
- Fig. 40. Mesial section of embryo similar to the last.  $\times 41$ .

*Pinguicula grandiflora.*

- Fig. 41. Embryo. Front view.  $\times 41$ .
- Fig. 42. Embryo. Back view.  $\times 41$ .

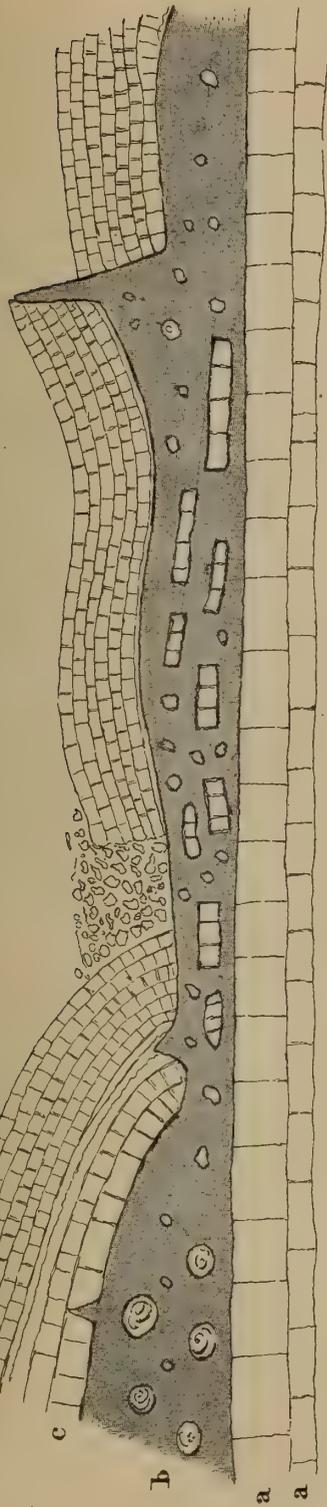
*Pinguicula caudata.*

- Fig. 43. and 44. Views from both sides of one embryo, showing the presence of two cotyledons.  $\times 43$ .

*Utricularia minor.*

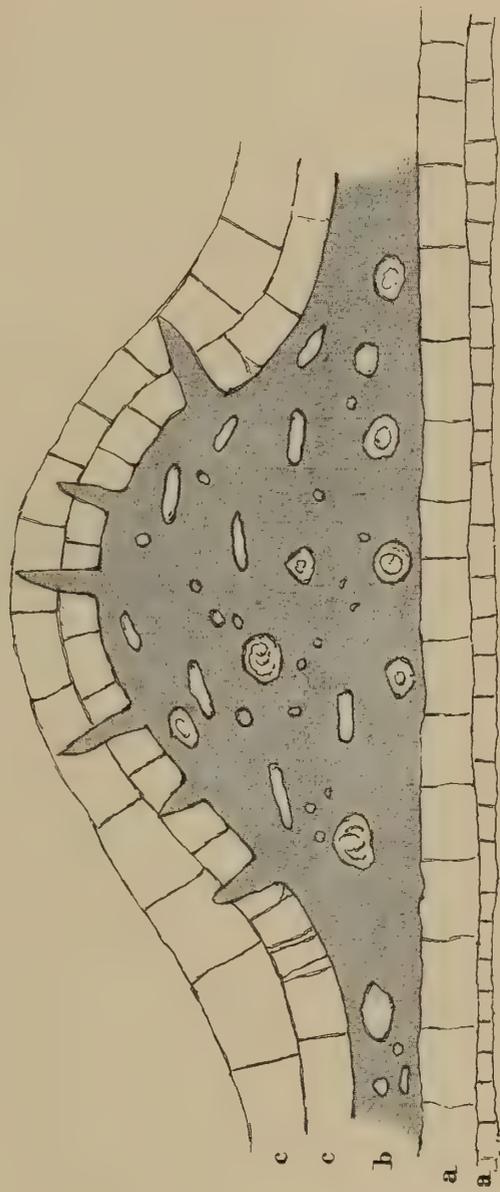
- Fig. 45. Embryo, showing *punctum vegetationis* (*pv*) surrounded by four very slight elevations (*c*) forming the somewhat incurved sides of a square.  $\times 43$ .





LINKSFIELD QUARRY LENGTH OF SECTION 80 FEET.

- a - Limestone in horizontal strata, rubbed & striated.
- b - Boulder Clay, containing besides the usual boulders portions of the Limestone c.
- c - Thin strata of Limestone, much disturbed and broken up
- d - Overlying Wealden strata also dislocated



LINKSFIELD QUARRY LENGTH OF SECTION ABOUT 25 FEET

- a Limestone in horizontal strata, striated
- b Boulder Clay, containing small bits of the Limestone rock c
- c Limestone strata, supposed to have been lifted up and dislocated by Boulder Clay



XIX.—*On the Boulder-Clay of Europe.* By DAVID MILNE HOME, Esq.  
(Plate XXXI.)

(Read 19th April 1869.)

“Boulder-clay” or “till,” abundant in Scotland, and occurring also in England, Ireland, and in some other parts of North-Western Europe, has long been, and still is, a puzzle to geologists.

Sir JAMES HALL, about fifty years ago, in this Society, was the first to draw attention to the deposit, by describing its composition, and endeavouring to explain its origin. He saw that it could not be included in either of the two great classes into which rocks were then divided. It was a deposit *sui generis*, bearing no resemblance to anything known, except a heap of rubbish, there being in the arrangement of its ingredients no regard to specific gravity or size.

Sir JAMES HALL ascribed the deposit to diluvial agency, and attempted to show how the transport of the boulders and pebbles in it, their rounded forms, and the abrasion of rocks covered by it, might all be accounted for, by supposing that great waves of the ocean had swept over the country from west to east, scattering debris in all quarters.\*

This *diluvial* theory was generally accepted, and relied on as satisfactory, until about the year 1840, when the “*glacier*” theory was started, suggested probably by the discovery that many of the shells found, if not in the boulder-clay itself, at all events in other pleistocene beds, alternating with it, bespoke an Arctic climate.

A strong impulse was given to this new theory by the publication of a magnificent work, on the Swiss Glaciers, by AGASSIZ, and by an account of a visit which was shortly afterwards made by that naturalist to Scotland, in company with the late Dr BUCKLAND. Both of these eminent men affirmed that they had seen unmistakable signs of glaciers in almost every valley they visited. Shortly afterwards, the late Principal FORBES, who had, by frequent visits to the Swiss glaciers, made himself well acquainted with their action, went to Skye, and discovered marks of ice on many of its rocks. He read a paper in this Society, describing these marks; and as the learned Principal was distin-

\* Sir JAMES HALL's theory is explained by him in the following paragraph:—“I imagine that a diluvial wave flowed at some remote period from a westerly or north-west direction, and broke over our island; that its magnitude was such, that a great body of its water crossing the ridge of country which separates the two coasts, overwhelmed the district, discharging itself into the German Ocean.”—(*Ed. R. S. Tr.* vol. vii. p. 202.)

guished for accurate observation and cautious deduction, his discovery in Skye added largely to the popularity of the glacier theory. The next quarter from which light came was Wales, where Professor RAMSAY recognised signs of ice action. He was followed by Dr CHAMBERS, the late Mr MACLAREN, and Mr JAMESON of Ellon, who severally pointed out localities in many of the Scotch counties.

But whilst generally adopting and helping to illustrate this theory, almost all of these geologists admitted that there were some phenomena of the boulder-clay which could not be explained by any imaginable local glacier; and they threw out the idea that icebergs or icefloes, which it was discovered carried in the Arctic and Antarctic regions enormous masses of rock and rubbish, might possibly have in former times done similar work in North-Western Europe.

By this time AGASSIZ himself appears to have become satisfied that many of the ascertained facts could not be explained on the theory of glaciers flowing down from isolated mountain ranges. Having gone to reside in America, he obtained there an opportunity of studying the phenomena on a much larger scale than either Switzerland or the whole of Northern Europe supplied, and was greatly struck by seeing that boulders were scattered over an area of the earth's surface, extending to nearly 1000 miles in every direction, and that these boulders generally had all been transported from one quarter, viz., the north. Having learnt, from the writings of MURCHISON and others, that the great mass of boulders in Russia and Poland had also come from the north, and that in some cases the parent rocks were more than 100 miles distant, he formally renounced the theory of local glaciers, and propounded the notion that gigantic glaciers, more than a mile in thickness, and derived from snow two or three miles deep, had been generated in the Arctic regions, and were by some cause made to move over the earth's surface towards the south, encasing great continents, filling sea-beds, rising up slopes of land, overtopping mountains, and pushing before them, with a colossal ice-foot, immense heaps of detritus. From his recent work on the Brazils, it appears that this enthusiastic naturalist contends that the huge glacier which passed over North America, reached even to the tropics.\*

I do not know or believe that this theory of AGASSIZ, in its full extent, has been adopted by any geologist in either America or Europe; but I am not sure that it is not, to a modified extent, adopted by some of our Scotch geologists.

Mr GEIKIE, in a very valuable paper on the "Glacial Drift of Scotland," says, "that the ice existed, not as mere local glaciers descending the chief valleys, but as one wide sheet covering the whole, or nearly the whole, country" (p. 78).

\* "Visit to the Brazils," p. 403.—AGASSIZ in this work contends for the existence of "a sheet of snow 10,000 or 15,000 feet in thickness, extending all over the northern and southern portions of the globe,—which in the end formed a northern and southern cap of ice moving towards the equator!"

“Down the whole of the west coast, from Cape Wrath to the Mull of Cantyre, one long expanse of ice filled up the fiords, and stretched out into the Atlantic. From the uplands of Wigtown and Galloway, the icy stream swept down into the valley of the Solway, and onward for Ireland. From the hills that border the lonely valley of Liddesdale, far away into the blue Cheviots, the same universal mantle of ice threw its folds athwart the hills and dales of the north of England.” (“Glacial Drift.” P. 84.)

The following passages in a later publication by Mr GEIKIE (“Scenery and Geology of Scotland,” 1865) may also be referred to:—

“The massive ice of the great Highland area came down into Strathmore, and kept steadily southward in such force as to mount over the chain of the Sidlaws, and even it would seem over the Ochils, until it went out to sea by the basin of the Forth.” (P. 300).

Referring to Scotch boulder-clay or till, Mr GEIKIE says, that “land ice has now given us the clue to the history of this remarkable superficial deposit, as will be afterwards pointed out; its internal structure, and its striated stones, show it to be the result of the abrasion carried on by the ice-sheet, as it moved over the land.” (P. 183).

“The high grounds of the interior receive a constant accession of snow; and the accumulated mass, pressing down the valleys, goes out to sea in long wide walls of ice.” “The moraine-rubbish of this great ice-sheet gathers into the thick deposit known as *boulder-clay*.” (P. 345).

The Rev. R. B. WATSON, in a paper on the “Drift-beds of Arran,” read in this Society in January 1864, says, that the phenomena indicated the existence not of glaciers merely, but of a massive *ice-cake*, “more universal than even in Southern Greenland now. Beneath this ice-cake the soil, and all of life it supported, would be gradually harried away to the sea; any traces of it left being nests of debris niched into corners, ground over and disturbed in every conceivable way by the ice above.” (P. 537 “Roy. Soc. Trans.” vol. xxiii.) “This being so, we are entitled to say that the boulder-clay is the result of land glaciation.” (P. 538).

Dr BRYCE of Glasgow, shortly after the publication of Mr WATSON’s paper, went to Arran to examine the sections described in it; and he concurs in holding that the circumstances proved “for the boulder-clay an origin on *land*.” (“Lond. Geol. Journal” for 1864, p. 211.)

The most recently published paper on boulder-clay, with which I am acquainted, is by the Rev. Dr THOMAS BROWN, who read in this Society an account of the “Arctic Shell-clay of Elie and Errol.” In this instructive paper he has a chapter on boulder-clay, which he says, both at Elie and Errol, lies beneath the Arctic shell-clay, and rests immediately on the rock. He states his opinion of its origin thus:—“It would seem that this lowest deposit, so long an enigma,

has at last yielded up its secret. It is a *land* deposit, formed at the period when *Scotland*, like Spitzbergen, lay beneath an *immense covering of ice*, which wrapped the whole face of the country, hill and dale. Underneath such a covering, possibly thousands of feet in thickness, the rocks would be ground down, and the boulder-clay formed. Thus the absence of fossils is accounted for; inasmuch as none of our usual forms of life could exist beneath such an ice-sheet; and thus we see also how the clay is so peculiarly hard and untractable." (P. 630).

I have briefly sketched the various theories relating to boulder-clay, to show the difficulty of the subject, and have referred more particularly to the views of the latest writers whose geological experience and knowledge are held in just repute.

It is, therefore, with considerable hesitation that I venture to call in question the soundness of these views, and I would not have done so, had it not been that some observations, bearing on the subject, do not appear to me to have received sufficient consideration. Most of the observations to which I allude are to be found scattered through different publications, and have never yet been brought together, so as to throw a combined light on the question;—I am able also to adduce some observations of my own, not yet published.

I shall advert, first, to the difficulties which beset the theory that our Scotch boulder-clay "is a *land* deposit," the product of *glaciers*; and will afterwards state the reasons which lead me to believe that it has been formed at the bottom of the sea—by the action of floating ice.

I disavow any originality in presenting the iceberg theory. Moreover, it has this presumption against it, that, having been formerly adopted by Mr GEIKIE, he has lately intimated that he has had to abandon it, because, as he says, "though the iceberg hypothesis is generally the accepted explanation of the phenomena of striated rocks and boulder-clay, its untenableness seems to me completely established." ("Glacial Drift," p. 10.)

Notwithstanding this very decided condemnation, I think there are good grounds for upholding the correctness of the iceberg hypothesis.

That there are some points not altogether explained by it, I will not deny; but that there are insuperable difficulties with which the glacier hypothesis has to contend, I shall now proceed to show—

1. If the boulder-clay was formed, as is alleged, by the action of glaciers; if it consists of debris derived from the rocks which the ice grinds down in its passage over them, and which are pushed forward by its ice-foot, we would see boulder-clay now forming in those countries where glaciers are in action. But it has never been alleged that in Switzerland, Norway, or Upper India, whose glaciers have been described by competent observers, anything like boulder-clay is seen to be produced. I have been twice in Switzerland; and, being anxious to watch the effects of glaciers on the rocks, made them a subject of study, and penetrated

under three glaciers near their lower extremities, without discovering anything like boulder-clay. Great abrasion of rocks there was undoubtedly. Blocks and pebbles under the ice I saw in abundance, all grinding, and many of them scoring the rocks. Much sediment there was, flowing out from under the ice. But what became of this sediment? It was carried off into rivers and lakes, there to form beds of mud or sand—none having any resemblance to boulder-clay. The terminal moraines of glaciers, no doubt, resembled it in one feature—want of stratification; but the absence from these moraines, of clay, hard, tough, and compact, showed that the deposits were essentially different.

It is no small confirmation of my own testimony on this point, that AGASSIZ, when he visited Scotland to search for the signs of ancient glaciers, avowed that he had never seen boulder-clay before he saw it in Scotland.\*

2. The next difficulty with which the glacier theory has to contend is, the prevalence of boulder-clay in districts where it is scarcely possible to suppose that glaciers could have existed, or, if they did exist, could have had to do with the production of the deposit.

Thus, in the flat districts of Norfolk, and in the still flatter districts of Denmark and North Holland, boulder-clay is found. But there are no mountains in or near these districts, where any glaciers could have been formed. The same remark has been made by Mr CUMMING of the boulder-clay in the Isle of Man †

Even in those parts where there are both boulder-clay and mountains, as in the Highlands of Scotland, it appears that the boulder-clay is derived from a quarter the very opposite from that where a glacier may have existed. Along the coasts of Western Ross-shire and Caithness, this deposit abounds, and has been studied by Mr JAMESON of Ellon and Dr ROBERT CHAMBERS—both of whom at first advocated the theory, that its formation could be accounted for by glacier-action. Mr JAMESON says—“The distribution of this dark grey mud harmonises with the supposition that the transport of it has been from the N.W.; and a movement of ice, from the N.W. to the S.E. across Caithness, is totally at variance with the notion of the scratches having been caused by glacier-action proceeding from the interior of the country towards the present coast.” ‡ And he adds in a footnote, that the phenomena “indicate a movement of ice from the N.W., where there is now nothing but open sea for an immense distance,” and “all suggestive of *marine* conditions.”

3. The next difficulty to the glacier theory is suggested by the immense extent of earth's surface over which the transporting agent has moved in one and the same direction.

\* See Edin. Phil. Journal for 1842, p. 227; and Geological Researches, by JAMES SMITH of Jordanhall, p. 12.

† “Isle of Man,” by CUMMING, p. 248.

‡ Proceedings of the Geological Society of London for 1866, p. 269.

If the boulder-clay was produced by glaciers, its transport would be coincident with the direction in which the glaciers moved—that is, in directions parallel with the valleys from which they emerged. We should expect, therefore, to find that the boulder-clay, and the blocks embedded in it, indicated a movement and transport from every conceivable point. The boulder-clay and boulders found on the west coast of Scotland should indicate a movement from the eastward; on the north coast, from the southward; and on the east coast, from the westward.

But is this the true state of the case, as shown by most recent observation? The phenomena of the boulder-clay show in all parts, not of Scotland only, but of Ireland, England, and even of the adjoining districts of North-Western Europe, a general movement from the north-westward. That exceptions to the rule exist I admit, and an explanation of these I shall afterwards offer; but I affirm that there is a general and prevailing direction over the wide area just mentioned, and that direction is from W.N.W. or N.N.W.

Before, however, giving proofs of this position, let us see what are the signs of transport on which geologists are agreed.

(1.) Mr GEIKIE has pointed out a relation between the *colour* of the boulder-clay and the rocks of the districts adjoining the deposit—as indicating transport. Thus he says—“The main mass of the boulder-clay, in the basin of the Forth for instance, consists of the comminuted debris of the carboniferous and other rocks which form the framework of that district. We can also gather that this loose fragmentary material has moved (there?) from west to east. In the upper part of the basin of the Firth of Forth, the coal-fields are covered with *red* boulder-clay, abounding in fragments of the rocks that lie towards the N.W., and deriving its prevalent tint from the waste of the Old Red Sandstone which stretches up to the foot of the Highland mountains.”\*

The late HUGH MILLER had previously pointed out how the pale oolitic rocks of Brora and Golspie are covered by a *yellow* boulder-clay, and the flagstones of Caithness are covered by a boulder-clay of a *grey* leaden colour. So also Mr CUMMING showed how, in the Black Isle, the boulder-clay has the colour of the *red* rocks there; whilst to the westward, the colour changes to a colour in correspondence with that of the slaty rocks. The same author points out how, in the Isle of Man, the colour of the boulder-clay is *blue* near the limestone rocks, and *red* near the Old Red Sandstone rocks; and how in each case these rocks are in the same direction from the boulder-clay, as if a current had swept over the rocks to provide materials for the clay.†

Mr NICOL has pointed out the same relationship in Cantyre.‡

\* Glacial Drift, p. 43.

† Lond. Geolog. Journal for 1850, vol. vi.—Isle of Man, pp. 115, 247.

‡ Lond. Geolog. Journal for 1852, vol. viii. p. 417.

Arguing from this test, the late Dr FLEMING showed very clearly that in the neighbourhood of Edinburgh, the boulder-clay "had been in motion from west to east."\*

These views are important, as proving also that geologists agree in holding that whatever may be the case with regard to the erratic blocks in the boulder-clay, the boulder-clay itself has been derived chiefly from the rocks over or near which it lies, and (to use Mr GEIKIE'S expression) "consists of the comminuted debris of" these rocks.

(2.) Another well-established indication of the direction in which the boulder-clay has moved, is afforded by the *striations* and *groovings* of the *rocks* covered by the deposit. Mr JAMESON, after stating various facts bearing on this point, says, "all this shows that the boulder earth, with its embedded fragments, was pushed along by the same agent that scored the rocky bed on which it lies."† So also Mr GEIKIE, after mentioning other examples, says—"Here it will be seen that the direction of transport of the boulder-clay exactly coincides with the trend of the groovings and striations on the rocks below,"‡

(3.) Farther evidence bearing on the same point is afforded by the *nature* of the boulders or erratics embedded in the boulder-clay; for when an examination of the rocks composing them has led to a discovery of the locality from which they have apparently been transported, the direction in most cases coincides with that of the striations on the rocks, and with the direction of the movement of the boulder-clay as indicated by its colour. Accordingly, AGASSIZ does not hesitate to admit, that the striæ on the rocks are due to the same cause which transported the blocks.§

This remark applies not merely to blocks in the boulder-clay, but to erratic or transported blocks in other positions, whether on rocky knolls or on beds of gravel. Whenever the quarter from which they have come has been clearly ascertained, it is found that the direction of their transport agrees with that of striæ on rocks in the neighbourhood.

There is another circumstance, not undeserving of consideration, long familiar to geologists, viz., that when erratics are of such a shape that their length greatly exceeds their width, their longer axis generally lies in the direction of their transport.

I believe, therefore, in common with other geologists, that the movement of the boulder-clay, whenever that has been ascertained, the transport of boulders, and the striations, groovings, and smoothings of the rocks, are due to one and the same agent; and hence the phenomena to which reference has just been made,

\* Lithology of Edinburgh.

† Lond. Geolog. Journal for 1866, p. 167.

‡ Glacial Drift, p. 45.

§ Edin. Phil. Journ. for 1842, vol. xxxiii. p. 223.

can be competently relied on to ascertain the quarter from which that agent has come.

What, then, is the evidence on this point afforded by these phenomena? Let me enumerate the localities where the direction has been clearly ascertained.

*Caithness.*—Mr JAMESON, in his instructive paper on this subject, enumerates about twenty localities in this county, where “the glacial markings on the rocks showed a pretty uniform direction over the whole district from the N.W.”

Mr JAMESON also examined the directions in which “the dark grey mud”—by which he designates the boulder-clay—derived, as he considered, “from the Caithness flags, had moved,” and he found that it also had moved from the N.W. He found that the Caithness flags—situated in the N.W. of the county—were themselves covered by “drift of a reddish-brown colour,” derived probably from the north-westward.\*

*Ross-shire and Argyleshire.*—Dr CHAMBERS mentions that “near Rhiconish we find striæ coming from the coast—*i.e.*, from the N.W., and passing across a high moor, with no regard whatever to the inequalities of the ground. A little further north, at Loch Laxford, a fine surface is marked with striation from the N.W., being *across the valley* in which it occurs. At an opening in the bold gneissic coast, *which looks out upon the Pentland Firth*, there are strong markings in a direction from N.N.W.”†

In the small Isle of Kerrara, opposite to Oban, and also in the Island of Mull, Dr CHAMBERS found striation, pointing in the one case N. 68° W., and in the other N. 60° W.‡

*Perthshire.*—The lofty mountain of Schehallion has been examined by both Dr CHAMBERS and Mr JAMESON. Dr CHAMBERS§ found striæ on it at a height of above 3000 feet, pointing W. 30° N.; and Mr JAMESON satisfied himself that the striæ he saw on the same mountain must have been made “not by ice flowing down the sides of the hill, but by ice pressing over it from the north.”|| He adds—“On the Perthshire hills, between Blair-Atholl and Dunkeld, I found ice-worn surfaces of rock at elevations of 2200 feet, as if caused by ice passing over them from the N.W., and transplanted boulders at even greater heights.”

*Forfarshire.*—Sir CHARLES LYELL, in the year 1842, pointed out how the till and its embedded boulders had been transported from the N.W.

Dr HOWDEN, of Montrose, has lately published a paper in the “Transactions of the Edinburgh Geological Society,” in which he observes “that the general lie of the range of hills is W.S.W. to E.N.E., while the direction of the glacial groov-

\* Lond. Geolog. Journ. for 1866, p. 268.

† Edin. New Phil. Journ. for 1852, vol. liv.

‡ *Ibid.*

§ Proceed. Geolog. Soc. of London for January 1865.

|| *Ibid.*

ings is from W. by N. to E. by S., so that the polishing agency must have crossed the hills at an angle of 30°."

*Dumbarton and Renfrew shires.*—Dr CHAMBERS mentions that on the sandstone plateau between Campsie and Stirling, the striations on the rocks were W. 20° N.

Having myself visited the moors, about three miles N.W. of Milngavie, in the parish of Baldernock, I found the white sandstone rocks ground down and flattened in large patches, with striæ and rents on them, indicating a movement from W.N.W. and N.W. (magnetic.)

The late Mr SMITH of Jordanhall, in his Geological Researches, after enumerating several localities where boulders of various kinds of rock had been examined by him in Clydesdale, observes—"In these cases, the bearing of the supposed parent rocks is N.W.; but in all of them the intervening space is intersected by deep arms of the sea and steep mountain ranges (p. 13). One of the boulders was found in the boulder-clay near Airdrie, the nearest granite rock being at Cruachan, about 60 miles N.W. of Airdrie." Mr SMITH adds—"I never yet saw or heard of an erratic block in the valley of the Clyde, whose course could be traced, that did not come in an opposite direction to the flow of the river. We can trace their course, not from the mountains to the sea, but from the sea to the mountains" (p. 131).

*Edinburghshire.*—The general direction of the movement in this district has been very accurately ascertained by Sir JAMES HALL, Mr MACLAREN, Dr FLEMING, ROBERT CHAMBERS, Mr NICOL, HUGH MILLER, Mr GEIKIE, and myself. All concur in representing that the movement has been from points varying between W. by S. and N.W., the most prevalent being from W. by N. (magnetic). The evidence of this is well stated by Mr GEIKIE in the following passage in his Memoir of the Geological Survey, No. 32:—"The parallelism of the striations throughout the district show that the floating ice must have moved in a pretty uniform direction; and that it was from the west, is clear by the striation of the western face of the hills, the great depth of the drift on their eastern sides, and by the fact that the transported boulders, when traceable to their parent rock, have been carried from west to east." Mr GEIKIE then specifies several of these boulders on the Pentland Hills, and one in particular of mica slate, first noticed by Mr MACLAREN, weighing eight or ten tons, and at a height of 1060 feet above the sea, which he says had "undoubtedly been transported from Cantyre or the Grampians." These boulders Mr GEIKIE at that time considered to be "ice-borne blocks dropped on the submarine slopes of the Pentlands." Whether he *now* thinks that they were brought by the agency of a glacier, I do not know.

*Stirlingshire and Lanarkshire.*—On the west side of *Damyat* (one of the Ochils) I found, at a height of from 500 to 600 feet, many patches of hard conglomerate rock, ground down and striated by an agent which had come from the

N.W. and N.N.W. At these places the general slope of the county is down towards the N.W. Therefore the agent which ground down these hard conglomerate rocks must have moved up hill.

On the trap hill of Croy, near *Kilsyth*, I lately found a conglomerate boulder, which probably came from the hills of that rock, situated between Dumbarton and Callander; and on the felspar hill at Stonebyres, I found pebbles of coal sandstone and clay ironstone, which apparently had come from the west.

Sir JAMES HALL describes a sandstone rock in Torwood, near Stirling, smoothed and striated—the direction of the striations being N. 50° W. (Ed. R. S. Tr. vol. vii. p. 200.)

*Berwickshire*.—In the parish of Eyemouth there is a brickwork of boulder-clay, in which lumps of water-worn coal and ironstone are occasionally found. In sinking a well lately on the farm of Blackhill (Coldingham parish) through boulder-clay, lumps of water-worn coal were found. The nearest place from which these erratics could have been transported is East Lothian, situated to the N.W.—the Lammermuir range of hills intervening.

In the last-mentioned parish, lumps of hematite have been picked up on the surface of the ground, resembling extremely the hematite worked on the Garlton Hills, in East Lothian, situated about 30 miles to the W.N.W.

In the parish of Dunse,\* there is a rounded boulder of mica slate, about one ton in weight, which must have come from the Highlands of Scotland.

In the parish of Hutton, there is a brickwork situated on a mass of boulder-clay, containing occasionally rounded pebbles and boulders. One of the boulders is a mass of blue greenstone, weighing about eleven tons, and angular in shape. The nearest parent rock is on Borthwick Hill, near Dunse, situated about ten miles N.W. from the boulder; and its longer axis points in that direction. In the same brickwork there are smaller boulders of greywacke, old conglomerate, and chert, all of which have most probably come from the Lammermuir Hills, situated to the north and west. An angular block of coal sandstone has also been excavated which adjoined the large greenstone boulder. No such sandstone is known in Berwickshire. I know of no place nearer than Mid-Lothian where this peculiar sandstone occurs in strata. It is of a yellow colour, and rather soft in texture. As the block is of an angular shape, its transportation could have been effected only by drift ice.

In *Liddesdale*, I found boulders of granite, which probably came from the granite hills of Dumfries and Ayrshire;† and within these few weeks I have found in *Northumberland*, north of Hexham, several granite boulders, probably from the same quarter.

\* This boulder was pointed out to me by Mr STEVENSON of Dunse.

† Geology of Roxburghshire, Roy. Soc. Trans. vol. xv. p. 402.

*Arran*.—Dr BRYCE\* has pointed out that the rocks on the N.W. sides of many of the hills have been denuded and smoothed, whilst on all other sides they are rough.

*Kirkcudbright*.—Mr HAY CUNNINGHAM states that “large rounded fragments of granites and syenites are abundantly scattered over the county, and so arranged as to indicate that they have been dispersed by a force proceeding from the N.W.”†

In *Northumberland*, the following table compiled from the reports of Mr TAIT of Alnwick, Secretary to the Berwickshire Naturalists' Club, ‡ shows the quarter from which the transporting agent moved, judging by the striations and groovings:—

Locality.	Nature of Surface.	True Bearings.
Ratcheugh.	Limestone Rock below Boulder-clay.	N.
Do.	Blocks in Boulder-clay.	N.W.
Belsay.	Limestone Rock.	N.N.W.
Sea-shore, Dun- stanborough. }	Limestone Rock below Boulder-clay.	W.N.W.
Swinhoe.	... .. do.	N. 50° E.
Belford.	... .. do.	N.W. by W.
Sea-shore, Birling.	Sandstone Rock under	do.
St Abb's Head.	Porphyry under	do.
Farne Islands.	Basalt under	do.
Alnwick.	Limestone under	do.

The plains of *Yorkshire* are strewed over with blocks transported from Cumberland, one of which is the well-known boulder of shap-granite, now standing in one of the streets of the town of Darlington.

Near *Liverpool*, the direction of the striations on the rocks is between N. 15° W. and N. 42° W.

In *Cheshire*, the direction is N. 30° W.§

In *Wales* and *Somersetshire*, chalk flints occur in the drift, which must have come from the county of Antrim, Ireland—*i.e.*, from N.W.

In *Norfolk* there are two boulder-clays, separated by a bed of sand containing sea-shells. The upper boulder-clay, as the late Mr TRIMMER showed,|| contains fragments of oolite rocks, which must have come from the westward, passing over a ridge of chalk rocks, which, however, do not indicate any abrasion. Mr TRIMMER, taking into view the levels of the country, held it impossible to ascribe

\* Geology of Clydesdale, p. 271.

† Highland Society's Transactions for 1843.

‡ Transactions of Berwickshire Naturalists' Club, vol. v. pp. 233, 372.

§ Lond. Geol. Journ. for 1862, vol. xviii. p. 377.

|| Lond. Geol. Journ. for 1858, vol. xiv.

the transport of the oolitic blocks to glaciers. Floating ice alone, in his opinion, afforded a solution.

In the *Isle of Man*, Mr CUMMING found chalk flints in the boulder-clay, which he thought must have come from the county of Antrim, in Ireland, situated to the N.W. ; and in the drift gravels of the island there were pebbles, which he could only refer to rocks also situated to the N.W.\*

In *Ireland*, the general direction of the transported boulders is the same as in Scotland and England. Sir RICHARD GRIFFITH † says—" If we look to the distribution of erratic blocks, as indicative of the direction of the currents by which they were distributed, we find in Ireland generally that they were carried from N.W. to S.E., though the current was often modified by the opposition of mountain ridges."

" The prevailing direction of our mountain ridges is N.E. and S.W., viz., at right angles to the supposed direction of the current ; and, as might be expected, we find the gravel banks and detritus distributed on the N.W. declivities of the hills, and intruding into the interior valleys."

A later observer, DU NOYER, has identified the boulders lying on the mountains near Cork with the granite rocks of Galway, situated on the N.W. coast of Ireland, and has shown that the striæ on the smoothed rocks have the same direction. ‡

In the *Shetland Islands*, an examination was instituted by Mr PEACH, at the request of Sir RODERICK MURCHISON, into the drift phenomena. Mr PEACH found on the hard primitive rock of the islands, clear evidence of grinding and polishing. The general inference which he drew was, that the agent, whatever it was, must have passed over the islands from the northward. The only exact bearings stated in his report were taken in the island of Unst, the most northern of the group, containing about 36 square miles, and having one hill on it about 500 feet high. The ruts in the rocks there all pointed W.N.W. ; and the side of the hill facing that quarter was (he says) polished to a depth from its top of about 150 feet. §

In the *Faroe Islands*, on the N.W. coast, the late Mr ALLAN, || when he visited them with Sir GEORGE MACKENZIE in the year 1812, was struck with a rocky hill, the surface of which appeared " to have been worn down by the friction of heavy bodies " over it. " The rock was scooped and scratched in a very wonderful degree, not only on the horizontal surface, but also on a vertical one of 30 to 40 feet, which had been opposed to the current, and presented the same scooped and polished appearance with the rest of the rock." Mr ALLAN says, " it would be

\* Lond. Geolog. Soc. August 1846, pp. 336 and 342.

† British Assoc. Rep. for 1863, vol. xiii. p. 51.

‡ Geologist for 1862, p. 246.

§ British Assoc. Rep. for 1864.

|| Edin. Roy. Soc. Trans. for 1815, vol. vii. pp. 244-265.

curious to investigate, whether this smoothness could be traced to any external cause such as that observed by Sir JAMES HALL, on Corstorphine Hill." At an early period geologists were unacquainted with the abrading effects of ice. From Mr ALLAN'S description of the markings on the hill at Eide, and from its situation on the N.W. coast of Faroe, it is not difficult to see the agency of icebergs.

In *Iceland* the striæ, as Dr CHAMBERS states in his instructive little book,\* run N. 30° W. (true). Having endeavoured, through my friend, Mr R. M. SMITH of Leith, who has correspondents in the island, to obtain farther information regarding the markings on the rocks, I have had sent to me by Mr SMITH the following extract from a letter by Dr HJALTELIN, of Reykavik, Knight of the Dannebrog, and principal physician in the island:—"The diluvial scratches are to be seen everywhere in the south part of our country. They run in lines parallel to one another, and can scarcely be occasioned by the action of rain or water. Their direction is very much against this view. You see them on the slope of the hills, not following the declivity of the rocks, but everywhere running in the well-known direction from N.W. to S.E., in spite of the declivity. Many of these scratches are very unequal, and seem evidently to have been produced by a hard material gliding over the rocks in the aforesaid direction. It must furthermore be remarked, that these furrows, which are unequal both in depth and diameter, could hardly have such a regular parallel direction, if occasioned by water or rain.

"These scratches are all round here in our mountains. In the lower flat lava-fields, some deep and broad irregular scratches may also be seen; but they have no constant direction, and seem to have been produced by pieces 'of hard material gliding over the lava when still in a soft condition.'" †

In *Sweden*, the markings on the rocks show a movement generally from the N.N.W., which is also the direction of the ösars or elongated gravel ridges, so abundant in that country.

In *Finland*, on the Gulf of Bothnia, and on the Lake of Ladoga, in Russia, the direction is N.W.

In *Denmark* (as FORSCHAMMER shows), the markings on the rocks show a movement from W. 25° N. (true). The following case mentioned by him at Gothenburg (situated near the southern extremity of Sweden) leaves little doubt regarding the nature of the agent which made the markings. There was a large furrow or rut on a rock, the prolongation of which rut had been arrested or prevented by a boulder lying on the rock and firmly jammed. The boulder was about 3 feet thick vertically; and on its upper surface there was a rut, which being exactly in a line with the rut on the rock, seemed to have been made by the same agent,

\* Voyage to Iceland and the Faroe Isles.

† Letter from Dr HJALTELIN, Knight of the Dannebrog, and Chief Physician in Iceland.

which agent must, therefore, have risen up from the rock about 3 feet, in order to score the boulder.\*

If the foregoing enumeration of localities correctly represents the general direction of the markings on the rocks, it is impossible to avoid the conclusion that they have been made by one and the same agent, and over a very large area of the earth's surface. How can it be conceived that glaciers should over that large area have all moved in the same direction? Objections to the glacier theory are suggested even by cases of isolated boulders.† But the objections become infinitely stronger, when it appears that over an area of North-Western Europe, comprising Iceland, Faroe, Shetland, Scotland, Ireland, a great part of England, and on some of the adjoining continental countries, the agent which affected the boulder-clay, transporting blocks and striating rocks, moved almost everywhere in nearly the same direction, and came chiefly from that quarter where there is only the ocean.

The late Principal FORBES, much as he was inclined to uphold the agency of glaciers, felt the force of these objections, and makes the following confession in his work on the Glaciers of Norway (p. 241):—"I hesitate to ascribe everything to glaciers. In fact, there appears to me to be situations along the coast of Norway, where the action of abrasion having been parallel with the coast, the movement of a glacier would be inconceivable. The general parallelism of the striæ, observed by BÖTHLINK and others, over a large area of country, not coincident with the general fall of the ground, would seem, if confirmed, to be equally inexplicable on the pure glacier hypothesis. The continuation of the striæ across table-lands, and over *cols*, is of the like ambiguous character. I have never hesitated to express, on similar grounds, doubts as to the universal application of the usual glacier theory to the phenomena of our own islands, which, on a small scale, are the counterpart of those of Norway. For, though perfectly satisfied that our hills were in former times the seat of glaciers which even approached the sea-level, I find the utmost difficulty in explaining, by such an hypothesis alone, the facts which occur even in the immediate vicinity of Edinburgh."

In another part of the same work, Principal FORBES threw out a surmise of the kind of agent which seemed to him probable. Referring to the range of hills on the west coast of Norway (p. 190), he says that these bore "the whole brunt of *forces which appear to have come from the north*, and not only

\* Lond. Geolog. Journal for 1845, vol. i. p. 376.

† Thus Mr MACLAREN says—"I have pointed out a boulder of mica slate in the Pentland Hills, weighing 8 or 10 tons, which must have come 50 miles at least. It lies on a steep acclivity 1000 feet above the sea; and it must have passed over extensive tracts of country from 500 to 800 feet lower than the spot on which it rests. Even were all Scotland converted into a *mer de glace*, like Greenland, no glacier could carry the boulder (and there are many such) from its parent rock, in Perthshire or Argyleshire, to the Pentlands."—*Select Writings*, vol. ii. p. 115.

defended the entire north of Europe from the shock, but probably furnished by their abrasion the materials, of which the low grounds of the Continent of Europe are mainly composed. In this general disposition of the mountains of Norway, we see a strong analogy to the west coasts of our own islands. It appears almost certain, that *a common cause has devastated the western shores of nearly every continent.*"

It is very evident what this "*common cause*" alluded to by Principal FORBES, as having left its mark on the western shores of North-Western Europe, must have been nothing less than the ocean itself. Sir JAMES HALL was, on the limited body of facts known in his day, led to the opinion that a resistless rush of waters over the country from the westward would explain the phenomena; and, down to a very recent period, attempts have been made to show how boulders could be carried by what are called waves of translation. These views have now been generally abandoned; and in place of them, it has been suggested that oceanic currents, with floating ice, and flowing over the submerged land, would afford a better explanation—an explanation strongly supported by the great extent of area over which the transporting agent has moved.

But in stating that the agent has moved over this large extent of area in the same direction, let me repeat that there are cases where the direction of the striæ and the transport of boulders point to a different quarter than the north-west. The percentage of these cases is so small, as not to affect the argument based on the generality of that direction, and on the extent of area over which it prevails.

Moreover, whilst some of these exceptional markings are undoubtedly indicative of local glaciers,—the probable epoch of which will be afterwards referred to,—others are not inconsistent with, but, on the contrary, are corroborative of the theory of oceanic currents. Thus, in the great glen of Scotland, the lines of striæ and the course of the transported boulders show a movement from S.W.\* So also in the estuary of the Forth, as well as in the trough which crosses Scotland along the south slopes of the Kilsyth and Campsie hills, the striæ and many of the boulders indicate a movement from W. by S. and W. by N. Now, in each of these cases, the deviation from the general or normal N.W. direction coincides with the range of the valley where it occurs; and it is not unreasonable to suppose that the obstruction, caused by adjoining hills of considerable height and extent, would modify the direction of the current so that it should flow with some approach to parallelism with them.

Sir JAMES HALL, in his paper, brings out this point very clearly, when he says "that the direction in the neighbourhood of Edinburgh may have been occasioned by the local influence of the estuary, since the direction of the stream

\* Lond. Geolog. Society's Proceedings for April 1849, p. 13.

before entering it, and after quitting it, is nearly from N.W. to S.E." Whilst he shows from numerous examples, that the direction in the Lothians was from W.  $\frac{1}{2}$  S., he shows that near Stirling it was from N.  $50^{\circ}$  W., and near St Abb's Head from N.  $35^{\circ}$  W.\* (true bearings).

4. The next point bearing on this question which I wish to put, is the character of the *fauna* found in the boulder-clay; which, being *marine*, afford strong evidence not only adverse to the theory that it is a land deposit, but favourable to the theory that it is a sea deposit.

The following enumeration of localities where boulder-clay has been found containing sea-shells is not complete; but it is sufficiently extensive to establish the fact:—

Near *Airdrie* (Lanarkshire), "in the till itself," the late Mr SMITH "found broken and water-worn fragments of shells irregularly dispersed in it, and amongst them the *Cyprina islandica* and a large species of *Balanus*." †

In *Wigtownshire*, "in the genuine till or brown sandy unstratified clay, with blocks of transported rocks interspersed through it," Mr MOORE found "one perfect valve of *Astarte compressa*." ‡

In *Aberdeenshire*, various species of sea-shells have been found in the boulder-clay by both Dr CHAMBERS and Mr JAMESON.§

In *Caithness*, at several places sea-shells and other marine *testacea* have been found in this deposit by Mr JAMESON and Mr PEACH. ||

This point was seen to be of so much importance that a special examination of the Caithness boulder-clay was undertaken by Mr PEACH and two other gentlemen. They not only discovered in it many species of sea-shells, but by washing it, and examining with the microscope, they discovered no less than ten or twelve genera of *Foraminifera*, *Entomostraca*, and other minute marine organisms. In a paper read by Mr PEACH before the British Association in 1864, and published in their Transactions, it is stated that he and Mr ANDERSON had "washed boulder-clays from many localities extending from near John O'Groat's to beyond Wick, and all the samples tried yielded more or fewer of these animals, from whatever part of the deposit the clay was taken." He adds, that "he had not previously found two valves of a shell united in the clay. He had, however, since got an *Anomia* with both valves in place. It occurred in boulder-clay containing the usual rubbed stones and broken shells. Mr ANDERSON has also a piece of shell on which is a cluster of young *Balani*." Mr PEACH gives a list of no less than "eighty-three species of shells, &c., from the boulder-clay of Caithness." ¶

In the same volume of the British Association Reports there is a list of thirteen

\* Ed. R. S. Tr. vol. vii. p. 200.

† Researches, p. 141.

‡ SMITH'S Researches, p. 143.

§ Proceedings of the Lond. Geolog. Society for 1866, pp. 274-5.

|| *Ibid.* p. 267.

¶ Brit. Assoc. Reports for 1864, p. 62.

species of marine shells from boulder-clay at *Scarborough* and *Whitby*, on the Yorkshire coast. These shells were discovered by Mr LACKENBY and Mr JEFFREYS, both recognised authorities.\*

The Rev. Mr CROSSKEY, Vice-President of the Glasgow Geological Society, well known for his knowledge of drift deposits, states that he found boulder-clay near *Sunderland*, "containing fragments of broken shells and many *Entomostraca* and *Foraminifera*." † That gentleman adds that he had found shells in boulder-clay on the banks of the *Mersey* and on the coasts of *Ireland*.

In boulder-clay, near *Tynemouth* (Northumberland), fragments of *Cyprina islandica* have been found by two accurate observers, Mr HOWSE and Mr BINNIE of Manchester. ‡

Dr THOMAS BROWN, in the paper recently published in the Transactions of this Society, § has given a list of above twenty species of sea-shells found at Errol (*Perthshire*), and at Elie (*Fifeshire*). Dr BROWN mentions (p. 630), that at both places the shells were in a bed resting on the boulder-clay. I had an opportunity lately, in company with Dr BROWN, of examining the deposit at Elie containing these shells, and found that it consisted of a hard or tough clay of a dark grey colour, presenting no stratification, and containing abundance of hard pebbles and boulders, all rounded and some of them scratched. It had the usual appearance of boulder-clay, and I expressed this opinion to Dr BROWN.

The Elie deposit I have not seen; but from the account given of it in Dr BROWN'S paper, and also by Mr JAMESON, I cannot doubt that it also is a true boulder-clay. In the section which Dr BROWN gives in his paper, he represents boulders in the deposit; and he expressly says that the shells "are found clustering around and beneath the enclosed boulders, a fact which seems to show that at the time these shells lived, this part of the sea-bottom must have been swept by a strong current." Dr BROWN adds, that he had obtained from the Errol deposit "portions of the skeleton of a seal." Mr JAMESON says, that in the deposit at Errol containing Arctic shells—being the same bed mentioned in Dr BROWN'S paper—he found many of the included boulders "glacially scratched—occasionally one may be found with *barnacles* on it;" and he adds, that "*Entomostraca* of the genus *Cythere* also occur." ||

In *Canada*, where till or boulder-clay abounds, marine shells have been found in the deposit. (*Amer. Journ. of Science* for 1866, vol. lxxxvii. p. 235.)

The fact of sea-shells, of various species, having thus been found in great numbers, and at places far distant from each other, in the boulder-clay, seems so conclusive as to the marine origin of the deposit, as to render further evi-

\* *Brit. Assoc. Reports* for 1864, p. 58; *R. S. E. Tr.* vol. xxiv. p. 617.

† *Trans. Glasg. Geol. Soc.* vol. iii. p. 150.

‡ *Berwickshire Nat. Club*, vol. v. p. 238.

§ *Vol.* xxiv.

|| *Proceed. of Lond. Geolog. Society* for January 1865, pp. 175 and 196.

dence superfluous. It therefore may simply be mentioned, that confirmatory evidence is afforded by numerous cases of boulder-clay alternating with strata, the marine character of which is indisputable. Localities are mentioned by Mr GEIKIE\* as occurring in Roxburghshire, Lanarkshire, and Ayrshire, where beds of unstratified boulder-clay, 30 to 40 feet thick, alternate with beds about the same thickness of stratified clay and stratified sand, the former sometimes containing marine shells. In such cases, the boulder-clay virtually forms part of the series.

5. Several observers, who have found marine shells in the boulder-clay, have been struck with their broken or fragmentary condition. This feature is not observable in the stratified or laminated clay beds where the same shells occur. In the brick clays of Lanark, Renfrew, and Ayr shires they are found perfect in form, and apparently in their natural position. But in boulder-clay, the same shells have been mutilated and smashed, so that it is difficult to identify the species.† Is it not a fair inference from this fact, that the beds in which, at the bottom of the sea, these shells had lived, must have been disturbed and deranged by some intrusive body of great weight and power, which both crushed the shells and obliterated all traces of stratification or lamination in the structure of the beds? It seems to me that such effects would result from the intrusion of ice-

\* *Glacial Drift*, pp. 54 to 65.

† Thus Dr WATSON, in describing the boulder-clay of *Arran*, says that the shells in it "are very much broken. The shells may often be found crushed, yet with each fragment in its own place. Some of the large specimens of *Cyprina*, though unbroken, are indented, as by a sudden violent blow. The whole condition of the shells suggests that heavy stones have been dashed down upon them." Dr BRYCE also notices that the Arctic shells found by him in *Arran* were "in single valves or in a fragmentary state, yet not so small but that the species can be determined."—*Geology of Arran*, p. 168.

The shells in the boulder-clay of *Caithness* have been examined by a great number of competent geologists, who all give the same testimony. Mr PEACH describes the shells so "broken" and "rubbed" he could find only one entire shell. Messrs CROSSKEY and ROBERTSON of Glasgow, having gone to *Caithness* on purpose to examine the boulder-clay there, describe it as "a hard and compact mass, with striated and polished boulders, being in appearance similar to that in the west of Scotland. The shells are thinly interspersed from top to bottom, and are of a water-worn and fragmentary character. They appear equally distributed, as if the whole mass had been mixed up and kneaded together."—*Geolog. Society of Glasgow Trans.* vol. iii. p. 126. Mr JAMESON of Ellon says that the drift-beds of *Caithness* contain "remains of sea-shells all through them, and these are broken, rubbed, and scratched, and evidently by the same agency that marked the rocks and boulders." His theory to account for the facts, is, that "much floating ice seems to have passed over the district from the N.W., which crushed and destroyed these marine beds, broke the shells, and mixed them up with other superficial debris into that mass of rough pebbly mud which now overspreads the surface."—*Proceed. of Lond. Geol. Society* for 1865, pp. 176-7.

Mr JAMESON has also the following statement regarding a deposit of boulder-clay near *Paisley* which he examined. He says—"I sometimes found, on heaving up a boulder, a number of young crushed mussel-shells beneath it, as if they had been squashed by the fall of the stone. The clay around also occasionally exhibited black stains, as if from the decay of sea-weed that had been attached to the stone."

At the various places where the Rev. Mr CROSSKEY found sea-shells in boulder-clay, along the coasts of Scotland, England, and Ireland, the shells were "very fragmentary, and even single valves are seldom found whole."—*Glasg. Geol. Soc. Trans.* vol. iii. p. 151.

bergs, the lower portion of which penetrated the sea-bottom, pushing before them boulders and pebbles, and pressing the sediment into greater compactness.

Besides the generally mutilated condition of the sea-shells in the boulder-clay, there is another circumstance, first pointed out by the late EDWARD FORBES, which tends in the same direction. On examining the shells found in the drift-beds of Wales, he observed that they belonged to different zones of life. These drift-beds presented "a confused mixture of fragments of species from all depths, both littoral and such as invariably live at a depth of many fathoms; inhabitants some of muddy grounds, some of sandy, some of rocky. Deep and shallow water species could not have lived together, or have been thrown up on one shore." His conclusion, therefore, was, that this confused and unnatural mixture "indicated the action of some disturbing influence, through the agency of icebergs, or a wave of translation, or of both combined."

The beds to which this observation applied was, it is true, not boulder-clay or till, but mud, gravel, and sand, "in the lowest beds of which were small and large boulders of transported rocks polished and scored."\* The position of these drift-beds was 1360 feet above the sea.

If icebergs acted on these drift-beds, as EDWARD FORBES inferred for the reasons mentioned by him, icebergs could have acted in like manner on the materials of boulder-clay.

Those who look upon the boulder-clay as a land deposit, meet the fact of marine shells being found in it thus:—They say that the glaciers which formed the deposit reached the coast, as now in Norway and Greenland, and pushed detrital matter out into the sea, where it became occupied by testacea. This answer is not satisfactory, because the testacea are almost invariably found in a mutilated state. These animals must have been bred and grown in a sea-bottom, which, whilst they lived, was undisturbed; and if the disturbance of their dwellings was due to the protrusion of glaciers, this would be admitting the marine origin of the deposit. Moreover, if the climate was so severe as to bring glaciers to the coast, icebergs would also abound in the adjoining seas; so that the question would then be, whether the effects were more likely due to the protrusion of glaciers, at the mouths of valleys, or to icebergs drifting in the sea and grating along the bottom. On the former supposition, boulder-clay would be formed only at particular spots, viz., at the mouths of valleys which reached the sea. On the latter supposition, boulder-clay would be formed much more extensively. The great abundance and continuity of the deposit in Northern Britain is therefore better accounted for by the iceberg than by the glacier hypothesis.

The advocates of the glacier theory of boulder-clay have also referred to the fact that terrestrial remains occur in the deposit, from which an inference

\* Notes of a Ramble through Wales, by W. S. SYMONDS, 1864, p. 12.

is drawn, that the deposit must have been formed, not in the sea, but in the land, it being supposed that these remains were drifted into the boulder-clay by rivers.\* But rivers flow into the sea, as well as into lakes. It is true that the boulder-clay near Glasgow and near Falkirk, at both of which places elephants' tusks were found, presented no marine shells. But it is equally true that elephants' tusks have been found in what are allowed to be sea-beds. At Kilmaurs (Ayrshire), two tusks of an elephant were found in a bed of stratified mud 9 inches thick, which was overlaid by a bed of sand containing sea-shells, these shells being covered by boulder-clay. (Journ. Lond. Geolog. Soc. vol. xx. p. 217). In Dumbartonshire, the bones of a rein-deer were taken out of a bed of laminated clay, associated with sea-shells.† (Edin. Phil. Journ. new series, vol. vi. p. 105.)

Mr GEIKIE, in support of his view, endeavours to explain the association of sea-shells and the bones of terrestrial animals in boulder-clay, by suggesting that the mass of earth and stones may have been pushed forward by a glacier "close to the sea-shore, and sea-shells might either be thrown up by high tides over the bones previous to their entombment, or be deposited above them during the slow sinking of the land." He adds—"I mention this as a possibility, in order that no difficulty need be felt in harmonising such a fact with the hypothesis that the boulder-clay is a deposit from land ice, and not from icebergs." ("On the Glacial Drift of Scotland," p. 94.)

The possibility of the occurrence here suggested I admit. Its probability is not so clear. But it is quite clear that the occurrence of elephants' bones at two places in boulder-clay, where no sea-shells were found, is no conclusive proof that the deposit must have been formed on the land, when, at two other places, similar bones were found in pleistocene beds, which must have been at the bottom of the sea when the bones were drifted into them.

6. I now pass to other facts which indicate that the boulder-clay not only has been disturbed and intruded on by some foreign agent, but has, in some cases, been moved *en masse* by some agent of tremendous power.

Thus Mr CUMMING, in his "Memoir on the Isle of Man" ("Proc. Lond. Geol. Soc." 4th Jan. 1854, p. 213), says—"There are appearances, as if the *boulder-clay* had been forced violently amongst the different beds of limestone. Fragments of the latter are torn up and carried forward, and these remain angular, though much scratched, at no great distance in the mass of clay which now covers the limestone beds."

The late Dr FLEMING ("Lithology of Edinburgh," p. 60) pointed out "near Gilmerton, a sandstone quarry, where the outcrop of the rocks is seen, covered

\* GEIKIE on Glacial Drift, p. 93.

† Bones of elephants, rhinoceros, &c., are found in Siberia very generally associated with Arctic sea-shells. (LYELL, "Principles," i. 183; Quart. Journ. Lond. Geolog. Society, 1st Feb. 1848.)

by a sandy boulder-clay, *which, having been in motion, has squeezed or bent the ends of the sandstone and shale towards the S.E.*" Two other localities are mentioned by this observer, where the outcropping strata of shale had been in like manner broken off and carried towards the S.E., by some agent pressing down upon them.

Dr HOWDEN, in describing the superficial deposits of Forfarshire ("Edin. Geol. Soc." vol. i. p. 139), says that "in the Brechin Quarry, and in other localities, where the strata are nearly horizontal, especially if they consist of thin laminæ, the rock has been *broken up into shivers*, large detached masses being *embedded in the (boulder) clay.*" Dr HOWDEN suggests no explanation, but it is evident that here also some agent must have ploughed up the strata, and disturbed the covering of clay.

I might refer also to the curious foldings observed in beds of clay, and even of sand, which both Sir CHARLES LYELL and Mr GEIKIE admit cannot be explained in any other way, than by supposing that icebergs or heavy masses of floating ice had pushed them out of their original position.\*

But the most remarkable case, where boulder-clay is shown to have been pushed and pressed forward *en masse*, was described some years ago by two most competent observers, Captain BRICKENDEN and Mr MARTINS of Elgin, observers who wrote independent reports, the former in the "Proceedings of the Geological Society of London," † the other in the "Edinburgh Philosophical Journal" ‡ some years afterwards. Being much impressed with the importance of the facts related by both observers, and wishing to obtain further information, I wrote to the only one whose address I could discover, Mr MARTINS, and received from him a letter, in which the following passages occur:—

"The Linksfield strata consist of a series of bands of limestone, shales, and blue clay. These have obtained the name of *Wealden*, from the fossils in them. Under these bands there lies a great deposit of limestone, called *Cornstone*. In some places, the Wealden bands are separated from the Cornstone by intercalated boulder-clay, having all the characteristics of boulder-clay met with throughout the country. It has the same tenacity, and the same want of stratification, and contains the usual travelled pebbles and blocks. The only difference observable is, that the clay when intercalated has a purplish tinge, evidently acquired by contact with the blue clay of the Wealden. It also contains fragments of the limestone from the band lying immediately above it. In some instances, large

\* LYELL, "Antiquity of Man," p. 138. Mr GEIKIE ("Glacial Drift," p. 119), after alluding to cases where "beds of clay were fairly bent back upon each other," says, "such contortion must be due to powerful pressure. It may have been produced *by masses of ice standing here, and pushed onward, partly by their own impetus, partly by the action of winds or currents.* The compression to which such a weight of ice would give rise, would probably be quite sufficient to corrugate beds of clay and sand."

† Proceed. of Geol. Society for 1851, vol. vii. p. 289.

‡ E. Ph. J. for 1856, vol. iv. p. 222.

flakes of the laminated blue clay were lying in it. This mass of *intruded* (boulder) clay always presents great irregularities in a section. At one place it is scarcely a foot thick, allowing the Wealden beds to rest nearly on the Cornstone. At a short distance it rises abruptly to the height of ten feet, in the form of a cone, from the apex of which several narrow bands stretch up through the fissures of the overlying mass, looking like veins of red granite among crystalline rocks. At one place the boulder-clay formed a rounded mass, and the superincumbent bands of limestone were folded neatly over it, so as to present the appearance of a stone arch. By the movement, a number of fissures had been caused in the overlying bands, into which the clay *had been forced up*. Some of these rents were 6 feet in length."

"It may be noticed that the surface of the Cornstone, when cleared of the till, is found finely smoothed and polished; any hollows on the rock are also smooth and polished. The Cornstone strata are not in the least disturbed."

Mr MARTINS had the goodness to send with his letter two or three sections, showing the relative positions of the boulder-clay to the rocks above and below it. These sections, it is right to add, were made by Mr MARTINS from memory, as the quarry had ceased to be used, and was filled with rubbish. The sections, therefore, can be taken only as giving a pictorial representation of what is described in Mr MARTINS' letter.\*

Both Captain BRICKENDEN and Mr MARTINS express their conviction, produced by a study of the sections when they were exposed, that the boulder-clay had been *forced in* between the upper Wealden bands and the lower Cornstone rock. Captain BRICKENDEN notices particularly the *polished* and *striated* surface of the Cornstone, caused by the passage and attrition of the overlying boulder-clay. He states that the direction of the striations was N.W. and S.E., and he inferred from the appearances, that the boulder-clay had been thrust in from the N.W. When he visited the quarry, the exposed boulder-clay had "a bright red colour," which made the embedded fragments of the purple Wealden rocks all the more striking. The explanation suggested by Captain BRICKENDEN is precisely the theory which it is the object of the present Memoir to support, viz., that the materials of the boulder-clay "had been subjected to the action of *vast and extensive masses of ice*, which by continuing to press onwards the accumulations of clay retained beneath it, had, by a force superior to that which the beds above could offer in resistance, eventually produced the phenomenon."

If it be said that a glacier could have propelled and pushed the boulder-clay between the rocks, quite as well as masses of floating ice, the question would be, whether, in the places referred to, where the boulder-clay has apparently been pushed forward *en masse*, there was any probability of a glacier having existed.

\* Two of these sections are given on Plate XXXI.

The low-lying, flat country of Elgin is, to say the least, most unfavourable for the glacier theory; and the Isle of Man is, from the absence of mountains, equally unfavourable.

The facts and views set forth in the preceding paragraphs, show that the materials of the boulder-clay have been disturbed, intruded on, pushed forward, and heavily pressed on by some extraneous agent; and if it be allowed that these materials, when so acted on, formed a sea-bottom, very little doubt can exist that floating ice was the agent.

7. But it will be asked, whether similar effects are now observable in the Arctic regions, where there are icebergs and floes drifted about by the winds and currents? Can it be shown that they do work on the sea-bottom or shores at all analogous to the appearances presented by our boulder-clay and drift-beds?

In all the channels and estuaries of the Arctic regions, we know that the sea is constantly covered with floating ice in every variety of form. As icebergs, they often strand in places where the sea is 1200 feet deep. On one occasion, the keeper of the lighthouse at Belle Isle, near the mouth of the St Lawrence, in latitude 50°, counted no less than 496 icebergs, some of them 200 feet high and half-a-mile long. About 100 of them were stranded, or were grating over the submarine banks.\*

It is not difficult to conceive what must be the effect on a sea-bottom, whatever the materials, of icebergs having a size greater than the hill of Arthur Seat. Soft materials would be so disturbed and ploughed through, that any appearance of regular bedding would be obliterated, testacea would be crushed, whilst hard fragments of rock would be pushed forward and rounded by the enormous friction.

This inference as to the disturbance and tearing up of the sea-bottom is confirmed by Dr SUTHERLAND, a surgeon in one of the Arctic expeditions. He says that in Davis' Straits, the icebergs, by their action on the sea-bottom, produce "*whole rafts of submarine forests*" of sea-weed, which float on the surface of the sea; and when the sea-bottom in these straits is dredged, little else than "*broken shells*" are brought up.†

The disturbance and dislocation of the submarine beds where icebergs abound, is further evidenced by the large amount of muddy sediment raised, discolouring the sea not only to the surface, but for many miles round.‡

MESSRS DEASE and SIMPSON, in the account they give of their Arctic discoveries in the year 1838, describe a long low spit, composed of gravel and coarse sand, in some places more than a quarter of a mile wide, the formation of which they did not hesitate to attribute to the action of floating ice—judging by what they saw done by ice.

\* See a short paper on this subject by Principal DAWSON, of Montreal, in the "Canadian Naturalist."

† Lond. Geolog. Journal, vol. ix. p. 306.

‡ DE LA BÈCHE, Geolog. Observer, p. 266.

Dr HAYES, in his account of a visit to the West Coast of Greenland in 1867,\* says, "where the current is swift, and the ice is pressed down upon the land with great force and rapidity, the rocks are worn away until they are as *smooth and polished as the surface of a table*." The bearing of this remark on the innumerable smooth and polished rock surfaces in Scotland and the north of England needs not be pointed out.

Dr HAYES mentions another effect of floating ice. He says that "a shelf of ice glued to the shore forms a winter girdle of all the Arctic coasts. It is usually broken away towards the close of every summer, when the masses of rock which have been hurled down upon it (during the previous eight months) from the cliffs above *are carried away and dropped in the sea*. The amount of rock thus transported is immense; and yet it falls far *short of what is carried by icebergs*" (p. 403).

Much to the same effect, on both of these points, Dr WALLICH mentions that, when dredging off Labrador at a depth of from 10 to 15 fathoms, he found the sea-bottom to consist "wholly of uncovered rock or of boulders"—"owing (as he adds) to the *long continued action of drift ice and currents*." †

Another effect produced by floating ice has been observed—the formation of *ruts and strice* on the smooth surfaces of rocks. Many competent observers have given evidence on this point. ‡

The foregoing statements refer to what is now seen going on, wherever there are icebergs and icefloes, and they show that these agents, if they existed in Scottish seas, at a former epoch of the world, must have had the power of producing most of the phenomena connected with our pleistocene deposits.

It is not unimportant to remark, in further confirmation of this view, that in the Arctic regions there exist boulder-clay and boulders pretty high above the sea, just as in Scotland, and that all the Arctic travellers who have paid attention to the subject do not doubt that the chief agents in producing them were icebergs and shore ice.

Thus Dr SUTHERLAND found in Barrow's Straits, up to a height of about 1000 feet, great numbers of boulders, all (he says) clearly transported by coast-ice previous to the elevation of the land,—just as he saw them being transported in that way along the existing shore of Greenland.

Mr LAMONT, in his Memoir on Spitzbergen, takes notice of several places where icebergs had evidently left their footprints, when the land was submerged. At one spot, about 20 feet above the present sea-level, he found a trench, about 100 yards long by 3 or 4 feet deep, formed among boulders, and caused, as he believed, by an iceberg drifting through them. At another spot, he found what

\* The Open Polar Sea.

† North Atlantic Sea-Bed, p. 40.

‡ Sir CHARLES LYELL—(1) Travels in North America, vol. ii. p. 173; (2) Lond. Geolog. Journal for 1849, Trapolli in Scandinavia; Lond. Geolog. Journal for 1868, xcvi.

had the appearance of a gigantic causeway of boulders, caused apparently by icebergs sliding over them, and levelling them.\*

It also deserves notice that true boulder-clay or till exists in the Arctic regions, and in districts where there is much less probability of glaciers than of icebergs having been at work. Sir JOHN RICHARDSON evidently describes this deposit when he mentions "*a tenacious and somewhat slaty blue clay, containing many boulder-stones,*" on the western shores of Hudson's Bay,—a country very little elevated above the sea, and possessing no mountains where glaciers could be formed.†

Nor is it irrelevant to notice the occurrence of boulder-clay in the Antarctic regions, and the opinion formed by that eminent naturalist, Mr DARWIN, as to its origin. After describing "great masses of mud of a dark colour, full of boulders of primitive rocks derived from mountains situated to the W. or S.W. about 60 miles distant," he says, that "the deposit in all respects resembles the till of Scotland;" and adds, that "at present the oceanic currents off Cape Horn set from the west; so that if the ancient currents had the same direction, the phenomena would be explained by floating ice."‡

It thus appears that both in Arctic and in the Antarctic regions, where floating ice has abounded, boulder-clay, boulders, and polished rock surfaces exist. These phenomena do not occur in warmer regions of the earth. Wherever they do occur, there are indications of the sea having stood much higher than at present, so that ice could have drifted at the necessary level; whilst, on the other hand, in many districts there is a total want of the conditions necessary for the formation and for the movement of glaciers in the required direction.

8. In the previous part of this Memoir, I have attempted to show—1st, That glaciers were not the agents to which boulder-clay owes its origin. 2d, That an examination of the deposits, containing, in numerous localities, sea-shells generally mutilated, suggests a submarine origin. 3d, That the way in which the deposit has been driven forwards, and pushed between older rocks, indicates pressure by some agent of enormous weight and magnitude. 4th, That icebergs and shore-ice would probably answer these conditions, and are seen now in the Arctic regions producing similar effects.

Assuming that the facts adduced at all events establish the probability of the theory, that icebergs and shore-ice would account for most of the drift phenomena in Great Britain, I proceed to offer a few remarks as to the circumstances and condition of Great Britain at that time.

Most geologists are agreed that, at the period of the boulder-clay, the sea must have stood greatly higher upon the land than at present. Beds of sea-

\* Lond. Geolog. Journal for 1868, vol. xvi. p. 433.

† FRANKLIN'S Journey in 1823, pp. 499, 501, 583.

‡ Phil. Journal for 1841, vol. xix. p. 530.

shells occur in the pleistocene beds of Lanarkshire, at a height of 526 feet above the sea; and as one of these shells is the *Cyprina islandica*, which requires for healthful existence a depth of at least 30 fathoms, that Lanarkshire deposit implies a submergence of more than 700 feet.

But in Wales, sea-shells of a similar character have been found in drift-beds at a height of no less than 1600 feet above the sea. And there are in many parts of England and Scotland beds of clay, sand, and gravel, at a height of nearly 2500 feet above the sea, which, judging from their stratification and materials, must have been marine.

If, therefore, the whole of the British Islands were submerged to the depth of 2500 feet lower than they are at present, they must have presented little else than an archipelago of islands,—few of which would, at their highest points, be more than 1500 feet above the sea.

As many of the shells found in these pleistocene beds are of an exclusively Arctic type, the sea, during the period of submergence now referred to, was favourable for the presence of drifting ice,—assuming, in the meantime, the existence of some current to bring the ice.

But it is not Great Britain only which was submerged. In Sweden, sea-shells of the same Arctic type have been found in drift-beds to the height of 800 feet, there being also beds, apparently marine, which occur at a still greater height.

These Swedish shell-beds have furnished one or two instructive facts bearing on the process of submergence, which probably apply to Northern Europe generally. Some of the beds are occupied almost exclusively by shells which lived in shallow water. These are in some places covered by beds containing shells of deep-water habits,—indicative not only of a submergence of the country, but a submergence to a considerable extent. Farther, it has been ascertained that these deep-water shells belong to much more Arctic types than those of shallow water which preceded them,—indicating that as the submergence went on, the cold was increasing. Then, again, other shell-beds have been discovered at a lower level, and evidently, from their geological relations, of a more recent date than those above mentioned, in which the Arctic shells are fewer in number and species,—a fact which suggests that, as the land emerged from beneath the waters, the climate improved.\*

It is, however, not in Sweden only that these Arctic shells are found on the Continent. Sir RODERICK MURCHISON, in his great work on the Geology of Russia, has shown that the drift-lands of that country are full of them, implying a general submergence of the whole of Northern Europe under the waters of an Arctic sea, reaching as far south as about latitude 51°.

\* These interesting and instructive facts will be found stated in a Memoir, by Mr GYWN JEFFREYS, in the British Association Reports for 1863; and also in a paper, by Professor SARS of Christiania, in the Edinburgh Phil. Journal for 1863.

Assuming, then, that icebergs and shore-ice prevailed in the sea which covered the British Islands and other parts of Northern Europe, how would these operate as agents in the production of boulder-clay? In some places, boulder-clay lies over stratified beds; in other places, boulder-clay is covered by them. In the former case, it may be supposed that an iceberg pierced through the sea-bottom only to a certain depth, leaving the part next to the rocks untouched. In the latter case, it may be supposed that more sediment was subsequently deposited by currents over the disturbed beds, and remained undisturbed. There are cases where, in one section, there have been found as many as three layers of boulder-clay, each from 20 to 30 feet thick, alternating with laminated beds of clay and sand. In such cases it is only necessary to suppose that icebergs came, drifted by currents, at different periods, some being larger and deeper in the water than others.

If the glacier theory be adopted, which assumes that the boulder-clay was formed on the land at the end of a glacier, or under its mass, then to account for these alternations of marine beds with boulder-clay, there must have been as many oscillations of the land below and above the sea as there are layers of boulder-clay—a supposition surely very improbable.

9. But a difficulty here suggests itself. Where did these icebergs come from? They could not have been generated by Scotch or English glaciers, if, when the land was submerged, there were no mountains higher than 1500 feet.

Where, then, was the high land to give birth to glaciers from which these icebergs came; and was there a current in the ocean so strong and extensive as to bring these icebergs over North-Western Europe, and in the direction indicated by the transported boulders and rock surface striations?

That the physical geography of the Northern Hemisphere must have been totally different from what it now is, is plain from the circumstance that the climate was so different. Perhaps the colder climate was brought about by the same conditions, which would suit the formation of icebergs, and the existence of a great current from the north-west by which they were drifted. The problem is to ascertain what circumstances would produce an Arctic temperature in North-Western Europe, and so far south as latitude 51°.

Labrador is in the latitude of Great Britain. What are the circumstances which give to that country a mean annual temperature of 25°, and a mid-winter temperature of—50°? Two causes co-operate—an Arctic current, loaded with icebergs, which flows past its shores; and proximity to the high land of Greenland, whose snow and ice chill the atmosphere.

From this fact, is it not probable that North-Western Europe, when it possessed a Labrador climate, was indebted for it to similar conditions?

One thing is certain—the Gulf Stream could not then have flowed along its

present course. But its absence would cause the winter temperature of Scotland to fall only by  $28^{\circ}$ ; and a winter temperature of  $10^{\circ}$  would not give to us a Labrador climate. Other conditions must therefore be sought for.

At present the Arctic current, which flows into the North Atlantic, is strong enough to carry icebergs even farther south than latitude  $50^{\circ}$ . They have been sometimes seen in latitude  $40^{\circ}$ . Of course, that Arctic current cannot cross the Atlantic and float icebergs on Great Britain, because the Gulf Stream would intercept it. But suppose the Gulf Stream not to run, as it now does, in a N.E. direction towards Norway. Suppose that, by the Isthmus of Panama being 300 feet lower than at present, the equatorial current, instead of being deflected by the American coast northwards, were to flow into the Pacific, and find its way through Behring's Straits, where there is now a current running from the Pacific, the Arctic current which now flows into the North Atlantic would not only have no Gulf Stream to interrupt it in its progress towards Europe, but would be immensely augmented in volume and speed. The stream passing through Behring's Straits would carry with it a tendency to move eastwards, having acquired that tendency in equatorial regions by the earth's diurnal rotation.

If high land nearer than Greenland is thought necessary, evidence is not wanting to justify that supposition.

In the first place, it is well ascertained that Greenland at its southern extremity has long been sinking, whilst its northern parts are rising.\* Dr KANE and Dr HAYES endeavoured to find where the axis of oscillation is situated. The one gives  $76^{\circ}$  of latitude, the other  $77^{\circ}$  of latitude. Both observers were struck with the fact, that whilst to the north of this supposed axis, lines of raised beaches were visible, none were visible to the south of it. Dr KANE saw and counted no less than forty-one beach lines, at a part of the coast in latitude  $78^{\circ} 30'$ , or about 150 miles to the north of the axis; the highest being 480 feet above the sea. Now it is not unreasonable to suppose, that on the south side of the axis of oscillation, the sinking would be at the same rate as the rising on the north; in which case, what is now the southern extremity of the continent, which is in latitude  $60^{\circ}$ , and therefore about 1000 miles from the axis of oscillation, must, before the sinking began, have been 3200 feet higher than it is at present, and a great deal of what is now sea-bottom to the S. and S.E. of Greenland must have been dry land.

The probability of changes having occurred in the bed of the North Atlantic is all the greater on account of the volcanic convulsions to which it has frequently been subject, of which not only Iceland is a proof, but the igneous

\* Dr HJALTELIN of Iceland, in his letter to Mr R. M. SMITH, quoted on page 667, mentions—“ I have seen the secular elevation of the northern shores of this island; and it is not unlikely that the north coast of Greenland is in a similar state.” Therefore it is probable that the north extremity of Iceland rose up simultaneously with North Greenland.

rocks in the Western Hebrides and the north of Ireland, which are certainly more recent than the chalk, and perhaps belong to the glacial period. On the shores of the Baltic, it has been made out that during this period there were great fractures in the earth's crust.

Whilst there are geological reasons for assuming the existence of high land in the North Atlantic Ocean, now submerged, there are physiological reasons in favour both of high land and of a great Arctic current from the north-west. It was the late Professor EDWARD FORBES who first drew prominent attention to the light thrown on the past history of the earth, by reference to the migrations of the fauna and flora of a country. His remarks, as applicable to Great Britain, are these—"There could not always have been such a separating abyss between Northern Europe and Boreal America as now divides them. The sea, through a great part, must have been a shallow sea; and somewhere, probably far to the north, there must have been *either a connection, or such a proximity, of land* as would account for the transmission of a non-migratory terrestrial\* and a littoral marine fauna." †

In another passage he says—"It is strongly impressed on my mind that the close of the glacial period was marked by the gradual *submergence* of some great northern *land*, along the coasts of which the littoral mollusks, aided by favouring currents, migrated; whilst a common flora ‡ became diffused over its hills and plains. Although I have made icebergs and icefloes the chief agents in the transportation of flora southwards, I cannot but think that so complete a transmission of that flora as we find on the Scottish mountains, was aided perhaps mainly by land to the north now submerged." ("Memoirs of the Geological Survey of Great Britain," vol. i.)

10. In considering the claims of the two theories which have been proposed for explaining the boulder-clay deposit and other drift phenomena, it is proper to keep in view, that whilst there are many of these phenomena which are susceptible of explanation on either theory, there are others again which, whilst

\* Allusion is probably here made to the remains of the woolly-haired elephant, rhinoceros, musk ox, rein-deer, black bear, and polar bear having been found in pleistocene beds in various parts of Great Britain. If, as is believed, these animals belong naturally to North America, how did they reach the small island of Britain?

† In a list of sea-shells given by Mr JAMESON of Ellon, as found in the boulder-clay and other pleistocene beds of Scotland, amounting altogether to 137, he represents 134 as now living in the Arctic circle, 60 in North-Eastern America, 26 in the North Pacific, and 82 in British seas. The number now living in the Arctic circle, North-Eastern America, and North Pacific, but not in British seas, is 52.

‡ Professor E. FORBES mentions, in illustration of this point, the *Eriocaulon septangulare*, "known in Europe only in the Hebrides, and at Connemara, in the west of Ireland. Elsewhere," he says, "it is an inhabitant of Boreal America, which is its true native country, and from whence, by means of transport, it has in all probability been introduced naturally into the British Isles." Professor BALFOUR has given to me the names of the following additional plants, natives of Labrador and Canada, which are found in Skye and on the west coast of Ireland, but nowhere else in Europe, viz., *Neottia gemmipara* and *Sisyrrinchium anceps*.

difficult of explanation on the glacier theory, are very intelligible on the other. A few of these difficulties will now be referred to.

(1.) It has been mentioned, as a result of the examination of the sea-shells in the pleistocene beds of Sweden, that the period of greatest cold was when the land was most deeply submerged. If this be the case, which of the two theories is most reconcilable with it?

When the land was most deeply submerged, the mountains would be elevated above the sea less than at any other period; and therefore circumstances would not be favourable for the formation of glaciers.

On the other hand, circumstances would be especially favourable for the drifting of icebergs among the archipelago of the British Islands.

(2.) The unequal distribution of boulder-clay over North-Western Europe deserves a passing remark. The deposit is much more abundant in Scotland than in any other country. Whilst it exists in both England and Ireland, it is chiefly in the northern and midland counties. In the southern parts of both England and Ireland it is hardly known.

Then in Denmark, the beds which are there called boulder-clay appear not to have been so disturbed as in Scotland. There are beds of clay which contain boulders and pebbles, evidently transported, and which also contain Arctic shells; but these are not described to be in a fragmentary or mutilated condition. These shells are described as belonging to species which are known to inhabit shallow water; and it is added by FORSCHAMMER,\* that there are extensive beds of sand, containing boulders and pebbles, which seem to belong to the same epoch as the boulder-clay. These beds of sand he also looks on as indications of a shallow sea.

Why should there be in the south of England and Ireland a less development of boulder-clay? May it not be that the icebergs melted before reaching so far south?

Why should there be little or none of the true "till" in Denmark? May it not be that the icebergs, brought by a north-west Arctic current, were intercepted by the Scotch archipelago? and if any drifted towards Denmark, would not the shallowness of the sea prevent them floating over and disturbing the banks of mud and sand forming the sea-bottom there?

Shore ice alone probably floated over these Danish waters, carrying boulders and pebbles, and spreading them on the submarine banks.

Whilst some such explanation of the unequal distribution of boulder-clay is suggested by the theory of water-borne ice, it is not easy to draw any explanation from the glacier theory.

(3.) Many persons have been perplexed at finding a ridge dividing two valleys, forming what is sometimes called in the Highlands a *col*, consisting of rock smoothed

\* Journ. of London Geological Society for 1845, vol. i. p. 373.

and striated by the action of ice. Thus, for example, the late Mr MACLAREN points out that the ridge which divides the Gareloch from Loch Long, about 450 feet above the sea, as also two other ridges to the eastward, the one 700 feet, and the other about 1700 feet above the sea, present rocks the surfaces of which are "smoothed and rounded off." Mr MACLAREN remarks upon this fact—"A glacier lodged within the valley would grind off the asperities of the rocks at its bottom; but what smoothed the very tops of the ridges? Is it not probable that it was icebergs?"\*

The Duke of ARGYLE was struck with the same appearances on the ridge of hills dividing Loch Fine and Loch Awe, and at a height of about 1800 feet above the sea. In a letter addressed to the late Principal FORBES,† His Grace observes, "In this case glacier action is impossible. Even if this hill had been the seat of a glacier, it could only have been snow, so near the summit. The only explanation which seems to me possible is, that this peak, when subject to the grinding force, was a rocky islet above the surface of a glacial sea, and that floating icebergs drifting from the N.E. were constantly grinding upon its sides."

In Arran, as Dr BOOG WATSON points out, there are several ridges between adjoining valleys which are smoothed in a like remarkable manner. He has no doubt that they were smoothed by ice; but he leans to the opinion that glaciers may have produced the effect, by overflowing the sides of the valleys in which they were formed.

(4.) Another remarkable phenomenon is the position of isolated boulders on narrow ridges of hills, or ledges of rock, from which to all appearance the slightest force could dislodge them. The wonder is how these boulders could have been placed in such precarious positions.

The Duke of ARGYLE takes notice of a number of these boulders as being on the hills about Loch Fine and Loch Awe, adding, that it is much less difficult to account for their transportation on the supposition of floating ice than of glaciers.

Professor RAMSAY, in his "Ancient Glaciers of Wales," gives a representation of several of these boulders perched on the very edges of cliffs; and I can, from personal observation, as well as sketches made by myself in Wales, confirm Professor RAMSAY's account of the singular appearance which some of these "blocs percés" present. One of these boulders, of angular shape (being 27 feet long, 15 feet high, and 6 feet broad), and weighing about 180 tons, is stated by Professor RAMSAY as being‡ "on the very *crest* of the slaty ridge of" a mountain, about 2000 feet above the sea. "The parent rock is at least a mile distant." What says Professor RAMSAY to the question, how this boulder was transported? "I am aware it has not been customary to consider accumulations at so great an

\* Edin. New Phil. Journ. for 1846, vol. xl. p. 141.

† Proceedings Roy. Soc. Edin. vol. iii. p. 461.

‡ Ancient Glaciers of Wales, p. 80.

elevation as belonging to glacial *marine* deposits. But when we consider their continuity with the shell-bearing strata,\* their regular smoothly sloping outline, and add to this the travelled boulders and masses of rock on the summits of hills and ridges 2300 feet high, it seems impossible to resist the conclusion that the whole is of *marine* origin, and due to the operation of one set of causes extending over a definite period." †

The conclusion which Professor RAMSAY drew from these phenomena was, "that the blocks of stone that now strew our continents and islands were chiefly dropped by the same agency—icebergs—that is now sowing the Western Atlantic with earth and boulders derived from the mountains and coasts of Greenland, where glaciers descend to the sea." †

There is another fact connected with the position of boulders which has often arrested my attention. They are more frequently found in clusters, at or near the tops of hills of moderate height, than anywhere else. On the hill of Croy, near Kilsyth, and on several hills to the west of Dunfermline, examples occur. Almost all the very large boulders, which are known to me, are situated near rising ground, and on the east side of it—as in the case of the Clochodrick stone in Renfrewshire, the great conglomerate boulder near Doune, the Carlin stone in Dunmore Park, and the Auld Wives' Lift, near Milngavie. §

Floating ice would ground most frequently on islets or shallow places, and discharge its cargo there on melting. Glaciers occupying chiefly the lowest parts of a valley would discharge their cargoes at the bottom. Therefore if boulders, either singly or in clusters, most frequently occupy crests or ridges of hills, they afford evidence more of icebergs than of glaciers.

(5.) There is, however, one phenomenon of a perplexing kind, which I admit cannot be easily explained by either of the two theories. I allude to the boulders whose present position has been ascertained to be higher than that of the parent rock. Such cases have been made out in the Isle of Man, Cumberland, and Roxburghshire. If floating ice will not explain these cases, still less will land ice, which, being moved by gravitation, must carry everything to a lower level. On the other hand, instances are on record of stones and gravel being raised to a higher level by means of floating ice. Sir CHARLES LYELL states, that on the coast of Norway sheets of ice with pebbles and moderately-sized boulders have, during a storm, been known to be driven up fully 50 feet above the sea-level. It

\* Professor RAMSAY mentions (p. 96), that sea-shells were found by him at a height of 1300 feet above the sea, "two miles west of Snowdon, on a sloping plain of drift charged with erratic blocks, one of which, of great size, is known as *Maen-bras*, or the large stone."

† Proceed. Lond. Geolog. Society, vol. viii. p. 373.

‡ Anc. Gl. of Wales, p. 92.

§ DE LUC (as quoted by Sir JAMES HALL, Ed. R. S. Tr. vol. vii. p. 160) says, "that the granitic blocks lying in the district between Berlin and the Baltic, occur frequently, and almost constantly, in very numerous assemblages, upon the summits of the sandy hills with which that country is interspersed, whilst none are to be met with in the intervening valleys."

is also related that Sir JAMES ROSS once saw an iceberg capsize, bringing up mud and stones to a height of more than 100 feet from the sea-bottom. Perhaps, therefore, the anomalous position of some boulders, in respect of being above the level of the parent rock, may, on the iceberg theory, admit of some explanation. On the glacier theory they admit of none.

(6.) The chief objection to the views which they have submitted in this Memoir, may be found in the following paragraph :—

“ The iceberg hypothesis will not account for the phenomena. We cannot conceive of a set of ice-rafts moving for ages *in one persistent direction* within a given area of the sea. A group of huge bergs in high latitudes often exhibits, on the contrary, a scene of the *wildest confusion*. If we could examine some parts of the sea-bottom off the Greenland coast, we should find them bruised and scored *in every direction* by the grounding of the bewildered icefloes.”\*

Whilst, in this passage, it is admitted that rocks can be smoothed and striated by icebergs and icefloes, it is said that the “*persistent direction*” in which the agents must have moved to produce the “phenomena,” indicates some other agency than icebergs, because these icebergs would not have moved “for ages in one persistent direction.”

The opinion thus expressed is not supported by evidence, and, moreover, is inconsistent with all the probabilities of the case. It is true we cannot see the markings made on the rocks which form the sea-bottom off Greenland, but, as there is an Arctic current always flowing past that coast out of the Arctic circle, loaded with icebergs and shore ice, the great probability is, that the scorings are not “*in every direction*,” but generally in a direction coincident with that of the current.

At the epoch of the boulder-clay in Scotland, I have assumed that there was a north-west Arctic current which flowed down on Great Britain, a current of greater strength and extent than any of the existing Arctic currents ; and I have suggested some reasons for the probable existence of such a current. If icebergs were drifted by it over the submerged land, is it not presumable that the scorings on the rocks would be all approximately “*in one persistent direction*?”

At all events, is there not more likelihood of this one persistent direction being produced—keeping in view the great extent of the area on the earth’s surface over which it prevails—by an oceanic current loaded with ice, than by any imaginable system of glaciers or ice-cakes ?

11. In a former part of this Memoir I stated, that whilst the evidence from various sources showed that the agency concerned in abrading, striating, and transporting, had moved generally in a south-east direction, there were exceptions, which, however, did not militate against my views as to the origin of boulder-clay.

\* Glacial Drift, by A. GEIKIE, p. 75.

These exceptions appear to indicate two agencies—one of a general, the other of a local character.

(1.) It has been already noticed, that in the Norfolk cliffs there is a boulder-clay, the lowest of all the drift-beds, which lies directly on the chalk rocks. In this boulder-clay there are boulders which have been identified by Sir CHARLES LYELL, Dr MITCHELL,\* and other geologists, with the rocks of Sweden and Norway. Mr GEIKIE mentions ("Geology of Scotland," p. 303) that Professor RAMSAY and he found Scandinavian rocks at the mouth of the Tees. No one doubts that these boulders must have been transported on floating ice, and by a current from the N.E. or N.N.E. AGASSIZ † himself allows the correctness of this view, observing that the "Swedish blocks on the coast of England may have been transported on floating ice." And again—"The Norwegian blocks found on the coast of England have been correctly assigned by LYELL to a similar origin, viz., to masses of ice set afloat."

It is not merely on the Norfolk coast that these traces of a north or north-easterly current, with floating ice, exist. Mr TATE of Alnwick informs me in a letter (1st January 1869) that, at Alnmouth, "he took the direction of the striæ on two blocks as they lay in the clay, each block being about 3 feet long. In one, the striæ were from N. to S., in the other, from N.N.E. to S.S.W. (true bearings)." In his "History of Alnwick," published since the date of his letter, Mr TATE states, that at Abberwick, four miles west of Alnwick, there is a block of grey granite *like that at Aberdeen*, which he refers to in proof of his remark that there are in the district boulders which have been transported great distances. The same accurate observer, in a note on the Farne Islands, in the sixth volume of the "Transactions of the Berwickshire Naturalists' Club," mentions, "that the surface of the whole of these islands had been ground and smoothed by the passage of a powerful agent. Besides the smoothed surface, and rounded little rock knolls, there are ruts or narrow hollows of some length, whose sides and bottom are smoothed and striated. From the slope of the dressings, it appeared that the agent had moved from the northward, which is not from the land, but from the sea, and nearly parallel with the coast. On these islands a larger area of glaciated surface is exposed than in any other part of the north of England."

The late Dr FLEMING, in his "Lithology of Edinburgh," p. 77, notices the occurrence of flints in the drift beds of Fifeshire and Aberdeenshire, which he infers, and with much probability, came from the chalk rocks of Denmark.

It is also a fact of no small significance, that in the Shetland Isles ‡ the stria-

\* Proceed. Lond. Geolog. Society for Nov. 1838.

† Proceed. Lond. Geolog. Society for Nov. 1838, pp. 179, 331.

‡ HIBBERT, Edin. Journ. of Science for 1831, vol. iv.

tions on the smoothed rocks run N. 47° E., and in the Faroe Islands\* N.E. 88°, in each case pointing to Scandinavia.†

It is, therefore, highly probable that Scandinavia was, at a very early period, cased in ice, and that icebergs from its glaciers flowed off towards the west and south, loaded with boulders and pebbles, and leaving portions of these materials on the east coasts of England and Scotland. Such, as it strikes me, is the most probable explanation of the north and north-east agency which is manifested on our coasts.

At all events, no one has proposed to explain the transport of the Norwegian and Swedish blocks to England by glaciers. Floating ice, in some way or other, is the only imaginable agent.

If this be a right conclusion, it adds no little weight to the supposition that the north-west agency, which is so much more clearly manifested, is of a similar description.

(2.) But I freely admit that there are in England and Scotland manifestations also of glacier action. Having been twice in Wales, and once in Cumberland, to study the subject of these rock markings, I saw evidences of glaciers in both districts. I have also seen glacial markings in the valley now occupied by Loch Doon and the River Doon, in Ayrshire. I think that at Loch Skeen, and in the valley north of Moffat, there are similar markings, and I cannot doubt that they exist in Skye, as they were recognised there by the late Principal FORBES.

The glacier markings, in several of the valleys where I have studied them, appear to me more recent than the markings which belong to the general north-west agency. In the valley of the Doon, and also in the valleys of Capel Curig and Llanberis in Wales, it is not difficult to distinguish between the two sets. In these valleys, the polished and striated rocks of the glacier are invariably low down, and only a little way up the sides; whilst the *roches moutonnées* and the transported boulders, due, as I believe, to iceberg agency, are at a higher level. Striations of the rocks, at these higher levels, are seldom or ever visible, unless protected by boulder-clay‡ from the influence of the atmosphere.

This distinction between two sets of drift phenomena, belonging to different epochs, also arrested the attention of Dr ROBERT CHAMBERS. He pointed out some small valleys in the bosoms of which local glaciers had left their marks. "But (he adds) on the summits and high slopes of the hills, and on the portions of the gneissic platform not connected with valleys, there are traces of an independent and, I believe, earlier glaciation."§ He then specifies a spot where the

\* CHAMBERS on "Faroe and Iceland," p. 28.

† In Finmark and Northern Russia blocks have been found which have also been referred to Scandinavia as their source.

‡ At St Abb's Head, the rocks, about 200 feet above the sea, were found striated, when the boulder-clay was removed from them, but at no other places.

§ Edin. New Phil. Journal for 1853, vol. liv. p. 250.

two sets of glacial striæ are seen crossing each other. "The strong normal streaks athwart the hill from the N.W., a direction in which no local or limited mass of ice could move, are chequered with fainter streaks, produced by this simple down hill movement, which happens to be from W.S.W."

12. The changes in the relative levels of sea and land, and the other events referred to in this paper, may be briefly summarised thus—

(1.) A period existed subsequent to the epoch of the lowest Norfolk boulder-clay before referred to, when the area now forming the British Isles was connected with Continental Europe on the one hand, and with North-Eastern Europe on the other, so as to permit terrestrial animals and flora to migrate from both continents.

At this period the climate was colder than at present, yet not so severe as to prevent the growth of Scotch fir, spruce, yew, oak, and beech, the remains of these trees having in England been found under the upper boulder-clay.\*

(2.) Afterwards a great part of North-Western Europe was submerged, so that in Scotland, and a considerable part of England and Ireland, mountains less than 2500 feet above the present sea-level disappeared.

Most of the land animals which had inhabited the country (including elephants, rhinoceros, rein-deer, musk, ox, &c.) would perish by starvation and drowning.

The climate became colder, so as to be suited for mollusks and other marine animals of an Arctic type.

Shore-ice was formed along the coasts, and icebergs would be drifted by a current from the north-west over North-Western Europe in great numbers, stranding and grating along the sea-bottom.

It was at this time, probably, that boulders were transported and lodged on the slopes of hills at great distances from the parent rocks; that rock surfaces, especially on the ridge and crests of hills, were smoothed, and that the beds of mud, gravel, and sand covering the rocks below the sea, were ploughed into and frequently changed into the tenacious, unstratified deposit called boulder-clay.

The Arctic current which brought these icebergs, if it flowed from Behring's Straits eastwards across Hudson's Bay, might have aided in the transport of North Pacific mollusks and Labrador plants to Great Britain.

(3.) The next change was the elevation of Britain and the adjoining continental districts to such a height, that what is now sea between Great Britain and the Continent and Ireland, became dry land, allowing again a migration of plants and animals.

The land probably rose high enough to admit the formation of glaciers in the principal valleys, in which case much of the boulder-clay previously formed when

\* Sir CH. LYELL gives proof that the forest and lignite beds of Cromer were preceded and followed by a period of glacial cold. These forest and lignite beds "underlie the great mass of glacial drift, in part unstratified, and containing boulders and angular blocks transported from great distances."—*Princ.* vol. i. p. 197.

the land was submerged, would be pushed out, and moraines would be formed at the mouths of these valleys.

By this time the general climate was improved, owing possibly to a rise in South America, whereby the Gulf Stream was made to take its present course.

Forests of pine, beach, and oak, again appeared in the British islands. Remains of these, under the present level of the sea, have been found round all our coasts. In the estuary of the Tay the submarine forest and peat-beds are situated on a blue clay. In the Firth of Forth, Dr BROWN has pointed out that at Elie, the submarine forest lies above the Arctic shell-bed.\*

(4.) The land again sunk, though probably not to the extent to which it was previously submerged—perhaps not more than to about 1000 feet above the present sea-level.

It was probably during this period that the submarine banks and spits of gravel and sand, called *kaims* in Scotland, were formed. Some of these are at a height of 750 feet above the sea (Berwickshire, Mid-Lothian, &c.)

The forests previously existing below that level would, of course, be destroyed, and be covered over with sediment. On the Tay, the submarine forest and peat-beds are covered with beds of clay and sand containing various species of sea-shells of the existing species.† The submarine forest in the Firth of Forth, according to Dr BROWN'S account of it, underlies a bed of clay full of the *Scrobicularia*.‡

During this submergence, any moraines formed by glaciers, situated in the submerged parts of the country, would probably be levelled by submarine currents. The only mounds I have seen in Scotland, which I thought were moraines, are at Loch Skeen, at a height of about 1700 feet above the sea.

(5.) The land again emerged from the sea, with intervals of suspension, when the old beach lines were formed, described by Dr ROBERT CHAMBERS in "Ancient Sea Margins."

\* Trans. Roy. Soc. Edin. vol. xxiv. p. 633.

† Newer Pliocene Geology, by SMITH, p. 35.

‡ Roy. Soc. Tr. vol. xxiv. pp. 619 and 633.

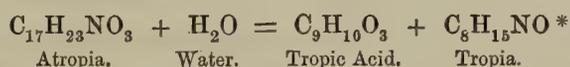


XX.—*On the Connection between Chemical Constitution and Physiological Action.*  
 Part II.—*On the Physiological Action of the Ammonium Bases derived from*  
*Atropia and Conia.* By Dr A. CRUM BROWN and Dr THOMAS R. FRASER.

(Read 18th January 1869.)

ATROPIA.

Atropia is a nitrile base, obtained from *Atropa Belladonna*. All we know of its constitution is, that by the action of strong acids and bases it is decomposed, in accordance with the equation—



So that atropia may be considered as tropia, in which one atom of hydrogen has been replaced by trotyl, the radical of tropic acid.

Atropia has a somewhat complicated physiological action, for it directly influences the functions of the cerebro-spinal and sympathetic nervous systems. The principal effects produced by it on the former system are paralysis of the sensory and motor nerves, and excitation of the spinal cord. By its action on the sympathetic nerves, it influences the contraction of the unstriped muscles; but as the mechanism of this action is by no means exactly defined, we shall merely allude to it in our comparison of the actions of the methyl and ethyl derivatives, with those of the alkaloid itself. In addition to these general actions, atropia influences, in a special manner, the functions of the vagi nerves and of the iris, suspending the cardiac inhibitory power of the former, and producing contraction of the latter.

To cause death in the lower animals, it is necessary that atropia be administered in comparatively large doses, even when it is exhibited by subcutaneous injection. Thus, the minimum fatal dose of sulphate of atropia for a dog, weighing eight or nine pounds, is about fifteen grains; for a full-grown rabbit, more than fifteen grains; and for a frog, a dose equivalent to the  $\frac{1}{1000}$ th or the  $\frac{1}{900}$ th of its weight.

*Iodide of methyl-atropium.*—Iodide of methyl acts very readily on atropia; a good deal of heat is produced; and after the reaction is over, the iodide of

\* KRAUT, "Annalen du Ch. u. Ph." band cxxviii. 1863, p. 280; band cxxxiii. 1865, p. 87; band cxlviii. 1868, p. 236. LOSSEN, *ibid.* band cxxxi. 1864, p. 43; band cxxxviii. 1866, p. 230.

methyl-atropium remains as a white mass. From this, the excess of iodide of methyl is removed by a current of air, and the dry salt dissolved in water, filtered, and evaporated at a temperature not exceeding 40° C. The concentrated solution thus obtained, on cooling deposits the salt in prismatic crystals, apparently belonging to the monoclinic system; sometimes, part of the salt separates as a heavy oil, which soon crystallises. These crystals have the composition  $C_{17}H_{23}NO_3CH_3I$ . They are tolerably stable, bearing a temperature of 100° C. without much alteration. When they are powdered, or when their solution is warmed, a pleasant fruity smell is observed.\*

Pursuing the plan adopted in our former communication, we shall, in the first place, describe the effects of this substance when it is exhibited by subcutaneous injection. As it is tolerably soluble in warm water, we were enabled to administer sufficiently large doses in the form of solution. In the previous part of this research, we found that the chemical addition of iodide, or sulphate of methyl, or of ethyl, greatly diminishes the lethal† activity of strychnia, brucia, thebaia, codeia, morphia, and nicotia. We have now to announce that a similar operation performed on atropia, in place of diminishing, considerably increases the lethal activity of this alkaloid. In our experiments with iodide of methyl-atropium, we were somewhat surprised to find that a dog was rapidly killed by the subcutaneous injection of ten grains, and that a rabbit survived for but a short period after the administration of three. We shall first describe the experiment referred to on a dog, as it illustrates not only the difference between the lethal activity of iodide of methyl-atropium and that of atropia, but also some of the more prominent differences between the symptoms produced by these two substances.

EXPERIMENT I.—A solution of ten grains of iodide of methyl-atropium, in about one hundred minims of warm distilled water, was injected under the skin of a healthy English terrier, weighing eight pounds and six ounces. In a few minutes, there was some difficulty in performing voluntary movements, and in ten minutes this was more marked. Soon after, the anterior extremities became gradually more and more weak, until they could no longer support the body, and the dog subsided on the chest, with the muzzle resting on the floor. In thirteen minutes, it fell over on the side in a state of flaccid helplessness; the respirations became somewhat laboured and shallow, and their frequency diminished until, at twenty-three minutes, only an infrequent gasp occurred. There were now some faint twitches in the *panniculus carnosus* muscle and in the muscles of the limbs; and irritation of the skin still excited feeble reflex move-

\* We shall give details of the chemical relations of the methyl derivatives of atropia on some other occasion.

† We have employed the phrase "lethal activity" as a substitute for the French "l'activité toxique," or death-producing action.

ments. With the exception of these rare gasps, and of a continuance of the cardiac contractions, at the rate of 100 beats in the minute, the animal appeared to be quite dead at twenty-seven minutes after the injection; for even the sensibility of the skin, conjunctiva, and cornea was at this time suspended. The respiratory gasps, however, continued at the rate of five or six in the minute, until thirty-two minutes after the administration of the poison, when death occurred.

In autopsy, it was found, five minutes after death, that the heart was beating at the rate of 96 per minute. The conductivity of the motor nerves and the contractility of the muscles were retained for several minutes afterwards.

The dog, which was the subject of this experiment, had received, some weeks previously, ten grains of sulphate of atropia; and it will be seen from the following account of the experiment, that this dose produced in it some of the more prominent effects of atropia-poisoning.

EXPERIMENT XXII.—Ten grains of sulphate of atropia was dissolved in fifty minims of distilled water, and injected under the skin of the dog that was used, some weeks afterwards, in Experiment I. Omitting many details of the earlier symptoms that were observed, it is sufficient for our present purpose to mention, that in five minutes, there was evident impairment of vision; that in seven minutes, some efforts were made to vomit; that in twelve minutes, urine was voided; that in thirteen minutes, partial paralysis was decidedly present; and that in thirty-eight minutes, frequent spasmodic starts and marked exaggeration of the reflex excitability coexisted with considerable loss of voluntary motor power. After this time, certain effects were observed that contrast in a remarkable manner with those observed in the previous experiment. Gradually the paralysis became more marked until the dog was unable to support itself on its limbs; and the spasmodic action acquired a greater prominence, so that, in a short time, it produced violent tetanic convulsions of an opisthotonic character. The first of these convulsions occurred at fifty-two minutes, and it was succeeded by a series, following each other at intervals of eight or nine minutes, until four hours and ten minutes after the administration, when the observations were interrupted. At nine hours, the dog was still affected with considerable paralysis, but no tetanic convulsions now occurred, though spasmodic starts and exaggeration of the reflex excitability had not yet disappeared. On the following morning, the dog was running about, and it ultimately recovered perfectly.

These two experiments appear to show that the chemical addition of iodide of methyl to atropia increases the lethal activity, and removes the convulsant action of this alkaloid. These changes have been carefully examined in many experiments on rabbits and frogs. In rabbits, we have ascertained that a dose of two and a-half grains produces marked paralytic symptoms, which do not terminate in death; while three grains is a sufficient dose to kill a large animal.

The special symptoms that were observed with these doses will be best described by a short narration of each experiment.

EXPERIMENT VII.—Two grains and a-half of iodide of methyl-atropium was dissolved in fifty-five minims of slightly warm distilled water, and one-half of the solution was injected under the skin at each flank of a rabbit, weighing three pounds and thirteen ounces and a half. Before the administration, the pupils measured  $\frac{1}{50}$ ths  $\times$   $\frac{1}{50}$ ths of an inch, and at six minutes after it, the size of the pupils had increased to  $\frac{1}{50}$ ths  $\times$   $\frac{1}{50}$ ths of an inch. This was the first symptom observed. At sixteen minutes, there was evident difficulty in retaining a normal posture, and soon after the fore-legs yielded, and the rabbit lay on the chest, with the lower jaw resting on the table. At twenty-four minutes, some uneasy movements were executed, during which the body was pushed forward, in the position last described, by the use of the posterior extremities alone. There was now a succession of very slight fibrillary twitches of the muscles of the head, body, and limbs. At fifty minutes, the rabbit lay altogether on the abdomen and chest, with the lower jaw still resting on the table, and it was obvious that the posterior extremities had become powerless like the anterior. The respirations were now shallow and abdominal, at the rate of 68 per minute; the pupils were dilated to  $\frac{1}{50}$ ths  $\times$   $\frac{1}{50}$ ths; the common sensibility appeared to be suspended; and paralysis had so far advanced that the rabbit lay flaccid on the abdomen and chest, with the head resting on the side. This condition continued for about fifteen minutes, when the head was again raised from the side, and, for short periods, even supported normally by the neck muscles. The symptoms then slowly disappeared until a normal condition was reassumed.

EXPERIMENT VIII.—In this experiment, the rabbit weighed three pounds and ten ounces, and it received, by injection under the skin of the two flanks, three grains of iodide of methyl-atropium dissolved in sixty minims of slightly warm distilled water. Dilatation of the pupils appeared in five minutes, and this symptom was soon succeeded by trembling and unsteady movements. In fifteen minutes, the head sunk until it rested on the chin; and in twenty minutes, the paralysis had become so severe that the limbs were unable to support the body. In twenty-eight minutes, the respiratory movements had diminished in number to thirty-four in the minute, while they had become laboured and abdominal in character. The rabbit was now lying on the side in a completely flaccid state. In fifty-four minutes, the respirations were so weak and shallow that it was somewhat difficult to determine the rate of their occurrence. In fifty-six minutes, merely an occasional gasp occurred, and this was frequently accompanied by weak, successive tremors. In fifty-eight minutes, all movement ceased, and death took place.

In the autopsy, it was found that, at three minutes after death, the conductivity of the motor nerves and the contractility of the muscles were retained; and that, at six minutes after death, the heart was motionless and distended.

These experiments are sufficient to illustrate the physiological effects that are produced in rabbits by the subcutaneous administration of iodide of methyl-atropium. They likewise show—and the result is confirmed by other experiments briefly described in the table at the end of this paper—that iodide of methyl-atropium is a much more active poison for rabbits than any salt of atropia. We have already mentioned that the minimum fatal dose, by subcutaneous administration, of even so soluble a salt as the sulphate of atropia, is greater than fifteen grains; whereas it is proved by Experiment VIII. that three grains of iodide of methyl-atropium, administered subcutaneously, is a fatal dose for a rabbit.

We have not succeeded in obtaining any data by which to compare the relative activity of these substances when given to rabbits by the stomach. We have given in this manner as large a dose of both as thirty grains, but have observed no obvious symptom with either substance, except dilatation of the pupils.

Though iodide of methyl-atropium is tolerably soluble in water, it is less so than sulphate of atropia. In Part I. of this investigation we have mentioned as a condition which it is advisable to fulfil, “that the substance is equally suitable for absorption into the system before and after the change.”\* In conformity with this condition, we have examined, with considerable care and detail, the poisonous activity and physiological action of the sulphate of methyl-atropium, a much more soluble salt than the iodide, and, therefore, a more suitable substance for comparison with sulphate of atropia.

*Sulphate of methyl-atropium* ( $(C_{17}H_{23}NO_3CH_3)_2SO_4$ ).—This salt was prepared from the iodide by the method formerly described for the preparation of the sulphates of methyl-strychnium, methyl-brucium, &c. It is a white, crystalline substance, very deliquescent, and very soluble in cold water.

Apparently on account of its greater solubility, it is a rather more active salt than the iodide; and both in rabbits and frogs its lethal activity was, accordingly, found to be much greater than that of sulphate of atropia.

We administered it to rabbits by injecting it under the skin, and also by introducing it into the stomach. The symptoms produced by the former method of administration are in character exactly the same as those produced by the iodide, as will be seen from the following detailed account of several of our experiments.

EXPERIMENT XXXII.—We dissolved two grains of sulphate of methyl-atropium in twenty-five minims of distilled water, and injected the solution under the skin at the right flank of a rabbit, weighing three pounds and seven ounces and a-half. In seven minutes, the animal moved about in an uneasy manner,

\* Transactions of the Roy. Soc. of Edinburgh, vol. xxv. part 1, 1867–68, p. 153.

and, soon, some weakness of the limbs was observed. This weakness increased until the limbs were no longer able to support the body; and, in fourteen minutes, the rabbit subsided on the abdomen and chest, with the lower jaw resting on the table. There were now some slight twitches in several of the muscles of the chest and thighs, and the respiratory movements were weak, though they occurred at the rate of sixty-two in the minute. During other seven minutes, voluntary movements could not be performed; but at the end of this period, some unsteady trembling movements occurred. In twenty-four minutes, the rabbit succeeded in raising the head, though only for a few seconds. It continued at short intervals to raise the head, until increasing strength at length enabled it to support the head normally by the neck muscles. In thirty minutes, the partial paralysis had so far disappeared, that the rabbit succeeded in raising the body on the limbs, and in assuming a natural sitting posture.

Before the administration of sulphate of methyl-atropium, the pupils measured  $\frac{9}{50}$ ths  $\times$   $\frac{7}{50}$ ths of an inch, and seven minutes thereafter they had become dilated to  $\frac{1}{50}$ ths  $\times$   $\frac{6}{50}$ ths.

In the next experiment we administered a fatal dose.

EXPERIMENT XXXIV.—Two grains and a-half of sulphate of methyl-atropium, dissolved in twenty minims of distilled water, was injected under the skin at both flanks of a rabbit, weighing three pounds and half an ounce. In two minutes, there were some uneasy restless movements; in two minutes and a half, slight twitches occurred in the limbs; in three minutes, the rabbit had great difficulty in going about, and weakness of the limbs was manifested by frequent stumbles; and in four minutes, paralysis had so far advanced that the limbs were unable to support the body. In four minutes and a half, the animal lay flaccid on the side, with shallow and infrequent respirations, and now and then a feeble jerking contraction of the diaphragm accompanied inspiration. Soon, the respirations were so feeble as to be hardly recognisable, and they altogether ceased at six minutes after the injection.

In the autopsy, it was found that the sciatic nerves retained their conductivity at ten minutes after death, and that the heart's contractions were rhythmical, and at the rate of thirty in the minute, at eleven minutes.

In this experiment, also, the pupils were greatly dilated a few minutes after the administration of the poison.

The account we have given of these two experiments shows that the effects of the sulphate of methyl-atropium are exactly the same as those of the iodide, the former salt, however, being more active as a poison than the latter.

In order to obtain some data by which to compare the action on rabbits of iodide and sulphate of methyl-atropium with that of sulphate of atropia, we made many experiments in which large doses of sulphate of atropia were administered by subcutaneous injection; but we found that this method of

exhibition usually failed to produce any serious symptom, even when so large a dose as fifteen grains was given. In this experiment (Experiment XXIII.), the symptoms were merely dilatation of the pupils with impaired vision, increase in the rapidity of the cardiac and respiratory movements, diuresis and catharsis, general excitement and slight spasms, and languor. In a few minutes, many of these effects had disappeared, and the rabbit recovered perfectly.

These experiments render it apparent that the action of the methyl derivatives of atropia differs in several striking respects from that of the natural base.

We have seen from Experiments XXII. and XXIII. that large doses of atropia produce diuretic and cathartic effects in both dogs and rabbits, effects that are universally recognised among the symptoms of atropia action. These are not produced by the methyl derivatives.

We have also seen, and our observations agree with those of many previous experimenters, that when a salt of atropia is administered in a large dose to a dog, the predominant symptoms are those of paralysis coexisting with convulsions. The experiments we have now described show that convulsions are never produced by the salts of methyl-atropium, but that the predominating symptoms of their action are those of paralysis alone. It is, therefore, obvious that by the chemical addition of iodide or sulphate of methyl, some important change has been effected in the action of atropia, by which its power to produce convulsions has been removed. The determination of the exact nature of this change can be conveniently effected only by experiments on frogs, for the causation of the convulsive symptoms that appear in mammals has not yet been referred with certainty to any special organ or structure.

One of us has shown, in a paper published in this volume of the Transactions, that when a dose of a salt of atropia near the minimum fatal is given to a frog, a distinctly defined stage of paralysis is in the first place produced, which lasts for many hours, or for several days; and that this stage is succeeded by one in which violent convulsive and tetanic symptoms are present. Further, it is demonstrated in that paper that the convulsive and tetanic symptoms which characterise the second stage, are due to an action of atropia on the spinal cord; in fact, to an action that may with propriety be likened to that of strychnia. From our knowledge of these facts, we are enabled to examine if this strychnia-like action of atropia is possessed by the salts of its methyl derivative. For this purpose, we have made numerous experiments on frogs, of which the following are examples.

EXPERIMENT XXXVI.—Two minims of a solution of two-tenths of a grain of sulphate of methyl-atropium, in forty minims of distilled water, was diluted with two minims of distilled water, and the four minims of solution thus obtained, containing one-hundredth of a grain of sulphate of methyl-atropium, was injected under the skin of a frog, weighing 230 grains. In six minutes, the frog had

difficulty in jumping, and the anterior extremities were somewhat feeble, for they could not properly support the chest. In ten minutes, progression was accomplished by vigorous pushing movements of the posterior extremities, the loss of power being so decided that jumping was impossible. In twenty minutes, the frog lay on the abdomen and chest, but still the condition was not one of complete flaccidity, for the posterior extremities were flexed, and retained their proper tone, while the anterior partially supported the head and upper part of the chest. At this time, the respiratory movements were confined to the muscles of the throat, and reflex contractions of a vigorous character followed slight irritations of the skin. In thirty-five minutes, the paralysis was still more decided, for irritation now produced merely a series of interrupted and weak movements in the extremities; but, otherwise, the frog was in much the same state as that last noted. It continued thus for other twenty minutes, when the paralysis became less severe. A normal posture was assumed, and, by-and-by, vigorous voluntary movements were performed. In about two hours after the administration, the frog was in a normal condition.

It is of interest to observe that these marked symptoms were produced by a dose equivalent to only the  $\frac{1}{32000}$ th of the weight of the frog, while such a dose of sulphate of atropia produces no obvious effect in frogs.

EXPERIMENT XXXVII.—We injected under the skin at the right flank of a frog, weighing 407 grains, one-twentieth of a grain of sulphate of methyl-atropium, dissolved in four minims of distilled water. Very soon after, the movements were performed with some difficulty; and in five minutes, the anterior extremities were sprawling, and the frog was unable to jump. In eleven minutes, the frog was in a flaccid state on the abdomen, chest, and lower jaw; and but feeble reflex contractions could be excited. In fifteen minutes, the reflex function was suspended, and all respiratory movements had disappeared. In twenty-five minutes, a sciatic nerve was exposed and subjected to galvanic stimulation, but no muscular contractions were thereby produced, although direct galvanic stimulation of the muscles caused vigorous contractions. The cardiac impulse was at this time ascertained to be pretty strong, and the beats at the rate of thirty-two per minute. During the two following days, the frog remained in this condition. On the fourth day, however, it was found that the motor nerves had recovered their conductivity, but still the reflex function of the spinal cord was suspended. On the fifth day, the latter function was again present, and, indeed, the action of the poison had now so far disappeared that the frog had resumed a normal posture, and jumped freely when stimulated. There was no symptom whatever on the following day.

The dose given in this experiment was equivalent to the  $\frac{1}{8140}$ th of the weight of the frog used. To produce complete paralysis of the motor nerves for more than two days with sulphate of atropia, it is necessary to exhibit a dose of

about seven times the relative weight. The symptoms following the paralysis produced by sulphate of atropia would, however, be very different from those just described; for in place of a gradual recovery to normality, violent convulsive and tetanic symptoms would appear, and probably continue for several days, before perfect recovery took place. This experiment, therefore, shows in the most satisfactory manner that sulphate of methyl-atropium, administered in a dose rather less than the minimum fatal, does not cause any convulsant action in frogs.

In the next experiment, a dose about the minimum fatal was given.

EXPERIMENT XL.—One-tenth of a grain of sulphate of methyl-atropium was dissolved in four minims of distilled water, and injected under the skin at the right flank of a frog, which weighed 460 grains. Symptoms followed with great rapidity; for in two minutes, the frog could not jump, and the anterior extremities were extended almost powerlessly at right angles to the body, while the respirations were extremely feeble and infrequent. In five minutes, the latter had entirely ceased, and, now, only feeble twitches of the toes could be excited by rather severe irritation of the skin, the limbs being perfectly flaccid and motionless. In nine minutes, irritation caused no reflex movement whatever, and the cardiac contractions were at the rate of thirty beats in the minute. In twenty-nine minutes, it was ascertained that the motor conductivity of the sciatic nerves was suspended, while idio-muscular contractility was still retained. During the two following days, the state of the frog was the same as that last described. On the fourth day, however, it was impossible to discover any cardiac impulse. The muscles still contracted vigorously when they were directly galvanised, and they continued to do so until the seventh day, when *rigor mortis* set in.

We learn from this experiment, that a dose of sulphate of methyl-atropium equivalent to the  $\frac{1}{4600}$ th of the weight of a frog, is sufficient to produce a fatal result. As we have already mentioned, the minimum fatal dose of sulphate of atropia for frogs is about the  $\frac{1}{900}$ th or the  $\frac{1}{1000}$ th of the weight of the animal, but after such doses death is usually preceded by a stage of tetanus. This stage was entirely absent in the experiment with sulphate of methyl-atropium.

As, however, it might be supposed that sulphate of methyl-atropium will cause convulsive and tetanic symptoms if it be given in the same relative proportion as is required to produce these symptoms with sulphate of atropia—viz., in a dose equivalent to about the  $\frac{1}{1000}$ th of the frog's weight—an experiment was performed to meet this supposition.\*

EXPERIMENT XLIV.—A solution, containing four-tenths of a grain of sulphate of methyl-atropium, dissolved in five minims of distilled water, was injected

\* The frogs used in Experiments XXXVI., XXXVII., and XL. had been kept in the laboratory for more than two months before the performance of each experiment. The convulsive and tetanic effects of atropia appear to be more readily produced in frogs that have been thus kept, than in those recently obtained from their natural habitat.

under the skin at the right flank of a frog, weighing 461 grains. The usual paralytic symptoms very quickly supervened; and in seven minutes and thirty seconds, it was ascertained by galvanic stimulation that the motor conductivity of the sciatic nerves was suspended, while muscular contractility was retained. On the following day, this condition of the motor nerves and of the muscles continued, and the heart's contractions were found to be occurring at the rate of thirty in the minute. On the third day, the body was slightly rigid, galvanism of the nerves and muscles caused no contraction, and the heart was motionless.

Similar experiments were made with iodide of methyl-atropium, and no convulsive symptoms were produced by this salt. It was found that its poisonous activity for frogs is less than that of the corresponding sulphate, though considerably greater than that of sulphate of atropia, being equivalent to the  $\frac{1}{2000}$ th of the frog's weight. Short details of these experiments will be found in the Tabular Summary.

Having thus determined, by our experiments on dogs, rabbits, and frogs, that the salts of methyl-atropium do not possess the convulsant action of atropia, it is important that we should next ascertain by what action the paralytic symptoms of the salts of methyl-atropium are produced. Before doing this, however, it may be of advantage to show in what manner atropia itself produces paralysis.

The mechanism of the paralytic action of atropia is a complicated one, for there is good reason to suppose that it consists of actions on the sensory and motor nerves, and probably, also, on the spinal cord.\* The following experiment illustrates the order in which several of these actions are produced.

EXPERIMENT XXV.—The sciatic artery and vein were ligatured at the upper part of the right thigh of a frog, weighing 215 grains; and, a few minutes afterwards, one-fourth of a grain of sulphate of atropia, dissolved in four minims of distilled water, was injected under the skin at the left flank. In eight minutes, a slight degree of paralysis was present, but the frog was able to perform somewhat imperfect jumping movements until thirty-five minutes. In forty minutes, however, it lay flaccid on the abdomen, with the head resting on the table, and, now, irritation of the skin of *any region* caused no other effect than a number of pretty vigorous movements in both posterior extremities, of rather greater energy in the non-poisoned (right) than in the poisoned. These reflex contractions could likewise be excited by gently touching the skin, in the poisoned as well as in the non-poisoned regions. The heart was now contracting at the rate of twenty-six beats in the minute. With the exception of a gradual diminution in the rate of the heart's contractions, no notable change occurred in the state of the animal until one hour and twenty minutes after the injection. At this time, gentle

\* Authorities differ somewhat in their interpretation of the relations of these actions, some considering that the motor nerves are paralysed more rapidly than the sensory (BOTKIN, &c.), and others that the sensory are paralysed more rapidly than the motor (LEMATTRE, MEURIOT, &c.).

stimulation of the skin of the poisoned region caused no movement whatever, but feeble movements could still be excited in both poisoned and non-poisoned regions by strong stimulation; and the sensibility of the non-poisoned region was in much the same condition. In one hour and forty-five minutes, it was impossible to excite any reflex movement whatever. Galvanic stimulation of the right (non-poisoned) sciatic nerve was followed by vigorous movements restricted to the right leg; and, when this stimulation was applied to the left (poisoned) sciatic nerve, similar movements were produced in the left leg, and nowhere else. The heart was now contracting at the rate of twenty beats per minute. This state of suspension of the reflex function, with retention of conductivity in the motor nerves and of contractility in the muscles, continued until at least three hours after the administration of the poison, when the observations were interrupted. On the following morning, it was found that the conductivity of the poisoned sciatic nerve was suspended, while the poisoned muscles contracted when directly stimulated by an interrupted current. The conductivity of the non-poisoned sciatic nerve was still retained.

We learn from this experiment, that although a large dose of atropia quickly produces in frogs a condition of marked paralysis, the conductivity of the sensory and motor nerves and the reflex function of the spinal cord are not completely suspended until considerable intervals after the administration. Of these special paralytic actions, that on the motor nerves appears to be the last to be effected; indeed, in this experiment an interval of least an hour and fifteen minutes elapsed between the complete suspension of the reflex function and that of conductivity of these nerves.

We shall now endeavour to discover if the salts of the methyl derivative of atropia produce their paralytic symptoms by the same actions as atropia does.

EXPERIMENT XLIII.—Having ligatured the artery and veins at the upper third of the right thigh of a frog that weighed 235 grains, we injected under the skin of the left flank one-tenth of a grain of sulphate of methyl-atropium, in four minims of distilled water. Paralytic symptoms followed with great rapidity: so that in five minutes and thirty seconds, the frog was motionless, excepting that vigorous spontaneous movements frequently occurred in the right (non-poisoned) posterior extremity; and stimulation of the skin of any part, even though severe, *did not produce the faintest muscular contraction in the poisoned region, although it produced strong contractions in the non-poisoned (right) posterior extremity.* In six minutes, it was ascertained that the heart was contracting at the rate of forty beats in the minute. Frequent observations were made, and it was found that no change whatever occurred during the subsequent three hours—the conductivity of the poisoned sensory (afferent) nerves, and the reflex function of the spinal cord being retained, while the conductivity of the poisoned motor nerves was completely suspended. On the two following days, this condition

was still present, except that on the third day, the rate of the heart's contractions had diminished to thirty-six in the minute. Death, with commencing rigidity, occurred on the fourth day.

The dose given in this experiment was greatly above the minimum fatal. We shall now describe the effects of a dose that was considerably below the minimum fatal.

EXPERIMENT XXXVIII.—The blood-vessels at the upper third of the right thigh were ligatured in a frog, weighing 379 grains, and immediately afterwards a solution of one-twentieth of a grain of sulphate of methyl-atropium, in four minims of distilled water, was injected under the skin at the left flank. In three minutes, the respiration had ceased, and the frog was lying on the abdomen, perfectly flaccid and motionless in the poisoned region; but retaining the normal tone in the non-poisoned posterior extremity, where spontaneous vigorous movements frequently occurred. Irritation of the skin of any region did not cause any movement in the poisoned region, but it caused energetic contractions in the non-poisoned. In ten minutes, the left (poisoned) sciatic nerve was subjected to galvanic stimulation, with the result that *no movement was thereby caused in the left posterior extremity or in any part to which the poison had access, while energetic reflex contractions were caused in the right (non-poisoned) posterior extremity.* The poisoned muscles freely contracted when directly stimulated. It was found that the cardiac impulse was, at this time, powerful, while contractions occurred forty-four times in the minute. Repeated observation showed that the conditions of the poisoned heart, spinal cord, nerves, and muscles, and of the non-poisoned nerves and muscles, described as being present at ten minutes after the injection, continued unchanged during the succeeding three hours. On the following day, the frog had resumed a normal posture. It moved and jumped about actively, and there was now no symptom present.

We have made experiments similar to these with iodide of methyl-atropium, and the same general results were obtained.

It has thus been shown, in the most satisfactory manner, that the salts of methyl-atropium produce their paralytic effects in a very different manner from atropia. The former substances do not appear to influence the sensory nerves or the spinal cord, but they act solely on the motor nerves. We have seen that this last action is possessed by atropia also, though in a comparatively feeble degree; and the experiment we have described confirms the opinion of previous observers, that it is primarily restricted to the peripheral terminations of these nerves. The evidence contained in the experiments we have narrated with sulphate of methyl-atropium is in favour of the paralysis produced by this substance being likewise due to an action on the peripheral terminations of the motor nerves, and the following experiment clearly proves that such is the case.

EXPERIMENT XLII.—The right gastrocnemius muscle of a frog, weighing

312 grains, was exposed, the blood-vessels that entered it were carefully ligatured, and all the connections of the muscle divided, except its origin and insertion. Immediately afterwards, a solution of one-tenth of a grain of sulphate of methyl-atropium, in five minims of distilled water, was injected under the skin of the back. Paralytic effects were quickly produced. In fifteen minutes, the left sciatic nerve was exposed and stimulated by galvanism, with the result that while no contraction was produced in the left limb, energetic movements occurred in the right. The right sciatic nerve was then exposed and subjected to galvanic stimulation; energetic movements occurred in the right leg, which were ascertained to be entirely caused by contractions of the gastrocnemius muscle (non-poisoned); and no movement occurred elsewhere. On directly galvanising the poisoned muscles it was found that their contractility was still retained.

We learn from Experiments XLIII. and XXXVIII. that sulphate of methyl-atropium does not paralyse the motor nerve trunks. We further learn from Experiment XLII. that certain terminations of a motor nerve protected from the direct action of this poison are not paralysed, while other terminations exposed to its action are very quickly paralysed. It is, therefore, apparent that the paralysis of the motor nerves, which this substance so energetically produces, is due to an action that is restricted to their peripheral terminations.

The valuable and interesting researches of BOTKIN,\* VON BEZOLD and BLOEBAUM,† MEURIOT,‡ and others, have shown that atropia exerts a paralyzing influence on the inhibitory cardiac branches of the vagi nerves. When administered, even in very small doses, this substance so completely paralyses these nerves, that powerful galvanic stimulation of the main trunk of one of the vagi does not produce stoppage of the heart's action, or even appreciably diminish the rate of its contractions. It seemed important that we should determine if the methyl and ethyl derivatives of atropia possess this remarkable action.

EXPERIMENT XXIX.—The two vagi nerves were exposed in the neck of a rabbit, weighing three pounds and eight ounces. On subjecting each vagus separately to galvanic stimulation of a certain strength, obtained by the use of DU BOIS REYMOND'S induction apparatus, it was found that total stoppage of the heart's contractions resulted on each occasion, during the ten seconds the galvanic stimulation was applied. A solution containing half-a-grain of sulphate of methyl-atropium, in fifteen minims of distilled water, was injected under the skin of the abdomen.

\* VIRCHOW'S Archiv. Bd. xxiv. 1862, p. 89.

† Untersuchungen aus dem Physiologischen Laboratorium in Würzburg, 1tes heft, 1867, p. 43.

‡ De la Méthod Physiologique en Thérapeutique et de ses Applications à l'étude de la Bella-donne, 1868, p. 76.

5	minutes	after the injection,	the heart was contracting	28	times in 10 seconds.
7	"	"	"	28	"
7	"	and 10 seconds	"	the right vagus was galvanised* for ten seconds, and the heart continued to contract, during the galvanism,	28 times in 10 seconds.
10	"	"	"	the heart was contracting	29 times in 10 seconds.
19	"	"	"	30	"
20	"	"	"	the left vagus was galvanised for ten seconds, and the heart continued to contract, during the galvanism,	30 times in 10 seconds.
20	"	and 20 seconds	"	the heart was contracting	30 times in 10 seconds.
26	"	"	"	30	"
26	"	and 20 seconds	"	the right vagus was galvanised for ten seconds, and the heart continued to contract, during the galvanism,	30 times in 10 seconds.

No general symptoms of the action of sulphate of methyl-atropium were developed during this period, the dose that was administered being but small.

The paralytic action on the inhibitory cardiac branches of the vagi, which this experiment clearly exhibits, would appear to be a very powerful one; for it was not counteracted, within twenty minutes, by half a grain of extract of physostigma subcutaneously administered, nor, within thirty minutes, by a second dose of three-fourths of a grain of extract of physostigma, administered twenty minutes after the first.

In other similar experiments on rabbits, we succeeded in completely paralyzing the vagi nerves with one-tenth and with one-twentieth of a grain of iodide of methyl-atropium, and with one-tenth of a grain of iodide of ethyl-atropium.

We have seen, from Experiments VII., VIII., XXXII., and XXXIV., that iodide and sulphate of methyl-atropium, when acting through the blood, produce marked dilatation of the pupils. A number of experiments were made to determine whether the topical application of these salts to the conjunctiva similarly affects the pupil, and, thus, further exhibits a similarity in action to atropia and its salts.

The largest dose we applied was the  $\frac{1}{500}$ th of a grain.

EXPERIMENT CXI.—This dose, dissolved in one minim of distilled water, was applied to the left eyeball of a rabbit, and it caused extreme dilatation of the left pupil ( $\frac{2}{5}$ ths  $\times$   $\frac{1}{5}$ ths of an inch) in less than five minutes, which lasted for more than three days. The left pupil was of normal size on the sixth day.

In order to test the delicacy of this reaction, we made the following experiments:—

EXPERIMENT CXII.—One minim of a solution of one grain of sulphate of methyl-atropium, in 1000 minims of distilled water ( $= \frac{1}{1000}$ th of a grain of sulphate of methyl-atropium), was placed on the *right* eyeball of a rabbit.

\* Throughout the experiment the strength of the galvanic current was the same as that which produced stoppage of the heart's contractions before the administration of sulphate of methyl-atropium.

Before the application, the <i>right</i> pupil measured	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths,	and the left,	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	of an inch.
8 minutes after the application,	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
13 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
17 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
55 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
2 hours	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
On the 2d day	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
" 3d "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
" 4th "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
" 5th "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"

EXPERIMENT CXIII.—One minim of a solution of one grain of sulphate of methyl-atropium, in 5000 minims of distilled water ( $=\frac{1}{5000}$ th of a grain of sulphate of methyl-atropium), was paced on the *left* eyeball of a rabbit.

Before the application, the <i>left</i> pupil measured	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths,	and the right,	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	of an inch.
13 minutes after the application,	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
15 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
17 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
20 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
2 hours	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
22 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"

EXPERIMENT CXIV.—One minim of a solution of one grain of sulphate of methyl-atropium, in 20,000 minims of distilled water ( $=\frac{1}{20000}$ th of a grain of sulphate of methyl-atropium), was placed on the *right* eyeball of a rabbit.

Before the application, the <i>right</i> pupil measured	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths,	and the left,	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	of an inch.
35 minutes after the application,	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
45 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
1 hour 10 minutes	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
25 hours	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"

EXPERIMENT CXV.—One minim of a solution of one grain of sulphate of methyl-atropium, in 50,000 minims of distilled water ( $=\frac{1}{50000}$ th of a grain of sulphate of methyl-atropium), was placed on the *right* eyeball of a young rabbit.

Before the application, the <i>right</i> pupil measured	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths,	and the left,	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	of an inch.
42 minutes after the application,	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
1 hour 5 minutes	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
1 " 10 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
2 hours	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
22 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"

EXPERIMENT CXVI.—One minim of a solution of one grain of sulphate of methyl-atropium in 100,000 minims of distilled water ( $=\frac{1}{100000}$ th of a grain of sulphate of methyl-atropium) was placed on the *right* eyeball of a rabbit.

Before the application, the <i>right</i> pupil measured	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths,	and the left,	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	of an inch.
39 minutes after the application,	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
1 hour	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
1 " 30 minutes	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
2 hours 10 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"
22 "	"	"	$\frac{1}{50}$ ths $\times$ $\frac{1}{50}$ ths	"

It seemed to us unnecessary to proceed further in our examination of the delicacy of this action on the iris. The researches of DE RUYTER have placed at our disposal a number of experiments with sulphate of atropia, similarly applied in extremely dilute solutions. From these researches we learn that a drop of a solution containing the  $\frac{1}{1281600}$ th of a grain of sulphate of atropia is capable of producing dilatation of the pupil in a dog, which lasts for eighteen hours.\* Comparing this result with that obtained in our experiment with the  $\frac{1}{100000}$ th of a grain of sulphate of methyl-atropium, we are justified in considering that the addition of sulphate of methyl to atropia does not diminish the mydriatic action of this alkaloid to any marked extent.

*Iodide of ethyl-atropium* ( $C_{17}H_{23}NO_3CH_3I$ ).—Our investigation also includes an examination of the physiological action of this ethyl derivative of atropia. The results of this examination prove that this substance acts in precisely the same manner as the previously described methyl derivatives.

Iodide of ethyl acts readily on atropia, but not so energetically as iodide of methyl. In preparing the iodide of ethyl-atropium, atropia was treated with a considerable excess of iodide of ethyl, in a sealed tube, at  $100^\circ C.$ , for an hour. The remainder of the process is the same as in the case of the methyl derivative, which in general appearance and character it closely resembles.

We found that two grains of this substance, administered by subcutaneous injection, is a poisonous dose for a full-grown rabbit.

EXPERIMENT XLVIII.—In a rabbit, weighing three pounds and seven ounces, it was found that the right pupil, under exposure to a full light, had a diameter of  $\frac{13}{50}$ ths  $\times$   $\frac{2}{50}$ ths of an inch, and that the respirations were irregular and at the rate of twenty-four in ten seconds.

A solution containing two grains of iodide of ethyl-atropium, in one hundred and twenty minims of slightly warmed distilled water, was then injected under the skin at the back of the rabbit. In two minutes, the pupils measured  $\frac{15}{50}$ ths  $\times$   $\frac{4}{50}$ ths. In three minutes, the respirations occurred regularly at the rate of twenty per ten seconds; but there was no other symptom present. In six minutes, some faint quivers occurred, and a slight degree of paralysis was present. The latter gradually increased in severity until, in sixteen minutes, the rabbit was unable to move about, and lay on the abdomen with the head resting on the table in an utterly flaccid state. The respirations were now shallow and somewhat jerking in character, and they occurred at the rate of fourteen per ten seconds. In twenty-two minutes, the respirations were extremely feeble, and at the rate of only nine per ten seconds. After this, they quickly diminished in number, until they altogether ceased in twenty-four minutes after the injection. At the time of the occurrence of death, the pupils measured  $\frac{15}{50}$ ths  $\times$   $\frac{4}{50}$ ths of an inch.

\* Quoted by MEURIOT, *op. cit.* p. 118, from *Nederlandsch Lancet*, 1853.

In the autopsy, it was found that the sciatic nerves retained their afferent (motor) and efferent (sensor) conductivity for at least fifteen minutes after death, and that the heart had ceased to contract previous to twenty minutes after this event.

These general effects very closely resemble those that have been described in the experiments with the iodide of methyl-atropium. We shall see from the following experiment that the effects on frogs of this ethyl derivative are likewise the same in character with those of the methyl derivatives.

EXPERIMENT LII.—We injected under the skin of the left flank of a frog, weighing 290 grains, three-twentieths of a grain of iodide of ethyl-atropium, dissolved in eight minims of distilled water. In six minutes, the frog was perfectly motionless and flaccid; and so complete was the paralysis, that even severe irritation of the skin did not cause any reflex movement. The heart was now contracting in regular rhythm, at the rate of thirty-nine beats in the minute. In seven minutes, the right sciatic nerve was exposed, and it was found, by galvanic stimulation, that its motor conductivity was completely suspended. The muscles, at this time, contracted vigorously when the electrodes were applied to their surface. This condition of the motor nerves and of the muscles was retained for other two days; but on the second day the cardiac impulse was weak, and the beats occurred at the diminished rate of twenty in the minute, while, on the third day, no cardiac impulse could be observed. On the fourth day, the muscles had become rigid.

These symptoms are in all essential characters the same as those we have described, with fatal doses of iodide of methyl-atropium. We have besides, given a dose considerably below the minimum fatal, and have observed a temporary stage of complete paralysis of the motor nerves, which was recovered from without the occurrence of the slightest spasmodic or convulsive symptoms. This ethyl derivative of atropia, therefore, resembles the methyl derivatives, in that it does not possess the well-marked convulsant action of atropia.

We have seen from the last experiment that the paralysis is accompanied by total suspension of the conductivity of the motor nerves. It is important that we should now discover whether the sensory nerves and spinal cord are also implicated in the production of this paralysis. This may readily be determined by experiments in which certain limited regions are protected from the direct action of the poison—as has been done in experiments with the methyl derivatives of atropia.

EXPERIMENT LI.—Having ligatured the blood-vessels in the lower third of the right thigh of a frog, weighing 301 grains, we injected three-twentieths of a grain of iodide of ethyl-atropium, dissolved in ten minims of distilled water, under the skin of the left flank. Paralysis very quickly supervened; and in two minutes, stimulation of the skin anywhere was followed by vigorous movements of the

right (non-poisoned) leg, but no movement occurred in any part of the poisoned region. It was likewise observed that frequent spontaneous movements of an energetic character occurred in the right (non-poisoned) leg. In twelve minutes, the symptoms were as last described, and the heart's contractions occurred thirty-eight times in the minute. In thirteen minutes, the left sciatic nerve was exposed, and on being stimulated by galvanism, *it was found that the conductivity of its motor fibres was suspended, while that of its sensory fibres continued*; no movement occurring in the left (poisoned) posterior extremity, although the muscles of that limb yet retained their contractility unimpaired, while vigorous reflex movements occurred in the right (non-poisoned) posterior extremity.

The occurrence of these reflex movements shows that the reflex function of the spinal cord is not destroyed by the direct action of this poison. It is obvious, from the details we have just given, that the sensory nerve fibres and the striped muscles are likewise unaffected; while the motor nerve fibres are powerfully affected. The paralytic effects of iodide of ethyl-atropium, like those of iodide and sulphate of methyl-atropium, are, therefore, caused entirely by an action on the motor nerves. The last experiment further shows that the trunks of the motor nerves are unaffected, while their peripheral portions are paralysed. It will be seen from the next experiment that the ultimate terminations of these nerves in the muscles are the portions of the periphery that are affected.

EXPERIMENT L.—In a frog, weighing 450 grains, the left gastrocnemius muscle was carefully dissected from all its connections, excepting its origin and insertion and the nerve fibres that entered it; and its blood-vessels were ligatured and divided. Two-tenths of a grain of iodide of ethyl-atropium, dissolved in ten minims of distilled water, was then injected under the skin at the right flank. In ten minutes, the frog was completely flaccid and motionless; and, when the skin anywhere was irritated, no movement occurred in the poisoned region, while well marked movements occurred in the left leg and foot. In twelve minutes, the right sciatic nerve was subjected to galvanic stimulation, with the result that while no movement occurred in the right leg, vigorous contractions occurred in the left. The left sciatic nerve was then similarly stimulated, and vigorous movements followed in the left leg, but nowhere else. *It was seen that these movements were due solely to contractions of the left gastrocnemius muscle, which was protected from the direct action of the poison.*

The results we have obtained from these experiments are of an extremely interesting character. They clearly prove that the ammonium bases derived from atropia possess an action which is very different from that of atropia itself. The latter substance produces paralysis chiefly by affecting the motor centres and the sensory nerves; it produces convulsions by stimulating the spinal cord; and it produces diuresis and catharsis by influencing the urinary apparatus and the intestinal functions. The salts of the ammonium basis possess none of these

actions. They, however, retain the dilating action of atropia on the pupil, and the paralysing action on the cardiac inhibitory branches of the vagi, and on the spinal motor nerves.

This last action, though resembling that of atropia in *character*, differs greatly from it in *degree*. While in atropia, this action has only a secondary prominence, and, in the presence of other and more potent paralysing actions, only a subsidiary influence in causing paralysis and death; in the methyl and ethyl derivatives, it assumes the prominence of the sole paralysis-producing action, and the primary cause of the poisonous activity of these substances.

As it is shown by our experiments that the poisonous activity of the methyl and ethyl derivatives is much greater than that of the salts of atropia, it is apparent that the paralysing action of the former on the motor nerve terminations must be very much greater than that of the latter.

In the following Table, we summarise the chief details of a few of our experiments, so as clearly to exhibit the difference of poisonous activity.

No. of Experiment.	Substance Employed.	Animal and its Weight.	Dose, by subcutaneous administration.	Relation of Dose to Weight of Animal.	Effect.
I.	Iodide of methyl-atropium.	Dog, 8 lbs. 6 oz.	10 grs. (containing 6·6 grs. of atropia).	$\frac{1}{8883}d.$	Decided paralysis in 10 minutes, accompanied with very faint twitchings; and death in 32 minutes.
XXII.	Sulphate of atropia.	Dog (same dog as in Expt. I.)	10 grs. (containing 8·48 grs. of atropia).	$\frac{1}{8883}d.$	Diuresis in 12 minutes; partial paralysis in 13 minutes; spasms in 38 minutes; decided paralysis in 48 minutes; tetanic convulsions in 52 minutes, and until 3 hours and 18 minutes; and followed by recovery.
VIII.	Iodide of methyl-atropium.	Rabbit, 3 lbs. 10 oz.	3 grs. (containing 2 grs. of atropia).	$\frac{1}{2228}th.$	Decided paralysis in 15 minutes; and death in 58 minutes.
XXIII.	Sulphate of atropia.	Do., 2 lbs. 5 oz.	15 grs. (containing 13·12 grs. of atropia).	$\frac{1}{1740}th.$	Diuresis, catharsis, and langour; followed by recovery in more than 3 hours and less than 9.
XXXIV.	Sulphate of methyl-atropium.	Do., 3 lbs. 0½ oz.	2·5 grs. (containing 2·05 grs. of atropia).	$\frac{1}{10108}th.$	Slight paralysis and feeble twitches in 3 minutes; decided paralysis in 4 minutes; and death in 6 minutes.
XL.	Do.	Frog, 460 grs.	0·1 gr. (containing 0·08 gr. of atropia).	$\frac{1}{4600}th.$	Decided paralysis in 2 minutes; complete paralysis in 9 minutes; and death on the 7th day.
XXIV.	Sulphate of atropia.	Do., 490 grs.	0·5 gr. (containing 0·42 gr. of atropia).	$\frac{1}{980}th.$	Incomplete paralysis 1st and 2d days; complete paralysis 3d day; tetanus 3d to 5th days; and recovery 7th day.
XLVIII.	Iodide of ethyl-atropium.	Rabbit, 3 lbs. 7 oz.	2 grs. (containing 1·27 gr. of atropia).	$\frac{1}{2380}th.$	Slight paralysis and tremors in 6 minutes; decided paralysis in 16 minutes; and death in 24 minutes.

## CONIA.

This substance is obtained from *Conium maculatum* (hemlock), and has been shown by VON PLANTA and KEKULÉ\* to be a variable mixture of two bases, to which they give the names of "Conia" and "Methyl-conia." These bases resemble one another very closely in physical properties. Their composition is represented by the formulæ  $C_8H_{15}N$  and  $C_9H_{17}N$ . The chemists above named investigated very completely the action of iodide of ethyl on conia, and proved that "conia" (or, as it is called in the present paper, *normal conia*) is an imide base, and that "methyl-conia" is a nitrile base.

The substances examined in the present paper are:—

1st, *Conia*—samples of which were obtained from Messrs DUNCAN & FLOCKHART, MACFARLAN & Co., and MORSON. We are also indebted to Dr CHRISTISON for the opportunity of examining the action of a specimen of conia, which he prepared in 1835.

2d, *Methyl-conia*—prepared from hydriodate of methyl-conia, produced by the union of iodide of methyl and *normal conia*. Our experiments were made with the hydrochlorate of this substance.

3d, *Iodide of dimethyl-conium*—obtained by the union of iodide of methyl and methyl-conia contained in conia, as obtained from the plant.

*Conia*.—The careful and elaborate investigations of CHRISTISON,† SCHROFF,‡ Von PRAAG,§ KÖLLIKER,|| and GUTTMANN,¶ have rendered important service to our knowledge of the effects and mode of action of conia. From the results obtained by these authors, it is now certainly established that this alkaloid is a poison of great activity, and that it produces marked paralytic and less obvious spasmodic symptoms. The former symptoms have been shown to depend principally on an action on the peripheral terminations of the motor nerves; but the causation of the latter is as yet unknown. It has also been ascertained, chiefly by the investigations of KÖLLIKER and GUTTMANN, that conia does not directly influence the functions of the sensory nerves, striped muscles, or heart.

In a general manner, our experiments confirm the above results; but they also prove that considerable differences occur both in the nature of the action and in the lethal activity of various samples of conia. In these respects, we observed the most marked differences between the conia prepared by Dr CHRISTISON and

\* Annalen der Chemie und Pharmacie, bd. lxxxix. 1854, p. 129.

† Transactions Roy. Soc. of Edinburgh, vol. xiii. 1837, pp. 398–415.

‡ Wochenblatt der Gesellschaft der Aerzte zu Wien, 1856; and Lehrbuch der Pharmacologie, 1869, p. 531.

§ Journ. f. Pharm., i. 44.

|| VIRCHOW'S Archiv. bd. x. 1856, p. 238.

¶ Berliner Klinische Wochenschrift, 1866, pp. 45, 55, 71, 81.

that obtained from Mr MORSON; and as these, therefore, represent the extremes among our samples, we shall describe in detail their action only.

Our experiments were made with the hydrochlorate, which we obtained as a nearly colourless, imperfectly crystalline, and deliquescent substance. The difference of activity between the hydrochlorate of Dr CHRISTISON'S conia and that of Mr MORSON was so great, that while two-tenths of a grain of the former speedily caused death in a full-grown rabbit, this dose of the latter did not produce any distinct effect, one grain being the smallest fatal dose for a rabbit. The symptoms that are produced in mammals by the different samples are very similar in character. The more prominent of these are stiffness of the limbs, causing difficulty in moving about; spasmodic starts; distinct increase of reflex excitability; gradually increasing paralysis, with diminution, and, afterwards, disappearance of the increased reflex excitability; and, finally, death by asphyxia. The exact causation of the paralytic symptoms differs, however, in a remarkable manner in different samples of conia; and the nature of this difference will be shown in the detailed descriptions that follow.

We shall consider, in the first place, the action of *hydrochlorate of Dr CHRISTISON'S conia*. The following experiment illustrates the symptoms in mammals.

EXPERIMENT LIV.—Two-tenths of a grain of hydrochlorate of Dr CHRISTISON'S conia, dissolved in four minims of distilled water, was administered by subcutaneous injection to a rabbit, weighing three pounds and six ounces and-a-half. In two minutes and thirty seconds, the limbs became somewhat stiff and abnormally extended, so that the body was raised and an awkward posture assumed. In three minutes, a slight touch of any part of the skin caused a sudden spasmodic start; and soon after a series of starts in rapid succession occurred spontaneously, during which the limbs were still stiffly extended. In eight minutes, these starts ceased, and the limbs assumed a nearly normal position; but the rabbit had now considerable difficulty in moving about, the limbs being slightly paralysed. In sixteen minutes, the rabbit lay down and rested in a crouching attitude, on the abdomen and chest. Soon after the neck muscles were unable properly to support the head, which frequently subsided on the table; but there was now no distinct evidence of exaggeration of reflex activity. In twenty-five minutes, the paralysis was so decided, that even a sitting posture could not be maintained, and the rabbit lay on the side. The respirations were infrequent and laboured, while common sensibility seemed to be unimpaired, and the heart was ascertained to be contracting with nearly normal force and rapidity. In twenty-eight minutes, some convulsive movements occurred in the body and limbs, and now the respirations were so weak as to be scarcely observable. The convulsive movements continued for other two minutes, but at the end of this period they consisted of extremely feeble spasmodic starts. In thirty-one minutes, the sensibility of the conjunctiva and cornea had dis-

appeared, the respirations had become mere infrequent gasps, but the heart was contracting at the rate of 120 beats in the minute. Death occurred thirty-two minutes after the administration. The pupils were frequently observed; they retained the same diameter during the experiment as they had immediately before it, but on the occurrence of death they contracted considerably.

After death, galvanic stimulation of the left sciatic nerve caused active movements in the left leg, and also well-marked reflex movements in the right. The exposed heart was contracting, six minutes after death, in proper rhythm, and at the rate of 100 beats in the minute.

In our experiments with frogs, we found that a dose equivalent to the  $\frac{1}{4400}$ th of the weight of the animal was sufficient to cause death. In the two experiments with Dr CHRISTISON'S conia, which we shall now describe, somewhat larger doses than that above mentioned were administered, the complete physiological action being but slowly developed with small doses.

EXPERIMENT LXI.—One-tenth of a grain of hydrochlorate of Dr CHRISTISON'S conia was dissolved in four minims of distilled water, and injected under the skin at the right flank of a frog, weighing 300 grains. The frog jumped about actively until five minutes after the administration, when it appeared to experience some difficulty in moving about, and it was observed that this difficulty was chiefly due to tonic spasm of the anterior extremities. This spasm, though by no means powerful, was sufficient to retain the extremities in a constrained perpendicular position, and in extreme extension, during the five minutes that succeeded its first appearance. In ten minutes, the frog was unable to jump, and it lay on the abdomen and chest; while the respirations had now ceased. In twenty-five minutes, it was perfectly flaccid, and the head rested on the table, but the heart's impulse was still well marked, and the rate of its contractions was forty per minute. At frequent intervals, the two posterior extremities were somewhat suddenly pushed out to extreme extension, and after remaining in this position, for one or two seconds, again partially flexed. In fifty minutes, these extension movements of the posterior extremities ceased, and irritation of the skin now caused merely faint twitches of the toes. In one hour and thirty minutes, it was impossible to excite any reflex movement whatever; and on applying galvanic stimulation to the trunk of a sciatic nerve, it was found that the motor conductivity was completely suspended. The heart was at this time contracting thirty-seven times in the minute, and the contractility of the striped muscles was unimpaired. On the following day, the frog was still in a flaccid and motionless state. The heart was contracting twenty-two times in the minute, and the nerves and muscles were in the condition last described. On the morning of the third day, rigor mortis was established.

In the next experiment, one limb was protected from the direct influence of the poison.

EXPERIMENT LXV.—Having ligatured the sciatic artery and the two principal veins at the middle of the right thigh in a frog weighing 195 grains, we injected six-tenths of a grain of hydrochlorate of Dr CHRISTISON'S conia, dissolved in four minims of distilled water, under the skin of the left flank. In two minutes, stiffness occurred in the anterior extremities. They gradually became curved inwards until the fore-paws were pressed against each other, and they were retained in this position by tonic spasm, the frog having apparently no voluntary control over them. Jumping movements could not now be accomplished, but the frog pushed itself about by vigorous contractions of the posterior extremities. In five minutes, there was marked weakness on the left posterior extremity, the right remaining unaffected. In eight minutes, the stiff incurvation of the anterior extremities had disappeared; and, now, the animal was flaccid everywhere, except in the right posterior extremity. In nine minutes, irritation of the poisoned skin was followed by barely perceptible twitches in the toes of the left posterior extremity, and extremely vigorous movements of the whole right posterior extremity. Occasionally, the right posterior extremity was extended stiffly, and retained thus for one or two seconds, the movements presenting a somewhat spasmodic appearance. In thirty minutes, the reflex contractions that followed irritation of the skin were confined to the right posterior extremity; and the heart was now contracting at the rate of twenty-four beats in the minute. In thirty-four minutes, the left sciatic nerve was exposed, the necessary dissection causing vigorous movements in the right leg, and on stimulating the nerve by an interrupted galvanic current, it was found that its motor conductivity was completely suspended, while its sensory (efferent) conductivity was retained; no movements occurring in the left posterior extremity, while energetic contractions occurred in the right (non-poisoned) posterior extremity. The contractility of the poisoned muscles was still unimpaired. Irritation of the skin in the poisoned region excited reflex movements of the right (non-poisoned) posterior extremity until two hours and fifteen minutes after complete paralysis had occurred in the poisoned motor nerves; but ten minutes after this, reflex movements could not be excited. The frog did not recover from the poisoning.

A considerable interval occurred, therefore, between the complete suspension of conductivity in the motor nerves and the loss of the reflex function of the spinal cord; and, accordingly, it is evident that the condition of paralysis and flaccidity caused by Dr CHRISTISON'S conia is mainly dependent on its action on the motor nerves. The experiment further shows that this paralysing action is restricted, in the first place, at least, to the peripheral terminations of the motor nerves.

In the last two experiments, we have shown that certain slight spasmodic symptoms are produced in frogs by conia. It is probable that these represent the more violent convulsions that occur in mammals, and to which we have drawn

attention in the description of an experiment on a rabbit (Experiment LIV.). Both in frogs and in mammals those spasmodic symptoms appear at an early stage in the poisoning.

Although our main object, in describing the action of *hydrochlorate of Mr MORSON'S conia*, is to point out certain peculiarities in the mode in which it produces paralysis, it may be advisable that we should also give some evidence in support of the assertion that its lethal activity is much less than that of the hydrochlorate of Dr CHRISTISON'S conia. We shall thus be able to show clearly that both the nature of the action and the lethal activity of various specimens of conia may differ considerably, while the symptoms produced by them are very similar in character.

In the following experiment, a dose below the minimum fatal was given.

EXPERIMENT LXVII.—We dissolved seven-tenths of a grain of hydrochlorate of Mr MORSON'S conia in fifteen minims of distilled water, and injected the solution under the skin at the back of a rabbit, weighing three pounds and three ounces and-a-quarter. The animal remained quiet until six minutes, when it moved about in an excited manner, and during these movements it was observed that the four limbs were abnormally and stiffly extended. This stiff extension of the limbs gradually became more marked, until it seriously impeded the movements of the rabbit. In fourteen minutes, a slight touch of the skin excited a sudden spasmodic start of the whole body. In twenty-five minutes, the stiffness of the limbs had greatly diminished, and now it was obvious that a slight degree of paralysis was present. From this time, these symptoms gradually but slowly disappeared; and the rabbit was jumping about actively one hour after the injection.

In the next experiment, the dose was a fatal one.

EXPERIMENT LXVIII.—One grain of hydrochlorate of Mr MORSON'S conia, dissolved in twenty minims of distilled water, was injected under the skin at the right side of a rabbit, weighing four pounds and one ounce. The symptoms were very similar to those observed with two-tenths of a grain of hydrochlorate of Dr CHRISTISON'S conia. Stiffness of the limbs and tremors occurred in six minutes; evidence of exaggeration of the reflex activity was obtained in eight minutes; decided paralysis was present in thirteen minutes; and, after the occurrence of a number of attacks of convulsive tremors, a condition of flaccid motionlessness, interrupted by infrequent respiratory gasps, supervened, which terminated in death, thirty-three minutes after the administration.

The general character of the symptoms produced by Mr MORSON'S conia in frogs was likewise found to be the same as that produced by Dr CHRISTISON'S conia; and, in proof of this, we shall briefly describe an experiment with a fatal dose of the former.

EXPERIMENT LXXVI.—We injected three-tenths of a grain of hydrochlorate of

Mr MORSON'S conia, dissolved in four minims of distilled water, under the skin at the right flank of a frog, weighing 140 grains. The effects were very speedily produced; for in less than two minutes, the frog was quite unable to jump, and a decided degree of general paralysis was present. In four minutes, some stiffness was present in the anterior extremities and the fingers, causing the latter to be continuously and stiffly elevated until eight minutes after the poisoning. At nine minutes, the frog was in a flaccid state, but, still, somewhat vigorous movements were spontaneously made in the posterior extremities. These consisted at first of extension and flexion movements of a normal character; soon, however, they became spasmodic, the extension being prolonged; and, at fourteen minutes, they assumed an almost tetanic character, extreme extension being maintained on each occasion for nearly two seconds. In twenty-five minutes, the spontaneous movements of the posterior extremities were extremely feeble, and in thirty minutes, they altogether ceased. In thirty-five minutes, the frog was perfectly flaccid and motionless, and irritation, even of a severe character, failed to excite any reflex movement whatever. The heart was now contracting in normal rhythm, at the rate of twenty-four beats in the minute.

On the following morning, the frog was dead and in rigor. In this experiment, the dose (equivalent to the  $\frac{1}{67}$ th of the frog's weight) was considerably above the minimum fatal. Our experiments have shown that in frogs, as in mammals, this sample of conia is much less active than that of Dr CHRISTISON, for the smallest dose which we have found to produce death is equivalent to the  $\frac{1}{850}$ th of the weight of the frog used.

The last experiment shows that after a fatal dose of Mr MORSON'S conia the predominant symptoms are those of paralysis. We shall now describe some experiments performed for the purpose of determining by what action or actions this paralysis is produced.

EXPERIMENT LXXI.—We ligatured the blood-vessels in the right thigh of a frog, weighing 112 grains, and, immediately afterwards, injected one-tenth of a grain of hydrochlorate of Mr MORSON'S conia, dissolved in five minims of distilled water, under the skin of the left flank. Complete general paralysis was quickly produced in the poisoned regions. In twenty-seven minutes after the injection, irritation of the skin caused active and apparently tetanic reflex movements of the right posterior extremity, but it failed to cause any movement in the parts to which the poison had access. In one hour, the left sciatic nerve was exposed, and subjected to galvanic stimulation; with the result, that no movement whatever was thereby excited in the left posterior extremity, or in any poisoned part, while pretty active reflex movements were excited in the right posterior extremity. This condition continued until one hour and twenty minutes after the injection, but at this time the reflex movements that were excited in the right posterior extremity were extremely feeble. In one hour and thirty-five minutes,

it was impossible to excite any reflex movement, although strong irritations were applied to the skin of both the poisoned and non-poisoned regions, and to the left sciatic nerve. At this time, the contractility of the muscles was unimpaired, the conductivity of the right (non-poisoned) sciatic nerve was retained, and the contractions of the heart were at the rate of twenty in the minute.

It is obvious that, in this experiment, the motor nerves were completely paralysed before the reflex function of the spinal cord was suspended.

EXPERIMENT LXXVIII.—After ligaturing the blood-vessels in the right thigh of a frog, weighing 110 grains, we injected a solution containing three-tenths of a grain of hydrochlorate of Mr MORSON'S conia, under the skin at the left flank. In forty-nine minutes, no reflex movements could be produced by irritation of the skin, whether of the poisoned or non-poisoned regions. The left sciatic nerve was now exposed; and, on galvanising its trunk, it was found that feeble twitches occurred in the toes of the left (poisoned) posterior extremity, while no reflex movements occurred in the right (non-poisoned) posterior extremity, or in any part. It was ascertained, at the same time, that the muscles everywhere contracted freely when directly stimulated, that the right sciatic nerve retained its functional activity, and that the heart's beats were occurring at the rate of eighteen in the minute. The condition of retained, though impaired, conductivity of the poisoned motor nerves, of retained conductivity of the non-poisoned (right) sciatic nerve, of apparently unimpaired contractility of the muscles, coexisting with complete suspension of the reflex function of the spinal cord, continued until one hour and ten minutes after the administration of the poison. At one hour and fourteen minutes, however, the left (poisoned) sciatic nerve was found to be completely paralysed.

We learn from this experiment that Mr MORSON'S conia may so energetically affect the spinal cord, as to suspend its reflex function, before the motor nerves are completely paralysed. The motor nerves, certainly, were affected at an early stage, and, even before the suspension of the reflex function of the spinal cord, their conductivity was so far impaired, that merely very feeble twitches could be excited by galvanising them. The general paralysis that was present in the poisoned region was, no doubt, to a considerable extent due to their impaired activity. Still, the action of this substance on the reflex function of the spinal cord was, at least, as important a cause of paralysis as the action on the motor nerves. In Experiment LXXI., likewise, both actions co-operated in the production of the paralysis, but the motor nerves were paralysed in it before the reflex function of the spinal cord was completely suspended.

These two experiments represent two varieties of action, which we have observed in our experiments with Mr MORSON'S conia. In both, the motor nerves and the spinal cord were markedly affected; but in the one, complete loss of function occurred in the motor nerves before it occurred in the spinal cord, and

in the other, complete loss of function occurred in the spinal cord before it occurred in the motor nerves.

The following Table contains a short account of these experiments:—

No. of Experiment.	Weight of Frog.	Relation of Weight of Frog to Dose.	Dose.	Effect.
LXXI.	112 grs.	$\frac{1}{1120}$ th.	0·1 gr.	Complete paralysis of <i>motor nerves</i> in 1 hour. " " <i>reflex function of spinal cord</i> in 1 ho. 35 min.
LXXII.	200 grs.	$\frac{1}{1000}$ th.	0·2 gr.	Complete paralysis of <i>motor nerves</i> in 40 minutes. " " <i>reflex function of spinal cord</i> in 1 ho. 10 min.
LXXIII.	256 grs.	$\frac{1}{256}$ d.	0·3 gr.	Complete paralysis of <i>motor nerves</i> in 1 hour 25 minutes. " " <i>reflex function of spinal cord</i> in 1 ho. 25 min.
LXXIV.	290 grs.	$\frac{1}{290}$ th.	0·36 gr.	Complete paralysis of <i>motor nerves</i> in 18 minutes. " " <i>reflex function of spinal cord</i> in from 2 to 18 hours.
LXXV.	110 grs.	$\frac{1}{550}$ th.	0·2 gr.	Complete paralysis of <i>reflex function of spinal cord</i> in 47 minutes. " " <i>motor nerves</i> in from 1 ho. 20 min. to 22 ho.
LXXVI.	140 grs.	$\frac{1}{140}$ th.	0·3 gr.	Complete paralysis of <i>reflex function of spinal cord</i> in 35 minutes. " " <i>motor nerves</i> in from 1 ho. 20 min. to 21 ho.
LXXVII.	110 grs.	$\frac{1}{385}$ th.	0·3 gr.	Complete paralysis of <i>motor nerves</i> in 30 minutes. " " <i>reflex function of spinal cord</i> in 1 ho. 30 min.
LXXVIII.	110 grs.	$\frac{1}{385}$ th.	0·3 gr.	Complete paralysis of <i>reflex function of spinal cord</i> in 49 minutes. " " <i>motor nerves</i> in 1 hour 14 minutes.
LXXIX.	115 grs.	$\frac{1}{287}$ th.	0·4 gr.	Complete paralysis of <i>reflex function of spinal cord</i> in 28 minutes. " " <i>motor nerves</i> in 35 minutes.

In the experiments in this Table, in which doses between the  $\frac{1}{1120}$ th and the  $\frac{1}{805}$ th of the frog's weight were given, the complete paralysis of the motor nerves occurred *before* the complete paralysis of the reflex function of the spinal cord; and in the experiments in which doses between the  $\frac{1}{550}$ th and the  $\frac{1}{287}$ th were given (excepting Experiment LXXVII.), the complete paralysis of the motor nerves occurred *after* that of the reflex function of the spinal cord. As we have already said, these two actions are, however, of nearly equal energy; for, at the time when the one has been completed, the other is usually nearly so. Experiment LXXVII., in which a dose equivalent to the  $\frac{1}{385}$ th was administered, conspicuously illustrates this nearly simultaneous progress, by its occurrence as an exception to the order in which the two actions are usually completed after such a dose.

We have accordingly shown that Mr MORSON'S conia differs from that of Dr CHRISTISON, both in lethal activity and in mode of action. We shall endeavour to explain these differences in a subsequent portion of this paper; the explanation of the varieties in the mechanism of the paralyzing action of Mr MORSON'S conia being dependent on results obtained by our experiments with methyl-conia.

*Hydrochlorate of methyl-conia* ( $C_8H_{14}(CH_3)NHCl$ ).—Iodide of methyl acts readily upon conia, producing a syrupy or crystalline substance, which is a mixture

of hydriodate of methyl-conia and iodide of dimethyl-conium—the former produced from the normal conia, and the latter from the methyl-conia. If the conia be free from water, this action is very rapid, and as heat is developed it is necessary that the vessel should be kept cool; if the conia contain water, the chemical change is very slowly effected. Caustic potash is added to the mixture, and it decomposes the hydriodate of methyl-conia, setting the base free as an oil, while it leaves the iodide of dimethyl-conium unacted upon. The methyl-conia was converted, after separation, into a hydrochlorate, which is extremely deliquescent, and has a brownish, semicrystalline appearance.

We found that this substance possesses a poisonous (lethal) activity, considerably greater than that of Mr MORSON'S conia, but nearly equal to that of Dr CHRISTISON'S conia; for two-tenths of a grain, exhibited by subcutaneous injection, speedily caused death in a rabbit, and a dose, equivalent to the  $\frac{1}{4560}$ th of the weight of the animal, is about the minimum fatal dose for a frog. The general character of the symptoms is likewise similar to that of Dr CHRISTISON'S conia, and, therefore, to that also of Mr MORSON'S; but the causation of these symptoms rather resembles that of the latter than of the former conia. Paralysis is the main symptom; and a careful examination, by experiments on frogs, of the mechanism by which this symptom is produced, showed that it is a result of actions on the motor nerves and spinal cord, and that with large doses the former action is completed before the latter, while with small doses the latter action is completed before the former.

We shall, in the first place, describe the symptoms that appeared in a rabbit, after the administration of a fatal dose.

EXPERIMENT LXXXI.—Two-tenths of a grain of hydrochlorate of methyl-conia was dissolved in twenty-five minims of distilled water, and injected under the skin at the right flank of a healthy rabbit, weighing two pounds and ten ounces and-a-half. The rabbit moved about in a normal manner until four minutes after the injection, when the movements became constrained, and it was observed that this was owing to stiff extension of the four limbs. A slight touch of the animal caused a series of rapid tremors, during which, as well as at other times, the body was elevated on the stiffly extended limbs. This somewhat remarkable condition continued without change until ten minutes, when the stiffness of the posterior extremities disappeared; but, in place of assuming a normally flexed position, these extremities became flaccidly abducted; and, when the animal moved about, they trailed behind it in a somewhat powerless manner. In eighteen minutes, the symptoms of exaggerated reflex activity, and the spasmodic extension of the anterior extremities had disappeared; and, now, there was so great a degree of general paralysis present, that the rabbit was unable to move about, and it lay quietly on the abdomen and chest. In nineteen minutes, the neck muscles could no longer continuously support the head, which, soon after, rested on the table. In twenty

minutes, the respiratory movements were laboured, and they occurred only twenty-four times in the minute. The rabbit now lay on the side, quite flaccid and powerless; and, at times, a series of slight tremors occurred. The respirations gradually became weaker and less frequent, the common sensibility disappeared, and death occurred, twenty-two minutes after the administration.

Three minutes after death, the exposed heart was contracting in normal rhythm, at the rate of seventy-four beats in the minute; and it was ascertained that the conductivity of the afferent and efferent nerve fibres of the sciatic nerves, the reflex function of the spinal cord, and the contractility of the striped muscles were still retained.

This description is sufficient to show that in rabbits hydrochlorate of methyl-conia produces very similar effects to hydrochlorate of conia. That this similarity also occurs in frogs will be seen from the following experiment.

EXPERIMENT LXXXVI.—A solution containing six-hundredths of a grain of hydrochlorate of methyl-conia, in five minims of distilled water, was injected under the skin at the right flank of a frog, weighing 185 grains. In ten minutes, a slight degree of stiffness, with rigid elevation of the fingers, was present in the anterior extremities, but the frog still jumped about actively. Gradually the movements became less energetic; some sprawling occurred; and, soon, the frog lay on the abdomen and chest, quite unable to jump or move about. In twenty minutes, the power of voluntary movement was completely lost, and irritation of the skin caused but feeble reflex twitches in both posterior extremities. The frog remained in this state until forty-seven minutes after the administration; but in fifty minutes, the most severe stimulation of the skin was unable to excite any reflex movement whatever. The right sciatic nerve was now exposed and galvanised; twitches were thereby excited in the right toes, but these were unaccompanied by any movement in the left posterior extremity or elsewhere. At this time the cardiac impulse was of fair strength, and the contractions of the heart were occurring at the rate of forty in the minute.

On the morning of the following day, the frog was dead and in rigor.

These symptoms agree closely in their general character with those described after corresponding doses of hydrochlorate of Dr CHRISTISON'S conia (Experiment LXI.), and of Mr MORSON'S conia (Experiment LXXVI.); but the slight spasmodic symptoms that appeared in the anterior extremities were not invariably observed in our other experiments with this substance. Paralysis is shown to be the predominant symptom, and the causation of this paralysis, after the small fatal dose exhibited in this experiment, appears to be due to an abolition of the reflex function of the spinal cord, rather than to a suspension of the conductivity of motor nerves. The action of hydrochlorate of methyl-conia, therefore, apparently resembles that of hydrochlorate of MORSON'S conia; and we shall see from the following experiments that the special variations pointed out as occurring with

different doses of conia obtained from that chemist, occur also with different doses of hydrochlorate of methyl-conia.

EXPERIMENT LXXXVIII.—Immediately after ligaturing the blood-vessels at the upper part of the right thigh in a frog, weighing 140 grains, we injected a solution, containing one-tenth of a grain of hydrochlorate of methyl-conia, under the skin at the left flank. In thirteen minutes, the frog was flaccid, and no voluntary movements occurred in the poisoned region; but vigorous movements, consisting of extreme and somewhat spasmodic extensions, occurred at frequent intervals in the right (non-poisoned) posterior extremity. Irritation of the skin in the poisoned region now caused merely feeble twitches in the left (poisoned) posterior extremity, and energetic movements in the right posterior. In fifty minutes, however, no reflex movement could be excited anywhere by irritation of the skin. The left sciatic nerve was exposed and subjected to galvanic stimulation, with the result that, while well-marked movements occurred in the left posterior extremity, no movement occurred in the right (non-poisoned). It was at the same time ascertained that the motor conductivity of the right sciatic nerve was not appreciably impaired, even in that part of the trunk exposed to the direct action of the poison; that the poisoned muscles retained their contractility; and that the heart was contracting, in normal rhythm, at the rate of twenty beats per minute. Several observations were made during the succeeding fifty minutes, but no change had occurred during this time, with the exception of a slight diminution in the rate of the heart's contractions.

On the following morning, the frog was dead.

EXPERIMENT XCI.—The blood-vessels were tied at the lowest third of the right thigh of a frog, weighing 200 grains, and two-tenths of a grain of hydrochlorate of methyl-conia, dissolved in four minims of distilled water, was then injected under the skin at the left flank. The first symptom that was observed occurred in three minutes, and consisted of a stiff extension of the anterior extremities, causing unnatural elevation of the thorax. After a few seconds, this symptom was modified to the extent that the anterior extremities became rigidly incurved, with the fore-paws in contact with each other. Vigorous jumping movements were still attempted, but as the anterior extremities took no part in these, they were very imperfect, and frequently resulted in the frog falling on one side. In ten minutes, this spasmodic condition of the anterior extremities disappeared, and now the frog lay flaccid on the lower jaw, chest, and abdomen. The power of voluntary movement seemed to be suspended in the poisoned region, but it was retained in the non-poisoned (right posterior extremity), where vigorous and somewhat spasmodic movements of extreme extension frequently occurred. In thirty-three minutes, irritation of the skin caused energetic reflex movements in the right posterior extremity, but no movement in any part of the poisoned region. In thirty-four minutes, the left sciatic nerve was exposed, and

it was found that its motor conductivity was completely suspended—galvanism of its trunk causing no contractions in the left posterior extremity; while its sensory conductivity was retained—galvanism causing energetic reflex movements in the right (non-poisoned) posterior extremity. The heart was now contracting twenty-four times in the minute; and the contractility of the striped muscles was apparently unimpaired. It was possible to excite reflex movements in the right posterior extremity by stimulating the skin of the poisoned region, until one hour and forty minutes after the administration. Very soon after this time, the activity of the reflex function was completely suspended. Irritation of the skin in the poisoned and non-poisoned regions, as well as galvanic stimulation of the poisoned (left) sciatic nerve, caused no movement, notwithstanding that the non-poisoned (right) sciatic nerve and muscles, and even the trunk of the right sciatic nerve above the position of the ligatures, retained their functional activity.

These two experiments are selected from nine which were made on frogs with different relative doses, and in which distinct evidence was obtained of the primary cause of the paralysis. The conductivity of the motor nerves was suspended before the reflex function was abolished in experiments in which doses were administered, equivalent to the  $\frac{1}{350}$ th, the  $\frac{1}{875}$ th, the  $\frac{1}{960}$ th, the  $\frac{1}{1000}$ th, and the  $\frac{1}{1100}$ th of the weight of the frog employed; while the activity of the reflex function was abolished before the motor nerves were paralysed in experiments in which doses were administered, equivalent to the  $\frac{1}{1335}$ th, the  $\frac{1}{1400}$ th, the  $\frac{1}{1950}$ th, and the  $\frac{1}{3083}$ d of the weight of the frog.

The details we have narrated of Experiments LXXXVIII. and XCI. demonstrate that paralysis of the motor nerves is due to an action on their peripheral terminations; as well when this paralysis precedes the abolition of the reflex function, as when it occurs subsequently thereto. It would appear that the abolition of the reflex function depends, at least in part, on an action on the spinal cord; for these experiments show that, after its occurrence, irritation of the skin of a region protected from the direct action of the poison, or galvanic stimulation of the trunk of a mixed nerve likewise protected from the direct action of the poison, does not cause any reflex movement, notwithstanding that the motor nerves and muscles everywhere retain their functional activity.

We have accordingly shown that conia and methyl-conia produce very similar symptoms; the more prominent of which are spasms and paralysis.

Our analysis of the mode in which the paralysis is produced, has resulted in proving its dependence on an action on the motor nerves and on the spinal cord. The rate at which each of these actions is produced by the substances examined (estimating this by the time of completion) varies in a remarkable, and, at first sight, perplexing manner. In the case of the conia prepared by Dr CHRISTISON, the former of these actions is the more powerful; while in that prepared by Mr MORSON, and in methyl-conia, the two are nearly equally prominent. In a series

of experiments on frogs with varying doses, it was found that Dr CHRISTISON'S conia invariably produced complete paralysis of the motor nerves before that of the reflex function of the spinal cord; that Mr MORSON'S conia usually produced complete paralysis of the motor nerves before that of the reflex function of the spinal cord in those experiments of the series where the dose was small, and complete paralysis of the reflex function of the spinal cord before that of the motor nerves where the dose was large; and that methyl-conia produced complete paralysis of the reflex function of the spinal cord before that of the motor nerves in those experiments where the dose was small, and complete paralysis of the motor nerves before that of the reflex function of the spinal cord where the dose was large.

As already mentioned, our chemical examination of the two specimens of conia proved that that of Dr CHRISTISON contains a much smaller proportion of methyl-conia than that of Mr MORSON. Our physiological examination has confirmed this result; for the action of the latter specimen of conia more closely resembles that of methyl-conia than the former. In other words, the conia containing the smallest proportion of methyl-conia acts most purely as a paralyser of motor nerves. It seems a legitimate deduction from this, that conia altogether free from methyl-conia (*i.e.*, *normal conia*) will be free also from all spinal action, and will, accordingly, produce paralysis solely by influencing the motor nerves.\*

Our experiments have shown that the lethal activity of Dr CHRISTISON'S conia is considerably greater than that of Mr MORSON'S. The comparatively feeble potency of the latter cannot be explained by its containing a large proportion of methyl-conia, for the activity of this substance is about the same as that of Dr CHRISTISON'S conia; it may be due to the presence of ammonia.

*Iodide of dimethyl-conium.*—When a moderately dilute solution of caustic potash is added to the mixture of iodide of dimethyl-conium and hydriodate of methyl-conia, the latter salt, as stated above, is decomposed, while the former remains in solution, and may be purified by crystallisation from strong aqueous, caustic potash. It is tolerably soluble in hot solutions of caustic potash, but on cooling the solution, it separates in the form of colourless silky needles. It is readily soluble in water, and its composition may be represented by the formula,  $C_8H_{14}(CH_3)NCH_3I$ .

In various experiments, we have administered to rabbits, by subcutaneous injection, doses of half-a-grain, two grains, two grains and-a-half, three, four, and five grains. No obvious effects were produced by half-a-grain, or by two grains; slight temporary paralysis was produced by two grains and-a-half, and death by three, four, and five grains respectively. It is, therefore, obvious that the

\* We have not as yet succeeded in obtaining a pure specimen of *normal conia*; and the quantities of ordinary conia at our disposal have not been sufficient to enable us to attempt a separation of normal conia from methyl-conia.

lethal activity of iodide of dimethyl-conium is greatly less than that of either conia or methyl-conia. That the character of the symptoms it produces is also different, will be seen from the following experiment.

EXPERIMENT XCVIII.—Having dissolved three grains of iodide of dimethyl-conium in forty minims of distilled water, we injected the solution under the skin of a rabbit, weighing four pounds. The animal remained sitting quietly for more than half an hour, during which time no symptom was observed. In thirty-two minutes, however, it became restless, and faint tremors occurred. Soon, it had difficulty in moving about; and after some endeavours to maintain a sitting posture, it lay down on the abdomen and chest. In forty-one minutes, the head rested on the table; and at this time the respirations were shallow, and at the increased rate of 144 in the minute. The rabbit remained quietly in this position until one hour and four minutes, when it succeeded, after some efforts, in rising on its limbs, but, being unable to support itself thus, it again lay down on the abdomen and chest, with the head resting on the table. The respirations were now eighty-four in the minute. In one hour and eleven minutes, slight tremors again occurred, and then the rabbit became perfectly flaccid, and the respirations infrequent and laboured. In one hour and twelve minutes, the respirations were mere gasps, occurring at the rate of about twelve in the minute; and soon after they became so shallow as to be hardly visible. In one hour and fifteen minutes, a few twitches occurred in the muscles of the face, and in a few seconds the rabbit was dead.

In the autopsy, the motor nerves and muscles were found active, twelve minutes after death; but at this time the exposed heart was found to be contracting irregularly and feebly.

In this experiment, we frequently tested the reflex excitability, but never observed the slightest evidence of its being increased.

We shall now briefly describe the experiment in which we administered five grains.

EXPERIMENT C.—A solution, containing five grains of iodide of dimethyl-conium, in fifty minims of distilled water, was injected under the skin of a rabbit, which weighed three pounds and six ounces and-a-half. As in the previous experiment, the first effects observed were a number of restless, uneasy movements, which occurred in eleven minutes. Soon afterwards, paralytic symptoms appeared; and in twenty minutes, these had so far advanced that the rabbit lay flaccid on the abdomen, chest, and lower jaw, while irritation of the skin was followed by extremely feeble movements of the head, or one or other of the extremities. In twenty-four minutes, the head fell over on the side, and rested thus on the table; and the respirations were infrequent, shallow, and laboured. After this, the respirations became greatly more infrequent and laboured, until they altogether ceased, thirty-one minutes after the injection of the poison.

In this experiment, likewise, we failed in discovering the slightest evidence of exaggeration in the reflex excitability, or any spasmodic symptom.

These descriptions are sufficient to show that iodide of dimethyl-conium acts simply as a paralysing agent, and that it does not produce any spasmodic effects in rabbits.

The general symptoms that appear in frogs after the administration of a fatal dose are illustrated in the following experiment.

EXPERIMENT CVII.—Having dissolved one-tenth of a grain of iodide of dimethyl-conium in four minims of distilled water, we injected the solution under the skin at the right flank of a frog, weighing 150 grains. In two minutes and thirty seconds, a slight degree of paralysis was observed in the anterior extremities, which were scarcely able to support the chest; and the jumping movements were now less active than before. Quickly, the paralysis became more decided; until at six minutes, the frog was lying on the abdomen and the lower jaw. The respiratory movements of the chest had now ceased, while those of the throat continued for several minutes longer. In nine minutes, irritation of the skin produced merely feeble movements in the posterior extremities; and in thirty minutes, it was impossible to excite any reflex movement whatever, even by severe irritation of the skin. The right sciatic nerve was now exposed in the thigh, and stimulated by an interrupted galvanic current, but no muscular contractions were thereby produced, although the muscles contracted actively when the electrodes were directly applied to their surfaces. At this time, the heart's impulse was of fair strength, and the beats occurred twenty-two times in the minute.

On the following day, the frog was found to be in the condition last noted; but on the third day, the contractility of the muscles had disappeared, and the heart's contractions had ceased.

In many other experiments on frogs, the same general phenomena were observed. The spasmodic symptoms to which we have drawn attention in our description of the effects of conia and of methyl-conia were entirely absent in our experiments with iodide of dimethyl-conium; and, accordingly, the symptoms we observed were those of paralysis only. We made several experiments to determine what structures are influenced in the production of this paralysis.

EXPERIMENT CVIII.—Immediately after ligaturing the blood-vessels at the upper part of the right thigh of a frog, weighing 192 grains, we injected three-twentieths of a grain of iodide of dimethyl-conium, dissolved in four minims of distilled water, under the skin of the left flank. In one minute thereafter, the movements of the frog had become somewhat feeble, the poisoned extremities being obviously weakened. In two minutes and thirty seconds, the frog lay on the abdomen and lower jaw, apparently unable to execute any voluntary movements with any part of the body except the right (non-poisoned) posterior

extremity, and there were no respiratory movements whatever. In nine minutes, stimulation of the skin in any region was followed by energetic reflex movements in the right posterior extremity, but no movements occurred in the poisoned region. In fourteen minutes, the left sciatic nerve was stimulated by an interrupted galvanic current, and, although active reflex movements of the right (non-poisoned) posterior extremity were thereby excited, no movement occurred in the left (poisoned) posterior extremity, or in any other part of the poisoned region. The heart's impulse was, at this time, found to be of fair strength, and occurring forty-two times in the minute; and the muscles contracted vigorously on direct stimulation. In three hours, the condition of the frog was the same as last noted, excepting that the rate of the heart's contractions had diminished to thirty-eight in the minute. The observations were now interrupted until the following morning, when the frog was found dead and in rigor.

In many other similar experiments with different doses of this substance, the symptoms and mode of action were exactly the same as in the last experiment. They show that the paralysis produced by dimethyl-conium is dependent on an action on the motor nerves, primarily restricted to the peripheral terminations. Even after the administration of a fatal dose, we have never observed any action on the spinal cord, beyond its necessary implication in the progress towards death. On the other hand, in experiments where doses below the minimum fatal, and therefore considerably smaller than in Experiments CVII. and CVIII., were given, the condition of complete paralysis of the peripheral terminations of the motor nerves existed along with retained functional activity of the spinal cord and sensory nerves, for periods protracted over many hours. Thus, in an experiment where the dose was equivalent to the  $\frac{1}{2300}$ th of the frog's weight (Experiment CIII.), the poisoned motor nerves remained completely paralysed for more than twenty-six hours, while, during this time, the poisoned sensory nerves and the spinal cord retained their functional activity.

We conclude from our experiments, that in physiological action iodide of dimethyl-conium differs from conia and methyl-conia in being entirely free from spasmodic and spinal-paralysing actions.

It is shown in the following Table of minimum fatal doses, that iodide of dimethyl-conium is much less active than either conia or methyl-conia:—

No. of Experiment.	Substance Employed.	Animal and its Weight.	Dose, by subcutaneous injection.	Relation of Dose to Weight of Animal.	Effect.
LIII.	Hydrochlorate of Dr CHRISTISON'S conia.	Rabbit, 3 lbs. 14½ oz.	0·1 gr.	$\frac{1}{372370}$ th.	Slight degree of stiffness in the limbs, followed by recovery.
LIV.	Do.	Do., 3 lbs. 6½ oz.	0·2 gr.	$\frac{1}{110180}$ th.	Death, in 32 minutes.
LXVI.	Hydrochlorate of Mr MORSON'S conia.	Do., 2 lbs. 12 oz.	0·2 gr.	$\frac{1}{88280}$ th.	None.
LXVIII.	Do.	Do., 4 lbs. 1 oz.	1 gr.	$\frac{1}{37370}$ th.	Death, in 33 minutes.
LXXX.	Hydrochlorate of methyl-conia.	Do., 3 lbs. 14½ oz.	0·1 gr.	$\frac{1}{372370}$ th.	None.
LXXXI.	Do.	Do., 2 lbs. 10½ oz.	0·2 gr.	$\frac{1}{87680}$ th.	Death, in 22 minutes.
XCVII.	Iodide of dimethyl-conium.	Do., 3 lbs. 6½ oz.	2·5 grs.	$\frac{1}{8834}$ th.	Slight paralysis, followed by recovery.
XCVIII.	Do.	Do., 4 lbs.	3 grs.	$\frac{1}{8833}$ d.	Death, in 1 hour and 15 minutes.

[This investigation into the physiological action of atropia and its methyl and ethyl derivatives, and of conia and its methyl derivatives, was commenced in July 1867; but, after performing a number of experiments, we considered it advisable to postpone the further examination of these substances until we had finished that portion of our researches which is published in Vol. XXV. Part 1 of the "Transactions." Although an abstract was read before the Society on the 18th of January 1869, this paper was not delivered to the Secretary for publication until the month of October.]

The subjoined Tabular Summary contains the leading facts of all the Experiments included in the present part of this investigation.

TABULAR SUMMARY OF EXPERIMENTS.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
I.	Iodide of methyl-atropium.	Dog, 8 lbs. 6 oz.	Subcutaneously.	10 grs.	Death, preceded by paralysis, &c.*	6 minutes.	26 minutes.	Same dog as survived 10 grs. of sulphate of atropia (Experiment XXII.)
II.	Do.	Rabbit, 2 lbs. 5 oz.	Do.	0.05 gr.	Paralysis of the vagi nerves, followed by recovery.			
III.	Do.	Do., 2 lbs. 7 oz.	Do.	0.1 gr.	Do. do.			
IV.	Do.	Do., 3 lbs. 9 oz.	By injection into a facial vein.	0.5 gr.	Paralysis, followed by recovery.			
V.	Do.	Do., 3 lbs. 10 oz.	Subcutaneously.	2 grs.	Slight tremors and paralysis, followed by recovery.	8 minutes.	1 hour and 20 minutes.	
VI.	Do.	Do., 3 lbs. 9 oz.	By injection into a facial vein.	2 grs.	Paralysis and slight tremors, followed by recovery.	2 minutes.	About 29 min.	Same rabbit as in Experiment IV.
VII.	Do.	Do., 3 lbs. 13½ oz.	Subcutaneously.	2.5 grs.	Paralysis and fibrillary twitches, followed by recovery.	16 minutes.	More than 1 hour and 50 minutes.	
VIII.	Do.	Do., 3 lbs. 10 oz.	Do.	3 grs.	Death, preceded by paralysis.	6 minutes.	52 minutes.	Same rabbit as in Experiment V.
IX.	Do.	Do., 4 lbs. 13 oz.	Do.	4 grs.	Death, preceded by paralysis and slight tremors.	10 minutes.	44 minutes.	
X.	Do.	Do., 4 lbs. 2 oz.	Do.	8 grs.	Death, preceded by paralysis and fibrillary twitches.	3 minutes.	6 minutes.	
XI.	Do.	Frog, 392 grs.	Do.	0.005 gr.	Incomplete general paralysis, followed by recovery.	3 minutes.	About 1 hour and 20 min.	
XII.	Do.	Do., 465 grs.	Do.	0.01 gr.	Complete paralysis of motor nerves, followed by recovery.	1 minute.	More than 3 hours.	
XIII.	Do.	Do., 455 grs.	Do.	0.025 gr.	Complete paralysis of motor nerves, followed by recovery.	Less than 4 minutes.	More than 6 hours and less than 18.	
XIV.	Do.	Do., 482 grs.	Do.	0.05 gr.	Complete paralysis of motor nerves, followed by recovery.	1 minute.	More than 26 hours and less than 40.	

\* Dilatation of the pupils was observed in all the experiments on mammals with atropia and its methyl and ethyl derivatives.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
XV.	Iodide of methyl-atropium.	Frog, 422 grs.	Subcutaneously.	0.1 gr.	Complete paralysis of poisoned motor nerves, followed by recovery.	Less than 2 minutes.	More than 72 hours and less than 94.	Vessels of the right posterior extremity were ligatured.
XVI.	Do.	Do., 204 grs.	Do.	0.1 gr.	Death, preceded by complete paralysis of motor nerves.	Less than 2 minutes.	About 6 days.	Interval of regained motility before death.
XVII.	Do.	Do., 300 grs.	Do.	0.15 gr.	Death, preceded by complete paralysis of the poisoned motor nerves.	Less than 3 minutes.	About 3 days.	Do., vessels of the left posterior extremity were ligatured.
XVIII.	Do.	Do., 300 grs.	Do.	0.2 gr.	Death, preceded by complete paralysis of the poisoned motor nerves.	Not noted.	Not noted.	Vessels of the right gastrocnemius muscle were ligatured.
XIX.	Do.	Do., 260 grs.	Do.	0.3 gr.	Death, preceded by complete paralysis of motor nerves.	About 2 min.	About 6 days.	
XX.	Do.	Do., 398 grs.	Do.	0.5 gr.	Death, preceded by complete paralysis of motor nerves.	Not noted.	About 2 days.	
XXI.	Do.	Do., 238 grs.	Do.	0.3 gr.	Death, preceded by complete paralysis of motor nerves.	Not noted.	About 2 days.	
XXII.	Sulphate of atropia.	Dog, 8 lbs. 6 oz.	Do.	10 grs.	Paralysis, spasms, and frequent tetanic convulsions, followed by recovery.	About 7 min.	More than 9 hours and less than 24.	Same dog as in Experiment I.
XXIII.	Do.	Rabbit, 2 lbs. 5 oz.	Do.	15 grs.	Diuresis, catharsis, and languor, followed by recovery.	About 15 min.	More than 3 hours and less than 9.	
XXIV.	Do.	Frog, 490 grs.	Do.	0.5 gr.	Paralysis and tetanic convulsions, followed by recovery.	Less than 15 minutes.	About 6 days.	
XXV.	Do.	Do., 215 grs.	Do.	0.25 gr.	Death, preceded by paralysis.	8 minutes.	Not noted.	Vessels of one posterior extremity were tied.
XXVI.	Iodide of methyl-atropium.	Rabbit, 4 lbs.	By stomach.	20 grs.	None (except on pupils).			
XXVII.	Do.	Do., 3 lbs. 12 oz.	Do.	30 grs.	None (except on pupils).			
XXVIII.	Sulphate of atropia.	Do. 3 lbs. 9½ oz.	Do.	30 grs.	None (except on pupils).			

TABULAR SUMMARY OF EXPERIMENTS—continued.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
XXIX.	Sulphate of methyl-atropium.	Rabbit, 3 lbs. 8 oz.	Subcutaneously.	0.5 gr.	Paralysis of the vagi nerves, &c., followed by recovery.	Less than 7 minutes.	Not noted.	
XXX.	Do.	Do., 3 lbs. 3 oz.	By injection into a facial vein.	0.5 gr.	Death.	Almost instantaneous.	Less than 5 minutes.	
XXXI.	Do.	Do. 4 lbs.	Subcutaneously.	1.5 gr.	Very slight paralytic symptoms, followed by recovery.	About 40 minutes.	Less than 15 minutes.	
XXXII.	Do.	Do., 3 lbs. 7½ oz.	Do.	2 grs.	Paralysis and slight tremors, followed by recovery.	About 7 minutes.	More than 23 minutes.	
XXXIII.	Do.	Do., 2 lbs. 7 oz.	Do.	2 grs.	Death, preceded by slight tremors and paralysis.	Not noted.	About 40 minutes.	Same rabbit as in Experiment III. The 2d experiment was performed immediately after the 1st.
XXXIV.	Do.	Do., 3 lbs. ½ oz.	Do.	2.5 grs.	Death, preceded by slight tremors and paralysis.	2 minutes.	4 minutes.	
XXXV.	Do.	Do., 3 lbs. 8 oz.	Do.	3 grs.	Death, preceded by slight tremors and paralysis.	Less than 3 minutes.	5 minutes.	
XXXVI.	Do.	Frog, 250 grs.	Do.	0.01 gr.	Incomplete paralysis, followed by recovery.	About 6 minutes.	About 2 hours.	
XXXVII.	Do.	Do., 407 grs.	Do.	0.05 gr.	Partial general paralysis, and complete paralysis of the motor nerves, followed by recovery.	About 4 minutes.	4 days.	
XXXVIII.	Do.	Do., 379 grs.	Do.	0.05 gr.	Incomplete general paralysis, complete paralysis of the poisoned motor nerves, followed by recovery.	1 minute.	More than 3 hours and less than 20.	Vessels of the right posterior extremity were ligatured.
XXXIX.	Do.	Do., 270 grs.	Do.	0.05 gr.	Incomplete general paralysis, complete paralysis of the motor nerves, followed by recovery.	About 3 minutes.	More than 3 hours and less than 23.	

## TABULAR SUMMARY OF EXPERIMENTS—continued.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
XL.	Sulphate of methyl-atropium.	Frog, 460 grs.	Subcutaneously.	0.1 gr.	Death, preceded by incomplete general paralysis and complete paralysis of the motor nerves.	Less than 2 minutes.	About 6 days.	
XLI.	Do.	Do., 320 grs.	Do.	0.1 gr.	Death, preceded by incomplete general paralysis and complete paralysis of the poisoned motor nerves.	Less than 2 minutes.	About 5 days.	Vessels of the right posterior extremity were ligatured.
XLII.	Do.	Do., 312 grs.	Do.	0.1 gr.	Death, preceded by incomplete general paralysis, followed by complete paralysis of the poisoned motor nerves.	Not noted.	Not noted.	Vessels of the right gastrocnemius muscle were ligatured.
XLIII.	Do.	Do., 235 grs.	Do.	0.1 gr.	Death, preceded by incomplete general paralysis, followed by complete paralysis of the poisoned motor nerves.	Less than 2 minutes.	About 3 days.	Vessels of the right posterior extremity were ligatured.
XLIV.	Do.	Do., 461 grs.	Do.	0.4 gr.	Death, preceded by incomplete general paralysis, followed by complete paralysis of the motor nerves.	About 1 minute.	About 2 days.	
XLV.	Do.	Rabbit, 3 lbs. 2 oz.	By stomach.	20 grs.	None (except on pupils).			
XLVI.	Iodide of ethyl-atropium.	Do., 2 lbs. 4 oz.	Subcutaneously.	0.1 gr.	Paralysis of the inhibitory cardiac branches of the vagi nerves, followed by recovery.	Less than 8 minutes.	Not noted.	
XLVII.	Do.	Do., 3 lbs. 8½ oz.	Do.	2 grs.	Death, preceded by slight tremors and paralysis.	About 8 minutes.	21 minutes.	
XLVIII.	Do.	Do., 3 lbs. 7 oz.	Do.	2 grs.	Death, preceded by slight tremors and paralysis.	6 minutes.	18 minutes.	

TABULAR SUMMARY OF EXPERIMENTS—continued.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
XLIX.	Iodide of ethyl-atropium.	Frog, 321 grs.	Subcutaneously.	0.05 gr.	Complete paralysis of the motor nerves, followed by recovery.	Not noted.	Less than 20 hours.	
L.	Do.	Do., 450 grs.	Do.	0.2 gr.	Death, preceded by complete paralysis of the poisoned motor nerves.	Not noted.	Not noted.	Vessels of the left gastrocnemius muscle were ligatured.
LI.	Do.	Do., 301 grs.	Do.	0.15 gr.	Death, preceded by incomplete general paralysis, followed by complete paralysis of the poisoned motor nerves.	Less than 2 minutes.	More than 30 hours and less than 44.	Vessels of the right posterior extremity were ligatured.
LII.	Do.	Do., 290 grs.	Do.	0.15 gr.	Death, preceded by incomplete general paralysis, followed by complete paralysis of the motor nerves.	2 minutes.	About 3 days.	
LIII.	Hydrochlorate of Dr CHRISTISON'S conia.	Rabbit, 3 lbs. 14½ oz.	Do.	0.1 gr.	Faint degree of stiffness in the animal's movements, followed by recovery.	10 minutes.	20 minutes.	
LIV.	Do.	Do., 3 lbs. 6½ oz.	Do.	0.2 gr.	Death, preceded by stiffness, exaggeration of reflex function, spasms, and paralysis.	2 minutes and 30 seconds.	29 minutes and 30 seconds.	
LV.	Do.	Do., 3 lbs. 6½ oz.	Do.	0.25 gr.	Death, preceded by exaggeration of the reflex function, spasms, and paralysis.	Less than 6 minutes.	15 minutes.	
LVI.	Do.	Do., 3 lbs. 3 oz.	Do.	0.5 gr.	Death, preceded by stiffness in the four limbs, exaggeration of the reflex function, spasms, and paralysis.	1 minute.	9 minutes.	
LVII.	Do.	Do., 3 lbs. 8½ oz.	Do.	1 gr.	Death, preceded by paralysis and spasms.	Less than 30 seconds.	4 minutes.	

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
LVIII.	Hydrochlorate of Dr CHRISTISON'S conia.	Frog, 220 grs.	Subcutaneously.	0·05 gr.	Death, preceded by paralysis.	Not noted.	Several days.	
LIX.	Do.	Do., 364 grs.	Do.	0·1 gr.	Death, preceded by stiffness of the fingers, paralysis, and spasmodic movements of the posterior extremities.	About 7 minutes.	Several days.	
LX.	Do.	Do., 180 grs.	Do.	0·05 gr.	Stiffness of the anterior extremities, and general paralysis, followed by recovery.	5 minutes.	3 days.	
LXI.	Do.	Do., 300 grs.	Do.	0·1 gr.	Death, preceded by tonic spasm in the anterior extremities, incomplete general paralysis, spasmodic movements of the posterior extremities, and complete paralysis of the motor nerves.	5 minutes.	Less than 2 days.	
LXII.	Do.	Do., 220 grs.	Do.	0·1 gr.	Death, preceded by incomplete general paralysis, followed by complete paralysis of the poisoned motor nerves.	5 minutes.	Less than 3 days.	Vessels of the right posterior extremity were ligatured.
LXIII.	Do.	Do., 202 grs.	Do.	0·1 gr.	Death, preceded by tonic spasm of the anterior extremities and paralysis.	2 minutes.	More than 3 hours and less than 20.	
LXIV.	Do.	Do., 170 grs.	Do.	0·2 gr.	Death, preceded by tonic spasm of the anterior extremities, incomplete general paralysis, spasmodic movements of the posterior extremities, and complete paralysis of the poisoned motor nerves.	Less than 4 minutes.	More than 2 hours and less than 19.	Vessels of the right posterior extremity were ligatured.

TABULAR SUMMARY OF EXPERIMENTS—continued.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
LXV.	Hydrochlorate of Dr CHRISTISON'S conia.	Frog, 195 grs.	Subcutaneously.	0·6 gr.	Death, preceded by tonic spasm of the anterior extremities, incomplete general paralysis, and complete paralysis of the poisoned motor nerves.	2 minutes.	More than 3 hours and less than 20.	Vessels of the right posterior extremity were ligatured.
LXVI.	Hydrochlorate of Mr MORSON'S conia.	Rabbit, 2 lbs. 12 oz.	Do.	0·2 gr.	None.			
LXVII.	Do.	Do., 3 lbs. 3½ oz.	Do.	0·7 gr.	Spasms, exaggeration of the reflex activity, and partial paralysis, followed by recovery.	6 minutes.	About 40 minutes.	
LXVIII.	Do.	Do., 4 lbs. 1 oz.	Do.	1 gr.	Death, preceded by tremors, spasms, exaggeration of the reflex activity, and reflex activity, and paralysis.	6 minutes.	27 minutes.	
LXIX.	Do.	Frog, 380 grs.	Do.	0·05 gr.	Slight paralysis, followed by recovery.	7 minutes.	About 3 hours.	
LXX.	Do.	Do., 218 grs.	Do.	0·05 gr.	Incomplete paralysis, followed by recovery.	6 minutes.	Several hours.	
LXXI.	Do.	Do., 112 grs.	Do.	0·1 gr.	Slight tonic spasm of anterior extremities, complete paralysis of the motor nerves and of the reflex function of the spinal cord.	3 minutes.	More than 24 hours.	Vessels of the right posterior extremity were ligatured.
LXXII.	Do.	Do., 200 grs.	Do.	0·2 gr.	Complete paralysis of the motor nerves and of the reflex function of the spinal cord.	4 minutes at least.	Not noted.	
LXXIII.	Do.	Do., 256 grs.	Do.	0·3 gr.	Death, preceded by tonic spasm and complete paralysis.	4 minutes.	More than 30 hours and less than 44.	

## TABULAR SUMMARY OF EXPERIMENTS—continued.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
LXXIV.	Hydrochlorate of Mr Mosson's conia.	Frog, 290 grs.	Subcutaneously.	0·36 gr.	Death, preceded by complete paralysis.	2 minutes.	More than 24 hours and less than 42.	Vessels of the right posterior extremity were ligatured.
LXXV.	Do.	Do., 110 grs.	Do.	0·2 gr.	Death, preceded by complete paralysis, &c.	2 minutes.	About 2 days.	Do.
LXXVI.	Do.	Do., 140 grs.	Do.	0·3 gr.	Death, preceded by tonic spasm and complete paralysis.	Less than 2 minutes.	More than 2 hours and less than 21.	
LXXVII.	Do.	Do., 110 grs.	Do.	0·3 gr.	Death, preceded by complete paralysis, &c.	About 1 minute.	Not noted.	Do.
LXXVIII.	Do.	Do., 110 grs.	Do.	0·3 gr.	Death, preceded by complete paralysis, &c.	2 minutes.	Not noted.	Do.
LXXIX.	Do.	Do., 115 grs.	Do.	0·4 gr.	Death, preceded by tonic spasm and complete paralysis.	2 minutes.	More than 4 hours and less than 20.	
LXXX.	Hydrochlorate of methyl-conia.	Rabbit, 3 lbs. 14½ oz.	Do.	0·1 gr.	None.			
LXXXI.	Do.	Do., 2 lbs. 10½ oz.	Do.	0·2 gr.	Death, preceded by stiffness of the limbs, exaggeration of the reflex activity, paralysis, and tremors.	4 minutes.	18 minutes.	
LXXXII.	Do.	Do., 3 lbs. 9 oz.	Do.	0·5 gr.	Death, preceded by spasms and paralysis.	5 minutes.	13 minutes.	
LXXXIII.	Do.	Do., 4 lbs. 1 oz.	Do.	1 gr.	Death, preceded by stiffness of the limbs, exaggeration of the reflex activity, paralysis, and spasms.	1 minute 30 seconds.	7 minutes.	
LXXXIV.	Do.	Frog, 249 grs.	Do.	0·05 gr.	Tonic spasm, incomplete and then complete paralysis, followed by recovery.	18 minutes.	More than 30 hours and less than 42.	
LXXXV.	Do.	Do., 228 grs.	Do.	0·05 gr.	Death, preceded by paralysis.	7 minutes.	About 2 days.	

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
LXXXVI.	Hydrochlorate of methyl-conia.	Frog, 185 grs.	Subcutaneously.	0.06 gr.	Death, preceded by tonic spasm and paralysis.	Less than 10 minutes.	More than 3 hours and less than 20.	
LXXXVII.	Do.	Do., 195 grs.	Do.	0.1 gr.	Death, preceded by paralysis.	Less than 7 minutes.	More than 4 hours and less than 23.	
LXXXVIII.	Do.	Do., 140 grs.	Do.	0.1 gr.	Death, preceded by clonic spasms (?) in the non-poisoned region, and by paralysis in the poisoned region.	Less than 7 minutes.	More than 2 hours and less than 22.	Vessels of the right posterior extremity were ligatured.
LXXXIX.	Do.	Do., 187 grs.	Do.	0.14 gr.	Death, preceded by paralysis in the poisoned region.	Less than 9 minutes.	More than 5 hours and less than 20.	Do.
XC.	Do.	Do., 165 grs.	Do.	0.15 gr.	Death, preceded by clonic spasms in the non-poisoned region, and by paralysis in the poisoned region.	5 minutes.	More than 2 hours and less than 24.	Do.
XCI.	Do.	Do., 200 grs.	Do.	0.2 gr.	Death, preceded by tonic and clonic (?) spasms, and by paralysis in the poisoned region.	3 minutes.	More than 3 hours and less than 19.	Do.
XCII.	Do.	Do., 192 grs.	Do.	0.2 gr.	Death, preceded by paralysis in the poisoned region.	2 minutes 30 seconds.	More than 2 hours and less than 22.	Do.
XCIII.	Do.	Do., 175 grs.	Do.	0.2 gr.	Death, preceded by clonic spasms in the non-poisoned region, and by paralysis in the poisoned region.	Less than 10 minutes.	More than 4 hours and less than 18.	Do.
XCIV.	Do.	Do., 210 grs.	Do.	0.6 gr.	Death, preceded by clonic spasms in the non-poisoned region, and by paralysis in the poisoned region.	3 minutes.	More than 3 hours and less than 16.	Do.

TABULAR SUMMARY OF EXPERIMENTS—continued.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
XCV.	Iodide of dimethyl-conium.	Rabbit, 3 lbs. 8 oz.	Subcutaneously.	0.5 gr.	None.			
XCVI.	Do.	Do., 4 lbs. 1½ oz.	Do.	2 grs.	None.			
XCVII.	Do.	Do., 3 lbs. 6½ oz.	Do.	2.5 grs.	Slight paralysis, followed by recovery.	29 minutes.	Less than 23 minutes.	
XCVIII.	Do.	Do., 4 lbs.	Do.	3 grs.	Death, preceded by general paralysis.	32 minutes.	43 minutes.	
XCIX.	Do.	Do., 4 lbs. 1½ oz.	Do.	4 grs.	Death, preceded by general paralysis.	11 minutes.	26 minutes.	
C.	Do.	Do., 3 lbs. 6½ oz.	Do.	5 grs.	Death, preceded by general paralysis.	11 minutes.	20 minutes.	Same rabbit as in Experiment XCVII.; an interval of 24 hours elapsed between each experiment.
CI.	Do.	Frog, 235 grs.	Do.	0.05 gr.	Complete paralysis of the motor nerves, &c., followed by recovery.	3 minutes.	More than 24 hours and less than 41.	
CII.	Do.	Do., 210 grs.	Do.	0.05 gr.	Complete paralysis of the poisoned motor nerves, followed by recovery.	2 minutes.	More than 30 hours and less than 41.	Vessels of left posterior extremity were ligatured.
CIII.	Do.	Do., 230 grs.	Do.	0.1 gr.	Complete paralysis of the poisoned motor nerves, followed by recovery.	About 2 minutes.	More than 49 hours and less than 66.	Vessels of right posterior extremity were ligatured.
CIV.	Do.	Do., 225 grs.	Do.	0.1 gr.	Complete paralysis of the motor nerves, followed by recovery.	2 minutes.	About 5 days.	
CV.	Do.	Do., 190 grs.	Do.	0.1 gr.	Complete paralysis of the motor nerves, followed by recovery.	1 minute 30 seconds.	More than 26 hours and less than 40.	
CVI.	Do.	Do., 155 grs.	Do.	0.1 gr.	Death, preceded by complete paralysis.	3 minutes.	Several days.	
CVII.	Do.	Do., 150 grs.	Do.	0.1 gr.	Death, preceded by complete paralysis.	2 minutes 30 seconds.	More than 30 hours and less than 41.	

TABULAR SUMMARY OF EXPERIMENTS—*continued*.

Number of Experiment.	Substance Employed.	Animal and its Weight.	Method of Exhibition.	Dose.	Effect.	Interval between administration and commencement of Symptoms.	Duration of Symptoms.	Notes.
CVIII.	Iodide of dimethyl-conium.	Frog, 192 grs.	Subcutaneously.	0.15 gr.	Death, preceded by complete paralysis.	1 minute.	More than 3 hours and less than 19.	Vessels of right posterior extremity were ligatured.
CIX.	Do.	Do., 127 grs.	Do.	0.1 gr	Death, preceded by complete paralysis.	2 minutes.	More than 5 hours and less than 20.	Do.
CX.	Do.	Do., 140 grs.	Do.	0.2 gr.	Death, preceded by complete paralysis.	3 minutes.	Not noted.	

\*\* CXXI.-CXXVI. include the experiments illustrative of the topical action of sulphate of methyl-atropium on the iris.



PROCEEDINGS

OF THE

STATUTORY GENERAL MEETINGS,

AND

LIST OF MEMBERS ELECTED AT THE ORDINARY MEETINGS,  
SINCE JANUARY 6, 1868,

WITH

LIST OF DONATIONS TO THE LIBRARY,  
FROM NOV. 25, 1867, TO NOV. 22, 1869.

# PROCEEDINGS, &c.

*Monday, 25th November 1867.*

At a Statutory General Meeting, Professor LYON PLAYFAIR, Vice-President, in the Chair, the Minutes of the Statutory Meeting of 26th November 1866 were read and confirmed.

The following Office-Bearers were elected for 1867-68 :—

Principal Sir DAVID BREWSTER, K.H., LL.D., D.C.L., President.

His Grace the DUKE of ARGYLL, Honorary Vice-President, having filled the Office of President.

Principal FORBES,

Professor INNES,

Professor LYON PLAYFAIR, C.B.,

DAVID MILNE HOME, Esq.,

Dr CHRISTISON,

Professor KELLAND,

Dr JOHN HUTTON BALFOUR, General Secretary.

Dr GEORGE JAMES ALLMAN,

Professor TAIT,

DAVID SMITH, Esq., Treasurer.

Dr MACLAGAN, Curator of Library and Museum.

} Vice-Presidents.

} Secretaries to the Ordinary Meetings.

## COUNCILLORS.

Dr A. CRUM BROWN.

Dr BURT.

Dr MATTHEWS DUNCAN.

WILLIAM TURNER, M.B.

Dr JOHN MUIR.

Rev. THOMAS BROWN.

JAMES SANDERSON, Esq.

HON. Lord NEAVES.

R. W. THOMSON, C.E.

GEORGE ROBERTSON, C.E.

Professor PIAZZI SMYTH.

PATRICK DUDGEON, Esq. of Cargen.

The following List of Honorary Fellows was submitted before being printed in the billet of the first ordinary meeting :—

## I. FOREIGN.

Professor BENJAMIN PEIRCE, Director of the United States Survey.

M. COUSIN DE REMUSAT, Paris.

FRIEDRICH WÖHLER, Gottingen.

## II. BRITISH.

JAMES PRESCOTT JOULE, LL.D., Manchester.

CHARLES WHEATSTONE, D.C.L., London.

Professor PLAYFAIR conveyed to the Society thanks from M. CHEVREUL for his election as an Honorary Fellow.

The Treasurer gave in his annual printed Report, certified by the Auditor.

On the motion of Dr BURT, GEORGE AULDJO JAMIESON, Esq., was elected Auditor for the year 1867-68.

It was announced from the chair that the Council had awarded the Keith Prize for the biennial period ending April 1867 to Professor C. PIAZZI SMYTH, for his paper on "Recent Measurements made at the Great Pyramid," published in the Transactions.

On the recommendation of the Council, it was agreed that a ballot should take place at next meeting for Dr ROBERT DAUN, who had resigned his seat in 1845, and who now desired to be re-elected.

The Meeting then adjourned.

(Signed) D. MILNE HOME, *V.-P.*

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*Monday, 23d November 1868.*

At a Statutory General Meeting, DAVID MILNE HOME, Esq., Vice-President, in the Chair, the Minutes of the Statutory Meeting of 25th November 1867 were read and confirmed.

The following Office-Bearers were elected for 1868-69 :—

Professor CHRISTISON, M.D.,	President.	
His Grace the DUKE of ARGYLL,		} Honorary Vice-Presidents.
JAMES DAVID FORBES, LL.D.,		
Professor C. INNES,		} Vice-Presidents.
Professor LYON PLAYFAIR, C.B.		
D. MILNE HOME, Esq.,		
Professor KELLAND,		
The Hon. Lord NEAVES,		
Professor Sir WILLIAM THOMSON,		
Dr JOHN HUTTON BALFOUR,	General Secretary.	
Dr GEORGE JAMES ALLMAN,		} Secretaries to Ordinary Meetings.
Professor TAIT,		
DAVID SMITH, Esq.,	Treasurer.	
Dr MACLAGAN,	Curator of Library and Museum.	

## COUNCILLORS.

Dr JOHN MUIR.

Rev. THOMAS BROWN.

JAMES SANDERSON, Esq.

R. W. THOMSON, C.E.

GEORGE ROBERTSON, C.E.

Professor PIAZZI SMYTH.

PATRICK DUDGEON, Esq. of Cargen.

Dr HUGH CLEGHORN.

W. DITTMAR, Esq.

Dr JAMES M'BAIN, Surgeon, R.N.

Dr WILLIAM ROBERTSON.

THOMAS STEVENSON, C.E.

The following List of Honorary Fellows was submitted before being printed in the billet of the first ordinary meeting :—

GUSTAV ROBERT KIRCHHOFF, Professor of Physics in the University of Heidelberg.

RUDOLPH VIRCHOW, Professor of Pathological Anatomy in the University of Berlin.

The SECRETARY announced that the Council had awarded the Makdougall Brisbane Prize for the biennial period 1866-68 to Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER for their conjunct paper on the Connection between Chemical Constitution and Physiological Action, which had been printed in the Transactions.

The SECRETARY announced that the Council had awarded the Neill Prize for the triennial period 1865-68 to Dr WILLIAM CARMICHAEL M'INTOSH for his paper on the British Nemertean and on some New British Annelids, which was submitted to the Society last session, and is to be printed in the Transactions.

The Meeting then adjourned.

(Signed) PHILIP KELLAND, *V.-P.*

## LIST OF MEMBERS ELECTED.

*December 2, 1867.*

JOHN F. M'LENNAN, Esq., Advocate. Dr ROBERT DAUN (Re-admitted).

*January 6, 1868.*

Rev. Dr DAVID AITKEN. Dr ROBERT M. FERGUSON.

*February 3, 1868.*

J. W. LAIDLAY, Esq. of Seacliff. W. WILLIAMS, Esq.

*March 2, 1868.*J. SAMSON GAMGEE, Esq. Rev. D. T. K. DRUMMOND.  
Rev. JOSEPH TAYLOR GOODSIR. Major J. H. M. SHAW STEWART, R.E., Madras.*March 16, 1868.*

JOHN J. STEVENSON, Esq. Rev. JAMES F. MONTGOMERY.

*April 6, 1868.*JOHN DICK PEDDIE, Esq., Architect. Col. SEATON GUTHRIE.  
SAMUEL RALEIGH, Esq. Dr THOMAS SMITH MACCALL.*April 20, 1868.*Rev. Dr THOMAS GUTHRIE. THOMAS KEY, Esq.  
ADAM GILLIES SMITH, Esq., C.A. JOHN MACMILLAN, Esq., M.A.*December 21, 1868.*OLIVER G. MILLER, Esq. WILLIAM DICKSON, Esq.  
ALEXANDER BUCHAN, Esq. PROFESSOR H. C. FLEEMING JENKIN.  
JOHN LEVESON DOUGLAS STEWART, Esq. JOHN PENDER, Esq.  
of Nateby Hall.*January 4, 1869.*ISAAC ANDERSON-HENRY, Esq. of Woodend. GEORGE ELDER, Esq.  
Sir CHARLES A. HARTLEY, C.E. DAVID MACGIBBON, Esq., Architect.  
Rev. THOMAS MELVILLE RAVEN, M.A. ALEXANDER HOWE, Esq., W.S.  
Viscount WALDEN. PROFESSOR ALEXANDER DICKSON.*January 18, 1869.*Dr W. C. M'INTOSH. Dr HENRY MARSHALL.  
Dr WILLIAM RUTHERFORD.*February 1, 1869.*

Dr R. CRAIG MACLAGAN.

## LIST OF MEMBERS ELECTED.

*February 15, 1869.*

JAMES DEWAR, Esq.

*March 1, 1869.*

Rev. H. CALDERWOOD, LL.D.

*March 15, 1869.*

Principal Sir ALEXANDER GRANT, Bart, LL.D.

Captain T. P. WHITE, Royal Engineers.

*April 5, 1869.*

Dr JOHN WILSON JOHNSTON.

*May 17, 1869.*

ROBERT HENRY BOW, Esq., C.E.

*May 31, 1869.*

MAURICE LOTHIAN, Esq. of St Catherine's.

JOHN M'LAREN, Esq., Advocate.

# ALPHABETICAL LIST OF THE ORDINARY FELLOWS OF THE SOCIETY.

*Corrected up to 1st November 1869.*

N.B.—Those marked \* are Annual Contributors.

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- 1864 \*Dr Andrew Wood, 9 Darnaway Street
- 1855 Dr Wright, Cheltenham 300
- 1864 \*Robert S. Wyld, Esq., W.S., 19 Inverleith Row
- 1861 \*James Young, Esq., Limefield, Mid-Calder
- 1863 \*Dr John Young, Professor of Natural History, Glasgow 303

*Fellows elected between the commencement of the Session and the 1st January of the following year are entered under the latter date, by which their Subscriptions are regulated:—Thus, Fellows elected in December 1867 have the date of 1868 prefixed to their names.*

LIST OF THE PRESENT ORDINARY MEMBERS,

*Corrected up to November 1, 1869.*

IN THE ORDER OF THEIR ELECTION.

PRESIDENT.

Dr CHRISTISON.

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HIS GRACE THE DUKE OF ARGYLL, K.T.

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- 1818 Patrick Miller, M.D., *The Grove, Mount Radford, Exeter.*  
1820 Charles Babbage, Esq., F.R.S., Lond.  
    Sir John F. W. Herschel, Bart., F.R.S., Lond.  
    William Macdonald, M.D., F.R.C.P.E., *Professor of Natural History, St Andrews.*  
1822 Sir W. C. Trevelyan, Bart., *Wallington, Northumberland.*  
1823 Captain Thomas David Stuart, *of the Hon. East India Company's Service.*  
    Warren Hastings Anderson, Esq.  
    Liscombe John Curtis, Esq., *Ingsdon-House, Devonshire.*  
    Robert Christison, M.D., *Professor of Materia Medica.*  
1824 Robert E. Grant, M.D., *Professor of Comparative Anatomy, University College, London.*  
1827 Very Rev. Edward Bannerman Ramsay, M.A. Camb., LL.D.  
1828 John Forster, Esq., *Architect, Liverpool.*  
    David Milne Home, Esq., *Advocate, of Milne-Graden and Wedderburn.*  
1829 A. Colyar, Esq.  
    Right Hon. Sir William Gibson-Craig, Bart. *of Riccarton.*  
    Right Hon. Lord Colonsay.  
    Venerable Archdeacon Sinclair, *Kensington.*  
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    James Syme, D.C.L. Oxon., M.D. Dub., M.D. Bonn.  
    Thomas Barnes, M.D., *Carlisle.*  
1832 Montgomery Robertson, M.D.  
1833 Rear-Admiral Sir Alexander Milne, R.N.  
    His Grace the Duke of Buccleuch, K.G., *Dalkeith Palace.*  
    Alexander Hamilton, LL.B., W.S.

Date of  
Election.

- 1834 Mungo Ponton, Esq., W.S., *Clifton, Bristol.*  
Isaac Wilson, M.D., F.R.S., Lond.  
William Sharpey, M.D., LL.D., F.R.S., *Professor of Anatomy, University College, London.*
- 1835 John Hutton Balfour, A.M., M.D., F.R.S., *Professor of Medicine and Botany.*  
William Brown, Esq., F.R.C.S.E.
- 1836 David Rhind, Esq., *Architect.*
- 1837 John Scott Russell, A.M., *London.*  
Archibald Smith, M.A., Camb., F.R.S., *Lincoln's Inn, London.*  
Richard Parnell, M.D.  
Peter D. Handyside, M.D., F.R.C.S.E.
- 1839 David Smith, Esq., W.S.  
Adam Hunter, M.D., F.R.C.S.E.  
Rev. Philip Kelland, A.M., F.R.S., *Professor of Mathematics.*  
Francis Brown Douglas, Esq., *Advocate.*
- 1840 Alan A. Maconochie Welwood, Esq., *of Meadowbank and Pitliver.*  
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James Mackenzie, Esq.
- 1841 John Miller, Esq., *of Leithen.*  
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- 1842 James Thomson, Esq., *Civil Engineer, London.*  
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Charles Piazzì Smyth, Esq., F.R.S., *Professor of Practical Astronomy.*

Date of  
Election.

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 James Nicol, Esq., *Professor of Natural History, Aberdeen.*  
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 Moses Stephen, Esq., *of Bellahouston.*
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 Patrick James Stirling, Esq.
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 Honourable Bouverie F. Primrose.  
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 The Most Noble the Marquis of Tweeddale, K.T., *Yester House.*  
 Edward Sang, Esq.
- 1850 William John Macquorn Rankine, LL.D., F.R.S., *Professor of Civil Engineering,  
 University, Glasgow.*  
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 Sheridan Muspratt, M.D., *Liverpool.*  
 James Stark, M.D., F.R.C.P.E. (Re-admitted.)  
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 Andrew Fleming, M.D., *Bengal.*
- 1853 James Watson, M.D., *Bath.*  
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 Græme Reid Mercer, Esq.
- 1854 John Addington Symonds, M.D., *Clifton, Bristol.*  
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 Sir James Coxe, M.D., F.R.C.P.E.

Date of  
Election.

- 1854 Ernest Bonar, Esq.
- 1855 Stevenson Macadam, Ph.D.  
Robert Etheridge, Esq., *Clifton, Bristol.*  
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Thomas Wright, M.D., *Cheltenham.*  
James Hay, Esq.  
R. M. Smith, Esq.
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