TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XL. PART I.—FOR THE SESSION 1900–1.

CONTENTS.

EDINBURGH:

PUBLISHED BY ROBERT GRANT & SON, 107 PRINCES STREET,

AND WILLIAMS & NORGATE, 14 HENRIETTA STREET, COVENT GARDEN, LONDON.

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TRANSACTIONS.

I.—*A Contribution to the Life-Histories of the Cod and Whiting.*

By Arthur T. Masterman, M.A., D.Sc. (Lond. and St. And.), F.R.S.E.

(With Three Plates.)

(Read 5th June 1899.)

I. The Cod.

Considering the abundance of this gadoid, it is a matter for surprise that our knowledge of its life-history is not more complete. The work of Prof. Sars upon the cod in Norwegian waters is well known, and need not be referred to in any detail here. It is, however, important to avoid the assumption that his account will apply in every detail to the cod of British waters.

In 1883 Prof. M'Intosh * showed that the cod spawned in early spring and that spawning was effected outside the territorial limit. Dr Fulton,† from an examination of a number of captured specimens, was led to the same conclusion; and further, that the cod, whilst having an extended spawning period reaching from January to June, must be regarded as having a main period of February, March, and April, the great majority of individuals spawning in the month of March. Thus whilst 40 per cent. of the fish examined in March were mature, the proportion fell to 16 per cent. for February and April, and 10 per cent. for May. In estimating the growth of this species, by the method pursued below, it is probably most correct to regard the true spawning period as including the months of February, March, April, and May. A study of the distribution of pelagic eggs off the Frith of Forth ‡ leads to a similar conclusion. From this latter source the spawning period of the cod can be defined as extending from

* *Trawling Commission Report*, 1884.

† *Tenth Annual Report Fishery Board*, pp. 232–243.

‡ *Fifteenth Annual Report Scottish Fishery Board*, pp. 222, 223.

the third week in February to the second week in May, with a maximum in early April.

The same proofs are to be obtained of the statement that cod, on the East Coast of Scotland, spawn some considerable distance from shore outside the three-mile limit, Prof. M'Intosh's statement to this effect being corroborated by Dr Fulton in examining the distribution of the spawning adults, and by the study of the distribution of the eggs. We therefore know that the young larval cod commence their larval history floating in the surface water, at or rather near the surface, at some considerable distance from land. In the egg and early larval stages they would appear, by the nature of the case, to be subject to the physical environment of currents, etc.; but on the assumption of the post-larval stage, there is no reason why definite migration on the part of the young cod should not be instituted.

This was pointed out in 1884 by Prof. M'Intosh * in the following terms:—"It is evident that the two areas, offshore and inshore, are dependent on each other, and that legislation confined to one might not be followed by much benefit. Many inshore grounds, for instance, depend on the offing for a supply of the eggs and young of the cod, haddock, whiting, coal fish, and pollack; whilst the offshore is fed by a variable stream of the larger young of these fishes from the laminarian region of the inshore." By drifting or by definite migration, or possibly by one method succeeding the other, such an ontogenetic migration must be effected, for the ordinary habitat of the young cod, somewhat over one inch in length, is inshore amongst the rock-pools. In the period between hatching and this stage, the young must have effected a migration from the offshore pelagic region to the inshore littoral. The details alone of this migration appear to require determining.

Prof. M'Intosh had in his possession a large collection of preserved material, extending over several years, and consisting of various gadoids of the post-larval and adolescent stages, and has kindly allowed me to go over them with a view to finding diagnostic stages in the life-history of some of the gadoids, and, secondly, of elucidating their early growth and migration.

Table I.† contains a list of most of the specimens, arranged in size and order of occurrence, which were identified as stages of the cod. It will be noticed that there occurs a complete series of young forms, gradating no more than 1 mm. between each, from 4 mm. to 46 mm., and that after this another series, varying no more than 2 mm., extends to 60 mm. It is quite possible to figure the whole series, but only in the very early stages is a repetition at intervals of 1 mm. found necessary; for the young cod, long before it reaches 46 mm., is well known. The stages of special interest will be referred to later. The ontogenetic migration must of necessity be gradual, and cannot be regarded as passing through its various stages with any absolute lines of division. As in most natural phenomena, the majority of the species probably

* *Report Trawling Commission*, p. 76.

† See note on page 3.

conform to a more or less definite sequence, whilst others, tailing off at either end of the average, will constitute the transitional forms.

For mere convenience' sake, we have in prior papers* divided the habitat into—Bottom, in moderate depths, indicated by (B); surface-water in offshore and the water near the surface, by (S); midwater, or the regions of the water extending from about two fathoms to the bottom, in moderate depths, by (M); and lastly, the littoral district and the sea floor therein as (L).

It is evident that these are purely arbitrary divisions, and, as has been indicated elsewhere,† it is not unlikely that physical changes may cause a temporary migration from surface to midwater, and even to the bottom. Such changes are known to take place in the case of other pelagic forms, and there is apparently no reason why the pelagic eggs and larvæ should form an exception. This phenomenon, however, does not invalidate the demonstration of the true ontogenetic migration, and the eggs, for example, of the cod, will still be acknowledged as normally of pelagic surface-habitat in spite of the fact that under certain physical conditions some may be found at or near the bottom.

Again, it must be remembered that the division of two fathoms or otherwise between the midwater and the surface is a purely arbitrary one, and that hence there are no doubt several post-larval forms placed amongst the midwater, which might be more accurately located amongst the surface section.

If the surface-forms, as indicated in Table I.,‡ be arranged as in Table II., with their lengths in mm., forming the abscissæ, and the number in each haul the co-ordinates, then a distinct curve results. It will be seen that the early post-larval stages, at 4 and 5 mm., swarm in great numbers in the surface-habitat, and that as growth proceeds, there is a steady diminution in numbers till a length of 13 mm. is reached. The occurrence of solitary individuals carries the curve on to a length of 17 mm., after which the young cod no longer appear at the surface.

This marked and rapid decrease in number of young cod may be due to several factors. Firstly, a great number of them, as development proceeds, forsake the surface-water and move downwards through the midwater, where they appear in the diagram under a separate curve. Secondly, as development proceeds, the young cod gradually become more and more dispersed over the inshore water, so that although the same net was employed throughout, and approximately the same districts were worked over, yet the number of fishes in each haul steadily decreases, even if we make a combined curve of all the four here depicted. At the same time, one can scarcely explain the great decrease as being entirely due to a distribution or scattering of the fry, and one is

* *Annals and Mag. Nat. Hist.*, vol. xvi., Oct. 1895.

† 14*th Annual Scottish Fishery Board Report*, 1896; 15*th Annual S. F. B. Report*, 1897.

‡ The tables were exhibited at the reading of the paper, but have unfortunately disappeared since the work was handed in for publication.—P. G. T., *Sec. R.S.E.*

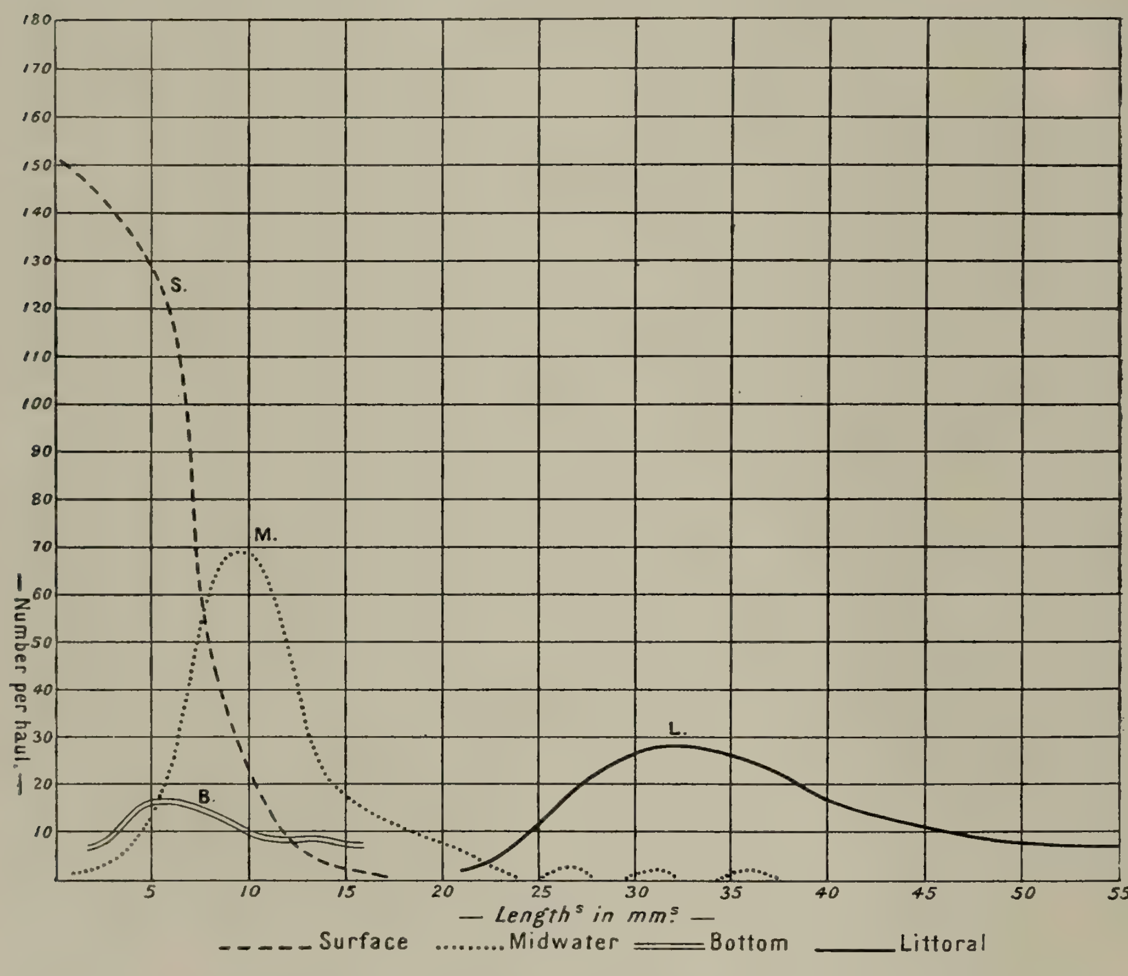
Table II. here given is a reproduction of the original to show this graphic method of illustrating the ontogenetic migration of food-fishes, and as an explanation of the text. The figures on this table are not absolutely accurate, except in so far as they have been taken from the text.—A. T. M., *March* 1900.

inclined to reservedly ascribe a great deal of it to the destructive agencies at work amongst the early cod, which we know must take place as a deduction from the high fecundity.*

The midwater curve in Table II. commences with a solitary specimen or two in the early 4 and 5 mm. stages, and steadily increases till a length of 9 mm. is

TABLE II.

Cod.



reached. From this point to the stage represented by a length of 24 mm., there is a gradual but very steady decrease in the number per haul, a decrease closely parallel to that of the surface curve. After 24 mm., from whatever cause, with the exception of an occasional isolated specimen at 33 and 38 mm., no more young cod appear in the midwater.

From about 22 mm. onwards there appears the littoral curve. At this size the

* *Natural Science*, 1896.

young cod are found in the neighbourhood of the shore. Their numbers per haul are seen to increase up to a length of about 33 mm., and then to steadily decrease till the curve is carried off the table. An inspection of Table I. will show that the littoral forms continue to occur with less and less frequency, up to 134 mm., before which size the tow-net is no longer an appropriate method of capture.

One other curve has to be noticed, namely, that of the bottom tow-net. This is at all times very small, although the bottom tow-net greatly exceeds the surface net in size of aperture, at least in the case of those captured in St Andrews Bay. The curve does not approach that of the midwater net in height, and is mainly carried on by one's and two's. Secondly, it will be found that its curve is almost exactly parallel to that of the combined curve of surface and midwater. For these reasons one is, I think, justified in regarding this curve as due to "incidental" specimens. These may occur in two ways. Firstly, a certain small proportion of the surface and midwater forms may exceptionally and precociously move to the bottom; and, secondly, the captured specimens may never have been upon the bottom at all. The bottom tow-net is a large trawl-like piece of apparatus, and, after its journey along the bottom, it is hauled up with open mouth through the midwater and surface. The parallel course of this curve to that of the combined surface and midwater curves would favour this view, *i.e.*, that it is due to a series of samples of the surface and midwater regions, taken by the bottom tow-net acting as a vertical net upon being hauled up.

It is quite possible that both the factors above referred to conduce to cause the occurrence of young cod in the bottom net, but in either case they would fall under the incidental category. They cease to occur after 19 mm., just before the midwater forms become littoral.

In tracing out the ontogenetic migration from these data, we have the two special relations of the migration to deal with.

In a horizontal direction we have the pelagic eggs, as a starting-point, floating in the offshore spawning areas. A study of the distribution of these eggs leads us to obtain evidence that a large proportion of them drift in towards the shore, and in addition, as all the specimens in Table I. have been caught in the districts between the spawning grounds and the shore, there is plenty of positive evidence that young cod do migrate in the course of development, from the offshore spawning areas to the littoral district of the East Coast. Whether others in any quantity migrate seawards, and if they do so, whether they survive and come to maturity, has yet to be proved.

As regards the vertical migration, the surface curve does not exactly merge into that of the midwater net, but from 4 mm. onwards the post-larval cod evidently commence to leave the surface water and move downwards into the midwater. The two curves cross at about 8 mm., and at about 9 mm. the young fishes occur in greatest numbers per haul.

From this point onwards to about 18 mm. the surface and midwater forms both decrease in number, so that it is improbable that the former owe their decrease mainly to a migration to midwater, except in so far as the surface curve decreases more rapidly than that of the midwater. At 23 mm. the decreasing midwater curve is met by the littoral, which from here to 32 mm. increases by increments from the midwater, and thereafter the results of distribution and of destruction by enemies cause a slow but sure decrease. The extreme limits of the surface forms are, from the egg to 18 mm.; those of the midwater, from 4 to 39 mm.; and those of the littoral, from 23 mm. onwards. The intersections of the respective curves are at 8·5 mm. and 22·5 mm. respectively, and these must be regarded as representing the epochs at which the *average* normally developing cod passes from one division to another.

It may be noted that the migration downwards from the surface to the midwater is more abrupt and effected more rapidly than that from the midwater to the littoral region, and this is especially so when it is recollected that growth is more rapid during the former period.

The general facts of this migration have, of course, been well known to scientists for several years, but we are further enabled to say that the average cod lives in the surface water till a length of 8–9 mm. is reached, when it moves down into the midwater. Here it remains till the proximity of land is reached and it reaches its bottom habitat at about 22 to 23 mm. in length.

There is no evidence in support of the idea that the young cod reach the bottom at any great distance from land and then migrate inward along the bottom; in fact, the study of the bottom-net curve in Table II. shows evidence to the contrary.

The egg and larval stages of the cod have been fully described* and are well known. In the St Andrews laboratory the young larvæ have occasionally been kept alive till all the yolk has disappeared. The length of one of these is given by Prof. M'Intosh as 4 mm., and even 4·5 mm. Following upon a description of this stage, the same authority describes briefly a number of transition forms which were caught in the tow-net. In the present paper several of the stages described by him are figured for the first time, and such intermediate stages are added as seem necessary to make a complete series from the close of the larval epoch to the young cod of one inch or more, from which latter stage there is no difficulty in identification. Fig. 1 represents an early post-larval form of 4·5 mm. (in spirit), probably one of the 5 mm. (fresh) specimens noticed by Prof. M'Intosh. The characteristic larval bands of black pigment have been reduced to two, which are the two post-anal ones. The ventral elements of these two have become fused by the appearance of pigment spots between them. A median ventral line of pigment now extends from the anus nearly to the tip of the tail. This pigment is superficial, and lies at the base of the marginal fin. Dorsally are the upper elements of the two bars and a few scattered pigment spots lying over the brain. There is no trace of the two anterior bars. With the addition of the intense black pigment of the eyes, this is all the pigmentation discernible in spirit specimens. On the other hand, on clearing, one can readily see a mass of black

* *Trans. Roy. Soc. Edin.*, vol. xxxv. pl. iii. pp. 812–822.

pigment in the peritoneal dorsal wall of the abdomen, especially above the swim-bladder, which at this stage is nearly circular.

When the mouth is closed the mandible forms a very prominent angle, and its lower border is very nearly vertical. This character, which is more or less marked in most gadoid and other pelagic larvæ, is probably to be traced to the method of feeding. A gaping mouth at the extreme front end with no inclination upwards or downwards would best subserve the capture of pelagic crustacea, as well as the function of respiration. The coloration at this and later stages consists of a diffuse greenish-yellow tinge, especially prominent* over the head and back.

In fig. 2 a length (in spirit) of 5·2 mm. is reached. The external pigmentation has increased, the dorsal elements of the post-anal bars approaching one another. A few spots upon the mandible and the commencement of a row along the lateral line may be recognised. The internal pigmentation has increased, and includes additions on the liver and pericardium. This stage carries the series on to fig. 3, in which it is evident that considerable progress has been effected. This stage has been described by Prof. M'Intosh.† In the external pigmentation we may notice the first appearance of a fine mid-ventral line along the median edge from the anus to the throat. The dorsal elements of the two larval post-anal bars have now united to form a median dorsal line. Apart from the few scattered spots on the head, the young cod now has an external pigmentation of four more or less prominent longitudinal lines, a mid dorsal from above the anus to the tail, a mid ventral from the throat past the anus to the tail, and a pair of short lateral-line rows in the post-anal portion. Whilst the transversely-barred condition of the larval cod is almost unique, this longitudinally-barred condition of the post-larval stage, whatever its significance, is of very general occurrence in young teleosteans.

So far as is possible, the pigment of the bars is, as it were, made use of to form the lines; the rest disappears.

At 7 mm. (spirit), fig. 4, the fin-rays are very evident in the tail, whilst the marginal fin is reduced. The external pigment system is similar to that in the preceding stage, but intensified. A characteristic blotch just behind and over the pectoral fin is evident. It is to be noticed that the dorsal and ventral lines do not here, nor at any stage, reach actually to the tail, but end suddenly at some distance therefrom. The internal pigment is much as before, and tends to spread over the swim-bladder, which is still nearly spherical.

Figs. 5 and 6 carry the series on by easy stages. Increase in intensity of the pigmentation is marked. In fig. 6 the dorsal and ventral lines are very marked, and the lateral lines have extended. In this stage the notochord no longer reaches to the tip of the caudal fin, which now becomes pronounced in shape. The internal pigment has become very dense in the abdominal cavity, and a line extends over the dorsal surface of the neural cord.

* *Loc. cit.*, p. 818. † *Loc. cit.*, p. 818.

In both figures the change in inclination of the mandible can be seen, the distance from the eye to the tip of the snout becoming proportionally greater. The swim-bladder has become oval in shape, and the notochord is less bent in its course dorsally to it. A stage closely similar to this (fig. 6) is figured and described elsewhere,* from which it appears that in the living condition there are numerous yellow chromatophores intermingled with the black, giving the animal a greenish translucence.

Fig. 7 shows great progress in the acquisition of adult characters, though little growth in length has been effected (11 mm.). At about 9 mm., or the length of fig. 6, the surface-water is forsaken and the progress through the midwater is gradually effected. This alteration in habitat may be correlated with the striking difference in structure. The caudal fin is now fully developed, the tip of the notochord, now no longer median, being deflected upwards.

The whole head is cod-like, and the angle of the lower jaw is at about 45° to the perpendicular. A minute trace of the barbel has appeared, and a dense mass of black pigment covers the brain. The dorsal and ventral lines have moved slightly from the median line and are now clearly paired, and the former reaches to the head. The lateral line is sharply defined. From the vent forwards the ventral line is small and median. Patches of pigment cover the jaws and snout.

All the adult fins are well differentiated, though still more or less continuous. As has already been pointed out, the fin rays of the first dorsal are later in appearance than those of the others. A cleared specimen shows the abdominal masses and the supra-neural line of the internal pigmentation.

A whole series of specimens carry us on to the stage in fig. 8, with a length of 19 mm. It will at once be seen that the changes in external structures effected from 9 mm. to 11 mm. are considerably more marked than those which have taken place in the growth from 11 mm. to 19 mm. Beyond the appearance of teeth, the further reduction of the tip of the notochord, the further development of the first dorsal and the ventral fins, there is little to note. The characteristic external pigmentation is still preserved in all its details. The lateral pigment line reaches to the level of the middle of the second dorsal.

A specimen of $\frac{15}{16}$ inch described by Prof. M'Intosh appears to agree very closely with the usually occurring stages from this specimen (fig. 8) up to about 22 mm., whilst these can easily be followed up through intermediate forms to fig. 9, in which the young cod has reached 24 mm. in length (spirit). This would probably be little short of one inch long in the fresh state. The head is now unmistakably that of a cod, and whilst it is to be noted that the arrangement of the external pigmentation is still very constant, apart from the fact that the mid ventral line from throat to anus is no longer distinguishable, the proportions have changed. The angle of the jaw has receded to well under the eye, and the ventrals have moved forwards. The young fish has at this

* *Loc. cit.*, pl. xix. fig. 2.

stage commenced to take up its littoral habitat, and a rapid development of the external pigment will give rise to the well-known 'tessellated' condition.

Whilst it is hard to correlate the characteristic pigment lines and markings in the early stages with the environment, we are probably justified in regarding the recession of the lower jaw and the forward movement of the ventral fins with the adoption of a ground-feeding habit.

II. The Whiting.

The data for elucidation of the life-history of the whiting are not so abundant as those for the cod, but they are sufficient to enable us to determine, in a general way, the form taken by the ontogenetic migration in this case.

From observations at St Andrews and elsewhere, it has long been known that the whiting has a very extended spawning period, from early March to the third week in August. A study of the occurrence of the eggs in the Frith of Forth district gave a period from the third week of February to the middle of July (*Fifteenth S.F.B. Report*, p. 227). The whiting begins its spawning period at much the same time as the cod, but whilst the latter does not extend beyond mid-May, the former continues till mid-July. Hence with a spawning period of no less than five months, the young whiting caught at any one time might be expected to show great diversity in size.

A reference to Table III.* will show this to be the case. On 11th July 1895, enormous numbers of young whiting were caught off the Frith of Forth, which ranged from 7 mm. in length to 56 mm. The greatest numbers occur at from 15 mm. to 38 mm., so that they form a maximum in the middle and taper off at each end. This also agrees with the facts of the spawning phenomena, and there can be little doubt that we have here a case of diversity in size, almost entirely due to a diversity in age, which in its turn is due to a prolonged spawning period. Thus, whilst we would be inclined to regard the whiting of 7 mm. as a few weeks old, that of 58 mm. might be five months old as a maximum.

The distribution of the eggs and spawning adults both tend to show that the whiting, whilst differing in minor features, agrees in general with the cod in its spawning areas. The egg is pelagic, and the life-cycle thus commences from much the same starting-point as in the case of the cod.

Up to the present there has been difficulty in obtaining the early post-larval stages of the whiting. The larvæ appear to be of normal surface-habitat, but on the acquirement of the post-larval stage with free locomotion they appear to avoid capture, and it is not till the length of 9 to 10 mm. that they are found with any frequency. They then occur in the midwater far offshore. This would lead us to suppose that they take an opposite course to the young cod, and instead of moving inshore they pass seawards. Their occurrence in shoals is also very diagnostic. Thus in the table will be

* See note ‡ on page 3.

seen a small shoal on 28th June, a huge shoal on 11th July, another on 21st July, and yet another on 9th August. This peculiar migration of the whiting to the deeper off-shore water during its earlier post-larval period has been already alluded to by Prof. M'Intosh.* Later on, in mid-July, the young whiting commence to move inwards "from their retreats in the offshore waters, therefore it is probable that the young whiting pass to the inshore waters when between 50 and 80 mm." By September they appear in great numbers in the littoral region, frequently the mouths of estuaries, some occurring with a length of 130 mm. (or 5 inches). Onwards through the winter they occur in increasing size, and on the return of spring they move off to deep water again. These facts, already described elsewhere, are confirmed very clearly by the table (Table III.).†

The ontogenetic migration of the whiting would appear somewhat as follows:—The surface-water is forsaken at, or near, the close of the larval period and the midwater is reached. A migration seawards is then effected into deep offshore waters, where the young whiting remains till a length of about 60 mm., when an inshore migration, still in the midwater, commences, and the littoral region is soon reached, at an average length of about 70–80 mm.

The oldest whiting which was reared in confinement at St Andrews laboratory is figured in pl. xvii. fig. 12 of the *Development and Life-Histories of Teleostean Fishes.*‡ A full description of this form is added in the text (p. 825). In this Prof. M'Intosh remarks, "so far as present observations go, the young whiting appears to be recognisable as such when from 9 to 12 mm. in length." The post-larval whiting above referred to appears to have been about 23 mm. long, so that there is a considerable gap. Figs. 10 and 11 illustrate two specimens caught in the midwater which I would tentatively assign to the whiting, though I have grave doubts concerning the propriety of so doing. My reasons *pro* and *con* will appear in the description. In fig. 10, little over $6\frac{1}{2}$ mm. in length, the black pigmentation is sparse. It consists of the usual gadoid external mid-dorsal line, more or less separated into two by the median fin, and a similar ventral line, but the latter terminates at the anus and the former over the swim-bladder, whilst neither extends to the tail. Of the internal pigment, the mass overlying the swim-bladder is present, as in other gadoids, and, in addition, there is a line of pigment above and below the notochord. There is no trace of pigment either upon the head or scattered over the surface of the body.

The pigmentation of the above-mentioned post-larval whiting is described by Prof. M'Intosh in the following terms:—"Black pigment spots arranged in a double series along the edges of the muscle-plates, the inner row in each case being somewhat fainter. A dense pigment band exists in the sub-notochordal region of the abdomen, and scattered spots occur generally over the surface." We may add that the dorsal median line is continued forward on to the head, in which region there are also addi-

* *Fifteenth Scottish Fishery Board Report*, p. 203.

† See note ‡ on page 3.

‡ *Trans. Royal Soc. Edin.*, vol. xxxv.

tional pigment spots. The two "somewhat fainter" lines appear to correspond to the two internal lines in fig. 10 above and below the notochord. These lines are therefore common both to fig. 10 and the young whiting, whereas they certainly are not found in the cod or green cod. All four lines terminate posteriorly in a similarly abrupt way in both the whiting and in fig. 10, but the mass of pigment found on the head in the former is entirely absent in the latter. In this respect fig. 10 differs from other gadoids as well. Lastly, the mid ventral line below the abdomen in the whiting, together with several scattered spots on the head, are absent in the form here described. Thus it differs essentially from the earlier whiting in the absence of several diagnostic elements of the black pigmentation. A sojourn for nine years in spirit, with imperfect preservation at the outset, may account for a good deal of this.

Both fig. 10 and fig. 11 are distinctly more advanced than the stages of similar length in the cod (*cf.* with figs. 3, 4, and 5). Thus fig. 10 falls between figs. 3 and 4 in size, and its mandibular inclination to the perpendicular is, if anything, more than in the latter, whereas the oval shape of the swim-bladder puts it on a par with a cod of 8 mm., with which it is also comparable in the extent of the median dorsal fin and the structure of the eye. Fig. 11, which is 7·45 mm. long, and evidently the same species as the preceding, shows even greater contrast with the cod of 8 mm. (fig. 5). In point of fact, it is little behind the cod of 11 mm. (fig. 7).

In the cod of 9 mm. the tail is still perfectly symmetrical though the fin is bifurcated, but in this specimen the notochord is deflected and the hypural elements have appeared. All the fins but the first dorsal are well formed, and the mandibular angle is comparable to that of fig. 7.

All these characters point, with some probability, to these two specimens belonging to a smaller species than the cod, and so far it agrees with the whiting. The poor-cod and the bib appear to be the only other possibilities (see below).

At 9 mm. (fig. 12) the young whiting is clearly distinguishable. The pigmentation is very thick and abundant, and its general distribution agrees very exactly with that of the larval whiting. The external masses on the head, the dorsal and ventral lines, and the internal abdominal masses, are all to be recognised, whilst a dorso-lateral and a ventro-lateral line are also present, and a ventral line along the abdomen.

The fins are continuous, but the positions of the dorsal and anal fins are outlined and a certain number of rays may be recognised. Prof. M'Intosh * described a whiting of 9 mm., which is at a slightly earlier stage than this specimen. In the latter the posterior end of the notochord is bent upwards, which is not the case in the former.

The young whiting of 10 mm. and 11 mm. does not differ in essential features from the above. The head becomes rather less prominent and the body becomes thicker, obscuring the internal pigmentation. Up to 11 mm. the young whiting has, as remarked

* *Fifteenth Scottish Fishery Board Report*, p. 201.

by the above author, "more black pigment in the postero-lateral region," and, we may add, on the head (*cf.* figs. 12 and 6), than the young cod of a similar stage.

At 12 mm. (fig. 13) the ventrals have made their appearance as a minute pair of papillæ almost exactly below the pectorals. "Groups of black pigment corpuscles are distributed along the base of the dorsal and the anal fins as well as over the brain, and a similar series exists along the median ventral line of the abdomen. Black specks also occur along the pre-maxillaries and the mandible" (M'Intosh). Over each side of the post-anal portion of the body are finely scattered black specks, which become more prominent in slightly later stages. Lastly, there is a series of delicate black bars across the caudal rays which make a curved line of pigment on the caudal fin. Fig. 13 is very characteristic of this stage, and there is some little change in specimens leading up to 15 and 16 mm. With increase in size the fins become separated by disappearance of the intermediate embryonic fin and further growth of the true fin-rays. The posterior tip of the notochord disappears and the caudal curve of pigment becomes fainter. The ventral fins move slowly forward; dots of black pigment appear over the median fins and the lateral black specks become large and more numerous. These and other lesser changes result in a stage, at 16 mm., of the appearance represented at fig. 14. The same general distribution of pigment as in fig. 13 can be recognised, and these two figures could not fail to serve for the identification of the young whiting from 11 to 20 mm. or more in length.

Along with the forward movement of the ventral fin appears that of the anus. Thus in fig. 13 the latter is situated immediately below the commencing separation between first and second dorsal fin, whereas in fig. 14 it is well below the middle of the former. With the forward progress of the anus, the first anal fin becomes longer, and new rays are continually added at its front end, so that the 18 of fig. 13 becomes 21 in fig. 14, and by the time the stage of fig. 15 is reached, their number has increased to 28.

At about 16 mm. there appears a series of little black touches at the bases of each ray in all the median fins. These are very symmetrical, and are quite distinct from the little specks scattered on the fins *between* the rays. They are not found in the cod, and are absent in the known stages of the haddock. They are, however, of constant occurrence in the whiting in every stage, from 16 mm. by gradations of 1 mm. up to 25 mm. and beyond. After this they appear to be lost in the general body pigment. The caudal curve becomes fainter after 16 mm., and disappears at about 18 mm. As will be seen by the accompanying Table, every stage with a gradation of 1 mm. from 7 to 25 mm. has been examined, and there is little to note in the way of change from 16 mm. onwards. The forward progress of the ventral fins and of the anus, with its anal fins, continues, so that in fig. 15 the ventral fins, now elongated, are below the occipital region of the head, and the anus is vertically below the first eighth of the first dorsal fin. The lateral pigment is greatly increased, and the dorso- and ventro-lateral lines of fig. 14 have merged into it. At 20 mm. "traces of the median ventral black line are still visible. The black pigment corpuscles along the sides often present a more or less longitudinally linear arrangement. No scales are developed. A minute papilla in the median line of the mandible indicates a barbel" (M'Intosh). These remarks apply with equal force to the 24–25 mm. stage (fig. 15).

Numerous lesser details may be noticed by a comparison of figs. 13, 14, and 15. These three figures should be sufficient to ensure identification at any stage up to 25 mm. or more.

From this stage onwards, Prof. M'Intosh thus described the young whiting.* He figures the young fish at 26 mm. (pl. vi. figs. 1 and 2), at 30 mm. (fig. 3), and at 40 mm. (fig. 4), and carries his description up to the stage of 70 mm., after which the adult characters are assumed.

It has already been noted that the early cod is, at the same size, at an earlier stage in development than the young whiting. This was evident in the stages up to 9 mm., and is strikingly illustrated by a comparison of figs. 12 and 6. Again, the figure of a cod at 11 mm. may be compared with figs. 13 and 14, and in some respects it is at an earlier stage than the whiting of 9 mm. To this rule there is one exception, namely, the barbel. This organ can be first clearly made out at about 10–11 mm. in the cod, whereas it does not make its appearance in the whiting till a length of 19–20 mm. is reached. This is a very marked exception, and one is tempted to connect it with the fact that whilst the cod retains its barbel throughout life, in the whiting it is vestigial only, and forms an "embryonic" organ.

Apart from differences in pigmentation, the young whiting from 15 mm. upwards can at once be distinguished from the young cod, and probably from all other gadoids, by the condition of the anal fin and the forward growth of the anus. The differences of external body-form are sufficiently striking, but in this, and especially in the proportional sizes of various parts, it is very dangerous to draw conclusions from spirit-specimens alone. We are safe to notice, however, that the head is proportionally larger and heavier in the young whiting up to 25 mm. than in a cod of a similar size, the reverse condition holding in the adult.

Fig. 15 has an outline of the body much nearer that of an adult cod than whiting, so that the whiting may be said, in this and other features, such as the presence of a barbel, to pass through a "cod" stage.

Lastly, in pigmentation (black) there are important differences.

In the larval and early post-larval cod are the transverse bars, diagnostic of the species, and the external pigment-line of the lateral line. This appears posteriorly very early and progresses forward gradually, and is a constant character throughout the stages here described.

In the whiting, on the other hand, the internal supra-neural and sub-notochordal lines, the scattered specks over the fin-membranes and over the post-anal part of the

* *Fifteenth Report Scottish Fishery Board*, pp. 202, 203.

trunk, occurring from 11 mm. onwards, and the regular dots at the bases of the fin-rays, from 15 mm. onwards, are to be specially noted.

Both species have, in common, the patches on the dorsal surface of the head, the internal masses of pigment in the dorsal abdominal wall, the few specks over the jaws and the dorsal and ventral external lines, the latter continued forward along the ventral surface of the abdomen. These are probably all more or less gadoid features.

EXPLANATION OF PLATES.

Plate I.

Fig. 1. Lateral view of post-larval cod 4·5 mm. long × about 16 (viewed as transparent object).
Fig. 2. „ „ „ 5·2 „ „ „ „ „ „
Fig. 3. „ „ „ 6 „ „ „ „ „ „
Fig. 4. „ „ „ 7 „ „ „ „ „ „
Fig. 5. „ „ „ 8 „ „ „ „ „ „
Fig. 6. „ „ „ 9 „ „ „ „ „ „

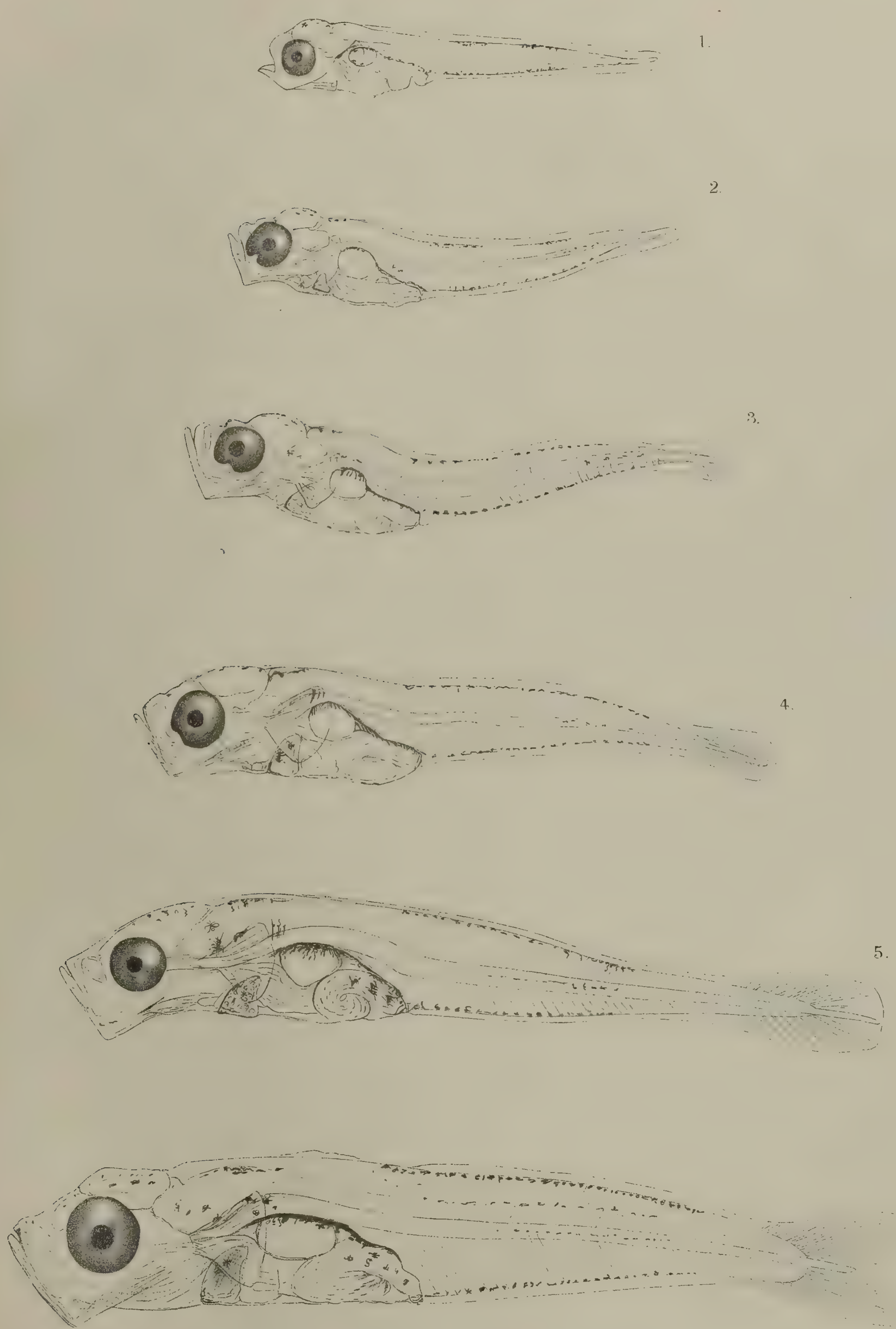
Plate II.

Fig. 7. Lateral view of young cod 11 mm. long × 11 (viewed as opaque object).
Fig. 8. „ „ „ 19 „ × 10 „ „ „
Fig. 9. „ „ „ 24 „ × 10 „ „ „

Plate III.

Fig. 10. Lateral view of post-larval gadoid (probably whiting) 6·6 mm. long × 15 (viewed as a transparent object).
Fig. 11. „ „ „ „ „ 7·45 „ „ „ „
Fig. 12. „ „ „ whiting 9 „ × 17 „ „ „
Fig. 13. „ „ „ „ 12 „ × 10 (viewed as an opaque object).
Fig. 14. „ young „ 16 „ × 10 „ „ „
Fig. 15. „ „ „ „ 24–25 mm. long × about 8 „ „

D^r. MASTERMAN ON LARVAL COD AND WHITING.—PLATE I.



1.

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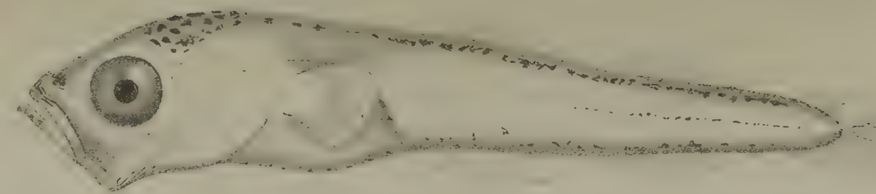
M^cFarlane & Erskine, Lith^rs. Edin^r.

Dr Masterman on Larval Cod and Whiting. Plate II



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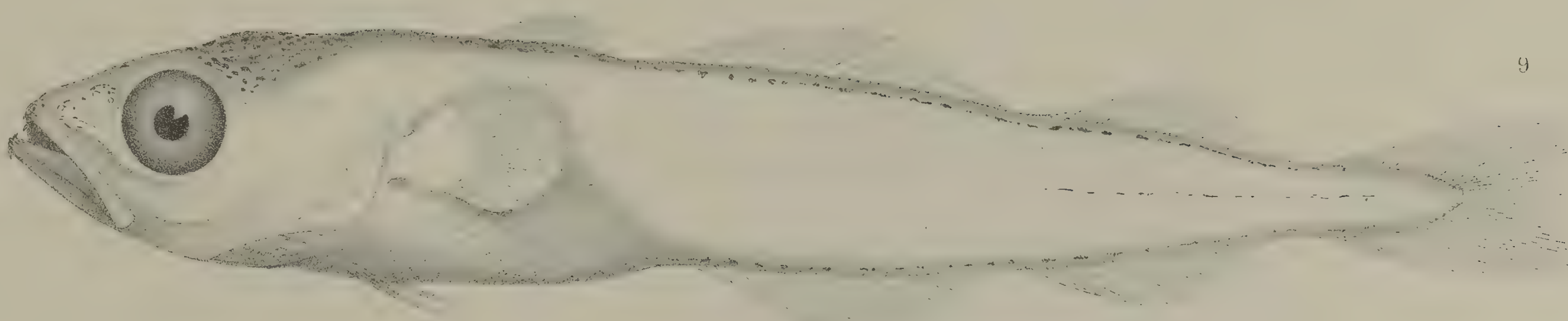

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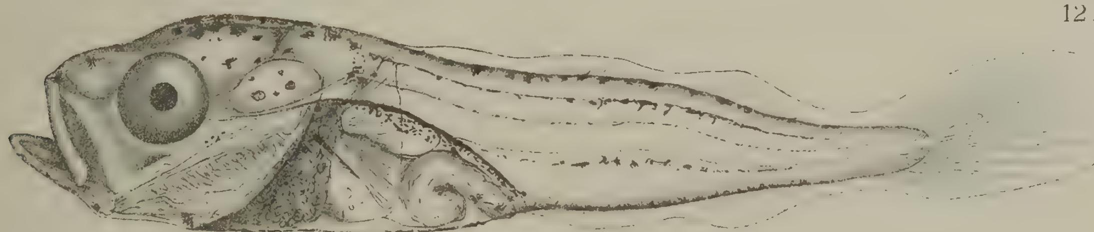
Dr MASTERMAN ON LARVAL COD AND WHITING.—PLATE III.

10.

11



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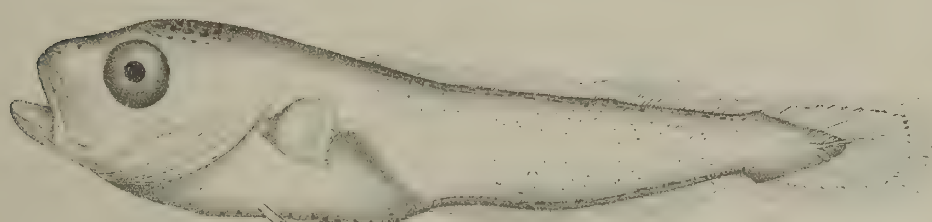
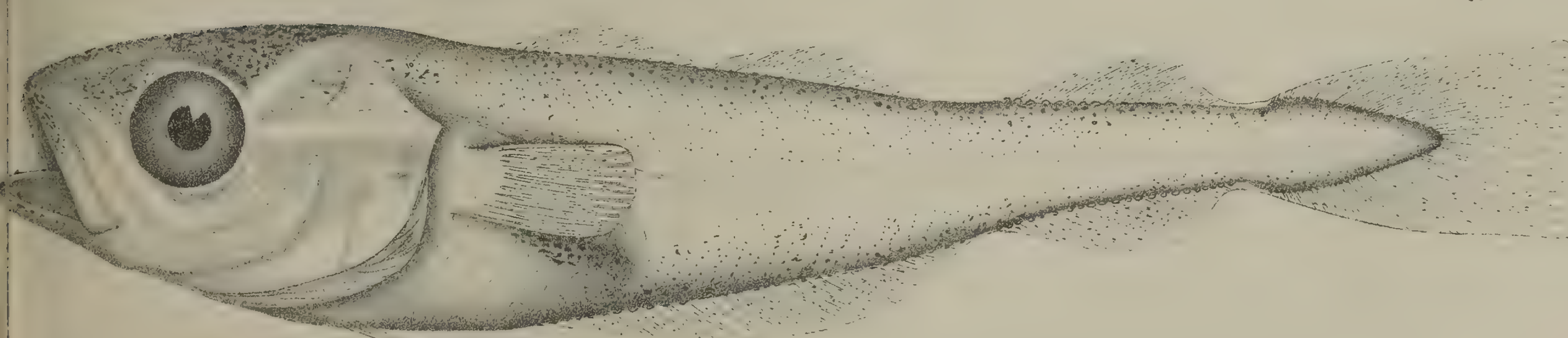


13.

14



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A. T. Masterman, del. ad nat.

McFarlane & Erskine, Lithrs Edinr

II.—*Two Historical Fallacies: Heather Beer and Uisge Beithe.*

By Robert C. Maclagan, F.R.S.E.

(Read 8th January 1900.)

Using Bellenden's translation of Boece, he tells us, "Attoure in all the desertis and muris of this realme growis ane herbe, namit hadder, but any seid, richt nutritive baith to beistis and fowlis; specialie to beis. This herbe, in the moneth of July, has ane floure of purpure hew, als sweit as huny. The Pichtis maid this herbe, sum time, ane richt delicus and hailsum drink. Nochtheless, the maner of the making of it is perist, be exterminioun of the said Pichtis out of Scotland; for they schew nevir the craft of the making of this drink bot to thair awin blud."

As the Picts left no written records, and as they were exterminated by Kenneth MacAlpin, it may be permitted to wonder how Boece came by his information (*Description of Albion*, p. 45). When we come down to Martin, about the year 1700, we find (*Western Islands*, p. 196) that the Picts refused to communicate their information to the Scots, "and so 'tis quite lost." In Lightfoot's *Flora Scotica*, 1777, we find: "Formerly the young tops of the heather are said to have been used alone to brew a kind of ale, and even now I was informed that the inhabitants of Islay and Jura still continue to brew a very potable liquor by mixing two-thirds of the tops of heather to one-third of malt."

The tradition of heather ale has stuck closely to Islay, where it says heather was grown for brewing ale. The story in the island is as follows:—

There are a number of remarkable little plots of ground by the side of the main road leading from Bridgend to Loch Gorm, and the remains of old dykes by which the plots were enclosed. Local tradition says it was here the Fein had a brewery where they made heather ale, and that the small plots were their heather-growing grounds.

Boece's tradition appears in two forms in the island. One is:—

A man lived in Balinaby who made heather ale for sale. His profits were so great that the neighbours wanted to find out the secret, which he would not reveal. At last they seized his son and urged him to tell them, but he would not, and they put him to death. They next took the father, but he continued obstinate, and they killed him also.

The other form of the tradition points out a flat stone in the old churchyard of Kildalton (exactly the opposite side of Islay from Balinaby) which is said to cover the grave of a father and his son who were both hanged together at the end of the church. This man and his sons were engaged making heather ale, which no one but themselves knew how to make. Some persons, wishing to find out the secret, bribed one of the sons, and the father, to prevent disclosure, with the help of his other son, put him to death. For this unnatural deed both the murderers were publicly hanged and their bodies buried together in one grave, over which the flat stone was placed.

Another local legend says that the time of the loss of how to make heather ale in Islay was, when the island was invaded by the Danes. This would make it appear that it was to the Danes that the secret was refused, and not to the Scots.

There are persons who still believe in the possibility of making a fermented drink from heather, and even a President of the Royal Society has given the legend a fresh lease of life. In the *Monastery*, Sir Walter Scott says: "During the meal Prince Charlie in vain attempted to engage his youthful companion in carousal, or, at least, in conversation. Halbert Glendinning pleaded fatigue, and expressed himself unwilling to take any liquor stronger than the heather ale, which was at that time frequently used at meals."

In Ireland, on the other hand, on the authority of a gardener from the vicinity of Lough Neagh, the Danes get the credit of making the heather beer. It is said there, that in cutting turf for peats, when they have stripped off the peat, they find heather laid out as if to dry, and it is believed it had been laid out with the intention of using it for making beer.

Heather ale is still manufactured in the north of Scotland, and people tell you that they have drank it. I had it on the authority of a lady that she had drank it in Banffshire and found it delicious; that it sparkled like ginger beer. One writer on Highland matters tells that he drank it as lately as 1840, but my lady friend's recollections are much less ancient than that. Seeing, then, that it was still in use so short a time ago, it seemed probable that some person would have a recollection of how it was made; and a diligent inquirer from a Gaelic source got the following, which seems to have all the accuracy one could expect from a practical brewer of it:—

"Take of the tops of heather as much as is required, put in a boiler, cover with water, and boil for three-quarters of an hour. Strain the liquid off, and allow it to cool to 70 degrees, add a teacupful and a quarter of yeast to the gallon of liquid, put in a crock or cask, covered with a cloth, and in two days it may be bunged down."

The quality of the heather, however, seemed to require a greater accuracy of description. The reciter was again interviewed, and said that the heather required to be gathered when in bloom, that the boiling was continued according to the strength of infusion required, but that there was no other test but that of the individual taste of the operator.

Now, as Martin told us that in his day the heather was mixed with malt, the question was put whether or no sugar was used, and he said "No," there was enough sugar in the heather. This reads, according to subsequent knowledge, like a mere tissue of falsehood, and yet I believe it was merely a repetition of a traditional story, though where the 70 degrees came in is a little hard to diagnose.

Heather honey is known to us all, and it seems quite natural that an infusion of that honey-producing plant should be fermentable. Indeed, the extract from Boece in which he calls attention to the liking for heather by "beis" seems to indicate the origin of the tradition. The locality of inquiry, then, was changed to the inland north of Scotland, where there could be no doubt that an acceptable fluid had been quite recently used under the title of heather beer. And from Miss Paull of the Manse of Tullynessle was got a recipe from "a woman who makes it often, says it is very good and supposed to be very strengthening." Here is the recipe:—

½ peck of malt.
1 oz. hops.
3 gallons of water.
Twa guid gowpenfu's of heather blossom.
1 lb. sugar or treacle.
Small teacupful of yeast.

Put the malt, hops, and heather blossom in a bag, and boil in the water for two hours. Add the sugar or treacle and strain; let it stand till lukewarm, then add the yeast. Let it stand till the third day, skim it, and then bottle it. The malt may be omitted if preferred. If the ale is wished sweet, more sugar must be added.

As the user of this recipe was quite willing to make some, I got a few bottles as a sample. It was not well brewed, was exceedingly sweet, and certainly had a curious taste, no doubt the result of the heather added to it. In fact, it was a poor sample of sugar beer with heather instead of ginger.

The heather employed was to be gathered in full bloom, was to be by preference not bell heather, and might be kept some time before using.

Another authority, who hails from Glen Urquhart, was equally positive that there they used "deep heather, the under part of the stems, bits that have not got the sun. You simply boil it a long time, sweeten it with syrup or sugar, add barm, and bottle it."

Finally I got a recipe, holograph, of the manufacturer:—

2 lbs. of heather bloom.
¼ lb. hops.
2 oz. ground ginger.
3 lbs. syrup.

Boil all together in 2 gallons of water for half an hour. Strain and add other 2 gallons of water, and when it is cold as new milk, add half a cupful of barm. Cover it up for twelve hours. Skim the top, pour it off gently to keep the barm that has sunk to the bottom, then bottle and cork firmly.

There is a reason for confusion between the heather top, that is the flowering stem of the heather, and the heather bloom, as both in Gaelic are called *barr*.

All my first information pointed to the use of the heather top, and so I collected with considerable care a quantity of the finest flowering stems of the common heather I could get. These were subjected to analysis for sugar in the laboratory under the care of Dr HUNTER STEWART. 100 grammes were digested for six hours in streaming steam 100 C.

The total bulk of the decoction so made was 1500 cubic centimetres. It contained ·1666 per cent. glucose. Another 100 grammes yielded 2·49 grammes glucose. These results show, then, that, decocted with steam, carefully selected heather tips yield practically $2\frac{1}{2}$ per cent. glucose. This seemed to give colour to the traditional possibility of fermenting a decoction of heather flowers.

Seeing, however, that the bees gather their honey from the flower itself, and not from the stick, a quantity of the bloom, as free as possible from all contamination with woody matter or leaves, was analysed at Granton, under the superintendence of Mr IRVINE, with the astonishing result that it yielded 17 per cent. of a substance which reduced Fehling's solution, and which by ordinary tests would appear to be sugar. To put this practically out of doubt, a decoction was made and yeast added to ferment it. The result was a negative one; there was no alcoholic fermentation and no disengagement of carbonic acid, so that there was no evidence of fermentable sugar being present in any proportion.

Having failed in the laboratory, it seemed advisable to have a trial made by a practical brewer, and Mr ANDREW MELVIN of the Boroughloch Brewery willingly undertook the experiment.

I supplied him with a quantity of pure heather blooms. He extracted 4 lbs. of this with 6 gallons of water. To give an idea of the bulk of the heather flower, 2 oz. very nearly equalled the bulk of one and a half imperial pints.

To the extract obtained yeast was added; at the end of ten days a fresh supply of yeast, artificial warmth being maintained and the cask well rolled. There was no appearance of fermentation. I examined it myself two days later, and its condition was unaltered. The fluid was of a fine dark beer colour, perhaps a little more inclined to red. When drawn from the top, it was bright and showed about $1\frac{1}{2}$ degrees of the saccharometer, a slight shade higher than before the yeast was added, which Mr MELVIN suggested might have arisen from a little wort adhering to the pressed yeast.

When the extract was first made, it had a marked woody flavour, but after the treatment above described this had entirely disappeared. This seems quite a satisfactory experiment from a brewer's point of view, and proves that a fairly strong infusion of carefully picked heather flowers will not ferment. This entirely agrees with the Granton laboratory experiment, which we should note was made alongside a solution of grape sugar of the same proportions as the pseudo-glucose, the grape sugar fermenting while the heather infusion remained unaltered.

Mr MELVIN'S extract was made in a copper jacket pan. The heather was added in two quantities, the brewer conducting the experiment thinking that the first quantity added was not sufficient. When I examined it, it had a perceptibly worty smell. It had no sweet taste, nor was thick in the mouth, nor indeed had it any special taste which one could be led to connect with the presence in it of heather flowers.

As we could not ferment a decoction of heather flowers, it seemed right to settle whether a decoction of malt could be fermented with heather. Some malt wort had heather flowers added to it and was carefully handled in the Boroughloch Brewery, in hopes that a good result would be got. After six days there was no appearance of an alcoholic fermentation, the heather blooms themselves were covered with a green mould where they floated upon the top of the wort, and the smell was by no means pleasant, a certain quantity of acetous fermentation being evidently present. After examining the sample myself it was corked and kept in a warm place, but no alcoholic fermentation took place.

It is difficult to believe that anyone who has repeated these stories ever made even a simple infusion of the flowers and tasted them. Such an infusion as strong as it can be made smells of the heather tops, but its taste is slightly bitter, with what one might describe as a leathery flavour; it is not in the least degree sweet, as would be the case if it contained any quantity at all of glucose.

Having now proved that beer could not be made from heather alone, and that the heather was not of itself a ferment, and regarding the recipes for heather ale which were the results of practical experience, Mr Melvin and I came to the conclusion that it could do nothing else, if it had any value at all, but act as a flavouring matter and preservative like hops. Mr Melvin then made the following experiment:—

Four gallons malt wort, sp. gr. 100, with four gallons of water, were boiled with heather flowers, total quantity being $2\frac{1}{2}$ lbs. The mixture was strained and the filtrate boiled for another half hour. The fluid smelt strongly of heather, and had an agreeable taste. It was now rapidly cooled to expose it as little as possible to air germs, and at a temperature of 69° Fahr. was poured into a 6-gallon cask, the quantity being made up to fill the cask of cooled boiled malt wort, and a pint of yeast well mixed into it. As it worked, the cask was kept carefully filled so as to allow the yeast to work thoroughly out of the beer. For the first twenty-four hours the fermentation was active and had an agreeable smell. When the fermentation was complete, the heather beer produced was bottled, and the result, though not perhaps adapted for exhibition, was a fairly potable liquor, with a roughish woody flavour peculiar to itself, and no doubt the effect of the heather which had been put into it. Before bottling, however, a sample was taken of the yeast from the latter workings and microscopically examined. The yeast cells were well defined, healthy, and vigorous-looking, but the field was decidedly impure, bacteria being present in such alarming quantity, from a brewer's point of view, that the beer's keeping quality was very doubtful. It was accepted that these bacteria came chiefly from the heather, as the yeast used was a very pure sample. Mr Melvin was of opinion that we had used too little heather for even such a small quantity as 6 gallons, and it was determined to make another brewing with fresh heather, guided by the experience already gained. In the following year, then, this intention was carried out, and a cask of heather ale of a highly satisfactory appearance was prepared and ready for bottling. Then the brewery unfortunately took fire, and afterwards all that was found of the heather beer that could be recognised were two hoops of the barrel and some charred staves. It had been fined and stacked with some special samples, of which as little remained. As the question was not one of flavouring malt, but of fermenting heather itself, and it being clearly proved that this was impossible, it was unnecessary to make further experiments. Nor has the result of our trials led Mr MELVIN to introduce to his consumers heather ale.

The truth is that the heather harvest is troublesome and not productive. To gather a pint of the flower, carefully stripped from the stalk, takes about one hour of diligent work. One of my gatherers was of opinion that the man who suffered martyrdom rather than tell the secret of how the heather ale was made deserved a monument by a grateful posterity. If the few bottles of beer we had got caused so much labour and expense, to what would it have been possible to compare the slavery of those unfortunate wives from whom their lords demanded it in bucketsful?

The whole story of heather beer has, I should fancy, arisen from a preconception as to the presence of honey in the heather flowers as we know it in the comb. The experiments so far go to prove that honey as such does not exist in the flower, and that bees are something more than mere gatherers so far as honey is concerned; but if the heather flower is extracted with ether the residuum on evaporation is ordinary beeswax, showing that this product exists already before gathering.

How misconceptions arise was proved by one who, seeing the heather being gathered and asking what it was for, said, "I will tell you who made heather ale not long ago: Mrs J. of E." "Who told you?" "Dr J., her son, was bad with asthma, and it is used as a cure for that."

As Dr J. was a professional brother, I took the liberty of writing to try to get at the bottom of the matter, and in a few words it turned out that from a paragraph in a paper, which stated that heather tea was a cure for asthma, A. had been dosed with it even by his own account with but little perceptible benefit. A. says that whether it would make beer or not he does not know, but that it was bitter enough to act, if it acted at all, as a tonic.

It was suggested that, by oxidation or other changes induced after plucking, the fermentable honey in the heather flowers might have altered. To see if this could possibly be the case, an attempt was made to ferment a decoction made of heather flowers gathered within thirty-six hours. There was no more evidence of fermentation under these circumstances than when the flowers had been kept for some time.

Further, to exclude any source of error in a statement that heather will not make ale in any way, experiments were made with a solution of honey, gravity by saccharometer 1056, to try if old blooms, or perfectly fresh, would act as a ferment in what might be considered their natural sugar. Neither gave any evidence of fermentation, while a parallel experiment with the same honey solution and ordinary yeast fermented successfully.

A supposititious and fabulous stimulant is also said to be procurable from the sap of the birch tree. So convinced are many persons of the possibility of the formation of such a liquor that they maintain that the Gaelic for whisky, *uisgè beatha*, aqua vitæ, is a corruption of *uisge beithe*, birch water. Hooker, in his *British Flora*, informs us that a wine is made from Betula Alba in Scotland, and other authorities speak of its rich, sugary, plentiful spring sap which makes a beer, a wine, and a vinegar.

As in the case of heather beer, the use of this sap is referred to old times. "At a very remote period Highlanders made incisions in birch trees in spring, and collected the juice which fermented and became a gentle stimulant" (Paper by a Supervisor of Excise, *Celtic Magazine*, vol. xi. p. 381). "Most of us when boys have had our favourite birch tree, and enjoyed the *fion*, wine."

A small matter delights boys. A native of Killin, in Perthshire, says that they made fissures in birch trees and sucked the juice with their mouths. One fissure would yield enough for a whole day. He used to go back and back to it. A deeper and wider hole was scooped at the bottom of the cut in which the sap accumulated. Others again peel the bark and scrape off and chew the white inner bark, which is very sappy. This goes by the name of *Snothach*, which simply means the sap. Lightfoot gives the following recipe, which, however, falls back on sugar for the source of the alcohol. He settles the question of self-fermentation by hard boiling. He says: "In the beginning of March, when the sap is rising, and before the leaves shoot out, bore holes in the bodies of the larger trees and put fossets therein, made of elder stick with the pith taken out, and then put any vessels under to receive the liquor. If the tree be large you may tap it in four or five places at a time without hurting it, and thus from several trees you gain several gallons of juice in a day. If you have not enough in one day, bottle up close what you have till you get a sufficient quantity for your purpose, but the sooner it is used the better. Boil the sap as long as any scum rises, skimming it all the time. To every gallon of liquor put four pounds of sugar and boil it afterwards half an hour, skimming it well; then put it into an open tub to cool, and when cold run it into your cask; when it has done working bung it up close, and keep it three months. Then either bottle it off or draw it out of the cask after it is a year old. This is a generous and agreeable liquor, and would be a happy substitute in the room of the poisonous whisky" (*Trans. Gael. Soc. Inv.*, vol. vii. p. 136).

This is one of the most imaginative prescriptions with the circumstance that has been written. The generous liquor can only have been simple syrup.

The method of gathering the sap, and the knowledge shown of the time for doing so, limits exactly the practical knowledge of Lightfoot's informant. The birch sap does not run when the leaf begins to bud; that is evidence of the cessation of the flow.

Samples were got from the central district of Perthshire and from the island of Islay for purposes of analysis. The Islay sending was first to arrive. On drawing one of the bottles and tasting it, it was so entirely free from taste of any sort, and so limpid, that the conclusion was at once formed that rain water had been supplied, and gave the credit to the friend acting as a collector of playing a trick, but on inquiry the forester who had gathered it, a thoroughly reliable man, described precautions to prevent any dilution by rain, and was prepared to guarantee that the sap was free from mixture of any kind. An analysis was then made in the Public Health Laboratory in the University, reported as follows :—

"It was slightly opalescent, with a faint acid reaction. Its specific gravity was 1002, water 1000. It contained no sugar, its total solids being 0·302 per cent. The solid matter was almost entirely vegetable albumen, and contained neither starch nor dextrine."

The Perthshire sample arrived in Edinburgh in the beginning of April, which accurately corresponds with the time of the year mentioned by Lightfoot as the best in which to gather it. It was very carefully analysed at Granton. It was also slightly opalescent, and it was impossible to filter it clear. At 59° Fahr. it had a specific gravity of 1003·09. It yielded on evaporation over the water bath 0·6910 solid matter per cent. On examination this was found to consist of extractive matter 5·420 per 1000 grammes (or grammes per litre); fermentable sugar, 0·440 per 1000; tartaric acid, 0·455; ash, lime, magnesia, and alkalies, 0·290.

In addition to the solid matter there was a small quantity of alcohol present, 0·244 per 1000; carbonic acid, 0·293 per 1000.

This alcohol must have been the result of a natural fermentation of the sugar in the sap, and allowing for this, the amount per cent. of sugar in the sap, including what had been fermented naturally, and what was fermentable, was 0·680 grammes per litre, or 0·0680 per cent.

The determination of the sugar was made by fermentation with dry German yeast; the tartaric acid by the silver mirror. The extractive matter was found to reduce copper, but only being fermentable to the small extent mentioned, we may conclude that the most of it was dextrine; this, however, does not agree with the first analysis, which gave no sugar, nor starch, nor dextrine, and consequently the whole was supposed to be vegetable albumen.

From a practical point of view, and granting what is possible, that samples of birch sap may vary in the richness of their contents, it never could of itself yield anything active either as a drink or as a medicine. Lightfoot's recipe would come out probably quite the same if so much water were used in place of the sap.

In fact, the conclusions reached are, that though heather flowers, if infused in malt, or added to a fermentable syrup, may give a flavour to the product, it is in itself of no value as a source of ale or beer.

Birch sap is equally useless, and can only give satisfaction as a drink or a medicine from an imaginative point of view.

III.—*On the Eliminant of a Set of General Ternary Quadrics.*—(Part II.)

By Thomas Muir, LL.D.

(Read December 4, 1899.)

(26) Of the various determinant forms thus far obtained the most promising is that of § 8 or that of § 14; and to these it is desirable now to return in order to obtain an expression for the eliminant in the ordinary non-determinant notation. In doing so it will also be well to make a slight change in the coefficients of the three quadrics, viz., to write f, g, h for $2f$, $2g$, $2h$, as in this way the diversity in the cofactors of the determinants occurring in the last three rows of either form of the eliminant disappears.

Using first the result of § 8, we have therefore as the eliminant of

$$\left.\begin{aligned} a_1x^2 + b_1y^2 + c_1z^2 + f_1yz + g_1zx + h_1xy &= 0 \\ a_2x^2 + b_2y^2 + c_2z^2 + f_2yz + g_2zx + h_2xy &= 0 \\ a_3x^2 + b_3y^2 + c_3z^2 + f_3yz + g_3zx + h_3xy &= 0 \end{aligned}\right\}$$

the determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 & f_1 & g_1 & h_1 \\ a_2 & b_2 & c_2 & f_2 & g_2 & h_2 \\ a_3 & b_3 & c_3 & f_3 & g_3 & h_3 \\ \cdot & [5] & -[3] & [8]+[8'] & [6] & [0] \\ -[1] & \cdot & [6] & [0] & [9]+[9'] & [4] \\ [4] & -[2] & \cdot & [5] & [0] & [7]+[7'] \end{vmatrix},$$

where

$$\begin{aligned} [0] &= |\,a_1b_2c_3\,|, \\ [1],\ [2],\ [3] &= |\,a_1g_2h_3\,|,\quad |\,b_1h_2f_3\,|,\quad |\,c_1f_2g_3\,|, \\ [4],\ [5],\ [6] &= |\,a_1b_2g_3\,|,\quad |\,b_1c_2h_3\,|,\quad |\,c_1a_2f_3\,|, \\ [7],\ [8],\ [9] &= |\,a_1b_2f_3\,|,\quad |\,b_1c_2g_3\,|,\quad |\,c_1a_2h_3\,|, \\ [7'],\ [8'],\ [9'] &= |\,b_1g_2h_3\,|,\quad |\,c_1h_2f_3\,|,\quad |\,a_1f_2g_3\,|. \end{aligned}$$

Now, as the minors formed from the first three rows of this determinant of the 6th order are the set of twenty to which belong the thirteen determinants appearing in the other three rows, it follows that if we take the expansion in terms of minors formed from the first three rows and their complementaries, we shall obtain for the eliminant an expression consisting of terms each of which is the product of four of the twenty determinants [0], [1], [2], ... Doing this, and bearing in mind the existence of triads due to the cyclo-symmetry, we have as a preliminary form of the development

$$
\begin{aligned}
&[0]\begin{vmatrix} [8]+[8'] & [6] & [0] \\ [0] & [9]+[9'] & [4] \\ [5] & [0] & [7]+[7'] \end{vmatrix} \\
&-\overset{\circ}{\Sigma}[7]\begin{vmatrix} -[3] & [6] & [0] \\ [6] & [9]+[9'] & [4] \\ . & [0] & [7]+[7'] \end{vmatrix}
+\overset{\circ}{\Sigma}[4]\begin{vmatrix} -[3] & [8]+[8'] & [0] \\ [6] & [0] & [4] \\ . & [5] & [7]+[7'] \end{vmatrix}
-\overset{\circ}{\Sigma}[10]\begin{vmatrix} -[3] & [8]+[8'] & [6] \\ [6] & [0] & [9]+[9'] \\ . & [5] & [0] \end{vmatrix} \\
&+\overset{\circ}{\Sigma}[9']\begin{vmatrix} [5] & -[3] & [0] \\ . & [6] & [4] \\ -[2] & . & [7]+[7'] \end{vmatrix}
-\overset{\circ}{\Sigma}[4']\begin{vmatrix} [5] & -[3] & [6] \\ . & [6] & [9]+[9'] \\ -[2] & . & [0] \end{vmatrix}
+\overset{\circ}{\Sigma}[1]\begin{vmatrix} [5] & -[3] & [8]+[8'] \\ . & [6] & [0] \\ -[2] & . & [5] \end{vmatrix} \\
&-[0']\begin{vmatrix} . & [5] & -[3] \\ -[1] & . & [6] \\ [4] & -[2] & . \end{vmatrix}.
\end{aligned}
$$

The development of the first of these eight terms, if we agree to drop the rectangular brackets in [0], [1], ... ,

$$
\begin{aligned}
&= 0(8+8')(9+9')(7+7') + 0000 + 0456 - \overset{\circ}{\Sigma}004(8+8'), \\
&= \left.\begin{array}{r} 0789 + 0897' + 089'7 + 089'7' + 08'97 + 08'97' + 08'9'7 + 08'9'7' + 0000 + 0456 \\ - \overset{\circ}{\Sigma}0048 - \overset{\circ}{\Sigma}0048' \end{array}\right\}, \\
&= 0789 + \overset{\circ}{\Sigma}0789' + \overset{\circ}{\Sigma}078'9' + 07'8'9' + 0000 + 0456 - \overset{\circ}{\Sigma}0048 - \overset{\circ}{\Sigma}0048';
\end{aligned}
$$

that of each of the others is directly evident; and the collected and simplified whole is

$$
\begin{array}{lllll}
0000 & & \qquad & +\overset{\circ}{\Sigma}1268 & \\
+\overset{\circ}{\Sigma}001\overline{11} & & & +\overset{\circ}{\Sigma}1268' & +\overset{\circ}{\Sigma}1268' \\
-\overset{\circ}{\Sigma}0048 & -\ \overset{\circ}{\Sigma}0048 & & -\overset{\circ}{\Sigma}126'8 & \\
-\overset{\circ}{\Sigma}0048' & & & -\overset{\circ}{\Sigma}126'8' & \\
+\overset{\circ}{\Sigma}0123 & & & +\overset{\circ}{\Sigma}1556 & +\overset{\circ}{\Sigma}1556 \\
-\overset{\circ}{\Sigma}0158 & -\ \overset{\circ}{\Sigma}0158 & & -\overset{\circ}{\Sigma}1556' & \\
+\overset{\circ}{\Sigma}0456 & +\ \ 0456 & & -\overset{\circ}{\Sigma}167\overline{11} & \\
-\overset{\circ}{\Sigma}0456' & & & -\overset{\circ}{\Sigma}167'\overline{11} & \\
+\overset{\circ}{\Sigma}049\overline{11} & & & +\overset{\circ}{\Sigma}1788 & \\
+\overset{\circ}{\Sigma}049'\overline{11} & & & +\overset{\circ}{\Sigma}1788' & \\
+\ \ 0789 & & & +\overset{\circ}{\Sigma}17'88 & \\
+\overset{\circ}{\Sigma}0789' & & & +\overset{\circ}{\Sigma}17'88' & \\
+\overset{\circ}{\Sigma}078'9' & & & +\overset{\circ}{\Sigma}4488 & \\
+\ \ 07'8'9' & & & +\overset{\circ}{\Sigma}4488' & \\
+\ \ 0'123 & & & -\overset{\circ}{\Sigma}446\overline{11} & \\
-\ \ 0'456 & & & -\overset{\circ}{\Sigma}4589 & \\
 & & & -\overset{\circ}{\Sigma}4589'. &
\end{array}
$$

(27) Taking now the result of § 14, viz.,

$$
\begin{vmatrix}
a_1 & b_1 & c_1 & f_1 & g_1 & h_1 \\
a_2 & b_2 & c_2 & f_2 & g_2 & h_2 \\
a_3 & b_3 & c_3 & f_3 & g_3 & h_3 \\
. & -2 & 8 & 5-5' & 0 & 7 \\
9 & . & -3 & 8 & 6-6' & 0 \\
-1 & 7 & . & 0 & 9 & 4-4'
\end{vmatrix},
$$

and expanding in exactly the same way, we have

$$\begin{array}{lll}
\quad 0000 & & +\mathring{\Sigma}1268 \\
+\mathring{\Sigma}001\overline{11} & & -\mathring{\Sigma}126'8 - \mathring{\Sigma}126'8 \\
-\mathring{\Sigma}0048 - \mathring{\Sigma}0048 & & +\mathring{\Sigma}1268' \\
+\mathring{\Sigma}004'8 & & -\mathring{\Sigma}126'8' \\
+\mathring{\Sigma}0123 & & +\mathring{\Sigma}1556 \\
-\mathring{\Sigma}0158 - \mathring{\Sigma}0158 & & -\mathring{\Sigma}1556' \\
+\ \ 0456 & & -\mathring{\Sigma}155'6 \\
-\mathring{\Sigma}0456' & & +\mathring{\Sigma}155'6' \\
+\mathring{\Sigma}045'6' & & -\mathring{\Sigma}167\overline{11} \\
-\ \ 04'5'6' & & +\mathring{\Sigma}16'7\overline{11} \\
+\mathring{\Sigma}049\overline{11} & & +\mathring{\Sigma}1788 + \mathring{\Sigma}1788 \\
-\mathring{\Sigma}04'9\overline{11} & & +\mathring{\Sigma}17'88 \\
+\mathring{\Sigma}0789 \ + \ 0789 & & +\mathring{\Sigma}4488 \\
+\mathring{\Sigma}0789' & & -\mathring{\Sigma}44'88 \\
-\ \ 0'789 & & -\mathring{\Sigma}4589 \\
+\ \ 0'123 & & +\mathring{\Sigma}4'589 \\
 & & -\mathring{\Sigma}778\overline{12}.
\end{array}$$

(28) The terms common to the two expressions are

$$\begin{array}{lll}
\quad 0000 & & +\mathring{\Sigma}1268 \\
+\mathring{\Sigma}001\overline{11} & & -\mathring{\Sigma}126'8 \\
-\mathring{\Sigma}0048 - \mathring{\Sigma}0048 & & +\mathring{\Sigma}1268' \\
+\mathring{\Sigma}0123 & & -\mathring{\Sigma}126'8' \\
-\mathring{\Sigma}0158 - \mathring{\Sigma}0158 & & +\mathring{\Sigma}1556 \\
+\ \ 0456 & & -\mathring{\Sigma}1556' \\
-\mathring{\Sigma}0456' & & -\mathring{\Sigma}167\overline{11} \\
+\mathring{\Sigma}049\overline{11} & & +\mathring{\Sigma}1788 \\
+\ \ 0789 & & +\mathring{\Sigma}17'88 \\
+\mathring{\Sigma}0789' & & +\mathring{\Sigma}4488 \\
+\ \ 0'123 & & -\mathring{\Sigma}4589.
\end{array}$$

The remaining terms in each case are fourteen in number, and of course the aggregate of the one group must be equal to the aggregate of the other: that is to say, it must be possible to show that

$$\begin{array}{lcl}
-\mathring{\Sigma}0048' & = & \mathring{\Sigma}004'8 \\
+\mathring{\Sigma}0456 & & +\mathring{\Sigma}045'6' \\
+\mathring{\Sigma}049'\overline{11} & & -\ \ 04'5'6' \\
+\mathring{\Sigma}078'9' & & -\mathring{\Sigma}04'9\overline{11} \\
+\ \ 07'8'9' & & +\mathring{\Sigma}0789 \\
-\ \ 0'456 & & -\ \ 0'789 \\
+\mathring{\Sigma}1268' & & -\mathring{\Sigma}126'8 \\
+\mathring{\Sigma}1556 & & -\mathring{\Sigma}155'6
\end{array}$$

$$\begin{array}{lcl}
-\mathring{\Sigma}167'\overline{11} & [=] & +\mathring{\Sigma}155'6' \\
+\mathring{\Sigma}1788' & & +\mathring{\Sigma}16'7\overline{11} \\
+\mathring{\Sigma}17'88' & & +\mathring{\Sigma}1788 \\
+\mathring{\Sigma}4488' & & -\mathring{\Sigma}44'88 \\
-\mathring{\Sigma}446\overline{11} & & +\mathring{\Sigma}4'589 \\
-\mathring{\Sigma}4589 & & -\mathring{\Sigma}778\overline{12}\,.
\end{array}$$

Fortunately the process by which this is accomplished brings to light a simpler expression than either of the two.

(29) In the first place it can be shown that the aggregate of the first two terms on the left is equal to the aggregate of the first and fifth on the right, the single term $\mathring{\Sigma}049\overline{11}$ being an equivalent for either. As a matter of fact, we have, by a well-known elementary theorem,

$$\begin{array}{rcl}
a_1b_2c_3\,|\cdot|\,c_1h_2f_3\,| & = & |\,h_1b_2c_3\,|\cdot|\,c_1a_2f_3\,| \;+\; |\,a_1h_2c_3\,|\cdot|\,c_1b_2f_3\,|\,, \\
\textit{i.e.,}\qquad 08' & = & 56-9\overline{11}\,,
\end{array}$$

and therefore, on multiplying by -04,

$$-0048' = -0456 + 049\overline{11}\,,$$

and consequently

$$-\mathring{\Sigma}0048' + \mathring{\Sigma}0456 = \mathring{\Sigma}049\overline{11}\,;$$

and the fact that

$$\mathring{\Sigma}004'8 + \mathring{\Sigma}0789 = \mathring{\Sigma}049\overline{11}$$

follows in exactly the same way from the identity

$$04' = 6\overline{10} - 79\,.$$

(30) There are four other pairs of identities like this, the full collection being

$$\begin{array}{rcrcl}
-\mathring{\Sigma}0048' + \mathring{\Sigma}0456 & = & \mathring{\Sigma}049\overline{11} & = & \mathring{\Sigma}004'8 + \mathring{\Sigma}0789\,, \\
\mathring{\Sigma}049'\overline{11} - \mathring{\Sigma}446\overline{11} & = & -\mathring{\Sigma}47\overline{11}\,\overline{12} & = & -\mathring{\Sigma}04'9\overline{11} - \mathring{\Sigma}778\overline{12}\,, \\
\mathring{\Sigma}1556 + \mathring{\Sigma}4488' & = & \mathring{\Sigma}446'\overline{11} & = & \mathring{\Sigma}045'6' + \mathring{\Sigma}4'589\,, \\
\mathring{\Sigma}078'9' - \mathring{\Sigma}4589' & = & -\mathring{\Sigma}778'\overline{12} & = & \mathring{\Sigma}1788 - \mathring{\Sigma}44'88\,, \\
\mathring{\Sigma}1268' + \mathring{\Sigma}17'88' & = & \mathring{\Sigma}16'7\overline{11} & = & -\mathring{\Sigma}126'8 + \mathring{\Sigma}155'6'.
\end{array}$$

Further it can be shown that

$$-\mathring{\Sigma}167'\overline{11} + \mathring{\Sigma}1788' = \mathring{\Sigma}16'7\overline{11} - \mathring{\Sigma}155'6\,;$$

but unfortunately in this case there is no simple equivalent which can be substituted for either. This difficulty, however, can be overcome by taking one of the terms common to the two original expansions, viz. $\mathring{\Sigma}1268$, and adding it to each side, for then we have

$$\begin{aligned} -\mathring{\Sigma}167'\overline{11} + \mathring{\Sigma}1788' + \mathring{\Sigma}1268 &= +\mathring{\Sigma}16'7\overline{11} - \mathring{\Sigma}155'6 + \mathring{\Sigma}1268, \\ &= -\mathring{\Sigma}167'\overline{11} + \mathring{\Sigma}16'7\overline{11}. \end{aligned}$$

It is thus seen that in each of the original expansions an aggregate of thirteen terms may be supplanted by an aggregate of seven, viz.

$$\mathring{\Sigma}049\overline{11} - \mathring{\Sigma}47\overline{11}\,\overline{12} + \mathring{\Sigma}446'\overline{11} - \mathring{\Sigma}778'\overline{12} + \mathring{\Sigma}16'7'\overline{11} - \mathring{\Sigma}167'\overline{11}^{*} + \mathring{\Sigma}16'7\overline{11}^{*};$$

and that it only remains to prove the equality of

$$07'8'9' - 0'456 \qquad \text{and} \qquad -04'5'6' - 0'789,$$

and if possible to find for either of them a simpler equivalent.

Beginning with the left-hand side we derive from

$$07' = 45 - 8\overline{10}, \qquad 0'6 = 8'9' + 34',$$

in the same manner as before the identities

$$\begin{aligned} \mathring{\Sigma}07'8'9' &= \mathring{\Sigma}458'9' - \mathring{\Sigma}88'9'\overline{10}, \\ -\mathring{\Sigma}0'456 &= -\mathring{\Sigma}458'9' - \mathring{\Sigma}344'5, \end{aligned}$$

* For each of the terms $\mathring{\Sigma}167'\overline{11}$, $\mathring{\Sigma}16'7\overline{11}$ an alternative form is available, by reason of the existence of a curious kind of identity of which there are three instances, viz.:—

$$\begin{aligned} \mathring{\Sigma}167'\overline{11} &= \mathring{\Sigma}159'\overline{11}, \\ \mathring{\Sigma}16'7\overline{11} &= \mathring{\Sigma}148'\overline{11}, \\ \mathring{\Sigma}44'88' &= \mathring{\Sigma}4'589'. \end{aligned}$$

The mode of establishing these may be illustrated by proving the last of the three.

By a well-known theorem we have

$$\begin{aligned} |a_1b_2f_3|\,|c_1h_2g_3| &= |g_1b_2f_3|\,|c_1h_2a_3| + |a_1g_2f_3|\,|c_1h_2b_3| + |a_1b_2g_3|\,|c_1h_2f_3|, \\ \textit{i.e.,}\quad 76' &= 5'9 - 9'5 + 48', \\ \text{or}\quad 76' - 95' &= 48' - 59', \end{aligned}$$

where, be it observed, each side consists of two terms of a triad. Multiplying, then, both sides by the remaining term of either triad, say by 84', we have

$$84'(76'-95') = 84'(48'-59'),$$

and therefore by cyclical substitution

$$95'(84'-76') = 95'(59'-67'),$$

and

$$76'(95'-84') = 76'(67'-48').$$

From these by addition there results

$$0 = \mathring{\Sigma}84'48' - \mathring{\Sigma}84'59'$$

or

$$\mathring{\Sigma}4'589' = \mathring{\Sigma}44'88'.$$

The three fundamental identities which can be treated in this manner are

$$76' - 95' = 48' - 59' = 1\,\overline{11} - 2\,\overline{12},$$

or, of course, their derivatives by cyclical substitution.

and therefore by addition obtain

$$\begin{aligned}\hat{\Sigma}07'8'9' - \hat{\Sigma}0'456 &= -\hat{\Sigma}88'9'\overline{10} - \hat{\Sigma}344'5\,, \\ &= -\hat{\Sigma}77'8'\overline{12} - \hat{\Sigma}155'6\,.\end{aligned}$$

Again, from the identities,

$$06' = 5\overline{12} - 89\,, \qquad 0'7 = -29' + 4'5',$$

we derive

$$\begin{aligned}-\hat{\Sigma}04'5'6' &= -\hat{\Sigma}4'5'5\overline{12} + \hat{\Sigma}4'5'89\,, \\ -\hat{\Sigma}0'789 &= \hat{\Sigma}2899' - \hat{\Sigma}4'5'89\,,\end{aligned}$$

and thence by addition

$$-\hat{\Sigma}04'5'6' - \hat{\Sigma}0'789 = -\hat{\Sigma}44'6'\overline{11} + \hat{\Sigma}1788'.$$

Now the two alternative forms thus obtained, viz.

$$-\hat{\Sigma}77'8'\overline{12} - \hat{\Sigma}155'6\,, \qquad -\hat{\Sigma}44'6'\overline{11} + \hat{\Sigma}1788',$$

though no simpler than the original, are readily seen to be equal, because

$$\hat{\Sigma}1788' + \hat{\Sigma}77'8'\overline{12} = \hat{\Sigma}46'78' = \hat{\Sigma}4'589'$$

and

$$-\hat{\Sigma}44'6'\overline{11} - \hat{\Sigma}155'6 = \hat{\Sigma}5'67'9 = \hat{\Sigma}4'589'.$$

(31) The simplified form of the eliminant to which we are thus led contains twenty-one of the twenty-two terms given in § 28 as being common to the two original expansions, and nine others which take the place of the fifteen remaining; and if, further, we substitute for $-\hat{\Sigma}0456' - \hat{\Sigma}4589$ its equivalent $-\hat{\Sigma}446\overline{11}$, we have finally

$$\begin{array}{ll}
\ 0000 & -\hat{\Sigma}126'8' \\
+\hat{\Sigma}00\overline{11}\,\overline{11} & +\hat{\Sigma}1556 \\
-\hat{\Sigma}0048\ -\hat{\Sigma}0048 & -\hat{\Sigma}1556' \\
+\hat{\Sigma}0123 & -\hat{\Sigma}167\overline{11} \\
-\hat{\Sigma}0158\ -\hat{\Sigma}0158 & -\hat{\Sigma}167'\overline{11} \\
+\ 0456 & +\hat{\Sigma}16'7\overline{11} \\
+\hat{\Sigma}049\overline{11}+\hat{\Sigma}049\overline{11} & +\hat{\Sigma}16'7'\overline{11} \\
+\ 0789 & +\hat{\Sigma}1788 \\
+\hat{\Sigma}0789' & +\hat{\Sigma}17'88 \\
+\ 07'8'9' & -\hat{\Sigma}446\overline{11} \\
+\ 0'123 & +\hat{\Sigma}446'\overline{11} \\
-\ 0'456 & +\hat{\Sigma}4488 \\
+\hat{\Sigma}1268' & -\hat{\Sigma}47\overline{11}\,\overline{12} \\
-\hat{\Sigma}126'8 & -\hat{\Sigma}778'\overline{12}\,.
\end{array}$$

Possibly there may be found modifications of this expression which are at least equally compact. There certainly will be unlimited variety if the number of terms be not restricted as here, since for almost every one of the terms a substitution of two or more similar terms is possible. The only terms, indeed, which cannot be replaced are 0000, $-2\mathring{\Sigma}0048$, $\mathring{\Sigma}4488$.

(32) Leaving now the subject of these eliminants of high order,—a subject which, as we have seen, originated with SYLVESTER,—let us ascertain what is possible in the direction of attaining the eliminant in the form of a determinant of an order *lower than the sixth.*

In one of the special cases already referred to it has been shown * that from the three original equations in x^2, y^2, z^2, yz, zx, xy we were able to deduce a set of three in yz, zx, xy, a set in x^2, y^2, z^2, and a set in x, y, z, and thus to obtain expressions for the eliminant in the form of a determinant of the *third* order. That is to say, instead of having in our equations all the possible facients of the second degree, viz., both those of the type x^2 and those of the type yz, we succeeded in confining ourselves to equations having only one type of facient.

In the same case it was also shown that there could be deduced a set of four equations in x^2y, y^2z, z^2x, xyz; a set in xy^2, yz^2, zx^2, xyz, and a set in x^3, y^3, z^3, xyz: and that in this way expressions could be obtained for the eliminant in the form of a determinant of the *fourth* order. Here, where the facients are of the third degree, there are four types of them, x^3, x^2y, xy^2, xyz; and SYLVESTER, just as in the case of the facients of the second degree, used the whole of them and thus saddled himself with a determinant of the tenth order. The reduction to the seventh order made in § 10 was due, it may be noted, to the elimination of one of the four types, the set of facients implicitly retained being x^2y, y^2z, z^2x, xy^2, yz^2, zx^2, xyz.

We shall now see whether the processes applied to this special case can be extended to the general problem at present before us.

(33) When the set of facients does not possess the cyclo-symmetry apparent in each of the sets just spoken of, it is scarcely reasonable to expect that the resulting eliminant will be simple or elegant in form. If, therefore, we seek to obtain the eliminant as a determinant of the *fifth* order—a course which would necessitate the use of a set of facients like y^2, z^2, yz, zx, xy, or x^2, y^2, z^2, yz, zx,—we must be prepared for more or less irregularity and complexity. It will be found, nevertheless, that this fifth-order form is full of interest.

Taking the set y^2, z^2, yz, zx, xy, we examine our collection of derived quadrics having co-efficients of the third degree, viz.:—

[TABLE

* MUIR, T., "Further Note on a Problem of Sylvester's in Elimination," *Proc. Roy. Soc. Edin.*, xx. pp. 371–382.

Source.	x^2	y^2	z^2	yz	zx	xy	Temporary Name.
$\lvert u_1b_2c_3 \rvert$	0			$\overline{11}$	8	5	A_1
$\lvert u_1c_2a_3 \rvert$		0		6	$\overline{12}$	9	A_2
$\lvert u_1a_2b_3 \rvert$			0	7	4	$\overline{10}$	A_3
$\lvert u_1b_2f_3 \rvert$	7		-11		$-5'$	-2	B_1
$\lvert u_1c_2g_3 \rvert$	-12	8		-3		$-6'$	B_2
$\lvert u_1a_2h_3 \rvert$		-10	9	$-4'$	-1		B_3
$\lvert u_1b_2g_3 \rvert$	4		-8	$5'$		$7'$	C_1
$\lvert u_1c_2h_3 \rvert$	-9	5		$8'$	$6'$		C_2
$\lvert u_1a_2f_3 \rvert$		-7	6		$9'$	$4'$	C_3
$\lvert u_1b_2h_3 \rvert$	10		-5	2	$-7'$		D_1
$\lvert u_1c_2f_3 \rvert$	-6	11			3	$-8'$	D_2
$\lvert u_1a_2g_3 \rvert$		-4	12	$-9'$		1	D_3
$\lvert u_1f_2g_3 \rvert$	$9'$	$-5'$	3			$0'$	E_1
$\lvert u_1g_2h_3 \rvert$	1	$7'$	$-6'$	$0'$			E_2
$\lvert u_1h_2f_3 \rvert$	$-4'$	2	$8'$		$0'$		E_3
$\lvert u_1\ \ b_2y+f_2z\ \ c_3 \rvert \div x$		5	-3	$8+8'$	6	0	F_1
$\lvert u_1\ \ c_2z+g_2x\ \ a_3 \rvert \div y$	-1		6	0	$9+9'$	4	F_2
$\lvert u_1\ \ a_2x+h_2y\ \ b_3 \rvert \div z$	4	-2		5	0	$7+7'$	F_3
$\lvert u_1\ \ b_2\ \ f_3y+c_3z \rvert \div x$		-2	8	$5-5'$	0	7	G_1
$\lvert u_1\ \ c_2\ \ g_3z+a_3x \rvert \div y$	9		-3	8	$6-6'$	0	G_2
$\lvert u_1\ \ a_2\ \ h_3x+b_3y \rvert \div z$	-1	7		0	9	$4-4'$	G_3

and we find that there are *seven* of them which do not contain a term in x^2. Of these, however, B_3, C_3, D_3 are each derivable from A_2 and A_3, the connecting equations being

$$\begin{aligned} 9A_3 - \overline{10}A_2 &= 0B_3\,, \\ 6A_3 - 7A_2 &= 0C_3\,, \\ \overline{12}A_3 - 4A_2 &= 0D_3\,. \end{aligned}$$

It is not possible, therefore, to use B_3, C_3, D_3 along with A_2, A_3 in a process of dialytic elimination, and we are thus left with only four available equations, viz., A_2, A_3, F_1, G_1. On examining whether any pair of the remaining quadrics may be readily used to obtain a quadric free of x^2, we see that the possible pairs are

$$C_1\,,F_3\,;\quad C_2\,,G_2\,;\quad E_2\,,F_2\,;\quad E_2\,,G_3\,;\quad F_2\,,G_3\,;$$

but that because of the relations

$$\begin{aligned} -C_1 + F_3 &= G_1, \\ C_2 + G_2 &= F_1, \\ F_2 - G_3 &= C_3 = (6A_3 - 7A_2) \div 0, \end{aligned}$$

we are reduced to the use of the pairs E_2, F_2 and E_2, G_3, which by addition give the quadrics

$$\begin{array}{l} . + \quad 7'\,y^2 + (6-6')z^2 + (0+0')yz + (9+9')zx + \quad 4\ xy, \\ . + (7+7')y^2 + \ (-6')z^2 + (0+0')yz + \quad 9\ zx + (4-4')xy. \end{array}$$

Further, since the difference of these two is C_3, *i.e.*, $(6A_3-7A_2)\div 0$, it is immaterial which we use along with A_2, A_3, F_1, G_1 for the purpose of dialytically eliminating y^2, z^2, yz, zx, xy. Taking the former of the two, therefore, we have the set

$$\begin{array}{rrrrrl} 7'y^2 + & (6-6')z^2 + & (0+0')yz + & (9+9')zx + & 4xy, & \qquad E_2+F_2 \\ -2\,y^2 + & 8\ z^2 + & (5-5')yz + & 0\ zx + & 7xy, & \qquad G_1 \\ 5\,y^2 + & (-3)\,z^2 + & (8+8')yz + & 6\ zx + & 0xy, & \qquad F_1 \\ .\ + & 0\ z^2 + & 7\ yz + & 4\ zx + & \overline{10}xy, & \qquad A_3 \\ 0\,y^2 + & .\ \ + & 6\ yz + & 12\ zx + & 9xy, & \qquad A_2 \end{array}$$

from which there results the eliminant

$$\left|\begin{array}{ccccc} 7' & 6-6' & 0+0' & 9+9' & 4 \\ -2 & 8 & 5-5' & 0 & 7 \\ 5 & -3 & 8+8' & 6 & 0 \\ . & 0 & 7 & 4 & \overline{10} \\ 0 & . & 6 & \overline{12} & 9 \end{array}\right|.$$

The extraneous factor contained in it is readily ascertained to be 0, by trying to express the determinant as an aggregate of products of complementary minors, one minor of each product being formed from the elements of the last two rows, *e.g.*

$$\left|\begin{array}{cc} 4 & \overline{10} \\ \overline{12} & 9 \end{array}\right| = -01, \qquad \left|\begin{array}{cc} 7 & \overline{10} \\ 6 & 9 \end{array}\right| = -04', \qquad \ldots\ldots$$

Of course the cyclical substitution gives two other similar forms of the result.

(34) When the h's vanish, the ten determinants

$$0',\ 1,\ 2,\ 4',\ 5,\ 6',\ 7',\ 8',\ 9,\ \overline{10}$$

vanish also, and this eliminant of the fifth order degenerates into

$$\left|\begin{array}{cccc} 6 & 0 & 9' & 4 \\ 8 & -5' & 0 & 7 \\ -3 & 8 & 6 & 0 \\ 0 & 7 & 4 & . \end{array}\right|$$

which by translation of the last row or column to the first place is seen to be axi-symmetric. We thus have the following theorem *:—

The eliminant of the equations

$$\left.\begin{aligned} a_1x^2 + b_1y^2 + c_1z^2 + f_1yz + g_1zx &= 0 \\ a_2x^2 + b_2y^2 + c_2z^2 + f_2yz + g_2zx &= 0 \\ a_3x^2 + b_3y^2 + c_3z^2 + f_3yz + g_3zx &= 0 \end{aligned}\right\} ,$$

or the expression which equated to zero gives the condition that the loci of

$$\left.\begin{aligned} a_1x^2 + b_1y^2 + g_1x + f_1y + c_1 &= 0 \\ a_2x^2 + b_2y^2 + g_2x + f_2y + c_2 &= 0 \\ a_3x^2 + b_3y^2 + g_3x + f_3y + c_3 &= 0 \end{aligned}\right\}$$

have a point in common, is

$$\begin{Vmatrix} \cdot & |a_1b_2g_3| & |a_1b_2f_3| & |a_1b_2c_3| \\ |a_1b_2g_3| & |a_1f_2g_3| & |a_1b_2c_3| & |a_1f_2c_3| \\ |a_1b_2f_3| & |a_1b_2c_3| & |b_1f_2g_3| & |b_1c_2g_3| \\ |a_1b_2c_3| & |a_1f_2c_3| & |b_1c_2g_3| & |f_1c_2g_3| \end{Vmatrix} .$$

(35) Let us now pass from the eliminants of the fifth order to those of the third.

Taking the original set of three quadrics, and a derived quadric which is known not to be an aggregate of multiples of these, say F_1 of § 33, we have

$$\left.\begin{aligned} a_1x^2 + b_1y^2 + c_1z^2 + f_1yz + g_1zx + h_1xy &= 0 \\ a_2x^2 + b_2y^2 + c_2z^2 + f_2yz + g_2zx + h_2xy &= 0 \\ a_3x^2 + b_3y^2 + c_3z^2 + f_3yz + g_3zx + h_3xy &= 0 \\ 5y^2 - 3z^2 + (8+8')\,yz + 6zx + 0xy &= 0 \end{aligned}\right\} ,$$

and therefore by elimination of x^2, y^2, z^2,

$$\begin{vmatrix} a_1 & b_1 & c_1 & f_1yz + g_1zx + h_1xy \\ a_2 & b_2 & c_2 & f_2yz + g_2zx + h_2xy \\ a_3 & b_3 & c_3 & f_3yz + g_3zx + h_3xy \\ \cdot & 5 & -3 & (8+8')yz + 6zx + 0xy \end{vmatrix} = 0 ,$$

or

$$\begin{vmatrix} a_1 & b_1 & c_1 & f_1 \\ a_2 & b_2 & c_2 & f_2 \\ a_3 & b_3 & c_3 & f_3 \\ \cdot & 5 & -3 & 8+8' \end{vmatrix} yz + \begin{vmatrix} a_1 & b_1 & c_1 & g_1 \\ a_2 & b_2 & c_2 & g_2 \\ a_3 & b_3 & c_3 & g_3 \\ \cdot & 5 & -3 & 6 \end{vmatrix} zx + \begin{vmatrix} a_1 & b_1 & c_1 & h_1 \\ a_2 & b_2 & c_2 & h_2 \\ a_3 & b_3 & c_3 & h_3 \\ \cdot & 5 & -3 & 0 \end{vmatrix} xy = 0.$$

Expanding each determinant here in terms of the elements of the last row and their complementary minors, we change the equation into

$$\{0(8+8')+37-56\}yz + \{06+34-5\overline{12}\}zx + \{00+3\overline{10}-59\}xy = 0 ,$$

* The result obtained by Lord M'Laren in his paper on "Symmetrical Solution of the Ellipse-Glissette Elimination Problem," in the *Proc. Roy. Soc. Edin.*, xxii. pp. 379–387, is the particular case of this where f_1, f_3, g_2, g_3 are made to vanish and a_1, a_2, a_3 are put equal to b_2, b_1, b_3 respectively.

or, since $08' - 56 = -9\bar{1}\bar{1}$, into

$$(08+37-9\bar{1}\bar{1})yz + (06+34-5\bar{1}\bar{2})zx + (00+3\bar{1}\bar{0}-59)xy = 0.$$

From this by cyclical substitution two other equations are obtained, and thence the eliminant

$$\left|\begin{array}{lll} 08+37-9\bar{1}\bar{1} & 06+34-5\bar{1}\bar{2} & 00+3\bar{1}\bar{0}-59 \\ 00+1\bar{1}\bar{1}-67 & 09+18-7\bar{1}\bar{2} & 04+15-6\bar{1}\bar{0} \\ 05+26-4\bar{1}\bar{1} & 00+2\bar{1}\bar{2}-48 & 07+29-8\bar{1}\bar{0} \end{array}\right|. \qquad (\gamma_1)$$

(36) From the same set of four equations, by the elimination of yz, zx, xy, we find in exactly the same way

$$\{09'-4'6+1(8+8')\}x^2 + \{-50'-05'+26+7'(8+8')\}y^2 + \{30'+03+68'-6'(8+8')\}z^2 = 0,$$

and thence the eliminant

$$\left|\begin{array}{rrr} 09'-4'6+1(8+8') & -05'+2(6-6')+7'8 & (0+0')3+68'-6'(8+8') \\ (0+0')1+49'-4'(9+9') & 07'-5'4+2(9+9') & -06'+3(4-4')+8'9 \\ -04'+1(5-5')+9'7 & (0+0')2+57'-5'(7+7') & 08'-6'5+3(7+7') \end{array}\right|. \qquad (\gamma_2)$$

(37) The obtaining of a set of three equations in x, y, z may be viewed, of course, as the obtaining of a set in x^2, xy, xz; or the obtaining of a set in xy, y^2, zy; or the obtaining of a set in xz, yz, z^2. Consequently from the set of four equations in x^2, y^2, z^2, yz, zx, xy which we have been using a three-fold form of result is possible.

In the first place by the elimination of y^2, z^2, yz and subsequent division by x we obtain the equation

$$\{56+37-(8+8')0\}x + \{58'+32-(8+8')5+0\bar{1}\bar{1}\}y + \{-53+35'-(8+8')8+6\bar{1}\bar{1}\}z = 0,$$

or, since $56-08' = 9\bar{1}\bar{1}$ and $35+88' = 6'\bar{1}\bar{1}$,

$$\{9\bar{1}\bar{1}-37-08\}x + \{32-58+0\bar{1}\bar{1}\}y + \{6\bar{1}\bar{1}+35'-88-6'\bar{1}\bar{1}\}z = 0,$$

and thence the eliminant

$$\left|\begin{array}{lll} 9\bar{1}\bar{1}-37-08 & 32-58+0\bar{1}\bar{1} & 6\bar{1}\bar{1}+35'-88-6'\bar{1}\bar{1} \\ 4\bar{1}\bar{2}+16'-99-4'\bar{1}\bar{2} & 7\bar{1}\bar{2}-18-09 & 13-69+0\bar{1}\bar{2} \\ 21-47+0\bar{1}\bar{0} & 5\bar{1}\bar{0}+24'-77-5'\bar{1}\bar{0} & 8\bar{1}\bar{0}-29-07 \end{array}\right|. \qquad (\gamma_3)$$

In the second place by the elimination of x^2, z^2, xz and subsequent division by y we obtain the equation

$$\{-0\bar{1}\bar{2}+69-31\}x + \{-5\bar{1}\bar{2}+60+34\}y + \{-(8+8')\bar{1}\bar{2}+66+39'\}z =$$

and thence the eliminant

$$\left|\begin{array}{lll} -0\bar{1}\bar{2}+69-31 & -5\bar{1}\bar{2}+60+34 & -(8+8')12+66+39' \\ -(9+9')\bar{1}\bar{0}+44+17' & -0\bar{1}\bar{0}+47-12 & -6\bar{1}\bar{0}+40+15 \\ -4\bar{1}\bar{1}+50+26 & -(7+7')\bar{1}\bar{1}+55+28' & -0\bar{1}\bar{1}+58-23 \end{array}\right|. \qquad (\gamma_4)$$

Lastly by the elimination of x^2, y^2, xy and subsequent division by z we obtain the equation

$$\{6\bar{1}\bar{0}-04-51\}x + \{(8+8')\bar{1}\bar{0}-07-4'5\}y + \{-3\bar{1}\bar{0}-00+59\}z = 0,$$

and thence the eliminant

$$\begin{vmatrix} 04+\ 15-6\bar{1}\bar{0} & 07+\ 29-8\bar{1}\bar{0} & 00+3\bar{1}\bar{0}-\ 59 \\ 00+1\bar{1}\bar{1}-\ 67 & 05+\ 26-4\bar{1}\bar{1} & 08+\ 37-9\bar{1}\bar{1} \\ 09+\ 18-7\bar{1}\bar{2} & 00+2\bar{1}\bar{2}-\ 48 & 06+\ 34-5\bar{1}\bar{2} \end{vmatrix} . \qquad (\gamma_5)$$

(38) Each of these five determinants γ_1, γ_2, γ_3, γ_4, γ_5 is of the 18th degree in the coefficients of the original quadrics, and must, therefore, contain an extraneous factor of the 6th degree. It will be seen that the first and last are essentially the same, the coefficients of the equations connecting yz, zx, xy being the same as the coefficients of one of the sets of equations connecting x, y, z; that the third and fourth are more complicated; and that the second is still more so.

At the outset the separation of the extraneous factor seemed likely to be a matter of considerable difficulty; a method, however, was fortunately hit upon which effects it in every case with comparative ease. This will be fully understood from the following application of it to the case of γ_1 or γ_5:—

Looking at any column of γ_5 we observe that the first terms of the three trinomials composing the column have a factor in common, that the second terms have also a common factor, but that the same cannot be said of the third terms. In the case of the first column, for example, the three third terms are -610, -67, -712, where analogy would have led us to expect either a 7 in the first term or a 6 in the last. This difficulty, however, can be overcome by writing

$$\text{either}\quad 79 + 04' \quad\text{for}\quad 6\bar{1}\bar{0}$$

$$\text{or}\quad 46 - 09' \quad\text{for}\quad 7\bar{1}\bar{2}.$$

Taking the latter alternative the determinant becomes

$$\begin{vmatrix} 00\qquad +1\bar{1}\bar{1}-\ 67 & 05\qquad +\ 26-4\bar{1}\bar{1} & 0(8+8')+\ 37-\ 56 \\ 0(9+9')+\ 18-\ 64 & 00\qquad +2\bar{1}\bar{2}-\ 48 & 06\qquad +\ 34-51\bar{2} \\ 04\qquad +\ 15-6\bar{1}\bar{0} & 0(7+7')+\ 29-\ 45 & 00\qquad +3\bar{1}\bar{0}-\ 59 \end{vmatrix} ;$$

and, as a consequence, when we proceed to express it as an aggregate of 27 determinants with monomial elements, each of the 27 can have a factor removed from each of its columns. Further, when the said factors have been removed, 9 of the 27 must have two of their columns alike, and may therefore be neglected. Of the remaining 18 it will be found that 3 are symmetrical with respect to the cyclical substitution, and that 15 can be grouped in triads. The following condensed form of the expansion is thus readily obtainable:—

$$
\begin{aligned}
&000\begin{vmatrix} 0 & 5 & 8+8' \\ 9+9' & 0 & 6 \\ 4 & 7+7' & 0 \end{vmatrix} + \overset{\circ}{\sum}001\begin{vmatrix} 0 & 6 & 8 \\ 7+7' & 0 & 5 \\ 5 & 8+8' & \overline{11} \end{vmatrix} - \overset{\circ}{\sum}004\begin{vmatrix} 0 & 4 & 5 \\ 8+8' & 0 & \overline{11} \\ 6 & 9+9' & 8 \end{vmatrix} \\
&+\overset{\circ}{\sum}012\begin{vmatrix} 0 & 5 & 9 \\ 8+8' & \overline{11} & 6 \\ 6 & 8 & \overline{12} \end{vmatrix} - \overset{\circ}{\sum}015\begin{vmatrix} 0 & \overline{12} & 8 \\ 7+7' & 9 & 5 \\ 5 & 6 & \overline{11} \end{vmatrix} + \overset{\circ}{\sum}045\begin{vmatrix} 0 & 11 & 6 \\ 9+9' & 8 & \overline{12} \\ 4 & 5 & 9 \end{vmatrix} \\
&+(123-456)\begin{vmatrix} 7 & \overline{11} & 6 \\ 4 & 8 & \overline{12} \\ \overline{10} & 5 & 9 \end{vmatrix}.
\end{aligned}
$$

Now of the seven complex terms here it is seen that three have 00 for a factor, that 0 is a factor of other three, and that in the last 0 does not appear at all. A little investigation, however, suffices to show that 00 is a factor of every one. Thus, taking the first of the second three, we have

$$
\begin{aligned}
\overset{\circ}{\sum}012\begin{vmatrix} 0 & 5 & 9 \\ 8+8' & \overline{11} & 6 \\ 6 & 8 & \overline{12} \end{vmatrix} &= \overset{\circ}{\sum}012\{0(\overline{11}\,\overline{12}-68) - (8+8')(5\overline{12}-89) + 6(56-9\overline{11})\}, \\
&= \overset{\circ}{\sum}012\{0{\cdot}03 - (8+8'){\cdot}06' + 6{\cdot}08'\}, \\
&= \overset{\circ}{\sum}0012(03-6'8-6'8'+68'):
\end{aligned}
$$

and similarly

$$
-\overset{\circ}{\sum}015\begin{vmatrix} 0 & \overline{12} & 8 \\ 7+7' & 9 & 5 \\ 5 & 6 & \overline{11} \end{vmatrix} = -\overset{\circ}{\sum}0015(56'-37-37'-08'),
$$

and

$$
\overset{\circ}{\sum}045\begin{vmatrix} 0 & \overline{11} & 6 \\ 9+9' & 8 & \overline{12} \\ 4 & 5 & 9 \end{vmatrix} = \overset{\circ}{\sum}0045(34+8'9+8'9'-06').
$$

The seventh and last, which is the most interesting, may be dealt with in the same way, but the resulting identity, viz.

$$
\begin{vmatrix} 7 & \overline{11} & 6 \\ 4 & 8 & \overline{12} \\ \overline{10} & 5 & 9 \end{vmatrix} = 000',
$$

is a known theorem regarding compound determinants.*

* See Muir's "Determinants," p. 216, ex. 7. A more general theorem is obtained thus :—

$$
\begin{aligned}
\begin{vmatrix} |a_1l_2f_3| & |b_1m_2f_3| & |c_1n_2f_3| \\ |a_1l_2g_3| & |b_1m_2g_3| & |c_1n_2g_3| \\ |a_1l_2h_3| & |b_1m_2h_3| & |c_1n_2h_3| \end{vmatrix} &= |a_1l_2f_3|\{|b_1m_2g_3|\,|c_1n_2h_3| - |b_1m_2h_3|\,|c_1n_2g_3|\} \\
&\quad - |a_1l_2g_3|\{|b_1m_2f_3|\,|c_1n_2h_3| - |b_1m_2h_3|\,|c_1n_2f_3|\} \\
&\quad + |a_1l_2h_3|\{|b_1m_2f_3|\,|c_1n_2g_3| - |b_1m_2g_3|\,|c_1n_2f_3|\}
\end{aligned}
$$

and this expanded form may, by the use of the theorem

$$
|b_1m_2g_3|\,|c_1n_2h_3| = |b_1m_2c_3|\,|g_1n_2h_3| + |b_1m_2n_3|\,|c_1g_2h_3| + |b_1m_2h_3|\,|c_1n_2g_3|
$$

Setting the factor 00 aside we consequently obtain the eliminant in the form

$$0\begin{vmatrix} 0 & 5 & 8+8' \\ 9+9' & 0 & 6 \\ 4 & 7+7' & 0 \end{vmatrix} + \mathring{\Sigma}1\begin{vmatrix} 0 & 6 & 8 \\ 7+7' & 0 & 5 \\ 5 & 8+8' & \overline{11} \end{vmatrix} - \mathring{\Sigma}4\begin{vmatrix} 0 & 4 & 5 \\ 8+8' & 0 & 11 \\ 6 & 9+9' & 8 \end{vmatrix}$$

$$\begin{aligned} &+ 12(03-6'8-6'8'+68') - 15(56'-37-37'-08') \\ &+ 45(34+8'9+8'9'-06') + (123-456)0', \end{aligned}$$

which, on the development of the determinants, and substitution of

$$\begin{array}{lcl} -\mathring{\Sigma}0456 \;\; +\mathring{\Sigma}049\,\overline{11} & \text{for} & -\mathring{\Sigma}0048', \\ -\mathring{\Sigma}455\,\overline{12} \;\; +\mathring{\Sigma}4589 & \text{for} & -\mathring{\Sigma}0456', \\ \mathring{\Sigma}446\,\overline{11} \;\; -\mathring{\Sigma}47\,\overline{11}\,\overline{12} & \text{for} & \mathring{\Sigma}049'\overline{11}, \\ \mathring{\Sigma}4678' \;\; -\mathring{\Sigma}778'\overline{12} & \text{for} & \mathring{\Sigma}078'9', \\ \mathring{\Sigma}16'7\overline{11} \;\; -\mathring{\Sigma}1788' & \text{for} & \mathring{\Sigma}1268, \\ \mathring{\Sigma}16'7'\overline{11} \;\; -\mathring{\Sigma}1268' & \text{for} & \mathring{\Sigma}17'88', \\ \mathring{\Sigma}446'\overline{11} \;\; -\mathring{\Sigma}1556 & \text{for} & \mathring{\Sigma}4488', \end{array}$$

gives the result of § 31. Denoting, therefore, the eliminant of § 31 by E we have

$$\gamma_1 = 00 \cdot \mathrm{E} = \gamma_5 .$$

In the same way it can be shown that

$$\begin{aligned} \gamma_2 &= 0'0' \cdot \mathrm{E}, \\ \gamma_3 &= (2\,\overline{12}+6'7) \cdot \mathrm{E}, \\ \gamma_4 &= (-3\,\overline{10}+67') \cdot \mathrm{E}. \end{aligned}$$

(39) The extraneous factors in the cases of γ_3 and γ_4 are in appearance a little peculiar, as they give no evidence of being symmetrical with respect to the circular substitution. That they really possess this symmetry is, however, readily

be changed into

$$\begin{aligned} &|a_1l_2f_3| \cdot \{|b_1m_2c_3|\,|g_1n_2h_3| + |b_1m_2n_3|\,|c_1g_2h_3|\} \\ - &|a_1l_2g_3| \cdot \{|b_1m_2c_3|\,|f_1n_2h_3| + |b_1m_2n_3|\,|c_1f_2h_3|\} \\ + &|a_1l_2h_3| \cdot \{|b_1m_2c_3|\,|f_1n_2g_3| + |b_1m_2n_3|\,|c_1f_2g_3|\} \end{aligned}$$

and thus into

$$\begin{aligned} &|b_1m_2c_3| \cdot \{|a_1l_2f_3|\,|g_1n_2h_3| - |a_1l_2g_3|\,|f_1n_2h_3| + |a_1l_2h_3|\,|f_1n_2g_3|\} \\ + &|b_1m_2n_3| \cdot \{|a_1l_2f_3|\,|c_1g_2h_3| - |a_1l_2g_3|\,|c_1f_2h_3| + |a_1l_2h_3|\,|c_1f_2g_3|\} : \end{aligned}$$

so that by a second use of the said theorem we have

$$\begin{aligned} - &|b_1m_2c_3|\,|a_1l_2n_3|\,|f_1g_2h_3| \\ + &|b_1m_2n_3|\,|a_1l_2c_3|\,|f_1g_2h_3| \end{aligned}$$

and finally

$$|f_1g_2h_3| \begin{vmatrix} |b_1m_2n_3| & |a_1l_2n_3| \\ |b_1m_2c_3| & |a_1l_2c_3| \end{vmatrix},$$

or by a third use of the same theorem

$$|f_1g_2h_3| \begin{vmatrix} |b_1m_2a_3| & |c_1n_2a_3| \\ |b_1m_2l_3| & |c_1n_2l_3| \end{vmatrix}.$$

established by obtaining for them equivalent expressions which bear the symmetry on their faces.

Of such expressions three at least are useful in the process of separating E from its co-factor. These are, in the case of γ_3,

$$\tfrac{1}{3}(\mathring{\Sigma}1\,\overline{11}+\mathring{\Sigma}4'8)\,,\quad \frac{\overline{10}\,\overline{11}\,\overline{12}-7\,8\,9}{0}\,,\quad \frac{1\,2\,3+4'5'6'}{0'}.$$

The first arises from the theorem repeatedly used in the note to § 36,—a theorem which ensures the identity of

$$2\,\overline{12}+6'7,\quad 3\,\overline{10}+4'8,\quad 1\,\overline{11}+5'9\,,$$

and therefore gives

$$2\,\overline{12}+6'7 = \tfrac{1}{3}(\mathring{\Sigma}1\,\overline{11}+\mathring{\Sigma}4'8).$$

The second and third have essentially the same source, for from it we obtain

$$\begin{aligned} 02\overline{12}+06'7 &= (\overline{10}\,\overline{11}-57)\overline{12}+(5\,\overline{12}-89)7\,,\\ &= \overline{10}\,\overline{11}\,\overline{12}-7\,8\,9\,; \end{aligned}$$

and

$$\begin{aligned} 0'2\overline{12}+0'6'7 &= 2(13+6'9')+6'(4'5'-29')\,,\\ &= 1\,2\,3+4'5'6'. \end{aligned}$$

The similar expressions for the extraneous factor $67'-310$, which occurs in the case of γ_4, are

$$\tfrac{1}{3}(\mathring{\Sigma}1\,\overline{11}-\mathring{\Sigma}48')\,,\quad \frac{\overline{10}\,\overline{11}\,\overline{12}-4\,5\,6}{0}\,,\quad \frac{1\,2\,3-7'8'9'}{0'}\,;$$

and it may be noted in passing that there is a third triad of such equivalent expressions, viz.

$$\tfrac{1}{3}(\mathring{\Sigma}4'8+\mathring{\Sigma}48')\,,\quad \frac{4\,5\,6-7\,8\,9}{0}\,,\quad \frac{4'5'6'+7'8'9'}{0'}\,,$$

which are got from the two previous triads by subtraction.

(40) To each of these triads, however, a fourth member may be added, as there exist symmetrical expressions of a quite different kind which can be proved equal to $2\,\overline{12}+6'7,\quad 67'-3\,\overline{10},\quad 6'7+59'$ respectively.

The origin of this is an identity in compound determinants, viz.

$$\begin{vmatrix} |\alpha_1\xi_2| & |\alpha_2\xi_3| & |\alpha_3\xi_1| \\ |\beta_1\eta_2| & |\beta_2\eta_3| & |\beta_3\eta_1| \\ |\gamma_1\zeta_2| & |\gamma_2\zeta_3| & |\gamma_3\zeta_1| \end{vmatrix} = \begin{vmatrix} |\alpha_1\xi_2\eta_3| & |\alpha_1\xi_2\beta_3| \\ |\gamma_1\zeta_2\eta_3| & |\gamma_1\zeta_2\beta_3| \end{vmatrix} = \begin{vmatrix} |\beta_1\eta_2\zeta_3| & |\beta_1\eta_2\gamma_3| \\ |\alpha_1\xi_2\zeta_3| & |\alpha_1\xi_2\gamma_3| \end{vmatrix}$$

$$= \begin{vmatrix} |\gamma_1\zeta_2\xi_3| & |\gamma_1\zeta_2\alpha_3| \\ |\beta_1\eta_2\xi_3| & |\beta_1\eta_2\alpha_3| \end{vmatrix}\,;$$

the first part of which is established by multiplying the determinant on the left by $|\eta_3\beta_1|$ in the form

$$\begin{vmatrix} \eta_3 & \beta_3 & \cdot \\ \eta_1 & \beta_1 & \cdot \\ \eta_2 & \beta_2 & 1 \end{vmatrix},$$

and then removing the same factor from the product; and the others by making a cyclical change in the rows of the original determinant and using the part already proved.

Thus, putting $\alpha, \beta, \gamma, \xi, \eta, \zeta = a, b, c, f, g, h$, we have

$$\begin{vmatrix} |a_1f_2| & |a_2f_3| & |a_3f_1| \\ |b_1g_2| & |b_2g_3| & |b_3g_1| \\ |c_1h_2| & |c_2h_3| & |c_3h_1| \end{vmatrix} = 59' + 6'7 = 67' + 4'8 = 48' + 5'9;$$

putting $\alpha, \beta, \gamma, \xi, \eta, \zeta = a, b, c, g, h, f$, we have

$$-\begin{vmatrix} |a_1g_2| & |a_2g_3| & |a_3g_1| \\ |b_1h_2| & |b_2h_3| & |b_3h_1| \\ |c_1f_2| & |c_2f_3| & |c_3f_1| \end{vmatrix} = 1\overline{11} - 48' = 2\overline{12} - 59' = 3\overline{10} - 67';$$

and putting $\alpha, \beta, \gamma, \xi, \eta, \zeta = a, b, c, h, f, g$, we have

$$-\begin{vmatrix} |a_1h_2| & |a_2h_3| & |a_3h_1| \\ |b_1f_2| & |b_2f_3| & |b_3f_1| \\ |c_1g_2| & |c_2g_3| & |c_3g_1| \end{vmatrix} = 4'8 + 3\overline{10} = 5'9 + 1\overline{11} = 6'7 + 2\overline{12}.$$

What is still more interesting is the fact that, when ξ, η, ζ are made equal to β, γ, α, the above theorem in compound determinants degenerates into the theorem regarding the adjugate; and that we thus obtain

$$\begin{vmatrix} |a_1b_2| & |a_2b_3| & |a_3b_1| \\ |b_1c_2| & |b_2c_3| & |b_3c_1| \\ |c_1a_2| & |c_2a_3| & |c_3a_1| \end{vmatrix} = 00, \text{ and } \begin{vmatrix} |f_1g_2| & |f_2g_3| & |f_3g_1| \\ |g_1h_2| & |g_2h_3| & |g_3h_1| \\ |h_1f_2| & |h_2f_3| & |h_3f_1| \end{vmatrix} = 0'0'$$

The diversity in the extraneous factors is thus seen to disappear entirely, the results being

$$\gamma_1 = \big|\, |a_1b_2| \quad |b_2c_3| \quad |c_3a_1| \,\big| \cdot \mathrm{E} = \gamma_5,$$

$$\gamma_2 = \big|\, |f_1g_2| \quad |g_2h_3| \quad |h_3f_1| \,\big| \cdot \mathrm{E},$$

$$\gamma_3 = -\big|\, |a_1h_2| \quad |b_2f_3| \quad |c_3g_1| \,\big| \cdot \mathrm{E},$$

$$\gamma_4 = -\big|\, a_1g_2 \quad |b_2h_3| \quad |c_3f_1| \,\big| \cdot \mathrm{E}.$$

IV.—*On the Convection of Heat by Air Currents.* By Prof. A. Crichton Mitchell. (With a Plate.)

(Read March 6, 1899, and December 18, 1899.)

1. The present paper deals with a series of experiments made in the Physical Laboratory, Edinburgh University, from January to October 1899, with the object of determining the convective loss of heat from a cooling body owing to the action of currents of air.

2. As some of the results obtained have a bearing on the history of the laws of cooling, it is necessary to refer to at least one of the investigations which have been made on the subject. The first to give any definite statement regarding the law of cooling was Newton. In a paper * communicated to the Royal Society of London, his experiments on cooling are detailed, and his conclusion stated as follows :—

"*Nam calor quem ferrum calefactum corporibus frigidis sibi contiguis dato tempore communicat, hoc est Calor, quem ferrum dato tempore amittit, est ut Calor totus ferri.*"

Since this appeared in 1701 it has been known as Newton's Law of Cooling, and has generally been reproduced in some such form as "The rate of cooling of a body at any temperature is proportional to the difference between that temperature and the temperature of the surroundings of the body." Or more shortly, "Rate of cooling is proportional to temperature excess."

But at the end of his paper Newton makes the following important statement :—

"*Locavi autem ferrum non in œre tranquillo, sed in vento uniformiter spirante, ut œr a ferro calefactus semper abriperetur a vento, et œr frigidus in locum ejus uniformi cum motu succederet. . . . Sic enim œris partes œquales œqualibus temporibus calefactus sunt, et calorem conceperunt calori ferri proportionalem.*"

Strangely enough, nearly all subsequent references to Newton's Law of Cooling omit any mention of its most important qualification, viz., that the cooling body is placed in a current of air moving with uniform speed. The only clear exception is Fourier's remarks on Newton's Law in § xxxi. of his paper,† "Questions sur la théorie physique de la Chaleur rayonnante," where he points out that the cooling of bodies in still air, or rather in air which has no other movement than that resulting from change in density,

* "Scala graduum caloris et frigoris," *Phil. Trans.*, April 1701, vol. xxii. p. 824. Also in Newton's Works, Horsley's edition, 1782, vol. iv.

† *Ann. de Ch. et de Physique,* 1817, vi. pp. 259–303.

follows a different law which NEWTON has not taken into consideration. That FOURIER correctly appreciated the importance of the condition attached to Newton's Law of Cooling, viz., that the body was placed "non in ære tranquillo, sed in vento uniformiter spirante," is seen from the exceedingly careful manner in which he defined* the coefficient h, the 'conductibilité extérieure' of a body.

But the long succession of commentators, from MARTINE onwards, have practically criticised Newton's Law on the assumption that the cooling body was placed in (so-called) still air. DULONG and PETIT only refer to its most important condition as the action of a constant cooling cause.

So far as I am aware, LESLIE† was the only one who attempted to realise experimentally the conditions required in any investigation whose object is to determine the accuracy or otherwise of Newton's Law of Cooling. A metallic vessel, containing water at a temperature above that of the air, was tied to the end of a string and whirled in a circle for a definite period of time, after which the diminution in temperature of the water was noted. From an experimental method of this kind, little could be expected by way of accurate result.

3. In nearly all the experiments (excepting those of LESLIE referred to above) hitherto made to determine the law of cooling, the cooling body has been placed either in (so-called) still or free air, or in a vessel containing air whose pressure is different from that of the atmosphere, or in a (so-called) vacuum. It has always appeared to me that experiments of the kind must lead to results of a doubtful type, owing to the indefinite character of the conditions under which they are made. The shape of the cooling body must undoubtedly affect the direction, as the size will affect the speed, of the currents of air by means of which convection is carried on;‡ and, as a consequence, experiments made with cooling bodies of different shapes and sizes are not comparable so far as the determination of emissivity is concerned. The unsteadiness of so-called still air must also affect the results, for, as will be shown later in this paper, currents of air of speed so low as to make them almost imperceptible to the unaided senses, are sufficient to exercise an appreciable effect on the rate of cooling of a body. It would, in short, appear as if a carefully-defined unit of 'convectivity' (to coin a word for convection analogous to that for conduction) were required.

4. The method adopted in the present inquiry consisted in exposing a heated body to the cooling action of currents of air of different speeds, determining the temperature of the body at successive intervals of time, and thereby estimating the rate of cooling at given excesses of temperature at different speeds of the air current.

The apparatus employed is represented in fig. 5, and consisted essentially of the

* *Théorie Analytique de la Chaleur*, ch. i. sect. ii.

† LESLIE, *An Experimental Enquiry into the Nature and Propagation of Heat*, London, 1804, p. 279.

‡ See PORTER, *Phil. Mag.*, xxxix. 268–279.

following parts :—(1) The heated body; (2) arrangements for producing a current of air whose speed could be varied; (3) an instrument for registering the speed of the air current; (4) means for determination of the temperature of the body from time to time.

The body experimented upon was a copper ball, 2 inches in diameter. A circular hole, $\frac{3}{8}$ inch in diameter and $1\frac{1}{4}$ inch in depth, was bored radially so as to admit a thermo-electric junction. Opposite to this hole a copper hook was screwed into the ball, by which means the ball was suspended. Previous to each experiment the surface of the ball was cleaned, and then carefully blackened by exposing it to the sooty flame of a turpentine lamp. Care was taken to blacken the ball as nearly as possible in the same way for each experiment.* It was then heated in one of Fletcher's circular gas furnaces; a piece of fine wire gauze being placed between the ball and the flame from the air-gas jet, to prevent the flame either burning or blowing off the lampblack deposited on the surface (see fig. 1 in section and fig. 2 in plan). During the process of heating, the wire suspending the ball was twisted so as to make the ball rotate rapidly about its vertical diameter, in order to prevent any one side of the ball being heated more highly than another. For each experiment, the ball was heated for the same time, viz., fifteen minutes, and when taken out of the gas furnace its temperature was approximately 400° Centigrade.

The arrangements for producing a steady current of air consisted of one of the Blackman Ventilating Company's fans (fig. 5, E), 32 inches diameter, fitted into a triangular frame, GF, in the side of an air-tight wooden box, A B C D, whose dimensions were 5 feet in length, 6 feet in breadth, and 6 feet in height. The fan had a pulley, H, on its outer side, and by means of a belt passing over this to a shafting driven by a gas engine the fan was made to revolve at a sufficiently high speed, and thereby exhaust the air in the box. Into a circular hole in the box, on the side opposite to that holding the fan, one end of a tinned iron tube, KL, 5 feet long and 6 inches in diameter, was fitted. When the fan revolved air was drawn into the box through the tube. The speed of the fan was the same for all the experiments, but in order to obtain different speeds of air through the tube a movable slit (fig. 3 longitudinal section, fig. 4 transverse section) was placed at the end of the tube where it entered the box; and by widening or narrowing the slit the speed of the air current could be increased or decreased between the limits of 10 and 1000 metres per minute.

It was necessary to obtain a current the stream lines in which were parallel to the axis of the tube. That such was obtained was proved by allowing smoke (tobacco smoke or sal-ammoniac fumes) to be drawn into the tube at L when the fan was working, and noticing the direction taken by it in passing through the tube. The necessity for such a precaution was noticed during some preliminary experiments made with a fan of 12 inches diameter fitted to the wider end of a conical tube. The

* It is very desirable that some coating be found, similar to that of soot or lampblack, which will not readily rub off, or be affected by steam or exposure to a high temperature.

diameter of this tube at the wider end was the same as that of the fan, and at its narrower end 6 inches. But this small fan had been so constructed that it could only drive air *into* the tube. I found that when this was done the air driven through the tube had no steady motion ; that, in fact, its path resembled a spiral.

The large wooden box into which the 30-inch fan was fitted was necessary for two reasons. First, in order to get the most work out of such a fan for a given rate of revolution it is necessary that it be allowed (what is termed by ventilating engineers) 'free feed' and 'free discharge.' In other words, it must catch up air from a free space, and must discharge it into a free space. The wooden box fulfilled these conditions sufficiently for all practical purposes. Second, the box acted as a regulator of the speed of the air current through the tube when this might tend to vary owing to any slight irregularity of the gas engine which drove the fan.

The end of the tube farthest from the fan was surrounded for a distance of $1\frac{1}{2}$ feet by a water jacket, M, into which water from a cistern entered at Q, and was discharged at R. The temperature of the water entering the jacket was, in all the experiments, nearly that of the air passing through the tubes. In any case where the temperatures of air and water differed by more than 1° Centigrade the results of the experiments were not employed in the final deduction of results.

The copper ball, after being heated, was so suspended by its hook from a loop of copper wire fastened to the inner surface of the tube that its centre was in the axis of the tube, and in the middle section, P, of the jacketed portion of the tube.

The instrument employed to determine the speed of the current of air was an aluminium fan anemometer, constructed by Richard Frères of Paris. It was placed in the position N shown in fig. 5, and so rested on three guides that its centre was in the axis of the tube. It was found that with a given speed of the fan, the speed recorded when the ball was hanging in the tube was lower than when the ball was not in the tube. This difference, which was due to the ball disturbing the steady motion of the air through the tube, was greater at higher speeds. Allowance was made for it by directly observing its amount for each speed at which an experiment was made.

The temperature of the copper ball was ascertained by means of a thermo-electric junction of iron and German silver, which passed through a pumice-stone plug placed in the circular hole bored in the ball. The junction of the two wires forming the circuit was as nearly as possible at the centre of the ball. A Thomson's reflecting galvanometer was included in the circuit. The value of unit deflection on the scale was ascertained after each experiment by noting the deflection produced by placing the junction in steam issuing from boiling water, and also noting the temperature of the mercury pool, which formed the cold junction of the circuit.

During the process of cooling, readings of the deflections on the galvanometer scale were taken from time to time. As a rule, unit deflection on the scale represented a difference in temperature of 2° Centigrade. The error in reading the scale did not likely exceed one-tenth of a division, that is, 0°·2 Centigrade. The interval of time between any two readings was generally chosen, so that the corresponding decrease in deflection was not less than ten divisions. The error in any one estimate of the rate of cooling during a given interval would therefore not exceed 1 per cent.

5. FOURIER'S equation for the motion of heat in a solid homogeneous sphere is

$$\frac{dv}{dt} = \frac{K}{CD}\left(\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr}\right)$$

with the condition that at the surface ($r = R$)

$$K\frac{dv}{dr} + hv = 0$$

The solution for $t = 0$ must, of course, be satisfied by the function of v and r representing the initial distribution. The solution for the particular case of uniform initial distribution is

$$v = \frac{2hR}{K}\sum_{e=1}^{e=\infty}\frac{\sin\frac{er}{R}}{\frac{er}{R}}\;\frac{\epsilon^{-\frac{K}{CD}\left(\frac{e}{r}\right)^2 t}}{e\operatorname{cosec} e - \cos e}$$

where e_1, e_2, etc., are the roots of the equation

$$\frac{eR}{\tan eR} = 1 - \frac{h}{K}R.$$

After some time has elapsed, the first term in the series for v is the only one of practical value. Further, if R be small, or the ratio $\frac{h}{K}$ be small, the temperature throughout the sphere will be uniform. To test this, we may take Macfarlane's determination * of the emissivity of a black surface and the value of K (thermal, not thermometric, conductivity) for copper to be 0·95 in C.G.S. units. With these assumptions it will be found that the difference between the temperature for $r = 0$ and $r = 1$ when $t = 600$ is within the limits of experimental error. It may, therefore, be assumed that the temperature at the centre of the ball is that of the ball generally.

6. The experimental results obtained may be regarded in two ways. *First,* their bearing on the law of cooling, stated by NEWTON; *second,* the rate of cooling for a given excess of temperature with different speeds of the current of air.

As regards the first of these, NEWTON gives no details as to the speed of the current of air in which he placed the cooling body, and merely states that it was placed "in

* *Proc. Roy. Soc.*, xxxii. 465.

vento uniformiter spirante." It is unlikely that the air current was produced artificially; most probably it was simply a breeze of wind which NEWTON believed to be blowing with uniform speed. Let us assume its speed to have been about eight miles per hour, and examine the results of an experiment made on the copper ball at or near that speed. The following table gives the rates of cooling in a current of air whose speed was 271 metres per minute (8·09 miles per hour):—

	Temperature Excess.				
	40	80	120	160	200
Rates of cooling (degrees Centigrade per minute).	3·9	8·0	12·0	16·2	20·6

From these figures it follows that the rate of cooling in a current of air whose speed is 217 metres per minute is proportional to the temperature excess, up to at least 120° of temperature excess, beyond which it proceeds according to some law involving terms higher than those of simple proportion.

Let us now take an experiment at a higher speed. At 1031 metres per minute the rates of cooling were as shown below:—

	Temperature Excess.				
	40	80	120	160	200
Rates of cooling (degrees Centigrade per minute).	8·9	17·8	26·7	35·6	44·5

Hence the rate of cooling at the higher speed of 1031 metres per minute is proportional to temperature excess up to at least the higher temperature excess of 200°. Other experiments confirm this result, and justify the following general statement—at least within the limits of 200° temperature excess, and an air-current speed of 1000 metres per minute:—

When a heated body is placed in a current of air of uniform speed the rate of cooling is proportional to the temperature excess, up to a temperature excess which increases with increasing speed of the current of air.

Another way—and from a historical point of view the more interesting way—of stating the result is, that

Newton's Law of Cooling is accurate under the conditions premised by him, provided the speed of the current of air passing the surface of the cooling body be sufficient.

It is necessary that this result should if possible be explained. I do not, of course, mean that the Law of Cooling generally is represented as above. Cooling is not the result of one action, but of at least three—radiation, convection, and conduction. Leaving the last out of consideration meanwhile, the law of radiation is known to involve higher powers of the temperature excess than the first. The amount of heat carried off by convection may be assumed to be proportional to temperature excess. Consequently, if the amount of heat radiated during unit time at a given temperature excess remain constant and comparatively small, while the amount lost by convection be considerably increased, the total rate of cooling due to both causes will become more nearly proportional to temperature excess. A rough estimate, deduced from results given later on, may be made of the relative amounts of heat lost by radiation and convection. When the copper ball was exposed to an air current of a speed of 1000 metres per minute, the rate of cooling, at a temperature excess of 50°, was $2\frac{1}{2}$° per minute due to radiation, and 7° per minute due to convection.

This seems to me an explanation of the result quoted above.

I would suggest that FORBES' method of determining thermal conductivity might be improved by the application of the above result. The bar experimented upon might serve for both parts—statical and cooling—of the experiment, and in both it might be exposed to a current of air passing *across* its breadth. The amount of heat lost by cooling during the statical experiment might then be determined with greater exactness than in the usual way.

7. As regards the rate of cooling at a given excess of temperature with different speeds of the cooling current of air, the results obtained are given in tabular form in the Appendix to this paper. Considering the many minute points in regard to which the experimental conditions might vary, the results are fairly concordant.

For a temperature excess of 50° the rates of cooling at different speeds of the air current were plotted as ordinates in a curve, the abscissæ in which were the different speeds. For 80° and 100° the same was done. The curves are shown in fig. 6.

A glance at these curves will show that the rate of cooling at a given temperature excess increases with the speed; at first almost proportionally to the speed, but afterwards more slowly. With regard to this peculiarity I would offer the following remarks by way of possible explanation.

Were the copper ball exposed to an air current whose speed gradually increased, there will obviously be reached a speed at which the motion of the air will cease to be steady motion; vortices will be formed, or the motion will become turbulent. It is reasonable to suppose that, until this speed is reached, the rate of cooling at a given excess will be proportional to speed. Beyond that speed, there being less and less steady motion of air past the surface of the ball, the cooling effect of the current will be less than proportional to its increase in speed. In this way the change in curvature of the curve might be explained. I attempted to determine the speed at which the motion of the air ceases to be steady, by allowing sal-ammoniac fumes to be drawn into the tube, gradually increasing the speed of the air current, and observing the direction taken by the stream lines of fumes in passing round the ball. The results were, however, indecisive. Perhaps a better indication as to whether such a critical speed really existed is afforded in another way. I have already stated that the speed recorded by the anemometer when the ball was hanging in the tube was less than when the ball was not in the tube, and that this difference varied with the speed. If now a curve be drawn whose ordinates represent this difference, and whose abscissæ represent the speeds with the ball in the tube, it will be seen that when a speed of about 450 metres per minute is reached the ordinates begin to increase much more quickly. This may be due to the motion of the air in passing, and after having passed, the ball ceasing to be steady at this speed, and therefore recording less in the anemometer. If, now, the curves in fig. 6 be examined, it will be seen that up to about the same speed the rate of cooling is nearly proportional to speed, but beyond that it is less than proportional to speed. I am therefore inclined to think that the change in curvature in the curves of fig. 6 is due to the motion of the air ceasing to be steady at or about a speed of 450 metres per minute.

Of course it is to be remembered that these results apply only to a cooling body of the shape and size of that used in this investigation. Were either shape or size different, the results would be different.

An improved method of experiment would be to heat a strip of platinum foil by means of an electric current, allow it to cool while exposed to a current of air passing across its breadth, and ascertain its temperature from time to time by means of its electrical resistance. It would then be possible to determine in absolute measure the amount of heat lost in unit time from unit surface for different excesses of temperature and for different speeds of air. No question regarding the character of the motion, steady or otherwise, of the air would then be involved.

There is another aspect of the question which deserves consideration. If the speed of the air current were increased enormously, friction between the air and the ball would generate heat, and thereby lessen the rate of cooling. That such would be the case is known from the behaviour of meteors in passing through the earth's atmosphere. Now, if the friction between highly rarefied air and a meteoric body moving at, say, 20 miles per second, is sufficient to render the body incandescent, the amount of heat similarly generated in air at the earth's surface with a speed of 45 miles per hour (a speed attained easily by the apparatus employed) may be sufficiently large to be measurable by experimental means. I tried to detect any result of this kind by the following experiment. The ball was placed in the tube, and allowed to remain there for forty-eight hours, so that its temperature might become the same as that of its surroundings and of the air in the room.* A Boys' radio-micrometer was fitted up in such a position that any rise in temperature of the surface of the ball might be at once detected. A current of air with a speed of about 45 miles per hour was then allowed to pass along the tube for ten minutes, after which the screen between the ball and

* The experiments described in this paper were conducted in an underground cellar, in which the diurnal variation of temperature is scarcely noticeable.

radio-micrometer was removed. Unfortunately, the circuit of the radio-micrometer was so delicately suspended that the vibration of the flooring, owing to the working of the gas engine, caused the spot of light on the scale to oscillate irregularly. I am not prepared to say whether the deflections observed were or were not partially due to a heating of the ball by the current of air. The time at my disposal, being furlough from India, did not allow of further inquiry in this direction.

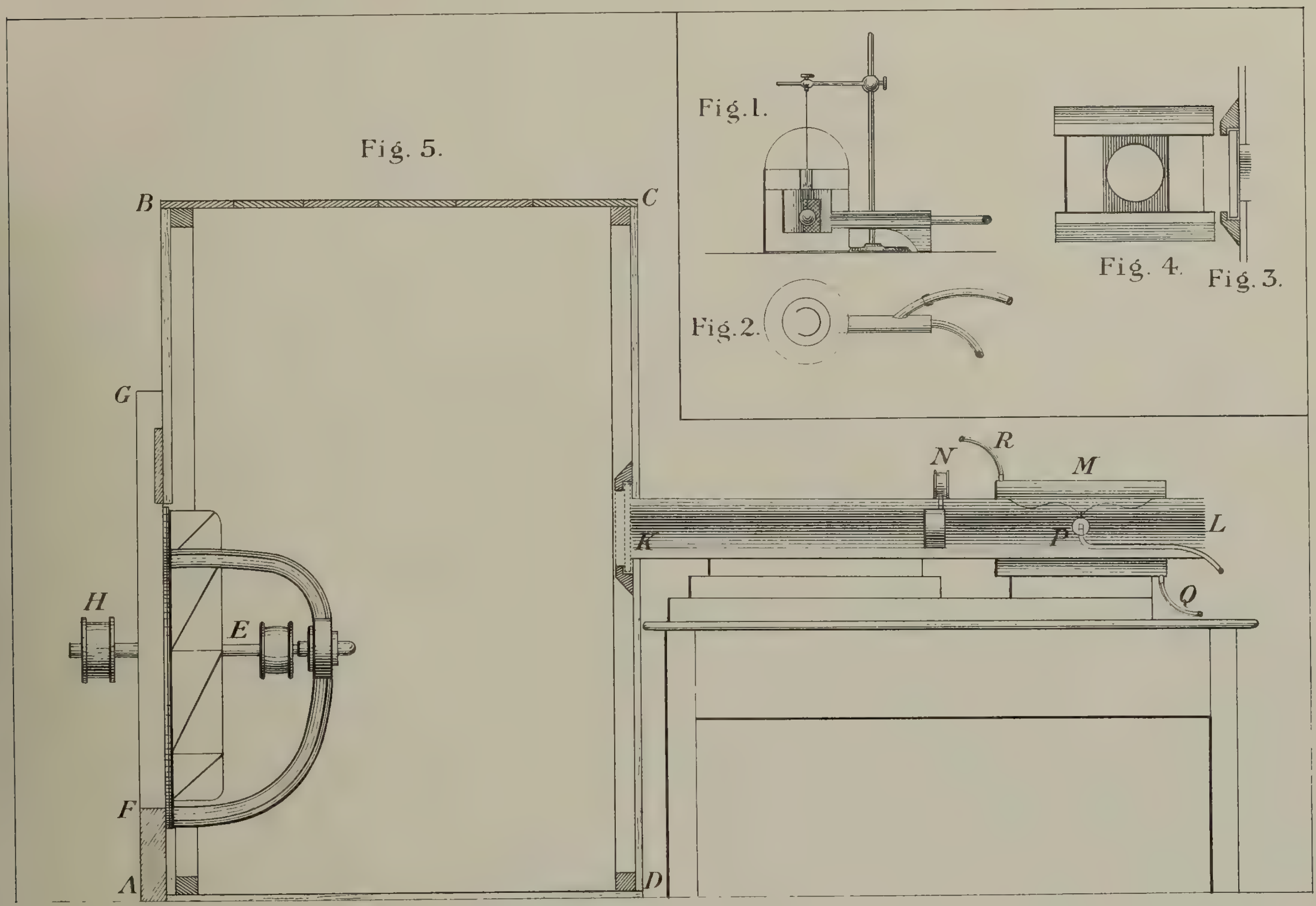
I have to acknowledge with many thanks Professor Tait's kindness in allowing me to make use of his laboratory and to draw upon its resources for the purpose of this investigation, and for otherwise assisting me with advice regarding it. I am also indebted to my friend Mr Robert Dickinson for some of the drawings published along with this paper. The Edinburgh University Court sanctioned a grant from the Moray Bequest for the purchase of apparatus required for the work.

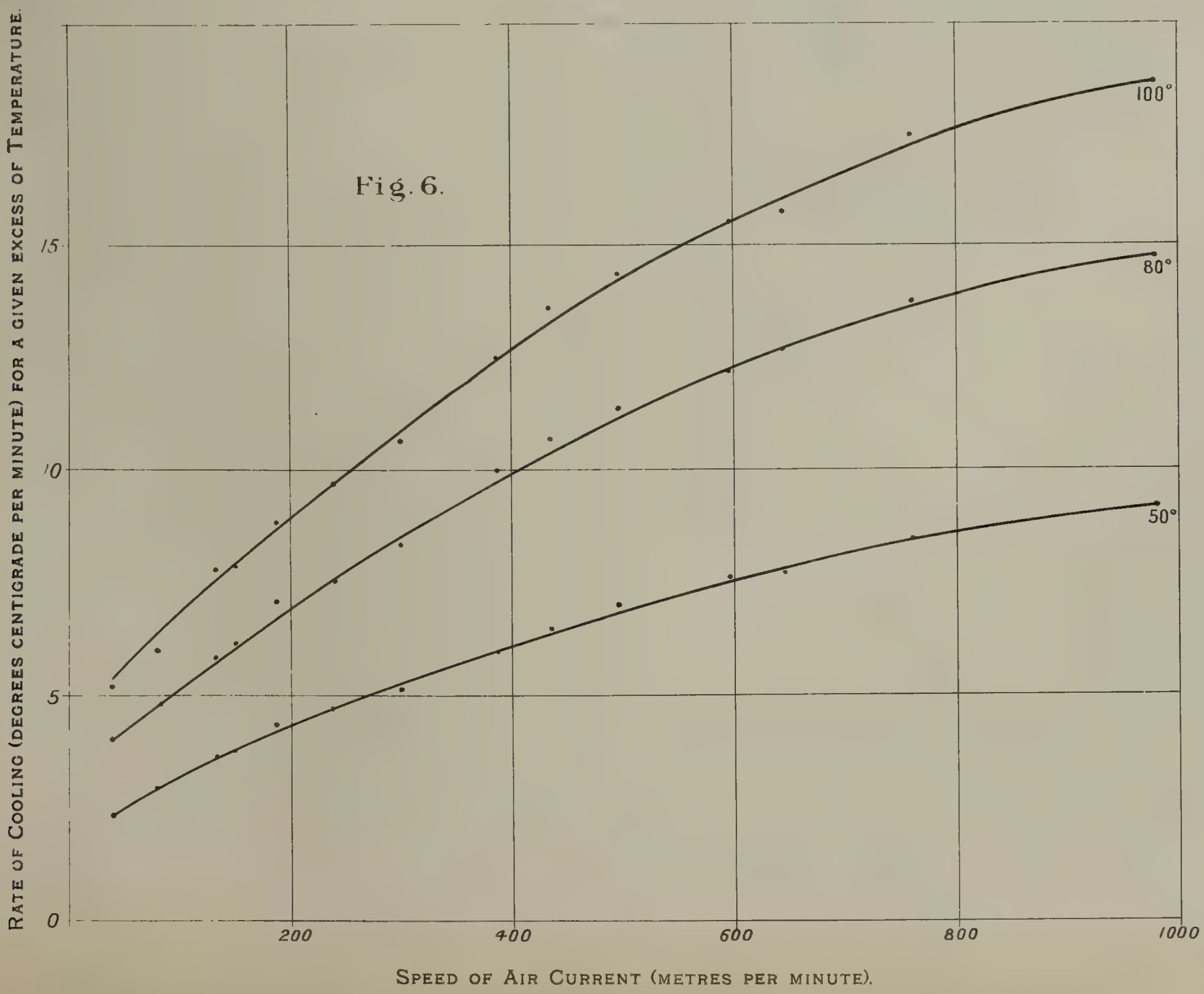
APPENDIX.

Rates of Cooling (Degrees Centigrade per Minute) at different Temperature Excesses and for different Speeds of the Air Current.

No. of Experiment,	37	2	9	11	12	27	28	17	19	18	21	29	25	26
Speed of Air Current,	41	79	133	149	187	240	301	388	436	497	597	643	758	976
10	0·47	0·58	0·72	0·71	0·90	0·97	1·07	1·20	1·18	1·34	1·37	1·44	1·35	1·80
20	0·94	1·16	1·44	1·45	1·76	1·83	2·00	2·37	2·42	2·82	2·91	2·99	2·87	3·62
30	1·43	1·76	2·18	2·23	2·67	2·83	3·11	3·51	3·76	4·23	4·45	4·57	4·97	5·40
40	1·93	2·37	2·91	3·04	3·53	3·81	4·18	4·71	5·15	5·63	6·00	6·21	6·69	7·34
50	2·38	2·98	3·68	3·81	4·43	4·73	5·20	6·02	6·45	7·05	7·63	7·77	8·44	9·21
60	2·98	3·59	4·39	4·65	5·29	5·66	6·06	7·29	7·90	8·50	9·21	9·34	10·27	11·10
70	3·58	4·19	5·15	5·44	6·19	6·64	7·37	8·65	9·31	9·94	10·83	11·07	11·98	12·95
80	4·09	4·79	5·89	6·19	7·14	7·63	8·44	9·99	10·72	11·37	12·24	12·66	13·75	14·77
90	4·64	5·39	6·83	7·01	7·98	8·65	9·56	11·30	12·14	12·86	14·02	14·20	15·68	16·67
100	5·22	5·99	7·77	7·91	8·86	9·72	10·69	12·55	13·61	14·33	15·55	15·81	17·47	18·67

Dr. Crichton Mitchell on the Convection of Heat by Air Currents.





V.—*A Development of a Pfaffian having a Vacant Minor.*

By Thomas Muir, LL.D.

(Read March 19, 1900.)

(1) The simplest possible case of the development referred to in the title is one which is seen to follow instantly from the determinant definition of a Pfaffian. Thus, by the said definition,

$$\left|\begin{array}{ccccc} \cdot & \cdot & a_4 & a_5 & a_6 \\ & \cdot & b_4 & b_5 & b_6 \\ & & c_4 & c_5 & c_6 \\ & & & d_5 & d_6 \\ & & & & e_6 \end{array}\right| = \sqrt{\left|\begin{array}{cccccc} \cdot & \cdot & \cdot & a_4 & a_5 & a_6 \\ \cdot & \cdot & \cdot & b_4 & b_5 & b_6 \\ \cdot & \cdot & \cdot & c_4 & c_5 & c_6 \\ -a_4 & -b_4 & -c_4 & \cdot & d_5 & d_6 \\ -a_5 & -b_5 & -c_5 & -d_5 & \cdot & e_6 \\ -a_6 & -b_6 & -c_6 & -d_6 & -e_6 & \cdot \end{array}\right|},$$

and therefore

$$= \sqrt{\left\{ - \left|\begin{array}{ccc} a_4 & a_5 & a_6 \\ b_4 & b_5 & b_6 \\ c_4 & c_5 & c_6 \end{array}\right| \cdot \left|\begin{array}{ccc} -a_4 & -a_5 & -a_6 \\ -b_4 & -b_5 & -b_6 \\ -c_4 & -c_5 & -c_6 \end{array}\right| \right\}},$$

$$= \sqrt{|\, a_4 \ \ b_5 \ \ c_6 \,|^2},$$

$$= \pm |\, a_4 \ \ b_5 \ \ c_6 \,| ;$$

the result reached being that a Pfaffian with certain zero elements is expressible as a determinant. It should be noted that, as in many instances where this form of definition is used, there is an ambiguity as to sign : it is of less importance to note that here the appropriate sign is $-$.

(2) Another case may readily be established by using either as a definition or as a proved theorem the recurrent law of formation which, in the original notation of Jacobi and Cayley, is exemplified by the identity

$$\begin{aligned}[123456] = 12[3456] - 13[2456] &+ 14[2356] \\ &- 15[2346] + 16[2345].\end{aligned}$$

Thus when there is only one zero element and the Pfaffian is of the 3rd order, we have

$$\left|\begin{array}{ccccc} \cdot & a_3 & a_4 & a_5 & a_6 \\ & b_3 & b_4 & b_5 & b_6 \\ & & c_4 & c_5 & c_6 \\ & & & d_5 & d_6 \\ & & & & e_6 \end{array}\right| = e_6 \left|\begin{array}{ccc} \cdot & a_3 & a_4 \\ & b_3 & b_4 \\ & & c_4 \end{array}\right| - d_6 \left|\begin{array}{ccc} \cdot & a_3 & a_5 \\ & b_3 & b_5 \\ & & c_5 \end{array}\right|$$

$$+ c_6 \left|\begin{array}{ccc} \cdot & a_4 & a_5 \\ & b_4 & b_5 \\ & & d_5 \end{array}\right| - b_6 \left|\begin{array}{ccc} a_3 & a_4 & a_5 \\ & c_4 & c_5 \\ & & d_5 \end{array}\right|$$

$$+ a_6 \left|\begin{array}{ccc} b_3 & b_4 & b_5 \\ & c_4 & c_5 \\ & & d_5 \end{array}\right| .$$

But, by the previous case, the 1st, 2nd, and 3rd terms of this expansion are equal to

$$-e_6|a_3b_4|, \qquad +d_6|a_3b_5|, \qquad -c_6|a_4b_5|,$$

respectively ; and the remainder

$$= - \left| \begin{array}{ccc} a_3b_6 & a_4b_6 & a_5b_6 \\ & c_4 & c_5 \\ & & d_5 \end{array} \right| + \left| \begin{array}{ccc} a_6b_3 & a_6b_4 & a_6b_5 \\ & c_4 & c_5 \\ & & d_5 \end{array} \right| ,$$

$$= \left| \begin{array}{ccc} a_6b_3 - a_3b_6 & a_6b_4 - a_4b_6 & a_6b_5 - a_5b_6 \\ & c_4 & c_5 \\ & & d_5 \end{array} \right| ,$$

$$= - \ |a_3b_6|d_5 \ + \ |a_4b_6|c_5 \ - \ |a_5b_6|c_4 .$$

Consequently we have the result

$$\left| \begin{array}{ccccc} \cdot & a_3 & a_4 & a_5 & a_6 \\ & b_3 & b_4 & b_5 & b_6 \\ & & c_4 & c_5 & c_6 \\ & & & d_5 & d_6 \\ & & & & e_6 \end{array} \right| \begin{array}{l} = - \ |a_3b_4|e_6 \ + \ |a_3b_5|d_6 \ - \ |a_4b_5|c_6 \\ \quad - \ |a_3b_6|d_5 \ + \ |a_4b_6|c_5 \ - \ |a_5b_6|c_4, \end{array}$$

where the first factors on the right are the set of six ($C_{4,2}$) two-lined determinants formable from

$$\begin{array}{cccc} a_3 & a_4 & a_5 & a_6 \\ b_3 & b_4 & b_5 & b_6 , \end{array}$$

and their cofactors are the remaining six $(3+2+1)$ elements

$$\begin{array}{ccc} c_4 & c_5 & c_6 \\ & d_5 & d_6 \\ & & e_6 \end{array}$$

of the Pfaffian.

(3) Had the given Pfaffian been of a higher order than the 3rd, it is clear that the cofactors of the two-lined determinants could not have been linear. It will now be seen by considering another case that in general they are themselves Pfaffians, and that consequently in the case just dealt with they have in strictness to be viewed not as elements but as Pfaffian minors of the 1st order. The expansion to which we are leading may thus be described as an aggregate of terms each of which is a product of a determinant and a Pfaffian.

As before we have

$$\left| \begin{array}{ccccccc} \cdot & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 \\ & & c_4 & c_5 & c_6 & c_7 & c_8 \\ & & & d_5 & d_6 & d_7 & d_8 \\ & & & & e_6 & e_7 & e_8 \\ & & & & & f_7 & f_8 \\ & & & & & & g_8 \end{array} \right|$$

$$= g_8 \left| \begin{array}{ccccc} \cdot & a_3 & a_4 & a_5 & a_6 \\ & b_3 & b_4 & b_5 & b_6 \\ & & c_4 & c_5 & c_6 \\ & & & d_5 & d_6 \\ & & & & e_6 \end{array} \right| - f_8 \left| \begin{array}{ccccc} \cdot & a_3 & a_4 & a_5 & a_7 \\ & b_3 & b_4 & b_5 & b_7 \\ & & c_4 & c_5 & c_7 \\ & & & d_5 & d_7 \\ & & & & e_7 \end{array} \right| + \ldots + c_8 \left| \begin{array}{ccccc} \cdot & a_4 & a_5 & a_6 & a_7 \\ & b_4 & b_5 & b_6 & b_7 \\ & & d_5 & d_6 & d_7 \\ & & & e_6 & e_7 \\ & & & & f_7 \end{array} \right|$$

$$- b_8 \left| \begin{array}{ccccc} a_3 & a_4 & a_5 & a_6 & a_7 \\ & c_4 & c_5 & c_6 & c_7 \\ & & d_5 & d_6 & d_7 \\ & & & e_6 & e_7 \\ & & & & f_7 \end{array} \right| + a_8 \left| \begin{array}{ccccc} b_3 & b_4 & b_5 & b_6 & b_7 \\ & c_4 & c_5 & c_6 & c_7 \\ & & d_5 & d_6 & d_7 \\ & & & e_6 & e_7 \\ & & & & f_7 \end{array} \right| ,$$

where, again, all the terms on the right except the last two can be dealt with by using the preceding case, the result of such use being an expression consisting of 30 (*i.e.*, 5×6) terms of the form

$$- g_8 |a_3 b_4| e_6 .$$

On examination, however, it will be found that these can be grouped in sets of three by reason of the fact that each of the ten (*i.e.*, $C_{5,2}$) two-lined determinants appearing in the expression occurs three times. For example, the determinant $|a_3 b_4|$ occurs the first time, as we have just seen, with the cofactor $-g_8 e_6$, the second time with $f_8 e_7$, and the third time with $-e_8 f_7$; its full cofactor thus being

$$-(g_8 e_6 - f_8 e_7 + e_8 f_7) \qquad \text{or} \qquad - \left| \begin{array}{ccc} e_6 & e_7 & e_8 \\ & f_7 & f_8 \\ & & g_8 \end{array} \right| .$$

By condensation in this way, therefore, there is obtained from the first five terms of the development with which we started an expression consisting of ten terms of the form

$$- |a_3 b_4| \cdot \left| \begin{array}{ccc} e_6 & e_7 & e_8 \\ & f_7 & f_8 \\ & & g_8 \end{array} \right| .$$

As for the remaining two terms of the said development, the same reasoning as before gives their aggregate

$$= \left| \begin{array}{ccccc} a_8 b_3 - a_3 b_8 & a_8 b_4 - a_4 b_8 & a_8 b_5 - a_5 b_8 & a_8 b_6 - a_6 b_8 & a_8 b_7 - a_7 b_8 \\ & c_4 & c_5 & c_6 & c_7 \\ & & d_5 & d_6 & d_7 \\ & & & e_6 & e_7 \\ & & & & f_7 \end{array} \right| ,$$

$$= - |a_3 b_8| \cdot \left| \begin{array}{ccc} d_5 & d_6 & d_7 \\ & e_6 & e_7 \\ & & f_7 \end{array} \right| + |a_4 b_8| \cdot \left| \begin{array}{ccc} c_5 & c_6 & c_7 \\ & e_6 & e_7 \\ & & f_7 \end{array} \right| - \ldots - |a_7 b_8| \cdot \left| \begin{array}{ccc} c_4 & c_5 & c_6 \\ & d_5 & d_6 \\ & & e_6 \end{array} \right| .$$

We thus have as a final result

$$\left|\begin{array}{ccccccc} \cdot & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 \\ & & c_4 & c_5 & c_6 & c_7 & c_8 \\ & & & d_5 & d_6 & d_7 & d_8 \\ & & & & e_6 & e_7 & e_8 \\ & & & & & f_7 & f_8 \\ & & & & & & g_8 \end{array}\right| = - |a_3b_4| . \left|\begin{array}{ccc} e_6 & e_7 & e_8 \\ & f_7 & f_8 \\ & & g_8 \end{array}\right| + \dots - |a_7b_8| . \left|\begin{array}{ccc} c_4 & c_5 & c_6 \\ & d_5 & d_6 \\ & & e_6 \end{array}\right| ,$$

where the first factors of the terms on the right are the fifteen (*i.e.*, $C_{6,2}$) two-lined determinants formable from

$$\begin{array}{cccccc} a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ b_3 & b_4 & b_5 & b_6 & b_7 & b_8 \end{array}$$

and their cofactors are the fifteen (*i.e.*, $5+4+3+2+1$) principal minors of the Pfaffian

$$\left|\begin{array}{ccccc} c_4 & c_5 & c_6 & c_7 & c_8 \\ & d_5 & d_6 & d_7 & d_8 \\ & & e_6 & e_7 & e_8 \\ & & & f_7 & f_8 \\ & & & & g_8 \end{array}\right| ,$$

the first determinant $| a_3b_4 |$ going along with the complementary minor of the first element c_4 of the Pfaffian, the second determinant $| a_3b_5 |$ going along with the complementary minor of the second element c_5 of the Pfaffian, and so on in every case.

(4) For the full investigation of the general theorem thus shadowed forth, neither of the definitions here employed is well suited : what is needed is a definition prescribing the mode of formation of the terms from the elements and the mode of determining the sign of each term—a definition, that is to say, similar to that ordinarily used for a determinant. The following will be found to satisfy these requirements :—

If $n(2n-1)$ *elements be each numbered by a pair of integers in order of magnitude, and be arranged in semiquadrate form, thus—*

$$\begin{array}{cccccc} 12 & 13 & 14 & \dots & 1,2n \\ & 23 & 24 & \dots & 2,2n \\ & & \dots & \dots & \dots \\ & & & & 2n-1,2n \end{array}$$

and all possible terms be taken which are products of n *elements whose united place-numbers include all the integers from* 1 *to* 2n, *the sign of each term being taken + or − according as the number of inverted-pairs in the series of integers specifying the term is even or odd; then, the function which is the aggregate of these terms is called a Pfaffian of the* n^{th} *order, and is denoted by the semiquadrate collection of elements bounded by two straight lines, a shorter on the left and a longer on the right.*

For example, the Pfaffian of the 2nd order,

$$\left|\begin{array}{ccc} 12 & 13 & 14 \\ & 23 & 24 \\ & & 34 \end{array}\right| = 12.34 - 13.24 + 14.23 .$$

(5) Instead of the elements being viewed as forming $2n-1$ rows of $2n-1$, $2n-2, \ldots, 1$ elements respectively, and at the same time $2n-1$ columns of 1, $2, \ldots, 2n-1$ elements respectively, they may, without alteration of position, be viewed as situated at the intersections of $2n$ frame-lines each containing $2n-1$ elements, the r^{th} frame-line being in every case made up of the r^{th} row and the $(r-1)^{\text{th}}$ column: and from this point of view the two integers used to specify an element are the numbers of the two frame-lines in which the latter is situated. This will be apparent from a glance at the following diagram of frame-lines for a Pfaffian of the 3rd order :—

$$\begin{array}{cccccccl}
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\
\rightarrow & 12 & 13 & 14 & 15 & 16 & \rightarrow & \text{1st frame-line} \\
 & & 23 & 24 & 25 & 26 & \rightarrow & \text{2nd frame-line} \\
 & & & 34 & 35 & 36 & \rightarrow & \text{3rd frame-line} \\
 & & & & 45 & 46 & \rightarrow & \text{4th frame-line} \\
 & & & & & 56 & \rightarrow & \text{5th frame-line} \\
 & & & & & \downarrow & & \\
 & & & & & \text{6th frame-line.} & &
\end{array}$$

It follows also that the rule for the formation of the terms is equivalent to a direction that all possible products of n elements are to be taken, no two elements in any product being from the same frame-line.

Thus, if in trying to form a term of the Pfaffian of the 3rd order whose elements are $a, b, c, \ldots, m, n, o$, we took, to commence with, the element a in the first row, we should thereby be debarred not only from taking anything else from this row (which is the first frame-line) but also from the second row (which is a part of the second frame-line), because a is an element in both, its place-name being 12. Our choice, consequently, would then be from among the elements left after deletion of these two lines, *i.e.*, from

$$
\begin{array}{c|ccccc|c}
\cdots & a & b & c & d & e & \cdots \\
& & f & g & h & i & \cdots \\
& & & j & k & l & \\
& & & & m & n & \\
& & & & & o & ;
\end{array}
$$

and if we next decided on taking k, which is in the 3rd and 5th frame-lines, we could not thereafter take anything else from these lines—that is to say, we could not take j or l from the 3rd frame-line, or m or o from the 5th. We should thus be left with

$$akn$$

as a term; and the number of inverted-pairs in the series 12 35 46 made up of the place-numbers of the chosen elements being 1, the sign would be negative.

(6) It is of interest to note, in passing, that the term composed of the 1st, 3rd, 5th, . . . elements in the hypotenuse of the semiquadrate array is always +, whatever the order of the Pfaffian may be, because the number of inverted-pairs in 12 34 56 . . . . is zero. Also, that the same is true of the term composed of the elements lying on the line which bisects the hypotenuse at right angles, because the series then to be considered is

$$1,2n, \quad 2,2n-1, \quad 3,2n-2, \; \ldots\ldots \; n-1,n \, ;$$

and it is manifest that none of the pairs beginning with 1, 2, 3, . . . , $n-1$ can be inverted, and that, while those beginning with $2n$, $2n-1$, . . . , are all of them inverted, the number is in each case even.

(7) With these preliminaries before us, let us now consider a fourth case of the theorem sought to be established, say the case where the Pfaffian is of the 5th order and the zero elements are in the places 12, 13, 23—*i.e.*, the Pfaffian

$$
\left|\begin{array}{ccccccccc}
\cdot & \cdot & 14 & 15 & 16 & 17 & 18 & 19 & 1t \\
& \cdot & 24 & 25 & 26 & 27 & 28 & 29 & 2t \\
& & 34 & 35 & 36 & 37 & 38 & 39 & 3t \\
& & & 45 & 46 & 47 & 48 & 49 & 4t \\
& & & & 56 & 57 & 58 & 59 & 5t \\
& & & & & 67 & 68 & 69 & 6t \\
& & & & & & 78 & 79 & 7t \\
& & & & & & & 89 & 8t \\
& & & & & & & & 9t
\end{array}\right| .
$$

Here the first three frame-lines, when freed of the portions containing zero elements, constitute a rectangular array of 3 rows and 7 columns, from which thirty-five (*i.e.*, $C_{7,3}$) determinants of the 3rd order are formable; and the initial proposition to be made good is that every one of the 210 terms of these thirty-five determinants is a portion of a term of the given Pfaffian. To do this we have only got to put in mental contiguity for a moment the definitions of a determinant and a Pfaffian: for each determinant term being required to consist of three elements taken from the rectangular array referred to, no two of which must belong to the same row or to the same column, complies with the requirement regarding the first three elements needed to form part of a term of the Pfaffian, viz., that they must be chosen from the first three frame-lines but that at the same time no two of them must belong to the same frame-line. The next proposition is that all the six terms of any particular one of the thirty-five determinants require the same cofactor in order that terms of the Pfaffian may be produced. This is made clear by considering the fact that all of the six terms have their elements taken from the same set of frame-lines, and that the remaining two elements in each case must be chosen from the elements left when the said set has been deleted from the Pfaffian. Thus, if the particular determinant were

$$\begin{vmatrix} 15 & 17 & 18 \\ 25 & 27 & 28 \\ 35 & 37 & 38 \end{vmatrix},$$

each of its terms could only become a term of the Pfaffian by having annexed to it two elements selected from those left when the 1st, 2nd, 3rd, 5th, 7th, 8th frame-lines of the Pfaffian have been deleted—that is to say, from

$$\begin{matrix} 46 & 49 & 4t \\ & 69 & 6t \\ & & 9t. \end{matrix}$$

We are thus prepared to advance a third proposition derived from the previous two, viz., that, apart from the question of sign, all the eighteen terms of each one of the products of the form

$$\begin{vmatrix} 15 & 17 & 18 \\ 25 & 27 & 28 \\ 35 & 37 & 38 \end{vmatrix} \cdot \begin{vmatrix} 46 & 49 & 4t \\ & 69 & 6t \\ & & 9t \end{vmatrix}$$

are terms of the Pfaffian. This means that 35×18, *i.e.*, 630, terms of the Pfaffian are accounted for. Now the total number of terms in a Pfaffian of the 5th order is 1.3.5.7.9; the number of these which will vanish when the element 12 vanishes is 1.3.5.7; the additional number which will vanish when 13 vanishes is 1.3.5.7; and the additional number which will vanish when 23 vanishes is 1.3.5.7. It follows, therefore, that the total number of non-vanishing terms in the Pfaffian under discussion ought to be

$$1.3.5.7.9 - 3(1.3.5.7),$$

i.e., $1.3.5.7(9 - 3)$,

i.e., 35×18;

and this is exactly the number obtained from our development.

(8) The only matters remaining now for consideration are those which concern the *signs* of the terms thus obtained, it being necessary for our purpose (1) to establish the fact that the eighteen signs of any product of the form

$$\begin{vmatrix} 15 & 17 & 18 \\ 25 & 27 & 28 \\ 35 & 37 & 38 \end{vmatrix} . \begin{vmatrix} 46 & 49 & 4t \\ & 69 & 6t \\ & & 9t \end{vmatrix}$$

are either all right or all wrong, and (2) to formulate a rule for distinguishing between products of these two kinds, so that the sign + may be prefixed to the one and − to the other.

Now if no sign precede the product, the sign of any of the eighteen terms is determinable from the sign of the portion of the term which comes from the determinant factor and the sign of the portion which comes from the Pfaffian factor; and the former being dependent upon the number of inverted-pairs in the series of column-numbers specifying the elements of the first part of the term, and the latter upon the number of inverted-pairs in the series of frame-line numbers specifying the elements of the second part of the term, it is clear that if σ be the sum of the said two numbers of inverted pairs the sign of the complete term will be $(-)^{\sigma}$. On the other hand, the sign which it *ought* to bear as a term of the parent Pfaffian is fixed by the number ν of inverted-pairs in the series of integers specifying the frame-lines of *all* the elements composing it. What is wanted, therefore, is a comparison of this number ν with σ; and, if we can show that $\nu - \sigma$ is constant for all the eighteen terms, it will follow, of course, that the eighteen signs are either all right or all wrong. For example, in the product

$$\begin{vmatrix} 15 & 17 & 18 \\ 25 & 27 & 28 \\ 35 & 37 & 38 \end{vmatrix} . \begin{vmatrix} 46 & 49 & 4t \\ & 69 & 6t \\ & & 9t \end{vmatrix}$$

$-17.25.38$ is a term of the determinant and $-49.6t$ a term of the Pfaffian, the sign − in the one case being fixed by the number of inverted-pairs in 758 and in the other by the number in $496t$, whereas the sign of the resulting term $17.25.38.49.6t$ of the Pfaffian is fixed by the number of inverted-pairs in $172538496t$.

For ease in making the necessary comparison let us use

$$\mathrm{I}(\alpha\beta\gamma\ldots.)$$

to stand for the number of inversions found in the pairs of integers obtainable by placing each integer in front of those which follow it, and

$$\mathrm{I}(\alpha\beta\gamma\ldots.\ ;\ \alpha'\beta'\gamma'\ldots.)$$

for the number of inversions found in the pairs obtainable by placing each integer of the first group in front of each integer of the second group. In the former, $\mathrm{I}(\alpha\beta\gamma\ldots)$, the order in which we write the integers of the group is all-important; in the latter, $\mathrm{I}(\alpha\beta\gamma\ldots;\ \alpha'\beta'\gamma'\ldots)$, the order is of no consequence so long as we do not mix the two groups.

With this notation a fundamental proposition regarding inverted-pairs, which bears directly on the subject in hand, can be stated very simply, viz.,

$$\begin{aligned}\mathrm{I}(\alpha\beta\gamma\ldots\ \alpha'\beta'\gamma'\ldots) = \mathrm{I}(\alpha\beta\gamma\ldots) &+ \mathrm{I}(\alpha'\beta'\gamma'\ldots)\\ &+ \mathrm{I}(\alpha\beta\gamma\ldots;\ \alpha'\beta'\gamma'\ldots),\end{aligned}$$

and it is immediately evident therefrom that whatever interchanges may be made in the group $\alpha\beta\gamma\ldots$ or in the group $\alpha'\beta'\gamma'\ldots$

$$\mathrm{I}(\alpha\beta\gamma\ldots\ \alpha'\beta'\gamma'\ldots) - \mathrm{I}(\alpha\beta\gamma\ldots) - \mathrm{I}(\alpha'\beta'\gamma'\ldots)$$

will remain constant.

As applied to the special groups of integers connected with the Pfaffian term above-mentioned this ensures that

$$\mathrm{I}(172538496t) - \mathrm{I}(172538) - \mathrm{I}(496t)$$

will not alter by reason of any interchanges taking place in the group 172538 or in the group $496t$. If in addition the integers 123 in the first group be excluded from interchange, we shall have

$$\begin{aligned}\mathrm{I}(172538) &= \mathrm{I}(7\,;\,23) + \mathrm{I}(5\,;\,2) + \mathrm{I}(758),\\ &= 3 + \mathrm{I}(758),\end{aligned}$$

through all interchanges in 758. By substitution it therefore follows that the number

$$\mathrm{I}(172538496t) - \mathrm{I}(758) - \mathrm{I}(496t)$$

will remain constant while interchanges take place in 758 or in $496t$; and this is equivalent to saying that the terms of the parent Pfaffian which are obtainable from the product

$$\begin{vmatrix} 15 & 17 & 18 \\ 25 & 27 & 28 \\ 35 & 37 & 38 \end{vmatrix} \cdot \begin{vmatrix} 46 & 49 & 4t \\ & 69 & 6t \\ & & 9t \end{vmatrix}$$

are either all correctly or all incorrectly signed.

If the said constant be even, the sign which ought to precede the product will of course be +, in the other possibility −. As a matter of practice, however, the easiest way of determining the sign to be placed in front of any product is to make the sign such that one of the terms obtainable from the product shall be correctly signed, and for this purpose the facts given in § 6 will be found useful. If the whole expansion be wanted, and the products be arranged in the natural order—that is to say, in such a way that the series of determinant factors shall begin with $|\,14\ \ 25\ \ 36\,|$

and end with | 18 29 3t |, the sign to be prefixed to any product is easily known from that of the preceding product.

(9) The number of different forms of this new development which are possible in the case of a Pfaffian of the n^{th} order is of course the number of partitions of the integer n into two integers, the first of the latter corresponding to the order of the determinant factors in the development, and the other to the order of the Pfaffian cofactors. For example, in the case of the Pfaffian of the 5th order we shall have the five developments

(α)	+ 12.| 3456789t |	− 13.| 2456789t |	+	($C_{9,1}$ terms),
(β)	− | 13 24 |.| 56789t |	+ | 13 25 |.| 46789t |	−	($C_{8,2}$ terms),
(γ)	− | 14 25 36 |.| 789t |	+ | 14 25 37 |.| 689t |	−	($C_{7,3}$ terms),
(δ)	+ | 15 26 37 48 |.9t	− | 15 26 37 49 |.8t	+	($C_{6,4}$ terms),
(ϵ)	+ | 16 27 38 49 5t |			($C_{5,5}$ term),

the parent Pfaffian containing no zero elements in the first case, 1 in the second, 1 + 2 in the third, 1 + 2 + 3 in the fourth, and 1 + 2 + 3 + 4 in the fifth.

The single elements in the first development may be viewed as determinants of the 1st order and in the fourth development as Pfaffians of that order.

VI.—*Contributions to the Craniology of the People of the Empire of India. Part II. The Aborigines of Chúta Nágpúr and of the Central Provinces, the People of Orissa, the Veddahs and Negritos.* By Professor Sir WM. TURNER, K.C.B., D.C.L., F.R.S. (With Four Plates.)

(Read July 2, 1900.)

It is my intention in this, the second part of my memoir on the Craniology of the Races of India, to give the results of my examination of skulls obtained from the districts occupied by the aboriginal tribes in Chúta Nágpúr, the Central Provinces, the people in the province of Orissa, and to compare them with the skulls of some other aboriginal people.

The majority of the specimens described belong to the Indian Museum, Calcutta, and through the courtesy of the Trustees I was permitted to have them on loan for purposes of study. Many of these crania had been those of persons who had died in jail. The names, tribes, and castes, and not unfrequently the age, stature, and other physical characters, had been recorded in the prison books, and were embodied in the lists which were sent to me along with the skulls by the authorities of the museum. Several of these skulls were especially interesting, as having been presented to the museum by Colonel DALTON, the author of the valuable treatise on the *Ethnology of Bengal.* Other specimens in the museum had been obtained from the Medical College, Calcutta, and several were presented by Professor D. B. SMITH ; in all probability they were from bodies which had been used for anatomical purposes. Mr W. H. P. DRIVER also had presented a series of crania from Ranchi.

In addition, I have received specimens from former students holding appointments in the Indian Medical Service, and I take this opportunity of acknowledging their courtesy in presenting them to me.

The descriptions in this part of my contribution to Indian Craniology are based on the examination of one hundred and one skulls, and the measurements are recorded in the series of Tables.

The works which I have chiefly consulted in drawing up the account of the geographical distribution and tribal characters of the aborigines, are Colonel DALTON'S *Descriptive Ethnology of Bengal*, Calcutta, 1872 ; Sir W. W. HUNTER'S *Statistical Account of Bengal* and *Imperial Gazetteer of India* ; Sir H. M. ELLIOT'S *Memoirs of the Races of the North-West Provinces of India*, edited by JOHN BEAMES, London, 1869 ; *The Tribes and Castes of Bengal, Ethnographic Glossary, and Anthropometric Data*, Calcutta, 1891, by H. H. RISLEY, I.C.S. ; *The Tribes and Castes of the North-Western Provinces and Oudh*, by W. CROOKE, B.A., B.C.S., Calcutta, 1896 ; "India," by Sir RICHARD TEMPLE in *Chambers's Encyclopædia* ; *Census of India*, 1891, General Report by Census Commissioner J. A. BAINES, I.C.S. ; *Report on the Lower Provinces* *of Bengal and their Feudatories*, by C. J. O'Donnell, M.A., I.C.S.; *Report on the Central Provinces and Feudatories*, by B. Robertson, I.C.S.; *Reports on Anthropology in Bulletin of Madras Government Museum*, Madras, 1897–1900, by Edgar Thurston; *The Distribution of the Negritos*, by A. B. Meyer, M.D., Dresden, 1899.

Aborigines.

Before I enter on the description of the craniological characters of the different aboriginal tribes, it will be useful to say something of the geographical position of the districts in which they live, and of the distribution and physical characteristics of the people of each tribe.

Chúta Nágpúr is a division of Bengal situated to the south of Mirzápur, in the North-West Provinces, and to the north and east of the Central Provinces. It contains, amongst others, the districts of Singbhúm, Manbhúm, Hazáríbágh and the tributary state of Sargúja, from all of which skulls had been obtained. In the Lohárdagá district is the town of Ránchi, where there is an important jail, from which had been procured the crania of some prisoners who had been executed or had died of disease—many of whom were natives of the adjoining villages. The country is broken up into hills, valleys, and raised plateaux. Hindus form the largest element of the population, but interspersed among them are semi-Hinduised natives and aboriginal tribes.

The Central Provinces are a large territory which extends as far south as the Godavery River, the Nizam's dominions, and the north part of the Madras Presidency. Skulls have been examined from Bastár, Ráipur, and other districts in the provinces. The country is diversified and contains tablelands, which in some parts are 2000 feet high, ranges of hills, valleys, and wide plains. The Hindus are the preponderating element amongst the people, but numbers of aborigines are to be found, especially on the Sátpura plateau and in the hill districts of the feudatory state of Bastár.

Orissa is an extensive province on the west side of the Bay of Bengal, and is bounded on the west by Chúta Nágpúr and the Central Provinces. Along the coast line it possesses a border of alluvial land, but the interior is an undulating country intersected by ranges of hills, the highest peaks of which are from 3000 to 4000 feet. Hindus constitute the mass of the people, but the aborigines and semi-Hinduised aboriginal tribes form an important element. Skulls have been obtained from Keunjhar, Kandh-mals, Cuttack, and other parts of Orissa.

In the several provinces under consideration the Hindus occupy and cultivate the valleys and more fertile lands. The aboriginal tribes live in the hills and on the higher plateaux, and preserve more or less completely their religion and tribal customs. Where the Hindus have come into immediate contact with the aborigines, the latter, whilst retaining to some extent their ancient forms of faith and customs, have, in other respects, adopted the Hindu religion and modes of thought.

Writers on the philology and ethnology of the people of India have distinguished, by the names Dravidian and Kolarian, two groups of languages spoken by the aboriginal tribes who occupy the hill ranges in the Central Provinces, Chúta Nágpúr, Orissa, extending also into Western Bengal and Southern India. The name Dravidian was given to the southern of the two linguistic groups by Bishop CALDWELL, and many writers have attached to it an ethnological value. This group of languages is most extensively represented in the Madras Presidency, where it forms the south Dravidian group, known as Telugu, Tamil, Kanarese, and Malayalám; but it also extends into the hill ranges in the Central Provinces and Orissa, as the north Dravidian group spoken by the Gonds, Túlús, Oráons, Kharwárs, Mál-Paháriás, and Kandhs. The Kolarian group of languages, as it has been named by Sir GEORGE CAMPBELL,* prevails amongst the tribes which lie to the north of those who speak Dravidian, and who occupy the hill tracts of Western Bengal and Central India. The Santals, Múndas, Hos, Kols, Korwás, and Bhils are the principal tribes to employ the languages of this group. It by no means, however, follows that tribes speaking a Kolarian dialect are ethnically distinct from those who speak Dravidian, as it is not uncommon to find that a tribe possessing the physical characteristics of the Dravidians is classed linguistically as Kolarian. The division, therefore, into these two linguistic groups has a philological rather than an ethnological significance. Dravidian dialects are apparently spoken by about one-fifth of the population of India; Kolarian by about one-tenth.

Gond. TABLE I.

These people are regarded on linguistic grounds as Dravidian. They inhabit an extensive tract of country formerly known as Gondwáná, which extended from the Vindhyan mountains to the Godavery, and which now constitutes a large part of the Central Provinces. They are found also in the southern part of Chúta Nágpúr and a small number in Orissa. They occupy the tableland of Sátpurá and the hill country from Mandla to Asirgarh, as well as Koreá, Sirgúja, and Udaipur. They were a brave and independent people before the rise of the Mogul Empire. Whilst some still retain their independence and original faith, others have been subjugated and have become either Hinduised or Mahomedans. Colonel DALTON considers the Márias who inhabit dense jungles in Bastár, Chanda, and other southern dependencies to be the best type of the primitive aboriginal Gond.† Along with the Rev. G. HISLOP, he describes the wild Gonds as having flat noses, distended nostrils, thick lips, dark skin, scanty beard and moustache, and straight, black hair; sometimes the hair is said to be short, crisp, and curly, but quite distinct from the woolly hair of the negro. In some instances the head is shaved, leaving only a top-knot, but more frequently the hair is matted and

* Races of India. *Journ. Ethno. Soc.*, London. N.S. Vol. I. p. 130, 1869.

† See also Chanda Settlement *Report*; Colonel Glasfurd's *Report* on Bastár; Mr Robertson's Census *Report*, 1891.

untidy. The Gonds are about the same height as the Márias and Bhatras, but are larger and heavier in build than the Oráons or Kols. They are scantily clothed and the women are tattooed. The dead are cremated and the ashes are then buried, but it is said that the women and children are buried without being cremated. The grave is dug so that the head lies to the south and the feet to the north. In character, the Gonds are reserved, sullen, and suspicious, and the Márias are a shy, timid people. They are totemistic and exogamous. They practise both infant and adult marriage, and widows remarry. The unmarried young men sleep in a common dormitory, and in some villages there is a similar provision for the unmarried young women. Dalton says that they are indifferent cultivators, and careless about the appearance of their houses. The Gonds, who are not Hinduised, worship their own deities and the spirits of the forests in which they live. From the *Census Report* of 1891, it would appear that 1,379,580 people were returned as speaking the Gond branch of the northern Dravidian group of languages, though the actual numerical strength of the Gonds is said to be 2,897,591.

The Edinburgh University Anatomical Museum contains four skulls of Gonds from the Godavery district, though the exact locality is not known. They had originally been in the collection of the late Dr Handyside, and were marked "wild tribes called Gōttch or Gōnd, from Godavery district of Central India." They were all adults, though the wisdom teeth were not erupted in D; three were presumably males and one a female.

Norma Verticalis.—The crania had a marked family likeness. They were elongated, narrow, with vertical sides, and dolichocephalic in form and proportions. In the males the parietal eminences were feeble, in the female (C) they were more projecting and gave greater relative breadth to the cranium. In both sexes they were situated considerably in front of the occipital point. The vault of the skull was somewhat roof-shaped, but not ridged in the sagittal line. The skulls were cryptozygous or nearly so. In three specimens the stephanic diameter was greater than the asterionic.

Norma Lateralis.—The skulls rested behind on the cerebellar part of the occiput. The glabella and supra-orbital ridges, although visible, were not prominent even in the men. The forehead in the males only slightly receded; in the female it bulged slightly forward. The antero-posterior curve of the cranial vault rose gently to the vertex, and from the obelion it sloped downwards and backwards into the occipital squama, which projected behind the inion. There was no sign of parieto-occipital flattening. The frontal longitudinal arc in each skull was slightly less than the parietal, but always considerably in excess of the occipital arc.

The nasal bones were of moderate size, with the bridge not prominent and concave forwards; the fronto-nasal suture was not depressed, and the nasal spine of the superior maxillæ was moderate. The junction of the side walls and floor of the anterior nares was rounded, and in three specimens the floor of the nose was separated from the incisive region of the maxilla by a low ridge. The canine and incisor fossæ were of moderate depth. The teeth were fully erupted except in D, in which the wisdoms had not appeared, and they were in good order except in B, in which the crowns were much worn. No skull was metopic, but the other cranial sutures were distinct and denticulated. In two skulls Wormian bones were in the lambdoidal suture, and in one also in the parieto-mastoid suture. In all, the ali-sphenoid and parietal articulated at the pterion, but in C the junction was very narrow; in B a very small epipteric bone was present in the suture. The muscular ridges and processes were not strong except in A. No skull had a 3rd occipital condyle or an exostosis in the external auditory meatus, or a subdivision of the malar bone. One skull had a pair of short para-mastoid processes: two had infra-orbital sutures. The interzygomatic breadth of the face invariably exceeded the intermalar, stephanic, and asterionic breadth; in A the interzygomatic breadth was slightly in excess of the parieto-squamous, and in B they were almost equal.

The lower jaw was moderate in size and with a deep symphysis in B; the chin was prominent; the coronoid height did not greatly exceed the condyloid. The intergonial width and gonio-symphysial length closely approximated to each other.

The mean cephalic index was 71·2 and the range of variation was from 69·4 to 75. The crania were therefore dolichocephalic. The greatest length of the crania ranged from 176 to 180 mm., and the mean was 177·5; the greatest breadth ranged from 123 to 132 mm., and the mean was 126·5. The vertical index was 76, and the range of variation was from 74·6 to 77·2. The crania were metriocephalic. The actual height of the skulls ranged from 132 to 139 mm., and the mean was 135. In each skull the basi-bregmatic height was greater than the parieto-squamous breadth.

The nasio-mental length ranged from 98 to 112 mm., with a mean of 104 mm.; the interzygomatic breadth ranged from 118 to 128 mm., with a mean of 121·5. The complete facial index ranged from 79·7 to 91·8, with a mean of 84·8; the skulls, therefore, were chamæprosopic or low-faced. The maxillary or upper facial index ranged from 46·9 to 53·4, with a mean of 50·2; in the proportion of its upper region, the face was in the lowest term of the leptoprosopic group.

The mean gnathic index was 99·8, and the range of variation was from 96·9 to 104·4; the skulls, therefore, on the average, were mesognathous, though one was orthognathous and another prognathous. The mean nasal index was 53·4, and the range of variation was from 48·9 to 56·8; though the mean was just within the platyrhine group, two of the crania were mesorhine. The mean orbital index was 83, and the range of variation was from 81·1 to 83·8. All the orbits were microseme. The mean palato-maxillary index was 114·5, and the range of variation was from 105·3 to 122; the greatest palato-maxillary length was 56 mm. and the greatest breadth was 61 mm.; the skulls were in the mean mesuranic, though one was dolichuranic and two brachyuranic.

The mean cubic capacity of the four crania was 1274·5 cub. cent., *i.e.*, microcephalic, to which category each cranium belonged.

Table I.

Dravidian Tribes.

	Gond.				Oráon.			Pahária, Bírbhúm.		Kharwár Bogta.	Kandh.	
					Somra, Konka Village.	Jura, Lalpur Village.	Chandoa Village.	Rampoojar.	Dhobia.	Bahadur.		Judisther-Jani, Bhatpara, Orissa.
Collection number,	E.U.A.M. A.	B.	C.	D.	I.M. 608	I.M. 610	I.M. 601	I.M. 559	I.M. 558	I.M. 551	I.M. 556	E.U.A.M.
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	50	Aged.	29	Ad.	25
Sex,	M.	M.	F.	M.	M.	M.	F. (?)	M.	M.	M.	F.	M.
Cubic capacity,	1238	1250	1295	1315	1420	1430	1250	1246	1206	1305	1070	1325
Glabello-occipital length,	180	177	176	177	186	189	175	176	178	175	158	172
Basi-bregmatic height,	139	132	134	135	130	136	127	124	128	128	123	140
Vertical Index,	*77·2*	*74·6*	*76·1*	*76·3*	*69·9*	*72·*	*72·6*	*70·5*	*71·9*	*73·1*	*77·4*	*81·4*
Minimum frontal diameter,	92	92	91	89	91	90	92	88	91	85	92	92
Stephanic,	110	110	105	101	104	104	105	102	102	101	115	106
Asterionic,	102	101	100	103	103	106	104	108	108	104	90	106
Greatest parieto-squamous breadth,	125p.	123s.	132p.	126p.	132s.	129s.	132p.	135s.	128s.	128s.	133p.	135s.
Cephalic Index,	*69·4*	*69·5*	*75·0*	*71·2*	*71·*	*68·3*	*75·4*	*76·7*	*71·9*	*73·1*	*84·2*	*78·5*
Horizontal circumference,	500	493	488	488	503	518	480	497	498	490	463	483
Frontal longitudinal arc,	135	132	130	130	130	128	118	127	118	123	110	119
Parietal „ „	140	243 (Parietal and Occipital)	132	131	126	147	234 (Parietal and Occipital)	118	130	124	127	120
Occipital „ „	103		108	114	121	110		109	112	115	105	119
Total „ „	378	375	370	375	377	385	352	354	360	362	342	358
Vertical transverse arc,	298	298	298	299	305	304	290	292	280	294	287	296
Length of foramen magnum,	29	34	32	33	30	35	33	35	33	37	28	37
Basi-nasal length,	104	91	95	97	103	101	95	96	98	91	88	99
Basi-alveolar length,	102	95	95	94	98	...	91	95	...	84	89	95
Gnathic Index,	*98·1*	*104·4*	*100·*	*96·9*	*95·1*	...	*95·8*	*99·*	...	*92·3*	*101·1*	*96·*
Interzygomatic breadth,	128	122	118	118	127	130	123	129	134	121	115	128
Intermalar „	117	113	109	109	115	124	108	111	124	112	106	116
Nasio-mental length,	102	112	...	98	108	126	...	...	...	107	...	108
Nasio-alveolar „	60	64	63	57	64	...	61	64	...	62	51	64
Complete Facial Index,	*79·7*	*91·8*	...	*83·*	*85·*	*96*	...	...	...	*88·*	...	*84·3*
Nasal height,	47	46	44	43	48	50	46	48	45	47	37	47
Nasal width,	23	24	25	24	26	27	22	25	26	23	25	25
Nasal Index,	*48·9*	*52·2*	*56·8*	*55·8*	*54·2*	*54·*	*47·8*	*52·1*	*57·8*	*48·9*	*67·6*	*53·2*
Orbital width,	36	37	37	37	37	37	36	39	41	38	35	40
Orbital height,	30	30	31	31	31	33	30	32	31	35	27	32
Orbital Index,	*83·3*	*81·1*	*83·8*	*83·8*	*83·8*	*89·2*	*83·3*	*82·*	*75·6*	*92·1*	*77·1*	*80·*
Palato-maxillary length,	55	56	50	50	52	...	48	51	...	49	50	52
Palato-maxillary breadth,	61	59	61	60	67	72	55	...	...	65	60	68
Palato-maxillary Index,	*110·9*	*105·3*	*122·*	*120·*	*128·8*	...	*114·5*	...	...	*132·6*	*120·*	*130·7*
Lower jaw. Symphysial height,	29	35	...	28	27	...	...	...	...	27	...	33
Lower jaw. Coronoid „	69	63	...	55	57	72	...	54	...	53	...	62
Lower jaw. Condyloid „	60	61	...	53	60	67	...	59	...	50	...	61
Lower jaw. Gonio-symphysial length,	88	87	...	84	80	90	...	77	...	78	...	81
Lower jaw. Inter-gonial width, outside,	89	87	...	79	89	106	...	86	...	89	...	100
Lower jaw. Breadth of ascending ramus,	32	32	...	31	33	34	...	30	...	27	...	35

Note.—In the Tables, as in Part I., I.M. signifies Indian Museum; E.U.A.M., Edinburgh University Anatomical Museum; H.T., Henderson Trust-Collection; T.C.D., Trinity College, Dublin.

Oráon. Table I.

The Oráons, or Uráons, are a Dravidian tribe in Chúta Nágpúr, especially in the tributary states of Sirgúja and Jashpúr, but scattered also in Singbhúm, Manbhúm, and Hazáríbágh. The tradition in the tribe is that they migrated from the west coast of India. Dalton states that the skin is a dark brown approaching black; the hair is long, black, coarse, and inclined to be frizzy; the jaws are projecting; the lips are thick; the forehead is low, narrow, and not receding; the eyes are bright but not oblique; the expression is pleasing; and the upper face displays intelligence. Dalton gives the height of a young man as 5 feet 2 inches, and that of four girls between 12 and 16 years as ranging from 4 feet $7\frac{1}{2}$ inches to 5 feet $\frac{1}{2}$ inch. The dress of the men is a long strip of cloth adjusted about the middle of the body, but giving free play to the limbs, and a girdle of cord is about the waist. The hair is gathered into a knot at the back of the head, in the knot are combs and ornaments of brass and glass; bright brass chains dangle from the ears. The women wear a waist-cloth, and when more civilised, a cotton dress, and ornament themselves with beads and copper or brass rings. They have tattoo marks on the brow and temple, and on the arms and back. The unmarried men sleep in a bachelor house, the Dhúmkúria, and it is probable that the young women have a similar arrangement. Adult marriage is practised, and widows may remarry. The dead are cremated, and the ashes are collected in an earthern vessel, which for a time is suspended to a post in front of the house of the deceased, but is subsequently buried. They eat flesh as well as vegetables. They worship a supreme being as represented by the sun. In the General Report on the Census of India, 1891, it is stated that 368,222 speak the tribal language, but that the numerical strength of the Oráons is 523,258.

Three skulls in the Indian Museum, obtained from the neighbourhood of Ranchi, are marked Oráon or Uráon: No. 601, from the village of Chandoa, 30 miles from Ranchi; No. 606 from Konka village; and No. 610 marked Jura from Lalpur village. They were presented by Mr W. H. P. Driver. They are all adult; I regarded two as males, but the sex of the skull from Chandoa was more doubtful.

In their general form they were elongated and ovoid, and with vertical sides, and resembled in general form the skulls of the Múnda race, also from Ranchi, to be described in a subsequent section. One was hyper-dolichocephalic, and the parietal longitudinal arc greatly exceeded the frontal and occipital; another was dolichocephalic with the frontal arc a little the longest; the third slightly exceeded the upper numerical limit of the dolichocephalic, and in it the parietal and occipital arcs could not be properly differentiated. In two specimens the basi-bregmatic diameter was less than the parieto-squamous, but in the hyper-dolichocephalic skull it was greater. The face was orthognathous. In two specimens the nose was platyrhine; in the third it was leptorhine. In two skulls the orbital proportions were microseme, in the third just within the megaseme group. The palato-maxillary index in one was mesuranic, in another brachyuranic. The face in one was chamæprosopic, in the other leptoprosopic. In the two males the mean capacity of the cranium was relatively high for an aboriginal race, viz., 1425 c.c.; in the possible female skull the capacity was 1250 c.c.

Málê Pahāriá or Hillmen of Rájmahál. Table I.

Dalton, in the *Ethnology of Bengal*, devotes a section in his chapter on the Dravidian tribes to the aborigines who inhabit the Rájmahál Hills. This range extends from the banks of the Ganges to the Bráhmani river and the boundary of the Bírbhúm district, and is in the Santál Parganás district of Bengal. He also states that in the Rámgarh Hills of the Bírbhúm district, and at the foot of the Rájmahál Hills, are villages occupied by a tribe who call themselves Mál-Paháriás,—the precise affinities of which it is somewhat difficult to determine. As two skulls of aborigines marked Paháriás from Bírbhúm have come under my observation, it is convenient, from their possible Dravidian affinities, to consider them in this section. The Málers are short in stature, face oval, nose not prominent but broad below, and with the nares circular rather than elliptical; lips full, eyes not oblique. They dress as well as the peasants of the plains, and the women wear a white skirt, a gay coloured square of silk over the right shoulder and tied under the left arm. The hair is collected into a knot behind the head, with two long locks hanging over the ears. They are apparently exogamous. Marriage is either infant or adult, and widows can remarry. A special house is provided for the bachelors, and another for the unmarried girls. They worship the sun and their ancestors, and believe in the transmigration of souls. The dead are sometimes buried, though, Mr Risley says, more usually cremated. They are hunters, but they also practise jhúm cultivation. They eat flesh as well as vegetables, and drink a fermented liquor. The numerical strength of the tribe is said to be 18,506, though 30,838 use the tribal language.

In the collection in the Indian Museum are the skulls of two men, Nos. 558, 559, from Bírbhúm, both of whom had died in the prison hospital. No. 558, marked Dhobia Pahāriá, was that of a man said to be 80 years old, with an edentulous upper jaw; he had sustained a comminuted fracture of the frontal bone, the pieces of which had subsequently united. No. 559, also marked Pahāriá, was named Rampoojar, and aged 50.

The skulls were not roof-shaped, but were somewhat flattened at the vertex, and the outline was ovoid in the *norma verticalis*, though the cranium in one was not specially elongated, and the side walls bulged somewhat in the squamous region. In No. 558 the length-breadth index was 71·9, dolichocephalic, and the parietal longitudinal are greatly exceeded both the frontal and occipital; the vertical index corresponded with the cephalic. In No. 559 the length-breadth index was 76·7 in the lower term of the mesaticephalic group; in this skull the frontal longitudinal arc greatly exceeded the parietal and occipital; the vertical index was much below the cephalic. The glabella and supra-orbital ridges were more prominent in the aged than in the younger man. In both the forehead slightly receded. In the old skull the parieto-occipital region was asymmetrical as if from artificial pressure, but in the other it had a gentle slope backwards. The nasion was not depressed, and the bridge of the nose, concave from above downwards, was distinct, though less so in the old man. The nose was platyrhine in the old skull, 57·8, and nearly so in the adult—viz., 52·1, in which also the upper jaw was mesognathous. In both the orbital index was mesoseme. The muscular ridges were stronger in the aged skull, which was markedly phænozygous, and wide both in the interzygomatic and intermalar diameters; it rested behind on the mastoids. The adult cranium was nearly cryptozygous, and rested behind on the occipital bone. In both the cubic capacity was small, the mean of the two being 1226 c.c.

Kharwár. TABLE I.

In Chúta Nágpúr and Southern Behar is a non-Aryan tribe named Kharwár, who speak a Kolarian tongue. The Bhogtas are the most important division of the tribe. DALTON states that the Kharwárs are mixed up with the Cheros, living in the same district, with whom they claim affinity. Both have become proselytes to Hinduism. When visited in 1794 by Captain J. T. BLUNT, they were seen to be nearly naked, and armed with bows, arrows, and hatchets. BUCHANAN found that whilst some were land-owners and others labourers, there were others again who were obviously primitive in habits, and represented the aboriginal inhabitants. The low Kharwárs are said by DALTON to resemble strongly the Santals. The skin was very dark, nose low and pyramidal-shaped, lips thick and protuberant, zygomata so prominent that the temples were hollow. Another observer says that the hair was black and straight. The facial type is much more refined in the land-owning class, owing to intermarriage with high castes. The women are tattooed as in other Dravidian tribes. The Kharwárs are totemistic, and marriage within the same sect is forbidden. They have in a large measure adopted the Hindu practice of infant marriage; in the more primitive tribes the marriage of widows is permitted. Some of the clans continue to offer sacrifices to spirits. They practise cremation, and throw the ashes into a running stream. They will not eat flesh, but cultivate the soil for grain. According to the Census Report for 1891, their numerical strength was 112,298, but only 7651 spoke the tribal language.

The Indian Museum contains a skull, No. 551, of a man named Bahadur of the Bhogta division of the Kharwár tribe. He came from Gola, Hazáribágh, Chúta Nágpúr. He was reported as 29 years old, 5 feet 0·5 inch high; eyes brown, not very almond shaped; beard very scanty, slight moustache, no whiskers; lips everted; nose pyramidal; cheek bones prominent. He died of phthisis, and is said to have been a poor example of his race. The skull was presented by Dr J. WOOD.

The cranium was an elongated ovoid, though the sides were not so vertical as in many dolichocephalic skulls of the aborigines; the parieto-squamous diameter was considerably greater than the stephanic; a low sagittal ridge was associated with a moderate slope outwards to the parietal eminences. The length-breadth index, 73·1, was dolichocephalic, and the frontal and parietal longitudinal arcs were almost of the same length; the breadth and height were equal. The forehead was retreating; the glabella and supra-orbital ridges were moderate. The slope downwards from the obelion was steeper than in the more dolichocephalic crania; the occipital squama was prominent and projected behind the inion. The nasion was not depressed; the bridge of the nose was sharp and laterally compressed; the nasal spine of the superior maxillæ was strong, and a sharp ridge separated the floor of the nose from the incisive region of the jaw. The nasal index, 48·9, was almost leptorhine, and the gnathic index, 92·3, was orthognathic. The orbital index, 92·1, showed the height of the orbit to be almost equal to its breadth; the palato-alveolar arch, 132·6, was strongly brachyuranic. In its complete facial index, 88, the face was chamæprosopic. The upper wisdom teeth were fully erupted, the lower were appearing; the upper incisive fossæ were deep. The skull was not metopic; there were no Wormian bones. A small epipteric bone was in the left pterion. The hard palate was strongly arched; the occipital condyles were flattened; the left jugular foramen was partially blocked by a growth from the petrous-temporal; the left jugal process was tuberculated. The lower jaw was feeble. The skull was cryptozygous, and rested behind on its lower occipital surface. The cubic capacity was 1305 c.c., and the cranium was microcephalic.

Kandh. Table I.

The Kandhs, Kondhs, or Khonds are regarded as Dravidians. The name signifies mountaineer, and they constitute one of the most important aboriginal tribes in Orissa, where they occupy an elevated plateau, intersected by ranges of hills called Kandhmals; but they are also scattered through the tributary states of Orissa. An interesting account of the people and their customs has been given both by Major Macpherson and by Colonel Dalton. The latter writer states that the men are physically a fine race, more so than the Gonds, Bhuiyás, and Páns. They are as tall as the average Hindu, and not much darker in complexion. He regards them as a mixed race, a blend of the Kol, Gond, and Aryan. They worship their own deities, one of the most important being the earth-god or goddess. They are an agricultural people, and before they came under British influence they made human sacrifices to the earth-goddess, and practised female infanticide. Their clothing is scanty, and consists of a waistcloth passed between the thighs. The long hair is tied into a horn-like projection between the eyes. The cheeks and forehead are tattooed. The Kandhs practise cremation. The unmarried young men have a common dormitory, and the girls also have a house assigned to them. Marriage is between adults, and not during infancy; widows may remarry. They are inveterate drunkards. In the Census Report for 1891, 627,388 persons are returned as Kandhs, though only 320,071 speak the tribal language.

I have had the opportunity of examining two skulls said to be those of Kandhs. One was presented to me by a former pupil, now Major Wm. B. Bannerman, M.D. It was that of a man named Judisther Jani, an inhabitant of the village of Bhatpara, in the Khonda subdivision of the commissionership of Orissa. The man had been hanged for murder in the jail at Cuttack. Another specimen, No. 556, in the Indian Museum, was presented by Dr W. D. Stewart, and was obtained from the Kandhmals. It was that of a woman said to be 18 years old, and 5 feet 1 inch in stature.

The male skull was that of an adult. The teeth were more worn in the upper jaw than in the lower. The sutures were unossified, and if it had not been for the worn condition of the molars, one would have regarded the man as about 30 years of age.

In the *norma verticalis* the skull was broadly ovoid with no sagittal ridge, and with a moderate slope from the suture to the parietal eminences. In the proportion of length and breadth the cranium was mesaticephalic, 78·5, and nearer therefore to the brachycephalic than the dolichocephalic standard. The parietal arc was only 1 mm. longer than either the frontal or occipital. The height was greater than the breadth, and the vertical index was 81·4, akrocephalic.

In the *norma lateralis* the glabella and supra-orbital ridges were moderate, the forehead was slightly receding, the vertex was moderately arched, and the slope backwards into the occipital squama was gentle. A slight want of symmetry was noticed in the occipital squama, but not sufficient to lead one to infer that there had been intentional parieto-occipital flattening. The skull was cryptozygous, and rested behind on the occipital condyles. The nasion was not depressed; the nasal bones were slender, and the osseous bridge was depressed and slightly concave. The nasal spine of the superior maxillæ was moderate, and the floor of the nose passed into the incisive region of the upper jaw without the interposition of a dividing ridge. The upper jaw was orthognathic. The complete facial index was 84·3,—*i.e.*, chamæprosopic; the nasal index was platyrhine, and the orbital index was microseme. The palate was remarkably deep and brachyuranic. The lower jaw was well formed and with a strong chin. A large epipteric bone was in each pterion. The cubic capacity of the cranium was microcephalic, 1325 c.c.

The female skull, No. 556, from the Kandhmals, was that of a young woman, and the wisdom teeth were not erupted. A slight transverse constriction was seen behind the coronal suture. Its breadth was great in relation to the length. The parieto-occipital region was steepish but not flattened; the cephalic index, 84·2, placed it amongst the brachycephalic. The parietal arc was much longer than either the frontal or occipital. The vertex was flattened; the frontal and parietal eminences were prominent, the forehead was vertical, all of which are sexual characters. The height was considerably below the breadth, and the vertical index was 77·4. The bridge of the nose was wide and flattened; the anterior nares were wide and rounded at the junction of the side walls with the floor; the nasal index was strongly platyrhine. The upper jaw was mesognathous, the orbital index was microseme, and the palate was brachyuranic. The cranial capacity was only 1070 c.c. The skull was cryptozygous.

Nágesar or Kisán. TABLE II.

The Nágesars are a Dravidian tribe found in Sirgúja, Jashpúr, Palámau, and Lohárdagá in Chúta Nágpúr. DALTON says that in appearance they resemble the Kols, but not the best type, the Santal rather than the Ho. They are not, however, marked with a *godna* or arrow, and the women are not tattooed. DALTON describes them as ill-favoured, the forehead receding, narrow and low; the nose short, broad at the base and with a truncated appearance; the front teeth and jaws project, tilt up the lip and the end of the nose, and give a prognathic character. The skin is deep brown to black; the stature is short. They are totemistic and practise adult marriage. They offer sacrifices to the sun and other deities, but many of them worship the tiger—like the Santals—and they also adore their ancestors.

The Indian Museum contains the skull (No. 405) of a man æt. 30, of the Nágesar tribe from Chúta Nágpúr. He was a Dacoit named Lukroo, who died in prison. The skull was presented by Lieut.-Col. DALTON.

The cranium in the *norma verticalis* was an elongated ovoid with vertical sides, a ridge-like sagittal region with a steep slope downwards and outwards to the parietal eminences. The cephalic index was only 67·8, and the skull was hyper-dolichocephalic. The basi-bregmatic height materially exceeded the breadth, and the vertical index was 73·3. The glabella and supra-orbital ridges were moderate; the forehead somewhat receded; the parieto-occipital region sloped gradually backwards; the occipital squama was rounded and projected behind the inion. The nasion was shallow; the bridge of the nose was almost vertical and inclined to be flattened; the nasal spine of the superior maxillæ was feeble, and the anterior nares rounded off into the incisive region of the upper jaw. The nasal index, 53·2, was platyrhine, but the gnathic index, 96·9, was orthognathous. The complete facial index was 80·6, *i.e.*, low-faced or chamæprosopic. The height of the orbit was materially below the breadth, and the index, 84·2, placed the orbit almost in the microseme group. The palato-maxillary index, 111·1, was almost dolichuranic. The teeth were fully erupted and showed signs of wear; the canine fossæ were deep. The skull was not metopic, and the other sutures were not ossified; a small inter-parietal bone and smaller Wormian bones were in the lambdoid region. In the left pterion were two epipteric bones, and the right alisphenoid was pointed. The os planum of the ethmoid was pointed in front. A pterygo-sphenoid foramen was present on the right side. The muscular ridges were moderate. A third condyle was not present, and the right jugal process was tuberculated. The cubic capacity of the cranium was only 1252 c.c., therefore distinctly microcephalic.

TABLE II.

Dravidian Tribes.

	Nágesar. Lukroo.	Bhuiyá.			Korwá. Fukeera.	Tamil from Madras.	
	I.M.	I.M.	I.M.	I.M.	I.M.	E.U.A.M.	
Collection number,	405	441	439	438	404	...	...
Age,	30	Adult.	Adult.	Adult.	28	Ad.	Ad.
Sex,	M.	M.	M.	F.	M.	M.	M.
Cubic capacity,	1252	1438	1330	1255	...	1150	1240
Glabello-occipital length,	180	189	175	177	186	181	181
Basi-bregmatic height,	132	136	142	131	137	131	132
Vertical Index,	*73·3*	*72·0*	*81·1*	*74·0*	*73·7*	*72·4*	*72·9*
Minimum frontal diameter,	89	95	94	89	91	90	87
Stephanic „ „	108	116	112	110	105	99	102
Asterionic „ „	102	106	109	96	107	101	95
Greatest parieto-squamous breadth,	122s.	132s.	130s.	133p.	128p.	121s.	130s.
Cephalic Index,	*67·8*	*69·8*	*74·3*	*75·1*	*68·8*	*66·9*	*71·8*
Horizontal circumference,	495	520	492	490	511	490	495
Frontal longitudinal arc,	120	130	128	130	130	129	123
Occipital „ „	243	253	234	125	138	125	130
Parietal „ „				109	104	105	108
Total „ „	363	383	362	364	372	359	361
Vertical transverse arc,	288	302	313	305	300	273	287
Length of foramen magnum,	33	36	34	31	36	34	35
Basi-nasal length,	98	105	103	95	105	104	105
Basi-alveolar length,	95	100	103	...	99	95	101
Gnathic Index,	*96·9*	*95·2*	*100·*	...	*94·3*	*91·3*	*96·2*
Interzygomatic breadth,	124	131	133	115	126	123	123
Intermalar „	113	117	122	103	117	115	114
Nasio-mental length,	100	...	...	...	106	105	...
Nasio-alveolar „	62	67	65	...	62	59	61
Complete facial Index,	*80·6*	...	...	...	*84·*	*85·3*	...
Nasal height,	47	50	50	...	46	47	47
Nasal width,	25	26	25	...	27	27	25
Nasal Index,	*53·2*	*52·*	*50·*	...	*58·7*	*57·4*	*53·2*
Orbital width,	38	38	38	40	39	39	36
Orbital height,	32	29	31	38	28	29	30
Orbital Index,	*84·2*	*76·3*	*81·6*	*95·*	*71·8*	*74·4*	*83·3*
Palato-maxillary length,	54	56	54	...	54	53	53
Palato-maxillary breadth,	60	65	66	...	65	60	62
Palato-maxillary Index,	*111·1*	*116·*	*122·2*	...	*120·*	*113·2*	*116·*
Lower jaw. Symphysial height,	30	...	...	...	29	28	...
Lower jaw. Coronoid „	60	...	...	...	57	62	...
Lower jaw. Condyloid „	59	...	...	...	54	61	...
Lower jaw. Gonio-symphysial length,	80	...	...	...	90	83	...
Lower jaw. Inter-gonial width, outside,	93	...	...	...	96	91	...
Lower jaw. Breadth of ascending ramus,	29	...	...	...	29	34	...

Bhuiyá. TABLE II.

In addition to the name Bhuiyá, these people are known by other appellations. Colonel DALTON uses as an alternative Bhúniyá, Mr BUCHANAN HAMILTON calls them Bhungiyá, Mr RISLEY adopts the form Bhuiyá, but gives a number of synonyms; Mr W. CROOKE also names them Bhuiyá. Mr RISLEY considers the name to mean "children of the soil," and that it is not employed as a definite tribal designation, but as implying a status or connection with the land. Bhuiyá is said to be a Sanskrit word, used over India from Assam to Rajputáná and from Madras to Behar, associated with some claim to land, a fact which Mr RISLEY regards as strongly supporting his contention. Mr O'DONNELL, in his Census Report, p. 42, states that Bhuiyá, from Bhui, land, is in Hindu terminology synonymous with autocthon. Colonel DALTON considers that in some parts of Chúta Nágpúr the name has a tribal significance, and he links them with the Dravidians. He says that the lowest type have swarthy, almost black skins, and coarse negro-like features. In the Keunjhar hills they are apparently the dominant aboriginal people, and are described by DALTON as having the skin varying from deep chocolate to tawny; very large mouths; thick, projecting lips; low, narrow foreheads; eyes dark, well-shaped; hair abundant on head but not on face; stature short, averaging 5 feet 2 inches. The higher types found in Gangpur and Bonai are dark brown in colour; hair black, straight, abundant on head, scanty on face; stature moderate; cheek and jaw bones projecting; face broad and square; nose rather retroussé, not very broad at the root; mouth and teeth well formed; eyes straight, not large or deeply set.

In the tributary States the girls seldom marry before puberty, but in other parts the marriage age is twelve, and in the land-holding class during infancy. In some places the unmarried men have a common domicile, and the girls also have a house set apart for them. Widows may marry again. The wealthier classes are properly clothed, but amongst the more primitive people the raiment is very scanty. The women are tattooed. The dead are cremated and the ashes are thrown into an adjoining stream. They eat pork and fowls, but not the flesh of the cow or buffalo. Many of the Bhuiyás are Hinduised, others worship their ancestors. Mr CROOKE states that the rules of succession do not differ from those of cognate Dravidian tribes.

The Indian Museum contains three adult crania marked Bhuiyá from Keunjhar in the Orissa Hills, presented by Dr W. D. STEWART in 1868. Two of these, Nos. 439, 441, were males; one, No. 438, was that of a woman.

When examined in the *norma verticalis* the general form was an elongated ovoid, but the greater projection of the parietal eminences in the woman's skull raised its breadth to 133 mm., which in relation to the length gave it a cephalic index 75·1. In the two male skulls the index was 69·8 and 74·3 respectively; both were dolichocephalic. In the woman's skull and in one of the men the vertex was comparatively flat; in the other man it was more roof-shaped, and the antero-posterior curve was higher at the vertex. The backward slope to the occipital point was more prolonged in the other crania. In the men the basi-bregmatic height exceeded the greatest breadth. In the woman it was somewhat less, and the greater parietal projection gave a pentagonal outline to the cranium, in which the frontal longitudinal arc was the longest. In the two men the large Wormian bones in the lambdoidal suture interfered with the measurements of the parietal and occipital longitudinal arcs. In two skulls a faint transverse depression behind the coronal suture indicated that a band had been worn during infancy. The forehead in the woman and in one man was almost vertical, but receded somewhat in the other male. The skulls were cryptozygous or nearly so, and rested behind on the occiput. The glabella and supra-orbital ridges only slightly projected. The nasion was not much depressed; the nose had a definite bridge, concave forwards; the nasal spine of the superior maxillæ was moderate. The nasal index in the two men was mesorhine, 52 and 50 respectively; in the woman's skull the face was broken. In the men the orbital index was microseme, in the woman megaseme; the palato-maxillary index was brachyuranic; the gnathic index in one male was orthognathous, in the other mesognathous. The teeth were erupted, though in one male the wisdoms were not fully in place. The cranial sutures were unossified; epipteric bones were seen in two crania. In one male, stunted paramastoid processes were present. In the female skull each occipital condyle was almost equally divided by a constriction into an anterior and a posterior area. The cubic capacity of the female skull was 1255 c.c., and the mean of the two males was 1384 c.c.

Korwá. Table II.

The Korwás are a Dravidian tribe living in Chúta Nágpúr, in the districts of Sargúja, Jashpúr, and Palámau, and claiming to be the aboriginal inhabitants. By some linguists the word Korwá is regarded as another form of Kol. They lead a nomadic life in the highlands, and armed with bows and arrows, are hunters and flesh eaters rather than agriculturists; though to some extent they are cultivators, and clear the ground by burning the jungle. Dalton states that they are the most savage looking of the Kolarian group of tribes. They are strongly built and active; the skin is dark brown, the face is broad, the forehead narrow, the hair is long and tangled, though in a figure of a man reproduced by Mr Crooke, the head is shaven; they grow a beard and moustache. The more savage of the Korwás have black skins, flat faces, projecting chins, and tawny hair. In stature, the men of the Sargúja Korwás averaged 5 feet 3 inches, the women 4 feet 9 inches; but the men living on the Khúria plateau were somewhat taller; one measured 5 feet 8 inches. Both sexes are scantily clothed. They worship the tribal god Râja Chandol, and offer sacrifices to it, but the Sargúja tribe sacrifice to the spirits of their ancestors. They are totemistic, and apparently marriage is prohibited within the sept using the same totem. Mr Crooke says the marriage age for boys is twelve and ten for girls; widows may remarry. Some families cremate, others bury the dead. Mr O'Donnell, in his Report on the Census of the Lower Provinces of Bengal, gives 79,954 persons as speaking the Korwá dialect of the Kolarian group of languages.

The Indian Museum contains a skull (No. 404) of a man of the Korwá tribe, 28 years old, named Fukeera, from Sargúja, Chúta Nágpúr. He died in prison, and the skull was presented by Lieut.-Colonel Dalton.

The cranium in the *norma verticalis* was an elongated ovoid, very narrow, somewhat roof-like in the sagittal region, and with the sides of the skull almost vertical. The length-breadth index was only 68·8, and the skull was hyper-dolichocephalic. The parietal longitudinal arc was more than the frontal and much longer than the occipital. The basi-bregmatic height materially exceeded the greatest breadth, and the vertical index was 73·7. The parieto-occipital region sloped gently downwards, and the occipital squama was rounded and projected behind the inion. The glabella was moderate and the forehead was somewhat retreating. The nasion was shallow; the bridge of the nose was slightly projecting and vertically concave. The nasal spine of the superior maxillæ was distinct, and a sharp border separated the floor of the nose from the incisive region of the upper jaw. The nasal index was 58·7, distinctly platyrhine; the gnathic index, 94·3, was that of an orthognathous jaw. The orbital index, 71·8, was mesoseme, and the palato-maxillary index, 120, was brachyuranic. The complete facial index, 84, placed it in the low-faced group, chamæprosopic. The teeth were fully erupted, but not much worn; the canine fossæ were depressed. Small Wormian bones were in the lambdoidal suture. The skull was phænozygous, and rested behind on the occipital bone.

Múnda, Ho, or Larkha Kol. Table III.

The Múndas are a large non-Aryan tribe, occupying the plateau in Chúta Nágpúr which attains an elevation of 3000 feet. On linguistic grounds they are classed as Kolarian. Mr Risley states that the name Múnda is of Sanskrit origin, and is applied to the headman of the tribe or village; it is also used generally as a tribal name. As regards their language, physical characteristics, and customs, the Múndas, Hos, Bhúmij, Korwá, Kharriás and Santals are closely allied, and from speaking the languages of the Kolarian group, they are frequently classed together as Kols or Coles. There is a difference of opinion as to the derivation and meaning of the term Kol. It has been regarded as signifying pig, and used by the Indo-Aryans as a term of contempt applied to the aborigines; but it is now, on the authority of Dalton, considered to be derived from the Mundâri word Ho, or Horo, which means a man. According to tradition, the Kols were the earliest settlers in the valley of the Ganges.

Dalton in his account of the Múndas regards the Hos or Larkha (fighting) Kols as so closely allied to them, that they are often included together in the same descriptive sentence. He states that the Múndas are located in Singbhúm, Chúta Nágpúr, and in the territory known as Kolhán. The Hos admit that they are of the same family as the Múndas, and that they came from Chúta Nágpúr. DALTON considers that from their isolation and independence, they furnish the best illustration of the characteristics of the Mundâris. They are physically a much finer people than the Bhúmij, Santals, or Kharriás. The men are 5 feet 5 or 6 inches in height, the women 5 feet 2 inches; they have an erect carriage. The skin has a brownish coppery tint; the eyes are dark brown; the hair is black, straight or wavy. Many have high noses, oval faces, and young girls are sometimes seen with delicate features, finely chiselled straight noses, so that there may be an admixture of Aryan blood. DALTON has also met some with strongly marked Mongolian features and a dark skin like the Santals.

The clothing is reduced to a minimum, and often consists only of a loin-cloth brought between the thighs and fastened in front to a girdle. The women wear the hair collected into a knot touching the back of the right ear and decorated with flowers. Marriage is between adults and is exogamous, and widow marriage is permitted. The national emblem is a *godna* or arrow. The dead are cremated and the ashes are buried, the spot being marked by a large grave-stone, and often a megalithic monument is set up outside the village. They are active and courageous, truthful and sensitive to wrong. They cultivate the ground, but eat also fowls and the flesh of pigs. They worship the sun and several other deities. In the general Report on the Census of 1891, it is stated that the Múnda, Ho, Kol, Kur, and Korwá people number 1,109,157 by tribe, and that of these 840,282 speak the tribal language.

In the series of skulls lent to me by the Indian Museum, six specimens are marked Kol or Cole. One of these, No. 31, from Singbhúm, designated Larkha Kol, was presented by Colonel DALTON; another, No. 557, from the Kandhmals, marked Pan Cole, said to be 42 years old, height 5 feet 8 inches, and of dark complexion, was presented by Dr W. B. STEWART. Nos. 440, 442, and 444, also presented by Dr STEWART, were from Keunjhar, Orissa. No. 24, named Phugooa, given by Colonel DALTON, was from Moorgoo, Chúta Nágpúr; the age was said to be 65, the stature 5 feet 5 inches; hair of head straight, grey, that of face scanty; eyes regular; food rice, flesh, and vegetables.

In the same museum were nine skulls, marked Múnda from Chúta Nágpúr. Of these, No. 25 is said to have been in height 5 feet 4 inches; hair black, coarse, straight; eyes large, black, straight; food rice, flesh, vegetables; whilst No. 26 was 34 years old; height 5 feet 5 inches; hair black, coarse; eyes large, black, straight; food as above; they were presented by Colonel DALTON. The others were collected in or near Ranchi by Mr W. H. P. DRIVER. Dr HEDLEY WOOD has presented to me the skull of a woman aged 24, also obtained at Ranchi.

Sixteen crania marked Múnda or Kol have therefore come under observation; thirteen of which are apparently those of men and three those of women. They are all adults, with the exception of No. 25, said to be that of a youth of 18, in which, though the wisdom teeth were not erupted, the basi-cranial synchondrosis was ossified.

Table III.

Múnda, Kol.

										Kol.	Larkha Kol.	Kol.				
	Dholeja Múnda, Jurobaree.	Dhirhoo Múnda. Kakadeeh.	Hochar. Lodha Village.	Biphaiya. Madkom Village.	Mangra Múnda. Ranchi. Old Town.	Debia Múnda, Lalpur Village.	Lellie Múnda. Konka Village.	Gonda Múnda. Lalpur Village.	Somari Múnda, Ranchi.	Phugooa. Moorgoo.	Singbhúm. Mora.	Keunjhar.			Pan Cole.	Jattia Múnda. Bhowro Village.
	I.M.	I.M.	I.M.	I.M. *	I.M.	I.M.	I.M.	I.M.	Metopic. E.U.A.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M. *
Collection number,	25	26	603	605	606	612	607	611	...	24	31	440	442	444	557	604
Age,	18	32	45	Ad.	Ad.	Ad.	Ad.	Ad.	24	Ad.	Adult.	Aged.	Adult.	Adult.	42	Ad.
Sex,	M.	M.	M.	M.	M.	M.	F.	F.	F.	M.	M.	M.	M.	M.	M.	M.
Cubic capacity,	1248	1210	1375	1430	1315	1310	1000	1110	1180	1306	1215	1470	1176	1220	1388	1200
Glabello-occipital length,	176	179	180	191	183	180	165	168	170	182	175	182	176	178	191	164
Basi-bregmatic height,	132	128	131	141	133	129	130	126	128	130	130	138	130	130	126	132
Vertical Index,	*75·*	*71·5*	*72·8*	*73·8*	*72·7*	*71·7*	*78·8*	*75·*	*75·3*	*71·4*	*74·3*	*75·8*	*73·9*	*73·*	*66·*	*80·5*
Minimum frontal diameter,	89	89	96	97	89	90	88	92	91	94	90	101	94	95	97	93
Stephanic,	102	105	102	109	99	100	89	97	109	111	110	121	107	110	118	107
Asterionic,	105	100	104	109	99	105	89	93	102	104	106	106	101	104	110	99
Greatest parieto-squamous breadth,	123s.	127s.	134s.	130s.	131p.	129p.	112s.	122s.	125	132s.	132s.	137s.	127s.	127s.	141s.	132s.
Cephalic Index,	*69·9*	*70·9*	*74·4*	*68·1*	*71·6*	*71·7*	*68·*	*72·6*	*73·5*	*72·5*	*75·4*	*75·3*	*72·2*	*71·3*	*73·8*	*80·5*
Horizontal circumference,	488	491	493	521	498	491	448	460	480	506	492	515	495	497	534	470
Frontal longitudinal arc,	125	112	124	135	124	120	112	116	120	130	122	130	122	124	130	118
Parietal ,, ,,	131	125	124	133	{257	136	120	130	127	129	121	132	130	136	113	117
Occipital ,, ,,	111	118	112	130		110	103	99	101	112	116	120	108	104	128	108
Total ,, ,,	367	355	360	398	381	366	335	345	348	371	359	382	360	364	371	343
Vertical transverse arc,	278	285	301	312	287	291	277	280	237	298	290	312	287	295	298	300
Length of foramen magnum,	35	35	33	31	36	32	31	32	32	35	31	34	31	33	34	33
Basi-nasal length,	91	101	101	102	94	96	98	93	95	98	95	101	101	101	101	97
Basi-alveolar length,	88	95	93	102	93	96	95	88	94	93	86	100	99	...	99	95
Gnathic Index,	*96·7*	*94·1*	*92·1*	*100·*	*98·9*	*100·*	*96·9*	*94·6*	*98·9*	*94·9*	*90·5*	*99·*	*98·*	...	*98·*	*97·9*
Interzygomatic breadth,	123	122	128	130	131	128	115	120	125	125	131	133	...	...	133	126
Intermalar ,,	112	109	118	122	120	118	109	110	114	104	119	120	...	...	120	115
Nasio-mental length,	102	107	115	117	106	103	98	...	...	...	107	...	...	...	...	100
Nasio-alveolar ,,	59	62	62	70	60	...	54	56	64	60	63	68	63	...	63	58
Complete Facial Index,	*82·9*	*87·7*	*89·8*	*90.*	*80·9*	*80·4*	*85·2*	...	...	...	*81·6*	...	...	...	...	...
Nasal height,	45	47	47	52	48	45	41	42	47	46	50	54	48	...	47	43
Nasal width,	23	26	23	27	25	25	24	22	25	23	26	25	22	...	27	24
Nasal Index,	*51·1*	*55·3*	*48·9*	*51·9*	*52·1*	*55·5*	*58·5*	*52·4*	*53·2*	*50·*	*52·*	*46·3*	*45·8*	...	*57·4*	*55·8*
Orbital width,	35	39	39	40	37	36	37	37	36	37	40	40	39	...	40	38
Orbital height,	34	31	30	30	29	31	30	34	33	32	32	33	34	...	30	30
Orbital Index,	*97·1*	*79·5*	*76·9*	*75·*	*78·4*	*86·1*	*81·1*	*91·9*	*91·7*	*86·5*	*80·*	*82·5*	*87·2*	...	*75·*	*78·9*
Palato-maxillary length,	49	53	49	58	56	...	51	45	50	54	47	60	55	...	53	51
Palato-maxillary breadth,	62	66	63	68	64	...	62	...	60	64	66	69	65	...	63	59
Palato-maxillary Index,	*126·*	*124·*	*128·5*	*117·2*	*114·2*	...	*121·5*	...	*120·*	*118·*	*140·*	*115·*	*118·*	...	*118·*	*115·6*
Lower jaw. Symphysial height,	30	29	31	32	31	...	27	...	...	...	30	...	...	...	...	26
Lower jaw. Coronoid ,,	60	62	65	66	...	63	53	...	...	...	66	...	...	...	...	52
Lower jaw. Condyloid ,,	60	60	63	64	...	59	54	...	...	...	61	...	...	...	...	58
Lower jaw. Gonio-symphysial length,	85	86	86	91	86	88	80	...	...	...	85	...	...	...	...	80
Lower jaw. Inter-gonial width outside,	85	91	95	99	92	93	93	...	...	...	91	...	...	...	...	86
Lower jaw. Breadth of ascending ramus,	30	30	37	35	35	31	33	...	...	...	32	...	...	...	...	35

* With Skeletons.

Of the sixteen crania, No. 604, stated in the museum list to be Jattia Múnda, of Bhowro village, near Ranchi, differed so greatly in the form and proportions of the cranium from the others, that it will be described in a separate paragraph (p. 79). The following description applies therefore to fifteen skulls, and of these No. 444 consisted only of the calvaria. The lower jaw was absent in several specimens.

The crania presented in the *norma verticalis* an elongated ovoid form, with steep sides and moderate parietal eminences. The sagittal region showed no special ridge or flattening, nor was the slope outwards to the parietal eminence, though distinct, so marked as one sees in some aborigines. In the males, the glabello-occipital length ranged from 165 to 191 mm., and the greatest breadth from 123 to 141 mm. In three crania the cephalic index was below 70, hyper-dolichocephalic; in ten it ranged from 70 to 75, dolichocephalic; in two it was between 75 and 76, essentially dolichocephalic, though numerically in the mesaticephalic group. The mean cephalic index of the fifteen crania was 72. In these skulls the occipital was the smallest of the three longitudinal arcs, except in one specimen where it exceeded the frontal; usually the parietal had the longest arc, but in five specimens the frontal was the longer. In the males, the basi-bregmatic height ranged from 126 to 141 mm.; in the females from 126 to 130 mm. The mean vertical index in the fifteen crania was 73·4, *i.e.*, metriocephalic. The basi-bregmatic height exceeded the greatest breadth in ten skulls; in four it was less, and in two the diameters were equal.

The forehead in the men did not much recede, and the skull sloped gently backwards in the parieto-occipital region; as a rule, the occipital squama was rounded, and projected behind the inion. The glabella and supra-orbital ridges were moderate; as a rule the nasion was not depressed. The nasal bones were not large, and the bridge was either feeble or only moderately projected. The nasal spine of the superior maxillæ was moderate. In some specimens a ridge demarcated the floor of the nose from the incisive region; but as a rule, they rounded off into each other. The mean nasal index in fourteen skulls was 52·1, high in the mesorhine series; but in the individual specimens, whilst five were markedly platyrhine, seven were mesorhine, and two were leptorhine; eight skulls were microseme, and the mean orbital index of the series was 83·5, *i.e.*, microseme; but the range of variation was considerable, so that three were in the megaseme group and three were mesoseme. The mean gnathic index of the series was 96·6, *i.e.*, orthognathous; no specimen was prognathous, and only six were mesognathous. In the male skulls the greatest interzygomatic breadth was 133 mm., but the mean of ten specimens was only 128·4, which is materially below the measurements of the face breadth given in Part I. of this memoir, in the Chinese, Burmese, Nágás, and Esquimaux. Owing to the lower jaw being absent in several specimens, the nasio-mental diameter could only be taken in eight skulls, all of which were chamæprosopic, and the mean of the series was 84·8. The mean palato-maxillary index was brachy-uranic, 121·7, and only one skull was below the lowest term of that group.

One skull, that of a woman, was metopic. The sutures were as a rule distinct, though in some they were more or less obliterated. The skull marked Pan Cole had a transverse depression behind the coronal suture as if from wearing a band during infancy. Wormian bones were present in the lambdoidal suture in several specimens. Three crania had a single epipteric bone, and in No. 25 the squamous temporal articulated with the frontal; no skull had a third condyle, but in No. 557 each occipital condyle was divided into an anterior and a posterior facet. The jugal processes were sometimes tuberculated. The crania rested behind either on the mastoids or occipital region. Several specimens showed the infra-orbital suture, and in one of these the superior maxilla and sphenoid articulated at the anterior end of the spheno-maxillary fissure. The cranial capacity ranged in twelve men from 1176 to 1470 c.c.; the mean of the series was 1305 c.c., and seven specimens exceeded the mean. In the three women the range was from 1000 to 1180, and the mean was 1097 c.c.

No. 605, Biphaiya Múnda from Madkom village, near Ranchi, was accompanied by a skeleton, the bones of which I have examined.

Pelvis.—The chief measurements of the pelvis are given below. The alæ were expanded and the iliac fossa were not translucent; the subpubic angle was relatively wide; the pectineal lines were not knife-like; there was a shallow præauricular sulcus. The breadth-height index of the entire pelvis was 81·2. The transverse diameter of the brim was much more than the conjugate, and the brim index was 86·7, *i.e.*, platypellic.* The sacrum consisted of five vertebræ, and the sacral index was 110·5, *i.e.*, platyhieric.

Measurements of Pelvis.

Collection, Indian Museum,	No. 605	No. 604				
Sex,	M.	M.				
1. Breadth of pelvis,	245mm	246mm				
2. Height of pelvis,	199	178				
3. *Breadth-height Index,*	*81·2*	*72·3*				
4. Between ant. sup. iliac spines,	215	222				
5. Between post. sup. iliac spines,	82	73				
6. Between ischial tubera,	136	116				
7. Vertical diameter of obturator foramen,	45	48				
8. Transverse diameter of obturator foramen,	33	29				
9. *Obturator Index,*	*73·3*	*60·4*				
10. Subpubic angle,	83°	62°				
11. Transverse diameter of brim,	113	114				
12. Conjugate diameter of brim,	98	87				
13. *Pelvic Index,*	*86·7*	*76·2*				
14. Length of sacrum,	95	4 vert.				
15. Breadth of sacrum,	105	102				
16. *Sacral Index,*	*110·5*	...				

True Vertebræ.—The cervical vertebræ were normal. Of the twelve dorsal vertebræ 1st to 8th were normal. The 9th had a small costal articular facet at the upper border, but none at the lower border of the side of the body. The 10th, 11th, and 12th had

* For the meaning of this and several other descriptive terms used to denote proportion between certain diameters of the skeleton, see my Report on Human Skeletons, in Challenger *Reports*, Part XLVII., 1886.

each a large single costal facet at the side of the body. The transverse process of the 10th dorsal had no costal facet, and those of the 11th and 12th had the usual three tubercles. The lumbar vertebræ were of the customary shape. The vertical diameter of their bodies in front and behind was as follows :—

	A.V.D.	P.V.D.	Indices.
1st lumbar, . . .	24 mm.	25 mm.	104
2nd „ . . .	23 „	26 „	113
3rd „ . . .	23 „	26 „	113
4th „ . . .	20 „	22 „	110
5th „ . . .	21 „	21 „	100
	Total 111 mm.	Total 120 mm.	Mean 108·1

In this skeleton the upper four vertebræ had the posterior vertical diameter longer than the anterior. It is customary to find the antero-vertical diameter of the 5th vertebra longer than the postero-vertical, but in this specimen they were equal. The mean general index of the series of five vertebræ was as high as 108·1, which places the lumbar spine in the koilorachic group.

Upper Limb.—Clavicles slender, right 138 mm., left 136 mm. long; subclavian groove shallow. Scapulæ: right, 143 mm. long, 105 broad, index 73·4; left, 150 mm. long, 106 broad, index 70·6. Supra-scapular notch shallow and wide, but with a distinct border. Humerus with shallow musculo-spiral groove and moderate muscular impressions, no supra-condylar process or inter-condylar foramen. Bones of forearm not specially noticeable. Radio-humeral index, 83·3 or dolichokerkic.

	Right.	Left.
Humerus, head to trochlea, ...	307 mm.	312 mm.
Radius to tip of styloid, ...	256 „	255 „
„ base „	252 „	250 „
Ulna to tip of styloid, ...	281 „	281 „
„ articular surface, ...	276 „	278 „

Lower Limb.—Femur with linea aspera and external condylar ridge fairly well marked, also the trochanters and gluteal ridge; no platymery; articular area on internal condyle prolonged forwards and lying in the same transverse plane as the origin of the inner head of the gastrocnemius; popliteal surface plano-concave. Tibia with the head retroverted; condylar surfaces with shallow concavities; antero-posterior diameter of shaft of right tibia in plane of nutrient foramen, 36 mm.; transverse diameter in same plane 25 mm.; index of platyknemia 69·4; the corresponding diameters of the left bone were 37 and 24 mm. No articular facet on the front of lower end of left tibia continuous with the astragalar area was seen, but a slight indication of one was present in the right bone. The fibulæ had strong oblique ridges and a deep concavity for the origin of the tibialis posticus.

No. 604, referred to on p. 77, and marked Jattia Múnda from Bhowro village, is so different in configuration from the other Múnda crania that there can, I think, be little doubt that it has been erroneously named by the collector. The skull is brachycephalic, 80·5, in its proportions and form. It was rounded in outline when seen from the norma verticalis, and comparatively flattened on the vertex. The frontal and parietal eminences were distinct, and the skull sloped steeply downwards in the parieto-occipital region, where it was unsymmetrical and flattened on the left side. The frontal longitudinal arc was the longest; the basi-bregmatic diameter was the same as the parieto-squamous. The upper jaw was orthognathous, the nasal index was platyrhine, the orbital index was microseme, and the palato-maxillary index was barely brachy-uranic. Although I have given the measurements of the lower jaw sent with the skull, I doubt if it really belonged to it. The cranial capacity was 1200 c.c.

The skull was accompanied by other bones of the skeleton.

Pelvis.—The measurements of the pelvis are given on page 78. The iliac fossæ were translucent, and the alæ were expanded; the subpubic angle was acute; the obturator foramen had a long vertical diameter. The pelvis was broad in relation to the height, and the index was 72·3. The transverse diameter of the pelvic brim greatly exceeded the conjugate, and the brim index, 76·2, was platypellic. The pectineal lines were knife-like; the præauricular sulcus was faintly marked. Only four sacral vertebræ were present; the body of the 4th was oval like a normal 5th, and its laminæ formed two sacral cornua and did not meet behind in a spine. The base of the sacrum had on the right of its body an articular surface for the right transverse process of the 5th lumbar vertebra.

True Vertebræ.—The cervical vertebræ were normal. The dorsi-lumbar vertebræ were eighteen in number. The 10th had a single facet on the side of the body for a part of the head of the 10th rib; the 11th and 12th had single facets for their corresponding ribs, they had both rudimentary transverse processes, and the inferior articular processes of the 12th dorsal were convex, and looked forwards and outwards. There were six vertebræ between the 12th dorsal and 1st sacral. The first of these approximated in shape to the 12th dorsal; its transverse processes were rudimentary, and showed the superior, inferior and external tubercles. On the side of the pedicle, immediately in front of the external tubercle, was a smooth facet 2 mm. in diameter, apparently for the head of a rudimentary rib; its articular processes had the characters of a lumbar vertebra. The remaining five vertebræ had the customary lumbar characters, and the right transverse process of the lowest was divided by a deep furrow into a non-articular part, and an articular part which was jointed to the base of the sacrum. The vertical diameters of the bodies of these vertebræ, in front and behind, was as follows:—

	Ant. V. Diam.	Post. V. Diam.	Index.
Dorsi-lumbar, . .	23 mm.	26 mm.	113
1st lumbar, . . .	24 „	26 „	108·3
2nd „ . . .	24 „	25 „	104·1
3rd „ . . .	23 „	24 „	104·3
4th „ . . .	22 „	22 „	100·
5th „ . . .	24 „	21 „	87·5
	Total 117 mm.	Total 118 mm.	Mean 100·8

In this skeleton the 4th lumbar body showed an equality in the vertical diameters; in those higher up the posterior diameter exceeded the anterior, whilst in the lowest, the anterior was distinctly greater than the posterior diameter. The mean general index of the series of five vertebræ was 100·8, and the lumbar spine was in the orthorachic group.

Upper Limb.—The bones of the upper limb were slender, and the muscular markings were feeble. The Humerus had neither supra-condyloid process nor inter-condylar foramen; the musculo-spiral groove was shallow, and the shaft had only a slight twist. The right radio-humeral index was 75·3, mesatikerkic.

	Right.	Left.
Humerus, head to trochlea, ...	292 mm.	288 mm.
Radius, head to tip of styloid, ...	220 „	222 „
„ „ base „	217 „	216 „
Ulna, olecranon to tip of styloid, ...	238 „	240 „
„ „ lower articular surface, . . .	234 „	236 „

The Clavicles were: right bone, 129 mm., left, 134 mm. long; their subclavian grooves were scarcely marked. The right Scapula was 129 mm. long and 91 broad, index 70·5; the left was 129 mm. and 94 broad, index 72·9; the supra-scapular notch was shallow and not differentiated from the superior border by a sharp margin.

Lower Limb.—The bones of the lower limb were also slender. In the Femur the trochanters and gluteal ridges were fairly marked, but there was no platymery. The linea aspera and external condylar ridge were distinct, the popliteal triangle was flattened or faintly concave; the inner condylar articular surface was prolonged backwards and in the same transverse plane as the place of origin of the inner head of the gastrocnemius. The head of the Tibia was slightly retroverted; the lower articular end was not prolonged on the front of the bone. The antero-posterior diameter of the shaft in the plane of the nutrient foramen was for the right bone, 28 mm.; for the left, 27 mm.; the transverse diameter was in each bone 21 mm.; the index of platyknemia was 75 in the right tibia.

In No. 605 the tibio-femoral index 86·4 was dolichoknemic; in No. 604 the index was 82·96, practically also dolichoknemic, *i.e.*, with a relatively long tibia.

	No. 604.		No. 605.	
	Right.	Left.	Right.	Left.
Femur, maximum length, . . .	410 mm.	410 mm.	445 mm.	445 mm.
„ oblique length, . . .	405 „	408 „	441 „	441 „
Tibia, condylar surface to tip of malleolus, .	343 „	336 „	395 „	393 „
„ „ „ astragalar surface, .	336 „	332 „	381 „	381 „
Fibula, maximum length, . . .	341 „	339 „	377 „	378 „

Bhúmij. TABLE IV.

The Bhúmij is a non-Aryan tribe living in the Manbhúm and Singbhúm districts of Chúta Nágpúr as well as in Western Bengal. They are regarded as the original inhabitants, and are located by DALTON in the country between the Kasai and Subarnarekhá rivers. They have been classed on linguistic grounds as Kolarian; most authorities regard them as closely allied to, and probably identical with, the Múndas, with whom they associate and intermarry. DALTON says that their appearance is inferior to that of the best of the Múndas and to the Hos of Singbhúm. They are short, but strongly built. The skin ranges in colour from a light brown to a dark chocolate. They build commodious houses and practise adult marriage. The divisions of the tribe are totemistic, and the marriage of adults is exogamous, as amongst the Múndas; widows may remarry. The dead are cremated, and the body is laid upon the pyre with the head to the south; the ashes are buried under gravestones, which are sometimes of large size. They are agriculturists, but they eat fowls and drink fermented liquors. They worship the sun as well as minor deities. Their numbers do not appear to have been separately recorded in the General Report on the Census of 1891, but in the special census of the lower provinces of Bengal and their Feudatories, Mr C. J. O'DONNELL gives a total of 306,473.

I have examined two skulls of the Bhúmij tribe, both adult males, collected at Mánbhúm. One in the Indian Museum, No. 18, is named Aunundo Bhoomiz; in the list supplied to me he is said to have been 40 years of age, 5 feet 3 inches in height, hair and eyes black, whiskers small. The other, a male named Karnai, aged 30, was presented to me by Dr J. J. HEDLEY WOOD.

In both specimens the cranium was long, relatively narrow, and roof-shaped in the sagitto-parietal region. The parietal eminences were well in front of the occipital point which projected behind the inion; the side walls of the cranium were almost vertical. In one skull the length-breadth index was 72·7, in the other 70·9; both were dolichocephalic. In one the frontal and parietal longitudinal arcs were equal and in excess of the occipital; in the other the frontal arc was the longest. In one the basi-bregmatic diameter was less than the greatest breadth; in the other it was slightly longer. The glabella and supra-orbital ridges were moderately projecting; the forehead slightly receded; the antero-posterior curve of the vault rose gradually to the vertex, and then sloped gently downwards to the occipital squama. In neither skull was any sign of parieto-occipital flattening. The nasion was somewhat depressed; the nasal bones were short, concave forwards, and only feebly projecting. The nasal spine of the superior maxillæ was moderate, and the floor of the nose was separated by a slight ridge from the incisive surface of the jaw.

The nasal index in both specimens was in the higher mesorhine group; the gnathic index in both was orthognathous; one skull was mesoseme, the other megaseme; the

Table IV.

Bhúmij and Turí Races.

	Bhúmij. Mánbhúm.		Mánbhúm. Race unknown. Scapho-cephalic.		Turí.				
	Aunundo Bhoomiz.	Karnai.			Bitna. Surungee.	Sookeam. Teerrah.			
	I.M.	E.U.A.M.	I.M.		I.M.	I.M.			
Collection number,	18	...	407		22	23			
Age,	40	30	Ad.		28	35			
Sex,	M.	M.	M.		M.	M.			
Cubic capacity,	1414	1235	1410		1280	1435			
Glabello-occipital length,	183	182	194		183	188			
Basi-bregmatic height,	131	130	137		132	132			
Vertical Index,	*71·6*	*71·4*	*70·6*		*72·1*	*70·2*			
Minimum frontal diameter,	94	89	93		93	95			
Stephanic,	115	113	107		110	111			
Asterionic,	105	100	114		99	109			
Greatest parieto-squamous breadth,	133s.	129	125s.		133p.	135s.			
Cephalic Index,	*72·7*	*70·9*	*64·4*		*72·7*	*71·8*			
Horizontal circumference,	517	502	520		514	518			
Frontal longitudinal arc,	135	130	137		128	127			
Parietal „ „	135	125	143		131	118			
Occipital „ „	114	103	124		106	129			
Total „ „	384	363	404		365	374			
Vertical transverse arc,	302	233	298		308	304			
Length of foramen magnum,	36	34	32		...	38			
Basi-nasal length,	95	101	102		101	99			
Basi-alveolar length,	92	93	94		102	101			
Gnathic Index,	*96·8*	*92·1*	*92·2*		*101*	*102*			
Interzygomatic breadth,	129	126	125		124	134			
Intermalar „	110	119	117		115	121			
Nasio-mental length,	120	113	109		...	110			
Nasio-alveolar „	66	64	65		61	66			
Complete Facial Index,	*93*	*89·7*	...		...	*82*			
Nasal height,	50	46	48		44	46			
Nasal width,	26	24	24		27	24			
Nasal Index,	*52*	*52·2*	*50*		*61·4*	*52·2*			
Orbital width,	38	35	37		41	39			
Orbital height,	33	32	29		31	30			
Orbital Index,	*86·8*	*91·4*	*78·4*		*75·6*	*76·9*			
Palato-maxillary length,	53	52	52		56	58			
Palato-maxillary breadth,	61	69	61		65	65			
Palato-maxillary Index,	*115*	*132·7*	*117·3*		*116·*	*112·*			
Lower jaw. Symphysial height,	35	32	29		...	32			
Lower jaw. Coronoid „	65	55	64		...	64			
Lower jaw. Condyloid „	58	59	65		...	65			
Lower jaw. Gonio-symphysial length,	91	82	89		...	96			
Lower jaw. Inter-gonial width outside,	96	95	96		...	98			
Lower jaw. Breadth of ascending ramus,	30	33	37		...	38			

palato-maxillary index in one was brachyuranic, in the other mesuranic. In one the complete facial index was chamæprosopic, in the other high-faced leptoprosopic. The teeth were somewhat worn from use; the canine and incisor fossæ were deep. The cranial sutures were distinct. In one there were no irregular ossifications; in the other the right pterion had a large epipteric bone. The muscular ridges and processes were well marked. In one the cubic capacity, 1414, was mesocephalic; in the other, 1235 c.c., microcephalic. The lower jaw was well proportioned and possessed a square chin.

Another skull from Manbhúm, an adult male, No. 407 in the Indian Museum, is marked "race unknown." It is a characteristic specimen of a scaphocephalic cranium. Although not known to be a Bhúmij, yet as it came from Manbhúm, it is convenient to describe its characters here. The skull was greatly elongated and narrow, strongly keeled in the sagittal region, and with the suture obliterated; the lambdoidal suture was almost completely obliterated, but the coronal and the lateral longitudinal sutures were to all appearance unossified. The glabella and supra-orbital ridges were prominent, and the nasion was depressed. The nasal bones were short and prominent. The canine and incisive fossæ were deep. The nasal spine of the superior maxillæ was moderate. The dimensions of the skull are given in Table IV. The modifications in shape produced by the premature closure of the sagittal and lambdoidal sutures have, however, deprived this skull of any ethnic significance. It will be seen from the Table that owing to the elongation of the cranium and the diminished parieto-squamous breadth, the length-breadth index is only 64·4. The cubic capacity, 1410 c.c., is apparently not affected by the cranial deformity.

Turi. Table IV.

The Turis are a non-Aryan tribe or caste, living in Chúta Nágpúr. In his account of these people Mr Risley states that they are without doubt a Hinduised offshoot of the Múndas. He adduces in support of this opinion the following:—They use amongst themselves a dialect of Mundari; their totems correspond closely with those in force amongst the Múndas; their original religion is closely akin to the form of animism current among the Múndas.

The Turis are cultivators and makers of baskets. They are, like the Múndas and Oráons, lax in articles of food. Each sub-caste is strictly endogamous. Girls usually marry as adults and widows can marry again. The caste is small, and in 1881 numbered apparently about 30,000 persons.

Two crania, marked Turi, are in the Indian Museum. No. 22 is that of Bitna, from Surungee. He was 28 years old; 5 feet 4 inches high; hair black, straight; eyes black, small; no beard or whiskers. No. 23, Sookeam, was from Teerrah. He was 35 years old; 5 feet 3 inches high; hair black, straight; eyes black; no beard or whiskers. Both men had been hanged in Ranchi jail as murderers.

The skulls resembled each other in the *norma verticalis* ; they were elongated ovoids, with distinct parietal eminences, and with a moderate slope outwards from the sagittal suture. They were both dolichocephalic, the mean length-breadth index being 72·2 ; in Bitna the parietal arc was a little longer than the frontal, but in Sookeam the occipital arc had the unusual diameter, 129 mm., and was longer than either the frontal or parietal. In each skull the basi-bregmatic height was slightly less than the breadth. The forehead was moderately receding, and the glabella and supra-orbital ridges were not prominent ; the crania sloped gently backwards and downwards from the obelion ; the occipital squama was rounded and prominent. The upper jaw slightly projected, and the gnathic index, mesognathous, was 101 and 102. The nasion was shallow ; the bridge of the nose was concave vertically ; the nasal spine of the superior maxillæ was moderate, and the anterior nares were rounded at the junction of the side-wall and floor. The nasal index in Bitna was markedly platyrhine ; in Sookeam it was mesorhine, and in his skull the face was chamæprosopic. In both skulls the orbital index was microseme ; in one the palato-maxillary index was mesuranic, in the other in the lower term of the brachuranic group. In No. 22 the arch of the palate was much deeper than in No. 23. Both crania were barely cryptozygous, and they rested behind on the cerebellar part of the occiput. In Bitna the wisdoms were erupted, in the other skull in process of eruption ; the incisor fossæ were deeper than the canine. The frontal suture was closed, but the other sutures were not ossified. In No. 23, small Wormian bones were in the lambdoidal suture, but there were no other special abnormalities. The muscular ridges were fairly developed. In Bitna the cranial capacity was only 1280 c.c., *i.e.*, microcephalic, whilst in Sookeam the capacity, 1435 c.c., placed it high in the mesocephalic group.

Juang. Table V.

The Juangs are a non-Aryan tribe living in the hill districts of Dhekanál and Keunjhar, two of the tributary states of Orissa. Dalton groups them with the Kolarians on account of some affinities of language, but he also says that, whilst they have adopted many Uriyá words, they employ vocables which cannot be connected with any Aryan, Kolarian, or Dravidian language. They are a primitive people, and claim to be the autochthones in Keunjhar. They are remarkably shy and timid. The stature of the men is somewhat less than 5 feet, that of the women about 4 feet 8 inches ; the forehead is upright, but narrow and low ; nasal bones depressed, alæ of nose spreading ; mouth large, lips thick, upper jaw rarely prognathous, chin receding ; hair coarse and frizzly ; prevailing colour of skin a reddish brown ; the jaw is flat, and the cheek bones are strongly projecting. The women tattoo the forehead and temples. Those seen by Dalton were not clothed, but wore a girdle composed of several strings of beads from which depended scanty curtains of leaves. The men wear a small cotton loin cloth. They had no knowledge of metals or pottery. They cremate the dead, and place the body on the bier with the head to the south ; the ashes are thrown into a running stream. Their huts are low, and measure about 6 feet by 8; but the boys of the village occupy a common dormitory. Marriage takes place between adults, and widows may remarry. They are exogamous. They are semi-nomadic in their habits, cultivate the ground sparingly, and eat all kinds of flesh. Little is known of their religious creed, and they make sacrifices to the sun and earth. 11,171 persons were said in 1891 to speak the tribal language.

The Indian Museum contains two skulls from Keunjhar in the Orissa hills, stated in the MS. Catalogue to be those of Juangs. They were presented by Dr Stewart in 1868. The larger skull, No. 443, is that of a man. The smaller, No. 446, is that of a woman. The male skull in the *norma verticalis* was an elongated ovoid, sloping steeply from the sagittal suture to the parietal eminences, below which the side walls of the skull were almost vertical. The cephalic index was 73·2, and the skull was dolichocephalic in form and proportions. In both, the parietal longitudinal arc exceeded the frontal. The height in the male was greater than the breadth, and the vertical index was 79·3. The glabella and supra-orbital ridges were moderate, the forehead was not specially receding, the slope from the obelion was not precipitous, and the occipital squama above the inion was not prominent, but there was no evidence of parieto-occipital flattening. The fronto-nasal suture was shallow; the nasal bones were short, narrow, concave forwards, and only slightly projecting. The canine and incisor fossæ were not specially marked; the skull was barely cryptozygous, it rested behind on the mastoids. The occipital bone sloped steeply upwards from the foramen magnum to the inion. The muscular ridges and processes were moderate; the sutures were simple and often with two small Wormian bones in the lambdoidal suture. The parieto-sphenoid articulations were broad. The sockets of the teeth were broken, and there were no marked osseous irregularities.

The female skull was much smaller; it was more flattened on the vertex than the male. Proportionally it was not so elongated, and its cephalic index was 77·4. The height was a little less than the breadth, and the vertical index was 76·2. The forehead was more vertical, and the glabella and supra-orbital ridges were feeble; the occipital squama above the inion was more projecting, and below the inion it was not so steep as in the male skull. There was no evidence of parieto-occipital flattening. The nasal bones were larger than in the man, but the bridge of the nose had a similar curvature. The canine fossæ were more hollowed out, and the teeth were much worn down. The cranial sutures were in process of obliteration; small Wormian bones were present in the lambdoidal suture; the parieto-sphenoid articulation was moderately broad. The mastoids were very feeble. The skull was cryptozygous, and rested behind on the occipital condyles.

Both crania were orthognathous and platyrhine. The proportions of the orbit in the woman were microseme, and in the man megaseme. The cranial capacity of the woman was very low, 1030 c.c.; but in the man it reached 1420 c.c.

TABLE V.

Juangs, and various Tribes or Castes.

	Juang.		Koydwar. Nagooloo.	Bunjana.	Bhima.		Ahir-Goálá. Teetoo. Puttea.	Teli.		Kámár. Bhudny. Hazári-bágh.	Lohár. Ranchi.
								Pittoria Village.	Soromoo. Raipur.		
	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	E.U.A.M.	E.U.A.M.	I.M.
Collection number,	443	445	284	285	602	599	27	598	...	...	600
Age,	Ad.	Ad.	50	40	Ad.	Ad.	25	Ad.	23	Ad.	Ad.
Sex,	M.	F.	M.	M.	M.	F.	M.	M.	F.	F.	F.
Cubic capacity,	1420	1030	1267	1292	1270	1170	1328	1370	1005	1230	1240
Glabello-occipital length,	179	164	181	166	180	178	183	184	168	173	170
Basi-bregmatic height,	142	125	126	131	132	125	135	138	110	128	131
Vertical Index,	*79·3*	*76·2*	*69·6*	*78·9*	*73·3*	*70·2*	*73·8*	*75·*	*65·8*	*74·*	*77·1*
Minimum frontal diameter,	95	87	87	93	89	92	89	95	89	90	90
Stephanic,	109	110	112	112	102	103	102	109	104	102	107
Asterionic,	103	94	104	106	100	98	100	108	94	100	103
Greatest parieto - squamous breadth,	131p.	127s.	129s.	142s.	130s.	123p.	125p.	134s.	121	128	130s.
Cephalic Index,	*73·2*	*77·4*	*71·3*	*85·5*	*72·2*	*69·1*	*68·3*	*72·8*	*72·*	*74·*	*76·5*
Horizontal circumference,	500	465	498	495	494	491	502	508	474	481	473
Frontal longitudinal arc,	120	120	128	121	122	130	126	125	114	125	126
Parietal " "	135	125	120	120	126	130	137	{ 253	120	118	120
Occipital " "	115	96	110	103	113	96	107	...	105	105	106
Total " "	370	341	358	344	361	356	370	378	339	348	352
Vertical transverse arc,	310	283	290	305	296	285	292	292	269	281	288
Length of foramen magnum,	33	30	35	37	35	36	41	36	27	34	34
Basi-nasal length,	106	93	100	98	102	91	96	102	91	100	96
Basi-alveolar length,	103 ?	86	96	93	99	90	87	97	95	95	89
Gnathic Index,	*97·2*	*92·5*	*96·*	*94·9*	*97·1*	*98·9*	*90·6*	*95·1*	*104·4*	*95·*	*92·7*
Interzygomatic breadth,	126	121	119	131	131	122	124	134	119	123	123
Intermalar,	116	107	109	120	118	112	111	120	114	112	112
Nasio-mental length,	...	...	100	105	104	...	106	...	...	...	106
Nasio - mental complete facial Index,	...	...	*84·*	*80·*	...	...	...	...	...	...	60
Nasio-alveolar length,	...	60	60	63	64	65	63	67	58	59	...
Maxillary upper facial Index,	...	...	*50·4*	*48·*	*79·4*	...	*85·*	...	...	...	*86·1*
Nasal height,	47	44	48	49	50	46	47	47	42	45	46
Nasal width,	25	24	27	26	21	23	25	25	24	24	22
Nasal Index,	*53·2*	*54·5*	*56·3*	*53·1*	*42·*	*50·*	*53·2*	*53·2*	*57·1*	*53·3*	*47·8*
Orbital width,	39	38	38	38	36	39	39	39	36	36	37
Orbital height,	35	31	29	35	32	36	34	31	31	30	32
Orbital Index,	*89·7*	*81·6*	*76·3*	*92·1*	*88·9*	*92·3*	*87·2*	*79·5*	*86·1*	*83·3*	*86·5*
Palato-maxillary length,	...	47	52	50	53	54	52	53	54	52	49
Palato-maxillary breadth,	...	55	59	61	60	61	63	65	65	64	58
Palato-maxillary Index,	...	*117·*	*113·4*	*122·*	*113·2*	*112·9*	*121·1*	*122·6*	*120·*	*123·*	*118·3*
Lower jaw. Symphysial height,	...	...	28	27	22	...	30	...	...	...	28
Lower jaw. Coronoid "	...	...	57	60	65	...	61	...	...	...	51
Lower jaw. Condyloid "	...	...	58	63	60	...	56	...	...	...	48
Lower jaw. Gonio-symphysial length,	...	...	83	82	80	...	89	...	...	...	81
Lower jaw. Inter-gonial width,	...	...	95	88	94	...	101	...	...	...	92
Lower jaw. Breadth of ascending ramus,	...	...	27	28	33	...	30	...	...	...	34

Bhima. Table V.

Two skulls, Nos. 599, 602, presented to the Indian Museum by Mr W. H. P. Driver, are marked Bhima race. The former is apparently that of a woman, and the latter that of a man who died in Ranchi. I have had a difficulty in determining the tribe, caste, or race known as Bhima. I find, however, that Mr Robertson, in his *Report*, p. 183, speaks of Bhimas as vagrants who form a small sub-division of the Gonds; but it is possible that it may be a mis-spelling of Bhaina, a tribe living along the southern border of Chúta Nágpúr.

The general form of the skulls in the *norma verticalis* was an elongated oval with the sides of the cranium steep, the parietal eminences not very bulging. The sagittal region was not ridged, and the slope downwards to the parietal eminences was not very steep. The slope from the obelion to the occipital point was gradual; the occipital squama moderately projected. In both, the length-breadth index was dolichocephalic; in the male the parietal longitudinal arc exceeded the frontal; in the female they were equal. In each skull the basi-bregmatic diameter was greater than the parieto-squamous, and the vertical index was higher than the cephalic. The forehead did not much recede, and the glabella and supra-orbital ridges showed no special projection. The nasal bones had not much prominence, and the bridge was concave in the vertical direction; the nasal spine of the superior maxillæ was relatively small. In the male the anterior nares were narrow, and the index was leptorhine; in the woman it was mesorhine. In the man the upper jaw was orthognathous, in the woman mesognathous. In the man the orbital index was mesoseme, in the woman megaseme. In both the palato-maxillary index was mesuranic. The cubic capacity was microcephalic, 1270 and 1170 c.c. respectively.

Koydwar. Table V.

The Indian Museum contains the skull, No. 284, of a man named Nagooloo, 50 years old, from Bijji, Bastar State, Central Provinces. He is said to have been of short stature; skin black; hair black and soft; eyes dirty brown; a moustache; food rice, flesh, fish, vegetables. He is stated in the list sent to me to be of the Koydwar race. It is possible that this term may be a mis-spelling for Kotwári, a term applied to the caste which performs the service of village watchman.

The skull was elongated and ovoid in the *norma verticalis*; the sides were moderately steep, the sagittal region was not ridged, the parietal eminences were much in advance of the occipital point, and the occipital squama was rounded and prominent. The length-breadth index was 71·3, and the skull was dolichocephalic. The frontal longitudinal arc was the longest. The basi-bregmatic height was a little below the greatest breadth, and the vertical index, 69·6, was tapeinocephalic. The anterior nares were wide, and the nasal index, 56·3, was distinctly platyrhine. The upper jaw was orthognathic, the gnathic index being only 96. The orbits were low, and the index was 76·3. The palato-alveolar arch was mesuranic. The complete facial index, 84, was chamæprosopic. The teeth were much worn. The sutures of the cranial vault were nearly obliterated. The skull was cryptozygous. The cranial capacity was 1267 c.c.

Bunjana. Table V.

A skull in the Indian Museum, No. 285, from the Central Provinces from Koromankiai near Bastar, marked Bunjana, is probably that of a Banjára or Bunjára. It is that of a man æt. 40, 5 feet 3 inches high; he had skin dark brown; hair grey; eyes dirty brown; a moustache; food, rice, mutton, vegetables. The Bunjáras are a nomadic class, engaged in the occupation of carrying goods by pack-bullocks.

This skull did not possess an elongated oval form. When seen from the *norma verticalis* it was more rounded, and its greatest length was only 166 mm. The parieto-occipital region was flattened, and as it was not symmetrical, it is probable that artificial pressure had been applied during infancy. The length-breadth index was 85·5 and the skull was hyper-brachycephalic. The frontal longitudinal arc was 1 mm. longer than the parietal. The basi-bregmatic height was much less than the greatest breadth, and the vertical index was 78·9. The anterior nares were wide, and the index was platyrhine. The upper jaw was orthognathous. The height and width of the orbits were almost equal, and the index was megaseme. The palato-maxillary index was brachyuranic, and the palate had a wide horse-shoe shape. The face was chamæprosopic, and the complete facial index was only 80. The teeth were much worn and stained with betel. The cranial sutures were distinct; small Wormian bones were present in the lambdoidal suture; the pterion was normal. The skull was cryptozygous. The cranial capacity was 1292 c.c.

Kámár and Lohár. Table V.

These names are applied to castes who manufacture articles in metal. The Kámárs work in metals generally; the Lohárs are the blacksmiths or workers in iron. The Kámárs are found in Bengal and Behar;* the Lohárs in Western Bengal, Behar, and Chúta Nágpúr. Mr Risley considers that these caste names express only a similarity in occupations, and do not indicate uniformity in race. He also states that the lohár or blacksmith is a recognised official in a Kol village community. Each caste is probably composed of persons belonging to different tribes, some of which are probably indigenous to the locality, whilst others have migrated into the district in which they live, so that they may include Aryans, Aborigines, and crosses between Aryan and non-

* Mr Robertson, in his *Report* on the Census in the Central Provinces, p. 190, states that in Raipur a tribe of people named Kámár live in remote jungles on fruits and small game, and although in some provinces, as Bengal, the term is an occupational one, it includes both aborigines and non-aboriginal people.

Aryan people. He supports this view by citing the prevalence of different social customs as well as religious differences. Some are orthodox Hindus, others worship gods not included in the Hindu mythology. As regards marriage, both infant and adult marriage prevail ; widow marriage is allowed by some, but forbidden by others. Some groups permit marriage within the group, whilst others are exogamous. Cremation is practised by the Kámárs.

I have examined the skull of a Kámár named Bhudny, from Hazáribágh, said to be a woman, presented to me by Dr J. J. Hedley Wood ; also that of a Lohár, who died at Ranchi, No. 600 in the India Museum.

The Kámár skull, ovoid in its general form, was long in relation to the breadth ; its sides were vertical, but it was not so roof-shaped as in some of the dolichocephali ; the length-breadth index was 74, and the frontal longitudinal arc was the longest. The basi-bregmatic corresponded with the greatest parieto-squamous diameter. The projection of the glabella and supra-orbital ridges gave one the impression of a male rather than a female cranium, but the forehead receded very slightly, and the vertex was inclined to be flattened ; the parieto-occipital region sloped gently into a rounded occipital squama. The nasion was a little depressed ; the bridge of the nose was concave, but projected at the tip ; the nasal spine of the superior maxillæ was moderate, and a low ridge separated the floor of the nose from the incisive region. The anterior nares were large and platyrhine, the upper jaw was orthognathous ; the orbits were mesoseme, and the palate was brachyuranic. There was no lower jaw. The teeth were only slightly worn, though some were carious ; the canine and incisive fossæ were deep. The sutures were unossified ; from their condition and that of the teeth the age was probably about 30. There were no Wormian bones, but a large epipteric was in each pterion ; with this exception no osseous irregularities were observed. The cranial capacity was 1230 c.c.

The Lohár skull was probably that of a female. Its breadth bore to the length a proportion which placed the cranium in the lower term, 76·5, of the mesaticephalic group, and the greatest breadth was about the squamous suture ; the frontal longitudinal arc was the longest. The height was somewhat greater than the breadth, and the vertical index was 77·1. The left parieto-occipital region was a little flattened. The nasal bones had but little projection, and the bridge was concave vertically ; the nasal spine of the superior maxillæ was small. The nose was relatively narrow and with a leptorhine index ; the upper jaw was orthognathous, the orbit was mesoseme, and the palate was brachyuranic. The face was chamæprosopic. The cranial capacity, 1240 c.c., was microcephalic.

Ahír-Goálá. Table V.

The Goálás or Gopas are the pastoral caste of India, extensively diffused in the North-West Provinces, the valley of the Ganges, Behar, Orissa, and Chúta Nágpúr. The name Ahír is applied to the whole caste in North-Western India ; but in the south and east it is apparently restricted to one of its divisions, the entire caste being named Goálá. The Ahír or Goálá, whose duty it is to look after the cattle, is, according to Mr RISLEY, one of the recognised officials of a Kol village community. Colonel DALTON groups the Ahírs as Aryans, but in the mountainous districts of Orissa and Chúta Nágpúr, they seem to have had incorporated with them a proportion of the aboriginal inhabitants, who have become Hinduised. In consequence of this intermixture, the physical characters of the caste vary in different localities. DALTON states that the Mathurábásis have high, sharp and delicate features, and light brown skins quite of the Aryan type; whilst the Magadhas have coarse features, the skin is dark in colour, the hands and feet are large, and the difference between them and the Kol-speaking people of Singbhúm is not distinguishable. The intermixture also affects the customs of the caste. Marriage usually takes place in infancy, though in Chúta Nágpúr adult marriage is permitted, and in the hill districts the marriage of widows is sanctioned. RISLEY states that in Chúta Nágpúr a man may not marry a woman of his own totem. Cremation is practised on the dead bodies of married persons, but not on those of children. In religion they are Hindus, and observe the usual festivals. The Ahírs and Goálás together numbered, in 1891, about eleven and a half millions of people.

In the Indian Museum is a skull, No. 27, marked Ahír, Goálá caste, which was presented in 1863 by Lieut.-Col. DALTON; the man, Teetoo, from Puttea, was hanged in Ranchi jail. He is said to have been 25 years old, 5 feet 2 inches high; hair black, long, coarse; eyes black, set straight in the face; food, rice, vegetables, and flesh.

The cranium, seen in the *norma verticalis*, was a very elongated ovoid, the sides vertical, with a slight sagittal ridge, and a slope outwards to the parietal eminences. The length-breadth index was 68·3, and the skull was hyper-dolichocephalic; the parietal longitudinal arc was much longer than either the frontal or occipital; the basi-bregmatic height exceeded considerably the breadth, and the vertical index was 73·8. The glabella and supra-orbital ridges were moderate; the forehead was somewhat retreating; the parieto-occipital region sloped gently backwards, and was flattened from side to side; the occipital squama was not prominent, and projected very little behind the inion. The nasion was slightly depressed; the bridge of the nose was not prominent, and was concave from above downwards. The nasal spine of the superior maxillæ was distinct, and a sharp ridge separated the floor of the nose from the incisive region. The nasal index was 53·2, *i.e.*, platyrhine; the gnathic index, 90·6, markedly orthognathous; the orbital index, 87·2, was mesoseme; the palato-maxillary index, 121·1, was brachyuranic; the complete facial index was 85, so that the face was chamæprosopic. The teeth were all erupted and a little worn; the incisive fossæ in the upper jaw were deep, and the canine fossæ were well marked. The cranial sutures were simple, and showed signs of commencing ossification. No Wormian bones were present, but a large epipteric bone was seen in each pterion. The jugal processes were tuberculated. The lower jaw was well developed. The skull was phænozygous and rested behind on the mastoid-temporals. The cubic capacity of the cranium, 1328 c.c., placed it in the microcephalic group.

Teli. Table V.

The Teli or Tili is a banking, trading, and oil-pressing caste in Bengal, Behar, and Orissa. Some are Hindus, others Mahommedans in religion. In Bengal, amongst the richer classes, they permit infant marriage and forbid the marriages of widows. In Orissa, again, they adhere more to aboriginal customs; they hold, says Mr Risley, totems in reverence. Infant marriage is not essential, and widow marriage is allowed. They cremate the dead. They number from 4,000,000 to 5,000,000 of people.

Two crania of this caste have come under my observation; one, No. 598 in the Indian Museum, a male from the village Pittoria, near Ranchi, Chúta Nágpúr; the other a female, presented to me by Dr Hedley Wood, from Raipur in the Central Provinces. The general form in the *norma verticalis* was the elongated ovoid so frequently referred to in the preceding descriptions of the dolichocephalic crania of the aborigines; this form being associated with vertical sides and a rounded occipital squama. The length-breadth index in the man was 72·8, and the basi-bregmatic diameter exceeded the parieto-squamous; in the woman the index was 72; the basi-bregmatic height was much below the parieto-squamous diameter, and the parietal longitudinal arc was longer than the frontal. The forehead was not receding; the glabella and supra-orbital ridges were not prominent. The nasal bones were not projecting, and the bridge was flattened; the nasal spine of the superior maxillæ was moderate; a ridge marked off the floor of the nose from the incisive region of the upper jaw; the anterior nares were wide, and the index in each specimen was platyrhine. In the Teli man, the upper jaw was orthognathous, in the woman prognathous. In the man the orbital index was microseme, in the woman mesoseme. In both, the palato-maxillary index was just within the brachyuranic group. In the woman's skull there were no osseous irregularities. The cranial capacity in the man was 1370 c.c.; in the woman it was only 1005 c.c.

Uriyá.

In addition to the crania described in the preceding part of this memoir, which are definitely associated with particular races, tribes, or castes, the Indian Museum contains a number of skulls from Orissa, marked in the catalogue Ooriá or Uriyá. Uriyá is a linguistic term, which expresses a particular derivative of Sanskrit. It is the mother tongue of a very large percentage, said to be 95·1 per cent., of the Hindu population of Orissa, of those who inhabit the plains as distinguished from the aborigines who live in the mountains, and the name of the language is given to the people who speak it. As the aborigines of this province speak either Dravidian or Kolarian, the Uriyá tongue of the Hindu population in Orissa contains a mixture of archaic forms and words derived from those languages. Uriyá-speaking people form a considerable proportion of the class of domestic servants in the north-east of India, which probably accounts for the number of crania in the Indian Museum marked Uriyá, most of which had been obtained from the medical school of Calcutta.

I have examined thirty skulls from the Indian Museum, marked Uriyá in the list sent to me, and in addition I have received from my friend Major Bannerman, M.D., two specimens which he had collected at Baghmari village in Orissa.

The crania were by no means a homogeneous series, but varied materially in form and proportions, so that it would be impossible to draw up a description which would be generally applicable. If we take the proportion of length and breadth to guide us in our examination, we shall find that the crania can readily be arranged in three groups. The larger number, seventeen in all, have the length-breadth index below 75, and in form and proportions are dolichocephalic; in ten skulls the corresponding index is between 75 and 80, mesaticephalic; whilst in five crania this index was upwards of 80, brachycephalic.

Dolichocephalic Series.—Of the seventeen crania belonging to this group, fifteen were apparently males and two females. They were all adults, with perhaps two exceptions about 20 and 21 years of age. When examined in the *norma verticalis*, they were seen to be elongated and ovoid in outline, with side walls approaching the vertical and with no great difference between the frontal and parietal transverse diameters. The parietal eminences were fairly marked. As a rule, the sagittal line was not raised above the general plane of the vertex, and the slope from it to the parietal eminence was moderate. In the majority the parieto-occipital region sloped gently backwards and downwards, but in four specimens it was inclined more abruptly, and in three of these it showed a want of symmetry, as if modified by artificial pressure. In No. 232 this character was most distinct, and in it was also seen a transverse post-coronal depression, as if from wearing a tight band during infancy. In No. 42, the elongated form was exaggerated and the skull was hyper-dolichocephalic; the sagittal suture was unossified, but the right parieto-mastoid and adjoining parieto-squamous were closed. The mean cephalic index of the series was 72·2. The male skulls in the greatest length ranged from 171 to 194 mm., but the majority were between 180 and 187 mm. In the greatest breadth they ranged from 124 to 139 mm., but the majority were between 127 and 134 mm. In no specimen was the occipital arc the longest; in several, the frontal and parietal longitudinal arcs were equal or almost equal; in a few, the frontal materially exceeded the parietal, in others the proportion was reversed. The mean vertical index of the series was 75·4, and in only three crania was the basibregmatic height less than the greatest breadth. (Table VI.)

The glabella and supra-orbital ridges had, as a rule, but little prominence, though well marked in the man from Baghmari village. In the men the forehead was slightly receding, but in the women it was almost vertical. The nasion was only slightly depressed; as a rule, the bridge of the nose projected forwards, but in a few it was not prominent. The nasal spine of the superior maxillæ was distinct as a rule, and the floor of the nose was separated from the incisive region of the maxilla by a sharp ridge.

The nasal index in sixteen skulls ranged from 45·8 to 56, and the mean was 51·6, *i.e.*, mesorhine, to which group eight specimens belonged; of the remainder, two were leptorhine, and six were platyrhine. The projection of the upper jaw was orthognathous, the mean gnathic index of fifteen crania being 96·2; no specimen was prognathous, and only four were mesognathous. The orbits were measured in sixteen crania, and the mean index was 85·6, mesoseme, to which group eight specimens belonged; five specimens were microseme and only three were megaseme. The palato-maxillary index showed a great range of variation, and indicated marked differences in the relative length and breadth of the palate and alveolar arch; five specimens were dolichuranic, six were mesuranic, six were brachyuranic; in several specimens the palate had a high arch. The nasio-mental diameter was taken in only seven skulls, in five of which the proportion between that diameter and the interzygomatic breadth was chamæprosopic, in the remaining two, leptoprosopic.

The cranial sutures were simple and, as a rule, not ossified. In ten skulls the lambdoidal suture contained Wormian bones, and in one of these they were numerous. In seven crania an epipteric bone or bones was present either on one or both sides, but in none did the squamous-temporal and frontal directly articulate. No skull was metopic. In No. 414 the basi-cranial synchondrosis was not ossified, and the upper wisdom teeth were not erupted; in the right orbit a slender process of the orbital plate of the superior maxilla ascended between the os planum and the lachrymal to articulate with the frontal. I have previously recorded examples of this variation in human crania in Bush, Papuan, and Lushai skulls.* Several specimens retained the infra-orbital suture. The muscular ridges and processes were not strongly marked. The skulls were cryptozygous. No specimen showed a paramastoid process, third condyl or auditory exostosis. In three crania the wisdom teeth had not appeared. The mean cranial capacity of fifteen male skulls was 1370 c.c., mesocephalic; and the range was from 1138 c.c. to 1660 c.c. The mean capacity of two female skulls was 1370 c.c.

Mesaticephalic Series.—Of the ten crania which belonged to this series, seven were apparently males and three females. They were all adult except No. 20, in which, though the wisdom teeth were erupted, the basi-cranial synchondrosis was not ossified.

Of these specimens, seven had a cephalic index between 75 and 77·5, whilst three ranged from 77·6 to 79·6. Those with the lower indices showed no great difference in the general form of the cranium from the dolichocephalic group, whilst those in the higher series approximated to the brachycephalic, to be next described. (Table VII.)

Two skulls were so steep and vertical in the parieto-occipital region as to give the impression that they had been artificially flattened. In four skulls the basi-bregmatic height was less than the greatest breadth; in three it was greater; in three they were equal. In all, the occipital longitudinal arc was less than either the parietal or frontal; in four the frontal exceeded the parietal; in four the opposite condition existed.

The glabella and supra-orbital ridges were moderate, but in No. 130 they were

* *Trans. Roy. Soc.*, Edinburgh, 1899, vol. xxxix. p. 712.

Table VI.

Uriyá.—Dolichocephali.

	Gopaul. Cuttack. Orissa.	Hurmah. Orissa.	Bhurruth. Orissa.	Hindu. Uriyá.	Bhoobun. Hindu. Uriyá.	Bhurruth. Uriyá.	Panoo. Uriyá.	Bipoo. Orissa.	Dudeah. Orissa.	Uriyá. Orissa.	Uriyá. Orissa.	Uriyá.	Naran. Hindu. Uriyá.	Bozeunto. Balasore. Orissa.	Baghmari. Orissa.	Uriyá. Orissa.	Baghmari. Orissa.
	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	I.M.	E.U.A.M.	I.M.	E.U.A.M.
Collection number,	2	149	54	20	23	32	33	179	232	409	412	419	24	42	...	414	...
Age,	28	23	45	27	23	38	24	30	18	Ad.	Ad.	Ad.	65	20	Ad.	Ab. 21	Ad.
Sex,	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M.	F.	F.
Cubic capacity,	1345	1455	1308	1448	1350	1305	1232	1300	1348	1660	1485	1138	1334	1406	1440	1396	1130
Glabello-occipital length,	181	180	174	187	177	180	176	175	177	194	180	171	182	186	184	182	168
Basi-bregmatic height,	143	146	130	140	130	126	137	138	134	138	146	129	135	136	135	138	122
Vertical Index,	*79·0*	*81·1*	*74·7*	*74·9*	*73·4*	*70·0*	*77·8*	*78·9*	*75·7*	*71·1*	*81·1*	*75·4*	*74·2*	*73·1*	*73·4*	*75·8*	*72·6*
Minimum frontal diameter,	92	95	90	95	91	89	99	96	92	97	91	95	101	91	99	92	92
Stephanic,	106	114	112	109	109	113	114	109	108	113	112	107	113	106	105	109	102
Asterionic,	100	100	100	109	109	99	97	103	104	108	108	98	100	98	106	96	95
Greatest parieto-squamous breadth,	127p.	133s.	127s.	134s.	129p.	133p.	127s.	130p.	128p.	139s.	132p.	127s.	129s.	124p.	134s.	128p.	125p.
Cephalic Index,	*70·2*	*73·9*	*73·0*	*71·7*	*72·9*	*73·9*	*72·2*	*74·3*	*72·3*	*71·6*	*73·3*	*74·3*	*70·9*	*66·7*	*72·8*	*70·3*	*74·4*
Horizontal circumference,	495	501	486	516	488	499	491	488	488	538	507	485	505	502	510	500	464
Frontal longitudinal arc,	130	130	130	137	129	134	137	124	130	130	147	128	128	138	137	135	128
Parietal ,, ,,	130	248	130	139	128	125	124	130	138	130	124	126	131	142	134	134	116
Occipital ,, ,,	110		99	116	113	114	107	105	99	127	110	99	109	112	109	103	101
Total ,, ,,	370	378	359	392	370	373	368	359	367	387	381	353	368	392	380	372	345
Vertical transverse arc,	310	314	298	309	302	303	308	300	302	320	322	288	302	302	306	309	279
Length of foramen magnum,	35	35	32	33	34	32	33	34	31	36	34	29	33	35	36	35	31
Basi-nasal length,	106	104	102	101	94	95	102	102	101	111	107	98	102	99	101	102	93
Basi-alveolar length,	95	96	100	98	94	94	98	97	96	111	100	95	...	96	98	97	93
Gnathic Index,	*89·6*	*92·3*	...	*97*	*100*	*98·9*	*96·1*	*95·1*	*95·*	*100·*	*93·5*	*96·9*	...	*97·*	*97·*	*95·1*	*100·*
Interzygomatic breadth,	126	133	124	127	114	123	126	128	130	133	125	123	125	123	131	118	122
Intermalar ,,	114	116	112	116	103	111	116	120	118	121	112	113	115	111	120	107	113
Nasio-mental length,	124	119	...	...	...	...	...	114	105	...	...	...	...	108	122	...	104
Nasio-alveolar,	73	66	62	66	62	64	65	66	60	72	67	62	...	64	69	59	56
Complete Facial Index,	*98·4*	*89·4*	...	...	...	...	...	*89·*	*80·*	...	...	...	...	*87·8*	*93·1*	...	83·6
Nasal height,	51	46	46	46	47	49	48	48	47	51	51	46	53	45	50	46	43
Nasal width,	24	25	26	25	23	24	25	22	23	28	26	24	27	23	28	25	24
Nasal Index,	*47·1*	*54·3*	...	*54·3*	*48·9*	*49*	*52·1*	*45·8*	*48·9*	*54·8*	*51·*	*52·2*	*50·9*	*51·1*	*56·*	*54·3*	*55·8*
Orbital width,	37	36	35	38	36	38	38	37	37	40	39	39	39	38	40	37	35
Orbital height,	36	33	29	32	31	31	29	32	32	33	32	32	35	33	34	32	30
Orbital Index,	*97·3*	*91·7*	...	*84·2*	*86·1*	*81·6*	*76·3*	*86·5*	*86·5*	*82·5*	*82·*	*82·*	*89·7*	*6·8*	*85·*	*86·5*	*85·7*
Palato-maxillary length,	53	54	55	57	54	53	55	56	52	63	54	55	51	3	52	52	50
Palato-maxillary breadth,	65	65	62	59	58	57	63	63	58	71	57	68	60	56	70	63	60
Palato-maxillary Index,	*120·7*	*120·3*	*112·7*	*103·5*	*107·4*	*107·5*	*114·5*	*112·5*	*111·5*	*112·7*	*105·5*	*123·6*	*117·6*	*105·6*	*134·6*	*121·*	*120·*
Symphysial height,	33	35	...	33	28	28	27	30	29	36	31	29	27	30	37	25	30
Coronoid ,,	69	60	...	70	56	67	54	59	57	62	58	61	66	67	60	64	69
Condyloid ,,	63	60	...	67	57	60	54	63	52	65	64	57	59		59	59	69
Gonio-symphysial length,	92	85	...	89	79	90	87	92	84	87	92	82	82	87	86	84	87
Inter-gonial width, outside,	105	100	...	...	...	...	...	98	92	...	...	...	...	96	98	...	93
Breadth of ascending ramus,	33	32	...	38	29	40	33	35	29	34	29	34	29	34	35	33	33

strongly marked. The forehead only slightly receded. The nasal bones were prominent and with usually a good bridge, but in No. 65 the bridge was flattened. The nasal spine of the superior maxillæ was moderate; in some specimens the nasal floor was separated from the incisive region by a ridge; in others, as in No. 65, they rounded off into each other. In seven specimens the nasal index was mesorhine; in one, leptorhine; in two, platyrhine. In six crania the upper jaw was orthognathous; in two, mesognathous, and in one, No. 65, prognathous. In six the orbital index was mesoseme; in three, microseme; in one, megaseme. As regards the relative length and breadth of the palato-alveolar arch, five specimens were mesuranic, one was dolichuranic, two were brachyuranic. The four crania in which the length of the entire face could be taken, were practically leptoprosopic or high faced.

The cranial sutures, as a rule, were simple; in four skulls, Wormian bones were present in the lambdoidal suture, and in one of these, No. 65, the right half of the upper occipital was an independent bone; in No. 415, two sutural bones were in the sagittal behind the obelion. In one skull on both sides, and in another on the left side, the squamous temporal articulated with the frontal; in three crania, epipteric bones were present, in two of these on both sides, in one on one side. Three skulls showed the infra-orbital suture. One skull, No. 98, was edentulous; in one, the teeth were stained with betel. No skull was metopic, or possessed a third condyl, paramastoid process or auditory exostosis. They were cryptozygous, and mostly rested behind on the occipital bone. The muscular ridges and processes were not strong. The mean cranial capacity in the men was 1336 c.c., and ranged from 1212 to 1530 c.c.; in the three women, the mean capacity was 1176 c.c.

Brachycephalic Series.—Five of the crania marked Uriyá were brachycephalic in form and proportions. Three were apparently males and two females. (Table VIII.)

In the *norma verticalis* the crania were rounded, and the male skulls, with one exception, had a less glabello-occipital length than the shortest male skull in the dolichocephalic group; whilst the female skulls were shorter than the female dolichocephalic Uriyás. The sagittal region was not ridged, and the crania generally were more flattened at the vertex than in the dolichocephali. The parietal eminences were prominent, especially in No. 38, and in the *norma occipitalis* the skulls had a pentagonal form. In four crania there was evidence of parieto-occipital flattening, more particularly in the hyper-brachycephalic skull, No. 417, in which the parieto-occipital region was almost vertical; the pressure had produced in two skulls an unsymmetrical projection to the right, and in two others to the left. In No. 38 the occipital region was rounded, and projected behind the inion. The cephalic index ranged from 80 to 88·2, and the mean was 83·7. In all, the occipital longitudinal arc was the shortest; in three, the frontal arc was longer than the parietal; in two, the parietal was the longer. In all, the basi-bregmatic diameter was less than the parieto-squamous, and the mean vertical index was 79·2.

The glabella and supra-orbital ridges were feeble; the forehead was almost vertical;

Table VII.

Uriyá.—Mesaticephali.

	Matu. Hindu. Orissa.	Bho-blanee. Hindu. Orissa.	Gally. Orissa.	Bassu. Orissa.	Orissa.	Orissa.	Orissa.	Orissa.	Orissa.	Orissa.
Collection number,	I.M. 65	I.M. 76	I.M. 98	I.M. 130	I.M. 199	I.M. 413	I.M. 415	I.M. 410	I.M. 416	I.M. 418
Age,	20	30	70	50	38	Ad.	Ad.	Ad.	Ad.	Ad.
Sex,	M.	M.	M.	M.	M.	M.	M.	F.	F.	F. (?)
Cubic capacity,	1260	1212	1205	1530	1408	1336	1405	1270	1110	1150
Glabello-occipital length,	174	169	167	185	175	173	179	170	168	168
Basi-bregmatic height,	132	128	130	150	132	134	128	136	128	132
Vertical Index,	*75·9*	*75·7*	*77·8*	*81·1*	*75·4*	*77·5*	*71·5*	*80·*	*76·2*	*78·6*
Minimum frontal diameter,	94	95	88	100	98	94	90	91	91	88
Stephanic,	114	115	104	120	115	108	111	109	108	103
Asterionic,	103	95	102	115	98	104	105	101	90	102
Greatest parieto-squamous breadth,	137p.	128p.	133p.	141s.	135p.	134s.	135p.	132p.	128p.	128p.
Cephalic Index,	*78·7*	*75·7*	*79·6*	*76·2*	*77·1*	*77·5*	*75·5*	*77·6*	*76·2*	*76·2*
Horizontal circumference,	494	480	475	518	493	495	500	480	472	470
Frontal longitudinal arc,	120	122	130	138	131	124	130	124	124	119
Parietal ,, ,,	243	236	115	125	134	118	134	131	118	120
Occipital ,, ,,			111	114	100	112	112	107	99	109
Total ,, ,,	363	358	356	377	365	354	376	362	341	348
Vertical transverse arc,	312	285	309	319	315	298	308	303	293	295
Length of foramen magnum,	29	31	31	33	37	33	34	35	31	30
Basi-nasal length,	97	96	93	106	97	99	93	98	100	99
Basi-alveolar length,	100	97	...	95	96	95	88	94	91	95
Gnathic Index,	*103·1*	*101·*	...	*89·6*	*99·*	*96·*	*94·6*	*95·9*	*91·*	*96·*
Interzygomatic breadth,	124	117	118	133	126	124	123	117	116	118
Intermalar ,,	116	110	106	120	117	116	116	107	104	110
Nasio-mental length,	109	116	...	120	120	...	...	...	...	...
Nasio-alveolar ,,	61	68	...	69	66	68	60	63	61	63
Complete Facial Index,	*89·5*	*99·*	...	*90·*	*95·*	...	...	...	...	...
Nasal height,	46	50	43	52	50	49	46	46	46	47
Nasal width,	23	24	20	25	25	26	23	23	23	27
Nasal Index,	*50·*	*48·*	*46·5*	*48·2*	*50·*	*53·1*	*50·*	*50·*	*50·*	*57·4*
Orbital width,	38	37	36	39	39	37	37	36	38	38
Orbital height,	31	30	32	35	35	33	34	30	34	32
Orbital Index,	*81·6*	*81·1*	*88·9*	*89·7*	*89·7*	*89·2*	*91·9*	*83·3*	*89·5*	*84·2*
Palato-maxillary length,	59	53	...	52	56	54	51	53	49	52
Palato-maxillary breadth,	63	61	...	64	64	63	60	59	57	63
Palato-maxillary Index,	*106·7*	*115·*	...	*123·*	*114·3*	*116·6*	*117·6*	*111·3*	*116·3*	*121·*
Lower jaw. Symphysial height,	28	31	...	28	36	29	33	28	25	29
Lower jaw. Coronoid ,,	56	63	57	61	63	62	60	59	58	61
Lower jaw. Condyloid ,,	58	66	58	66	63	59	56	48	52	54
Lower jaw. Gonio-symphysial length,	90	83	72	97	87	80	87	85	76	81
Lower jaw. Inter-gonial width, outside,	92	91	...	105	90	...	...	...	...	...
Lower jaw. Breadth of ascending ramus,	42	33	27	35	30	35	35	30	31	33

Table VIII.

Uriyá.—Brachycephali.

	Hindu. Orissa.	Siplo. Hindu. Orissa.	Orissa.	Orissa.	Puttonez. Hindu. Cuttack, Orissa.			
Collection number,	I.M. 4	I.M. 129	I.M. 411	I.M. 417	I.M. 38			
Age,	20	32	Ad.	...	40			
Sex,	M.	M.	M.	F.	F.			
Cubic capacity,	...	1148	1200	1118	1240			
Glabello-occipital length,	173	161	163	152	167			
Basi-bregmatic height,	139	128	126	126	127			
Vertical Index,	*80·3*	*79·5*	*77·3*	*82·9*	*76·*			
Minimum frontal diameter,	82	90	88	88	83			
Stephanic,	116	112	109	106	104			
Asterionic,	106	97	99	93	97			
Greatest parieto-squamous breadth,	140p.	138p.	135p.	134p.	135p.			
Cephalic Index,	*80·9*	*85·7*	*82·8*	*88·2*	*80·8*			
Horizontal circumference,	488	478	473	452	466			
Frontal longitudinal arc,	130	124	131	117	118			
Parietal ,, ,,	128	117	120	119	125			
Occipital ,, ,,	105	100	99	92	112			
Total ,, ,,	363	341	350	328	355			
Vertical transverse arc,	314	302	307	300	290			
Length of foramen magnum,	37	29	32	34	33			
Basi-nasal length,	101	96	94	88	87			
Basi-alveolar length,	95	96	96	86	87			
Gnathic Index,	*94·1*	*100·*	*102·1*	*97·7*	*100·*			
Interzygomatic breadth,	123	122	120	110	115			
Intermalar ,,	109	112	109	100	102			
Nasio-mental length,	109	96	...	...	96			
Nasio-alveolar ,,	62	60	64	56	58			
Complete Facial Index,	*88·6*	*78·6*	...	...	*83·4*			
Nasal height,	48	48	45	43	43			
Nasal width,	24	24	24	19	22			
Nasal Index,	*50·*	*50·*	*53·3*	*44·2*	*51·2*			
Orbital width,	36	37	33	34	36			
Orbital height,	31	30	29	31	30			
Orbital Index,	*86·1*	*81·1*	*87·9*	*91·2*	*83·3*			
Palato-maxillary length,	50	56	58	49	48			
Palato-maxillary breadth,	61	61	60	56	58			
Palato-maxillary Index,	*122·*	*109·*	*104·4*	*114·3*	*120·8*			
Lower jaw. Symphysial height,	28	25	33	30	24			
Lower jaw. Coronoid ,,	53	60	58	53	60			
Lower jaw. Condyloid ,,	51	59	56	54	53			
Lower jaw. Gonio-symphysial length,	86	90	81	73	78			
Lower jaw. Inter-gonial width, outside,	95	90	...	...	79			
Lower jaw. Breadth of ascending ramus,	35	35	34	28	28			

the nasion was not depressed; the bridge of the nose was not very prominent; the nasal spine of the superior maxillæ was moderate; the floor of the nose in some specimens was separated from the incisive region by a sharp ridge. The mean nasal index was 49·7, mesorhine, to which group three specimens belonged: one was leptorhine, one platyrhine. The mean gnathic index was 98·7, mesognathous, to which group three specimens belonged, but two were orthognathous. The mean orbital index was 85·9, mesoseme, to which group two skulls belonged; one was megaseme; two were microseme. The relative length and breadth of the palato-alveolar arch showed great variation: two skulls were dolichuranic; one mesuranic; two brachyuranic. In all the face was chamæprosopic.

No skull was metopic. The cranial sutures were simple. In two specimens the lambdoidal suture contained Wormian bones; in one there was a right epipteric bone; in two the infra-orbital suture was present. The crania were cryptozygous. The mean cranial capacity of two males was 1174 c.c., and of two females 1179 c.c.; each skull was microcephalic.

Comparison of Aboriginal Crania.

Before proceeding to consider the relations, as regards race, which the Dravidian and Kolarian-speaking tribes bear to each other, it will be advisable to examine the evidence of the possible presence in India of a people more ancient even than the present wild tribes of the hill districts. From time to time objects have been found, which, from the material of their construction and the simplicity of the workmanship, would point to the existence in India of people who manufactured and employed tools and implements of stone.

In 1842 Dr W. H. Primrose found at Lingsoo-goor,* near a tumulus on which the mess-house of the Hyderabad contingent was built, knives and arrow heads made of cornelian, jasper, agate, and chalcedony.

In 1863 Mr R. Bruce Foote discovered in the Madras Presidency, *in situ*, in beds of a red ferruginous clay mingled with sand and gravel, and at an elevation of 300 feet above the sea, chipped implements formed of quartzite.† Stone implements have also been obtained by other collectors in Orissa, Mirzapore, Jubbulpoor, and the South Mahratta country. Although formed of quartzite and not of flint, Sir John Evans‡ considers that, as far as general form is concerned, they are identical with the implements from European river-drifts, and he regards them as belonging to palæolithic times. Mr F. Swynnerton states§ that quartzite implements of palæolithic type have been found on the surface of the ground at Raipur.

Sir John Evans has recorded a worked arrow head from India in the possession of Professor Buckman which belonged to the late Stone age. A number of arrow heads, with

* Meadows Taylor in *Journ. Ethno. Soc.*, London, N.S., vol. i. p. 175, 1869.

† *Geological Magazine*, vol. xi. p. 503.

‡ *Ancient Stone Implements*, 2nd ed., p. 651, London, 1897.

§ *Journ. Anthro. Inst.*, 1899, vol. ii. p. 141.

stone beads, a celt, a perforated stone and other objects, formed of chert, chalcedony, rock crystal, and quartz have been found by Mr W. H. P. DRIVER at Ranchi in Chúta Nágpúr. They have been described and figured by Professor J. WOOD-MASON.* The place where they were found had obviously been a neolithic settlement. Mr SWYNNERTON has described roughly chipped fragments of jasper and chert in the gravel of the Sourrka river, from the alluvium of the plain in which the city of Gwalior is built.

We can scarcely expect to trace a direct continuity between the present aborigines and those prehistoric men who manufactured the primitive palæolithic implements. It is, however, worthy of consideration if some of the existing hill tribes may not be the descendants of the people of neolithic times.

Of the hill tribes referred to in the earlier pages of this memoir the Juangs are without doubt the most primitive. Colonel DALTON speaks decidedly on this point, and regards them as representatives of the Stone age. Until strangers came amongst them, they had no knowledge of metals, they had no word in their language to designate iron or other metals, and they employed implements made of stone. They could neither spin nor weave, nor had they the simplest knowledge of pottery. They wore no clothes but leaves, and were remarkably shy and timid. Although their language is in part Kolarian, like that of the Hos and Santals, they have many words which cannot be connected with the languages now spoken by other people in India, and the people themselves claim to be the autochthones in Keunjhar.

Like other primitive people they are of low stature; they have thick lips and, according to DALTON, coarse frizzly hair, though the two girls drawn from photographs in his great work do not support this statement, as the hair is long and wavy. The colour of the skin is not black, but reddish brown.

In an account which Dr SHORTT has given† of the Juangs, Juags, or leaf wearers of Orissa, met with by him in the tributary Mahals of Cuttach, he states that the head is well formed and globular, the forehead expanded, the cheek bones high, nasal ridge depressed and wide, lips fleshy, chin pointed, face triangular or wedge-shaped; eyes large and expressive, a character which scarcely conforms to the Mongolian type of countenance which he ascribes to the Juangs. The hair is copious and long on the head, moustache and beard scanty. He attaches importance to the large proportion of persons in whom the lower jaw is 'underhung.' The average stature of the men is 5 feet $1\frac{1}{2}$ inches, of the women 5 feet.‡

If the two skulls in the Indian Museum which I have measured are genuine specimens of the Juang race, it will be seen that whilst the male is dolichocephalic, the index

* *Journal Asiatic Soc. Bengal*, vol. lvii. part xi., 1888.

† *Journ. of Anthropo. Soc.*, p. cxxxvi. in *Anthropological Review*, vol. iii., 1865.

‡ M. J. WALHOUSE has described, *Journ. Anth. Inst.*, 1875, vol. iv. p. 369, a leaf wearing tribe, named Korăgar, in South Canara, on the western coast of India. The leaves are worn by the women, a survival, apparently, of a habit prior to the use of raiment, but outside the clothes. The people are black skinned, thick lipped, nose broad and flat, hair rough and bushy. The men, he says, seldom exceed in stature 5 feet 6 inches, but this is probably too high an estimate of their stature.

of the female is about the middle of the mesaticephalic group; both were orthognathous and platyrhine. The breadth in the malar and zygomatic regions was not so great as to give the impression that the face was markedly broad; but from the absence of the lower jaw the proportion between the length and breadth of the entire face could not be obtained. The general dimensions of the woman's skull were small, and its cranial capacity, 1030, was in the lowest category of human skulls. In the man, however, the capacity was higher than is customary in the skulls of savage races. If we are to regard these people, and some of the primitive tribes in Southern India described by Mr EDGAR THURSTON, as præ-Dravidian, there is no evidence that they are Negritos.

It is customary, in speaking of the existing natives of India, to consider that they belong to four ethnic types—Mongolian, Kolarian, Dravidian and Aryan or Indo-Aryan. The possibility of the presence of a Negrito element should also be made the subject of enquiry.

The Mongolians or Tibeto-Burmans are found on the northern and eastern confines of India, and on the east of the Bay of Bengal. I have described representative people of this type in Part I. of this Memoir.*

The Kolarians and Dravidians, on account of linguistic differences, have been by many writers regarded as two distinct ethnic types. It has been assumed that the Kolarian invaders had preceded the Dravidian, and had migrated into India through the north-east passes. The Dravidians, again, are stated to have found their way into the Punjab by the north-west passes, and to have spread into Central and Southern India, though others have conjectured that they came from the south and east.† They are regarded as older inhabitants than the Aryans, who are thought to have entered India, something more than 4000 years ago, from the Hindu Kush, the Pamir plateau, and the high valley of Cashmere. The aborigines of the hill districts in Southern India, the Central Provinces and the Lower Provinces of Bengal, have been described as in part Kolarians and in part Dravidians.

Mr B. H. HODGSON, in his essay on the Kocch, Bódo and Dhimal tribes,‡ uses the term Tamulian as equivalent to aboriginal, and, whilst the people of the sub-Himalayan district belong to the Tibetan stock, and those further east to the Chinese, he regards those to the south as Tamulian, and as represented by the Kols, Bhils, Gonds, Oráons and Múndas. He is of opinion that amongst the Tamulians the physical type is essentially the same in all the tribes.

During the last ten years, and principally through the influence of the writings of Mr H. H. RISLEY,§ the distinction between Kolarian and Dravidian-speaking tribes has come to be regarded as only linguistic, and not as representing differences in physical type. "The Málé of the Rahjmahal hills," he says, "and the Oráons of Chota Nagpore, both of whom speak languages classed as Dravidian, are identical in point of physique

* *Trans. Roy. Soc. Edin.*, vol. xxxix., 1899.

† Sir W. W. HUNTER'S *Indian Empire* and THURSTON'S *Madras Bulletin*, 1899, p. 195.

‡ Calcutta, 1847.

§ *The Tribes and Castes of Bengal*, 1891.

with the Múndas and Santals, who are classed on linguistic grounds as Kolarian." He does away with the term 'Kolarian' as having an ethnic significance, and he includes both sets of people under the common term 'Dravidian.' Mr RISLEY's conclusions were arrived at after a series of anthropological examinations and measurements, conducted under his supervision, on about 6000 living persons in Bengal, the North-Western Provinces and the Punjab. He defines the Dravidian type as follows:—Head usually inclined to be dolichocephalic; nose thick and broad, so that the formula of its platyrhine index is higher than in any known race except the Negro; facial angle comparatively low; lips thick; face wide and fleshy; features coarse and irregular; average stature ranges from 156·2 to 162·1 cm. (5 feet 1 inch to 5 feet 3 inches); figure squat; limbs sturdy. The colour of the skin varies from very dark brown to a shade closely approaching black. The term Dravidian, as employed by RISLEY, has a similar meaning, as regards the tribes which it embraces, to the term Tamulian suggested by Mr HODGSON.

Mr RISLEY defines also the Aryan type in India, and as by contrast it brings out more clearly the Dravidian characters, I append it:—Head relatively long (dolichocephalic); nose straight, finely cut (leptorhine); face long, symmetrically narrow; forehead well developed, features regular; facial angle high; stature fairly high, ranging from 171·6 in the Sikhs (5 feet 7 inches) to 165·6 (5 feet 5 inches) in the Brahmins of Bengal; build of figure well proportioned, slender rather than massive. The colour of the skin is a very light transparent brown, though with various gradations.

I have had no opportunities of measuring the heads of living natives of India, but I propose to summarise the chief characters of the crania measured in Tables I.–IV. Unfortunately, some of the tribes are only sparsely represented, as regards the number of skulls, but the entire collection gives one a fair amount of material for comparison. The Gond, Oráon, Pahária, Karwár, Nágesar, Korwá and Bhuiyá tribes, who are Dravidians in the earlier and restricted use of that term, contribute collectively fifteen crania.* The Múnda, Bhúmij and Turi tribes belong to the old Kolarian group, and contribute collectively nineteen specimens.†

If we take the fifteen skulls in the first or proper Dravidian group, we find that the highest length-breadth index was 76·7. In six crania the index was below 70, hyper-dolichocephalic; in five crania it was between 70 and 75, dolichocephalic; in four crania it was between 75 and 76·7, *i.e.*, in the division of the mesaticephalic which approximates to the dolichocephalic group.‡ The customary type was therefore dolichocephalic.

* I have not included in this number the two Bhima skulls, which possibly may be a sub-division of the Gonds, with which, in their form and proportions, they indeed closely correspond. As there may be a doubt as to their racial position, I thought it advisable to exclude them.

† I have not included in this number I.M. No. 407 (Table IV.), which is deformed from scaphocephalus, nor I.M. No. 604, Jattia Múnda.

‡ I have discussed the relations of mesaticephalic skulls to dolichocephalic and brachycephalic crania in Part I. of this Memoir in *Trans. Roy. Soc. Edinburgh*, vol. xxxix. part iii. p. 744.

In the description which I have written of these crania, it is noted that in the *norma verticalis* they were elongated and ovoid; the sides vertical, or nearly so; the vertex roof-shaped, though not ridged in the sagittal region; the forehead only slightly receding; the parieto-occipital region not flattened, and the occipital squama rounded and projecting behind the inion. The muscular ridges and processes were not strong, so that the outer table was comparatively smooth, and the skulls were not characterised by their weight.

In nine crania the basi-bregmatic height exceeded the greatest breadth; in four the height was less than the breadth; in two they were equal. In these skulls, as is so frequently found in the dolichocephali, the height was usually greater than the breadth.

In the *norma facialis* the glabella and supra-orbital ridges were not prominent, and the nasion was not depressed. In seven specimens the anterior nares were wide in relation to their height, and the nasal index was platyrhine; in six specimens the proportion of width was not quite so great and the index was in the mesorhine group, but usually in its upper term; one specimen had a leptorhine index which expressed a relatively narrow nose; the customary type was therefore platyrhine. In seven specimens the upper jaw was orthognathous; in four, in the lower term of the mesognathous series; one specimen only was prognathic; the customary type of jaw, therefore, was orthognathic. In eleven skulls the orbit was microseme; in one, mesoseme; in three, megaseme; the orbit was usually low, therefore, in relation to its breadth. In the relative proportion of the length and breadth of the palato-alveolar arch only one specimen was dolichuranic; three were mesuranic, seven were brachyuranic; the type form therefore was that of a wide horseshoe. In the determination of the length and breadth of the entire face, the lower jaw was present in nine skulls, in seven of which the complete facial index was below 90, which places them in the chamæprosopic, or low-faced group, *i.e.*, a face which is broad in relation to its length.

In Table II. I have given the cranial measurements of two Tamil-speaking male natives of Madras, who may be regarded as representing the south Dravidian branch. They were both dolichocephalic, and the height exceeded the breadth. The glabella and supra-orbital ridges, and the depression at the nasion, were somewhat more pronounced than in the skulls of the northern Dravidian tribes. In both, the upper jaw was orthognathous, the nose was platyrhine, the orbit was microseme, and the palato-alveolar arch in one was mesuranic, in the other brachyuranic. In the skull with a lower jaw the face was chamæprosopic. The characters were distinctly Dravidian.

In the series of seventeen crania under analysis, including the Tamils but excluding those marked Kandh, the cubic capacity of thirteen male skulls ranged from 1438 to 1150 c.c., of which three were above 1400, three were between 1300 and 1400, six were between 1200 and 1300, and one was 1150 c.c.; the mean of the series was 1294 c.c. Of the four women, three were between 1200 and 1300, and one was only 1070; the mean of the series was 1217 c.c.

In making this analysis of the crania I have purposely excluded the two marked Kandh. In one of these the length-breadth index was 84·2, brachycephalic; in the other, 78·5. If the Kandhs are to be regarded as an unmixed Dravidian people, the high index in each instance leads one to think that the specimens may have been misnamed, and are not genuine examples of the race. If the tribe consists, however, as Dalton supposes, of a mixture of races, these crania, more especially the brachycephalic specimen, may indicate the presence of a brachycephalic strain, which intermingled with the Dravidian would tend to modify the original dolichocephalic type. It should be stated that the nasal index in each skull was platyrhine, and in the brachycephalic specimen strongly so; the orbital index was microseme; the palato-maxillary index was brachyuranic; in neither was the upper jaw prognathic, and in the only one with a lower jaw the face was chamæprosopic. In the facial characters the skulls marked Kandh corresponded with the Dravidian type.

We may now proceed to the analysis of the skulls belonging to Kolarian-speaking tribes. One specimen, No. 604, Indian museum, marked Jattia Múnda of Bhowro village, near Ranchi, had a cephalic index, 80·5, but as in the configuration of the cranium it differed so much from the other Múndas I have excluded it from the general description. The following observations apply therefore to nineteen skulls.

In three crania the length-breadth index was below 70, *i.e.*, hyper-dolichocephalic; in fourteen specimens it was between 70 and 75, dolichocephalic; in two specimens, between 75 and 76, which, although not numerically, yet in form and essential characters were dolichocephalic. In general form, the crania were elongated and ovoid, with steep side walls, moderate parietal eminences, no special ridging in the sagittal region, and, with the slope outwards to the parietal eminences, not very steep. The forehead was not markedly receding, indeed often approaching the vertical; the parieto-occipital slope was gradual; the occipital squama was, as a rule, rounded, and projected behind the inion. The muscular ridges and processes were fairly marked, and the skulls had no unusual weight.

The basi-bregmatic height exceeded the greatest breadth in twelve crania; it was less than the breadth in six, and in one they were equal.

In the *norma facialis* the glabella and supra-orbital ridges moderately projected, and the nasion was only slightly depressed. In six specimens the anterior nares were wide, and the nasal index was platyrhine; in ten specimens the nose was mesorhine, and in all of these, with one exception, with the index above 50; two specimens had a narrow leptorhine index.* In nine specimens the upper jaw was orthognathous; eight specimens were mesognathous; no face was prognathous. Ten specimens had a low microseme orbit; four were mesoseme; four had a high megaseme orbit. In no skull was the palato-alveolar arch so elongated as to be dolichuranic; three were mesuranic; the rest were brachyuranic. The lower jaw was present in eleven of the nineteen skulls,

* It is not unlikely that in the living person the nose may have, on account of the lateral extension of the alæ, a more strongly marked platyrhine character than would be obtainable from the width of the anterior nares in the skull itself.

in nine of which the proportion of the breadth to the length of the face was low or chamæprosopic; in the remaining two the complete facial index was 90 and 93 respectively, and the face was within the leptoprosopic division.

In the Kolarian group the cranial capacity of the men ranged from 1470 to 1176 c.c.; of these four were above 1400, five were between 1300 and 1400, six were between 1200 and 1300, and one was below 1200 c.c.; the mean of the series was 1314 c.c. The three women's skulls had a mean capacity of 1097 c.c., and the lowest measured only 1000 c.c.

If we compare the characters of the skull in the Dravidian with the Kolarian group, we shall find that they correspond in essential particulars. In both, the type of cranium in form and proportion was dolichocephalic; the anterior nares were platyrhine, or in the higher term of the mesorhine group; the presence of a leptorhine index was altogether exceptional; the upper jaw was usually orthognathous; only one of the thirty-six skulls was prognathous; as a rule the orbit was low or microseme, the palato-alveolar arch was brachyuranic. In both groups also the face was chamæprosopic, *i.e.*, the interzygomatic width was great in proportion to the length of the face. If we take the cranial capacities of the two groups together, the men have a mean 1304 c.c., the women 1157 c.c.

Judging, therefore, from the characters of the skull, one would draw the conclusion that there is no difference of moment in the form and proportion of this part of the skeleton between the Dravidian and Kolarian types, and support is given to the view of their essential structural unity as advocated by Mr Risley. For descriptive purposes both groups of skulls may be classed therefore as Dravidian.

Many ethnologists of great eminence have regarded the aborigines of Australia as closely associated with the Dravidians of India. Some also consider the Dravidians to be a branch of the great Caucasian stock, and affiliated therefore to Europeans. If these two hypotheses are to be regarded as sound, a relationship between the aboriginal Australian and the European would be established through the Dravidian people of India.

The affinities between the Dravidians and Australians have been based upon the employment of certain words by both people, apparently derived from common roots; by the use of the boomerang, similar to the well-known Australian weapon, by some Dravidian tribes; by the Indian peninsula having possibly had in a previous geologic epoch a land connection with the Austro-Malayan Archipelago, and by certain correspondences in the physical type of the two people.

Both Dravidians and Australians have dark skins approximating to black; dark eyes; black hair, either straight, wavy, or curly, but not woolly or frizzly; thick lips; low nose with wide nostrils; usually short stature, though the Australians are somewhat taller than the Dravidians.

When the skulls are compared with each other, whilst they correspond in some particulars, they differ in others.* In both races the general form and proportions are

* I may refer to my *Challenger Report* on Human Crania, part xxix., 1884, for an analysis of the characters of the skulls of the Australian aborigines.

dolichocephalic, but in the Australians the crania are absolutely longer than in the Dravidians, owing in part to the prominence of the glabella. In the Australians it is not unusual for the adult male to have the glabello-occipital diameter approaching, or even a little more than, 200 mm., whilst in the male Dravidians measured in Tables I.–IV. only two specimens reached 191 mm. The Australian skull is heavier, and the outer table is coarser and rougher than in the Dravidian; the forehead also is much more receding; the sagittal region is frequently ridged, and the slope outwards to the parietal eminence is steeper. The Australians in the *norma facialis* have the glabella and supra-orbital ridges much more projecting; the nasion more depressed; the jaws heavier; the upper jaw usually prognathous, sometimes remarkably so; the teeth larger and coarser, so as to deserve the name macrodont. The coarser character of the skull, especially in the temporal region, the heavier jaws and the large strong teeth, point to the use of a coarser food by the Australians, for which a more powerful masticatory apparatus is required. On the other hand, both Australian and Dravidian crania have the nasal index platyrhine or mesorhine; the occurrence of a long, narrow, or leptorhine nose being so exceptional, that its presence indicates that the skull has probably been incorrectly named, or is not of a pure race. In both races also the males have usually a microseme orbit; but whilst the Australians have customarily a long, dolichuranic palato-alveolar arch, in the Dravidians it is broader in relation to the length, and frequently brachyuranic.

As regards the cranial capacity of the Australians, whilst the range in the thirty-nine male skulls which I have measured was from 1514 c.c. to 1044, the mean was only 1280 c.c., which is somewhat less than the general Dravidian mean 1314 c.c. In the female Australians, twenty-four women ranged from 1240 to 930, and had a mean 1115·6 c.c., which is also less than the Dravidian mean 1157 obtained from seven female crania. It should be stated that of the series of sixty-three Australian skulls, eight men were less than 1200 c.c., and only four above 1400 c.c.; whilst of the women only three were above 1200 c.c., and ten were below 1100 c.c.

By a careful comparison of Australian and Dravidian crania, there ought not to be much difficulty in distinguishing one from the other. The comparative study of the characters of the two series of crania has not led me to the conclusion that they can be adduced in support of the theory of the unity of the two people.

The skulls which belonged to the Koydwar, Kámár, Ahír-Goálá and Teli castes or tribes were dolichocephalic, platyrhine, and, with one exception, orthognathic, characters which they shared with the Dravidian crania. It is not unlikely that in these castes there is a strong Dravidian element. The Bhima skulls, though dolichocephalic and either orthognathous or mesognathous, were not platyrhine. The Bunjana skull, on the other hand, was hyper-brachycephalic, though the jaw was orthognathous, and the nose was platyrhine. The Lohár skull was mesaticephalic and orthognathic, but the nasal index was leptorhine, and in so far pointed to a predominance of Aryan blood. The specimens were too few to enable one to draw a general conclusion on the cranial characters of these tribes or castes.

As already stated, the skulls of the Uriyá group presented considerable variations in the cephalic index, and in the configuration of the skull. In the dolichocephalic series about one-third were platyrhine in the nasal index, the others were mesorhine or leptorhine; in the majority the upper jaw was orthognathous, and no skull was prognathous. In the mesaticephalic series the majority were mesorhine, only two were platyrhine, and one was leptorhine; the upper jaw was usually orthognathous, and only one was prognathous. The brachycephalic series was represented by only five specimens, three of which were mesorhine, one platyrhine, and one leptorhine; as regards the upper jaw, no specimen was prognathous.

As many of these crania were derived by the Indian Museum from the Medical School in Calcutta, it may have happened that no proper history of the dead had been obtained, and that, in consequence, the skulls had not been accurately identified.* If we grant that they had all belonged to the Uriyá-speaking people, the inference seems obvious that the community of language would by no means express unity of race.

It would seem, therefore, that in the Uriyás some crania partook of Dravidian, others of Aryan characters, and from the presence of a proportion of brachycephalic skulls, there might also have been a trace of Mongolian or other brachycephalic intermixture. As regards the Uriyá group, it is probable that a considerable Dravidian element is contributed through the presence of tribes of Hinduised aborigines, intermingled with the people who possess a strain of Aryan blood.

I will now proceed to the consideration of the Veddahs, the aboriginal hill tribe in Ceylon, of the Mincopies, the aborigines in the Andaman Islands, and of the hill tribes in the Malay peninsula.

Veddahs. Table IX.

In the study of the aboriginal dolichocephalic tribes in and near the Indian peninsula, we should not overlook the aborigines known as Veddahs or Weddas, who live in the hill districts of the adjoining island of Ceylon. Various travellers in Ceylon, of whom may be especially mentioned Robert Knox,† John Davy,‡ C. Pridham,§ Sir Emerson Tennent,|| B. F. Hartshorne,¶ John Bailey,** and C. S. V. Stevens,†† have given accounts of these people and the districts in which they live. George

* The crania marked Uriyá, Orissa, in the Tables, are those which had been obtained from the Medical College. It will be seen that specimens so marked occur in each of the three groups tabulated in VI., VII., VIII.

† *Historical Relation of the Island of Ceylon.* London, 1817.

‡ *Account of the Interior of Ceylon and of its Inhabitants.* London, 1821.

§ *Ceylon and its Dependencies.* London, 1849.

|| *Ceylon.* London, 1859.

¶ *Fortnightly Review*, London, 1876, vol. xix.

** *Trans. Ethnol. Soc.*, London, 1863.

†† *Overland Times of Ceylon*, Nov. 6th, 1886.

Busk described four specimens of their crania in 1862,* which, along with three others, had their chief measurements recorded by Sir Wm. Flower in his catalogue of crania in the Hunterian Museum. MM. De Quatrefages and Hamy figured a skull in the *Crania Ethnica*, Pl. LVIII. Barnard Davis has also recorded, in the *Thesaurus Craniorum*,† the measures of ten Veddah skulls. George Rolleston exhibited to the British Association in 1872‡ photographs of jungle Veddahs, and also three skulls of this people in the Oxford Museum. Virchow has described§ three Veddah skulls, and has discussed the ethnological relations of the people. Arthur Thomson has given an account‖ of the osteology of the Veddahs, and has described, along with the other bones of the skeleton, the characters of nine skulls in the Oxford Museum. He has also included in his tables of measurement three skulls measured by Virchow, fifteen in the Museum of the Royal College of Surgeons of England, and eleven in the collection of Barnard Davis. Much the most complete description of the habits, distribution, and physical characters of the Veddahs, and, indeed, of the natives generally of Ceylon, is contained in the monumental work on that island by Paul and Fritz Sarasin,¶ who record, in addition to an account of the skeleton generally, the measurements of eighteen male and four female skulls from the interior of the island, and four male and four female skulls from the coast districts; also some young and imperfect crania.

As regards the external physical characters of the Veddahs, the Sarasins have contributed the fullest and most carefully analytical description, which I have summarised as follows:—The colour of the face in men varies from a deep brown to one with shades of lighter brown; they have never seen a pure black skin, and those that seem to be black, when closely examined are distinctly brown. The skin of the breast is more frequently an opaque brown, though it may have a medium or reddish-brown shade. In women there is not the same range in the brown tint, and on the whole the skin is a clearer brown. The eyes have a brownish-black or opaque brown colour. The hair of the head is black, coarse, wavy, tangled, and hanging down to the shoulders or the back; that of the beard and moustache is black and sparse. On the body the hair is also sparse, though on the legs it may be abundant. The face is tolerably broad and not high, the mean index of sixteen men being 80·7, *i.e.*, low-faced, chamæprosopic: the chin is pointed. The eyebrows are not strong, the eyes are generally large, and there is no fold of skin connecting the eyelids at the inner canthus (epikanthus), as in the Mongols. The nose has a deep pit in men at the root, the bridge is not strong, and the alæ have considerable breadth; in women the nose is flatter than in men. The lips are large and the jaws are orthognathic.

* *Proc. Linn. Soc.*, 1862, vol. vi.

† *Thesaurus Craniorum*, 1867.

‡ *Scientific Papers and Addresses*, vol. i., Oxford, 1884, edited by W. Turner.

§ "Ueber die Weddas von Ceylon," *Abh. der K. Akad. der Wiss. zu Berlin*, 1881.

‖ *Journ. Anth. Inst.*, Nov. 1889.

¶ *Ergebnisse naturwissenschaftlicher Forschungen auf Ceylon*, 3d Band, *die Weddas von Ceylon.* Wiesbaden, 1892–93.

The stature is low; in the Veddahs of the central district, where the race is probably the purest, the mean height of twenty-four men was 1533 mm. (5 feet $\frac{1}{3}$ inch), of eleven women 1433 mm. (4 feet 7 inches); that of twenty-four men from the coast district was 1588 mm. (5 feet 2 inches), of ten women 1494 mm. (4 feet 9 inches), whilst fourteen men from the district of Wewatte were 1607 mm. (5 feet $2\frac{3}{4}$ inches) in height. In the sea-coast and Wewatte districts there has probably been some intermixture with Singhalese, Tamils, or even Indo-Arabians, which would affect both the stature and other physical characters of the Veddahs.

As regards the Dravidian Tamils of Ceylon, the SARASINS have also described their external physical characters. They are a bigger people than the Veddahs; the mean stature of the men was 1653 mm. (5 feet 4 inches) and of the women 1545 mm. (5 feet $\frac{3}{4}$ inch). The pigmentation of the skin was deeper in the lower than the higher castes. In about one-half the men examined the skin of the face was a medium, rarely a red-tinted, brown; in the other half a brighter brown shading into yellow: in the women a more opaque brown prevailed. The eyes were an opaque brown. The hair was black and scarcely differed from the hair of the Veddahs, though it was perhaps coarser and had a greater tendency to curl. The supra-orbital region was often well developed in the men. The face was oval and proportionately higher and narrower than in the Veddahs. The eyes were large and without an epikanthus. The nose had a stronger bridge than in the Veddahs, and the alæ were not so wide. The lips were thick. The teeth were strongly developed, and the jaws were more projecting than in the Veddahs.

I have examined and measured nine Veddah crania which have not previously been described. Three of these belonged to the Henderson Trust Collection, now in the Edinburgh University Museum; they were presented in 1827 by the Rev. G. LYON and were probably the earliest examples of the race to reach this country. One was presented to me about twenty years ago by the late Dr KRIEKENBECK of Colombo; the man had died in jail; the skull is metopic, a rare condition in dolichocephalic savages. One from Batticaloa, in the east of Ceylon, was presented by H. THWAITES, Esq. In one skull, No. 555 in the Indian Museum, the face was broken. Of the three others, two have been for some years in the Museum of Trinity College, Dublin, and another, also in Dublin, came from Batticaloa. I have to thank Professor CUNNINGHAM for permission to examine them. The skulls were all adults; to all appearance seven were men and two probably women.

When examined in the *norma verticalis* the crania were seen to be elongated antero-posteriorly; the side walls were almost vertical; the vertex in some specimens was roof-shaped, but not keeled in the sagittal region, and in others the vertex was more flattened; the parietal eminences were distinct. The skull sloped gently backwards as a rule into the occipital region, and the occipital point usually projected definitely behind the inion; there was no evidence of parieto-occipital flattening. In three of the skulls the length-breadth index ranged from 66·5 to

Table IX.

Veddah.

	Henderson Trust.			E.U.A.M.		I.M.	Trinity College, Dublin.		
Collection number,	143	145	144	Batticaloa.	Metopic.	555	...	...	Batticaloa.
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.
Sex,	M.	M.	F.	M.	M.	M.	M.	M.	F.
Cubic capacity,	...	1226	1090	1090	1100	1170	1262	1362	1088
Glabello-occipital length,	177ap.	170	174	167	180	180	175	185	174
Basi-bregmatic height,	130	129	131	127	126	130	137	139	127
Vertical Index,	*73·4*	*75·9*	*75·3*	*76·*	*70·*	*72·*	*78·*	*75·*	*73·*
Minimum frontal diameter,	93	87	88	93	93	91	93	94	89
Stephanic,	108	104	100	98	99	96	109	113	100
Asterionic,	103	98	99	97	100	103	98	101	101
Greatest parieto-squamous breadth,	125ap.	128s.	121p.	127s.	121s.	128s.	125s.	123s.	127s.
Cephalic Index,	*70·6*	*75·3*	*69·5*	*76·*	*67·*	*71·*	*71·4*	*66·5*	*73·*
Horizontal circumference,	492ap.	478	475	477	500	497	490	510	485
Frontal longitudinal arc,	...	123	120	120	120	130	130	132	120
Parietal „ „	130 }	233	128	111	130	128	122	145	110
Occipital „ „	104 }		105	110	113	114	112	110	113
Total „ „	...	356	353	341	363	372	364	387	343
Vertical transverse arc,	292	288	281	289	278	292	295	302	288
Length of foramen magnum,	37	34	33	32	29	35	34	33	32
Basi-nasal length,	94	91	97	97	98	96	97	101	100
Basi-alveolar length,	96	89	88	98	100	...	92ap.	90ap.	93
Gnathic Index,	*102·1*	*97·8*	*90·7*	*101·*	*102·*	...	*94·8*	*89·*	*93·*
Interzygomatic breadth,	131	120	111	129	121	...	126	117	123
Intermalar „	117	109	103	116	116	...	112	108	113
Nasio-mental length,	...	...	...	107	117	...	...	...	...
Nasio-alveolar „	59	56	55	66	64	...	52	60	58
Complete Facial Index,	...	...	...	*82·9*	*96·7*	...	...	...	...
Nasal height,	43	44	42	45	46	...	42	45	44
Nasal width,	25	23	26ap.	22	22	...	25	23	25
Nasal Index,	*58·1*	*52·3*	*61·9*	*48·9*	*47·8*	...	*59·5*	*51·*	*56·8*
Orbital width,	41	36	37	38	36	36	39	38	35
Orbital height,	29	30	32	31	30	30	31	34	33
Orbital Index,	*70·7*	*83·3*	*86·5*	*81·6*	*83·*	*83·*	*79·5*	*89·5*	*94·3*
Palato-maxillary length,	50	50	47	54	54	...	52	45ap.	50
Palato-maxillary breadth,	66	58	53	64	63	...	56	57	63
Palato-maxillary Index,	*132·*	*116·*	*112·7*	*118·5*	*116·6*	...	*107·9*	*126·6*	*126·*
Lower jaw. Symphysial height,	...	...	...	28	33	...	...	...	...
Lower jaw. Coronoid „	...	...	...	65	61	...	...	...	58
Lower jaw. Condyloid „	...	...	...	59	61	...	...	...	48
Lower jaw. Gonio-symphysial length,	...	...	...	90	95	...	...	...	88
Lower jaw. Inter-gonial width,	...	...	...	82	82	...	...	...	80
Lower jaw. Breadth of ascending ramus,	...	...	...	33	34	...	...	...	39

69·5, hyperdolichocephalic; in four the index was from 70·6 to 73, dolichocephalic; in the remaining two it was 75·3 and 76. The mean of the series was 71·1. In seven skulls the basi-bregmatic diameter exceeded the greatest breadth; in two they were equal: the mean vertical index of the series was 74·3. In one skull the occipital longitudinal arc was a little longer than the parietal, but not so long as the frontal arc; in four skulls the frontal arc exceeded the parietal; in three the opposite condition was seen. With one exception the crania were cryptozygous.

When looked at in the *norma lateralis*, the glabella and supra-orbital ridges projected only slightly, the forehead was sometimes nearly vertical, at others receded a little. The nasion was depressed in one specimen, but not in the others. The nasal bones were usually small, not prominent and concave forwards. The nasal spine of the superior maxillæ was distinct, and the floor of the nose was separated from the incisive region by a ridge. The mean nasal index was 54·4 platyrhine, and of the individual skulls four were markedly platyrhine, three were mesorhine, and one on the boundary between leptorhine and mesorhine. The orbits varied in the relation of width and height; six were low, microseme; two were high, megaseme; one was mesoseme; the mean index, 83·5, was microseme. In no specimen was the upper jaw prognathous, five were orthognathous, and three were mesognathous; the mean gnathic index, 96·3, was orthognathous.

The nasio-mental diameter could be measured in only two skulls, in one of which the complete facial index was chamæprosopic, in the other high-faced or leptoprosopic. The mean palato-maxillary index was 119·5, *i.e.*, brachyuranic, and with two exceptions, one dolichuranic, the other mesuranic, the other skulls belonged to the brachyuranic group.

The teeth had been fully erupted in all the skulls except a wisdom tooth in No. 143; the crowns were mostly betel stained, and the grinding surfaces of the molars were worn flat. The sutures were, as a rule, distinct, and one was metopic; though in one the sagittal was partially obliterated. In two crania the lambdoidal suture contained small Wormian bones. One had a right epipteric bone, but in none was the squamous temporal in articulation with the frontal.

The cranial capacity in both sexes was low, the mean of six men was only 1201 c.c., and the range was from 1090 to 1362 c.c.; the mean of two women was only 1089 c.c. The lower jaw was present in only three specimens, in each of which the chin was well marked; the body of the bone was deep, for the lodgment of the fangs of the teeth and the angle was well marked.

I may now briefly state the chief cranial characters of the specimens described by previous observers. Arthur Thomson has embodied in a table the measurements made by Busk, Virchow, Flower, Barnard Davis, and himself. Of the thirty-seven skulls included in that table fourteen had a length-breadth index below 70, fourteen were between 70 and 75, five were from 75 to 77·5, one was 78, and three were from 80·6 to 82·9. All the skulls, with four exceptions, were definitely dolichocephalic or in the lower terms of the mesaticephalic group. Of the four exceptional specimens, one with the index 82·9 from Bintenne of Badulla (R.C.S. Eng. No. 676) is said to be unsymmetrically distorted from occipital pressure, which had doubtless affected the relation of length to breadth ; another, from Batticaloa, measured by Virchow, with an index 80·6, is said to be evidently abnormal, probably from an artificial or accidental deformity in the occipital region.

This series of skulls confirms what I have previously had occasion to point out in the study of crania, that in the dolichocephalic crania of savage races the basi-bregmatic height usually exceeds the greatest breadth. Thus, of thirty-six skulls in THOMSON's table, in which both breadth and height are recorded, the height exceeded the breadth in thirty-one, and it was equal to the breadth in one specimen. In only four crania was the height less than the breadth, and in three of these the length-breadth index was above 80, and the skull was brachycephalic.

The seventeen skulls in THOMSON's table in which the proportions of the upper jaws were measured were all orthognathous. Of the twenty-two skulls in which the proportions of the nose were measured, ten were platyrhine, seven were mesorhine, and only five were leptorhine. The orbital index was variable ; in six specimens it was microseme, in eight mesoseme, in eight megaseme. The palato-alveolar index in eight skulls measured exceeded 120 in only one specimen.

As regards the cranial capacity it is difficult to make a precise statement, as the methods used by different observers in its determination were not uniform, and the results cannot be strictly compared with each other. It may suffice to state that the capacity in one woman's skull is said to be as low as 960 c.c. ; in eleven other women the range of capacity was 1025 to 1442 c.c., and the mean was 1230 c.c., *i.e.*, microcephalic. In twenty men the range was from 1140 to 1611 and the mean was 1336 c.c., also microcephalic. In both sexes the mean was materially higher than in the skulls which I measured, and several skulls exceeded considerably that with the highest capacity, 1362 c.c. in my series, an excess which may perhaps partly be due to the methods employed yielding a larger result than is obtained by the plan which I am in the habit of following, which I believe to be more exact.*

If we now examine the series of thirty skulls measured by the Messrs SARASIN, we find that the mean length-breadth index of the Veddahs from the interior was 70·5 for seventeen men, and 69·1 for four women ; whilst the corresponding index of four men from the coast was 76·5, and of four women 73. No skull was brachycephalic, but in five the index was from 75·9 to 79·8. In each group, except in that of the men from the coast, the height exceeded the breadth. The mean complete facial index in each group was near the upper limit of the chamæprosopic division. The mean gnathic index in each group was orthognathous, and no specimen was prognathous, and only a small minority was mesognathous. The mean nasal index was in the higher mesorhine series ; only four specimens were leptorhine, but thirteen were platyrhine. Fifteen specimens were

* I have described my method in *Challenger Reports*, part xxix. p. 9, 1884. By the method of BROCA, followed by so many craniologists, the capacity is overstated.

megaseme, and the mean orbital index of the series came just within the megaseme division, but four specimens were microseme. In the relative proportions of the length and breadth of the palato-alveolar arch the mean index fell just within the brachy-uranic division. As regards the cranial capacity, the mean of twenty-two men was 1277 c.c., and of ten women 1139 c.c.

Seventy-six skulls ascribed to Veddahs have now been studied and described by experienced craniologists. With very few exceptions they were elongated, with the sides approaching the vertical, the sagittal line not keeled, or only slightly so; relatively narrow, and the length-breadth index was dolichocephalic, frequently hyper-dolichocephalic. It is known that some of the skulls in which the index exceeded 75 or 76 were from natives who had lived on the coast, where the possibility of an admixture of blood with other races is probable. The basi-bregmatic height in almost every case exceeded the greatest breadth.

The face was broad in relation to the height. The nose was platyrhine or mesorhine, seldom leptorhine. The upper jaw was orthognathous. The orbit was variable in the proportions of height and breadth, but tended to a relatively high vertical diameter. The palato-alveolar arch was moderately elongated. The cranial capacity was low.

If these characters be compared with those previously given, as found in the Dravidian group, they will be found to correspond in many respects. In both the crania were dolichocephalic in form and proportions; in both the height as a rule exceeded the breadth. The glabella and supra-orbital ridges did not strongly project, the forehead was not specially retreating, and in many specimens approached the vertical; the occipital squama was usually convex, and projected behind the inion. The face was low in relation to the breadth; the nasion was seldom much depressed; the anterior nares were platyrhine or mesorhine, rarely leptorhine; the upper jaw was orthognathous, occasionally mesognathous, not prognathous; the orbits varied in the proportion of width and height; the palato-alveolar arch also varied, though the index seldom much exceeded 120, and the breadth was not greatly in excess of the length. The cranial capacity was microcephalic in both Veddahs and Dravidians, though the former were, on the whole, of smaller capacity than the latter. It is difficult, therefore, to lay down a series of characters in which the Veddah and Dravidian skulls differed from each other.

Andaman Islanders. Table X.

I have stated on p. 101 that the possibility of the presence of a Negrito element in the people of India has to be enquired into. Considerable attention has been given to this subject by several ethnologists, and opinions both affirmative of and adverse to the affinity between the black races of India and the Negritos have been expressed. Mr O'Donnell in his *Census Report* has indeed used the term Negritic as if it were synonymous with Dravidian, and has indicated (p. 264) a route along which he thinks a

Negrito race could have reached southern India and passed to south-eastern Asia and Australia.

That a Negrito race is scattered in the Philippine Islands is well established, and that similar people exist in other islands of the great eastern Archipelago, and in a few localities on the adjacent continent, has been asserted by eminent authorities. There can be no doubt that the Mincopies, or natives of the Andaman Islands in the Bay of Bengal, have the Negrito characters of low stature, very dark skin approaching black, with woolly or frizzly black hair growing in short, close curls. The proximity of these islands to the Indian peninsula has seemed to indicate that a Negrito population had preceded in India the present dark-skinned Dravidian race, and that traces of their existence can be still found in the aboriginal people. Although some writers have referred to black, frizzly or woolly-haired tribes in certain of the mountainous districts in India, the evidence on this head is by no means conclusive, and it may be a question if the terms woolly or frizzly may not have been loosely used to characterise the wavy hair which has been seen in individuals of some of the aboriginal tribes. The statements which have been made in regard to this question have been carefully analysed by A. B. MEYER, in his *Memoir on the Distribution of the Negritos*,* and he has come to the conclusion that the present state of our knowledge does not permit a judgment to be given that the aboriginal people of India were Negritos. As bearing on this matter, I may state that DALTON, in his *Ethnology of Bengal*, figures a Santal with curly hair, quite distinct, however, from the short, close locks of the natives of the Andaman Islands. In his portraits of the Juangs and Korwás, two tribes short in stature and primitive in habits, the hair is long, more or less matted, but not curly. Messrs FORBES WATSON and Sir J. W. KAYE have reproduced † photographs of a Santal, Kurumbas, Yenadies, a jungle tribe of Chingleput, a Toda and a Kandh with curly tangled hair. EDGAR THURSTON, in his description of the short, broad-nosed tribes of Southern India, figures Kadirs from the Ānaimalai Hills, in whom the hair was curly, relatively long, and projecting from the head, not unlike the "mop" of the Papuans. He also gives portraits of Paniyans from Malabar and Kurumbas from the Nilgiri Hills, in whom the hair had a similar character. These tribes or races are primitive in their habits, and the stature does not apparently exceed 5 feet 2 inches. Wavy and curly black hair are, he says, in the south Dravidians common types; but he had seen no head of hair to which the term woolly could be correctly applied.‡ The wavy or curly character seems to be no more marked than the curly locks not unfrequently seen in the white races.

I need not dwell upon the physical characters and the customs of the people of the Andaman Islands, as they have been described in considerable detail by J. MOUAT,§ E. H. MAN,|| DE QUATREFAGES,¶ and E. S. BRANDER.**

* Dresden, 1899.

† *The People of India*, 10 vols., 1868, *e. s.* London. India Museum.

‡ *Madras Bulletin*, vol. ii. No. 3, p. 187, 1899.

§ *Adventures in Andaman Islands.* London, 1861.

|| *Journ. Anthrop. Inst.*, xiv., 1885.

¶ *Les Pygmées*, Paris, 1887 ; and in conjunction with M. Hamy, *Crania Ethnica*, p. 184.

** *Proc. Roy. Soc. Edin.*, 1880, p. 415.

The University Anatomical Museum contains the skulls of six Andaman Islanders, presented, along with other bones of the skeleton, by Drs J. Dougal, J. S. Forrester, D. D. Cunningham, and Colonel Cadell, V.C. In the Museum of the Royal College of Surgeons of Edinburgh is another skeleton.* Of the seven skulls, two had not quite reached maturity; the others were adult, of these three apparently were women and two men.

When looked at in the *norma verticalis* the skulls were seen to be flattened at the vertex, and the vault had a low curve; they were relatively wider in the parietal regions, the eminences in which were distinctly marked even in the men's skulls. The stephanic diameter was much below the parietal, and its relatively short breadth contributed to give a characteristic contour to the cranium. Although there was no appearance of parieto-occipital flattening, the slope behind the obelion was somewhat abrupt, and the parietal eminences were much closer to the occipital than to the frontal pole of the cranium. With one exception the skulls were cryptozygous. The crania ranged in length from 173 to 158 mm., in greatest breadth from 141 to 128 mm. The mean length-breadth index was 81·5, brachycephalic, and the range was from 78·6 to 88·7. In each skull the basi-bregmatic height was, as is customary in brachycephalic crania, distinctly less than the greatest breadth, and the mean vertical index was 75·7. With one exception the occipital longitudinal arc was the shortest, but there was no constancy in the relative proportions of the frontal and parietal arcs.

In the *norma lateralis* the glabella and supra-orbital ridges were feeble in the males and scarcely marked in the female skulls; the forehead was vertical in the women and very slightly receding in the men; the frontal eminences were distinct. The nasion was not depressed, the nasal bones were not prominent except in one specimen, and were flattened across the bridge. In two skulls the nasal index was mesorhine, the rest were platyrhine, and the mean index was 55. One orbit was high in relation to the width, three were much lower, and the others were intermediate, the mean index of the series, 85·5, was mesorhine. The upper jaw in its degree of projection was in two cases orthognathous, in one prognathous, in the rest mesognathous, the mean of the series was 99·8, mesognathous. The face in each specimen was chamæprosopic, and the mean complete facial index was 80·5.

The nasal spine of the superior maxillæ was moderate, and the floor of the nose was usually separated from the incisive region by a ridge. The teeth had mostly erupted, but in some of the specimens the wisdoms were not complete, and in one of these the right upper canine and right lower central incisor were concealed in the jaws. In the older skulls the crowns were worn from use. In the younger skulls the sutures were well denticulated, but in the older they were beginning to be obliterated. One was metopic, and in it the frontal transverse diameters much exceeded those in the other skulls. In one specimen a large Wormian bone constituted the upper part of the

* The bones of five of the skeletons, exclusive of the skulls, were described by me in the *Challenger Reports*, Zoology, vol. xvi. part xlvii., 1886.

TABLE X.

Andaman Islanders—Sakai.

	Andaman Islanders.							Sakai.	
	Edin. Univ. Anat. Museum.						E.R.C.S.	Ed. U. A. M.	
								Kampar.	Pahang.
Collection number,	No. 6	No. 1	No. 5	No. 2	No. 3	No. 4	...	...	...
Age,	Ad.	21 ?	Ad.	Ad.	Ad.	23 ?	Ad.	Ad.	Ad.
Sex,	M.	M.	M.	F.	F.	F.	F.	M.	M.
Cubic capacity,	1080	1255	1270	1080	1190	1153	1090	1155	1385
Glabello-occipital length,	158	159	173	166	161	159	164	169	175
Basi-bregmatic height,	125	123	127	122	119	125	121	130ap.	134
Vertical Index,	*79·1*	*77·4*	*73·4*	*73·5*	*73·9*	*78·6*	*73·8*	*76·5*	*76·6*
Minimum frontal diameter,	89	90	102	90	88	90	87	91	94
Stephanic,	100	111	122	107	104	109	99	95	106
Asterionic,	97	102	96	99	95	97	91	99	106
Greatest parieto-squamous breadth,	128p.	141p.	136p.	131p.	130s.	131p.	132p.	126s.	139s.
Cephalic Index,	*81·*	*88·7*	*78·6*	*78·9*	*80·7*	*82·4*	*80·5*	*74·6*	*79·4*
Horizontal circumference,	462	468	493	475	468	465	467	473	505
Frontal-longitudinal arc,	115	113	123	111	121	117	112	112	120
Parietal „ „	115	102	135	113	113	120	125	127	128
Occipital „ „	102	...	103	111	103	100	101	108	111
Total „ „	332	345	361	335	337	337	338	347	359
Vertical transverse arc,	288	300	304	291	288	295	270	276	295
Length of foramen magnum,	29	31	32	33	29	34	30	36	37
Basi-nasal length,	90	83	94	93	90	89	92	93	98
Basi-alveolar length,	91	82	91	90	91	89	96	89	93ap
Gnathic Index,	*101·1*	*98·8*	*96·8*	*96·8*	*101·1*	*100·*	*104·3*	*95·7*	*94·9* ap
Interzygomatic breadth,	121	112	128	123	118	115	119	116	...
Intermalar breadth,	113	103	118	112	106	103	111	108	...
Nasio-mental length,	99	92	103	96	88	92	...	...	...
Nasio-alveolar „	58	53	62	59	54	55	56	40	...
Complete Facial Index,	*82·*	*82·1*	*80·4*	*78·*	*74·5*	*80·*	...	...	...
Nasal height,	43	40	45	43	41	41	44	41	51
Nasal width,	22	20	24	25	21	23	25	24	26
Nasal Index,	*51·2*	*50·*	*53·3*	*58·1*	*51·2*	*56·1*	*56·8*	*58·5*	*51·*
Orbital width,	37	35	37	36	37	37	36	36	...
Orbital height,	32	32	32	30	30	31	31	28	...
Orbital Index,	*86·5*	*91·4*	*86·5*	*83·3*	*81·1*	*83·8*	*86·1*	*78·*	...
Palato-maxillary length,	50	47	50	52	49	50	53	47	...
Palato-maxillary breadth,	62	56	64	60	53	56	59	59	...
Palato-maxillary Index,	*124·*	*119·1*	*128·*	*115·4*	*108·1*	*112·*	*111·3*	*125·*	...
Lower jaw. Symphysial height,	26	22	23	23	23	25	24	...	...
Lower jaw. Coronoid „	59	49	49	53	53	51	55	...	...
Lower jaw. Condyloid „	54	53	51	52	47	47	51	...	...
Lower jaw. Gonio-symphysial length,	85	77	91	85	85	80	89	...	...
Lower jaw. Inter-gonial width,	85	80	94	82	76	78	84	...	...
Lower jaw. Breadth of ascending ramus,	31	27	35 Metopic	34	37	36	35	...	...

occipital squama. One skull had an epipteric bone on each side; another had on the left side a broad articulation of the squamous temporal with the frontal, and on the right both an epipteric bone and a direct temporo-frontal articulation. In one the os planum of the ethmoid was so narrowed in front that the orbital plate of the maxilla almost articulated with the frontal; this specimen approached therefore the condition of direct fronto-maxillary articulation, such as I have previously referred to on page 94. In three skulls indications of an infra-orbital suture were present. The lower jaw had a feeble chin and shallow symphysis, the vertical diameter of the body of the bone was moderate, the coronoid process was short, and the sigmoid notch shallow. The cubic capacity of the crania was small; the males ranged from 1080 to 1270, with a mean 1202 c.c.: the females from 1080 to 1153, with a mean of 1106 c.c.

Although OWEN and BUSK had described a few crania, the late Sir WM. FLOWER made the most extensive research into the characters of the Andaman skull that has yet been conducted. He described* a series forty-eight in number, six of which were metopic, and as one of my specimens had the same character, it is obvious that a persistent frontal suture is not uncommon in the crania of this race. The mean length-breadth index of his specimens was 82·8. The height was less than the breadth, and the length-height index was 77·7. The mean gnathic index was 100 in the men, 102 in the women. The mean nasal index was 51·1, and the orbital index, though variable, had a mean 90·9. Both in FLOWER'S series and in mine the length-breadth index was brachycephalic; the height was distinctly below the breadth; the upper jaw was mesognathous; the nasal index was mesorhine or platyrhine; the orbits were mesoseme or megaseme; the cranial capacity was microcephalic. The number of specimens examined is so large as to justify one in saying that the leading characters of the cranium in these people have now been ascertained.

The series of Dravidian crania described in this Memoir differ in essential particulars from those of the Andaman Islanders, and the eye at once recognises the differential features, both in form and proportion. The measurements made by Mr THURSTON of the heads of the hill tribes in the Madras Presidency have shown the great majority to have a length-breadth index below 75, though a few ranged from 75 to 77·5; the south Dravidians, like those further north, have, therefore, heads of dolichocephalic proportions. Did we accept the view that a brachycephalic Negrito people preceded the Dravidians in the occupation of India, we could not, I consider, regard the latter, either in cranial configuration or external physical characters, as the direct descendants of the former. It might be argued that had there been a previous Negrito people, some amount of intermixture in times past of the two races might have taken place, which might have led to the production of a wavy or curly character of the hair such as has been seen in the tribes referred to on p. 114, and to the occasional presence of a skull tending to

* *Journ. Anthrop. Inst.*, Nov. 1879, vol. ix., and Nov. 1884.

brachycephalic proportions in some of the existing aboriginal Dravidian tribes, but the direct evidence of either a past or a present Negrito population in India has yet to be obtained.*

Sakai. TABLE X.

The name Sakai is given to aboriginal people dwelling in the hill regions in the Malay peninsula. Since the early part of the century certain tribes called Semangs have been described in Kedah to the north of Pînang and in Tringânû on the east coast. ANDERSON speaks of a native of Kedah as 4 ft. 6 in. in height, the hair woolly and tufted, the skin jet black, the lips thick, the nose flat, the belly protuberant as in the Andaman Islanders. J. R. LOGAN states that a tribe of Semangs in the hills opposite Pînang have a stature from 4 ft. 8 in. to 4 ft. 10 in., the nose with depressed root and spreading alæ, the skin dark brown though sometimes lighter, but black where most exposed.† The Russian traveller, v. MIKLUCHO-MACLAY, became acquainted with people named Orang Sakai in his journey through Johore in 1874–75. He stated that the hair consisted of very fine curls, arranged in a compact mass projecting for a short distance from the head, and formed a good guide to the purity of the race.‡ He regarded the people as Melanesians, though they approached the Negritos of the Philippines. The height of the men varied from 1450 to 1650 mm. (4 ft. 7 in. to 5 ft. 4 in.), and the heads were mesocephalic to brachycephalic. M. DE QUATREFAGES figured § from photographs natives, said to be Sakais from Perak, in one of whom the hair was smooth and in two others was frizzled. Mr ABRAHAM HALE has seen the Sakai people in the Kintah district of Perak, and has given an account || of many of their customs. He states that an ancient race the Semangs are also found in Perak, on the right bank of the Perak river, whilst the Sakais inhabit the left bank.

HALE did not describe the physical characters of the Sakai, but stated that their primitive dress consisted of bark cloth twisted round the waist and drawn between the thighs. The nasal septum was pierced to wear a porcupine quill or a bone, and the ears were often pierced. The women had the hair standing out from the head in a great mop; they wore bracelets, and ornamented the face and breast with red figures. The Kelantan Sakais from the north-east were finer-looking men than those in the Kintah district.

At the instigation of Professor VIRCHOW, Mr VAUGHAN STEVENS travelled in the eastern

* After this Memoir was in type I received, through the courtesy of Major BANNERMAN, M.D., the *Madras Christian College Magazine* for September and October 1900, in which is an article by Mr C. HAYAVADAWA RAU, B.A., on the origin of the Servile Classes and Hill Tribes of South India. In this article Mr RAU discusses, from the physical, social, linguistic and intellectual points of view, the Negrito theory of the origin of the Dravidians, and regards the theory as untenable. He draws the inference that all the indigenous tribes found by the Aryan immigrants in Southern India belonged substantially to one and the same Dravidian race.

† These accounts are abstracted in G. W. EARL's work on the Native Races of the Indian Archipelago, London, 1853.

‡ *Verh. der Berliner Ges. für Anth.*, etc., 1876 and 1891, p. 837; *Journ. of Straits Branch of Royal Asiatic Soc.*, 1878.

§ *Les Pygmées*, Paris, 1887, pp. 54, 55.

|| *Journal of Anth. Institute*, vol. xv. p. 285, 1886.

part of the Malay peninsula. He sent to Berlin specimens of the hair of a tribe which he called Blandass or Belendas, a name which he seems to use as synonymous with Sakai.* VIRCHOW states that the hair varied in length from 59 to 26 cm.; it was ebony in colour, the more slender examples being paler, and in a child pale reddish brown. In no specimen was it curly or spirally twisted, though at the tip it bent into a crescentic form. At a later date STEVENS forwarded specimens of the hair and a skull from the Panghan tribe (Panggan), on the east side of the peninsula. The men cut the hair close to the scalp, but left a tuft at the top of the occiput. The tuft was said to be of 'peppercorn' shape, and only 5 to 10 mm. above the scalp. The hair was black, slender and spirally twisted as in the Negrito, and could at once be distinguished from the hair of the Belendas tribe. The Semang tribe of Perak on the western side have apparently a similar tuft of hair, possessing the same character. VIRCHOW figures the skull, which was short, broad and high, hypsibrachycephalic; the length-breadth index being 81·5, the length-height 76·9. The glabella and supra-orbital ridges were prominent. The face was broad and low, chamæprosopic; the orbital index 80, was microseme; the nose was short, with a low bridge, mesorhine; the upper jaw was strongly prognathic; the cranial capacity was 1370 c.c.

In 1897 Dr R. MARTIN undertook a journey through the Malay peninsula with the object of seeing the wild tribes in the interior.† He distinguished the appearance of the Semangs, who live especially in the north and in part in the Siamese provinces, from the Sakais, who are found especially in Perak, Selangor, and the west of Pehang. The Semangs, he says, had black skins, black frizzled hair, and were doubtless closely allied to the Negritos of the Philippines. In the Sakai the skin of the breast and body was reddish brown in tint, whilst on the face it was a medium brown with yellowish brown shades; the hair was black, but with a brownish shimmer in certain lights, and throughout was wavy, which distinguished it from the frizzled hair of the Semang, and from the stiff hair of all Mongols, including the Malays. The stature of the Sakai men averaged about 1500 mm. (4 ft. 9 in.), that of the women 1420 mm. (4 ft. 6 in.). The head, from numerous measurements, had a mean length-breadth index 79; the face was broad, but pointed at the chin, mesoprosopic in its proportions, the nose had slight projection, but with broad alæ, which were deeper than the septum; the tegumentary part of the lips, especially the upper, was thick. They painted the face and breast with red, white and black spots, put hollow cylinders of bamboo into the ears and filled them with grasses, which formed a green frame around the face of the women. The men bored the nasal septum and passed through it a piece of wood or porcupine quill.

I am indebted to Mr NELSON ANNANDALE, who travelled in 1899 in the northern part of the peninsula, for photographs of a Sakai youth aged about 15, who lived in the Aring district, a hilly country in Kelantan, in the centre of the peninsula. He had

* *Verh. der Berliner Ges. für Anth.*, etc., November 1891, July and October 1892.

† *Mitteil. der Naturwiss. Ges. in Winterthur*, Heft ii., 1900.

been captured by the Malays as a child, and had been circumcised and brought up as a Mahommedan. His skin was dark, approaching black ; the forehead was almost vertical, the nose was short, with a low flattened bridge and wide alæ, the upper lip was thick and prominent, the facial configuration was negroid, but the hair, instead of being woolly or frizzled, was straight, and apparently three or four inches long.

In March 1891 I received from my former pupil, the late Dr W. Duncan Scott, an imperfect skeleton, which he believed to be that of a Sakai, with a letter giving an interesting account of the people. Dr Scott had accompanied his chief, Mr Abraham Hale, in his visit to a tribe of Sakais inhabiting the hill-tops above the Kintah river at a place called Tanjang Keukong. Dr Scott is the officer referred to by Mr Hale in the appendix to his account of these people.* Dr Scott writes as follows :—The Sakais occupy the hill country in the Malay peninsula as far south as the north end of Johore. The skull and bones were found in a valley watered by the Kampar river, a tributary of the Kintah river, about 25 miles from Batu Gajah. The hills are inhabited by scattered groups of Sakais. The bones were found on a rude platform, about 6 feet from the ground, in a lean-to hut under the shelter of a hill. The hut was made of boughs of trees, and the bones were further protected by a sort of cage of branches.†

The Sakais, he says, were an active, well-proportioned people, with stout muscular limbs, and of a sturdier make than the Malays. Their stature was probably on the average about 5 feet 2 inches, though some may be 5 feet 3 or 4 inches. The skin was lighter in colour than in the Malay, and but little deeper in tint than in the Chinese, though rather brown than yellow, and those who lived in the hills were lighter than those who occupied the low ground. The features, on the whole, were broad, but not markedly so, and the lips were not especially thick. The hair was black, and in those seen by Dr Scott was inclined to be long, wavy, reaching to the shoulders ; but in some tribes he says that it was stiff, slightly curled, and stood out like a mop around the head, whilst in the people who lived more to the south it was in short corkscrew-like curls. The eyes, as far as he recollects, were dark brown. The gait was peculiar, with a step and swing from the hip.

The younger women wore the Malay sarong round the waist and over the breasts ; the older women were generally content with a sarong or piece of bark cloth or fringe of fibrous roots around the waist, and with necklaces of shells, seeds, or monkeys' teeth. The men wore a loin-cloth made of bark, and on festive occasions they wound a strip of bark round the head. Many of the men ornamented the face with a white patch on the cheek, and the girls had the face covered with red and brown streaks. They carried on the back a light basket of rattan to hold fruit or small animals taken in the jungle. They obtained iron choppers, or parangs, from the Malays, but could not smelt the

* *Journ. Anthrop. Inst.*, vol. xv. p. 299, 1886.

† Mr Nelson Annandale has kindly given me photographs which he took of a Sakai rock shelter in Patalung which resembles the hut described by Dr Scott.

ore. Their weapons were spears of bamboo and the sumpitan with poisoned darts. Dr Scott also wrote an account of their religion, houses, dances, etc., but as this closely corresponded with the description already in print by Mr Hale, it is unnecessary to reproduce it.

The skull presented to me by Dr Scott is, I think, that of a man, apparently about middle life; the lower jaw is unfortunately absent.

In the *norma verticalis* the outline was broadly ovoid, with almost vertical side walls, not ridged, but flattened in the sagittal region; the parietal eminences were not prominent, and the skull was without the marked disproportion between the breadth of the frontal and parietal regions seen in the Andaman crania. The length-breadth index was 74·6, and the skull was dolichocephalic. The vertical index was 76·5, and the height was more than the breadth. The parietal longitudinal arc was much the longest. A shallow, vertical-transverse constriction, as if from the pressure of a band during infancy, was immediately behind the coronal suture. The parieto-occipital slope passed gradually downwards, and the occipital squama was rounded.

The glabella and supra-orbital ridges were distinct but not excessive, the forehead only slightly receded, and the frontal eminences were not prominent. The nasion was a little depressed; the nasal bones were small, concave forwards, and projected feebly at the tip. The nasal spine of the superior maxillæ was short. The anterior nares were wide, and the nasal index, 58·5, was strongly platyrhine. The floor of the nose and the incisive region of the jaw were separated by a shallow ridge. The upper jaw was orthognathous. The orbital index, 78, was microseme. Although the absence of the lower jaw prevented the complete facial index being taken, the short nasio-alveolar diameter, as compared with the interzygomatic breadth index, 34·5, gave a low chamæprosopic character to the face. The palato-maxillary region was broad in relation to the length, and the index was 125.

The teeth were not much though several had been lost during life, and the sockets were absorbed; their cro were smaller than in the Andaman Islanders. The sutural denticulations were short and relatively simple. A small Wormian bone was in the left parieto-mastoid suture, and in the left pterion was a large epipteric bone. The left jugal process was tuberculated. The mastoids were feeble, and the skull rested behind on the posterior border of the foramen magnum. The cranium was phænozygous. The cranial capacity was microcephalic.

From the examination of the bones of the skeleton, especially those of the limbs, it was evident that the person had been of low stature. The atlas was the only true vertebra which reached me.

Pelvis.—It was small in general dimensions: the alæ were not expanded or very translucent: the pectineal lines were not knife-like: the præ-auricular sulcus was distinct. The sacrum had a feeble anterior concavity: its index, 102, was platyhieric, but the length was almost equal to the breadth. The conjugate diameter of the pelvic brim was distinctly greater than the transverse, and the brim index, 108·5, was highly dolicho-pellic. The highest indices which I had previously recorded* were in a male Australian 116, a male Bushman 109, and a male Malay 105. The highest brim index in the male Andaman pelves which I have measured was 102. The want of expansion in the iliac fossæ was shown by the small breadth between the crests of these bones. The width of the pubic arch, with its angle 80°, gave a feminine aspect to the pelvis which led me at first to doubt, notwithstanding the cranial characters, if the skeleton were that of a male. Of the numerous pelves which I have measured in the female sex, no specimen up to this time has shown the conjugate diameter to exceed the transverse, whilst in the males of savage races this is not unfrequent. In the Sakai pelvis the conjugate was so much in excess that I regard it as confirmatory evidence of the skeleton being of the male sex. I may also state that in a male pelvis in each of the following races I have measured the subpubic angle as follows:—Andaman, 78°; Chinese, 76°; Malay, 76°; Bush, 72°.

Measurements of Pelvis.

	mm.
1. Breadth of pelvis,	211
2. Height of pelvis,	164
3. *Breadth-Height Index,*	*77·7*
4. Between ant. sup. iliac spines,	193
5. Between post. sup. iliac spines,	80
6. Between ischial tubera,	126
7. Vertical diameter of obturator foramen,	38
8. Transverse diameter of obturator foramen,	31
9. *Obturator Index,*	*81·6*
10. Subpubic angle,	80°
11. Transverse diameter of brim,	106
12. Conjugate diameter of brim,	115
13. *Pelvic or Brim Index,*	*108·5*
14. Intertuberal diameter,	107
15. Depth of pelvic cavity,	72
16. Length of sacrum,	94
17. Breadth of sacrum,	96
18. *Sacral Index,*	*102*

Upper Limb.—The Clavicles were slender bones, feebly curved, and with faintly-marked grooves for the subclavius muscles. The right bone was 120 mm., the left 123 mm. long. The Scapulæ were small in their dimensions, with well-marked muscular impressions indicative of strong muscles; the axillary border was concave in its long diameter, the supra-scapular notch was shallow. The right bone was 122 mm. long and 83 broad, its scapular index was 68; the left bone was 123 mm. long and 80 broad, its index was 65. The mean index of the two scapulæ was 66·5, which is less than the mean of 69·8 obtained by FLOWER and GARSON from twenty-one scapulæ of Andaman Islanders, and of 70·6 by myself from six scapulæ of that race. The Humeri had strong muscular impressions and distinct musculo-spiral grooves; no intercondylar

* *Challenger Report* on Human Skeletons, part xlvii., 1886.

foramen or supra-condylar process was present. The bones of the forearm, though short, were well-proportioned and with distinct muscular impressions, but the styloid processes were feeble.

The dimensions were as follows :—

	Right.	Left.
Humerus, head to trochlea, ...	253 mm.	246 mm.
Radius to tip of styloid, ...	203 „	201 „
„ base „	200 „	199 „
Ulna to tip of styloid, ...	...	222 „
„ articular surface, ...	...	222 „

The radio-humeral index was 80·2, or dolichokerkic,* a proportion which these bones have in common with the Andaman Islanders and with the Negritos measured by MEYER and TÜNGEL and by HAMY, which expresses that the forearm was in its relation to the upper arm proportionately longer than is found in Europeans.

Lower Limb.—The right femur, tibia, fibula and tarsal bones had been sent to me. The Femur, though small, was well-proportioned, and with strong muscular impressions. The head showed the slight prolongation of the articular surface on to the upper part of the anterior surface of the neck, which I have elsewhere named the extensor area.† The upper end of the anterior intertrochanteric line was unusually strong, and indicated that the ilio-femoral ligament which takes so important a part in the maintenance of the erect attitude had been well developed. The gluteal ridge and the linea aspera were strongly marked. The flattening of the upper third of the shaft which I described in some aboriginal femora,‡ and which MANOUVRIER has subsequently termed platymery, was not present, and there was no external infra-trochanteric ridge distinct from the gluteal ridge. The transverse diameter of the upper third of the shaft was 23 mm., the antero-posterior 18 mm., and the index of platymery was 78. The transverse diameter of the shaft opposite the nutrient foramen was 20 mm., the antero-posterior diameter was 23 mm., and the pilastric index was 115, which expresses the relatively strong projection of the linea aspera. The articular surface of the internal condyl was not specially prolonged upwards and backwards.

The Tibia was well-proportioned. The head was somewhat retroverted ; the internal condylar surface was concave, the external was plano-concave. The shaft was not platyknemic; its breadth in the middle was 18 mm., its antero-posterior diameter 22 mm., and the index was 81·8. At its lower end the tibia had a well-marked astragalar articular facet, prolonged to the front of the bone. Associated with this was a corresponding prolongation of the upper articular surface on the astragalus, which was almost continuous with the anterior convex surface for the scaphoid. So well defined was this additional tibio-astragalar articulation that, as ARTHUR THOMSON and HAVELOCK CHARLES have shown, the ankle joint must have been frequently acutely flexed as takes

* For the use of this term see my *Challenger Report* on Human Skeletons, part xlvii., 1886.

† Address to section of Anthropology in British Association *Reports*, Toronto, 1897.

‡ *Challenger Report*, *op. cit.*, page 97.

place in the attitude of squatting.* The Fibula was well marked in its surfaces, borders, and muscular impressions.

The long bones had the following dimensions :—

	Right.
Femur, maximum length,	368 mm.
,, oblique length,	365 ,,
Tibia condylar surface to tip of malleolus,	299 ,,
,, ,, astragalar surface,	295 ,,
Fibula, maximum length,	299 ,,

It will be observed that the tibia and fibula are of the same length. The tibio-femoral index, calculated from the oblique length of the femur and the condylo-astragalar length of the tibia, was 80·9, and the leg, therefore, scarcely reached dolichoknemic proportions; a result similar to that which I obtained from the measurement of four skeletons of Andaman Islanders, in which the mean index was 81·2. An index of the relative length of the upper and lower limbs, called intermembral index, has been obtained by the following formula $\frac{\text{humerus} + \text{radius} \times 100}{\text{femur} + \text{tibia}}$, in which the maximum length of the bones was taken. In the Sakai skeleton this index was 68·3, which is somewhat less than the mean 68·9 obtained some years ago by FLOWER and myself from the measurement of a number of skeletons of Andaman Islanders. In both these people this index is relatively low, and points to the bones of the shaft of the upper limb, being short in comparison with those of the lower limb. The relative lengths of the humerus and femur have been calculated by the formula $\frac{\text{humerus} \times 100}{\text{femur}}$, and the index is 68·7, a number which is less than the mean 70 obtained by FLOWER and myself in the Andaman Islanders.

With the view of obtaining an estimate of the stature of the person whose skeleton I had examined, I compared the length of the femur and tibia with that of the corresponding bones of a male Andaman islander in the University Museum,† whose articulated skeleton was 4 feet $6\frac{1}{2}$ inches (1385 mm.) in height. The oblique length of the femur in this skeleton was 385 mm. ($15\frac{1}{10}$ ins.), and the condylo-astragalar length of the tibia was 322 mm. ($12\frac{6}{10}$ ins.)—together, 707 mm. ($27\frac{7}{10}$ ins.): whilst in the Sakai skeleton the corresponding diameters in the two bones were together only 660 mm. (26 ins.), which points to a stature about two inches less than that of the Andaman islander.

A short time after the receipt of the skeleton of the Sakai, Dr DUNCAN SCOTT presented me with a skull found in the jungle in Ulu Pahang, on the eastern sea-board of the Malay peninsula, immediately to the north of Johore. The collector from whom Dr SCOTT received it could not say whether it was a Sakai or a Malay, but

* *Journal of Anat. and Phys.*, 1889–1894.

† I am indebted to Colonel CADELL, V.C., for the gift of this skeleton, which I have had articulated.

thought from the locality that it was the former. Although there is a doubt as to the race, I have thought well to give a brief description of it.

The skull had been injured, and there was no lower jaw; it was obviously that of a man; the loss of teeth and the absorption of the sockets gave the impression of an aged person, but the cranial sutures were unossified and scarcely denticulated. In the right coronal were two sutural bones, in the left pterion a small epipteric, and in the lambdoidal suture several small Wormian bones. In the *norma verticalis* the cranium was broadly ovoid, raised along the sagittal line, and sloping rapidly down to the parietal eminences, below which the sides were somewhat convex. Its length-breadth index was 79·4, a little below the brachycephalic numerical limit, and the vertical index was only 76·6,—so that in the proportions of length and breadth to height, it had the brachycephalic rather than the dolichocephalic character. The parietal was the longest of the longitudinal arcs. The actual length of this skull was 6 mm. more than the one just described, but its breadth was 13 mm. greater, which accounted for the higher length-breadth index. The parieto-occipital slope was gradual, and not more or less abrupt than one sees in the more characteristic brachycephalic crania; the occipital squama did not project much behind the inion.

The glabella and supra-orbital ridges were feeble; the frontal eminences were scarcely marked; the forehead receded a little; the nasion was not depressed; the nasal bones slightly projected, and the bridge was shallow; the anterior nares were wide, but the height of the nose, 51 mm., brought the index into the mesorhine group; the nasal spine of the superior maxillæ was feeble. The absorption of the incisive alveoli made it impossible to determine the original projection of the jaw, and the gnathic index, 94·9, is only approximative. The broken zygomata prevented the width of the face from being taken. The cranial capacity was mesocephalic.

Although much remains to be done to complete our knowledge of the inhabitants of the Malay peninsula, it is obvious that in addition to the Malays, who dwell on the sea-coast, and the Siamese invaders in the northern provinces, whose appearance in the peninsula is probably of relatively recent date, the hill-districts are peopled by tribes who, in their external characters and cranial configuration, differ from each other. From the preceding narrative it will be seen, that whilst some tribes named Semangs and Panghans have the black skin and frizzly hair characteristic of the Negritos, in other tribes the skin is not so dark, and the hair, though black, is not frizzly or woolly, but is relatively straight and several inches long. Travellers do not always differentiate by descriptive names the straighter-haired from the frizzly-haired people, and by some the name Sakai is employed to designate both varieties of aborigines who dwell in the hilly and jungly districts. If the frizzly-haired, black-skinned Negrito people are the aboriginal inhabitants, those with straighter hair doubtless also represent an ancient race. The question, however, naturally arises, whether there may not have been in the course of centuries an intermixture and cohabitation of the Negrito race with the straight-haired Malays from the sea-board, as well as with the straight-haired Siamese who have entered the peninsula from the north, so as to lead to a modification in the physical characteristics of the people and the production in certain districts of a mixed race.

As regards the cranial configuration, the skull of the frizzly-haired Panghan, described by Professor Virchow, was brachycephalic; and the figure which he has reproduced obviously represents a type of skull resembling that of the Andaman Islanders. The skull form, therefore, confirms the view of the presence of a Negrito people in the Malay peninsula.

Of the two skulls which I have described, the one from the Kintah district, from its locality and the nature of the interment, must be regarded as of an aboriginal race and not a Malay. The skull was dolichocephalic, a proportion which belongs neither to the Negrito nor to the Malay. From Dr Scott's description of the people, to whom he gives the general name of "Sakai," it would seem that the hill-tribes in this district had long and not frizzly hair, a skin not black but lighter in colour than the Malay, which, conjoined with the dolichocephalic skull, gave race characters differing materially from the Negritos. These people, however, have, like the Negritos, a low stature. The skull from Pahang, on the other hand, differed so materially in its proportions and general appearance from the Kintah specimen, that it cannot, I think, have belonged to the same tribe or race,—the proportion of the length-breadth index, though numerically mesaticephalic, 79·4, was essentially brachycephalic, though the parieto-occipital slope was not abruptly steep. In the form of the vertex and the proportions of the nose it differed from the Kintah skull, but its injured condition did not admit of a complete comparison being made. I hesitate, therefore, to give an opinion on the race to which it had belonged.

From the consideration of the whole question there seems to be little doubt that in the hill regions of the Malay peninsula two aboriginal races are met with, distinguished from each other by the colour of the skin, the characters of the hair, and the form of the cranium, though both possess in common a low stature.

EXPLANATION OF PLATES IV.–VII.

The Plates and Figures are numbered in sequence with those in Part I. of this Memoir.

For the Photographs from which the figures are reproduced I am indebted to Mr W. E. Carnegie Dickson, B.Sc.

FIG. 15. Gond, Godavery District, Central India, profile. Table I.
„ 16. The Same, full face. Table I.
„ 17. The Same, vertex. Table I.
„ 18. Kandh, Khoorda, Orissa, profile. Table I.
„ 19. The Same, full face. Table I.
„ 20. Bhúmij Tribe, Mánbhúm, ♂ æt. 30, profile. Table IV.
„ 21. The Same, full face. Table IV.
„ 22. The Same, vertex. Table IV.
„ 23. Tamil-speaking native of Madras, profile. Table II.
„ 24. The Same, full face. Table II.
„ 25. Uriyá, Baghmari Village, Orissa, profile. Table VI.
„ 26. The Same, full face. Table VI.
„ 27. Veddah, metopic skull, male, ♂ profile. Table IX.
„ 28. Veddah, Batticaloa, E. Coast of Ceylon, ♂ full face. Table IX.
„ 29. Múnda, Ranchi ♀, æt. 24, profile. Table III.
„ 30. Andaman Islander, ♂ profile. Table X.
„ 31. The Same, full face. Table X.
„ 32. The Same, vertex. Table X.
„ 33. Sakai, Malay peninsula, profile. Table X.
„ 34. The Same, full face. Table X.
„ 35. Section through skull of Juang, ♂, page 128. I.M., No. 443, Table V.
„ 36. Section through skull of Múnda, ♂, page 128. I.M., No. 26, Table III.

The Antero-posterior almost mesial sections show the contour of the crania and the radial measurements.

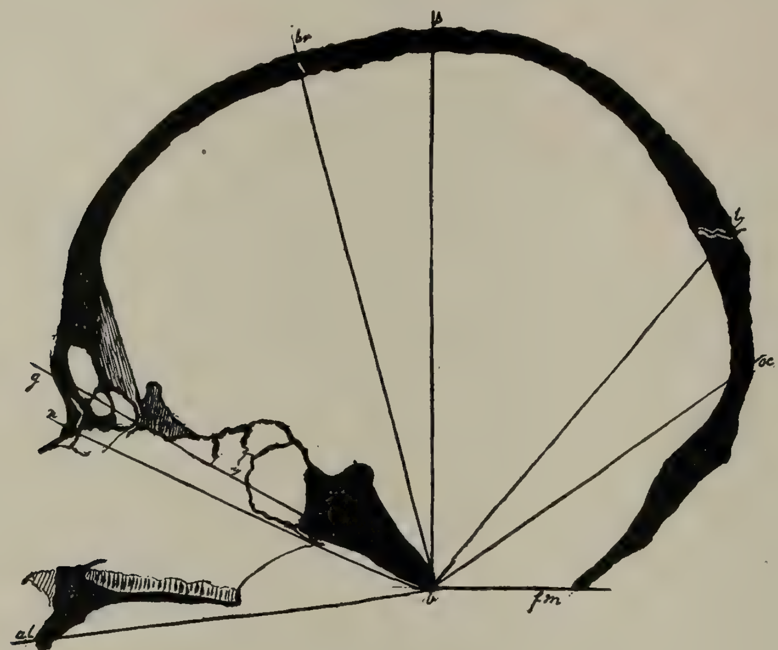


FIG. 35.—Juang.

f.m. plane of foramen magnum.

b. the basion: the lines drawn from which to the points on the circumference are radial from that point, and measure in millimètres as follows:—

	Juang.	Múnda.
b.al. basi-alveolar radius,	103	95
b.n. basi-nasal "	106	101
b.g. basi-glabellar "	111	111
b.br. basi-bregmatic "	142	128
b.p. a radial line perpendicular to the plane of the foramen magnum,	145	133
b.l. basi-lambdal radius,	120	116
b.oc. basion to occipital point,	101	94

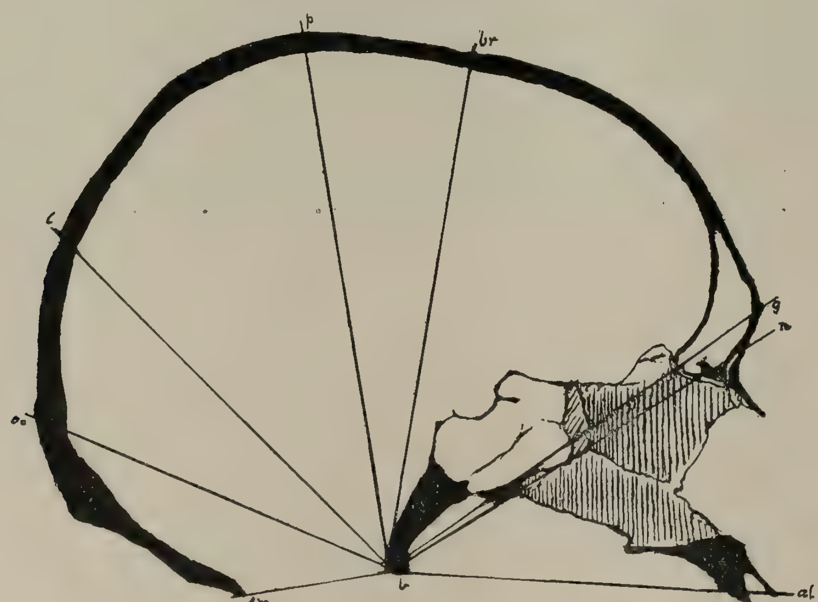


FIG. 36.—Múnda.

CONTENTS OF PARTS I. AND II.

SIR WILLIAM TURNER on "Craniology of People of India."—PLATE IV.

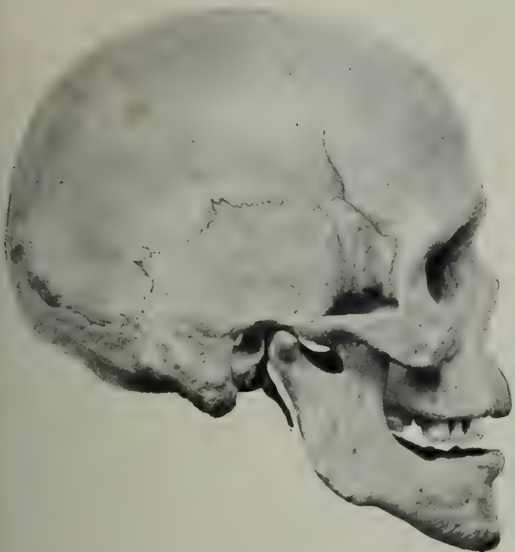


FIG. 15.—Gond.

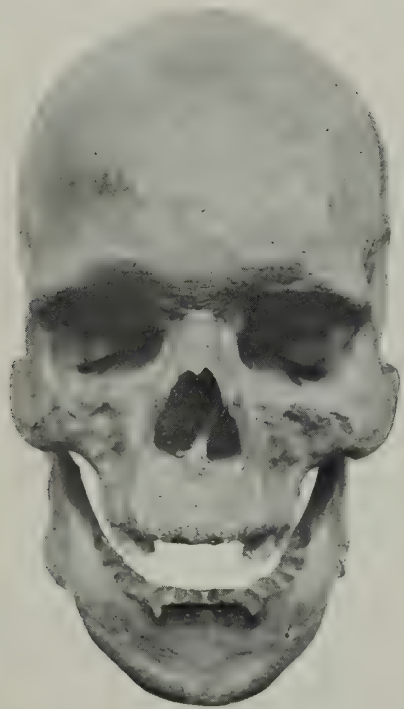


FIG. 16.—Gond

FIG. 17.—Gond.

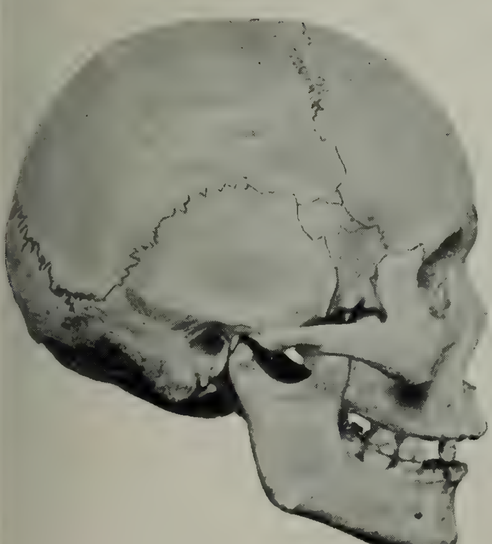


FIG. 18.—Kondh.

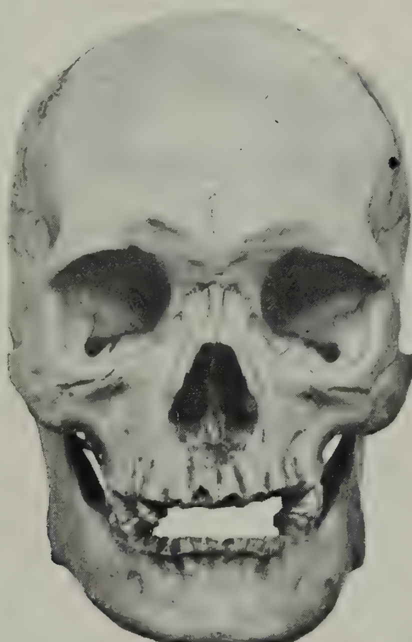


FIG. 19.—Kondh.

Sir William Turner on "Craniology of People of India."—Plate V.

Fig. 20.—Bhúmij.

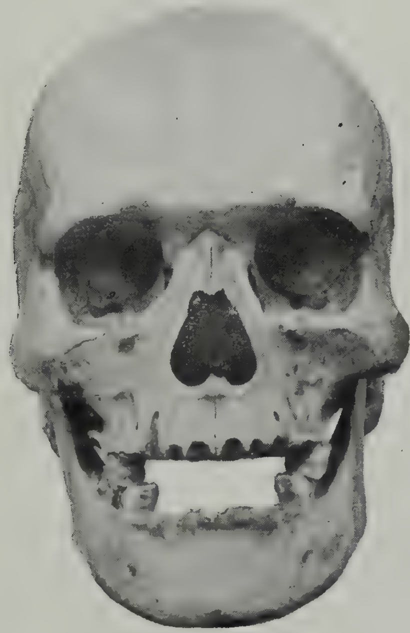


Fig. 21.—Bhúmij.

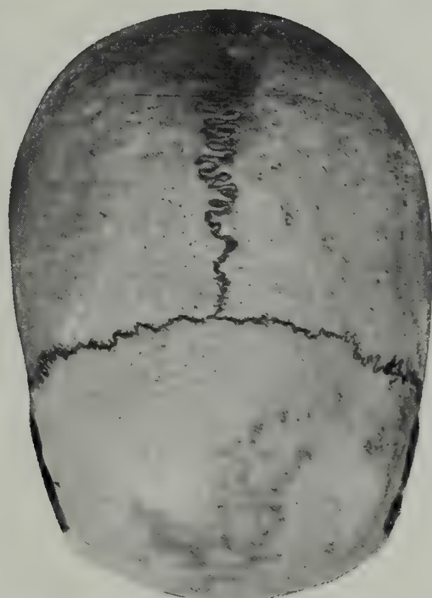


Fig. 22.—Bhúmij.

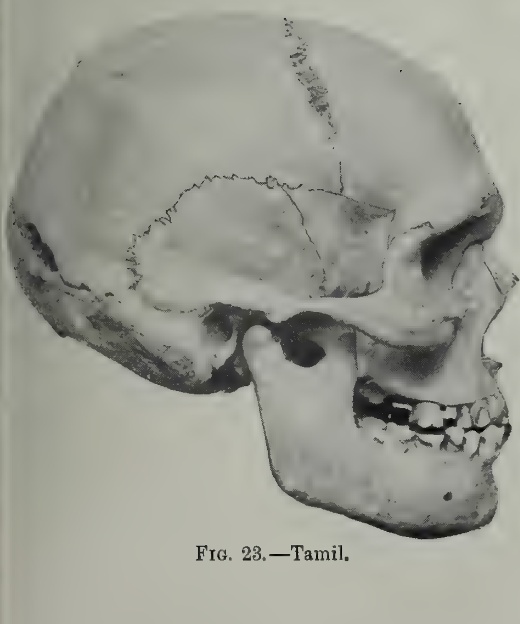


Fig. 23.—Tamil.

Fig. 24.—Tamil.

SIR WILLIAM TURNER on "Craniology of People of India."—PLATE VI.

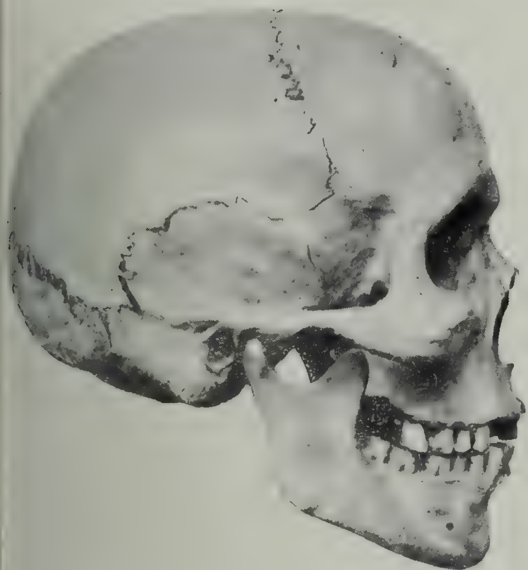


FIG. 25.—Uriyá.

FIG. 26.—Uriyá.

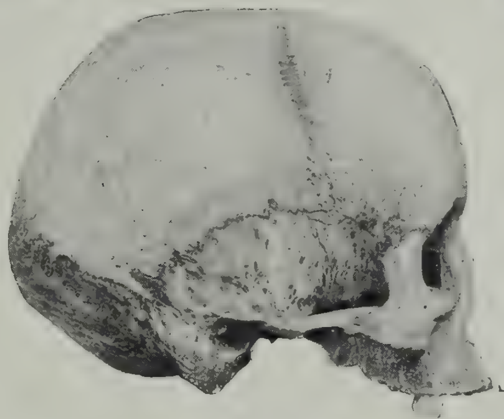


FIG. 29.—Múnda.

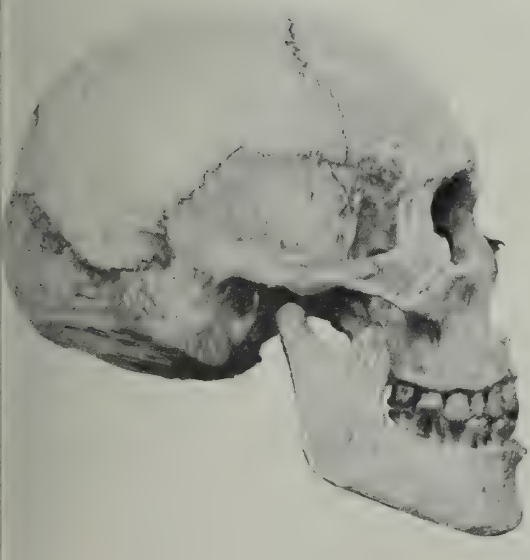


FIG. 27.—Veddah.

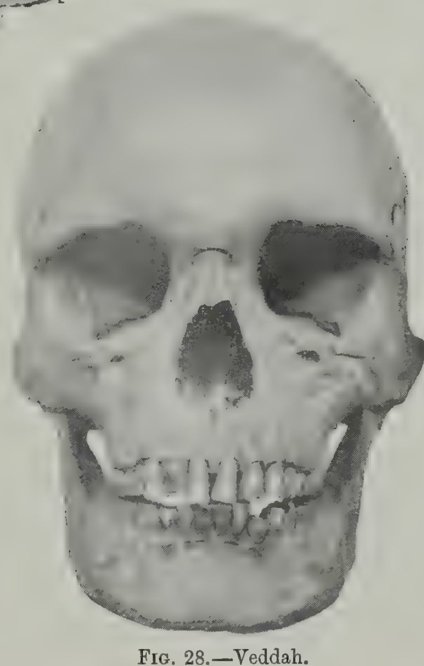


FIG. 28.—Veddah.

Sir William Turner on "Craniology of People of India."—Plate VII.

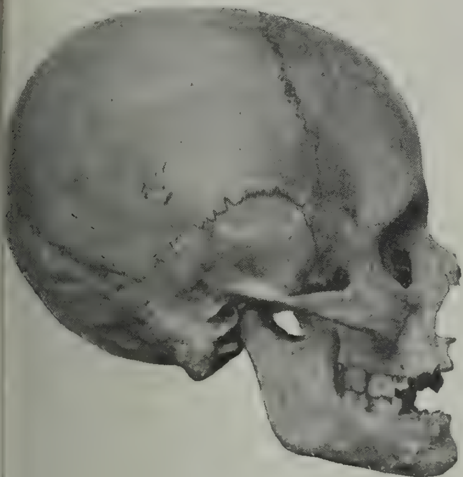


Fig. 30.—Andaman.

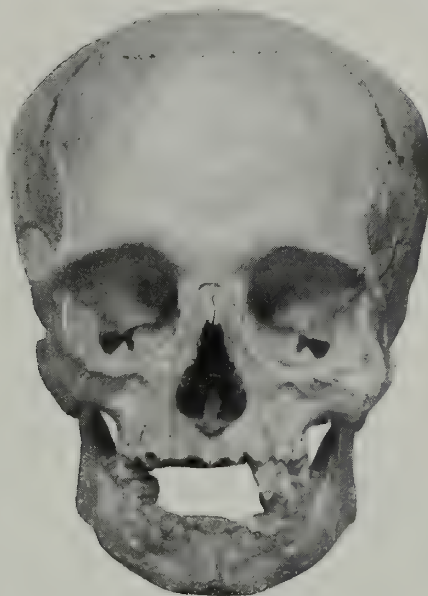


Fig. 31.—Andaman.

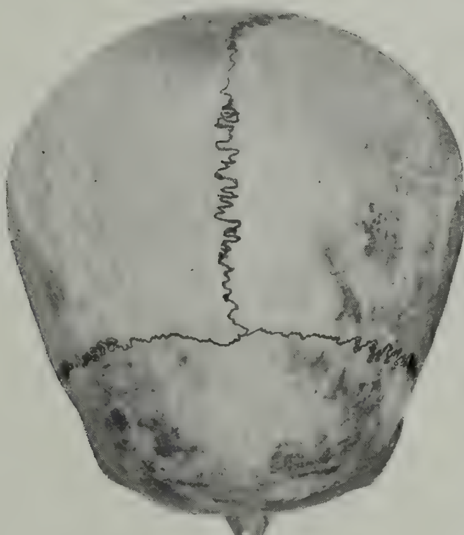


Fig. 32.—Andaman.

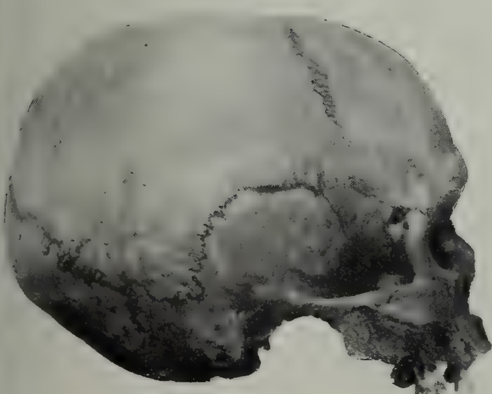


Fig. 33.—Sakai.

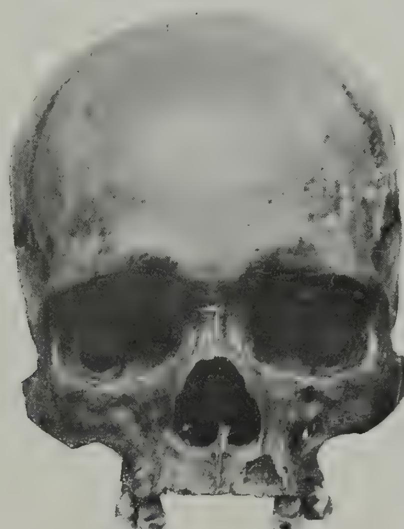


Fig. 34.—Sakai.

VII.—*Notes on the Dynamics of Cyclones and Anticyclones.* By John Aitken, F.R.S. (With a Plate.)

PART I.

(Read March 5, 1900.)

The vertical movements of the earth's atmosphere from which the energy is derived which causes the horizontal movements of the air which we call winds, and by means of which the moisture evaporated from the surface of land and water is collected and carried to the higher regions of the atmosphere, where it is condensed to cloud and again distributed in the form of rain over the earth's surface, are of great interest, and a thorough knowledge of the laws governing these vertical movements is necessary to enable us to arrive at a correct forecast of the coming weather over any area.

In the present communication I do not intend entering on a review of the work which has already been done in this field. Many explanations have been offered of the movements of cyclones and anticyclones as a whole, and of the winds within their areas, but any detailed reference to these would far exceed the limits of these notes, and would, I fear, only complicate matters. In what I have to say there will necessarily be much that is old, and I am sorry I must leave to the reader the task of finding out what is new, as in a subject of this kind, on which so much has already been written, it is impossible to say whether any particular point has not been referred to before by some other writer. And further, I shall confine my remarks to what takes place over our area and Western Europe, so as to avoid unnecessary verbal complications; but the principles can be easily applied to other areas in the Northern hemisphere, and to the reversed direction of circulation in the cyclones and anticyclones in the Southern hemisphere.

At the outset it will be as well for me to make a few elementary remarks on the formation of cyclonic movements, as I find that many who take an interest in Meteorology have rather hazy ideas of how the vortex motion in cyclones is produced. All that some seem to think necessary is to have an area of low pressure and that the air will rush towards it in spiral paths, just as they see water in a wash-hand basin forming a vortex movement whenever the plug is withdrawn and the water allowed to run away. Now it must be clearly understood that no vortex will form in air or water that is at rest before the low-pressure area is formed; the air or water under these conditions will flow to the low-pressure centre along radial paths and not in spiral ones. The above statement requires qualification. When it is said the air or water is at rest it is not meant that it is at rest absolutely, which would be an impossibility in this rotating, revolving, and space-travelling world of ours. All that is necessary is that the water or air be at rest relatively to the centre of low pressure—that is, that the centre of low pressure and the air surrounding it are all moving in the same direction and at the same velocity. Another qualification necessary in the case of the air on the earth's surface is, that the area of low pressure be very small, otherwise the different parts of the area will have different rates of movement owing to the earth's rotation causing its surface and the air near it to move faster near the equator than near the poles, so that the air to the south of the low-pressure centre moves more quickly in an easterly direction than the air to the north of it—that is, supposing there is no wind.

Let us now return to the question of the cause of the cyclonic movement. That motion relatively to the low-pressure centre is necessary to produce the spiral approach of the air or water to the centre is easily illustrated. Take a circular vessel—say 40 cm. in diameter and 15 cm. deep—filled with water. The vessel should have an opening in the centre which can be closed from the outside, so that it can be opened without disturbing the water in the vessel. Put some sawdust, or similar substance, in the water and mix it with it. Now leave it at rest till all movement in the water ceases, then open the outlet in the bottom of the vessel and allow the water to run away, when it will be seen that the sawdust suspended in the water moves towards the outlet flowing from all directions, and at all depths, in radial lines, without a trace of vortex movement.

Let the conditions be now changed and a slight circular motion be given to the water before the outlet is opened. A well-formed cyclone will now be formed by the outflowing water, and the quicker the initial movement the greater will be the number of turnings in the spiral path of the water before it arrives at the outlet, and the deeper the hollow in the surface of the water over the outlet, while the direction of the spiral movement will be the same as the initial movement given to the water. The depression of the surface of the water at the centre of the vortex is extremely interesting. If the water before the outlet be opened be not perfectly at rest, a faint depression will always be detected over the outlet, having in its centre a quickly-rotating vortex of very small area. If, however, the initial motion be greater—but it need not be quick—then the depression deepens and forms an air-tube extending to the bottom of the vessel, showing that the slow initial circular motion has enabled the low-pressure area to generate a velocity at the centre of the cyclone sufficient to enable the centrifugal force of the water at that part to withstand the hydrostatic pressure at the bottom of the vessel.

On examining these water vortexes there is a point that strikes one as very remarkable—namely, the great increase in the velocity of the water as it approaches the centre of movement. Not only is the angular velocity greatly increased owing to the shorter path required to complete a revolution near the centre, but the absolute velocity is also greatly increased. Perhaps this point can be more easily seen by making the experiment in another form, and using solids in place of liquids. Suspend two balls, A A, fig. 1, by long fine wires, either from the same point of suspension, or, to reduce the complications due to the effects of gravity owing to the balls falling a short distance as they approach the centre, suspend the balls from the opposite ends of a short beam B, suspended by fine wires, C C, as shown in the figure. The upper ends of the wires are fixed to a swivel D, to enable the balls to be put into circular movement round their common centre. To each of the balls a short cord E E is attached. These cords are passed through a small ring F, which is suspended at the level of the balls by the wire G fixed to the swivel D. The two cords E E are joined below to another swivel H, which has a cord attached to its lower end, to allow the balls to rotate round G, whilst the lower cord is held in the hand. If we now put the balls into a slow circular movement round their common centre, and take hold of the lower central cord below the swivel H, note the rate of revolution and pull the cord downwards. This will draw the balls towards the centre of motion, in the same way as the particles of water are drawn into the centre of the vortex. When this is done it will be seen that the initial slow motion is rapidly quickened, and that when the balls are near the centre they are whirling round each other at a great velocity.

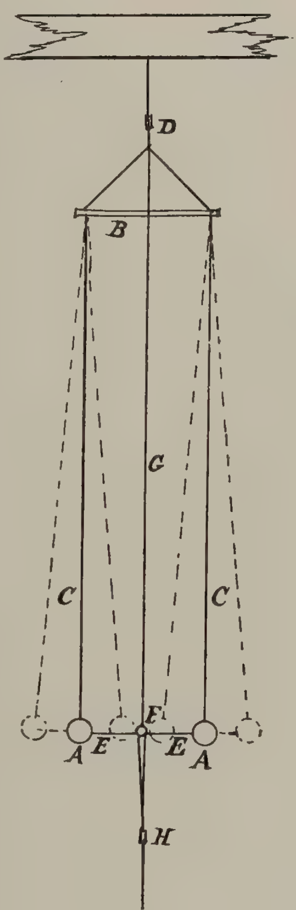


FIG. 1.

Another way of making this experiment is to remove the small ring F and its suspending wire, and tie the balls with a cord of about 50 cm. long, then attach to this cord, at a point halfway between the balls, a small rod held vertically. If the balls be now made to rotate round the rod, and the rod be prevented from turning, the balls will gradually wind themselves towards the centre, and it will be noticed when making the experiment in this way that whilst the angular velocity increases, the absolute velocity does not, showing that the mere fact of the balls approaching the centre has nothing to do with the increase in the velocity observed in the previous experiment. If now, in place of holding the centre rod firm, we cause it to rotate on its vertical axis so as to wind the balls towards the centre, it will now be seen that the absolute as well as the angular velocity is greatly increased, as was seen in the previous experiment, in which the balls were drawn together by means of the cords E E. The balls now fly round so quickly the eye can hardly follow them.

Further, it will be noticed that the balls offer a rapidly-increasing resistance to the centripetal force. This to a certain extent is to be expected, because as the balls approach the centre the centrifugal force increases; if the velocity is constant it is double what it was at first by the time the balls are drawn half way to the centre. But on account of the increased velocity which we cannot avoid giving the balls in the process, they are enabled to offer still greater resistance to the centripetal force. This, which might be called a secondary result, comes in and assists the centrifugal force in offering resistance to the change. There are plenty of similar results in physical phenomena, where these secondary results assist in resisting the change, as for instance, when we compress air. Suppose, as in the above case, we reduce the volume to one-half, then we have doubled the pressure, which seems fair enough. But the air, with almost human-like dislike to compulsion, heats in the process, and by so doing helps to resist the compression till it has had time to cool. These remarks may suggest something as to the great power of vortex movements for storing energy.

The cause of the great increase in the velocity in the balls when they are drawn towards the centre is due to work having been done on them. When the balls are revolving round their common centre, held by the cord at a fixed distance, no energy is communicated to them by the tightened cord, because the force acts at right angles to the direction of movement. But when the balls move in a spiral path, then only one component of the radial force acts at right angles to the direction of motion, whilst another is in the direction of the movement, thus tending to increase its velocity. The same explanation applies to the increased velocity of air or water approaching the centre of cyclones.

This resistance offered by the spirally-moving air to be drawn into the centre of the cyclone has one important effect. It enables the cyclone to develop more energy than if the air moved inwards radially. When the air moves in radially the supply is so abundant that a great fall of pressure cannot take place, but when the inflow of the air is retarded by the centrifugal force, a greater fall of pressure results, and the energy of the cyclone is increased; and in addition to this we shall see later that it adds greatly to the efficiency of cyclones considered as circulating engines.

It may perhaps be as well that I give my reasons for saying that whilst the vortex motion gives rise to strong currents—that is, increase of velocity—it yet retards the inflow, and by so doing it will allow a lower pressure to be formed in the centre. This point can be most easily illustrated by means of water vortexes. The circular vessel already referred to was used, the outlet being a short pipe fixed in the centre of the vessel. In each experiment the vessel was filled with water to the same depth, and the time observed that the water took to empty when it had no motion before starting to empty—that is, emptied out without vortex motion—and when a slight movement was given so as to cause it to form a cyclone when emptying. The result was the water took a half longer in the latter case than in the former, showing a great retardation due to the cyclonic motion, and this retardation results in a lowering of the pressure at the centre, and it also prevents the low-pressure area being so rapidly filled up. Care was of course taken in the above experiments that the pipe carrying away the water was always full in both cases, so that the head of water should be the same in both. This was done by placing a glass disc over the outlet and at some distance above it, and making the outlet pipe discharge below water. By these means air was prevented entering either end of the discharge pipe.

For making experiment on the cyclonic movements in air, I have found the following piece of apparatus useful. To produce the up-draught a thin metal tube 15 cm. diameter and fully 2 m. high was used. At the lower end of the tube is fixed a circular disc 76 cm. diameter. The disc is supported on three legs, 15 cm. high, thus leaving an air space of 15 cm. between the disc and the table on which it rests. To produce the up-draught three small jets of gas are fitted inside the tube near the lower end. To study the circulation produced by the apparatus, a number of small light vanes supported on small stands were used, so that they could be put to show the direction of the air currents anywhere within the area affected. These vanes show the directions of the circulation at the different parts of the area affected by the cyclone, but for studying the variations in the circulation at different heights from the surface of the table, the fumes from hydrochloric acid and ammonia were found to be more useful. These fumes are very suitable for the purpose, as they have only a very slight proper motion of their own, rising only very slowly, so that their own movements do not interfere much with the cyclonic motions. When working with fumes a large plate of glass should be placed below the apparatus to give freedom for experimenting, and the fumes are best made by placing pieces of paper on the glass and dropping the ammonia and acid on them. These papers, being close to the surface, do not give rise to local eddies.

Before lighting the gas to make an experiment, some fumes should be made under the tube, to see if there are any air currents in the room. Suppose there are none, and the fumes rise slowly without drifting in any direction, the gas should now be lighted and a number of centres of fumes should be started at different points under and round the tube, either by dropping the acid and ammonia on separate pieces of paper, or by putting them in watch glasses. If we observe the fumes under these conditions, it will be seen that they rise and move radially towards the hot chimney, moving upwards in even curves, but showing no tendency to rotate round the centre. An element is still wanting to produce the cyclonic motion, the air must be given some initial movement before the up-draught will take the spiral form. Suppose that on first testing the air on the table, we found that it was not free from movement, but that there was a slight current blowing across it. If this current be equally strong at all points, it will not be of much use in generating cyclonic movement, but if we put up a screen so as to cut off the current from one side of the area and allow it to blow on the other, then we have the conditions necessary for producing vortex motion under the chimney. If we examine the fumes rising under these new conditions, it will be seen they no longer move radially but are in violent cyclonic motion, swirling round and round in the direction given by the tangential current, the rising fumes forming graceful ascending spirals. So strong is the circular motion that at times the gas in the chimney is heard flaring as in a strong wind.

If there is no cyclonic motion formed when the gas is lit, any light objects lying on the table are not disturbed by the radially moving air, but if a good cyclonic circulation is set up, then any light bodies, such as sawdust, paper, or tufts of cotton wool, which happen to be lying under the chimney, are seen to be lifted up and tossed about, generally getting thrown out of the centre of the cyclone by their centrifugal force, but the cotton wool is frequently drawn up the centre and discharged at the top of the tube. Should there be no current in the room suitable for producing the tangential motion, then the cyclone may be formed by blowing by artificial means across one side of the area, or over two or more sides if in correct directions.

Let us now study more closely the movements of the air under the draught tube. Suppose we have it arranged so that a gentle current is blowing across one half of the cyclonic area, and the air is forming a cyclone under the tube, we shall now place the small vanes—already referred to—within the area of the cyclone, to show the directions of the movements of the air—that is, the winds at the different parts of the cyclone. The result is roughly shown in fig. 2. The air current over the table is supposed to be blowing in the direction of the arrow at the side, and to be strongest on that side of the area, and the cyclonic movement is roughly indicated by the arrows in the diagram. It is impossible to get these directions satisfactorily indicated by the vanes, owing to the difficulty of keeping the cross current constant, and the amount of curving of the spirally-moving air is constantly changing with every change in this tangential component. The arrows, however, show that on the side where the tangential current is strongest the in-draught is most tangential, whilst on the opposite side the incoming air moves more radially, or, to put it more generally, the air in the right-hand front of cyclones in our area moves more tangentially than the opposite left-hand rear position. It should be noticed here that these remarks apply to the case where the centre of the cyclone is fixed. Some modification will be necessary when applying them to cyclones in which the centre of low pressure is in motion.

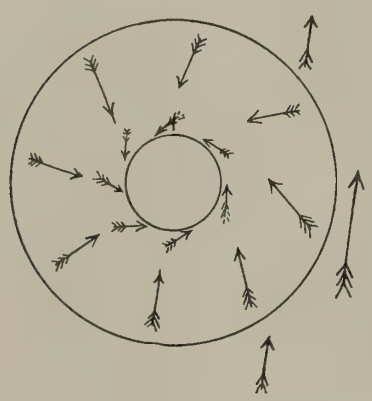


FIG. 2.

We may learn something further with this apparatus if we use fumes to study the movements of the air over the different sections of the cyclonic area. One very marked result which will be observed is that the air—as shown by the fumes—near the surface of the table at all parts of the area moves much more radially than the air higher up, and also that the air lying on the surface of the table is drawn into the very core of the cyclone, up which it rises in a rapidly-circling path of small diameter, whilst the air higher up comes towards the centre along a rising wider spiral path, and forms the outer lining of the cyclonic tube. The lower air keeps near the surface till it arrives near the centre of the cyclone and then rises, making many more revolutions than the outer air for the same amount of ascent, as roughly indicated in fig. 3. This in-draught of the lower air into the core of the cyclone is one of great importance in connection with meteorological phenomena, and will be referred to later on. The reason for the lower air being drawn into the core is very obvious. The air near the ground, or surface of the table in this case, has less tangential movement than the air higher up, owing to its motion being retarded by friction. The result of this is that whilst the greater centrifugal force of the upper air keeps it back against the low pressure in the cyclone, the lower air as it moves nearly radially offers but little resistance to the in-draught. Hence the lowest stratum of air being at the lower end of the cyclonic tube, it is drawn into the very centre.

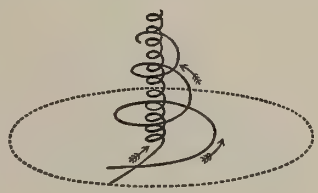


FIG. 3.

Anyone can make similar experiments to these without a special draught tube, and study for themselves the conditions necessary for the formation of cyclonic circulation. All that is necessary is a good fire, with a free-going chimney, and a wet cloth. The cloth is hung up in front of the fire and pretty near it, so as to cause a liberal amount of condensed steam to rise from its surface. Probably without some arrangement of the draughts in the room no cyclone will be formed, and the vapour will rise vertically, keeping close to the wet cloth. But if the room has a door or a window in the wall at right angles to the fireplace, and at the side nearest the fireplace, so as to cause the air coming from it to make a cross current past the fire, then a cyclone will be formed and the steam on the wet cloth will be seen circling round, and when the cyclone is well formed, all the steam is collected into the centre of the cyclone and forms a white pillar extending from the cloth to the chimney. If the cross-draught be equally strong at top and bottom of the wet cloth, it will be necessary to screen the current from either the top or bottom of the cyclonic area. Should there be no suitable draught in the room, then an artificial one may be made in any direction across one side of the wet area.

We have seen from these experiments that no cyclone can form without some tangential movement in the air entering the area of low pressure. The next question one naturally asks is—Has this tangential motion any other effect on the cyclone? From the fact that some of the air entering the cyclone has motion in a particular direction, we would naturally expect that unless an equal amount of air, moving in the opposite direction, also entered at the same time on the other side, that the whole system would move in the direction of the greatest tangential force. If we examine the cyclone produced in our artificial conditions with the draught tube we shall find that this is the case. The lower end of the cyclone bends away from under the centre of the apparatus, moving in the direction of the tangential current.

This point can, however, be better illustrated by means of water vortexes. Using the circular vessel as before, and in order to make the tangential current on one side stronger than on the other, all that is necessary is either to put the outlet for the water excentric to the walls of the vessel, or more easily by taking a strip of thin metal, having a breadth the same as the depth of the vessel and of a length of a little more than the diameter, and bend it into its place in the circular vessel, as shown in fig. 4. When the water now runs out it all passes round the vessel, and as the passage between the outlet, that is the centre of the cyclone, and the temporary division is narrower than at any other place, the water has to pass this part at a much greater velocity than at any other place, and the result is, the top of the vortex no longer remains over the outlet, but travels in the direction of the quickest moving water, showing that, as might be expected, the quickest moving water tends to carry the centre of the vortex along with it. See fig. 4, where the small dotted circle represents the outlet and the full circle the top of the vortex, which is carried by the water to the right, the direction of movement of the water being shown by the arrows. In addition to moving in the direction of the strongest current, the centre seems also to be attracted towards the quickly-moving area, but to this point I shall not further refer at present.

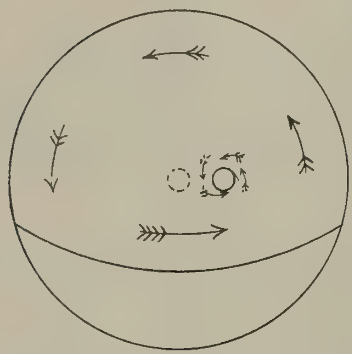


Fig. 4.

Turning now from these experimental observations, let us see how far they help us to understand the phenomena in cylones and anticyclones in our atmosphere. At the outset I may say that in forecasting attention seems to have been given too exclusively to what takes place in cyclonic areas, and too little to the part played by anticyclones. We have been looking too much on the cyclone as the active member of this dual partnership. But I think we will have to admit that the anticyclone is not the sleeping partner it is generally supposed to be, whose only duty is to follow and fill up the depressions made by the active partner. A closer examination of the part played by the anticyclone will show that it also is an active though silent partner in the firm, and that it initiates and keeps up its own circulation, and collects and forwards the material with which the more showy partner, the cyclone, makes its display. The anticyclone also in a great measure directs the path of the cyclone and adds much to its cyclonic motion. In fact the vertical circulation seems to be kept up by the partners in this dual system playing into each other's hands, and neither could work efficiently without the other.

Let us now look at what the effect would be if there were no cyclones or anticyclones in our atmosphere. Suppose a large area, say, some hundreds of miles in diameter, on the earth's surface over which the air was still, suppose further that the sky was cloudless and a summer's sun was shining. After a time the conditions would become unstable and columns of air would begin to ascend at many points all over the area wherever the air happened to be most highly heated and charged with vapour. The result of this would be a disorderly mob of ascending currents without leader, organization, or combined effort. These small isolated attempts at vertical movements could never give rise to anything like a systematized and law-abiding circulation, though they might result in local disturbances such as thunderstorms and rain.

Let us now suppose the conditions are changed and that an anticyclone is blowing at one side of our supposed hot area, and note the change. When the upper air descends to the earth's surface, it spreads out in all directions, and if the earth had no motion of rotation the air would spread in radial paths from the centre—that is, supposing the descending column of air had no motion of its own, which is very unlikely. But as little is known on this subject, we must at present neglect it, and may safely do so, as probably even if it has motion it will not greatly affect the result. Suppose that the descending air arrives at the surface of the earth and spreads outwards in all directions, then, owing to the earth's rotation, the air in place of moving radially from the centre, goes in spiral paths. This is owing to the air that moves in a northerly direction blowing over a part of the earth that has a slower easterly motion than the descending air which has come from the south, and the wind seems over the northern part of the anticyclone to blow more or less from the west. On the other hand, the air moving southwards of the centre flows over an area going more quickly eastwards than the descending air, and the wind over that part of the anticyclone seems to blow more or less from an easterly direction, whilst the air moving east and west from the centre would appear only to have its velocity altered, were it not for other causes which come into play, and make the circulation from the anticyclone to be, in a general way, spiral all round the centre. As the air to the north of the centre moves eastward, and that to the south westwards, the circulation in the anticyclone is right-handed—that is, in the direction of the hands of a watch with its face upwards.

One effect of spiral circulation, to which we have already referred in the experimental part of the paper, is that the air near the surface of the ground where its motion is retarded by friction moves more radially than the upper air. This causes the higher air currents to cross the lower at a greater or less angle, and has the effect of checking the rising of the lower air even when lighter than the upper. In our atmosphere the case is more complicated than in the experiments, owing to the earth's rotation. Over certain areas of the anticyclone the upper and lower airs may be moving in nearly the same direction: this will be the case when the upper air is moving in nearly the same direction as the surface of the earth at the place. But even when the direction of the upper and lower airs is the same, the difference in velocity of the two has some influence in checking the lower air from rising.

Let us now see what the effect of such an organized circulation is in our atmosphere. Let the sketch fig. 5 represent an imaginary anticyclone, which we have shown circular, though the shape is generally far from being so regular. If such a system be established over our imaginary area of sun-heated air, then it will be seen that the anticyclonic winds will prevent the formation of local cyclones, and drive all the moist hot air to its circumference, keeping it near the ground. Suppose, now, that the hot air begins to form a cyclone at the outside of the anticyclone, as shown in fig. 6, it will be evident it has a supply of hot air collected for it by the anticyclone. But not only has the anticyclone collected the material for making the cyclone, but it also supplies the cyclone with the tangential force necessary for producing the spiral circulation so well known in cyclones.

The explanation generally given of the spiral movement in cyclones is, that the air drawn in from the south has a greater amount of eastward motion than the centre of the cyclone, whilst the air from the north has less, and that these oppositely moving airs from the two directions cause the whole system to circulate round the centre. This, no doubt, is one cause of the circular movement in cyclones, but it is far from

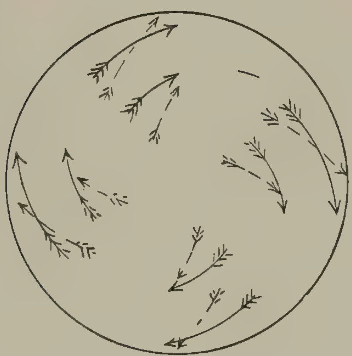


Fig. 5.

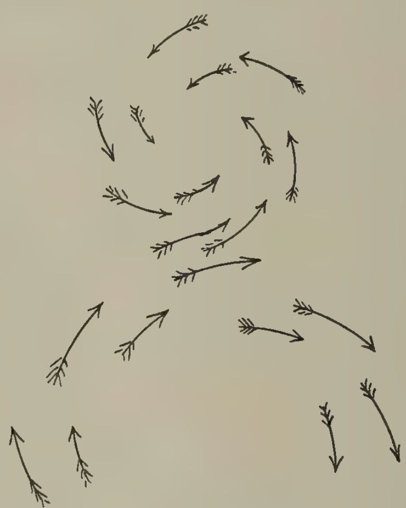


Fig. 6.

being the only one. This tangential action—due to the earth's rotation—only comes into play after the air has travelled a considerable distance towards the centre, whereas the air from the anticyclone already has a considerable tangential movement over the area between the cyclone and the anticyclone. The true explanation would rather appear to be that the cyclone and the anticyclone form one system; the anticyclone forces its air tangentially into the cyclone, whilst the cyclone draws air tangentially from the anticyclone. The earth's rotation is the original source of the rotatory movements, and starts the machine, but both intensify the initial motion; they are, so to speak, geared into each other and kept in motion by the earth's rotation, but both capable, when conditions are favourable, of developing energy on their own account and increasing their rate of rotation.

We saw in the experimental part that a cyclone in the conditions represented in fig. 6—that is, with strong winds blowing tangentially on one side—cannot remain in the same place, but must move in the direction of the strongest winds. It would be fatal for a cyclone to remain long in one place; it would soon use up all the supplies, and when all the hot moist air had been drawn in, it would be starved and get weaker, and the depression soon get filled up. So the cyclone has to move on for fresh supplies, and if our experimental illustration is correct, it will move along in the direction of the strongest tangential winds.

Now, if we look at the Synoptic Charts issued by the Meteorological Office, we shall see that they support this explanation, that these tangential winds are the principal cause of the movement of the cyclone as a whole, that the centre of depression moves in the direction and nearly parallel to these winds; or, in other words, the cyclone moves nearly parallel with the isobars at the side on which the barometric gradient is steepest. Further observation will show that the greater the difference in the barometric gradients on the two sides, the quicker is the advance of the cyclone, due to the tangential energy on the steep side being greater than on the other. The examination of these charts will also show that, whenever the isobars surrounding the cyclone become equally spaced on all sides, that is, when the barometric gradient is the same all round the centre—or even on the front and sides—that the cyclone travels very slowly and is generally in a dying condition. The air under these conditions comes to the cyclone with equal velocities from all directions, and there is no tendency for the centre to move. The tangential force is still in great abundance, the winds blowing in from all directions systematically all round, but there is no unbalanced tangential force, and the centre remains over the same area, and as the hot moist air—the source of the energy—is soon used up, the winds tend to fall and the depression to fill up. This seems to be the history of many of the cyclones with circular and concentrically placed isobars, though in winter they seem occasionally to linger for a time, their winds then seem to depend a good deal on the energy of surrounding anticyclones. It also seems possible that the movements of such cyclones may be determined a good deal by differences of temperature over the anticyclonic areas.

In illustration of these points we shall now refer to some actual examples taken from the Synoptic Charts issued weekly by the Meteorological Office, London. These charts show the distribution of pressure and direction of winds over Europe from observations taken at 8 a.m. and 6 p.m., the day interval being thus 10 hours, whilst the night one is 14 hours. There are charts also giving the temperatures at 8 a.m. Plate I. is a reproduction of four of these charts, showing the distribution of pressure over our area during the passage of two cyclones,—the one a typical quickly-moving one, and the other a slowly-progressing one. In the charts for 10th December 1898 we have the characteristic distribution of pressure, as shown by the isobars, for a quickly-moving cyclone; whilst the charts for 9th December 1897 show the form of isobars associated with slowly-moving ones. In the 1898 cyclone it will be observed that the barometric gradient to the south of the centre of the cyclone was very steep in the 8 a.m. chart, as shown by the closeness of the isobars on that side, whilst the gradients on the other side and in front were easy. The strong winds, as the form of the isobars would lead us to expect, are all to the south of the centre, and are blowing from the west, and the result was that the centre of this cyclone, which was over the Shetland Isles at 8 a.m., travelled to the east coast of Sweden by 6 p.m., a distance of 700 miles. During the following night the centre moved in an E.N.E. direction at a considerable velocity, but at a slower rate than during the day, as the barometric gradients at 6 p.m. would lead us to expect.

Turn now to the chart for the 9th December 1897. Here it will be seen that the barometric gradients are nearly equal all round, and strong winds are blowing in all directions systematically round the centre. Now note the slow rate of progress. This cyclone appeared on the N.W. of Scotland on the evening of the 8th December. Next morning at 8 o'clock it was in the place shown in the chart, having travelled a distance of 175 miles in the 14 hours. When this cyclone first appeared the barometric gradient was steeper to the south than on any other side. This, no doubt, was the cause of the advance of 170 miles during the night. It will, however, be seen from the 6 p.m. chart, that the centre moved very little during the day, somewhere about 75 miles, and the whole of the following night it was still over the same place, having apparently become stationary, as it was still there on the morning of the 10th, but the gradients were getting easier, the depression getting filled up. These two cyclones were selected—out of many having similar histories—to illustrate the point under discussion, and they were selected because they travelled over exactly the same track, and at the same time of the year, so that we should only have the question of the conditions shown by the shape of isobars to consider.

To illustrate this point still further, the following table gives in brief the histories of a few of the cyclones during the last two years. The table shows the date and hour when the cyclone first appeared on the chart. Then there is given the situation of the centre of the cyclone, followed by the character of the isobars or distribution of gradient, and, lastly, the distance travelled in miles.

1898.

Date.	Hour.	Position of centre of Cyclone	Isobars.	Miles.
Mar. 1	8 a.m.	North-east of Scotland, ...	Not much difference.	...
	6 p.m.	North Sea, ...	" "	150
" 2	8 a.m.	"	" "	150
	6 p.m.	Denmark, ...	" filling up.	200
" 3	8 a.m.	"	" "	60
	6 p.m.	S. Baltic, ...	" "	130
" 4	8 a.m.	N. Germany, ...	" "	130
Oct. 16	8 a.m.	Off entrance to English Channel, . . .	Nearly equal.	...
	6 p.m.	" "	" "	100
" 17	8 a.m.	At " "	" "	90
	6 p.m.	" "	" "	50
" 18	8 a.m.	West of London, ...	" filling up.	200
	6 p.m.	Midland counties, ...	" "	90
" 19	8 a.m.	Border of Wales, ...	" "	50
	6 p.m.	N. Wales, ...	" "	75

1898.

Date.	Hour.	Position of centre of Cyclone.	Isobars.	Miles.
Nov. 23	8 a.m.	Over Ireland, ...	Nearly equal all round.	...
	6 p.m.	St George's Channel, ...	„ „	190
„ 24	8 a.m.	Entrance to English Channel, . . .	„ „	150
	6 p.m.	„ „	„ „	75
„ 25	8 a.m.	Over Cornwall, ...	„ „	40
	6 p.m.	Off Land's End, ...	„ „	120
„ 26	8 a.m.	Off Cape Finisterre, ...	„ „	110
Dec. 2	8 a.m.	South of Norway, ...	Steep to south.	...
	6 p.m.	North of Christiania, ...	„	230
„ 3	8 p.m.	East coast of Sweden, ...	„	350
	6 p.m.	Over Finland, ...	„	500
		1899.		
Jan. 12	8 a.m.	Off north-west of Ireland, ...	Steep to south.	...
	6 p.m.	Off north-east of England, ...	„	450
„ 13	8 a.m.	Over south of Gothland, ...	„	600
	6 p.m.	Over Baltic provinces, ...	„	400
Mar. 28	6 p.m.	Off west coast of Scotland, ...	Steep to south.	...
„ 29	8 a.m.	Off west coast of Norway, ...	„	600
	6 p.m.	Over Norway and Sweden, ...	„	400
April 6	6 p.m.	Off north-west of Ireland, ...	Steepest to south.	...
„ 7	8 a.m.	Over north of England, ...	Steepest to south-west.	375
	6 p.m.	Over south of North Sea, ...	„ „	200
„ 8	8 a.m.	East of Schleswig, ...	„ to south.	200
	6 p.m.	Over Holstein, ...	Nearly equal all round.	70
„ 9	8 a.m.	South of Denmark, ...	„ „	120
April 9	6 p.m.	North-west of Ireland, ...	Steep to south.	...
„ 10	8 a.m.	Off east coast of England, ...	„	430
	6 p.m.	East of Holstein, ...	Becoming equal all round.	250
„ 11	8 a.m.	East of Denmark, ...	„ „	220
	6 p.m.	South of Sweden, ...	„ „	200
„ 12	8 a.m.	Over Stockholm, ...	„ „	100
	6 p.m.	South of Gulf of Bothnia, ...	„ „	100
Aug. 16	6 p.m.	Off north of Scotland, ...	Steep to south.	...
„ 17	8 a.m.	Over the Skager Rack, ...	„	420
	6 p.m.	Over south of Sweden, ...	Becoming equal all round.	300
„ 18	8 a.m.	Over the north of Baltic, ...	„ „	125
	6 p.m.	Entrance to Gulf of Finland, ...	„ „	60
„ 19	8 a.m.	„ „	„ „	20

1899.

Date.	Hour.	Position of centre of Cyclone.	Isobars.	Miles.
Oct. 23	6 p.m.	West of Norway, ...	Steep to south-west.	...
,, 24	8 a.m.	Over Norway and Sweden, ...	,, ,,	400
	6 p.m.	Over entrance of Gulf of Finland, . .	Becoming equal all round.	400
,, 25	8 a.m.	South of Gulf of Finland, ...	,, ,,	150
	6 p.m.	West of Moscow, ...	,, ,,	300
,, 26	8 a.m.	,, ,, ...	,, ,,	50
	6 p.m.	,, ,, ...	,, ,,	50

The history of all these cyclones bears out the conclusion that, when the barometric gradient is steep on one side, the cyclone travels quickly and nearly parallel with the isobars on the steep side, and that, as the gradients become equal all round the centre, the advance becomes slow.

Dr Buchan, in his Introductory Text-Book of Meteorology, points out that the isobars in cyclones are frequently elliptical. Now that is just the form we would expect them to take when the barometric gradient was steep on one side only —that is, with strong winds blowing in one direction. The centrifugal force of these strong winds will tend to draw out the front of the cyclone, and make the length of the ellipse to point in the direction in which the centre of the storm is moving. When the winds are equally strong on all sides there is little tendency for the isobars to depart from the circular form, which an examination of the Synoptic Charts will show to be the case.

There is another effect of the inertia of the winds blowing along the steepest gradient which will be observed in these charts. Whilst the air from the anticyclone on the side next the cyclone is drawn into the cyclonic vortex, the air nearer the centre of the anticyclone passes on in the general anticyclonic circulation, and this quickly-moving air in the anticyclonic area seems to force back the high pressure in front. The effect is to aid the advance of the cyclone by lowering the pressure in front of it. This effect will be seen in the chart for the 10th December 1898 (Plate I.). On the morning of the 9th the shape of the isobars was similar to that shown in the chart for the 10th, but the furthest north point of the 29·9 isobar, instead of being near the eastern limit of our area, was a little to the north of Scotland, and it was driven eastwards to the position shown on the morning of the 10th. The curve of this isobar kept much the same form as it travelled eastwards.

We showed in the experimental part that the air currents at different elevations coming to the centre of the cyclone move in different paths, the air near the ground moving more radially than the air higher up, and it may be asked—Is there anything corresponding to this in the cyclones in the atmosphere? To get an answer to this question, the air currents in a number of cyclones have been examined. The position of the centre of the depression was taken from the Synoptic Charts, and the air currents were taken from the observations made by Messrs Bolam & Redpath at Leith, along with those made by myself. The direction of the lower current is given by the winds, whilst the direction of the upper current is shown by the cloud movement. The result of this examination of the cyclones in the atmosphere shows this feature in quite a distinct manner. In all cases in which there was any difference in the recorded directions of the upper and lower currents, the lower current flowed more directly towards the centre of the depression than the upper. It was also noticed that the upper current was frequently almost quite tangential, apparently not pointing at all to the centre of the depression. The observations used in investigating this point were almost all taken on the side of the depression on which the barometric gradient was steepest and winds strongest, and it would seem to indicate that the inflow towards the centre on this side—particularly of the upper air—is less than on the other sides. This result is indicated in the experimental part of the paper, where it is shown that the air on the side of the cyclone which receives the greatest tangential force curves inwards less than the air on the other sides.

The upper currents moving more tangentially than the lower ones has the effect, when a depression is passing any station, of making the upper currents appear to veer in advance of the lower. For instance, with a depression to the north of the point of observation, the wind will be south of west, whilst the cloud carry will be about west, and when the depression has passed eastwards the wind will change to west, whilst the carry will have some north in it. It may be as well to note here that it is possible that the upper part of the cyclonic column may travel in advance of the lower, owing to the friction on the earth's surface acting as a drag on the lower end. If this be the case it will in part explain the veering of the upper currents in advance of the lower.

We shall now turn our attention to anticyclones, and see what part they play in the vertical circulation of our atmosphere, as too little attention has, I think, been given to them. The sun's heat seems always to have been looked upon as the great source of the energy in our winds, to the exclusion of the effects of cold. It is as if, in studying a steam-engine, we had devoted our attention to the boiler and furnace, and neglected the condenser. The engine would no doubt work without the condenser, though not so efficiently, but it may be doubted whether heat alone would work a cyclone without the cold-driven anticyclone. Its absence, at least, would be a greater loss to the vertical circulation than the loss of the condenser to the engine; unless, indeed, the engine was working at a very low pressure.

It is well known that the mean pressure over continental areas is high during winter and low during summer. That is just as the sun's rays during summer, by heating the air over continental areas, gives rise to cyclonic conditions; so the earth's radiation during winter gives rise to anticyclonic conditions, and this cooling of the air seems to be as true a cause of vertical circulation as the heating of it, though perhaps not to the same extent. If this be the case, then we ought to give more attention to the anticyclone than we have been giving, as a study of the area under its influence will probably assist in forecasting the movements of the cyclone and the weather conditions generally.

An examination of the weather charts for the winter months over our area shows that when the temperature is low over any part of it, particularly over the Spanish Peninsula and over the north-east of Europe, that as the temperature falls the pressure rises—that is, whenever the temperature falls below the mean, an anticyclone forms over the cold area, increasing in pressure as the cold strengthens. And, on the other hand, when the temperature begins to rise the pressure falls. It must be admitted that there are difficulties in studying this subject, as there is not at present sufficient information to enable us thoroughly to study the point. We would require a complete record of the clouds over the area, so as to enable us to judge how far the earth's radiation initiates the change. But I think it may be accepted that, as a rule, any extreme cold over our area is generally accompanied by a small amount of cloud, and that the cold is due to radiation. An examination of the weather charts shows that, on some occasions, when the directive force of the cyclone was small, a rise of temperature in the anticyclonic area, by weakening the high pressure, caused the cyclone to move towards it. On the other hand, a decrease in temperature in the anticyclone may force the cyclone back. These charts also show that, if the temperature over both the Spanish Peninsula and the north-east of Europe falls very low at the same time, then the whole of Europe comes under anticyclonic conditions, the cyclones being driven from our area.

Let us now turn to the general question of cyclonic circulation, and see if what has been stated helps us to understand some of the other phenomena associated with cyclones and anticyclones. We have seen that the upper winds, circling from the anticyclones and to the cyclones, by moving more quickly, and by moving at an angle across the lower air, tends to prevent the latter rising, even though it be the lighter. The effect of this, as already pointed out, is to drive the hot moist air lying near the earth's surface to the circumference of the anticyclone, where it is picked up by the cyclone, and as the spirally moving cyclonic winds also tend to prevent the lower air rising, the hot, moist air is swept into the front of the low-pressure area, and the upper winds here cross the lower at a considerable angle, the hot moist air is compelled to keep near the ground, and, further, the air near the ground having a less tangential force, it is drawn into the centre of the depression, where it is drawn up, and as we saw in the experiments, forms the core of the cyclone. By this method of arranging the air in layers, the cyclone gets the best efficiency out of its material, the pressure falling from the circumference to the centre, whilst the temperature rises towards the centre of low pressure.

The effect of cyclonic circulation in keeping the hot moist air near the ground explains why the air in front of a cyclone is always hotter and moister than the air in the rear. The lower air sent forward by the anticyclone is swept off its circumference by the cyclone and drawn into the cyclone curving round the front of it, and in its passage it gets added to it all the hot moist air in front of the cyclonic area. After the cyclone is past it is evident that, as it has drawn up the surface air over the area of its track, that what was previously the higher, drier, and purer air will have taken its place; hence the coolness, freshness, and purity of the air after the disturbance has passed. The cyclone has brought down to us the air we would have got if, before it passed, we had ascended a considerable distance from the earth's surface.

The above seems to give a fair explanation of the presence of the hot moist air in front of cyclones. Still, some may feel inclined to say that is not the whole explanation, and that there is something else in the air over that area which gives it a peculiar heavy feel—something, in fact, to which no hygroscope is sensitive. To explain these peculiar sensations produced by this air, there is a point to which reference might be made. It is one which I do not think has ever been investigated or even referred to, and it is one which may help to explain the peculiar physiological effects experienced under these conditions. In front of a cyclone where the pressure is falling there will be a considerable amount of air rising from the soil and rocks underneath the surface. This air will come charged with moisture, and more or less changed in its composition by contact with the soil; it will also bring with it any impure gases there may be in the soil. That such air does rise there can be no doubt. If we observe the surface of the ground while the barometer is falling, after a slight fall of snow which has come before the frost, so that the temperature of the ground is not below the freezing point, we shall see little bare spots where the snow has melted. As these little patches are scattered all over bare ground, gravel walks, etc., where the subsoil is uniform, they can hardly be due to heat conducted upwards, but rather seem to be the result of hot air rising from the ground. The fact that the ground under these bare patches is more porous than elsewhere also points to the rising air as the cause of the melting. Though moist air rises from under the soil with a falling barometer, yet it is difficult to get any satisfactory way of measuring its amount. We can easily find out the air space in any sample of soil, and a few tests have been made in this direction. In place of taking the air capacity, the unoccupied space was measured by means of water. A cylindrical vessel that held 17 litres of water was used. In making the tests this vessel was filled with the soil, firmly tramping in layer by layer; water was then added till it showed on the top. In making this measurement it is necessary to fill the vessel with water from below, otherwise it will not penetrate all through the soil. In the tests this was done by thrusting a small pipe down through the centre of the soil, and pouring in the water at the top of the pipe. Well-packed garden soil, tested in winter, when it was wet, gave the following result. In one case 3 litres of water were required to fill up the air spaces, and in another it required 3·4 litres, so that something between $\frac{1}{5}$ to $\frac{1}{6}$ of garden soil is air space. A clay soil would probably have less air space. When sand was treated in the same way, it required a little over 5 litres in one case and a little under 5 in another, which shows that about $\frac{3}{10}$ of sand is air space. The garden soil in the test vessel would probably be as closely packed as the soil in the garden, but the sand would not be so firm as it was in its undisturbed condition, as I found it possible, by continued shaking while in the water, to make it more compact, and bring fully $\frac{1}{2}$ a litre of water to the surface.

These figures seem to indicate that about $\frac{1}{6}$ of cultivated soil is air space, and about $\frac{1}{4}$ is air space in consolidated sand. With these figures it would be easy to calculate the amount of air that each cubic foot would send to the surface with a given fall of the barometer. But this will not help us much, as it is difficult to get the air space in the lower undisturbed strata and find the mean depth from which the air rises. From the porosity of many soils it is evident that a considerable amount of air must ascend and descend with every change of pressure, and this air will have properties different from the air in the general circulation, and may give rise to certain physiological effects.

PART II.

(Read May 7, 1900.)

In the experiments described in the first part of this paper, the cyclonic movements were produced under very artificial conditions, the centre of low pressure being kept in a fixed position and the ascending column of air protected by solid walls. It seemed, however, that if the views set forth in Part I. were correct, that there was no reason why these experimental illustrations might not be extended, and made without much apparatus, and in free air, so as to allow us to study the motion of the centre of the depression as well as the spiral movements of the air towards it. On trial I found this could be easily done, and soon had the pleasure of seeing small cyclones forming and travelling across the experimental area, and the spirally inflowing air was seen moving towards the onward moving centre of low pressure.

The apparatus required for these experiments is very simple, and consists of a small platform, the surface of which can be heated for supplying the hot air required to make the cyclones. On this hot surface a wet piece of cloth or paper may be laid, or fumes may be formed, as in the previous experiments, to enable the eye to follow the movements of the air. Over this heated area, under certain conditions, cyclones are seen to form and travel across its surface. To enable the experimental surface to be heated, it was made in the form of a shallow tin box, closed on all sides and provided with two small pipes, one for the entrance of steam to heat it, and another for draining away the condensed water. This box is 75 cm. square and 1 cm. deep. This heated platform is placed on a table, and the boiler for producing steam to heat it at some distance away, so as to prevent the heat of the boiler producing disturbing currents near the experimental area.

When we watch this hot area we will see the steam rising from its wet surface, and according to the draughts in the room, or absence of them, it will either rise up in an irregular manner, or be drifted across the hot area to the one side, but showing in neither case any tendency to form cyclonic movements. Suppose the air in the room to be perfectly still and the hot air ascending vertically. If we now blow gently over one side of the hot area, a cyclone at once forms which collects the hot air into its core and carries it to a considerable height. This cyclone soon dies away if we cease to supply the tangential energy.

Let us now see what the effect is if there is a draught across the hot area, and we alter the conditions by placing a screen across the current so as to shield one side of the hot area whilst it is allowed to blow over the other. A cyclone will now at once be formed at a short distance from the edge of the screen—a sort of eddy, in fact. It does not, however, remain in its place, but at once begins to travel across the hot area, in the same direction as the cross air current, rotating in the direction given by this tangential current, and rising to a considerable height above the platform.

It is not necessary that the screen protecting the one side of the platform should have a vertical edge, though such an edge is best suited for making eddies. The edge of the screen may be shaped like a magnified comb, as shown in fig. 7. When so shaped the screen exerts very little effect on the current at the tips of the teeth, but more near their base, thus allowing a stronger current to blow over one side of the area than over the other. If this screen be put up across the current a cyclone will at once be seen forming near the points, and so soon as formed it begins to travel away across the hot area in the direction of the strongest tangential air current. Before this cyclone has gone far another forms near the screen, and it in turn passes, following the course of the first, and so on, cyclone following cyclone so long as there is a cross current and a heated area.

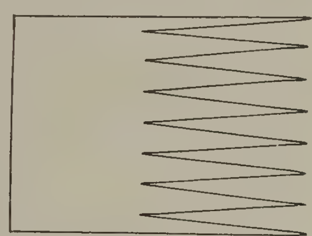


Fig. 7.

It may be as well to point out here that though there may be a slight tendency for eddies to form at the edge of the screen, in the above experiment, yet this tendency is very slight and has but little to do with the formation of the cyclones. If we repeat the experiment, without heating the experimental area, and using fumes to show the movements of the air, it will be found that only very slight eddying effects can be detected. Any that are formed with a screen having a vertical edge are small, irregular and broken, and frequently cannot be traced at all, and when the comb-like obstruction is used no eddying effect is produced unless there is an ascending column of air. The question of the necessity of any eddying effect of the screen in starting the cyclonic movement in these experiments is disposed of by the experiment previously described, in which it is shown that cyclones are formed in still air when a horizontal current is made by blowing across one side of the ascending column, and without the intervention of a screen. The screen in these experiments is only used as a convenient method of protecting one side of the hot area from the horizontal current. If we wish to be more realistic we may crush up a piece of paper roughly into the form of a mountain ridge and put it in the place of the screen, with the result that cyclones go on forming as before.

When the air from the natural draught in the room flows over the hot area at the same velocity at all parts, the rising steam does not ascend far, but keeps close to the hot surface, and is irregular in its movements. But when a cyclone is formed, most of the steam is collected into a rapidly whirling vertical column, which ascends to a considerable height—to a metre or more above the hot surface, and often presenting the appearance of a well-defined column, as it rises through the clearer air.

In making these experimental cyclones, it was noticed there must be a definite relation between the amount of heating and the velocity of the cross current. If the heating be slight the cross current must be slow, otherwise the cyclonic movement will not be properly formed owing to the weakness of the ascent, before it is swept off the experimental area ; whilst with hotter air a stronger current may be permitted, with the result that a more violent cyclonic movement is produced, which penetrates the upper air to a greater height.

The use of steam in these experiments is evidently open to many objections, other than the great amount of heat required to evaporate the water. Although it is water that plays a part in these experimental cyclones as well as in those in nature, yet in the two cases the part played by the water is reversed. In nature the water goes up as vapour, and when it condenses it liberates a great amount of heat, which prevents the temperature of the air falling so much as it would if no condensation took place, so aiding in the ascent of the column. But in the experimental cyclones this action is reversed, much of the water goes up in the cloudy or condensed form, and as in its ascent it gets mixed with more air, the water particles get evaporated and heat is absorbed. So that whilst the water taken up as vapour in the cyclones in nature increases the energy of the motion, it checks the movements in the artificial ones. Condensed water vapour or what we often call steam is thus not the best thing to use for showing these cyclonic movements, but it has the advantage that it is produced all over the experimental area, so that we can by means of it see the beginning and trace the movements of the cyclones. Fumes of hydrochloric acid and ammonia may be used, but in that case it is necessary to cover the experimental area with a sheet of glass, which is almost certain to crack with the heat. If the glass plate is not used the acid and ammonia may be put in watch glasses. Working with fumes we get a much greater violence of movement and greater ascent, owing to the air being more highly heated, but as the liquids evaporate very rapidly they require frequent renewal, and the rising fumes are patchy, thus allowing the core of the cyclone to be often invisible, and they do not often show a well-defined core of fumes.

These experiments are sometimes difficult to repeat with any degree of certainty, so much depends on the cross currents that happen to be in the room, which vary with changes in the outside temperature, winds, etc., and inside heating. For this reason it has been found advantageous to use some method of making a cross current that would be completely under control. An ordinary screw-shaped fan of 75 cm. diameter does very well for the purpose. Owing to a fan of this construction not delivering its air in a uniform horizontal flow, but with an irregular somewhat spiral motion, it was placed at a distance of about $1\frac{1}{2}$ metres from the hot area, and into the space between the fan and the hot area was fitted a horizontal surface, so as to cause the air to acquire a horizontal flow before arriving at the hot area. With this apparatus no difficulty has been experienced in repeating the experiments.

These miniature cyclones illustrate many of the points referred to in the first part of this paper. The slower rise of the air over the hot area when the circulation is uniform across it, that is, so to speak, anticyclonic, and the collecting together of the lower hot air, and the drawing of it up by the cyclone, and the power of the whirling column to penetrate the higher regions, is well shown. These small cyclones also illustrate the dependence of the spiral movements on the tangential current both for their velocity as well as direction of rotation. They also illustrate that the direction and the rate at which the cyclone as a whole travels is dependent on the direction and velocity of this tangential current.

If the cyclones be watched for a time whilst the draught in the room is not constant in direction or force, it will be seen that the stronger the tangential current the quicker the centre of depression moves along with it, and the more rapid is the rate of rotation, and if the draught slows down and reverses, then the direction of the movement of the centre of the cyclone also reverses, and it goes back on its track. Further, if there is no tangential force, as when there are no draughts across the area, then no cyclone is formed, and if formed during a temporary cross current it will die away and cease to rotate when the cross current ceases. Again, cross currents of equal force on opposite sides of hot areas keep the cyclone in one place and rapidly rotating.

It is not necessary to make special apparatus for making these experimental cyclones, as it will be found that a simple arrangement first tried is sufficient for the purpose. It consisted of a large sheet of tin plate; a thick sheet of iron would have been better. This was covered with two or three sheets of thick paper well soaked in water. This is placed over a stove or other means of heating it sufficiently to cause steam to rise freely from its surface. When steaming freely the wet surface is removed to a table, to be away from the draughts rising from the stove, and one side of the hot surface is protected from cross draughts by means of a screen. When this is done the cyclones will be seen travelling over the steaming surface.

It seems probable that the small cyclones often seen in nature are produced under similar conditions to those above described. These small natural cyclones are often seen on dusty roads whilst the sun is shining, the whirling column having a core of dusty air, and the centre of motion travelling along the road, tossing up the dust and other light objects as it moves along. On one occasion I had the opportunity of seeing one of these small cyclones in a large park in early spring. The wind, which was very light, was cold and from the north, but the sun was very hot. The movements of the cyclone were easily seen as it tossed up the dry leaves in its path, whirling them up to a height of four or five feet and scattering them all round. The path of the cyclone was due south—that is, in the direction of the wind—the distance travelled being more than a hundred yards. As there were a number of large trees in the park, it was easy to see that the necessary conditions were present for starting and carrying on the cyclonic movement. It seems probable that these small cyclones are of more frequent occurrence than we are aware of, but owing to the absence of light bodies—such as dust, leaves, etc.—there is nothing to indicate their presence.

[ADDITIONAL NOTE. Received February 19, and read March 4, 1901.]

If both the direction of revolution and the direction of movement of cylones as a whole are determined by the anticyclones, it may be asked—Why do cyclones over our area almost always move in an easterly or north-easterly direction, and so seldom in other directions? and why do these north-easterly moving cyclones generally travel so much quicker than those moving in other directions? The reason for so many cyclones moving in a north-easterly direction would appear to be due to there being generally a high-pressure area to the south of us, and accompanied with this, there is generally hot moist air coming from the Atlantic and blowing to the north of the high-pressure area, and the cyclones form in this hot moist air and travel in the direction of the winds blowing out of the anticyclone lying to the south. That is, the frequency of these north-easterly moving cyclones is due to the conditions being more generally favourable for the formation of cyclones on the north-west side of the anticyclones, than on any of the other sides.

Turning now to the question of why these easterly moving cyclones should travel, as a rule, so much more quickly than those moving in other directions. It has been pointed out that the cyclone is formed in the low-pressure area between two anticyclones, and that it moves in the direction in which it receives the strongest tangential current. If a cyclone were situated between two equally strong anticyclones, it would receive equally strong winds on opposite sides, and it would not move, as it would receive equal amounts of air from both anticyclones, and as these equal amounts of air would have equal and opposite velocities, there would be no tendency for the centre of the cyclone to travel. But a little consideration will show us that the air to the north-western side of anticyclones, in our area, will generally have a greater velocity than the air on any of the other sides. The high-pressure areas which regulate the movements of cyclones over our area are situated to the south-west of Spain, and over Siberia and northern Asia. Now, the air descending from high elevations over these areas has, in all probability, come from the upper general air circulation, and was previously moving from the equator towards the north pole. It will therefore have a greater easterly rate of motion than the surface of the earth where it descends, on account of it coming from a lower latitude, and further, it will also have a northerly motion. The result of this is, that the air descending from the upper parts of the atmosphere, while it tends to move spirally outwards all round over the high-pressure area, yet owing to it having a northerly motion and a greater easterly motion than the surface of the earth where it descends, the air moving to the northwards and eastwards will have a much greater velocity relatively to the earth than the air which moves southwards and westwards. Further, over Europe the high-pressure area to the north is always much weaker than the one to the south, and is sometimes absent, so that the cyclone is always more influenced by the anticyclone to the south than the one to the north, and it thus receives its north-easterly motion from the strong winds on the north-west side of the southern anticyclone. These remarks only explain in detail how cyclones are affected by the general easterly drift of the atmosphere over our area.

An examination of the weather charts, however, shows that if a cyclone receives its strongest winds from any side of an anticyclone, it will move in the direction of these winds. If, for instance, it receives its strongest indraught from an easterly direction on the south side of the anticyclone, it will move to the west, or it may move northwards along the west side of the anticyclone, or southwards along its east side, but as the winds on these sides are generally feeble, the cyclonic movements are generally slow.

There is another point to which reference should be made here, as it has lately assumed considerable importance. For long it was held that cyclones were due to a low-pressure area being formed by a rising column of hot moist air towards which the surrounding air flowed in spirally; while lately it has been contended that cyclones are only secondary effects due to the interaction of air currents, and are, in fact, eddies in the atmosphere formed by the general circulation of the earth's atmosphere. These two theories thus attribute the energy of cyclones to two different causes. According to the first theory the cyclone receives its energy from the hot and moist air; whilst according to the other it gets its energy from the general air circulation. The one theory we might call the convectional theory, the other the dynamical or driven theory. The question then comes to be, which of these two theories seems to be the most probable. To many it may seem difficult to imagine how the dynamical theory ever originated. The diameter of cyclones is so very great it is difficult to imagine any way in which the air currents in the atmosphere could produce eddies hundreds of miles in diameter.

Are there not, however, differences in these two forms of cyclonic movement by means of which we can distinguish the one from the other, and say whether the cyclones in our atmosphere are convectionally or dynamically driven ? I think there are. One way in which we may distinguish between the two kinds is in the direction of the circulation. In a convectionally driven cyclone the circulation is spirally *inwards*, whereas in a dynamically driven one we would expect it to be spirally *outwards*. In a dynamically driven cyclone in our atmosphere we would be entitled to expect an inward current near the surface of the earth, owing to the velocity there being retarded by friction ; the walls of the cyclone would therefore be weak at that part, and air would be drawn in at the surface of the ground, and it would also probably be drawn in at the top. Now, is there any evidence of a general outward circulation in cyclones with in-draught at top and bottom ?

For information on this point we cannot do better than turn to the valuable and very important results obtained by observations on clouds made by means of the nephoscope and theodolite, and published in the *Report of the International Cloud Observations*, prepared under the direction of WILLIS L. MOORE, Chief of the Weather Bureau, by FRANK H. BEGELOW, M.A., and published last year by the Weather Bureau, Washington, U.S.A. If we turn to the series of diagrams in the above Report showing the direction and velocity of the circulation in cyclones, we shall see that at only one cloud level does the circulation show any tendency to be outwards. The following is a summary of the results taken from the Report, pp. 435–6 : "Beginning with the surface, it is seen (1) that all the vectors have an inward component up to the cumulus, the inner increasing in strength ; (2) that they continue inward in the levels from the strato-cumulus to the cirro-cumulus, except that there is a slight tendency to turn outwards on the exterior circle, the alto-stratus again showing average conformity with the others ; (3) the tendency is still inward in the cirro-stratus and cirrus level in the interior, but apparently lawless over the outer parts." From the above it will be seen that at all levels except the strato-cumulus the circulation is inwards. This tendency to outward circulation at the one level is evidently due to the cyclone having at the strato-cumulus level its greatest rate of rotation, as shown by the diagrams, and this high velocity of rotation at this level would appear to be due to the air in the anticyclone also having its maximum velocity at the same level. Though the evidence here shows the direction of almost the whole of the circulation to be towards the centre, and thus points to the circulation being convectionally driven, yet as there may be some hesitation in accepting this evidence as final on the point, as all the circulation is not inwards to the top, I shall therefore now proceed to point out another and more definite way in which we can distinguish between dynamically and convectionally driven cyclones.

In a convectionally driven cyclone the velocity of movement of the air, both absolute and angular, *increases* from the outside towards the centre, whereas in a dynamically driven one the reverse is the case, the air in the outer parts moving quicker than the air in the inner. Judged by this test the evidence is entirely in favour of the theory that cyclones are convectionally driven. An examination of the diagrams in the *Report of the International Cloud Observations* above referred to show in a marked way that in cyclones the velocity of the wind increases towards the centre. These charts do not show any great increase of velocity towards the centre of the winds at the surface of the earth, but they show that as we ascend to greater elevations the increased velocity becomes more marked, and is very great at the strato-cumulus level, and up through the cirro-cumulus, alto-cumulus, alto-stratus, and into the cirro-cumulus level, a result we would not expect to find if cyclones were dynamically driven.

There is another point in connection with the source of energy in cyclones to which reference may be made here, and which seems to support the above conclusion. If cyclones are convectionally driven we would expect them to tend to form and move over areas where the air is relatively hot, or moist, or both. If we examine Dr Buchan's maps of the isobars and winds of the world, and his corresponding maps of the isotherms, and also Dunwoody's maps of the storm tracks of the world, all in Bartholomew's *Physical Atlas*, Vol. III., we will find this supposition receives considerable support. Confining our attention to the area over Europe and the eastern side of the Atlantic, it will be seen that in winter the general air circulation over this area is from the south-west, blowing out of the high-pressure area situated at this season to the south-west of Spain. It is this warm south-westerly wind that carries the high temperature at this season northwards to Iceland and Scandinavia, and gives rise to the upcurving over this area of the isothermal lines shown in the maps, and this hot moist air, driven northwards between the cold air on both sides, would seem to be the cause and source of energy of the cyclones which traverse this area during the winter months. The maps of the storm tracks show that the storms coming towards our area from the western Atlantic move generally along one track till they pass Newfoundland, where the track divides in two, and the storms travel over one of the two routes. A number of them go north-eastwards over Scandinavia, whilst the others move across England and eastwards over the Continent. Confining our attention to the more northerly route, it will be seen from the maps that during winter this track is at its furthest north position. In summer the conditions become somewhat changed, the general circulation does not blow so much from the south, the winds being at this season about west-south-westerly, and the isothermal lines no longer curve northwards to the extent they did in winter, and the track of the storms is now situated at its furthest south position, the relatively hot and moist air being now more to the south than it was in winter. From this it will be seen that the track of the cyclones over our area moves northwards as the isothermal lines curve northwards with the approach of winter, and again coming southwards when the isotherms tend to have an east and westerly direction, the storm track thus keeping over the relatively hot and moist area.

The movements of the southern storm track seem to be governed by the same law. In winter it attains its most southerly position, drawn southwards apparently by the hot moist air of the Mediterranean, over which area the storms move at this season, but when summer comes the continental area to the north becomes warmer than the Mediterranean, and the track of the storms moves northwards, and in summer is across continental Europe. It will be noticed that these two storm tracks move in opposite directions at the same season, the northern track moving south in summer and north in winter, while the southern one moves north in summer and south in winter.

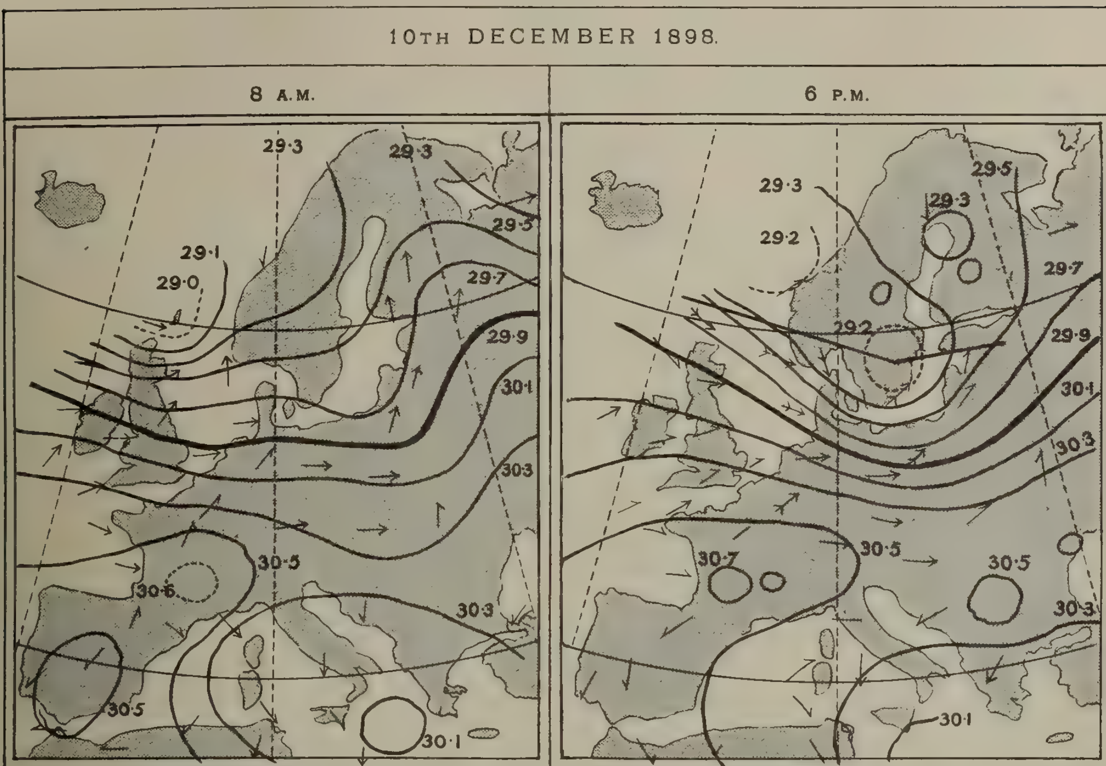
It thus appears that storms tend to form and move over areas in which there is a supply of hot moist air, and to change their tracks so as always to follow the changes in the position of the best supply, and it is difficult to understand why this should be so, unless the hot moist air is the cause and source of energy in the cyclone.

Another consideration which leads us to suppose that convection currents play an important part in cyclones is the greater violence of the winds over cyclonic than anticyclonic areas, a result we would not expect to find unless some source of energy came into action in the cyclonic area. All these considerations point to the conclusion that however important the action of anticyclones may be in the formation of cyclones, yet cyclones are, to a very large extent, convectionally driven.

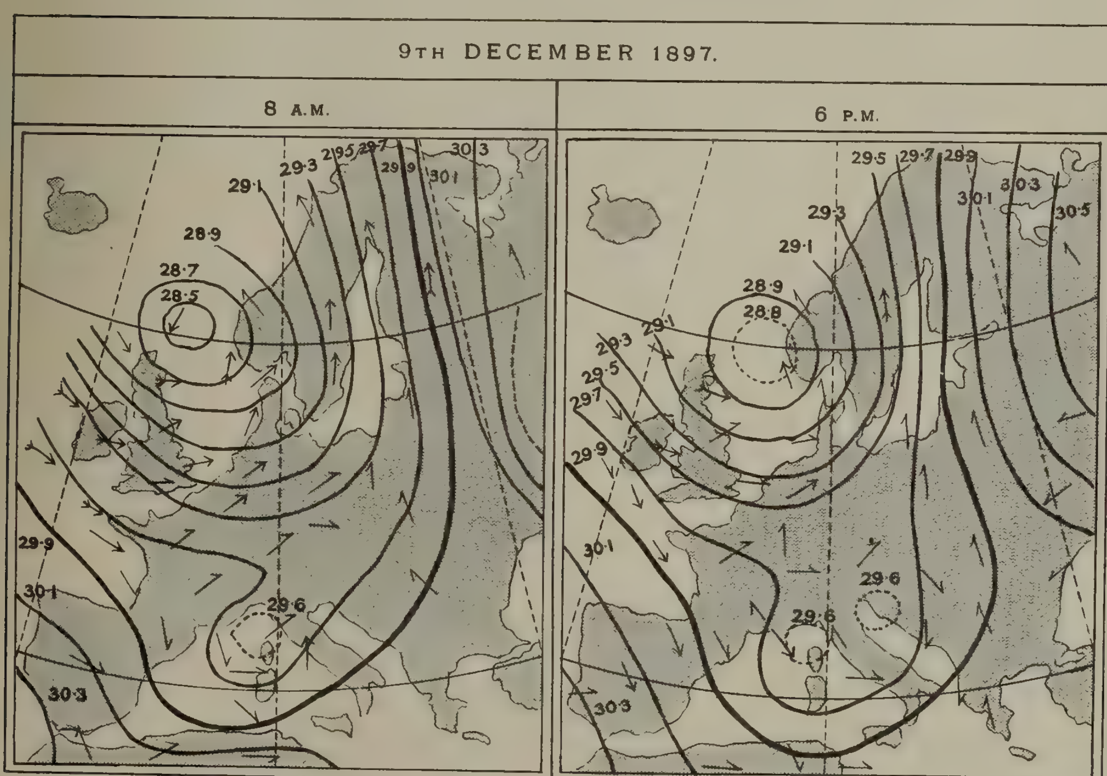
The general circulation over our area, as pointed out, is more from the south during winter than during the summer months. This change is partly brought about by the weakening during the latter season of the anticyclone to the south-west of Europe, but apparently much more to the disappearance of the anticyclone over Siberia and Northern Asia. In winter, Eastern Europe and Northern Asia are covered by a large and well-marked high-pressure area, and it would appear that it is this high-pressure area to the north-east which causes the anticyclonic circulation to turn more to the northwards in winter than in summer, so that the high winter air temperature and hot Gulf Stream water which carry the high winter temperature to Iceland and Scandinavia would appear to be greatly due to the high-pressure area over Siberia and Northern Asia. I fear I must apologise for adding one more theory to the many explaining the high winter temperature of the north-west of Europe. It seems strange that the cold over the northern parts of Europe and Asia should play any part in the abnormally high winter temperature of the sea and the lands surrounding the north-western parts of the Atlantic.

Mr. Aitken on the Dynamics of Cyclones.

ARRANGEMENT OF ISOBARS IN QUICK MOVING CYCLONES.



ARRANGEMENT OF ISOBARS IN SLOW MOVING CYCLONES.



A RITCHIE & SON, EDIN^R

VIII.—*Observations of the Edinburgh Rock Thermometers.* By THOMAS HEATH, B.A., Assistant Astronomer, Royal Observatory, Edinburgh. (With Four Plates.)

(Read February 18, 1901.)

THE NEW ROCK THERMOMETERS.

The accompanying Tables I. and II. contain the readings from May 1888 to December 1899 of the new set of deep rock thermometers erected in June 1879 at the Calton Hill Observatory, to replace the old set which were destroyed in September 1876. The tables are in continuation of, and are similarly arranged to, those published by the Royal Society of Edinburgh in Vol. XXXV. part 3 of the *Transactions*, along with a paper by the late Prof. PIAZZI SMYTH. The tables published with Prof. SMYTH's paper contain the observations made between October 1879 and April 1888, being the beginning of the series with the new set of thermometers.

The construction, testing, and placing in position of these thermometers have already been described at length in a previous paper by Prof. SMYTH, published in Vol. XXIX. part 2 of the *Transactions*. It is unnecessary therefore to do more than recapitulate here the principal facts concerned in the mounting of the instruments. The contract for the construction of the thermometers was given to Messrs Adie & Sons, Opticians, Edinburgh, and the actual work was carried out by their foreman, the late Mr Thomas Wedderburn, under the immediate superintendence of Mr R. Adie. The thermometers, when originally placed in the bore-hole in the rock of the Calton Hill, were arranged as follows, counting from the surface of the rock to the centres of their respective bulbs :—

t_1,	at	a depth	of	250	inches.
t_2,	,,	,,		125	,,
t_3,	,,	,,		50	,,
t_4,	,,	,,		25	,,

There was also a similarly constructed thermometer placed with its bulb an inch underground, and an air thermometer was hung inside the box with its bulb a few inches above the surface. All, except the air thermometer, had 30 inches of tube, of wider calibre than the rest of the stem, attached to their upper ends above the surface, for scale readings. In their construction care was taken to make the tubes and bulbs in all particulars similar to the old thermometers, broken pieces of which had been retained for patterns. The stems are, however, shorter than the old ones. After each thermometer had been put in place, fine sand was poured into the hole in sufficient quantity to receive the next shorter thermometer, till the whole set was in place. In the case of the old set the mouth of the hole was closed with plaster of Paris, or some such material. This plan was not adopted with the new set, as it was thought possible that the hard material had something to do with the unaccountable fracture in 1861 of the second longest of the old set. After the new thermometers had been some time in position, it was noticed that the sand was gradually settling down, and that the shorter thermometers were slowly sinking. In 1880, as measured by Prof. PIAZZI SMYTH, their bulbs were then below the surface of the rock,

t_1 250 inches, t_2 127·5 inches, t_3 57·0 inches, and t_4 34·5 inches.

In May 1889 they were again measured by Prof. COPELAND, and found to be below the surface,

t_1 $251\frac{5}{8}$ inches, t_2 $130\frac{5}{8}$ inches, t_3 $58\frac{11}{16}$ inches, t_4 $35\frac{15}{16}$ inches.

In August 1900 their positions were once more measured, and they appeared to have sunk still further, their bulbs being then below the surface,

t_1 $251\frac{3}{4}$ inches, t_2 $133\frac{1}{2}$ inches, t_3 $60\frac{1}{4}$ inches, t_4 $37\frac{1}{2}$ inches.

Altogether from the date of their establishment in the rock up to last year, the thermometers have sunk,

t_1 $1\frac{3}{4}$ inch, t_2 $8\frac{1}{2}$ inches, t_3 $10\frac{1}{4}$ inches, t_4 $12\frac{1}{2}$ inches.

Whether anything can be done, without risk to the safety of the instruments, to replace them in their original position or to prevent them sinking further into the bore-hole, is a question which will have to be taken into consideration at an early date.

Plate I. gives a graphic representation of the rise and fall of temperature of the four thermometers, as shown by the annual means taken quarterly. These are compared with the shaded air temperature curve and the rainfall in Scotland, and with the sunspot curve. The last-named curve has been plotted from the sunspot numbers according to the late R. WOLF, and A. WOLFER. It is remarkable that the series of maximum temperatures shown for 1882 correspond to a period in the sunspot cycle which is further removed from the sunspot minimum than has ever occurred with this series of observations, including those of both the old and new thermometers. Another remarkable feature is a dip in the curve at 1898, which goes through all the thermometers, affecting even t_1 to a slight extent. It is no doubt connected with the rise in the sunspot curve in the same year. The air temperature and rainfall curves have been laid down from the table printed on page 186, the numbers used being the quadruple annual means for quarterly periods. The sunspot curve has been plotted from the following table :—

SUN-SPOT NUMBERS ACCORDING TO R. WOLF AND A. WOLFER.

1877,	11·1	1883,	65·3	1889,	6·3	1895,	64·0
1878,	3·8	1884,	63·3	1890,	7·1	1896,	41·8
1879,	7·7	1885,	49·9	1891,	35·6	1897,	26·2
1880,	31·5	1886,	26·1	1892,	73·0	1898,	26·7
1881,	54·4	1887,	13·5	1893,	84·9	1899,	12·1
1882,	58·1	1888,	6·7	1894,	78·0		

TABLE I.—*The Edinburgh Royal Observatory Rock Thermometers.*
Original Reading.

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1888.										
May 7	45·25		42·49		41·80		42·55		49·6	55·6
14	45·16	45·14	42·66	42·82	43·00	43·54	44·45	45·13	44·5	46·2
21	45·10		42·92		43·98		46·23		50·7	56·8
28	45·04		43·22		45·39		47·30		45·0	46·3
June 4	44·99		43·64		45·61		46·00		42·8	45·0
11	44·98	44·98	44·00	44·11	45·48	46·21	46·79	47·54	49·7	55·4
18	44·98		44·25		46·48		48·37		52·2	55·4
25	44·98		44·54		47·29		49·00		49·2	50·3
July 2	45·00		44·89		48·08		49·76		51·6	53·7
9	45·05		45·27		48·28		49·39		53·0	57·2
16	45·09	45·10	45·59	45·57	48·52	48·80	50·31	50·34	50·5	49·8
23	45·14		45·90		49·31		51·38		54·4	57·6
30	45·22		46·21		49·80		50·88		49·5	51·2
Aug. 6	45·31		46·54		49·62		50·56		50·9	54·4
13	45·40	45·45	46·78	46·87	49·98	49·94	51·40	51·12	54·1	57·5
20	45·50		46·99		49·96		50·71		52·4	54·0
27	45·60		47·18		50·20		51·80		53·9	56·5
Sept. 3	45·71		47·36		50·18		50·97		52·8	55·1
10	45·80	45·86	47·50	47·55	50·08	49·92	50·30	50·39	48·0	49·9
17	45·92		47·64		49·77		50·40		51·4	56·0
24	46·01		47·70		49·63		49·90		49·1	49·9
Oct. 1	46·08		47·72		49·09		48·61		41·8	42·0
8	46·19		47·73		47·63		45·58		44·4	48·8
15	46·27	46·26	47·60	47·55	47·20	47·68	46·19	47·13	46·3	51·0
22	46·34		47·41		47·08		46·77		44·1	43·0
29	46·42		47·29		47·40		48·51		48·9	50·0
Nov. 5	46·45		47·21		47·32		46·33		42·9	43·3
12	46·48	46·50	47·16	47·06	46·25	46·22	44·65	45·16	42·2	44·5
19	46·54		47·04		45·91		44·80		46·4	50·5
26	46·55		46·84		45·40		44·85		40·6	38·3
Dec. 3	46·60		46·68		44·60		42·95		47·1	52·6
10	46·59		46·44		45·10		44·40		39·5	39·3
17	46·58	46·58	46·30	46·28	44·18	44·16	42·40	42·65	40·6	42·2
24	46·58		46·09		43·80		42·90		41·9	42·6
31	46·55		45·87		43·10		40·60		36·0	39·0
1889.										
Jan. 7	46·52		45·64		42·30		40·60		35·3	32·3
14	46·49	46·47	45·35	45·23	42·00	42·02	40·10	40·62	36·8	37·9
21	46·46		45·09		41·80		40·80		39·7	41·9
28	46·40		44·85		42·00		41·00		40·2	43·9
Feb. 4	46·34		44·66		42·20		40·80		35·5	35·7
11	46·25	46·24	44·53	44·41	41·30	41·20	38·80	39·52	30·8	30·7
18	46·23		44·35		40·40		39·00		44·8	50·0
25	46·14		44·09		40·90		39·50		35·7	35·6
Mar. 4	46·06		43·90		40·20		37·90		31·6	31·5
11	46·00	45·96	43·70	43·59	39·70	40·20	38·00	39·08	35·2	37·3
18	45·92		43·45		40·30		40·10		38·9	38·3
25	45·85		43·30		40·60		40·30		45·4	47·0

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1889.										
April 1 . . .	45·77		43·24		41·30		41·60		42·2	45·5
8 . . .	45·66		43·23		41·30		40·40		38·5	39·5
15 . . .	45·58	45·59	43·24	43·23	41·10	41·62	40·20	41·61	39·5	42·5
22 . . .	45·51		43·20		42·00		42·95		43·3	45·7
29 . . .	45·44		43 25		42·40		42·90		43·7	47·3
May 6 . . .	45·38		43·35		43·15		44·41		45·5	46·8
13 . . .	45·32	45·31	43·50	43·66	44·15	44·63	45·48	46·21	46·6	48·5
20 . . .	45·29		43·76		44·82		45·95		49·0	54·6
27 . . .	45·25		44·04		46·39		49·02		51·5	53·8
June 3 . . .	45·22		44·38		47·18		49·01		51·9	54·6
10 . . .	45·21	45·23	44·79	44·99	48·10	48·40	50·00	50·52	49·5	53·1
17 . . .	45·24		45·21		48·72		50·99		55·4	62·0
24 . . .	45·26		45·60		49·59		52·10		53·4	54·3
July 1 . . .	45·31		46·03		50·58		53·45		55·9	60·8
8 . . .	45·37		46·46		51·28		53·58		51·9	55·6
15 . . .	45·45	45·47	46·94	46·85	51·16	50·97	52·48	52·76	52·7	55·0
22 . . .	45·54		47·28		50·90		51·82		52·6	56·8
29 . . .	45·66		47·56		50·94		52·48		56·2	60·0
Aug. 5 . . .	45 77		47·73		51·82		54·00		54·8	54·2
12 . . .	45·89	45·95	48·02	48·12	51·82	51·69	52·90	52·88	51·8	54·3
19 . . .	46·02		48·28		51·70		52·82		54·2	56·8
26 . . .	46·13		48·47		51·41		51 80		49·5	52·8
Sept. 2 . . .	46·27		48·61		51·28		52·10		53·1	54·6
9 . . .	46·39		48·72		51·43		52·35		55·8	60·9
16 . . .	46·50	46·49	48·78	48·79	51·78	51·04	52·70	51·18	49·8	50·3
23 . . .	46·59		48·88		51·00		49·90		42·8	44·9
30 . . .	46·71		48·94		49·69		48·87		47·2	48·3
Oct. 7 . . .	46·81		48·82		49·10		48·25		47·5	49·0
14 . . .	46·92	46·95	48·66	48·56	48 35	48·22	46·98	47·11	44·2	45·0
21 . . .	47·00		48·49		47·90		47·21		47·3	48·0
28 . . .	47·07		48·28		47·51		46·00		42·0	42·6
Nov. 4 . . .	47·12		48·08		46·90		45·55		43·0	41·9
11 . . .	47·19	47·18	47·91	47·82	46·75	46·77	46·55	45·86	47·2	48·3
18 . . .	47·20		47·72		46·90		45·85		43·7	45·3
25 . . .	47·20		47·57		46·51		45·49		40·7	39·3
Dec. 2 . . .	47·21		47·44		45·10		42·75		40·5	41·0
9 . . .	47·24		47·20		44·15		41.80		45·2	49·7
16 . . .	47·22	47·20	46·88	46·87	43·55	43·91	41·30	41·75	42·4	45·4
23 . . .	47·18		46·55		43·55		41·60		38·7	40·9
30 . . .	47 14		46·28		43·20		41·30		36·2	37·0
1890.										
Jan. 6 . . .	47·11		46·04		42·90		41·20		41·2	44·8
13 . . .	47·05	47·01	45·78	45·71	43·10	42·95	42·40	41·72	43·0	44·7
20 . . .	46·97		45·58		43·40		42·60		36·9	36·8
27 . . .	46·90		45·45		42·40		40·70		37·4	38·8
Feb. 3 . . .	46·85		45·27		42·30		41·50		41·8	45·1
10 . . .	46·76	46·73	45·06	44·96	42·10	41·52	40·00	39·85	35·4	35·9
17 . . .	46·69		44·88		41·00		38·70		37·2	41·0
24 . . .	46·62		44·64		40·70		39·20		40·7	43·4
Mar. 3 . . .	46·52		44·37		40·70		38·80		32·5	33·0
10 . . .	46·46		44·17		40·70		39·30		40·7	47·9
17 . . .	46·36	46·36	43·98	44·07	41·55	41·39	41·70	40·68	40·8	41·0
24 . . .	46·28		43·91		41·80		41·20		40·2	43·4
31 . . .	46·19		43·90		42·20		42·40		41·5	43·9

TABLE I.—*continued.*

Date.	t_1		t_2		t_3		t_4		t_5	Air.
	Each Reading.	Monthly Mean.	Each Reading.	Monthly Mean.	Each Reading.	Monthly Mean.	Each Reading.	Monthly Mean.	Surface Theorem. Each Reading.	Each Reading.
1890.										
April 7	46·11		43·91		42·75		43·19		43·2	44·7
14	46·02	46·00	43·97	43·98	42·75	42·79	41·90	42·53	39·2	43·9
21	45·97		44·02		42·60		42·08		44·8	50·3
28	45·89		44·03		43·05		42·95		41·5	44·1
May 5	45·83		44·08		43·80		44·75		46·4	48·2
12	45·80	45·78	44·19	44·31	44·48	44·88	45·09	46·05	46·7	51·2
19	45·75		44·38		45·11		46·30		48·6	48·5
26	45·73		44·58		46·14		48·05		47·9	50·8
June 2	45·70		44·84		46·65		47·58		49·3	52·0
9	45·69		45·13		47·35		48·85		53·0	58·1
16	45·68	45·70	45·43	45·44	48·01	47·97	49·29	49·42	55·3	58·9
23	45·70		45·74		48·61		50·50		54·4	57·6
30	45·74		46·05		49·23		50·86		52·4	54·2
July 7	45·77		46·36		49·20		50·25		50·4	53·8
14	45·82	45·87	46·63	46·73	49·36	49·78	50·57	51·27	55·1	56·7
21	45·90		46·86		49·98		51·70		56·8	62·5
28	45·97		47·03		50·59		52·55		54·9	58·1
Aug. 4	46·05		47·38		51·10		53·17		58·3	62·9
11	46·12	46·16	47·63	47·79	51·78	51·49	53·80	52·88	55·6	55·9
18	46·19		47·95		51·70		52·76		53·0	55·9
25	46·28		48·22		51·40		51·79		50·3	53·2
Sept. 1	46·38		48·39		50·61		50·30		49·4	52·0
8	46·52		48·50		50·85		52·19		55·6	60·3
15	46·62	46·61	48·55	48·58	51·23	51·11	52·33	51·97	54·7	58·4
22	46·72		48·66		51·47		52·51		53·5	57·5
29	46·81		48·78		51·37		52·51		52·8	54·6
Oct. 6	46·90		48·89		50·88		51·34		51·5	52·6
13	47·00	47·02	48·95	48·90	50·58	50·13	51·16	49·75	52·3	53·9
20	47·06		48·92		49·95		48·68		44·0	43·8
27	47·13		48·84		49·12		47·81		37·2	36·7
Nov. 3	47·22		48·70		47·88		46·59		40·3	39·0
10	47·30	47·32	48·48	48·32	47·02	46·77	45·15	45·33	39·4	40·7
17	47·37		48·20		46·10		44·29		40·6	40·7
24	47·41		47·88		46·10		45·28		39·9	39·0
Dec. 1	47·47		47·65		44·85		41·80		44·8	51·2
8	47·44		47·32		44·40		42·20		36·9	36·9
15	47·43	47·41	47·02	46·98	43·45	43·33	40·50	40·58	36·0	38·9
22	47·38		46·65		42·35		39·20		32·0	32·1
29	47·35		46·28		41·60		39·20		34·2	33·8
1891.										
Jan. 5	47·30		45·88		41·20		39·10		33·1	33·3
12	47·27	47·20	45·52	45·34	40·60	40·68	38·30	38·52	40·8	44·9
19	47·15		45·14		40·70		38·40		32·9	34·0
26	47·09		44·84		40·20		38·30		39·8	44·4
Feb. 2	46·99		44·54		40·80		39·90		41·7	46·7
9	46·87	46·82	44·35	44·34	41·50	41·39	41·20	40·62	37·2	38·4
16	46·77		44·27		41·55		40·80		41·0	43·2
23	46·67		44·22		41·70		40·60		40·0	43·2
Mar. 2	46·55		44·17		41·60		41·00		43·1	43·4
9	46·42		44·08		41·75		40·30		31·6	30·7
16	46·34	46·35	44·04	43·95	40·60	40·85	38·50	39·56	36·4	38·3
23	46·27		43·85		40·10		38·90		38·4	41·3
30	46·18		43·62		40·20		39·10		34·6	36·3

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1891.										
April 6	46·10		43·45		40·20		39·10		37·6	37·6
13	46·04	45·98	43·31	43·30	40·40	40·90	40·00	40·68	38·7	41·8
20	45·95		43·22		41·10		41·20		39·9	40·8
27	45·84		43·21		41·90		42·40		42·4	45·9
May 4	45·77		43·27		42·60		42·90		42·3	46·6
11	45·70	45·66	43·42	43·53	43·15	43·51	43·93	44·03	46·2	50·0
18	45·62		43·60		44·08		44·30		42·8	45·4
25	45·57		43·84		44·20		45·00		44·8	46·2
June 1	45·53		44·04		44·65		45·67		46·8	46·3
8	45·50		44·24		45·20		45·88		46·4	48·0
15	45·49	45·50	44·47	44·51	46·00	46·44	48·00	48·32	50·9	54·3
22	45·49		44·73		47·51		50·65		53·7	56·5
29	45·50		45·09		48·83		51·41		57·1	62·2
July 6	45·49		45·53		49·62		51·70		54·7	59·1
13	45·53	45·57	45·99	46·18	49·98	50·36	52·01	52·56	56·3	61·5
20	45·59		46·40		50·68		53·26		57·2	60·0
27	45·66		46·79		51·18		53·27		55·0	56·0
Aug. 3	45·74		47·15		51·11		52·68		53·6	55·5
10	45·83		47·46		51·17		52·70		53·4	55·0
17	45·93	45·94	47·72	47·69	51·29	51·27	52·75	52·48	53·6	55·8
24	46·04		47·95		51·48		52·68		52·7	55·3
31	46·15		48·15		51·28		51·59		51·4	55·9
Sept. 7	46·25		48·31		50·81		51·28		50·2	53·5
14	46·37	46·42	48·42	48·45	50·99	50·86	52·73	51·63	56·3	55·7
21	46·47		48·46		51·15		51·92		46·8	46·7
28	46·60		48·62		50·59		50·58		51·7	56·3
Oct. 5	46·70		48·64		50·18		50·30		51·0	52·6
12	46·79	46·83	48·63	48·58	49·90	49·29	49·59	48·56	47·6	49·2
19	46·88		48·59		49·00		47·75		46·4	48·2
26	46·97		48·48		48·10		46·59		44·6	47·0
Nov. 2	47·04		48·29		47·15		45·22		44·7	46·4
9	47·10		48·06		46·63		45·30		42·5	44·8
16	47·15	47·13	47·80	47·79	45·98	45·86	44·27	43·99	41·8	43·7
23	47·16		47·52		45·35		43·60		36·6	34·9
30	47·18		47·26		44·19		41·55		37·7	39·7
Dec. 7	47·19		46·94		44·05		43·00		39·4	39·0
14	47·19	47·17	46·63	46·48	43·65	43·09	41·10	41·22	36·2	36·9
21	47·17		46·34		42·65		40·70		37·9	38·4
28	47·12		46·02		42·00		40·10		37·1	37·0
1892.										
Jan. 4	47·06		45·67		41·60		39·60		32·9	31·9
11	46·99	46·96	45·34	45·16	40·90	40·58	38·00	38·15	31·8	31·0
18	46·93		45·01		40·00		37·00		34·6	37·3
25	46·84		44·64		39·80		38·00		36·6	39·1
Feb. 1	46·75		44·29		40·60		40·30		40·9	41·3
8	46·66		44·10		40·50		39·20		40·2	42·4
15	46·54	46·54	43·96	43·98	41·10	40·52	40·50	39·28	36·2	34·9
22	46·44		43·85		40·40		37·90		31·9	34·9
29	46·31		43·68		40·00		38·50		35·7	35·8
Mar. 7	46·20		43·44		39·60		37·50		33·0	33·0
14	46·09	46·05	43·25	43·11	39·00	39·48	37·00	38·18	31·8	32·5
21	46·01		42·99		39·30		39·20		36·7	38·3
28	45·91		42·78		40·00		39·00		32·2	32·9

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1892.										
April 4	45·80		42·69		39·90		40·10		41·9	46·0
11	45·70	45·65	42·65	42·70	41·00	40·74	41·10	40·75	40·3	41·9
18	45·60		42·71		40·80		39·70		35·5	38·4
25	45·51		42·74		41·25		42·10		43·2	46·2
May 2	45·43		42·82		41·85		41·90		43·6	48·9
9	45·35		42·94		42·45		43·10		46·7	51·8
16	45·30	45·30	43·09	43·17	43·70	43·57	45·65	44·63	47·7	50·8
23	45·24		43·34		44·45		45·10		45·8	50·2
30	45·20		43·66		45·40		47·42		52·3	59·4
June 6	45·16		43·98		46·50		48·17		50·8	55·3
13	45·15	45·15	44·34	44·55	47·60	47·35	49·09	48·78	44·7	46·8
20	45·13		44·76		47·42		48·25		47·4	49·1
27	45·17		45·11		47·86		49·61		55·4	59·2
July 4	45·20		45·40		48·87		50·93		52·7	52·2
11	45·24	45·29	45·74	45·90	49·22	49·33	50·88	51·14	52·3	52·0
18	45·31		46·07		49·45		50·96		51·6	52·9
25	45·40		46·38		49·78		51·80		54·9	54·5
Aug. 1	45·49		46·66		50·45		52·53		56·8	58·5
8	45·56		46·94		50·90		52·48		51·3	50·0
15	45·65	45·67	47·28	47·23	50·72	50·87	52·29	52·44	54·7	58·6
22	45·78		47·54		50·93		52·39		59·4	64·4
29	45·88		47·71		51·37		52·49		49·5	51·0
Sept. 5	45·98		47·97		50·78		50·69		48·2	52·0
12	46·10	46·16	48·10	48·10	50·26	50·25	50·58	50·35	51·6	55·3
19	46·22		48·15		50·20		50·55		52·4	55·9
26	46·32		48·18		49·76		49·59		49·5	54·3
Oct. 3	46·40		48·15		49·08		48·00		46·1	48·6
10	46·51		48·11		48·30		47·10		44·2	46·2
17	46·58	46·57	47·95	47·90	47·48	47·29	46·19	45·90	40·3	40·5
24	46·65		47·76		46·33		44·05		37·9	37·3
31	46·72		47·53		45·26		44·16		41·7	42·9
Nov. 7	46·77		47·21		45·09		44·20		41·7	42·9
14	46·80	46·80	46·94	46·87	45·20	44·86	44·50	43·59	42·8	42·3
21	46·81		46·75		44·90		43·05		41·4	42·0
28	46·83		46·59		44·25		42·60		45·5	50·3
Dec. 5	46·78		46·32		43·45		40·30		31·4	28·8
12	46·76	46·75	46·06	45·84	41·90	42·21	38·90	39·92	35·8	36·8
19	46·77		45·69		41·55		40·80		43·0	44·8
26	46·69		45·28		41·95		39·70		31·5	29·4
1893.										
Jan. 2	46·63		45·04		40·64		37·75		31·5	28·2
9	46·57		44·72		39·80		36·70		31·7	34·3
16	46·50	46·50	44·35	44·36	39·20	39·88	36·70	37·81	35·2	38·8
23	46·44		43·99		39·50		38·30		42·9	47·3
30	46·34		43·69		40·25		39·60		41·8	47·1
Feb. 6	46·23		43·52		40·80		40·40		39·7	43·0
13	46·10	46·06	43·45	43·42	41·00	40·80	40·10	39·91	34·4	33·5
20	46·03		43·41		40·70		40·35		43·0	44·9
27	45·90		43·32		40·70		38·80		33·2	34·8
Mar. 6	45·80		43·24		39·90		39·20		42·9	45·7
13	45·71	45·67	43·05	43·08	40·85	40·69	40·90	40·21	41·8	43·8
20	45·64		43·02		40·90		39·40		39·9	46·3
27	45·54		43·02		41·10		41·35		41·0	41·0

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1893.										
April 3 . . .	45·47	45·36	42·98	43·11	41·60	42·52	42·05	43·01	45·1	49·3
10 . . .	45·39		43·04		42·40		42·75		42·7	43·9
17 . . .	45·32		43·13		42·80		43·05		38·4	39·6
24 . . .	45·25		43·28		43·30		44·20		44·9	49·1
May 1 . . .	45·19	45·15	43·44	43·97	44·20	45·73	45·09	47·41	43·0	46·0
8 . . .	45·17		43·69		44·60		45·95		47·5	48·7
15 . . .	45·14		43·93		45·69		47·98		51·3	52·1
22 . . .	45·12		44·23		46·66		48·44		50·7	55·5
29 . . .	45·12		44·57		47·48		49·61		50·7	48·1
June 5 . . .	45·14	45·18	44·94	45·50	47·99	49·38	49·85	51·64	52·8	57·4
12 . . .	45·14		45·29		48·76		50·90		54·7	58·5
19 . . .	45·20		45·67		49·95		53·67		62·5	67·4
26 . . .	45·25		46·10		50·84		52·15		51·2	54·4
July 3 . . .	45·32	45·51	46·56	47·20	50·80	51·25	52·81	52·92	56·6	57·8
10 . . .	45·40		46·90		51·10		53·10		56·0	58·8
17 . . .	45·50		47·22		51·26		52·70		53·9	57·0
24 . . .	45·62		47·54		51·50		53·06		57·9	62·4
31 . . .	45·73		47·77		51·59		52·95		53·1	57·0
Aug. 7 . . .	45·85	46·04	48·01	48·37	51·52	52·55	52·47	54·33	54·5	58·0
14 . . .	45·99		48·25		52·20		54·56		60·5	65·9
21 . . .	46·10		48·45		53·39		56·20		58·7	60·8
28 . . .	46·22		48·76		53·10		54·09		52·4	55·8
Sept. 4 . . .	46·35	46·52	49·05	49·20	52·70	52·02	53·88	52·15	55·2	59·1
11 . . .	46·46		49·17		52·51		52·61		48·4	51·3
18 . . .	46·58		49·30		51·89		52·40		53·4	56·7
25 . . .	46·69		49·28		51·00		49·72		45·0	44·6
Oct. 2 . . .	46·83	47·04	49·28	48·95	50·01	49·09	49·20	48·42	48·4	52·6
9 . . .	46·94		49·12		49·20		48·18		44·8	46·6
16 . . .	47·06		48·97		48·69		48·39		53·4	56·7
23 . . .	47·15		48·76		49·04		49·21		47·0	49·0
30 . . .	47·20		48·62		48·51		47·12		38·9	38·9
Nov. 6 . . .	47·26	47·32	48·50	48·02	46·92	45·41	44·00	43·03	36·5	37·0
13 . . .	47·31		48·22		45·60		43·60		40·0	41·0
20 . . .	47·34		47·85		45·05		42·51		46·6	37·8
27 . . .	47·37		47·50		44·06		42·00		37·6	42·8
Dec. 4 . . .	47·39	47·34	47·16	46·72	44·17	43·56	42·09	41·90	42·9	47·0
11 . . .	47·35		46·83		43·94		41·71		37·0	37·9
18 . . .	47·32		46·59		43·10		42·00		41·5	44·7
25 . . .	47·28		46·29		43·05		41·80		40·3	40·4
1894.										
Jan. 1 . . .	47·21	47·07	46·03	45·60	43·30	42·23	42·61	40·69	37·8	36·1
8 . . .	47·13		45·81		42·40		39·20		30·7	30·8
15 . . .	47·08		45·60		41·59		40·49		38·6	37·9
22 . . .	47·00		45·48		42·10		41·15		38·1	39·1
29 . . .	46·93		45·08		41·75		40·00		34·4	35·1
Feb. 5 . . .	46·85	46·70	44·87	44·56	41·21	41·17	40·02	39·69	39·0	40·0
12 . . .	46·75		44·64		41·71		40·60		38·3	38·1
19 . . .	46·65		44·46		41·05		39·04		33·0	33·0
26 . . .	46·57		44·27		40·70		39·09		38·2	43·9
Mar. 5 . . .	46·45	46·31	44·05	43·81	40·79	40·94	39·61	40·18	37·2	39·0
12 . . .	46·36		43·86		40·89		39·99		36·6	39·1
19 . . .	46·28		43·74		40·60		39·50		43·9	48·7
26 . . .	46·17		43·61		41·50		41·60		41·2	43·9

TABLE I.—*continued.*

Date.	t_1		t_2		t_3		t_4		t_5	Air.
	Each Reading.	Monthly Mean.	Each Reading.	Monthly Mean.	Each Reading.	Monthly Mean.	Each Reading.	Monthly Mean.	Surface Theorem. Each Reading.	Each Reading.
1894.										
April 2	46·08		43·60		42·00		42·50		43·1	44·3
9	45·99		43·62		42·51		42·44		42·0	44·0
16	45·91	45·91	43·70	43·75	43·15	43·07	43·60	43·54	44·8	47·3
23	45·82		43·83		43·67		44·29		43·8	44·6
30	45·77		43·99		44·03		44·89		47·7	50·1
May 7	45·69		44·13		44·52		44·79		44·4	46·8
14	45·50	45·59	44·31	44·37	44·79	44·78	45·51	45·06	45·2	45·9
21	45·60		44·46		44·89		44·59		40·5	42·8
28	45·58		44·59		44·93		45·37		43·6	43·8
June 4	45·59		44·74		45·35		46·28		48·4	49·5
11	45·57	45·58	44·87	45·01	46·10	46·69	47·39	48·24	49·0	52·3
18	45·57		45·09		47·22		49·30		50·7	54·0
25	45·58		45·35		48·10		49·97		51·4	51·8
July 2	45·60		45·70		49·21		52·00		56·3	58·9
9	45·65		46·20		50·44		53·06		55·6	59·1
16	45·68	45·70	46·52	46·52	50·75	50·51	52·76	52·69	54·1	55·0
23	45·75		46·92		50·96		52·54		53·5	57·6
30	45·82		47·26		51·20		53·10		54·9	57·0
Aug. 6	45·91		47·57		51·80		53·50		54·9	58·4
13	46·02	46·07	47·89	47·99	51·84	51·61	53·29	52·67	54·3	58·0
20	46·11		48·15		51·60		52·34		51·6	55·9
27	46·23		48·36		51·19		51·55		51·0	54·0
Sept. 3	46·34		48·46		51·02		51·55		48·9	50·2
10	46·46	46·51	48·55	48·54	50·54	50·59	50·35	50·89	49·0	52·0
17	46·55		48·57		50·37		51·00		51·1	51·2
24	46·68		48·60		50·44		50·66		49·9	50·5
Oct. 1	46·78		48·62		49·91		49·40		48·4	50·5
8	46·86		48·61		49·48		49·09		48·7	50·4
15	46·93	46·93	48·55	48·52	49·40	48·86	49·26	47·85	44·5	46·0
22	47·00		48·46		48·34		46·29		39·0	41·0
29	47·09		48·35		47·15		45·22		42·9	46·0
Nov. 5	47·15		48·11		47·00		46·75		48·6	51·4
12	47·19	47·20	47·88	47·82	47·06	46·61	46·00	45·57	43·1	45·0
19	47·24		47·75		46·37		44·90		45·0	47·0
26	47·23		47·54		46·00		44·64		40·6	40·0
Dec. 3	47·25		47·34		45·39		43·89		38·4	36·1
10	47·27		47·15		44·71		42·64		43·1	48·9
17	47·25	47·23	46·90	46·90	44·81	44·56	43·49	42·78	39·2	44·5
24	47·22		46·67		44·10		42·40		40·7	41·8
31	47·17		46·45		43·81		41·50		33·0	32·0
1895.										
Jan. 7	47·13		46·22		42·39		39·21		32·9	34·0
14	47·09	47·05	45·90	45·65	41·06	40·91	37·50	37·89	31·7	36·1
21	47·02		45·46		40·30		37·85		32·4	32·4
28	46·94		45·01		39·91		37·00		31·2	26·0
Feb. 4	46·89		44·61		39·20		36·40		31·8	35·0
11	46·76	46·72	44·20	44·01	38·56	37·84	35·25	35·49	25·2	25·0
18	46·67		43·83		37·59		35·30		30·8	30·9
25	46·56		43·40		36·00		35·00		31·7	33·3
Mar. 4	46·44		42·91		36·00		35·06		31·9	36·1
11	46·30	46·23	42·66	42·56	38·50	38·07	37·00	37·71	35·1	35·0
18	46·16		42·42		38·30		38·60		39·5	42·8
25	46 02		42·25		39·50		40·20		40·2	41·6

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1895.										
April 1 . . .	45·85		42·25		40·00		39·40		38·1	39·0
8 . . .	45·70		42·32		40·10		39·50		37·1	41·0
15 . . .	45·57	45·59	42·35	42·39	40·70	40·96	40·90	41·16	39·9	41·2
22 . . .	45·47		42·42		41·29		42·28		48·5	53·8
Tuesday 30 . . .	45·35		42·59		42·71		43·70		46·2	50·0
May 6 . . .	45·27		42·80		43·34		44·50		47·5	50·0
13 . . .	45·21	45·17	43·08	43·25	44·48	44·59	46·46	45·81	52·8	59·0
20 . . .	45·12		43·35		45·20		45·89		43·7	45·1
27 . . .	45·09		43·77		45·35		46·40		51·6	59·5
June 3 . . .	45·06		44·06		46·67		49·18		51·3	52·4
10 . . .	45·07	45·07	44·42	44·65	47·85	47·80	50·60	49·85	54·7	57·9
17 . . .	45·07		44·84		48·32		49·54		49·6	52·0
24 . . .	45·09		45·28		48·36		50·08		54·1	55·9
July 1 . . .	45·13		45·61		49·46		51·69		54·3	57·8
8 . . .	45·20		45·99		50·15		52·60		59·0	64·8
15 . . .	45·28	45·29	46·37	46·36	50·60	50·33	52·16	52·06	54·0	57·9
22 . . .	45·38		46·74		50·70		52·20		54·2	58·4
29 . . .	45·47		47·08		50·76		51·61		52·8	54·7
Aug. 5 . . .	45·57		47·33		50·66		51·75		53·4	54·5
12 . . .	45·68	45·76	47·56	47·67	50·95	51·25	52·51	52·71	54·2	55·9
19 . . .	45·84		47·78		51·38		53·49		58·9	61·4
26 . . .	45·95		48·01		52·03		53·10		52·7	56·0
Sept. 2 . . .	46·08		48·30		51·84		53·05		55·1	58·2
9 . . .	46·19		48·50		51·79		52·70		53·9	57·0
Tuesday 17 . . .	46·35	46·33	48·69	48·63	51·69	51·70	52·51	52·63	55·4	57·1
23 . . .	46·45		48·78		51·57		51·89		51·8	56·4
30 . . .	46·58		48·88		51·60		52·98		53·0	54·9
Oct. 7 . . .	46·67		48·94		50·93		49·84		44·8	46·0
14 . . .	46·80	46·83	49·00	48·86	49·75	49·15	49·09	47·49	48·9	48·4
21 . . .	46·88		48·86		48·80		47·05		41·1	39·2
28 . . .	46·98		48·65		47·12		44·00		35·7	35·7
Nov. 4 . . .	47·09		48·37		45·75		43·30		40·2	40·7
11 . . .	47·18	47·18	47·99	47·83	45·65	45·41	44·30	43·64	44·2	45·9
18 . . .	47·22		47·60		45·27		43·45		38·3	39·3
25 . . .	47·25		47·36		44·96		43·50		40·2	41·3
Dec. 2 . . .	47·27		47·10		44·75		43·60		40·1	43·4
9 . . .	47·26		46·87		44·30		41·75		39·1	44·6
16 . . .	47·22	47·22	46·63	46·60	43·30	43·42	41·20	41·24	36·5	35·6
23 . . .	47·19		46·34		42·75		40·25		36·0	37·0
30 . . .	47·16		46·05		42·00		39·41		39·5	41·0
1896.										
Jan. 6 . . .	47·11		45·68		42·36		41·61		39·5	40·9
13 . . .	47·04	47·01	45·42	45·35	42·25	42·09	40·30	40·89	36·4	36·5
20 . . .	46·96		45·24		41·95		41·00		37·8	38·6
27 . . .	46·91		45·07		41·80		40·63		44·9	50·8
Feb. 3 . . .	46·80		44·87		42·14		41·09		39·2	43·3
10 . . .	46·72	46·67	44·75	44·72	42·33	42·59	42·10	41·97	42·8	44·5
17 . . .	46·63		44·65		43·00		42·94		43·3	45·8
24 . . .	46·52		44·62		42·89		41·75		34·7	34·1
Mar. 2 . . .	46·45		44·58		42·05		40·76		38·5	40·0
9 . . .	46·37		44·49		41·74		40·49		41·1	42·5
16 . . .	46·29	46·30	44·35	44·34	41·80	41·91	40·40	40·93	39·5	42·0
Tuesday 24 . . .	46·23		44·18		41·80		41·50		45·1	49·5
Monday 30 . . .	46·14		44·08		42·15		41·50		37·2	37·9

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1896.										
April 6	46·09		44·07		42·30		42·70		47·3	51·8
13	46·00	45·98	44·06	44·12	43·50	43·41	43·59	43·90	40·3	42·9
20	45·94		44·15		43·44		43·75		48·0	52·5
27	45·87		44·22		44·40		45·55		49·0	51·9
May 4	45·82		44·37		44·75		45·15		47·7	54·0
11	45·79	45·77	44·56	44·71	45·56	46·16	47·24	47·61	52·3	62·5
18	45·75		44·77		46·90		49·00		52·7	54·6
25	45·72		45·16		47·41		49·05		51·4	55·0
June 1	45·71		45·39		48·34		50·50		52·4	55·4
8	45·71		45·74		48·70		49·59		50·8	53·2
15	45·74	45·75	46·09	46·06	48·98	49·28	50·94	51·12	55·3	58·0
22	45·76		46·38		50·00		51·90		52·9	56·0
29	45·83		46·69		50·36		52·65		54·7	57·5
July 6	45·91		47·02		50·70		52·22		57·3	62·0
13	45·98	46·03	47·33	47·46	50·98	51·30	52·95	53·03	57·6	61·3
20	46·08		47·62		51·63		53·85		61·5	66·4
27	46·14		47·88		51·90		53·10		52·2	53·9
Aug. 3	46·25		48·22		51·70		52·83		54·7	57·6
10	46·35		48·40		51·66		52·69		51·8	55·9
17	46·46	46·47	48·56	48·55	51·77	51·67	52·85	52·67	52·8	55·0
24	46·58		48·73		51·74		52·85		55·6	58·0
31	46·69		48·86		51·49		52·14		52·7	55·0
Sept. 7	46·80		48·97		51·60		52·46		52·3	52·0
14	46·92	46·97	49·07	49·09	51·64	51·14	52·54	51·61	54·5	57·6
21	47·03		49·13		51·00		51·87		51·7	53·0
28	47·12		49·19		50·30		49·57		46·8	49·0
Oct. 5	47·21		49·11		49·84		49·62		44·4	44·1
Tuesday 13	47·30	47·34	48·97	48·86	48·70	48·16	46·50	46·45	38·4	39·1
Monday 19	47·39		48·82		47·60		45·85		42·5	42·1
26	47·45		48·55		46·50		43·82		37·6	38·5
Nov. 2	47·52		48·24		45·40		43·10		39·5	40·3
9	47·55		47·86		44·70		42·49		36·3	39·4
16	47·58	47·55	47·50	47·55	44·66	44·76	43·38	43·12	42·9	44·5
23	47·60		47·22		44·47		43·30		47·9	48·8
30	47·52		46·94		44·56		43·35		34·7	31·0
Dec. 7	47·51		46·75		43·91		42·46		40·0	40·0
14	47·46	47·42	46·54	46·39	43·70	43·04	42·10	41·15	37·5	37·6
21	47·39		46·30		42·69		39·74		34·7	35·6
28	47·34		45·99		41·85		40·30		37·8	37·5
1897.										
Jan. 4	47·28		45·66		42·21		40·96		40·7	41·9
11	47·19	47·13	45·40	45·29	42·09	41·52	40·20	39·43	35·0	35·8
18	47·08		45·16		41·30		38·80		32·5	33·0
25	46·96		44·92		40·50		37·75		32·2	38·1
Feb. 1	46·86		44·61		39·70		39·00		32·0	33·6
8	46·76	46·70	44·27	44·11	39·10	39·56	37·00	38·55	36·4	40·6
15	46·65		43·95		39·25		37·99		38·2	39·3
22	46·54		43·63		40·20		40·20		44·2	48·7
Mar. 1	46·37		43·50		41·40		41·49		39·2	40·1
8	46·23		43·45		40·90		39·80		36·1	36·3
15	46·10	46·11	43·42	43·42	40·80	41·14	39·70	40·68	36·2	36·1
22	45·98		43·29		40·80		40·50		44·5	48·7
29	45·85		43·45		41·82		41·89		36·5	37·1

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1897.										
April 5	45·74		43·26		41·00		39·28		35·5	38·9
12	45·65	45·60	43·20	43·19	40·66	41·14	40·00	40·53	42·0	47·0
19	45·55		43·13		41·20		41·15		42·8	45·9
26	45·45		43·19		41·70		41·69		43·3	46·5
May 3	45·36		43·10		42·29		42·88		42·5	43·6
10	45·33		43·21		42·80		43·38		44·9	49·0
17	45·25	45·26	43·36	43·39	43·23	43·55	44·26	44·47	48·5	53·6
24	45·20		43·52		44·41		45·72		46·9	48·1
31	45·16		43·76		45·02		46·11		51·4	56·3
June 7	45·10		44·05		45·89		47·83		47·4	47·7
14	45·10	45·09	44·33	44·50	46·50	46·77	48·80	48·47	55·0	55·9
21	45·09		44·65		47·09		47·53		50·1	56·9
28	45·09		44·96		47·60		49·72		52·7	53·0
July 5	45·14		45·26		48·61		50·72		55·6	60·2
12	45·15	45·20	45·60	45·78	48·98	49·57	50·63	51·79	56·0	60·8
19	45·22		45·94		49·91		52·52		56·5	61·9
26	45·28		46·32		50·77		53·31		56·1	59·4
Aug. 2	45·37		46·74		51·36		53·85		58·6	62·7
9	45·46		47·13		52·21		54·88		57·9	61·9
16	45·55	45·57	47·52	47·48	52·40	51·99	53·98	53·79	54·1	57·7
23	45·67		47·88		52·06		53·09		54·5	57·1
30	45·79		48·15		51·90		53·14		55·0	58·1
Sept. 6	45·91		48·33		51·50		51·31		49·6	52·6
13	46·06	46·08	48·48	48·42	50·42	50·48	49·90	50·16	54·0	59·7
20	46·16		48·45		50·31		49·81		47·6	52·0
27	46·19		48·41		49·70		49·61		49·5	53·0
Oct. 4	46·40		48·34		49·60		49·53		46·9	47·3
11	46·49	46·54	48·30	48·25	49·23	48·88	48·83	48·45	47·0	49·7
18	46·60		48·25		48·29		47·58		51·2	55·0
25	46·67		48·10		48·40		47·86		45·8	48·0
Nov. 1	46·74		47·99		47·85		47·30		46·2	46·7
8	46·79		47·86		47·46		47·41		46·1	48·8
15	46·81	46·82	47·70	47·72	47·39	47·22	47·30	46·76	37·5	35·0
22	46·89		47·61		46·80		46·51		46·9	48·9
29	46·88		47·44		46·60		45·30		37·7	37·1
Dec. 6	46·92		47·31		45·22		42·95		41·9	44·5
13	46·91	46·92	47·05	46·90	44·45	44·24	42·30	42·76	36·6	38·2
20	46·92		46·75		44·05		42·60		39·2	39·4
27	46·94		46·50		43·22		43·20		44·4	37·9
1898.										
Jan. 3	46·90		46·18		43·22		41·82		39·9	39·1
10	46·87		45·90		43·13		41·86		39·6	42·0
17	46·82	46·82	45·69	45·75	43·38	43·47	42·51	42·60	43·4	48·1
24	46·79		45·55		43·71		43·00		43·3	46·5
31	46·72		45·45		43·90		43·82		44·4	45·0
Feb. 7	46·63		45·36		43·86		42·00		35·0	36·0
14	46·59	46·55	45·67	45·30	43·09	42·87	41·98	41·67	40·1	43·3
21	46·52		45·16		42·80		40·90		32·7	30·9
28	46·47		45·02		41·73		41·81		39·6	39·0
Mar. 7	46·42		44·80		41·25		39·30		35·1	34·8
14	46·37	46·34	44·56	44·49	41·18	41·63	40·39	40·55	39·9	40·2
21	46·32		44·34		42·00		41·90		39·4	42·5
28	46·23		44·26		42·10		40·63		36·3	37·9

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1898.										
April 4	46·17		44·19		41·55		40·10		38·9	42·0
11	46·12	46·08	44·09	44·11	42·20	42·61	42·90	42·60	45·4	49·6
18	46·05		44·05		42·98		43·19		44·6	49·8
25	45·98		44·12		43·70		44·20		44·6	46·0
May 2	45·92		44·23		43·98		44·23		45·3	47·2
9	45·86		44·35		44·45		45·30		49·1	51·6
16	45·81	45·83	44·50	44·50	44·75	44·70	44·73	45·21	42·9	45·8
23	45·78		44·64		44·90		45·40		46·2	46·9
30	45·76		44·76		45·43		46·37		46·1	49·2
June 6	45·76		44·93		45·79		46·65		50·3	61·1
13	45·74	45·75	45·10	45·27	47·09	47·41	48·90	49·04	49·9	51·0
20	45·75		45·37		48·05		50·30		54·4	57·0
27	45·75		45·69		48·71		50·32		52·1	52·5
July 4	45·77		46·05		49·39		51·14		51·8	55·2
11	45·83	45·85	46·39	46·55	49·88	50·22	52·02	52·19	57·5	61·5
18	45·86		46·70		50·64		52·68		56·1	59·0
25	45·93		47·05		50·96		52·92		54·2	58·8
Aug. 1	46·01		47·36		51·10		52·63		55·8	58·8
8	46·07		47·66		51·39		52·60		53·2	55·1
15	46·18	46·20	47·95	47·91	51·60	51·60	53·70	53·07	57·2	59·0
22	46·38		48·20		51·91		53·39		57·5	62·1
29	46·36		48·38		52·00		53·02		50·0	52·0
Sept. 5	46·51		48·65		51·61		52·96		59·9	67·0
12	46·59	46·65	48·75	48·86	52·49	52·05	54·10	53·08	52·7	54·0
19	46·70		48·95		52·33		53·88		50·5	50·1
26	46·80		49·10		51·76		51·39		47·5	48·4
Oct. 3	46·93		49·22		50·86		50·94		55·0	57·1
10	47·02		49·17		50·96		51·00		49·2	50·0
17	47·12	47·12	49·14	49·11	51·16	50·32	50·06	49·92	45·3	45·5
24	47·22		49·09		49·48		49·15		48·0	50·0
31	47·30		48·95		49·15		48·45		46·6	47·9
Nov. 7	47·40		48·83		48·50		47·09		46·0	48·3
14	47·46	47·46	48·66	48·52	47·70	47·39	46·17	45·77	43·2	45·9
21	47·49		48·44		47·38		46·20		40·4	40·4
28	47·50		48·17		46·00		43·63		35·1	33·0
Dec. 5	47·59		47·95		43·18		43·68		48·9	54·0
12	47·60	47·58	47·61	47·51	45·28	44·62	44·12	43·88	47·4	52·0
19	47·56		47·32		45·25		44·23		39·4	39·1
26	47·55		47·15		44·78		43·50		46·0	47·2
1899.										
Jan. 2	47·50		46·90		44·40		42·03		36·5	37·0
9	47·49		46·72		43·53		44·23		42·2	45·8
16	47·43	47·42	46·42	46·40	43·08	43·12	41·10	41·55	40·6	42·0
23	47·36		46·12		42·72		41·46		35·9	34·2
30	47·32		45·83		41·89		38·95		34·5	35·9
Feb. 6	47·22		45·52		41·05		38·22		32·8	33·1
13	47·17	47·11	45·19	45·06	40·95	41·24	40·28	39·76	40·2	42·2
20	47·08		44·86		41·57		41·00		38·3	39·0
27	46·97		44·68		41·39		39·52		32·7	33·0
Mar. 6	46·88		44·54		41·29		40·13		38·9	43·9
13	46·79	46·72	44·41	44·36	41·39	41·47	41·05	40·33	43·2	48·0
20	46·65		44·26		42·13		41·52		33·7	33·3
27	46·58		44·25		41·06		38·63		38·6	45·7

TABLE I.—*continued.*

Date.	t_1 Each Reading.	t_1 Monthly Mean.	t_2 Each Reading.	t_2 Monthly Mean.	t_3 Each Reading.	t_3 Monthly Mean.	t_4 Each Reading.	t_4 Monthly Mean.	t_5 Surface Theorem. Each Reading.	Air. Each Reading.
1899.										
April 3 . . .	46·48		44·09		41·31		41·19		43·9	49·2
10 . . .	46·37	46·35	43·98	44·00	42·12	41·78	41·58	41·21	39·4	39·0
17 . . .	46·31		43·97		41·90		41·03		37·6	38·6
24 . . .	46·23		43·96		41·78		41·05		44·6	52·1
May 1 . . .	46·13		43·92		42·72		43·02		40·2	39·4
8 . . .	46·07		43·97		42·98		42·99		44·0	47·7
15 . . .	46·01	46·00	44·04	44·08	43·62	43·57	44·01	43·87	44·7	45·7
22 . . .	45·92		44·14		44·18		44·50		42·8	43·1
29 . . .	45·89		44·33		44·34		44·85		49·9	57·7
June 5 . . .	45·86		44·47		45·92		48·31		56·1	61·6
12 . . .	45·84	45·83	44·76	45·01	47·68	48·04	54·80	51·61	57·9	68·9
19 . . .	45·80		45·16		49·01		51·80		53·7	55·6
26 . . .	45·82		45·66		49·57		51·55		57·3	62·2
July 3 . . .	45·83		46·14		50·03		51·50		54·5	59·0
10 . . .	45·87		46·55		50·55		53·02		56·5	57·3
17 . . .	45·93	45·95	46·93	46·91	51·05	51·00	53·16	52·97	58·3	61·9
24 . . .	46·01		47·29		51·52		53·40		57·6	62·3
31 . . .	46·10		47·64		51·83		53·78		61·2	66·8
Aug. 7 . . .	46·19		47·94		52·58		54·71		54·4	54·7
14 . . .	46·30	46·36	48·27	48·41	52·55	52·83	54·46	54·80	58·1	60·0
21 . . .	46·43		48·60		52·80		54·29		59·4	65·9
28 . . .	46·53		48·84		53·38		55·74		57·8	59·8
Sept. 4 . . .	46·66		49·13		53·26		54·26		57·2	61·9
11 . . .	46·78	46·83	49·35	49·38	52·97	52·51	53·72	53·66	55·0	62·0
18 . . .	46·88		49·48		52·44		52·65		49·1	51·4
25 . . .	47·02		49·56		51·39		54·00		48·2	51·0
Oct. 2 . . .	47·14		49·53		50·10		48·85		46·8	49·2
9 . . .	47·27		49·39		49·42		48·20		47·7	51·7
16 . . .	47·38	47·36	49·18	49·19	49·19	49·16	47·68	48·03	44·3	48·7
23 . . .	47·47		49·02		48·72		47·85		47·4	50·1
30 . . .	47·52		48·82		48·38		47·58		45·2	46·7
Nov. 6 . . .	47·59		48·64		47·92		47·02		45·4	48·6
13 . . .	47·64	47·64	48·48	48·37	47·47	47·24	45·89	46·09	47·2	50·7
20 . . .	47·65		48·27		46·87		45·26		44·3	43·6
27 . . .	47·69		48·09		46·69		46·19		48·8	50·6
Dec. 4 . . .	47·69		47·89		46·93		45·62		46·6	51·6
11 . . .	47·65	47·65	47·70	47·58	46·13	45·16	43·65	42·71	33·7	32·4
18 . . .	47·64		47·53		44·26		41·02		35·8	37·0
25 . . .	47·62		47·19		43·30		40·55		34·9	34·5

TABLE II.—*The Edinburgh Royal Observatory Rock Thermometers. Monthly, Quarterly, and Annual Means.*

Date.	t_1			t_2			t_3			t_4		
	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.
1888.												
January, .	46·80			44·84			41·25			39·78		
February,	46·38	46·39		44·02	43·98		40·25	40·26		38·47	38·67	
March, .	45·99			43·08			39·28			37·75		
			46·03			45·28			44·54			44·20
April, .	45·54			42·40			39·83			39·67		
May, .	45·14	45·22		42·82	43·11		43·54	43·19		45·13	44·11	
June, .	44·98			44·11			46·21			47·54		
			45·88			45·18			44·76			44·60
July, .	45·10			45·57			48·80			50·34		
August, .	45·45	45·47		46·87	46·66		49·94	49·55		51·12	50·62	
September,	45·86			47·55			49·92			50·39		
			45·84			45·28			44·98			44·86
October, .	46·26			47·55			47·68			47·13		
November,	46·50	46·45		47·06	46·96		46·22	46·02		45·16	44·98	
December,	46·58			46·28			44·16			42·65		
			45·88			45·50			45·40			45·36
1889.												
January, .	46·47			45·23			42·02			40·62		
February,	46·24	46·22		44·41	44·41		41·20	41·14		39·52	39·74	
March, .	45·96			43·59			40·20			39·08		
			46·00			45·81			45·82			45·78
April, .	45·59			43·23			41·62			41·61		
May, .	45·31	45·38		43·66	43·96		44·63	44·88		46·21	46·11	
June, .	45·23			44·99			48·40			50·52		
			46·17			46·01			45·89			45·76
July, .	45·47			46·85			50·97			52·76		
August, .	45·95	45·97		48·12	47·92		51·69	51·23		52·88	52·27	
September,	46·49			48·79			51·04			51·18		
			46·29			46·14			46·09			46·01
October, .	46·95			48·56			48·22			47·11		
November,	47·18	47·11		47·82	47·75		46·77	46·30		45·86	44·91	
December,	47·20			46·87			43·91			41·75		
			46·40			46·29			46·17			45·98
1890.												
January, .	47·01			45·71			42·95			41·72		
February,	46·73	46·70		44·96	44·91		41·52	41·95		39·85	40·75	
March, .	46·36			44·07			41·39			40·68		
			46·46			46·24			46·06			45·92
April, .	46·00			43·98			42·79			42·53		
May, .	45·78	45·83		44·31	44·58		44·88	45·21		46·05	46·00	
June, .	45·70			45·44			47·97			49·42		
			46·50			46·32			46·17			46·00
July, .	45·87			46·73			49·78			51·27		
August, .	46·16	46·21		47·79	47·70		51·49	50·79		52·88	52·04	
September,	46·61			48·58			51·11			51·97		
			46·52			46·22			45·93			45·71
October, .	47·02			48·90			50·13			49·75		
November,	47·32	47·25		48·32	48·07		46·77	46·74		45·33	45·22	
December,	47·41			46·98			43·33			40·58		
			46·49			46·02			45·53			45·29
1891.												
January, .	47·20			45·34			40·68			38·52		
February,	46·82	46·79		44·34	44·54		41·39	40·97		40·62	39·57	
March, .	46·35			43·95			40·85			39·56		
			46·43			45·96			45·54			45·34
April, .	45·98			43·30			40·90			40·68		
May, .	45·66	45·71		43·53	43·78		43·51	43·62		44·03	44·34	
June, .	45·50			44·51			46·44			48·32		
			46·38			45·84			45·38			45·18
July, .	45·57			46·18			50·36			52·56		
August, .	45·94	45·98		47·69	47·44		51·27	50·83		52·48	52·22	
September,	46·42			48·45			50·86			51·63		
			46·31			45·73			45·18			44·92
October, .	46·83			48·58			49·29			48·56		
November,	47·13	47·04		47·79	47·62		45·86	46·08		43·99	44·59	
December,	47·17			46·48			43·09			41·22		
			46·23			45·65			45·25			45·02

TABLE II.—*continued.*

Date.	t_1			t_2			t_3			t_4		
	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.
1892.												
January, .	46·96			45·16			40·58			38·15		
February,	46·54	46·52		43·98	44·08		40·52	40·19		39·28	38·54	
March, .	46·05			43·11			39·48			38·18		
			46·16			45·56			45·08			44·79
April, .	45·65			42·70			40·74			40·75		
May, .	45·30	45·37		43·17	43·47		43·57	43·89		44·63	44·72	
June, .	45·15			44·55			47·35			48·78		
			46·08			45·37			44·76			44·43
July, .	45·29			45·90			49·33			51·14		
August,	45·67	45·71		47·23	47·08		50·87	50·15		52·44	51·31	
September,	46·16			48·10			50·25			50·35		
			45·97			45·26			44·82			44·62
October, .	46·57			47·90			47·29			45·90		
November,	46·80	46·71		46·87	46·87		44·86	44·79		43·59	43·14	
December,	46·75			45·84			42·21			39·92		
			45·93			45·44			45·32			45·28
1893.												
January, .	46·50			44·36			39·88			37·81		
February,	46·06	46·08		43·42	43·62		40·80	40·46		39·91	39·31	
March, .	45·67			43·08			40·69			40·21		
			46·01			45·74			45·77			45·73
April, .	45·36			43·11			42·52			43·01		
May, .	45·15	45·23		43·97	44·19		45·73	45·88		47·41	47·35	
June, .	45·18			45·50			49·38			51·64		
			46·14			45·99			46·08			46·06
July, .	45·51			47·20			51·25			52·92		
August, .	46·04	46·02		48·37	48·26		52·55	51·94		54·33	53·13	
September,	46·52			49·20			52·02			52·15		
			46·29			46·25			46·32			46·28
October, .	47·04			48·95			49·09			48·42		
November,	47·32	47·23		48·02	47·90		45·41	46·02		43·03	44·45	
December,	47·34			46·72			43·56			41·90		
			46·41			46·30			46·06			45·84
1894.												
January, .	47·07			45·60			42·23			40·69		
February,	46·70	46·69		44·56	44·66		41·17	41·45		39·69	40·19	
March, .	46·31			43·81			40·94			40·18		
			46·42			46·16			45·80			45·58
April, .	45·91			43·75			43·07			43·54		
May, .	45·59	45·69		44·37	44·38		44·78	44·85		45·06	45·61	
June, .	45·58			45·01			46·69			48·24		
			46·40			46·12			45·97			45·82
July, .	45·70			46·52			50·51			52·69		
August, .	46·07	46·09		47·99	47·68		51·61	50·90		52·67	52·08	
September,	46·51			48·54			50·59			50·89		
			46·39			45·97			45·34			45·03
October, .	46·93			48·52			48·86			47·85		
November,	47·20	47·12		47·82	47·75		46·61	46·68		45·57	45·40	
December,	47·23			46·90			44·56			42·78		
			46·29			45·73			45·24			45·03
1895.												
January, .	47·05			45·65			40·91			37·89		
February,	46·72	46·67		44·01	44·07		37·84	38·94		35·49	37·03	
March, .	46·23			42·56			38·07			37·71		
			46·22			45·70			45·29			45·13
April, .	45·59			42·39			40·96			41·16		
May, .	45·17	45·28		43·25	43·43		44·59	44·45		45·81	45·61	
June, .	45·07			44·65			47·80			49·85		
			46·20			45·70			45·12			44·81
July, .	45·29			46·36			50·33			52·06		
August, .	45·76	45·79		47·67	47·55		51·25	51·09		52·71	52·47	
September,	46·33			48·63			51·70			52·63		
			46·20			45·88			45·93			45·86
October, .	46·83			48·86			49·15			47·49		
November,	47·18	47·08		47·83	47·76		45·41	45·99		43·64	44·12	
December,	47·22			46·60			43·42			41·24		
			46·34			46·27			46·39			46·35

TABLE II.—*continued.*

Date.	t_1			t_2			t_3			t_4		
	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.	Monthly.	Quarterly.	Annual.
1896.												
January, .	47·01			45·35			42·09			40·89		
February,	46·67	46·66		44·72	44·80		42·59	42·20		41·97	41·26	
March, .	46·30			44·34			41·91			40·93		
			46·52			46·47			46·46			46·34
April, .	45·98			44·12			43·41			43·90		
May, .	45·77	45·83		44·71	44·96		46·16	46·28		47·61	47·54	
June, .	45·75			46·06			49·28			51·12		
			46·60			46·43			46·29			46·20
July, .	46·03			47·46			51·30			53·03		
August, .	46·47	46·49		48·55	48·37		51·67	51·37		52·67	52·44	
September,	46·97			49·09			51·14			51·61		
			46·60			46·30			45·93			45·78
October, .	47·34			48·86			48·16			46·45		
November,	47·55	47·44		47·55	47·60		44·76	45·32		43·12	43·57	
December,	47·42			46·39			43·04			41·15		
			46·48			45·98			45·31			45·01
1897.												
January, .	47·13			45·29			41·52			39·43		
February,	46·70	46·65		44·11	44·27		39·56	40·74		38·55	39·55	
March, .	46·11			43·42			41·14			40·68		
			46·26			45·70			45·14			44·88
April, .	45·60			43·19			41·14			40·53		
May, .	45·26	45·32		43·39	43·69		43·55	43·82		44·47	44·49	
June, .	45·09			44·50			46·77			48·47		
			46·09			45·70			45·50			45·48
July, .	45·20			45·78			49·57			51·79		
August, .	45·57	45·62		47·48	47·23		51·99	50·68		53·79	51·91	
September,	46·08			48·42			50·48			50·16		
			46·07			45·93			45·99			46·00
October, .	46·54			48·25			48·88			48·45		
November,	46·82	46·76		47·72	47·62		47·22	46·78		46·76	45·99	
December,	46·92			46·90			44·24			42·76		
			46·21			46·16			46·26			46·28
1898.												
January, .	46·82			45·75			43·47			42·60		
February,	46·55	46·57		45·30	45·18		42·87	42·66		41·67	41·61	
March, .	46·34			44·49			41·63			40·55		
			46·36			46·30			46·41			46·50
April, .	46·08			44·11			42·61			42·60		
May, .	45·83	45·89		44·50	44·63		44·70	44·91		45·21	45·62	
June, .	45·75			45·27			47·41			49·04		
			46·52			46·49			46·58			46·63
July, .	45·85			46·55			50·22			52·19		
August, .	46·20	46·23		47·91	47·77		51·60	51·29		53·07	52·78	
September,	46·65			48·86			52·05			53·08		
			46·65			46·51			46·39			46·37
October, .	47·12			49·11			50·32			49·92		
November,	47·46	47·39		48·52	48·38		47·39	47·44		45·77	46·52	
December,	47·58			47·51			44·62			43·88		
			46·69			46·44			46·28			46·35
1899.												
January, .	47·42			46·40			43·12			41·55		
February,	47·11	47·08		45·06	45·27		41·24	41·94		39·76	40·55	
March, .	46·72			44·36			41·47			40·33		
			46·73			46·56			46·49			46·61
April, .	46·35			44·00			41·78			41·21		
May, .	46·00	46·06		44·08	44·36		43·57	44·46		43·87	45·56	
June, .	45·83			45·01			48·04			51·61		
			46·77			46·56			46·43			46·38
July, .	45·95			46·91			51·00			52·97		
August, .	46·36	46·38		48·41	48·23		52·83	52·11		54·80	53·81	
September,	46·83			49·38			52·51			53·66		
October, .	47·36			49·19			49·16			48·03		
November,	47·64	47·55		48·37	48·38		47·24	47·19		46·09	45·61	
December,	47·65			47·58			45·16			42·71		

The Annual Curves.

Complete tables of the whole of the observations made with the old set of rock thermometers during the forty years from 1837 to 1876 inclusive will be found in Volumes XI., XII., XIII., and XIV. of the *Edinburgh Astronomical Observations.* This set of thermometers was erected at the Calton Hill in 1837 at the expense of the British Association for the Advancement of Science. At the same time two other sets were put up by the Association, one in the sandstone rock of Craigleith Quarry and another at the Experimental Gardens, now part of the Botanic Gardens. All three sets were established chiefly at the recommendation of the late Prof. J. D. Forbes. The two sets at Craigleith Quarry and at the Botanic Gardens were regularly observed for five years, and the results, along with those of the Calton Hill set for the same five years, are to be found in an interesting paper by Prof. Forbes, published in Vol. XVI. of the *Transactions of the Royal Society of Edinburgh.* The destruction of both these valuable sets of instruments appears to have taken place soon after the five years' observations had been secured. The Craigleith thermometers were maliciously destroyed in May 1842, and the set at the Experimental Gardens was broken by a storm in the winter of 1844–5. The sole remaining set, that in the rock of the Calton Hill, existed complete up to 1861, when t_2, or the second deepest thermometer, was found broken off at the surface of the rock during the severe frost of the winter of 1860–61. The remaining three of this set were destroyed in September 1876 by an unfortunate Portuguese sailor, whom, on his arrest by the police, it was found necessary to place in a lunatic asylum.

A complete description of the construction and erection of these thermometers, and a discussion of the method of determining the corrections of their readings, will be found in the paper by Prof. Forbes mentioned above, where he further discusses the results of the observations, and deduces the mean temperatures for the five years, the rate of increase of temperature with depth, and the ranges at different depths, showing how these depend on the varying conductivity of the strata at the three stations. The forms of the annual curves are then discussed, and their equations determined for each of the four thermometers at the three stations. The curves are shown graphically in a plate, and from these curves the epochs of maximum and minimum temperatures were obtained by interpolation. To obtain the value of the ratio $\sqrt{\frac{k}{c}}$, where k is the conductivity of the soil and c the specific heat, Prof. Forbes makes use of the equation log. $\Delta = \mathrm{A} + \mathrm{B}p$, where Δ is the thermometric range at a depth p in French feet, A and B constants, of which B is always negative. The determination of these constants Prof. Forbes looked upon as the primary object of his investigation. A is equal to the logarithm of the thermometric range at the surface, or where $p = 0$. B determines the rate of diminution of the range below the surface, being smaller as the heat descends more readily, or as the conductivity is greater. This coefficient B was shown by

FOURIER to be equal to $\sqrt{\frac{\pi}{a}}$ log.e, where $a = \sqrt{\frac{k}{c}}$. Hence, if A and B could be determined from the curves showing the range at different depths, and if c, the specific heat, were determined by laboratory experiment, the conductivity of the strata at the three stations could be deduced. Specimens of the three varieties of strata were submitted to M. REGNAULT of Paris, who determined their specific heats by experiment, and from these values, combined with the values of B obtained from the curves, the values of k the conducting power of the strata were computed, "which," Prof. FORBES remarks, "has rarely been so accurately determined for any form of matter."

This research of Prof. FORBES has a special interest, to which no later investigation of the rock thermometer observations in Edinburgh can aspire, inasmuch as the existence of the three sets enabled him to compare the different circumstances depending on the locality of the instruments, more particularly the relative conducting powers of the different rocks or soils in which they were buried. The destruction of two of the sets prevents any redetermination of these quantities by this method from a longer series of observations so far as these two stations are concerned, and the loss at a later date of t_2 at Calton Hill still further restricted the material for investigation. After the Calton Hill set, however, had been in existence for a further period of thirteen years, Lord KELVIN (then Prof. WILLIAM THOMSON) and Prof. EVERETT, in papers read before the Royal Society of Edinburgh on the 30th April 1860, re-discussed the whole of the physical phenomena concerned, both from the theoretical and practical points of view. Lord KELVIN, specially, shows how the theory of periodic variations can be applied to the particular case of terrestrial temperature. As Prof. FORBES had already done, he adopts FOURIER's solution of the problem, and applies it in a more elaborate form than Prof. FORBES had attempted. FOURIER's solution of the problem of the deduction of the conductivity of the strata from the retardation of epoch, and the amplitude at different depths, may be stated in Lord KELVIN's own words: "If the temperature at any point of an infinite plane, in a solid extending infinitely in all directions, be subjected to a simple harmonic variation, the temperature throughout the solid on each side of the plane will follow everywhere according to the simple harmonic law, with epochs retarded equally, and with amplitudes diminished in a constant proportion for equal augmentations of distance. The retardation of epoch expressed in circular measure (arc divided by radius) is equal to the diminution of the Napierian logarithm of the amplitude; and the amount of each per unit of distance is equal to $\sqrt{\frac{\pi c}{\mathrm{T}k}}$, if c denote the capacity for heat of a unit bulk of the substance, and k its conductivity.

"Hence if the complex harmonic functions expressing the varying temperature at two different depths be determined, and each term of the first be compared with the corresponding term of the second, the value of $\sqrt{\frac{\pi c}{\mathrm{T}k}}$ may be determined either by dividing the difference of the Napierian logarithms of the amplitudes, or the difference of the epochs by the distance between the points. The comparison of each term in the one series with the corresponding term in the other gives us, therefore, two determinations of the value of $\sqrt{\frac{\pi c}{k}}$, which should agree perfectly, if (1) the data were perfectly accurate, if (2) the isothermal surfaces throughout were parallel planes, and if (3) the specific heat and conductivity of the soil were everywhere and always constant."

By the method thus indicated Lord KELVIN applied the general theory—1st, to the five years' observations at the three stations; 2nd, to the thirteen years' observations at Calton Hill alone. The result he brought out was that the figures representing the conducting power of the rock at Calton Hill, the values of $\sqrt{\frac{\pi c}{k}}$, as deduced from the diminution of amplitude, and from the retardation of epoch, appear to diminish as the deeper thermometers are approached. There are thus outstanding discrepancies from FOURIER'S theory, which supposes that the values should come out alike for all depths. Lord KELVIN states his opinion that "there can be no doubt but that this discrepance is not attributable to errors of observation, and it must therefore be owing to deviation in the natural circumstances from those assumed for the foundation of the mathematical formula." Later on he says: "I can only infer that the residual discrepancies . . . . are not with any probability attributable to variation of conductivity and specific heat in the rock, and conclude that they are to be explained by irregularities, physical and formal, on the surface." Some of these irregularities he specifies, the ground rising slightly to the east and falling abruptly at a distance of about 15 yards on the west, the immediate surface being flat, partly covered with grass, partly with gravel.

It thus appeared to be of great interest to see whether the reduction of the whole series of forty years' observations of the old thermometers, and the series of twenty years of the new thermometers erected in 1879 would bring out a similar or a more satisfactory result. In carrying out this work I have availed myself largely of the elegant methods of procedure detailed in the paper by Prof. EVERETT mentioned already. The readings of the thermometers have throughout been made once a week, on Mondays at noon, and the corrected readings for 1837–76, which were published in the *Edinburgh Astronomical Observations*, have now been taken out and arranged in four series of ten years, under each Monday of the year. The means of the columns so formed give the temperature of each thermometer for the mean date of the Monday to which it belongs. The average of the four series of ten years were then taken as the final temperatures from which the annual curves of the old set of thermometers were obtained.

In computing the equations of these curves, or the harmonic function of the temperature at any time represented by the fraction of the year, I have followed Prof. EVERETT in dividing the year into twelve parts instead of thirty-two, the division Lord KELVIN adopted. I have done so for two reasons—(1) because the labour involved is much less, the equations being solved by a simple method of elimination, and the accuracy attainable is sufficient for the purpose ; (2) because the fifty-two weekly mean temperatures could be divided up into twelve lots, eight of four each, and four of five each, the means of which would not differ much in weight, and these monthly means could be used in solving the equations, without having to resort to the necessity of interpolating values from the curves themselves, a method which is not so accurate, unless the curves are drawn on a very large scale. The year was then divided into twelve equal parts, starting from the mean date of the first Monday. This date for the forty years is January 4·10. If to this we add $\frac{365 \cdot 25}{12} = 30 \cdot 4375$ days, and consider February as consisting of 28·25 days, we arrive at the following twelve equi-mensual dates.

January,	4·10	July,	5·48
February,	3·54	August,	4·93
March,	5·72	September,	4·35
April,	5·16	October,	4·79
May,	5·60	November,	4·22
June,	5·04	December,	4·66

As, however, the mean dates of the four or five weekly mean temperatures used in forming the twelve monthly means did not generally come out quite the same as the equi-mensual dates, it was necessary to apply a small correction depending on the rate of increase or decrease of temperature at the time. This correction is always small, its greatest values being $+0^{\circ} \cdot 015$ in the case of t_1 and $+0^{\circ} \cdot 208$ for t_4. The following table gives the monthly mean temperatures thus deduced, and their dates for the old thermometers from forty years' observations, except in the case of t_2, of which only twenty-four years' observations had been secured at the date of its fracture.

OLD THERMOMETERS.

Equi-mensual Date.		t_1	t_2	t_3	t_4
		°F.	°F.	°F.	°F.
January,	4·10	47·876	47·358	44·324	41·346
February,	3·54	47·789	46·297	42·876	40·150
March,	5·72	47·572	45·386	42·169	40·003
April,	5·16	47·275	44·811	42·382	41·407
May,	5·60	46·975	44·737	43·868	44·535
June,	5·04	46·736	45·275	46·214	48·462
July,	5·48	46·629	46·376	49·007	52·164
August,	4·93	46·690	47·672	51·073	53·945
September,	4·35	46·922	48·756	51·701	53·416
October,	4·79	47·256	49·292	50·858	50·829
November,	4·22	47·582	49·163	48·800	46·789
December,	4·66	47·813	48·412	46·168	43·330
	$T_0 =$	47·2596	46·9612	46·6200	46·3647

The new thermometers were similarly treated for the twenty years' observation available at the end of 1899. It is a somewhat unfortunate circumstance, however, that there exists no record of the relative capacity of the different parts of the tube of which these new thermometers are made, so far as could be discovered after enquiry. This appears to be due to the lamented deaths of Mr Richard Adie, the head of the firm of Messrs Adie & Sons, and of Mr Thomas Wedderburn, who was their responsible manager when they held the contract for the construction of the thermometers. It is known, however, that the new instruments were made with, as nearly as possible, the same size of tube as the old ones. We may therefore be considered justified in supposing that the relative capacities of the capillary parts and the wide parts of the tubes are, approximately, the same in the new set as they were in the old, and that the only difference of importance is to be found in the different lengths of the two sets. By far the greater part of the total corrections applied to the old readings was that depending on the difference of temperature of the bulb and of the wide part of the tube above ground, or the correction for the temperature of the air, and only a small part, never more than 0·03 of a degree F. for the longest thermometer, was due to the temperature of the stem. The corrections to be applied to the new set may then without risk of important error be assumed the same as those of the old.

The following table gives the values of the monthly mean temperatures and the equi-mensual dates for the new thermometers from the twenty years' readings, 1880–1899.

NEW THERMOMETERS.

Equi-mensual Dates.		t_1	t_2	t_3	t_4
		°F.	°F.	°F.	°F.
January,	3·95	47·097	45·788	42·082	40·128
February,	3·39	46·767	44·597	40·949	39·465
March,	5·57	46·343	43·780	40·534	39·421
April,	5·01	45·920	43·314	41·154	40·748
May,	5·45	45·553	43·470	43·211	43·853
June,	4·89	45·352	44·395	46·382	48·045
July,	5·32	45·405	45·873	49·545	51·638
August,	4·77	45·716	47·341	51·170	52·815
September,	4·20	46·198	48·358	51·158	52·295
October,	4·64	46·700	48·681	49·815	49·387
November,	4·08	47·088	48·149	46·963	45·541
December,	4·51	47·238	47·273	44·386	42·477
	$T_0 =$	46·2814	45·9182	45·6124	45·4844

From the numbers in this table the equations to the curves for the new thermometers have been formed.

In the reduction of the readings of the new thermometers, the corrections have not been applied to the single readings, nor to the monthly, quarterly, and annual means deduced from them (Tables I. and II.). They have, however, been applied to the weekly averages of twenty years used in the graphical drawing of the annual curves. The corrections used for this purpose are the means of the corrections applied to the old thermometers in the corresponding weeks over the ten years, 1837–46. The similarity between the two sets of curves would seem to afford a sufficient assurance that the corrections thus applied can be only inappreciably in error, allowance being of course made, in comparing two curves, for the fact that the length of the longest of the new tubes is only 250 inches, while that of the longest old one was 307 inches, the other new tubes being also shorter than the corresponding ones of the old set. This difference of length has, as has been said, but slight effect on the amount of the correction to the readings, only about a maximum of 0·006 of a degree F. It has however, a considerable influence on the form of the curve. As might have been anticipated, a glance at Plates II. and III. will show, that in the new set the annual range, is greater than in the old, but the retardation of epoch is less. While, then, it is to be regretted that the corrections of the new thermometers cannot be determined with the same order of accuracy and confidence with which those of the old set were worked out by Prof. FORBES (Vol. XVI., *Transactions*, Royal Society, Edinburgh), it is reassuring to be able to say, in the words of the late Prof. PIAZZI SMYTH, that "although these new thermometers cannot compete with the old ones as instruments of the most delicate Natural Philosophy chamber problems, I have been much pleased to find that, step by step, they have shown their full sufficiency to keep up the differential historical record of superannual cycles of temperatures."

Having already no less than forty years' observations of the old set, with a complete theory of their corrections, rigorously applied to each individual reading, and having worked out the forms and the equations of the annual curves, and deduced therefrom the conductivity of the rock, it would appear to be almost superfluous to attempt to derive the same quantities from the twenty years' readings of the new set, when the result, owing to the difficulty involved in the corrections, must necessarily be somewhat less reliable. It was thought that this part of the work should be undertaken, however, in the hope that a comparison of the results would form a more or less crucial test of the propriety of the corrections which have been applied, and of the value of the new set for carrying on the "historical record" alluded to by Prof. PIAZZI SMYTH. The value of the conductivity of the soil can be derived either from the difference of the Napierian logarithms of the amplitude at different depths, or from the difference of the epochs. Now the corrections take the positive sign in the winter months and the negative sign in the summer. Hence in the case of the deepest thermometers in each set, which have their highest readings in winter and their lowest in summer, the range is increased ; whereas, with the other three of each set the range is more or less diminished by the application of the corrections. But the epochs are not similarly affected, because the corrections follow a very smooth curve. Hence, if the two determinations of the conductivity by means of the new thermometers agree fairly well with one another, and with the similar determinations by means of the old set, we may reasonably conclude that the corrections have not been appreciably erroneous, and that the new thermometers may be confidently accepted as fairly competent to take the place of the old ones, allowance being made for the difference of the depths of the two sets.

EQUATIONS TO THE CURVES.

We may put the equation to the curve in the form

$$T = T_0 + a_1 \sin(2\pi f + r_1) + a_2 \sin(4\pi f + r_2) + \&c.$$

where T_0 is the mean temperature of the year, a_1 and a_2 the half-amplitude of the annual and semi-annual terms respectively, f the fraction of the year, and r_1 and r_2 the retardation of phase of the same two terms. If we put $\alpha_1 = a_1 \sin r_1$, and $\beta_1 = a_1 \cos r_1$, then $\sqrt{(\alpha_1^2+\beta_1^2)} = a_1$ and $\frac{\alpha_1}{\beta_1} = \tan r_1$. Substituting these values of a_1 and r_1, and similar values of a_2 and r_2, we reduce the equation to the form

$$T = T_0 + (\alpha_1 \cos 2\pi f + \beta_1 \sin 2\pi f) + (\alpha_2 \cos 4\pi f + \beta_2 \sin 4\pi f) + \&c.$$

Giving to f the successive values 0, $\frac{1}{12}$, $\frac{2}{12}$, ... $\frac{11}{12}$, we form the following twelve equations:—

$$\begin{aligned}
T - T_0 &= +\alpha_1 \quad\quad 0 \quad\quad +\alpha_2 \quad\quad 0\\
&= +\alpha_1 \cos 30^\circ + \beta_1 \sin 30^\circ + \alpha_2 \cos 60^\circ + \beta_2 \sin 60^\circ\\
&= +\alpha_1 \cos 60^\circ + \beta_1 \sin 60^\circ - \alpha_2 \cos 60^\circ + \beta_2 \sin 60^\circ\\
&= \quad 0 \quad\quad +\beta_1 \quad\quad -\alpha_2 \quad\quad 0\\
&= -\alpha_1 \cos 60^\circ + \beta_1 \sin 60^\circ - \alpha_2 \cos 60^\circ - \beta_2 \sin 60^\circ\\
&= -\alpha_1 \cos 30^\circ + \beta_1 \sin 30^\circ + \alpha_2 \cos 60^\circ - \beta_2 \sin 60^\circ\\
&= -\alpha_1 \quad\quad 0 \quad\quad +\alpha_2 \quad\quad 0\\
&= -\alpha_1 \cos 30^\circ - \beta_1 \sin 30^\circ + \alpha_2 \cos 60^\circ + \beta_2 \sin 60^\circ\\
&= -\alpha_1 \cos 60^\circ - \beta_1 \sin 60^\circ - \alpha_2 \cos 60^\circ + \beta_2 \sin 60^\circ\\
&= \quad 0 \quad\quad -\beta_1 \quad\quad -\alpha_2 \quad\quad 0\\
&= +\alpha_1 \cos 60^\circ - \beta_1 \sin 60^\circ - \alpha_2 \cos 60^\circ - \beta_2 \sin 60^\circ\\
&= +\alpha_1 \cos 30^\circ - \beta_1 \sin 30^\circ + \alpha_2 \cos 60^\circ - \beta_2 \sin 60^\circ
\end{aligned}$$

The eight sets of values of $T - T_0$ are contained in the two tables on pp. 177 and 178 for the old and new thermometers respectively. In working out the equations, Prof. EVERETT's method of elimination has been followed, and the results are shown in the following tables:—

	Old.				New.			
	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
α_1	+0·6265	+0·5060	−2·3535	−5·4648	+0·8458	+0·0135	−3·5798	−5·5810
β_1	+0·0112	−2·2480	−4·1797	−4·5537	−0·4003	−2·6992	−4·2038	−4·1902
α_2	−0·0063	−0·0938	−0·0022	+0·3077	−0·0288	−0·0682	+0·1947	+0·4128
β_2	−0·0193	+0·0757	+0·3996	+0·6347	−0·0298	+0·1140	+0·4140	+0·5890

From these values of α_1, β_1, α_2, and β_2, the corresponding values of a_1, r_1, a_2, and r_2 were deduced.

	Old.				New.			
	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
a_1	0·627	2·304	4·796	7·113	0·936	2·699	5·522	6·979
r_1	88°·58′·6	167°·18′·9	209°·23′·0	230°·11′·8	115°·18′·6	179°·42′·3	220°·25′·0	233°·6′·0
a_2	0·020	0·120	0·400	0·705	0·041	0·133	0·458	0·719
r_2	198°·4′·6	308°·54′·2	359°·42′·0	25°·51′·8	224°·1′·3	329°·6′·5	25°·11′·0	35°·1′·5

Substituting these values in the original general equation, we have the following eight equations to the annual curves for the two sets of thermometers.

For old t_1 $\quad T = 47°·260 + 0·627 \sin(2\pi f + 88°·58′·6) + 0·020 \sin(4\pi f + 198°·4′·6) + \&c.$

old t_2 $\quad T = 46\ ·961 + 2·304 \sin(2\pi f + 167\ ·18\ ·9) + 0·120 \sin(4\pi f + 308°·54′·3) + \&c.$

old t_3 $\quad T = 46\ ·620 + 4·796 \sin(2\pi f + 209\ ·23\ ·0) + 0·400 \sin(4\pi f + 359°·42′·0) + \&c.$

old t_4 $\quad T = 46\ ·365 + 7·113 \sin(2\pi f + 230\ ·11\ ·8) + 0·705 \sin(4\pi f + 25°·51′·8) + \&c.$

new t_1 $\quad T = 46\ ·281 + 0·936 \sin(2\pi f + 115\ ·18\ ·6) + 0·041 \sin(4\pi f + 224°·1′·3) + \&c.$

new t_2 $\quad T = 45\ ·918 + 2·699 \sin(2\pi f + 179\ ·42\ ·3) + 0·133 \sin(4\pi f + 329\ ·\ \ 6\ ·5) + \&c.$

new t_3 $\quad T = 45\ ·612 + 5·522 \sin(2\pi f + 220\ ·25\ ·0) + 0·458 \sin(4\pi f + 25\ ·11\ ·0) + \&c.$

new t_4 $\quad T = 45\ ·484 + 6·979 \sin(2\pi f + 233\ ·\ \ 6\ ·0) + 0·719 \sin(4\pi f + 35\ ·\ \ 1\ ·5) + \&c.$

The maximum and minimum values of the annual and semi-annual terms in these equations will be found by making the various sines successively equal to $+1$ and -1, or the angles concerned equal to $\pm 90°$. The values of f, the fraction of the year (reckoning from the mean date of the first Monday) corresponding to the maximum and minimum values of the several terms, were deduced from the equations so formed. The following is a synopsis of the eight thermometers arranged in order of depth, showing the amplitudes and dates of maxima and minima of the annual and semi-annual terms respectively.

ANNUAL TERM.

	Feet.	Semi-Amplitude.	Dates of	
			Maximum.	Minimum.
Old t_1 . .	25·6	0·627	January 5·1	July 6·7
New t_1 . .	21·0	0·936	December 9·5	June 9·9
Old t_2 . .	12·8	2·304	October 17·9	April 18·3
New t_2 . .	11·1	2·699	October 5·2	April 5·6
Old t_3 . .	6·4	4·796	September 5·2	March 6·6
New t_3 . .	5·0	5·522	August 24·9	February 23·3
Old t_4 . .	3·2	7·113	August 15·1	February 13·5
New t_4 . .	3·1	6·979	August 12·0	February 10·4

SEMI-ANNUAL TERM.

	Feet.	Semi-Amplitude.	Dates of			
			Maxima.		Minima.	
Old t_1 . .	25·6	0·020	May 11·9,	November 10·5	February 9·6,	August 11·3
New t_1 . .	21·0	0·041	April 28·6,	October 28·2	January 27·3,	July 28·9
Old t_2 . .	12·8	0·120	March 16·7,	September 15·3	December 15·6,	June 16·0
New t_2 . .	11·1	0·133	March 6·3,	September 4·9	December 5·2,	June 5·6
Old t_3 . .	6·4	0·400	February 18·9,	August 20·5	November 19·9,	May 21·2
New t_3 . .	5·0	0·458	February 5·8,	August 7·4	November 6·7,	May 8·1
Old t_4 . .	3·2	0·705	February 5·7,	August 7·3	November 6·6,	May 8·0
New t_4 . .	3·1	0·719	February 0·8,	August 2·5	November 1·7,	May 3·2

We are now in a position to deduce the conducting power of the rock, both from the Napierian logarithms of the amplitudes, and from the retardation of phase, expressed in circular measure. We have the two values of $\sqrt{\frac{\pi c}{\mathrm{T}k}} = \Delta \log_e a = \Delta r$, where T is the period, one year for the annual term, and half a year for the semi-annual.

The following table gives the values of r_1 and r_2 in circular measure, and the Napierian logarithms of a_1 and a_2.

	Depth.	r_1	r_2	Log$_e$ a_1	Log$_e$ a_2
	Feet.				
Old t_1	25·6	1·553	3·457	−0·467	−3·912
t_2	12·8	2·920	5·391	+0·835	−2·120
t_3	6·4	3·654	6·278	+1·568	−0·916
t_4	3·2	4·018	6·735	+1·962	−0·350
New t_1	21·0	2·013	3·910	−0·066	−3·194
t_2	11·1	3·136	5·744	+0·993	−2·017
t_3	5·0	3·847	6·723	+1·709	−0·781
t_4	3·1	4·068	6·894	+1·943	−0·329

If we now take every possible combination of two thermometers from the four in each set, and divide the differences of their retardations of phase, and of the logarithms of their amplitudes, by the difference of depth in feet, we get the following two tables showing the values of $\sqrt{\frac{\pi c}{k}}$ for the annual and the semi-annual terms.

VALUES OF $\sqrt{\frac{\pi c}{k}}$ FROM THE ANNUAL TERM.

	Diff. of Depth.	Δr_1	$\sqrt{\frac{\pi c}{k}}$	$\Delta \log_e a_1$	$\sqrt{\frac{\pi c}{k}}$
Old.	Feet.				
t_1 and t_2	12·8	1·367	0·107	1·302	0·102
t_1 and t_3	19·2	2·101	0·109	2·035	0·106
t_1 and t_4	22·4	2·465	0·110	2·429	0·108
t_2 and t_3	6·4	0·734	0·115	0·733	0·115
t_2 and t_4	9·6	1·098	0·114	1·127	0·117
t_3 and t_4	3·2	0·364	0·114	0·394	0·123
		Mean	0·1115	Mean	0·1118
New.					
t_1 and t_2	9·9	1·123	0·113	1·059	0·103
t_1 and t_3	16·0	1·834	0·115	1·775	0·111
t_1 and t_4	17·9	2·055	0·115	2·009	0·112
t_2 and t_3	6·1	0·711	0·117	0·716	0·117
t_2 and t_4	8·0	0·932	0·116	0·950	0·119
t_3 and t_4	1·9	0·221	0·116	0·234	0·123
		Mean	0·1153	Mean	0.1148

VALUES OF $\sqrt{\frac{\pi c}{k}}$ FROM THE SEMI-ANNUAL TERM.

	Diff. of Depths.	Δr_2	$\sqrt{\frac{2\pi c}{k}}$	$\sqrt{\frac{\pi c}{k}}$	$\Delta \log_e a_2$	$\sqrt{\frac{2\pi c}{k}}$	$\sqrt{\frac{\pi c}{k}}$
Old.	Feet.						
t_1 and t_2	12·8	1·934	0·151	0·107	1·792	0·140	0·099
t_1 and t_3	19·2	2·821	0·146	0·103	2·996	0·156	0·110
t_1 and t_4	22·4	3·278	0·146	0·103	3·562	0·159	0·112
t_2 and t_3	6·4	0·887	0·139	0·098	1·204	0·188	0·133
t_2 and t_4	9·6	1·344	0·140	0·099	1·770	0·184	0·130
t_3 and t_4	3·2	0·457	0·143	0·101	0·566	0·177	0·125
			Mean	0·1018		Mean	0·1182
New.							
t_1 and t_2	9·9	1·834	0·185	0·131	1·177	0·119	0·084
t_1 and t_3	16·0	2·813	0·176	0·124	2·413	0·151	0·107
t_1 and t_4	17·9	2·984	0·167	0·118	2·865	0·160	0·113
t_2 and t_3	6·1	0·979	0·160	0·113	1·236	0·203	0·144
t_2 and t_4	8·0	1·150	0·144	0·102	1·688	0·211	0·149
t_3 and t_4	1·9	0·171	0·090	0·064	0·452	0·238	0·168
			Mean	0·1087		Mean	0·1275

The values of $\sqrt{\frac{\pi c}{k}}$ deduced from the epoch and amplitude of the annual term agree amongst themselves much better than the values derived from the semi-annual term. In the case of the latter, however, the coefficients are so small that any better agreement could hardly be expected. It will be seen from these tables that all the four mean values derived from the new thermometers are somewhat greater than those from the old, and that the two values from the old and new derived from the amplitudes of the semi-annual term are both greater, and by nearly the same amount, than those derived from the retardation of phase. The results from the annual term specially show also a distinct tendency to decrease as the deeper thermometers are approached, a tendency which was pointed out by Lord KELVIN in his paper referred to. This will be more clearly seen from the following table, showing the results from both sets of thermometers and from both the terms of the equations, arranged in the order of the means of the depths of the two thermometers concerned in each case.

VALUES OF $\sqrt{\frac{\pi c}{k}}$ IN ORDER OF MEAN DEPTH.

	Mean Depth.	Annual Term.		Semi-Annual Term.		Means.
		From Phase.	From Amplitude.	From Phase.	From Amplitude.	
Old t_1 and t_2	19·2	0·107	0·102	0·107	0·099	0·104
Old t_1 and t_3	16·0	·109	·106	·103	·110	·107
New t_1 and t_2	16·0	·113	·107	·131	·084	·109
Old t_1 and t_4	14·4	·110	·108	·103	·112	·108
New t_1 and t_3	13·0	·115	·111	·124	·107	·114
New t_1 and t_4	12·0	·115	·112	·118	·113	·115
Old t_2 and t_3	9·6	·115	·115	·098	·133	·115
Old t_2 and t_4	8·0	·114	·117	·099	·130	·115
New t_2 and t_3	8·0	·117	·117	·113	·144	·123
New t_2 and t_4	7·1	·116	·119	·102	·149	·121
Old t_3 and t_4	4·8	·114	·123	·101	·125	·116
New t_3 and t_4	4·0	·116	·123	·064	·168	·118

The tendency of the value of $\sqrt{\frac{\pi c}{k}}$ to decrease with increasing depth is very apparent from about 7 feet deep, to the lowest depths in question. The shorter thermometers of each set, however, show strange irregularities, which seem difficult to explain, except on the supposition that owing to surface conditions different from those assumed as the foundation of the harmonic law, they do not follow that law with so great a degree of exactness as the deeper ones. In the case of the results from new t_3 and t_4, some part of the irregularities may also be due to the sinking of these thermometers, to which I have already referred, since their establishment in the rock. The general tendency of the deeper thermometers to give smaller values is, however, apparent, and seems to suggest that the conducting power of the rock increases with depth.

From these values of $\sqrt{\frac{\pi c}{k}}$, the value of $\frac{k}{c}$, the conductivity of the rock expressed "in terms of the thermal capacity of a cubic foot of its own substance" can now be deduced, and finally the value of k, the conductivity expressed in terms of the thermal capacity of a cubic foot of water. In the following table these are compared with the values deduced by Prof. FORBES, Lord KELVIN, and Prof. EVERETT. As the earlier computations have been all referred to the French foot as the unit of measure, it has been necessary to reduce them to the English foot.* The value of c, the specific heat of the rock per unit of volume used, is 0·5283, the number determined by M. REGNAULT at Prof. FORBES' request.

	$\sqrt{\frac{\pi c}{k}}$		$\frac{k}{c}$		k	
	Per French Foot.	Per English Foot.	Per French Foot.	Per English Foot.	Per French Foot.	Per English Foot.
Prof. Forbes, . .	0·1152		(236·7)		(125·0)	142·0
Lord Kelvin, . .	0·1156		235·1	267·0	(124·2)	141·1
Prof. Everett, . .	0·1174		(227·9)		(120·4)	(136·8)
Old Thermometers, .		0·1116		252·2		133·2
New Thermometers, .		0·1150		237·5		125·7

The numbers enclosed in brackets are not given in the papers of the authors opposite whose names they are placed.

From the dates of the maxima and minima of the various thermometers we can determine the time necessary for heat to pass through 1 foot of the rock of Calton Hill.

	Mean Depth. Feet.	Days.	
From old t_1 and t_2	19·2	6·2	6·6 days.
new t_1 and t_2	16·0	6·6	
old t_2 and t_3	9·6	6·7	
new t_2 and t_3	8·0	6·8	
old t_3 and t_4	4·8	6·6	
new t_3 and t_4	4·0	6·8	

To determine the mean annual range at different depths, I have plotted the curve, Plate IV., showing the ranges taken from the equations to the annual curves. According to theory the range decreases geometrically as the depth increases in arithmetical progression, or the curve is a logarithmic curve. Hence we have log. $R = A + Bp$, where R is the range at a depth p in feet, A is evidently the logarithm of the range at the surface, where $p = 0$ and B is a constant fixing the rate of decrease of the range below the surface. This surface range I have taken $= 20°$, or the mean value shown by the curves. Hence log. $20 = A = 1·30103$. From the point on the curve where the range

* French foot = English foot × 1·06575 ; $(1·06575)^2 = 1·13582$.

is 10°, we have $1 = A + Bp_{10}$, and $p_{10} = 6{\cdot}2$ feet. From these equations we find $B = -0{\cdot}0485$.

Again p_1, the depth where the range is 1° F. $= -\frac{A}{B} = 26{\cdot}8$ feet.

$$\text{Similarly } p_{\cdot 1} = -\frac{1+A}{B} = 47{\cdot}4 \text{ feet.}$$

$$p_{\cdot 01} = -\frac{2+A}{B} = 68{\cdot}1 \text{ feet.}$$

Rainfall and Mean Shade Temperature from Scottish Meteorological Society's Returns, published in the Registrar General's Quarterly Reports.

	1888.		1889.		1890.		1891.		1892.		1893.	
	Rain.	Mean Temp.	Rain.	Mean Temp.	Rain.	Mean Temp.	Rain.	Mean Temp.	Rain.	Mean Temp.	Rain.	Mean Temp.
January, .	2·97	38·5	2·55	39·5	5·80	41·0	2·67	36·1	3·42	35·8	2·22	36·3
February, .	1·61	35·3	2·92	36·8	1·08	37·8	1·26	41·6	2·44	36·8	4·01	38·3
March, . .	3·39	35·7	2·32	39·6	3·38	41·6	3·35	37·0	1·15	36·3	1·54	42·7
April, . .	2·14	42·1	3·01	42·8	1·44	43·6	1·20	41·3	1·22	43·2	1·40	47·2
May, . .	2·84	48·3	2·24	53·1	2·19	51·0	2·28	46·8	3·65	49·2	1·88	52·4
June, . .	2·54	52·0	0·99	57·1	3·96	53·9	1·14	55·6	3·11	52·8	2·12	57·0
July, . .	4·69	53·7	2·82	55·9	3·79	54·7	2·87	57·3	2·56	54·8	3·35	57·6
August, .	2·66	54·5	5·40	55·9	3·90	55·2	5·51	55·6	5·34	55·6	3·56	59·4
September, .	1·34	51·7	1·70	52·1	3·84	56·4	5·16	54·3	4·01	50·2	3·45	51·8
October, .	2·91	46·8	3·99	45·6	4·45	48·4	4·31	46·7	4·69	42·8	4·51	47·0
November, .	6·23	42·9	2·11	43·1	6·77	41·0	3·72	40·5	3·41	42·1	3·66	39·5
December, .	3·33	40·5	3·39	39·0	1·67	35·2	5·80	38·5	1·98	34·0	4·67	40·5
	36·65	45·2	33·44	46·7	42·27	46·6	39·27	45·9	36·98	44·5	36·37	47·5

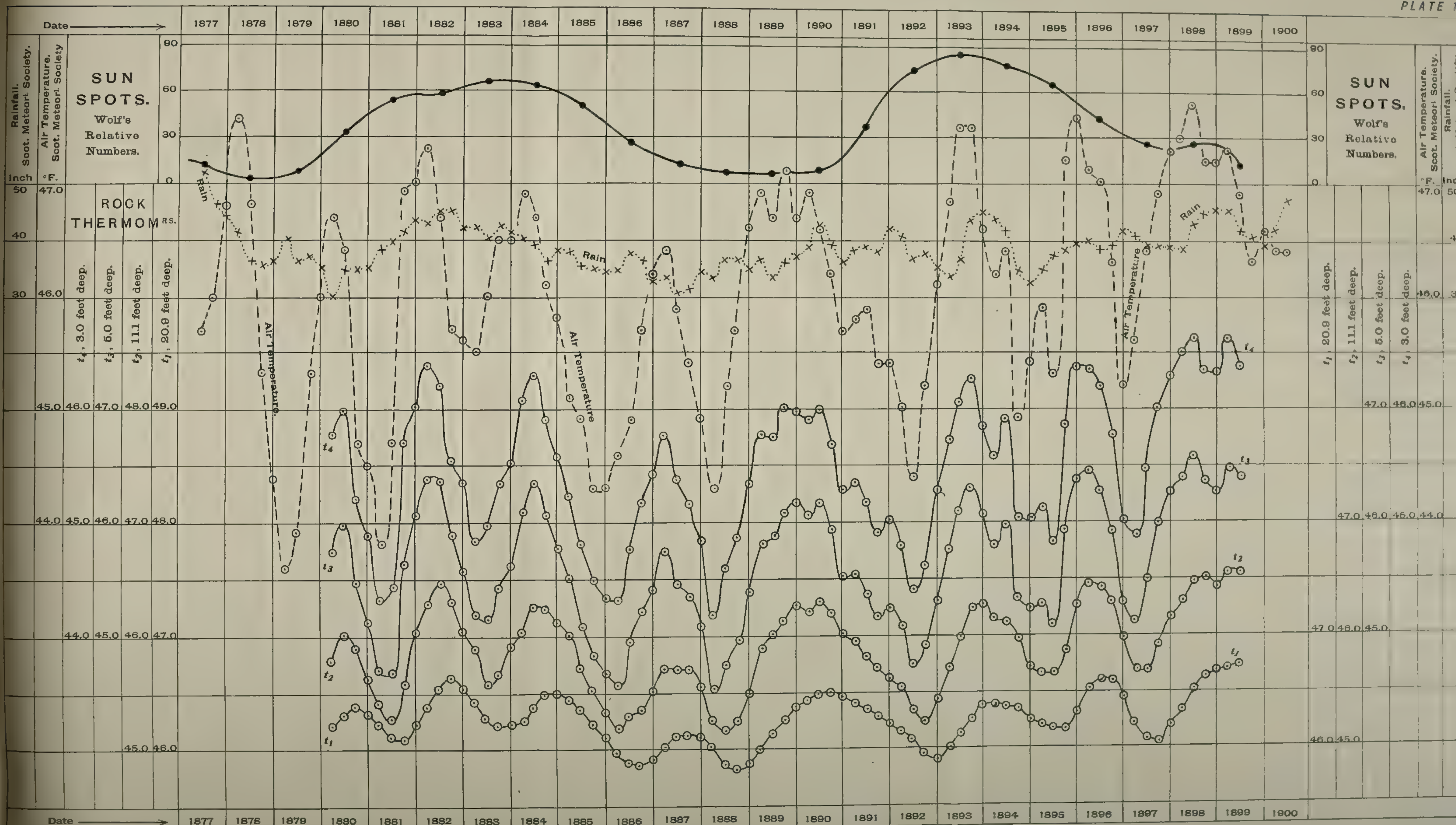
	1894.		1895.		1896.		1897.		1898.		1899.	
	Rain.	Mean Temp.	Rain.	Mean Temp.	Rain.	Mean Temp.	Rain.	Mean Temp.	Rain.	Mean Temp.	Rain.	Mean Temp.
January, .	4·85	36·6	2·76	30·8	2·53	39·7	1·75	34·2	3·14	42·9	5·02	36·6
February, .	7·05	38·6	1·17	29·0	2·00	41·4	2·84	39·4	3·90	38·2	2·69	38·5
March, . .	2·73	42·4	3·55	39·6	4·12	41·2	4·89	40·4	2·27	39·6	3·44	40·2
April, . .	1·43	47·1	2·16	45·1	2·03	47·0	2·17	41·9	3·61	45·6	3·87	43·0
May, . .	3·46	45·2	0·90	51·8	0·98	53·0	2·29	47·3	2·62	47·7	3·29	46·3
June, . .	2·50	53·5	2·14	55·0	3·39	56·2	4·10	54·3	2·32	54·4	1·80	57·4
July, . .	3·64	58·2	4·37	55·5	3·89	56·6	2·44	58·2	1·52	56·5	3·46	59·1
August, .	4·78	55·4	5·22	57·8	2·75	55·0	4·59	58·9	4·36	57·3	1·34	60·1
September, .	0·53	51·2	1·75	57·2	4·87	52·4	3·46	51·0	3·73	56·0	4·59	51·9
October, .	3·34	45·2	4·61	42·8	4·62	42·2	2·47	48·0	4·42	50·1	3·10	47·8
November, .	3·63	44·3	4·44	41·8	1·90	41·5	3·24	44·6	5·05	41·7	4·85	46·4
December, .	3·84	39·6	4·38	37·4	5·61	37·6	5·37	38·6	6·07	42·6	4·16	35·6
	41·78	46·4	37·45	45·3	38·69	47·0	39·61	46·4	43·01	47·7	41·61	46·9

ROYAL OBSERVATORY, EDINBURGH. ROCK THERMOMETERS.

Compared with the Sun-Spot Curve, Scottish Air Temperature and Rainfall.

Quadruple Annual Means of Temperatures from 1880 to 1899, Cleared from the Effects of Annual Range.

PLATE 1.



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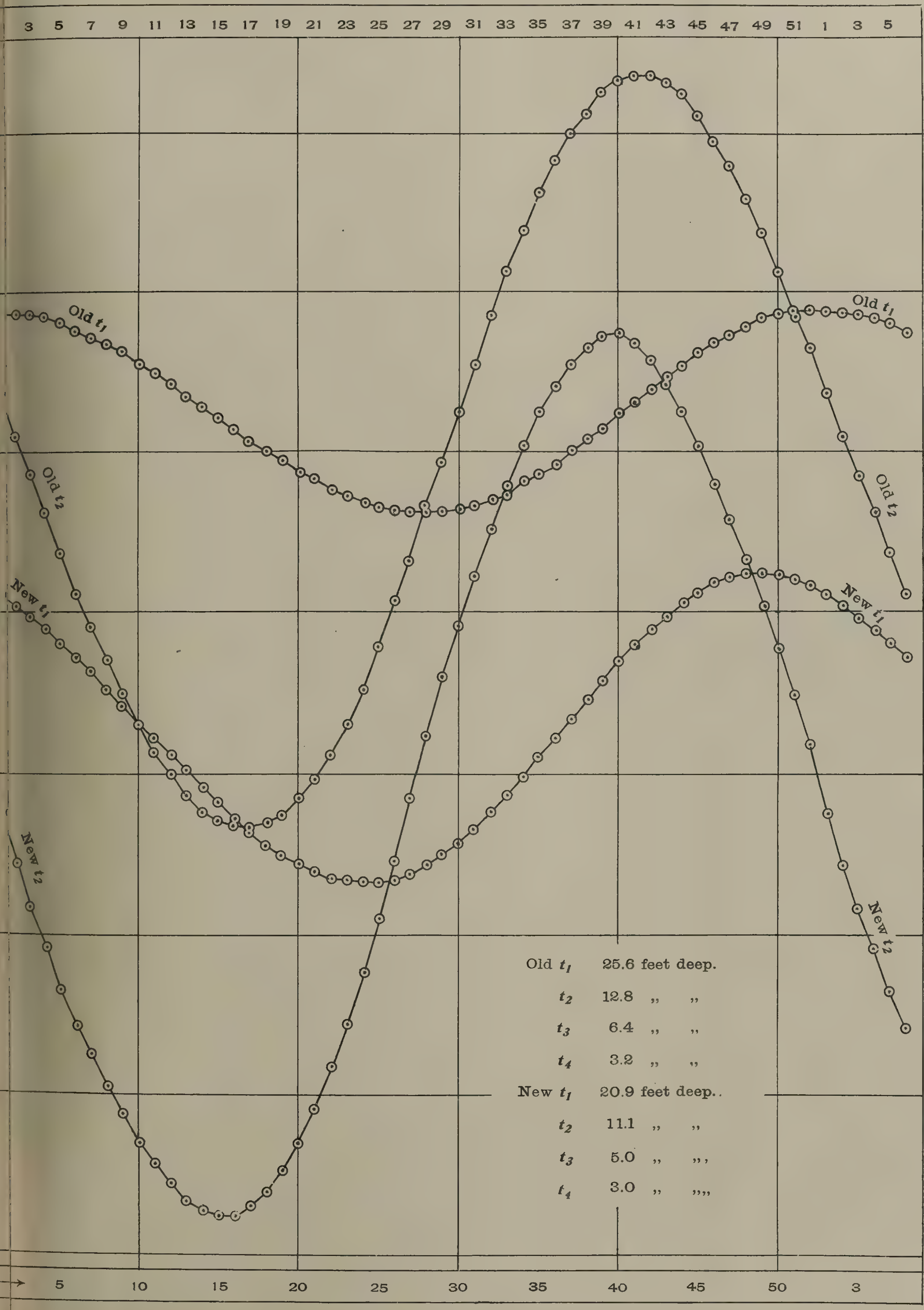
EDINBURGH ROCK THERMOMETERS. ANNUAL CURVES.

Old t_1 and t_2, and New t_1 and t_2.

PLATE 2.

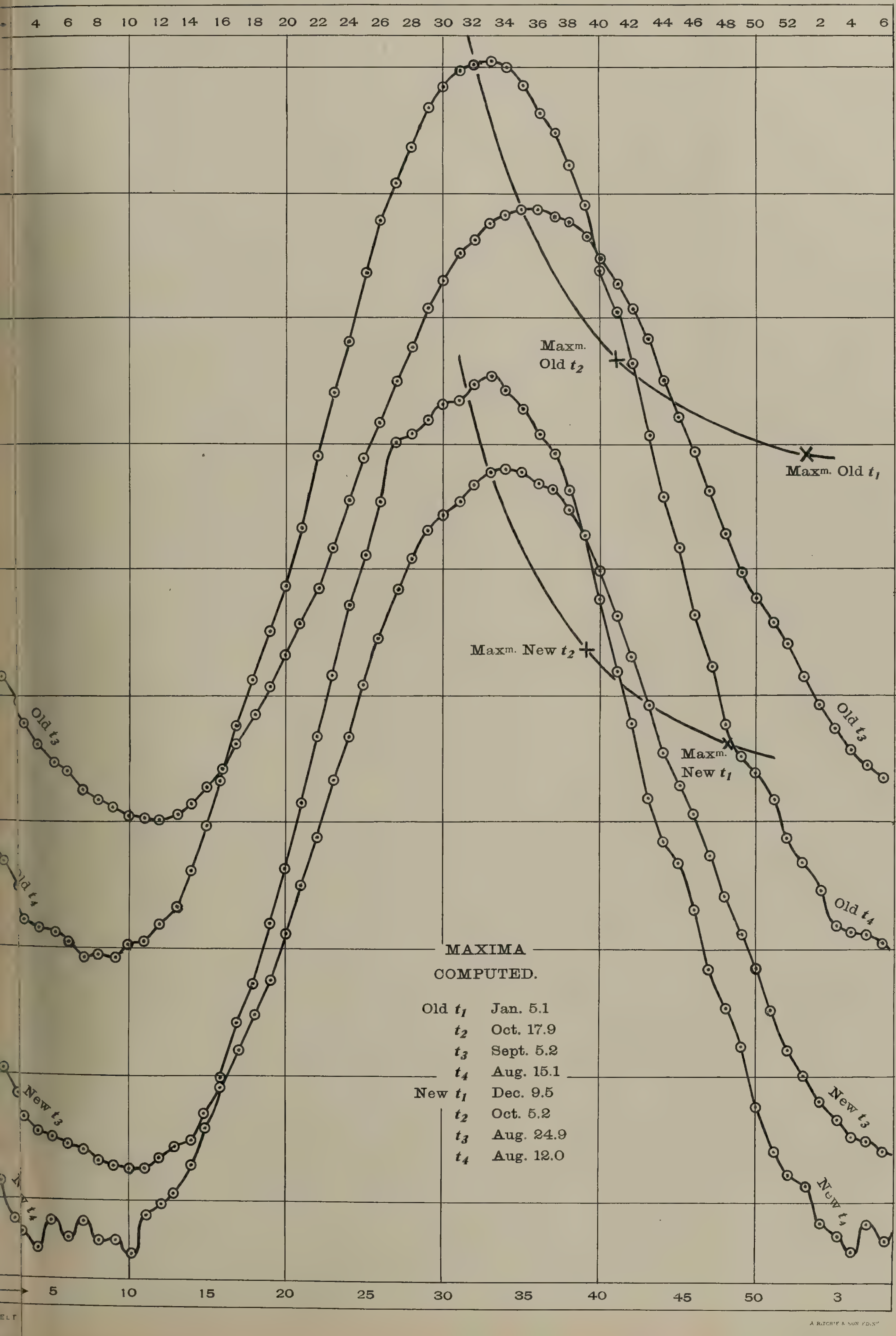


A. Ritchie & Son Edin.

EDINBURGH ROCK THERMOMETERS. ANNUAL CURVES.

Old t_3 and t_4, and New t_3 and t_4.

PLATE 3.



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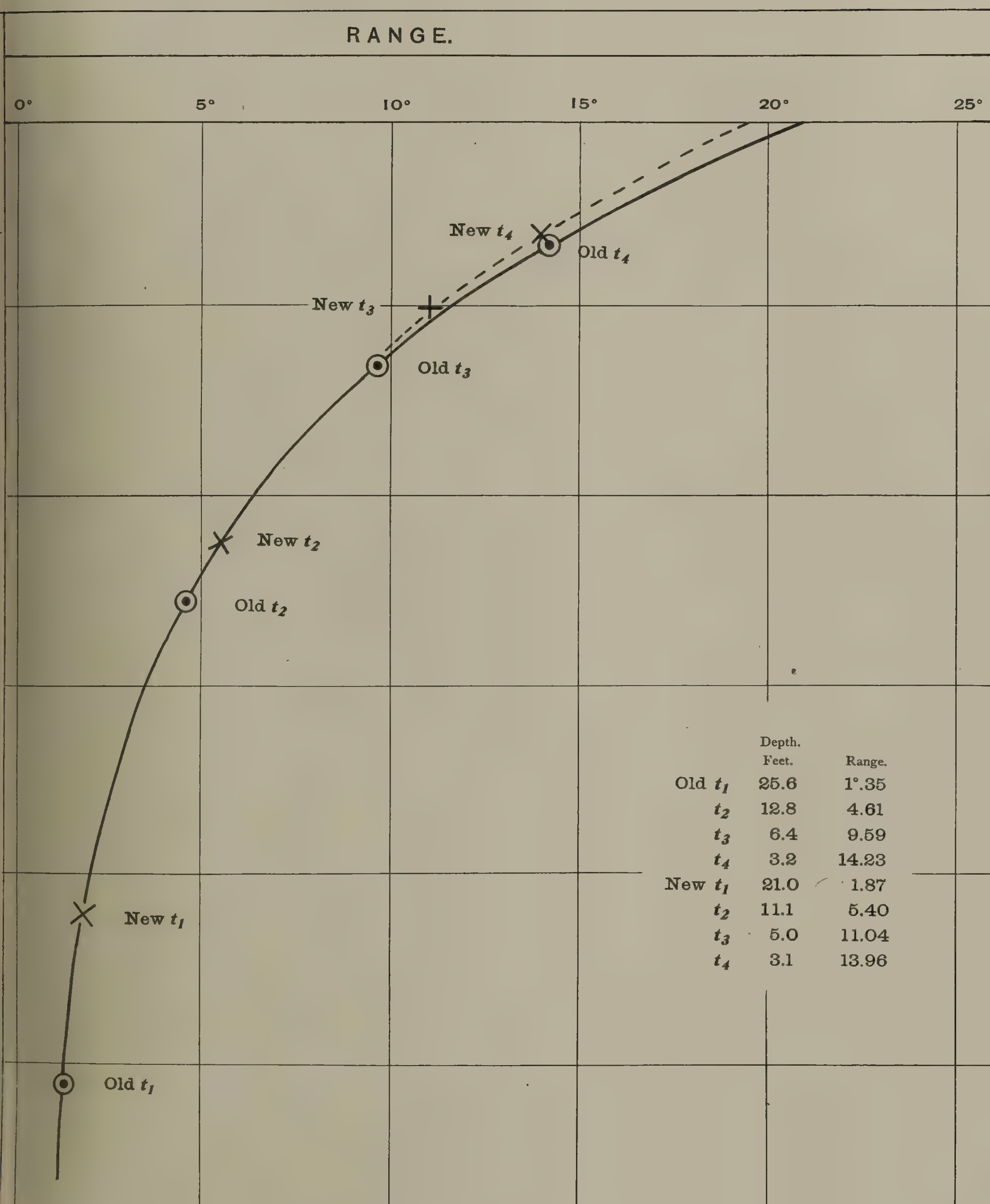
EDINBURGH ROCK THERMOMETERS.

RANGE AT DIFFERENT DEPTHS.

PLATE 4.



	Depth. Feet.	Range.
Old t_1	25.6	1°.35
t_2	12.8	4.61
t_3	6.4	9.59
t_4	3.2	14.23
New t_1	21.0	1.87
t_2	11.1	5.40
t_3	5.0	11.04
t_4	3.1	13.96

A RITCHIE & SON EDIN

IX.—*Some Identities connected with Alternants, and with Elliptic Functions.*

By Thomas Muir, LL.D.

(Read December 3, 1900.)

(1) Cayley in his paper* entitled "Note sur l'addition des fonctions elliptiques" obtains among other similar things an expression for

$$S(u+v+w\ldots)$$

in terms of

$$Su,\ Sv,\ Sw,\ldots$$

where

$$Su = \sqrt{k}\cdot\mathrm{sinam}\,\frac{u}{\sqrt{k}}.$$

The form of the expression is the quotient of two determinants, and as the expression becomes useless for such cases as $u=v$, $u=w$, ... on account of the simultaneous vanishing of numerator and denominator, he is led to seek a means of throwing out the common evanescent factors. In doing so there is brought to light the need for the existence of certain identities, viz.,
in connection with the numerator and denominator for the case of three arcs the respective identities

$$\begin{vmatrix}1 & a & A\\ 1 & b & B\\ 1 & c & C\end{vmatrix}(B+C)(C+A)(A+B) = \begin{vmatrix}1 & a & A^2\\ 1 & b & B^2\\ 1 & c & C^2\end{vmatrix}(A^2+B^2+C^2+BC+CA+AB) - \begin{vmatrix}1 & a & A^4\\ 1 & b & B^4\\ 1 & c & C^4\end{vmatrix},$$

$$\begin{vmatrix}1 & a & aA\\ 1 & b & bB\\ 1 & c & cC\end{vmatrix}(B+C)(C+A)(A+B) = \begin{vmatrix}1 & a & aA^2\\ 1 & b & bB^2\\ 1 & c & cC^2\end{vmatrix}(A^2+B^2+C^2+BC+CA+AB) - \begin{vmatrix}1 & a & aA^4\\ 1 & b & bB^4\\ 1 & c & cC^4\end{vmatrix}$$

in connection with the numerator for the case of four arcs

$$\begin{vmatrix}1 & a & a^2 & aA\\ 1 & b & b^2 & bB\\ 1 & c & c^2 & cC\\ 1 & d & d^2 & dD\end{vmatrix}(A+B)(A+C)(A+D)(B+C)(B+D)(C+D)$$

$$= \begin{vmatrix}1 & a & a^2 & aA^2\\ 1 & b & b^2 & bB^2\\ 1 & c & c^2 & cC^2\\ 1 & d & d^2 & dD^2\end{vmatrix}M - \begin{vmatrix}1 & a & a^2 & aA^4\\ 1 & b & b^2 & bB^4\\ 1 & c & c^2 & cC^4\\ 1 & d & d^2 & dD^4\end{vmatrix}N + \begin{vmatrix}1 & a & a^2 & aA^6\\ 1 & b & b^2 & bB^6\\ 1 & c & c^2 & cC^6\\ 1 & d & d^2 & dD^6\end{vmatrix}P,$$

* *Crelle's Journ.*, xli. pp. 57–65 ; or *Collected Math. Papers*, i. pp. 540–549.

where

$$\begin{aligned} M^* &= 2AB^2C^2 + \ldots + A^3B^2 + \ldots + A^3BC + \ldots + 3A^2BCD + \ldots, \\ N &= (A+B+C+D)(A^2+B^2+C^2+D^2) + (ABC+BCD+CDA+DAB), \\ P &= A+B+C+D; \end{aligned}$$

and in connection with the denominator for the case of four arcs

$$\begin{vmatrix} 1 & a & A & aA \\ 1 & b & B & bB \\ 1 & c & C & cC \\ 1 & d & D & dD \end{vmatrix} (A+B)(A+C)(A+D)(B+C)(B+D)(C+D)$$

$$= \begin{vmatrix} 1 & a & A^2 & aA^2 \\ 1 & b & B^2 & bB^2 \\ 1 & c & C^2 & cC^2 \\ 1 & d & D^2 & dD^2 \end{vmatrix} (A^2B^2 + \ldots + ABC^2 + \ldots + 2ABCD) - \begin{vmatrix} 1 & a & A^4 & aA^4 \\ 1 & b & B^4 & bB^4 \\ 1 & c & C^4 & cC^4 \\ 1 & d & D^4 & dD^4 \end{vmatrix}.$$

For convenience of reference these four identities may be denoted by (N_3), (D_3), (N_4), (D_4) respectively. No proof is given of them by CAYLEY, and after stating them he adds "mais je n'ai pas encore trouvé la loi générale de ces équations."

The object of the present paper is to do something to supply these wants.

(2) In all the identities the determinants are seen to be multiplied by symmetric functions of as many letters as the determinants have rows or columns. A general theorem for the performance of such multiplications is thus seen to be desirable, and the following has been found. It is a generalisation of a theorem given in 1879 in the *Transactions Roy. Soc. Edinburgh*, xxix. p. 53.

The product of a determinant of the nth *order by a single symmetric function of* n *quantities* $\alpha, \beta, \gamma, \ldots$ *is equal to the sum of as many determinants as there are terms in the function, each determinant of the sum being got from the given determinant and a term* $\alpha^x\beta^y\gamma^z \ldots$ *of the function by multiplying each element of the first row (or column) of the given determinant by the* xth *power of the corresponding one of the* n *quantities, each element of the second row (or column) by the* yth *power of the corresponding one of the* n *quantities, and so on.*

For example,

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} (A^2B+A^2C+B^2C+B^2A+C^2A+C^2B)$$

$$= \begin{vmatrix} a_1A^2 & a_2A & a_3 \\ b_1B^2 & b_2B & b_3 \\ c_1C^2 & c_2C & c_3 \end{vmatrix} + \begin{vmatrix} a_1A^2 & a_2 & a_3A \\ b_1B^2 & b_2 & b_3B \\ c_1C^2 & c_2 & c_3C \end{vmatrix} + \begin{vmatrix} a_1 & a_2A^2 & a_3A \\ b_1 & b_2B^2 & b_3B \\ c_1 & c_2C^2 & c_3C \end{vmatrix}$$

$$+ \begin{vmatrix} a_1A & a_2A^2 & a_3 \\ b_1B & b_2B^2 & b_3 \\ c_1C & c_2C^2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1A & a_2 & a_3A^2 \\ b_1B & b_2 & b_3B^2 \\ c_1C & c_2 & c_3C^2 \end{vmatrix} + \begin{vmatrix} a_1 & a_2A & a_3A^2 \\ b_1 & b_2B & b_3B^2 \\ c_1 & c_2C & c_3C^2 \end{vmatrix}.$$

The essence of the proof lies in the fact that if we single out for consideration

* These cofactors are incorrectly printed both in the original journal and in the collection, and unfortunately the mistake consists in putting small letters in place of capitals.

any term of the determinant on the left,—the term $-a_3b_2c_1$ say,—the corresponding term of each of the determinants on the right is $-a_3b_2c_1$ multiplied by a term of the symmetric function, the multiplying term being different for every determinant.*

Applying this theorem to the left-hand member of CAYLEY's first identity, and for shortness' sake writing only the first line of each determinant, we have

$$\begin{aligned} |\mathrm{A}^2\, a\mathrm{A}\ \mathrm{A}| &+ |\mathrm{A}^2\, a\ \mathrm{A}^2| + |1\ a\mathrm{A}^2\ \mathrm{A}^2| + |\mathrm{A}\ a\mathrm{A}^2\ \mathrm{A}| \\ &+ |\mathrm{A}\ a\ \mathrm{A}^3| + |1\ a\mathrm{A}\ \mathrm{A}^3| + 2|\mathrm{A}\ a\mathrm{A}\ \mathrm{A}^2|, \end{aligned}$$

which, as the first and last terms combine into one, and the second and fourth terms vanish, reduces to

$$-|\mathrm{A}^2\, a\mathrm{A}\ \mathrm{A}| + |1\ a\mathrm{A}^2\ \mathrm{A}^2| + |\mathrm{A}\ a\ \mathrm{A}^3| + |1\ a\mathrm{A}\ \mathrm{A}^3|.$$

Similarly the right-hand side becomes

$$\begin{aligned} |\mathrm{A}^2\, a\ \mathrm{A}^2| &+ |1\ a\mathrm{A}^2\ \mathrm{A}^2| + |1\ a\ \mathrm{A}^4| + |1\ a\mathrm{A}\ \mathrm{A}^3| \\ &+ |\mathrm{A}\ a\ \mathrm{A}^3| + |\mathrm{A}\ a\mathrm{A}\ \mathrm{A}^2| - |1\ a\ \mathrm{A}^4|, \end{aligned}$$

which in like manner reduces to

$$|1\ a\mathrm{A}^2\ \mathrm{A}^2| + |1\ a\mathrm{A}\ \mathrm{A}^3| + |\mathrm{A}\ a\ \mathrm{A}^3| + |\mathrm{A}\ a\mathrm{A}\ \mathrm{A}^2|.$$

And as these two results are the same, the identity is established.

(3) The second identity is readily proved in the same way, both sides being equal to

$$|\mathrm{A}^2\, a\ a\mathrm{A}^2| + |\mathrm{A}\ a\ a\mathrm{A}^3| + |1\ a\mathrm{A}\ a\mathrm{A}^3| + |\mathrm{A}\ a\mathrm{A}\ a\mathrm{A}^2|.$$

It is however essentially the same identity as the first, as may be seen on writing a^{-1}, b^{-1}, c^{-1} for a, b, c respectively in either of them.

(4) The same method of course suffices for the proving of the third and fourth

* Another theorem on the same subject may be illustrated by the same example, viz.:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} (\mathrm{A}^2\mathrm{B}+\mathrm{A}^2\mathrm{C}+\mathrm{B}^2\mathrm{A}+\mathrm{B}^2\mathrm{C}+\mathrm{C}^2\mathrm{A}+\mathrm{C}^2\mathrm{B})$$

$$= \begin{vmatrix} a_1 & a_2 & a_3\mathrm{A}^2\mathrm{B} \\ b_1 & b_2 & b_3\mathrm{B}^2\mathrm{C} \\ c_1 & c_2 & c_3\mathrm{C}^2\mathrm{A} \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3\mathrm{A}^2\mathrm{C} \\ b_1 & b_2 & b_3\mathrm{B}^2\mathrm{A} \\ c_1 & c_2 & c_3\mathrm{C}^2\mathrm{B} \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3\mathrm{B}^2\mathrm{A} \\ b_1 & b_2 & b_3\mathrm{C}^2\mathrm{B} \\ c_1 & c_2 & c_3\mathrm{A}^2\mathrm{C} \end{vmatrix}$$

$$+ \begin{vmatrix} a_1 & a_2 & a_3\mathrm{B}^2\mathrm{C} \\ b_1 & b_2 & b_3\mathrm{C}^2\mathrm{A} \\ c_1 & c_2 & c_3\mathrm{A}^2\mathrm{B} \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3\mathrm{C}^2\mathrm{A} \\ b_1 & b_2 & b_3\mathrm{A}^2\mathrm{B} \\ c_1 & c_2 & c_3\mathrm{B}^2\mathrm{C} \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3\mathrm{C}^2\mathrm{B} \\ b_1 & b_2 & b_3\mathrm{A}^2\mathrm{C} \\ c_1 & c_2 & c_3\mathrm{B}^2\mathrm{A} \end{vmatrix}.$$

Here only one row or column of the original determinant is multiplied, the multipliers being complete terms of the symmetric function. Each multiplying term, it will be observed, is used three times, and occurs in a different position every time; for example, the cofactor of $\mathrm{A}^2\mathrm{B}$ on the right is

$$\begin{vmatrix} . & . & a_3 \\ b_1 & b_2 & . \\ c_1 & c_2 & . \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & . \\ b_1 & b_2 & . \\ . & . & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & . \\ . & . & b_3 \\ c_1 & c_2 & . \end{vmatrix},$$

which is equal to $|a_1b_2c_3|$, as it should be.

identities, but the labour of writing out the products is very considerable. For example, the multiplier

$$(A+B)(A+C)(A+D)(B+C)(B+D)(C+D)$$

or

$$\Sigma A^3B^2C \ + \ 2\Sigma A^3BCD \ + \ 2\Sigma A^2B^2C^2 \ + \ 4\Sigma A^2B^2CD$$

on the left-hand side of (N_4) gives rise to $24+4+4+6$ determinant terms, which however reduce to 22, viz. :

$$\begin{array}{llll}
\; |A^3\, aA\;\; a^2\;\; aA^3| & +\; |A^3\;\; a\;\; a^2A^2\, aA^2| & +\; |A^3\;\; a\;\; a^2A\;\; aA^3| & +\; |A^3\, aA\;\; a^2A\, aA^2| \\
+\; |A^2\, aA\;\; a^2\;\; aA^4| & +\; |A^2\;\; a\;\; a^2A\;\; aA^4| & +\; |A^2\;\; a\;\; a^2A^3\, aA^2| & +\; |A^2\, aA^2\;\; a^2\;\; aA^3| \\
+2\, |A^2\;\; a\;\; a^2A^2\, aA^3| & +\, 2\, |A^2\, aA\, a^2A^2\, aA^2| & +\, 3\, |A^2\, aA\;\; a^2A\;\; aA^3| & \\
+\; |A\;\; aA^2\;\; a^2\;\; aA^4| & +\; |A\;\; a\;\; a^2A^3\, aA^3| & +\; |A\;\; a\;\; a^2A^2\, aA^4| & \\
+\; |A\;\; aA\;\; a^2A^3\, aA^2| & +\, 2\, |A\;\; aA\, a^2A\;\; aA^4| & +\, 2\, |A\;\; aA^2\, a^2A\;\; aA^3| & +\, 3\, |A\, aA\;\; a^2A^2\, aA^3| \\
+\; |1\;\; aA^2\, a^2A\;\; aA^4| & +\; |1\;\; aA\, a^2A^3\, aA^3| & +\; |1\;\; aA\;\; a^2A^2\, aA^4| & +\; |1\;\; aA^2\, a^2A^2\, aA^3|\, ;
\end{array}$$

and the result of multiplication on the right-hand side is 64 determinant terms which reduce to the same 22.

In the case of (D_4) still further reduction on both sides is possible, viz., to 13 terms; but it is quite clear that little is to be hoped for from this method when determinants of higher order than the fourth come to be dealt with.

(5) A most important simplification of the form of the identities is suggested on noticing that the multipliers in (N_3),

$$(B+C)(C+A)(A+B) \ \text{ and } \ A^2 + B^2 + C^2 + BC + CA + AB\, ,$$

are each expressible as the quotient of an alternant by the difference-product of A, B, C, viz. :

$$(B+C)(C+A)(A+B) = \begin{vmatrix} 1 & A^2 & A^4 \\ 1 & B^2 & B^4 \\ 1 & C^2 & C^4 \end{vmatrix} \div \begin{vmatrix} 1 & A & A^2 \\ 1 & B & B^2 \\ 1 & C & C^2 \end{vmatrix},$$

and

$$A^2 + B^2 + C^2 + BC + CA + AB = \begin{vmatrix} 1 & A & A^4 \\ 1 & B & B^4 \\ 1 & C & C^4 \end{vmatrix} \div \begin{vmatrix} 1 & A & A^2 \\ 1 & B & B^2 \\ 1 & C & C^2 \end{vmatrix}.$$

When these new expressions are substituted, multiplication of both sides by the common denominator $|A^0B^1C^2|$ gives (N_3) in the form

$$\begin{vmatrix} 1 & a & A \\ 1 & b & B \\ 1 & c & C \end{vmatrix} . \begin{vmatrix} 1 & A^2 & A^4 \\ 1 & B^2 & B^4 \\ 1 & C^2 & C^4 \end{vmatrix} - \begin{vmatrix} 1 & a & A^2 \\ 1 & b & B^2 \\ 1 & c & C^2 \end{vmatrix} . \begin{vmatrix} 1 & A & A^4 \\ 1 & B & B^4 \\ 1 & C & C^4 \end{vmatrix} + \begin{vmatrix} 1 & a & A^4 \\ 1 & b & B^4 \\ 1 & c & C^4 \end{vmatrix} . \begin{vmatrix} 1 & A & A^2 \\ 1 & B & B^2 \\ 1 & C & C^2 \end{vmatrix} = 0,$$

and (D_3) in the form

$$\begin{vmatrix} 1 & a & aA \\ 1 & b & bB \\ 1 & c & cC \end{vmatrix} . \begin{vmatrix} 1 & A^2 & A^4 \\ 1 & B^2 & B^4 \\ 1 & C^2 & C^4 \end{vmatrix} - \begin{vmatrix} 1 & a & aA^2 \\ 1 & b & bB^2 \\ 1 & c & cC^2 \end{vmatrix} . \begin{vmatrix} 1 & A & A^4 \\ 1 & B & B^4 \\ 1 & C & C^4 \end{vmatrix} + \begin{vmatrix} 1 & a & aA^4 \\ 1 & b & bB^4 \\ 1 & c & cC^4 \end{vmatrix} . \begin{vmatrix} 1 & A & A^2 \\ 1 & B & B^2 \\ 1 & C & C^2 \end{vmatrix} = 0.$$

This transformation makes a totally different mode of proof possible : for, seeking for a vanishing determinant of the sixth order which can be expanded into the left-hand member of (N_3) by LAPLACE'S theorem, we easily obtain

$$\begin{vmatrix} 1 & a & A & A^2 & A^4 & 1 \\ 1 & b & B & B^2 & B^4 & 1 \\ 1 & c & C & C^2 & C^4 & 1 \\ . & . & A & A^2 & A^4 & 1 \\ . & . & B & B^2 & B^4 & 1 \\ . & . & C & C^2 & C^4 & 1 \end{vmatrix}.$$

Similarly the manifest identity

$$\begin{vmatrix} 1 & a & aA & aA^2 & aA^4 & a \\ 1 & b & bB & bB^2 & bB^4 & b \\ 1 & c & cC & cC^2 & cC^4 & c \\ . & . & A & A^2 & A^4 & 1 \\ . & . & B & B^2 & B^4 & 1 \\ . & . & C & C^2 & C^4 & 1 \end{vmatrix} = 0$$

gives at once (D_3).

(6) Here, as is usual, a really appropriate proof makes generalisation easy. It is readily seen that the vanishing of the latter determinant of the sixth order is not dependent on the values of the elements in the first half of the first and second columns. Substituting therefore h, k, l for 1, 1, 1 and m, n, r for a, b, c in these columns we still have

$$\begin{vmatrix} h & m & aA & aA^2 & aA^4 & a \\ k & n & bB & bB^2 & bB^4 & b \\ l & r & cC & cC^2 & cC^4 & c \\ . & . & A & A^2 & A^4 & 1 \\ . & . & B & B^2 & B^4 & 1 \\ . & . & C & C^2 & C^4 & 1 \end{vmatrix} = \begin{vmatrix} h & m & . & . & . & . \\ k & n & . & . & . & . \\ l & r & . & . & . & . \\ . & . & A & A^2 & A^4 & 1 \\ . & . & B & B^2 & B^4 & 1 \\ . & . & C & C^2 & C^4 & 1 \end{vmatrix},$$

$$= 0,$$

and therefore

$$\begin{vmatrix} h & m & aA \\ k & n & bB \\ l & r & cC \end{vmatrix} . \begin{vmatrix} 1 & A^2 & A^4 \\ 1 & B^2 & B^4 \\ 1 & C^2 & C^4 \end{vmatrix} - \begin{vmatrix} h & m & aA^2 \\ k & n & bB^2 \\ l & r & cC^2 \end{vmatrix} . \begin{vmatrix} 1 & A & A^4 \\ 1 & B & B^4 \\ 1 & C & C^4 \end{vmatrix}$$

$$+ \begin{vmatrix} h & m & aA^4 \\ k & n & bB^4 \\ l & r & cC^4 \end{vmatrix} . \begin{vmatrix} 1 & A & A^2 \\ 1 & B & B^2 \\ 1 & C & C^2 \end{vmatrix} - \begin{vmatrix} h & m & a \\ k & n & b \\ l & r & c \end{vmatrix} . \begin{vmatrix} A & A^2 & A^4 \\ B & B^2 & B^4 \\ C & C^2 & C^4 \end{vmatrix} = 0.$$

On putting $h, k, l = 1, 1, 1$ and $m, n, r = a, b, c$, we have (D_3), and on putting $h, k, l = 1, 1, 1$ and $a, b, c = 1, 1, 1$ we have (N_3).

A still further generalisation lies in the change of the exponents of A, B, C into suffixes, the vanishing of the determinant of the sixth order being independent of the meaning assigned to A^n.

(7) Turning now to (N_4) and (D_4) we find the same alteration of their form possible. From the theory of alternants it is known that

$$
\begin{aligned}
|A^0B^2C^4D^6| \div |A^0B^1C^2D^3| &= (D+C)(D+B)(D+A)(C+B)(C+A)(B+A),\\
|A^0B^1C^2D^6| \div |A^0B^1C^2D^3| &= \Sigma A^3 + \Sigma A^2B + \Sigma ABC,\\
|A^0B^1C^4D^6| \div |A^0B^1C^2D^3| &= \Sigma A^3B^2 + \Sigma A^3BC + 2\Sigma A^2B^2C + 3\Sigma A^2BCD,\\
|A^0B^1C^2D^4| \div |A^0B^1C^2D^3| &= A + B + C + D,\\
|A^0B^1C^4D^5| \div |A^0B^1C^2D^3| &= \Sigma A^2B^2 + \Sigma A^2BC + 2ABCD.
\end{aligned}
$$

Substituting for these expressions on the right and then multiplying by $|A^0B^1C^2D^3|$ we find (N_4) in the form

$$
\begin{aligned}
0 = &\begin{vmatrix} 1 & a & a^2 & aA \\ 1 & b & b^2 & bB \\ 1 & c & c^2 & cC \\ 1 & d & d^2 & dD \end{vmatrix} \cdot \begin{vmatrix} 1 & A^2 & A^4 & A^6 \\ 1 & B^2 & B^4 & B^6 \\ 1 & C^2 & C^4 & C^6 \\ 1 & D^2 & D^4 & D^6 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 & aA^2 \\ 1 & b & b^2 & bB^2 \\ 1 & c & c^2 & cC^2 \\ 1 & d & d^2 & dD^2 \end{vmatrix} \cdot \begin{vmatrix} 1 & A & A^4 & A^6 \\ 1 & B & B^4 & B^6 \\ 1 & C & C^4 & C^6 \\ 1 & D & D^4 & D^6 \end{vmatrix} \\
+ &\begin{vmatrix} 1 & a & a^2 & aA^4 \\ 1 & b & b^2 & bB^4 \\ 1 & c & c^2 & cC^4 \\ 1 & d & d^2 & dD^4 \end{vmatrix} \cdot \begin{vmatrix} 1 & A & A^2 & A^6 \\ 1 & B & B^2 & B^6 \\ 1 & C & C^2 & C^6 \\ 1 & D & D^2 & D^6 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 & aA^6 \\ 1 & b & b^2 & bB^6 \\ 1 & c & c^2 & cC^6 \\ 1 & d & d^2 & dD^6 \end{vmatrix} \cdot \begin{vmatrix} 1 & A & A^2 & A^4 \\ 1 & B & B^2 & B^4 \\ 1 & C & C^2 & C^4 \\ 1 & D & D^2 & D^4 \end{vmatrix},
\end{aligned}
$$

and (D_4) in the form

$$
\begin{aligned}
0 = &\begin{vmatrix} 1 & a & A & aA \\ 1 & b & B & bB \\ 1 & c & C & cC \\ 1 & d & D & dD \end{vmatrix} \cdot \begin{vmatrix} 1 & A^2 & A^4 & A^6 \\ 1 & B^2 & B^4 & B^6 \\ 1 & C^2 & C^4 & C^6 \\ 1 & D^2 & D^4 & D^6 \end{vmatrix} - \begin{vmatrix} 1 & a & A^2 & aA^2 \\ 1 & b & B^2 & bB^2 \\ 1 & c & C^2 & cC^2 \\ 1 & d & D^2 & dD^2 \end{vmatrix} \cdot \begin{vmatrix} 1 & A & A^4 & A^5 \\ 1 & B & B^4 & B^5 \\ 1 & C & C^4 & C^5 \\ 1 & D & D^4 & D^5 \end{vmatrix} \\
&+ \begin{vmatrix} 1 & a & A^4 & aA^4 \\ 1 & b & B^4 & bB^4 \\ 1 & c & C^4 & cC^4 \\ 1 & d & D^4 & dD^4 \end{vmatrix} \cdot \begin{vmatrix} 1 & A & A^2 & A^3 \\ 1 & B & B^2 & B^3 \\ 1 & C & C^2 & C^3 \\ 1 & D & D^2 & D^3 \end{vmatrix}.
\end{aligned}
$$

(8) In its new form (N_4) can be as easily proved and generalised as (N_3). Proceeding at once to the generalisation, we have clearly

$$
\begin{vmatrix}
m_1 & m_2 & m_3 & aA_1 & aA_2 & aA_4 & aA_6 & aA_0 \\
n_1 & n_2 & n_3 & bB_1 & bB_2 & bB_4 & bB_6 & bB_0 \\
r_1 & r_2 & r_3 & cC_1 & cC_2 & cC_4 & cC_6 & cC_0 \\
s_1 & s_2 & s_3 & dD_1 & dD_2 & dD_4 & dD_6 & dD_0 \\
\cdot & \cdot & \cdot & A_1 & A_2 & A_4 & A_6 & A_0 \\
\cdot & \cdot & \cdot & B_1 & B_2 & B_4 & B_6 & B_0 \\
\cdot & \cdot & \cdot & C_1 & C_2 & C_4 & C_6 & C_0 \\
\cdot & \cdot & \cdot & D_1 & D_2 & D_4 & D_6 & D_0
\end{vmatrix} = 0,
$$

and therefore

$$
0 = -\begin{vmatrix} m_1 & m_2 & m_3 & aA_1 \\ n_1 & n_2 & n_3 & bB_1 \\ r_1 & r_2 & r_3 & cC_1 \\ s_1 & s_2 & s_3 & dD_1 \end{vmatrix} \cdot \begin{vmatrix} A_0 & A_2 & A_4 & A_6 \\ B_0 & B_2 & B_4 & B_6 \\ C_0 & C_2 & C_4 & C_6 \\ D_0 & D_2 & D_4 & D_6 \end{vmatrix} + \begin{vmatrix} m_1 & m_2 & m_3 & aA_2 \\ n_1 & n_2 & n_3 & bB_2 \\ r_1 & r_2 & r_3 & cC_2 \\ s_1 & s_2 & s_3 & dD_2 \end{vmatrix} \cdot \begin{vmatrix} A_0 & A_1 & A_4 & A_6 \\ B_0 & B_1 & B_4 & B_6 \\ C_0 & C_1 & C_4 & C_6 \\ D_0 & D_1 & D_4 & D_6 \end{vmatrix}
$$

$$
-\begin{vmatrix} m_1 & m_2 & m_3 & aA_4 \\ n_1 & n_2 & n_3 & bB_4 \\ r_1 & r_2 & r_3 & cC_4 \\ s_1 & s_2 & s_3 & dD_4 \end{vmatrix} . \begin{vmatrix} A_0 & A_1 & A_2 & A_6 \\ B_0 & B_1 & B_2 & B_6 \\ C_0 & C_1 & C_2 & C_6 \\ D_0 & D_1 & D_2 & D_6 \end{vmatrix} + \begin{vmatrix} m_1 & m_2 & m_3 & aA_6 \\ n_1 & n_2 & n_3 & bB_6 \\ r_1 & r_2 & r_3 & cC_6 \\ s_1 & s_2 & s_3 & dD_6 \end{vmatrix} . \begin{vmatrix} A_0 & A_1 & A_2 & A_4 \\ B_0 & B_1 & B_2 & B_4 \\ C_0 & C_1 & C_2 & C_4 \\ D_0 & D_1 & D_2 & D_4 \end{vmatrix}
$$

$$
+ \begin{vmatrix} m_1 & m_2 & m_3 & aA_0 \\ n_1 & n_2 & n_3 & bB_0 \\ r_1 & r_2 & r_3 & cC_0 \\ s_1 & s_2 & s_3 & dD_0 \end{vmatrix} . \begin{vmatrix} A_1 & A_2 & A_4 & A_6 \\ B_1 & B_2 & B_4 & B_6 \\ C_1 & C_2 & C_4 & C_6 \\ D_1 & D_2 & D_4 & D_6 \end{vmatrix} .
$$

Changing the suffixes of the capital letters into exponents, and putting

$$
\begin{aligned} m_1 , n_1 , r_1 , s_1 &= 1 , 1 , 1 , 1 , \\ m_2 , n_2 , r_2 , s_2 &= a , b , c , d , \\ m_3 , n_3 , r_3 , s_3 &= a^2, b^2, c^2, d^2, \end{aligned}
$$

we obtain (N_4).

(9) The insertion of particular values in the same general identity will not however give (D_4), or anything resembling it. In fact it would seem that (D_4), although from other points of view simpler than (N_4), cannot be proved in this way at all,—that is to say, from a vanishing determinant of the 8th order by the use of LAPLACE's expansion-theorem,—the first factors of the determinant products containing as many as *eight* columns,

$$
\begin{matrix} 1 & a & A & aA & A^2 & aA^2 & A^4 & aA^4 \\ 1 & b & B & bB & B^2 & bB^2 & B^4 & bB^4 \\ 1 & c & C & cC & C^2 & cC^2 & C^4 & cC^4 \\ 1 & d & D & dD & D^2 & dD^2 & D^4 & dD^4 ; \end{matrix}
$$

and the second factors as many as *seven*,

$$
\begin{matrix} 1 & A & A^2 & A^3 & A^4 & A^5 & A^6 \\ 1 & B & B^2 & B^3 & B^4 & B^5 & B^6 \\ 1 & C & C^2 & C^3 & C^4 & C^5 & C^6 \\ 1 & D & D^2 & D^3 & D^4 & D^5 & D^6 ; \end{matrix}
$$

so that if a determinant of the 8th order were formed with these, the number of resulting products instead of being fewer than in the case of (N_4) would be far greater.

(10) There is still a third form which the identities may be made to take, and from which it was reasonable to expect something in the way of suggestion for a new mode of proof. This is obtained from the previous form,—that is to say, the form which we have just been considering,—by performing the determinant multiplications there indicated. In the original form the identities consisted of terms each of which was the product of a determinant and a symmetric function: then they were changed into vanishing aggregates of products of pairs of determinants: and now by a further change they appear as vanishing aggregates of single determinants.

A little examination shows that it is essential in performing the determinant multiplications that the lines multiplied in the process shall both be *columns*;—that, for example, instead of multiplying

$$\begin{vmatrix} 1 & a & A \\ 1 & b & B \\ 1 & c & C \end{vmatrix} \quad \text{by} \quad \begin{vmatrix} 1 & A^2 & A^4 \\ 1 & B^2 & B^4 \\ 1 & C^2 & C^4 \end{vmatrix}$$

which, strictly speaking, would give

$$\begin{vmatrix} 1+a+A & A^2+aB^2+AC^2 & A^4+aB^4+AC^4 \\ 1+b+B & A^2+bB^2+BC^2 & A^4+bB^4+BC^4 \\ 1+c+C & A^2+cB^2+\ C^3 & A^4+cB^4+\ C^5 \end{vmatrix},$$

we first change the multiplicand into its conjugate with the result that the product takes the form

$$\begin{vmatrix} 3 & A^2+B^2+C^2 & A^4+B^4+C^4 \\ a+b+c & aA^2+bB^2+cC^2 & aA^4+bB^4+cC^4 \\ A+B+C & A^3+B^3+C^3 & A^5+B^5+C^5 \end{vmatrix}.$$

Doing this (N_3) becomes

$$\begin{vmatrix} 3 & \Sigma A^2 & \Sigma A^4 \\ \Sigma a & \Sigma aA^2 & \Sigma aA^4 \\ \Sigma A & \Sigma A^3 & \Sigma A^5 \end{vmatrix} - \begin{vmatrix} 3 & \Sigma A & \Sigma A^4 \\ \Sigma a & \Sigma aA & \Sigma aA^4 \\ \Sigma A^2 & \Sigma A^3 & \Sigma A^6 \end{vmatrix} + \begin{vmatrix} 3 & \Sigma A & \Sigma A^2 \\ \Sigma a & \Sigma aA & \Sigma aA^2 \\ \Sigma A^4 & \Sigma A^5 & \Sigma A^6 \end{vmatrix} = 0,$$

and (D_3) becomes

$$\begin{vmatrix} 3 & \Sigma A^2 & \Sigma A^4 \\ \Sigma a & \Sigma aA^2 & \Sigma aA^4 \\ \Sigma aA & \Sigma aA^3 & \Sigma aA^5 \end{vmatrix} - \begin{vmatrix} 3 & \Sigma A & \Sigma A^4 \\ \Sigma a & \Sigma aA & \Sigma aA^4 \\ \Sigma aA^2 & \Sigma aA^3 & \Sigma aA^6 \end{vmatrix} + \begin{vmatrix} 3 & \Sigma A & \Sigma A^2 \\ \Sigma a & \Sigma aA & \Sigma aA^2 \\ \Sigma aA^4 & \Sigma aA^5 & \Sigma aA^6 \end{vmatrix} = 0.$$

(11) Now in the former of these although three determinants occur each with nine elements, the number of *different* elements is only two more than the number in a single determinant. Putting therefore $|a_1b_2c_3|$ in place of the first determinant and ξ_1, ξ_2 in place of the two additional elements found elsewhere, let us investigate the aggregate thus formed, viz. the aggregate

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} - \begin{vmatrix} a_1 & c_1 & a_3 \\ b_1 & \xi_1 & b_3 \\ a_2 & c_2 & \xi_2 \end{vmatrix} + \begin{vmatrix} a_1 & c_1 & a_2 \\ b_1 & \xi_1 & b_2 \\ a_3 & c_3 & \xi_2 \end{vmatrix}.$$

Clearly the terms involving c_2 in the first determinant are cancelled by those in the second, the terms involving c_3 in the first by those in the third, and the terms involving ξ_2 in the second by those in the third. The aggregate may thus be legitimately replaced by

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & \cdot & \cdot \end{vmatrix} - \begin{vmatrix} a_1 & c_1 & a_3 \\ b_1 & \xi_1 & b_3 \\ a_2 & \cdot & \cdot \end{vmatrix} + \begin{vmatrix} a_1 & c_1 & a_2 \\ b_1 & \xi_1 & b_2 \\ a_3 & \cdot & \cdot \end{vmatrix},$$

and therefore by

$$c_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - a_2\begin{vmatrix} c_1 & a_3 \\ \xi_1 & b_3 \end{vmatrix} + a_3\begin{vmatrix} c_1 & a_2 \\ \xi_1 & b_2 \end{vmatrix}$$

which manifestly vanishes.

(12) The identity thus obtained is of course more general than (N_3), because it holds when the eleven different elements involved in it are quite independent of one another. Passing over (D_3) let us at once try the process on the very general theorem of § 6 which includes both (N_3) and (D_3). The aggregate then to be considered is

$$\begin{vmatrix} \Sigma h & \Sigma h\mathrm{A}^2 & \Sigma h\mathrm{A}^4 \\ \Sigma m & \Sigma m\mathrm{A}^2 & \Sigma m\mathrm{A}^4 \\ \Sigma a\mathrm{A} & \Sigma a\mathrm{A}^3 & \Sigma a\mathrm{A}^5 \end{vmatrix} - \begin{vmatrix} \Sigma h & \Sigma h\mathrm{A} & \Sigma h\mathrm{A}^4 \\ \Sigma m & \Sigma m\mathrm{A} & \Sigma m\mathrm{A}^4 \\ \Sigma a\mathrm{A}^2 & \Sigma a\mathrm{A}^3 & \Sigma a\mathrm{A}^6 \end{vmatrix}$$
$$+ \begin{vmatrix} \Sigma h & \Sigma h\mathrm{A} & \Sigma h\mathrm{A}^2 \\ \Sigma m & \Sigma m\mathrm{A} & \Sigma m\mathrm{A}^2 \\ \Sigma a\mathrm{A}^4 & \Sigma a\mathrm{A}^5 & \Sigma a\mathrm{A}^6 \end{vmatrix} - \begin{vmatrix} \Sigma h\mathrm{A} & \Sigma h\mathrm{A}^2 & \Sigma h\mathrm{A}^4 \\ \Sigma m\mathrm{A} & \Sigma m\mathrm{A}^2 & \Sigma m\mathrm{A}^4 \\ \Sigma a\mathrm{A} & \Sigma a\mathrm{A}^2 & \Sigma a\mathrm{A}^4 \end{vmatrix},$$

where the number of different elements is now five more than the number in any one of its four determinants. Replacing these five elements by $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$, and the first determinant by $|a_1b_2c_3|$ we obtain

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} - \begin{vmatrix} a_1 & \xi_1 & a_3 \\ b_1 & \xi_2 & b_3 \\ \xi_3 & c_2 & \xi_4 \end{vmatrix} + \begin{vmatrix} a_1 & \xi_1 & a_2 \\ b_1 & \xi_2 & b_2 \\ \xi_5 & c_3 & \xi_4 \end{vmatrix} - \begin{vmatrix} \xi_1 & a_2 & a_3 \\ \xi_2 & b_2 & b_3 \\ c_1 & \xi_3 & \xi_5 \end{vmatrix},$$

and see that the terms in c_1, c_2, c_3 in the first determinant are cancelled by those in c_1, c_2, c_3 in the 4th, 2nd, 3rd determinants respectively; so that the aggregate reduces to

$$- \begin{vmatrix} a_1 & \xi_1 & a_3 \\ b_1 & \xi_2 & b_3 \\ \xi_3 & \cdot & \xi_4 \end{vmatrix} + \begin{vmatrix} a_1 & \xi_1 & a_2 \\ b_1 & \xi_2 & b_2 \\ \xi_5 & \cdot & \xi_4 \end{vmatrix} - \begin{vmatrix} \xi_1 & a_2 & a_3 \\ \xi_2 & b_2 & b_3 \\ \cdot & \xi_3 & \xi_5 \end{vmatrix},$$

which by reason of the terms in ξ_3, ξ_4, ξ_5 cancelling themselves reduces further to zero.

Strange to say, the general vanishing aggregate to which we have thus been led for the purpose of establishing CAYLEY's identities (N_3), (D_3) is essentially the same as that to which a study of KRONECKER's theorem regarding the minors of an axisymmetric determinant brought me in 1888 (see *Proc. Roy. Soc. Edinb.*, xv. pp. 96–98). As a foundation of two important theorems of so diverse a character the said vanishing aggregate becomes of considerable interest.

(13) Turning now to (N_4) and (D_4) let us take the latter first, because although it proved recalcitrant to the previous method, it ought to yield more readily than (N_4) to the present method, if it yield at all. The new form of it is

$$\begin{vmatrix} 4 & \Sigma a & \Sigma\mathrm{A} & \Sigma a\mathrm{A} \\ \Sigma\mathrm{A}^2 & \Sigma a\mathrm{A}^2 & \Sigma\mathrm{A}^3 & \Sigma a\mathrm{A}^3 \\ \Sigma\mathrm{A}^4 & \Sigma a\mathrm{A}^4 & \Sigma\mathrm{A}^5 & \Sigma a\mathrm{A}^5 \\ \Sigma\mathrm{A}^6 & \Sigma a\mathrm{A}^6 & \Sigma\mathrm{A}^7 & \Sigma a\mathrm{A}^7 \end{vmatrix} + \begin{vmatrix} 4 & \Sigma a & \Sigma\mathrm{A}^4 & \Sigma a\mathrm{A}^4 \\ \Sigma\mathrm{A} & \Sigma a\mathrm{A} & \Sigma\mathrm{A}^5 & \Sigma a\mathrm{A}^5 \\ \Sigma\mathrm{A}^2 & \Sigma a\mathrm{A}^2 & \Sigma\mathrm{A}^6 & \Sigma a\mathrm{A}^6 \\ \Sigma\mathrm{A}^3 & \Sigma a\mathrm{A}^3 & \Sigma\mathrm{A}^7 & \Sigma a\mathrm{A}^7 \end{vmatrix}$$
$$- \begin{vmatrix} 4 & \Sigma a & \Sigma\mathrm{A}^2 & \Sigma a\mathrm{A}^2 \\ \Sigma\mathrm{A} & \Sigma a\mathrm{A} & \Sigma\mathrm{A}^3 & \Sigma a\mathrm{A}^3 \\ \Sigma\mathrm{A}^4 & \Sigma a\mathrm{A}^4 & \Sigma\mathrm{A}^6 & \Sigma a\mathrm{A}^6 \\ \Sigma\mathrm{A}^5 & \Sigma a\mathrm{A}^5 & \Sigma\mathrm{A}^7 & \Sigma a\mathrm{A}^7 \end{vmatrix} = 0,$$

where, be it observed, the number of different elements in the three determinants is only 16, that is to say, exactly the number in any single one of them. Replacing therefore the first of them by $|a_1b_2c_3d_4|$ we obtain for investigation the more general aggregate

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & c_1 & c_2 \\ a_3 & a_4 & c_3 & c_4 \\ b_1 & b_2 & d_1 & d_2 \\ b_3 & b_4 & d_3 & d_4 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 & b_1 & b_2 \\ a_3 & a_4 & b_3 & b_4 \\ c_1 & c_2 & d_1 & d_2 \\ c_3 & c_4 & d_3 & d_4 \end{vmatrix}.$$

Expressing each of the three determinants as an aggregate of products of complementary minors formed from the first two columns and the last two columns we see that

the 1st product of the 1st determinant			
.. 2nd	.....	..... 1st	.....
.. 5th	.....	..... 1st	.....
.. 6th	.....	..... 1st	.....
.. 1st	.....	..... 2nd	.....
.. 6th	.....	..... 2nd	.....

are cancelled by

the 2nd product of the 2nd determinant			
.. 2nd	.....	..... 3rd	.....
.. 5th	.....	..... 3rd	.....
.. 5th	.....	..... 2nd	.....
.. 1st	.....	..... 3rd	.....
.. 6th	.....	..... 3rd	.....

respectively. Only six products are thus left, viz., the 3rd and 4th of each determinant, the aggregate of the six being

$$\begin{vmatrix} a_1 & a_2 \\ d_1 & d_2 \end{vmatrix} \cdot \begin{vmatrix} b_3 & b_4 \\ c_3 & c_4 \end{vmatrix} + \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \cdot \begin{vmatrix} a_3 & a_4 \\ d_3 & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 \\ b_3 & b_4 \end{vmatrix} \cdot \begin{vmatrix} c_3 & c_4 \\ d_1 & d_2 \end{vmatrix}$$
$$+ \begin{vmatrix} a_3 & a_4 \\ b_1 & b_2 \end{vmatrix} \cdot \begin{vmatrix} c_1 & c_2 \\ d_3 & d_4 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 \\ c_3 & c_4 \end{vmatrix} \cdot \begin{vmatrix} b_3 & b_4 \\ d_1 & d_2 \end{vmatrix} - \begin{vmatrix} a_3 & a_4 \\ c_1 & c_2 \end{vmatrix} \cdot \begin{vmatrix} b_1 & b_2 \\ d_3 & d_4 \end{vmatrix},$$

which is nothing more than the double of zero in the form of the well-known vanishing aggregate

$$|\alpha_1\beta_2| \cdot |\alpha_3\beta_4| - |\alpha_1\beta_3| \cdot |\alpha_2\beta_4| + |\alpha_1\beta_4| \cdot |\alpha_2\beta_3|.$$

(14) The new form of (N_4) is

$$\begin{vmatrix} 4 & \Sigma A^2 & \Sigma A^4 & \Sigma A^6 \\ \Sigma a & \Sigma aA^2 & \Sigma aA^4 & \Sigma aA^6 \\ \Sigma a^2 & \Sigma a^2A^2 & \Sigma a^2A^4 & \Sigma a^2A^6 \\ \Sigma aA & \Sigma aA^3 & \Sigma aA^5 & \Sigma aA^7 \end{vmatrix} - \begin{vmatrix} 4 & \Sigma A & \Sigma A^4 & \Sigma A^6 \\ \Sigma a & \Sigma aA & \Sigma aA^4 & \Sigma aA^6 \\ \Sigma a^2 & \Sigma a^2A & \Sigma a^2A^4 & \Sigma a^2A^6 \\ \Sigma aA^2 & \Sigma aA^3 & \Sigma aA^6 & \Sigma aA^8 \end{vmatrix}$$
$$+ \begin{vmatrix} 4 & \Sigma A & \Sigma A^2 & \Sigma A^6 \\ \Sigma a & \Sigma aA & \Sigma aA^2 & \Sigma aA^6 \\ \Sigma a^2 & \Sigma a^2A & \Sigma a^2A^2 & \Sigma a^2A^6 \\ \Sigma aA^4 & \Sigma aA^5 & \Sigma aA^6 & \Sigma aA^{10} \end{vmatrix} - \begin{vmatrix} 4 & \Sigma A & \Sigma A^2 & \Sigma A^4 \\ \Sigma a & \Sigma aA & \Sigma aA^2 & \Sigma aA^4 \\ \Sigma a^2 & \Sigma a^2A & \Sigma a^2A^2 & \Sigma a^2A^4 \\ \Sigma aA^6 & \Sigma aA^7 & \Sigma aA^8 & \Sigma aA^{10} \end{vmatrix} = 0,$$

where now the number of different elements is 20,—that is to say, *four* more than the number in any single determinant. Replacing these four elements by $\xi_1, \xi_2, \xi_3, \xi_4,$

and the first determinant by $|a_1b_2c_3d_4|$ we obtain for investigation the more general aggregate

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} - \begin{vmatrix} a_1 & \xi_1 & a_3 & a_4 \\ b_1 & d_1 & b_3 & b_4 \\ c_1 & \xi_2 & c_3 & c_4 \\ b_2 & d_2 & b_4 & \xi_3 \end{vmatrix}$$

$$+ \begin{vmatrix} a_1 & \xi_1 & a_2 & a_4 \\ b_1 & d_1 & b_2 & b_4 \\ c_1 & \xi_2 & c_2 & c_4 \\ b_3 & d_3 & b_4 & \xi_4 \end{vmatrix} - \begin{vmatrix} a_1 & \xi_1 & a_2 & a_3 \\ b_1 & d_1 & b_2 & b_3 \\ c_1 & \xi_2 & c_2 & c_3 \\ b_4 & d_4 & \xi_3 & \xi_4 \end{vmatrix}.$$

Now it is readily seen that

the cofactor of d_2 in the 1st determinant
..... d_3 ... 1st .....
..... d_4 ... 1st .....
..... ξ_3 ... 2nd .....
..... b_4 ... 2nd .....
..... ξ_4 ... 3rd .....

are cancelled by

the cofactor of d_2 in the 2nd determinant
..... d_3 ... 3rd .....
..... d_4 ... 4th .....
..... ξ_3 ... 4th .....
..... b_4 ... 3rd .....
..... ξ_4 ... 4th .....

respectively. All therefore that is left in each determinant is one element accompanied by its cofactor, viz., the element in the place (41); so that the aggregate with which we started is reduced to

$$-d_1\begin{vmatrix} a_2 & a_3 & a_4 \\ b_2 & b_3 & b_4 \\ c_2 & c_3 & c_4 \end{vmatrix} + b_2\begin{vmatrix} \xi_1 & a_3 & a_4 \\ d_1 & b_3 & b_4 \\ \xi_2 & c_3 & c_4 \end{vmatrix} - b_3\begin{vmatrix} \xi_1 & a_2 & a_4 \\ d_1 & b_2 & b_4 \\ \xi_2 & c_2 & c_4 \end{vmatrix} + b_4\begin{vmatrix} \xi_1 & a_2 & a_3 \\ d_1 & b_2 & b_3 \\ \xi_2 & c_2 & c_3 \end{vmatrix}$$

which again is clearly equivalent to

$$\begin{vmatrix} \xi_1 & a_2 & a_3 & a_4 \\ d_1 & b_2 & b_3 & b_4 \\ \xi_2 & c_2 & c_3 & c_4 \\ d_1 & b_2 & b_3 & b_4 \end{vmatrix}$$

and therefore vanishes.

(15) This identity is not new, being simply a special instance of the fourth-order case of the theorem above referred to as having been used in proving KRONECKER'S relation between the minors of an axisymmetric determinant. (N_3), (D_3), (N_4) are thus seen to be dependent on the same theorem. The full expression for $|a_1b_2c_3d_4|$ is

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_5 \\ b_1 & b_2 & b_3 & \beta_5 \\ c_1 & c_2 & c_3 & \gamma_5 \\ D_{14} & D_{24} & D_{34} & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_5 & a_4 \\ b_1 & b_2 & \beta_5 & b_4 \\ c_1 & c_2 & \gamma_5 & c_4 \\ D_{13} & D_{23} & d_3 & D_{34} \end{vmatrix} + \begin{vmatrix} a_1 & a_5 & a_3 & a_4 \\ b_1 & \beta_5 & b_3 & b_4 \\ c_1 & \gamma_5 & c_3 & c_4 \\ D_{12} & d_2 & D_{23} & D_{24} \end{vmatrix} + \begin{vmatrix} a_5 & a_2 & a_3 & a_4 \\ \beta_5 & b_2 & b_3 & b_4 \\ \gamma_5 & c_2 & c_3 & c_4 \\ d_1 & D_{12} & D_{13} & D_{14} \end{vmatrix},$$

where the elements which may be all different are

$$\begin{array}{lll} \text{the } [n^2, \text{ which here is}] & 4^2 \text{ elements of} & |\, a_1 b_2 c_3 d_4 \,|, \\ \text{the } [n-1, \text{ which here is}] & 3 \text{ elements} & \alpha_5, \beta_5, \gamma_5, \\ \text{and the } [\tfrac{1}{2}n(n-1), \text{ which here is}] & 6 \text{ elements} & D_{12}, D_{13}, D_{14}, \\ & & \quad\;\, D_{23}, D_{24}, \\ & & \qquad\quad\; D_{34}. \end{array}$$

This degenerates into the identity of the preceding paragraph if we put $D_{12}, D_{13}, D_{14} = b_2, b_3, b_4$; $D_{23} = b_4$; $\beta_5 = d_1$.

(16) Having thus the general fourth-order theorem of which (N_4) is a case, it is natural to inquire whether there be a corresponding general fourth-order theorem of which (D_4) is a case. The following new result regarding the sum of two determinants of the fourth order gives the answer to this inquiry :—

If $|\, a_1 b_2 c_3 d_4 \,|$, $|\, \alpha_1 \beta_2 \gamma_3 \delta_4 \,|$ *be any two fourth-order determinants, their sum is equal to the sum of six determinants, each of which contains a pair of complementary minors from each of the originals, viz., the sum*

$$\begin{vmatrix} a_1 & a_2 & \gamma_3 & \gamma_4 \\ b_1 & b_2 & \delta_3 & \delta_4 \\ \alpha_1 & \alpha_2 & c_3 & c_4 \\ \beta_1 & \beta_2 & d_3 & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & \beta_3 & \beta_4 \\ \alpha_1 & \alpha_2 & b_3 & b_4 \\ c_1 & c_2 & \delta_3 & \delta_4 \\ \gamma_1 & \gamma_2 & d_3 & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & \alpha_3 & \alpha_4 \\ \beta_1 & \beta_2 & b_3 & b_4 \\ \gamma_1 & \gamma_2 & c_3 & c_4 \\ d_1 & d_2 & \delta_3 & \delta_4 \end{vmatrix}$$
$$+ \begin{vmatrix} \alpha_1 & \alpha_2 & a_3 & a_4 \\ b_1 & b_2 & \beta_3 & \beta_4 \\ c_1 & c_2 & \gamma_3 & \gamma_4 \\ \delta_1 & \delta_2 & d_3 & d_4 \end{vmatrix} + \begin{vmatrix} \beta_1 & \beta_2 & a_3 & a_4 \\ b_1 & b_2 & \alpha_3 & \alpha_4 \\ \delta_1 & \delta_2 & c_3 & c_4 \\ d_1 & d_2 & \gamma_3 & \gamma_4 \end{vmatrix} + \begin{vmatrix} \gamma_1 & \gamma_2 & a_3 & a_4 \\ \delta_1 & \delta_2 & b_3 & b_4 \\ c_1 & c_2 & \alpha_3 & \alpha_4 \\ d_1 & d_2 & \beta_3 & \beta_4 \end{vmatrix}.$$

Expressing each of the six as an aggregate of products of complementary minors, we have the sum equal to an aggregate of thirty-six products, viz. :

$$\begin{aligned}
&\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\begin{vmatrix} c_3 & c_4 \\ d_3 & d_4 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 \\ \alpha_1 & \alpha_2 \end{vmatrix}\begin{vmatrix} \delta_3 & \delta_4 \\ d_3 & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 \\ \beta_1 & \beta_2 \end{vmatrix}\begin{vmatrix} \delta_3 & \delta_4 \\ c_3 & c_4 \end{vmatrix} + \begin{vmatrix} b_1 & b_2 \\ \alpha_1 & \alpha_2 \end{vmatrix}\begin{vmatrix} \gamma_3 & \gamma_4 \\ d_3 & d_4 \end{vmatrix} - \begin{vmatrix} b_1 & b_2 \\ \beta_1 & \beta_2 \end{vmatrix}\begin{vmatrix} \gamma_3 & \gamma_4 \\ c_3 & c_4 \end{vmatrix} + \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix}\begin{vmatrix} \gamma_3 & \gamma_4 \\ \delta_3 & \delta_4 \end{vmatrix} \\
+ &\begin{vmatrix} a_1 & a_2 \\ \alpha_1 & \alpha_2 \end{vmatrix}\begin{vmatrix} \delta_3 & \delta_4 \\ d_3 & d_4 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 \\ c_1 & c_2 \end{vmatrix}\begin{vmatrix} b_3 & b_4 \\ d_3 & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 \\ \gamma_1 & \gamma_2 \end{vmatrix}\begin{vmatrix} b_3 & b_4 \\ \delta_3 & \delta_4 \end{vmatrix} + \begin{vmatrix} \alpha_1 & \alpha_2 \\ c_1 & c_2 \end{vmatrix}\begin{vmatrix} \beta_3 & \beta_4 \\ d_3 & d_4 \end{vmatrix} - \begin{vmatrix} \alpha_1 & \alpha_2 \\ \gamma_1 & \gamma_2 \end{vmatrix}\begin{vmatrix} \beta_3 & \beta_4 \\ \delta_3 & \delta_4 \end{vmatrix} + \begin{vmatrix} c_1 & c_2 \\ \gamma_1 & \gamma_2 \end{vmatrix}\begin{vmatrix} \beta_3 & \beta_4 \\ b_3 & b_4 \end{vmatrix} \\
+ &\begin{vmatrix} a_1 & a_2 \\ \beta_1 & \beta_2 \end{vmatrix}\begin{vmatrix} c_3 & c_4 \\ \delta_3 & \delta_4 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 \\ \gamma_1 & \gamma_2 \end{vmatrix}\begin{vmatrix} b_3 & b_4 \\ \delta_3 & \delta_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} b_3 & b_4 \\ c_3 & c_4 \end{vmatrix} + \begin{vmatrix} \beta_1 & \beta_2 \\ \gamma_1 & \gamma_2 \end{vmatrix}\begin{vmatrix} \alpha_3 & \alpha_4 \\ \delta_3 & \delta_4 \end{vmatrix} - \begin{vmatrix} \beta_1 & \beta_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} \alpha_3 & \alpha_4 \\ c_3 & c_4 \end{vmatrix} + \begin{vmatrix} \gamma_1 & \gamma_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} \alpha_3 & \alpha_4 \\ b_3 & b_4 \end{vmatrix} \\
+ &\begin{vmatrix} \alpha_1 & \alpha_2 \\ b_1 & b_2 \end{vmatrix}\begin{vmatrix} \gamma_3 & \gamma_4 \\ d_3 & d_4 \end{vmatrix} - \begin{vmatrix} \alpha_1 & \alpha_2 \\ c_1 & c_2 \end{vmatrix}\begin{vmatrix} \beta_3 & \beta_4 \\ d_3 & d_4 \end{vmatrix} + \begin{vmatrix} \alpha_1 & \alpha_2 \\ \delta_1 & \delta_2 \end{vmatrix}\begin{vmatrix} \beta_3 & \beta_4 \\ \gamma_3 & \gamma_4 \end{vmatrix} + \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ d_3 & d_4 \end{vmatrix} - \begin{vmatrix} b_1 & b_2 \\ \delta_1 & \delta_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ \gamma_3 & \gamma_4 \end{vmatrix} + \begin{vmatrix} c_1 & c_2 \\ \delta_1 & \delta_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ \beta_3 & \beta_4 \end{vmatrix} \\
+ &\begin{vmatrix} \beta_1 & \beta_2 \\ b_1 & b_2 \end{vmatrix}\begin{vmatrix} c_3 & c_4 \\ \gamma_3 & \gamma_4 \end{vmatrix} - \begin{vmatrix} \beta_1 & \beta_2 \\ \delta_1 & \delta_2 \end{vmatrix}\begin{vmatrix} \alpha_3 & \alpha_4 \\ \gamma_3 & \gamma_4 \end{vmatrix} + \begin{vmatrix} \beta_1 & \beta_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} \alpha_3 & \alpha_4 \\ c_3 & c_4 \end{vmatrix} + \begin{vmatrix} b_1 & b_2 \\ \delta_1 & \delta_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ \gamma_3 & \gamma_4 \end{vmatrix} - \begin{vmatrix} b_1 & b_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ c_3 & c_4 \end{vmatrix} + \begin{vmatrix} \delta_1 & \delta_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ \alpha_3 & \alpha_4 \end{vmatrix} \\
+ &\begin{vmatrix} \gamma_1 & \gamma_2 \\ \delta_1 & \delta_2 \end{vmatrix}\begin{vmatrix} \alpha_3 & \alpha_4 \\ \beta_3 & \beta_4 \end{vmatrix} - \begin{vmatrix} \gamma_1 & \gamma_2 \\ c_1 & c_2 \end{vmatrix}\begin{vmatrix} b_3 & b_4 \\ \beta_3 & \beta_4 \end{vmatrix} + \begin{vmatrix} \gamma_1 & \gamma_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} b_3 & b_4 \\ \alpha_3 & \alpha_4 \end{vmatrix} + \begin{vmatrix} \delta_1 & \delta_2 \\ c_1 & c_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ \beta_3 & \beta_4 \end{vmatrix} - \begin{vmatrix} \delta_1 & \delta_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ \alpha_3 & \alpha_4 \end{vmatrix} + \begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}\begin{vmatrix} a_3 & a_4 \\ b_3 & b_4 \end{vmatrix}.
\end{aligned}$$

A little examination of this square array brings out the fact that all its terms cancel each other with the exception of those which are situated in either diagonal, and that the cancellation takes place in a pleasingly symmetrical fashion, each non-diagonal term and its conjugately situated fellow annihilating one another. Further, the aggregate of the six terms in the principal diagonal is seen to be

$$|\, a_1 \quad b_2 \quad c_3 \quad d_4 \,|$$

and the aggregate of the six terms in the secondary diagonal to be

$$|\, \alpha_1 \quad \beta_2 \quad \gamma_3 \quad \delta_4 \,| ;$$

so that the theorem is established.

Now turning to the identity of § 13, which, as we have noted, has no elements different from the sixteen composing a single one of the determinants involved in it, we see at once that if it is to be included in that just found, the elements of the second parent determinant of the latter must be the same as the elements of the first. Making them the same even in form we obtain the nugatory result

$$2\,|\, a_1b_2c_3d_4 \,| \;=\; 2\,|\, a_1b_2c_3d_4 \,| .$$

But making the consanguinity less pronounced, the second parent being of the form $|a_3b_4c_1d_2|$, we find that only the 3rd and 4th of the progeny are of no account, and that the 6th and 5th are the same as the 1st and 2nd respectively: so that after division by 2 there results

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & c_1 & c_2 \\ b_1 & b_2 & d_1 & d_2 \\ a_3 & a_4 & c_3 & c_4 \\ b_3 & b_4 & d_3 & d_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & b_1 & b_2 \\ a_3 & a_4 & b_3 & b_4 \\ c_1 & c_2 & d_1 & d_2 \\ c_3 & c_4 & d_3 & d_4 \end{vmatrix}$$

which completely agrees with the identity of § 13.

(17) A fourth method of investigation consists in making transformations which result in the segregation of the capital letters from the small letters. As, however, nothing new results from it in the case of (N_3), (D_3), (N_4), its application to (D_4) is all that need be given.

The aggregate to be considered in this case is

$$\begin{vmatrix} 1 & a & A & aA \\ 1 & b & B & bB \\ 1 & c & C & cC \\ 1 & d & D & dD \end{vmatrix} \cdot |\, A^0B^2C^4D^6 \,| - \begin{vmatrix} 1 & a & A & aA^2 \\ 1 & b & B & bB^2 \\ 1 & c & C & cC^2 \\ 1 & d & D & dD^2 \end{vmatrix} \cdot |\, A^0B^1C^4D^5 \,| + \begin{vmatrix} 1 & a & A^4 & aA^4 \\ 1 & b & B^4 & bB^4 \\ 1 & c & C^4 & cC^4 \\ 1 & d & D^4 & dD^4 \end{vmatrix} \cdot |\, A^0B^1C^2D^3 \,| .$$

Now each of the first factors of the three determinant products here appearing is of the form

$$\begin{vmatrix} 1 & a & A^\omega & aA^\omega \\ 1 & b & B^\omega & bB^\omega \\ 1 & c & C^\omega & cC^\omega \\ 1 & d & D^\omega & dD^\omega \end{vmatrix}$$

which is easily seen to be transformable into

$$\begin{vmatrix} 1 & a \\ 1 & b \end{vmatrix} \cdot \begin{vmatrix} 1 & c \\ 1 & d \end{vmatrix} \cdot (C^\omega D^\omega + A^\omega B^\omega) - \begin{vmatrix} 1 & a \\ 1 & c \end{vmatrix} \cdot \begin{vmatrix} 1 & b \\ 1 & d \end{vmatrix} (B^\omega D^\omega + A^\omega C^\omega) + \begin{vmatrix} 1 & a \\ 1 & d \end{vmatrix} \cdot \begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix} \cdot (B^\omega C^\omega + A^\omega D^\omega).$$

The aggregate under consideration can thus be changed into

$$\begin{aligned} &\begin{vmatrix} 1 & a \\ 1 & b \end{vmatrix} \cdot \begin{vmatrix} 1 & c \\ 1 & d \end{vmatrix} \Big\{ (CD+AB) \cdot | A^0B^2C^4D^6 | - (C^2D^2+A^2B^2) \cdot | A^0B^1C^4D^5 | + (C^4D^4+A^4B^4) \cdot | A^0B^1C^2D^3 | \Big\} \\ - &\begin{vmatrix} 1 & a \\ 1 & c \end{vmatrix} \cdot \begin{vmatrix} 1 & b \\ 1 & d \end{vmatrix} \cdot \Big\{ (BD+AC) \cdot | A^0B^2C^4D^6 | - (B^2D^2+A^2C^2) \cdot | A^0B^1C^4D^5 | + (B^4D^4+A^4C^4) \cdot | A^0B^1C^2D^3 | \Big\} \\ + &\begin{vmatrix} 1 & a \\ 1 & d \end{vmatrix} \cdot \begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix} \cdot \Big\{ (BC+AB) \cdot | A^0B^2C^4D^6 | - (B^2C^2+A^2B^2) \cdot | A^0B^1C^4D^5 | + (B^4C^4+A^4B^4) \cdot | A^0B^1C^2D^3 | \Big\}; \end{aligned}$$

and this can only vanish identically when the three expressions enclosed in { } are equal, for the cofactor of bd is the difference between the first and third, and the cofactor of bc is the difference between the first and the second. Now the sum of the said three expressions is

$$| A^0B^2C^4D^6 | \cdot \Sigma AB - | A^0B^1C^4D^5 | \cdot \Sigma A^2B^2 + | A^0B^1C^2D^3 | \cdot \Sigma A^4B^4,$$

and this by the theorem for the multiplication of an alternant by a symmetric function of its variables is found to be

$$3 | A^0B^3C^5D^6 | + 3 | AB^2C^4D^7 | .$$

Consequently each of the three is equal to

$$| A^0B^3C^5D^6 | + | AB^2C^4D^7 | .*$$

The aggregate above reached can thus be changed into

$$\left\{ \begin{vmatrix} 1 & a \\ 1 & b \end{vmatrix} \begin{vmatrix} 1 & c \\ 1 & d \end{vmatrix} - \begin{vmatrix} 1 & a \\ 1 & c \end{vmatrix} \begin{vmatrix} 1 & b \\ 1 & d \end{vmatrix} + \begin{vmatrix} 1 & a \\ 1 & d \end{vmatrix} \begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix} \right\} \cdot \Big(| A^0B^3C^5D^6 | + | AB^2C^4D^7 | \Big),$$

and therefore into

$$\tfrac{1}{2} \begin{vmatrix} 1 & a & 1 & a \\ 1 & b & 1 & b \\ 1 & c & 1 & c \\ 1 & d & 1 & d \end{vmatrix} \cdot \Big(| A^0B^3C^5D^6 | + | AB^2C^4D^7 | \Big),$$

where the vanishing factor is what each of the first factors in the original aggregate becomes when we put $A = B = C = D = 1$.

* A direct proof that

$$\begin{aligned} | A^0B^2C^4D^6 | \cdot (AB+CD) - | A^0B^1C^4D^5 | \cdot (A^2B^2+C^2D^2) + | A^0B^1C^2D^3 | \cdot (A^4B^4+C^4D^4) \\ = | A^0B^3C^5D^6 | + | A^1B^2C^4D^7 | \end{aligned}$$

is obtainable from the theorem above given regarding the sum of two fourth-order determinants, the parents being the two determinants on the right, and the progeny the six determinants obtainable on the left by performing the multiplications indicated, viz.:

$$\begin{vmatrix} A & A^3 & A^5 & A^7 \\ B & B^3 & B^5 & B^7 \\ 1 & C^2 & C^4 & C^6 \\ 1 & D^2 & D^4 & D^6 \end{vmatrix}, \begin{vmatrix} 1 & A^2 & A^4 & A^6 \\ 1 & B^2 & B^4 & B^6 \\ C & C^3 & C^5 & C^7 \\ D & D^3 & D^5 & D^7 \end{vmatrix}, \ldots .$$

The corresponding results for (N_3), (D_3), (N_4) are

$$\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} \,|\,\mathrm{A}^1\mathrm{B}^2\mathrm{C}^4\,|, \qquad \begin{vmatrix} 1 & a & a \\ 1 & b & b \\ 1 & c & c \end{vmatrix} \,|\,\mathrm{A}^1\mathrm{B}^2\mathrm{C}^4\,|, \qquad \begin{vmatrix} 1 & a & 1 & a \\ 1 & b & 1 & b \\ 1 & c & 1 & c \\ 1 & d & 1 & d \end{vmatrix} \,|\,\mathrm{A}^1\mathrm{B}^2\mathrm{C}^4\mathrm{D}^6\,|,$$

as may also be seen from §§ 6, 8.

(18) The problem of finding, for determinants of a higher order than the fourth, identities similar to CAYLEY's I do not at present enter upon: like CAYLEY, "je n'ai pas encore trouvé la loi générale de ces équations." I content myself with stating the problem in as simple a form as possible. For the fifth and sixth orders it is :—

To express each of the products

$$|\,1 \quad a \quad a^2 \quad \mathrm{A} \quad a\mathrm{A}\,| \,.\, |\,\mathrm{A}^0\mathrm{B}^2\mathrm{C}^4\mathrm{D}^6\mathrm{E}^8\,|$$
$$|\,1 \quad a \quad a^2 \quad a^3 \quad a\mathrm{A} \quad a^2\mathrm{A}\,| \,.\, |\,\mathrm{A}^0\mathrm{B}^2\mathrm{C}^4\mathrm{D}^6\mathrm{E}^8\mathrm{F}^{10}\,|,$$
$$|\,1 \quad a \quad a^2 \quad \mathrm{A} \quad a\mathrm{A} \quad a^2\mathrm{A}\,| \,.\, |\,\mathrm{A}^0\mathrm{B}^2\mathrm{C}^4\mathrm{D}^6\mathrm{E}^8\mathrm{F}^{10}\,|,$$

as aggregates of products of a similar kind, the first factor of each product on the right of the identity being formable from the corresponding factor on the left by replacing A *by an even power of* A, B *by the same even power of* B, *and so on.*

For example, and more definitely, to determine α, β, γ, δ, ϵ, ζ, η, θ, so that

$$\begin{aligned} |\,1 \quad a \quad a^2 \quad \mathrm{A} \quad a\mathrm{A}\,| \cdot |\,\mathrm{A}^0\mathrm{B}^2\mathrm{C}^4\mathrm{D}^6\mathrm{E}^8\,| &= |\,1 \quad a \quad a^2 \quad \mathrm{A}^2 \quad a\mathrm{A}^2\,| \cdot |\,\mathrm{A}^0\mathrm{B}^\alpha\mathrm{C}^\beta\mathrm{D}^\gamma\mathrm{E}^\delta\,| \\ &\pm |\,1 \quad a \quad a^2 \quad \mathrm{A}^4 \quad a\mathrm{A}^4\,| \cdot |\,\mathrm{A}^0\mathrm{B}^\epsilon\mathrm{C}^\zeta\mathrm{D}^\eta\mathrm{E}^\theta\,| \\ &\pm |\,1 \quad a \quad a^2 \quad \mathrm{A}^6 \quad a\mathrm{A}^6\,| \cdot |\,\mathrm{A}^0\mathrm{B}^1\mathrm{C}^2\mathrm{D}^3\mathrm{E}^4\,|. \end{aligned}$$

X.—*The Hessian of a General Determinant.*

By Thomas Muir, LL.D.

(Read January 21, 1901.)

(1) There being n^2 independent variables in a general n-line determinant, the Hessian of the determinant with respect to the said variables must be a determinant with n^2 lines: and as a general n-line determinant has all its terms linear in the elements involved, it follows that each element of the Hessian being a second differential-quotient cannot be of a higher degree in the variables than the $(n-2)^{\text{th}}$, and that consequently the degree of the Hessian itself cannot exceed $n^2(n-2)$. The object of the present short paper is to show that this degree is attained by the $n(n-2)^{\text{th}}$ power of the given determinant being a factor of the Hessian.

(2) Beginning with the case of $n=3$, let the originating determinant be

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix},$$

so that single differentiation with respect to the nine elements produces

$$\begin{array}{ccc} |b_2c_3|, & -|b_1c_3|, & |b_1c_2| \\ -|a_2c_3|, & |a_1c_3|, & -|a_1c_2| \\ |a_2b_3|, & -|a_1b_3|, & |a_1b_2| \end{array} \quad \text{or} \quad \begin{array}{ccc} A_1, & A_2, & A_3 \\ B_1, & B_2, & B_3 \\ C_1, & C_2, & C_3 \end{array} \quad \text{say.}$$

These being each differentiated in the same way, *i.e.* with respect to each of the nine original elements, we obtain the eighty-one elements of the Hessian: and if we agree to take the independent variables in the order, a_1, a_2, a_3; b_1, b_2, b_3; c_1, c_2, c_3, the Hessian itself will be

$$\begin{vmatrix} \cdot & \cdot & \cdot & \cdot & c_3 & -c_2 & \cdot & -b_3 & b_2 \\ \cdot & \cdot & \cdot & -c_3 & \cdot & c_1 & b_3 & \cdot & -b_1 \\ \cdot & \cdot & \cdot & c_2 & -c_1 & \cdot & -b_2 & b_1 & \cdot \\ \cdot & -c_3 & c_2 & \cdot & \cdot & \cdot & \cdot & a_3 & -a_2 \\ c_3 & \cdot & -c_1 & \cdot & \cdot & \cdot & -a_3 & \cdot & a_1 \\ -c_2 & c_1 & \cdot & \cdot & \cdot & \cdot & a_2 & -a_1 & \cdot \\ \cdot & b_3 & -b_2 & \cdot & -a_3 & a_2 & \cdot & \cdot & \cdot \\ -b_3 & \cdot & b_1 & a_3 & \cdot & -a_1 & \cdot & \cdot & \cdot \\ b_2 & -b_1 & \cdot & -a_2 & a_1 & \cdot & \cdot & \cdot & \cdot \end{vmatrix}.$$

By way of remembering its constitution we should think of it as divided into three equal parts by two left-to-right lines, and at the same time into three equal parts by two top-to-bottom lines. The nine small square arrays thus resulting may then be symbolized by

$$\begin{array}{ccc} \cdot & \gamma & -\beta \\ -\gamma & \cdot & \alpha \\ \beta & -\alpha & \cdot \end{array},$$

each of the three-line matrices α, β, γ being zero-axial skew, and composed of elements from only one row of the original determinant.

Now if we multiply this by $|a_1b_2c_3|^3$ in the form

$$\left|\begin{array}{ccccccccc} a_1 & a_2 & a_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_1 & b_2 & b_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_1 & c_2 & c_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & a_1 & a_2 & a_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & b_1 & b_2 & b_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & c_1 & c_2 & c_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_1 & a_2 & a_3 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & b_1 & b_2 & b_3 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c_1 & c_2 & c_3 \end{array}\right|,$$

we obtain

$$\left|\begin{array}{ccccccccc} \cdot & \cdot & \cdot & -B_1 & A_1 & \cdot & -C_1 & \cdot & A_1 \\ \cdot & \cdot & \cdot & -B_2 & A_2 & \cdot & -C_2 & \cdot & A_2 \\ \cdot & \cdot & \cdot & -B_3 & A_3 & \cdot & -C_3 & \cdot & A_3 \\ B_1 & -A_1 & \cdot & \cdot & \cdot & \cdot & \cdot & -C_1 & B_1 \\ B_2 & -A_2 & \cdot & \cdot & \cdot & \cdot & \cdot & -C_2 & B_2 \\ B_3 & -A_3 & \cdot & \cdot & \cdot & \cdot & \cdot & -C_3 & B_3 \\ C_1 & \cdot & -A_1 & \cdot & C_1 & -B_1 & \cdot & \cdot & \cdot \\ C_2 & \cdot & -A_2 & \cdot & C_2 & -B_2 & \cdot & \cdot & \cdot \\ C_3 & \cdot & -A_3 & \cdot & C_3 & -B_3 & \cdot & \cdot & \cdot \end{array}\right|,$$

which if divided like the multiplicand into nine three-line minors takes the form

$$\begin{array}{ccc} \cdot & \gamma' & \beta' \\ -\gamma' & \cdot & \alpha' \\ -\beta' & -\alpha' & \cdot \end{array}.$$

It is better however to write the three zero minors differently, viz.—

$$\begin{array}{cccc} A_1 & -A_1 & \cdot & \cdot \\ A_2 & -A_2 & \cdot & \cdot \\ A_3 & -A_3 & \cdot & \cdot \end{array}, \qquad \begin{array}{cccc} \cdot & B_1 & -B_1 & \cdot \\ \cdot & B_2 & -B_2 & \cdot \\ \cdot & B_3 & -B_3 & \cdot \end{array}, \qquad \begin{array}{cccc} \cdot & \cdot & C_1 & -C_1 \\ \cdot & \cdot & C_2 & -C_2 \\ \cdot & \cdot & C_3 & -C_3 \end{array},$$

for then we are able to state more easily the law of formation of the nine. Thus, we may then say that the three

$$\cdot \qquad \gamma' \qquad \beta'$$

have $-A_1$, $-A_2$, $-A_3$; $-B_1$, $-B_2$, $-B_3$; $-C_1$, $-C_2$, $-C_3$ respectively in their first columns, and A_1, A_2, A_3 in a column of each, the particular column being the first of the first, the second of the second, and the third of the third. Similarly, the next three

$$-\gamma' \qquad \cdot \qquad \alpha'$$

have $-A_1$, $-A_2$, $-A_3$; $-B_1$, $-B_2$, $-B_3$; $-C_1$, $-C_2$, $-C_3$ respectively in their second columns, and B_1, B_2, B_3, in a column of each; and so in the remaining case. The product-determinant has thus two kinds of columns, viz., those with three non-zero elements, and those with six. Of the former kind there are six, and of the latter three. Further, when there are only three non-zero elements in a column they are all negative, and when there are six they are all positive. Adding together the three columns which have six non-zero elements, we can remove the factor 2 from the resulting column: and if we then diminish the two other columns by this altered column, we shall change them into columns with only three non-zero elements like the rest. Our result then will take the form

$$2\begin{vmatrix} A_1 & \cdot & \cdot & -B_1 & \cdot & \cdot & -C_1 & \cdot & \cdot \\ A_2 & \cdot & \cdot & -B_2 & \cdot & \cdot & -C_2 & \cdot & \cdot \\ A_3 & \cdot & \cdot & -B_3 & \cdot & \cdot & -C_3 & \cdot & \cdot \\ B_1 & -A_1 & \cdot & \cdot & -B_1 & \cdot & \cdot & -C_1 & \cdot \\ B_2 & -A_2 & \cdot & \cdot & -B_2 & \cdot & \cdot & -C_2 & \cdot \\ B_3 & -A_3 & \cdot & \cdot & -B_3 & \cdot & \cdot & -C_3 & \cdot \\ C_1 & \cdot & -A_1 & \cdot & \cdot & -B_1 & \cdot & \cdot & -C_1 \\ C_2 & \cdot & -A_2 & \cdot & \cdot & -B_2 & \cdot & \cdot & -C_2 \\ C_3 & \cdot & -A_3 & \cdot & \cdot & -B_3 & \cdot & \cdot & -C_3 \end{vmatrix}$$

which is easily changed further into

$$(-)^3\,2\begin{vmatrix} A_1 & B_1 & C_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_2 & B_2 & C_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_3 & B_3 & C_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ B_1 & \cdot & \cdot & A_1 & -B_1 & C_1 & \cdot & \cdot & \cdot \\ B_2 & \cdot & \cdot & A_2 & -B_2 & C_2 & \cdot & \cdot & \cdot \\ B_3 & \cdot & \cdot & A_2 & -B_2 & C_3 & \cdot & \cdot & \cdot \\ C_1 & \cdot & \cdot & \cdot & \cdot & \cdot & A_1 & B_1 & -C_1 \\ C_2 & \cdot & \cdot & \cdot & \cdot & \cdot & A_2 & B_2 & -C_2 \\ C_3 & \cdot & \cdot & \cdot & \cdot & \cdot & A_3 & B_3 & -C_3 \end{vmatrix}$$

by interchanging the 2nd, 3rd, 6th columns with the 4th, 7th, 8th respectively. We are thus led to the equation

$$H(|a_1b_2c_3|)\cdot|a_1b_2c_3|^3 = (-)^5\,2\,|A_1B_2C_3|^3,$$

and therefore finally to

$$H(|a_1b_2c_3|) = -2\,|a_1b_2c_3|^3.$$

(3) Taking next the case of $n=4$, the originating determinant being

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix},$$

we obtain, from single differentiation with respect to the sixteen elements,

$$\begin{array}{rrrr} |b_2c_3d_4|, & -|b_1c_3d_4|, & |b_1c_2d_4|, & -|b_1c_2d_3, \\ -|a_2c_3d_4|, & |a_1c_3d_4|, & -|a_1c_2d_4|, & |a_1c_2d_3|, \\ |a_2b_3d_4|, & -|a_1b_3d_4|, & |a_1b_2d_4|, & -|a_1b_2d_3|, \\ -|a_2b_3c_4|, & |a_1b_3c_4|, & -|a_1b_2c_4|, & |a_1b_2c_3|, \end{array}$$

or say

$$\begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ B_1 & B_2 & B_3 & B_4 \\ C_1 & C_2 & C_3 & C_4 \\ D_1 & D_2 & D_3 & D_4 . \end{array}$$

These being each differentiated in the same way, *i.e.* with respect to each of the sixteen original elements, we obtain the 256 elements of the Hessian ; and find the latter, when partitioned into 4×4 minor matrices of the 4th order, to be of the form

$$\begin{array}{cccc} \cdot & (\gamma,\delta) & -(\beta,\delta) & (\beta,\gamma) \\ -(\gamma,\delta) & \cdot & (\alpha,\delta) & -(\alpha,\gamma) \\ (\beta,\delta) & -(\alpha,\delta) & \cdot & (\alpha,\beta) \\ -(\beta,\gamma) & (\alpha,\gamma) & -(\alpha,\beta) & \cdot \end{array}$$

where (α,β), (α,γ), . . . are all exactly alike,—for example

$$(\alpha,\beta) = \left\{ \begin{array}{cccc} \cdot & |a_3b_4| & -|a_2b_4| & |a_2b_3| \\ -|a_3b_4| & \cdot & |a_1b_4| & -|a_1b_3| \\ |a_2b_4| & -|a_1b_4| & \cdot & |a_1b_2| \\ -|a_2b_3| & |a_1b_3| & -|a_1b_2| & \cdot \end{array} \right\},$$

—being thus all zero-axial skew like their parent matrix.*

Multiplying the Hessian by $|a_1b_2c_3d_4|^4$ we obtain as before a product which, when partitioned like the multiplicand, is of the form

$$\begin{array}{cccc} \cdot & (\beta_1\alpha_2)' & (\gamma_1\alpha_3)' & (\delta_1\alpha_4)' \\ -(\beta_1\alpha_2)' & \cdot & (\gamma_2\beta_3)' & (\delta_2\beta_4)' \\ -(\gamma_1\alpha_3)' & -(\gamma_2\beta_3)' & \cdot & (\delta_3\gamma_4)' \\ -(\delta_1\alpha_4)' & -(\delta_2\beta_4)' & -(\delta_3\gamma_4)' & \cdot \end{array}$$

where

$$(\beta_1\alpha_2)' = \left\{ \begin{array}{cccc} -B_1 & A_1 & \cdot & \cdot \\ -B_2 & A_2 & \cdot & \cdot \\ -B_3 & A_3 & \cdot & \cdot \\ -B_4 & A_4 & \cdot & \cdot \end{array} \right\}$$

the suffix of α indicating the column in which the positive A's appear, and the suffix of β the column in which the negative B's appear. Any one of the four lines of matrices, say the line

$$\cdot \quad (\beta_1\alpha_2)' \quad (\gamma_1\alpha_3)' \quad (\delta_1\alpha_4)',$$

may consequently be described in language exactly similar to that employed in the case

* Note, too, that the determinant of every matrix vanishes, being, in the case exemplified, the square of the vanishing Pfaffian

$$|a_1b_2|\,|a_3b_4| - |a_1b_3|\,|a_2b_4| + |a_1b_4|\,|a_2b_3|.$$

where $n=3$; and thus the product determinant comes to be transformable into 3 times a determinant whose first four columns are

$$\begin{array}{cccc}
A_1 & . & . & . \\
A_2 & . & . & . \\
A_3 & . & . & . \\
A_4 & . & . & . \\
. & -A_1 & . & . \\
. & -A_2 & . & . \\
. & -A_3 & . & . \\
. & -A_4 & . & . \\
. & . & -A_1 & . \\
. & . & -A_2 & . \\
. & . & -A_3 & . \\
. & . & -A_4 & . \\
. & . & . & -A_1 \\
. & . & . & -A_2 \\
. & . & . & -A_3 \\
. & . & . & -A_4
\end{array}$$

these being followed by four columns containing B's similarly placed but all negative, these again by four columns containing negative C's, and these by four columns containing negative D's. Interchanging columns as before, with the necessary accompaniment of $(3+2+1)+3$ changes of sign, we obtain

$$\begin{aligned} H(|a_1b_2c_3d_4|) \cdot |a_1b_2c_3d_4|^4 &= (-)^9 3\,|A_1B_2C_3D_4|^4, \\ &= (-)^9 3\,|a_1b_2c_3d_4|^{12}, \end{aligned}$$

and therefore

$$H(|a_1b_2c_3d_4|) = -3\,|a_1b_2c_3d_4|^8.$$

(4) The natural generalisation for an originating determinant of order n is thus fully legitimised, the number of requisite changes of sign then being

$$\{(n-1)+(n-2)+\ldots+2+1\} + (n-1),$$

which of course is the same as

$$(n-2) + (n-3) + \ldots + 2+1,$$

i.e.,

$$\tfrac{1}{2}(n-1)(n-2):$$

and the resulting equation is

$$H(|a_{1n}|) \cdot |a_{1n}|^n = (-)^{\frac{1}{2}(n-1)(n-2)} \cdot (n-1) \cdot |A_{1n}|^n,$$

so that

$$\begin{aligned} H(|a_{1n}|) &= (-)^{\frac{1}{2}(n-1)(n-2)} \cdot (n-1) \cdot |a_{1n}|^{n(n-1)-n}, \\ &= (-1)^{\frac{1}{2}(n-1)(n-2)} \cdot (n-1) \cdot |a_{1n}|^{n(n-2)}. \end{aligned}$$

XI.—*The Differentiation of a Continuant.*

By Thomas Muir, LL.D.

(Read January 21, 1901.)

(1) The continuant

$$\begin{vmatrix} a_1 & b_1 & \cdot & \cdot \\ -1 & a_2 & b_2 & \cdot \\ \cdot & -1 & a_3 & b_3 \\ \cdot & \cdot & -1 & a_4 \end{vmatrix}$$

is conveniently denoted by

$$\mathrm{K}\left(\begin{array}{c} \;\; b_1 \;\; b_2 \;\; b_3 \;\; \\ a_1 \;\; a_2 \;\; a_3 \;\; a_4 \end{array}\right) \quad \text{or} \quad \left(\begin{array}{c} \;\; b_1 \;\; b_2 \;\; b_3 \;\; \\ a_1 \;\; a_2 \;\; a_3 \;\; a_4 \end{array}\right);$$

and, when the elements b_1, b_2, b_3 of the variable minor diagonal are each equal to 1, a further simplification is obtained by leaving them out ;—thus,

$$\mathrm{K}(a,b,c,d) \quad \text{or} \quad (a,b,c,d)$$

stands for

$$abcd + ab + ad + cd + 1.$$

(2) The general continuant, K, of the n^{th} order is a function of not more than $2n-1$ independent variables,—n in the principal diagonal and $n-1$ in the variable minor diagonal. We thus may be expected to know the differential-quotient of the continuant with respect to an element in either of these diagonals.

Taking first the case where the element is in the minor diagonal, b_h say, we use the known identity

$$\left(\begin{array}{c} \;\; b_1 \; \cdots\cdots \; b_{h-1} \;\; b_h \; \cdots\cdots \; b_{n-1} \;\; \\ a_1 \;\; a_2 \; \ldots \; a_{h-1} \;\; a_h \;\; a_{h+1} \; \ldots \; a_{n-1} \;\; a_n \end{array}\right) = \left(\begin{array}{c} \;\; b_1 \; \cdots\cdots \; b_{h-1} \;\; \\ a_1 \;\; a_2 \; \ldots \; a_{h-1} \;\; a_h \end{array}\right) \cdot \left(\begin{array}{c} \;\; b_{h+1} \; \cdots \; b_{n-1} \;\; \\ a_{h+1} \; \ldots \; a_{n-1} a_n \end{array}\right)$$

$$+ \; b_h \left(\begin{array}{c} \;\; b_1 \; \cdots \;\; \\ a_1 \;\; a_2 \; \ldots \; a_{h-1} \end{array}\right) \cdot \left(\begin{array}{c} \;\; \cdots \; b_{n-1} \;\; \\ a_{h+2} \; \ldots \; a_{n-} \;\; a_n \end{array}\right)$$

to separate out the element in question, and thus see at a glance that

$$\frac{\partial \mathrm{K}}{\partial b_h} = \left(\begin{array}{c} \;\; b_1 \; \cdots \;\; \\ a_1 \;\; a_2 \; \ldots \; a_{h-1} \end{array}\right) \left(\begin{array}{c} \;\; b_{n-1} \;\; \\ a_{h+2} \; \ldots \; a_{n-1} \; a_n \end{array}\right).$$

In other words, the differential-quotient of K with respect to b_h is obtained mechanically by taking K, deleting b_{h-1}, a_h, b_h, a_{h+1}, b_{h+1} from it, and inserting the brackets)(instead.

Secondly, taking the case where the element is in the principal diagonal, a_h say, we use the same identity at the outset, but go a step further and alter

$$\left(\begin{array}{c} \;\; b_1 \; \cdots\cdots \; b_{h-1} \;\; \\ a_1 \;\; a_2 \; \ldots \; a_{h-1} \;\; a_h \end{array}\right) \quad \text{into} \quad \left(\begin{array}{c} \;\; b_1 \; \cdots\cdots \;\; \\ a_1 \;\; a_2 \; \ldots \; a_{h-2} \end{array}\right) b_{h-1} + \left(\begin{array}{c} \;\; b_1 \; \cdots\cdots \;\; \\ a_1 \;\; a_2 \; \ldots \; a_{h-1} \end{array}\right) a_h$$

for the purpose, as before, of separating out the element concerned, thus obtaining

$$\frac{\partial K}{\partial a_h} = \left(\begin{matrix} b_1 \;\ldots\ldots & \\ a_1 \;\; a_2 \;\ldots\; a_{h-1} \end{matrix}\right)\left(\begin{matrix} \ldots\ldots\; b_{n-1} \\ a_{h+1} \;\ldots\; a_{n-1} \;\; a_n \end{matrix}\right).$$

In other words, the differential-quotient of K with respect to a_h is obtained mechanically by taking K, deleting b_{h-1}, a_h, b_h, and inserting the brackets)(instead.

When one of the end elements is used as independent variable the result is of course simpler: thus

$$\frac{\partial K}{\partial a_1} = \left(\begin{matrix} b_2 \;\ldots\ldots\; b_{n-1} \\ a_2 \;\; a_3 \;\ldots\; a_{n-1} \;\; a_n \end{matrix}\right),$$

$$\frac{\partial K}{\partial b_{n-1}} = \left(\begin{matrix} b_1 \;\ldots\ldots\; b_{n-3} \\ a_1 \;\; a_2 \;\ldots\; a_{n-3} \;\; a_{n-2} \end{matrix}\right).$$

(3) It is thus seen that in whatever diagonal the independent variable is situated the differential-quotient of K is in general the product of two coaxial minors of K, and that when the independent variables are in the same row the differential-quotients have a factor in common. As a result of this

$$\frac{\partial K}{\partial a_h} \div \frac{\partial K}{\partial b_h} = \frac{\left(\begin{matrix} \ldots\ldots\; b_{n-1} \\ a_{h+1} \;\ldots\; a_{n-1} \;\; a_n \end{matrix}\right)}{\left(\begin{matrix} \ldots\ldots\; b_{n-1} \\ a_{h+2} \;\ldots\; a_{n-1} \;\; a_n \end{matrix}\right)},$$

$$= a_{h+1} + \cfrac{b_{h+1}}{a_{h+2} + \cfrac{b_{h+2}}{a_{h+3} + \cdots + \cfrac{b_{n-1}}{a_n}}}.$$

(4) If we differentiate $\partial K/\partial a_h$ with respect to another element of the main diagonal, that element must be in one of the two factors of $\partial K/\partial a_h$,

$$\left(\begin{matrix} b_1 \;\ldots\ldots & \\ a_1 \;\; a_2 \;\ldots\; a_{h-1} \end{matrix}\right), \qquad \left(\begin{matrix} \ldots\ldots\; b_{n-1} \\ a_{h+1} \;\ldots\; a_{n-1} \;\; a_n \end{matrix}\right),$$

and not in the other, so that the result of the second differentiation is in general to give an expression of three factors. Thus if

$$K = \left(\begin{matrix} \alpha \;\beta\; \gamma\; \delta\; \epsilon\; \zeta \\ a \;\; b \;\; c \;\; d \;\; e \;\; f \;\; g \end{matrix}\right)$$

we have

$$\frac{\partial^2 K}{\partial c.\partial f} = \left(\begin{matrix} \alpha \\ a \;\; b \end{matrix}\right) \cdot \left(\begin{matrix} \delta \\ d \;\; e \end{matrix}\right) \cdot g.$$

When the two elements used as independent variables belong to the minor diagonal, the like consequences ensue: it has to be noted however that if they be consecutive the result is 0, because when we differentiate with respect to b_h, neither b_{h-1} nor b_{h+1} appears in the result.

(5) As each term of the continuant is linear in each of the elements occurring in it,

the differential-quotient with respect to any element is the same as the cofactor of that element. Hence instead of

$$\frac{\partial K}{\partial a_h}, \qquad \frac{\partial^2 K}{\partial a_h \partial a_{h+1}},$$

we may write

$$\text{cof } a_h, \qquad \text{cof } a_h a_{h+1}.$$

(6) When the two elements used as independent variables are consecutive in the main diagonal, the result is the same as is got by one differentiation with respect to an element of the minor diagonal. For, taking $\partial K/\partial a_h$ as above, we have

$$\begin{aligned}\frac{\partial^2 K}{\partial a_h \partial a_{h+1}} &= \begin{pmatrix} b_1 \cdot \cdot \cdot \cdot \cdot & \\ a_1 \quad a_2 \ \ldots \ a_{h-1} \end{pmatrix} \begin{pmatrix} \ldots \ b_{n-1} & \\ a_{h+2} \ \ldots \ a_{n-1} \quad a_n \end{pmatrix}, \\ &= \frac{\partial K}{\partial b_h}.\end{aligned}$$

Corroboration of this is found in the fact that one of the two-line minors of K is

$$\begin{vmatrix} a_h & b_h \\ -1 & a_{h+1} \end{vmatrix} \quad \text{or} \quad a_h a_{h+1} + b_h,$$

thus ensuring that the cofactor of $a_h a_{h+1}$ is the same as the cofactor of b_h.

(7) When the continuant concerned has unit elements in the minor diagonal the rule for mechanically obtaining the differential-quotient is still simpler, the only element to be deleted being that which is taken as the independent variable. Thus if

$$K = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$$

we have

$$\frac{\partial K}{\partial a_5} = (a_1, a_2, a_3, a_4)\,(a_6, a_7),$$

and

$$\frac{\partial^2 K}{\partial a_5 \partial a_3} = (a_1, a_2) \cdot (a_4) \cdot (a_6, a_7).$$

(8) The foregoing considerations have been suggested by a curious theorem which has turned up recently in the course of an investigation connected with Hessians.

The Hessian, it will be remembered, is always axisymmetric : and, if the originating function be linear in each of the variables, its second differential-quotient with respect to any variable will be 0,—that is to say, the Hessian will be *zero-axial* as well as axisymmetric. In this case, consequently, there arises a semi-quadrate array which seems worthy of study.

(9) Taking the case where the originating function is $K(a,b,c,d)$, *i.e.* $abcd + ad + cd + ab + 1$ we have

$$\begin{vmatrix} \frac{\partial^2 K}{\partial a.\partial b} & \frac{\partial^2 K}{\partial a.\partial c} & \frac{\partial^2 K}{\partial a.\partial d} \\ & \frac{\partial^2 K}{\partial b.\partial c} & \frac{\partial^2 K}{\partial b.\partial d} \\ & & \frac{\partial^2 K}{\partial c.\partial d} \end{vmatrix} = \begin{vmatrix} cd+1 & bd & bc+1 \\ & ad & ac \\ & & ab+1 \end{vmatrix},$$

$$
\begin{aligned}
&= (cd+1)(ab+1) \;-\; bd.ac \;+\; (bc+1)\,ad\,, \\
&= abcd + ab + cd + 1 - abcd + abcd + ad\,, \\
&= \mathrm{K}(a,b,c,d)\,.
\end{aligned}
$$

Again, taking the case where the originating function is

$$\mathrm{K}\,(a,b,c,d,e,f,)$$

we have our Hessian-like Pfaffian

$$
\left|\begin{array}{ccccc}
\dfrac{\partial^2\mathrm{K}}{\partial a.\partial b} & \dfrac{\partial^2\mathrm{K}}{\partial a.\partial c} & \dfrac{\partial^2\mathrm{K}}{\partial a.\partial d} & \dfrac{\partial^2\mathrm{K}}{\partial a.\partial e} & \dfrac{\partial^2\mathrm{K}}{\partial a.\partial f} \\
 & \dfrac{\partial^2\mathrm{K}}{\partial b.\partial c} & \dfrac{\partial^2\mathrm{K}}{\partial b.\partial d} & \dfrac{\partial^2\mathrm{K}}{\partial b.\partial e} & \dfrac{\partial^2\mathrm{K}}{\partial b.\partial f} \\
 & & \dfrac{\partial^2\mathrm{K}}{\partial c.\partial d} & \dfrac{\partial^2\mathrm{K}}{\partial c.\partial e} & \dfrac{\partial^2\mathrm{K}}{\partial c.\partial f} \\
 & & & \dfrac{\partial^2\mathrm{K}}{\partial d.\partial e} & \dfrac{\partial^2\mathrm{K}}{\partial d.\partial f} \\
 & & & & \dfrac{\partial^2\mathrm{K}}{\partial e.\partial f}
\end{array}\right|
$$

$$
= \left|\begin{array}{ccccc}
(c,d,e,f) & b(d,e,f) & (b,c)\,(e,f) & (b,c,d)f & (b,c,d,e) \\
 & a(d,e,f) & ac(e,f) & a(c,d)f & a(c,d,e) \\
 & & (a,b)\,(e,f) & (a,b)df & (a,b)\,(d,e) \\
 & & & (a,b,c)f & (a,b,c)e \\
 & & & & (a,b,c,d)
\end{array}\right|
$$

$$
\begin{aligned}
= (c,d,e,f) &\left|\begin{array}{ccc}
(a,b)\,(e,f) & (a,b)df & (a,b)\,(d,e) \\
 & (a,b,c)f & (a,b,c)e \\
 & & (a,b,c,d)
\end{array}\right| \\
- \; b(d,e,f) &\left|\begin{array}{ccc}
ac(e,f) & a(c,d)f & a(c,d,e) \\
 & (a,b,c)f & (a,b,c)e \\
 & & (a,b,c,d)
\end{array}\right| \\
+ \; \cdot \; \cdot \; \cdot \; \cdot & \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \\
+ \; (b,c,d,e) &\left|\begin{array}{ccc}
a(d,e,f) & ac(e,f) & a(c,d)f \\
 & (a,b)\,(e,f) & (a,b)df \\
 & & (a,b,c)f
\end{array}\right| .
\end{aligned}
$$

Now it is easily verified that the five minor Pfaffians here are equal to

$$(a,b)\cdot\mathrm{K}, \quad ac\cdot\mathrm{K}, \quad ad\cdot\mathrm{K}, \quad ae\cdot\mathrm{K}, \quad af\cdot\mathrm{K}\,;$$

consequently the original Pfaffian

$$
\begin{aligned}
&= \mathrm{K}\cdot[\,(a,b)\,(c,d,e,f) - abc(d,e,f) + a(b,c)d(e,f) - a(b,c,d)ef + a(b,c,d,e)f\,]\,, \\
&= \mathrm{K}\cdot[\,(a,b)\,(c,d,e,f) - \{abcd(e,f) + abcf\} + \{abcd(e,f) + ad(e,f)\} - a(b,c,d)ef + \{a(b,c,d)ef + a(b,c)f\}\,]\,, \\
&= \mathrm{K}\cdot[\,(a,b)\,(c,d,e,f) - abcf + ad(e,f) + a(b,c)f\,]\,, \\
&= \mathrm{K}\cdot[\,(a,b)\,(c,d,e,f) + ad(e,f) + af\,]\,, \\
&= \mathrm{K}\cdot(a,b,c,d,e,f)\,, \\
&= \mathrm{K}^2.
\end{aligned}
$$

(10) These two cases raise the presumption that when the originating function is

$$\mathrm{K}(a_1,\, a_2,\, a_3,\, \ldots\, ,\, a_{2n})$$

the semi-Hessian of K will be

$$K^{n-1}.$$

It is clear however that before we can test this the auxiliary theorems which we have used in proving the first two cases must be generalised.

(11) The first of these auxiliary theorems is that which involves the fact that all our minor Pfaffians of the second order contain K as a factor. What the theorem itself actually is becomes more apparent if we adopt the 'cofactor' notation instead of the notation of differential-quotients: for then the instances are—

$$\left|\begin{array}{ccc} \text{cof } cd & \text{cof } ce & \text{cof } cf \\ & \text{cof } de & \text{cof } df \\ & & \text{cof } ef \end{array}\right| = \text{K}\cdot (a,b),$$

$$\left|\begin{array}{ccc} \text{cof } bd & \text{cof } be & \text{cof } bf \\ & \text{cof } de & \text{cof } df \\ & & \text{cof } ef \end{array}\right| = \text{K}\cdot ac,$$

$$\cdots\cdots\cdots\cdots\cdots;$$

or

$$\text{cof } cd\,.\,\text{cof } ef - \text{cof } ce\,.\,\text{cof } df + \text{cof } cf\,.\,\text{cof } de = \text{K}\cdot (a,b),$$
$$\text{cof } bd\,.\,\text{cof } ef - \text{cof } be\,.\,\text{cof } df + \text{cof } bf\,.\,\text{cof } de = \text{K}\cdot ac,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$

(12) The general theorem here exemplified is—

If K *be a simple continuant of any number of elements* $a_1, a_2, \ldots, a_n$, *and* $\alpha, \beta, \gamma, \delta$ *be the suffixes of any four elements taken in the order in which they occur, then*

$$\text{cof } a_\alpha a_\beta\,.\,\text{cof } a_\gamma a_\delta - \text{cof } a_\alpha a_\gamma\,.\,\text{cof } a_\beta a_\delta + \text{cof } a_\alpha a_\delta\,.\,\text{cof } a_\beta a_\gamma$$
$$= \text{K}\cdot\text{cof } a_\alpha a_\beta a_\gamma a_\delta.$$

In proof of this we note in the first place that the expression on the left

$$\begin{aligned} = & (a_1,\ldots,a_{\alpha-1})(a_{\alpha+1},\ldots,a_{\beta-1})(a_{\beta+1},\ldots,a_n)\cdot(a_1,\ldots,a_{\gamma-1})(a_{\gamma+1},\ldots,a_{\delta-1})(a_{\delta+1},\ldots,a_n) \\ & -(a_1,\ldots,a_{\alpha-1})(a_{\alpha+1},\ldots,a_{\gamma-1})(a_{\gamma+1},\ldots,a_n)\cdot(a_1,\ldots,a_{\beta-1})(a_{\beta+1},\ldots,a_{\delta-1})(a_{\delta+1},\ldots,a_n) \\ & +(a_1,\ldots,a_{\alpha-1})(a_{\alpha+1},\ldots,a_{\delta-1})(a_{\delta+1},\ldots,a_n)\cdot(a_1,\ldots,a_{\beta-1})(a_{\beta+1},\ldots,a_{\gamma-1})(a_{\gamma+1},\ldots,a_n), \end{aligned}$$

which, on account of there being two factors common to all the terms,

$$\begin{aligned} = (a_1,\ldots,a_{\alpha-1})(a_{\delta+1},\ldots,a_n)\big[& (a_{\alpha+1},\ldots,a_{\beta-1})(a_{\beta+1},\ldots,a_n)(a_1,\ldots,a_{\gamma-1})(a_{\gamma+1},\ldots,a_{\delta-1}) \\ & -(a_{\alpha+1},\ldots,a_{\gamma-1})(a_{\gamma+1},\ldots,a_n)(a_1,\ldots,a_{\beta-1})(a_{\beta+1},\ldots,a_{\delta-1}) \\ & +(a_{\alpha+1},\ldots,a_{\delta-1})(a_1,\ldots,a_{\beta-1})(a_{\beta+1},\ldots,a_{\gamma-1})(a_{\gamma+1},\ldots,a_n)\big]. \end{aligned}$$

Again, the last two terms inside the rectangular brackets here have two factors in common, their aggregate thus being

$$= (a_1,\ldots,a_{\beta-1})(a_{\gamma+1},\ldots,a_n)\left\{-(a_{\alpha+1},\ldots,a_{\gamma-1})(a_{\beta+1},\ldots,a_{\delta-1})+(a_{\alpha+1},\ldots,a_{\delta-1})(a_{\beta+1},\ldots,a_{\gamma-1})\right\},$$

which, by an extension of the general theorem used in § 2

$$= (a_1,\ldots,a_{\beta-1})(a_{\gamma+1},\ldots,a_n)\cdot(-1)^{\gamma-\beta+1}(a_{\alpha+1},\ldots,a_{\beta-1})(a_{\gamma+1},\ldots,a_{\delta-1});$$

and as this has two factors in common with the first term inside the rectangular brackets, the original expression takes the form

$$(a_1, \ldots, a_{\alpha-1})(a_{\delta+1}, \ldots, a_n)(a_{\alpha+1}, \ldots, a_{\beta-1})(a_{\gamma+1}, \ldots, a_{\delta-1})$$
$$\cdot \left\{(a_{\beta+1}, \ldots, a_n)(a_1, \ldots, a_{\gamma-1}) + (-1)^{\gamma-\beta+1}(a_1, \ldots, a_{\beta-1})(a_{\gamma+1}, \ldots, a_n)\right\}.$$

Making a second use of the above-mentioned general theorem we can substitute

$$(a_1, \ldots, a_n)(a_{\beta+1}, \ldots, a_{\gamma-1})$$

for the expression inside the double-curved brackets. The original expression is thus resolved into six factors, one of which is K, and the others

$$(a_1, \ldots, a_{\alpha-1}),\quad (a_{\alpha+1}, \ldots, a_{\beta-1}),\quad (a_{\beta+1}, \ldots, a_{\gamma-1}),\quad (a_{\gamma+1}, \ldots, a_{\delta-1}),\quad (a_{\delta+1}, \ldots, a_n),$$

the product of which, from §§ 4, 5, we know to be

$$\text{cof } a_\alpha a_\beta a_\gamma a_\delta .$$

(13) The next auxiliary theorem used in § 9 is that which affirms that

$$(a,b)(c,d) - abcd + a(b,c)d = (a,b,c,d)\,*$$

and

$$(a,b)(c,d,e,f) - abc(d,e,f) + a(b,c)d(e,f) - a(b,c,d)ef + a(b,c,d,e)f = (a,b,c,d,e,f);$$

or that

$$\text{cof } ab \,.\, \text{cof } cd - \text{cof } ac \,.\, \text{cof } bd + \text{cof } ad \,.\, \text{cof } bc = (a,b,c,d)$$

and

$$\left.\begin{array}{rl} & \text{cof } ab \,.\, \text{cof } cdef \\ - & \text{cof } ac \,.\, \text{cof } bdef \\ + & \text{cof } ad \,.\, \text{cof } bcef \\ - & \text{cof } ae \,.\, \text{cof } bcdf \\ + & \text{cof } af \,.\, \text{cof } bcde \end{array}\right\} = (a,b,c,d,e,f).$$

The general theorem which includes these concerns $(a_1, a_2, \ldots, a_{2n})$, and is

$$\left.\begin{array}{rl} & \text{cof } a_1a_2 \,.\, \text{cof } a_3a_4a_5 \ldots a_{2n} \\ - & \text{cof } a_1a_3 \,.\, \text{cof } a_2a_4a_5 \ldots a_{2n} \\ + & \text{cof } a_1a_4 \,.\, \text{cof } a_2a_3a_5 \ldots a_{2n} \\ - & \ldots\ldots\ldots\ldots \end{array}\right\} = (a_1, a_2, \ldots, a_{2n});$$

and there is a corresponding theorem for the case where the number of elements is odd, viz.,

$$\left.\begin{array}{rl} & \text{cof } a_1a_2 \,.\, \text{cof } a_3a_4a_5 \ldots a_{2n+1} \\ - & \text{cof } a_1a_3 \,.\, \text{cof } a_2a_4a_5 \ldots a_{2n+1} \\ + & \ldots\ldots\ldots\ldots \end{array}\right\} = (a_3, a_4, \ldots, a_{2n+1}).$$

(14) The two theorems may therefore be enunciated together thus:—

If cof x *denote the cofactor of* x *in the continuant* K $(a_1, a_2, \ldots, a_n)$, *then*

$$\sum(-1)^r \text{cof } a_1a_r \,.\, \text{cof } a_1a_2 \ldots a_n/a_1a_r = \text{K},$$
$$\textit{or} \ = \text{cof } a_1a_2,$$

according as n *is even or odd.*

* It should be noted that this is also a case of the *first* auxiliary theorem.

By way of proof we note that the first of the $n-1$ terms on the left is

$$(a_1, a_2)(a_3, a_4, \ldots, a_n),$$

and that therefore we may write instead of it,

$$K - a_1(a_4, \ldots, a_n).$$

Then, as the second of the $n-1$ terms is

$$-a_1a_2a_3(a_4, \ldots, a_n)$$

the aggregate of the first two is

$$K - a_1(a_2, a_3)(a_4, \ldots, a_n).$$

Similarly, the third term having the factor (a_2, a_3) in common with this aggregate, the aggregate of the first three is found to be

$$K - a_1(a_2, a_3)(a_6, \ldots, a_n).$$

This process being continued it is seen that the aggregates of odd numbers of terms follow one law, and the aggregates of even numbers another law, viz.,

No. of Terms.	Aggregate.	No. of Terms.	Aggregate.
1	$K - a_1(a_4, \ldots, a_n)$,	2	$K - a_1(a_2, a_3)(a_4, \ldots, a_n)$,
3	$K - a_1(a_2, a_3)(a_6, \ldots, a_n)$,	4	$K - a_1(a_2, \ldots, a_5)(a_6, \ldots, a_n)$,
5	$K - a_1(a_2, \ldots, a_5)(a_8, \ldots, a_n)$,	6	$K - a_1(a_2, \ldots, a_7)(a_8, \ldots, a_n)$,
. . . .	. . . .	. . . .	. . . .

From the first column we learn that when n is odd the aggregate of the $n-1$ terms is

$$K - a_1(a_2, \ldots, a_n)$$
$$\text{i.e.,} \quad (a_3, \ldots, a_n),$$

as was to be proved.

Also, from the second column it is seen that when n is even the aggregate of $n-2$ terms (*i.e.*, all the terms except the last) is

$$K - a_1, (a_2, \ldots, a_{n-1})\, a_n$$

to which if we add the last, viz., $+a_1(a_2, \ldots, a_{n-1})\, a_n$, we obtain the aggregate

$$K$$

as desired.

(15) There is however a more elementary theorem which leads easily up to this, and which can be more readily proved in the same way. It has to be noted too, that from some points of view it is of greater interest than that to which it leads. It is :—

If cof a_r *stand for the cofactor of* a_r *in the continuant* $K(a_1, a_2, \ldots, a_n)$, *then*

$$a_1 \operatorname{cof} a_1 - a_2 \operatorname{cof} a_2 + a_3 \operatorname{cof} a_3 - \ldots = K \text{ or } 0$$

according as n *is odd or even.*

To condense the proof we may take advantage of the fact that $(a_1, \ldots, a_n) = (a_n, \ldots, a_1)$, and sum the half of the terms from the beginning and the half from the end. Thus, when n is even, equal to $2m$, the aggregate of the first m terms is found to be

$$K - (a_1, \ldots, a_m)(a_{m+1}, \ldots, a_{2m}) \quad \text{or} \quad K - (a_1, \ldots, a_{m-1})(a_{m+2}, \ldots, a_{2m}),$$

according as m is even or odd: and therefore the aggregate of the last m terms is

$$-\mathrm{K} + (a_{2m}, \ldots, a_{m+1})(a_m, \ldots, a_1) \qquad \text{or} \qquad -\mathrm{K} + (a_{2m}, \ldots, a_{m+2})(a_{m-1}, \ldots, a_1)$$

according as m is even or odd: from which we see that in either case the gross aggregate is 0, as was to be proved.

Instead of using the cofactor notation 'cof' we might have taken a hint from a usage in the exposition of the theory of general determinants, viz., where the cofactor of a_{rs} in $|a_{1n}|$ is denoted by A_{rs}. Our theorem would then stand thus:—

If $\mathrm{a}_1, \mathrm{a}_2, \ldots, \mathrm{a}_n$ *be the elements of a simple continuant* K, *and* $\mathrm{A}_1, \mathrm{A}_2, \ldots, \mathrm{A}_n$ *their respective cofactors, then*

$$a_1\mathrm{A}_1 - a_2\mathrm{A}_2 + a_3\mathrm{A}_3 - \ldots = \mathrm{K} \text{ or } 0$$

according as n *is odd or even.*

Without any notation for cofactors at all, it may however be neatly written as follows:—

$$\begin{array}{l} a_1(a_2, a_3, \ldots, a_n) \\ -a_2(a_3, \ldots, a_n)a_1 \\ +a_3(a_4, \ldots, a_n)(a_1, a_2) \\ -a_4(a_5, \ldots, a_n)(a_1, a_2, a_3) \\ \quad \cdots\cdots\cdots\cdots \\ (-)^{n-1}a_n(a_1a_2a_3 \ldots\ldots a_{n-1}) = (a_1, a_2, \ldots, a_n) \quad \text{or} \quad 0, \end{array}$$

the seemingly cyclical permutation of the elements arising from the act of reversing the order of the two parts of each cofactor.

It is when stated in this last form that its suitableness for proving the theorem of § 13 is most readily recognised. Thus, to take the second instance there given, we have

$$\begin{aligned} &(a,b)(c,d,e,f) - abc(d,e,f) + a(b,c)d(e,f) - a(b,c,d)ef + a(b,c,d,e)f \\ &= (c,d,e,f) - a\{\, b(c,d,e,f) \\ &\qquad\qquad - c(d,e,f)b \\ &\qquad\qquad + d(e,f)(b,c) \\ &\qquad\qquad - e(f)(b,c,d) \\ &\qquad\qquad + f(b,c,d,e) \,\}, \\ &= (c,d,e,f) + a(b,c,d,e,f), \\ &= (a,b,c,d,e,f). \end{aligned}$$

(16) The next point to be noticed is the effect of changing (a,b) into ab, (b,c) into bc, or (c,d) into cd in the identity

$$\left|\begin{matrix} (a,b) & ac & ad \\ & (b,c) & bd \\ & & (c,d) \end{matrix}\right| = (a,b,c,d).$$

Such a change is clearly equivalent to subtracting (c,d), ad, or (a,b),—and therefore the right-hand side must be simultaneously changed into

$$\begin{array}{ll} & a(b,c,d), \\ & (a,b)(c,d), \\ \text{or} & (a,b,c)d. \end{array}$$

(17) We are now prepared to show that

$$\left|\begin{array}{ccccc} (a,b) & ac & ad & ae & af \\ & (b,c) & bd & be & bf \\ & & (c,d) & ce & cf \\ & & & (d,e) & df \\ & & & & (e,f) \end{array}\right| = (a,b,c,d,e,f).$$

For, expanding the Pfaffian in terms of the elements of the first frame-line and their complementary minors, we have

$$(a,b)\left|\begin{array}{ccc} (c,d) & ce & cf \\ & (d,e) & df \\ & & (e,f) \end{array}\right| - ac\left|\begin{array}{ccc} bd, & be, & bf \\ & (d,e) & df \\ & & (e,f) \end{array}\right| + ad\left|\begin{array}{ccc} (b,c) & be & bf \\ & ce & cf \\ & & (e,f) \end{array}\right|$$

$$- ae\left|\begin{array}{ccc} (b,c) & bd & bf \\ & (c,d) & cf \\ & & df \end{array}\right| + af\left|\begin{array}{ccc} (b,c) & bd & be \\ & (c,d) & ce \\ & & (d,e) \end{array}\right|,$$

which by § 16 is equal to

$$(a,b)\,(c,d,e,f) - ac\,.\,b(d,e,f) + ad\,.\,(b,c)\,(e,f) - ae\,.\,(b,c,d)f + af(b,c,d,e),$$

and therefore by § 14 equal to

$$(a,b,c,d,e,f).$$

(18) The effect of changing (a,b) into ab on the left of the preceding identity is to subtract

$$\left|\begin{array}{ccc} (c,d) & ce & cf \\ & (d,e) & df \\ & & (e,f) \end{array}\right| \quad \text{i.e.,}\quad (c,d,e,f),$$

and as the right-hand side equals

$$a(b,c,d,e,f) + (c,d,e,f),$$

the result is

$$a(b,c,d,e,f).$$

The other changes taken in order lead to

$$\begin{gathered} (a,b)\,(c,d,e,f), \\ (a,b,c)\,(d,e,f), \\ (a,b,c,d)\,(e,f), \\ (a,b,c,d,e)f. \end{gathered}$$

(19) In this way we reach the perfectly general theorem*

$$\left|\begin{array}{ccccc} (a_1, a_2) & a_1a_3 & a_1a_4 & \dots & a_1a_{2n} \\ & (a_2, a_3) & a_2a_4 & \dots & a_2a_{2n} \\ & & (a_3, a_4) & \dots & a_3a_{2n} \\ & & & \dots\dots & \\ & & & & (a_{2n-1}, a_{2n}) \end{array}\right| = (a_1, a_2, \dots, a_{2n}),$$

with which is connected the series of identities resulting from the change of any one of

* It is worth noting that the elements of the Pfaffian are the two-line coaxial minors of the equivalent continuant.

the hypotenuse elements from (a_r, a_{r+1}) into $a_r a_{r+1}$, the corresponding change on the right-hand being from $(a_1, a_2, \ldots, a_r, a_{r+1}, \ldots, a_{2n})$ into

$$(a_1, a_2, \ldots, a_r)(a_{r+1}, \ldots, a_{2n}).$$

(20) Now as the Law of Complementaries holds in regard to Pfaffians as well as in regard to determinants, let us apply it to find the Complementary of the general theorem just reached. Instead of each element of the Pfaffian we have to substitute its complementary minor; and as the value of the Pfaffian is $(a_1, a_2, \ldots, a_{2n})$ the complementary minor of any element is nothing else than its cofactor in $(a_1, a_2, \ldots, a_{2n})$. On the right-hand side the complementary minor of $(a_1, a_2, \ldots, a_{2n})$ is 1, and therefore we have to annex to it a power of $(a_1, a_2, \ldots, a_{2n})$ of the same degree as the altered Pfaffian on the left, viz., the degree $n(2n-2)$. We thus have the theorem

$$\left|\begin{array}{ccccc} \text{cof}\,(a_1,a_2) & \text{cof}\,a_1a_3 & \text{cof}\,a_1a_4 & \ldots & \text{cof}\,a_1a_{2n} \\ & \text{cof}\,(a_2,a_3) & \text{cof}\,a_2a_4 & \ldots & \text{cof}\,a_2a_{2n} \\ & & \text{cof}\,(a_3,a_4) & \ldots & \text{cof}\,a_3a_{2n} \\ & & & \ldots & \ldots \\ & & & & \text{cof}\,(a_{2n-1},a_{2n}) \end{array}\right| = (a_1, a_2, \ldots, a_{2n})^{n-1},$$

which is the generalisation surmised in § 10 to exist by reason of the two cases established in § 9.

(21) Now, however, that we have accomplished the purpose with which we started, it is important to notice that the fundamental theorem of the whole is that given in § 19. Any other mode of establishing it is thus of interest; and the known quantitative relation between Pfaffians and determinants suggests that by substituting determinants for squares of Pfaffians, the theorem in determinants thus resulting may be easy of proof,—in other words, that instead of proving, for example, that

$$\left|\begin{array}{ccc} (a,b) & ac & ad \\ & (b,c) & bd \\ & & (c,d) \end{array}\right| = (a,b,c,d),$$

we may seek to prove the identity resulting from this by squaring both sides, viz., the identity

$$\left|\begin{array}{cccc} . & (a,b) & ac & ad \\ -(a,b) & . & (b,c) & bd \\ -\ ac & -(b,c) & . & (c,d) \\ -\ ad & bd & -(c,d) & . \end{array}\right| = \left|\begin{array}{cccc} a & 1 & . & . \\ -1 & b & 1 & . \\ . & -1 & c & 1 \\ . & . & -1 & d \end{array}\right|^2.$$

(22) Beginning then on the left-hand side, but taking a determinant of higher order so as to leave no doubt of the generality of the process, viz., the determinant

$$\left|\begin{array}{cccccc} . & (a,b) & ac & ad & ae & af \\ -(a,b) & . & (b,c) & bd & be & bf \\ -\ ac & -(b,c) & . & (c,d) & ce & cf \\ -\ ad & -\ bd & -(c,d) & . & (d,e) & df \\ -\ ae & -\ be & -\ ce & -(d,e) & . & (e,f) \\ -\ af & -\ bf & -\ cf & -\ df & -(e,f) & . \end{array}\right|$$

let us first perform on it the operations

$$\begin{aligned} &(\text{row})_1 - \frac{a}{b}(\text{row})_2, \\ &(\text{row})_2 - \frac{b}{c}(\text{row})_3, \\ &\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ &(\text{row})_5 - \frac{e}{f}(\text{row})_6, \end{aligned}$$

and then, to preserve the skewness, the corresponding operations

$$\begin{aligned} &(\text{col})_1 - \frac{a}{b}(\text{col})_2, \\ &(\text{col})_2 - \frac{b}{c}(\text{col})_3, \\ &\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ &(\text{col})_5 - \frac{e}{f}(\text{col})_6. \end{aligned}$$

The result of this is a zero-axial skew determinant which is the square of

$$\left|\begin{array}{ccccc} \frac{(a,b,c)}{c} & -\frac{a}{b} & \cdot & \cdot & \cdot \\ & \frac{(b,c,d)}{d} & -\frac{b}{c} & \cdot & \cdot \\ & & \frac{(c,d,e)}{e} & -\frac{c}{d} & \cdot \\ & & & \frac{(d,e,f)}{f} & -\frac{d}{e} \\ & & & & \frac{(e,f)}{1} \end{array}\right|.$$

Developing the Pfaffian in terms of the elements of the first frame-line and their complementary minors, we see that it

$$\begin{aligned} &= \frac{(a,b,c)}{c}\left\{\frac{(c,d,e)}{e}\cdot(e,f) - \frac{c}{e}\right\} - \frac{a}{b}\cdot\frac{b}{c}\cdot\frac{(e,f)}{1} \\ &= \frac{(a,b,c)}{c}\cdot(c,d,e,f) - \frac{a}{c}(e,f) \\ &= (a,b,c,d,e,f), \end{aligned}$$

as was to be proved.

(23) If, on the other hand, we begin with $(a, b, c, d, e, f)^2$ we must try to transform its equal factors in such a way that the subsequent application of the multiplication-theorem may produce the left-hand member.

Writing the factors in the form

$$\left|\begin{array}{cccccc} a & 1 & \cdot & \cdot & \cdot & \cdot \\ -1 & b & 1 & \cdot & \cdot & \cdot \\ \cdot & -1 & c & 1 & \cdot & \cdot \\ \cdot & \cdot & -1 & d & 1 & \cdot \\ \cdot & \cdot & \cdot & -1 & e & 1 \\ \cdot & \cdot & \cdot & \cdot & -1 & f \end{array}\right|, \quad \left|\begin{array}{cccccc} 1 & -a & \cdot & \cdot & \cdot & \cdot \\ b & 1 & \cdot & -1 & \cdot & \cdot \\ -1 & \cdot & 1 & -c & \cdot & \cdot \\ \cdot & \cdot & d & 1 & \cdot & -1 \\ \cdot & \cdot & -1 & \cdot & 1 & -e \\ \cdot & \cdot & \cdot & \cdot & f & 1 \end{array}\right|,$$

the second being got from the first by changing the signs of the 1st, 3rd, 5th columns and then exchanging the latter with the 2nd, 4th, 6th columns respectively, we find the product to be

$$\begin{vmatrix} . & ab+1 & -a & . & . & . \\ -ab-1 & . & 2 & d & -1 & . \\ a & -2 & . & cd+1 & -c & . \\ . & -d & -cd-1 & . & 2 & f \\ . & 1 & c & -2 & . & ef+1 \\ . & . & . & -f & -ef-1 & . \end{vmatrix} .$$

This is zero-axial skew, and a fair approximation to the form required; but the further changes necessary are too many in number to be worth noting.

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MDCCCCII.

Price Thirty-two Shillings and Sixpence.

XII.—*Ice-Erosion in the Cuillin Hills, Skye.* By Alfred Harker, M.A., F.G.S., Fellow of St John's College, Cambridge; H.M. Geological Survey of Scotland. Communicated by John Horne, F.R.S. (With a Map.)

(Read 20th May 1901.)

I. General Account of the Glaciation of Central Skye.

(i.) *Introduction.*

Since the publication in 1846 of a brief but valuable memoir by J. D. Forbes, in which that author drew attention to "the traces of ancient glaciers" in the Cuillin Hills, that district has remained almost unnoticed by glacial geologists for half a century.* This neglect is doubtless attributable chiefly to the difficulty of access to the mountains, a consequence of their peculiar configuration, which in turn is closely bound up with the glacial history of the district. The present contribution is the outcome of observations made during the years 1895–1900 in mapping the central part of Skye for the Geological Survey of Scotland.† In traversing the mountains day after day throughout several successive seasons, the writer has been struck especially by the impressive evidence which they present of glacial erosion as the dominant factor in their sculpture, and to enforce this is the chief object of the present communication.

* J. D. Forbes, "Notes on the Topography and Geology of the Cuchullin Hills in Skye, and on the Traces of Ancient Glaciers which they Present," *Edin. New Phil. Journ.*, vol. xl. pp. 76–99, pl. iv., v., 1846. Among scattered notices of later date, see A. Geikie, "On the Phenomena of the Glacial Drift of Scotland," *Trans. Geol. Soc. Glasg.*, vol. i. part ii., 1863; and J. Geikie, *The Great Ice Age*; also for the adjacent Red Hills, T. G. Bonney, "On a Cirque in the Syenite Hills of Skye," *Geol. Mag.* 1871, pp. 535–540.

† Some of these observations have been briefly recorded in the *Summary of Progress* for 1897 and 1898. See also Harker, "Glaciated Valleys in the Cuillins, Skye," *Geol. Mag.* 1899, pp. 196–199. In the following pages the subject is developed more fully than would be convenient in an official publication.

Concerning the general orography and 'solid' geology of central Skye a few words will suffice in this place. The Cuillins are built essentially of a great laccolitic mass of gabbro, enclosing patches of metamorphosed basaltic lavas and traversed by countless dykes and sheets also of basic rocks. The main range, rising in many places more than 3000 feet, has roughly the form of a semicircular arc, with its concavity to the east. A lower branch ridge (Druim nan Ramh, etc.) runs S.E., enclosing the basin of Coruisk with outlet south-eastward to the sea-loch Scavaig. Further east is the Blath-bheinn range, running nearly N. to S., which is also of gabbro, but is cut off by Strath na Creitheach, which drains southward by Camasunary. The interior of the northern Cuillins is drained by the Sligachan River, which turns northward and finds an outlet in the sea-loch Sligachan. East of Glen Sligachan is a range of hills composed of granite and granophyre, the beginning of a tract of like rocks which extends nearly to Broadford. These 'Red Hills' are almost always less than 2500 feet in altitude, and they have smooth rounded forms which contrast very markedly with the bold peaks and acute ridge-lines of the Cuillins. Outside the two mountain-groups, the central portion of Skye is built essentially of basalt, in the form of greatly eroded plateaux rarely rising more than 1500 feet above sea-level. The older stratified rocks (Torridonian and Jurassic), upon which these basalts rest, are exposed only in places along the coast. The coast-line is highly irregular in outline, long sea-lochs running up in some places almost or quite to the base of the mountains.

It is greatly to be regretted that we have as yet no satisfactory map of the Cuillins, the more so since what here follows deals in great part with the detailed topography of the district. An accurate and carefully contoured map of this the finest of all the mountain-groups of Britain would be eminently interesting to the physical geographer. The original Ordnance Survey* was made at a time when much of the mountain district was considered inaccessible; and, although later issues embody numerous corrections, they still leave much to be desired. Contour-lines are drawn on the one-inch map only, and they cannot pretend to more than approximate accuracy.

(ii.) *Independent Ice-Cap of the Skye Mountains.*

We proceed to a general view of the glaciation of the Cuillins and adjacent country, as preparatory to a closer consideration of our special subject. What at once challenges interest is the fact that the Skye mountains, at the stage of maximum glaciation, sustained a small local ice-cap, round which the great Scottish ice-sheet flowed on

* The maps illustrating the present paper are, Scotland (one-inch), sheets 70 and 71; Skye (six-inch), sheets 38, 39, 44, 45, 49, and 50. A reduced copy of the six-inch map of the Cuillins, with additional names and heights, is given in No. 25 of the *Scottish Mountaineering Club Journal*, issued January 1898. For some corrections of the topography of the ridges, see a rough sketch map by C. Pilkington, pub. 1890 (Manchester); also W. Douglas, *S.M.C.J.*, vol. iv. pp. 209–213, 1897; and A. Harker, *ibid.*, vol. vi. pp. 1–13, 1900.

both sides. That the glaciation of this part of the island was strictly local, has been recognised by other observers, *e.g.* by Professor JAS. GEIKIE.* The relations are roughly exhibited upon the small sketch map (fig. 1), where the movement of the Scottish ice in south-eastern Skye and on the neighbouring mainland is laid down from information kindly communicated by my colleague Mr C. T. CLOUGH, while data concerning the north-eastern part of the area are afforded by the published sheet 81 of the Geological Map of Scotland. The south-eastern portion of the island, which includes

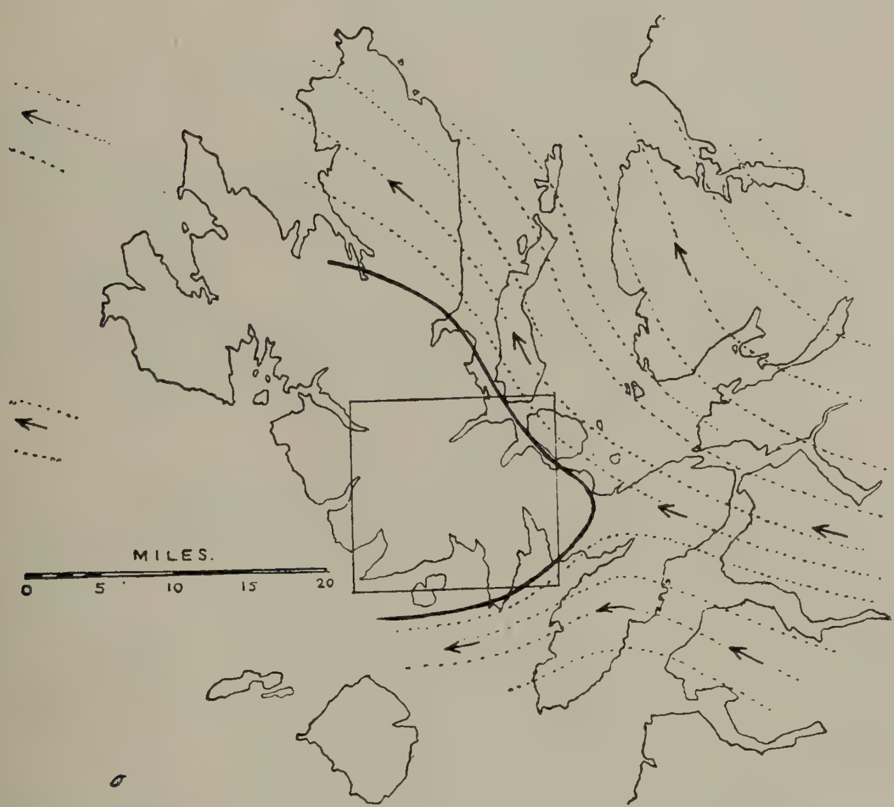


FIG. 1.—Sketch map to show the relation of the Skye ice-cap to the Scottish ice-sheet. The heavy line indicates approximately the boundary between the native and foreign ice at the stage of maximum glaciation. The arrows give the direction of movement. The rectangular area marked out is that included in the detailed map below. [The latter has subsequently been extended a little farther, both eastward and westward.]

few considerable elevations, was completely overridden by ice from the mainland of Scotland; but the ice generated upon the Cuillins and the Red Hills was always powerful enough to defend its own small territory against the Scottish invasion.† Of this we have ample proof both in the direction of the glacial striæ and in the absence

* "The lofty Coolin Mountains of Skye . . . . formed of themselves a centre of dispersion, but the northern parts of the island were overflowed by the ice that crept out from the great glens of Ross."—*The Great Ice Age*, p. 83 of third edition, 1894.

† It would perhaps be more accurate to say that if, at an early stage, the Scottish ice did obtain a footing among the Skye mountains, it has left absolutely no trace of its occupation; and the episode, if it ever occurred, is not to be reckoned with as a factor in the glaciation of the area as it is now to be studied.

of foreign boulders from the area. The exceptions to this latter generalisation are of a kind which go to emphasize the rule. Boulders of rocks foreign to the district (which farther S.E. occur at all altitudes) are found in central Skye only near the shore; usually about high-water mark, but occasionally up to 50 or even 75 feet above sea-level. It is to be noted that occasional relics of the 'hundred foot' raised beach prove that the land stood lower by that amount about the close of the glacial period. Since these foreign erratics are never embedded in the boulder-clay, but lie exposed on the surface, there is no difficulty in supposing that they have been transported by floating ice at a late stage of the Glacial period.*

Both the Cuillins and the Red Hills afforded gathering-ground for the ice. The latter, though less lofty, are not less extensive than the former; and it appears from the thick accumulations of drift on the edge of the granite tract, and from the wide dispersal of granite boulders, that this group of hills played, in some respects, almost as important a part as the other. The nature of the granite of the Red Hills, however, does not lend itself to the preservation of glacial scorings, while the generally uniform character of the rock makes it impossible to trace the movement of the ice in detail by the distribution of boulders. For these and other reasons it is much less easy to obtain precise data in the Red Hills than in the Cuillins with their adjacent basaltic tract, and it is to these latter that we shall confine our attention. It is not to be understood that the Cuillins and the Red Hills were in any sense two distinct centres of glaciation. They formed together a single gathering-ground about 12 miles long in an E. to W. direction, with a breadth of 5 or 6 miles and an area of roughly 40 square miles.

During the maximum glaciation the central part of Skye was not merely a feeding-ground for glaciers: it carried a true ice-cap, under which the mountains were wholly buried. The evidence of this is cumulative, and is implicitly involved in much of what follows. We may note in this place, however, that such a conclusion appears inevitable from the consideration that the local ice was able to withstand the pressure of the ice-sheet from the mainland. It is certain that throughout a long time the two were in equilibrium along their line of confluence, indicated approximately in the small sketch map (fig. 1). Here the thickness of the Skye ice must have been equal to that of the Scottish, *i.e.*, probably not less than 3000 feet. A very moderate rate of rise from here towards the mountains would suffice to carry the surface well above the highest summits. The ice was presumably thickest over the broad double strath which divides the Cuillins on the west from Blath-bheinn and the Red Hills on the east, and is formed by the lower portions of the Camasunary and Sligachan valleys, the one running south and the other north. The watershed dividing these two valleys

* This remark applies to the occasional boulders, some of large size, found in places along and above the sea-lochs. Where the present coast-line lies near what was the boundary of the Scottish ice, as in many places between Broadford and Loch Ainort, we find on the beach more numerous fragments of foreign rocks, doubtless washed out of the ground-moraine of the Scottish ice-sheet.

is only 250 feet above sea-level, and over this point we may conceive the summit of the ice-cap to have been situated. Some such supposition is necessary to account for the enormous volume of what we may style the interior ice-drainage of the Cuillin district. The ice-streams which found outlet by the three principal interior valleys already mentioned—viz., Coruisk, Camasunary, and Sligachan—were clearly of far greater volume than those which drained the exterior valleys of the Cuillins. The Sligachan ice-stream, for instance (see map below), on emerging from its valley, spread out fan-like through an angle of at least 120°, crossing several minor watersheds and over-riding the hills (sometimes 1300 or 1400 feet high) as well as the lower ground. Its left wing swept round the northern end of the Cuillins into the head of Glen Brittle, penning into the narrow space between there and the mountains the ice from all the northern and north-western corries. Its right wing, moving northward, was for some distance strong enough to prevent the Scottish ice (with that from the eastern Red Hills) from encroaching upon the coast of Skye. All this bespeaks a very great thickness farther back, in the comparatively restricted valley and on the central watershed. Other features in the movement of the ice of the Cuillins point to the same conclusion, and we shall see that the boulders in the drift accumulations afford strong confirmatory evidence.

(iii.) *Movement of Ice during the Great Glaciation.*

The movement of the ice in and immediately around the Cuillins during the stage of maximum glaciation is sufficiently indicated on the large map below. The chief data are the directions of striæ on rock surfaces and the distribution of boulders of recognisable rock-types. These two criteria supplement one another, the former being of most service among the mountains and the latter on the lower ground; while various other circumstances, such as the moulding of exposed crags, afford additional information. The striæ necessarily give the direction of movement of the lower layers of the ice only, and we shall see that this is also true in great measure of the dispersal of boulders.

It is seen that within the mountain area proper, the natural outward flow was, in general, closely guided, as regards the lower layers of the ice, by the form of the ground. The main ridge-line of the Cuillins and the higher parts of the principal branch ridges everywhere acted, for these lower layers, as an ice-shed. Some of the branch ridges, however, were over-ridden. The most striking example of this is afforded by the ice from Lota Corrie and the upper part of Harta Corrie. These form the interior basin of the northern Cuillins, and now supply the head-waters of the Sligachan River, which turns through a complete semicircle before running northward to Sligachan. The ice-drainage took a more direct line, and found an outlet southward, a large part of it crossing obliquely the ridge of Druim nan Ramh into the Coruisk basin. In following this course part of the lower layers of the ice would have to face an upward gradient of about 1 in 4 for a distance of nearly three-quarters of a mile. Such behaviour would be incomprehensible in a valley glacier, but it is intelligible on the supposition that the Sligachan valley near Loch Dubh watershed was occupied by ice extending higher than the summits of the Cuillins.

Immediately outside the mountain area proper, a new and potent factor came into play, viz., the pressure of the Scottish ice-sheet coming from the east. Hence we find a sharp westward diversion of the great ice-streams from the Sligachan, Camasunary, and Coruisk valleys and of the other branches, such as that from the east side of Blath-bheinn. The interior ice-drainage in turn bore back that from the outer side of the Cuillins, as already remarked, so that, with increasing distance from its source, the movement of the native ice conformed more and more to that of the great Scottish ice-sheet. In the Red Hills the relations must have been more peculiar, for there some important valleys opened directly upon the flank of the invading Scottish ice. It is probable that at the climax of glaciation no point in the Isle of Skye rose above the ice. It is certain, at least, that for many miles from the Cuillins hill and valley were alike buried, and the form of the ground exerted only a very partial control over the direction of flow. The mid-stream line of the ice from Glen Sligachan crossed three watersheds in its nearly semicircular course from Sligachan to Loch Eynort; while the left wing of the same ice-stream found its way over the pass Bealach a' Mhàim, to the north of Bruach na Frithe, part of the base of the stream rising for this purpose about 1000 feet in a distance of three miles.

As already remarked, the direct evidence concerning the direction of flow of the ice is applicable to the lower portion only. It is not improbable that the upper layers followed in some places a somewhat different course. In the mountain area their movement would be less directly influenced by the configuration of the land-surface, and might more nearly realise the ideal radial outflow. Elsewhere the direction of movement might cross a valley occupied by ice either stagnant or flowing down the valley. There is reason to believe that something of this kind occurred at one time in the case of Loch Sligachan. Since in discussing glacial erosion we are concerned with the basal portion of the ice only, it is not necessary to pursue this question.

The foregoing account concerns the principal glaciation of the area only. Here, as in some other parts of Britain, there was a later and minor glaciation, taking the form, not of an ice-cap, but of glaciers occupying the valleys. At this stage the obstruction offered by the Scottish ice-sheet had been removed, and the Skye ice was free to follow a course more in accordance with the local topography, as is partly shown by the second set of arrows on the map.

II. Physical Features of the Cuillin Hills as a typical district of Ice-Erosion.

(iv.) *General Considerations.*

Viewing ice-work in its dual aspect—destructive and constructive—we may expect to find beneath an independent ice-cap, an inner area of glacial erosion and an outer area of glacial accumulation. Allowing for a broad intermediate belt, in which the two processes have operated either successively or simultaneously side by side, we may recognise an approximate partition of this kind in the central part of Skye. The presence of ice-worn and striated surfaces does indeed show that no part of the area surveyed was quite beyond the province of glacial erosion; but with increasing distance from the mountains these signs become less frequent, while the mantle of drift becomes more persistent and uniform. In the mountain district itself, on the other hand, the drift disappears, and the evidence of important glacial erosion is displayed in the most striking manner. In this generalised statement, and still more in the detailed features to be described, we have a further confirmation of our supposition that the ice attained its greatest thickness over the mountain area proper. It cannot be doubted that, other conditions being the same, erosion would be most active at places where the pressure at the base of the ice was greatest, and the pressure at different places would bear approximately a direct ratio to the thickness of the ice.

In discussing the phenomena of glacial erosion it is, then, to the Cuillins that we must turn as the principal theatre of operations. Several causes contribute to make this group of mountains a model of a well-marked type of glaciated topography. Highly complex as it is in detailed structure, it may be broadly regarded as carved out of a single unbroken rock-mass—viz., a great laccolite of gabbro. Another element of simplicity arises from the independence of this centre of glaciation. The ground has not been over-ridden, as in some other districts, by a foreign ice-sheet crossing ridge and valley indifferently. The pre-Glacial surface-relief was strongly marked, and the movement of the ice was guided, with few exceptions, by the form of the ground, so that it exercised throughout the whole time a *cumulative* effect as regards developing the characteristic forms in their simplest expression. It is a further advantage that the actual shape of the ground is everywhere clearly exhibited. The mountains themselves are of perfectly naked rock; the same is true of all the higher corries, except in so far as they are encumbered with screes; and even in the lower corries and main valleys the drift is never so thick as to obscure the true form. Again, it is to be remarked that the effects of ice and frost-action remain practically without modification by later agencies, the sum total of post-Glacial erosion being almost a negligible quantity.* Finally, it is

* See Harker, "Notes on Subaërial Erosion in the Isle of Skye," *Geol. Mag.* 1899, pp. 485–491.

essential to some of the considerations to be adduced below to note that the pre-Glacial drainage-system of Skye was a fully matured one. The igneous rocks are of Eocene age, and perhaps in part later, but they had undergone an enormous amount of erosion during the latter half of Tertiary time, prior to the advent of the Glacial epoch. Bearing in mind that, under the conditions now existing, erosion is practically at a standstill, it is difficult to resist the inference that during the carving out of the pre-Glacial valleys the land stood somewhat higher than it does to-day. At the close of the glaciation it stood about 100 feet lower than at present. These and other considerations may suffice to assure us that at the time when glacial conditions were initiated the land had not experienced any recent elevation: the drainage-system was a fully established one, with valleys adapted to the streams which flowed in them and with the normal adjustment of tributaries to principal valleys. The point is of importance in comparing the types of topography due to ice- and water-erosion respectively, for some of the most instructive points of contrast do not hold good in their full degree unless this condition is realised.

It is part of the plan of the present contribution to confine attention to the special phenomena of the selected area, which, as has been indicated, has peculiar claims to be regarded as a type, and accordingly few references will be made to other districts. Still less is it within our province to discuss in its generality the much-vexed question of the degree of importance to be attached to ice as an erosive agent. Having regard to the mechanical element in erosion only, it is manifest that a sand-grain gripped in the sole of a glacier, or of an ice-sheet thousands of feet in thickness, must be incomparably more efficient as a graving-tool than the same grain rolled along the bed of a stream. The question, therefore, from the *a priori* point of view, turns upon the *rate* of working and the *duration* of the requisite conditions. As regards the former point, it is to be observed that, where a groove has been cut in a rock-surface by abrasion by an individual sand-grain dragged along it, the time required to cut the groove was obviously the time taken by the grain in travelling the length of the groove. Assigning even a low rate of flow to the ice, and allowing for the sand-grain lagging behind the ice in its movement, this consideration still suggests that the removal of material thus effected by ice well supplied with rock-débris in its lower layers must be a rapid process as compared with anything that can be effected by the agency of running water. The duration of the Ice Age is a question upon which we cannot enter in this place.

It is beyond doubt that the carving out of the mountain and valley system of the Cuillins is, as regards its broad outlines, the result of aqueous erosion during the latter half of Tertiary time; but it is no less certain that the actual details of the relief, as we now see them, are to be credited to the action of ice and frost during the Glacial period. If we have regard to the total amount of material removed, we must recognise water-erosion as the chief factor in the result; but from the point of view of earth-sculpture it is to glacial erosion that we must assign the more important *rôle.* The latter agent, following upon the former, has replaced the forms characteristic of aqueous erosion by those proper to itself; and it is in the peculiar topography thus developed that we find the most convincing proofs of the important part played by glacial erosion in this district.

The most palpable evidence of the abrasive power of ice, fortified by included débris, is seen in the rounded, grooved, striated, and polished rock-surfaces throughout the gabbro mountains. Excluding only the higher parts of the summit-ridges, where this characteristic appearance has been obliterated by subsequent frost-weathering, it is scarcely too much to say that almost every square foot of the surface bears in this way the stamp of glaciation. Very striking are the shores of Loch na Creitheach and Loch Coruisk and the sea-loch Scavaig, localities known to many tourists; but an equally remarkable display is seen on the floor of any of the higher corries or on a steep smooth mountain-slope such as the west faces of Blath-bheinn, Sgùrr na Stri, and Druim nan Ramh. The same smoothing, fluting, and polishing is found on the vertical walls of gullies and on the undercut and overhanging rock-surfaces, which are not uncommon in some parts of the Cuillins. It is clear that the ice has been in close contact, throughout its whole extent, with the subjacent rocks, and has forced its way into hollows and openings, vertically and horizontally, in a fashion which argues effective plasticity in its lower layers. The conditions were totally different from those which obtain beneath an Alpine glacier near its termination.

In this connection it is instructive to turn from the mountains proper to the belt of country a little beyond them, where the ice has evidently had much less erosive power. Here we find that the bottoms of certain deep gorges have escaped glacial erosion. The best example is the gorge of Allt Coire na Banachdich, just below Eas Mòr, where for some distance the gabbro is so rotten that it can be dug with a spade. Something similar is seen in the gorge of Allt a' Coire Ghreadaidh. These few places preserve the only relics in the district of the pre-Glacial weathered surface.

Another difference between the mountains and the bordering tract is seen in the form of the roches moutonnées. In the sub-montane belt these exhibit the well-known contrast between "Stossseite" and "Leeseite," but among the mountains each knoll and ridge is commonly as well rounded and polished on the lee as on the weather side. It is not to be supposed, however, that in the former case the craggy shape of the lee side is an indication that ice-erosion has not operated on it: only the mode of operation was different, taking the form of fracture instead of abrasion. This point will be considered later.

The impression of a smooth rounded outline upon every prominence, implying as it does the removal of a considerable amount of material, is enough to show that the work of the ice was something more than a mere excoriation of the surface. It is very far, however, from affording a measure of the actual amount of glacial erosion. An idea of this is to be gained only by an analysis of the physical features of the mountain-district, distinguishing those due to glacial from those due to aqueous action. In the writer's opinion, such analysis is to a great extent practicable, and an essay towards it is offered in the following divisions of this section. The mechanics of ice in bulk presents in its entirety a difficult physical problem, as yet unsolved. It follows that we have no basis for a full *a priori* discussion of the mechanism of glacial erosion, and the attempts hitherto made on this line are necessarily inconclusive. M'Gee's investigation,* for example, involves in many parts conflicting elements, the relative magnitudes of which are not known. But although we have no working theory of ice-erosion, we have a theory of water-erosion which is complete in most essentials and is amply justified by the results to which it leads. It supplies us with most important laws built up on a few simple principles. These fundamental principles are proper to water but alien to ice, and this must be true of the consequences deduced from them. Thus, although it may be difficult to lay down *a priori* the laws of ice-erosion, it is not difficult to see in many cases how they must differ from the laws of water-erosion; and different laws will find their expression in different topographic forms. This is the point of view to be adopted here. The differences in question fall conveniently under five heads.

(v.) *Independence of Physical Features and Geological Structure.*

A study of the actual topography of the Cuillins shows that, under the conditions that there prevailed, ice-erosion is controlled in a much less degree than water-erosion by lithological differences and geological structure.

One consideration which would lead us to anticipate this difference is sufficiently evident; while in the case of water-action the process is effected by the co-operation of chemical with mechanical disintegration, in ice-action the chemical factor is minimised or wholly in abeyance. We shall have to notice below, other circumstances which conduce to simplicity of form on glaciated land-surfaces, and so tend to overrule the expression of geological constitution in surface-relief.

The general principle propounded is beautifully illustrated in the Cuillins. This group of mountains, remarkably simple in constitution in a broad view, is in detail highly complex. On the summit-ridges and on many parts of the higher slopes this complexity of structure expresses itself in the form of the ground. Many of the dykes give rise to gullies and notches, and the parallel intrusive sheets of dolerite impart something of a step-like character to the slopes. All this becomes most marked in those places which have suffered most from frost-weathering after the epoch of ice-moulding. On the floors of the corries, and in the main valleys, *i.e.* in places where the maximum effects of glacial erosion have been experienced and the resulting surface remains intact, the appearance is very different. Here we see gabbro, basaltic lavas,

* W. J. M'Gee, "Glacial Cañons," *Journ. of Geol.*, vol. ii. pp. 350–364, 1894.

dykes, and sheets eroded down to a common level and figuring upon a single smooth flat rock-surface. Throughout the interior of the mountain area indeed, all the conspicuous features—precipices, ridges, barriers, basins, etc.—are carved out of the rock-complex in a fashion wholly irrespective of lithological differences or geological structure. Only on the outskirts of the mountains do we find exceptions, which thus go to emphasize the rule. This is seen, for instance, on the southern face of Gars-bheinn, and still more clearly in the form of the corries to the east of that mountain (see Ordnance Map), where the juxtaposition of gabbro and basaltic lavas has given rise to some bold escarpments. Here, where glacial erosion has played a less dominant part in shaping the existing land-surface, geological structure has asserted itself in the usual manner.

(vi.) *Forms of the Valleys, and Relation of Tributaries to Principal Streams.*

We may now proceed to recall some of the more obvious differences between ice and water which may be expected to aid us in discriminating the effects produced by these two agents of erosion. Running water in a valley concentrates its direct action in great part upon certain narrow channels, viz., the main stream and its tributaries. The courses of the tributaries, and even of the main stream in different parts of its course, make various angles with the general direction of the valley. If now we suppose the valley to become occupied by ice moving down it, the conditions are greatly changed. A glacier fills a large part of the width of the valley ; an ice-cap, as in the Cuillins at the principal glaciation, more than fills the whole valley. Moreover, we may, as a first approximation to the truth, consider this body of ice as moving down-stream *as a whole.* The differential movement which takes place within the mass imports a considerable modification of this broad view, but does not destroy its validity for our argument : though ice is not a ‘ rigid ’ substance, it is rigid in comparison with water.

This different manner in which the eroding force is applied must produce results which can in part be foreseen. We must expect a tendency to *simplification* of the form of the valley in ground-plan and in cross-section (the longitudinal profile falls under other rules). Lateral erosion—unfettered here by any consideration of ‘ base level ’—will come into play to reduce or destroy projecting spurs, to straighten curved reaches, to plane away the subsidiary ridges which separate adjacent minor tributaries, etc. ; and the result of such action, if continued, will be to widen the floor of the valley and to straighten and steepen its walls.

The valleys of the Cuillins are straight in ground plan,* and this straightness extends also in very great measure to the slopes which bound them. Transversely they show a flattening of the floor and a steepness of the bounding walls, which give

* Harta Corrie is the only exception, and here we have already seen that, at the maximum glaciation, the ice when it reached the curve no longer followed the direction of the valley.

the characteristic canal-shaped or 'U-shaped' cross-section (figs. 2 and 5) recognised in many other glaciated areas. The persistence of this form of cross-section along the middle and principal part of the valley's course, in conjunction with the straightness and smoothness of the sides, already adverted to, results in something like a semi-cylindrical shape in this portion of the valley, when most typically developed. Since it is difficult to visualise the actual shape of a valley from a contoured map alone, a rough attempt is made in figs. 3 and 4 to render it in another fashion. The appearance as seen on the ground is quite marked. One may describe it roughly by saying that the valleys give the impression of trenching unduly upon the dividing ridges and being too large, and in particular too wide, for the district.

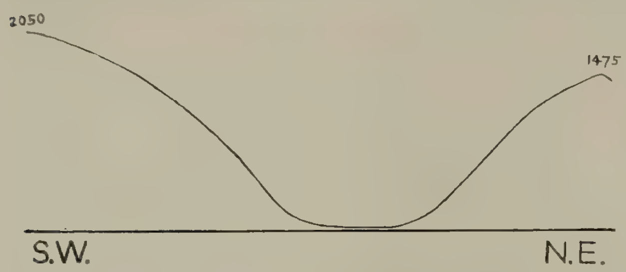


FIG. 2.—Transverse section of the Coruisk valley about ½ mile above the head of the loch; scale, about 2¼ inches to a mile. The horizontal line shows the sea-level (O.D.).

This and the other sections are drawn from the contoured Ordnance Map with the aid of additional altitudes taken with the aneroid. They are drawn to true scale as regards horizontal and vertical distances, but they cannot pretend to close accuracy in detail. [The original drawings have unfortunately been reduced on no settled scale.]

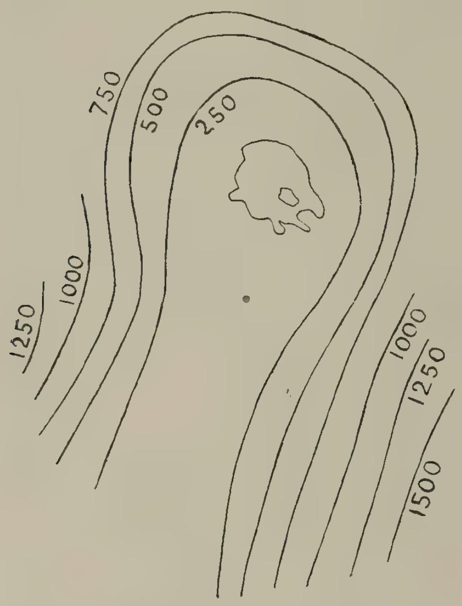


FIG. 3.—Ground Plan of Coir' a' Ghrunnda.

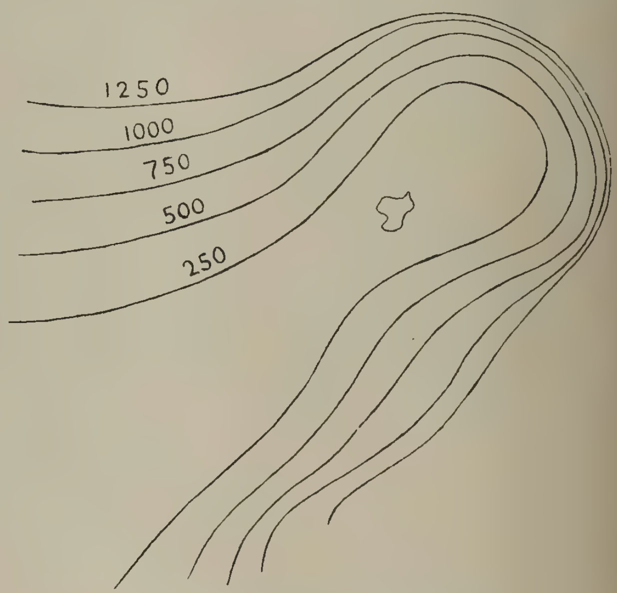


FIG. 4.—Ground Plan of Coire Labain.

The lines in these two figures are not contour-lines of the ordinary kind. They are intended to show elevations, not above sea-level, but above the neighbouring floor of the valley; thus eliminating the inclination of the valley-floor itself. The figures bring out well the form of the cirque in which any of these valleys heads, as described below. Of the two, Coir' a' Ghrunnda has a very elevated cirque and a semicylindrical middle course; Coire Labain has a larger cirque at a somewhat lower level, and a rather more open valley below. Scale, about 3 inches to a mile.

It remains to consider the tributary glens in their relation to the trunk valley to which they belong. In so far as the ice filling the latter moves as a solid body, it must tend to pond back the smaller tributary ice-streams, at least when these debouch in directions making high angles with the trend of the main valley. We must expect that this tendency will become effective in varying degree, depending inversely upon the magnitude of the tributaries. The smallest of these will be completely blocked; the larger may be only partially checked, or at least they will surrender their freedom of exit later in the waxing stage of glaciation and recover it sooner in the waning stage; the largest will be able to assert themselves throughout. For these reasons—apart from the consideration of the varying thickness of the ice as affecting the rate of erosive action—we must look for much more erosion in the main valley than in the smaller tributary glens. Moreover, while the bottom of one of these latter is occupied by ice which is not effectively eroding, the upper parts of its bounding slopes may be reduced by the action of ice moving athwart the direction of the glen.

These points seem to be illustrated in the Cuillin district. In the first place the sides of the main valleys are formed in some cases by long straight mountain-slopes broken only by narrow gullies. The west face of Blath-bheinn and the south-west slope of Druim nan Ramh are good examples, in Strath na Creitheach and Coruisk respectively. The ice-worn surfaces inside these gullies prove that they are not of post-Glacial age; and they seem to be the relics of larger glens which have been almost obliterated, just as an inscription cut in a stone slab is obliterated by grinding away the surface.* The walls of the gully never curve towards the wall of the main valley to form a continuous surface with it. The pre-Glacial glens in these places, though larger than the gullies which now represent them, were evidently of very small dimensions. They must have been wholly aborted as channels during the great glaciation. In this connection it is instructive to contrast the slope of Druim nan Ramh overlooking Coruisk with the opposite side of the same main valley.

A different and very interesting case occurs where a tributary glen of larger dimensions than the preceding, but still small in comparison with the trunk valley, has come steeply down to join the trunk in a direction making a high angle with it. Here the lower part of the tributary glen may have been greatly reduced or wholly obliterated by the planing process, while its head has been on the other hand developed into a cirque in the manner to be discussed later. Of this, numerous examples, with various modifying circumstances, are found in the Cuillins, the most remarkable being Coir' an Lochain, overlooking Coruisk. Here we have a corrie 500 yards across and going back 1000 yards, the tarn on the floor of the corrie being at an altitude of over 1800 feet. The stream draining this, on emerging from the corrie, plunges abruptly over a steep slope, fully 1000 feet high, consisting of smooth glaciated rocks in which there is, for the most part, no sort of channel. The striæ which everywhere mark this steep slope, and may be seen through the water which cascades over it, are parallel to the direction of Coruisk and at right angles to the

* Something comparable with this has been described by Dr W. T. Blanford in the Great Glen of Scotland ("On a Particular Form of Surface, apparently the Result of Glacial Erosion, seen on Loch Lochy and Elsewhere," *Quart. Journ. Geol. Soc.*, vol. lvi. pp. 198–203, pl. ix., 1900). There are, however, considerable differences between the two cases.

tributary stream itself. Higher up, however, we find the striæ emerging from the corrie in the natural direction and then curving away into the direction of the Coruisk valley. In this case it seems impossible to doubt that there has been a considerable pre-Glacial valley, the lower half of which has been completely planed away. The Coir' an Lochain ice was too powerful a body to be ponded back, and it accordingly swept out to join the main stream; but it did this *at a high level*, the lower part of the valley thus ceasing to operate as a channel and being thereupon gradually ground out of existence by the action of the Coruisk ice streaming directly across it. Only in this way can we explain the situation of this and other high-level niches in the Cuillins, though the amphitheatral form to which they so generally tend involves another element not yet considered.

What has just been described as illustrated by Coir' an Lochain has much in common with what Davis,* following Gilbert, has termed 'hanging valleys.'

These are simply tributary valleys which debouch at levels considerably above the floor of the trunk valley, into which they therefore drain by cascades of some height. They seem to be a characteristic feature of some glaciated districts, and are explained by the greater amount of ice-erosion in the main valley as compared with its tributaries. If the pre-Glacial stream of Coir' an Lochain had been larger, and its gradient more moderate, something more closely comparable with the typical hanging valley might have resulted; but this and other examples which might be cited in the Cuillins are better described as 'hanging' cirques, or, as we have already called them, high-level niches. Our small area does not comprise many tributaries of more than very moderate dimensions. The best example is Tairneilear, which debouches some 250 or 300 feet above the floor of Coir' a' Mhadaidh or Coire na Creiche, and may be taken as a fairly typical hanging valley.

Professor Davis lays stress especially upon the deepening of the main valley. Either deepening or widening may conceivably bring about the result, though the former at a greater cost of total erosion. In the middle and lower courses of valleys like those of the Cuillins we have already seen reason to attach special importance to glacial erosion in the lateral direction, but we shall see that in some circumstances there has also been a considerable amount of erosion in the vertical sense.

(vii.) *Cirques: Character of Ridges.*

We come next to what is perhaps the most striking characteristic of the surface-relief of the Cuillins, viz., the evidence of excessive erosion in the upper parts of all the valleys. Quite apart from what has been described in other countries, we should be led by general considerations to connect this peculiarity with glacial erosion. As we

* W. M. Davis, "Glacial Erosion in the Valley of the Ticino," *Appalachia*, vol. ix. pp. 136–156, pl. xv., xvi., 1900; "Glacial Erosion in France, Switzerland, and Norway," *Proc. Bost. Soc. Nat. Hist.*, vol. xxix. pp. 273–322, pl. 1–3, 1900.

have already remarked, the efficiency of erosive action at the lower surface of ice well supplied with rock-débris may be expected to increase, *cæteris paribus*, with the pressure, and therefore with the thickness of the superincumbent mass. A glacier, unlike a river, comes into being full-grown ; and the ice-cap which covered the Skye mountains during the great glaciation was, as we have urged, thickest towards the centre.

As we pass up any of the valleys of the Cuillins, we find that the U-shape becomes more pronounced and the concave sweep of the transverse section more regular (fig. 5). Further, the valley does not contract towards its head, but shows a decided expansion (figs. 3 and 4). These points may be made out to some extent on the contoured Ordnance Map; and even in our map it is noticeable that the concave portions of the 2000-feet line have the sweeping curve of bays. In some places these contrast with sharper indentations of the 1000-feet line, but this point would come out more clearly if the line corresponding with 1500 feet were drawn.

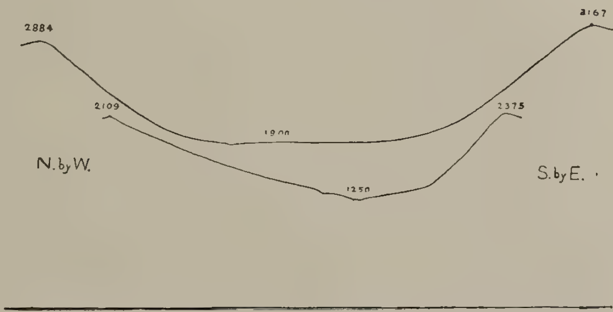


Fig. 5.—Two transverse sections of Coir' a' Ghreadaidh ; scale, about $2\frac{1}{2}$ inches to a mile.

The expansion, becoming more marked towards the head of the valley, culminates in a cirque or typical corrie in the strict sense.* In vertical section the simple cirque presents a flowing concave curve up to the actual crest-lines of the bounding ridges, and the form is the same in longitudinal as in transverse section (fig. 6). More accurately, these terms cease to have any meaning ; for there is no longer any

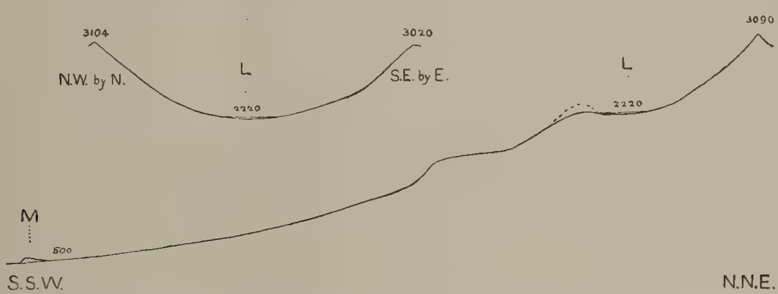


Fig. 6.—Longitudinal profile of the floor of Coir' a' Ghrunnda ; scale, about $2\frac{1}{2}$ inches to a mile. Also another section across the cirque which forms the head of the valley.

L is the tarn at the bottom of the cirque ; M the crescentic moraine opposite the mouth of the valley, described below.

'Thalweg,' or rather the whole surface of the cirque may be regarded as the Thalweg. If part of the water which courses down the slopes collects into gullies or other channels, these are to be regarded rather as incidents not essential to the typical cirque. This presents the same sweeping concave form in horizontal as in vertical section, and may be pictured simply as the half, or rather more than the half, of a hemispherical bowl.

* Gaelic *coirè*, a cauldron : the term is, however, loosely applied in common usage to the whole valley.

A common incident of the cirque is a small rock-basin on its floor, occupied of course by a tarn. The altitudes of the principal examples in the Cuillins are as follows :—

Coir' a' Bhasteir, ...	2250 feet.
Coir' a' Ghrunnda, . . .	2220 ,,
Coir' an Lochain, ...	1815 ,,
Coire Labain, ...	1805 ,,

These high-level tarn-basins, a consequence of that excessive erosion in the head portion of the valleys to which we have adverted, are of different significance from the elongated lake-basins to be noticed below. They are of small dimensions, and approximate to the circular form. Though we have not sounded any of them, it is clear that they are of comparatively small depth. In a cirque approaching most nearly to the ideal form the tarn occupies the exact centre; but if there is any tendency to elongation in the direction of the valley, the tarn is found a little further down (compare figs. 3 and 4).

The concave upward sweep of the cirque continues, as has been said, to the actual crest-line. Hence arises the characteristic *cuspate* form in cross-section of the main ridge of the Cuillins, and the adjoining portions of the chief branch ridges; a form illustrated by a rather extreme example in fig. 7. In every place the ridges are very narrow and the slopes very steep. Here we must make a remark of considerable importance in this connection. The erosion in the higher corries of which we have been speaking was of course dependent upon an adequate supply of abrading material at the under surface of the ice. On the principal ridges, which we have shown acted as ice-sheds, ice-erosion necessarily failed for want of a tool to work with. Hence, as the excavation of the cirques proceeded, the dividing ridges were left standing out in more and more salient relief. Thus arises that knife-edge form of the ridges which makes them the delight of climbers. Hence, too, the peculiarly unbroken character of the main ridge as a whole. From Sgùrr nan Gillean to Gars-bheinn it extends seven miles, and nowhere presents anything that can be called a pass in the usual sense. Although only the higher peaks rise above 3000 feet, the ridge never falls below 2500. A stranger ascending one of the valleys, where he looks for a pass at the head, is confronted by a precipitous rock-face, viz., the back wall of the cirque.

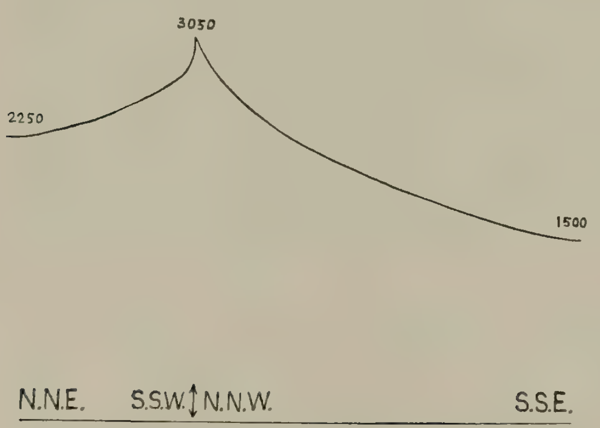


FIG. 7.—Section across the Basteir ridge from Coir' a' Bhasteir on the north to Lota Corrie on the south; scale, about $4\frac{1}{2}$ inches to a mile.

Just as each portion of the main watershed—once the ice-shed—is merely the cuspate septum left by the excavation of two opposed cirques, so each culminating peak is in general the triangular pyramid left in the midst of three such cirques. The form is not strictly pyramidal, for the cirques are concave in horizontal as well as in vertical section, so that the ground plan of each peak comes to have the outline of a tricuspate curve which is highly characteristic. This is well shown by the 3000 feet contour-line on the map given below. Bidein Druim nan Ramh and Sgùrr a' Ghreadaidh, standing each in the midst of four corries, are four-cusped instead of three-cusped. The former of these mountains and the subsidiary spurs of the latter do not reach 3000 feet, but their shapes are partly indicated on the map by the 2000-feet line.

(viii.) *Longitudinal Profile of Valleys: Lake-Basins.*

We have next to examine the form of the valleys in longitudinal section along the actual main drainage-line or 'Thalweg'; and we observe at the outset that this line is by no means always concave upward, nor does its declivity show anything like a steady diminution from the head to the outlet of the valley. If we may take the Cuillin district as a type, it appears that ice-erosion does not, like water-erosion, work constantly towards the establishment of an even gradient along a valley in which it operates. It tends, not to reduce, but to exaggerate within certain limits the more marked inequalities of the longitudinal profile; and in some circumstances it may set up a negative gradient in a certain portion of the valley.

The bed of a river which has attained a mature state maintains a steady gradient so long as the volume of water is unchanged, and the gradient diminishes down-stream in a manner proportioned inversely to the increasing volume. This law is a consequence of the relations which necessarily subsist between declivity, velocity, volume, and load. Perhaps the clearest presentation of the argument is that in Gilbert's *Geology of the Henry Mountains*; and a glance over his treatment of the subject is sufficient to show that these fundamental principles in the case of water have no counterpart in the case of ice. On the other hand, it appears that in ice-erosion certain other principles will come into operation which are peculiar to this agency. Thus we may expect that, other conditions being the same, erosion will be most efficient where the pressure below the ice is greatest, *i.e.*, where the thickness is greatest. If the longitudinal profile of a valley be of irregular form, while the upper surface of the ice declines steadily, the thickness will be greater over parts which have re-entrant or concave forms than over adjacent parts which have salient or convex forms; and, on the principle laid down, differential erosion will operate so as to exaggerate the inequalities. The original inequalities postulated must have a certain magnitude. On the other hand, the condition that the upper surface of the ice declines steadily implies that they must not be too great in proportion to the thickness of the ice. Within these limits it appears that the steady gradient, which for water-erosion is the stable form, is for ice-erosion unstable, since any departure from it leads to a further departure. There must, of course, be a limit to the action described, viz., when the lower layers of the ice begin to be ponded in the lee of a strong feature, and the upper layers slide over them.

These remarks receive striking illustration in the Cuillin Hills. Excluding the three main interior valleys, which in no part reach more than a very moderate altitude, all the glens show a very remarkable configuration. The longitudinal profile consists of two or three stretches of moderate slope divided by relatively steep drops, over which the water cascades. Where two such drops occur, as in all the longer valleys, the upper one is both higher and steeper than the lower. The heads of some of the glens, such as Coire na Banachdich, Coir' a' Ghrunnda, and Coire nan Laogh, are almost inaccessible from below; and to an observer coming into the district for the first time this stepped or storeyed form of the valleys is one of its most conspicuous peculiarities. The more or less precipitous drops, which constitute the steps and separate the successive storeys, are in no instance connected with anything in the geological structure of the ground, and there is no correspondence as regards levels between even adjacent valleys. In typical examples the descent is usually 200 to 400 feet, with an average gradient which varies in different cases from 30 to about 70 vertical in 100 horizontal. The best example of a valley divided into three parts by two steep drops is the one made up of Lota Corrie and the upper part of Harta Corrie.* Coir' a' Ghrunnda (fig. 6) illustrates a different case, the upper part of the Corrie being cut off by two sharp drops close together, giving a total fall of about 850 feet in a horizontal distance of 2100 feet.

Apart from superficial accumulations which may complicate the conditions—a case with which we are not concerned in the Cuillins—a negative or reversed gradient in the 'Thalweg' implies, of course, a rock-basin, and is, as Ramsay long ago pointed out, a result which cannot be arrived at by aqueous erosion. There is, in the opinion of the present writer, some danger of attaching too much weight to rock-basins as phenomena indicative of ice-action, with the result of diverting attention from other phenomena equally characteristic and often of a larger order. That the floor of a valley is lower at a certain place than at another place farther down-stream is of interest because it is an *absolute* criterion, not one of degree; but to dwell unduly upon this is to treat the rock-basin as an isolated phenomenon instead of what it is, viz., an integral part of the valley. It is an incident, depending not merely on glacial erosion, but on glacial erosion operating under certain local conditions. The requisite conditions may be realised in more than one way; and in a classification of the characteristic forms of ice-erosion according to their origin and essential significance different rock-basins would fall under different heads.

Of the three main interior valleys of the Cuillins, two, viz., those of Coruisk and Camasunary (Strath na Creitheach), contain elongated rock-basins. The determining

* See section, *Geol. Mag.* 1899, p. 197, fig. 2.

condition in both cases was the same, a marked constriction of the valley towards its lower end, which must have occasioned a certain heaping up of the ice in that part. In Coruisk the constriction was caused by the Sgùrr Dubh ridge running out eastward from the main range; in the Camasunary valley the same effect was produced by the convergence southward of the flanking ridges, Blath-bheinn on the east and Druim an Eidhne, Sgùrr an Eidhne, and Sgùrr na Stri on the west. The third main valley, that of Sligachan, opens out towards its lower end, and there is accordingly no rock-basin.

With the kind co-operation of Mr T. A. FALCON, a series of about 150 soundings have been taken in Loch Coruisk, and the results are embodied in the rough contoured map given in fig. 8. Owing to practical difficulties,* the soundings are rather deficient

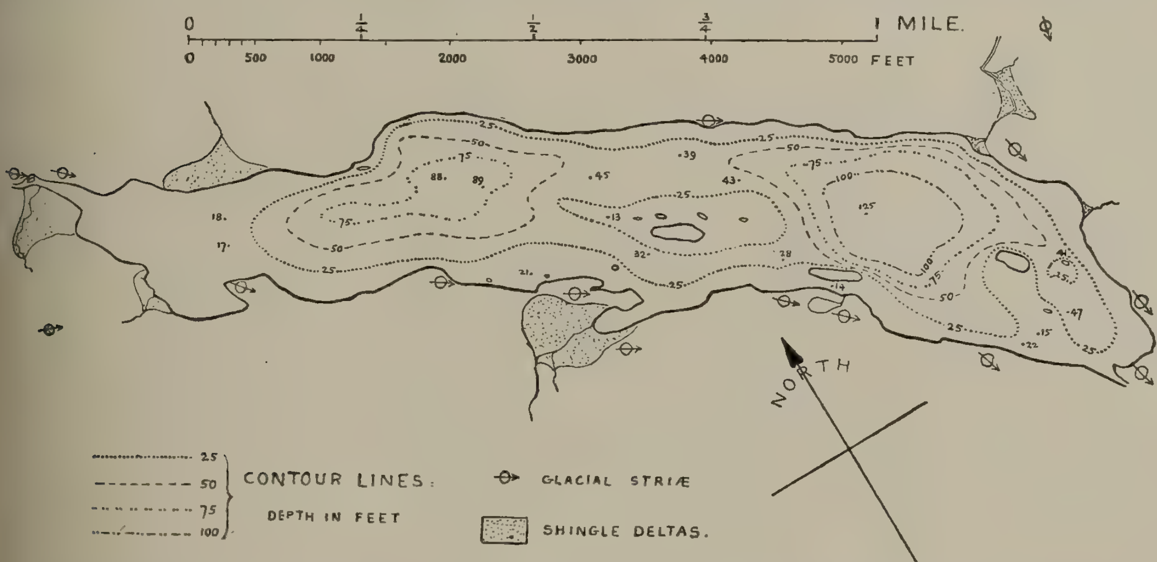


FIG. 8.—Bathymetric map of Loch Coruisk, plotted from 150 soundings taken by T. A. FALCON and A. HARKER. The actual soundings are given only in places where the contour-lines alone do not suffice to render the form of the bottom. The water-surface is 26 feet above sea-level.

in places, and unfortunately so in the deeper part of the loch; but the general form of the bottom is rendered with sufficient accuracy for our purpose. It will be seen that there are in reality two basins, the maximum depth being nearly 90 feet in the upper one and 125 feet in the lower, which latter goes nearly 100 feet below the sea-level (O.D.). These two basins are separated by a shallower area, never exceeding 40 feet in depth and rising in several places into small islets. The bottom is in general of bare rock. The shallow head of the loch is partly filled by a flat shingle delta, and the bottom is also shingly in places along the south-western shore; but these accumulations are nowhere in such force as to prevent the soundings giving very closely the true shape of the rock-basin. This seems to differ in no respect, as regards detailed sculpture, from the shape of the valley bottom where it does not happen to be covered

* The principal difficulty was in finding sufficiently calm weather. In a wind it was found impossible to keep the boat in place while a sounding was made and the position determined.

by water; but the soundings taken are not numerous enough to bring this out in the lower and deeper of the two basins.

In the Camasunary valley there are two lakes, the upper and smaller Loch an Athain and the lower and larger Loch na Creitheach. The latter is visibly rock-bound except at its head, where a gravel flat intervenes, which extends up to Loch an Athain and some three-quarters of a mile further. The form of the rock-surface is thus obscured, but it seems almost certain that the two lakes lie in separate basins. The proprietor, Mr R. L. Thomson, had some soundings taken, and has kindly communicated the result. The greatest depth found in Loch na Creitheach was 91 feet, at a point about one-third up the loch from the S. end and a quarter across from the W. side. As the water-surface is 85 feet above the Ordnance Datum, the loch descends about 6 feet below this. Loch an Athain (111 feet above O.D.) has only about half the depth of its larger neighbour.

(ix.) *Asymmetric Element in the Surface-Relief.*

The last peculiarity to be mentioned in the topography of the Cuillins is one which is very evident when once pointed out. It is observable, not in the central, but in the peripheral parts of the mountain-area; and takes the form of a decided asymmetry, as between the northerly and southerly aspects, in the transverse section of any element of the relief (ridge or valley) which has something of an E. to W. trend. The northward-facing slopes are invariably steeper than those facing in the opposite direction. There is nothing in the geological structure of the ground to account for this, and it is certainly too prevalent a phenomenon to be dismissed as fortuitous. The only reason which suggests itself for this dependence of the character of a mountain-slope upon its aspect, is one which connects it with the direction of incidence of solar radiation.

The asymmetric character is almost lost in the central parts of the area, but declares itself more and more as we approach the margin. This is shown, *e.g.*, in the two sections across Coir' a' Ghreadaidh given above (fig. 5); but the point perhaps comes out more clearly if we consider, not the valleys, but the ridges, *i.e.*, those of the exterior branch ridges which have (as is mostly the case) something of an easterly or westerly direction. Any one of these in its higher or proximal portion, where it forms the boundary between two cirques, shows the same general shape as the main ridge. Farther away it changes its character. It may or may not abate something of its steepness, and in some cases it becomes round-backed, though still with relatively steep flanks; but in every case the contrast between the two faces declares itself in the distal portion of the ridge (fig. 9). The westerly spurs of the Cuillins are Sgùrr Thuilm, Sgùrr nan Gobhar with its offshoot An Diallaid, Sgùrr Dearg (west ridge), and Sgùrr Sgùmain, to which we may add the west ridge of Gars-bheinn. On the opposite side of the gabbro mountains we have Belig, Sgùrr nan Each, and the easterly spurs of Clach Glas and Blath-bheinn. Each of these ten branches has a precipitous face towards the north and a less steep slope towards the south, though the chances against such a coincidence as an accident are more than a thousand to one. It may be noticed, too, that the only parts of the main ridge of the Cuillins which have the E. to W. direction, viz., the Basteir and Sgùrr a' Mhadaidh, are also steeper on the north side than on the south (fig. 7).

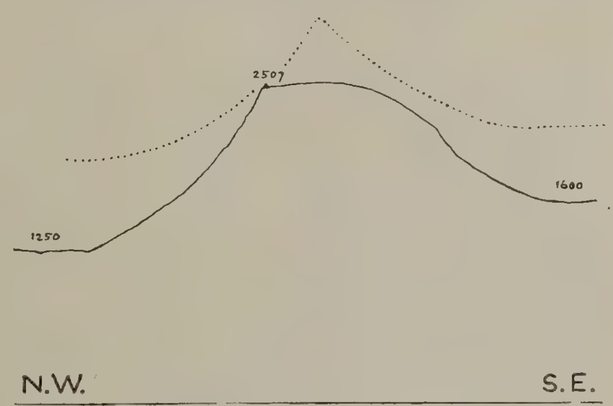


Fig. 9.—Transverse section of lower part of the Sgùrr Sgùmain ridge, to show its asymmetric form. The dotted line gives for comparison the symmetrical cuspate cross-section of the higher part of the ridge. Scale, about 3½ inches to a mile.

The distribution of the cirques or true corries in the Cuillins and the adjacent Red Hills also suggests a connection with the direction of sunshine. Helland* long ago pointed out that in Norway most of the cirques face northward, or towards some point of north; and something of the same kind is to be noticed in the Skye mountains, as appears from the following table giving the aspects of 52 cirques:—

W.N.W.	4		E.S.E.	1	
N.W.	2		S.E.	3	
N.N.W.	3		S.S.E.	1	
N.	8		S.	1	
N.N.E.	4	20	S.S.W.	2	5
N.E.	8		S.W.	2	
E.N.E.	2		W.S.W.	1	
E.	6		W.	4	
	37			15	

The inequality here, though decided, is not overwhelming, and except in connection with the foregoing remarks would not call for notice. It should be remembered, however, that we have included here the interior of the district, where the influence of aspect is much less evident than towards the exterior. If we separate the interior from the exterior corries, we find—

	N., etc.	S., etc.
Interior . .	18	12
Exterior . .	19	3

Here the prevalence of the northerly aspect in the second group of cirques is very marked. The basaltic plateaux beyond the mountain district owe their form almost everywhere to their geological structure, but even here there are some significant exceptions. An Cruachan, for instance, a hill to the west of Glen Brittle, presents a

* *Quart. Journ. Geol. Soc.*, vol. xxxiii. pp. 162, 163, 1877.

bold range of cliff, partly enclosing a corrie, to the north and a gentler slope to the south, though the dip, which is northerly, would lead us to anticipate the reverse.

If, as I believe, the asymmetric character of the ridges and valleys in the outer parts of the mountain area is really significant, and depends upon the different aspects of the slopes relatively to the sun, it seems clear that this influence was exerted, not at the stage of maximum glaciation, but when the ice-cap had shrunk so as to occupy the valleys alone, and during the later glaciation, which was effected by glaciers only. In the preceding sections I have not attempted to apportion the work of erosion between the ice-cap and the glaciers. I have in more than one place postulated a great thickness of ice over the centre of the area, and it is to be inferred that some of the most characteristic features of the district, especially in its interior part, were developed under such conditions; but we may still allow no inconsiderable *rôle* as agents of erosion to the glaciers, more particularly in the middle and lower courses of the mountain-glens. It is possible that a more thorough analysis might enable us to discriminate, by means of the resulting surface-relief, between the sculpture of a valley by a limb of an ice-cap on the one hand and by a glacier on the other, both being assumed to move down the valley. At present we are only concerned in distinguishing these two jointly from a foreign ice-sheet forced across the face of the country with scant regard to the pre-existing form of the ground. From the foregoing brief examination of the peculiar type of surface-configuration presented by the Cuillins, it has been made sufficiently clear that most of the *positive* characteristics, *i.e.*, those involving the development as distinguished from the obliteration of features of relief, depend upon the fact that the flow of the ice followed, in general and upon a broad scale, the natural direction of drainage. That in other circumstances the results might be very different is suggested by several of the phenomena described above. A good illustration is afforded by the small pre-Glacial glen of Allt a' Coir' an Lochain, a type of numerous others in the district. Here, as we have seen, the movement of the ice in the upper part of the valley was down-stream, but in the lower part directly at right angles to it. The result has been that the head of the glen is enlarged and the lower portion completely obliterated.

III. The Glacial Accumulations and their Testimony to Ice- and Frost-Erosion.

(x.) *Drift Deposits.*

We turn now from erosion to the complementary process of deposition; but, in accordance with the general design of this contribution, the glacial accumulations will be considered chiefly as throwing light upon the subject of ice-erosion. Drift deposits are found, though not as an unbroken sheet, over the whole of the tract that lies beyond the mountains. The continuous deposits—*i.e.*, excluding scattered erratics—

ascend usually to heights of 800 to 1000 feet, or sometimes as much as 1300 feet, on the higher moorland hills; only in places near the coast does the drift-line sink much lower, and even locally to sea-level. In the mountain-valleys the drift, always with diminished thickness, runs up in tongues to altitudes of 1000 to 1500 feet, and exceptionally 1750 feet.

We are here referring to the tract within which the glaciation was strictly local. The limits of this tract are partly laid down in the small sketch-map given above (fig. 1). Within it, the boulders are wholly derived from the tract itself, thus presenting a marked contrast to the area outside the line drawn. In the south-eastern part of Skye, Mr Clough has found foreign boulders even up to the highest summit (Sgùrr na Coinnich, 2400 feet). The same observer notes along Loch Eishort a certain intermingling of boulders from different sources, which may be taken to indicate some oscillation of the line where the native and foreign ice marched together. Such oscillations, depending on the balance of varying pressures, are to be expected. On the north-eastern side of our area the domain of the native ice is delimited with sufficient closeness. Especially marked is the contrast between Scalpay and the neighbouring part of Skye. On the smaller island, which was overflowed by the great ice-sheet from the mainland, boulders of Scottish extraction occur at all altitudes, and in the boulder-clay no less than on the surface, while further points of difference are apparent in the nature and distribution of the drift and in the form of the ground. Raasay evidently falls under the same head. Our detailed survey of Skye has not progressed far enough northward to trace the boundary-line in that direction, but the breaking in of the Scottish ice over the northern part of the island seems to be sufficiently established.

A closer examination of the materials of the local drift affords much information relative to the movement of the Skye ice, and also leads to observations which have a very direct bearing upon the amount and the mechanics of glacial erosion. Since in the area chiefly involved the number of local rocks which can be readily identified is not great, it becomes necessary to take note of the relative proportions in which different rocks enter into the composition of the accumulations. The writer has found that for this purpose general impressions are not to be trusted, and he has followed as far as possible the statistical method. As an illustration of the use of this, as well as for some of the results obtained, we will follow the course of what we have already referred to as the mid-stream line of the Glen Sligachan ice (see map below). To lay down this line, about a hundred convenient stations were selected, and at each station from 200 to 500 boulders were taken without selection from the drift and the percentages of different rocks estimated.

Looking down Glen Sligachan from the watershed at Loch Dubh, the observer has the Red Hills on his right and the Cuillins on his left. The former are essentially of granite, the latter of gabbro; the line of junction of the two rocks running for some distance along the bottom of the valley. Accordingly granite boulders preponderate in the drift on the right side of the valley and gabbro boulders on the left. A line drawn down the valley through places where the two rocks are equally represented is what we have styled the mid-stream line of that branch of the ice which took this direction. This line can be drawn with considerable precision, since the relative proportions of the two rocks vary rather rapidly as we cross the floor of the valley. Thus, at or near Loch Dubh, the percentages of granite and gabbro are 76 and 15 at 70 yards to the right (*i.e.*, E.) of the line, 86 and 4 at 250 yards, and 98 and 2 at 450 yards; while to the left (or W.) of the line there is an equally rapid change in the opposite sense. The 100 per cent. is made up by boulders of basalt (including dolerite) and, at this place, volcanic agglomerate. We have not always found it possible to discriminate with certainty between the basalt lavas and the basalt and dolerite dykes and sheets, but at this place the latter must supply the chief contribution.

Proceeding down the glen, we find that the line can be traced throughout with sufficient accuracy. Near the outfall of Allt Coire Riabhach, for instance, the percentages of granite and gabbro are 55 and 36 to the right and 7 and 67 to the left, the two spots being only 40 yards apart. Here, as in other places where it is most sharply defined, the line coincides exactly with the junction of granite and gabbro in place. Indeed, there are many circumstances which seem to indicate that the immediately subjacent rocks have contributed an important part of the boulders. This becomes very clear when, about $1\frac{1}{4}$ mile above Sligachan, we come on to the basaltic lavas. At once the proportion of those miscellaneous basaltic boulders which we have grouped together as the third element begins to increase rapidly, and 700 or 800 yards lower down they already make up 65 to 80 per cent. of the whole, instead of 10 per cent. or less. It is clear, too, that the bulk of these basalt boulders are of the lava type, and they must indisputably have been torn from the floor of this broad open strath.

From Sligachan bridge, being now clear of the mountains, the line curves away in accordance with the general westward deflection already considered, sweeping round by Loch Mòr na Caiplaich into the Drynoch valley, where for some distance it runs very near the Dunvegan highroad. The percentage of basaltic boulders has now risen to 90 or 95 or even 98 or 99, mostly amygdaloidal lavas, so that it becomes necessary to examine a larger number of boulders in order to determine the proportions of granite and gabbro with sufficient exactness.* Before reaching Drynoch our line takes a curve to the south-west, rising to about 500 feet and coming down to the valley of the Vikisgill Burn. Crossing this, it passes southward up a tributary valley, Allt na Creadha, over a third watershed (about 550 feet) into the valley of Allt nam Fitheach, and so south-west to the sea at Loch Eynort, having traversed a semicircle from Sligachan.

To the left of the line as thus traced, gabbro boulders are in force, while those of granite become rare; though a few of the latter are still found as much as $2\frac{1}{2}$ miles

* Farther west the basalt has often been discarded and only the granite and gabbro boulders counted.

away, at the upper bridge over the Brittle River, which point they have reached by way of Bealach a' Mhaìm. To the right of the line the reverse is observed. Two miles away, about the head of the Eynort River, the granite boulders are about ten times as numerous as those of gabbro, and beyond this the latter are rare. Granite is plentiful between Loch Eynort and Loch Harport, boulders up to 2 feet in diameter occurring, for instance, on the top of Preshal Beg (1160 feet) near Talisker. It is, of course, not to be assumed that all the granite necessarily comes from Glen Sligachan and the hills overlooking it; for that part of the ice from the central and eastern Red Hills which found an outlet to the north must have been forced westward by the Scottish ice-sheet before the latter itself effected a landing on the northern part of Skye. That the Sligachan ice spread over a very wide tract is nevertheless proved by boulders of certain easily recognised rocks; *e.g.*, a pitchstone from Glamaig, a granophyre crowded with ovoid patches of a basic rock from Glamaig and Sròn a' Bhealain, and a granite enclosing gabbro débris from Marsco.

There is no need here to trace out in the above fashion all the other branches of the ice from the Cuillins. Their course cannot usually be followed with the same precision as above over the low ground; unless, indeed, boulders of some local and distinctive rock be available for the purpose, such as the rhyolite, etc., of Fionn-choire and the picrite of Coir' a' Ghrunnda. Wherever the test can be applied, we find a very close correspondence between the transport of boulders and the direction of striæ; which would not always be the case if the boulders had been to any important extent carried on the ice or in its upper part. The manner in which the basaltic boulders always become increasingly abundant as soon as we pass from the gabbro to basalt in place is especially striking. Since it is certain that no basalt stood out above the ice, this proves that, at least in the belt of country bordering the mountains, a very considerable amount of erosion of the basaltic ground went on beneath the ice. Further, this erosion, in so far as we have direct evidence of it, was effected not merely by abrasion but by fracture of the surface rocks.

This last point calls for further remark, for the importance of ice-action in tearing away pieces of the subjacent rocks is one of the most salient facts of glacial erosion in our area. It is emphasized by the large proportion which boulders bear to matrix in all the glacial accumulations in the vicinity of the mountains. To such action also we must ascribe the rough craggy surface (certainly not the pre-Glacial surface) seen often on the lee side of a *roche moutonnée* just outside the mountains. In the interior of the mountain-area, as already remarked, this is not usually seen, and the inference suggested is that fracture of rocks is not so readily effected under a great pressure of ice. If there be truth in this, it is still only one of the factors which determine whether a rock shall yield by grinding down or by tearing away. The nature of the rock is doubtless another factor, and an important one, especially as regards the presence or absence of joints or other lines of weakness. In this connection it is to be noticed that in the drift of the mountain-area proper boulders from the basic dykes and sheets intersecting the gabbro invariably play an unduly prominent part as compared with the boulders of gabbro. The composition of the drift in the lower corries is, indeed, very remarkable. Tairneilear is a good example, since here the question is not complicated by patches of basaltic lavas enclosed in the gabbro. The bare surface of the corrie is composed of gabbro intersected by dykes and sheets of basalt and dolerite. These minor intrusions, though very numerous, make up but a small part of the whole: probably one-twentieth would be an over-estimate. When, however, towards the mouth of the corrie, we come upon the drift, we find that boulders of these rocks make up, not one-twentieth, but from one-third to one-half of the total boulders. Clearly some selective influence has operated. Since the average constitution of the drift must be the same as that of the rocks in place, we conclude that the jointed and brittle rocks of the minor intrusions have mostly broken away and formed boulders, while the more massive gabbro has in much greater measure been ground down and gone into the matrix.

Hitherto we have made no classification of the drift accumulations. The more or less continuous deposits with which we have been dealing, seem to belong wholly to the time when the flow of the Skye ice outside the mountain-area was diverted in the manner already described by the pressure of the Scottish ice-sheet. Among them we may, however, distinguish two types: one corresponding with the phase of maximum glaciation, when central Skye was covered by a continuous ice-cap; the other connected with the waning phase of this principal glaciation, when only fragmentary relics of the ice-cap remained. The most widely spread type of drift in our area has the ordinary characters of a ground-moraine. In the valleys and on the lower slopes of the basalt country, where it is best displayed, it imparts to the landscape the familiar gently undulating appearance with smooth flowing outlines. Here it consists of a reddish sandy clay enclosing numerous small boulders and some large ones, often planed and scratched. Elsewhere its composition varies to some extent, the local element being always important. The second type is what we have called in mapping the country the 'hummocky' drift, and it seems to answer to the 'kettle-moraine' of some American geologists. The ground is closely studded with circular mounds, like tumuli, usually from 10 or 15 feet to 50 or 60 feet in height, only rarely showing any linear or other arrangement. The finer material, which may be regarded as a matrix, is commonly reduced to a minimum, the bulk of the accumulation consisting of boulders, mostly subangular but rarely scratched. It is often noticeable that the larger boulders occur towards the summit of the mound. The hollows of the irregular surface frequently hold tarns, from 300 or 400 yards long down to mere pools, those in the neighbourhood of Sligachan being good examples. Loch an Fhir-bhallaich, near Glen Brittle, illustrates another kind of tarn, occurring above and outside the margin of an area of kettle-moraine, but held in by it.

The hummocky drift has a much more restricted distribution than the smooth, and this distribution is a significant one. The patches of hummocky drift lie constantly within the area occupied by the smooth ; they do not extend so far from the mountains, and on the other hand they do not enter the mountain-tract itself except along the floors of the wide and level interior valleys. This type of drift is indeed confined to open places, usually near the mountains, in parts where the slope of the surface is gentle, and it is found in such places especially where the flow of the ice has been obstructed or checked by extraneous interference. The finest development is in the Red Hills tract, and more particularly at such places as Luib and Strollamus, where considerable branches of the native ice abutted directly upon the flank of the Scottish ice-sheet. In the part of the island which we have chiefly considered, the largest area of hummocky drift is that near Sligachan, extending from Harta Corrie to near Garadubh, on the Portree road, a distance of ten miles. Arms of this extend up the valley of Allt Dearg Mòr and over into the Drynoch valley for distances of 1 mile and $1\frac{1}{2}$ miles respectively, and there is a detached area in Coire Reidh na Loch. Patches occur again in Strath na Creitheach and the open part of Coire Riabhach, between Druim nan Ramh and Druim an Eidhne, and others on the west side of the Cuillins, viz., south of Coir' a' Ghrunnda and between Allt Coire Labain and Allt Coire na Banachdich. The most remote isolated occurrence observed is a small patch at the head of the Talisker valley.

The phenomena in the Skye district, viewed generally, seem to find their simplest explanation on the supposition that the waxing and waning of glacial conditions have been controlled less by secular changes in the mean temperature than by variations in the amount of precipitation over the area. This question could not be profitably discussed as a local one. But whatever the causes which terminated the maximum glaciation of central Skye, it cannot be doubted that the ice-cap shrank away from the mountains as well as from its margin. As the steep summit-ridges emerged, there was added a new element of ice-transport by the accumulation of material upon the upper surface of the ice ; an element increasing in relative importance as the erosive effect at the lower surface became feebler. We interpret the hummocky drift as the material—superglacial, englacial, and infraglacial—finally deposited by stranded portions of the confluent glaciers, cut off from their supply behind and melting as they stood. All the features of its composition, no less than its distribution, seem to accord best with this view. The absence of all ordinary morainic accumulations referable to this epoch suggests that the disappearance of the ice was a somewhat rapid event. It is clear, from the dispersal of boulders, that no change in the direction of the ice-drainage took place from the maximum glaciation to the close of this phase, such lines as that already noticed marking the mid-stream of the Sligachan branch being traceable uninterruptedly through the smooth and the hummocky drift alike.

(xi.) *Later Glaciers and Frost-Erosion.*

In numerous parts of Scotland geologists have recognised two more or less distinct glaciations, the first a general one and the second a local. In the central part of Skye also there is clear evidence of two glaciations, though—since foreign ice never obtained a footing here—they cannot be distinguished in those terms. That the second glaciation here corresponds with the local glaciation in adjacent parts of Scotland, and was in a general sense contemporaneous with it, is evident from the map given below, on which the flow of the ice during this later stage is indicated by a second set of arrows. These have been inserted only in places where the evidence was quite clear; but they suffice to show that the movement was very different on many parts of the lower ground from that during the principal glaciation, and that the difference was due to the withdrawal of the Scottish ice-sheet. Instead of the general diversion westward, upon which we commented before, there is now a simple radiate outflow from the mountains to the sea. This reversion to what we may consider the natural direction of ice-drainage for the district, consequent upon the removal of the constraint from without, involved, at many places outside the mountain area, considerable departures from the former directions of flow. Thus, to the north-west of the Cuillins, the new line of movement, directed towards Loch Bracadale, was at right angles to the old. In the valley of Allt Dearg Mòr the direction of flow was directly reversed. In the earlier glaciation the ice had moved up the valley, bringing boulders from Glen Sligachan; in the later glaciation it moved in the natural direction, carrying down the rhyolitic and other rocks of Fionn-choire and Fhinn-choire, which are found in abundance along the burn and to heights of 60 or 70 feet above it. The large extent of country overrun by the later glaciers and the way in which, in certain cases, they overflowed some of the lower watersheds, prove that they were of very considerable magnitude, but it is clear that they were greatly inferior to the ice-sheet of the earlier glaciation.

The erosive action exerted by these glaciers was mainly confined to the valleys of the mountain area, and was almost negligible at a distance from the mountains. Here it has often failed to obliterate the scorings made by the earlier glaciation, and has usually caused but little disturbance in the older drift accumulations over which it passed. Even quite near to the mountains, patches of hummocky drift have retained most of their characteristic appearance in places where they have certainly been over-ridden by the later glaciers. In the lower parts of the mountain valleys themselves, however, the earlier drift accumulations seem to have suffered erosion in many places. In certain cases, too, there are drift-ridges between the mouths of adjacent valleys, which seem to have been originally of the nature of drumlins, but to have been scarped and moulded by the later glaciers.

The accumulations referable to this later glaciation are represented on the lower ground mainly by a vast number of erratics scattered over the tracks of the glaciers and lying thickly along certain lines and in certain places. In attempting to map these out accurately, considerable difficulty might be experienced in some parts in separating the later from the earlier deposits without suspicion of possible error. Where, however, characteristic rocks are available for tracing out the lines of movement no doubt can exist. For instance, as we emerge from Glen Sligachan, we find that the surface erratics are of granite to the right and of gabbro to the left, with only a narrow belt of intermingling, and we can draw a line accordingly, as already done for the earlier drift. We find that, instead of curving away westward, this line runs straight on towards Portree, only a little on the east side of the highroad; showing that these erratics were brought down at a time when the Scottish ice-sheet had withdrawn, or at least had ceased to press heavily upon the coast of Skye.

To the west of this line and of the mountains, gabbro is constantly the principal element in these accumulations, and remains so to all distances. There are, however, basaltic lavas, from the patches enclosed in the gabbro mass and from the lower slopes, and representatives of the doleritic and other rocks of the minor intrusions of the Cuillins; besides peridotites and other locally distributed types, which are found along lines leading from their several places. The blocks are not planed or scored. Many of them are of considerable size, and some very large ones are found in the lower part of Coire Labain, in Coire na Creiche, near the mouth of Harta Corrie, and elsewhere. The largest are usually of gabbro, picrite, and other massive rocks; the more jointed rocks of the dykes and sheets have broken into smaller blocks; the laminated rhyolite, with a slate-like fracture, is usually represented by small fragments.

Only in a few places do these later glacial accumulations assume anything like the form of typical moraines. There is, however, one remarkable exception, which is of sufficient interest to demand notice. It occurs opposite the mouth of Coir' a' Ghrunnda, and was noticed by J. D. Forbes in his paper already mentioned.* Here we have a perfect crescentic moraine, measuring 900 yards from horn to horn and 800 yards from that line to the front. The material consists in great part of large blocks, chiefly of gabbro, but including also picrite and the other rocks of Coir' a' Ghrunnda. The front portion is a curved ridge 150 yards across and 50 feet in height; but towards the two horns the height diminishes and the width increases, until the ridge is represented only by a belt of closely scattered blocks. The moraine lies partly upon a patch of hummocky drift, which has preserved much of its characteristic surface relief.

To the later glaciation must be attributed the perched blocks which are conspicuous objects on the bare slopes of some of the Cuillin valleys, as noticed by Sir A. Geikie. Good examples are seen on the western side of Sgùrr na Stri, towards Lochs Coruisk and Scavaig, and on the lower slopes of Sgùrr a' Coir' an Lochain. In the latter place

* On Forbes' small sketch-map Coir' a' Ghrunnda appears as "Bottomless Corry." The moraine is marked with the letter E on our map; see also fig. 6.

there are some of peridotite, which, in virtue of the excessively rough surface presented by that rock, have been able to take a very remarkable posture (fig. 10).

Although the glaciers while in the mountain-glens may have exercised a considerable erosive action upon their beds, it seems evident that the blocks of gabbro and other rocks which constitute most of the accumulations which can be confidently referred to the later glaciation were only transported, not detached, by the ice. They were broken away from the parent rock on slopes overlooking the glaciers by subaërial agency, in which frost must have been the most important factor. To an observer approaching the Cuillins by any of the principal glens, or still better by Loch Scavaig, one of the first things to strike the eye is the strong contrast between the smooth rounded form of the lower slopes and the splintered shapes of many parts of the higher ridges. It might be hastily inferred that these latter have never been submerged beneath ice; but such an explanation would soon be found to break down when applied in detail, and we have already seen from other considerations that it is inadmissible. There are two reasons for the higher ridges and summits not showing the effects of glaciation in the same way as the corries below. Firstly the ridges, acting as ice-sheds at the time of the maximum glaciation, escaped erosion owing to the lack of rock-débris in the ice overlying them, which left it almost powerless; and secondly, the same ridges, exposed above the ice-surface during the later glaciation, were then subjected to the splintering and shattering action of frost.

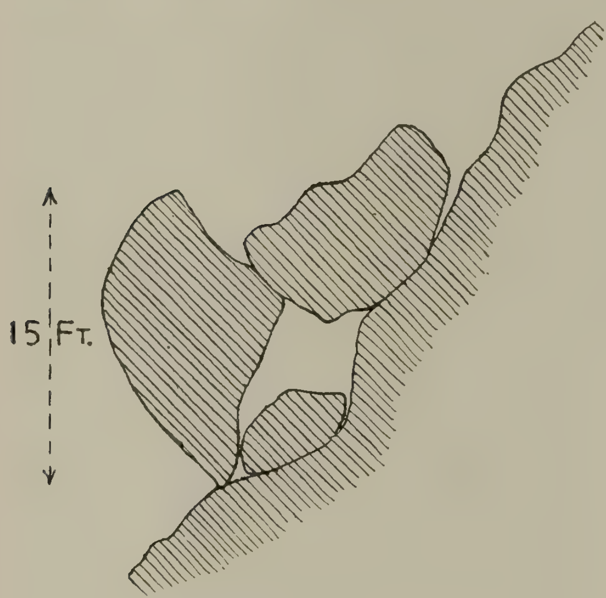


FIG. 10.—Perched blocks on the lower slope of Sgùrr a' Coir' an Lochain, towards Coruisk.

The operation of what we may call frost-erosion was not confined merely to the time when the valleys were occupied by the later glaciers. The requisite conditions, viz., a sufficiency of moisture and an air-temperature fluctuating above and below the freezing-point, must have existed during a part of the interval between the disappearance of the ice-cap and the birth of the later glaciers, and certainly continued for some time after these glaciers had vacated at least the upper parts of the mountain glens. This is part of the evidence, already alluded to, which goes to show that, in determining the glaciation of this part of the country, variations in the amount of precipitation were of greater moment than variations in mean annual temperature. The proof that the later glaciers withdrew from at least the upper parts of the glens while a severe temperature still prevailed, is afforded by a class of accumulations not hitherto mentioned, viz., the huge taluses which are so conspicuous a feature of almost all the higher corries of the Cuillins. These have not the character of screes resulting from modern subaërial waste; and indeed, despite an increased elevation amounting to 100 feet, the actual waste under existing conditions is exceedingly small.* Certainly it is inadequate to account for more than a very small fraction of the material which chokes the heads of many of the glens.

While the great taluses, composed mainly of blocks of gabbro and the associated rocks, on the slopes and much of the débris on the floors of the corries have clearly reached their present situations by falling, rolling, and sliding, probably assisted in part by snow-slopes, it is often impossible to divide these accumulations from the similar material farther down-stream, which has doubtless been ice-borne, probably on the tail of a dwindling glacier. A good instance of this difficulty is seen in An Garbh-choire, the glen to the south of the Sgùrr Dubh ridge. The whole length of the valley, about a mile, is rendered almost impassable by the blocks, great and small, here chiefly of peridotites, by which it is covered. These cannot be separated distinctly from the more scattered blocks over the little plateau between the mouth of the glen and Allt a' Chaoich. In the lower part of the valley the peridotite blocks must have travelled down, for the slopes on both sides are of gabbro; but the higher part of the accumulation is merely a great talus streaming down from the steep main ridge. Something of the same kind is seen in Coireachan Ruadha, where the taluses of gabbro and peridotite blocks are on a large scale. Among the outer corries of the Cuillins, Coire Labain is a good example of the large amount of material detached from the ridges by frost-weathering at a late epoch. On the slopes round its head are three or four large taluses, more or less confluent, the one from the gap between Sgùrr Alaisdair and Sgùrr Tearlach coming down some 1200 or 1300 feet to the floor of the corrie.

It is perhaps significant that Coir' a' Ghrunnda, which lies in the heart of the highest mountains, and at a considerably higher level than its neighbours, has comparatively little talus. We have seen that this valley differs from the others in having below it a large and well developed crescentic moraine. We may conjecture that the upper part of the Coir' a' Ghrunnda glacier survived after the heads of the neighbouring glens had been vacated, and that the moraine in this case corresponds with the taluses in the other corries.

In conclusion, it should be remarked that frost and other subaërial agents have had no share in developing the characteristic forms of the mountains and valleys as described in the foregoing section; their effect has often been to undo in some measure the work of ice-erosion, viewed from the standpoint of topographic forms. The slopes which hem in the cirques, for instance, have the familiar 'glaciated' surface far up towards the summit ridges, and it is these latter which have suffered from later destructive action. The boldly salient form and unbroken character of the ridges must have been more remarkable when they first emerged from the declining ice-cap than they are at the present time.

* See *Geol. Mag.* 1899, pp. 485–491.

EXPLANATION OF MAP.

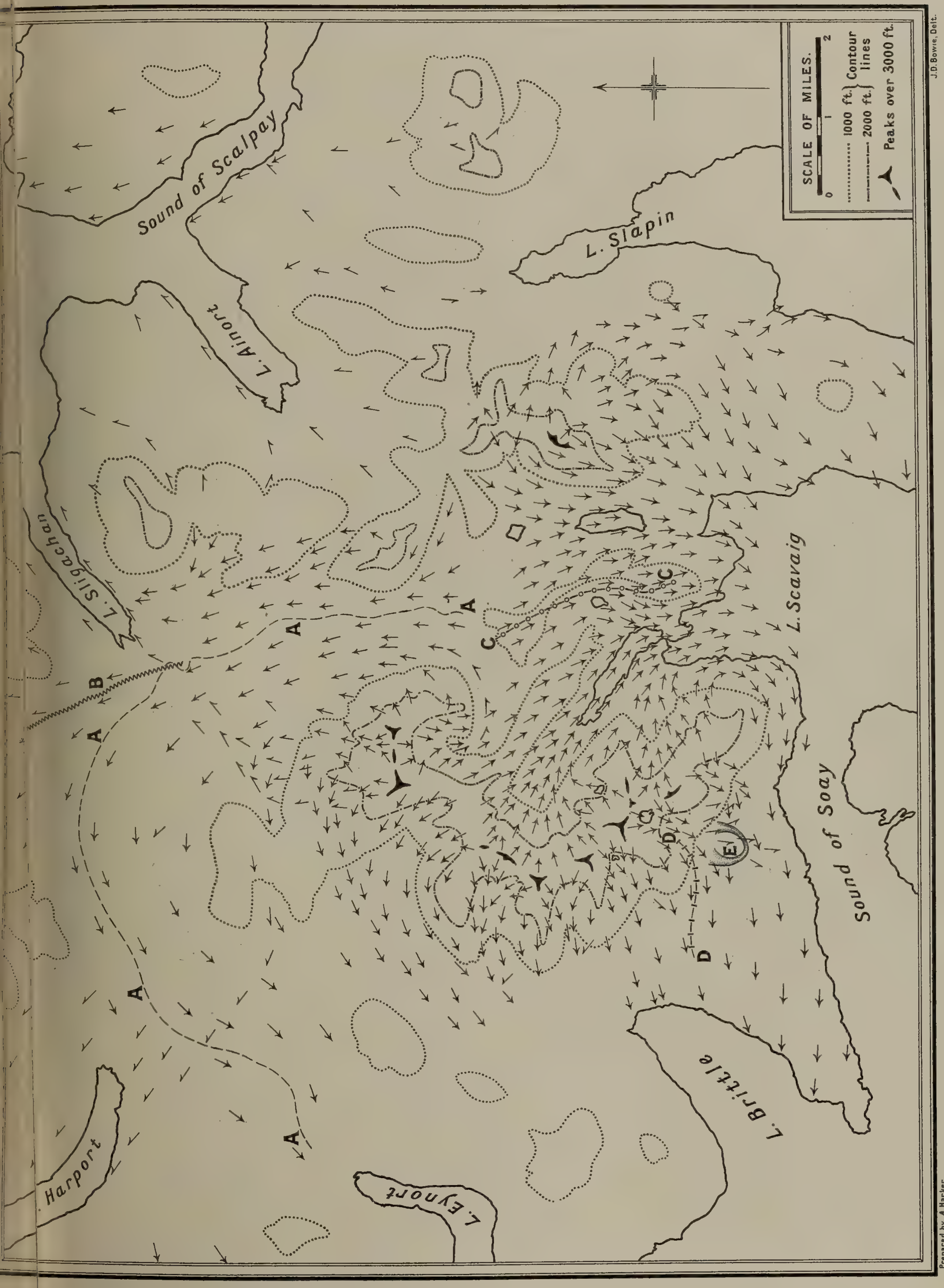
This sketch-map shows the central part of Skye on the scale of about $\frac{3}{5}$ inch to a mile. The general character of the surface relief is roughly indicated by the contour-lines corresponding with altitudes of 1000, 2000, and 3000 feet above sea-level.

The doubly-barbed arrows indicate the movement of the ice during the stage of maximum glaciation, as deduced from the striated surfaces, the dispersal of boulders, etc. These arrows belong in the north-eastern corner of the map to the ice-sheet from the Scottish mainland, but elsewhere to the native ice-cap. In the Red Hills, which occupy much of the eastern portion of the map, the data are in general insufficient to determine the movement of the ice with any precision.

The singly-barbed arrows given in several parts of the map indicate the movement of the native ice during the later stages of glaciation, when the disturbing factor of the Scottish ice-sheet was removed.

Certain lines drawn on the map are explained as follows:—A A A is the mid-stream-line of the Glen Sligachan ice-stream. Among the boulders in the drift, granite preponderates over gabbro to the right of this line and the reverse to the left. B is the corresponding line for the surface-erratics, belonging to a later stage of the glaciation; in Glen Sligachan itself this line is practically coincident with the preceding, but diverges from it at Sligachan Bridge. C C is the western limit of granite boulders carried by the southward-moving ice. D D is the northern limit of peridotite and picrite boulders from Coir' a' Ghrunnda and An Sgùman. E is the moraine of Coir' a' Ghrunnda.

ICE EROSION IN THE CULLIN HILLS, SKYE.



XIII.—*The General Form of the Involutive* 1-1 *Quadric Transformation in a Plane.* By Charles Tweedie, M.A., B.Sc.

(Read 15th July 1901.)

§ 1. In a communication read before the Society, 3rd December 1900, Dr Muir discusses the generalisation, for more than two pairs of variables, of the proposition that: If

$$x=(\eta^2-\xi)/(1-\xi\eta);\ \ y=(\xi^2-\eta)/(1-\xi\eta);$$

then

$$\xi=(y^2-x)/(1-xy);\ \ \eta=(x^2-y)/(1-xy).$$

If we interpret (x, y) and (ξ, η) as points in a plane, it is manifest that the transformation thereby obtained is a Cremona transformation. It has the special property of being reciprocal or involutive in character; *i.e.*, if the point P is transformed into Q, then the repetition of the same transformation on Q transforms Q into P. Symbolically, if the transformation is denoted by T, $T(P)=Q$, and $T(Q)=T^2(P)=P$; so that $T^2=1$, and $T=T^{-1}$. Moreover, if the locus of P (x, y) is a straight line, the locus of Q (ξ, η) is in general a conic.

§ 2. The object of this note is to find the most general bilinear transformation connecting two points (x, y), (ξ, η) of the form

$$\left.\begin{aligned}L_1\xi+M_1\eta+N_1=0\\ L_2\xi+M_2\eta+N_2=0\end{aligned}\right\}\ \text{I.}$$

(L_1, etc., being linear functions of x and y) which possesses this property; *i.e.*, the most general 1-1 transformation which is involutive in character and in which to a straight line corresponds a conic.

This problem has already been discussed by Czuber (*Monatshefte für Mathematik und Physik*, 1894), but unfortunately his discussion is not free from error, and one of the best-known transformations of the kind—the so-called Hirst transformation—entirely escapes his observation. Moreover, he describes the above transformation (I.) as the most general 1-1 point transformation, which is by no means the case, as it is not difficult to frame 1-1 point transformations in which to a straight line corresponds a curve of higher degree than the second (*v.* Salmon's *Higher Plane Curves*). In his paper, however, he discusses a very large variety of degenerate cases, and this enables me to dispense with these entirely and to discuss only the leading case in which to a straight line corresponds a conic.

§ 3. The transformation (I.) would appear to be the most general 1-1 quadric transformation. On solving for ξ and η we deduce

$$\xi=\frac{M_1N_2-M_2N_1}{L_1M_2-L_2M_1}$$

$$\eta=\frac{N_1L_2-N_2L_1}{L_1M_2-L_2M_1}.$$

The conics $M_1N_2 - M_2N_1 = 0$; $N_1L_2 - N_2L_1 = 0$; $L_1M_2 - L_2M_1 = 0$ have three points in common, and this is the characteristic of the quadric Cremona transformation (*v.* SALMON).

Conversely, any three conics having three common points may be so represented. Let A B C P Q, A B C Q R, A B C R P be three such conics (where no two of the points P Q R are coincident). Let $L_1 = 0$, $L_2 = 0$ denote two lines through P; $M_1 = 0$, $M_2 = 0$ two lines through Q; $N_1 = 0$, $N_2 = 0$ two lines through R.

The conic A B C P Q may be represented as the intersection of corresponding rays of the pencils

$$\begin{aligned} L_1 - aL_2 &= 0 \\ M_1 - aM_2 &= 0. \end{aligned}$$

Similarly, the conic A B C Q R may be obtained from

$$\begin{aligned} M_1 - aM_2 &= 0 \\ N_1 - aN_2 &= 0, \end{aligned}$$

N_1 and N_2 being lines suitably chosen through R.

Now the two pencils of lines

$$\begin{aligned} L_1 - aL_2 &= 0 \\ N_1 - aN_2 &= 0 \end{aligned}$$

furnish a conic which clearly passes through A, B, C, also through the centres P and R, and therefore through the five points A B C P R, *i.e.*, they furnish the conic A B C P R. The values of a corresponding to A, B, C are found by expressing the condition that the three equations in x and y

$$L_1 - aL_2 = 0; \ M_1 - aM_2 = 0; \ N_1 - aN_2 = 0$$

be consistent. When the cubic for a has two equal roots, two of the points coincide and the conics have contact of the first order; when all three roots coincide the conics have contact of the second order—all at a common point. A point P may also coincide with A, say, in a given direction, but Q cannot coincide with A at the same time.

It may be noted that a linear construction for any number of points on the third conic is hereby indicated, the first two conics being given. Let S be any point on the first conic, and let Q S meet the second conic in T. Then R T and P S intersect on the third conic in U.

§ 4. Let the bilinear equations in x, y, ξ, η be

$$\begin{aligned} &(1) \quad \xi(A_1x + B_1y + C_1) + \eta(A_2x + B_2y + C_2) + A_3x + B_3y + C_3 = 0 \\ &(2) \quad \xi(a_1x + \beta_1y + \gamma_1) + \eta(a_2x + \beta_2y + \gamma_2) + a_3x + \beta_3y + \gamma_3 = 0. \end{aligned}$$

If these equations are involutive, then the interchange of ξ and x, η and y, must give two equations which lead to identical solutions with (1) and (2). Hence the result of the substitutions must be to replace (1) and (2) by two equations,

$$\begin{aligned} k(1) + l(2) &= 0 \\ m(1) + n(2) &= 0 \end{aligned}$$

where $kn - lm$ is distinct from zero.

The transformed equations are

$$\begin{array}{ll}(1)' & x(A_1\xi+B_1\eta+C_1)+y(A_2\xi+B_2\eta+C_2)+A_3\xi+B_3\eta+C_3=0\\ (2)' & x(a_1\xi+\ \text{etc.}\ )+\qquad \text{etc.} \qquad =0.\end{array}$$

Case 1. If

$$\begin{aligned}(1)&\equiv(1)'\\(2)&\equiv(2)'\end{aligned}$$

there must exist the following equations in the coefficients of (1)—

$$\begin{array}{lll}A_1=A_1, & A_2=B_1, & A_3=C_1,\\ B_1=A_2, & B_2=B_2, & B_3=C_2,\\ C_1=A_3, & C_2=B_3, & C_3=C_3,\end{array}$$

so that (1) may be written as

I. $$A_1x\xi+B_2y\eta+C_3+C_1(x+\xi)+C_2(y+\eta)+B_1(\xi y+x\eta)=0.$$

But if $A_1=B_1=C_1=0$, we may reverse signs throughout and deduce

II. $$C_1(x\pm\xi)+C_2(y\pm\eta)+B_1(\xi y\pm x\eta)=0.$$

Similar results hold for the second equation. (I. is practically the only case discussed by Czuber.)

Case 2. Nothing new is gained by supposing

$$(1)'\equiv\pm(2)\ ;\ (2)\equiv\pm(1).$$

For the solutions of $(1)=0$ and $(2)=0$ are those of

$$(1)+(2)=0\ ;\ (1)-(2)=0,$$

and the transformation transforms these latter equations into themselves, so that there is a reduction to the preceding case.

Case 3. More generally, no new result is obtained by supposing

$$\begin{aligned}(1)'&\equiv k(1)+l(2) \quad \text{(i.)}\\(2)'&\equiv m(1)+n(2) \quad \text{(ii.)}\ ;\end{aligned}$$

for, since the repetition of the transformation gives again $(1)=0$ and $(2)=0$, there would result

$$\begin{aligned}(1)&\equiv k\{k(1)+l(2)\}+l\{m(1)+n(2)\}\\(2)&\equiv m\{k(1)+l(2)\}+n\{m(1)+n(2)\}\end{aligned}$$

i.e.,

$$\begin{aligned}(1)&\equiv(k^2+lm)(1)+l(k+n)(2)\\(2)&\equiv m(k+n)(1)+(lm+n^2)(2),\end{aligned}$$

giving

$$\left.\begin{aligned}k^2+lm&=1\\l(k+n)&=0\\lm+n^2&=1\\m(k+n)&=0\end{aligned}\right\}\text{(iii.)}.$$

If $l\neq0$, $m\neq0$, these equations reduce to

$$k+n=0\ ;\ k^2+lm=1.$$

Now it is possible to determine a and b such that

$$a\{k(1)+l(2)\}+b\{m(1)-k(2)\}\equiv a(1)+b(2),$$
$$a'\{k(1)+l(2)\}+b'\{m(1)-k(2)\}\equiv-\{a'(1)+b'(2)\};$$

for these lead to

$$\left.\begin{aligned} ak+bm&=a \\ al-bk&=b \end{aligned}\right\}\text{(A)},\qquad \left.\begin{aligned} a'k+b'm&=-a' \\ a'l-b'k&=-b' \end{aligned}\right\}\text{(B)},$$

pairs of equations which are consistent, since $k^2+lm=1$. Also $\dfrac{a}{a'}$ can not be equal to $\dfrac{b}{b'}$.

Hence the original equations may be replaced by

$$a(1)+b(2)=0$$
$$a'(1)+b'(2)=0,$$

so that the discussion again reduces to that of Case (1).

§ 5. If $l=0$, $m=0$, then $k=\pm1$; $n=\pm1$, a case already discussed.

If $l\neq0$, $m=0$, there result, when $k=+1$,

$$(1)'\equiv(1)+l(2)$$
$$(2)'\equiv\quad-(2).$$

From the identity $(2)'\equiv-(2)$ it follows that (2) has the form

$$x-\xi+\mathrm{K}(y-\eta)+\mathrm{L}(x\eta-y\xi)=0,\ (\mathrm{K}\text{ and }\mathrm{L}\text{ constants}).$$

The other identity leads to

$$\xi(\mathrm{A}_1x+\mathrm{B}_1y+\mathrm{C}_1)+\eta(\mathrm{A}_2x+\mathrm{B}_2y+\mathrm{C}_2)+\mathrm{A}_3x+\mathrm{B}_3y+\mathrm{C}_3$$
$$\equiv\Sigma x(\mathrm{A}_1\xi+\mathrm{B}_1\eta+\mathrm{C}_1)+l[x-\xi+\mathrm{K}(y-\eta)+\mathrm{L}(x\eta-y\xi)].$$

The comparison of coefficients shows that the corresponding equation is

(1) $$\xi(\mathrm{A}_1x+\mathrm{B}_1y+\mathrm{C}_1)+\eta(\mathrm{B}_1x+\mathrm{B}_2y+\mathrm{C}_2)+\mathrm{C}_1x+\mathrm{C}_2y+\mathrm{C}_3+l(\mathrm{L}x\eta+x+\mathrm{K}y)=0;$$

while (2) is

$$x-\xi+\mathrm{K}(y-\eta)+\mathrm{L}(x\eta-y\xi)=0.$$

Now, so far as solutions of (1) and (2) are concerned,

$$x+\mathrm{K}y+\mathrm{L}x\eta=\xi+\mathrm{K}\eta+\mathrm{L}y\xi$$
$$=\tfrac{1}{2}\{x+\xi+\mathrm{K}(y+\eta)+\mathrm{L}(x\eta+y\xi)\}.$$

Therefore for (1) may be substituted

$$\xi(\mathrm{A}_1x+\ \ldots\ \ )+\text{etc.}+\frac{l}{2}[x+\xi+\mathrm{K}(y+\eta)+\mathrm{L}(x\eta+y\xi)]=0,$$

an equation collaterally symmetrical in $xy\,|\,\xi\eta$, and therefore of a type already discussed.

Finally, if $l\neq0$, $m=0$, $k=-1$, we obtain

$$(1)'=-(1)+l(2)$$
$$(2)'=+(2),$$

the solutions for which are the same as for

$$(1)-\frac{l}{2}(2)=0$$

$$(2)=0,$$

which are of a type already discussed.

§ 6. The analysis therefore leads to the conclusion that when the transformation is involutive the bilinear equations may be reduced to one or other of the types I. and II.

If we consider (ξ, η) as a fixed point, the equation I. is simply its polar with respect to the conic

$$A_1x^2+2B_1xy+B_2y^2+2C_1x+2C_2y+C_3=0;$$

whereas the equation

$$C_1(x-\xi)+C_2(y-\eta)+B_1(\xi y-x\eta)=0$$

is simply the equation to the straight line joining (ξ, η) to the fixed point $\left(-\frac{C_2}{B_1}, \frac{C_1}{B_1}\right)$.

Hence the theorem :—

The most general transformation of the nature of a quadric inversion, in which to a straight line corresponds a conic, may be obtained as a point transformation in which :—

First.—*To a point corresponds the intersection of its polars with respect to two fixed conics; or*

Second.—*To a point corresponds the intersection of its polar with respect to a fixed conic with the straight line joining the point to a fixed point.*

The case in which there are two equations of the form II. simply corresponds to the identical transformation. Naturally there are various degenerate cases, for many of which CZUBER's paper may be profitably consulted. The ordinary inversion is a particular case of the second transformation in which the conic is a circle, while the fixed point is its centre.

Both transformations have already been discussed geometrically, the first by BELTRAMI in 1863, in his well-known memoir, "Intorno alle coniche di nove Punti" (*Mem. della Acad. di Bologna*, Tomo II.); the second by HIRST ("Quadric Inversion of Plane Curves," *Proc. R. S. L.*, 1865).

HIRST never mentions the Beltrami transformations, although BELTRAMI had already shown how to obtain certain Hirst transformations, such as the ordinary inversion. CZUBER (*l.c.*), in his analytical discussion, has omitted the Hirst transformations entirely.

§ 7. The two transformations present several points of contrast, and that of BELTRAMI would appear to be the more symmetrical.

In the Beltrami transformation let the two conics be represented by

$$\begin{aligned} ax^2+by^2+cz^2&=0\\ lx^2+my^2+nz^2&=0, \end{aligned}$$

as referred to their common self-conjugate triangle X Y Z.

To any point (ξ, η, ζ) corresponds the point given by

$$\begin{aligned} ax\xi+by\eta+cz\zeta&=0\\ lx\xi+my\eta+nz\zeta&=0, \end{aligned}$$

therefore

$$x:y:z=(bn-cm)\eta\zeta:(cl-an)\zeta\xi:(am-bl)\xi\eta$$

and similarly

$$\xi:\eta:\zeta=(bn-cm)yz: \quad \text{etc.} \quad : \quad \text{etc.}$$

There are *four* self-corresponding points, the points A, B, C, D, in which the two conics cut, so that X, Y, Z are the intersections of pairs of opposite sides of this quadrangle.

To a straight line

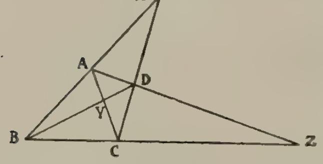
$$px+qy+rz=0$$

corresponds the conic

$$\Sigma p(bn-cm)\eta\zeta=0,$$

i.e., a conic through the principal points X, Y, Z. This conic is not degenerate unless a coefficient is zero. (This would always happen if $b/c=m/n$.)

If $p=0$, *i.e.*, if the line passes through X, the conic breaks up into the line Y Z (which corresponds to X) and a line through X. Thus, if to the point P corresponds P′, to X P corresponds X P′, and inversely; so that the lines through a principal point



X are paired in involution, the two self-corresponding lines of the involution being simply the two sides of A B C D that pass through X, viz., A B and C D. Any two corresponding lines through X are therefore harmonically separated by X A B and X D C.

If a line X P cuts B C in P, the ray X P′ corresponding to X P therefore cuts B C in P′, which is the harmonic conjugate of P with respect to B and C. Hence Beltrami's theorem: To a straight line not passing through a principal point corresponds a conic through the following nine points—the three points X, Y, Z, and the harmonic conjugates of all such points P in which the straight line cuts the six sides of the quadrangle A B C D.

The discussion is simplified by noting that the conics of reference may be replaced by any two conics of the system through A B C D, and in particular by two pairs of opposite sides of A B C D, especially when all these points are real.

Beltrami also proves that to a curve of degree n, passing α times through X, β times through Y, γ times through Z, there corresponds a curve of degree n', passing α' times through X, β' times through Y, γ' times through Z, where

$$n'=2n-\alpha-\beta-\gamma;$$
$$n'-\alpha'=n-\alpha;\ n'-\beta'=n-\beta;\ n'-\gamma'=n-\gamma;$$
$$n=2n'-\alpha'-\beta'-\gamma'.$$

One case is worthy of note. To a conic through two principal points corresponds a conic through the same points. If the conics intersect in P and P′, P and P′ are corresponding points and are coincident only when the conic passes through a vertex of A B C D, in which case the conics touch at that point. Hence any non-degenerate conic through two principal points and through two of the points A B C D must correspond to itself, for two conics cannot have four common points and contact at two of these points without coinciding. The points on such a conic are paired in an involution, and therefore the joins of corresponding points are concurrent, the centre of the involution being the intersections of the tangents at these two self-corresponding points through which the conic passes.

§ 8. In the Hirst transformation all points on the fixed conic are self-corresponding points, and the three principal points are X, the given fixed point, and the points of contact Y, Z of the tangents from X to the conic. To a straight line through X corresponds a line through X, but to a line through Y a line through Z, and *vice versa*, while the numerical equations for two corresponding curves are

$$n' = 2n - \alpha - \beta - \gamma ;$$
$$n' - \alpha' = n - \alpha ;\ n' - \beta' = n - \gamma ;\ n' - \gamma' = n - \beta ;$$
$$n = 2n' - \alpha' - \beta' - \gamma',$$

so that the Beltrami transformation is the more symmetrical.

In the Hirst transformation the points on any line through X are paired in an involution which is hyperbolic or elliptic according as the line cuts or does not cut the fundamental conic.

Also in order that a conic shall transform into a conic it must pass through two of the points X, Y, Z. Hence, if a conic transforms into itself it must pass through the two points Y and Z, for to a conic through X and Y corresponds a conic through X and Z, so that, if self-corresponding, it would pass through X, Y, Z, which is impossible.

If the self-corresponding conic through Y and Z cut the fundamental conic again in P and Q, since P and Q are self-corresponding points, it follows that X P and X Q are tangents to the new conic. The points on it are paired in involution, the centre of involution being, of course, the point X.

§ 9. If we take the fundamental conic to be

$$x^2 - yz = 0$$

it is easy to prove that the correspondence gives

$$x : y : z = \frac{1}{\xi} : \frac{1}{\zeta} : \frac{1}{\eta},$$

and the self-corresponding conics are given by

$$x^2 + yz + x(\mathrm{B}y + \mathrm{C}z) = 0.$$

There are therefore a two-fold infinity of such conics. Such a conic is over-specified, for it passes through Y, Z, P, Q, and is tangent to X P and to X Q. This suggests the following theorem :—

"If two conics cut in four points Y Z P Q, and if the pole of Y Z with respect to one conic is on the tangent at P to the second conic, it is also on the tangent at Q to the second conic and is the pole of P Q with respect to the second conic."

This proposition may be verified analytically as follows :

Let $x^2 - ayz = 0$ be the equation in trilinear co-ordinates to one of the conics, so that the pole of $x = 0$ with respect to it is the point X.

Let P Q be the line $2x + \mathrm{B}y + \mathrm{C}z = 0$.

The equation to any conic through the four points is

$$k(x^2 - ayz) + x(2x + \mathrm{B}y + \mathrm{C}z) = 0.$$

Let (ξ, η, ζ) be the co-ordinates of P. The equation to the tangent at P is

$$x(2k\xi+4\xi+B\eta+C\zeta)+y(\text{etc.})+z(\text{etc.})=0.$$

Hence, if this line pass through X,

$$2k\xi+4\xi+B\eta+C\zeta=0;$$

but

$$2\xi+B\eta+C\zeta=0$$

$$\therefore\ 2k\xi+2\xi \qquad =0$$

Hence $k=-1$, and the conic has for equation

$$x^2+ayz+x(By+Cz)=0,$$

while there is no distinction between the points P and Q. The theorem is therefore established. Various sub-cases arise according to the relative positions of the four common points.

The tangents at Y and Z to the second conic are given by

$$az+Bx=0 \quad (1)$$
$$ay+Cx=0 \quad (2),$$

and the tangent at P to the first conic is

$$2x\xi-ay\zeta-az\eta=0 \quad (3).$$

These tangent lines are concurrent, provided

$$2\xi+B\eta+C\zeta=0,$$

which is the case.

Hence the theorem :—

"If two conics cut in Y, Z, P, Q, and if the pole of Y Z with respect to the first conic coincides with the pole of P Q with respect to the second conic, then the pole of Y Z with respect to the second conic coincides with the pole of P Q with respect to the first conic."

Naturally both statements admit of reciprocation. In a sense, they are particular cases of the theorem that the eight tangents to two conics at their common points in general envelop a curve of the second class.

§ 10. It may be noted that one of the three canonical forms of the 1-1 quadric transformation, as given in Miss Scott's *Modern Analytical Geometry*,

$$x:y:z=\frac{1}{\xi}:\frac{1}{\eta}:\frac{1}{\zeta}$$

is a Beltrami transformation and not really a Hirst transformation.

It corresponds to

$$x\xi=y\eta=z\zeta,$$

so that (x, y, z) is the point of intersection of the polars of (ξ, η, ζ) with respect to the two degenerate conics

$$x^2-y^2=0\,;\ x^2-z^2=0.$$

The other two are Hirst transformations.

§ 11. Numerous examples of either transformation are to be found in the elementary geometry. One example of each is given.

If the base A A′ of a triangle A A′ C is kept fixed, the orthocentre P of the triangle is such that C is the orthocentre of the triangle A A′ P. Hence to C corresponds P and to P the point C. Moreover, if C moves in a straight line, the locus of P is in general a conic. The transformation is therefore one of the kind in question.

Take A A′ as the x-axis, so that A and A′ are the points $(a, 0)$, $(-a, 0)$. Let C be the point (ξ, η); then P has for co-ordinates

$$x=\xi;\ y=-(\xi^2-a^2)/\eta.$$

Hence the two pairs of co-ordinates are connected by the relation

$$x-\xi=0\ ;\ y\eta+x\xi-a^2=0.$$

Hence the straight line C P passes through the point at infinity in a direction perpendicular to A A′, and P is on the polar of C with respect to the circle whose diameter is A A′. Hence the transformation is a Hirst transformation.

The analysis also leads to the known proposition that the three lines found by taking the polar of each vertex of a triangle with respect to the circle which has for diameter the opposite side are concurrent in the orthocentre of the triangle.

§ 12. A Beltrami transformation is furnished by the following theorem of Professor Chrystal's, and its generalisation, viz.: "A circle meets the side B C of a triangle A B C in D and D′, C A in E and E′, and A B in F and F′. If A D, B E, C F be concurrent, then A D′, B E′, C F′ are also concurrent.

This is included in the following: P and P′ are two points taken in the side O A of the triangle O A B, and Q Q′ on the side O B such that O P.O P′ $=\rho$, O Q.O Q′ $=\sigma$, where ρ and σ are constants. A Q and B P meet in S; A Q′ and B P′ meet in S′. If A B is met by O S in R and by O S′ in R′, it follows that the six points P P′ Q Q′ R R′ lie on a conic. Moreover, to S corresponds a unique point S′ and inversely to S′ corresponds S, so that the transformation from S to S′ is involutive. Also if S move on a straight line S′ in general traces out a conic. The transformation ought therefore to be either a Hirst transformation or a Beltrami transformation.

To obtain the transformation, let O A and O B be taken as axes, and let O A $=a$, O B $=b$, O P $=\alpha$, O Q $=\beta$ (so that α and β vary).

Then A Q and B P have for equations

$$\left.\begin{aligned}\frac{x}{a}+\frac{y}{\beta}=1\\ \frac{x}{\alpha}+\frac{y}{b}=1\end{aligned}\right\}\quad \text{(i.)}$$

Hence, if S be the point (x, y),

$$\alpha=\frac{bx}{b-y};\quad \beta=\frac{ay}{a-x}.\quad \text{(ii.)}$$

Similarly if S′ be the point (ξ, η),

$$\alpha=\rho\frac{b-\eta}{b\xi} ; \beta=\sigma\frac{a-\xi}{a\eta} \quad \text{(iii.)}$$

$$\therefore \quad \frac{bx}{b-y} = \rho\frac{b-\eta}{b\xi} ; \frac{ay}{a-x} = \sigma\frac{a-\xi}{a\eta} \quad \text{(iv.)}$$

$$i.e., \quad b^2x\xi-\rho(b-y)(b-\eta)=0 ; a^2y\eta-\sigma(a-\xi)(a-\eta)=0. \quad \text{(v.)}$$

Hence (x, y) is the point of intersection of the polars of (ξ, η) with respect to the two degenerate conics :—

$$b^2x^2-\rho(b-y)^2=0 ; a^2y^2-\sigma(a-x)^2=0. \quad \text{(vi.)}$$

These conics determine a quadrilateral X Y Z W, which is such that the intercepts cut off by its sides on O A and O B are bisected at O, while the two pairs of lines pass through A and B respectively. For these conics may be substituted any two conics through X Y Z W.

Two of the principal points are A and B. The third principal point is not O, but the intersection C of X Z and Y W which are given by

$$x^2\sigma(\rho\sigma-a^2b^2)-y^2\rho(\rho\sigma-a^2b^2)-2a\rho\sigma(\sigma-b^2)x+2b\rho\sigma(\rho-a^2)y+\rho\sigma(a^2\sigma-b^2\rho)=0.$$

(A B C is the self-conjugate triangle of the system of conics.)

The lines through the origin parallel to these are given by

$$x^2\sigma-y^2\rho=0.$$

They are therefore parallel to the sides of the parallelogram K L M N, where K L M N are the points in which O A and O B are cut by the sides of X Y Z W. Hence the third line-pair X Z and Y W are such that each makes on the axes intercepts the square of whose ratio is $\rho : \sigma$. When $\rho=\sigma$, *i.e.*, when the circles of inversion coincide, the parallelogram is a rectangle, and each line of the third line-pair makes equal intercepts on the axes.

In the transformation as a Beltrami transformation the point O has no important *rôle.* In the general Beltrami transformation the lines through B are paired in involution, and they therefore determine a point-range in involution on any straight line such as O A. Similarly, the lines through A determine an involution on O B, and we have here the particular case in which the centres of involution of the point-ranges coincide in the point common to the two ranges. If A and B are real, and if L N pass through B, the locus of its middle point O is a conic passing through A, and similarly for O as the middle point of K M through A. Hence when A and B are real there may exist real points in finite number possessing the property in question.

The transformation suggests the apparent generalisation :—A and B are fixed points in the plane of two involutive point-ranges P P′ and Q Q′, where P P′, Q Q′ denote corresponding points of the respective involutions. The lines joining A and B to P P′ and Q Q′ determine a quadrilateral S T S′ T′ in which S corresponds to S′ (or T to T′) in a 1-1 involutive quadric transformation.

XIV.—*Apparatus for Measuring Strain and Applying Stress, with an Account of some Experiments on the Behaviour of Iron and Steel under Stress.* By E. G. Coker, D.Sc. Communicated by Dr C. G. Knott. (With Eight Plates.)

(Read 3rd June 1901.)

CONTENTS.

I.—Introduction.

The behaviour of metals under stress has long been the subject of investigation, both by mathematicians and physicists, so that the laws of strength are tolerably complete. Owing to the importance of iron and steel in construction, these materials have been subjected to very extensive tests, particularly in simple tension and compression.

Numerous tests of cylindrical iron and steel bars in torsion are also available, the bulk of these being tests to destruction of samples of material used in actual machines and structures designed by engineers. In such tests scientific accuracy is not of much importance, the chief consideration being the obtaining of sufficient data for use in design. The most accurate torsional work upon iron and steel has been the work of physicists, and nearly all their investigations have been conducted upon specimens of very small sectional area; the reasons for this, no doubt, being that such specimens in the form of wires are easily obtainable, and of great uniformity in size and quality, while large test pieces are costly to prepare, and, moreover, cause considerable difficulty in testing, because of the magnitude of the forces involved. Owing to the mode of manufacture, the physical properties of wire often differ to a considerable extent from turned specimens of iron and steel. These differences may be caused by the hardening effect of the drawing, minute cracks in the wires, want of roundness, and the like. It therefore appeared probable that experiments on the lines indicated by physicists would be of some service, and it was with this idea that the investigation was commenced.

The chief difficulty in the accurate investigation of the torsional properties of metal bars lies in the lack of suitable apparatus for the work; and after reviewing the chief machines available for measuring strain and applying torque—to all of which there seemed some objection—it was resolved to design and construct special appliances for the work.

Attention was first directed to the design and construction of a self-contained instrument for measuring strains, which should be sufficiently accurate to measure strains of one second of arc; and after some experiments an instrument was constructed which satisfied these conditions.* A modification of this was used in the work of this paper, and is described in Section II.

In most machines for applying torque, the construction is such that the weigh-lever can only be used for torsion in one direction, and the ends of the specimen are fixed, so that it is impossible, for instance, to apply a bending moment and torque, or a tension and torque, together.

A machine was therefore constructed to allow of torque in either direction, and also permit of the application of a uniform bending moment and a pure torque, to give a combined stress. A separate device was constructed for giving the combined stresses of tension and torsion.

II.—Description of the Apparatus.

1. *Instrument for Measuring Strains.*

In making measurements of small strains it is a great advantage to use an instrument which will read directly, and which is self-contained and wholly supported on the specimen under test, thereby avoiding external scales, the positions of which with respect to the specimen may be changed by a disturbing element, such as slipping of the grips, applied bending moment, and the like. In order to meet these conditions, an instrument was designed for the purpose of these experiments, and is shown in sectional elevation by fig. 1, and in side elevation by fig. 2. It consists of a graduated circle A mounted upon a chuck plate B, provided with three centring screws adjustable by hand. Upon the Vernier plate J, an arm O carries an extension K, upon which is secured a frame X, carrying a thick wire P. The movement of the wire is observed by a reading microscope carried in the sleeve R of an arm S mounted upon the short cylinder C, which latter is gripped upon the test bar by screws L.

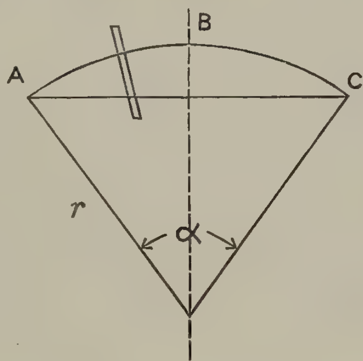
The reading microscope has an eye-piece T provided with a glass scale, and a right-

* Coker, "On Instruments for Measuring Small Torsional Strains." *Phil. Mag.*, December 1899.

angled prism is interposed between this and the objective W, so that readings can be easily taken. The tube Q is free to slide or rotate in its guide R, but, in order to readily focus the wire, this latter is carried in a frame X, pivoted upon the Vernier plate J, and adjusted by a screw V.

The microscope arm S is secured to the cylinder C by a divided collar, the two halves of which are pivoted on one side, and the free ends clamped by screws.

If it is desirable to turn the telescope round or to release it altogether, the screw may be thrown out of engagement. Readings are taken from one edge of the thick wire, and as this edge is very distinct, it fatigues the eye much less than a spider line or scratch upon glass, which latter have the further disadvantage of being of appreciable thickness. Fig. 5 shows the appearance of the field of view of the reading microscope and the wire P.



No appreciable error is caused by the fact that the divisions upon the glass scale of the microscope are linear measurements, while the movement of the wire is in a circle. For if ABC be the path of the wire and AC the chord, then the error is the difference between the arc ABC and its chord AC when the angle α is a small quantity of the first order.

$$\textit{I.e.}\quad \triangle = r\alpha - 2r\sin\frac{\alpha}{2}$$

$$= r\alpha - 2r\left(\frac{\alpha}{2} - \frac{\alpha^3}{2^3\lfloor 3} + \frac{\alpha^5}{2^5\lfloor 5} - \text{etc.}\right)$$

$$= r\left(\frac{\alpha^3}{2^2\lfloor 3} - \frac{\alpha^5}{2^4\lfloor 5} + \text{etc.}\right)$$

a small quantity of the third order, and therefore negligible. In practice this is shown to be the case, as no difference can be observed between the parallelism of the wire P and scale for any position of the former. The reading of the microscope scale may therefore be taken as directly proportional to the angular displacement, and the calibration is effected by moving the wire through a definite angle and noting the equivalent reading of the micrometer eye-piece.

It is essential that the graduated circle be set accurately upon the bar, with its plane perpendicular thereto and its centre coinciding with the longitudinal axis of the bar. An arrangement was devised to effect this, consisting of a pair of divided collars a, the halves pivoted together at b, and secured by nuts c. The collars are wedge-shaped, in radial section to engage with correspondingly wide-angled grooves, upon the chuck plate and cylinder only the angled sides being in contact, so that the collars are readily fixed or freed when required. The lower halves d of the divided collars are connected by one or more distance pieces e, so that when the former grip their respective grooves each piece has one degree of freedom with respect to the clamp, and this is sufficiently suppressed by the frictional grip of the collars, thereby causing the parts to act as one rigid whole for setting the instrument on the bar.

Both main pieces are chucked by set-screws, and with a little experience this can be effected as accurately as by self-centring chucks.*

In nearly all machines for applying torque some bending is also present, and it is therefore necessary to eliminate any possible errors due to this cause.

If the bar is bent in the plane containing its centre line and the observation wire, it has the effect of causing new parts of the wire to come opposite the scale, but no error in reading is caused thereby. If, however, the bar is bent in a plane at right angles to this, the effect of the bending will be read as an addition to, or subtraction from the twist. Bending in any other plane may be resolved into components in these two planes, and it is therefore only necessary to eliminate the error due to bending in a plane perpendicular to the plane of the paper.

The error may be got rid of by using two reading microscopes set opposite to one another, and a mean reading taken, but as this doubles the labour of observation it is inconvenient. Another plan is to arrange the wire mid-way between the sections gripped by the set-screws: then if

$2a$ be the length under measurement,
$\theta =$ angle of bending at first section,
$\phi =$ angle of bending at second section,
error in reading becomes $a\ (\sin\theta + \sin\phi)$;

and if θ and ϕ are equal and opposite the error vanishes. A specimen stressed by two equal and opposite couples applied at its ends bends into the arc of a circle, and fulfils the necessary condition for the equality of θ and ϕ, and in the application of torque this condition has been fulfilled. As a matter of precaution the observation wire is always set in the plane of bending.

In order to test the accuracy of the instrument, torsion tests were made (I.) with no bending, (II.) with bending moment of known amount. As an example the following may be quoted :—

Turned bar of rivet steel . .	Torsion arm = 15·00.
Diameter 0·662 . . .	Calibration.
Length under test . . .	1 min. = 54·4 divisions.

* *Loc. cit. ante.*

TABLE I.

Torque in inch lbs.	No Bending Moment.		Bending Moment 480 inch lbs.		Bending Moment 800 inch lbs.		Bending Moment 1120 inch lbs.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	0		0		0		0	
		-51		-51		-50		-50
7·5	51		51		50		50	
		-51		-50		-51		-50
15·0	102		101		101		100	
		-51		-51		-49		-50
22·5	153		152		150		150	
		-51		-51		-50		-51
30·0	204		203		200		201	
		-52		-51		-51		-51
37·5	256		254		251		252	
		-51		-52		-50		-51
45·0	307		306		301		303	
		-50		-50		-51		-51
52·5	357		356		352		354	
		-51		-50		-52		-50
60·0	408		406		404		404	
		-51		-51		-51		-49
67·5	459		457		455		453	
		-50		-50		-50		-50
75·0	509		507		505		503	
0	2		1		1		2	

The differences caused by bending will be discussed in Section XI.

2. *Machine for applying Torque and Bending.*

The apparatus used for applying twist in either direction, and for the combined stresses of twist and bending, is shown in side elevation by fig. 6, in plan by fig. 7, and in sectional end elevation by fig. 8. The machine was specially constructed for the work of this paper, and consists essentially of two similar and equal castings *A*, bored axially to receive double-coned spindles *B*, the outer ends of which project through the castings and are secured by nuts *C*. At the weigh-lever end the cone is secured to the casting by studs *D*, but at the other end, in order to take up the twist upon the specimen, the cone is gripped partly by the back-nut *C*, and partly by a plate *E* pressed against its face by studs.

Each casting is bored at right angles to the axis to receive arms *F G*, of which the former are used for hanging weights therefrom to give the torque, while the latter one, *G*, carries a link *H*, having an adjusting screw *I*, and nut *J*, whereby the weigh-levers *F* can always be brought to the horizontal position, the final adjustment being made with a sensitive level *K*; while the other carries a balance weight *L*. The ends of the specimen M under test are secured in grips NN upon the projecting ends of the cones B.

In order to obtain a pure torque and a pure bending moment, both acting at the same time, each casting is supported by a ring (fig. 9) encircling the spindles B, and furnished with friction rollers P, running in grooves in the spindles B. The rings have bearings Q, turning in stirrups R, these latter being hung from a horizontal bar S, by adjustable vertical hangers T. With this arrangement we get a pure torque of a known amount throughout the specimen.

Bending Moment.—Into the outer ends of the nuts C are screwed projecting arms V, of known length and carrying weights at their ends. These put a bending couple upon the specimen without shear, the arm of the couple being the distance of the weight from the hanger T. With this arrangement simple twist and simple bending, or any combination thereof, can be applied to a specimen with ease. The specimen is free to take up its own position of equilibrium, since it is imperfectly constrained—(the specimen can be easily rocked about even when fully loaded, but always comes back to the first position after a few oscillations)—and the conditions of stress are accurately known.

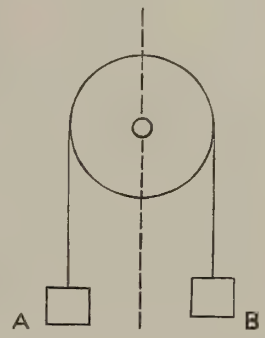
Corrections: Twist.—The results of tests on the friction of the roller bearing show that the friction is so small a quantity as not to introduce any sensible error.

Bending.—The friction error is that due to the stirrups embracing the bearings Q, which latter were made large purposely. No experiments were made in which the bending moment varied during the experiment; consequently it was sufficient in each case to calculate the error due to friction for the particular load applied, and to make the small corrections necessary. This has been done in all cases.

3. *Apparatus for applying Torque and Tension.*

The apparatus for applying torque and tension is shown in general elevation by fig. 11, and in plan by fig. 12. A detailed section of some of the parts with the measuring instrument fixed to the specimen is shown by fig. 13.

The specimen A was screwed into a turned piece B, having a slotted hole above, and tapped to receive a screw C, centred in a corresponding depression in a piece of tool steel D resting upon a plate E, which latter was carried by four bolts F, depending from a cast-iron beam G mounted upon two pillars H. Below, the specimen was screwed into a turned piece J, carrying a sleeve and pulley L, the lower end of the piece being fitted to receive a nut M and hanger N for weights. The torque was applied by weights attached to fine bands made of clock spring steel, which latter were attached to the pulley at convenient points, and passed over guide-pulleys O mounted on ball-bearings. The applied torque was balanced above by a double-ended lever P, keyed to the piece B, so that its axis passed through the point of suspension, and the lever being furnished with screws at its outer ends, these latter pressed equally against the pillars H. The weights used for applying the torque were made by Oertling, or were copies therefrom, and the twelve 200-pound weights for applying the tension were standard weights forming part of the equipment of the 100-ton Buckton testing machine in the laboratory. The method of suspension ensures that all the tension load is evenly distributed in the section of the specimen, and there is no correction for friction, as the load is a dead-weight one.



In applying the torque a small correction must be made for the friction of the pulleys. This was determined as follows :—

The pulley was first balanced by winding lead strips round its arms until it would stand in any position, or when rotated by a smart pull, continue to revolve several minutes. Next, equal weights, A B, were attached to the spring steel tape passing over the pulley with the ends vertical, and the additional mass required to just start the pulley in one direction was determined. The weights were then reversed, and the additional mass required was again determined, the mean value of the two being taken.

Let T_1 be the tension on one side.
T_2 ,, ,, the other side.
T_3 = tension horizontally.

We have

$$\frac{T_2}{T_1} = \epsilon^{\mu\pi}$$

$$\frac{T_3}{T_1} = \epsilon^{\mu\frac{\pi}{2}}$$

very approximately ; from which we get by a simple transformation,

$$T_3 = \sqrt{T_1T_2}.$$

This value was calculated throughout the range. As an example the following numbers may be quoted for the left-hand pulley :—

TABLE II.

T_1 pounds.	$T_3=\sqrt{T_1T_2}$	T_1 pounds.	$T_3=\sqrt{T_1T_2}$
2	2·0078	12	12·0118
4	4·008	14	14·012
6	6·0088	16	16·013
8	8·0095	18	18·014
10	10·0113	20	20·015

The correction for the other pulley was slightly less.

III.—Method of Experimenting.

The diameter of the bar to be tested was first ascertained by a micrometer caliper, the mean of several readings being taken. The measuring instrument was then applied, and the calibration value of the readings ascertained. The bar was then placed in the testing machine, and the balance weights adjusted to give zero torque. If bending moment had to be applied, this was effected before the final adjustment of the reading microscope, care being taken to bring the wire midway between the two sets of clamping screws, and also to set the wire in the most favourable position for taking accurate readings, viz., the plane in which bending takes place. Unless readings are taken at equal intervals of time, the time effect of a stress will show itself, and it is therefore very necessary that the separate loadings be at equal intervals. It was found that the most convenient interval was one and a half minutes, this being necessary to bring the weigh-beam back to its zero position ; and all readings were taken with this interval, except where otherwise stated.

IV.—The Form of the Stress-strain Curve.

Before taking up the detailed examination of the relation of stress to strain, it is of interest to consider the stress-strain curve as a whole.

Fig. 15 shows the general nature of the stress-strain curve for a wrought-iron or steel bar of circular section when subjected to a gradually increasing torque. Starting from no torque, and gradually increasing the load upon the bar, the relation between stress to strain is found to be a linear one until near the point a, when the defect from linearity is first noticed in the gradual creeping up of the readings, the whole twist upon the bar being at the rate of from 1° to 2° per inch of length.

At a the yield-point occurs, and there is a large increase in the strain, with no increase in the loading. The material has also changed from a nearly perfectly elastic to a semi-plastic condition, and the bar when released from load will no longer go back to zero, but shows a very considerable set. The material has also hardened by the process, and the curve rises at first quickly to a point c, and then more slowly until fracture occurs at d ; the strain then being generally considerably more than one hundred-fold the strain within the perfectly elastic condition.

V.—The Form of the Curve at the Yield-point.

The first sign of deviation from the linear law indicates the failure of the elastic state at the fibres most severely strained—viz., the outer ones—and a semi-plastic condition is entered upon, which, as the loading proceeds, extends inwardly until a more or less uniform shear stress is established throughout the section. The passage from the one state to the other in a solid specimen requires a certain range of stress, so that the diagram at the point a exhibits a well-marked rounding.

If we call q_o the maximum shear at the outer surface,

$$r_o = \text{radius of the specimen.}$$
$$r_p = \text{radius to which plasticity extends.}$$

Then the resistance of the bar to torque is the sum of the resistances due to (1) the still elastic core, (2) the semi-plastic shell, and may be represented by

$$\mathrm{T} = 2\pi q_o \frac{1}{r_o}\int_o^{r_p} r^3 dr + 2\pi q_o \int_{r_p}^{r_o} r^2 dr$$

where r = any radius

$$= \frac{\pi}{2} q_o \frac{r_p^4}{r_o} + 2\pi q_o \left(\frac{r_o^3}{3} - \frac{r_p^3}{3}\right)$$

up to the point where perfect elasticity prevails $r_o = r_p$ and

$$\mathrm{T} = \frac{\pi}{2} q_o r_o^3;$$

but when the specimen is wholly plastic

$$r_p = 0,$$

and we get

$$\mathrm{T} = \frac{2}{3}\pi q_o r_o^3,$$

which is $\frac{4}{3}$ of the value at the elastic limit.

It would therefore appear that if the bar changes from the elastic to the plastic condition at the yield-point, the maximum torque will be four thirds of that value at which the first-marked deviation from perfect elasticity occurs.

The result of experiment shows a fair agreement with this conclusion. In the example of a wrought-iron specimen quoted in the next section, Table III., col. I., the first-marked deviation occurs below 375 inch pounds, while the material failed at 525 inch pounds, giving a ratio of 1·4.

Taking another case for the steel specimen quoted in the same section, Table IV., col. I., the first deviation occurred below 675 inch pounds, and failure took place at 870 inch pounds, corresponding roughly to a ratio of 1·29, which is very close to $\frac{4}{3}$.

Having regard to the difficulty of observing exactly the first sign of failure, it seems probable that the conditions assumed are not far from the truth.

VI.—Recovery of Elasticity with Time.

If a bar of iron or steel be subjected to a torque causing a permanent strain in it, the condition of the bar becomes quite different; it no longer obeys Hooke's law, and the strain for a given stress is now greater than before the increase, being more marked at the higher loads. As an example we may take that of a turned wrought-iron bar of length between centres of 4·00 inches; diameter 0·472; calibration value of readings 1 min. = 12·85 divisions. The following results were obtained :—

TABLE III.

Column I.

Torque in inch lbs.	Reading.	Δ
0	0	
		243
75	243	
		243
150	486	
		244
225	730	
		242
300	972	
		254
375	1226	
		268
450	1484	
		140
480	1624	
		160
510	1784	
525	Went off scale	

The load was now removed, and immediately afterwards re-applied, with the following result :—

TABLE III.—*continued.*

Column II.

Torque in inch lbs.	Reading.	Δ
0	0	
		245
75	245	
		255
150	500	
		262
225	762	
		269
300	1031	
		289
375	1320	
		249
300	1071	
		249
225	822	
		259
150	563	
		265
75	298	
		272
0	26	

It will be seen at once that the bar exhibits quite different qualities from that shown before. The stress is now no longer proportional to the strain, and the curve showing the relation between the two no longer returns upon itself, but forms a looped figure.

Similar results are obtainable from bars subjected to tensional stress.* †

If the bar be tested again after a short interval of time, recovery will be found to be very marked. At the end of one hour a test of the bar gave the results shown by column III., there being a marked falling off of the increments at the higher loads. Thus the strain caused by increasing the torque from 300 inch pounds to 375 inch pounds now caused only 272 units of strain, instead of 289; and similarly at the end of three hours we find a further decrease to 268 units. The recovery of the bar was tested at suitable intervals of time, as shown in the annexed table, the effect becoming less apparent as the time increased; but practically perfect recovery was reached at the end of two days, and very little change was noticeable after this time.

TABLE III.—*continued.*

Column III. (One hour afterwards.)

Torque in inch lbs.	Reading.	Δ
0	0	
		246
75	246	
		252
150	498	
		255
225	753	
		266
300	1019	
		272
375	1291	
		248
300	1043	
		250
225	793	
		257
150	536	
		260
75	276	
		269
0	7	

The results may be shown graphically by direct plotting, but it is more convenient to adopt the plan of subtracting from each reading a number proportional to the torque, and plot the new set of readings thus obtained. The method is due to Prof. EWING, and is used in fig. 16, the diminution in the case being 200 units of scale reading for

* EWING, "On the Measurement of Small Strains in the Testing of Materials and Structures." *Proc. Royal Society*, May 1898.

† MUIR, "On the Recovery of Iron from Overstrain." *Phil. Trans.*, 1899.

TABLE III.—*continued.*

Torque in inch lbs.	Col. IV. 3 hours after.		Col. V. 6 hours after.		Col. VI. 12 hours after.		Col. VII. 1 day after.		Col. VIII. 2 days after.		Col. IX. 2 days after.		Col. X. 5 days after.		Col. XI. 6 days after.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	0		0		0		0		0		0		0		0	
		245		246		245		245		243		245		245		244
75	245		246		245		245		243		245		245		244	
		246		244		243		243		245		247		244		245
150	491		490		488		488		488		492		489		489	
		252		249		245		246		247		246		245		244
225	743		739		733		734		735		738		734		733	
		256		256		247		244		245		245		246		246
300	999		995		980		978		980		983		979		979	
		265		260		253		249		246		244		246		246
375	1264		1255		1233		1227		1226		1127		1225		1225	
		245		245		242		244		247		245		246		247
300	1019		1010		991		983		979		972		979		978	
		249		248		246		243		243		244		245		245
225	770		762		745		740		736		728		734		733	
		254		250		247		243		247		247		245		245
150	516		512		498		497		489		481		489		488	
		254		255		249		246		245		247		246		245
75	252		257		249		251		244		234		243		243	
		259		256		248		246		245		244		243		243
0	-7		1		1		0		-1		-10		0		0	

a torque of 75 inch pounds. A time-recovery curve, fig. 17, has been plotted, the ordinates of which correspond to the reading under a torque of 375 inch pounds and the abscissæ are times; this curve shows in a marked manner how rapid is the recovery at first.

As a means of comparison with the last bar, a steel bar was now tested, the specimen being classed as machinery steel, *i.e.*, semi-mild. Length under test 4·00; diameter 0·425; calibration 1 min. = 12·85 divisions.

The following results were obtained :—

Torque in inch lbs.	Reading.	Difference.
0	0	
75	385	385
150	770	385
225	1157	387
300	1543	386
375	1930	387
450	2323	393
525	2710	387
600	3103	393
675	3503	400
750	3920	420
780	4231	311
810	4675	444
825	4929	254
840	5309	380
855	6009	700
870	Went off scale	

The load was then removed, and again applied by increments of 75 inch pounds until a limit of 750 inch pounds was reached, the load being afterwards reduced by 75 inch pounds to nothing. Tests were made at intervals of time, as recorded in Table IV., which latter shows that the recovery is much slower in the former case; and even at the end of nineteen days the specimen showed signs of the initial overstrain.

The curves, fig. 18, showing the results of the experiments were plotted by the indirect method previously described, 350 units being subtracted for an increment of

TABLE IV.

Torque in inch lbs.	Col. I.		Col. II. Immediately after I.		Col. III. 1 hour after I.		Col. IV. 3 hours after I.		Col. V. 6 hours after I.		Col. VI. 1 day after I.		Col. VII. 3 days after I.		Col. VIII. 6 days after I.		Col. IX. 19 days after I.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	0		0		0		0		0		0		0		0		0	
75	385	385	395	395	395	395	392	392	390	390	390	390	390	390	389	389	389	389
150	770	385	792	397	790	395	784	392	783	393	778	388	777	387	780	391	778	389
225	1157	387	1202	410	1195	405	1187	403	1184	401	1175	397	1170	393	1168	388	1167	389
300	1543	386	1612	410	1611	416	1593	406	1590	406	1579	404	1564	394	1557	389	1555	388
375	1930	387	2026	414	2023	412	2007	414	1997	407	1988	409	1968	404	1957	400	1944	389
450	2320	390	2448	422	2441	418	2429	422	2412	415	2407	419	2380	412	2360	403	2337	393
525	2707	387	2860	412	2859	418	2844	415	2829	417	2822	415	2795	415	2770	410	2728	391
600	3100	393	3302	442	3284	425	3268	424	3244	415	3242	420	3216	421	3189	419	3128	400
675	3500	400	3746	444	3706	422	3609	431	3674	430	3669	427	3643	427	3616	427	3533	405
750	3920	420	4206	460	4166	460	4137	438	4108	434	4103	434	4077	424	4038	422	3940	407
675			3822	384	3768	398	3738	399	3715	393	3706	397	3694	383	3643	395	3549	391
600			3416	406	3370	398	3339	399	3315	400	3308	398	3292	402	3245	398	3159	390
525			3009	407	2964	406	2936	403	2911	404	2907	401	2891	401	2845	400	2766	393
450			2509	410	2548	416	2528	408	2503	408	2497	410	2485	406	2446	399	2374	392
375			2192	407	2135	413	2117	411	2092	411	2087	410	2080	405	2044	402	1981	393
300			1777	415	1724	411	1702	415	1679	413	1675	412	1672	408	1638	406	1587	394
225			1359	418	1305	419	1286	416	1266	413	1261	414	1260	412	1229	409	1191	396
150			935	424	889	416	864	422	840	426	837	424	839	421	814	415	793	398
75			497	438			433	431	414	426	415	422	416	423	407	417	393	400
0			52	445			3	430	– 14	428	– 9	424	– 1		– 11	418	– 4	397

75 inch pounds of torque. The rate of recovery is also shown by fig. 19, corresponding to fig. 17 of the former case. The readings at 750 inch pounds are plotted as ordinates, and the times as abscissæ. The difference between these latter curves is very apparent. From the diagrams it is apparent that the bar recovers very rapidly at first like the wrought iron; but this rate of recovery soon slackens and becomes less and less apparent as the time increases, and unless a very considerable time is given the recovery does not become complete (*cp.* Table XV., col. X.).

VII.—The Position of the Yield-point as affected by previous Stress.

The effect of a previous stress upon the properties of a bar has been explained in Section VI., and it remains to point out that overstrain in one direction has a very considerable influence upon the yield-point or curve, separating the elastic from the plastic stage; in fact it disappears, but gradually reappears again—the recovery in the case of the iron being practically complete in two days, while for the steel nineteen days effects partial recovery only. In both cases, if sufficient time be given, the yield-curve will assume a definite position above the last position, and this rise is augmented by every overstrain. As an example, we may take that of a wrought-iron bar, having a length under test of 4·00 inches; diameter 0·420 inches; calibration 1 min. = 12·8 divisions.

The bar was subjected to stress extending beyond the yield-point, and afterwards left to rest for a minimum period of $1\frac{1}{2}$ days, when the load was repeated. Eight tests were made, and each time there was a perceptible rise in the yield-curve. The observations were plotted with each curve spaced 1000 divisions from its neighbour. The curves are given in fig. 20, and require no further explanation.

VIII.—Twist in alternately opposite Directions.

It has long been a common assumption that the limits of elasticity for a bar subjected to torsion lie equally distant from the position of no torque, and this is no doubt true for a specimen not previously strained.

Apparently the first theoretical discussion of the problem is that by James Thomson,* and in his original paper he makes the further assumption "that the limits of elasticity in a substance which has already been strained beyond its limits of elasticity are equal on the two sides of the shape which it has when in equilibrium without disturbing force." This note, added in October 1877, goes on to say: "A supposition which may be true or may not be true. Experiment is urgently needed to test it, for its truth or falseness is a matter of much importance in the theory of elasticity."

The paper further points out that these assumptions lead to the important result that if a wire be overstrained, its strength to resist torsion in the original direction is twice that in the other direction.

* *Cambridge and Dublin Mathematical Journal*, 1848; and article "Elasticity," *Enc. Brit.*

From the mathematical point of view, THOMSON'S conclusions may be arrived at as follows :—

If a specimen be subjected to stress sufficient to cause a uniform shear throughout, and then be released, we have a new distribution of shear throughout the section, which may be expressed by

$$\text{Shear} = q_o - ar$$

where q_o = original shear at the external radius.
r = any radius.
a = a constant.

Since the bar is in equilibrium, we must have

$$\int_o^{r_o} (q_o - ar) 2\pi r^2 dr = 0$$

giving

$$a = \frac{4}{3} \frac{q_o}{r_o}.$$

Thus the shear in the bar is given by the expression

$$q = q_o\left(1 - \frac{4}{3} \frac{r}{r_o}\right).$$

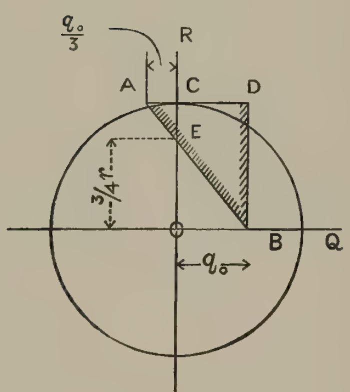


FIG. 21.

The distribution may be shown graphically, as in fig. 21, by a line AB, where OB = q_o and AC = $-\frac{q_o}{3}$.

This line evidently crosses the axis OR at the distance OE = $\frac{3}{4}r$, and gives a point on the circle of no stress.

Clearly, if no change has taken place in the limits of elasticity, the maximum shear is q_o at the centre, and evidently the bar will now stand a torque given by the expression

$$T = \int_o^{r_o} \left[\frac{4}{3} \frac{q_o}{r_o} r + q_o\left(1 - \frac{4}{3} \frac{r_o}{r}\right)\right] 2\pi r^2 dr = \frac{2}{3} \pi q_o r_o^3;$$

while in the opposite direction the torque will be given by the expression

$$T = \int_o^{r_o} \left[\frac{2}{3} \frac{q_o}{r_o} r - q_o\left(1 - \frac{4}{3} \frac{r}{r_o}\right)\right] 2\pi r^2 dr = \frac{1}{3} \pi q_o r_o^3.$$

It remains to be seen if the assumptions are justified.

In order to examine this point, a wrought-iron specimen was taken, having a length under test of 4·00 inches; diameter 0·634; calibration value 1 min. = 12·76 divisions of the scale.

The specimen was set in the machine so that torque could be applied in either direction, and observations were made of the strains for loads, which in turn caused permanent set in both the positive and negative directions.

These readings are plotted in fig. 22, and from an inspection of this it is apparent that the stress-strain curve was approximately linear before the yield-point was reached. The return curve was less so, but as soon as torsion was applied in the negative direction the linearity disappeared, and the strains, though irregular, became greater and greater as the torque increased. The material finally gave way under a torque of about 1100 inch pounds. The torque was now reversed, and the stress-strain curve became approximately linear until the zero torque was reached, from which point the curve began to bend over to the left, until a torque of 1175 inch pounds was reached.

In order to roughly test the behaviour of the specimen still further, the applied torque was continued, but no strain measurements were taken.

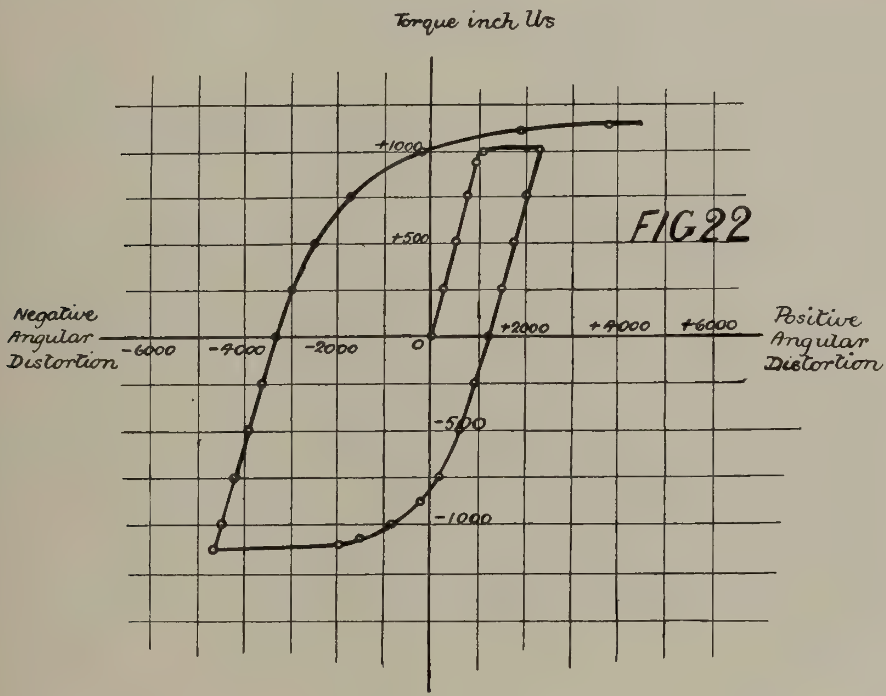


FIG 22

Each complete cycle produced a hardening effect on the bar, widening its limits of endurance each time until a final limit of 1750 inch-pounds of torque was reached after fourteen reversals of the stress. The bar was now cracked in several places along the minute seams of impurities, and further experiment seemed useless.

This experiment demonstrates that the limits of elasticity do not remain in their original positions, and, further, it shows that stress carried beyond the elastic limit in one direction reduces the other limit to zero. The conclusion derived from the theory above, that the bar is twice as strong to resist torsion in the original direction as in the other, is also not borne out by the experiment.

A second bar of wrought iron was next examined in the same manner, only four cycles being performed, of which the first two are shown in fig. 23 and the last two in fig. 24. These curves exhibit the same general properties as the one described above. It is evident from figs. 23 and 24 that, after the first reversal of stress, there is no perceivable yield-point, all such critical points being absent. The commonly received idea that raising the elastic limit in one direction lowers it in the contrary direction does not hold good here, since all critical points vanish.

The further development of the idea that the distance apart of the limits is a constant, appears to have no physical basis for the torsion of iron.

IX.—The Influence of Tension on Torsion.

1. *Tension within the Elastic Limit, and Torsion within the Elastic Limit.*

Among the notable experiments made upon the influence of tension upon torsion are those of M'Farlane upon steel pianoforte wire.*

From the article it does not appear that any experiments were made to ascertain whether tension within the elastic limit has any influence upon torsion within the elastic limit, but in any case it was thought worth while to make the experiments, as specimens of much larger diameter could be dealt with.

The first specimen tried had the comparatively large diameter of $\frac{3}{4}$ inch, and the maximum tension load which could be applied was 3000 pounds. Repeated experiments failed to show any difference in the torsional properties of the bar, whether loaded or unloaded.

A second bar was then prepared, having a diameter of $\frac{1}{2}$ an inch, and the experiment was repeated with tension loads varying from 200 to 3000 pounds, the latter corresponding to a stress of 15,300 pounds per square inch. The diameter of the pulley was 41·62 inches, so that a weight of one pound in each pan corresponded to a torque of 41·62 inch pounds.

The length of the specimen was 8 inches, and the calibration value gave one division of the scale = 16·85 seconds.

The following Table gives a summary of the results obtained :—

Table V.

Tension Load on Specimen. Lbs.	Mean Value of Reading corresponding to 1 lb. in each pair = 41·62 inch lbs.		Mean of Columns 2 and 3.
	Torque increasing.	Torque diminishing.	
200	59·00	58·98	58·99
600	59·00	58·74	58·87
1000	58·56	58·70	58·63
2000	58·59	58·70	58·65
3000	58·75	58·65	58·70

* *Enc. Brit.* Art. "Elasticity."

As will be seen from the last column, the values obtained differ very little, in no case varying more than one-half of one per cent.

The above experiments were carried out for me during the latter part of 1898 and the beginning of 1899 by Mr Colpitts—then a student in the Civil Engineering Department of the University. As a test of the accuracy of these results, a third bar was prepared, having a diameter of 0·375 inch; length 8·00. A new objective was fitted to the measuring instrument, rendering it much more sensitive. The calibration value gave 1 minute of arc = 62·4 divisions of the scale. This necessitated low torques, in order to prevent the observation wire from passing out of the field of view. A new pulley was used, having a diameter of 20·82 inches, and weighing with hangers, etc., 120 pounds.

A test was first made with no tension load beyond that of the pulley, and immediately afterwards a load of 2400 pounds was applied, corresponding to an increased load of 21,700 pounds per square inch. The readings obtained are shown in Table VI.

Note.—Considerable difficulty was experienced in making accurate readings when the tensional load was applied, as the time of vibration of the heavy weights with respect to the specimen was so large that any accidental motion due to the putting on or taking off the weights was very difficult to damp out.

A deep four-armed vane was attached to the hanger and dipped into a water-trough on the floor; this effected a great improvement, but did not wholly counteract the vibration. The readings obtained in Table VI. show a slight diminution when a tensional load is applied, but owing to the difficulties of observation mentioned above, the author feels he cannot lay much stress upon them. The observations were repeated with very nearly the same result. As will be seen in the next section, bending affects the angular distortion in the same manner.

Table VI.

Load in each Pan in lbs.	No Tension except Weight of Pulley.		Tension, 2400 lbs. + Weight of Pulley.	
	Reading.	Difference.	Reading.	Difference.
0·4	0		0	
		196		194
0·5	196		194	
		195		195
0·6	391		389	
		195		193
0·7	586		582	
		197		195
0·8	783		777	
		196		196
0·9	979		973	
		195		194
1·0	1174		1167	

2. *Effect of Tensional Stress on the Yield-point.*

The only experiments upon the yield-point appear to be those of M'FARLANE.* These showed that a tension lowered the yield-point. Reasoning from this result, Lord KELVIN concludes that a compression stress would raise it, but no experiments appear to have been made to verify this conclusion.

In order to examine the effect of tension at or about the yield-point, a bar of wrought iron was taken and cut into two parts; one specimen was turned truly parallel to a convenient diameter (0·424 inches), and the second was made exactly the same size. Both specimens were tested and found to be perfectly cylindrical, as far as could be ascertained by a micrometer gauge.

The first specimen was then tested in the ordinary way, with the result shown in column I., Table VII. The first noticeable deviation occurred when each pan was loaded with a weight of 16 pounds—corresponding to a torque of 333 inch-pounds, the maximum torque being 385 inch pounds.

The second bar was then stressed; but before the tension load was applied a preliminary reading was taken—to see whether the readings agreed with those from the first specimen, and, as will be seen (col. II.), the agreement is very close.

The specimen was now loaded with an additional 2400 pounds—corresponding to an increase of stress of 17,900 pounds per square inch—and a torque applied by increments. As shown by column III., a slight deviation was noticed at 333 inch-pounds, and failure was accomplished by a torque of 360 inch-pounds.

This result shows in a marked way the lowering of the yield-point by tension, and confirms M'FARLANE's experiments.

* *Loc. cit. ante.*

Table VII.

Diameter, 0·424″. Length, 4·00″. 12·8 divisions of scale = 1 min. of arc.

Load in each Pan. Lbs.	Column I.		Column II.	
	No Tension.		No Tension.	
	Reading.	Δ	Reading.	Δ
2	0		0	
		157		155
3	157		155	
		155		157
4	312		312	
		157		156
5	469		468	
		156		157
6	625		625	
		156		156
7	781		781	
		154		156
8	935		937	
		155		157
9	1090		1094	
		157		157
10	1247		1250	
		156		157
11	1403		1407	
		156		157
12	1559		1564	
		157		
13	1716			
		156		
14	1872			
		157		
15	2029			
		159		
16	2188			
		168		
17	2356			
		120		
17·5	2476			
		950		
18	3426			
		400		
18·2	3826			
18·5	Went off scale			

Column III.		
Load in each Pan. Lbs.	Tension 2400 lbs.	
	Reading.	Δ
2	0	
		156
3	156	
		155
4	311	
		156
5	467	
		157
6	624	
		158
7	782	
		156
8	938	
		156
9	1094	
		157
10	1251	
		155
11	1406	
		158
12	1564	
		158
13	1722	
		156
14	1878	
		156
15	2034	
		81
15·5	2115	
		106
16·0	2221	
		120
16·5	2341	
		472
17·0	2813	
17·3	Went off scale	

The results of Table VII. are plotted and are shown on fig. 25.

3. *Tension beyond the Elastic Limit.*

A machinery steel bar turned to a mean diameter of 0·537, with a length of 8·00 under test, was chosen. The bar was first placed in the torsion machine, and gave the results under test shown by col. I., Table VIII. The mean twist for 75 inch pounds was 278 divisions. The specimen was then set in the tension grips of the Buckton Testing Machine, and a gradually increased load applied, until a permanent set of 0·18 inches was produced. Immediately after, a fresh test in torsion was made, the results obtained being shown by col. II. The results are also plotted in fig. 26, from which it will be seen that the effect of the tensional overstrain has entirely altered the properties of the material under torsion. The strain is no longer proportional to the stress; the deviation being even more marked than in the case of specimens upset by a previous overstrain by torsion.

Table VIII.—*Steel Specimen.*

Diameter, 0·537″. Length, 8·00″. 12·75 divisions of scale = 1 min. of arc.

Column I. No Tension.			Column II. Specimen permanently lengthened 0·18 inch.		
Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ
0	0		0	0	
		278			299
75	278		75	299	
		279			301
150	557		150	600	
		278			321
225	835		225	921	
		277			339
300	1112		300	1260	
		277			364
375	1390		375	1624	
					406
			450	2030	
					484
			525	2514	
					612
			600	3126	
					292
			525	2834	
					298
			450	2536	
					300
			375	2236	
					300
			300	1936	
					305
			225	1631	
					310
			150	1321	
					311
			75	1010	
					310
			0	700	

X.—Effect of Torsion on Tension.

The effect of torsion upon the properties of a bar subjected to tensional stress was only examined below the elastic limit.

The measuring instrument used was of the Ewing type,* each unit of extension representing $\frac{1}{50000}$ inch upon an 8-inch length; diameter of bar, 0·498 inch.

A series of tests were made, beginning with no tension and increasing by equal increments until the yield-point of the material was reached. The results are recorded in Table IX., and it will be seen that no difference was observable, whether the bar was twisted or not, provided the elasticity of the bar remained unimpaired.

Table IX.

Loads. Lbs.	No Torsion.		Torque of 141 inch lbs.		Torque of 282 inch lbs.		Torque of 423 inch lbs.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
200	0		0		0		0	
		15		15		16		15
400	15		15		16		15	
		14		15		15		16
600	29		30		31		31	
		14		14		15		14
800	43		44		46		45	
		16		15		15		15
1000	59		59		61		60	
		14		15		14		15
1200	73		74		75		75	
		14		15		14		15
1400	87		89		89		90	
		15		14		14		14
1600	102		103		103		104	
		14		15		14		14
1800	116		118		117		118	
		15		15		14		16
2000	131		133		131		134	
		15		14		15		13
2200	146		147		146		147	
		15		14		14		15
2400	161		161		160		162	

XI.—Effect of Bending on Torsion.

One of the most interesting cases of stress which occurs in practice is that of torsion combined with bending, a subject which has received little or no attention from the experimental side. The apparatus described in Section II. enables uniform twist and uniform bending to be applied to a bar in any proportions, and the torsional strain to be accurately measured without reference to any external body; so that the bar can

* On Measurements of Small Strains in the Testing of Materials and Structures. By J. A. Ewing, F.R.S. *Proc. Royal Society*, May 1895.

assume its own position of equilibrium without affecting the readings of the measuring instrument.

Attention has been directed to the influence of bending on the torsional properties of a bar.

1. *Bending within the Elastic Limit, and Torsion within the Elastic Limit.*

In a previous section it has been shown that the effect of a tension produced little or no effect upon the torsional properties of a bar while in the elastic state. It might be expected, therefore, that a bending action which results in a varying tension and compression upon the longitudinal fibres would have but little effect upon the strain. As an example we may take the case quoted in Section II. of a rivet steel bar, in which an increase in the bending moment caused a slight diminution of the strain per unit torque.

Similar decrements were found in every case of the same type; as, for example, in the case of a semi-mild steel bar of diameter 0·869 inch, and of length 8·00 inches under test.

The unit reading corresponds to $\frac{1}{54}$ minute of arc.

TABLE X.

Torque in lbs.	No Bending Moment.		640 inch lbs.	
	Reading.	Difference.	Reading.	Difference.
0	0		0	
		166		163
75	166		163	
		166		164
150	332		327	
		166		166
225	498		493	
		165		166
300	663		659	
		167		165
375	830		824	

2. *Bending beyond the Elastic Limit.*

A more interesting case was that of a steel bar in which the bending moment was gradually increased until a permanent set was given to the bar in the plane of bending. The readings obtained are shown in Table XI., and a summary of the results in Table XII. It will be noticed that when the bar is bent the true value of the torque is given by its apparent value multiplied by cos θ, where θ = angle of bending of the weigh-lever about its axis.

The results show that the slight diminution in the readings within the elastic limit is followed by a much greater rise when the yield-point is reached.

Table XI.

Diameter of bar, 0·511″. Length, 8·00″. 54·7 divisions = 1 min. of arc.

Torque in inch lbs.	No Bending Moment.		Bending Moment 224 inch lbs.		Bending Moment 377 inch lbs.		Bending Moment 529 inch lbs.		Bending Moment 752 inch lbs. *		Bending Moment 922 inch lbs.		Bending Moment 1092 inch lbs.		Bending Moment 1432 inch lbs.		Bending Moment 1772 inch lbs.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	0		0		0		0		0		0		0		0		0	
		137		137		136		135		135		135		136		138		139
7·5	137		137		136		135		135		135		136		138		139	
		137		136		135		136		136		136		137		139		140
15·0	274		273		271		271		271		271		273		277		279	
		136		136		137		136		136		137		137		138		140
22·5	410		409		408		407		407		408		410		415		419	
		136		135		136		135		135		135		136		137		141
30·0	546		544		544		542		542		543		546		552		560	
		136		136		135		135		134		137		137		138		140
37·5	682		680		679		677		676		680		683		690		702	
		136		137		135		135		135		137		137		138		140
45·0	818		817		814		812		811		817		820		828		842	
		136		135		136		135		137		137		138		139		141
52·5	954		952		950		947		948		954		958		967		983	
		137		136		136		136		136		136		137		138		140
60·0	1091		1088		1086		1083		1084		1090		1095		1105		1123	

* Slight permanent set in the bar due to bending.

TABLE XII.

Diameter of bar, 0·511″. Length, 8·00″. 54·7 divisions = 1 min. of arc.

Bending Moment.	Readings for Torque of 60 inch lbs.	L. of Bending θ at Weigh-lever.	Cos θ.	Corrected Reading = Reading × cos θ.
0	1091	0°	1·000	1091
224	1088	30′	·999	1088
337	1086	48′	·999	1086
529	1083	1° 54′	·999	1083
752*	1084	2° 24′	·999	1084
922	1090	3° 10′	·998	1088
1092	1095	3° 20′	·998	1092
1432	1105	4° 30′	·997	1102
1772	1123	7° 0′	·993	1115

* Permanent set due to bending.

3. *Effect of Bending upon the Yield-point.*

The experiment was performed in a similar manner to that in Section IX. Two bars cut from the same rod were turned up to exactly the same size. One of them was tested in the ordinary manner, and the other was subjected to bending moment below the elastic limit, and then twisted beyond the yield-point. Each bar was 0·489 inch in diameter, the length under test being 4 inches, and 12·85 divisions on the scale corresponded to 1 minute of arc. The readings are recorded in Table XIII., in which column I. refers to the first specimen, and the remaining columns to the second specimen. The readings in column II. were made to check the correctness of the setting of the measuring instrument. The readings show a remarkable lowering of the yield-point for a bending moment of 668 inch pounds; the reason for which was not at first apparent, until it was noticed that the specimen took a permanent set, the ends being bent to a considerable degree.

At first sight this might appear to be a mere time effect; but in the author's opinion the probable cause was the increase of stress, due to the torque applied later. Apparently the maximum stress due to bending was of itself insufficient to cause yield, but the application of a further torque caused the principal stresses to assume the values

$$p_1 = \frac{p_n}{2} + \sqrt{\frac{p_n^2}{4} + q^2}$$

$$p_2 = \frac{p_n}{2} - \sqrt{\frac{p_n^2}{4} + q^2}$$

where p_n = normal stress due to bending.
q = shear stress due to applied torque.

If we adopt RANKINE'S theory of maximum stress, then p_1 in this case passed the working limit of the material, and a set resulted.

On the maximum strain theory of ST VENANT,

if e_1 = principal strain,
m = POISSON'S ratio,

then

$$\mathrm{E}e_1 = p_1 - \frac{p_2}{m}$$

$$= \frac{m-1}{2m} p_n + \frac{m+1}{2m} \sqrt{p_n^2 + 4q^2};$$

and since m has a value between 3 and 4 for steel, it is clear that the addition of a shear stress q would cause an increase in the value of e, which, if below the limit before, might increase sufficiently to cause failure.

The relation of stress to strain after the permanent set is clearly shown by a further test indicated in the table, column IV. There is now considerable hysteresis in the relation of stress to strain.

TABLE XIII.—*Combined Bending and Twist.*

	Bar I.		Bar II.							
	Col. I.		Col. II.		Col. III.			Col. IV.		
Torque in inch lbs.	No Bending Moment.		No Bending Moment.		Torque in inch lbs.	Bending Moment 668 inch lbs.		Torque in inch lbs.	3 hours after last Test.	
	Reading.	Δ	Reading.	Δ		Reading.	Δ		Reading.	Δ
0	0		0		0	0		0	0	
		195		196			196			201
75	195		196		75	196		75	201	
		195		196			196			202
150	390		392		150	392		150	403	
		197		193			198			207
225	587		585		225	590		225	610	
		197		197			215			208
300	784		782		300	805		300	818	
		196					765			266
375	980				375	1570		375	1084	
		125								194
420	1105				After 2 minutes, went off scale entirely			300	890	
		44								201
435	1149							225	689	
		42								203
450	1191							150	486	
		42								202
465	1233							75	284	
		43								208
480	1276							0	76	
		46								
495	1322									
		46								
510	1368									
		50								
525	1418									
		50								
540	1468									
		48								
555	1516									
		62								
570	1578									
		98								
585	1676									
		700								
605	2376									
		1100								
615	3476									

The author has not been able to find any other experiments bearing upon the position of the yield-point as affected by bending. The yield-point is, however, known to be lowered by tension, as mentioned previously.

A case which bears considerable resemblance to the case of permanent set last quoted is one by M'FARLANE,* who has shown that if a wire is twisted to nearly its limit of torsional elasticity, an increase in pull will cause the torque to give the wire a permanent set. This latter case can be easily explained in the same manner as the one described above.

XII.—EFFECT OF ANNEALING.

It has long been known that iron and mild steel stressed beyond the limit of elasticity regain their elastic properties when heated to a red heat and allowed to cool slowly. The process may be repeated many times without apparently changing the elastic properties of the material. The yield-point, however, is found to alter in position as the annealings proceed. In a particular case † a mild steel bar, which in an ordinary test would give an extension of 25 per cent. upon a ten-inch length, and a yield-point of about 18 tons per square inch, was stretched approximately $\frac{1}{4}$ inch, and annealed in the ordinary manner after each operation. Throughout the experiment the bar appeared to recover its elastic properties after each annealing, and finally broke with a total extension of approximately 100 per cent. The yield-point remained fairly constant, except at the end, when it experienced a rise.

Copper treated in the same manner has been drawn out by the author to considerably more than double its length in this way without causing fracture. Remarkable advances in our knowledge of annealing have been recently obtained by MUIR,‡ acting upon a suggestion of Professor EWING.

MUIR has shown that comparatively low temperatures, such as boiling water, will restore a strained bar to its elastic condition. The yield-point, however, alters during the process, and is always higher than in the original condition.

After a few applications of stress, followed by heating in boiling water, or even water at 50° C., the bar fractures, with a total extension not very different from a bar stressed to breaking without special treatment. The annealing at low temperatures, therefore, appears to be less complete than that at a high temperature.

In order to discover what effect a temperature of 100° C. would exert on a bar overstrained by a torque, the steel bar which had been used for experiments on the recovery of elasticity with time (see Section VI., Table IV.) was selected, and after being boiled for fifteen minutes was overstrained, giving results (col. X., Table XIV.) practically identical with those of col. I., Table IV., for the first part of the curve. As in practice it is troublesome to get exactly the same calibration value for each setting of the instrument, this latter (stripped of the reading microscope and wire holder) remained on the bar during the heating, and the labour of comparing readings whose unit values differed by

* Art. "Elasticity," *Enc. Brit.*, Part 21.

† COKER, "Note on the Endurance of Steel Bars subjected to Repetitions of Tensional Stress." *Proc. Inst. C.E.*, 1899.

‡ "The Recovery of Iron from Overstrain." By JAMES MUIR. *Phil. Trans.*, 1899.

a small amount was thereby avoided. Each stress operation causing overstrain was succeeded by a heating in water at 100° C. for fifteen minutes, and in all the bar was stressed eight times. The readings obtained are given in columns X.-XVII. in Table XIV., and are plotted in the ordinary manner, fig. 29, the curves being spaced 100 units apart for convenience. As might be expected, the curves show a general agreement with those obtained by MUIR, having regard to the fact that the stress is non-uniform.

TABLE XIV.

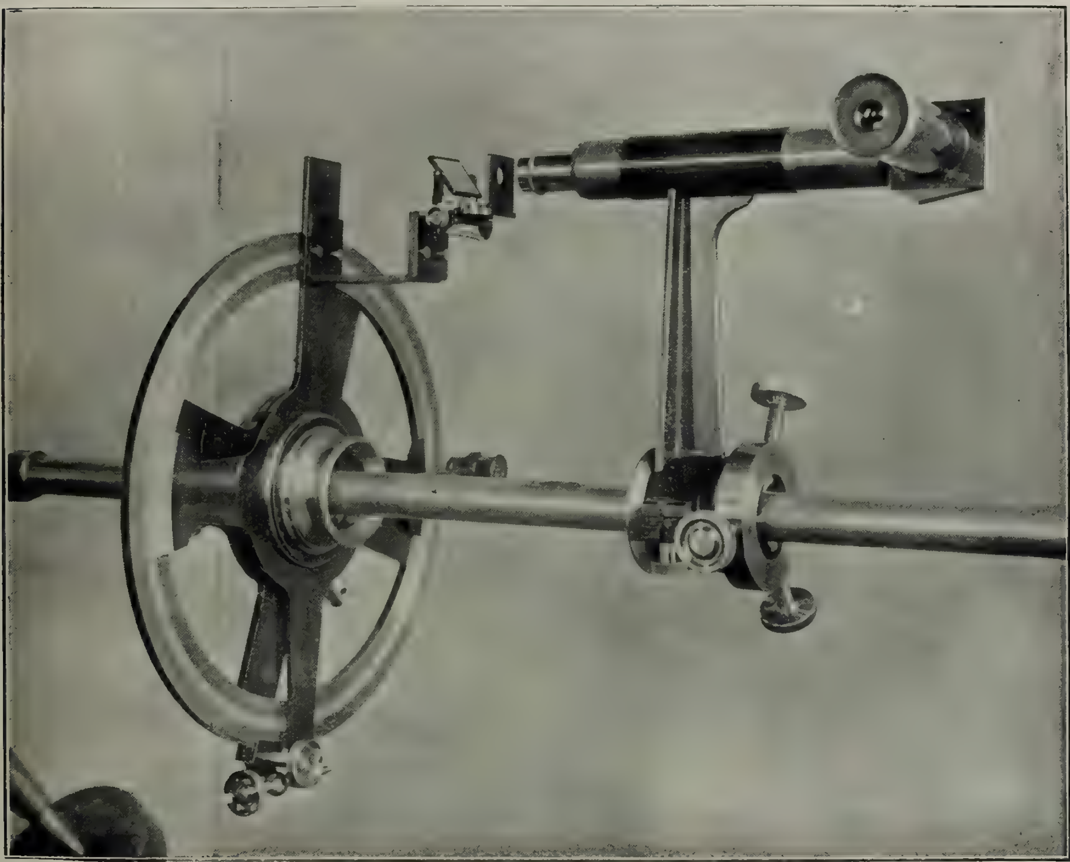
Column X.			Column XI.			Column XII.			Column XIII.		
Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ
0	0		0	0		0	0		0	0	
		385			390			389			386
75	385		75	390		75	389		75	386	
		385			390			387			389
150	770		150	780		150	776		150	775	
		388			389			389			389
225	1158		225	1169		225	1165		225	1164	
		387			389			389			388
300	1545		300	1558		300	1554		300	1552	
		389			388			387			389
350	1934		375	1946		375	1941		375	1941	
		392			391			389			389
450	2326		450	2337		450	2330		450	2330	
		389			390			388			392
525	2715		525	2727		525	2718		525	2722	
		395			388			389			388
600	3110		600	3115		600	3107		600	3110	
		400			392			389			389
675	3510		675	3507		675	3496		675	3499	
		404			390			386			394
750	3914		750	3897		750	3882		750	3893	
		426			394			391			402
825	4340		825	4291		825	4273		825	4295	
		463			410			398			402
900	4803		900	4701		900	4671		900	4697	
					617			419			414
930	Went off scale		975	5318		975	5090		975	5111	
					260			303			475
			990	5578		1025	5393		1050	5586	
			1015	Went off scale		1050	Went off scale		1080	Went off scale	

TABLE XIV.—*continued.*

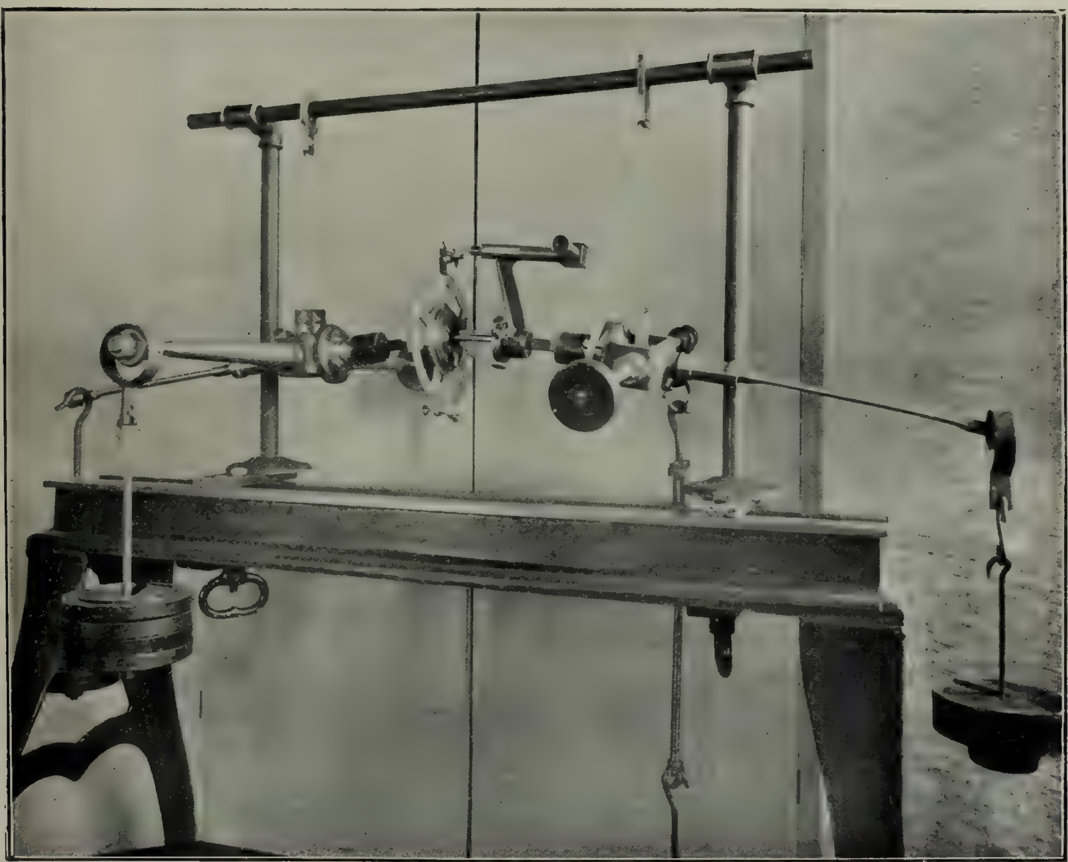
Column XIV.			Column XV.			Column XVI.			Column XVII.		
Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ	Torque in inch lbs.	Reading.	Δ
0	0		0	0		0	0		0	0	
		386			387			387			388
75	386		75	387		75	387		75	388	
		387			387			387			389
150	773		150	774		150	773		150	777	
		384			386			387			391
225	1157		225	1160		225	1159		225	1168	
		386			387			385			389
300	1543		300	1547		300	1544		300	1557	
		388			386			386			389
375	1931		375	1933		375	1930		375	1946	
		387			387			387			389
450	2318		450	2320		450	2317		450	2335	
		388			388			387			390
525	2706		525	2708		525	2704		525	2725	
		386			391			389			393
600	3092		600	3099		600	3093		600	3118	
		389			397			389			395
675	3481		675	3496		675	3482		675	3513	
		388			395			392			396
750	3869		750	3891		750	3874		750	3909	
		392			402			394			401
825	4261		825	4293		825	4268		825	4310	
		392			412			398			409
900	4653		900	4705		900	4666		900	4719	
		403			425			410			410
975	5056		975	5130		975	5076		975	5129	
		430			445			437			411
1050	5486		1050	5575		1050	5513		1050	5540	
		197			522						458
1080	5683		1125	6097		1105	Went off scale		1125	5998	
1110	Went off scale		1155	Went off scale					1200	Went off scale	

In conclusion the author desires to express his thanks to Prof. BOVEY, Dean of the Faculty of Applied Science, M'Gill University, who placed the resources of the Testing Laboratory of the Civil Engineering Department at his disposal, and also to Mr WITHYCOMBE, Mechanical Superintendent, who gave much help in the preparation of the apparatus

Mr E. G. COKER on an "Apparatus for measuring Strain and applying Stress."—PLATE I.

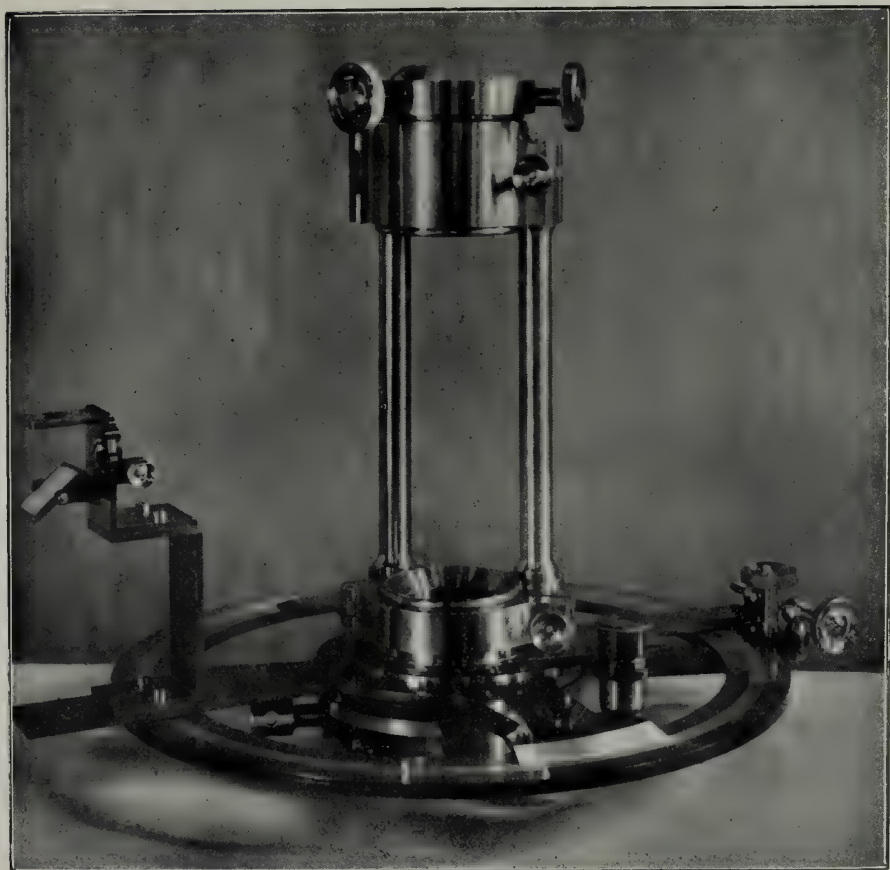


Instrument for measuring angular distortion upon the specimen.

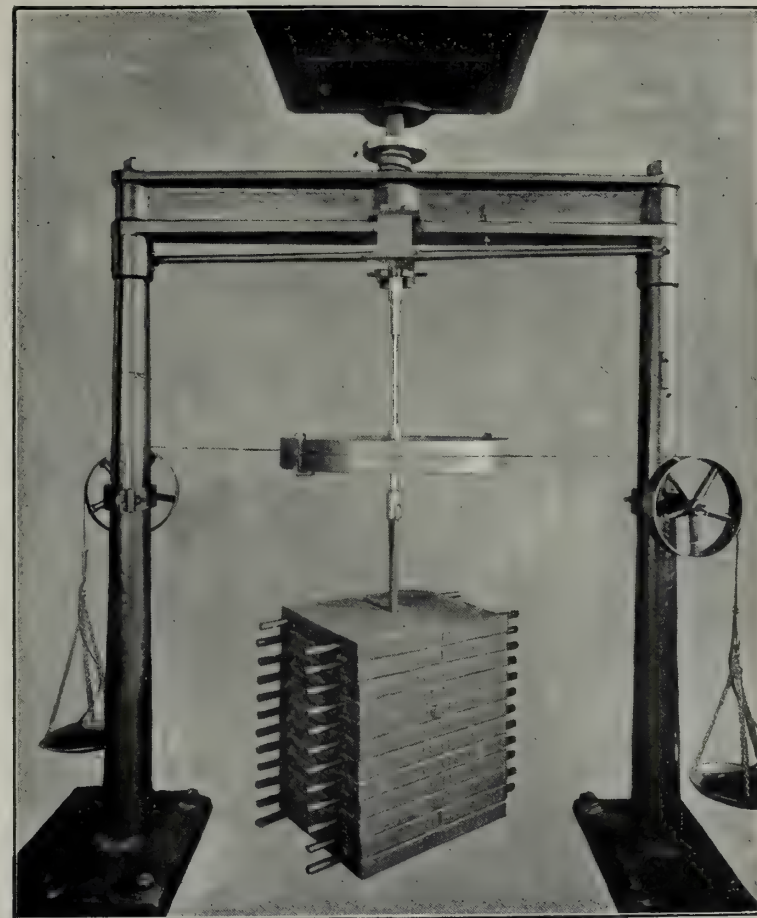


Apparatus for applying the combined stresses of Bending and Torsion.

Mr E. G. Coker on an "Apparatus for measuring Strain and applying Stress."—Plate II.

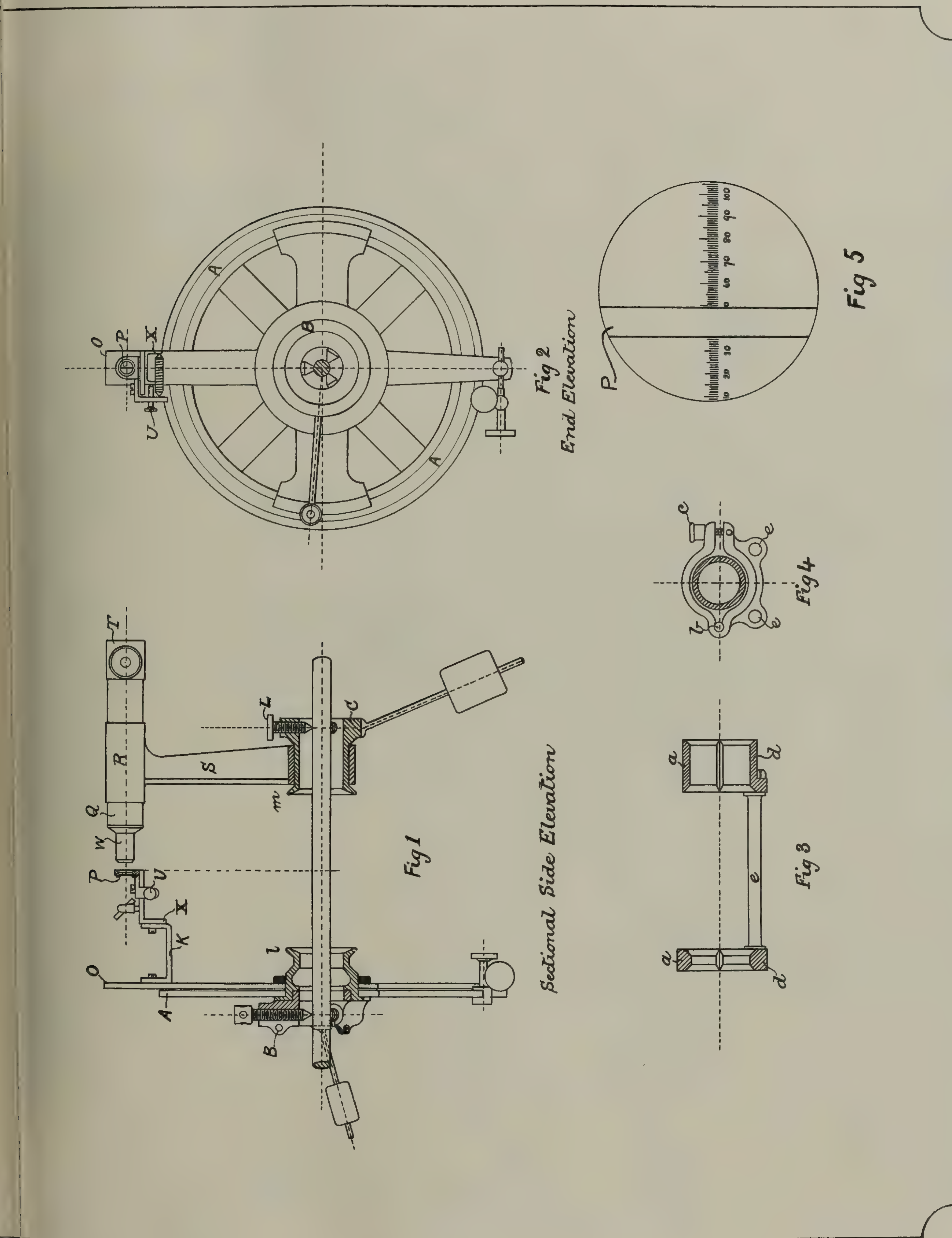


Instrument for measuring angular distortion, with the clamp in position for setting the pieces correctly on the specimen.

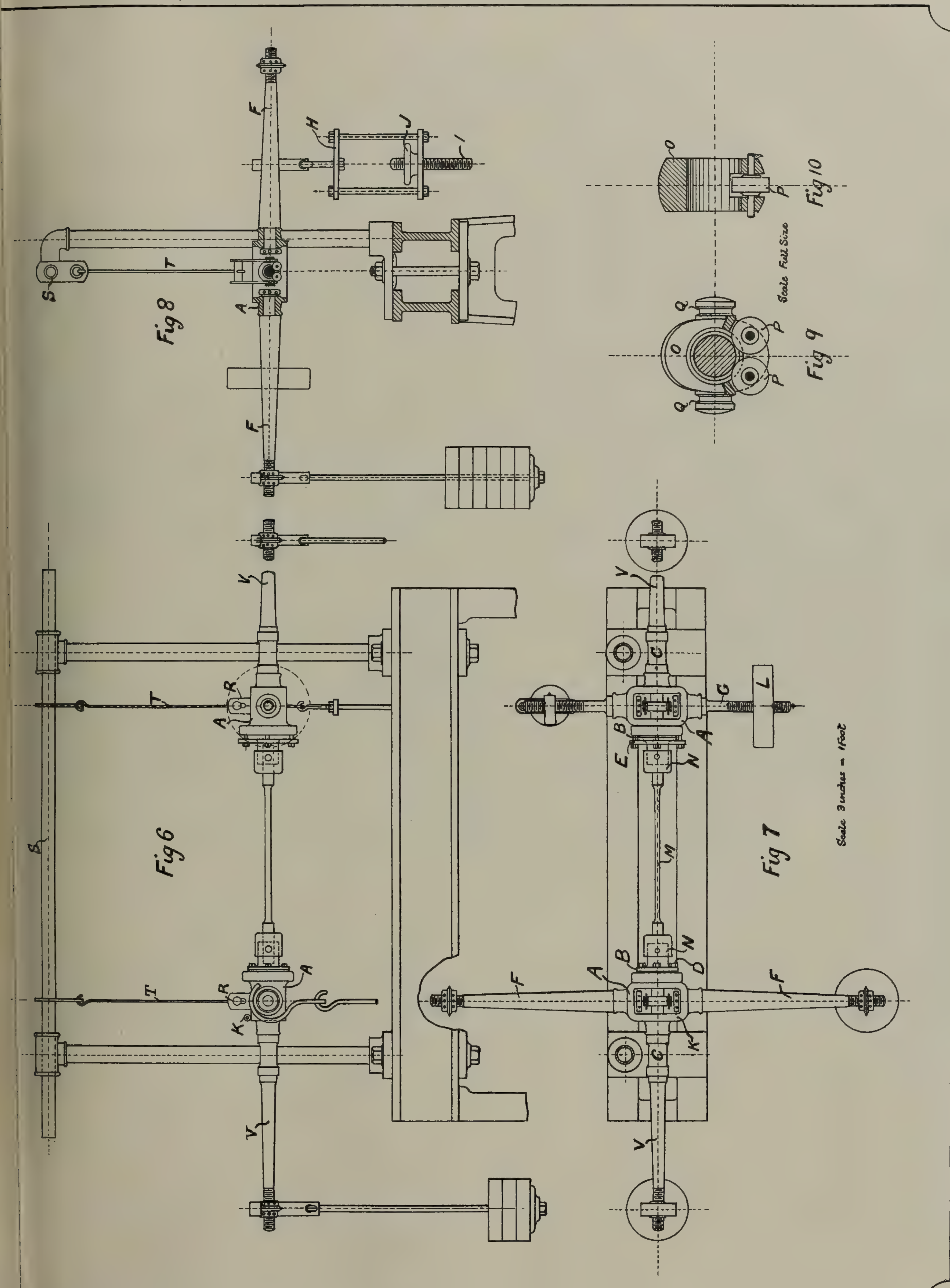


Apparatus for applying the combined stresses of Tension and Torsion.

DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE III.



DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE IV.



DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE V.

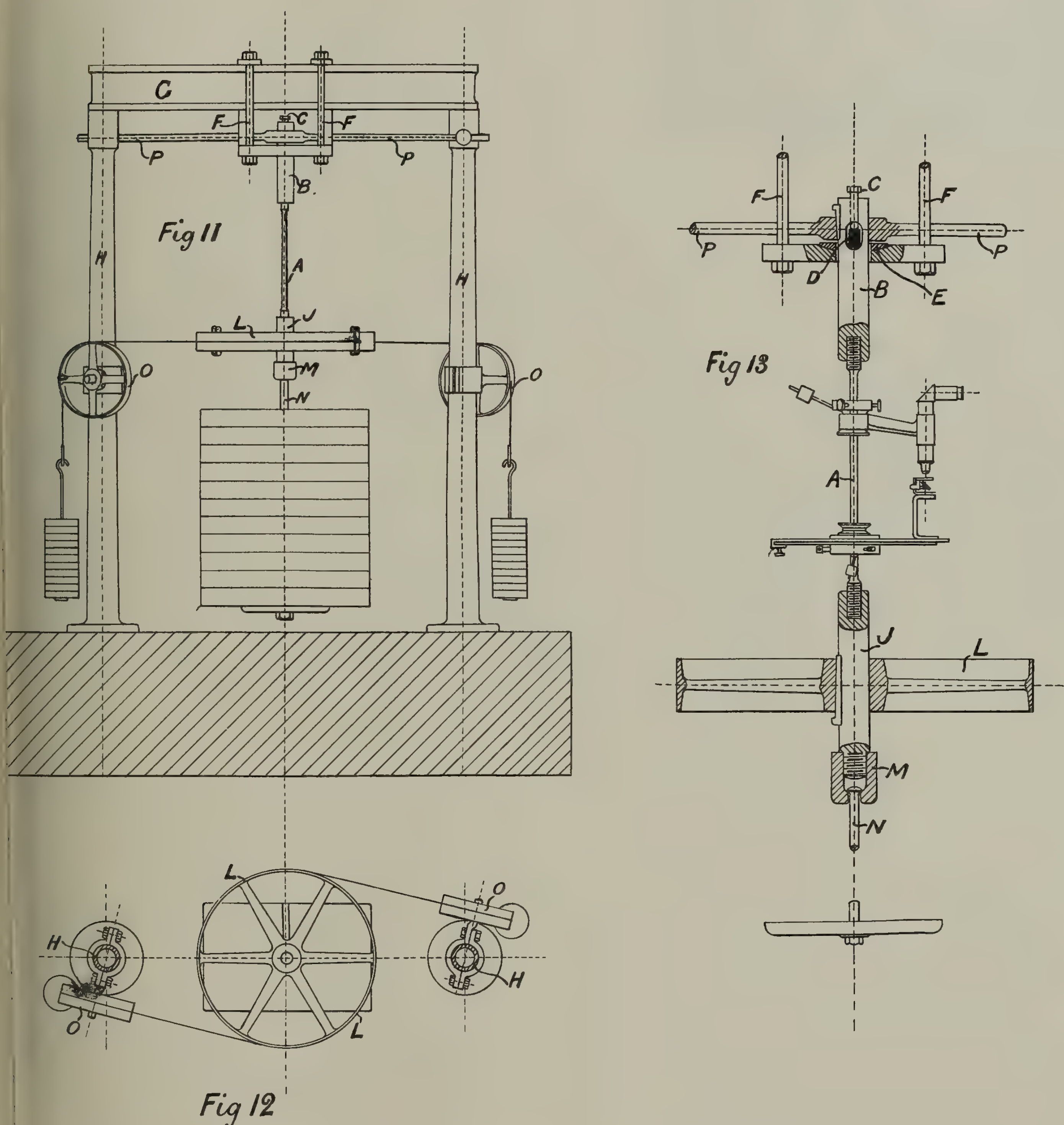
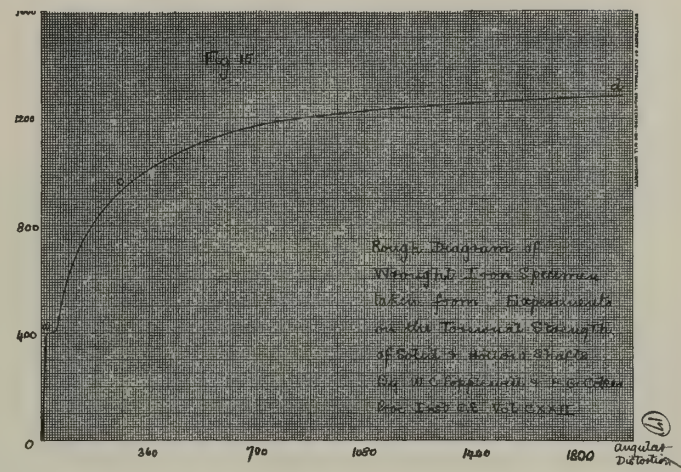


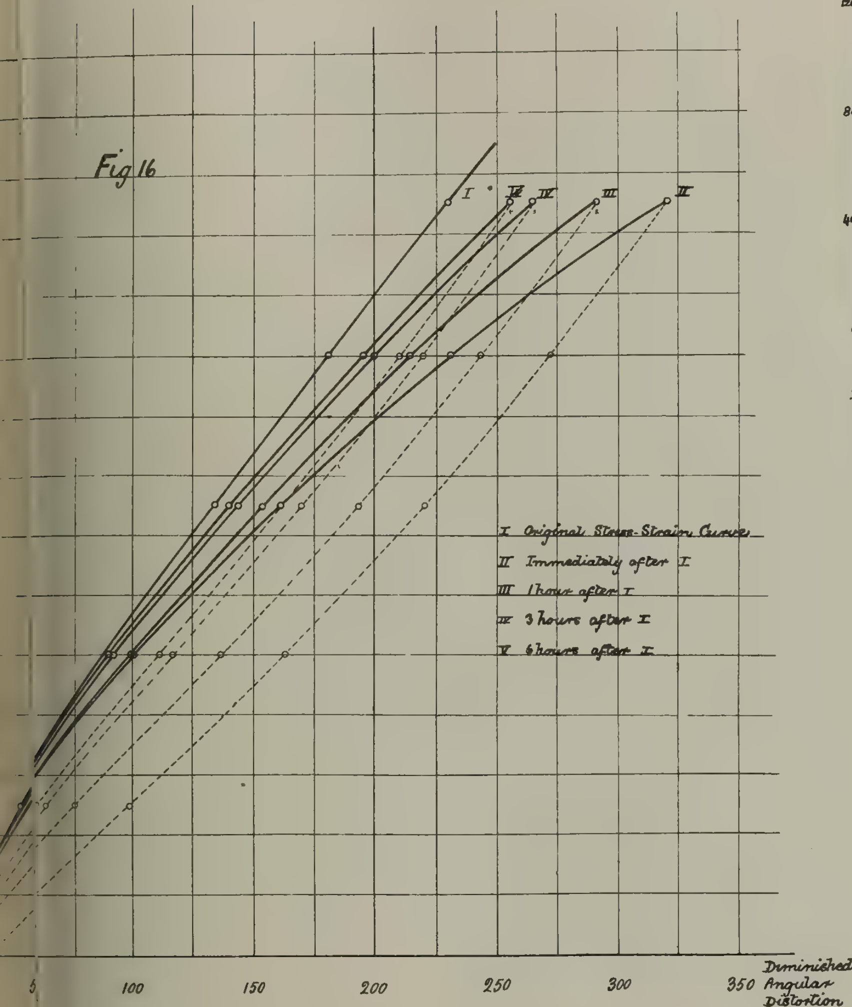
Fig 11

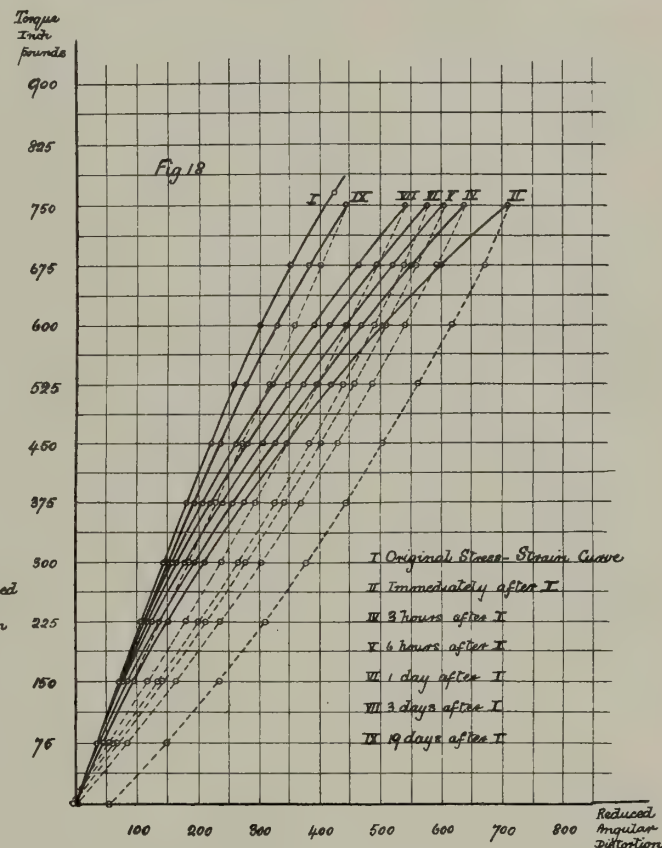
Fig 13

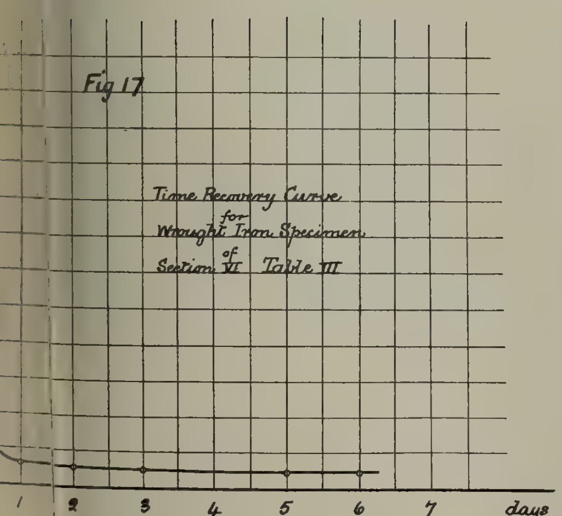
Fig 12

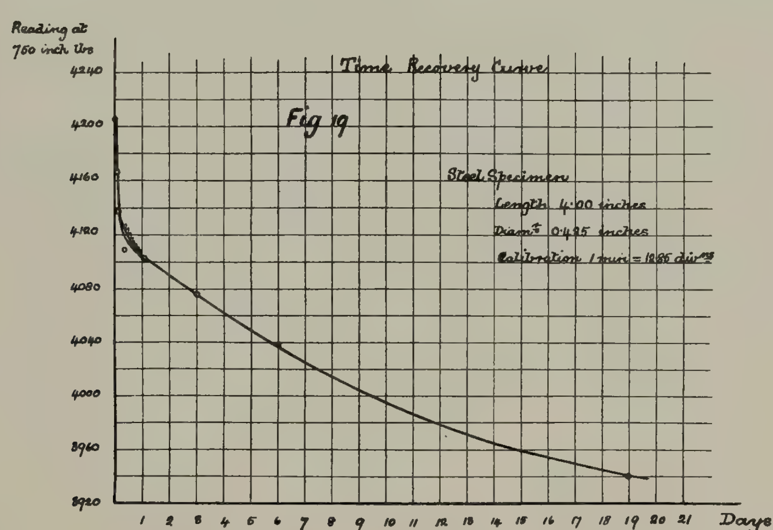
Dr. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—Plate VI.



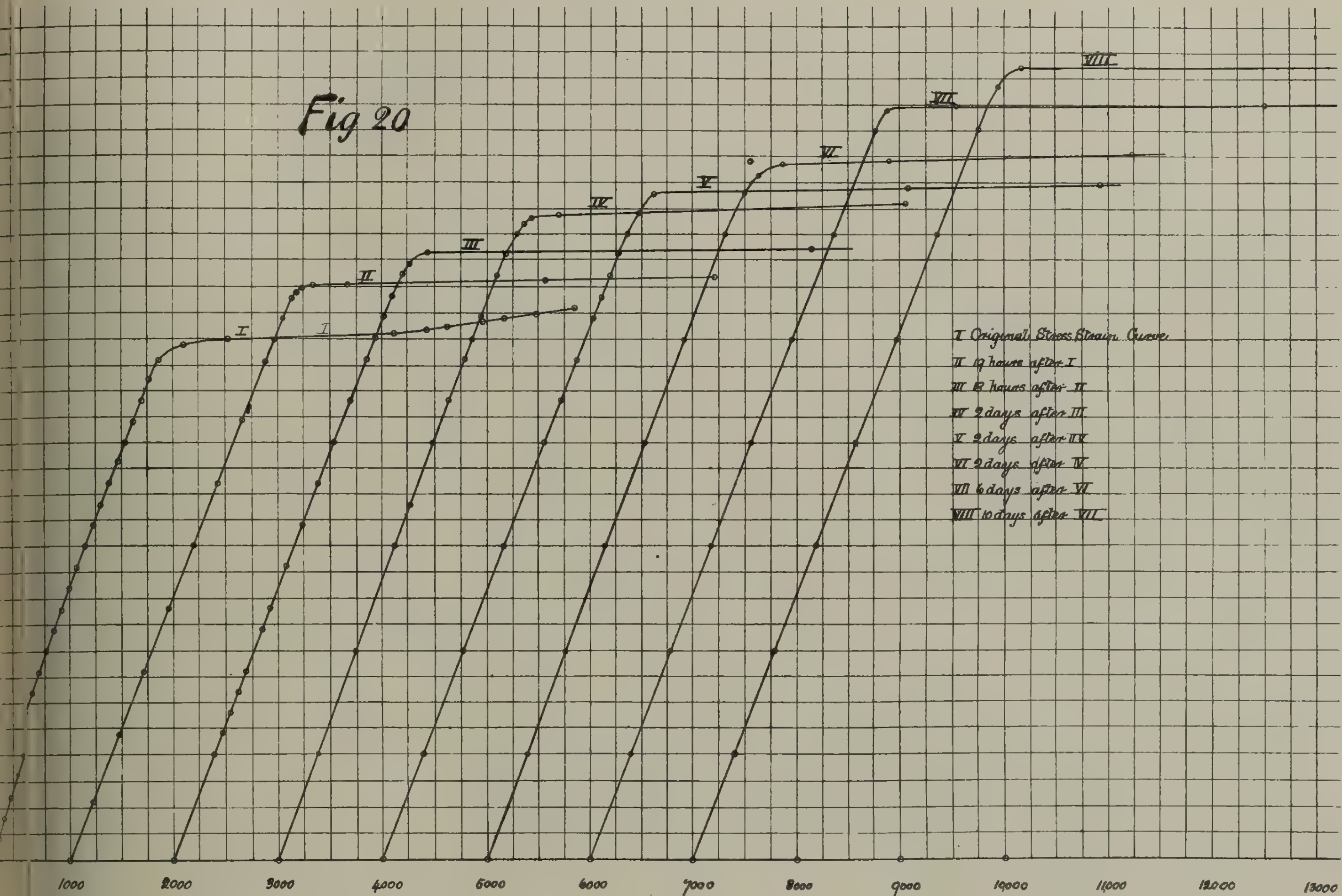


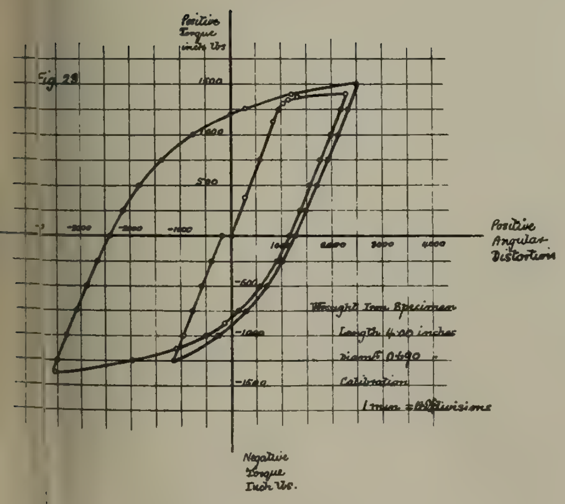


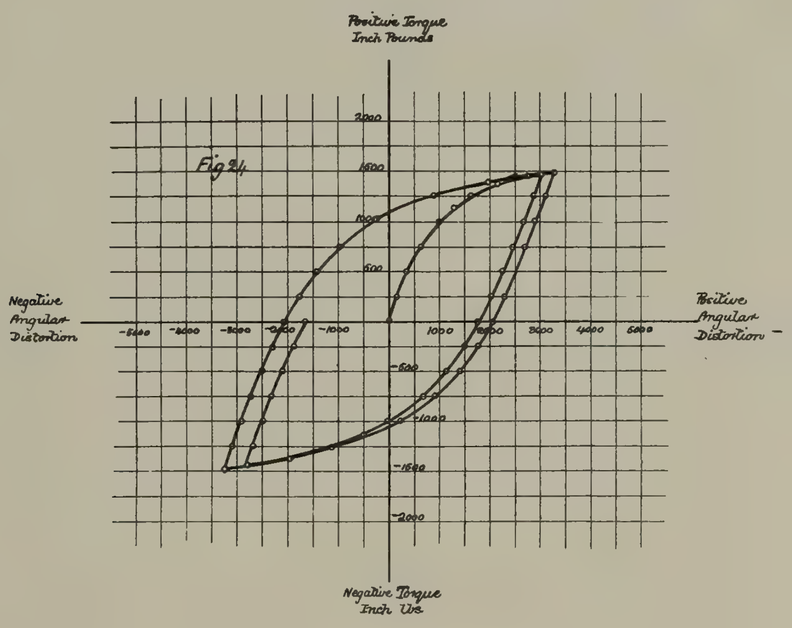




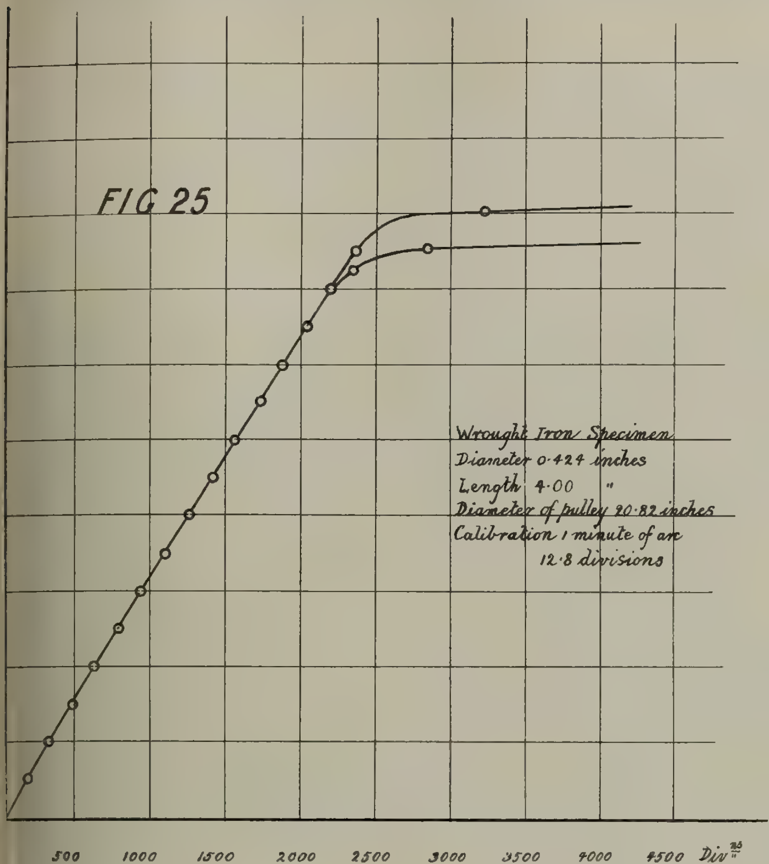
DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE VII.

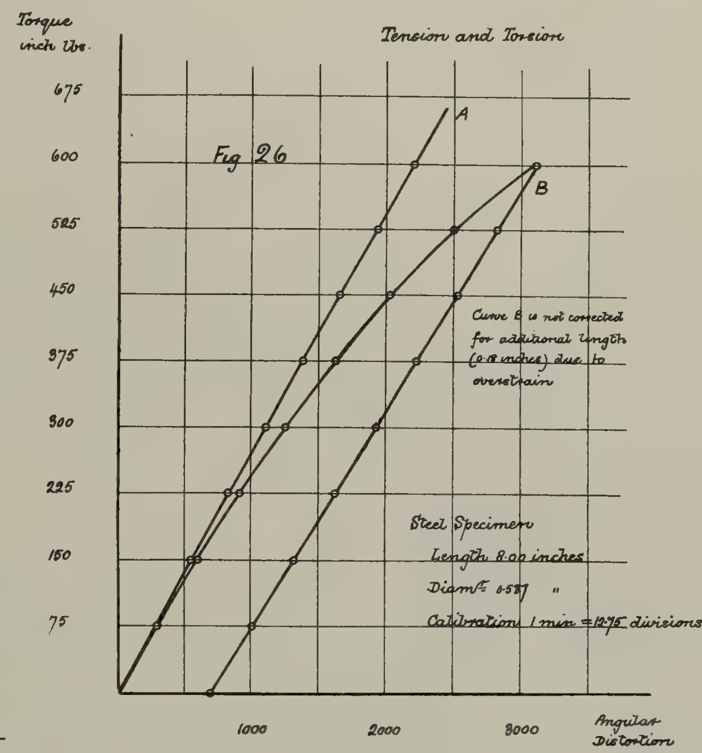


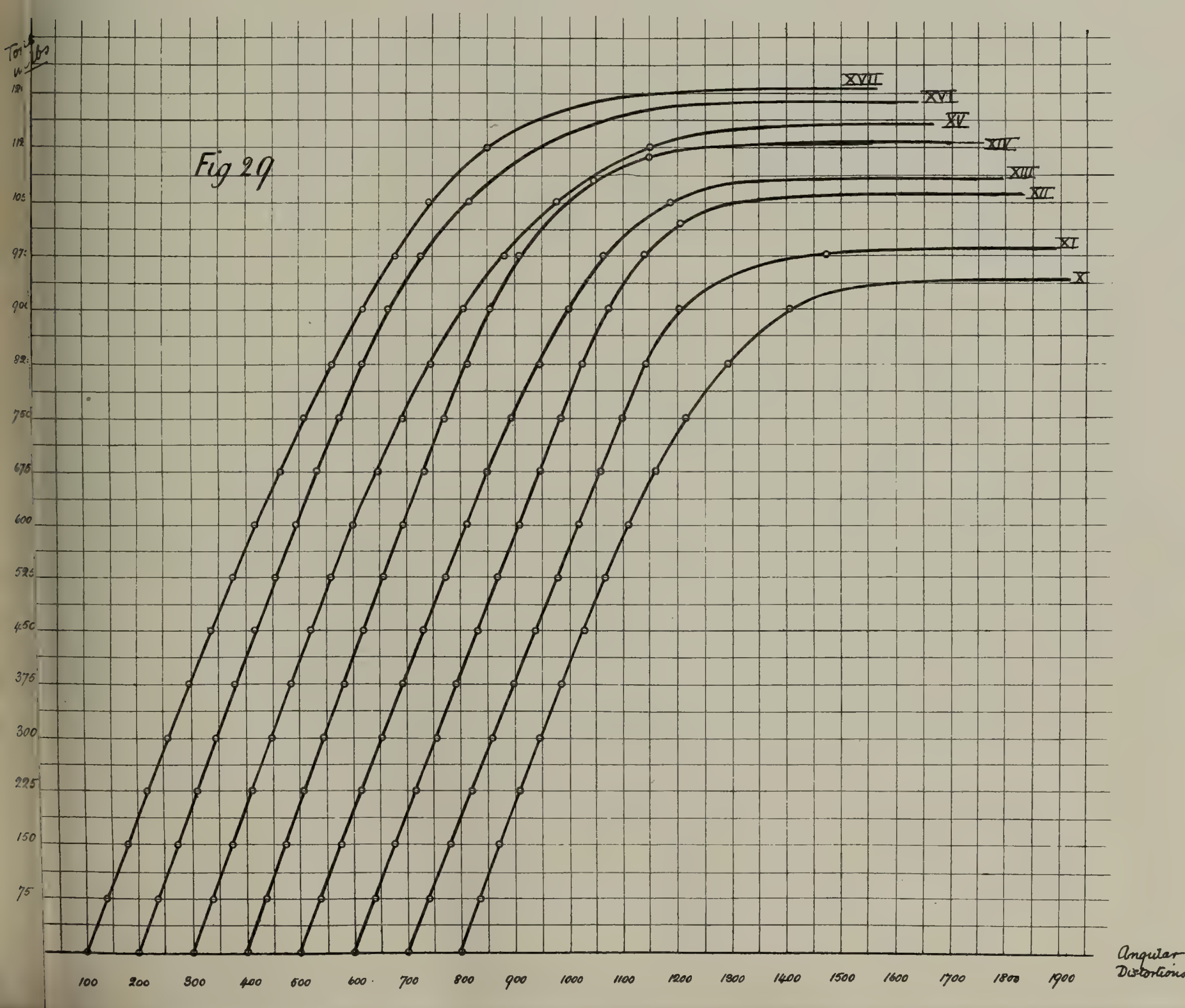




DR. COKER: APPARATUS FOR MEASURING STRAIN AND APPLYING STRESS.—PLATE VIII.







XV.—*On the Anatomy of a Collection of Slugs from N.W. Borneo; with a List of the Species recorded from that Region.* By Walter E. Collinge, Lecturer on Zoology and Comparative Anatomy in the University of Birmingham. Communicated by Professor W. C. M'Intosh. (With Three Plates.)

(Read 3rd June 1901.)

CONTENTS.

I. Introduction.

In the early part of 1900 I received from J. H. Ponsonby, Esq., a small collection of land molluscs from N.W. Borneo, with a request that I would investigate and report upon the same. This collection, the property of the Sarawak Museum, proved, on examination, to contain examples of two new genera (*Wiegmannia* and *Isselentia*) of great interest. In the case of the former genus there were only two specimens, each belonging to a different species; and as more material was very desirable, Mr Ponsonby very kindly invited the authorities of the Museum to send over, if possible, a further collection. This, in due course, arrived, but contained duplicates of *Isselentia* only, the remaining specimens being all new species, excepting one, which proved to be the *Damayantia dilecta* of Issel, which, I believe, has not previously been found since described in 1874.

As there seems no immediate likelihood of obtaining further material, and so very little is known of the slugs of this region, the results of the examination of the collection are now set forth, together with a list of the species of slugs which have been recorded from Borneo.

I need scarcely say how deeply indebted I am to the great kindness of Mr PONSONBY, and to the generous spirit shown by Mr R. SHELFORD and the authorities of the Sarawak Museum. My thanks are also tendered to Mr EDGAR A. SMITH, of the British Museum, for the many facilities he has given me for examining specimens in the collections under his charge. Finally, I wish to express my thanks to the Council of the Birmingham Natural History and Philosophical Society for defraying the expenses connected with the drawing of the accompanying figures.

II. THE BORNEAN SLUG-FAUNA.

It seems surprising that the Slug-fauna of Borneo has hitherto received so little attention. An island known to possess so rich a molluscan fauna, so far as the shelled forms are concerned, could not fail, one would think, to exhibit a wealth and variety of slug-like species. It does not, however, of necessity follow that the one always accompanies the other, at least so far as our present knowledge goes; but this possibly may be due to the fact that very little serious collecting has been undertaken for those forms in which the shell is either absent or inconspicuous. As a case in point, I may instance the Indian and Chinese faunas. In the former region a very rich fauna of land-molluscs had been described long before any number of slugs were known. Up to the present time upwards of forty species are known, and I have collections in my possession, awaiting investigation, in which there are at least another eight or nine new species. An equally rich fauna of land-molluscs is found in China, but up to the present only about a dozen species of slugs are known from that region.

One would presuppose, from the natural conditions of this island, that very many slug-like genera would be preesnt, and more careful investigation in Sarawak and the remainder of the island will, I am inclined to think, reveal a series of such of unusual interest.

ISSEL (6), in 1874, in his well-known work, recorded six species, viz.:—

Veronicella hasselti, v. Marts.	*Parmarion becarii*, Issel.
Veronicella bleekeri, Kerfst.	*Parmarion doriæ*, Issel.
Veronicella wallacei, Issel.	*Damayantia dilecta*, Issel.

It is open to question if the two species placed in the genus *Parmarion* are rightly assigned. COCKERELL (1) has placed the *P. becarii* in the genus *Ibycus*, Heynemann, with the *P. doriæ* as a variety. From ISSEL'S figures (6, T. iv. figs. 7–11), I am inclined to think that they both belong to the genus *Wiegmannia*. Nothing, however, being known of the internal structure of these forms, it is exceedingly difficult to say, with any certainty, what they are. Possibly the genus *Parmarion*, *sens. str.*, does not occur in Borneo; certainly nothing yet has been described from this region which agrees with the genus as known from Java. If this view should ultimately prove to be correct, then the Bornean slug-fauna would show a closer agreement with that of the Philippines, rather than with that of Java.

Damayantia is another genus peculiar to Borneo. Hitherto it has been known only from ISSEL'S description; it is now for the first time re-described, with some account of the anatomy and an emended description of *D. dilecta*, Issel. In the form of the mantle this genus undoubtedly shows affinities with the genera *Philippinella*, Mlldff. (8), and *Parmunculus*, Cllge. (2).

In 1895 (3), in conjunction with Lieut.-Col. H. H. GODWIN-AUSTEN, I described a new species of *Damayantia* from Borneo, and two new species of *Microparmarion*, Simr. All three, however, were generically wrongly assigned. For the latter two SIMROTH (13) has constituted a new genus, *Collingea*, and in this the former must now be placed. In the same year SCHEPMANN (10) described two new species—*Parmarion goedhuisi* and *Microparmarion litteratus*. Unfortunately no particulars of their internal structure were given, so that it is difficult to say if they are generically rightly named.

WIEGMANN (14), in 1898, described two further species—*Parmarion maculosus* and *P.? dubius*. This latter I have in the present paper included in the new genus *Wiegmannia*. The former is perhaps the only species which approaches in its structure the true Parmarions, though it may possibly prove to be more nearly related to *Wiegmannia*.

The remaining species all belong to the genus *Veronicella*, excepting one, the *Oncidium nigrum* of PLATE (9).

III. THE GENUS DAMAYANTIA, Issel.

This genus was founded by ISSEL in 1874 (6) upon three specimens which he named *D. dilecta*. Accompanying the description, three excellent figures of the external features are given, but no account of the internal structure; and as there is only one specimen in the present collection, I am, as yet, unable to supply this very desirable information. A new species, *D. carinata*, is now described, and I am able to give some details of its internal structure.

ISSEL'S original description* is incomplete, and in some points incorrect; while some of the characters set forth are undoubtedly due to the contraction produced by the alcohol. His three specimens measured respectively 24, 17·5, and 10·5 millim. in length. The specimen I have examined was 28 millim. long. All the *D. carinata* were about 25 millim.†

* For comparison ISSEL'S description is here reprinted. "Mollusco terrestre privo di conchiglia e di limacella Mantello convertito in sacco viscerale e collocato alla parte anteriore del corpo. Apertura respiratoria situata a destra del mantello ed un po' all' innanzi. Orifizio genitale posto al lato destro del corpo. Testa munita di 4 tentacoli. Muso claviforme. Bocca sprovvista (?) di mandibola. Codo fornita di poro muccoso."

† For a very careful translation of those parts of ISSEL'S work relating to the Slugs of Borneo, I am indebted to my colleague, Professor C. BÈVENOT.

Damayantia, Issel (em. Cllge.).

Damayantia, Issel: *Moll. Born.*, 1874, p. 389.

Animal limaciform, long and tapering posteriorly, dorsum sharply keeled and definitely marked off from the posterior portion of the body; anteriorly the dorsum is marked with two lateral grooves. Mantle, which completely covers the shell, exhibits a well-defined right and left keel, the right one overlapping the left posteriorly. Tentacles four. Visceral mass situated anteriorly, and lying more to the right side than the left. Rugæ somewhat rhomboidal in shape, absent in the region of the mantle. Peripodial groove well defined. An obliquely placed, oval caudal mucous pore present. Generative orifice on the right side, slightly below and behind the right lower tentacle. Respiratory orifice on the right side of the mantle. Foot-fringe well marked. Foot-sole not divisible into median and lateral planes. Shell very small, almost entirely membranaceous. Receptaculum seminis sessile. Dart with solid calcareous head, at the base of which is a small opening.

Damayantia is undoubtedly related to the genus *Philippinella* of Möllendorff (8).

1. *Damayantia dilecta*, Issel.

Damayantia dilecta, Issel: *Moll. Born.*, 1874, p. 390, T. iv. figs. 4–6.

Pl. I. figs. 1–3.

Animal yellowish-brown. Mantle completely covers the shell, minutely spotted with black. Keels well developed on the postero-lateral portions of the visceral mass and overlap one another posteriorly on the median line. The dorsum is sharply keeled. Rugæ small anteriorly, postero-laterally large and somewhat rhomboidal, absent on mantle. Caudal mucous pore large and overlapped by the extremity of the tail. Peripodial groove well defined. Foot-fringe yellowish in the anterior region, brownish posteriorly; lineoles faint and set very closely. Foot-sole white, narrow, divided into median and lateral planes.

Length (in alcohol) 28 millim.

Hab.—Mt. Penrissen, 2800–3500 feet. 1 specimen.

Issel (6, p. 28) mentions the presence of two longitudinal and medial furrows on the top part of the head, and at the sides two polygonous tubercles. In the specimen examined these features were not discernible.

2. *Damayantia carinata*, n. sp.

Pl. I. figs. 4, 5; Pl. II. figs. 22, 23.

Animal greyish-brown, postero-laterally a faint dark band runs from the posterior end of the visceral mass to the tail; lateral grooves well defined. Mantle completely covers the shell; posteriorly the two keels meet on the visceral mass, the right overlapping the left one. Posteriorly the body is sharply keeled, the keel being broken at irregular intervals, giving it a jagged or toothed appearance. Rugæ small and indistinct, excepting on the postero-lateral portion, where they stand out conspicuously. Caudal mucous pore small. Peripodial groove distinct. Foot-fringe same colour as body with very faint, closely set, sepia-coloured lineoles. Foot-sole almost white, narrow.

Length (in alcohol) 25·5 millim.

Shell membranaceous, thin, almost transparent, slight indication of apical whorl; striæ faint; ventrally there is a thin calcareous portion toward the apex.

Maj. diam. 6·8 ; min. diam. 5·5 millim., about.

Hab.—Kuching, Mt. Penrissen, and Mt. Santubong, N.W. Borneo.

Generative Organs.—(Pl. II. figs. 22, 23.)

The male organ opens into the vestibule as a narrow tube, just beyond which it becomes enlarged and forms an ovoid sac, giving place again to a short tube-like portion which distally again becomes sac-like. At the distal end there is a short diverticulum. The retractor muscle is inserted on the right side at the distal end, almost opposite to which the short vas deferens connects the prostatic canal with the penis. The receptaculum seminis is a simple, pear-shaped, sessile sac, covered in life by the bend of the large dart-gland. There is a well-developed vagina; the free oviduct is extremely short. The common duct is richly folded. The dart-gland is very large and has a sharp **S**-shaped bend at about the middle of its length; distally there is a short retractor muscle. The dart (Pl. II. fig. 23) is a hollow tube with a solid calcareous head; at the base of the head is a small lateral opening.

IV. The Genus Wiegmannia, n. gen.

As already pointed out, Wiegmann in 1898 described a slug-like mollusc from Borneo, to which he gave the name *Parmarion ? dubius*. In the present collection there are four specimens which must be classed in the same genus as *P. ? dubius*. From the external characters and the internal structure, it is clear that they cannot be placed in the genus *Parmarion*, Fisch., or *Microparmarion*, Simr. I therefore propose a new genus for their reception, and have much pleasure in associating with it the name of Herrn Fritz Wiegmann of Jena, whose anatomical studies have so largely added to our knowledge of the mollusca of the Malayan Archipelago.

In connection with Wiegmann's work (14), I may perhaps be permitted to point out that the *Parmarion flavescens* of Keferstein is not a *Parmarion* at all, but a true *Urocyclus* (7); further, that the *Parmarion extranea*, Fer., undoubtedly belongs to the genus *Girasia*, *sens. str.*, agreeing, as it does, with the Indian forms, although it is extremely doubtful if the species figured by Semper is the *extranea* of Ferussac (*cf.* Godwin-Austen, 4, pp. 217–218). Semper imagined that the structure of

Urocyclus flavescens, Kerfst., agreed pretty well with that of *Parmarion pupillaris*, Humb., and *Girasia extranea*, Fer.; but I am not of this opinion (*cf.* SEMPER, *Reis. Arch. Philip.*, p. 11). The literature relating to the Asiatic and Malayan slugs and slug-like molluscs abounds in similar inaccuracies. The two forms (*Urocyclus flavescens*, Kerfst., and *Parmarion pupillaris*, Humb.) are widely separated from one another, externally, anatomically, and geographically.

WIEGMANNIA, n. gen.

Animal Parmarion-like. Anteriorly the dorsum is marked with two lateral grooves, which, commencing from the sides of the head, converge towards the median line, and then pass to the right and left respectively. There is also a conspicuous row of rugæ passing between these two lateral grooves in the mid dorsal line. Posteriorly the dorsum is keeled. The mantle shows faint traces of a keel, and has a thin shell border more or less covering the borders of the shell. Visceral mass large and lying upon a depression of the dorsum. Generative orifice immediately behind and below the right lower tentacle. Tail truncate, with large slit-like mucous pore which extends to the foot-sole. Dart-gland and sac large; dart with solid calcareous tip. Penis has a small diverticulum. Receptaculum seminis sessile.

Shell a thin membranaceous sac, covering the posterior border of the visceral mass.

1. *Wiegmannia dubius*, Wgm.

Parmarion ? dubius, Wgm.: *Abhandl. d. Senck. naturf. Gesell.*, 1898, Bd. ii. p. 105, T. xxi. figs. 27–40; T. xxii. figs. 1–6.

For purposes of comparison, I have reproduced WIEGMANN'S figures of the external portion of the head and parts of the generative organs (Pl. II. figs. 24–26), from which, I think, it will at once be evident that this species belongs to the same genus as the following specimens.

One point to be noted is that WIEGMANN failed to find in either of his specimens any dart. He writes (14, p. 298): "Während nämlich die weibliche Anhangsdrüse bei der Species von Java, ebenso wie bei *P. pupillaris* nach SEMPER, nut einem Kalkigen Pfeile von sehr charakteristischen Form versehen ist, fehlt dieser gänzlich den beiden vorliegenden Tieren von Borneo, bei welchen die Pfeildrüse in einen durchbohrten fleischigen Papille endigt." Judging from the figure of this fleshy papilla (14, Taf. xxi. fig. 40, and reproduced here on Pl. II. fig. 26), it has all the characters of a fundus, showing a dart in course of formation.

2. *Wiegmannia gigas*, n. sp.

Pl. I. figs. 6–8; Pl. II. figs. 27, 28.

Animal greyish-brown, with few blackish blotches on the sides of the body posteriorly. Head and tentacles dark blue, lateral grooves prominent, median line of rugæ well marked. Mantle finely spotted with black; posteriorly does not cover the visceral mass; has a thin shell-border and faint trace of a keel. Extremity of foot truncate. Posterior portion of dorsum bluntly keeled. Rugæ ill defined, fairly large laterally. Sulci blackish. Caudal mucous pore a longitudinal vertical slit extending to the foot-sole. Peripodial groove distinct. Foot-fringe same colour as the body with faint black lineoles. Foot-sole yellowish-brown, with two faint chocolate-coloured bands between median and lateral planes; lateral planes marked by transverse lines, median plane papillated.

Length (in alcohol) 50 millim., foot-sole 10 millim. Shell dark amber-coloured, membranaceous, faint trace of apical whorl.

Hab.—Kuching, N.W. Borneo.

This fine species is the largest I have seen of the genus. The visceral mass is considerably larger than in either of the two following species, and the keel on the mantle is only very feebly developed.

Generative Organs.—(Pl. II. figs. 27, 28.)

The vestibule is a large, spacious cavity, into which the penis opens on the right side. This latter organ is very characteristic of the genus, differing in its length, peculiar form, and the presence of a diverticulum, from the same organ in *Parmarion* and *Microparmarion.* In the present species it is folded upon itself at a distance of about one-third from its proximal end; then forming a loop-like portion it enters the distal third; at the distal end of the loop-like portion, a short retractor muscle is inserted, and at the commencement of the distal third there is a short diverticulum. I looked carefully for any trace of calcareous granules here, but did not succeed in finding any. Gradually tapering to a fine tube, the penis now passes imperceptibly into the long vas deferens, which joins the prostatic portion of the common duct on its left side (Pl. II. fig. 27). The receptaculum seminis is a large, pear-shaped, sessile sac, and has, in this species, a short retractor muscle attached to its free end (Pl. II. fig. 27). The vagina is a short tubular cavity with the small opening of the receptaculum seminis on the right side—when looked at from the anterior end—and the larger opening of the free-oviduct on the left. This latter organ is rather more than three times the length of the vagina; it is coiled upon itself, making a single turn, and then passes into the larger, richly convoluted oviducal portion of the common duct, which is also folded upon itself toward the anterior end. A similar condition obtains in all the three new species here described. The albumen gland is large, as is also the flattened, elongated hermaphrodite gland, which latter has a comparatively short and slightly convoluted duct. The dart-gland is a large and conspicuous organ, lying on the left ventral side. It has the usual fold at about its middle, and a short retractor muscle at its distal end (Pl. II. fig. 27). The dart is smaller than in either of the two following species; it measures 3·7 millim. in length, is slightly curved, and the body, externally, is not differentiated from the head, which is a solid calcareous tip.

3. *Wiegmannia ponsonbyi*, n. sp.

Pl. I. figs. 9, 10 ; Pl. II. figs. 29, 30.

Animal yellowish-brown, with few, almost black, blotches and spots. Head almost black ; lateral grooves and median line of rugæ well marked. Mantle same colour as body, comes upon all sides of the visceral mass, and has a thin shell-border and fairly well-developed keel. Extremity of foot truncate. Posterior portion of dorsum bluntly keeled. Rugæ faintly marked. Sulci blackish. Caudal mucous pore a longitudinal vertical slit extending to the foot-sole. Peripodial groove prominent. Foot-fringe same colour as the body ; lineoles black. Foot-sole almost black anteriorly, posteriorly same colour as the body ; divided into median and lateral planes.

Length (in alcohol) 42 millim.

Shell same as in *W. gigas*, only smaller and reddish-brown in colour.

Hab.—Kuching, N.W. Borneo.

Generative Organs.—(Pl. II. figs. 29, 30.)

The external form of the penis differs considerably from that of *W. gigas* or *W. dubius*, Wgm. ; it is much shorter and is not folded to anything like the same extent. From the vestibule as far as the diverticulum it is uniform in circumference ; opposite the diverticulum there is a small retractor muscle inserted. The distal portion of the penis gradually tapers, giving place to the vas deferens (Pl. II. fig. 29). The receptaculum seminis is small, somewhat pear-shaped, and opens into the right side of the vagina. This latter organ is much longer than in the preceding species and exhibits a slight constriction just beyond its anterior third. The free-oviduct, on the other hand, is very short. The dart-gland is similar to that in *W. gigas*, only larger and not so uniform in shape, exhibiting a series of constrictions and dilatations in the anterior (Pl. II. fig. 29, *d.s.*). Structurally the dart is the same as that in *W. gigas*, but in this species the one present was much more fragile, and a little over twice the length of that found in the preceding species (Pl. II. fig. 30).

4. *Wiegmannia borneensis*, n. sp.

Pl. I. figs. 11, 12 ; Pl. II. figs. 31, 32.

Animal brownish-yellow with faint blackish mottling on the fore part of the head and dorsum, lateral grooves and median line of rugæ well marked. Mantle same colour as body with dark mottling ; comes upon all sides of the visceral mass ; has a thin shell-border ; keel more conspicuous posteriorly. Extremity of foot truncate. Posterior portion of dorsum keeled. Rugæ large. Sulci sepia coloured. Caudal mucous pore a vertical slit extending to the foot-sole. Peripodial groove distinct. Foot-fringe same colour as the body, with faintly coloured lineoles. Foot-sole brownish-yellow with median and lateral planes.

Length (in alcohol) 49 millim.

Shell a thin membranaceous sac, reddish-brown in colour, with very faint lines of growth ; apical portion distinct.

Hab.—Kuching, N.W. Borneo.

Generative Organs.—(Pl. II. figs. 31, 32.)

The generative organs agree more closely with those of *W. gigas* than with those of *W. ponsonbyi*. The vestibule is sac-like, and the vagina long, as in *W. ponsonbyi*. The penis is folded upon itself at a distance of about one-third from its proximal end ; this and the middle portion form a fairly wide tube, which now gradually tapers until it passes into the vas deferens. At the point where the retractor muscle is inserted (Pl. II. fig. 31, *div.*) there is a small diverticulum. The receptaculum seminis is a somewhat ovoid-shaped sac opening on the right side of the vagina. The free-oviduct is proportionally not so long as in *W. gigas*, but longer than in *W. ponsonbyi*. The dart-gland is very similar in shape to that in *W. gigas*, but the dart-sac contained a dart more like that described for *W. ponsonbyi*, differing, however, in possessing a more perfectly developed head, with a longer, solid, calcareous tip (Pl. II. fig. 32).

5. *Wiegmannia*, sp.

A small, bluish-grey form, measuring 14 millim. in length (in alcohol) ; may possibly be a further new species. The mantle border is finely spotted, and posteriorly it rises around the visceral mass, and has a well-developed keel encircling it. I await further material before naming the specimen.

Hab.—Mt. Penrissen, 2800–3500 feet.

V. The Genus Collingea, Simr.

Collingea, Simr. : *Zool. Jahrb.* (*Abth. f. Syst.*), 1898, Bd. ii. p. 168.

In 1895 (3), I described, in conjunction with Lieut.-Col. H. H. Godwin-Austen, a slug-like mollusc from the Poeh Mountains, Sarawak, to which the name *Damayantia smithi* was given. At that time I had not seen Issel's description (6) and figures of *D. dilecta* ; but Lieut.-Col. Godwin-Austen was of opinion that the specimens from the Poeh Mountains belonged to Issel's genus. Having recently seen a specimen of *D. dilecta* and compared it with Issel's description and figures, I have no hesitation in at once removing the specimen named *D. smithi* from that genus.

Through the kindness of Mr Edgar A. Smith, I have had the opportunity of re-examining this very interesting mollusc, and am now able to give an emended description of it and some further particulars respecting its internal structure.

Unfortunately, a very serious error was made at the time it was originally described.

Mr EDGAR A. SMITH sent me three specimens. One of these GODWIN-AUSTEN figured (3, pl. xi. figs. 1–6), which undoubtedly belongs to the genus *Collingea*, Simr. One of the remaining two I dissected, and described and figured the generative organs (*cf.* 3, pl. xi. figs. 9–11); but, unfortunately, this was very distinct from the one which GODWIN-AUSTEN figured, and on re-examining it I find that it and its fellow belong to the genus *Isselentia* here described (p. 305).

The specimen of *Collingea smithi* had been opened, and I have made a careful examination of the generative organs, and figures of these are now given for the first time (Pl. II. figs. 34–36). The peculiar handle-like extension of the penis (the Henkel of SIMROTH) at once characterises this species as belonging to the genus *Collingea*.

GODWIN-AUSTEN has given (5, pp. 55–57) what he terms an amended description of both the animal and anatomy of what was originally termed *Damayantia smithi*. The description of the animal, of course, applies to *Collingea smithi*, whilst the anatomical account applies, in so far as it is correct, to *Isselentia globosa* (p. 305). It is doubtful if GODWIN-AUSTEN refers to a true *Damayantia*, especially as he compares these two molluscs with *D. dilecta*. ISSEL's figures (6, T. iv. figs. 4–6) show how different the genus is from *C. smithi* or *I. globosa*.

This author, on p. 58, writes: "I illustrate the anatomy of *Microparmarion* with my original drawings (*those in the* P. Z. S., *1895, being copies of them**)." The figures in the *P. Z. S.* paper, which are credited to me, were made by me from the dissections.

1. *Collingea smithi*, Cllge. and Godw.-Aust.

Pl. II. figs. 33–36.

Damayantia smithi, Cllge. and Godw.-Aust.: *Proc. Zool. Soc.*, 1895, p. 242, pl. xi. figs. 1–4.
Damayantia smithi, Godwin-Austen: *Moll. of India*, 1898, vol. ii. p. 55, pl. lxxiii. figs. 1–7*d*.

Animal: body yellowish, with dark blue or bluish-brown mottling on the sides in the posterior region.† Mantle yellowish-grey with irregular dark blue or black mottling; has a thin shell-border and distinct lateral keel. Extremity of foot truncate. Posterior portion of dorsum sharply keeled. Rugæ large. Caudal mucous pore large, but does not extend to the foot-sole. Peripodial groove well marked. Foot-fringe yellowish-brown with faintly marked lineoles. Foot-sole darker than the foot-fringe and divided into median and lateral planes.

Length (in alcohol) 28 millim.; breadth of foot-sole 4·5 millim.

Shell oval, membranaceous, thin, and shiny; apical whorl distinct (Pl. II. fig. 33).

Hab.—Poeh Mountain (3500 feet), Sarawak (A. H. EVERETT).

Type in collection of British Museum.

Generative Organs.—(Pl. II. figs. 34–36.)

The vagina is a wide, sac-like cavity, on the left side of which the receptaculum

* The italics are mine.—W. E. C.

† Originally described as a "very dark blue or black streak runs along the side of the foot posteriorly, crossing it diagonally downwards to the mucous pore." The figure is wrong in showing this.

seminis opens. This is an irregular-shaped sac with a short duct. The penis is a thick, muscular-walled tube, narrow at its proximal end, but increasing in size as it nears the distal end, where it makes a sharp bend to the right, this distal extremity forming a sac-like extension (Pl. II. fig. 34, 35). On the left-hand side is a loop-like extension—the Henkel of Simroth. The vas deferens leaves the distal extremity of the penis on its ventral side (Pl. II. fig. 36, *v.d.*). The retractor muscle also is inserted on the ventral side, at the point where the penis makes a sharp bend to the right, (Pl. II. fig. 36, *r.m.*). The dart-gland (Pl. II. fig. 34, *d.gl.*) is a large, muscular organ; distally it presents a swollen appearance which occupies about one-half; the proximal half is tube-like.

Compared with the three known species of *Collingea*, viz., *C. strubelli*, Simr., *C. pollonerai*, Cllge. and Godw.-Aust., and *C. simrothi*, Cllge. and Godw.-Aust., this species approaches most nearly to *C. simrothi*. It differs considerably from *C. strubelli* (12) in the form of the penis and dart-gland, and in the same manner from *C. pollonerai*.

VI. The Genus Isselentia, n. gen.

Animal slug-like. The mantle anteriorly forms two wing-like appendages lying on each side of the visceral mass, inner borders plicated or folded, comes up around the posterior and lateral borders of the visceral mass; shell-borders thin. Dorsum posteriorly keeled. Generative orifice behind right lower tentacle. Small caudal mucous pore. Foot-sole divided into median and lateral planes. Viscera elevated into the dorsal hump, the body-cavity not extending beyond it into the tail, which is solid.

Generative system: penis with or without diverticulum. Sessile receptaculum seminis. Dart calcareous, with small laterally placed aperture.

1. *Isselentia plicata*, n. sp.

Pl. II. figs. 13–15*b*; Pl. III. figs. 37–49.

Animal yellowish with dark blue dorsum posteriorly, head bluish. Mantle reddish-yellow, with small blackish spots and blotches; shell-border thin. Posterior portion of dorsum exhibits a wavy keel of a deep yellow colour. Rugæ more or less ovoid. Caudal mucous pore small and partly hidden by the extremity of the dorsum. Peripodial groove very prominent. Foot-fringe yellow, with fine, very closely set lineoles. Foot-sole yellowish, divided into median and lateral planes.

Length (in alcohol) 26 millim.

Shell amber-coloured; a thin membranaceous sac; apical whorl distinct.

Hab.—Mt. Penrissen and Mt. Santubong.

Generative Organs.—(Pl. III. figs. 37–49.)

The penis is of considerable length; it has a somewhat globose proximal portion, followed by a narrower portion, again expanding and narrowing, and terminating by a sharp bend, gives place to a somewhat conical head. The whole of the distal end is covered with connective tissue, so that at first sight it has the appearance shown in fig. 37 (Pl. III.). When, however, this is dissected away, the **S**-shaped bend is seen, the tube becoming gradually larger as it nears the point where the retractor muscle is inserted (Pl. III. fig. 38). The external wall exhibits a series of ring-like constrictions; one of these immediately beyond the retractor muscle is much deeper and sharply divides the "head" into two parts, viz., that already described and the conical portion beyond, which has similar constrictions. This gradually tapers off into the vas deferens (Pl. III. fig. 37). The vagina is a long, wide tube, having an opening on its dorsal wall for the small, twisted receptaculum seminis, and a larger opening at its posterior end for the small globular free-oviduct. The first portion of the common duct is sharply coiled upon itself. The dart-gland (Pl. III. fig. 37, *d.gl.*) when viewed externally exhibits a globose distal portion, to which a small retractor muscle is attached, a middle tube-like portion, forming the bend, and a dart-sac, the proximal portion. Three specimens were examined. In the first the dart was immature, and had the peculiar bent form shown in fig. 40 (Pl. III.). In the second specimen this was in much the same condition. In the third, however, a well-formed dart was present, measuring 4 millim. in length. The dart is situated at the posterior end of the proximal sac-like portion. Externally the dart is covered by a calcareous sheath, which has a small, laterally placed, oval-shaped aperture. The head is not differentiated from the body, which is almost straight and about the same thickness throughout (Pl. III. figs. 39–42).

Under a high power of the microscope, the calcareous layer is seen to consist of an outer structureless layer, and an inner one which, when looked at longitudinally, has the appearance of short dark and light bands (Pl. III. fig. 41). This inner layer is more conspicuous in the region of the head than elsewhere. The basal end or annulus (Pl. III. fig. 43) fits into a groove; and so far as I could make out the structure, which proved very difficult, the internal cavity of the dart is continuous with that of the expanded distal portion of the dart-gland (Pl. III. fig. 44).

The dart-gland consists of a thin external sheath of connective tissue, within which is a longitudinal muscular layer, and then a layer of circular muscle fibres with a few radial fibres intermixed, some of these latter extending as far as, and into, the longitudinal layer. The central cavity is bounded by an epithelial lining (Pl. III. figs. 45 and 47). Longitudinal and transverse sections were made of both the proximal and distal portions. The former has a glandular lining, and when looked at in surface view, the wall has the appearance of being studded with a series of bluntly pointed papillæ (Pl. III. fig. 49). In longitudinal section these are seen to consist of an outer epithelial layer of cuboid cells, and an inner layer of almost circular cells (Pl. III. figs. 46 and 48). In the distal portion the epithelial lining consists of columnar cells, and the cavity contains a larger series of exceedingly minute particles (calcareous?) imbedded in a jelly-like matrix. Sections cut by a freezing microtome were stained in an aqueous solution of magenta, but the matrix remained unstained. Others were treated with

5 and 10 per cent. solutions of hydrochloric acid, but no effect was obtained. Strong hydrochloric acid caused the matrix to coagulate.

2. *Isselentia globosa*, n. sp.

Pl. III. fig. 50.

Animal smaller but not at all unlike *I. plicata*; the ground colour, however, is lighter, and the posterior portions of the dorsum considerably lighter. The plications of the mantle lobes are only slightly developed.

Hab.—Poeh Mountain (3500 feet), Sarawak (A. H. Everett).

Type in collection of British Museum. Two specimens.

When recently examining these two specimens, I felt inclined to refer them to *I. plicata*, but an examination of the generative organs shows that they exhibit some important differences.

Generative Organs.—(Pl. III. fig. 50.)

The vestibule is small. The penis consists of a sac-like portion, above which it becomes suddenly constricted and then dilates into a bulbous head. From the distal portion of the penis, above the vas deferens, is a short diverticulum, partially covered by the strong retractor muscle (Pl. III. fig. 50). From the side of the bulbous head of the penis the vas deferens passes off as a thick tube, narrowing gradually as it approaches the prostatic portion of the common duct. The receptaculum seminis is somewhat ovoid and sessile, and opens into the vagina; to the right of this is the opening of the free-oviduct. The first portion of this organ is thrown into a series of constrictions. The oviduct is a wide tube and densely folded, the prostatic and oviducal portions terminating in a bulbous head lying immediately in front of the globular albumen gland. The hermaphrodite gland is almost circular and flattened, showing a slight fold or indentation in the centre. The dart-gland is a large, thick, muscular-walled tube, making a sharp **S**-shaped bend distally. Just below this is the dart-sac, which contains a calcareous dart similar in shape to that in *I. plicata* (Pl. III. fig. 39), but whether or not it is perforated I cannot say, as the head had been broken away.

VII. The Genus Veronicella, Blainv.

1. *Veronicella shelfordiana*, n. sp.

Pl. I. figs. 16, 17.

Animal dark-brown dorsally, with dense yellow spotting and median dorsal yellowish-brown stripe. Hyponotum and foot-sole light-brown.

Length (in alcohol) 20 millim.; foot-sole 2·5 millim. broad; hyponotum 4 millim. broad. Female generative orifice on the right side, 3 millim. from the foot-sole, 11·5 millim. from the right lower tentacle, and 8·5 millim. from the posterior end of the body.

Hab.—Kuching, N.W. Borneo.

I have pleasure in associating with this handsome species the name of Mr SHELFORD of the Sarawak Museum.

2. *Veronicella exima*, n. sp.

Pl. I. figs. 18, 19.

Animal yellowish-brown dorsally, densely and minutely speckled with black, leaving a clear unicolourous margin and broad medio-dorsal line. Hyponotum yellowish-brown; foot-sole brown.

Length (in alcohol) 22·5 millim.; foot-sole 2 millim. broad; hyponotum 3·5 millim. broad. Female generative orifice on the right side, 3 millim. from the foot-sole, 12·5 millim. from the right lower tentacle, and 12 millim. from the posterior end of the body.

Hab.—Kuching, N.W. Borneo.

VIII. THE GENUS ONCHIDIUM, Buchan. (em. Plate).

1. *Onchidium ponsonbyi*, n. sp.

Pl. I. figs. 20, 21.

Animal dirty green dorsally with large, irregularly distributed black spots. Hyponotum dark greenish-blue; foot-sole dirty yellow.

Length (in alcohol) 30 millim.; hyponotum 10 millim.; foot-sole 8 millim.

Hab.—Mt. Penrissen (2800-3500 feet).

It gives me much pleasure to associate with this very fine species the name of Mr PONSONBY.

IX. SUMMARY AND CONCLUSION.

From an examination of the foregoing specimens it is, as yet, difficult to generalise as to their affinities or relationships with allied genera, for our knowledge of their structure and specific variation is too fragmentary. Further, it is of little use comparing the form and structure of the generative organs of such genera as *Wiegmannia* and *Isselentia* with *Parmarion*, *Microparmarion*, etc., for our knowledge of the structure of these latter genera is not much more complete. I sincerely hope, however, that with the invaluable aid of Mr SHELFORD and other naturalists, I shall be able, before long, to treat of the general anatomy of many of the Bornean slugs in much greater detail. The present communication must be regarded more in the light of a preliminary notice of species, which, as further material is obtained, will receive more exhaustive treatment.

X. List of Species of Slugs recorded from Borneo.

Damayantia, Issel: *Ann. Mus. Civ. Genova*, 1874, vi. p. 26.
1. *D. dilecta*, Issel: *Ibid.* p. 27, T. iv. figs. 4–6.
2. *D. carinata*, Cllge.: *Ante*, p. 298.

Wiegmannia, Cllge.: *Ante*, p. 299.
3. *W. dubius*, Wgm.: *Abhandl. d. Senckenb. naturf. Gesell.*, 1898, p. 305, T. xxi. figs. 27–40, T. xxii. fig. 1–6.
4. *W. gigas*, Cllge.: *Ante*, p. 300.
5. *W. ponsonbyi*, Cllge.: *Ante*, p. 302.
6. *W. borneensis*, Cllge.: *Ante*, p. 302.
7. *W.*, sp.: *Ante*, p. 303.

Collingea, Simr.: *Zool. Jahrb.* (*Abth. f. Syst.*), 1898, Bd. ii. p. 168.
8. *C. pollonerai*, Cllge. and Godw.-Aust.: *Proc. Zool. Soc.*, 1895, p. 244, pls. xii., xiii., figs. 13–25.
9. *C. simrothi*, Cllge. and Godw.-Aust.: *Ibid.* p. 246, pls. xii., xiii., figs. 26–35.
10. *C. smithi*, Cllge. and Godw.-Aust.: *Ibid.* p. 242.

Ibycus, Heyn.: *Mal. Blätt.*, 1862, p. 142, pl. 1, fig. 3.
11. *I. beccarii*, Issel: *Ann. Mus. Civ. Genova*, 1874, vi. p. 23, T. iv. figs. 9–11.
12. *I. doriæ*, Issel: *Ibid.* p. 25, T. iv. figs. 7, 8.

Parmarion, P. Fischer: *Actes. Soc. Linn. Bordeaux*, 1855.
13. *P. maculosus*, Wgm.: *Abhandl. d. Senckenb. naturf. Gesell*, 1898, p. 299, T. xxi. figs. 8–26.
14. *P. goedhuisi*, Schepm.: *Notes fr. Leyden Mus.*, 1895, vol. xvii. p. 146, pl. 2, figs. 1*a*–1*c*.

Microparmarion, Simr.: *Zool. Ergebnisse*, 1893, Bd. iii. p. 104.
15. *M. litteratus*, Schepm.: *Notes fr. Leyden Mus.*, 1895, vol. xvii. p. 148, pl. 2, figs. 2*a*–2*c*.

Isselentia, Cllge.: *Ante*, p. 305.
16. *I. plicata*, Cllge.: *Ante*, p. 305.
17. *I. globosa*, Cllge.: *Ante*, p. 307.

Veronicella, Blainv.: *Journ. de Physique*, 1817, p. 440, pl. vi. figs. 1, 2.
18. *V. bleekeri*, Kerfst.: *Zeit. f. wiss. Zool.*, 1865, Bd. xv. p. 125, T. ix. figs. 1–7.
19. *V. hasselti*, v. Marts.: *Die Landschnecken Preuss. Exped. nach Ost-Asien*, 1867, p. 176, T. v. figs. 2, 4.
20. *V. flava*, Heyn.: *Jahrb. Deutsch. Malad. Gessel.*, 1885, Bd. xii.
21. *V. idæ*, Semp.: *Reisen im Arch. Philip.*, 1885, Heft. vii. p. 321.
22. *V. borneensis*, Simr.: *Abhandl. d. Senckenb. naturf. Gesell.*, 1897, Bd. xxiv. p. 142, T. xiv. figs. 8, 15, 16, 17.
23. *V. wallacei*, Issel: *Ann. Mus. Civ. Genova*, 1874, vi. p. 22, T. iv. figs. 1–3.
24. *V. shelfordiana*, Cllge.: *Ante*, p. 307.
25. *V. exima*, Cllge.: *Ante*, p. 308.

Onchidium, Buchan. (em. Plate): *Trans. Linn. Soc.*, 1800, vol. v. p. 132.
26. *O. nigrum*, Plate: *Zool. Jahrb.* (*Abth. f. Morph.*), 1893, Bd. 7, p. 188, T. 8, fig. 31*a*; T. 10, fig. 53; T. 11, fig. 75.
27. *O. ponsonbyi*, Cllge.: *Ante*, p. 308.

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10. Schepmann, M. M., "The Mollusca of the Dutch Scientific Borneo Expedition, with description of the new species," *Notes from the Leyden Mus.*, 1895, vol. xvii. pp. 145–162, pls. 2–4.
11. Semper, C., *Reisen im Archipel der Philippinen*, 1870, Th. II. Bd. iii. "Landmollusken."
12. Simroth, Heinrich, "Ueber einige Parmarion-Arten," *Zool. Ergebnisse einer Reise in Niederländisch Ost-Indien*, 1893, Bd. iii. pp. 100–111, T. vii., viii.
13. Simroth, Heinrich, "Ueber die Gattungen *Parmacochlea*, *Parmarion* und *Microparmarion*," *Zool. Jahrb. (Abth. f. Syst.)*, 1898, Bd. ii. pp. 151–172, T. 15.
14. Wiegmann, F., "Landmollusken (Stylommatophoren) Zootomischer Teil," *Abhandl. d. Senckenb. naturf. Gesell.*, 1898, pp. 289–557, Taf. xxi.–xxxi.

XII. Reference Letters.

alb.gl. Albumen gland.
ap. Aperture of dart.
c.d. Cavity of dart.
c.d.gl. Cavity of dart-gland.
c.e. Columnar epithelium.
c.t. Connective tissue.
c.m. Circular muscle fibres.
d. Dart.
d.gl. Dart-gland.
d.s. Dart sac.
div. Diverticulum of penis.
ep. Epithelium.
f.ov. Free-oviduct.
gl. Gland cells.
H. Henkel.
h.d. Hermaphrodite duct.
h.gl. Hermaphrodite gland.
i.c.l. Inner calcareous layer.
l.m. Longitudinal muscle fibres.
l.p. Lateral plane of foot-sole.
m.p. Median plane of foot-sole.
o.c.l. Outer calcareous layer.
ov. Oviduct.
p. Penis.
pr. Prostate.
r.m. Retractor muscle.
r.m.f. Radial muscle fibres.
r.s. Receptaculum seminis.
v. Vestibule.
v.d. Vas deferens.
vg. Vagina.

XIII. Description of Plates.

Plate I.

Fig. 1. *Damayantia dilecta*, Issel. Lateral view, × 2.
Fig. 2. " " " Dorsal view, × 2.
Fig. 3. " " " Dorsal view of shell, × $1\frac{1}{2}$.
Fig. 4. *Damayantia carinata*, n. sp. Lateral view, × 2.
Fig. 5. " " Dorsal view, × 2.
Fig. 6. *Wiegmannia gigas*, n.sp. Lateral view, × 1.
Fig. 7. " " Dorsal view of the head, showing lateral grooves and median row of rugæ, × 2.
Fig. 8. " " Foot-sole showing median and lateral planes, × 3.
Fig. 9. *Wiegmannia ponsonbyi*, n. sp. Lateral view, × 1.
Fig. 10. " " Dorsal view of the head, × 2.
Fig. 11. *Wiegmannia borneensis*, n. sp. Lateral view, × 1.
Fig. 12. " " Dorsal view of the head, × 2.
Fig. 13. *Isselentia plicata*, n. sp. View from the right side, × 2.
Fig. 14. " " View from the left side, × 2.
Fig. 15. " " Dorsal view, × 2.
Figs. 15*a*, *b*. " " Dorsal and ventral view of shell, × 1.
Fig. 16. *Veronicella shelfordiana*, n. sp., as seen from above, × 2.
Fig. 17. " " as seen from below, × 2.
Fig. 18. *Veronicella exima*, n. sp., as seen from above, × 2.
Fig. 19. " " as seen from below, × 2.
Fig. 20. *Onchidium ponsonbyi*, n. sp., as seen from above, × 1.
Fig. 21. " " as seen from below, × 1.

Plate II.

Fig. 22. *Damayantia carinata*, n. sp. Generative organs.
Fig. 23. " " Dart, × 16.
Fig. 24. } *Wiegmannia dubius*, Wgm. Parts of the terminal ducts of the generative organs (after Wiegmann).
Fig. 25. }
Fig. 26. " " Dart (after Wiegmann).
Fig. 27. *Wiegmannia gigas*, n. sp. Generative organs.
Fig. 28. " " Dart, × 16.
Fig. 29. *Wiegmannia ponsonbyi*, n. sp. Generative organs.
Fig. 30. " " Dart, × 16.
Fig. 31. *Wiegmannia borneensis*, n. sp. Generative organs.
Fig. 32. " " Dart, × 18.
Fig. 33. *Collingea smithi*, Cllge. and Godw.-Aust. Shell, × 2·5.
Fig. 34. " " " Generative organs.
Fig. 35. " " " Dorsal view of penis, etc.
Fig. 36. " " " Ventral view of penis, etc.

PLATE III.

Fig. 37.	*Isselentia plicata*, n. sp.	Generative organs.
Fig. 38.	,, ,,	Distal end of penis, enlarged.
Fig. 39.	,, ,,	Dart, × 15.
Fig. 40.	,, ,,	Immature dart.
Fig. 41.	,, ,,	Head of dart, highly magnified.
Fig. 42.	,, ,,	Portion of immature dart, showing bends in calcareous sheath.
Fig. 43.	,, ,,	Diagrammatic view of fundus.
Fig. 44.	,, ,,	Distal end of dart-gland showing cavity.
Fig. 45.	,, ,,	Diagrammatic view of a transverse section through the distal portion of the dart-gland.
Fig. 46.	,, ,,	Longitudinal section through the proximal portion of the dart-gland.
Fig. 47.	,, ,,	Transverse section through the distal portion of the dart-gland.
Fig. 48.	,, ,,	Epithelial and gland cells from the proximal portion of the dart-gland.
Fig. 49.	,, ,,	Surface view of the internal wall of the dart-gland, proximal portion.
Fig. 50.	*Isselentia globosa*, n. sp.	Generative organs.

E DEL. E PINX.

W. GRIGGS, LITH.

W. GRIGGS, LITH.

XVI.—*The True Shape, Relations, and Structure of the Alimentary Viscera of the Porpoise (Phocæna communis), as displayed by the Formal Method.* (With Lantern Demonstration of their Microscopic Structure.) By David Hepburn, M.D., F.R.S.E., Lecturer on Regional Anatomy, and David Waterston, M.A., M.D., F.R.S.E., Demonstrator of Anatomy, University of Edinburgh. (With Three Plates.)

(Read July 15, 1901.)

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Introductory.—Among the toothed whales (Odontoceti) the porpoise is the best-known representative of those members of the genus Delphinus or true dolphins which present a rounded muzzle as distinguished from a snout, and consequently it has already frequently been subjected to anatomical examination of a more or less detailed character. As in the case of all the Cetacea, however, the rapidity with which decomposition affects the various tissues and organs has hitherto proved a barrier to a prolonged and systematic examination of them, while, even under the most favourable conditions, the increasing putridity of the carcase has seriously militated against the recording of accurate observations.

Accordingly, when a porpoise, which had been captured in some fishing nets twenty-four hours previously, came into our possession last December, we took immediate steps for its perfect preservation, so that its dissection might be conducted at leisure under conditions of comfort as well as accuracy, and its various tissues "fixed" for reliable examination by the microscope.

Method of Preparation.—We therefore first recorded the measurements and external appearances of the animal, and then, having placed it upon its back in such a way as to remove all pressure from its dorsal fin, we opened a comparatively small vessel on its ventral aspect between the anus and the tail. Into this vessel—a small vein immediately under cover of the blubber—we tied a fine canula having a lumen which would admit the stilette of a dissector's blowpipe. Through this vessel we injected, by means of a gravitation pressure of about four feet, about two gallons of an arsenical preservative to which 10 per cent. of formaldehyde had been added. The fluid took several hours to run into the animal, the body meanwhile becoming firm and rigid, but not in any way distended or deformed. The success of this method of preservation has been apparent at every stage of the dissection. After seven months, the carcase is still absolutely devoid of unpleasant odour, and no trace of decomposition is visible anywhere. The blood has been everywhere coagulated in the vessels, which are thus filled with a natural solid injection mass. The viscera have been fixed so as to retain their natural shapes and relationships. The tissues are all in perfect condition for undergoing further treatment in preparation for the microscope, with the exception of a slight desquamation of a few superficial cells from the mucous membrane of the alimentary canal. In our examination of this animal, therefore, we are in a position to present observations, made under conditions which we believe to be unique, which give results of a thoroughly reliable kind, besides being in many respects entirely novel and which no doubt account for differences between our results and those of former observers.

External Appearances.—The animal, which was a male, presented the characteristic features of its genus as regards its rounded muzzle, its teeth, the comparatively high position of its fore limbs upon its sides, the colour of its body and appendages, the tuberculated border of its dorsal fin, etc. The following measurements were recorded before the preservative fluid was injected:—

Length from tip of muzzle to centre of tail,	49½ inches	1·257 metres.
,, from muzzle to vent,	34½ ,,	·876 ,,
,, from vent to centre of tail,	15 ,,	·381 ,,
,, of oral cleft,	4 ,,	·102 ,,
,, from muzzle to anterior edge of root of flipper,	10½ ,,	·267 ,,
,, from muzzle to dorsal fin,	22¾ ,,	·578 ,,
,, of flipper,	6¾ ,,	·172 ,,
Width of tail,	10½ ,,	·267 ,,
Girth of tail at root,	6 ,,	·153 ,,
Distance between genito-urinary cleft and vent,	11½ ,,	·292 ,,
Length of base of dorsal fin,	8½ ,,	·216 ,,
Height of dorsal fin,	4½ ,,	·114 ,,
Distance from external angle of eye to external auditory meatus,	2 ,,	·051 ,,

There were nowhere any traces of hair, and no evidence of an external ear, while the external auditory meatus would not admit the stem of a wax vesta. As the adult porpoise is usually from four to five feet long, although it may reach a length of six feet, the present specimen may be regarded as slightly under its full growth.

In the present communication we desire to confine our attention to that part of the alimentary system contained within the cavity of the abdomen.

Cavity of Abdomen.—On opening the abdominal cavity by removing the ventral wall, it was found that the general shape of the cavity was oval, the anterior end was more obtuse than the posterior, and the greatest transverse diameter was opposite the lower or hinder border of the liver. The upper (anterior) part contained the liver and greater part of the stomach, while the posterior part was filled by the convoluted mass of the intestine.

The border of the liver extended across the cavity from a point opposite the eighth right costo-chondral junction to a corresponding point on the left side, in an almost transverse direction ; and near the mesial plane of the body there was a triangular notch which was occupied by a small portion of the wall of one of the chambers of the stomach, while another small part of the stomach wall projected from below the border of the left lobe, but only for three-quarters of an inch. The coils of intestine of were practically the same calibre throughout, and were suspended from the dorsal wall by a single mesial mesentery. There was no sign of a vermiform appendix, or cæcum, and no part of the tube showed to the naked eye any of the appearances characteristic of the large, as distinguished from the small, intestine.

The abdominal cavity measured 36·5 cms. in its long axis, and 19·5 cms. in its maximum transverse diameter, and at the posterior end it narrowed suddenly, and opened by a definite constricted orifice into an elongated tubular chamber, the representative of a pelvic cavity (Pl. I. figs. 1 and 2). The aperture was formed by a projecting margin of peritoneal membrane, over which the vasa deferentia turned in their course to the urethra. The diameters of this orifice were 2·5 cms. in the saggital axis, by 1·5 cm. in the transverse, and the depth of the chamber was 7 cms. This tubular peritoneal recess passed between the pelvic bones ventrally, and the vertebral column dorsally, and formed the lining membrane of a chamber, which, from its position, contents, and boundaries, we regard as the representative of a pelvic cavity. The projecting margin on either side corresponded to the brim of a true pelvis. The relations of the viscera associated with the peritoneum confirm the analogy, as the urinary bladder lay between the peritoneum of its ventral wall and the pelvic bones and interpubic ligament, while the rectum descended in relation to the dorsal wall, being supported by a mesentery at its upper or anterior part, but gradually losing its peritoneal investment to end in the anal canal. Hitherto, this arrangement of the peritoneum does not seem to have been recognised as a pelvic cavity, although Turner * in a description of the posterior end of the abdominal cavity of Risso's Dolphin (*Grampus griseus*), refers to the peritoneum as forming "four cæcal pouches," of which the dorso-mesial one apparently corresponds to the pelvic cavity which we have described.

Peritoneal Folds and Reflections.—The falciform ligament of the liver and the Ligamentum teres were both distinct, and were almost mesial in position. The former disappeared into a vertical slit, 5 cms. in length, in the ventral surface of the liver.

* Sir Wm. Turner, *Jour. of Anat. and Phys.*, vol. xxvi. p. 264.

A peritoneal fold of the nature of a great omentum was represented by a short fold, not more than 6 cms. in length, depending from the curvature of the stomach, where it appeared from under cover of the liver, but did not contain any quantity of fatty tissue. The coils of intestine were suspended from the posterior or dorsal abdominal wall by a single mesial fold or mesentery, in which near the root there were large masses of lymphatic tissue, which diminished in size towards the caudal end. The lower end of the intestine passed into the pelvic chamber mesially, and was suspended by a continuation of the mesentery.

The absence of any intestinal coil corresponding in position to the colon rendered the arrangement of the peritoneum about the stomach and pancreas somewhat unusual.

The great omentum, already described as depending from the stomach, was composed of four layers of peritoneum—two ventral and two dorsal. On tearing through the ventral layers, a lesser sac of peritoneum was opened, whose boundaries were as follows :—

Ventrally, the wall of the stomach; dorsally, pancreas; on the left, the first chamber of the stomach; and on the right side, the last part of the stomach and the duodenum.

No aperture corresponding to a foramen of Winslow could be found, and the arrangement of the peritoneal membrane was more clearly brought out by examining it from behind, after removing the viscera *en bloc.*

The lesser sac was then seen to be completely closed in by the peritoneum, which had the following attachments :—

The anterior layers of the great omentum, attached to the greater curvature of chamber No. 2 of the stomach, were prolonged on the left side to the posterior surface of chamber No. 1, from which they were reflected off along an oblique line from the centre of its ventral border to the spleen.

Above the spleen, the two layers separated to envelop the œsophagus. The two posterior or dorsal layers pass dorsally to the border of the pancreas, where they diverge, one passing on the anterior ventral surface, and the other to the inferior caudal surface of that viscus.

On the right side, the layers passed on to the duodenum, and then backwards on to the posterior abdominal wall and pancreas, and thus on the right side the lesser sac became completely closed.

The apex of the pancreas was in contact with the under surface of the liver.

The absence of an aperture into the lesser sac * may be explained by reference to the relative positions of the stomach, liver, and pancreas. The pancreas, instead of lying

* In a monograph entitled, "Recherches sur le développement de la cavité hépato-entérique de l'Axolotl et de l'arrière cavité du péritoine chez les mammifères (Lapin), par Albert Brachet, *Archives de Biologie*, tome XIII., 1893, pp. 559–618 (Plates XXIV. to XXVII.), the following passage occurs :—" La fermeture de cet hiatus" (de Winslow) "chez l'amphibien, provient de ce que le bord postérieur du méso-latéral est très peu étendu, et que dans son intérieur ne pénètre pas le foie. Ce bord, se continuant dans le mésoduodénum à son extrémité inférieure, se soude peu à peu à lui, de bas en haut. L'union entre les deux, progressant dans ce sens, amène l'occlusion de l'hiatus."

behind the lesser sac, with its head in the concavity of the duodenum, was found between the duodenum and the liver projecting to the left side, so as to be related to the first and second chambers of the stomach within the folds of the lesser omentum. The alteration in the peritoneal relations is best realised by supposing the pancreas to have moved to the right and forwards (ventrally), and then upwards (anteriorly) between the upper border of the duodenum and the liver, thereby separating the layers of the gastro-hepatic omentum, and pressing its posterior or dorsal layer into contact with the peritoneum covering the dorsal wall of the abdomen, until these two opposite walls of the lesser sac have fused together. That there has been an alteration of this kind is shown by the fact that the bile-duct has no free course, but from the under surface of the liver enters the head of the pancreas and remains in it, until it pierces the wall of the duodenum, and by the further fact that there is a reflection of peritoneum from the under surface of the liver to the back of the head of the pancreas, completely occluding any communication between the sac which lies on the dorsal aspect of the stomach and the general or great peritoneal cavity. Furthermore, that part of the liver bounded by the obliterated ductus venosus and the inferior vena cava (Lobulus spigelii), was situated on the dorsal aspect of the liver, and was entirely devoid of peritoneal covering. On the other hand, the Lobulus caudatus, situated on the dorsal aspect of the hilum of the liver, was associated with a blind digital process of peritoneum.

Between the layers forming the ventral part of the omentum, there was a curious arrangement of lymphoid tissue. There wss a large number of bloodvessels in the peritoneal membrane, and these vessels passed through small nodules of lymphoid tissue, so that they produced the appearance of a bunch of grapes on a stalk, and the arrangement closely resembled the condition present in the human spleen when the vessels are isolated from the spleen tissue. A similar condition was not found in the other parts of the peritoneum.

Stomach.—As is well known, the stomach of this animal has been the subject of much debate. Observers have agreed that it presented four compartments, but they have differed as to the homologies and the sequence of these chambers, and, therefore, their special interest in the present instance is due to the fact of their having been fixed in a normal position so that they show their natural shapes, and consequently their homologies can readily be understood.

The different chambers composing the complex stomach had the following positions and relations:—1. The first and second chambers lay side by side, the first being situated on the dorsal aspect of the other compartments, and somewhat to the right side of the second compartment. *The first compartment* received the œsophagus at its anterior end, and formed a somewhat conical bag, not unlike a cardiac ventricle, measuring about five and a half inches (14 cms.) in the antero-posterior or long diameter, and three and a half inches (9 cms.) in the dorso-ventral direction. The axis of the chamber was a direct continuation of that of the œsophagus. It had two borders and two surfaces. The one border was dorsal in position, straight in character, and carried, about its middle, a small spleen and an accessory mass of lymphoid tissue. The other border, ventral in position, was curved, and was in two parts. The anterior part was blended with the coats of the second chamber, while the posterior half was free, and united only by a peritoneal fold to the second chamber in the anterior half.

The surfaces were left dorsal, and right ventral. The former was free and entirely invested by peritoneum. The latter was flat, and in its upper part was in contact with the left surface of the pancreas, which here projected into the angle between chambers 1 and 2. This surface was crossed obliquely by the line of reflection of the peritoneum. The interior was filled with food consisting of fish which had been crushed and triturated so that the flesh was entirely removed from the bones, while the latter were, for the most part, disarticulated from each other. The food was moist, but not at all wet or pulpy, so that the treatment to which it had been subjected in this chamber might be called "dry mastication." The interior of this "kau-magen" presented appearances which quite corresponded with its apparent functions. Its walls were covered by a thick, white, almost porcellaneous lining, which was everywhere thrown into bold, prominent rugæ. On the ventral wall of the chamber, and immediately behind the œsophageal opening, the rugæ were thrown into a cauliflower-like projection, and in the centre of this mass a careful examination revealed the outlet of the chamber. The inlet and outlet were thus remarkably close to each other, but, besides being situated at right angles to each other, the manner in which the outlet was concealed within a mass of prominent rugæ made it quite impossible for food to enter indiscriminately from the œsophagus into the second, as well as the first, compartment. In the first instance, it was only possible for food to enter the first chamber. To the naked eye, the lining membrane was continuous with, and of the same nature as, the lining membrane of the œsophagus.

Under the microscope (Pl. II. fig. 3), the lining membrane was found to be situated upon a well-marked layer of loose areolar tissue in which numerous capillary blood-vessels ramified. Its vertical thickness measured almost 1 cm. The free surface was extensively but not deeply corrugated, while its deep surface was interrupted by numerous narrow clefts extending towards the free surface for varying distances, but usually not more than half-way. The whole arrangement was closely suggestive of the rete malpighi and stratified epithelial layers of skin, while the areolar tissue and vascular prolongations, which occupied the subjacent furrows or clefts, bore a close resemblance to the arrangement of the papillæ of the true skin. In the deeper layers, the cells of this lining membrane presented rounded nuclei, which stained deeply. Nearer to the free surface the nuclei stained less distinctly, but their rounded character was well maintained. Comparatively close to the surface, the nuclei became markedly flattened, and, at the same time, the cell stratification became pronounced. At this level also, and onwards to the free surface, the tissue was distinctly paler because it absorbed less of the staining agent. At no part of the membrane was there any trace of any arrangement for secretion.

There has been much discussion as to whether this compartment should be regarded as a part of the stomach, or merely a post-diaphragmatic diverticulum of the œsophagus. We have already shown that it carries the gastro-splenic omentum, and possesses the general peritoneal relations which one associates with the stomach, and there can be no doubt that it acts simply as a triturator of the food. Moreover, since the outlet is situated at right angles to the inlet, it is highly improbable that food could pass from the inlet to the outlet without first of all making its way through the triturator. Again, as the food which was found in the compartment was not digested, there appears no reason to suppose that any digesting takes place in this chamber as the result of the regurgitation of gastric juice into it from the second compartment.

In consideration of the nature of the teeth with which the porpoise is provided—viz., tearing teeth and not grinding teeth—it appears highly advantageous that the stomach should be so specialised as to supply the necessary grinding or triturating apparatus through the action of a chamber which is able to crumble food exactly as a piece of bread might be reduced to powder by the crushing action of the hand. We do not regard the similarity between its lining membrane and that of the œsophagus as of itself a sufficient reason for concluding that it is a diverticulum of the œsophagus, from the fact that in a one-chambered stomach the œsophageal lining membrane may be prolonged for some distance upon the interior of the stomach—*e.g.*, in the pig; while from the general, but, more especially, the splenic, relations of this first compartment, we are of opinion that it must be regarded as an undoubted specialisation of the stomach, and not of the œsophagus. The peritoneal mesenteric or omental connection between this chamber and the spleen gives a strong argument for recognising the chamber as stomach, especially as the spleen always develops within the mesentery which attaches the primitive stomach, and not the œsophagus, to the dorsal wall of the abdominal cavity. That this association of the spleen with the first compartment of cetacean stomachs is not peculiar to the porpoise, has been shown by Sir Wm. Turner, who has recorded * a similar arrangement in the stomachs of *Hyperoodon rostratus*, *Delphinus delphis*, *Delphinus* (*Lagenorhynchus*) *albirostris*, *Monodon monoceros*, and *Grampus griseus*. We think that the term "kau-magen," or masticatory stomach, would fairly express its function and its morphology.

The *second compartment*, which was situated on the ventral aspect of the first, and formed an acute angle with it, had very much the shape of the human stomach, and presented two surfaces, two borders, and two apertures, but the chief axis lay more in the long axis of the abdominal cavity than does that of the human stomach. The ventral surface, rounded and convex, was in contact with the liver and the anterior abdominal wall. The dorsal surface looked towards the lesser peritoneal sac, and was in contact with the pancreas and chamber 1. The greater curvature measured 16 cms., and had an omentum attached to it; the lesser border measured 12·5 cms. In contact

* Turner, *Jour. of Anat. and Phys.*, vol. xxiii. pp. 466–492, vol. xxvi. pp. 258–270.

with this border, from left to right, were the pancreas, a peritoneal fold, and the recurved tubular part of the stomach for a distance of 6 cms. The apertures were situated, not at the ends, but in the walls or curvature. When opened, it was empty, and its cavity was only slightly smaller than that of the first chamber. With the exception of the external layer of muscle, the mucous and muscular coats were raised into thick prominent rugæ, separated from each other by deep intervening furrows. Although these rugæ were indented by mutual pressure, yet they resembled cerebral convolutions on a small scale. Their general disposition was in the long axis of the cavity, but, though running for the most part parallel to each other, they converged around the inlet and outlet of the chamber.

As already indicated, the inlet was placed upon the dorsal aspect of the chamber, and was at right angles to the line of entrance of the œsophagus into the first compartment. The appearances might be simulated by making a tight constriction round a one-chambered stomach close below its œsophageal orifice, and then applying the long contiguous sides of the two parts closely together. However, the sharp angle between the two compartments was occupied by the loose folds of the great omentum, so that neither of the chambers was prevented from distending, by its peritoneal relationships. Thread parasites adhered to the lining membrane, which was smooth and of a rusty brown colour.

Under the microscope, the secreting mucous membrane was from 2 to 3 mms. thick (Pl. II. fig. 4 (A)), and it closely invested the strong muscular ridges, which, as already indicated, resembled cerebral convolutions or large columnæ carneæ. It consisted essentially of tubular glands (Pl. II. fig 4 (B)) supported upon a very distinct muscularis mucosæ, which sent definite prolongations, accompanied by a fine fibrous stroma, among the closely arranged tubules. The intertubular tissue was plentifully provided with capillary vessels.

The glands occupied the entire thickness of the mucous membrane, and each gland presented two parts, distinguishable from each other by structural differences. First, each gland, from its mouth inwards for a distance of ·04 mm., consisted of a delicate basement membrane lined by short columnar cells, which surrounded a circular lumen —the duct of the gland—and stained after the manner characteristic of duct cells. The cells which had covered the free surface of the mucous membrane had been mostly desquamated, but so far as could be ascertained, they were similar to those lining the ducts just described. The gland proper constituted the remainder and second part of the secreting apparatus. Each tubule appeared to become branched as it passed inwards from the surface, but, judging from the fact that transverse sections of the tubules were not found in the deeper levels of the mucous membrane, the amount of tortuosity was not great. Each tubule (Pl. II. fig. 4 (B)) presented magnificent examples of central and parietal cells. The central cells were large nucleated polyhedral cells, set closely together and bounding the irregular lumen of the tubule. Everywhere they stained very deeply. The parietal cells formed a continuous single layer of cells, globular in shape and set in intimate contact with the basement membrane, which sent prolongations inwards between them towards the central cells. As a result, each of the parietal cells occupied a recess or pocket exactly as if it lay in one compartment of a honeycomb. The parietal cells were not quite so large as the central cells, and from the fact that the former separated quite readily from the latter, one judged that there was no very definite uniting medium. The products of the activity of the parietal cells would find their way into the lumen of the tubule through fine interstices between the central cells, and these interstices were visible or concealed according to the direction of the section. Each parietal cell might be regarded as a unicellular secreting gland. They were filled with coarse eosinophile granules. It is important to note that these cells formed an unbroken layer, and that they were situated next to the blood supply, and that, therefore, they intervened between the central cells and their direct blood supply. In presenting a continuous layer of parietal cells, these tubular glands differ from those with which they correspond in such animals as the dog, the bat, and even in man, in all of which they only occur at intervals in the walls of the tubules. It is also of interest to note that these oxyntic cells are definitely restricted to one compartment of the stomach of the porpoise, whereas in the single-chambered stomachs of the other mammals above referred to, they are specially characteristic of the fundus, although not exclusively confined to that region.

The intertubular stroma everywhere showed very well marked capillary vessels, and throughout the stroma, more especially towards the free side of the mucous membrane, numbers of lymph cells were scattered.

From the ventral aspect of this compartment, and about one inch (2·5 cms.) in front of its most dependent part, a small constricted passage, wide enough to transmit the handle of a rod 5 mms. in diameter, led into the *third compartment* (Pl. III. fig. 10B). This passage was quite concealed among the thick muscular rugæ of the second chamber, and its exact position would scarcely be suspected from an examination of the second chamber. Viewed from the third compartment, however, it presented the appearance of a small firmly constricted aperture opening backwards. We feel assured that this is nothing more than a mural passage from the second to the third chamber. It has been regarded by Jungklaus as a separate chamber, supplying in itself the third chamber, which has long been sought for in order to complete the analogy between the stomach of the porpoise and that of other cetacea. But the necessity for doing this disappears, since the method we have employed reveals the presence of a distinct globular third chamber, entirely separate from the second and fourth compartments, and outside of the mural passage, which is required as an outlet from the second to the third compartments. The third compartment was situated somewhat behind and on the ventral aspect of the second, to which it was closely adherent by the surface next the inlet without the intervention of peritoneum; but on its dorsal and ventral surfaces the peritoneum was prolonged from the second compartment without interruption, and, moreover, it had the great omentum attached to the hinder border of its free surface. This globular chamber was marked off from the distal tubular part by a constriction visible externally, and well marked internally. The walls of this chamber were little more than one-sixteenth of an inch (less than 2 mms.) in thickness. The lining mucous membrane was pale, very slightly rugose, and at frequent intervals it presented pin-point depressions surrounded by slightly raised rings of the mucous membrane. This chamber was found empty, but it appeared capable of holding, without distension, material equal in bulk to the size of a small orange. While the inlet opened backwards to the second compartment, the outlet opened forwards into the fourth compartment. The adjacent margins of these two openings were about three-eighths of an inch (9 mms.) apart. The outlet was not so firmly constricted as the inlet, and could transmit the tip of a little finger without being unduly stretched.

Examined microscopically (Pl. II. fig. 5), the mucous membrane was found to rest upon a thick layer of muscularis mucosæ, which everywhere sent prolongations into the intertubular intervals. As in the preceding compartment, the surface layer of epithelium had been desquamated, but otherwise the tissue was in a satisfactory condition. It formed a layer averaging about 5 mms. in thickness, and throughout it presented large numbers of tubular glands. At intervals in its deeper half spherical nodules of lymphoid tissue appeared, while all through the intertubular stroma, which was considerable in amount, large numbers of lymph corpuscles were visible. These were most numerous in the immediate vicinity of the lymph nodule. The tubular glands appeared to be fairly simple in their arrangement, and in all probability they do not always branch as they descend into the substance of the mucous membrane. When they do divide, it is probably not oftener than once. Towards their deeper ends they appear to follow a sinuous course, and they may there be somewhat convoluted. The end next the surface tends to be straighter and less wavy. A delicate basement membrane supported the cells which lined the glands. These cells were somewhat cubical in shape, and, while their nuclei were always quite distinct, yet those pertaining to the superficial part of the tubules stained much more deeply than those belonging to the deeper part of the gland. A transverse section of the deep end of a tube examined under a higher power (Pl. II. fig. 6) revealed a small circular lumen surrounded by a close-fitting layer of nucleated cells. There was no trace of any arrangement corresponding to parietal cells. The tubules of this and of the succeeding compartment may very fairly be likened to the pyloric glands of other mammals, but instead of being scattered among oxyntic glands and only predominating near, or being exclusively found in the vicinity of the pylorus, they are restricted to the third and fourth compartments, and are not intermingled with oxyntic glands.

Vessels of different sizes were readily visible in the submucous layer, but the penetration of capillaries into the intertubular stroma was not very great, as in the case of the second compartment. It would therefore appear as if the vascularity of this mucous membrane was not a prominent characteristic of its structure.

The succeeding or *fourth compartment* was shaped like an inverted V ($\wedge$), of which the proximal limb was rather shorter than the distal one. A small amount of constriction was observed at the angle of the Λ, while at its termination it was bent slightly forwards. It would be possible to describe this segment of the stomach as *two* chambers, but that seems to be uncalled for, in view of the fact that the microscopic characters of its mucous membrane did not, in any special manner, differ from those detailed for the third compartment. Indeed, there is reason to believe that this entire segment is, in reality, one chamber of nearly uniform cylindrical appearance, capable of being divided into a larger or smaller number of subdivisions by means of septa or circular constrictions more or less pronounced. It is this chamber which is so frequently divided in the narwhal. They might very well correspond to the pyloric half of the human stomach subdivided by constrictions. The proximal limb of the Λ-shaped compartment was situated in front of the third chamber and in close apposition with the lesser curvature of the second, to which it was intimately connected by direct prolongations of the peritoneum upon their free surfaces. The great omentum was also continued into the angle formed by the limbs of the V. Like the preceding compartment, it was empty. From its inlet to the slight constriction at the angle of the V measured three and a half inches (9 cms.) in length by one and a half (38 mms.) in width. The general form of this segment of the chamber was an elongated oval.

Its walls, as regards thickness, were similar to those of the preceding chamber. The mucous membrane was pale, showing only a faint amount of rugosity; thereby indicating that it did not require to undergo much distension. At intervals pin-point depressions like those already noted in the preceding chamber were observed. At the point of the V—*i.e.*, at the acute angle—there was a certain narrowing of the lumen which might be described as constriction, or might be referred to the mere acute bending. The microscopic features of the mucous membrane did not differ in any marked way from those already described in connection with the third compartment, but the tubules were probably rather shorter, and the mucous membrane slightly thinner.

The distal limb of the inverted V extended to the pylorus. It virtually formed a cylindrical tube measuring about five inches (13 cms.) in length, and directed backwards and to the right. Within one inch (2·5 cms.) of the pylorus it underwent a slight dilatation on its hinder aspect, forming an Antrum pylori. At the same time it turned forwards with considerable abruptness, to end at the pylorus. Along its anterior and somewhat concave aspect it afforded attachment for the gastro-hepatic omentum. Between the laminæ of this omentum, masses of lymphatic gland were found towards its left end, and the head of the pancreas occupied a similar position in relation to the pylorus at the right end

The mucous membrane was very slightly rugose, and presented numerous pin-hole depressions, each of which, as in the places already mentioned, was surrounded by a slightly raised ring of the mucous membrane. Microscopically the mucous membrane did not present any variation upon what has been stated in connection with the last two compartments (Pl. III. figs. 7A and B). Apparently the pin-hole depressions may be associated with the nodules of lymphoid tissue which are everywhere embedded in the mucous membrane of the second and subsequent compartments. Whether these nodules communicate directly with the lumen of the intestine we cannot say, but we have not found any trace of surface epithelium superficial to these nodules in any part of the alimentary canal, where they occur embedded in the mucous membrane. There is plentiful evidence that these nodules push their way entirely through the stratum of tubular and intertubular tissue of which the mucous membrane consists. If they are covered over by a layer of surface cells, as is generally supposed, there are certainly no glandular structures between them and the lumen of the canal. In their whole structure they present a remarkable similarity to the faucial tonsils of man, and they may serve a similar purpose.

The Pylorus.—The pyloric orifice was a tightly constricted passage which, with some pressure, would admit a rod 5 mms. in diameter. It was situated in the middle of a projection somewhat like an exaggerated nipple. The actual orifice was found on the summit of this nipple, which was directed into the interior of the compartment backwards and to the right side.

Every one of these constricted passages was placed as if with the definite object of making the onward progress of the contents as difficult as possible, and only attainable after the most complete circuit of the compartment. The length of this canal was rather more than half an inch (16 mms.). In its general arrangement and appearance it resembled the canal which formed the passage of communication from the second to the third compartments. Still no one has ever proposed that the pyloric passage should be adduced as a separate chamber, and so we regard JUNGKLAUS' statement upon the third chamber as an error in homology.

The Duodenum.—Beyond the pylorus, the intestine commenced with a marked dilatation, whose diameter at first resembled that of the adjoining part of the stomach. Very soon, however, it slightly increased in size, but after a course of two and a half inches (6·5 cms.), it suddenly dwindled and assumed the general characters of the remainder of the intestine.

Immediately before this dilatation became reduced to the proportions of the ordinary bowel, it received the bile-duct which penetrated its dorsal surface. Further, the pancreas was closely applied to this portion of the canal, but the pancreatic duct formed a tributary of the bile-duct, and so did not show a separate opening through the wall. This disposition of the pancreatic ducts was not surprising, since the bile-duct passed through, and was therefore surrounded by the head of the pancreas. From this disposition of these important ducts we are justified in regarding this section of the canal as the duodenum, which therefore corresponds to what are called the first and second parts of the human duodenum.

On being opened, the duodenum was seen to be lined by a mucous membrane considerably darker in colour than those which had lined the preceding compartments of the stomach. Whether this colour should be attributed to staining by bile we cannot say, as there was no gall-bladder, and therefore we saw no bile. It was thrown into prominent rugæ, and presented a miniature copy of the interior of the second stomach chamber. Its capacity was about equal to that of the third compartment of the stomach. The opening of the bile-duct was situated on a papilla (*Papilla voteri*) on the dorsal wall of the duodenum, about half an inch (12 mms.) from the termination. This termination was not marked by any valvular constriction, but was merely indicated by a sudden reduction in calibre to the shape and proportions of the intestinal tube.

From such a distinct and precise disposition of this portion of the canal there can be no reasonable doubt that, so far as this animal is concerned, the duodenum should not be regarded as a section of the intestine, but rather as a separate and special chamber within which the liquid or peptonised food, having left the stomach, is subjected to the action of the biliary and pancreatic secretions. It was a contention of the late Professor GOODSIR that the duodenum of man ought to be considered as a separate segment of the bowel, on account of its attachments, structure, and functions. On these points, so far as the porpoise is concerned, this view is quite clearly supported.

Microscopically, the mucous membrane was remarkable for its negative as well as for its positive characters. Villi were entirely wanting from its surface, and there were no trace of Brunner's glands.

The rugæ already mentioned formed longitudinal ridges of muscularis mucosæ covered by the lining mucous membrane, which consisted of tubules similar to those found in the adjoining intestine.

The surface epithelium was, for the most part, denuded, but on some remaining patches the cells were short and cubical. Each gland was comparatively straight, the duct being lined by short columnar epithelium, while the deeper part of the gland presented a considerable number of chalice cells.

The intestine, which commenced at the end of the duodenum, extended to the anal aperture as a tube of uniform appearance and supported by a single mesentery. In calibre it appeared to be also fairly uniform, and although some parts were more firmly contracted than others, there was no outward evidence of division into large and small bowel. In its general appearance it resembled small bowel, and its least contracted parts were not larger than the diameter of an average digit. No diverticulum or appendix occurred anywhere. It measured rather more than fifty feet in length—*i.e.*, nearly twelve times the length of the animal.

When we consider that the length of the human intestine is only from four to five times as long as the individual, we are probably justified in associating the unusual length of the intestine of the porpoise with the provision of an extended absorbing surface in compensation for the absence of villi from its lining mucous membrane. Transverse sections made at intervals along the entire length of the tube revealed the presence of eight or nine longitudinal and projecting folds which occupied the greater part of the lumen. These left very little free lumen, and in those places where the bowel was firmly contracted, the cut face appeared almost solid by reason of the longitudinal projections. In the first half or thereby of the bowel, these projections were fairly evenly distributed round the interior of the tube (Pl. III. fig. 8A), but in the lower or hinder half those upon the mesenteric side of the gut were considerably reduced in size to make room for an increased prominence of those upon the side opposite to the mesentery (Pl. III. fig. 9A).

The microscopic appearances of the mucous membrane differed considerably in the upper and lower parts. At no point were villi discovered, but in the upper or anterior half the glandular arrangements were almost exactly similar to those of the duodenal mucous membrane (Pl. III. fig. 8B), except that in the bowel the glands were somewhat shorter than in the duodenum. Chalice cells were also prominent appearances. In the lower part of the bowel, the first noteworthy feature was the difference in the size of those ridges attached on the side next to the mesentery, as compared with the size of those on the side opposite to the mesentery. The latter set consisted of four thick and long projections. They occupied half of the circumference of the tube by their bases, but their projecting ends filled considerably more than half of the interior. The mucous membrane which lined the entire tube was very similar to that which has already been described for the upper part of the bowel, but the number of chalice cells now so greatly preponderated that the mucous membrane appeared like a network. The outstanding feature of the sections was the presence of globular masses of lymph tissue situated in the submucous layer (Pl. III. fig. 9B), but sending prolongations through the mucous covering apparently to discharge upon the free surface. We did not find a layer of epithelium covering those lymph nodules, but, as in other parts of the bowel, it may have been desquamated. This disposition of lymph tissue was confined to the three trenches which separated the four large ridges from each other, and these lymphoid patches were strictly limited to one side of the intestine, and no similar arrangement occurred on the mesenteric side of the intestinal wall. Nine or ten of these lymphoid masses were visible on the sides of a single ridge in each microscopic section, and the total amount of lymph tissue thus arranged must have been very great.

The pancreas, lying between the liver and stomach, was pyramidal in shape, measuring 8 cms. transversely, 7 cms. antero-posteriorly, and 6 cms. in height. The base was posterior, and looked towards the lesser sac, while the blunted apex was in contact with the under surface of the liver.

The left side was in contact with the first chamber of the stomach, the dorsal surface with the dorsal abdominal wall, but covered by peritoneum; the right surface was in contact with the terminal part of the stomach, pylorus, and duodenum; and the anterior surface, continuous with the right surface, looked towards the lesser sac, and was crossed by the tubular part of the stomach.

The tissue of which the gland was composed was folded round the bile-duct, which, therefore, passed through the gland.

The liver had two large lobes—right and left—which were prolonged anteriorly into two conical projections, between which there was a triangular depressed area, mesial in position.

These two projections coincided with the hollowed-out bases of the lungs, while the heart occupied the mesial depression, the diaphragm intervening in both cases.

There was no gall-bladder, and therefore no quadrate lobe to the liver, but indications of a spigelian and a caudate lobe have already been noted.

The microscopic structure of the liver, spleen, and pancreas did not materially differ from the appearances of those organs in man. The sections of the pancreas displayed the characteristic cells associated with acini and ducts.

From an examination of the sections, we are inclined to think that the cells described as "central acinar cells" are merely sectional views of the cells which line the commencement of ducts that chance to lie upon the side of the section next to the observer; and further, that the so-called paranucleus or "nebenkern" results from the same structures lying on the side farthest from the observer, and therefore less distinct. Otherwise it seems that in such clear and distinct sections these special structures should appear more frequently than they do.

It ought to be recorded that the tape-worm—Bothriocephalus latus—was found in the intestine. This parasite has not hitherto been observed in the alimentary canal of a marine mammal.

Conclusions.

(1) Although the porpoise, like other cetacea, is a mammal without hind limbs, and although its innominate bones are reduced to a couple of slender rods, yet, in the peritoneal arrangements associated with the posterior end of the abdominal cavity, there is evidence of a pelvic cavity which has not hitherto been recognised as such.

(2) From the present observations, we agree with those previous observers who have described the stomach of the porpoise as four-chambered, and we are in harmony with them as regards the homologies of the first and second compartments, although we regard the first compartment as developed from the primitive stomach, and not as a dilatation of the post-diaphragmatic œsophagus.

(3) We do not regard the "canal" or mural "passage," which lead onwards through the wall of the second compartment, as the third compartment, but look upon this "canal" as the inevitable association of the thick wall of the second compartment. Moreover, it is not quite so long as the "passage" of communication between the first and second chambers, and but little longer than the pyloric "passage," neither of which have ever been regarded as homologous with separate compartments.

(4) We find the third compartment in a distinct chamber beyond the wall of the second, and clearly demarcated from the fourth, both by its somewhat spherical shape and by its constricted outlet.

(5) Although Jungklaus and other writers support the view that the third compartment of the adult porpoise is the mural "canal" or "passage" already referred to,

yet, in his plate, Jungklaus figures several stomachs of fœtal porpoises which apparently entirely agree with our description of the adult stomach.

We are therefore forced to conclude that, by our method of preservation, we have been able to retain the normal appearances and shapes of these stomach chambers, which have hitherto either been lost, or of such uncertain characters as to lead to error or difficulty in the determination of their true homologies.

(6) The fourth compartment, being tubular or cylindrical, is distinctly marked off from the spherical third chamber on the one hand, and from the duodenum on the other. The acute bend near the middle of this chamber, and the pre-pyloric dilatation (Antrum pylori), show how readily it might be still further subdivided.

(7) From these considerations, it would seem as if the stomachs of all cetacea were constructed upon a common plan of subdivision into a series of chambers, with such variations as regards the number, size, and particular shapes of the compartments as are explicable by reference to the porpoise, and are probably due to differences in the characters of the teeth and the nature of the food determining the presence or absence of that particular compartment which we have called the "kau-magen" or masticatory stomach, and which in the case of the porpoise forms the first of the series of compartments. Further, the homologies of these compartments among different members of the Cetacea should be established by their structure and anatomical relations rather than by numerical sequence.

(8) We regard the duodenum as that dilated part of the alimentary canal between the pylorus and a point immediately beyond the common entrance of the bile and pancreatic ducts.

(9) Intestine proper commences at the termination of the duodenum, and is suspended throughout in the mesial peritoneal mesentery.

(10) The small size of the spleen is probably compensated for by the unusual amount of lymphoid tissue distributed in the omentum and at different parts in the wall of the alimentary canal, especially towards its lower end.

(11) Throughout the entire length of the alimentary canal, villi were absent.

LITERATURE.

The literature on the Cetacean Stomach is very extensive, but it has been brought up to date by Dr Friedrich Jungklaus, who quotes 63 memoirs in his paper "Der Magen der Cetaceen," *Jenaische Zeitschrift für Naturwissenschaft*, xxxii., 1898.

Lehrbuch der Vergleichenden Mikroskopischen Anatomie der Wirbeltiere, Oppel (Jena), 1896.

EXPLANATION OF FIGURES.

Fig. 1. Inlet of pelvic cavity of male porpoise (viewed from abdominal cavity).

Fig. 2. The hinder part of the abdominal cavity viewed from the side, showing inlet of pelvic cavity.

Fig. 3. Section of first compartment of stomach of porpoise stained in hæmatoxylin-eosin. × 96. Zeiss, oc. 4, obj. BB.

Fig. 4 (A). Outline of section of second compartment of stomach of porpoise, × 20, to show thickness of mucous membrane.

Fig. 4 (B). Section of second compartment of stomach of porpoise stained in Benda's fluid. × 270. Zeiss, oc. 4, obj. DD, to show structure of secreting tubules.

Fig. 5. Section of third compartment of stomach of porpoise stained in hæmatoxylin-eosin. × 96. Zeiss, oc. 4, obj. BB.

Fig. 6. Section of a tubule (deep end) of third compartment stained in hæmatoxylin-eosin. × 270. Zeiss, oc. 4, obj. DD.

Fig. 7A. Section of fourth compartment stained in hæmatoxylin-picro-fuchsin. × 96. Zeiss, oc. 4, obj. BB.

Fig. 7B. T.S. of one tubule. × 470. Zeiss, oc. 4, obj. E.

Fig. 8. T.S. section of upper end of intestine stained in eosin-methyl-blue. (A) natural size. (B) × 96. Zeiss, oc. 4, obj. BB.

Fig. 9A. T.S. of lower end of intestine, natural size.

Fig. 9B. One ruga in transverse section stained in hæmatoxylin-eosin. × 20. Zeiss, oc. 4 : Beck, obj. 2″.

Fig. 10. Scheme of compartments of stomach of porpoise.

A. *After Jungklaus.*

O. Œsophagus.
I. First compartment.
G. Epithelial boundary between first and second compartments.
II. Second compartment.
III. Third ,,
IV. Fourth ,,
P. Pylorus.
Ad. Ampulla duodenalis.
P.v. Papilla vateri.
D. Duodenum.

B. *Hepburn and Waterston.*

O. Œsophagus.
I. First compartment (Kaumagen).
II. Second ,,
III. Third ,,
IV. (*a*) Proximal part of fourth compartment.
(*b*) Distal ,, ,, ,,
(*c*) Antrum pylori.
P. Pylorus.
D. Duodenum.
P.v. Common entrance of bile and pancreatic ducts, Papilla vateri.
Int. Intestine.

We desire to express our indebtedness to Sir WM. TURNER and Professor SCHÄFER r much friendly criticism and suggestion during the preparation of this paper.

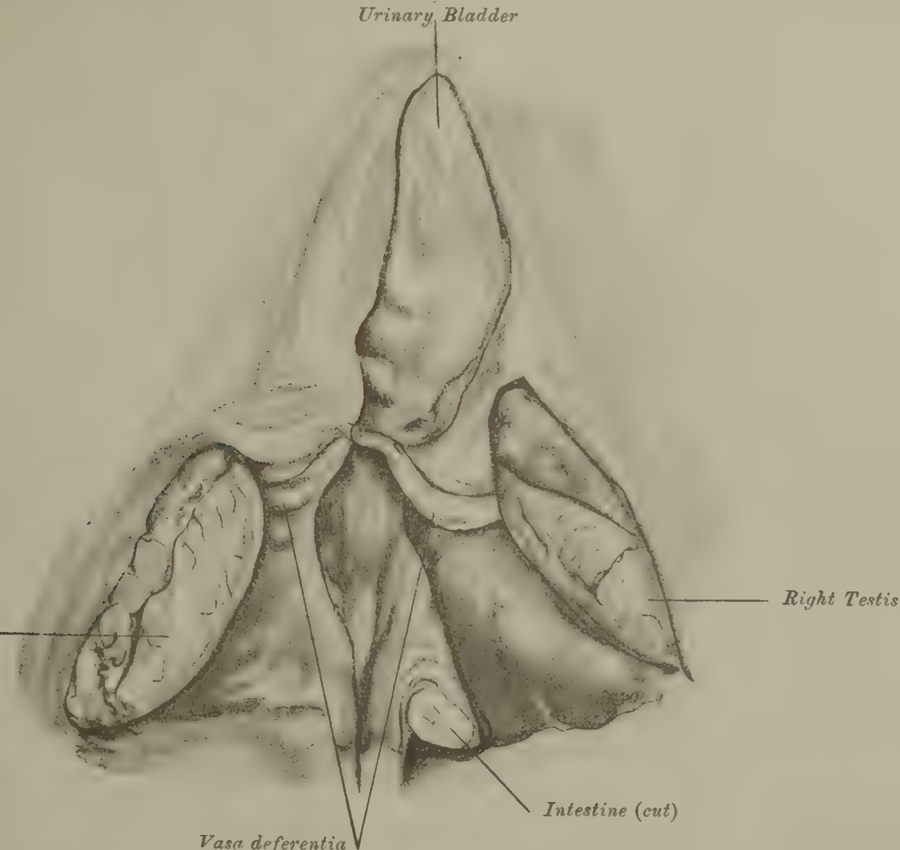


Fig. 1. INLET OF PELVIC CAVITY OF MALE PORPOISE VIEWED FROM ABDOMINAL CAVITY.

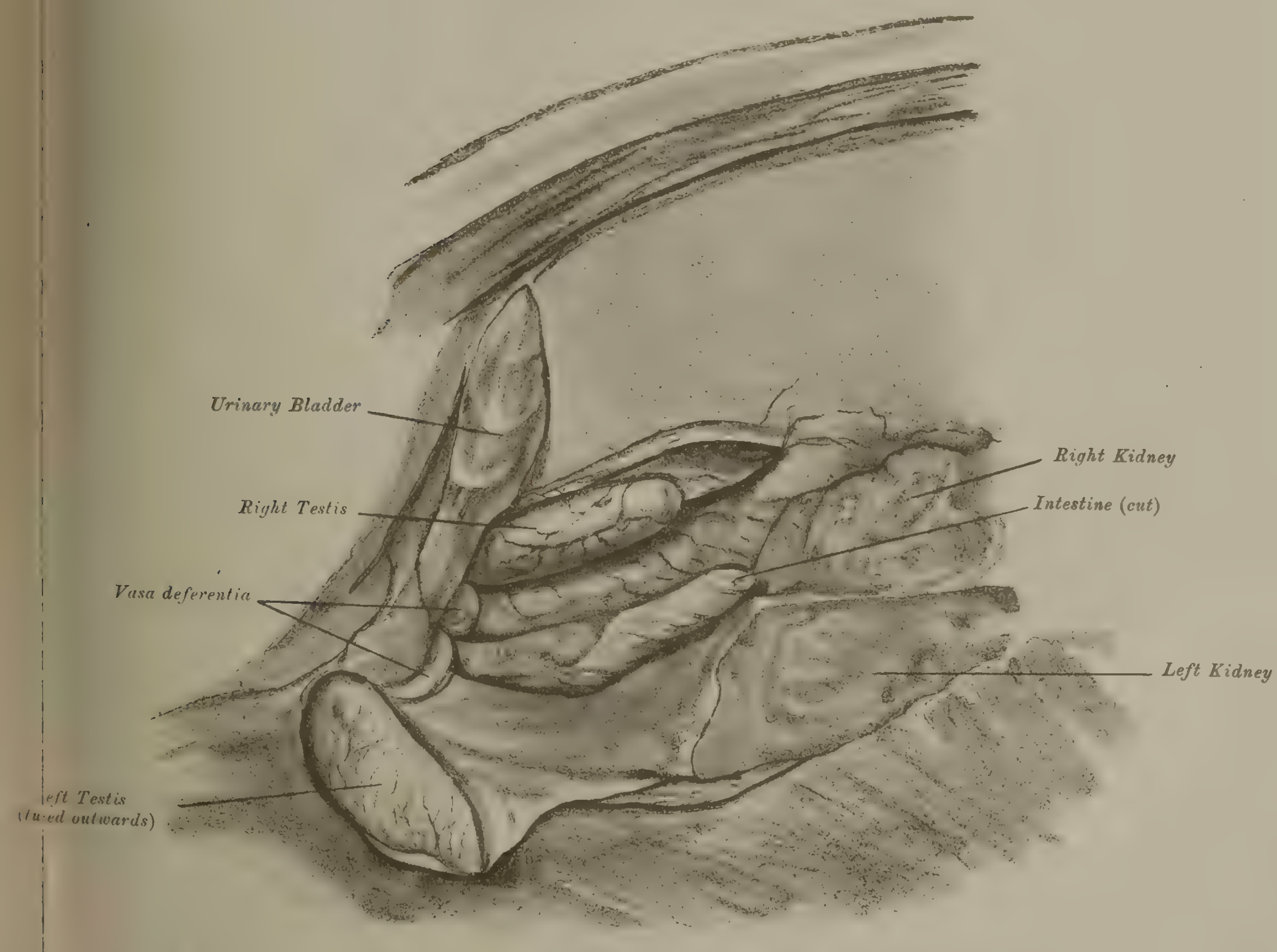


Fig. 2. THE HINDER PART OF THE ABDOMINAL CAVITY VIEWED FROM THE SIDE.

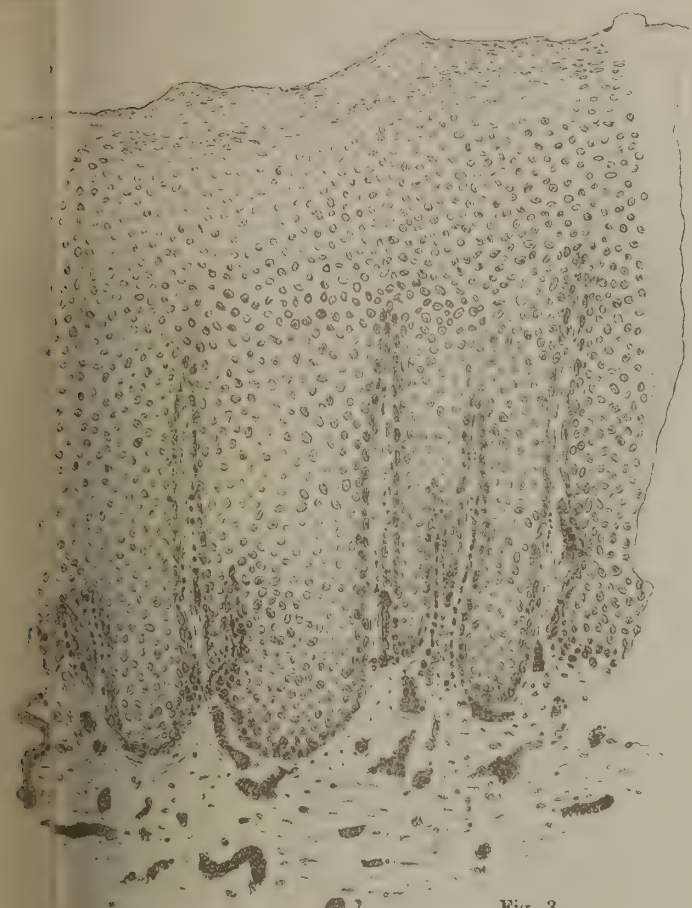


Fig. 3.

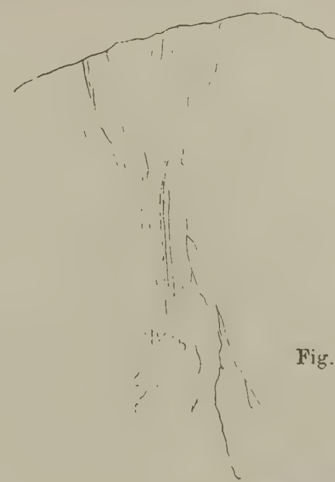


Fig. 4 (A).

Fig. 4 (B).

Fig. 5.

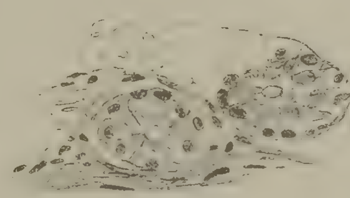


Fig. 6.

D. DEL AD NAT. W. GRIGGS. LITH.

Fig. 7 (A).

Fig. 8 (A).

Fig. 8 (B).

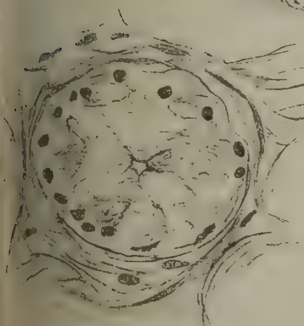


Fig. 7 (B).

Fig. 9 (A).

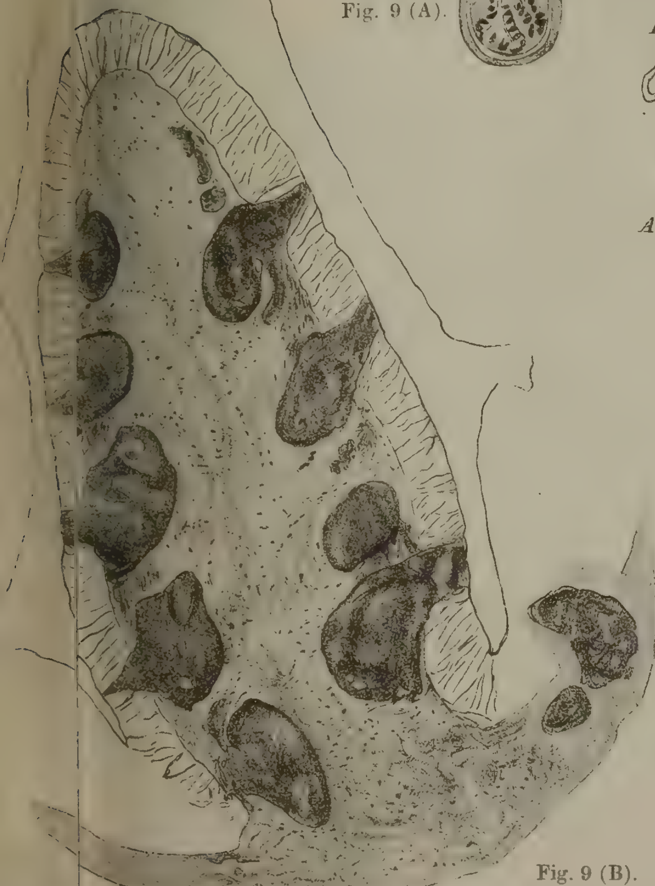


Fig. 9 (B).

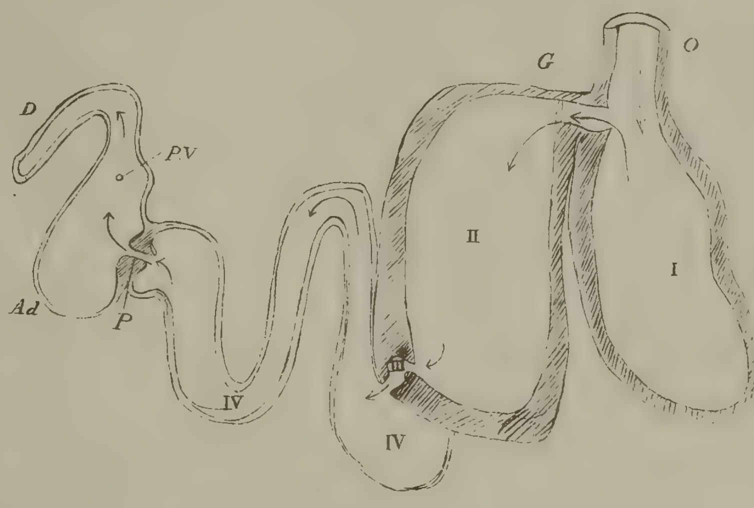


Fig. 10 (A).

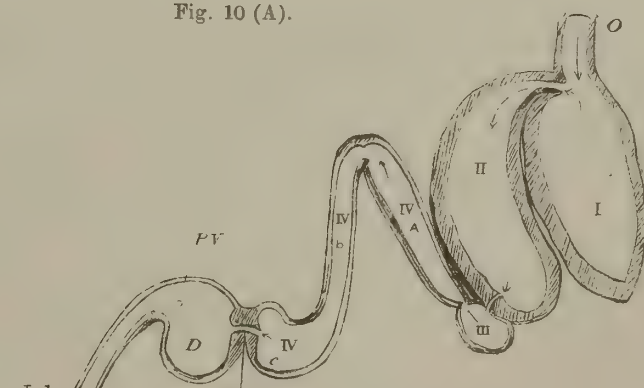


Fig. 10 (B).

XVII.—*On the Primary Structure of certain Palæozoic Stems with the Dadoxylon Type of Wood.* By D. H. Scott, M.A., Ph.D., F.R.S., Hon. Keeper of the Jodrell Laboratory, Royal Botanic Gardens, Kew. *Communicated by* Professor I. Bayley Balfour, F.R.S. (With Six Plates.)

(Read January 6, 1902. Issued separately April 7, 1902.)

In a Note published in the *Annals of Botany* for December 1899,* I gave some account of the structure of two stems from the Lower Carboniferous of Scotland, provisionally named *Araucarioxylon fasciculare*, sp. nov., and *A. antiquum*, Kr. (Witham, sp.).

In the present paper these stems are described fully, with the help of illustrations, and others, presenting similar points of interest, are added. The species dealt with are the following:—

Calamopitys fascicularis (*Araucarioxylon fasciculare* of the Note).
Calamopitys beinertiana (*Araucarioxylon beinertianum*, Kr.), (Göpp., sp.).

Pitys antiqua, Witham (*Araucarioxylon antiquum*, Kr. of the Note).
Pitys Withami (*Pinites Withami*, Lindl. & Hutt.).
Pitys primæva, Witham.

Dadoxylon Spenceri, sp. nov.

The reasons for the nomenclature adopted will be given in each case when the structure has been described. It was stated in the Preliminary Note that the two stems there described would certainly require generic separation on the basis of their primary characters (*l.c.*, p. 619). This has now been done, but I have succeeded in avoiding the creation of any new genus, for in the one case the characters appeared to indicate the genus *Calamopitys* of Unger as the appropriate one, while in the other the use of Witham's old generic name, in an emended sense, seemed to meet the case.

Where the old genus *Dadoxylon* or *Araucarioxylon* has to be kept up, I agree with Knowlton † and Zeiller ‡ that the former name is to be preferred in the case of Palæozoic woods, restricting *Araucarioxylon* to Secondary or Tertiary specimens, which may more probably be referred to true Araucarieæ. The use of the name *Araucarioxylon* for Palæozoic specimens, probably belonging to the Cordaiteæ, or to some other family equally remote from true Araucarieæ, is likely to mislead, and though I employed this name in the Preliminary Note, I now think it better to avoid it.

* "On the Primary Wood of certain *Araucarioxylons*," *Ann. Bot.*, vol. xiii. p. 615.
† "A Revision of the Genus *Araucarioxylon* of Kraus," *Proc. U.S. Nat. Mus.*, vol. xii. p. 601, 1890.
‡ *Éléments de Paléobotanique*, p. 279, 1900.

The importance of the fossils now to be described consists in their showing the primary structure of the wood; in all of them there is proof of the existence, within the zone of secondary wood, of distinct strands of primary xylem, resembling more or less closely those which have long been known in the stems of the Lyginodendreæ and Poroxyleæ. This structure co-exists with secondary wood of the *Dadoxylon* type, in some cases agreeing exactly with the wood which is known to have belonged to the stems of Cordaiteæ.* Thus these fossils tend to establish a connection between the stems of Palæozoic Gymnosperms and those of certain Cycadofilices, and so to throw new light on the question of the Filicinean origin of the Gymnospermous Phanerogams. The subject of the course of the leaf-traces is closely connected with that of the primary wood-strands, and is dealt with below so far as the material afforded data. Other characters are considered incidentally. At the close of the paper the theoretical bearings of the structural features in question are discussed.

I. Calamopitys, Unger.

1. *Calamopitys fascicularis*, sp. n.

This is the stem described in my Note of 1899 under the provisional name of *Araucarioxylon fasciculare.* The reasons for now placing the fossil in Unger's genus will become apparent when the structure has been described. As mentioned in the Preliminary Note, two specimens have been examined; the first which came under investigation is in the collection of Mr Kidston, and was found in 1898 by Mr John Renwick at the Loch Humphrey Burn in the Kilpatrick Hills, Dumbartonshire. The horizon is that of the Calciferous Sandstone Series. Mr Kidston very kindly placed his sections at my disposal for investigation, and also allowed me to have some additional sections cut from the remainder of the block.

The second specimen is one of which the sections are preserved in the Williamson Collection under the generic name *Dadoxylon.* The structure is identical with that of Mr Kidston's stem (*cf.* Pl. I. phot. 1 and Pl. III. fig. 1). The Williamson specimen is described in his MS. catalogue as being derived from the Carboniferous Limestone near Haltwhistle. Thus both specimens are of Lower Carboniferous age. This plant, like the other species to be described, combines an Araucarian type of secondary wood with the presence of distinct primary strands of xylem around the pith. It is characterised by the small diameter of the pith, the small number and relatively large size of the primary xylem-strands, the simple leaf-traces, and the narrow medullary rays, giving the secondary wood a Cordaitean character.

The diameter of the pith is only about 2 mm. in Mr Kidston's specimen, and about 3 mm. in the Williamson example. The whole specimen reaches a maximum diameter

* On the Cordaiteæ, see Grand' Eury, *Flore carbonifère du Département de la Loire*, 1877; Renault, *Structure comparée de quelques Tiges de la 'Flore carbonifère*, 1879; *Cours de Bot. Fossile*, t. i., 1881. A general account of the family is given in Solms-Laubach's *Fossil Botany*, chap. v., Eng. transl., 1891, and in my *Studies in Fossil Botany*, Lecture XII., 1900.

of nearly 3 cm. in the former, and about 2 cm. in the latter, but these dimensions are of no significance, as the wood is manifestly incomplete. In neither specimen is anything beyond the wood preserved.

In Mr Kidston's specimen the pith is complete, though somewhat contracted (see fig. 1); consequently all the xylem-strands can be recognised, and their course traced in successive transverse sections. In the Williamson stem the pith has perished, and the smaller xylem-strands are obscure; for details of the wood and larger primary strands, however, this specimen is rather superior to the other. Fig. 1, from one of the transverse sections of Mr Kidston's specimen, gives a good idea of the general structure. The parenchymatous pith, which, owing to shrinkage, has partly separated from the surrounding wood, has, in itself, a very uniform structure; the peripheral cells are narrower and have rather thinner walls than those towards the centre (compare the longitudinal section in fig. 3); some of the larger elements are filled with dense carbonaceous contents, which may indicate that they had a secretory function during life. Around the pith a number of xylem-strands are disposed, forming an irregular ring. Eight of these strands are wholly or partially embedded in the pith; a ninth strand (B), much the largest of all, is passing out into the zone of wood. It is a constant rule, holding good for all the sections of both specimens, that the outgoing bundles are those which attain the maximum dimensions (*cf.* phot. 2, from the Williamson specimen). A, the next largest strand, is in close contact with the secondary wood, and will be the next to pass out above, as is shown by the comparison of successive transverse sections. Most of the smaller strands are actually embedded in the pith, and are separated from the inner edge of the wood by several layers of parenchyma (*cf.* the longitudinal section, fig. 3). We have here an approach to a condition which we shall find existing, in a much more marked degree, in *Pitys antiqua*.

There are in all eight transverse sections of Mr Kidston's specimen. They were cut at different times, and I have no record of their order, but have been able to determine it with practical certainty by careful comparison of the peculiarities of the individual sections as to detailed structure, position and form of cracks, exact state of preservation, etc. The succession of the sections having been thus ascertained, it became possible to determine the course of the xylem-strands. The *camera lucida* diagrams in the text, 1 to 4, prepared for me by Mr L. A. Boodle, F.L.S., are taken from a series of four consecutive sections, sufficient to fix the essential points in the distribution of the strands. The series is from above downwards. The top section of the four (diagram 1) shows three principal bundles, markedly larger than the rest. One of them, C, is still far out in the secondary wood; another, B, of equal or even greater size, has just reached, with its inner edge, the periphery of the pith; the third, A, which is much smaller, though still far exceeding the other circum-medullary strands in size, has already entered the pith. These three leaf-traces may be taken as fixing the position of the three orthostichies, A, B and C, on which the leaves supplied by these traces were inserted. A, being cut lowest down in its course, belongs to the uppermost leaf of the three, B to the next, and C to the lowest. In the next section (diagram 2) the same three orthostichies are represented. The bundle C has here moved up nearly to the pith, but is still separated from it by a mass of radially arranged parenchyma. B has moved but little inwards, and has scarcely changed. A is much smaller than before, and has shifted in watch-hand direction, approaching an adjacent small strand (a) on that side. In the next section below (diagram 3) a new bundle, D, makes its appearance out in the wood, between A and B; it evidently comes from a leaf next below C, and thus gives the position of a new orthostichy. C has here just reached the edge of the pith and B is projecting further into it. A is now scarcely

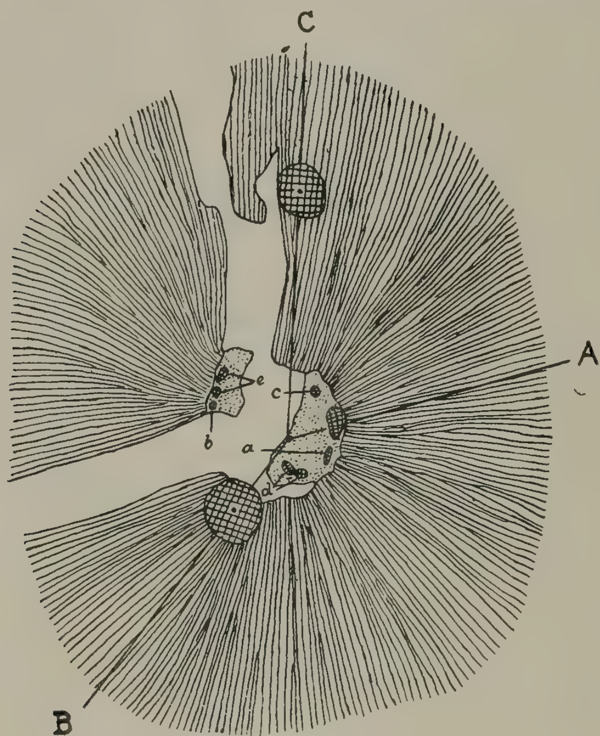


DIAGRAM 1 (K. 788).

larger than its fellow circum-medullary strands; it has shifted a little further and is now joining the adjacent strand, a; it has also become embedded in the tissue of the pith.

In the lowest section of the series (diagram 4) a new strand again, E, is entering through the wood on the left, between B and C, thus fixing the position of the fifth orthostichy, and clearly belonging to the lowest of the five leaves which are represented by their traces in this series. D has here reached the edge of the pith; C is beginning to enter it, and is somewhat reduced in size; B is much smaller than before, and has entered the pith sufficiently to have been drawn away from the wood by the contraction of the former. A, no longer distinguished by size, is still fusing with the adjacent strand, a.

The whole arrangement clearly points to a 2/5 phyllotaxis. The three successive traces, which are alone recognisable in diagrams 1 and 2, are separated by angles which correspond roughly to a 2/5 between A and B, and B and C, and about 1/5 between C and A. Where a new trace, D, makes its appearance (diagram 3) it bisects the 2/5 gap A—B, and again where a fifth trace, E, appears (diagram 4) it bisects in like manner the other 2/5 gap, B—C. It may be mentioned that all the leaf-traces shown in the other sections, not figured, are likewise referable to the same five orthostichies, A, B, C, D, E, and follow the same order of succession. No other than a 2/5 arrangement would account for the facts. The irregularities in divergence which occur are easily explained by the distortion due to cracks, and to the contraction of the pith.

The course of the smaller strands, *i.e.*, of the lower ends of the leaf-traces where they have become medullary, has not been completely made out, but some light has

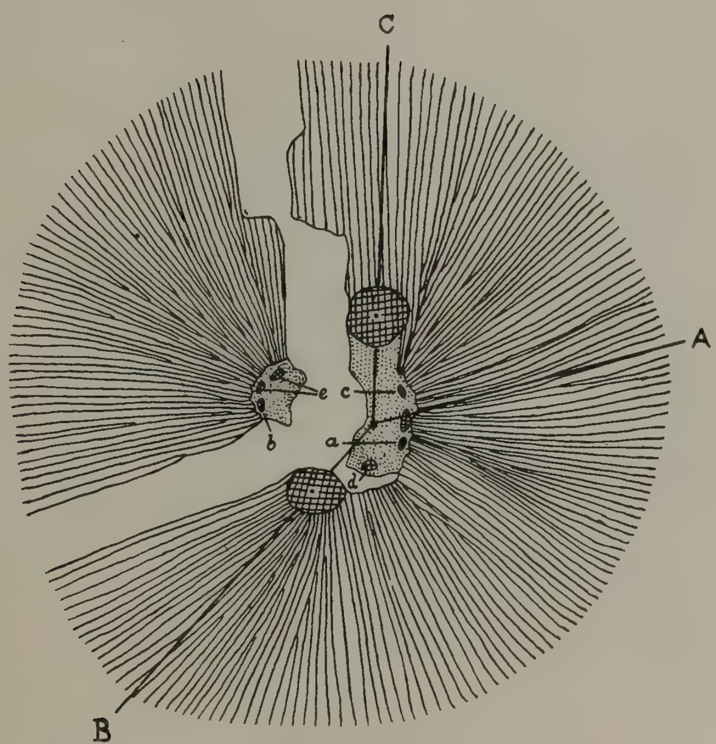


DIAGRAM 2 (K. 628).

been thrown on it. Thus the entering leaf-trace A, after it has become embedded in the pith and has diminished much in size, obviously united with the adjoining bundle (a) on the kathodic side (diagrams 3 and 4). The arrangement of the strands indicates that this was a general rule. The last leaf-trace to enter, above A, would have lain on the orthostichy E. In this position we see, in diagram 1, two small bundles (e) which may well be the reduced leaf-trace with its reparatory strand. Lower down (diagram 3) these two bundles are fusing, and in the lowest section (diagram 4) they are completely fused.

The leaf-trace still further above would have been on the orthostichy D. Two small bundles in this position (d) are already uniting in the uppermost section (diagram 1), and in the next below (diagram 2) their union is complete.

Considering next the leaf-traces which enter below A, the small strand b on the kathodic side of B is presumably the one destined to unite with that leaf-trace (diagrams 1–3); in the lowest section (diagram 4) it has disappeared, which may be due to its fusion with B, or more probably to mere destruction of tissue; for the fusion, according to the analogy of other strands, would not be likely to take place so high up. Lastly, the remaining small bundle, *c*, which appears in all four sections, is in all probability the reparatory strand ready to unite with C, which it is already approaching in the lowest section (diagram 4). In other words, if we trace the course of the bundles from below upwards, we may say that each circum-medullary strand branches

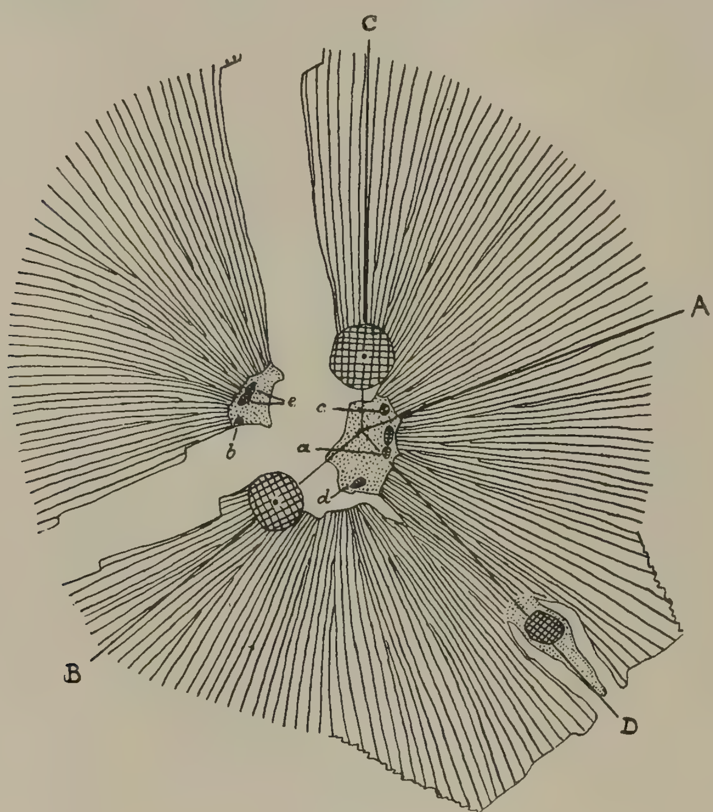


DIAGRAM 3 (K. 629).

at regular intervals; the one branch, that on the anodic side, becomes the leaf-trace and passes out, while the other continues its course up the stem as a reparatory strand, until the next leaf of the orthostichy has to be supplied. It is, however, probable that subsidiary unions between the bundles also occurred.

The course of the bundles, so far as it has been determined, thus appears to be identical with that found in the stem of *Lyginodendron oldhamium*.*

The position of the xylem-strands in the three transverse sections of the WILLIAMSON specimens also points to a 2/5 phyllotaxis. The order of the sections from above downwards appears to be :—1378, 1380, 1379.

The internodes of the stem were probably short, as is indicated by the rapid succession

* WILLIAMSON and SCOTT, "Further Observations on the Organization of the Fossil Plants of the Coal-Measures." Pt. III. *Lyginodendron and Heterangium*, *Phil. Trans. Roy. Soc.*, vol. 186 (1895), B, p. 711.

of the outgoing leaf-traces. Sometimes, as shown in diagram 3, as many as three successive traces, passing out through the wood, are seen in the same transverse section. In the lower part of its course, as we have seen, the leaf-trace bundle passes very gradually outwards, diverging but little from the vertical direction. When it has once fairly entered the wood, however, it curves more rapidly (owing to the growth in thickness of this zone), so as to assume a more nearly horizontal course, and is consequently cut in an approximately transverse plane when intersected by a tangential section of the wood; such a section, from the WILLIAMSON specimen, is represented in Pl. IV. fig. 7.

From the course of these strands, as described above, there can be no doubt that

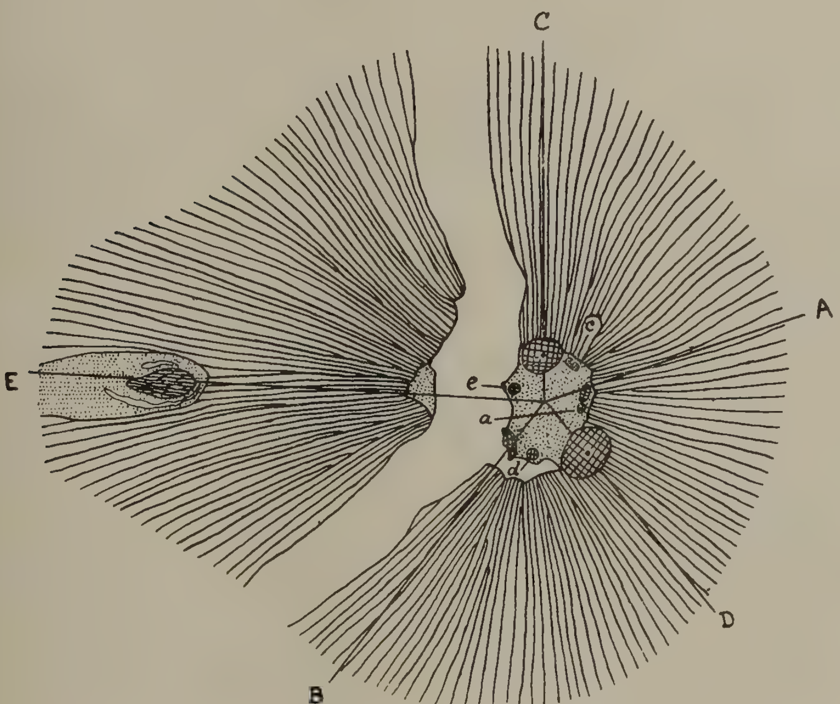


DIAGRAM 4 (K. 540c).

they represent the leaf-traces, or rather their xylem-constituent, passing out to leaves with a 2/5 phyllotaxis. On a superficial view, it might perhaps be supposed that the larger strands belonged to branches, but more careful observation shows every grade of transition between the larger and smaller strands, and proves their identical nature (see diagrams 1–4, and figs. 1 and 2). It is evident that the bundle, as it approached its point of exit from the pith, increased rapidly in size, attaining its full dimensions where it began to pass outwards. A similar increase in size, though perhaps less striking, occurs in the outgoing strands of *Lyginodendron* * and *Poroxylon*.†

The structure of the primary xylem-strands is most obvious where they attain their

* WILLIAMSON and SCOTT, *loc. cit.*, Pl. 21, fig. 1.

† BERTRAND et RENAULT, "Recherches sur les Poroxylons," *Arch. Bot. du Nord de la France*, 3me Année, 1886, figs. 198, 199, etc.

largest dimensions. The general contour is here nearly circular, and the smallest elements are placed near the centre, where they form a small group, accompanied by a little parenchyma (phot. 2, fig. 1, B). In some bundles the small elements form two distinct groups, separated by parenchyma; this is found chiefly where the strand is well advanced on its outward course, as for example in that shown in fig. 7, from a tangential section, where the strand is seen passing through the secondary wood. Oblique sections show that the small central tracheides are spirally thickened; I have not seen a satisfactory longitudinal section through one of the larger strands, but in the small bundle represented in radial section in fig. 3, the spirals in the interior of the strand are evident. As regards the large strands, there can be no doubt that the structure is *mesarch*, the protoxylem lying about at the centre of the whole strand, and probably separating into two groups as the bundle passed outwards. The tracheides towards the periphery of the primary strand have pitted walls, like those of the secondary wood, but are of larger size, reaching a diameter of 0·1 mm. or more. Between these large elements and the central protoxylem, transitional, scalariform or reticulate forms of sculpturing occur.

As the xylem-strand is followed downwards at the margin of the pith, it rapidly diminishes in size, and its elements become smaller (see A in fig. 1). Lower down, the strand passes deeper into the pith, so as to become separated from the inner margin of the secondary wood by a few (about 2–6) layers of parenchyma (fig. 1). In the lower part of its course, the arrangement of the elements of the bundle undergoes some change; the larger tracheides come to be limited to the outer side of the strand, and the spiral elements lie further inwards. A good example of a bundle fairly low down in its course is shown, in transverse section, in fig. 2. The structure is still mesarch; the protoxylem, however, is beginning to approach the inner edge of the strand; the xylem is interrupted at several points by parenchymatous elements. The small strand shown in radial longitudinal section, in fig. 3, has its spiral elements much nearer the inner than the outer side.

A similar structure is seen in some of the smaller bundles in the transverse section represented in fig. 1. Thus, as the bundle is traced downwards, the centripetal part of the xylem diminishes, but it does not appear that a purely endarch structure was ever attained. In the bundles of *Poroxylon*, according to Messrs BERTRAND and RENAULT, the centripetal xylem disappears altogether towards the lower end of each bundle.* The change of structure in *Calamopitys fascicularis* is in the same direction, but has not gone so far.

Broadly speaking, the secondary wood has the same structure as in the stem of a *Cordaites*; the medullary rays are narrow, and the pitting of the tracheides is of the usual Araucarian type. The structure is well shown in the radial section represented in fig. 4, where the pits are seen to be arranged in three or four rows on the radial walls. A small part, represented more highly magnified in fig. 5, shows the hexagonal bordered

* *Loc. cit.*, p. 306.

pits, with the narrow, slit-like, more or less inclined pore, very clearly. The borders of the pits are sometimes beautifully shown in section, where the wood is cut tangentially, as represented in Pl. IV. fig. 6 (*b.p.*), from the WILLIAMSON specimen.

The medullary rays, which have the usual muriform arrangement of their elements (fig. 4), are, for the most part, one cell only in thickness, but often become two cells thick in places (see figs. 6 and 7). Some are of considerable height (up to sixteen cells or more), while others are only one or two cells high (fig. 6). The outgoing leaf-trace is accompanied by a considerable amount of parenchyma, especially on the upper side * (fig. 7). The medullary rays in the neighbourhood of the leaf-trace are irregular, and generally shorter and broader than elsewhere.

The pits adjacent to the medullary rays are bordered only on the side towards the tracheide—the usual structure in all such cases (see fig. 6, *m.r.*).

The chief peculiarity of the secondary wood is in its innermost region, near the pith, where the elements have an unusual form and arrangement. The tracheides here are broad and short, often with horizontal terminal walls, which thus appear in surface view when seen in a transverse section (*cf.* figs. 2 and 3). Their course is tortuous and irregular; the maximum diameter is usually in the radial direction (see figs. 2 and 3). The pits on their walls, though in more numerous series than elsewhere, are of the usual form; the arrangement of the tracheides, so far as any regularity can be traced, is in radial series, and the medullary rays pass between them; towards the exterior the structure passes over rapidly into that of the normal wood. This peculiarity of the inner zone of wood is common to both the specimens investigated. There seems to be no doubt that the short tracheides in question belong to the secondary wood; they resemble the primary tracheides found by Mr SEWARD in his new genus *Megaloxylon*,† and may probably have served, as he believes to have been the case in that plant, for the storage rather than for the conduction of water.

The chief results arrived at from the investigation of *Calamopitys fascicularis* are the following:—

(1.) The small pith (2–3 mm. in diameter) is surrounded by a ring of distinct primary xylem-strands, eight or nine in number, with mesarch structure.

(2.) These strands are the xylem-constituent of the leaf-traces; they attain their maximum diameter (·8 mm.–1 mm.) when they are about to leave the pith and to pass out through the secondary wood. Below this point they rapidly diminish in diameter, and each unites with the adjacent strand on its kathodic side.

(3.) The outgoing strands are arranged on five orthostichies, corresponding to a 2/5 phyllotaxis. In passing through the wood, each leaf-trace is represented by a single strand.

(4.) The secondary wood has the typical Araucarian or Cordaitean structure, with

* The orientation of fig. 7 has been determined by comparison with a transverse section of the same specimen, in which the parenchyma accompanying a leaf-trace is found on the *inner* (= upper) side of the strand.

† SEWARD, "Notes on the Binney Collection of Coal-Measure Plants." Part II. *Megaloxylon*. *Proc. Cambridge Phil. Soc.*, vol. x, 1899, p. 158.

medullary rays one, or at most two cells in thickness. The inner part of the wood consists of short broad tracheides, with a tortuous course.

The reasons for placing this species in the genus *Calamopitys* of UNGER may now be briefly considered. This genus was established by UNGER on the species *C. Saturni* in 1856,* but our present accurate knowledge of its structure is due entirely to the recent work of Count SOLMS-LAUBACH,† who has further shown that UNGER's *Stigmaria annularis* was also a *Calamopitys*, scarcely distinct from the original species. The generic name *Calamopitys*, which expressed UNGER's view of the Calamarian affinities of his fossil, is entirely inappropriate, and the real relationships of the genus have proved to lie in quite a different direction. The old name is kept up simply in order to avoid burdening the synonymy with a new one.

In the specimens of *Calamopitys Saturni* there is a small pith (only about 1–2 mm. in diameter) surrounded by an irregular tracheal zone, reduced or perhaps wholly interrupted at certain points, and forming enlarged nests between them, each such nest having a central group of small elements, presumably the protoxylem. This zone of primary xylem is surrounded by the secondary wood, the tracheides of which have small narrow circular pits ranged in several rows on their radial walls. The medullary rays are usually pluriseriate. Some remains of the phloem have been found, and the cortex is well shown; in its inner part it consists of parenchyma, while towards the periphery it contains parallel bands of hypodermal fibres, thus having the well-known 'Sparganum' structure. In the cortex the leaf-trace bundles are also found; their course has been followed with great completeness, in successive transverse sections, by Count SOLMS-LAUBACH, who finds that a single bundle leaves the wood, and at first (as in *Lyginodendron*) is accompanied by secondary xylem. The leaf-trace divides into two on entering the cortex, then into four, and finally into six; the six resulting bundles enter one of the leaf-bases which are found attached to the stem. The leaf-stalk has the structure of *Kalymma*, and as *Kalymma* is known to have branched, the inference is that the leaves of *Calamopitys* were compound. Count SOLMS-LAUBACH has shown beyond doubt that the phyllotaxis was 2/5, or extremely near it. In the form referred to *C. annularis*, the primary wood is more extensive, and apparently more continuous; in some of the specimens the pith attains a diameter of 7 mm. In other respects there is no important difference between *C. annularis* and *C. Saturni*, and it is not even certain that the species were really distinct. Both forms belong to the Culm, or Lower Carboniferous, of Central Germany, and are thus of similar horizon to that of the British species.

In comparing the German species of *Calamopitys* with our own fossil, we are unfortunately restricted to characters presented by the pith and wood, for these parts are alone preserved in the British specimens. Count SOLMS-LAUBACH most kindly lent

* RICHTER u. UNGER, *Beitrag z. Palæont. d. Thüringer Waldes, Denkschr. d. K. K. Akad. zu Wien, math. naturw. Cl.* Bd. xi., 1856.

† *Pflanzenreste des Unterculm v. Saalfeld in Thüringen—Abh. d. K. Preuss. Geol. Landesanstalt*, Heft 23, 1896, p. 63, Taf. IV.

me some sections of *C. annularis* for comparison with our own, and on visiting Strasburg I was able to examine a number of other sections both of that species and of *C. Saturni*. Neither of the German fossils is specifically identical with the British form, but I could find no grounds on which to base a generic distinction. A small specimen of *C. annularis* closely resembled Mr KIDSTON'S specimen of *C. fascicularis* in the character of its tissues, and might, on superficial examination of the sections, have been taken for a part of the same stem. It differed, however, in the more continuous primary xylem, and the less marked enlargement of the outgoing xylem-strands, as compared with the others. In this specimen the rays were usually two cells thick, but often one cell only in thickness near the pith. There was thus little difference in this respect from the British form. In the large specimens, both of *C. annularis* and *C. Saturni*, the rays are wider. The pith appeared to present no marked difference from that of *C. fascicularis*. Count SOLMS-LAUBACH has pointed out that in *C. Saturni* a xylem-strand is sometimes found embedded in the pith, without direct contact with the secondary wood, though not far removed from it (*loc. cit.*, p. 72). I have observed the same thing in one of his sections of *C. annularis*, and this, as we have seen, is characteristic of all the smaller xylem-strands in our *C. fascicularis*. *C. Saturni*, in the greater separation of the individual circum-medullary strands, approaches nearer to our species than does the form *C. annularis*. The preservation of the Thuringian specimens is, however, such that the exact limits of the xylem-strands are much more difficult to make out than in the British fossil, especially Mr KIDSTON'S specimen. In the shortness of the internodes *C. Saturni* also agrees with *C. fascicularis*, and, as we have seen, the phyllotaxis and general course of the xylem-strands were the same, so far as the evidence available can show.

On the whole, taking into consideration all the characters available for comparison I feel no doubt that the genus *Calamopitys* is that in which our fossil may most naturally be placed. The form and relative dimensions of the xylem-strands, and the usually uniseriate rays, serve to characterise *C. fascicularis* specifically.

2. *Calamopitys beinertiana* (Goepp. sp.).

The investigation of this species is based on a specimen collected by Mr KIDSTON in September 1900 at Norham Bridge on the Tweed; the horizon (Calciferous Sandstone Series) is similar to that of the Dumbartonshire specimen of *C. fascicularis*. Mr KIDSTON had numerous sections prepared by Mr LOMAX from his specimen, and from these he himself determined the main points in its structure, and identified the species with the *Araucarites beinertianus* of GOEPPERT.* He then very kindly lent me the sections for further investigation, with a view to the inclusion of the species in the present communication.

The specimen is a rather large one, about 4 cm. in diameter in its present incomplete

* This identification has since been confirmed, as will be explained below, by comparison with authentic sections of *A. beinertianus*, for the loan of which I am indebted to Count SOLMS-LAUBACH.

state (see phot. 3); the main stem shows nothing outside the wood, and probably not the full thickness of that, but the transverse sections also contain a detached fragment which has the bark attached, though in a poor state of preservation. The main piece is fairly well preserved, but in places the tissues appear to have suffered from maceration before petrifaction took place. The most important region—the zone immediately surrounding the pith—is a good deal damaged, but the chief features of its structure are sufficiently plain, as the figures show. The pith, 13–15 mm. in diameter, has a very characteristic appearance, owing to the presence of conspicuous masses of dark-coloured cells, much resembling the 'sclerotic nests' in the pith of *Lyginodendron Oldhamium* * (see phots. 3 and 4). The nests here consist of rather thick-walled cells, containing carbonaceous matter, which may probably have been derived from the disorganised inner layers of a cell-wall originally much thicker than it at present appears. In the middle of each nest there is a small irregular group of very dark cells; the more peripheral elements of the nest are squarish cells, arranged in series radiating in all directions from the central group. These radial series are continued out into the surrounding thin-walled pith, and are no doubt the result of growth and cell-division subsequent to the first origin of the sclerotic nest. Such a structure is commonly met with in recent plants, around groups of hard tissue differentiated in the midst of an actively growing parenchymatous matrix. Similar cell-divisions occur around the sclerotic nests of *Lyginodendron*, but not to the same extent as in the present species. The general resemblance in the pith of the two plants is sufficiently striking.

The most important point, however, is the presence of a number of primary xylem-strands around the pith, adjoining the inner edge of the secondary wood. The bundles are small compared with the size of the pith, though some of them reach a diameter of about ·75 mm. (see Pl. IV. fig. 8). The larger strands are just entering the wood; those which remain at the periphery of the pith are smaller (see fig. 9). It appears, therefore, that here, as in *C. fascicularis*, the strand attained its maximum size just before passing out towards a leaf. As would be expected from the large dimensions of the pith, the xylem-strands are numerous; I was able, in spite of the imperfect preservation, to count seventeen strands which were clear enough for the position of the protoxylem to be recognised. No doubt there were others besides, too obscure to be identified. In parts of the periphery of the pith the primary xylem appears to be almost continuous, for the inner edge of the wood is here formed of irregularly arranged tracheides, larger than those of the secondary zone. This lateral confluence of the primary xylem-groups, though not amounting to continuity all round the pith, recalls the structure found by Count Solms-Laubach in *Calamopitys annularis*.† The primary strands of *C. beinertianus* bear a strong resemblance to those of *C. fascicularis*, as is evident if we compare the large outgoing xylem-bundle shown in fig. 8 with the corresponding large strands

* Williamson and Scott, *loc. cit.*, p. 717; Pl. 18, phots. 1 and 4; Pl. 21, fig. 1.

† *Loc. cit.*, p. 74.

of the former species represented in fig. 1 and phot. 2; or again, if we compare the smaller xylem-strands of the two species (fig. 9 with fig. 2). In *C. beinertianus*, as in the former species, the outgoing strand has an almost circular transverse section, with the smallest elements towards the centre (fig. 8). As the strand figured is damaged at the back, we cannot be quite certain whether the structure was strictly mesarch, *i.e.*, whether there were tracheides all round the protoxylem. From the evidence of another outgoing strand, however, it is probable that this was the case, so that the resemblance to *C. fascicularis* appears to be complete, so far as these larger strands are concerned. I have also carefully examined all the smaller xylem-strands shown in the transverse sections; some of them, like the outgoing bundles, may be mesarch, but the majority are certainly endarch; some have a structure which may be described as hippocrepiform-endarch (see fig. 9); that is to say, the ring of tracheides is interrupted at the back of the bundle, so that on the side towards the pith the protoxylem is in contact with thin-walled tissue. There is very little real difference between this structure and that of the smaller mesarch strands of *C. fascicularis* (*cf.* fig. 2); there also the xylem, or rather the tracheal tissue, is interrupted, but not so regularly on the inner side of the strand. Similar differences occur among the bundles in the stem of *Osmunda.**

This partial assumption of endarch structure is of interest, as marking the first step in that disappearance of centripetal xylem which characterises the later types of Gymnospermous stem.

In some of the longitudinal sections the spiral tracheides of the protoxylem are quite distinct, as in the bundle shown in fig. 10. Here the protoxylem is adjacent to parenchyma, but the poorly preserved element still further to the interior is a tracheide. The section, however, as shown by the direction of the medullary rays, was not accurately radial, so most probably this was a 'hippocrepiform endarch' bundle, one of the flanking tracheides appearing on the inner side of the strand in consequence of the deviation of the section from the radial plane. The primary tracheides show the usual transitions through reticulate to pitted structure. The walls of the largest of the primary xylem-elements have numerous rows of hexagonal bordered pits, sometimes beautifully preserved (see fig. 11). The largest primary tracheides are as much as ·1 mm. in diameter; those of the secondary wood seldom exceed ·05 mm.

The pitting of the secondary wood, usually imperfectly preserved, but well shown at a few places, is limited, as usual, to the radial walls. The pits are most often in two rows only; sometimes they are scattered, and even when in contact do not usually assume a regular hexagonal outline, though sometimes there is an approach to this form. The bordered structure of the pit with a narrow slit-like pore is evident in the better preserved parts of the wood. Examples of medullary rays, as seen in tangential section, are shown in fig. 12. They are nearly always one cell only in thickness; cases where the ray is locally two or more cells thick (as in fig. 12, B) are very rare, and appear to be connected with some irregularity in the course of the tracheides. The great differences in

* ZENETTI, *Bot. Zeitung*, p. 57, woodcut 2, and p. 62, woodcut 3, 1895.

the height of the rays are sufficiently illustrated by the comparison of figs. 12, A, and 12, C. Small rays two cells in height are common, and rays only one cell high also occur. In the rays of greater height there is often considerable variation in the dimensions of the constituent cells (see fig. 12, A). The secondary wood, as a whole, has quite a Coniferous character, and thus offers a striking contrast to the primary structure.

In some of the tangential sections a large leaf-trace bundle, accompanied by parenchyma, is seen passing out through the wood (see Pl. V. fig. 13). The strand is rather obscure, as it is cut obliquely; the smallest elements are central, with some xylem and parenchyma next them, and the structure therefore probably mesarch. The pits on the larger tracheides of the strand can be recognised. It is important to note that in this part of its course the leaf-trace is represented by a *single* strand and not by a pair of bundles, thus agreeing with *C. fascicularis*.

The fragment of stem with bark attached has been already mentioned. The bark, which is about 5 mm. thick, is shown in transverse section, and consists of alternate darker and lighter tangential bands of tissue. The whole mass evidently represents a regular scale-bark; in some of the darker bands radially arranged peridermic cells can be recognised; the intermediate softer tissue may be partly phloem; the larger cells to the outside between the periderm-bands may have belonged to the primary cortex. At one place a broad band of thick-walled periderm is well preserved; at another, a few displaced fibres can be recognised; but, on the whole, the preservation is too imperfect to justify a more detailed description.

As mentioned above, the specimen just described was identified by Mr KIDSTON with the *Araucarites beinertianus* of GOEPPERT,* with which the characters of the secondary wood agree, as shown by GOEPPERT and STENZEL'S diagnosis (1888). "Ar[aucariæ] ligni, stratis concentricis haud conspicuis, tracheidis amplis punctatis, punctis 1-, 2-, rarius 3-serialibus spiraliter dispositis approximatis aut subcontinguis rotundatis, radiis medullaribus latis 1-, rarius 2-serialibus e cellulis crassis 1–10, rarius pluribus superpositis formatis" (*loc. cit.*, p. 30). The figures, cited in the footnote, also agree very well with our specimen. On characters of the secondary wood alone, however, one might have hesitated in affirming identity, but Mr KIDSTON'S determination has now been fully confirmed by comparison with sections of the Falkenberg plant, very kindly lent me by Count SOLMS-LAUBACH, in one of which the pith and primary wood are included. The section in question is shown in an excellent photograph (Plate VII. fig. 10) among the illustrations to Count SOLMS-LAUBACH'S second paper on the Falkenberg Culm-fossils,† though on too small a scale for details to be exhibited. The identity of the

* *Araucarioxylon beinertianum*, Kr. (Göpp., sp.), 1870–72; *Araucarioxylon beinertianum*, Kraus in Schimper, *Traité d. paléont. Végét.*, vol. ii. p. 381, 1850; *Araucarites beinertianus*, Göpp., *Monog. d. foss. Coniferen.*, p. 233, pl. xlii. figs. 1–3; pl. xliii. fig. 1, 1852; *Araucarites beinertianus*, Göpp., *Foss. Flora d. Uebergangs. Form.*, p. 254, pl. xxxv. figs. 1–4, 1888; *Araucarites beinertianus*, Göpp. u. Stenzel, *Nacht z. Kennt. d. Coniferenhölzer d. palæoz. Form.*, p. 30, pl. iv. figs. 36–39; *Araucarites beinertianus*, Göpp., *Revision d. foss. Conif.*, p. 11 (*Bot. Centrabl.*, 1881, vol. v. p. 396).

† "Ueber die in den Kalksteinen des Kulm von Glätzisch-Falkenberg in Schlesien erhaltenen structurbietenden Pflanzenreste, II.," *Bot. Zeit.*, 1893, p. 207.

specimen with the *Araucarites beinertianus* of GOEPPERT was established by Count SOLMS-LAUBACH in conjunction with STENZEL, whom he consulted.

The preservation is decidedly better than that of the Tweed specimen. The pith (about 8 mm. in diameter) contains three sclerotic nests,* and agrees in every respect with that of our plant. At three points distinct mesarch strands of primary wood are present; two of these belong to outgoing leaf-traces, while the third, which is smaller, has not yet begun to pass out. At several other places small strands of tracheides, apparently primary, can be recognised; most of these were no doubt endarch in their development, and they show no clear cases of mesarch arrangement. The secondary wood, in both transverse and longitudinal section, shows essentially the same structure as in the Tweed specimen, except perhaps that biseriate medullary rays are somewhat more frequent.

Mr KIDSTON, who has also examined Count SOLMS-LAUBACH'S sections, agrees with me that they remove all doubt as to the specific identity of the British specimen with the *Araucarites beinertianus* of GOEPPERT.

The chief results relating to *C. beinertiana* are the following :—

(1) The relatively large pith contains 'sclerotic nests' resembling those in the pith of *Lyginodendron*.

(2) Around the pith, and in contact with the secondary wood, numerous primary xylem-strands, sometimes laterally confluent with one another, are disposed.

(3) The primary strands attain their largest size where they enter the wood; at this point they resemble the corresponding bundles in *C. fascicularis*, and are of mesarch structure.

(4) The small strands are more usually endarch, and sometimes of a horse-shoe form, the opening being turned towards the pith, and the protoxylem lying in the concavity of the strand.

(5) A single strand constitutes the leaf-trace, where it enters the secondary wood.

(6) The secondary wood has a regular Cordaitean structure, with medullary rays seldom more than one cell thick.

(7) A scale-bark was formed on the older stems.

The reasons for placing this species in the genus *Calamopitys* are apparent at once from the foregoing description. The detailed structure of both primary and secondary wood is so closely similar in the two species, that if *C. fascicularis* is rightly placed in UNGER'S genus, it is impossible to doubt that the other species must accompany it. Naturally, both determinations, though resting, as it seems to me, on good grounds, must be regarded as provisional until the characters of the cortex and leaf-bases are known. In the meantime, it is interesting to note that in the same beds which yielded the specimen of *C. beinertiana*, Mr KIDSTON found a petiole—provisionally named by him *Rachiopteris multifascicularis*—which presents very much the same structure as the small *Kalymma*-like leaf-bases borne on the stem of *Calamopitys Saturni*.

* SOLMS-LAUBACH, *loc. cit.*, p. 208.

Distinctive specific characters of *C. beinertiana* are to be found in the large size and peculiar structure of the pith, the relatively small extent of the primary xylem, the frequent endarch structure of the smaller primary strands, and the usually somewhat scattered arrangement of the pits on the secondary tracheides.

II. Pitys, Witham, emend.

1. *Pitys antiqua*, Witham.

The structure of this stem has been investigated principally in a specimen from Mr Kidston's collection (sections 598A–598H), collected by Mr B. N. Peach, F.R.S., at Lennel Braes in Berwickshire, in 1883. The specimen, like those already described, is of Lower Carboniferous age (Calciferous Sandstone Series). Its specific identity has been established by comparison with Witham's type-specimen, as will be explained below.

In this case, also, I am indebted to Mr Kidston for calling my attention to the peculiarities of the fossil, and lending me his sections for investigation.

The stem is remarkable for its large pith, which in the specimen collected by Mr Peach, has a diameter of 22 mm. (phot. 5). In a section from another specimen, also from Lennel Braes (No. 221 in Mr Kidston's collection) the pith as preserved is as much as 34 mm. in diameter, and may be incomplete. The structure of the pith is characteristic; it consists of large, but very short cells, their width, which usually exceeds their height, being from ·15 to ·2 mm. (see figs. 14, 15, and 16). Many of the pith-cells are filled with dense carbonaceous contents, suggesting that they may have been secretory elements. Although they do not differ in form from the surrounding, comparatively empty cells, yet their somewhat regular arrangement, and the fact that their relative frequency is unaffected by the state of preservation, may indicate that some real differentiation existed during life.

A conspicuous feature of the pith is the presence of large, horizontal, lenticular gaps in its tissue (see phot. 6). These gaps are largest in the outer part of the pith, though they are present to some extent all through it. They appear to be due to a shrivelling of the tissues, for the cells between the larger gaps are flatter than elsewhere, and have a collapsed look. The gaps extend out into the principal medullary rays. Their presence gives the pith, as seen in radial section, an appearance not unlike that of the well-known discoid pith of *Cordaites*, but much less regular. It may be doubted whether the resemblance is more than a superficial one. The Cordaitean discoid structure, as shown, for example, in *Cordaites Brandlingi*,* is strikingly regular, and appears to have been due to rupture during a normal process of growth; the gaps are strictly limited to the middle part of the pith, the peripheral zone being uninterrupted; neither do the cells show any signs of collapse. In all these points the pith of *Pitys antiqua* is different; here the imperfectly discoid structure has much

* Scott, *Studies in Fossil Botany*, fig. 137, A; Renault, *Cours de Bot. Fossile*, vol. i. pl. 12, fig. 12.

more the appearance of having been caused by unequal contraction of the tissue, perhaps after death.* Where the specimen is badly preserved, the gaps are much exaggerated. The pith has evidently undergone dilatation, as shown by the increase in width of the medullary rays at their inner ends, and by the marked horizontal elongation of many of the pith-cells (Pl. II. phots. 8 and 9, Pl. V. figs. 14–16). The latter feature is especially conspicuous around the primary xylem-strands, from which the dilated medullary cells usually radiate out in all directions (phots. 8 and 9, figs. 14 and 15). This is a familiar phenomenon wherever lignified strands occur in the midst of an actively-growing cellular tissue, as, for example, in fleshy roots.

The chief point of interest in the stem of *Pitys antiqua* is the presence of numerous strands of primary xylem around the pith, and for the most part embedded in its tissue. Their distribution is shown in diagram 5, prepared by Mr L. A. Boodle, in which all

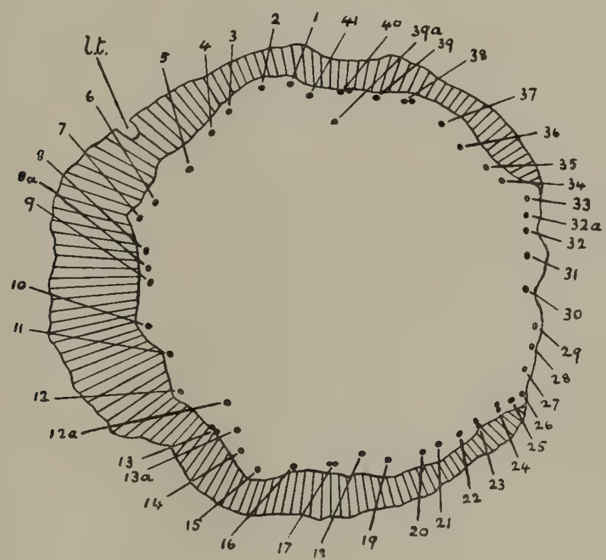


Diagram 5.

the strands shown in a transverse section have been accurately plotted, in their exact position, with the aid of the *camera lucida*. The total number of xylem-strands present in this section was 46. It will be noticed that, with few exceptions, they are separated by an appreciable interval from the inner edge of the secondary wood. Actual contact is only shown at three points, namely, in the case of the strands numbered 13, 23, 40. All the others are separated from the wood by distances ranging from ·3 mm. to 1·8 mm.† The xylem-strands themselves vary in diameter from about ·15 mm. to about ·3 mm., so their distance from the wood almost always exceeds their own diameter, and is often many times as great. Strands 13 and 40 are double (*cf.* Pl. V. fig. 14 for strand 13); we know, from the evidence of successive transverse sections, that strand 13 passed

* Similar lacunæ are present in the central tissue (primary wood) of *Megaloxylon*. See Seward, "Notes on the Binney Collection of Coal-Measure Plants"; Part ii., *Megaloxylon*. *Proc. Cambridge Phil. Soc.*, vol. x., 1899.

† Of course only that part of the section in which the secondary wood is present is taken into consideration.

out into the secondary wood a little higher up the stem. The gap shown at *l.t.* on diagram 5 (see also phot. 5), clearly marks the course of another outgoing leaf-trace. The strand 23 also seems to be double, but here the tissue is damaged. From the somewhat slender evidence available, it seems probable that contact between a primary strand and the secondary wood only occurs at points where the former is about to pass outwards, presumably on its way to a leaf. It also appears that, at the point of exit from the pith, the leaf-trace was a double strand, but its two branches, as we shall see, re-united in passing through the wood. Double or paired strands may also occur elsewhere, independently of outgoing leaf-traces (see, for example, phot. 8, representing strand 38 in diagram 5). The diagram is taken from a section higher up the stem, where the two strands have approached nearer to each other.

Before further considering the distribution of the xylem-strands, it will be well to describe their structure. The strands, as already mentioned, are small, indeed very small in comparison with the size of the pith. Most commonly their maximum diameter is about ·25 mm.; the sectional form of the strand is usually elliptical, the major axis lying in the tangential direction. A good typical example is shown, in transverse section, in fig. 15, which represents the strand numbered 3 in diagram 5. The smallest elements (*px*) lie near the middle of the strand; towards its periphery the tracheides become considerably larger, about equalling the innermost elements of the secondary wood in size. A few parenchymatous cells occur among the primary tracheides, especially near the middle of the group. Phot. 8 shows the same structure in each strand of the paired bundle numbered 38. Of the two strands shown in phot. 9, one (No. 14) shows the usual arrangement of the elements; the other (13A) is less regular. Longitudinal sections show the nature of the elements. Fig. 16 is from a tangential section (shown as a whole in phot. 7) which passes through the periphery of the pith, and here cuts through the middle of a xylem-strand. The more central tracheides have an evident spiral thickening; the narrowest among them are no doubt the actual protoxylem. The peripheral tracheides of the strand are larger, and their cells definitely reticulated, the lines of reticulation having a spiral course (fig. 16, *r. t.*). Close examination shows that, in the outer tracheides, each mesh is bordered, so that the reticulation is passing over into a system of spirally arranged, bordered pits. Similar elements, one of which is shown in detail in fig. 17, occur at the inner margin of the secondary wood, where, however, no true spiral tracheides have been detected.*

Fig. 16 also shows the xylem-parenchyma, with possible secretory sacs, within the primary bundle.

The evidence thus indicates that the primary xylem-strands of *Pitys antiqua* have a mesarch structure, differentiation having begun at a point near the middle of the strand, as indicated by the position of the spiral tracheides. The mesarch structure holds good

* I find that perfectly similar elements have been described by ROTHERT, in recent Conifers, under the name of "Gemischte Gefässe." See his "Tracheiden u. Harzgänge im Mark von *Cephalotaxus-Arten*," *Ber. d. Deutsch. Bot. Gesellsch.*, Bd. 17, 1899, p. 284.

for the great majority of the strands; in a few of the smallest the protoxylem may lie on one side or the other (*e.g.*, the strand 13A shown in phot. 9).

At the point where a double bundle comes into contact with the secondary wood previous to passing out into the latter, the structure is less regular. Fig. 14 shows this in the case of the double strand numbered 13. The smallest elements of the two xylem-strands are here directed towards each other, and some of them abut directly on a wedge of secondary wood. In a section cut a little further up the stem, where this bundle is shown entering the wood on its outward course, it appears as a single strand, the two half-bundles having re-united. It is possible that their temporary separation may have been due merely to the intrusion of dilated parenchyma.

The similar outgoing strand between bundles 5 and 6 (*l.t.* in diagram 5 and phot. 5), though obscure, and difficult to distinguish from the adjacent secondary wood, is clearly a single one. It is remarkable how small were the dimensions of the leaf-trace (as we must assume it to have been), at least as regards its primary xylem. Possibly it was supplemented on its outward course by an arc of secondary wood, as was the case in *Poroxylon*,* but at present we have no information as to any of the external tissues of our fossil. It will be noticed that behind each of the outgoing strands 13 and 40 (diagram 5) there is a small xylem-bundle deeply embedded in the pith (12A and 39A respectively); the strand 5 stands in a similar relation to the leaf-trace, *l.t.* The strand 12A is shown in detail in fig. 14. In this case there is a second deep-seated strand near by (13A, shown in phot. 9). In the uppermost of the five successive transverse sections of the stem which we possess, where the strand 13 is beginning to enter the wood, the two deep-seated strands, 12A and 13A, are approaching each other as if about to fuse, but there is no evidence to show whether such a fusion was of general occurrence. In any case it is natural to regard the deep-seated strand behind the leaf-trace as a reparatory strand, destined to constitute or contribute to the next outgoing bundle of the same orthostichy.

The phyllotaxis was no doubt a spiral one, and very probably complex, as suggested by the large number of primary strands.

There is evidence that the primary strands occasionally branched and anastomosed with one another. This is best seen in a tangential section, passing through the outer part of the pith, and touching in places on the secondary wood, represented in phot. 7. Several of the primary xylem-strands are shown; the double strand, which corresponds almost exactly in size and position with some of those shown in transverse section (*cf.* fig. 14), is in contact with the innermost tracheides of the secondary wood, and may probably represent a leaf-trace about to pass out. An oblique anastomosis between the strands of this pair is present.

Another strand appears to be branching, and at several places single tracheides are seen diverging from the xylem-strands in various directions, probably to form a connection with others.

* *Cf. Pitys Withami*, below, p. 355.

Such isolated tracheides, either between two xylem-strands, or between a xylem-strand and the secondary wood, are not infrequently met with in the transverse sections, as shown in phot. 9, between the strands numbered 14 and 13A in diagram 5. It is probable that these elements served to keep up communication between different strands, though in some cases their separation from the bundle to which they belonged may have been an accidental effort of the dilatation of the adjacent parenchyma.

The wide separation between most of the primary xylem-strands and the secondary wood presents a considerable difficulty, which exists also, though in a less degree, in the case of *Calamopitys fascicularis*, and probably other species of that genus. In *Pitys antiqua*, as mentioned above, the actual distance ranges from about a third of a millimetre to almost two millimetres. The average interval, it is true, is only about half a millimetre, but this is equal to twice the average diameter of the xylem-strand itself. Cases of contact are rare, and probably limited to bundles approaching their exit. Apart from these special cases, we find that the number of pith-cells intervening between a primary strand and the wood varies from one up to about twenty, averaging five or six.

The question arises : What could have been the primary structure of a stem in which such an arrangement prevailed? If, as all the evidence indicates, the process of secondary growth was of the normal type, the secondary wood being intercalated between the primary xylem and the phloem, it follows that in the primary condition the xylem and phloem of each bundle must have been widely separated, and that to a very unequal extent in different bundles, unless indeed the isolation of the xylem-strands can be explained by subsequent dilatation of the parenchyma. We have already seen that this took place to a considerable extent; in some cases the apparent doubling of a xylem-strand has evidently been brought about by tangential dilatation of its own fascicular parenchyma, and the isolation of single tracheides may sometimes have been due to a similar cause. But I have found no evidence that in *Pitys antiqua* the dilatation was greater between the xylem-strand and the secondary wood than elsewhere. It rather appears that the *relative* position of the two has remained approximately constant, though the *absolute* distance between them has no doubt been increased by the general extension of the parenchyma. If this were so, there must have been from the first an unusual separation, varying in different bundles, between xylem and phloem, and the cambium must have arisen towards the phloem-side of the intervening tissue, thus leaving the primary xylem more or less isolated. The tissue between primary and secondary wood would, on this view, have originally been fascicular, but have become assimilated to the pith by subsequent dilatation. The position of the specially deep-seated xylem-strands, behind the outgoing leaf-traces, remains a difficulty, and I do not think that the whole question will be solved until young stems are discovered. Some analogy for the separation of primary from secondary wood is afforded by the peduncles of certain Cycads (especially *Stangeria*) where the centripetal is often rather remote from the centrifugal wood.* A similar separation is of common occurrence in roots, especially those of Gymnosperms.

Two other explanations are theoretically possible, though I believe really untenable. We might suppose that the cambium was extrafascicular, the xylem-strands thus representing complete vascular bundles. I have not, however, found the least indication of phloem in connection with them, and the preservation is sufficiently good to leave little doubt that none existed, and that the xylem-strands were surrounded on all sides by parenchyma only.

The other possibility is that the xylem-strands may not have constituted the primary xylem of the leaf-trace system, but may have formed a merely accessory system of medullary strands, remotely comparable to the medullary bundles of *Encephalartos*, *Macrozamia*, or even, as some might suggest, to the star-rings of Medulloseæ. The absence of phloem is an obvious objection to this view also, and even apart from this, the facts that the xylem-strands pass out through the secondary wood, and that they are the only part of the wood where spiral elements occur, seem fairly decisive for their primary nature, as part of the main leaf-trace system of the plant. The very remarkable medullary xylem found by ROTHERT in a Conifer, which he refers to *Cephalotaxus Koraiana*, offers a certain analogy with that of our fossil, but, in the case of this recent plant, no spiral elements occur among the medullary tracheides.†

The general structure of the secondary wood of *Pitys antiqua* has long been known,‡ but as Mr KIDSTON'S specimen (No. 598) is probably the best-preserved of any hitherto investigated, a short description may be given, more especially as the existing accounts are inaccurate in various points.

The chief generic character of *Pitys*, as at present defined—the wide medullary rays—is well exhibited. The principal rays are usually much wider at their inner ends than elsewhere. A ray ·3 mm. in tangential width at its junction with the pith may diminish to a tenth of that width in a distance of little more than a millimetre (fig. 14, phot. 8). The difference depends more on the width of the individual ray-cells than on their number, which remains nearly constant, and is evidently due, in great part, to dilatation occurring during the early stages of wood-formation. The gaps in the pith, referred to above (p. 346), run out for some distance into the principal medullary rays. Close to the pith, where the rays are wide, the nature of these gaps, as mere tears in the tissue, is evident; further out, as the rays become narrower, the gaps assume a more regular form, and sometimes strongly suggest the resin-canals in the rays of Abietineæ (see Pl. VI. fig. 20, *g*). This appearance is in all probability

* SCOTT, "Anatomical Characters presented by the Peduncle of Cycadaceæ," Pl. XX. figs. 1–5, *Ann. Bot.*, vol. xi., 1897.

† ROTHERT, *l.c.* It is interesting to find (p. 285), that ROTHERT'S "gemischte Gefässe" occur in the medullary as well as in the normal wood of his *Cephalotaxus*, just as is the case in *Pitys antiqua*.

‡ WITHAM of Lartington, *Internal Structure of Fossil Vegetables*, Edinburgh, 1833, pp. 25–27, 37, 38, 71, pl. iii.; pl. iv., figs. 1–7; pl. vii., figs. 9–12; pl. viii., figs. 1–3; pl. xvi., figs. 9, 10. Little appears to have been added by later writers.

deceptive, and the spaces due simply to the contraction of the surrounding tissue. Traced out into the wood, the rays soon assume the ordinary structure; their tissue, as seen in radial section, has the usual muriform character (see Pl. V. fig. 18). The cells usually retain some remains of their contents, often in the form of definite granules. Secondary rays appear between the principal ones, as best shown in specimens where the wood attains some considerable thickness.

Seen in tangential section (Pl. VI. fig. 19) the larger rays are found usually to reach a width of four cells, sometimes five or even six, and are often of great height, seventy cells or upwards in some cases. Among the large rays, however, much smaller ones occur, only one cell thick, and of small height, sometimes reduced to a single cell. At one or two places an isolated strand of xylem-parenchyma was observed, consisting of a row of vertically elongated cells, at least three times as high as those of the rays. This tissue is, however, extremely scanty, and in most sections is not shown.

The tracheides are exquisitely preserved. Those at the inner edge of the secondary wood are transitional between the reticulated and pitted forms; the spirally arranged reticulations are distinctly bordered (fig. 17). Further out in the wood, the spiral arrangement becomes less marked, and the pits assume the characteristic hexagonal form (fig. 18). They are ranged in four or five ranks, on the radial walls of the large tracheides, and are usually in contact with one another throughout. Occasionally, however, especially near the ends of the tracheides, the pits are more scattered, as WITHAM described them,* and may be even reduced to a single row, leaving the rest of the wall bare. The border of each pit is, as a rule, perfectly preserved, enclosing a narrow slit or pore, usually in an inclined position (fig. 18). A more perfect example of typical '*Araucarioxylon*' structure than that presented by this wood could not be imagined.

The tangential walls of the tracheides are, as a rule, without pits, but exceptions occur, especially in the inner layers of the wood. A good example is shown in fig. 20, where a number of pits (*t.p.*) are seen on the tangential wall of a tracheide. Where they occur, they are less crowded than those on the radial walls, and do not cover the whole surface. Tangential pits are of common occurrence in the wood of the Coniferæ, especially in the first-formed layers, and in the tracheides of the autumn wood.†

We may sum up the chief anatomical characters presented by the stem of *Pitys antiqua* as follows:—

(1) The pith is large (22 mm. to 34 mm. or more in the specimens examined) and consists of short-celled parenchyma, interrupted by extensive horizontal lacunæ, probably due to shrinkage of tissue.

(2) Around the periphery of the pith, and usually at some little distance from the inner edge of the secondary wood, are a large number (40–50) of small primary xylem-strands, which occasionally anastomose with one another. The central elements of each strand are spiral tracheides, indicating a mesarch structure.

* *L.c.*, p. 38.

† *L.c.*, STRASBURGER, "Leitungsbahnen," *Histologische Beiträge*, iii. p. 9, 1891.

(3) At certain points the primary xylem-strands come into contact with the secondary wood, and pass out through it; these outgoing strands no doubt represent the leaf-traces.

(4) The secondary wood is traversed by numerous medullary rays, the larger of which are usually four cells or more thick. The principal rays are much dilated towards their junction with the pith. In addition to the rays, vertical strands of xylem-parenchyma occur, but very sparingly.

(5) The secondary tracheides have, on their radial walls, several rows of bordered pits, usually contiguous and hexagonal. Tangential pits also occur not infrequently. No true spiral elements are present at the inner edge of the secondary wood.

The identification of Mr Kidston's specimen, No. 598, on which the description above is based, with the *Pitys antiqua* of Witham, rests on a comparison with sections of Witham's type-specimen, kindly lent me by Prof. Bayley Balfour, F.R.S., and Mr Kidston. The specimen from which these sections were cut is the large stem shown in transverse section, reduced to half natural size, in Witham's *Internal Structure*, Plate III. The two sections sent me from Edinburgh are originals of Witham's, while those lent by Mr Kidston (Nos. 217–220 in his Collection) are better and more modern preparations from the same specimen. In all these only the secondary wood is shown. The preservation is not equal to that of the stem (No. 598) collected by Mr Peach, but, allowing for this difference, the structure essentially agrees. The locality, Lennel Braes on the Tweed, is the same. The medullary rays in Witham's specimen are more scattered, and sometimes broader than in No. 598, attaining an extreme width of seven cells, as against five, or rarely six, in the latter, but these differences may well be due to the much greater size of the Witham stem. The dimensions of the elements agree very nearly. Where the pitting is well shown in radial section, the arrangement corresponds with that in No. 598. At many places the pits are in 3–5 rows, covering the whole wall of the tracheide, closely packed and hexagonal in outline, quite like those shown in fig. 18, except that, as the preservation is not so good, the outline of the borders is less sharp. In other places the pits are more scattered and rounded in outline, as also occurs in the other specimen. There is a section in Mr Kidston's collection (No. 221) which, he tells me, may be from a different specimen. This is the section referred to above (p. 346) as showing a pith at least 34 mm. in diameter. The state of preservation is similar to that of the type-specimen, with which its secondary wood exactly agrees. The pith is lacunar, and primary strands of xylem are present, just as in No. 598. A transverse section of a branch, figured by Witham (Pl. VII., fig. 11), with a pith more than 3 cm. in diameter, has quite the anatomical habit of our fossil (*cf.* phot. 5).

Considering the identical locality, I feel no doubt that the specimen on which the description given in the preceding pages is based, is referable to Witham's species, *Pitys antiqua*, his 'Lennel Braes Tree.'

2. *Pitys Withami* (Lindl. et Hutt., sp.).

I take this species to include the *P. medullaris* of LINDLEY and HUTTON, which WITHAM himself regarded as probably identical with the former.* There seems no object in keeping up the two specific names, as the characters on which *P. medullaris* was separated—the large pith, and the appearance of concentric markings (probably not annual rings) in the wood, are common to many stems of the *Dadoxylon* group, and of no diagnostic value. Both were included in the old genus *Pinites* of LINDLEY and HUTTON, which, as employed by those authors, has long since been abandoned. The distinction drawn by WITHAM † between *Pinites* and *Pitys*, and based on the round, separate pits in the case of *Pitys*, and the hexagonal contiguous pits in that of *Pinites*, has no value, as both conditions are found in different parts of the same specimen. *Pitys Withami* is, in fact, a closely-allied species to *P. antiqua*. The pitting on the tracheides is identical (if equally well-preserved specimens are compared), and there is no constant difference in the size of the elements. The medullary rays are, however, on the whole narrower in *P. Withami* than in *P. antiqua*, rarely exceeding four cells in width in the former. The point of interest for our present purpose is that *Pitys Withami* shows the same primary anatomical structure as *P. antiqua*, having, like that species, a ring of primary xylem-strands disposed round the pith. My observations were made on an original section of WITHAM'S (figured in *Internal Structure*, Pl. VI., figs. 5–8), lent me by Professor BAYLEY BALFOUR. This section was from the branch to which LINDLEY and HUTTON gave the name of *P. medullaris*; it is represented as a whole in Pl. II. phot. 10. The pith, which measures 19 × 10 mm., shows essentially the same structure (so far as exhibited in transverse section) as that of *P. antiqua*, but much of the tissue is altered in appearance by the infiltration of some dark-brown substance. The preservation is tolerably good, but towards the wood, where the cells become smaller, the structure is obscure, partly owing to the section not being sufficiently thin. Yet at several places small strands of thick-walled elements can be distinguished lying near the outer margin of the pith, a little within the ring of secondary wood. These groups agree so closely, in appearance and position, with the primary xylem-strands of *P. antiqua*, that we cannot doubt their identical nature. The strand figured (Pl. VI. fig. 21) appears to be a double one.

The secondary wood is described by WITHAM (*l.c.*, p. 32) as showing "decided indications of five concentric layers." The layers are marked by tangential bands of somewhat flattened elements (see WITHAM, Pl. VI., fig. 8),‡ and the bands, so far as the section extends, are fairly, though not completely, continuous. There is thus a certain resemblance to the annual rings of recent Coniferæ, but very much less marked and regular in the fossil, so that (considering the inconstancy of such markings in Palæozoic

* *Internal Structure*, pp. 36 and 42. *Pitys Withami* was founded on the well-known Craigleith trees discovered in 1826 and 1831 in the Craigleith Quarry, near Edinburgh.

† Who, however, himself had some doubts as to the generic value of the distinction. *L.c.*, p. 39.

‡ This figure gives a fair idea of the relative forms of the elements, but the dark shading makes the rings appear more conspicuous than they really are.

stems) it would be extremely rash to draw any inference as to a seasonal periodicity of growth.

At one place a bundle, no doubt a leaf-trace, is clearly shown, passing out through the wood (see phot. 10, *l.t.*). A very definite arc of secondary wood forms part of the outgoing leaf-trace, and is sharply marked off from the general wood of the stem. This observation confirms the conclusion, drawn from *Pitys antiqua*, that in this genus the leaf-traces, on leaving the pith, were single strands.

So far as the evidence extends, there was thus a complete agreement in the primary structure of the stem between *Pitys Withami* and *P. antiqua*.

3. *Pitys primæva*, Witham.*

This species, one of WITHAM'S Tweed Mill fossils,† is a very distinct form, as shown by the great width of the medullary rays, which are commonly seven cells in thickness and often more, and of a decidedly broader and shorter form, in tangential section, than those of *P. antiqua* or *P. Withami* (see fig. 22). The tracheides, also, are larger, and the pitting slightly different, the pits of *P. primæva* being less crowded than those of the other species. Hence the hexagonal form is less marked and a circular outline more frequent in the pits of *P. primæva* than in those of its congeners.

As regards the question of primary xylem-strands, the material at my disposal was not favourable, as I have seen no sections passing through the pith of a main stem. In one case, however, a tangential section happens to cut transversely across the base of a lateral branch (Pl. II. phot. 11). The pith of the branch is only partly preserved; what remains of it resembles that of *P. antiqua*. At two places in the pith, near the wood, I observed a group of small, rather thick-walled elements, similar to the tracheides of the secondary xylem. At one point the spiral band of a tracheide could be recognised. The pith-cells are elongated radially around the groups in question, and the whole appearance (allowing for the imperfect preservation) is in all respects similar to that of the primary xylem-strands in *P. antiqua*.

The same tangential section is also of interest from another point of view, for it appears to throw light on the problematic fossil described by WILLIAMSON under the name of *Lyginodendron* (?) *anomalum*.‡ In the section of *P. primæva* (phot. 11), the medullary rays near the lateral branch have a form very different from that which characterises them elsewhere (see fig. 23, and compare with fig. 22). They are shorter than usual, and at the same time much wider, so as sometimes to assume a nearly circular form, as seen in the tangential section. These exaggerated medullary rays constitute in this region the great mass of the wood, the strands of tracheides merely forming a sinuous network between them. The appearance is almost identical with

* Sections of two specimens, one from the River Irthing, Northumberland, the other from Juniper Green, Midlothian, were lent me by Mr KIDSTON for investigation. Both are from the Calciferous Sandstones.

† *L.c.*, pp. 38, 71, pl. viii., figs. 4–6.

‡ "Organization of Fossil Plants of Coal-Measures," Part IX., *Phil. Trans.*, 1878, pt. ii. p. 352, pl. 25, figs. 90–92.

that presented by the tangential section of *Lyginodendron anomalum* (Williamson, *loc. cit.*, fig. 92), except that in the latter the dilated medullary rays are on a still larger scale.

In *Pitys primæva* the dilated rays are limited to the neighbourhood of the lateral branch, becoming normal at a greater distance from it.

The puzzling structure of *Lyginodendron anomalum*, which is only known as a fragment of calcified wood from the volcanic ash of Arran, was correctly interpreted by Williamson, who says (*l.c.*, p. 352): "These cell-masses are in fact huge medullary rays of a most extraordinary form." No stem, however, has hitherto been known, presenting the same peculiarity in so extreme a degree. Mr Seward, who observed a somewhat similar enlargement of the rays near the outgoing leaf-trace, in the stem named by him *Lyginodendron robustum*, made the following suggestion: "Such a comparison suggests the probability that the shorter and broader medullary rays and the more irregular course of the tracheides may not represent the normal character of the stem from which the Arran fragment was obtained, but that these appearances may be the result of some disturbing influence in the secondary wood." * In the specimen of *Pitys primæva* just described, we have a striking confirmation of Mr Seward's suggestion. Under the 'disturbing influence' of the presence of a lateral branch, the wood of this plant assumes the same peculiar structure which characterises the Arran fragment, while elsewhere retaining the ordinary organisation.

In other respects, and notably in the pitting of the tracheides, there is a close, though not an absolutely exact agreement between the wood of *Pitys primæva* and that of the Arran fossil, which is of similar Lower Carboniferous age. It is possible that the two are specifically identical, the specimen known as *Lyginodendron anomalum* being simply a fragment of a large stem of *Pitys primæva*, from a part affected by the presence of some bulky appendage. Until further evidence is obtained, it may be well to keep up Williamson's specific name, but the genus is presumably *Pitys* rather than *Lyginodendron*.

The Genus *Pitys*.

Apart from the doubtful fragment last mentioned, all the species of *Pitys*, as limited by Goeppert,† prove to be characterised by the presence of small primary strands of xylem around the pith. Goeppert enumerates four species, *P. Withami*, *P. medullaris*, *P. antiqua*, and *P. primæva*. The first two are not really distinct,‡ and the name *medullaris* therefore disappears from the list. The remaining three, *P. Withami*, *P. antiqua*, and *P. primæva*, have all been found to possess the primary xylem-strands, which I propose to make a character of the genus.

* Seward, "A Contribution to our Knowledge of *Lyginodendron*," *Ann. of Bot.*, vol. xi. p. 81, 1897.

† Goeppert, "Revision meiner Arbeiten über die Stämme der fossilen Coniferen, etc.," *Bot. Centralbl.*, Bd. v., 1881, p. 403.

‡ Goeppert, *l.c.*, p. 404, conjectures that all four may have to be united. This, however, is not borne out by comparison of the specimens; *P. primæva* is a very distinct form, and *P. Withami* and *P. antiqua*, though allied, are not identical.

Taking, with free modification, the more essential parts of GOEPPERT's generic diagnosis,* and adding the newly discovered primary characters, we may define the genus as follows :—

Pitys, Witham, emend.

Trunks arboreal, with wood resembling that of recent Araucarieæ.

Trunks composed of a large central medulla, and a zone of wood, in which the presence of concentric layers is inconstant.

Medulla surrounded by numerous small strands of primary xylem, which pass out through the wood. Tracheides with bordered pits ; pits in three or four series, on the radial walls, spirally arranged, usually contiguous and hexagonal ; sometimes occurring on the tangential walls also. Larger medullary rays formed of 2, 3, 4, or more series of cells.

The old genus *Pitys*, rightly revived by GOEPPERT, turns out to be quite neatly characterised by its numerous small mesarch strands of primary xylem, as well as by the large pith, and multiseriate rays, which have hitherto served to distinguish it.

III. *Dadoxylon Spenceri*, sp. nov.

This stem was found by the late Mr JAMES SPENCER in the Horse Bridge Clough at Hebden Bridge, about six miles to the west of Halifax.† I am only acquainted with one specimen of importance. A second specimen, from the same locality, appears to have a similar structure, but the preservation is too imperfect for determination.

Sections of the good specimen are preserved in the WILLIAMSON Collection, and also in Mr SPENCER's private collection, which, since his death, has come into my possession. The horizon is given by Mr SPENCER as that of the Yoredale Rocks, while WILLIAMSON, who described the specimen in 1879, speaks of it as derived from "the Marine Ganister Bed near Halifax." ‡ This apparent contradiction, involving a doubt whether the fossil belonged to the Lower or Upper Carboniferous, led me to make inquiries of my friend Mr KIDSTON, who referred the question to Dr E. D. WELLBURN, of Sowerby Bridge, who has made a special study of the Geology of the district. Dr WELLBURN's conclusion is that the rocks at Horse Bridge Clough are not the equivalents of the Yoredales of Phillips, but are a newer group of rocks belonging to the Upper Carboniferous division. It thus appears that the true horizon of the specimen cannot be very different from that assigned to it by WILLIAMSON ; *Dadoxylon Spenceri* is consequently the only stem of Upper Carboniferous age dealt with in the present paper.

WILLIAMSON, in his description of the specimen, calls attention especially to the

* *L.c.*, p. 403.

† The discovery of the prolific plant-bed in which the *Dadoxylon* occurred is recorded by Mr SPENCER in his paper on "The Yoredale and Millstone Grit Rocks," published posthumously in the *Proc. Yorkshire Geol. and Polytechnic Soc.*, New Series, vol. 13, 1898 (see p. 378).

‡ "Organization of Fossil Plants of Coal-Measures," Part X., *Phil. Trans.*, 1880, pt. ii. p. 516, Pl. 20, fig. 60.

pairs of vascular bundles which pass out, almost horizontally, through the woody zone (WILLIAMSON, *l.c.*, fig. 60 ; *cf.* Pl. II. phot. 12 in the present paper). He was at that time inclined to believe that they supplied paired branches of the stem (*l.c.*, p. 517), though in a previous Memoir, in discussing similar structures in other Dadoxylons, he said "either two bundles went to one leaf with a double midrib, or the leaves were arranged in pairs."* On a later occasion, WILLIAMSON† compared these paired strands in *Dadoxylon* with the closely similar double leaf-traces of the recent *Ginkgo.* There can be no doubt that in the fossil, as in the recent stem, each pair of bundles represents the trace of a single leaf.

The stem, as preserved, is about 1·8 cm. in diameter ; nothing beyond the secondary wood is shown. The pith, which has a diameter of from 5 to 6 mm., is obtusely pentagonal, the prominent angles evidently corresponding to the points of exit of the paired leaf-trace bundles. At two places in photograph 12, the dark bands, marking the position of a double leaf-trace, are seen passing out from the two sides of a truncated angle of the pith.

The pith, which is not very well preserved, consists of a fairly uniform parenchymatous tissue, the cells averaging about ·07 mm. in diameter, but becoming smaller towards the periphery. They are for the most part filled with dark, carbonaceous contents ; at some places a larger carbonaceous mass is seen, suggesting the presence of a secretory space, but this appearance may merely be due to disorganisation.

The secondary wood is very dense, and has typical *Dadoxylon* characters. The tracheides are narrow, averaging ·025 mm. in radial and rather less in tangential diameter. Their radial walls bear multiseriate bordered pits, closely packed. The medullary rays are small, one to eight cells in height, and hardly ever more than one cell thick. In rare cases a ray two cells thick in the middle may be met with. The absolute width of the ray is from ·01–·015 mm. Fig. 25 gives a fair idea of the aspect of the wood in tangential section.

The interest of the specimen lies in the fact that distinct strands of primary xylem occur at the inner margin of the secondary wood, and in contact with it. In one of the transverse sections (No. 1381 in the WILLIAMSON Collection) five of these strands are present ; the other transverse section (now No. 1504 in my collection) only shows four, the tissues having been destroyed where the fifth would lie. Some of these strands are double, others single. Phot. 13 represents a double strand ; the two groups are each about ·15 mm. in tangential width, and are separated from each other by a space nearly equal to their diameter. The smallest elements lie towards the middle of each strand. Fig. 24 represents on a larger scale another bundle, in which the two strands are partly fused, the whole having a diameter of about ·3 mm. ; the smallest elements are in two groups, accompanied by parenchyma, and no doubt represent the protoxylem. Towards the outer side the irregularly grouped primary tracheides pass over gradually

* Part VIII., *Phil. Trans.*, 1877, vol. 167, part i. p. 231.

† Part XII., *Phil. Trans.*, 1883, pt. ii. p. 469.

into the radial series of the secondary wood. I have not been able to observe a longitudinal section showing these xylem-strands, but in one case a displaced tracheide, belonging to the more external part of the group, showed a spiral thickening.

The xylem-strands, being both small and few in number, form only an inconspicuous feature in the general structure, but none the less they are perfectly distinct, especially where approaching their exit from the pith, and here represent mesarch primary wood-strands, quite comparable to those of *Lyginodendron*, though on a smaller scale.

Although there is no series of successive sections in which the course of the bundles could be followed, there is little doubt that the distinctly double strands are those near their exit, preparing to furnish the twin-bundles of a leaf-trace, while the fused or single strands are cut lower down in their course. The arrangement of the bundles indicates a 2/5 phyllotaxis as probable. The stem differs from *Lyginodendron* in the fact that the leaf-trace divides into two before leaving the pith, and that it passes out almost horizontally. In the former point our fossil resembles *Poroxylon*, in the latter it agrees with some of the other *Dadoxylons* figured by WILLIAMSON.* The elements of the outgoing leaf-traces are seen in one of the transverse sections, and have spiral or reticulated sculpturing. The leaf-traces are also seen in pairs in the tangential sections; the small strand of transversely cut tracheides, constituting the trace itself, is accompanied by contorted elements belonging to the secondary wood, such as are usually associated with outgoing bundles. In some cases the irregular secondary elements have insinuated themselves among those belonging to the leaf-trace.

A *Dadoxylon* from Brazil, probably of Permian age, described by ZEILLER † under the name of *D. Pedroi*, bears some slight resemblance to our fossil. The pith, as shown in transverse section, has three prominent angles, recalling the five similar projections in our specimen, and, like them, marking the lines along which lateral appendages were inserted. The medullary rays are usually uniseriate, and the secondary wood agrees fairly well with that of *D. Spenceri*. The pith, however, is enormously larger than in our fossil, having a diameter of 37 or 38 mm. as against 5 or 6 mm. The structure appears to have been purely endarch, as the author shows the spiral elements on the inner edge of the wood. There is thus little to connect *D. Pedroi* with our fossil, except a certain general similarity in the transverse section.

The stem of *Metacordaites Rigolloti* of RENAULT ‡ is at once distinguished from that of our plant by the fact that each leaf-trace, in passing through the wood, consisted of five bundles, not to mention other differences.

I have named the Hebden Bridge stem *Dadoxylon Spenceri*, in commemoration of the discoverer, to whom we owe so many valuable contributions to Carboniferous Palæobotany. I have not thought it necessary to found a new genus for its reception, as the primary xylem-strands are much reduced, and the stem in other respects agrees

* *L.c.*, Part viii., Plate 9, figs. 44 and 46.

† "Note sur la Flore fossile des Gisements houillers de Rio Grande do Sul," *Bull. Soc. Geol. de France*, Sér. 3, t. 23, 1895, p. 619. Text-figs. 8–19; pl. ix., fig. 4.

‡ B. RENAULT, "Note sur le Genre Metacordaïte," *Soc. d'Hist. Nat. d'Autun*, 1896.

closely with that of other *Dadoxylons* in which primary wood has not yet been distinguished.

The pentagonal pith, double leaf-traces, small and few primary xylem-strands, and dense wood, with almost wholly uniseriate medullary rays, may serve to characterise the species.

The importance of *Dadoxylon Spenceri* lies in its being, on the one hand, a typical *Dadoxylon*, with the type of secondary wood which we know to have belonged to *Cordaites*, while, on the other hand, it shows, in a reduced form, primary xylem comparable to that of *Lyginodendron* or *Poroxylon*. It suggests, perhaps more strongly than any of the other species described, a truly Gymnospermous stem, which may well have belonged to one of the Cordaiteæ, but which still retains the last relics of the primary wood-structure characteristic of the Poroxyleæ and Lyginodendreæ.

Summary and Conclusions.

The principal result of the present investigation has been to show that in a number of stems of Palæozoic age (most of them from the Lower, but one from the Upper, Carboniferous) with secondary wood of the well-known *Dadoxylon* structure, distinct, usually mesarch, strands of primary xylem, forming the downward continuation of the leaf-traces, were present around the pith. Thus the anatomical structure, of which we may take *Lyginodendron oldhamium* as the type, proves to have been widely distributed among Palæozoic plants, and to have extended to stems which, on the basis of other characters, would have been referred with some probability to the Cordaiteæ.

The stems examined appear to range themselves naturally in three groups, as indicated by the generic names employed.

The *Calamopitys* group is characterised by the relatively large dimensions and distinct mesarch structure of those primary xylem-strands which are about to pass out from the pith, while the same strands, lower down in their course, are reduced in size, and in some cases assume endarch structure, owing to the dying out of the centripetal wood. A single strand passed out from the pith to form the leaf-trace. The pith is solid, with no trace of discoid structure; it is very variable in size; in *C. fascicularis* and in some species of *C. Saturni*, it is remarkably small (1–3 mm. in diameter). The secondary wood has the typical *Dadoxylon* structure; the medullary rays are in many cases one, or at most two cells in thickness (*C. fascicularis, C. beinertiana*; some specimens of *C. annularis*); in *C. Saturni* pluriseriate rays appear to prevail.

The characters of the cortex and leaf-bases are known in *C. Saturni* and *C. annularis*, but not as yet in the species occurring in Britain. The repeated subdivision of the leaf-trace in passing through the cortex is one of the most important characters exhibited by the Thuringian specimens. The reference of *C. fascicularis* and

C. beinertiana to the genus, though I think highly probable, must be regarded as provisional until the structure of these species is more completely known.

In the *Pitys* group the numerous small xylem-strands disposed around a large pith are characteristic. The strands, except the outgoing ones, are more or less deeply embedded in the pith; as a general rule their structure is mesarch. Here also a single strand passed out through the wood, though it may appear as a double bundle immediately below its exit from the pith. The pith is large in all known specimens (1–5 cm.)*, and may show some approach to discoid structure (*P. antiqua*), but is probably not really comparable to the *Sternbergia* pith of the true Cordaiteæ. The secondary wood has the typical characters of the *Pissadendron* sub-genus of *Dadoxylon*, with rather wide elements; the larger medullary rays are always pluriseriate (4-7 cells or more thick). Nothing is known as yet of the cortical structure or of the appendages.

Lastly, there is the type of *Dadoxylon Spenceri*. In the one specimen known there are a few small primary xylem-strands scattered at the margin of the pith, and closely applied to the secondary wood. When clearly seen, these strands are found to be of mesarch structure. They pass out *in pairs*, each pair presumably constituting a single leaf-trace. The pith is of moderate dimensions (5–6 mm. in diameter), and probably not discoid. The secondary wood is once more of the usual *Dadoxylon* character, but very dense, consisting of small tracheides, with very narrow medullary rays, scarcely ever more than one cell in thickness. In this form—at present isolated—the primary bundles are less conspicuous features than in either of the previous groups. The double leaf-trace is a striking character shared with other stems, both fossil and recent (*Ginkgo*), in which no such primary strands appear. For these reasons I have not thought it well at present to found a new genus for the *D. Spenceri* type.

We have now to consider the probable affinities of these various groups.

The stems referred to *Calamopitys*, even on the characters shown in the incomplete specimens by which the British species are represented, are strongly suggestive of Cycadofilices, owing to the great development of the primary xylem-strands, and the marked similarity to the structure of *Lyginodendron*.

The additional characters present in the Thuringian specimens of the forms described by Count Solms-Laubach seem decisive on this point. The cortical structure, the large leaf-bases, with the characters of *Kalymma*, the course and structure of the leaf-trace bundles, all point in the same direction. The petioles, known as *Kalymma*, were no doubt those of compound, fern-like leaves. As mentioned above, it is extremely probable that the petiole, named *Rachiopteris multifascicularis* by Mr Kidston, belonged to one or other of our *Calamopitys* stems.

The genus *Calamopitys* has clear affinities with *Lyginodendron*, but differs in the structure of the petiole, which shows some approach to that of a *Myeloxylon*. As, however, I have no new observations to record on this part of the structure, I will pass on to one or two points on which the British species have thrown additional light. It is

* A specimen of *Pitys antiqua*, with a pith two inches in diameter, is mentioned by Witham, *l.c.*, p. 27.

interesting to find that, while the bundles approaching their exit are so distinctly mesarch in structure, those lower down in their course tend to lose their centripetal wood, so that, in *C. beinertiana*, most of them become actually endarch. These facts may indicate that the *Calamopitys* group had advanced rather further towards the usual stem-structure of Gymnosperms than was the case with *Lyginodendron* or even *Poroxylon.* Another point is the narrow-rayed secondary wood, quite Cordaitean or Araucarian in structure, co-existing with primary characters pointing to the Cycadofilices. The width of the rays, however, varies a good deal within the genus, and sometimes even within a single species (*C. annularis*) and is clearly a character on which too much stress has been laid by palæobotanists. On the whole, *Calamopitys* may be regarded as decidedly the most primitive of the three groups dealt with.

The *Pitys* stems are known to have belonged to tall branching trees.* We know of no Cycads or Cycadofilices with at all a similar habit, nor is there any evidence that the Coniferæ existed at so early a period. The only known family to which these trees could be referred is that of the Cordaiteæ, leaves of which have been found at a similar horizon.† The species of *Pitys* differ from stems, traced with certainty to true Cordaiteæ, in the broad medullary rays, the non-discoid pith (for the slight approach to discoid structure which they exhibit is of doubtful value), and in the presence of the primary xylem-strands. On the whole, I am disposed to regard the genus *Pitys* as a primitive member of the Cordaitean family, retaining some of the characters of an earlier stock. The mesarch xylem-strands, in spite of their reduced size, and the peculiarities of their arrangement, are evidently comparable to those of *Lyginodendron* or *Calamopitys.* Thus the *Pitys* trees appear to afford a new link, so far as stem-structure is concerned, between the Cycadofilices of the family Lyginodendreæ and the true Cordaiteæ. Such a connection was already indicated by the structure of *Poroxylon*, but that genus, from its later horizon (Permian), has a less direct bearing on the question of the origin of the Cordaiteæ.

Lastly, *Dadoxylon Spenceri*, with its dense wood and double leaf-traces, appears to stand near the typical Cordaiteæ (though its pith seems not to have been discoid), and also, as WILLIAMSON pointed out, strongly suggests the recent genus *Ginkgo*, which itself may have Cordaitean affinities.‡ The primary xylem-strands of this species are small in size and few in number, but though so much reduced, have essentially the same structure as in *Lyginodendron.* The fossil indicates that in the period of the Upper Carboniferous deposits, stems which in other respects had attained a typically Gymnospermous character, had not quite lost the primitive form of wood, which we can trace back, through the Cycadofilices, to the Ferns.

* The trunk of the Craigleith tree (*P. Withami*) found in 1830 was 47 feet in length, and at the top still had a diameter of about 1½ feet. WITHAM, *Internal Structure*, p. 29.

† KIDSTON, "On the various Divisions of the British Carboniferous Rocks," *Proc. Roy. Phys. Soc. Edin.*, 1894, p. 255.

‡ SEWARD and GOWAN, "The Maidenhair Tree, *Ginkgo biloba*," *Ann of Bot.*, vol. xiv., 1900, pp. 137 and 146.

EXPLANATION OF PLATES I.–VI.

Plates I. and II., Photographs, taken direct from the sections, by Mr L. A. Boodle, F.L.S.

Plate I.

Photographs 1 and 2. *Calamopitys fascicularis.*

Phot. 1. Transverse section, showing the pith (only preserved in its outer portion) and the surrounding secondary wood. A, B, C, arrows pointing to the principal primary xylem-strands. A, rather small xylem-strand, at the border of the pith; B, large strand (shown in detail in phot. 2) just entering the secondary wood; C, strand passing out through the wood. (See p. 332.) × about 10. From the Williamson specimen. (W.* 1380.)

Phot. 2. Transverse section, showing the xylem-strand B, more magnified, with its central protoxylem; *p*, part of pith. (See p. 338.) × about 30. (W. 1380.)

Phots. 3 and 4. *Calamopitys beinertiana.*

Phot. 3. General transverse section, showing the large pith, containing several sclerotic nests, and the surrounding wood. (See p. 342.) × about 2½. (K. 677.)

Phot. 4. Radial section, showing the pith, *p*, and part of the wood, x^2. The pith contains several sclerotic nests. (See p. 342.) × about 4. (K. 677[F].)

Phots. 5–7. *Pitys antiqua.*

Phot. 5. General transverse section, showing the large pith, surrounded by secondary wood. *l.t.*, a leaf-trace, passing out through the wood. (See p. 346.) × about 3. (K. 598[A].)

Phot. 6. General radial section, showing the pith, *p*, with horizontal gaps in its tissue, and on one side the secondary wood, x^2. *v.b.*, a primary xylem-strand. (See p. 346.) × 3¼. (K. 598[E].)

Phot. 7. General tangential section, passing through the outer part of the pith, and the inner secondary wood, x^2. x^2, wood in oblique section; $x^{2\prime}$, portion of wood in tangential section. Six primary xylem-strands, *v.b.*, are shown; *v.b.*′, a double strand in contact with the secondary wood, $x^{2\prime}$. *v.b.* 2, the primary strand shown in detail in fig. 16. (See p. 349.) × 4½. (K. 598[D].)

Plate II.

Photographs 8 and 9. *Pitys antiqua.*

Phot. 8. Transverse section, showing part of pith, *p*, and secondary wood, x^2. *v.b.*, *v.b.*, double primary xylem-strand, embedded in the pith, numbered 38 in diagram 5, in the text. (See p. 347.) × 35. (K. 598[C].)

Phot. 9. Transverse section showing pith, *p*, and wood, x^2. *v.b.* 13[a], and *v.b.* 14, two primary xylem-strands, indicated by arrows, and numbered as in diagram 5, with isolated tracheides between them. (See pp. 348 and 350.) × 350. (K. 598[C].)

Phot. 10. *Pitys Withami.* General transverse section, showing large pith, surrounded by secondary wood. *l.t.*, arrow pointing to outgoing leaf-trace bundle. Note the concentric rings in the secondary wood. (See p. 354.) × about 3. From one of Witham's original sections in the Edinburgh Botanical Collections, marked "Craigleith, 1831," and figured in his *Internal Structure of Fossil Vegetables*, pl. vi., figs. 5–8.

Phot. 11. *Pitys primæva.* Tangential section through secondary wood, showing base of a branch, *br.* *m.r.*′, region in which the dilated medullary rays, shown in detail in fig. 23, occur. In other parts the rays show the ordinary structure, as shown in fig. 22. (See p. 355.) × about 4. (K. 216.)

Phots. 12 and 13. *Dadoxylon Spenceri.*

Phot. 12. General transverse section, showing the obtusely pentagonal pith, and the surrounding wood. *l.t.*, *l.t.*, two double leaf-traces passing out almost horizontally through the wood. *v.b.*, arrow pointing towards the angle of the pith where the double xylem-strand shown in phot. 13 is present. (See p. 358.) × 4½. (S. 1504—from Spencer Collection.)

Phot. 13. Transverse section showing secondary wood, and outer border of pith, *p*. *v.b.*, pair of mesarch primary xylem-strands, in contact with the secondary wood. (See p. 358.) × about 50. (S. 1504.)

* The letter W. indicates the Williamson Collection, K. Mr Kidston's Collection, and S. that of the author.

PLATES III.–VI.

Original drawings, made, under the author's supervision, by Mr G. T. GWILLIAM.

PLATE III.

Calamopitys fascicularis.

Fig. 1. Transverse section, showing the contracted pith, primary xylem-strands, and inner part of the secondary wood. From a section adjacent to that from which diagram 1, p. 334, was drawn, and showing practically identical structure. The strands are lettered as in the diagram. A, large xylem-strand in contact with secondary wood, x^2; *px.* protoxylem; B, still larger strand, beginning to pass out into the wood; *px*, protoxylem; C, direction of outgoing leaf-trace shown in diagram 1; *a*, *b*, *c*, reparatory strands of the outgoing bundles A, B, and C; *d*, strands of the orthostichy D, cut low down in their course; *e*, similar strands of the orthostichy E. (See p. 333.) × 24. (K. 540[A].)

Fig. 2. Transverse section, showing one of the smaller xylem-strands, *v.b.* (this is the larger of the two marked *e* in diagram 2, p. 335); *px*, protoxylem. x^2, secondary wood, showing pits on the horizontal walls of the dilated tracheides (*cf.* fig. 3). (See p. 338.) × 140. (K. 628.)

Fig. 3. Radial section, from the border of the pith and wood. *p*, pith; *v.b.*, primary xylem-strand; *px*, its protoxylem; *p′*, circum-medullary tissue; x^2, inner elements of secondary wood, showing the short, irregular tracheides (*cf.* fig. 2). (See pp. 338 and 339.) × about 100. From a new section of Mr KIDSTON's specimen.

Fig. 4. Radial section of the secondary wood, showing the tracheides, with multiseriate bordered pits, and the medullary rays. (See p. 338.) about × 120. (K. 540[I].)

Fig. 5. Tracheides in radial section, more enlarged, to show the hexagonal pits, with slit-like pores. (See p. 338.) × about 300. (K. 540[I]).

PLATE IV.

Figs. 6 and 7. *Calamopitys fascicularis.*

Fig. 6. Tangential section through the secondary wood, showing tracheides and medullary rays, *m.r.* *b.p.*, bordered pits of the tracheides in section. (See p. 339.) × about 120. From the WILLIAMSON specimen (W. 1392.)

Fig. 7. Tangential section through the wood, showing an outgoing leaf-trace. *px.*, protoxylem of the leaf-trace; *pa.*, parenchyma, accompanying the leaf-trace; *m.r.*, medullary rays. (See p. 337.) × about 30. (W. 1392.)

Figs. 8–12. *Calamopitys beinertiana.*

Fig. 8. Transverse section, showing a large primary xylem-strand, *v.b.*, just entering the secondary wood, x^2, from the pith. *px*, protoxylem of the primary strand. (See p. 342.) × 60. (K. 677.)

Fig. 9. Transverse section, showing a smaller xylem-strand, *v.b.*, at the inner margin of the wood, x^2. *px*, protoxylem of the strand, which is here of the "hippocrepiform-endarch" type. (See p. 343.) × 60. (K. 677.)

Fig. 10. Somewhat oblique radial section of a primary xylem-strand. *px*, spiral protoxylem-element of the strand. The ill-preserved element to the right is a tracheide. (See p. 343.) × 100. (K. 677[H].)

Fig. 11. Part of a large primary tracheide, showing the numerous rows of hexagonal bordered pits. (See p. 343.) × about 200. (K. 677[G].)

Fig. 12. Tangential sections from the secondary wood—

A, showing part of a long medullary ray. × 70. (K. 677[K].)

B, showing a 2–3-seriate ray with irregular adjacent tracheides. × 90. (K. 677[K].)

C, showing several short rays. × 90. (K. 677[M].) (See p. 343.)

PLATE V.

Fig. 13. *Calamopitys beinertiana.*

Approximately tangential section of the secondary wood, passing obliquely through a large outgoing leaf-trace, accompanied by parenchyma. (See p. 344.) × about 50. (K. 677[L].)

Figs. 14–18. *Pitys antiqua.*

Fig. 14. Transverse section, showing part of the pith and secondary wood, x^2. *v.b.* 13, double primary xylem-strand, in contact with the secondary wood; *v.b.* 12*a*, xylem-strand, deep in the pith, perhaps a reparatory strand. (*Cf.* diagram 5 in text, p. 347.) × 40. (K. 598[C].)

Fig. 15. Transverse section. *v.b.*, primary xylem-strand, embedded in the pith; *px*, protoxylem of the strand; x^2, part of secondary wood, with broad medullary rays, *m.r.* The xylem-strand is that numbered 3 in diagram 5. (See p. 348.) × about 110. (K. 598[A].)

Fig. 16. Tangential section, passing medianly through a primary xylem-strand (numbered *v.b.* 2 in photograph 7) embedded in the pith. *px*, protoxylem-elements of the strand; *r.t.*, a reticulated tracheide; the meshes of the reticulation are bordered, in reality, as in the element shown in fig. 17. (See p. 348.) × 140. (K. 598[D].)

Fig. 17. Part of one of the innermost tracheides of the secondary wood, showing transition from reticulations to bordered pits. (See p. 348.) × 200. (K. 598[E].)

Fig. 18. Radial section through the secondary wood, showing a medullary ray, and tracheides with multiseriate, hexagonal, bordered pits. (See p. 352.) × 120. (K. 598[E].)

Plate VI.

Figs. 19 and 20. *Pitys antiqua.*

Fig. 19. Tangential section through the secondary wood, showing tracheides, and medullary rays of various dimensions. The large ray to the right has gaps in its tissue. (See p. 351.) × 32. (K. 598[G].)

Fig. 20. Tangential section, from inner part of secondary wood. *m.r.*, medullary rays; the larger of the two contains a definite lacuna, *g.*; *t.p.*, bordered pits, on the *tangential* wall of a tracheide. The pits to the right are on radial walls seen obliquely. (See p. 352.) × 200. (K. 598[H].)

Fig. 21. *Pitys Withami.* Transverse section, showing a primary xylem-strand, *v.b.*, embedded in the pith. x^2, inner part of secondary wood. (See p. 354.) × about 130. From Witham's original section, shown in phot. 10.

Figs. 22 and 23. *Pitys primæva.*

Fig. 22. Tangential section of the secondary wood, to show the ordinary form of the medullary rays, large and small. (See p. 355.) × 36. (K. 211.)

Fig. 23. Tangential section of the secondary wood near the branch (represented in phot. 11), to show the dilated form of the rays, and tortuous course of the tracheides, in this region. (See p. 355.) × 36. (K. 216.)

Figs. 24 and 25, *Dadoxylon Spenceri.*

Fig. 24. Transverse section showing a double primary xylem-strand, *v.b.*, in contact with the secondary wood, x^2; *px*, one of the protoxylem-groups of the primary strand. *p.p.*, pith. (See p. 358.) × 150. (W. 1381.)

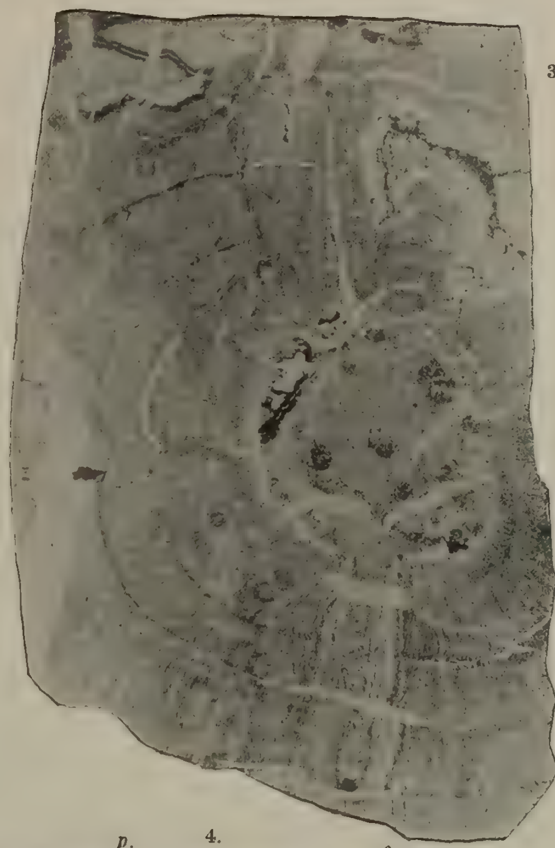
Fig. 25. Tangential section through the secondary wood, showing the tracheides, *t*, and the narrow medullary rays, *m.r.* (See p. 358.) × 120. (W. 1382.)

SCOTT: PALÆOZOIC STEMS—PLATE I.

1.

2.

3.



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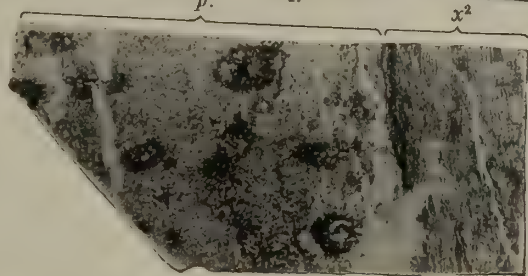

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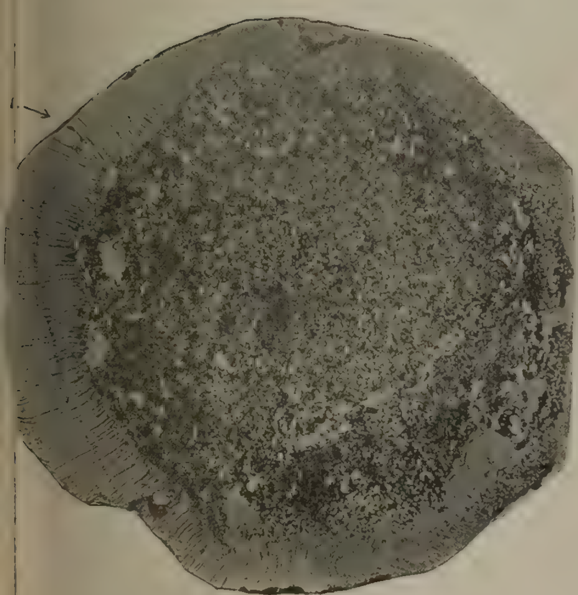

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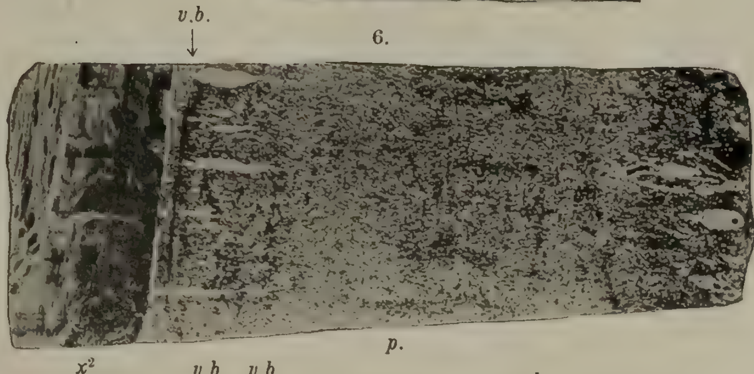

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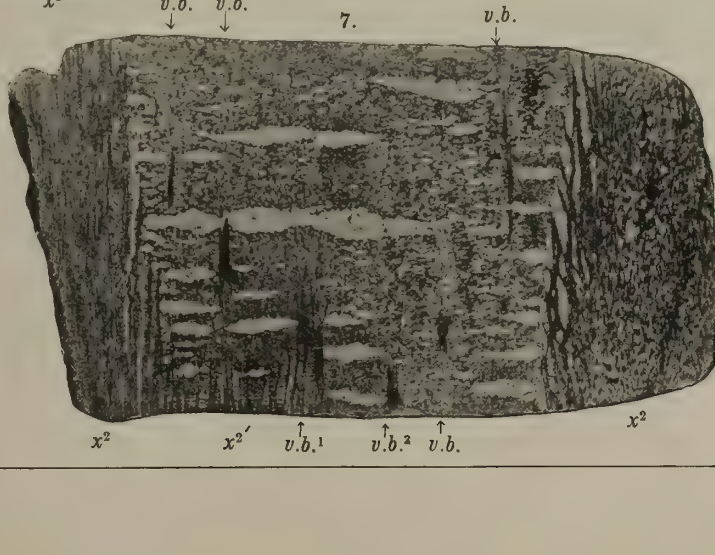

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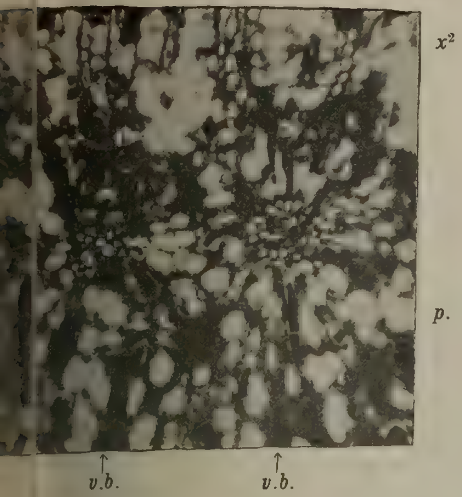

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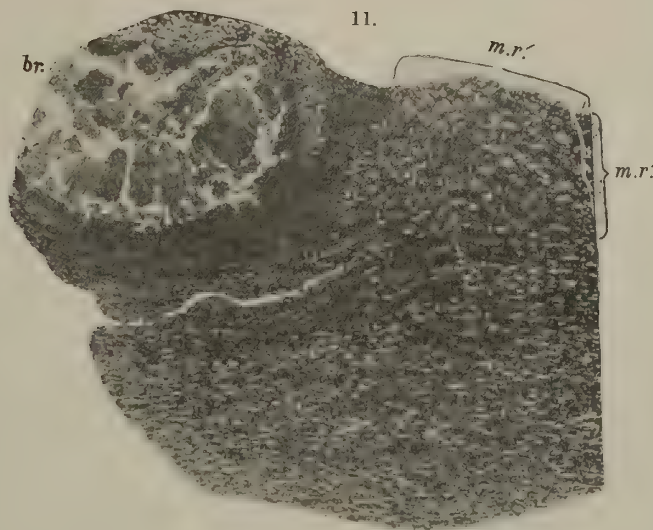

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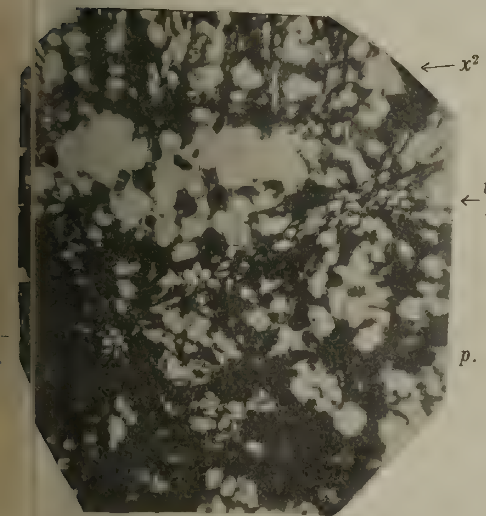

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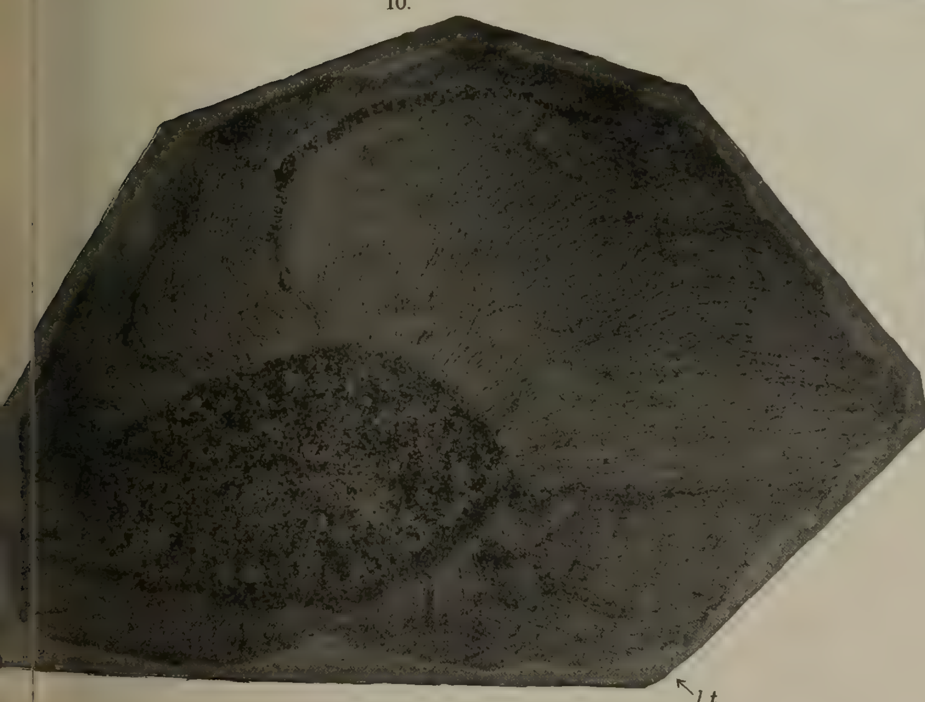

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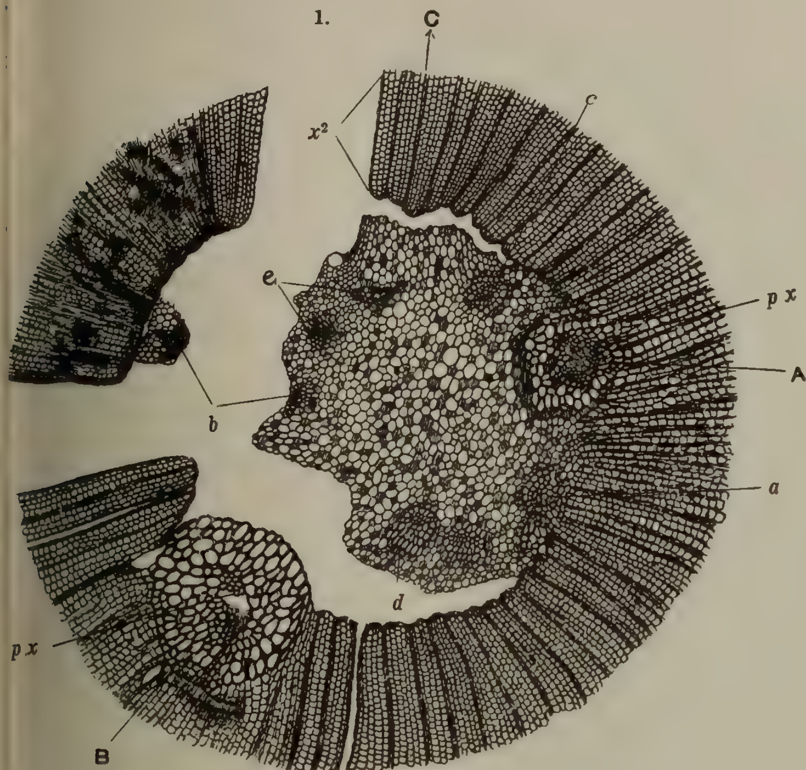

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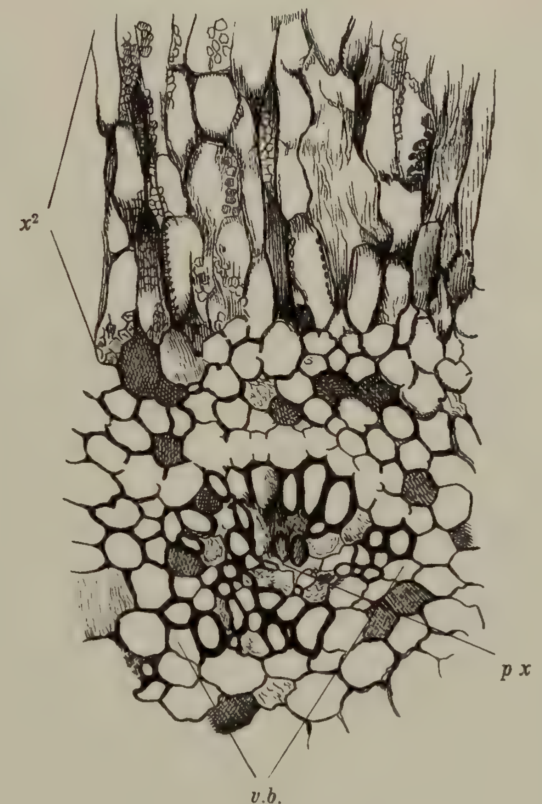

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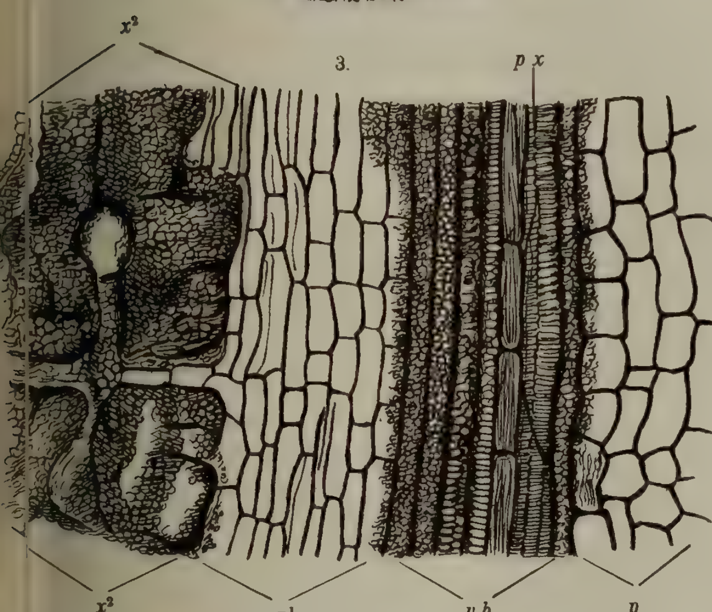

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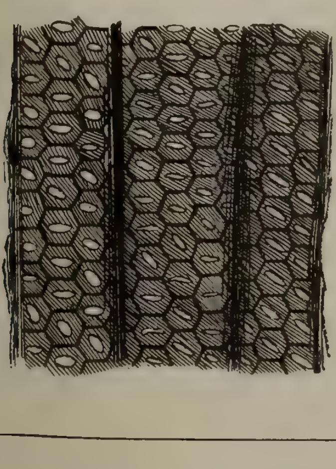
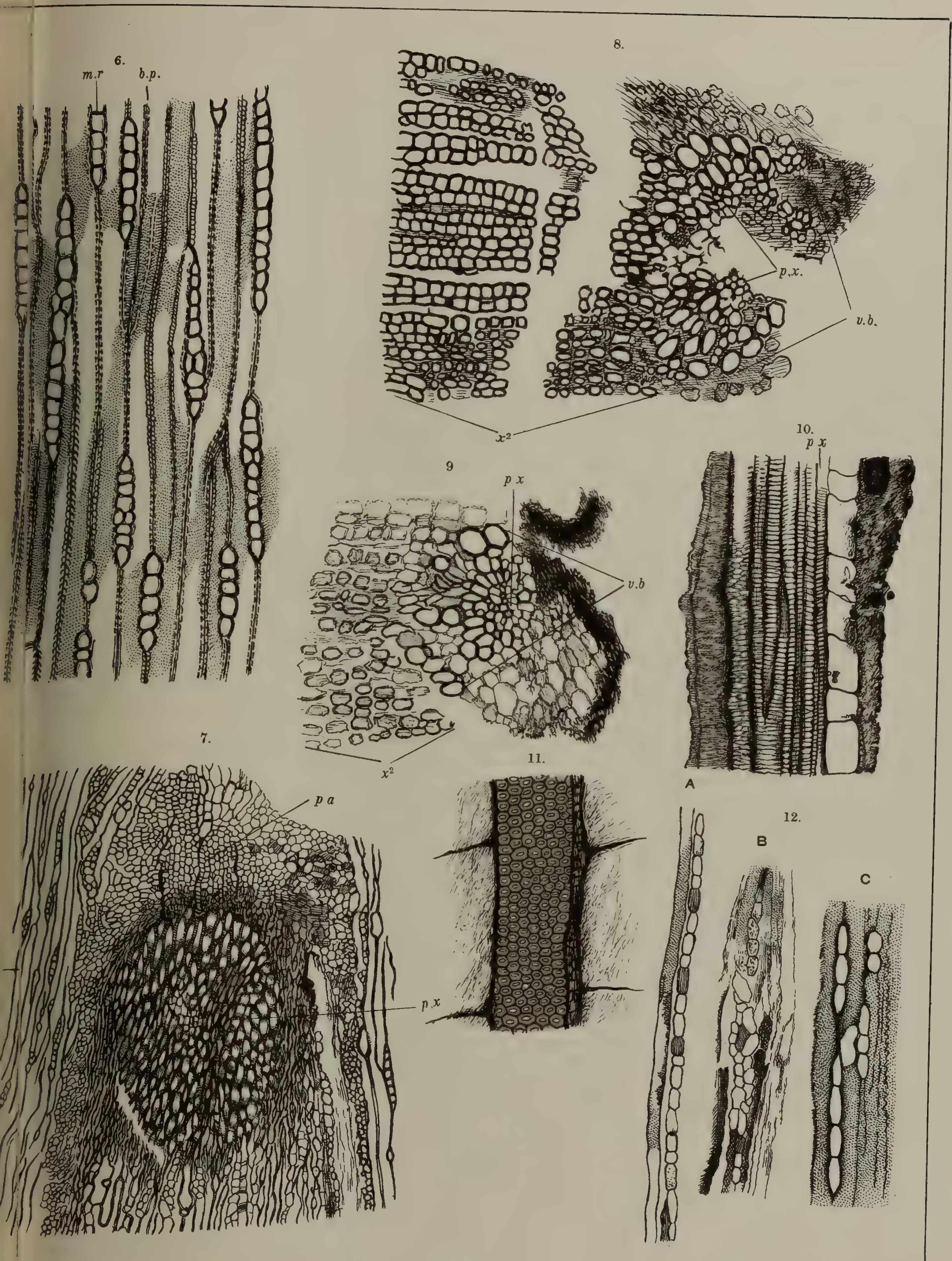
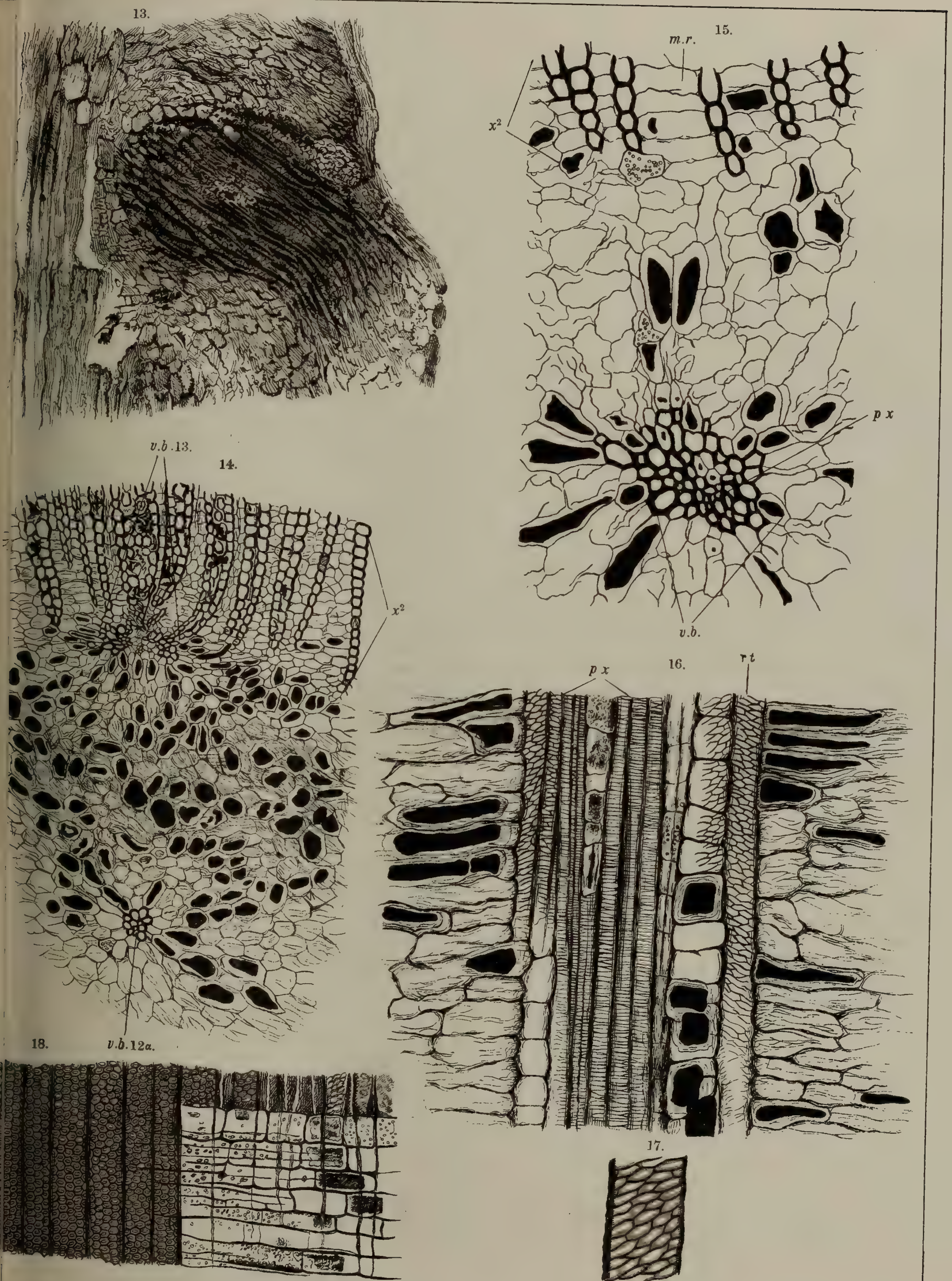


SCOTT: PALÆOZOIC STEMS—PLATE IV.



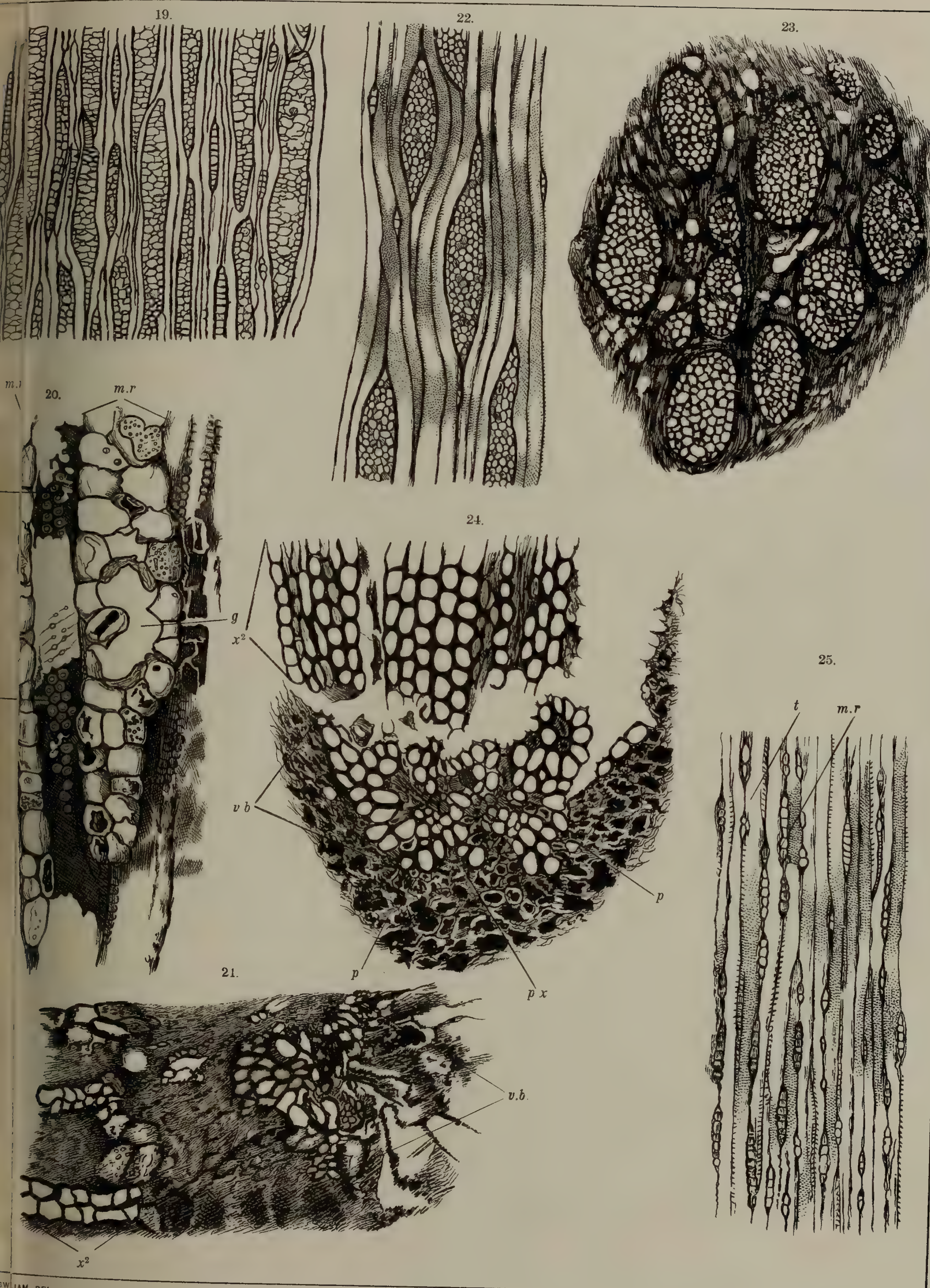
SWILLIAM, DEL.

W. GRIGGS, LITH.



M. DEL.

W. GRIGGS, LITH.



WLIAM, DEL.

W. GRIGGS, LITH.

XVIII.—*On a Possible Stridulating Organ in the Mosquito.* (*Anopheles maculipennis*, Meig.) By A. E. Shipley, M.A., and Edwin Wilson, F.E.S. *Communicated by* Sir John Murray, K.C.B. (With a Plate.)

(Read 3rd March 1902. Issued separately 24th April 1902.)

At the base of the wings of *Anopheles maculipennis*, Meig, is a small quadrilateral area (fig. 2, Plate) which has escaped the notice of the systematist, since the nervures in which he is interested lie almost entirely beyond the area in question, and which has escaped the notice of the morphologist, because it presents to him but a confused outline of meaningless ridges and intervening depressions of no structural importance. If, however, the well-preserved wing of a mosquito, whilst still attached to the thorax, be examined, a very curious apparatus can be made out, lying across the centre of the said quadrilateral area which is bounded posteriorly by the alula.* This apparatus lies at about equal distances from the anterior and the posterior edge of the wing, and divides the base of the wings roughly into two halves.

The arrangement of the apparatus is not quite constant in different specimens; details vary, but the following points can in all cases be made out. It should, however, be mentioned that the apparatus is very minute; that it lies very close to the thorax, hence, if the wings be cut off, it is almost certainly destroyed, and thus there is small chance of seeing it in a detached wing; that it is only visible when the wings are extended, a position in which it is not always easy to mount the insect; and finally, that when the wings are at rest and folded back over the abdomen, the details of the apparatus are obscured and become confused in a mass of ridges and furrows, from which they can hardly be disentangled by the eye. The presence of the striated scales on some of the ridges of this part of the wing still further serves to obscure the parts in question.

The structure of the apparatus is very complex, but can be easily realised from an inspection of fig. 1, Plate, which represents the right half of the thorax of *Anopheles maculipennis* with the base of the right wing and the right halter. To bring out the structures of the parts in question, we have coloured them, and it will be seen at a glance that they fall into two series, an anterior one, which we have coloured blue, and a posterior set of structures, which we have coloured yellow.

The anterior system consists of a slightly movable bar A, which covers a thickening in the tissue of the wing. This last-named ultimately passes into the sub-costal nervure. The bar extends from the articulation of the wing to the humeral

* Nuttall and Shipley, *J. Hygiene*, i., 1901, p. 474.

cross-nervure,* and, as we have stated, we consider it to be slightly movable. On its hinder, free edge the bar carries a series of thirteen to fifteen well-marked teeth, which, under certain circumstances, rasp across a series of ridges borne on the second part of the system, which we now proceed to describe.

This is coloured yellow in our Plate. It consists primarily of a chitinous blade B, shaped something like a butcher's knife, which is ridged on its upper surface with thirteen to fifteen sharply-defined elevations placed slightly obliquely. It is against these ridges that the teeth of the bar mentioned above rasp. The relative position of the bar and the blade is not constant. The latter is usually lying partly under the former, but at times it is free from it, and it seems as though it were hinged at the handle and can oscillate through a few degrees, around an axis at the base of the proximal end. But it cannot rotate very far. A little way behind the back of the blade is an upstanding flange of the membranous tissue of the wing, which we have called the trough C. This trough catches and holds the blade, and prevents the latter being thrown too far back.

It is worthy of note that the wing area between C and A, over which the blade travels, is quite smooth and entirely free from the minute hairs or prominences which cover the rest of the wing. A similar freedom from these processes is found beneath the next region of the apparatus to be described, and this, in our opinion, is also slightly movable. These are the only two regions of the entire wing which are quite smooth and free from hairs.

The base of the blade or handle is held in position partly by being clamped by the overlying bar A, and partly by a chitinous flap of the wing marked H in our figure. Underneath this is a knob or process which may act as a fulcrum on which the knife rotates.

Running from the extreme proximal end of the knife handle to the posterior inner angle of the wing are a series of sclerites of irregular outline, roughly recalling in shape a number of knuckle bones. The most distal of these, D, seems to send a process into a hollow at the extreme base of the knife, and may very probably serve to throw the latter in, and out of, gear. The middle sclerite E seems to be articulated with the proximal F by a very distinct joint, and the proximal runs into the thorax, just ventral to the scutellum, and forms the attachment of the wing to the body at the posterior edge.

The bases of the middle sclerites are overshadowed by a flange G^1 which is supported by the thickened ridge G which forms the outer limit of the squama.

It must be remembered that the attachment of the wing is not in one horizontal plane, but the line of junction has a slightly oblique course. The posterior edge of the base of the wing is slightly dorsal to that of the anterior end of the same. Hence it seems probable that, as the wings are raised and depressed, there will be a shearing action between the slightly movable toothed bar A and the ridged blade

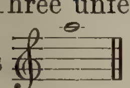
* *Vide* Nuttall and Shipley, *J. Hygiene*, i., 1901, p. 475.

B, and we think that this movement of these two ridged surfaces, one on the other, is responsible for some of the characteristic buzzing of the mosquito.

The exact cause of the note of a mosquito has been a matter of much dispute. The following extract from a paper published last January by one of us, in collaboration with Dr G. H. F. NUTTALL, describes some experiments made last year at Cambridge on this subject :—

' According to HOWARD * the sound during flight is " apparently produced, as with ' flies and other dipterous insects, not by the rapid vibration of the wings, but by the ' vibrations of a chitinous process in the large tracheæ just behind the thoracic spiracles. ' These vibrations are produced by the air during respiration." He furthermore states ' that the sound produced in its flight is higher in *Culex* than it is in *Anopheles*, adding ' that " the villain in the play has usually a bass voice."

' Our experiments do not support HOWARD'S assertion with regard to the wing not ' producing a note, for we have found by cutting off more and more of the wing that ' the sound decreased in volume, the note rising progressively. When the wing was cut ' off quite closely, a very high-pitched note of slight intensity remained, this, as we ' supposed, being produced by an internal apparatus such as HOWARD indicates. It may, ' however, be due to respiratory movements which are exaggerated through the efforts ' at flight. The sound is not produced by the insect in repose. We found that the males ' gave a higher-pitched note than the females, and that the note was higher in both ' sexes when they had fed ; the greater the meal, the higher the note. Of four unfed ' females, three gave notes within a quarter of a tone of 264 (*i.e.* of 240 to 270 vibra- ' tions), the fourth female gave an abnormally low note of about 175 vibrations. Four ' other females were arranged in the order of the distension of the abdomen by food, ' the last being largely distended ; these gave notes corresponding roughly to 264—281 ' —297—317 vibrations, or according to the musical scale, the notes :

' Three unfed males gave exactly the same note, viz. corresponding to 880 vibra- ' tions  ; immediately after feeding one gave the note A♯, another which had fed ' well B♮. The unfed males were more closely concordant than the unfed females, the ' latter varying over about a semitone. Mr J. W. CAPSTICK, M.A., Fellow of Trinity ' College, Cambridge, to whom we are greatly indebted for making these ear determina- ' tions for us by means of tuning-forks, was not certain that the note given by the ' males was not one of 440 vibrations. Overtones were obviously strong, and it sounded ' at times as if there were a faint note of 440 vibrations overshadowed by a strong one ' of 880.

* *Mosquitoes*, by L. O. HOWARD. New York : M'Clure, Phillips & Co., 1901, p. 14.

'The obvious explanation of the higher note given off by the males is that their 'wings are markedly narrower and shorter than those of the females. Although a 'female *Culex pipiens* gave a higher-pitched note than a female *A. maculipennis*, we 'are not at all sure that it was not simply due to the smaller size of the former insect. 'The male of this species of *Culex* certainly gave a higher-pitched note than the 'female.' *

In the experiments described above, the note increased in pitch as the wings were shortened, until a very short stump was left. As long as these stumps were left on the body, a note was heard. These stumps would certainly include the stridulating apparatus figured on Plate, which lies very close to the thorax at the base of the wings. Dr NUTTALL assures us that when these stumps were removed, all perceptible sound ceased. Thus it appears, that as long as the very small portion of the wing which includes the stridulating organ remains on the body, a note is heard, but when this be removed no sound is perceived. This is evidence that the sound proceeds from the base of the wings, and not from the spiracles or other parts of the thorax.

Mons. J. PEREZ † has carefully gone into the question of the production of sound in the Diptera. He shows, experimentally, that the stigmata take no part in the production of sound. "Les causes du bourdonnement résident certainement dans les ailes." He, too, points out that if the wings are cut short the note becomes more acute, until the 'timbre' resembles that of certain interruptors which break, and make an electric conductor. This sound we should attribute to the stridulator described above. Mons. PEREZ definitely states that both in the Diptera and in the Hymenoptera, the buzzing is due to two causes, "l'une, les vibrations dont l'articulation de l'aile est le siége et qui constituent le vrai bourdonnement; l'autre, le frottement des ailes contre l'air, effet qui modifie plus or moins le premier." The apparatus we have described is, we believe, the mechanism by means of which his first vibrations are produced.

In the same periodical ‡ Mons. JOUSSET DE BELLESME confirms the statement that both Dipterous and Hymenopterous insects emit two sounds, one deep and one acute, and states that the latter is usually the octave of the former. It is this double note which gives rise to the peculiar buzzing associated with these two orders of insects. Mons. DE BELLESME, like Mons. PEREZ, discards the view that acute sounds are due to any action of the issuing air in the stigmata, and attributes it to the vibrations of the pieces of the thorax which support the wing, and which are moved by the muscles of flight. It is usually stated that these muscles are not inserted into the wing, but into the sides of the thorax, to which the wing is so attached, that when the lateral walls of this part of the body are deformed by the action of the muscles, the wings move up and down. But whether this be the case or not, it is clear that the vibrations of the sides of the thorax caused by the muscles of flight—and causing the vibrations of the wing—will synchronise in number with these wing vibrations, and will give forth the same note. The existence of the higher note—'usually the octave' of the one produced by

* *J. Hygiene*, ii., 1902, 77 *f.* † *C. R. Ac. Paris*, lxxxvii., 1878, p. 378. ‡ *Ibid.*, p. 535.

the wing vibrations—is unexplained by this view. It is, however, easily explicable if such a stridulating organ as we have described at the base of the wing in *Anopheles maculipennis* be found in other Diptera and in Hymenopterous insects.

On Plate we have thought it well to figure the upper surface of the halter as seen under a high magnification. The drawing shows the hinge on which the halter quivers, the basal-papilla-plate, as WEINLAND * calls it, and the same author's 'skapale'-papilla-plate of the upper side. The papillæ of the basal-papilla-plate are continued on to its under surface, and only a few are shown in our drawing.

There is little doubt that the main functions of the halteres is that of balancing and orientating the insect. They may, however, have some secondary function; in some flies they are known to vibrate with extreme rapidity. It is just possible that in these rapid vibrations the papillæ of the concave surface rubbing against those of the convex basal plate may produce a note. As long ago as 1764, VON GLEICHEN-RUSSWORM † observed that when the halteres of the common house-fly are removed, the amount of the buzzing is diminished. This, however, in all probability, is due to the diminished activity of the wings. On the other hand, Mr STANLEY J. GARDINER informs us that he has noticed that mosquitoes still continue to give forth a faint note even when their wings are quite at rest, and this may possibly be caused by the halteres.

* *Zeitschr. wiss. Zool.*, li., 1891, p. 55.

† *Geschichte der gemeinen Stubenfliege*, Nuremberg, 1764.

[EXPLANATION.

EXPLANATION OF PLATE.

Fig. 1. Right half of thorax of *Anopheles maculipennis*, Meig, with base of right wing and right halter. Magnified about 90–100.

Fig. 2. The same magnified about 15 to show the area which bears the stridulator. This is coloured blue.

A. Toothed bar.
A^1. The teeth which rasp on the ridges borne by B.
B. Blade bearing the ridges R.
C. Trough which limits the movements of B.
D. Distal chitinous sclerite bearing a claw which works in.
D^1. The hollow at the base of B.
E. Intermediate sclerite between D and F.
E^1. Hinge between E and F.
F. Proximal sclerite which is inserted into the thorax beneath the scutellum.
F^1. Process on F.
G. Thickened edge of squama.
G^1. Fold of squama overhanging E and D.
H. Process of chitin which overhangs the base of the blade.
K. Distal end of halter.
L. Concave area covered with papillæ.
M. Knob.
M^1. Hinge.
N. Papillæ on knob.

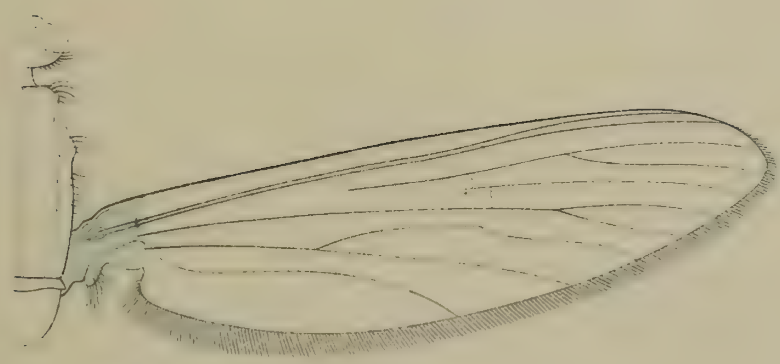


FIG. 2.

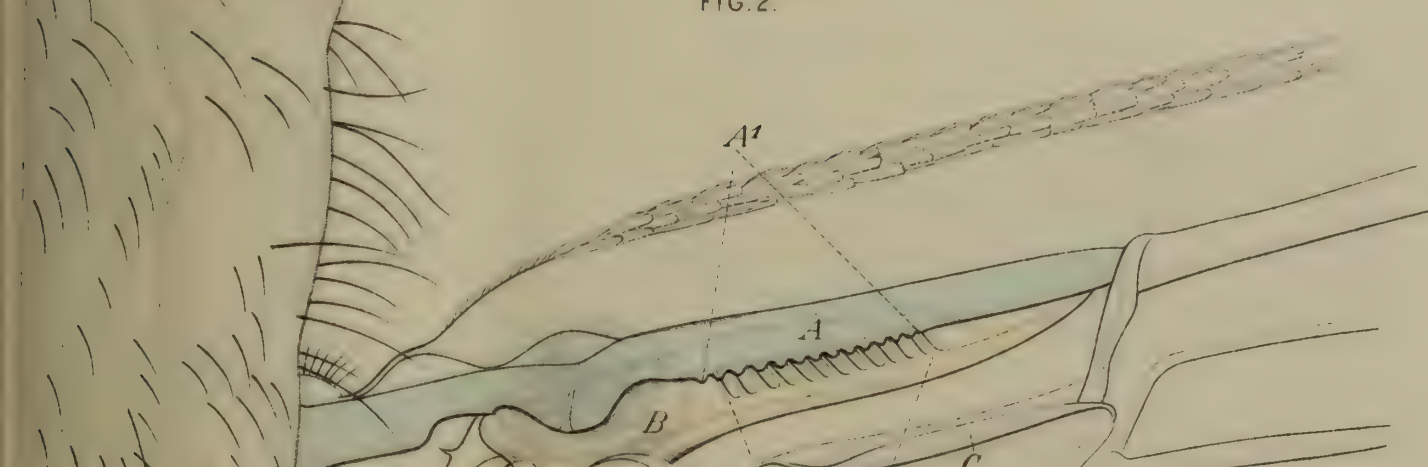



FIG. 1

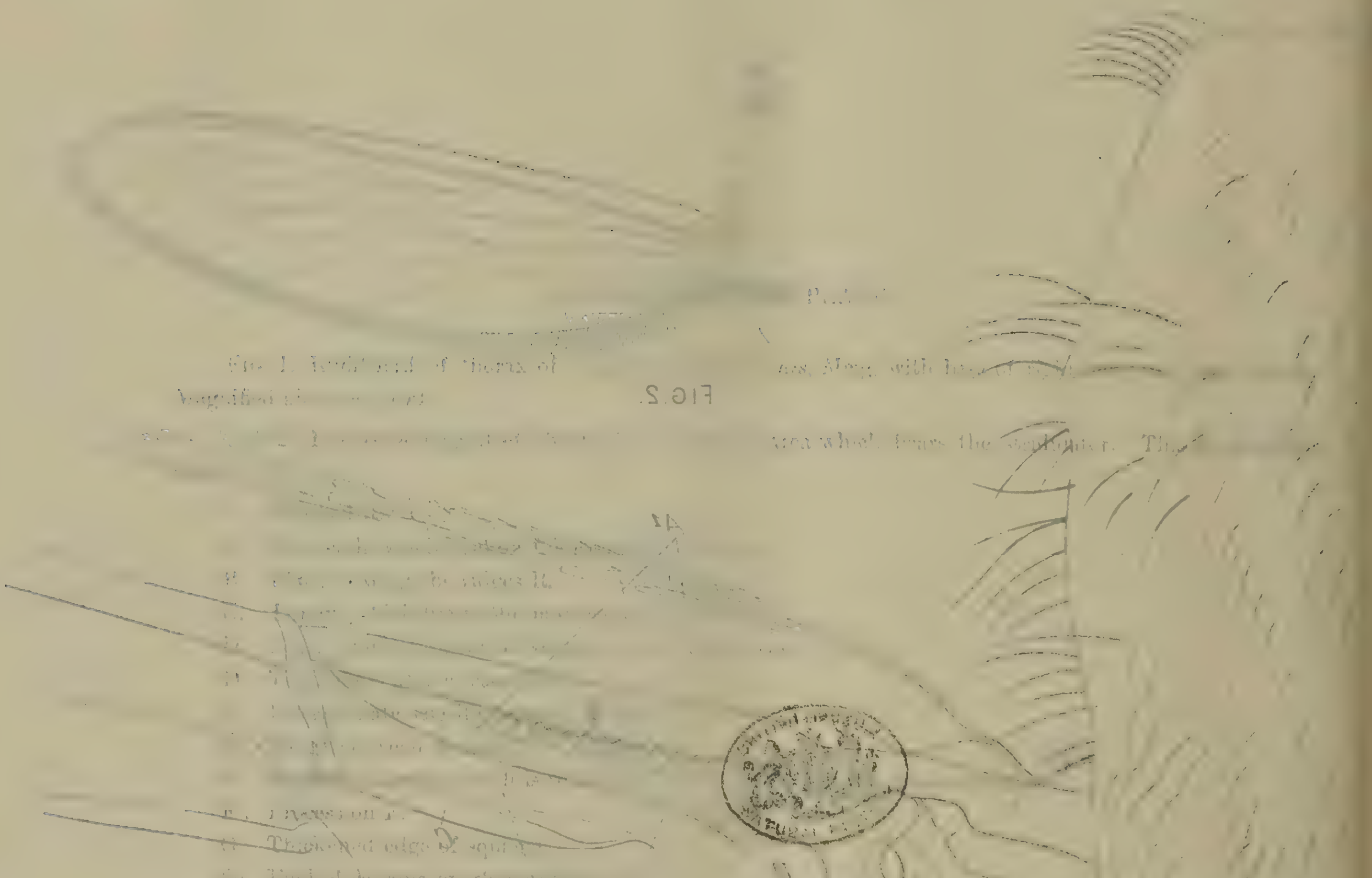


FIG. 2.

SHIPLEY AND WILSON: STRIDULATING ORGAN IN THE MOSQUITO.

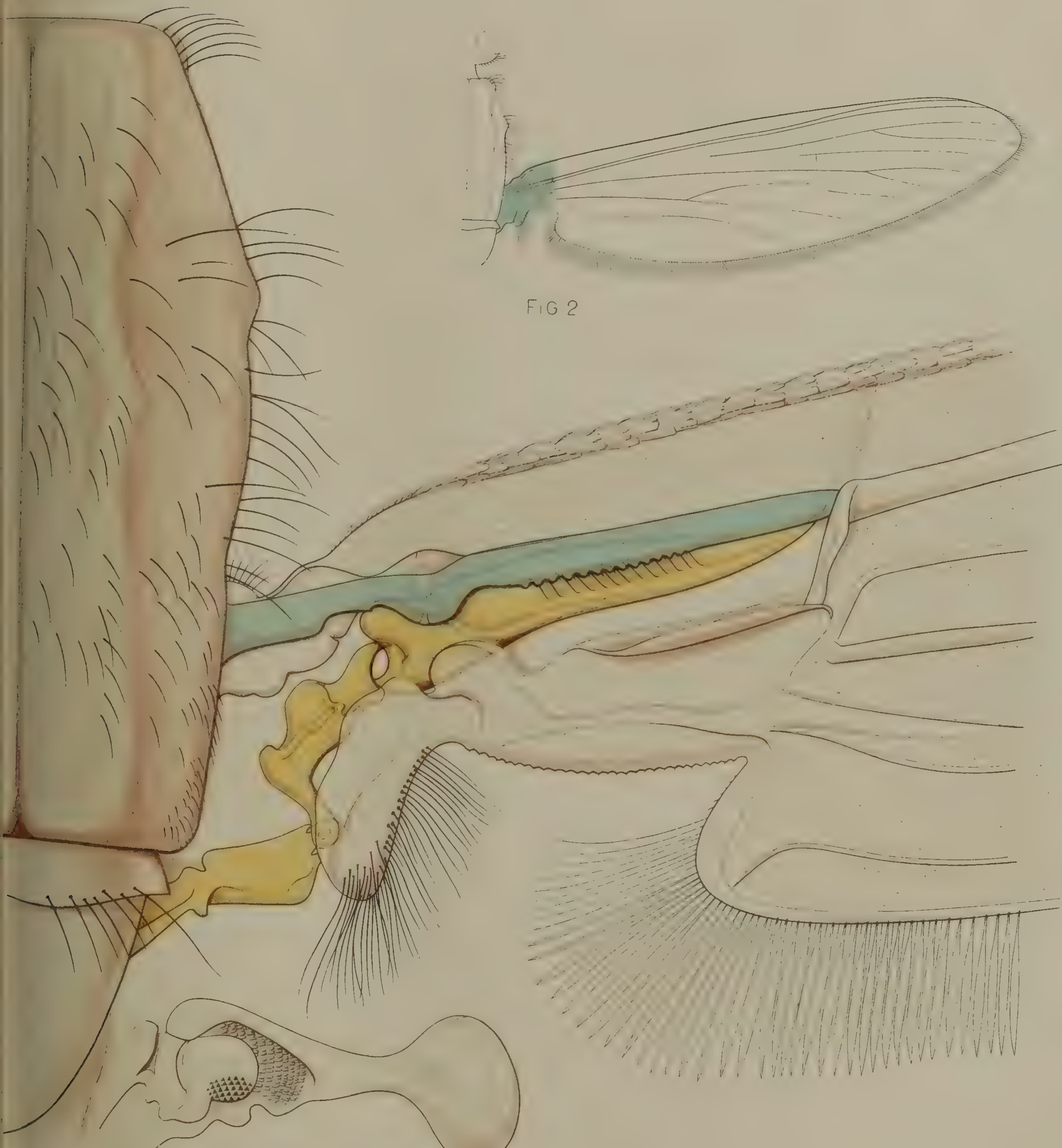


FIG 2

FIG 1

Edwin Wilson del et lith. Cambridge.

XIX.—*The Early Development of* Cribrella oculata (Forbes), *with Remarks on Echinoderm Development.* By Arthur T. Masterman, M.A., D.Sc., F.R.S.E., Lecturer in Zoology in the School of Medicine of the Royal Colleges, Edinburgh. (With Five Plates.)

(*From the Laboratory of the Royal College of Physicians, Edinburgh.*)

(Read March 3, 1902. Issued separately May 27, 1902.)

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The breeding habits of this asterid have been noted by M. Sars,* who has described the external features of the developmental stages. He observed the peculiar habit of 'brooding.' In the spring of 1840 he found several of the parents at very low-tide-mark fixed to stones in their brooding position. The species he examined is described as *Echinaster sanguinolentus*, and is apparently the same species, though he speaks of the 'blood-red' embryo, whereas my specimens were a light orange. They conform exactly to the tint of the oral surface of the parent. The time of breeding of *Cribrella oculata* at St Andrews appears to extend from February to April inclusive, and throughout this period the parents are found fairly plentifully, adhering to the under surface of rocks, nursing the masses of eggs or young under their bodies. This habit of protecting its young is also pursued by an allied species, *Asterias mülleri*, which is equally abundant in the same locality, but the breeding of which is effected earlier, *i.e.*, in November and December. I hope to proceed with the study of this species in the near future.

Ovaries.—The ovaries of *Cribrella oculata* do not differ in essential structure from those of the common *Asterias rubens*, though they contain fewer and larger eggs. The

* Sars, M., *Fauna Littoralis Norvegiæ*, Erste Lieferung, Christiana, 1846, p. 47, Tab. 8, f. 1–37.

apertures of the ovaries to the exterior are, however, situated upon the oral surface of the animal, an adaptation to the habit of 'carrying' the eggs which was conjectured by SARS. A series of sections through the parent at once demonstrates the small inter-radial openings just at the edge of the oral disc. As the ovary becomes ripe the eggs gradually increase in size and their protoplasm becomes filled with an enormous quantity of small yolk-granules of a pale yellow hue. These granules, as far as I can ascertain, are deposited in the substance of each ovum by intra-cellular secretion, and are not produced by the disintegration of adjacent ova nor by the secretory activity of follicular cells.

BROODING HABITS.—The phenomena of oviposition and of fertilisation have escaped my observation, and the present account commences with the fertilised eggs and their subsequent history. Each female lays from thirty to fifty eggs, which are found in the so-called 'brood-chamber'; SARS found from ten to thirty. The female attaches itself by the tube-feet of the distal half of each arm to the surface of the rock, and then bends the proximal half of each arm at right angles upwards, at the same time arching its oral disc. In this way is produced a bell-shaped brood-cavity, into which the eggs are shed and retained. In this position the mother starfishes are found adhering to the perpendicular face of rocks, or more commonly hanging downwards under overhanging ledges. They do not occur very far above low-tide-mark, but large numbers are found completely uncovered by the tide. In every case they appear to be capable of retaining water in the brood-cavity, the necessity for which may account for this predilection for hanging downwards. So far, the habits are very similar in *Asterias mülleri*, but there are slight differences to be noted. The habit of 'brooding' is not so intense in *Cribrella*, and the slightest disturbance at any stage serves as a pretext for the female to forsake her duties. It is, however, fair to add that the forsaken eggs or larvæ do not appear to suffer by such treatment, and proceed in their development in a similar way to those still retained. In *Asterias mülleri* the maternal instincts are stronger, and the parent will retain her eggs in confinement under considerable vicissitudes. Similarly, the eggs and larvæ of the former are never at any time mechanically attached to the parent nor to each other, but in the latter each larva attaches itself firmly to its ruptured egg-membrane, and the whole forms a solid ball of larvæ, which is firmly retained *in situ* by the parent. So much is this the case that the late larvæ can only be separated at great risk of mechanical injury, whereas in *Cribrella* the embryos can be washed out by a pipette. During the period of 'brooding' the parent can obviously obtain no food, and in consequence the internal organs undergo some remarkable changes which cannot be here described. If the eggs of a *Cribrella* be removed no amount of force will serve to straighten out the arms, breakage always taking place, but if the animal be left undisturbed on its aboral surface it will in the course of about half-an-hour or so assume a perfectly normal shape, and shortly after will greedily devour any food presented to it. I have never yet succeeded in coaxing a female to re-commence her maternal duties after the eggs have once been washed out of her brood-cavity. If a *Cribrella* fixes itself to the side of a glass vessel, the interior of the brood-cavity is visible and the progress of development can be watched. After hatching, the uniformly ciliated larvæ can be seen slowly moving about or rotating in the chamber, and occasionally they may attach themselves in the later stages to the glass or to the buccal membrane of the mother by their sucker. When the eggs are set free early in development all the larvæ sooner or later adhere to the glass by their suckers, until the young starfish can creep about by their tube-feet. Whether any of the larvæ leave the brood-chamber of the mother in natural conditions I do not know, but these facts appear to indicate that in the event of a parent either accidentally or intentionally discontinuing her protective functions, each larva would be quite capable of proceeding in its development independently of its parent. Through the courtesy of Dr Appellöff of the Bergen Museum I have been enabled to see a collection of embryos belonging to two species of *Solaster*, the larval forms of which appear to be closely similar to those of *Cribrella*, though I understand that in these instances there is no 'brooding.' *Asterina gibbosa* and *Ophiura brevispina* appear to be other instances of a similar type of development. In all these species there is a large amount of yolk which serves for the nourishment of the larva, in some instances throughout the whole metamorphosis; the larva is either uniformly ciliated or banded, and is not pelagic, but lives at or near the bottom, in many cases actually attaching itself to a foreign body. This type of development we may term the 'demersal,' as it appears to have the same relationship to the 'pelagic' type with its *bipinnaria* or *brachiolaria* as the demersal types of development have to the pelagic. Similar divergent types can be noticed in the *Enteropneusta* (*Tornaria* and Bateson's larva). Other instances of the demersal type have been described in the *Echinodermata*, and it appears to be of sufficiently frequent occurrence to forbid our regarding the *bipinnaria* and other pelagic larvæ as being by any means the rule amongst asterids.

It is not uncommon to find at the present day certain statements regarding these 'brooding' types which would imply that the parent contributes nourishment to the young during the progress of development. This view has undoubtedly been held by several writers in the past, and the pre-oral lobe of *Asterias mülleri* has even been described as a 'placenta' (Wyville Thomson). Neither in *Asterias mülleri* nor in *Cribrella oculata* is there the slightest evidence that the connection between parent and young is anything more intimate than a purely mechanical one, from which a certain amount of protection is obtained. It has already been stated above that in *Cribrella* (and the same applies to *A. mülleri*) the development proceeds perfectly normally after the eggs are removed from the parent, which could hardly be the case if they were thereby deprived of even a part of their nutriment.

Methods.

The eggs and larvæ were preserved immediately on being taken from the mother. One series was fixed in corrosive-acetic and passed gradationally through alcohols:

another was placed in a mixture of alcohol, acetic acid, and formalin, in the proportions of

70 per cent. alcohol, . . .	90 parts.
Glacial acetic, ...	3 ,,
Formalin, ...	7 ,,

This mixture, recommended to me originally by my friend Mr E. J. Bles of Cambridge, I have found invaluable for a variety of histological purposes. For marine collecting it is very handy, and specimens may be kept in it till required for sectioning. (Another series was preserved with osmic acid, but these were not successful.) Both series were excellently preserved, and gave equally good results. Sections were cut in paraffin and celloidin, but as the paraffin alone appeared to be perfectly satisfactory, I finally abandoned the use of celloidin as being less expeditious. In all pelagic echinoderms celloidin appears to be essential, but these larvæ certainly did not present the least difficulty in manipulation.

A large amount of labour and time were expended in searching for transition stages, as it was felt that unless an absolutely complete series were forthcoming, the work could hardly mark an advance upon previous efforts.

I hope the sections actually figured will be sufficient to show the completely serial nature of the work, though all the stages have been worked through in all three planes of orientation, as also the transition stages. Naturally, the somewhat revolutionary result in regard to the fate of the dextral cœlomic elements has led me to hesitate in publishing my conclusions, especially as badly orientated specimens deceived me at first into supposing that *Cribrella* agreed with the usually accepted account of other asterids. Whether the conclusions drawn from the development of *Cribrella* are accepted by authorities on the group or not, the facts as described here may, I hope, be relied upon as accurate; and I have, by using general names in the descriptive part, tried to keep separate the two parts of the paper, *i.e.*, the facts and the conclusions which are drawn therefrom.

The development from the fertilised egg onwards may be conveniently divided into three parts :—

(1) *The embryonic period up to the time of hatching*, comprising the phenomena of segmentation, gastrulation and formation of the ciliated demersal larva.

(2) *The period of larval life;* the earlier part free, and the later more or less attached by the sucker.

(3) *The post-larval period*, during which the larval body becomes of secondary import to that of the young starfish. This might be termed the period of metamorphosis, except that there is no real metamorphosis or reconstruction. It is one of the most important features of an echinoderm development such as this, that there is no real histolytic or metamorphic stage, and that larval organs are merely moulded into those of the adult, if we except the reduction of the pre-oral lobe.

Upon the actual duration of development it is difficult to speak with certainty

Like SARS, I have found that in confinement about eight to ten days may easily be occupied by the embryonic period till hatching, whilst the larval period may perhaps be put down as about six to ten days. Of the duration of the post-larval period I cannot speak with any certainty, but the whole development is undoubtedly prolonged.

PART I. EMBRYONIC PERIOD.

If the forty or more eggs belonging to one parent be emptied out of her brood-cavity, the first feature to be noticed is their great variation in size. They all appear as bright orange spheres, and the majority have a diameter of about 1 mm., but some may be as large as 1·2 mm., whilst others are down to ·8 mm. A similar variation is found in the latest young stages and at all intermediate stages, so it is legitimate to suppose that the abnormality in respect to size is not necessarily prohibitive of normal development.

Again, the eggs, as a rule, exhibit great diversity in their stage of development, a fact observed previously by SARS. Some may be not yet segmented, whilst others may have nearly completed their embryonic stages, and all intermediate conditions between these two may be found. I have only very rarely found an embryonic egg amongst the parents in which the majority are free larvæ, and these clutches, in which the young are more advanced (*i.e.*, in second and third periods), usually show far greater uniformity in structure. This is probably to be explained by the fact that the early embryonic stages occupy a much shorter time than the late embryonic and early larval, so that early inequalities are reduced to a minimum as development proceeds.

Each egg is enclosed in a very delicate egg-membrane, in which it remains till the end of this period.

Segmentation.—Taking into account the large quantity of yolk, one might perhaps expect that there would be a modification of the total equal segmentation which is usually regarded as typical of the *Echinodermata*, but one could hardly have anticipated the actual facts of the case.

These are so extraordinary that although the main features were known to me four years ago, I have hesitated to publish them until I had gone over the fresh material for several seasons, and even now the results, from this very peculiarity, leave a feeling of dissatisfaction. To sum up, we may say that there is no definite type of segmentation whatever. The eggs present the widest possible varieties, which only meet on common ground at the end of segmentation when the egg is in all cases converted into a solid morula, with approximately equal cells throughout. In one type which is by no means uncommon, the first plane of segmentation is horizontal (fig. 7), and divides the whole egg into two unequal segments. This is followed by two perpendicular planes producing an eight-celled stage with a small blastocœle cavity (fig. 8). This type reminds one forcibly of the egg of the frog, and is of the holoblastic unequal type. After this stage, however, there are no more planes formed with definite relations to the axes of the egg, but each octant commences to divide in the most irregular fashion. For some time one can recognise traces of the former inequality in the cells of the lower pole (fig. 9), but soon the lower cells become divided up in the same way as the upper into cells of the same size and the morula is produced. This type, with small variations, must be regarded as the normal, if such a term can be applied at all. It is that described and figured externally by Sars in his Norwegian species. In a second common type the segmentation commences at one pole with the formation of a cap of small cells. Radiating outwards and downwards from this pole small cells are, as it were, cut out of the yolk by the formation first of nuclei by mitotic division, and then of cell-walls enclosing them (fig. 11). As a rule, these are formed most rapidly at the surface of the egg, the cells gradually coming to envelop the central mass of yolk; but as the formation of cells always proceeds into the deeper parts before the whole yolk is enveloped, and as the contained yolk does not differ in structure from the outer layer of cells, but is destined very soon to be similarly segmented, we cannot compare this type to the true superficial. This method of formation is often found succeeding the eight-celled stage of the first type. Of other types we may note the uncommon one in which the egg divides into two equal blastomeres, and the formation of small cells commences at the two poles and spreads round therefrom.

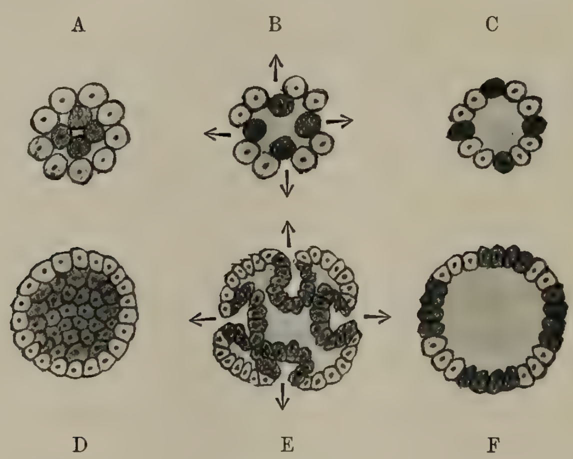
But in whatever manner the segmentation proceeds, the ultimate result is always a solid morula (fig. 3) of very characteristic appearance. The cells are all of the same size throughout and proceed to the very centre. Fig. 14 is a median section through this stage. It will be seen that each cell has a nucleus at its centre, and the minute spherules of yolk are arranged in a radiating manner, separated by fine strands of cytoplasm (fig. 15). The cell-walls are clearly distinguishable, but the cytoplasm tends to gather at the periphery of the cell, thus forming a definite meshwork throughout the egg. On the average there are about twelve cells from the edge of the egg to the centre.

A fairly regular type of segmentation, with the production of a monoblastic blastula, has been described in so many echinoderms of all five classes (the examples are so numerous that it is needless to particularise), that *Cribrella* must in this particular be regarded as a remarkable exception. Even so close an ally as *Asterina* has a perfectly definite blastula stage with no morula (Ludwig, MacBride). At the same time it will be evident later that *Ophiura brevispina* may possibly have a morula stage similar to the above, and possibly also *Solaster*. Future work can alone determine these points.

Gastrulation.—The first question which would naturally occur to any observer of this morula is—How is gastrulation to be effected? The answer to this query has been by no means easy to obtain, and has involved an amount of labour which is not commensurate with the result. At the same time it has its points of interest. Externally we notice the following changes. The morula gradually loses its reticulate surface, the slight grooves separating the cells gradually becoming obliterated, until the embryo appears merely as a sphere with a slightly roughened surface. In this condition it remains for a considerable time, its general appearance differing little from that of the unsegmented egg. Slowly, however, there appear on the surface certain irregular but shallow grooves (fig. 4) which gradually extend from one part of the egg to another. The process is tedious, and the appearance of the grooved embryos has so strong a likeness to an unhealthy histolytic change that I invariably threw away my developing embryos at this stage, feeling convinced that abnormal changes had ensued. One set which were, however, kept over presented later a number of complete and perfectly definite gastrulæ (fig. 6), with large and gaping blastopore. Investigation of sections, compared with similar stages derived from embryos preserved immediately after catching, has convinced me that we are dealing with perfectly normal and definite stages. The furrowed embryo (fig. 5) is an extremely common stage to find in the brood-cavity of the parent. I have been enabled to recognise an exactly similar stage in two species of *Solaster.* Fig. 16 shows the cells commencing to arrange themselves in definite relation to the furrows, which are seen in section as grooves. The cell-walls are becoming more distinct, and the peripheral cytoplasm is uniting in definite directions into cytoplasmic tracts which pass inwards from the furrows and divide up the cells into intermediate groups. From this stage onwards I have numerous transition stages to the condition shown in fig. 17, in which the furrowed condition is at its maximum. The changes consist of a definite loss of cell-walls and reduction of the whole embryo to a syncytial mass of vacuolated cytoplasm and numerous nuclei. The peripheral cytoplasm of the cells has formed even more distinct tracts leading outwards to the furrows, giving the appearance of a main branch, the tract carrying numerous delicate lateral branches of cytoplasm passing out between the nuclei, which still have yolk-granules arranged in definite radial relation to them. These cytoplasmic tracts are commonly, as in the present instance, arranged somewhat symmetrically, but the external view of the embryos shows that there is considerable variation in this respect. In the centre of each cell-aggregate may be seen a cavity in course of formation ; it is produced by the cells drawing away from each other in all directions, and is usually intersected by thin plasmic strands. In a similar way there is a central cavity produced by the separation of the cell-aggregates from each other. This cavity increases in size, and eventually becomes continuous with the cavities inside each cell-aggregate. Like the latter it is intersected by a faintly staining plasma. Along the tracts there are numerous nuclei, which can be seen to be more or less ranged in a row on each side of the tract.

These are the structural facts ascertained by serial sections. Their interpretation is open to question, and the conclusion to which I have been led may possibly be erroneous. There is little question that rapid and important re-arrangement of the cells (as represented in later stages by the nuclei) is taking place, and that in the last stage described the nuclei in the tracts are migrating either outwards or inwards.

The production of the furrows and the appearance of the tracts can only be due to a movement, either inwards or outwards. At first I was inclined to believe that it is a case of formation of the diploblastic stage by multiple ingression, but my opinion now is that it is an interesting process which we may term egression, and that the cells (or nuclei) are really migrating outwards to the periphery. The reasons for this conclusion are these :—The process is immediately succeeded by complete and total gastrulation in which the outer layer is well-formed, whilst the blastocœle space and the archenteric cavity are devoid of cells and nuclei (fig. 18): thus, if it is a process of multiple ingression, why is it followed by true gastrulation? Again, it is noticeable that as the migration proceeds, the outer layer becomes more and more defined, the number of nuclei increases, and the cell-walls become better developed,



1.—Diagram of the Process of Egression.—(A) Section through a simple morula, inner cells shaded. (B) Section through the egression stage. (C) Section through the blastula, with the former inner cells shaded. (D) Section through a more complex morula, *e.g.*, *Cribrella*. (E) The egression stage. (F) The fully formed blastula; former inner cells are shaded.

whereas if it were a case of ingression the reverse should hold. Similarly, the progress of the migration reduces the number of the internal nuclei.

For these reasons I would hold that a peculiar process of cellular egression is instituted with the evident purpose of producing a blastula from the morula.

In the simple morula of a few cells, such as occurs commonly in development, the single internal cells must be assumed, during the course of segmentation, to wander outwards and take up their place between the outer cells, producing a typical blastula. In such a complex morula as that of *Cribrella* the large quantity of internal cells necessitate the cellular egression being conducted in a more complex manner, involving the formation of cell-aggregates, which roll outwards and thus bring their constituent cells to the surface. The accompanying figure will illustrate the process.

We may assume that a morula stage is a cœnogenetic derivative of the blastula produced by conditions of economy of space, in obedience to the same laws of growth as produce the invaginated heads in cystic stages of *Tœnia*. In each case an organ which belongs normally to the outer surface is first produced in the interior, and is afterwards rapidly translated to its normal position by egression.

Immediately following upon the process of egression is that of invagination. The final result of complete egression is the production of a monoblastic embryo corresponding to a blastula, but this stage, if attained at all, is very rare. All gastrulæ which I have examined show, as in fig. 6, the egression-grooves radiating outwards over the surface of the embryo. The blastopore is at first very large and wide, but soon narrows considerably to the condition found in fig. 6. From this stage onwards its edges gradually approximate, until it is reduced to a small circular hole at the hind-end of the embryo. In many cases it persists in this condition through stage A of the larva, but usually it closes at the hatching period.

A section through the gastrula (fig. 18) shows the epiblast in places still exhibiting evidence of the process of egression. Between it and the hypoblast is a definite archicœle cavity, which is practically free of cells. The hypoblast itself is many-layered. Cell-production is evidently proceeding rapidly, in earlier stages mainly on the inner surface of the hypoblast, but later upon the outer. The cells produced from the former fall free into the archicœle cavity, and give rise to a protoplasmic network similar to the mesenchyme of other larvæ (*Antedon*), whilst those produced upon the latter fill up the archenteric cavity with a network of protoplasm which is histologically identical with the true mesenchyme. There does not appear to be any special centre for the production of mesenchyme, nor anything in the nature of pole-cells. For the mass of protoplasm and nuclei, corresponding in origin and in histological structure with the mesenchyme, but filling the archenteric cavity, the name of 'hypenchyme' is suggested.

We may now examine in greater detail the last embryonic stage, as shown in fig. 19. The embryo is spherical, or nearly so, as in many cases the anterior end can be recognised by a very slight attenuation. At the posterior end is seen a small aperture, the blastopore. A median section is seen in fig. 26. The epiblast is now completely consolidated, and there is no trace of egression; each epiblast cell (fig. 27) is of great length, and scarcely wider than its own nucleus. The nuclei are aggregated in a zone about half-way along the cell, whilst scattered throughout the cytoplasm are numerous yolk-globules. These globules are small and few in number outside the nucleus, but are large and numerous in the inner parts of the cells. This arrangement results in the epiblast having a darker appearance in its peripheral part when examined under the low power. The cell-walls are perfectly definite, though very delicate; and the same remark applies to the basal wall dividing the epiblast from the mesenchyme. Externally the epiblast is covered by a thick cuticle of a pale yellow refractive appearance. At the blastopore the epiblast is merged into the hypoblast, and the cuticle is disintegrated into small fragments, which lie loose in the opening.

The hypoblast is now in the shape of a club with a large head and a constricted stalk. The structure is the same throughout, and is shown in fig. 28. Each cell is compressed, but is about half as long and twice as broad as an epiblast cell. The nuclei are aggregated towards the inner end centripetally, and the cytoplasm lying inside them is singularly free from yolk-globules. On the other hand, the part of the cell lying outside the nucleus is filled with very large yolk-granules, most of them exceeding in size those found elsewhere. The contrast between the outer and inner portions of these cells is in consequence very striking, and in all later stages the mesenteron and to a lesser extent the cœlom have their inner limits clearly indicated by their darker appearance. The cell-walls are like those of the epiblast, but naturally there is a basal wall at each end separating the hypoblast from the mesenchyme distally and from the hypenchyme proximally. Each of these consists of a meshwork of cytoplasm, through which are scattered numerous nuclei, which are usually slightly smaller than those of the epiblast or hypoblast. Yolk-globules are numerous and arranged indiscriminately. As a rule, they are smaller than those of the hypoblast, and average about the same as those of the epiblast. The mesenchyme completely fills the archicœle cavity, and the hypenchyme fills the archenteron to the very lips of the blastopore. The hypoblast walls are constricted below the 'head' and lie in contact, but this condition is not universal. The 'head' portion is probably the anterior cœlom of the next stage, and the constricted part is the mesenteron. Such is the last embryonic stage of *Cribrella*, which, although solid itself, has been produced from a solid morula by a normal process of invagination.

Part II. Larval Period.

The larval period lasts from the hatching epoch to the time when the larva fixes itself permanently, in the examples which are freed from the brood-cavity. The exact duration of this larval life, as already remarked, is hard to determine, but it is probably on the average about seven to nine days.

I have, for comparison with other species such as *Asterina*, and in order to facilitate description, divided this larval period into four stages, clearly recognised by the external appearance.

External Structure. (*Stages A–D.*)

It may be well to glance at these four stages externally before proceeding to the internal structure of each.

Stage A—(fig. 20).—The newly hatched larva is nearly spherical, though the main antero-posterior axis is distinctly elongated and the anterior end is somewhat pointed. At the blunter posterior end there can still be discerned a small epiblastic pit marking the final closure of the blastopore, though the pit is blind at its inner end. This blastoporic pit is slightly inclined towards the ventral surface. The larva is uniformly ciliated, and moves gently forwards through the water when set free from the brood-cavity. It invariably moves with the anterior or tapering end forwards. The larva, like the egg, is of a uniform warm orange tint, and has this appearance throughout its larval life.

Stage B—(figs. 21 and 22).—This stage is reached by a somewhat rapid elongation of the anterior part of the larva, causing a marked difference in its general appearance. There is now no difficulty in recognising its points of symmetry; and as it also becomes somewhat flattened from side to side (fig. 22), it always rests on one side or the other after death. In life it has similar habits to stage A. In some specimens there is still a trace of the blastoporic pit, but in most there is none.

Stage C—(figs. 23, 24 and 25).—Little increase in size has been effected by this stage, but the anterior end has now become boldly indicated by the presence of a median dorsal process at the anterior end of the larva, quickly followed by the protrusion of a median ventral process at its base. These processes are evidently parts of the pre-oral lobe, and together constitute a homologue of the so-called 'larval organ' of the *Asterina* larva. This term appears to be objectionable, as tending to emphasise an unnatural distinction between the 'larval' anterior part and the 'adult' posterior. Nothing is more clearly indicated by recent echinoderm research than the fact that the whole organism is a larva at its early stages, becoming almost in its entirety the adult organism, a portion of the pre-oral lobe alone being lost. The conception of an adult growing upon a fostering larval portion and eventually metamorphosing is not in accordance with the facts, at least of asterid development. A clear recognition of these facts will focus the questions of orientation to which reference will be made later.

Stage D—(figs. 46, 47, 48, 49).—In this stage the ventral pre-oral process divides into two. These two form with the dorsal process a tripod, upon which the larva rests after fixation (fig. 48). These three processes, a further development of the 'larval organ' in *Asterina* (Ludwig), are apparently homologous with the three brachiolar arms of the *Brachiolaria*. Ludwig found that some specimens of *Asterina* had the ventral part of the larval organ partially forked, producing an even closer resemblance to the three processes in *Cribrella*. MacBride does not agree with Ludwig's comparison, and regards the adhesive disc as being the homologue of the larval organ.

The outline of the larva is now more defined, with a slight constriction or 'neck' behind the pre-oral processes. The cilia on these processes appear longer and more numerous than those on the posterior part of the body. At this stage the larva is still perfectly plano-symmetric. It swims actively, head first, and with its ventral surface downwards. Larvæ at this stage may be readily kept in a small vessel for several days; they travel about very slowly and methodically, just clear of the bottom, and very commonly fix themselves temporarily to the side of the vessel, occasionally to the bottom; in several cases they also lie on their right side for a short time preparatory to renewing their movements. It is a little difficult to reconcile these active and fixative habits with their natural surroundings. It is quite common to find mothers with a brood of young starfishes as advanced as stage F, yet the brood-cavity is too confined for the execution of any movements at all comparable to those performed by the emancipated larvæ. One can hardly suppose that the mother, in natural surroundings, allows her offspring to make excursions out of the brood-chamber like chickens out of a hen-coop, and there is no indication of this habit in mothers kept under observation. On the other hand, it is quite possible that a certain proportion of the mothers permanently set free their offspring during the larval stages, in which cases there is every reason to suppose that normal development would proceed, though a higher mortality may be incurred. In any case *Cribrella* is evidently, from an evolutional point of view, just acquiring the habit of brooding, and at present the later larvæ certainly appear more suitably equipped for free life than for quiescent existence within the brood-chamber. Probably the ciliary action is still of great functional importance in the aeration of the brood-chamber.

After stage D, the free life (in aquaria) comes to an end. Fixation is effected, as in fig. 48, by the pre-oral processes, and indications of the pentamerous axial symmetry commence to appear. It must be said that a large number do not fix themselves, but lie on their right side; these do not appear to be detrimentally affected, but pass on to later stages without any obvious abnormality.

Before leaving stage D, it may be observed that many of the larvæ show indications of a division of the dorsal pre-oral process into two (fig. 47). This is interesting when we notice that the larva of *Asterias mülleri* at a corresponding stage has the dorsal process completely divided into two, so that the anterior end of the larva bears four equal tentacle-like processes radiating to the four points of the compass. *Asterina* (with two processes), *Cribrella* (with three), and *Asterias mülleri* (with four) thus present a continuous series in the evolution of the pre-oral lobe. At no stage in the larval period is there a mouth or anus, and the first opening to appear is the water-pore, which is recognisable early in stage E (sometimes in D) as a small aperture in the centre of the right side. We may correlate the absence of any ciliated bands with the absence of mouth and with the presence of yolk. At no period does the young *Cribrella* depend upon the outside world for food till it is adolescent, hence the nutritive bands of the *Bipinnaria* are not required, and the powerful motor bands of *Antedon* larva are equally useless. The yolk is, as has been seen, in the last embryonic stage, evidently scattered throughout the cells in the form of granules of varying size, enabling development to proceed independently of the capture or acquirement of food. But although this is the case, there is to be discerned no essential difference in the course of development which would justify the application of a term like 'direct' development, the term 'indirect' being similarly applied to the process in *Bipinnaria*. I have carefully studied both processes, and confess that the distinction thus emphasised does not appear to be valid. The presence of lecithal nutrition, like that of hæmal nutrition in viviparous forms, no doubt accounts for certain marked adaptations not unlike those characteristic of parasites, such as loss of alimentary, sensory and motor organs, but there can be little question that the larval 'entity' is evident both in time and space in these demersal types equally with the pelagic. In the former the pre-oral lobe becomes specialised for swimming (cf. *Bipinnaria asterigera*), and in the latter for fixation, but there is little to choose in the matter of 'directness' of development between the two types. The term 'larva' has been applied freely to these free stages, but it may be noted that its use will to many appear to be unjustifiable. If the difference between an 'embryo' and a 'larva' is to be defined in terms of alimentation, and a larva be defined as a young organism which obtains its own nutriment, then *Cribrella* has no larva at all, and its development is purely embryonic. A similar remark applies to the early tadpole stages of the frog and to the young *Amphioxus*. If, on the other hand, we define the larva by the criterion of locomotion, a larva being a young animal capable of independent and free movement, then the young stages of *Cribrella* here described are true larvæ. Although the former definition has been customary, I think that for many reasons the latter recommends itself as being more easily defined and a more natural distinction.

Internal Structure. (*Stages A–D.*)

Stage A.—A nearly median horizontal section (fig. 29) through stage A reveals a structure closely similar to that of the last embryonic stage, but the hypoblastic sac has become isolated and is assuming definite shape. A constriction towards the centre gives it a dumbbell shape. The anterior portion thus defined may be termed the *anterior cœlom*; its walls are already perceptibly thinner than those of the rest of the hypoblastic sac. Dorsally it grows backwards slightly over the constriction, and on either side it is produced backwards into processes which are precisely similar. The left of these horns will produce the lining epithelium of the water-vascular system, and from its position we may recognise it as the left lateral cœlom. Its fellow is the right lateral cœlom. The middle constriction becomes in later stages the true mesenteron, and the posterior portion we may term the *posterior cœlom*: it also shows slight indications of right and left horns, which later give rise to the right and left posterior cœloms. A more ventral section shows the epiblastic pit, the last trace of the blastopore, and it also cuts the anterior and posterior cœloms separately. Histologically there is little change from the last embryonic stage. The hypoblast cells are more numerous and more clearly defined, and their arrangement more definite. As in the embryo, the archenteric cavity is completely filled with hypenchyme of the same reticulate nature.

Stage B—(figs. 30–35)—shows several important advances upon stage A. As is to be expected, the anterior cœlom is now large and expanded anteriorly, the two lateral cœloms (figs. L. 2, R. 2, 30, 31) are now sharply defined, and dorsally their cavities communicate only by a narrow canal with that of the anterior cœlom. In the median dorsal line the posterior wall of the anterior cœlom commences to push backwards as a small vesicle, not unlike the hydrocœle, but smaller (fig. 31, *c*): it is here called the central cœlom. From this small pouch (which becomes the cardiac vesicle ('*herzblase*,' pericardium) of the adult), there passes a groove downwards (fig. 32) to the mesenteron, which is still merely a canal connecting the two cœloms. At the level of the mesenteron, the lateral cœloms are still widely open to the anterior cœlom. In fig. 34 is still seen a trace of the blastopore as an epiblastic thickening, occasionally with a short lumen. It is important to notice that the '*anlage*' of the central cœlom is median, for although it is in fig. 31 very slightly to the right, the fact that the epiblastic pit (fig. 34) also appears slightly to the right, which is not actually the case in a properly orientated larva, justifies us in this opinion. Fig. 35 is a median coronal section of a late B stage; in it the mesenteron is seen constricting off posteriorly, and the elongated anterior cœlom is evident.

Stage C is an important epoch in the larval history. In it there appear the first indications of asymmetry. Figs. 36 and 37 are dorsal and ventral views respectively of the entire larva viewed as a transparent object after being cleared in oil of cloves. Such views are difficult to obtain owing to the thickness of the epiblast, and some amount of optical distortion is inevitable, but if they be carefully compared with a series of transverse sections (figs. 38–45) a clear idea is easily obtained. The anterior cœlom has increased greatly forwards and backwards. At its anterior end it is assuming a triangular shape preparatory to trifurcating into the pre-oral processes not yet developed (fig. 24). Posteriorly its two lateral cœloms (L. 2, R. 2) have grown backwards very rapidly, the right one to the level of the posterior border of the mesenteron or thereabouts (figs. 36 and 53), and the left one still further (figs. 36 and 45); this more extensive growth of the left lateral cœlom has dragged the opening of the mesenteron into the anterior cœlom downwards to the left (fig. 40), a character shown in every specimen examined. Not only has the left lateral cœlom extended further backwards, but it is also clearly seen in the sections to be of larger size. The posterior wall of the anterior cœlom still extends slightly dorsally over the mesenteron, but the central cœlom has now commenced to travel to the right, its distal end lying over the right lateral cœlom (fig. 42). But the most marked asymmetry is produced by the posterior cœlom. Its left portion has grown forwards dorsally and ventrally in a pair of long horns which lie almost exactly in the median line. The dorsal horn (D.L. 3) is the shorter and runs forward over the mesenteron to about the level of the central cœlom, nearly up to the posterior wall of the anterior cœlom. It appears to actually squeeze the central cœlom to the right (fig. 36). The ventral horn (V.L. 3) passes forwards below the mesenteron (figs. 37 and 40) till it touches the ventral wall of the anterior cœlom. Still it pushes forwards between the epiblast and the latter, till it extends over the middle line of the larva. The rapid growth of this left posterior cœlom disguises the right posterior cœlom altogether, which may, however, still be recognised (fig. 36) as a small process (R. 3). Thus the asymmetry of the posterior cœlom is even more marked than that of the anterior.

Of histological differentiation there is little to note. The mesenteron is becoming sharply defined by its thicker walls, whilst the cœlomic walls do not materially differ from each other. The hypenchyme varies greatly; in some cases it is still quite intact, but in many it is seen to be rapidly breaking up into isolated fragments. The specimen figured was almost freed from its presence, but this is somewhat exceptional.

Before leaving this stage we should emphasise the fact that internal asymmetry is clearly commencing, evinced chiefly in the predominant growth of the left lateral cœlom, and the forward extension of the left posterior cœlom in marked contrast to the right. The total result is that the organs belonging to the left side fill up more than half the larva and throw the median central cœlom to the right.

Stage D—(figs. 46–76).—Stage D is the culminating point of the larval entity, as already described. Externally it is perfectly symmetrical and its orientation is easy. The anterior end is indicated by the trilobed pre-oral lobe, the dorsal surface by the single largest of the processes, and the lateral surfaces by the flattening from side to side which soon becomes evident (figs. 71–76). It is the internal structure of this stage which must be relied upon to give indications of the structure of the hypothetical bilateral ancestor of the *Echinodermata.*

A general idea of the internal structure may be derived from an inspection of 'cleared' specimens. Fig. 50 shows a dorsal view of one, and the large and spacious pre-oral cœlom, throwing processes into the three parts of the pre-oral lobe, can be recognised. The anterior cœlom is now constricted into its four elements of pre-oral cœlom, central cœlom, and two lateral cœloms, which all still communicate by small passages. On the right side can be recognised the central cœlom, and dorsal to it is the commencement of the pore-canal (*p.c.*) growing out as a pocket towards the right side of the larva. Late stages of D often show the canal freely opening to the exterior in the centre of the right side. In the centre of the larval body is the dark mass of the mesenteron, covered by the dorsal horn of the left posterior cœlom (D.L. 3). Fig. 51 is a ventral view, and here we note the presence of the right posterior cœlom (R. 3), small but distinct, and lying internally and ventrally to the right lateral cœlom. In the lateral view (fig. 52) the openings of mesenteron into anterior and posterior cœloms respectively can be recognised ; the full extent of the left posterior cœlom and the stone-canal, as a groove along the posterior border of the anterior cœlom, are to be noticed.

Hence at this stage all the cavities of the body are in free communication. The pre-oral cœlom has connected with it the three pre-oral processes, the two lateral cœloms, the central cœlom and the pore-canal, while the posterior cœlom has its left portion with dorsal and ventral horns, and its right portion with its ventral horn only.

It is now not difficult to follow the series of sections. Figs. 58 to 62 illustrate a transverse series taken through a specimen which might be termed C/D or a very early D. It can be compared, section for section, with the series (figs. 38–45) of stage C. In fig. 58 we see that the pre-oral cœlom shows indications not only of the two ventro-lateral pre-oral processes, but a pair of grooves which will later form processes into the divided median process seen in stage E. In fig. 59 the opening of the pre-oral cœlom into the mesenteron is seen towards the left, as in fig. 40. The central cœlom is not much different from stage C, but the left lateral cœlom has increased in size, spreading out dorsally and ventrally, and pushing the left posterior cœlom over toward the right.

Figs. 61 and 62 show clearly the origin of the right posterior cœlom, which can be seen in fig. 61, and even in fig. 50, as a long process lying inside the right lateral cœlom.

The vertical series (figs. 53–57) exhibits a slightly later stage, besides showing by its different orientation some special points. Fig. 53 is on the extreme right of the larva, and shows only the right lateral and right posterior cœloms. The next fig. shows the latter opening into the main part of the posterior cœlom, and also a portion of the left posterior cœlom, which, as we already have seen, invades the right side. Dorsally to the right lateral cœlom is the tip of the pore-canal (*p.c.*). In fig. 55 the mesenteron opens into the posterior cœlom, the left dorsal and ventral horns of which are cut throughout their length. The pore-canal here can be traced into the stone-canal (*s.c.*) of fig. 56. In this figure the mesenteron opens into the anterior cœlom, and from this and the next figure the wide extent of the left lateral cœlom may be gathered.

This completes the structure of stage D, the two other series mentioned being really at the D/E stage, marking the commencement of changes leading to the adult-structure.

The larva as thus indicated has a curious mixture of characters. It is specially to be emphasised that, with all its distortions and cœnogenetic features, it does not show the slightest trace of axial symmetry. The only deviation from the true bilateral symmetry is in the nature of a left-handed or sinistral asymmetry ; thus the left lateral cœlom (which obviously will become the hydrocœle) is much larger than its fellow, though in stage B they were identical in size and appearance. Again, the left posterior cœlom is larger than its fellow, and extends dorsally as well as ventrally. Lastly, we have seen that the central cœlom has travelled to the right side, and (inferentially) the pore-canal has shifted its aperture from the left to the right.

General names have been used for these various parts of the cœlom because there are at least two possible interpretations of their true homologies, and the use of these merely descriptive names enables the account of their changes to be kept apart from the interpretation. However, to facilitate further description, it may be stated that the left lateral cœlom becomes the 'hydrocœle,' and hence the water-vascular system; the right lateral cœlom forms the epigastric cœlom (Goto); the right and left posterior cœloms become the 'hypogastric cœlom' (Goto); the central cœlom forms the cardiac vesicle of the adult, whilst the pre-oral cœlom remains, in part, as the axial sinus.

Part III. Post-Larval Period.

The post-larval period consists of two intermediate stages (E and F), in which the larval entity gradually gives place to that of the young starfish. The external character of one stage of the young *Cribrella*, *i.e.*, stage G, is here added, but the fourth or adolescent period must, as regards its complete description, be left to a subsequent memoir, material for the completion of this part being at present wanting.

Stage E is shown in figs. 77 and 78, a view from the right and left side respectively. The body is seen to be expanded in a vertical plane and correspondingly flattened laterally. The expansion is most evident dorso-ventrally, converting the body of the larva into a disc. The pre-oral lobe is not so prominent, and the median process is more widely separated from the ventro-lateral processes. An anterior view (fig. 79) shows the formation of a sucker between the three processes, on the ventral surface of the pre-oral lobe. This sucker is used for attaching the larva as described, a mode of fixation of a different nature from the temporary habit indulged in by the larva of stage D. In the latter case the larva appears to adhere by the tips of the three pre-oral processes.

This figure indicates the commencing torsion of the body or disc upon the stalk or pre-oral lobe. This torsion will be alluded to later.

In fig. 78 the first indication of axial symmetry is seen in the five-lobed condition of the hydrocœle.

Stage F—(figs. 105–106)—shows considerable progress. The destruction of the larva is indicated by the degenerating pre-oral lobe, though the sucker is further elaborated and the median process is very commonly split into two. Progress in the young starfish is exhibited by the rapidly developing groups of tube-feet, whilst dorsally the adult body is beginning to show a five-lobed condition. The five radii of the hydrocœle are here numbered in the order of their usual development, namely, from the 'mouth-region' counter-clock-wise. Thence the antero-ventral is 1, the postero-ventral is 2, the posterior is 3, the postero-dorsal is 4, and the antero-dorsal is 5.

We may now notice the internal changes from D to F inclusive. The transverse larval series (figs. 71–76), is through a very important stage, D/E, illustrating the commencement of progress to the adult organisation. Compared with the similar series of stage D, there does not at first sight appear to be much difference, but we can see (in fig. 71) that anteriorly the termination of the ventral horn of the left posterior cœlom is expanding vertically and horizontally into two prominent processes; the horizontal process passes under the pre-oral cœlom over to the right-hand side, whilst the vertical process hangs downwards towards the ventral surface of the larva. On the left of the figure the left lateral cœlom has thrown out two processes anteriorly, which are the first and fifth radii. An inspection of the remaining figures shows the second, third and fourth, all well marked. The left lateral cœlom is therefore now axially symmetric, forming a star with five blunt processes. We may now call it the hydrocœle. The water-pore now opens to the exterior in the middle of the right side, between figs. 73 and 74. Lastly, the two posterior sections, *i.e.* figs. 75 and 76, show the right and left posterior cœloms fusing with one another. This fusion first takes place at the distal end of the right element, and gradually the double wall becomes thinned out backwards till the two are merged into one cavity. In fig. 76 the remaining septum is still intact posteriorly, though in fig. 75 it has broken down further forward. This fusion of right and left posterior cœlom is extremely important from the point of view of homology, and a large amount of labour and time have been devoted to corroborating the process. It has been confirmed in vertical and horizontal series, and, however disappointing a result from the point of view of generalisation, it must be accepted as the true course of events.

The horizontal series of figs. 63 to 70 illustrates a slightly later stage of D/E. In it may be noted the five elements of the hydrocœle—the pore-canal, the right lateral cœlom, the central cœlom with constricted aperture, and the completely fused right and left posterior cœloms. The series is chiefly useful for comparison with the next series through stage E. The stone-canal evidently (fig. 63) commences opposite the inter-radius between 4 and 5. The right lateral cœlom is closely comparable to the left, and does not extend quite so far as the latter. The mesenteron (fig. 66) is still perfectly oval in shape, with an opening at each end into the cœlom.

We are now in a position to follow the structures in stage E, as shown in the horizontal series (larval orientation) of figs. 79 to 86. In fig. 63 the dorsal horn of the left posterior cœlom is closely pressed against the posterior wall of the pre-oral cœlom.

In fig. 79 the contiguous walls have broken down, and the two cavities remain confluent for some time. In fig. 80 the two dorsal radii (4 and 5) of the hydrocœle have become sharply defined, and the pore-canal now opens freely to the exterior on the right side of the larva. Slight traces of the degenerated hypenchyme fill its lumen. Slightly below and anterior to this (fig. 81) is the central cœlom, still communicating by a small fissure with the anterior cœlom. Posteriorly to the pore-canal, on the right side, can be seen the right posterior cœlom, which in fig. 82 communicates still by a wide aperture with the anterior cœlom below the pore canal.

In (fig. 82) the ventral horn of the left posterior cœlom comes into view on the right side anteriorly, and the mesenteron is seen to still open into the posterior cœlom by a small aperture. Between this section and that in fig. 83 the anterior end of the mesenteron should, judging by D/E, open into the anterior cœlom, but such is not the case. The anterior end constricts till the opening is completely closed, and all connection between the cavities of anterior cœlom and of mesenteron is lost. At the same time the mesenteron, which in D/E was somewhat sausage-shaped (fig. 66), bends into a V dorsally, bringing the anterior and posterior ends close together dorsally (both are cut across in fig. 83). The right side of the mesenteron then becomes flattened (figs. 84 and 85), giving the mesenteron a semicircular outline in section. Covering the whole of its right surface is the right lateral cœlom, which thus also appears semi-circular in section, though hemispherical in shape. In fig. 83 we again recognise the anterior process of the left posterior cœlom, with its process extending from right to left immediately ventral to the pre-oral cœlom. In fig. 85 this process reaches almost to the left side of the larva. Figs. 86 and 87 show the progress in axial symmetry. Not only have 1 and 2 radii of the hydrocœle advanced considerably, but the left posterior cœlom has pushed out two radial extensions lying over their respective hydrocœlic elements.

Altogether stage E gives us some great advances upon D. The formation of the adult entity is marked chiefly—(1) by the general squeezing together and consolidation of the mass of organs, so that their future relations are fixed; (2) by the rapidly developing pentamerous axial symmetry of the hydrocœle from one to five, followed by that of the left posterior cœlom; (3) by the curving of the larval mesenteron in a vertical (larval) plane, and the complementary flattening of the contiguous surfaces of mesenteron and right posterior cœlom. These internal changes are sufficiently great to throw the balance over towards the adult side, and from this onwards it is clear that we are dealing with a starfish, and not with a bilateral animal. Hence it is only right that the nomenclature should be changed as far as may be. The term 'pre-oral cœlom' may still be retained, but the following changes are already sufficiently clear:

Stage D (*Bilateral*).		*Stage E* (*Axial*).
Left lateral cœlom	=	Hydrocœle.
Right „ „	=	Epigastric cœlom (Goto). (Aboral cœlom.)
Left posterior cœlom } fused Right posterior cœlom }	=	Hypogastric cœlom (Goto).
Central cœlom	=	Dorsal sac (Bury).
Right side	=	Aboral surface.
Left side	=	Oral surface.

We can observe that the hydrocœle passes from a circular disc to a rayed disc, and does not form a ring till later by perforation of its centre; further, the radius 3 is exactly posterior, or as nearly so as it is possible to identify. The hypogastric cœlom, however, is from the outset curved: in fig. 52 it is already seen in early D (*cf.* fig. 57) to be curved into a half-circle, with its dorsal and ventral horns. In stage E its ventral horn evidently lies over radii 1 and 2, and is continued forwards and upwards till it is just at the base of 5, though it is shut out from the dorsal surface of this radius by the still persistent pre-oral cœlom. The dorsal horn extends round till it lies at the base of radius 4, beyond which it comes in contact with the pre-oral cœlom. Hence radii 4 and 5 are not yet completely covered by their share of the hypogastric cœlom. The hypogastric cœlom is enabled to extend as far round as this owing to the contracting of the pre-oral cœlom in the 'neck' region (see diagram 2, p. 400).

If we number the radii of the hypogastric cœlom, we see that the most anterior one lies at the base of hydrocœlic radius 5, the next two ventral, II. and III., are over radii 1 and 2 respectively, IV. is over radius 3, and the last, *i.e.* V., is very nearly over radius 4 of the hydrocœle. Thus we already have the curious twisting phenomenon of the oral or hydrocœle element upon the aboral or cœlomic part which has been described for *Asterina* by Ludwig and for several other asterids, though said to be absent in *Bipinnaria asterigera* by Bury. In most cases, however, this twist takes place at a later stage, when its progress can be actually observed externally.

Stage F.—We may now pass to stage F, as shown in figs. 90 and 91. Here a considerable advance has been made internally. Histologically, there is still remarkably little progress. The epiblast still remains the same columnar epithelium, the mesenchyme presents the same reticular structure, and the epithelium of the various organs is the same as in the last stage, with the exception of the greater thickness of the mesenteron. Figs. 92 to 100 give a series orientated as in the former series of stage E, *i.e.*, horizontal longitudinal to larva and through radius 3 of the adult.

In fig. 96 the small aperture from the hypogastric cœlom still leads into the basal part of the pre-oral cœlom dorsally to the stone-canal; the pre-oral cœlom now consists of a small basal part, the axial sinus, connected by a long canal or neck to the more spacious part still persisting in the pre-oral lobe. Radius 5 of the hydrocœle shows the first pair of tube-feet and the radial vessel clearly defined. In fig. 97 the dorsal sac is seen to be quite separated from the anterior cœlom, and to be (also in figs. 98 and 99) alongside of the pore-canal. Close to its anterior (larval) border is the tip of the hypogastric cœlom; in figs. 81 and 82 these two parts are seen to be in close proximity, and here they actually touch. Recalling to mind Goto's statement that the dorsal sac in *Asterina arises* from this process of the hypogastric cœlom (or posterior cœlom), it is well to notice this contiguity here, and the manner in which it is produced. A comparison of figs. 97 and 98 shows that the neck or canal of the pre-oral cœlom is now lying in a bay or groove on the termination of the hypogastric cœlom, which is pushing round both orally and aborally. At the oral end of the axial sinus is the perihæmal cavity 4/5 growing orally as a crescentic process (*cf.* fig. 98). We may note the very close approximation of the two ends of the hypogastric cœlom, which are now only separated by the breadth of the axial sinus in the inter-radius.

Fig. 98 shows the passage between mesenteron and hypogastric cœlom just closing, whilst the epigastric cœlom also comes into view. In fig. 99 two consecutive tube-feet of radius 3 are cut, their fellows being seen in fig. 102; four tube-feet are also present in 2 and 1. The hypogastric cœlom is seen to be giving off the perihæmal cœlom 4/3 growing orally to the hydrocœle ring, as in fig. 100. Aborally to this is the oral cœlom (MacBride) arising as a process in inter-radius 4/3 from the hypogastric cœlom. Another rudiment of the oral cœlom is seen in fig. 101 as a similar process protruding between mesenteron and hydrocœle in inter-radius 5/1, another in fig. 103 growing out into inter-radius 2/1, and in the same figure a fourth growing out into inter-radius 2/3. In other words, the oral cœlom arises at this stage from four inter-radial processes from the hypogastric cœlom, each corresponding in position (aborally thereto) to the respective four perihæmal processes. There is at this stage no trace of the element of the oral cœlom corresponding to inter-radius 5/4, the hypogastric cœlom being absent from this inter-radius. MacBride and Goto find the oral cœlom originating from the hypogastric cœlom, but, so far as I am aware, do not remark upon its origin from inter-radial elements in *Asterina* or *Asterias pallida*. The perihæmal elements 5/4 and 4/3 have been noticed: fig. 101 shows the commencement of 5/1; fig 102 shows 3/2 well advanced; and 2/1 is seen in figs. 103 and 104. Like the hydrocœlic radii, the perihæmal elements show progressive development—1/2, 2/3 being very marked, 3/4 and 4/5 slightly less formed, and 5/1 only just arising. In a similar manner, the oral cœlom element of 1/2 is the best developed (fig. 102), 2/3 is next (fig. 102), 3/4 is at an earlier stage (not evident from appearance in section, but 2/3 runs throughout four or more sections, whilst 3/4 only extends through about 3 or 4), 4/5 is absent at this stage, and 5 or 1 is only just arising (fig. 101).

The hydrocœle is now a ring, formed by an approximation of its oral and aboral walls in the centre, and a breaking down of the same. This process has been described in other asterids, and it does not appear necessary to figure sections illustrating the point. The mesenteron has commenced to push out a process orally towards the surface through the lumen of the hydrocœle ring (fig. 102), but only after the lumen has been formed; it does not at this stage reach the epiblast. The epigastric cœlom has become of more definitive shape, arching over the mesenteron aborally, and forming a definite circular mesentery with the hypogastric cœlom (figs. 100 to 104).

By the close of this stage the young starfish may be said to be definitely formed. The organs that arise *ab initio* in stages E and F, such as the perihæmal elements and the oral cœlom, all appear as axially symmetric organs, and the elements of these two arise in a regular cyclical succession, agreeing with that of the hydrocœlic and hypogastric radii. Up to the end of stage F there is no appearance of the skeleton, the mesenchyme is still undifferentiated, and the epiblast shows no special developments. The histology of the various organs which have been considered is very little altered from that in stage D or earlier. Evidently the mesenteron has increased enormously in thickness of wall, and certain of the cœlomic elements, such as the dorsal sac and the epigastric cœlom, have their walls distinctly thinned; but the hydrocœle has thick walls, especially in the tube-feet, and the cells of the hydrocœle are always more regularly arranged than those of the other parts.

In the Adolescent Period, commencing with stage G, we have to deal with the completion of the perihæmal and oral cœloms, the formation of the ovoid gland, the skeletal and genital systems, and the further development of the alimentary system. It appears more desirable to pause here than to give an incomplete account of such an important part of the development, especially as the account here given is more or less complete in itself.

General Considerations.

Comparisons with other types.—After an extensive search amongst the copious literature of the *Echinodermata*, I have failed to find any authentic instance of a development at all resembling that of *Cribrella.* The total equal segmentation with production of a typical blastula, followed by complete invagination to form a normal gastrula, has been described in so many pelagic types, and also in the demersal types, *Asterina* and *Antedon*, that it must be regarded as the normal mode of development for *Echinodermata.* The development here described must therefore be considered as a remarkable exception. There is every reason to suppose that a similar mode of development holds in the case of *Solaster*, and also in *Ophiura brevispina.* In the latter species, GRAVE* has described certain stages of the larval development. The larva is of a free-swimming, demersal type, in some respects resembling that of *Antedon.* His first note was somewhat unintelligible from lack of data, and his fuller paper* does not clear up the main points. He naively states that all his earlier stages were destroyed, as he did not know that any special interest would be found, and he commences with the early larval stage, corresponding to the last embryonic stage of *Cribrella.* This larva exhibits an archenteron filled with hypenchyme, and its appearance leads the author to conjecture that invagination does not take place, and that "the larva before gastrulation is a solid, planula-like affair, and later the archenteron is formed by a splitting away of the central core. In the same way the plug of cells is probably formed by the hollowing out of the solid archenteron" (pp. 87, 88). In the light of the singular resemblance indicated, it is the more regrettable that GRAVE did not examine the early stages, for the resemblance serves to cast grave doubt on the 'probabilities' suggested.

A strict regularity of segmentation appears to hold principally in the *Holothuroidea* (SELENKA† and CLARK‡) producing an equal segmentation. On the other hand, certain echinids, *e.g.* *Strongylocentrotus lividus* (SELENKA) and *Echinocardium cordatum* (FLEISCHMANN§) appear to have a more unequal segmentation after the division into four blastomeres. At the same time a hollow blastula is produced, although the cells are unequal at the two poles. Slight inequalities have also been observed in *Asteroidea* (LUDWIG||), but these are apparently the nearest approaches to the condition in *Cribrella.* The unequal segmentation, followed by the production of a solid morula, appears, with our present knowledge, to be unique for echinoderms. The same remark applies to the process of multiple egression and invagination, with the formation of hypenchyme. For somewhat similar types of early development we have to go to the *Alcyonaria* and *Zoantharia.* In each of these groups a solid morula is very common, and in numerous instances the later larva is described as having an archenteron, filled with a mass of cytoplasm and nuclei of a hypenchymatous character. It appears to have been usual to conclude that the solid morula was converted by delamination into the solid planula, but JOURDAN¶ has noted that in *Actinia equina* there is complete invagination, followed by a filling up of the archenteron by a mass of yolk granules, the origin of which is doubtful; and KOWALEWSKI** describes in a species of *Astraea* a larva whose archenteron

* GRAVE, CASWELL, *Johns Hopkins Univ. Memoirs*, iv.

† SELENKA, E., *Zeitschr. wiss. Zool.*, Bd. xxvii.

‡ CLARK, H. L., *Johns Hopkins Univ. Memoirs*, iv.

§ FLEISCHMANN, A., *Zeitschr. wiss. Zool.*, Bd. xlvi., 1888.

|| LUDWIG, H., "Entwicklungsgeschichte der *Asterina gibbosa*," *Zeitsch. w. Zool.*, vol. xxxvii., 1882.

¶ JOURDAN, E., *Ann. Sci.*, Nat. Series, 6, T. x., 1879–80.

** KOWALEWSKI, A., *Jahresb. Anat. u. Physiol.*, 1873.

is filled with hypenchyme of a very similar nature to that of the embryonic *Cribrella*. Again, APPELLÖFF,* in a recent work on development of certain *Actiniæ*, finds yolk elements in the archenteric cavity, and there are numerous other instances of the same kind. That the early development of *Cribrella* should approach that of *Alcyonaria* and *Zoantharia* much more nearly than that of its echinoderm allies is of itself sufficiently remarkable, but I think it quite probable that other echinoderms will be found to have a development not unlike that described here.

Comparison with other Asterids.

Leaving out the earlier stages, we are in a position to compare this development, in its larval stages, step by step with that described by other workers. It is equally undesirable and impossible to pass in review the mass of embryological work in relation to the asterids. Without underrating the value of the earlier workers, we may, without much error of commission, take as our basis of comparison the work of LUDWIG † on *Asterina* (which will always remain the classic memoir upon asterid development), of MACBRIDE‡ upon the same species, of GOTO § upon *Asterias pallida* and on *Asterina*,|| and of BURY ¶ upon *Bipinnaria asterigera* and other types. In the case of an asterid with a demersal larva like *Cribrella*, namely, *Asterina*, we have the extremely accurate observations of LUDWIG and the exact application of the modern methods of differential staining and serial sections by MACBRIDE and GOTO; yet it cannot be said that the last word has been said, especially with regard to the fate of the body-cavities. Hence the study of an allied form like *Cribrella* should at least assist to make matters more definite.

We may review the structure of the bilateral larva of *Cribrella*, after which it will be possible to discuss how far its structure is of phyletic significance. If we glance again at stage D we notice an organism slightly elongated antero-posteriorly and slightly compressed bilaterally. At the anterior end the part which we can recognise as the pre-oral lobe, by analogy with other forms having a mouth, is produced into a dorsal and two central processes. Behind is a slight neck, followed by a disc-shaped body. Externally the symmetry is perfectly bilateral. Coming to the interior, we find a central mesenteron, around which are arranged certain cœlomic pouches, all of which are destined later to separate completely from it. We recollect that one section of these have all arisen from a large anterior cœlom, and another section of them from a posterior cœlom; the former have never fused or joined up with the latter, and the two series have remained perfectly distinct throughout the development. It is this feature, unique amongst known asterids, which constitutes the main value of the *Cribrella* larva from

* APPELLÖFF, A., Bergen Museum, Aarbog, 1900.

† LUDWIG, H., "Entwicklungsgeschicte der *Asterina gibbosa*," *Zeitsch. f. w. Zool.*, vol. xxxvii., 1882.

‡ MACBRIDE, E. W., "Variations in the Larva of *Asterina gibbosa*," *Philos. Soc. Camb.*, May 1894.

§ GOTO, S., *Journal of the College of Science*, Imp. Univ. Tokyo, Japan, vol. x. pt. iii., 1898.

|| GOTO, S., *ibid.*, vol. xii. pt. iii., 1898; and *Annotationes Zool. Japonenses*, vol. ii. pars iii., 1898.

¶ BURY, H., "Studies in the Embryology of Echinoderms," *Quart. Journ. Micr. Sci.*, April 1889.

** BURY, H., "The Metamorphosis of Echinoderms," *Quart. Journ. Micr. Sci.*, vol. xxxviii. 95–96.

a morphological standpoint. Of the elements derived from the anterior cœlom we can recognise two lateral cœloms right and left. These, from their very first origin, have been exactly equivalent in size, shape, and relationships. Stages A, B and C all indicate clearly the development of these two lateral cœloms, and the only discernible difference between them is the more rapid growth, and hence the larger size, of the left one. It seems difficult to deny to these two a morphological equivalence, and yet the left becomes the water-vascular system, and the right persists as the epigastric cœlom, *i.e.*, the body-cavity of the starfish which lies over the aboral part of the stomach. The conclusion seems to be that *the water-vascular system and the epigastric cœlom are homologous structures, and have been derived from equivalent portions of the cœlom.* These two portions, from their position in the larva, seem to be a pair of cœlomic segments lying immediately behind the mouth.

The central cœlom is lying on the right side, but we have seen that its origin is median in stage B, and the pore-canal, although its rudiment now lies in the middle line, belongs, as we know from other larvæ, to the left side. The fourth part of the anterior cœlom is that filling the cavity of the pre-oral lobe or the pre-oral cœlom.

The posterior cœlom, like the anterior, is at first perfectly symmetrical, but the asymmetry is even more striking than in the anterior. Quite early the left posterior element, with its dorsal and ventral horns, becomes much more conspicuous than the small right element with only a ventral horn. We have already seen that in the stage D/E these two fuse together to form the hypogastric cœlom or main body-cavity of the starfish. How far are these two comparable to the right and left posterior cœloms or cœlomic sacs of *Asterina* and *Asterias pallida*? The only possible reason for doubt arises from their very different fate. The only way in which to reconcile the conclusions here arrived at with the work of MacBride and Bury on the fate of the body-cavities is to suppose that the posterior cœlom, with its one dorsal and two ventral diverticula, is merely the *left posterior cœlom* arising alone at the posterior end of the mesenteron, whilst the right lateral cœlom of *Cribrella* is the *right posterior* cœlom of *Asterina*, but arising from the anterior cœlom and forming the epigastric cœlom. Such a reasoning appears to me a great distortion of the facts in order to fit a special need. Had it been incontrovertibly proved that in other asterids the left and right posterior cœloms become the hypogastric and epigastric cœloms, then it might be advisable to regard *Cribrella* as one of nature's wilful freaks, somewhat like Kleinenberg's *Lopadorhynchus*, but the point turns upon what we regard as 'posterior cœlom.' In *Cribrella* the posterior element cannot possibly be confused with the anterior, because they arise at opposite ends of the mesenteron, and are apart throughout the development. In other asterids, as far as is now known, there is either, as in *Asterina*, a large anterior cœlom with two arms from which the posterior cœlom is constricted off, or, as in *Asterias*, there is a large paired cœlom, which in each case has a posterior element cut off from it. In each type the posterior elements join together across the middle line posteriorly, thus obscuring the limits of right and left elements. We therefore have in such types a difficulty at the anterior ends of the posterior elements in defining their exact separation from the anterior, and at their posterior ends we have still more difficulty in distinguishing between right and left. These impediments have been recognised recently by GOTO * in both *Asterina* and *Asterias*, and his conclusions on this point are, I think, largely in agreement with mine, though not exactly in his way of stating them. It is only fair to say that I have not examined *Asterina* nor *Asterias pallida*, and that MACBRIDE, who has worked exhaustively upon the former type, casts doubts upon GOTO's results. GOTO ascribes MACBRIDE's conflicting results to the use of osmic acid as a fixing reagent, after which there is always a difficulty in staining. MACBRIDE replies by indicating the uselessness of corrosive-acetic and glycerine, and the necessity of celloidin imbedding for pelagic larvæ, and so on. I confess that all this nicety of technique is somewhat beyond me ; and not having worked on either *Asterina* or a complete series of *Asterias*, I cannot presume to judge between these two workers. I can only indicate that with regard to the origin and fate of the body-cavities and the central cœlom, GOTO's work agrees more closely with the facts as in *Cribrella*. As regards questions of symmetry and orientation, my facts agree with MACBRIDE's account of *Asterina*.

In *Asterias pallida* GOTO finds a pair of 'enterocœles,' right and left, which early fuse anteriorly, producing a stage of a large cœlom with an unpaired pre-oral part and a pair of lateral wings closely similar to that in *Asterina*. The left wing gives rise to the hydrocœle and a posterior portion which he calls the left posterior enterocœle ; the right gives off a dorsal portion, the epigastric cœlom, and a residue which fuses with the left posterior cœlom to form the hypogastric cœlom of the adult. A comparison of GOTO's figures with fig. 53 will show that my right lateral cœlom lies over the right posterior cœlom in the same position as GOTO's epigastric cœlom. Hence the total result of the cœlomic changes in *Asterias pallida* is that the left side produces *a hydrocœle and a residue*, the right produces *an epigastric cœlom and a residue*, and the *two residues fuse to form a hypogastric cœlom.* This result is disguised by GOTO, because he terms the whole of the right wing the right posterior cœlom, whereas from my point of view it contains the lateral element as well as the posterior.

If GOTO's results be looked at in this light, they agree absolutely with mine as to the fate of the body-cavities ; GOTO found the epigastric cœlom arising from '*the dorsal posterior corner*' of the right wing (right posterior cœlom of GOTO), a position so closely agreeing with that in *Cribrella*, clearly brought about by the growth of the right posterior cœlom forwards and ventrally to the epigastric cœlom. Only this difficulty could have prevented GOTO from recognising the homology of the hydrocœle and the epigastric cœlom as left and right elements. In a subsequent examination of *Asterina gibbosa*,† GOTO found the same arrangement and fate of the body-cavities.

* GOTO, S., *Journal of the College of Science*, Imp. Univ. Tokyo, Japan, vol. x. part iii., 1898 ; vol. xii. part iii., 1898.

† GOTO, S., *ibid.*, vol. xii. part iii., 1898. We may note that he very clearly indicates the epigastric cœlom to be formed of one-half of the right posterior enterocœle.

Central cœlom.—We now come to the question of the small sac-like portion of the cœlom which is constricted off from the posterior wall of the anterior cœlom in stage B, here termed the central cœlom. This is evidently the 'dorsal sac' of BURY,* the 'schizocœle vesicle' of FIELD,† and the 'right hydrocœle' of MACBRIDE.‡ It agrees closely in origin and appearance with the last of these. MACBRIDE finds that the right hydrocœle arises in *Asterina* on the dorsal border of the right cœlomic wing, a position taken up by the central cœlom in *Cribrella* soon after its first appearance. Apart from its origin, it is evident that if the epigastric cœlom is to be regarded as the homologue of the hydrocœle, it is impossible for the central cœlom to also occupy this relationship. As regards its origin, if we may suppose that it is a little later and more modified in development in *Asterina*, then MACBRIDE'S facts of development would fall into line with mine. GOTO § finds the dorsal sac arising as a diverticulum from the left posterior cœlom, or rather from the junction of left anterior and posterior cœloms. This is very nearly the position in which I find it, and I think that a median organ such as this could quite easily be dragged over to the left by the more rapid growth of the left cœlom (hydrocœle), just as in stage C of *Cribrella* the anterior opening of the mesenteron is dragged over to the left. MACBRIDE'S evidence for a right hydrocœle obtained from abnormal larvæ is weakened by the fact that it is impossible to tell from what part of the cœlom the abnormal right hydrocœle is developed: it does not appear impossible from some of his figures that the right anterior cœlom gives off the right hydrocœle comparable to the epigastric cœlom.

In any case I think the facts, either according to MACBRIDE or GOTO, are not opposed to regarding the central cœlom as a primarily median organ.

Pre-oral cœlom.—The structure of the remainder of the anterior cœlom, and its conversion into pre-oral cœlom, axial sinus, pore-canal and stone-canal agree so closely with the researches of BURY, MACBRIDE and GOTO that further remark is unnecessary except to further point out that the development of the perihæmal cœlom 4/5 from the axial sinus agrees also with MACBRIDE, confirmed by GOTO.

We may sum up the origin and fate of the cœlomic elements in *Cribrella* as follows:—

Larva (Stage D). *Starfish* (Stage F).

Anterior cœlom < Right lateral cœlom = Epigastric cœlom.
Pre-oral cœlom = Part lost; remainder gives axial sinus, pore-canal, stone-canal and perihæmal 4/5.
Central cœlom = Pericardium.
Left lateral cœlom = Hydrocœle.

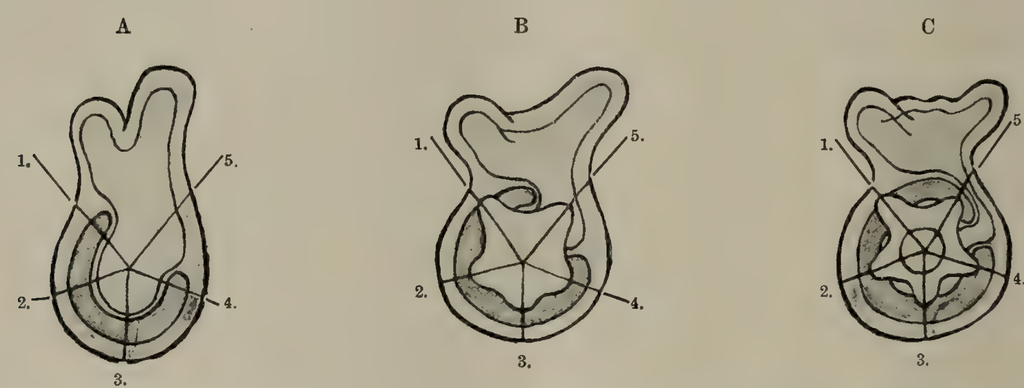
Posterior cœlom < Right posterior cœlom > Hypogastric cœlom.
Left " " Oral cœlom and four perihæmals.

* BURY, H., "Sudies in the Embryology of Echinoderms," *Quart. Journ. Micr. Sci.*, April 1889.
† FIELD, G. W., "The Larva of *Asterias vulgaris*," *Quart. Journ. Micr. Sci.*, vol. xxxiv., 1894.
‡ MACBRIDE, E. W., "The Development of *Asterina gibbosa*," *Quart. Journ. Micr. Sci.*, xxxviii., 1895–6.
§ GOTO, S., *Journal of the College of Science*, Imp. Univ. Tokyo, Japan, vol. x. part iii., 1898.

Relation of Larva to Adult.

To anyone who has carefully followed the description of the various stages to F it will be evident that there is a certain definite relation between the symmetry of the larva and that of the adult. The sagittal plane of the larva, at least up to stage D, *i.e.* as long as the larval entity can be recognised, is as nearly as possible at right angles to the axis of symmetry of the adult, or, in other words, is parallel to the oral and aboral planes or surfaces of the adult. Nothing is more clearly indicated in the processes of growth than the fact that the left and right sides of the larva become respectively the oral and aboral portions of the adult. In some of the transverse sections there is a small angular deviation from this, but no more than can be accounted for partly by distortion and partly by the knowledge that organisms can never be regulated by goniometers. At later stages, from F onwards, or after fixation, it is quite true that the pre-oral lobe of the larva becomes twisted somewhat upon the disc, usually in such a manner as to bring the right lateral process backwards towards the right side (larval) or the aboral side of the adult. At the same time, the disc is also twisted round in the same direction to an even greater angle (fig. 90). The process can really be described as follows:—the two lateral processes remain fixed to the foreign object, but the dorsal process, and with it the whole disc, undergoes a twist through about 15° or so, in a counter-clock-wise direction when viewed from above. I believe this to be purely a mechanical torsion connected with the tendency of the disc to bend over to the oral side, which increases till the whole disc touches the substratum and the attachment is lost. Many of the advanced stages are entirely without this torsion, as in fig. 91, though in this case the sucker can be seen adapting its shape to the 'pull' of the disc over towards the oral side. I have inquired very carefully into this point, as GOTO has made some somewhat startling statements concerning the relationships of larval and adult planes, as discovered by him in *Asterias pallida* and in *Asterina gibbosa*. As will be seen above, *Cribrella* does not lend any confirmation to his view. It is true that all pelagic types of asterid larvæ which I have examined usually have the disc in later stages at the extreme posterior end of the larva, but the mere weight of the growing disc should sufficiently account for this. GOTO takes as a criterion of the larval planes the position of the three brachiolar arms, and whenever these rotate in any direction, he assumes that the larval planes have also been altered in position. This seems to me a fallacious method of determination, for the pre-oral lobe has different functions from the rest of the body. If a 'sandwich-man' parades the street with a board on his dorsal and ventral surfaces, we do not regard these boards as lateral in position because he turns his head to one side, but GOTO's determination of the larval planes by the position of the brachiolar arms appears to be based on much the same kind of deduction. GOTO's own figures of both species clearly show the same fact as those of many previous workers, *i.e.*, that the hydrocœle and oral structures are moulded from the left side of the larva, the aboral from the right, and no amount of twisting of the degenerating pre-oral lobe at a later stage can nullify this fact.

Recollecting that the sagittal larval plane is parallel to the oral and aboral adult planes, the larval left corresponding to the former and the larval right to the latter, we may go on to inquire the further relation of the madreporic plane of the adult. By this is understood a plane passing through the axis of symmetry, through the madreporic inter-radius and through the opposite radius. This is often referred to as the plane of bilateral symmetry. I have numbered the hydrocœle radii in the order of their appearance, the antero-ventral being 1, the most posterior being 3, and the antero-dorsal being 5. This is the reverse order to MACBRIDE's in *Asterina*, but naturally No. 3 corresponds in each case: it agrees with the order adopted by LUDWIG* and GOTO.† *The adult bilateral plane passes through* 4/5 *inter-radius across to the postero-ventral radius*, i.e. *No.* 2. This will be clear from a consideration of the following:—In stage D the figures 50, 52 and 57 all show clearly that the pre-oral



2.—DIAGRAM of Stages D, E and F of *Cribrella* larvæ seen from the left side, illustrating the changes in the posterior cœlom and its relationships to the hydrocœle and pre-oral cœlom. (A) Stage D, showing the posterior cœlom extending in a curve from 4/5 to 1 (or about 250°). (B) Stage E, showing the posterior cœlom extending from 4/5 to 5/1 (or about 290°); all the extension is in the ventral arm, the dorsal remaining still in 4/5. (C) Stage F, showing the posterior cœlom now extending through about 340°, a small canal of pre-oral cœlom alone separating the two ends; as before, all the growth has been effected by the ventral arm. The dorsal arm gives off an aboral rudiment (V.) over the hydrocœlic radius 4. The next (IV.) is formed at the posterior end over 3, and the other three, III., II., and I., are formed by the ventral arm over 2, 1 and 5 respectively; hence an unconformability is produced, but there is no torsion.

cœlom has a posterior wall abutting against the dorsal horn of the left posterior cœlom. Eventually, in stage F the latter acquires an aperture into the former, but the relation of these two never changes. Along this posterior wall of the pre-oral cœlom is formed the stone-canal, which passes into the pore-canal on the right and into the hydrocœle on the left (*cf.* fig. 63). Fig. 64 shows that immediately the hydrocœle becomes radiate, the stone-canal opens into it in the 4/5 inter-radius, and the relationships of these parts remain the same throughout. On the other hand, the left posterior cœlom lies round in a crescent with its ventral horn pressing (in figs. 55 and 52) against the

* LUDWIG, H., "Entwicklungsgeschichte der *Asterina gibbosa*," *Zeitsch. f. w. Zool.*, vol. xxxvii., 1882.

† GOTO, S., *Journal of the College of Science*, Imp. Univ. Tokyo, Japan, vol. x. part iii., 1898.

wall of the pre-oral cœlom in the ‘neck region,’ but in this case the position of these parts does not remain constant. In fig. 52 the tip of the ventral horn is opposite the future radius 1, but in stage E it bends upwards, pushing the ventral wall of the pre-oral cœlom upwards and backwards. Hence in figs. 83 and 84 it is seen to have reached the base of radius 5, and in stage F it forms an aboral radial process lying over radius 5 (figs. 96, 97, 98). In doing this it has pushed the ventral wall of the pre-oral cœlom so far upwards, and later backwards, towards the dorsal wall, that this pre-oral cœlom, instead of, as in stage D (figs. 56 and 57), extending dorso-ventrally from inter-radius 4/5 to radius 1 (with an aperture of about 160°), is now reduced to a narrow canal lying in inter-radius 4/5, from which it runs forwards to the main bulk of the pre-oral cœlom in the pre-oral lobe. (See A, B and C, diagram 2.)

Thus the inter-radius of the stone-canal is 4/5 of the larva, and the plane of adult bilateral symmetry is along inter-radius 4/5—radius 2 of the larva. This plane passes, as we have seen, at right angles to the sagittal plane of the larva, and it is at an angle of 72° to the larval coronal plane, which naturally passes along radius 3 and inter-radius 1/5.

The dorsal horn of the hypogastric cœlom forms the aboral radial process over hydrocœlic radius 4, the posterior portion over radius 3, and the ventral horn, as is clearly indicated in figs. 86 and 87, forms the processes complementary to 2 and 1. The last is the latest, as it cannot be formed by the ventral horn till it reaches radius 5 as detailed above. From this it is clear that the ventral horn provides three aboral processes and the dorsal one; hence if we number the aboral processes clockwise, similarly to the hydrocœlic, we find that I. lies over 5, II. over 1, III. over 2, IV. over 3, and V. over 1. Thus is produced the appearance of the torsion actually observed in so many asterids by previous workers. Had the dorsal horn grown forwards equally with the ventral horn, the stone-canal would have been formed in inter-radius 1/5, the dorsal horn would have supplied the two dorsal aboral elements, and there would have been no want of conformity in the numbering. It should be noted that there is, at least in *Cribrella*, no true torsion at all; the hypogastric ring does not twist as a whole; its dorsal termination is fixed throughout, and its ventral part merely moves round by elongation till it reaches radius 5. In other asterids at present described the left posterior cœlom lies loose as it is constricted off from the anterior cœlom, and is not attached to the posterior end of the mesenteron; it is therefore free to rotate, and does so till the same end is attained as in *Cribrella.* I am inclined to think that *Cribrella* is primitive in this respect, and that therefore the torsion in asterids of oral and aboral elements upon each other through 72° is primarily due to these two facts—(1) the true madreporic plane lies through inter-radius 4/5 on the dorsal side of the larva, compelling the hypogastric cœlom, in assuming axial symmetry, to grow only ventrally; (2) in species with free hypogastric cœlom this part grows symmetrically in dorsal and ventral horns, and then has to twist through 72° in order to bring the same elements in opposition as in *Cribrella*, in which there is no torsion. I have here used the terms radius and inter-radius to make the process more clear; the more accurate expression in speaking of stage D is to describe the stone-canal as lying some little distance along the dorsal surface, but its position can be better identified as in an imaginary 4/5 inter-radius. From the above it will be seen that if we place a starfish (*e.g.*, *Cribrella*) on its oral surface, the larval sagittal plane will be horizontal and the larval right side uppermost; if we wish to determine the larval antero-posterior axis, we must place the starfish with the madreporic inter-radius away from us, and the opposite arm towards us; the antero-posterior axis of the larva will then start anteriorly in the inter-radius immediately to the left of the madreporic inter-radius, and pass along the radius opposite it, immediately to the right of the main radius.

Theoretical Considerations.

How far does the development of *Cribrella* justify us in drawing conclusions in regard to questions of phylogeny? The fact that the cœlom arises by separate anterior and posterior elements places this species in a unique position amongst asterids, and the great length of the development, uninterrupted by larval feeding or the vicissitudes of pelagic life, is favourable to a correct repetition of the phyletic stages. Naturally, the loss of mouth and anus must be regarded as secondary. The objection may be urged that it is rash to attempt phyletic speculations upon an experience of one type only, but as long as one is not led into matters which are specially connected with other groups, this objection does not appear to apply. There must have been an unbroken chain of living organisms from the cœlenterate to the modern starfish. Does *Cribrella* help us to realise the structure of the early links?

The Bilateral Ancestor.

From the preceding it will not be difficult to sketch the main features of the bilateral ancestor. We are certainly justified in regarding as precocious and cœnogenetic phenomena the enlargement of the organs of the left side, and in correcting this asymmetry we naturally bring the central cœlom back to the median line and the pore-canal to the left. Again, we regard the absence of mouth or anus and the persistent continuity of the various elements of the cœlom with the mesenteron as purely embryonic characters. We are then left with a bilateral organism with a large pre-oral lobe, with a simple alimentary canal, a ventral mouth below the pre-oral lobe, and a posterior anus. The cœlom was segmented into a pre-oral unpaired part, a small central vesicle over the œsophagus, a pair of lateral cœloms and a pair of posterior cœloms. Locomotion was probably effected by cilia, in all probability arranged in definite tracts in relation to the mouth. I think there are few who will not allow this much as a legitimate inference from the facts. When we commence to inquire whether the lateral cavities possessed tentacles we find all conjecture. If they did, it is obvious that the right lateral cavity has lost them. The only evidence in favour of this view consists in (1) MacBride's description of abnormal larvæ in which a starred hydrocœle was present on each side. (2) My own discovery of *Cribrella* larvæ like fig. 95, in which the right and left sides have exchanged form and function, resulting in an enantiomorph of the normal. Such enantiomorphs could not be recognised as such after the pre-oral lobe had been lost, and may therefore, like enantiomorphic flounders, be fairly common in nature. (3) Inferences from such types as *Actinotrocha* and *Cephalodiscus.*

However, it seems safer, with present evidence, to suppose that the bilateral ancestor was sufficiently provided with ciliated tracts for feeding purposes, and that the tentacles were of later origin. Unquestionably the nervous and vascular systems were in evidence, but for the present we may leave these out of consideration.

We owe the identification of the anterior cœlom and its relationship to the axial sinus and water-pore to Bury, and it is therefore peculiar that he should, in constructing his prototype, have assumed a primitive *bilateral* condition for this organ; he also supposed that the left very early became reduced to a mere remnant, and that the right had disappeared altogether. There does not appear to be any satisfactory evidence for supposing that this cœlom was ever paired.

Again, to Bury we owe the discovery of the 'dorsal sac,' a median pulsating sac over the œsophagus, subsequently observed by Field, and stated by him to arise as a schizocœle. There can be little question that the central cœlom here described is the same organ.*

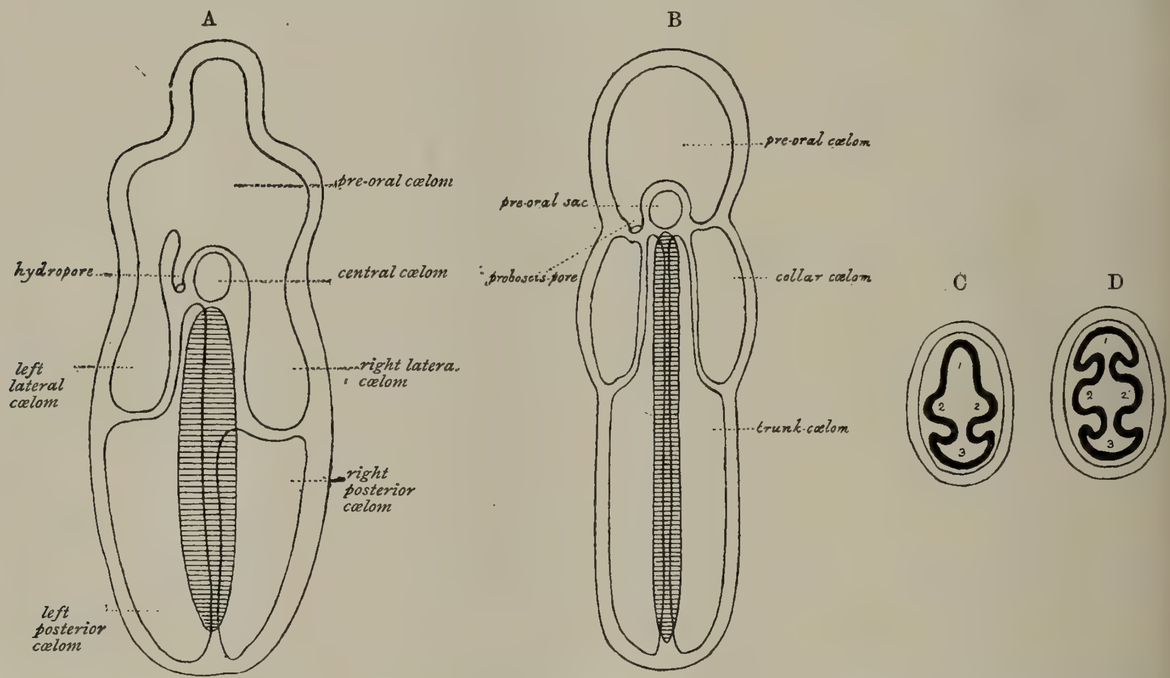
This bilateral ancestor can be directly compared to *Balanoglossus.* The anterior cœlom (its pre-oral part) has already been compared by Bury to the proboscis cavity of *Balanoglossus*, the water-pore to a left proboscis pore, and the central cœlom (dorsal sac) to the pericardium (*herzblase*). These homologies I adopt from him, with the reservation with regard to the unpaired nature of the anterior cœlom. Beyond this I cannot go with Bury. He makes a special attack against the assumption that a fixed stage is necessary to account for the axial symmetry of echinoderms. I have elsewhere † attempted to gather together the various laws regulating the symmetry of animals, and have tried to show that they depend on the inter-action between an organism and its environment along definite lines. Such being the case, it seems that an 'ancestor' must be constructed, not only having ontogenetic changes in mind, but also taking into consideration the probable surroundings of the animal at each period. For example, Bury proposes to show that a fixed stage is not a necessary assumption, and proceeds to construct an ideal ancestor (bilateral) in which the œsophagus is already formed into a closed ring with five tentacles. This ancestor, as he himself points out, closely resembles the 'Pentactæa' conception of Semon. In each case the whole question of the hydrocœle ring is avoided by starting with this assumption. The 'ancestor' of Bury

* This will be confirmed later by a description of its later changes. In the adult it has the relationship of a heart-vesicle or pericardial vesicle to the blood-vascular system, the presence of which, I hope, as indicated elsewhere, to show in detail later (*Proc. Royal Physical Soc.*, March 1901).

† Masterman, A. T., *Natural Science*, January 1899.

and of SEMON brings us no nearer any other phyla of the animal kingdom; and if the structure of an early ancestor does not fulfil this condition, its credentials are open to question, and its construction does not seem even to possess the virtue of utility. In fact, the *Pentactœa* (quite apart from other objections urged by BURY, CLARK, LUDWIG and others) appears to me an instance of precocious segregation, in which the typical echinoderm axo-symmetric character of a hydrocœle ring has been grafted on to the simple character of an early bilateral ancestor. Such an anachronism occurs in ontogeny, but not in phylogeny.

The equivalence of the hydrocœle and epigastric cœlom in *Cribrella* leads us to regard their parent organs, the left and right lateral cœloms, as being comparable to the two collar-cavities of *Balanoglossus*, and the right and left posterior cœloms as the homologues of the two trunk-cavities. Going further afield, we may recognise in the pre-oral cœlom, the lateral and posterior cœloms,—the protocœle, the mesocœles,



3.—DIAGRAMS showing relationships of *Cribrella* larva (stage D) to *Balanoglossus*. (A) Dorsal view of *Cribrella* larva (stage D), with the parts named. The water-pore and the central cœlom are supposed to be pushed back into their true position. (B) A dorsal view of a young *Balanoglossus*, with the parts named. (C) Diagrammatic coronal section of larva of *Cribrella* at stage B. (D) Diagrammatic coronal section through larva of *Balanoglossus*. The hypoblast is indicated black in each case.

and the opisthocœles respectively of the archi-cœlomate, the central cœlom being also comparable to the archimeric heart.* Our bilateral ancestor, as derived from stage D, therefore takes us to the junction of echinoderms with numerous triploblastic *Metazoa*, with archimeric segmentation. The 'ancestor' as here outlined would unquestionably take its place in this group, and probably also would be placed beside *Balanoglossus* in the *Archi-Chorda* (*Hemichorda*).

It will be seen that this 'bilateral' ancestor differs from that of such authors as

* *Proc. Roy. Soc. Edin.*, Dec. 1898.

MacBride and Bather chiefly in the presence of the right lateral cœlom (later the epigastric cœlom), and its comparison with the right collar-cavity of *Balanoglossus.* MacBride compares the latter with his right hydrocœle, the vestigial remnant of which he finds in the central cœlom. I agree with Bury in regarding this as a median organ.

Transition to Echinoderms.

The subject of the phyletic history of Echinoderms is at present in the state in which the speculations have so out-distanced the progress of research—and the same remark appears to apply to most branches of biology—that I shall merely indicate in the briefest way what further light the facts here narrated help to throw upon the ancestry of the echinoderms. Bather * has recently reviewed the leading theories of echinoderm descent in an able and suggestive paper, and has put forward his own views in a clear and concise manner. The earlier history up to the conception of a bilateral ancestor so resembles my own views of the descent of *Archi-cœlomata,*† in which I have suggested the inclusion of *Echinodermata,* that we are necessarily in agreement, but naturally I now desire to make the reservations here expressed in regard to the homology of the epigastric and hypogastric cœlom. Whether Bather will be prepared to accept this view on the evidence submitted remains to be seen.

We may now trace the steps by which this archi-cœlomate ancestor was transformed into an axially symmetric echinoderm. We are at present concerned with asterid ontogeny, and here there are two evident changes of symmetry, which succeed but overlap each other; it is important to keep these, together with their respective causes, carefully apart. The first of these changes, already apparent in stage D of *Cribrella,* is the transition to left-handed asymmetry. The organs of the left side become predominant, the left lateral cœlom becomes radiate, the left posterior cœlom elongates dorsally and ventrally, and finally, at a very late stage, the œsophagus and mouth appear on the left side. The second change is the assumption of axial symmetry. This appears later than the asymmetry, and commences in stage E by the radiate growth of the hydrocœle, followed by that of the hypogastric cœlom. It terminates in the formation of the starfish. Hence the starfish owes its underlying bilateral symmetry to its free-swimming period, its oral-aboral arrangement to its period of asymmetry, and its axial symmetry to its period of axial fixation.

1. *Asymmetry.*—This asymmetry has been recognised by many. Quite early Bütschli ‡ attempted to account for its appearance by assuming a fixation by the right side. MacBride supposes that, upon fixation, the factors inducing bilateral symmetry are in abeyance and permit the organism to run riot; by accident, the riot develops on the left side. But we must recollect that bilateral symmetry is included in axial. If

* Bather, F. A., *Journ. London Coll. Soc.*, viii. (pp. 21–33), May 1901.

† Masterman, A. T., *Proc. Roy. Soc. Edin.*, Dec. 1898, and *Quart. Journ. Micr. Sci.*, Aug. 1897.

‡ Bütschli, O., "Versuch der Ableitung des Echinoderms aus einer bilateralen Urform," *Zeitsch. f. w. Zool.*, vol. liii.

we take any longitudinal plane through the main axis of, *e.g.*, *Hydra*, the factors making for symmetry are the same on each side. Axial factors are therefore merely superposed upon bilateral, the latter not being removed. BURY* takes up a rather peculiar position. His 'ancestor' moves about, mouth downwards, on the sea-floor, apparently intermittently fixed by the five tentacles. The axial symmetry proceeds to develop spontaneously, with apparently no suggested reason. The body then, for some reason unexplained, rolls over to the left, the anatomical result being that the mouth becomes situated on the left side of the larva. BURY, with a remarkably able piece of reasoning, backed by numerous ontogenetic facts, shows the necessity for assuming this migration of the mouth to the left side, but naturally he brings the hydrocœle ring round with it.

Now, in asterids the hydrocœle is on the left (its plane parallel to that of the median larval axis) from the outset. As BURY remarks—(*loc. cit.*, p. 102)—"the two planes being at right angles to one another in ophiurids; inclined at a lesser angle in *Brachiolaria* and *Crinoids*; and parallel from the first in the *Bipinnaria*." BURY starts with the anomalous hydrocœle ring encircling the œsophagus and at right angles to the median larval axis in his ancestor (also SEMON), and he is then led into the position of regarding the asterid larvæ as being in this respect the most specialised, and the ophiurids as being the most primitive. As already remarked, the hydrocœle is essentially a left lateral organ, as shown by BURY himself, even in ophiurids, and there is really no evidence for assuming a primitive annular arrangement on the ventral side of the body.

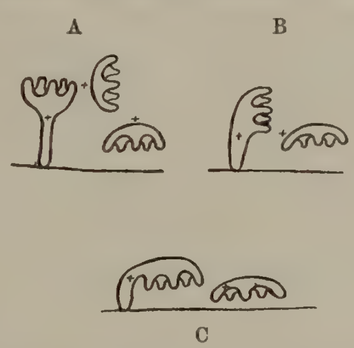
I think we may assume, between the period of free-swimming life and that of fixation, a period during which the organism was gradually forsaking its pelagic life and contracting the habit of lying on its right side on the sea-floor. Its diet was unquestionably microscopic floating organisms obtained by ciliary action, and the universal distribution of such would make this habit possible. At the same time, such a change of position would bring the body into relationship with the factors of symmetry at right angles to that previously existing. As in the analogous instance of pleuronectids, the heterogeneity would be bilateral rather than dorso-ventral, and the mouth would move, with food from above, into the middle of the upper side, the left side of the larva. The tentacles were evolved round the mouth in relation to the already existing ciliated tracts, as in *Tornaria*, and connected with the cilio-trophic function. The hydrocœle tentacles do not appear to me to have served as organs of support (BURY), and I hardly see the *modus vivendi* of a larva with the mouth downwards (BURY, MACBRIDE). We may perhaps allow that the tentacles would tend, for purposes of nutrition, to be arranged around the mouth, the more so as the free life was gradually given up. During this stage the pre-oral lobe became more and more frequently an organ of fixation, but not in a permanent degree till the sinistral asymmetry was permanently instituted.

Axial symmetry.—Fixation by the pre-oral lobe appears to me to be one of the most clearly indicated phyletic facts in Echinoderm ontogeny. We know that axial symmetry can be induced only under circumstances in which the surroundings are

* BURY, H., "The Metamorphosis of Echinoderms," *Quart. Journ. Micr. Sci.*, vol. xxxviii. 95–96.

similar around an axis, and such only holds in drifting pelagic organisms and in sedentary types. It is hardly conceivable that a free-swimming bilateral animal like the archi-cœlomate would degenerate to a drifting type; at least we have no analogy for such a case, and no indications of such in ontogeny; the developing star in bipinnariæ is painfully out of place in its pelagic surroundings. Hence we must fall back on the assumption of a fixed stage to account for the radial symmetry. Extremely sluggish animals may have certain organs axially arranged in respect to some aperture, such as the branchiæ of *Doris*, but for such fundamental axo-symmetry as that of echinoderms fixation is indispensable. BURY promised an "attempt to show that such an assumption is neither embryologically sound nor necessary as a basis for phylogenetic speculation" (p. 95), but he follows this up by starting with a 'pentactula' ancestor, in which the axial symmetry is already of a very marked character (the ring hydrocœle), and he builds the axial symmetry of the other organs around this ring by simple processes of growth. Naturally, if we start with an axo-symmetric ancestor, the origin of which we do not discuss, it is unnecessary to assume a fixed period for further progress.

After fixation, the mouth and hydrocœle migrate up the left side till they reach the posterior end of the larva; a process very clearly indicated in *Antedon*, in many *Ascidiæ* and *Polyzoa*. The fixed period has evidently lasted longest in the Crinoids. I may only say that it appears to me easy to reconcile the position of the pre-oral lobe in asterids and crinoids. MACBRIDE (*loc. cit.*) argues a very wide divergence between the Asteroids and Crinoids, because the pre-oral lobe bends on to the oral side in the former, but on to the aboral side in the latter. He states that the anterior cœlom, bent orally, becomes surrounded both by the left cœlomic sac and the hydrocœle (p. 392). The pre-oral lobe is certainly not encircled by the hydrocœle in *Cribrella*, and does not appear to be so from MACBRIDE'S figures. It appears that in asterids the axial sinus lies in a mesentery formed by the dorsal and ventral horns of the left posterior cœlom (left cœlomic sac), and hence is not enveloped at all by the hydrocœle or the posterior sac, as the rupture between axial sinus and pre-oral cœlom occurs distally in the madreporic inter-radius; if this is the case, the moving of the pre-oral lobe orally or aborally along the inter-radius is a very small matter, anatomically speaking.



4.—DIAGRAM to illustrate the evolution of the *Asteroidea* from a fixed ancestor. (A) The fixed ancestor, with the adult detaching itself from the stalk, as in *Antedon*. (B) A type with the development abbreviated; the whole course of phylogeny is not repeated, the young leaving the stalk at an earlier stage. (C) A type, such as *Asterina* or *Cribrella*, in which a still greater abbreviation takes place, the disc never facing upwards. The + indicates the cicatrix of pre-oral lobe.

If we assume that this has been the true course of events, it is clear that the asterids, whilst repeating the stage of fixation, do not repeat the passage of the organs to the posterior end of the larva and back again. Any vestigial trace of this movement would be indicated as a deflection of the body upon the pre-oral lobe over towards the right-hand side of the larva and back again. As a matter of fact, GOTO maintains that in *Asterina* he has recognised just such a dextral bending, which is, however, largely disguised by the sinistral bending which follows it, and so eventually brings the oral surface down to the substratum. This 'rocking' of the 'body' on its stalk and pre-oral lobe has no other meaning than the vestige of a more complete rotation through 180° and backwards. In addition, it is a common phenomenon in pelagic larvæ for the disc to be at first directed with its oral face posteriorly.

Without considering other points of view, I think this is to be expected from the conditions of existence in the two orders. Diagram 4 will illustrate the point. In A the fixed asterid breaks away from its stalk, turns over through 180°, and takes up its position on the sea-floor. If this process takes place earlier in development (B), the asterid never looks upwards, but drops off obliquely; and if it is still further hastened (C), the pre-oral lobe never reaches the aboral surface, but is first lateral, and then is bent orally, as actually takes place. *Antedon*, with its longer period of sedentary life (phyletically), has not shortened matters to this extent. This may not be the true explanation, and MACBRIDE may be perfectly correct in his inference, but such an explanation should at least be borne in mind.

The principal organs affected by the axial symmetry in stages E and F are the radiate hydrocœle, the disc-shaped epigastric cœlom, the annular hypogastric cœlom, the pentamerous perihæmal and oral rudiments.

So far, therefore, as we can read phylogeny into ontogeny, we may find in the embryonic stages of *Cribrella* the well-marked cœlenterate in the gastrula; in stages A and B of the larval period we have the cœlenterate with cœlomic pouches, very nearly conforming to a tetramerous axial symmetry, and comparable to those in the larva of *Balanoglossus*. In stage C the bilateral archi-cœlomate asserts itself, embryonically sketched. This culminates in stage D, in which there are already forcible indications of the creeping habit, with its sinistral asymmetry. We may recall the actual resting habit of the living larvæ at this stage. Fixation next follows, and in stages E and F the axial symmetry rapidly appears. The disappearance of the median sagittal mesentery and the radiation of the hydrocœle lead the way, and the other processes follow in rapid succession.

It should be noted that, on the interpretation here shown, the adult circular mesentery is not a median mesentery of the bilateral stage, but is a primary mesentery, between the posterior and right lateral cœloms.

Comparison with other Groups of Echinoderms.

With present knowledge it is perhaps premature to attempt a comparison in a detailed degree with other groups, but there are a few points to which some reference should be made.

A study of a selected asterid, crinoid, ophiurid, echinid, and holothurian, not to mention the comparison of *Cribrella* with *Asterina*, convinces us that it is impossible to base homologies upon the method of origin of the several cœlomic elements. Hence our only hope in phyletic conclusions lies in the homology of adult cœlomic cavities. Thus we assume, until the contrary is proved, that the hydrocœle is homologous in all cases, and for similar reasons we strive to prove the homology of the epigastric with the aboral cœlom and the other cavities in a similar manner.

MacBride (*loc. cit.*) has already advocated the view that asterids are to be regarded as likely to show primitive characters in their development, and he relies on such characters as the ectodermal nervous system, the retention of the pre-oral lobe and the period of fixation, summing up with the remark, "It is, however, difficult for me to see how anyone can doubt that the asterids are the least modified group of the Echinoderms" (p. 398). My own belief to this effect led me originally to undertake the task here published, and I believe myself that we have in stage D of *Cribrella* the *nearest approximation to the bilateral ancestor of the Echinoderms yet described.* We might have added to MacBride's dictum the fact that in asterids the hydrocœle is developed normally on the left side and remains there, whereas in other groups the hydrocœle moves on to the ventral surface. As Bury has incidentally shown, the proof for the former existence of a *true* bilateral ancestor comparable to animals outside the *Echinodermata* rests upon the assumption that asterids are the least modified of all the division, a conclusion to which he was logically led by starting with his pentactula-like prototype.

If, then, we grant that asterids are the most primitive, especially in their development, and that *Cribrella* is the most primitive, in respect of the arrangement of the cœlomic cavities, that has yet been described, we can proceed to the inferences in regard to the other orders. Let us speak of the pre-oral cœlom (or protocœle) as 1,* the left and right lateral (or mesocœles) are 2 and 2′ respectively, and the posterior cœlom (or opisthocœle (paired)) is 3.

Ophiuroidea.

Grave † has, as already indicated, published a paper on an ophiurid, *Ophiura brevispina*, which has a demersal larva not unlike that of *Cribrella*, and also having a remarkable resemblance to the larva of *Antedon.* We have already alluded to the resemblance in its earlier stages to the former. In following out the fate of the body-cavities, Grave finds, apparently beyond dispute, that the cœlom arises as a large anterior cœlom and a posterior cœlom.

* On the nomenclature usually adopted, 1 = anterior cœlom,
2 = hydrocœle,
2′ = right hydrocœle,
3, 3′ = left and right enterocœles.

† Grave, Caswell, *Johns Hopkins Univ. Memoirs*, iv.

The latter is unpaired, and he makes the following remark:—"In most Echinoderms the posterior enterocœles originate as paired structures; and if the statements of BURY and MACBRIDE are correct, that the left posterior enterocœle of the larva forms the hypogastric cavity of the adult, and the right posterior enterocœle goes to form the epigastric cœlom, then, according to this, the large ventral pouch, which I regard as the fused right and left posterior enterocœles, really represents the left only, because it takes no part in the formation of the epigastric body-cavity of the adult ophiurid, but, with the left, does pass directly into the hypogastric" (p. 88).

Further he remarks, "As to the origin of this structure" (the epigastric enterocœle) "I have no direct observations to give, but certain facts have led me to believe that it is formed from the right anterior enterocœle . . . . Against such an interpretation as the above there is the fact that in no other case has the epigastric enterocœle been observed to take its origin from the right anterior pouch" (p. 90).

I must add that GRAVE's figures, so far as they go, carry out these assertions; and it is much to be hoped that he will be able to give a more exhaustive account of these points.

Crinoidea.

If we pass to the Crinoids we notice at once that *Cribrella* has this in common with *Antedon*, as described by BURY* and SEELIGER,† that the cœlom arises by separate anterior and posterior rudiments. It is well known that the posterior cœlom in *Antedon* divides into right and left parts, which form aboral (epigastric) and oral (hypogastric) cœloms respectively. This disagreement with *Cribrella* can only be reconciled in one of two ways. Firstly, we may assume that the hypogastric and epigastric cœloms of the starfish are respectively homologous with the oral and aboral cœloms of the crinoid, as has been done by most zoologists, in which case, in the light of the present work, we cannot allow that the posterior cœlom of *Cribrella* is the same as the posterior cœlom of *Antedon*, the former being $3+3'$ or left and right opisthocœles, whereas the latter would be $3+3'$ together and $2'$ (the right lateral cœlom or right mesomere). This seems a peculiar view to take, but we may recollect that the anterior cœlom of *Antedon* is $1+2$ or parietal canal (pre-oral cœlom) and hydrocœle, hence it is quite a different organ from the anterior cœlom of *Cribrella* under any view, for it has no right element at all. Again, unless we make this assumption, we must acknowledge that *Antedon* has no representative whatever of the right lateral cœlom. Whichever way we look at the matter, the anterior cœlom of *Antedon* is *asymmetric* (formed of a median and a sinistral element), therefore there is no inherent improbability in the posterior cœlom being equally asymmetric, and consisting of a median posterior and a dextral element. Lastly, the cœlomic development in *Holothuroidea* shows that the combined origin of contiguous cœlomic elements is an actual fact.

* *Phil. Trans. Roy. Soc. Lond.*, vol. clxxix., 1888.

† SEELIGER, O., *Zool. Jahrb.*, Abth. Anat., 4, Bd. 6.

The second method of reconciliation is to suppose that the anterior and posterior cœloms are homologous in the two forms, and that the asterids and the crinoids are divergent from the very fixation of the bilateral stage. Although this view, as we noticed above, has been advocated on other grounds, I do not think we should accept such a drastic resource until all else has failed.

Hence in *Antedon* (Bury and Seeliger) the first division is into 1 + 2 + mesenteron and 2′ + 3, according to our interpretation.

The anterior then divides into 1 + 2 and the mesenteron, whilst the posterior divides into 2′ and 3, 2′ to its own dextral side, to become aboral cœlom, and 3 to the left posterior end of the larva to form the oral cœlom.

Archenteron
- anterior cœlom
  - 1 + 2
  - mesenteron
- posterior cœlom (2′ + 3)
  - 2′ (aboral cœlom)
  - 3 (oral cœlom)

If Bury's or Seeliger's memoirs be read over carefully with this interpretation kept in view, several strange features in the early development seem thereby to be explained. Thus Bury first pointed out that the two elements of the posterior cœlom become partially separated into a dumb-bell shape, the connecting stalk passing completely through the annular mesenteron. Such a remarkable feature might well be caused by a persistent connection between posterior cœlom (3) and right lateral cœlom (2′) pushing itself forwards into the mesenteron. Again, we find that the fully developed bilateral larva of *Antedon*, as figured by Bury (*e.g.*, fig. 17), shows the right lateral cœlom ('right body-cavity of Bury') lying on the right side *exactly opposite* the hydrocœle, whilst the posterior cœlom (left body-cavity) is exactly posterior to the mesenteron (*cf.* also fig. 20). His earlier stages (figs. 7, 8 and 9) * and also those of Seeliger appear to favour this view. On such an interpretation the aboral cœlom of Crinoids = the epigastric cœlom of asterids = the right lateral cœlom of the bilateral ancestor, from which equation there follow many evident interesting deductions which cannot be pursued here.

Holothuroidea.

In Holothurians we have *Synapta* described by Semon † and confirmed by Clark ‡. Here we have the first division into mesenteron and anterior cœlom, which must have in itself the elements 1 + 2 + 2′ + 3. A division into two leaves the vestigial pre-oral cœlom (anterior cœlom of Bury) and the hydrocœle together (1 + 2) on the one hand, and the other elements in one sac, namely, 2′ + 3. This division therefore agrees with that of *Antedon*, for we have now (1) mesenteron, (2) 1 + 2, (3) 2′ + 3. The latter sac, corresponding with the 'posterior' cœlom of *Antedon*, then divides into 2′ and 3, 2′ forming the right body-cavity and 3 the left.

* These figures seem to have their true axes of symmetry at 45° to those assumed by Bury.

† Semon, R., "Die Entwicklung der *Synapta digitata* und die Stammes geschichte der Echinodermen," *Jen. Zeitsch. f. Naturwiss*, vol. xxii., 1884.

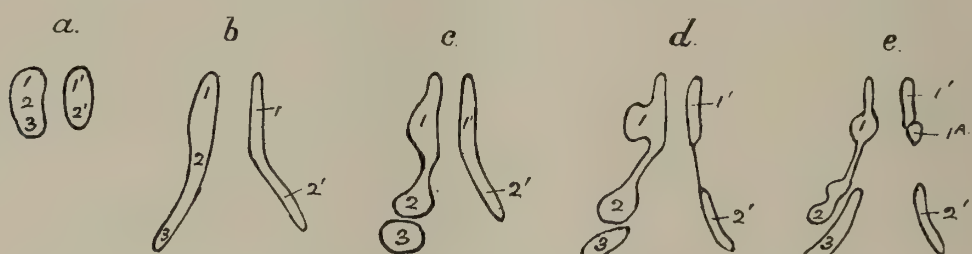
‡ Clark, H. L., *Johns Hopkins Univ. Memoirs*, iv.

Archenteron — mesenteron
Archenteron — cœlom — anterior cœlom (1 + 2)
cœlom — posterior cœlom (2′ + 3) — 2′ (right body-cavity)
posterior cœlom (2′ + 3) — 3 (left body-cavity)

Echinoidea.

For comparison with the *Echinoidea* we may take the work of MacBride upon *Echinus esculentus*, a preliminary note upon which has been recently published.* He finds a single anterior cœlomic rudiment from which all the parts of the cœlom are derived. This divides into left and right portions. The left is the larger, and on the eighth to ninth day it divides into two parts, the anterior forming the left anterior cœlom and hydrocœle, united by a constricted stone-canal, whilst the posterior forms the left posterior cœlom.

On the right side the cœlomic vesicle divides somewhat later into right anterior cœlom and right posterior cœlom. Lastly, at about sixteen days, a small solid outgrowth from the right anterior cœlom grows dorsally, acquires a cavity, and takes up its position as a closed vesicle dorsal to the œsophagus (fig. 3).



5.—The accompanying Diagram is copied from MacBride's work in its outlines, and shows the progressive changes in the right and left cœlomic vesicles. The numbers have, however, been added by myself to indicate the manner in which the facts described in this work lead me to interpret the changes in *Echinus esculentus.*

In this view it is clear that the posterior cœlom (3) in *Echinus* is never divided whilst the anterior cœlom is paired (1 + 1′), the two parts of which are presumably fused at a later stage.

The position of the central cœlom (1*a*) is as in MacBride's figure, but, judging by his fig. 3 and his description, it would be more accurate to draw it further into the median line, when its homology would be even more clear. We may therefore see that *Echinus* does not differ essentially from *Cribrella* in the segmentation of its cœlom, the chief divergence being the paired origin of the pre-oral cœlom, which is closely paralleled by the pelagic asterids, such as *Asterias pallida.*

We may assume that the right and left posterior cœloms are comparable to the similarly named parts of *Asterina*, but MacBride does not at present indicate the fate of the right anterior cœlom. The small vesicle he compares to the "right hydrocœle" of *Asterina*, and it is evidently homologous to the organ so named by him. If we compare it to the central cœlom of *Cribrella*, we notice that it differs therefrom by its origin from the right-hand side. If the right anterior cœlom is to be regarded as the

* "The Development of *Echinus esculentus*," *Proc. Royal Soc.*, vol. lxix. No. 455.

right half of the anterior cœlom, as found in the unpaired condition (in which case it would fuse with the left anterior cœlom later in development), then it is evident that a median organ arising from the anterior cœlom, like the central cœlom, must in *Echinus* arise either from the right or the left half of the anterior cœlom, and as it moves to the right side after origin in *Cribrella*, we should expect to find it originating from this side in *Echinus*. After origin it appears to move into a median dorsal position on the œsophagus (*m.v.*, fig. 3), where it has been noticed in pelagic larvæ by BURY.

TABLE *showing relationships of Cœlomic Elements in Various Types.*

Balanoglossus.	Cribrella Larva.	Asterid.	Crinoid.	Holothurian.
Proboscis cavity.	Pre-oral cœlom.	Axial sinus, etc.	Parietal canal.	(Vestigial.)
Left collar-cavity.	Left lateral cœlom.	Hydrocœle.	Hydrocœle.	Hydrocœle.
Right collar-cavity.	Right lateral cœlom.	Epigastric cœlom.	Aboral cœlom.	Right body-cavity.
Trunk cavities.	Posterior cœlom (L + R).	Hypogastric cœlom.	Oral cœlom.	Left body-cavity.

[6.—DIAGRAM

6.—DIAGRAM illustrating the Segmentation of the Cœlom in *Echinodermata.*

1.1′ = left and right elements of pre-oral cœlom (1. only is pre-oral cœlom).

2.2′ = left and right lateral cœlom (hydrocœle and aboral cœlom respectively).

3.3′ = left and right posterior cœlom (3. only is posterior cœlom).

The small round vesicle in 1, 3, and 6 is the central cœlom.

1. As seen in development of *Cribrella* ; the perfect bilaterality is characteristic.

2. As seen in *Ophiura brevispina,* according to GRAVE. There is some doubt about 1′ and 2′, but the presence of anterior and posterior cœlomic rudiments appears certain.

3. As seen in *Echinus esculentus,* according to MACBRIDE. This should be compared with diagram 5. The main fact of impŏrtance is the early formation of right and left, rather than anterior and posterior elements. Asymmetrical origin of the true posterior cœlem (3) seems indicated.

4. As seen in *Synapta digitata,* according to SEMON (CLARK). The asymmetry is even more pronounced. 2′ and 3 are at one time together.

5. As seen in *Antedon rosacea*, according to BURY (SEELIGER). Here the posterior element is $2'+3$, and divides as in *Synapta*.

6. As in *Asterina gibbosa*, according to GOTO. Here the posterior cœlom (3) is said to be composed of a left element from the left side and a right from the right, which fuse together. According to MACBRIDE, *Asterina* more closely resembles *Echinus* (No. 3), the posterior cœlom coming from the left side only.

7. As in *Asterias pallida*, according to GOTO. Here the bilateral division is instituted; and if GOTO's work on this form be confirmed, it gives us one of the most symmetrical types.

The diagrams are based upon the structural facts as narrated by the various authorities, though there is no evidence at present that any of them would agree with the interpretations of the changes as here given.

This diagram clearly shows that closely allied species or groups may diverge widely in their ontogenetic stages, not only externally and in such processes as segmentation and gastrulation, but in the methods of production of segments of the mesoblast.

Conclusion.

We are thus led to the conclusion that the third body-cavities of the bilateral ancestor have very early in the history of the Echinoderms fused together, and that the only trace of their division is seen in such embryonic stages as the larva of *Cribrella* and the primary separation of the posterior cavities in *Asterina* and *Asterias*, which soon meet and fuse posteriorly. The median mesentery of Holothurians as well as the circular transverse mesentery of other groups appears to be the line of junction between right lateral cœlom and the two posterior cœloms.

In concluding, I must take this opportunity of expressing my sense of gratitude to my friend Prof. M'INTOSH, at whose suggestion I commenced this work several years ago, and to whom I owe much help and encouragement in its prosecution. I must also express a sense of indebtedness to Dr NOËL PATON and the Committee of the Royal College of Physicians Laboratory for the facilities which have been freely placed in my hands for the last two years.

SUMMARY.

The principal points dealt with may be tabulated somewhat as follows:—

Embryonic Period.

1. Segmentation very variable, but always culminating in a solid morula of equal cells.
2. A process of multicellular egression reduces the morula to a blastula, upon which gastrulation takes place.
3. The archenteron becomes filled with hypenchyme and blastopore closes.
4. Archenteron divides into mesenteron, anterior cœlom, and posterior cœlom. Embryo set free, with uniform coat of cilia.

Larval Period.

1. *Stage A.*—Similar to embryo. Anterior cœlom grows laterally into two lateral cœloms.
2. *Stage B.*—Posterior cœlom becomes differentiated into left and right. Larva elongates and anterior cœlom fills pre-oral lobe. Central cœlom developed from posterior wall of anterior cœlom.

3. *Stage C.*—Development of pre-oral lobe with dorsal and ventral processes. Forward growth of dorsal and ventral horns of left posterior cœlom, and backward growth of left and right lateral cœloms.

4. *Stage D.*—Culminating stages of bilateral larva. Dorsal process partially and ventral completely divided into two. Occasional fixation. Development of pore-canal. Forward growth of right posterior cœlom. Enlargement of sinistral elements.

Post-larval Period.

1. *Stage E.*—Fixation. Fusion of right and left posterior cœloms to form hypogastric cœlom; right lateral cœlom becomes a disc-shaped epigastric cœlom; water-pore opens on right side (aboral surface); central cœlom becomes 'dorsal sac'; left lateral cœlom becomes hydrocœle and gives off five radii, developing in order 1, 2, 3, 4, 5; 1 and 2 ventral, 3 posterior, and 4 and 5 dorsal. Hypogastric cœlom forms aboral radii over 1 and 2, partially over 3; opening of dorsal horn of hypogastric cœlom into anterior cœlom.

2. *Stage F.*—Degeneration of pre-oral lobe by infoldings. Further development of hydrocœle into ring radii with one pair of tube-feet, often two; hypogastric cœlom forms aboral radius 4, and tip of ventral horn progresses to base of 5; anterior cœlom forms pre-oral cœlom (in pre-oral lobe) and axial sinus; perihæmal elements arise in inter-radii, 4/5 from axial sinus, the remainder from hypogastric cœlom; three (or four) elements ($\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$) of the oral cœlom appear from hypogastric cœlom in numerical order.

Adolescent Period.

Stage G.—Further development of the star. Changes to be followed in later paper.

Relationship of larva to adult.

Larval sagittal plane corresponds to discal plane of starfish.

Left side of larva = oral surface of starfish.

Right ,, ,, ,, = aboral ,, ,, ,,

Coronal longitudinal plane of larva is at 72° to plane of adult through madreporic inter-radius. In development of hydrocœle and hypogastric cœlom there is no torsion, but, owing to extension of ventral horn only of hypogastric cœlom, there is a union of I. with 5, II. with 1, III. with 2, and so on.

General considerations.

Bilateral ancestor closely similar to *Balanoglossus.* Subjected to period of creeping life on right side, hence sinistral *asymmetry.* Followed by fixed period, hence axial symmetry.

LIST OF ILLUSTRATIONS.

Plate I.

Figs. 1–3. Three segmenting eggs, external view. (× 28.)

Fig. 4. An egg at completion of segmentation. (× 32.) The egression tracts are commencing to appear.

Fig. 5. A later embryo, with numerous ingression tracts in the form of grooves covering the surface. (× 32.)

Fig. 6. A gastrula stage, with gaping blastopore and vestiges of four egression tracts. (× 32.)

Fig. 7. Median section through a stage with two blastomeres. Notice the many nuclei. (× 32.)

Fig. 8. Median section through the unequal total type, with small archicœle. (× 32.)

Figs. 9 and 10. Median sections of similar types at various stages. (× 32.)

Fig. 11. Median section through another type of segmentation, partially superficial, but indefinite. (× 32.)

Figs. 12, 13. Median sections through a very common type of segmentation, with slight inequality in fig. 12, which disappears in fig. 13. (× 47.)

Fig. 14. Median section through the morula, to which all the types of segmentation eventually lead. (× 78.)

Fig. 15. A single cell of the morula, showing the radiate yoke-granules. (× 370.)

Fig. 16. A later morula, in which the cell-walls are becoming indistinct, and the cells are commencing to arrange themselves into aggregates producing egression tracts. (× 47.)

Fig. 17. The transition stage of a syncytial mass, with egression tracts; a central blastocœle and peripheral blastocœles are becoming evident. (× 47.)

Fig. 18. The gastrula in median section. (× 47.)

Plate II.

Fig. 19. The embryo immediately before hatching. (× 28.)

Fig. 20. The free larva, stage A. (× 28.)

Fig. 21. The free larva, stage B. (× 28.)

Fig. 22. The same as fig. 21, anterior view.

Fig. 23. Stage C, from right side. (× 28.)

Fig. 24. The same, but slightly later. (× 28.)

Fig. 25. The same, anterior view. (× 28.)

Fig. 26. Median section through the embryo in fig. 19. (× 34.)

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Fig. 29. A median coronal section of stage A. (× 57.)

Figs. 30–34. A coronal selected series through stage A/B. (× 28.)

Fig. 35. A median coronal section through stage B. (× 28.)

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Plate III.

Figs. 46–49. A sinistral, ventral, dextral and anterior view, respectively, of stage D. Fig. 48 shows the method of temporary fixation.

Figs. 50–52. A dorsal, ventral and sinistral view, respectively, of stage D, viewed as transparent objects. (Oil of cloves.) (× 40.)

Figs. 53–57. A sagittal series through stage D. (× 28.) (The selected sections are Nos. 20, 22, 30, 40, 42.)

Figs. 58–62. A transverse series through stage C/D. (The selected sections are Nos. 5, 30, 49, 54, 56.) (× 28.)

Figs. 63–70. A coronal series through stage D/E. (× 28.) The selected sections are Nos. 26, 32, 36, 49, 58, 52, 67 and 71.)

Figs. 71–76. A transverse series through stage D/E. (× 28.) The selected sections are Nos. 19, 30, 37, 43, 57, 66.)

PLATE IV.

Figs. 77–78. Dextral and sinistral views, respectively, of stage E. (× 28.)

Figs. 79–87. A coronal series through stage E. (× 28). (The selected sections are Nos. 54, 60, 64, 73, 83, 90, 100, 116, 129.)

Figs. 88, 89. Dextral and sinistral (aboral and oral) views, respectively, of stage F. (× 28.)

Fig. 90. Anterior view of stage F. (× 28.)

Figs. 91, 92. Aboral and oral views of stage G. (× 28.)

Fig. 93. Anterior view (larval) of stage G. (× 34.)

Fig. 94. Another specimen of stage G, showing quadrifid pre-oral lobe. (× 28.)

Fig. 95. An enantiomorphic specimen, oral but right side. (× 28.) Stage G/H.

PLATE V.

Figs. 96–104. A selected series of coronal sections through stage F. (× 57.) (The selected sections are Nos. 48, 53, 56, 60, 64, 76, 85, 90, 97.)

Figs. 105–106. Left and right views of stage F, seen as transparent objects, cleared with oil of cloves. (× 34).

Figs. 107–111. Diagrams illustrating the suggested homologies and fates of the cœlomic elements in *Asteroidea* and *Crinoidea*. 1 = pre-oral cœlom. 1*a* = central cœlom. 2 = left lateral cœlom or hydrocœle. 2′ = right lateral cœlom. 3 = posterior cœlom. The details of each cœlomic element have been omitted, and the figures are at most approximate diagrams.

Fig. 107. Dorsal view of the asymmetric (sinistral) ancestor of *Echinodermata*. The mouth is on the left side. The stage D/E of *Cribrella*, except for absence of mouth and anus, is closely similar to this stage; it is also represented by the larva of *Antedon*, which, however, has no mouth, and its hydrocœle is deflected towards the ventral surface. This stage is slightly later than that shown in Diagram 3 (A).

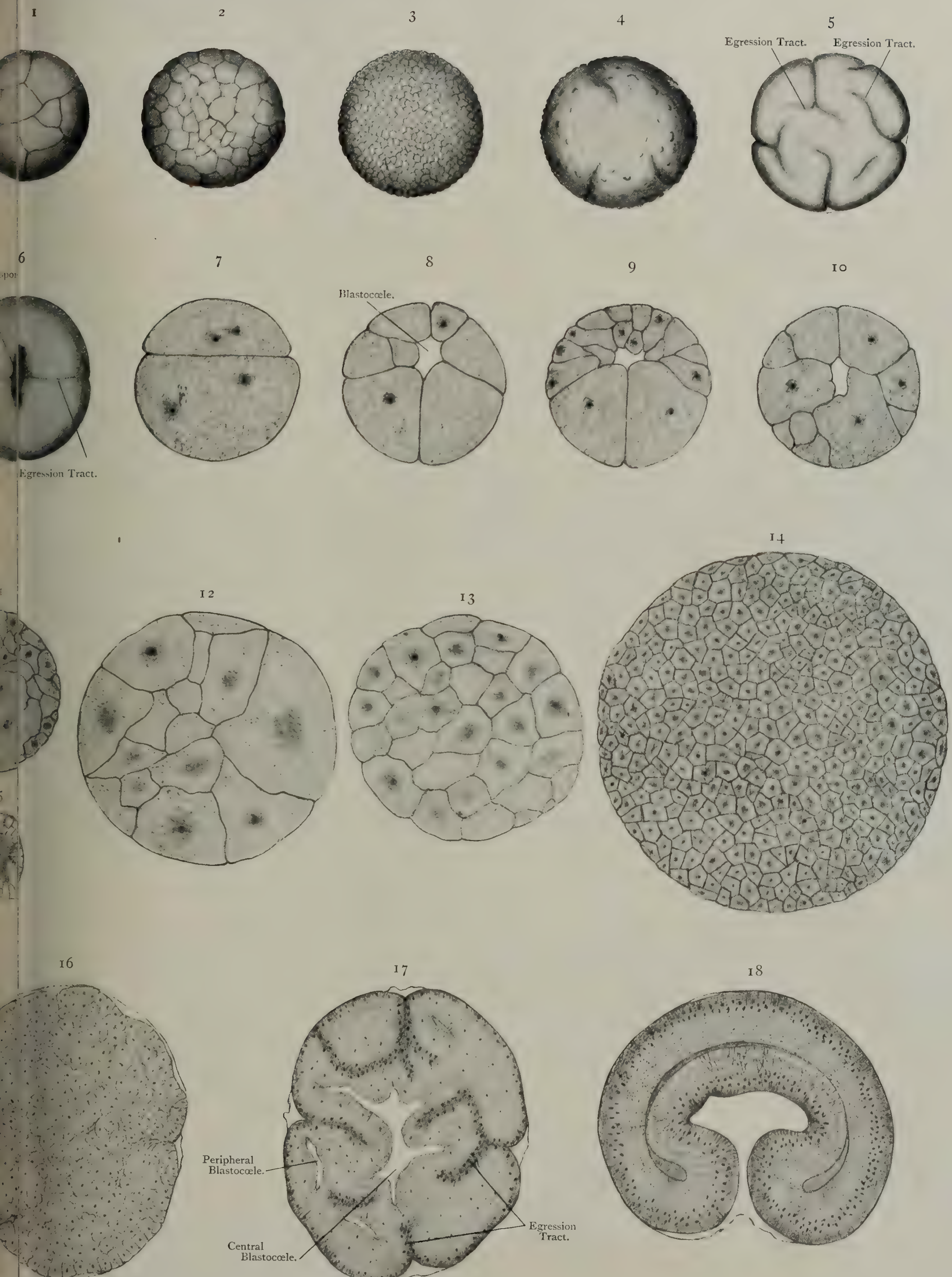
Fig. 108. The early axo-symmetric stage; the sedentary habit is sufficiently pronounced to reflect itself in axo-symmetry; the mouth has commenced its migration towards the posterior end; the hydrocœle commences to surround the œsophagus, the posterior cœlom also commences to encircle the stomach. *N.B.*—No attempt is made to indicate the 4/5 position of the stone-canal.

Fig. 109. The ancestor of the *Crinoidea*; the mouth is now terminal, the œsophagus is encircled by the hydrocœle, the stomach by the posterior cœlom, which becomes the 'oral cœlom'; the right lateral cœlom, now the aboral cœlom, is growing out into the stalk to form the chambered organ; the pre-oral cœlom remains as the 'parietal canal.' It should be noted that the mesentery between posterior and right lateral cœloms crosses diagonally, as has been indicated in *Antedon* (BURY) and *Asterina* (LUDWIG).

Fig. 110. The fully formed 'cystid' stage of *Antedon*.

Fig. 111. The early asterid stage, set free at a stage intermediate between stages B and C (above). The posterior cœlom is here seen to form the hypogastric, and the right lateral becomes the epigastric cœlom. The pre-oral cœlom remains, in part, as the axial sinus, in the mesentery of the hypogastric cœlom, alongside of the stone-canal.

MASTERMAN: EARLY DEVELOPMENT OF CRIBRELLA OCULATA.—PLATE I.



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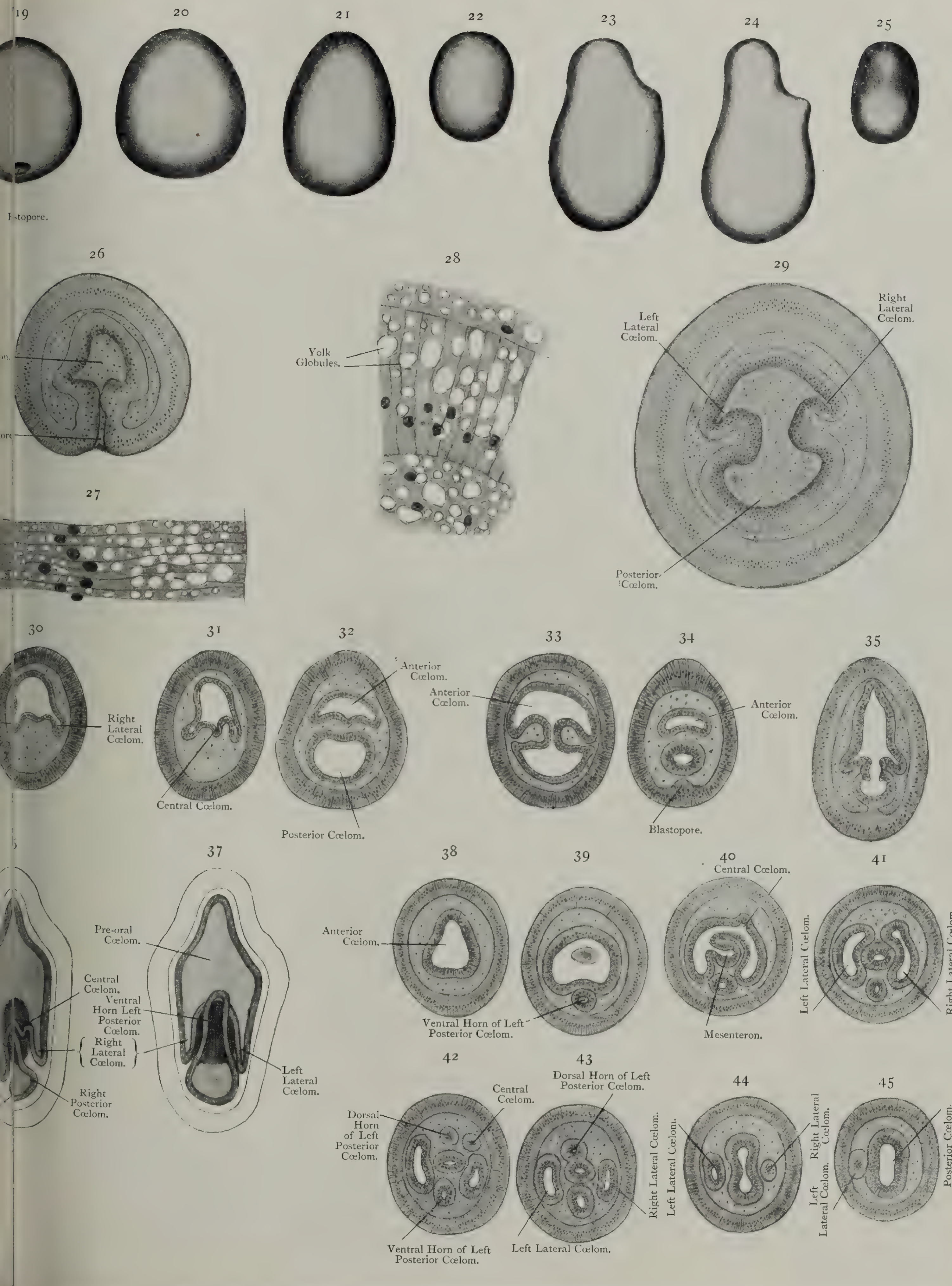
MASTERMAN: EARLY DEVELOPMENT OF CRIBRELLA OCULATA—PLATE II.



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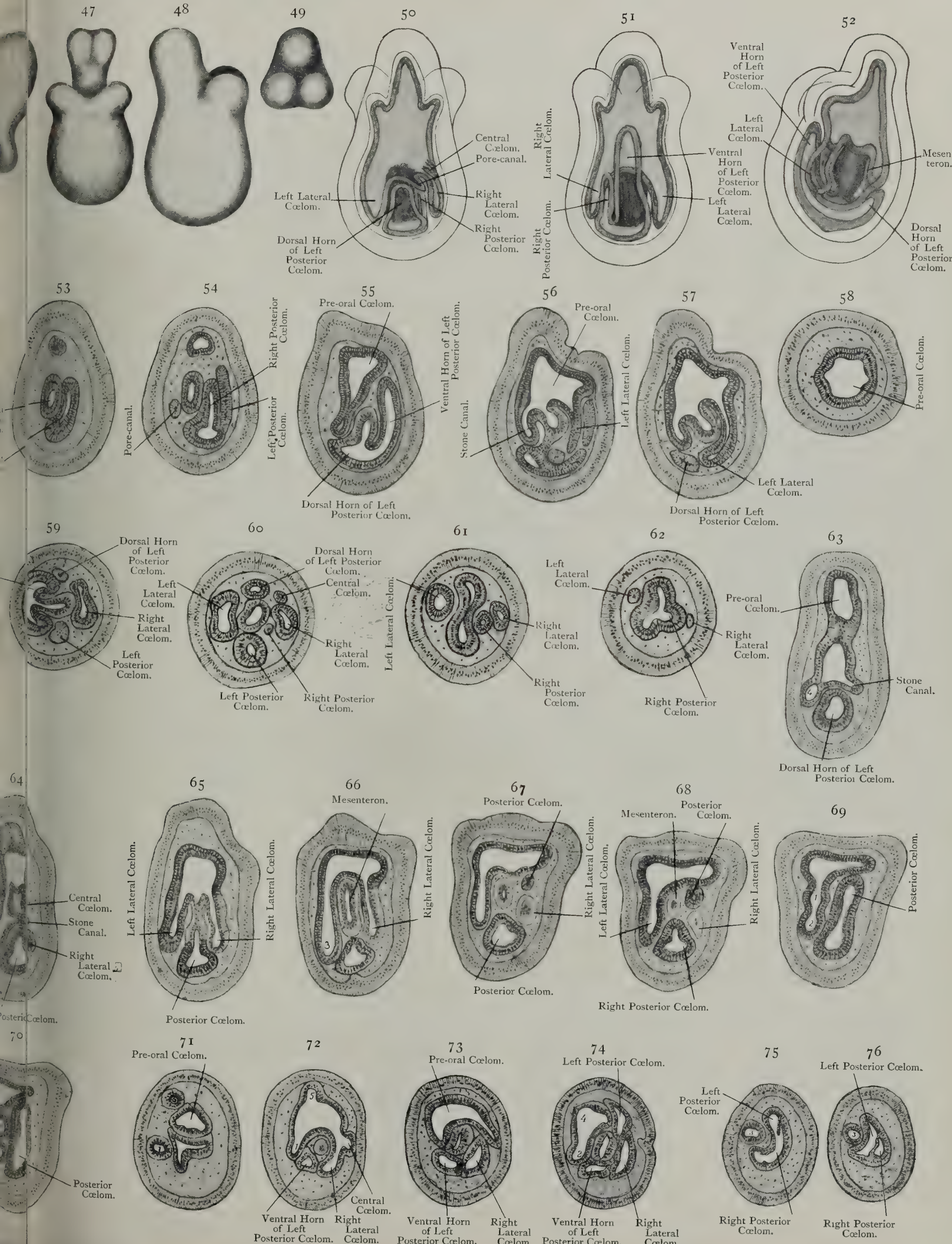
MASTERMAN: EARLY DEVELOPMENT OF CRIBRELLA OCULATA—PLATE III.



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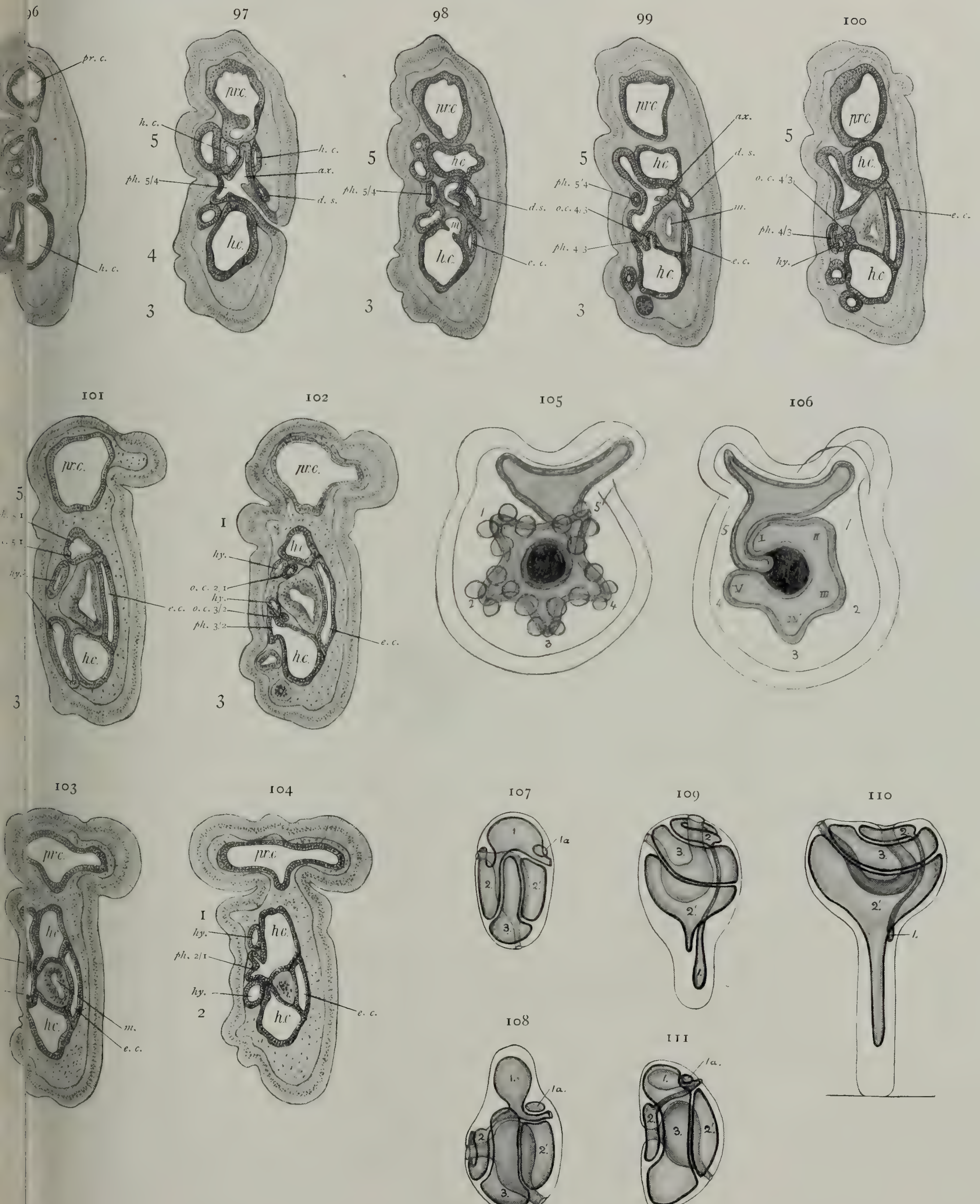
MASTERMAN: EARLY DEVELOPMENT OF CRIBRELLA OCULATA—PLATE V.



a. x. Axial Sinus. *d. s.* Dorsal Sac. *e. c.* Epigastric Cœlom. *h. c.* Hypogastric Cœlom. *hy.* Hydrocœle.
m. Mesenteron. *o. c.* The Oral Cœlom Elements. *pr. c.* Pre-oral Cœlom. *ph.* Peri-hæmal Elements.

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XX.—*A Bathymetrical and Geological Study of the Lakes of Snowdonia and Eastern Carnarvonshire.* By T. J. Jehu, M.B., B.Sc. (Edin.), M.A. (Camb.), F.G.S., late Heriot Fellow of the University of Edinburgh. *Communicated by* Professor James Geikie. (With Eight Plates.)

(Read December 16, 1901. Issued separately June 16, 1902.)

CONTENTS.

I. Introduction.

The study of lakes has received more attention on the Continent than it has in our own country. The inland waters of France and Switzerland have been most carefully surveyed, and in America accurate soundings of many of the lakes have been made by the Geological Surveys. But until recent years this work has been almost altogether neglected in Britain; the Government had considered it to be outside the function of the Ordnance Survey, and though of importance to geological research, it has not been undertaken by the Geological Survey. The absence of adequate knowledge concerning the forms of the basins occupied by the lakes has been a serious obstacle to the geological inquiry as to the mode of origin of these basins. But recently, in the English Lake District and in Scotland, this obstacle has been removed to a great extent through the work of geographers, who have carried out a very complete bathymetrical survey of many of the lakes of those regions; and the importance of this work has been recognised by geologists. But in North Wales not only had no attempt been made to ascertain the configuration of the lake-beds, but in many cases even the depths of the lakes remained unknown.

Messrs J. E. Marr and R. H. Adie, in a paper on "The Lakes of Snowdon" (*Geol. Mag.*, 1898, p. 51), expressed a wish that "some one would do for the lakes of North Wales what Dr Mill has so admirably performed in the case of those of English Lake-land." The writer has made an effort to supply this want, and the results of his work are given in this memoir. Though the survey may not be so complete as that carried out by Dr Mill on the English Lakes, it is hoped that it will be sufficiently accurate to throw light on the question of the mode of origin of these lakes.

II. Previous Work on British Lakes.

1. *Scotland.*—Previous to the year 1883 the only fresh-water lakes of which a systematic survey had been taken were Lochs Lomond and Awe. The work was carried out in the interests of navigation, and bathymetrical charts of these two lochs were published by the Hydrographic department of the Admiralty.

During the years 1883–84 representations were made to the Government by the Royal Society of Edinburgh and by the Royal Society of London, urging upon it the desirability, in the interests of science, of executing a bathymetrical survey of the inland waters of the United Kingdom. But unfortunately the Government declined to carry out the work.

In the year 1888, Mr J. E. Grant-Wilson published in the *Scottish Geographical Magazine* (vol. iv. p. 251) a paper entitled "A Bathymetrical Survey of the Chief Perthshire Lochs and their Relation to the Glaciation of that District." The lakes he deals with are Lochs Tay, Earn, Rannoch, and Tummel, of which he gives coloured maps and soundings with some sections. His conclusions are that all these lochs lie in rock-basins, and that Loch Tay, a part of the bottom of which is situated below sea-level, belongs to the type known as a deflection-basin.

In 1900 Sir John Murray and the late Mr Fred Pullar published in the *Geographical Journal* (vol. xv. p. 309) an account of "A Bathymetrical Survey of the Fresh-water Lochs of Scotland." It deals with the lochs of the Trossachs and Callander district, namely, Lochs Katrine, Arklet, Achray, Vennachar, Drunkie, Lubnaig, Voil, and Doine. The paper is illustrated by coloured maps, with the subaqueous contour-lines marked. The only lake of which the bottom reaches below sea-level is Loch Katrine. Appended to this paper are some notes contributed by Messrs Peach and Horne on the geology and glaciation of the district. The conclusion arrived at is that "the soundings of the various lakes in the basin of the Forth above Callander, when viewed in connection with the geological structure and glacial phenomena of that area, furnish strong evidence in support of the theory of their excavation by ice-action."

2. *The Lake District.*—During 1874–75, two papers were published in the *Quart. Journ. Geol. Soc.* (vol. xxx. p. 96; vol. xxxi. p. 152) by the late Mr Clifton Ward on the "Origin of the Lake-Basins of Cumberland." In the first paper he discussed Derwent-water, Bassenthwaite, Buttermere, Crummock, and Lowes-water; in the second, Wastwater, Grasmere, Windermere, Coniston-water, and Codale, Easdale, and other tarns. Plans of the lakes were given, with the positions of the soundings marked by numbers, indicating the depths in feet at those points. Sections were also added to illustrate the form of the valleys, the depth of the lakes, the height of the mountains, and the probable thickness of the ice. Ward found that some of the larger lakes had depths reaching below sea-level. When, however, the true dimensions are laid down to scale, he was struck by "the insignificance of the hollows in which the lakes lie as

compared with the elevations of the surrounding ground." They are seen to be "but shallow grooves at the bottom of the valleys in which they occur, and their depth is small as compared with the thickness of the ice which moved over these spots." From these and other considerations he concludes that "the immediate cause of these lake-basins was the onward movement of the old glaciers, ploughing up their beds to this slight depth, in the way Professor Ramsay's theory suggests."

As to the mountain tarns, these appeared to him to be due "sometimes wholly to glacial erosion, sometimes to this combined with a moraine dam, and occasionally to the pounding back of rocks by moraines alone, or moraine-like mounds, at the foot of snow-slopes."

Dr H. R. Mill gives in the *Geographical Journal* for 1895 (vol. vi. pp. 46, 135) an account of his work in this district under the title of "Bathymetrical Survey of the English Lakes." The paper is illustrated by coloured maps, giving the subaqueous and land-contours and also representative sections of the lakes.

He found that the lakes conform to two main types:—

(1) The shallow, exemplified in Derwent-water and Bassenthwaite. They only average 18 feet in depth, and their average depth is only 25 per cent. of the maximum depth. These two lakes formerly formed one sheet of water. Their beds "may be roughly described as an undulating plain, grooved and ridged into shallow hollows, and low shoals running parallel to the long axis of the lake."

(2) The deep type, "the shallowest of which has an average depth of 40 feet, and in which the average depth varies from 36 to 61 per cent. of the maximum depth, showing a steep-sided character." This type is exemplified in Crummock-water, Buttermere, Wastwater, Coniston-water, Windermere, Hawes-water, and Ullswater. Ennerdale shows the characteristics of the two types, being deep in its upper sections and shallow in its lower part. He adds that "the typical form of this class of lake is a steep-sided, flat-bottomed trough, diversified along the slopes by the still deeper conical mounds of débris thrown down at the mouths of streams."

The deepest parts of the lakes are, in several cases, found to reach below sea-level.

In the Introduction to his paper, Dr Mill shows that the mountain and valley system of the Lake District presents a simple radial symmetry bearing no relation to the present geological structure; on the contrary, the drainage-lines "bear testimony to an earlier and simpler structure, when a dome of vanished rocks spread over the area, the dissected skeleton of which now alone remains." The valleys are arranged like the spokes of an irregular wheel passing out from the centre of the ancient dome; and each valley possesses one or more lakes, actual or extinct.

Mr J. E. Marr has devoted much attention to the Tarns and Lake-basins of the Lake country. In 1895–96 he published papers in the *Quart. Journ. Geol. Soc.* (vol. li. p. 35; vol. lii. p. 12) on "The Tarns of Lake-land." He points out that no one doubts the occurrence of moraine-dammed lakes; and should the exit immediately over-lie the old river bed, such a lake or tarn will necessarily have a comparatively brief existence.

But cases may occur in which the lowest point of the drift-basin does not lie vertically above the bottom of the old moraine-filled valley. "In such a case the stream would cut down rapidly until it reached the level of the rock, and then, in the majority of instances, the stream would cut sideways along the junction between drift and solid rock until, when the stream reached its original position, the lake would be drained. But if a subsidiary range of rock lay between the position attained by the stream issuing from the lake and the position of the former valley-bottom, denudation would be retarded to so great an extent that the lakelet would become much more permanent, and its depth would be the difference between the height above sea-level of the bottom of the old moraine-filled valley and that of the present exit." Hence it is quite possible that the tarns, though really moraine-blocked, should have their exit over rock, and so this in itself is no proof of the existence of a true rock-basin. To prove conclusively the existence of a true rock-basin, it becomes therefore necessary to show that there is no drift-filled valley by which the water might have escaped before the filling up with drift took place. An examination of the tarns of Lake-land has convinced Mr Marr that in many cases, and possibly in all, a valley of this kind did exist.

In 1896 Mr Marr published a paper in the *Proc. Geol. Assoc.*, "On the Lake Basins of Lake-land" (vol. xiv. p. 273). It is there argued that Dr Mill's detailed observations on the lakes are antagonistic to the theory that the lake-basins owe their formation to ice-erosion. Mr Marr remarks that "the subaqueous scenery of the lakes presents several difficulties, on the supposition that they are formed by ice-erosion. Each of these difficulties may not be absolutely fatal to the theory in itself, but taken together, they seem to me to furnish a mass of evidence which cannot be got over. On the contrary, the scenery can be readily accounted for on the supposition that the lakes are due to the damming-up of river-eroded valleys, which have had a certain amount of material deposited upon their floors, both before and after the process of conversion into lakes." He gives instances of drift-filled valleys in England, one of which occurred in the outskirts of the lake-district itself, in the Furness district, where there is an indication of a buried valley at least 450 feet below sea-level. He proceeds to consider separately each of the lakes surveyed by Dr Mill, and argues that in no case can we exclude the possibility of a drift-barrier sufficient to account for the existence of that lake, and that consequently there is no proof of the existence of rock-basins in Lake-land.

3. *Wales.*—No systematic survey of the fresh-water lakes of North Wales had up to this time been carried out. It is true that the late Sir Andrew Ramsay, in his memoir on "The Geology of North Wales," stated that the Rev. W. T. Kingsley had been sounding the lakes of North Wales with a view of proving them to be true rock-bound basins, but the results of this work have never been published.

Sir Andrew Ramsay, whose name is associated with the theory of the glacial erosion of lake-basins, has referred to many of the lakes of Snowdonia and Eastern Carnarvonshire in his well-known work on *The Old Glaciers of Switzerland and North Wales*, where the glacial phenomena of this region were so fully and admirably described. While holding that many of the lakes owe their origin entirely to ice-erosion, he allows that some of them are, partially at least, dammed-up by moraines.

Mr W. W. Watts contributed "Notes on some Tarns near Snowdon" to the *Geol. Mag.*, 1895 (p. 565). The tarns dealt with are Ffynnon Frech and Ffynnon Felen in Cwm Glas, and Glaslyn and Llydaw in Cwm Dyli. He concludes that the lower lake in Cwm Glas is certainly confined in a rock-basin, and that Glaslyn and Llydaw, unless much shallower than is generally supposed to be the case, also lie in rock-basins.

Some of the lakes of Eastern Carnarvonshire are discussed by Mr Brend in a short paper which appeared in the *Geol. Mag.*, 1897 (p. 404). He took soundings in a few of them, and figures are given indicating the greatest depth in feet obtained in each case. He concludes that in most of them it is impossible to exclude the possibility of a drift-dam; but, judging from the surroundings, Crafnant almost certainly lies in a rock-basin.

Messrs Marr and Adie are the joint authors of a paper on "The Lakes of Snowdon," already referred to. They discuss the origin of both the lower valley-lakes and of the upland tarns of Snowdon. The conclusion arrived at is that in no case can there be found evidence sufficient to prove the existence of a rock-basin, and that all probably owe their origin to the filling up of a depression by drift.

Mr J. R. Dakyns published a paper in the *Geol. Mag.*, 1900 (p. 58), on "Some Snowdon Tarns," in which the views put forth by Messrs Marr and Adie as to their mode of origin are stoutly controverted.

The discussion carried on with regard to the mode of origin of the Snowdonian lakes has been hampered by the lack of positive knowledge as to their depths. Arguments are sometimes advanced on the assumption that a lake has only a depth of a certain number of feet. And Messrs Marr and Adie in their paper express a strong desire that accurate soundings of these lakes should be made.

III. The Orographical Features of the District.

The lakes dealt with in this memoir are all situated in the mountainous region of Carnarvonshire. Within this area occur the highest mountains in Wales. The general distribution of the lakes and the physical features of the region are shown in the contoured map appended to the memoir (Plate III.). The height of the land is indicated, and the lakes dealt with in this memoir are coloured blue. The district comprises the Snowdon region and the part of Carnarvonshire lying to the east of this region. It is unequalled in Wales for the height of its mountains and the wildness of its scenery. A glance at the map will show that within this district we can distinguish three mountain-tracts, which, in the course of time, have become separated by valleys trending from south-east to north-west. To the west the great mass of Snowdon, with its many peaks and buttresses, stands isolated from all others. It is bounded on three sides by valleys; on the north-east lies the narrow pass of Llanberis, at the lower end of which are situated the two lakes Padarn and Peris; on the south-east the deep Vale of Gwynant, with its two lakes Llyn Gwynant and Llyn-y-Ddinas; the boundary to the south-west is formed by Nant Colwyn and the Gwyrfai Valley, the latter of which holds the lakes of Llyn-y-Gader and Cwellyn. The highest peak of Snowdon—Y Wyddfa—towers to a height of 3560 feet above sea-level. Standing on the summit we can see that Snowdon is penetrated on all sides by wild upland-valleys or cwms. These vast hollows are six in number, and many of them harbour lakes. They are tributary to the bigger valleys below; Cwm-glas-bach and Cwm Glas, opening north-eastwards on to the Pass; Cwm Dyli and Cwm-y-Llan, south-eastwards on to the Vale of Gwynant; Cwm-y-Clogwyn, westwards above the Vale of Gwyrfai; and Cwm Brwynog, towards the north. Soundings have been taken of two of the lakes, Llyn Llydaw and Llyn Glaslyn, lying in Cwm Dyli.

To the north-east of the Pass of Llanberis is another mountain mass, comprising several heights. The Glyders, y Tryfan, y Garn, and Elidyr Fawr, all rise to heights of over 3000 feet; and Y Foel Goch, Moel Perfedd, and Elidyr Fach, exceed 2500 feet. This area is bounded on the south-east by Nant-y-Gwrhyd, at the lower end of which are the lakes of Capel Curig, and on the north-east by Nant Francon and Nant-y-Benglog, at the head of which is situated Llyn Ogwen. The bottom of Nant Francon, for some miles below the head, probably marks the site of an old lake. A great number of cwms or corries open eastwards out to Nant Francon and Nant-y-Benglog, and in one of these, above the top of the Pass, lies Llyn Idwal.

To the north-east of the Nant Francon and Nant-y-Benglog depression, is situated the third and largest mountain tract. It is bounded on the south by the Vale of Llugwy, and on the east by the Vale of Conway. This tract presents a very rugged and wild aspect. It includes several high mountains, the most noteworthy of which are Carnedd Dafydd, Carnedd Llewelyn, and Y Foel Fras, all rising to heights of over 3000 feet. This tract is very rich in lakes, many of which occur at a very high altitude. The larger ones are described in this memoir, and include Dulyn, Eigiau, Cawlyd, Crafnant, and Geirionydd. Dulyn is a corrie-lake. The others lie in valleys, some of which have cirque-like heads. They all drain north-eastwards into the Vale of Conway.

IV. Methods.

The methods of carrying on the work were similar to those employed by Dr Mill in his survey of the English lakes. All the soundings were taken from rowing-boats, which on most of the lakes were hired without difficulty. But on some of the lakes lying at high elevations, no boat was to be found; in such a case arrangements had to be made for the conveyance to the lake of a boat from the nearest place where it was possible to get one. This often proved to be no easy matter, for some of the lakes are situated among the mountains in out-of-the-way places, and at a high altitude. A cart or waggon would bring the boat part of the way up, and for the rest of the journey it had to be carried on the shoulders of several men. Sometimes after succeeding in getting a boat on to such a lake, it was vexatious to have to wait for several days for the wind to abate sufficiently to proceed with the work.

A proper sounding-line was used made of cord which was water-laid, and about a quarter of an inch in diameter. To this was attached a 5 lb. lead. The line was marked at every five and ten feet. For the tens, tufts of coloured worsted—red, green, black, brown, and purple—were used, and tassels of white worsted to mark the fives. The correctness of the markings were verified each day, and allowance made for any stretching or shrinking of the sounding-line.

The method was to row across the lake along definite lines from point to point, and the shore-ends of each section were usually well-marked positions easily noted, and determined at the time on the 6-inch Ordnance Map. As the lakes are all narrow and comparatively small, there was no difficulty in seeing the object on the opposite shore, towards which we steered in as straight a course as possible. All the rowing was done by boatmen who were trained for some time for the work. Soundings were taken at every tenth stroke, and, when near the shore-line, often at every fifth stroke. The lines along which soundings were taken were marked at the time on the 6-inch Ordnance Survey maps. The position of each sounding on the section lines were afterwards indicated by dots.

Maps have been prepared on a scale of 6 inches (since reduced to 3 inches) to the mile, on which the details regarding the survey of each of the lakes are shown. The positions of the soundings are indicated by dots, and the lakes are contoured according to depths. The land contours are also marked up to heights of 1000 feet above sea-level. These are taken from the Ordnance Survey maps. Where the heights exceed 1000 feet, bench marks are put in where possible with the figures obtained from the Survey maps. Longitudinal and cross-sections of each lake are also given (Plate VIII.); these are on a scale of 6 inches to the mile. As originally prepared, the sections also showed the depth exaggerated fifteen times, and so conveyed a better idea of the forms of the depressions; but it has not been possible to reproduce these on Plate VIII.

A map is also appended showing the orography of the district in which the lakes occur.

All the contoured maps have been prepared by Mr T. A. Brock of Cambridge, to whom I am also indebted for working out the calculations connected with each lake.

A few photographs illustrating the character of the scenery in the neighbourhood of the lakes and valleys are also appended.

For most part the work of sounding had to be carried out single-handed, but I have to thank Mr Phillip Lake, M.A., of St John's College, Cambridge, for assistance in the survey of Gwynant and Dinas; and Mr H. Woods, M.A., of St John's College, for assistance in the work of Llyniau Mymbyr and Cawlyd.

V. Description of the Individual Lakes and their Surroundings.

The largest and most magnificent of the cwms of Snowdon—Cwm Dyli—penetrates the mountain mass from the south-east. It has the form of a huge rocky amphitheatre, bounded by the precipitous cliffs of Lliwedd, Y Wyddfa, Crib-y-Ddysgyl and Crib Goch. Enclosed within this vast hollow are three lakes—Llyn Teyrn is the lowest, Llydaw lies in the middle, and Glaslyn is the highest.

1. *Llyn Teyrn* is a small tarn somewhat oval in outline, and lying at a height of a little over 1200 feet above sea-level. It appears to be shallow, and the streamlet draining the lake flows out over drift.

2. *Llyn Llydaw* is a beautiful sheet of water rather more than a mile in length, lying in the heart of this upland valley, and taking an oblique direction across it. The lake has an irregular and somewhat winding outline; the axis runs first from west to east, then turns towards the north-east, and finally bends back again to the east. The water of Llyn Llydaw is of a markedly green or light blue colour, and this is usually ascribed here to the presence of copper, as mines for this mineral are worked close to the lake. But according to Messrs Marr and Adie, not a trace of copper was found on analysis either in the water of this lake or in that of Glaslyn.

Only one important stream—the Afon Glaslyn—enters the lake; it comes from the tarn higher up the cwm, and flows into Llydaw at its north corner, bifurcating into two branches a short distance above its junction with the lake. Other tiny streamlets run into Llydaw on either side. During dry weather these are mere rills, but after heavy rain they become temporarily swollen into rushing torrents, and carry much rock-rubbish from the steeper slopes into the lake. The river Glaslyn flows out at a point on the south-eastern shore about a third of the length from the lower end of the lake, and runs south-eastwards, making its way eventually by a waterfall down to Nant Gwynant.

In its upper reaches the lake has a fairly regular outline, but lower down the shore-line becomes more irregular, owing to the immense quantities of morainic material scattered about, especially on the south-eastern side. The remains of a great moraine can be traced near the outlet; portions of it occur as islets, composed of boulders and loose material, opposite the mouth of the stream; a continuation of the same moraine can be traced on the northern side opposite in the shape of huge mounds.

A causeway crosses the lake at its narrowest part, and for the construction of this the surface of the lake was, according to the late Sir Andrew Ramsay, lowered as much as 16 feet. Beyond the causeway the lake widens out again considerably.

The work done on Llydaw occupied the 21st, 22nd, and 23rd of August 1900. The weather conditions were, on the whole, favourable, but at times the wind was troublesome, making it difficult to steer straight courses from side to side, and during the last day heavy showers made the work unpleasant.

The altitude of the water-surface, as determined by the Ordnance Survey, was 1415·7 feet above sea-level. The lake has a total length of 1950 yards, with a maximum width of 460 yards. The mean breadth is 301 yards, being 15 per cent. of the length. The area covered by the waters of the lake is approximately 587,200 square yards. In all, twenty-six sections were taken, the total number of soundings being 292. The greatest depth recorded was 190 feet, and this was found near the upper end of the lake. The mean depth is estimated to be about 77·4 feet, and this is 41 per cent. of the maximum depth. The bulk of water contained in the lake is approximately 409 million cubic feet.

The general configuration of the subaqueous scenery can be gathered from an examination of the contoured map of Llydaw with the accompanying sections (Plates IV. and VIII.). The lake is very deep in its upper reaches and shallows as traced downwards. The area within each contour-line is broadest at its upper end, and tapers rapidly as traced downwards. The 60 feet contour-line ends just opposite the point of exit of the river Glaslyn. Beyond this point no depths of over 60 feet were registered. The lake is shallowest on either side of the causeway and for some distance out from this. Beyond the causeway, towards the eastern end of the lake, we met with a considerable area over 40 feet, the greatest depth recorded within this area being 55 feet.

We may divide the lake roughly into two portions, the one lying to the south-west of the place where the river Glaslyn emerges from the lake, and the other lying to the north-east of this. The first half forms a deep basin and the second is comparatively shallow. It is quite evident, from the amount of earthy material, stones, and blocks of all sizes, which cumber the ground, and can be seen at places below the surface of the water, that the portion of the lake immediately opposite the point of exit of Glaslyn and that at the north-east of this owes its shallowness largely to morainic material scattered along the lake bottom. On the shores bordering that portion of the lake we meet with the remains of the huge moraine mentioned above. When we get clear of the remains of the big moraine, the lake deepens again, as shown on the map, in that portion of the lake lying to the east of the causeway. So that were it not for this morainic material, it is probable that the 40 feet contour-line would extend all the way to the eastern end of the lake, and that the 60 feet contour-line would extend a considerable way in that direction also, and the 80 feet line to a less extent. This would make the configuration of the lake basin comparatively simple. The deepest hollow lies in the upper third, and the lake shallows gradually towards the lower end. These points are seen clearly in the sections, which show that Llydaw lies in a deep, somewhat trough-shaped basin. Its gradual shallowing, as the region of the moraine is approached, is well brought out in the longitudinal section. As much as 40 per cent. of the total superficial area is covered by water attaining depths of over 100 feet, and 64 per cent. of the total area is enclosed within the 40 feet subaqueous line. Only 18·8 per cent. of the total superficial area corresponds to depths below 20 feet.

In the upper and deeper part of the lake the isobaths are seen to cling closely to the shore-line, and the slope to the bottom is very steep from the head of the lake and from the sides, as shown by the sections given on Plate VIII.

3. *Glaslyn*, the highest of the lakes of Snowdon, lies at an altitude of nearly 2000 feet above the level of the sea. The old and correct name is Ffynon Las (The Green Well), so called on account of the bluish-green colour of its waters.

The lake lies east and west, and is encircled on three sides by precipitous rocks. On the west it is surmounted by the great precipice of Y Wyddfa, which has an almost sheer fall from the top of Snowdon to the head of the lake; on the north it is bounded by Crib-y-Ddysgyl and Crib Goch, and on the south and south-east by Y Crimau and the ridge of Y Gribbin. The outlet is towards the east, whence a streamlet issues and rushes down a steep rocky slope to make its way below to Llyn Llydaw. The lake is considerably longer than it is broad, and its margins are very regular, with the exception that it is indented by a bay on the north side, and that it narrows towards the exit on the east. The path from Penygwryd to the summit of Snowdon follows the northern side of the lake. A streamlet flows in at its north-east corner, and its point of entrance has, owing to rock-rubbish carried down from the copper-workings above, been shifted down lake a little since the preparation of the Ordnance Survey map.

The soundings on Glaslyn were made on 24th August 1900, under most favourable conditions, the weather being beautifully calm, and not a ripple disturbing the surface of the waters. The soundings were all taken by myself, and the rowing done by an excellent boatman. We were able to steer perfectly straight courses from side to side, and so the work done on this lake was more accurate and complete than that done on any other of the lakes of this group. Situated high up in one of the deepest recesses of Snowdon, the surface of this gloomy lake is, according to the Ordnance Survey maps, 1970·7 feet above the sea-level. It has a total length of 535 yards, with a maximum breadth of 275 yards. Its mean breadth is 197 yards, being about 37 per cent. of the length. Its waters cover an area of about 105,600 square yards. The number of soundings taken in Glaslyn was 107. The greatest depth registered was 127 feet, and this is approximately in the centre of the lake. The mean depth has been calculated to be 62·6 feet, and this is 49 per cent. of the maximum depth. The bulk of water contained in the lake is estimated at 59,500,000 cubic feet.

An examination of the map (Plate IV) shows that the lake is deepest in the middle, and that the contour-lines follow each other with great regularity. The relation between the length and the breadth of the area enclosed within each contour-line is very similar to that of the length and breadth of the lake as a whole. The bay on the northern side causes the 20 feet contour-line to curve outwards more or less parallel to the shore-line, and the 40 feet and 60 feet contour-lines to curve out to a less extent. The contour-lines marking greater depths are unaffected by the bay. There is a curious curving inwards of the contour-lines from a point on the southern shore; all the lines are affected in a similar way, but the indentation is especially marked on the 120 feet contour-line. The cause of this is not obvious; but it may be noted that just at this point on the margin of the lake, the bare rock rises clear of the thin covering of drift lying on the slope of the Gribbin, along the south-east margin of the lake.

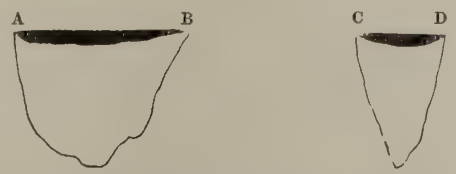


Fig. 1.—Sections of Glaslyn. The black portion shows the true slopes; the outline shows the slopes exaggerated ten times. A–B, longitudinal; C–D, cross section.

Three sections are shown (Plate VIII.): a longitudinal section A–B, and two cross sections C–D and E–F. They serve to make clear that the waters of Glaslyn lie in a deep basin-shaped hollow, about twice as long as it is broad, with sides steeply sloping, especially from the western and southern margins. The great basin-like depression is seen not to extend into the bay, and it terminates some distance away from the exit at the narrow end of the lake. Nearly 20 per cent. of the total area lies within the 100 feet contour-line; one half of the total area is within the 60 feet line, and only 15·7 per cent. of the area corresponds to depths below 20 feet.

4. *The Lakes of Llanberis.*—The two lakes—Llyn Padarn and Llyn Peris—lie at the foot of the well-known Pass of Llanberis. The Pass is a long narrow valley or glen, wild and stony in aspect, running from south-east to north-west, and bounded on both sides by some of the loftiest mountains in Wales. On the left or south-west the great mass of Snowdon rises to a height of 3570 feet above sea-level, and its two precipitous shoulders, Crib Goch and Llechog, immediately overhang the Pass; on the right, tower Y Glyder Fawr, Y Garn, and Elidyr Fawr—all of which attain heights of over 3000 feet.

Down the Pass rushes a brawling stream bearing the name of Afon Nant Peris, and, after a course of about four miles, during which it is joined by tributary streamlets coming down from Cwm Glas and Cwm Glas-bach on the left and from the slopes of Y Glyder on the right, it enters an alluvial stretch at Gwastad-Nant, and a little short of a mile lower down loses itself in Llyn Peris. During its course along the alluvial flat, it is joined by two streamlets from the right, one issuing from Cwm-gafar, and the other from Cwm Dudodyn, the latter joining the main stream just above its entrance into the lake.

Llyn Peris is over a mile in length, and trends in the same direction as the Pass. The slopes of Elidyr Fawr come down rather steeply to the right margin, and Pen Careg-y-Fran towers up from the left. From its lower end a stream called Afon-y-Bala makes its way over a flat strip of ground, about a quarter of a mile in length, to Llyn Padarn. Llyn Padarn, the axis of which keeps in the same direction as that of Peris, is two miles long, and from its lower end the river Seiont escapes to make its way over undulating and drift-covered lowlands to the Menai Straits.

The alluvial flat lying just below Dolbadarn Castle, and separating Peris from Padarn, is made up for the most part of material brought down by the Afon Llwch from Cwm Dwythwch and the heights of Snowdon. It is evident that at no distant date back Llyn Padarn and Llyn Peris formed one continuous sheet of water, and that they are now isolated owing to the deposition of this tongue of sediment carried down by the tributary stream from the left. Another fact pointing to the same conclusion is that the water-surfaces of both lakes are at the same elevation above sea-level, the Ordnance Survey map giving that of Peris at 339·5 feet, and that of Padarn at 339·6 feet.

The distance from the lower end of Llyn Padarn up to the head of Llyn Peris is over three miles. The length of the original lake must, however, have been much greater. For not only is there an alluvial expanse rising very gently from the head of Llyn Peris, and stretching up in the direction of the Pass as far as Gwastad-Nant, but a marshy flat, partly under water and often flooded over after heavy rains, extends from the foot of Llyn Padarn down below Cwm-y-Glo. So, originally, the lake extended without a break all the way from this point up to Gwastad-Nant, a distance of between four and five miles. From a point at a height above Gwastad-Nant, one can see right along both lakes to the lower end of Llyn Padarn (Plate II. fig. 1). Their main axis, running from south-east to north-west, lies in the same line as the Pass, and the lakes evidently fill up the lower end of the valley. At Pen-y-Pass, close to the Gorphwysfa Hotel, a bench-mark indicates an elevation of 1178 feet above sea-level. The height of the alluvial flat, at a point just below Gwastad-Nant, is given on the Ordnance map as 360 feet. So the fall is steep from the top of the Pass to the head of the alluvial stretch, after which it becomes very gentle, falling only to 339·5 feet at the water-surface of Llyn Peris and continuing the same in Llyn Padarn. Up above, the Pass is narrow, but at the head of the alluvial strip the valley begins to widen a little. Below Dolbadarn Castle the hills recede from the left side of the valley, which therefore broadens in this region, but further down the valley narrows somewhat again, and it becomes still narrower at the lower end of Llyn Padarn. The river escapes from Llyn Padarn between two rocky knobs on to the marshy flats of Cwm-y-Glo.

The alluvium lying between the two lakes extends back for a considerable distance along the course of the Afon Llwch. This stream now enters Llyn Peris at its lower left corner. The hills, which slope down to the lake from a great height at the upper end of Llyn Peris and along its right bank, and also on the right bank of Llyn Padarn for some distance below its head, gradually sink to lower levels as traced down to the lakes. The slope down to the margin of the lake is usually steeper on the right side. But here and there, especially in Peris, rocky knobs dip steeply into the water from the opposite or left side.

Above Llyn Peris and the upper end of Llyn Padarn, on the right side, are the great Dinorwic Slate Quarries. The working of these quarries has sadly disfigured the face of nature. The mountain-side has been torn open, and the vast accumulations of slate-rubbish heaped up along the margins of the lakes have helped to mar the beauty of the scene, and in places have quite obliterated the original form of the shore-line. Smaller quarries are also worked opposite the middle of Llyn Padarn on the western side. Owing to the fact that the lakes are used as receptacles for the slate-rubbish, quantities of this material must cover a considerable area of the lake bottom. The sounding operations were at times rendered difficult, owing to the danger of approaching near to these rubbish-heaps, for loose slabs and blocks continually roll over and come tumbling down into the water.

The survey of these lakes was carried out on August 14th, 15th, 16th, and 17th, 1900. The weather was favourable, and we were able to steer straight courses without much difficulty. The rowing was done by a boatman from Llanberis, who proved intelligent, and readily adapted himself to the work. All the soundings were taken by myself.

5. *Llyn Peris.*—This is the upper and smaller of the two lakes. It has an area of about 598,720 square yards, and as its total length is 1930 yards, its average breadth is 310 yards.

The lake terminates somewhat abruptly at both ends, the head having a square-cut form, and the lower end being a little more rounded. The outline of the lake-margin is rather irregular, rocky promontories jutting out into the water here and there on both sides.

Two well-marked bays occur, one on the left side, about the middle of the lake, and the other on the opposite side, but a little higher up. The lake is widest opposite to the bays, the maximum breadth being 500 yards across the middle of the lake. The slope of the ground is generally steep along both margins; but the 400 contour-line retreats for some distance opposite the bay on the left side, and on the right side near the head of the lake, and again a little above its lower end.

The only streams of any importance flowing into the lake are Afon Nant Peris at the head, and Afon Llwch at the lower end. The outline of the shore below the slate quarries is now quite hidden by the rubbish thrown down from above, and mounds of this material project well into the lake. It was impossible to take sections across Llyn Peris in this region, and so long oblique sections had to be taken from points low down to points high up the lake on the opposite side. The total number of soundings taken in Llyn Peris was 246. The volume deduced from the soundings was 344 million cubic feet. The greatest depth recorded was 114 feet, and this was reached at two places not far from the lower end of the lake. The mean depth is calculated to be 63·9 feet, being 56 per cent. of the maximum depth. The 100 feet subaqueous contour-line encloses two distinct areas, one lying along the middle of the lake, and the other not far above its lower termination. These two areas taper towards each other, and it is quite possible that they were originally continuous, and have now become separated, owing to the slipping of slate-rubbish into the middle of the lake, which is constantly occurring. All the other subaqueous contour-lines are continuous. A large part of the lake is over 80 feet deep, as much as 42 per cent. of the whole superficial area being enclosed within the 80 feet contour-line. The 60 feet contour-line encloses 60 per cent. of the superficial area, and only 17 per cent. corresponds to depths of water below 20 feet. Its contour-lines lie close to each other along the margins and at the lower end, indicating that the slope is steep in those regions; but, as traced towards the head of the lake, the lines are seen to be separated by greater distances, indicating that the slope is here more gentle. As a rule, the contour-lines follow the irregularities of the shore-line, but it is worthy of note that they are but slightly affected by the bay on the south-west side.

Representative sections of Llyn Peris are given (Plate VIII.). The section A–B is along the length of the lake; the other two are cross-sections; C–D across the deep area near the lower end, and E–F at a place not far from the head of the lake. A study of the map (Plate IV.) and of these sections will show that the configuration of the lake-bottom is simple. The longitudinal section brings out very clearly the difference in steepness between the slopes at the two ends of the lake. The slopes on both sides are also very steep, as shown by the cross-sections, and for some distance the bottom has a form approaching that of a flat plain, which rises gently towards the upper end of the lake. The slight irregularities are in all probability partly due to the irregular distribution of slate-rubbish on the lake-bottom. The lake consists of a long flat-bottomed trough with steep sides, the flatness of the bottom becoming, however, less marked as we approach the head of the lake.

6. *Llyn Padarn.*—This lake lies a little further down the valley, being separated from Llyn Peris by about a quarter of a mile of low alluvial ground. The rocky hills of Yr Allt Wen, Fachwen, and Clegir border the lake on the right or northern side, and the slope down to the margin is steep at the upper end, becomes more gentle towards the middle, and again increases a little lower down. On the left the hills fall back, and the slope down to the lake margin is gentle from the head of the lake for a considerable distance below the town of Llanberis, after which it becomes steeper as far as the lower end of the railway tunnel. Just below the tunnel a depression, occupied by alluvium, leaves the lake margin and winds round a rocky mass at the lower end of the lake to join the main valley again above Cwm-y-Glo. The shore-line is fairly even on the Dinorwic side, but is more irregular on the Llanberis side, and is in parts obliterated by accumulations of slate-rubbish, carried down from the quarries on the left side of the lake. Not far from the lower end, on its right side, a rocky island and some rocky promontories are seen to be beautifully glaciated. In addition to Afon-y-Bala, which flows in at the head of the lake, two small tributaries run in just above Llanberis, and another enters from the Dinorwic side, at a point more than a third of the way down the lake. Llyn Padarn is broad at its head, becomes narrower just below Llanberis, and then broadens out again for a short distance; after this, it narrows once more, maintaining a nearly uniform breadth to a point a little below the railway tunnel, then tapers considerably to the termination at the bridge, under which escapes the river Seiont.

The lake is 3530 yards long, and has a maximum breadth of 595 yards. Its waters cover an area of 1,340,800 square yards. The mean breadth is 380 yards, and this is 11 per cent. of its length. During the survey nineteen sections were taken, the total number of soundings amounting to 286. The deepest sounding obtained was 94 feet, just opposite the town of Llanberis. So Padarn is nowhere as deep as Peris. The mean depth is 52·4 feet, and this is 56 per cent. of the maximum depth. The total bulk of water contained in the lake is approximately 632 million cubic feet. The upper half of the lake is deeper than the lower half. The contour-lines for the most part follow the inflexions of its coast lines, broadening out at head of the lake, and narrowing where the lake narrows. The 80 feet contour-line encloses 13·5 per cent. of the total superficial area. It has a remarkable form—a broad swollen head tapering to a narrow channel, which runs nearly half way down the length of the lake. This channel bulges out a little about its middle. Nearly 50 per cent. of the total superficial area of the lake is enclosed within the 60 feet contour-line, and as much as 70 per cent. lies within the 40 feet contour-line. Only 18 per cent. corresponds to depths below 20 feet.

A longitudinal section of the lake is shown along the line A–B; and in addition three cross-sections are given—G–H near the head and across the deepest part, E–F about the middle, and C–D from a point just above the upper end of the railway tunnel at a point a little lower down on the opposite side (Plate VIII.). These with the map (Plate IV.) show that the configuration of this lake-basin is again simple. The same characteristic trough-shaped form is shown with steeply sloping sides and a more or less flat bottom. As one would expect, the slope is steeper on the Dinorwic side, corresponding to the steeper slope of the land on that side. The longitudinal section shows the steepness of the slope at the upper end of the lake, and also proves that the deep basin ends at a point opposite the lower end of the railway tunnel, beyond which the lake is comparatively shallow.

7. *Llyn Gwynant.*—This beautiful lake is less than a mile long, its total length being 1540 yards. It has a maximum width of 515 yards. Its waters cover an area of about 545,800 square yards, and from this its mean breadth has been calculated to be 354 yards, which is 23 per cent. of the total length. The lake lies in the Vale of Gwynant, at a distance of about three miles from the head, and its main axis follows the trend of that valley, running from north-east to south-west. From its upper end an alluvial flat stretches up towards the head of the valley as far as Gwastad Agnes. This flat consists almost entirely of meadow-land, and along it winds the river Glaslyn, which, however, keeps pretty close to the Snowdon side, and enters the lake at the angle between its head and the right margin. Several other small streams join the lake, the majority of which come down from the hills on the left or south-east side. The right or north-west side of Gwynant is very uniform in outline, the margin of the lake running in almost a straight line. The south-eastern or left side is more irregular, the lake-margin sweeping round in a wide curve from the entrance of the river, so as to

cause the lake to broaden in its upper half, and then approaching nearer the opposite side, and causing the lake to narrow in its lower half, and especially near its termination. The hillsides are well wooded in places along the lake-margins. The ground rises steeply from the right bank to a height of over 1000 feet, the contour-lines following each other rapidly, but the interval between the 300 feet and 400 feet lines widens so as to form a shelf running above the lake. The opposite coast is flatter for some distance from the lake-margin, after which the hills again rise steeply to heights of over 1000 feet. Towards the lower end the hills close in from both sides, and from a distance appear to shut in the lake completely. An exit is found, however, from the narrow square-cut end, and the river goes on its way down a lovely glen to reach Llyn Dinas lower down the valley.

The work of sounding was done in July 1900 by Mr Philip Lake and myself. Finding a convenient bench-mark near the lake-margin, we used an Abney's level to determine the altitude of the water-surface. We found it to be 215·5 feet. This is a little less than that found by the Ordnance Survey in February 1886, which is given on the map as 216·9 feet; the water no doubt standing a little higher in winter than in summer.

The total number of soundings taken was 105. The greatest depth found was 54 feet. The mean depth is 18·6 feet, being 36 per cent. of the maximum depth. The volume of water contained in the lake is estimated to be approximately 91 million cubic feet. The deepest water lies in an irregular narrow depression defined by the 50 feet line. This area is found about half way down the lake, but close to the right margin, where the rocks slope at a high angle into the water. All the contour-lines follow each other closely off the right bank, but they widen out in all other directions. The areas enclosed within the 40 feet, 30 feet, and 20 feet contour-lines narrow towards their upper ends and broaden out towards their lower ends, so as to approach nearer to the left bank where the lake begins to narrow. Except at this place, the loch is shallow on the left side, and it is also shallow at the head and for a long distance from its lower end. Over 40 per cent. of the area of the lake has a depth of under 10 feet, and only 13·6 per cent. of the area has a depth of over 40 feet.

Three sections are shown with the map (Plates V. and VIII.). The longitudinal section A–B shows that the basin is deepest about the middle in that direction, and that it shallows rapidly towards the lower end, and more gently towards the upper end. The cross-section E–F brings out very clearly the fact that the deeper water lies close under the right bank, and that the lake shallows rapidly towards the left bank. C–D is a section taken across the lake where it begins to narrow, and shows that its slope becomes comparatively steep here from the left side also. An isolated sounding of over 30 feet was obtained a short distance from the left bank, along the line C–D, causing the irregularity shown in the section.

Llyn Dinas.—This lake, little less beautiful than Gwynant, lies a mile and a half farther down the valley, and its main axis runs in the same direction—from north-east to south-west. Much of the valley between the two lakes is occupied by drift, through which the river is gradually cutting its way. The valley broadens at some distance above the lake, and the river Glaslyn here flows over low ground, and enters the lake at the apex of a large delta. The lake is broadest in its upper half, and narrows as we approach its lower end. It is not as long as Gwynant, its exact length being 1240 yards. Its maximum width is 420 yards, and it covers an area of about 371,920 square yards. The mean breadth is 300 yards, and this is 24 per cent. of the length. Several tributary streamlets join the lake from the left side, the largest of which is the Afon Goch, entering near its foot. The shore-line is, on the whole, very even, the only irregularities worth noticing being those formed by the delta at the head, and a small grassy promontory jutting into the lake from the left side, opposite the middle of the lake.

The hills rise steeply on the Snowdon side to a height of over 1000 feet. On the opposite side the coast is flatter for some distance from the head, after which the hill-sides again rise steeply. Near the end of the lake the valley becomes more contracted, and the water flows out by a weir through a narrow strait between rocky hills. The upper part of the lake is very shallow, and water-plants advance into the lake for a long distance from its upper left hand corner. It is evident that the lake is being gradually silted up towards its head.

In the work on Llyn Dinas I had again the assistance of Mr Philip Lake. The total number of soundings taken was 61. A bench-mark being found near the lake-margin, we were able, by means of an Abney's level, to ascertain the elevation of the water-surface to be 175·45 feet, as against 175·9 feet obtained by the Ordnance Survey in June 1886.

Llyn Dinas is a shallow lake, the greatest depth recorded being only 30 feet. The small area attaining this depth lies near the middle of the lake. The mean depth is but 12·9 feet; this is 43 per cent. of the maximum depth. The bulk of water in the lake is calculated to be about 43 million cubic feet. A glance at the map (Plate V.) will show that the configuration of the lake-bottom is very simple. A large proportion of the upper half of the lake is under 10 feet in depth. The deeper areas extend more into the lower half, and have a somewhat oval shape. Of the total superficial area as much as 48·8 per cent. corresponds to depths under 10 feet, and only 27·6 per cent. corresponds to depths of over 20 feet.

The longitudinal section given (Plate VIII.) shows that most of the water lies in the lower two-thirds of the lake, the upper third being very shallow. The slope is gentle from both sides, and the cross-sections C–D, E–F, give pan-shaped figures. The lake-bottom was very muddy, and often the lead was only pulled out with difficulty. The lake is, in all probability, getting silted up rather rapidly.

9. *Llyn Cwellyn.*—Snowdon is bounded on the west by the Gwyrfai Valley. Near the head of this valley lies a small and shallow lake bearing the name of Llyn-y-Gader. The river Gwyrfai issues from this lake and runs in a northerly direction, to lose itself

a little more than a mile lower down in the waters of Llyn Cwellyn. Llyn Cwellyn, or Cawellyn as it ought to be spelt, is a fine lake, nearly a mile and a quarter in length, with Snowdon rising on one side and Y Mynydd Mawr on the other. The main axis lies in the same direction as the valley—from south-east to north-west. This lake is wider than any of the other lakes of Snowdon, and a fairly uniform breadth is maintained from its head downwards to near its lower end, where it narrows a little. The river Gwyrfai flows in at the upper end, a short distance from the left margin. Other tiny streamlets run in further to the right, and several, coming down the slopes of Snowdon on the one side and of the Mynydd Mawr on the other, also find their way into the lake. The river Gwyrfai escapes again at the foot of the lake, and runs for about half a mile over an alluvial flat as far as Nant Mill, where it falls over rock. An alluvial flat also extends from the head of the lake for a quarter of a mile up its valley, so originally the lake must have been quite two miles long. At present the length of the lake is 2120 yards, and its greatest width is 665 yards. The area covered by its waters is 1,069,600 square yards, and its mean breadth is 505 yards, this being 24 per cent. of the length. The hill sides rise steeply from the left shore to heights of over 1000 feet. On the right the slope is a little more gentle, and the 500 feet contour-line lies farther off from the shore. The lake-margin on the Mynydd Mawr side is nearly straight, but on the Snowdon side the shore-line is more wavy and irregular. The valley narrows towards the lower end of the lake, and at Nant Mill, where the alluvial flat ends, the hills close in from both sides, a rocky barrier extending right across the valley, over which the river falls in a cascade.

Llyn Cwellyn abounds with the red char, a fish said to be peculiar to Alpine lakes. This fish also thrives in the lakes of Llanberis, but does not occur in any of the other lakes of this district.

The surface of Llyn Cwellyn is, according to the Ordnance Survey Maps, 463·6 feet above sea-level. The lake was sounded on 28th August 1900. The conditions for carrying on the work were most favourable until towards the close of the day, when a strong wind arose, blowing up the lake, in consequence of which the courses taken across the lake near its upper end deviated somewhat from a straight line.

Twelve sections were made, and these included in all 217 soundings. The greatest depth observed was 122 feet, and this was found about a fourth of the way from the lower end of the lake. The bulk of the water contained in the lake is estimated at 713 million cubic feet, and the mean depth at 74 feet, being 61 per cent. of the maximum depth; so Llyn Cwellyn may be regarded as a relatively deep lake. It will be observed, from an examination of the map (Plate V.), that the deeper part of the lake approaches much nearer the lower than it does the upper end. The 120 feet depression has an elongated form, is widest at its lower end, and tapers to a narrow channel as traced upwards. It corresponds to only 2·3 per cent. of the total superficial area of the lake. The 110 feet depression is over half a mile in length, and is of a considerable extent. It is broadest down lake, and tapers up lake obliquely towards the Mynydd Mawr shore.

The area enclosed within the 100 feet contour line corresponds to as much as 35 per cent. of the total superficial area of the lake. The area with depths of over 60 feet is estimated at about 64·5 per cent. of the total area. The area between the shore-line and the 20 feet contour-line amounts to only 12·7 of the entire superficial area of the lake.

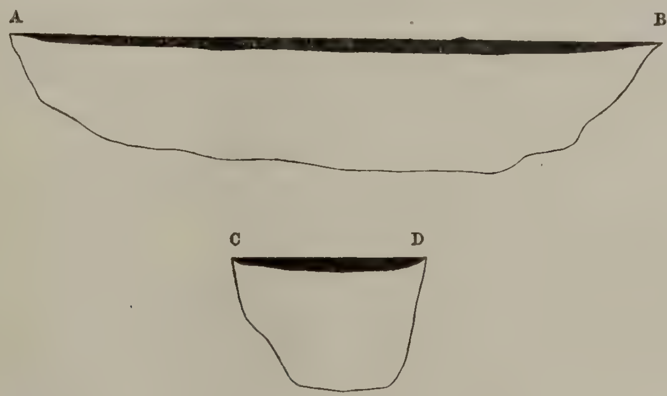


FIG. 2.—Sections of Cwellyn. The black portion shows the true slopes; the outline shows the slopes exaggerated ten times. A–B longitudinal; C–D cross section.

A comparison of the map with the sections (fig. 2 and Plate VIII.) makes it clear that the lake has the form of a simple trough, with steeply sloping walls and a nearly flat floor. For the greater part of the way the slope is steeper on the Mynydd Mawr side, but towards the lower end of the lake, where the section C–D is taken, the slope is somewhat steeper on the Snowdon side. The floor slopes up steeply at both ends, as shown in the longitudinal section A–B, and the deeper water is seen to lie towards the foot of the lake. C–D is a section taken across the deepest part, and shows how the bottom of the lake forms a nearly flat plain. E–F is a section across the middle of the lake, where the slope is steeper on its left side than it is on the right. The section G–H is taken obliquely, and the flatness of the bottom is not so well shown. Maps and sections bring out clearly the fact that this lake-basin has a pronounced chaldron structure. In this it resembles the lakes of Llanberis, which lie on the other side of Snowdon.

10. *Llyn Idwal.*—This gloomy lake lies in the great hollow of Cwm Idwal, a high mountain valley at the head of Nant Francon, and penetrating into the mountains on the south-west. It is backed by the steep sides of Y Glyder Fawr and Y Garn, and the rocks towering upwards at the far end of the cwm are cleft into a great chasm known as Y Twll-Du, or the Devil's Kitchen, which is famous for its Alpine plants. Below the Devil's Kitchen, in the bottom of the cwm, lies Llyn Idwal, a small lake nearly half a mile in length, the main axis of which runs north and south. The main features of this valley were described over half a century ago by DARWIN, and again more fully by RAMSAY nearly twenty years later. Evidences of the former existence of glaciers are to be seen on all sides.

A huge terminal moraine dams back the water at the lower end of the lake. Long green mounds, arranged one within the other, skirt the western side, and are probably the remains of lateral moraines. Remnants of smaller terminal moraines also occur at the upper end of the lake, projecting into the water where the lake narrows. On the east lies a slope of drift, through which the bare rock is often seen rising. The lake is broadest at the lower end, where it has a fairly even outline, but as followed upwards, it narrows and becomes more irregular until near the head it is quite constricted. Several small streamlets trickle down the face of the cliffs at the head of the cwm, and find their way into the lake. After heavy rains the volume of water coming down is largely increased. A few others run into the lake from the slopes on the east and west sides. The river Idwal flows out at a point on the western side, a short distance from the lower end of the lake, and runs north-eastwards towards the head of Nant Francon.

The lake lies at a high altitude, the elevation of the water-surface above sea-level being, according to the Ordnance Survey Map, 1223·4 feet. When I visited the lake in July 1900, the water was abnormally low, as shown by the markings on the rocks. This was confirmed by the boatman, who knew Idwal well. The length of the lake is 846 yards, and it has a maximum width of 340 yards. The area covered by the water is about 159,300 square yards, and the mean breadth is estimated at 188 yards, being 22 per cent. of the length. The number of soundings taken was 81.

The bottom proved to be very irregular; in places it was very muddy, but over a large part of it boulders of all sizes seemed to be scattered about, and this must have interfered to some extent with the soundings, for the lead, coming in contact with a huge boulder, would at that point indicate less than the real depth.

The greatest depth registered was 36 feet, and this was obtained in two places. The total volume of water is calculated to be 16 million cubic feet, and the mean depth 11 feet, being 31 per cent. of the maximum depth. The greater part of the lake is extremely shallow, 57·3 per cent. of the total area corresponding to depths under 10 feet. A glance at the map (Plate V.) will show that the deepest part lies close to the west shore, a little below the middle of the lake, and from this shore, opposite the depression, a rocky knob dips steeply into the water. The areas enclosed within the 20 feet and 10 feet contour lines lie in the lower half of the lake; but a small depression, over 10 feet in depth, lies in the narrow part, near the head of the lake.

A longitudinal section is given (Plate VIII.) along the axis of maximum depth, and shows clearly that the hollow is deepest in its lower half. In the neck, which separates off the narrow part at the head of the lake, the depth is nowhere more than 3 feet.

Two cross-sections are given, C–D at the lower end of the lake, and E–F at its middle. At the lower end, the sides slope gently from both banks to a nearly flat bottom. The configuration of the lake bottom, in the deeper area near the middle, shows more irregularity. Section E–F shows an abrupt descent from the western shore into deep water; that from the eastern shore is more irregular, a small rocky islet lying at a short distance off the lake-margin, and a ridge rising from the deeper water, a little more than midway across the lake. The irregularities found in the deeper part of the lake are due either to small rocky knobs projecting upwards from the lake bottom, or to the irregular distribution of boulders, possibly to both.

11. *Llyn Ogwen.*—This lake is situated at the head of the Pass of Nant Francon, and lies at an altitude of nearly 1000 feet above sea-level. It is a fine sheet of water, nearly a mile in length, and much celebrated for its trout. The main axis of the lake runs due east and west. It is bounded on both sides by lofty mountains. On the northern side Y Braich Ddu, a shoulder of Carnedd Dafydd, slopes down from a height of over 3000 feet to the water's edge; at the south, the rugged mass of Y Tryfan lifts its peaked crown to a height almost as great, and beyond lie the Glyders. The lake is broadest at the eastern end, and narrows towards the west, where the hills press in from both sides to form a gorge. Looking along the lake from the eastern end, one would think that it was shut in to the west by lofty hills, which appear to stretch right across the valley. But at the foot of the lake the gorge becomes still narrower, and then terminates abruptly, the outflowing river plunging over rock down the Falls of Ogwen into the deep and flat valley of Nant Francon, which lies below. Nant Francon signifies "the Glen of the Beavers": these animals formerly existed in Wales, and their skin was in much demand in the tenth century, as shown by the laws of the king, Howell Dda. From the upper end of the lake an alluvial tract stretches eastwards for about half a mile, and a few hundred yards further lies the watershed, which only rises to an elevation of 1050 feet. So the lake was originally considerably larger, and it is quite possible that formerly it drained eastwards towards Capel Curig, as has been suggested by Brend.* The present watershed is marked by a mound of drift. But at the head of the alluvial tract, bare rock is seen stretching most of the way across the valley as a low ridge.

The surface of the lake, according to the Ordnance Survey map, stands at an altitude of 984·4 feet above sea-level. The exact length of the lake is 1700 yards, and its maximum breadth 415 yards. The total area covered by its waters is approximately 456,400 square yards, and the mean breadth 268 yards, being 16 per cent. of the length. The Afon Dena enters the lake at its head near the left margin. Another streamlet, coming down from Ffynnon Lloer, a small mountain tarn lying well up on the Carnedd Dafydd, flows into the head of the lake at the opposite corner. The only other stream worth noting is one coming down from Cwm Bochlwyd on the left, and running into Llyn Ogwen near its lower end, its point of entrance being marked by a delta. The Holyhead road runs along the left margin; and the cliffs above the road are beautifully glaciated, and on them are stranded a number of erratics.

A well-marked promontory, made up chiefly of huge blocks and boulders which have rolled down from the slopes above, projects into the water from the southern side opposite the middle of the lake. The northern shore-line is more regular, and the hill-sides rise steeply from the lake-margin.

* *Geol. Mag.* 1897, p. 404.

The work on Ogwen was carried out in June 1901, and the weather being beautifully calm, a very complete survey was taken. Ten sections were run, including 102 soundings. The lake is remarkable for its extreme shallowness, the greatest depth registered being only 10 feet. The total bulk of water comes to about 28 million cubic feet, and the mean depth is 6·8 feet, this being 68 per cent. of the maximum depth. A glance at the map (Plate VI.) will show that practically the whole lake is under 10 feet, that depth only being obtained in two small and isolated areas. Contour-lines have been put in to mark depths of 8 feet and 5 feet. The configuration of the lake bottom is extremely simple ; 47·6 per cent. of the total superficial area is enclosed within the 8 feet line. The area attaining this depth lies mostly in the eastern half of the lake, and is broadest near the head, narrows as traced westwards, and terminates about a third of the way from the foot of the lake. Except along the margins and opposite the points where the streams flow in at the head, depths of five feet are everywhere attained.

A thick yellow mud extends like a carpet from near the foot of the lake to near the upper or eastern end, where the mud is replaced by very fine sand, and afterwards, as the mouth of the river Dena is approached, by coarser sand and gravel. Under the water along the margins lies a row of boulders, and at a few places rushes and other water-plants extend into the lake.

The longitudinal and cross-sections given (Plate VIII.) bring out clearly the remarkable flatness of the lake bottom. It has the form of a shallow pan. Llyn Ogwen is evidently becoming filled up gradually and evenly. Were the lake to disappear entirely, a flat meadow-like expanse would be left with the stream meandering through it.

12. *Llyniau Mymbyr.*—These two lakes are situated at the lower end of Nant-y-Gwrhyd, a bare and open valley extending from Penygwrhyd, near the base of Snowdon, eastwards for a distance of four miles to Capel Curig. To the north lie the slopes of Y Glyder-fach, and the vale is bounded on the south by the base of Moel Siabod. The bottom of the valley is largely covered with drift, which extends upwards on the hill-slopes for some distance. At Capel Curig, Nant-y-Gwrhyd is met by Nant-y-Benglog coming from the direction of Llyn Ogwen in the north, and the two join to form Dyffryn Llugwy, or the Vale of the Llugwy, which runs down to Bettws-y-Coed. The lakes stretch upwards from the lower end of Nant-y-Gwrhyd for a mile up the valley. The many peaks of Snowdon rise grandly in the distance, and the view of that mountain obtained from these lakes is unsurpassed. The lakes are now separated by a tongue of land, covered with grass, which seems to be made up of boulders and rock-rubbish. This is not a stream delta, but appears to have been formed by the slipping down of rock material from the hill-sides to the left. The stream joining the two lakes runs to one side just under the southern margin.

The river Gwrhyd flows into the upper lake on the left side near the upper end at the apex of a delta. Another streamlet from the south joins the lake at its head. There are no other tributary streams of any importance, but a few tiny rills run in from the left side, and one coming down from Moel Siabod enters the lower lake near its foot. The valley narrows a little towards the outlet of the lower lake, and the stream issuing from it joins the Afon Llugwy just below Capel Curig.

The hills rise to heights of over 1000 feet on both sides of the valley opposite the lakes, but the slope from the left shore is, for a short distance from the lake, a little less steep than is that from the right shore. The land rises to 700 feet just at the head of the upper lake, and the Gwrhyd river makes its way down by means of a waterfall. The upper lake is the broader, and the lower lake narrows rapidly towards its foot.

These lakes were sounded in June 1901, and for the work I had the assistance of Mr H. Woods, M.A., St John's College, Cambridge.

The upper lake has a length of 760 yards, and a maximum breadth of 412 yards. The total area covered by its waters is about 221,520 square yards, and the mean breadth 291 yards, being 38 per cent. of the length. The elevation of the water above sea-level is 588 feet. The total number of soundings taken was 63. The lake is nowhere of any great depth, 29 feet being the deepest sounding obtained. The total volume of water is approximately 22 million cubic feet, and the mean depth comes to 11 feet, being 38 per cent. of the maximum depth. The deepest area lies near the middle of the lake, and is almost as broad as long. It is enclosed within the 20 feet contour-line, and corresponds to 15·6 per cent. of the total superficial area of the lake. The 10 feet subaqueous line encloses an area somewhat similar in form, but having an irregular prolongation towards the head of the lake. 45 per cent. of the total area lies within the 10 feet line, leaving 55 per cent. for depths under 10 feet. Three sections are given with the map (Plates VI. and VIII.). They show that the form of the lake bottom is fairly simple, the sides sloping gently from both banks. Huge boulders lie in the water at the foot of the lake, just off the projecting tongue of land.

The lower lake is longer but narrower than the upper lake. It has a total length of 880 yards, and its greatest width is 330 yards. The lake covers an area of approximately 174,000 square yards, and its mean breadth is 198 yards, being 22·5 per cent. of its length. The surface stands at practically the same level as that of the upper lake, the elevation given on the Survey map being 587·9 feet. In all, 38 soundings were taken. The greatest depth recorded is 29 feet, the same as that obtained in the upper lake. The mean depth is only 9·7 feet; this is 33 per cent. of the maximum depth. The total bulk of water is estimated to be approximately 15 million cubic feet. The deepest area lies a little lower than the head of the lake, and is enclosed within the 20 feet contour-line. It is smaller in extent than the area of similar depth in the upper lake, and corresponds to only 9·8 of the total area.

The 10 feet contour-line is very irregular, the area lying within it being broadest towards the head, and narrowing into a channel as followed towards the lower end of the lake. As much as 62 per cent. of the total area is covered by water under 10 feet deep. The longitudinal section L–M shows that the hollow is deepest at its upper end, and that it is very shallow near the foot of the lake. The bottom is more irregular, as shown by the cross-section N–O, P–Q, than is that of the upper lake.

The channel between the two lakes is, for the greater part, shallow and full of huge boulders, but at one point near the south side the sounding-line indicated a depth of 22 feet. This is not marked on the map.

13. *Llyn Crafnant.*—This lake is situated in the wild hilly country between Capel Curig and Trefriw. In form it is long and narrow, running from south-west to north-east, and draining into the valley of the Conway.

The surroundings of Crafnant are very striking, high hills appearing to shut it in on all sides. Above the head of the lake the hills sweep around to form an irregular amphitheatre, and Crafnant is widest in its upper reaches. Its width lessens gradually as traced downwards, the hills encroaching from both sides, and finally converging so as to render the lower end of the lake extremely narrow. The outflowing stream makes its escape through a narrow gap or gorge between the hills. The hills rise to the heights of over 1000 feet from the lake sides, but the slope is steeper from the left margin than it is from the right. Near the head, especially towards the south side, the hills fall back for a considerable distance. The main tributary stream—the Afon Hendre—flows in at the head of the lake ; other streamlets run in on both sides, the largest of which is one coming down Ceunant Parc on the left, near the upper end.

Crafnant has at present an area of 297,300 square yards. The total length is 1390 yards, and so its average breadth comes to 214 yards, being 15 per cent. of its length. The lake is widest not far from its upper end, reaching a maximum breadth of 320 yards. Near its foot it narrows to about 50 yards. At its upper end the lake is undergoing a silting-up process, and an alluvial tract now extends upwards for about a quarter of a mile from its head, so that formerly the lake covered a considerably greater area.

The axis of Llyn Crafnant is nearly straight, and the shore-lines do not show any prominent irregularities.

The work on Crafnant occupied the 10th and 11th of August 1900. During the second day the wind was at times exceedingly troublesome. According to the Ordnance Survey in May 1887, the surface of the water was 602·5 feet above sea-level. More soundings were taken in this lake in proportion to its size than in any of the other lakes, the total number amounting to 230. The greatest depth recorded was 71 feet approximately near the middle of the lake. The total bulk of water is estimated to be about 83 million cubic feet, and the mean depth comes to 31 feet, this being 44 per cent. of the maximum depth. The deepest area, with depths of over 60 feet, lies a little above the middle of the lake, at about equal distances from both margins. It corresponds to 9·4 per cent. of the total superficial area, and is somewhat pear-shaped in form, the broadest part being near its lower end. Nearly 36 per cent. of the superficies lies within the 40 feet contour-line, and this area, widest opposite the middle of the lake, extends to within nearly equal distances of both ends. Its outline is irregular, bulging out in the centre, and narrowing to form somewhat blunt terminations both ways.

The 20 feet line, enclosing 64·6 of the whole area, extends to within 100 yards of the foot of the lake, but at the other end leaves an interval of about 250 yards between it and the head. It is closely followed by the 10 feet contour-line, only 22·9 per cent. of the total area lying between these two lines, leaving 22·5 per cent. of the area for depths under 10 feet, and of this the greater part lies at the head of the lake, which is very shallow. Not far from the foot of the lake a ridge rises from the bottom, and, on very low water, appears as a small rocky islet at the surface.

An examination of the map and of the sections given (Plates VI. and VIII.) will show that the shape of the lake bottom is somewhat complicated. The 10 feet and 20 feet subaqueous lines keep fairly close to the shore-line except at the upper end. The isobath of 40 feet diverges considerably from the land, and its form is in no way dependent on that of the coast-line. The 60 feet isobath is again quite peculiar in outline, showing no relationship to the trend of the other subaqueous lines.

The longitudinal section A–B shows how the accumulation of sediment has shallowed the upper part of the lake as compared with the lower part. The main depression is seen to lie about midway along the lake. Just where section C–D is taken, the slope is a little greater on the left side, but a little lower down it becomes much greater from the right side. Opposite the middle, where the section E–F is taken, the slope from both sides to the deepest part is about the same. G–H is a section taken towards the lower end of the lake, showing the ridge rising nearly midway between the two sides.

14. *Llyn Geirionydd.*—This is the most easterly of the long, narrow lakes situated in the country lying north of Capel Curig, and, like the others, it drains into the valley of the Conway. Its surface lies at an altitude of 616·4 feet above sea-level. The main axis of the lake runs nearly north and south. It has a total length of 1348 yards, and a maximum breadth of 356 yards. The mean breadth is 221 yards, and this is 16 per cent. of the length. The area covered by its waters is approximately 297,600 square yards.

On both sides of the lake the ground rises steeply to heights of over 1000 feet, the steepness being somewhat more marked on the left or western side than it is on the right or eastern side.

An alluvial flat extends a little way up from the head of the lake. The chief tributary stream enters the lake at its head, other smaller ones flow in a little lower down. The Afon Geirionydd makes its escape from the lake at its foot. The right margin is very regular along its whole length, but there is a marked irregularity on the left margin in the shape of a bay, which occurs about a third of the way from the lower end.

The work on Geirionydd was carried out in July 1901. A small punt had to be brought up on a cart from Trefriw, and this proved to be not very well adapted for the purpose. At the time when the soundings were taken the water was exceptionally low; small rocky islets stood well out of the water at the lower end. These were beautifully glaciated on the side facing up the lake, but were rough on the side facing down lake.

In all, nine sections were run, including 122 soundings. The greatest depth obtained was 48 feet; this was found in three places close to each other, about a quarter of the way down the lake from the head. The mean depth is 21·3 feet, being 44 per cent. of the maximum depth. The total bulk of water contained in the lake is approximately 57 million cubic feet. Geirionydd is deepest in its upper half.

The area enclosed within the 40 feet subaqueous line lies a little nearer to the left margin than it does to the right. It is club-shaped in form, with the broad end up lake, and the narrow end terminating about mid-way from both ends near the middle of the lake. This area corresponds to 9·7 per cent. of the total superficial area of the lake. The area enclosed within the 30 feet contour-line has a length of about 650 yards, and in all encloses 27·8 per cent. of the total area. It does not extend as far down as the bay which occurs on the western shore. The area within the 20 feet line has a length of about 1000 yards, and extends to within 200 yards of the foot of the lake. This contour-line is unaffected by the bay on the west side. Over half of the total area of the lake lies within this line.

The 10 feet contour-line keeps close to the shore-line along the right and left banks, and follows the curve of the shore-line at the bay; it extends to within 100 yards of the two ends of the lake. The area between the shore and the 10 feet line is estimated to be 23·8 of the total superficial area.

An examination of the map and sections (Plates VI. and VIII.) will show that opposite the deepest part the slope of the lake-bottom is steeper from the western shore than it is from the eastern. But in the lower reaches, opposite the bay, the steepest slope is from the eastern bank. The hollow in which Geirionydd lies is trough-like, with somewhat steep sides and an approximately flat bottom.

15. *Llyn Cawlyd.*—This is the largest of the numerous lakes situated in that mountainous district lying between Bangor, Capel Curig and Conway, and it is the deepest of all the lakes dealt with in this paper. Its upper end is about two miles due north of Capel Curig, and the lake, which is over a mile and a half in length, trends from south-west to north-east, and drains into the valley of the Conway. Cawlyd lies almost parallel with Geirionydd and Crafnant, but is situated at a much higher elevation, being considerably over 1000 feet above sea-level. The lake is situated at the head of a long narrow valley, and the mountains rise steeply from both banks to heights of over 2000 feet. The slope of the hill on the right or southern side is very steep, and along a great part of its length it is flanked by screes. On the right or northern side the slope is not quite so steep, and about half-way down the lake, the hills fall back a little, leaving a band of more gently rising ground immediately bordering the lake at the lower end. The hollow in which the lake lies, ends abruptly at the upper end, the ground rising fairly rapidly to the watershed, which is only about 300 feet above the level of the lake. Cawlyd is very narrow at its upper end, broadens a little towards the middle, but narrows somewhat again towards the outlet. The shore-line on both sides is very regular, there being no marked promontories or bays. There is no alluvial tract in this case stretching upwards from the head of the lake. Cawlyd is remark-

able in that no streams of any importance run into it. Several tiny streamlets are marked on the map as flowing in from the left side, and the largest of these occurs near the lower end. Those marked as entering the head of the lake are the merest rills. The right side is almost altogether free from water-courses. But after heavy rains, no doubt water comes flowing down the mountain slopes, especially on the left side. From the foot of the lake escapes the Afon Ddu, which pursues its way north-eastwards through a gently sloping valley largely covered with drift.

The surface of Llyn Cawlyd is, according to the Ordnance Survey map (April 1887), 1164·6 feet above sea-level. It has a total length of 2855 yards, with a maximum width of 495 yards. The mean breadth is 335 yards, being 12 per cent. of the length. Its waters cover an area of about 957,800 square yards. The lake was surveyed on 25th June 1901, under the most favourable weather conditions. All the soundings were taken by myself, and the rowing was done by an excellent boatman. During the work on this lake, I was accompanied by Mr H. Woods, M.A., St John's College, Cambridge. The total number of soundings taken was 99. The greatest depth observed was 222 feet. The volume of water contained in the lake is estimated to be approximately 941 million cubic feet, and the mean depth at 109·1 feet, or 49 per cent. of the maximum depth. An examination of the map (Plate VII.) will show that the configuration of the floor of the lake is extremely regular and simple. The deepest area is approximately in the centre of the lake, and has the form of a long narrow depression, with depths exceeding 200 feet. Within this a still narrower depression, with depths over 220 feet, is marked on the map. The 200 feet depression has a maximum width of about 180 yards, and a length of a little over 1000 yards, and it has an area corresponding to nearly 16 per cent. of the total superficial area of the lake.

The 100 feet depression is over 2000 yards in length, with a maximum breadth of about 340 yards. The area enclosed between the 100 feet and 200 feet contour-lines is 35·6 per cent. of the entire area of the lake; this, added to the area lying within the 200 feet contour-line, gives over 50 per cent. of the total area for depths of over 100 feet.

The 40 feet depression is 2400 yards in length, with a maximum width of about 430 yards; the area enclosed between the 40 feet and 100 feet contour-lines is about 25 per cent. of the total area, and this added to the area lying within the 100 feet subaqueous line, gives over 75 per cent. of the entire area of the lake for depths of over 40 feet. The area lying between the 20 feet contour-line and the coast corresponds to only 23·9 per cent. of the entire area.

There is a comparatively shallow area near the foot of the lake, and the soundings gave indications of a subaqueous ridge rising from the lake bottom not far from the lower end.

Most of the contour lines follow pretty closely the outline of the lake, and so the form of the floor is very simple. It is a steep and narrow trough. Along the axis it deepens rapidly at first, and then more slowly to 200 feet a little more than a half of a mile from the head, maintains this depth for more than half a mile further, the ramp rising more rapidly towards the lower end of the lake until the 40 feet line is reached, after which the slope becomes gentle.

At the sides the isobaths follow each other rapidly, and the slope is very steep from both banks.

Representative sections of Cawlyd are given (Plate VIII.). The longitudinal section is along the axis of maximum depth. E–F is a section across the middle of the lake, C–D towards the upper end, and G–H towards the lower end. At the upper end the slope is steeper from the right bank; at the lower end it is steeper from the left bank.

16. *Llyn Dulyn.*—This lake is situated in the heart of the wild and mountainous district of Carnarvonshire, lying to the north-east of Nant Francon. Being very difficult of access, it is but rarely visited. Its dark and gloomy waters lie in a deep basin-shaped hollow, near the south-eastern base of Y Foel Fras, a mountain attaining a height of over 3000 feet. Just to the south-west of Llyn Dulyn the bare and rounded top of Carnedd Llewelyn is seen rising to a height of 3482 feet, which almost rivals that of Snowdon. Bare rock bounds the lake on all sides except the east; here a steep grassy slope rises, largely covered with boulders, but this appears to be merely due to a thin covering of drift, and the rock probably lies not far from the surface. On the western side the precipice is remarkable, the rocky face coming down into the water, almost perpendicularly, from a height of several hundred feet. Sheep often fall down this precipice, and a shepherd told the writer that he had counted as many as twenty dead bodies at one time. Several tiny streamlets come trickling down the mountain-sides into the lake, which drains into the valley of the Conway by means of Yr Afon Dulyn. This stream escapes at the south-eastern corner, flowing out over rock. Just at this corner there is a small bay, which is, however, shallow.

Llyn Dulyn is one of the highest mountain tarns in Wales, its surface lying, according to the Ordnance Survey (April 1900), 1747·2 feet above sea-level. It has a total length of 560 yards, with a maximum width of 380 yards. The mean breadth is 297 yards, being 53 per cent. of the length. The waters cover an area of about 166,520 square yards. The survey of this lake was carried out in July 1901 under fairly favourable conditions. The total number of soundings taken was 94. The lake proved to be remarkably deep, a depth of 189 feet being attained in one place. The mean depth is very great, being as much as 104·2 feet; this is 55 per cent. of the maximum depth. The bulk of water contained in the lake is estimated to be about 156 million cubic feet.

An examination of the map (Plate VI.) will show that the deepest area lies approximately in the middle of the lake. This area is enclosed within the 180 feet line, and corresponds to 10 per cent. of the total superficial area of the lake. The 160 feet subaqueous line encloses a somewhat heart-shaped area, with its broadest part towards the north; towards the west the left corner of the broad end approaches very near to the lake margin. 14·5 per cent. of the total superficial area lies between the 160 feet and the 180 feet subaqueous lines, so that in all 24·5 per cent. of the total area is covered by water attaining depths of over 160 feet. The 140 feet and 120 feet contour-lines follow the 160 feet line closely on all sides except to the north, where the interval between them becomes greater. The 100 feet contour-line encloses 52·8 per cent. of the total area, so that more than half of the lake is over 100 feet deep. The remaining contour-lines follow pretty closely the outline of the lake margin, and lie very near each other on the eastern, northern, and especially the western margins. On the south side there is a somewhat greater interval between them, indicating a slope which is not quite so steep. The 40 feet line encloses in all 82·2 per cent. of the total area, leaving only 17·8 per cent. for depths under 40 feet. The slope to the lake bottom is steep from all sides, but is especially so from the eastern and western margins opposite the middle of the lake. The slope is remarkable along the face of the great precipice which falls sheer into the water on the western margin. Here a depth of 55 feet was obtained at a distance of only 3 feet from the face of the rock.

The steepness of the sides and the form of the lake bottom are brought out clearly in the sections which are given (Plate VIII.). The waters of Llyn Dulyn are seen to fill up a deep funnel-shaped basin.

17. *Llyn Eigiau.*—This lake is situated about a mile and a half due north of Llyn Cawlyd, in the lower part of Cwm Eigiau. The main axis runs nearly north and south. It is probable that the lake formerly covered a much larger area, for an alluvial flat is found extending far up into Cwm Eigiau, and low marshy ground borders the lake on the east, and extends outwards for a considerable distance from the lake margin. On the left or west side the precipitous rocks of Craig Eigiau rise steeply from the water's edge to a height of nearly 2400 feet. The river Eigiau, coming down the cwm, enters the lake near its head, at the right hand corner, where a small delta has been formed. Just beyond this, to the right is a shallow bay covered with rushes, and from this bay escapes the river Porthlwyd; it makes its way north-eastwards into the Vale of Conway. A few other small rills enter the lake on the west side, and a tiny streamlet flows out at the foot. The lake is widest in its upper reaches; at a point a little above the middle it becomes constricted, owing to a promontory jutting out from the west side. To the north of this it broadens out a little again, to become very narrow once more near the foot.

The height of the surface of the water above sea-level is, according to the Ordnance Survey map (1887), 1219 feet. The total length of the lake is 1690 yards, and it has a maximum width of 520 yards. The main breadth is 255 yards, this being 15 per cent. of the length. Its waters cover an area of about 431,200 square yards.

The soundings were taken in July 1901 under most favourable conditions. It was sometimes difficult to fix points accurately on the lake margins, owing to the rather imperfect character of the Ordnance Survey map of this region. The total number of soundings taken was 90. The lake was found to be shallow, the greatest depth found being only 32 feet. The mean depth is only 9·2 feet, and this is 29 per cent. of the maximum depth. The total bulk of water contained in the lake is estimated to be about 36 million cubic feet.

Llyn Eigiau is deepest about a quarter of a mile from the upper end. The area enclosed within the 30 feet line is exceedingly small, amounting to only ·3 per cent. of the total superficial area. The 20 feet contour-line gives but a small oval-shaped area, which encloses only 2·7 per cent. of the total area. An isolated sounding of 22 feet was obtained also near the western margin, at a distance of less than a quarter of a mile from the foot of the lake. As much as 60·8 per cent. of the total area corresponds to depths under 10 feet. The 10 feet subaqueous line encloses an area which is smaller at the head, becomes constricted at the neck, and passes as a somewhat narrow channel to end at a distance of about 330 yards from the foot of the lake.

The bottom was found to be largely covered with boulders and stones, and at places rushes encroach well into the lake from the margins. Eigiau is evidently becoming gradually silted up, and shows a stage in the process somewhat less advanced than that seen in the case of Llyn Ogwen. Sections are given with the map (Plates VII. and VIII.), which help to make clear the configuration of the lake bottom.

VI. General Summary of the Characters of the Lakes.

For convenience of reference and comparison, the more important details concerning the different lakes are gathered together into three Tables. In Table I. are arranged the statistics for each of the sixteen lakes with regard to the number of soundings, length, maximum and mean breadth, ratio of average breadth to length, elevation of the surface of the water above sea-level, the maximum and mean depth, the ratio of mean to maximum depth, the superficial area of the surface, and the estimated volume of water contained in each lake. In Table II. all the measurements are given again in metric units, but the ratios are not repeated. Table III. shows the percentage of superficial area corresponding to different depths of water in the case of each lake. The arrangement of depths in the different lakes is shown at a glance, and an indication given of the form of the depressions in which the lakes lie.

It will be noticed that five of the lakes, namely, Dinas, Gwynant, Cwellyn, Padarn, and Peris, lie at comparatively low levels, having the surface of their waters at elevations of less than 500 feet above sea-level. All these lie in the valleys which immediately encircle the base of Snowdon. Their positions relative to the valleys in which they lie differ considerably, Padarn and Peris occurring at the foot of a valley just before it opens out on to the low grounds; Cwellyn lying high up its valley, at no great distance from the head; and Gwynant and Dinas being situated along the course of another valley, the former at a considerable distance below the head, and the latter still farther down.

TABLE I.—*Statistics of Welsh Lakes.*

Name.	Number of Soundings.	Length (yards).	Breadth (yards).		Mean Breadth per cent. of Length.	Elevation of Surface above Sea Level (feet).	Depth (feet).		Mean Depth per cent. of Max.	Area (square yards).	Volume, Million Cubic Feet.
			Max.	Mean.			Max.	Mean.			
Llyn Gwynant, ...	105	1540	515	354	23	215·5*	54	18·6	36	545,800	91
Llyn Dinas, ...	61	1240	420	300	24	175·45*	30	12·9	43	371,920	43
Llyn Idwal, ...	81	846	340	188	22	1223·4	36	11·0	31	159,300	16
Llyn Crafnant, ...	230	1390	320	214	15	602·5	71	31·0	44	297,300	83
Llyn Padarn, ...	286	3530	595	380	11	339·6	94	52·4	56	1,340,800	632
Llyn Peris, ...	246	1930	500	310	16	339·5	114	63·9	56	598,720	344
Llyn Llydaw, ...	292	1950	460	301	15	1415·9	190	77·4	41	587,200	409
Llyn Glaslyn, ...	107	535	275	197	37	1970·7	127	62·6	49	105,600	59½
Llyn Cwellyn, ...	217	2120	665	505	24	463·6	122	74·1	61	1,069,600	713
Llyn Ogwen, ...	102	1700	415	268	16	984·4	10	6·8	68	456,400	28
Llyn Cawlyd, ...	99	2855	495	335	12	1164·6	222	109·1	49	957,800	941
Llyniau Mymbyr, { upper lake, .	63	760	412	291	38	588·0	29	11·0	38	221,520	22
Llyniau Mymbyr, { lower lake, .	38	880	330	198	22½	587·9	29	9·7	33	174,000	15
Llyn Eigiau, ...	90	1690	520	255	15	1219·0	32	9·2	29	431,200	36
Llyn Geirionydd, . . .	122	1348	356	221	16	616·4	48	21·3	44	297,600	57
Llyn Dulyn, ...	94	560	380	297	53	1747·2	189	104·2	55	166,520	156

* Elevations found at time of sounding. Other elevations taken from the Ordnance Survey.

TABLE II.—*Statistics of Welsh Lakes in Metric Units.*

Name.	Length (metres).	Breadth (metres).		Elevation of Surface above Sea Level (metres).	Depth (metres).		Area (square metres).	Volume, Million cubic metres.
		Max.	Mean.		Max.	Mean.		
Llyn Gwynant, ...	1408	471	324	65·7	16·5	5·7	456,344	2·6
Llyn Dinas, ...	1134	384	274	53·5	9·1	3·9	310,963	1·2
Llyn Idwal, ...	774	311	172	372·9	11·0	3·4	133,191	·4
Llyn Crafnant, ...	1271	293	196	183·6	21·6	9·4	248,573	2·3
Llyn Padarn, ...	3228	544	347	103·5	28·7	16·0	1,121,045	17·9
Llyn Peris, ...	1765	457	283	103·5	34·7	19·5	500,591	9·7
Llyn Llydaw, ...	1783	421	275	431·6	57·9	23·6	490,959	11·6
Llyn Glaslyn, ...	489	251	180	600·7	38·7	19·1	88,292	1·7
Llyn Cwellyn, ...	1938	608	462	141·3	37·2	22·6	894,294	20·2
Llyn Ogwen, ...	1554	379	245	300·0	3·0	2·1	381,597	·8
Llyn Cawlyd, ...	2611	453	306	355·0	67·7	33·3	800,818	26·6
Llyniau Mymbyr, { upper lake, . .	695	377	266	179·2	8·8	3·4	185,213	·6
Llyniau Mymbyr, { lower lake, . .	805	302	181	179·2	8·8	2·95	145,482	·4
Llyn Eigiau, ...	1545	475	233	371·5	9·8	2·8	360,527	1·0
Llyn Geirionydd, ...	1233	326	202	187·9	14·6	6·5	248,824	1·6
Llyn Dulyn, ...	512	347	272	532·5	57·6	31·8	139,228	4·4

TABLE III.—*Welsh Lakes.*

Showing the Percentage of Superficial Area corresponding to different Depths of Water.

Feet,	0 to 10.	10 to 20.	20 to 30.	30 to 40.	40 to 50.	50 to 60.	60 to 80.	80 to 100.	100 to 120.	120 to 140.	140 to 160.	160 to 180.	180 to 200.	200 to 220.	Over 220.
Metres,	0 to 3·05.	3·05 to 0·1.	6·1 to 9·1.	9·1 to 12·2.	12·2 to 15·2.	15·2 to 18·3.	18·3 to 24·4.	24·4 to 30·5.	30·5 to 36·6.	36·6 to 42·7.	42·7 to 48·8.	48·8 to 54·9.	54·9 to 61·0.	61·0 to 67·1.	Over 67·1.
Llyn Ogwen,*	100														
Llyniau Mymbyr, upper lake,	55·0	29·3	15·6												
Llyniau Mymbyr, lower lake,	62·0	28·3	9·8												
Llyn Dinas,	48·8	23·7	26·2	1·4											
Llyn Eigiau,	60·8	36·4	2·4	·3											
Llyn Idwal,	57·3	28·0	11·1	3·6											
Llyn Geirionydd,	23·8	25·8	22·4	18·1	9·7										
Llyn Gwynant,	40·9	19·1	17·8	8·5	11·8	1·8									
Llyn Crafnant,	22·5	12·9	29·2		26·0		9·4								
Llyn Padarn,	18·1		12·1		21·2		35·2	13·5							
Llyn Peris,	17·2		11·0		12·2		17·4	26·4	15·7						
Llyn Cwellyn,	12·7		8·9		13·8		12·2	17·5	32·5	2·3					
Llyn Glaslyn,	15·7		16·9		15·2		15·8	17·0	14·5	4·9					
Llyn Dulyn,	10·2		7·6		7·9		9 6	11·9	11·0	8·4	8·9	14·5	10·0		
Llyn Llydaw,	18·8		17·0		13·5		8·8	8·0	5·9	7·2	7·4	10·0	3·4		
Llyn Cawlyd,	14·8		9·1		7·7		8·0	9·2	6·9	6·8	7·0	6·4	8·5	12·5	3·3

* The percentages for Llyn Ogwen are, 0 to 5 feet, 21·9 ; 5 to 8, 30·5 ; 8 to 10, 46·8 ; 10 feet, ·8.

In Llyniau Mymbyr (Capel Curig), which lie at an elevation of over 500 feet, and originally formed one sheet of water, we meet an example of a lake occurring at the lower end of a mountain valley (Nant-y-Gwrhyd), just above its junction with another mountain valley (Nant-y-Benglog). Below the lake these two mountain valleys unite to form the Vale of Llugwy.

Llyn Ogwen, lying at an elevation of nearly 1000 feet, is an example of a mountain valley lake situated right at the head of the valley.

Of the upland valley-lakes found in the wild mountain tract to the north-east of Capel Curig, Crafnant and Geirionydd have their surfaces at an altitude of only a little over 600 feet; but Cawlyd and Eigiau are at much higher levels, their surfaces standing at an elevation of over 1000 feet above the sea. All these lie in long, narrow valleys: Cawlyd high up in its valley, and extending almost right up to the head; Eigiau at a considerable distance below the head of a cwm of the same name; and Crafnant and Geirionydd, about midway between the heads of their respective upland valleys and the place where these unite lower down to form one valley.

Cirque, or corrie, or cwm lakes are common in the district, and three, or perhaps four, of these are dealt with in this paper. Idwal has its surface at an elevation of over 1200 feet, while Glaslyn and Dulyn are at heights not far short of 2000 feet. Llyn Llydaw may be looked upon as partly a cwm or corrie lake and partly an upland-valley lake. It lies at an elevation of over 1400 feet.

The lakes of Caernarvonshire are all comparatively small, the great majority of them being under a mile in length. The largest of them are those dealt with in this memoir. The double lake of Llanberis, Padarn and Peris, has, if we regard both as one sheet of water, a total length of over three miles. Of the others, only three, namely, Cawlyd, Cwellyn and Llydaw, exceed a mile in length. The widest of all the lakes is Cwellyn, with a maximum breadth of 665 yards and a mean breadth of 505 yards. The lakes may be arranged according to their depths into three main divisions—the shallow, the intermediate and the deep.

The *Shallow Lakes* are six in number, including Ogwen, Llyniau Mymbyr (2), Dinas, Eigiau, and Idwal. In all these the average or mean depth is under 15 feet, and in none of them does the maximum depth reach 40 feet.

The *Lakes of Intermediate Depths* include but three—Gwynant, Geirionydd and Crafnant; they have a mean depth of over 15 feet but under 35 feet, and a maximum depth of between 40 and 75 feet.

In all the lakes belonging to the above two divisions (except Ogwen, the bottom of which has been converted into a flat plain by the deposition of sediment), the ratio of the average to the maximum depth is considerably under 50 per cent.

The *Deep Lakes* comprise all the others, seven in all, namely, Padarn, Peris, Cwellyn, Glaslyn, Llydaw, Dulyn and Cawlyd. These have a maximum depth over 75 feet (all except Padarn are over 100 feet), and a mean or average depth exceeding 50 feet. In this division the mean depth approaches, indeed in most of the lakes exceeds, half of the extreme depth. In Llydaw, it is true that for the lake as a whole the ratio of the mean to the maximum depth falls to 41 per cent., but this is due to the shallowing caused by morainic material towards the lower end; in the upper deep basin the ratio is much greater than this. PENCK and MILL have pointed out that where, as in the lakes of this division, the ratio of the average to the minimum depth is great, the hollow presents a pronounced chaldron structure. All the deep lakes lie in depressions which have steeply sloping walls, as is well brought out in Table III., which shows the large percentage of areas at great depths; this is also illustrated in the sections which are appended with the maps.

It is interesting to note that the deepest lakes often occur at very high elevations. One of the most remarkable facts brought to light is the great depth of some of the cwm or corrie lakes. They usually lie in crater-like hollows with somewhat circular outlines. Glaslyn, with a total length of 535 yards and a mean breadth of 197 yards, has a maximum depth of 127 feet and a mean depth of 62·6 feet, which is nearly half of the maximum depth. Dulyn is still more striking; with a total length of 560 yards, and a mean width of 297 yards, it has a maximum depth of 189 feet, and a mean depth of 104 feet, this being 55 per cent. of the maximum depth. Thus it is seen that these cwm-lakes are remarkably deep in proportion to their extent, much more so than is the case with the valley-lakes at low levels. Idwal is shallow, and does not show the characteristics of the other cwm lakes; it lies at a much lower level, and its shallowness is partly due to the great amount of morainic débris scattered over its floor, and partly to the silting-up process which has been going on. Rock-falls are continually taking place at the head of the cwm, and the detritus is gradually carried into the lake. Llydaw lies in a cirque-valley, the upper end of the lake resting in a corrie, and it is here that the basin is deepest, a depth of 190 feet being obtained, and formerly, before the artificial lowering of the lake took place, it exceeded 200 feet.

Of the valley-lakes the most striking is Llyn Cawlyd, which lies at a high level. It has a maximum depth of over 200 feet, and a mean depth of over 100 feet. It is deeper than any of the other lakes sounded. The only other lakes which exceed 100 feet in depth are Peris and Cwellyn, which lie at the base of Snowdon.

Padarn, Peris and Cwellyn lie in hollows which are trough-like with steeply sloping sides and a somewhat flat floor.

The other valley-lakes present more basin-shaped cross-sections. The deep cwm-lakes lie in deep basin-shaped hollows with steep sides. Ogwen shows a pan-shaped structure on cross-section, and this is characteristic of a lake which is in process of being silted up evenly.

The deep cwm-lakes are deepest approximately in the centre. But it is noteworthy that in the valley-lakes the deepest part of the basin often lies in the upper portion towards the head. This is the case in Llydaw, Eigiau and Geirionydd. Of the Llanberis lakes, too, the higher one is the deeper, and in Padarn (the lower lake) we find the deeper area near the head. The deepest part is approximately central in Gwynant, Dinas, Crafnant, Cawlyd and Llyniau Mymbyr. In Cwellyn only do we find the greatest depth situated towards the low end of the lake, and in this case the lake is very deep all along to near the head.

All the different stages in the existence of lakes are exemplified amongst those found in Carnarvonshire. Idwal, Eigiau, Llyniau Mymbyr, and Dinas are gradually becoming filled up, the deposition of sediment forming shallows over which vegetation pushes its way from the lake-margins. The sections across certain portions of those lakes show a tendency to a shallow pan-shaped structure. This structure, however, is most beautifully seen in the case of Ogwen, which is in the last stage of its existence. This lake is filling up very evenly, a carpet of thick mud or clay being spread over the bottom. Were the water finally to disappear, a flat meadow-like expanse would be left.

Traces of old lakes which have finally disappeared are abundant in this district. Their place is now often marked by a turbary or peat-moss, as may be seen in Nant-y-Benglog, between Ogwen and Capel Curig. Alluvial flats are also common, marking the sites of old lakes which have become drained at the exit. Some flats occur along the Vale of Gwynant and at Beddgelert. But the most remarkable is that which occurs on the bottom of Nant Francon between Ogwen bank and the falls of Ogwen (Plate II. fig. 2). The valley, for a distance of almost three miles, presents a remarkably flat bottom, with steeply-sloping sides, and this flatness is maintained right up to the head, where the river Ogwen descends over a steep rocky escarpment from the lake at the head of Nant Francon. The photograph shows how the river meanders through the flat meadows which now mark the site of the old lake. The lake must have been nearly equal in extent to the double lake of Llanberis; but, judging from the shape of the ground, it was probably deeper. The depression in which the lake rested had here also the form of a simple trough, with steeply-sloping walls, and a nearly flat floor.

Comparing the lakes of Snowdonia and of eastern Carnarvonshire with the English lakes described by Dr Mill, we note that the latter are all valley-lakes, occurring at a comparatively low level, only one of which, namely, Haweswater, has its surface at an elevation of over 500 feet. The English lakes are generally much larger in area than are the Welsh lakes. No large lakes comparable in extent to Windermere and Ulleswater exist in Caernarvonshire. The only lake which exceeds two miles in length is that of Llanberis. But though the Welsh lakes are so much smaller in size, some of them rival in depth even the largest of the English lakes; and when considered in proportion to their extent, their depth is far more striking.

In some of the English lakes a portion of the bottom lies below sea-level : in none of the Welsh lakes is the depth sufficiently great to bring any part of their bottom below the level of the sea. But the deeper Welsh lakes are, as a rule, situated at much higher levels than are the English lakes described by Dr Mill.

VII. Origin of the Lakes.

The frequency of lakes in regions which have been well glaciated has often been noted, and it is generally agreed that this is something more than a mere coincidence. That part of North Wales with which we are concerned in this memoir is remarkable for the evidences which are there displayed of a past Glacial Epoch; and a glance at a general map of the district will show that its surface is sprinkled over with lakes. The close association subsisting between the glaciation of a district and the development of lake-basins suggests a causal connection between the one and the other; the basins in some way or other owe their origin to glacial action. The late Sir Andrew Ramsay, while admitting that some of the lakes of North Wales are at least partially dammed up by moraines, held that many occupy basins scooped out of the solid rock by glaciers. It is now forty years since Ramsay first advanced his theory of ice-erosion to account for the formation of lake-basins lying in the path of the old glaciers. Arguments in support of the theory were brought forward by Tyndall, A. R. Wallace, and other writers; and it was adopted by the late Mr Clifton Ward to explain the origin of the lake-basins of Cumberland, and by Professor James Geikie to explain many of those of Scotland. Other geologists have been unable to accept Ramsay's hypothesis, and a keen controversy has been carried on for many years concerning the power of glaciers to erode their beds. As this question is of vital importance in discussing the possible modes of origin of the lake-basins in such a glaciated region as North Wales, it becomes necessary for us to obtain clear ideas as to what power a glacier has to excavate hollows in the solid rock. That glaciers often abrade, smooth, and polish the rocks over which they flow is admitted by all. But there is no such unanimity regarding the capacity of ice to excavate rock-basins.

According to some writers, the erosive power of glaciers is insignificant; others admit that it may be sufficient under favourable circumstances for the production of small rock-basins, but are opposed to the view that the larger lake-basins have been formed in this way. Professor Bonney, for instance, maintains that the hypothesis of glacial erosion will not suffice to explain the origin of the great Swiss and Italian lakes, but thinks it possible that smaller lake-basins, such as some of those in the Lepontine Alps, have been produced in this way; and M. Delabecque holds similar views with regard to the lakes of France.

In connection with the Swiss lakes, Mortillet and Gastaldi, many years ago, suggested that the work done by glaciers has been the ploughing out of the alluvium which in pre-glacial times filled the valleys. Again, it is quite possible that before the advent of the Glacial Epoch, the rocks of a region like North Wales had decayed and rotted to a considerable depth owing to their exposure for a vast lapse of time to sub-aerial weathering agents. Such sub-aerial disintegration has been observed to occur especially among crystalline schists and eruptive rocks. Many instances are given by Sir A. Geikie in his *Text-Book of Geology* (ed. 3, p. 350). He states that in Brazil, the crystalline rocks are sometimes decayed to a depth of more than 300 feet; and in parts of North America "the depth of disintegration appears gradually to increase southward from the limits where the country has been glaciated by ice-sheets during the Glacial Epoch." A similar superficial decay has been observed in our own country in the granite and phyllite of Cornwall and Devon, which lay outside the limits of glacial action. In many places they show a deep cover of rotted rock, and so "afford some indication of what may have been elsewhere the condition of Britain before the period of glaciation." This rotting of the solid rock through the prolonged operations of the weathering processes is a fact of some importance. The rocks would waste irregularly according to their varying powers of resistance. Some lakes may lie in hollows scooped out of the decomposed material by glaciers; or a hollow thus started by sub-aerial waste may subsequently get enlarged and deepened by the moving ice. The relative hardness of the rocks, on the one hand, and the thickness of the ice, on the other, would have an important effect on the work done.

Mr J. E. Marr, in his work on the *Scientific Study of Scenery*, has pointed out that in a region where the climate is humid, small rock-basins may be formed by the unequal weathering of rocks which are covered by vegetation. He has observed in the English Lake-District little basins of this character, a few feet or yards in diameter, and in every stage of formation.

But during recent years important facts have been brought to light which go to show that glaciers are powerful agents of erosion, and are actually able to quarry the beds over which they flow. Much of this new evidence is summarised by Professor James Geikie in his recent book on *Earth-Sculpture*. The lines of evidence are both indirect and direct. The indirect evidence is obtained by a study of ground-moraines. Professor Geikie shows that the existence of ground-moraines does not depend on the presence of superficial moraines. It may be true that in some of the Swiss glaciers much of the infra-glacial detritus comes chiefly from superficial sources, but this cannot be the case in Norway and Greenland. The Norwegian glaciers, as compared with those of the Alps, are almost devoid of rock-débris on their surface. Nevertheless they extrude ground-moraines. This is even more clearly the case in Greenland, where the ice-sheet, though free from superficial moraines, shows ground-moraines where it terminates on land. "Nansen, for example, tells us that at Austmannatjern, where he left the inland ice after his famous traverse, enormous accumulations of moraines were seen. These were of true infra-glacial origin, consisting largely of blunted and striated stones, which could only have been transported by the ice as ground-moraine. No 'Nunatakkr' occurred within the *mer de glace* near this place, and not a vestige of surface moraine was visible." Holst, Drygalski, and Chamberlin have also made similar observations. The stones and boulders of such a ground-moraine must therefore have an infra-glacial origin; they have been torn away from the rocky bed over which the glacier or ice-sheet flows.

Direct evidence is also obtainable to show that glaciers not only abrade their beds, but also shatter and disrupt rock-masses. This rock-shattering has been observed by Professor SIMONY on the bed of one of the Dachstein glaciers during the temporary retreat of the ice. Similar phenomena have also been recorded by MM. PENCK, BRÜCHNER, and BALTZER at the Uebergossenen Alm and other places. The débris formed by the breaking up of the rock had been incorporated in the ground-moraine lower down.

But while such observations prove that moving ice is sometimes a great eroding agent, capable of crushing and shattering the rocks over which it flows, instances have also been recorded of glaciers moving over beds of gravel, sand, and other unconsolidated material without scooping them away, or even disturbing the accumulations to any great extent. This shows that a glacier varies in its action, being, under certain conditions, a disturbing agent, and, under others, a preserving agent.

Professor W. M. DAVIS, of Harvard, has recently made an important contribution to glacial geology, by bringing forward fresh and striking evidence in favour of the excavating powers of a glacier. This is all the more interesting in view of the fact that Professor DAVIS had hitherto been a doubter of the erosive power of ice. In a paper which appeared in *Appalachia*, March 1900, he gives an account of his observations on "Glacial Erosion in the Valley of the Ticino," and he deals again with the same subject in the *Proc. of the Boston Soc. of Nat. Hist.*, July 1900, where the question of the competence of glaciers to deepen and widen their valleys is more fully discussed. He was struck by the fact that in the Ticino Valley the side valleys are strongly out of accord with the main valley. "There is no nice adjustment of declivities, for the side valleys mouth several hundred feet up on the wall of the main valley, and the side streams cascade down in sharp-cut shallow clefts from their high perches to the main valley floor." The walls of the main valley are steep, and run sub-parallel for miles together; no spurs enter the valley floor, and so it lacks the successively overlapping profiles seen in normal valleys, and instead of being V-shaped, it is U-shaped on cross section. At a height of 400 to 600 metres above the main valley floor, a bench of gentler slope leans back from the top of the basal cliffs. "The benches seem to be the remnants of the lower slopes of an ancient wide open valley, in whose floor the present cliff-shaped deeper valley has been eroded, and this supposition is confirmed when it is found that the high-standing lateral valleys are systematically related to the ancient rather than to the modern valley floor." He shows conclusively that the crossing spurs of the ancient river valley have been destroyed by glacial erosion, and that a glacier has deepened and widened the main valley so as to give it the form of a trough, with smooth and steep sides. The term "over-deepened" has been applied by PENCK to valleys of this kind. DAVIS adds that "the deepening of a glaciated valley for a good part of its length is thus seen to be a general result of glacial erosion; it is accompanied throughout by discordant or hanging lateral valleys, and the production of a lake in the distal portions of such a valley is but a subordinate result of glacial erosion." At the time when the valleys were filled with ice, the surface of the trunk and branch glaciers must have been nicely gradated in relation to each other. The discordance of the hanging valleys and the main valley, seen to-day, is due to the fact that the bed of the main glacier has been worn deeper than the beds of the side glaciers.

It ought not to be forgotten that as long ago as 1893 Dr A. R. WALLACE had, in an article which appeared in the *Fortnightly Review*, suggested that erosion had been carried on more rapidly in the main valleys. Referring to large lake-basins, he says: "On looking at the maps of any of these lakes, one cannot but see that the lake *surface*, not the lake *bottom*, represents approximately the level of the pre-glacial valley, and that the lateral streams and torrents enter the lake in the way they do because they could only erode their channels down to the level of the old valley before the ice overwhelmed it."

Somewhat similar phenomena to those described by DAVIS had already been observed in Norway, and are referred to in Professor GEIKIE's *Earth-Sculpture*. In that country we have a markedly ice-worn plateau land, intersected by deep, chasm-like, fiord valleys, which are U-shaped in cross section. The erosion of the main or fiord valleys is greatly in advance of that of the lateral valleys. The tributary streams, after winding through the plateau land in broad and shallow valleys, suddenly cascade down the precipitous walls into the fiord. The explanation given by Dr RICHTER is that while at certain stages of the Glacial Epoch glaciers and glacier streams were deepening the main valleys and making their walls steeper, erosion was practically at a standstill in the side valleys, which lay buried under the firn and ice of the plateau.

In 1901 a paper was communicated to the Royal Society of Edinburgh by Mr ALFRED HARKER, on "Ice-erosion in the Cuillin Hills, Skye" (*Transactions*, vol. xl. part ii. No. 12). In that region, also, straight steep-sided valleys, giving U-shaped cross sections, were in evidence, and are attributed to the work of local glaciers as distinguished from merely aqueous erosion.

The fact that glaciers are thus proved to be powerful agents of erosion, capable not only of quarrying their beds at certain spots where the conditions are favourable, but also of deepening and widening for miles together the main valleys along which they flow, simplifies greatly the discussion of the origin of lake-basins in such a strongly glaciated region as that of North Wales. Indeed, the lack of similar evidences of erosion in such an area would be a fact so strange that it would need some explanation. It becomes natural, therefore, for us to expect to find lakes resting in basins hollowed out of the valley floor, and also, possibly, to find indications that some of the main valleys have been over-deepened as compared with the side valleys.

The district with which we are concerned presents a highly complicated geological structure. It is made up of a heterogeneous assemblage of sedimentary and aqueous rocks of Cambrian and Ordovician Age. Slates, grits, and sandy and calcareous beds are found interstratified with felspathic ashes and lavas, and intruded into these are sills and bosses of greenstone, felspathic porphyry, and other massive igneous rocks.

It is owing to this intermingling of rocks of varying character, and to the unequal way in which they have resisted the agents of denudation, that the country now presents such a rugged and wild aspect. The principal orographical features of the district had been determined before the advent of the Ice Age. A sufficient interval of time had already elapsed for the ordinary agents of denudation to carve out the valleys and to sculpture the rocks into something like their present diversified forms before the glaciers began to do their work. All the main valleys are, therefore, pre-glacial, and have merely been modified by subsequent events.

The signs of glaciation which abound in this district have been very fully described and interpreted in the works of Sir ANDREW RAMSAY, and so are well known to all geologists. The Snowdonian mountains were the nurseries of great glaciers, which crept down the valleys on to the low grounds. The ice-streams flowing down Nant Francon, the Pass of Llanberis, the Gwyrfai Valley, and the Nantlle Valley, became confluent on the low grounds which lie between the mountains and the Menai Straits, and were met there by the vast ice-sheet which, coming from the north, overwhelmed Anglesey, and skirted the North Wales coast to pass on towards Cardigan Bay. On the other side of Snowdon a huge glacier moved down the Vale of Gwynant towards Beddgelert and Aberglaslyn, and another passed eastwards along Nant-y-Gwrhyd to Capel Curig, where it was met by a glacier coming down Nant-y-Benglog, the united stream then continuing its course towards Bettws-y-Coed.

The striæ which can be seen on the rock surfaces always trend in the directions in which these main valleys run. The main streams were joined by tributary streams coming down from the cwms and upland valleys. The highest peak of Snowdon—Y Wyddfa—formed a centre from which radiated six glaciers, which made their way down to join the ice-streams in the bigger valleys below. The Nant Francon glacier was augmented by tributary glaciers coming down from a number of cwms which overhang the main valley on the left, and lower down was joined on the opposite side by the bigger tributary streams of Cwm Llafar and Afon-Gaseg, which emerged from the north-western flanks of the Carneddau and Y Foel Fras. South of Nant Francon, at the head of Nant-y-Benglog, a few cwms also occur on both sides, and from these tributary streams moved to join the mass of ice creeping towards Capel Curig; some of the ice moving in this direction was probably diverged over the low col above Llyn Cawlyd into the valley where the lake now lies. The mountainous country lying to the north-east of Capel Curig also gave birth to glaciers which moved north-eastwards towards the Vale of Conway.

Thus, at a certain period, every valley was filled by its own special glacier, those in the cwms and upland valleys going to feed those in the bigger valleys, and these again moving outwards in all directions from the mountains to spread out as fans over the lowlands. The track of the old glaciers is marked at many places by smoothed, rounded and striated rocks, the striæ indicating the direction of the ice-flow. The cwms are especially rich in the remains of glacial action, and huge moraines are there seen in all stages of preservation. These mark the final retreat of the glaciers after the ice had disappeared from the low grounds.

In Wales, as in other glaciated countries, erosion must have been carried on most rapidly along the chief lines of ice-flow. At certain stages of the Glacial Epoch the glaciers in the main valleys must have been engaged in deepening and widening the channels along which they moved, while the tributary valleys and cwms were protected underneath a mantle of *névé* and snow. One is therefore not surprised to find that the more important valleys are at places over-deepened as compared with the lateral valleys, and now have a trough-like form with a flat bottom and steep cliff-like walls, giving a U-shaped figure in cross-section. Features of this kind are well seen in Nant Francon, where the valley from a little above Ogwen Bank right up to its head at Ogwen Falls is remarkably flat and deep, and at the sides the walls rise steeply from the broad floor to heights of over 1000 feet. On the western side six tributary cwms mouth at a height of several hundred feet above the valley floor, reminding one of the hanging valleys of the Ticino district, described by DAVIS. A study of the ground shows that the valley, at the time of maximum glaciation, was filled up to the level of these cwms with ice. In his little book on *The Old Glaciers of Switzerland and North Wales*, RAMSAY writes thus: "Taking Nant Francon in connection with its branching valleys, an attentive consideration of all the circumstances has led me to think that it was so filled with ice that the mouths of the minor valleys, to a height of from 700 to 1000 feet above the river, were over-ridden by the main stream of ice, which flowed across the lower end of the spurs that branch from the crested ridge on the west." A photograph of Nant Francon is given (Plate II. fig. 2), showing a view of this straight flat-bottomed valley as seen from the heights above Llyn Ogwen. It has already been stated that the bottom of this valley was for a time probably occupied by a lake, which has now disappeared by becoming drained at the exit.

At Pen-y-Gwrhyd, a low water-shed separates the Vale of Gwrhyd from the Vale of Gwynant. It is interesting to compare these two valleys. The Vale of Gwrhyd has a comparatively gentle slope from Pen-y-Gwrhyd to Capel Curig; but the descent from the watershed to the bottom of the Vale of Gwynant, near its head, is very steep. This is accounted for by the fact that the mass of ice moving down the Vale of Gwynant was much greater than that moving eastwards towards Capel Curig. For not only did much of the snow which gathered on the slopes to the south of Gorphwysfa (at the head of the Pass of Llanberis) pass down the Vale of Gwynant, but the huge glacier coming down from the great upland valley of Cwm Dyli entered the Vale of Gwynant near its head, and so greatly increased the thickness of the ice in that region. The valley above Llyn Gwynant is broad and flat and deep, with walls rising sharply at the sides. We have here again an approach to that U-shaped form which is so characteristic of a deepened valley. The level of the valley floor in pre-glacial times is possibly indicated by the shelf of Penmaen Brith, which passes above Llyn Gwynant on the right side. Lower down, Cwm-y-Llan opens out at some distance above the present valley floor. No cwms of any importance open out along the course of Nant-y-Gwrhyd.

Many examples of such **U**-shaped valleys occur also further south in Merionethshire, one of the most remarkable of which is the Vale of Festiniog.

The Vale of Conway, which marks the eastern border of Carnarvonshire, presents similar features for a long distance below Bettws-y-Coed. This valley lies along a line of fault, but it has all the appearance of having been over-deepened by moving ice. The side-walls are steep, being remarkably precipitous on the left or Carnarvonshire side. Here the tributary valleys join the main valley at a considerable height above its present floor, and we see, too, a series of streams—Afon Crafnant, Afon Ddu, Afon Porthulwyd and Afon Dulyn, cascading down the precipitous walls to join the river Conway below.

If glaciers have thus in Wales, also, eroded the channels along which they flowed, the excavation of rock-basins below the general level of the valley floor at certain places where the conditions were especially favourable, need no longer excite surprise or be looked upon as anything more than subordinate incidents in the general history of ice-erosion. Turning to the lakes themselves, we shall consider first those lying at low levels in the valleys which encircle the foot of Snowdon. The lakes occupy in their respective valleys just those positions in which the glaciers might be expected to have carried on most actively the work of erosion.

The Lakes of Llanberis.—These lakes lie at the distal end of the steep and narrow Pass of Llanberis, occupying the lower reaches of the valley for a distance of over three miles. We may consider the lakes as forming one sheet of water, and originally this was of greater extent, as shown by the alluvial tract stretching up from the head of Llyn Peris to Gwastad-Nant. From the top of the Pass down to this point the incline is very great, but below Gwastad-Nant the valley loses its steepness. Looking down from the summit of one of the spurs in the Pass, one can see the alluvial stretch, and the surface of the lakes extending as a straight flat plain as far as the lower end of Llyn Padarn (Plate II. fig. 1). The valley is held in on either side by steep rocky hills, and it narrows towards the foot of the lake. Nowhere in North Wales are the signs of glacial action so striking as on the rocky slopes which border the lakes. The rocks are smoothed, rounded, and striated from a height of several hundred feet above the lakes right down to the water's edge, and the striæ all run north-westerly—in the direction of the valley. Here, if anywhere, we have all the conditions requisite for the hollowing-out of a rock-basin. The glacier, as it emerged from the steeper part of the valley and attained the more gently inclined reaches below Gwastad-Nant, would exert greater pressure upon its rocky bed, and as the ice would be thus retarded in its flow, this pressure would be still further increased by the heaping up of the ice to form a mass of greater thickness. Mr Marr has suggested that the strip of low ground occupied by alluvium, which leaves the lake on the western side near the lower end, may mark the site of a drift-filled depression. But there is no evidence to show that the drift occurs here to any great thickness. The form of the hollow in which the rock lies, favours the view that we have here a rock-basin. The contoured maps and sections show that Padarn and Peris occupy a long trough-like depression, with a somewhat flat floor and steep cliff-like walls, giving in cross-section a U-shaped figure. The trough is deepest towards the upper end, and shallows gradually as traced downwards. This is just what we should expect, for the erosive power of the glacier would be most marked at the upper end, and its amount would tend to diminish downwards. The aspect presented by the lower reaches of the Pass suggest the possibility that the valley has here been over-deepened considerably as compared with what it was in pre-glacial times, and that the trough in which the lakes lie was dug out of the floor of this already over-deepened valley.

Llyn Cwellyn.—Here, again, we have a similar U-shaped, trough-like depression, which is deep along its whole extent. The valley, which is wide towards the upper end, narrows below the foot at Nant-Mill, and this would form an obstacle to the flow of the glacier, and so favour the erosion of a hollow just above. A barrier of rock extends right across the valley to Nant-Mill, over which the stream from the lake runs in a waterfall. Mr Marr, assuming the lake to be only 50 feet deep, thought he could find traces of a buried gorge at the upper end of the lake, along which the waters of Llyn Cwellyn might formerly have been carried away in the opposite direction. No proof has been offered that such a gorge occurs here; and Mr Dakyns, in his paper on "Some Snowdon Tarns," has shown that its existence is extremely improbable. And the further fact that the lake proves to be so much deeper than had been supposed (122 feet, as against 50 feet), militates against that hypothesis. The depth of the lake and the general form of the hollow are more in consonance with the view that we have here a rock-basin excavated in the floor of the valley by glacial action.

Llyn Gwynant and Llyn Dinas.—We have already seen that the Vale of Gwynant has in all probability been broadened and deepened along a great part of its course. But at two places the valley becomes constricted, owing to the mutual approach of its flanks. Above the upper constriction lies Llyn Gwynant, and above the lower lies Llyn Dinas. So that these lakes occupy just those positions in the valley where the glacier would be retarded in its flow by these obstacles, and where, therefore, the conditions for erosion were most favourable. The hollows in which they lie are rapidly becoming silted up, and so the lakes are more shallow. In Gwynant, the streams coming in on the left side have caused the lake to shallow more rapidly there than on the right side, as shown in the cross-section E–F (Plate VIII.). (The section C–D is probably more typical of the original form of the hollow.) Dinas is still shallower, and shows a more advanced stage in the process of silting up. The bottom of the valley between Gwynant and Dinas is largely covered by drift. The exit from Dinas is almost certainly over rock; but in order to avoid the hypothesis of a rock-basin, Mr Marr pointed to the possibility of a drift-filled depression, leaving the lake on the right side a little above the exit, and passing round a rocky knob into the road. Though one cannot exclude the possibility of these lakes being drift-dammed, the more probable and natural hypothesis is that they lie in rock-basins which have to a great extent been filled up by the deposition of sediment.

Two of the upland valley lakes of Snowdon were sounded, namely, Glaslyn and Llydaw, both of which lie in Cwm Dyli. The depths obtained in the two lakes were remarkably great.

Glaslyn, situated at an altitude of 1970 feet above sea-level, occupies a cup-shaped depression just under the great precipice of Y Wyddfa, and is a typical corrie lake With regard to the lake Mr Watts says : " Glaslyn is bounded on all sides by live rock, except at and near its outlet. This exit is over moraine, which, however, is evidently not very deep, for rock makes its appearance just below, and in such a way as to almost compel belief in a complete rock bar. Beside the present course of the effluent stream is a parallel strip of moraine running down towards Llyn Llydaw, but living rock soon makes its appearance in this in such a way as to show that if there is any old channel in this direction, it must be exceedingly narrow and tortuous. Thus, if this lake is not contained in a true rock-basin it must be very shallow, or else must have found exit by a gorge quite as narrow as those found at the end of some of the Swiss glaciers." Mr Marr, visiting the lake later, fastened on the " parallel strip of moraine running down towards Llyn Llydaw " as possibly marking the site of a gorge now filled up with drift. Some time afterwards Mr Dakyns pointed out, in his paper on " Some Snowdon Tarns," that solid rock can be seen in the bed of the stream a little below the lake, and " at the old mill, seventy yards from the lake, solid rock extends right across the stream from the Gribbin on one side to the cliffy ground on the other, along which the road from the mines runs." The writer can confirm this statement, at any rate so far as to say that the interval where solid rock cannot be seen is so small that it can easily be stepped over. This point is 40 feet below the level of the lake. The soundings give a greatest depth of 127 feet, and this proves pretty conclusively that the water is held in a rock basin. The contoured map and sections show that the basin has steeply sloping sides, and that the deep water does not extend into the bay on the north side. On each side of the entrance into the small bay are seen sections of what looks like a morainic mound. The cirque or cwm in which such a lake lies is not the work of glaciers, but it is the cup-shaped depression lying at the bottom of such a cirque that has been ground out by the ice.

Such corrie-lakes or tarns are often very deep in proportion to their extent, and such is the case with Glaslyn. Many niche-like cirques occur which have not a basin or cup-shaped depression in their floor ; such a niche is seen to the north above Glaslyn, and is shown in the photograph of Glaslyn appended (Plate I. fig. 1). It is during the early and late stages of the Glacial Epoch that the small local glaciers in the cwms would hollow out their floors. Owing to the slope of the cirques, the ice would be able to concentrate its power upon a small area, and this accounts for the great depths obtained in some of these cirque-basins. That the cirque in which Glaslyn lies underwent severe glaciation is shown by the rounded and smoothed faces of the rocks forming the spur of Y Gribbin at a considerable height above the lake on the south side, and also by similar features seen on the rocks on the opposite side of the lake above the outlet.

Llyn Llydaw lies in Cwm Dyli, at a lower level than Glaslyn, the altitude of its surface being 1415 feet above sea-level. This lake is generally admitted to be in part dammed up by glacial deposits ; but while Mr MARR thinks that it is probably altogether held up by such deposits, Mr WATTS and Mr DAKYNS believe that it lies in a rock-basin. The soundings show clearly that there is a deep corrie or cirque-like basin at the upper end. The shallowing lower down is largely due to the morainic material which is scattered about. Mr WATTS found live rock at a depth of 40 or 50 feet below the level of the lake in the stream which issues from it. Mr MARR traced another depression by which the lake may formerly have been drained, but in this depression Mr DAKYNS found solid rock 40 feet below the level of the lake. As Llydaw has been found to be nearly 200 feet deep, we can only conclude that while its level may be raised by a barrier of morainic material, the water is, for the most part, resting in a rock-basin. The fact that the greatest depth occurs quite at the head of the lake is not in consonance with the view that we have here an upland valley which has been drowned by means of a barrier of drift keeping back the water which drains it.

Other examples of cwm or corrie lakes dealt with in this memoir are Llyn Idwal and Llyn Dulyn.

Llyn Idwal lies in Cwm Idwal, above the head of Nant Francon, at an elevation of 1223 feet. It is very shallow, the greatest depth obtained being only 36 feet. The floor of the lake is very irregular, small rocky knobs rising up here and there, and much morainic débris and boulders lying scattered about. It was probably at one time much deeper but has been gradually getting filled up by rock-falls from the neighbouring heights, and by the washing in of morainic detritus and the deposition of sediment. RAMSAY, not knowing the depth or the form of the lake-bottom, was of opinion that the lake was partly formed by a dam of terminal moraine, and was partly retained in a rock-basin. But the mass of drift crossing the valley at the foot of the lake seems to be of sufficient depth to account for its formation. The configuration of the lake bottom, also, as shown in the contoured maps and sections, supports this view. There is no deep cup-shaped depression such as is found in Glaslyn, but an irregular floor with rocky knobs jutting up here and there. Therefore, although it was once probably deeper, the evidence seems to show that we have here not a rock-basin, but a barrier-basin, the floor of which may have been modified to some extent by glacial erosion.

Llyn Dulyn, situated at the southern base of Y Foel Fras, has the surface of its waters at an elevation of 1747 feet. It occupies a deep basin-shaped hollow at the foot of a mural precipice. Rock can be traced all round the lake-margin, except on the eastern side. Here the ground, rising steeply from the water, is covered with drift. There is no reason to think that the drift is of any great thickness. At the south corner the outflowing stream has cut through it, and runs out over solid rock. Dulyn is excessively deep, especially when the depth is considered in proportion to its extent. A sounding of 189 feet was obtained in one place. The map and sections show that the form of the hollow is that of a deep basin or cup, such as a glacier would excavate on the floor of a cirque. This water, therefore, in all probability, occupies a rock-basin.

Just south of Dulyn is situated another small tarn, named *Melynllyn*, the surface of which reaches an elevation of over 2000 feet. No soundings were taken in this lake; but it appears to be very deep, and is interesting in that an unmistakable morainic mound can be traced running along its eastern border. It is probably partly a rock-basin and partly a barrier-basin.

Llyniau Mymbyr.—These lakes lie at the distal end of Nant-y-Gwrhyd, and may be considered as one sheet of water. The valley narrows towards the outlet, and hence the flow of the ice would be here retarded. The greatest depth obtained was only 29 feet —this was found in both the higher and lower portions. Though the possibility of a drift-dam cannot be excluded, it is more probable that the shallow depression has been formed by the erosive action of the glacier moving down the valley.

Llyn Ogwen.—This lake lies at the head of a valley at an elevation of 984 feet. Though nearly a mile long, it has nowhere a depth of over 10 feet. At one time it may have been much deeper, but it is now shallowed, owing to the great amount of sediment which has been carried into it. Rock can be traced right across the valley at the exit, and the stream from the lake runs over this in a waterfall. Rocky cliffs border the lake on either side, but at the head an alluvial flat extends upwards towards the watershed. Before reaching the watershed, which consists of drift, live rock can be followed across the valley, except for an interval of about 40 yards wide near Pont Bodesi. It is possible that Ogwen once drained in the opposite direction towards Capel Curig. But during the Glacial Epoch a great thickness of ice must have accumulated on the site of the lake, for the glaciers of Cwm Bochlwyd and of Cwm Idwal descended to swell the mass already gathered there. Consequently there would be a tendency to erode a shallow basin under the ice. In Ogwen we have probably an example of a basin produced at the head of a valley by glacial erosion during a late stage of the Glacial Epoch, when the ice had retreated from the lower reaches of the valley.

To the north-east of Capel Curig a number of long, narrow upland-valleys run from south-west to north-east and drain into the valley of the Conway. In their upper reaches these valleys are held in at the sides by high hills, and their floors are occupied by lakes. Lower down, the valleys are wider, and are largely covered with drift. The streams, on reaching the boundary of this upland district to the east, fall with a steep descent into the broad and deep Vale of Conway.

Of these lakes *Llyn Crafnant* almost certainly lies in a rock-basin. The valley in which the lake lies is cirque-like at its head, the hills rising to heights of 1500 feet, and their elevation is still greater along the sides of the lakes. The surface of the water stands at an altitude of 602 feet. At the foot, the hills on either side approach each other so as almost completely to shut in the lake. But a narrow gorge is left by which the stream escapes from the lake. Rock can be seen along the margins of the stream, but its bed is filled with large boulders. The water has a rapid fall through the gorge, and so would probably have carried away any drift which lay in its path.

The lake has a greatest depth of 70 feet, and the deeper contour-lines are of very irregular outline, and do not conform to the trend of the lake margins or to the land contour-lines above. The snows gathering in the cirque-like head of the valley would give birth to a glacier whose motion would be so much impeded at the narrow gorge that the ice would heap up to a great thickness, and exert a greatly increased pressure on the valley floor, and so excavate a rock-basin.

Llyn Geirionydd is possibly in part drift-dammed, for though live rock can be seen in the stream at some distance below the exit, a considerable portion of the valley floor near the sides is covered with drift. The lake proved to be shallow, the greatest depth being only 48 feet. This occurs near the head of the lake. There is an irregularity on the western margin, but the 20 feet contour-line is not affected by it. When the water is low, rocky knobs can be seen near the foot of the lake. These are severely glaciated on their upper sides. The configuration of the valley would favour the work of ice-erosion, and so it is not unlikely that its waters are partly retained in a rock-basin.

Llyn Cawlyd lies at an elevation of 1164 feet. It occupies the higher reaches of the valley, and is held in at the sides by high mountains. The watershed at the head of the lake is low, and some of the ice moving down in the direction of Nant-y-Benglog was probably diverted into this valley, where the lake lies. The valley is very narrow above, but lower down it widens, and so the ice-stream which was confined within a narrow space above would spread out here, and its erosive power would be much diminished. Below the lake the valley slopes gently eastwards, and is covered with drift. The thickness of the drift is not known, but at a level of about 100 feet below the lake, masses of rock appear in the valley floor. The lake attains a depth of 222 feet, being the deepest in the district. Its floor is very regular, and has the form of a long trough, deepest in the middle, and basin-shaped in cross-section. The probability is that the valley is partly blocked by drift, but that this is not of sufficient thickness to account for the great depth of the lake. Ice-erosion will also have played an important part in the formation of the lake-basin.

Llyn Eigiau is situated at an altitude of 1217 feet in the lower reaches of Cwm Eigiau, just where the valley widens. It is very shallow, the greatest depth obtained being only 32 feet. Drift and alluvium cover the valley floor at the foot of the lake and along its eastern shore. This drift is probably sufficient to account for the present depth of the lake.

When the lakes are thus studied in their relation to the glaciation of the district, and in the light of recent observations on the eroding power of glaciers, we are driven to conclude that whilst some may be partly rock-basins and partly barrier-basins, and a few simply barrier-basins, most of them lie in rock-basins.

(This memoir embodies the results of research work carried on by the author as Heriot Fellow of the University of Edinburgh.)

EXPLANATION OF PLATES.

Plate I.

The upper figure shows Glaslyn, with the great precipice of Y Wyddfa above.
The lower figure shows Gwynant, with the alluvial flat extending above the lake.

Plate II.

The upper figure shows the Lakes of Llanberis, with the alluvial flat stretching up to Gwastad-Nant.
The lower figure shows the remarkably flat-bottomed valley of Nant Francon.

Plate III.

Shows the general distribution of the lakes and the physical features of the district. The lakes described in the text are coloured blue.

The next four Plates give the details concerning each lake. In each case the contour lines and the greatest depth are marked in feet. In some cases the figures are indistinct, but they run thus :—

Plate IV.

Llydaw—20, 60, 80, 100, 120, 140, 160, 180. Greatest depth, 190.
Glaslyn—20, 40, 60, 80, 100, 120. Greatest depth, 127.
Padarn—20, 40. 60, 80. Greatest depth, 94.
Peris—20, 40, 60, 80, 100. Greatest depth, 114.

Plate V.

Gwynant—10, 20, 30, 40, 50. Greatest depth, 54.
Dinas—10, 20, 30. Greatest depth, 30.
Cwellyn—20, 40, 60, 80, 100, 110, 120. Greatest depth, 122
Idwal—10, 20, 30. Greatest depth, 36.

Plate VI.

Ogwen—5, 8, 10. Greatest depth, 10.
Crafnant—10, 20, 40, 60. Greatest depth, 71.
Mymbyr—Upper Lake—10, 20. Greatest depth, 29.
,, Lower Lake—10, 20. Greatest depth, 29.
Geirionydd—10, 20, 30, 40. Greatest depth, 48.
Dulyn—20, 40, 60, 80, 100, 120, 140, 160, 180. Greatest depth, 189.

Plate VII.

Eigiau—10, 20, 30. Greatest depth, 32.
Cawlyd—20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220. Greatest depth, 222.

Plate VIII.

Shows sections of the lakes on a scale of 6 inches to the mile

By permission of The Photochrom Co. Ltd.]

Glaslyn, with the peak of Snowdon.

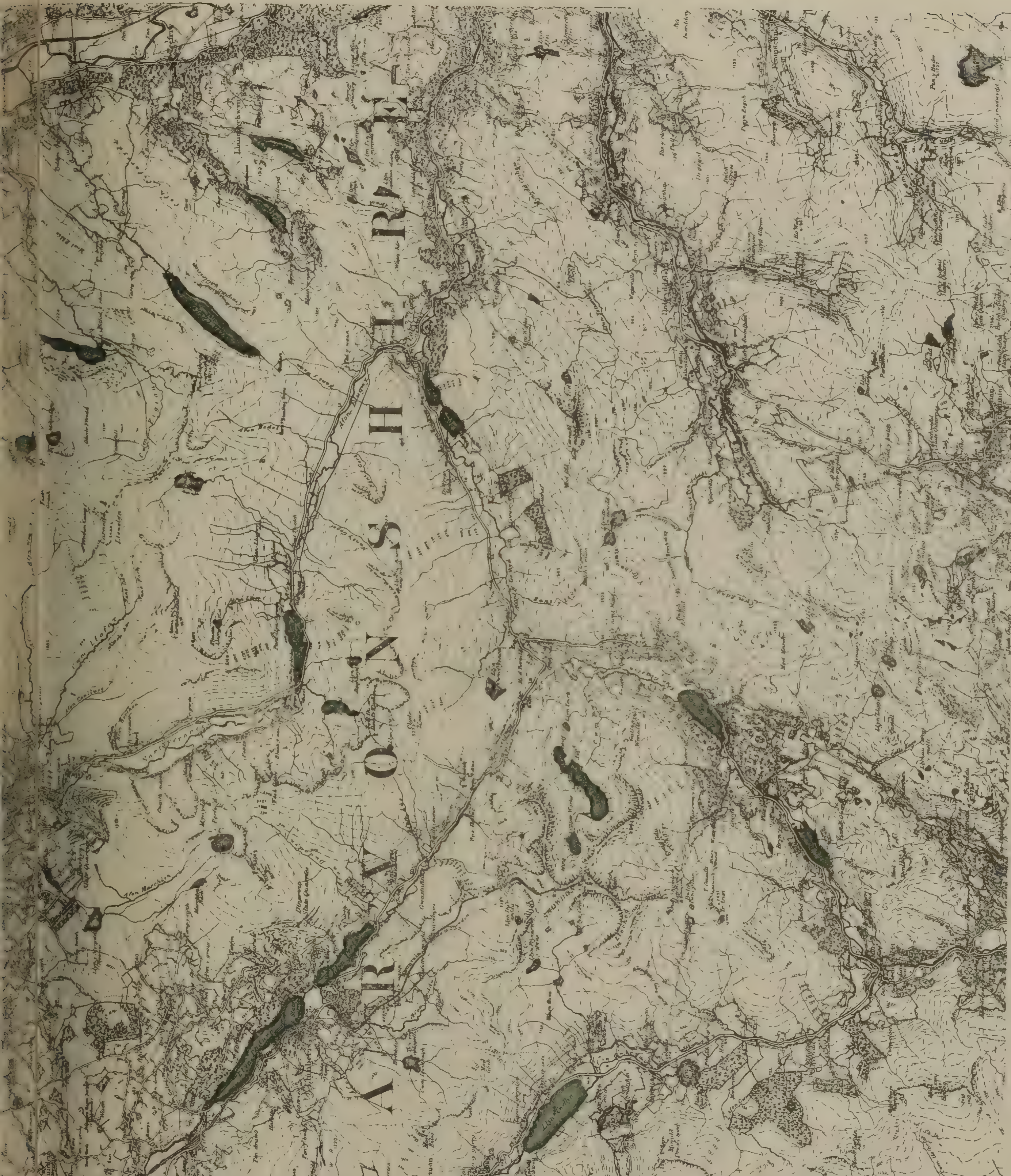
By permission of The Photochrom Co Ltd.]

Gwynant, looking down the valley.

y permission of The Photochrom Co. Ltd.] Lakes of Llanberis from the Pass.

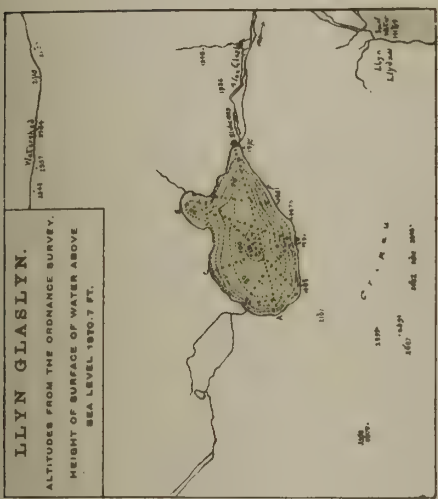
permission of The Photochrom Co. Ltd.] Nant Francon, looking down the valley.

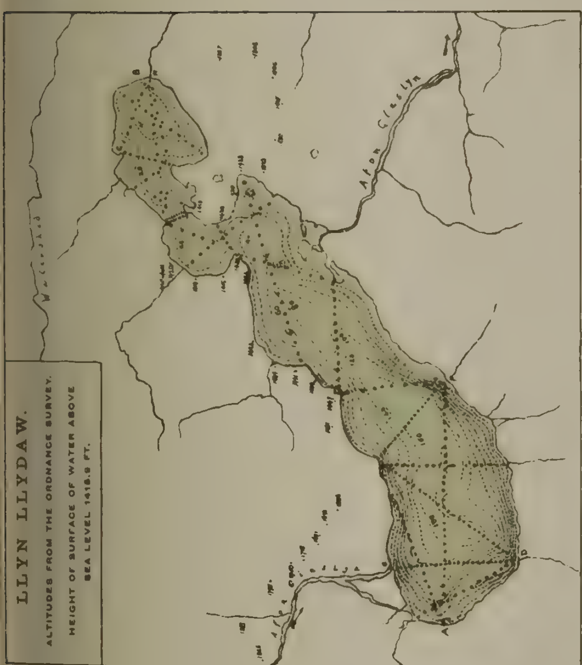
JEHU: BATHYMETRICAL AND GEOLOGICAL SURVEY OF WELSH LAKES—PLATE III.



Scale three-fifths of an inch to 1 mile.

EHU: BATHYMETRICAL AND GEOLOGICAL SURVEY OF WELSH LAKES—PLATE IV.

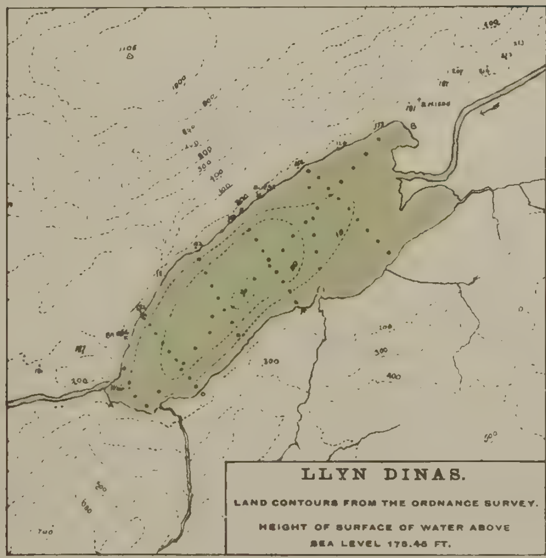


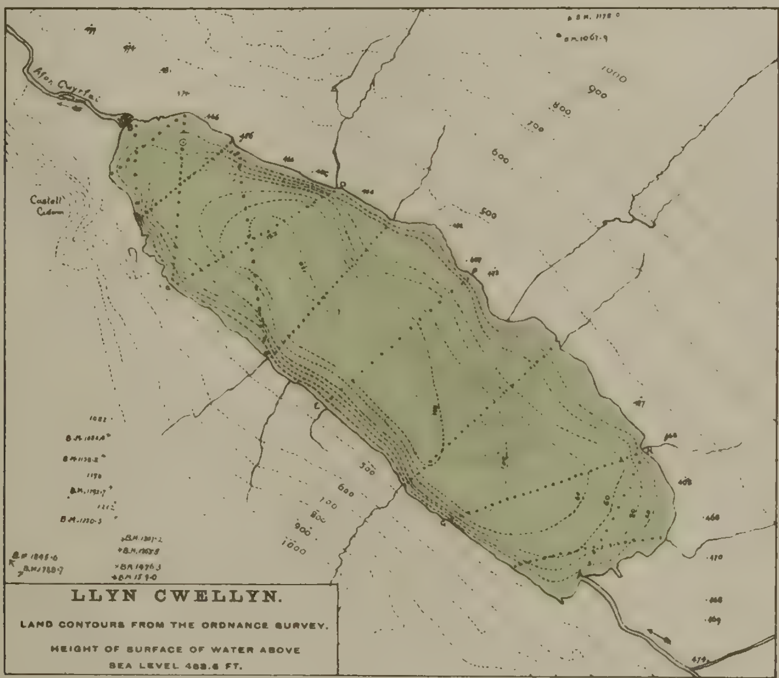


Scale 3 inches to 1 mile.

Depths in feet.

JEHU: BATHYMETRICAL AND GEOLOGICAL SURVEY OF WELSH LAKES—PLATE V.

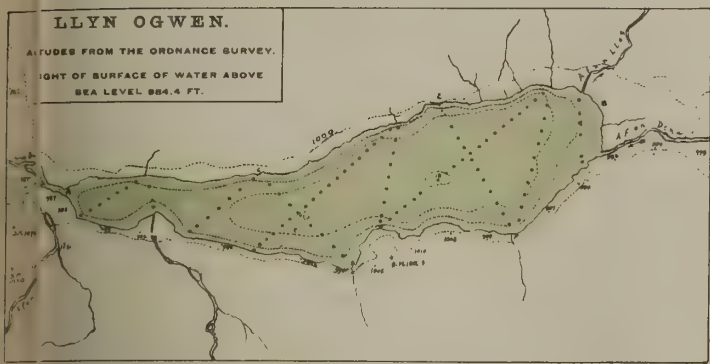


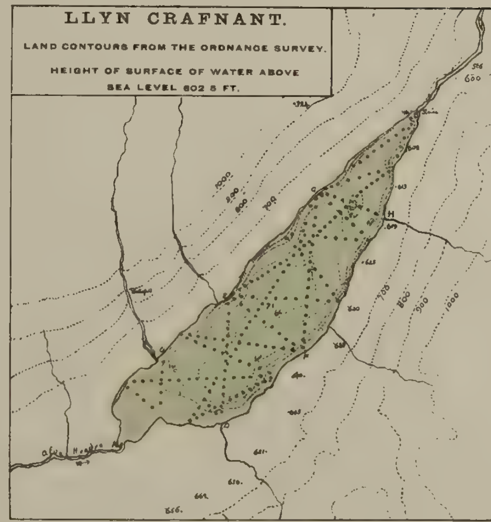


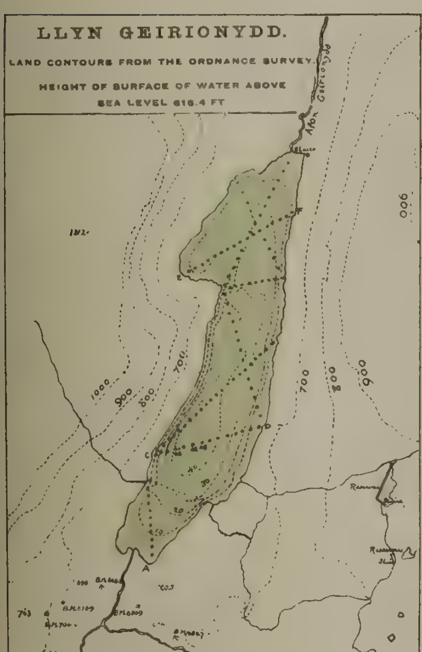
Scale 3 inches to 1 mile.

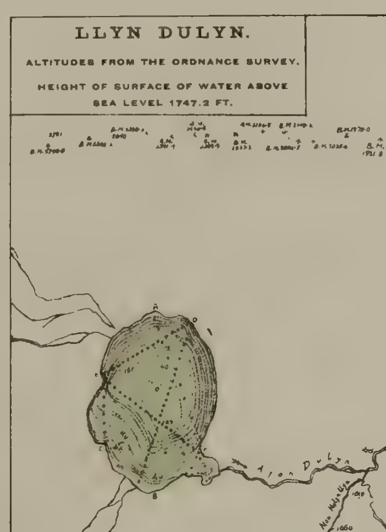
Depths in feet.

JEHU: BATHYMETRICAL AND GEOLOGICAL SURVEY OF WELSH LAKES—PLATE VI.





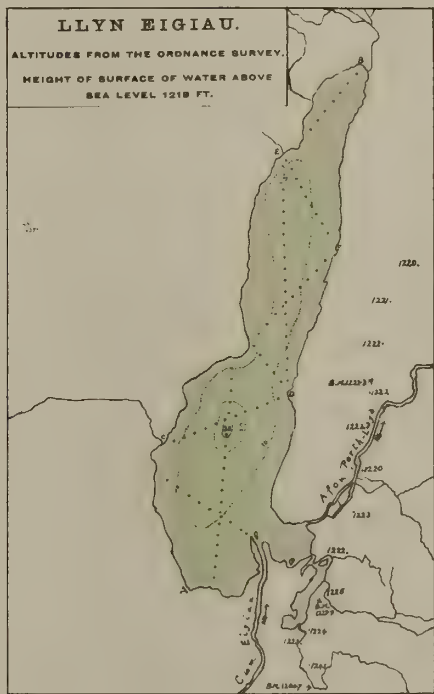


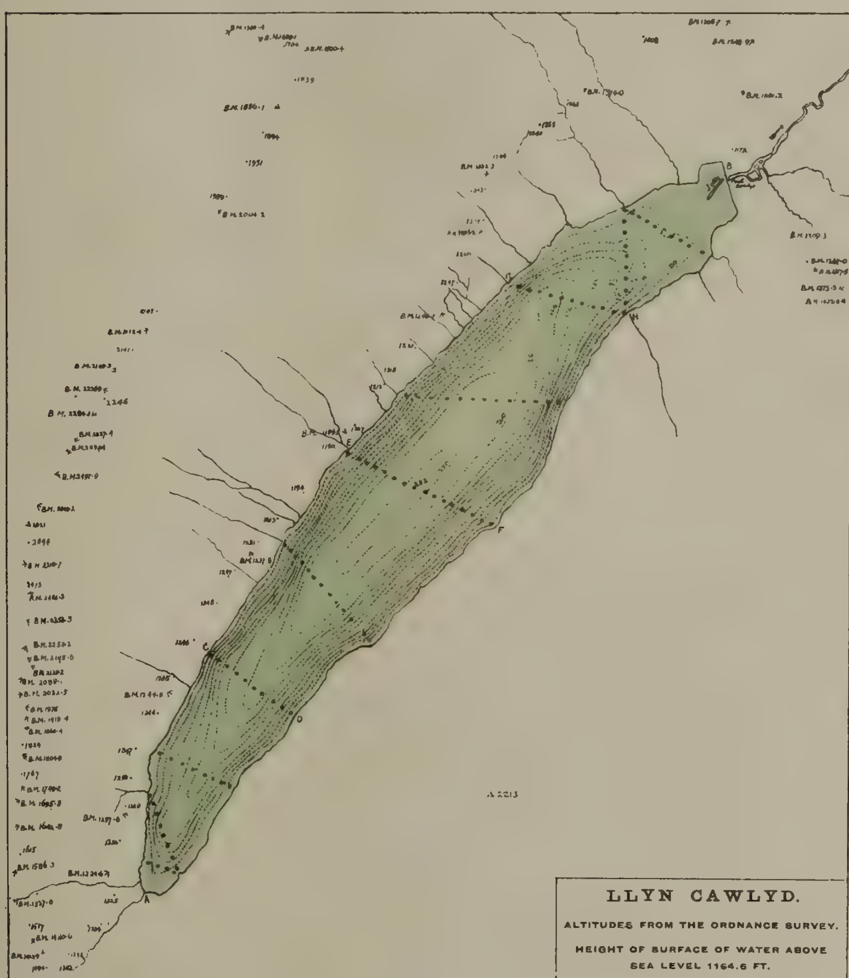


Scale 3 inches to 1 mile.

Depths in feet.

JEHU: BATHYMETRICAL AND GEOLOGICAL SURVEY OF WELSH LAKES—PLATE VII.





Scale 3 inches to 1 mile.

Depths in feet.

JEHU: BATHYMETRICAL AND GEOLOGICAL SURVEY OF WELSH LAKES—PLATE VIII.

Scale 6 inches to 1 mile.

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XXXI.	4 4 0	3 3 0
XXXII. Part 1.	1 0 0	0 16 0
„ Part 2.	0 18 0	0 13 6
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„ Part 2.	2 2 0	1 11 0
„ Part 3.	0 12 0	0 9 6
XXXIV.	2 2 0	1 11 0
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„ Part 3.	1 0 0	0 16 0
XXXVII. Part 1.	1 14 6	1 5 6
„ Part 2.	1 1 0	0 16 0
„ Part 3.	0 16 0	0 12 0
„ Part 4.	0 7 6	0 5 8
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„ Part 2.	1 5 0	0 19 0
„ Part 3.	1 10 0	1 3 0
„ Part 4.	0 7 6	0 5 8
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CONTENTS.

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MDCCCCIII.

Price Twenty-Six Shillings.

XXI.—*The Meteorology of Edinburgh.* By R. C. Mossman, F.R.S.E., F.R. Met. Soc. (With a Plate.)

(Read 19th May 1902.)

PART III.

In my previous papers on "The Meteorology of Edinburgh"* the data referring to local climate were reduced down to the end of 1896. In the time that has elapsed since their publication a number of memoirs from places on the Continent and elsewhere have appeared in which the data, mostly dealing with long periods, were brought down to the end of 1900. In order to facilitate the comparison of the Edinburgh record with those referred to, I have completed new monthly and annual averages for the ten years 1891–1900, the fifty years 1851–1900, and for longer periods, which embrace 137 years in the case of mean temperature, 131 years for mean barometric pressure, 130 years for the non-instrumental phenomena, and 124 years for rainfall (see Tables I. to IV.). The values previously published have been carefully examined, and a number of errors which escaped detection at the time eliminated. A Table of Errata (see Table XXIX.) is appended. Tables V. and VI. show respectively the mean monthly and annual departure from the normal of the mean maximum and minimum temperatures for the fifty years 1851–1900. A few of the values formerly published (see Table XVIII., *Trans.*, vol. xxxix. pp. 130–133) are about 0°·6 too high, but they have been corrected. The years affected are mostly between 1851 and 1860. Table VII. shows the Decennial monthly and annual means of the mean daily maximum and minimum temperatures, and the mean and extreme values of daily range. Table VIII. gives the extreme mean monthly temperatures from 1764 to 1900, and similar values for mean maximum and mean minimum temperatures during the period 1851–1900. Table IX. gives a general summary of the mean and extreme annual values of some of the more important climatic elements in Edinburgh from 1851 to 1900, while Table X. continues to 1900 the monthly and annual values of temperature, pressure, and rainfall. In Table XI. is given the monthly frequency of heavy rains exceeding an inch in twenty-four hours during ninety-six years in which the rainfall was observed daily. The chronological list of remarkable atmospheric and celestial phenomena contains notices collected since the publication of my former catalogue. The more important epidemics observed in Edinburgh from 1497 to 1900 are also given, as an intimate relation is known to exist between weather and the health of the community. The

* Part I., *Trans. Roy. Soc. Edin.*, vol. xxxviii. pp. 681–755 ; Part II., vol. xxxix. pp. 63–207.

condensed results of some investigations dealing with various phases of local climate have been included. A table showing the price of wheat in Edinburgh from 1801 to 1900 is also given, as the relation of local weather to wheat prices down to about 1872 is of an intimate nature. Access to the records of grain prices in the Teind Office was obtained through the courtesy of Mr NENION ELLIOT, S.S.C.

Bright Sunshine, 1890–1900.

Table XII. shows the total bright sunshine recorded in Edinburgh from August 1890 to December 1900. The station was in the Newington district during the whole period, except in the last two months of 1900, when the Royal Botanic Garden record was utilised. The sunniest month in this period was July 1897, with 232 hours' bright sunshine, or 44 per cent. of the possible, and the dullest December, 1890, with only seven hours, or 3 per cent. of the possible. The greatest percentage of possible sunshine recorded in any month was for March 1894—viz. 47 per cent. The annual mean shows 26 per cent. of the possible, the sunniest month being May, with 34 per cent., and the dullest December, with only 10 per cent. Table XIII. shows the distribution of sunshine through the hours of the day on the mean of ten years. Of the total sunshine registered, 47 per cent. is recorded before noon, and 53 per cent. after noon. Except in May, June and July the sunniest hour is either that ending with noon or 1 P.M. In June the maximum is not reached till between 2 and 3 P.M. In this month and in May the mornings are generally cloudy, with a good deal of fog and haze accompanying the annual maximum of easterly winds.

In Table XIII. the days are analysed with reference to the percentage of possible sunshine recorded.

Partial Droughts in Edinburgh, 1857–1900. (*Table XIV.*)

In the *Transactions* (vol. xxxix. p. 146) I gave a list of absolute droughts, which the late Mr SYMONS defined as periods of more than fourteen days entirely without rain. Partial droughts are much more difficult to define; in a vague way we know what is meant, but directly any attempt is made to put it into words difficulties arise. Mr SYMONS has defined a partial drought as a period of "more than twenty-eight consecutive days, the aggregate rainfall of which does not exceed one-hundredth of an inch per day." During the forty-four years ending with 1900, twenty-nine such dry spells were recorded, their mean durations being thirty-seven days. The longest drought was sixty-four days, from April 24 to June 26, 1859. The seasonal variation shows a well marked maximum in June, with 23 per cent. of all the cases recorded. There is a marked fall in May, droughts being then comparatively rare. This may be due to the prevalence of rainstorms caused by the predominating easterly winds. The annual minimum occurs in

October. Entering the number of days during which partial droughts prevailed in the different months we obtain the following values:—

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Days, . . .	26	98	118	177	85	243	139	83	9	4	55	37	1074
Per cent., . .	2	9	11	17	8	23	13	8	1	0	5	3	100

In the construction of the above table, a drought beginning, say, on June 15 and terminating on July 20 would have 16 days entered to June and 20 to July. The complete absence of spells of dry weather in September and October indicates how unsettled the general weather of these months is. In October the frequency of rainy days is at a maximum, a result of a rapid autumnal fall of temperature and the comparative absence of high-pressure systems.

Decennial Means of Solar Radiation for each Day in the Year.
(Tables XV. and XVI.)

The data utilised in this discussion have been derived from daily observations made during the ten years ending with 1898, at my station in the Southside of Edinburgh, 254 feet above mean sea-level. The instrument employed was a self-registering solar radiation thermometer, black bulb *in vacuo* freely exposed to the sun's rays at all seasons, and mounted on a wooden post at a height of 4 feet over a grass plot. The method adopted was to compare the readings of this instrument day by day during the period with the corresponding readings of a maximum thermometer placed in a Stevenson's screen, and enter the excess of the black bulb readings on a form ruled with 365 columns, one for each day in the year. If the shade maximum, for example, was 70°·6 and the black bulb maximum 100°·2, the difference, viz. 29°·6, would represent the solar excess for the day. What the black bulb *in vacuo* really records is open to doubt, as no two instruments exposed under similar conditions register the same, the temperature recorded varying with the height above the ground, the perfection of the vacuum, the density of the glass, and the thickness of the superficial film of lamp-black covering the bulb of the thermometer. All that is claimed for the data discussed is, that they have been derived from instruments exposed as described during the period under review, so that the records are strictly comparable *inter se*. The mean annual excess of the freely exposed black bulb instrument over that placed in the screen is 34°·0, the maximum being 48°·9 in May and the minimum 7°·6 in December. The most rapid increase in solar radiation takes place in February and March, and the most marked decrease in November and December. The greatest excess for any day in the year is 55°·5 on May 27, and the least 5°·1 on December 18. The maximum excess thus occurs nearly a month before the longest day, while the minimum is registered about the time of the winter solstice. The following diagram (see Plate) shows the excess or defect from the annual mean for each day in the year. By "mean line" is meant the average excess of 34°·0, which is the mean of the 365 daily values. All the values exceeding 34° are placed above the mean line, while those under it are less than the mean value. In order to obtain an approximation to a smooth curve, and thus eliminate irregularities due to the comparative shortness of the period discussed, the values have been treated by Bloxam's method of taking the mean of continuous five-day groups. For example, the value for January 3, viz. 25°·4, is not the actual departure from the normal for that day, but the mean of the five-day period January 1 to 5, that of the 4th of January the mean of the five-day group January 2 to 6, and so on. The effect of this method is to reduce considerably the saw-like character of the curve, and thus place the facts in a clear light. To obtain an approximate idea of the relations subsisting between solar radiation and the other climatic elements, the above curve should be compared with Plates I. and II. appended to Part I. of "The Meteorology of Edinburgh" (*Trans.* vol. xxxviii., part iii.). Although the daily means of solar radiation now discussed are for a period of but ten years, yet they will be found to offer many resemblances to the curves of other elements deduced from much longer periods of observation, which are given in my previous work. The following are the more important characteristics of the curve.

In January solar radiation falls to a minimum about the 3rd of the month, and thereafter increases in a somewhat irregular manner till March 23, when a well marked maximum is reached. There is a decided diminution in the values of black bulb excess during the first ten days of April, which is associated with a fall of pressure and a maximum of easterly winds. About this time the day values of shade temperature exhibit a slight fall, doubtless due to the cloudy weather which accompanies winds from the east. Solar radiation increases rapidly after the 4th of April, and culminates in a maximum about the end of the month, when pressure is high and north-west winds in excess. In May there is a general fall in the values until the 14th, the minimum being associated with a high barometer which accompanies the annual maximum of easterly winds. After the 15th solar radiation increases rapidly till the 23rd, when the excess of the black bulb over the shade temperature is greater than at any other period of the year. During the first four days of June there is a decided tendency for cloudy weather, and a rapid diminution in solar radiation occurs. At this time east winds are frequent. From June 9 to August 2 the curve is comparatively featureless; maxima are, however, shown about June 21 and July 8 and 21, and minima about July 11 and 27. The period in August characterised by a great excess of rainfall known as the "Lammas floods" shows a marked diminution in the intensity of solar radiation, which falls to a minimum about the 9th. The effect of this heavy precipitation is evidently to greatly increase the transparency and diathermancy of the air, as a rapid rise in the black bulb excess follows the copious rainfall, a maximum in the radiation values being shown about the 16th. From August 29 to September 7 another wet period prevails, and the excess of the black bulb over the shade temperature diminishes rapidly. A slight rise follows, but the values remain low till the 16th. Between September 17 and October 9 the values do not show much variation, but there is a very noticeable fall in the ten days succeeding October 9. This period has more rainy days than any other time of the year. After a slight increase from the 20th to the 25th of October solar radiation diminishes till the annual minimum at the end of December. A few and unimportant interruptions in the general decline take place from time to time.

Non-Instrumental Phenomena with Different Winds in Edinburgh from 1857 *to* 1900. (*Tables XVII. to XXVI.*)

The observations discussed are those taken during the forty-four years 1857–1900 at the Edinburgh stations of the Scottish Meteorological Society. The place of observation during nearly the whole time was in the Newington district, so that the various registers are strictly comparable. The daily weather notes for the period under review were read, and notices of snow, hail, gales, thunderstorms, lightning without thunder, fog, solar and lunar halos and auroras extracted along with the direction of the wind at the time. Tables were then prepared showing the number of times these phenomena occurred, with winds from the N., N.E., E., and so on, for the eight principal directions, and with calms. These tables, although necessary as an essential preliminary, are of little use without a table (see Table XXV.) showing the number of days the wind blew from the various directions, it being quite evident that the number of gales experienced with a West wind will depend, other causes being equal, on the frequency with which the wind in question prevailed. For example, the mere statement that 320 gales were experienced from the South-West and 66 from the East during the period under review conveys but little information, but when we have ascertained that the South-West wind blew on 2118 days, and the East wind on 2542 days, we are able to say that 15·1 per cent. of the days characterised by a South-West wind were stormy, but only 2·6 per cent. of the days on which the East wind blew. The more prominent results are expressed in percentages in Table XXVI.

Snow. (*Table XVII.*)

In discussing statistics of snow the values were computed for the six months November to April as very few cases were observed in October and May. The snowiest direction is N.E. with 25·6 per cent. of snowy days, closely followed by North with 21·5, and East with 16·5 per cent.; the lowest value is with S.W. winds, only 3·4 per cent. being accompanied by snow. It is evident that the temperature of the air has a great deal to do with the occurrence of snow, the coldest winds, as was to be expected, being those with which snow most frequently falls.

Hail. (*Table XVIII.*)

Hail showers may be divided into three sections: (1) the spring maximum, which extends from February to May, this being almost entirely made up of cases of "graupel" or soft hail; (2) cases associated with thunderstorms, from June to September; and (3) ill-defined falls, principally "graupel", from October to January. Looking at the annual values, it will be seen that hail is most frequent with N.W., N., and N.E. winds, a large number of cases occurring with the N.W. and northerly winds which blow in the rear of a cyclonic disturbance. Very few cases are recorded with south and south-east winds. The spring maximum differs but little from the annual values, the only pronounced feature being that hail with northerly winds is 3 per cent. greater than with N.E. winds, whereas in the case of the annual values, N. and N.E. winds give practically the same percentage.

Gales. (*Table XIX.*)

On the mean of the year, gales are most frequently recorded with winds from the S.W. and W., while very few occur when the direction is North. The barometric gradient is thus much steeper with a S.W. than with a North wind. The seasonal results present few features of interest, with one important exception, viz., in winter, when there is a well marked secondary maximum with winds from the N.E. Those N.E. gales cause great damage and loss of life on the East Coast among the seafaring community.

Thunderstorms. (*Table XX.*)

An examination of the summer thunderstorms, from May to August, shows that they occur most frequently with south-east and south winds, which prevail in the south-east quadrant of the shallow depressions with which the thunderstorm is associated. Very few cases are recorded with south-west and northerly winds or when the air is calm, as the thunderstorm is usually accompanied by a squall. The winter thunderstorms and other electrical phenomena almost always take place with west or south-west winds which accompany the deep cyclonic storms so prevalent at this season.

Fog. (*Table XXI.*)

Fog, as is to be expected, occurs most frequently when the air is calm, but the values with N.E., E., and S.E. winds are also high; indeed, nearly all the fogs in spring and summer are associated with winds blowing off the North Sea. The few cases observed with S.W., W., and N.W. winds take place in winter during hard frost, just before a thaw.

Solar and Lunar Halos. (*Tables XXII. and XXIII.*)

These optical phenomena are commonly observed when the surface wind is from the west, calms however having a still higher percentage. They frequently herald a deep barometric depression, and are thus of considerable forecasting value. With easterly winds halos are rarely observed.

Aurora Borealis. (*Table XXIV.*)

From an examination of the data, it cannot be said that any connection can be traced between displays of the aurora and the direction of the surface winds at the time of their appearance.

Price of Wheat per Imperial Quarter in Edinburgh from 1801 *to* 1900.

Table XXVII. shows the fiars prices of the first quality of wheat from 1801 to 1900. The data have been obtained from the Teind Office, through the courtesy of Mr Nenion Elliot, S.S.C., and from Oliver & Boyd's Almanac. The highest price of wheat during the period was 120/ in 1812, and the lowest 25/4 in 1893. A comparatively full account of the political, economic, meteorological and other causes affecting the price of wheat locally will be found in my paper, "On the Price of Wheat at Haddington from 1627 to 1897."* Table XXVIII. shows the harvest dates at the farm of Liberton, near Edinburgh, from 1812–1836.

In Table XXIX. will be found a list of Errata contained in Part II. of "The Meteorology of Edinburgh," *Trans.*, vol. xxxix. p. 63.

* *Accountants' Magazine*, February 1900.

APPENDIX OF REMARKABLE ATMOSPHERIC AND CELESTIAL PHENOMENA AND EPIDEMICS.

In Part II. of "The Meteorology of Edinburgh" (*Trans.*, vol. xxxix., part i. pp. 93–108) I gave a catalogue of phenomenal atmospheric vicissitudes observed in Edinburgh from 1575 to 1895. The following list brings the catalogue down to the end of 1900, and also contains a considerable number of notices of unusual phenomena collected since the publication of the above memoir. As before, except in those cases where the phenomenon was of a very remarkable character, the extracts from newspapers, books, and observers' manuscript notes appear in a highly condensed form. A list of all the more important epidemics is also given, the information being largely derived from Dr Creighton's exhaustive *History of Epidemics in Britain*. In order to keep the catalogue within reasonable limits, notices of phenomena that can be readily perceived from an inspection of the tables in the paper have been excluded.

Year.	Phenomenon, or Epidemic.	REMARKS.
1133	Eclipse	August 2. Total solar eclipse. In London nearly nine-tenths of the sun's disc were obscured.
1185	Eclipse	May 1. In Scotland, Tycho Brahé says this solar eclipse was total. In London, between eight and nine tenths of the sun's upper limb covered soon after 2 o'clock.
1330	Eclipse	July 16. Eclipse of the sun, total for a very short time across Scotland.
1433	Eclipse	June 17. This eclipse was total across Scotland, including Edinburgh, also in Northumberland. The total phase occurred about 3 P.M., and was referred to for generations afterwards as the "black hour."
1497	Epidemic	"A contagious sickness callit grangore" breaks out; the infected are sent to Inchkeith.
1541–42	Severe Winter	"And this same yeir ane storm callit the evill stoorme began the fourteine day of Yuill, and continued untill the tenth day of Appryll thairefter." Lindsay's *Chronicles of Scotland*, vol. i., Edinburgh, 1814.
1545	Plague	Prevalent in the town previous to June 24.
1562	Influenza	An epidemic of very characteristic influenza was prevalent towards the end of the year, which was known as "the newe acqayntance."
1568	Plague	Great plague, probably the most severe that Edinburgh experienced, commenced 8th September, having been brought by "ane called James Dalgleish, merchant." The infected families were compelled to remove out of the town, and were lodged in wretched huts hastily erected on the Burgh-moor. Plague continued until February 1569, and carried off 2500 of the inhabitants.
1574	Plague	Plague came to Leith on October 14, and was first observed in Edinburgh on October 24.
1585	Plague	The infection broke out in the Flesh Mercat Close on May 1, and continued through the winter until January 1586. Trade greatly depressed.
1587	Plague	Plague in Edinburgh about November 4.
1598	Dearth	June 6. Oatmeal sold for 6s. the peck.
1602	Plague	Plague from February 4 to end of April. The sick were removed to temporary lodgings in the land of Sciennes, belonging to Napier of Merchiston.
1604	Plague	In April a case was recorded, the infection spreading rapidly in May. In July the epidemic became so severe that people fled the city. The infection reappeared towards the end of July 1605.

Year.	Phenomenon, or Epidemic.	REMARKS.
1607–8	Frost	A vehement frost continued from Martinmas 1607 till 20th of February 1608. "The sea froze so far as it ebbed, and sundry went into ships upon ice and played at *chamiare* a mile within the sea-mark. Sundry passed over the Forth, a mile above Alloa and Airth, to the great admiration of aged men, who had never seen the like in their days." Rivers and springs were frozen, the young trees were killed, and birds and beasts perished in great number.
1624	Plague	Plague discovered to be in several houses on November 23, and the session of the law courts adjourned till January 8.
1644–47	Plague	Plague severe. The plague-stricken were housed in huts in the King's Park, below Salisbury Crags. This was the last epidemic of plague in Edinburgh.
1652	Early Harvest	Corn was shorn in June and harvest finished in August without rain, storm, or tempest.
1654	Eclipse	August 12. Solar eclipse, very nearly total. At Inverness total about 8.49 A.M.
1659	Rainstorm	September 1 to 4. Great storm of wind and rain commenced on the evening of September 1, and continued for three days and nights. Several houses "in and upon" the Water of Leith, with eleven mills belonging to Edinburgh and five belonging to Heriot's Hospital, destroyed.
1669	Early Barometric Observations	SINCLAIR, who was Professor of Natural Philosophy in the University of Glasgow, made observations of the height of the barometer, or "baroscope" as he names it. The place of observation was in the vicinity of Dalkeith, where he observed a height of 29·9 inches in December 1669, the lowest being 27·9 inches in March following.
1681	Storms	Very severe storms of wind during the winter.
1698	Dearth	Very late harvest; corn in many places was not reaped till January 1699, and the snow beaten off it. Bread made of it fell in pieces and tasted sweet like malt.
1699	Eclipse	September 23. Solar eclipse. Total for 10 or 15 seconds at 9 A.M. in the north of Caithness. This was the last total solar eclipse in Scotland. JOHN MARR, "professor of navigation," calculated that the greatest obscuration at Edinburgh took place at 9.25 A.M., the digits eclipsed being 11.
1707–8	Frost and Snow-storms	The frost lasted from early in October till end of April; not so severe as in England, but with much snow, which began on January 25, 1708, and "fell for several days together."
1724	Eclipse	May 22. Partial eclipse of the sun, total in England.
1731	Dysentery	Epidemic in the autumn.
1732–33	Influenza	Horses were "attacked with running at the nose and coughs towards the end of October and beginning of November." Similar symptoms began suddenly among men on December 17. The epidemic in a week became general, very few escaping, and remained in the city until the middle of January 1733. It was remarked that the boys of Heriot's Hospital and the prisoners in the gaol escaped. The effect on the death-rate was well marked, the number of burials in November 1732 being 89, in December 109 interments took place, the number reaching a maximum in January 1733 with 214. By February the number of burials had fallen to 135. The principal victims were "poor, old, and consumptive people."
1734	Dysentery	In the autumn there was a severe epidemic, "being fatal to some and very tedious to others."
1735	Measles	The epidemic began in June and became universal in December.
1735–36	Fever	From October 1735 to February 1736 there was an epidemic of relapsing fever.
1737	Eclipse	March 1. Annular eclipse at Edinburgh. MACLAURIN, who observed there, reported:—"A little before the annulus was complete, a remarkable speck of pale light appeared near the middle of the part of the moon's circumference that was not yet come upon the disc of the sun. During the appearance of the annulus the direct light of the sun was still con-

Year.	Phenomenon, or Epidemic.	REMARKS.
1737	Eclipse—*contd.*	siderable, but the places that were shaded from his light appeared gloomy. Venus appeared plainly, and continued visible long after the annulus had dissolved, and I am told that other stars were seen by some." It was remarked by Lord ABERDOUR that a narrow streak of dusky red light coloured the dark edge of the moon immediately before the ring was completed and after it was dissolved.
1740	Dearth	May 16. In consequence of the high price of provisions, due to a bad harvest in 1739, a mob attack Leith and Bell's mills; the military having been called out, fire on the crowd, and wound three of them, one of whom died.
1740–41	Smallpox	Severe outbreak, the mortality in 1740 being 274, and in 1741, 206, or 17 per cent. of the total deaths from all causes.
1741	Whooping-cough	The deaths from whooping-cough, which in 1740 numbered 26, rose in 1741 to 101, the population at the time being 30,000.
1743	Influenza	Severe epidemic of influenza in the spring, the weekly burials being trebled in consequence.
1744	Storm	February 19. Hurricane, which untiled several houses and removed the lead roof of the Tron Church.
1744	Thunderstorm	August 13. "We have had these three days past great rains in this neighbourhood, and yesterday from half-an-hour after 11 till near 3 afternoon one continued deluge, with hailstones of an extraordinary shape and great size. The water rose in the Parliament Close to such a height that it broke down the new stairs leading to the Cowgate in floods sufficient to set any mill agoing. In the Cowgate all the low houses were filled with water, so that a boat might have rowed in the streets. From 10 in the forenoon till near 3 afternoon there were a great many and most terrible peals of thunder with lightning which broke upon the Castle in several places, damaged the roof of the great lodging over the Half-moon, also the carved stone thistle that stood above one of the large windows. Three soldiers in one room were hurt and several others beat off their legs. James Campbell, gunner, suffered most, and lies dangerously ill; the blow he received has left a fair impression of a star on his shoulder bone. A house or two in the Grassmarket was also damaged, the windows of a house at Fountainbridge were quite demolished, the walls of the church of Liberton rent, etc. Though the oldest people living do not remember to have seen a storm so dreadful, yet we hear that it did not extend above a very few miles from this city."—*Caledonian Mercury*, August 14, 1744. A waterspout burst on the west side of Arthur's Seat, dividing into two portions, one of which tore up the channel under the Lion's Head known as the "Guttit Haddie," while the other flowed down the west side and inundated the village of Duddingston, carrying away the gable of the most westerly cottage, and flooding the loch over the adjacent meadows. The hailstones measured 5 inches round, and entirely threshed the standing corn.
1747	Drought	Autumn hot and dry, the rivers lower than ever known.
1748	Hot Summer	Summer exceptionally warm and dry.
1748	Eclipse	July 14. Annular eclipse observed by Lord MORTON and SHORT from Aberdour Castle, just out of annularity. A brown light was seen to stretch along the circumference of the moon from each of the cusps. Venus was visible to the east of the sun. There was no great darkness, the sky appearing of a "faint, languid colour."
1753	Snowstorm	February 15. Edinburgh roads almost blocked with snow.
1758	Influenza	First noticed with east winds from the 16th to 20th September. In October it became general, not one in six or seven escaping.
1758	Meteor	November 26. Equal to size of moon, travelled from south to north. Of great brilliancy. Seen all over Great Britain.
1759	Smallpox	Out of 1136 deaths in the year, 232 were from smallpox.
1762	Smallpox	The deaths from smallpox numbered 274 out of a total mortality for the year of 1305.
1775	Influenza	Epidemic became prevalent towards end of the year.

Year.	Phenomenon, or Epidemic.	REMARKS.
1782	Influenza	The epidemic appeared about the middle of May, and was at its height in July during the haymaking.
1783	Meteor	August 18. A round, well-defined meteor seen at 9 P.M. in zenith. It was of a greenish colour, and cast a shade upon the ground of a similar tint. A tail of considerable length attended it.
1786	Gale	July 27. Between 10 A.M. and 3 P.M. it blew almost a hurricane. It was so violent as to unroof houses, root up trees, and to destroy the fruit in the gardens. "So severe a storm has seldom happened here at this season."
1787	Meteor	September 17. At 8.30 large meteor seen. Brightly luminous, larger than apparent magnitude of the sun, and of an elliptic figure. It was seen for two minutes travelling from west to east, when it burst behind a cloud.
1788	Influenza	The epidemic in Edinburgh, but apparently not of a virulent type.
1793	Eclipse	September 5. Solar eclipse, annular about the Shetlands and Orkneys.
1803	Influenza	Epidemic in April, there being a considerable increase in the burials in Greyfriars Churchyard.
1803	Meteor	November 13. Equal to moon in size. Travelled from E.S.E. to W.N.W. Also seen in London 23 miles high when first observed. Velocity 8 miles a second.
1804–5	Scarlet Fever	Broke out in 1804 among the boys in Heriot's Hospital, and in the city generally in 1805.
1807	Snow	April 16 and 17. Ground covered with snow to the depth of several inches, and long icicles hanging from the roofs of houses.
1807–8	Measles	Began at Edinburgh in the winter of 1807 and lasted till the summer of 1808. An observer remarked, "I believe that the present epidemic has been more general in this place and its vicinity than ever happened within the remembrance of any medical man at present living, and I am sorry to say it has been very fatal." A feature of the epidemic was the number of adults attacked.
1808	Meteor	October 17. A large and bright meteor was seen about 8.15 P.M. Its course was nearly from S.E. to N.W. When first observed, it appeared to descend obliquely, but when near the earth seemed to fly off in a horizontal direction, increasing in brilliancy and of a red colour.
1809	Snow and Frost	April 16. Heavy showers of snow and hail, with strong easterly wind. Blossom of the apricot, peach, and plum trees destroyed. Hard frost continues nightly till the 21st.
1811	Lightning	December 4. Several flashes of lightning were observed in the evening.
1817–18	Smallpox	Epidemic in the spring of 1817, and again in 1818.
1817–19	Relapsing Fever	During these years a severe epidemic of relapsing fever continued in Edinburgh. A special fever hospital was opened at Queensberry House, the cases admitted to December 1, 1819, together with the fever cases at the Royal Infirmary, numbered 3110, of whom 138 died, or 4·4 per cent. The epidemic was attributed to the destitution occasioned by the failure of the 1816 crop, and subsequent high prices.
1820	Eclipse	September 27. Great solar eclipse. (In *Trans.*, vol. xxxix. p. 101, the date is wrongly given as January 7.)
1825	Meteor	November 14. Travelled from E. to W., quick motion. Tail long and large.
1826	Fog	January 15. Thick fog. Moisture congealed in beautiful ice pellicles on every object. At 10 A.M. the temperature was 15°·5.
1826	Aurora	January 21. Beautiful auroral display, "somewhat resembling the famous arch of 19th March 1825." The moon was extremely bright, but the aurora was of such tenuity that stars of the sixth and seventh magnitude could be seen through it.
1826	Meteor	April 9. At 8.25 P.M. a meteor was observed to move through an arc of 7° to 10°.
1826	Meteor	April 29. Brilliant meteor observed at 9.35 P.M., and another at 11.13 P.M.

Year.	Phenomenon, or Epidemic.	REMARKS.
1826	Waterspout	May 14. At 10.30 A.M. the unusual phenomenon of an aerial waterspout was observed in the neighbourhood of Edinburgh. A light dusky cloud of a funnel shape was seen in the N.W., clearly relieved from a darker cloud behind. It was evidently transfusing the contents of a very dense and dark black cloud into one immediately below, with which it formed the only connection. At this time the lower extremity of the waterspout was bent from the direction of the wind, being N.E.; the upper cloud was moving in a contrary direction. In a very few minutes it reached its greatest distinctness, and a manifest transfusion of the contents was taking place, the column presenting the appearance of smoke or steam, and the undulation at the edges was scarcely perceptible. The undulations at the edges gradually increasing in distinctness, the waterspout grew less elongated, and the bottom turned in a direction contrary to the wind, which below still remained in the same point. The cloud above had now become less dense, while the one below increased in blackness; it then became short, with a broad conical termination. During this change the currents descending on the east and ascending on the west side presented at the bottom the appearance of violent ebullition. The waterspout then merged into the cloud above, and in about twenty minutes after being first observed it wholly disappeared. A heavy downpour of a local character accompanied the phenomenon. (See *Edinburgh Jour. of Science*, vol. ix. p. 131.)
1826	Meteor	August 26. Travelled from S.W. to N.E., fireball like a rocket. 45°, arc; train; burst.
1827	Aurora	September 9. Remarkable aurora, consisting of variously shooting threads of light, forming an arch round Ursa Major, and at each extremity shooting downwards with great beauty, although the moon shone at the time. Some highly electrified cirrus clouds were observed in the zenith.
1827–28	Relapsing Fever	"Like that of 1817–19, the epidemic arose in Edinburgh during a protracted period of want of work and low wages among the labouring classes and tradespeople; it prevailed only among the working classes and unemployed poor—in the Fountainbridge and West Port districts, the Grassmarket 'closes,' the Cowgate, and the narrow 'wynds' descending on either side of the long sloping back of the High Street and Canongate." —CHRISTISON.
1828	Dysentery	Outbreak began about the end of July; attacks of it were numerous among the patients admitted to the Infirmary.
1828	Typhoid	A remarkable outbreak of typhoid among the richer classes of the west-end of Edinburgh, known as "the New-Town Epidemic."
1828	Meteor	November 21. A meteor of such great brilliancy that it was seen during bright sunshine.
1829	Enteric Fever	"A very few cases of enteric fever were dissected in Edinburgh."—CHRISTISON.
1830–31	Smallpox	In the winter of 1830–31 this disease was unusually prevalent and fatal, the epidemic dying out in May 1831.
1832	Cholera	The Asiatic cholera first appears on January 27, but up to the 8th of February there had been only 8 cases, with four deaths. From the middle of February to the middle of June the new cases ranged, as a rule, from five to ten or fifteen a day, with an occasional increase, as, for example, on the 29th of April, when twenty-six persons were seized. There was a marked diminution in the end of May and beginning of June, after which the disease became more common. The maximum occurred in the week ending 7th October, when there were 214 cases, the maximum in one day being 45 on October 4. The epidemic was practically over before the middle of November. The total number of persons attacked was 1886, of whom 1065 died, the population of Edinburgh at the time being 136,301.
1833	Influenza	Epidemic in April and May.

Year.	Phenomenon, or Epidemic.	REMARKS.
1833	Whirlwind	July 29. The temperature about 4 P.M. was about 75° in shade, and the atmosphere felt sultry. Suddenly, about 5 P.M. the thermometer fell 20°, and immediately there rose a violent wind, which for a brief time blew furiously, the dust rising above Arthur's Seat like a brown-coloured haze.
1835	Scarlet Fever	This epidemic was remarkable for the large number of malignant cases.
1836	Eclipse	May 1. An annular eclipse of the sun was observed at 3 P.M. The day being Sunday, the greater number of the churches in the city had no afternoon service in consequence. HENDERSON states that previous to the formation of the annulus an arc of faint reddish light was seen.
1836–39	Typhus Fever	Epidemic of typhus, culminating in 1838, when 2244 cases were admitted into the Infirmary. This was the highest number of cases admitted in the period 1831–1840, the next highest being 1394 in the year 1832.
1839	Lunar Rainbow	August 20. Lunar rainbow at 8 P.M.
1839	Meteor	November 8. Twice size of moon, motion N.W. to S.E. Travelled downwards. Increased gradually from size of Venus.
1841	Lunar Rainbow	October 30. Lunar rainbow.
1843–44	Relapsing Fever	The fever was first observed in February 1843; it increased rapidly until October, when the number of cases admitted into the Infirmary amounted to 638. During several months from 30 to 50 cases were daily refused admission. The total number of cases in Edinburgh, according to ALISON's calculation, was 9000. The fever abated about the end of February 1844.
1844	Meteor	April 11. Large dark-red-coloured fireball observed, travelled from N. to S.
1848	Meteor	January 9. Meteor, one-third diameter of moon, travelled slowly from north to south, and burst while expanding itself.
1848	Cholera	The disease broke out almost simultaneously at Newhaven and Edinburgh on the 1st and 2nd of October, and at Leith on the 9th. The epidemic lasted until the 18th of January 1849, the number of persons attacked being 801, of whom 448 died. A cholera hospital was opened in Surgeon Square on the 28th of October, and 248 cases admitted up to 14th December, of whom 154 died. "Of the whole 248 cases, the Grassmarket sent 42, the Cowgate 37, the Canongate 33, College Wynd 16, High Street 14, and numerous scattered localities of the New and Old towns one or more cases each."
1848–49	Influenza	Severe epidemic, no less than 15 per cent. of the total cases admitted to the Royal Infirmary suffering from this disease.
1852	Meteor	July 12. Meteor, two-thirds diameter of moon, travelled towards N.E.
1853–54	Cholera	On the 16th September 1853 the old Cholera Hospital in Surgeon Square was opened, but only 45 cases were received until it was closed at the beginning of June 1854. The real epidemic began in the autumn of 1854; the hospital was again opened on 24th August, the number of admissions up to the end of November being 198.
1855	Meteor	December 11. Round meteor of a violet colour seen. Tail, and sparks thrown off.
1870	Mirage	July 22. Remarkable mirage seen in the Firth of Forth. The day was very hot and sultry, and there was a peculiarity about the atmosphere which is seldom observed in this country. About midday a thin, clear and transparent kind of vapour, through which the surrounding objects began to make their appearance in the most fantastic and grotesque shapes imaginable, settled over the sea. The phantasmagoria were principally confined to the mouth of the Firth, but at one time the phenomena embraced the whole of the Fife coast, the towns and villages being high up on the horizon with remarkable distinctness. Though the whole coast seemed at least half way up the horizon, the appearances presented by the towns were very different, some of them having the houses inverted, while others appeared in their natural position. The Bass Rock, the Isle of May,

Year.	Phenomenon, or Epidemic.	REMARKS.
1870	Mirage—*contd.*	and the rocks around Dunbar harbour assumed extraordinary forms. The Bass, which at one time seemed to lie flat upon the sea, suddenly shot up into a tall spiral column, apparently ten times its usual height, surrounded by battlements rising tier on tier, and presenting a most imposing spectacle. The Isle of May, in the course of the afternoon, underwent an almost innumerable series of transformations. At one time it was apparently as round as a circle, at another seemingly drawn out for miles against the horizon, now flat upon the water, then rising to ten times its usual height. Occasionally portions appeared to break off and sail away, then to return and unite again, all within the space of a few minutes. Vessels in the offing appeared double, one on the water and another inverted in the air, and in one instance three figures of one vessel were distinctly visible. The fishing-boats proceeding to sea in the evening underwent the same transformations when only a few yards off the shore, the double appearance being distinctly visible within a certain distance. The rocks at the harbour also seemed to play fantastic tricks, opening and shutting, rising and falling, with apparent regularity. These extraordinary phenomena lasted from midday till nightfall.
1870	Aurora	August 19. Sky very clear at 1 A.M. Auroral bank of light strong on N.W. horizon.
1870	Aurora	August 20. At midnight a beam of aurora vertical in N.W.
1873	Gale	September 13. Severe gale at 2.30 P.M. Building in course of erection and almost completed, 100 feet long, 75 feet wide and 24 feet in height, belonging to A. B. Fleming & Co., printing-ink manufacturers, blown down.
1877	Storm	January 1 and 2. Severe N.E. gale, with high tide. Nearly the whole of the sea-wall between Portobello and Joppa washed away.
1877	Snowstorm with Thunder	October 11. Gale and snowstorm, accompanied with several peals of thunder, in forenoon.
1884	Meteors	November 14. A number of bright meteors observed.
1886	Snow-squall	January 31. Violent snow-squall from 7.55 to 8.8 A.M. The melted snow yielded 0·15 inch of water.
1886	Aurora	March 30. At 9 P.M. a most magnificent and extensive display of aurora, first stationary, with bifurcate streamers N.W. and E.S.E., then serpentine curves, with pulsations of varying brightness from horizon to zenith.
1886	Aurora	November 19.
1886	Aurora	December 21.
1888	Earthquake	February 2. Slight shock of earthquake.
1889–90	Influenza	The epidemic breaks out on December 20, reaching a maximum in the second week of 1890. Epidemic over by end of February.
1891	Influenza	Slight epidemic from April 12 to July 12.
1891–92	Influenza	Influenza prevailed in epidemic form from 25th October 1891 to 13th February 1892.
1893	Influenza	Epidemic from March 11 to April 28.
1893–94	Influenza	Epidemic from 15th October 1893 to 31st January 1894.
1895	Influenza	Severe epidemic from February 11 to March 31. In the three weeks ending with March 23 the average rate of mortality in Edinburgh was 43·7 per 1000, being the highest for at least twenty years.
1896	Early Snow	October 10.
1897	Rain from a cloudless sky	February 25. At 9 P.M. the sky was cloudless, but the air was saturated, rain falling heavily. A strong gale was blowing at the time.
1896–97	Sunless weather	During the ten months ending with June 1897, the total bright sunshine amounted to only 712 hours, or 20 per cent. of the possible.
1898	Thunderstorm	February 6. A thunderstorm, with heavy snow and hail, was experienced at 3.31 P.M.
1898	Aurora	March 15. Brilliant aurora.
1898	Rainstorm	August 30. During a thunderstorm 1·42 inch of rain fell in four hours.

Year.	Phenomenon.	REMARKS.
1898	Heat Wave	September 3–17. The mean temperature of these fifteen days was 63°·6 or 8°·5 in excess of the normal. From the 4th to the 6th the maximum temperatures were over 80°, being 82°, 82°·8, and 80°, while the minima were 59°, 57°·6, and 62°·6. The mean temperature was 70°·5 on the 4th, 70°·2 on the 5th, and 71°·3 on the 6th, this being the only occasion in eighty years on which each of three consecutive days had a mean temperature in excess of 70°. The nocturnal warmth from the 3rd to the 8th was very unusual, the mean of all the minima being 60°·2; the air was unusually damp, which made the heat very oppressive.
1899	Auroras	February 12 and 23.
1899	Silver Thaw	February 26. Rain fell, with the temperature below the freezing point, covering trees, telegraph wires, etc. with a transparent sheet of hard ice.
1899	Cold Wave	March 19–25. The mean temperature for the week was 32°·6, or 8°·6 below the normal.
1899	Aurora	March 21.
1899	Drought	May 24 to June 17. During these twenty-five days no rain fell, the drought being the most severe since the summer of 1869.
1900	Thunderstorm and Aurora	January 19. Thunderstorm, with heavy rain and hail, at 5.45 P.M.; a bright aurora was seen at 6·15 P.M.
1900	Snowstorm	February 15. South-east gale, with heavy snowdrift.
1900	Snowstorm	February 20. Heavy snow during night; eight inches fell.
1900	Eclipse	May 28. Partial solar eclipse, observed under fair conditions.
1900	Thunderstorm	June 12. Severe thunderstorm, with great darkness and heavy rain and hail. The storm was the worst experienced for sixteen years. Several houses struck.
1900	Rainstorm	August 6. Two inches of rain fell in 16 hours, with squally E. and N.E. wind.
1900	Thunderstorm and Waterspout	August 23. Severe thunderstorm, with very heavy rain and hail, from 5.30 to 6.20 P.M., 0·62 inch falling in eighteen minutes. A well-defined waterspout was seen at Duddingston about 6 P.M., remaining visible for fully ten minutes, when the lower portion slowly melted away, the upper part being absorbed into the clouds. It presented much the same appearance as the one seen at Edinburgh on May 14, 1826, described in the *Edin. Jour. of Science*, vol. ix. p. 131, a graphic representation of which is shown in plate ii. appended to that volume.
1900	Meteor	September 2. Brilliant meteor seen at 6.55 P.M. in south, of a blue colour. Travelled straight down.
1900	Gale	December 20–21. Severe S.W. gale. Anemometer at Blackford Hill recorded a mean velocity of 60 miles per hour, rising to 93 miles in the hour ending midnight of 20th. Many trees blown down in Edinburgh and vicinity.
1900	Wet Autumn	Autumn very wet, the rainfall in the Newington district in October being 4·87 inches, in November 5·42 inches, and in December 4·43 inches, a total of 14·72 inches for the three months, or 96 per cent. above the average. October was the wettest since 1864, and November the wettest since 1772.

TABLE I.

Mean and Extreme Values for Periods Exceeding 50 Years.

Element.	Period Covered by Observations.	Years.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
			ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
Barometric Pressure:															
Mean (Corrected and Reduced to 32° and Sea-level),	1770–1900	131	29·820	29·811	29·864	29·892	29·940	29·933	29·880	29·876	29·875	29·813	29·803	29·798	29·859
Mean (Corrected and Reduced to 32° and Sea-level),	1840–1900	61	29·807	29·863	29·859	29·895	29·938	29·931	29·885	29·863	29·894	29·829	29·831	29·820	29·868
Mean of Monthly Maxima,	,, ,,	,,	30·485	30·477	30·482	30·400	30·420	30·345	30·293	30·290	30·417	30·410	30·476	30·472	30·701
Mean of Monthly Minima,	,, ,,	,,	28·896	29·035	29·049	29·215	29·315	29·393	29·349	29·285	29·225	28·972	28·905	28·874	28·511
Mean Monthly Range, .	,, ,,	,,	1·589	1·442	1·433	1·185	1·105	0·952	0·944	1·005	1·192	1·468	1·571	1·598	2·190
Temperature in Shade:															
Mean Monthly, . . .	1764–1900	137	36°·9	38°·3	40°·3	44°·8	49°·9	55°·7	58°·6	57°·8	53°·6	47°·3	41°·0	38°·4	46°·9
,, ,, . . .	1840–1900	61	37·8	38·7	40·6	45·0	49·8	55·8	58·5	57·8	54·0	47·2	41·8	38·9	47·2
Mean of all the Highest, .	,, ,,	,,	42·6	43·9	46·6	52·1	57·5	63·5	65·7	65·0	60·8	52·9	46·8	43·5	53·4
,, ,, Lowest, .	,, ,,	,,	33·1	33·6	34·6	38·0	42·1	48·0	51·3	50·6	47·1	41·4	36·9	34·3	40·9
,, Daily Range, . .	,, ,,	,,	9·5	10·3	12·0	14·1	15·4	15·5	14·4	14 4	13 7	11·5	9·9	9·2	12·5
,, Temperature, . .	,, ,,	,,	37·8	38·8	40·6	45·0	49·8	55·8	58·5	57·8	54 0	47·2	41·8	38·9	47·2
,, of all the Absolute Maxima,	,, ,,	,,	52·1	53·1	57·5	63·9	69·7	74·7	76·5	74·7	70·0	63·1	56·4	54·2	79·3
,, of all the Absolute Minima,	,, ,,	,,	21·4	23·3	24·5	29·6	33·3	39·9	43·4	42·7	38·1	30·6	26·7	23·2	17·3
,, Monthly Range, . .	,, ,,	,,	30·7	29·8	33·0	34·3	36·4	34·8	33·1	32·0	31·9	32·5	29·7	31·0	62·0
,, Daily Variability of Temperature,	,, ,,	,,	3·20	2·91	2·83	2·72	2·81	2·79	2·53	2·53	2·63	2·91	3·14	3·22	2·85
,, Number of Days with Frost, . .	1803–1831 1840–1851 1857–1900	85	14·5	11·3	11·2	4·6	0·8	0·0	0·0	0·0	0·1	2·3	7·4	11·7	63·9
			ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
Rainfall, ...	1770–76, 1780 1785–1900	124	1·94	1·69	1·53	1·49	1·91	2·19	2·79	2·80	2·39	2·51	2·41	2·25	25·90
Non-Instrumental Phenomena:															
Mean Number of Days:															
Thunderstorms, . .	1771–1900	130	0·12	0·09	0·09	0·25	0·80	1·36	1·84	1·25	0·46	0·15	0·07	0·05	6·53
Snow, ...	,, ,,	,,	4·9	4·5	4·8	1·6	0·4	0·0	0·0	0·0	0·0	0·3	1·4	3·1	21·0
Hail, ...	,, ,,	,,	0·6	0·8	1·7	2·1	1·4	0·4	0·3	0·2	0·3	0·6	0·7	0·6	9·7
Gales, ...	,, ,,	,,	4·4	3·6	3·2	1·8	1·2	1·0	0·8	1·2	1·9	2·8	3·0	3·8	28·7
Mist or Fog, . .	,, ,,	,,	1·3	0·7	1·1	1·2	1·6	1·8	1·1	1·2	1·4	1·4	1·3	1·2	15·3
Total Number of Days:															
Lightning without Thunder,	1807–1835 1868–1900	62	9	5	2	4	9	3	12	7	19	7	5	9	91
Aurora, . .	1773–1781 1800–1900	110	33	44	49	33	12	1	6	21	44	47	41	18	349

Table II.

Mean Monthly Values for the 50 *Years* 1851–1900 *and for Periods Exceeding* 10 *and Less than* 50 *Years.*

Means 1851–1900.

Element.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Year.
Mean Barometric Pressure:	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
Corrected and Reduced to 32° and Sea-level,	29·812	29·879	29·850	29·908	29·945	29·941	29·891	29·868	29·888	29·845	29·853	29·806	29·874
Mean Monthly Range,	1·593	1·435	1·428	1·221	1·007	0·929	0·909	0·987	1·152	1·483	1·561	1·597	2·204
Temperature in Shade. 250 feet above Sea:													
Mean of all the Highest,	42°·3	43°·6	45°·9	51°·6	57°·0	63°·1	65°·6	64°·6	60°·4	52°·8	46°·4	43°·2	53°·0
,, ,, Lowest,	33·6	33·8	34·6	38·2	42·3	48·2	51·3	50·9	47·3	41·7	37·0	34·4	41·1
,, Daily Range,	8·7	9·8	11·3	13·4	14·7	14·9	14·3	13·7	13·1	11·1	9·4	8·8	11·9
,, Temperature,	38·0	38·7	40·2	44·9	49·7	55·6	58·4	57·8	53·8	47·2	41·7	38·8	47·1
,, of all the Absolute Maxima,	51·9	52·9	56·6	63·3	69·3	74·3	76·6	74·6	69·4	62·6	56·1	53·9	79·2
,, ,, ,, Minima,	22·5	23·9	24·9	30·0	33·6	40·3	44·2	43·5	38·7	31·0	27·2	23·2	18·2
,, Monthly Range,	29·4	29·0	31·7	33·3	35·7	34·0	32·4	31·1	30·7	31·6	28·9	30·7	61·0
,, Daily Variability,	3·10	2·85	2·74	2·70	2·77	2·78	2·50	2·43	2·51	2·83	3·07	3·25	2·79
Direction of Wind (from observations made at 9 a.m. and 9 p.m.): Mean Number of Days it blew from certain Directions, viz.:	*Mean Number of Days.*												
N.,	1·4	1·5	2·1	1·5	1·6	1·6	1·4	0·9	1·3	1·4	1·7	1·3	17·7
N.E.,	1·4	1·4	2·6	3·3	3·7	2·9	2·4	2·0	2·0	1·5	1·5	1·2	25·9
E.,	2·6	3·3	4·3	7·1	7·8	6·6	5·2	5·1	4·0	4·0	3·3	2·6	55·9
S.E.,	2·1	2·0	2·0	2·1	2·0	1·7	1·1	1·4	1·3	1·9	1·8	1·7	21·1
S.,	3·1	1·9	2·0	1·5	1·8	1·6	1·6	1·5	1·9	2·4	2·0	2·3	23·6
S.W.,	6·1	4·6	3·8	2·7	2·6	3·3	3·6	4·4	4·2	4·2	4·9	6·1	50·5
W.,	10·9	9·8	9·8	7·7	7·7	8·9	12·4	12·4	11·5	11·7	10·4	11·8	125·0
N.W.,	2·3	2·3	3·4	2·9	2·6	2·2	2·4	2·0	2·2	2·7	3·0	2·6	30·6
Calm or Var.,	1·1	1·2	1·0	1·2	1·2	1·2	0·9	1·3	1·6	1·2	1·4	1·4	14·7
Rainfall at Charlotte Square,	ins. 2·09	ins. 1·68	ins. 1·68	ins. 1·49	ins. 1·90	ins. 2·31	ins. 2·76	ins. 3·00	ins. 2·47	ins. 2·44	ins. 2·29	ins. 2·31	ins. 26·42
Non-Instrumental Phenomena: Mean Number of Days of—	*Mean Number of Days.*												
Thunderstorms,	0·20	0·14	0·16	0·32	0·98	1·68	2·18	1·48	0·56	0·34	0·10	0·08	8·22
Snow,	3·7	4·2	4·9	1·0	0·4	0·0	0·0	0·0	0·0	0·3	1·2	3·0	18·7
Hail,	0·7	0·6	1·2	1·4	1·4	0·5	0·4	0·4	0·3	0·7	0·6	0·5	8·7
Gales,	4·1	2·8	2·6	1·1	0·9	0·8	0·7	0·8	1·7	2·3	2·4	3·5	23·7
Mist or Fog,	1·8	1·2	1·3	1·2	1·2	1·8	1·1	1·3	1·2	1·9	1·8	1·7	17·5
Total Number of Auroras,	14	21	30	17	12	1	4	12	23	27	21	9	19·1

TABLE II.—*continued.*

Showing Mean Monthly and Annual Values for Periods Exceeding 10 *and Less than* 50 *Years.*

Element.	Period.	Years.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Mean Daily Temperature— Max. and Min., . . .	1857–1900	44													
Mean of Absolute Highest, .	,,	,,	47°·4	48°·1	50°·0	53°·5	58°·7	64°·1	66°·3	65°·7	61°·7	56°·6	50°·9	49°·2	68°·3
,, ,, Lowest, .	,,	,,	28·3	29·9	31·7	37·6	41·4	47·6	52·0	51·4	46·9	37·9	32·8	28·4	24·9
,, Range, ...	,,	,,	19·1	18·2	18·3	15·9	17·3	16·5	14·3	14·3	14·8	18·7	18·1	20·8	43·4
Max. Daily Range of Temperature,	,,	,,	23·3	24·0	31·4	39·0	36·0	32·6	33·3	31·0	30·0	25·6	26·3	29·0	39·0
Mean Relative Humidity Saturation = 100 (Hours 9 and 9),	1862–1900	39	86·5	86·0	83·7	80·4	78·1	77·7	78·9	81·5	82·6	85·7	86·7	86·1	82·8
Days with 0·01 inch or more of Rain,	1877–1900	24	16	14	15	14	14	14	17	19	16	17	17	17	190
Total Number of Solar Halos observed,	1857–1900	44	23	18	35	34	18	14	16	20	16	18	13	10	235
Total Number of Lunar Halos observed,	1857–1900	44	46	22	42	23	12	6	3	2	20	29	36	39	280

Showing Mean Monthly and Annual Values of the … *1900.*

	Barometric Pressure Corrected and Reduced to 32° and Mean Sea-level. Mean Monthly Pressure.	Monthly Range.	Temperature in Shade. 250 Feet above Sea-level. Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Mean Temperature.	Monthly Range.	Mean Daily Temp. Variability.	Relative Humidity Saturation = to 100. Hours 9 A.M. and 9 P.M.	Mean Amount of Cloud 0–10. Hours 9 A.M. and 9 P.M.	Rainfall. Charlotte Square.	Blacket Place.	Mean Number of Days with 0·01 in. or more of Rain.	Sunshine. Mean Number of Hours.	Mean Percentage of Possible.	Mean Number of Sunless Days.	Black Bulb in Vacuo. Mean Black Bulb Maximum.	Mean Excess over Shade Maximum.
	ins.	ins.	°	°	°	°	°	°			ins.	ins.					°	°
January,	29·891	1·552	42·1	33·7	8·4	37·9	28·3	2·99	83·5	6·5	1·28	1·63	17	32	15	14	56·1	14·0
February,	·880	1·457	44·1	33·8	10·3	38·9	31·7	3·05	83·1	5·9	1·80	2·05	13	62	23	7	71·6	27·5
March,	·832	1·416	47·0	35·0	12·0	41·0	33·9	2·79	80·4	5·9	1·63	1·86	15	104	28	5	84·7	37·7
April,	·926	1·083	52·9	38·6	14·3	45·7	34·4	2·66	78·4	6·7	1·22	1·35	13	132	31	4	97·3	44·4
May,	·985	1·158	57·9	42·5	15·4	50·2	35·3	2·56	77·3	7·0	1·85	2·02	14	170	34	4	107·0	49·1
June,	·982	0·974	63·5	48·7	14·8	56·1	36·3	2·97	79·1	7·3	2·44	2·50	15	146	28	5	110·4	46·9
July,	·928	0·961	65·2	51·6	13·6	58·4	30·7	2·46	79·3	7·6	2·46	2·92	15	148	28	4	112·4	47·2
August,	861	0·999	65·6	51·6	14·0	58·6	32·0	2·34	81·3	7·3	3·35	3·77	18	140	30	3	111·3	45·7
September,	·892	1·169	61·3	47·8	13·5	54·6	33·3	2·65	81·9	6·1	2·07	2·27	17	109	29	4	101·1	39·8
October,	·817	1·582	52·9	41·2	11·7	47·0	34·0	2·94	84·0	6·0	2·69	3·01	17	82	26	7	84·4	31·5
November,	·892	1·763	47·9	39·2	8·7	43·5	28·3	3·14	85·1	6·8	2·12	2·43	18	38	15	14	65·9	18·0
December,	·756	1·855	44·0	35·6	8·4	39·8	30·0	3·20	84·5	6·5	2·56	2·78	19	22	10	18	53·4	9·4
Year,	29·887	2·274	53·7	41·6	12·1	47·7	62·9	2·81	81·5	6·6	25·47	28·59	191	1185	26	89	88·0	34·3

	Minimum on Grass. Mean.	Mean Difference from Shade	Direction of Wind from Observations made at 9 A.M. and 9 P.M. Mean Number of Days it Blew from Different Directions. N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or or Var.	Mean Wind Velocity Miles per hour.	Mean Number of Days Certain Phenomena were Observed. Snow.	Hail.	Gales.	Thunder-storms.	Lightning without Thunder.*	Mist or Fog.	Auroras.*	Frost in Shade.
January,	29·9°	3·8°	2·2	0·9	3·8	1·4	1·4	3·7	13·6	2·0	2·0	7·0	4·5	2·0	2·4	0·3	0	0·6	1	12
February,	29·8	4·0	1·2	0·8	3·4	2·0	1·4	1·7	12·5	2·1	2·9	6·8	4·5	1·2	1·9	0·4	1	1·1	4	10
March,	31·1	3·9	2·1	2·0	4·1	1·2	1·5	4·0	11·6	2·3	2·2	7·1	5·6	2·3	2·4	0·1	1	0·7	10	9
April,	35·0	3·6	1·5	2·1	7·5	1·9	1·6	2·1	8·9	2·6	1·8	5·7	1·3	1·7	0·4	0·3	2	1·0	6	3
May,	39·3	3·2	2·6	2·4	8·9	1·0	1·3	1·7	8·3	2·8	2·0	5·4	0·3	2·0	0·3	0·9	4	1·3	2	0
June,	46·5	2·2	1·6	1·9	9·5	1·9	1·3	1·6	9·0	1·9	1·3	4·4	0·0	1·0	0·3	1·6	1	2·8	0	0
July,	49·3	2·3	1·6	1·9	6·5	0·6	1·3	1·2	14·2	1·7	2·0	5·2	0·0	0·6	0·3	2·6	0	0·7	3	0
August,	48·8	2·8	1·5	1·2	5·4	1·0	0·7	3·1	13·1	2·0	3·0	5·1	0·0	0·8	0·3	2·2	1	0·5	1	0
September,	44·2	3·6	1·4	1·2	4·0	0·9	0·7	2·1	14·7	2·0	3·0	6·2	0·0	0·4	1·3	0·3	3	1·0	3	0
October,	37·3	3·9	2·5	0·9	3·3	1·9	1·5	2·5	13·2	2·2	3 0	6 2	0·7	0·8	1·3	0·3	1	0·9	4	2
November,	35·2	4·0	1·3	1·3	4·3	2·0	2·1	3·4	12·0	1·5	2·1	6·9	1·0	0·7	1·7	0·2	0	1·6	5	3
December,	31·6	4·0	0·7	0·9	2·7	2·0	1·7	4·9	12·7	2·4	3·0	7·4	3·0	0·2	2·7	0·0	3	1·1	1	9
Year,	38·2	3·4	20·2	17·5	63·4	17·8	16·5	32·0	143·8	25·5	28·3	6·1	20·9	13·7	15·3	9·2	17	13·3	40	48

* Total cases recorded.

Table IV.

Showing the Departure of the Monthly and Annual Means for the Decennium 1891–1900 from those of the Period 1851–1900.

Element.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
Mean Barometric Pressure: Corrected and Reduced to 32° and Mean Sea-level,	**·079**	**·001**	*·018*	**·018**	**·040**	**·041**	**·037**	*·007*	**·004**	*·028*	**·039**	*·050*	**·013**
Mean Monthly Range,	*·041*	**·022**	*·012*	*·138*	**·151**	**·045**	**·052**	**·012**	**·017**	**·099**	**·202**	**·258**	**·070**
Temperature in Shade:	°	°	°	°	°	°	°	°	°	°	°	°	°
Mean of all the Highest,	*0·2*	**0·5**	**1·1**	**1·3**	**0·9**	**0·4**	*0·4*	**1·0**	**0·9**	**0·1**	**1·5**	**0·8**	**0·7**
,, ,, Lowest,	**0·1**	*0·0*	**0·4**	**0·4**	**0·2**	**0·5**	**0·3**	**0·7**	**0·5**	*0·5*	**2·2**	**1·2**	**0·5**
,, Daily Range,	*0·3*	**0·5**	**0·7**	**0·9**	**0·7**	*0·1*	*0·7*	**0·3**	**0·4**	**0·6**	*0·7*	*0·4*	**0·2**
,, Temperature,	*0·1*	**0·2**	**0·8**	**0·8**	**0·5**	**0·5**	*0·0*	**0·6**	**0·8**	*0·2*	**1·8**	**1·0**	**0·6**
,, Monthly Range,	*1·1*	**2·7**	**2·2**	**1·1**	*0·4*	**2·3**	*1·7*	**0·9**	**2·6**	**2·4**	*0·6*	*0·7*	**1·9**
,, Daily Variability,	*0·11*	**0·20**	**0·05**	*0·04*	*0·21*	**0·19**	*0·04*	*0·09*	**0·14**	**0·11**	**0·07**	*0·05*	**0·02**
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
Rainfall at Charlotte Square,	*0·81*	**0·12**	*0·05*	*0·27*	*0·05*	**0·13**	*0·30*	**0·35**	*0·40*	**0·25**	*0·17*	**0·24**	*0·95*
,, ,, ,,	%	%	%	%	%	%	%	%	%	%	%	%	%
,, ,, ,,	*39*	**7**	*3*	*18*	*3*	**6**	*11*	**12**	*16*	**10**	*7*	**11**	*4*
Non-Instrumental Phenomena:	days.	days.	days.	days.	days.	days.	days.	days.	days.	days.	days.	days.	days.
Thunderstorms,	**0·1**	**0·3**	*0·1*	*0·0*	*0·1*	**0·1**	**0·4**	**0·7**	*0·3*	*0·0*	**0·1**	*0·1*	**1·0**
Snow,	**0·8**	**0·3**	**0·7**	**0·3**	*0·1*	*0·0*	*0·0*	*0·0*	*0·0*	**0·4**	*0·2*	*0·0*	**2·2**
Hail,	**1·3**	**0·6**	**1·1**	**0·3**	**0·6**	**0·5**	**0·2**	**0·4**	**0·1**	**0·1**	**0·1**	*0·3*	**4·0**
Gales,	*1·7*	*0·9*	*0·2*	*0·7*	*0·6*	*0·5*	*0·4*	*0·5*	*0·4*	*1·0*	*0·7*	*0·8*	*8·4*
Mist or Fog,	*1·2*	*0·1*	*0·6*	*0·2*	**0·1**	**1·0**	*0·4*	*0·8*	*0·2*	*1·0*	*0·2*	*0·6*	*4·2*

Note.—The heavy type indicates that the values for 1891–1900 were above those for the long period; light type, that they were below the 50 years' mean.

Table V.

Showing the Departure from the Average (1851–1900) of the Monthly and Annual Mean Maximum Temperature.

	Jan.	Feb.	Mar.	Apr.	May	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1851, . .	**2°·4**	**2°·2**	**0°·9**	*0°·3*	**2°·2**	**1°·3**	*1°·9*	*0°·9*	*0°·2*	**3°·2**	*3°·9*	**1°·3**	**0°·6**
1852, . .	**0·1**	**2·5**	*0·5*	**5·3**	**1·7**	*0·4*	**6·0**	**2·8**	**1·4**	*0·6*	**1·3**	**4·3**	**2·0**
1853, . .	**1·2**	*5·8*	*3·3*	*0·5*	*0·7*	**2·1**	*0·3*	*0·8*	*1·0*	*0·1*	**1·4**	*2·0*	*0·7*
1854, . .	*2·1*	**2·0**	**5·2**	**0·9**	**0·8**	**0·3**	**0·3**	**3·3**	**2·6**	**0·8**	**0·5**	**2·1**	**1·5**
1855, . .	*1·6*	*8·5*	*3·1*	**1·4**	*4·5*	**2·1**	**2·7**	**2·0**	**0·1**	**0·3**	*1·4*	*2·0*	*0·9*
1856, . .	*2·6*	**1·8**	*0·3*	*0·5*	*3·4*	**2·1**	**1·1**	**0·2**	*2·7*	**1·8**	**1·3**	**1·5**	**0·1**
1857, . .	**0·2**	**2·3**	*0·3*	*0·7*	**1·3**	**1·3**	**3·8**	**2·7**	**3·1**	**3·5**	**4·2**	**7·5**	**2·4**
1858, . .	**3·3**	*2·4*	**0·5**	*1·5*	**1·5**	**2·4**	*4·1*	*1·9*	**1·0**	*1·8*	*2·8*	**0·9**	*0·4*
1859, . .	**2·9**	**1·7**	**3·1**	*3·8*	**0·9**	*2·3*	**0·3**	**0·5**	*0·7*	*2·0*	*2·5*	*6·7*	*0·7*
1860, . .	*3·8*	*5·3*	*1·8*	*4·8*	**0·5**	*6·4*	*0·5*	*2·7*	*1·4*	*1·2*	*4·1*	*6·3*	*3·1*
1861, . .	*2·1*	*0·2*	**2·4**	*1·6*	**0·5**	*2·6*	*1·8*	**0·1**	*1·7*	**1·6**	*3·0*	*2·5*	*0·9*
1862, . .	*0·4*	**0·8**	*3·8*	*0·0*	**1·0**	*3·7*	*4·9*	*2·8*	*1·6*	**0·9**	*4·9*	**4·1**	*1·2*
1863, . .	**1·2**	**3·4**	**2·6**	*0·2*	*1·0*	**0·7**	**1·0**	*2·1*	*5·7*	*1·0*	**2·7**	**3·6**	**0·5**
1864, . .	*1·2*	*6·0*	*3·4*	**1·5**	**3·0**	*1·9*	*1·6*	*2·0*	*0·8*	*1·8*	**0·3**	**0·7**	*1·1*
1865, . .	*2·7*	*5·6*	*4·0*	**1·8**	*0·1*	**3·0**	**0·9**	*1·8*	**4·7**	*1·0*	**0·3**	**4·9**	**0·1**
1866, . .	**2·9**	*1·7*	*2·9*	*2·7*	*0·4*	**1·9**	*0·1*	*2·4*	*0·5*	**1·5**	**0·1**	**3·2**	*0·0*
1867, . .	*5·3*	**3·6**	*4·5*	**0·4**	*3·6*	**0·8**	*3·3*	**0·4**	*0·7*	*0·5*	*0·6*	**1·1**	*1·0*
1868, . .	*0·1*	**3·6**	**3·8**	**1·2**	**3·4**	**2·0**	**5·0**	**2·2**	**0·1**	*1·9*	*3·0*	**2·2**	**1·6**
1869, . .	**2·7**	**3·9**	*1·9*	**4·6**	*3·6*	*0·5*	**3·3**	**1·0**	**0·3**	**0·1**	**0·7**	*1·4*	**0·8**
1870, . .	*1·7*	*4·0*	**0·2**	**4·2**	**2·9**	**1·4**	**2·4**	**2·5**	**2·7**	*0·0*	*1·5*	*4·5*	**0·4**
1871, . .	*3·9*	**1·9**	**3·5**	*3·3*	**2·3**	*2·0*	**0·3**	**4·0**	*0·9*	**0·7**	*2·7*	**0·6**	**0·1**
1872, . .	**1·7**	**2·5**	**1·6**	*0·5*	*3·5*	**0·7**	*0·0*	**1·9**	*2·8*	*3·1*	*0·5*	**0·6**	*0·4*
1873, . .	**2·3**	*3·2*	*2·7*	*1·3*	*1·9*	*0·6*	**1·3**	*0·8*	*1·6*	*0·6*	**1·8**	**3·6**	*0·3*
1874, . .	**3·1**	**1·9**	**4·8**	**2·0**	*5·5*	**0·4**	**0·9**	*1·6*	*0·2*	**0·8**	*0·5*	*6·8*	*0·0*
1875, . .	**3·5**	*2·7*	*0·4*	**1·6**	**2·2**	*0·6*	*1·3*	*0·1*	*0·0*	*0·7*	*2·0*	**1·0**	**0·1**
1876, . .	**3·0**	*1·5*	*1·1*	*1·3*	*1·2*	**1·0**	**0·9**	*0·8*	*2·7*	**1·8**	*1·6*	**0·5**	*0·2*
1877, . .	**1·4**	**3·6**	*1·5*	*4·1*	*3·7*	**1·5**	*1·1*	*3·0*	*0·1*	**1·1**	**1·9**	**1·4**	*0·2*
1878, . .	**0·9**	**3·5**	**1·4**	**1·1**	**2·0**	**2·1**	**3·7**	**1·8**	**0·3**	**2·1**	*3·5*	*7·5*	**0·7**
1879, . .	*6·5*	*4·7*	*2·9*	*5·1*	*2·8*	*4·7*	*5·9*	*1·2*	*1·3*	*0·0*	*1·7*	*2·6*	*3·2*
1880, . .	*1·7*	**4·6**	**2·1**	**1·0**	**0·1**	**0·1**	*1·2*	**3·6**	**1·4**	*3·4*	*1·7*	*1·5*	**0·4**
1881, . .	*8·2*	*3·7*	*1·3*	*1·4*	**2·3**	*0·6*	*0·3*	*2·5*	*1·0*	*2·5*	**5·2**	**0·5**	*1·1*
1882, . .	**3·9**	**5·1**	**4·3**	*1·3*	**0·9**	*2·0*	*1·7*	*1·2*	*0·7*	**0·9**	*1·5*	*4·8*	**0·2**
1883, . .	**1·6**	**2·5**	*3·3*	**1·2**	*0·9*	*2·8*	*2·7*	*0·4*	**0·3**	**1·5**	**0·1**	**2·9**	*0·0*
1884, . .	**3·5**	**2·1**	**2·6**	**0·5**	**0·8**	*0·9*	*2·7*	**0·7**	**0·1**	**0·9**	**0·1**	*0·9*	**0·6**
1885, . .	*1·7*	**2·3**	**0·8**	**1·0**	*3·7*	*2·1*	**2·9**	*2·9*	*0·6*	*3·5*	*0·5*	**0·5**	*0·6*
1886, . .	*3·1*	*4·1*	*3·2*	*1·3*	*1·8*	*0·4*	**0·5**	**0·9**	*0·8*	**2·7**	**4·2**	*3·2*	*0·8*
1887, . .	**1·7**	**2·0**	*1·1*	*1·8*	**0·1**	**4·0**	**3·9**	**1·4**	*1·3*	*1·7*	*2·8*	*2·5*	**0·2**
1888, . .	**0·9**	*3·7*	*4·5*	*2·0*	**1·6**	*3·5*	*3·8*	*1·7*	*0·7*	**1·1**	**0·6**	**2·4**	*1·1*
1889, . .	**1·7**	*1·3*	**0·1**	*3·5*	**2·6**	**3·5**	**2·2**	*1·4*	*0·8*	*1·4*	**2·8**	**1·4**	**0·5**
1890, . .	**4·4**	*0·7*	**2·7**	**1·0**	**0·9**	*0·6*	*1·3*	*1·1*	**5·8**	**2·3**	**0·7**	*4·5*	**0·8**
1891, . .	*1·5*	**6·7**	*1·5*	*1·4*	*0·9*	*1·5*	**0·1**	*1·5*	**1·8**	**1·0**	*0·6*	**0·8**	**0·2**
1892, . .	*1·5*	*1·3*	*2·2*	**0·4**	**1·6**	*1·1*	*3·2*	*0·8*	*2·2*	*3·7*	**1·7**	*3·8*	*1·3*
1893, . .	*0·8*	**1·2**	**5·4**	**4·5**	**4·5**	**3·7**	*1·2*	**5·0**	**0·2**	**2·6**	*1·1*	**3·1**	**2·3**
1894, . .	**0·5**	**2·1**	**5·4**	**3·0**	*2·8*	*1·1*	**0·1**	*0·9*	*2·1*	*0·2*	**4·2**	**2·4**	**0·9**
1895, . .	*6·1*	*6·5*	*0·0*	**1·0**	**5·0**	**2·2**	*1·6*	**1·2**	**6·0**	*2·7*	**1·4**	*1·3*	*0·1*
1896, . .	**3·2**	**3·4**	**2·9**	**4·1**	**7·0**	*0·3*	*0·5*	*0·8*	*1·6*	*4·1*	*0·4*	*0·5*	**1·1**
1897, . .	*3·6*	**2·0**	**0·9**	*2·1*	*1·5*	*2·5*	**1·6**	**3·3**	**0·3**	**3·2**	**3·0**	**1·3**	**0·5**
1898, . .	**6·2**	**1·1**	**0·9**	**2·2**	*1·0*	**0·6**	**1·3**	**1·7**	**4·6**	**2·8**	**0·8**	**5·0**	**2·2**
1899, . .	**0·6**	**1·2**	**1·0**	*1·1*	*3·7*	**3·3**	**0·4**	**5·4**	**0·3**	**2·9**	**5·6**	*3·1*	**1·1**
1900, . .	**0·7**	*4·8*	*1·9*	**2·6**	**0·7**	**0·3**	**2·2**	*2·1*	**1·9**	*0·8*	**0·6**	**4·6**	**0·3**
Mean, 1851–1900,	42·3	43·6	45·9	51·6	57·0	63·1	65·6	64·6	60·4	52·8	46·4	43·2	53·0

Note.—The heavy type indicates an excess, and the italic type a defect.

Table VI.

Showing the Departure from the Average (1851–1900) *of the Monthly and Annual Mean Minimum Temperature.*

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1851, . .	2·9	1·9	0·6	*2·0*	*0·7*	*1·9*	*2·8*	*1·4*	*2·5*	2·7	*4·9*	2·5	*0·5*
1852, . .	2·0	0·6	0·2	2·1	0·9	*1·3*	5·1	1·7	*1·0*	*1·2*	0·8	2·5	1·0
1853, . .	0·6	*3·9*	*1·7*	2·0	*0·0*	3·0	1·8	0·9	0·7	2·5	1·7	*1·6*	0·5
1854, . .	*1·0*	0·5	4·8	2·1	1·7	1·5	2·1	2·3	3·7	0·6	0·2	0·2	1·6
1855, . .	*0·2*	*7·8*	*2·3*	*0·7*	*2·4*	1·2	3·6	0·9	0·5	*0·0*	0·6	*1·0*	*0·5*
1856, . .	0·3	3·6	2·1	2·2	0·3	4·2	0·9	1·5	0·4	6·5	1·0	1·8	2·1
1857, . .	*0·4*	2·5	1·6	0·8	1·8	*1·1*	1·2	3·2	3·0	4·7	5·3	8·0	2·1
1858, . .	2·0	*2·6*	0·6	*2·6*	1·0	3·3	*1·8*	0·2	0·9	*3·3*	*1·6*	0·6	*0·3*
1859, . .	1·0	0·5	2·8	*4·8*	*0·4*	*1·9*	*0·0*	*2·0*	*1·5*	*1·5*	*3·2*	*4·3*	*1·3*
1860, . .	*3·1*	*4·9*	*1·8*	*3·3*	1·4	*1·9*	*1·4*	*2·9*	*3·9*	*0·5*	*2·5*	*4·4*	*2·4*
1861, . .	*1·2*	0·9	0·3	*0·6*	0·1	1·1	*1·8*	1·1	1·0	2·8	*4·4*	*1·0*	*0·1*
1862, . .	0·7	2·4	*1·9*	0·2	0·9	*0·9*	*3·0*	0·2	*0·8*	*0·9*	*4·9*	3·2	*0·4*
1863, . .	0·3	1·5	2·8	*0·4*	0·6	*0·7*	*2·0*	*1·4*	*2·1*	0·1	2·1	1·7	0·2
1864, . .	*2·7*	*4·8*	*2·7*	1·3	*0·3*	*1·2*	*1·3*	*3·8*	*0·9*	*2·0*	*1·1*	0·4	*1·6*
1865, . .	*3·2*	*5·1*	*3·4*	*0·2*	1·9	*0·2*	*0·0*	*0·7*	3·8	*1·6*	*1·3*	3·9	*0·5*
1866, . .	1·3	*0·6*	*3·1*	*1·7*	*2·9*	0·7	*0·8*	*0·8*	*0·7*	2·8	0·7	2·8	*0·2*
1867, . .	*5·0*	4·6	*2·1*	2·7	*0·0*	1·0	*1·3*	2·1	1·9	0·2	0·9	2·0	0·6
1868, . .	0·3	4·0	3·8	2·3	3·6	1·4	2·4	2·0	1·6	*1·6*	*1·6*	3·1	1·9
1869, . .	1·9	3·4	*2·7*	1·2	*4·7*	*2·0*	*0·3*	*2·3*	1·1	0·9	*0·1*	*2·4*	*0·5*
1870, . .	*1·2*	*3·7*	*2·3*	3·6	2·2	2·6	2·2	*0·4*	0·1	*0·2*	*1·9*	*2·7*	*0·1*
1871, . .	*1·3*	4·3	2·6	*0·8*	*0·9*	*3·9*	*1·1*	0·1	*1·6*	*0·8*	*4·3*	*2·8*	*0·9*
1872, . .	*0·6*	1·4	0·8	*0·2*	*2·9*	2·8	1·3	0·2	*0·7*	*0·3*	*0·1*	0·8	0·2
1873, . .	3·1	*2·2*	0·1	0·9	*0·7*	*0·9*	1·8	1·1	*1·3*	*3·3*	*0·1*	2·9	0·1
1874, . .	2·4	*1·2*	3·9	2·6	*0·5*	*0·5*	2·4	*0·4*	*0·7*	0·4	0·3	*7·1*	0·2
1875, . .	2·4	*0·0*	1·1	1·9	3·3	*0·0*	*1·6*	1·8	0·3	0·9	*1·6*	1·5	0·9
1876, . .	2·0	*1·7*	*1·5*	0·3	*0·4*	*1·4*	0·6	0·3	1·2	4·6	*0·2*	2·8	0·5
1877, . .	1·2	1·4	*2·2*	*2·5*	*2·7*	0·5	*0·1*	*1·8*	*5·4*	*0·8*	1·3	1·0	*0·8*
1878, . .	0·2	4·0	0·1	1·0	1·4	0·1	1·3	0·3	1·7	1·2	*3·9*	*8·2*	*0·1*
1879, . .	*6·9*	*3·1*	*2·7*	*3·5*	*4·0*	*1·7*	*2·3*	*1·4*	*2·5*	*3·2*	*2·0*	*5·5*	*3·2*
1880, . .	*0·1*	4·4	0·2	0·9	*0·1*	*0·4*	*0·4*	2·4	1·8	*3·2*	*2·9*	*0·6*	0·2
1881, . .	*9·5*	*2·2*	*2·2*	*2·7*	1·0	*1·1*	0·2	*2·5*	0·9	*3·2*	4·0	*0·7*	*1·5*
1882, . .	4·1	4·2	3·9	*0·3*	*0·2*	*0·7*	0·4	*0·1*	*3·6*	1·5	*2·0*	*5·6*	0·1
1883, . .	0·7	2·6	*3·7*	0·7	*0·8*	*0·9*	*1·9*	*0·1*	1·1	*0·7*	*0·3*	1·7	*0·1*
1884, . .	3·2	1·8	1·5	*1·3*	*0·8*	*0·5*	*0·1*	0·7	1·7	1·9	*0·0*	*0·8*	0·6
1885, . .	*0·3*	1·6	*0·6*	0·3	*2·4*	0·2	0·3	*3·0*	*2·1*	*3·3*	*1·0*	*0·1*	*0·9*
1886, . .	*3·2*	*2·8*	*1·3*	*1·1*	*1·3*	*2·3*	*0·5*	*0·2*	0·7	3·8	2·0	*4·1*	*0·9*
1887, . .	0·7	*0·0*	*1·1*	*2·8*	*1·2*	1·6	1·9	*0·8*	*1·2*	*2·6*	*0·8*	*1·6*	*0·6*
1888, . .	1·8	*1·9*	*3·0*	*2·8*	*0·4*	*4·3*	*2·7*	*2·3*	*2·0*	0·2	2·3	2·7	*1·0*
1889, . .	1·9	*1·4*	0·1	*0·3*	2·2	0·8	*2·3*	*0·1*	*0·9*	*0·9*	1·8	0·2	0·1
1890, . .	3·2	*1·1*	2·4	*1·7*	1·7	*0·5*	*2·5*	*1·2*	3·1	2·1	*1·0*	*2·5*	0·2
1891, . .	*0·9*	2·6	*2·6*	*2·7*	*2·3*	0·1	0·8	*0·3*	2·5	0·2	0·1	1·0	*0·1*
1892, . .	*1·2*	*1·2*	*3·7*	*2·5*	0·8	*1·4*	*1·9*	*0·4*	*1·7*	*3·1*	1·1	*4·2*	*1·6*
1893, . .	*0·0*	1·1	1·9	1·6	4·2	2·3	*0·2*	2·8	*0·6*	0·8	*1·0*	3·4	1·4
1894, . .	*0·3*	1·3	2·3	2·5	*2·3*	*0·8*	0·9	*0·8*	*1·3*	*1·6*	4·3	1·8	0·5
1895, . .	*6·1*	*8·4*	1·6	1·2	1·8	*0·2*	*1·0*	2·2	3·5	*3·3*	2·0	*0·9*	*0·6*
1896, . .	3·3	3·8	1·4	3·7	3·4	2·4	0·5	*1·1*	0·5	*3·9*	1·1	0·8	1·3
1897, . .	*2·3*	3·3	2·2	*2·4*	*1·5*	*0·6*	*0·2*	2·1	*1·9*	0·7	4·8	0·8	0·4
1898, . .	7·1	1·2	0·4	3·0	*1·2*	0·1	*1·4*	0·4	3·4	4·8	0·8	5·3	2·0
1899, . .	*0·1*	0·6	1·7	*0·7*	*1·6*	2·2	2·5	2·5	*0·0*	0·8	5·8	*2·7*	0·9
1900, . .	1·9	*4·4*	*1·6*	0·2	1·1	1·2	2·9	*0·1*	0·6	*0·4*	2·1	6·4	0·8
Mean, 1851–1900,	33·6	33·8	34·6	38·2	42·3	48·2	51·3	50·9	47·3	41·7	37·0	34·4	41·1

Note.—The heavy type indicates an excess, and the italic type a defect.

Table VII.

Decennial Means of Mean Daily Maximum, Minimum, and Daily Range of Temperature, 1851–1900. *Mean Daily Maximum Temperature.*

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1851–60, .	42°·5	42°·6	45°·9	51°·1	57°·0	63°·3	66°·3	65°·1	60°·6	53°·2	45°·7	43°·3	53°·0
1861–70, .	41·6	43·4	44·7	52·5	57·2	63·2	65·7	64·1	60·1	52·6	45·5	44·3	52·9
1871–80, .	42·7	44·2	46·4	50·6	55·8	62·9	65·4	64·6	59·6	52·7	45·4	42·1	52·7
1881–90, .	42·8	43·6	45·6	50·8	57·3	62·6	65·3	63·8	60·4	52·8	47·3	42·4	52·9
1891–1900, .	42·1	44·1	47·0	52·9	57·9	63·5	65·2	65·6	61·3	52·9	47·9	44·0	53·7
Means, 1851–1900,	42·3	43·6	45·9	51·6	57·0	63·1	65·6	64·6	60·4	52·8	46·4	43·2	53·0

Mean Daily Minimum Temperature.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1851–60, .	34°·0	32°·8	35°·3	37°·8	42°·7	48°·7	52°·2	51°·4	47°·3	42°·8	36°·7	34°·8	41°·4
1861–70, .	32·7	34·2	33·5	39·0	42·5	48·4	50·7	50·5	47·8	41·8	35·8	35·5	41·0
1871–80, .	33·8	34·5	34·8	38·3	41·5	47·6	51·5	51·2	46·7	41·3	35·6	32·9	40·8
1881–90, .	33·8	33·9	34·2	37·1	42·1	47·4	50·6	49·9	47·1	41·6	37·5	33·3	40·7
1891–1900, .	33·7	33·8	35·0	38·6	42·5	48·7	51·6	51·6	47·8	41·2	39·2	35·6	41·6
Means, 1851–1900,	33·6	33·8	34·6	38·2	42·3	48·2	51·3	50·9	47·3	41·7	37·0	34·4	41·1

Mean Daily Range of Temperature.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1851–60, .	8°·5	9°·8	10°·6	13°·3	14°·3	14°·6	14°·1	13°·7	13°·3	10°·4	9°·0	8°·5	11°·6
1861–70, .	8·9	9·2	11·2	13·5	14·7	14·8	15·0	13·6	12·3	10·8	9·7	8·8	11·9
1871–80, .	8·9	9·7	11·6	12·3	14·3	15·3	13·9	13·4	12·9	11·4	9·8	9·2	11·8
1881–90, .	9·0	9·7	11·4	13·7	15·2	15·2	14·7	13·9	13·3	11·2	9·8	9·1	12·2
1891–1900, .	8·4	10·3	12·0	14·3	15·4	14·8	13·6	14·0	13·5	11·7	8·7	8·4	12·1
Means, 1851–1900, .	8·7	9·8	11·3	13·4	14·7	14·9	14·3	13·7	13·1	11·1	9·4	8·8	11·9

Mean Daily Range of Temperature.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Max., . .	11·0	13·9	14·8	16·8	18·0	18·1	18·8	17·6	18·4	14·3	11·6	12·2	13·2
Year, . .	1872	1891	1893	1869	1864	1851,'64	1889	1871	1877	1879	1886	1871	1869
Min., . .	5·8	7·1	8·5	10·2	9·7	10·4	10·7	10·7	9·5	6·4	7·4	6·4	9·9
Year, . .	1856	1875	1873	1889	1874	1860	1879	1862	1863	1856	1855,'87	1859	1856
	5·2	6·8	6·3	6·6	8·3	7·7	8·1	6·9	8·9	7·9	6·2	5·8	3·3

Table VIII.

Abstract of Temperature Observations.

	Mean Temperature 1764–1900.					Mean Maximum Temperature 1851–1900.					Mean Minimum Temperature 1851–1900.				
	Highest.	Year.	Lowest.	Year.	Range.	Highest.	Year.	Lowest.	Year.	Range.	Highest.	Year.	Lowest.	Year.	Range.
January,	44°·6	1898	26°·5	1814	18°·1	48°·5	1898	34°·1	1881	14°·4	40°·7	1898	24°·1	1881	16°·6
February,	47·2	1779	29·8	1838	17·4	50·3	1891	35·1	1855	15·2	38·8	1868	25·4	1895	13·4
March,	46·5	1779, 1845	34·2	1785	12·3	51·3	1893,'94	41·4	1867,'88	9·9	39·4	1854	30·9	1883,'92	8·5
April,	49·8	1792, 1798	38·9	1837	10·9	56·9	1852	46·5	1879	10·4	41·9	1896	33·4	1859	8·5
May,	55·8	1833	45·1	1810	10·7	64·0	1896	51·5	1874	12·5	46·5	1893	37·6	1869	8·9
June,	61·9	1846	51·5	1860	10·4	67·1	1887	56·5	1860	10·6	52·4	1856	43·9	1888	8·5
July,	65·2	1779	54·4	1879	10·8	71·6	1852	59·7	1879	11·9	56·4	1852	48·3	1862	8·1
August,	63·7	1779	52·6	1830	11·1	70·0	1899	61·6	1877	8·4	54·1	1857	47·1	1864	7·0
September,	59·5	1846	48·2	1807	11·3	66·4	1895	54·7	1863	11·7	51·1	1865	41·9	1877	9·2
October,	52·7	1831	42·0	1817	10·7	56·3	1857	48·7	1896	7·6	48·2	1856	37·8	1896	10·4
November,	47·4	1899	34·0	1807	13·4	52·0	1899	41·5	1862	10·5	42·8	1899	32·1	1851,'62	10·7
December,	47·8	1843	31·0	1878	16·8	51·1	1857	35·7	1878	15·4	42·4	1857	26·2	1878	16·2
Year.	49·6	1779, 1846	43·8	1879	5·8	55·4	1857	49·8	1879	5·6	43·2	1856,'57	37·9	1879	5·3

TABLE IX.

General Summary of the Mean and Extreme Annual Values in Edinburgh from 1851 *to* 1900.

Year.	Barometric Pressure at 32° and Mean Sea-level. Maximum.	Minimum.	Range.	Mean Pressure.	Temperature in Shade 4 ft. above Grass and 250 ft. above Sea. Max.	Date.	Min.	Date.	Annual Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Mean Temperature.	Mean Daily Variability of Temperature.	Rainfall at Charlotte Square.	Non-Instrumental Phenomena. Days of Snow.	Hail.	Gales.	Thunderstorms.	Frost in Shade.	Mist or Fog.	Auroras.
	ins.	ins.	ins.	ins.	°		°		°	°	°	°	°	°	ins.							
1851	30·637	28·851	1·786	29·905	75·0	June 29	23·0	December 3	52·0	53·6	40·6	13·0	47·1	2·87	22·78	4	4	34	2	57	11	0
1852	·797	·051	2·746	·796	85·0	July 6	24·0	November 30	60·0	55·0	42·1	12·9	48·6	2·77	31·51	6	11	23	14	?	16	2
1853	·517	·707	1·810	·818	75·5	June 23	19·0	February 11	56·5	52·3	41·6	10·7	46·9	2·77	25·63	23	7	20	7	?	23	0
1854	·902	·676	2·226	·850	75·0	July 12	18·5	January 2	56·5	54·5	42·7	11·8	48·6	2·82	20·89	13	9	25	9	?	4	0
1855	·653	·740	1·913	·872	79·5	June 7, Sept. 9	14·5	February 16	65·0	52·1	40·6	11·5	46·3	3·03	20·34	33	3	17	6	?	30	0
1856	·750	·606	2·144	·846	81·9	August 2	18·6	December 3	63·3	53·1	43·2	9·9	48·1	2·80	28·48	3	2	5	4	?	18	0
1857	·810	·474	2·336	·926	81·6	June 28	20·0	January 29	61·6	55·4	43·7	11·7	49·5	2·67	24·92	11	4	16	7	35	21	0
1858	·693	·798	1·895	·918	78·2	June 16	20·5	March 8	57·7	52·6	40·8	11·8	46·7	2·66	24·35	14	1	7	6	59	26	0
1859	·740	·455	2·285	·827	78·7	July 12	18·7	December 19	60·0	52·3	39·8	12·5	46·1	2·56	25·97	17	6	19	2	76	20	6
1860	·782	·464	2·318	·783	71·7	July 16	5·0	December 24	66·7	49·9	38·7	11·2	44·3	2·50	33·45	23	8	10	8	86	28	4
1861	·583	·852	1·731	·848	71·1	June 14	16·3	January 8	54·8	52·1	41·0	11·1	46·5	2·55	28·62	18	6	22	6	65	31	1
1862	·693	·621	2·072	·830	70·0	April 30	22·0	March 3	48·0	51·8	40·7	11·1	46·2	2·70	33·92	16	4	22	4	51	16	2
1863	·756	·646	2·110	·844	75·2	July 11, 12	24·5	December 28	50·7	53·5	41·3	12·2	47·4	2·77	25·63	8	8	30	5	40	15	7
1864	·776	·636	2·140	·962	79·0	May 18, July 18	18·0	March 10	61·0	51·9	39·5	12·4	45·7	2·55	28·09	21	6	23	4	78	16	0
1865	·856	·466	2·390	·923	78·0	June 22	18·0	February 15, 17	60·0	53·1	40·6	12·5	46·8	2·72	23·65	21	3	12	2	80	20	0
1866	·616	·846	1·770	·830	82·7	July 12	20·0	March 5	62·7	53·0	40·9	12·1	46·9	2·93	27·23	9	9	3	4	61	20	1
1867	·846	·556	2·290	·883	76·7	July 10, Aug. 14	16·0	January 1	60·7	52·0	41·7	10·3	46·9	2·77	31·04	31	2	26	10	62	11	6
1868	·606	·446	2·160	·811	87·7	August 5	23·0	November 7	64·7	54·6	43·0	11·6	48·8	3·08	28·57	11	5	36	12	47	9	3
1869	·670	·331	2·339	·862	80·7	August 28	20·0	December 2	60·7	53·8	40·6	13·2	47·2	3·18	22·23	15	2	23	4	84	15	5
1870	·673	·482	2·191	·938	84·7	July 23	16·0	December 23	58·7	53·4	41·0	12·4	47·2	2·68	22·11	32	2	9	3	82	33	19
1871	·616	·493	2·123	·919	79·7	August 11	23·0	November 13	56·7	53·1	40·2	12·9	46·7	2·92	26·87	17	7	16	9	75	31	21
1872	·432	·215	2·217	·737	79·7	July 5	23·0	March 26	56·7	52·6	41·3	11·3	47·0	2·70	38·96	11	3	27	20	54	35	8
1873	·623	·232	2·391	·882	82·9	July 21	22·2	February 24	60·7	52·7	41·2	11·4	47·0	2·81	28·19	12	9	20	11	43	19	12
1874	·879	·647	2·232	·861	81·3	July 18	13·6	December 29	67·7	53·0	41·3	11·7	47·1	3·05	25·76	21	13	17	14	57	9	10
1875	·732	·630	2·102	·950	76·8	July 7	14·0	January 1	62·8	53·1	42·0	11·1	47·6	2·68	24·35	20	7	19	6	39	24	4
1876	·677	·260	2·417	·838	86·7	July 16	21·3	February 14	65·4	52·8	41·6	11·2	47·2	2·98	35·65	36	12	16	7	45	24	0
1877	·627	·255	2·372	·794	72·0	June 14	22·4	February 28	49·6	52·8	40·3	12·5	46·5	2·78	35·80	13	6	35	10	51	21	2
1878	·685	·823	1·862	·876	83·7	July 20	9·0	December 14	74·7	53·7	41·0	12·7	47·4	2·61	24·90	19	10	21	8	79	26	1
1879	·685	·871	1·814	·905	78·0	August 12	7·5	December 4	70·5	49·8	37·9	11·9	43·8	2·90	28·52	31	4	30	9	108	24	0
1880	·650	·642	2·008	·928	77·6	August 11	20·6	November 21	57·0	53·4	41·3	12·1	47·3	2·70	24·86	12	10	38	12	54	28	12

TABLE IX.—*continued.*

Year.	Barometric Pressure at 32° and Mean Sea-level. Maximum.	Minimum.	Range.	Mean Pressure.	Temperature in Shade 4 ft. above Grass and 250 ft. above Sea. Max.	Date.	Min.	Date.	Annual Range.	Mean of all the Highest.	Mean of all the Lowest.	Mean Daily Range.	Mean Temperature.	Mean Daily Variability of Temperature.	Rainfall at Charlotte Square.	Non-Instrumental Phenomena. Days of Snow.	Hail.	Gales.	Thunderstorms.	Frost in Shade.	Mist or Fog.	Auroras.
1881	30·778	28·171	2·607	29·887	79·2	May 30, 31	9·4	January 17	69·8	51·9	39·6	12·3	45·8	2·79	28·22	25	16	57	8	83	18	2
1882	·866	·817	2·049	·835	81·0	August 12	6·4	December 15	74·6	53·2	41·2	12·0	47·2	2·57	30·23	14	11	43	15	42	15	5
1883	·734	·594	2·140	·868	75·0	July 3	24·5	March 15	50·5	53·0	41·0	12·0	47·0	2·60	22·33	27	15	53	13	38	16	2
1884	·736	27·451	3·285	·888	79·9	June 27	23·0	November 30	56·9	53·6	41·7	11·9	47·7	2·97	24·66	13	13	69	15	45	11	3
1885	·638	28·349	2·289	·842	82·2	July 24	19·8	November 18	62·4	52·4	40·2	12·2	46·3	2·87	17·58	14	12	55	4	58	7	5
1886	·661	27·651	3·010	·829	80·7	July 2	12·2	January 19	68·5	52·2	40·2	12·0	46·2	2·88	26·05	37	4	22	11	64	12	7
1887	·700	28·779	1·922	·946	83·2	June 18	21·0	February 9	72·2	53·2	40·5	12·7	46·8	2·90	19·80	19	5	24	7	71	16	0
1888	·691	·859	1·832	·912	76·8	May 19	18·3	February 16	58·5	51·9	40·1	11·8	46·0	2·91	24·86	25	8	28	5	62	10	1
1889	·733	·660	2·073	·889	78·4	June 26	20·8	March 4	57·6	53·5	41·2	12·3	47·4	2·83	22·31	13	16	25	9	45	9	0
1890	·735	·660	2·075	·872	76·0	September 8	23·2	December 14	52·8	53·8	41·3	12·5	47·6	2·86	26·77	18	17	39	7	50	8	0
1891	·795	·363	2·432	·860	79·8	September 12	20·3	March 9	59·5	53·2	41·0	12·2	47·1	2·76	24·23	17	15	31	5	56	10	7
1892	·621	·708	1·913	·864	80·1	June 9	14·0	February 19	66·1	51·7	39·5	12·2	45·6	2·97	22·41	33	21	13	5	81	7	9
1893	·653	·510	2·143	·902	85·9	June 18	15·0	January 6	70·9	55·3	42·5	12·8	48·9	2·87	20·93	17	7	20	13	38	7	2
1894	·759	·121	2·638	·872	77·5	July 6	13·9	January 6	63·6	53·9	41·6	12·3	47·8	2·78	28·25	15	22	25	15	37	11	8
1895	·775	·357	2·418	·870	78·3	June 26, Sept. 25	11·9	February 8	66·4	52·9	40·5	12·4	46·7	2·59	26·72	30	18	11	16	72	24	7
1896	31·071	·299	2·772	·942	78·1	May 11	23·8	December 1	54·3	54·1	42·4	11·7	48·2	2·88	23·59	10	9	8	4	38	14	1
1897	30·653	·685	1·968	·886	81·0	July 15	20·8	January 25	60·2	53·5	41·5	12·0	47·5	2·79	20·58	16	12	13	7	45	27	0
1898	·528	·638	1·890	·882	82·8	September 5	20·3	November 29	62·5	55·2	43·1	12·1	49·2	2·90	24·34	14	16	14	10	26	10	2
1899	·772	·456	2·316	·902	83·1	August 24	18·3	December 15	64·8	54·1	42·0	12·1	48·1	3·00	29·90	15	7	6	7	47	8	3
1900	·643	·391	2·252	·845	77·4	July 10	17·0	February 8	60·4	53·3	41·9	11·4	47·6	2·77	33·64	24	10	12	10	45	15	1
Means 1851–1900	30·710	28·506	2·204	29·874	79·2	July 12	18·1	January 11	61·1	53·0	41·1	11·9	47·1	2·79	26·42	18·7	9·7	23·7	8·2	58*	17·5	3·8
Highest	31·071	28·871	3·285	29·962	87·7		24·5		74·7	55·4	43·7	13·2	49·5	3·18	38·96	37	22	69	20	108*	35	21
Year	1896	1879	1884	1864	1868	August 5	1863, 1883	December 28, March 15	1878	1857	1857	1869	1857	1869	1872	1886	1894	1884	1872	1879	1872	1871
Lowest	30·432	27·451	1·731	29·737	70·0		5·0		48·0	49·8	37·9	9·9	43·8	2·50	17·58	3	1	3	2	26*	4	0
Year	1872	1884	1861	1872	1862	April 30	1860	December 24	1862	1879	1879	1856	1879	1860	1885	1856	1858	1866	1851,'59,'65	1898	1854	vari's
Range	0·639	1·420	1·554	0·225	17·7		19·5		26·7	5·6	5·8	3·3	5·7	0·68	21·38	34	21	66	18	82*	31	21

* For 44 years 1857–1900.

TABLE X.

Showing the Mean Barometric Pressure, Mean Temperature, and Rainfall in Edinburgh during the Period 1897–1900. Mean Barometric Pressure corrected to 32° and reduced to Mean Sea-level.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1897, . .	29·933	29·938	29·482	29·837	29·949	30·007	29·961	29·696	29·900	30·106	30·100	29·724	29·886
1898, . .	30·108	29·752	29·908	29·805	29·834	29·937	30·058	29·899	29·992	29·808	29·775	29·775	29·882
1899, . .	29·680	29·795	29·958	29·736	30·031	30·057	30·024	30·082	29·724	29·956	29·944	29·830	29·902
1900, . .	29·801	29·546	30·078	29·860	29·936	29·888	29·930	29·952	30·008	29·811	29·697	29·639	29·845

Mean Temperature Height above Mean Sea-level 250 feet.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1897, . .	35°·0	41°·4	41°·8	42°·6	48°·2	54°·1	59°·2	60°·4	53°·0	49°·2	45°·6	39°·8	47°·5
1898, . .	44·6*	39·8	40·9	47·5	48·6	56·0	58·4	58·8	57·8	51·0	42·5	44·0	49·2
1899, . .	38·2	39·6	41·6	44·0	47·0	58·4	59·7	61·7	54·0	49·1	47·4*	35·9	48·1
1900, . .	39·2	34·1	38·5	46·3	50·6	56·4	61·0	56·6	55·1	46·7	43·1	44·3	47·6

Mean Maximum Temperature.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1897, . .	38°·7	45°·6	46°·8	49°·5	55°·5	60°·6	67°·2	67°·9	60°·7	56°·0	49°·4	44°·5	53°·5
1898, . .	48·5	44·7	46·8	53·8	56·0	63·7	66·9	66·3	65·0	55·6	47·2	48·2	55·2
1899, . .	42·9	44·8	46·9	50·5	53·3	66·4	66·0	70·0	60·7	55·7	52·0	40·1	54·1
1900, . .	43·0	38·8	44·0	54·2	57·7	63·4	67·8	62·5	62·3	52·0	47·0	47·8	53·3

Mean Minimum Temperature.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1897, . .	31°·3	37°·1	36°·8	35°·8	40°·8	47°·6	51°·1	53°·0	45°·4	42°·4	41°·8	35°·2	41°·5
1898, . .	40·7	35·0	35·0	41·2	41·1	48·3	49·9	51·3	50·7	46·5	37·8	39·7	43·1
1899, . .	33·5	34·4	36·3	37·5	40·7	50·4	53·8	53·4	47·3	42·5	42·8	31·7	42·0
1900, . .	35·5	29·4	33·0	38·4	43·4	49·4	54·2	50·8	47·9	41·3	39·1	40·8	41·9

Mean Daily Variability of Temperature.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1897, . .	2°·2	2°·3	2°·3	2°·4	2°·5	4°·7	3°·5	1°·9	2°·3	2°·8	2°·9	3°·7	2°·79
1898, . .	2·7	3·2	3·0	2·7	2·4	2·2	2·6	2·5	3·7	2·6	3·8	3·4	2·90
1899, . .	3·4	3·2	3·5	3·0	2·1	3·2	2·2	2·5	2·5	3·4	3·2	3·2	3·00
1900, . .	2·4	2·7	2·0	3·2	3·0	2·9	2·5	2·4	3·0	3·4	3·0	2·7	2·77

Rainfall at Charlotte Square.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
1897, . .	·67	1·12	2·33	·79	1·60	3·20	1·50	3·10	1·36	1·02	1·74	2·15	20·58
1898, . .	·82	·86	1·19	1·24	2·05	1·24	1·41	3·50	2·04	3·30	3·34	3·35	24·34
1899, . .	3·35	1·41	2·06	2·50	3·88	3·20	1·60	·73	4·03	1·00	2·93	3·21	29·90
1900, . .	1·79	2·96	1·35	1·48	1·25	3·00	3·09	4·68	2·06	4·01	5·02	2·95	33·64

* Absolute highest during period 1764–1900.

Table XI.

Falls of Rain of 1·00 *Inch or more in* 24 *Hours. Years* 1770–1776, 1780, 1785–1817, 1824–1831, 1854–1900. *Total,* 96 *Years.*

	1·00–1·49.	1·50–1·99.	2·00–2·49.	2·50–2·99.	3·00 or more.	Total.
January,	6	1	0	0	0	7
February,	6	1	0	0	0	7
March,	4	1	0	0	0	5
April,	4	2	0	0	0	6
May,	15	1	0	0	0	16
June,	13	1	0	0	0	14
July,	13	7	2	1	0	23
August,	21	12	2	0	0	35
September,	13	3	2	0	3	21
October,	11	6	2	1	0	20
November,	10	3	2	1	0	16
December,	4	0	0	0	1	5
Year,	120	38	10	3	4	175
Per cent.	68	22	6	2	2	100

Table XII.

Showing the Results Obtained from the Campbell-Stokes Sunshine Recorder in Edinburgh from August 1890 *to December* 1900. *Hours of Bright Sunshine Recorded.*

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1890,	...	...	...	...	...	...	...	121	123	72	40	7	
1891,	29	57	86	132	136	127	150	121	128	108	41	34	1149
1892,	43	62	122	162	161	175	120	152	100	64	40	37	1238
1893,	30	63	137	165	138	185	114	178	114	120	36	33	1313
1894,	40	53	171	111	170	143	132	134	91	64	75	17	1201
1895,	36	72	65	124	201	203	139	101	158	100	44	19	1262
1896,	37	53	109	146	227	137	148	144	60	58	22	10	1151
1897,	24	47	78	128	202	83	232	181	121	94	35	25	1250
1898,	23	90	96	94	158	143	192	144	102	64	27	9	1142
1899,	27	61	97	113	166	149	118	165	104	85	43	12	1140
1900,	35	59	75	143	144	115	138	79	117	65	22	21	1013
Means, 1891–1900,	32	62	104	132	170	146	148	140	109	82	38	22	1185

Mean Percentage of Total Possible Sunshine.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1890,	...	...	...	...	...	...	...	26	31	22	16	3	
1891,	12	21	23	31	25	24	28	26	33	35	17	16	26
1892,	18	23	33	38	32	34	23	33	26	20	16	17	28
1893,	13	24	37	39	27	35	21	38	29	37	15	15	29
1894,	17	20	47	26	33	27	25	29	24	20	30	8	27
1895,	15	27	18	29	40	39	26	22	41	31	18	9	28
1896,	16	20	30	34	45	27	28	31	16	18	9	5	26
1897,	10	18	21	30	40	16	44	39	32	29	14	12	28
1898,	10	34	26	22	31	27	36	31	26	20	11	4	26
1899,	12	23	27	27	33	28	22	36	27	26	18	6	25
1900,	15	22	21	34	29	22	26	17	31	20	9	10	23
Means, 1891–1900,	15	23	28	31	34	28	28	30	29	26	15	10	26

Table XIII.

Mean Amount of Bright Sunshine for Hour ending Greenwich Time during 10 *Years, October* 1890 *to September* 1900.

	A.M.								P.M.							
	5	6	7	8	9	10	11	Noon.	1	2	3	4	5	6	7	8
	Hours.	Hours.	Hours.	Hours.	Hours.	Hours.	Hours	Hours.	Hours.	Hours.	Hours.	Hours.	Hours.	Hours.	Hours.	Hours.
January,	...	...	...	...	...	0·6	5·2	7·1	**7·6**	6·9	4·3	0·6	...	...	...	...
February,	...	...	...	0·2	1·9	5·6	9·5	**10·4**	9·8	10·0	8·8	4·5	1·0	...	...	...
March,	...	...	0·5	3·6	8·8	11·6	**13·2**	**13·2**	12·8	11·8	10·5	9·8	6·3	1·4	...	...
April,	...	0·8	4·2	7·5	10·3	12·4	13·3	**14·0**	13·4	12·7	12·8	11·8	10·1	7·2	1·3	0·1
May,	0·9	6·7	9·6	10·8	11·2	13·0	14·0	14·4	14·5	**14·6**	13·8	13·4	12·5	11·9	7·8	1·4
June,	2·4	6·4	8·6	9·4	9·4	10·8	10·2	11·4	11·6	11·2	**11·8**	10·8	10·8	10·4	8·5	2·6
July,	1·9	6·6	7·9	9·7	10·6	11·1	11·4	11·9	12·2	**12·3**	11·8	11·3	10·8	9·7	7·6	1·6
August,	0·2	2·4	7·2	10·1	11·8	12·3	12·5	**12·8**	12·5	12·7	12·5	11·2	10·2	7·7	3·4	0·1
September,	...	...	0·8	4·8	9·5	11·7	12·1	11·7	**12·3**	11·9	11·3	10·2	8·6	4·0	0·4	...
October,	...	...	...	0·6	4·6	9·8	11·2	**12·6**	12·2	12·1	10·3	7·3	2·1	...	...	...
November,	...	...	...	...	...	2·4	7·1	**9·0**	8·5	6·8	5·2	1·1	0·1	...	...	..
December,	...	...	...	...	...	0·4	2·6	5·0	**6·1**	4·7	1·7	0·1	...	...	...	...
Spring,	0·9	7·5	14·3	21·9	30·3	37·0	40·5	**41·6**	40·7	39·1	37·1	35·0	28·9	20·5	9·1	1·5
Summer,	4·5	15·4	23·7	29·2	31·8	34·2	34·1	36·1	**36·3**	36·2	36·1	33·3	31·8	27·8	19·5	4·3
Autumn,	...	...	0·8	5·4	14·1	23·9	30·4	**33·3**	33·0	30·8	26·8	18·6	10·8	4·0	0·4	...
Winter,	...	...	...	0·2	1·9	6·6	17·3	22·5	**23·5**	21·6	14·8	5·2	1·0	...	...	...
Year,	5·4	22·9	38·8	56·7	78·1	101·7	122·3	**133·5**	**133·5**	127·7	114·8	92·1	72·5	52·3	29·0	5·8

Table XIII.

Showing the Number of Sunless and Sunny Days in Edinburgh for 10 *Years, October* 1890 *to September* 1900.

Per cent. of possible duration.

Month.	Sunless.	1–10.	11–20.	21–30.	31–40.	41–50.	51–60.	61–70.	71–80.	+80.	Maximum per cent.
January,	139	60	21	27	19	24	16	3	1	0	72
February,	73	52	30	31	26	27	18	15	9	1	82
March,	51	55	38	40	31	23	26	22	19	5	85
April,	45	46	33	22	42	38	31	24	17	2	84
May,	36	51	30	30	33	44	28	19	24	15	88
June,	54	57	29	31	29	36	23	19	12	10	87
July,	40	52	43	39	42	35	23	17	15	4	84
August,	28	57	52	36	32	28	32	28	15	2	84
September,	40	65	32	29	29	29	30	29	15	2	81
October,	66	61	21	37	35	22	38	18	12	0	80
November,	130	49	19	29	20	27	8	12	6	0	72
December,	187	46	17	19	14	12	7	5	3	0	73
Total,	889	651	365	370	352	345	280	211	148	41	88

Table XIV.

Partial Droughts in Edinburgh from 1857–1900.

Note.—Definition of Partial Drought is "a period of more than 28 consecutive days the aggregate rainfall of which does not exceed 0·01 inch per day."

Commenced.	Terminated.	Days.	Amount.
			ins.
February 6, 1857	March 14, 1857	37	·28
July 6, 1857	August 5, 1857	31	·30
March 4, 1858	April 24, 1858	52	·52
April 24, 1859	June 26, 1859	64	·60
April 1, 1860	May 10, 1860	40	·40
June 19, 1863	July 17, 1863	29	·28
December 13, 1863	January 15, 1864	34	·26
June 4, 1865	July 6, 1865	33	·20
June 9, 1867	July 10, 1867	32	·29
October 28, 1867	November 29, 1867	33	·30
May 27, 1868	July 4, 1868	39	·37
June 16, 1869	July 27, 1869	42	·37
July 17, 1870	August 23, 1870	38	·24
January 21, 1873	February 25, 1873	36	·36
March 25, 1873	May 2, 1873	39	·39
April 15, 1874	May 14, 1874	30	·27
March 10, 1875	April 26, 1875	48	·38
August 3, 1880	September 9, 1880	38	·33
May 25, 1885	June 23, 1885	30	·28
June 14, 1886	July 13, 1886	30	·30
May 27, 1887	July 2, 1887	37	·36
June 9, 1889	July 8, 1889	30	·22
November 5, 1889	December 18, 1889	44	·37
February 4, 1891	March 9, 1891	34	·30
March 26, 1891	April 30, 1891	36	·31
March 7, 1893	April 14, 1893	39	·34
February 4, 1895	March 7, 1895	32	·31
May 21, 1899	June 19, 1899	30	·21
July 21, 1899	August 26, 1899	37	·35

Table XV.

Showing the Mean Excess of the Black Bulb in Vacuo over the Shade Temperature for each Day in the Year from 1889–1898.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	8°·8	22°·8	36°·7	41°·6	52°·4	52°·4	48°·5	47°·3	35°·4	39°·0	22°·5	14°·0
2	11·0	19·9	34·3	43·3	43·5	40·0	46·1	48·5	40·2	32·0	23·6	11·5
3	8·4	26·0	36·9	37·2	48·4	37·0	37·4	49·4	44·2	30·5	14·6	12·3
4	5·8	23·8	36·7	34·2	51·2	48·3	48·0	46·8	41·2	34·7	18·6	9·7
5	9·0	25·1	35·1	39·5	43·9	41·5	47·9	45·6	41·3	30·6	26·0	7·5
6	9·1	22·3	35·5	34·7	52·2	41·9	52·0	49·4	40·0	39·1	24·4	9·6
7	7·0	20·1	31·4	37·4	48·0	45·6	48·8	52·3	45·0	32·1	15·8	6·0
8	9·5	27·9	39·4	44·4	44·7	42·9	49·5	42·5	42·8	35·5	18·5	6·0
9	10·6	29·2	35·9	45·2	49·0	48·4	50·9	42·2	39·3	29·7	21·3	12·6
10	12·7	24·1	36·9	43·0	48·2	42·5	44·4	45·2	45·6	37·9	21·2	5·4
11	11·6	30·4	32·2	44·4	44·0	44·6	41·4	37·6	39·3	34·4	20·2	10·5
12	10·7	26·7	36·7	37·4	48·2	47·2	44·2	43·2	43·9	34·9	19·4	8·0
13	11·4	22·0	42·9	48·3	48·0	52·1	49·3	47·2	37·1	29·4	19·0	8·5
14	10·9	26·1	37·5	48·4	41·3	49·4	49·7	49·7	32·4	30·7	14·5	4·3
15	11·3	26·3	32·0	45·8	43·7	44·3	51·0	49·6	37·3	22·5	24·1	7·4
16	15·8	28·0	40·7	42·8	41·9	46·5	42·7	52·0	41·6	32·3	13·2	5·7
17	6·5	27·8	34·6	42·3	51·1	48·5	45·7	53·7	32·4	28·0	17·8	5·2
18	19·5	31·6	40·2	46·7	42·5	47·9	46·7	48·7	41·8	29·7	14·7	5·1
19	11·0	28·6	38·0	47·1	45·2	53·1	50·8	47·2	41·6	31·1	20·1	6·3
20	13·2	29·2	35·1	39·6	49·4	46·8	46·9	41·5	35·4	22·1	17·2	5·8
21	14·5	27·1	37·6	52·4	52·8	54·0	45·5	45·3	38·2	20·2	13·0	7·2
22	22·1	25·9	38·7	50·8	47·2	50·8	52·3	49·4	37·2	28·2	14·5	6·7
23	17·8	35·0	42·4	45·5	53·4	42·5	54·2	49·2	34·8	27·1	17·0	6·7
24	17·4	29·2	44·3	44·9	55·2	53·1	51·3	44·3	39·2	26·5	7·4	6·8
25	16·0	27·3	32·3	54·1	55·4	52·0	35·3	51·9	38·4	33·1	9·2	5·6
26	18·9	25·4	47·0	49·9	47·0	47·6	46·1	47·2	36·4	34·2	14·8	6·2
27	22·4	32·0	49·6	43·2	55·5	51·6	44·8	48·2	35·8	31·6	11·1	5·7
28	19·9	33·5	33·9	49·4	42·6	42·5	46·4	48·3	42·5	21·1	12·2	5·8
29	21·3		36·2	50·5	53·5	55·2	49·2	47·0	27·9	27·5	17·3	7·8
30	21·9		39·6	50·3	53·8	49·7	48·6	48·0	36·0	26·0	9·6	7·5
31	21·7		45·9		53·2		49·8	41·8		25·5		9·3
	13·8	26·9	37·9	44·5	48·9	47·3	47·3	47·1	38·8	30·2	17·1	7·6

The Mean of the Twelve Monthly Values is 34°·0.

Table XVI.

Showing the Smoothed Departure from the Annual Mean of the Maximum Black Bulb in Vacuo over the Shade Maxima for each Day in the Year from 1889 *to* 1898.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	*25°·0*	*11°·5*	**0°·7**	**7°·5**	**15°·4**	**13°·3**	**13°·4**	**14°·7**	**9°·9**	*0°·9*	*11°·6*	*21°·1*
2	*25·3*	*11·1*	**1·6**	**6·4**	**15·2**	**12·2**	**11·9**	**14·4**	**6·6**	**0·4**	*13·0*	*22·6*
3	*25·4*	*10·5*	**1·9**	**5·2**	**13·9**	**9·8**	**11·6**	**13·5**	**6·5**	*0·6*	*12·9*	*23·0*
4	*25·3*	*10·6*	**1·7**	**3·8**	**13·8**	**7·7**	**12·3**	**13·9**	**7·4**	*0·6*	*12·6*	*23·9*
5	*26·1*	*10·5*	**1·1**	**2·6**	**14·7**	**8·9**	**12·8**	**14·7**	**8·3**	*0·6*	*14·1*	*25·0*
6	*25·9*	*10·2*	**1·6**	**4·0**	**14·0**	**10·0**	**15·2**	**13·3**	**8·1**	**0·4**	*13·3*	*26·2*
7	*25·0*	*9·1*	**1·5**	**6·2**	**13·6**	**10·0**	**15·8**	**12·4**	**7·7**	*0·6*	*12·8*	*25·7*
8	*24·2*	*9·3*	**1·8**	**6·9**	**14·4**	**10·3**	**15·1**	**12·3**	**8·5**	**0·9**	*13·8*	*26·1*
9	*23·7*	*7·7*	**1·2**	**8·9**	**12·8**	**10·8**	**13·0**	**10·0**	**8·4**	*0·1*	*14·6*	*25·9*
10	*23·0*	*6·3*	**2·2**	**9·0**	**12·8**	**11·1**	**12·1**	**8·1**	**8·2**	**0·5**	*13·9*	*25·5*
11	*22·6*	*7·5*	**2·5**	**9·7**	**13·5**	**13·0**	**12·0**	**9·1**	**7·0**	*0·7*	*13·8*	*25·0*
12	*22·5*	*8·1*	**3·2**	**10·3**	**11·9**	**13·2**	**11·8**	**10·6**	**5·7**	*0·5*	*15·1*	*26·7*
13	*22·8*	*7·7*	**2·3**	**10·9**	**11·0**	**13·5**	**13·1**	**11·5**	**4·0**	*3·6*	*14·6*	*26·3*
14	*22·0*	*8·2*	**4·0**	**10·5**	**10·6**	**13·9**	**13·4**	**14·3**	**4·5**	*4·0*	*16·0*	*27·2*
15	*22·8*	*8·0*	**3·5**	**11·5**	**11·2**	**14·1**	**13·7**	**16·4**	**2·2**	*5·4*	*16·3*	*27·8*
16	*21·2*	*6·0*	**3·0**	**11·2**	**10·1**	**13·3**	**13·2**	**16·7**	**3·1**	*5·4*	*17·1*	*28·5*
17	*21·2*	*5·5*	**3·1**	**10·9**	**10·9**	**14·1**	**13·4**	**16·2**	**4·9**	*5·3*	*16·0*	*28·1*
18	*20·8*	*5·0*	**3·7**	**9·7**	**12·0**	**14·6**	**12·6**	**14·6**	**4·6**	*5·4*	*17·4*	*28·4*
19	*21·1*	*5·1*	**3·1**	**11·6**	**14·2**	**16·1**	**13·1**	**13·3**	**3·9**	*7·8*	*17·4*	*28·0*
20	*19·9*	*5·5*	**3·9**	**13·3**	**13·4**	**16·5**	**14·4**	**12·4**	**4·8**	*7·7*	*18·1*	*27·7*
21	*20·3*	*4·8*	**4·3**	**13·1**	**15·6**	**15·4**	**15·9**	**12·5**	**3·4**	*3·3*	*17·6*	*27·3*
22	*19·0*	*4·7*	**5·6**	**12·6**	**17·6**	**15·4**	**16·0**	**11·9**	**3·0**	*9·2*	*20·2*	*27·2*
23	*18·4*	*5·1*	**5·0**	**15·5**	**18·8**	**16·7**	**13·7**	**13·8**	**3·6**	*7·0*	*21·8*	*27·3*
24	*17·6*	*5·4*	**6·9**	**15·1**	**17·6**	**15·2**	**13·8**	**14·4**	**3·2**	*4·2*	*21·4*	*27·6*
25	*15·5*	*4·2*	**9·1**	**13·5**	**19·3**	**15·4**	**12·3**	**14·2**	**2·9**	*3·5*	*20·3*	*27·8*
26	*15·1*	*4·5*	**7·4**	**14·3**	**17·1**	**15·4**	**10·8**	**14·0**	**4·5**	*4·7*	*23·1*	*28·0*
27	*14·3*	*3·0*	**5·8**	**15·4**	**16·8**	**15·8**	**10·4**	**14·5**	**2·2**	*4·6*	*21·1*	*27·8*
28	*13·1*	*1·6*	**7·3**	**14·7**	**16·5**	**15·3**	**13·1**	**13·7**	**1·7**	*5·9*	*21·0*	*27·4*
29	*12·6*		**7·1**	**15·2**	**17·7**	**15·5**	**13·8**	**12·7**	**2·2**	*7·7*	*21·2*	*26·8*
30	*12·5*		**5·4**	**15·2**	**17·1**	**14·4**	**14·3**	**10·1**	**1·5**	*9·5*	*21·1*	*26·0*
31	*12·5*		**7·3**		**16·6**		**14·7**	**8·5**		*8·8*		*25·1*

Note.—Heavy type indicates an excess, light type a defect.

Table XVII.

Showing the Number of Times Snow fell with each Wind from 1857 *to* 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Total.
January,	14	25	33	12	9	10	46	16	2	167
February,	12	21	55	21	8	9	33	15	5	179
March,	27	38	48	20	5	7	47	28	6	226
April,	9	10	13	3	0	2	7	4	1	49
May,	3	5	2	0	0	1	0	2	0	13
October,	4	2	1	0	0	0	4	2	0	13
November,	12	10	3	4	2	5	9	9	1	55
December,	11	22	22	10	8	8	41	14	4	140
Total,	92	133	177	70	32	42	187	90	19	842

Note.—No snow fell in the months of June, July, August and September during the period 1857–1900.

Table XVIII.

Showing the Number of Times Hail fell with each Wind from 1857 *to* 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Total.
January,	4	6	11	0	1	3	5	3	0	33
February,	1	11	5	0	0	1	10	2	0	30
March,	7	9	5	3	0	2	18	7	2	53
April,	15	8	5	0	0	5	15	15	1	64
May,	7	7	5	3	2	1	15	7	2	49
June,	0	0	3	2	0	1	12	2	1	21
July,	1	0	3	1	0	2	3	6	1	17
August,	1	2	5	0	0	2	9	2	0	21
September,	1	1	3	0	1	1	7	3	1	18
October,	7	8	1	0	0	1	12	7	0	36
November,	1	8	7	0	0	5	4	0	1	26
December,	1	7	5	0	1	3	6	1	0	24
Total,	46	67	58	9	5	27	116	55	9	392

Table XIX.

Showing the Number of Times Gales occurred with each Wind from 1857 *to* 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Total.
January,	1	5	5	2	21	61	81	4	180
February,	0	6	4	4	12	46	53	4	129
March,	2	2	8	5	3	31	66	6	123
April,	1	5	8	0	5	10	22	0	51
May,	0	4	5	0	3	6	21	3	42
June,	0	0	3	2	0	11	19	0	35
July,	0	2	4	0	2	7	13	0	28
August,	0	3	3	0	2	9	20	2	39
September,	3	2	4	3	4	20	38	3	77
October,	1	8	6	3	3	30	35	9	95
November,	4	5	9	7	1	36	50	1	113
December,	0	8	7	1	8	53	69	7	153
Total,	12	50	66	27	64	320	487	39	1065

Table XX.

Showing the Number of Times Thunderstorms occurred with each Wind from 1857 *to* 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Total.
January,	0	0	1	0	1	1	4	0	0	7
February,	0	1	0	0	0	1	4	1	0	7
March,	0	1	1	0	0	1	4	0	1	8
April,	0	0	0	1	1	1	4	4	3	14
May,	1	7	2	8	7	2	11	1	3	42
June,	2	3	26	7	5	7	14	2	7	73
July,	4	15	24	5	7	7	25	10	5	102
August,	2	8	21	3	1	5	19	4	6	69
September,	0	1	5	1	3	2	9	2	3	26
October,	0	0	2	2	0	2	10	1	0	17
November,	0	1	0	0	0	0	4	0	0	5
December,	0	1	0	0	1	0	0	1	0	3
Total,	9	38	82	27	26	29	108	26	28	373

Table XXI.

Showing the Number of Times Fog occurred with each Wind from 1857 to 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Total.
January,	0	11	12	9	7	7	13	3	9	71
February,	1	7	12	7	4	2	7	4	12	56
March,	2	4	21	8	2	1	3	3	6	50
April,	2	8	34	7	0	1	1	1	3	57
May,	1	7	38	0	0	1	3	0	5	55
June,	3	16	54	9	1	1	2	1	2	89
July,	0	14	33	3	1	0	0	0	0	51
August,	1	9	36	2	2	2	0	1	6	59
September,	5	10	19	5	2	0	1	1	12	55
October,	0	14	22	12	13	5	8	2	10	86
November,	5	4	23	15	10	4	10	4	9	84
December,	3	6	13	10	7	7	13	7	20	86
Total,	23	110	317	87	49	31	61	27	94	799

TABLE XXII.

Showing the Number of Solar Halos Recorded with each Wind from 1857 to 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Total.
January,	0	0	0	0	1	4	14	4	0	23
February,	0	1	0	0	3	4	8	1	1	18
March,	4	0	5	2	2	5	12	2	3	35
April,	3	3	2	2	1	2	9	6	6	34
May,	1	1	2	0	0	2	7	3	2	18
June,	0	2	2	0	1	0	6	0	3	14
July,	0	1	1	0	1	0	10	3	0	16
August,	0	0	2	1	1	3	8	2	3	20
September,	1	1	0	1	2	3	6	2	0	16
October,	0	1	0	2	1	0	11	3	0	18
November,	0	0	0	0	0	2	10	0	1	13
December,	1	0	0	0	0	0	8	0	1	10
Total,	10	10	14	8	13	25	109	26	20	235

TABLE XXIII.

Showing the Number of Lunar Halos Recorded with each Wind from 1857 to 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Total.
January,	0	2	2	3	2	8	18	3	8	46
February,	1	1	0	1	0	3	12	3	1	22
March,	1	1	5	3	3	5	16	4	4	42
April,	1	0	3	1	1	4	8	3	2	23
May,	1	1	2	0	1	0	5	1	1	12
June,	0	0	0	0	2	0	0	3	1	6
July,	0	0	0	0	1	0	2	0	0	3
August,	0	0	0	0	0	0	2	0	0	2
September,	2	1	2	0	0	2	9	1	3	20
October,	5	0	2	1	2	3	10	4	2	29
November,	2	0	2	1	4	4	15	3	5	36
December,	0	1	2	2	2	3	20	1	8	39
Total,	13	7	20	12	18	32	117	26	35	280

Table XXIV.

Showing the Number of Auroras Recorded with each Wind from 1857 *to* 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.	Total.
January,	1	0	0	2	1	1	8	0	0	13
February,	0	0	2	1	3	3	7	1	1	18
March,	1	1	5	1	0	0	10	5	1	24
April,	0	3	4	4	0	0	1	2	1	15
May,	0	0	3	1	1	1	1	2	2	11
June,	0	0	1	0	0	0	0	0	0	1
July,	0	0	0	0	0	0	0	0	0	0
August,	1	3	3	0	0	1	1	1	0	10
September,	2	1	1	1	2	1	7	3	2	20
October,	7	2	0	0	1	4	8	6	0	28
November,	3	0	1	1	1	3	11	1	1	22
December,	0	2	0	0	0	1	1	4	0	8
Total,	15	12	20	11	9	15	55	25	8	170

Table XXV.

Showing the Number of Days of each Wind in Edinburgh from 1857 *to* 1900.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
January,	59	63	113	89	123	259	510	100	48
February,	54	61	146	94	84	207	444	99	54
March,	90	115	194	85	73	165	452	145	45
April,	74	139	324	93	63	125	326	123	53
May,	68	154	354	89	74	113	334	118	60
June,	67	134	298	71	63	142	386	103	56
July,	60	100	229	51	68	142	570	104	40
August,	40	88	236	64	62	179	554	79	62
September,	52	82	179	59	88	176	515	96	73
October,	59	64	190	87	96	165	532	120	51
November,	67	63	157	84	84	211	477	120	57
December.	52	51	122	72	101	234	549	115	68
Total,	742	1114	2542	938	979	2118	5649	1322	667

Table XXVI.

Showing the Percentage Frequency of Non-Instrumental Phenomena with different Winds from 1857 *to* 1900.

Snow.—November to April, 6 *months.*

N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var
21·5	25·6	16·5	13·5	6·1	3·4	6·6	12·2	5·8

Gales.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
Spring,	1·4	4·1	3·0	3·3	9·1	17·5	11·5	2·7
Summer,	0·0	1·5	1·4	0·9	2·4	6·0	4·1	0·9
Autumn,	2·7	5·6	2·1	2·9	3·7	11·3	5·8	4·7
Winter,	2·8	10·2	5·4	4·1	9·7	21·3	13·0	3·6
Year,	1·6	4·5	2·6	2·9	6·5	15·1	8·6	2·9

Summer Thunderstorms.—May to August.

N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
3·8	6·9	6·5	8·4	7·5	3·6	3·7	4·2	1·0

Hail.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
February to May,	10·5	7·5	2·0	1·7	0·7	1·5	3·7	6·4	2·4
June to September,	1·4	0·7	1·5	1·2	0·4	0·9	1·5	3·4	1·3
October to January,	5·5	12·0	4·1	0·0	0·5	1·4	1·3	2·4	0·4
Year,	6·2	6·0	2·3	1·0	0·5	1·3	2·1	4·2	1·3

Table XXVI.—*continued.*

Fog.

	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
Spring,	2·3	6·0	10·9	8·1	2·7	0·8	0·9	2·2	13·8
Summer,	2·1	9·5	14·2	5·7	0·9	0·5	0·4	0·3	4·5
Autumn,	4·0	14·1	12·7	9·0	6·9	1·3	0·6	1·3	15·1
Winter,	4·5	11·9	12·3	13·9	7·8	2·6	2·3	4·2	22·0
Year,	3·1	9·9	12·5	9·3	5·0	1·5	1·1	2·0	14·1

Solar Halos.—Annual Values.

N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1·3	0·9	0·6	0·9	1·3	1·2	1·9	2·0	3·0

Lunar Halos.—Annual Values.

N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm or Var.
1·7	0·6	0·8	1·3	1·8	1·5	2·1	2·0	5·2

TABLE XXVII.

Price of Wheat (First Quality) in Edinburgh from 1801 *to* 1900.—
Per Imperial Quarter.

Year.	Price.		Year.	Price.		Year.	Price.		Year.	Price.		Year.	Price.	
	s.	*d.*		*s.*	*d.*		*s.*	*d.*		*s.*	*d.*		*s.*	*d.*
1801 ...	59	0	1821 ...	56	0	1841 ...	53	6	1861 ...	56	4	1881 ...	33	4
1802 ...	53	0	1822 ...	41	0	1842 ...	45	0	1862 ...	39	10	1882 ...	36	7
1803 ...	52	6	1823 ...	58	0	1843 ...	51	6	1863 ...	36	11	1883 ...	35	2
1804 ...	84	0	1824 ...	63	10	1844 ...	43	4	1864 ...	34	0	1884 ...	29	9
1805 ...	60	0	1825 ...	60	8	1845 ...	44	10	1865 ...	46	4	1885 ...	28	1
1806 ...	76	0	1826 ...	56	6	1846 ...	64	0	1866 ...	56	9	1886 ...	26	6
1807 ...	65	0	1827 ...	48	8	1847 ...	51	6	1867 ...	61	0	1887 ...	28	8
1808 ...	95	0	1828 ...	74	0	1848 ...	44	7	1868 ...	46	0	1888 ...	26	8
1809 ...	84	0	1829 ...	52	8	1849 ...	33	6	1869 ...	40	3	1889 ...	29	4
1810 ...	78	0	1830 ...	58	8	1850 ...	36	4	1870 ...	51	0	1890 ...	30	6
1811 ...	95	0	1831 ...	57	4	1851 ...	37	6	1871 ...	50	1	1891 ...	36	0
1812 ...	120	0	1832 ...	49	0	1852 ...	46	0	1872 ...	34	0	1892 ...	26	10
1813 ...	71	0	1833 ...	48	0	1853 ...	62	4	1873 ...	55	2	1893 ...	25	4
1814 ...	59	0	1834 ...	40	8	1854 ...	68	1	1874 ...	40	3	1894 ...	21	7
1815 ...	50	0	1835 ...	33	6	1855 ...	70	9	1875 ...	40	0	1895 ...	26	6
1816 ...	79	0	1836 ...	54	0	1856 ...	40	0	1876 ...	42	2	1896 ...	30	11
1817 ...	80	0	1837 ...	54	7	1857 ...	38	4	1877 ...	35	9	1897 ...	33	1
1818 ...	75	0	1838 ...	65	0	1858 ...	40	1	1878 ...	38	0	1898 ...	27	6
1819 ...	65	0	1839 ...	56	0	1859 ...	44	8	1879 ...	37	10	1899 ...	26	6
1820 ...	61	0	1840 ...	56	8	1860 ...	47	8	1880 ...	42	0	1900 ...	26	4

Table XXVIII.

Harvest Dates at the Farm of Liberton, near Edinburgh, from 1812 to 1836.

Year.	Began Cutting.		All In.	
1812, ...	September	10	October	10
1813, ...	August	22	September	11
1814, ...	"	24	"	19
1815, ...	"	25	"	18
1816, ...	September	20	October	10
1817, ...	"	1	"	3
1818, ...	August	21	September	11
1819, ...	"	14	"	8
1820, ...	"	12	"	8
1821, ...	"	20	"	13
1822, ...	"	20	"	5
1823, ...	September	2	"	26
1824, ...	August	12	"	4
1825, ...	"	8	"	5
1826, ...	July	17	August	8
1827, ...	August	18	September	14
1828, ...	"	11	"	5
1829, ...	"	21	"	22
1830, ...	"	30	"	25
1831, ...	"	8	August	29
1832, ...	"	13	September	7
1833, ...	"	13	"	2
1834, ...	"	5	August	23
1835, ...	"	4	September	8
1836, ...	"	23	"	22

Table XXIX.

Errata in Part II. of the "Meteorology of Edinburgh,"
Trans., vol. xxxix. part i.

Table I. p. 110. February 1856, *for* 39·803 inches *read* 29·803 inches.

Table II. p. 112. October 1853, *for* 30·972 *read* 29·972.

Table V. p. 115. Range of Highest Pressure, 1840–1896.
Month November, *for* 2·676 ins. *read* 0·676 ins.
Range of Lowest Pressure, 1840–1896.
Month December, *for* 0·933 ins. *read* 1·933 ins.

Table VII. Showing the Mean Temperature of the Air in Edinburgh from 1764–1896, pp. 117 and 118.

July	1840	*for*	59°·9	*read*	56°·0.
Year	1858	„	46°·2	„	46°·7.
September	1882	„	57°·0	„	51°·7.
April	1885	„	43°·7	„	45°·5.
Year	1885	„	46°·2	„	46°·3.
April	1886	„	42°·6	„	43°·7.
April	1887	„	42°·8	„	42°·6.

Decennial Means for April 1881–1890, *for* 43°·7 *read* 44°·0.
Decennial Means for September 1881–1890, *for* 54°·3 *read* 53°·7.

Table XIII. p. 124. Annual Range 1772, *for* 44° *read* 54°.

Table XV. p. 127. Highest for December, *for* 37°·0 *read* 37°·4.
Range for December, *for* 24°·6 *read* 25°·0.

Table XVIII. p. 130. A number of the values of mean maximum temperature and mean minimum temperature between the years 1851 and 1860, and in 1862, 1873, and 1876, are a little too high.

These values have been corrected and revised, and are shown in the form of departures from the mean of the 50 years 1851–1900 in Tables V. and VI. of this paper. The other columns in Table XVIII. are unaffected.

Table XIX. p. 134. Warmest June, *for* 61°·4 June 1826 *read* 61°·9 1846.
Range Column, *for* 9°·9 *read* 10°·4.

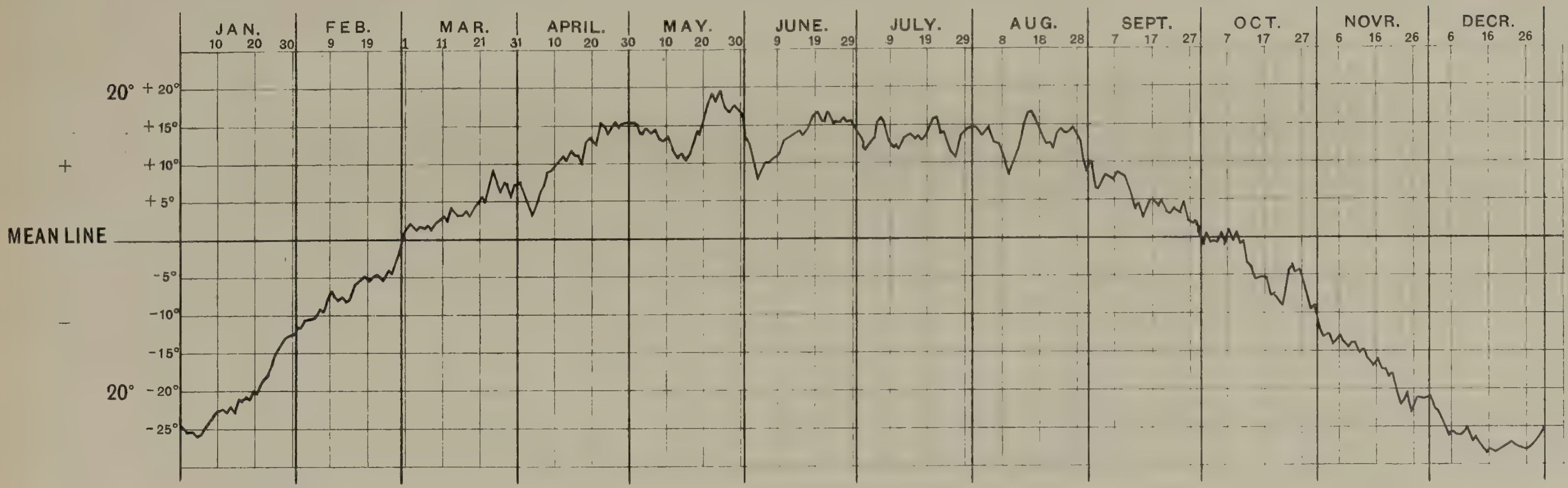
Table XXIX. p. 145. Rainfall for December 1896, *for* 2·25 ins. *read* 2·59 ins.
Rainfall for year 1896, *for* 23·35 ins. *read* 23·59 ins.

Table XLIII, p. 180. December 1886, *for* 0 *read* 1, and for year *read* 7 *instead of* 6.

Table LV. p. 187. Year 1779, *for* 1°·9 *read* 1°·1.

(*Issued separately September* 5, 1902.)

SHOWING THE DEPARTURE FROM THE ANNUAL MEAN OF DAILY VALUES OF SOLAR RADIATION AT EDINBURGH, FROM 1889 TO 1898



R. C. MOSSMAN, DELT.

THE VALUES HAVE BEEN SMOOTHED BY CONTINUOUS FIVE-DAY GROUPS.

A. RITCHIE & SON EDIN^R

XXII.—*Vanishing Aggregates of Secondary Minors of a Persymmetric Determinant.* By Thomas Muir, LL.D.

(Read 19th May 1902.)

(1) The persymmetric determinant of the n^{th} order

$$\begin{vmatrix} a_1 & a_2 & a_3 & \dots\dots & a_n \\ a_2 & a_3 & a_4 & \dots\dots & a_{n+1} \\ a_3 & a_4 & a_5 & \dots\dots & a_{n+2} \\ \cdot & \cdot & \cdot & \cdot\cdot\cdot\cdot\cdot & \cdot \\ a_n & a_{n+1} & a_{n+2} & \dots\dots & a_{2n-1} \end{vmatrix}$$

being such that in every case the element in the place r, s is the same as the element in the place $r-1$, $s+1$, and therefore having only $2n-1$ independent elements, viz., the elements $a_1, a_2, \dots, a_{2n-1}$ forming the first row and last column, is conveniently denoted by

$$P(a_1, a_2, \dots, a_{2n-1}).$$

As it is a special form of axisymmetric determinant, any known relation between minors of the latter must of course hold in regard to the corresponding minors of the former. Now in the case of the axisymmetric determinant

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix}_{r,s=s,r}$$

it was shown in April 1897 that there existed between three of its secondary minors the relation

$$\begin{vmatrix} 1 & 3 & 5 \\ 1 & 2 & 4 \end{vmatrix} - \begin{vmatrix} 1 & 2 & 5 \\ 1 & 3 & 4 \end{vmatrix} - \begin{vmatrix} 1 & 4 & 5 \\ 1 & 2 & 3 \end{vmatrix} = 0;$$

and it was pointed out how this and other similar theorems might be generalised.* A few months later there was published in Italy a paper the object of which was to establish the following theorem :—*If in a persymmetric determinant the* 1^{st} *and* r^{th} *rows be deleted and the* 1^{st} *and* s^{th} *columns, the secondary minor so obtained is equal to the sum of two similar secondary minors, one got by deleting the* 1^{st} *and* $(r+1)^{th}$ *rows of the original and the* 1^{st} *and* $(s-1)^{th}$ *columns, the other by deleting the* 1^{st} *and* 2^{nd} *rows and the* $(s-1)^{th}$ *and* r^{th} *columns.*† If we indicate by

$$\begin{vmatrix} \alpha, \beta, \gamma, \dots \\ \zeta, \eta, \theta, \dots \end{vmatrix}$$

* Muir, Th., "The automorphic linear transformation of a quadric," *Trans. Roy. Soc. Edin.*, xxxix. [pp. 209–230] p. 226.

† Cazzaniga, Tito, "Relazioni fra i minori di un determinante di Hankel," *Rendiconti del R. Ist. Lomb. di sc., e lett.*, serie ii. vol. xxxi. (1898) pp. 610–614.

the minor obtained by deletion of the α^{th}, β^{th}, γ^{th}, ... rows, and ζ^{th}, η^{th}, θ^{th}, ..., columns,—in other words, the complementary minor of

$$\left| \begin{matrix} \alpha, \beta, \gamma, \dots\dots \\ \zeta, \eta, \theta, \dots\dots \end{matrix} \right| ,$$

the two results will be found readily comparable. The first of them, relative to an axisymmetric determinant of the 5^{th} order, would then become

$$\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

or, with an appearance of greater generality,

$$\begin{vmatrix} \alpha & \gamma \\ \beta & \delta \end{vmatrix} = \begin{vmatrix} \alpha & \delta \\ \beta & \gamma \end{vmatrix} + \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix},$$

where α, β, γ, δ are a set of four integers chosen from 1, 2, 3, 4, 5; and the second, relative to any axisymmetric determinant, would become

$$\begin{vmatrix} 1 & r \\ 1 & s \end{vmatrix} = \begin{vmatrix} 1 & r+1 \\ 1 & s-1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ s-1 & r \end{vmatrix}.$$

As neither of these is included in the other, I have been induced to make a fuller investigation, which has resulted, among other things, in the generalisation of both, and the discovery of the nature of the connection between the two as thus generalised.

(2) It is seen that in both theorems one determinant is expressed as the sum of two others, and the presumption is thus raised that they are cases of the general theorem published in 1888, which shows how any determinant of the n^{th} order can be expressed as the sum of n determinants of the same order, the specialisation introduced having the effect of reducing the n determinants to 2. As regards 4^{th} order determinants the said general theorem gives

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = \begin{vmatrix} a_1 & P_1 & P_2 & P_3 \\ Q_1 & b_2 & b_3 & b_4 \\ Q_2 & c_2 & c_3 & c_4 \\ Q_3 & d_2 & d_3 & d_4 \end{vmatrix} + \begin{vmatrix} P_1 & a_2 & P_4 & P_5 \\ b_1 & Q_1 & b_3 & b_4 \\ c_1 & Q_2 & c_3 & c_4 \\ d_1 & Q_3 & d_3 & d_4 \end{vmatrix}$$
$$+ \begin{vmatrix} P_2 & P_4 & a_3 & P_6 \\ b_1 & b_2 & Q_1 & b_4 \\ c_1 & c_2 & Q_2 & c_4 \\ d_1 & d_2 & Q_3 & d_4 \end{vmatrix} + \begin{vmatrix} P_3 & P_5 & P_6 & a_4 \\ b_1 & b_2 & b_3 & Q_1 \\ c_1 & c_2 & c_3 & Q_2 \\ d_1 & d_2 & d_3 & Q_3 \end{vmatrix},$$

where there occur on the right nine elements which do not appear on the left, viz., $P_1, P_2, \dots, P_6$ and Q_1, Q_2, Q_3. Now it is manifest that the fourth determinant on the right will vanish by making

$$P_3, P_5, P_6, Q_3 = d_1, d_2, d_3, a_4,$$

and the third by making

$$P_2, P_4, Q_2, P_6 = c_1, c_2, a_3, c_4.$$

Doing this, and noting the two conditions in regard to P_6, we have the identity

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & c_4 & d_4 \end{vmatrix} = \begin{vmatrix} a_1 & P & c_1 & d_1 \\ Q & b_2 & b_3 & b_4 \\ a_3 & c_2 & c_3 & c_4 \\ a_4 & d_2 & c_4 & d_4 \end{vmatrix} + \begin{vmatrix} P & a_2 & c_2 & d_2 \\ b_1 & Q & b_3 & b_4 \\ c_1 & a_3 & c_3 & c_4 \\ d_1 & a_4 & c_4 & d_4 \end{vmatrix}$$

—a result which, when found, is readily verified by examining the cofactors of a_1, b_1, c_1, d_1, P.

Similarly in the case of the identity of the 5th order,

$$\begin{aligned} |\, a_1 b_2 c_3 d_4 e_5 \,| &= \begin{vmatrix} a_1 & P_1 & P_2 & P_3 & P_4 \\ Q_1 & b_2 & b_3 & b_4 & b_5 \\ Q_2 & c_2 & c_3 & c_4 & c_5 \\ Q_3 & d_2 & d_3 & d_4 & d_5 \\ Q_4 & e_2 & e_3 & e_4 & e_5 \end{vmatrix} + \begin{vmatrix} P_1 & a_2 & P_5 & P_6 & P_7 \\ b_1 & Q_1 & b_3 & b_4 & b_5 \\ c_1 & Q_2 & c_3 & c_4 & c_5 \\ d_1 & Q_3 & d_3 & d_4 & d_5 \\ e_1 & Q_4 & e_3 & e_4 & e_5 \end{vmatrix} + \begin{vmatrix} P_2 & P_5 & a_3 & P_8 & P_9 \\ b_1 & b_2 & Q_1 & b_4 & b_5 \\ c_1 & c_2 & Q_2 & c_4 & c_5 \\ d_1 & d_2 & Q_3 & d_4 & d_5 \\ e_1 & e_2 & Q_4 & e_4 & e_5 \end{vmatrix} \\ &\quad + \begin{vmatrix} P_3 & P_6 & P_8 & a_4 & P_{10} \\ b_1 & b_2 & b_3 & Q_1 & b_5 \\ c_1 & c_2 & c_3 & Q_2 & c_5 \\ d_1 & d_2 & d_3 & Q_3 & d_5 \\ e_1 & e_2 & e_3 & Q_4 & e_5 \end{vmatrix} + \begin{vmatrix} P_4 & P_7 & P_9 & P_{10} & a_5 \\ b_1 & b_2 & b_3 & b_4 & Q_1 \\ c_1 & c_2 & c_3 & c_4 & Q_2 \\ d_1 & d_2 & d_3 & d_4 & Q_3 \\ e_1 & e_2 & e_3 & e_4 & Q_4 \end{vmatrix}, \end{aligned}$$

by putting

$$\begin{aligned} P_4, P_7, P_9, P_{10}, Q_4 &= e_1, e_2, e_3, e_4, a_5, \\ P_3, P_6, P_8, Q_3, P_{10} &= d_1, d_2, d_3, a_4, d_5, \\ P_2, P_5, Q_2, P_8, P_9 &= c_1, c_2, a_3, c_4, c_5, \end{aligned}$$

and noting the two conditions in regard to P_8, P_9, P_{10}, we obtain

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ b_1 & b_2 & b_3 & b_4 & b_5 \\ c_1 & c_2 & c_3 & c_4 & c_5 \\ d_1 & d_2 & c_4 & d_4 & d_5 \\ e_1 & e_2 & e_5 & d_5 & e_5 \end{vmatrix} = \begin{vmatrix} a_1 & P & c_1 & d_1 & e_1 \\ Q & b_2 & b_3 & b_4 & b_5 \\ a_3 & c_2 & c_3 & c_4 & c_5 \\ a_4 & d_2 & c_4 & d_4 & d_5 \\ a_5 & e_2 & c_5 & d_5 & e_5 \end{vmatrix} + \begin{vmatrix} P & a_2 & c_2 & d_2 & e_2 \\ b_1 & Q & b_3 & b_4 & b_5 \\ c_1 & a_3 & c_3 & c_4 & c_5 \\ d_1 & a_4 & c_4 & d_4 & d_5 \\ e_1 & a_5 & c_5 & d_5 & e_5 \end{vmatrix},$$

which, again, can be readily verified by examining the cofactors of $a_1, b_1, c_1, d_1, e_1, P$. The cofactors of e_5, it is worth noting in passing, are the three determinants of the previous case.

The general theorem thus reached may be formally enunciated as follows:—

If the elements of the determinant $|\mathrm{a}_{1n}|$ *be any whatever subject to the condition that the last coaxial minor of the* $(\mathrm{n}-2)^{th}$ *order is axisymmetric, it can be expressed as the sum of two others formed from it by putting* $\begin{smallmatrix} \mathrm{a}_{11} & \mathrm{P} \\ \mathrm{Q} & \mathrm{a}_{22} \end{smallmatrix}$ *in the one and* $\begin{smallmatrix} \mathrm{P} & \mathrm{a}_{12} \\ \mathrm{a}_{21} & \mathrm{Q} \end{smallmatrix}$ *in the other in place of* $\begin{smallmatrix} \mathrm{a}_{11} & \mathrm{a}_{12} \\ \mathrm{a}_{21} & \mathrm{a}_{22} \end{smallmatrix}$, *and making the new complementary minors of* $\mathrm{a}_{11}, \mathrm{a}_{12}$ *the same as in the original and the new complementary minors of* $\mathrm{a}_{21}, \mathrm{a}_{22}$ *the conjugates of those in the original.*

If in either or both of the determinants on the right rows were changed into columns the enunciation would be similar.

By applying the theorem to one of the determinants on the right, the original determinant may be expressed as the sum of three: *e.g.*, in the case of the 4^{th} order we have

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & c_4 & d_4 \end{vmatrix} = \begin{vmatrix} a_1 & \mathrm{P} & c_1 & d_1 \\ \mathrm{Q} & b_2 & b_3 & b_4 \\ a_3 & c_2 & c_3 & c_4 \\ a_4 & d_2 & c_4 & d_4 \end{vmatrix} + \begin{vmatrix} \mathrm{P} & \mathrm{X} & c_1 & d_1 \\ \mathrm{Y} & \mathrm{Q} & b_3 & b_4 \\ c_2 & a_3 & c_3 & c_4 \\ d_2 & a_4 & c_4 & d_4 \end{vmatrix} + \begin{vmatrix} \mathrm{X} & a_2 & a_3 & a_4 \\ b_1 & \mathrm{Y} & c_1 & d_1 \\ b_3 & c_2 & c_3 & c_4 \\ b_4 & d_2 & c_4 & d_4 \end{vmatrix}.$$

(3) We are now in a position to enunciate and prove the first generalisation above referred to, which is—

Secondary minors of any axisymmetric determinant of order higher than the 3^{rd} *are connected by the relation*

$$\begin{vmatrix} 1 & k \\ h & l \end{vmatrix} = \begin{vmatrix} 1 & l \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & h \\ k & l \end{vmatrix} \qquad \text{(A)}$$

where $l > k > h > 1$.

If we indicate the minors by means, not of omitted lines but of those retained, the identity takes the form

$$\begin{vmatrix} 2, \ldots, \ldots, k-1, k+1, \ldots\ldots, n \\ 1, \ldots, h-1, h+1, \ldots, l-1, l+1, \ldots, n \end{vmatrix} = \begin{vmatrix} 2, \ldots\ldots, l-1, l+1, \ldots, n \\ 1, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, n \end{vmatrix}$$
$$+ \begin{vmatrix} 2, \ldots, h-1, h+1, \ldots\ldots, n \\ 1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \end{vmatrix},$$

or, by transposition of certain rows and of certain columns, and the omission of the common sign-factor $(-1)^{h+k+l-7}$,

$$\begin{vmatrix} h, l, 2, 3, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \\ 1, k, 2, 3, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \end{vmatrix}$$
$$= \begin{vmatrix} h, k, 2, 3, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \\ 1, l, 2, 3, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \end{vmatrix}$$
$$+ \begin{vmatrix} k, l, 2, 3, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \\ 1, h, 2, 3, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \end{vmatrix}.$$

As thus written the three determinants have the same final secondary minor, viz.,

$$\begin{vmatrix} 2, 3, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \\ 2, 3, \ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \end{vmatrix},$$

which is clearly axisymmetric; the cofactors of $(h, 1)$ and $(l, 1)$ in the first determinant, viz.,

$$\begin{vmatrix} l, 2, 3, \ldots\ldots \\ k, 2, 3, \ldots\ldots \end{vmatrix} \text{ and } \begin{vmatrix} h, 2, 3, \ldots\ldots \\ k, 2, 3, \ldots\ldots \end{vmatrix},$$

are the conjugates of the cofactors of the same elements in the second determinant and third determinant respectively: the cofactors of (h, k), (l, k) on the left are exactly the same as their cofactors on the right; and the two new elements introduced into the one determinant on the right, viz. $(k, 1)$, (h, l), are the same as those introduced into the other. It follows therefore from the theorem of the preceding paragraph that (A) is true.

(4) Deleting the final secondary minor from each of the determinants in (A) as last written we obtain three determinants

$$\begin{vmatrix} (h,1) & (h,k) \\ (l,1) & (l,k) \end{vmatrix}, \quad \begin{vmatrix} (h,1) & (h,l) \\ (k,1) & (k,l) \end{vmatrix}, \quad \begin{vmatrix} (k,1) & (k,h) \\ (l,1) & (l,h) \end{vmatrix},$$

and find at a glance that the first is still equal to the sum of the two last. It thus appears that the latter simple identity, which we recognise to be the first case of KRONECKER's relation between n-line minors of a $2n$-line axisymmetric determinant, has an Extensional, viz., (A).

(5) Before proceeding to the second generalisation it is desirable to state a few elementary properties of persymmetric determinants. These are :—

(a) *If the first row and last column, or first column and last row, of an* n-*line persymmetric determinant* $P(a_1, a_2, \ldots, a_{2n-1})$ *be deleted, the resulting primary minor is persymmetric, viz.,* $P(a_2, a_3, \ldots, a_{2n-2})$.

(b) *If the first row and first column be deleted the resulting primary minor is persymmetric, viz.,* $P(a_3, a_4, \ldots, a_{2n-1})$.

(c) *If the last row and last column be deleted, the resulting primary minor is persymmetric, viz.,* $P(a_1, a_2, \ldots, a_{2n-3})$.

(d) *If any one of the theorems* (a), (b), (c) *be applied to any one of the persymmetric determinants thus resulting, secondary minors will be obtained which are persymmetric.*

(e) *If* $\alpha, \beta, \ldots, \lambda$ *be* $2m-1$ *numbers in equidifferent progression chosen from* $1, 2, \ldots, 2n-1$, *then* $P(a_\alpha, a_\beta, \ldots, a_\lambda)$ *is an* m-*line persymmetric minor of* $P(a_1, a_2, \ldots, a_{2n-1})$.

(f) *If any rows and the corresponding columns be deleted from a persymmetric minor of* $P(a_1, a_2, \ldots, a_{2n-1})$ *the result is an axisymmetric minor of the latter.* For example,

$$\begin{vmatrix} 1, & h+1, & k+1, & l+1 \\ h, & k, & l, & n \end{vmatrix}$$

is axisymmetric; for $\begin{vmatrix} 1 \\ n \end{vmatrix}$ being persymmetric, if we delete its h^{th}, k^{th}, l^{th} rows, which are the $(h+1)^{\text{th}}$, $(k+1)^{\text{th}}$, $(l+1)^{\text{th}}$ rows of the original, and its h^{th}, k^{th}, l^{th} columns, which bear the same numbers in the original, the result will be axisymmetric. The numbers of all the rows retained in the result, it may be noted, are each greater by 1 than the numbers of the columns.

(6) With these before us the second general theorem is readily dealt with. It is—

Secondary minors of any persymmetric determinant of order higher than the third are connected by the relation

$$\begin{vmatrix} 1 & k+1 \\ h & l \end{vmatrix} = \begin{vmatrix} 1 & l+1 \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & h+1 \\ k & l \end{vmatrix} \qquad \text{(B)}$$

where $l > k > h > 0$.

The three minors, if written in the other mode of notation, take the form

$$\left|\begin{matrix} 2,3,\ldots\ldots\ldots, k, k+2, \ldots\ldots\ldots\ldots, n \\ 1,2,\ldots, h-1, h+1, \ldots, l-1, l+1, \ldots, n \end{matrix}\right|,$$

$$\left|\begin{matrix} 2,3,\ldots\ldots\ldots\ldots\ldots\ldots\ldots, l, l+2, \ldots, n \\ 1,2,\ldots, h-1, h+1, \ldots, k-1, k+1, \ldots, n \end{matrix}\right|,$$

$$\left|\begin{matrix} 2,3,\ldots, h, h+2, \ldots\ldots\ldots\ldots\ldots\ldots, n \\ 1,2,\ldots, k-1, k+1, \ldots, l-1, l+1, \ldots, n \end{matrix}\right|;$$

and by the transposition of certain rows and of certain columns become

$$\left|\begin{array}{lllllll} h+1, l+1, & 2,3,\ldots, h, h+2, & \ldots, k, k+2, & \ldots, l, l+2, & \ldots, n \\ \;k\;\;,\;\; n\;, & 1,2,\ldots, h-1, h+1, & \ldots, k-1, k+1, & \ldots, l-1, l+1, & \ldots, n-1 \end{array}\right|,$$

$$\left|\begin{array}{lllllll} h+1, k+1, & 2,3,\ldots, h, h+2, & \ldots, k, k+2, & \ldots, l, l+2, & \ldots, n \\ \;l\;\;,\;\; n\;, & 1,2,\ldots, h-1, h+1, & \ldots, k-1, k+1, & \ldots, l-1, l+1, & \ldots, n-1 \end{array}\right|,$$

$$\left|\begin{array}{lllllll} k+1, l+1, & 2,3,\ldots, k, h+2, & \ldots, k, k+2, & \ldots, l, l+2, & \ldots, n \\ \;h\;\;,\;\; n\;, & 1,2,\ldots, h-1, h+1, & \ldots, k-1, k+1, & \ldots, l-1, l+1, & \ldots, n-1 \end{array}\right|,$$

if the common sign-factor $(-1)^{h+k+l+n-10}$ be deleted. In the matter of their last secondary minor they are now identical; and the said minor, which in the shorter notation is

$$\left|\begin{matrix} 1, & h+1, & k+1, & l+1 \\ h, & k, & l, & n \end{matrix}\right|,$$

is seen to be axisymmetric. To apply the proposition of § 2 for the purpose of establishing our theorem (B), it therefore only remains to take the three complementary minors of this common secondary minor, viz., the minors

$$\left|\begin{matrix} h+1, & l+1 \\ k, & n \end{matrix}\right|, \quad \left|\begin{matrix} h+1, & k+1 \\ l, & n \end{matrix}\right|, \quad \left|\begin{matrix} k+1, & l+1 \\ h, & n \end{matrix}\right|,$$

and prove that the cofactors of the elements in the first are equal to the cofactors of the same elements in the second or third, and that the two remaining elements of the second are the same as the two remaining elements of the third. Now the first element in the first, $(h+1, k)$, has the cofactor

$$\left|\begin{matrix} l+1, & 2,3,\ldots\ldots \\ n & ,1,2,\ldots\ldots \end{matrix}\right|,$$

and the first element in the third $(k+1, h)$, which because of the persymmetry is equal to $(h+1, k)$, has the same cofactor: similarly, it is seen that the element $(l+1, k)$ in the first has the same cofactor as the element $(k+1, l)$ in the second. Again, the cofactors of $(h+1, n)$ in the first and second being

$$-\left|\begin{matrix} l+1, & 2,3,\ldots\ldots \\ k & ,1,2,\ldots\ldots \end{matrix}\right|, \quad -\left|\begin{matrix} k+1, & 2,3,\ldots\ldots \\ l & ,1,2,\ldots\ldots \end{matrix}\right|,$$

are by reason of the persymmetry the conjugates of each other: similarly with the cofactors

$$\left|\begin{matrix} h+1, & 2,3,\ldots\ldots \\ k & ,1,2,\ldots\ldots \end{matrix}\right|, \quad \left|\begin{matrix} k+1, & 2,3,\ldots\ldots \\ h & ,1,2,\ldots\ldots \end{matrix}\right|$$

of $(l+1\,,n)$ in the first and third. Lastly, the two remaining elements of the second $(h+1\,,l)$ and $(k+1\,,n)$ are manifestly equal to the two remaining elements of the third $(l+1\,,h)$ and $(k+1\,,n)$. The theorem (B) is thus established, the basis of it being exactly the same as that of (A), viz., the proposition of § 2.

(7) If we delete the common secondary minor from each of the determinants in (B), what remains is still an identity, viz.,

$$\begin{vmatrix} (h+1\,,k) & (h+1\,,n) \\ (l+1\,,k) & (l+1\,,n) \end{vmatrix} = \begin{vmatrix} (h+1\,,l) & (h+1\,,n) \\ (k+1\,,l) & (k+1\,,n) \end{vmatrix} + \begin{vmatrix} (k+1\,,h) & (k+1\,,n) \\ (l+1\,,h) & (l+1\,,n) \end{vmatrix}.$$

On this account (B) like (A) may be viewed as an extensional of a very simple identity.

When in (B) we put $h=1$ we obtain the theorem of § 1 due to Pascal and Cazzaniga.

(8) The conjugate determinant of $P(a_1\,,a_2\,,\,.\,.\,.\,,\,a_{2n-1})$—that is, the determinant obtained by rotating P through an angle of 180° with the diagonal $a_1\,,a_3\,,\,.\,.\,.\,,\,a_{2n-1}$ as axis—is exactly the same in both appearance and substance as P. The secondary conjugate, however—that is, the determinant obtained by the same rotation with the secondary diagonal $a_n\,,a_n\,,\,.\,.\,.\,,\,a_n$ as axis—is only the same in substance, being in form the persymmetric determinant $P(a_{2n-1}\,,a_{2n-2}\,,\,.\,.\,.\,,\,a_1)$, or P′ say. Any theorem regarding the minors of the former is thus also applicable to the latter; and, as every minor of the latter is replaceable by a minor of the former, the theorems (A) and (B) as applied to $P(a_{2n-1}\,,a_{2n-3}\,,\,.\,.\,.\,,\,a_1)$ must also be theorems in regard to $P(a_1\,,a_2\,,\,.\,.\,.\,,\,a_{2n-1})$. More definitely, since the r^{th} row or column from the beginning of P′ is the reverse of the $(n+1-r)^{\text{th}}$ column or row from the beginning of P, any theorem in regard to P′ will become a theorem in regard to P if we change the numbers of the rows and columns accordingly. In addition to (A) and (B), therefore, we have two theorems which we may call the secondary conjugates of these, viz.,

$$\left\{\begin{aligned} &\begin{vmatrix} n+1-l & n+1-h \\ n+1-k & n \end{vmatrix} = \begin{vmatrix} n+1-k & n+1-h \\ n+1-l & n \end{vmatrix} + \begin{vmatrix} n+1-l & n+1-k \\ n+1-h & n \end{vmatrix}, \\ &\begin{vmatrix} n+1-l & n+1-h \\ n-k & n \end{vmatrix} = \begin{vmatrix} n+1-k & n+1-h \\ n-l & n \end{vmatrix} + \begin{vmatrix} n+1-l & n+1-k \\ n-h & n \end{vmatrix}, \end{aligned}\right.$$

or

$$\left\{\begin{aligned} &\begin{vmatrix} l' & h' \\ k' & n \end{vmatrix} = \begin{vmatrix} k' & h' \\ l' & n \end{vmatrix} + \begin{vmatrix} l' & k' \\ h' & n \end{vmatrix} \qquad \text{(A}'\text{)} \\ &\begin{vmatrix} l' & h' \\ k'-1 & n \end{vmatrix} = \begin{vmatrix} k' & h' \\ l'-1 & n \end{vmatrix} + \begin{vmatrix} l' & k' \\ h'-1 & n \end{vmatrix} \qquad \text{(B}'\text{)}. \end{aligned}\right.$$

(9) Again, since any identity connecting $a_1\,,a_2\,,\,.\,.\,.\,,\,a_{2n-1}$ will remain an identity if the suffix of each a be diminished by unity; and, since in the case of a minor of $P(a_1\,,a_2\,,\,.\,.\,.\,,\,a_{2n-1})$ this unit-diminution of the suffixes can be made by unit-diminution of the numbers of the rows or columns involved, it follows that from any

relation between those minors of P which do not involve both the first row and the first column there will arise another relation which we may call the Diminutive of the former.

With this in view let us return to theorem (A), viz.,

$$\begin{vmatrix} 1 & k \\ h & l \end{vmatrix} = \begin{vmatrix} 1 & l \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & h \\ k & l \end{vmatrix}.$$

The rows of P which occur in the minor on the left are those whose numbers are

$$2\,,\,3\,,\,\ldots\,,\,k-1\,,\,k+1\,,\,\ldots\,,\,n\,,$$

to each of which unit-diminution can be applied, the result being

$$1\,,\,2\,,\,\ldots\,,\,k-2\,,\,k\,,\,\ldots\,,\,n-1$$

and the minor itself becoming

$$\begin{vmatrix} k-1 & n \\ h & l \end{vmatrix}.$$

Treating the other minors in the same way, we have for the Diminutive of (A)

$$\begin{vmatrix} k-1 & n \\ h & l \end{vmatrix} = \begin{vmatrix} l-1 & n \\ h & k \end{vmatrix} + \begin{vmatrix} h-1 & n \\ k & l \end{vmatrix}.$$

In similar fashion there is obtained

$$\begin{vmatrix} k & n \\ h & l \end{vmatrix} = \begin{vmatrix} l & n \\ h & k \end{vmatrix} + \begin{vmatrix} h & n \\ k & l \end{vmatrix}$$

as the Diminutive of (B).

By the change of rows into columns the two take the more convenient form

$$\left\{\begin{aligned} \begin{vmatrix} h & l \\ k-1 & n \end{vmatrix} &= \begin{vmatrix} h & k \\ l-1 & n \end{vmatrix} + \begin{vmatrix} k & l \\ h-1 & n \end{vmatrix} \qquad (\alpha) \\ \begin{vmatrix} h & l \\ k & n \end{vmatrix} &= \begin{vmatrix} h & l \\ k & n \end{vmatrix} + \begin{vmatrix} k & l \\ h & n \end{vmatrix} \qquad (\beta). \end{aligned}\right.$$

(10) When the six identities (A), (B), (A′), (B′), (α), (β) are collected, a glance suffices to bring out the fact that (α) is the same as (B′), and (β) the same as (A′): in other words, the Diminutive of either of the two original theorems is the Secondary Conjugate of the other.

This being the case, only one of the original theorems needs to be separately established: for if (A) be proved we can obtain at once from it its Secondary Conjugate (A′), that is (β), and from (β) by the principle of unit-increase we can derive (B). Further, as (A) may be derived (§ 4) from a self-evident identity connecting three two-line minors of an axisymmetric determinant of the fourth order by proving the applicability of the Law of Extensible Minors to axisymmetric determinants, it is seen that the six theorems can be traced back to a very humble origin.

(11) Since from (A) we have

$$\begin{vmatrix} 1 & k+1 \\ h & l \end{vmatrix} = \begin{vmatrix} 1 & l \\ h & k+1 \end{vmatrix} + \begin{vmatrix} 1 & h \\ k+1 & l \end{vmatrix},$$

and from (B)

$$\begin{vmatrix} 1 & k+1 \\ h & l \end{vmatrix} = \begin{vmatrix} 1 & l+1 \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & h+1 \\ k & l \end{vmatrix},$$

it follows that *In any persymmetric determinant the aggregate*

$$\begin{vmatrix} 1 & l \\ h & k+1 \end{vmatrix} - \begin{vmatrix} 1 & l+1 \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & h \\ k+1 & l \end{vmatrix} - \begin{vmatrix} 1 & h+1 \\ k & l \end{vmatrix}$$

vanishes, it being remembered that $l > k+1$, $k > h > 1$.

Along with this, of course, goes its 'secondary conjugate,' viz., *In any persymmetric determinant of the* n^{th} *order the aggregate*

$$\begin{vmatrix} k-1 & h \\ l & n \end{vmatrix} - \begin{vmatrix} k & h \\ l-1 & n \end{vmatrix} + \begin{vmatrix} l & k-1 \\ h & n \end{vmatrix} - \begin{vmatrix} l & k \\ h-1 & n \end{vmatrix}$$

vanishes, it being remembered that $l < k-1$, $k < h < n$.

(12) In tabulating the various instances of the theorems (A), (B), (A′), (B′) in connection with any particular determinant, caution is sometimes necessary to prevent the appearance of the same instance a second time in a different form. This is due to the fact that more than two secondary minors of a persymmetric determinant may be equal. On account of the axisymmetry, every minor that is not coaxial is equal to its conjugate, and therefore may be said to occur twice in the determinant: but this occasions no difficulty in performing the tabulation referred to if in specifying minors we agree to make the lowest-numbered line always a *row*. To avoid the difficulty which does arise when the same minor happens to be found in more than two positions the following theorem is worth noting:—

In any persymmetric determinant of the n^{th} *order the secondary minor got by deleting the* 1^{st} *and* h^{th} *rows and* k^{th} *and* n^{th} *columns is the same as the minor got by deleting the* 1^{st} *and* $(k+1)^{th}$ *rows and* $(h-1)^{th}$ *and* n^{th} *columns: that is, in symbols*

$$\begin{vmatrix} 1 & h \\ k & n \end{vmatrix} = \begin{vmatrix} 1 & k+1 \\ h-1 & n \end{vmatrix}.$$

By way of proof we note (1) that the deletion of the 1^{st} row and n^{th} column of $P(a_1, a_2, \ldots, a_{2n-1})$ gives us $P(a_2, a_3, \ldots, a_{2-n2})$: (2) that this latter determinant being axisymmetric the deletion of its $(h-1)^{th}$ row and k^{th} column produces the same effect as the deletion of its k^{th} row and $(h-1)^{th}$ column: and (3) that the $(h-1)^{th}$ row and k^{th} column here are the h^{th} row and k^{th} column of the original, and the k^{th} row and $(h-1)^{th}$ column the $(k+1)^{th}$ row and $(h-1)^{th}$ column of the original.

Obvious extensions of this, not needed for our present purpose, are

$$\begin{vmatrix} 1 & h & p \\ k & q & n \end{vmatrix} = \begin{vmatrix} 1 & k+1 & q+1 \\ h-1 & p-1 & n \end{vmatrix},$$

$$\begin{vmatrix} 1 & 2 & h \\ k & n-1 & n \end{vmatrix} = \begin{vmatrix} 1 & 2 & k+2 \\ h-2 & n-1 & n \end{vmatrix};$$

and the general identity based on the persymmetry of

$$\left|\begin{matrix} 1\,,2\,,3\,, \ldots\ldots\ldots\ldots, m \\ n-m+1\,, n-m+2\,, \ldots, n \end{matrix}\right|$$

is

$$\left|\begin{matrix} 1\,,2\,, \ldots, m\,, h_1\,, h_2\,, \ldots\ldots\ldots\ldots, h_s \\ k_1\,, k_2\,, \ldots, k_s\,, n-m+1\,, n-m+2\,, \ldots, n \end{matrix}\right|$$

$$= \left|\begin{matrix} 1\,,2\,, \ldots\ldots\ldots\ldots\ldots\ldots m\,, k_1+m\,, k_2+m\,, \ldots, k_s+m \\ h_1-m\,, h_2-m\,, \ldots, h_s-m\,, n-m+1\,, n-m+2\,, \ldots\ldots, n \end{matrix}\right| .$$

As, however, there are other ways of preserving persymmetry (§ 5, *e*) than by deleting the first m rows and the last m columns, there must be similar identities not herein included.

(13) *The secondary minors of a persymmetric determinant of the* n[th] *order which have equivalent fellow minors other than their conjugates are* $\frac{1}{2}(n-2)(n-1)$ *in number.*

If in giving values to h and k we have counted the cases where $h \ngtr k$, no fresh case will be got by taking $h > k\,, = k+a$, say ; because the identity

$$\left|\begin{matrix} 1 & k+a \\ k & n \end{matrix}\right| = \left|\begin{matrix} 1 & k+1 \\ k+a-1 & n \end{matrix}\right|$$

would already have been reached in the form

$$\left|\begin{matrix} 1 & k+1 \\ k+a-1 & n \end{matrix}\right| = \left|\begin{matrix} 1 & k+a \\ k & n \end{matrix}\right| .$$

Further, it is clear that $h>1$ and $k<n$. The possible cases are thus—

$$\begin{matrix} h=2\,, & k=2\,,\ 3\,,\ \ldots, n-1 \\ h=3\,, & k=\ 3\,,\ \ldots, n-1\,, \end{matrix}$$

so that the total number of them is

$$(n-2) + (n-3) + \cdots + 1 \qquad i.e.,\ \tfrac{1}{2}(n-2)(n-1).$$

(14) When $h=1$ the identity (B) can be applied to the first minor of its right-hand member, the said minor being previously replaced by its conjugate ; and the repetition of the operation leads finally to

$$\left|\begin{matrix} 1 & k+1 \\ 1 & l \end{matrix}\right| = \left|\begin{matrix} 1 & 2 \\ k & l \end{matrix}\right| + \left|\begin{matrix} 1 & 2 \\ k-1 & l+1 \end{matrix}\right| + \left|\begin{matrix} 1 & 2 \\ k-2 & l+2 \end{matrix}\right| + \cdots\cdot$$

as is already known.

A similar action is possible in the case of the identity (A) when $l=n$, if the last minor on the right be replaced by an equivalent fellow minor other than its conjugate in accordance with § 12. We have in fact

$$\begin{aligned}
\begin{vmatrix} 1 & k \\ h & n \end{vmatrix} &= \begin{vmatrix} 1 & n \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & h \\ k & n \end{vmatrix}, \\
&= \begin{vmatrix} 1 & n \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & k+1 \\ h-1 & n \end{vmatrix} \quad \text{by § 12}, \\
&= \begin{vmatrix} 1 & n \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & n \\ h-1 & k+1 \end{vmatrix} + \begin{vmatrix} 1 & h-1 \\ k+1 & n \end{vmatrix} \quad \text{by (A)}, \\
&= \cdots\cdots\cdots\cdots\cdots\cdots \\
&= \begin{vmatrix} 1 & n \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & n \\ h-1 & k+1 \end{vmatrix} + \begin{vmatrix} 1 & n \\ h-2 & k+2 \end{vmatrix} + \cdots\cdots
\end{aligned}$$

In connection with these results the last sentence of § 2 should not be forgotten.

(15) A second point to be observed when tabulating instances of (A), (B), (A′), (B′) is that the secondary conjugate of an identity may not be a different identity.

Taking (A) and (A′), viz.,

$$\begin{vmatrix} 1 & k \\ h & l \end{vmatrix} = \begin{vmatrix} 1 & l \\ h & k \end{vmatrix} + \begin{vmatrix} 1 & h \\ k & l \end{vmatrix},$$

$$\begin{vmatrix} l' & h' \\ k' & n \end{vmatrix} = \begin{vmatrix} k' & h' \\ l' & n \end{vmatrix} + \begin{vmatrix} l' & k' \\ h' & n \end{vmatrix},$$

we see that as there is one integer fixed in the first, viz., 1, and one integer in the second, viz., n, the two cannot have a common instance unless $l' = 1$ and $l = n$. In the next place, the secondary conjugate of

$$\begin{vmatrix} 1 & k \\ h & n \end{vmatrix} \quad \text{being} \quad \begin{vmatrix} 1 & n+1-h \\ n+1-k & n \end{vmatrix},$$

the condition that the two may be identical is clearly

$$h + k = n + 1.$$

Lastly, it is seen that if this condition be fulfilled, the minors on the right of (A) will also be their own secondary conjugates. It follows therefore that the instances common to (A) and (A′) are included in

$$\begin{vmatrix} 1 & n-m \\ 1+m & n \end{vmatrix} = \begin{vmatrix} 1 & n \\ 1+m & n-m \end{vmatrix} + \begin{vmatrix} 1 & 1+m \\ n-m & n \end{vmatrix} \qquad \text{(a)};$$

and since in the selection of m here all that is necessary is that $1+m < n-m$, *i.e.*, that

$$2m < n-1,$$

we conclude that—

In the case of an axisymmetric determinant of the $(2\mathrm{n})^{th}$ *or* $(2\mathrm{n}+1)^{th}$ *order the number of instances in which the identity* (A) *is its own secondary conjugate is* $\mathrm{n}-1$, *the form then being* (a).

A similar examination of (B) and (B′) leads to the conclusion that no instance of (B) is its own secondary conjugate.

(16) Let us now return to the fundamental theorem of § 2, the establishment of which was there made dependent on a previously known theorem for the expression of

a determinant of the n^{th} order as the sum of n determinants of the same order. Since the cofactors of a_1, a_2, b_1, b_2, P, Q manifestly cancel themselves, an alternative proof will be obtained if we can show that the expression

$$\begin{vmatrix} \cdot & \cdot & a_3 & a_4 & a_5 \\ \cdot & \cdot & b_3 & b_4 & b_5 \\ c_1 & c_2 & c_3 & c_4 & c_5 \\ d_1 & d_2 & c_4 & d_4 & d_5 \\ e_1 & e_2 & e_5 & d_5 & e_5 \end{vmatrix} - \begin{vmatrix} \cdot & \cdot & c_1 & d_1 & e_1 \\ \cdot & \cdot & b_3 & b_4 & b_5 \\ a_3 & c_2 & c_3 & c_4 & c_5 \\ a_4 & d_2 & c_4 & d_4 & d_5 \\ a_5 & e_2 & c_5 & d_5 & e_5 \end{vmatrix} - \begin{vmatrix} \cdot & \cdot & c_2 & d_2 & e_2 \\ \cdot & \cdot & b_3 & b_4 & b_5 \\ c_1 & a_3 & c_3 & c_4 & c_5 \\ d_1 & a_4 & c_4 & d_4 & d_5 \\ e_1 & a_5 & c_5 & d_5 & e_5 \end{vmatrix}$$

vanishes. This, however, leads to the consideration of the more general expression in which the complementary of the minor of zero elements is not axisymmetric, say the expression

$$\begin{vmatrix} \cdot & \cdot & B_1 & B_2 & B_3 \\ \cdot & \cdot & C_1 & C_2 & C_3 \\ A_1 & x_1 & \alpha_1 & \beta_1 & \gamma_1 \\ A_2 & x_2 & \alpha_2 & \beta_2 & \gamma_2 \\ A_3 & x_3 & \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} - \begin{vmatrix} \cdot & \cdot & A_1 & A_2 & A_3 \\ \cdot & \cdot & C_1 & C_2 & C_3 \\ B_1 & x_1 & \alpha_1 & \beta_1 & \gamma_1 \\ B_2 & x_2 & \alpha_2 & \beta_2 & \gamma_2 \\ B_3 & x_3 & \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} + \begin{vmatrix} \cdot & \cdot & A_1 & A_2 & A_3 \\ \cdot & \cdot & B_1 & B_2 & B_3 \\ C_1 & x_1 & \alpha_1 & \beta_1 & \gamma_1 \\ C_2 & x_2 & \alpha_2 & \beta_2 & \gamma_2 \\ C_3 & x_3 & \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}.$$

By LAPLACE's expansion-theorem this is clearly equal to

$$\begin{aligned} & |B_1C_2|\cdot|A_1x_2\gamma_3| - |B_1C_3|\cdot|A_1x_2\beta_3| + |B_2C_3|\cdot|A_1x_2\alpha_3| \\ - & |A_1C_2|\cdot|B_1x_2\gamma_3| + |A_1C_3|\cdot|B_1x_2\beta_3| - |A_2C_3|\cdot|B_1x_2\alpha_3| \\ + & |A_1B_2|\cdot|C_1x_2\gamma_3| - |A_1B_3|\cdot|C_1x_2\beta_3| + |A_2B_3|\cdot|C_1x_2\alpha_3|, \end{aligned}$$

and therefore by recombination, the addition being now performed on columns, so to speak,

$$\begin{aligned} &= \begin{vmatrix} A_1 & B_1 & C_1 & \cdot & \cdot \\ A_2 & B_2 & C_2 & \cdot & \cdot \\ A_1 & B_1 & C_1 & x_1 & \gamma_1 \\ A_2 & B_2 & C_2 & x_2 & \gamma_2 \\ A_3 & B_3 & C_3 & x_3 & \gamma_3 \end{vmatrix} - \begin{vmatrix} A_1 & B_1 & C_1 & \cdot & \cdot \\ A_3 & B_3 & C_3 & \cdot & \cdot \\ A_1 & B_1 & C_1 & x_1 & \beta_1 \\ A_2 & B_2 & C_2 & x_2 & \beta_2 \\ A_3 & B_3 & C_3 & x_3 & \beta_3 \end{vmatrix} + \begin{vmatrix} A_2 & B_2 & C_2 & \cdot & \cdot \\ A_3 & B_3 & C_3 & \cdot & \cdot \\ A_1 & B_1 & C_1 & x_1 & \alpha_1 \\ A_2 & B_2 & C_2 & x_2 & \alpha_2 \\ A_3 & B_3 & C_3 & x_3 & \alpha_3 \end{vmatrix}, \\ &= |A_1B_2C_3|\cdot|x_1\gamma_2| - |A_1B_2C_3|\cdot|x_1\beta_2| + |A_1B_2C_3|\cdot|x_2\alpha_3|, \\ &= |A_1B_2C_3|\cdot\{-x_1(\beta_3-\gamma_2) + x_2(\alpha_3-\gamma_1) - x_3(\alpha_2-\beta_1)\}. \end{aligned}$$

It thus appears that the aggregate set for consideration has $|A_1\ B_2\ C_3|$ for a factor, and that the cofactor—and therefore the whole aggregate—vanishes when $|\alpha_1\ \beta_2\ \gamma_3|$ is axisymmetric.

This curious proposition led to an investigation which resulted in the following series of theorems, the first of which is new only in form.

(17) *If to* n − 2 *given columns having* n *elements each there be appended the* r^{th} *column of a given determinant of the* n^{th} *order, and from the resulting array the* r^{th} *row be deleted, there being thus produced a square array of the* $(n-1)^{th}$ *order the determinant of which is* D_r, *then the aggregate*

$$\sum_{r=1}^{r=n} (-1)^{r-1} D_r$$

will vanish when the given determinant is axisymmetric.

For example, when the two given arrays are

$$\begin{matrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{matrix} \qquad\qquad \begin{matrix} \alpha_1 & \beta_1 & \gamma_1 & \delta_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2 \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 \\ \alpha_4 & \beta_4 & \gamma_4 & \delta_4 , \end{matrix}$$

the aggregate in question is

$$| x_2 y_3 \alpha_4 | - | x_1 y_3 \beta_4 | + | x_1 y_2 \gamma_4 | - | x_1 y_2 \delta_3 | .$$

To establish the general theorem it will suffice to show that in the said aggregate the cofactor of the element p_q differs only in sign from the cofactor of the element q_p, and that the element r_r does not occur. Now the latter statement is manifestly true, for the appending of the r^{th} column of the determinant to the given non-quadrate array, A say, is immediately followed by deleting from the latter the r^{th} row, so that the element r_r is thus struck out. Secondly, the term in which the element p_q occurs is $(-1)^{p-1}D_p$ where it occupies the place $(q-1, n-1)$ and has for its cofactor $(-1)^{p-1}(-1)^{q-1+n-1}$ multiplied by the determinant of the array got by deleting the p^{th} and q^{th} rows from A: similarly it is seen that the term in which the element q_p occurs is $(-1)^{q-1}D_q$ where it occupies the place $(p, n-1)$ and has for cofactor $(-1)^{q-1}(-1)^{p+n-1}$ multiplied by the same determinant: thus the two cofactors differ only in sign, as was to be proved.

If $A_{(p,q}$ denote the determinant of the array produced by deleting the p^{th} and q^{th} rows of A, we may write our result in the form

$$\sum_{r=1}^{r=n} (-1)^{r-1} D_r = \sum_{p<q}^{q=2,\ldots,n} A_{(p.q} \cdot \{p_q - q_p\} (-1)^{n+p+q-1} .$$

For example, the aggregate above instanced, viz.,

$$\begin{aligned} & | x_2 y_3 \alpha_4 | - | x_1 y_3 \beta_4 | + | x_1 y_2 \gamma_4 | - | x_1 y_2 \delta_3 | \\ = \; & | x_3 y_4 | (\alpha_2 - \beta_1) - | x_2 y_4 | (\alpha_3 - \gamma_1) + | x_2 y_3 | (\alpha_4 - \delta_1) \\ & \qquad + | x_1 y_4 | (\beta_3 - \gamma_2) - | x_1 y_3 | (\beta_4 - \delta_2) \\ & \qquad\qquad + | x_1 y_2 | (\gamma_4 - \delta_3) . \end{aligned}$$

(18) A suitable notation for such aggregates is obtained by writing the column-letters of the non-quadrate array without their row-suffixes; following this up, after a separation mark, by the principal term of the given determinant; and enclosing the whole in rectangular brackets. Thus, the particular instance just used would then be

$$[x y \,;\, \alpha_1 \beta_2 \gamma_3 \delta_4] ,$$

from which any one of its terms is got by simply taking the letters on the left along with one of the letters on the right and affixing to these three the suffixes of the remaining letters on the right. The instance immediately preceding this is

$$[x \,;\, \alpha_1 \beta_2 \gamma_3] ,$$

standing for

$$| x_2 \alpha_3 | - | x_1 \beta_3 | + | x_1 \gamma_2 | ,$$

and vanishing when $| \alpha_1 \beta_2 \gamma_3 |$ is axisymmetric.

With the help of this notation the theorem of § 17 can be enunciated in an exceedingly simple way, viz.,

$$\text{“}\begin{bmatrix} 1, 2, \ldots, n-2; n-1, n, \ldots, 2n-2 \\ n-1, n, \ldots, 2n-2 \end{bmatrix} = 0 \text{ when } \begin{vmatrix} n-1, n, \ldots, 2n-2 \\ n-1, n, \ldots, 2n-2 \end{vmatrix} \text{ is axisymmetric.”}$$

(19) *If to* n $-2-a$ *given columns of* n *elements each there be appended the* c_1^{th}, c_2^{th}, . . . , c_a^{th} *and* r^{th} *columns of a given determinant of the* n^{th} *order, and from the resulting array the* r^{th} *row be deleted, there being thus produced a square array of the* $(n-1)^{th}$ *order the determinant of which is* D_r, *then the aggregate*

$$\sum_{r=1}^{r=n} (-1)^{r-1} D_r$$

will vanish when the given determinant is axisymmetric.

By way of proof it is only necessary to note in the first place, that if we increase the $n-2-a$ columns of the non-quadrate array by repeating a of the columns which occur in the quadrate array, the former array will consist of $n-2$ columns, and the theorem of the preceding paragraph will be applicable: and in the second place, that when this application is made, the result obtained is the more general theorem just enunciated.

Of course the number of terms in the aggregate will not now be n but $n-a$, as D_r will have two identical columns, and therefore will vanish when r equals any one of the series $c_1, c_2, \ldots, c_a$.

Further, the number of identities connected with the two arrays is now not 1 by $C_{n,a}$.

(20) Although the theorem of § 18 in the form there given is so readily deduced from that of § 17, it being possible to view it as the particular case of the latter where a columns are common to the two given arrays, the state of matters is very different when we seek, as has been done at the end of § 17, to obtain the development of $\sum(-1)^{r-1}D_r$ in terms of the minors of the $(n-2-a)^{\text{th}}$ order formable from A. The theorem of § 18 is then unmistakably the more general, that of § 17 being merely the case where $a=0$. The law of the development, too, is then not by any means lying so close to the surface, and almost requires for its proper expression the use of a series of new integral functions connected with the theorem referred to in the third sentence of § 1.

(21) There has first to be recalled the well-known theorem of KRONECKER, sometimes insufficiently described as giving a linear relation between minors of an axisymmetric determinant, but which really asserts that certain aggregates of $n+1$ minors of the n^{th} order belonging to a determinant of the $(2n)^{\text{th}}$ order vanish when the latter determinant is axisymmetric. For example, when $n=3$, it asserts that

$$\begin{vmatrix} 1\,2\,3 \\ 4\,5\,6 \end{vmatrix} - \begin{vmatrix} 1\,2\,4 \\ 3\,5\,6 \end{vmatrix} + \begin{vmatrix} 1\,2\,5 \\ 3\,4\,6 \end{vmatrix} - \begin{vmatrix} 1\,2\,6 \\ 3\,4\,5 \end{vmatrix} = 0,$$

$$\begin{vmatrix} 1\,3\,2 \\ 4\,5\,6 \end{vmatrix} - \begin{vmatrix} 1\,3\,4 \\ 2\,5\,6 \end{vmatrix} + \begin{vmatrix} 1\,3\,5 \\ 2\,4\,6 \end{vmatrix} - \begin{vmatrix} 1\,3\,6 \\ 2\,4\,5 \end{vmatrix} = 0,$$

$$\cdots\cdots\cdots\cdots\cdots$$

when $\begin{vmatrix} 1\,2\,3\,4\,5\,6 \\ 1\,2\,3\,4\,5\,6 \end{vmatrix}$ is axisymmetric.* Now this theorem was extended by me in 1897, the nature of the generalisation being made clear and at the same time easily rememberable by introducing the notation $\vdots\;\;\vdots$ of § 1, for then we pass from Kronecker's to the more general theorem by simply writing $\vdots\;\;\vdots$ for $|\;\;|$. It is the aggregates of this more general theorem which we are now called on to consider, viz., such aggregates as

$$\vdots\begin{matrix} 1\,2 \\ 3\,4 \end{matrix}\vdots - \vdots\begin{matrix} 1\,3 \\ 2\,4 \end{matrix}\vdots + \vdots\begin{matrix} 1\,4 \\ 2\,3 \end{matrix}\vdots,$$

$$\vdots\begin{matrix} 1\,2\,3 \\ 4\,5\,6 \end{matrix}\vdots - \vdots\begin{matrix} 1\,2\,4 \\ 3\,5\,6 \end{matrix}\vdots + \vdots\begin{matrix} 1\,2\,5 \\ 3\,4\,6 \end{matrix}\vdots - \vdots\begin{matrix} 1\,2\,6 \\ 3\,4\,5 \end{matrix}\vdots,$$

$$\vdots\begin{matrix} 1\,2\,3\,4 \\ 5\,6\,7\,8 \end{matrix}\vdots - \vdots\begin{matrix} 1\,2\,3\,5 \\ 4\,6\,7\,8 \end{matrix}\vdots + \vdots\begin{matrix} 1\,2\,3\,6 \\ 4\,5\,7\,8 \end{matrix}\vdots - \vdots\begin{matrix} 1\,2\,3\,7 \\ 4\,5\,6\,8 \end{matrix}\vdots + \vdots\begin{matrix} 1\,2\,3\,8 \\ 4\,5\,6\,7 \end{matrix}\vdots,$$

$$\cdots\cdots\cdots\cdots\cdots$$

to which may be prefixed $\vdots\begin{matrix} 1 \\ 2 \end{matrix}\vdots - \vdots\begin{matrix} 2 \\ 1 \end{matrix}\vdots$, viz., the difference between a primary minor and its conjugate.

(22) A convenient short notation for these functions is obtained by writing the variable line-numbers immediately after the invariable, and putting a semicolon to separate the two groups: for example,

$$\left[1;\,2,3,4\right],\quad \left[1,2;\,3,4,5,6\right],\quad \left[1,2,3;\,4,5,6,7,8\right],\ \cdots$$

The following are their fundamental properties:—

(a) *The functions are alternating with respect to any two of the variable line-numbers, or any two of the invariable: and therefore vanish when two numbers of either group are identical.*

(b) *If one of the invariable line-numbers be the same as one of the variable, the function is reducible to one of the next lower order.*

For example, when, y in $\left[a, b;\; x, y, z, w\right]$ is equal to a we have by definition

* As I have already pointed out elsewhere (*Proc. Roy. Soc. Edin.*, xxiii. p. 147), it is not necessary for the truth of only one of these identities that the whole of $\begin{vmatrix} 1\,2\,3\,4\,5\,6 \\ 1\,2\,3\,4\,5\,6 \end{vmatrix}$ be axisymmetric, but merely a minor of it. For example, the first of the two here given, which manifestly may be written

$$\begin{bmatrix} 1\,2\,;\,3\,4\,5\,6 \\ 3\,4\,5\,6 \end{bmatrix},$$

holds if $\begin{vmatrix} 3\,4\,5\,6 \\ 3\,4\,5\,6 \end{vmatrix}$ be axisymmetric. This explains the insertion of the theorem of § 17, which is really Kronecker's in an amended form.

$$
\left[\, a, b;\, x, a, z, w \,\right] = \begin{vmatrix} a & b & x \\ a & z & w \end{vmatrix} - \begin{vmatrix} a & b & a \\ x & z & w \end{vmatrix} + \begin{vmatrix} a & b & z \\ x & a & w \end{vmatrix} - \begin{vmatrix} a & b & w \\ x & a & z \end{vmatrix},
$$

$$
= \begin{vmatrix} a & b & x \\ a & z & w \end{vmatrix} - 0 - \begin{vmatrix} a & b & z \\ a & x & w \end{vmatrix} + \begin{vmatrix} a & b & w \\ a & x & z \end{vmatrix},
$$

which is simply

$$
\begin{vmatrix} b & x \\ z & w \end{vmatrix} - \begin{vmatrix} b & z \\ x & w \end{vmatrix} + \begin{vmatrix} b & w \\ x & z \end{vmatrix},
$$

or

$$
\left[\, b;\, x, z, w \,\right],
$$

in regard to a determinant of the next lower order.

(23) With these auxiliaries let us now return to the development of $\sum(-1^{r-1}D_r$ in § 18, beginning with the instance where the two arrays are

$$
\begin{matrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_5 & y_5 \\ x_6 & y_6 \end{matrix} \qquad \begin{matrix} \alpha_1 & \beta_1 & \gamma_1 & \delta_1 & \epsilon_1 & \zeta_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2 & \epsilon_2 & \zeta_2 \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 & \epsilon_3 & \zeta_3 \\ \alpha_4 & \beta_4 & \gamma_4 & \delta_4 & \epsilon_4 & \zeta_4 \\ \alpha_5 & \beta_5 & \gamma_5 & \delta_5 & \epsilon_5 & \zeta_5 \\ \alpha_6 & \beta_6 & \gamma_6 & \delta_6 & \epsilon_6 & \zeta_6 , \end{matrix}
$$

and where therefore the first aggregate is

$$
|\, x_1 y_2 \alpha_4 \beta_5 \gamma_6 \,| - |\, x_1 y_2 \alpha_3 \beta_5 \delta_6 \,| + |\, x_1 y_2 \alpha_3 \beta_4 \epsilon_6 \,| - |\, x_1 y_2 \alpha_3 \beta_4 \zeta_5 \,| .
$$

Expanding each of the four determinants here in terms of minors of the first two columns and the complementaries of these minors, we obtain

$$
\begin{aligned}
& |\, x_1 y_2 \,| \cdot \{ |\, \alpha_4 \beta_5 \gamma_6 \,| - |\, \alpha_3 \beta_5 \delta_6 \,| + |\, \alpha_3 \beta_4 \epsilon_6 \,| - |\, \alpha_3 \beta_4 \zeta_5 \,| \} \\
+ & |\, x_1 y_3 \,| \cdot \{ |\, \alpha_2 \beta_5 \delta_6 \,| - |\, \alpha_2 \beta_4 \epsilon_6 \,| + |\, \alpha_2 \beta_4 \zeta_5 \,| \} \\
+ & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\
+ & |\, x_5 y_6 \,| \cdot \{ |\, \alpha_1 \beta_2 \gamma_4 \,| - |\, \alpha_1 \beta_2 \delta_3 \,| \}.
\end{aligned}
$$

The cofactor of $|\, x_1\, y_2 \,|$ may also be written in the form

$$
\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} - \begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{vmatrix} - \begin{vmatrix} 1 & 2 & 6 \\ 3 & 4 & 5 \end{vmatrix},
$$

and therefore is equal to

$$
\left[\, 1, 2;\, 3, 4, 5, 6 \,\right].
$$

Similarly we have

$$
\text{cof} \begin{vmatrix} x_1\, y_3 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 4 \\ 3 & 5 & 6 \end{vmatrix} - \begin{vmatrix} 1 & 3 & 5 \\ 3 & 4 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 3 & 6 \\ 3 & 4 & 5 \end{vmatrix},
$$

$$
= - \left[\, 1, 3;\, 3, 4, 5, 6 \,\right];
$$

and finally

$$
\text{cof} \begin{vmatrix} x_5\, y_6 \end{vmatrix} = \begin{vmatrix} 3 & 5 & 6 \\ 4 & 5 & 6 \end{vmatrix} - \begin{vmatrix} 4 & 5 & 6 \\ 3 & 5 & 6 \end{vmatrix},
$$

$$
= \left[\, 5, 6;\, 3, 4, 5, 6 \,\right].
$$

We are thus brought to the following interesting identity :—

$$\left| x_1 y_2 \alpha_4 \beta_5 \gamma_6 \right| - \left| x_1 y_2 \alpha_3 \beta_5 \delta_6 \right| + \left| x_1 y_2 \alpha_3 \beta_4 \epsilon_6 \right| - \left| x_1 y_2 \alpha_3 \beta_4 \zeta_5 \right|$$

$$= \sum_{p<q}^{q=2,\ldots,6} \left| x_p y_q \right| \cdot \left[p, q ; 3, 4, 5, 6 \right] (-1)^{p+q-1}.$$

In this, by reason of the mere presence of a KRONECKER-MUIR* aggregate, $\left[p, q : 3, 4, 5, 6 \right]$, as a factor in each term on the right, is included the fact that the left-hand member vanishes when $\left| \alpha_1 \beta_2 \gamma_3 \delta_4 \epsilon_5 \zeta_6 \right|$ is axisymmetric.

(24) Had the square array in the preceding paragraph been of the 5th order we should have had

$$\left| x_1 y_3 \alpha_4 \beta_5 \right| - \left| x_1 y_2 \alpha_4 \gamma_5 \right| + \left| x_1 y_2 \alpha_3 \delta_5 \right| - \left| x_1 y_2 \alpha_3 \epsilon_4 \right|$$

$$= \sum_{p<q}^{q=2,\ldots,5} \left| x_p y_q \right| \cdot \left[p, q ; 2, 3, 4, 5 \right] (-1)^{p+q-1} ;$$

but then the restrictions in the values of p and q being such that at least one of the two must be the same as one of the variable group $2, 3, 4, 5$, the aggregate $\left[p, q ; 2, 3, 4, 5 \right]$ reduces to one of the next lower order.

Similarly when the square array is of the 4th order we have

$$\left| x_2 y_3 \alpha_4 \right| - \left| x_1 y_3 \beta_4 \right| + \left| x_1 y_2 \gamma_4 \right| - \left| x_1 y_2 \delta_3 \right|$$

$$= \sum_{p<q}^{q=2,3,4} \left| x_p y_q \right| \cdot \left[p, q ; 1, 2, 3, 4 \right] (-1)^{p+q-1} ;$$

but as in every case p is one of the integers $1, 2, 3, 4$, and q another, the aggregate is reducible in its order to the extent of *two* steps, thus becoming simply the difference between a minor and its conjugate, as we have already seen (end of § 17).

(25) Taking next an instance where the non-quadrate array has three columns, the square array being of the 8th order say, we may show exactly as in § 21 that

$$\left| x_1 y_2 z_3 \alpha_5 \beta_6 \gamma_7 \delta_8 \right| - \left| x_1 y_2 z_3 \alpha_4 \beta_6 \gamma_7 \epsilon_8 \right| + \left| x_1 y_2 z_3 \alpha_4 \beta_5 \gamma_7 \zeta_8 \right|$$

$$- \left| x_1 y_2 z_3 \alpha_4 \beta_5 \gamma_6 \eta_8 \right| + \left| x_1 y_2 z_3 \alpha_4 \beta_5 \gamma_6 \theta_7 \right|$$

$$= \sum_{p<q<r}^{r=3,\ldots,8} \left| x_p y_q z_r \right| \cdot \left[p, q, r ; 4, 5, 6, 7, 8 \right] (-1)^{p+q+r},$$

* So called perforce, as the less general expressions *e.g.* $\begin{bmatrix} 1 \; 2 \; ; \; 3\,4\,5\,6 \\ 3\,4\,5\,6 \end{bmatrix}$ first brought to light by KRONECKER already bear his name.

where the members of the aggregate $\overline{\underline{p , q , r ; 4 , 5 , 6 , 7 , 8}}$ are minors of the determinant $| \alpha_1 \beta_2 \gamma_3 \delta_4 \epsilon_5 \zeta_6 \eta_7 \theta_8 |$, and therefore all vanish when the latter is axisymmetric. The general form of the development of the aggregate of § 18 is thus apparent.

(26) The case where the quadrate array is of the $(2m)^{\text{th}}$ order and the non-quadrate array consists of $m-1$ columns is specially interesting. We have then to increase the latter array by $m-1$ columns repeated from the square array in order to make § 17 applicable, and the theorem takes the form of an extensional of KRONECKER'S *without any apparent corresponding extension of the condition of axisymmetry.* Thus KRONECKER'S four-line identity is

$$0 = | \alpha_5 \beta_6 \gamma_7 \delta_8 | - | \alpha_4 \beta_6 \gamma_7 \epsilon_8 | + | \alpha_4 \beta_5 \gamma_7 \zeta_8 | - | \alpha_4 \beta_5 \gamma_6 \eta_8 | + | \alpha_4 \beta_5 \gamma_6 \theta_7 |$$

with the condition that $| \alpha_1 \beta_2 \ldots \theta_8 |$ be axisymmetric, and, as the preceding paragraph shows, we can extend each minor to the 7^{th} order by prefixing $x_1 y_2 z_3$ without any condition in regard to the 24 introduced elements

$$\begin{array}{l} x_1 , x_2 , \ldots , x_8 \\ y_1 , y_2 , \ldots , y_8 \\ z_1 , z_2 , \ldots , z_8 . \end{array}$$

Equating cofactors of x_1 in the two sides of this extended identity we have a similar identity, and equating cofactors of y_2 in the two sides of the result thus derived we have a third.

If we use a different notation for KRONECKER'S aggregates, denoting the aggregate

$$\begin{vmatrix} 1\,2\,3 \\ 4\,5\,6 \end{vmatrix} - \begin{vmatrix} 1\,2\,4 \\ 3\,5\,6 \end{vmatrix} + \begin{vmatrix} 1\,2\,5 \\ 3\,4\,6 \end{vmatrix} - \begin{vmatrix} 1\,2\,6 \\ 3\,4\,5 \end{vmatrix}$$

say, by

$$\sum \begin{vmatrix} 1\,2\,\overline{3} \\ \underline{4\,5\,6} \end{vmatrix}$$

where underlining and overlining indicate variability, the matter may be put more satisfactorily. The proposition then is that besides the vanishing of $\sum \begin{vmatrix} 1\,2\,\overline{3} \\ \underline{4\,5\,6} \end{vmatrix}$ when the determinant $\begin{vmatrix} 1\,2\,3\,4\,5\,6 \\ 1\,2\,3\,4\,5\,6 \end{vmatrix}$ is axisymmetric, we also have

$$\left\{\begin{array}{l} \sum \begin{vmatrix} x\,1\,2\,\overline{3} \\ 1\,\underline{4\,5\,6} \end{vmatrix} = 0 , \\ \sum \begin{vmatrix} x\,1\,2\,\overline{3} \\ 2\,\underline{4\,5\,6} \end{vmatrix} = 0 , \end{array}\right.$$

$$\sum \begin{vmatrix} x\,y\,1\,2\,\overline{3} \\ 1\,2\,\underline{4\,5\,6} \end{vmatrix} = 0 ,$$

apart altogether from any condition in regard to the elements

$$\begin{array}{l} x_1 \;\; x_2 \;\; \ldots , x_6 \\ y_1 , y_2 \;\; \ldots , y_6 . \end{array}$$

The general theorem involved may without symbols be formally enunciated as follows :—

A vanishing Kronecker aggregate of m*-line minors may have each minor extended to any order lower than the* (2m)th *without impairing the identity, provided the introduced column-numbers be included among the invariable row-numbers previously existing.*

(27) The need for the two restrictions, (1) that the order of the extended minors be lower than the $(2m)^{\text{th}}$, and (2) that the introduced column-numbers be not different from numbers found among the original set of invariable row-numbers, will be tolerably apparent from the consideration of one special case, say the case of $\Sigma\begin{vmatrix} x\,y\,1\,2\,\bar{3} \\ 1\,2\,\underline{4\,5\,6} \end{vmatrix}$.

Here if we put α, β in place of the column numbers 1, 2 we have a more general aggregate for investigation. By expansion in terms of minors of the first two rows and the complementaries of these there is obtained

$$\begin{aligned}\Sigma\begin{vmatrix} x\,y\,1\,2\,\bar{3} \\ \alpha\,\beta\,\underline{4\,5\,6} \end{vmatrix} = {}& \begin{vmatrix} x\,y \\ \alpha\,\beta \end{vmatrix}\cdot\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \underline{4\,5\,6} \end{vmatrix} - \begin{vmatrix} x\,y \\ \alpha\,4 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \beta\,\underline{5\,6} \end{vmatrix} + \begin{vmatrix} x\,y \\ \alpha\,5 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \beta\,\underline{4\,6} \end{vmatrix} - \begin{vmatrix} x\,y \\ \alpha\,6 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \beta\,\underline{4\,5} \end{vmatrix} \\ & + \begin{vmatrix} x\,y \\ \beta\,4 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \alpha\,\underline{5\,6} \end{vmatrix} - \begin{vmatrix} x\,y \\ \beta\,5 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \alpha\,\underline{4\,6} \end{vmatrix} + \begin{vmatrix} x\,y \\ \beta\,6 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \alpha\,\underline{4\,5} \end{vmatrix} \\ & + \begin{vmatrix} x\,y \\ 4\,5 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \alpha\,\beta\,\underline{6} \end{vmatrix} - \begin{vmatrix} x\,y \\ 4\,6 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \alpha\,\beta\,\underline{5} \end{vmatrix} \\ & + \begin{vmatrix} x\,y \\ 5\,6 \end{vmatrix}\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \alpha\,\beta\,\underline{4} \end{vmatrix}.\end{aligned}$$

Now in the first line of this development $\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \underline{4\,5\,6} \end{vmatrix}$ vanishes, but the similar factors of the other terms do not unless $\beta = 1$ or 2 : in the second line arises the like condition $\alpha = 1$ or 2 ; and in the remaining lines both conditions at once. Our only possible result of the 5$^{\text{th}}$ order is thus

$$\Sigma\begin{vmatrix} x\,y\,1\,2\,\bar{3} \\ 1\,2\,\underline{4\,5\,6} \end{vmatrix} = 0.$$

(28) As one might expect from a remembrance of the opening lines of § 20, the theorem may also be looked upon as a special case of KRONECKER'S. Thus from KRONECKER as amended we have

$$\Sigma\begin{vmatrix} x\,y\,1\,2\,\bar{3} \\ \underline{7\,8\,4\,5\,6} \end{vmatrix} = 0$$

provided that $\begin{vmatrix} 3\,7\,8\,4\,5\,6 \\ 3\,7\,8\,4\,5\,6 \end{vmatrix}$ be axisymmetric ; and the change of 7 into 1 and 8 into 2 gives us

$$\Sigma\begin{vmatrix} x\,y\,1\,2\,\bar{3} \\ \underline{1\,2\,4\,5\,6} \end{vmatrix} \quad i.e. \quad \Sigma\begin{vmatrix} x\,y\,1\,2\,\bar{3} \\ 1\,2\,\underline{4\,5\,6} \end{vmatrix} = 0$$

when $\begin{vmatrix} 3\,1\,2\,4\,5\,6 \\ 3\,1\,2\,4\,5\,6 \end{vmatrix}$ is axisymmetric. As the condition for the vanishing of $\Sigma\begin{vmatrix} 1\,2\,\bar{3} \\ \underline{4\,5\,6} \end{vmatrix}$ is merely that $\begin{vmatrix} 3\,4\,5\,6 \\ 3\,4\,5\,6 \end{vmatrix}$ be axisymmetric, we thus see that the extension referred to cannot be made without a simultaneous extension of the condition of axisymmetry. This accounts for the use of the word "apparent" in the italicised clause near the beginning of § 26.

For the same reason the notation for Kronecker aggregates given in § 18 may be used to express the theorems of §§ 23, 24, 25. For example the first of these may be written—

$$\begin{aligned}[\, x\, y\, \alpha\, \beta;\ \alpha_1 \beta_2 \gamma_3 \delta_4 \epsilon_5 \zeta_6 \,] = \ & |\, x_1 y_2 \,| \cdot [\, \alpha \beta;\ \gamma_3 \delta_4 \epsilon_5 \zeta_6 \,] - |\, x_1 y_3 \,| \cdot [\, \alpha \beta;\ \beta_2 \delta_4 \epsilon_5 \zeta_6 \,] \\ & + \ \ldots\ldots\ldots \ + |\, x_5 y_6 \,| \cdot [\, \alpha \beta;\ \alpha_1 \beta_2 \gamma_3 \delta_4 \,],\end{aligned}$$

where the occurrence of one or more letters on both sides of the semicolon in [ ] results in the vanishing of one or more terms of the aggregate so denoted.

(29) We can now return better equipped to consider the theorem of § 16, from which we were led aside by the need for investigating certain auxiliary results. And, first of all, the said theorem can be generalised as follows :—

If two determinants of the n^{th} *order* δ, δ' *and an array* X *consisting of* $n-2$ *columns of* n *elements each be taken, and a new determinant of the* $(2n-1)^{th}$ *order,* Δ_r, *be formed with zero elements in the places of its first coaxial minor of the* $(n-1)^{th}$ *order, with the conjugate of* δ' *for the complementary minor, with the array* X *and the* r^{th} *row of* δ *in the as yet unappropriated left-hand bottom space, and with the remaining rows of* δ *in the conjugate space,—then*

$$\sum_{r=1}^{r=n} \Delta_r (-1)^{r-1}$$

is equal to δ *multiplied by an expression which vanishes when* δ' *is axisymmetric.*

The mode of proof employed in § 16 applies generally and need not be rehearsed. As for the vanishing cofactor of δ it is nothing more than a Kronecker aggregate in the form which appears in § 17.

(30) If we take one of the minors formed from X by the deletion of two rows, and equate its cofactor on the one side of the identity of § 27 to its cofactor on the other we have the following theorem :—

If a determinant δ *of the* n^{th} *order and an array* γ *consisting of* n *rows and two columns be given, and from this a determinant* Δ_r *of the* $(n+1)^{th}$ *order be formed by taking all the rows of* δ *except the* r^{th}, *placing under them as rows the two columns of* γ, *and prefixing a column consisting of* $n-1$ *zeros and the two last elements of the* r^{th} *row of* δ,*—then*

$$\sum_{r=1}^{r=n} \Delta_r (-1)^{r-1}$$

is equal to δ *multiplied by the difference between the* $(n-1)^{th}$ *element of the* 2^{nd} *column of* γ *and the* n^{th} *element of the first column.*

Thus, in the case of $n=3$ we have

$$\begin{vmatrix} \cdot & B_1 & B_2 & B_3 \\ \cdot & C_1 & C_2 & C_3 \\ A_2 & \alpha_2 & \beta_2 & \gamma_2 \\ A_3 & \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} - \begin{vmatrix} \cdot & A_1 & A_2 & A_3 \\ \cdot & C_1 & C_2 & C_3 \\ B_2 & \alpha_2 & \beta_2 & \gamma_2 \\ B_3 & \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} + \begin{vmatrix} \cdot & A_1 & A_2 & A_3 \\ \cdot & B_1 & B_2 & B_3 \\ C_2 & \alpha_2 & \beta_2 & \gamma_2 \\ C_3 & \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} = |A_1 B_2 C_3| (\beta_3 - \gamma_2).$$

(31) A still more general theorem than that of § 27 is as follows :—

If two arrays B, B′ *be taken,* B *containing* p + 1 *rows and* p + q − r *columns and* B′ *containing* p + q − 1 *rows and* q *columns, and a new determinant,* Δ_ρ, *of the* $(p+q)^{th}$ *order be formed having for its first* p *rows all the rows of* B *except the* ρ^{th} *each preceded by* r *zeros, and for the first column of the remaining space the last* q *elements of the* ρ^{th} *row of* B, *and for the other columns the* p + q − 1 *rows of* B′,*—then*

$$\sum_{\rho=1}^{\rho=p+1} \Delta_\rho(-1)^{\rho-1}$$

vanishes if the last coaxial minor of the q^{th} *order in* Δ *be axisymmetric.*

Thus, when

$$B \equiv \begin{Bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \\ B_1 & B_2 & B_3 & B_4 & B_5 \\ C_1 & C_2 & C_3 & C_4 & C_5 \\ D_1 & D_2 & D_3 & D_4 & D_5 \end{Bmatrix}$$

$$B' \equiv \begin{Bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ \delta_1 & \delta_2 & \delta_3 & \delta_4 \\ \epsilon_1 & \epsilon_2 & \epsilon_3 & \epsilon_4 \\ \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 \end{Bmatrix}$$

—that is to say, when $p=3$, $q=4$, $r=2$,—the four-termed aggregate

$$\sum \begin{vmatrix} \cdot & \cdot & B_1 & B_2 & B_3 & B_4 & B_5 \\ \cdot & \cdot & C_1 & C_2 & C_3 & C_4 & C_5 \\ \cdot & \cdot & D_1 & D_2 & D_3 & D_4 & D_5 \\ A_2 & \alpha_1 & \beta_1 & \gamma_1 & \delta_1 & \epsilon_1 & \zeta_1 \\ A_3 & \alpha_2 & \beta_2 & \gamma_2 & \delta_2 & \epsilon_2 & \zeta_2 \\ A_4 & \alpha_3 & \beta_3 & \gamma_3 & \delta_3 & \epsilon_3 & \zeta_3 \\ A_5 & \alpha_4 & \beta_4 & \gamma_4 & \delta_4 & \epsilon_4 & \zeta_4 \end{vmatrix}$$

vanishes when $|\gamma_1 \delta_2 \epsilon_3 \zeta_4|$ is axisymmetric.

Developing each of the seven-line determinants of the aggregate in terms of minors formed from the first three rows and minors formed from the last four rows, we obtain

$$\sum |B_1 C_2 D_3| \cdot \begin{vmatrix} A_2 & \alpha_1 & \epsilon_1 & \zeta_1 \\ A_3 & \alpha_2 & \epsilon_2 & \zeta_2 \\ A_4 & \alpha_3 & \epsilon_3 & \zeta_3 \\ A_5 & \alpha_4 & \epsilon_4 & \zeta_4 \end{vmatrix} - \sum |B_1 C_2 D_4| \cdot \begin{vmatrix} A_2 & \alpha_1 & \delta_1 & \zeta_1 \\ A_3 & \alpha_2 & \delta_2 & \zeta_2 \\ A_4 & \alpha_3 & \delta_3 & \zeta_3 \\ A_5 & \alpha_4 & \delta_4 & \zeta_4 \end{vmatrix} + \cdots$$

Of this expansion the first term is, clearly,

$$= - \begin{vmatrix} A_1 & B_1 & C_1 & D_1 & \cdot & \cdot & \cdot \\ A_2 & B_2 & C_2 & D_2 & \cdot & \cdot & \cdot \\ A_3 & B_3 & C_3 & D_3 & \cdot & \cdot & \cdot \\ A_2 & B_2 & C_2 & D_2 & \alpha_1 & \epsilon_1 & \zeta_1 \\ A_3 & B_3 & C_3 & D_3 & \alpha_2 & \epsilon_2 & \zeta_2 \\ A_4 & B_4 & C_4 & D_4 & \alpha_3 & \epsilon_3 & \zeta_3 \\ A_5 & B_5 & C_5 & D_5 & \alpha_4 & \epsilon_4 & \zeta_4 \end{vmatrix} = - \begin{vmatrix} A_1 & B_1 & C_1 & D_1 & & & \\ A_2 & B_2 & C_2 & D_2 & & & \\ A_3 & B_3 & C_3 & D_3 & & & \\ \cdot & \cdot & \cdot & \cdot & \alpha_1 & \epsilon_1 & \zeta_1 \\ \cdot & \cdot & \cdot & \cdot & \alpha_2 & \epsilon_2 & \zeta_2 \\ A_4 & B_4 & C_4 & D_4 & \alpha_3 & \epsilon_3 & \zeta_3 \\ A_5 & B_5 & C_5 & D_5 & \alpha_4 & \epsilon_4 & \zeta_4 \end{vmatrix}$$

$$= - | A_1 B_2 C_3 D_4 | \cdot | \alpha_1 \epsilon_2 \zeta_4 | + | A_1 B_2 C_3 D_5 | \cdot | \alpha_1 \epsilon_2 \zeta_3 | ;$$

the second

$$= | A_1 B_2 C_3 D_2 | \cdot | \alpha_1 \delta_3 \zeta_4 | - | A_1 B_2 C_4 D_5 | \cdot | \alpha_1 \delta_2 \zeta_3 | ;$$

and similarly for the eight others. If the series of products thus obtained be collected, the result will be found to be

$$\begin{array}{l} \ | A_2 B_3 C_4 D_5 | \cdot \{ | \alpha_2 \beta_3 \gamma_4 | - | \alpha_1 \beta_3 \delta_4 | + | \alpha_1 \beta_2 \epsilon_4 | - | \alpha_1 \beta_2 \zeta_3 | \} \\ - | A_1 B_3 C_4 D_5 | \cdot \{ \qquad\qquad - | \alpha_1 \gamma_3 \delta_4 | + | \alpha_1 \gamma_2 \epsilon_4 | - | \alpha_1 \gamma_2 \zeta_3 | \} \\ + | A_1 B_2 C_4 D_5 | \cdot \{ | \alpha_2 \delta_3 \gamma_4 | \qquad\qquad + | \alpha_1 \delta_2 \epsilon_4 | - | \alpha_1 \delta_2 \zeta_3 | \} \\ - | A_1 B_2 C_3 D_5 | \cdot \{ | \alpha_2 \epsilon_3 \gamma_4 | - | \alpha_1 \epsilon_3 \delta_4 | \qquad\qquad - | \alpha_1 \epsilon_2 \zeta_3 | \} \\ + | A_1 B_2 C_3 D_4 | \cdot \{ | \alpha_2 \zeta_3 \gamma_4 | - | \alpha_1 \zeta_3 \delta_4 | + | \alpha_1 \zeta_2 \epsilon_4 | \qquad\qquad \} , \end{array}$$

which, in the notation of § 18, is readily seen to be

$$\begin{array}{l} \ | A_2 B_3 C_4 D_5 | \cdot [\alpha \beta ; \gamma_1 \delta_2 \epsilon_3 \zeta_4] - | A_1 B_3 C_4 D_5 | \cdot [\alpha \gamma ; \gamma_1 \delta_2 \epsilon_3 \zeta_4] \\ + | A_1 B_2 C_4 D_5 | \cdot [\alpha \delta ; \gamma_1 \delta_2 \epsilon_3 \zeta_4] - | A_1 B_2 C_3 D_5 | \cdot [\alpha \epsilon ; \gamma_1 \delta_2 \epsilon_3 \zeta_4] \\ + \ A_1 B_2 C_3 D_4 | \cdot [\alpha \zeta ; \gamma_1 \delta_2 \epsilon_3 \zeta_4] , \end{array}$$

where every one of the second factors is a KRONECKER aggregate which vanishes when $| \gamma_1 \delta_2 \epsilon_3 \zeta_4 |$ is axisymmetric. The object aimed at is thus attained.

(32) We may even yet generalise further by substituting for "the last q elements" in the enunciation of § 29 the words "any q elements"; but then the minor whose axisymmetry is necessary is not the last coaxial minor but that minor of the last q rows of Δ whose elements are in the same columns with the elements of B corresponding to the particular q elements chosen at the outset. Thus

$$\sum \begin{vmatrix} \cdot & B_1 & B_2 & B_3 & B_4 & B_5 \\ \cdot & C_1 & C_2 & C_3 & C_4 & C_5 \\ \cdot & D_1 & D_2 & D_3 & D_4 & D_5 \\ A_2 & \alpha_2 & \beta_2 & \gamma_2 & \delta_2 & \epsilon_2 \\ A_3 & \alpha_3 & \beta_3 & \gamma_3 & \delta_3 & \epsilon_3 \\ A_4 & \alpha_4 & \beta_4 & \gamma_4 & \delta_4 & \epsilon_4 \end{vmatrix} \begin{array}{l} = - | A_2 B_3 C_4 D_5 | \cdot [\alpha ; \beta_2 \gamma_3 \delta_4] \\ \ \ + | A_1 B_3 C_4 D_5 | \cdot [\beta ; \beta_2 \gamma_3 \delta_4] \\ \ \ - | A_1 B_2 C_4 D_5 | \cdot [\gamma ; \beta_2 \gamma_3 \delta_4] \\ \ \ + | A_1 B_2 C_3 D_5 | \cdot [\delta ; \beta_2 \gamma_3 \delta_4] \\ \ \ - | A_1 B_2 C_3 D_4 | \cdot [\epsilon ; \beta_2 \gamma_3 \delta_4] , \end{array}$$

and therefore vanishes when $| \beta_2 \gamma_3 \delta_4 |$ is axisymmetric.

Further, by equating in this the cofactor of ϵ_4 on the left with the cofactor on the right we have

$$\sum \begin{vmatrix} \cdot & B_1 & B_2 & B_3 & B_4 \\ \cdot & C_1 & C_2 & C_3 & C_4 \\ \cdot & D_1 & D_2 & D_3 & D_4 \\ A_2 & a_2 & \beta_2 & \gamma_2 & \delta_2 \\ A_3 & a_3 & \beta_3 & \gamma_3 & \delta_3 \end{vmatrix} = |\, A_1 B_2 C_3 D_4 \,| \cdot (\beta_3 - \gamma_2),$$

which vanishes when $|\, \beta_2 \gamma_3 \,|$ is axisymmetric.

(33) When $p = r$ the array of zeros in Δ is square, and therefore also the complementary array. Thus,

$$\sum \begin{vmatrix} \cdot & \cdot & B_1 & B_2 & B_3 & B_4 \\ \cdot & \cdot & C_1 & C_2 & C_3 & C_4 \\ A_1 & x_1 & a_1 & \beta_1 & \gamma_1 & \delta_1 \\ A_2 & x_2 & a_2 & \beta_2 & \gamma_2 & \delta_2 \\ A_3 & x_3 & a_3 & \beta_3 & \gamma_3 & \delta_3 \\ A_4 & x_4 & a_4 & \beta_4 & \gamma_4 & \delta_4 \end{vmatrix} = \begin{array}{l} \quad |\, A_2 B_3 C_4 \,| \cdot [\, x\, a\, ;\ a_1 \beta_2 \gamma_3 \delta_4 \,] \\ - |\, A_1 B_3 C_4 \,| \cdot [\, x\, \beta\, ;\ a_1 \beta_2 \gamma_3 \delta_4 \,] \\ + |\, A_1 B_2 C_4 \,| \cdot [\, x\, \gamma\, ;\ a_1 \beta_2 \gamma_3 \delta_4 \,] \\ - |\, A_1 B_2 C_3 \,| \cdot [\, x\, \delta\, ;\ a_1 \beta_2 \gamma_3 \delta_4 \,] \end{array}$$

which vanishes when $|\, a_1 \beta_2 \gamma_3 \delta_4 \,|$ is axisymmetric.

When $p = n - 1$, $q = n$, $r = n - 1$ the theorem degenerates into that of § 27.*

* The proof desired at the end of § 10 will be found in a paper sent to the Edinburgh Math. Soc. on 29th Jan. 1902, and at present being printed as part of Vol. xx. of the *Proceedings* of that Society, under the title "The applicability of the Law of Extensible Minors to determinants of special form."

(*Issued separately September* 26, 1902.)

XXIII.—*Change of Electric Resistance of Nickel due to Magnetisation at Different Temperatures.* By Professor C. G. Knott, D.Sc.

(Read 16th June and 19th July 1902. Issued separately December 31, 1902.)

Since Lord Kelvin showed that the resistance of the magnetic metals was influenced by the state of magnetisation of these substances, a good deal of work has been done in investigating more carefully the extent and nature of the relation. It does not appear, however, that any attempt has been made to discover how change of temperature affects this influence. Twelve years ago in Japan I set one of my Japanese students to the construction of an apparatus suitable for making this research. A few preliminary experiments were made; and leaving Japan shortly after, I brought the apparatus with me in the hope of continuing the work in this country. Not till this summer, however, did I find leisure to return to the problem. The results obtained I wish now to lay before the Society.

The apparatus was constructed with a view to compensate as far as possible for the inevitable changes of temperature accompanying the application of magnetising forces by means of currents through coils. Two nickel wires of exactly the same length and resistance were made into two similar flat coils which then serve as anchor ring cores for the magnetising coils to be wound round. The magnetising coil round each nickel was wound in two distinct portions with exactly the same number of turns. When these two portions were joined together end to end so that the current flowed in both in the same direction round the nickel core the nickel was of course magnetised. But when they were connected so that the current in one flowed round the nickel in a direction opposite to that in which the current flowed in the other, then the nickel was subjected to no magnetising force at all. Thus by simply reversing the current in one portion we could at once pass from strong magnetising force to none, and yet keep the heating effect of the current practically unaltered. In the experiments, as conducted, the two nickel wires formed neighbouring arms of a Wheatstone Bridge, and the same current was passed round both, but generally in such a manner that only one was magnetised. When the current was passed so as to magnetise neither, there was observed no abrupt change of resistance at make and break of the current circulating in the copper coils. This showed that the magnetic compensation when the two portions of the coils acted in different directions was perfect.

The nickel wires forming the cores of the anchor rings I shall distinguish by the letters M and N. Their ends were attached to the ends of stout nickel rods, three in all, one forming the junction where M and N met. These rods were lashed to a central brass pillar covered with asbestos cloth, so that all were carefully insulated one from the other; and this whole compact arrangement could be lifted in and out of the air- or oil-bath in which the nickel wires were raised to different temperatures. The two anchor rings lay close together, and the four terminals of the pairs of coils were led up the side of the pillar and connected to commutators so that either, neither, or both nickels could be magnetised or not by the passing current as occasion required. From the upper ends of the nickel rods nickel wires were led to form the other arms of the Wheatstone Bridge and give the necessary connections to the galvanometer and to the single cell used for testing the resistance. It was advisable to make the connections with nickel wire so as to prevent as far as possible thermoelectric currents.

The resistances which formed the other arms of the Wheatstone Bridge will be distinguished as m and n; and the balance of the galvanometer was produced when $\mathrm{M}n = \mathrm{N}m$.

The battery branch joined the junction of M and N to the junction of m and n; and the galvanometer branch passed from the junction $\mathrm{M}m$ to the junction $\mathrm{N}n$. A small part of m was measured off and connected in multiple arc, when necessary, with a variable resistance R supplied by a resistance box of the dial construction and capable of giving resistances from 1 to 10,000 ohms. By this means the resistance of m could be rapidly adjusted within certain limits to any required value. I shall call the part of m so used as a shunt with R the resistance x.

At the beginning of each day's experiment, and occasionally throughout the experiment, the galvanometer readings were calibrated by measuring the deflection produced by a slight change in the resistance m. Suppose, for example, that the deflection produced on reversal of the galvanometer circuit was a when the shunt resistance was R, and that it became a' when the shunt was R′, then the change of resistance corresponding to the change of deflection $a - a'$ is easily shown to be measured by the quantity $x^2\ ((x+\mathrm{R})^{-1} - (x+\mathrm{R}')^{-1})$. Since the cell used was a secondary battery of very small resistance acting through a resistance of 10 ohms, we could safely assume the electromotive force to remain constant for one day's experiment. Indeed, day after day with the adjusting magnet of the galvanometer in the same position the calibration experiments gave the same results to within a half per cent.

In the final series of experiments, the coils were immersed in olive oil, which was kept at a constant temperature by being surrounded by a second vessel filled to a sufficient height with water, whose temperature was kept steady by a suitably adjusted Bunsen flame. In this way very satisfactory observations were obtained at temperatures 12°, 57°, and 93° C. The attempt to reach higher temperatures in this series of experiments failed, for it was found that the oil began to conduct at about 120° C. Moreover, since in the earlier experiments in the air-bath the temperature had once or twice been taken above 160°, the silk covering on the outside wires began to give way; and finally the insulation broke down first in the coil M and then in the coil N. That is to say, the nickel core came into electric contact at some one or two points, and the comparatively strong magnetising current gave rise to a difference of potential in the one arm of the

Wheatstone Bridge circuits, which of course utterly prevented all possibility of resistance measurements being made.

The resistances of the wire x and the various branches of the Wheatstone Bridge were as follows, the ohm employed being the unit of the resistance box used, that is, the original British Association unit. Since the results are, strictly speaking, relative, there is no need reducing to the present legal ohm.

Branch or Wire.	Resistance at 10° C.
M,	2·379
N,	2·366
m,	3·082
n,	3·089
x,	0·3015
Galvanometer,	2·24
Battery,	5

The magnetising current was measured on a Kelvin long range galvanometer, which was calibrated *in situ* by comparison with a Kelvin ampere balance.

The total length of nickel wire forming each core, excluding the ends outside the anchor ring, was 459 cm. in the case of M and 458 cm. in the case of N. The total number of loops in each was twenty. The total number of turns of copper wire in the magnetising coil was 846 in M and 840 in N. Hence to reduce the current in amperes to strength of field in the heart of the anchor ring spaces we must multiply by the factors 46·4 and 46·2 respectively.

Let us now consider the theory of the mode of experimenting and of the method of deducing the final quantitative results. In the Wheatstone Bridge arrangement the value of the current I through the galvanometer is given by the equation

$$\mathrm{DI} = \mathrm{E}\,(\mathrm{M}n - m\mathrm{N})$$

where E is the electromotive force of the battery and D is the well-known determinant involving the values of the resistances of the six conductors which make up the bridge. Two of these, M and N, were subjected to various temperatures, so that the value of D varied correspondingly. The changes of resistance in the M and N branches due to the magnetisation were too slight to affect the value of D appreciably. At the very most these changes are of the order of 2 in 500, and in the great majority of cases much less. They are therefore within the errors of observation.

When the galvanometer was gauged in the manner already described the wires were at the ordinary temperature of the room. The temperature was then brought to the point required and an approximate balance produced on the galvanometer. Thereafter the experiment consisted in comparing the deflections produced (1) when the magnetising current was applied so as to magnetise the one nickel wire, and (2) when no magnetising current was passing.

Let $d\mathrm{I}$ be the change of deflection due to a change dn in n produced in the operation by which the galvanometer is calibrated; and let D be the value of the determinant. Then

$$\mathrm{D}d\mathrm{I} = \mathrm{EM}dn\,.$$

Let dM be the change of the deflection when M changes by amount dM due to magnetisation; and let D_1 be the value of determinant. Then

$$D_1 di = E n dM .$$

Finally let dj be the change of deflection due to change $dndN$ when N is magnetised; and let D_2 be the value of the determinant. Then

$$D_2 dj = -E m dN .$$

The values of D_1 and D_2 will depend upon the particular temperature at which the experiment is for the moment being conducted. They can easily be calculated for the different cases which arise. The quantities I, i, j, are proportional to the observed

RESISTANCE CHANGES OF THE BRANCH M.

Temperature C.	Current in Amperes.	dM/M.	M.	dM.
11	1·616	780	2·361	0·0184
12·3	1·304	658	2·374	156
13·3	1·143	596	2·384	142
12·9	·981	528	2·380	126
13·4	·831	449	2·385	107
13	·57	302	2·381	72
12·8	·427	208	2·379	50
12·3	·346	148	2·374	35
12	·248	73	2·371	17
57	1·616	811	2·874	233
57·7	1·379	712	2·881	205
57·4	1·131	619	2·878	178
57·6	·854	489	2·88	141
57·4	·704	414	2·878	119
,,	·571	329	,,	95
,,	·514	291	,,	837
,,	·398	203	,,	583
57·6	·329	149	2·88	429
,,	·283	108	,,	312
,,	·237	77	,,	212
,,	·185	43	,,	125
94	1·512	752	3·267	246
92·6	1·258	689	3·251	224
93·3	1·021	595	3·258	194
93	·889	536	3·255	174
93·8	·767	351	3·265	154
93·7	·635	403	3·263	131
93·2	·375	211	3·257	688
93	·306	152	3·253	495
91·5	·237	88	3·239	284

deflections. The increment dn is known. Hence dividing the second and third equations by the first we obtain equations from which the increments dM and dN can be calculated. The expressions are, when reduced to their most convenient forms,

$$\frac{dM}{M} = \frac{D_1}{D} \cdot \frac{dn}{n} \cdot \frac{di}{dI} ; \quad \frac{dN}{N} = \frac{D_2}{D} \cdot \frac{dn}{n} \cdot \frac{dj}{dI} .$$

Since only the ratios of the currents enter into these expressions, they may be assumed to be proportional to the deflections. Thus every term is known on the right hand sides of the equations and the ratios dM/M and dN/N can be calculated. Finally, multiplying these proportional changes by the actual resistances of the corresponding branches at the moment of the experiment, we get the total changes of resistance in ohms. These are the changes due to the magnetisation of the nickel portions of the branches M and N which are included within the magnetising coils. The results on carrying out these various operations are shown in the accompanying tables, of which the first and second give the resistance changes of the branches M and N at the various temperatures at which the last and most satisfactory series of experiments were carried out.

RESISTANCE CHANGES OF THE BRANCH N.

Temperature C.	Current in Amperes.	dN/N.	N.	dN.
12·3	1·616	732	2·373	0·0174
12·5	1·304	611	2·375	145
14	1·143	540	2·391	129
13·4	·981	469	2·384	112
13·7	·831	387	2·389	92
13·6	·57	249	2·387	594
12·8	·427	167	2·378	399
12·1	·346	119	2·371	282
12	·248	63	2·37	149
56·6	1·616	813	2·863	233
58·1	1·379	678	2·878	195
56·9	1·131	579	2·866	166
57·6	·854	435	2·873	125
57·4	·704	355	2·871	102
„	·571	277	„	794
„	·514	238	„	684
„	·398	164	„	472
57·6	·329	120	2·873	346
„	·283	87	„	249
„	·237	65	„	187
„	·185	31	„	111
95·8	1·512	769	3·268	254
95·4	1·258	653	3·263	213
95·2	1·021	546	3·26	178
93·6	·889	478	3·256	155
93·8	·767	416	3·259	136
93·4	·635	338	3·254	110
93·0	·375	168	3·25	546
92·9	·306	119	3·248	388
91·9	·237	68	3·237	221

In these tables the resistances headed M and N are the resistances of the whole branch in the Wheatstone Bridge containing the nickel wire. But the changes dM and dN due to the effect of magnetisation are produced almost entirely in the parts of the nickel wires enclosed in the anchor ring. Hence these changes should be compared, not with the total resistances of the branches, but with the resistances of the nickel within the anchor rings. After the arrangement was taken to pieces the resistances of the nickel wires without connections were carefully measured at a temperature of 13°·8 C. with the following results:—

Wire.	Resistance.	
	Entire Length.	Length within Anchor Ring.
M, . .	2·355	2·248
N, . .	2·361	2·253

From these and other data already referred to the resistances of the parts of the nickel wires which were subjected to the magnetising forces could be calculated for the various temperatures at which experiments were made. The results are given in the succeeding table. To bring out the nature of the phenomena more clearly, definite values of current at regular intervals are chosen, the corresponding changes of resistance being taken from the curves obtained by plotting against each other the values in the second and fifth columns of the preceding tables.

The first column contains the value of the magnetising current; and then follow six columns of resistance changes, three for each wire, each headed by its appropriate temperature and the resistance of the wire in the unmagnetised condition. This temperature is the mean of the various temperatures in each set of experiments given in the earlier tables.

RESISTANCE CHANGES IN THE NICKEL WIRES *M* AND *N*.

	Resistance of M			Resistance of N		
	At 12°·6.	At 57°·5.	At 93°·1.	At 12°·9.	At 57°·4.	At 93°·9.
Current	2·236	2·715	3·073	2·244	2·712	3·083
1·5	+·0174	+·0219	+·0245	+·0164	+·0215	+·0252
1·4	166	207	240	155	199	236
1·3	157	197	229	145	187	219
1·2	148	186	217	135	175	206
1·1	138	173	205	125	162	191
1·0	128	160	191	113	148	175
·9	118	147	176	102	132	159
·8	104	135	160	895	118	141
·7	90	118	143	775	102	123
·6	76	100	125	637	844	102
·5	615	805	102	500	664	828
·4	452	584	754	360	477	606
·3	255	358	481	217	292	373
·25	170	241	330	150	209	245

Dividing the numbers in each column by the corresponding resistance of the unmagnetised wire and multiplying by 100,000, we obtain the following increments of resistance due to the magnetisation of wires of nickel, the resistances in the unmagnetised condition being 100,000 ohms at each temperature. In the second column of this table the values of the magnetic fields in the heart of the coils are also given.

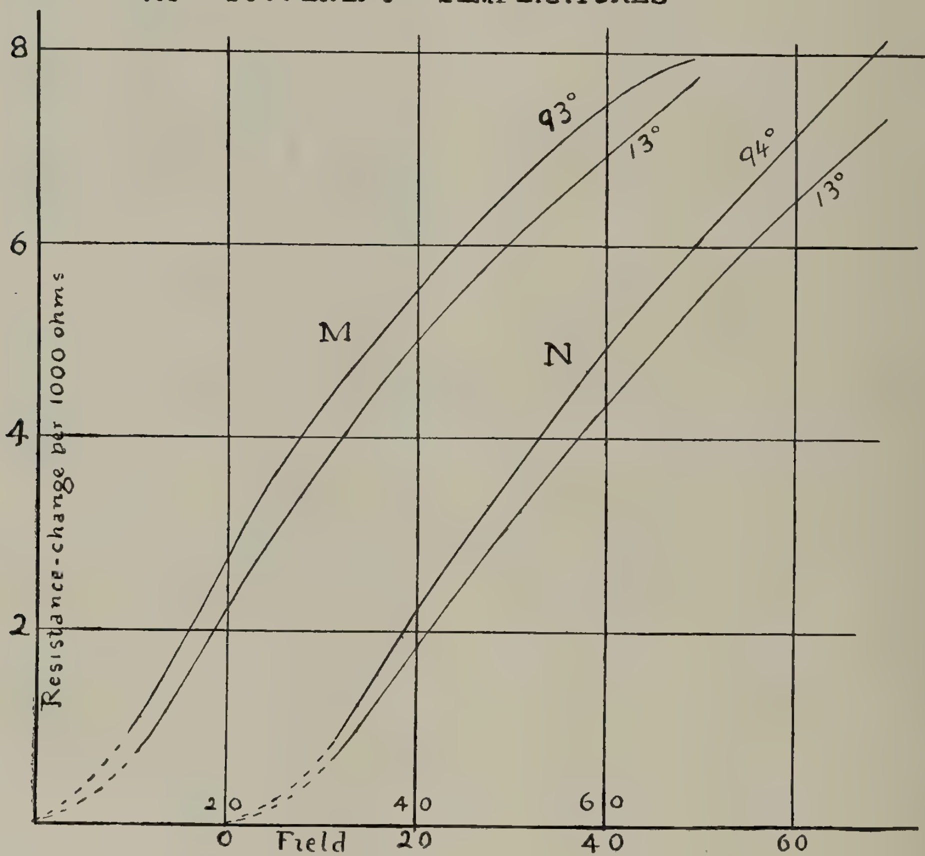
Current.	Field.	Increments of Resistance of 100,000 ohms of Wire M			Wire N		
		At 12°·6.	At 57°·5.	At 93°·1.	At 12°·9.	At 57°·4.	At 93°·9.
1·5	69·6	777	807	793	732	793	818
1·4	65	739	762	781	693	735	766
1·3	60·3	701	725	745	648	690	711
1·2	55·7	659	685	706	603	646	668
1·1	51	616	637	667	556	598	620
1·0	46·4	571	589	622	503	546	568
·9	41·8	525	541	573	456	487	516
·8	37·1	462	489	521	400	436	457
·7	32·5	402	434	466	346	376	399
·6	27·8	339	368	408	284	311	331
·5	23·2	275	296	332	223	245	268
·4	18·6	202	215	245	161	176	196
·3	13·9	118	132	156	97	108	121
·25	11·6	76	89	108	67	77	80

The general results embodied in these tables may be gathered at a glance from the graphs on page 542, in which, however, the curves for the intermediate temperatures are not given. A slight inspection of the numbers shows that the omitted curve lies exactly between the other two for the same wire.

Thus it appears that up to a temperature of 100° C. the effect of rise of temperature is to increase the total change of resistance due to the application of a given field.

It will be noticed that in the case of the wire M, the increase of resistance in the field, 69·6, is somewhat less at 93°·1 than at 57°·5. But the exception is only apparent, and is due to an error of observation which is unavoidable under the conditions of the experiment. It is this same error which gives the different forms to the M and N curves in the higher fields. The curves for the M wire become distinctly more convex upwards as the field attains its higher values, whereas the curves for the N wire tend rather to become concave. Now it must be remembered that the two nickel wires were cut originally from the same piece and had been subjected throughout their history to the same treatment. Nevertheless they did not behave exactly the same as regards their rate of heating when a given current was passed through the surrounding coils. Because of some slight difference in the winding and the general form of the coils, the one wire heated more quickly than the other. Hence when a strong current was used and passed round the wires so as to magnetise neither, a gradually growing deflection was obtained on the galvanometer. Similarly, when the current was broken, the one wire cooled more quickly, and again a gradually changing deflection was obtained. In like manner when the current was so applied as to magnetise the one wire, the sudden deflection due to the effect of the magnetisation on the resistance was followed by a gradually growing deflection due to the different rates of heating in the coils. The reading of the deflection could not be taken at once, but a few seconds had to elapse before the spot of light ceased oscillating to and fro about its mean position. The method of observation was to take the mean of the last distinct oscillation after the magnetisation had been produced in the one wire, then break the current, take the mean of the last distinct oscillation, and again make the current, and once more take the mean of the last distinct oscillation, and so on until the operation was complete.

RESISTANCE OF NICKEL IN VARIOUS FIELDS AT DIFFERENT TEMPERATURES



Now the rates of heating when the current was passing were not related in the same way as the rates of cooling when the current ceased to act. A little consideration will show, what was clearly demonstrated by the experiment, that the effect will be to increase the real deflection in the case of the one wire, and to decrease it in the case of the other. In other words, the measured changes of resistance of the wire M in the higher fields are less than the real values because of an uncompensated temperature effect, in virtue of which the measured changes of resistance of the wire N are greater than the real values. As the magnetising current and field are diminished this disturbance rapidly becomes insignificant, since the heating effect is as the square of the current. The only way of obviating this source of error would be to leave the current on until temperatures were perfectly steady in both wires, then reverse the current suddenly in one of the two coils round one of the wires so as to produce magnetisation or destroy it as the case might be, the heating effect remaining unchanged. This method would have entailed a great expenditure of electric energy in the higher fields, and was not thought of until the insulation in the anchor ring N broke down. It is clear that too much stress must not be laid upon the readings in the highest fields. Probably the best average results will be obtained by combining the readings for the two wires, as in the following table, in which, corresponding to their appropriate fields, the means of the numbers for the two wires are tabulated in columns headed by the mean of the nearly equal temperatures.

CHANGE OF RESISTANCE OF 100,000 OHMS OF NICKEL WIRE
(MEAN FOR THE TWO WIRES).

Field.	At 12°·7.		At 57°·5.		At 93°·5.		ddR/dt.	
		Diff.		Diff.		Diff.	At 35°.	At 75°.
69·5	755	39	800	51	806	32	1·100	0·017
65	716	41	749	41	774	46	0·74	·70
60·3	675	44	708	42	728	41	·74	·56
55·7	631	45	666	48	687	43	·78	·58
51	586	49	618	50	644	49	·71	·72
46·4	537	46	568	54	595	50	·69	·75
41·6	491	60	514	51	545	56	·51	·86
37·1	431	57	463	58	489	56	·71	·72
32·5	374	62	405	65	433	63	·69	·78
27·8	312	63	340	69	370	70	·63	·83
23·2	249	67	271	75	300	79	·49	·81
18·6	182	74	196	76	221	82	·31	·69
13·9	108		120		139		·27	·53
11·6	72		83		94		·25	·31

In addition to the resistance changes with their differences, two columns are given showing the average rate of change, due to rise of temperature, in the value of the resistance change in given fields. These average rates of change may be taken to be the rates of change at temperatures 35° C. and 75° C., which were the means of 12°·7 and 57°·5 and of 57°·5 and 93°·5 respectively.

Distinguishing the rate of change of resistance per unit increase of field at constant temperature, and the rate of change, per unit increase of temperature, of the increase of resistance in a given magnetic field by the somewhat elliptic terms "magnetic change rate," and "thermal change rate," we may state the results embodied in these tables in the following words :—

(1) The magnetic change rate of resistance of nickel increases steadily with increase of field, but at a somewhat slower rate as the field increases;

(2) The magnetic change rate increases unmistakably, with rise of temperature up to 100° C. and probably to higher temperatures;

(3) The thermal change rate is fairly constant throughout, tending to diminish a little in the lower fields.

In the earlier set of experiments, in which the apparatus was heated in an air-bath, the temperature was pushed up to 170° C., beyond which it was dangerous to go in case the silk insulation should break down. As a matter of fact it was because of this heating that the insulation did ultimately break down and prevented the experiments in their later and improved form being carried up to even such a perfectly safe temperature as 130°. These earlier experiments, however, gave the same general results as the later, and indicate that up to 170° in the lower fields at least the magnetic change goes on increasing. There was a good deal of uncertainty in these first experiments as to the temperature of the nickel wires, especially in the higher fields, and the numbers are not deemed sufficiently accurate to be placed alongside of the numbers obtained in the later form of experiment.

In the lower fields there was evidence of hysteresis, the change of resistance at break of the magnetising current being somewhat less than at make; but in fields above thirty or forty there was no measurable hysteresis. As a general rule the field was applied cyclically through several cycles in alternate directions before the observations were taken. Occasionally, however, the magnetising current was applied and removed and applied again in the same direction instead of being reversed; that is to say, instead of the cycle being $(+, 0, -, 0, +)$ it was either $(+, 0, +)$ or $(-, 0, -)$. In such cases it was invariably found that the accompanying change of resistance was greater under the cycle $(+, 0, +)$ or $(-, 0, -)$ than under the complete reversing cycle $(+, 0, -, 0, +)$.

If, following Professor J. J. Thomson's theory of electric corpuscles, we try to get at the significance of these results, we arrive at some interesting conclusions. These negatively charged corpuscles of mass, very small compared to the mass of the neutral molecule or of the positively charged cormolecule, are regarded as the agents which carry the charge through the conducting material. The process is analogous to the process of conduction of heat in gases according to the generally accepted kinetic theory; and Professor J. J. Thomson has given reasons for believing that the process is due to the impacts of the corpuscles on the neutral molecules, the result of such a collision in any particular case being the entanglement of the impinging corpuscle with the neutral molecule and the instantaneous setting free of another corpuscle which moves on with its charge to the next collision. The effect of an externally applied electric force is to give direction to the motion of the momentarily dissociated corpuscles. The rate at which the charges are carried, that is, the amount of electricity conducted, will depend upon the velocities of the corpuscles and their free paths, and also upon their state of aggregation. Anything which increases their velocities in a particular direction will tend to increase the conductivity and to diminish the resistance. Anything which tends to diminish the free path will increase the resistance.

Consider the case of the nickel with a current passing through it. Here the corpuscles will be driven against what is conventionally known as the direction of the current. Let now a longitudinal magnetic field be applied. Its effect will be to give the charged corpuscles a helical motion about the axis of the wire, and to drive them out towards the outer surface of the wire. This will tend to increased aggregation of the corpuscles with corresponding diminution of free path, and the result will be an increase of resistance. Professor J. J. Thomson has shown that the manner in which resistance increases with temperature in the case of metals must depend upon the effect of change of temperature on the rate of dissociation of the molecules, and requires the assumption that the effective collisions are those which occur between the negatively charged corpuscles and the neutral molecules. In a recent paper by Mr J. Patterson (*Phil. Mag.*, June 1902) some calculations are made along the lines of this theory in connection with experiments upon the change of resistance of certain non-magnetic metals when placed in powerful magnetic fields. The problem is evidently quite a different one in the case of the magnetic metals in which measurable changes of resistance occur in low fields.

Whatever be the ultimate structure of a molecule of iron or nickel it is almost essential that it must be magnetic, the effect of an applied magnetic force being to break up closed chains of molecules and give them a definite set more or less in the direction of magnetisation. If we suppose these magnetic molecules to owe their magnetic condition to whirls of electrified particles, it would appear that there was greater chance for a corpuscle to be driven out of its whirl by a magnetic force acting perpendicular to its axis than by a magnetic force acting parallel to its axis. Hence it is quite conceivable that under a transverse force the resistance of the magnetic metals should diminish, while under a longitudinal force it should (comparatively speaking) greatly increase.

In connection with this whole inquiry, an exceedingly interesting question will be, How will the magnetic effect be affected when the temperature is raised so high that the nickel ceases to be magnetic in the ordinary significance of the term? This question I hope soon to answer by means of an improved apparatus, capable of being raised to very high temperatures.

XXIV.—*A Contribution to the Craniology of the People of Scotland.* Part I., *Anatomical.* By Professor Sir William Turner, K.C.B., D.C.L., F.R.S. (With Five Plates.)

(Read November 3, 1902. Issued separately February 10, 1903.)

Up to the present time no systematic account of the cranial characters of the people of Scotland has been published. Incidental references to, and measurements of, a limited number of Scottish skulls may indeed be found in the writings of various authors, as in Sir Daniel Wilson's *Prehistoric Annals of Scotland*, in Drs Davis and Thurnam's *Crania Britannica*, and in Professor Cleland's *Memoir on Variations in the Human Skull.** Measurements of five Scottish crania are recorded by Sir W. H. Flower in the Osteological Catalogue (Man) of the Museum of the Royal College of Surgeons of England, four of which were found amongst the ruins of an ancient Culdee Monastery at St Andrews, and the fifth is said to be that of a Highlander. Dr Barnard Davis, in his *Thesaurus Craniorum*, gives the measurements of a somewhat larger number, six of which were from Caithness, and one is stated to be a Scottish Highlander. The same skulls have been remeasured and described by Dr J. G. Garson.†

A number of years ago I began to form a collection with the view of studying the characters of the skull in the Scottish people; but the acquisition of authentic examples from definite localities is a slow process, and time is required to obtain sufficient specimens to enable one to form a general conception of their form and proportions. Every teacher of Anatomy has, of course, the material provided by his practical rooms, but the greater number of these crania are of necessity cut in pieces in the course of the dissection; as a rule also, so little is known of the history of the waifs and strays of humanity who come within the provisions of the Anatomy Act, that in many instances it is difficult to ascertain their nationality or race, though presumably in Edinburgh the majority would naturally be Scotch. As belonging also to the pauper part of the community, one cannot obtain from the study of their skulls a due conception of the cranial type of the educated and well-to-do classes. It is therefore to a limited extent only that I have employed in this investigation specimens from the dissecting room, and not unless I could ascertain either the name of the person, or from other satisfactory reason feel tolerably certain that the skull was that of a Scot.

I have consequently looked elsewhere for additions to my material, and have been fortunate to obtain, through the kind interest taken in the subject by several former pupils, and from other friendly sources, skulls from known localities, from Shetland in the North to Wigtonshire in the South, and from Iona in the West to Dunbar in the East.

* *Philosophical Transactions*, 1869.

† Osteology of the ancient inhabitants of the Orkney Islands. *Journal Anthropological Institute*, vol. xiii., 1883.

As might be expected, Edinburgh and the counties of Fife, East Lothian, and Mid-Lothian have furnished me with a considerable proportion of the specimens.

The collection long located in Edinburgh in the Phrenological Museum through an arrangement made with the Henderson Trustees having been deposited in the Anatomical Museum of the University, has also been made available for my purpose, though I have not included in my tables some specimens in it which, though probably Scottish, wanted a precise statement of locality. Several of the crania belonging to this collection are of great interest, and whenever the locality was definitely stated, to ensure that the skull was Scottish, it has been examined and noted. Altogether one hundred and seventy-six skulls have been studied. Comparatively few had the lower jaw attached to the cranium. Unfortunately, many were more or less injured, especially in the facial region, so that the proportions of that part of the skull have been estimated from a smaller number of examples than were available for the study of the cranial box. As the majority of the skulls described have been obtained in the counties south of the Clyde and the Tay, this memoir is more especially descriptive of the cranial characters of the people of lowland Scotland.

Fifeshire. Table I. Plates I., II., V.

The skulls from Fifeshire were from two localities.

a. The greater number were obtained during the operations connected with the rearrangement, some years ago, of the interior of a parish church in the landward part of the county. In removing the pavement and the subjacent earth, quantities of human bones were exposed. Fifteen skulls came into my possession, and although some were injured, the majority were in good order. To only one specimen was the lower jaw attached. From the place of interment being within the church, it is to be presumed that the skulls belonged to parishioners of the better class. It is probable that the interments were made in the eighteenth century, when intramural burials were not uncommon, and it is doubtful if any had been later than the earlier part of the century following. From these specimens, therefore, one may form a good idea of the cranial characters of the educated people of this part of the county. Measurements of the crania, distinguished by the letter M, are given in Table I.

They were apparently twelve males and three females, and were, with two exceptions, the skulls of persons in middle or even more advanced life; the specimen measured in the first column of that table was known to be from a man æt. 66, and the ages of the other specimens are approximately stated. In four specimens the face was much injured. In the skulls where the upper jaw was uninjured, the teeth were in part worn down and decayed, though on the whole they were well preserved. In the majority of the skulls, the sutures of the vault were in process of obliteration; but two crania, a male and a female, were metopic.

Norma verticalis.—The skulls were characteristic examples of "well-filled" crania.* As a rule, they were broadly ovoid, and the transverse diameter of the vault of the cranium was so marked that the outline was rounded, and the majority of the skulls were obviously either brachycephalic or approaching thereto. There was no sagittal keel or ridge, the parietal bone sloped gently downwards and outwards from the sagittal suture to the parietal eminence, and the transverse arc of the cranium behind the bregma was a low rounded arch. The side walls were not vertical, and in the majority of the specimens the greatest transverse diameter was near the squamous suture. In only five of the crania were the zygomatic arches unbroken, and in all but one the arches were concealed or cryptozygous. In all the specimens the parieto-squamous breadth was greater than the interzygomatic. The stephanic diameter exceeded the asterionic with two exceptions, and in one of these the diameters were equal.

Norma lateralis.—In the males the glabella and supraorbital ridges were distinct, and the forehead had a slight backward slope. From a little in front of the obelion the skull sloped downwards and backwards into the occipital squama, which projected in a marked manner behind the inion, so that the fossa for the occipital lobes of the cerebrum considerably overlapped the cerebellum. With three exceptions the crania rested behind on the conceptacula cerebelli, which indicated a considerable convexity of the inferior surface of the hemispheres of the cerebellum. In the exceptional crania the skulls rested behind on the tips of the mastoid processes. In the profile outline of the face the nasal bones projected forwards, and formed a well-marked bridge to the nose, which, in some specimens, was very prominent. The nasal spine of the superior maxillæ was also as a rule strongly projecting. In ten males the maximum total longitudinal arc was 384 mm., the minimum 361 mm., and the mean 373·9 mm.; in three females the maximum was 370, the minimum 346, and the mean 356·3. There was no constant rule, either for the entire series of these crania, or for the different sexes, as to the relative proportion of the frontal, parietal and occipital longitudinal arcs, though it was more usual for the frontal arc to be longer than either the parietal or occipital, and for the parietal to be longer than the occipital. But in four specimens the parietal exceeded the frontal; in four, the occipital was either equal to or exceeded the parietal, and in three the occipital was either equal to or exceeded the frontal.

The maximum glabello-occipital length in the twelve male skulls was 193 mm., the minimum 179 mm., and the mean 185 mm. In three females the maximum length was 181, the minimum 172, and the mean 176·3 mm. Of ten male skulls the maximum basi-bregmatic height was 137 mm., the minimum 125 mm., and the mean 130·3 mm.; whilst two females were respectively 120 and 121 mm. The maximum parieto-squamous diameter in the eleven males was 158 mm., the minimum was 136 mm., the mean being 149·6 mm.; whilst in three females the maximum was 141, the

* This term is adopted from Professor Cleland's "Memoir on Variations in the Human Skull" (*op. cit.*).

TABLE I.—*Fifeshire.*

Collection,	Metopic Ma.	Mb.	Mc.	Md.	Me.	Mf.	Mg.	Mh.	Mi.	Mm.	Mn.	Mo.	Ml.	Mk.	Mp.	H.T. D. 578	H.T. D. 5
Age,	66	abt. 60	abt. 60	abt. 50	abt. 70	abt. 60	60–70	60–70	30–40	60–70	60–70	60–70	abt. 30	abt. 50	abt. 60	Ad.	Adu
Sex,	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	F.	F.	F.	M.	F.
Cubic capacity,	1636	1380	1598	1490	1442	1515	...	...	...	1495	...	...	1275	...	1182	1700	120
Glabello-occipital length,	189	179	193	186	186	180	182	181	190	183	179	192	181	176	172	201	176
Basi-bregmatic height,	129	133	134	127	125	128	132	130	128	137	...	...	120	...	121	141	135
Vertical Index,	*68·3*	*74·3*	*69·4*	*68·3*	*67·2*	*71·1*	*72·5*	*71·8*	*67·4*	*74·9*	...	...	*66·3*	...	*70·3*	*70·1*	*76*
Minimum frontal diameter,	112	98	98	94	98	96	100	...	94	96	93	94	93	96	88	101	91
Stephanic ,, ,,	128	117	126	120	126	130	123	123	116	125	95	117	111	111	108	...	95
Asterionic diameter,	124	112	126	111	113	109	113	117	109	116	106	110	110	94	102	115	99
Greatest parieto-squamous breadth,	157s.	149s.	151s.	146s.	158s.	152s.	145s.	151s.	...	156s.	136	145	141s.	131s.	137p.	144s.	128
Cephalic Index,	*83·1*	*83·2*	*78·2*	*78·5*	*84·9*	*84·4*	*79·7*	*83·4*	...	*85·2*	*76·*	*75·5*	*77·9*	*74·4*	*79·7*	*71·6*	*72*
Horizontal circumference,	560	528	548	537	550	528	524	...	545ap	530	505	540	515	500	497	555	485
Frontal longitudinal arc,	125	124	138	128	129	130	119	132	132	132	130	127	133	122	118	140	119
Parietal ,, ,,	130	122	128	116	128	123	} 242	233 }	123	138	110	129	109	127	115	135	128
Occipital ,, ,,	125	116	118	129	125	121			123	112	...	...	128	97	120	125	106
Total ,, ,,	380	362	384	371	382	374	361	365	378	382	...	...	370	346	353	400	353
Vertical transverse arc,	322	303	324	308	327	324	306	320	310	325	310	333	297	288	300	320	280
Basal transverse diameter,	138	134	124	129	130	131	131	124	126	127	107	121	120	112	108	128	111
Vertical transverse circumference,	460	437	448	437	457	455	437	444	436	452	417	454	417	400	408	448	391
Length of foramen magnum,	32	34	36	35	31	30	34	36	33	35	...	...	29	...	31	37	33
Basi-nasal length,	108	98	106	101	96	99	104	98	101	96	...	...	91	...	...	109	96
Basi-alveolar length,	100	91	104	96	87	91	105	99	95	...	...	...	88	...	...	108	...
Gnathic Index,	*92·6*	*92·9*	*98·1*	*95·*	*90·7*	*91·9*	*101·*	*101·*	*94·1*	...	...	...	*96·7*	...	...	*99·1*	...
Total longitudinal circumference,	520	494	526	507	509	503	499	499	512	513	...	...	490	...	...	516	482
Interzygomatic breadth,	...	139	134	...	...	135	...	136	...	...	...	...	123	...	...	139	...
Intermalar, ,,	134	122	116	112	114	117	...	122	115	...	...	...	109	106	..	123	...
Nasio-mental length,	...	...	...	...	...	...	...	...	114	...	...	...	...	...	...	135	...
Complete Facial Index,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	*97·*	..
Nasio-alveolar length,	79	76	77	75	75	71	64	76	73	...	...	...	63	64	...	82	..
Maxillo-facial Index,	...	*54·6*	*57·4*	...	...	*52·6*	...	*55·8*	...	...	...	...	...	...	...	*59·*	..
Nasal height,	56	54	56	53	53	52	48	51	52	...	...	...	48	48	...	60	..
Nasal width,	26	25	23	24	21	25	23	25	21	...	...	...	23	24	...	24	..
Nasal Index,	*46·4*	*46·3*	*41·1*	*45·3*	*39·6*	*48·2*	*47·9*	*49·*	*40·4*	...	...	...	*47·9*	*50·*	...	*40·*	..
Orbital width,	43	40	38	39	40	39	40	40	39	...	...	...	38	38	...	37	..
Orbital height,	36	32	33	34	34	34	34	35	34	...	...	...	30	34	...	35	..
Orbital Index,	*83·7*	*80·*	*86·8*	*87·2*	*85·*	*87·2*	*85·*	*87·5*	*87·2*	...	...	...	*78·9*	*89·5*	...	*94·6*	..
Palato-alveolar length,	58	56	58	55	51	48	56	56	50	...	...	...	51	49	...	58	..
Palato-alveolar breadth,	63	71	63	59	57	53	...	69	59	...	...	...	57	55	...	65	..
Palato-alveolar Index,	*108·*	*126·*	*108·*	*107·*	*111·*	*110·*	...	*123·*	*118·*	...	...	...	*111·*	*112·*	...	*112·*	..
Lower jaw: Symphysial height,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	35	..
Lower jaw: Coronoid ,,	...	...	...	...	...	...	...	...	63	...	...	...	...	...	...	72	..
Lower jaw: Condyloid ,,	...	...	...	...	...	...	...	...	64	...	...	...	..	...	...	72	..
Lower jaw: Gonio-symphysial length,	...	...	...	...	...	...	...	...	86	...	...	...	...	...	...	102	..
Lower jaw: Inter-gonial width,	...	...	...	...	...	...	...	...	90	...	...	...	...	...	...	110	..
Lower jaw: Breadth of ascending ramus,	...	...	...	...	...	...	...	...	29	...	...	...	..	...	...	40	..

minimum 131, and the mean 136·3 mm. Of eleven male skulls the maximum horizontal circumference was 560 mm., the minimum 505 mm., and the mean 535·9 mm.; of three females the maximum was 515, the minimum 497, and the mean 504 mm. The maximum intracranial capacity in seven males was 1636 c.c., the minimum 1380 c.c., and the mean 1508 c.c.: two females were respectively 1182 and 1275 c.c.

Some of the crania exhibited individual peculiarities which require to be noted. In several males the inion and superior curved occipital lines were very distinct, but the temporal ridges were only moderate. Two female crania possessed an epipteric bone or bones in each pterion, and in a third an epipteric was present only in the right pterion. The ali-sphenoido-parietal articulation was usually wide, though occasionally it was attenuated. In one skull the right squamous-temporal articulated with the frontal. In six specimens a small Wormian bone, or bones, was present in the lambdoidal suture. In one skull the right external pterygoid plate was sutured with a bony plate from the spine of the sphenoid, and enclosed a large pterygo-spinous foramen, and in another specimen these two processes were almost united with each other on the right side. Two crania possessed rudimentary paramastoid processes, but no specimen had a third occipital condyle. Indications of an infraorbital suture in process of obliteration were seen in some of the skulls.

I shall now pass to the consideration of the indices obtained by a comparison with each other of two diameters. In eleven male crania the length-breadth or cephalic index presented a range of variation from a maximum of 85·2 to a minimum of 75·5, the mean being 81·1. No skull was below 75, but one was 75·5 and another 76; three ranged from 78·2 to 79·7; six were very distinctly brachycephalic, ranging from 83·1 to 85·2. The three female crania ranged from 74·4 to 79·7, with a mean of 77·3; two were mesaticephalic, and one was near the higher limit of the dolichocephalic group.

The length-height or vertical index ranged in ten males from 67·4 to 74·9 and the mean was 70·5. In two females the mean index was 68·3. In the females the height was not only absolutely less than in the males, but it was also less in relation to the length of the skull, so that the mean vertical index was less than the mean cephalic index. In every specimen, indeed, in which both the height and breadth could be measured, the breadth of the cranium materially exceeded the height. The crania were in the mean, tapeinocephalic (chamæcephalic), and only two specimens were metriocephalic.*

The projection of the upper jaw, as estimated by the gnathic index, ranged in nine males from 90·7 to 101; only three specimens were mesognathous and the rest were orthognathous.

In one female this index was 96·7, the mean was orthognathous, and only three specimens were mesognathous.

* I continue to use this term in preference to orthocephalic, as recommended by the German craniologists in the Frankfurt agreement, for the reasons given in my *Challenger Report*, 1884, note, p. 5.

The nasal index ranged in nine males from 39·6 to 49, and the mean was 44·9. With two exceptions the whole series was leptorhine, or with a long and narrow nose. The mean of two females was 48·9. The floor of the nose was separated from the incisive region by a well-defined and often a sharp ridge of bone.

The orbits were large, and a mean index of 85.5 was obtained in nine males, the variations being from 80 to 87·5. Two specimens only were microseme and seven were mesoseme. The two females were respectively 89·5 and 78·9.

The relative length and breadth of the palato-alveolar arch varied materially in those specimens which were sufficiently perfect to allow of the arch to be measured. Of eight male skulls three were dolichuranic, one was brachyuranic, two were hyperbrachyuranic, and two were mesuranic: collectively they gave a mesuranic mean 113·9, although a minority only of the specimens were in this group. Two female crania in which the arch was complete were mesuranic.

Either the absence of the lower jaw or the injured zygomatic arches prevented the proportions of length and breadth of the entire face from being taken, but in four specimens the length of the upper jaw was compared with the interzygomatic breadth and a mean maxillo-facial index 55·1 was computed; the face was therefore leptoprosopic.

b. In the collection belonging to the Henderson Trust are two normal skulls from the Abbey Church of Dunfermline, in the west of Fife, measurements of which are given in Table I., and they are distinguished by the letter D.

One of these skulls was a male aged apparently about 50. It was well proportioned, distinctly dolichocephalic, 71·6, and of unusually large cranial capacity, 1700 c.c. In its general configuration it bore some resemblance to the cast of the skull ascribed to King Robert the Bruce, and indeed was identical with that cast in the glabello-occipital diameter; but its transverse diameter was not so great by several millimetres, and it differed also in some other characters and dimensions. It gave one the impression of having belonged to a man of power and capacity. It is marked H.T. 578 D. in Table I., and is figured in Pl. I., fig. 1, 2, 3. From the measurements it will be seen that the height of the cranium, 141 mm., though one of the most lofty of the series, was 3 mm. less than the parieto-squamous diameter. As the glabello-occipital length was 201 mm., it is one of the longest crania examined in the series, and the horizontal circumference was considerably above the average. In the *norma verticalis* the cranium was ovoid, and the lateral walls were wider in the region of the squamous sutures than opposite the parietal eminences. The vertex had not so flat a transverse arc as in the series M, and the upper parietal region sloped a little more abruptly outwards to the parietal eminences, which were not prominent; there was no sagittal ridge. The occipital squama projected distinctly behind the inion and superior curved line. The glabella and supraorbital ridges were distinct, and the frontal bone sloped backwards and upwards. The skull was cryptozygous and rested behind on the cerebellar fossæ of the occipital bone. The bridge of the nose was not specially prominent: the nose was strongly leptorhine. The orbit was only 2 mm. wider than high. Though the breadth of the face was considerable, its vertical diameter was so pronounced, owing to a well-developed lower jaw and an unusual vertical diameter of the superior maxillæ, that the complete index 97 was hyperleptoprosopic. The majority of the teeth were in place, not decayed and comparatively little worn. The hard palate had considerable depth. The gnathic index was in the lower mesognathic group. There were no special modifications in ossification, except that the right posterior condyloid foramen was unusually large, and tunnelled forwards so as to open into the jugular foramen. The left posterior condyloid foramen was absent.

The other skull from Dunfermline, H.T. 582 D., was wanting in the facial bones. It was evidently an adult female. It was elongated ovoid in shape, with a length-breadth index of 72·7, and in it the basi-bregmatic height was so much greater than the parieto-squamous breadth, that the vertical index was distinctly more than the cephalic. The parietal longitudinal arc was considerably longer than either the frontal or occipital. The cranium was not flattened at the vertex, and sloped steeply downwards from the sagittal ridge to the parietal eminences, below which the side walls were vertical. The glabella was feeble, the forehead was almost vertical. The parieto-occipital slope was gradual: the occipital squama was rounded, and projected behind the inion. There were no Wormian or epipteric bones.

East Lothian. Table II.

The skulls from East Lothian were obtained from three localities.

a. The larger number were procured in the course of extensive alterations connected with remodelling the interior of the nave of an old abbey church in the landward part of the county. The pavement and about 18 inches to 2 feet of earth were removed, and the bones were found principally at the bases of the pillars which supported the roof of the nave, and beneath where the pulpit stood. Without doubt they belonged to the better classes in the parish. It is said that the last interment within the nave was in 1795.

Thirteen skulls were collected, several of which were injured either in the cranium or face, and in none did the lower jaw accompany the skull. They were apparently ten males and three females. In only four specimens were the facial bones sufficiently entire to enable me to take the face measurements. In several the foramen magnum was so injured that the height of the cranium could not be taken. Two specimens were metopic. In five the sutures of the cranial vault were very distinct; but in the other crania they were in process of obliteration, so much so in two cases that the persons were obviously advanced in years. The teeth, as a rule, were lost; in one specimen they were flattened from use; in another they were not much worn. In at least two skulls the crania were obliquely distorted, as if from post-mortem pressure. The measurements of the crania are given in Table II., and the skulls are distinguished by the initial letter H.

TABLE II.—*East Lothian.*

	Abbey Church.													Dunbar.	North Berwick.	
Collection,	Hb.	Hi.	Hl.	Hm.	Ha.	Hc.	Hd.	Hf.	Metopic. He.	Hk.	Metopic. Hg.	Hh.	Ho.	Metopic. Ht. 41.	Ht. 44.	Ht 3
Age,	Adult.	Ad.	Ad.	Ad.	Aged.	Aged.	Ad.	Aged.	Ad.	Aged.	Ad.	Aged.	Aged.	Ad.	Aged.	Ag.
Sex,	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	F.	F.	F.	M.	M.	[illegible]
Cubic capacity,	...	...	1305	...	...	...	...	...	...	...	...	...	...	...	1600	...
Glabello-occipital length,	184	182	175	180	185	194	181	180	182	190	182	186	184	182	188	17
Basi-bregmatic height,	121	130	135	129	134	...	...	...	...	...	...	123	...	125	133	...
Vertical Index,	*65·8*	*71·4*	*77·1*	*71·7*	*72·4*	...	...	...	...	...	...	*66·1*	...	*68·7*	*70·7*	
Minimum frontal diameter,	90	95	93	101	106	101	94	101	99	91	93	91	92	99	106	[illegible]
Stephanic frontal diameter,	116	106	105	124	121	119	105	121	125	113	104	...	106	109	117	[illegible]
Asterionic diameter,	112	105	112	114	130	...	...	112	...	...	...	106	...	97	116	...
Greatest parieto-squamous breadth,	151s.	139s.	137s.	146ap.	159s.	140s.	152s.	149s.	157s.	142ap.	144s.	147s.	140s.	141s.	150s.	14
Cephalic Index,	*82·1*	*76·4*	*78·3*	*81·1*	*85·9*	*72·2*	*82·9*	*82·8*	*86·3*	*74·7*	*79·1*	*79·0*	*76·1*	*77·5*	*79·8*	*8*
Horizontal circumference,	524	517	500	...	550	535	525	525	...	...	520	526	...	508	548	50
Frontal longitudinal arc,	122	130	122	130	128	133	120	129	130	145	131	125	130	115	133	12
Parietal ,, ,,	124	130	133	120	130	131	118	132	120	140	125	120	125	116	125	12
Occipital ,, ,,	115	111	116	108	116	116	107	107	...	...	...	120	...	126	119	...
Total ,, ,,	361	371	371	358	374	380	345	368	...	...	...	365	...	357	377	...
Vertical transverse arc,	310	315	302	...	340	...	...	318	...	...	...	310	...	289	310	...
Basal transverse diameter,	125	111	124	...	141	...	...	127	...	...	...	125	...	119	134	...
Vertical transverse circumference,	435	426	426	...	481	...	...	445	...	...	...	435	...	408	444	...
Length of foramen magnum,	37	30	31	35	...	...	...	...	...	...	...	37	...	32	35	...
Basi-nasal length,	96	102	95	101	104	...	...	...	...	...	...	101	...	100	105	...
Basi-alveolar length,	95	99	98	97	...	...	...	...	...	...	...	...	...	91	...	...
Gnathic Index,	*99·*	*97·1*	*103·2*	*96·*	...	...	...	...	...	...	...	...	...	*91*	...	...
Total longitudinal circumference,	494	503	497	...	...	...	...	...	...	...	...	503	...	489	517	...
Interzygomatic breadth,	137	117	131	...	...	...	...	...	...	...	...	...	...	...	...	...
Intermalar,	115	108	119	...	...	...	...	...	...	...	...	...	...	117	...	...
Nasio-mental length,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Nasio-alveolar length,	74	64	69	73	...	...	...	...	...	...	...	...	...	67	...	...
Maxillo-facial Index,	*54·*	*54·5*	*52·6*	...	...	...	...	...	...	...	...	...	...	...	...	...
Nasal height,	54	47	52	53	...	...	...	...	...	...	...	...	...	51	...	...
Nasal width,	22	26	25	23	...	...	...	...	...	...	...	...	...	24	...	...
Nasal Index,	*40·7*	*55·3*	*48·2*	*43·4*	...	...	...	...	...	...	...	...	...	*47·1*	...	...
Orbital width,	38	34	36	40	...	...	...	...	...	...	...	...	...	38	...	...
Orbital height,	34	28	30	37	...	...	...	...	...	...	...	...	...	31	...	...
Orbital Index,	*89·5*	*82·4*	*83·3*	*92·5*	...	...	...	...	...	...	...	...	...	*81·6*	...	...
Palato-alveolar length,	52	48	54	56	...	...	...	...	...	...	...	...	...	46	...	...
Palato-alveolar breadth,	61	50	57	64	...	...	...	...	...	...	...	...	...	60	...	...
Palato-alveolar Index,	*117·3*	*104·1*	*105·5*	*110·7*	...	...	...	...	...	...	...	...	...	*130·*	...	...

Norma verticalis.—The skulls were well filled, and were not keeled or ridged in the sagittal region. The majority were so broadly ovoid as to be rounded in outline, and obviously of brachycephalic proportions. The parietal eminences in a few cases projected, but, as a rule, they had no special prominence, and the vault of the skull sloped gently downwards and outwards from the sagittal suture to the parietal eminence. The side walls were not vertical. The four specimens in which zygomatic arches were present were cryptozygous, and the interzygomatic diameter exceeded the intermalar, stephanic and asterionic; except in one skull, the stephanic diameter exceeded the asterionic.

Norma lateralis.—The glabella and supraorbital ridges were, as a rule, moderately prominent in the male skulls, but in two specimens they were more strongly marked. The forehead had a gentle slope backwards; the vertex was, as a rule, flattened; the curve from the postero-parietal into the occipital regions was usually gradual, but in two instances it was much more abrupt; the occipital squama above the superior curved line did not as a rule project much behind the inion. Four specimens sufficiently perfect to be tested rested behind on the conceptacula cerebelli. The nasal bones when present were prominent, and the bridge slightly concave; but in a female skull they were small and flattened below the fronto-nasal suture. The maxillo-nasal spine was moderately prominent, and the floor of the nose was separated from the incisive region by a sharp ridge. The maximum longitudinal arc in the males was 380 mm., the minimum was 345, and the mean of nine specimens was 366 mm. The occipital arc was always shorter than either the frontal or parietal, but the frontal arc in some exceeded the parietal, whilst in others the reverse was seen. The basi-nasal diameter could be taken in only a few skulls; the maximum was 104 mm., the minimum 95 mm., and the mean of six specimens was 99·8.

The maximum transverse diameter was near the squamous suture; in the male skulls it was 159 mm., and the minimum was 137 mm., whilst the mean was 147; in two females the maximum was 144, the minimum 136. The maximum glabello-occipital length in the ten male skulls was 194 mm., the minimum 175, whilst the mean was 183 mm. The basi-bregmatic height was measured in six specimens; in five, presumably males, it ranged from 121 to 135 mm.; whilst another, possibly a female, was 123 mm. The horizontal circumference had a maximum of 550 mm. and a minimum of 500, whilst the mean of seven males was 525 mm. The maximum vertical transverse circumference in the males was 481 mm., the minimum was 426 mm., and the mean of five males was 442·6 mm. The maximum longitudinal circumference in three males was 503 mm., the minimum 494, and the mean was 498 mm.

Few individual peculiarities were found in these crania. In one specimen the right squamous-temporal articulated at the pterion directly with the frontal; there were no epipteric bones. In one specimen Wormian bones were present in the lambdoidal suture; another had a sutural bone in the sagittal suture, 13 mm. behind the bregma. In one skull a vertical transverse depression was behind the coronal suture. No skull had a third occipital condyle or paroccipital process, but in two crania the under surface of the jugal process was tuberculated. In one specimen an infraorbital suture was visible.

The length-breadth or cephalic index ranged in thirteen skulls from 86·3 to 72·2; six crania were upwards of 80, two being hyperbrachycephalic; three were between 78·3 and 79·1, approaching brachycephalic; two were 76·4 and 76·1 respectively, two had the index below 75 and were dolichocephalic. The mean length-breadth index of the series of skulls was 79·7, *i.e.*, approximately brachycephalic.

The length-height or vertical index could be taken in only six crania. In five males it ranged from 72·4 to 65·8, the mean of the series being 71·7. In no skull was the basi-bregmatic diameter equal to the parieto-squamous in the same specimen; but in one male the breadth only exceeded the height by 2 mm.; the cephalic index was always greater than the vertical index.

The projection of the upper jaw, as estimated by the gnathic index, ranged in the four males that could be measured from 103·2 to 96, and the mean was 98·8; the mean was mesognathous; in a female skull it was 103·2, barely prognathous. In one specimen the nasal height was short and the nares wide, so as to give the platyrhine index 55·3, but three other specimens had a mean leptorhine index 44. The orbital index of two males placed them in the megaseme class with the orbits high in relation to the width, whilst in two others the index was microseme, the width being proportionally greater than the height.

The palato-alveolar index had in the four skulls measured a considerable range of variation; two specimens were dolichuranic, one mesuranic, and one brachyuranic; the mean of the series, 109·4, was dolichuranic.

The absence of the lower jaw prevented one from taking the complete facial index, but in each of the four specimens the maxillo-facial index was leptoprosopic, and the mean of the series was 53·9.

Owing to injury to the cranial box, the internal capacity could only be taken in one specimen, 1305 c.c.

b. North Berwick.—Two crania from North Berwick are in the collection of the Henderson Trust. They were found in the old churchyard nearly fifty years ago, and are referred to in the *Prehistoric Annals of Scotland.** In both the facial bones had been broken away. They were from persons well advanced in life, and the sutures of the cranial vault had almost disappeared in the outer table. One was much larger than the other, and was obviously a male. It was a good example of a well-filled skull. The forehead was capacious, and the frontal suture had not quite disappeared. The vertex was flattened, and the descent from the obelion to the occipital squama was abrupt, but without evidence of parieto-occipital flattening. The skull was essentially brachycephalic, though the index, 79·8, was fractionally below the lower limit which custom

* 1st edition, p. 175, 1851.

assigns to that type of cranium. The basi-bregmatic diameter was much less than the greatest breadth. The cranial capacity was 1600 c.c.

The smaller of the two crania was apparently a female. It was narrow in the frontal region, and gradually widened backwards to the parietal eminences, where the cranium had the greatest transverse diameter. The flattening at the vertex was also well marked in this skull. In the relations of length and breadth it was distinctly brachycephalic, 80·6. An injury to the base prevented one from taking the basi-bregmatic diameter and internal capacity.

c. Dunbar.—The skull, an adult male, was obtained at a burial-place in Dunbar, and is numbered 41 in the catalogue of the Henderson Trust. It was broadly ovoid in form and sloped gently backwards and downwards in the parieto-occipital region. The cephalic index, 77·5, was mesaticephalic; the basi-bregmatic diameter was much below the parieto-squamous. The face was orthognathic, the nose was leptorhine, the orbit was wide in relation to the height, and the palate was hyperbrachyuranic. The skull was metopic, but the most striking peculiarity was a double parietal bone on the left side.* The intraparietal suture was strongly denticulated, and completely divided the left bone into two unequal moieties. The upper part was 106 mm. in antero-posterior diameter and 78 mm. in vertical diameter; the lower part was 104 mm. in antero-posterior and 38 mm. in its least vertical diameter. At the anterior or coronal end of the dividing suture two small sutural bones were interposed between it and the coronal suture, and the skull was depressed somewhat in this region. The lambdoidal and coronal sutures were strongly denticulated, and a small epipteric bone was situated in each pterion. All these sutures were distinctly marked on the inner table, though much more feebly denticulated than in the outer table, and it was observed that small sutural bones were differentiated in the inner table both at the lambdoidal end of the intraparietal suture and within the lambdoidal suture, additional to those already referred to in the exterior of the skull. A short paracondyloid process projected downwards from the under surface of the left jugal process.

Mid-Lothian. Tables III., IV., V., VI., VII. Plates II., V.

Collections of skulls were obtained from different localities in the county of Mid-Lothian. They may conveniently be arranged in three groups:—

a. Those collected in churches and churchyards in rural districts.

b. Those obtained from a church and churchyard near the sea coast.

c. Those obtained from interments in Edinburgh or its immediate vicinity.

a. Of those gathered in rural districts seven were procured from a churchyard on the western border of the county. They are distinguished in Table III. by the letter R.

* I have described a similar condition in the right parietal of an Admiralty Islander in *Challenger Reports*, 1884, part xxix. plate iv. p. 57, and in the right parietal of an Australian in *Journal of Anatomy and Physiology*, vol. xxv. pp. 462–473.

Two skulls from villages on the northern slope of the Pentlands are marked B and C in the same Table. Four skulls from Lasswade, marked L in this Table, have been for a number of years in the collection of the Henderson Trust.

The nine skulls marked R, B, and C are from late interments, and I have little doubt are fairly representative of the country people of the western part of the county. I propose to look at them as a group. They were apparently eight males and one aged female. The sex of some of the persons was known to my correspondent from whom I obtained the specimens. The lower jaw was present in four skulls. Three crania were metopic. In five specimens, including the old female, the sutures were in process of being obliterated. The woman's skull was edentulous, and the measurements of the lower jaw, when compared with those in which the teeth were mostly present, show the effect produced by absorption of the alveolar border. In two specimens the teeth were much worn from use, in others they were much decayed; but in two the cusps of the molars were distinct.

Norma verticalis.—Four skulls were broadly ovoid, but the others were longer in relation to the breadth. They were all well filled, the transverse arc at the vertex was low and rounded, and there was no sagittal keel or ridge. In one a sagittal vertical transverse constriction was behind the coronal suture. In two metopic specimens the parietal and frontal eminences were prominent. The skulls were cryptozygous. The interzygomatic diameter exceeded the intermalar, stephanic and asterionic; the stephanic diameter exceeded the asterionic, but in three cases by only 2, 3 and 4 mm. respectively. The side walls were not vertical, and the maximum transverse diameter, except in one metopic skull, was in the region of the squamous suture.

Norma lateralis.—The forehead in most of the skulls had only a slight slope backwards, and the glabella and supraorbital ridges were moderately prominent. One of the three metopic crania had the widest minimum frontal diameter in the group; another was the third, and another the fourth in width in the same region. Owing to the flattening at the summit of the skull the antero-posterior curve of the vertex was moderate; the parieto-occipital region did not slope steeply downwards, though in some skulls it was a little more abrupt than in others: in a few crania the occipital squama did not project much behind the inion. The skulls rested behind on the conceptacula cerebelli, and in no specimen did the mastoids touch the table. The nasal bones were well formed, and with the bridge prominent and slightly concave. The maxillo-nasal spine was well marked, and the nasal floor was separated by a sharp ridge from the incisive region of the jaw. The maximum longitudinal arc in the males was 397 mm., the minimum 371, and the mean was 382·1 mm. The occipital arc, except in two skulls, was shorter than either the frontal or parietal. In four males the frontal arc was shorter than the parietal; in four the reverse was seen. The maximum basi-nasal diameter was 105 mm., the minimum 96, and the mean of seven males was 101.

The maximum transverse diameter was 155 mm., the minimum 136 mm., and the mean was 143·9 mm. The maximum glabello-occipital length was 194 mm., the

TABLE III.—*Mid-Lothian Rural Districts.*

Collection,	Rw.	Metopic. Ry.	Rt.	Rx.	Rz.	Metopic. Rv.	Rs.	C.	Metopic B.	L. Ht. 66	L. Ht.383	L. Ht.386	L. Ht.566
Age,	Adult.	Adult.	Ad.	Ad.	Ad.	Ad.	Aged.	Ad.	Ad		Aged.	Adult.	Aged.
Sex,	M.	M.	M.	M.	M.	M.	F.	M.	M.	M.	M.	M.	F.
Cubic capacity,	...	1452	1545	1440	1500	1420	1405	1590	1470	1400	1800	1660	1625
Glabello-occipital length,	194	189	188	181	187	185	181	191	187	191	204	196	193
Basi-bregmatic height,	141	127	137	134	135	130	131	142	129	133	145	138	140
Vertical Index,	*72·7*	*67·2*	*72·9*	*74·*	*72·2*	*70·3*	*72·4*	*75·4*	*69·*	*69·6*	*71·1*	*70·4*	*72·5*
Minimum frontal diameter,	96	102	103	99	88	104	95	95	99	94	104	97	106
Stephanic frontal diameter,	129	129	117	130	118	124	117	120	115	104	...	118	119
Asterionic diameter,	126	108	108	110	110	108	113	118	112	107	111	114	113
Greatest parieto-squamous breadth,	155s.	136s.	144s.	145s.	142s.	140p.	144s.	148s.	141s.	137p.	150s.	143s.	153s.
Cephalic Index,	*79·9*	*72·*	*76·6*	*80·1*	*75·9*	*75·7*	*79·6*	*77·5*	*75·4*	*71·7*	*73·5*	*73·*	*79·3*
Horizontal circumference,	553	535	528	522	530	532	523	548	527	532	572	546	550
Frontal longitudinal arc,	140	148	129	123	130	130	126	146	127	130	141	134	139
Parietal ,, ,,	124	132	134	131	133	136	127	134	120	130	140	128	135
Occipital ,, ,,	133	111	119	117	125	109	116	115	121	120	132	124	126
Total ,, ,,	397	381	382	371	388	375	369	395	368	380	413	383	400
Vertical transverse arc,	332	305	318	312	313	304	311	320	295	298	320	305	329
Basal transverse diameter,	131	119	116	122	119	123	117	128	...	121	136	127	130
Vertical transverse circumference,	463	424	434	434	432	427	428	448	...	419	456	432	459
Length of foramen magnum,	38	36	32	34	35	35	36	37	36	33	38	41	32
Basi-nasal length,	102	96	105	99	100	100	95	104	102	104	108	103	105
Basi-alveolar length,	100	98	95	91	94	98	...	91	105	...	...	97	91
Gnathic Index,	*98·*	*102·1*	*90·5*	*91·9*	*94·*	*98·*	...	*87·5*	*102·9*	...	...	*94·2*	*86·7*
Total longitudinal circumference,	537	513	519	504	523	510	500	536	506	517	559	527	537
Interzygomatic breadth,	136	134	135	...	125	...	121	132	138	131	143	131	135
Intermalar ,,	115	121	121	...	112	112	107	115	121	117	127	112	118
Nasio-mental length,	122	122	136	...	...	...	...	...	...	...	...	...	...
Complete Facial Index,	*89·7*	*91·*	*100·*	...	...	...	...	...	...	...	...	...	...
Nasio-alveolar length,	75	72	75	74	67	71	...	75	73	...	...	74	70
Maxillo-facial Index,	*55·*	*53·7*	*55·5*	...	*53·6*	...	...	*56·8*	*52·9*	...	...	*56·5*	*51·8*
Nasal height,	57	50	53	53	50	51	54	54	52	52	58	56	52
Nasal width,	23	22	24	21	23	22	26	22	24	28	22	24	24
Nasal Index,	*40·3*	*44·*	*45·3*	*39·6*	*46·*	*43·1*	*48·1*	*40·7*	*46·*	*53·5*	*37·9*	*42·8*	*46·*
Orbital width,	41	39	40	37	38	41	38	40	38	39	40	40	41
Orbital height,	36	31	36	34	31	33	31	33	34	33	38	37	37
Orbital Index,	*87·8*	*79·5*	*90·*	*91·9*	*81·6*	*80·5*	*81·6*	*82·5*	*89·5*	*84·6*	*95·*	*92·5*	*90·2*
Palato-alveolar length,	56	55	58	54	53	54	...	52	58	...	...	54	52
Palato-alveolar breadth,	60	59	57	...	64	...	...	61	...	...	...	...	60
Palato-alveolar Index,	*107·*	*107·*	*98·2*	...	*120·*	...	...	*117·*	...	...	...	...	*115·3*
Lower jaw. Symphysial height,	33	36	35	...	...	...	28	...	...	...	...	...	...
Lower jaw. Coronoid ,,	73	70	71	...	...	...	66	...	...	...	...	...	...
Lower jaw. Condyloid ,,	77	68	73	...	...	...	62	...	...	...	...	...	...
Lower jaw. Gonio-symphysial length,	83	94	92	...	...	...	82	...	...	...	...	...	...
Lower jaw. Inter-gonial width,	103	99	100	...	...	...	83	...	...	...	...	...	...
Lower jaw. Breadth of ascending ramus,	34	45	34	...	...	...	31	...	...	...	...	...	...

minimum 181, and the mean of the seven male skulls was 187·8. The basi-bregmatic height ranged in the males from 144 to 127 mm., and the mean of the series was 134·4 mm. The horizontal circumference had a maximum 553 mm., a minimum 522, and the mean was 534·3 mm. The maximum vertical transverse circumference was 463 mm., the minimum 419, and the mean 436·9 mm. The maximum total longitudinal circumference was 536 mm., the minimum 504, and the mean 519·2. The intra-cranial capacity ranged in the males from 1590 to 1420 cub. cent., and the mean was 1431 cub. cent. The crania were, with two exceptions, in the megacephalic group. The only female skull, like two of the males, was mesocephalic in its capacity.

Few individual peculiarities were seen in these skulls. Two skulls had each a right epipteric bone and one a left: in one of the metopic skulls the right squamous-temporal just touched the frontal; in another skull the alisphenoid had a narrow articulation with the parietal. Small Wormian bones were present in three crania in the lambdoidal suture. There was no condylus tertius or paramastoid process, though in one the jugal process was strongly tuberculated. The remains of an infraorbital suture were seen in these crania. In two the external pterygoid plate was prolonged for some distance backwards into a pointed process, due to partial ossification of the pterygo-spinous ligament.

The length-breadth or cephalic index of eight males ranged from 80·1 to 72; one skull was brachycephalic, 80·1; three were between 77·5 and 79·9; one only was dolichocephalic and three ranged from 75·4 to 76·6. The mean was 76·6, which places the series in the mesaticephalic group. The cephalic index of the female was 79·6.

The length-height or vertical index ranged in the males from 75·4 to 67·2, and the mean of the group was 71·7. The parieto-squamous diameter exceeded the basi-bregmatic in each specimen, so that the cephalic index was greater than the vertical.

The projection of the upper jaw, as estimated by the gnathic index, ranged from 102·4 to 87·5, and the mean of the series was 94·6. The average index was therefore orthognathic, and only three specimens were mesognathic. In each specimen the nose was narrow or leptorhine, and the mean of the series was 43. The orbital index ranged from 91·9 to 79·5; the mean of the series was 85, *i.e.*, mesoseme. Of the eight males three were megaseme, one was mesoseme, whilst four were microseme. The palato-alveolar index could only be determined in five males, the mean of which was 109·8, *i.e.*, on the verge of being mesuranic; but three of the skulls were dolichuranic, the other two brachyuranic.

The complete facial index could only be taken in three skulls, which yielded a mean of 93·5, *i.e.*, leptoprosopic, with high and relatively narrow faces; a conclusion as to the character of the face which was borne out in the skulls where the lower jaw was absent, by the proportion of the length of the upper jaw to the interzygomatic breadth, in which the mean maxillo-facial index was 54·6, also leptoprosopic.

The four crania from Lasswade, marked L in Table III., formed a part of the Henderson Trust collection. They were obtained about forty years ago. Three were undoubted male skulls. The fourth skull was said to be that of a woman. Its dimensions and internal capacity were considerably greater than the average of the female sex ; a shallow transverse depression was situated immediately behind the coronal suture, probably due to the wearing in infancy of a band across the head. In its general form this skull corresponded to the crania from Fife, which approached most closely to the brachycephalic character. Two male crania, again, were longer, more capacious, but not so wide, and had a more ovoid form, with dolichocephalic proportions. They were not so well filled, and had a ridge-like elevation in the sagittal region. The glabella was prominent, and the nasion considerably depressed ; the bridge of the nose also was moderately projecting. A third male skull, said to be that of a musician in his sixty-seventh year, was much smaller in its dimensions than those just referred to. It possessed a strong glabella and supraorbital ridges, a deep depression at the nasion, a very prominent nose and large nasal bones. It was distinctly dolichocephalic, with an index of 71·7, and in its *norma verticalis* was an elongated ovoid, and not so well filled as the crania previously described. The nasal index was platyrhine ; the orbital index was mesoseme, and the cranial capacity was 1400 c.c. The upper jaw was edentulous, but the denticulations of the cranial sutures were well marked on the outer table. This skull possessed a large left epipteric bone and some small Wormian bones in the lambdoidal suture.

No. 566 was free from Wormian bones, but in 386 a small ossicle was in the coronal and others in the lambdoidal suture. The sutures in 383 were so much ossified that it was impossible to say whether ossicula had at one time been present. None of the Lasswade skulls had a third occipital condyle ; in two the under surface of the jugal process was tuberculated : in one were faint indications of an infraorbital suture.

In each of the four crania the vertical index was less than the cephalic ; the jaw was orthognathous ; the maxillo-facial index was lepto-prosopic. In three crania the nasal index was leptorhine ; the orbital index was megaseme ; the cranial capacity had the unusual average of 1698 c.c. In all four crania the occipital longitudinal arc was the smallest and the frontal was the longest.

b. The series of crania from a village near the sea coast were mostly from intramural interments, and were found lying loose in the earth below the flooring of a church ; two specimens were, however, from graveyard burials. The series is marked I. in Table IV.

This collection of skulls consisted apparently of eight males and seven females, in only one of which the lower jaw was present : several of the specimens were so much broken that only partial measurements could be taken. One skull, a female, was, judging from the dentition and the unossified basi-cranial synchondrosis, between 18 and 20 years. The others were all adults, and at least three specimens were advanced in life. No skull was metopic, but a calvaria, too much broken to be included in the measurements in the Table, had an open frontal suture.

Norma verticalis.—Many of the skulls were broadly ovoid. Two females were

Table IV.—*Mid-Lothian. Sea-Coast Village.*

Collection,	Ia.	Ib.	Ic.	Id.	Ie.	Ig.	Ih.	Ii.	Ik.	Il.	Im.	In.	Io.	Ip.
Age,	Ad.	Ad.	Ad.	Aged.	Aged.	Aged.	Ad.	Ad.	Ad.	Aged.	18–20	Ad.	Ad.	Ad.
Sex,	M.	M.	M.	M.	M.	M.	M.	F.	F.	F.	F.	F.	F.	F.
Cubic capacity,	1580	1465	...	1400	...	...	1745	1290	1300	...	1495	..	1380	1130
Glabello-occipital length,	200	188	193	175	195	190	199	180	189	169	183	193	186	169
Basi-bregmatic height,	144	131	126	128	...	...	132	121	121	123	134	131	123	118
Vertical Index,	*72·0*	*69·7*	*65·3*	*73·1*	...	...	*66·3*	*67·2*	*64·0*	*72·8*	*73·2*	*67·9*	*66·1*	*69·8*
Minimum frontal diameter,	97	91	95	96	98	101	100	99	89	88	93	92	95	89
Stephanic, ,, ,,	111	110	128	118	113	110	119	108	110	109	114	108	114	97
Asterionic diameter,	112	100	106	111	116	111	119	107	105	109	107	100	106	101
Greatest parieto squamous breadth,	147p.	138s.	157s.	143s.	150s.	142s.	150s.	134s.	131s.	138s.	143s.	138s.	141s.	131s.
Cephalic Index,	*73·5*	*73·4*	*81·3*	*81·7*	*76·9*	*74·7*	*75·4*	*74·4*	*69·3*	*81·7*	*78·1*	*71·5*	*75·8*	*77·5*
Horizontal circumference,	550	520	555	510	545	531	560	510	518	487	515	...	522	483
Frontal longitudinal arc,	136	130	148	122	140	...	126	120	118	231	124	128	133	122
Parietal ,, ,,	123	123	119	241	125	...	134	122	125		123	133	108	112
Occipital ,, ,,	139	122	119		...	...	134	115	123	112	118	121	131	107
Total ,, ,,	398	375	386	363	...	...	394	357	366	343	365	382	372	341
Vertical transverse arc,	328	310	334	308	310	300	320	272	275	283	308	...	308	277
Basal transverse diameter,	125	114	130	121	131	121	127	119	115	119	115	...	114	111
Vertical transverse circumference,	453	424	464	429	441	421	447	391	390	402	423	...	422	383
Length of foramen magnum,	37	34	34	34	...	...	36	39	34	34	35	35	35	31
Basi-nasal length,	106	100	104	94	...	...	106	99	99	95	101	102	98	90
Basi-alveolar length,	103	95	93	...	...	...	105	95	95	...	91	...	...	...
Gnathic Index,	*97·2*	*95·*	*89·4*	...	...	...	*99·1*	*96·0*	*96·0*	...	*90·1*	...	...	...
Total longitudinal circumference,	541	509	524	491	...	...	536	495	499	472	501	...	505	462
Interzygomatic breadth,	134	...	141	...	...	...	134	...	117	...	...	...	...	...
Intermalar,	121	109	113	...	...	...	117	...	104	...	111	...	...	...
Nasio-mental length,	...	117	...	...	...	...	...	...	...	...	...	...	...	...
Complete Facial Index,	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Nasio-alveolar length,	81	73	71	...	...	...	72	67	67	...	67	...	...	...
Maxillo-facial Index,	*60·4*	...	*50·3*	..	...	...	*53·7*	...	*57·2*	...	...	...	...	...
Nasal height,	57	54	53	...	...	...	53	51	47	...	48	...	...	...
Nasal width,	27	22	22	...	...	...	23	22	22	...	21	...	...	...
Nasal Index,	*47·4*	*40·7*	*41·5*	...	...	...	*43·4*	*43·1*	*46·8*	...	*43·8*	...	...	...
Orbital width,	39	38	39	...	...	...	40	37	35	...	36	...	...	...
Orbital height,	38	34	36	...	...	...	33	35	31	...	32	...	...	...
Orbital Index,	*97·4*	*89·5*	*92·3*	...	...	...	*82·5*	*94·6*	*88·6*	...	*88·9*	...	...	...
Palato-alveolar length,	57	55	50	...	...	...	56	53	51	...	48	...	...	...
Palato-alveolar breadth,	60	58	...	...	...	...	58	58	57	...	60	...	...	...
Palato-alveolar Index,	*100·5*	*105·4*	...	...	...	...	*103·5*	*109·4*	*111·7*	...	*125*	...	...	...
Lower jaw. Symphysial height,	...	30	...	...	...	...	...	...	...	...	...	...	...	...
Lower jaw. Coronoid ,,	...	68	...	...	...	...	...	...	...	...	...	...	...	...
Lower jaw. Condyloid ,,	...	70	...	...	...	...	...	...	...	...	...	...	...	...
Lower jaw. Gonio-symphysial length,	...	85	...	...	...	...	...	...	...	...	...	...	...	...
Lower jaw. Inter-gonial width,	...	96	...	...	...	...	...	...	...	...	...	...	...	...
Lower jaw. Breadth of ascending ramus,	...	37	...	...	...	...	...	...	...	...	...	...	...	...

more elongated and narrow in proportion to the length; one had a cephalic index 71·5, the other 69·3. Two males and one female, on the other hand, were much wider in proportion to the length, and had a cephalic index between 81 and 82: in one of these (*c*) the width of the skull, 157 mm., was strikingly marked, being one of the broadest measured in this investigation. The skulls generally were well filled, the parietal eminences were not protuberant, as a rule no sagittal ridge, and with the vertex rounded in the vertical transverse arc. The side walls were usually not vertical, but were convex in the squamous regions. In the female crania the frontal eminences were moderately prominent. In the few skulls in which it could be measured, the interzygomatic diameter exceeded the intermalar, stephanic and asterionic. In eight specimens the stephanic diameter exceeded the asterionic: in four the reverse was the case: in two they were equal: with one exception the greatest breadth was in the squamous region. The crania were cryptozygous. In one skull the occipital longitudinal arc was the longest, in one the parietal was the shortest, in three it was the longest, but in the greater number the frontal arc was the longest.

Norma lateralis.—In the female skulls the forehead approached the vertical. In the males it receded somewhat, and the glabella and supraorbital ridges were distinctly marked. In the skulls generally, the slope in the parieto-occipital region was moderate, but in the specimens whose proportions were brachycephalic it was more abrupt, and in the skull *d* (cephalic index 81·7) the occipital squama did not project behind the inion, and had a slope upwards and forwards not unlike that seen in the well known Neanderthal skull. Most of the skulls rested behind on the conceptacula cerebelli. The nasal bones were well formed, as a rule moderately projecting, but in *h* more strongly so, and the bridge usually was slightly concave upwards. Except where the glabella was most marked, the nasion was not depressed. As a rule the maxillo-nasal spine was well seen, and a sharp edge separated the floor of the nose from the incisive surface of the superior maxilla. The crania were remarkably free from sutural bones, which showed themselves but seldom in the lambdoidal suture as small denticles; two specimens had small epipteric bones in the pterion. On the right side of *b* the squamous-temporal articulated by a broad tongue directly with the frontal, an arrangement which was obviously due to the fusion of a large epipteric bone with the squamous-temporal. There was considerable variety in the breadth of the alisphenoido-parietal articulation. No third condyle or par-occipital process was seen, but the jugal process was frequently tuberculated. The infraorbital suture had disappeared.

Six of these crania were 190 mm. or upwards in glabello-occipital diameter, and one reached 200 mm. Eight exceeded 140 mm. in greatest breadth, and of these three were 150 mm. or upwards. Six crania had the cephalic index below 75, were dolichocephalic; three were between 75 and 76·9, approximating to the dolichocephalic numerical standard; three were above 80, brachycephalic, and two were 77·5 and 78·1 respectively, *i.e.* approximating to the brachycephalic group. The lowest vertical index was 64, and the highest was 73·2, and in each skull this index was less than the cephalic.

In seven males the mean horizontal circumference was 538 mm., and the mean vertical transverse circumference was 439·8, whilst in five males the mean longitudinal circumference was 520 mm. One skull was mesognathous, the six others capable of being measured were orthognathous. In each skull the nasal index was leptorhine; five orbits were megaseme, two were mesoseme, one microseme. Two skulls had hyperdolichuranic palates, two dolichuranic, one hyperbrachyuranic, two mesuranic. The complete facial index could not be obtained, but as the maxillo-facial index was in each specimen leptoprosopic the proportions for the entire region were without doubt high-faced.

c. The skulls obtained from interments either in Edinburgh or Leith were thirty-three in number, and many of them belonged to the collection of the Henderson Trust. Nine are referred to in WILSON's *Prehistoric Annals of Scotland*,* and of these six were obtained in 1832 in the course of excavating the site of the Law Courts which were built on ground which had at one time been a city cemetery, situated between St Giles' Church and the Cowgate. It was probably some time after the Reformation before this was disused as a place of burial. One skull was found at the top of the Vennel whilst digging the foundations of a school on the site of the old wall built after the battle of Flodden, and another was procured in 1830 on the northern slope of the Castle Hill. Two skulls were found in 1854 at St Leonard's Hill, where at one time there is believed to have been a cemetery. Nine skulls came from Leith, and were obtained in 1831 in the course of making an excavation in Constitution Street.† It was thought at the time that they might be the remains of persons who fell at the siege of Leith by the English in 1559, but from the appearance of the skulls it is not likely that they are so old. As a part of Constitution Street was carried through the churchyard of South Leith, they had probably been interred there at a more recent period. Fourteen skulls belonged to the University collection. Two were obtained from beneath the pavement of St Giles' during the course of the recent remodelling of its interior; a third was presented by Sir ARTHUR MITCHELL, and had been procured in the grounds of St Roque, which at one time had been the site of an old ecclesiastical establishment. Nine were obtained in the course of recent excavations which interfered with the former burying-ground of the Kirk o' Field; one was found in digging the foundations of the Solicitors' Library, and one was from a disused burial-ground.

This series of thirty-three crania from burial-grounds in Edinburgh and Leith exhibited differences in form and proportions, which is only what might be expected from the mixing of nationalities in large centres of population, especially when one is a seaport town.

I have regarded nineteen crania as males and fourteen as females, and of the latter one (H.T. 408) was about ten years old, judging from the dentition. The lower jaw was absent in the majority of the skulls. Five crania were metopic, and in some others the frontal suture was not fully obliterated near the nasion. In several specimens the

* 1851, pp. 166, 176.; also *Phrenological Journal and Miscellany*, vol. viii. p. 185.

† *Phrenological Journal and Miscellany*, vol. vii. p. 287.

sutures of the vault were to a large extent obliterated. Three were edentulous or nearly so. In several the facial bones were broken away and the condition of the teeth could not be ascertained. Wormian bones were present in eleven crania in the lambdoidal suture; one had a fontanelle bone behind the bregma. Two crania had epipteric bones on both sides; four on one side only. One had a normal left pterion, but on the right side a broad tongue-like process of the squamous-temporal articulated with the frontal bone. No skull had a third occipital condyle or a par-occipital process, though occasionally the jugal process was tuberculated on its under surface; a complete pterygo-spinous foramen was not present, but in one specimen a process from the external pterygoid almost reached the spine of the sphenoid.

In five male crania the glabella and supraorbital ridges were well marked, but in the others they were moderate. The forehead as a rule had only a slight backward slope. The vertex was not ridged in the sagittal region, and in the majority the slope outwards to the parietal eminences was not steep, and the skulls had a well filled appearance. Many of the specimens sloped rapidly downwards and backwards in the parieto-occipital region, and in these crania the *norma verticalis* had a broadly ovoid outline. In others again the curve was much longer and the skulls were elongated and ovoid. In the hyperdolichocephalic skull, H.T. 406, the occipital squama projected considerably behind the inion and superior curved line. H.T. 32 and 412, which I have regarded as female skulls, showed a broad transverse depression immediately behind the coronal suture, probably produced by wearing a band across the head during infancy and early childhood.

More than one half of the skulls possessed considerable breadth both absolutely and relatively to the length, and eighteen crania had a length-breadth index of 77·5 and upwards. In eight of these the index was upwards of 80 and in two was above 85, *i.e.*, hyperbrachycephalic. These crania were therefore either brachycephalic or in the highest term of the mesaticephalic group, and this character was shown not only by their numerical proportions but by their general configuration.

Seven crania had a length-breadth index below 75, one of which was as low as 69·8, whilst six skulls ranged from 75 to 77·4. Less than one half were dolichocephalic, or in the lower term of the mesaticephalic series, and of these one was hyperdolichocephalic.

The vertical index in one skull was the same as the cephalic index; in the others it was less, and in many instances considerably below it. The highest vertical index was 76·0, and the cephalic index of the same skull was 85·4. The lowest vertical index was 64·8, and the cephalic index of the same skull was 69·8. Eleven crania had the vertical index below 70 and were therefore chamæcephalic; two were above 75, hypsicephalic; twelve were between 70 and 75, metriocephalic.

In seven crania the occipital arc was longer than the parietal, in one it was longer than the frontal: in twenty-two the frontal arc was longer than the parietal, but in nine the proportion was reversed. The mean horizontal circumference in the males was 520 mm., in the females 502 mm.: the mean vertical transverse circumference in the males

Table V.—*Edinburgh.*

	Old Cemetery.								Vennel.	Castle Hill.	St. Leonards.		St. Rocque.
Collection,	Ht. 30.	Ht. 31.	Ht. 34.	Ht. 35.	Metopic. E.U.A.M.	Ht. 32.	Ht. 33.	E.U.A.M.	Ht. 28.	Ht. 29.	Ht. 47.	Ht. 46.	E.U.A.M.
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Aged.	Ad.	Ad.
Sex,	M.	M.	M.	M.	M.	F.	F.	F.	F.	M.	M.	F.	F.
Cubic capacity,	1390	...	1410	...	1505	1270	...	...	...	...	...	1540	1100
Glabello-occipital length,	182	180	175	183	184	175	161	169	188	189	194	174	169
Basi-bregmatic height,	126	...	127	132	125	124	120	...	130	...	...	132	122
Vertical Index,	*69·2*	...	*72·6*	*72·1*	*67·9*	*70·9*	*74·5*	...	*69·1*	...	...	*75·9*	*72·2*
Minimum frontal diameter,	91	95	94	95	102	95	83	88	102	98	104	94	86
Stephanic frontal diameter,	107	105	115	115	116	105	98	105	112	111	118	122	106
Asterionic diameter,	111	107	115	101	110	105	105	97	112	118	118	110	100
Greatest parieto-squamous breadth,	143s.	142s.	148s.	141p.	148s.	138p.	133s.	138p.	142s.	143s.	145s.	153s.	135
Cephalic Index,	*78·6*	*78·9*	*84·6*	*77·0*	*80·4*	*78·9*	*82·6*	*81·7*	*75·5*	*75·7*	*74·7*	*87·9*	*79·9*
Horizontal circumference,	512	524	512	512	538	494	470	486	525	533	542	518	488
Frontal longitudinal arc,	128	125	125	135	138	128	111	123	128	132	131	132	121
Parietal ,, ,,	115	125	114	122	125	126	107	120	...	...	141	118	112
Occipital ,, ,,	116	123	116	114	113	104	111	109	...	...	104	110	114
Total ,, ,,	359	373	355	371	376	358	329	352	...	...	376	360	347
Vertical transverse arc,	290	300	300	304	313	295	272	295	304	298	310	311	298
Basal transverse diameter,	127	...	129	118	125	111	109	115	121	133	125	132	111
Vertical transverse circumference,	417	...	429	422	438	406	381	410	425	431	435	443	409
Length of foramen magnum,	34	...	32	35	35	31	33	...	...	...	...	33	31
Basi-nasal length,	99	...	93	101	95	92	89	...	98	...	...	99	93
Basi-alveolar length,	93	...	...	...	89	...	...	...	...	...	...	91	93
Gnathic Index,	*93·9*	...	...	...	*93·7*	...	...	...	...	...	...	*91·9*	*100·*
Total longitudinal circumference,	492	...	498	507	506	481	451	...	...	...	...	492	471
Interzygomatic breadth,	130	...	132	...	134	...	...	118	...	...	...	...	...
Intermalar,	114	...	...	...	118	...	...	101	...	...	...	116	...
Nasio-mental length,	...	...	...	...	...	...	...	...	...	...	...	...	...
Complete Facial Index,	...	...	...	...	...	...	...	...	...	...	...	...	...
Nasio-alveolar length,	68	...	...	...	67	...	...	70	...	...	...	66	60
Maxillo-facial Index,	*52·3*	...	...	...	*50·*	...	...	*59·3*	...	...	...	...	...
Nasal height,	52	...	...	...	52	...	...	50	...	...	...	...	46
Nasal width,	24	...	...	...	23	...	...	24	...	...	...	...	22
Nasal Index,	*46·0*	...	...	...	*44·2*	...	...	*48·0*	...	...	...	...	*47·8*
Orbital width,	40	...	...	...	38	...	...	35	...	...	...	38	38
Orbital height,	35	...	...	...	37	...	...	34	...	...	...	35	30
Orbital Index,	*87·5*	...	...	...	*97·4*	...	...	*97·1*	...	...	...	*92·1*	*78·9*
Palato-alveolar length,	54	...	...	...	50	...	...	52	...	...	...	...	...
Palato-alveolar breadth,	62	...	...	...	63	...	...	58	...	...	...	...	...
Palato-alveolar Index,	*114·8*	...	...	...	*126·*	...	...	*111·5*	...	...	...	...	...

Table VI.—*Edinburgh.*

	Kirk o'Field—E.U.A.M.									Cowgate.	Buccleugh Parish.
Collection,	Metopic. A.	B.	C.	D.	E.	Metopic. F.	G.	H.	I.		
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	20	Ad.	Ad.	Ad.	Ad.
Sex,	F.	F.	M.	F.	M.	F.	F.	M.	M.	M.	M.
Cubic capacity,	1315	1320	...	1250	...	1300	1390	1460	...	1300	1360
Glabello-occipital length,	185	175	186	180	176	179	178	187	192	171	183
Basi-bregmatic height,	125	120	133	130	127	130	124	130	...	130	121
Vertical Index,	*67·6*	*68·6*	*71·5*	*72·2*	*72·2*	*72·6*	*69·7*	*69·5*	...	*76·*	*66·1*
Minimum frontal diameter,	92	88	96	90	90	91	88	91	102	92	95
Stephanic frontal diameter,	99	103	110	105	99	114	111	102	112	113	116
Asterionic diameter,	103	106	107	104	97	101	98	112	108	106	106
Greatest parieto-squamous breadth,	134	138	133	135	...	131	139	139s.	145s.	146s.	141s.
Cephalic Index,	*72·4*	*78·9*	*71·5*	*75·*	...	*73·2*	*78·1*	*74·3*	*75·5*	*85·4*	*77·*
Horizontal circumference,	516	499	515	504	...	504	505	526	545	500	515
Frontal longitudinal arc,	122	128	132	127	113	123	128	126	128	125	133
Parietal ,, ,,	137	123	130	122	122	124	130	136	140	114	124
Occipital ,, ,,	118	102	111	111	115	115	112	116	113	120	114
Total ,, ,,	377	353	373	360	350	362	370	378	381	359	371
Vertical transverse arc,	286	296	...	287	...	289	298	296	311	310	303
Basal transverse diameter,	113	120	...	116	...	112	114	123	128	121	121
Vertical transverse circumference,	399	416	...	403	...	401	412	419	439	431	424
Length of foramen magnum,	36	36	35	36	35	36	33	35	...	35	32
Basi-nasal length,	92	91	102	97	98	98	87	101	...	93	95
Basi-alveolar length,	87	...	...	86	...	95	80	96	...	90	89
Gnathic Index,	*94·6*	...	...	*88·7*	...	*96·9*	*91·9*	*95·*	...	*96·8*	*93·7*
Total longitudinal circumference,	505	480	510	493	483	496	490	514	...	487	498
Interzygomatic breadth,	117	...	...	...	...	120	116	129	133	128	...
Intermalar ,,	105	...	...	...	...	106	104	118	120	115	...
Nasio-mental length,	...	...	...	...	...	102	107	121	...	...	...
Complete Facial Index,	...	...	...	...	...	*85·*	*92·2*	*93·8*	...	...	...
Nasio-alveolar length,	60	66	...	...	...	64	64	72	70	67	69
Maxillo-facial Index,	*51·2*	...	...	...	...	*53·3*	*55·1*	*55·8*	*52·6*	*52·3*	...
Nasal height,	46	51	...	49	49	50	58	52	49	51	52
Nasal width,	21	21	...	...	19	21	20	24	22	24	24
Nasal Index,	*45·7*	*41·2*	...	...	*38·8*	*42·*	*34·5*	*46·*	*44·9*	*47·1*	*46·*
Orbital width,	35	...	...	35	39	37	36	37	38	38	37
Orbital height,	29	...	...	33	31	33	32	32	34	30	35
Orbital Index,	*82·9*	...	...	*94·3*	*79·5*	*89·2*	*88·9*	*86·5*	*89·5*	*78·9*	*94·6*
Palato-alveolar length,	46	...	...	...	...	50	45	54	52	51	...
Palato-alveolar breadth,	...	...	...	...	...	55	55	61	64	60	...
Palato-alveolar Index,	...	...	...	...	...	*100·*	*122·2*	*112·9*	*123·*	*117·6*	...
Lower jaw. Symphysial height,	...	...	...	...	...	27	26	32	...	...	...
Lower jaw. Coronoid ,,	...	...	...	...	...	60	49	74	...	...	...
Lower jaw. Condyloid ,,	...	...	...	...	...	58	51	73	...	...	...
Lower jaw. Gonio-symphysial length,	...	...	...	...	...	82	80	93	...	...	...
Lower jaw. Inter-gonial width,	...	...	...	...	...	92	85	102	...	...	...
Lower jaw. Breadth of ascending ramus,	...	...	...	...	...	33	30	37	...	...	...

TABLE VII.—*Leith.*

Collection,	Metopic. Ht. 36	Ht. 404	Ht. 406	Ht. 414	Ht. 416	Ht. 418	Ht. 408	Ht. 411	Metopic. Ht. 412
Age,	Ad.	Ad.	Aged.	Ad.	Ad.	Ad.	about 10	Ad.	Aged.
Sex,	M.	M.	M.	M.	M.	M.	F.	F.	F.
Cubic capacity,	...	...	1450	1295	...	...	1490	1140	...
Glabello-occipital length,	176	180	199	177	179	171	179	171	180
Basi-bregmatic height,	126	132	129	132	...	...	125	118	...
Vertical Index,	*71·6*	*73·3*	*64·8*	*74·6*	...	...	*69·8*	*69·0*	...
Minimum frontal diameter,	97	92	96	96	101	89	93	90	98
Stephanic „ „	120	114	109	113	110	117	120	102	122
Asterionic diameter,	111	110	112	104	116	102	106	102	106
Greatest parieto-squamous breadth,	144p.	141s.	139s.	137s.	150s.	135s.	141s.	134s.	134s.
Cephalic Index,	*81·8*	*78·3*	*69·8*	*77·4*	*83·8*	*78·9*	*78·8*	*78·4*	*74·4*
Horizontal circumference,	510	515	546	508	517	490	514	490	512
Frontal longitudinal arc,	124	124	133	120	121	125	132	120	120
Parietal „ „	121	123	130	123	114	114	122	108	128
Occipital „ „	112	112	120	112	116	...	115	109	104
Total „ „	357	359	383	355	351	...	369	337	352
Vertical transverse arc,	300	298	297	297	314	296	297	278	297
Basal transverse diameter,	118	124	120	119	132	113	120	118	111
Vertical transverse circumference,	418	422	417	416	446	409	417	396	408
Length of foramen magnum,	36	34	35	33	...	...	30	34	...
Basi-nasal length,	95	101	105	101	...	...	92	88	...
Basi-alveolar length,	...	...	...	86	...	...	89	84	...
Gnathic Index,	...	...	...	*85·1*	...	...	*96·7*	*95·4*	...
Total longitudinal circumference,	488	494	523	489	...	...	441	459	...
Interzygomatic breadth,	...	131	132	133	...	...	123	125	115
Intermalar,	...	114	120	119	...	...	108	110	103
Nasio-mental length,	...	...	...	...	...	...	...	...	...
Complete Facial Index,	...	...	...	...	...	...	...	...	...
Nasio-alveolar length,	...	...	...	67	...	...	70	60	67
Maxillo-facial Index,	...	...	...	*50·3*	...	...	*56·9*	*48·*	*58·*
Nasal height,	...	52	50	47	...	...	52	44	48
Nasal width,	...	22	24	21	...	...	22	...	21
Nasal Index,	...	*42·3*	*48·0*	*44·7*	...	...	*42·3*	...	*43·8*
Orbital width,	...	38	40	39	...	...	37	39	38
Orbital height,	...	31	33	30	...	...	37	31	35
Orbital Index,	...	*81·6*	*82·5*	*76·9*	...	...	*100·*	*79·5*	*92·1*
Palato-alveolar length,	...	...	...	48	...	...	50	47	55
Palato alveolar breadth,	...	...	...	...	...	...	54	...	52
Palato-alveolar Index,	...	...	...	...	...	...	*108·*	...	*94·5*

was 425·8, in the females 409 mm. : the mean longitudinal circumference in the males was 499, in the females 478 mm.

In many of the skulls the upper jaw was injured, but in fourteen the gnathic index could be computed. With one exception, in which it reached 100, this index was below 98, orthognathous. Eighteen crania admitted of nasal measurements, and in only two specimens did the nasal index reach 48, so that the nose was leptorhine. In twenty crania the orbital index was obtained ; in nine it was above 89, *i.e.*, megaseme ; in eight it was below 84, microseme ; in the others it was intermediate, *i.e.*, mesoseme.

In ten skulls the palato-alveolar index was computed ; two were hyperdolichuranic ; one was dolichuranic ; one was brachyuranic ; three were hyperbrachyuranic ; the remainder between 110 and 115 were mesuranic.

The complete facial index could only be computed in three skulls, in two of which it was upwards of 90, high faced or leptoprosopic. This character of the face was supported by the measurements of thirteen skulls in which the maxillo-facial index was computed, in all of which, with one exception, it was leptoprosopic.

The cranial capacity could be taken in only eighteen skulls ; in eight males it ranged from 1295 to 1505, with a mean 1396·2 c.c. : in nine adult females from 1100 to 1540, with a mean 1291·6 c.c.

Linlithgowshire. Table VIII.

The collection contains only three skulls from West Lothian ; one was from Linlithgow, one from South Queensferry, and a third from the eastern border of the county. They are all adults ; one is a male, two are females. Although too few in number on which to base a general statement, their more salient characters may be noted.

The female skulls were broadly ovoid in the *norma verticalis*, the vertex was flattened, the forehead smooth, in one almost vertical, in the other somewhat retreating. In one the frontal bone was metopic and marked with grooves, running upwards from the supraorbital foramina ; there was no parieto-occipital flattening in either. The cephalic index was 79·4 and 82·7, brachycephalic in form, though one was fractionally below the numerical standard of that group. The vertical index in both was less than the cephalic. The face was orthognathous, and the maxillo-facial index was leptoprosopic. The anterior nares were relatively wide in one in which the nasal index was mesorhine, but in the other it was leptorhine ; the orbits were mesoseme and microseme respectively, and the palato-alveolar arch was mesuranic and brachyuranic. The skulls showed no unusual ossifications.

The male skull was an elongated ovoid in the *norma verticalis*. The vertex was not ridged, low-arched and with a gentle slope from the sagittal suture to the parietal eminences. The glabella and supraorbital ridges were moderate, the forehead was only slightly retreating. There was no parieto-occipital flattening. The skull was cryptozygous, 200 mm. in greatest length, 145 mm. in breadth, with a cephalic index 72·5 ; the

TABLE VIII.

	West Lothian.			Stirlingshire.		Lanarkshire.		Peebles.			Roxburghshire.	
	Linlithgow.	Queensferry.	Almond.	Kilsyth.	Kilsyth.	Bothwell.	New Lanark	Linton.	Wiston.	Wiston.	Kelso.	
	Metopic.			Bal. Col.	Metopic.				E.U.A.M.		E.U.A.M.	
Collection number,					Ht. 27.			Ht. 40.	A.	B.		
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Advanced.	Ad.	Ad.	Ad.	Aged.	Ad.
Sex,	F.	M.	F.	M.	M.	M.	M.	F.	F.	F.	M.	F.
Cubic capacity,	1320	...	1285	...	...	1310	1580	...	1310	...	1565	1505
Glabello-occipital length,	175	200	173	167	183	177	201	168	167	173	189	190
Basi-bregmatic height,	129	142	121	129	134	120	128	122	128	127	136	130
Vertical Index,	*73·7*	*71·*	*69·9*	*77·2*	*73·2*	*67·8*	*63·7*	*72·6*	*76·6*	*73·4*	*72·*	*68·4*
Minimum frontal diameter,	98	101	90	94	92	92	95	86	95	92	94	91
Stephanic frontal diameter,	113	116	109	111	122	109	120	105	110	107	113	115
Asterionic diameter,	106	117	106	110	106	107	101	96	114	...	114	109
Greatest parieto-squamous breadth,	139s.	145	143	140s.	143s.	132s.	137s.	131s.	143s.	135s.	144s.	145s.
Cephalic Index,	*79·4*	*72·5*	*82·7*	*83·8*	*78·1*	*74·6*	*68·2*	*78·*	*85·6*	*78·*	*76 2*	*76·3*
Horizontal circumference,	508	549	497	...	...	499	548	470	498	...	530	531
Frontal longitudinal arc,	125	132	125	120	...	124	138	120	123	128	135	126
Parietal ,, ,,	119	133	113	108	...	118	133	102	120	112	133	135
Occipital ,, ,,	113	137	109	110	...	114	120	119	118	114	120	114
Total ,, ,,	357	399	347	338	373	356	391	341	361	354	388	375
Vertical tranverse arc,	288	314	289	290	305	286	308	281	306	...	306	306
Basal transverse diameter,	124	128	118	120	119	112	117	111	...	...	123	118
Vertical tranverse circumference,	412	442	407	410	424	398	425	392	...	...	429	424
Length of foramen magnum,	33	38	36	35	35	34	36	33	32	...	32	37
Basi-nasal length,	98	110	89	95	99	96	105	86	89	100	107	102
Basi-alveolar length,	86	102	82	95	96	87	98	...	...	...	...	96
Gnathic Index,	*87·8*	*92·7*	*92·1*	*100·*	*97·*	*90·6*	*93·3*	...	...	...	...	*94·1*
Total longitudinal circumference,	488	547	472	468	507	486	532	...	...	...	527	514
Interzygomatic breadth,	125	135	125	129	125	120	126	...	...	...	132	124
Intermalar, ,,	117	117	...	118	107	105	110	...	...	...	116	108
Nasio-mental length,	...	...	111	106 *ap.*	...	115	...	...	...	...	...	...
Complete Facial Index,	...	...	*88·8*	*82·1*	...	*95·8*	...	...	...	...	...	...
Nasio-alveolar length,	67	73	69	64	70	68	76	...	...	...	...	70
Maxillo-facial Index,	*53·6*	*54·*	*55·2*	*49·6*	*56·*	*56·6*	*60·3*	...	...	...	...	*56·4*
Nasal height,	50	54	50	48	49	55	56	...	...	...	47	52
Nasal width,	26	21	19	24	20	21	23	...	...	...	24	20
Nasal Index,	*52·*	*38·9*	*38·*	*50·*	*40·8*	*38·2*	*41 1*	...	...	...	*51·1*	*38·6*
Orbital width,	39	41	39	38	36	41	40	...	...	...	37	36
Orbital height,	34	35	32	28	33	36	36	...	...	...	32	37
Orbital Index,	*87·2*	*85·4*	*82·*	*73·7*	*91·7*	*87·8*	*90·*	...	...	...	*86·5*	*102·8*
Palato-alveolar length,	48	57	47	53	50	50	55	...	...	..	...	57
Palato-alveolar breadth,	55	...	55	56	59	61	...	...	...	...	...	56
Palato-alveolar Index,	*114·5*	..	*117·*	*118·*	*118·*	*120·*	...	...	...	...	...	*98·2*
Lower jaw. Symphysial height,	...	...	31	...	...	28	...	...	...	...	...	...
Lower jaw. Coronoid,	...	...	61	70	...	62	...	...	...	...	...	...
Lower jaw. Condyloid,	...	...	62	74	...	60	...	...	...	...	...	...
Lower jaw. Gonio-symphysial length,	...	...	73	...	...	76	...	...	...	...	...	...
Lower jaw. Inter-gonial width,	...	...	94	99	...	93	...	...	..	...	...	...
Lower jaw. Breadth of ascending ramus,	...	...	30	35	...	34	...	...	...	...	...	...

vertical index, 71, was less than the cephalic. The occipital arc was longer than the frontal and parietal. The nasion was not depressed; the nasal bridge was moderately projecting; the maxillo-nasal spine was distinct, and a sharp ridge separated the floor of the nose from the incisive region. The upper jaw was orthognathous; the maxillo-facial index was leptoprosopic; the nose was elongated and leptorhine; the orbits were mesoseme. The skull had the massive character, with the ample dimensions which one sees in so many adult Scotchmen. Except that the hard palate was deeper than usual, the skull showed no special variations in ossification.

Stirlingshire. Table VIII.

Two crania were found in a moss at Kilsyth; one belongs to the Henderson Trust collection, No. 27; the other to the Ballingall collection in the University Museum. They are both stained almost black by the peat in which they were lying, and they are injured as if from swordcuts received in battle. The skull in the Henderson Trust collection is referred to in Wilson's *Prehistoric Annals.* They were adult males, and one was metopic.

Norma verticalis.—One cranium was rounded in outline and brachycephalic, with the cephalic index 83·8; the other was a little more elongated, and the index was 78·1. In both, the basi-bregmatic diameter was much below the parieto-squamous breadth. The vertex was flattened, the slope outwards to the parietal eminences was gentle and the side walls were somewhat bulging; the parieto-occipital slope was abrupt, but the occipital squama was not flattened.

Norma lateralis.—The glabella and supraorbital ridges were distinct but not specially prominent; the forehead was only slightly retreating. The bridge of the nose was moderately projecting; the nasal floor was separated from the incisive region by a definite crest. In the brachycephalic skull the nasal index was mesorhine, the orbital index microseme, the gnathic index mesognathous, the palato-maxillary index brachyuranic; the complete facial and maxillo-facial indices were chamæprosopic. In the other skull the corresponding indices were leptorhine, megaseme, orthognathous, brachyuranic and leptoprosopic.

Lanarkshire. Table VIII.

The collection contains two skulls from this county, one from the parish of Bothwell and the other from New Lanark. They were both males; the Lanark skull was advanced in years; in the Bothwell specimen the sutures were undergoing obliteration.

Norma verticalis.—Both were elongated ovoids, not specially flattened on the vertex, sloping downwards to the parietal eminences and with vertical sides. Both were good examples of dolichocephalic crania; but the length of the skull from New Lanark, 201 mm., was promoted by the Wormian bones in the lambdoidal suture being set obliquely, so that the occipital squama formed a shelf-like projection. The shelf added several millimetres to the absolute length of the skull and influenced the cephalic index, the total longitudinal arc and the longitudinal circumference. The Museum contains other four crania not described in this memoir, two of which were obtained in the dissecting-room, in which a similar shelf-like character is present.

In both the Lanarkshire skulls the basi-bregmatic height was below the greatest breadth. In both the face was orthognathic and leptoprosopic; the nose was leptorhine, the orbit was megaseme; in the Bothwell specimen the palato-alveolar arch was brachyuranic.

Peeblesshire. Table VIII.

Three imperfect adult and apparently female skulls from this county are in the collection; one from Linton (H.T. 40) was found in a moss, but it is not peat-stained; the two others are from the parish of Wiston. In one skull the length-breadth index was hyperbrachycephalic, 85·6; in the two others in the higher mesaticephalic group, 78. In all three the basi-bregmatic diameter was materially below the parieto-squamous breadth. The cranial measurements were on a small scale, indicative of the sex. The face in each skull was so much injured that the facial measurements could not be taken.

Roxburghshire. Table VIII.

The collection contains two skulls which were obtained in Butts Lane, Kelso, in 1864, during the construction of a system of sewage. One belonged to a skeleton lying at full length in a grave formed of slabs of stone loosely placed together. The other was got in close proximity to this grave, but the finder could not tell me if it were in a similar grave, or was in a collection of human bones unenclosed in coffins in the surrounding earth. One was that of a man advanced in years, with the alveolar arch absorbed; the other was apparently that of a woman.*

In their general form, as seen from the *norma verticalis*, the crania were elongated ovoid, with somewhat bulging side walls, with no sagittal ridge, with the postero-parietal region steep, and with a convex occipital squama. The proportions of length and breadth were almost identical; the cephalic index, 76·2 and 76·3 respectively, was in the lower mesaticephalic group. In each skull the basi-bregmatic height was much less than the greatest breadth. In the female the face was orthognathous, leptoprosopic, leptorhine, megaseme, and hyperdolichuranic; in the male the corresponding indices, so far as they could be computed, were mesorhine and mesoseme. In each skull the cranial capacity was more than 1500 c.c.

* A description of the find is given by me in *Proc. Scot. Soc. Antiquaries*, June 1865, vol. vi. p. 245.

Renfrewshire. TABLES IX., X. PLATES II., III., V.

Twenty-one skulls from Renfrewshire, including the town of Paisley, were examined; eleven were apparently males and ten females. The majority were in adult life; one was about twenty years of age; several, to judge either from the obliteration of the sutures of the cranial vault, or the loss of teeth and the absorption of the alveoli, or from the presence of both these conditions, were advanced in years. Three skulls were metopic.

Norma verticalis.—The crania were not uniform in appearance; six were relatively broadly ovoid, whilst the rest were more elongated in relation to the breadth. As a rule the vertex was low and rounded in the transverse arc, and sloped gently outwards to the parietal eminences; but in three specimens the sagittal line was somewhat ridged, and the slope outwards from it was more abrupt. The side walls were slightly bulging, and the greatest breadth was near the squamous suture; in each skull the greatest breadth exceeded the interzygomatic diameter. The relative diameter in the stephanic and asterionic regions varied in different skulls. There was no parieto-occipital flattening, but the skulls varied in the slope of that region, and in the amount of projection of the occipital squama. The crania were cryptozygous.

Norma lateralis.—In the males the glabella and supraorbital ridges were distinct, and the forehead sloped backwards, in some slightly, in others to a greater extent. In the females the forehead approached the vertical, and the region of the frontal air sinuses was relatively smooth. The bridge of the nose was usually prominent and frequently concave upwards. The nasion was depressed in only three male skulls. The maxillo-nasal spine was distinct, sometimes very prominent, and a sharp ridge separated the floor of the nose from the incisive region of the upper jaw. In one skull the longitudinal occipital arc was longer than either the frontal and parietal, in one longer than the frontal, in one longer than the parietal; there was no constancy in the relative length of the frontal and parietal arcs.

In the male crania the glabello-occipital length ranged from 180 to 201 mm., and the mean was 190 mm.; the breadth ranged from 130 to 153 mm., with a mean 142·8 mm.; the basi-bregmatic diameter ranged from 121 to 143 mm., and the mean was 133·5 mm. The mean horizontal circumference was 531 mm.; the mean vertical transverse circumference was 429·9 mm.; the mean longitudinal circumference was 523 mm. In the female crania the corresponding dimensions were as follows:—Range of glabello-occipital diameter from 169 to 188 mm., with the mean 177 mm.; the breadth ranged from 130 to 142 mm., with a mean 135·7; range of basi-bregmatic diameter from 121 to 133 mm., with the mean 127 mm.; mean horizontal circumference 501·6, mean vertical transverse circumference 406, the mean longitudinal circumference 489·9. In the males the cranial capacity ranged from 1230 to 1855 c.c., with the mean 1526 c.c.; in the females the range was from 1180 to 1490 c.c., and the mean was 1300·5 c.c.

TABLE IX.—*Renfrewshire, including Paisley. Males.*

Collection number,	A.	B.	U.	W.	G.	Metopic. H.	I.	Metopic. K.	L.	M.	N.
Age,	Advanced.	Ad.	Ab. 25.	Ad.	Ad.	Ad.	Ad.	Aged.	Aged.	Ad.	Aged.
Sex,	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.
Cubic capacity,	1845	1410	1470	1385	1710	1605	1365	1855	1230	1535	1380ap.
Glabello-occipital length,	201	180	183	186	196	192	191	201	184	195	183
Basi-bregmatic height,	143	136	131	136	143	137	131	136	121	126	129
Vertical Index,	*71·1*	*75·6*	*71·6*	*73·1*	*73·*	*71·4*	*68·6*	*67·7*	*65·8*	*64·6*	*70·5*
Minimum frontal diameter,	103	...	90	87	96	101	96	103	96	101	93
Stephanic diameter,	119	115	120	106	112	116	104	122	99	108	111
Asterionic diameter	118	103	119	106	112	113	106	113	111	116	113
Greatest parieto-squamous breadth,	153	137	142	139	149	146	137	150	130	143	145
Cephalic Index,	*76·1*	*76·1*	*77·6*	*74·7*	*76·*	*76·*	*71·7*	*74·6*	*70·7*	*73·3*	*79·2*
Horizontal circumference,	560	507	520	512	550	545	520	568	506	534	521
Frontal longitudinal arc,	139	137	137	136	143	129	127	134	132	130	128
Parietal ,, ,,	} 267	127	132	125	138	141	128	137	132	120	118
Occipital ,, ,,		104	119	114	117	123	120	133	109	120	...
Total ,, ,,	406	368	388	375	398	393	375	404	373	370	...
Vertical transverse arc,	326	309	306	298	311	323	304	325	284	288	291
Basal transverse diameter,	133	115	125	121	132	121	118	126	116	129	128
Vertical transverse circumference,	459	424	431	419	443	444	422	451	400	417	419
Length of foramen magnum,	38	35	35	38	37	32	33	38	32	41	...
Basi-nasal length,	109	101	91	98	108	101	106	106	97	104	95
Basi-alveolar length,	...	97	81	91	99(ap.)	92	98	...	...	104	...
Gnathic Index,	...	*96·*	*89·*	*92·9*	*91·7*	*91·1*	*92·5*	...	...	*100·*	...
Total longitudinal circumference,	553	504	514	511	543	526	514	548	502	515	...
Interzygomatic breadth,	...	126	120	121	138	126	127	...	124	138	...
Intermalar ,,	...	...	111	110	118	112	116	...	114	124	...
Nasio-mental length,	...	...	112	115	126	...	...	...	...	...	...
Complete Facial Index,	...	...	*93·3*	*95·*	*91·3*	...	...	...	...	...	...
Nasio-alveolar length,	...	72	70	68	...	75	72	...	...	84	...
Maxillo-facial Index,	...	*57·1*	*58·3*	*56·2*	...	*59·5*	*56·6*	...	...	*60·8*	...
Nasal height,	...	52	46	49	53	53	53	57	48	59	...
Nasal width,	...	22	19	20	26	...	25	...	24	27	...
Nasal Index,	...	*42·3*	*41·3*	*40·8*	*49·1*	...	*47·2*	...	*50·*	*45·8*	...
Orbital width,	...	42	36	36	41	39	41	39	38	44	37
Orbital height,	...	38	32	33	38	35	32	37	31	40	34
Orbital Index,	...	*90·5*	*88·9*	*91·7*	*92·7*	*89·7*	*78·*	*94·9*	*81·6*	*90·9*	*91·9*
Palato-alveolar length,	...	55	52	52	...	55	55	...	51	62	...
Palato-alveolar breadth,	...	60	58	59	...	61	65	...	59	63	...
Palato-alveolar Index,	...	*109·*	*111·5*	*113·4*	...	*110·9*	*118·*	...	*115·6*	*101·6*	...
Lower jaw. Symphysial height,	...	...	33	32	32	...	...	...	...	...	31
Lower jaw. Coronoid ,,	...	...	55	71	56	...	...	...	...	...	57
Lower jaw. Condyloid ,,	...	...	56	67	62	...	...	...	...	...	55
Lower jaw. Gonio-symphysial length,	...	...	78	83	95	...	...	...	...	...	88
Lower jaw. Inter-gonial width,	...	...	88	96	110	...	...	...	...	...	95
Lower jaw. Breadth of ascending ramus,	...	...	33	35	39	...	...	...	...	...	30

TABLE X.—*Renfrewshire, including Paisley. Females.*

Collection number,	C.	D.	E.	F.	O.	P.	Q.	R.	Metopic. S.	T.
Age,	Ad.	Ad.	Ad.	Ad.	Aged.	Ad.	abt. 20	Ad.	Ad.	Aged.
Sex,	F.	F.	F.	F.	F.	F.	F.	F.	F.	F.
Cubic capacity,	1330	1230	1375	1250	1200	1180	1490	1310	...	1340
Glabello-occipital length,	188	169	180	188	171	176	178	172	176	180
Basi-bregmatic height,	126	125	123	128	121	126	133	129	...	131
Vertical Index,	*67·*	*74·*	*68·3*	*68·1*	*70·8*	*71·6*	*74·7*	*75·*	...	*72·8*
Minimum frontal diameter,	91	88	89	91	92	87	85	93	92	88
Stephanic diameter,	109	100	111	111	103	99	101	102	105	105
Asterionic diameter,	109	105	108	113	114	95	112	100	110	112
Greatest parieto-squamous breadth,	140p.	132	136	136	136(s.)	130s.	142s.	134(p.)	133(s.)	138s.
Cephalic Index,	*74·5*	*78·1*	*75·6*	*72·3*	*79·5*	*73·9*	*79·8*	*77·9*	*75·6*	*76·7*
Horizontal circumference,	523	486	507	517	483	488	508	490	505	509
Frontal longitudinal arc,	135	114	123	125	117	113	125	118	123	129
Parietal „ „	239	122	128	243	109	123	123	132	126	126
Occipital „ „		107	119		118	119	121	98	...	118
Total „ „	374	343	370	368	344	355	369	348	...	373
Vertical transverse arc,	295	284	300	289	280	275	300	290	286	302
Basal transverse diameter,	121	119	112	114	118	110	119	113	119	114
Vertical transverse circumference,	416	403	412	403	398	385	419	403	405	416
Length of foramen magnum,	34	36	35	33	30	37	39	33	...	31
Basi-nasal length,	101	92	87	100	94	93	93	100	...	97
Basi-alveolar length,	102	89	79	93	...	93	85	98	...	...
Gnathic Index,	*101·*	*96·7*	*90·8*	*93·*	...	*100*	*91·4*	*98·*	...	...
Total longitudinal circumference,	509	471	492	501	468	485	501	481	...	501
Interzygomatic breadth,	...	116	120	123	122	116	120	119	122	120
Intermalar „	...	102	106	107	108	100	104	105	105	106
Nasio-mental length,	...	...	...	...	...	...	...	...	...	...
Complete Facial Index,	...	...	...	...	...	...	...	...	...	...
Nasio-alveolar length,	70	68	66	71	...	72	60	67	74	...
Maxillo-facial Index,	...	*58·6*	*55·*	*57·7*	...	*61·8*	*50·*	*54·7*	*60·6*	...
Nasal height,	49	51	49	55	52	54	46	46	54	50
Nasal width,	...	22	21	22	21	21	21	21	19	24
Nasal Index,	...	*43·1*	*42·9*	*40·*	*40·4*	*38·9*	*45·7*	*45·7*	*35·2*	*48·*
Orbital width,	39	35	37	38	36	37	36	36	38	36
Orbital height,	35	34	36	36	33	32	29	31	35	32
Orbital Index,	*89·7*	*97·1*	*97·3*	*94·7*	*91 7*	*86·5*	*80·6*	*86·1*	*92·1*	*88·9*
Palato-alveolar length,	59	48	48	51	...	52(ap.)	47	55	57	...
Palato-alveolar breadth,	61	60	57	...	...	61	60	58	60	...
Palato-alveolar Index,	*103·4*	*125·*	*118·7*	...	...	*117·3*	*127·6*	*105·4*	*105·2*	...

The relative proportions of the cranium and of the face, as determined by their respective indices, in the two sexes, were as follows: — The length-breadth index ranged from 70·7 to 79·8, and the mean of twenty-one crania was 75·7; eight crania were between 70·7 and 74·9, *i.e.* were dolichocephalic; seven were from 75·6 to 76·7, *i.e.* in the lower half of the mesaticephalic group, and approached therefore to the dolichocephalic; six were from 77·6 to 79·8, *i.e.* approached the brachycephalic, and of these three almost reached the index of 80. The mean vertical index was 70·7, and in each skull the basi-bregmatic height was less than the greatest breadth. The mean gnathic index of thirteen skulls was 94, *i.e.* orthognathous, but in three of these the index was higher, mesognathous. The mean nasal index was only 43·5, so that the narrow, elongated, leptorhine nose was well pronounced; only two specimens exceeded 48, and were in the lower mesorhine group. The rounded form of the orbit generally was shown by the mean megaseme index 89·7, though it should be stated that in three crania the index was in the microseme group. The mean palato-alveolar index was 113, *i.e.* mesuranic; the range in the index was considerable, for ten specimens were dolichuranic; two were hyperdolichuranic; two were hyperbrachyuranic, four were brachyuranic, and only four were mesuranic; the mean mesuranic index did not represent the proportions in individual skulls, but the mean between the extreme proportions. From the absence of the lower jaw, or from the changes due to alveolar absorption, the complete facial index was only obtainable in three specimens, in all of which the index was leptoprosopic or high faced. It was possible to compute in thirteen specimens the maxillo-facial index, or the proportion between the interzygomatic diameter and the height of the upper jaw, the mean of which index was 57·4, which places them high in the leptoprosopic group. In only one specimen was the index as low as 50.

The sutures, when not obliterated, had as a rule well-marked denticulations, and sutural bones were infrequent. In one specimen a small interparietal bone was seen, three had small Wormians in the lambdoidal suture, one had an anterior fontanelle bone, one a small sutural bone in the left half of the coronal suture, one a small sutural bone in the squamous suture. One skull had a left epipteric bone, two had each a right epipteric, and in one skull the squamous-temporal articulated with the frontal on the left side and almost did so on the right side. In more than one the alisphenoid had a very narrow articulation with the parietal. No skull had a third condyle or par-occipital process, though in some the jugal process was tuberculated; in one, each occipital condyle was transverse, divided into two facets; in another a pair of short, sharp tubercles projected downwards from the basi-occipital antero-internal to the condyles.

Ayrshire. Table XI.

Three crania were obtained from the county of Ayr. One, No. 24 in the Henderson Trust collection, was from Kirk Alloway, the others were from intramural interments in the town of Ayr. They were all adults; one was a male, the others apparently females.

In the *norma verticalis* two were seen to be elongated ovoids, whilst the third was more rounded in form. They were all low arched at the vertex, but they varied in the steepness of the slope from the sagittal suture to the parietal eminences. The side walls of the cranium were slightly bulging. In the more elongated crania, the backward slope from the squamous suture to the projecting occipital squama was so marked as to give a distinct pentagonal outline to the cranium. They were cryptozygous. The diameter in the parieto-squamous region was almost the same in each, but one skull was absolutely so short as to be brachycephalic, whilst in the others the cephalic index was 75 and 75·9 respectively. In each skull the vertical index was materially below the cephalic.

In the male skull the glabella and supraorbital ridges were distinct, the forehead was somewhat retreating, and the nasion was depressed. In the females these characters were much less pronounced. In one the nose was prominent, in the other less so, and the floor was separated from the incisive region by a sharp ridge. Two skulls were orthognathous, one mesognathous, and the face was leptoprosopic; two skulls were leptorhine, one mesorhine; in two the orbits were megaseme, in one microseme, in two the palate was brachyuranic, in one hyperbrachyuranic.

One female skull was metopic, in the other the sagittal and lambdoidal sutures were almost obliterated; no Wormian or epipteric bones were present. A third condyle was not present, but in two the jugal process was tuberculated on its inferior surface.

Wigtonshire. Table XI. Plate III.

Four skulls in the collection are from Kirkmadrine, Wigtonshire. From the conditions under which they were found there is reason to believe that they were from interments made more than a century ago. They doubtless represent the cranial characters of the people of Galloway of that period. They were adult crania, two apparently males and two females.

Norma verticalis.—They were elongated and ovoid, and in no instance was the breadth proportionally large in relation to the length. The vertex was somewhat rounded in the transverse arc, there was no sagittal ridge, and the slope outwards to the parietal eminences was gentle. In two crania the side walls were nearly vertical, in the other two they were somewhat bulging. The parietals sloped downwards behind the obelion, and the occipital squama was convex backwards. The crania were cryptozygous. In the two men the parieto-squamous exceeded the interzygomatic diameter. In two skulls the stephanic exceeded the asterionic, in the other two the proportions were reversed.

Norma lateralis.—In the men the glabella and supraorbital ridges were distinct; in the women they were feeble; in all the forehead slightly retreated and the arch of

Table XI.

	Wigtonshire.				Ayrshire.		
	Kirkmadrine.				Ayr.		Kirkalloway.
Collection number,	A.	B.	C.	D.	E.U.A.M.	E.U.A.M.	Metopic. H.T. 24
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.
Sex,	M.	M.	F.	F.	M.	F.	F.
Cubic capacity,	1650	1490	1210	1120	1350	...	1410
Glabello-occipital length,	198	187	181	182	187	170	188
Basi-bregmatic height,	131	129	124	119	123	132	130
Vertical Index,	*66·2*	*69·*	*68·5*	*65·4*	*65·8*	*77·6*	*69·1*
Minimum frontal diameter,	103	91	91	91	96	96	100
Stephanic diameter,	118	110	101	112	115	105	110
Asterionic diameter,	112	113	107	109	119	107	110
Greatest parieto-squamous breadth,	146s.	143s.	130s.	138s.	142s.	142s.	141s.
Cephalic Index,	*73·7*	*76·5*	*71·8*	*75·8*	*75·9*	*83·5*	*75·*
Horizontal circumference,	547	530	501	513	525	500	531
Frontal longitudinal arc,	133	129	118	124	130	123	118
Parietal " "	138	126	120	123	116	} 229	130
Occipital " "	113	112	113	111	116		115
Total " "	384	367	351	358	362	352	363
Vertical transverse arc,	305	302	286	288	300	302	302
Basal transverse diameter,	128	125	114	112	129	126	119
Vertical transverse circumference,	433	427	400	400	429	428	421
Length of foramen magnum,	37	38	36	32	33	35	38
Basi-nasal length,	106	106	100	96	100	96	102
Basi-alveolar length,	100	...	100	89	100	91	96
Gnathic Index,	*94·3*	...	*100·*	*92·7*	*100·*	*94·8*	*94·1*
Total longitudinal circumference,	527	511	487	486	495	483	503
Interzygomatic breadth,	141	133	...	...	133	126	125
Intermalar "	127	114	113	...	117	114	113
Nasio-mental length,	...	...	...	...	...	114	...
Complete Facial Index,	...	...	...	...	...	*90·4*	...
Nasio-alveolar length,	78	...	67	65	72	65	74
Maxillo-facial Index,	*55·3*	...	...	...	*54·*	*51·6*	*59·*
Nasal height,	58	51	48	49	55	48	57
Nasal width,	25	23	21	25	24	25	22
Nasal Index,	*43·1*	*45·1*	*43·8*	*51·0*	*43·6*	*52·1*	*38·6*
Orbital width,	42	40	35	36	37	39	38
Orbital height,	37	35	29	35	35	32	34
Orbital Index,	*88·1*	*87·5*	*82·9*	*97·2*	*94·6*	*82·0*	*89·5*
Palato-alveolar length,	56	52ap.	52	49	53	49	50
Palato-alveolar breadth,	68	63	60	63	63	64	60
Palato-alveolar Index,	*121·4*	*121·1*	*115·3*	*128·5*	*118·8*	*130·6*	*120·*
Lower jaw. Symphysial height,	...	...	...	...	...	31	...
Lower jaw. Coronoid "	...	...	...	...	...	66	...
Lower jaw. Condyloid "	...	...	...	...	...	60	...
Lower jaw. Gonio-symphysial length,	...	...	...	...	...	85	...
Lower jaw. Inter-gonial width,	...	...	...	...	...	97	...
Lower jaw. Breadth of ascending ramus,	...	...	...	...	...	34	...

the vault was flattened. The nose was moderately prominent, and in a male the nasion was depressed; the maxillo-nasal spine was distinct, and the floor of the nose was separated from the incisive region by a crest. The occipital longitudinal arc was in each skull the shortest; in two the frontal arc was the longest, in two the parietal exceeded the frontal.

The male crania in the glabello-occipital diameter were 198 and 187 respectively, the female 181 and 182; the basi-bregmatic diameter in the men was 131 and 129, in the women 124 and 119 mm. The maximum parieto-squamous diameter was 146 mm., the minimum (a female) 130 mm., and the mean in the men was 144·5, in the women 134 mm. In the two men the mean horizontal circumference was 538 mm., the mean vertical transverse circumference was 430; the mean longitudinal circumference was 519 mm.; the mean cubic capacity was 1570 c.c.

The cephalic index in the four crania ranged from 71·8 to 76·5 and the mean was 74·4; the skulls may be regarded as dolichocephalic, though two slightly exceeded the upper limit of that group. The mean vertical index was 67·2, and in each skull the basi-bregmatic height was less than the greatest breadth. The mean gnathic index was orthognathous, though one female skull slightly exceeded the upper limit of that group. The complete facial index was 55·3, high-faced or leptoprosopic. One nasal region had a low mesorhine index, the others were leptorhine. One orbit was microseme, two mesoseme, one megaseme. In two skulls the palato-alveolar index was brachyuranic, in two hyperbrachyuranic.

With the exception of small Wormian bones in the lambdoidal suture in two skulls, and very simple sutures of the vault in a female skull, no special variations in the ossification were observed.

Forfarshire. Table XII.

The collection of the Henderson Trust contains two skulls (H.T. 37, 39), both of which were found in 1833 under the foundation of the steeple of the old church in Montrose. No. 37, referred to in the *Prehistoric Annals of Scotland*, is a large male skull which, judging from the cranial sutures, is of a person advanced in years; the facial bones are broken away. No. 39 is of much smaller capacity, with the alveolar arcade absorbed, but with the sutures distinct; it is apparently a male.

Norma verticalis.—No. 37 was rounded in outline and flattened in the parieto-occipital region. No. 39 was more elongated and with the occipital squama convex. They were both flattened on the vertex, and sloped gently from the sagittal suture to the parietal eminences, with the side walls slightly bulging. No. 37 was hyperbrachycephalic, with cephalic index 87·2; No. 39 closely approached an index of 80. In both the basi-bregmatic diameter was much below the greatest breadth.

In both the glabella and supraorbital ridges were only feebly projecting; the forehead only slightly receded. In No. 39 the nasion was not depressed, and the bridge of the nose had scarcely any projection; the nasal index was leptorhine. The orbits were rounded, megaseme. The absence of a lower jaw and the absorption of the maxillo-alveolar arcade prevented the facial proportions from being taken.

Banffshire and Kincardineshire. Table XII.

The skull from Banff is from the village of Gamrie, but I have no record of the conditions under which it was found; from its appearance, I judge it to have been buried for a considerable period. It was a male, somewhat advanced in years.

In the *norma verticalis* the outline was broadly ovoid, flattened on the vertex, sloping gently outwards to the parietal eminences; the postero-parietal region sloped downwards and backwards, but the occipital squama was convex. The cephalic index, 80·3, was brachycephalic; the basi-bregmatic height was much below the greatest breadth.

The glabella and supraorbital ridges were well marked, the forehead was somewhat retreating, the bridge of the nose was injured, but the part remaining had not much projection; the nasal index was leptorhine; the orbits were megaseme; the face was broad, but the absence of the lower jaw and the broken maxillo-alveolar arch prevented me from obtaining the proportions of the face. The cranial capacity, 1630 c.c., was much above the average of Europeans.

Some years ago I had the opportunity of seeing several skulls from the parish of Fordoun in Kincardineshire. They were so imperfect that very few measurements could be taken, and in only three specimens was it possible to obtain the relation of length to breadth; the cephalic index ranged from 79·9 to 84, so that they were of the brachycephalic type. They were apparently male skulls, and although their internal capacities could not be taken, it would seem from the external dimensions that *a* and *b* had possessed a good amount of brain space. The glabella and supraorbital ridges were moderate, the forehead only slightly retreated; in *a* the vertex was somewhat ridged, but in *b* and *c* not so. The postero-parietal region was flattened from above downwards and backwards, but the occipital squama was convex; *b* had several Wormian bones in the lambdoidal suture.

Caithness. Table XII.

The collection contains three crania from Caithness; one, H. T. No. 45, was found at Knockstanger, on the site of a battle fought between the Mackays and Sinclairs in 1437: it is referred to in Wilson's *Prehistoric Annals of Scotland.* A second, perhaps from the same locality, was originally in the collection of Professor Alexander Monro, *tertius*, and it has the characters of a female skull. A third skull, a female, was from the "Burial Mound" at Keiss, and formed one of a series excavated by Mr Samuel Laing

TABLE XII.—*North-Eastern Counties.*

	Caithness.			Banff.	Kincardineshire.			Forfarshire.	
	Knockstanger.		Keiss.	Gamrie.	Fordoun.			Montrose.	
Collection number,	Ht. 45.	B. 17.	E.U.A.M.	E.U.A.M.	A.	B.	C.	Ht. 37.	Ht. 39.
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Aged.	Ad.	Aged.	Aged.
Sex,	M.	F.	F.	M.	M.	M.	M.	M.	M.
Cubic capacity,	...	1285	1460	1630	...	...	...	...	1240
Glabello-occipital length,	192	182	183	188	184	180	175	180	182
Basi-bregmatic height,	130	127	130	139	...	126	...	140	117
Vertical Index,	*67·7*	*69·8*	*71·0*	*73·9*	...	*70·*	...	*77·8*	*64·3*
Minimum frontal diameter,	98	95	88	102	106	103	98	106	97
Stephanic diameter,	105	111	101	125	111	120	106	126	...
Asterionic diameter,	114	114	114	108	130	...	...	116	105
Greatest parieto - squamous breadth,	142s.	136s.	134	151s.	147	144	147	157	145s.
Cephalic Index,	*74·*	*74·7*	*73·2*	*80·3*	*79·9*	*80·*	*84·*	*87·2*	*79·7*
Horizontal circumference,	535	510	513	544	542	525	...	540	523
Frontal longitudinal arc,	120	130	120	138	130	247	...	142	130
Parietal " "	124	120	128	133	244		117	148	104
Occipital " "	124	124	123	129		101	...	94	126
Total " "	368	374	371	400	374	348	...	384	360
Vertical transverse arc,	293	288	294	336	320	...	...	326	295
Basal transverse diameter,	127	116	113	127	135	...	...	134	119
Vertical transverse circumference,	420	404	407	463	455	...	...	460	414
Length of foramen magnum,	37	32	34	34	...	37	...	33	35
Basi-nasal length,	109	93	97	99	...	105	...	100	91
Basi-alveolar length,	...	89	94	...	...	...	...	...	...
Gnathic Index,	...	*95·7*	*96·9*	...	...	...	...	...	...
Total longitudinal circumference,	524	499	502	533	...	490	...	517	486
Interzygomatic breadth,	...	...	122	133ap.	...	...	...	...	...
Intermalar "	...	...	105	121	...	...	...	...	...
Nasio-mental length,	...	...	110	...	...	...	...	...	...
Complete Facial Index,	...	...	*90·1*	...	...	...	...	...	...
Nasio-alveolar length,	...	69	66	...	...	...	...	...	...
Maxillo-facial Index,	...	...	*54·*	...	...	...	...	...	...
Nasal height,	...	47	51	48	...	...	...	...	51
Nasal width,	...	22	21	23	...	...	...	...	22
Nasal Index,	...	*46·8*	*41·2*	*47·9*	...	...	...	...	*43·1*
Orbital width,	...	39	37	41	...	...	...	...	38
Orbital height,	...	38	35	34	...	...	...	...	35
Orbital Index,	...	*97·4*	*94·6*	*82·9*	...	...	...	...	*92·1*
Palato-alveolar length,	...	50	53	...	...	...	...	...	...
Palato-alveolar breadth,	...	58	60	...	...	...	...	...	...
Palato-alveolar Index,	...	*116*	*113*	...	...	...	...	...	...
Lower jaw. Symphysial height,	...	...	27	...	...	...	...	...	...
Lower jaw. Coronoid "	...	...	55	...	...	...	...	...	...
Lower jaw. Condyloid, "	...	...	58	...	...	...	...	...	...
Lower jaw. Gonio-symphysial length,	...	...	84	...	...	...	...	...	...
Lower jaw. Inter-gonial width,	...	...	97	...	...	...	...	...	...
Lower jaw. Breadth of ascending ramus,	...	...	32	...	...	...	...	...	...

and described by Professor HUXLEY.* They were from persons in the later stage of adult life.

Norma verticalis.—The crania were elongated ovoids, with a tendency to be ridged and roof-like in the sagittal region, and sloped distinctly downwards from the sagittal suture to the parietal eminences; they were flattened in the postero-parietal region, and in two the side walls were vertical. In each skull the length-breadth index was below 75 and therefore dolichocephalic; the mean of the series was 73·9. In each skull also the basi-bregmatic diameter was below the greatest breadth. In two crania the occipital longitudinal arc was greater than the frontal but less than the parietal; in two the parietal was greater than the frontal. In the Keiss skull the occipital squama bulged backwards.

The glabella and supraorbital ridges were not very prominent; in the females the forehead was almost vertical, in the male slightly retreating. The nasion was not depressed, the bridge of the nose present in the Keiss cranium was sharp and aquiline, and in it also the maxillo-nasal spine was long, and a distinct crest separated the floor of the nose from the incisive region. In the male Knockstanger skull the face was broken away, but in the other skulls the nasal index was leptorhine; the orbits were megaseme and the upper jaw was orthognathous. In the Keiss specimen the face was leptoprosopic and the palato-alveolar arch was mesuranic, the angle of the lower jaw was gently rounded, and the symphysis was somewhat pointed; in the other female skull the arch was brachyuranic.

Shetland Islands. TABLE XIII. PLATES III., IV., V.

Five male skulls were collected in Shetland. Two were from a parish in the north-west of the mainland, one from a parish in its southern part, and two from the neighbourhood of Lerwick. They were from persons in the later stage of adult life. In one the teeth were all shed and the alveoli absorbed, in two others many of the alveoli were absorbed, and in the other two the crowns were worn and flattened. Two crania were metopic, and in all the sutures of the vault were visible in the outer table.

The *norma verticalis* was broadly ovoid, though in two specimens the relative breadth was not so great as in the others. In three the vertex was low-arched from side to side; no sagittal ridge, and the slope outwards to the parietal eminences was gentle; the side walls were somewhat bulging. In one the occipital squama was convex, in the others it

* See Laing and Huxley's *Prehistoric Remains in Caithness* (London, 1866), in which I gave a detailed description of this skull. Several skulls from this Burial Mound are described by Professor Huxley: they varied in the cephalic index from 70 to 78. The so-called mound was on the natural terrace of sand and shingle parallel and close to the sea beach, and was scarcely elevated above the surface of the terrace. Stones were found in two of the graves which Mr Laing regarded as rude stone implements, and he associated the burials with the early stone period. The bodies had been buried in the extended position in long graves covered with flat stones, whilst the walls were formed of unhewn flagstones, a mode of burial which is known to have prevailed during the Christian era, and examples of which are not uncommon on the sea shore. It is questionable if these burials had the antiquity which Mr Laing has ascribed to them. See also *Proc. Scottish Soc. Antiquaries*, vol. vii. p. 38, 1870.

TABLE XIII.—*Shetland.*

	Northmavine.		St. Ninians.	Lerwick.	
Collection number,	Metopic. E.U.A.M.	E.U.A.M.	E.U.A.M.	Metopic. E.U.A.M.	E.U.A.M.
Age,	Ad.	Ad.	Ad.	Ad.	Aged.
Sex,	M.	M.	M.	M.	M.
Cubic capacity,	1615	1770	1630	1560	1580
Glabello-occipital length,	190	201	189	180	192
Basi-bregmatic height,	142	141	138	138	140
Vertical Index,	*74·7*	*70·1*	*73·0*	*76·7*	*72·9*
Minimum frontal diameter,	106	101	103	104	102
Stephanic diameter,	125	114	125	117	130
Asterionic diameter,	119	120	120	110	117
Greatest parieto - squamous breadth,	147s.	151s.	150s.	155s.	146s.
Cephalic Index,	*77·4*	*75·1*	*79·4*	*86·1*	*76·0*
Horizontal circumference,	551	568	538	538	544
Frontal longitudinal arc,	139	140	134	119	139
Parietal „ „	134	140	123	115	130
Occipital „ „	112	127	125	126	126
Total „ „	385	407	382	360	395
Vertical transverse arc,	320	337	318	326	333
Basal transverse diameter,	125	129	126	130	118
Vertical transverse circumference,	445	466	444	456	451
Length of foramen magnum,	37	36	36	33	29
Basi-nasal length,	107	110	103	106	103
Basi-alveolar length,	...	...	95	97	...
Gnathic Index,	...	...	*92·2*	*91·5*	...
Total longitudinal circumference,	529	553	521	499	527
Interzygomatic breadth,	138	140	136	140	...
Intermalar „	123	125	121	121	...
Nasio-mental length,	...	137	120	...	...
Nasio-mental complete facial Index,	...	*98·*	*88·2*	...	...
Nasio-alveolar length,	...	...	69	73	74(ap.)
Maxillo-facial Index,	...	...	*50·7*	*52·*	...
Nasal height,	58	56	53	57	57
Nasal width,	22	24	22	25	23
Nasal Index,	*37·9*	*42·8*	*41·5*	*43·8*	*40·3*
Orbital width,	38	41	41	40	40
Orbital height,	34	38	35	37	39
Orbital Index,	*89·5*	*92·7*	*85·4*	*92·5*	*97·5*
Palato-maxillary length,	...	...	57	55	...
Palato maxillary breadth,	...	...	61	...	...
Palato-maxillary Index,	...	...	*107·*	...	...
Lower jaw. Symphysial height,	34	37	29	...	...
Lower jaw. Coronoid „	61	72	60	...	...
Lower jaw. Condyloid „	64	72	57	...	...
Lower jaw. Gonio symphysial length,	93	105	92	...	...
Lower jaw. Inter-gonial width,	...	114	111	...	...
Lower jaw. Breadth of ascending ramus,	40	41	33	...	...

was more flattened in continuation with the flatness of the parietal region behind the obelion. They were all cryptozygous. The parieto-squamous diameter in each skull exceeded the interzygomatic. With one exception the stephanic exceeded the asterionic diameter.

Norma lateralis.—The glabella and supraorbital ridges were well marked; the forehead sloped slightly backward. The nose was prominent and with a strong bridge, moderately concave forwards; the nasion was somewhat depressed; the maxillo-nasal spine was distinct and the floor of the nose was separated from the incisive region by a definite crest. The occipital arc in one skull was longer than the parietal; in another longer than either frontal or parietal; in four skulls the frontal arc exceeded the parietal, in one they were equal.

The crania ranged in glabello-occipital diameter from 180 to 201 mm. and the mean was 190·4 mm.; the basi-bregmatic diameter ranged from 138 to 142 mm., and the mean was 139·8 mm. The maximum parieto-squamous diameter was 155 mm., minimum 146, and the mean was 149·8 mm. The mean horizontal circumference was 547·8 mm., the mean vertical transverse circumference was 452·4, the mean longitudinal circumference was 525·8 mm.; the cubic capacity ranged from 1560 to 1770 c.c., and the mean was 1631 c.c. Both in external dimensions and internal capacity the Shetland skulls were characterised by their magnitude.

The cephalic index ranged from 75·1 to 86·1; one was hyperbrachycephalic, the others were mesaticephalic, though two approached the dolichocephalic standard; the mean of the series was 78·8, *i.e.* mesaticephalic. The mean vertical index was 73·4, and in each skull the basi-bregmatic height was less than the greatest breadth.

Owing to the alveolar absorption, the gnathic index could only be computed in two skulls, in both of which it was orthognathous. In each skull the nose was narrow or leptorhine; in one the orbit was mesoseme, in the others rounded or megaseme. The facial index was in the leptoprosopic or high-faced group.

As regards individual peculiarities, two crania were metopic, one had a large interparietal bone, the left third of which had a separate ossification; three had several small Wormian bones in the lambdoidal suture. One metopic skull had a small left epipteric bone. No skull had a third occipital condyl or par-occipital process, though in two the jugal processes were tuberculated.

Perthshire. Table XIV.

Only one skull has been obtained from this large county. It came from the Bridge of Garry, at the foot of the pass of Killiecrankie. It is that of a man advanced in years, but unfortunately is so much injured that only a few measurements could be taken. The sutures of the cranial vault were ossified, and the teeth were much worn and flattened on the crowns. The cranium in the occipital and right parietal regions was marked with incisions extending through the outer table into the diplöe as if from sword cuts, and it is possible that the man may have been one of the combatants at the battle of Killiecrankie.

Seen in the *norma verticalis* the cranium was greatly elongated, and relatively narrow; the cephalic index was only 69·7, hyperdolichocephalic; the base of the cranium was much broken, and none of the measurements from the basion could be taken.

The glabella and supraorbital ridges were distinct; the forehead only slightly receded; the cranial vault formed a lofty curve; the post-parietal region sloped gently downwards, and the occipital squama was convex.

Argyllshire. Table XIV. Plate V.

In the collection of the Henderson Trust are three skulls from Argyllshire; two (Nos. 25 and 26, H. T.) were found near Crutchingman on Loch Tarbert, Kintyre, and one of these is referred to in the *Prehistoric Annals* as having been dug up in a cave, near to where tradition affirms that a battle was fought between the natives and the Northmen. The third skull, H. T. 5, was dug out of the sand on the sea-beach at Larnahinden, where a party of "Danes" are said to have landed and been defeated. No. 25 has the characters of a female skull. They were adults, and with the crowns of the teeth flattened from use.

Norma verticalis.—These skulls were elongated ovoids, elevated on the line of the sagittal suture, sloping steeply outwards to the parietal eminences, with vertical sides, with the postero-parietal region sloping gently downwards and the occipital squama convex. The crania were characteristically dolichocephalic, the mean index being 70·6. The basi-bregmatic height was a little less than the greatest breadth. In one, the inter-zygomatic breadth exceeded the parieto-squamous, in another it was slightly below it.

The glabella and supraorbital ridges were strong in the male skulls, and the nasion was depressed; the bridge of the nose was prominent, and its floor was separated from the incisive region by a crest. The forehead was retreating in the male. In one, the gnathic index was orthognathous; in two, in the lower mesognathic group; in one, the face was leptoprosopic, in another chamæprosopic. In all three the nose was leptorhine. The orbits varied in the proportion of height and width, one being in each division of the group. Two of the palato-alveolar arches were dolichuranic, one was mesuranic. Two specimens had a right epipteric bone; two had small Wormian bones in the lambdoidal suture, and in one of these a minute sutural bone was at the anterior end of the sagittal suture.

Ross and Sutherland. Table XIV. Plate IV.

An adult male skull was obtained from each of these counties; that from Ross was metopic, and the alveolar border of the lower jaw was, to a large extent, absorbed; that

Table XIV.—*Highlands and Islands.*

	Argyllshire.			Perthshire.	Sutherlandshire.	Ross-shire.	Hebrides.						
	Loch Tarbert, Kintyre.	Lamahinden.	Loch Tarbert, Kintyre.	Killecrankie.			Iona.					South Uist.	Stornoway.
						Metopic.				Metopic.			
Collection,	H.T. 26	H.T. 5	H.T. 25	E.U.A.M.	E.U.A.M.	E.U.A.M.	H.T. 48	H.T. 49	H.T. 50	H.T. 53	H.T. 51	E.U.A.M.	
Age,	Ad.	Ad.	Ad.	Adv.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	60	68
Sex,	M.	M.	F.	M.	M.	M.	M.	M.	M.	M.	F.	F.	F.
Cubic capacity,	1570	...	1435	...	1510	1415	...	1410	1390	...	1380	1260	1470
Glabello-occipital length,	196	189	190	198	192	191	189	179	181	186	183	181	184
Basi-bregmatic height,	135	131	132	...	133	139	132	135	135	...	122	121	124
Vertical Index,	*68·9*	*69·3*	*69·5*	...	*69·3*	*72·8*	*69·8*	*75·5*	*74·6*	...	*66·7*	*66·9*	*67·4*
Minimum frontal diameter,	102	96	95	108	97	96	100	95	96	100	96	96	93
Stephanic ,, ,,	105	107	101	108	116	117	105	118	104	107	102	102	121
Asterionic diameter,	107	109	107	110	108	114	113	105	115	103	112	107	103
Greatest parieto-squamous breadth,	138s.	132p.	136s.	138s.	144p.	139s.	142s.	142s.	143s.	138s.	141p.	136s.	137p.
Cephalic Index,	*70·4*	*69·8*	*71·6*	*69·7*	*75·0*	*72·8*	*75·1*	*79·3*	*79·0*	*74·2*	*77·0*	*75·1*	*74·5*
Horizontal circumference,	542	518	522	533	532	521	...	...	518	524	515	508	521
Frontal longitudinal arc,	130	136	130	...	142	128	135	127	123	124	122	120	132
Parietal ,, ,,	138	127	133	...	125	131	128	128	128	122	130	120	243
Occipital ,, ,,	122	104	124	...	118	113	114	110	114	...	120	116	
Total ,, ,,	390	367	387	...	385	372	377	365	365	...	372	356	375
Vertical transverse arc,	307	297	300	305	305	300	310	307	307	...	...	282	302
Basal transverse diameter,	123	...	121	132	120	119	121	124	127	...	...	122	107
Vertical transverse circumference,	430	...	421	437	425	419	411	431	434	...	...	404	409
Length of foramen magnum,	35	36	35	...	37	36	35	35	39	...	38	35	35
Basi-nasal length,	104	104	97	...	102	104	101	99	99	...	90	100	90
Basi-alveolar length,	99	103	96	...	98	97	...	...	...	...	...	100	...
Gnathic Index,	*95·2*	*99·*	*99·0*	...	*96·1*	*93·3*	...	...	...	...	...	*100*	...
Total longitudinal circumference,	529	...	519	...	524	512	513	499	503	...	500	491	...
Interzygomatic breadth,	134	135	...	...	128	130	...	...	...	...	...	132	116
Intermalar ,,	119	118	115	...	112	112	...	...	...	...	...	119	98
Nasio-mental length,	...	113	...	...	121	119	...	...	...	...	...	109	...
Complete Facial Index,	...	*83·7*	...	...	*94·5*	*91·5*	...	...	...	...	...	*82·5*	...
Nasio-alveolar length,	72	63	70	...	71	76	...	...	...	...	...	64	...
Maxillo-facial Index,	*53·7*	*46·6*	...	...	*55·4*	*58·4*	...	...	...	...	...	*48·5*	...
Nasal height,	53	50	49	...	50	55	...	...	...	...	...	49	51
Nasal width,	23	22	26	...	22	22	...	...	...	...	...	24	2[illegible]
Nasal Index,	*43·4*	*44·*	*53·1*	...	*44·*	*40·*	...	...	...	...	...	*49·*	*39·[illegible]*
Orbital width,	40	38	38	...	38	39	...	...	...	...	...	41	3[illegible]
Orbital height,	33	34	32	...	35	32	...	...	...	...	...	30	3[illegible]
Orbital Index,	*82·5*	*89·5*	*84·2*	...	*92·1*	*82·*	...	...	...	...	...	*73·2*	*102[illegible]*
Palato-alveolar length,	57	55	52	...	59	56	...	...	...	...	...	57	...
Palato-alveolar breadth,	62	60	59	...	64	...	...	...	...	...	...	60	...
Palato-alveolar Index,	*108·7*	*109·*	*113·4*	...	*108·4*	...	...	...	...	...	...	*105·2*	...
Lower jaw. Symphysial height,	...	29	...	...	35	36	...	...	...	...	...	30	...
Lower jaw. Coronoid ,,	...	68	...	...	60	77	...	...	...	...	...	66	5[illegible]
Lower jaw. Condyloid ,,	...	66	...	...	64	76	...	...	...	...	...	70	5[illegible]
Lower jaw. Gonio-symphysial length,	...	91	...	...	94	92	...	...	...	...	...	93	...
Lower jaw. Inter-gonial width,	...	99	...	...	102	102	...	...	...	...	...	101	...
Lower jaw. Breadth of ascending ramus,	...	33	...	...	32	39	...	...	...	...	...	39	...

from Sutherland had the teeth much more complete, but the cranial sutures were in process of obliteration.

In the *norma verticalis* each skull had an elongated ovoid outline, though one was proportionately wider than the other, and the cephalic indices were respectively 75 and 72·8, dolichocephalic; the sagittal line was ridged, and the side walls were bulging, but the Sutherland specimen had, as is unusual in the male skull, the greatest breadth in the parietal region. In the Sutherland cranium the height was materially less than the breadth, but in that from Ross these dimensions were equal. In both, the occipital squama bulged backwards, especially in the Ross specimen. In both, the interzygomatic diameter was less than the greatest breadth of the cranium; they were cryptozygous. In both, the glabella and supraorbital ridges moderately projected, and the forehead had a slight backward slope. In the Ross cranium, a slight vertical transverse depression, as if from a constricting band in infancy, was behind the coronal suture. In both, the occipital longitudinal arc was the shortest; in one, the frontal arc exceeded the parietal; in the other, the opposite condition was met with.

The nasion was moderately depressed, the bridge of the nose was prominent, the anterior nares were narrow, the maxillo-nasal spine, especially in the Sutherland cranium, was projecting; the nasal index was leptorhine. The upper jaw in both was orthognathous. The face was elongated and relatively narrow (leptoprosopic), both in the complete facial and maxillo-facial indices. In one, the orbital index was microseme, in the other megaseme. In the Sutherland specimen the palato-alveolar index was dolichuranic. The cranial capacity was 1415 and 1510 cub. cent. respectively.

The Ross cranium had small Wormian bones in the lambdoidal suture, also one in the left half of the coronal suture, and a very narrow parieto-sphenoid articulation. The cranial bones generally were thin and translucent. The Sutherland cranium was free from sutural or other ossific variations.

Hebrides. Table XIV.

In April 1833, Mr Donald Gregory presented to the Phrenological Society of Edinburgh* six skulls as those of "Druids from the Hebrides" (Henderson Trust collection, Nos. 48–53). In commenting on these specimens, Sir Daniel Wilson† states that one was brought from Harris, and that the others were no doubt obtained during excavations carried on by the Iona Club in the island of Iona, in the ancient cemetery called "Relig Oran." Iona, he says, is sometimes called the isle of Druids, and the designation affixed by Mr Gregory to these crania only signified that he believed them to have belonged to the native population prior to the landing of St Columba and the introduction of Christianity in the sixth century. It is to be regretted that in each skull the cranium only has been preserved, and in No. 52 it has been so much injured

* *Phrenological Journal*, vol. ix. p. 86, 1836.
† *Prehistoric Annals of Scotland*, first edition, p. 173, 1851.

as to make it impossible to obtain measurements which can be relied on. From the condition of the sutures, the skulls were obviously in the later stage of adult life. The sex characters were not strongly pronounced, but it is probable that the majority were of the male sex; one skull was metopic.

Norma verticalis.—In their general form, owing to their well-marked parietal eminences, and the mode in which the skull inclined backwards to the occipital squama, the crania had a pentagonal outline. They varied, however, in the proportion of length to breadth, and two were more elongated than the rest. The vertex was not flattened, and, though not ridged, it had a tendency to be elevated in the line of the sagittal suture. The descent from the obelion to the lambdoidal suture was gradual, and the postero-parietal region was obliquely flattened. The mean length of the crania was 183·6 mm., the mean height was 131 mm., the mean breadth was 141 mm., the mean horizontal circumference was 519 mm., the mean vertical transverse circumference was 425 mm., the mean longitudinal circumference was 503 mm. The cephalic index ranged from 74·2 to 79·3; three crania were either dolichocephalic or approximated thereto, whilst two approached the brachycephalic standard; the mean index of the series was 76·9, mesaticephalic. In each cranium the basi-bregmatic height was less than the greatest breadth.

The glabella and supraorbital ridges were moderate in projection, the forehead was slightly retreating, the frontal eminences were distinct. One cranium had a single Wormian ossicle in the lambdoidal suture, and another had a small one near the posterior end of the sagittal suture. No facial measurements could be taken.

I have recently received two female skulls, one a native of South Uist with the teeth in good condition, another born in Stornoway with the teeth all shed and the alveoli absorbed. The crania were elongated and ovoid, of the dolichocephalic type. One, with the index 75·1, fractionally exceeded the upper term of that group; the other had a long slope backwards and downwards in the post-parietal regions, associated with a strong development of Wormian bones in the lambdoidal suture, which gave to that region something of the shelf-like character referred to in the description of the skull from New Lanark (p. 572). In both, the breadth exceeded the height. The facial proportions in the skull from South Uist were mesognathous, chamæprosopic, mesorhine, microseme, dolichuranic. In the Stornoway skull only the nasal and orbital indices could be taken, which were respectively leptorhine and megaseme; the orbit had the unusual relation of being higher than wide; in this skull also each squamous-temporal articulated at the pterion with the frontal bone.

Practical Rooms. Table XV.

Sixteen skulls were obtained from the dissecting-room. The names of ten persons were known, and three of these, Haggart, Howison, and Gordon, were executed for murder from sixty to seventy years ago. The remaining six, though the names were

TABLE XV.—*Crania from Practical Rooms, etc.*

	Haggart.	Howison.	Hart.	Turnbull.	Miller.	Smith.	D.R. A.	D.R. B.	D.R. F.	D.R. C.	Gordon.	D.R. D.	D.R. E.	Wilson.	Jamieson.	Wyllie.
Collection,	Edinburgh University Anatomical Museum.										Ht. 317	E.U.A.M.				
Age,	Ad.	Ad.	23	49	Metopic. 36	60	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	69	58	70
Sex,	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	M.	F.	F.
Cubic capacity,	1530	...	...	1245	1650	1480	1350	1450	1510	1450	1395	1350	1510	1470	...	...
Glabello-occipital length,	186	188	196	181	190	191	184	188	187	182	175	188	188	196	184	187
Basi-bregmatic height,	136	137	140	136	142	132	127	133	143	137	139	125	120	130	127	135
Vertical Index,	*73·1*	*72·9*	*71·4*	*75·1*	*74·7*	*69·1*	*69·*	*70·7*	*76·5*	*75·3*	*79·4*	*66·5*	*63·8*	*66·3*	*69·*	*72·2*
Minimum frontal diameter,	92	92	104	92	103	95	96	94	98	94	96	93	99	96	97	90
Stephanic ,, ,,	117	125	124	115	132	121	121	124	119	104	...	113	121	112	112	114
Asterionic diameter,	117	113	116	110	115	117	109	107	109	101	105	115	112	111	109	109
Greatest parieto-squamous breadth,	139s.	147s.	139s.	137s.	142s.	143s.	144s.	139s.	143	141s.	140s.	145s.	147s.	146s.	140s.	138s.
Cephalic Index,	*74·7*	*78·2*	*70·9*	*75·7*	*74·7*	*74·9*	*78·3*	*73·9*	*76·5*	*77·5*	*80·0*	*77·1*	*78·2*	*74·5*	*76·1*	*73·8*
Horizontal circumference,	520	534	542	512	545	540	525	530	532	515	502	528	533	542	515	527
Frontal longitudinal arc,	138	143	140	125	144	130	126	137	128	130	134	134	126	133	128	127
Parietal ,, ,,	134	251	144	134	132	121	123	128	132	123	127	122	126	115	118	140
Occipital ,, ,,	110		111	113	124	122	113	123	123	112	102	122	109	120	116	116
Total ,, ,,	382	394	395	372	400	373	362	388	383	365	363	378	361	368	362	383
Vertical transverse arc,	314	329	318	305	322	291	294	308	320	313	304	300	295	300	294	302
Basal transverse diameter,	116	123	126	121	127	131	130	117	...	119	...	126	131	131	...	117
Vertical transverse circumference,	430	452	444	426	449	422	424	425	...	432	...	426	426	431	..	419
Length of foramen magnum,	38	37	39	32	37	37	36	33	37	36	29	29	39	39	38	38
Basi-nasal length,	98	100	106	102	101	104	101	96	105	100	104	98	103	107	94	98
Basi-alveolar length,	92	88	102	...	98	100	99	89	102	83	102	93	101	...	...	96
Gnathic Index,	*93·9*	*88·*	*96·2*	...	*97·*	*96·2*	*98·*	*92·7*	*97·1*	*83·*	*98·1*	*94·9*	*98·1*	...	...	*98·*
Total longitudinal circumference,	518	531	540	506	538	514	499	517	525	501	496	505	503	514	494	519
Interzygomatic breadth,	122	140	135	125	132	142	137	130	135	128	...	128	131	144	124	120
Intermalar ,,	105	119	125	111	115	124	121	120	116	114	101	111	118	...	107	104
Nasio-mental length,	118	...	...	...	...	132	125	...	...	121	...	...	104	...	...	...
Complete facial Index,	*96·7*	...	...	...	...	*93·*	*91·2*	...	...	*94·5*	...	...	*79·3*	...	...	...
Nasio-alveolar length,	65	67	69	70	72	80	73	72	75	70	65	68	61	...	...	68
Maxillo facial Index,	*53·3*	*48·*	*51·1*	*56·*	*54·5*	*56·3*	*53·2*	*55·3*	*55·5*	*54·7*	...	*53·1*	*46·5*	...	...	*56·6*
Nasal height,	50	51	46	51	52	59	53	54	53	51	47	48	50	...	50	50
Nasal width,	20	23	23	23	24	25	24	24	26	22	24	21	27	...	21	20
Nasal Index,	*40·*	*45·1*	*50·*	*45·1*	*46·*	*42·4*	*45·3*	*44·4*	*49·1*	*43·1*	*51·1*	*43·8*	*54·*	...	*42·*	*40·*
Orbital width,	35	42	46	39	38	39	39	39	39	40	37	39	40	40	40	40
Orbital height,	30	34	40	32	34	41	35	32	35	33	31	34	31	35	38	34
Orbital Index,	*85·7*	*81·*	*87·*	*82·*	*89·5*	*105·1*	*89·7*	*82·*	*89·7*	*82·5*	*83·8*	*87·2*	*77·5*	*87·5*	*95·*	*85·*
Palato-alveolar length,	51	50	57	...	57	62	55	49	57	46	54	54	54	...	...	55
Palato-alveolar breadth,	57	...	62	56	60	...	67	60	68	57	55	63	66	...	...	60
Palato-alveolar Index,	*111·*	...	*108·*	...	*105·*	...	*121·*	*122·*	*119·*	*100·8*	*101·8*	*116·6*	*122·2*	...	...	*109·*
Lower jaw. Symphysial height,	30	33	34	...	...	35	37	...	...	32	...	...	26	...	31	...
Lower jaw. Coronoid ,,	59	60	70	...	...	74	63	...	...	62	...	...	59	...	55	...
Lower jaw. Condyloid ,,	61	65	65	...	...	75	68	...	...	63	...	...	58	...	55	...
Lower jaw. Gonio-symphysial length,	90	...	...	...	...	93	93	...	...	83	...	...	85	...	85	...
Lower jaw. Inter-gonial width,	98	...	...	...	...	99	108	...	...	96	...	...	98	...	95	...
Lower jaw. Breadth of ascending ramus,	33	31	42	...	...	36	37	...	...	27	...	..	31	...	33	...

unknown, were, it is believed, natives of Scotland, but I have no information of the part of the country in which they were born.

Fourteen were males and two females. They were all adults, and for the most part in the prime of life; but in some specimens the sutures of the vault were in process of obliteration, the alveolar arches were partially absorbed, and the teeth, when present, were flattened on the crowns from use.

When viewed in the *norma verticalis*, six of the crania were seen to be well filled, but the others could not be regarded as examples of this form of skull, for three were ridged in the sagittal region, and in these, as in several others, there was a marked slope from the sagittal suture to the parietal eminences. Several of the crania had an elongated ovoid outline and were dolichocephalic in proportions; others again were more broadly ovoid, and were from their proportions in the higher terms of the mesaticephalic series; in one specimen the cranium was brachycephalic. Only one skull was metopic, with a stephanic diameter of 130 mm., and its cephalic index was 74·7. The basi-bregmatic diameter exceeded the greatest breadth in only one specimen; in two they were equal, and in five the breadth was not more than 5 mm. in excess of the height. The glabella and supraorbital ridges were well marked, and in three crania unusually so. The frontal bone sloped somewhat backward. The occipital squama projected behind the inion, though in general not to any extent. The downward slope of the more posterior half of the parietal bone was not as a rule steep. The greatest breadth of the crania was in the squamous region. In all the specimens the frontal longitudinal arc exceeded the occipital. In ten the frontal arc exceeded the parietal, in one they were equal, and in the remainder the parietal exceeded the frontal. In two specimens the occipital arc exceeded the parietal, in one they were equal; in the remainder the parietal exceeded the occipital. The stephanic diameter exceeded the asterionic with two exceptions, in one of which these diameters were equal, and in another the asterionic was slightly greater. The interzygomatic breadth in each case was less than the parieto-squamous.

The bridge of the nose, as a rule, projected forwards, and the nasion was not much depressed. The nose had the elongated narrow leptorhine proportions, with four exceptions, in one of which the index was platyrhine. In six skulls the orbital index was microseme, in five it was megaseme, in the remainder mesoseme.

The palato-alveolar index varied from a hyperdolichuranic, 101·8, to a hyperbrachyuranic index, 122·2. In all the specimens the upper jaw was orthognathous, with two exceptions, in which the index was 98·1, a fraction above the orthognathic term.

With two exceptions the face was leptoprosopic. The mean cubic capacity of the crania of fourteen men was 1449 c.c.

As a rule the skulls rested behind on the conceptacula. No specimen had a third occipital condyl, or a para-mastoid process, though the jugal process was occasionally tuberculated. In two crania indications of an infraorbital suture could be recognised. One skull had a pair of epipteric bones, another had one on the right side, a third had one on the left; as a rule the parieto-sphenoid articulation was broad. Eight crania had small Wormian bones in the lambdoidal suture.

Variations in Ossification.

The skulls examined were regarded as 117 males and 59 females. In considering the variations in ossification the sutures naturally call for special attention. In a number of crania the sutures were in process of ossification, and in a smaller proportion they were, owing to age, to a considerable extent obliterated. No specimen showed characters which indicated that a premature closure of the sutures had taken place. In twenty-three crania, sixteen males and seven females, the frontal suture was distinct, and the metopic condition was present in 13 per cent. of the series. It furnished an example of the persistence during adult life of the fœtal division of the bone into right and left lateral halves.

The skull from Dunbar (p. 557) showed on the left side an intraparietal suture which divided the parietal bone into an upper and a lower segment. It is probable that in many, if not in every skull, the parietal bone ossifies from two centres, an upper and a lower, and the presence of this suture is a persistent condition of the plane of separation between these centres.

The infraorbital suture persisted in a number of the crania; but an example of a division of a malar bone into two parts by a suture was not noticed.

A number of examples of sutural bones was recognised. As usual, the lambdoidal suture was their most frequent seat, and no fewer than forty-nine crania had Wormian bones in this suture: in two specimens they were set obliquely so as to form a shelf, which projected the occipital squama backwards, and gave increased length to the cranium. In two skulls an intraparietal bone was present in the parieto-occipital region.

The pterion is a part of the cranium to which anthropologists have given much attention. The series showed every variety from a broad sphenoido-parietal suture to one so pointed that the ali-sphenoid barely touched the parietal angle. The frontal articulated in four crania with the right squamous-temporal, in two with the left, and in one skull the temporo-frontal articulation was present on both sides. In ten crania an epipteric bone was present in the right pterion, in seven in the left, and in eight on both sides. One skull had a sutural bone in the squamous suture. In two crania a small sutural bone occupied the region of the anterior fontanelle, and three others had separate ossifications further back in the sagittal suture. Two crania had each a small bone in the left half of the coronal suture.

In no skull was the occipital bone seen to possess a third or middle condyl. Three had small par-occipital processes, and in several the under surface of the jugal process was tuberculated. In one skull the spine of the sphenoid articulated with the external pterygoid plate of the same bone, and enclosed a pterygo-spinous foramen; and an approximation to this condition, though without actual junction of the plates of bone, occurred in two other specimens, owing to ossification of the pterygo-spinous ligament.

Five crania showed a vertical transverse depression parallel to and immediately behind the coronal suture, a condition which is usually regarded as due to a tight band or ribbon having been worn across this part of the head in infancy and early childhood. Attention was especially drawn to this character by the late PAUL BROCA, who recognised it as a common occurrence in the heads of the people of France living in and near Toulouse, where the practice of wearing such a band prevails, and to this appearance the name *la déformation toulousaine* has been applied.

In my Report on Human Crania in the *Challenger Reports*, part xxix., 1884, to which I have several times referred in this memoir, I have summarised the observations made on the variations in ossification noted in 143 crania of aboriginal people therein described, *e.g.* from South Africa, South America, Australia and the islands of the Pacific. When compared with the series of Scottish skulls several interesting points of difference may be noted. The absence of the metopic condition of the frontal was remarked in the aboriginal series, although I have since seen it in the skull of a Veddah and in an Andaman islander, and FLOWER has observed metopism in six Andaman crania. The squamous-temporal articulated with the frontal in ten of the aboriginal skulls, which is a distinctly larger proportion than the seven cases I have noted in the Scottish crania. The observations of RANKE and VIRCHOW on German skulls, of CALORI on Italian, and of WENZEL GRUBER on Sclavonic crania, give something less than 2 per cent. of cases of temporo-frontal articulation, which is not so high as in the Scottish skulls. On the other hand, the aboriginal series had epipteric bones in sixteen crania, *i.e.* 11 per cent., whilst in the Scottish skulls they were present in twenty-five specimens, about 14 per cent., which is a larger proportion. No third occipital condyl was seen in a Scottish skull, whilst four aboriginal crania had this character. Only one Scottish skull had a pterygo-spinous foramen, which was noticed in three aboriginal crania. Exostoses in the external auditory meatus, so common in the Pacific Islanders, had no representative in the Scottish skulls. Wormian bones in the lambdoidal suture were not uncommon in both series, but the presence of sutural bones in the coronal and sagittal sutures was perhaps somewhat more frequent in the Scottish crania.

It would seem, therefore, that whilst some forms of variation in cranial ossification are more freqnent in aboriginal crania, others again are more numerous in a civilised people like the natives of Scotland.

General Survey of the Characters of Scottish Skulls.

In the preceding sections the characters of the skulls obtained in the several Scottish counties have been described in some detail. In this chapter it is intended to look at them as a whole, with the view of elucidating the form, dimensions, and proportions which prevailed in the crania generally. I have endeavoured to group them according to sex; and though in the great majority I have succeeded in distinguishing the skulls of the men from those of the women, it is not unlikely that, like other craniologists, I have had to deal with a few specimens in which the sex characters were wanting in precision, and consequently a skull may possibly have been ascribed to the wrong sex. If we grant that this has occurred in a small minority, yet from the numerous specimens at my disposal, in which the sex could confidently be stated, the general conclusions cannot have been materially affected.

I propose, in the first instance, to analyse the dimensions, proportions, and form of the cranial box, and afterwards to consider those of the face.

The Cranial Box.

The shape of the cranium, from its influence on the form of the head and from its association with the brain contained in its cavity, has attracted attention from the earliest periods of craniological research. Since the time of ANDERS RETZIUS the relations of the length to the breadth and the grouping of skulls into those in which the cranium is relatively narrow and elongated, and those in which it is more rounded in form, have been regarded as of great importance in the recognition of racial distinctions. According to modern methods the character of the cranium can be determined by combining observations on its shape with exact measurements. The measurements are taken with callipers in straight lines between certain definite points, in order to determine the length, breadth, and height of the exterior of the box; with a graduated tape over the curved walls of the outer table so as to determine its arcs and circumferences, and with shot to estimate its internal capacity. The points of measurement in the straight lines are indicated by the terms employed in the Tables. The measurements of the curved surfaces, whilst agreeing with the methods pursued in my memoirs in the *Challenger Reports* * and in my two memoirs on Indian crania † in regard to the horizontal circumference, the vertical transverse arc, and the frontal, parietal, occipital, and total longitudinal arcs, have in this memoir been somewhat amplified so as to yield a vertical transverse circumference and a total longitudinal circumference, dimensions which for the first time are definitely stated in my Tables. The vertical transverse circumference is obtained by measuring with callipers a basal transverse diameter between opposite supra-auricular points, and adding this to the vertical transverse arc. The data for obtaining a total longitudinal circumference existed in the Tables in my previous memoirs, and consisted of the total longitudinal arc, the antero-posterior diameter of the foramen magnum, and the basi-nasal diameter; in this memoir the respective measurements have been added together and stated collectively in the Tables. The capacity of the cranial cavity has been taken by the method described in my *Challenger Report*, 1884, and the additional experience of its

* *Zoology*, part xxix., 1884, and part xlvii., 1886.
† *Trans. Roy. Soc. Edin.*, part i., 1899; part ii., 1901.

accuracy which I have had since that date has added to my confidence in the method as giving a close approximation to the real capacity, and not an exaggerated statement of the cubage, such as is obtained by the well-known method of PAUL BROCA.

Speaking generally, and subject of course to occasional exceptions, we may say that the Scottish cranium is large and capacious ; the vertex is seldom keeled or roof-like, but has a low rounded arch in the vertical transverse plane at and behind the bregma, and with a gentle slope from the sagittal suture to the parietal eminences. The side walls are not vertical, and bulge slightly outwards in the parieto-squamous region, so that the greatest breadth is usually at or near the squamous suture. The occipital squama bulges behind the inion, and the slope from the obelion is downwards and backwards, so as to give in the *norma verticalis* an obliquely flattened character to the postparietal region, but without occasioning a vertical parieto-occipital flattening such as is found in many normal brachycephalic crania, or in those in which artificial compression is employed in infancy. Owing to the width in the parieto-squamous region and the projecting occipital squama (probole) in many crania, their outline is more or less pentagonal, the frontal region forming one boundary, the sides of the cranium as far back as the parietal eminences forming two others, and the remaining two sides are the walls from the parietal eminences to the most projecting part of the occiput. In men the glabella and supraorbital ridges are fairly but not strongly pronounced, the forehead only slightly recedes from the vertical plane, and the nasion is scarcely depressed.

Length.—The glabello-occipital or maximum length was measured in one hundred and seventy-six crania, viz., one hundred and seventeen men and fifty-nine women. In the men the longest skull was 204 mm., and eight were 200 mm. and upwards ; thirty-three were from 190 to 199 mm., so that nearly one-fourth of these crania were above 190 mm. in greatest length. The shortest skull in the men was 167 mm., and only sixteen crania were below 180 mm. in their greatest length. The longest skull in the women was 193 mm., and only three crania were 190 mm. and upwards ; the shortest woman's skull was 161 mm. ; and eight crania were below 170 mm. The mean length of the male crania was 186·6 mm., that of the female crania was 178·7 mm.

The projection of the glabella was not, even when most prominent, equal to what one sees in the long skulls of so many Australian and other black people, and consequently the length of the Scottish skull indicated a cranial cavity and a brain longer than existed in the dolichocephalic black races. Owing, however, to the depth of the frontal sinuses and the thickness of the frontal and occipital bones the cranial length from the glabella to the occipital point is appreciably greater, especially in the male sex, than the long diameter of the cerebrum. In order to eliminate the frontal sinus with the consequent projection of the glabella from the comparison, and to associate the length of the skull more closely with the length of the cranial cavity and the cerebrum, it was suggested by Dr ROLLESTON * that the point to be selected in front for taking the cranial length should be the ophryon, a point immediately above

* In Greenwell's *British Barrows*, p. 506, 1877, and in vol. i. *Scientific Papers and Addresses*, edited by W. Turner.

the glabella. The observations of A. Logan Turner * have shown that the frontal sinus is not limited to the region of the glabella and supraorbital ridges, but extends in a large proportion of skulls above the ophryon, so that the influence of the sinus in adding to the cranial length is by no means eliminated by selecting the ophryo-occipital in preference to the glabello-occipital diameter. (Figs. 25, 26, Pl. V.)

Breadth.—The greatest parieto-squamous breadth was obtained in one hundred and seventy-four crania, viz., one hundred and fourteen men and sixty women. In the men the broadest skull was 159 mm., and twenty-four crania were between 150 and 159 mm. The narrowest male skull was 130 mm., and twenty-six skulls ranged from 130 to 139 mm. In the women the broadest skull was 153 mm., two specimens being of that diameter. The narrowest skull was 128 mm., and thirty-six specimens ranged from 130 to 139 mm., whilst nineteen were between 140 and 150 mm. The mean breadth of the male crania was 149·3 mm., that of the female was 138 mm. This diameter approximates to the greatest breadth of the cerebrum in each individual.

In addition to the parieto-squamous breadth the tables contain two breadth measurements of the frontal region, as well as the asterionic diameter which gives the breadth of the occipital bone between its lateral angles. As a general rule the frontal stephanic diameter materially exceeded the minimum frontal, though in a few instances it was not more than from 2 to 8 mm. greater. These dimensions give an approximation to the width of the frontal lobes of the cerebrum. Twenty-three crania had a persistent frontal suture, viz. sixteen males and seven females. The metopic crania as a rule exceeded in their frontal diameter the skulls of the corresponding sex from the same locality in which the frontal suture was ossified, and confirmed the view entertained by many craniologists that persistence of the frontal suture contributes to an increase in the transverse diameter of the skull and brain in that region.

The asterionic diameter, except in one skull, was greater than the minimum frontal, but as a rule it was less than the stephanic, though there were several exceptions. This diameter may be regarded as giving an indication of the breadth of the cerebellum.

Cephalic Index.—As is well known, this index expresses the relation which the greatest parieto-squamous breadth of a skull bears to its maximum length, the length being regarded as = 100, and the formula is as follows:

$$\frac{\text{greatest breadth} \times 100}{\text{maximum length}}.$$

The index was obtained in one hundred and seventy-four skulls, one hundred and fourteen of which were males and sixty were females. The index showed a great range of variation from 87·2 to 68·2. The mean length-breadth index in the men was 77·4, in the women 77·2. Both sexes, taken collectively, had essentially the same mean index, and were in the middle of the mesaticephalic group. If we follow the customary arbitrary grouping of crania according to the length-breadth or cephalic index, we find

* *Accessory Sinuses of the Nose*, p. 105. Edin., 1901.

that forty-nine skulls were below 75, *i.e.* dolichocephalic; ninety skulls were between 75 and 79·9, *i.e.* mesaticephalic (mesocephalic); thirty-five skulls were 80 or upwards, *i.e.* brachycephalic

Although it is a matter of convenience to accept a mesaticephalic group, interposed between the more extreme dolichocephalic and brachycephalic forms, it should be kept in mind, as I have stated in my memoir on Indian craniology,* that if we take 77·5 as marking a division of this group into two sections, the skulls which have an index between 77·5 and 80 approach in their characters more closely to the brachycephalic, whilst those that range from 77·5 to 75, on the other hand, are more allied to the dolichocephalic type. In these crania forty-five mesaticephali had their indices from 77·5 to 79·9, in no fewer than eighteen of which the index was between 79 and 80, brachycephalic therefore in form, though they were fractionally below its lowest numerical limit.

It is obvious, therefore, that a strong brachycephalic strain pervades the population of Scotland at the present time, as in no fewer than fifty-three crania of this series the index was 79 or upwards, either numerically brachycephalic or closely approximating thereto. If expressed in percentages we may say that 20% were numerically brachycephalic, and an additional 10% had a cephalic index from 79 to 79·9; on the other hand, 28% were dolichocephalic, and in 42% the index ranged from 75 to 79.

The relative proportion of the more rounded to the more elongated heads varied, however, materially in the different counties. Of the sixteen skulls from Fife six had the index above 80, one of which was hyperbrachycephalic, two were 79·7, three were between 77·5 and 79. In the Lothians, including Edinburgh and Leith, of seventy-nine skulls twenty had the length-breadth index 80 and upwards, and of these four were hyperbrachycephalic; eight crania also ranged from 79 to 79·9 and were thus essentially brachycephalic, whilst fourteen ranged from 77·5 to 78·9. In the group of nine skulls from Stirlingshire, Lanarkshire, Peebles, and Roxburghshire, two had a length-breadth index above 80, and one of these was hyperbrachycephalic, and three others were 78 or 78·1. The Renfrewshire group of twenty-one crania, on the other hand, had no specimen with an index as high as 80, though three were between 79 and 80, and three were from 77·5 to 79. The three skulls from Ayrshire had one brachycephalic example. Of the six skulls from the north-eastern counties of Forfar, Kincardine, and Banff, four had the length-breadth index 80 or upwards, and one of these was hyperbrachycephalic; the remaining two were 79·7 and 79·9 respectively, and were essentially brachycephalic. In the five crania from Shetland, one was hyperbrachycephalic, and another had the index 79·4. Of the five crania from Iona the two highest were 79 and 79·3 respectively. In the miscellaneous series of sixteen crania from the dissecting-room, only one had an index 80, no specimen was between 79 and 80, and four were from 77·5 to 79.

Our attention should now be directed to the distribution of dolichocephalic crania in the different counties; and along with those whose index is below 75, we shall

* *Trans. Roy. Soc. Edin.*, vol. xxxix. p. 744, 1899.

consider the crania in the mesaticephalic group with an index between 75 and 77·4. In the Fifeshire group only three had the length-breadth index below 75, and two were 75·5 and 76 respectively. Of seventy-nine skulls from the Lothians twenty were below 75, and two of these were hyperdolichocephalic, while sixteen ranged from 75 to 77·4. Two crania from Lanark were below 75, and one of these was hyperdolichocephalic; two from Roxburgh were 76·2 and 76·3 respectively. In the Renfrewshire group eight skulls were dolichocephalic, and seven were between 75 and 76·7. Two of the three Ayrshire skulls were 75 and 75·9 respectively. Two of the four Wigtonshire were dolichocephalic, the other two were 75·8 and 76·5 respectively. No skull from Shetland was below 75, but three were from 75·1 to 77·4. In three crania from Caithness and six from the Highland counties of Argyll, Perth, Ross and Sutherland, the length-breadth index was in no instance above 75, and two of these were hyper-dolichocephalic. Five of the seven crania from the Hebrides ranged from 74·2 to 77. Seven of the dissecting-room series were below 75, and four ranged from 75·7 to 77·1.

From this analysis of the cephalic indices in the crania under observation it would appear that a brachycephalic type of skull prevailed in Fife, in the Lothians, in the north-east counties of Forfar, Kincardine and Banff; and it occurred to some extent in Shetland, in Ayr, in the border county of Peebles, and in Stirlingshire.

The dolichocephalic type of skull was feebly represented in Fife; it was proportionately more numerous in the Lothians, in which district are included the skulls from Edinburgh and Leith; it was represented in Lanark, Ayr, Shetland and the Hebrides. It formed the prevailing type in Wigtonshire, in Caithness, in the skulls from the Highland counties, and in the important series of skulls from Renfrewshire. Whilst examples of this type occurred generally throughout the series, it may be noted that only five hyper-dolichocephali, *i.e.* skulls with the index below 70, were measured, but that eight hyper-brachycephalic crania, *i.e.* with the index 85 and upwards, occurred in the series.

In the study of the Scottish brachycephalic crania I have been led to compare them with crania of some other races measured by me some years ago, which had numerically this type of head. The comparison has been made with twenty-four male Burmese skulls * and with eight skulls of male Sandwich Islanders † described in previous memoirs, in each of which the cephalic index was 80 or upwards. The mean length-breadth index in the Burmese brachycephali was 84·2. The shortest skull in this group was 158 mm., the longest was 184, and the mean length was 171·8 mm. The parieto-squamous breadth ranged from 139 to 153 mm., and the mean breadth was 144·7 mm. In the Sandwich Islands brachycephali the mean length-breadth index was 83·8; the length ranged from 169 to 184 mm., and the mean was 176·5 mm.; the breadth ranged from 142 to 155 mm., and the mean was 148 mm. In twenty-seven male brachycephalic Scottish skulls the mean length-breadth index was 83·2, almost the

* See my memoir on Indian Crania, part i., *Trans. Roy. Soc. Edin.*, vol. xxxix., 1899.

† See *Challenger Reports*, "Zoology," part xxix., 1884, pp. 64 and 66, and part xlvii., 1886, p. 125.

same as that of the Burmese. The length ranged from 167 to 193 mm., and the mean was 180·3 mm.; the breadth ranged from 140 to 159 mm., and the mean was 150 mm. The mean length of the Scottish brachycephalic crania exceeded, therefore, by several millimetres the length of the brachycephalic Burmese and Sandwich Islanders. The greater length in the Scottish brachycephali was associated with a backward projection of the occipital squama, which contrasted with the almost vertical post-parieto-occipital region in the Burmese, Siamese and brachycephalic Sandwich Islanders. For the production of a high index in skulls of this type, the breadth required to be proportionately increased, and the Scottish brachycephalic crania both in length and breadth were larger and more capacious than the brachycephalic Burmese and Sandwich Islanders.

Height.—The distance from the basion to the bregma was taken as expressing the height of the cranium, and it was measured in one hundred and fifty specimens, ninety-eight of which were males and fifty-two females. In the men the highest skull was 145 mm.; fifteen skulls were between 140 and 145, fifty between 130 and 140, and thirty-four below 130, the lowest being only 117 mm. in height. The mean height of the male skulls was 132·4 mm. In the women the highest skull was 140 mm., the lowest was 118 mm., and the mean was 126. If we compare the height of the male Scottish crania with that of the male Burmese already referred to, we find that the mean height in the latter people was 135 mm., a somewhat greater figure than in the Scottish specimens.

Vertical Index.—This index expresses the relation which the basi-bregmatic height bears to the maximum length, which is regarded as = 100, and is computed by the formula

$$\frac{\text{basi-bregmatic height} \times 100}{\text{maximum length}}.$$

The index was obtained in one hundred and fifty crania, ninety-eight of which were men and fifty-two women. It was subject to a great range of variation, from 63·7 to 79·4. The mean vertical index in the men was 70·9, in the women 70·5; both were metriocephalic,* and the sexual difference was very slight, though slightly in favour of the men. The number of skulls with vertical index 75 and upwards was seventeen; thus a small proportion only were hypsicephalic or high skulls; sixty-five crania on the other hand had the vertical index below 70, *i.e.*, were low skulls, chamæcephalic or tapeinocephalic; the remainder had the index between 70 and 75 and were metriocephalic, which, as above stated, was the mean of the entire series.

Breadth-Height Index.—The relations of the length to the breadth and to the height of the cranium have long been recognised as important subjects of investigation

* I prefer, for the reasons stated in my *Challenger Report*, 1884, to employ the descriptive term metriocephalic rather than orthocephalic, as recommended by the German craniologists in the Frankfurt agreement (*Archiv für Anthropologie*, Bd. xv. p. 1, 1884). In this memoir I have, however, adopted the numerical subdivision of the group which they have suggested, viz., chamæcephalic up to 70, metriocephalic (orthocephalic) 70·1–75; hypsicephalic, 75·1 and upwards.

in the study of the racial characters of skulls, but the relations of the breadth and height to each other have not had an equal attention given to them.

In my *Challenger Report* (1884) I pointed out that in the brachycephalic crania from New Guinea and other Pacific Islands, the breadth was as a rule greater than the height, whilst in the dolichocephalic Papuans the opposite condition prevailed. In subsequent memoirs, more especially those on Indian craniology, I called attention to the relations of these diameters in several Asiatic races. In his work on the accessory sinuses of the nose already quoted, A. Logan Turner has recorded the proportion of breadth to height in a large number of crania, European and exotic.

In order to express numerically the relations of the breadth and height of the cranium to each other, an index may be computed by the following formula:

$$\frac{\text{basi-bregmatic height} \times 100}{\text{parieto-squamous breadth}},$$

the breadth being regarded as 100. The data for obtaining the index exists in the Tables.

When the index exceeds 100, the height is greater than the breadth, and the skull is *hypsistenocephalic*,* *i.e.* a high narrow skull: when the index is less than 100, the breadth is greater than the height and the skull is *platychamæcephalic*, *i.e.* a wide low skull.

From the measurements which I have made of the breadth and height of the cranium in many races of men, I have ascertained that in some the height usually exceeded the breadth, whilst in others the breadth exceeded the height.† In well-pronounced dolichocephalic races like the Esquimaux, the Melanesians, the Dravidians, Veddahs and the Australians generally, as a rule the height was greater than the breadth, and the crania were hypsistenocephalic. In the brachycephalic crania of the Burmese, Siamese, Chinese, Andaman Islanders, and brown Polynesians, on the other hand, the breadth as a rule was greater than the height and the crania were platychamæcephalic.

In the series of one hundred and fifty Scottish crania in which both the breadth and height were measured, in only two skulls was the height greater than the breadth, and in four others they were equal. In all the rest, whether the cephalic index was high or low, the vertical diameter was less than the breadth. A striking feature of the Scottish crania, therefore, was the preponderance of the cephalic index over the vertical index, notwithstanding the considerable number of dolichocephalic skulls in the series, and in this respect the crania favoured the brachycephalic rather than the dolichocephalic type. The Scottish skulls are platychamæcephalic.

Horizontal Circumference.—This measurement was taken in one hundred and sixty-three skulls, one hundred and eight of which were males and fifty-five females.

* Dr Barnard Davis introduced the term hypsistenocephalic to designate the high, narrow dolichocephalic crania of natives of islands in the Western Pacific (*Natuurkundige Verhandelingen*, Deel. xxiv., Haarlem, 1866), and I propose that it should have a more general application, as in the text. The term platychamæcephalic is now suggested to designate wide and low crania.

† See my memoir in *Challenger Report*, 1884; also on New Guinea Skulls in *Proc. Roy. Soc. Edinburgh*, July 1899, and on Indian Crania in *Trans. Roy. Soc. Edinburgh*, 1899 and 1901.

The maximum male skull was 572 mm., the minimum was 490 mm., and the mean was 531 mm. The maximum female skull was 550 mm., the minimum was 470 mm., and the mean was 506 mm.

Vertical Transverse Circumference.—This measurement was made in one hundred and fifty-three skulls, of which one hundred and three were males and fifty were females. The maximum male skull was 464 mm., the minimum was 398 mm., and the mean was 434 mm. The maximum female skull was 459 mm., the minimum was 381 mm., and the mean was 409·6 mm.

Total Longitudinal Circumference.—This dimension was taken in one hundred and thirty-nine crania, of which ninety-six were males and forty-three were females. The maximum male skull was 559 mm., the minimum was 468 mm., and the mean was 513·2 mm. The maximum female skull was 537 mm., the minimum was 441 mm., and the mean was 488·8 mm. The high longitudinal circumference was found in those skulls in which the glabello-occipital length was 200 mm. or approaching thereto, whilst in the skulls in which this diameter was small the longitudinal circumference was relatively low.

The total longitudinal arc was much the most important factor in this measurement, and the skulls were sufficiently numerous to enable me to ascertain the relative lengths of the frontal, parietal and occipital arcs, which collectively form the total longitudinal arc. In the series of skulls in which the arcs were measured, it was found that the occipital arc in thirteen specimens was greater than the frontal and in one hundred and thirty-one it was less: in twenty-six it was greater than the parietal and in one hundred and twelve it was less. It is the rule, therefore, for the frontal and parietal longitudinal arcs to exceed the occipital, though exceptions to the rule occur in recognisable numbers. The relative arcs of the frontal and parietal bones were measured in one hundred and fifty-eight crania; in ninety-six the frontal arc was longer than the parietal, in fifty-five the parietal was longer than the frontal, and in seven they were equal. It is obvious, therefore, that as so much variation occurs in the relative length of the longitudinal arcs, they have no appreciable value as race characters in the Scottish skulls, and the variation occurred in both the brachycephalic and dolichocephalic types. The longest occipital arc was 139 mm., the shortest 94 mm.; the longest frontal arc was 148 mm., the shortest 111 mm.; the longest parietal arc was 148 mm., the shortest 102 mm.*

From a comparison of the three circumferential measurements it will be seen that the horizontal circumference is the greatest, for it includes both the glabello-occipital and parieto-squamous diameters, which are the longest diameters in the Scottish crania. The vertical transverse circumference, again, is the shortest, as the basi-bregmatic diameter is the shortest of the three dimensions in the Scottish crania. The total longitudinal circumference ranks intermediate, for it includes only one of the two longer diameters.

Cubic Capacity.—The internal capacity of the cranium was taken with shot in

* For the relations of the longitudinal arc to the base line of the cranium, see p. 610.

accordance with the method which I described in 1884.* One hundred and fifteen crania were cubed; seventy-three were males and forty-two were females. The maximum capacity in the male skulls was 1855 c.c., the minimum was 1230 c.c., and the mean was 1478 c.c. Thirty-three skulls were more than 1500 c.c., and of these seven were 1700 and upwards, nine were between 1600 and 1700, seventeen were between 1500 and 1600; further, twenty-two were between 1400 and 1500, sixteen were between 1300 and 1400, and four were below 1300 c.c. The maximum capacity in the female was 1625 c.c., the minimum was 1100, and the mean was 1322 c.c. Only three female skulls were above 1500 c.c., eight were between 1400 and 1500, sixteen were between 1300 and 1400, eighteen were below 1300, and of these six were below 1200 c.c. The general result approximates to what has been observed in the crania of other races and peoples, that the female skull is about 10 per cent. less capacious than the male. If I had employed Broca's method, by which the cubic contents of so many races have been taken by anthropologists in France and elsewhere, the average for both sexes would have been considerably higher. It is possible, however, from the Tables compiled by E. Schmidt,† to state the cubic contents of the Scottish crania approximately in the terms of Broca's method, according to which the mean capacity of the males would have been about 1570 c.c. and that of the females about 1400 c.c. The Scottish male skull therefore is, according to Broca's method of cubage, somewhat in excess of the mean 1500 c.c. ascribed to the crania of European men.

In twenty-five dolichocephalic crania the mean capacity was 1516 c.c., and in twenty-one crania approximating to the dolichocephali in which the cephalic index was from 75 to 77·4 the mean capacity was 1519 c.c. In thirteen brachycephalic skulls the mean capacity was 1469 c.c., and in fifteen, in which the cephalic index ranged from 77·5 to 79·9, the mean capacity was 1452 c.c. A claim has been made by people whose crania have brachycephalic proportions that a brachycephalic head is higher in its type than a dolichocephalic. So far as the quality of type is expressed by the amount of cranial capacity, the skulls of the people of Scotland do not sustain this claim, as those with dolichocephalic proportions had a distinctly greater mean capacity than the brachycephali.

In addition to these more general statements, the Tables enable us to form some estimate of the existence of differences in the capacity of skulls from various districts of Scotland, though in many localities the number measured was too small on which to generalise. In the male skulls from Fife, Mid-Lothian, Shetland and Renfrewshire, the average in each group was, according to my measurements, somewhat more than 1500 c.c.; in East Lothian and Wigtonshire it was slightly lower than 1500; in the skulls from Edinburgh and Leith, West Lothian, the North-Eastern Counties, the Highland Counties and the Dissecting-room, the mean again was still lower. In making this statement I do not draw any inference that the difference in cranial capacity had a

* *Challenger Reports*, "Zoology," part xxix., 1884.

† *Archiv für Anthropologie*, supplement, vol. xiii. p. 53, 1882.

definite relation to the intellectual endowment of the people in these localities. Many other factors than the volume of the cranial cavity have to be taken into consideration in the estimation of the intellectual power either of individuals or of a collection of individuals belonging to the same people or race.

In the comparison of different races with each other there is, however, evidence that those in which the mean cranial capacity is low are intellectually inferior to the races whose mean capacity is on a distinctly higher scale.

If we take as an example the aboriginal Australians who are recognised as a race incapable, apparently, of intellectual improvement beyond their present condition, my measurements have shown that in thirty-nine men the mean cranial capacity was 1280 c.c., whilst twenty-four women were only 1156 c.c. Of the men, eight had a smaller capacity than 1200 c.c., and four only were above 1400 c.c.; whilst in the women ten were below 1100, and only three were 1200 c.c. and upwards.

The differences between the capacities of the native Australians and the Scottish skulls are much more than can be accounted for by variations in the stature and muscularity of the two peoples, and undoubtedly express a size and quality of brain associated with differences in the intelligence and the mental capabilities of the two races.

The Face.

All craniologists from the time of PRICHARD and RETZIUS have agreed in stating that in the study of the face it is important to determine the degree of forward projection of the upper jaw and to decide if the face is orthognathic or prognathic.

Gnathic Index.—In this memoir I have adopted the method followed by Sir WM. H. FLOWER and compared the length from basion to nasion with that from basion to the alveolar point. The basi-nasal length was taken in one hundred and forty-nine skulls, and ranged in the males from 91 mm. to 110 mm., and the mean was 101·4 mm.; whilst in the females it ranged from 86 to 105, with a mean of 95·3 mm. The basi-alveolar length ranged in sixty-seven males from 81 mm. to 108 mm., and the mean was 96 mm.; whilst in thirty-one females it ranged from 79 to 102, with a mean of 91 mm.

The gnathic index was computed as follows:

$$\frac{\text{basi-alveolar length} \times 100}{\text{basi-nasal length}}.$$

Whilst the index gives the numerical relation between the two diameters, it does not necessarily express the relative projection of the upper jaw beyond the profile outline of the face, for in many skulls the nasion is depressed below the plane of the glabella and of the forehead generally.

The gnathic index was computed in ninety-seven skulls, sixty-six of which were men and thirty-one women. It ranged from 85·1 to 103·2, and the mean in the men was 94·5, in the women 94·8. If we take FLOWER's subdivision of the group, and regard an index 103 as marking the lowest limit of prognathism, only one specimen came into that category. If an index 98 be taken as marking the upper limit of orthognathism, seventy-two skulls belonged to this group, whilst twenty-four had indices from 98 to 103 and were mesognathous. The Scottish skulls are therefore characterised by an almost complete absence of prognathism.

It is sometimes stated that in the same race or people the women show a relatively greater prognathic character than the men. This can scarcely be said of the Scottish skulls, for the difference between the two sexes was only fractional, so that for all practical purposes they may be regarded as identical.

Orbital Index.—Broca paid much attention to the determination of the height and width of the orbit and to the computation of an index of their relative proportions. The width was measured from the dacryon, or point of junction of the frontal, lachrymal and ascending process of the maxilla, to the most distant point on the edge of the outer border of the orbit. These measurements were taken in one hundred and twenty-four skulls. The greatest width in eighty-four males was 46 mm., the least was 35 mm., and the mean was 39 mm.; in forty females the greatest width was 41 mm., the least was 35, and the mean was 37·4 mm. The greatest height in the males was 41 mm., the least was 28 mm., and the mean was 34 mm.; in the females the greatest height was 37 mm., the least was 29 mm., and the mean was 33 mm.

The orbital index is obtained as follows:

$$\frac{\text{orbital height} \times 100}{\text{orbital width}}.$$

The index was computed in one hundred and twenty-five skulls, of which eighty-four were men and forty-one were women. It ranged from 73·7 to 105·1, and the mean was 86·4.

In grouping skulls in their orbital and nasal indices I have in this, as in my previous craniological memoirs, adopted the terms employed by Broca and Flower, as well as their numerical divisions of the groups. An orbit is said to be microseme when the height is low in relation to the width and the index is below 84. Thirty-three skulls came into this group. On the other hand, when the height and width closely approximate so that the base is rounded and the index is 89 and upwards, the orbit is megaseme, and to this group fifty-seven specimens belonged, and in three of these the index was 100 or upwards. Orbits are named mesoseme when the index is between 84 and 89, and thirty-three skulls fell into this category. In Scottish skulls the rule was for the orbit to be high in relation to the width, and somewhat rounded in outline, though exceptions not unfrequently occurred. My observations on the orbital index in the skulls of numerous races have satisfied me that it presents a great range of variation in the same race, and that it possesses only a secondary value as a race character.

Nasal Index.—The relation between the height of the nose, measured from the nasion to the lower border of the apertura pyriformis, and the greatest width of that aperture, constitutes one of the most important anthropological characters of the face.

In eighty-four male skulls the height ranged from 60 mm. to 46 mm., and the mean was 53·5 mm.; in thirty-eight females the range was from 57 mm. to 44 mm., and the mean was 49·9 mm. In eighty-two males the nasal width ranged from 28 mm. to 19 mm., and the mean was 23·1 mm. In thirty-five females the range was from 26 mm. to 19 mm. with a mean of 22·1 mm. The nasal index expresses the numerical relation between the width and height, and is computed as follows, the height being = 100:

$$\frac{\text{nasal width} \times 100}{\text{nasal height}}.$$

The index was obtained in one hundred and twenty-three specimens, eighty-one males and forty-two females. It ranged from 55·3 to 34·5; the mean was 42·5, and with few exceptions the height was more than twice the width. If with Broca and Flower we regard all skulls in which the nasal index is 53 and upwards as platyrhine, *i.e.* with the pyriform aperture wide in relation to the height of the nose, only four specimens exhibited this character. On the other hand, in ninety-three skulls the anterior nares were narrow and elongated, and the nasal index below 48 was leptorhine, and in fourteen of these specimens the index was below 40. The remaining twenty-six skulls had the index ranging from 48 to 53 and formed an intermediate or mesorhine group. The occurrence of wide nostrils in the Scottish face may be regarded therefore as accidental, and due perhaps to intermixture, through an ancestor, of a strain of some race in which a platyrhine nose was an ethnic character. The four platyrhine specimens were one in each of the East Lothian, Mid-Lothian, Highland and Dissecting-room groups. The customary form of nose in Scotland is long, relatively narrow, with a well-marked bridge, and projecting so that the type of face is prosopic, which means that the nose distinctly projects beyond a line drawn between the anterior part of the two malar bones.

Facial Indices.—An important character which has been systematically studied by Kollmann is the relation between the length and breadth of the face in different crania. The length or height of the entire face is measured from the nasion to the lower border of the symphysis menti, whilst the breadth is between the most projecting parts of the two zygomata. In twenty-one male skulls measured, the longest face was 137 mm., the shortest was 104 mm., and the mean was 120·7 mm.; in six females, the mean length was 108·8. In sixty-eight male skulls the greatest breadth was 144 mm., the least was 117 mm., and the mean was 132·2 mm. In thirty female skulls the greatest breadth was 135 mm., the least was 115 mm., and the mean was 121·5 mm. With one exception, in which the length and breadth were equal, the breadth of the face exceeded the length.

A *complete or nasio-mental facial index* can be computed as follows:

$$\frac{\text{nasio-mental length} \times 100}{\text{interzygomatic breadth}}.$$

As so frequently happens in craniological collections, the lower jaw had been preserved in only a small number of the skulls, and the complete facial index could only be taken in twenty-six specimens, twenty-one males and five females; the mean of the series was 90: that of the males was 92·3, that of the females 87·8.

Kollmann divides skulls and heads into two groups according to the relation of the length to the breadth of the face. When the index is 90·1 or upwards the face, he says, is leptoprosopic, high (long) or narrow faced; when the index is below 90·1 it is chamæprosopic, low or broad faced. In the study of the proportions of the face, and in grouping skulls in accordance with their facial indices, it is useful, as in the other relative proportions of the skull, to have a group intermediate between the two more extreme forms. We may appropriately speak, therefore, of a third or mesoprosopic group, and include in it those skulls and heads in which the index ranges from 85 to 90, both inclusive. The chamæprosopic group under this arrangement would consequently be limited to those heads in which the index is below 85. In the series of Scottish crania under consideration eighteen were leptoprosopic, four were mesoprosopic, and only four were chamæprosopic in my more limited use of the term.

To obtain as far as possible an idea of the relation between the length and breadth of the face in skulls where the lower jaw is absent, Kollmann has suggested that the interzygomatic breadth should be compared with the length of the superior maxilla measured from the nasion to the alveolar point between the two central incisors. Seventy-nine crania were measured in these diameters, viz., fifty-six males and twenty-three females.

The male crania ranged in the maxillary length from 61 mm. to 84, and the mean was 71·6 mm. The female crania ranged from 60 to 74 mm., and the mean was 67 mm. An index, which may be appropriately named *maxillo-facial*, can be computed as follows:

$$\frac{\text{nasio-alveolar length} \times 100}{\text{interzygomatic breadth}}.$$

The maxillo-facial index was taken in seventy-nine skulls, fifty-five of which were males and twenty-four females. It ranged from 61·8 to 46·5, and the mean was 54·6.

In grouping crania under the maxillo-facial index, Kollmann employs the same terms, leptoprosopic and chamæprosopic, as in the divisions of the complete facial index, but the numerical limits of the two groups, owing to the length representing only a segment of the complete face, are necessarily different. When the maxillo-facial index is 50·1 and upwards, he regards it as leptoprosopic; when 50 or less, it is chamæprosopic. In this memoir I have retained the numerical limit of the leptoprosopic group, and find that with seven exceptions all the skulls belonged to it, and that in five leptoprosopic specimens the index ranged from 60·3 to 61·8. If a division of the chamæprosopic group of Kollmann into mesoprosopic and chamæprosopic were adopted for the maxillo-facial as I have suggested for the complete facial index, and 45 were taken as the lower numerical limit of the mesoprosopic group, the seven exceptional skulls above referred to would fall into that group. No skull, therefore, in its maxillo-facial index was chamæprosopic in this more restricted use of that term, and the general type of the face in the Scottish crania is leptoprosopic.

The facial indices may be grouped as follows:

	Complete facial.[1]	Maxillo-facial.
Leptoprosopic,	90·1 and upwards,	50·1 and upwards.
Mesoprosopic,	85 to 90,	45 to 50.
Chamæprosopic,	below 85,	below 45.

[1] The complete or nasio-mental facial index corresponds, in the diameters from which the index is computed, with the zygomatic facial index of KOLLMANN. The maxillo-facial index corresponds with the upper facial index of KOLLMANN in the points of measurements.

A low or chamæprosopic maxillo-facial index necessarily depends on the upper jaw being short, in relation to the breadth of the face, and for the production of a chamæprosopic complete facial index in both the upper and lower jaws being relatively short. A relatively short upper jaw necessarily also affects both the height of the nose and the height of the orbit, so that one would expect to find a chamæprosopic face associated with a low and possibly a platyrhine nose and with a low or microseme orbit. The Scottish face is therefore long and narrow in comparison with the broad, squat faces in the Mongolian and some other types of head. In the Esquimaux, for example, the mean interzygomatic diameter in eighteen males was 138·0 mm., whilst in the Scotsmen it was only 132·2 mm.

Palato-alveolar or Palato-maxillary Index.—Anthropologists concur in considering that the relations between the length and breadth of the hard palate in the races of men should be enquired into. BROCA* and VIRCHOW† limited the measurements in this region to the hard palate itself, and computed an index which has been named staphylin or palatal. FLOWER‡ modified and improved these measurements by including the alveolar arch, and computed an index which he termed maxillary. In my *Challenger Report* § I suggested that the terms palato-maxillary or palato-alveolar were to be preferred, as expressing more fully the parts measured and the index which is computed from them. The length is taken from the alveolar point to the midpoint of a line drawn between the hinder ends of the alveolar borders, and the width is between the outer part of the alveolar arch opposite the second upper molar tooth. The palato-alveolar length in fifty-five males ranged from 46 to 62 mm., and the mean was 55·6 mm.; in twenty-eight females the range was from 45 to 59 mm., with a mean 51 mm. The palato-alveolar breadth in the males ranged from 50 to 71 mm., and the mean was 60·9; in the females the range was from 52 to 64 mm., with a mean of 58·3 mm.

* *Instructions craniologiques*, p. 77.

† *Archiv für Anthropologie*, Bd. xv. s. 5, 1884.

‡ "Cranial Characters of Fiji Islanders," *Journ. Anthrop. Inst.*, November 1880.

§ 1884, p. 7, and *Journ. Anat. and Phys.*, vol. xvi. p. 135, October 1881.

The palato-alveolar index was computed as follows:

$$\frac{\text{palato-alveolar breadth} \times 100}{\text{palato-alveolar length}}.$$

In my *Challenger Report* I suggested that relatively long palato-alveolar regions with an index below 110 should be named dolichuranic; relatively wide palates with an index above 115, brachyuranic; and those with an intermediate index between 110 and 115, mesuranic. As skulls exhibit, however, a wide range in the index in this region, I now make the further suggestion that when the index falls below 105 it should be called hyperdolichuranic; where it exceeds 120, hyperbrachyuranic. The divisions of the group may be expressed in tabular form as follows:

Hyperdolichuranic,	below 105.
Dolichuranic,	105 to 110.
Mesuranic,	110 to 115.
Brachyuranic,	115 to 120.
Hyperbrachyuranic,	above 120.

In this series of Scottish skulls nineteen were hyperbrachyuranic; seventeen were brachyuranic; fifteen were mesuranic; twenty were dolichuranic; and eleven were hyperdolichuranic. In only three specimens, two females and a male, was the length greater than the breadth; but in twenty-eight skulls the length was considerable in relation to the breadth, though not greater, so that the palate had an elongated appearance. As a rule, however, the breadth of the region was materially greater than the length, and the form of the palato-alveolar arch was that of a wide horseshoe.

Lower Jaw.—This bone had been preserved in only thirty-five skulls, twenty-six of which were males. In several of these, many teeth had been lost during life and their alveoli absorbed, so that the form of the bone had been more or less modified. Where the teeth had been in great part preserved, the body of the jaw had in the male sex a vertical diameter at the symphysis, ranging from 26 to 37 mm., and with a well-defined chin; the ascending ramus was broad and the angle was pronounced. The entire jaw had in most specimens a massive appearance, which had materially contributed to give character to the face, and from the marked vertical diameter of the body of the bone had constituted an important factor in giving to the entire face a length which placed it distinctly in the leptoprosopic group. The condyloid and coronoid diameters of the jaws varied in relative length in the series: in seventeen specimens the height from the angle to the top of the condyl was greater than to the tip of the coronoid, whilst in thirteen the coronoid height was longer, and in three specimens they were equal. The intergonial width ranged in the male jaws from 88 to 114 mm., and the mean of twenty-four specimens was 100 mm., a diameter between the angles of the jaw materially below the interzygomatic, intermalar and stephanic breadths, but distinctly higher than the minimum frontal diameter.

To assist the reader in obtaining a bird's-eye view of the dimensions and proportions of the constituent parts of the Scottish skulls studied in this memoir, I have prepared Table XVI., in which I have stated for both sexes the maximum and minimum dimensions, as well as the mean of the principal measurements in the series of skulls, together with the maximum, minimum and mean of the respective indices. I have also, by way of comparison, included in the Table the mean diameters and indices of a number of skulls of male aboriginal Australians which I have measured.

TABLE XVI.

	Scottish Skulls.						*Australians.*
	Females.			Males.			Males.
	Max.	Min.	Mean.	Max.	Min.	Mean.	Mean.
Cubic capacity,	1625	1100	1322	1855	1230	1478	1280
Glabello-occipital length,	193	161	178·7	204	167	186·6	191·3
Basi-bregmatic height,	140	118	126	145	117	132·4	135
Vertical index,	*77·6*	*64·*	*70·5*	*79·4*	*63·7*	*70·9*	*70·6*
Greatest parieto-squamous breadth,	153	128	138	159	130	149·3	132
Cephalic index,	*87·9*	*69·3*	*77·2*	*87·2*	*68·2*	*77·4*	*69·*
Horizontal circumference,	550	470	506	572	490	531	530
Vertical transverse circumference,	459	381	409·6	464	398	434	...
Basi-nasal length,	105	86	95·3	110	91	101·4	...
Basi-alveolar length,	102	79	91	108	81	96	...
Gnathic index,	*100·*	*86·7*	*94·8*	*103·2*	*83·*	*94·5*	*100·6*
Total longitudinal circumference,	537	441	488·8	559	468	513·2	...
Interzygomatic breadth,	135	115	121·5	144	117	132·2	...
Nasio-mental length,	114	102	108·8	137	104	120·7	...
Complete facial index,	*92·2*	*82·5*	*87·8*	*100·*	*79·3*	*92·3*	...
Nasio-alveolar length,	74	60	67	84	61	71·6	...
Maxillo-facial index,	*61·8*	*48·*	*55·1*	*60·8*	*46·5*	*54·3*	...
Nasal height,	57	44	49·9	60	46	53·5	...
Nasal width,	26	19	22·1	28	19	23·1	...
Nasal index,	*54·*	*34·5*	*44·4*	*55·3*	*37·9*	*38·9*	*57·*
Orbital width,	41	35	37·4	46	35	39	...
Orbital height,	37	29	33	41	28	34	...
Orbital index,	*102·8*	*73·7*	*84·6*	*105·1*	*76·9*	*87·2*	*81·8*
Palato-alveolar length,	59	45	51	62	46	55·6	...
Palato-alveolar breadth,	64	52	58·3	71	50	60·9	...
Palato-alveolar index,	*130·6*	*94·5*	*109·8*	*130·*	*98·2*	*113·*	*109·*

Sagittal Sections.

In my *Challenger Report*, 1884, I gave a figure of at least one characteristic specimen of each group of aboriginal skulls, in which a skull had been bisected longitudinally and vertically immediately to one side of the septum nasi and the mesial plane of the cranial cavity, and in Part II. of the memoir on Indian Crania I produced two figures of a similar kind. In my description of these figures in the *Challenger Report* I stated that I was in accordance with Professors HUXLEY * and CLELAND † in regard to the import-

* *Jour. of Anat. and Phys.*, Nov. 1866, vol. i.

† *Ibid.*, July 1877, vol. xi., and Memoir on Variations already cited.

ance of the study of sections of this kind. In my present memoir I have pursued a similar method and have figured in Plate V. a series of sections.

The basion (b) has been selected as a centre, and from it radii have been drawn to definite points on the periphery of the skull. The only radius which requires explanation is the perpendicular (p), which is named from being drawn from the basion perpendicular to the plane of the foramen magnum. The perpendicular radius reaches the vertex usually more than an inch behind the bregma, and its upper limit approximates to the upper end of the fissure of Rolando, and indicates the posterior boundary of the frontal lobe of the cerebrum. The part of the cavity in relation to the cranial vault which lies in front of the perpendicular radius may be regarded as occupied by the frontal lobe, whilst that which lies behind the same radius and above the plane of the tentorium contains the parietal, occipital and temporo-sphenoidal lobes. The length of the several radii in the five Scottish skulls bisected and measured is given in Table XVII.

Table XVII.

Radii and other Lines.	Fife, Mh. C. Ix. 83·4.	Mid-Lothian, Rx. C. Ix. 80·1	Mid-Lothian, C. C. Ix. 77·5.	Northmaven, Shetland. C. Ix. 75·1.	Kintyre. C. Ix. 70·4.
	mm.	mm.	mm.	mm.	mm.
Basi-occipital,	109	104	114	116	112
Basi-lambdal,	114	117	120	123	119
Perpendicular,	131	136	142	145	135
Basi-bregmatic,	130	134	142	141	134
Basi-glabellar,	108	105	110	118	114
Basi-nasal,	98	99	104	110	104
Basi-alveolar,	99	91	91	97	99
From perpendicular radius to anterior pole of cranial cavity,	81	82	90	92	94
From perpendicular radius to posterior pole of cranial cavity,	77	78	80	86	85
Basi-occipito-sphenoid axis,	61	67	66	70	61
Cribriform axis,	31	24	30	29	31
Sphenoido-ethmoid angle,	137°	144°	136°	147°	137°
Spheno-maxillary line,	80	72	75	73	72
Spheno-maxillary angle,	91°	85°	76°	88°	92°
Base line,	134	133	141	146	139
Total longitudinal arc,	365	371	395	407	390

It will be seen that in each skull the distance from the perpendicular radius to the anterior part of the cranial cavity in which the anterior pole of the cerebrum is lodged, is longer than to the corresponding point behind, in which the occipital pole of the cerebrum is situated. The two crania in the table with brachycephalic proportions, Fife Mh. and Mid-Lothian Rx., show a closer approximation in the amount of cerebral space in front of and behind the perpendicular radius than is the case with the three mesaticephalic and dolichocephalic skulls. In the dolichocephalic Fuegian, Admiralty Island and Oahuan skulls measured in my *Challenger Report*, p. 120, the brain space behind the perpendicular radius was greater than that in front, but the contrary was the case in the mesaticephalic and brachycephalic skulls recorded in the same Report.

In addition to the radial lines drawn on the figures, the dimensions of which are given in Table XVII., lines have been drawn to express other relations. Thus the line *s* is parallel with the dorsum sellæ and cuts the plane of the foramen magnum at an obtuse angle,—named in my *Challenger Report* the foramino-sellar angle. The line *o.s.* is drawn from the basion through the basi-occipital, and the body of the sphenoid to the sphenoido-ethmoid articulation. It is the basi-occipito-sphenoid axis, and corresponds with the basi-cranial axis of HUXLEY. The line *c.r.*, or cribriform axis, is in the plane of the cribriform plate of the ethmoid bone, is drawn through the sphenoido-ethmoid and ethmo-frontal sutures, and its length is the distance between these sutures. It is intersected by the basi-cranial axis, and forms with it the sphenoido-ethmoid angle or basi-ethmoid angle of HUXLEY. If the inclination of the basi-occipito-sphenoid axis were a constant quantity, variations in this angle would express the degree of departure of the cribriform plate from the horizontal plane; but as the basi-cranial axis is not of uniform obliquity in the human skull, the angle may be modified by its degree of inclination, as well as by that of the cribriform axis. The difference in the angle in the series of crania was not more than 8°.

A line drawn from the sphenoido-ethmoid angle to the most projecting part of the superior maxilla is the spheno-maxillary line, and it forms with the basi-occipito-sphenoid axis, the spheno-maxillary angle. If this axis had been constant in its obliquity the angle would have been necessarily more open in prognathic jaws, but with this, as with the sphenoido-ethmoid angle, variations in the angle are also produced by modifications in the obliquity of the basi-cranial axis. In determining the value of this angle the obliquity of both the factors, which by their intersection form it, requires to be considered. The maximum difference in this angle in the five crania was 16°.

Of the three factors which collectively make up the longitudinal circumference, two, viz., the length of the foramen magnum and the basi-nasal diameter, together form the base line as defined by CLELAND. The total longitudinal arc constitutes the third factor, and the tables of measurements of the respective skulls give the data from which the relation of the base line to that arc in each specimen can be easily computed.

In the five male skulls specified in Table XVII. the relation of the base line to the longitudinal arc was as 1 to 2·78. In a larger series of seventeen male skulls from Fife and Mid-Lothian the relation was as 1 to 2·8. In a series of twenty male Australian aboriginals the base line was to the arc as 1 to 2·72, which gives, therefore, a smaller proportion of arc. In the Scottish skulls the mean length of the base line was 134·3 mm. and that of the arc was 376·5, whilst in the dolichocephalic Australians the mean base line was 139·8 mm. and that of the arc was 380·4.

I have measured the arc and base line in the skulls of five adult male gorillas, and found the mean base line to be 163·8 mm. and the mean arc 311·6, the proportion of base line to arc being as 1 to 1·9. The increase in the proportion of the base line to the longitudinal arc in the human skull may be regarded, therefore, as marking a stage of approximation to a lower mammalian type.

Summary.

The customary characters of the Scottish skulls may be summarised as follows :—

The crania were generally capacious, with the vertical transverse arc rounded behind the bregma, and they were not vertically flattened in the parieto-occipital region. The mean length-breadth index was mesaticephalic, but many specimens were dolichocephalic, and others brachycephalic. The mean vertical index was metriocephalic, though a considerable proportion were chamæcephalic. The breadth was greater than the height, and the crania were platychamæcephalic. The mean cubic capacity in the males was 1478 c.c., in the females 1322 c.c. The face was usually orthognathous, sometimes mesognathous; the nose was prominent, long and narrow, leptorhine; the orbits had usually the vertical diameter high in relation to the transverse, mesoseme or megoseme; the face was high in relation to the width, leptoprosopic; the palato-alveolar arch varied in the relations of length and breadth, but the form was frequently that of a wide horse-shoe. The lower jaw had a well-defined angle, the body of the bone was massive in the males, and with a pronounced chin.

I have restricted myself in Part I. of this memoir to the consideration of the anatomical characters of Scottish skulls as seen in the people of modern times along with a few which are perhaps mediæval in date. To complete the subject it will be necessary that skulls obtained in prehistoric burials in Scotland should be carefully examined. For this purpose I have collected from time to time, as opportunities occurred, specimens from different parts of Scotland, and have prepared descriptions which I hope to communicate as a second part of the memoir to the Society before the end of the session. In Part II. will also be discussed the characters of Scottish crania and heads in their general ethnographical relations to prehistoric races in Britain, and to the people of the adjoining part of the Continent of Europe.

EXPLANATION OF PLATES I.–IV.

The crania in these plates were photographed with much care by my friend and former pupil, W. E. Carnegie Dickson, B.Sc., M.B., to whom I was indebted for the series of photographs published in illustration of my "Memoirs on Indian Craniology." The process blocks were prepared from Dr Dickson's negatives by Messrs M. & T. Scott, Craigmillar Park.

Fig. 1. Profile of male skull, Dunfermline. Table I., H. T. 578.
Fig. 2. *Norma facialis* of the same skull.
Fig. 3. Vertex view of same skull.
Fig. 4. Profile of male skull from inland parish, Fife. Table I., Me.
Fig. 5. Vertex view of same skull.
Fig. 6. Profile of male skull from Fife. Table I., Ma.
Fig. 7. Vertex view of same skull.
Fig. 8. Profile of male skull, country parish, Mid-Lothian. Table III., Rt.
Fig. 9. Vertex view of same skull.
Fig. 10. Facial aspect of female skull from Paisley. Table X.
Fig. 11. Vertex view of same skull.
Fig. 12. Profile of male skull, country parish, Renfrewshire. Table IX., W.
Fig. 13. Profile of male skull, Kirkmadrine, Wigtonshire. Table XI., A.
Fig. 14. Vertex view of same skull.
Fig. 15. Vertex view of male skull, St Ninian's, Shetland.
Fig. 16. Facial view of same skull.
Fig. 17. Profile view of same skull.
Fig. 18. Profile of male skull from Sutherlandshire. Table XIV.
Fig. 19. Facial view of same skull.
Fig. 20. Vertex view of same skull.

EXPLANATION OF PLATE V.

Fig. 21. Vertical transverse section through a cranium from Renfrewshire. The section was made through the occipital condyles, mastoid processes, and the vertex about 1½ inch behind the bregma. The specimen gives a favourable view of the rounded low arch of the vertex when seen in the transverse plane. In this and the other sections I took a careful impression of the cut surface, which was subsequently reduced by photography for the process block.

Figs. 22–26. Antero-posterior almost mesial sections through skulls on which radial lines have been drawn from the basion to points on the periphery of the skull. *b.*, basion; *fm.*, plane of foramen magnum; *b.al.*, basi-alveolar radius; *b.n.*, basi-nasal radius; *b.gl.*, basi-glabellar radius; *b.br.*, basi-bregmatic radius; *b.p.*, perpendicular radius; *b.l.*, basi-lambdal radius; *b.oc.*, basi-occipital radius; *s.m.*, spheno-maxillary line; *cr.*, the cribriform axis, a line parallel to the cribriform plate; *o.s.*, basi-occipito-sphenoid axis; *s.*, plane of dorsum sellæ.

Fig. 22. Section of male brachycephalic skull from Fife. Table I., Mh. C. Ix. 83·4.
Fig. 23. Section of male brachycephalic skull from Mid-Lothian. Table III., Rx. C. Ix. 80·1.
Fig. 24. Section of male mesaticephalic skull from Mid-Lothian. Table III., C. C. Ix. 77·5.
Fig. 25. Section of male mesaticephalic skull from Northmavine, Shetland. Table XIII. C. Ix. 75·1.
Fig. 26. Section of male dolichocephalic skull from Kintyre. Table XIV., H.T. 26. C. Ix. 70·4.

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Part I.

Sir William Turner on "Craniology of the People of Scotland."—Plate I.

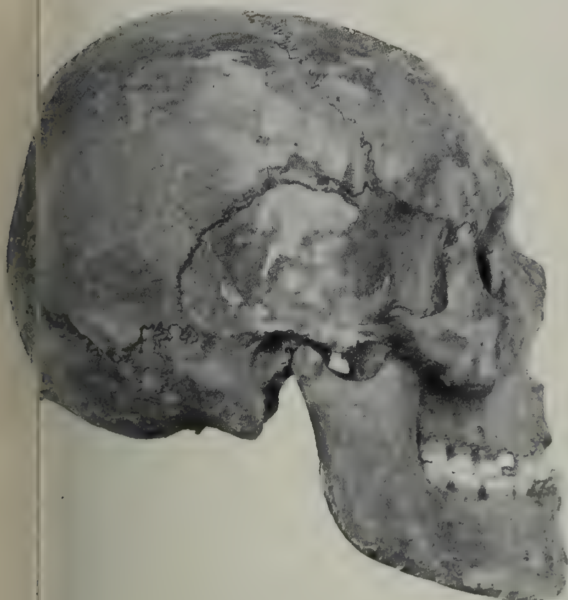


Fig. 1.—Dunfermline.

Fig. 2.—Dunfermline.

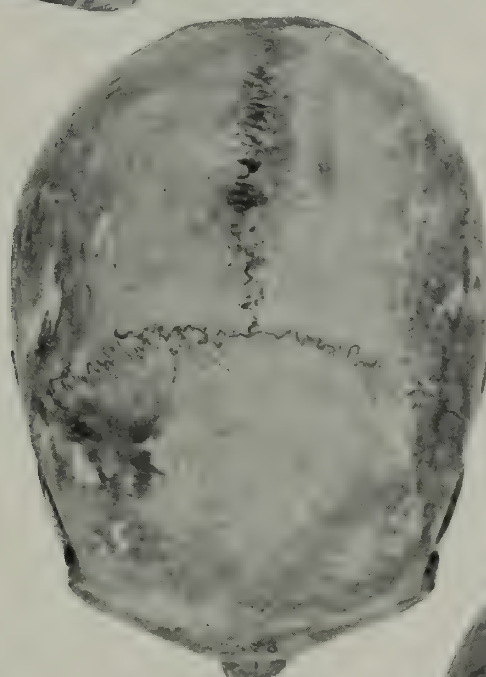


Fig. 3.—Dunfermline.

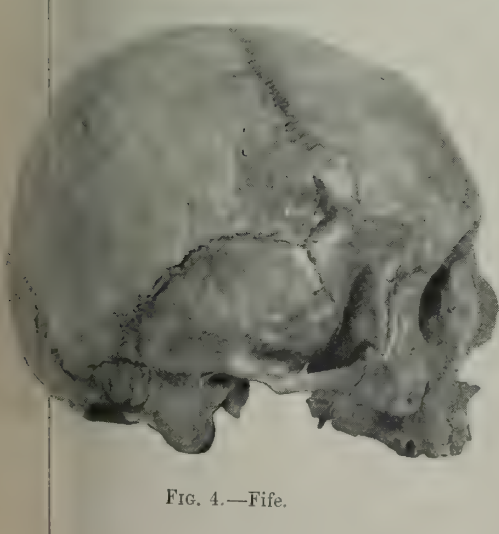


Fig. 4.—Fife.

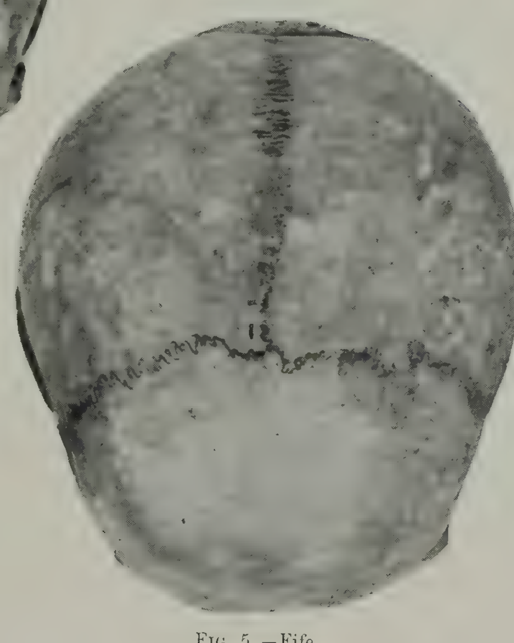


Fig. 5.—Fife.

SIR WILLIAM TURNER on "Craniology of the People of Scotland."—PLATE II.

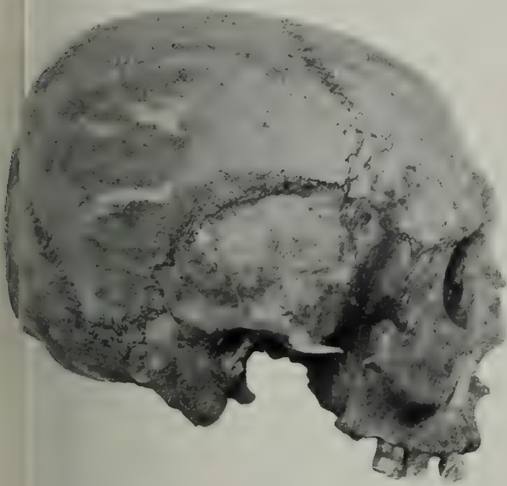


FIG. 6.—Fife.

FIG. 8—Mid-Lothian.

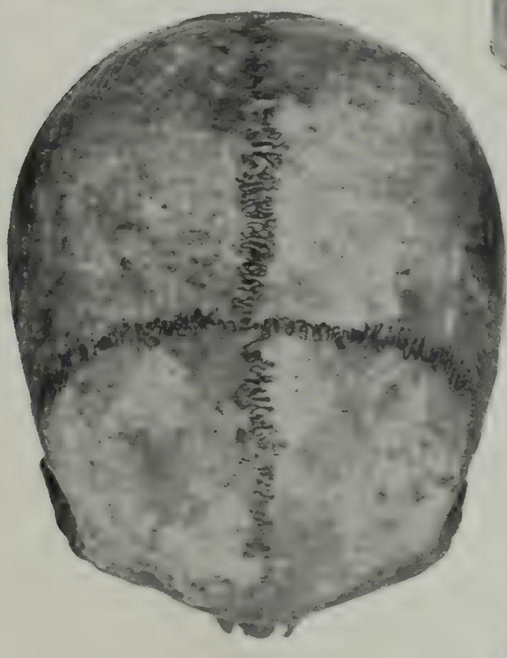


FIG. 7.—Fife.

FIG. 10.—Paisley.

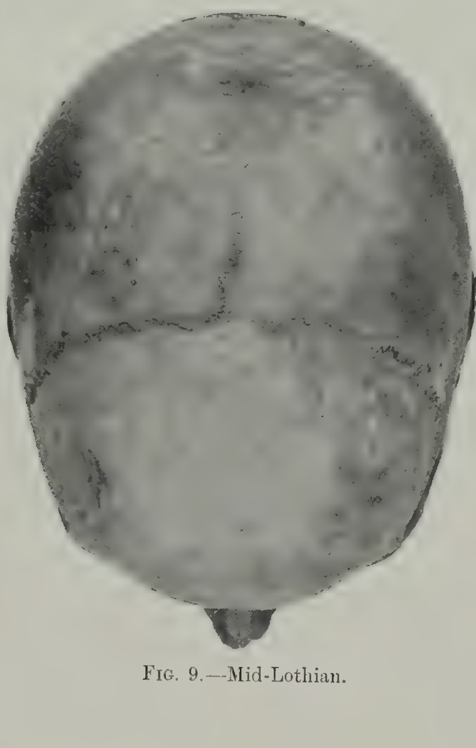


FIG. 9.—Mid-Lothian.

Sir William Turner on "Craniology of the People of Scotland."—Plate III.

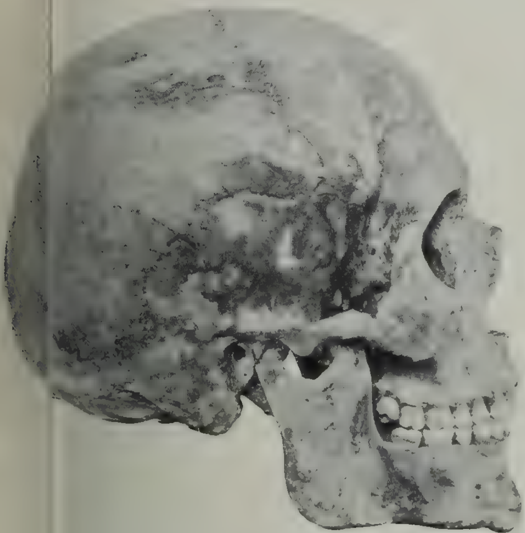


Fig. 12.—Renfrewshire.

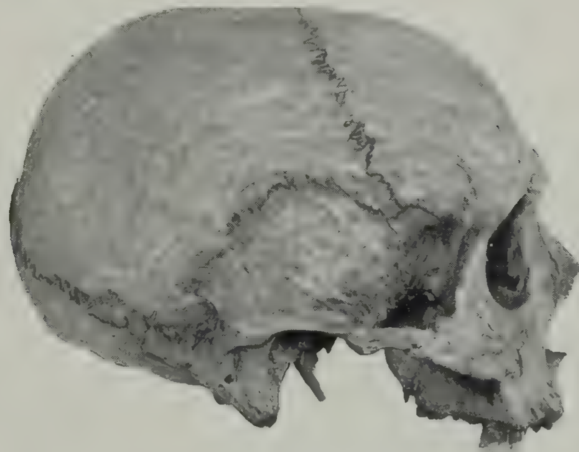


Fig. 13.—Wigtonshire.

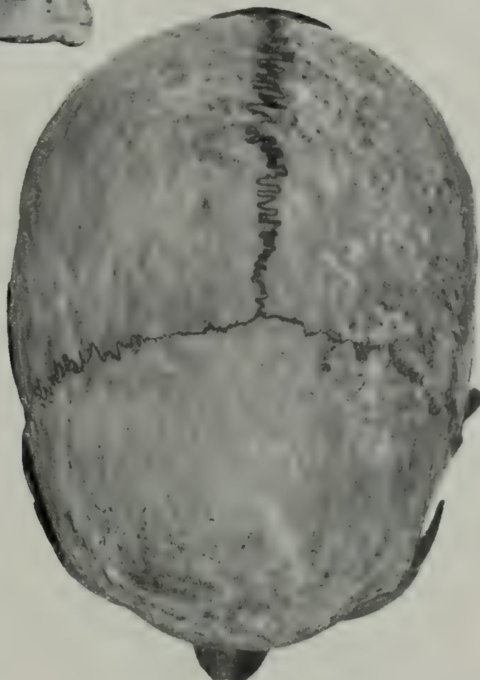


Fig. 14.—Wigtonshire.

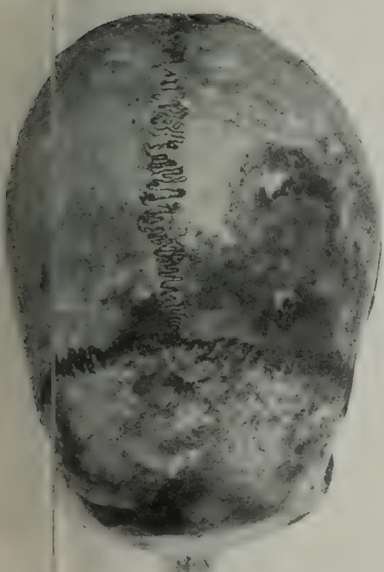


Fig. 11.—Paisley.

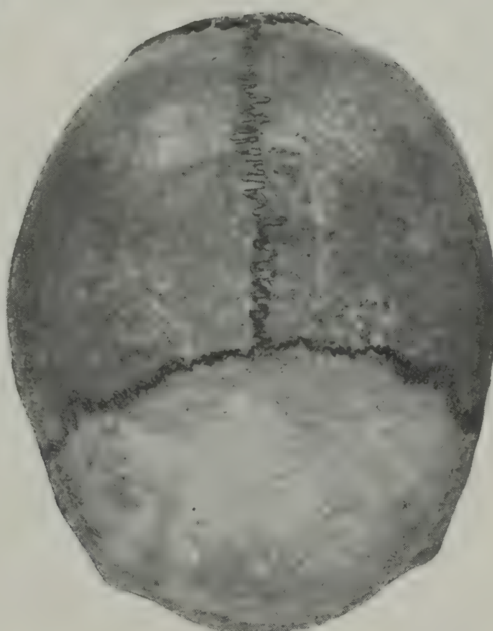


Fig. 15.—Shetland.

SIR WILLIAM TURNER on "Craniology of the People of Scotland."—PLATE IV.

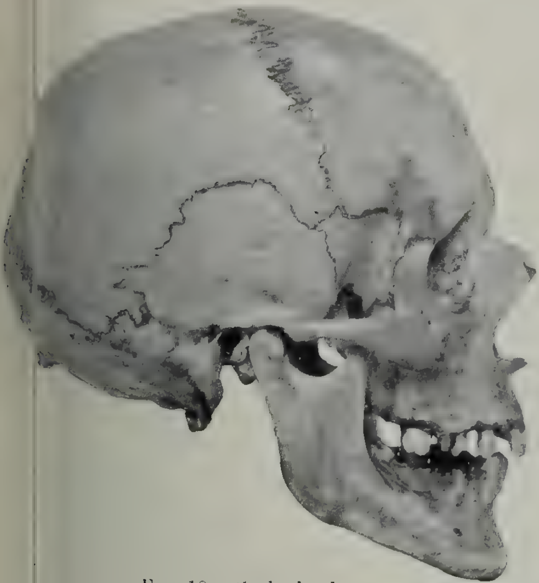


FIG. 18.—Sutherland.

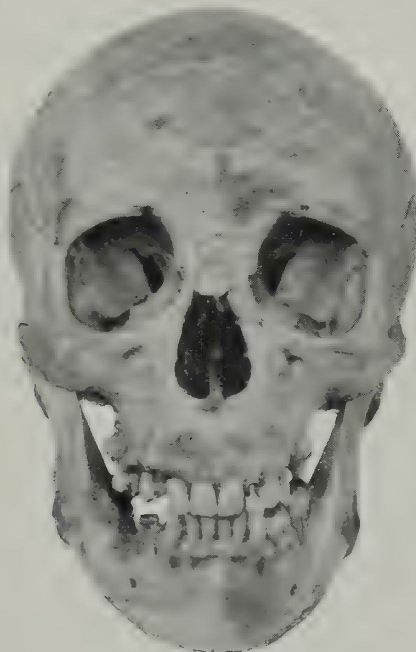


FIG. 19.—Sutherland.

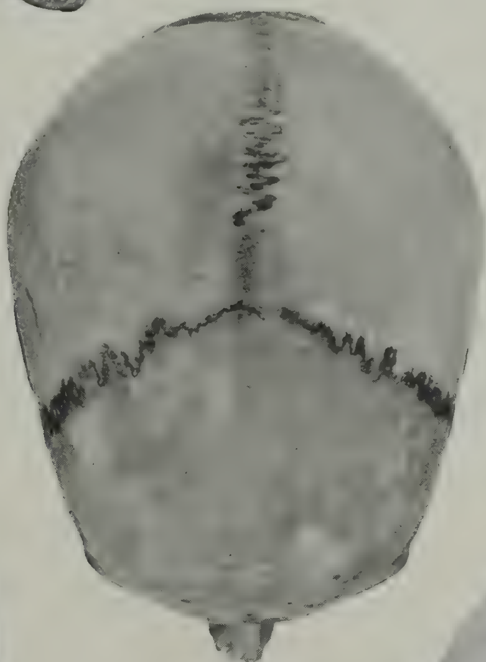


FIG. 20.—Sutherland.

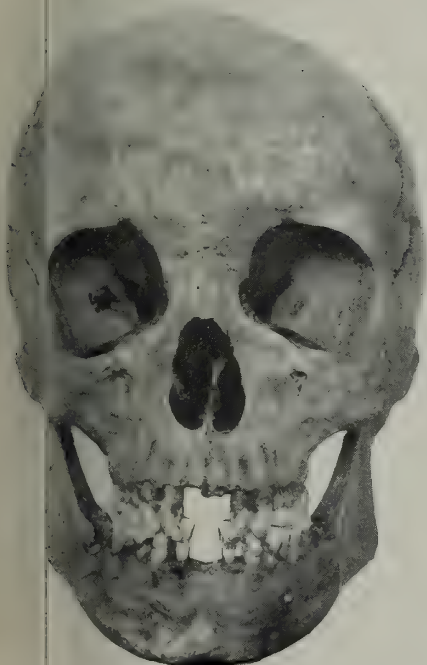


FIG. 16.—Shetland.

FIG. 17.—Shetland.

Sir William Turner on "Craniology of the People of Scotland."—Plate V.

Fig. 21.—Renfrewshire.

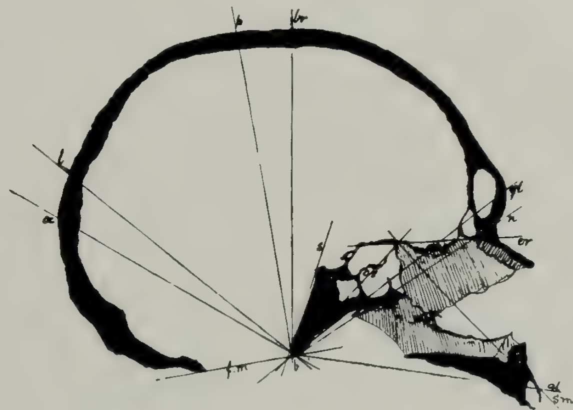


Fig. 22.—Fifeshire.

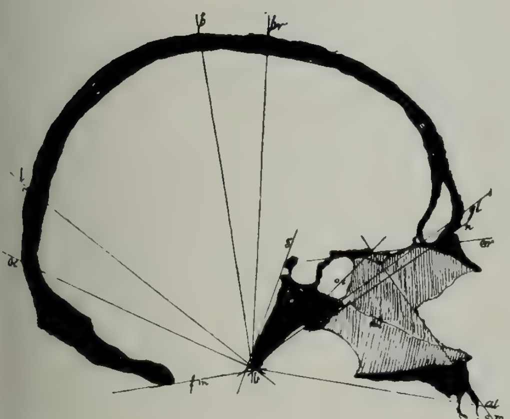


Fig. 23.—Mid-Lothian.

Fig. 24.—Mid-Lothian.

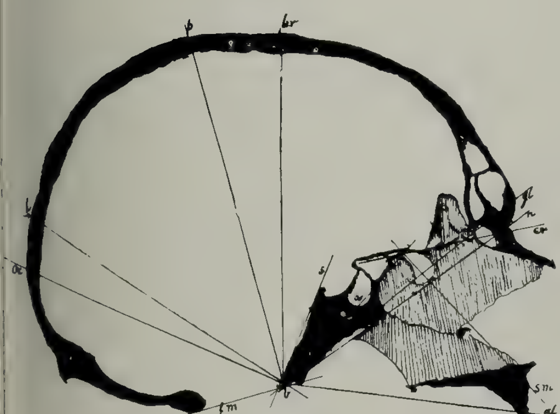


Fig. 25.—Shetland.

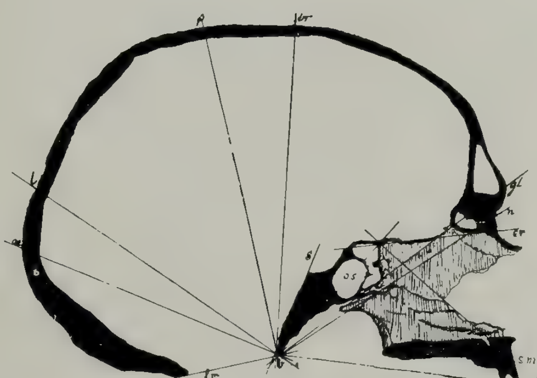


Fig. 26.—Kintyre.

XXV.—*The Generating Function of the Reciprocal of a Determinant.* By Thomas Muir, LL.D.

(MS. received 8th September 1902. Read 3rd November 1902. Issued separately March 6, 1903.)

(1) This is a subject to which very little study has been directed. The first to enunciate any proposition regarding it was Jacobi; but the solitary result which he reached received no attention from mathematicians,—certainly no fruitful attention,—during seventy years following the publication of it.

Jacobi was concerned with a problem regarding the partition of a fraction with composite denominator $(u_1 - t_1)$ $(u_2 - t_2)$ . . . into other fractions whose denominators are factors of the original, where u_1 , u_2 , . . . are linear homogeneous functions of one and the same set of variables. The specific character of the partition was only definable by viewing the given fraction $(u_1 - t_1)^{-1}(u_2 - t_2)^{-1}$ . . . as expanded in series form, it being required that each partial fraction should be the aggregate of a certain set of terms in this series. Of course the question of the *order* of the terms in each factor of the original denominator had to be attended to at the outset, since the expansion for $(a_1x + b_1y + c_1z - t)^{-1}$ is not the same as for $(b_1y + c_1z + a_1x - t)^{-1}$. Now one general proposition to which Jacobi was led in the course of this investigation was that *the coefficient of* $x_1^{-1}x_2^{-1}x_3^{-1}$ . . . *in the expansion of* $u_1^{-1}u_2^{-1}u_3^{-1}$ . . ., *where*

$$\begin{aligned} u_1 &= a_1x_1 + a_2x_2 + a_3x_3 + \dots \\ u_2 &= b_1x_1 + b_2x_2 + b_3x_3 + \dots \\ u_3 &= c_1x_1 + c_2x_2 + c_3x_3 + \dots \\ &\dots\dots\dots\dots \end{aligned}$$

is $|\mathrm{a}_1\mathrm{b}_2\mathrm{c}_3 \dots|^{-1}$, *provided that in every case the first term of* u_r *is that containing* x_r.

This is the proposition with which we are here concerned.

(2) Jacobi gave no proof of it. He considered first the case of $(ax + by)^{-1}$ $(b'y + a'x)^{-1}$, passing thence to $(ax + by - t)^{-1}$ $(b'y + a'x - t')^{-1}$: next he dealt with $(ax + by + cz)^{-1}(b'y + c'z + a'x)^{-1}(c''z + a''x + b''y)^{-1}$, from which he passed readily as before to $(ax + by + cz - t)^{-1}$ . . . . : and then he found the work so forbidding that he turned to other things,—"Ad quatuor pluresve variabiles haec extendere non lubet, cum iam pro tribus tam prolixa exstiterint. Progredimur ad alia" (p. 354).*

(3) Doubtless it is true that often in the theory of determinants the treatment of the first two cases of a theorem is not only a sufficient guide to the generalisation of the

* C. G. J. Jacobi, "Exercitatio algebraica circa discerptionem singularem fractionum, quae plures variabiles involvunt," *Crelle's Journ.*, v. pp. 344–364 (year 1829).

theorem, but virtually involves the generalisation, and makes the enunciation of the latter a mere matter of form. Let us see, however, whether it be so in this particular instance.

In the first case JACOBI, abruptly inserting $| a_1 b_2 |$ into the numerator of the given fraction, says that

$$\frac{a_1b_2 - a_2b_1}{(a_1x_1 + a_2x_2)(b_2x_2 + b_1x_1)} = \frac{a_1}{x_2} \cdot \frac{1}{a_1x_1 + a_2x_2} - \frac{b_1}{x_1} \cdot \frac{1}{b_2x_2 + b_1x_1},$$

also that

$$\frac{a_1}{x_2} \cdot \frac{1}{a_1x_1 + a_2x_2} = \frac{1}{x_1x_2} - \frac{1}{x_1} \cdot \frac{a_2}{a_1x_1 + a_2x_2},$$

and that therefore

$$\frac{a_1b_2 - a_2b_1}{(a_1x_1 + a_2x_2)(b_2x_2 + b_1x_1)} = \frac{1}{x_1x_2} - \frac{1}{x_1} \cdot \frac{a_2}{a_1x_1 + a_2x_2} - \frac{1}{x_2} \cdot \frac{b_1}{b_2x_2 + b_1x_1}, \quad . \quad . \quad . \quad . \quad . \quad (a)$$

a glance being then all that is necessary to make clear that the expansion of the last two fractions on the right consists of terms involving negative powers of one variable and positive powers of the other, and consequently that the cofactor of $x_1^{-1}x_2^{-1}$ in $(a_1x_1 + a_2x_2)^{-1}(b_2x_2 + b_1x_1)^{-1}$ is $(a_1b_2 - a_2b_1)^{-1}$.

In the second case, after certain preliminaries regarding subsidiary functions which are necessary for the expression of his final result but which have nothing to do with the theorem at present before us, he with equal abruptness throws down the "easily verified" identity

$$\begin{aligned} | a_1b_2c_3 | \cdot x_1x_2x_3 = u_1u_2u_3 &- x_1u_1(b_1c_1x_1 + c_1b_2x_2 + b_1c_3x_3) \\ &- x_2u_2(c_2a_2x_2 + a_2c_3x_3 + c_2a_1x_1) \\ &- x_3u_3(a_3b_3x_3 + b_3a_1x_1 + a_3b_2x_2), \end{aligned}$$

and dividing by $x_1x_2x_3 \cdot u_1u_2u_3$ obtains of course

$$\begin{aligned} \frac{| a_1b_2c_3 |}{(a_1x_1 + \ldots)(b_2x_2 + \ldots)(c_3x_3 + \ldots)} = \frac{1}{x_1x_2x_3} &- \frac{b_1c_1x_1 + c_1b_2x_2 + b_1c_3x_3}{x_2x_3(b_2x_2 + \ldots)(c_3x_3 + \ldots)} \quad . \quad . \quad . \quad (b) \\ &- \frac{c_2a_2x_2 + a_2c_3x_3 + c_2a_1x_1}{x_3x_1(c_3x_3 + \ldots)(a_1x_1 + \ldots)} \\ &- \frac{a_3b_3x_3 + b_3a_1x_1 + a_3b_2x_2}{x_1x_2(a_1x_1 + \ldots)(b_2x_2 + \ldots)} \end{aligned}$$

This is the analogue to (a) in the previous case, and reasoning with it exactly as with (a) he concludes that the cofactor of $x_1^{-1}x_2^{-1}x_3^{-1}$ in $(a_1x_1 + a_2x_2 + a_3x_3)^{-1}$ $(b_2x_2 + \ldots)^{-1}(c_3x_3 + \ldots)^{-1}$ is $| a_1b_2c_3 |^{-1}$.

(4) The natural commentary upon this is that there is a certain arbitrariness in the mode of procedure in both cases: that there is no similarity until the final identity is reached: and that the result of this similarity is to force upon us the conviction that for the establishment of the next case of the theorem there must be an identity (c) similar to (b) and (a). Now an examination of the last two fractions in (a) and the last three fractions in (b) shows that each set forms a cycle, the generating substitutions in (a) being

$$\begin{pmatrix} a, b \\ b, a \end{pmatrix}, \quad \begin{pmatrix} 1, 2 \\ 2, 1 \end{pmatrix};$$

and in (b)

$$\begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}.$$

This, of course, is as it should be, because the remaining terms of the two identities are unaltered by the substitution: but the observation derives additional importance from the fact that the general application of it does away with the necessity for any stipulation as to the order of the terms in $u_1, u_2, \ldots$, for, having $u_1 \equiv a_1x_1 + a_2x_2 + a_3x_3$ cyclical substitution gives $u_2 \equiv b_2x_2 + b_3x_3 + b_1x_1$ and not $\equiv b_1x_1 + b_2x_2 + b_3x_3$. The identities (a) and (b) are thus more appropriately written

$$\frac{|a_1b_2|}{u_1u_2} = \frac{1}{x_1x_2} - \overset{\circ}{\sum}\frac{b_1}{x_2u_2},$$

$$\frac{|a_1b_2c_3|}{u_1u_2u_3} = \frac{1}{x_1x_2x_3} - \overset{\circ}{\sum}\frac{b_1c_1x_1 + c_1b_2x_2 + b_1c_3x_3}{x_2u_2 \cdot x_3u_3};$$

or if we fall back to the stage which JACOBI views in the second case as being fundamental

$$|a_1b_2|\,x_1x_2 = u_1u_2 - \overset{\circ}{\sum}x_1u_1 \cdot b_1,$$

$$|a_1b_2c_3|\,x_1x_2x_3 = u_1u_2u_3 - \overset{\circ}{\sum}x_1u_1 \cdot (b_1c_1x_1 + c_1b_2x_2 + b_1c_3x_3).$$

The simple question with which we are therefore brought face to face is as to the existence of a general identity of the latter kind.

(5) By way of answer to this an examination of the next case may be found sufficient. Is there, then, a quadric function, F_1, of x_1, x_2, x_3, x_4 such that

$$|a_1b_2c_3d_4| \cdot x_1x_2x_3x_4 = u_1u_2u_3u_4 - \overset{\circ}{\sum}x_1u_1 \cdot F_1\ ?$$

So probable is it made by analogy, by enumeration of terms and by actual experiment, that a searching examination cannot be dispensed with.

Denoting the ten possible coefficients of F_1 by $1_{11}, 1_{12}, \ldots, 1_{44}$ so that

$$\overset{\circ}{\sum}x_1u_1 \cdot F_1 \equiv x_1x_1 \cdot \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1_{11} & 1_{12} & 1_{13} & 1_{14} & x_1 \\ & 1_{22} & 1_{23} & 1_{24} & x_2 \\ & & 1_{33} & 1_{34} & x_3 \\ & & & 1_{44} & x_4 \end{array} + x_2u_2 \cdot \begin{array}{cccc|c} x_2 & x_3 & x_4 & x_5 & \\ \hline 2_{22} & 2_{23} & 2_{24} & 2_{21} & x_2 \\ & 2_{33} & 2_{34} & 2_{31} & x_3 \\ & & 2_{44} & 2_{41} & x_4 \\ & & & 2_{11} & x_1 \end{array} + \cdots$$

we have for the determination of the 40 unknowns the set of 35 equations obtained by taking the coefficients on the right-hand side of the supposed identity and equating to zero the coefficients of $x_1^4, x_2^4, \ldots, x_1^3x_2, \ldots$, and to $|a_1b_2c_3d_4|$ the coefficient of $x_1x_2x_3x_4$.* The first four equations,—a cyclic set,—

$$a_1b_1c_1d_1 - a_11_{11} = 0, \quad a_2b_2c_2d_2 - a_22_{22} = 0, \quad \ldots \qquad (1)_1, (1)_2, \ldots$$

* In view of the insufficiency in the number of equations it may be noted that in the preceding case there is a redundancy, viz., ten equations to nine unknowns, but no inconsistency.

give at once for 1_{11}, 2_{22}, 3_{33}, 4_{44} the values

$$b_1c_1d_1 ,\quad c_2d_2a_2 ,\quad d_3a_3b_3 ,\quad a_4b_4c_4 .$$

Taking next the coefficient of $x_1{}^3x_2$, we have

$$a_1 1_{12} + a_2 1_{11} + b_1 2_{11} = a_1b_1c_1d_2 + a_1b_1c_2d_1 + a_1b_2c_1d_1 + a_2b_1c_1d_1 ,$$

which, in consequence of the previous determination of 1_{11}, is reduced by a term on both sides. Consideration, therefore, of this coefficient and the eleven others like it will be seen to lead to the twelve equations

$$\begin{array}{rcll}
a_1 1_{12} + b_1 2_{11} &=& a_1b_1c_1d_2 + a_1b_1c_2d_1 + a_1b_2c_1d_1 , & (2)_1 \\
b_2 2_{23} + c_2 3_{22} &=& b_2c_2d_2a_3 + b_2c_2d_3a_2 + b_2c_3d_2a_2 , & (2)_2 \\
c_3 3_{34} + d_3 4_{33} &=& c_3d_3a_3b_4 + c_3d_3a_4b_3 + c_3d_4a_3b_3 , & (2)_3 \\
d_4 4_{14} + a_4 1_{44} &=& d_4a_4b_4c_1 + d_4a_4b_1c_4 + d_4a_1b_4c_4 , & (2)_4 \\[4pt]
a_1 1_{13} + c_1 3_{11} &=& a_1b_1c_1d_3 + a_1b_1c_3d_1 + a_1b_3c_1d_1 , & (3)_1 \\
b_2 2_{24} + d_2 4_{22} &=& b_2c_2d_2a_4 + b_2c_2d_4a_2 + b_2c_4d_2a_2 , & (3)_2 \\
c_3 3_{13} + a_3 1_{33} &=& c_3d_3a_3b_1 + c_3d_3a_1b_3 + c_3d_1a_3b_3 , & (3)_3 \\
d_4 4_{24} + b_4 2_{44} &=& d_4a_4b_4c_2 + d_4a_4b_2c_4 + d_4a_2b_4c_4 , & (3)_4 \\[4pt]
a_1 1_{14} + d_1 4_{11} &=& a_1b_1c_1d_4 + a_1b_1c_4d_1 + a_1b_4c_1d_1 & (4)_1 \\
b_2 2_{12} + a_2 1_{22} &=& b_2c_2d_2a_1 + b_2c_2d_1a_2 + b_2c_1d_2a_2 & (4)_2 \\
c_3 3_{23} + b_3 2_{33} &=& c_3d_3a_3b_2 + c_3d_3a_2b_3 + c_3d_2a_3b_3 & (4)_3 \\
d_4 4_{34} + c_4 3_{44} &=& d_4a_4b_4c_3 + d_4a_4b_3c_4 + d_4a_3b_4c_4 & (4)_4
\end{array}$$

Dealing next with the coefficients of $x_1{}^2x_2{}^2$, $x_1{}^2x_3{}^2$, . . . we obtain

$$\begin{array}{rcll}
a_1 1_{22} + a_2 1_{12} + b_2 2_{11} + b_1 2_{12} &=& a_1b_1c_2d_2 + a_1c_1b_2d_2 + a_1d_1b_2c_2 + b_1c_1a_2d_2 + b_1d_1a_2c_2 + c_1d_1a_2b_2 & (5)_1 \\
b_2 2_{33} + b_3 2_{23} + c_3 3_{22} + c_2 3_{23} &=& b_2c_2d_3a_3 + b_2d_2c_3a_3 + b_2a_2c_3d_3 + c_2d_2b_3a_3 + c_2a_2b_3d_3 + d_2a_2b_3c_3 & (5)_2 \\
c_3 3_{44} + c_4 3_{34} + d_4 4_{33} + d_3 4_{34} &=& c_3d_3a_4b_4 + c_3a_3d_4b_4 + c_3b_3d_4a_4 + d_3a_3c_4b_4 + d_3b_3c_4a_4 + a_3b_3c_4d_4 & (5)_3 \\
d_4 4_{11} + d_1 4_{14} + a_1 1_{44} + a_4 1_{14} &=& d_4a_4b_1c_1 + d_4b_4a_1c_1 + d_4c_4a_1b_1 + a_4b_4d_1c_1 + a_4c_4d_1b_1 + b_4c_4d_1a_1 & (5)_4 \\
a_1 1_{33} + a_3 1_{13} + c_3 3_{11} + c_1 3_{13} &=& a_1b_1c_3d_3 + a_1c_1b_3d_3 + a_1d_1b_3c_3 + b_1c_1a_3d_3 + b_1d_1a_3c_3 + c_1d_1a_3b_3 & (6)_1 \\
b_2 2_{44} + b_4 2_{24} + d_4 4_{22} + d_2 4_{24} &=& b_2c_2d_4a_4 + b_2d_2c_4a_4 + b_2a_2c_4d_4 + c_2d_2b_4a_4 + c_2a_2b_4d_4 + d_2a_2b_4c_4 & (6)_2
\end{array}$$

These six, it will be found, do not contain any other unknowns than those contained in the previous set of twelve; and, what is still more important, the unknowns contained in any one of the set of six are exactly those contained in two of the set of twelve. For the purposes of solution, therefore, it is better to view the eighteen as belonging to six sets of three, viz.,

$$\begin{array}{lcl}
(5)_1 ,\ (2)_1 ,\ (4)_2 & & \\
(5)_2 ,\ (2)_2 ,\ (4)_3 & \text{and} & (6)_1 ,\ (3)_1 ,\ (3)_3 \\
(5)_3 ,\ (2)_3 ,\ (4)_4 & & (6)_2 ,\ (3)_2 ,\ (3)_4 . \\
(5)_4 ,\ (2)_4 ,\ (4)_1 & &
\end{array}$$

Taking the first triad, viz.,

$$\left.\begin{array}{rcl}
a_1 1_{22} + a_2 1_{12} + b_2 2_{11} + b_1 2_{12} &=& a_1b_1c_2d_2 + \ldots \\
a_1 1_{12} + b_1 2_{11} \qquad\qquad &=& a_1b_1c_1d_2 + a_1b_1c_2d_1 + a_1b_2c_1d_1 \\
a_2 1_{22} \qquad\qquad\qquad + b_2 2_{12} &=& b_2c_2d_2a_1 + b_2c_2d_1a_2 + b_2c_1d_2a_2
\end{array}\right\}$$

we see that there are four possible solutions of the third equation; and, as substitution for 1_{22} and 2_{12} in the first equation produces an equation like the second, it follows that there are not more than four solutions possible for the triad. These are

$$\left.\begin{array}{lcl} 1_{22} &=& 0, \\ 2_{12} &=& c_2d_2a_1+c_2d_1a_2+c_1d_2a_2 \\ 1_{12} &=& b_2c_1d_1 \\ 2_{11} &=& a_1c_1d_2+a_1c_2d_1 \end{array}\right\}\left\{\begin{array}{l} b_2c_2d_1 \\ c_2d_2a_1+c_1d_2a_2 \\ b_1c_2d_1+b_2c_1d_1 \\ a_1c_1d_2 \end{array}\right\}\left\{\begin{array}{l} b_2c_1d_2 \\ c_2d_2a_1+c_2d_1a_2 \\ b_1c_1d_2+b_2c_1d \\ a_1c_2d_1 \end{array}\right\}\left\{\begin{array}{l} b_2c_2d_1+b_2c_1d_2 \\ c_2d_2a_1 \\ b_1c_1d_2+b_1c_2d_1+b_2c_1d_1 \\ 0. \end{array}\right.$$

The solutions of the next three triads are got by cyclic substitution.

The fifth triad in the same way gives us

$$\left.\begin{array}{lcl} 1_{33} &=& 0 \\ 3_{13} &=& a_3b_1d_3+a_1b_3d_3+a_3b_3d_1 \\ 1_{13} &=& b_1c_3d_1 \\ 3_{11} &=& a_1b_1d_3+a_1b_3d_1 \end{array}\right\}\left\{\begin{array}{l} b_1c_3d_3 \\ a_1b_3d_3+a_3b_3d_1 \\ b_1c_1d_3+b_1d_1c_3 \\ a_1d_1b_3 \end{array}\right\}\left\{\begin{array}{l} b_3c_3d_1 \\ a_3b_1d_3+a_1b_3d_3 \\ b_1d_1c_3+c_1d_1b_3 \\ a_1b_1d_3 \end{array}\right\}\left\{\begin{array}{l} b_1c_3d_3+b_3c_3d_1 \\ a_1b_3d_3 \\ b_1c_1d_3+b_1d_1c_3+b_3c_1d_1 \\ 0. \end{array}\right.$$

and the solutions of the sixth are got therefrom by cyclical substitution.

Here, however, a hitch occurs, for if we proceed with the cyclical substitution we find that while in the matter of the equations we are led back, as we ought to be, from the sixth triad to the fifth, the same is not true of the solutions. In fact, the solutions of the fifth triad alone are immediately convincing of the hopelessness of our quest; for pairs of the unknowns in that triad, *e.g.*, 1_{33}, 3_{11} belong to the same cycle, and the value of the second of the pair should therefore be obtainable from that of the first by cyclical substitution, which is not the case.

(6) A different mode of procedure from JACOBI's being thus necessary for the establishment of his theorem, I would substitute the following, which, though ostensibly concerned only with the special case which we have just been examining, is of perfectly general application.

Since the sum of the elements of every one of its columns is zero, the determinant

$$\begin{vmatrix} u_1-a_2x_2 & -b_2x_2 & -c_2x_2 & -d_2x_2 \\ -a_3x_3 & u_2-b_3x_3 & -c_3x_3 & -d_3x_3 \\ -a_4x_4 & -b_4x_4 & u_3-c_4x_4 & -d_4x_4 \\ -a_1x_1 & -b_1x_1 & -c_1x_1 & u_4-d_1x_1 \end{vmatrix}$$

vanishes identically. Expanding it in a series arranged according to products of the u's we have therefore

$$\begin{aligned} 0 = u_1u_2u_3u_4 - \sum u_1u_2u_3 \cdot d_1x_1 + \sum u_1u_2 \cdot | c_4d_1 | \cdot x_4x_1 \\ - \sum u_1 \cdot | b_3c_4d_1 | \cdot x_3x_4x_1 + | a_2b_3c_4d_1 | \cdot x_2x_3x_4x_1 \end{aligned}$$

and consequently

$$\begin{aligned} | a_1b_2c_3d_4 | \cdot x_1x_2x_3x_4 = u_1u_2u_3u_4 - \sum^{\circ} u_1u_2u_3 \cdot d_1x_1 + \sum^{\circ} u_1u_2 \cdot | c_4d_1 | \cdot x_4x_1 \\ + \sum u_1u_3 \cdot | b_3d_1 | \cdot x_3x_1 - \sum u_1 | b_3c_4d_1 \cdot x_3x_4x_1 \end{aligned}$$

On dividing by $x_1x_2x_3x_4 \cdot u_1u_2u_3u_4$ this is changed into

$$\frac{| a_1b_2c_3d_4 |}{u_1u_2u_3u_4} = \frac{1}{x_1x_2x_3x_4} - \sum^{\circ} \frac{d_1}{u_4 \cdot x_2x_3x_4} + \sum^{\circ} \frac{| c_4d_1 |}{u_3u_4 \cdot x_2x_3} + \sum^{\circ} \frac{| b_3d_1}{u_2n_4 \cdot x_2x_4} - \sum^{\circ} \frac{| b_3c_4d_1}{n_2n_3n_4 \cdot x_1}$$

and as the expansion of each of the four fractions under the sign of summation contains in every term a positive power of x_1, it follows that the cofactor of $x_1^{-1}x_2^{-1}x_3^{-1}x_4^{-1}$ in the expansion of $|a_1b_2c_3d_4|\ 1u_1u_2u_3u_4$ is 1, as was to be proved.

(7) The construction of the vanishing determinant on which the proof rests is easily completed when the diagonal elements are given; in every case, however, there is more than one set of diagonal elements suitable for the purpose. The requirements are (1) that the elements chosen from $|a_1b_2c_3\ldots|$ to appear in the diagonal of the vanishing determinant shall form a positive or negative term of $|a_1b_2c_3\ldots|$ according as the number of these elements is odd or even: and (2) that no suffix shall be in the place which it occupied in the original diagonal.

The first requirement is due to the need for having the sign of $|a_1b_2c_3\ldots|.x_1x_2x_3\ldots$ negative at the end of the expansion of the vanishing determinant: and the second to the need for having a positive power of one of the x's in every term, except one, of the expansion of the subsequent fractions.

Both requirements are seen to be satisfied by taking the suffixes 1, 2, 3, ..., n and moving either the last to the first place or the first to the last place: for then the number of inverted-pairs in the suffixes is $n-1$ and no element is in its original place. In the case where $n=3$, these are the only possibilities: in the case where $n=4$ there are, however, four others, viz., a_2, b_4, c_1, d_3; a_3, b_1, c_4, d_2; a_3, b_4, c_2, d_1; a_4, b_3, c_1, d_2.

(8) For the purpose of clearing up JACOBI's mode of procedure, let us examine the new proof as applied to his second case. The two vanishing determinants are

$$\begin{vmatrix} u_1-a_2x_2 & -b_2x_2 & -c_2x_2 \\ -a_3x_3 & u_2-b_3x_3 & -c_3x_3 \\ -a_1x_1 & -b_1x_1 & u_3-c_1x_1 \end{vmatrix}, \quad \begin{vmatrix} u_1-a_3x_3 & -b_3x_3 & -c_3x_3 \\ -a_1x_1 & u_2-b_1x_1 & -c_1x_1 \\ -a_2x_2 & -b_2x_2 & u_3-c_2x_2 \end{vmatrix},$$

their expansions being

$$u_1u_2u_3 - \overset{\circ}{\sum} u_1u_2\cdot c_1x_1 + \overset{\circ}{\sum} u_1\cdot|b_3c_1|\cdot x_3x_1 - |a_2b_3c_1|\cdot x_2x_3x_1,$$

$$u_1u_2u_3 - \overset{\circ}{\sum} u_1u_2\cdot c_2x_2 + \overset{\circ}{\sum} u_1\cdot|b_1c_2|.x_1x_2 - |a_3b_1c_2|\cdot x_3x_1x_2;$$

and the resulting identities

$$\frac{|a_1b_2c_3|}{u_1u_2u_3} = \frac{1}{x_1x_2x_3} - \overset{\circ}{\sum}\frac{c_1}{u_3\cdot x_2x_3} + \overset{\circ}{\sum}\frac{|b_3c_1|}{u_2u_3\cdot x_2},$$

$$\frac{|a_1b_2c_3|}{u_1u_2u_3} = \frac{1}{x_1x_2x_3} - \overset{\circ}{\sum}\frac{c_2}{u_3\cdot x_1x_3} + \overset{\circ}{\sum}\frac{|b_1c_2|}{u_2u_3\cdot x_3},$$

the one being as suitable as the other for establishing the theorem regarding the coefficient of $x_1^{-1}x_2^{-1}x_3^{-1}$. If now, however, we combine the last two terms of each, there results

$$\frac{|a_1b_2c_3|}{u_1u_2u_3} = \frac{1}{x_1x_2x_3} - \overset{\circ}{\sum}\frac{b_1c_1x_1+b_2c_1x_2+b_1c_3x_3}{x_2x_3\cdot u_2u_3},$$

$$\frac{|a_1b_2c_3|}{u_1u_2u_3} = \frac{1}{x_1x_2x_3} - \overset{\circ}{\sum}\frac{b_2c_1x_1+b_2c_2x_2+b_3c_2x_3}{x_1x_3\cdot u_2u_3},$$

one of which, as might have been expected, is identical with JACOBI'S, and therefore not less suitable than in its previous form. In the other case combination has proved harmful, the reason being that then it is not the same x which occurs with positive index in the expansions of the two fractions concerned.*

An examination of the corresponding fractions in § 6 will attest the accuracy of the result reached in § 5.

(9) JACOBI'S theorem that

the coefficient of $x_1^{-1}x_2^{-1}x_3^{-1}\cdots$ *in the expansion of* $u_1^{-1}u_2^{-1}u_3^{-1}\cdots$ *is* $|a_1b_2c_3\cdots|^{-1}$,

has a curious analogue in the theorem that

the coefficient of $x_1x_2x_3\cdots$ *in the expansion of* $u_1u_2u_3\cdots$ *is* $\overset{+}{|}a_1b_2c_3\cdots\overset{+}{|}$,

where the permanent $\overset{+}{|}a_1b_2c_3\ldots\overset{+}{|}$ is a function differing from the determinant $|a_1b_2c_3\ldots|$ in having all its terms positive.† I now propose to show that this is not a solitary instance of such dualism,—a dualism which surely warrants the two theorems being called *per-reciprocals* of one another: but before doing so it is desirable to change the point of view somewhat, so as to have the theorems in a wider field.

(10) If to each linear homogeneous function u_r of $x_1, x_2, \ldots$ there be prefixed as a factor another letter, τ say, with the same suffix, the terms in $\tau_r u_r$ consist of one in which the suffix of τ is the same as the suffix of the x with which it is associated, and others where this similarity does not exist: in other words, $\tau_r u_r$ is partitionable into $a_r\tau_r x_r$ and $\tau_r(u_r - a_r x_r)$. Terms in which each x is accompanied by a τ with the same suffix may be conveniently spoken of as *conjugal* terms, and the operation of taking the aggregate of the conjugal terms existing in the expansion of any function of $\tau_1u_1, \tau_2u_2, \ldots$ may be denoted by T. It is of course clear that

$$\begin{aligned}\mathrm{T}(\phi_1+\phi_2+\cdots) &= \mathrm{T}(\phi_1)+\mathrm{T}(\phi_2)+\cdots,\\ \mathrm{T}(\mathrm{C}\phi) &= \mathrm{CT}(\phi),\end{aligned}$$

where C is a constant with respect to the τ's and x's : and it is convenient to consider

$$\mathrm{T}(\mathrm{C}) = \mathrm{C}.$$

Now, returning to the first of the two identities in § 8, and dividing by $\tau_1\tau_2\tau_3$, we have

$$\frac{|a_1b_2c_3|}{\tau_1u_1\cdot\tau_2u_2\cdot\tau_3u_3} = \frac{1}{\tau_1x_1\cdot\tau_2x_2\cdot\tau_3x_3} - \overset{\circ}{\sum}\frac{c_1}{u_3\cdot\tau_1\cdot\tau_2x_2\cdot\tau_3x_3} + \overset{\circ}{\sum}\frac{|b_3c_1|}{\tau_1\tau_3\cdot u_2u_3\cdot\tau_2x_2};$$

and, as in the expansion of the fractions under the sign of summation τ_1 is negative and x_1 positive, it follows that the aggregate of the conjugal terms in the expansions on the right is $1/\tau_1x_1\cdot\tau_2x_2\cdot\tau_3x_3$, and therefore that the aggregate of the conjugal

* Had we taken as our vanishing determinant that corresponding to the only other positive term of $|a_1b_2c_3|$ we should have found the resulting identity objectionable in both forms.

† *Proceedings Roy. Soc. Edin.*, xi. pp. 409–418 (Sess. 1881–2).

terms in the expansion of the reciprocal of $\tau_1u_1 \cdot \tau_2u_2 \cdot \tau_3u_3$ is the reciprocal of $\tau_1x_1 \cdot \tau_2x_2 \cdot \tau_3x_3 \cdot | a_1b_2c_3 |$

$$i.e., \quad \mathrm{T}(\tau_1u_1 \cdot \tau_2u_2 \cdot \tau_3u_3)^{-1} = | a_1b_2c_3 |^{-1} \cdot (\tau_1x_1 \cdot \tau_2x_2 \cdot \tau_3x_3)^{-1}.$$

The similar variant of the other theorem of § 9 is

$$\mathrm{T}(\tau_1u_1 \cdot \tau_2u_2 \cdot \tau_3u_3) = \overset{+}{|} a_1b_2c_3 \overset{+}{|} \cdot (\tau_1x_1 \cdot \tau_2x_2 \cdot \tau_3x_3).$$

(11) Knowing the truth of this second theorem in its general form, viz.:

$$\mathrm{T}(\tau_hu_h \cdot \tau_ku_k \cdot \tau_lu_l \cdot \cdots) = \overset{+}{|} h_hk_kl_l \cdots \overset{+}{|} \cdot (\tau_hx_h \cdot \tau_kx_k \cdot \tau_lx_l \cdots)$$

we can readily find the aggregate of the conjugal terms in any series

$$\mathrm{C}_1\phi_1 + \mathrm{C}_2\phi_2 + \mathrm{C}_3\phi_3 + \cdots$$

where the ϕ's are of the form $\tau_hu_h \cdot \tau_ku_k \cdot \tau_lu_l \cdot \cdots$. For example:

$$\begin{aligned} &\mathrm{T}(\mathrm{C}\tau_1u_1 \cdot \tau_2u_2 + \mathrm{D}\tau_2u_2 \cdot \tau_3u_3 + \mathrm{E}\tau_3u_3 \cdot \tau_1u_1) \\ &= \mathrm{C}\overset{+}{|} a_1b_2 \overset{+}{|} \cdot \tau_1x_1 \cdot \tau_2x_2 + \mathrm{D}\overset{+}{|} b_2c_3 \overset{+}{|} \cdot \tau_2x_2 \cdot \tau_3x_3 + \mathrm{E}\overset{+}{|} c_3a_1 \overset{+}{|} \cdot \tau_3x_3 \cdot \tau_1x_1. \end{aligned}$$

(12) A conspicuous instance of such a series is the expansion of $(m_1-\tau_1u_1)$ $(m_2-\tau_2u_2)$ $(m_3-\tau_3u_3)\ldots$. Restricting the number of factors to three, merely for simplicity's sake in writing, we have the aggregate of the conjugal terms of this product

$$\begin{aligned} &= \mathrm{T}[m_1m_2m_3 - (m_1m_2 \cdot \tau_3u_3 + \cdots) + (m_1 \cdot \tau_2u_2 \cdot \tau_3u_3 + \cdots) - \tau_1u_1 \cdot \tau_2u_2 \cdot \tau_3u_3] \\ &= m_1m_2m_3 - (m_1m_2 \cdot c_3\tau_3x_3 + \cdots) + (m_1 \cdot \overset{+}{|} b_2c_3 \overset{+}{|} \cdot \tau_2x_2 \cdot \tau_3x_3 + \cdots) - \overset{+}{|} a_1b_2c_3 \overset{+}{|} \tau_1x_1 \cdot \tau_2x_2 \cdot \tau_3x_3 \\ &= \overset{+}{\left|\begin{array}{ccc} m_1 - a_1 \cdot \tau_1x_1 & -b_1 \cdot \tau_1x_1 & -c_1 \cdot \tau_1x_1 \\ -a_2 \cdot \tau_2x_2 & m_2 - b_2 \cdot \tau_2x_2 & -c_2 \cdot \tau_2x_2 \\ -a_3 \cdot \tau_3x_3 & -b_3 \cdot \tau_3x_3 & m_3 - c_3 \ \tau_3x_3 \end{array}\right|}^{+}, \end{aligned}$$

the reason for the last step being the fact that the law for the expression of a determinant with binomial elements in terms of determinants with monomial elements applies to the case of permanents also.

(13) This theorem has corresponding to it a per-reciprocal theorem, viz.,

$$\mathrm{T}\{(m_1-\tau_1u_1)(m_2-\tau_2u_2)(m_3-\tau_3u_3)\}^{-1} = \left|\begin{array}{ccc} m_1 - a_1 \ \tau_1x_1 & -b_1 \cdot \tau_1x_1 & -c_1 \cdot \tau_1x_1 \\ -a_2 \cdot \tau_2x_2 & m_2 - b_2 \cdot \tau_2x_2 & -c_2 \cdot \tau_2x_2 \\ -a_3 \cdot \tau_3x_3 & -b_3 \cdot \tau_3x_3 & m_3 - c_3 \cdot \tau_3x_3 \end{array}\right|^{-1} \cdot (\tau_1x_1 \cdot \tau_2x_2 \cdot \tau_3x_3)^{-1}$$

By way of proof let it be noted that the determinant, Δ say, whose reciprocal appears on the right may be written

$$\left|\begin{array}{ccc} (m_1-\tau_1u_1)+\tau_1(u_1-a_1x_1) & -\tau_1 \cdot b_1x_1 & -\tau_1 \cdot c_1x_1 \\ -\tau_2 \cdot a_2x_2 & (m_2-\tau_2u_2)+\tau_2(u_2-b_2x_2) & -\tau_2 \cdot c_2x_2 \\ -\tau_3 \cdot a_3x_3 & -\tau_3 \cdot b_3x_3 & (m_3-\tau_3u_3)+\tau_3(u_3-c_3x_3) \end{array}\right|,$$

and that this when expanded becomes

$$(m_1-\tau_1u_1)(m_2-\tau_2u_2)(m_3-\tau_3u_3) + \sum^{\circ}(m_1-\tau_1u_1)(m_2-\tau_2u_2)\cdot\tau_3(u_3-c_3x_3)$$
$$+ \sum^{\circ}(m_1-\tau_1u_1)\cdot\tau_2\tau_3\cdot\begin{vmatrix} u_2-b_2x_2 & -c_2x_2 \\ -b_3x_3 & u_3-c_3x_3 \end{vmatrix}$$
$$+ \tau_1\tau_2\tau_3\begin{vmatrix} u_1-a_1x_1 & -b_1x_1 & -c_1x_1 \\ -a_2x_2 & u_2-b_2x_2 & -c_2x_2 \\ -a_3x_3 & -b_3x_3 & u_3-c_3x_3 \end{vmatrix},$$

where the addition of the elements of each column shows that the three-line determinant vanishes and that the two-line determinant has x_1 for a factor. As a consequence we have

$$\frac{\Delta}{(m_1-\tau_1u_1)(m_2-\tau_2u_2)(m_3-\tau_3u_3)} = 1 + \sum^{\circ}\frac{\tau_3(u_3-c_3x_3)}{m_3-\tau_3u_3} + \sum^{\circ}\frac{\tau_2\tau_3(b_1c_1x_1+b_3c_1x_3+b_1c_2x_2)\cdot x_1}{(m_2-\tau_2u_2)(m_3-\tau_3u_3)}$$

and thus see that, since the first fraction on the right has for a factor τ_3 without x_3, and the second has x_1 without τ_1, there results

$$\mathsf{T}\left\{\frac{\Delta}{(m_1-\tau_1u_1)(m^8-\tau^6u^6)(m_3-\tau_3u_3)}\right\} = 1,$$

as was to be proved.

(14) Putting $m_1=m_2=m_3=\ldots\ldots=0$ in the theorem of the preceding paragraph we obtain JACOBI'S case of 1829 with which we started; and putting $m_1=m_2=m_3=\ldots=1$ we obtain the case which is the subject of a memoir by Major MACMAHON, printed in the *Transactions* of the Royal Society of London for 1893.*

The new per-reciprocal theorem of § 12 of course degenerates in like fashion.

(15) A still wider generalisation, however, is possible. All that is requisite is to make use of a simple fact employed by JACOBI in the memoir above noted, viz., that if $\omega_1, \omega_2, \ldots$ be what $u_1, u_2, \ldots$ become when $x_1, x_2, \ldots$ are changed into $\xi_1, \xi_2, \ldots$, —in other words, if $\xi_1, \xi_2, \ldots$ be the values of $x_1, x_2, \ldots$ which satisfy the equations $u_1=\omega_1, u_2=\omega_2, \ldots$,—then any identity connecting $u_1, u_2, \ldots$ and the quantities of which $u_1, u_2, \ldots$ are functions will still remain an identity if in every instance x_r be changed into $x_r-\xi_r$ and u_r into $u_r-\omega_r$. As a consequence of this action we obtain from the result of § 12 the theorem,—

In the expansion of

$$\{m_1-\tau_1(u_1-\omega_1)\}\cdot\{m_2-\tau_2(u_2-\omega_2)\}\ldots\ldots$$

the aggregate of the terms which are such that every τ *occurring in them is accompanied by the corresponding* $x-\xi$, *and vice-versa*, is

$$\begin{vmatrix} m_1-a_1\tau_1(x_1-\xi_1) & -b_1\tau_1(x_1-\xi_1) & -c_1\tau_1(x_1-\xi_1) & \ldots\ldots \\ -a_2\tau_2(x_2-\xi_2) & m_2-b_2\tau_2(x_2-\xi_2) & -c_2\tau_2(x_2-\xi_2) & \ldots\ldots \\ -a_3\tau_3(x_3-\xi_3) & -b_3\tau_3(x_3-\xi_3) & m_3-c_3\tau_3(x_3-\xi_3) & \ldots\ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}^{+}_{+}.$$

* MACMAHON, P. A., "A certain class of generating functions in the theory of numbers," *Trans. Roy. Soc.*, clxxxv. pp. 111–160.

Similarly from § 13 there is obtained the theorem,

In the expansion of

$$\{m_1 - \tau_1(u_1 - \omega_1\}^{-1} \cdot \{m_2 - \tau_2(u_2 - \omega_2)\}^{-1} \cdot \cdot \cdot \cdot$$

the aggregate of the terms which are such that every τ *occurring in them is accompanied by the corresponding* $x - \xi$, *and vice-versa, is*

$$\begin{vmatrix} m_1 - a_1\tau_1(x_1 - \xi_1) & -b_1\tau_1(x_1 - \xi_1) & -c_1\tau_1(x_1 - \xi_1) & \cdots\cdots \\ -a_2\tau_2(x_2 - \xi_2) & m_2 - b_2\tau_2(x_2 - \xi_2) & -c_2\tau_2(x_2 - \xi_2) & \cdots\cdots \\ -a_3\tau_3(x_3 - \xi_3) & -b_3\tau_3(x_3 - \xi_3) & m_3 - c_3\tau_3(x_3 - \xi_3) & \cdots\cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}^{-1}.$$

(16) Leaving the subject of conjugal aggregates at this point, let us return for a moment to the question of § 5. It will be remembered that JACOBI'S first two cases of a supposed general theorem were

$$|\, a_1 b_2 \,| \cdot x_1 x_2 = u_1 u_2 - \sum^{\circ} u_1 \cdot b_1 x_1 ,$$

$$|\, a_1 b_2 c_3 \,| \cdot x_1 x_2 x_3 = u_1 u_2 u_3 - \sum^{\circ} u_1 \cdot (b_1 c_1 x_1 + b_2 c_1 x_2 + b_1 c_3 x_3) \cdot x_1 ;$$

and that for three different reasons it was difficult to avoid a strong presumption that the form of the general theorem must be

$$|\, a_1 b_2 c_3 \ldots \,| \cdot x_1 x_2 \ldots x_n = u_1 u_2 \ldots u_n - \sum^{\circ} u_1 \cdot \mathrm{F}_1 \cdot x_1$$

where F_1 is an integral homogeneous function of the $(n-2)^{\text{th}}$ degree in $x_1, x_2, \ldots x_n$. A fourth reason will now incidentally appear, it being inseparable from certain curious results in determinants which were discovered in consequence of it, and which deserve on their own account to be chronicled.

Early in the investigation it was observed that the expression following $\overset{\circ}{\Sigma} u_1$ in the second case above is equal to the determinant

$$\begin{vmatrix} u_3 - c_2 x_2 & -c_3 x_3 \\ -b_2 x_2 & u_2 - b_3 x_3 \end{vmatrix},$$

and on turning to the first case it was seen that the corresponding expression there, viz., $b_1 x_1$, being equal to $u_2 - b_2 x_2$, could be considered to be of the same form. The question thus arose whether or not there might be a similar determinant of the third order having (1) x_1 for a factor, (2) a quadratic function of x_1, x_2, x_3, x_4 for the cofactor, (3) the terms of this function sixteen in number and all positive, and (4) these terms such as to ensure an identity; and, strange to say, determinants possessing the first three of these properties were found to be realisable.

(17) *If* $\mathrm{u}_1 \equiv \mathrm{a}_1\mathrm{x}_1 + \ldots + \mathrm{a}_4\mathrm{x}_4$, $\mathrm{u}_2 \equiv \mathrm{b}_1\mathrm{x}_1 + \ldots \mathrm{b}_4\mathrm{x}_4, \ldots$ *each of the six determinants.*

$$\begin{vmatrix} u_2 - b_2x_2 & -b_3x_3 & -b_4x_4 \\ -c_2x_2 & u_3 - c_3x_3 & -c_4x_4 \\ -d_2x_2 & -d_3x_3 & u_4 - d_4x_4 \end{vmatrix}, \quad \begin{vmatrix} u_2 - b_2x_2 & -b_4x_4 & -b_3x_3 \\ -c_2x_2 & u_3 - c_4x_4 & -c_3x_3 \\ -d_2x_2 & -d_4x_4 & u_4 - d_3x_3 \end{vmatrix}, \quad \begin{vmatrix} u_2 - b_3x_3 & -b_2x_2 & -b_4x_4 \\ -c_3x_3 & u_3 - c_2x_2 & -c_4x_4 \\ -d_3x_3 & -d_2x_2 & u_4 - d_4x_4 \end{vmatrix}$$

$$\begin{vmatrix} u_2 - b_3x_3 & -b_4x_4 & -b_2x_2 \\ -c_3x_3 & u_3 - c_4x_4 & -c_2x_2 \\ -d_3x_3 & -d_4x_4 & u_4 - d_2x_2 \end{vmatrix}, \quad \begin{vmatrix} u_2 - b_4x_4 & -b_2x_2 & -b_3x_3 \\ -c_4x_3 & u_3 - c_2x_2 & -c_3x_3 \\ -d_4x_4 & -d_2x_2 & u_4 - d_3x_3 \end{vmatrix}, \quad \begin{vmatrix} u_2 - b_4x_4 & -b_3x_3 & -b_2x_2 \\ -c_4x_4 & u_3 - c_3x_3 & -c_2x_2 \\ -d_4x_4 & -d_3x_3 & u_4 - d_2x_2 \end{vmatrix},$$

is presentable in the form $x_1 \cdot F_1$, *where* F_1 *is a complete quadric function of* x_1, x_2, x_3, x_4, *consisting of sixteen positive terms, the coefficient of* x_r^2 *being monomial and that of* $x_r x_s$ *binomial.*

That x_1 is a factor is readily seen by adding the elements of each row; thus, in the case of the first determinant of the six we have at once

$$x_1 \cdot \begin{vmatrix} b_1 & -b_3x_3 & -b_4x_4 \\ c_1 & c_1x_1 + c_2x_2 + c_4x_4 & -c_4x_4 \\ d_1 & -d_3x_3 & d_1x_1 + d_2x_2 + d_3x_3 \end{vmatrix}$$

But here the cofactor of b_1 consists of eight positive terms of the kind promised, the two $+c_4d_3x_4x_3$, $-c_4d_3x_4x_3$ cancelling each other: the cofactor of c_1 has four of similar kind, and the cofactor d_1 also four. The result consequently can be written

$$x_1 \cdot \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline b_1c_1d_1 & b_1c_2d_1 & b_1c_1d_3 & b_1c_4d_1 & x_1 \\ b_1c_1d_2 & b_1c_2d_2 & b_1c_2d_2 & b_1c_4d_2 & x_2 \\ b_3c_1d_1 & b_3c_1d_2 & b_3c_1d_3 & b_4c_1d_3 & x_3 \\ b_4c_1d_1 & b_4c_2d_1 & b_3c_4d_1 & b_4c_4d_1 & x_4 \end{array}.$$

(18) If the elements of each row in the quadrate array here obtained be divided by the diagonal element of that row, there results the more interesting form

$$x_1 \cdot \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & \frac{c_2}{c_1} & \frac{d_3}{d_1} & \frac{c_4}{c_1} & x_1 \cdot b_1c_1d_1 \\ \frac{c_1}{c_2} & 1 & \frac{d_3}{d_2} & \frac{c_4}{c_2} & x_2 \cdot b_1c_2d_2 \\ \frac{d_1}{d_3} & \frac{d_2}{d_3} & 1 & \frac{b_4}{b_3} & x_3 \cdot b_3c_1d_3 \\ \frac{c_1}{c_4} & \frac{c_2}{c_4} & \frac{b_3}{b_4} & 1 & x_4 \cdot b_4c_4d_1 \end{array},$$

of which the quadrate array is inverso-symmetric.*

The existence of this peculiar symmetry gives us the means of deriving the twelve non-diagonal elements from those in the diagonal; and as the latter four are easily obtained we can thus write out the full expansion without any delay upon intermediate work. Let us take as an example the third of the six determinants, viz.,

$$\begin{vmatrix} u_2 - b_3x_3 & -b_2x_2 & -b_4x_4 \\ -c_3x_3 & u_3 - c_2x_2 & -c_4x_4 \\ -d_3x_3 & -d_2x_2 & u_4 - d_4x_4 \end{vmatrix}.$$

* The first to draw attention to determinants of such array was probably JOSEPH HORNER: see his "Notes on Determinants" in the *Quart. Journ. of Math.*, viii. pp. 157–162 (year 1865).

We note that here as in the five other cases (1) the letters of the coefficients are b, c, d, (2) that the suffixes of these letters are the suffixes of the associated x's; and we note that (3) in this particular case, as a glance at the elements suffices to show, b_3, c_2, d_4 do not occur. Now the diagonal elements of the quadrate array of the equivalent bipartite form are the coefficients of

$$x_1 \cdot x_1x_1 , \quad x_1 \cdot x_2x_2 , \quad x_1 \cdot x_3x_3 , \quad x_1 \cdot x_4x_4$$

and therefore in accordance with the conditions just noted must be

$$b_1c_1d_1 , \quad c_1b_2d_2 , \quad b_1c_3d_3 , \quad d_1b_4c_4 .$$

Next we note that because of the inverso-symmetry (4) the product of any two conjugate elements of the quadrate array is equal to the product of the two diagonal elements which occupy the same rows and columns. Using this we take the product of any two diagonal elements, say the 3rd and 4th, viz.,

$$b_1c_3d_3 \cdot b_4c_4d_1$$

and rearrange the suffixes so as to have two sets of 1 , 3 , 4 , these being the suffixes of $x_1 \cdot x_3x_4$, $x_1 \cdot x_4x_3$. The result $b_1c_4d_3$, $b_4c_3d_1$ and others similarly obtained gives us

$$x_1 \cdot \begin{array}{cccc|l} x_1 & x_2 & x_3 & x_4 & \\ \hline b_1c_1d_1 & b_1c_1d_2 & b_1c_1d_3 & b_1c_4d_1 & x_1 \\ b_2c_1d_1 & b_2c_1d_2 & b_2c_1d_3 & b_4c_1d_2 & x_2 \\ b_1c_3d_1 & b_1c_3d_2 & b_1c_3d_3 & b_1c_4d_3 & x_3 \\ b_4c_1d_1 & b_2c_4d_1 & b_4c_3d_1 & b_4c_4d_1 & x_4 , \end{array}$$

whence we have as before

$$x_1 \cdot \begin{array}{cccc|l} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & \frac{d_2}{d_1} & \frac{d_3}{d_1} & \frac{c_4}{c_1} & x_1 \cdot b_1c_1d_1 \\ \frac{d_1}{d_2} & 1 & \frac{d_3}{d_2} & \frac{b_4}{b_2} & x_2 \cdot b_2c_1d_2 \\ \frac{d_1}{d_3} & \frac{d_2}{d_3} & 1 & \frac{c_4}{c_3} & x_3 \cdot b_1c_3d_3 \\ \frac{c_1}{c_4} & \frac{b_2}{b_4} & \frac{c_3}{c_4} & 1 & x_4 \cdot b_4c_4d_1 . \end{array}$$

(19) It deserves to be noted in passing that in no case is the inverso-symmetry unique, any one of the non-diagonal elements of the first row of the array being replaceable by another quotient, provided the corresponding change be made in the first column. Thus, instead of d_2/d_1 in the case just dealt with, we may write b_2/b_1 provided we alter the conjugate element into b_1/b_2.

(20) *If* $u_1 , u_2 , \ldots , u_5$ *be used to stand for* $a_1x_1 + \ldots + a_5x_5$, $b_1x_1 + \ldots + b_5x_5$, $\ldots$ $e_1x_1 + \ldots + e_5x_5$, *there are twenty-four determinants of the type*

$$\begin{vmatrix} u_2 - b_2x_2 & - b_3x_3 & - b_4x_4 & - b_5x_5 \\ - c_2x_2 & u_3 - c_3x_3 & - c_4x_4 & - c_5x_5 \\ - d_2x_2 & - d_3x_3 & u_4 - d_4x_4 & - d_5x_5 \\ - e_2x_2 & - e_3x_3 & - e_4x_4 & u_5 - e_5x_5 \end{vmatrix}$$

which are all presentable in the form $\mathrm{x}_1 \cdot \mathrm{F}_1$, *where* F_1 *is a complete cubic in* $\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_5$, *consisting of* 125 *positive terms, viz., in the case of the determinant specified, five terms of the form* $\mathrm{b}_1\mathrm{c}_1\mathrm{d}_1\mathrm{e}_1 \cdot \mathrm{x}_1^3$, *twenty compound terms of the form*

$$x_1^2x_2 \cdot (b_1c_1d_1e_2 + b_1c_1d_2e_1 + b_1c_2d_1e_1),$$

and ten compound terms of the form

$$x_1x_2x_3 \cdot (b_1c_1d_2e_3 + b_1c_1d_3e_2 + b_1c_2d_1e_3 + b_1c_2d_3e_1 + b_3c_1d_1e_2 + b_3c_1d_2e_1).$$

By adding all the other columns to the first column we see at once that the determinant is equal to

$$\begin{vmatrix} b_1 & -b_3x_3 & -b_4x_4 & -b_5x_5 \\ c_1 & c_1x_1+c_2x_2+c_4x_4+c_5x_5 & -c_4x_4 & -c_5x_5 \\ d_1 & -d_3x_3 & d_1x_1+d_2x_2+d_3x_3+d_5x_5 & -d_5x_5 \\ e_1 & -e_3x_3 & -e_4x_4 & e_1x_1+e_2x_2+e_3x_3+e_4x_4 \end{vmatrix},$$

and by treating the cofactor of b_1 here in the same way we can separate it into two parts of the same kind as the cofactors of b_3, b_4, b_5. F_1 is thus seen to be the sum of five similar parts, viz.,

$$b_1x_1 \cdot \begin{vmatrix} c_1 & -c_4x_4 & -c_5x_5 \\ d_1 & d_1x_1+d_2x_2+d_3x_3+d_5x_5 & -d_5x_5 \\ e_1 & -e_4x_4 & e_1x_1+e_2x_2+e_3x_3+e_4x_4 \end{vmatrix} + \ldots\ldots$$

Taking the first of these and dealing with it after the manner followed in the preceding paragraph we find it expressible as the sum of twenty-five positive terms, viz.,

$$b_1x_1 \cdot \begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline c_1d_1e_1 & c_1d_1e_2 & c_1d_1e_3 & c_1d_1e_4 & c_5d_1e_1 & x_1 \\ c_1d_2e_1 & c_1d_2e_2 & c_1d_2e_3 & c_1d_2e_4 & c_1d_5e_2 & x_2 \\ c_1d_3e_1 & c_1d_3e_2 & c_1d_3e_3 & c_1d_3e_4 & c_1d_5e_3 & x_3 \\ c_4d_1e_1 & c_4d_1e_2 & c_4d_1e_3 & c_4d_1e_4 & c_5d_1e_4 & x_4 \\ c_1d_5e_1 & c_5d_2e_1 & c_5d_3e_1 & c_4d_5e_1 & c_5d_5e_1 & x_5, \end{array}$$

and the like is true of the four others. Removing the b at the beginning of each of these five expressions and attaching it to every element of the square array to which it belongs, we obtain a variant form consisting of x_1 multiplied by a quadric in $x_1, x_2, \ldots, x_5$ with coefficients of the form $b_rc_sd_te_u$, x_2 multiplied by a similar quadric, x_3 multiplied by a similar quadric, and so on. It follows therefore that the natural way to represent the result is to use space of three dimensions, each square array being made to overlie the one before it. We should thus have x_1, x_2, x_3, x_4, x_5 placed at equal distances along each of three mutually perpendicular straight lines, and the 125 coefficients disposed within the cube of which these lines are concurrent edges. As $x_1\,x_2$ would arise from $x_1x_1x_2$ or $x_1x_2x_1$ or $x_2x_1x_1$ to each of which a coefficient attaches, the coefficient of $x_1^2\,x_2$ would clearly be three-termed: and for a similar reason the coefficient of $x_1x_2x_3$ would be six-termed. Our theorem is thus established.

(21) The first set of twenty-five positive terms specified in the preceding paragraph may be put in the form

$$
b_1x_1 \cdot \begin{array}{ccccc|l}
x_1 & x_2 & x_3 & x_4 & x_5 & \\
\hline
1 & \frac{e_2}{e_1} & \frac{e_3}{e_1} & \frac{e_4}{e_1} & \frac{d_5}{d_1} & x_1 \cdot c_1d_1e_1 \\
\frac{e_1}{e_2} & 1 & \frac{e_3}{e_2} & \frac{e_4}{e_2} & \frac{d_5}{d_2} & x_2 \cdot c_1d_2e_2 \\
\frac{e_1}{e_3} & \frac{e_2}{e_3} & 1 & \frac{e_4}{e_3} & \frac{d_5}{d_3} & x_3 \cdot c_1d_3e_3 \\
\frac{e_1}{e_4} & \frac{e_2}{e_4} & \frac{e_3}{e_4} & 1 & \frac{c_5}{c_4} & x_4 \cdot c_4d_1e_4 \\
\frac{d_1}{d_5} & \frac{d_2}{d_5} & \frac{d_3}{d_5} & \frac{c_4}{c_5} & 1 & x_5 \cdot c_5d_5e_1
\end{array}
$$

where the square array is inverso-symmetric. Each of the four other sets can be similarly represented: but it is curious to note that the five arrays so obtained are neither all alike nor all different. In fact, if we use E to stand for that just written, the second and third sets of twenty-five terms are respectively

$$
b_1x_2 \cdot \begin{array}{ccccc|l}
x_1 & x_2 & x_3 & x_4 & x_5 & \\
\hline
 & & & & & x_1 \cdot c_2d_1e_1 \\
 & & & & & x_2 \cdot c_2d_2e_2 \\
 & & \mathrm{E} & & & x_3 \cdot c_2d_3e_3 \\
 & & & & & x_4 \cdot c_4d_2e_4 \\
 & & & & & x_5 \cdot c_5d_5e_2 ,
\end{array}
\qquad
b_3x_3 \cdot \begin{array}{ccccc|l}
x_1 & x_2 & x_3 & x_4 & x_5 & \\
\hline
 & & & & & x_1 \cdot c_1d_1e_1 \\
 & & & & & x_2 \cdot c_1d_2e_2 \\
 & & \mathrm{E} & & & x_3 \cdot c_1d_3e_3 \\
 & & & & & x_4 \cdot c_4d_1e_4 \\
 & & & & & x_5 \cdot c_5d_5e_1 ,
\end{array}
$$

while the fourth and fifth are

$$
b_4x_4 \cdot \begin{array}{ccccc|l}
x_1 & x_2 & x_3 & x_4 & x_5 & \\
\hline
1 & \frac{e_2}{e_1} & \frac{e_3}{e_1} & \frac{e_4}{e_1} & \frac{d_5}{d_1} & x_1 \cdot c_1d_1e_1 \\
\frac{e_1}{e_2} & 1 & \frac{e_3}{e_2} & \frac{e_4}{e_2} & \frac{c_5}{c_2} & x_2 \cdot c_2d_1e_2 \\
\frac{e_1}{e_3} & \frac{e_2}{e_3} & 1 & \frac{e_4}{e_3} & \frac{d_5}{d_3} & x_3 \cdot c_1d_3e_3 \\
\frac{e_1}{e_4} & \frac{e_2}{e_4} & \frac{e_3}{e_4} & 1 & \frac{c_5}{c_4} & x_4 \cdot c_4d_1e_4 \\
\frac{d_1}{d_5} & \frac{c_2}{c_5} & \frac{d_3}{d_5} & \frac{c_4}{c_5} & 1 & x_5 \cdot c_5d_5e_1 ,
\end{array}
\qquad
b_5x_5 \cdot \begin{array}{ccccc|l}
x_1 & x_2 & x_3 & x_4 & x_5 & \\
\hline
1 & \frac{d_2}{d_1} & \frac{e_3}{e_1} & \frac{e_4}{e_1} & \frac{d_5}{d_1} & x_1 \cdot c_1d_1e_1 \\
\frac{d_1}{d_2} & 1 & \frac{d_3}{d_2} & \frac{c_4}{c_2} & \frac{d_5}{d_2} & x_2 \cdot c_2d_2e_1 \\
\frac{e_1}{e_3} & \frac{d_2}{d_3} & 1 & \frac{e_4}{e_3} & \frac{d_5}{d_3} & x_3 \cdot c_1d_3e_3 \\
\frac{e_1}{e_4} & \frac{c_2}{c_4} & \frac{e_3}{e_4} & 1 & \frac{c_5}{c_4} & x_4 \cdot c_4d_1e_4 \\
\frac{d_1}{d_5} & \frac{d_2}{d_5} & \frac{d_3}{d_5} & \frac{c_4}{c_5} & 1 & x_5 \cdot c_5d_5e_1 ,
\end{array}
$$

the square array of the fourth differing from E in the place (2 , 5) and that of the fifth in the places (1 , 2) , (2 , 3) , (2 , 4).

The cofactors of b_1x_1 and b_3x_3, it will be seen, are identical,—a fact which would have appeared in the preceding paragraph if the expressions for them as determinants of the third order had been given in the case of both.

(22) The general character of the theorem which includes §§ 17, 20 as special cases is readily apparent, the number of terms in F_1 for the next case being 6^4, and generally n^{n-2}; and this is exactly the number which the supposed theorem of Jacobi required, as the following considerations will show:—The expansion of $u_1u_2 \ldots u_n$ is an aggregate of n^n terms all positive: and as in $|\, a_1b_2c_3 \ldots \,| \cdot x_1x_2 \ldots x_n$ there are $\frac{1}{2}(n!)$ positive terms

and $\frac{1}{2}(n!)$ negative terms, all of which appear in the previous aggregate,—where in fact the cofactor of $x_1x_2 \ldots x_n$ is $\overset{+}{|} a_1b_2c_3 \ldots \overset{+}{|}$,—it follows that the sum or difference of the two is, like the former, an aggregate of n^n positive terms, of which however $\frac{1}{2}(n!)$ are twins. If therefore this sum or difference is to be represented in the form $\overset{s}{\Sigma} u_1 \cdot D_1$, the number of terms in $u_1 \cdot D_1$ must be n^{n-1} and the number in D_1 must be n^{n-2}.

(23) The fact that the determinants of the type appearing in §§ 17, 20 have their terms all positive and n^{n-2} in number is a simple deduction from one of the following pair of theorems :—

(a) *In the final expansion of the determinant*

$$\begin{vmatrix} a_1+\ldots+a_z & -a_1 & -a_2 & \ldots\ldots \\ -\beta_1 & \beta_1+\ldots+\beta_z & -\beta_2 & \ldots\ldots \\ -\gamma_1 & -\gamma_2 & \gamma_1+\ldots+\gamma_z & \ldots\ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}_n$$

all the terms are positive, and the number of them is

$$(z-n+1)(z+1)^{n-1}.$$

(b) *In the final expansion of the determinant*

$$\begin{vmatrix} a_s & -a_1 & -a_2 & \ldots\ldots \\ \beta_s & \beta_1+\ldots+\beta_z & -\beta_2 & \ldots\ldots \\ \gamma_s & -\gamma_2 & \gamma_1+\ldots+\gamma_z & \ldots\ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}_n$$

all the terms are positive, and the number of them is

$$(z+1)^{n-1}.$$

If for a particular value of n the two theorems be established, it is easy to show that the second will hold for the next higher value of n, and then from this to show that the first will also hold for this higher value. In the one case the determinant is expressed in terms of the elements of the first column and the complementary minors of these elements : in the other, each element of the first column is increased by all the elements in the same row with it, and the determinant then partitioned into determinants with monomial elements in the first column.

XXVI.—*Magnetic Shielding in Hollow Iron Cylinders and Superposed Inductions in Iron.* By James Russell, F.R.S.E. (With Six Plates.)

(Read February 3 and July 7 and 21, 1902. Issued separately April 21, 1903.)

CONTENTS.

Introductory.

§ 1. The shielding effect which exists within hollow iron spheres and cylinders, when placed in a uniform magnetic field, has been previously investigated. Poisson, Stefan,* Maxwell, and among others, and more recently, Professors Rücker † and Du Bois,‡ have contributed to the subject. If

$$g = \frac{\text{External Magnetic Field}}{\text{Internal Magnetic Field}}$$

$$n = \text{thickness of the shell}$$

$$\mathrm{R} = \text{outside radius,}$$

and the ratio of n to R be of the order of 1 to 100, it appears that the following expressions may be taken as sufficiently correct :

$$g-1 = \frac{2}{3}\,\frac{d}{\mathrm{R}}(u-2) \quad \text{(sphere)}$$

$$g-1 = \frac{1}{2}\,\frac{d}{\mathrm{R}}(u-2) \quad \text{(cylinder).}$$

* *Wied. Ann.*, xvii. p. 928 (1882).

† *Phil. Mag.* (5), vol. xxxvii. p. 95 (1894).

‡ *Electrician*, "Magnetic Shielding," vol. xl. pp. 218, 316, 511, 652, 814, and vol. xli. p. 108 (1897–98).

Experimental investigations of the shielding ratio g have been made by STEFAN and DU BOIS using the methods of oscillations and deflections. Dr KNOTT recently gave an account before this Society, of exploring the field within a hollow iron sphere by comparing the twist produced in a nickel wire per unit current and magnetised longitudinally in various fields. Closely connected with this subject is the distribution of the pull between copper conductors and the armatures, when these conductors are sunk in tunnels. Mr MORDEY * has measured the mechanical pull exerted by a conductor sunk in a slotted armature, and Professor DU BOIS and the writer the side thrust experienced by iron rings or cylinders when shielding a magnetic field from conductors. My measurements of this thrust, under various conditions of magnetisation, led me to undertake this investigation. It may also be mentioned that iron shields have been utilised in protecting delicate instruments from strong magnetic fields. Some of Lord KELVIN'S special types of galvanometers afford examples of this.

§ 2. At an early stage of this investigation, and when dealing with forces acting upon the iron in addition to that due to the transverse field, the subject of superposed magnetic inductions necessarily came to the front. WERNER VON SIEMENS,† SCHULTZE,‡ KENNELLY,§ Messrs EVERSHED and VIGNOLES,§ KNOTT,|| Messrs GEROSA and FINZI,¶ and CROOK,** and doubtless others, have experimentally investigated this subject. In sections 14 to 29, the permeability of iron to magnetising forces at right angles to each other will be dealt with independent of and uncomplicated by shielding in hollow iron cylinders.

I must here express my thanks to Dr PEDDIE for the ready advice given me at all times during the course of these experiments.

Objects of Investigation.

§ 3. *First. Magnetic Shielding.*—The present contribution to this subject is confined to the shielding which exists within thin hollow iron *cylinders*, when placed in a uniform magnetic field, the lines of force of this field being at right angles to the axis of the cylinder.

I endeavour to determine experimentally the shielding ratio between the transverse field and the field within the cylinder under the following conditions, viz. :—

(a) When no other magnetising force is acting upon the iron than that due to the transverse field.

(b) When a circular magnetising force is acting upon the iron in addition to that due to the transverse field.

* *Electrician*, vol. xxxix. p. 190.

† *Scientific and Technical Papers of Werner von Siemens.* English edition, vol. i. pp. 353 to 372 (1892).

‡ *Wied. Ann.*, xxiv. p. 663 (1885).

§ "On the Permeability of Iron at Right Angles to the Direction of Magnetisation," *Electrician*, vol. xxv. pp. 111, 141 and 158.

|| *Proc. R.S.E.*, vol. xviii. p. 124.

¶ *Magnetic Induction in Iron*, EWING, 3rd ed., p. 330.

** *American Journal of Science*, vol. clxi. p. 365 (1901).

(*c*) When a longitudinal magnetising force is acting upon the iron in addition to that due to the transverse field,

and to co-ordinate the results obtained with the magnetic induction in the iron under the conditions. I also endeavour to show how far, and under what conditions, the experimental results approximate to theoretical formulæ which assume the permeability to be uniform all round the shield and the absence of retentivity and coercive force in the iron.

§ 4. *Second. Superposed Magnetic Inductions.*—I also deal with the superposition of two magnetising forces at right angles to each other, and the co-ordination of the two components of the resultant magnetic induction under various conditions of field superpositions.

Third. Magnetic æolotropy of demagnetised iron.—My experiments also show the unequal directional permeability of iron after having been magnetised and then demagnetised by decreasing reversals; and a possible explanation is offered, based upon the molecular theory of induction.

Apparatus and Experimental Methods.

§ 5. Two iron cylinders or shields were experimented with. They were made by folding thin sheet iron round a roller, bevelling and overlapping the edges about one-third cm. The edges were clamped together and soldered in position. The shields were uniformly wound with wire, threading it in and out so that they could be magnetised circularly, *i.e.*, in a closed magnetic circuit. An exploring coil was also threaded on, at the position relative to the transverse field shown in fig. III., to enable determinations of the magnetic induction to be made. In addition to this, the shields were also wound with four layers of wire (one layer only being shown in fig. III*a*.), which formed an external solenoid, so that the iron could be magnetised longitudinally. Also with a few turns of finer wire widely wound in a central position, and shown only in fig. III*a*., so as to measure an average longitudinal induction in the iron extending to about an inch on each side of the centres of the cylindrical shields, and corresponding with the length of the rotating inductor within the shields to be immediately described.

The dimensions of the shields are as follows :—

	Shield A.	Shield B.
Length, ...	13·2 cms.	13·2 cms.
Outside ratius, R, . . .	1·435 ,,	1·45 ,,
Thickness, d, . . .	0·0233 ,,	0·029 ,,
Ratio, $\frac{d}{R}$	0·0162 ,,	0·02 ,,

Shield A was made from transformer iron, and was heated to redness after being bent into cylindrical form; shield B from common tinned iron and not annealed in any way.

§ 6. Figs. I. and II. show the apparatus used. S S is one of the iron shields fitted in a vertical position on a suitable framework F F, with large rectangular coils C and C, one on each side, after the manner of a Helmholtz tangent galvanometer. The mutual axis of the coils was at right angles to the magnetic meridian, and the value of the transverse field in the central region to be occupied by the shield was determined experimentally in C.G.S. units per unit current in the coils. In order to determine the magnetic field within the shield, a rectangular coil or inductor D wound with fine wire was placed within the shield and so pivoted that, on releasing a catch K, the inductor was caused to rotate by means of an elastic band half a revolution. This inductor was in connection with a ballistic galvanometer, which was calibrated from day to day for these shielding measurements by the rotation of the inductor D, in the known magnetic field, when the shield was altogether removed. The magnetic axis of the rotating inductor coincided with that of the two large coils C C and C′ C′ in its two stationary positions.

This apparatus was used in connection with two sources of current (E.P.S. cells), so that the transverse and circular fields could be varied independently the one of the other. The accessories were modifications of those in use for the determination of the magnetic properties of iron by the ballistic method arranged to suit the special requirements of the experiments. Both fields could be increased by steps from zero to a maximum, reversed at each step, carried from a positive maximum by decreasing steps through zero to a negative maximum, or the maximum currents could during reversals be continuously decreased to zero. The whole was arranged so that one observer could conveniently manipulate in any given sequence the necessary apparatus, and at the same time read the ballistic galvanometer, which was sufficiently removed from disturbing fields. A shunted Weston ammeter was used for current measurements, and the long solenoid and measured current method for the calibration of the galvanometer, as also for the determination of the value of the transverse field above referred to.

§ 7. In the experimental determinations of the shielding ratios, the following magnetic fields require to be known: The transverse field, undisturbed by the presence of the iron shield, due to the coils C C and C′ C′, is known experimentally in terms of the current strength. The circular field which, when acting alone, is the true magnetising force in the iron, is obtained from the known ampère turns per cm. of circumference of the shields.* The other fields are those within the shield, which are determined by the rotation of the inductor D. They are, first, the weakened or shielded field existing within the shield, when the transverse field is maintained by a current in the coils C C and C′ C′; and second, the negative residual field within the shield due to polarity when the transverse field is withdrawn. In addition to these two fields there is a third magnetic field within the shield due to the overlapped joint and possible want of

* The longitudinal field is also obtained from the known ampère turns per cm. of length, but is not the true value of H acting in the iron, as the demagnetising factor is considerable.

homogeneity in the iron when the shield is circularly magnetised. Let these fields be denoted by the following symbols :—

Transverse Field	H_t.
Circular Field	H_c.
Longitudinal Field	H_l.
Weakened Shielded Field	$(H)_w$.
Negative Residual Field	$(H)_r$.
Leakage Field due to circular magnetisation	$(H)_p$.

Let the experimentally determined shielding ratio be

$$\frac{H_t}{(H)_w}=g'$$

Brackets surrounding H denote, in all cases, magnetic fields within the shield. The last-mentioned fields $(H)_r$ and $(H)_p$ will be discussed later. Meanwhile let

$$\frac{H_t}{(H)_w-\{-(H)_r\}}=g''$$

Theoretical Shielding Ratios for Shields A and B.

§ 8. Neglecting the negative quantity bracketed with μ (μ being large, any inaccuracy thus introduced is within the limits of error in the experimental determinations of the shielding ratio) in the formula (cylinder) given in § 1, we may write

$$g=\tfrac{1}{2}\frac{d}{R}\mu+1$$

The shielding ratio, minus unity, is thus for all practical purposes proportional to the geometrical factor and to the permeability. The usual definition of permeability is the actual ratio,

$$\mu=\frac{B}{H}$$

it being assumed that the magnetic force H at any point produces the induction B at that point. This may be called the ratio permeability. But it may also be defined as the rate at which B is changing with respect to H, which is given by the tangent of the B–H curve at any point.

$$\mu=\frac{dB}{dH}$$

This is called the differential permeability. Two definitions of permeability are thus available, and substituting these for μ in the above formula we obtain two different values of the theoretical shielding ratios. Let

$$\tfrac{1}{2}\frac{d}{R}\cdot\frac{B}{H}+1=(g) \qquad \text{and} \qquad \tfrac{1}{2}\frac{d}{R}\cdot\frac{dB}{dH}+1=((g))$$

The symbol (g) will therefore mean the theoretical shielding ratio regarded as the above function of the ratio permeability, and the symbol $((g))$ the theoretical shielding ratio regarded as the same function of the differential permeability. Substituting the values given in § 5 for $\frac{d}{R}$ in above formulæ, we obtain for

$$\text{Shield A. } (g) = \cdot 0081 \ \frac{B}{H} + 1$$

$$\text{,,} \quad ((g)) = \cdot 0081 \ \frac{dB}{dH} + 1$$

and for

$$\text{Shield B. } (g) = \cdot 01 \ \frac{B}{H} + 1$$

$$\text{,,} \quad ((g)) = \cdot 01 \ \frac{dB}{dH} + 1$$

§ 9. In order to assign numerical values for B/H and dB/dH, the magnetic properties of the shields require to be known. Fig. IV. shows for shields A and B, first, the B–H curves, in full lines, when the magnetising force Hc (the true force in the iron) is increased by increments from zero (Table I., columns 1 and 2); second, the corresponding values of the shielding ratios (g), in dash line curves (Table I., columns 6, calculated from the values of B/H given in columns 5); and third, the corresponding values of the shielding ratios $((g))$, in dotted curves (Table I., columns 8, calculated from the values of dB/dH given in columns 7).

Fig. VII. on the other hand shows for shields A and B, first, the usual hysteresis loops in full line curves, when the magnetising force Hc carries the induction B round complete cycles (Table II., columns 1 and 2); and, second, the corresponding values of the shielding ratio $((g))$ in dotted curves (Table II., columns 4, calculated from the values of dB/dH given in columns 3), which is obviously the only ratio available.

When comparison falls to be made between the theoretical shielding ratio and those determined experimentally, reference will be made to these tables and curves.

The hysteresis loss may be given here, expressed in ergs per cycle per cub. cm. for

	Max. Induction B.	$\frac{1}{4\pi}\int B dH$ (ergs)	Hysteretic Constant (η).
Shield A	9600	3500	·0015
Shield B	9750	6700	·0028

I. (*a*) *Experimental Shielding Ratios.—Transverse Field increased by increments from Zero.*

§ 10. Table V. gives the experimental data under the first above-mentioned conditions, viz., with increasing values of the transverse field H_t. The iron shields having been previously demagnetised, H_t was increased from about 4 C.G.S. units to a maximum of 130 C.G.S. units. At each step H_t was repeatedly reversed in order to secure a stable magnetic condition, and one which could be repeated with a minimum of error. The first column gives the values of the transverse field, the second column the corresponding values of the weakened field within the shield obtained by the rotation of the inductor, and the third column the experimental shielding ratio g' $[H_t/(H)_w]$. In fig. VI. these ratios for shields A and B are plotted as ordinates against the corresponding values of H_t.

These curves now fall to be compared with the theoretical shielding ratios. The induction round the sides of the shields is not uniform (see fig. XXXIV. (1)), and obviously reaches a maximum on both sides of the shields, where the iron is cut by a plane which is normal to the transverse field, and which contains the axis of the cylindrical shield. The induction at one of these positions, m, was measured by means of the exploring coil shown in fig. III., and is plotted in fig. V. (full line curves) from columns 1 and 2 of Table III. When the induction is carried beyond the initial stages, Du Bois remarks that "the difficulty is to assign the corresponding mean value of the true magnetising force, which is less than the impressed and greater than the shielded field, though probably nearer the latter." * No difficulty, however, exists in assigning values for each of the theoretical ratios (g) and $((g))$ corresponding to the induction at the maximum positions indicated, and it remains to be seen how far either of these ratios approximates to the experimental shielding ratios (fig. VI.). By taking a sufficient number of points on the induction curves (fig. V.), the same values of induction in the B–H curves of fig. IV., where H is the true magnetising force in the iron, supply on the same vertical ordinates corresponding values of the theoretical ratios (g) and $((g))$ from the dash and dotted curves respectively. These values will be found in the third and fourth columns of Table III., and are plotted against the corresponding values of the transverse field in fig. V.

§ 11. It is at once evident that the $((g))$ curves (dotted lines) reach considerably higher values in the earlier stages of induction, and lower values in the latter stages of induction, when compared with the experimental curves of fig. VI. On the other hand, the (g) curves (dash lines) are very similar to those of fig. VI., attain, within the limits of experimental error, the same maximum and minimum values, although, it must be noted, at somewhat lower values of the transverse field. But the theoretical shielding values have been based upon the maximum measurements of induction round the shields, which

* Du Bois, *Electrician*, "Magnetic Shielding," vol. xl. p. 815 (1898). Fig. X. shows the relation between the shielded field and the true magnetising force in the iron at the position of maximum induction.

are necessarily higher than what might be called the effective average induction which might be fairly assumed in order to arrive at the theoretical shielding ratios. Trial shows that if the ordinates of the induction curves fig. v. are reduced one seventh, the theoretical shielding values based thereon very closely approximate to the experimental ratio curves of fig. vi. The dash lines give a fair idea of the approximation.

We therefore conclude that when the transverse field is increased to a maximum of 130 C.G.S. units, the shielding ratio g' approximates to

$$g' = \frac{1}{2}\frac{d}{R} \cdot \frac{KB}{H} + 1$$

where B is the maximum induction and K is a constant somewhat less than unity. Thus the shielding ratio, minus unity, is approximately proportional to the ratio permeability and not to the differential permeability.

I. (*a*) *Experimental Shielding Ratios.—Transverse Field decreased by steps from a maximum.*

§ 12. Table VI. gives the measurements obtained for both shields under the above conditions when the transverse field is decreased from a maximum of about 80 C.G.S. units, and also when it is carried through zero to a negative maximum. In figs. viii. and ix. the values of the weakened shielded field $(H)_w$ (full line curves) and of the negative residual field $(H)_r$ (dotted line curves) given in Table VI. are plotted against the corresponding values of H_t for shields A and B respectively. It will be noticed that (with the exception of small values of H_t immediately after changing sign), the measurements of the residual field $(H)_r$ are of the opposite sign when compared with the measurements of the transverse field $(H)_t$. This is obviously due to the residual magnetisation in the sides of the iron shields completing its circuit partly within the shields, when the transverse force sustaining this magnetisation has been withdrawn. With the first shield experimented with, viz., shield B, measurements of the residual field were taken throughout, but they have not been retained in these tables, with the exception of those given in Table VI., as results did not appear to justify the extra complication introduced, and they moreover made the work unnecessarily laborious, more especially when the circular field was introduced, as after each determination the iron shield had to be demagnetised in order to retain the same magnetic sequence. With the second shield A, therefore, the residual field $(H)_r$ was not measured except with the decreasing values of H_t given in Table VI.

In fig. ix. the value of $(H)_w$ (Table V.) (faint full line curve), and the values of $(H)_r$ (not given in these tables) (faint dash line curve) have been added when the transverse field is increased from zero. Dealing therefore with shield B in the first place (fig. ix.), we observe that with values of H_t decreasing from about 80 C.G.S. units, the field within the shield in the neighbourhood of $H_t = 50$ becomes zero; while with an increasing field at the same value, the field within the shield is nearly 4 C.G.S. units. In both cases the residual fields, after H_t has been withdrawn, do not differ greatly from each other, being 1·4 and 1·9 C.G.S. units respectively. With a decreasing field less than 50, $(H)_w$ became negative, while between the limits of $H_t = 20$ and $H_t = 0$, $(H)_w$ is equal to $(H)_r$ in magnitude and sign. The same curves for shield A (fig. VIII.) show the same peculiarities, the lower values of the full line curve $(H)_w$ denoting better shielding, or a higher permeability; and the lower values of the dotted line curves $(H)_r$ denoting less coercive force. If the g' ratios $(H)_t/(H)_w$ be considered, the shielding becomes infinite when $H_t = 50$ and 55 C.G.S. units for shields B and A respectively, and when H_t is further decreased, the ratio between the transverse field and the shielded field becomes negative. If, on the other hand, the residual field be taken into account, and the g'' ratio

$$\frac{H_t}{(H)_w - \{-(H)_r\}}$$

be considered, the shielding ratios increase very rapidly, as H_t is decreased and becomes infinite between the limits of $H_t = 20$ and $H_t = 0$. But it must be noted that the g'' ratios never become negative. It is obvious that any comparison with the theoretical ratios is debarred, the theoretical conditions as to absence of retentivity and coercive force not being fulfilled.

§ 13. Before leaving this subject attention may be called to the third $(H)_r$ columns Table VI. (figs. VIII. and IX.). It will be noted that for both shields the residual field $(H)_r$ is less when the transverse field is reduced to zero at one step, than when it is reduced by a series of steps to zero. Although the difference is not large it is sufficiently well marked, and illustrates the known fact that sudden withdrawal of the magnetising force produces the greater internal commotion and so lessens residual magnetisation in the same way that vibration would.

Table IV. shows the ratio which exists between the transverse field, disturbed H_t' and undisturbed H_t by the presence of the iron shield for various values of the transverse field. (See § 47).

II. *Superposed Magnetic Inductions in Iron.*—(a) *Magnetising Forces increasing from Zero.*

§ 14. At a later stage of this investigation it was found that the value of the transverse field being equal to 20·9 C.G.S. units, the shielding ratio was 12 per cent. higher when the shields had been previously demagnetised by decreasing reversals of the circular field, than when they had been previously demagnetised by decreasing reversals of the longitudinal field.* By way of elucidating this point, various experiments were made; and these fields being at right angles to one another, the

* See § 54; also §§ 30–37 on Magnetic Æolotropy of Demagnetised Iron.

subject of cross magnetisation, or, more correctly, of superposed magnetic inductions in iron, was investigated, apart from the subject of shielding in hollow iron cylinders. It has been thought advisable to introduce this now, although the experiments were made at a later date than those following on magnetic shielding. Their bearing on this latter subject, however, will become evident the moment more magnetic forces (circular or longitudinal) than that due to the transverse field fall to be considered.

In § 2 various references to previous workers in this subject will be found. The general results appear to have been that, apart from vibration effects and under steady forces, the permeability of iron to a given magnetising force is lowered, if, at the same time, it be subjected to a magnetising force at right angles. So far as known to the writer, the two components of the resultant magnetic induction have not been co-ordinated the one with the other, when two magnetising forces at right angles to each other are superposed the one upon the other.

§ 15. Shield B was in the first place used for these experiments. The two primary windings (§ 5) supplied the means whereby two magnetising forces, circular and longitudinal, could act upon the iron. It may be repeated that the circular force H_c is, when acting alone, the true magnetising force in the iron. The longitudinal force is its calculated value due to the ampère turns of the solenoid merely.

Table VIII. gives the results of experiments, when upon increments from zero of the circular force H_c is superposed (CL * conditions) and repeatedly reversed (CLL * conditions) four different values of the longitudinal force H_l. It will suffice to describe the two sets of readings when $H_l = 3$ C.G.S. units, the lowest value used. In the first set the ballistic galvanometer is connected with the exploring coil measuring either circular induction or the circular component of the resultant induction. The first increment of $H_c = {\cdot}96$ C.G.S. units gives a throw of ·9 scale divisions of the galvanometer (B.G.). The superposition of H_l gives an additional throw of ·6 scale divisions in the same direction, making a total of 1·5 scale divisions. The next column, B, converts the results into C.G.S. units of induction. H_l is now repeatedly reversed, the ballistic throw being noted at each reversal. The final total is 2·2 scale divisions, or, as in the last column, $B = 850$. The iron is now demagnetised by decreasing reversals of the circular field, and the second increment of $H_c = 1{\cdot}6$, made from zero, resulting in a throw of 2·4 scale divisions. The superpositions of H_l are made as before, and so on until the maximum increment of H_c has been reached. It will be noted that the first few reversals of H_l causes the circular component of the induction to increase, and this irrespective of whether H_l be positive or negative, but this effect soon ceases, and further reversals produce no increase of the induction. It is, of course, necessary that the iron be demagnetised before each increment of H_c. In the third, fifth, and seventh sets the same operations are followed for higher values of the longitudinal field, viz., $H_l = 11{\cdot}9$, $21{\cdot}5$, and $39{\cdot}6$ C.G.S. units respectively.

* See footnote on following page.

The same sequence of observations as above is repeated for the second set of readings, with this difference, that the galvanometer is now connected with the exploring coil which measures the longitudinal component of the induction. The figures given under the column headed "total" might in this instance have shown both the final positive and immediately previous negative reading. Instead of this, however, the average of both these readings has simply been given. Column B, therefore, is the average of the final values of the induction which is alternately positive and negative. Owing to the large air circuit embraced by the exploring coil, the measured induction is too large by about 7 per cent. This has not been corrected. The iron being demagnetised at each increment of H_c, the resulting induction produces no longitudinal component, and no throw in the galvanometer was observed. The effect of the circular induction is, however, plainly shown when H_l is superposed, and the readings then obtained ought to be compared with those in the fourth, sixth, and eighth sets, which differ only in that higher values of the longitudinal field are used, viz., $H_l = 11·9$, $21·5$, and $39·6$ C.G.S. units respectively.

§ 16. Table VII. gives the results of experiments when, upon the induction due to four different values of the longitudinal field, the circular field, between the limits of $H_c = 0$ and $Hc = 13$ C.G.S. units, is first superposed (LC* conditions) and then repeatedly reversed (LCC* conditions). During the first, third, fifth, and seventh sets of readings, the galvanometer is connected with the exploring coil measuring circular induction or the circular component of the induction, and during the second, fourth, sixth, and eighth sets, with the exploring coil measuring the longitudinal induction or the longitudinal component of the induction. The four different values of H_l are the same as in Table VIII., but the order in which the two fields are superposed is reversed. After the readings for any single value of H_c have been completed, the iron is demagnetised by decreasing reversals of the circular field.

§ 17. The measurements given in Tables VII. and VIII. are plotted in figs. XI., XII., and XIII. for the values of $H_l = 3$, $21·5$, and $39·6$ C.G.S. units, omitting those for $H_l = 11·9$ as superfluous owing to their intermediate character. In these three figs. the dotted curves represent the normal induction curves when the circular force and the longitudinal force are each acting alone. Necessarily the latter induction, being due to a force kept at a constant value, is represented by straight lines parallel to the horizontal axes. The other four curves, marked B_c, measure the circular components of the induction due to H_c; and the other four curves, marked B_l, the longitudinal component of the induction due to H_l. In all cases the heavy full lines measure the component of the induction in the direction of the force first

* These letters refer to the circular and longitudinal fields respectively. The order in which they occur signifies the order in which they are superposed the one upon the other. Thus "CL conditions" means that the circular field H_c acts first upon the iron, and the longitudinal field, second. "LC conditions" has, necessarily, the opposite signification. CC and LL mean that the circular and longitudinal fields respectively are repeatedly reversed. § 38 may be referred to. The definitions there given are immediately applicable, by substituting L (longitudinal field) for T (transverse field).

acting, when the magnetising force at right angles is superposed ; and the heavy dash curves, the component of the induction in the direction of the force first acting after the superposed force has been repeatedly reversed. On the other hand, the faint full lines measure the component of the induction in the direction of the magnetising force last acting, and which has been superposed ; and the faint dash curves measure the component of the induction in the direction of the force last acting and which has been repeatedly reversed. Reference to the tables (VII. and VIII.) shows that the heavy full line curves, CL (or LC), when taken along with the faint full line curves, CL (or LC), co-ordinate the circular and longitudinal components of the resultant induction under the CL (or LC) conditions. Also that the heavy dash curves, CLL (or LCC), taken along with the faint dash curves, CLL (or LCC), co-ordinate the circular and longitudinal components of the resultant induction under the CLL (or LCC) conditions.

§ 18. It will be observed that both the faint line curves, CL and LC, are near their origin above and afterwards decidedly below the normal curves (dotted lines). As the first effect is small, it was thought advisable to repeat the measurements of the circular component of the induction under the LC conditions with shield A, limiting the values of the circular field between $H_c = 0$ and $H_c = 2{\cdot}2$ C.G.S. units. Table IX. gives these results obtained in a slightly different manner from those given in Tables VII. and VIII. Four sets of readings were taken, increasing the longitudinal force by increments from zero, and at each increment superposing the circular field, kept at a constant value for each set. The second column gives the measurements of the longitudinal force, which, as will be observed from the third column, produces no throw of the galvanometer connected so as to measure B_c. The circular field, at the particular value given in the first column for each set of readings, being now superposed, the circular components of the resultant inductions are obtained in scale divisions, fourth column, and in C.G.S. units, last column. In fig. XIV. the circular component B_c is plotted against the superposed force H_c, for four different values of the longitudinal field, viz. :—$H_l = 0$, 6·5, 16·4, and 41 C.G.S. units. By taking readings when $H_l = 0$, the normal B–H curve (dotted line) is obtained with a minimum of error, due to altered conditions when compared with the full line curves. Fig. XIV. thus shows on an enlarged scale, and for another quality of iron (shield A), the earlier positions of the B_c curves under the LC conditions. It is at once evident that the initial effect mentioned at the beginning of this section is confirmed.

Figs. XI., XII. and XIII. show that the effects of field superposition are essentially the same in the B_c as compared with the B_l curves, although more pronounced in the former than in the latter case. This difference is fully accounted for by the fact that in the B_l curves the longitudinal induction completes its circuit through a large air reluctance.

§ 19. The distinction between "circular" and "longitudinal" may thus be regarded as arbitrary. The conclusions now to be drawn can be equally well or better stated in terms of the order of field superposition. Of the two magnetising forces at right angles to each other, let H_1 be the force first acting, H_2 the force superposed. Each force acting alone produces the normal B–H induction curve. Let B_1 and B_2 be the two components of the resultant induction in the directions of H_1 and H_2 respectively.

When H_2 is superposed upon a pre-existing induction due to H_1, the B_1 component always lies above the B_2 component. Repeated reversals of H_2 accentuate this result; B_1 is further increased, and concurrently with this B_2 is further lowered. For low fields, the B_1 component is considerably *above the normal induction curve*, but as the fields are increased a point (depending upon the relative strengths of both fields) is reached where the curves cross, the B_1 component afterwards falling below the normal curve. This first increase of B_1 at the early stages of induction is well marked and evidently reaches a maximum in fig. XII. (CL and CLL, B_c curves); while in fig. XI. with a lower value, and in fig. XIII. with a higher value of the superposed force, the effect is in both cases somewhat reduced.

Messrs GEROSA and FINZI (*loc. cit.*) have shown that an alternating current in an iron wire increases, within due limits, the permeability of the iron to longitudinal magnetisation. The effect is regarded as due to the violent agitation of the molecules brought about by the rapid reversals of the circular magnetisation, due to the alternating current in the wire, which corresponds to the reversals of the force H_2 in my experiments. It must be noted, however, that repeated reversals of H_2 merely accentuate the very considerable molecular disturbance caused by the *first superposition* of the *unidirectional* force H_2, in increasing the permeability of the iron at the earlier stages of induction, to the constant force H_1 first acting.

§ 20. Also the superposition of H_2 lowers the B_2 component below the normal induction curve, with this exception, that at low values of H_1 the superposition of the second force H_2 increases the B_2 component above the normal induction curve. This is, however, a relatively small effect. The subsequent lowering of the B_2 component is one of the more obvious effects of field superposition, and has been noted by SIEMENS, SCHULTZE, Messrs EVERSHED & VIGNOLES, KNOTT, Messrs GEROSA and FINZI and others (*see* § 2). The first initial increase of B_2, however, must be taken into account in stating these results of field superposition. So far as these experiments are available, it follows that for any two values of the force first acting (including $H_1=0$) the curves measuring the B_2 induction component cross, that for the lower value of H_1 being at first below and afterwards above that for the higher value of H_1.

§ 21. In this connection it is interesting to quote the remarks of Messrs EVERSHED and VIGNOLES (*loc. cit.*): "Starting with the idea of Mr SWINBURNE, that it should be possible on the WEBER hypothesis to get increased permeability in iron by magnetising it first in one direction, and then in another at right angles to it, we proceeded to investigate the point." They conclude as follows:—"Our results, in short, show, as far as they go, (1) that no such increase of permeability as anticipated by Mr SWINBURNE on the WEBER hypothesis actually takes place; and (2) that no increase of permeability in one direction takes place when a constant transverse magnetising force is maintained." They add: "The experiments described have nothing whatever to do with the initial part of the force induction curve for iron to which so large an amount of attention has been devoted, and which is of so little practical importance."

But it is sufficiently obvious that any conclusions based upon experiments in which strong magnetising forces are used cannot be legitimately used as against the WEBER hypothesis, which explains induction at high values, on the assumption that the rotating molecular magnets have already placed themselves well in line with the resultant of the strong forces at right angles to each other. The results of the experiments on superposed inductions given in the preceding sections are quite in harmony with and satisfactorily explained by the usually accepted theory that induction consists in the definite orientation of molecular magnets. Such experiments, however, to have any bearing on the subject, must be conducted during the process of this molecular turning, and not when the turning is nearly completed. This subject is again considered when dealing with the magnetic æolotropy of demagnetised iron (see § 30), and it is also shown (see § 36) that a sufficiently well marked increase of permeability actually exists when the iron has been left with residual magnetisation at right angles to that of the subsequently applied magnetising forces.

II. *Superposed Magnetic Inductions in Iron.*—(b) *Cyclic Conditions.*

§ 22. In the preceding sections, the two components of the final resultant induction were considered, when upon an induction due to a magnetising force first acting, repeated reversals of a force at right angles were superposed. These components, however, were co-ordinated, the one with the other, only at the extreme positions of the superposed reversals. In the experiments now to be described, each superposed reversal is a cyclic process performed in a series of steps. The component of the induction due to the force first acting (H_c or H_l as the case may be) can thus be traced at each step of the superposed cycle or cycles.

The longitudinal induction component being measured, Table XI. gives the measurements obtained, when upon a longitudinal induction of $B_l = 7300$ sustained a field of $H_l = 20{\cdot}4$ C.G.S. units, the superposed circular force is first increased by increments from zero to a maximum of $H_c = 12$ C.G.S. units, and then carries the B_c component round complete cycles between positive and negative extremes of approximately $B_c =$ 10,000. The first and fifth columns give the values of the magnetising force H_c, the second and sixth columns the corresponding readings of the galvanometer in scale divisions measuring B_l as above mentioned. In the fourth and eighth columns, the cyclic changes of B_l are given in C.G.S. units. In the last four columns, a sufficient number of intermediate points are taken to determine more fully the shape of the curve after the changes have become cyclic. All the B.G. columns give the average results of two distinct sets of readings for the same values of H_c, the galvanometer connections being reversed at the beginning of the second set.

In fig. XV. the values of B_l thus obtained are plotted against the values of the superposed cyclic force H_c. On this force being increased by steps to a maximum, the B_l component at first rises above, but finally falls somewhat below its original value when $H_c = 0$. This curve is approximately the same as the B_l curve given in fig. XII. (for a slightly higher value of B_l) under the LC conditions, the ordinates being for present purposes expanded five times. H_c now carries the B_c component round a complete cycle, and it is at once seen that the B_l component responds. After a few reversals the curve closes upon itself and becomes cyclic. Fig. XV. thus shows (1) the B_l curve when H_c is increased from zero to a maximum (LC conditions); (2) the process by which the higher values of the B_l component are reached under the LCC conditions at the extremes of the H_c reversals; and (3) the changes which occur in the B_l component of the resultant induction after the process has become cyclic, under the influence of the superposed cyclic force H_c.

§ 23. Obviously the pre-existing induction upon which a force at right angles is superposed may be at any stage of the hysteresis loop. The case where the induction is at one of the extremes of this loop, sustained by a magnetising force from zero, has just been given. In figs. XVI. and XVII. the varying cyclic force is superposed upon points of this B–H cycle, where B is changing most rapidly with respect to H, and where consequently it might be expected a superposed force would have a maximum effect upon the pre-existing induction. In the former fig. H_l is still the force first acting, the induction being left at a small negative value of $B_l = -296$. In the latter fig. H_c is the force first acting, the induction being in this case left at a small positive value of $B_c = 1760$. The experimental data for figs. XVI. and XVII. will be found in Tables XII. and X. respectively. At the top of each table the negative values of the force first acting are given in C.G.S. units, and also the positive maximum values from which they were reduced. The various columns have the same signification as in Table XI. with this exception, that in Table XII. both sets of B.G. readings have been tabulated, from which the average readings have been derived.

Both figs. may be discussed together. As in fig. XV. the ordinates measure the component of the induction in the direction of the force first acting; the abscissæ, the values of the alternating force which is superposed in steps. In both cases, we observe that, *in the first place*, the superposition of the cyclic force causes the ordinate values rapidly to decrease, or rather to increase negatively, and to approximate to the values which would have held had the induction due to the force first acting been a negative increment superposed upon zero induction. Compare the B_l ordinate value attained at the extremes of the H_c cycle (fig. XVI.) with the B_l curve of fig. XI. under the LCC conditions at a value of $H_c = 12$. Also the ordinate value attained at the extremes of the H_l reversals (fig. XVII.), viz., $B_c = -4900$, may be transferred to fig. VII. at a position on the horizontal axis of $H = -2$ C.G.S. units, and compared with the B–H cycle for shield B. Both those comparisons show that the effects of hysteresis due to the force first acting have practically disappeared.

In both figs. (XVI. and XVII.), we observe, *in the second place*, that the ordinate changes measuring the induction component in the direction of the force first acting finally become cyclic (as in fig. XV.), the curves returning upon themselves after a few reversals of the superposed force.

§ 24. But these figs. (XVI. and XVII.) also show points of difference. The ordinate changes in the latter fig. cover a much wider range than the ordinate change in the former. This is accounted for by the fact, that in fig. XVII. the varying force is superposed upon an induction due to H_c, which is the true magnetising force in the iron. The difference between the particular point selected on the descending limb of the B_c–H_c cycle, and the maximum negative values of B_c reached at the extremes of the final cyclic changes, when a varying force is superposed, is of the order of $B_c = 6500$. When, however, the longitudinal force is that first acting upon the iron, the hysteresis loop is sheared over, and the ordinate distance between its descending limbs and the maximum negative values of B_l reached at the extremes of the superposed B_c cycles must be relatively small. This is exemplified in fig. XVI.

The same differences between the maximum and minimum values in the direction of the force first acting also appear when the ordinate changes have become cyclic. In fig. XVII., with the force first acting $= 2$ C.G.S. units, the ordinate change is $B_c = 1330$, while in fig. XVI., with the force first acting $= 2{\cdot}8$ C.G.S. units, the ordinate change only amounts to $B_l = 230$. In fig. XV. the cyclic change of $B_l = 950$, but in this instance the value of the force first acting is larger, viz. :—$H_l = 20{\cdot}4$ C.G.S. units. On the other hand, the points of inflection of the final cyclic curves are much more sharply marked in figs. XV. and XVI. than they are in fig. XVII., where the numerical values of the superposed varying force H_l due to the solenoid are greatly in excess of the true magnetising force acting in the iron. The points of inflection are therefore flattened. In figs. XV. and XVI. the superposed alternating force is H_c (which when acting alone is the true magnetising force in the iron). The maximum values of dB_c/dH_c are therefore rapidly passed in reference to the numerical value on the horizontal axis. The differences discussed in this section are therefore non-essential, and due to both magnetic circuits not being wholly completed in the iron. They are differences of degree only.

§ 25. Consequently the conclusions now to be drawn may be expressed as before (§ 19) in terms of the order of field superposition. Of the two magnetising forces at right angles to each other, let H_1 be the force first acting, H_2 the force superposed, and let B_1 and B_2 be the two components of the resultant induction in the direction of H_1 and H_2 respectively. We have seen that when the superposed magnetising force H_2, first increased from zero and then repeatedly reversed, carries the B_2 induction component in steps round a series of cycles, the B_1 component due to H_1 kept at a constant value, is found to respond by passing through a series of changes or oscillations in the process of likewise becoming cyclic.

The extent of the changes which occur in the ordinate values of B_1 during the transition process depends upon the particular point which has been selected on B_1–H_1 hysteresis loop upon which to superpose the cyclic alternations of H_2 and also upon the maximum value of these alternations. This ordinate change of B_1 may vary from zero to a maximum. The conditions which prevail when the change is zero have already been stated in § 19, and this occurs at the crossing of the curves there described. Fig. XV. illustrates such a position, the final ordinate values of B_1 at the extremes of the cyclic alternations of H_2 being almost the same as when H_2 is zero. If H_1 had been taken weaker, the final ordinate values would have shown an increase, if stronger a decrease (see figs. XII. and XIII.). The ordinate changes are a maximum (see fig. XVII.) when B_1 has in the first instance been decreased from a maximum, or in other words when a suitable intermediate point on the hysteresis loop has been selected upon which to superpose the reversals of H_2. The effect of hysteresis due to the first magnetising process are in this instance almost completely obliterated, and the final values reached by B_1 at the extremes of the H_2 reversals approximate to the values they would have assumed had repeated reversals of H_2 been superposed upon B_1 (increased from zero induction) and supported by H_1, the force first acting.

§ 26. But the ordinate changes of B_1 also depend upon the maximum values of the superposed alternating force. Various maximum values limiting the amplitude of H_2 were not tried, but it appears from a comparison of the CLL, B_c curves of figs. XI. and XII. that if the maximum values of the superposed reversals had been reduced, the ordinate changes would likewise have been less. In other words, the disappearance of the effects of hysteresis due to the force first acting would have been less complete.

Indeed, Mr ZENO CROOK * has shown that an alternating current of 10 ampères per sq. mm. in an iron rod reduces the area of the hysteresis loop due to a cyclic variation of the longitudinal force to a greater extent than an alternating current of 2·5 ampères per sq. mm. The experiments of Messrs GEROSA and FINZI † already referred to are well known. They have shown the almost total collapse of the usual hysteresis loop in an iron wire under the above-mentioned conditions. The effect is ascribed to the violent agitation of the molecules brought about by the rapid reversals of the circular magnetisation, due to the alternating current in the wire, and which corresponds in the experiments described above to the alternations (in steps) of the superposed force H_2.

§ 27. The experiments carry the matter a step further. Figs. XV., XVI. and XVII. show not only the total ordinate change, but *the rate of change* under the various conditions, as the superposed force H_2 is being repeatedly reversed. This rate of change of the B_1 induction component is seen to be least at the extreme values of the superposed reversals, and greatest when the molecules are left freer to obey the constant force H_1, as H_2 approximates to zero. The effect of the *first superposition* of H_2 in comparison with subsequent reversals in producing change of B_1 is very marked. As has been remarked

* *American Journal of Science*, vol. clxi. p. 365, 1901.

† "Magnetic Induction in Iron," EWING, 3rd ed., p. 332, 1900.

in § 19, reversals of H_2 merely accentuate the very considerable molecular disturbance caused by the superposition of the unidirectional force at right angles. It also appears probable, judging from the readings given in the first, third, fifth and seventh sets of determinations given in Table VIII., that had the amplitude of the H_2 reversals been less, the relatively large effect of the first superposition would have been somewhat reduced in comparison with subsequent reversals. We know that the total ordinate change of B_1 would have been reduced; and, in any case, it is interesting to observe that it is the first superposition, followed by a very few reversals of the superposed force, which produces the total ordinate change of B_1. Further alternations of H_2 are powerless to effect more than a partial collapse of the hysteresis loop due to the force first acting when the amplitude of these alternations is insufficient.

§ 28. The above results obtained with superposed forces slowly alternating may be compared with similar effects due to high frequency discharges. Mr E. RUTHERFORD,* in his experiments on the demagnetising effect of such discharges on needles magnetised to saturation, found that after a steady state was reached "the passage of further discharges has no apparent effect on the magnetisation of the needle." He obtained the further result, that the magnetometer deflection, measuring demagnetisation, "was approximately proportional to the magnetic force acting upon the needle, provided the magnetic force was well below the value required to completely demagnetise the steel."

Mr RUTHERFORD was able to detect high frequency waves in free space at a distance of over half a mile by means of a detector constructed on above principle, and recently Signor MARCONI † has made use of the extreme sensitiveness of that part of B–H induction cycle where B is changing most rapidly with respect to H, as a detector of electric waves for space telegraphy.

§ 29. As the ordinate changes of B_1 above described reach their final values at the extremes of the superposed reversals, the curves become closed upon themselves, and further changes of the B_1 induction component are strictly cyclic. This final change of the B_1 component falls to be co-ordinated with the cyclic change of the B_2 component impressed upon the iron by the alternating superposed force H_2. The cyclic values of B_2 were measured under the same conditions as prevail in fig. XV. In this case B_2 corresponds with the circular magnetising force. It was found that the B_2/H_2 cycle, when the force first acting upon the iron was $H_l = 20{\cdot}4$ C.G.S. units, differed somewhat from the same cycle when $H_l = 0$,—or, in other words, from the B–H cycle given in fig. VII. for shield B. The maximum induction values at the extremes of the cycle were reduced by about ten per cent. The positions on the cycle at which the values of H_2 changed most rapidly with respect to H_2 were not materially altered, and occurred as the horizonal axis was crossed, as in the normal cycle (fig. VII.). But the maximum

* *Phil. Trans.*, "A Magnetic Detector of Electric Waves," vol. clxxxix., 1897.

† Paper read before Royal Society 12th June 1902, and entitled "A Magnetic Detector of Electric Waves which can be employed as a Receiver for Space Telegraphy."

values of dB_2/dH_2, as given by the tangent of the curves at the points referred to, were reduced from the normal of 6000, as given in Table II. for shield B, to a value of about 3500. The B–H curve for shield B given in fig. VII., if slightly sheared over, sufficiently well represents the cyclic changes of the B_2 induction component due to the superposed alternating force H_2. (If, however, the values of H_1 are unduly increased, the B_2–H_2 cycle becomes lop-sided. This was not further investigated.

If comparison be now made between this curve and the cyclic changes of the B_1 component, it is at once seen that the lowest values of B_1 occur at the positive and negative extremes of the superposed force H_2; and the highest at those points of the B_2 cycle, where B_2 is changing most rapidly with respect to H_2. The maximum and minimum values of B_1 thus each occur twice during one complete reversal of the superposed force, and the similarity of the B_1 cycle to the effects of repeated positive and negative twisting of an iron rod or wire upon its longitudinal magnetisation may also be noted.

We may therefore conclude that the final cyclic changes of the B_1 induction component in the direction of the magnetising force H_1 kept at a constant value, follow at least approximately the changing permeability (dB_2/dH_2) impressed upon the iron by the superposed alternating force H_2 carrying the B_2 round its complete cycle. The effects of the previous history of the iron may be wiped out, but hysteresis appears in another form.

III. *Magnetic Æolotropy of Demagnetised Iron.*

§ 30. It is well recognised that, however completely iron once magnetised may be apparently demagnetised by magnetic means, it is not in the same molecular condition as it was previous to its having been magnetised, or after it has been raised to a high temperature. EWING gives a very marked example of this (*Magnetic Induction in Iron*, 3rd ed., p. 98), where, by subjecting iron to a certain sequence of magnetising forces, it is possible to leave the iron at zero magnetisation and in zero field. Re-application of the magnetising force, however, shows "a striking want of directional symmetry when it is subsequently magnetised" in a positive or negative direction. It is pointed out by way of contrast that iron which has been made neutral by the process of demagnetising by decreasing reversals, does not show this directional difference as to whether the subsequent magnetising force be positive or negative.

But here again we know that the molecular condition in which the iron is left even by this process must be very different from that of originally unmagnetised iron, or iron demagnetised by the action of heat. The following example may be given, also from EWING (*loc. cit.*, pp. 340–341): When a magnetising force is applied and then repeatedly reversed, "successive repetitions of the process give a gradually diminishing range of magnetic change,"—"but when demagnetised by reversals after the magnetising force has been raised to a high value" this property is lost. "In this respect, then, a wire *demagnetised by reversals* differs from the same wire in its *primitive annealed state.*"

It has already been stated (§ 14) that the shielding to an external transverse field is higher when the hollow iron cylinders or shields have been previously demagnetised by decreasing reversals of the circular as compared with the longitudinal field (see § 54). This is the case immediately in point, as it raised the question whether iron, after having been demagnetised by decreasing reversals, shows directional permeability (not as to subsequent forces being positive or negative, but) to subsequent forces at right angles to that used in the demagnetising process, or whether the effects were spurious, depending upon other causes entirely.

§ 31. In the preliminary experiments, the shields A and B were dispensed with. A sheet of transformer iron was taken, and a piece cut from it in the form of a cross. The two arms were equal and measured about six inches in length by three-quarters of an inch in breadth. The ends of one of the arms were bevelled, bent round so as to meet, overlapped and secured in position, forming a sheet iron ring. The ends of the other arm were bent in the opposite direction and formed into a second ring similar to the first. These rings, after having been heated to redness in a bunsen flame, were wound with wire through which independent currents could be sent, thus forming two magnetic circuits, M and N. The piece of sheet iron, three-quarters of an inch square and not wound with wire, was necessarily common to both circuits. An exploring coil of fine wire was also wound round one of the circuits, M, close up to this mutual square of iron. It is evident that this mutual square could be demagnetised by decreasing reversals of either circuit, and that these are at right angles to one another.

The magnetic circuit M was now demagnetised by decreasing reversals. The magnetising force was then re-applied by increments and the induction measured at each step by means of the exploring coil and ballistic galvanometer. The circuit M was again demagnetised, followed in this instance by decreasing reversals of the magnetising force in the circuit N. This left the square common to both circuits demagnetised by means of a force at right angles to M. The induction in the M circuit was again measured as before. It was found that for low values of the magnetising force the induction was lower when the iron square common to both circuits had been demagnetised by the force at right angles. (The same result was obtained when the iron circuit M was opened out and did not form a closed magnetic circuit.) These experiments were repeated many times, always with consistent results. The difference, which was of the order of 5% (the iron circuits being closed), entirely disappeared as the induction increased. It ought to be borne in mind that the mutual square formed only a small part of the magnetic reluctance of the circuit.

These preliminary experiments, made under different conditions, confirmed the supposition that the observed differences in the shielding ratio were essential and not spurious effects, and were due to a difference in the permeability of the iron at the early stages of induction, determined by the direction of the magnetic force used to demagnetise the iron by decreasing reversals.

§ 32. Shields A and B were now reverted to, and in order to wipe out as far as possible the previous magnetic history of the iron, the shields were demagnetised by repeated reversals of both the circular and the longitudinal magnetising forces. The first four columns of Table XIII. give the measurements obtained due to increments of the circular magnetising force, following upon demagnetisation leaving the iron without apparent polarity, under the influence of decreasing reversals of the longitudinal field last applied. The second four columns, on the other hand, give the measurements due to increments of the circular magnetising force following upon demagnetisation obtained under the influence of decreasing reversals of the circular field last applied. In the first case the iron had been demagnetised by a force at right angles to the subsequent magnetising force, and the results are plotted as dotted line curves in fig. XVIII. for both qualities of iron. In the second case the iron had been demagnetised by a force acting in the same direction (positive and negative) as the subsequent magnetising force, and the results are plotted as full line curves in the same fig. for both qualities of iron—these curves being the normal B–H curves of induction. The differences between the two curves are sufficiently well marked, confirming the results obtained under the conditions described in the section immediately preceding, for a different quality of iron.

§ 33. We may therefore conclude that during the early stages of induction iron is more permeable to a re-application of a magnetising force in the same direction (whether positive or negative) as that used in the immediately preceding demagnetising process, than it is to a force (whether positive or negative) at right angles to that used in the immediately preceding demagnetising process. The maximum difference did not exceed 500 C.G.S. magnetic units, but as this occurs at low values of the B–H curve, it amounts to a very considerable proportion of the induction. The full line curves exceed the dotted curves by about 30 to 35 per cent., and this maximum corresponds approximately with the period of maximum permeability (dBd/H) in the specimens of iron used. When, however, the magnetising force is increased, the two curves rapidly approximate and appear to become one.

Theoretical Considerations.

§ 34. Ewing's modification of Weber's theory of magnetic induction assumes (with Weber) that the molecules are magnets pointing in all directions in unmagnetised iron, but falling into line and showing polarity in the mass when a magnetising force is applied. It dispenses with any arbitrary forces controlling the molecular magnets, and explains that the constraint which " proceeds only from their mutual action as magnets evidently suffices to explain generally the characteristics of the magnetising process." In order to arrive at some possible explanation of this directional permeability at the early stages of induction, consider the demagnetisation of iron by decreasing reversals. If during this demagnetising process the induction were measured we would obtain a series of decreasing hysteresis loops, the curves of which would not be closed upon themselves. Their extreme positive and negative values would continuously decrease, until they vanished in the zeros of H and B. At the first positive value of B, the molecular magnets may be supposed for the most part to be well in line with the magnetising force H. On this being reversed to a somewhat less negative value, the measured induction would be less. To what does this correspond in terms of the molecular magnets? The force being again reversed to a still less positive value, the measured induction would again prove to be less, and the same question arises, and so on until the reversals of H and B are as small as you choose to make them.

In unmagnetised iron the resultant axes in all stable molecular groups are on the average equally distributed in all directions, and on the average the stability is the same in all directions. The turning moment tending to turn such a group when the axis makes an angle θ with the force H is

$$m = a \text{ H} \sin \theta.$$

During the process of demagnetisation by reversals, and as the decreasing, repeatedly reversed force dies down, the force which is just able to turn a group into a stable position θ which is just retained when the force is reversed, is smaller and smaller as the angle θ increases. Assume that it is proportional to $\cos \theta$, then

$$m = a\text{K} \cos \theta \sin \theta = \tfrac{1}{2} a\text{K} \sin 2\,\theta$$

and m is a maximum when $\frac{dm}{d\theta}$ is equated to zero

$$\frac{dm}{d\theta} = a\text{K} \cos 2\,\theta = 0$$

$$\text{or when } \theta = \frac{\pi}{4}.$$

The turning moment is greatest when the angle $\theta = 45°$, and it vanishes when $\theta = 0°$ and when $\theta = 90°$.

§ 35. Now repeat the demagnetising process, the magnetic axis of the molecules coinciding on the whole with the direction of H. On reversing to a somewhat less negative value of H, the majority of the molecules will topple over, but some will be left. On reversing again, the lower positive value of H will again fail to topple over all the molecules, but still the majority will topple over under the influence of the deflecting force $\text{H} = \text{K} \cos \theta$. And so on the process will go, leaving molecules or groups of molecules distributed at all angles between $\theta = 0$ and $\theta = 90°$. But the turning moment, m, being a maximum when $\theta = 45°$, it follows that at each reversal the molecule or group of molecules left stranded which the force H has failed to topple over will be wider spaced the nearer the angle in which they are left approximates to 45°, and closer spaced the nearer the angle in which they are left approximates to $\theta = 90°$ or to $\theta = 0°$, because as these angles are approached the turning moment is getting less and less.

There being no reason to suppose that the molecular magnets topple over more in one direction than another, it follows that the molecules will tend to point less towards a zone making an angle of 45° with the magnetising force, more towards the magnetising or polar force H, and also more towards the equatorial regions. (See diagrams 1 and 2, fig. XIX.). We may therefore conclude that on the whole, owing to the greater number of groups on the equatorial side of the 45° latitude line, the molecular magnets will tend to point in a plane or equatorial belt at right angles to the demagnetising force. But the pull exercised by the demagnetising force H is greatest on those molecules lying equatorially; hence it follows that if the direction of the subsequent magnetising force be rotated, the number of molecules lying equatorially is reduced and the pull becomes less and less as the rotation approximates to 90°. Hence we have obtained a possible explanation, in terms of molecular theory, why iron is not isotropic to a magnetising force at right angles to that used in the demagnetising process.

§ 36. At this stage a further experiment suggested itself. The iron shields were thoroughly demagnetised, first by decreasing reversals of H_l and then of H_c, and, second, subjected to a maximum magnetising force H_l, which on being withdrawn left the iron with residual magnetisation. This amounted to $B_l = 760$ C.G.S. units for shield A and to $B_l = 800$ C.G.S. units for shield B. The last four columns of Table XIII. supply the data when the B–H curve due to H_c increased by increments is superposed upon this residual magnetisation at right angles to the magnetising force. In fig. XVIII. these measurements are plotted as dash line curves (3), and at the earlier stages of induction the increase of the permeability is marked when compared with either of the two other curves. We have thus for the two qualities of iron experimented with a sequence of three curves, all obtained by increasing the magnetising force H_c by increments from zero. In each case, however, the iron had been left in a different molecular condition, unsupported by any external magnetic field. If we consider the full line curves obtained by the re-application of the same directional force as that used in the immediately preceding demagnetising process, the usual induction curve under normal conditions; then iron demagnetised by a force at right angles to that subsequently applied is *less permeable*, and iron left with residual polarity due to a force at right angles to that subsequently applied is *more permeable* than iron magnetised under the normal conditions. Fig. XIX. shows graphically and without further explanation why the pull on the molecular magnets, and consequently the permeability at low values of H, should be in an ascending order for curves 1, 2 and 3 (fig. XVIII.).

§ 37. In conclusion, it must be borne in mind that a possible explanation of the general nature of the effects is all that is attempted. Thus in discussing the molecular condition of iron demagnetised by decreasing reversals, it has been assumed that $H = K \cos \theta$. A different law for H would give a different angle for the maximum turning moment. Perhaps a point worth noting is this, that unless it can be shown that the turning moment is the same for all values of the angle θ during the process of demagnetising by reversals, it appears to the writer that iron demagnetised in this way is bound to exhibit æolotropic properties at the early stages of induction.

I. (*b*) *Shielding Ratios with Transverse and Circular Fields.*

§ 38. The addition of the circular magnetising force H_c supplies a nearer approach to uniform magnetisation than when the transverse field is the only force acting upon the iron, more especially when the values of H_t are small. We have consequently a nearer approach to uniform values of B/H or dB/dH all round the shield, one of the conditions assumed in the theoretical shielding formulæ.

When the circular field is increased from zero, preliminary experiments showed that the order and manner in which the one field is superimposed upon the other affected the shielding ratio to an enormous extent.* The following sequences were tried, the value of the transverse field being in all cases equal to 20·9 C.G.S. units.

First.—When upon a pre-existing induction due to the transverse field were superposed increments of the circular field ascending from zero to a maximum.	TC.
Second.—When upon a pre-existing induction due to the transverse field were superposed repeated reversals of the circular field ascending from zero to a maximum.	TCC.
Third.—When upon a pre-existing induction due to the circular field ascending from zero to a maximum was superposed at each increment the (uni-directional) transverse field.	CT.
Fourth.—When upon a pre-existing induction due to the circular field ascending from zero to a maximum were superposed at each increment repeated reversals of the transverse field.	CTT.
Fifth.—When upon a pre-existing induction due to the transverse field, the superposed circular field carries the iron in steps round a complete magnetic cycle or series of cycles.	Cyclic.

Let the above first four sequences of superposition of magnetic fields be referred to, for the sake of brevity, by the letters to their right, T and C signifying transverse and circular fields respectively, and also the order in which they are impressed on the iron shields. CC and TT mean that these fields are repeatedly reversed. Further, let these symbols be used to indicate not only the conditions themselves, but also the shielding ratio curves obtained under these conditions.

§ 39. Figs. XX. and XXI. show the shielding ratio curves obtained under the above

* It is of interest to observe that it has also been noted that the amount of twist produced in an iron wire by a given combination of circular and longitudinal magnetisms, depends largely upon the order in which these are applied. See "Magnetism and Twist in Iron and Nickel," by Dr Knott, *Trans. R.S.E.*, vol. xxxvi. p. 490, 1891.

first four sequences for shields A and B, plotted as ordinates against values of H_c as abscissæ. When $H_c = 0$, the curves have their origin at values corresponding to those plotted in fig. VI. when the transverse field is the only force acting upon the iron, which in this case is equal to 20·9 C.G.S. units. As these curves leave their origin in the vertical axis, a general comparison of figs. XX. and XXI. with the theoretical shielding ratios $((g))$ given in fig. IV. shows that the differences which exist in the magnetic qualities of the two shields A and B are well brought out in the shielding ratios. For shield A they rise higher, attain their maximum values earlier, and ultimately descend to lower values than for shield B. Leaving out of consideration meanwhile the curves obtained under the CTT conditions given above, these remarks are undoubtedly true for those obtained under the TC and TCC conditions, and also, although to a less extent, for those obtained under the CT conditions. Indeed, more careful examination shows that, when increments of the circular field are superimposed upon a pre-existing induction due to the transverse field (TC condition), the shielding ratio curves coincide (except near their origin) with the theoretical $((g))$ curves, within the limits of experimental error.

§ 40. As the conclusions arrived at in this investigation, when both transverse and circular fields act upon the iron, and indicated in the last sentence of § 39, are not in harmony with the inductions drawn from the experiments of STEFAN and DU BOIS, I quote from the latter's recent contributions on the subject. He writes as follows* :—

> "One of the thick wrought-iron cylinders was uniformly wound with wire, by means of which a *circular* field of several C.G.S. units could be excited. Besides the corresponding circular magnetisation this produced considerable polarity, owing to the welded joint and general want of homogeneity. Notwithstanding this and the high permeability corresponding to the circular field, the shielding ratio proved practically unaffected by it. This result lends support to the assumption, made throughout, that shielding against moderate disturbing fields depends, within due limits, upon the constant permeability for small increments or decrements of force superimposed upon any condition of magnetisation."

In a footnote he adds :—

> "This result is given by the late Professor STEFAN without proof, and is probably empirical."

This quotation states the problem clearly. The distinction here drawn or implied between the permeability of the iron to the circular field and the permeability of the iron to the transverse field is a valid one. It can hardly be maintained that the above conclusion, even if correct, could be arrived at otherwise than empirically. The problem is therefore one for experimental evidence. STEFAN'S results are stated to be given without proof; and whether DU BOIS' data are sufficient or insufficient to establish his conclusions will become evident as we proceed.

* *Electrician*, "Magnetic Shielding," vol. xl. p. 654 (1898).

§ 41. As already stated, the shielding effects plotted in figs. XX. and XXI. were obtained with the transverse field equal to 20·9 C.G.S. units. Table XV. supplies the data for shield A and Table XVI. for shield B, when the transverse field = 4·37, 20·9, 33·6 and 64·25 C.G.S. units. With the weakest transverse field, viz., $H_t = 4{\cdot}37$, it was found necessary to eliminate the disturbing effect of the leakage field $(H)_p$ within the shield, due to the overlapped joint, when circularly magnetised. By the rotation of the inductor, this field could be measured, the overlapped joint and the coil of the inductor being in the same plane. It was found to amount to nearly 1 C.G.S. unit when the induction in the iron was $B = 12{,}500$. By rotating the shield 90^c and making the final adjustments in azimuth by experiment, the effect of this leakage field when the inductor was rotated could be reduced to a minimum. This adjustment was made for both shields before the measurements tabulated in Tables XV. and XVI. were made. In addition to this, each set of measurements (H_c being increased from zero to a maximum) were repeated twice, with the circular magnetising force first in one direction and second in the opposite direction, the average readings being given in the second columns $(H)_w$.

§ 42. Table XIV. gives in full detail the various sequences of measurements adopted under the conditions of superposition of transverse and circular fields applicable to Tables XV. and XVI. The measurements under the TC and TCC conditions do not appear to call for any remark; but, under the CT and CTT conditions, attention may be called to the necessity of demagnetising the shields after the readings taken with each + or − increment of the circular field, in order that the superposition of the transverse field upon increments of H_c from zero (magnetisation) might be maintained throughout. The experimental data given in Table XV. (shield A) and Table XVI. (shield B) are plotted as shielding ratio curves (with the exception of the TCC curves) for the four selected values of the transverse field, viz., $H_c = 4{\cdot}37$, 20·9, 33· and 64·25 C.G.S. units.

In fig.	XXII.	for Shield A	under the TC conditions.
„ fig.	XXIII.	„ „ B	
In fig.	XXIV.	for Shield A	under the CT conditions.
„ fig.	XXV.	„ „ B	
In fig.	XXVI.	for Shield A	under the CTT conditions.
„ fig.	XXVII.	„ „ B	

Inspection of these curves shows that for each condition of superposition of fields, the differences which exist in their initial values, due to the transverse field alone, diminish usually at first and always finally as the circular magnetising force increases; and that the curves given in figs. XX. and XXI., when $H_c = 20{\cdot}9$, may be regarded as typical so long as the values of the transverse field are not too high. The *differences* which exist between the four types of curves corresponding to the TC, TCC, CT, and CTT conditions are sufficiently evident, and are a maximum when the values of dB/dH due to the circular force are a maximum for shields A and B (see fig. IV.). Thus the maximum shielding ratios when the circular field is simply superimposed upon the transverse (TC conditions) are more than double the shielding ratios which obtain when the transverse is superimposed upon the circular (TC conditions); and the maximum shielding ratios under the TCC conditions are as much as six or seven times as great as the shielding ratios which obtain for the same field values under the CTT conditions. Finally, in whatever way the two magnetic fields are superimposed the one upon the other, all the shielding ratio curves seem to approximate, as the circular magnetising force is further and further increased to the same minimum asymptotic value. Obviously the shielding ratio curves might be plotted for various fixed values of H_c against values of H_t as abscissæ. In figs. XXXVI. and XXXVII.* this has been done for shield A under two conditions of field superposition, viz., TC and CTT conditions. The curve where $H_c = 0$ is the same for both figs.

Co-ordination of Shielding Ratios with Magnetic Induction.

§ 43. The question may now be introduced whether the very large differences in the values of the shielding ratios, under the various conditions, can in any way be co-ordinated with the magnetic induction in the iron shields. Reference to fig. III. shows that when both the magnetising forces H_t and H_c act upon the iron, the induction will be a maximum on one side of the shield and a minimum on the other. In the figure, the left side, *m*, is that of maximum induction, and the right side, *n*, that of minimum induction.

Table XVIII. gives the inductions at positions *m* and *n* of the shields A and B when increments of the circular field are superimposed upon a pre-existing induction due to the transverse field (TC condition); distinct sets of experiments being made for two values of the latter, viz., $H_t = 4{\cdot}37$ (not tabulated) and $H_t = 20{\cdot}9$ C.G.S. units. In the third and sixth columns the measurements are in scale divisions of the ballistic galvanometer. In figs. XXVIII. and XXIX. the values of B at these positions of maximum and minimum inductions are plotted as ordinates against the corresponding values of H_c as abscissæ, for shields A and B respectively. The curves *m* and *n* are the induction curves at positions *m* and *n*, fig. III., when $H_t = 20{\cdot}9$; and the dash line curves the induction curves at same positions when $H_t = 4{\cdot}37$ C.G.S. units.

Table XVII., on the other hand, gives the inductions at positions *m* and *n* of the shields A and B, when upon a pre-existing induction due to the circular field between the limits of $H_c = 0$ and $H_c = 9$ C.G.S. units, the transverse field is superimposed and then repeatedly reversed. The first superposition of H_t corresponds to the CT conditions, the repeated reversals of H_t to the CTT conditions, separate sets of experiments being made for the two values of the transverse field given above. In figs. XXX. and XXXI.* the final values of B (and therefore the inductions under the CTT conditions) at the positions of maximum and minimum inductions are plotted against the corresponding values of the circular field for shields A and B respectively. As before, the *m* and *n* curves are the induction curves at positions *m* and *n*, fig. III.,

* In fig. XXXVII. "shield B" should read "shield A"; and in fig. XXXI. "shield A" should read "shield B." See Plate 5.

when $H_t = 20{\cdot}9$, and the dash lines the induction curves at same positions when $H_t = 4{\cdot}37$ C.G.S. units (not tabulated).

The addition of the normal B–H curves (see Table I. or fig. IV.) in dotted lines to these figures facilitates comparisons being made between the induction curves obtained under the TC conditions (fig. XXVIII., shield A, fig. XXIX., shield B) and those obtained under the CTT conditions (fig. XXX., shield A, fig. XXXI., shield B). The differences of the induction curves under these two conditions of superimposed fields are very apparent, and it is now proposed to determine what part of the induction is linked with the transverse field and what part is circular and wholly completed in the iron shields.

§ 44. Let the four diagrams of fig. XXXIV. represent in section one of the iron shields placed in a uniform transverse field (1) when H_c is zero, (2) when H_c produces zero magnetisation at position n, (3) when H_c just causes the induction at n to change sign, and (4) when H_c is increased sufficiently to produce wholly circular induction. Also let the following conventions hold, (a) that the transverse field, and consequently the shielded field, be considered positive, and (b) that the induction in the sides of the iron shield be considered positive if the magnetic lines are clockwise, and negative if anti-clockwise, relative to the circular section of the shield. Further, the total magnetic flux per cm. of length of the shield

$$\text{At position } m = dB_m$$
$$\text{At position } n = dB_n$$
$$\left.\begin{array}{c}\text{Across the}\\ \text{shielded space}\end{array}\right\} = 2\,(R-d)\,(H)_w$$

where d and R are the thickness and outside radius of either of the two shields. Then it follows that whether B_n be negative (1) or zero (2) or positive (3 and 4)

$$dB_m + 2(R-d)\,(H)_w - dB_n = N_t$$

where N_t is the total number of magnetic lined linked or continuous with the transverse field per cm. of length of the shield.

It is also evident that when the flux at n has changed sign (thus becoming positive) and is not less than the total flux across the shielded space (3 and 4),

$$dB_n - 2(R-d)\,(H)_w = N_c,$$

where N_c is the total number of circular magnetic lines wholly completed in the iron per cm. of length of the shield.

The values of $(H)_w$ given in Table XV. *for shield A* under the TC and CTT conditions, H_t being equal to $20{\cdot}9$, are now multiplied by $2(R-d)$, which for shield $A = 2{\cdot}82$ (see § 5), and the total magnetic flux across the shielded space thus obtained, plotted (dot and dash line curves) against the corresponding values of the circular field, to an ordinate scale on the right of figs. XXVIII. and XXX. respectively. In like manner, the values of $(H)_w$ given in Table XVI. *for shield B* under the TC and CTT conditions, H_t being equal to $20{\cdot}9$, are multiplied by $2(R-d)$, which for shield $B = 2{\cdot}84$ (see § 5), and the total magnetic flux across the shielded space thus obtained plotted (dot and dash line curves), to an ordinate scale on the right of figs XXIX. and XXXI. respectively. These scales having been made equal to the induction scales on the left, multiplied by d (for shield A, $d = \cdot 0233$, and for shield B, $d = \cdot 029$), it follows that all the curves of figs. XXVIII. to XXXI. measure total magnetic flux when referred to the ordinate scales on the right.

§ 45. Hence by means of the simple formula for N_c given immediately above may be derived from the full line curves of figs. XXVIII. and XXX. (or, better still, from the Tables XVIII. and XVII.), taken in conjunction with the dot and dash line curves of these figs., the full line curves TC and CTT of fig. XXXII. (shield A), which will measure the total number of circular magnetic lines wholly completed in and round the iron, under the TC and CTT conditions respectively. Also, in a similar way, by means of the formula for N_t may be derived the dash line curves TC and CTT of fig. XXXII. (shield A), which will measure the total number of magnetic lines which are continuous with those of the transverse field, under the TC and CTT conditions respectively. Fig. XXXIII. gives the corresponding curves for shield B, obtained in the same way as those of fig. XXXII. for shield A. In both cases the transverse field $H_t = 20 \cdot 9$ C.G.S. units. The ordinate scales on the right have been retained, but it is apparent that the curves can only correctly be referred to the flux scales on the left.

§ 46. It would therefore appear that under the CTT conditions the wholly circular induction is *greater*, and for low values of the circular force, very much greater, while concurrently with this the magnetic lines continuous with the transverse field are somewhat *less* than under the TC conditions. When the transverse field is equal to $4 \cdot 37$ C.G.S. units, the induction curves for both sides of the shields A and B (dash line curves of figs. XXX. and XXXI.) are, except near their origin, above the normal (dotted line) curve when $H_t = 0$. Reference to Table XVII. shows that this increase is brought about by the creeping up of the induction upon each reversal of the transverse field, superimposed upon a pre-existing induction to the circular field.

The effect is evidently one of molecular vibration. The transverse field superimposed in one direction upon the pre-existing circular induction due to H_c supplies the first molecular tap which increases the induction in one side of the shield more than it diminishes it on the other. When, however, H_t is repeatedly reversed, the increase in the one side of the shield finally becomes equal to the decrease in the other side, so that under these repeated molecular taps the induction becomes stable at considerably increased values (subject to the induction not being pushed to extreme values by either magnetising force). This result merely confirms that of other experimenters working with cross magnetising force (H_c and H_t act in the same plane). EWING,* summarising a paper by G. G. GEROSA and G. FINZI, writes : "When a rapidly alternating current of moderate strength traversed the wire, the susceptibility to longitudinal

* *Magnetic Inductions in Iron*, 3rd edition, p. 330.

magnetisation was notably increased; the magnetisation curve was found in that case to lie above the normal curve everywhere except in the region of strongest magnetisation."

§ 47. The difference between the dash line curves of figs. XXXII. and XXXIII. speaks to a decrease of the magnetic lines converging upon the iron (which as we have seen accompanies the increase of the circular induction) unber the CTT conditions, as compared with the somewhat greater number of magnetic lines converging upon the iron (which accompanies the decrease of the circular induction) under the TC conditions.

But we know that when a very long cylindrical rod of circular section is placed in a uniform transverse field, the induction B is given by

$$\frac{B}{H_t} = \frac{2\mu}{\mu+1}$$

so that when μ is large the induction in the rod approximates to twice the value of the undisturbed transverse field. Further, that if the long rod be a hollow cylinder, the number of lines of the transverse field converging upon the iron will be considerably less than the above ratio as the wall of the cylinder is reduced in thickness. Reference may be made to the lines of force diagram given by Du Bois in the articles already referred to.* But Table IV. (§ 13) shows that for both shields the lines of force converging upon the iron is practically the same as if the shields had been solid. This discrepancy between theory and experiment is fully accounted for by the fact that the conditions of great length relative to thickness are not fulfilled—the ratios of length to diameter for both shields being about 4·5 to 1.

In figs. XXXII. and XXXIII. the dotted horizontal lines measure the transverse field undisturbed by the presence of the iron shields, or $H_t = 20·9$ C.G.S. units. Comparison with the dash line curves shows that under the TC conditions the ratio which at its origin is approximately equal to two, shows a slight increase at low values of H_c, but that as the circular field is further increased the ratio tends to fall below two. Under the CTT conditions, on the other hand, there is no such initial rise, but the ratio falls slowly but continuously as H_c is increased.

These facts point to the conclusion that, although the dominating factor in determining the number of magnetic lines of the transverse field converging upon the iron is for large values of μ the geometrical factor, still the permeability of the iron to the transverse field is to a certain extent at least distinctly seen to respond to the permeability impressed upon the iron by the superposed circular field. On the other hand, the effect of repeated reversals of the transverse field superposed upon the circular is to abnormally increase the induction due to the latter (full line curves CTT), and concurrently with this to decrease somewhat the permeability of the shields to the transverse field (dash line curves CTT).

* *Electrician*, vol. xl. p. 513.

Conclusions under TC *conditions.*

§ 48. We may now proceed to discuss more fully the shielding ratio curves given on Plate IV.

It was found that under the TC conditions the shielding ratio curves coincide (except near their origin) with the theoretical curves $((g))$ (§ 39), and that when various values of the transverse field are used, the differences which exist in the initial values of the shielding ratio diminish finally as the circular field increases (§ 42).

Circles have been added to fig. XXII. (shield A) and fig. XXIII. (shield B) at various points corresponding to the theoretical shielding ratio $((g))$ curves of fig. IV. in order to show to what extent the theoretical and actual shielding ratios approximate the one to the other for various values of the transverse field. When $H_c = 0$, and the transverse field is the only magnetising force in the iron, the initial shielding ratios, minus unity, are proportional to the values of B/H (see § 11), but as the circular field is increased the quantities rapidly become proportional to the values of dB/dH where H is the circular magnetising force; always provided that the values of H_t are not pushed too high (see fig. XXII. (shield A), when $H_t = 64 \cdot 25$ and 80). As the values of the transverse field are reduced, this approximation takes place at lower values of H_c, and reference to the curves shows that when $H_t = 4 \cdot 37$, the lowest value experimented with, the theoretical and experimentally determined shielding ratios are, within the limits of error, identical for all values of H_c greater than 0·5 C.G.S. units.

But we have just seen that the permeability of the iron to the transverse field, under the same conditions, likewise responds to the superposed circular force H_c. Hence the following two effects are observed under the TC conditions of field superposition. First, the number of magnetic lines of the transverse field converging upon iron follows, although necessarily in a limited way as shown in § 46, the permeability of the iron impressed upon the shields by the superposed circular force H_c. Second, the experimentally determined ratios between the undisturbed transverse field and the weakened fields within the shields, diminished by unity, have been found to be proportional to the permeability, where μ is defined as the rate at which B_c is changing with respect to the superposed force H_c.

These two effects are co-related the one with the other under the given conditions. But the theoretical formula assumes (§ 3) the permeability of the iron to be uniform all round the shields, and the absence of retentivity and coercive force in the iron. Hence we may conclude that when values of the circular force increased from zero are superposed upon a pre-existing induction due to the transverse field, the theoretical conditions hold; and that the experimentally determined shielding ratios are not effected by the phenomena usually associated with hysteresis in iron.

The TC conditions may therefore fairly be regarded as the *normal conditions* of field superposition under which theoretical conditions are for all practical purposes fully satisfied.

The diagrams given under fig. XXXIV. are intended to represent graphically first the increase and then the decrease of the shielding ratio under the TC conditions. (1) shows the iron shield under the influence of the transverse field only; (2) the superposition of a sufficient circular force to produce zero magnetisation at n, with an increase of the shielding ratio; (3) a further increase of H_c and a further increase of the shielding ratio. In (4), however, a further increase of H_c lowers the shielding ratio. It is an interesting deduction from these lines of force diagrams that, when the flux at n has changed sign and is not less than the total flux across the shielded space, the magnetic lines of the transverse field cannot be represented as being continuous with the magnetic lines crossing the shielded space. See (3) and (4), also fig. XXXV. (2). These lines of force diagrams are limited to a representation of the changes which occur in the shell of, and the internal shielded space within, the shields. Owing to the few number of lines that can be used without a sacrifice of clearness, they do not show the variation of the number of magnetic lines converging upon the iron which we have seen follows upon the superposition of the circular magnetising force. Du Bois' lines of force diagrams (*loc. cit.*) may be referred to as showing the effect which various thicknesses of the cylindrical shells have upon the number of magnetic lines converging upon the iron.

We therefore conclude that when increments of the circular field are superposed upon a pre-existing induction due to the transverse field (TC conditions) the shielding ratio approximates nearer and nearer to

$$g' = \frac{1}{2}\,\frac{d}{R}\cdot\frac{dB_c}{dH_c} + 1 = ((g))$$

as the value of the transverse field is reduced. Thus the shielding ratio minus unity under the above conditions of field superposition is within the limits of experimental error proportional to the permeability impressed upon the iron by the superposed circular field; μ being defined as the differential permeability dB_c/dH_c and not the ratio permeability as was found to be the case when the transverse field alone is acting upon the iron (§ 11).

Conclusions under CTT *conditions.*

§ 49. Reference to figs. XX. and XXI. shows that the shielding ratio curves under the CTT conditions differ from those obtained under the TC conditions, and that when various values of the transverse field are used, figs. XXVI. and XXVII., the *differences* which exist in the initial values of the shielding ratios diminish at a much slower rate as H_c increases under the CTT as compared with the TC conditions. No apparent connection appears to exist between the shielding curves, which descend from their maximum values in the vertical axis as the circular magnetising force increases, and the values of dB/dH, which increase as the circular magnetisation creeps up during the repeated reversals of the transverse field. The difference in the number of magnetic lines converging upon the iron appears, however, to point to the conclusion that the permeability of the shields is somewhat decreased to the force which is not constant, *i.e.*, to the transverse field. We must, however, look for an explanation of the great diminution in the values of the shielding ratio to the effects following upon the differences of field superposition as compared with what we have called the normal or TC conditions. Under these conditions the pre-existing induction due to H_t is lightly held, the demagnetising factor being, as we have seen, very large. But under the CTT conditions now under consideration, the circular induction, being first in possession of and forming a closed magnetic circuit on the iron, is rigidly held.

The superposed repeated reversals of the transverse field have a double effect: (first) the circular magnetisation is increased as a result of molecular vibration (§ 46); and (second) the induction due to the transverse field, determined principally by the geometrical factor (§ 47), ousts from their position in the m side of the shield the circular magnetic lines, abnormally increased during reversals of H_t, and compels them to complete their circuit across the central shielded space. In other words, the shielding effect is reduced. Diagrams (1) and (2), fig. XXXV., are intended to graphically represent this result. (1) shows the shield circularly magnetised by the force H_c, the heavy circular line representing any given number of magnetic lines. The superposition of repeated reversals of the transverse field increases the circular induction (a vibration effect), and (2) shows that four of them are compelled to cross the shielded space, thus lowering the shielding ratio in comparison with diagram (1), fig. XXXIV., where the transverse field alone is acting, or in comparison with diagrams (2), (3) and (4), fig. XXXIV., where the transverse field is that first acting, and H_c the force superposed.

The conclusion therefore is that when repeated reversals of the transverse field are superposed upon a pre-existing induction due to the circular magnetising force (CTT conditions), the permeability of the iron is increased to the force which remains constant, viz., the circular field, and reduced to the force which is being reversed, viz., the transverse field. Hence the shielding ratio also is reduced, and does not bear any simple relationship to the values either of B/H or dB/dH due to the circular force; but is a complicated function of the magnetic properties of the iron, embracing permeability, the phenomena associated with hysteresis, and the effects of molecular vibration, all as determined by the particular mode of field superposition under the CTT conditions.

Conclusions under CT *conditions.*

§ 50. The shielding ratio curves obtained when the transverse field is simply superposed upon a pre-existing induction due to H_c (CT conditions), fig. XXIV. (shield A) and fig. XXV. (shield B), lie between those obtained under the TC and CTT conditions. They more nearly resemble the former, but approximate more nearly the latter in value. It is of special interest to note that as H_t is increased the maximum of each curve occurs nearer the vertical axis, thus approximating more nearly to the CTT curves, which have their maxima in the vertical axis. This effect is marked in the case of both shields. The superposition of the unidirectional transverse field is evidently the first step in the vibrational process (§ 46), which only becomes final after repeated reversals of H_t—CTT conditions. The stronger the first molecular tap (increased values of H_t) the nearer is the approach to the final conditions for the same values of the transverse field.

We may therefore conclude that the results arrived at in the immediately preceding paragraph in reference to the CTT conditions are to a certain extent applicable here, but that the effects usually associated with hysteresis, and with molecular vibration, are not so marked, where the transverse field is merely superposed, as to wipe out all apparent connection between the shielding ratios and the value of dB/dH due to the circular field.

Conclusions under TCC *conditions.*

§ 51. The TCC curves have not been plotted for the various values of H_t. Reference to Tables XV. and XVI. shows that when repeated reversals of the circular field are superposed upon a pre-existing induction due to the transverse field, the shielding ratio curves (TCC) given in figs. XX. and XXI., when $H_t = 20{\cdot}9$, are also typical for higher values of H_t. When it was found that repeated reversals of H_t increased the circular magnetisation and diminished the shielding ratio, it was obvious that repeated reversals of H_c would probably increase the permeability of the shield to the magnetising force which remained constant, *i.e.*, the transverse field, and that the shielding ratio would also increase. This deduction was confirmed by the above experimental results. In figs. XX. and XXI. the shielding ratio curves are, at the early stages of induction and when the values of dB_c/dH_c are a maximum, seen to be greatly increased above their normal and, as we have seen, their theoretical values under the TC conditions (§ 48). Repeated reversals of H_c therefore increase the permeability of the iron to the constant transverse force, and concurrently with this, the wholly circular induction was found to be slightly less that under the TC conditions (see figs. XXXII. and XXXIII.). On H_c being further increased the shielding ratio curves rapidly descend and coincide with the curves under the TC conditions, until they reach their minimum asymptotic values. There appears to be a slight tendency for these curves to cross, as is seen in fig. XX., but this difference is within the limits of experimental error.

(When $H_t = 4{\cdot}37$ the increase of the shielding ratio was not obtained (see Table XV.) for shield A, and only slightly obtained (see Table XVI.) for shield B. Owing to the low values of the shielded field measured in comparison with the higher value of the leakage field due to H_c, as also the smallness of the galvanometer deflections, it does not necessarily follow that the same increase in the shielding ratio would not have been obtained, if more perfectly homogeneous shields had been available.)

We may therefore conclude that repeated reversals of the circular field superposed upon a pre-existing induction due to the transverse field (TCC conditions) abnormally increases the shielding ratio, at the period of maximum permeability due to H_c, but that after this period is passed the shielding ratio, minus unity, rapidly becomes proportional to the values of dB_c/dH_c, as was found to be the case under the TC conditions (§ 48).

Conclusions under Cyclic conditions.

§ 52. Table XIX. supplies the data for both shields when, upon a pre-existing induction due to the transverse field of $H_t = 20{\cdot}9$ C.G.S. units, the superposed circular field carries the iron round a complete magnetic cycle. The circular field, after being repeatedly reversed, starts at a positive maximum, is reduced by steps to zero, increased by steps to a negative maximum, again reduced to zero, and finally increased to a positive maximum. At each step the shielded field is measured by means of the rotating inductor. The first columns give the average readings of H_c between positive and negative maxima, and between negative and positive maxima, the second and third columns the corresponding values of the weakened fields within the cylinders and the shielding ratios respectively. The values of the shielding ratios are plotted as dash line curves in fig. VII., and their close connection with the theoretical curves ((g)) (fig. VII.) obtained from the corresponding values of dB/dH (Table II.) impressed upon the iron by the circular field is at once apparent. The theoretical curves take very different values at their extreme positions, according as the B–H curve is descending from or ascending to the maximum values of B. This difference the experimentally determined ratio curves (g', fig. VII.) necessarily fail to show, and comparison may also be made between Tables XIX. and II. on this point. The theoretical values ((g)) given where B has ascended to a maximum correspond very closely with the experimentally determined ratios, being practically values obtained under the TCC conditions. The high maximum values of the shielding ratios obtained at the maximum values of dB_c/dH_c are very marked. For shield A the theoretical maximum value is 97, the experimentally determinal maximum value 500, when in both cases $H_c = -1{\cdot}2$ C.G.S. units. For shield B, the theoretical maximum value is 61, the experimentally determined maximum value 131, when in both cases $H_c = -2{\cdot}3$ C.G.S. units. These cycles in reality show the change of the shielding ratio that takes place as the circular field is varied between its extreme positions under the TCC conditions. We may therefore conclude that an alternating circular force superposed upon a pre-existing induction due to the transverse field, abnormally increases the permeability of the iron to the constant force H_t, so that the shielding ratio minus unity is greatly in excess even of the high values of dB_c/dH_c which occur during the superposed cyclic process ; but that, as H_c ascends to its positive or negative cyclic extremes, the shielding ratio approximates to the values which obtain under the TCC conditions.

I. (c) *Shielding Ratios with Transverse and Longitudinal Fields.*

§ 53. Table XX. supplies measurements for both shields under the four conditions of field superpositions, the value of the transverse field being $H_t = 20{\cdot}9$ C.G.S. units. The letters TL, TLL, LT, and LTT have the same signification as in § 38, L (longitudinal) taking the place of C (circular). The sequences observed in these measurements are also those which were observed in the measurements recorded in Tables XV. and XVI. (figs. XX. and XXI.) and detailed in Table XIV. By substituting L and H_l for C and H_c respectively, they are at once made fully applicable for the determinations given in Table XX. The curves are plotted as full lines in fig. XXXVIII. for shield A, and in fig. XXXIX. for shield B. They differ from fig. XX. and XXI. only in the fact that the longitudinal has replaced the circular magnetising force.

§ 54. It was observed that the origin of the new curves in the vertical axis was lower for both shields by at least unity, when compared with those previously discussed (figs. XX. and XXI.). The only known difference between the magnetic conditions of the shields appeared to be this, that in the case of the new curves the iron was systematically demagnetised by decreasing reversals of the longitudinal field, but in the case of the old curves by decreasing reversals of the circular field. It was found, as a matter of experiment, that the differences in the initial values of the shielding ratios depended on whether the shields had been demagnetised by the circular or by the longitudinal magnetising forces. This fact formed the starting point of experiments being made on the magnetic æolotropy of demagnetised iron, which have been incorporated in this paper, together with others dealing with superposed magnetic inductions in iron, §§ 14 to 37, to which the reader is referred.

§ 55. Comparison of figs. XXVIII. and XXIX. with figs. XX. and XXI. at once shows that the same general relationship exists *inter se* between the four curves resulting from the four methods of field superposition in both cases. Also that the maximum value for shield A of, say, the TL curve is at least approximately the same as the maximum value of the TC curve (fig. XX.); but that the maximum value for shield B of the TL curve is distinctly less than the maximum value of the TC curve (fig. XIX.). When shield B, however, was demagnetised by decreasing reversals of the *circular field*, not only was the initial value of the shielding ratio raised, but the new curve assumed higher values as it receded from the vertical axis. This is shown by the dot and dash curve TL, fig. XXIX., plotted from the last three columns of Table XXI., and its maximum value is seen to be practically the same as the TC curve of fig. XXI. Experiments were not made with shield A under the same conditions of demagnetisation. Table XXI., however, supplies for shield A the shielding ratios obtained under the TL conditions for additional values of the transverse field, viz., $H_t = 33{\cdot}6$ and $64{\cdot}25$ C.G.S units. The former practically coincides with that already plotted when $H_t = 20{\cdot}9$ units. The latter, however, is plotted in fig. XXVIII., which shows distinctly lower values of the shielding ratio. Compare this with the curve obtained under the same conditions, but with the circular field, H_t being also equal to 64·25 units. The difference is marked.

§ 56. These curves now fall to be compared with the theoretical shielding ratios. The calculated value of H_l being that due to the solenoid, and not the true magnetising force in the iron, it was necessary to measure the longitudinal induction in the shields by means of the exploring coil (§ 5). As mentioned at the end of § 15, the induction measured in this way required correction. This was done graphically by means of data determined experimentally. By taking a sufficient number of points on the induction curves the same values of induction in the B–H curves of fig. IV. furnished corresponding values of the theoretical ratios $((g))$ and (g) from the dotted and dash curves respectively. These values have been transferred to fig. XXXVIII. (shield A). The circles are the values obtained for the $((g))$ ratios, and the crosses those obtained for the (g) ratios. It is at once apparent that no simple relationship exists between the permeability (either $d\mathrm{B}/d\mathrm{B}$ or B/H) of the iron due to the longitudinal magnetising force and the experimentally determined shielding ratios. The same remarks apply with equal force to shield B, where the same general features hold. All the curves given for shield A are typical of shield B.

SUMMARY OF CONCLUSIONS.

I. *Magnetic Shielding in Hollow Iron Cylinders.*

(*a*) When no other magnetising force is acting upon the iron than that due to the transverse field increased by increments from zero, the theoretical formula for the shielding ratio (g) given in § 8 is very approximately fulfilled within the limits of the transverse field used in the experimental determinations, viz., between $H_t = 4{\cdot}4$ and 130 C.G.S. units.

The shielding ratio minus unity is proportional to what may be called the ratio permeability B/H, and not to the differential permeability $d\mathrm{B}/d\mathrm{H}$.

If, however, the transverse field is decreased from a maximum, theoretical formulæ are not applicable, the conditions as to absence of retentivity and coercive force not being fulfilled.

(*b*) When a circular magnetising force is acting upon the iron in addition to that due to the transverse field, the order and manner in which the one field is superposed upon the other affects the shielding ratio to an enormous extent.

First.—When upon a pre-existing induction due to the transverse field increments of the circular force ascending from zero to a maximum are superposed, the theoretical formula for the shielding ratio $((g))$ given in § 8 is fulfilled if the value of H_t be not unduly increased. The shielding ratio minus unity is proportional to the permeability impressed upon the iron by the circular field; μ being defined as the differential permeability $d\mathrm{B}_c/d\mathrm{H}_c$, and not the ratio permeability as was found to be the case where the transverse field alone is acting upon the iron.

Second.—Repeated reversals of the circular force superposed upon a pre-existing induction due to the transverse field abnormally increase the shielding ratio at the period of maximum permeability due to the circular force; but after this period is passed the shielding ratio minus unity rapidly becomes proportional to the values of dB_c/dH_c as was found to be the case under the "First" conditions.

Third.—When upon a pre-existing induction due to the circular magnetising force ascending from zero to a maximum, is superposed at each increment a given value of the transverse field, the shielding ratio curves, although showing a well-marked initial rise, assume throughout much lower values, and the theoretical formulæ given in § 8 are not applicable. The permeability of the iron is increased to the circular, and lowered to the transverse force, with this exception, that at low values of the former the permeability of the iron to the transverse field is slightly increased. The superposition of the transverse field appears to be the first step in a vibrational process, which only becomes final under the "Fourth" conditions of field superposition, but which in itself is not sufficient to wipe out all apparent connection between the shielding ratios and the values of dB/dH, due to the circular field. It may be noted that as the transverse field is increased, the maximum of each curve occurs nearer the vertical axis.

Fourth.—When repeated reversals of the transverse field are superposed upon a pre-existing induction, due to the circular force ascending from zero to a maximum, no apparent connection appears to exist between the shielding ratio which descends from their maximum values in the vertical axis, and the theoretical formulæ of § 8. The permeability of the iron is increased to the force which remains constant, viz., the circular field, and reduced to the force which is being reversed, viz., the transverse field. Hence the shielding ratio is also reduced, and does not bear any simple relationship to the values either of B/H or dB/dH, due to the circular field; but is a complicated function of the magnetic properties of the iron, embracing permeability, the phenomena associated with hysteresis and the effects of molecular vibration, all as determined by the particular method of field superposition.

Fifth.—When upon a pre-existing induction due to the transverse field the superposed circular force carries the iron in steps round a complete magnetic cycle, the force which is varied increases the permeability of the iron to the constant transverse field, and the maximum shielding ratios are abnormally increased in comparison with the theoretical values obtained from even the high values of dB_c/dH_c, which prevail during the superposed cycle. Although exact numerical correspondence with the theoretical formula ((g)) given in § 8 could not, under the conditions as to hysteresis and molecular vibrational effects, be anticipated, the connection between the experimental shielding ratio curves and the changing values of dB_c/dH_c impressed upon the iron by the cyclic circular field is yet sufficiently striking.

(c) When a longitudinal magnetising force is acting upon the iron in addition to that due to the transverse field, the order and manner of field superposition affects the shielding ratio to an enormous extent, and the same general relationship exists as was the case with the transverse and circular fields. There was in no case, however, the same numerical correspondence with the theoretical formulæ.

II.—*Superposed Magnetic Inductions in Iron.*

Of the two magnetising forces *at right angles* to each other, let H_1 be the force first acting, H_2 the force superposed. Each force acting alone produces the normal B–H induction curve. Also let B_1 and B_2 be the two components of the resultant induction in the direction of H_1 and H_2 respectively.

First.—When upon a pre-existing induction due to H_1 increments of H_2 ascending from zero to a maximum are superposed, the B_1 component of the resultant induction always lies above the B_2 component. For low fields the B_1 curve is above the normal induction curve; but as the fields are increased a point is reached where the curves cross, the B_1 component then falling below the normal curve. The first increase is a relatively large effect, and appears to be due to molecular disturbance caused by the superposition of H_2 increasing the permeability of the iron to the constant force H_1; the final decrease to lowered permeability.

Second.—When repeated reversals of H_2 are superposed upon a pre-existing induction due to H_1, the B_1 component lies still further above the B_2 component. For low fields the B_1 curve is also further increased above the normal curve; but as the fields are increased, a point is reached where the curves cross, the B_1 component then falling below the normal induction curve. The increase is the well recognised effect due to molecular vibration; the final decrease to lowered permeability. Within due limits if the effect of the first superposition of H_2 is large, repeated reversals have relatively less effect.

Third.—When upon a pre-existing induction due to H_1 increments of H_2 ascending from zero to a maximum are superposed, the B_2 component of the resultant induction always lies below the B_1 component—an effect of hysteresis. For low fields the B_2 curve lies above the normal induction curve, but as the fields are increased a point is reached where the curves cross, the B_2 component then falling below the normal curve. The first increase is relatively a small effect, and appears to be caused by the two fields acting together hastening the second or steep portion of the normal curve for both components. For any two values of H_1 (including $H_1=0$) the B_2 curves cross, the curve for the lower value of H_1 being at first below and afterwards above that for the higher value of H_1. The decrease of B_2 is a large effect and has been repeatedly observed.

Fourth.—When repeated reversals of H_2 are superposed upon a pre-existing induction due to H_1, the B_2 component lies still further below the B_1 component. The B_2 curve throughout appears to lie below the normal induction curve. If there exists an initial rise, it must occur at lower minimum values than those used in these experiments and close to the vertical axis.

Fifth.—When H_2 carries the B_2 induction component in steps round a series of complete cycles, the B_1 component due to H_1 kept at a constant value responds and passes through a series of changes in the process of likewise becoming strictly cyclic. The changes which take place during this preliminary transition period depend upon the previous magnetic history of the iron and the well known molecular vibrational effect due to the superposed reversals. If the amplitude of the H_2 reversals are below the critical value required to practically wipe out the effects of hysteresis due to H_1 no further apparent change is produced after the first superposition (a relatively large effect) followed by a few reversals of H_2. The values of B_1 at the extremes of the superposed reversals having become final, the molecular 'accommodation' which still goes on is cyclic.

The B_1 component due to H_1 kept at a constant value follows at least approximately the changing permeability impressed upon the iron by the superposed alternating force H_2. The minimum values of B_1 occur at the extremes of the H_2 reversals, and the maximum values of B_1 when B_2 is changing most rapidly with respect to H_2 or when dB_2/dH_2 is a maximum. The maximum and minimum values of the B_1 induction component thus occur twice during one complete reversal of H_2; in fact, the cyclic curve closely resembles the curve which represents the changes produced in the magnetisation of an iron wire when repeatedly twisted first in one direction and then in the opposite direction.

The B_2 induction component at the extremes of the H_2 reversals, and the maximum values of the permeability (dB_2/dH_2) which occur during these reversals, are both somewhat reduced in comparison with the normal B–H complete cycle when H_1 is not acting. Also, if the values of H_1 be unduly increased, a lop-sidedness of the superposed B_2–H_2 cycle results.

III.—*Magnetic Æolotropy of Demagnetised Iron.*

During the early stages of induction, iron is more permeable to a reapplication of a magnetising force in the same direction (positive or negative), as that used in the immediately preceding process of demagnetising by decreasing reversals, than it is to a force (positive or negative) at right angles to that used in the immediately preceding demagnetising process. For the two samples of iron used this difference is of the order of 30 per cent., but vanishes as the magnetising force is increased. By assuming that the deflecting force H is proportional to the cosine of the angle which H makes with the magnetic axis of any stable molecular group, a possible explanation of the general nature of the effect is afforded.

Iron left with residual polarity due to a force at right angles to that subsequently applied is *more permeable* during the early stages of induction, and iron demagnetised by a force at right angles to that subsequently applied is *less permeable* during the early stages of induction, than iron magnetised under normal conditions. This comparison has an obvious bearing on the molecular condition of demagnetised iron. The subject is being continued.

TABLE I.—*Magnetic Induction.—Permeability.—Theoretical Shielding Ratio.* (See Fig. IV.)

SHIELD A.								SHIELD B.							
Increments from Zero.		Derived from B-H curves, fig. IV.						Increments from Zero.		Derived from the B-H curves, fig. IV.					
H_c	B	H_c	B	$\frac{B}{H}$	(g)	$\frac{dB}{dH}$	$((g))$	H_c	B	H_c	B	$\frac{B}{H}$	(g)	$\frac{dB}{dH}$	$((g))$
·44	295	·6	500	800	7·5	1000	9	·94	380	1	400	400	5	830	9·3
·75	715	·75	750	1000	9·0	2000	17	1·45	832	2	1,750	875	9·8	1800	19
·97	1,305	1	1,300	1300	11·4	3000	25	2·06	1,840	2·5	2,700	1080	11·8	2000	21
1·7	4,050	1·3	3,000	2300	19·4	4500	37	2·33	2,360	3	3,700	1233	13·3	1660	17·7
3·0	7,200	2	5,225	2625	22·0	3000	25	2·77	3,296	4	5,400	1375	14·7	1330	14·3
4·15	8,900	3	7,200	2400	20·2			3·08	3,920	5	6,700	1340	14·4	1150	12·5
6·6	11,000	4	8,700	2175	18·4	1250	11	3·80	5,064	6	7,600	1260	13·6	850	9·5
9·3	12,600	6	10,600	1770	15·2	750	7·0	4·35	5,860	7	8,400	1200	13·0	700	8·0
13·2	13,600	9	12,400	1400	12·2	430	4·4	5·73	7,350	8	9,000	1125	12·2	600	7·0
15·7	14,100	12	13,300	1100	9·8	250	3·0	7·42	8,600	10	10,000	1000	11·0	470	5·7
18·3	14,300	15	14,000	933	8·5	160	2·3	9·4	9,745	15	11,800	787	8·9	280	3·8
23·0	14,650	20	14,500	725	6·8	77	1·6	15·2	11,900	20	13,000	650	7·5	200	3·0
49·9	15,700	25	14,750	588	5·7			21·2	13,300	25	13,900	556	6·5		
		30	15,000	500	5·0	50	1·4	29·7	14,500	30	14,500	483	5·8	110	2·1
		35	15,200	434	4·5			36·3	15,150	35	15,000	428	5·3		

TABLE II.—*Hysteresis Loop.—Theoretical Shielding Ratio.* (See Fig. VII.)

SHIELD A.				SHIELD B.			
Cyclic.		From B-H curve of fig. VII.		Cyclic.		From B-H curve of fig. VII.	
H_c	B	$\frac{dB}{dH}$	$((g))$	H_c	B	$\frac{dB}{dH}$	$((g))$
5	9600	125	2	9·4	9750	180	2·8
2·24	9020	500	5	5·72	9060	250	3·5
·66	7650	200	10·6	3·0	8300	470	5·7
·0	6920	636	14·3	1·7	7560	666	7·7
− ·38	6000			0	6180	1000	11
− ·66	5200	000	33	−1·04	4880	1600	17
− ·83	4430	7,000	57	−1·7	3420	3000	31
−1·2		12,000	97	−2·0	1960	5500	56
−1·37	−1520	9,000	73	−2·3		6000	61
−2·24	−5700	2,666	22·3	−2·54	− 960	5000	51
−2·92	−7080	1,666	14·3	−3·0	−2800	3000	31
−4·03	−8600	1,250	11	−4·23	−5450	1500	16
−5	−9600	1,000	9	−5·85	−7400	1000	11
				−8·15	−9200	600	7
				−9·4	−9750	500	6

TABLE III.—*Magnetic Induction.—Theoretical Shielding Ratio.* (See Fig. V.)

SHIELD A.				SHIELD B.			
B determined by reversals of H_t.		Derived from fig. IV.		B determined by reversals of H_t.		Derived from fig. IV.	
H_t	B	(g)	$((g))$	H_t	B	(g)	$((g))$
9·5	1,225	11	22	7·4	720	6	12·5
17·6	2,280	17	37	15·9	1,600	9	18
26·0	3,420	21	33	27	2,800	12	21
30	3,880	22·3	31	39	4,040	14	16·9
41	5,380	21·6	24	51·5	5,350	14·7	14·2
53·3	6,930	20·3	15·3	65	6,700	14·3	12
64·6	8,230	19	12	75·8	7,700	13·5	9·6
76	9,540	17	9	90	9,120	12	7
90	11,220	14·1	6·1	102	10,250	11	5·3
102	12,600	12·1	4·1	116	11,500	9	4·2
117	14,100	8	2	130	12,600	8	3·1
133	15,250	4·1	1·2				

TABLE IV.

Shield A.		Shield B.	
H_t	$\frac{H_t'}{H_t}$	H_t	$\frac{H_t'}{H_t}$
21	2·1	21	2·0
33·6	2·2	33·5	2·0
52·3	2·1	53·3	2·1
75·0	2·1	77·6	2·0
100	2·0	103	2·0
130	2·0	134	2·0

TABLE V.—*Shielding Ratios.—Transverse Field.* (See Fig. VI. and Fig. X.)

Shield A.			Shield B.		
H_t	$(H)_w$	g'	H_t	$(H)_w$	g'
4·4	·73	6	4·4	·98	4·5
21	1·40	15	21	2·38	8·8
33·6	1·71	19·6	33·5	2·96	11·3
52·3	2·32	22·5	53·3	3·94	13·5
64·2	3·05	21·0	64·0	4·38	14·6
75·0	3·93	19·1	77·6	5·61	13·8
89	5·21	17·1	91·7	7·10	12·9
100	6·43	15·5	103	8·52	12·1
115	8·95	12·8	117·5	10·70	11·0
130	14·45	9·0	134	14·25	9·4
H_t repeatedly reversed at each step.					

TABLE VI.—*Shielding Ratios.—Transverse Field.—Cycle.*

Shield A. (See Fig. VIII.)					Shield B. (See Fig. IX.)				
H_t	$(H)_w$	$(H)_r$	$(H)_w-(H)_r$	g''	H_t	$(H)_w$	$(H)_r$	$(H)_w-(H)_r$	g''
78·4	4·23	−1·11	5·34	14·6	78·5	5·7	−1·95	7·65	10·3
64·6	·863	−1·06	1·92	33·6	57·0	·895	−1·82	2·78	20·5
50·2	− ·343	−1·14	·80	63·0	39·4	− ·616	−1·91	1·30	30·3
35·8	− ·814	−1·17	·36	99·5	20·2	−1·95	−1·95	0	Infinite
23·1	−1·11	−1·11	0	Infinite	8·6	−2·13	−2·12	− ·01	,,
10·8	−1·27	−1·24	− ·03	,,	0				,,
0	−1·24	−1·24	0	,,	− 9	−2·39	− ·817	−1·57	5·7
−10·8	−1·32	·13	−1·45	7·5	−20	−2·65	·123	−2·77	7·2
−23·3	−1·48	·521	−2·00	11·6	−39·8	−3·40	1·16	−4·56	8·7
−36·0	−1·76	·814	−2·57	14·0	−58·2	−4·24	1·58	−5·82	10·0
−50·3	−2·28	·960	−3·24	15·5	−78	−5·55	2·08	−7·63	10·2
−64·6	−3·08	1·11	−4·19	15·4					
−78·4	−4·23	1·11	−5·34	14·6					

Sequence observed in above determinations.

H_t	$(H)_w$	$(H)_t$	$(H)_r$	H_t
Maximum value after repeated reversals, ...	Reading taken.	Put off.	Reading taken.	Repeatedly reversed at maximum value.
Reduced by first step, . . .	do.	do.	do.	do.
,, ,, and second step, .	do.	do.	do.	do.
,, ,, second and third step, and so on, until zero is reached, then increased by steps in same manner.	do.	do.	do.	do.

TABLE VII.—*Superposed Magnetic Inductions, LC and LCC Conditions.* SHIELD B.

				H_c superposed.			H_c repeatedly reversed.							
		H_c	BG	BG	Total.	B	BG						Total.	B
$H_t = 3$ C.G.S. See fig. XI.	B_c measured.	1·6	0	2·9		1,130	− 5·9	+ 5·7	− 5	+ 5·2	− 5		2·5	980
		3·3	0	10·3		4,030	− 20·8	+ 21·0	− 20·8	+ 20·8			10·4	4,070
		7·1	0	21·2		8,300	− 42·2	+ 42·0	− 42·2	+ 42·0			21·1	8,240
		9·8	0	24·7		9,650	− 49·8	+ 49·7	− 50·0	+ 50·0			25·0	9,760
		12·7	0	28·0		10,950	− 56·2	+ 55·0	− 55·5	+ 55·0			27·5	10,730
	B_t measured.	1·4	1·7	·2	1·9	900	·4	0	·1	0	0		2·4	1,130
		3·3	1·7	1·0	2·7	1,270	0	0					2·7	1,270
		7·2	1·7	1·1	2·8	1,320	·7	0	0				3·5	1,650
		9·9	1·7	·8	2·5	1,180	·2	0	0				2·7	1,270
		12·9	1·7	·9	2·6	1,230	0	·1	0	0			2·7	1,270
$H_t = 11·9$ C.G.S.	B_c measured.	1·7	0	3·7		1,250	− 5·9	+ 5·0	− 4·8	+ 4·7	− 4·6	+ 4·3	2·2	860
		3·3	0	10·4		4,070	− 19·9	+ 19·0	− 18·5	+ 18·8	− 18·4	+ 18·4	9·2	3,600
		7·1	0	20·2		7,900	− 40·6	+ 40·6	− 40·0	+ 40·0	− 39·9	+ 40·0	20·0	7,820
		9·9	0	24·2		9,450	− 48·8	?	?	+ 48·3	− 48·5		24·2	9,450
		12·9	0	27·4		10,700	− 55·5	+ 54·5	?	+ 54·8	− 54·4		27·3	10,660
	B_t measured.	1·7	8·8	·6	9·4	4,440	·3	0					9·7	4,580
		3·3	8·8	·7	9·5	4,480	·5	0					10·0	4,720
		7·2	8·8	·8	9·6	4,530	·6	0					10·2	4,800
		10·3	8·8	·3	9·1	4,300	·7	·1	0				9·9	4,670
		13·3	8·8	0	8·8	4,150	1·1	− 1	0	0			8·9	4,200
$H_t = 21·5$ C.G.S. See fig. XII.	B_c measured.	1·3	0	2·2		860	− 3·5	+ 3·0	− 2·2	+ 2·1			1·1	430
		2·6	0	6·2		2,420	− 11·2	+ 10·0	− 10·0	+ 9·7	− 9·8		4·9	1,920
		5·6	0	15·8		6,180	− 30·8	+ 30·1	− 30·9	+ 30·1			15·0	5,860
		7·8	0	20·3		7,940	− 40·0	+ 39·3	− 39·0	+ 39			19·5	7,620
		10·4	0	23·8		9,300	− 46·6	+ 46·6	− 46·5	+ 46·5			23·2	9,060
	B_t measured.	1·7	16·5	·2	16·7	7,900	·3	0	0				17·0	8,020
		3·3	16·5	·5	17·0	8,020	·7	0	0				17·7	8,350
		7·2	16·5	0	16·5	7,800	·7	0	0				17·2	8,100
		10·0	16·5	0	16·5	7,800	·2	0	0				16·7	7,800
		13·0	16·5	·8	15·7	7,400	0	0	0				15·7	7,400
$H_t = 39·6$ C.G.S. See fig. XIII.	B_c measured.	1·0	0	1·1		430	− 2·0	+ 1·7	− 1·7	+ 1·7			·85	330
		3·3	0	6·2		2,420	− 12·0	+ 11·2	− 10·7	+ 10·9			5·4	2,110
		5·8	0	12·5		4,900	− 24·8	+ 24·0	− 23·9	+ 23·8			11·9	4,650
		8·5	0	17·5		6,830	− 34·4	+ 34·0	− 33·9	+ 33·8			16·9	6,600
		10·0	0	20·1		7,850	− 39·7	+ 34·8	− 38·8	+ 38·7			19·4	7,580
	B_t measured.	1·3	28	0	28	13,200	0	0	0				28	13,200
		2·6	28	·2(?)	28·2	13,300	0	0	0				28·2	13,300
		5·7	28	·2	28·2	13,300	·6	0	0				28·8	13,600
		7·8	28	− ·2	27·8	13,100	1·0	− ·9	·2				28·1	13,250
		10·3	28	− ·8	27·2	12,800	1·2	− 1·2	0	0	0		27·2	12 800

TABLE VIII.—*Superposed Magnetic Inductions CL and CLL Conditions.* SHIELD B.

				H_l superposed.			H_l repeatedly reversed.		
		H_c	BG	BG	Total.	B	BG	Total.	B
$H_l = 3$ C.G.S. See fig. XI.	B_c measured.	·96	·9	·6	1·5	580	·7 0 0 0	2·2	850
		1·6	2·4	1·0	3·4	1,320	1·3 ·4 ·3 ·3 ·3 ·2 0	6·2	2,410
		2·2	5·1	1·3	6·4	2,490	1·9 ·7 ·6 0	9·6	3,720
		4·8	16·2	·8	17·0	6,600	1·0 ·4 ·4 ·3 ·1	19·2	7,450
		6·5	20·2	·8	20·8	8,100	1·0 ·3 0 0	22·1	8,600
		9·0	23·6	·6	24·2	9,500	·6 ·2 0 0	25·0	9,700
	B_l measured.	0	0	1·7		800	−3·1 +3·1 −3·1 +3·1	1·55	730
		1·7	0	1·8		850	−3·8 +3·2 −3·1 +3·1	1·55	730
		3·5	0	1·9		890	−3·4 +3·1 −3·1 +3·0	1·5	700
		5·25	0	1·75		820	−3·5 +3·0 −3·0 +2·9	1·5	700
		13·2	0	1·7		800	−3·1 +3·0 −2·8 +3·0	1·45	680
$H_l = 11·9$ C.G.S.	B_c measured.	1·1	1·2	2·3	3·5	1,360	2·1 ·8 ·7 0 ·3 0 ·2 0 ·1	9·7	3,000
		1·6	2·2	3·2	5·4	2,100	2·9 ·7 ·6 0 ·3 0 0	9·9	3,850
		2·2	5·1	3·7	8·8	3,420	2·2 ·6 ·4 0 0	12·0	4,650
		4·9	16·3	2·2	18·5	7,700	1·8 ·6 ·2 0 0	21·1	8,200
		8·7	24·0	1·1	25·1	9,750	1·0 0 0	26·1	10,150
		10·8	27·0	1·0	28·0	10,900	·7 0 0	28·7	11,130
		13·5	29·2	·9	30·1	11,700	·7 0 0	30·7	11,900
	B_l measured.	0	0	8·8		4,130	−17 ? +17·2 −17·3 +17·5	8·7	4,080
		1·7	0	8·9		4,180	−17·2 +17·0 −17·0 +17·1	8·5	4,000
		2·5	0	8·8		4,130	−17·2 +17·0 −17·0 +16·9	8·4	3,940
		5·3	0	8·2		3,850	−16·3 +16·0 −15·9 +16·1	8·0	3,760
		9·3	0	7·8		3,680	−16·0 +15·4 −15·4	7·7	3,620
		13·0	0	7·5		3,520	−15·0 +14·8 −14·7 +14·5 −14·6	7·3	3,430
$H_l = 21·5$ C.G.S. See fig. XII.	B_c measured.	·87	1·0	2·6	3·6	1,400	2·3 0 ·5 0 ·5 0 ·5 0	7·4	2,880
		1·4	1·8	3·5	5·3	2,060	3·0 0 ·9 0 ·6 −·3 ·1	9·6	3,730
		2·3	5·4	4·2	9·6	3,780	3·0 ·1 ·5 0 ·4 0 ·4 0 ·2	14·2	5,500
		3·8	12·3	2·5	14·6	5,750	2·0 0 ·2 0 ·2 0 ·2 0 ·2	17·6	6,850
		5·9	18·2	1·3	19·5	7,580	1·1 ·2 ·2 0 0	21·0	8,150
		7·6	22·2	1·1	23·3	9,050	·9 0 0 0 0	24·2	9,500
		10·5	26·3	·8	27·1	10,520	·7 0 0	27·8	10,800
		13·8	29·8	·2	30·0	11,680	·5 0 0	30·5	11,850
	B_l measured.	0	0	16·5		7,750	−33 +32·4 −32·4 +32·5	16·2	7,600
		·7	0	16·1		7,550	−32·4 +32·6 −32·0 +32·2	16·1	7,550
		1·75	0	16·0		7,500	−32·5 +32·3 −31·9 +32·0	16·0	7,500
		3·4	0	16·0		7,500	−31·8 +31·3 −31·2 +31·3	15·6	7,330
		5·5	0	15·7		7,360	−31·0 +30·8 −30·7 +30·8	15·4	7,150
		8·5	0	15·0		7,050	−30·0 +29·9 −29·8 +29·8	14·9	7,000
		10·5	0	14·2		6,660	−29·0 +28·9 −28·8 +28·8	14·4	6,760
		13·6	0	13·9		6,530	−28·2 +27·8 −27·7 +27·7	13·8	6,500
$H_l = 39·6$ C.G.S. See fig. XIII.	B_c measured.	1·1	1·1	2·0	3·1	1,210	1·7 ·6 ·6 0 0	6·0	2,330
		1·6	2·4	2·8	5·2	2,300	1·9 0 ·3 −·2 ·4 −·3 ·4	7·7	3,000
		2·2	4·9	2·4	7·3	2,840	2·0 ·3 ·2 −·2 ·5 0	10·1	3,930
		4·8	16·0	−1·1	14·9	5,800	1·1 0 0 0	16·0	6,220
		6·2	19·8	−1·8	18·0	7,000	·9 0 0 0	18·9	7,350
		8·8	24·2	−2·6	21·6	8,400	·8 0 0 0	22·4	8,700
		10·9	27·2	−3·0	24·2	9,500	·6 0 0 0	24·8	9,630
	B_l measured.	0	0	27·8		13,100	− ? +55·6 −55·3 +55·6	27·7	13,000
		1·3	0	27·8		13,100	−55 +55·2 −54·9 +55·0	27·5	12,900
		2·7	0	27·2		12,800	−54·7 +54·8 −54·4 +54·6	27·2	12,800
		5·7	0	26·5		12,500	−53·0 +53·0 −52·8 ? +53·0	26·5	12,500
		8·0	0	25·9		12,150	−52·0 +51·8 −51·2 +51·6	25·7	12,100
		10·4	0	24·8		11,600	−50·2 +50·3 −50 +50·0	25·0	11,700

TABLE IX.—*Superposed Magnetic Inductions. LC Conditions.* SHIELD A. (See Fig. XIV.)

	H_t	BG	BG	B_c
$H_c = \cdot 77$ C.G.S.	0	0	1·5	700
	6·5	0	2·1	965
	16·4	0		900
	41·3	0	1·1	506
$H_c = 1\cdot 3$ C.G.S.	0	0	5·4	2480
	6·4	0	5·8	2670
	16·4	0	4·7	2160
	40·4	0	2·8	1290
$H_c = 2\cdot 2$ C.G.S.	0	0	11·9	5470
	6·4	0	11·9	5470
	16·3	0	9·9	4550
	40·4	0	5·2	2390
$H_c = \cdot 48$ C.G.S.	0	0	·7	300
	6·5	0	1·0	460
	16·4	0	1·2	550
	40·9	0	·8	370

Iron demagnetised after each superposition of H_c.

TABLE X.—*Superposed Magnetic Induction.—Cyclic.* SHIELD B. (See Fig. XVII.)

$H_c = -2\cdot 0$ reduced from a maximum of $+11\cdot 8$ C.G.S. H_t superposed.

H_t	BG	Totals.	B_c	H_t	BG	Totals.	B_c
0	4·65	4·65	+1760	−30·7	+1·7	−11·05	−4180
− 3·4	−2·05	2·60	+ 985	−24·0	−0·65	−11·70	−4430
− 9·1	−3·05	− 0·45	− 170	− 9·1	−2·45	−14·15	−5360
−13·9	−2·00	− 2·45	− 930	0	−0·95	−15·10	−5720
−24·0	−2·05	− 4·50	−1700	9·1	−0·55	−15·65	−5930
−30·7	−0·50	− 5·00	−1900	30·7	+3·30	−12·35	−4680
−24·0	−0·65	− 5·65	−2140	0	−2·70	−15·05	−5700
−13·9	−2·00	− 7·65	−2900	− 3·4	−0·30	−15·35	−5820
− 9·1	−1·60	− 9·25	−3500	− 5·8	−0·10	−15·45	−5850
− 3·4	−1·35	−10·60	−4020	− 9·1	0	−15·45	−5850
0	−0·80	−11·40	−4320	−13·9	+0·40	−15·05	−5700
3·4	−0·80	−12·20	−4630	−30·7	+3·00	−12·05	−4570
5·8	−0·55	−12·75	−4830	− 9·1	−3·10	−15·15	−5740
9·1	−0·45	−13·20	−5000	0	−0·90	−16·05	−6080
11·6	0	−13·20	−5000	5·8	−0·40	−14·45	−6230
13·9	−0·10	−13·30	−5040	13·9	+0·40	−16·05	−6080
24·0	+1·00	−12·30	−4660	30·7	+3·10	−12·95	−4900
30·7	+1·10	−11·20	−4250	13·9	−2·00	−14·95	−5660
13·9	−1·90	−13·10	−4960	0	−1·10	−16·05	−6080
3·4	−1·45	−14·55	−5520	− 9·1	−0·40	−16·45	−6230
0	−0·30	−14·85	−5630	−30·7	+3·50	−12·95	−4900
− 9·1	0	−14·85	−5630	0	−2·90	−15·85	−6000
−24·0	+2·10	−12·75	−4830				

Table XI.—*Superposed Magnetic Induction.—Cyclic.* Shield B. (See Fig. xv.)

$H_l=20·4$ C.G.S. H_c superposed.								$H_l=20·4$ C.G.S. H_c superposed.			
H_c	BG	Totals.	B_l	H_c	BG	Totals.	B_l	H_c	BG	Totals.	B_l
0	96	96	7300	0	7·95	103·10	7840	11·2		not measured	
1·5	2·05	98·05	7455	− 3·1	2·55	105·65	8030	7·8	1·45	1·45	110
3·4	1·50	99·55	7570	−12·0	−10·75	94·90	7220	2·9	3·75	5·20	395
7·0	− 1·95	97·60	7420	0	8·30	103·20	7850	0	3·95	9·15	695
8·8	− 1·85	95·75	7270	3·1	2·45	105·65	8030	− 1·5	2·50	11·65	900
11·0	− 2·20	93·55	7100	12·0	−10·75	94·90	7220	− 2·1	·80	12·45	960
12·0	− ·70	92·85	7050	0	8·00	102·90	7820	− 3·0	− ·70	11·75	905
0	9·35	102·20	7760	− 3·1	2·25	105·15	8000	− 6·3	−5·10	6·65	505
− 3·1	2·90	105·10	7990	−12·0	−10·75	94·40	7170	− 8·0	−2·75	3·90	296
−12·0	−10·10	95·00	7220	0	8·30	102·70	7800	−11·2	−4·10	− ·20	−15
0	8·45	103·45	7860	3·1	2·70	105·40	8020	− 7·8	1·30	1·10	84
3·1	2·50	105·95	8050	12·0	−10·50	94·90	7220	− 3·0	3·75	4·85	370
12·0	−10·75	95·20	7230	0	7·95	102·85	7810	0	4·20	9·05	700
0	8·05	103·25	7850	− 3·1	2·45	105·30	8010	1·5	2·70	11·75	905
− 3·1	2·30	105·55	8020	−12·0	−10·60	94·70	7200	2·1	·80	12·55	965
−12·0	−10·80	94·75	7200	0	8·10	102·80	7810	3·0	− ·65	11·90	915
0	8·45	103·20	7850	3·1	2·65	105·45	8020	6·3	− 5·00	6·90	524
3·1	2·50	105·70	8030	12·0	−10·50	94·95	7220	8·0	− 2·70	4·20	320
12·0	−10·55	95·15	7230					11·2	− 4·05	·15	12

Table XII.—*Superposed Magnetic Induction.—Cyclic.* Shield B. (See Fig. xvi.)

$H_l=-2·8$ reduced from a maximum of about 44 C.G.S. H_c superposed.											
H_c	BG	BG	Average.	Totals.	B_l	H_c	BG	BG	Average.	Totals.	B_l
0	− 5·4	+ 2·4	− 3·9	− 3·9	− 296	3	− 1·2	+ 1·2	− 1·25	−15·15	− 1150
1·6			− 3·2	− 7·1	− 540	12	+ 2·8	− 2·9	+ 2·85	− 12·30	− 935
3·3			− 3·0	− 10·1	− 767	0	− 1·6	+ 1·6	− 1·60	− 13·90	− 1055
6·8			− 0·8	− 10·9	− 828	3	− 1·7	+ 1·8	− 1·75	− 15·65	− 1190
12·0			− 0·1	− 11·0	− 835	12	+ 2·9	− 3·0	+ 2·95	− 12·70	− 965
0	− 1·8	+ 1·9	− 1·85	− 12·85	− 976	0	− 1·2	+ 1·6	− 1·40	− 14·10	− 1070
3	− 1·5	+ 1·4	− 1·45	− 14·30	− 1090	3	− 1·0	+ 1·3	− 1·15	− 15·25	− 1160
12	+ 2·1	− 2·2	+ 2·15	− 12·15	− 923	12	+ 2·9	− 2·9	+ 2·90	− 12·35	− 940
0	− 1·4	+ 1·7	− 1·55	− 13·70	− 1040	0	− 1·3	+ 1·6	− 1·45	− 13·80	− 1050
3	− 1·9	+ 1·8	− 1·85	− 15·55	− 1180	3	− 1·7	+ 1·9	− 1·80	− 15·60	− 1185
12	+ 3·0	− 3·0	+ 3·00	− 12·55	− 953	12	+ 2·8	− 3·1	+ 2·95	− 12·65	− 960
0	− 1·2	+ 1·5	− 1·35	− 13·90	− 1055						

TABLE XIII.—*Magnetic Æolotropy.* (See Fig. XVIII.)

	Demagnetised by Decreasing Reversals. First of H_c and then of H_t.				Demagnetised by Decreasing Reversals. First of H_t and then of H_c.				Demagnetised by Decreasing Reversals of H_c. After this, a maximum value of H_t put on and then put off, leaving Residual B_t = 760 to 800 C.G.S.			
	H_c	BG	Totals.	B	H_c	BG	Totals.	B	H_c	BG	Totals.	B
SHIELD A.	·68	·75	·75	338	·68	1·10	1·10	496	·67	2·0	2·0	902
	1·0	1·05	1·80	812	1·0	1·65	2·75	1,240	1·0	2·4	4·4	1,980
	1·6	5·40	7·20	3,250	1·6	5·05	7·80	3,520	1·6	4·9	9·3	4,200
	2·7	7·15	14·35	6,470	2·7	6·40	14·20	6,400	2·7	5·9	15·2	6,860
	5·3	Increments	22·00	9,930	5·3	Increments	22·00	9,930	5·3	6·9	22·1	10,000
	9·2	from Zero.	27·90	12,600	9·2	from Zero.	27·90	12,600	9·2	5·8	27·9	12,600
SHIELD B.	1·5	1·65	1·65	640	1·5	2·20	2·20	850	1·5	3·45	3·45	1,340
	2·1	1·60	3·25	1,260	2·1	2·20	4·40	1,700	2·1	2·90	6·35	2,460
	3·0	5·40	8·65	3,350	3·0	4·90	9·30	3,610	3·0	4·80	11·15	4,330
	4·5	6·20	14·85	5,760	4·5	5·80	15·10	5,850	4·5	5·30	16·45	6,380
	6·8	5·75	20·60	8,000	6·8	5·55	20·65	8,000	6·8	5·10	21·55	8,350
	8·8	3·20	23·80	9,230	8·8	3·15	23·80	9,230	8·9	3·00	24·55	9,520

TABLE XIV.—*Sequence observed in the Determinations given in Tables XV. and XVI.*

TC Conditions.	Average of Third and Sixth Columns, reduced to C.G.S. Units = $(H)_w$.	H_t after Reversals.	H_c+ Superposed.	On Inductor being rotated.	H_t after Reversals.	H_c− Superposed.	On Inductor being rotated.
		Left on	0	Reading taken	Left on	0	Reading taken
			1st Increment	do.		1st Increment	do.
			2nd do.	do.		2nd do.	do.
			n^{th} do.	do.		n^{th} do.	do.
		H_t, put off, Shield demagnetised by Decreasing Reversals of H_c.			H_t, put off, Shield demagnetised by Decreasing Reversals of H_c.		

TCC Conditions.	Average of Third and Fifth Columns, reduced to C.G.S. Units = $(H)_w$.	H_t after Reversals.	H_c, repeatedly reversed, superposed.	On Inductor being rotated.	H_c	On Inductor being rotated.
		Left on	0	Reading taken		
			1st value, left +	do.	Reversed −	Reading taken
			2nd do. do. +	do.	do. −	do.
			n^{th} do. do. +	do.	do. −	do.
		H_t put off. Shield demagnetised by Decreasing Reversals of H_c.				

H_c Increments from Zero.	H_t	On Inductor being rotated.	H_t	On Inductor being rotated.	H_t	Shield by decreasing Reversals of H_c.
0			Repeatedly reversed	Reading taken	Put off	Demagnetised
1st Increment +	Superposed	Reading taken	Repeatedly reversed	Reading taken	Put off	Demagnetised
do. −	do.	do.	do.	do.	do.	do.
2nd Increment +	Superposed	Reading taken	Repeatedly reversed	Reading taken	Put off	Demagnetised
do. −	do.	do.	do.	do.	do.	do.
n^{th} Increment +	Superposed	Reading taken	Repeatedly reversed	Reading taken	Put off	Demagnetised
do. −	do.	do.	do.	do.	do.	do.
CT Conditions.			CTT Conditions.			
Average Readings of each + and − increment of H_c, reduced to C.G.S. units = $(H)_w$.						

TABLE XV.—*Shielding Ratios.—Transverse and Circular Fields.* SHIELD A. (See Plate IV.)

	$H_t=4{\cdot}37$.			$H_t=20{\cdot}9$.			$H_t=33{\cdot}6$.			$H_t=64{\cdot}25$.		
	H_c	$(H)_w$	g'	H_c	$(H)_w$	g'	H_c	$(H)_w$	g'	H_c	$(H)_w$	g'
TC Conditions.	0	·72	6·07	0	1·38	15·1	0	1·7	19·8	0	3·01	21·3
	·78	·237	18·4	·78	·72	29·0	·68	1·11	30·3	·57	2·57	25
	·96	·172	25·4	·96	·60	34·8	·93	·985	34·0	·9	2·31	27·8
	1·3	·125	35·0	1·3	·588	35·5	1·1	·936	35·8	1·0	2·26	28·4
	1·55	·147	29·8	1·55	·678	30·8	1·6	1·00	33·6	1·25	2·17	29·6
	2·13	·213	20·5	2·1	·925	22·6	2·4	1·62	20·8	1·5	2·12	30·3
	3·1	·319	13·7	3·1	1·40	14·9	4·2	2·87	11·7	2·4	2·48	25·9
	4·6	·442	9·9	4·6	2·06	10·1	8·2	6·4	5·25	4·4	4·88	13·1
	7·1	·687	6·4	7·1	3·24	6·5	14·5	11·73	2·86	7·4	9·75	6·6
	9·1	·940	4·6	9·0	4·50	4·65	18·0	14·6	2·3	12·2	15·8	4·07
	13·0	1·55	2·8	12·5	6·64	3·15						
	32·5	3·38	1·3	32·0	15·34	1·36						
TCC Conditions.	0	·72	6·07	0	1·38	15·1	0	1·7	19·8	0	3·01	21·3
	·78	·201	21·8	·77	·506	41·3	·68	·952	35·3	·56	2·32	27·7
	·96	·131	33·3	·95	·458	45·6	·93	·762	44	1·0	1·96	32·8
	1·3	·131	33·3	1·3	·442	47·3	1·1	·675	49·8	1·2	1·91	33·6
	1·5	·155	28·2	1·55	·515	37·0	1·6	·77	43·6	1·5	1·76	35·5
	2·13	·237	18·4	2·1	·900	23·2	2·4	1·45	23·2	2·4	2·16	29·8
	3·0	·355	12·6	3·1	1·45	14·4	4·1	2·78	12·1	4·3	4·42	14·5
	4·55	·449	9·7	4·5	2·18	10·05	8·2	6·4	5·25	7·2	9·1	7·07
	7·1	·736	5·9	7·0	3·36	6·2	14·5	11·9	2·8			
	9·0	1·01	4·3	9·0	4·70	4·5	18	14·9	2·25			
	12·7	1·62	2·7	12·6	6·78	3·1						
	32·3	3·40	1·3	31·8	15·38	1·36						
CT Conditions.	·44	·588	7·5	·51	1·22	17·1	·3	1·56	21·5	0	3·1	20·8
	·76	·361	12·1	·63	1·16	18·7	·5	1·58	21·3	·3	2·5	25·7
	1·25	·286	15·25	·83	1·10	19·0	·78	1·64	20·5	·6	2·5	25·7
	1·5	·311	14·0	1·15	1·2	17·4	1·3	1·92	17·5	·8	2·58	24·9
	2·0	·436	10	1·4	1·38	15·1	1·72	2·41	13·9	·9	2·68	24·0
	3·0	·646	6·75	1·8	1·74	12·0	3·5	4·24	7·9	1·4	3·08	20·9
	4·36	·906	4·8	3·8	3·24	6·5	5·5	6·25	5·4	1·75	3·35	19·2
	8·9	1·44	3·0	6·2	4·80	4·4	7·75	9·14	3·7	3·6	5·95	10·8
	12·65	2·21	2·0	8·9	7·22	2·9	17·3	17·5	1·92	5·75	9·8	6·5
	31·6	3·55	1·23	21·7	14·7	1·4				8·0	13·63	4·7
CTT Conditions.	·44	·772	5·7	·30	1·45	14·4	0	1·73	19·4	0	3·1	20·8
	·94	·915	4·8	·51	1·73	12·1	·3	1·82	18·4	·3	3·09	20·8
	1·5	1·09	4·0	·83	2·09	10·0	·5	2·04	16·5	·5	3·21	20
	2·0	1·33	3·3	1·15	2·44	8·5	·78	2·47	13·6	·9	3·55	18·1
	3·0	1·56	2·8	1·4	2·69	7·8	1·3	3·1	10·8	1·4	4·18	15·7
	4·4	1·81	2·4	1·8	3·08	6·8	1·72	3·58	9·4	1·75	4·46	14·8
	7·0	2·22	2·0	3·8	4·48	4·7	3·5	5·33	6·3	3·6	7·15	9·0
	12·6	3·05	1·4	6·2	6·01	3·5	5·5	7·4	4·5	5·75	10·75	6·0
	31·6	3·62	1·2	8·9	8·05	2·6	7·75	9·75	3·4	8·0	14·38	4·5
				21·7	15·15	1·38	17·3	16·9	1·99			

TABLE XVI.—*Shielding Ratios.—Transverse and Circular Fields.* SHIELD B. (See Plate IV.)

	$H_t = 4·37$.			$H_t = 20·9$.			$H_t = 33·6$.			$H_t = 64·25$.		
	H_c	$(H)_w$	g'	H_c	$(H)_w$	g'	H_c	$(H)_w$	g'	H_c	$(H)_w$	g'
TC Conditions.	0	·945	4·6	0	2·35	8·9	0	3·00	11·2	0	4·65	13·8
	1	·527	8·3	1	1·60	13·1	1·0	2·2	15·6	1·22	3·72	17·3
	1·6	·279	15·7	1·5	1·25	16·7	1·7	1·7	19·8	1·76	3·44	18·7
	2·3	·186	23·5	1·8	1·16	18·0	2·4	1·57	21·4	2·24	3·24	19·8
	3·0	·217	20·1	2·25	1·05	19·9	3·1	1·76	19·1	3·6	3·40	18·9
	5·0	·357	12·5	2·9	1·07	19·5	3·6	2·10	16·0	5·3	5·00	12·8
	6·5	·473	9·25	3·8	1·295	16·1	5·0	2·77	12·3	6·7	6·40	10·0
	8·0	·628	6·9	5·4	1·99	10·5	6·2	3·58	9·4			
	10·3	·922	4·7	7·8	3·01	7·0	7·5	4·41	7·6			
				10·1	4·0	5·2	8·3	4·90	6·8			
							9·5	5·73	5·8			
TCC Conditions.	0	·945	4·6	0	2·35	8·9	0	2·97	11·3	0	4·65	13·8
	1	·364	12	1	1·295	16·1	1·0	1·76	19·1	1·2	3·24	19·8
	1·6	·225	19·4	1·5	·935	22·4	1·67	1·30	25·8	1·7	2·85	22·5
	2·3	·17	25·8	1·8	·792	26·4	2·4	1·09	30·8	2·2	2·60	24·7
	3·0	·186	23·5	2·25	·730	28·8	3·0	1·41	23·8	3·5	2·59	24·8
	3·6	·256	17·1	2·99	·825	25·3	3·5	1·47	22·8	5·2	4·03	16·0
	5·0	·356	12·6	3·8	1·19	17·5	4·9	2·40	14·0	6·5	5·30	12·1
	6·4	·480	9·1	5·4	1·91	10·9	6·0	3·20	10·5			
	8·0	·635	6·9	7·8	2·99	7·0	7·4	4·08	8·2			
	9·0	·720	6·1	10·1	4·03	5·2	8·16	4·62	7·2			
	10·2	·890	4·9				9·4	5·56	6·0			
CT Conditions.	0	·972	4·5	0	2·39	8·75	0	2·92	11·5	0	4·68	13·7
	·94	·733	6·0	·97	2·11	9·9	1·16	2·58	13·0	1·15	4·17	15·4
	1·27	·637	6·9	1·54	1·84	11·35	1·82	2·54	13·25	1·82	4·53	14·2
	1·82	·486	9·0	2·29	1·86	11·25	2·54	2·94	11·4	2·11	4·65	13·8
	2·9	·462	9·5	2·9	2·15	9·7	3·22	3·48	9·7	2·6	5·15	12·5
	3·8	·55	8·0	3·8	2·75	7·6	4·25	4·40	7·64	3·5	6·43	10·0
	5·4	·765	5·7	5·4	3·70	5·65	5·9	6·05	5·5	5·8	9·33	6·9
	7·7	1·075	4·1	7·8	5·0	4·2	8·15	7·82	4·3	7·9	12·1	5·3
	9·9	1·32	3·3	10·0	6·07	3·4	10·25	9·33	3·6	10·8	16·6	3·9
CTT Conditions.	0	·972	4·5	0	2·39	8·75	1·165	3·71	9·05	0	4·68	13·7
	·5	1·02	4·28	·97	2·9	7·2	1·82	4·40	7·64	1·15	5·3	12·1
	1·27	1·13	4·13	1·5	3·4	6·1	2·12	4·77	7·05	1·82	6·03	10·6
	1·82	1·16	3·77	2·3	3·94	5·3	2·54	5·13	6·55	2·11	6·35	10·1
	2·9	1·31	3·34	2·9	4·40	4·8	3·22	6·07	5·5	2·6	7·06	9·1
	3·8	1·48	2·95	3·8	5·04	4·1	4·25	6·80	5·0	3·5	8·1	8·0
	5·4	1·72	2·54	5·5	5·91	3·5	5·9	8·10	4·2	5·8	10·8	6·0
	7·7	1·95	2·24	7·8	6·95	3·0	8·15	9·49	3·56	7·9	13·26	4·9
	9·9	2·09	2·1	10	7·85	2·7	10·25	10·95	3·07	10·8	16·25	4·0

TABLE XVII.—*Magnetic Induction—CTT Conditions.*

H_t=20·9 C.G.S.			H_t superposed and then repeatedly reversed.	Totals.		Average.		B.	
	H_c	BG	BG	*m*	*n*	*m*	*n*	*m*	*n*
Shield A. See fig. XXX.	·93	− 2·3	−11·8+11·7−13+11·8−12·2+11·8−12+11·8−11·9	16·1	4·3	15·6	3·8	6,780	1,650
		+ 2·3	+9·4−9·6+13−11·8+11·8−11·8+11·8	15·1	3·3				
	2·4	−13·9	−7·9+9·7−11+10·8−10·9+10·7	23 2	12·5	23·15	12·45	10,200	5,410
		+13·9	+7·8−9·7+11·2−10·9+11−10·8+10·7	23·1	12·4				
	5·1	−22·5	−6·9+9·5−10+10−10	29·9	20·0	29·9	20·0	13,000	8,700
		+22·4	+6·9−9·4+9·9−9·8+9·9	29·9	20·0				
	9·1	−28·9	−4·6+8·2−8·7+8·2−8·2	34·0	25·8	33·9	25·7	14,700	11,200
		+28·9	+4·7−8·2+8·2−8+8·2−8·3	33·8	25·6				
Shield B. See fig. XXXI.	2·1	+ 4·8	+8·6−8·2+11·8−10·3+10·9−10·4+10·8−10·5+10·6	18·1	7·6	18·35	7·95	6,860	2,970
		− 4·8	−8·3+8·3−12+10·1−11+10·3−10·8+10·3−10·6+10·3−10·4	18·6	8·3				
	4·0	+14·2	+7·6−8·2+10·7−9·9+9·9−9·9+10	24·3	14·4	23·95	14·05	8,950	5,250
		−14·2	−7·2+8·2−10·3+9·8−9·9	23·6	13·7				
	6·2	+20·2	+6·6−8·3+9·6−8·9+9·3−9·1+9·1	28·5	19·4	28·50	19·4	10,650	7,250
		−20·4	−6·6+8·2−9·6+9·1−9·2+9·1−9·2+9·1	28·5	19·4				
	9·0	+25·5	+5·9−8+8·8−8·4+8·8−8·7+8·5−8·5	32·4	23·9	32·0	23·6	11,960	8,820
		−25·2	−5·8+8−8·6+8·3−8·3+8·4−8·3	31·6	23·3				

TABLE XVIII.—*Magnetic Induction—TC Conditions.*

H_t=20·9 C.G.S. left on after repeated reversals.							
		m			*n*		
	H_c	BG	Totals.	B	BG	Totals.	B
Shield A. See fig. XXVIII.	0	12·75	6·30	2,750	12·6	− 6·3	−2750
	·9	3·1	9·40	4,090	2·4	− 3·9	−1700
	2·4	9·45	18·85	8,200	10·45	6·55	2850
	5·1	8·55	27·40	11,900	9·25	15·80	6870
	9·1	5·4	32·80	14,300	6·4	22·20	9650
Shield B. See fig. XXIX.	0	11·3	5·65	2,110	11·3	− 5·65	−2110
	2·1	5·1	10·75	4,020	4·7	− 0·95	− 355
	4·0	8·25	19·00	7,100	8·75	7·80	2920
	6·2	5·95	24·95	9,320	6·55	14·35	5360
	8·9	Increment from zero.	30·1	11,200	5·2	19·55	7300

TABLE XIX.—*Shielding Ratios*—H_t=20·9

SHIELD A. See Fig. VII.			SHIELD B. See Fig. VII.		
H_c	$(H)_w$	g'	H_c	$(H)_w$	g'
5·0	2·2	9·5	9·5	3·6	5·8
3·3	1·845	11·3	8·1	3·36	6·2
2·0	1·44	14·5	6·0	2·95	7·0
1·2	1·15	18·2	3·5	2·3	9·0
·75	·975	21·4	1·9	1·78	11·7
·44	·82	25·5	·98	1·56	13·3
0	·627	33·2	0	1·11	18·7
− ·44	·388	54·0	− ·98	·674	30·9
− ·75	·209	100	−1·9	·206	101
− ·92	·116	180	−2·3	·158	131
−1·2	·038	550	−3·0	·309	67·4
−1·6	·271	77	−3·5	·495	43·8
−2·0	·496	42	−4·8	1·22	17
−3·3	1·36	15·4	−6·0	1·9	10·[illegible]
−5·0	2·20	9·5	−7·4	2·57	8·1
			−8·2	3·02	6·[illegible]
			−9·5	3·54	5·[illegible]

TABLE XX.—*Shielding Ratios—Transverse and Longitudinal Fields* $H_t = 20·9$ C.G.S.

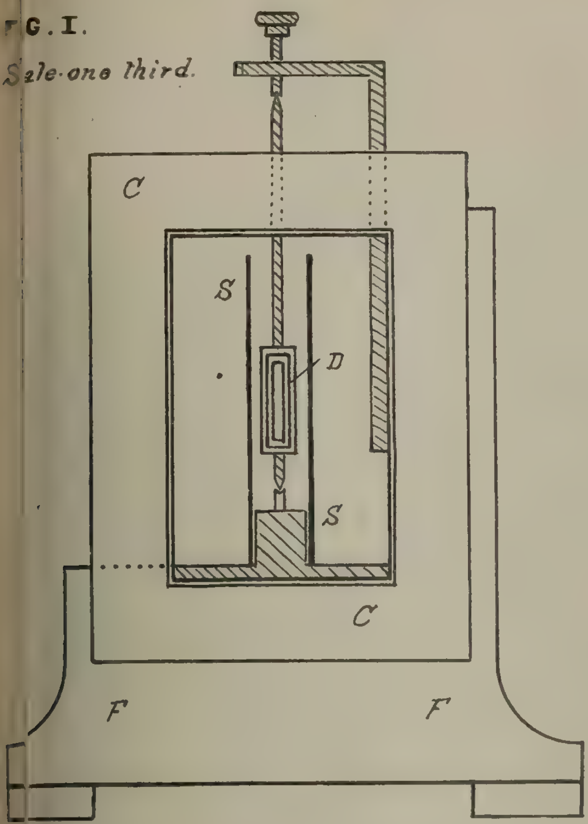
	TL Conditions.			TLL Conditions.			LT Conditions.			LTT Conditions.		
	H_t	$(H)_w$	g'	H_t	$(H)_w$	g'	H_t	$(H)_w$	g'	H_t	$(H)_w$	g'
Shield A. See fig. XXXVIII.	0	1·51	13·8	0	1·51	13·8	0	1·44	14·5	as under LT Conditions.	1·50	13·9
	3·1	·85	24·8	3·1	·535	39·1	3·3	1·33	15·7		1·58	13·2
	5·0	·688	30·4	5·3	·381	55·0	7·7	1·26	16·6		1·66	12·6
	7·1	·59	35·4	7·7	·365	57·3	9·2	1·30	16·1		1·71	12·2
	8·7	·55	38·0	15·4	·445	47·0	15·1	1·43	14·6		1·72	12·1
	13·5	·575	36·4	19·3	·543	38·5	20·2	1·55	13·5		?	
	18·0	·623	33·5	24·2	·632	33·1	23·0	1·68	12·4		2·03	10·3
	23·7	·737	28·4	31·5	·923	22·7	28·4	1·87	11·2		2·13	9·8
	30·0	·987	21·2	36·6	1·52	13·7	37·4	2·57	8·13		3·10	6·7
	36·5	1·51	13·8	45·3	2·42	8·6	46·2	3·62	5·78		4·03	5·2
	45·6	2·47	8·5	52·3	3·14	6·7	53·3	4·3	4·86		4·64	4·5
	53·1	3·22	6·5	59·5	3·72	5·6	60·1	4·9	4·23		5·2	4·0
	60·6	3·84	5·5									
Shield B. See fig. XXXIX.	0	2·75	7·6							as under LT Conditions.		
	2·2	2·14	9·8	2·7	1·75	12	2·7	2·56	8·16		2·78	7·6
	3·4	1·98	10·5	5·1	1·15	18·4	5·3	2·40	8·7		2·49	8·4?
	7·8	1·47	14·2	7·8	1·02	20·5	7·8	2·37	8·8		2·75	7·6
	9·6	1·385	15·1	9·5	·973	21·5	15·7	2·35	8·9		3·0	7·0
	15·7	1·195	17·5	15·6	·816	25·6	25·6	2·60	8·0		3·3	6·3
	20·5	1·19	17·5	20·4	·89	23·5	36·6	3·16	6·6		3·73	5·6
	25·6	1·25	16·7	25·6	1·01	20·7	59·5	4·63	4·5		5·11	4·1
	31·2	1·54	13·6	31·0	1·24	16·9						
	36·6	1·7	12·3	36·4	1·43	14·7						
	45·3	2·21	9·4	44·9	2·03	10·3						
	52·4	2·7	7·7	51·9	2·49	8·4						
	59·5	2·38	6·2	58·5	3·05	6·8						

TABLE XXI.—*Shielding Ratios—Transverse and Longitudinal Fields.*

	TL Conditions $H_t = 33·6$.				TL Conditions $H_t = 64·25$ C.G.S.				TL Conditions $H_t = 20·9$ C.G.S.		
	H_t	$(H)_w$	g'		H_t	$(H)_w$	g'		H_t	$(H)_w$	g'
Shield A. See fig. XXXVIII.	0	1·78	18·9	Shield A. See fig. XXXVIII.	0	3·45	18·6	Shield B demagnetised by circular field. See fig. XXXIX. broken line curve.	0	2·5	8·4
	3·2	1·13	29·6		2·1	2·54	25·3		2·7	1·82	11·5
	4·9	·945	35·5		4·7	2·08	30·9		5·0	1·49	14·0
	7·1	·85	39·5		9·0	1·94	33·2		7·7	1·27	16·5
	8·7	·85	39·5		13·3	2·08	30·9		9·2	1·16	18·0
	13·7	·87	38·6		16·4	2·24	28·7		15·0	1·00	21·0
	17·6	·96	35·0		21·3	2·56	25·0		19·4	1·05	20
	21·2	1·12	30·0		26·2	3·20	20·1		23·5	1·18	17·7
	25·3	1·28	26·2		30·1	4·10	15·7		30·0	1·36	15·4
	32·6	1·94	17·3		34·2	5·25	12·2		36·5	1·65	12·7
	39·6	3·0	11·2						45·1	2·12	9·9
	45·0	3·9	8·6						52·1	2·66	7·9
	50·0	4·7	7·1						59·1	3·24	6·5

MAGNETIC SHIELDING IN HOLLOW IRON CYLINDERS.—PLATE 1

FIG. I.
Scale one third.



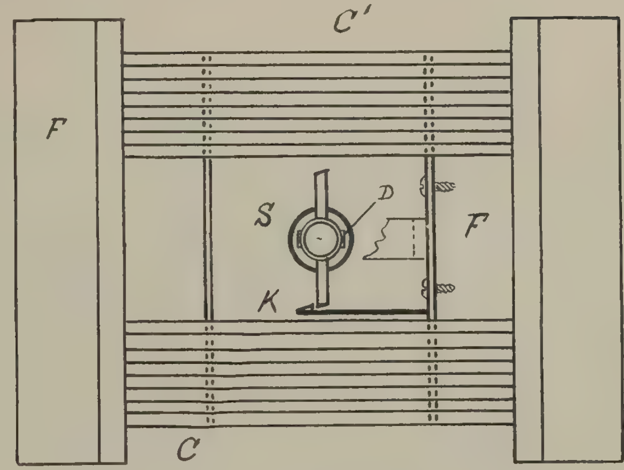
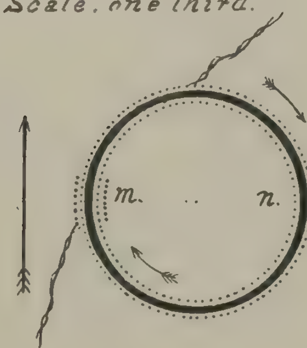


FIG. II.
Scale, one third.

FIG. III.
Scale Full size.



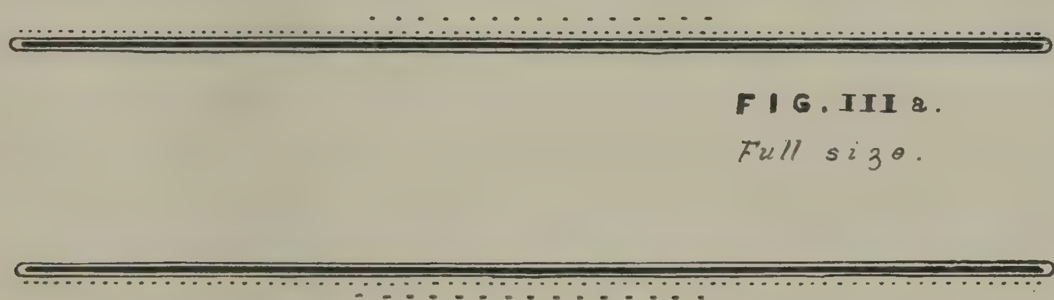


FIG. III a.
Full size.

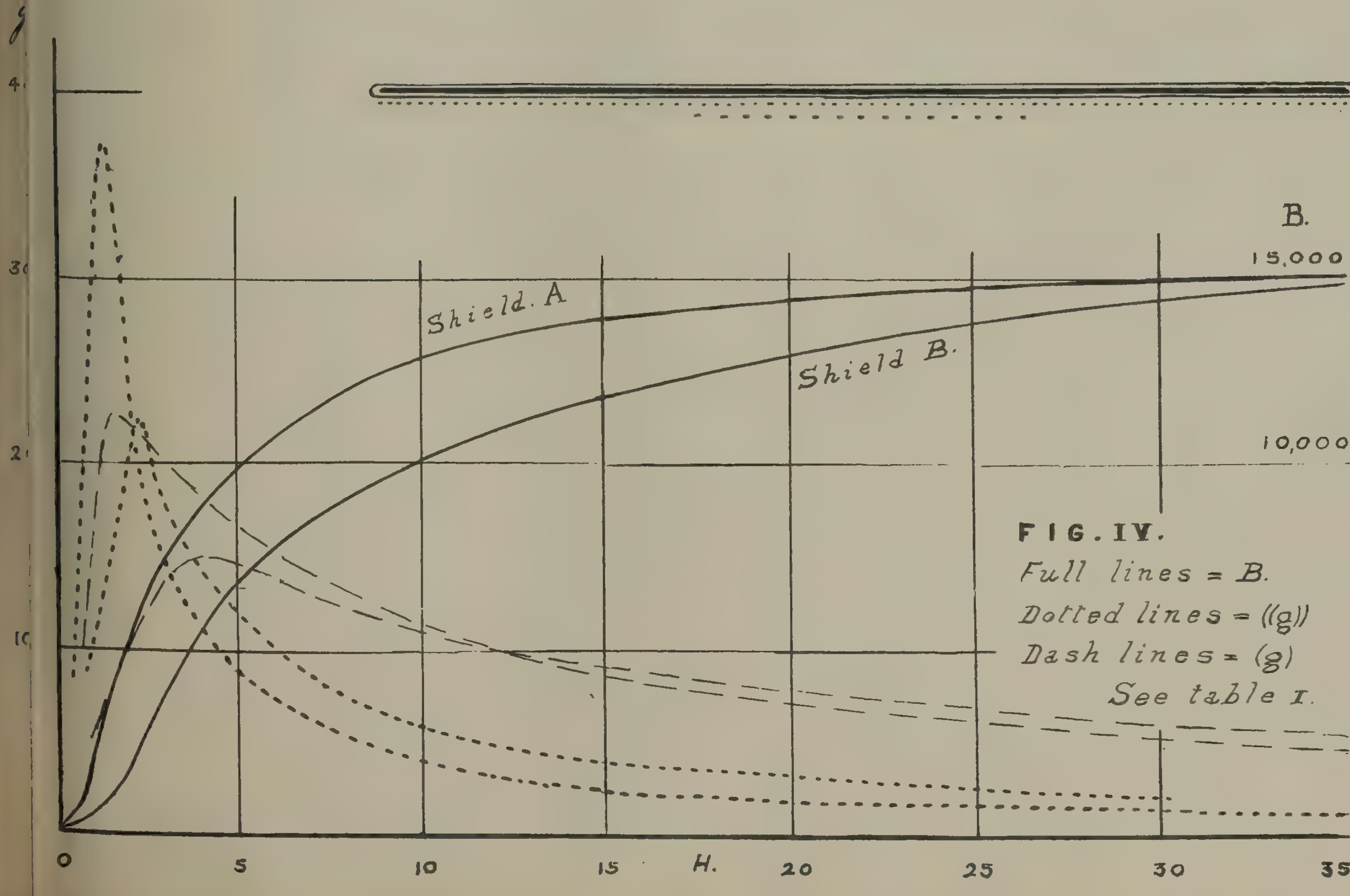
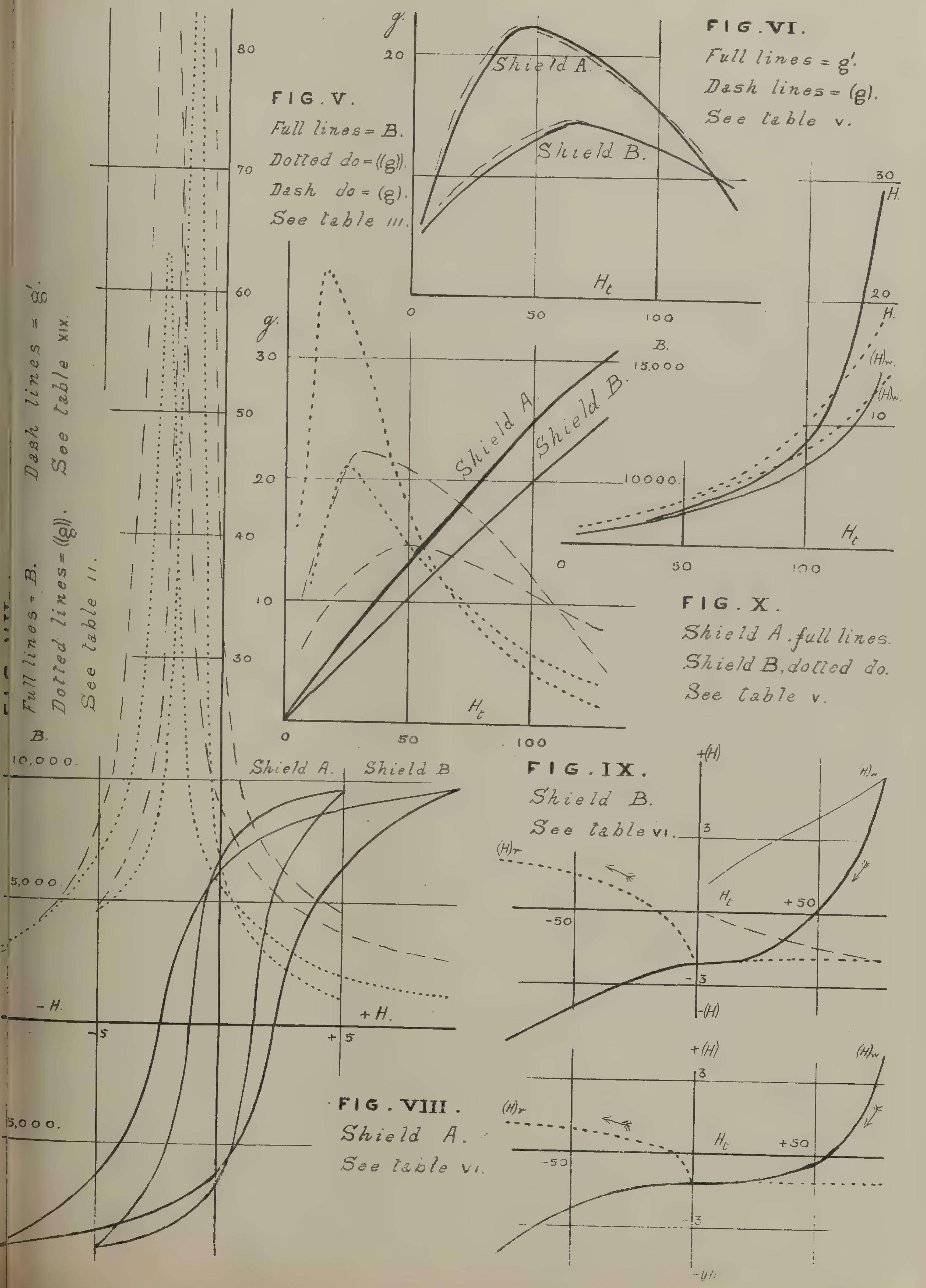


FIG. IV.
Full lines = B.
Dotted lines = ((g))
Dash lines = (g)
See table I.

MAGNETIC SHIELDING IN HOLLOW IRON CYLINDERS.—PLATE 2.



SUPERPOSED MAGNETIC INDUCTIONS IN IRON.—PLATE 3.

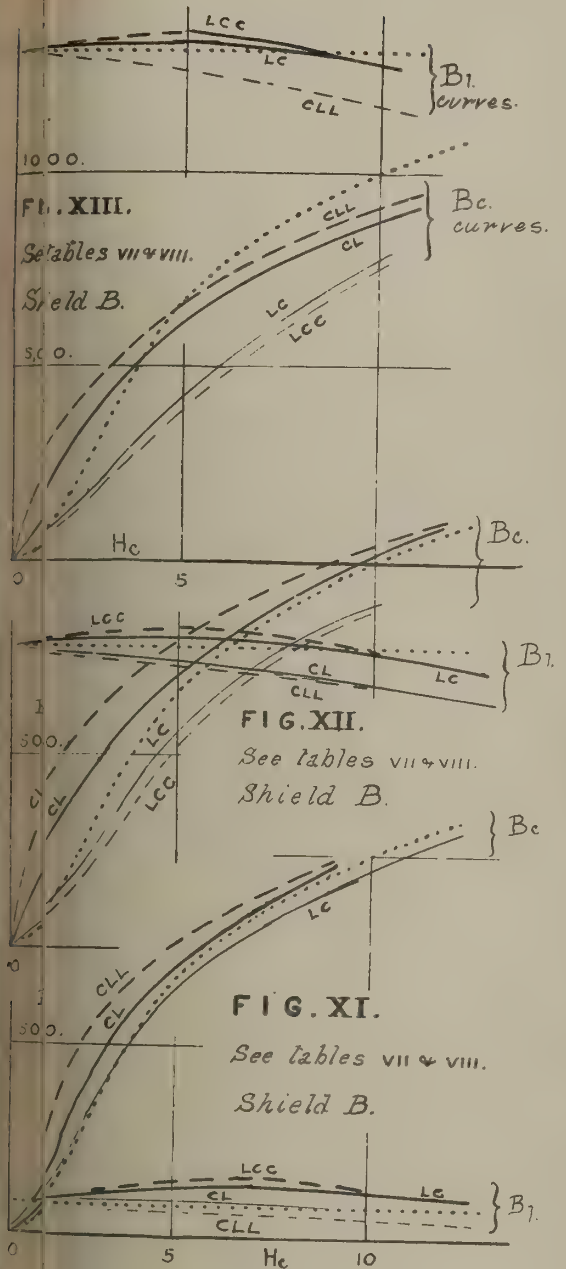


FIG. XIII. See tables VII & VIII. Shield B.

FIG. XII. See tables VII & VIII. Shield B.

FIG. XI. See tables VII & VIII. Shield B.

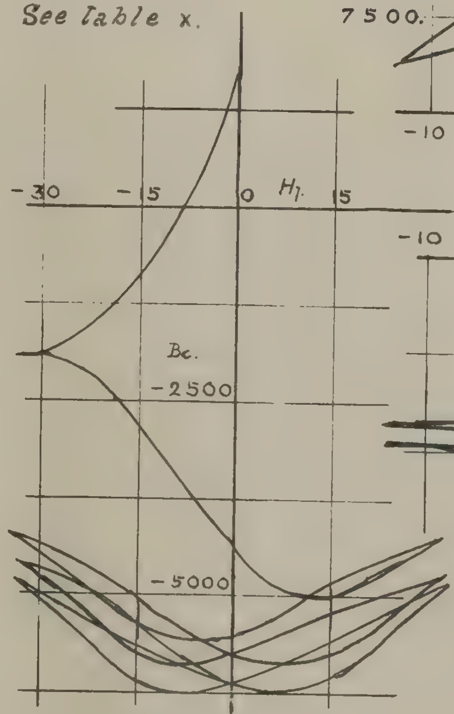


FIG. XVII. Shield B. See table X.

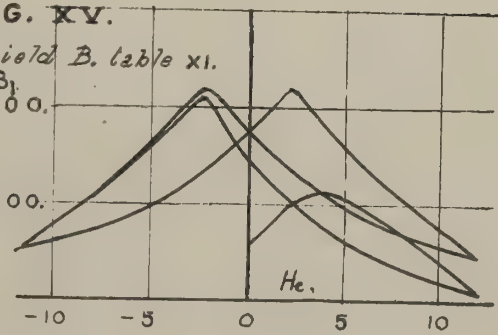


FIG. XV. Shield B. table XI.

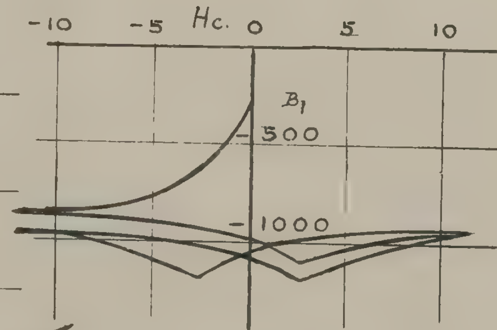


FIG. XVI. Shield B. table XII.

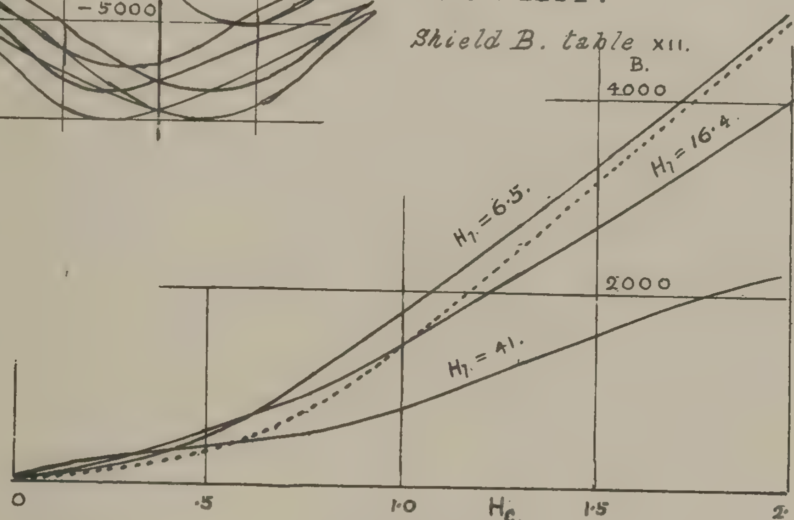


FIG. XIV. Shield A. See table IX.

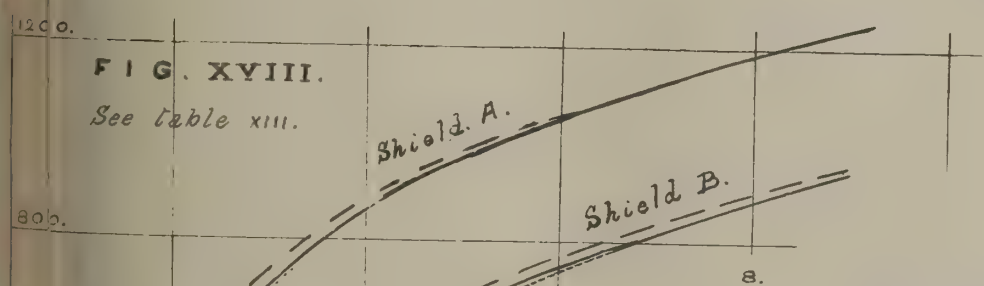
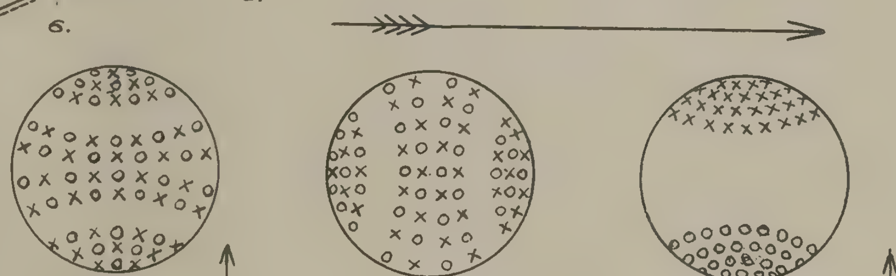


FIG. XVIII. See table XIII.

FIG. XIX.



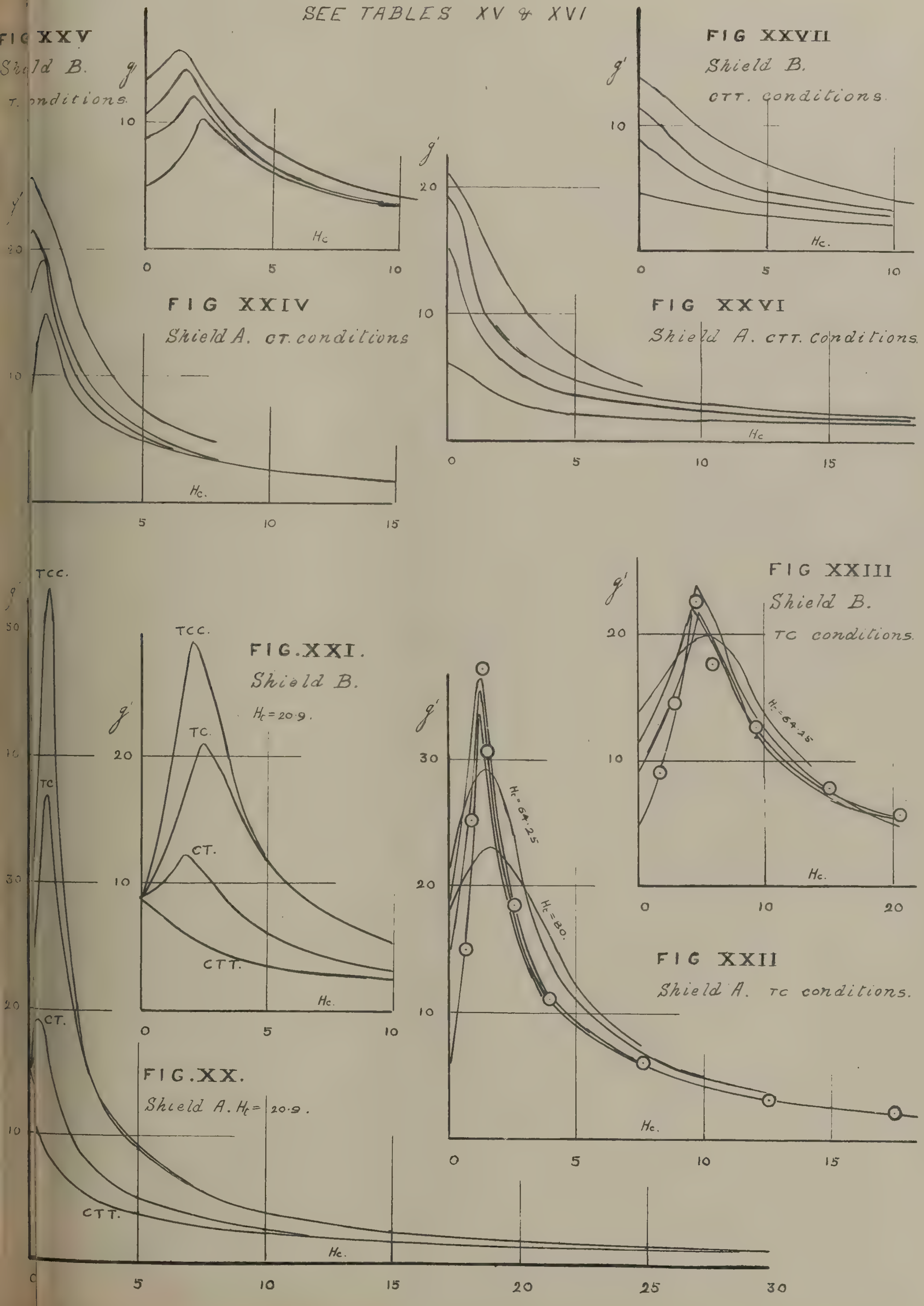
1. Demagnetised by H_1. Dotted line curves fig XVIII.

2. Demagnetised by Hc. Full line curves fig XVIII.

3. Left with residual magnetisation due to H_1. fig. XVIII.

Magnetic Shielding in Hollow Iron Cylinders.—Plate 4.

SEE TABLES XV & XVI



MAGNETIC SHIELDING IN HOLLOW IRON CYLINDERS.—PLATE 5.

FIG. XXXII.

Shield A

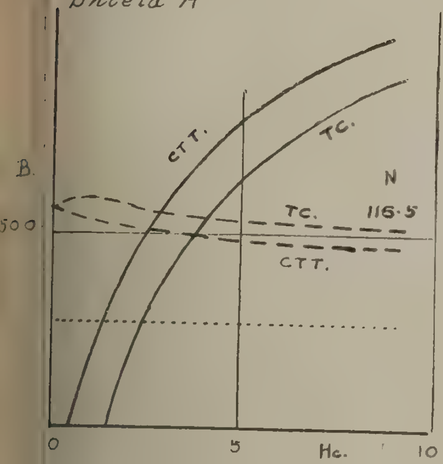


FIG. XXVIII.

Shield A. TC conditions.

See table XVIII.

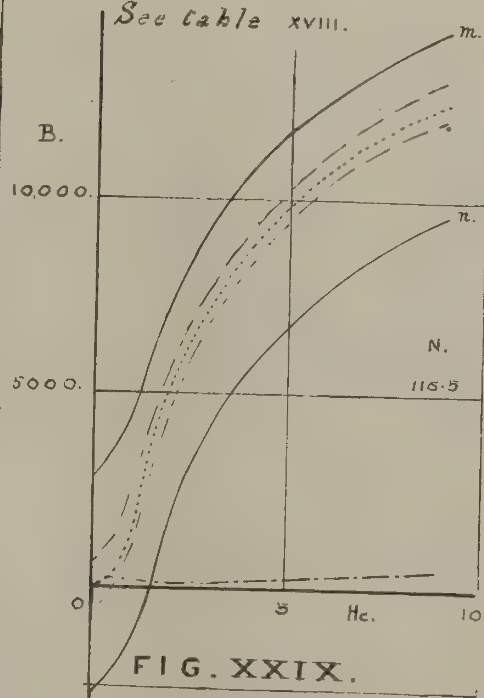


FIG. XXX.

Shield A. CTT conditions

See table XVII.

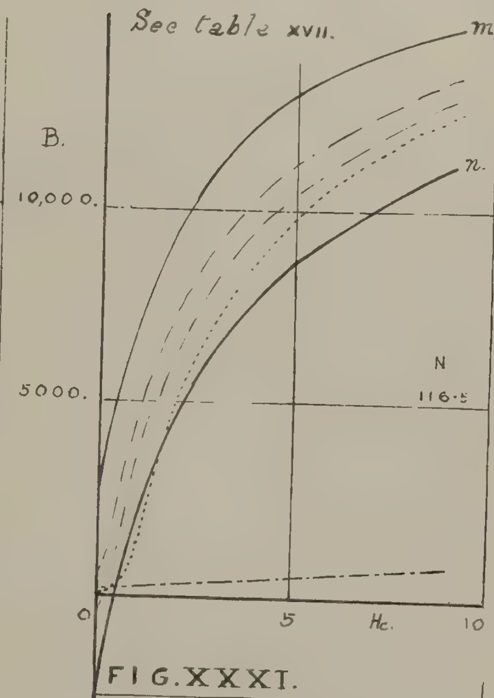


FIG. XXXIII.

Shield B.

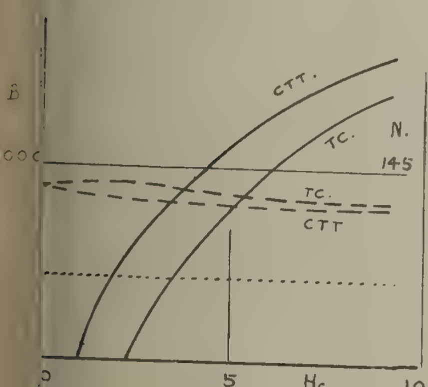


FIG. XXIX.

Shield B TC conditions

See table XVIII.

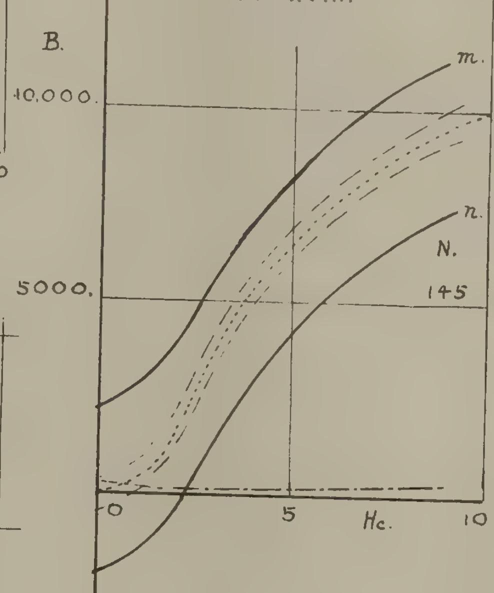
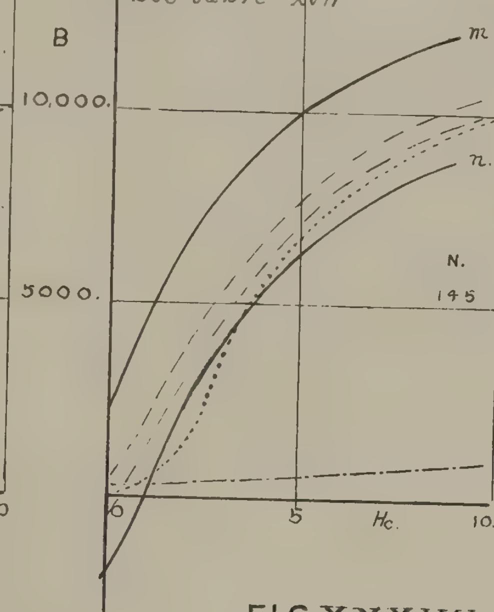


FIG. XXXI.

Shield A. CTT conditions.

See table XVII



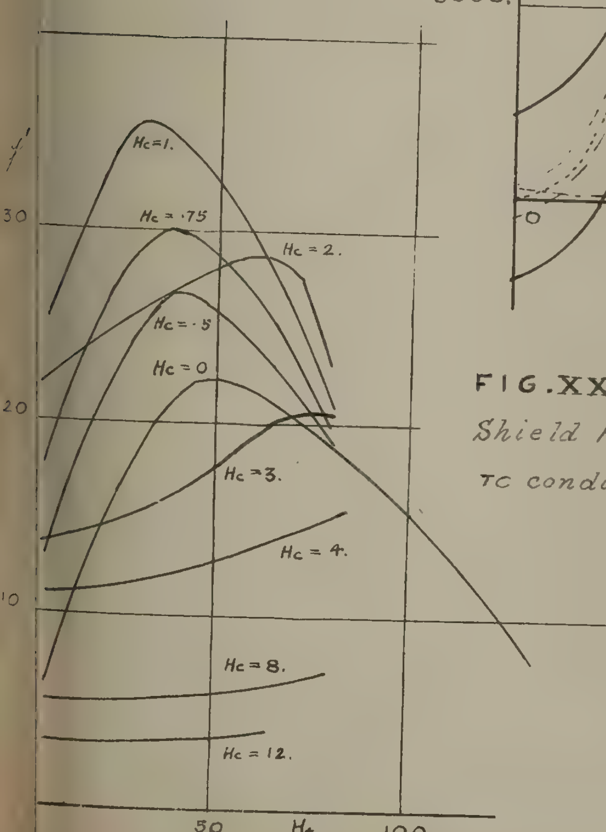


FIG. XXXVI.

Shield A.

TC conditions.

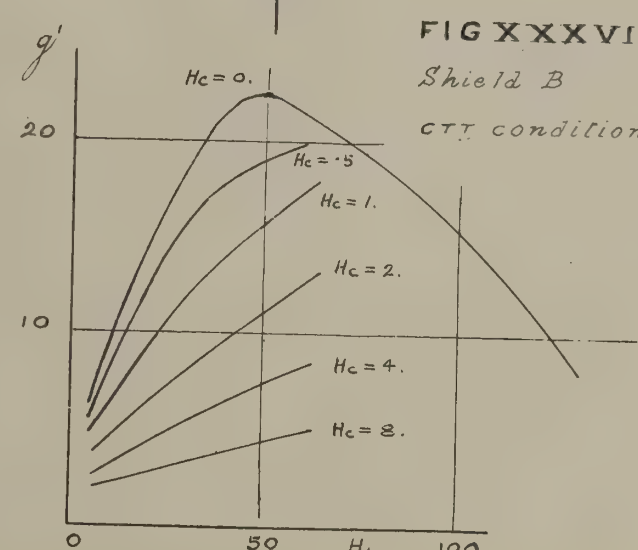


FIG XXXVII

Shield B

CTT conditions.

MAGNETIC SHIELDING IN HOLLOW IRON CYLINDERS.—PLATE 6

FIG XXXIV

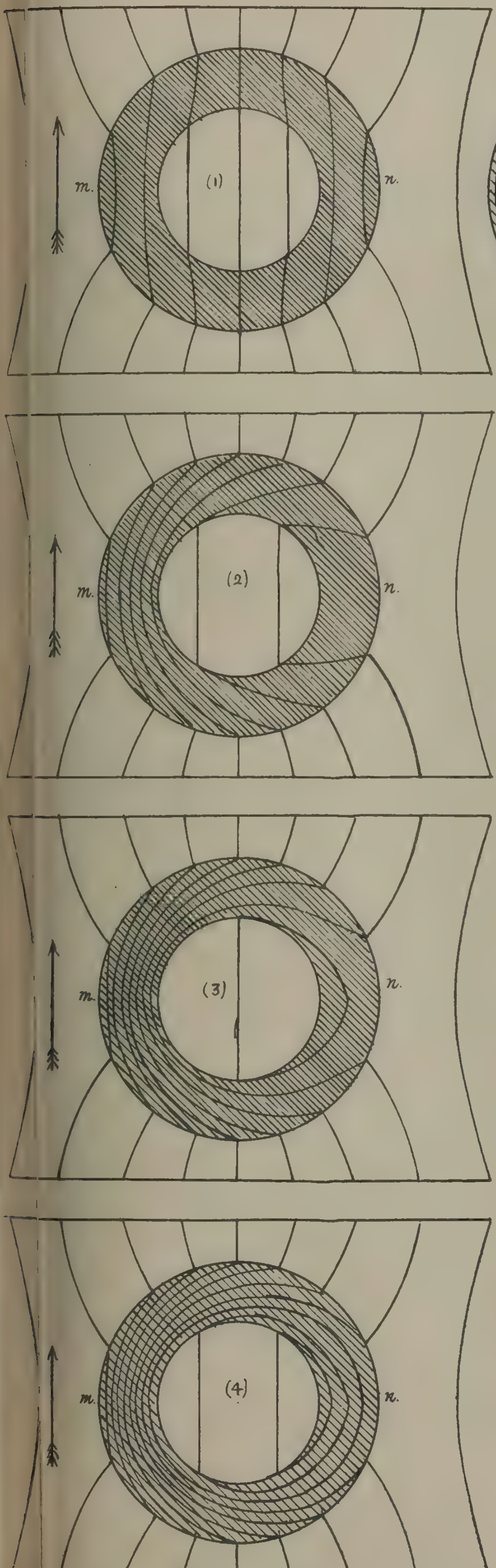


FIG XXXV

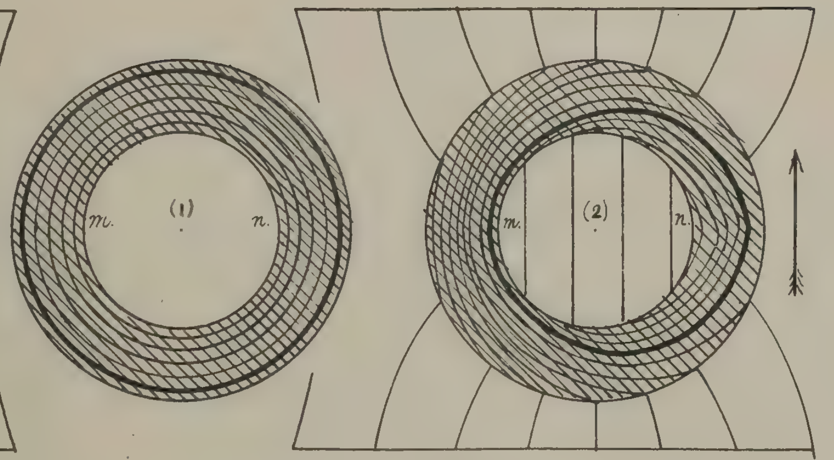


FIG XXXIX
Shield B.
Tables XX & XXI.

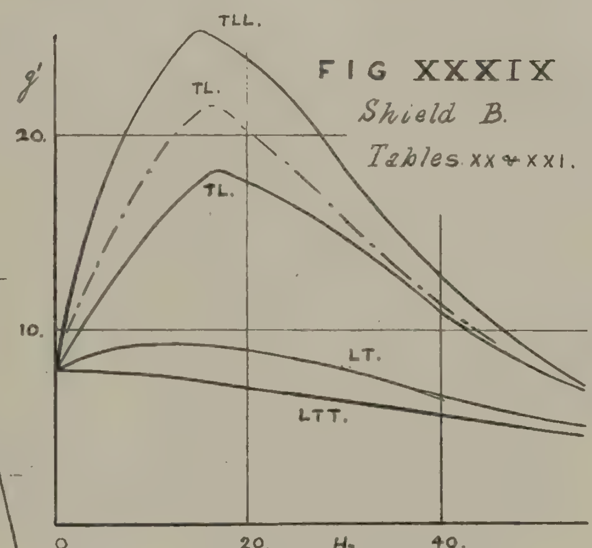
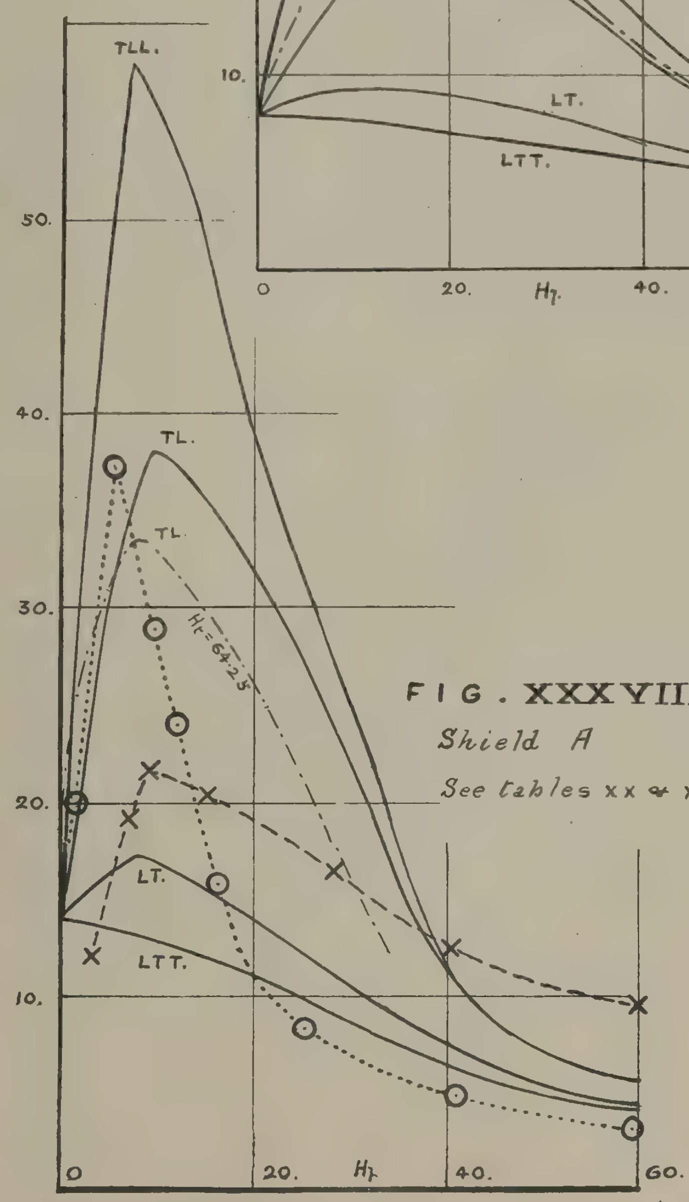


FIG. XXXVIII.
Shield A
See tables XX & XXI



XXVII.—*On the Effect of Temperature on the taking of Salmon with Rod and Fly in the River Spey at Gordon Castle in the Autumns of* 1898, 1899, 1900, *and* 1901.

By George Muirhead, Commissioner for His Grace the Duke of Richmond and Gordon, K.G. (With Four Plates.)

(MS. received April 25, 1903. Read June 1, 1903. Issued separately September 12, 1903.)

The Duke of Richmond and Gordon's salmon fishings on the Spey extend on both sides of the river from the Boat o' Brig, where the Highland Railway crosses the stream between Keith and Elgin, to the sea, a distance of about nine miles.

The river here varies in breadth from 50 to 140 yards, and in depth from 2 feet, where the stream is broad, to about 10 or 12 in some of the pools. Its course is rapid, the fall being about 16 feet to the mile, and its volume is quickly increased when there is much rain, or when snow melts in the Monadhliadh and Grampian Mountains.

There are many excellent pools for salmon fishing with the rod on the Gordon Castle water, and in them large numbers of fish are usually taken with the fly every year during the month of September and the first half of October.

His Grace's guests at Gordon Castle fish for salmon in the Spey there with rod and fly only, no other lure than the fly being used. They are all well-known and skilful salmon fishers, who generally visit Gordon Castle during the salmon rod-fishing season every year, and in the autumns of 1899 and 1900 they included His Royal Highness the Prince of Wales. The pools which are fished, and the arrangements connected with the fishing, were the same in each of the four seasons above mentioned.

The usual experience of salmon fishers with rod and fly is, that whilst the fish may take well on certain days, yet, on other days, which do not appear to differ from these in any way whatever, they can scarcely be induced to rise to it. This peculiarity was particularly observable in the Gordon Castle water in the autumn of 1897; and as it then occurred to me that the cause of it might be on account of changes in the temperature of the water from day to day, I made careful observations of the temperature of the river during the rod-fishing season in the four following years.

The temperature of the water in the Spey was taken twice a day near Gordon Castle at nine o'clock in the morning and four o'clock in the afternoon, being the most convenient hours for ascertaining approximately the lowest and the highest temperatures for the day, and the results are indicated by the dark blue line on the Diagrams. The red line shows the daily mean temperature of the air at the station of the Scottish Meteorological Society at Gordon Castle during the same period. The red columns indicate by their respective heights and the figures marked immediately above each of them the average number of salmon which were caught by each 'rod' daily during the above mentioned salmon rod-fishing season in each of the four years from 1898 to 1901 inclusive. The height of the water in the river daily is indicated by the blue tint on the Diagrams, on which are also shown the daily readings of the barometer, as well as the general state of the weather with regard to rainfall, sunshine, and wind. The average numbers of salmon taken daily by each 'rod' during each of the above mentioned four seasons were: in 1898, 1·26; in 1899, 1·31; in 1900, 1·46; and in 1901, 1·41,—the average for the four years being 1·36.

For the number of 'rods' which were fishing daily, and the number of salmon caught by them daily, I am indebted to His Grace the Duke of Richmond and Gordon, who personally keeps very accurate records of them.

TABLE *showing the highest and lowest weekly temperature, the amount of variation in the weekly temperature, and the average number of salmon caught by each 'rod' daily during each week of the rod-fishing season of*

1898.

Date.	Temperature.	Amount of Variation in Temperature.	Average Number of Salmon caught daily by each 'Rod.'
Sept. 3–10	53°–65°	12°	0·40
12–18	52–63	11	0·70
19–24	50–55	5	1·60
26–Oct. 1	52–48	4	1·94
3–9	58–51	7	1·38
10–15	52–47	5	1·59

TABLE *showing the highest and lowest weekly temperature, the amount of variation in the weekly temperature, and the average number of salmon caught by each 'rod' daily during each week of the rod-fishing season of*

1899.

Date.	Temperature.	Amount of Variation in Temperature.	Average Number of Salmon caught daily by each 'Rod.'
Sept. 4–9	55°–62°	7°	1·12
11–16	52–60	8	1·02
18–23	53–46	7	1·45
25–30	49–43	6	1·38
Oct. 2–7	48–44	4	1·64
9–14	53–44	9	0·78

TABLE *showing the highest and lowest weekly temperature, the amount of variation in the weekly temperature, and the average number of salmon caught by each 'rod' daily during each week of the rod-fishing season of*

1900.

Date.	Temperature.	Amount of Variation in Temperature.	Average Number of Salmon caught daily by each 'Rod.'
Sept. 1–8	...	...	No fishing.
10–15	...	...	do.
17–22	60°–52°	8°	1·13
24–29	54–48	6	1·38
Oct. 1–6	49–45	4	1·82
8–15	50–45	5	1·51

TABLE *showing the highest and lowest weekly temperature, the amount of variation in the weekly temperature, and the average number of salmon caught by each 'rod' daily during each week of the rod-fishing season of*

1901.

Date.	Temperature.	Amount of Variation in Temperature.	Average Number of Salmon caught daily by each 'Rod.'
Sept. 2–7	52°–58°	6°	0·68
9–14	60–55	5	0·76
16–21	50–57	7	1·46
23–28	55–60	5	1·37
30–Oct. 5	56–50	6	1·68
7–15	47–44	3	2·51

On referring to the foregoing Tables and to the Diagrams it will be found that salmon rod-fishing took place on 131 days, and that the average number of salmon caught by each 'rod' on each day during that period was 1·36; also, that the average weekly variation in the temperature was 6·36° Fah. It is shown also that when the weekly variation in temperature was only 3° the weekly average of salmon taken by each 'rod' in the day was as high as 2·51, and when the weekly temperature varied as much as 12° the weekly average of salmon taken by each 'rod' in the day fell to 0·40.

The following Table shows the whole series of weekly variations in temperature during the above mentioned period of 131 days, with the relative numbers of salmon taken daily by each 'rod.'

TABLE.

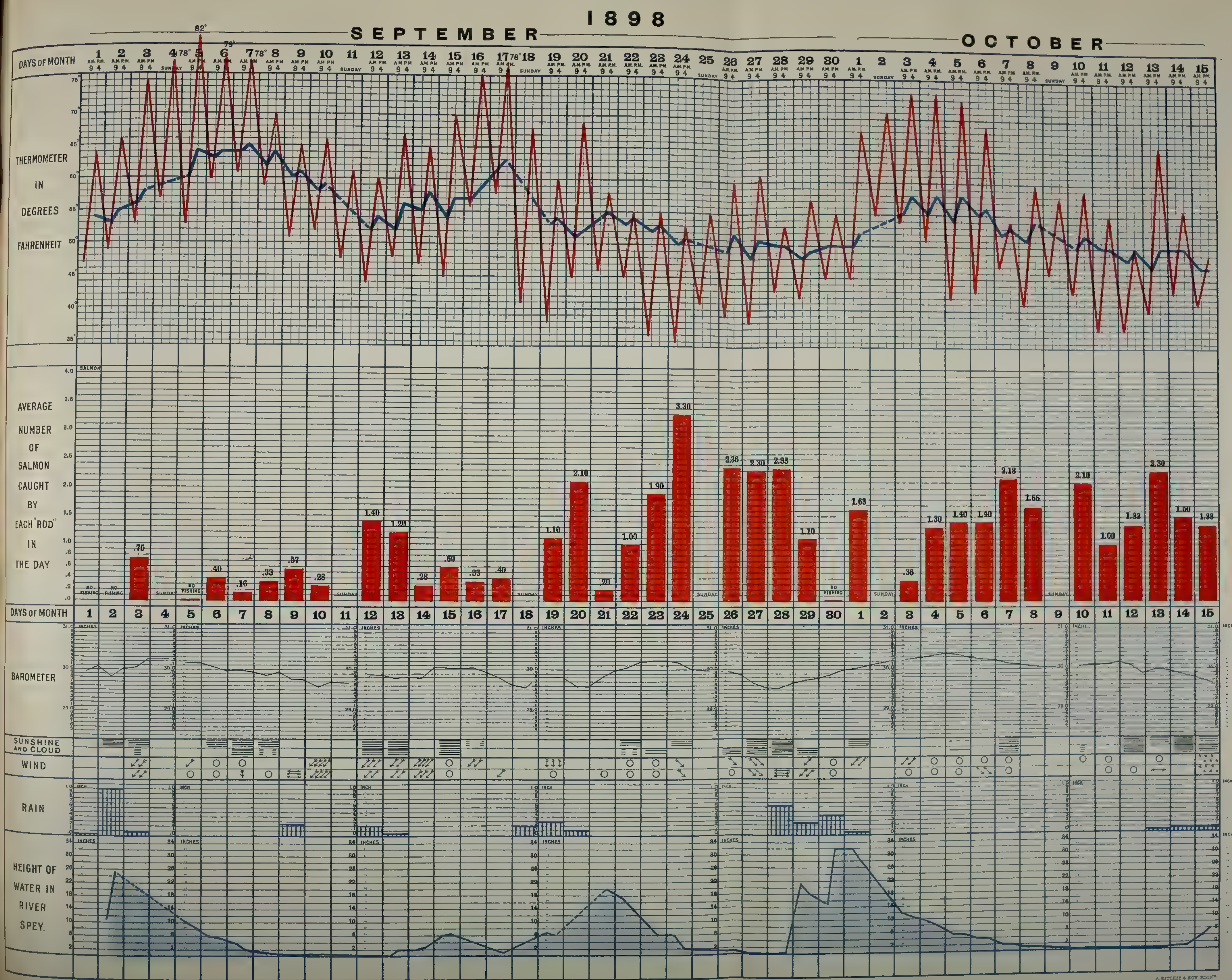
Average Weekly Variation in Temperature in Degrees Fahrenheit.	Average Number of Salmon taken daily by each 'Rod.'
3°	2·51
4	1·80
5	1·36
6	1·40
7	1·35
8	1·07
9	0·78
11	0·70
12	0·40

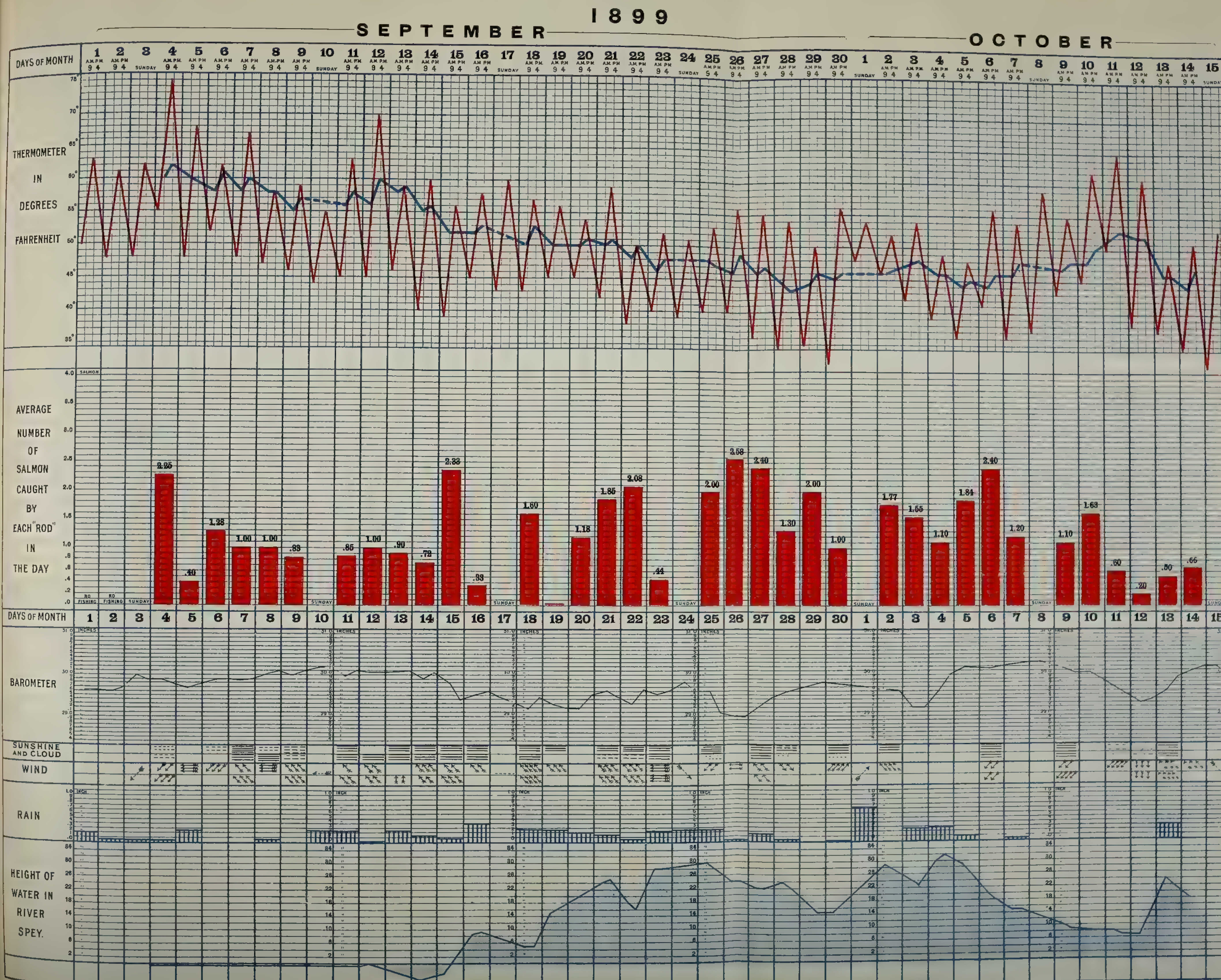
CONCLUSIONS.

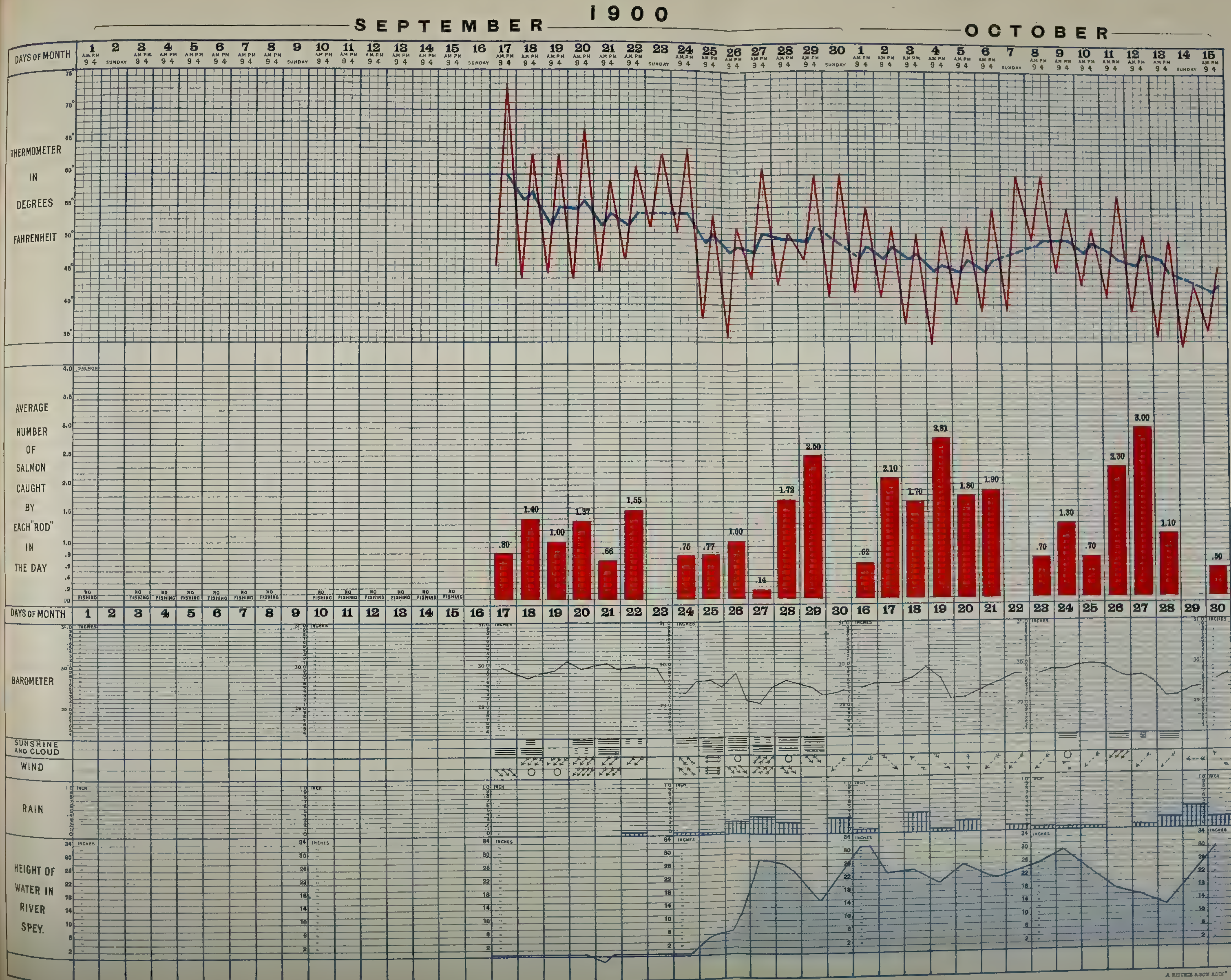
1. That salmon are influenced by variations in the temperature of the water in the taking of the artificial fly in the Spey at Gordon Castle during the rod-fishing season there.

2. That under ordinary conditions of river and weather they take the fly best there during a period when there is least variation in the temperature of the water of the river from day to day.

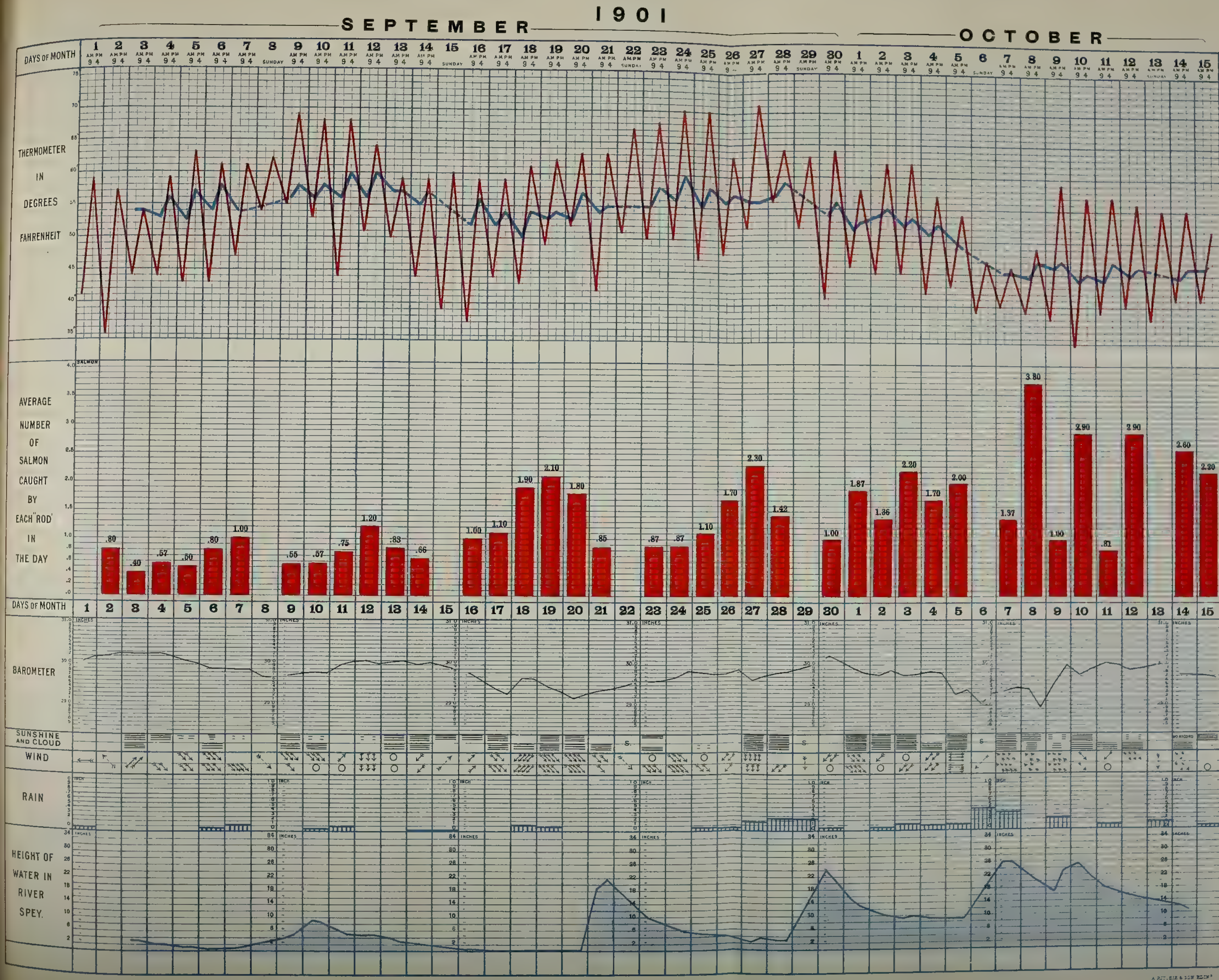
3. That under ordinary conditions of river and weather they do not take the fly well during a period when there is much variation in the temperature of the water of the river from day to day.


1898
SEPTEMBER
OCTOBER
DAYS OF MONTH
THERMOMETER IN DEGREES FAHRENHEIT
AVERAGE NUMBER OF SALMON CAUGHT BY EACH "ROD" IN THE DAY
BAROMETER
SUNSHINE AND CLOUD
WIND
RAIN
HEIGHT OF WATER IN RIVER SPEY.
A. Ritchie & Son Edin.


1899
SEPTEMBER
OCTOBER
DAYS OF MONTH
THERMOMETER IN DEGREES FAHRENHEIT
AVERAGE NUMBER OF SALMON CAUGHT BY EACH "ROD" IN THE DAY
2.25
.40
1.28
1.00
1.00
.83
.85
1.00
.90
.72
2.33
.33
1.60
1.18
1.85
2.08
.44
2.00
2.58
2.40
1.30
2.00
1.00
1.77
1.55
1.10
1.84
2.40
1.20
1.10
1.63
.60
.20
.50
.66
BAROMETER
SUNSHINE AND CLOUD
WIND
RAIN
HEIGHT OF WATER IN RIVER SPEY.



A. RITCHIE & SON EDIN.


1901
SEPTEMBER
OCTOBER
DAYS OF MONTH
THERMOMETER IN DEGREES FAHRENHEIT
AVERAGE NUMBER OF SALMON CAUGHT BY EACH "ROD" IN THE DAY
.80
.40
.57
.50
.80
1.00
.55
.57
.75
1.20
.83
.66
1.00
1.10
1.90
2.10
1.80
.85
.87
.87
1.10
1.70
2.30
1.42
1.00
1.87
1.36
2.20
1.70
2.00
1.37
3.80
1.00
2.90
.81
2.90
2.60
2.20
BAROMETER
SUNSHINE AND CLOUD
WIND
RAIN
HEIGHT OF WATER IN RIVER SPEY.

XXVIII.—*On the Distribution of Fossil Fish-remains in the Carboniferous Rocks of the Edinburgh District.* By Ramsay H. Traquair, M.D., LL.D., F.R.S., Keeper of the Natural History Collection in the Museum of Science and Art, Edinburgh. (With Two Plates.)

(Read July 1, 1901. Given in for publication May 6, 1903. Issued separately, October 16, 1903.)

The district in which the city of Edinburgh is situated was one of the first in Britain from which fish-remains of Carboniferous age were collected. It is now sixty-seven years ago since Agassiz described the fossil fishes which were discovered by Lord Greenock at Wardie, Dr Hibbert at Burdiehouse, and Professor Jameson at Burntisland. The list given from this region in the "Tableau Générale" at the beginning of the *Poissons Fossiles* comprises twenty-nine names, of which eight were *nomina nuda* and are not now verifiable, the original specimens being lost; one, *Diplodus minutus*, was described, but insufficiently, and the original is also lost; six are synonyms of others in the list; leaving fourteen good species, of which one, *Ptychacanthus sublævis*, is a synonym of a Selachian spine (*Tristychius arcuatus*), described and figured from the Glasgow district.

The list given by Salter in the Appendix to the "Geology of the Neighbourhood of Edinburgh" (*Mem. Geol. Survey, Scotland*, Sheet 32, 1861) is in the main a reproduction of that in the *Poissons Fossiles*, though it contains some additional species. Thirty-one names are given, of which four are of genera only; but of the rest, only eleven can be said to represent species which will "stand" at the present time. The richness in fishes of the Carboniferous rocks of the Edinburgh district had yet to be realised.

In 1890,* after many years' work, I published a *List of the Fossil Dipnoi and Ganoidei of Fife and the Lothians*, in which those of the Upper Old Red Sandstone were also included. Fifty species were here enumerated, and of these forty will stand as "good" for the district included in Sheet 32, with which we have to do in the present paper. Adding the Selachian form then omitted, and bringing the whole list up to date, we find that the Carboniferous Fish-fauna of the district in question, according to present knowledge, numbers no less than eighty-seven named species.

One feature of special interest in Scottish Carboniferous Palæontology is the opportunity afforded of comparing the plants and animals which lived under similar estuarine conditions during the deposition of the Lower and Upper divisions of the system respectively. This cannot be so readily done in England or Ireland, where the Upper Carboniferous rocks are mainly of "estuarine" or "lagoon" formation, and the Lower almost as exclusively marine in their origin, except in the extreme north. The case is,

* *Proc. Roy. Soc. Edin.*, vol. xvii. 1890, pp. 385–400.

however, different in Scotland, where the strata below the Millstone Grit, though containing intercalated beds of marine limestone, are largely similar to those of the overlying Coal Measures in their lithological character, and in the *facies* of their imbedded organic remains. This is a matter of great importance in connection with the question of Life-Zones in the Carboniferous system which has for some time back been engaging the attention of British geologists. Mr KIDSTON * has pointed out the dissimilarities between the Upper and Lower Carboniferous land flora in Great Britain, and I have on more than one occasion † drawn attention to the fact that different assemblages of estuarine fishes characterise the two great divisions of the strata of this period in our island.

Before proceeding with the enumeration of genera and species from the various horizons, it is, however, necessary to lay before the reader a general view of the relations of the strata of the Carboniferous area of the district in question, namely, that embraced in Sheet 32 of the Ordnance and Geological Surveys of Scotland. For that purpose I applied to the Director of the Geological Survey in Edinburgh, and received from him the general section of the Carboniferous rocks of the Lothians given in Plate I., and which was drawn up by Dr B. N. PEACH, F.R.S.

One thing which strikes the eye at the first glance is the small comparative thickness of the Upper Carboniferous series in this section, the Coal Measures with the Millstone Grit occupying only about 1500 feet, while the subjacent Lower Carboniferous attains a thickness of at least 7000 feet, and is probably still thicker in its lower part than here represented.

The Lower Carboniferous division commences above with the so-called "Carboniferous Limestone Series," which, however, consists principally of sandstones, shales, fireclays, ironstones, and coal-seams, some of the latter being of great economic value. Intercalated among these towards the top, and again towards the bottom of the series, are beds of marine limestone ; hence the threefold subdivision into Upper Limestone, Edge Coals, and Lower Limestones—the name of the middle group being derived from the circumstance that at Drum and Niddrie, in Midlothian, these strata are tilted up on "edge," so as in fact to be nearly vertical.

Below the Carboniferous Limestone Series comes the thickest part of the Lower Carboniferous division in this region ; namely, the "Calciferous Sandstones" of MACLAREN, the subordinate members of which are noted in the section, and will be dealt with in succession further on. This series is in the Edinburgh district characterised by a rarity of "marine" beds, the principal limestone, that of Burdiehouse, being also of estuarine origin, like the sandstones, shales, and ironstones which constitute the mass of the rocks here included. Most geologists are agreed that the Calciferous Sandstones of Scotland represent the lower portion of the "Carboniferous Limestone" of England and Ireland, though deposited under very different conditions.

* *Proc. Roy. Phys. Soc. Edin.*, vol. xii. 1893, p. 190 *et seq.*

† *Loc. cit.*, p. 386, 387. Also, *Geol. Mag.* (3), vol. i. 1884, p. 121.

The species noted in the following lists are all in the Edinburgh Museum of Science and Art, or in my private collection, with the exception of a few from the marine beds of the Lower Limestone group, which are in the collection of the Geological Survey of Scotland. For leave to include the latter I have to record my thanks to the Director of the Survey, Mr J. J. H. Teall, F.R.S.

LISTS OF SPECIES.

Calciferous Sandstone Series.

Beds below the Horizon of the Craigleith and Granton Sandstones.

No fish-remains whatever have been found in the *Craigmillar* or Red Sandstone Group, which seems to stand on the border-line between the Lower Carboniferous and the Upper Old Red. Nor have any occurred in the sandstones which in this district are reckoned by the Geological Survey to the last-named formation (Upper Old Red).

Ballagan Beds.—Scales referable to *Rhizodus*, along with a few of undeterminable palæoniscid type, were found in 1898 by Mr D. Tait, of the Geological Survey, in rocks exposed during excavations for the foundations of the new *Scotsman* Offices at the North Bridge, and are now in the collection of the Survey.

Arthur Seat Group.—A bed of stratified ash underlying basalt at St Anthony's Chapel has long been known to contain remains of fishes as well as of plants. In Hugh Miller's *Testimony of the Rocks* mention is made of the finding there of a tooth of a "carboniferous *Holoptychius*" (=*Rhizodus*), by the late Dr MacBain; and in the *Geological Survey Memoir*, on Sheet 32 (1861), reference is also made to the occurrence of "scales of *Rhizodus*" in the same bed. The late Mr David Grieve, of Edinburgh, had a considerable collection of these remains, but they were unfortunately lost sight of after his death; and as none have been collected since, it is not possible to give a critically determined list of them.

Abbey Hill Shales.—The Arthur Seat beds are overlaid by what are termed the Abbey Hill Shales, from which scales of *Eurynotus* and *Rhadinichthys*, found in boring for water at Abbeyhill, were recorded by Mr John Henderson in 1880.* Recently Mr J. G. Duncan has found in them, also at a cutting for a drain, a few palæoniscid scales (*Elonichthys*), and also a fragmentary plate of *Megalichthys*.

Craigleith and Granton Sandstones.—On a piece of dark indurated shale from strata at Lochend, which are supposed to belong to this horizon, is a pretty well preserved specimen of *Rhadinichthys ornatissimus* (Agass.), an easily recognised palæoniscid fish, which is very characteristic of the Oil-Shale Group, and even extends into

* *Trans. Geol. Soc. Edin.*, vol. iv. p. 34.

the Edge Coal Series above. The specimen was found by the late Dr MacBain, and by him presented to the Edinburgh Museum.

Wardie Shales.—Here we are first introduced to the richness in fish-remains of the Carboniferous rocks of the Edinburgh district. These shales directly overlie the Granton and Craigleith Sandstones, and are estuarine in character, though containing some marine bands with *Myalina*, *Schizodus*, *Lingula*, etc., but no *hinged* Brachiopods. They are extensively exposed on the shore between Granton Quarry and Wardie, the last-named locality being that from which Lord Greenock collected the fishes described and figured by Agassiz, and which has since that time afforded to other collectors, myself included, much fine additional material.

Most of the fishes which have been collected at this locality (Wardie) are contained in hard clay-ironstone nodules, of which a great quantity may be seen on the beach, both *in situ* and detached from the shaly matrix. Coprolites form, however, the nuclei of the immense majority of these nodules, so that the expenditure of much time and patience is necessary towards obtaining specimens of fishes.

The list from Wardie beach, brought up to date, is as follows :—

Pleuracanthus, sp.
Acanthodes sulcatus, Ag.
Megalichthys, sp.
Rhizodus Hibberti (Ag.).
Gonatodus punctatus (Ag.).
Elonichthys Robisoni (Hibb.).
" *striatus* (Ag.).
Rhadinichthys ornatissimus (Ag.).
" *carinatus* (Ag.).
" *brevis*, Traq.
" *ferox*, Traq.
Nematoptychius Greenocki (Ag.).
Eurynotus crenatus, Ag.
Wardichthys cyclosoma, Traq.

Notes on Agassiz's species.—*Amblypterus nemopterus*, Ag., is included in *Elonichthys Robisoni* (Hibb.) ; *Eurynotus fimbriatus*, Ag., is a synonym of *E. crenatus*, Ag.

In shales exposed in the banks of the Water of Leith at Woodhall, close above the marine band (*Schizodus*-bed) belonging to the Wardie Shale horizon, *Tristychius arcuatus*, Ag., *Elonichthys Robisoni*, and *Eurynotus crenatus* were collected by the late Mr John Henderson. The last-named fish has also occurred in the shale immediately overlying the sandstone at Craigleith Quarry.

Hailes and Redhall Sandstones and Shales.—The Wardie beds are succeeded in Midlothian by the Hailes and Redhall Sandstones with associated shales. At Hailes Quarry, shale overlying the sandstone contains clay-ironstone nodules which have yielded *Acanthodes sulcatus*, *Tristychius arcuatus*, and *Diplodus*, sp. Shales seen in the bed of the Water of Leith between Slateford and Colinton, and reckoned to this group, have also yielded *Rhadinichthys carinatus*, *Rh. ornatissimus*, and *Eurynotus crenatus.*

Pumpherston Shales.—The next fish-bearing horizon in ascending order with which I am acquainted in the Edinburgh district is that of the Oil Shales, which in Linlithgowshire occur at a depth of 800 feet below the Burdiehouse Limestone, and are worked for oil at Pumpherston and the Roman Camp.

In No. 3 seam, "curly," there is a comparatively thin band which contains the following fishes :—

Acanthodes, sp.
Ctenodus interruptus, Barkas.
Mesopoma macrocephalum, Traq.
Elonichthys Robisoni.
Rhadinichthys carinatus.
Eurynotus crenatus.

The smaller fishes are usually entire and beautifully preserved. Of *Ctenodus interruptus* only one specimen has occurred, namely, a parasphenoid bone with attached palatopterygoids and tooth-plates, for which the Museum is indebted to Major J. PACE CLEGHORNE of Broxburn.

Burdiehouse Limestone.—This well-known limestone, with which the name of HIBBERT will ever be associated, attracted notice in the course of the fourth decade of last century on account of its fossil vertebrate remains. Such fossils were at the time not much known from British Carboniferous rocks, and the Burdiehouse Limestone is in consequence referred to in nearly every text-book of geology. It is still extensively wrought at Burdiehouse village, but seems now to yield very little in the way of fishes. Limestones of approximately the same horizon and containing similar fossils are or have been worked at Raw Camps near Midcalder, South Queensferry, and Burntisland and Starley Burn in Fife.

At Burdiehouse itself the list is as follows :—

Pleuracanthus, sp.
Callopristodus pectinatus, (Ag.).
Gyracanthus rectus, Traq.
Sphenacanthus serrulatus (Ag.).
Tristychius arcuatus.
" *minor*, Portl.
Cynopodius crenulatus, Traq.
Megalichthys laticeps, Traq.
Rhizodus Hibberti.
" *ornatus*, Traq.
Uronemus lobatus, Ag.
Ctenodus, sp. scales.
Gonatodus, sp.
Elonichthys Robisoni.
" *striatus.*
Rhadinichthys ornatissimus.
" *carinatus.*
Nematoptychius Greenocki.
Eurynotus crenatus.

Remarks on Agassiz's species.—AGASSIZ'S *Palæoniscus striolatus* and *Pygopterus Bucklandi* are synonyms of *Elonichthys Robisoni*. His *Ctenoptychius denticulatus* is the same as his *Ct. pectinatus* (= *Callopristodus*, Traq.) *Phyllolepis tenuissimus* is, I believe, founded partly on scales of *Rhizodus Hibberti*, partly on those of a large *Ctenodus*, probably *Ct. interruptus*. *Ptychacanthus sublævis* is clearly founded on a worn specimen of *Tristychius arcuatus*, and the specimens referred by AGASSIZ to *Gyracanthus formosus* have been erected by the present writer into a separate species, *G. rectus*, Traq. *Cladodus Hibberti*, *Cl. parvus*, *Diplopterus Robertsoni*, *Ctenodus Robertsoni*, and *Pygopterus Jamesoni* are named from Burdiehouse in the "Tableau Générale," but not described; and as the originals seem to have been lost, the names must simply drop. The same fate must, I fear, also befall *Diplodus minutus*, for the description and figures are not sufficient for accurate identification, and the originals seem also to have been lost.

Mr SALTER observes in his "Description and List of Fossils" from the district

(*Mem. Geol. Survey*, Sheet 32, Appendix) that "the fossils found in the Burdiehouse Limestones are for the greater part distinct from those of other parts of the Lower Carboniferous series. This may be partly due to greater attention having been paid to this celebrated limestone by collectors." This explanation of a supposed fact has indeed proved to be true even in a more emphatic sense than is implied by the word "partly." As regards the fishes, at least, with which we are alone dealing in this paper, if we leave out of consideration the names cancelled above, all the species contained in the Burdiehouse list except *Megalichthys laticeps* and *Uronemus lobatus* occur in the Dunnet shale above, and many of them also in the Wardie shales below.

I have seen no fish-remains from the hard limestone representing the Burdiehouse deposit at Burntisland, but in the associated calcareous shales *Elonichthys Robisoni*, *Rhadinichthys ornatissimus*, *Rh. carinatus*, and *Eurynotus crenatus* are of frequent occurrence.

Dunnet Shale.—This is one of the most important seams of oil shale in this district, and at Straiton and Pentland, where it was until a few years ago extensively wrought by the Clippens Oil Company, it has yielded an extensive set of fish-remains. Few are found in the productive shale itself; they principally occur in the "roof" and in the "floor" of the seam, and are for the most part enclosed in nodules of clay ironstone. The list is as follows:—

Pleuracanthus, sp.
Diplodus, sp.
Cladodus, sp.
Sphenacanthus serrulatus.
Tristychius arcuatus.
" *minor.*
Euphyacanthus semistriatus.
Gyracanthus rectus.
Aganacanthus striatulus, Traq.
Cynopodius crenulatus.
Acanthodes sulcatus.
Megalichthys lævis, Traq.
Rhizodus ornatus.
Strepsodus striatulus, Traq.
Ctenodus interruptus.
Gonatodus macrolepis, Traq.
Elonichthys Robisoni.
" *striatus.*
" *pectinatus*, Traq.
Acrolepis semigranulosus, Traq.
Rhadinichthys ornatissimus.
" *carinatus.*
Nematoptychius Greenocki.
Eurynotus crenatus.

Of the twenty-one accurately determined species here recorded, twelve occur in the Burdiehouse Limestone below, which was considered to be a "fresh-water" limestone by Dr HIBBERT. Fragments of cephalopodous shells in the roof-shale bear witness to the fact that marine conditions were not far off, and that "estuarine" is rather a better word in the circumstances.

Regarding the fish-remains of the seams of oil shale which lie above the "Dunnet" and between it and the Lower marine limestones, I have no exact information, but no evidence that they were different in *facies* to those recorded above. Specimens of *Sphenacanthus serrulatus*, *Gyracanthus*, sp., *Acanthodes sulcatus*, *Nematoptychius Greenocki*, *Elonichthys Robisoni*, and *Elonichthys pectinatus* have been collected by the late Messrs C. W. PEACH and JOHN GIBSON, as well as by others, from the pit-heads at Oakbank, Addiewell, and West Calder; but the exact seams from which they were derived were not ascertained. They may, indeed, have been also from the "Dunnet."

Carboniferous Limestone Series.

The lower limit of the Carboniferous Limestone Series is placed by Sir A. Geikie at the Hurlet or Gilmerton Limestone, though purely marine beds also occur below this horizon, as at Kinghorn in Fife, where the first and second "Abden" Limestones are reckoned as still belonging to the Calciferous Sandstone series.* The late Mr John Henderson † also noted two such limestones, containing hinged brachiopods and crinoidal remains, at Stenhouse Mills in the Edinburgh district.

To Mr Cadell of Grange I am indebted for the section (Pl. II.) of the strata of the Carboniferous Limestone Series as developed at Gilmerton, Loanhead, and Niddrie, near Edinburgh. This section shows the relative position to each other of the various limestones, coals, and ironstones, and is specially useful for the purposes of this paper as the pits of this neighbourhood have yielded the greater number of the fish-remains collected from rocks of this series in the Edinburgh district.

Lower Limestone Group.

Marine Limestones.—Most of the fish-remains, all teeth and spines of marine Elasmobranchs, which have occurred in these limestones are from the quarries on the east side of the coal-field. The Museum collection is poor in these, but I have, as already said, to thank the Director of the Geological Survey for permission to include in my list a number of species which I have not seen except in the Scottish Survey collection. No exact information is, however, available as to the precise beds in the section from which they were derived—whether from the lowest limestone (No. 1, or "Hurlet"), or from those above (Nos. 2 and 3, or "Hosies"); the appended localities will, however, be so far useful. The list is meanwhile small, but may doubtless be much increased by future collecting.

Cladodus mirabilis, Ag., Mayfield.
,, *striatus*, Ag., Charlestown, Mayfield.
Petalodus acuminatus, Ag., Charlestown.
Ctenoptychius lobatus (R. Eth.), Charlestown.
,, *serratus* (Ag.), Cousland.
Petalorhynchus psittacinus (M'Coy), Mayfield.
Polyrhizodus magnus, M'Coy, Mayfield.
Pristodus falcatus, Davis, Charlestown.
Copodus planus (Davis), Brunstane.
Psammodus rugosus, Ag., Charlestown, Esperston.
Cochliodus contortus, Ag., Mayfield, Esperston.
Xystrodus striatus (M'Coy), Mayfield.
Pœcilodus Jonesii (M'Coy), Mayfield.
Psephodus magnus (M'Coy), Middleton.
Acondylacanthus Jenkinsoni (M'Coy), Mayfield.
Harpacanthus fimbriatus (Stock), Gilmerton.

It will be seen at a glance that not one of the above-quoted species occurs in any of the lists which I have given from the Calciferous Sandstone Series of the district, while all of them except *Harpacanthus fimbriatus* are well known from the Mountain Limestone of England, and, except *Pristodus falcatus*, of Ireland likewise. And that this

* "The Geology of Central and Western Fife," *Mem. Geol. Surv. Scotland*, 1900, pp. 73, 74.
† *Trans. Geol. Soc. Edin.*, vol. iv. 1882, pp. 217, 218.

striking difference on the one hand and similarity on the other is due to the conditions of deposit, is clearly shown in the next and following lists of species.

Gilmerton Ironstone.—Between the two lowest marine limestones (see Pl. II.) is a well-known coal-seam, the "North Greens," and not far above that is the "Gilmerton Ironstone," which yielded in times gone by a multitude of large bones and jaws of *Rhizodus Hibberti* and *Rh. ornatus*, as well as the originals of the labyrinthodonts *Pholidogaster pisciformis*, Huxley, *Loxomma Allmani*, Huxley, and *Macromerium Scoticum*, Lydekker. On my return to Edinburgh in 1874 I found that this ironstone was again being worked at Venturefair Pit, Gilmerton, and, with the aid of some of the miners, enlarged the list of fishes to the following :—

Acanthodes, sp.
Gonatodus macrolepis.
Elonichthys Robisoni.
„ *multistriatus*, Traq.
Nematoptychius Greenocki.
Eurynotus crenatus.
Megalichthys, sp.
Rhizodus Hibberti.
„ *ornatus.*
Sagenodus quinquecostatus, Traq.

Here it will be seen that the Elasmobranch fauna of the marine limestones is utterly wanting, and the estuarine fishes have returned. Of the eight specifically determined fishes of the above list, no less than six are found in the oil shales, including the Wardie beds, below, and consequently must have been living elsewhere in waters adapted to their organisation during the time when the Hurlet Limestone was deposited under true marine conditions on the same spot.

Edge Coal Group.

The next bed to be noticed is a "blackband" ironstone near the horizon of the North Coal which has been worked at Niddrie. It is accompanied by a hard grey micaceous shale with *Lingula squamiformis*, and consequently of marine or brackish water origin. This shale contains, moreover, an interesting set of fish-remains, as follows :—

Pleuroplax falcatus, Traq.
Oracanthus armigerus, Traq.
Tristychius arcuatus.
„ *minor.*
Euphyacanthus semistriatus.
Megalichthys, sp.
Rhizodopsis, sp.
Cœlacanthus Abdenensis, Traq.
Eurynotus crenatus.
Rhadinichthys, sp.

Here we have a fish-fauna which, especially in the presence of *Pleuroplax falcatus*, *Oracanthus armigerus*, and *Cœlacanthus Abdenensis*, reminds us at once of that of the Abden "Bone Bed" at Kinghorn, in Fife, though at a considerably higher horizon, for the Fifeshire bed is placed by Sir A. Geikie near the top of the Calciferous Sandstones, so that we have consequently the whole of the Lower Limestone Group between. It is clear that we have here another instance of the recurrence of similar forms under similar conditions, for the Abden bed is still more obviously of marine origin, and lies close below limestone containing hinged brachiopods, corals, etc.

Loanhead Ironstone, No. 1.—This ironstone (see Pl. II.) lies between "Brown's" coal and the "Glass" coal, nearly in the middle of the Edge Coal Group. Above it is also a shale with *Lingula squamiformis*, and, including the species found both in that and in the ironstone and accompanying "parrot," the list is as follows:—

Pleuroplax falcatus.
Pleuroplax, sp.
Oracanthus armigerus.
Tristychius arcuatus.
„ *minor.*
Acanthodes, sp.
Rhizodus Hibberti.
Cœlacanthus Abdenensis.
Ctenodus interruptus.
Elonichthys Robisoni.
Nematoptychius Greenocki.
Eurynotus crenatus.

All the species in the above list we have already noted from beds below, but it is of interest to find that along with the *Lingula* we have again the *Pleuroplax*, *Oracanthus*, and *Cœlacanthus* of the ironstone shale last mentioned.

Borough Lee Ironstone, or *Loanhead Ironstone, No. 2.*—This ironstone is situated above the "Great" Seam near the top of the Edge Coal Group. It is the most productive in species of fishes and fish-remains of the beds which has turned up in the Lothians, and has yielded no fewer than thirty-five species, some of which have not been found elsewhere. They are as follows:—

Diplodus parvulus, Traq.
Pleuracanthus elegans, Traq.
„ *gracillimus*, Traq.
„ *horridulus*, Traq.
„ *fastigiatus*, Davis.
Dicentrodus bicuspidatus, Traq.
Callopristodus pectinatus (Ag.).
Gyracanthus nobilis, Traq.
„ *Youngi*, Traq.
Aganacanthus striatulus, Traq.
Tristychius arcuatus.
„ *minor.*
Sphenacanthus serrulatus.
Cynopodius crenulatus.
Euctenius elegans, Traq.
Megalichthys, sp.
Rhizodopsis, sp.
Rhizodus Hibberti.
Strepsodus striatulus.
Uronemus splendens, Traq.
Ctenodus interruptus.
„ *angustulus*, Traq.
Sagenodus quinquecostatus.
Eurylepis Scoticus, Traq.
Gonatodus parvidens, Traq.
Drydenius insignis, Traq.
Elonichthys Robisoni.
„ *pectinatus.*
Rhadinichthys ornatissimus.
„ *carinatus.*
Nematoptychius Greenocki.
Cryphiolepis striatus, Traq.
Eurynotus microlepidotus, Traq.
Cochliodont spine, undet.

Leaving *Diplodus parvulus* and *Euctenius elegans* out of consideration, as they may have belonged to one or other of the *Pleuracanthi* here noted, we find in this list thirty named species, of which fifteen have already occurred in the strata below. Some—namely, *Tristychius arcuatus*, *Rhizodus Hibberti*, *Elonichthys Robisoni*, *Rhadinichthys carinatus*, *Nematoptychius Greenocki*—extend down even into the Wardie shales, in which all, except the first, are common species. On the other hand, however, fifteen species make their first appearance here, and of these six have not yet been found elsewhere.

Upper Limestone Group.

South Parrot Coal Shale.—Passing upwards, we reach the Upper Limestone Group at the "Index" Limestone (No. 4 in the section, Pl. II.). Lying between this limestone and No. 5, the "Calmy" or "Arden," but nearer to the former, is the South Parrot Coal-seam, at present extensively wrought at Niddrie. The roof-shale is of a glossy black colour, and contains here and there fragments of *Lepidodendron*, and on one piece I detected a *Lingula*. In general appearance, colour, and texture it closely resembles some Upper Carboniferous fish-bearing shales, both Scotch and English; but how different are the species of fishes contained in it! The following have been obtained up till now:—

Diplodus parvulus.
Pleuracanthus elegans.
„ *horridulus.*
„ *fastigiatus*, Davis.
„ sp.
Dicentrodus bicuspidatus, Traq.
Callopristodus pectinatus (Ag.).
Gyracanthus, sp.
Aganacanthus striatulus.
Sphenacanthus serrulatus.
Tristychius arcuatus.
„ *minor.*
Euphyacanthus semistriatus.
Cynopodius crenulatus.
Euctenius elegans.
Rhizodus Hibberti.
„ *ornatus.*
Strepsodus striatulus.
„ sp.
Ctenodus interruptus.
Sagenodus quinquecostatus.
Gonatodus parvidens.
Elonichthys pectinatus.
Rhadinichthys, sp.
Nematoptychius Greenocki.
Eurynotus crenatus.

Now, though a considerable thickness of strata, including one truly marine limestone, intervenes between this South Parrot Coal Shale and the Borough Lee Ironstone below, we are at once struck by the similarity of the two lists of fishes. Of the twenty-three named species occurring in the list given above, twenty-one are also found in the Borough Lee bed, the two not so accounted for being *Euphyacanthus semistriatus* and *Rhizodus ornatus*, both of which occur, however, in the Calciferous Sandstone Series below. The three undetermined species are: a *Gyracanthus*, represented by species too imperfect and eroded for correct identification; a *Strepsodus*, indicated by teeth which resemble in shape and markings those of the Upper Carboniferous *S. sauroides*, Binney, but probably new; and a small *Rhadinichthys*.

MILLSTONE GRIT.

No fish-remains have been found in the strata belonging to this group in the Edinburgh district, nor in Scotland in general, so far as I am aware.

COAL MEASURES.

The productive Coal-bearing strata of the Edinburgh district above the Millstone

Grit are, like those of other parts of Scotland, referred by Mr KIDSTON, on account of their fossil flora, to the Lower Coal Measures of England. Unfortunately few fish-bearing shales are or have been available to the collector in the rocks in question; in fact, the only one which has of late yielded such remains is that to which we shall now refer.

The "Four Foot" Coal at present worked at Niddrie is the lowest but one of the workable seams of the Upper Carboniferous in this neighbourhood, and it so happened that a few years ago an inclined shaft was being driven down along the dip of the seam, and a considerable amount of the roof-shale was brought to the surface. From it were obtained the following fish-remains :—

Diplodus gibbosus, Ag.
Janassa linguæformis.
Ctenoptychius apicalis, Ag.
Lepracanthus Colei.
Pleuroplax Rankinei.
Sphenacanthus, tooth.
Acanthodes Wardi, Egert.
Acanthodopsis Wardi.
Megalichthys Hibberti, Ag.
Megalichthys pygmæus, Traq.
Rhizodopsis sauroides, Williamson.
Strepsodus sauroides (Binney).
" *sulcidens.*
Cœlacanthus elegans, Newb.
Elonichthys Aitkeni, Traq.
Platysomus parvulus, Young.
" *Forsteri.*
Cheirodus, sp.

The difference between this list and those previously given is startling, as not one species found in the Calciferous Sandstone or the Carboniferous Limestone Series is here represented; but this is a matter to which we shall presently return.

Smeaton Colliery.—In former times many fish-remains were procured at Smeaton, near Dalkeith, from the roof-shale of a seam which, I understand, was the "Jewell." Unfortunately this colliery has been shut up for more than twenty years, so that no fresh material can be obtained, and the list which I am able to give is rather short. It only comprises the following :—

Pleuracanthus lævissimus, Ag.
Gyracanthus formosus, Ag.
Acanthodopsis Wardi.
Megalichthys Hibberti.
Rhizodopsis sauroides.
Strepsodus sulcidens.
Cœlacanthus elegans.

If the fish-remains marked "Dalkeith Coal Field," and collected many years ago, which one finds in various museums, including the British Museum and the Edinburgh Museum of Science and Art, came, as is very possible, also from this colliery, then we must add *Sphenacanthus hybodoides* (Egert.) and *Strepsodus sauroides* to the list. Sir PHILIP EGERTON indeed gave "the neighbourhood of Dalkeith" as the source of his "*Ctenacanthus*" *hybodoides* and *nodosus.*

RESULTS.

The palæontological record is of necessity imperfect, and our opportunities of collecting even the fossils which have been preserved are also limited by circumstances which are hardly within our control; indeed, they come largely under the operation of what we are pleased to designate as "chance." The foregoing lists cannot in any way be considered as containing a complete record of the fishes contained in the Carboniferous rocks of the district around Edinburgh. Nevertheless, it will be well, after having worked at these remains for nearly thirty years now, to put together the general results as to their distribution to which we are in the meantime conducted. These results are, in the main, already known to those who are specially interested in palæozoic ichthyology, but a more concise account as to how the matter stands at present may not be unwelcome to geologists in general.

Lower Carboniferous.

We have seen that the fish-remains found in the limestones of open sea origin at Mayfield, Charlestown, and other places are different from those occurring in the estuarine beds, and that they belong, in fact, to the marine fish-fauna characteristic of the Mountain Limestone of England and Ireland. This fauna consists mainly of Elasmobranch forms, Dipnoi and "Ganoids" being rare; while, on the other hand, the estuarine strata throughout the whole Carboniferous formation are characterised not only by fishes coming under the two latter designations, but also by a set of Elasmobranchs which differ specifically and for the most part also generically from those of the marine limestones.

Rarely do we find any commingling of these two faunæ; one notable exception being at East Kilbride, in Lanarkshire, where the shale overlying the Calderwood Cement Limestone contains such estuarine forms as *Rhizodus Hibberti*, *Strepsodus striatulus*, *Elonichthys Robisoni*, and *Rhadinichthys ornatissimus*, along with such typically marine species as *Petalodus acuminatus*, *Psephodus magnus*, *Pœcilodus Jonesii*, *Psammodus porosus*, etc.

As to the zonal question, I am not aware that any definite succession of forms can be established, either in Great Britain or in Ireland, on the fish-fauna of the marine limestones.

What, then, of the estuarine fishes in which the Lower Carboniferous rocks of the Edinburgh district are so rich?

Our observations must commence with the Craigleith and Wardie beds, as no specifically determinable fish-remains are available from the underlying strata; but from that horizon upwards this estuarine fish-fauna persists in its main features into the Upper Limestone Group, the South Parrot Coal shale being the highest bed in the Lower Carboniferous division from which fossil fishes have been in this region obtained. And an examination of the accompanying table of vertical distribution shows that it is scarcely possible to mark out satisfactory zones by means of the fishes, so far as the *Lower Carboniferous* rocks are concerned;—of the *Upper* we shall presently speak.

	Wardie Shales.	Pumpherston Shales.	Burdiehouse Limestone.	Dunnet Shale.	Lower Limestone Group.	Edge Coals.	Upper Limestone Group.	Coal Measures.
Elasmobranchii.								
Pleuracanthus lævissimus, Ag.	...	...	...	...	...	...	...	+
,, *elegans*, Traq.	...	...	...	...	...	+	+	...
,, *gracillimus*, Traq.	...	...	...	...	...	+	...	...
,, *horridulus*, Traq.	...	...	...	...	...	+	+	...
,, *fastigiatus* (Davis)	...	...	...	...	...	+	+	...
Diplodus gibbosus, Ag.	...	...	...	...	...	...	...	+
,, *parvulus*, Traq.	...	...	...	+	...	+	+	...
Cladodus mirabilis, Ag.	...	...	...	...	+	...	...	...
,, *striatus*, Ag.	...	...	...	...	+	...	...	...
Dicentrodus bicuspidatus, Traq.	...	...	...	...	...	+	+	...
Janassa linguæformis, Atthey	...	...	...	...	...	...	...	+
Petalorhynchus psittacinus (M'Coy).	...	...	...	...	+	...	...	...
Petalodus acuminatus (Agass.)	...	...	...	...	+	...	...	...
Ctenoptychius serratus (Ag.)	...	...	...	...	+	...	...	...
,, *lobatus* (R. Eth., jun.)	...	...	...	...	+	...	...	...
,, *apicalis*, Agass.	...	...	...	...	...	...	...	+
Polyrhizodus magnus, M'Coy	...	...	...	...	+	...	...	...
Callopristodus pectinatus (Ag.)	...	...	+	...	...	+	+	...
Pristodus falcatus, Davis	...	...	...	...	+	...	...	...
Copodus planus (Davis)	...	...	...	...	+	...	...	...
Psammodus rugosus, Agass.	...	...	...	...	+	...	...	...
Helodus simplex, Agass.	...	...	...	...	...	...	...	+
Pleuroplax Rankinei, Han. and Atthey	...	...	...	...	...	...	...	+
,, *falcatus*, Traq.	...	...	...	...	...	+	...	...
Xystrodus striatus, M'Coy	...	...	...	...	+	...	...	...
Cochliodus contortus, Ag.	...	...	...	...	+	...	...	...
Pœcilodus Jonesii (M'Coy)	...	...	...	...	+	...	...	...
Psephodus magnus (M'Coy)	...	...	...	...	+	...	...	...
Acondylacanthus Jenkinsoni (M'Coy)	...	...	...	...	+	...	...	...
Lepracanthus Colei, Owen	...	...	...	...	...	...	...	+
Tristychius arcuatus, Ag.	+	...	+	...	...	...	+	...
,, *minor*, Portlock	...	...	+	...	...	...	...	...
Euphyacanthus semistriatus, Traq.	...	...	...	+	...	...	+	...
Sphenacanthus serrulatus (Ag.)	...	...	+	+	...	+	+	...
,, *hybodoides* (Egert.)	...	...	...	...	...	...	...	+
Gyracanthus rectus, Traq.	...	...	+	+	...	...	...	...
,, *nobilis*, Traq.	...	...	...	...	...	+	...	...
,, *Youngi*, Traq.	...	...	...	...	...	+	...	...
,, *formosus*, Ag.	...	...	...	...	...	...	...	+
Cynopodius crenulatus, Traq.	...	...	+	+	...	+	+	...
Euctenius elegans, Traq.	...	...	...	...	...	+	+	...
Harpacanthus fimbriatus (Stock.)	...	...	...	...	+	...	...	...
,, *major*, Traq.	...	...	...	...	...	+	...	...
Acanthodes Wardi, Egert.	...	...	...	...	...	...	...	+
,, *sulcatus*, Agass.	+	?	+	+	...	?	?	...
Acanthodopsis Wardi, Han. and Atthey	...	...	...	...	...	...	...	+
Crossopterygii.								
Megalichthys Hibberti, Ag.	...	...	...	...	...	...	...	+
,, *laticeps*, Traq.	...	...	+	...	...	...	...	...

	Wardie Shales.	Pumpherston Shales.	Burdiehouse Limestone.	Dunnet Shale.	Lower Limestone Group.	Edge Coals.	Upper Limestone Group.	Coal Measures.
CROSSOPTERYGII—*continued.*								
Megalichthys lævis, Traq.	...	...	...	+	...	...	...	...
,, *pygmæus*, Traq.	...	...	...	...	...	...	...	+
,, sp. undet.	+	...	...	...	...	+	...	...
Rhizodopsis sauroides (Williamson)	...	...	...	...	...	...	...	+
Rhizodus Hibberti (Ag.)	+	...	+	...	+	+	+	...
,, *ornatus*, Traq.	...	...	+	+	...	...	+	...
Strepsodus sauroides (Binney)	...	...	...	...	...	...	...	+
,, *sulcatus*	...	...	...	...	...	...	...	+
,, *striatulus*	...	...	...	+	...	+	+	...
Cœlacanthus elegans	...	...	...	...	...	...	...	+
Cœlacanthus Abdenensis	...	...	...	...	...	+	...	...
DIPNOI.								
Uronemus lobatus, Ag.	...	...	+	...	...	...	...	...
,, *splendens*, Traq.	...	...	...	...	...	+	...	...
Ctenodus interruptus, Barkas	...	+	?	+	...	+	+	...
,, *cristatus*, Ag.	...	...	...	...	...	...	...	+
,, *angustulus*	...	...	...	...	...	+	...	...
Sagenodus quinquecostatus, Traq.	...	...	...	...	+	+	+	...
ACIPENSEROIDEI.								
Eurylepis Scoticus, Traq.	...	...	...	...	...	+	...	...
Gonatodus punctatus (Ag.)	+	...	?	...	...	...	...	...
,, *macrolepis*, Traq.	...	...	...	+	+	...	...	...
,, *parvidens*, Traq.	...	...	...	...	...	+	+	...
Drydenius insignis, Traq.	...	...	...	...	...	+	...	...
Elonichthys Robisoni (Hibbert)	+	+	+	+	+	+	...	...
,, *striatus* (Ag.)	+	...	+	+	...	...	...	...
,, *pectinatus*, Traq.	...	...	...	+	...	+	+	...
,, *multistriatus*, Traq.	...	...	...	...	+	+	...	...
,, *Aitkeni*, Traq.	...	...	...	...	...	...	...	+
Mesopoma macrocephalum, Traq.	...	+	...	...	...	...	...	...
Acrolepis semigranulosa, Traq.	...	...	...	+	...	...	...	...
Rhadinichthys ornatissimus (Ag.)	+	...	+	+	...	+	...	...
,, *carinatus* (Ag.)	+	+	+	+	...	+	?	...
,, *brevis*, Traq.	+	...	...	...	...	...	...	...
,, *ferox*, Traq.	+	...	...	...	...	...	...	...
Nematoptychius Greenocki (Ag.)	+	...	+	+	+	+	+	...
Cryphiolepis striatus, Traq.	...	...	...	...	...	+	...	...
Eurynotus crenatus, Ag.	+	+	+	+	+	+	+	...
,, *microlepidotus*, Traq.	...	...	...	...	...	+	...	...
Wardichthys cyclosoma, Traq.	+	...	...	...	...	...	...	...
Cheirodus granulosus (Young)	...	...	...	...	...	...	...	+
,, *crassus*, Traq.	...	...	...	...	...	+	...	...

NOTE.—A few of the genera range lower down than is indicated in the above list, which only includes specifically determined remains, and many specimens are not so determinable. *Rhizodus* ranges down to the Ballagan beds, *Megalichthys*, *Rhadinichthys*, and *Eurynotus* to the Abbeyhill shales, and *Pleuracanthus* (incl. *Diplodus*) to the Wardie shales. The remains of *Acanthodes*, *Megalichthys*, and *Rhizodopsis* occurring in the ironstones of the Carboniferous Limestone Series are also too fragmentary for secure specific identification.

Some amount of difference there is between the lists from the top and from the bottom of the fish-bearing series. For instance *Gyracanthus rectus* gives way to *G. nobilis* and *Youngi*; *Gonatodus punctatus* of Wardie is succeeded by *G. macrolepis* in the Dunnet shale and Gilmerton Ironstone, and by *G. parvidens* in the Borough Lee Ironstone and South Parrot shale. Some species have as yet only been found in certain beds, as *Wardichthys cyclosoma* and *Rhadinichthys ferox* at Wardie, *Uronemus lobatus* at Burdiehouse, *Elonichthys multistriatus* in the Gilmerton ironstone, *Uronemus splendens*, *Drydenius insignis*, *Cryphiolepis striatus* and others in the Borough Lee Ironstone. On the other hand, some of the commonest species appearing in the Wardie shales below pass up into the Upper Limestone Group, while others have a not much less extended range. This will be best understood by reference to the following table, in which the vertical ranges of twenty common species may be compared.

	Wardie Shale.	Pumpherston Shale.	Burdiehouse Limestone.	Dunnet Shale.	Borough Lee Limestone.	South Parrot Coal.
Acanthodes sulcatus	+	?	...	+	...	...
Elonichthys striatus	+	...	+	+	...	...
Rhadinichthys ornatissimus	+	...	+	+	+	...
,, *carinatus*	+	+	+	+	+	?
Elonichthys Robisoni	+	+	+	+	+	...
Tristychius arcuatus	+	...	+	+	+	+
Rhizodus Hibberti	+	...	+	...	+	+
Nematoptychius Greenocki	+	...	+	+	+	+
Eurynotus crenatus	+	+	+	+	+	+
Ctenodus interruptus	...	+	?	+	...	+
Sphenacanthus serrulatus	...	...	+	+	+	+
Rhizodus ornatus	...	...	+	+	...	+
Cynopodius crenulatus	...	...	+	+	+	+
Tristychius minor	...	...	+	+	+	+
Callopristodus pectinatus	...	...	+	...	+	+
Euphyacanthus semistriatus	...	...	...	+	...	+
Elonichthys pectinatus	...	...	...	+	+	+
Aganacanthus striatulus	...		...	+	+	+
Strepsodus striatulus	...	...	...	+	+	+

If a separate zone could be distinguished it would be one comprising the Borough Lee Ironstone and the South Parrot Coal shale, as these contain at least seven species which are not found in the strata below. But the fish-fauna of the Borough Lee stone is so closely linked with that of the Dunnet shale, and the latter again with those of Burdiehouse and Wardie, that the best plan seems to be for the present to look upon the Lower Carboniferous of the Central Valley of Scotland as forming one great life-zone so far as the fishes are concerned. For the Lower Carboniferous fishes of Fifeshire, Lanarkshire, and Ayrshire belong essentially to the same great assemblage of forms as those of the Lothians, though in the West the number of marine species is greater and of estuarine less.

But now a word must be said about the strange fact that a different Lower Carboniferous estuarine fish-fauna crops up when we get to the border country on the south side of the Southern Uplands. This is at all events the case in Eskdale, and I rather think that the fishes of the Calciferous Sandstone Series in Berwickshire will be found to vary considerably from those of the north side of the hills. But Berwickshire is not yet sufficiently searched, and I shall therefore confine the present remarks to the former locality, where, more than twenty years ago, a novel assemblage of fossil fishes was discovered by Mr MACCONOCHIE, collector to the Scottish Geological Survey, and entrusted to me by Sir A. GEIKIE for description. The material was also afterwards increased by collections made by Mr JEX, collector to Mr DAMON of Weymouth, and by Mr T. STOCK, the best part of these subsequent collections being now in the British Museum and in the Edinburgh Museum of Science and Art.

Subjoined is the list from Glencartholm on the Esk, near Langholm, with the inclusion of one interesting species from Tarras Foot, marked with an asterisk.

Chondrenchelys problematica, Traq.
Cladodus, sp.
Sphenacanthus costellatus, Traq.
Tristychius minor.
Acanthodes nitidus, A. S. Woodw.
Tarrasius problematicus, Traq.
Megalichthys, sp.
Strepsodus, 2 species.
Cœlacanthus Huxleyi, Traq.
Canobius Ramsayi, Traq.
„ *elegantulus*, Traq.
Mesopoma pulchellum, Traq.
„ *politum*, Traq.
Elonichthys serratus, Traq.
„ *pulcherrimus*, Traq.
Acrolepis ortholepis, Traq.
Cycloptychius concentricus, Traq.
Rhadinichthys fusiformis, Traq.
„ *Macconochii*, Traq.
„ *delicatulus*, Traq.
„ *tuberculatus*, Traq.
„ *angustulus*, Traq.
**Styracopterus fulcratus*, Traq.
Phanerosteon mirabile, Traq.
Eurynotus, 2 sp.
Platysomus superbus, Traq.
Mesolepis rhombus, Traq.
„ *tuberculatus*, Traq.
Cheirodopsis Geikiei, Traq.

The list is not yet complete for the entire region, as the few Liddesdale fishes are not yet completely worked up; but this assemblage from Glencartholm is sufficiently striking, seeing that only one named species in it, namely, *Tristychius minor*, is found in the Lower Carboniferous beds of Central Scotland. A land barrier may have been the cause of this striking difference in the fishes, as Mr KIDSTON finds that of the terrestrial plants of the deposit some are peculiar to the locality, but more are common to the Calciferous Sandstones. At all events we have not to do with strata deposited in a land-locked lake, seeing that marine shells (*Orthoceras*, *Conularia*, *Bellerophon*, *Aviculopecten*, etc.) occur close to the band which is most productive in fishes.

Several years ago Mr B. N. PEACH suggested to me that the Eskdale beds might belong to a lower horizon than those from which the Edinburgh list is derived, and it must be remembered that in the great thickness of strata below the Craigleith and Granton Sandstones fish-remains are extremely scanty, consisting only of a few isolated scales (teeth also at Arthur's Seat), which as yet are not specifically determinable. Should this prove to be the correct explanation of the case, then we should have a zonal distinction, and a very marked one too. But more work has yet to be done ere the problem can be said to be placed on a satisfactory basis.

Upper Carboniferous.

The list of fishes from the Upper Carboniferous strata of the Edinburgh district is as yet short, seeing that it contains only nineteen species, but as not one of these can safely be identified as occurring in the rocks below, we have evidently got into quite a new ichthyological stage.

This is, however, the same fish-fauna as that of the Upper Carboniferous of the rest of Britain, as will clearly be seen by comparing the following lists.

First, as to the West of Scotland, where the rocks of this division occupy a much greater area than in the East, and are more extensively worked for coal. I extract the following Upper Carboniferous species from my list of the Carboniferous Fishes of the West of Scotland, published in the *British Association Handbook*, Glasgow, 1901.

Diplodus gibbosus, Ag.
Pleuracanthus lævissimus, Ag.
„ *cylindricus* (Ag.).
„ *robustus*, Davis.
„ *alatus*, Davis.
„ *denticulatus*, Davis.
„ *Thomsoni*, Davis.
„ *tenuis*, Davis.
Janassa linguæformis, Atthey.
Ctenoptychius apicalis, Ag.
Callopristodus pectinatus (Ag.).
Helodus simplex, Ag.
Pleuroplax Rankinei (Ag.)
„ *Attheyi* (Barkas).
Sphenacanthus hybodoides (Egert.).
Euctenius unilateralis (Barkas).
Lepracanthus Colei, Owen.
Gyracanthus formosus, Ag.
Bertacanthus striatus, Traq.
„ *tuberculatus*, Traq.
Acanthodes Wardi, Egert.
Acanthodopsis Wardi, H. and Atth.
Megalichthys Hibberti, Ag.
„ *coccolepis*, Young.
„ *intermedius*, A. S. Woodward.
„ *pygmæus*, Traq.
Rhizodopsis sauroides (Williamson).
Strepsodus sauroides (Binney).
„ *sulcidens* (Atthey).
Cœlacanthus elegans, Newb.
Ctenodus cristatus, Ag.
Sagenodus inequalis, Owen.
Elonichthys Egertoni (Egert.)
„ *Aitkeni*, Traq.
Acrolepis Hopkinsi (M'Coy).
Rhadinichthys Grossarti, Traq.
„ *Monensis* (Egert.).
Cheirodus granulosus (Young).
Mesolepis, sp.
Platysomus parvulus, Young.
„ *Forsteri*, H. and Atthey.

Here we have forty-one species, including every one of those which occur in the Edinburgh district.

Passing next to the English Upper Carboniferous, we first note the fish-remains from the roof-shale of the Low Main seam at Newsham, near Newcastle-on-Tyne, belonging to the Lower Coal Measures, and which yielded to the labours of the late Messrs Hancock and Atthey quite an unusual number of species to come from a single bed. The following list of the species in the Atthey collection, now in the Museum at Newcastle, was made by myself a few years ago with the permission of the late Mr Howse, then curator of that museum :—

Pleuracanthus lævissimus.
„ *cylindricus.*
„ *robustus.*
„ *denticulatus.*
Diplodus gibbosus.
Gyracanthus formosus.
Sphenacanthus hybodoides.
Janassa linguæformis.
Callopristodus pectinatus.
Pleuroplax Rankinei.

Euctenius unilateralis.
Acanthodes Wardi.
Acanthodopsis Wardi.
Rhizodopsis sauroides.
Strepsodus sauroides.
„ *sulcidens.*
Megalichthys Hibberti.
„ *coccolepis.*
Cœlacanthus elegans.
Ctenodus cristatus.
Sagenodus inequalis.
Elonichthys semistriatus, Traq.
„ *Egertoni.*
Acrolepis Hopkinsi.
Eurylepis Anglica, Traq.
Rhadinichthys Wardi, Young.
„ *Hancocki*, Atthey.
Platysomus parvulus.
„ *Forsteri.*
„ *rotundus*, H. and Atthey.
Cheirodus striatus, H. and Atthey.

In no part of England has the fish-fauna of the Coal Measures been more sedulously collected and accurately determined than in North Staffordshire. The splendid collection made there by Mr John Ward, F.G.S., of Longton, and now in the British Museum, is an historical one, with which I have been for nearly thirty years familiar, and to the working out of which I have myself contributed. To Mr Ward personally, and to his work *On the Geological Features of the North Staffordshire Coal Fields*, I am indebted for the following lists :—

Lower Coal Measures (Cockshead Ironstone).

Helodus simplex.
Pleuroplax Rankinei.
Gyracanthus formosus.
Sphenacanthus hybodoides.
Acanthodes Wardi.
Megalichthys Hibberti.
Megalichthys coccolepis.
Cœlacanthus elegans.
Ctenodus cristatus.
Elonichthys, sp.
Mesolepis scalaris, Young.
Cheirodus granulosus (Young).

Middle Coal Measures (Ash or Rowhurst Ironstone and Coal up to below the Bassy Mine Ironstone).

Pleuracanthus lævissimus.
„ *robustus.*
„ *alatus*, Davis.
„ *Wardi*, Davis.
„ *cylindricus.*
Diplodus gibbosus.
„ *equilateralis*, Ward.
Janassa linguæformis.
Ctenoptychius apicalis.
Callopristodus pectinatus.
Helodus simplex.
Pleuroplax Rankinei.
Sphenacanthus hybodoides.
Gyracanthus formosus.
Euctenius unilateralis.
Listracanthus, sp.
Acanthodes Wardi.
„ *major*, Davis.
Acanthodopsis Wardi.
„ *microdon.*
Megalichthys Hibberti.
„ *coccolepis.*
„ *intermedius.*
„ *pygmæus.*
Rhizodopsis sauroides.
Strepsodus sauroides.
Cœlacanthus elegans.
Ctenodus cristatus.
Sagenodus inequalis.
Elonichthys Aitkeni.
„ *caudalis*, Traq.
„ *Egertoni.*
„ *microlepidotus.*
„ *semistriatus.*
„ *oblongus*, Traq.
Cycloptychius carbonarius, Young.
Acrolepis Hopkinsi.
Rhadinichthys Wardi.
„ *macrodon*, Traq.
„ *Planti.*
„ *Monensis.*
Eurylepis Anglica.
Gonatodus Molyneuxi, Traq.
Platysomus parvulus.
„ *rotundus.*
Mesolepis Wardi. Young.
„ *scalaris*, Young.
Cheirodus granulosus.

Transition Measures.—Mr Ward has given me the following list from those North Staffordshire measures, now generally known as "Transition Series":—

Acanthodes, sp.	*Megalichthys Hibberti.*
Ctenodus cristatus.	*Cœlacanthus elegans.*
„ *Murchisoni*, Ward (Ag., M.S.).	*Elonichthys Egertoni.*
Diplodus gibbosus.	„ sp.

It will be seen that in this, the "Potteries," Coal Field, the great repository of fossil fishes is the Middle Coal Measures, in which forty-eight species occur. But as of the forty-eight no less than thirty-three are found also in the Lower Coal Measures of Scotland and Northumberland, including most of the common and characteristic forms, we can hardly be said to have got into a new life-zone, so far as estuarine fishes are concerned.

In a paper "On the Fish-Fauna of the Yorkshire Coal Measures," published in 1901,* Mr E. D. Wellburn gives a list of seventy-eight species, a few of which are queried, while some are given as new, but not described. As they stand, however, the seventy-eight species may be tabulated as follows:—

Occurring in the Lower Coal Measures, ...	72
„ „ Middle, ...	44
„ „ Lower, but not in the Middle, . . .	34
„ „ Middle, but not in the Lower, . . .	6
Common to both Middle and Lower, ...	38

And the thirty-eight species common to the two divisions comprise nearly all the characteristic fishes known from the Lower and Middle Coal Measures in other parts of the island. They are:—

Pleuracanthus lævissimus.	*Megalichthys Hibberti.*
„ *cylindricus.*	„ *pygmæus.*
Diplodus gibbosus.	„ *coccolepis.*
„ *tenuis.*	„ *intermedius.*
Petalodus Hastingsi.	*Cœlacanthus Tingleyensis*, Davis.
Ctenoptychius apicalis.	„ *elegans.*
Callopristodus pectinatus.	„ *corrugatus*, Wellb.
Helodus simplex.	*Elonichthys Aitkeni.*
„ sp.	„ *Egertoni.*
Pleuroplax Rankinei.	„ *semistriatus.*
„ *Attheyi.*	*Acrolepis Hopkinsi.*
Sphenacanthus hybodoides.	*Rhadinichthys Monensis.*
Acanthodes Wardi.	*Gonatodus Molyneuxi.*
Gyracanthus formosus.	*Cycloptychius carbonarius.*
Euctenius unilateralis.	*Mesolepis Wardi.*
Ctenodus cristatus.	„ *scalaris.*
Rhizodopsis sauroides.	*Cheirodus granulosus.*
Strepsodus sauroides.	*Platysomus parvulus.*
„ *sulcidens.*	„ *Forsteri.*

Two of these, however, *Cœlacanthus Tingleyensis*, Davis, and *C. corrugatus*, Wellburn (undescribed), are as yet known only from Yorkshire, while *Petalodus Hastingsi* is a marine form characteristic of the Carboniferous Limestone of Armagh.

* *Proc. Yorks. Geol. and Polytechn. Soc.*, vol. xiv. pp. 159–174.

It is also interesting to note that nine species given by WELLBURN as occurring in the Lower but not in the Middle Coal Measures of Yorkshire are found in the list from the Middle Division in North Staffordshire. They are:—

Pleuracanthus Wardi.	*Elonichthys caudalis.*
Janassa linguæformis.	„ *oblongus.*
Acanthodes major.	*Rhadinichthys Planti.*
„ *Wardi.*	„ *macrodon.*
Sagenodus inequalis.	

We see, then, that Yorkshire corroborates the evidence of North Staffordshire, Northumberland, and Scotland in this, that nearly all the common Upper Carboniferous estuarine fishes being found in both the Lower and Middle Coal Measures, it is not possible to divide these strata into ichthyological life-zones at least. For the establishment of such zones must chiefly depend on the common and characteristic species, not on those which are rare, or, it may be, met with only in one locality or bed.

The fishes from the other English coal fields do not seem to have been so exhaustively collected, but I have seen nothing from any of them contrary to the above conclusions, namely, that in the Estuarine Fish-life of the Carboniferous System in Great Britain there are only two great chronological divisions, Upper and Lower. That is to say, keeping the true Upper Coal Measures out of view, for as yet, strange to say, we have no fish-remains from that series. Those from the "Transition" series of KIDSTON are ordinary Lower and Middle Coal Measure species; but it is, of course, very probable that, if we knew the fishes of the Upper Coal Measures, they might form a very different assemblage.

A word as to the Millstone Grit. According to Mr KIDSTON, the plants of this division are entirely Upper Carboniferous in aspect.—What of the fishes?

I have already stated that I have seen no determinable fish-remains from the Millstone Grit series in Scotland, but Mr WELLBURN has recorded a number from this series in Yorkshire and Lancashire, which, according to his determinations, are partly Lower Carboniferous marine species, partly Upper Carboniferous estuarine forms.* Judging from the latter, the evidence of the plants is corroborated, but the question also occurs to our minds, namely—Did the marine fish-fauna of the Carboniferous epoch change less rapidly than that of the estuaries and lagoons—or what was its condition after the latter had undergone so extensive a change as came in at least with the commencement of the Coal Measure period. We have as yet no answer to that question so far as Great Britain is concerned, though the marine Upper Carboniferous fish-remains of Miatschkowa, in Russia, are certainly different from those of the Carboniferous Limestone of other parts of Europe.

* *Geological Magazine* (4), vol. viii., 1901, pp. 216–222. In this paper Mr WELLBURN enumerates nineteen forms, of which four—*Cladodus mirabilis, Pristodus falcatus, Pœcilodus Jonesii, Urodus elongatus*—occur in the Lower Carboniferous marine beds; three—*Acanthodes Wardi, Strapsodus sulcatus, Elonichthys Aitkeni*—are Upper Carboniferous Estuarine species; one—*Acrolepis Hopkinsi*—is common to both divisions of the system. *Psephodus minutus* and *Euctenodopsis tenuis* are described as new, but the remaining nine are not determined specifically.

Reverting to the Estuarine fishes of the Carboniferous strata of Britain, we have seen in the preceding pages how great is the difference between the species which occur below and above the Millstone Grit. Only two species can with certainty be named as common to the two divisions, namely, *Callopristodus pectinatus* and *Acrolepis Hopkinsi*, though the list may yet be increased. For example, some of the fragmentary remains of *Megalichthys* and *Rhizodopsis* which occur in the Lower Carboniferous rocks of Scotland may, when more perfect material is obtained, turn out to be identical with *M. Hibberti* and *Rh. sauroides* of the Coal Measures, and I may add that it is hard to distinguish between *Sphenacanthus serrulatus* of the Lower and *Sph. hybodoides* of the Upper Carboniferous except in point of size. But the leading facts of the case would still be the same.

And the leading facts are these. We have in the Estuarine beds of the Lower Carboniferous series of the central valley of Scotland a fish fauna of which many of the species persist through thousands of feet of strata, and must therefore have lived for a very long time without change in their specific characters. Then, after the Millstone Grit, poor in fish-remains, is passed, we come to a new fauna, from which nearly all the Lower Carboniferous species, and with them also a number of genera, have disappeared, their place being taken by an Upper Carboniferous assemblage, which in its main features is characteristic not only of the Coal Measures of Scotland, but of the Lower and Middle Coal Measures of England, extending also into the Transition series of the latter country.

Why this sweeping change took place just about the time of the Millstone Grit we do not know; neither can we as yet explain the peculiar fish-fauna of the Lower Carboniferous Estuarine beds of the South of Scotland. It remains to be seen whether future investigation, for which there is abundance of room, will throw light upon those questions concerning which we are at present so much in the dark.

EXPLANATION OF THE PLATES.

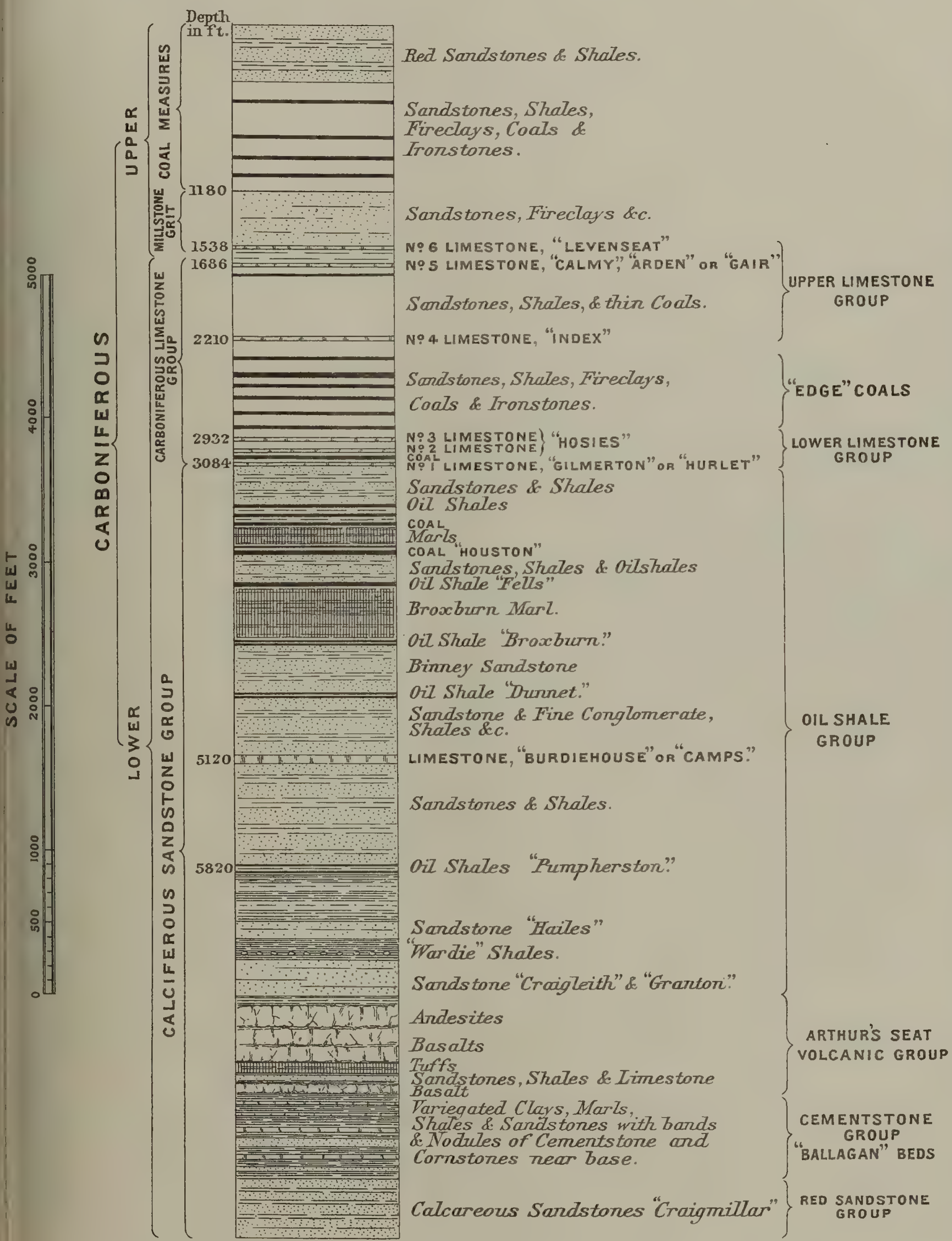
Plate I.

General Section of the Carboniferous Strata of the Edinburgh District, drawn up by Dr Peach, F.R.S., of the Geological Survey of Scotland.

Plate II.

General Section of the Carboniferous Limestone Series in the Midlothian Coalfield, by Messrs Geddes, Mining Engineers. Communicated to the Author by Mr H. M. Cadell of Grange.

DR. R. H. TRAQUAIR ON CARBONIFEROUS FISHES OF EDINBURGH DISTRICT.—PLATE I.



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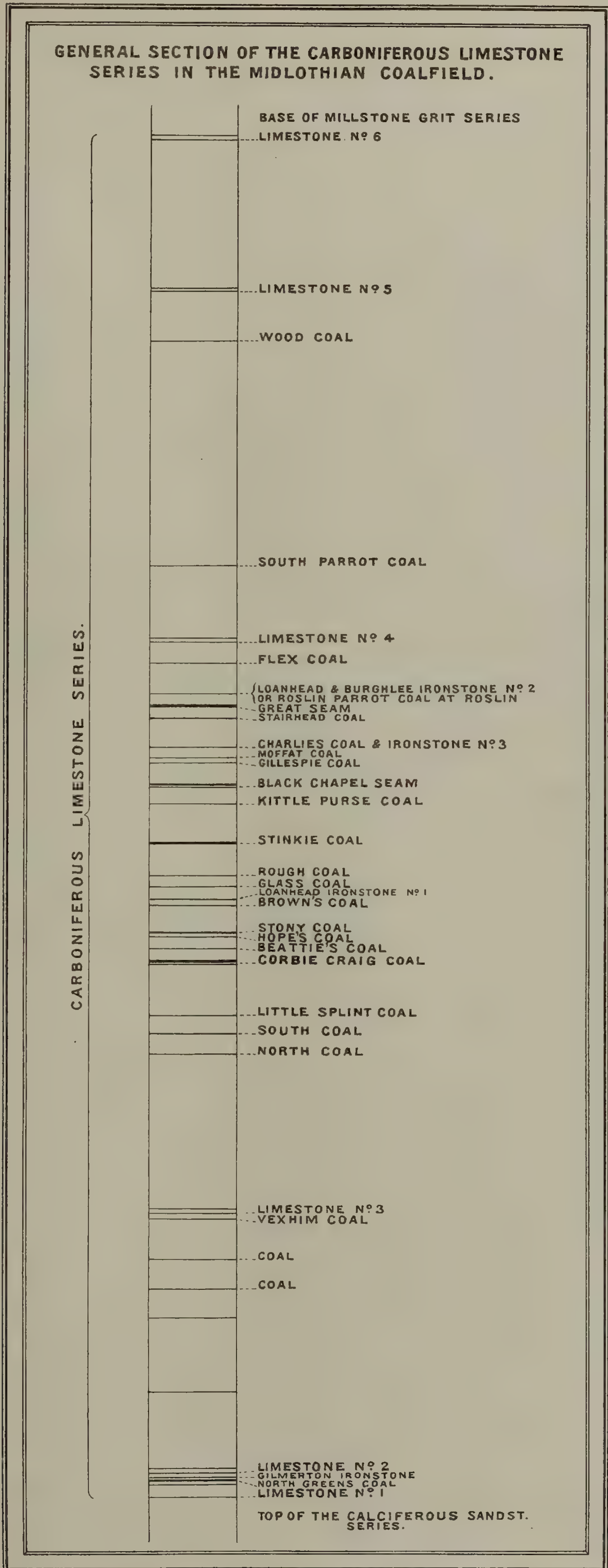
Dʳ R. H. TRAQUAIR ON CARBONIFEROUS FISHES OF EDINBURGH DISTRICT. —— PLATE II.



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XXIX.—*On the Applications of Quaternions in the Theory of Differential Equations.*

By J. H. Maclagan-Wedderburn.

(MS. received July 7, 1903. Read July 20, 1903. Issued separately October 30, 1903.)

The object of this paper is twofold: in the first place, to classify and sytematise vector differential equations; and in the second place, to show the applicability of quaternions to the theory of differential equations. So far as I am aware, the only paper of importance on the subject is one by Tait.* He, however, deals only with simple cases.

In the classification of forms I have followed in general the treatment adopted by Forsyth.† Reference will also occasionally be made to Jordan.‡ In what follows the order of the highest occurring differential coefficient will be called the *rank* of the equation; while the order of the equation will be used, as usual, to denote the order of the equivalent normal system.

Differential Equations of the First Rank.

1. *Variables Separable*—(Forsyth, § 13).—The general form is

$$\psi(\rho\,,\,d\rho)=\phi a\,.\,dt \qquad . \qquad . \qquad . \qquad (1)$$

where ψ is any vector function of ρ and $d\rho$, which is linear in $d\rho$ and ϕa is any vector function of t (a vector function of t can always be put in this form, a being any fixed constant vector, and ϕ a variable linear vector function).

The solution of the equation is obviously

$$\int\psi(\rho\,,\,d\rho)=\int dt\phi a+\beta$$

where β is an arbitrary constant vector, but the integration can only be carried out when $\psi(\rho d\rho)$ is of the form

$$\psi(\rho\,,\,d\rho)=\mathrm{S}d\rho\nabla\,.\,(\mathrm{F}\rho)\,.$$

A particular case is

$$\psi(\rho^{n-1}d\rho)=\phi a dt$$

* "Note on Linear Differential Equations," *Proc. R.S.E.*, 1870. *Scientific Papers*, vol. i. p. 153.

† *Treatise on Differential Equations*, 1888.

‡ *Cours d'Analyse*, i.–iii., 1887.

where ψ is a completely self-conjugate function.* The integral is

$$\frac{1}{n}\psi\rho^n = \int dt\phi\alpha + \beta\,.$$

2. *Homogeneous Equations*—(Forsyth, § 16).—The general linear homogeneous equation is

$$\phi d\rho = f(\rho\,,\,t) = t^n g\left(\frac{\rho}{t}\right) \quad . \quad . \quad . \quad (2)$$

where $\phi d\rho$ is a linear vector function of $d\rho$, whose constituents are homogeneous functions of ρ and t of the n^{th} degree and $f(\rho\,,\,t)$ is also a homogeneous vector function of ρ and t of the n^{th} degree.

Putting $\rho = t\sigma$ and expressing ϕ in terms of σ, we get

$$\phi\sigma + t\phi\dot{\sigma} = g(\sigma)$$

and consequently

$$\int \frac{\phi d\sigma}{g(\sigma) - \phi\sigma} - \log t = \mathrm{A}$$

where A is an arbitrary constant. $U\sigma$ is also arbitrary, making in all three arbitrary constants.

3. *Clairaut's Form*—(Forsyth, § 20).

$$\rho = t\dot{\rho} + f(\dot{\rho}) \quad . \quad . \quad . \quad (3)$$

where $f(\dot{\rho})$ is any vector function of $\dot{\rho}$.

The solution is evidently

$$\rho = t\alpha + f(\alpha)$$

where α is an arbitrary vector.

An allied form is

$$\rho = tf(\dot{\rho}) + g(\dot{\rho}) \quad . \quad . \quad . \quad (3')$$

Differentiating and putting ∇ for ${}_{\dot{\rho}}\nabla$ we have

$$\dot{\rho} = f(\dot{\rho}) - S\dot{\rho}\nabla\,.\,(tf(\dot{\rho}) + g(\dot{\rho}))$$

hence

$$dt - t\frac{Sd\dot{\rho}\nabla\,.\,f(\dot{\rho})}{f(\dot{\rho}) - \dot{\rho}} = \frac{Sd\dot{\rho}\nabla\,.\,g(\dot{\rho})}{f(\dot{\rho}) - \dot{\rho}}$$

and therefore

$$t = \mathrm{A}e^{\int\frac{Sd\dot{\rho}\nabla\,.\,f(\dot{\rho})}{f(\dot{\rho}) - \dot{\rho}}} + e^{\int\frac{Sd\dot{\rho}\nabla\,.\,f(\dot{\rho})}{f(\dot{\rho}) - \dot{\rho}}}\int e^{-\int\frac{Sd\dot{\rho}\nabla\,.\,f(\dot{\rho})}{f(\dot{\rho}) - \dot{\rho}}}\frac{Sd\dot{\rho}\nabla\,.\,g(\dot{\rho})}{f(\dot{\rho}) - \dot{\rho}}$$

4. *Linear Form*—(Forsyth, § 14. Tait, *Proc. R.S.E.*, 1870).—The general linear equation of the first rank may be written

$$\phi_0\dot{\rho} + \phi_1\rho = \psi\alpha \quad . \quad . \quad . \quad (4)$$

where ϕ_0 ϕ_1 and ψ are variable linear vector functions and α is a constant vector. For the present we will only consider the case where (4) may be written

$$\dot{\rho} + \phi_0^{-1}\phi_1\rho = \phi_0^{-1}\psi\alpha$$

* *Proc. R.S.E.*, xxiv., 1903, p. 410.

or say

$$\dot{\rho} + \phi\rho = \chi a \quad . \quad . \quad . \quad (5)$$

if the axes of ϕ are constant, we evidently have

$$\rho = e^{-\int \phi dt} \beta + e^{-\int \phi dt} \int e^{\int \phi dt} \chi a dt$$

the proof being the same as for ordinary equations.

If the axes of ϕ are not constant, this solution does not apply, for in general

$$de^{\phi} \neq d\phi e^{\phi}.$$

Consider first the case where the right-hand side of the equation is zero. The series

$$\rho = (1 - \int \phi dt + \int \phi \int \phi dt^2 - \int \phi \int \phi \int \phi dt^3 + \ldots)\beta$$

is evidently a solution, β being an arbitrary constant vector; for

$$d\rho = -\phi\rho dt.$$

When the right-hand side is not zero an obvious particular integral is

$$\rho = (1 - \int \phi dt + \int \phi \int \phi dt^2 - \ldots) \int \frac{1}{(1 - \int \phi dt + \int \phi \int \phi dt^2 \ldots)} \chi a dt$$

so that, putting

$$\mathrm{F}(\phi) = 1 + \phi + \int \phi\phi dt + \int \phi \int \phi\phi dt^2 + \cdots,$$

the complete solution of (5) is

$$= \mathrm{F}(-\int \phi dt)\beta + \mathrm{F}(-\int \phi dt) \int \{\mathrm{F}(-\int \phi dt)\}^{-1} \chi a dt \quad . \quad . \quad . \quad (6)$$

for it contains three independent arbitrary constants. This method of solution is sometimes of use in the theory of ordinary differential equations. For, if we know the quaternion solution of a system and then reduce the system by CAUCHY's method to an ordinary differential equation, the solution of this equation may be deduced from the quaternion solution of the system by resolving the latter along an appropriate axis. For instance, putting

$$\phi = -ai\mathrm{S}.j - bj\mathrm{S}.i$$

we find that the complete solution of

$$\ddot{x} + \mathrm{P}\dot{x} + \mathrm{Q}x = 0$$

is

$$x = \mathrm{A}(1 + \int a \int b dt^2 + \int a \int b \int a \int b dt^4 \ldots)$$
$$+ \mathrm{B}(\int a dt + \int a \int b \int a dt^3 + \int a \int b \int a \int b \int a dt^5 \ldots) \quad . \quad . \quad . \quad (7)$$

where

$$a = e^{-\int \mathrm{P} dt} \text{ and } b = -\mathrm{Q}e^{\int \mathrm{P} dt}.$$

This method has some interesting applications, which I propose to develop in a separate paper. In particular, it will be found to lead in an easy and natural manner to BESSEL's and NEUMANN's functions.

5. The linear equation of the first rank

$$\dot{\rho} + \phi\rho = 0$$

can be put in the interesting form

$$\frac{d\rho}{d\psi} + \rho = 0 \qquad . \quad . \quad . \qquad (8)$$

where

$$\psi = \int \phi dt$$

and $\frac{d}{d\psi}$ is defined as $(d\psi)^{-1}d\rho$; for

$$\frac{d}{dt} = \frac{d\psi}{dt}\,\frac{d}{d\psi} = \phi\frac{d}{d\psi}.$$

If the axes of ψ are constant, the solution of (8) is evidently

$$\rho = e^{-\psi}\beta$$

where β is an arbitrary constant vector.

In this case ψ may in many respects be treated as a scalar, for $d\psi$ and ψ are coaxial, and therefore commutative.

If the axes of ψ are not constant, the solution can be obtained in the form

$$\begin{aligned}\rho &= (1 - \int d\psi + \int d\psi \int d\psi \ \ldots \ )\beta \\ &= \mathrm{F}(-\psi)\beta .\end{aligned}$$

Another interesting case arises if we assume $\rho = \chi\alpha$ where α is an arbitrary constant vector, then

$$\dot{\chi} + \phi\chi = 0 \qquad . \quad . \quad . \qquad (9)$$

the solution of which is

$$\chi = \mathrm{F}(-\int \phi dt)\varpi$$

where ϖ is an arbitrary constant linear vector function.

A connected form is

$$\dot{\chi} + \chi\phi = 0 \qquad . \quad . \quad . \qquad (10)$$

the solution of which is

$$\begin{aligned}\chi &= \varpi(1 - \int \phi dt + \int\int \phi dt\phi dt - \int\int\int \phi dt\phi dt\phi dt \ \ldots \ ) \\ &= \varpi\mathrm{G}(-\int \phi dt) .\end{aligned}$$

Another equation of similar form is

$$\dot{\chi} + \phi\chi + \chi\phi = 0 .$$

Its solution is easily found to be

$$\chi = \mathrm{F}(-\int \phi dt)\mathrm{G}(-\int \phi dt) .$$

6. The following are some properties of

$$F(\phi)=1+\int d\phi+\int d\phi\int d\phi \ldots$$

and

$$G(\phi)=1+\int d\phi+\int\int d\phi d\phi \ldots$$

Taking conjugates in (10) we get

$$\dot{\chi}'+\phi'\chi'=0$$

which is of the same form as (9), hence *the conjugate of* $F(\phi)$ *is* $G(\phi')$.

It has been already shown that the solution of

$$\dot{\chi}+\phi\chi=0$$

is

$$\chi=F(-\int\phi dt)\varpi$$

put

$$\theta=\chi^{-1} \quad \text{then} \quad \dot{\chi}=-\theta^{-1}\dot{\theta}\theta^{-1}$$

$$\therefore \quad \dot{\theta}-\theta\phi=0$$

the solution of which is

$$\theta=\psi G(\int\phi dt)$$

ϖ and ψ being arbitrary constant linear vector functions. It follows immediately that

$$\{F(\phi)\}^{-1}=G(-\phi).$$

Similarly

$$\{G(\phi)\}^{-1}=F(-\phi).$$

This enables us to put the particular solution of the linear equation of the first rank in the simpler form

$$F(-\int\phi dt)\int G(\int\phi dt)\chi dt.$$

If $\chi=F(\phi)$ we have

$$\int\frac{d\chi}{\chi}=\phi=F^{-1}(\chi).$$

Put $1-\varpi$ for χ, then

$$F^{-1}(1-\varpi)=-\int d\varpi(1+\varpi+\varpi^2+\varpi^3 \ldots)$$

the series being convergent so long as the roots of ϖ are less than 1. A similar result is got on substituting $1+\varpi$ for χ.*

* *Note on the form of* e^ϕ.—If the elements of ϕ are $\begin{pmatrix} g_1 & g_2 & g_3 \\ \sigma_1 & \sigma_2 & \sigma_3 \end{pmatrix}$ we have

$$\phi^n=(g_1{}^n\sigma_1 S.\sigma_2\sigma_3+g_2\sigma_2 S.\sigma_3\sigma_1+g_3\sigma_3 S.\sigma_1\sigma_2)/S\sigma_1\sigma_2\sigma_3$$

$$\therefore \quad e^\phi=(e^{g_1}\sigma_1 S.\sigma_2\sigma_3+e^{g_2}\sigma_2 S.\sigma_3\sigma_1+e^{g_3}\sigma_3 S.\sigma_1\sigma_2)/S\sigma_1\sigma_2\sigma_3$$

and in general

$$f(\phi)=\Sigma f(g_1)\sigma_1\frac{S.\sigma_2\sigma_3}{S\sigma_1\sigma_2\sigma_3}$$

f can also be expanded in the form $A\phi^2+B\phi+C$ where A B and C are given by the equations

$$Ag_1{}^2+Bg_1+C=f(g_1) \quad Ag_2{}^2+Bg_2+C=f(g_2) \quad Ag_3{}^2+Bg_3+C=f(g_3).$$

—(See TAIT, *Quaternions*, 3rd ed., p. 124.)

The General Linear Differential Equation with Constant Coefficients.

7. *Preliminary Formulæ.*—Before proceeding further, it is necessary to explain a convention which allows linear vector functions to be separated freely from their arguments and to be treated as scalars. When a linear vector function is separated from its argument a suffix is attached to both, and the former treated as a scalar. The following examples define the notation and make its use obvious :—

$$V\phi\rho\sigma + V\rho\phi\sigma = (\phi_1 + \phi_2)V\rho_1\sigma_2$$
$$V\phi\rho\phi\sigma = \phi_1\phi_2 V\rho_1\sigma_2$$
$$\psi\phi\rho = \phi_1\psi_2\rho_{12} = \psi_2\phi_1\rho_{12}$$

$$\begin{vmatrix} \phi\alpha & \psi\alpha & \chi\alpha \\ \phi\beta & \psi\beta & \chi\beta \\ \phi\gamma & \psi\gamma & \chi\gamma \end{vmatrix} = \begin{vmatrix} \phi_1 & \psi_1 & \chi_1 \\ \phi_2 & \psi_2 & \chi_2 \\ \phi_3 & \psi_3 & \chi_3 \end{vmatrix} \alpha_1\beta_2\gamma_3$$
$$= \begin{vmatrix} \phi_1 & \phi_2 & \phi_3 \\ \psi_1 & \psi_2 & \psi_3 \\ \chi_1 & \chi_2 & \chi_3 \end{vmatrix} \alpha_1\beta_2\gamma_3 .$$

As it is often convenient to denote *different* functions by the addition of suffixes to the same symbol, care must be taken not to confuse the two uses. The meaning, however, is generally clear from the context.

8. In what follows $f(\phi)$ is used to denote any function of a linear vector function ϕ. The coefficients in $f(\phi)$ if not constant are supposed to act subsequently to ϕ, *i.e.*, $f(\phi)$ is of the form

$$\psi\phi^n + \chi\phi^{n-1} \cdots + \varpi + \cdots \theta\phi^{-m} \cdots$$

where ψ ϕ etc. are linear vector functions.

Theorem I.—(Forsyth, § 32).—

$$f(D)e^{t\phi} = f(\phi)e^{t\phi}$$

ϕ being any constant linear vector function and $D = \dfrac{d}{dt}$; for

$$D^n e^{t\phi} = \phi^n e^{t\phi} .$$

Theorem II.—(Forsyth, § 33).—

If X is any function of t (in general a linear vector function) we have

$$f(D)(Xe^{t\phi}) = f(D_2 + \phi_1)X_2 e_1^{t\phi}$$

for

$$D\{Xe^{t\phi}\} = (DX + X\phi)e^{t\phi} = (D_2 + \phi_1)X_2 e_1^{t\phi}$$

so that

$$D^n\{Xe^{t\phi}\} = (D_2 + \phi_1)^n X_2 e_1^{t\phi} .$$

Put

$$(D_2 + \phi_1)^n X_2 = Y$$

then

$$X=(D_2+\phi_1)^{-n}Y_2$$

$$\therefore \quad D^n.(D_2+\phi_1)^{-n}Y_2e_1^{t\phi}=Ye^{t\phi}$$

$$\therefore \quad (D_2+\phi_1)^{-n}Y_2e_1^{t\phi}=D^{-n}.Ye^{t\phi}$$

where, as X was arbitrary, Y may be any function of t. Similarly

$$f(D)e^{t\phi}X=f(D_2+\phi_1)e_1^{t\phi}X_2$$

also

$$f(D)X=f(D_2+\psi_1)(Xe^{-t\psi})_2e_1^{t\psi}$$
$$=f(D_2+\psi_1)e_1^{t\psi}(e^{-t\psi}X)_2.$$

THEOREM III.—(Forsyth, § 34).—

$$f(D^2)\begin{matrix}\sin\\ \cos\end{matrix}(t\phi+\psi)=f(-\phi^2)\begin{matrix}\sin\\ \cos\end{matrix}(t\phi+\psi).$$

THEOREM IV.—(Forsyth, § 35).—

$$f(tD)t^mX=t^mf(tD+m)X.$$

Similar theorems can be formulated when the independent variable is a linear vector function with constant axes, but they are not of sufficient importance to merit a separate discussion.

For instance

$$f(\phi D)\phi^mX=\phi^mf(\phi D+m)X$$

where $D=\dfrac{d}{d\phi}$.

PROPERTIES OF THE GENERAL LINEAR DIFFERENTIAL EQUATION.

9. The general linear differential equation of the n^{th} rank is of the form

$$\Omega\rho=(\phi_0D^n+\phi_1D^{n-1}+\phi_2D^{n-2}\ \ldots\ \phi_n)\rho=\phi a \qquad (11)$$

where $\phi_0\ \ldots\ \phi_n\ \phi$ are functions of t but not of ρ, and a is a constant vector.

If ρ_1 is any particular integral, putting $\rho=\sigma+\rho_1$ we find immediately

$$\Omega\sigma=0.$$

Then, if the general solution of this equation is $\sigma=\sigma_1$, that of (11) is $\rho=\sigma_1+\rho_1$.

If k independent particular integrals of (11) can be obtained, the order of the equation can be reduced by k (see JORDAN, *Cours d'Analyse*, i. iii., 1887, p. 138). The most important case is where $k=3$. In this case let the three particular integrals be α, β, and γ, and put

$$\rho=\alpha Si\sigma+\beta Sj\sigma+\gamma Sk\sigma=\psi\sigma$$

then

$$\Omega\rho=\{\phi_0\psi D^n+(\phi_1\psi+n\phi_0D\psi)D^{n-1}\cdots+\Omega\psi\}\sigma=\phi a \qquad (12)$$

in which, as $\Omega\psi=0$, we may regard $D\sigma$ as the dependent variable.

The second term of (12) may be removed by solving the linear equation

$$\phi_1\psi+n\phi_0\dot{\psi}=0.$$

10. If ϕ_0^{-1} is determinate, *i.e.*, if ϕ_0 is not degenerate, we have

$$(D^n + \phi_0^{-1}\phi_1 D^{n-1} \dots \phi_0^{-1}\phi_n)\rho = \phi_0^{-1}\phi a$$

and therefore, by CAUCHY'S existence theorem, the order of the equivalent system is $3n$.

If ϕ_0 is degenerate, we may proceed as follows. Suppose, in the first instance, that ϕ_0 is a planar linear vector function. It can then be put in the form

$$f\sigma_1 \frac{S\,.\,\sigma_2\sigma_3}{S\sigma_1\sigma_2\sigma_3} + g\sigma_2 \frac{S\,.\,\sigma_3\sigma_1}{S\sigma_1\sigma_2\sigma_3}.$$

Let ψ_r be defined by the equation

$$\phi_r = \left(\phi_0 + \sigma_3 \frac{S\,.\,\sigma_1\sigma_2}{S\sigma_1\sigma_2\sigma_3}\right)\psi_r \quad . \quad . \quad . \quad . \quad (13)$$

and let

$$\phi_r = \sigma_1 S\,.\,a_r + \sigma_2 S\,.\,\beta_r + \sigma_3 S\,.\,\gamma_r$$
$$\psi_r = \sigma_1 S\,.\,a'_r + \sigma_2 S\,.\,\beta'_r + \sigma_3 S\,.\,\gamma'_r\,.$$

Substituting in (13) we find

$$a_r = fa'_r \quad \beta'_r = g\beta'_r \quad \gamma_r = \gamma'_r\,.$$

Therefore

$$\Omega\rho = \phi a$$

is equivalent to

$$\phi_0(D^n + \psi_1 D^{n-1} + \psi_2 D^{n-2} \dots \psi_n)\rho = \phi_0\psi a$$

together with

$$\sigma_3 \frac{S\,.\,\sigma_1\sigma_2}{S\sigma_1\sigma_2\sigma_3}(\psi_1 D^{n-1} + \psi_2 D^{n-2} \dots \psi_n)\rho = \sigma_3 \frac{S\sigma_1\sigma_2\psi a}{S\sigma_1\sigma_2\sigma_3}.$$

If the complete solutions of

$$(D^n + \psi_1 D^{n-1} \dots + \psi_n)\rho = \psi a \quad . \quad . \quad . \quad (14)$$

and

$$(\psi_1 D^{n-1} + \psi_2 D^{n-2} \dots + \psi_n)\rho = \psi a \quad . \quad . \quad . \quad (15)$$

are ρ_1 and ρ_2 respectively, the complete solution of $\Omega\rho = \phi a$ is

$$\rho = \phi_0\rho_1 + \sigma_3 \frac{S\,.\,\sigma_1\sigma_2\rho_2}{S\sigma_1\sigma_1\sigma_3}.$$

The modifications, where γ, etc. vanish or when ϕ_0 is unidirectional, are obvious and not sufficiently important to merit a separate discussion. It is important to notice that this process is the same whether the coefficients are constant or variable, so that, if a method can be found for deducing the order of a system with constant coefficients, the same method is applicable in the case of variable coefficients if we treat the independent variable as a constant.

11. Treating D as a constant, we have identically

$$(\Omega^3 - m_1\Omega^2 + m_2\Omega - m_3)\rho = 0 \quad . \quad . \quad . \quad (16)$$

m_1, m_2, and m_3, therefore, when expanded in terms of D, constitute a set of differential invariants of Ω. It will be shown in the next section that the degree of m_3 in D is the order of the system. If $m_3 \equiv 0$ then Ω is degenerate and the order of the system is the degree of m_2 in D. Similarly, if also $m_2 \equiv 0$, the order of the system is the degree of m_1.

General Linear Equation with Constant Coefficients—(Forsyth, § 166 ; Tait, *Proc. R.S.E.*, 1870).

12. If the coefficients are constants, D is commutative with the ϕ's, therefore from (16) $m_3\rho = 0$ if $\Omega\rho = 0$, *i.e.*, any solution of $\Omega\rho = 0$, is also a solution of $m_3\rho = 0$. If $\begin{pmatrix} h_1 & h_2 & h_3 \\ \sigma_1 & \sigma_2 & \sigma_3 \end{pmatrix}$ are the elements of Ω (*i.e.*, the axes and latent roots),

$$\Omega\rho = h_1\sigma_1 \frac{S\sigma_2\sigma_3\rho}{S\sigma_1\sigma_2\sigma_3} + h_2\sigma_2 \frac{S\sigma_3\sigma_1\rho}{S\sigma_1\sigma_2\sigma_3} + h_3\sigma_3 \frac{S\sigma_1\sigma_2\rho}{S\sigma_1\sigma_2\sigma_3} = 0$$

hence

$$h_1 \frac{S\sigma_2\sigma_3\rho}{S\sigma_1\sigma_2\sigma_3} = 0 \qquad h_2 \frac{S\sigma_3\sigma_1\rho}{S\sigma_1\sigma_2\sigma_3} = 0 \qquad h_3 \frac{S\sigma_1\sigma_2\rho}{S\sigma_1\sigma_2\sigma_3} = 0$$

$$\therefore \quad \frac{S\sigma_2\sigma_3\rho}{S\sigma_1\sigma_2\sigma_3} = p_1 \qquad \frac{S\sigma_3\sigma_1\rho}{S\sigma_1\sigma_2\sigma_3} = p_2 \qquad \frac{S\sigma_1\sigma_2\rho}{S\sigma_1\sigma_2\sigma_3} = p_3$$

and therefore

$$\rho = \sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 p_3 \,.$$

The following proof is perhaps more satisfactory. Assume as a solution

$$\rho = e^{t\chi}a$$

where a is arbitrary and χ is a constant linear vector function. Differentiating and substituting in $\Omega\rho = 0$, we get

$$(\phi_0\chi^n + \phi_1\chi^{n-1} \dots \phi_n)a = 0 \qquad . \qquad . \qquad . \qquad (17)$$

If $\begin{Bmatrix} g_1 & g_2 & g_3 \\ \rho_1 & \rho_2 & \rho_3 \end{Bmatrix}$ are the elements of χ, we get immediately

$$(\phi_0 g_1{}^n + \phi_1 g_1{}^{n-1} + \dots\dots + \phi_n)\rho_1 = 0 \qquad . \qquad . \qquad (18)$$

or say

$$\Theta\rho_1 = 0 \,.$$

The form of (18) is independent of the suffix attached to ρ and g, showing that (17) is equivalent to only three scalar equations, not nine as might have been expected.

If $\begin{Bmatrix} h_1 & h_2 & h_3 \\ \sigma_1 & \sigma_2 & \sigma_3 \end{Bmatrix}$ are the elements of Θ, (18) is equivalent to

$$h_1 S\sigma_2\sigma_3\rho_1 = 0 \qquad h_2 S\sigma_3\sigma_1\rho_1 = 0 \qquad h_3 S\sigma_1\sigma_2\rho_1 = 0 \qquad . \qquad . \qquad . \qquad (19)$$

Suppose $h_2 \pm 0$, $h_3 \pm 0$, $h_1 = 0$, which gives the value of g, then $S\sigma_3\sigma_1\rho_1 = 0$ $S\sigma_1\sigma_2\rho_1 = 0$, therefore ρ_1 is parallel to σ_1 and $\rho = Ae^{gt}\sigma_1$ is a solution of $\Omega\rho = 0$, A being an arbitrary constant. A similar result follows if h_2 or h_3 respectively is zero. Therefore the order of the equation is the degree of $h_1\, h_2\, h_3$ in g,—*i.e.*, the degree of m_3 in D.

It remains to consider the following cases, first where h_1 h_2 and h_3, or any two of them, have one or more common factors; second, where h_1 h_2 or h_3 have repeated roots.

First, suppose $g = g_0$ is a common root of $h_1 = 0$, $h_2 = 0$, $h_3 = 0$, then $\rho = e^{g_0 t}\alpha$ is a solution of the equation, where α is an arbitrary vector. If g_0 is a common root of two only, say h_1 and h_2, we get from (19) $\rho_1 = e^{g_0 t}(\mathrm{A}\sigma_1 + \mathrm{B}\sigma_2)$, where A and B are arbitrary constants. In both cases the number of arbitrary constants is the same as before.

Secondly, if $h_1 = 0$ has an r-ple root, we have for the same value of g_1

$$h_1 = 0 \quad \frac{dh_1}{dg_1} = 0 \ \ldots \ \frac{d^{r-1}h_1}{dg_1} = 0 \, .$$

Therefore

$$\rho = e^{g_1 t}(\mathrm{A}_1 + \mathrm{A}_2 t \ \ldots \ \mathrm{A}_r t^{r-1})\sigma_1$$

is a solution of the differential equation, the number of arbitrary constants remaining the same as before.

It only remains to notice that there is always a direction σ_1 corresponding to any of the values of g_1; for suppose that when $g_1 = g_0$, σ_1 vanishes, then $(g_1 - g_0)$ must be a factor of $\mathrm{T}\sigma_1$, and as $\mathrm{T}\sigma_1$ is arbitrary, this factor may be neglected.

If in (17) we take conjugates we get

$$\chi'^{n}\phi_0' + \chi'^{n-1}\phi'_1 \ \ldots \ \phi'_n = 0 \, .$$

Equations of this type can therefore be solved by the above method. A similar investigation can be given, when the independent variable is a linear vector function, by assuming $\rho = \mathrm{F}(\phi\chi)\alpha$.

13. To find a particular solution of $\Omega\rho = \phi\alpha$ we have

$$\begin{aligned} \rho &= \Omega^{-1}\phi\alpha \\ &= \frac{1}{m_3}(\Omega^2 - m_1\Omega + m_2)\phi\alpha \, . \\ &= \psi\alpha \end{aligned}$$

The theorems enunciated in paragraph 7 are useful in this connection.

14. The following theorem is sometimes useful.* If ϕ is any linear vector function whose constituents are integral functions of D with constant coefficients,

$$\phi\Omega\rho = 0$$

is equivalent to

$$\Omega\rho = 0$$

if the third invariant, M_3, of ϕ does not contain D. For any solution of $\phi\Omega\rho = 0$ is solution of $\mathrm{M}_3\Omega\rho = 0$, *i.e.*, of $\Omega\rho = 0$, if M_3 does not contain D.

* See Prof. CHRYSTAL, "A Fundamental Theorem regarding the Equivalence of Systems of Ordinary Linear Differential Equations," *Trans. R.S.E.*, 1895.

15. When Ω is readily factorisable, the following method may be used:— Let

$$\Omega\rho = (\mathrm{D}-\phi)(\mathrm{D}-\psi)(\mathrm{D}-\epsilon)\ \ldots\ (\mathrm{D}-\theta)\rho = \varpi\alpha$$

$$\therefore\ (\mathrm{D}-\psi)(\mathrm{D}-\epsilon)\ \ldots\ (\mathrm{D}-\theta)\rho = \frac{1}{\mathrm{D}-\phi}\varpi\alpha + e^{t\phi}\beta$$

$$\begin{aligned}(\mathrm{D}-\epsilon)\ \ldots\ (\mathrm{D}-\theta)\rho &= (\mathrm{D}-\psi)^{-1}(\mathrm{D}-\phi)^{-1}\varpi\alpha \\ &\quad + (\mathrm{D}-\psi)^{-1}e^{t\phi}\beta + e^{t\psi}\gamma \\ &= (\mathrm{D}-\psi)^{-1}(\mathrm{D}-\phi)^{-1}\varpi\alpha + (\phi_1-\psi_2)^{-1}e^{t\phi}_{2}\beta \\ &\quad + e^{t\psi}\gamma\end{aligned}$$

and so on.

If $\phi\psi\ \ldots\ \theta$ are commutative we have evidently

$$\rho = \Sigma e^{t\phi}\alpha + \Omega^{-1}\varpi\alpha\,.$$

16. *Homogeneous Equations*—(Forsyth, § 55). The equation

$$\Omega\rho = (\phi_0\breve{\varpi}\varpi^n\mathrm{D}^n + \phi_1\breve{\varpi}\varpi^{n-1}\mathrm{D}^{n-1}\ \cdots\ + \phi_n)\rho = 0 \qquad (20)$$

where ϖ is a variable linear vector function with constant axes, can be reduced to the case of constant coefficients by changing the dependent variable to ϖ and then putting $\rho = \varpi\sigma$ and $\varpi = e^\theta$, which gives $\frac{d}{d\varpi} = e^{-\theta}\frac{d}{d\theta}$ and therefore

$$\varpi^n\frac{d^n\rho}{d\varpi^n} = \left(\frac{d}{d\theta} - n + 1\right)\left(\frac{d}{d\theta} - n + 2\right)\ \ldots\ \left(\frac{d}{d\theta} - 1\right)\frac{d}{d\theta}\,.$$

The more general substitution $\rho = e^{m\theta}\sigma\ \varpi = e^\theta$ is sometimes useful.

17. *Exact Differential Equations*—(Forsyth, §§ 56, 57).—The condition of exactness is found as in ordinary differential equations to be

$$\phi_n - \frac{d\phi_{n-1}}{dt} + \frac{d^2\phi_{n-2}}{dt^2}\ \ldots\ (-1)^n\frac{d^n\phi_0}{dt^n} = 0\,.$$

Linear Equation of the Second Rank with Variable Coefficients.

18. The general form is

$$\phi_0\ddot{\rho} + \phi_1\dot{\rho} + \phi_2\rho = \phi\alpha$$

where $\phi_0\ \phi_1\ \phi_2$ and ϕ are variable linear vector functions.

If three independent particular solutions α, β, γ of the subsidiary equation are known, the equation can be solved (*cf.* Forsyth, § 58).
For putting

$$\rho = (\alpha\mathrm{S}\,.\,i + \beta\mathrm{S}\,.\,j + \gamma\mathrm{S}\,.\,k)\sigma = \varpi\sigma$$

we get

$$\phi_0\varpi\ddot{\sigma} + (2\phi_0\dot{\varpi} + \phi_1\varpi)\dot{\sigma} + (\phi_0\ddot{\varpi} + \phi_1\dot{\varpi} + \phi_2\varpi)\sigma = \phi\alpha \qquad (21)$$

$$\therefore\quad \phi_0\varpi\ddot{\sigma} + (2\phi_0\dot{\varpi} + \phi_1\varpi)\dot{\sigma} = \phi\alpha$$

which is linear in $\dot{\sigma}$.

19. *Reduction to normal form.*—If ϕ_0 is not degenerate, we may, without loss of generality, put $\phi_0 = 1$. The equation may then be written

$$\ddot{\rho} + \phi_1\dot{\rho} + \phi_2\rho = \phi a \quad . \quad . \quad . \quad (22)$$

If in (21) ϖ satisfies the equation

$$\dot{\varpi} + \frac{1}{2}\phi_1\varpi = 0$$

we have

$$\ddot{\sigma} + \chi\sigma = \varpi^{-1}\phi a \quad . \quad . \quad (23)$$

where

$$\chi = \varpi^{-1}(\phi_2 - \frac{1}{2}\dot{\phi}_1 - \frac{1}{4}\phi_1^2)\varpi\,.$$

The reduction to this form may also be made by changing the independent variable. Let ϖ be the new variable. Then we have

$$\dot{\rho} = \dot{\varpi}\frac{d\rho}{d\varpi} \qquad \ddot{\rho} = \dot{\varpi}^2\frac{d^2\rho}{d\varpi^2} + \ddot{\varpi}\frac{d\rho}{d\varpi}\,.$$

Hence on substituting in (22) we get

$$\dot{\varpi}^2\frac{d^2\rho}{d\varpi^2} + (\ddot{\varpi} + \phi_1\dot{\varpi})\frac{d\rho}{d\varpi} + \phi_2\rho = \phi a$$

which if

$$\varpi = \int dt \mathrm{F}(-\int dt\phi_1)$$

reduces to

$$\frac{d^2\rho}{d\varpi^2} + \dot{\varpi}^{-2}\phi_2\rho = \dot{\varpi}^{-2}\phi a\,.$$

20. The following method of solution is analogous to that given in paragraph 4 for ordinary differential equations.

It may be verified by direct calculation that the solution of

$$\ddot{\rho} + \phi_1\dot{\rho} + \phi_2\rho = 0 \quad . \quad . \quad . \quad (24)$$

is

$$\rho = (1 + \int a\int b dt^2 + \int a\int b\int a\int b dt^4 \,.\,.\,.\,)\alpha$$
$$+ (\int a dt + \int a\int b\int a dt^3 + \,\cdots\,)\beta$$

where α and β are arbitrary constant vectors and

$$a = \mathrm{F}(-\int\phi_1 dt) \qquad b = -\mathrm{G}(\int\phi_1 dt)\phi_2\,.$$

21. If we put

$$\sigma = (\int b dt + \int b\int a\int b dt^3 \,.\,.\,.\,)\alpha$$
$$+ (1 + \int b\int a dt^2 \,.\,.\,.\,)\beta$$

we have

$$\dot{\rho} = a\sigma \qquad \dot{\sigma} = b$$

$$\therefore \quad \ddot{\sigma} = \dot{b}\rho + b\dot{\rho} = \dot{b}b^{-1}\dot{\sigma} + ba\sigma$$

$$= \{\mathrm{G}(\int\phi_1 dt)\phi_1\mathrm{F}(-\int\phi_1 dt) + \mathrm{G}(\int\phi_1 dt\dot{\phi}_2\phi_2{}^{-1}\mathrm{F}(-\int\phi_1 dt)\}\dot{\sigma}$$

$$-\mathrm{G}(\int\phi_1 dt)\phi_2\mathrm{F}(-\int\phi_1 dt)\sigma \quad . \quad . \quad . \quad (25)$$

If a solution of either of these equations can be found, the solution of the other is readily obtainable. In the same way in ordinary differential equations we can readily derive the solution of

$$\ddot{x} - (\mathrm{P} + \dot{\mathrm{Q}}/\mathrm{Q})\dot{x} + \mathrm{Q}x = 0$$

from the solution of

$$\ddot{x} + \mathrm{P}\dot{x} + \mathrm{Q}x = 0 .$$

22. The methods of solution by infinite series and by definite integrals, which are used in the theory of ordinary differential equations, are applicable, with a few changes, to the theory of vector equations. In this way we can, as in section 4, sometimes make the solution of an ordinary differential equation depend on the solution of a vector equation of lower rank.

XXX.—*The Lower Devonian Fishes of Gemünden.* By R. H. Traquair, M.D., LL.D., F.R.S., Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh. (With Seven Plates.)

(Read 16th March 1903 ; given in for publication 8th September 1903. Issued separately October 31, 1903.)

Gemünden is situated in Rhenish Prussia, about eighteen miles to the west and slightly also to the south of Bingen, in the district known as the "Hunsrück," and the rock in which the fishes to be described in this memoir occur is called the "Hunsrück Slates."

These Hunsrück slates belong to the Lower Devonian of the Rhenish area, and the position assigned to them by German geologists is as follows :—

Upper Devonian, . .	*Clymenia* Limestone and *Cypridina* Slates.
	Adorf Goniatite Limestone.
Middle Devonian, . .	*Stringocephalus* beds.
	Calceola beds.
	Zone of *Spirifer cultrijugatus*.
Lower Devonian, . .	Coblenz beds (Spirifer Sandstone).
	Hunsrück Slates.
	Taunus Quartzite.
	Sericitic Phyllite and (?) Gneiss of the Taunus.

The above table is quoted from Kayser and Lake's *Text-book of Comparative Geology*, as is also the following brief statement regarding the Hunsrück slates themselves.

"The Taunus quartzite is succeeded by the Hunsrück slates, a thick series of dark-coloured clay slates, including numerous layers of roofing-slate. They form the monotonous plateaux of Hunsrück and Taunus, but are also repeated in their characteristic form in the Venn and in the Ardennes. In these areas, however, they are for the most part represented by the Grauwacke of Montigny. The fauna of the Hunsrück schists (chief localities, Bundenbach and Gemünden in the Hunsrück, Caub on the Rhine, Alles on the Semois), unlike that of the rest of the Lower Devonian, consists chiefly of trilobites (*Phacops Ferdinandi*, *Homalonotus ornatus*, etc., *Cryphæus*, *Dalmanites* [*Odontochile*]), mailed fish, bivalves, cephalopods (*Orthoceras*, *Goniatites*), crinoids, and beautifully-preserved starfishes, whilst Brachiopods are almost entirely wanting."

The slate is often available for roofing purposes, as at Caub, Bacharach, Gemünden, and Bundenbach, and it is only through quarrying operations that any abundance of fossils has been found. The roofing-slate, or "Dachschiefer," is of a dark blackish-grey colour, sometimes with a tinge of purple, moderately hard, and splitting very evenly into layers, the surface of which has a certain silky appearance, due apparently to the presence of excessively minute scales of mica. The fossils are entirely converted into hard iron pyrites, and are, when found in the quarry, invariably covered with a layer of matrix, which has to be removed by the expenditure of much time and patience, the instruments used being pointed knives and a brush of fine brass wire, which has the advantage of being harder than the slate, but softer than the pyritised fossils. When properly prepared, these fossils, both invertebrate and vertebrate, form extremely striking and beautiful objects, their drawback being that their pyritised condition renders them quite unavailable for microscopic examination. Occasionally the pyrites is not confined to the substance of the organic remains themselves, but, at places, forms also a layer over their surfaces, which cannot possibly be cleaned off, and by which the appearance of the fossils, as well as their value as specimens, is much impaired.* Hitherto I have only seen this condition in the case of fishes from Gemünden.

Another point to be noticed as regards the condition of the fossils is the *deformation* to which they have mostly been subjected. In this way bilaterally symmetrical organisms, such as Trilobites and fishes, are seen to be almost invariably obliquely disturbed, one side being pushed in advance of the other, as is well shown in the specimens of *Drepanaspis* represented in Plates II., III., and IV.

It is perfectly clear from the contained invertebrate fossils that the Hunsrück slates are of marine origin, and consequently that their "mailed" fishes were inhabitants of the sea. This is, however, not strange when we remember that "mailed" fishes (*Pterichthys*, etc.) also occur in the Middle Devonian Limestones of the Eifel, in company with such purely marine fossils as crinoids and brachiopods. Strange it is, however, that, so far as I can ascertain, it is only in one locality, namely Gemünden, that fish remains have been found in these slates, while, on the other hand, Bundenbach is famed for the variety and beauty of its crinoids and starfishes.

It is also noteworthy that at Gemünden the overwhelming majority of the fish-remains belong to one species, namely, *Drepanaspis Gemündenensis* of Schlüter; the fact being that of the four other species obtained, each is as yet represented only by a single unique specimen. This reminds us of the condition in the Upper Old Red Sandstone at Nairn in the North of Scotland, where almost all the fish-remains (tolerably abundant in certain parts of the rock) belong to one species, namely, *Asterolepis maxima*, Agas., though the proportion of relics of other species is certainly higher than at Gemünden in relation to *Drepanaspis*.

My attention was first drawn to these fishes twelve years ago by Mr STÜRTZ, of Bonn, through whose agency, and also through that of Dr F. KRANTZ, I have succeeded in getting together an excellent collection for the Edinburgh Museum of Science and Art. To Professors BRANCO and JAEKEL I have to express my sincere thanks for permission to

* See Plate I. fig. 1 and Plate V. fig. 2, in which the scales on the side of the tail pedicle of *Drepanaspis* are in this manner concealed.

study the specimens in the Museum of Natural History in Berlin, and also to Professor SCHMEISSER for allowing me to figure three examples belonging to the collection of the Prussian Geological Survey.

Apart from my outline restorations of *Drepanaspis*, I am not aware that any of the vertebrate remains occurring at Gemünden have hitherto been figured.

DESCRIPTION OF SPECIES.

Order HETEROSTRACI, Lankester.

Family DREPANASPIDÆ, Traquair.

Head not externally marked off from body, both enclosed in a carapace of osseous plates, the surfaces of which are ornamented by stellate tubercles. Mouth terminal, unprovided with teeth, or with skeletal parts comparable to jaws. No paired limbs or limb-like appendages. Tail covered with angular sculptured scales, which assume the form of imbricating fulcra along the dorsal and ventral margins. Caudal fin heterocercal, covered with small scales, but without perceptible rays.

Genus *Drepanaspis*, Schlüter, 1887.

Mouth bounded below by a broad median mental plate ; a small perforation (sensory ?) on each side of the head on the ventral surface just within the margin of the carapace. A large median plate on the dorsal and ventral surfaces respectively, the space between these and the lateral or marginal plates being filled up by a mosaic of small polygonal plates. No dorsal fin, caudal not bilobate.

The only known species is :—

Drepanaspis Gemündenensis, Schlüter. Pl. I. figs. 1–3 ; Pls. II.–IV.

Drepanaspis Gemündenensis—Schlüter, *Sitzungsb. niederrhein. Ges.*, Bonn, 1887, p. 126.
" " A. S. Woodward, *Cat. Foss. Fishes Brit. Mus.*, pt. ii., 1891, p. 311.
" " R. H. Traquair, *Nature*, vol. liv., 1896, p. 263 ; *Trans. Roy. Soc.*, vol. xxxix., 1899, p. 844 ; *Geol. Mag.* (4), vol. vii., 1900, p. 153, figs. 1, 2, 3 ; *ib.*, vol. ix., 1902, p. 289, figs. 1, 2.

As this is the only known species of the genus, no specific diagnosis is necessary.

History.—The name *Drepanaspis Gemündenensis* was given in 1887 by Professor C. SCHLÜTER, of the University of Bonn, to some fragmentary remains from the Gemünden slate, which he apparently considered to indicate a fish allied to *Cephalaspis*. In Dr SMITH WOODWARD'S *Catalogue of the Fossil Fishes in the British Museum* (pt. ii., 1891, p. 311), the fish in question is only mentioned by name along with a number of other imperfectly known forms (*Aspidichthys*, *Anomalichthys*, etc.), which he considered as " perhaps for the most part " referable to the Coccosteidæ. However, in

1896, I briefly expressed the opinion that the affinities of Drepanaspis lay rather with the Pteraspidæ (*Nature*, vol. liv. p. 263).

To this opinion I adhered in my first detailed account of *Drepanaspis*, which was included in my Report on the Silurian fishes of Scotland published by the Royal Society of Edinburgh in December 1899. In that memoir I instituted the family Drepanaspidæ, and included it in the Heterostraci along with the Cœlolepidæ, Psammosteidæ, and Pteraspidæ.

In April 1900 I published in the *Geological Magazine* a paper entitled "Notes on *Drepanaspis Gemündenensis*," illustrated by an amended restoration of the dorsal

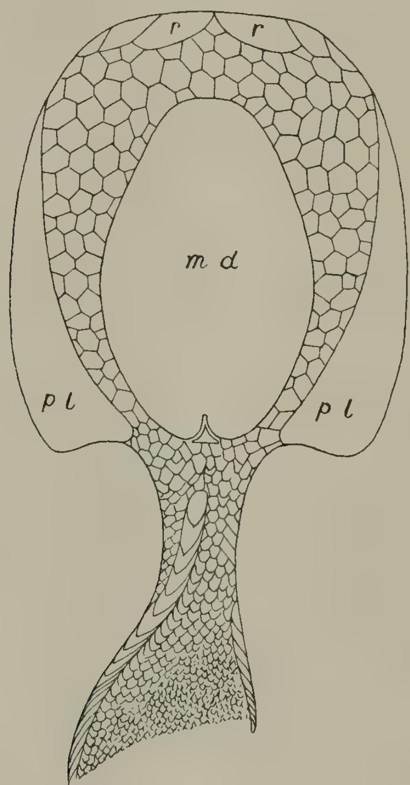


FIG. 1.—*Drepanaspis Gemündenensis*, Schlüter; restored outline of the dorsal aspect; the surface ornament omitted, and the tail twisted round so as to show the caudal fin in profile. *m.d.*, median dorsal plate; *p.l.*, postero-lateral plates; *r.*, rostral plates.

aspect, to which I now added one showing the form and arrangement of the plates on the ventral surface. Fresh material having, however, shown that the position of the sensory openings was still not quite correct, I again in 1902 published in the same journal some "Additional Notes" on the same creature, accompanied by restored figures properly amended as regards the point in question, though the form and arrangement of the scales on the sides of the tail still left something to be desired.

Description.—Nearly entire specimens are shown in Pl. I. fig. 1, Pl. II., and Pl. IV., while in text-figures 1 and 3 I have given restorations of the dorsal and ventral surfaces respectively. The carapace is broad, depressed, very obtusely rounded in front, and terminating behind in a prominent though rounded angle on each side. There is on the dorsal surface a large median plate (*m.d.*) of an ovate-hexagonal shape, the anterior margin being short and nearly straight, while the still shorter posterior one is acutely notched. The outer tuberculated surface of this plate is well seen in Pl. I. fig. 1 and in Pl. II., while in Pl. IV. it is exhibited from the smooth inner aspect. Each postero-lateral angle of the carapace is formed by a narrow, elongated, somewhat falciform plate (*p.l.*), the postero-lateral or cornual, which, tapering to a sharp point in front, forms most of the external margin of the shield. On the dorsal aspect of the carapace (Pl. I. fig. 1, Pl. II.) the entire contour of this plate is seen, the surface being closely covered with the characteristic stellate tubercles, but on the ventral side (Plates III. and IV.) only a thickened external margin is visible, except in dislocated specimens, which margin is marked with wavy ridges instead of tubercles, and obliquely bevelled off just at the postero-lateral angle. Internal to this peculiar thickened margin the inner surface of the plate is smooth, and in entire examples of the fish it is, of course, covered by the ventral plates (see Pl. III. on the left-hand side of the figure). The rest of the dorsal surface is formed by small polygonal plates, well seen in Pl. II., and not so distinctly in Pl. I. These remind us closely of the polygonal areas on the tessellated surface of many Psammostean shields, and, as in these, we often find a central tubercle which is larger and more prominent than the others with which the surface is closely covered. Lastly, at the anterior margin of the dorsal aspect of the carapace we find a few large rostral plates, also well marked in the specimen figured in Pl. II.; the form and number of these plates do not, however, appear to be constant in different examples of the fish.

An invariable appearance on the dorsal surface is a rounded pit on one of the plates on each side at the antero-external margin, and close in front of the forwardly-directed apex of the postero-lateral plate. This pit, conspicuously seen on the left side in Pl. II., I first interpreted as an orbit, a view which I had to abandon on finding that it had a non-perforated floor, which was also ornamented by the same tuberculation as the rest of the surface. The explanation of this phenomenon I subsequently ascertained, as will be seen further on.

Proceeding now to the ventral side of the carapace, an admirable view of its construction, though slightly imperfect behind, is afforded by the specimen represented in Pl. III., of which I also give a still more reduced pen-and-ink lettered sketch in fig. 2 in the text, while in the restoration fig. 3 is added the information derived from other specimens, especially that shown in Pl. V. fig. 1.

First we may note the mouth (*o.*), which is terminal, being placed at the anterior margin of the carapace. It is a transverse slit, the upper margin of which is formed by the anterior edges of the rostral plates (*r.*) already mentioned, while at each outer corner there is a small *external labial* plate (*e.l.*), the tubercles on which are rather larger than those elsewhere. The lower boundary of the oral slit is formed by the anterior margin of a broad pentagonal plate (*m.*), which we may call *mental*. Its anterior or oral margin, which is also the longest, is, however, not quite straight, as it shows a very obtuse angle in the middle; the two lateral margins are shorter and slightly convergent posteriorly; the postero-lateral margins follow, each at an obtuse angle to its respective lateral one, and they meet each other behind, also at an obtuse angle, which fits into the anterior notch or excavation of the plate next to be described. This, the great median ventral plate (*m.v.*), is of a broadly oval form, having a widely open notch or indentation in front, which, as aforesaid, receives the posterior angle of the mental plate. In the specimen shown in Pl. III., and of which text fig. 2 is a sketch, the hinder extremity of this plate is broken off, but in fig. 3 I have restored its contour from other specimens, among others that represented in Pl. V. fig. 1. It will

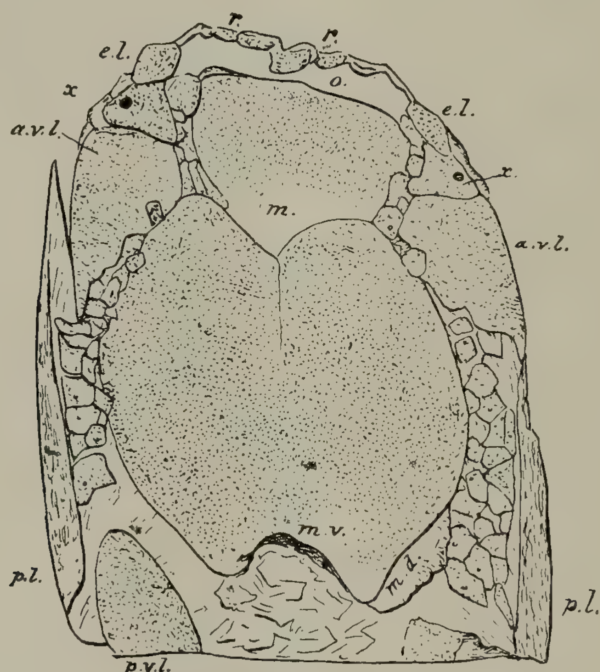


FIG. 2.—Sketch of a specimen of the carapace of *Drepanaspis Gemündenensis*, Schl. One-third natural size. *o.*, mouth; *r.*, rostral or upper labial plates; *e.l.*, external labial plate; *m.*, mental plate; *a.v.l.*, anterior ventro-lateral; *m.v.*, median ventral; *m.d.*, portion of inner surface of median dorsal; *p.v.l.*, posterior ventro-lateral; *p.l.*, postero-lateral or cornual plate.

therefore be seen that the posterior border of this plate presents a narrow bluntly-rounded median notch or indentation, the direction of which is continued for a little way in front by a longitudinal fold-like elevation of the surface. To this indentation I shall again refer in speaking of the scales of the caudal region.

Returning now to the front, we find on each margin of the carapace, immediately behind the small external labial plate, another and rather larger element of a trapezoidal shape (*x.*); transversely placed, with a short external margin, a longer internal one, while the posterior is the largest, and is directed nearly at a right angle to the middle line of the creature. This plate is seen *in situ* on both sides of the carapace in Pl. III., that of the left side in Pl. IV., while in Pl. II. that of the right side is seen detached from its place and turned round so as to show its sculptured surface. In Pl. I.

fig. 2 another detached example of this plate, a right one, is represented, also seen from the sculptured surface.

Closely within the outer narrow margin of this element x. is a small round opening which perforates the plate through and through, from the outer to the inner surface. Now, when we get a view of the inner surface as shown in Pl. I. fig. 3, we perceive that on this aspect of the plate the aforesaid opening is surrounded by a prominent thickened ring-like margin. Here we have the explanation of the rounded pit seen in a corresponding situation on the dorsal aspect of the carapace. It results, as is proved

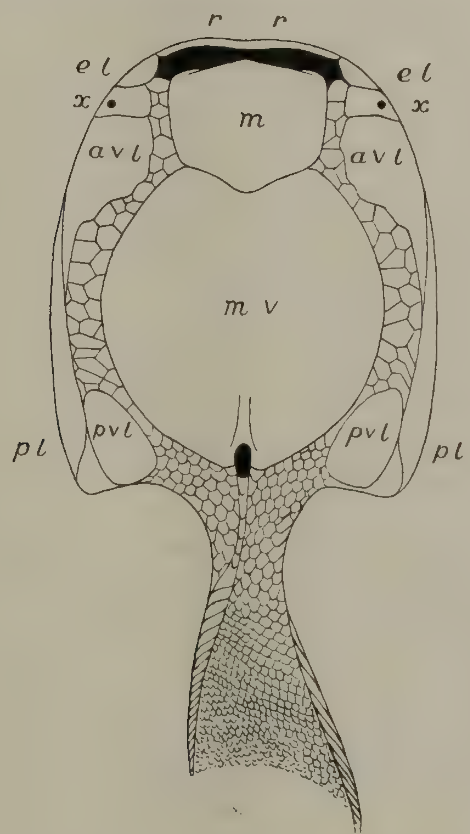


Fig. 3.—Restored outline of the ventral aspect of *Drepanaspis Gemündenensis*, Schl. Surface ornament omitted and the tail twisted round so as to appear in profile. Lettering as in Fig. 2, but the mouth in front and the cloacal opening behind are represented in black.

by specimens worked out from both sides, from the dorsal armature above being squeezed down on this ring, which therefore *shows through*, and produces the appearance of a shallow pit with elevated edges as seen on the left side in Pl. II. Why, in this specimen, is no pit seen on the right side? Clearly because the plate x. is on this side removed from its place—it is seen turned upside down, lying apart from, though close to, the edge of the carapace—and consequently the ring-like margin of its perforation could not operate, as above indicated, in producing the shallow pit in question.

As to the function of this perforation, it is to be noted that it and the plate which bears it occupy a quite similar position to the ocular plate and the supposed orbit o

Pteraspis,—only in *Drepanaspis* the plate and its round opening, though close to the margin, are nevertheless on the ventral surface of the carapace; a circumstance which rather militates against the idea of an eye having had its place there during life. But as these Hunsrück-slate specimens are all crushed absolutely flat, it is by no means certain that in the original uncompressed condition the opening did not look out to the side. It is, however, perfectly clear that a sense-organ of some sort is here indicated, and so we may safely apply to this plate and its round opening the term *sensory*.

Immediately behind this "sensory" plate is another and larger one (*a.v.l.*) of an approximately triangular form, a long irregularly-scolloped side internally, a gently-convex outer one, and an acutely-pointed posterior angle. The outer margin of this plate, which we may call *anterior ventro-lateral*, fits on below the anterior pointed extremity of the great postero-lateral plate (*p.l.*) so extensively seen on the dorsal surface, but, as already explained, appearing on the ventral aspect only as a thickened margin (see Pl. III. and text-figs. 2 and 3). These lateral elements on the ventral surface are on each side separated from the mental and median ventral plates by a series of small polygonal ones, as seen in figs. 2 and 3, but, just behind, there is an ovate-oblong one of considerable size (*p.v.l.*), which may be called posterior ventro-lateral. The space between this and the posterior external angle of the plate *p.l.* (left empty in the figures) seems in one specimen to be covered by another smaller one; any way, I think that in this region the branchial aperture must have been placed, though its position is as yet not exactly determined.

The tail, springing from the middle of the posterior margin of the carapace, is comparatively short, and terminates in a heterocercal though scarcely bilobed caudal fin, but there is no trace of any other fins or appendages, paired or unpaired. The mode of origin of the tail, as seen from above, is well shown in Pl. II. Here we see that the small polygonal plates behind the median dorsal pass into angular imbricating scales, the outer surface of which is covered by small sharp tubercles, which are longer than they are broad, and are arranged in concentric lines which are parallel with the free margins of the scale. But in the middle line, shortly behind the median dorsal plate, there developes a series of elongated median, acutely-pointed and imbricating fulcral scales, which, becoming in succession longer and more acute in contour, form a row along the dorsal margin of the tail. These dorsal fulcra are seen also in Pl. I. fig. 1, Pl. IV. right-hand side, Pl. V. figs. 1 and 2, but more especially in the latter.

On the ventral aspect the relations of the corresponding inferior row of fulcra are best seen in a specimen belonging to the Prussian Geological Survey, and represented in Pl. V. fig. 1. In this specimen, which I have used in the restoration of the region of the body here concerned (see text-fig. 3), the posterior notch of the median ventral plate seems to form, along with an elevated median scale just behind it, a narrow opening, which I take to be the orifice of the cloaca. This is succeeded in the backward direction by four narrow elevated scales, which pass into the median fulcra of the ventral margin of the tail and caudal fin. It will be seen, on inspecting Pl. V.

figs. 1 and 2, that the ventral fulcra are shorter and smaller than those of the dorsal margin.

Both sets exhibit an ornate sculpture, which consists of the tubercles of the ordinary scales, but more lengthened out, so much so in some cases as rather to be described as ridges (Pl. V. fig. 1).

The sides of the tail are clothed with angular imbricating scales, the external sculpture of which has been already noted; but a fact not recorded in my previous descriptions and restorations is, that between the carapace and the origin of the caudal fin there is at least one longitudinal row of scales which are considerably higher than broad,—the cause of my not having noted this before being that this part of the tail is usually more or less covered and obscured by pyritous deposit, as seen to a marked extent in Pl. I. fig. 1, Pl. IV., and Pl. V. figs. 1 and 2. Further on, the scales (Pl. V. fig. 1) become more equilateral and acutely rhombic, and ever smaller as we proceed backwards.

The caudal fin (Pl. I. fig. 1, Pl. IV., Pl. V. fig. 2) is *not bilobate*,—in that respect resembling that of *Cephalaspis* and *Pterichthys*. It is *heterocercal*, inasmuch as the dorsal apex passes further back than the ventral one, and the fulcra along that margin are larger than those along the other. There is, however, no definite line of demarcation between the scales of the body-prolongation and those of the fin-membrane, though the latter become gradually smaller.

The plates of the carapace are thin when compared with those of most other armoured fishes; and as they are entirely converted into iron pyrites, it is unfortunately quite impossible to study their microscopic structure. The creature attained a very considerable size, the largest entire specimen which I have seen being that represented in Pl. IV., and which measures $18\frac{1}{4}$ inches in length.

Observations.—I have nothing to add to or to alter in my opinion as to the more immediate affinities of *Drepanaspis* as expressed in my paper on the Silurian Fishes of Scotland. We have here a fish-like creature whose hard parts are entirely dermal; whose endoskeleton must therefore have been quite unossified in any part, as no traces of it can be found; and whose mouth, a simple transverse slit, shows no teeth nor anything which can be called a mandible. These characters assign to *Drepanaspis* a place in the Ostracodermi, and its indubitable resemblance to *Pteraspis* leads us to class it in the Heterostracous subdivision, although evidence from microscopic structure is unfortunately unavailable. But if it be allied to *Pteraspis*, it is also clearly related to *Psammosteus* and to *Thelodus*; and hence, in my "Silurian" memoir, I included in the Heterostraci not merely the Pteraspidæ, as formerly, but also the Drepanaspidæ, Psammosteidæ, and Cœlolepidæ, the last being looked on as the least, and the first as the most specialised member of the group. To enter again into the question of the origin of the Cœlolepidæ does not come into the scope of this paper, which is purely descriptive; but one cannot help remarking that the structure of Drepanaspis does not seem to lend much support to the idea of the evolution of the Ostracodermi from a Eurypterid ancestry.

OF UNCERTAIN SUB-CLASS.

ORDER ARTHRODIRA.

Family COCCOSTEIDÆ.

Genus *Coccosteus*, Agassiz.

Coccosteus angustus, Traquair. Pl. VI. figs. 1 and 2.

Coccosteus angustus, Traquair, *Nature*, vol. liv., 1896, p. 263; *Proc. Brit. Assoc.*, Belfast, 1902, p. 263.

Specific Characters.—Ventral cuirass narrow; median dorsal plate, with a median crest on the second fourth of its length.

Description.—Only one specimen has occurred, which has been worked from both sides by Mr STÜRTZ's assistants.

The ventral side, Pl. VI. fig. 1, is the more perfect, and shows, besides the ventral cuirass, the displaced suborbital and median dorsal plates. The ventral cuirass shows the unmistakable contour of that of *Coccosteus*, but the lines of demarcation between the individual plates cannot be made out, although a good many cracks, the result of crushing, are very distinctly marked. The position of the anterior median ventral plate is covered by a growth of pyrites, and the posterior one is also indistinguishable.

The anterior ventro-laterals are of the form usual in *Coccosteus*, and, as in *C. decipiens*, the tuberculation of the surface is fine in the centre of the plate, coarse along the anterior, inner, and posterior margins. Along the anterior external margin is seen the short, blunt, pectoral spine, which projects just a little outwards beyond the external angle, as it does in *C. decipiens.*

The part of the cuirass occupied by the posterior ventro-laterals is narrow, hence the specific name; but, as in *C. decipiens*, the left one overlaps the right.

Just in front of the cuirass is the displaced left maxilla or suborbital plate, showing the usual sensory groove bifurcating below into an anterior and posterior branch.

Then, lying close to the outer margin of the right posterior ventro-lateral is the median dorsal, torn from its place and turned upside down, so that we see its upper or external surface on the same side of the slab which displays the ventral plates. Its form is narrow-oblong, truncate in front, faintly pointed behind; its length is $1\frac{7}{8}$ and its breadth $\frac{13}{16}$ inch. In the middle line and in the second fourth of its length is a median crest, which gradually rises anteriorly, suddenly ceases posteriorly, and in the present specimen is bent over by crushing to the right side. The surface of the entire plate is ornamented by stellate tubercles, which toward the posterior extremity are arranged in concentric lines.

In fig. 2 of Plate VI. we have a view of the same specimen worked out from the dorsal side, whereby a certain amount is added to our knowledge of the species. In front is seen a portion of the upper surface of the cranial shield, showing very distinctly the median and external occipital plates. Behind this, the space which ought to have been occupied by the median dorsal is of course blank, but at the sides we have evidence of the lateral plates of the body-cuirass, though hardly in a condition for accurate description. Lastly, on the right side of the hinder part of the cuirass is seen the *inner* surface of the displaced median dorsal plate, which for obvious reasons is not so extensively worked out as on the outer aspect. It shows, however, very distinctly the downwardly-projecting median process, which attains so large a development in *Coccosteus decipiens*, and in *Homosteus* is represented by the "nail"-like process of HUGH MILLER.

Observations.—This is undoubtedly a true *Coccosteus*, and one of the oldest known species of the genus, two other Lower Devonian species being *C. occidentalis*, Newb., from the Corniferous of North America, and *C. hercynius*, H. von Meyer, from the Harz. The genus is therefore of wide geological range, as, besides being abundantly represented in the Middle Old Red Sandstone (Orcadian series) of Scotland, a species (*C. Canadensis*, A. S. Woodw.) has been found in the Upper Devonian of Canada, another (*C. magnus*, Traq.) in the Scottish Upper Old Red, and a third (*C. disjectus*, A. S. Woodw.) in the Upper Old Red of Ireland. The present species is distinguished from all others by the crest on the median dorsal plate.

Genus *Phlyctænaspis*, Traquair.

Phlyctenaspis Germanica, Traquair. Pl. I. fig. 4.

Phlyctænaspis Germanica, Traq., *Proc. Brit. Assoc.*, Belfast, 1902, p. 263.

In fig. 4, Pl. I., we have what appears to me to be the median portion of the cephalic shield of a species of *Phlyctænaspis* from the rostral plate in front to the posterior margin of the median occipital behind, but wanting the lateral portions to which the postorbital, marginal, and external occipital plates so largely contribute. That the lateral parts have been broken away is rendered certain by the fact that a portion of the posterior part of the outer margin of the shield still remains on the right side, and is seen to be joined behind to the occipital region.

No sutures are visible, but there are certain indications of the contour of *rostral* and *pre-orbital* plates similar to those of *Ph. Anglica*, Traq.* The posteriorly directed sensory groove on each *central* plate is also indicated, though the exact demarcation of the plates themselves cannot be deciphered.

The surface, where intact, is covered with stellate tubercles, which in proportion are much smaller than in *Ph. Anglica*, Traq., and not so closely placed as in *Ph. Acadica* (Whiteaves).

* R. H. TRAQUAIR, *Geol. Mag.* (3), vol. vii., 1890, pl. iii. figs. 3, 4. *Ann. and Mag. Nat. Hist.* (6), vol. xiv., 1894 p. 369, woodcut. A. S. WOODWARD, *Cat. Foss. Fishes, Brit. Mus.*, Pt. ii., pl. viii. figs. 5, 6.

Incertæ Sedis.

Family Gemuendinidæ.

Genus *Gemündina*, Traquair.

Gemündina Stürtzi, Traquair. Pl. VII.

Gemündina Stürtzi, Traq., *Proc. Brit. Assoc.*, Belfast, 1902, p. 263.

This is without doubt a vertebrate organism, but its affinities are so problematical, and the appearances present are so difficult, that it is not without much misgiving that I enter into its description at all.

The specimen is represented in Pl. VII. fig. 1, being what I take to be the ventral surface of the creature, while fig. 2, the same slab worked from the opposite side, shows what in that case must be the dorsal aspect.

The side shown in fig. 1, being the one first exposed and worked out, naturally presents all that remains of the contour of the fossil, as, bearing in mind the thinness and fragility of the remains, it was obviously impossible to lay bare the entire surface on the reverse side. Seen from this presumable ventral aspect, the fish measures eight inches from the tip of the snout to where the tail is cut off by the margin of the stone, and, judging from the rate of attenuation of that part, I should say that at least three inches more would be required to complete the original length.

The general contour is somewhat like that of a ray, there being on the left side (the right is imperfect) a lateral expanse like that of the pectoral fin in *Torpedo*, but there is no trace of a ventral. The distance between the outer convex margin of this expanse and the middle line of the fish is $2\frac{1}{8}$ inches, so that were the contour perfect on the left side the entire breadth would be $4\frac{1}{4}$ inches. The snout is bluntly rounded; the tail gradually narrows till it reaches the edge of the stone, and has still a breadth of $\frac{3}{4}$ inch where there cut off.

On examining the fossil (fig. 1) two sets of appearances are distinguishable:—firstly, clear evidence of dermal hard parts covering the entire or nearly the entire surface; secondly, markings which seem to indicate the presence of endoskeletal structures below the skin. We begin with the latter.

Immediately behind the rounded snout we see what is certainly strongly suggestive of the two rami of a lower jaw, the articular part of which would be $1\frac{1}{8}$ inch from the front. It is possible that this appearance may be deceptive, but whether or not homologous with a true mandible, the parts exhibited seem, in my opinion, to support the mouth. Behind these parts is now seen an area of a strongly convex-concave crescentic form, the convex aspect being anterior, the concave back posterior, each limb of the crescent ending in a right and left backwardly-directed pointed process. Just on the posterior concave margin of this crescent, and somewhat to the left of the middle line, we see a small ophiurid starfish firmly adherent to the surface of the fish. Within the concavity formed by the two points of the crescent we find other two curved elevations of the surface, concave towards the middle line and meeting behind. In the middle of the oval space thus enclosed is an oblong elevation, which is clearly of an axial nature.

With the exception of the last-mentioned feature, it is extremely hard to put any interpretation on the above-described appearances, which are also very well shown in Pl. VII. fig. 1. The anterior broad crescentic portion may possibly represent a branchiostegal or opercular flap, but what of the two curved elevations behind and within the deep concavity of the horns of that crescent? Have they to do with the shoulder-girdle?

Immediately behind the elliptical space enclosed by the prominent lines last referred to, and continuing the line of the axial elevation in its centre, is a raised band, $\frac{3}{8}$ inch broad in front, which runs straight back in the middle line for the whole remaining length of the fossil. This band is transversely segmented, the segments being very distinct in front, where fourteen may be counted in the extent of $1\frac{5}{8}$ inch; they get, however, more obscure further back, though still evident. The segments in front likewise show a median furrow, which is well exhibited in the plate.

I interpret this band as a vertebral column, consisting of calcified ring-vertebræ, the longitudinal median groove seen in the front segments being, according to this view, due to vertical crushing. But no evidence of apophyses, either neural or hæmal, can be discerned.

As to the *dermal* structures, almost the entire surface is studded with stellate tubercles, which are larger and more distantly placed in the anterior part of the fish, smaller and closer together behind. Indeed, in front, each of these tubercles seems to form the centre of a small polygonal plate, the rest of the area of which is minutely granulated. This appearance of a mosaic of defined polygonal areas gets lost posteriorly, but at the tail, for a couple of inches before it is cut off by the edge of the stone, we find on each side of the vertebral column a row of small, longitudinally-ridged and imbricating scutes. The fact that the segmented body, which I interpret as the vertebral column, is nearer the right than the left row of scutes, seems to me to be due to a slipping over of the skin to the left side, and therefore to prove that it is an endoskeletal structure.

As shown in fig. 2 of Pl. VII., the specimen has also been worked out from the opposite, or, from my point of view, the *dorsal* side; but, of course, in order that the fossil might hold together, the operator has, except in the extreme front, kept well within the margin, so that this aspect does not exhibit nearly such an extent of surface as the other. What we do see is unfortunately quite as problematic as before. Quite in front there certainly is what *appears* to be a wide-open mouth, close behind which are two oval markings, right and left, reminding us of eyes, but I rather suspect that *this*, at all events, is a deceptive appearance. The rest of the head and body part of the specimen, had best be left without any attempt to decipher it; suffice it to say that we have here abundance of stellate tubercles, smaller and more closely set than on the opposite side; their individuality is, however, in many places obscured by a growth of iron pyrites over their surfaces. And on the tail, as clearly shown in the figure, we have an arrangement of small scutes in longitudinal rows—at least four are shown in the specimen and in the figure—but it must be remembered that on this side of the slab only a small portion of the tail has been uncovered.

Observations.—Little can be said about the affinities of *Gemündina.* The first question which arises is whether or not it may be an Ostracoderm, seeing that the dermal hard parts, especially as seen on the assumed ventral surface, do certainly remind us of the small polygonal plates in *Drepanaspis* and the polygonal areas on many Psammostean shields, while the general contour of the creature is also suggestive of the Cœlolepidæ. But the presence of what seems to be the axial portion of a vertebral column, and of other indications of internal skeleton as described above, seems decisively to negative the idea of relationship in that direction. There remain only the Elasmobranchii and the Chimæroidii to choose from, as it is certainly not a Teleostome. My own feeling—for the idea rests more on feeling than on anything else—is to look upon Gemündina as being possibly a Chimæroid. It is unfortunate that the specimen is as yet unique, but it is to be hoped that additional material will in time be forthcoming to throw more light on the structure and affinities of this singular creature.

Hunsrückia problematica, Traq., n. gen. and sp. Pl. VI. fig. 3.

Undetermined Vertebral Column, Traq.,—*Proc. Brit. Assoc.*, Belfast, 1902, p. 263.

In Pl. VI. fig. 3 is represented, natural size, a series of neural arches and spines belonging to the vertebral column of an otherwise unknown fish.

We first observe a band of pyritous granules, varying from $\frac{1}{4}$ to $\frac{3}{8}$ inch in depth, lying in the position of the vertebral axis, but not showing any distinct signs of segmentation, and over this is the series of arches and spines, forty-eight in number. The spines are slender, elongated, and fused with the arches below so as to form a series of forked rods, which increase in length from the first, which measures $\frac{3}{8}$ inch, to the thirty-first, which attains a length of $1\frac{1}{16}$ inch, after which they again become somewhat shorter. The substance of the arches and spines is granular, as in the calcified cartilage of *Pleuracanthus* and other Selachii, a circumstance which leads one to suppose that the affinities of the creature to whom this vertebral column belonged were with the Elasmobranchs.

In my notice of the Gemünden fishes given to the British Association at Belfast in September 1902 I avoided giving any name to this vertebral column, but since that time I have come to the conclusion that it would be better if it were known by some special designation. I accordingly venture to call it *Hunsrückia problematica*, the generic name being adapted from the name of the district in which Gemünden is situated, namely, the "Hunsrück" or *Dog Back.*

I am indebted to my friend Professor JAEKEL for his good offices in procuring for me the loan of the above described specimen, which is in the collection of the Prussian Geological Survey, Berlin. In accordance with his request, I append an extract from a letter which I received from him shortly after the specimen arrived in Edinburgh.

"Ueber die Wirbelsäule, die ich Ihnen sandte, ist meines Wissens nichts gedruckt worden. Ich habe einen Vortrag darüber gehalten in dem ich auf Grund dieses Stückes betonte, dass die übliche Vorstellung einer langsamen Heranbildung der Wirbelsäule innerhalb der Fische irrig sei. Die von verschiedenen Typen des Devon vorliegenden Wirbelsäulen erfuhren bei den Fischen zunächst eine Reduction der Verknöcherung, um sich dann innerhalb der Fische neu auszubilden. Die vorliegende dem untersten Devon angehörende Wirbelsäule beweist, dass vor den bekannten Ganoiden, Haien, und andern Fischen, Vertebraten mit echter Wirbelsäule existirten. Diese Thatsache war mir deshalb bedeutungsvoll, weil sie die Wahrscheinlichkeit erhöht, dass jene ältesten Fische von terrestrischen Tetrapoden abstammen. Ich würde Ihnen sehr dankbar sein wenn Sie das in Ihrer Arbeit als meine Ansicht darüber benutzten." (*Letter dated* 30*th January* 1900.)

The scope of this paper being, as I have explained on a previous page, purely descriptive, I cannot enter into a discussion of the point raised in the above extract. I am however obliged to confess that, though I am sorry to disagree with Professor JAEKEL, I cannot as yet see my way to adopting the view that fishes are descended from terrestrial tetrapod animals, which have found in the water a new direction for their evolution.

EXPLANATION OF THE PLATES.

[All the figures in the following Plates have been reproduced from photographs taken from the specimens themselves, except in the case of fig. 1 Pl. I. and fig. 1 Pl. V., in which the photographs were, for convenience' sake, taken from plaster casts. In both of these cases, however, the original specimens were carefully studied.]

PLATE I.

Fig. 1. A nearly entire specimen of *Drepanaspis Gemündenensis*, dorsal view, the extreme front and a part of the right side of the carapace being, however, defective, as is also a portion of the circumference in the anterior part of the left side and the posterior external angle of the left dorso-lateral plate. The scales of the tail are obscured by a covering of pyrites as far as the caudal fin, which is well preserved. The present illustration has been taken from an excellent plaster cast of the original specimen which is contained in the collection of the Prussian Geological Survey in Berlin. Reduced by one-fifth.

Fig. 2. Isolated *sensory* or *ocular* plate of the right side, showing the tuberculation of the external surface, and the rounded perforation or opening close to the outer margin. Natural size.

Fig. 3. Sensory and anterior ventro-lateral plates seen from the inner surface, the former showing the thickened ring surrounding the opening on that aspect of the plate. Some of the small polygonal plates of the carapace are also seen scattered about on the upper part of the figure, one of which, seen from the tuberculated surface, is adherent to the anterior ventro-lateral. Natural size. Specimen in the Museum of Science and Art, Edinburgh.

Fig. 4. Central part of the cranial buckler of *Phlyctænaspis Germanica*, Traq. Natural size; original in the Museum of Science and Art, Edinburgh.

Plate II.

In this plate is shown a specimen of *Drepanaspis Gemündenensis*, dorsal view, which, with the exception of the caudal fin-membrane and a small portion lost out of the middle of the carapace, is tolerably complete. Note, on the left side, near the front, the rounded pit produced by the compression of some of the small dorsal plates down on the thickened ring around the inner opening of the perforation of the sensory plate. On the right side, as explained in the text, no such pit is to be seen in this specimen, the cause being that the sensory plate has been removed from its place and is seen lying close to the outer margin of the front of the carapace. This specimen also exhibits very well the gradual elevation and expansion of the fulcra of the dorsal margin of the tail as we proceed backwards from the posterior part of the carapace. Reduced by four-thirteenths; specimen belonging to the Museum of Science and Art, Edinburgh.

Plate III.

This plate shows the greater part of the carapace of *Drepanaspis Gemündenensis* seen from the ventral aspect. A full explanation of the details shown in this specimen is given at p. 727 in the description of text-figure 2, which is a reduced sketch of the original of this plate. Specimen in the Museum of Science and Art, Edinburgh.

Plate IV.

Here is represented, one-half natural size, the most entire specimen of *Drepanaspis Gemündenensis* with which I am acquainted. The fish lies on its back, the ventral surface being upwards. The mouth, in front, is very distinctly seen, as is also the sensory plate of the left side and its perforation, the left anterior ventro-lateral, and the outer thickened margin of the postero-lateral. The same plates are seen on the right side, but the sensory and anterior ventro-lateral have each a narrow portion cut off by the edge of the stone, and the posterior ventro-lateral is seen displaced over the projecting angle of the postero-lateral. The edge of the mental plate forming the lower margin of the mouth is quite clear, but the rest of the plate is obscured by pyrites, as are also most of the small polygonal plates. The great median ventral plate has, however, fallen out, and in its place is seen the internal smooth surface of the median dorsal, a condition which I have observed in more than one specimen lying in the same position as the present one. The scales of the tail are partly obscured by pyrites, but those on the caudal fin are well shown, as are also the marginal fulcra. The caudal fin is, however, not so complete as in the specimen figured in Pl. I. fig. 1. Specimen in the Edinburgh Museum of Science and Art.

Plate V.

Fig. 1. Specimen showing the posterior extremity of the median ventral plate and the commencement of the tail of *Drepanaspis Gemündenensis*. The raised median fold on the hinder part of the median ventral plate, and which ends at the marginal notch, is well seen, as is also the position of the supposed cloacal

opening, but the divisions between the four succeeding narrow elevated scales have not come out so well in the plaster cast from which the photograph was taken. In the proximal part of the tail the scales are covered by pyritous deposit, but further back they are well seen, their angular contour and bold sculpture being strongly marked. The commencement of the line of ventral fulcra is clearly exhibited; those of the dorsal set are somewhat disturbed proximally, but their form and sculpture are very sharply defined. The original specimen is in the collection of the Prussian Geological Survey in Berlin.

Fig. 2. This is a tail of the same species, with the caudal fin truncated and the lateral scales covered with a layer of pyrites, except on a small area behind; the fulcra are, however, well seen, and we may note their short, stout form at the commencement of each series, as well as the greater size of those on the dorsal aspect. Specimen in the Edinburgh Museum of Science and Art.

Plate VI.

Fig. 1. *Coccosteus angustus*, Traquair, seen from the ventral aspect (see text, p. 732). The only known specimen, and preserved in the Edinburgh Museum of Science and Art.

Fig. 2. The same specimen worked out from the dorsal aspect (see text, p. 732).

Fig. 3. *Hunsrückia problematica*, Traquair, portion of vertebral column (see text, p. 736). Specimen in the Collection of the Prussian Geological Survey, Berlin.

Plate VII.

Fig. 1. *Gemündina Stürtzi*, Traquair, ventral aspect (see p. 734) Specimen in the Edinburgh Museum of Science and Art.

Fig. 2. The same specimen worked out from the other side (see p. 737).

DR R. H. TRAQUAIR ON FOSSIL FISHES OF GEMUENDEN—PLATE I.

Fig. 1.
Reduced by One-fifth.

Fig. 3.
Natural Size.

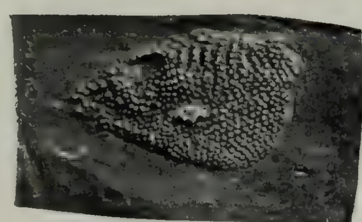


Fig. 2.
Natural Size.

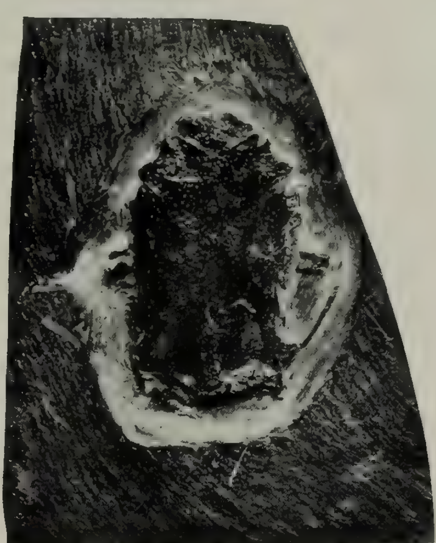
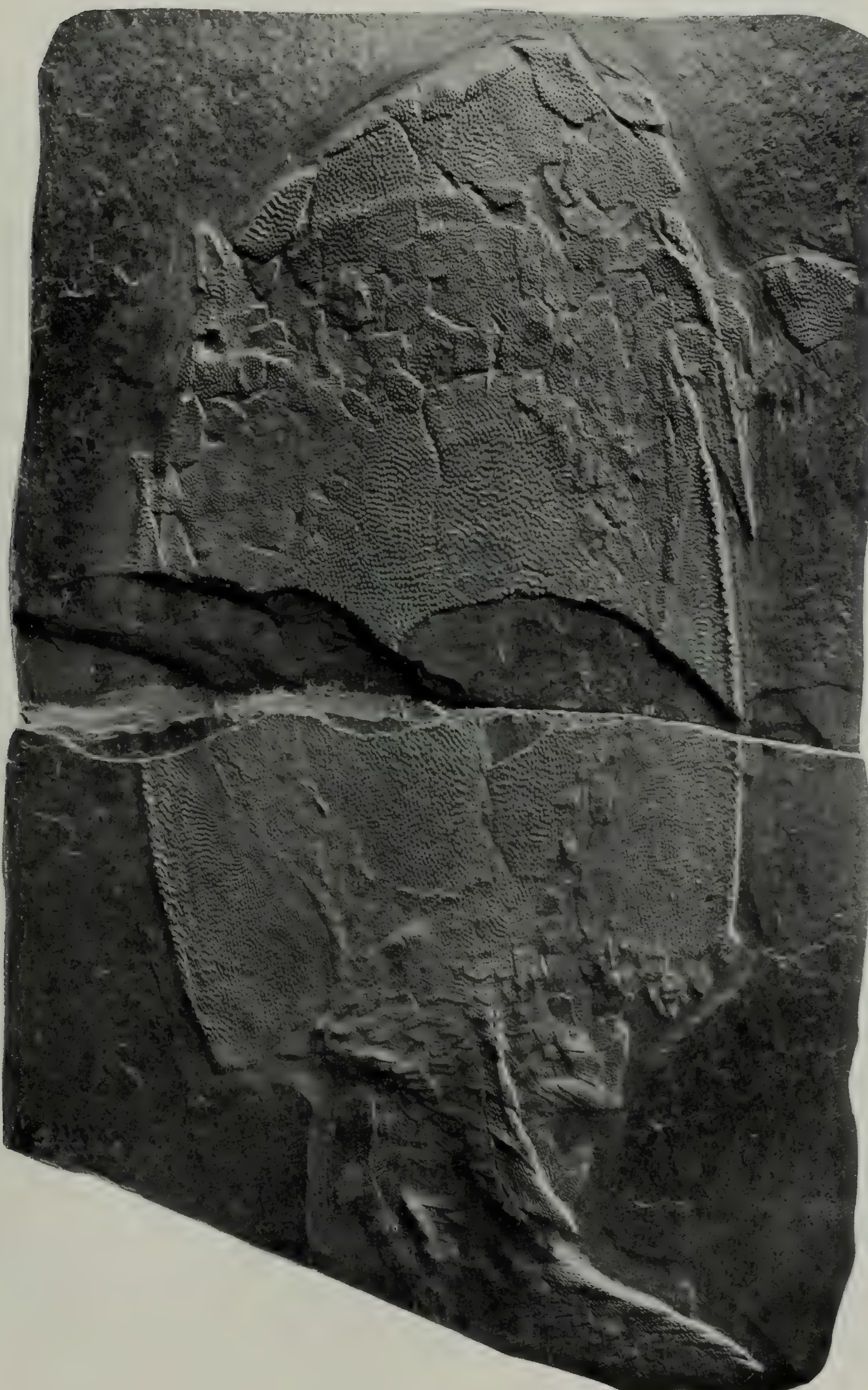


Fig. 4.
Natural Size.

M'Farlane & Erskine, Edinburgh.

DR R. H. TRAQUAIR ON FOSSIL FISHES OF GEMUENDEN—PLATE II.



Nine-thirteenths Natural Size.

M'Farlane & Erskine, Edinburgh.

DR R. H. TRAQUAIR ON FOSSIL FISHES OF GEMUENDEN—PLATE III.

Reduced by One-fifth.

M'Farlane & Erskine, Edinburgh.

DR R. H. TRAQUAIR ON FOSSIL FISHES OF GEMUENDEN—PLATE IV.

Reduced by One-half.

M'Farlane & Erskine, Edinburgh.

DR R. H. TRAQUAIR ON FOSSIL FISHES OF GEMUENDEN—PLATE V.

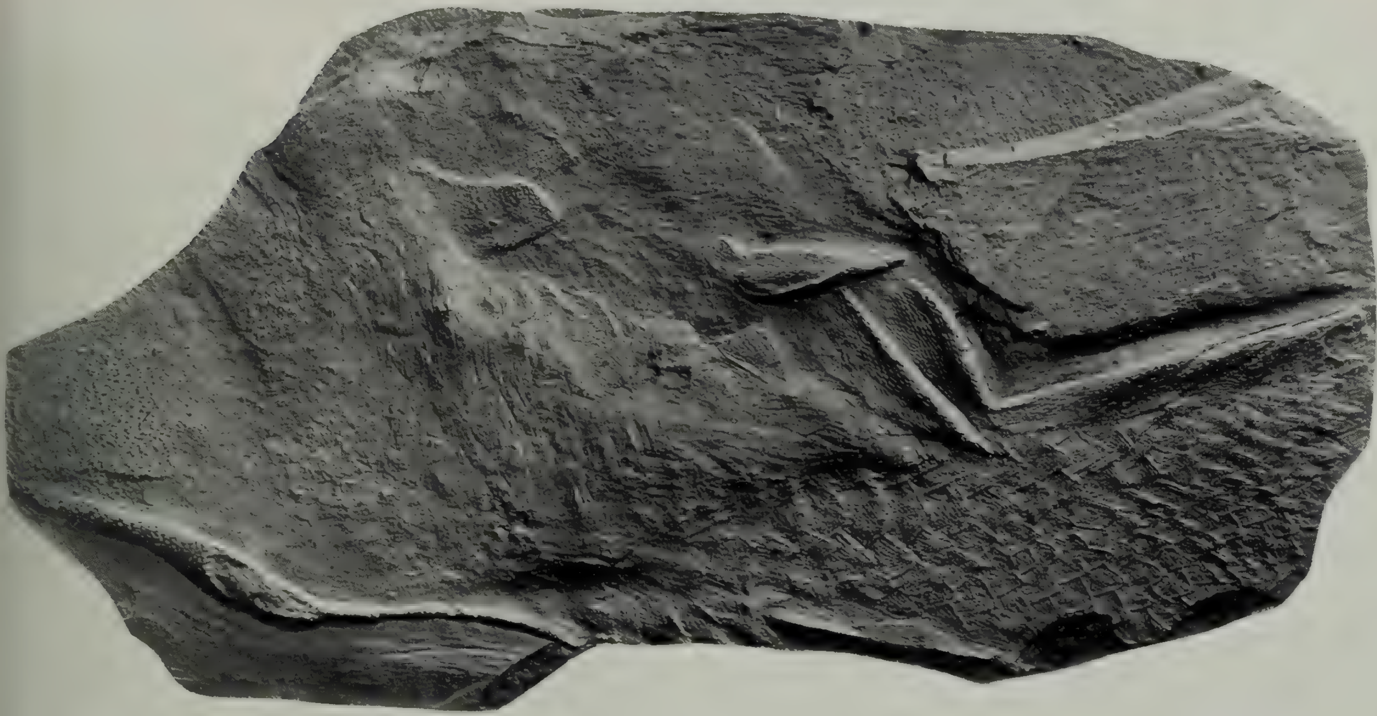


Fig. 1.

Seven-tenths Natural Size.

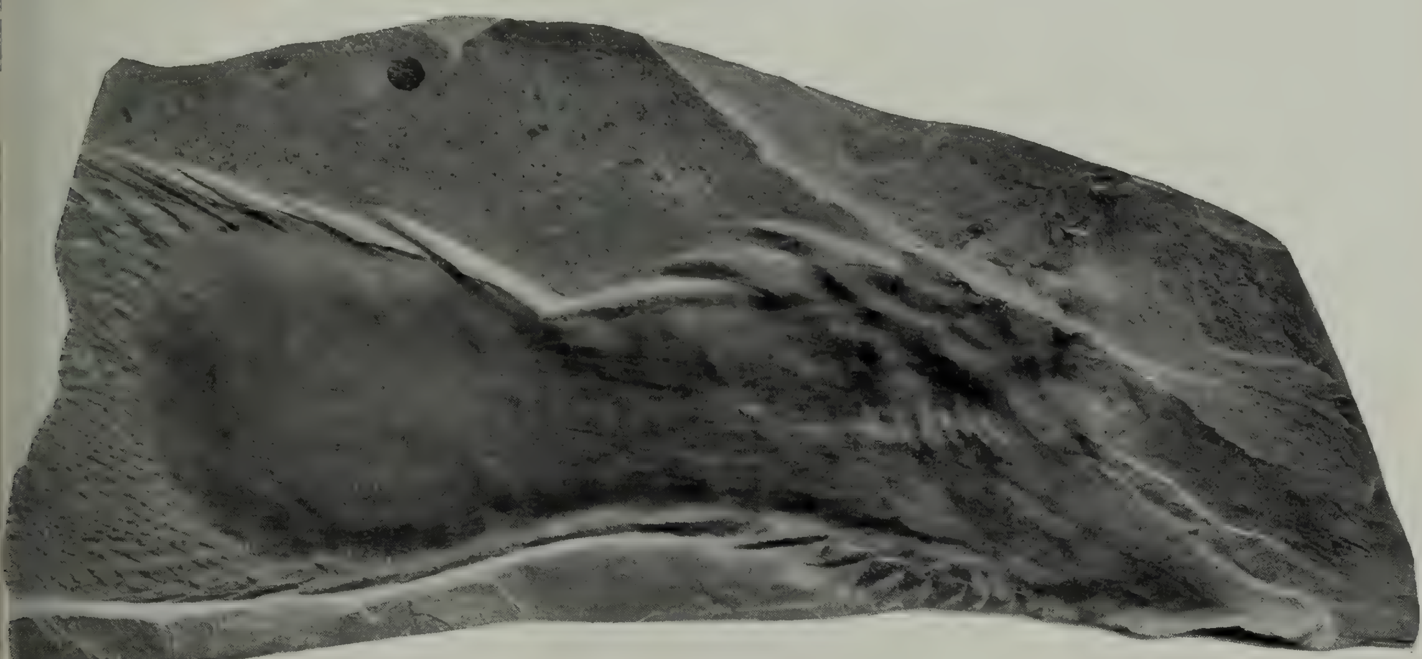


Fig. 2.

Seven-tenths Natural Size.

M'Farlane & Erskine, Edinburgh.

DR R. H. TRAQUAIR ON FOSSIL FISHES OF GEMUENDEN—PLATE VI.

Fig. 3.

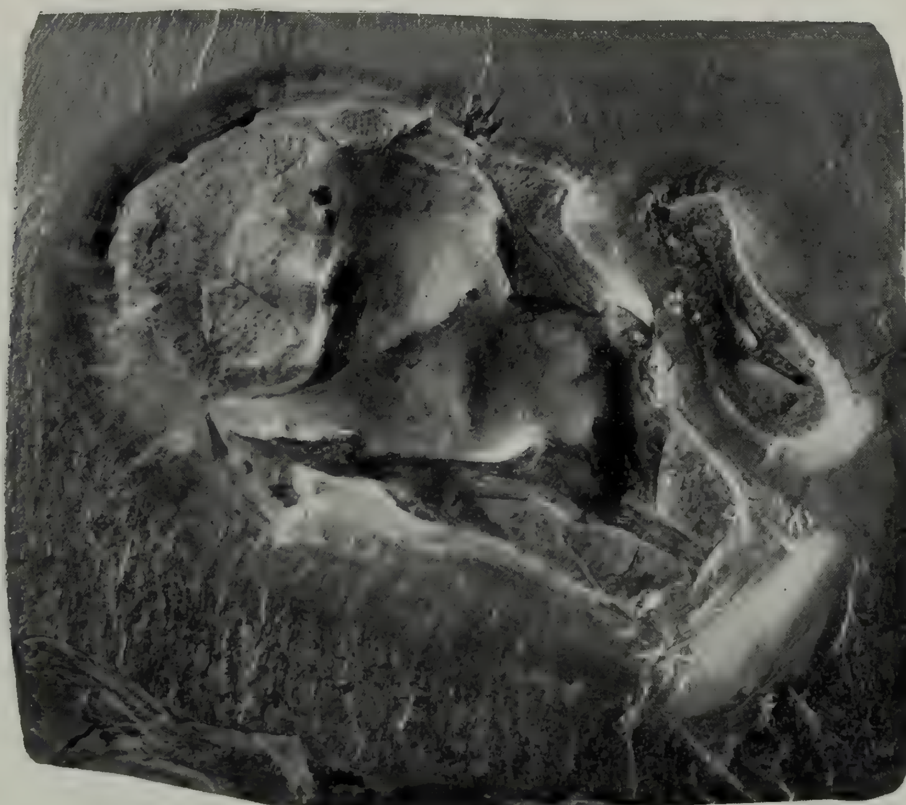


Fig. 2.

Natural Size.

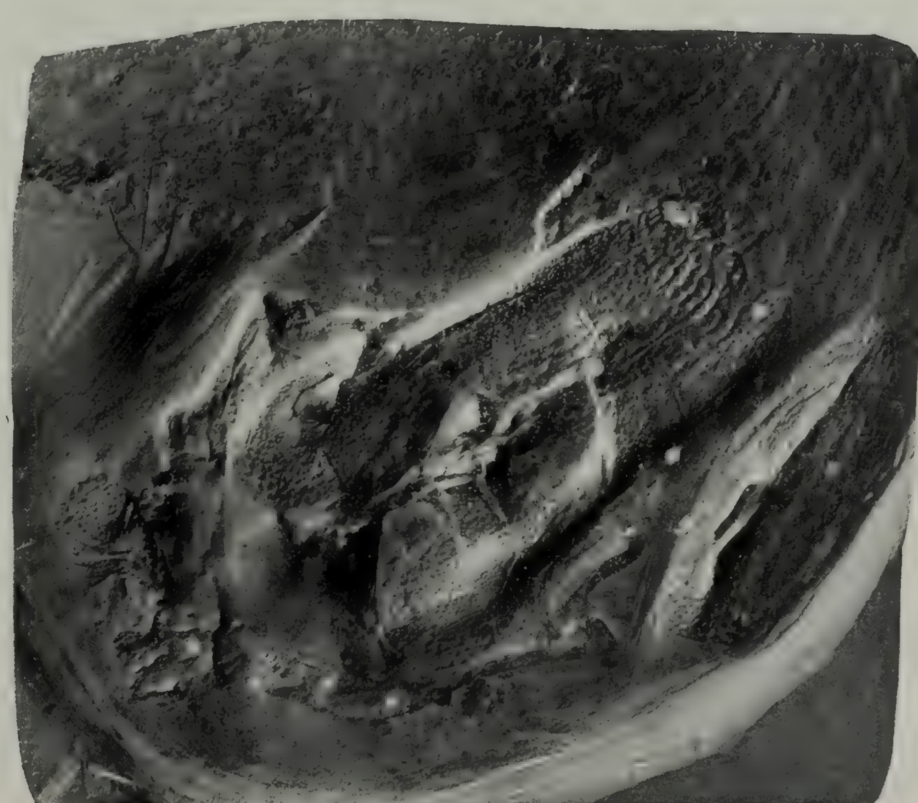


Fig. 1.

M'Farlane & Erskine, Edinburgh.

DR R. H. TRAQUAIR ON FOSSIL FISHES OF GEMUENDEN—PLATE VII.

Fig. 1.

Fig. 2.

Natural Size.

M'Farlane & Erskine, Edinburgh.

XXXI.—*The Fossil Plants of the Carboniferous Rocks of Canonbie, Dumfriesshire, and of Parts of Cumberland and Northumberland.* By Robert Kidston, F.R.S.L. & E., F.G.S. (With Five Plates.)

(Read June 15, 1903. Issued separately December 7, 1903.)

This communication deals with the fossil plants of the Calciferous Sandstone Series and Carboniferous Limestone Series of Eskdale and Liddesdale; of the Lower Coal Measures of Canonbie; of the Middle Coal Measures of Byre Burn, Canonbie; and of the Upper Coal Measures, Jockie's Syke, Cumberland; to which is added a list of the fossil plants of the Calciferous Sandstone Series of Cumberland and Northumberland, with the object of comparing them with those from the corresponding series in Scotland.

The geological structure of the areas from which the fossil plants were derived has been fully described by Dr John Horne, F.R.S., and Dr B. N. Peach, F.R.S.* It is therefore unnecessary for me to make any remarks on this part of the subject, so I have restricted my geological notes to a bare statement of the *horizons* of the localities from which the plants were collected.

I must here express my thanks and great indebtedness to Mr J. J. H. Teall, F.R.S., Director of the Geological Survey of the United Kingdom, for permission to use the Collections of Plants made by the Geological Survey. I wish also to acknowledge the valuable assistance I received from Mr A. Macconochie, by whom the greater portion of the specimens were collected on the Scotch side of the Border, and to Mr John Rhodes, of the Geological Survey of England, who collected almost all the specimens from the Lower Carboniferous Rocks of Cumberland and Northumberland.

I am also indebted to the late Mr Hugh Miller, F.R.S.E., for my knowledge of some of the horizons of the Lower Carboniferous Plants from Cumberland and Northumberland, and to Dr Horne and Dr Peach for the horizons of the Plants from the Lower Carboniferous of Dumfriesshire.

For the Lower Carboniferous Rocks of Berwickshire and some other districts which are included in the paper by Dr Horne and Dr Peach I have not given lists of the fossil plants, as these are reserved for another occasion.

The Fossil Plants of the Calciferous Sandstone Series of Eskdale and Liddesdale.

The great part of the specimens on which the list of the fossil plants of the Calciferous Sandstone Series of Eskdale and Liddesdale is founded was collected by

* "The Canonbie Coal Field: its Geological Structure and Relations to the Carboniferous Rocks of the North of England and Scotland." By B. N. Peach, LL.D., F.R.S., and J. Horne, LL.D., F.R.S. *Trans. Roy. Soc. Edin.*, vol. xl. p. 835.

Mr A. Macconochie, of the Geological Survey of Scotland, about twenty years ago, and on these I contributed a paper to this Society in 1883,* and to Mr Macconochie, I believe, is also due the credit of discovering all the localities from which fossil plants have been collected from this series in Dumfriesshire.

After the publication of my paper dealing with the Eskdale and Liddesdale fossil plants, some of Mr Macconochie's localities were visited by other collectors and a few specimens collected by Mr T. Stock, and a larger series, which was acquired by the Geological Department of the British Museum, from Glencartholm, were subsequently examined by me. The present list contains the results of all these collections, and I take this opportunity of correcting one or two of my earlier identifications which further investigation has shown to be inaccurate.

To save repetition, I give here a list of all the localities and horizons from which the specimens have been derived, as in the appended list of species the localities are given in a contracted form and the horizons are not stated, as at no one locality have the fossil plants been collected from more than one horizon.

I. *Locality.*—Bank of River Esk, Glencartholm, Eskdale.
Horizon.—"Carbonaceous Series" (="Scremerston Series").†

II. *Locality.*—Foot of Tarras Water, three miles south of Langham, Eskdale.
Horizon.—Top of the "Cementstone Series" (="Ballagan Series"), not many feet below the base of the Fell Sandstones.

III. *Locality.*—Archerbeck, above Millsteads, Canonbie.
Horizon.—Lawston Linn Coal Group (="Scremerston Series").†

IV. *Locality.*—Kershope Burn, Liddesdale.
Horizon.—"Carbonaceous Series" (="Scremerston Series").†

V. *Locality.*—Tweeden Burn (near Tweedenhead (?)), Liddesdale.
Horizon.—"Carbonaceous Series" (="Scremerston Series").†

VI. *Locality.*—Docken Beck, near Irvine House, South of Langholm.
Horizon.—Top of the "Cementstone Series" (="Ballagan Series").

VII. *Locality.*—Tinnis Burn, Liddesdale.
Horizon.—"Cementstone Series" (=Ballagan Series).

VIII. *Locality.*—Burn near Sauchtree, Liddesdale.
Horizon.—Not far above the base of the "Cementstone Series" (=Ballagan Series).

IX. *Locality.*—Left bank of Mein Water, below Johnstone Hall, two miles E. of Ecclefechan, Dumfriesshire—J. Bennie.
Horizon.—Cementstone Series (=Ballagan Series).

Algæ.

Bythotrephis, Hall.

1848. *Buthotrephis,* Hall, *Nat. Hist. of New York,* "Palæont. of New York," vol. i. p. 8.

* *Trans. Roy. Soc. Edin.*, vol. xxx. p. 531.

† In my original paper these localities were erroneously placed in the "Cementstone Series."

The genus *Bythotrephis* is now generally employed for the reception of palæzoic Algæ with simple or divided and branched fronds, the genus *Chondrites* being retained for those of similar character which occur in more recent rocks.

In the absence of any knowledge of the affinities of these fossils, this course has much to commend it.

Bythotrephis acicularis, Göppert, sp.

1852. *Confervites acicularis*, Göpp., *Foss. Flora d. Ubergangs*, p. 80, pl. xli. fig. 3.
1850–56. *Confervites acicularis*, Sandberger, *Vers. d. Rheinischen Schichten*, p. 422 pl. xxxviii. fig. 3.
1886. *Confervites acicularis*, Kidston, *Catal. Palæoz. Plants*, p. 21.
1894. *Bythotrephis acicularis*, Kidston, *Proc. Roy. Phys. Soc.*, vol. xii. p. 238.

Remarks.—The true nature of this fossil is very problematical. They appear to be of vegetable origin; and though they agree well with GÖPPERT's figure and description, they possess very little character on which to speak with certainty as to their identity.

Locality.—Glencartholm.

Bythotrephis plumosa, Kidston.

1883. *Chondrites plumosus*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 532, pl. xxx. fig. 3, pl. xxxii. fig. 2.
1894. *Bythotrephis plumosa*, Kidston, *Proc. Roy. Phys. Soc. Edin.*, vol. xii. p. 238.

Note.—Mr SEWARD has suggested that this fossil might be the fine roots of a water plant, but I rather incline to the view that it is algoidal.*

Locality.—Glencartholm.

Bythotrephis simplex, Kidston.

1883. *Chondrites simplex*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 533, pl. xxxi. fig. 14.
1894. *Bythotrephis simplex*, Kidston, *Proc. Roy. Phys. Soc. Edin.*, vol. xii. p. 238.

Locality.—Glencartholm.

Bythotrephis Scotica, Kidston.

(Plate I. figs. 1 and 2.)

1883. *Bythotrephis*, sp., Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 534 (woodcut).
1886. *Bythotrephis Scotica*, Kidston, *Catal. Palæoz. Plants*, p. 22.

Description.—Frond frequently dichotomising at an acute angle, segments of frond generally contracted at point of dichotomy.

Remarks.—When preparing the Report on the Fossil Plants from Eskdale and Liddesdale† I had only imperfect specimens of this species, but of the best a woodcut was given.

* Seward, *Fossil Plants*, p. 148, 1898.

† *Trans. Roy. Soc. Edin.*, vol. xxx. p. 531, 1883.

The Geological Department of the British Museum subsequently obtained a collection of fossil plants from Glencartholm, Eskdale, and among the specimens are the two examples of *Bythotrephis Scotica* shown on Plate I. figs 1 and 2.

The larger specimen, fig. 1, most probably represents a portion of a frond from near the base. All the segments are broken, and the lower part of the fossil is also incomplete. This specimen, which is fully six inches long, illustrates well the dichotomous division of the frond. The lower part of the fossil is half an inch broad, but about three-quarters of an inch from its base it swells out considerably, and is here nine-tenths of an inch wide, but this width most probably represents the measurement of two contiguous segments; and though the fossil does not here show any line of division, in all likelihood the separation of the segments extended further down, but from their close proximity the line of separation has been obliterated through pressure. These branchlets again dichotomise, the segments becoming more narrow, till at their upper extremity, where they are broken over, they are only one-fifth of an inch broad.

In the other example, fig. 2, which is only $2\frac{1}{2}$ inches long, the length of the branchlets between the bifurcations is not so great as in the previous example, and it is possibly a portion of a frond nearer the apex.

Immediately above the base of the fragment it divides into two branches, each of which again dichotomises. These attain rather more than an inch in length, when they give rise to a third set of dichotomous segments. The branchlets which arise from a dichotomy are slightly contracted at their base, and the summit of the segment from which they spring is also constricted. In no case have I been successful in observing the termination of a segment; but from the manner in which they regularly decrease in width, there is probably only a very small portion of the upper part of this specimen wanting.

As the segments are frequently bent over each other, the alga has evidently been of a flaccid nature, but as the fossils are represented on the matrix not only by a well defined carbonaceous stain, but have a quantity of carbonaceous matter adhering to them here and there, the plant must have originally possessed considerable consistency.

My thanks are due to Dr A. Smith Woodward, F.R.S., Keeper of the Department of Geology and Palæontology, British Museum, for permission to figure these specimens.

Locality.—Glencartholm.

Spirophyton, Hall.

1862. *Spirophyton,* Hall, *Contributions to Palæontology,—16th Rept. on the Cabinet of Nat. Hist.*, p. 79.

Spirophyton cauda-galli, Vanuxem, sp.

1842. (*Fucoides*) *cauda-galli,* Vanuxem, *Nat. Hist. of New York,* "Geol. of New York, part iii., Survey of the Third Geological District," p. 128, figs. 3 (1–2).

1863. *Spirophyton cauda-galli,* Hall, *Contributions to Palæont.,—16th Annual Rept. on the Cabinet of Nat. Hist.*, pp. 79–80, figs. 1–2.

Note.—This fossil appears to be characteristic of the uppermost beds of the Calciferous Sandstone series—those in a position immediately below the Hurlet Limestone or its equivalents and the Carboniferous Limestone series.

Localities and Horizon.—River Esk, a short distance above Gilnockie Bridge, Canonbie. A little below the Gilnockie Limestones, which contain the equivalent of the Hurlet Limestone.

Liddelwater, Penton Linns, Canonbie. A little below the Penton Linns Limestones (= Gilnockie Limestones), which contain the equivalents of the Hurlet Limestone.

Filicaceæ.

Sphenopterideæ.

Calymmatotheca, Stur.

Calymmatotheca bifida, L. and H., sp.

1832. *Sphenopteris bifida*, L. and H., *Fossil Flora*, vol i., pl. liii.
1836. *Sphenopteris bifida*, Hibbert, *Trans. Roy. Soc. Edin.*, vol. xiii. p. 177, pl. vi. figs. 1–2.
1857. *Sphenopteris bifida*, Miller, *Testimony of the Rocks*, p. 466, fig. 129.
1836. *Trichomanites bifidus*, Göpp., *Syst. fil. foss.*, p. 264, pl. xv. fig. 11.
1884. *Calymmatotheca (Sphenopteris) bifida*, Kidston, *Quart. Journ. Geol. Soc.*, vol. xl. p. 591.
1886. *Calymmatotheca bifida*, Kidston, *Catal. Palæoz. Plants*, p. 68.
1887. *Calymmatotheca bifida*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 140, pl. viii. figs. 1–5, 6*a*, pl. ix. figs. 16–17.
1894. *Calymmatotheca bifida*, Nathorst, *Foss. Flora d. Polarländer*, Erster Theil, Erste Lief., p. 19, pl. iii. figs. 1–3.
1901. *Calymmatotheca bifida*, Vaffier, *Étude géol. et paléont. du Carbon Infér. du Maconnais*, p. 104, pl. i. figs. 3, 3*a*.
1860. *Gleichenites rutæfolius*, Eichwald (*non* Gutbier), *Lethæa Rossica*, vol. i. p. 90, pl. ii. figs. 5–6. (*Figures inaccurate.*)
1883. *Sphenopteris rutæfolia*, Schmalhausen (*non* Gutbier), *Mém. Acad. Impér. d. Sciences de St Pétersbourg*, vii[e] sér., vol. xxxi. No. 13, p. 4, pl. i. figs. 1–5. (*Die Pflanzenreste der Steinkohlenformation am Ostlichen Abhange des Ural Gebirges.*)
1875. *Todea Lipoldi*, Stur, *Culm Flora*, Heft i. p. 71, pl. xi. fig. 8; Heft ii. p. 291.
1879. *Todea Lipoldi*, Schimper, *in Zittel, Handb. d. Palæont.*, ii. Abth., *Palæophytologie*, p. 107, fig. 75.
1876. *Sphenopteris (Trichomanites) frigida*, Heer, *Beitr. zur foss. Flora Spitzbergens*, p. 6, pl. i. figs. 1–6. (*Kongl. Svenska Vetenskaps-Akad. Handl.*, Band 14, No. 5.)
1883. *Staphylopteris Peachii*, Kidston (*non* Balfour), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 539, pl. xxxi. fig. 6.

Remarks.—The *Sphenopteris rutæfolia*, Eichwald (*non* Gutbier), as figured by Schmalhausen, is evidently the *Sphenopteris bifida* of Lindley and Hutton. Schmalhausen also points out that the figures of *Sphenopteris rutæfolia* originally given by Eichwald are very inaccurate, and that the plants he figures are identical with Eichwald's specimens.

The *Sphenopteris bifida*, Schmalhausen,* is, however, a distinct species from the *Sphenopteris bifida*, L. and H., and the name has evidently been adopted by SCHMALHAUSEN for his plant, which comes from the Permian, by an oversight.

Localities.—Glencartholm, Eskdale; Kershope Burn and Tweeden Burn, Liddesdale.

Sphenopteris, Brongniart.

Sphenopteris crassa, L. and H.

1835. *Sphenopteris crassa*, L. and H., *Fossil Flora*, vol. ii. p. 21, pl. clx.
1883. *Sphenopteris crassa*, Kidston, *Ann. and Mag. Nat. Hist.*, ser. 5, vol. xi. p. 117, pl. iv.
1883. *Sphenopteris crassa*, Kidston, *Proc. Roy. Phys. Soc.*, vol. vii. p. 235, pl. v.
1869. *Adiantites crassus*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 425.
1836. *Adiantites pachyrrachis*, Göpp., *Syst. fil. foss.*, p. 387.
1845. *Cyclopteris pachyrrachis*, Unger, *Synop. plant. foss.*, p. 56.
1850. *Cyclopteris adiantoides*, Unger, *Genera et species*, p. 100.
1837. *Sphenopteris linearis*, L. and H. (*non* Brongt.), *Fossil Flora*, vol. iii. pl. ccxxx.
1843. *Sphenopteris linearis*, Portlock (*non* Brongt.), *Rept. Geol. of Londonderry*, p. 594, pl. xxxviii. figs. 7–7*a*.
1883. *Sphenopteris linearis*, Kidston (*non* Brongt.), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 535.
1875. *Sphenopteris Kiowitzensis*, Stur, *Culm Flora*, Heft i. p. 32, pl. vi. fig. 8.
1877. *Calymmotheca Kiowitzensis*, Stur, *Culm Flora*, Heft ii. p. 151.

Remarks.—Some years ago the late Mr HOWSE, Curator of the Natural History Museum, Newcastle, called my attention to the specimen figured by LINDLEY and HUTTON under the name of *Sphenopteris linearis*, Brongt.† Their specimen is not BRONGNIART'S plant, but an exceedingly fine example of the upper portion of a frond of *Sphenopteris crassa*, but the plate is not a satisfactory rendering of the fossil.

Localities.—Docken Beck, near Langholm; Glencartholm, Eskdale; and Tinnis Burn, Liddesdale.

Sphenopteris pachyrrhachis, Göppert.

1852. *Sphenopteris pachyrrhachis*, Göpp., *Foss. Flora d. Ubergangs*, p. 143, pl. xiii. fig. 3.
1856. *Sphenopteris pachyrrhachis*, Sandberger, *Vers. d. rhein. Schichtensyst.*, p. 428, pl. xxxix. figs. 6–7.
1889. *Sphenopteris pachyrrhachis*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxv. p. 425.
1875. *Archæopteris pachyrrhachis*, Stur, *Culm Flora*, Heft i. p. 64, pl. viii. figs. 8–9.
1852. *Sphenopteris pachyrrhachis*, var. *stenophylla*, Göpp., *Foss. Flora d. Ubergangs*, p. 143, pl. xiii. figs. 4–5.
1869. *Sphenopteris pachyrrhachis*, var. *stenophylla*, Ludwig, *Foss. Pflanzen. a. d. paläolith. Form. etc.*, *Palæentographica*, vol. xvii. p. 119 (? pl. xxiii. figs. 2, 2*a*) (*non* pl. xxiii. fig. 3).

Remarks.—The specimen placed here is the terminal portion of a pinna. It agrees well with GÖPPERT'S figures of the var. *stenophylla*, which I believe to represent the upper portion of the frond. Reference has already been made to the difficulty of

* *Die Pflanzenreste d. Artinskischen u. Permischen Ablagerungen im Osten des Europäischen Russland*,—*Mem. du Comité géol.*, vol. ii. No. 4, p. 35, pl. ii. fig. 20, 1887.

† The *Sphenopteris linearis*, Brongt. (*non* Sternb.), is the *Calymmatotheca affinis*, L. and H., sp.

separating fragmentary examples of the upper part of *Sphenopteris crassa*, L. and H., and *Sphenopteris pachyrrhachis*, Göpp.

I now find I was in error in formerly regarding these two species as possibly being referable to *Sphenopteridium dissectum*, Göpp., sp.

Locality.—Glencartholm, Eskdale.

Sphenopteris obovata, L. and H.

1834. *Sphenopteris obovata*, L. and H., *Fossil Flora*, vol. ii. pl. cix.
1838. *Cyclopteris obovata*, Presl, *in Sternb.*, *Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 134.
1836. *Adiantites microphyllus*, Göpp., *Syst. fil. foss.*, p. 228.
1837. *Sphenopteris excelsa*, L. and H., *Fossil Flora*, vol. iii. pl. ccxii.
1883. *Sphenopteris excelsa*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 537, pl. xxx. fig. 2, pl. xxxi. figs. 7–8.

Remarks.—I have referred elsewhere to the almost certain error in the locality given by Lindley and Hutton for their *Sphenopteris obovata* and *Sphenopteris excelsa.** There is almost certain evidence to show that they came from the neighbourhood of Edinburgh, and this view was strongly held by the late Mr Howse, Newcastle.

From the examination of many specimens I am now perfectly satisfied as to the identity of *Sphenopteris excelsa* with *Sphenopteris obovata.*

Sphenopteris obovata, L. and H., was extremely common at Glencartholm.

Locality.—Glencartholm, Eskdale.

Sphenopteris Hibberti, L. and H., var.

1836. *Sphenopteris Hibberti*, L. and H., *Fossil Flora*, vol. iii. pl. clxxvii.
1883. *Sphenopteris Hibberti*, var. Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 538, pl. xxx. fig. 1.

Remarks.—It is quite possible that the fern I have figured as *Sphenopteris Hibberti*, var., may be a form of *Sphenopteris obovata*, L. and H. I have not yet met with any specimens which could be referred with certainty to *Sphenopteris Hibberti*, L. and H., and hitherto have been unable to discover the type of the species.

Locality.—Glencartholm, Eskdale.

Sphenopteris decomposita, Kidston.

1883. *Sphenopteris decomposita*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 538, pl. xxxii. figs. 1, 1*a*, 4, and 5.

Locality.—Glencartholm, Eskdale.

* *Proc. Roy. Phys. Soc. Edin.*, vol. x. pp. 368 and 380, 1891.

Sphenopteris Macconochiei, Kidston.

1883. *Eremopteris Macconochiei*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 540, pl. xxxii. figs. 3, 3*a*.
1894. *Sphenopteris Macconochiei*, Kidston, *Proc. Roy. Phys. Soc. Edin.*, vol. xii. p. 240.

Locality.—Glencartholm, Eskdale.

Sphenopteris, sp.

1883. *Sphenopteris Höninghausi*, Kidston (*non* Brongt.), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 538.

Locality.—Glencartholm, Eskdale.

Rhodea, Presl.

Rhodea Machaneki, Ettingshausen, sp.

1865. *Trichomanites Machanekii*, Ett., *Foss. Flora d. Mährisch-Schlesischen Dachschiefers* (*Denksch. d. Math. Naturwiss. Classe d. Akad. d. Wissensch.*), vol. xxv. p. 25, fig. 12.
1875. *Rhodea Machanekii*, Stur, *Culm Flora*, Heft i. p. 34.
1886. *Sphenopteris Machanekii*, Kidston, *Catal. Palæoz. Plants*, p. 82.
1883. *Sphenopteris furcata*, Kidston (*non* Brongt.), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 535.

Locality.—Glencartholm, Eskdale.

Rhacopteris, Schimper.

Rhacopteris inæquilatera, Göpp., sp.

1860. *Cyclopteris inæquilatera*, Göpp., *Foss. Flora d. Silur. Devon. u. unter Kohlenform.*, p. 72, pl. xxxvii. figs. 6–7.
1889. *Rhacopteris inæquilatera*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxv. p. 424.
1861. (?) *Adiantites Lindseæformis*, Bunbury, *Geol. Survey of Gt. Brit.*, "Geology of the Neighbourhood of Edinburgh," pp. 144 and 151, fig. 26.
1872. (?) *Adiantites Lindseæformis*, Balfour, *Introd. to Study of Palæont. Botany*, p. 41, fig. 22 *bis*.
1875. *Rhacopteris flabellifera*, Stur, *Culm Flora*, Heft i. p. 76, pl. vi. fig. 10.
1884. *Rhacopteris flabellifera*, Sterzel, ix. *Bericht d. Naturwiss. Gesell. zu Chemnitz*, p. 206, plate, fig. 1 (*Ueber d. Flora u. d. Geol. Alter. d. Kulmform. v. Chemitz-Hainichen*).

Localities.—Glencartholm and foot of Tarras Water, Eskdale.

Rhacopteris Geikiei, Kidston, sp.

1883. *Sphenopteris Geikiei*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 535, pl. xxx. fig. 5, pl. xxxi. fig. 9.
1886. *Rhacopteris Geikiei*, Kidston, *Catal. Palæoz. Plants*, p. 63.

Locality.—Glencartholm, Eskdale.

Neuropterideæ.

Cardiopteris, Schimper.

Cardiopteris polymorpha, Göppert, sp.

1860. *Cyclopteris polymorpha*, Göpp., *Foss. Flora d. Silur. Devon. u. unter. Kohlenform.*, p. 78, pl. xxxviii. figs. 5*a*–5*b*.
1862. *Cyclopteris polymorpha*, Schimper, *Terr. Trans. d. Vosges*, p. 339, pl. xxvii. figs. 1–7.
1873. *Cyclopteris polymorpha*, Feistmantel, *Zeitsch. d. deut. geol. Gesell.*, vol. xxv. p. 522, pl. xvi. figs. 21–23 (? fig. 24) (*Das Kohlenkalkvorkommen bei Rothwaltersdorf in der Grafschaft Glatz*).
1869. *Cardiopteris polymorpha*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 452.
1883. *Cardiopteris polymorpha*, Renault, *Cours d. botan. foss.*, vol. iii. p. 202, pl. xxxv. figs. 2–3.
1899. *Cardiopteris polymorpha*, Zeiller, *Flore foss. d. bassin houil. d'Héraclée*, p. 43, pl. iv. fig. 11 (*Mém. Soc. Géol. d. France. Paléont.*, No. 21).
1899. *Cardiopteris polymorpha*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 132, fig. 120.
1860. *Cyclopteris polymorpha*, var. *rotundifolia*, Göpp., *Foss. Flora d. Silur. Devon. u. unter. Kohlenform.*, p. 78, pl. xxxviii. figs. 6*a* and 6*b*.
1865. *Cyclopteris Hochstetteri*, Ettingshausen, *Foss. Flora d. Mähr.-Schles. Dachschiefers*, p. 21, pl. vi. fig. 3 (*Denksch. d. k. Akad. d. Wissen.*, Band xxv.).
1875. *Cardiopteris Hochstetteri*, Stur, *Culm Flora*, Heft i. p. 48, pl. xiv. fig. 2 (? fig. 3).
1866. *Cyclopteris flabellata*, Salter (*non* Brongt.), *Mem. Geol. Survey of Gt. Britain*, "Geol. of East Lothian," p. 73, fig. 23.
1877. *Cardiopteris*, Stur, *Culm Flora*, Heft ii. p. 288, pl. xi. fig. 6.
1884. *Cardiopteris*, Sterzel, ix. *Bericht d. Naturwiss. Gesell. zu. Chemnitz*, p. 211, pl., fig. 6.
1894. *Cardiopteris*, Nathorst, *Foss. Flora d. Polarländer.*, Erst. Theil, Erst. Lief., "Palæoz. Flora d. Arktischen Zone," p. 25, pl. iii. fig. 9.
1883. *Neuropteris* (*Cyclopteris*) *trichomanoides* (?), Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 541.
1883. *Neuropteris cordata*, Kidston (*non* Brongt.), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 541

Note.—The specimen originally identified as *Neuropteris cordata* is an imperfectly preserved and partially covered pinnule of *Cardiopteris polymorpha*, Göpp.

Locality.—Glencartholm, Eskdale.

Alcicornopterideæ.

Alcicornopteris, Kidston.

Alcicornopteris convoluta, Kidston.

1883. *Staphylopteris*, sp., Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 546, pl. xxxi. fig. 5.
1883. *Rhacophyllum Lactuca*, Kidston (*non* Presl), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 540.
1887. *Alcicornopteris convoluta*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 152, pl. viii. figs. 11–15.

Remarks.—A second species of *Alcicornopteris*—*Alcicornopteris Zeilleri*—has lately been described by VAFFIER,* who has been fortunate in discovering specimens showing the fructification attached to the ends of the branchlets. Dr VAFFIER regards the organ

* VAFFIER, *Etude géol. et paléont. du Carbonifère inférieur du Maconnais*, p. 124, pl. vi. fig. 5, pl. vii. figs. 1, 1*a*, 1*b*, 1*c*, 1*d*, 1*e*, 1*f* (*Ann. de l'Universite de Lyon, Nouv. Sér.*, i. *Sciences Médicine*, fasc. 7), 1901.

which terminates the fruiting branchlets as an indusium, but judging from his figures I would be more inclined to consider the narrow segments into which the terminal structure is divided as sporangia, and not an indusium split into segments. Assuming that this sporangial interpretation is correct, the arrangement of the sporangia in *Alcicornopteris* is somewhat similar in general appearance to that of *Calymmatotheca*, but the synangia are considerably larger than any known Calymmatothecous synangia. In Dr VAFFIER's figures the sporangial stalk expands at its summit and forms a saucer-shaped disc, from the margins of which—according to my interpretation of the structure—spring the narrow lanceolate sporangia, which are rather more than a quarter of an inch long. It is probable that the saucer-like expansion to which the sporangia seem to be attached is partially formed by their united bases.

It is a remarkable circumstance that though in a few localities the fruiting branches of *Alcicornopteris convoluta* are comparatively common, no trace of the fructification has ever been found, nor is there even any remains of the saucer-like base of the fructification attached to the branchlets. It would appear, then, that the whole structure became attached after maturity.

Localities.—Archerbeck, Canonbie; Docken Beck, Eskdale.

Eskdalia, Kidston, n.g.

Stems with smooth cortex bearing slightly distant, spirally arranged oval scars, with the vascular cicatrice placed towards the upper part of the scar. In the sub-epidermal condition the scar shows a central oval band occupying about a third of the area of the scar.

Remarks.—This genus is founded for the reception of the fossil I originally described as *Caulopteris minuta*, but the discovery of better preserved specimens has shown that it cannot be retained in the genus *Caulopteris*.

Eskdalia minuta, Kidston.

(Plate I. figs. 4–8.)

1883. *Caulopteris minuta*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 541, pl. xxxi. figs. 1, 1*a*.

Description.—Stem with smooth cortex bearing egg-shaped scars with slight lateral angles; rounded base and with narrower rounded apex having a small notch; immediately below the notch is a small circular scar. The vascular bundle appears as a narrow elongated ridge, placed about two-thirds above the base. On the young condition of the stem the scars are oval, and terminate a very slight elevation of the cortex. Specimens with the epidermal layer removed show in the scar a central oval band.

Remarks.—On the specimen I originally described the epidermal layer was removed from all the scars, and in this condition they show a central oval ring. On the scar figured in the earlier description * the ring shows a small notch at the summit, but as this is not a constant character, its occurrence may have been accidental.

On Plate I. of the present communication several small figures are given, which illustrate more fully the structure of this plant. Fig. 4 shows a small fragment of the cortex, natural size; at fig. 5 one of the scars is given, enlarged about two times, which shows the lateral angles and the small notch at the apex, immediately below which is a small circular point; this is better seen at the further enlarged fig. 6. Fig. 7 gives an outline sketch of a young stem, natural size, and one of the scars is enlarged at fig. 8. The lateral angles and notch are not observable here, but the scars are only 1 mm. long. On the specimen shown at fig. 4 the scars are 5 mm. long. The stems show no trace of aerial rootlets.

I think *Eskdalia* is probably a fern stem, but on this point I do not speak with any certainty.

Locality.—Glencartholm; and Kershope Burn, Liddesdale.

Equisetaceæ.

Asterocalamites, Schimper.

Asterocalamites scrobiculatus, Schlotheim, sp.

1820. *Calamites scrobiculatus*, Schlotheim, *Petrefactenkunde*, p. 402, pl. xx. fig. 4.
1826. *Bornia scrobiculata*, Sternb., *Essai flore monde prim.*, vol i. fasc. 4, p. xxviii.
1843. *Bornia scrobiculata*, Roemer, *Vers. d. Harzgebirges*, p. 1, pl. i. fig. 4.
1852. *Bornia scrobiculata*, Göpp., *Foss. Flora d. Ubergangs*, p. 131, pl. x. figs. 1–2.
1854. *Bornia scrobiculata*, Römer, *Palæont.*, vol. iii. p. 45, pl. vii. fig. 5.
1869. *Bornia scrobiculata*, Ludwig, *Palæont.*, vol. xvii. p. 116, pl. xxi. figs. 1, 1*a*, 2, 2*a*.
1880. *Asterocalamites scrobiculatus*, Zeiller, *Végét. foss. du terr. houil.*, p. 17, pl. clix. fig. 2.
1899. *Asterocalamites scrobiculatus*, Zeiller, *Flore foss. du bassin houil. d'Héraclée*, p. 58, pl. v. fig. 1.
1899. *Asterocalamites scrobiculatus*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 184, fig. 178.
1900. *Asterocalamites scrobiculatus*, Zeiller, *Éléments de paléobot.*, p. 159, fig. 112.
1901. *Asterocalamites scrobiculatus*, Vaffier, *Étude géol. et paléont. du Carbonifère inférieur du Maconnais*, p. 125, pl. viii. figs. 1, 1*a*, 1*b*, 1*c*, 1*d*.
1901. *Asterocalamites scrobiculatus*, Potonié, *Silur- und die Culm-Flora des Harzes u. des Magdeburgischen*, p. 86, figs. 46–51.
1898. *Archæocalamites scrobiculatus*, Seward, *Fossil Plants*, p. 386, fig. 103, p. 385.
1828. *Calamites radiatus*, Brongt., *Prodrome*, p. 37.
1828. *Calamites radiatus*, Brongt., *Hist. d. végét. foss.*, p. 122, pl. xxvi. figs. 1–2.
1880. *Calamites radiatus*, Rothpletz, *Flora u. Fauna d. Culmform. bei Hainichen in Sachsen,—Botan. Centralblatt*, p. 4, pl. i. figs. 1–5.
1833. *Equisetites radiatus*, Sternb., *Essai flore monde prim.*, vol. ii. fasc. 5–6, p. 46.
1862. *Calamites* (*Asterocalamites*) *radiatus*, Schimper, *Le terr. de Transition d. Vosges*, p. 321, pl. i.
1899. *Calamites* (*Asterocalamites*) *radiatus*, Hofmann and Ryba, *Leitpflanzen*, p. 23, pl. i. figs. 3–4.
1869. *Bornia radiata*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 335, pl. xxiv. figs. 1–9 (*non* fig. 10).

* *Trans. Roy. Soc. Edin.*, vol. xxx., pl. xxxi. fig. 1*a*.

1877. *Bornia radiata*, Schmalhausen, *Ein fernerer Beitrag. z. Kenntniss d. Ursastufe Ost-Sibiriens. Mélang. Phys. et Chim.*, vol. x. p. 738, pl. i. fig. 1 (? figs. 2, 3).
1896. *Bornia radiata*, Renault, *Bassin houil. et perm. d'Autun et d'Épinac. Flore foss.*, deux. part., p. 81, pl. xlii. figs. 2–4 (? fig. 1).
1875. *Archæocalamites radiatus*, Stur, *Culm Flora*, Heft i. p. 2, pl. i. figs. 3–8, pl. ii., pl. iii., pl. iv., pl. v. figs. 1–2; Heft ii. p. 180, pl. xix. figs. 1–6, pl. xx. figs. 1–2, pl. xxi. figs. 1, 1*b*, pl. xxii. fig. 1, 1877.
1880. *Archæocalamites radiatus*, Schimper, *in Zittel, Handb. d. Palæont.*, ii. Abth., *Palæophytologie*, p. 175, figs. 132–133.
1888. *Archæocalamites radiatus*, Toula, *Die Steinkohlen*, p. 203, pl. v. figs. 7 and 10.
1896. *Archæocalamites radiatus*, Solms-Laubach, *Abhandl. d. k. Preuss. Geol. Landesanstalt. Neue Folge*, Heft 23, p. 79, pl. v. figs. 1–2 (Ueber die zeinerzeit von Unger beschrieben structurbietenden Pflanzenreste des Unterculm von Saalfeld in Thüringen).
1900. *Archæocalamites radiatus*, Scott, *Studies in Fossil Botany*, p. 65, figs. 28–29.
1852. *Calamites transitionis*, Göpp., *Foss. Flora d. Ubergangs*, p. 116, pl. iii., pl. iv., and pl. xxxviii.
1853. *Calamites transitionis*, Geinitz, *Vers. d. Grauwackenform.*, part ii. p. 82, pl. xviii. figs. 6–7.
1854. *Calamites transitionis*, Geinitz, *Darstell. d. Flora d. Hainich-Ebersdorfer*, p. 30, pl. i. figs. 2–7.
1854. *Calamites transitionis*, Römer, *Beitr. z. geol. Kentniss. d. nord-west. Härzgebirges*, p. 45, pl. vii. fig. 4.
1850–55. *Calamites transitionis*, Sandberger, *Vers. d. Rhein. Schicht. in Nassau*, p. 426, pl. xxxix. figs. 1, 1*a*.
1860. *Calamites transitionis*, Eichwald, *Lethœa Rossica*, vol. i. p. 166, pl. xiii. figs. 1–2.
1864. *Calamites transitionis*, Richter (*pars*), *Der Kulm. in Thüringen. Zeitsch. d. Deut. geol. Gesell.*, vol. xvi p. 166, pl. v. figs. 7–8, pl. vi. figs. 1 (? 2), 3, 4.
1865. *Calamites transitionis*, Ett., *Foss. Flora d. Mährisch-Schlesischen Dachschiefers.*, p. 10, pl. i. fig. 4, pl. ii., pl. iii. figs. 2–5, pl. iv. figs. 1, 3, 4, and fig. p. 11.
1868. *Calamites transitionis*, Dawson, *Acad. Geol.*, 2nd ed., p. 536, fig. 186.
1869. *Calamites transitionis*, Ludwig, *Foss. Pflanzenreste a. d. palæolithischen Formationen, etc., Palæont.*, vol. xvii. p. 115, pl. xxi. figs. 4, 4*a*, 4*b*, 4*c*, 4*d*, 4*e*.
1870. *Calamites transitionis*, Römer, *Geol. v. Oberschlesien*, p. 54, pl. iv. figs. 1–3.
1873. *Calamites transitionis*, Feistmantel, *Kohlenkalkvorkommen bei Rothwaltersdorf in der Grafschaft Glatz., etc.* (*Zeitsch. d. deut. geol. Gesell.*, vol. xxv.), p. 491, pl. xiv. figs. 3–4.
1854. *Bornia transitionis*, Roemer, *Palæont.*, vol. iii. p. 45, pl. vii. fig. 7.
1852. *Calamites variolatus*, Göpp., *Foss. Flora d. Ubergangs*, p. 124, pl. v.
1852. *Stigmatocanna Volkmanniana*, Göpp., *Foss. Flora d. Ubergangs*, p. 126, pl. viii. pl. ix.
1852. *Bornia Jordaniana*, Göpp., *Foss. Flora d. Ubergangs*, p. 132, pl. x. fig. 3.
1860. *Calamites Sternbergii*, Eichwald, *Lethæa Rossica*, vol. i. p. 172, pl. xiv. fig. 3.
1862. *Calamites inornatus*, Dawson, Flora of the Devonian Period in N.-E. America, *Quart. Journ. Geol. Soc.*, vol. xviii. p. 310, pl. xvii. fig. 56.
1869. *Bornia inornata*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 336.
1865. *Calamites laticostatus*, Ett., *Foss. Flora d. Mährisch-Schlesischen Dachschiefers*, p. 12, pl. iii. fig. 1.
1869. *Bornia laticostata*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 336.
1865. (?) *Calamites tenuissimus*, Ett., *Foss. Flora d. Mährisch-Schlesischen Dachschiefers*, p. 16, p. 17, fig. 2, pl. i. figs. 1–2
1843. *Calamites cannæformis*, Roemer (*non* Schlotheim), *Vers. d. Harzgebirges*, p. 2, pl. i. fig. 7.

Foliage:—

1852. *Asterophyllites elegans*, Göpp., *Foss. Flora d. Ubergangs*, p. 133, pl. vi. fig. 11.
1854. *Sphenophyllum furcatum*, Geinitz, *Darstell. d. Flora d. Hainichen-Ebersdorfer*, p. 36, pl. i. figs. 10–12, pl. ii. figs. 1–2.
1869. *Asterophyllum furcatum*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 345.
1865. *Schizæa transitionis*, Ett., *Foss. Flora d. Mährisch-Schlesischen Dachschiefers*, p. 27, pl. vii. fig. 5.

1873. *Schizæa transitionis*, Feistmantel, *Kohlenkalkvorkommen bei Rothwaltersdorf, etc.*, p. 519, pl. xv. fig. 19 (*Zeitsch. d. deut. geol. Gesell.*, vol. xxv.).
1860. *Schizopteris Lactuca*, Göpp. (*non* Presl.), *Foss. Flora d. Silur. Devon. u. unter. Kohlenform.*, p. 79, pl. xxxviii. figs. 7–8.
1865. *Schizopteris Lactuca*, Ett., *Foss. Flora d. Mährisch-Schlesischen Dachschiefers*, p. 29, fig. 15.
1873. (?) *Asterophyllites equisetiformis*, Feistmantel (*non* Schlotheim), *Kohlenkalkvorkommen bei Rothwaltersdorf, etc.*, p. 498, pl. xiv. fig. 6.

Fructification :—

1841. *Pothocites Grantoni*, Paterson, *Trans. Bot. Soc. Edin.*, vol. i. p. 45, pl. iii.
1872. *Pothocites Grantoni*, Balfour, *Intro. to Study of Palæontol. Botany*, p. 67, fig. 54.
1883. *Pothocites Grantoni*, Kidston, *Ann. and Mag. Nat. Hist.*, ser. 5, vol. xi. p. 300, pl. ix. figs. 1–5.
1874. *Pothocites Patersoni*, R. Etheridge, jr., *Trans. Bot. Soc. Edin.*, vol. xii. p. 151.
1883. *Pothocites Patersoni*, Kidston, *Ann. and Mag. Nat. Hist.*, ser. 5, vol. xi. p. 302, pl. x. figs. 6, 7, 8, pl. xi. figs. 9, 10, pl. xii. fig. 14.
1883. *Pothocites Patersoni*, Williamson, *Proc. Roy. Institution of Gt. Brit.*, vol. x. part ii. p. 299, fig. 9.
1882. *Pothocites calamitoides*, Kidston, *Ann. and Mag. Nat. Hist.*, vol. x. p. 404.
1883. *Pothocites calamitoides*, Kidston, *Ann. and Mag. Nat. Hist.*, 5 ser., vol. xi. p. 305, pl. xii. figs. 13, 15, 16, 17.
1883. *Pothocites*, sp., Kidston, *Ann. and Mag. Nat. Hist.*, ser. 5, vol. xi. p. 304, pl. x. fig. 8.
1883. *Bornia radiata*, Kidston, *Trans. Bot. Soc. Edin.*, vol. xvi. p. 37, pl. i. figs. 1–5, pl. ii. figs. 6–8, pl. iii. figs. 9–10, pl. iv. figs. 13–17.
1873. *Asterophyllites spaniophyllus*, Feistmantel, *Kohlenkalkvorkommen bei Rothwaltersdorf, etc.*, p. 498, pl. xiv. fig. 5.

Note.—The most perfect specimen of the cone of *Asterocalamites scrobiculatus*, Schlotheim, sp., yet discovered, was collected by Mr T. Stock at Glencartholm. It was to this specimen I applied the name of *Pothocites calamitoides* before I recognised the identity of the specimen with the *Pothocites Grantoni*, Paterson.

Locality.—Glencartholm, Eskdale.

Volkmannia, Sternberg.

Volkmannia, sp.

Locality.—Glencartholm, Eskdale.

Pinnularia, Lindley and Hutton.

Pinnularia, sp.

Locality.—Left bank of Mein Water, two miles N.E. of Ecclefechan. Collected by Mr J. Bennie.

Lycopodiaceæ.

Lepidodendron, Sternberg.

Lepidodendron Veltheimii, Sternberg.

1820. "*Schuppenpflanze*," Rhode, *Beitr. z. Pflanz. d. Vorwelt*, p. 16, pl. iii.
1823. "*Vegetable Impression*," Allan, *Trans. Roy. Soc. Edin.*, vol. ix. p. 235, pl. xiv.
1826. *Lepidodendron Veltheimii*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. 48, pl. lii. fig. 3.
1886. *Lepidodendron Veltheimii*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 451, pl. lxvii. fig. 2.
1899. *Lepidodendron Veltheimii*, Potonié, *Lehrb. d. Pflanzenpal*, p. 222, fig. 217.
1901. *Lepidodendron Veltheimii*, Potonié, *Silur- u. d. Culm-Flora d. Harzes u. d. Magdeburgischen*, p. 116, figs. 72, 73, 74, 75A, 75B, 76.
1826. *Lepidodendron Veltheimianum*, Sternb., *Essai Flore monde prim.*, vol. i. fasc. 4. p. xii.
1865. *Lepidodendron Veltheimianum*, Heer (*pars*), *Urwelt Schweiz*, p. 7, fig. 2*a*, 2*b*.
1874. *Lepidodendron Veltheimianum*, Heer, *Beitr. z. Steinkohlen Flora d. Artischen Zone*, p. 4, pl. iv., pl. v. fig. 3.
1875. *Lepidodendron Veltheimianum*, Stur (*pars*), *Culm Flora*, Heft i. p. 79, Heft ii. (1877) p. 375, pl. xxxv. figs. 2–3, pl. xxxvi. figs. 5–6 (*non* figs. 8–10), pl. xxxvii. figs. 1–6, pl. xxxviii., pl. xxxix., figs. 3*a*, 3*b* (*non* figs. 1–2).
1882. *Lepidodendron Veltheimianum*, Renault (*pars*), *Cours d. botan. foss.*, vol. ii. p. 9, pl. v. fig. 1.
1885. *Lepidodendron Veltheimianum*, Kidston (*pars*), *Ann. and Mag. Nat. Hist.*, ser. 5, vol. xvi. p. 243, pl. iii., pl. iv. fig. 2 (*non* figs. 3–4), pl. iv. figs. 11, 11*a*, 11*b*.
1888. *Lepidodendron Veltheimianum*, Toula (*pars*), *Die Steinkohlen*, pp. 195, 196, 198, pl. iii. figs. 2, 7, 12, 15.
1899. *Lepidodendron Veltheimianum*, Hofmann and Ryba, *Leitpflanzen*, p. 79, pl. xv. figs. 7–9.
1900. *Lepidodendron Veltheimianum*, Scott, *Studies in Fossil Botany*, p. 120, fig. 49.
1902. *Lepidodendron Veltheimianum*, Kidston, *Proc. York. Geol. and Polytech. Soc.*, vol. xiv. part iii. pp. 347, 381, 383, pl. lvi. fig. 1, pl. lvii. fig. 1.
? . . *Lepidodendron Veltheimianum*, König, *Icones fossilium sectiles*, pl. xviii. fig. 236.
1828. *Stigmaria* (?) *Veltheimiana*, Brongt., *Prodrome*, p. 88.
1838. *Sagenaria Veltheimiana*, Presl., *in Sternb., Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 180, pl. lxviii. fig. 14.
1852. *Sagenaria Veltheimiana*, Göpp. (*pars*), *Foss. Flora d. Ubergangs*, p. 180, pl. xviii., pl. xix., pl. xxiii. figs. 1–3, pl. xxiv., pl. xliii. fig. 1.
1854. *Sagenaria Veltheimiana*, Geinitz, *Darstell. d. Flora d. Hainichen-Ebersdorfer u. d. Floehaer Kohlenbassins*, p. 51, pl. iv., pl. v., pl. vi. figs. 1, 1*a* (*non* figs. 2–3).
1854. *Sagenaria Veltheimiana*, Roemer, *Palæont.*, vol. iii. p. 46, pl. vii. fig. 14.
1860. *Sagenaria Veltheimiana*, Eichwald, *Lethæa Rossica*, vol. i. p. 119, pl. vii. figs. 2–6.
1862. *Sagenaria Veltheimiana*, Schimper, *Terr. d. transition d. Vosges*, p. 337, pl. xxi., pl. xxii., pl. xxiii., pl. xxiv., pl. xxv.
1862. *Sagenaria Veltheimiana*, Roemer, *Palæont.*, vol. ix. p. 10, pl. iii. fig. 6.
1873. *Sagenaria Veltheimiana*, Feistmantel, *Das Kohlenkalkvorkommen bei Rothwaltersdorf in der Grafschaft Glatz.* (*Zeitsch. d. deut. geol. Gesell.*, vol. xxv.), p. 529, pl. xvii. figs. 31–32.
1826. *Lepidodendron ornatissimum*, Sternb. (*pars*), *Essai flore monde prim.*, vol. i. fasc. 4, p. xii.
1837. *Lipidodendron ornatissimum*, Brongt., *Hist. d. végét. foss.*, vol. ii. pp. 70, 72, pl. xviii.
1837. *Selaginites patens*, Brongt., *Hist. d. végét. foss.*, vol. ii. p. 68, pl. xxvi.
1870. *Lepidodendron patens*, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 36.
1837. *Ulodendron Rhodii*, Buckland, *Geol. and Mineral.*, vol. ii. p. 93, pl. lvi. fig. 6.
1837. *Ulodendron Allanii*, Buckland, *Geol. and Mineral.*, vol. ii. p. 92, pl. lvi. fig. 3.
1838. *Ulodendron Rhodeanum*, Presl, *in Sternb.*, *Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 186.

1838. *Ulodendron ellipticum*, Presl, *in Sternb., Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 186, pl. xlv. fig. 2.

1854. *Sagenaria caudata*, Geinitz (*non* Presl), *Darst. d. Flora d. Hainichen-Ebersdorfer*, p. 53, pl. vi. fig. 4.

1862. *Sagenaria caudata*, Roemer (*non* Presl), *Beitr. z. geol. Kenntniss d. nordw. Harzgebirges*, p. 9, pl. iii. fig. 5.

1854. *Sagenaria geniculata*, Roemer, *Palæont.*, vol. iii. p. 46, pl. vii. fig. 13.

1870. *Lipidodendron geniculatum*, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 33.

1860. *Sagenaria confluens*, Eichwald, *Lethæa Rossica*, p. 121, pl. vii. fig. 1.

1860. *Sagenaria aculeata*, Göpp. (*non* Sternberg), *Foss. Flora d. Silur. Devon. u. unter Kohlenform.*, p. 95, pl. xxxix., pl. xl. figs. 1–3, pl. xli. fig. 1.

1873. *Sagenaria aculeata*, Feistmantel (*non* Sternberg), *Zeitsch. d. deut. geol. Gesell.*, vol. xxv. p. 531, pl. xvii. fig. 33.

1869. *Sagenaria elliptica*, Ludwig (*non* Göppert), *Palæont.*, vol. xvii. p. 122, pl. xxvi. figs. 1*a*, 1*b*, 1*c*, 1*d*.

1870. *Ulodendron commutatum*, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 40, pl. lxiii.

1870. *Ulodendron parmatus*, Carr. (*non* Steinhauer), *Monthly Mic. Journ.*, p. 152, pl. xliv. fig. 4.

1870. *Ulodendron ovale*, Carr., *Monthly Mic. Journ.*, p. 152, pl. xliv. fig. 1.

1876. *Lepidodendron Sternbergii*, Heer (*pars*) (*non* Brongt.), *Beitr. z. Foss. Flora Spitzbergens*, p. 11, pl. iii. figs. 1, 2, 5–18, 20, pl. iv. figs. 3–4

1876. *Lepidodendron selaginoides*, Heer (*non* Sternberg), *Beitr. z. Foss. Flora Spitzbergens*, p. 14, pl. iii. fig. 21.

1880. *Ulodendron minus*, Thomson (*non* L. and H.), *Trans. Edin. Geol. Soc.*, vol. iii. p. 341, pl. (i.) figs. 2–3.

1882. *Ulodendron majus*, Weiss (*non* L. and H.), *Aus d. Steink.*, p. 9, pl. vi. fig. 37 (zweiter abdr.).

Note.—Not common.

Locality.—Glencartholm, Eskdale.

Bothrodendron, Lindley and Hutton.

Bothrodendron Wükianum, Kidston.

1889. *Bothrodendron Wükianum*, Kidston (*pars*), *Ann. and Mag. Nat. Hist.*, ser. 6, vol. iv. p. 65, pl. iv. figs. 3–4.

1889. *Bothrodendron Wükianum*, Kidston (*pars*), *Proc. Roy. Phys. Soc. Edin.*, vol. x. p. 94, pl. iv. figs. 3, 4.

1893. *Sagillaria* (*Bothrodendron*) *Wükianum*, Weiss, *Die Sigillarien d. preuss. Steink.-u.-Rothl. Gebiete.*, ii. Gruppe, *Die Subsigillarien*, p. 57, pl. xxviii. figs. 111–112 (*Abhandl. d. König. Preuss. geol. Landesanstalt. Neue Folge*, Heft 2).

Note.—For remarks on this and the following species, see notes under *Bothrodendron Kidstoni*, Weiss, p. 823.

Locality.—Left bank of Mein Water, two miles N.E. of Ecclefechan. Collected by Mr J. Bennie.

Bothrodendron Kidstoni, Weiss.

1889. *Bothrodendron Wükianum*, Kidston (*pars*), *Ann. and Mag. Nat. Hist.*, ser. 6, vol. iv. p. 65, pl. iv. fig. 2.

1889. *Bothrodendron Wükianum*, Kidston (*pars*), *Proc. Roy. Phys. Soc. Edin.*, vol. x. p. 94, pl. iv. fig. 2.

1893. *Sigillaria* (*Bothrodendron*) *Kidstoni*, Weiss, *Die Sigillarien d. preuss. Steink.-u.-Rothl. Gebiete*, ii. Gruppe, *Die Subsigillarien*, p. 56, pl. xxviii. fig. 110.

Locality.—Left bank of Mein Water, two miles N.E. of Ecclefechan. Collected by Mr J. Bennie.

Lepidophyllum, Brongniart.

Lepidophyllum lanceolatum, L. and H.

1831. *Lepidophyllum lanceolatum*, L. and H., *Fossil Flora*, vol. i. pl. vii. figs. 3–4.
1855. *Lepidophyllum lanceolatum*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 50, pl. ii. fig. 8.
1858. *Lepidophyllum lanceolatum*, Lesqx., *in Rogers*, *Geol. of Pennsyl.*, vol. ii. part ii. p. 875, pl. xvii. fig. 1.
1869. *Lepidophyllum lanceolatum*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 141, pl. xxviii. fig. 10.
1879. *Lepidophyllum lanceolatum*, Lesqx., *Atlas to Coal Flora*, p. 14, pl. lxix. fig. 38.
1886. *Lepidophyllum lanceolatum*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 505, pl. lxxvii. figs. 7–8.
1899. *Lepidophyllum lanceolatum*, Zeiller, *Étude sur la flore foss. du bassin houil d'Héraclée*, p. 50, fig. 11, p. 75.
1900. *Lepidophyllum lanceolatum*, Zeiller, *Éléments de paléobot.*, p. 187, fig. 129.
1880. *Lepidostrobus lanceolatus*, Lesqx., *Coal Flora*, p. 436.
1890. *Lepidostrobus lanceolatus*, Kidston, *Trans. York. Nat. Union*, part xiv. p. 50.
1855. *Sagenaria dichotoma*, Geinitz (*pars*) (*non* Sternberg), *Vers. d. Steinkf. in Sachsen*, p. 34, pl. ii. figs. 6–8.
1855. *Lepidostrobus lepidophyllaceus*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 50, pl. ii. figs. 6–7.
1899. *Lepidostrobus variabilis*, Hofmann and Ryba (*pars*) (*non* L. and H.), *Leitpflanzen*, p. 86, pl. xvi. fig. 6.

Localities.—Glencartholm, Eskdale; and Tweeden Burn, Liddesdale.

Lepidostrobus, Brongniart.

Lepidostrobus variabilis, L. and H.

1831. *Lepidostrobus variabilis*, L. and H., *Fossil Flora*, vol. i. pl. x. pl. xi. (*figure to right only*).
1870. *Lepidostrobus variabilis*, Schimper (*pars*), *Traité d. paléont. végét.*, vol. ii. p. 61, pl. lviii. fig. 2*a* and fig. 5.
1838. *Lepidostrobus*, Brongt., *Hist. d. végét. foss.*, vol. ii., pl. xxii. figs. 5–7.

Remarks.—A very ill defined species, in which are placed a certain type of cone belonging most certainly to several species. I believe that the *Lepidostrobus ornatus*, Brongt.,* only represents a state of preservation of cones which as ordinary impressions would find a place under *Lepidostrobus variabilis*, L. and H.

Localities.—Sauchtree, Liddesdale; Glencartholm, Eskdale.

Lepidostrobus fimbriatus, Kidston.

1883. *Lepidostrobus fimbriatus*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 543, pl. xxxi. figs. 2–4.

Localities.—Glencartholm, Eskdale; Tweeden Burn, Liddesdale.

* Brongniart, *Prodrome*, p. 87.

Stigmaria, Brongniart.

Stigmaria ficoides, Sternberg, sp.

1720. *Lithophyllon*, Volkmann, *Silesia subterranea*, p. 106, pl. xi. fig. 1, add. pl. iv. fig. 9.
1793. Ure, *History of Rutherglen and East Kilbride*, p. 301, pl. xiii. figs. 2 and 7.
1804. Parkinson, *Organic Remains*, vol. i., pl. iii. fig. 1.
1809. *Phytolithus plantites*, Martin, *Petrificata Derbiensia*, pl. xi., pl. xii., pl. xii*.
1820. *Palmacites verucosus*, Schlotheim, *Petrefactenkunde*, p. 349, pl. xv. fig. 4.
1820. *Variolaria ficoides*, Sternb., *Essai flore monde prim.*, vol. i. fasc. i. pp. 23, 26, pl. xii. figs. 1–3.
1822. *Phytolithus verrucosus*, Parkinson, *Outlines of Oryctology*, p. 11, pl. i. figs. 1–2.
1825. *Ficoidites furcatus*, Artis, *Antedil. Phyt.*, pls. iii. iii.*bis*.
1825. *Ficoidites verrucosus*, Artis, *Antedil. Phyt.*, pl. x.
1825. *Ficoidites major*, Artis, *Antedil. Phyt.*, pl. xviii.
1828. *Stigmaria ficoides*, Brongt., *Class. d. végét. foss.*, pp. 28 and 89, pl. i. fig. 7.
1832. *Stigmaria ficoides*, L. and H., *Fossil Flora*, vol. i. pls. xxxi.–xxxvi. ; vol. iii., pl. clxvi. (1835).
1836. *Stigmaria ficoides*, Göpp., *Syst. fil. foss.*, p. 92, pl. xxxiii. fig. 7 (*var.*).
1837. *Stigmaria ficoides*, Buckland, *Geol. and Mineral.*, vol. i. p. 476 ; vol. ii., pl. lvi. figs. 8–11.
1839. *Stigmaria ficoides*, Brongt., "Observ. sur la structure intérieure du Sigillaria elegans comparée à celle des Lepidodendron et des Stigmaria," p. 426, pl. v. (xxix.), (*Archives du Mus. d'hist. nat.*, vol. i., Paris).
1841. *Stigmaria ficoides*, Göpp., *Gatt. d. foss. Pflanzen.*, Lief. 1–2, p. 13, pls. viii.–xv. (? pl. xvi.) (*includes vars.*).
1845. *Stigmaria ficoides*, Corda (*pars*), *Flora d. Vorwelt*, p. 32, pl. xii.
1848. *Stigmaria ficoides*, Hooker, *Mem. Geol. Survey of Gt. Brit.*, vol. ii. part ii. p. 431, pl. i. figs. 1–3, pl. ii. figs. 1–14.
1848. *Stigmaria ficoides*, Sauveur, *Végét. foss. d. terr. houil. de la Belgique*, pl. lxv. fig. 1.
1851. *Stigmaria ficoides*, Göpp. (*pars*), *Zeitsch. d. deut. geol. Gesell.*, vol. iii. p. 278, pl. xi. fig. 6, pl. xiii. figs. 7–9.
1852. *Stigmaria ficoides*, Göpp., *Foss. Flora d. Ubergangs*, p. 245, pl. xxxii. (*vars.*).
1852. *Stigmaria ficoides*, Bronn, *Lethæa Geog.*, vol. i. p. 137, pl. vi.' figs. 13–15, pl. vii. fig. 7.
1854. *Stigmaria ficoides*, Geinitz, *Darst. d. Flora d. Hainichen-Ebersdorfer*, p. 59, pl. xi. figs. 1–2.
1855. *Stigmaria ficoides*, Goldenberg (*pars*), *Flora Sarap. foss.*, Heft i. p. 36, pl. B. figs. 26–28, Heft iii. (1862) p. 17, pl. xiii. fig. 1.
1858. *Stigmaria ficoides*, Binney, *Quart. Journ. Geol. Soc.*, vol. xv. p. 76, pl. iv.
1862. *Stigmaria ficoides*, Roemer, *Palæontographica*, vol. ix. p. 10, pl. iii. fig. 7.
1862. *Stigmaria ficoides*, Schimper (*pars*), *Terr. d. transition d. Vosges*, p. 324, pl. ii., pl. iii., pl. v., pl. viii. (*includes vars.*).
1865. *Stigmaria ficoides*, Dawson, *Quart. Journ. Geol. Soc.*, vol. xxii. p. 148, pl. xii. figs. 83–85 (*vars.*).
1868. *Stigmaria ficoides*, Ebray, *Végét. foss. d. terr. d. transition d. Beaujolais*, p. 17, pls. i.–iv., pl. v. (upper fig.).
1869. *Stigmaria ficoides*, Roehl (*pars*), *Foss. Flora. d. Steink-Form. Westph.*, p. 119, pl. xxv.
1870. *Stigmaria ficoides*, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 114, pl. lxix. (*includes vars.*).
1871. *Stigmaria ficoides*, Heer (*pars*), *Foss. Flora d. Bären. Insel.*, p. 45, pl. viii. fig. 5*c*, pl. ix. fig. 5*a*, pl. xii. figs. 1–4, 6.
1872. *Stigmaria ficoides*, Balfour, *Introd. to Study of Palæontological Bot.*, p. 47, figs. 38–39, pl. iii. figs. 7–9.
1873. *Stigmaria ficoides*, Feistmantel, *Zeitsch. d. deut. Geol. Gesell.*, vol. xxv. p. 535, pl. xvii. fig. 37.
1874. *Stigmaria ficoides*, Heer, *Steinkf. d. Artischen Zone*, p. 5, pl. i. fig. 4, pl. ii., pl. iii.
1875. *Stigmaria ficoides*, Binney, *Palæont. Soc.*, pp. 139, 143, pl. xxi., pl. xxiv. ("Obser. Struct. Foss. Plants," part iv.).
1876. *Stigmaria ficoides*, Heer, *Foss. Flora Helv.*, p. 43, pl. xvi. fig. 9 (*var. vulgaris*).

1879. *Stigmaria ficoides*, Lesqx. (*pars*), *Coal Flora*, p. 514, pl. lxxiv. figs. 1–4, 8, 10, 11 (*includes vars.*
1880. *Stigmaria ficoides*, Schimper, *in Zittel, Handb. d. Palæont.*, ii. Abth., *Palæophyt.*, p. 207, fig. 157.
1880. *Stigmaria ficoides*, Zeiller, *Végét. foss. d. terr. houil.*, p. 140, pl. clxxiii. fig. 4 (*includes vars.*).
1881. *Stigmaria ficoides*, Renault, *Cours d. botan. foss.*, vol. i. p. 155, pl. xix. figs. 7–8 (*includes var.*).
1882. *Stigmaria ficoides*, Weiss, *Aus d. Steink.*, p. 9, pl. vi. fig. 40 (zweiter abdr.).
1883. *Stigmaria ficoides*, Schmalhausen, *Bull. Akad. Impér. Sc. St Pétersbourg*, vii^e. sér., vol. xxxi. p. 17, pl. iv. figs. 9–12.
1884. *Stigmaria ficoides*, Lesqx., *Indiana Dept. of Geol. and Nat. Hist. 13th Ann. Rept.*, part ii., *Palæont.*, p. 95, pl. xix. figs. 1–2 (*includes var.*).
1886. *Stigmaria ficoides*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 611, pl. xci. figs. 1–6 (*includes var.*).
1887. *Stigmaria ficoides*, Williamson, *Morph. and Histol. of Stigmaria ficoides*, *Palæont. Soc.*, pp. i.–iv. 1–62, pls. i.–xv. (text figs.) (*excl.* pl. xiii. figs. 71 and 78).
1888. *Stigmaria ficoides*, Toula, *Die Steinkohlen*, p. 199, pl. iv. figs. 2–5 (*includes var.*).
1888. *Stigmaria ficoides*, Schenk, *Die foss. Pflanzenreste*, p. 91, figs. 48–50.
1890. *Stigmaria ficoides*, Renault, *Flore foss. terr. houil. d. Commentry*, deux. part., p. 552, pl. lxi. fig. 7, pl. lxii. figs. 1–4.
1891. *Stigmaria ficoides*, Solms-Laubach, *Fossil Bot.*, p. 263, figs. 30–37*a*, 37*b*.
1893. *Stigmaria ficoides*, Sterzel, *Die Flora des Rothl. im Plauenschen Grunde bei Dresden*, p. 105, pl. x. fig. 3.
1894. *Stigmaria ficoides*, Nathorst, *Zur foss. Flora d. Polarländer*, Erst. Theil, Erst. Lief., *Zur Palæozoischen Flora d. Arktischen Zone*, p. 43, pl. viii. fig. 9, and pp. 44 (var. *minima*), 71.
1899. *Stigmaria ficoides*, Hofmann and Ryba, *Leitpflanzen*, p. 97, pl. xix. figs. 1–3, pl. xx. figs. 1–2.
1900. *Stigmaria ficoides*, Scott, *Studies in Fossil Botany*, p. 217, figs. 82–89.
1900. *Stigmaria ficoides*, Zeiller, *Éléments de Paléobotanique*, p. 200, fig. 139.
1901. *Stigmaria ficoides*, Vaffier, *Étude géol. et paléont. d. Carbon inférieur des Maconnais*, p. 143, pl. xii. figs. 1, 1*a*, 1*b*.
1901. *Stigmaria ficoides*, Potonié, *Silur- u. d. Culm-Flora d. Harzes. u. d. Maddeburgischen*, p. 100, fig. 59.
1902. *Stigmaria ficoides*, Kidston, *Proc. Yorks. Geol. Polytech. Soc.*, vol. xiv. part iii. p. 356, fig. 10, pl. lvi. fig. 3.
1838. *Stigmaria*, King, *Edin. New Phil. Journ.*, vol. xxxviii. pp. 119, 135, pl. v. figs. 1–2.
1835. *Caulopteris gracilis*, L. and H., *Foss. Flora*, vol. ii. p. 163, pl. cxli.
1862. *Stigmaria anabathra*, Goldenberg (*non* Corda ?), *Flora Saræp. foss.*, Heft iii. p. 19, pl. xi. fig. 7, pl. xiii. figs. 3–4, 9–11, 13–17 (*includes vars.*).
1872. *Stigmaria*, Williamson, *Phil. Trans.*, pp. 220, 234, 235, pl. xxix. figs. 44–46, pl. xxx. figs. 43, 47–49, 51, pl. xxxi. figs. 50, 52, 53.
1876. *Lepidophyllum caricinum*, Heer, *Foss. Flora Spitzbergens*, p. 14, pl. iii. fig. 26 (*Rootlets*).
1888. *Stigmaria*, Renault, *Les plants fossiles*, p. 293, fig. 38.
1890. *Stigmaria*, Potonié, *Jahrb. d. Königl. preuss. geol. Landesanstalt fur* 1889, p. 246, pls. xix.–xxii.
1894. "*Stigmarian Stool*," Kidston, *Trans. Manchester Geol. Soc.*, part xxi. vol. xxii. p. 639, figs.
1899. *Stigmaria*, Potonié, *Lehrb. d. Pflanzenpalæont.*, p. 210, figs. 202–204.
1899. *Stigmaria verrucosa*, White, *Foss. Flora of Lower Coal Meas. of Missouri*, p. 244.

Localities.—Glencartholm, Eskdale; Peel Burn, near Myredykes, Liddlehead, and Sauchtree, Liddesdale.

Stigmaria ficoides, var. undulata, Göppert.

1841. *Stigmaria ficoides*, var. *undulata*, Göpp., *Gatt. d. foss. Pflanzen.*, Lief. 1–2, pp. 13, 30, pl. ix. figs. 5–8 (? fig. 9).
1852. *Stigmaria ficoides*, var. *undulata*, Göpp., *Foss. Flora d. Ubergangs*, p. 245, pl. xxxii. fig. 2.

1884. *Stigmaria ficoides*, var. *undulata*, Lesqx., *Indiana Dept. of Geol. and Nat. Hist.* 13*th Ann. Rept.*, part *Palæont.*, ii., p. 96, pl. xix. fig. 3.

1873. *Stigmaria ficoides*, Feistmantel, *Zeitsch. d. deut. geol. Gesell.*, vol. xxxv. pp. 535, 540, pl. xvii. fig. 37.

1865. *Stigmaria*, with scars in rhomboidal areas, Dawson, *Quart. Journ. Geol. Soc.*, vol. xxii. p. 169, pl. xii. fig. 83.

Locality.—Left bank of Mein Water, two miles N.E. of Ecclefechan. Collected by Mr J. Bennie.

Cordaiteæ.

Cordaites, Unger.

Cordaites, sp.

Locality.—Foot of Tweeden Burn, Liddesdale.

Carpolithes, Schlotheim.

Carpolithes, sp.

1883. *Cardiocarpus*, sp., Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 545, pl. xxxii. fig. 6.

Locality.—Glencartholm, Eskdale.

Incertæ sedis.

Ptilophyton, Dawson.

1878. *Ptilophyton*, Dawson, *Canadian Naturalist*, vol. viii. No. 7, February ("Notes on some Scottish Devonian Plants," p. 7).

I do not think it can be claimed that the true systematic position of the genius *Ptilophyton*, Dawson, has yet been clearly determined. The late Sir William Dawson included five species in his genus *Ptilophyton*.* These were :—

1. *Ptilophyton pennæformis*, Göppert, sp.†
2. *Ptilophyton Vanuxemi*, Dawson, sp.‡
3. *Ptilophyton plumula*, Dawson, sp.§
4. *Ptilophyton Thomsoni*, Dawson.||
5. *Ptilophyton lineare*, Lx., sp.¶

* *Canadian Naturalist*, vol. viii. No. 7, Feby. 1878, and specially see *Geol. Survey of Canada,—Fossil Plants of the Erian (Devon.) and Upper Silurian Forms. of Canada*, part ii. p. 119, 1882.

† *Lycopodites pennæformis*, Göpp., *Foss. Flora d. Silur. Devon. u. unter. Kohlenf.*, p. 84, pl. xlii. fig. 2, 1860.

‡ *Lycopodites Vanuxemi*, Dawson, *Quart. Journ. Geol. Soc.*, vol. xviii. p. 314, pl. xvii. fig. 57, 1862.

§ *Lycopodites plumula*, Dawson, "Rept. Fossil Plants,—Low Carboniferous and Millstone Grit Forms, Canada," p. 24, pl. i. figs. 7–9, 1873 (*Geol. Survey of Canada*).

|| "Notes on some Scottish Devonian Plants," *Canadian Nat.*, vol. viii., Feby. 1878. See also figure of this specimen given by Carruthers, *Journal of Botany*, Nov. 1873, pl. cxxxvii. fig. 2.

¶ *Trochophyllum lineare*, Lesqx., *Coal Flora*, vol. i. p. 64, pl. iii. figs. 24, 25, 25*b*, 1879. *Ptilophyton lineare*, Lesqx., *ibid.*, vol. iii. p. 791, 1884. Dawson, *Geol. Survey of Canada*, "Fossil Plants of Erian (Devon.) and Upper Silur. Forms of Canada," part iii. p. 119, 1882.

Of these, numbers 2 and 4 are Devonian, the others are carboniferous.

Ptilophyton Thomsoni must, however, be excluded from the genus, as more perfect specimens show it to belong to a different class of plants from that with which the remaining species of *Ptilophyton* are supposed to have affinities. The late Sir WILLIAM DAWSON sums up the conclusions to which he had arrived as to the systematic position of Ptilophyton in the following paragraph:—"The species of *Ptylophyton* will thus constitute a peculiar group of aquatic plants belonging to the Devonian and Lower Carboniferous Periods, and perhaps allied to Lycopods and Pillworts in the organisation and fruit, but specially distinguished by their linear leaves serving as floats, and arranged pinnately on slender stems." *

The British Museum possesses a fine specimen † from Stonegun, near Thurso, which shows *Ptilophyton Thomsoni* terminating a stem 16 inches long, and rather less than quarter of an inch thick, and which shows the remains of lateral branches. Similar stems, not showing the terminal portion—the *Ptilophyton Thomsoni*—cannot be distinguished from stems of *Psilophyton*, and under the name of *Psilophyton Dechenianus*, Göpp., sp., I have recorded these, ‡ believing that plant to be synonymous with *Psilophyton robustius*, Dawson, but I now regard these two plants as specifically distinct.

From the further study of additional material within the last few years, I have now little doubt that the stem described as *Caulopteris* (?) *Peachii* by SALTER § is only the larger trunk which bore the branches I identified as *Psilophyton Dechenianus*, and whose terminal portion, as already stated, is the *Ptilophyton Thomsoni* of DAWSON. It is impossible, therefore, to include *Ptilophyton Thomsoni* in the genus *Ptilophyton* as defined by DAWSON.

The *Ptilophyton lineare*, Lesq., sp., I would also feel inclined to exclude from DAWSON'S genus, but not having seen any specimens of this plant it is unsafe to express any definite opinion on this point.

DAWSON and LESQUEREUX had apparently no doubt as to the vegetable nature of *Ptilophyton*, but their figures do not show all the characters they mention; and judging from the only examples of Ptilophyton which have come under my observation, and which are certainly the *Ptilophyton plumula*, Dawson, I must confess to still having some doubt as to the vegetable nature of the organisms included in this genus.

HALL || at one time described the fossils subsequently placed in *Ptilophyton* by DAWSON under the name of *Ptilophyton Vanuxemi* as perhaps crinoid tentacles, or more probably analogous to the *Sertularia*, and regarded them as animal structures.

* "Fossil Plants, Erian (Devon.) and Upper Silur. Forms. of Canada," part ii. p. 122, 1882.

† Geol. Department, Registration No. V1419.

‡ *Catal. of Palæozoic Plants in the British Museum*, p. 232.

§ *Caulopteris* (?) *Peachii*, Salter, *Quart. Journ. Geol. Soc.*, vol. xv. p. 408, fig. 14.

|| Vanuxem, *Nat. Hist. New York*, "Geol. of New York," part iii., Survey of the Third Geological District, p. 175, fig. 46, 1842; also Hall, *Nat. Hist. of New York*, "Geol. of New York," part iv., Geol. of Fourth Geol. District, p. 273, fig. 125, Albany, 1843.

Ptilophyton plumula, Dawson, sp.

1873. *Lycopodites plumula*, Dawson, "Rept. on Fossil Plants of Lower Carb. and Millstone Grit Forms of Canada" (*Geol. Survey of Canada*), p. 24, pl. i. figs. 7–9.
1878. *Ptilophyton plumula*, Dawson, *Canadian Nat.*, vol. viii. No. 7, Feby. ("Notes on Scottish Devonian Plants," p. 7).
1882. *Ptilophyton plumula*, Dawson, *Fossil Plants of Devon. and Upper Silur. Forms of Canada*, p. 121 (*Geol. Survey of Canada*).

Locality.—Glencartholm, Eskdale.

Schutzia, Geinitz.

Schutzia, sp.

1883. *Schutzia*, sp., Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 545, pl. xxxi. figs. 10–12.

Localities.—Tweeden Burn and Kershope Burn, Liddesdale.

Bythotrephis, Hall.

Bythotrephis gracilis, Hall.

(Plate I. fig. 3.)

1843. *Fucoides gracilis*, Hall, *Nat. Hist. of New York*, "Geol. of New York," part iv., Survey of Fourth Geol. District, p. 69, fig. 14.
1848. *Buthotrephis gracilis*, Hall, *Nat. Hist. of New York*, "Palæont. of New York," vol. i. p. 62, pl. xxi. fig. 1.
1852. *Buthotrephis gracilis*, Hall, *ibid.*, vol. ii. p. 18, pl. v. figs. 1*a*, 1*b*, 1*c*, 1*d*.
1884. *Buthotrephis gracilis*, Lesqx., *Indiana Dept. of Geol. and Nat. Hist. 13th Annual Report*, part ii. p. 30, pl. i. figs. 6, 7 (? fig. 1).
1869. *Bythotrephis gracilis*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 198.
1852. *Bythotrephis gracilis*, var. *intermedia*, Hall, *Nat. Hist. of New York*, "Palæont. of New York," vol. ii. p. 19, pl. v. figs. 2*a*, 2*b*.
1852. *Bythotrephis gracilis*, var. *crassa*, Hall, *ibid.*, vol. ii. p. 19, pl. v. figs. 3*a*, 3*b*, 3*c*, 3*d*.
1879. *Palæophycus gracilis*, Lesqx., *Coal Flora*, vol. i. p. 11 (? pl. B, figs. 9–10*a*).
1883. *Chondrites Targioni*, Kidston (*non* Brongt.), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 548.

Remarks.—The specimen figured on pl. i. fig. 3 was originally identified as *Chondrites Targioni*, Brongt.,* and there is really no character by which some of the forms of *Bythotrephis gracilis*, Hall, can be separated from *Chondrites Targioni*, Brongt., but experience has shown that the change which has taken place in the flora of the palæozoic rocks make it most improbable that any palæozoic species passes into the later strata which yields *Chondrites Targioni*. Hence, as already mentioned,† the palæozoic species of the type of *Chondrites Targioni* are now placed in a separate genus.

The form of the plant collected at Borron Point, Arbigland, agrees with HALL's

* *Hist. d. végét. foss.*, p. 56, pl. iv. figs. 2–6.

† *Ante*, p. 743.

Bythotrephis gracilis, var. *intermedia*, pl. v. fig. 2 (*l.c.*), though this differs little from some of the forms included under the var. *crassa*. The whole series of type and varieties described by Hall pass insensibly into each other.

Locality and Horizon.—From the Cementstone group of the Calciferous Sandstone series in limestone beds on the shore between the gardener's cottage and Borron Point, Arbigland, Kirkcudbrightshire.

Fossil Plants of the Carboniferous Limestone Series.

The only section of this series from which we were successful in collecting fossil plants is that exposed along the right bank of the river Esk, extending from the Gilnockie Limestones (which include the equivalent of the Hurlet Limestone) to immediately above Byre Burn Bridge, where the Carboniferous Limestone Series is faulted against the Middle Coal Measures of Byre Burn.

The plants collected from this series are from the following localities :—

A. Sandstone bed, river Esk, right bank, about 150 yards below Gilnockie Bridge.

B. Black Carbonaceous Shale, river Esk, right bank, about 200 yards above foot of Byre Burn.

C. Sandstone bed, river Esk, right bank, about 125 yards above foot of Byre Burn.

Algæ.

Spirophyton, Hall.

Spirophyton cauda-galli, Vanuxem, sp.

Spirophyton cauda-galli, Vanuxem, sp. See *ante*, p. 744.

Locality C.—Entirely filling a bed of sandstone.

Lycopodiaceæ.

Lepidodendron, Sternberg.

Lepidodendron Glincanum, Eichwald, sp.

(Plate II. figs. 20, 21 ; Plate III. figs. 27, 28 ; Plate IV. figs 37–40 ; Plate V. figs. 41–43.)

1860. *Sagenaria Glincana*, Eichwald, *Lethæa Rossica*, vol. i., Ancienne Période, p. 127, pl. v. figs. 21–22, pl. va. figs. 1–10.

1870. *Lepidodendron Glincanum*, Schimper, *Traité d. paléont végét.*, vol. ii. p. 34.

1883. *Lepidodendron Glincanum*, Schmalhausen, *Die Pflanzenreste der Steinkohlenformation am Ostlichen Abhange des Ural Gebirges* (*Mém. l'Acad. Impér. d. Sc. d. St Pétersbourg*, vii[e] sér., vol. xxxi. No. 13), p. 11, pl. ii. figs. 5–16, pl. iii. figs. 1–19, pl. iv. figs. 1–5.

Description.—" Leaves narrow, linear, sharp-pointed, spreading, seldom upright, bent in a sigmoid manner. Sporangial portion of bracts at right angles to the axis, free portion directed upwards, long and acicular. Leaf cushions on the young stems arranged in distinctly spiral columns; on the old stems the leaf cushions form distinct vertical rows. Leaf cushions with longitudinal keel, generally with a rhomboidal half-moon-shaped scar placed above the middle, and below the scar on each side occurs a somewhat bent little linear scar. Leaf cushions on the young stems generally touching each other, rhomboidal, almost quadrate to elongate rhomboidal, or even obovate, and with pointed or elongated ends. Leaf cushions in old age six-angled, fusiform to almost lyre-shaped, generally arranged in vertical rows, and with the extremities united to each other, seldom separate; the leaf cushions on neighbouring columns alternate with each other, and are separated by a more or less broad band of longitudinally wrinkled or reticulated bark."

Remarks.—The above description is that drawn up by SCHMALHAUSEN after a careful examination of numerous specimens, and may be regarded as a reply to the opinion expressed by STUR that *Lepidodendron Glincanum* belonged partly to *Lepidodendron Veltheimii*, Sternb., and partly to *Lepidodendron Volkmannianum*, Sternb.*

In addition to a large amount of material from different localities, SCHMALHAUSEN re-examined the specimens seen by STUR, and several of these are figured in his *Pflanzenreste der Steinkohlenformation am Östlichen Abhange des Ural Gebirges.*"

Certain forms of *Lep. Glincanum* have some resemblance to *Lepidodendron Veltheimii*, and others with vertical rows of leaf cushions remind one of *Lepidodendron Volkmannianum*, especially the figure of a somewhat imperfectly preserved specimen given by EICHWALD on his pl. v*a*. fig. 7; but notwithstanding this resemblance of certain specimens to these two species, it does not appear possible to refer *Lepidodendron Glincanum* to either *Lepidodendron Veltheimii* or *Lepidodendron Volkmannianum*, as proposed by STUR. Even if *Lepidodendron Glincanum*, as refigured and described by SCHMALHAUSEN, contains more than one species, it is not to either of these species that it can be referred, and it may well be that *Lepidodendron Glincanum*, as believed by SCHMALHAUSEN, may be a single but very variable species.

The most peculiar character connected with *Lepidodendron Glincanum* appears to be the two series into which the specimens can be divided, namely, those with spirally placed leaf cushions and those on which the leaf cushions are placed vertically, the latter condition being usually restricted to aged stems; still, SCHMALHAUSEN states that all these varieties are connected by intermediate forms.

SCHMALHAUSEN recognised the following varieties:—

A. *tessellatum*, pl. iii. figs. 1–4, 6.
B. *obovatum*, pl. iii. figs. 7, 7*a*, 8.
C. *rimosum*, pl. iii. figs. 9–15.
D. *sigillariiforme*, pl. iii. figs. 16–19, pl. iv. figs. 1–4.†

* *Ein Beitrag. zur Kenntniss d. Culm und Carbon Flora in Russland,—Verhandl. d. k. k. Geol. Reichsanstalt*, vol. xxviii., Jahrgang 1878, No. 11, p. 219.

† These references apply to SCHMALHAUSEN's paper already referred to.

All the specimens met with at Canonbie belong to those with spirally placed leaf cushions, with the exception of that given on Plate V. fig. 41, where the cushions form almost upright columns.

Description of Specimens:—

At Pl. II. fig. 21 are shown fragments of small branches. A portion of one of these is given enlarged two times at Pl. IV. fig. 37, and an outline of one of the cushions and leaf scar at fig. 38.

The leaf cushions are contiguous, rhomboidal, keeled, with lateral angles rounded; the leaf scar, which is placed above the centre of the cushion, is transversely rhomboidal, lateral angles pointed, upper and lower angles rounded but prominent. This corresponds to SCHMALHAUSEN'S var. *tessellata*, pl. iii. figs. 1–3, but the leaf scar on my specimens is a little larger.

Pl. II. fig. 20, of which a portion is enlarged two times on Pl. IV. fig. 39, and outline of the cushion and scar given at fig. 40.

The leaf cushions here are more fusiform, slightly separated, and provided with prolonged ends, which generally unite with the prolonged point of the cushions above and below in the same series, but sometimes the prolongations of the cushions do not unite, but the extremities pass each other laterally. This form corresponds to SCHMALHAUSEN'S var. *obovatum*, pl. iii. fig. 7. The leaf scar, which is placed above the centre, is also slightly larger than in the figures given by SCHMALHAUSEN, but I believe this may be accounted for by the better preservation of my specimens. The cortex between the cushions is feebly striated longitudinally.

Plate V. fig. 41, a portion of which is enlarged two times on Pl. V. fig. 42, and outline sketch of cushion and scar shown at fig. 43.

The cushions here are fusiform, with long slightly bent produced extremities, with a distinct tendency to form vertical columns. The produced points sometimes unite with neighbouring leaf cushions, sometimes pass each other at their extremities. The leaf scars are separated by a band of irregularly longitudinally striated cortex. This is the var. *rimosum*, and corresponds to the figure given by SCHMALHAUSEN on his pl. iii. fig. 12. This variety differs from *Lepidodendron rimosum*, Sternb., in the leaf scar being proportionally larger; it here occupies almost the complete width of the cushion, whereas in *Lepidodendron rimosum*, Sternb., it occupies less than half the width of the cushion. *Lepidodendron Glincanum*, var. *rimosum*, also differs in the coarser striation of the interfoliar cortex, which is apparently not ornamented with fine oblique striæ as in *Lepidodendron rimosum*, Sternb.

Pl. III. fig. 28. This specimen probably shows an older condition of the var. *tessellatum*, with quadrate leaf cushions. The leaf scars are effaced, and the specimen seems to have suffered from pressure.

Pl. III. fig. 27. This illustrates a still older conditition of the plant. Only the leaf cushions are shown, the leaf scar being quite obliterated, probably the result of age. The specimen shows very clearly the truncation of the leaf cushions, especially at their lower extremities. This is well seen on the cushions marked *a* and *b*, which are bluntly rounded, and that this is not the result of fracture or break is proved by the margin having a distinct and slightly upturned border. This example is approaching to the aged stems described by SCHMALHAUSEN with 'six-angled' leaf cushions.

The specimens just described record the first occurrence of *Lepidodendron Glincanum*, Eichwald, sp., in Britain, and which, as far as I am aware, has not been previously met with outside of Russia.

The bed from which the specimens were collected consisted largely of decorticated and flattened stems of *Lepidodendra* and *Sigillariæ*, with a few specimens of *Stigmaria*. The only plants contained in the bed which could be specifically identified were *Lepidodendron Glincanum*, Eich., sp., *Sigillaria Canobiana*, n.sp., and *Stigmaria ficoides*, Sternb., sp. The fragments of large decorticated trunks in all likelihood belonged to the two species of arborescent Lycopods with which they were associated.

Locality B.

Sigillaria, Brongniart.

Sigillaria Canobiana, Kidston, n.sp.*

(Plate III. fig. 26 ; Plate IV. figs. 29–35 ; Plate V. figs. 46–47.)

Description.—Stem ribbed, ribs expanded in neighbourhood of leaf scar ; leaf scars more or less distant, occupying whole width of rib, rhomboidal, upper and lower angles rounded, usually with a notch on the upper side, lateral angles prominent, cicatricules about the middle of scar, central punctiform, the two lateral lunate. Ribs ornamented with transverse wrinkles, especially distinct above the leaf scars, but becoming gradually less distinct upwards.

Description of Specimens.—This species was fairly plentiful, and the plates illustrate the various states and ages of the plant.

Plate III. fig. 26 ; Plate IV. figs. 29–30. The youngest condition met with is shown natural size at Plate III. fig. 26 ; a portion of this specimen enlarged two times is given on Plate IV. fig. 29, and an outline of a leaf scar at fig. 30. This example has not suffered from pressure, for the leaf scars are placed on gradually increasing elevations of the rib, from the summit of which the leaf scars slope back. The leaf scars are rhomboidal, with rounded upper and lower angles and prominent lateral angles. This is the only specimen I possess of this species on which the leaf scars have no distinct notch on their upper margin. The leaf scars are here separated by little more than their own height.

Plate IV. fig. 33, natural size, portion enlarged two times at fig. 34, and outline scar at fig. 35.

Although from the size of the leaf scars this is probably portion of an older specimen than the last, the leaf scars are rather closer. The ribs do not show the elevation of

* Called after the parish of Canobie, more usually designated Canonbie.

the leaf scar as in Pl. III. fig. 26, but the absence of this elevation on this and the succeeding examples is evidently the result of pressure. The leaf scars on this specimen are scarcely so wide as the rib, and from their lateral angles two little lines run downwards, and in some cases seem to bend inwards and meet, cutting off an oval area. At first sight one might think this specimen specifically distinct from the other examples of *Sigillaria Canobiana* described here, but the upper part of the fossil marked A shows the ordinary form of the species, though towards the middle, and especially to the left, the leaf scars become closer and appear as if separated into compartments, but at the base at B they again begin to assume the normal condition. Such approximation of the leaf scars on the stems of *Sigillariæ* is no uncommon occurrence.

Plate IV. fig. 31 is portion of another specimen enlarged two times, and fig. 32 shows an outline sketch of a leaf scar and portion of a rib. The distinctive characters of the species are well shown on this example. The general form of the leaf scar is rhomboidal, the upper margin is distinctly notched, the lower is rounded, while the lateral angles are prominent. The leaf scars occupy the whole width of the rib, which at this point widens out. Immediately below the leaf scar the rib narrows gradually till it reaches the scar beneath, where it slightly expands again, though it still remains considerably narrower than the leaf scar against which it terminates. The margin of the interfoliar region of the rib thus forms a sigmoid line, the sinus of whose curve is occupied by the inflated portion of the neighbouring rib. The transverse wrinkling on the surface of the rib, which extends almost over the whole width, is strongest immediately above the leaf scar, and becomes more feeble as it is traced upwards, till below the next higher leaf scar it is absent or only very slightly shown. The leaf scars are here more distant than on the specimens already described, being distant from each other about the space of two leaf scars.

Plate V. fig. 45, natural size, portion enlarged two times given at fig. 46, and outline sketch of scar at fig. 47.

The leaf scars on this specimen are more truly rhomboidal than on some of the examples already described, and are slightly more distantly placed. As in all cases except that shown on Plate IV. fig. 33, they occupy the whole width of the rib, which at their point of insertion is widened.

Fig. 45 illustrates well the inflation of the rib, which, a short distance below the scar, becomes contracted. The margin of the upper part of the rib between the leaf scars is convex, but it afterwards runs almost straight till it abuts on the scar below it. The sigmoid curve is not so prominent as in Plate IV. fig. 31. The transverse wrinkling on the surface of the interfoliar cortex is similar to that of the specimens already described.

Remarks.—There is no species of *Sigillaria*, as far as I am aware, with which *Sigillaria Canobiana* could be mistaken. Its nearest ally is probably *Sigillaria polyploca*, Boulay.* The ribs in this species are also inflated, but their widest part is a

* *Sigillaria polyploca*, Boulay, *Terr. houil. du Nord de la France*, p. 47, pl. ii. fig. 8, 1876. See also Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 450, pl. lxxxii. figs. 7–8.

little distance below the more elongated leaf scar, which does not occupy the whole width of the rib and which is surmounted by a distinct lunate scar. These characters at once distinguish the two species. *Sigillaria polyploca* appears to be restricted in its distribution to the Middle Coal Measures.

Sigillaria Canobiana is also easily known from *Sigillaria Youngiana*, Kidston,* which has much more contracted and expanded ribs, their width about midway between the leaf scars being only half the width of the rib shortly below the leaf scar, by the form and position of the leaf scar which is placed about the centre of the inflation, and by the delicate very short lines, mostly upright, with which the surface of the interfoliar cortex is ornamented.

Sigillaria Youngiana, which comes from the Possil Ironstone group of the Carboniferous Limestone series, and *Sigillaria Canobiana* are the only two ribbed *Sigillariæ* yet discovered in the Lower Carboniferous Rocks of Britain, and the first mentioned is only known from a single specimen.

Localities A and B.

Stigmaria, Brongniart.

Stigmaria ficoides, Sternb., sp.

Stigmaria ficoides, Sternb., sp. (See *ante*, p. 757.)

Locality B.—Common.

Stigmaria ficoides, Sternb., sp. var.

Locality B.

Stigmaria (? **Stigmariopsis**) **rimosiformis,** Kidston, n.sp.

(Plate II. fig. 15.)

Description.—Rootlet scars in structure and position as in *Stigmaria ficoides*; cortex ornamented with irregularly flexuous lines, which converge towards the rootlet scars, especially on the two sides facing the axis of the rhizome.

Remarks.—This *Stigmaria* has some similarity to the *Stigmaria ficoides*, var. *rimosa*, Goldenberg,† but in *Stigmaria rimosiformis* the ridges are more numerous, finer, and converge towards the scars, where they are thinner and closer than on the inter-rootlet portion of the cortex, whereas in *Stigmaria ficoides*, var. *rimosa*, Goldenberg, the ridges are coarser and more distant, and have the tendency to bend round the scars, not converging towards them.

It seems to me possible that both *Stigmaria ficoides*, var. *rimosa*, Gold., and

* *Sigillaria Youngiana*, Kidston, *Proc. Roy. Phys. Soc. Edin.*, vol. xii. p. 261, pl. vi. figs. 2, 2*a*.

† *Stigmaria Anabathra*, var. *rimosa*, Goldenberg, *Flora Saræp. foss.*, Heft iii. p. 19, pl. xiii. fig. 16.

‡ *See* Kidston, *Carboniferous Lycopods and Sphenophylls*,—*Trans. Nat. Hist. Soc. Glasgow*, vol. vi. (new series) p. 108, 1901.

Stigmaria rimosiformis may belong to the genus *Stigmariopsis*, Grand' Eury. The internal structure of these two rhizomes is unknown, but the ridges on the surface are probably the external indications of sclerenchymatous bands in the outer cortex, which is one of the characters of *Stigmariopsis*, but is a character not observed in any specimens of undoubted Stigmaria which have shown their internal organisation.

Locality A.

Fossil Plants of the Lower Coal Measures.

All the fossil plants from this division were collected from the coals at present being worked at Rowanburn.

In addition to the specimens collected by ourselves, Mr Kenneth Bowie showed us examples of *Calamites undulatus*, Sternb., and *Calamites Cistii*, Brongt.

Filicaceæ.

Sphenopteris, Brongniart.

Sphenopteris obtusiloba, Brongt.

1829. *Sphenopteris obtusiloba*, Brongt., *Hist. d. végét. foss.*, p. 204, pl. liii. fig. 2*.
1833. *Sphenopteris obtusiloba*, Sternb., *Essai flore monde prim.*, vol. ii. fasc. v.–vi. p. 63.
1848. *Sphenopteris obtusiloba*, Sauveur (*pars*), *Végét. foss. d. terr. houil. Belgique*, pl. xv. fig. 2 (*non* pl. xvi. fig. 3).
1854. *Sphenopteris obtusiloba*, Ettingshausen, *Steinkf. v. Radnitz*, p. 37, pl. xxi. fig. 2.
1869. *Sphenopteris obtusiloba*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 55, pl. xvi. fig. 10 (? fig. 11) (? pl. xxix. fig. 9).
1869. *Sphenopteris obtusiloba*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 399, pl. xxx. fig. 1.
1879. *Sphenopteris obtusiloba*, Schimper, *in Zittel, Handb. d. palæont.*, vol. ii. p. 108, fig. 77.
1879. *Sphenopteris obtusiloba*, Roemer, *Lethæa geog.*, vol. i. p. 169, pl. 51, figs. 1*a*, 1*b*.
1880. *Sphenopteris obtusiloba*, Zeiller, *Végét. foss. du terr. houil.*, p. 39, pl. clxii. figs. 1–2.
1882. *Sphenopteris obtusiloba*, Weiss, *Aus d. Steink.*, p. 13, pl. xi. fig. 67 (zweiter abdr.).
1883. *Sphenopteris obtusiloba*, Renault, *Cours d. botan. foss.*, vol. iii. p. 190, pl. xxxiii. figs. 5-6.
1886. *Sphenopteris obtusiloba*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 65, pl. iii. figs. 1–4, pl. iv. fig. 1, pl. v. figs. 1–2.
1893. *Sphenopteris obtusiloba*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxvii. p. 321, pl. i. fig. 1.
1899. *Sphenopteris obtusiloba*, Zeiller, *Étude sur la flore fossile du bassin houiller d'Héraclée* (*Asie Mineure*),—*Mém. Soc. géol. d. France*, No. 21, p. 5.
1899. *Sphenopteris obtusiloba*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 137, fig. 131.
1899. *Sphenopteris obtusiloba*, Hofmann and Ryba, *Leitpflanzen*, p. 38, pl. iii. fig. 23, pl. iv. figs. 1–2.
1901. *Sphenopteris obtusiloba*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. pp. 191, 205, pl. xxv. figs. 1, 1*a*.
1877. *Diplothmema obtusilobum*, Stur, *Culm Flora*, Heft ii. p. 230.
1885. *Diplothmema obtusilobum*, Stur, *Die Farne d. Carb. Flora*, vol i. p. 354, pl. xxv. fig. 8, pl. xxv.*b*, fig. 1.
1888. *Diplothmema obtusilobum*, Toula, *Die Steinkohlen*, p. 187, pl. i. figs. 7–8.
1836. *Cheilanthites obtusilobus*, Göpp., *Syst. fil. foss.*, p. 246.

1865. *Gymnogramme obtusiloba*, Ett., *Foss. Flora d. Mähr.-schles. Dachschiefers*, p. 22, excl. fig. 6.
1884. *Pseudopecopteris obtusiloba*, Lesqx., *Coal Flora*, vol. iii. p. 753.
1899. *Pseudopecopteris obtusiloba*, White, *Foss. Flora of the Lower Coal Measures of Missouri*, p. 24, pl. vii. figs. 1–3, pl. viii.
1833. *Sphenopteris irregularis*, Sternb., *Essai Flore monde prim.*, vol. ii. fasc. 5–6, p. 63, pl. xvii. fig. 4.
1862. *Sphenopteris irregularis*, Roemer, *Palæont.*, vol. ix. p. 24. pl. v. fig. 5.
1866. *Sphenopteris irregularis*, Andræ, *Vorwelt. Pflanzen.*, p. 24, pl. viii., pl. ix. fig. 1.
1869. *Sphenopteris irregularis*, Roehl, *Flora d. Steink.-Form. Westph.*, p. 56, pl. xvi. fig. 2, pl. xxxi. figs. 5–6.
1869. *Sphen.* (*Gymnogrammides*) *irregularis*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 373.
1836. *Cheilanthites irregularis*, Göpp., *Syst. fil foss.*, p. 247.
1877. *Diplothmema irregulare*, Stur, *Culm Flora*, Heft ii. p. 230.
1885. *Diplothmema irregulare*, Stur, *Die farne d. Carb. Flora*, vol. i. p. 296.
1880. *Pseudopecopteris irregularis*, Lesqx., *Coal Flora*, vol. i. p. 211 (?pl. lii. figs. 1–3, 8).
1830. *Sphenopteris trifoliolata*, Brongt. (*non* Artis), *Hist. d. végét. foss.*, p. 202, pl. liii. fig. 3.
1848. *Sphenopteris trifoliolata*, Sauveur (*non* Artis), *Végét. foss. d. terr. houil. Belgique*, pl. xix. fig. 2, pl. xxi.
1866. *Sphenopteris trifoliolata*, Andræ (*non* Artis), *Vorwelt Pflanzen.*, p. 28, pl. ix. figs 2–4.
1869. *Sphenopteris trifoliolata*, Roehl (*non* Artis), *Foss. Flora d. Steink.-Form. Westph.*, p. 65, pl. xvi. fig. 3.
1883. *Sphenopteris trifoliolata*, Renault (*non* Artis), *Cours d. botan. foss.*, vol. iii. p. 192, pl. xxxiii. figs. 7–8.
1886. *Sphenopteris trifoliolata*, Zeiller (*non* Artis), *Flore foss. bassin houil. d. Valen.*, p. 75, pl. i. figs. 1–4.
1899. *Sphenopteris trifoliolata*, Potonié (*non* Artis), *forma laxa, Potonié, Lehrb. d. Pflanzenpal.*, p. 137, fig. 130.
1848. *Sphenopteris grandifrons*, Sauveur, *Végét. foss. d. terr. houil. Belgique*, pl. xiv.
1883. *Sphenopteris Weissiana major*, Achepohl, *Niederrhein.-Westfäl. Steink. Gebirge*, p. 121, pl. xxxvii. figs. 14–18.
1883. *Sphenopteris Schillingsii*? Achepohl, *ibid.*, p. 89, *Ergänzungs Blatt*, iv. figs. 21–22.
1883. *Sphenopteris Andräi*, Achepohl, *ibid.*, p. 94, pl. xxxii. fig. 2, *Ergänzungs Blatt*, iv. fig. 37.
1883. *Sphenopteris Andräi major*, Achepohl, *ibid.*, p. 122, pl. xxxviii. fig. 4, *Ergänzungs Blatt*, iv. fig. 64.
1883. *Sphenopteris Weissiana*, Achepohl, *ibid.*, p. 114, pl. xxxiv. figs. 16–18, *Ergänzungs Blatt*, iv. fig 55.
1886. *Sphenopteris polyphylla*, Zeiller (*non* L. and H.), *Flore foss. bassin houil. d. Valen.*, p. 73, pl. i. fig. 4.

Locality.—Engine Pit, Rowanburn.

Horizon.—Shale associated with "Five-foot Coal."

Remarks.—Considerable confusion seems to exist amongst continental botanists as to the distinctive characters of *Sphenopteris trifoliolata*, Artis, sp., for, as far as known to me, none of the specimens which have been identified as that species by continental writers are referable to Artis' plant. *Sphenopteris trifoliolata*, Artis, sp., does not appear to have been met with out of Britain.

The pinnules of *Sphenopteris trifoliolata* are smaller and more distant than those of *Sphenopteris obtusiloba* and the plant has a much laxer growth. A trifoliata arrangement of the pinnules is frequent on some specimens of *Sphenopteris obtusiloba*,—a form probably only depending on its position on the frond,—and this is the plant which has invariably been mistaken for *Sphenopteris trifoliolata*, Artis, sp.

Sphenopteris trifoliolata, though not common, is not unfrequent in the Middle Coal

Measures of England, and I have examined a number of specimens collected by Mr Hemingway from the same horizon (Barnsley Thick Coal) as that from which the type was derived.

The plant figured by Zeiller as *Sphenopteris polyphylla*, L. and H., appears to me to be also referable to *Sphenopteris obtusiloba*; in any case it is not the *Sphenopteris polyphylla*, L. and H.,* which is rare in Britain, but the type specimen has fortunately been preserved.† The pinnules are much more obovate than in *Sphenopteris obtusiloba*, and the terminal pinnule is proportionally larger. *Sphenopteris polyphylla* has somewhat the appearance of an *Adiantites*. As the original figure of the species represented the plant more dense and heavier in growth than it really is, I refigured the type in 1892 in the *Proc. Roy. Phys. Soc. Edin.*, vol. xi. pl. ix. fig. 2.

(?) Sphenopteris Schützei, Stur, sp.

1885. *Hapalopteris Schützei*, Stur, *Farne d. Carbon-Flora d. Schatz. Schichten*, p. 56, pl. xli. figs. 1, 1*a*, 2, 3, 4.
1886. (?) *Sphenopteris* (*Hapalopteris*) *Schützei*, Kidston, *Trans. Geol. Soc. Glasgow*, vol. viii. p. 57, pl. iii. figs. 5, 5*a*, 5*b*.

Locality.—Blinkbonny Pit, Rowanburn.
Horizon.—Roof of Main Coal.
Note.—Only a small fragmentary specimen was found.

Eremopteris, Schimper.

Eremopteris artemisiæfolia, Sternb., sp.

1826. *Sphenopteris artemisiæfolia*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xv., pl. liv. fig. 1.
1828. *Sphenopteris artemisiæfolia*, Brongt., *Prodrome*, p. 50.
1829. *Sphenopteris artemisiæfolia*, Brongt., *Hist. d. végét. foss.*, p. 176, pl. xlvi., pl. xlvii. figs. 1–2.
1848. *Sphenopteris artemisiæfolia*, Sauveur (*pars*), *Végét. foss. d. terr. houil. Belgique*, pl. xx. fig. 3 (*non* figs. 1–2).
1836. *Gleichenites artemisiæfolius*, Göpp., *Syst. fil. foss.*, p. 184.
1869. *Eremopteris artemisiæfolia*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 416, pl. xxx. fig. 5.
1879. *Eremopteris artemisæfolia*, Lesqx., *Coal Flora*, p. 293, pl. liii. figs. 5–6.
1883. *Eremopteris artemisiæfolia*, Lesqx., *Indiana Dept. of Geol. and Nat. Hist.* 13*th Ann. Rept.*, part ii., *Palæont.*, p. 69, pl. xv. fig. 5.
1899. *Eremopteris artemisiæfolia*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 143, fig. 139.
1877. *Sphenopteris* (*Eremopteris*) *artemisiæfolia* (?), var., Lebour, *Illustr. of Fossil Plants*, pl. xxxiii.
1826. (?) *Sphenopteris laxa*, Sternb., *Essai flore monde prim.*, fasc. 2, p. 40; fasc. 4, p. xv. pl. xxxi. fig. 3.
1826. *Sphenopteris stricta*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xv., pl. lvi. fig. 3; vol. ii. p. 57.

* Lindley and Hutton, *Fossil Flora*, vol. ii., pl. cxlvii., 1835.
† In the Collection of the Geological Society, London.

1829. *Sphenopteris stricta,* Brongt., *Hist. d. végét. foss.,* p. 208, pl. xlviii. fig. 2.
1832. *Sphenopteris crithmifolia,* L. and H., *Fossil Flora,* vol. i. p. xlvi.
1877. *Sphenopteris,* sp., Lebour, *Illustr. of Fossil Plants,* pls. xxxiv., xxxv., xxxvi.
Asplenioides obtusum, König., *Icones foss. sectiles,* pl. xvi. fig. 199.

Locality.—Blinkbonny Pit, Rowanburn.
Horizon.—Shale over Main Coal.
Remark.—Apparently plentiful, but it appeared to be restricted to a single layer.

Mariopteris, Zeiller.

Mariopteris muricata, Schloth., sp.

1804. Schlotheim, *Flora d. Vorwelt,* pp. 54, 55, pl. xii. figs. 21 and 23.
1820. *Filicites muricatus,* Schlotheim, *Petrefactenkunde,* p. 409.
1826. *Pecopteris muricata,* Sternb., *Essai flore monde prim.,* vol. i. fasc. iv. p. xviii.
1832. *Pecopteris muricata,* Brongt., *Hist. d. végét. foss.,* p. 352, pl. xcv. figs. 3–4, pl. xcvii.
1848. *Pecopteris muricata,* Sauveur, *Végét. foss. d. terr. houil. Belgique,* pl. xliii. fig. 1, pl. xliv. fig. 2.
1876. *Pecopteris muricata,* Heer, *Flora foss. Helv.,* Lief. i. p. 33, pl. xv. fig. 3.
1836. *Alethopteris muricata,* Göpp., *Syst. fil. foss.,* p. 313.
1854. *Alethopteris muricata,* Ett., *Steinkohlf. v. Radnitz,* p. 43, pl. xiv. fig. 1.
1869. *Alethopteris muricata,* Roehl, *Foss. Flora d. Steink.-Form. Westph.,* p. 78, pl. xi. fig. 1.
1876. *Sphenopteris muricata,* Feistmantel, *Vers. d. Böhm. Kohlenab.,* p. 281, pl. lxv. fig. 3 (*excl. syn. Sph. acutifolia*).
1877. *Diplothmema muricatum,* Stur, *Culm Flora,* Heft ii. p. 230.
1885. *Diplothmema muricatum,* Stur, *Farne d. Carbon-Flora d. Schatz. Schichten,* p. 393, pl. xxi. figs. 1–5, pl. xxii. figs. 1–5, pl. xxiii. figs. 1–6.
1878. *Mariopteris muricata,* Zeiller, *Végét. foss. d. terr. houil.,* p. 71, pl. clxvii. fig. 5.
1879. *Mariopteris muricata,* Zeiller, *Bull. Soc. Géol. d. France,* 3ᵉ sér., vol. vii. p. 92.
1886. *Mariopteris muricata,* Zeiller, *Flore foss. bassin houil. d. Valen.,* p. 173, pl. xx. figs. 2–3, pl. xxi. fig. 1, pl. xxii. fig. 2.
1899. *Mariopteris muricata,* Zeiller, *Mém. Soc. Géol. d. France. Paléont., Mém.* 21, *Flore foss. bassin houil. d'Héraclée,* p. 32, pl. ii. figs. 14–15.
1899. *Mariopteris muricata,* Potonié, *Lehrb. d. Pflanzenpal.,* p. 140, fig. 135.
1899. *Mariopteris muricata,* Hofmann and Ryba, *Leitpflanzen,* p. 44, pl. vi. figs. 5 and 16.
1901. *Mariopteris muricata,* Kidston, *Proc. York. Geol. and Polytech. Soc.,* vol. xiv. pp. 195, 219, pl. xxxii. figs. 1, 1*a*.
1886. *Mariopteris muricata forma nervosa,* Zeiller, *Flore foss. bassin houil. d. Valen.,* p. 173, pl. xx. fig. 1, pl. xxii. fig. 1, pl. xxiii. fig. 1.
1886. *Mariopteris muricata,* var. *hirta,* Zeiller, *Flore foss. bassin houil. d. Valen.,* p. 182, pl. xx. fig. 4.
1879. *Pseudopecopteris muricata,* Lesqx., *Coal Flora,* vol. i. p. 203, pl. xxxvii. fig. 2.
1832. *Pecopteris nervosa,* Brongt., *Hist. d. végét. foss.,* p. 297, pl. xciv., pl. xcv. figs. 1–2.
1833. *Pecopteris nervosa,* L. and H., *Fossil Flora,* vol. ii. pl. xciv.
1848. *Pecopteris nervosa,* Sauveur, *Végét. foss. d. terr. houil. Belgique,* pl. xliv. fig. 1.
1869. *Pecopteris nervosa,* Schimper, *Traité d. paléont. végét.,* vol. i. p. 513, pl. xxx. figs. 6–7.
1876. *Pecopteris nervosa,* Heer, *Flora foss. Helv.,* Lief. i. p. 33, pl. xv. figs. 1–2.
1882. *Pecopteris nervosa,* Weiss, *Aus d Steink.,* p. 17, pl. xvi. fig. 98 (zweiter abdr.).
1882. *Pecopteris nervosa,* Achepohl, *Niederrh. Westfäl. Steink.,* pp. 74, 76, 90, pl. xxii. fig, 6, pl. xxiii. fig. 14, pl. xxviii. figs. 10–14.
1878. *Mariopteris nervosa,* Zeiller, *Végét. foss. d. terr. houil.,* p. 69, pl. clxvii. figs. 1–4.
1879. *Mariopteris nervosa,* Zeiller, *Bull. Soc. Géol. d. France,* 3ᵉ sér., vol. vii. p. 97, pl. v. figs. 1–2.

1899. *Mariopteris nervosa*, Hofmann and Ryba, *Leitpflanzen*, p. 44, pl. vii. fig. 7.
1836. *Alethopteris nervosa*, Göpp., *Syst. fil. foss.*. p. 312.
1855. *Alethopteris nervosa*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 30, pl. xxxiii. figs. 2–3.
1869. *Alethopteris nervosa*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 77, pl. xxxi. fig. 7.
1881. *Alethopteris nervosa*, Achepohl, *Niederrh. Westfäl. Steink.*, pp. 57, 64, pl. xvi. fig. 1, pl. xviii. figs. 15, 16.
1877. *Diplothmema nervosum*, Stur, *Culm Flora*, Heft ii. p. 230.
1885. *Diplothmema nervosum*, Stur, *Carbon-Flora d. Schatz. Schichten*, vol. i. p. 384, pl. xxiv. fig. 1, pl. xxv.*b* fig. 2.
1888. *Diplothmema nervosum*, Toula, *Die Steinkohlen*, p. 188, pl. i. figs. 12–13.
1879. *Pseudopecopteris nervosa*, Lesqx., *Coal Flora*, vol. i. p. 197, pl. xxxiv. fig. 1 (? figs. 2–3).
1832. *Pecopteris Sauveuri*, Brongt., *Hist. d. végét foss.*, p. 299, pl. xcv. fig. 5.
1836. *Alethopteris Sauveuri*, Göpp., *Syst. fil. foss.*, p. 311.
1885. *Diplothmema Sauveuri*, Stur, *Farne d. Carbon-Flora d. Schatz. Schichten*, vol. i. p. 380, pl. xxiv. fig. 2–4.
1899. *Mariopteris Sauveuri*, Frech, *Lethæa. geog.*, vol. ii., Lief. ii., Steinkf., pl. l.*a.* fig. 6.
1826. *Pecopteris incisa*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xx ; vol. ii. fasc. 5–6, pl. xxii. fig. 3, fasc. 7–8, p. 156.
1834. *Pecopteris laciniata*, L. and H., *Fossil Flora*, vol. ii., pl. cxxii.
1877. *Pecopteris laciniata*, Lebour, *Illustr. of Fossil Plants*, p. 59, pl. xxix.
1838. *Alethopteris Lindleyana*, Presl., *in Sternb.*, *Vers.*, vol. ii. fasc. 7–8, p. 145.
1848. *Pecopteris heterophylla*, Sauveur (*non* Brongt.), *Végét. foss. terr. houil. Belgique*, pl. xlvii.
1854. *Sphenopteris acutifolia*, Ett. (*non* Brongt.), *Steinkf. v. Radnitz*, p. 39, pl. xiv. fig. 2.
1862. *Pecopteris subnervosa*, Roemer, *Palæont.*, vol. ix. p. 36, pl. viii. fig. 11.
1869. *Pecopteris subnervosa*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 90, pl. xiii. fig. 5.
1877. *Sphenopteris macilenta*, var. Lebour., (*non* L. and H.), *Illustr. of Fossil Plants*, p. 39, pl. xix.
1877. *Neuropteris heterophylla*, Lebour (*non* Brongt.), *ibid.*, p. 29, pl. xiv.
1877. *Neuropteroid frond*, Lebour, *ibid.*, p. 31, pl. xv.
1877. *Pecopteris* (*Alethopteris*) *aquilina*, Lebour (*non* Schloth.), *ibid.*, p. 33, pl. xvi.
1877. *Pecopteris* (*Alethopteris*) *marginata*, Lebour (*non* Brongt.), *ibid.*, p. 35, pl. xvii.
1877. *Pecopteris serra*, Lebour (*non* L. and H.), *ibid.*, pl. xxii.
1882. *Odontopteris*, Achepohl, *Niederrh. Westfäl. Steink.*, pp. 93, 95, pl. xxxi. fig. 2, pl. xxxiii. figs. 4–5.
1882. *Odontopteris dentiformis*, Achepohl, *ibid.*, p. 93, pl. xxxi. fig. 6.
1882. *Odontopteris Reichiana*, Achepohl (*non* Gutbier), *ibid.*, p. 95, pl. xxxii. figs. 6–9.
1883. *Alethopteris conferta*, Achepohl (*non* Sternb.), *ibid.*, p. 117, pl. xxxv. fig. 10.
1883. *Alethopteris acuta*, Achepohl, *ibid.*, p. 118, pl. xxxvi. fig. 6.
1885. *Diplothmema hirtum*, Stur, *Die Farne d. Carbon-Flora d. Schatz.-Schichten*, p. 372, pl. xxxiv. fig. 1.

Locality.—Blinkbonny Pit, Rowanburn.
Horizon.—Roof of *Main Coal.*
Locality.—Engine Pit, Rowanburn.
Horizon.—Shale associated with *Five-foot Coal.*

Alethopteris, Sternberg.

Alethopteris lonchitica, Schloth., sp.

1709. Scheuchzer, *Herbarium diluvianum*, p. 11, pl. i. fig. 4.
1804. Schlotheim, *Flora d. Vorwelt*, p. 55, pl. xi. fig. 22.

1809. *Filicites (striatus)*, Martin, *Petrificata Derbiensia*, expl. to pl. x. figs. 1–4.
1820. *Filicites lonchiticus*, Schloth., *Petrefactenkunde*, p. 411.
1828. *Pecopteris lonchitica*, Brongt., *Prodrome*, p. 57.
1832 or 1833. *Pecopteris lonchitica*, Brongt., *Hist. d. végét. foss.*, p. 275, pl. lxxxiv. figs. 1–7.
1835. *Pecopteris lonchicita*, L. and H., *Fossil Flora*, vol. ii., pl. cliii.
1848. *Pecopteris lonchitica*, Sauveur (*pars*), *Végét. foss. terr. houil. Belgique*, pl. xli. figs. 1–2, pl. xlii. fig. 5 (*non* fig. 4).
1826. *Alethopteris lonchitidis*, Sternb., *Vers.*, i. fasc. iv. p. xxi ; *Vers.*, ii. fasc. vii.–viii. p. 142.
1854. *Alethopteris lonchitidis*, Geinitz, *Flora d. Hainich-Ebersdorfer*, p. 43, pl. xiv. figs. 1–2.
1860. *Alethopteris lonchitidis*, Eichwald, *Lethæa Rossica*, vol. i. p. 85, pl. ii. fig. 3.
1869. *Alethopteris lonchitidis*, Roehl (*pars*), *Foss. Flora d. Steink.-Form. Westph.*, p. 72, pl. xiv. fig. 2, pl. xxxi. fig. 4.
1881. *Alethopteris lonchitidis*, Achepohl, *Niederrh. Westfäl. Steink.*, p. 33, pl. viii. figs. 1, 11.
1842. *Alethopteris lonchitica*, Unger, *Neues Jahrb.*, p. 608.
1869. *Alethopteris lonchitica*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 554 (*refs. in part*).
1879. *Alethopteris lonchitica*, Lesqx., *Coal Flora*, vol. i. p. 177, pl. xxviii. fig. 7 (*refs. in part*).
1879. *Alethopteris lonchitica*, Schimper, *in Zittel, Handb. d. palæont. Palæophyt.*, p. 118, fig. 93 (1).
1883. *Alethopteris lonchitica*, Renault, *Cours. d. botan. foss.*, vol. iii. p. 156, pl. xxvii. figs. 5–6.
1886. *Alethopteris lonchitica*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 225, pl. xxxi. fig. 1.
1894. *Alethopteris lonchitica*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxvii. p. 594.
1899. *Alethopteris lonchitica*, Hofmann and Ryba, *Leitpflanzen*, p. 55, pl. viii. fig. 1, 1*a*.
1901. *Alethopteris lonchitica*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. pp. 195, 219, pl. xxxii. figs. 2–3.
1826. *Alethopteris vulgatior*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xxxi., pl. liii. fig. 2 ; ii. fasc. 7–8, p. 142.
1828. *Pecopteris blechnoides*, Brongt., *Prodrome*, p. 56.
1832 or 1833. *Pecopteris urophylla*, Brongt., *Hist. d. végét. foss.*, p. 290, pl. lxxxvi.
1838. *Alethopteris urophylla*, Sternb., *Vers.*, ii. fasc. 7–8, p. 143.
1869. *Alethopteris urophylla*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 75, pl. xxii. fig. 7.
1836. *Alethopteris Sternbergii*, Göpp., *Syst. fil. foss.*, p. 295.
1854. *Alethopteris Sternbergii*, Ett., *Steinkf. v. Radnitz*, p. 42, pl. xviii. fig. 4.

Locality—Blinkbonny Pit, Rowanburn.
Horizon.—Roof of *Main Coal.*
Locality.—Engine Pit, Rowanburn.
Horizon.—Shale associated with *Five-foot Coal.*

Neuropteris, Brongniart.

Neuropteris heterophylla, Brongt.

1709. *Lithosmunda minor*, Scheuchzer, *Herb. diluv.*, p. 15, pl. iv. fig. 3.
1760. *Lithosmunda minor*, Luid., *Lith. Brit. Ichnographia*, p. 12, pl. iv. fig. 189.
1809. *Phytolithus* (*osmundæ regalis*), Martin, *Petrificata Derbiensia*, pl. xix. figs. 1–3.
1822. *Filicites* (*Neuropteris*) *heterophyllus*, Brongt,, *Class. d. végét. foss.*, p. 33, pl. ii. fig. 6*a* and 6*b*.
1828. *Neuropteris heterophylla*, Brongt., *Prodrome*, p. 53.
1830. *Neuropteris heterophylla*, Brongt., *Hist. d. végét. foss.*, p. 243, pl. lxxi., pl. lxxii. fig. 2.
1833. *Neuropteris heterophylla*, Sternb., *Essai flore monde prim.*, vol. ii. fasc. 5–6, p. 72.
1837. *Neuropteris heterophylla*, L. and H., *Fossil Flora*, vol. iii., pl. cc. (*non* pl. clxxxiii.).
1848. *Neuropteris heterophylla*, Sauveur, *Végét. Foss. d. terr. houil. Belgique*, pl. xxix. figs. 3–4, pl. xxx. figs. 1–2.

1869. *Neuropteris heterophylla*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 37, pl. xvi. figs. 5, 7.
1876. *Neuropteris heterophylla*, Heer (*pars*), *Flora foss. Helv.*, Lief. i. p. 23, pl. iv. figs. 1–2 (? fig. 3, pl. v. fig. 4 (*non* pl. xii. fig. 10*b*).
1880. *Neuropteris heterophylla*, Zeiller, *Végét. foss. d. terr. houil.*, p. 49, pl. clxiv. fig. 1 (*non* fig. 2).
1882. *Neuropteris heterophylla*, Weiss, *Aus d. Steink.*, p. 15, pl. xiv. fig. 88 (zweiter abdr.).
1883. *Neuropteris heterophylla*, Renault, *Cours d. botan. foss.*, vol. iii. p. 170, pl. xxix. figs. 6–7.
1886. *Neuropteris heterophylla*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 261, pl. xliii. figs. 1–2, pl. xliv. fig. 1.
1887. *Neuropteris heterophylla*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 150, pl. viii. fig. 7.
1888. *Neuropteris heterophylla*, Zeiller, *Flore foss. terr. houil. d. Commentry*, part i. p. 257, pl. xxix. fig. 4.
1890. *Neuropteris heterophylla*, Zeiller, *Flore foss. bassin houil. et perm. d'Autun et d'Épinac*, p. 142, pl. xii. fig. 1.
1891. *Neuropteris heterophylla*, Kidston, *Trans. Geol. Soc. Glasgow*, vol. ix. p. 34, pl. iii. fig. 36.
1899. *Neuropteris heterophylla*, Zeiller, *Flore foss. d. bassin houil. d'Héraclée*, p. 44.
1899. *Neuropteris heterophylla*, Hofmann and Ryba, *Leitpflanzen*, p. 64, pl. ix. fig. 6, 6*a*, 7–10.
1900. *Neuropteris heterophylla*, Zeiller, *Éléménts de paléobot.*, p. 52, fig. 18, p. 107, fig. 81.
1830. *Neuropteris Loshii*, Brongt., *Hist. d. végét. foss.*, p. 242, pl. lxxii. fig. 1, pl. lxxiii.
1832. *Neuropteris Loshii*, L. and H., *Fossil Flora*, vol. i., pl. xlix. (*fig. inaccurate*).
1835. *Neuropteris Loshii*, Gutbier, *Vers. d. Zwick. Schwarz*, p. 55, pl. viii. fig. 6.
1848. *Neuropteris Loshii*, Sauveur, *Végét. foss. d. terr. houil. Belgique*, pl. xxxi. figs. 1–2.
1864. *Neuropteris Loshii*, Sandberger, *Flora d. ober. Steink. im badischen Schwarz*, p. 6, pl. iv. fig. 1.
1869. *Neuropteris Loshii*, Roehl, *Flora d. Steink.-Form. Westph.*, p. 37, pl. xvii.
1876. *Neuropteris Loshii*, Feistmantel, *Vers. d. böhm. Kohlenab.*, Abth. iii. p. 64, pl. xvii. fig. 3.
1880. *Neuropteris Loshii*, Lesqx., *Coal Flora*, vol. i. p. 98 (? pl. xi. figs. 1–4).
1830. *Cyclopteris trichomanoides*, Brongt., *Hist. d. Végét. foss.*, p. 217, pl. lxi. *bis*, fig. 4.
1835. *Cyclopteris trichomanoides*, Gutbier, *Vers. d. Zwick. Schwarzk.*, p. 45, pl. vi. fig. 1.
1855. *Cyclopteris trichomanoides*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 23, pl. xxviii. figs. 2–3.
1869. *Cyclopteris trichomanoides*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 44, pl. xvii. (? pl. xxix. fig. 10).
1883. *Cyclopteris trichomanoides*, Renault, *Cours d. botan. foss.*, vol. iii. p. 184, pl. xxx. fig. 5.
1888. *Cyclopteris trichomanoides*, Zeiller, *Flore foss. terr. houil. d. Commentry*, part i. p. 265, pl. xxiii. fig. 3.
1832. *Pecopteris adiantoides*, L. and H., *Fossil Flora*, vol. i., pl. xxxvii (*fig. inaccurate*).
1833. *Neuropteris Brongniarti*, Stern., *Essai flore monde prim.*, vol. ii. fasc. v.–vi. p. 72.
1830. *Cyclopteris obliqua*, Brongt., *Hist. d. végét. foss.*, p. 221, pl. lxi. fig. 3.
1833. *Cyclopteris obliqua*, L. and H., *Fossil Flora*, vol. ii. p. 25, pl. xc. A–B.
1835. *Cyclopteris inæqualis*, Gutbier, *Vers. d. Zwick. Schwarz*, p. 46, pl. vi. fig. 3.
1836. *Adiantites trichomanoides*, Göpp., *Syst. fil. foss.*, p. 220.
1836. *Adiantites obliquus*, Göpp., *Syst. fil. foss.*, p. 221.
1840. *Cyclopteris semiflabelliformis*, Morris, *in Prestwick*, *Trans. Geol. Soc. London*, 2nd ser., vol. v. p. 488, pl. xxxviii. fig. 7.
1848. *Otopteris cycloidea*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl xxvi. figs. 1–2 (?), pl. xxviii. fig. 3 ?
1848. *Otopteris reniformis*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. xxvi. fig. 3.
1862. *Odontopteris oblongifolia*, Roemer, *Palæontographica*, vol. ix. p. 31, pl. vii. fig. 1.
1869. *Odontopteris oblongifolia*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 43, pl. vii. fig. 1.
1869. *Odontopteris obtusiloba*, Roehl (*non* Naumann), *Foss. Flora d. Steink.-Form. Westph.*, p. 42, pl. xvi. figs. 12–15.
1869. *Odontopteris britannica*, Roehl (*non* Gutbier) (*pars*), *Foss. Flora d. Steink.-Form. Westph.*, p. 41, pl. xx. fig. 4.

Remarks.—M. GRAND' EURY suggests that *Cyclopteris trichomanoides*, Brongt., is founded on the large circular pinnules or foliar stipules of *Odontopteris minor*, Brongt., and *Odontopteris Reichiana*, Gutbier, with which he finds them associated.* M. ZEILLER, on the other hand, would refer *Cyclopteris trichomanoides* exclusively to *Odontopteris minor*, Brongt., as *Odontopteris Reichiana*, Gutbier, does not occur in the beds in which he finds the *Cyclopteris*.†

In Britain, *O. minor*, Brongt., has not yet been discovered, and *O. Reichiana*, Gutbier, is very rare, whereas *Cyclopteris trichomanoides* is very common, and almost invariably in the beds in which *Neuropteris heterophylla*, Brongt., occurs, and to which species I believe the British specimens of *Cyclopteris trichomanoides* should without doubt be referred. To the localities to which I refer, and in which the *C. trichomanoides* is especially abundant, the genus *Odontopteris* does not occur. The excellent specimen of *Neuropteris heterophylla* figured by ROEHL under the name of *Neuropteris Loshii* shows the cyclopteroid pinnules in position on the rachis. ‡

The true explanation of this difference of opinion may perhaps be, that several species bore rachial cyclopteroid pinnules, which, when removed from the parent frond, cannot be specifically distinguished.

Cyclopteris trichomanoides, Brongt., and *Cyclopteris obliqua*, Brongt., are both shown on the specimen figured by ROEHL, the oblique form of *C. obliqua* arising from its position and mode of insertion on the rachis.

Locality.—Blinkbonny Pit, Rowanburn.

Horizon.—Roof of *Main Coal*.

Locality.—Engine Pit, Rowanburn.

Horizon.—Shales associated with *Five-foot Coal*.

Neuropteris gigantea, Sternberg.

1823. *Osmunda gigantea*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 2, pp. 32, 37, pl. xxii.
1826. *Neuropteris gigantea*, Sternb., *Essai flore monde prim.*, vol. i. fasc. iv. p. xvi.
1830. *Neuropteris gigantea*, Brongt., *Hist. d. végét. foss.*, p. 240, pl. lxix.
1832. *Neuropteris gigantea*, L. and H., *Fossil Flora*, vol. i. pl. lii.
1848. *Neuropteris gigantea*, Sauveur, *Végét. foss. d. terr. houil. Belgique*, pl. xxxiii. fig. 1 (*non* pl. xxxi. figs. 3–4.
1886. *Neuropteris gigantea*, Zeiller, *Flore Foss. bassin houil. d. Valen.*, p. 258, pl. xlii. fig. 1.
1892. *Neuropteris gigantea*, Potonié, *Ueber einige Carbonfarne*, iii. Theil, p. 22, text, figs. 1–4, pl. ii. figs. 1–2, pl. iii. figs. 1–4, pl. iv. figs. 1–2. (*Jahrb. d. k. preuss. geol. Landesanstalt* for 1891.)
1899. *Neuropteris gigantea*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 113, fig. 101, p. 118, fig. 105, p. 153, fig. 150.
1899. *Neuropteris gigantea*, Hofmann and Ryba (*pars*), *Leitpflanzen*, p. 64, pl. ix. figs. 4, 4*a*–4*d* (*non* pl. viii. fig. 14, pl. ix. fig. 3).
1899. *Neuropteris gigantea*, Zeiller, *Flore foss. d. bassin houil. d. Héraclée*, p. 44, pl. iv. fig. 10.
1900. *Neuropteris gigantea*, Zeiller, *Éléments d. paléobot.*, p. 105, fig. 79.
1901. *Neuropteris gigantea*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. pp. 193, 211, 213, pl. xxviii. fig. 3, pl. xxix. fig. 4.

* *Flore Carbon d. Départ. d. l. Loire et d. Centre d. l. France*, p. 113.
† *Flore foss. terr. houil. d. Commentry*, p. 266.
‡ Roehl, *Foss. Flora d. Steink.-Form. Westph.*, pl. xvii.

1848. *Neuropteris flexuosa*, Sauveur (*non* Sternberg.), *Végét. foss. d. terr. houil. Belgique*, pl. xxxii. figs. 1–2 (? pl. xxxiii. fig. 2).
1892. *Neuropteris Zeilleri*, Potonié, *Ueber einige Carbonfarne*, iii. Theil, pp. 22, 32, fig. 5.
1899. *Neuropteris pseudogigantea*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 113, fig. 102.

Locality.—Blinkbonny Pit, Rowanburn.
Horizon.—Roof of *Main Coal.*
Locality.—Engine Pit, Rowanburn.
Horizon.—Shale associated with *Five-foot Coal.*

Neuropteris Blissii, Lesquereux.

1884. *Neuropteris Blissii*, Lesqx., *Coal Flora*, vol. iii. p. 737, pl. xcv. figs. 1, 1*a*.
1888. *Neuropteris Blissii*, Zeiller, *Flore foss. terr. houil. d. Commentry*, part i. p. 243, pl. xxviii., figs. 3–6.
1893. *Neuropteris Blissii*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxvii. p. 329, pl. i. figs. 3, 3*a*.

Locality.—Blinkbonny Pit, Rowanburn.
Horizon.—Roof of *Main Coal.*
Locality.—Engine Pit, Rowanburn.
Horizon.—Shale associated with *Five-foot Coal.*

Neuropteris obliqua, Brongniart, sp.

1832. *Pecopteris obliqua*, Brongt., *Hist. d. végét. foss.*, p. 320, pl. xcvi. figs. 1–4.
1838. *Alethopteris obliqua*, Presl., *in Sternb.*, *Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 144.
1874. *Odontopteris obliqua*, Stur, *Verhandl. der k. k. geol. Reichsanstalt*, No. 4, p. 80.
1883. *Odontopteris obliqua*, Zeiller, *Bull. Soc. Géol. d. France*, 3^{e} ser., vol. xii. p. 198.
1886. *Neuropteris obliqua*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 284, pl. xlviii. figs. 1, 2 (? fig. 3), figs. 4–7.
1893. *Neuropteris obliqua*, Arber, *Quart. Journ. Geol. Soc.*, vol. lix. p. 4, pl. i. fig. 2.
1883. *Odontopteris binervosa*, Achepohl, *Niederrh. Westfäl. Steinkohl.*, p. 118, pl. xxxvi. fig. 5.

Locality.—Engine Pit, Rowanburn.
Horizon.—Shale associated with *Five-foot Coal.*

Equisetaceæ.

Calamites, Suckow.

Group I.—Calamitina, Weiss.

Calamites (Calamitina) undulatus, Sternberg.

1826. *Calamites undulatus*, Sternb., *Ess. fl. monde prim.*, vol. i. fasc. 4, p. xxvi.; vol. ii. fasc. 5–6, p. 47, pl. i. fig. 2 (? pl. xx. fig. 8).
1828. *Calamites undulatus*, Brongt., *Hist. d. végét. foss.*, p. 127, pl. xvii. figs. 1–4.
1848. *Calamites undulatus*, Sauveur, *Végét. foss. d. terr. houil. Belgique*, pl. v. figs. 1–3, pl. viii. fig. 1.

1873. *Calamites undulatus*, Dawson, *Fossil Plants Low. Carb. and Millstone Grit Form. Canada*, p. 30, pl. viii. fig. 68 (? figs. 66, 67, 69–73).
1886. *Calamites undulatus*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 338, pl. liv. figs. 1, 4.
1888. *Calamites undulatus*, Seward, *Geol. Mag.*, Dec. iii., vol. v. p. 289, pl. ix.
1822. *Calamites decoratus*, Brongt. (*non* Schlotheim), *Class. végét. foss.*, pp. 17, 89, pl. i. fig. 2.
1828. *Calamites decoratus*, Brongt. (*pars*), *Hist. d. végét. foss.*, p. 423, pl. xiv. figs. 3–4 (*non* figs. 1–2).
1828. *Calamites decoratus*, Bischoff, *Kryptogam. Gewächse.*, pp. 51, 60, pl. vi. fig. 11.
1854. *Calamites communis*, Ett. (*pars*), *Steinkf. v. Radnitz*, p. 24, pl. iii. figs. 1 and 3, pl. iv. figs. 1 and 3.
1869. *Calamites cannæformis*, Roehl (*pars*) (*non* Schlotheim), *Foss. Flora d. Steink.-Form. Westph.*, p. 12, pl. ii. fig. 3.
1874. *Calamites cannæformis*, Feistmantel (*pars*) (*non* Schlotheim), *Vers. d. Böhm. Ablag.*, i. Abth. p. 109, pl. vii. fig. 3.
1883. *Calamites inæquus*, Achepohl, *Niederrh. Westfäl. Steinkohl.*, p. 114, pl. xxxiv. fig. 15.
1883. *Calamites duplex*, Achepohl, *ibid.*, p. 135, pl. xli. fig. 11.
1884. *Calamites* (*Stylocalamites*) *Suckowii*, var. *undulatus*, Weiss, *Steinkohlen Calamarien*, part ii. pp 129, 134, 135, pl. xvii. fig. 4.
1890. *Stylocalamites undulatus*, Kidston, *Trans. York. Nat. Union*, part xiv. p. 20.
1893. *Calamitina undulata*, Kidston, *Trans. York. Nat. Union*, part xviii. p. 99.

Locality.—Blinkbonny Pit, Rowanburn.
Horizon.—Shale above *Nine-foot Coal.*

Group III.—**Stylocalamites**, Weiss.

Calamites (Stylocalamites) Cistii, Brongt.

1828. *Calamites Cistii*, Brongt., *Hist. d. végét. foss.*, p. 129, pl. xx.
1855. *Calamites Cistii*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 7 (? *non* pl. xi. figs. 7–8, pl. xii. figs. 4–5, pl. xiii. fig. 7).
1876. *Calamites Cistii*, Heer, *Flora foss. Helv.*, Lief. i. p. 47, pl. xx. fig. 3 (? *non* figs. 1, 2, 4).
1877. *Calamites Cistii*, Grand' Eury, *Flore Carbon. d. Départ. de la Loire*, p. 19, pl. ii. figs. 2, 3 (? fig. 1).
1882. *Calamites Cistii*, Renault, *Cours d. botan. foss.*, vol. ii. p. 162, pl. xxiv. fig. 7.
1886. *Calamites Cistii*, Zeiller, *Flore foss. d. bassin houil. d. Valen.*, p. 242, pl. lvi. figs. 1–2.
1890. *Calamites Cistii*, Grand' Eury (? *pars*), *Géol. et paléont. du bassin houil. du Gard*, p. 217, pl. xv. fig. 1 (?).
1890. *Calamites Cistii*, Renault, *Flore foss. terr. houil. de Commentry*, part ii. p. 389, pl. xliii. fig. 4 (? pl. xliv. fig. 1), pl. lvii. fig. 4.
1899. *Calamites Cistii*, Hofmann and Ryba, *Leitpflanzen*, p. 25, pl. i. fig. 11.

Locality.—Blinkbonny Pit, Rowanburn.
Horizon.—Roof of *Main Coal.*

Calamites, sp.

Locality.—Engine Pit, Rowanburn.
Horizon.—Shale associated with *Five-foot Coal.*

Sphenophyllaceæ.

Sphenophyllum, Brongniart.

Sphenophyllum cuneifolium, Sternberg., sp.

1823. *Rotularia cuneifolia*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 2, pp. 33, 37, pl. xxvi. figs. 4*a*, 4*b*.
1880. *Sphenophyllum cuneifolium*, Zeiller, *Végét. foss. du terr. houil.*, p. 30, pl. clxi. figs. 1–2.
1882. *Sphenophyllum cuneifolium*, Renault, *Cours d. botan. foss.*, vol. ii. p. 87, pl. xiii. fig. 10.
1886. *Sphenophyllum cuneifolium*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 413, pl. lxii. fig. 1, pl. lxiii. figs. 1–10.
1893. *Sphenophyllum cuneifolium*, Zeiller, *Mém. Soc. géol. d. France, Paléont.*, vol. iv. No. 11, p. 12, pl. i. (iii.) figs. 1–4, pl. ii. (iv.) figs. 1–3, pl. iii. (v.) figs. 1–2.
1894. *Sphenophyllum cuneifolium*, Potonié, *Bericht. d. Deutschen bot. Gesell.*, vol. xii. Heft 4, p. 99, fig. 3.
1899. *Sphenophyllum cuneifolium*, Zeiller, *Étude sur la flore foss. d. bassin houil. d'Héraclée*, p. 56, pl. vi. figs. 6–7.
1899. *Sphenophyllum cuneifolium*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 176, fig. 172.
1900. *Sphenophyllum cuneifolium*, Zeiller, *Éléments de paléobot.*, p. 139, fig. 100.
1901. *Sphenophyllum cuneifolium*, Kidston, *Trans. Nat. Hist. Soc. Glasgow*, vol. vi. (new series), p. 124, fig. 21*a–b*, p. 121.
1902. *Sphenophyllum cuneifolium*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. p. 360, fig. 12 *a–b*.
1826. *Rotularia saxifragæfolium*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xxxii., pl. lv. fig. 4.
1848. *Sphenophyllum saxifragæfolium*, Göpp., *in Bronn, Index palæont.*, p. 1166.
1854. *Sphenophyllum saxifragæfolium*, Geinitz, *Flora d. Hainichen-Ebersdorfer*, p. 37, pl. xiv. figs. 7–10.
1855. *Sphenophyllum saxifragæfolium*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 13, pl. xx. figs. 8, 8*a*, (? figs. 9, 10).
1869. *Sphenophyllum saxifragæfolium*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 31, pl. iv. fig. 17 (? pl. iii. fig. 2*c*).
1880. *Sphenophyllum saxifragæfolium*, Zeiller, *Végét. foss. d. terr. houil.*, p. 31, pl. clxi. figs. 3–6.
1882. *Sphenophyllum saxifragæfolium*, Renault, *Cours d. botan. foss.*, vol. ii. p. 87, pl. xiii. figs. 11–14.
1882. *Sphenophyllum saxifragæfolium*, Weiss, *Aus d. Steink.*, p. 12, pl. x. fig. 62 (zweiter abdr.).
1826. *Rotularia polyphylla*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, pp. xxxii and 47, pl. l. fig. 4.
1826. *Rotularia pusilla*, Sternb., *Essai flore monde prim.*, vol. i. fasc. iv. p. xxxii.
1828. *Sphenophyllum pusillum*, Bischoff, *Die Kryptogram. Gewächse*, p. 90, pl. xiii. fig. 3.
1848. *Sphenophyllum pusillum*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxiv. fig. 4.
1831. *Sphenophyllum erosum*, L. and H., *Fossil Flora*, vol. i. pl. xiii.
1847. *Sphenophyllum erosum*, Bunbury, *Quart. Journ. Geol. Soc.*, vol. iii. p. 430, pl. xxiii. fig. 3*a–b*.
1864. *Sphenophyllum erosum*, Cœmans and Kickx., *Bull. Acad. Roy. Belgique*, vol. xviii. p. 149, pl. i. figs. 5 *a*, *b*, *c*.
1868. *Sphenophyllum erosum*, Dawson, *Acad. Geol.*, 2nd. ed. p. 480, fig. 165 *c*, *c'*, p. 444.
1869. *Sphenophyllum erosum*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 30, pl. iv. fig. 19.
1869. *Sphenophyllum erosum*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 341, pl. xxv. figs. 10–14.
1876. *Sphenophyllum erosum*, Heer, *Flora foss. Helv.*, 1 Lief. p. 53, pl. xix. figs. 11–13 (*non* fig. 14).

1880. *Sphenophyllum erosum*, Schimper, *in Zittel, Handb. d. palæont.*, Abth. ii. p. 179, fig. 135 (3 and 4).
1882. *Sphenophyllum erosum*, Weiss, *Aus d. Steink.*, p. 12, pl. x. fig 57 (zweiter abdr.).
1888. *Sphenophyllum erosum*, Dawson, *Geol. Hist. of Plants*, p. 122, fig. 45 *c*, *c'*.
1891. *Sphenophyllum erosum*, Newberry, *Journ. Cincinnati Soc. Nat. Hist.*, vol. xiii. p. 215, pl. xix. fig. 1.
1864. *Sphenophyllum erosum*, var. *saxifragæfolium*, Cœmans and Kickx., *Bull. Acad. Roy. Belgique*, vol. xviii. p. 151, pl. i. fig. 6 *a*, *b*, *c*, *d*.
1831. *Rotularia dichotoma*, Germar and Kaulfuss, *Act. Acad. Nat. Curios*, vol. xv. p. 226, pl. lxvi. fig. 4.
1850. *Sphenophyllym dichotomum*, Unger, *Genera et Species*, p. 71.
1887. *Sphenophyllm dichotomum*, Stur, *Calamarien d. Schatzlarer Schicht.*, p. 233, pl. xv. figs. 5 *a*, *b*, *c*, *d* ((?) pl. xiii.*b* fig. 2 (*in lower right angle of figure*).
1888. *Sphenophyllum dichotomum*, Toula, *Die Steinkohlen*, p. 204, pl. v. fig. 16 (? fig. 21).
1848. *Sphenophyllum multifidum*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxiv. figs 1–2.
1852. *Sphenophyllum Schlotheimii*, Ett. (*non* Brongt.), *Steinkf. v. Stradonitz*, p. 6, pl. vi. fig. 6.
1854. *Sphenophyllum Schlotheimii*, Ett. (*non* Brongt.) (*pars*), *Steinkf. v. Radnitz*, p. 30, pl. xi. figs. 1–3.
1874. *Sphenophyllum Schlotheimii*, Feistmantel (*pars*), *Vers. d. böhm. Ablager.*, Abth. 1, p. 133, pl. xviii. fig. 4 ((?) figs. 2, 3, 5, 6), pl. xix. figs. 2–3 ((?) fig. 1).
1855. *Sphenophyllum emarginatum*, Geinitz (*non* Brongt.) (*pars*), *Vers. d. Steinkf. in Sachsen*, p. 12, pl. xx. fig. 6.
1886. *Sphenophyllum emarginatum*, Sterzel (*non* Brongt.) (*pars*), *Flora d. Rothl. im Nordw. Sachsen, in Dames und Kayser. Palæont. Abhandl.*, vol. iii. Heft iv., Berlin, p. 23, pp. 26, 27, fig. 9 (? fig. 16).
1877. *Sphenophyllum costatulum*, Stur, *Calamarien d. Carbon-Flora d. Schatz. Schicht.*, p. 228, fig. 41, pl. vii.*b* fig. 5, pl. xiv.*b* fig. 6, pl. xv. fig. 6.
1888. *Sphenophyllum costatulum*, Toula, *Die Steinkohlen*, p. 204, pl. v. figs. 17–18.
1877. (?) *Calamites Sachsei*, Stur (*pars*), *Calamarien d. Carbon-Flora d. Schatz. Schicht.*, p. 180, pl. ix. fig. 3, pl. xi. figs. 2–6.
1887. (?) *Sphenophyllum Sachsei*, Stur, *ibid.*, p. 233, fig. 39.

Locality.—Engine Pit, Rowanburn.

Horizon.—Shale associated with *Five-foot Coal.*

Lycopodiaceæ.

Lepidodendron, Sternberg.

Lepidodendron aculeatum, Sternb.

1820. *Schuppenpflanze*, Rhode, *Beitr. z. Pflanzenkunde d. Vorwelt*, pp. 8, 9, pl. i. figs. 5–6.
1820. *Lepidodendron aculeatum*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 1, pp. 21 and 25, pl. vi. fig. 2, pl. viii. fig. 1 B, *a*, *b*; fasc. 2, p. 28, pl. xiv. figs. 1–4; fasc. 4, p. x.
1848. *Lepidodendron aculeatum*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxiii. fig. 4.
1870. *Lepidodendron aculeatum*, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 20, pl. lix. fig. 3, pl. lx. figs. 1–2 ((?) fig. 6).
1877. *Lepidodendron aculeatum*, Fairchild (*pars*), *Ann. New York Acad. Sc.*, vol. i. No. 3, p. 77, pl. v. figs. 1–4, pl. vi. figs. 1–4 (? fig. 5, ? *non* fig. 6), pl. vii. figs. 1–4 (? figs. 5–6), pl. viii. figs. 1–2 (? figs. 3–6), pl. ix. (? fig. 6, *non* figs. 1–5, 7).

1879. *Lepidodendron aculeatum*, Lesqx., *Coal Flora*, vol ii. p. 371, pl. lxiv. fig. 1.
1882. *Lepidodendron aculeatum*, Renault, *Cours d. botan. foss.*, vol. ii. p. 12, pl. i. fig. 7, pl. vi. fig. 4.
1836. *Lepidodendron aculeatum*, Zeiller, *Végét. foss. bassin houil. d. Valen.*, p. 435, pl. lxv. figs. 1–7.
1899. *Lepidodendron aculeatum*, Hofmann and Ryba, *Leitpflanzen*, p. 79, pl. xiv. figs. 8–10 (? fig. 11).
1899. *Lepidodendron aculeatum*, Zeiller, *Étude sur la flore foss. d bassin houil. d'Héraclée*, p. 72, pl. vi. fig. 6.
1900. *Lepidodendron aculeatum*, Zeiller, *Éléments de paléobot.* p. 180, fig. 123.
1902. *Lepidodendron aculeatum*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, pp. 345, 346, fig. 1; 371, pl. li. fig 1.
1903. *Lepidodendron aculeatum*, Arber, *Quart. Journ. Geol. Soc.*, vol. lix. p. 7, pl. i. fig. 4.
1838. *Sagenaria aculeata*, Presl., *in Sternb.*, *Vers.*, vol. ii. fasc. vii.–viii. p. 177, pl. lxviii. fig. 3.
1875. *Sagenaria aculeata*, Feistmantel (*pars*), *Vers. d. böhm. Ablager.*, Abth. ii. p. 34, pl. xii. fig. 1 (? *non* pl. xi. figs. 3, 4).
1820. *Lepidodendron crenatum*, Sternb., *Essai flore monde prim.*, vol. i. fasc. i. pp. 22, 25, pl. viii. fig. 2 B *a b*; fasc. iv. p. x.
1836. *Lepidodendron crenatum*, Göpp., *Syst. fil. foss.*, p. 465, pl. xlii. figs. 4–6.
1848. *Lepidodendron crenatum*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxiii. fig. 2.
1869. *Lepidodendron crenatum*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 128, pl. viii. fig. 2.
1822. *Sagenaria cœlata*, Brongt., *Class. d. végét. foss.*, pp. 24 and 89, pl. i. fig. 6.
1838. *Sagenaria cœlata*, Sternb., *Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 180.
1826. *Lepidodendron cœlatum*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xi.
1848. *Lepidodendron cœlatum*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxi. fig. 5.
1858. (?) *Lepidodendron conicum*, Lesqx., *in Roger's Geol. of Pennsyl.*, vol. ii. p. 874, pl. xv. fig. 3.
1858. *Lepidodendron modulatum*, Lesqx., *in Roger's Geol. of Pennsyl.*, vol. ii. p. 874, pl. xv. fig. i.
1860. *Lepidodendron modulatum*, Lesqx., *in 2nd Rept. of a Geol. Reconnoissance of Middle and South Counties of Arkansas*, p. 310, pl. iii. figs. 1, 1*a*.
1879. *Lepidodendron modulatum*, Lesqx., *Coal Flora*, vol. ii. p. 385, pl. lxiv. figs. 13, 14.
1860. *Lepidodendron Mekiston*, Wood, *Proc. Amer. Acad. Sc. Philadel.*, p. 239, pl. v. fig. 3.
1860. *Lepidodendron Lesquereuxi*, Wood, *ibid.*, p. 240, pl. v. fig. 4.
1860. *Lepidodendron Bordæ*, Wood, *ibid.*, p. 240, pl. vi. fig. 3.
1860. (?) *Lepidodendron Dikrocheilos*, Wood, *ibid.*, p. 239, pl. vi. fig. i.
1869. *Lepidodendron uræum*, Wood, *Trans. Amer. Phil. Soc.*, vol. xiii. p. 343, pl. ix. fig. 5.
1869. *Lepidodendron caudatum*, var. Rhoel (? Sternberg.), *Foss. Flora d. Steink.-Form. Westph.*, p. 130, pl. vi. fig. 7, pl. viii. fig. 7.
1875. (?) *Sagenaria distans*, Feistmantel, *Vers. d. böhm. Ablager.*, Abth. ii. p. 38, pl. xix. fig. 3.
1848. *Lepidodendron obovatum*, Sauveur (*non* Sternb.), *Végét. foss. terr. houil. Belgique*, pl. lxiii fig 3.
1875. *Lepidodendron obovatum*, Feistmantel (*non* Sternb.) (*pars*), *Vers. d. böhm. Ablager.*, Abth. ii. p. 30, pl. ix. fig. 2
1899. *Lepidodendron obovatam*, Hofmann and Ryba (*non* Sternb.) (*pars*), *Leitpflanzen*, p. 80, pl. xiv. figs. 4–5.
1870. *Lepidodendron Sternbergii*, Schimper (*non* Brongt.), *Traité d. paléont. végét.*, vol. ii. p. 19, pl lx. figs. 2 and 5.
1876. *Lepidodendron Sternbergii*, Roemer (*non* Brongt.), *Lethæa geog.*, vol. i. p. 212, fig. 27, pl. liii fig. 3.
1880 *Lepidodendron Sternbergii*, Schimper (*non* Brongt.), *in Zittel, Handb. d. palæont.*, ii. Abth. p. 190, fig. 140.
1888. *Lepidodendron Sternbergii*, Toula (*non* Brongt.), *Die Steinkohlen*, p. 197, pl. iii. fig. 17.
1882. *Lepidodendron dichotomum*, Weiss (*non* Sternb.), *Aus d Steink.*, p. 7, pl. iv. fig. 27 (zweiten abdr.).
1881. *Lepidodendron dichotomum Ajax*, Achepohl, *Niederrh. Westfäl. Steink.*, p. 54, pl. xv. figs. 1–2.
1881. *Lepidodendron dichotomum rhombiforme*, Achepohl, *ibid.*, p. 67, pl. xx. fig. 3.
1882. *Lepidodendron dichotomum transiens*, Achepohl, *ibid.*, p. 92, pl. xxx. fig. 4.
1883. *Leipodendron lamellosum*, Achepohl, *ibid.*, p. 134, pl. xl. fig. 15.
1844. *Lepidodendron*, King, *Edin. New Phil. Journ.*, vol. xxxvi. p. 273, pl. iv. figs. 2, 2*x*, 4.

Decorticated or Imperfectly Preserved Conditions.

1824. *Lepiodendron appendiculatum*, Sternb, *Essai flore monde prim.*, vol. i. fasc. 3, p. 43, pl. xxviii.; fasc. 4, p. xi.
1828 *Sigillaria appendiculata*, Brongt., *Prodrome*, p. 64.
1836. *Sigillaria appendiculata*, Brongt., *Hist. d. végét. foss.*, p. 420, pl. cxli. fig. 2.
1838. *Aspidiaria appendiculata*, Presl, *in Sternb.*, *Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 183.
1825. *Aphyllum cristatum*, Artis, *Antedil. Phyt.*, pl. xvi.
1838. *Aspidiaria cristata*, Presl, *in Stern.*, *Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 183.
1848. *Lepidodendron confluens*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxii. fig. 3.
1875. *Aspidiaria undulata*, Feistmantel, *Vers. d. böhm. Ablag.*, Abth. ii. p. 31 (? pl. x. figs. 1–4), pl. xi. fig. 1 (*non* fig. 2).

Locality.—Blinkbonny Pit, Rowanburn.
Horizon.—Roof of *Main Coal.*

Lepidodendron obovatum, Sternberg.

1820. *Lepidodendron obovatum*, Sternb, *Essai flore monde prim.*, vol. i. fasc. i. pp. 21, 25, pl. iv. fig. 1, pl. viii. fig. 1 A, *a*, *b*; fasc. 4, p. x.
1832. *Lepidodendron obovatum*, L. and H., *Fossil Flora*, vol. i. pl. xix.
1869. *Lepidodendron obovatum*, Roehl (*pars*), *Foss. Flora d. Steink.-Form. Westph.*, p. 129, pl. viii. fig. 8*b*.
1879. *Lepidodendron obovatum*, Lesqx., *Atlas to Coal Flora*, p. 12, pl. lxiv. fig. 3.
1882. *Lepidodendron obovatum*, Renault, *Cours d. botan. foss.*, vol. ii. p. 13, pl. vi. fig. 5.
1886. *Lepidodendron obovatum*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 442, pl. lxvi. figs. 1–8.
1888. *Lepidodendron obovatum*, Toula, *Die Steinkohlen*, p. 196, pl. iii. fig. 8.
1899. *Lepidodendron obovatum*, Hofmann and Ryba (*pars*), *Leitpflanzen*, p. 80, pl. xiv. figs. 6, 6*a*, pl. xv. fig. 1.
1899. *Lepidodendron obovatum*, Zeiller, *Étude sur la flore foss. bassin houil. d'Héraclée*, p. 73, pl. vi. fig. 11.
1838. *Sagenaria obovata*, Presl, *in Sternb.*, *Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 178, pl. lxviii. fig. 6.
1875. *Sagenaria obovata*, Feistmantel (*pars*), *Vers. d. böhm. Ablag.*, Abth. ii. p. 30, pl. ix. figs. 1 and 3 (*non* figs. 2 and 4).
1838. *Sagenaria rugosa*, Presl, *in Sternb.*, *Essai flore monde prim.*, vol. ii. fasc. 7–8, pl. lxviii. fig. 4.
1848. *Lepidodendron Rhodianum*, Sauveur (*non* Sternb.), *Végét. foss. terr. houil. Belgique*, pl. lxiii. fig. 1.
1850–57. *Lepidodendron clypeatum*, Lesqx., *Boston Journ. Nat. Hist.*, vol. iv. No. 4, p. 429.
1858. *Lepidodendron clypeatum*, Lesqx., *in Rogers*, *Geol. of Pennsyl.*, p. 875, pl. xv. fig. 5, pl. xvi. fig. 7.
1879. *Lepidodendron clypeatum*, Lesqx., *Coal Flora*, vol. ii. p. 380, pl. lxiv. fig. 16, 16*a* (*non* figs. 16*b*, 17, 18).
1860. *Lepidodendron venustum*, Wood, *Proc. Amer. Acad. Nat. Sc. Philad.*, p. 239, pl. v. fig. 2.
1869. *Lepidodendron venustum*, Wood, *Trans. Amer. Phil. Soc.*, vol. xiii. p. 346, pl. ix. fig. 1.
1860. *Lepidophloios irregularis*, Lesqx., *Second Rept. of a Geol. Reconnoissance of the Middle and Southern Counties of Arkansas*, p. 311, pl. iv. fig. 3.
1869. *Lepidodendron Sternbergii*, Roehl (*non* Brongt.), *Foss. Flora d. Steink.-Form. Westph.*, p. 127, pl. viii. fig. 8*a*.
1875. *Sagenaria aculeata*, Feist. (*non* Sternb.) (*pars*), *Vers. d. böhm. Ablag.*, Abth. ii. p. 34, pl. xi. figs. 3–4.
1880. *Lepidodendron dichotomum*, Lesqx. (*non* Sternb.), *Coal Flora*, vol. ii. p. 384, pl. lxiv. fig. 3.

Locality.—Engine Pit, Rowanburn.

Horizon.—Shale associated with *Five-foot Coal.*

Remarks.—From the examination of a small series of *Lepidodendron clypeatum* Lesqx., received from the late Mr R. D. Lacoe, I have satisfied myself that this species does not differ in any character from *Lepidodendron obovatum*, Sternb.

Lepidostrobus, Brongniart.

Lepidostrobus, sp.

Locality.—Engine Pit, Rowanburn.

Horizon.—Shale associated with *Five-foot Coal.*

Stigmaria, Brongniart.

Stigmaria ficoides, Sternberg, sp.

Stigmaria ficoides, Sternb., sp. See *ante*, p. 757.

Locality.—Engine Pit, Rowanburn.

Horizons.—Shale associated with *Five-foot Coal and floor of Seven-foot Seam.*

Cordaiteæ.

Cordaianthus, Grand' Eury.

Cordaianthus Pitcairniæ, L. and H., sp.

1833. *Antholithus Pitcairniæ*, L. and H., *Fossil Flora*, vol. ii., pl. lxxxii.
1877. *Botryoconus Pitcairniæ*, Grand' Eury, *Flore Carbon. d. Départ. de la Loire*, p. 280.
1881. *Cordaianthus Pitcairniæ*, Renault, *Cours d. botan. foss.*, vol. i. p. 94, pl. xiii. fig. 7.
1886. *Cordaianthus Pitcairniæ*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 639, pl. xciv. figs 4–5.
1900. *Samaropsis Pitcairniæ*, Zeiller, *Éléments d. Paléobot.*, p. 211, fig. 141.
1833. *Cardiocarpum acutum*, L. and H., *Fossil Flora*, vol. i., pl. lxxvi.
1872. *Cardiocarpon Lindleyi*, Carr., *Geol. Mag.*, vol. ix. p. 55, figs. 1–2.
1872. *Cardiocarpon Lindleyi*, Balfour, *Palæont. Bot.*, p. 65, figs. 51–52.
1879. *Cardiocarpon Lindleyi*, Roemer, *Lethæa Geog.*, vol. i. p. 247, fig. 36.
1874. *Antholithus Lindleyi*, Schimper, *Traité d. paléont. végét.*, vol. iii. p. 566, pl. cx. figs. 10–11.
1881. *Cordaianthus Lindleyi*, Renault, *Cours d. botan. foss.*, vol. i. p. 95, pl. xiii. fig. 9.

Locality.—Blinkbonny Pit, Rowanburn.

Horizon.—Roof of *Main Coal.*

The following Table shows the vertical distribution in Britain of the Lower Coal Measure plants from Rowanburn, Canonbie.

	U. C. M.	M. C. M.	L. C. M.
Sphenopteris obtusiloba, Brongt.,		×	×
(?) ,, *Schützei*, Stur, sp.,		?	×
Eremopteris artemisiæfolia, Sternb., sp.,		×	×
Mariopteris muricata, Schl., sp.,	×	×	×
Alethopteris lonchitica, Schl., sp.,	×	×	×
Neuropteris heterophylla, Brongt.,		×	×
,, *gigantea*, Sternb.,		×	×
,, *Blissii*, Lesqx.,			×
,, *obliqua*, Brongt., sp.,		×	×
Calamites (*Calamitina*) *undulatus*, Sternb.,	×	×	×
,, (*Stylocalamites*), *Cistii*, Brongt.,	×	×	×
Calamites, sp.,	×	×	×
Sphenophyllum cuneifolium, Sternb., sp.,		×	×
Lepidodendron aculeatum, Sternb.,	×	×	×
,, *obovatum*, Sternb.,		×	×
Lepidostrobus, sp.,	×	×	×
Stigmaria ficoides, Sternb., sp.,	×	×	×
Cordaianthus Pitcairniæ, L. and H., sp.,		×	×

Of the sixteen species under consideration, all occur in the Lower Coal Measures of Britain, fourteen occur in the Middle Coal Measures of Britain, excluding one of whose occurrence in that horizon there is some doubt, and six are common to all divisions of the Coal Measures. In all cases the majority of the Lower Coal Measure species are found in the Middle Coal Measures, but the Middle Coal Measures are distinguished from the Lower Coal Measures by the presence of species peculiar to that division, and these are entirely absent from the Rowanburn Coals, which contain a most typical Lower Coal Measure flora.

From the Upper Coal Measures the Rowanburn Coals are easily distinguished by the entire absence of all characteristic Upper Coal Measure species.

The fossil plants from Rowanburn leave, therefore, no doubt as to the series belonging to the Lower Coal Measures.

Fossil Plants of the Middle Coal Measures.

In the Middle Coal Measures several localities were found by Mr A. Macconochie which yielded fossil plants, and one bed which passes across the mouth of the Byre Burn a few yards above Byreburn Bridge, and which is again found on the River Esk about 30 yards below the junction of the Byre Burn with that river, was particularly rich in species.

The following are the localities in the Middle Coal Measures, Canonbie, from which fossil plants were collected, arranged in ascending series,—locality A being the lowest bed, and locality F being the highest, in which fossil plants were observed.

Locality A.—Byre Burn, underneath railway viaduct near Gilnockie railway station. In clayey sandy shale.

Locality B.—Byre Burn, about 300 yards above junction with river Esk. In soft grey shale.

Locality C.—Bed in stream a few yards above bridge at foot of Byre Burn.

Horizon.—Dark shale band lying between the *Three-quarter Coal* and *Main Coal of Byreburn.*

Locality D.—River Esk, left bank, about 30 yards below junction of Byre Burn, and a short distance above old engine-house.

Horizon.—Dark shale with ironstone band, lying between *Three-quarter Coal* and *Main Coal of Byreburn.* Localities C and D are different portions of the same band.

Locality E.—River Esk, right bank, below junction of Byre Burn, almost opposite Byreburn cottages. In soft shale.

Locality F.—River Esk, left bank, about 240 yards below junction of Byre Burn. In soft red-stained shale.

Filicaceæ.

Sphenopteris, Brongniart.

Sphenopteris obtusiloba, Brongt.

Sphenopteris obtusiloba, Brongt., *ante*, p. 768.

Locality A.

Sphenopteris Laurentii, Andræ.

1869. *Sphenopteris Laurentii*, Andræ, *Vorwelt Pflanzen.*, p. 39, pl. xiii. figs. 1–3.
1886. *Sphenopteris Laurentii*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 85, pl. vi. fig. 3, pl. ix. fig. 4.
1899. *Sphenopteris* (*Renaultia*) *Laurenti*, Zeiller, *Flore foss. d. bassin houil. d'Héraclée*, p. 16, pl. i. fig. 16.
1883. *Hapalopteris Laurentii*, Stur, *Morph. und Syst. d. Culm u. Carbon Farne*, p. 32.
1885. *Hapalopteris Laurentii*, Stur, *Carbon-Flora d. Schatz. Schichten*, vol. i. p. 36, pl. xliv. figs. 5–6.
1869. *Sphenopteris stipulata*, Andræ (*non* Gutbier), *Vorwelt Pflanzen.*, p. 40, pl. xiii. fig. 4.

Locality D.

Sphenopteris mixta, Schimper.

1869. *Sphenopteris mixta*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 382.
1870. *Sphenopteris mixta*, Lesqx., *Geol. Survey of Illin.*, vol. iv. p. 409, pl. xv. figs. 7–8.
1879. *Sphenopteris mixta*, Lesqx. (*pars*), *Coal Flora*, vol. i. p. 276 (*non* pl. liv. figs. 1–3).
1886. *Sphenopteris mixta*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 95, pl. xii. fig. 3.
1889. *Sphenopteris mixta*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxv. p. 405.

1899. *Sphenopteris mixta*, White, *Foss. Flora Lower Coal Meas. Missouri*, p. 35, pl. xi. fig. 3, pl. xii. figs. 1–2, pl. xiii. figs. 4–5.
1893. *Ovopteris mixta*, Potonié, *Flora d. Rothl. von Thüringen*, p. 44.
1866. *Sphenopteris rigida*, (?) Lesqx. (*non* Brongt.), *Geol. Survey Illin.*, vol. ii. p. 435, pl. xxxix. figs. 5–6.
1872. *Sphenopteris* (*Aneimioides*) *pulchra*, Marrat, *in Higgins*, *Proc. Liverpool Geol. Soc.*, session 13, 1871–72, p. 101, pl. viii. fig. 1.
1884. *Pseudopecopteris nummularia*, Lesqx. (*non* Gutbier), *Coal Flora*, vol. iii. p. 751, pl. ciii. figs 1–3.

Locality C.

Sphenopteris Hœninghausi, Brongt.

1828. *Sphenopteris Hœninghausi*, Brongt., *Prodrome*, p. 51.
1829. *Sphenopteris Hœninghausi*, Brongt., *Hist. d. végét. foss.*, vol. i. p. 199, pl. lii.
1848. *Sphenopteris Hœninghausi*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. xxii. fig. 2.
1865. *Sphenopteris Hœninghausi*, Andræ, *Vorwelt Pflanzen.*, p. 13, pls. iv–v.
1869. *Sphenopteris Hœninghausi*, Roehl, *Foss. Flora d. Stein.-Form. Westph.*, p. 54, pl. xiv. fig. 8 (? pl. xiii. fig. 3).
1869. *Sphenopteris Hœninghausi*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 385, pl. xxix.
1880. *Sphenopteris Hœninghausi*, Zeiller, *Végét. foss. d. terr. houil.*, p. 41, pl. clxii. figs. 4–5.
1880. *Sphenopteris Hœninghausi*, Lesqx., *Coal Flora*, vol. i. p. 288, pl. lv. fig. 5.
1880. *Sphenopteris Hœninghausi*, Achepohl, *Niederrh. Westfäl. Steink.*, p. 26, pl. vi. fig. 1; *Erganzungsblatt*, i. fig. 39.
1882. *Sphenopteris Hœninghausi*, Weiss, *Aus d. Steink.*, p. 13, pl. xi. figs. 68–69 (zweiter abdr.).
1883. *Sphenopteris Hœninghausi*, Renault, *Cours d. botan. foss.*, vol. ii. p. 191, pl. xxxii. figs. 1–3.
1891. *Sphenopteris Hœninghausi*, Kidston, *Trans. Geol. Soc. Glas.*, vol. ix. p. 48, pl. iv. fig. 44.
1899. *Sphenopteris Hœninghausi*, Hofmann and Ryba, *Leitpflanzen*, p. 41, pl. iv. figs. 7, 7*a*, 7*b*.
1901. *Sphenopteris Hœninghausi*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. p. 213, pl. xxix. fig. 5.
1886. *Sphenopteris* (*Calymmatotheca*) *Hœninghausi*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 82, pl. v. fig. 3, pl. vi. figs. 1–2.
1899. *Sphenopteris* (*Calymmatotheca* ?) *Hœninghausi*, Zeiller, *Étude flore foss. bassin houil. d'Héraclée*, p. 10.
1836. *Cheilanthites Hœninghausi*, Göpp., *Syst. fil. foss.*, p. 244.
1877. *Calymmotheca Hœninghausi*, Stur, *Culm Flora*, vol. ii. p. 266.
1883. *Calymmotheca Hœninghausi*, Stur, *Morph. u. Syst. d. Culm u. Carbonfarne*, p. 174.
1885. *Calymmotheca Hœninghausi*, Stur, *Carbon-Flora d. Schatz. Schicht.*, vol. i. p. 258, pl. xxx., pl. xxxi. figs. 1, 2, 3.
1888. *Calymmotheca* (*Sphenopteris*) *Hœninghausi*, Toula, *Die Steinkohlen*, p. 188, pl. i. fig. 14.
1826. *Sphenopteris asplenioides*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xvi; vol. ii. fasc. 5–6, p. 62.
1869. *Sphenopteris distans*, Roehl (*non* Sternb.), *Foss. Flora d. Steink.-Form. Westph.*, p. 54, pl. xv. fig. 9.
1869. ? *Sphenopteris elegans*, Roehl (*non* Brongt. ?), *Foss. Flora d. Steink.-Form. Westph.*, p. 52, pl. xv. fig. 8, pl. xxvi. fig. 5.

Remarks.—I believe the figures of *Sphenopteris distans* and *Sphenopteris elegans* given by ROEHL * belong to *Sphenopteris Hœninghausi*, Brongt., as already pointed out by STUR.† The figures are, however, not good.

* *Foss. Flora d. Steink.-Form. Westph.*, *l.c.*, in synonymy.
† *Carbon-Flora d. Schatz. Schicht.*, vol. i., *Die Farne*, p. 259.

FEISTMANTEL also gives a figure of *Sphenopteris Hœninghausi* which is probably referable to this species, but if so, the figure is not quite satisfactory.* The same remark may be made regarding the figure given by GEINITZ.† His enlargement is a somewhat inaccurate copy of BRONGNIART'S, pl. lii. fig. *a*.

The *Sphenopteris Hœninghausi*, Feistmantel,‡ is probably referable to the *Sphenopteris bermudensiformis*, Schl., sp. (= *Sphenopteris distans*, Sternb.).

POTONIÉ ‖ has proposed the union of *Calymmotheca Stangeri*, Stur, *Calymmotheca Larischi*, Stur, *Calymmotheca Schlehani*, Stur, and *Calymmotheca Rothschildi*, Stur, § with *Sphenopteris Hœninghausi*, treating them as varieties of BRONGNIART'S species. This view I am unable to accept, as STUR'S species appear to me to be essentially distinct from BRONGNIART'S *Sphenopteris Hœninghausi*.

The fern figured under the name of *Sphenopteris Hœninghausi* by LINDLEY and HUTTON ¶ is specifically distinct from BRONGNIART'S plant, and for it I have proposed the name of *Sphenopteris effusa*.** The specimen figured by LINDLEY and HUTTON is preserved in the Hutton Collection in the museum, Newcastle-on-Tyne.

The specimens on which *Sphenopteris Hœninghausi*, Brongt., is here recorded were received from the late Mr HUGH MILLER, F.R.S.E., some years after the publication of my earlier "Report on the Fossil Plants collected by the Geological Survey of Scotland in Eskdale and Liddesdale." †† They bear the date of April 1869, and are merely localised "Byre Burn."

Locality.—Byre Burn.

Sphenopteris multifida, L. and H.

1834. *Sphenopteris multifida*, L. and H., *Fossil Flora*, vol. ii. p. 113, pl. cxxiii.

Locality A.

Pecopteris, Brongniart.

Pecopteris, sp.

Locality D.

Mariopteris, Zeiller.

Mariopteris muricata, Schlotheim, sp.

Mariopteris muricata, Schl., sp. See *ante*, p. 771.

Locality A.

* *Vers. d. böhm. Ablager.*, Abth. iii. p. 57, pl. xvi. fig. 2, 1876.
† *Vers. d. Steinkf. in Sachsen*, pl. xxiii. figs. 5–6.
‡ *Zeitsch. d. deut. geol. Gesell.*, vol. xxv. pl. xiv. fig. 7.
§ All described in *Culm Flora*, vol. ii. 1877.
‖ *Ueber einige Carbonfarne*, ii. Theil, *Jahrb. d. König. preuss. geol. Landesanstalt für* 1890, p. 16, pls. vii.–ix., 1891.
¶ *Fossil Flora*, vol. iii. pl. cciv.
** *Catal. Palæoz. Plants*, p. 71, 1886.
†† *Trans. Roy. Soc. Edin.*, vol. xxx. p. 531, 1883.

Alethopteris, Sternberg.

Alethopteris lonchitica, Schlotheim, sp.

Alethopteris lonchitica, Schl., sp. See *ante*, p. 772.

Localities A, C, E.

Alethopteris Davreuxi, Brongt., sp.

1828. *Pecopteris Davreuxi*, Brongt., *Prodrome*, p. 57 (*excl. ref.*).
1832 or 1833. *Pecopteris Davreuxi*, Brongt., *Hist. d. végét. foss.*, p. 279, pl. lxxxviii. figs. 1–2.
1848. *Pecopteris Davreuxi*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. xlii. figs. 2–3.
1836. *Alethopteris Davreuxi*, Göpp., *Syst. fil. foss.*, p. 295.
1886. *Alethopteris Davreuxi*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 228, pl. xxxii. fig. 1.
1888. *Alethopteris Davreuxi*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 386, pl. xxiv. fig. 1.
1893. *Alethopteris Davreuxi*, Potonié, *Flora d. Rothl. v. Thüringen*, p. 102 (? pl. x. figs. 2–3).
1832 or 1833. *Pecopteris Dournaisii*, Brongt., *Hist. d. végét. foss.*, p. 282, pl. lxxxix. fig. 1 (? *non* fig. 2).
1836. *Alethopteris Dournaisii*, Göpp., *Syst. fil. foss.*, p. 298.
1848. *Pecopteris Hoffmanni*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. xxxvii. fig. 1.
1848. *Pecopteris rugosa*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. xxxvii. fig. 2.
1883. *Alethopteris Rungi*, Achepohl, *Niederrh. Westfäl. Steink.*, p. 135, pl. xli. fig. 10.
1883. *Alethopteris interrupta*, Achepohl, *Niederrh. Westfäl. Steink.*, p. 136, pl. xli. fig. 13.

Localities C, D.

Alethopteris Grandini, Brongt., sp.

1832 or 1833. *Pecopteris Grandini*, Brongt., *Hist. d. végét. foss.*, p. 286, pl. xci. figs. 1–4.
1876. *Pecopteris Grandini*, Heer, *Flora foss. Helv.*, p. 33, pl. xii. fig. 10*a*.
1836. *Alethopteris Grandini*, Göpp., *Syst. fil. foss.*, p. 299.
1883. *Alethopteris Grandini*, Renault, *Cours d. botan. foss.*, vol. iii. p. 157, pl. xxvii. figs. 3–4.
1886. *Alethopteris Grandini*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 237, pl. xxxviii. figs. 1–2.
1888. *Alethopteris Grandini*, Zeiller, *Flore foss. terr. houil. d. Commentry*, prem. partie, p. 203, pl. xxi. figs. 1–8.
1890. *Alethopteris Grandini*, Zeiller, *Flore foss. bassin houil. et perm. d'Autun et d'Épinac*, prem. partie, p. 114, pl. ix. figs. 6–7.
1899. *Alethopteris Grandini*, Hofmann and Ryba, *Leitflanzen*, p. 56, pl. viii. figs. 4, 4*a*, 5, 5*a*.

Localities C, D.

Neuropteris, Brongniart.

Neuropteris heterophylla, Brongt.

Neuropteris heterophylla, Brongt. See *ante*, p. 773.

Locality E.

Neuropteris gigantea, Sternb.

Neuropteris gigantea, Sternb. See *ante*, p. 775.

Localities B, F.

Equisetaceæ.

Calamites, Suckow.

Group I.—Calamitina, Weiss.

Calamites (Calamitina) undulatus, Sternb.

Calamites (Calamitina) undulatus, Sternb. See *ante*, p. 776.

Locality D.

Calamites (Calamitina) Schützei, Stur.

1881. *Calamites Schützei*, Stur, *Zur Morph. d. Calamarien*, p. 8 (*Sitzb. d. k. Akad. d. Wissensch.*, vol. lxxxiii., i. Abth., 1881, p. 416), pl. i. fig. 1.
1887. *Calamites Schützei*, Stur, *Calamarien d. Carbon-Flora d. Schatz. Schicht.* (*K. k. geol. Reichs. Abhandl.*, vol. xi. Abth. ii., Wien), p. 131, pl. iii. figs. 2, 2*b*, pl. iv., pl. iv.*b* fig. 1, pl. xvii. fig. 2, text figs. 33–38.
1888. *Calamites Schützei*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 350, pl. lv. fig. 4.
1884. *Calamites* (*Calamitina*) *varians*, cf. *Schützei*, Weiss, *Steinkohl. Calamarien*, part ii. p. 79, pl. xxi. fig. 5, p. 80, pl. xxvii. fig. 2.
1825. (?) *Calamites approximatus*, Artis (*non* Schloth.), *Antedil. Phyt:*, pl. iv.
1828. (?) *Calamites approximatus*, Brongt. (*pars*) (*non* Schloth.), *Hist. d. végét. foss.*, p. 133, pl. xv. figs. 7–8, pl. xxiv. fig. 1.
1833. (?) *Calamites approximatus*, L. and H., *Fossil Flora*, vol. i., pl. lxxvii.
1887. (?) *Calamites approximatus* Stur, (*pars*), *Calamarien d. Carbon.-Flora d Schatz. Schicht.*, p. 119, pl. viii. figs. 2–3, pl. xii. fig. 7.

Remarks.—Zeiller has clearly pointed out that the *Calamite* figured by several writers under the name of *Calamites approximatus* cannot be Schlotheim's plant which was described as being somewhat similar to his *Calamites cannæformis*, but with longer joints and narrower ribs.* What *Calamites cannæformis* really is is equally uncertain, but it is clear that the short jointed *Calamite* pith cast, which has so often been figured as *Calamites approximatus*, cannot be the described but unfigured plant to which Schlotheim applied the name. Zeiller thinks that the true *Calamites approximatus*, Schloth., may be identical with Brongniart's *Calamites Cistii*, but this cannot be satisfactorily determined.

There seems, therefore, no other course open than to let the name of *Calamites approximatus*, Schlotheim (*non* Artis and Brongt.), disappear from our lists, and slip into oblivion.

The question next arises,—To which species are the various specimens which have

* Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 352.

been figured and described as *Calamites approximatus* to be referred? Several of these seem to be referable to STUR's *Calamites Schützei*, such as those figured in part by BRONGNIART, by ARTIS, by LINDLEY and HUTTON, and in part by STUR.

The other forms with very short joints, such as those figured by BRONGNIART,* GEINITZ,† WEISS,‡ ARBER,§ and myself,|| offer considerable difficulty in referring them to any of the recognised species. STUR has proposed the name of *Calamites Waldenburgensis* for these, but includes under this name some which appear to belong to his *Calamites Schützei*.¶

These short jointed *Calamites* possibly do not represent a true species, but may be only a condition of growth of *Calamites varians*, Sternb.,** or of *Calamites Schützei*, Stur. In fact, *Calamites Schützei* appears to be very closely related to *Calamites varians*, Sternb., and may be only a form or variety of that species, and this view was that which I previously held; nor am I yet quite certain that this is not the correct opinion, for in many cases it is difficult to determine whether some specimens should be referred to *Calamites Schützei* or to *Calamites varians*, Sternb. Many of the specimens I have previously recorded as *Calamites varians* would, according to the present view of most botanists, be referred to *Calamites Schützei*, Stur.

For the short jointed forms, to which I have already referred, until their true position is determined, I would propose that STUR's *Calamites Waldenburgensis* be provisionally employed for their reception.

Locality D.

Calamites (Calamitina) pauciramis, Weiss.

(Plate IV. fig. 36; Plate V. fig. 44.)

1884. *Calamites* (*Calamitina*) *pauciramis*, Weiss, *Steinkohlen Calamarien*, part ii. p. 93, pl. xi. fig. 1.

This species is closely related to *Calamites* (*Calamitina*) *discifer*, Weiss,†† and may be specifically identical, the chief difference being that *C. discifer*, Weiss, has three scars on each branch-bearing node, whereas *C. pauciramis*, Weiss, has only two. The bark of *C. discifer* is smooth, while that of *C. pauciramis*, Weiss, is said to be ribbed. According to WEISS, the branch scars are borne on every third node.

I have only seen two fragments of *C. pauciramis*, which are given on Plate IV. fig. 36 and Plate V. fig. 44, natural size.

That shown on Plate IV. fig 36 is the more perfect, but is not sufficiently complete to show two branch-bearing nodes, only three internodes being seen, of which the lowest node bears the branch scar. On this specimen the smooth bark is covered with short

* *Hist. d. végét. foss.*, pl. xxiv. figs. 2–5.

† *Vers. d. Steinkf. in Sachsen*, pl. xii. fig. 3.

‡ *Steinkohl. Calamarien*, part 2, pl. xxv. fig. 1.

§ *Quart. Journ. Geol. Soc.*, vol. lix., pl. i. fig. 3.

|| *Trans. Roy. Soc. Edin.*, vol. xxxvii., pl. ii. figs. 5–6. *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv., pl. xxxv. fig. 2.

¶ STUR, *Calamarien d. Carbon-Flora d. Schatz. Schichten*, p. 119.

** *Essai flore monde prim.*, vol. ii. p. 50, pl. xii.

†† WEISS, *Steinkohlen Calamarien*, part ii. p. 91, pl. vii. fig. 3.

fine close lines, and shows the true outer surface of the stem, as proved by the cetaceous single nerved leaves being still attached to the uppermost and second nodes. The ribbed stem of the example figured by WEISS I am inclined to ascribe to its condition of preservation.

The specimen given on Plate V. fig. 44 is not quite so well preserved, but shows faint indications of the ribbing described by WEISS. The straight parallel lines seen passing over the scar and other parts of the surface of this specimen must not, however, be mistaken for ribbing, as they have been imparted to the stem by longitudinal cracks in the coaly matter which once adhered to it.

STUR * has united *C. discifer*, Weiss, *C. pauciramis*, Weiss, and *C. macrodiscus*, Weiss, to *C. Germarianus*, Göpp.,† but I do not think we are at present justified in uniting all these species under one name.

Calamites (Calamitina), sp.

Locality D.

Group II.—Eucalamites, Weiss.

Calamites (Eucalamites) ramosus, Artis.

1825. *Calamites ramosus*, Artis, *Antedil. Phyt.*, pl. ii.
1828. *Calamites ramosus*, Brongt., *Hist. d. végét. foss.*, p. 127, pl. xvii. figs. 5–6.
1848. *Calamites ramosus*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. ix. figs. 2–3.
1882. *Calamites ramosus*, Weiss, *Aus d. Steinkohl.*, p. 10, pl. viii. fig. 44 (zweiter abdr.).
1886. *Calamites ramosus*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 345, pl. lv. fig. 3, pl. lvi. fig. 3.
1886. *Calamites ramosus*, Kidston, *Trans. Geol. Soc. Glas.*, vol. viii. p. 51, pl. iii. fig. 1.
1887. *Calamites ramosus*, Stur (*pars*), *Calamarien d. Carbon-Flora d. Schatz. Schicht.*, p. 96, pl. xii. figs. 1–4 (*non* 5–6), pl. xii.*b* figs. 1–4 (? 5), 6, pl. xiii. figs. 1–9, pl. xiv. figs. 3–5, text figs. 1 (p. 4), 31 (p. 104), 32 (p. 105), (*non* fig. 2, p. 8).
1888. *Calamites ramosus*, Toula, *Die Steinkohlen*, p. 205, pl. v. fig. 24.
1891. *Calamites ramosus*, Kidston, *Proc. Roy. Phys. Soc. Edin.*, vol. x. p. 354.
1901. *Calamites ramosus*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. pp. 201, 229, pl. xxxvii. figs. 3–4.
1884. *Calamites* (*Eucalamites*) *ramosus*, Weiss, *Steinkohl. Calamarien*, part ii. p. 98, pl. ii. fig. 3, pl. v figs. 1–2, pl. vi., pl. vii. figs. 1–2, pl. viii. figs. 1, 2, 4, pl. ix. figs. 1–2, pl. x. fig. 1, pl. xx. figs. 1–2 (includes *Annularia ramosa* and *Calamostachys ramosa*).
1899. *Calamites* (*Eucalamites*) *ramosus*, Hofmann and Ryba, *Leitpflanzen*, p. 25, pl. i. fig. 8.
1824. *Calamites nodosus*, Sternb. (*non* Schloth.), *Essai flore monde prim.*, vol. i. fasc. 2, pp. 30 and 36, pl. xvii. fig. 2; fasc. 4, p. xxvii; fasc. 5–6, p. 48.
1831. *Calamites nodosus*, L. and H. (*pars*) (*non* Schloth.), *Fossil Flora*, vol. i. pl. xv. (not branch to right or fig. 2) (*non* pl. xvi.).
1877. *Calamites nodosus*, Lebour (*non* Schloth.), *Illustr. of Fossil Plants*, pp. 3, 7, pls. ii.–iii.
1833. *Calamites carinatus*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 3, pp. 40 and 44, pl. xxxii. fig. 1; fasc. 4, p. xxvii.
1854. *Calamites communis*, Ett. (*pars*), *Steinkf. v. Radnitz*, p. 24, pl. iii. fig. 2, pl. iv. fig. 4.

* STUR, *Die Calam. d. Carbon-Flora. d. Schatz. Schichten*, p. 174.
† GÖPPERT, *Foss. Flora d. Ubergangs*, p. 122, pl. xlii. fig. 1.

Foliage :—

1822. *Asterophyllites radiatus*, Brongt., *Class. d. végét. foss.*, p. 35, pl. ii. figs. 7*a*, 7*b*.
1828. *Annularia radiata*, Brongt., *Prodrome*, p. 156.
1848. *Annularia radiata*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxvii. fig. 2.
1869. *Annularia radiata*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 28, pl. iv. fig. 3 (fig. 4 ?).
1874. *Annularia radiata*, Feistmantel, *Vers. d. böhm. Ablager.*, p. 130, pl. xvii. figs. 2–4.
1877. *Annularia radiata*, Breton, *Étude stratig. d. terr. houil. d'Auchy-au-Bois*, pl. viii. (*pars*).
1880. *Annularia radiata*, Zeiller, *Végét. foss. du terr. houil.*, p. 24, pl. clx. fig. 1.
1882. *Annularia radiata*, Renault, *Cours. d. botan. foss.*, vol. ii. p. 133, pl. xx. fig. 4.
1886. *Annularia radiata*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 394, pl. lix. fig. 8, pl. lxi. figs. 1–2.
1899. *Annularia radiata*, Zeiller, *Flore foss. d. bassin houil. d'Héraclée*, p. 64, pl. v. fig. 15.
1899. *Annularia radiata*, Hofmann and Ryba, *Leitpflanzen*, p. 28, pl. ii. fig. 10.
1899. *Annularia radiata*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 221, fig. 197.
1899. *Annularia radiata*, Frech, *Leth. geog.*, Band 2, Lief. 2, *Steinkohlenform.*, pl. l.*a* fig. 1.
1901. *Annularia radiata*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. pp. 201, 229, pl. xxxvii. fig. 2.
1832. *Asterophyllites foliosa*, L. and H., *Fossil Flora*, vol. i., pl. xxv.
1855. *Asterophyllites foliosa*, Geinitz (*pars*), *Vers. d. Steinkf. in Sachsen*, p. 10, pl. xvi. fig. 3 (? 2) (*non* figs. 1 and 4).
1874. ? *Asterophyllites foliosa*, Feistmantel, *Vers. d. böhm. Ablager.*, p. 121, pl. xiv. figs. 2, 3, 4.
1848. *Annularia asterophylloides*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxvii. fig. 1.
1848. *Annularia patens*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. lxix. fig. 4.
1886. *Annularia patens*, Kidston, *Trans. Geol. Soc. Glasgow*, vol. viii. p. 53, pl. iii. fig. 2.
1851. *Annularia minuta*, Ett., *Haidinger's Naturwiss. Abhandl.*, vol. iv. Abth. i. p. 83, pl. x. figs. 1–2.
1890. ? *Annularia stellata*, Renault (? *non* Schlotheim), *Flore foss. terr. houil. d. Commentry*, part. ii., explan. to plates, p. 2, pl. xlvii. figs. 1–2.

Locality D (stems and foliage branches).

Group III.—Stylocalamites, Weiss.

Calamites (Stylocalamites) Suckowii, Brongt.

1784. *Calamites*, Suckow, *Acad. Elect. Theodoro-Palatinæ*, vol. v. p. 355, pl. xvi. fig. 2, pl. xix. figs. 8–9.
1828. *Calamites Suckowii*, Brongt., *Hist. d. végét. foss.*, p. 124 (pl. xiv. fig. 6 ?), pl. xv. figs. 1–6, pl. xvi. (fig. 1 ?) figs. 2, 3, 4.
1833. *Calamites Suckowii*, Sternb., *Essai flore monde prim.*, vol. ii. fasc. 5–6, p. 49.
1835. *Calamites Suckowii*, Gutbier, *Abdr. u. Vers. d. Zwick. Schwarzkohl.*, p. 17, pl. ii. fig. 1 (*non* fig. 2).
1848. *Calamites Suckowii*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. iii., pl. iv. figs. 1–2, pl. xi. fig. 3.
1851. *Calamites Suckowii*, Bronn, *Lethæa geog.*, vol. i. p. 101, pl. vi. figs. 1*a*, 1*b*.
1855. *Calamites Suckowii*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 6, pl. xiii. figs. 1, 3, 5, 6 (? 4).
1868. *Calamites Suckowii*, Dawson, *Acad. Geol.*, 2nd ed., p. 195, fig. 39, p. 442, fig. 163 A^2, A^4, p. 478.
1869. *Calamites Suckowii*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 9, pl. i. fig. 6, pl. ii. fig. 2.
1871. *Calamites Suckowii*, Weiss, *Foss. Flora d. jüngst. Stk. u. Rothl.*, p. 117, pl. xiii. fig. 5.
1874. *Calamites Suckowii*, Feistmantel (*pars*), *Vers. d. böhm. Ablager.*, Abth. i. p. 102, pl. ii. figs. 3–4, pl. iii. figs. 1–2, pl. iv. figs. 1–2, pl. v., pl. vi. fig. 1.

1876. *Calamites Suckowii*, Weiss, *Steinkohlen Calamarien*, part i. p. 123, pl. xix. fig. 1; part ii., 1884, p. 129, pl. ii. fig. 1, pl. iii. figs. 2, 3, pl. iv. fig. 1, pl. xxvii. fig. 3.
1879. *Calamites Suckowii*, Roemer, *Lethæa geog.*, vol. i. p. 144, pl. l. fig. 1.
1877. *Calamites Suckowii*, Grand' Eury, *Flore Carbon. du Départ. de la Loire*, p. 14, pl. i. figs. 1–6.
1880. *Calamites Suckowii*, Zeiller, *Végét. foss. d. terr. houil.*, p. 12, pl. clix. fig. 1.
1882. *Calamites Suckowii*, Weiss, *Aus d. Steink.*, p. 10, pl. vii. fig. 43 (zweiter abdr.).
1882. *Calamites Suckowii*, Renault, *Cours d. botan. foss.*, vol. ii. p. 159, pl. xxiv. figs. 3–5.
1886. *Calamites Suckowii*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 333, pl. liv. figs. 2–3, pl. lv. fig. 1.
1887. *Calamites Suckowii*, Stur (*pars*), *Calamarien d. Carbon-Flora d. Schatz. Schicht.*, p. 145, pl. iii. figs. 3–4, pl. v. figs. 5–6, pl. xvi. figs. 1–2 (*non* pl. i. fig. 3, pl. ix. fig. 2, pl. xiv. fig. 1).
1888. *Calamites Suckowii*, Toula, *Die Steinkohlen*, p. 202, pl. v. figs. 1, 2, 9 (*non* fig. 26).
1890. *Calamites Suckowii*, Renault, *Flore foss. terr. houil. d. Commentry*, p. 385, pl. xliii. figs. 1–3, pl. xliv. figs. 4–5.
1899. *Calamites Suckowii*, Hofmann and Ryba, *Leitpflanzen*, p. 24, pl. i. fig. 6.
1900. *Calamites Suckowii*, Zeiller, *Éléments de paléobot.*, p. 149, fig. 149.
1900. *Calamites Suckowii*, Scott, *Studies in Fossil Botany*, p. 15, fig. 2, p. 16, fig. 3.
1901. *Calamites Suckowii*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. pp. 200, 202, 215, 225, pl. xxx. fig. 1, pl. xxxv. fig. 3.
1833. *Calamites cannæformis*, L. and H. (*non* Schlotheim), *Fossil Flora*, vol. i., pl. lxxix.
1833. *Calamites æqualis*, Sternb., *Essai flore monde prim.*, vol. ii. fasc. 5–6, p. 49.
1848. *Calamites Artisii*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. vii. figs. 1–2.
1848. *Calamites nodosus*, Sauveur (*non* Schlotheim), *Végét. foss. terr. houil. Belgique*, pl. xii. fig. 3.
1874. *Calamites approximatus*, Feistmantel (*non* Schlotheim), *Vers. d. böhm. Ablager.*, Abth. i. p. 106, pl. vi. fig. 2, pl. vii. figs. 1–2.
1877. *Calamites cannæformis*, Lebour (*non* Schlotheim), *Illustr. of Fossil Plants*, pl. i.
1882. *Calamites irregularis*, Achepohl, *Niederrh. Westfäl. Steink.*, p. 89, pl. xxviii. fig. 2.
1898. *Calamites*, Seward, *Fossil Plants*, p. 323, fig. 82.
1825. (?) *Calamites decoratus*, Artis, *Antedil. Phyt.*, pl. xxiv.
1828. (?) *Calamites decoratus*, Brongt. (*pars*), *Hist. d. végét. foss.*, p. 123, pl. xiv. figs. 1–2 (*non* figs. 3–4).

Locality D.

Calamites (Stylocalamites) Cistii, Brongniart.

Calamites (Stylocalamites) Cistii, Brongt. See *ante*, p. 777.

Locality D.

Calamocladus, Schimper.

Calamocladus equisetiformis, Schlotheim, sp.

1709. Scheuchzer, *Herb. diluv.*, pl. i. fig. 5, pl. ii. fig. 1.
1793. Ure, *Rutherglen and East Kilbride*, pl. xii. fig. 4.
1804. Schlotheim, *Flora d. Vorwelt*, p. 30, pl. i. figs. 1–2, pl. ii. fig. 3.
1809. *Phytolithus* (*stellatus*), Martin, *Petrificata Derbiensia*, pl. xx. figs. 4–6 (*non* fig. 5).
1820. *Casuarinites equisetiformis*, Schloth., *Petrefactenkunde*, p. 397.
1826. *Bornia equisetiformis*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xxviii.
1841. *Bornia equisetiformis*, Steininger, *Geol. Beschr. d. Landes zw. Saar u. Rheine Nachtr.*, p. 12, fig. 13.
1828. *Asterophyllites equisetiformis*, Brongt., *Prodrome*, p. 158.

1845. *Asterophyllites equisetiformis*, Germar, *Vers. v. Wettin. u. Löbejun.*, p. 21, pl. viii.
1855. *Asterophyllites equisetiformis*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 8, pl. xvii. fig. 1 (? figs. 2–3).
1864. *Asterophyllites equisetiformis*, Göppert, *Foss. Flora d. perm. Form*, p. 36, pl. i. fig. 5.
1869. *Asterophyllites equisetiformis*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 22, pl. iii. fig. 5.
1871. *Asterophyllites equisetiformis*, Weiss., *Foss. Flora d. jüngst. Stk. u. Rothl.*, p. 126, pl. xii. fig. 2.
1874. *Asterophyllites equisetiformis*, Feistmantel (*pars*), *Vers. d. böhm. Ablager.*, Abth. i. p. 116, pl. x. fig. 2 (? fig. 1), pl. xi. (? pl. xii. fig. 2).
1879. *Asterophyllites equisetiformis*, Lesqx. (*pars*), *Coal Flora.* vol. i. p. 35, pl. ii. fig. 3.
1880. *Asterophyllites equisetiformis*, Zeiller, *Végét. foss. d. terr. houil.*, p. 19, pl. clix. fig. 3.
1882. *Asterophyllites equisetiformis*, Weiss, *Aus d. Steink.*, p. 10, pl. ix. fig. 45 (zweiter abdr.).
1882. *Asterophyllites equisetiformis*, Renault, *Cours d. botan. foss.*, vol. ii. p. 112, pl. xviii. fig. 1.
1883. *Asterophyllites equisetiformis*, Schenk, *in Richthofen's China*, vol. iv. p. 235, pl. xxxvii. fig. 3.
1883. *Asterophyllites equisetiformis*, Lesqx., *Indiana Dept. of Geol. and Nat. Hist. 13th Ann. Rept.*, part ii. p. 42, pl. vi. figs. 1–2.
1886. *Asterophyllites equisetiformis*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 368, pl. lviii. figs. 1–7.
1890. *Asterophyllites equisetiformis*, Renault, *Flore foss. bassin houil. d. Commentry*, part ii. p. 409, pl. xlviii. figs. 3, 4, 5, 7.
1899. *Asterophyllites equisetiformis*, White, *Foss. Flora Lower Coal Meas. Missouri*, p. 151, pl. lix. fig. 1c.
1899. *Asterophyllites equisetiformis*, Hofmann and Ryba, *Leitpflanzen*, p. 27, pl. ii. figs. 3–4.
1900. *Asterophyllites equisetiformis*, Zeiller, *Éléments d. paléobot.*, p. 161, fig. 113.
1869. *Calamocladus equisetiformis*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 324, pl. xxii. figs. 1, 2, 3.
1874. *Calamocladus equisetiformis*, Crépin, *Bull. Acad. roy. d. Belgique*, 2 sér., vol. xxxviii. p. 7, pl. ii. figs. 1–3.
1898. *Calamocladus equisetiformis*, Seward, *Fossil Plants*, p. 335, fig. 87.
1901. *Calamocladus equisetiformis*, Kidston, *Proc. York. Geol. and Polytech. Soc.*, vol. xiv. part. ii. pp. 203, 215, pl. xxx. fig. 3.
1836. *Hippurites longifolia*, L. and H., *Fossil Flora*, vol. iii. pls. cxc.–cxci.
1869. *Annularia calamitoides*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 349, pl. xxvi. fig. 1.
1876. *Calamocladus binervis*, Boulay, *Terr. houil. du nord de la France et ses végét. foss.*, p. 22, pl. ii. fig. 1.
1880. *Asterophyllum equisetiformia*, Schimper, *in Zittel, Handb. d. palæont.*, Abth. ii. p. 175, fig. 131.
1876. *Calamostachys*, Boulay, *Terr. houil. du nord de la France et ses végét. foss.*, p. 24, pl. i. figs. 1, 1 *bis*.
1876. *Calamostachys Germanica*, Weiss, *Steinkohl. Calamar.*, part i. p. 47, pl. xvi. figs. 3–4.
1883. *Calamostachys Germanica*, Schenk, *in Richthofen's China*, vol. iv. p. 233, pl. xxxvi. fig. 5.

Localities C, D.

Calamocladus charæformis, Sternberg, sp.

1826. *Bechera charæformis*, Sternb., *Essai flore monde prim.*, vol. i. fasc. iv. p. xxx., pl. lv. fig. 3 (? fig. 5).
1840. *Bechera charæformis*, Morris, *in Prestwick, Trans. Geol. Soc. London*, 2nd ser., vol. v., pl. xxxviii. fig. 2, and explanation to plate.
1845. *Asterophyllites charæformis*, Unger, *Synop. plant foss.*, p. 33.
1893. *Calamocladus charæformis*, Kidston, *Trans. York. Nat. Union*, part 18, p. 86.
1869. *Asterophyllites delicatuta*, Roehl (*non* Sternb.) (*pars*), *Vers. d. Steink.-Form. Westph.*, p. 26, pl. ii. fig. 6.
1875. *Asterophyllites* (?) *minutus*, Andrews, *Descr. of Fossil Plants from the Lower Carb. Strata of Ohio* (*Ohio Geol. Rept.*, vol. ii., *Geol. and Palæont.*), p. 424, pl. li. figs. 4–4*a*.
1860. *Asterophyllites gracilis*, Lesqx., in *2nd Rept. of a Geol. Reconnoissance of the Middle and South Counties of Arkansas*, p. 310, pl. ii. fig. 4, 4*a*.

1879. *Asterophyllites gracilis*, Lesqx. (*pars*), *Coal Flora Atlas*, p. 2, pl. ii. figs. 4, 5, text, vol. i. p. 42; vol. iii. p. 714, pl. xciii. figs. 3–4 (?) (*non* figs. 5–6).
1883. *Asterophyllites gracilis*, Lesqx. (*pars*), *Indiana Dept. of Geol. and Nat. Hist. 13th Ann. Rept.*, part ii. p. 43, pl. vi. figs. 4–5 (*non* fig. 6 ?).
1887. *Asterophyllites Roehli*, Stur (*pars*), *Calamar. d. Carbon-Flora d. Schatz. Schichten*, p. 209, pl. xiv. figs. 13 *a*, *b*, *c*, pl. xv.*b* fig. 3.
1890. *Calamocladus Roehli*, Kidston, *Trans. York. Nat. Union*, part 14, p. 22.

Locality D.

Palæostachya, Weiss.

Palæostachya Ettingshauseni, Kidston, n.sp.

1854. *Calamites communis*, Ett. (*pars*), *Steinkohlenf. v. Radnitz*, p. 24, pl. viii. figs. 1 and 4.
1869. *Volkmannia elongata*, Roehl (*non* Presl.), *Foss. Flora Steink.-Form. Westph.*, p. 19, pl. vii. fig. 1.
1869. *Calamostachys typica*, Schimper (*pars*), *Traité d. paléont. végét.*, vol. i. p. 328 (pl. xxiii. fig. 1 ?); vol. iii. p. 457.
1890. *Calamostachys typica*, Kidston, *Trans. York. Nat. Union*, part 14, p. 23.
1884. *Calamostachys Ludwigi*, Weiss (*pars*), *Steinkohl. Calamarien*, part ii. p. 163, pl. xviii. fig. 2 (*non* pl. xxii. figs. 1–8, pl. xxiii., pl. xxiv.).

Remarks.—For some time I have been under the impression that Schimper has included two distinct types of Calamitic fructification under the name of *Calamostachys typica*, one a true *Calamostachys*, the *C. Ludwigi*, Carr., and the other a *Palæostachya*.

A Calamitic cone, agreeing in every respect with those included in the above synonymy, is not uncommon in the Middle and Lower Coal Measures, but none of the figured specimens included above have, as far as I am aware, shown the position of the sporangiophore. A few examples which I have lately examined, especially one from Canonbie,* show that the sporangiophores spring from the axils of the bracts, and that these cones are typical members of the genus Palæostachya.

For these I propose the name of *Palæostachya Ettingshauseni*, after Dr Constantin von Ettingshausen, by whom this fossil was first figured and described.

Localities C, D. Very plentiful.

Paracalamostachys, Weiss.

(?) Paracalamostachys Williamsoniana, Weiss.

1884. *Paracalamostachys Williamsonia*, Weiss, *Steinkohlen Calamarien*, part. ii. p. 193, pl. xxii. fig. 9.
1869. *Calamodendron commune*, (?) Binney, *Mem. Lit. and Phil. Soc. Manchester*, 3rd ser., vol. iv. p. 218, pl. vi. fig. 2.

Remarks.—Several small Calamitic cones have been collected, which are most probably referable to this species, but their preservation is too imperfect to admit of a satisfactory determination.

Locality D.

* Reg. No. K/3130.

Lycopodiaceæ.

Lepidodendron, Sternberg.

(?) Lepidodendron lycopodioides, Sternberg.

1823. *Lepidodendron lycopodioides*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 2, pp. 29 and 35, pl. xvi. figs. 1, 2, and 4.
1882. *Lepidodendron lycopodioides*, Renault, *Cours d. botan. foss.*, p. 14, pl. v. fig. 8.
1886. *Lepidodendron lycopodioides*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 464, pl. lxix. figs. 2–3 (? pl. lxx. fig. 1).
1902. *Lepidodendron lycopodioides*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. part 2, p. 373, pl. lii. fig. 2.
1903. *Lepidodendron lycopodioides*, Arber, *Quart. Journ. Geol. Soc.*, vol. lix. p. 12, pl. ii. fig. 5.
1826. *Lycopodiolites elegans*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. viii.
1828. *Lepidodendron elegans*, Brongt., *Prodrone*, p. 85.
1823. *Lycopodiolites selaginoides*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 2, pp. 29 and 35; fasc. 4, p. viii, pl. xvi. fig. 3, pl. xvii. fig. 1.
1828. *Lycopodiolites selaginoides*, Bischoff, *Kryptogam. Gewächte*, p. 117, pl. xiii. fig. 4.
1828. *Lepidodendron selaginoides*, Brongt., *Prodrome*, p. 85.
1834. *Lepidodendron selaginoides*, L. and H., *Fossil Flora*, vol. ii., pl. cxiii.
1875. *Lycopodites selaginoides*, Feistmantel, *Vers. d. böhm. Ablager.*, Abth. ii. p. 10, pl. i. figs. 3–4 (? pl. ii.).

Remarks.—The specimen from Canonbie was fragmentary, but I have very little doubt as to its belonging to *Lepidodendron lycopodioides*, Sternb.

Lepidodendron lycopodioides, Sternb., is very common in the Middle Coal Measures, and from the examination of many specimens I have satisfied myself that *Lepidodendron selaginoides*, Sternb., is founded on the leafy and younger branches of *Lepidodendron lycopodioides*,—some of my specimens, collected by Mr. W. Hemingway, show the organic union of the two forms.

The *Lepidodendron elegans*, Brongt.,* and Lindley and Hutton,† have been referred by some botanists to *Lepidodendron lycopodioides*, Sternb., but, so far as one can learn the characters of the specimens from the figures given—the mode in which the foliage springs from the stems—suggests rather that they should be referred to *Lepidodendron ophiurus*, Brongt.

Locality F.

Sigillaria, Brongniart.

Sigillaria elegans, Sternberg, sp.

1826. *Favularia elegans*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, pp. xiv and 48, pl. lii. fig. 4.
1887. *Favularia elegans*, Weiss (*copied from Sternb.*), *Sigillarien d. preuss. Steink.*, i. *Gruppe Favularien*, p. 54, pl. xv. fig. 2.
1828. *Sigillaria elegans*, Brongt., *Prodrone*, p. 65.
1836. *Sigillaria elegans*, Brongt., *Hist. d. végét. foss.*, p. 438, pl. cxlvi. fig. 1, pl. clv., pl. clviii. fig. 1.
18—? *Sigillaria elegans*, König., *Icones foss. sectiles*, pl. xv. fig. 184.
1852. *Sigillaria elegans*, Bronn, *Lethæa geog.*, vol. i. part ii. p. 134, pl. vi. fig. 6.
1857. *Sigillaria elegans*, Goldenberg (*pars*), *Flora saræp. foss.*, Heft ii. p. 27, pl. vi. figs. 16–17 (*non* pl. v. figs. 6–13).

* *Hist. d. végét. foss.*, vol. ii., pl. xiv.

† *Fossil Flora*, vol. ii., pl. cxviii.

1869. *Sigillaria elegans*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 96, pl. viii. fig. 9, pl. xxviii. fig. 17 (? pl. xxviii. fig. 6).
1881. *Sigillaria elegans*, Renault, *Cours d. botan. foss.*, vol. i. p. 132, pl. xvii. fig. 4, 4*bis*.
1881. *Sigillaria elegans*, Achepohl, *Niederrh. Westfäl. Steink.*, p. 35, pl. ix. figs. 20–23; *Ergänzungsblatt*, ii., figs. 13–14.
1882. *Sigillaria elegans*, Weiss, *Aus d. Steink.*, p. 5, pl. i. fig. 2 (zweiter abdr.).
1886. *Sigillaria elegans*, Zeiller, *Flore foss bassin houil. d. Valen.*, p. 582, pl. lxxxvii. figs. 1–4.
1887. *Sigillaria elegans*, Weiss, *Sigillarien d. preuss. Steink.*, i. *Gruppe Favularien* (*Abhandl. d. geolog. Landesanstalt*, vol. vii. Heft 2), p. 32.
1887. *Sigillaria elegans*, Weiss (*copied from Brongt.*), *Sigillarien d. preuss. Steink.*, i. *Gruppe. Die Favularien*, pl. xv. fig. 5, 5*a*, 6, 6*a*, 7.
1899. *Sigillaria elegans*, Hofmann and Ryba, *Leitpflanzen*, p. 89, pl. xvi. fig. 10, 10*a*.
1899. *Sigillaria elegans*, Zeiller, *Flore foss. bassin houil. d'Héraclée*, p. 79, pl. vi. fig. 20.
1900. *Sigillaria elegans*, Zeiller, *Éléments d. paléobot.*, p. 192, fig. 134.
1901. *Sigillaria elegans*, Potonié, *in Engler-Prantl. Naturliche Pflanzenfam.*, p. 749, fig. 447.
1902. *Sigillaria elegans*, Kidston, *Proc. Yorks. Geog. and Polytech. Soc.*, vol. xiv. part iii. pp. 354, 385, pl. lviii. fig. 3.
1887. *Sigillaria elegans*, var. *regularis*, Weiss, *Sigillarien d. preuss. Steink.*, i. *Gruppe. Die Favularien*, p. 32, pl. x. figs. 38–39.
1887. *Sigillaria elegans*, var. *Brongniartiana*, Weiss, *ibid.*, p. 32, pl. x. figs. 40–50.
1887. *Sigillaria elegans*, var. *tenuimarginata*, Weiss, *ibid.*, p. 32, pl. x. figs. 41 and 43.
1887. *Sigillaria elegans*, var. *communis*, Weiss, *ibid.*, p. 32, pl. x. figs. 44, 45, 47, 48, pl. xi. fig. 60.
1887. *Sigillaria elegans*, var. *squamea*, Weiss, *ibid.*, p. 32, pl. x. figs. 46 and 49.
1836. *Sigillaria hexagona*, Brongt., *Hist. d. végét. foss.*, pl. clv. pl. clviii. fig. 1.
1857. *Sigillaria hexagona*, Goldenberg, *Flora saræp. foss.*, Heft ii., pl. vi. fig. 16.
1882. *Sigillaria hexagona*, Weiss, *Aus d. Steink.*, p. 5, pl. i. fig. 1 (zweiter abdr.).
1887. *Sigillaria hexagona*, Weiss (*copied from Bronginart*), *Sigillarien d. preuss. Steink.*, i. *Gruppe. Die Favularien*, p. 56 pl. xv. figs. 6, 6*a*, 7.
1888. *Sigillaria hexagona*, Schenk, *Die fossilen pflanzenreste*, p. 81, fig. 40².
1888. *Sigillaria hexagona*, Toula, *Die Steinkohlen*, p. 200, pl. iv. fig. 11.
1891. *Sigillaria hexagona*, Solms-Laubach, *Fossil Botany*, p. 243, fig. 26*b*.
1836. *Sigillaria minima*, Brongt., *Hist. d. végét. foss.*, p. 435, pl. clviii. fig. 2.
1857. *Sigillaria minima*, Goldenberg, *Flora saræp. foss.*, Heft ii. p. 26, pl. vi. fig. 15.
1870. *Sigillaria minima*, Schimper, *Atlas, Traité d. paléont. végét.*, p. 24, pl. lxviii. fig. 3.
1887. *Sigillaria minima*, Weiss (*copied from Brongt.*), *Sigillarien d. preuss. Steink.*, i. *Gruppe. Die Favularien*, p. 55, pl. xv. figs. 13, 13*a*.
1887. *Sigillaria minima*, Weiss (*copied from Schimper*), *ibid.*, p. 62, pl. xv. fig. 20.
1887. *Sigillaria elegantula*, Weiss, *Sigillarien d. preuss. Steink.*, i. *Gruppe. Die Favularien*, p. 44, pl. xiii. figs. 74–78.
1894. *Sigillaria elegantula*, Potonié, *Jahrb. d. k. preuss. geol. Landesanstalt für* 1893, pp. 31, 40, pl. iv. fig. 2.
1887. *Sigillaria elegantula*, var. *regularis*, Weiss, *ibid.*, p. 44, pl. xiii. fig. 74.
1887. *Sigillaria elegantula*, var. *subregularis*, Weiss, *ibid.*, p. 44, pl. xiii. figs. 75–76.
1887. *Sigillaria elegantula*, var. *imperfecta*, Weiss, *ibid.*, p. 44, pl. xiii. fig. 77.
1877. *Sigillaria elegantula*, var. *emarginata*, Weiss, *ibid.*, p. 44, pl. xiii. fig. 78.
1870. *Sigillaria tessellata*, Schimper (*non* Brongt.) (*pars*), *Traité d. paléont. végét.*, vol. ii. p. 81, pl. lxviii. fig. 3 (? pl. lxviii. fig. 1).
1872. *Favularia*, Williamson, *Phil. Trans.*, vol. clxxii. pp. 221, 235, pl. xxxi. fig. 58.

Remarks.—I possess specimens from the Middle Coal Measures of Yorkshire, collected by Mr W. Hemingway, which agree well with Brongniart's *Sigillaria minima*, but, except in the small size of the scars and their slightly more elongated form, they differ in no respect from *Sigillaria elegans*, of which I regard *Sigillaria minima* to be a young condition.

Sigillaria elegantula, Weiss, seems to me also to be only a form of *Sigillaria elegans*. The presence of the keels from the two lateral or basal angles of the leaf scar is not a constant character, which is one of the chief specific distinctions. Sometimes one keel only is present, and on a neighbouring leaf cushion they may be entirely absent. Even if the keels were always present, it seems too slight a character on which to found a species.

On a specimen of *Sigillaria elegantula* from Königsgrube, Aachen, received from the late Dr WEISS, few of the cushions show any trace of a keel descending from the lower angles of the leaf scar.

The specimen figured by POTONIÉ (*l.c.*) under the name of *Sigillaria elegantula* shows no sign of the keels, and seems to be a typical example of *Sigillaria elegans*.

Localities C, D. Not uncommon.

Pinakodendron, Weiss.

1893. *Pinakodendron*, Weiss, *Die Sigillarien d. preuss. Steinkohlen-und Rothliegenden Gebiete*, ii. Gruppe, *der Subsigillarien,—Abhandl. d. Köing. preuss. geol. Landesanstalt, Neue Folge*, Heft 2, p. 61.

Description.—Stems with small distant quincuncially placed leaf scars. Outer surface of cortex ornamented with very fine raised lines, which unite to form a fine net-like reticulation.

Remarks.—The genus is closely related to *Bothrodendron*, L. and H., but easily distinguished by the net-like reticulation of the outer surface of the cortex and the form of the leaf scars on the hitherto discovered species of the genus.

WEISS describes two species—*Pinakodendron musivum** and *Pinakodendron Ohmanni*.†

In *Pinakodendron musivum* the leaf scars are oval-upright and placed in a slight depression, which is surrounded on the lower margin by a slight ridge. The surface ornamentation both above and below the leaf scars is very fine and faint, and the space so occupied forms an elongated rhomboidal area, of which the centre is occupied by the leaf scar.

In *Pinakodendron Ohmanni* the leaf scar is of very peculiar form. It consists of an upper transversely elongated triangular area, apparently not always clearly defined, with two or three cicatricules placed immediately above a transverse ridge which separates this upper portion from the lower triangular shield-shaped part, which has a small circular depression at its basal extremity. I am inclined to regard the lower shield-shaped portion the leaf scar proper, and that the cicatricules above this are probably so-called ligule pits. Unless this be the explanation of the structure, the leaf scar is other-

* *l.c.*, p. 61, pl. iii. figs. 16, 16*a*.
† *l.c.*, p. 62, pl. iii. figs. 17, 17*a*, 18, 18*a*.

wise difficult to understand; and as pointing to this being the correct explanation, the upper shield is only present on the part shown at fig. 18*a*. The vascular impression is therefore not clearly seen on any of the specimens yet discovered.

The occurrence of the genus at Canonbie is an interesting addition to the British Palæozoic flora.

Pinakodendron Macconochiei, Kidston, n.sp.

(Plate I. figs. 9–11.)

Description.—Leaf scars distant, upright-oval, very small, about 2 mm. high and 1·50 mm. broad. Cortex ornamented with a very fine irregular meshwork formed of little ridges. Vascular cicatricules not visible.

Remarks.—The only specimen discovered is shown natural size on Pl. I. fig. 10, and a portion enlarged three times at fig. 9. A still further enlarged sketch of the ornamentation of the outer surface of the bark is given at fig. 11. The surface ornamentation becomes very fine immediately above and below the leaf scars.

Pinakodendron Macconochiei differs from both the species described by Weiss in the much more irrgular meshwork ornamentation of the cortex, and is further distinguished from *Pinakodendron musivum*, Weiss, by the absence of the surrounding elevated rim of the little semi-pit-like structure in which the leaf scar is placed, and from *Pinakodendron Ohmanni*, Weiss, by its oval leaf scar.

Locality D.

Stagmaria, Brongniart.

Stagmaria ficoides, Sternberg, sp.

Stagmaria ficoides, Sternb., sp. *Ante*, p. 757.

Locality D.—The ordinary form and another with closer and slightly small scars were collected.

Sphenophyllaceæ.

Sphenophyllum, Brongniart.

Sphenophyllum cuneifolium, Sternb., sp. *Ante*, p. 778.

Locality C.—*Sphenophyllum cuneifolium* and var. *saxifragæfolium*, were met with here.

Locality D.—Only *Sphenophyllum cuneifolium* was collected at this locality.

Cordaiteæ.

Cordaites, Unger.

Cordaites principalis, Germar, sp.

1848. *Flabellaria principalis*, Germar, *Vers. v. Wettin. u. Löbejun*, p. 55, pl. xxiii.
1869. *Flabellaria principalis*, Roehl, *Foss. Flora Steink.-Form. Westph.*, p. 163, pl. xx. figs. 1–2.
1855. *Cordaites principalis*, Geinitz (*pars*), *Vers. d. Steinkf. in Sachsen*, p. 41, pl. xxi. fig. 1, 2, 2*a*, 2*b*.
1864. *Cordaites principalis*, Göpp., *Foss. Flora. d. perm. Form.*, p. 159, pl. xxii. figs. 6–9.
1876. *Cordaites principalis*, Heer, *Flore foss. Helv.*, Lief. i. p. 55, pl. i. figs. 1*b*, 12–16.
1882. *Cordaites principalis*, Weiss, *Aus d. Steink.*, p. 19, pl. xx. fig. 114 (zweiter abdr.).
1883. *Cordaites principalis*, Schenk, *in Richthofen's China*, vol. iv. pp. 213, 228, pl. xxx. figs. 11–12, pl. xliv. figs. 3, 3*a*.
1886. *Cordaites principalis*, Sterzel, *Flora d. Rothl. in Nordw. Sachsen*, p. 32, pl. iii. figs. 6–9, pl. iv. figs. 1–3 (? figs. 4–5).
1886. *Cordaites principalis*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 629, pl. xciii. fig. 3, pl. xciv. fig. 1.
1893. *Cordaites principalis*, Potonié, *Flora d. Rothl. von Thüringen*, p. 210, pl. i. fig. 5.
1893. *Cordaites principalis*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxvii. p. 352, pl. ii. figs. 8, 8*a*, pl. iv. figs. 16–17.
1895. *Cordaites principalis*, Sterzel, *Flora d. Rothl. von Oppenau*, p. 308, pl. ix. figs. 6–8 (*Mitt. d. Badischen Geol. Landesanstalt*, iii. Band, 2 Heft).
1899. *Cordaites principalis*, Hofmann and Ryba, *Leitpflanzen*, p. 99, pl. xix. fig. 6, 6*a*.
1902. *Cordaites principalis*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. part 3, pp. 363, 375, 383, 397, pl. liii. fig. 1, pl. lvii. fig. 2, pl. lxiv. fig. 3.
1870. *Pycnophyllum principale*, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 191.
1833. *Knorria taxina*, L. and H., *Fossil Flora*, vol. ii. pl. xcv.A (*stem*).

Localities C, D. Frequent.

Cordaianthus, Grand' Eury.

(?) Cordaianthus Pitcairniæ, L. and H., sp.

Cordaianthus Pitcairniæ, L. and H., sp. See *ante*, p. 782.

Locality C.

Cordaianthus Volkmanni, Ettingshausen, sp.

1852. *Calamites Volkmanni*, Ett. (*pars*), *Steinkf. v. Stradonitz*, p. 5, pl. v. figs. 1–3 (*non* fig. 4, *non* pl. vi. figs. 1–2).
1886. *Cordaianthus Volkmanni*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 637, pl. xciv. fig. 6.
1874. *Antholithus parviflorus*, Schimper, *Traité d. paléont. végét.*, vol. iii. p. 567, pl. cx. figs. 1–3.

Locality D.

Cordaianthus, sp.

Locality D.

Cordaicarpus, Geinitz.

Cordaicarpus Cordai, Geinitz, sp.

(Plate I. figs. 12-13.)

1855. *Carpolithus Cordai*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 41, pl. xxi. figs. 7–16.
1876. *Carpolithus Cordai*, Boulay, *Terr. houil. du nord de la France*, p. 50, pl. i. fig. 4.
1861. *Cordaicarpon Cordai*, Geinitz, *Dyas.*, p. 150.
1886. *Cordaicarpus Cordai*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 645, pl. xciv. fig. 13.
1871. *Cyclocarpus Cordai*, Weiss, *Foss. Flore d. jüngst. Stk. u. Rothl.*, p. 207.
1886. *Cyclocarpus Cordai*, Sterzel, *Flora d. Rothl. im. nordw. Sachsen*, p. 68 (? pl. ix. fig. 4).
1899. *Cyclocarpus Cordai*, Hofmann and Ryba, *Leitpflanzen*, p. 101, pl. xix. figs. 15–18.

Description.—Seed smooth, lenticular in section, with a more or less circular contour, very feebly apiculate at the summit, and sometimes slightly cordate at the base, generally surrounded by a narrow keel or border. They vary in size from 7 mm. to 20 mm.

Remarks.—The narrow surrounding keel or border probably results from pressure, and may represent the pericarp by which the harder nucule was surrounded. Only a single specimen was met with at Canonbie, but the seed occurs at other localities in the Middle Coal Measures of England.

At Plate I. fig. 12 the seed is shown natural size, and at fig. 13 it is enlarged three times.

Locality C.

Carpolithes, Schlotheim.

Carpolithes, sp.

Locality D.

Artisia, Sternberg.

Artisia, sp.

Locality D.

The following table shows the vertical distribution of the fossil plants from the Middle Coal Measures of the Byre Burn series.

	U. C. M.	M. C. M.	L. C. M.
Sphenopteris obtusiloba, Brongt.,		×	×
,, *Laurenti*, Andræ,		×	×
,, *mixta*, Schimper,		×	
,, *Hæninghausi*, Brongt.,		×	×
,, *multifida*, L. and H.,		×	
Pecopteris, sp.,			
Mariopteris muricata, Schloth., sp.,	×	×	×
Alethopteris lonchitica, Schloth., sp.,	×	×	×
,, *Davreuxi*, Brongt., sp.,	×	×	
,, *Grandini*, Brongt., sp.,	×	×	
Neuropteris heterophylla, Brongt.,		×	×
,, *gigantea*, Sternb.,		×	×
Calamites undulatus, Sternb.,	×	×	×
,, *Schützei*, Stur,		×	×
,, *pauciramis*, Weiss,		× *	
,, *ramosus*, Artis,	×	×	×
,, *Suckowii*, Brongt.,	×	×	×
,, *Cistii*, Brongt.,	×	×	×
Calamocladus equisetiformis, Schloth., sp.,	×	×	×
,, *charæformis*, Sternb., sp.,		×	
Palæostachya Ettingshauseni, Kidston., n.sp.,		×	×
Cf. *Paracalamostachys Williamsoniana*, Weiss,		?	×
Annularia radiata, Brongt., see *Calamites romosus*, Artis,		×	×
Lepidodendron, cf. *lycopodioides*, Sternb.,		×	×
Sigillaria elegans, Sternb., sp.,		×	×
Pinakodendron Macconochiei, Kidston, n.sp.,		× *	
Stigmaria ficoides, Sternb., sp.,	×	×	×
Sphenophyllum cuneifolium, Sternb., sp.,		×	×
,, ,, var. *saxifragæfolium*, Stbg. (sp.),		×	×
Cordaites principalis, Germar, sp.,		×	×
Cordaianthus, cf. *Pitcairniæ*, L. and H., sp.,		×	×
,, *Volkmanni*, Ett., sp.,		×	
Cordaicarpus Cordai, Geinitz, sp.,		×	
Carpolithes, sp.,			
Sternbergia, sp.,			

Of the Byre Burn group of fossil plants a few extend into the Upper Coal Measures of England, but these belong to those species which are common to all three divisions of the Coal Measures, with the exception of *Alethopteris Davreuxi*, Brongt., sp., and *Alethopteris Grandini*, Brongt., sp., which were fairly plentiful at Localities C and D, which, however, as already mentioned, are different portions of the same bed. I have not previously met with *Alethopteris Grandini*, Brongt., sp., below the Upper Coal Measures.

A greater number of the Byre Burn plants also occur in the Lower Coal Measures, but after deducting these there are several left which are peculiar to the Middle Coal Measures, as far as at present known. These are *Sphenopteris mixta*, Schimper, *Sphenopteris multifida*, L. and H., *Calamites pauciramis*, Weiss, *Calamocladus charæformis*,

* These are the first British records, but their *horizon* on the Continent corresponds to our Middle Coal Measures.

Sternb., sp., *Pinakodendron Macconochi*, Kidston, n.sp., *Cordaianthus Volkmanni*, Ett., sp., and *Cordaicarpus Cordai*, Geinitz, sp. Though *Sphenopteris Laurenti*, Andræ, and *Sigillaria elegans*, Sternb., sp., also occur in the Lower Coal Measures, they are very rare in that horizon, and are much more characteristic of the Middle Coal Measures.

From the evidence afforded by the fossil plants, I have no hesitation in classing the Byre Burn group with the Middle Coal Measures of Britain.

Fossil Plants of the Upper Coal Measures. (*The Red Shales.*)

These occupy a considerable tract of ground, and are well exposed in many places in the parish of Canonbie and in the neighbouring part of Cumberland.

The series as a whole is extremely barren of fossils, though it offers many good sections along the river Esk and the Liddel Water; and though these sections were more or less carefully examined by Mr Macconochie and myself, they did not yield a single fossil plant.

At Jockie's Syke, in Cumberland, one mile east by north of Riddings Junction, where in 1879 Mr Macconochie found a few small specimens in this series, we were successful in discovering plant remains in three different bands.

The section here is, however, high up in the series, and for this portion of it the fossils indicate an Upper Coal Measure age.

It is very unfortunate that the lower portion of the series along the river Esk and Liddel Water is so barren (though plants may still be found in it, for the extent of ground to examine is considerable); for if the rocks form a continuous series with the Byre Burn Middle Coal Measures (though there may possibly be a fault between them), one would expect to find in the lower portion of the 'Red Shales' a transition flora composed partly of Middle and partly of Upper Coal Measure plants, similar to the series which occurs above the Middle Coal Measures in the Potteries Coalfield, and to which I have applied the name of the *Transition series.**

From the absence of any plant remains in the *lower beds* of these 'Red Shales,' I am unable to determine whether they belong to this Upper Transition series or to the Upper Coal Measures.

The flora of the upper beds, however, as developed in Jockie's Syke, have a distinct Upper Coal Measure facies; and though one misses from the list of plants collected there several of the *Pecopteris-Cyatheites* group, some do occur, but their preservation is so unsatisfactory that it is impossible to determine them specifically, as the nervation is seldom shown.

It is sometimes difficult to draw the exact line between the Upper Transition series

* Kidston, "On the various Divisions of British Carboniferous Rocks as determined by their Fossil Flora," *Proc. Roy. Phys. Soc. Edin.*, vol. xii. pp. 228–229, 1894. Also "Additional Records and Notes on the Fossil Flora of the Potteries Coalfield, North Staffordshire," *Trans. N. Staffordshire Field Club*, 1897, Stoke-upon-Trent.

and the Upper Coal Measures, for they are only the upper and lower ends of a continuous chain, but taking all the available evidence into consideration, we must class that portion of the 'Red Shales' which are met with at Jockie's Syke with the Upper Coal Measures.

The following are the localities from which the plants were collected in Jockie's Syke, one mile east by north of Riddings Junction, Cumberland :—

A. Jockie's Syke. The original locality discovered by Mr MACCONOCHIE in 1879, the exact position of which cannot now be determined.

B. Jockie's Syke. Streamlet entering on right of main stream, 30 yards south of North British Railway. In soft red and greenish shales.

C. Jockie's Syke. Main stream in Syke, about 125 yards above railway. In purplish shale.

D. Jockie's Syke. Main stream of Syke, about 300 yards above railway. In purplish clayey shale.

Filicaceæ.

Pecopteris, Brongniart.

(?) Pecopteris arborescens, Schlotheim, sp.

1804. Schlotheim, *Flora d. Vorwelt*, p.41, pl. viii. fig. 13.
1820. *Filicites arborescens*, Schloth., *Petrefactenkunde*, p. 404.
1826. *Pecopteris arborea*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xviii.
1828. *Pecopteris arborescens*, Brongt., *Prodrome*, p. 56.
1833. *Pecopteris arborescens*, Brongt., *Hist. d. végét. foss.*, p. 310, pl. cii. figs. 1–2, pl. ciii. figs. 2–3.
1838. *Pecopteris arborescens*, Sternb. (*pars*), *Essai flore monde prim.*, vol. ii. fasc. 7–8, p. 147.
1848. *Pecopteris arborescens*, Gutbier, *Vers. d. Rothl. in Sachsen*, p. 16, pl. ii. fig. 9.
1851. *Pecopteris arborescens*, Germar, *Vers. v. Weittin u. Löbejun*, p. 97, pl. xxxiv. figs. 1–3, pl. xxxv. figs. 5–7 (? fig. 4).
1865. *Pecopteris arborescens*, Heer, *Urwelt d. Schweiz*, p. 13, pl. i. fig. 8.
1876. *Pecopteris arborescens*, Roemer, *Lethæa geog.*, vol. i. p. 176, pl. lviii. fig. 3 (? pl. lii. fig. 4).
1877. *Pecopteris arborescens*, Grand' Eury, *Flore carbon. d. Départ. de la Loire*, p. 68, pl. viii. fig. 6.
1879. *Pecopteris arborescens*, Schimper, *in Zittel, Handb. d. Palæont.* ii. Abth., *Palæophyt.*, p. 127, fig. 103.
1880. *Pecopteris arborescens*, Zeiller, *Végét. foss. d. terr. houil.*, p. 81, pl. clxix. fig. 4.
1879. *Pecopteris arborescens*, Lesqx., *Coal Flora*, vol. i. p. 230, pl. xli. figs. 6–7.
1883. *Pecopteris arborescens*, Renault, *Cours d. botan. foss.*, vol. iii. p. 108, pl. xvii. figs. 1–2.
1888. *Pecopteris* (*Asterotheca*) *arborescens*, Zeiller, *Flore foss. bassin houil. d. Commentry*, p. iii. pl. xi. figs. 1–2.
1890. *Pecopteris arborescens*, Grand' Eury, *Bassin houil. du Gard*, p. 274, fig. D.
1890. *Pecopteris* (*Asterotheca*) *arborescens*, Zeiller, *Flore foss. bassin houil. et perm. d'Autun et d'Épinac*, i. part, p. 43, pl. viii. fig. 1.
1893. *Pecopteris* (*Scolecopteris*) *arborescens*, Sterzel, *Flora d. Rothl. im Plauen. Grunde bei Dresden*, p. 17, pl. i. figs. 16–17.
1893. *Pecopteris arborescens*, Potonié (*pars*), *Flora d. Rothl. v. Thüringen*, p. 57, pl. vi. figs. 6–7 (*non* fig. 5).
1899. *Pecopteris arborescens*, Hofmann and Ryba, *Leitpflanzen*, p. 50, pl. vi. figs. 6–8.
1901. *Pecopteris arborescens*, Kidston, *Proc. Yorks. Geol. and Polytech. Soc.*, vol. xiv. part ii. pp. 194, 209, pl. xxvii. fig. 3.

1836. *Cyatheites arborescens*, Göpp., *Syst. fil. foss.*, p. 321.
1876. *Cyatheites arborescens*, Feistmantel, *Vers. d. böhm. Ablager.*, iii. Abth., p. 70, pl. xviii. figs. 6, 6*a*.
1876. *Cyatheites arborescens*, Heer, *Foss. Flora Helv.*, p. 27, pl. viii. figs. 1–4.
1883. *Cyatheites arborescens*, Schenk, *in Richthofen's China*, vol. iv. p. 212, pl. xlv. figs. 14–16.
1873. *Cyatheites arborescens*, Feistmantel, *Zeitsch. d. d. deut. geol. Gesell.*, vol. xxv. p. 600, pl. xviii. figs. 15, 15*a*.
1869. *Cyathocarpus arborescens*, Weiss, *Foss. Flora d. jüngst. Stk. u. Rothl.*, p. 84.
1877. *Asterotheca arborescens*, Stur, *Culm Flora*, Heft ii. p. 293.
1883. *Scolecopteris arborescens*, Stur, *Morph. u. Syst. d. Culm u. Carbon Farne*, p. 102, fig. 20*a*, p. 122.
1885. *Scolecopteris arborescens*, Stur, *Carbon-Flora d. Schatz. Schichten, Die Farne*, p. 196, fig. 24*a*.
1861. *Cyatheites Schlotheimi*, Göpp. (*pars*), *Foss. Flora d. perm. Form.*, p. 120, pl. xv. fig. 1.

Remarks.—The specimens included here agree entirely with *Pecopteris arborescens* in size and form of the pinnules, but as the nervation is not clearly shown, the record is marked with a 'querry.'

Locality A.

Pecopteris (Cyatheites), sp.

Remarks.—Possibly more than one species is included here, but as the specimens are fragmentary and the nervation not shown, it is impossible to determine them specifically with any degree of certainty. It is rather remarkable that among the Pecopteris none of the specimens showed the nervation, whereas in some of the other ferns found with them the nervation was clearly seen.

Localities B, D.

Alethopteris, Sternberg.

Alethopteris aquilina, Schlotheim, sp.

1804. Schlotheim, *Flora d. Vorwelt*, p. 34, pl. iv. fig. 7, pl. v. fig. 8.
1820. *Filicites aquilinus*, Schloth., *Petrefactenkunde*, p. 405.
1828. *Pecopteris aquilina*, Brongt., *Prodrome*, p. 56.
1836. *Alethopteris aquilina*, Göpp., *Syst. fil. foss.*, p. 298.
1826. *Pecopteris affinis*, Sternb., *Essai flore monde prim.*, vol. i. fasc. iv. p. xx.
1828. *Pecopteris Schlotheimii*, Brongt., *Prodrome*, p. 57.

Remarks.—The examples from Jockie's Syke agree entirely with SCHLOTHEIM'S figures. I will not at present enter into the question of the identity or otherwise of BRONGNIART'S *Pecopteris aquilina* * with SCHLOTHEIM'S plant, as I hope at another time to publish some notes on this subject.

Localities B, C.

Alethopteris Grandini, Brongniart, sp.

Alethopteris Grandini, Brongt. See *ante*, p. 787.

Locality B.

* *Hist. d. végét. foss.*, p. 284, pl. xc.

Alethopteris Serlii, Brongniart, sp.

1804. Parkinson, *Organic Remains*, vol. i. pl. iv. fig. 6
1828. *Pecopteris Serlii*, Brongt., *Prodrome*, p. 57.
1832 or 1833. *Pecopteris Serlii*, Brongt., *Hist. d. végét. foss.*, p. 292, pl. lxxxv.
1837. *Pecopteris Serlii*, L. and H., *Fossil Flora*, vol. iii., pl. ccii.
1836. *Alethopteris Serlii*, Göpp., *Syst. fil. foss.*, p. 301, pl. xxi. figs. 6–7.
1862. *Alethopteris Serlii*, Roemer, *Palæont.*, vol. ix. p. 32, pl. viii. fig. 9.
1876. *Alethopteris Serlii*, Roemer, *Lethæa geog.*, vol. i. p. 181, pl. lii. figs. 2*a*, 2*b*.
1879. *Alethopteris Serlii*, Lesqx., *Coal Flora*, vol. i. p. 176, pl. xxix. figs. 1–5.
1880. *Alethopteris Serlii*, Zeiller, *Végét. foss. d. terr. houil.*, p. 75, pl. clxiii. figs. 1–2.
1882. *Alethopteris Serlii*, Weiss, *Aus d. Steink.*, p. 16, pl. xvi. fig. 97 (zweiter abdr.).
1883. *Alethopteris Serlii*, Renault, *Cours d. botan. foss.*, vol. iii. p. 157, pl. xxvii. fig. 7.
1886. *Alethopteris Serlii*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 234, pl. xxxvi. figs. 1–2, pl. xxxvii. figs. 1–2.
1888. *Alethopteris Serlii*, Toula, *Die Steinkohlen*, p. 189, pl. i. figs. 31–32.
1899. *Alethopteris Serlii*, White, *Fossil Flora Lower Coal Meas. of Missouri*, p. 117, pl. xxxvii. fig. 1.
1899. *Alethopteris Serlii*, Hofmann and Ryba, *Leitpflanzen*, p. 56, pl. viii. figs. 2, 3.
1899. *Alethopteris Serlii*, var. *Missouriensis*, White, *Foss. Flora Lower Coal Meas. of Missouri*, p. 118, pl. xxxvii. fig. 2, pl. xlii. fig. 5.
1848. *Pecopteris Hannonica*, Sauveur, *Végét. foss. terr. houil. Belgique*, pl. xxxviii.
1854. *Alethopteris Sternbergii*, Ettingshausen, *Steink. v. Radnitz*, p. 42, pl. xviii. fig. 4.
1869. (?) *Alethopteris irregularis*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 81, pl. xv. figs. 2, 14 and 15.
1879. *Alethopteris lonchitica*, Schimper (*non* Schloth.), *in Zittel, Handb. d. Palæont.*, ii. Abth., *Palæophyt.*, p. 118, fig. 93[1].

Localities B, C.

Alethopteris, sp.

Locality D.

Neuropteris, Brongniart.

Neuropteris ovata, Hoffmann.

1826. *Neuropteris ovata*, Hoffmann, *in Keferstein, Teuchland geognostisch-geolog. dargestellt.*, vol. iv. p. 158, pl. 1*b*, figs. 5, 6, 7 (*non* fig. 8).
1887. *Neuropteris ovata*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 359, pl. xxii. fig. 1, 1*a*.

Remarks.—The fern figured by Roemer under this name appears to be specifically distinct from Hoffmann's species.* The *Neuropteris ovata*, Germar,† is the *Callipteridium pteridum*, Schlotheim, sp.‡

Locality B.

Neuropteris flexuosa, Sternberg.

1824. *Neuropteris gigantea*, var. *β* Sternb., *Essai flore monde prim.*, vol. i. fasc. 3, p. 44, pl. xxxii. fig. 2.

* *Beitr. z. geol. Kenntniss d. nordwestl. Harzgebirges*, iv. Abth., *Palæont.*, vol. ix. p. 28, pl. vi. fig. 1.
† *Vers. von Wettin u. Löbejun*, p. 33, pl. xii., 1845.
‡ *Flora d. Vorwelt*, p. 59, pl. xiv. fig. 27, *Petrefactenkunde*, p. 406.

1826. *Neuropteris flexuosa*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xvi; vol. ii. fasc. 5–6, p. 71.
1828. *Neuropteris flexuosa*, Brongt., *Prodrome*, p. 53.
1830. *Neuropteris flexuosa*, Brongt., *Hist. d. végét. foss.*, p. 239, pl. lxv. figs. 2–3, pl. lxviii. fig. 2.
1835. *Neuropteris flexuosa*, Gutbier, *Vers. d. Zwick. Schwarzk.*, p. 56, pl. vii. figs. 1, 2, 5, 7 (? pl. vi. fig. 12, pl. vii. figs. 10–13).
1869. *Neuropteris flexuosa*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 35, pl. xii. fig. 3*a*, pl. xv. figs. 3 and 10 (? pl. iv. fig. 1*b*).
1869. *Neuropteris flexuosa*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 434, pl. xxx. figs. 12–13.
1870. *Neuropteris flexuosa*, Unger, *Sitzungsb. d. Math. Naturw. Classe*, vol. lx. i. Abth. p. 785 (pl. ii. figs. 1–2 ?).
1876. *Neuropteris flexuosa*, Feistmantel, *Vers. d. böhm. Ablager.*, Abth. iii. p. 64, pl. xvi. figs. 5–6.
1876. *Neuropteris flexuosa*, Heer (*pars*), *Foss. Flora Helv.*, Lief. i. p. 20, pl. iv. figs. 7–13, pl. v. fig. 3.
1876. *Neuropteris flexuosa*, Roemer, *Lethæa geog.*, vol. i. p. 183, pl. li. fig. 5*a*, 5*b*.
1880. *Neuropteris flexuosa*, Rothpletz, *Steinkohlf. an der Ost des Todi*, p. 5, pl. i. figs. 8–9.
1882. *Neuropteris flexuosa*, Weiss, *Aus d. Steink.*, p. 15, pl. xv. fig. 90 (zweiter abdr.).
1883. *Neuropteris flexuosa*, Renault, *Cours d botan foss.*, vol. iii. p. 169, pl. xxix. figs. 10–11.
1886. *Neuropteris flexuosa*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 277 (? pl. xlvi. fig. 2).
1888. *Neuropteris flexuosa*, Toula, *Die Steinkohlen*, p. 189, pl. i. figs. 34–35.
1830. *Neuropteris rotundifolia*, Brongt., *Hist. d. végét. foss.*, p. 238, pl. lxx. fig. 1.
1835. *Neuropteris rotundifolia*, Gutbier, *Vers. d. Zwick. Schwarzk.*, p. 56, pl. vii. figs. 3–4.
1879. *Neuropteris rotundifolia*, ? Lesquereux, *Coal Flora*, vol. i. p. 97, pl. xiii. fig. 8.
1888. *Neuropteris flexuosa*, var. *rotundifolia*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. part. ii. p. 359.
1880. *Neuropteris flexuosa*, var. *Grangeri*, Rothpletz (*non* Brongt.), *Steinkohlf. an der Ost des Todi*, p. 5, pl. i. fig. 7.

Localities A, B, C, D.

Neuropteris Scheuchzeri, Hoffmann.

1826. *Neuropteris Scheuchzeri*, Hoffm., *in Keferstein's Teuchland geognostisch geol. dargestellt*, vol. iv. p. 156, pl. i.*b* figs. 1–4.
1882. *Neuropteris Scheuchzeri*, Zeiller, *Flore houil. d. Asturies*, p. 6 (*Mém. Soc. Geol. du Nord*).
1886. *Neuropteris Scheuchzeri*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p. 251, pl. xli. figs. 1–3.
1888. *Neuropteris Scheuchzeri*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 356, pl. xxiii. figs. 1–2.
1899. *Neuropteris Scheuchzeri*, Zeiller, *Flore foss. d. bassin houil. d'Héraclée*, p. 42, pl. iv. fig. 9.
1899. *Neuropteris Scheuchzeri*, White, *Foss. Flora Lower Coal Meas. of Missouri*, p. 132, pl. xlii. fig. 3, pl. lxiv. fig. *d*.
1830. *Neuropteris angustifolia*, Brongt., *Hist. d. végét foss.*, p. 231, pl. lxiv. figs. 3–4.
1883. *Neuropteris angustifolia*, Lesqx., *Indiana Dept. of Geol. and Nat. Hist. 13th Ann. Rept.*, part ii., *Palæont.*, p. 52, pl. x. fig. 1.
1830. *Neuropteris acutifolia*, Brongt., *Hist. d. végét. foss.*, p. 231, pl. lxiv. figs. 6–7.
1835. *Neuropteris acutifolia*, Gutbier, *Vers. d. Zwick. Schwarzk.*, p. 52 (? pl. vii. fig. 6).
1854. *Neuropteris acutifolia*, Ett., *Foss. Flora v. Radnitz*, p. 32, pl. xviii. fig. 5.
1855. *Neuropteris acutifolia*, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 22 (? pl. xxvii. fig. 8).
1899. *Neuropteris acutifolia*, Hofmann and Ryba, *Leitpflanzen*, p. 65, pl. ix. fig. 11 (? figs. 1–2).
1847. *Neuropteris cordata*, var. *angustifolia*, Bunbury, *Quart. Journ. Geol. Soc.*, vol. iii. p. 424, pl. xxi. fig. 1*b*.
1832. *Neuropteris cordata*, L. and H. (*non* Brongt.), *Fossil Flora*, vol. i. pl. xli.
1847. *Neuropteris cordata*, Bunbury (*non* Brongt.), *Quart. Journ. Geol. Soc.*, vol. iii. pp. 423 and 437, pl. xxi. figs. 1*a*, 1*c*, 1*d*, 1*e*, 1*f*.

1868. *Neuropteris cordata*, Dawson (*non* Brongt.), *Acad. Geol.*, 2nd ed. pp. 482, 466, fig. 166*b*.
1888. *Neuropteris cordata*, Kidston (*non* Brongt.) (*pars*), *Catal. Palæoz. Plants*, p. 98.
1858. *Neuropteris hirsuta*, Lesqx., *in Rogers*, *Geol. of Pennsyl.*, vol. ii. p. 857, pl. iii. fig. 6, pl. iv. figs. 1–16.
1866. *Neuropteris hirsuta*, Lesqx., *Rept. Geol. Survey of Illin.*, vol. ii. p. 427, pl. xxxv. figs. 6–10.
1879. *Neuropteris hirsuta*, Lesqx., *Coal Flora*, vol. i. p. 88, pl. viii. figs. 1, 4, 5, 7, 9, 12.
1880. *Neuropteris hirsuta*, White, *Indiana 2nd Ann. Rept. Dept. Statistics and Geol.*, p. 520, pl. ix. figs. 1–3.
1862. *Dictyopteris cordata*, Roemer, *Palæont.*, vol. ix. p. 30, pl. vi. fig. 4.
1869. *Dictyopteris cordata*, Roehl, *Foss. Flora Steink.-Form. Westph.*, p. 50 (? pl. xv. fig. 6, pl. xxi. fig. 7*b*).
1862. *Dictyopteris Scheuchzeri*, Roemer, *Palæont.*, vol. ix. p. 30, pl. ix. fig. 1.
1869. *Dictyopteris Scheuchzeri*, Roehl, *Foss. Flora Steink.-Form. Westph.*, p. 49 (? pl. xxi. fig. 12).

Note.—*Neuropteris Scheuchzeri*, Hoffm., was the species most plentifully met with at Jockie's Syke.

Localities B, D.

Equisetaceæ.

Calamites, Suckow.

Calamites (Calamitina) undulata, Sternberg.

Calamites (Calamitina) undulata, Sternb. See *ante*, p. 776.

Locality B.

Calamites (Calamitina), sp.

Locality B.

Calamocladus, Schimper.

Calamocladus equisetiformis, Schlotheim, sp.

Calamocladus equisetiformis, Schloth., sp. See *ante*, p. 792.

Locality B.

Annularia, Sternberg.

Annularia radiata, Brongniart.

See **Calamites (Eucalamites) ramosus**, *ante*, p. 790.

Locality B. Frequent, but mostly confined to a single layer.

Annularia stellata, Schlotheim, sp.

1723. Scheuchzer, *Herb. diluv.*, pl. xiii. fig. 3.
1760. Luid, *Lith. Brit. Ichnographia*, p. 12, pl. v. fig. 201.
1804. Schlotheim, *Flora d. Vorwelt*, p. 32, pl. i. fig. 4.
1820. *Casuarinites stellatus*, Schloth., *Petrefactenkunde*, p. 397.
1826. *Bornia stellata*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. xxviii.

1860. *Annularia stellata*, Wood, *Proc. Acad. Nat. Sciences Phil.*, p. 236.
1880. *Annularia stellata*, Zeiller, *Végét. foss. d. terr. houil.*, p. 26, pl. clx. figs. 2–3.
1886. *Annularia stellata*, Zeiller, *Flore foss. bassin houil. d. Valen.*, p 398, pl. lxi. figs. 3–6.
1887. *Annularia stellata*, Stur, *Calamarien d Carbon-Flora d. Schatz. Schichten*, p. 55, pl. xiii.*b* fig. 3, pl. xiii*b*. *bis*, fig. 3.
1889. *Annularia stellata*, Renault (*pars*), *Flore foss. terr. houil. d. Commentry*, part ii. p. 398, pl. xlv. figs. (? 1) 2–7, pl. xlvi. figs. 1–6 (*non* pl. xlvii. figs. 1–2).
1892. *Annularia stellata*, Potonié, *Naturwiss. Wochenschrift*, No. 51, p. 520, figs. 1–2.
1893. *Annularia stellata*, Potonié, *Flora d. Rothl. v. Thüringen*, p. 162, pl. xxiv. figs. 1–6.
1896. *Annularia stellata*, Renault, *Bassin houil. et perm. d'Autun et d'Épinac.*, *fasc.* iv. *Flore foss. Deux. part.*, p. 67, pl. xxviii. figs. 1, 3, 4.
1898. *Annularia stellata*, Seward, *Fossil Plants*, p. 338, fig. 88.
1899. *Annularia stellata*, Frech, *Lethæa geog.*, i. Theil, *Lethœa palœoz.*, 2 Band, 2 Lief, *Steinkohlenform.*, pl. l.*b*. fig. 1.
1899. *Annularia stellata*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 200, fig. 195.
1899. *Annularia stellata*, White, *Foss. Flora of Lower Coal Meas. of Missouri*, p. 159, pl. xxiv. fig. 3*b*.
1828. *Annularia longifolia*, Brongt., *Prodrome*, p. 156.
1845. *Annularia longifolia*, Germar, *Vers. v. Wettin u. Löbejun*, p. 25, pl. ix. figs. 1–4.
1852. *Annularia longifolia*, Ettingshausen, *Stenkf. v. Stradonitz*, p. 8, pl. i. fig. 4 (*Abhandl. d. k. k. Geol. Reichs, Wien*, vol. i. Abth. 3, No. 4).
1855. *Annularia longifolia*, Geinitz, *Vers. d. Stenkf. in. Sachsen*, p. 10, pl. xviii. figs. 8–9 ; pl. xix. figs. 3–5 (? figs. 1–2).
1865. *Annularia longifolia*, Heer, *Urwelt d. Schweiz.*, p. 9, fig. 7.
1869. *Annularia longifolia*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 348, pl. xxvi. figs. 2–4 (? pl. xxii. fig. 5 (*non* figs. 6–10).
1869. *Annularia longifolia*, Roehl, *Foss. Flora d. Steink.-Form. Westph.*, p. 28, pl. iv. fig. 6 (? fig. 15).
1869. *Annularia longifolia*, Unger, *Anthracit-Lager in Kärnthen.*, p. 783, pl. i. fig. 9 (*Sitzb. d. k. Akad. d. Wissensch. Math. Naturw. cl.*, vol. lx., i. Abth., Wien).
1873. *Annularia longifolia*, Renault, *Ann. d. Sciences Nat.*, 5e sér., *Bot.*, vol. xviii. pp. 14 and 20, pls. xix.–xxiii.
1874. *Annularia longifolia*, Feistmantel, *Vers. d. böhm. Ablag.*, p. 127, pl. xv. figs. 3–4, pl. xvi. fig. 1.
1876. *Annularia longifolia*, Heer, *Flora foss. Helv.*, p. 51, pl. xix. figs. 4–5.
1876. *Annularia longifolia*, Roemer, *Lethœa geog.*, vol. i. p. 150, pl. l. fig. 8 (? fig. 9).
1878. *Annularia longifolia*, Renault, *Rech. sur la struc. et les affin. botan. d. végét. silicifiés*, p. 31, pls. i.–ii.
1879. *Annularia longifolia*, Lesqx., *Coal Flora*, vol. i. p. 45, pl. ii. figs. 1–2 (? pl. iii. fig. 10, *non* fig. 12).
1880. *Annularia longifolia*, Schimper, *in Zittel, Handbk. d. Palæont.*, ii. Abth., p. 167, p. 166, fig. 126.
1880. *Annularia longifolia*, C. A. White, *State of Indiana 2nd Ann. Rept. Statistics and Geol.*, p. 521, pl. xi. fig. 1 (? fig. 2).
1882. *Annularia longifolia*, Renault, *Cours d. botan. foss.*, vol. ii. p. 126, pl. xx. fig. 1, pl. xxi. figs. 1–6.
1882. *Annularia longifolia*, Weiss, *Aus d. Steink.*, p. 11, pl. ix. fig. 49 (zweiter abdr.).
1883. *Annularia longifolia*, Schenk, *in Richthofen's China*, pp. 231–233, pl. xxxiv. figs. 4, 6, 7, (*non* fig. 5), pl. xxxv. figs. 7, 7*a*, pl. xxxvi. figs. 1–4, pl. xxxix., pl. xli. fig. 6.
1883. *Annularia longifolia*, Lesqx., *Indiana Dept. of Geol. and Nat. Hist. 13th Ann. Rept.*, part ii., *Palœont.*, p. 44, pl. vii. figs. 1–2.
1899. *Annularia longifolia*, Hofmann and Ryba, *Leitpflanzen*, p. 28, pl. ii. fig. 9.
1886. *Annularia longifolia*, var. *stellata*, Sterzel, *Flora d. Rothl. im Nordw. Sachs.*, p. 58, pl. viii. fig. 3.
1823. *Annularia spinulosa*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 2, p. 36, pl. xix. fig. 4 ; fasc. 4, p. xxxi.
1826. *Annularia fertilis*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, pp. 47 and xxxi., pl. 51, fig. 2.
1852. *Annularia fertilis*, Bronn, *Lethæa geog.*, vol. i. part 2, p. 105, pl. viii. fig. 8.

1883. *Annularia mucronata*, Schenk, *in Richthofen's China*, vol. iv. p. 226, fig. 10, pl. xxx. fig. 10.
1887. *Annulara Geinitzii*, Stur, *Die Calamarien d. Carbon-Flora d. Schatz. Schichten*, pp. 52 and 215, pl. xvi.*b* figs. 1–3.
1888. *Annularia Geinitzii*, Toula, *Die Steinkohlen*, p. 204, pl. v. fig. 14.
1887. *Asterophyllites Westphalicus et Annularia Westphalica*, Stur, *Die Calamarien d. Carbon-Flora d. Schatz. Schichten*, p. 213, pl. xiii.*b*, fig. 2 (*a*), pl. iv.*b* fig. 4.
1834. *Asterophyllites equisetiformis*, L. and H. (*non* Schlotheim, sp.), *Fossil Flora*, vol. ii. pl. cxxiv.

Fructification :—

1709. Scheuchzer, *Herb. diluv.*, pl. ii. fig. 6.
1826. *Bruckmannia tuberculata*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, pp. 45 and xxix., pl. xlv. fig. 2.
1874. *Bruckmannia tuberculata*, Feistmantel, *Vers. d. böhm. Ablager.*, Abth. i., p. 128 (? pl. xvi. fig. 1) (*non* pl. xvi. figs. 2–3).
1877. *Bruckmannia tuberculata*, Grand' Eury, *Flore carbon. d. Départ. de la Loire*, pp. 44–45, pl. vi. fig. 4, 4′.
1828. *Asterophyllites tuberculata*, Brongt., *Prodrome*, p. 159.
1876. *Stachannularia tuberculata*, Weiss, *Steinkohlen Calamarien*, Heft i. p. 17, pl. i. figs. 2–5, pl. ii. figs. 1–3, 5 (*left-hand figure*), pl. iii. figs. 3–10, 12.
1882. *Stachannularia tuberculata*, Weiss, *Aus d. Steink.*, p. 11, pl. ix. fig. 50 (zweiter abdr.).
1884. *Calamostachys tuberculata*, Weiss, *Steinkohlen Calamarien*, Heft ii. p. 178.
1899. *Calamostachys* (*Stachannularia*) *tuberculata*, Hofmann and Ryba (*pars*), p. 30, pl. ii. figs. 12–13 (*non* fig. 14).

Locality B.

Lycopodiaceæ.

Lepidodendron, Sternberg.

Lepidodendron fusiforme, Corda, sp.

(Plate II. figs. 17–18 ; Plate III. figs. 22–25.)

1809. *Phytolithus* (*cancellatus*), Martin, *Petrificata Derbiensia*, pl. xiii. fig. 3.
1818. *Phytolithus cancellatus*, Steinhauer (*pars*), *Trans. Amer. Phil. Soc.*, p. 280, pl. vi. figs. 4–5.
1822. *Phytolithus cancellatus*, Parkinson, *Outlines of Oryctology*, p. 14, pl. i. fig. 5.
1845. *Sagenaria fusiformis*, Corda (? *pars*), *Flora d. Vorwelt*, p. 20, pl. vi. fig. 5.
1875. *Sagenaria fusiformis*, Feistmantel, *Vers. d. böhm. Ablager.*, Abth. ii., p. 38, pl. xix. fig. 2.
1850. *Lepidodendron fusiforme*, Corda, *Genera et Species*, p. 257.
1855. *Sagenaria rimosa*, Geinitz (*non* Sternb.) (*pars*), *Vers. d. Steinkf. in Sachsen*, p. 35, pl. iii. fig. 15.
1875. *Sagenaria rimosa*, Feistmantel, (*non* Sternb.) (*pars*), *Vers. d. böhm. Ablager.*, Abth. ii., p. 36, pl. xx. fig. 1.
1869. *Lepidodendron rimosum*, Roehl (*non* Sternb.) (*pars*), *Foss. Flora d. Steink.-Form. Westph.*, p. 132, pl. x. fig. 2.
1899. *Lepidodendron rimosum*, Hofmann and Ryba (*non* Sternb.) (*pars*), *Leitpflanzen*, p. 81, pl. xv. fig. 4.
1866. *Lepidodendron simplex*, Lesqx., *Rept. Geol. Survey of Illin.*, vol. ii. p. 454, pl. xlv. fig. 5.

Description.—Leaf cushions touching each other, and sometimes united in spiral series, broadly fusiform, ending in sharp, almost straight points, lateral angles rounded, keel very slight, with occasionally a few transverse notches on its lower portion ; leaf scar placed very slightly above the centre of the cushion—almost central—and occupying rather more than three-fifths of its width, rhomboidal or diamond-shaped, with upper angle rounded, lower angle sharp, lateral angles rounded or pointed, and from which two lines frequently descend; cicatricules three, punctiform, slightly below the centre of the scar.

Description of Specimens:—

Plate II. fig. 17 is a photograph, natural size, of a plaster of Paris cast from an impression from the "Coal Measures near Halifax, Yorkshire," in the collection of the Geological Department of the British Museum. An outline of a cushion is given at fig. 18.

This specimen is uncompressed, and the contiguous leaf cushions rise up towards the leaf scar, above which the cushions are rather more raised than below it, as the leaf scar slopes slightly downwards. The lower part of the cushion is slightly keeled with transverse notches, the upper portion of the cushion is smooth, without a keel. The leaf scar is almost central, rounded above and sharply pointed below, with lateral angles which are not very prominent, and from which extend two downward lines. The leaf scar sometimes appears as if very sharp-pointed on its lower margin, but this appearance is partly caused in the uncompressed condition by the central keel rising up to meet the lower angle of the leaf scar. The leaf cushions are straight, or if the points are twisted, the bend is so slight that it is scarcely observable.

My thanks are due to Dr A. Smith Woodward for permission to figure this example.

Plate III. fig. 25 shows a younger condition of the plant from Jockie's Syke, Cumberland, and a portion is enlarged two times at fig. 22, while a leaf cushion and leaf scar are seen at figs. 23 and 24.

The contiguous keeled leaf cushions have straight sides with slightly rounded lateral angles, and about their centre they bear a relatively large rhomboidal leaf scar.

Remarks.—This species, which was first figured by Martin in 1809 under the name of *Phytolithus cancellatus*, has been confused with other species of *Lepidodendra*, but especially with *Lepidodendron rimosum*, Sternb., but from *Lepidodendron fusiforme*, *Lepidodendron rimosum* differs in the leaf cushions being distant, more narrowly fusiform, with long tail-like prolongations from the ends of the cushions, which often unite with the neighbouring cushions of the same series; the leaf scar is also smaller in proportion to the width of the cushion, only occupying about one-third of the width, and the interfoliar cortex is ornamented with wavy lines crossed obliquely by fine striæ.

Lepidodendron simplex, Lesqx., appears to me to be referable to *Lepidodendron fusiforme*. His figure shows the leaf cushions united in spiral series, and the same character is seen in the specimen figured by Martin.

Lepidodendron fusiforme is not common in Britain, though it extends throughout the whole of the Coal Measures.

Locality B.

Lepidophyllum, Brongniart.

Lepidophyllum, sp.

Locality B.

Stigmaria, Brongniart.

Stigmaria ficoides, Sternberg, sp.

Stigmaria ficoides, Sternb, sp. See *ante*, p. 757.

Locality C.

The vertical distribution of these fossils is seen in the following table.

	U. C. M.	M. C. M.	L. C. M.
Cf. *Pecopteris arborescens*, Schloth., sp., ...	×		
Pecopteris (*Cyatheites*), sp., ...	×		
Alethopteris aquilina, Schl., sp., ...	×	×	
,, *Grandini*, Brongt., sp., ...	×	×	
,, *Serlii*, Brongt., sp., ...	×		
Neuropteris ovata, Hoffm., ...	×		
,, *flexuosa*, Sternb., ...	×		
,, *Scheuchzeri*, Hoffm., ...	×	×	
Calamites (*Calamitina*) *undulatus*, Sternb., ...	×	×	×
,, (*Calamitina*), sp., ...			
Calamocladus equisetiformis, Schl., sp., ...	×	×	×
Annularia radiata, Brongt. (= *Calamites ramosus*, Artis), . .	×	×	×
,, *stellata*, Schloth., sp., ...	×		
Lepidodendron fusiforme, Corda, sp., ...		×*	×
Lepidophyllum, sp., ...			
Stigmaria ficoides, Sternb. sp., ...	×	×	×

An analysis of the fossil plants found at Jockie's Syke shows that of the 14 names available for comparison, 13 are already known to occur in the Upper Coal Measures of England, and of these 6 are characteristic of that horizon, 7 are common to Upper and Middle Coal Measures, while 5 occur in the Lower Coal Measures of Britain, but of these 5, 4 are common to all the divisions of the Coal Measures, while the 5th, *Lepidodendron fusiforme*, Corda, sp., has only been previously seen in Britain in the Middle and Lower Coal Measures.

This analysis shows that at least the upper portion of the 'Red Shales' must be classed with the Upper Coal Measures, not only on account of the number of Upper Coal Measure species therein met with, but also on account of the absence of a single characteristic Middle Coal Measure species. This absence of Middle Coal Measure species makes it equally impossible to class these beds with the Upper Transition series.

The fact of *Lepidodendron fusiforme*, Corda, sp., occurring in the Upper Coal

* MARTIN's specimen probably comes from this horizon.

Measures, which has hitherto only been discovered in the Middle and Lower Coal Measures (where, however, it is extremely rare), does not vitiate the conclusions arrived at in regard to the age of this portion of the Red Shales specially under consideration, but merely shows that *Lepidodendron fusiforme* is one of those species which extends throughout the whole of the Coal Measures, as several of the *Lepidodendra* do. It is also the only plant discovered at Jockie's Syke which had not previously been found in the Upper Coal Measures of other areas.

The Fossil Plants from that portion of the Carboniferous Limestone Series of Northumberland and Cumberland, which is the Geological Equivalent of a portion of the Calciferous Sandstone Series of Scotland.

Rhodea moravica, Ett., sp.

The great majority of the specimens on which the following list is founded were collected by Mr John Rhodes, Fossil Collector to the Geological Survey of England. A few additional specimens were also collected by the late Mr Hugh Miller while surveying in the district under consideration, and by Mr A. Macconochie of the Scotch Geological Survey. A list of the plants contained in these collections was prepared in 1886, but was laid aside at the time. It is now published, with a few additions, with the double object of recording the fossil plants which occur in the Calciferous Sandstone series of the North of England, and also for the purpose of comparing them with those from the Calciferous Sandstone series of Dumfriesshire.

The relative positions of the geological horizons mentioned are shown in the following table of classification, supplied by Mr C. T. Clough, in which the highest horizon is placed at the top.

Calcareous Division.

Scremerston, or Carbonaceous Division.

Fell Sandstones.

Cementstone Series (= Ballagan Series) and Rothbury Limestones.

All these groups certainly belong to the Calciferous Sandstone Series of Scotland.

List of Fossil Plants.

Algæ.

Bythotrephis, Hall.

Bythotrephis acicularis, Göppert, sp.

Bythotrephis acicularis, Goppert, sp., see *ante*, p. 743.

Locality.—Lumby Law railway cutting, quarter mile north of Edlingham church, Northumberland.

Horizon.—Cementstone and Rothbury Limestone series. (Algæ limestone on thick sandstone, middle of cutting.)

Bythotrephis plumosa, Kidston, sp.

Bythotrephis plumosa, Kidston, sp. See *ante*, p. 743.

Locality.—Bull Cleuch, Kirk Beck, Bewcastle, Cumberland.

Horizon.—Calciferous Sandstone series, near base of Fell Sandstones.

Bythotrephis gracilis, Hall.

Bythotrephis gracilis, Hall. See *ante*, p. 743.

Remarks.—The specimen from White Line is preserved as an ochreous stain on an impure limestone. The form of the plant is that which HALL distinguishes as var. *crassa.*

Locality.—White Line, near Low House, Cumberland.

Horizon.—Low down in the Calciferous Sandstone series.

Filicaceæ.

Sphenopterideæ.

Calymmatotheca, Stur.

Calymmatotheca affinis, L. and H., sp.

1832. *Sphenopteris affinis*, L. and H., *Fossil Flora*, vol. i. pl. xlv.
1836. *Sphenopteris affinis*, Hibbert, *Trans. Roy. Soc. Edin.*, vol. xiii. p. 178 pl. vi. fig. 4, pl. v. *bis.*

1876. *Sphenopteris affinis*, Peach, *Trans. Bot. Soc. Edin.*. vol. xii. pp. 162, 187.
1877. *Sphenopteris affinis*, Peach, *Quart. Journ. Geol. Soc.*, vol. xxxiv. p. 132, pl. vii., pl. viii. figs. 5–7.
1879. *Sphenopteris affinis*, Schimper, *in Zittel, Handb. d. Palæont.*, ii. Abth., *Palæophytologie*, p. 106, fig. 74.
1886. *Calymmatotheca affinis*, Kidston, *Catal. palæoz. plants in Brit. Mus.*, p. 66.
1887. *Calymmatotheca affinis*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 145, pl. ix. figs. 18–22.
1901. *Calymmatotheca affinis*, Vaffier, *Étude géol. et paléont. du Carbon inférieur du Maconnais*, p. 102, pl. i. figs. 1, 1*a*. (Ann. de l'Université de Lyon, Nouv. Sér., i. Sciences, Médecine, fasc. 7.)
1829. *Sphenopteris linearis*, Brongt. (*non* Sternberg), *Hist. d. végét. foss.*, p. 175, pl. liv. fig. 1.
1836. *Sphenopteris linearis*, Hibbert, *Trans. Roy. Soc. Edin.*, vol. xiii. p. 178, pl. vi. fig. 3.
1836. *Cheilanthites linearis*, Göpp., *Syst. fil. foss.*, p. 232 (*excl. ref. Sternberg*).
1877. *Staphylopteris*? *Peachii*, Peach, *Quart. Journ. Geol. Soc.*, vol. xxxiv. p. 131, pl. viii. figs. 1–3 (? fig. 4).
1877. *Diplothmema affine*, Stur, *Culm Flora*, Heft ii. p. 230.

Remarks.—I was formerly under the opinion that some of the specimens of *Sphenopteris flexilis* and *Sphenopteris frigida*, as figured by HEER,* were referable to *Calymmatotheca affinis*, L. and H., sp.,† but NATHORST, who has carefully refigured this species, has shown that I was mistaken in this view.‡

The only record, therefore, for this species outside of Britain appears to be that by Dr VAFFIER, who has discovered *Calymmatotheca affinis*, L. and H., sp., in the Culm of Fuissé, Canton of Mâcon, France.

A very good restoration of *Calymmatotheca affinis* is given by HUGH MILLER as the frontispiece to his *Testimony of the Rocks* (edition 1857).

Locality.—Bull Cleuch, Kirkburn; Bewcastle, Cumberland.

Horizon.—Cementstone series.

Locality.—Warksburn, above the sun-cracked (?) calc grit.

Horizon.—In upper part of Calciferous Sandstone series—below base of Yoredales.

Calymmatotheca bifida, L. and H., sp.

Calymmatotheca bifida, L. and H., sp. See *ante*, p. 745.

Locality.—Shore section, Sandstone quarry, a little south of Sea Houses, near North Sunderland, Northumberland.

Horizon.—Calcareous Group,—rather above the Middle, *i.e.* about 50 feet below the Eelwell Limestone—Uppermost beds of Calciferous Sandstone series.

Locality.—Cranecleuch Burn, opposite Cranecleuch new houses, Whickhope Burn, North Tynedale, Northumberland.

Horizon.—Top of Fell Sandstones—(no line drawn here between Fell Sandstones and Carbonaceous group = bottom of Scremerston group.

* *Beitr. z. foss. Flora Spitzbergens*, Kongl. Svenska Vetenskaps Akad. Handl., Band 14, No. 5, 1876.
† *Catal. Pelæoz. Plants*, p. 67; also *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 145.
‡ *Foss. Flora d. Polarländer*, Erst. Theil, Erst. Lief., Kongl. Svenska Vetenskaps Akad., Band 26, No. 4, 1894.

Locality.—River Irthing, seven-eighth mile N. of Lampert, county boundary between Northumberland and Cumberland.

Horizon.—Carbonaceous group = Scremerston group.

Locality.—Bateinghope Burn, one mile from head of stream, Redesdale, Northumberland.

Horizon.—Cementstone series.

Locality.—Rigend Burn, Keilder, Northumberland.

Horizon.—Lower Limestone series. Carbonaceous group = Scremerston group.

Locality.—Foot of Sauchie Syke, Little Whickhope Burn, North Tynedale, Northumberland.

Horizon.—Carbonaceous group = Scremerston group.

Locality.—East bank of Lewis Burn, Barney's Cut, a little over quarter mile S.W. of Lewis Burn Bridge, North Tynedale, Northumberland.

Horizon.—Carbonaceous group, in Lewis Burn Coal series = Scremerston group.

Locality.—Lewis Burn, over 200 yards below Lewis Burn Colliery, North Tynedale, Northumberland.

Horizon.—Carbonaceous group = Scremerston group.

Locality.—Plashetts Burn, North Tyne, Northumberland.

Horizon.—Carbonaceous group = Scremerston group.

Locality.—Buck Burn, three-quarter mile N.W. of Willow Bog, Oakenshaw Burn, North Tynedale, Northumberland.

Horizon.—Upper part of Fell Sandstone group.

Locality.—River Irthing, one mile due north of Lampert.

Horizon.—Carbonaceous group = Scremerston group.

Locality.—Bothrigg Burn, near the head, one mile E. of Flat, Bewcastle, Cumberland.

Horizon.—Carbonaceous group = Scremerston group.

Locality.—River Irthing, two miles N.E. of Waterhead, Cumberland.

Horizon.—Carbonaceous group = Scremerston group.

Locality.—Whintingstone Burn, Black Line, Cumberland.

Horizon.—Lower Limestone series (Cementstone series ?).

Locality.—Whintingstone Burn, Clattering Ford, Bewcastle, Cumberland.

Horizon.—Lower Limestone series (Cementstone series ?).

Locality.—River Irthing, three-quarter mile E. of Waterhead, Cumberland.

Horizon.—Carbonaceous group = Scremerston group.

Locality.—Foot of streamlet, one mile S.E. of Wileysike, river Irthing, Cumberland.

Horizon.—Carbonaceous group = Scremerston group.

Sphenopteris, Brongniart.

Sphenopteris elegans, Brongt.

1720. *Fumaria officinalis, Siles. Subterr.*, p. 111, pl. xiv. fig. 2.
1804. Schlotheim, *Flora d. Vorwelt*, p. 49, pl. x. fig. 13 (*pars*).
1820. *Filicites adiantoides*, Schlotheim, *Petrefactenkunde*, p. 408, pl. xxi. fig. 2 (*pars*).
1820. *Farrnkraut*, Rhode, *Beitr. z. Pflanzenkunde d. Vorwelt*, p. 33, pl. viii. figs. 7–10.
1822. *Filicites* (*Sphenopteris*) *elegans*, Brongt., *Class. d. végét. foss.*, p. 33, pl. ii. figs. 2*a*, 2*b*.
1823. Sternberg, *Essai flore monde prim.*, vol. i. fasc. 2, p. 33, pl. xxiii. figs. 2*a*, 2*b*.
1826. *Sphenopteris elegans*, Sternb., *Essai flore monde prim.*, fasc. iv. p. 15, pl. xxiii. figs. 2*a*, 2*b*; vol. ii. fasc. v.–vi. p. 56 (*non* pl. xx. figs. 3–4*).
1828. *Sphenopteris elegans*, Brongt., *Prodrome*, p. 50.
1830. *Sphenopteris elegans*, Brongt., *Hist. d. végét. foss.*, p. 172, pl. liii. figs. 1–2.
1848. *Sphenopteris elegans*, Sauveur, *Végét. foss. de la Belgique*, pl. xvi. figs. 1–2.
1882. *Sphenopteris elegans*, Weiss, *Aus d. Steink.*, p. 13, pl. xi. fig. 71 (zweiter abdr.).
1899. *Sphenopteris elegans*, Hofmann and Ryba, *Leitpflanzen*, p. 38, pl. iii. figs. 18–20 (*non* fig. 17).
1899. *Sphenopteris elegans*, Frech, *Lethæa geog.*, vol. ii. Lief. 2, pl. xxxvii.*a* fig. 2.
1899. *Sphenopteris elegans*, Potonié, *Lehrb. d. Pflanzenpal.*, p. 137, fig. 128.
1836. *Cheilanthites elegans*, Göpp., *Syst. fil. foss.*, p. 233, pl. x. fig. 1, pl. xi. figs. 1–2.
1877. *Diplothmema elegans*, Stur, *Culm Flora*, Heft ii. p. 236, pl. xxx. fig. 5, pl. xxxi. figs. 1–6.
1899. *Diplothmema elegans*, Zeiller, *Mém. Soc. géol. d. France. Paléont.*, No. 21, *Flore foss. bassin houil. d'Héraclée*, p. 30, pl. iii. figs. 3–4.
1852. *Sphenopteris officinalis*, Giebel, *Deutschl. Petrefacten.*, p. 39.
1853. *Sphenopteris Johnstoniana*, Tate, *in Johnston, Nat. Hist. of Eastern Borders*, vol. i., *Botany*, p. 306, figs. 1–2.

Locality.—Budle, Northumberland ("In shale," Tate).

Horizon.—Top of Calciferous Sandstone series = Oil Shale group of Midlothian.

Locality.—Northern branch of Caller Cleuch, $\frac{5}{8}$ mile E. of Kielder Head, Northumberland.

Horizon.—Cementstone series.

Sphenopteris decomposita, Kidston.

Sphenopteris decomposita, Kidston. See *ante*, p. 747.

Locality.—Yate Burn, two miles S.S.W. of High Long House, Northumberland.

Horizon.—Calciferous Sandstone series (? Fell Sandstones).

Sphenopteris Dicksonioides, Göppert, sp.

1836. *Aspidites Dicksonioides*, Göpp., *Syst. fil. foss.*, p. 361, pl. xxviii.
1869. *Pecopteris Dicksonioides*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 521.
1877. *Diplothmema Dicksonioides*, Stur, *Culm Flora*, Heft ii. p. 248, pl. xxxiii. figs. 1–5.
1882. *Sphenopteris Dicksonioides*, Weiss, *Aus d. Steink.*, p. 12, pl. xi. figs. 65–66 (zweiter abdr.).

* The plant figured here appears to be the *Sphenopteris divaricata*, Göpp., sp.

1883. *Sphenopteris Dicksonioides*, Kidston, *Trans. Edin. Geol. Soc.*, vol. iv. p. 333.
1899. *Sphenopteris Dicksonioides*, Zeiller, *Mém. Soc. géol. d. France. Paléont.*, No. 21 (*Flore foss. d. bassin houil. d'Héraclée*), p. 6, pl. i. fig. 2.

Locality.—Shore section, W. of Budle, Chesterfield Slakes, Northumberland.

Horizon.—Calcareous group—about the middle—top of Calciferous Sandstone series = Oil Shale group of Midlothian.

Sphenopteris, sp.

(Plate II. fig. 16.)

Note.—Allied to *Sphenopteris foliolata*, Stur.*

Locality.—Shore section, W. of Budle, Chesterfield Slakes, Northumberland.

Horizon.—Calcareous group, about the middle—top of Calciferous Sandstone series = Oil Shale group of Midlothian.

Rhodea, Presl.

Rhodea moravica, Ettingshausen, sp.

(Woodcut, p. 812.)

1865. *Trichomanes moravicum*, Ett., *Foss. Flora d. Mähr.-Schles. Dachschiefers*, p. 24, fig. 9, pl. vi. fig. 4.
1869. *Sphenopteris* (*Trichom.*) *moravica*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 414.
1875. *Rhodea moravica*, Stur, *Culm Flora*, Heft i. p. 38, pl. x. figs. 3–7, pl. xi. fig. 1.
1891. *Rhodea moravica*, Vaffier, *Étude géol. et paléont. du Carbon inférieur du Maconnais*, p. 109, pl. iii. figs. 2, 2*a*, 2*b*, 2*c*, 2*d*.
1877. *Calymmotheca moravica*, Stur, *Culm Flora*, Heft ii. p. 278.

Remarks.—A few fragments of *Rhodea moravica*, Ett., sp., were collected at Budle. The type specimens, as figured by Ettingshausen, have suffered much from decay before fossilisation took place, hence his figures show little more than the veins of the pinnules, from which all the limb has decayed.

Stur, in his *Culm Flora*, more fully figures and describes the species. The pinnules are divided into 3–10 simple or bifid linear segments, each having a central vein. The limb of the pinnule forms a very narrow border to the veins. Though the Budle specimens are fragmentary, they show very well the form and segmentation of the pinnules.

Locality.—Shore section, W. of Budle, Chesterfield Slakes, Northumberland.

Horizon.—Calcareous group, about the middle—top of Calciferous Sandstone series = Oil Shale group of Midlothian.

* *Culm Flora*, Heft i. p. 22, pl. v. figs. 3–6, 1875.

Rhodea dissecta, Brongt., sp.

1839. *Sphenopteris dissecta*, Brongt., *Hist. d. végét. foss.*, p. 183, pl. xlix. figs. 2–3.
1877. *Diplothmema dissectum*, Stur, *Culm Flora*, Heft ii. p. 230.
1899. *Diplothmema dissectum*, Zeiller, *Étude sur la flore foss. du bassin houil. d'Héraclée*, p. 30, pl. iii. fig. 2.
1900. *Diplothmema dissectum*, Zeiller, *Éléments d. paléobotanique*, p. 87, fig. 58.
1899. *Rhodea dissecta*, Potonié, *Lehrb. d. Pflanzenpalæont.*, p. 135, fig. 125.
1899. *Rhodea dissecta*, Frech, *Lethæa geog.*, i. Theil, *Lethæa palæoz.*, 2 Band, 2 Lief., *Die Steinkohlenform.*, pl. xxxvii.*b* fig. 5, and explanation to figure.
1877. *Diplothmema Schützei*, Stur, *Culm Flora*, Heft ii. p. 234, pl. xxx. figs. 4 *a, b, c, d, e, f.*
1888. *Diplothmema Schützei*, Toula, *Die Steinkohlen*, p. 187, pl. i. fig. 10.

Locality.—Shore at Pit open water-level, Spittal, Tweedmouth, Northumberland.

Horizon.—Carbonaceous group = Scremerston group.

Rhodea patentissima, Ettingshausen, sp.

1865. *Hymenophyllites patentissima*, Ett., *Foss. Flora d. Mähr.-Schles. Dachschiefers*, p. 26, fig. 13, pl. vii. fig. 4.
1869. *Sphenopteris* (*Hymen.*) *patentissima*, Schimper, *Traité d. paléont. végét.*, vol i. p. 407.
1900. (?) *Sphenopteris patentissima*, White, *Stratigraphic Succession of the Fossil Floras of the Pottsville Formation in the Southern Anthracite Coal Field, Pennsylvania* (*20th Ann. Rept. U.S. Geol. Survey*, part ii., *Geol. and Palæont.*), p. 880, pl. clxxxviii. fig. 1.
1875. *Rhodea patentissima*, Stur, *Culm Flora*, Heft 1, p. 36, pl. ix. figs. 1–9.

Note.—Only a single specimen has been met with, and, though small, is sufficiently well preserved to permit of a satisfactory determination.

Locality.—Kirk Beck, White Line, Bewcastle, Cumberland.

Horizon.—Cementstone series.

Rhacopteris, Schimper.

Rhacopteris flabellata, Tate, sp.

1853. *Sphenopteris flabellata*, Tate, *in Johnston, Nat. Hist. of the Eastern Borders*, vol. i., *Botany*, p. 308, fig. 3.
1886. *Rhacopteris flabellata*, Kidston, *Catal. Palæoz. Plants Brit. Mus.*, p. 63.
1889. *Rhacopteris flabellata*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxv. p. 442, pl. i. fig. 2, pl. ii. figs. 4–6 (? fig. 7).
1865. *Noeggerathia*, sp., Gomes, *Flora foss. do terr. carbon. das visin. do Porto, Serra do Bussaco, etc.*, p. 32, pl. ii. figs. 1–2 (*Commissãs geol. de Portugal*).

Locality.—Budle, Northumberland (" In shale," Tate).

Horizon.—Top of Calciferous Sandstone series = Oil Shale group of Midlothian.

Rhacopteris subcuneata, Kidston.

1894. *Rhacopteris subcuneata*, Kidston, *Proc. Roy. Phys. Soc. Edin.*, vol. xii. p. 261, pl. v. fig. 2, pl. vi. fig. 1.

Locality.—East bank of Lewis Burn, Barney's Cut, a little over quarter mile S.W. of Lewis Burn Bridge, North Tynedale, Northumberland.

Horizon.—Lewis Burn Coal group—low down in Carbonaceous group = Scremerston group.

Adiantites, Göppert.

Adiantites antiquus, Ettingshausen, sp.

1865. *Adiantum antiquum*, Ett., *Foss. Flora d. Mährisch-Schlesischen Dachschiefers*, p. 22, fig. 7, pl. vii. fig. 1.
1875. *Adiantides antiquus*, Stur, *Culm Flora*, Heft i. p. 66, pl. xvi. figs. 4–6, pl. xvii. figs. 3–4.
1889. *Adiantides antiquus*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxv. p. 421, pl. i. fig. 1.

Locality.—Spithope Burn, Rede Water, Northumberland.

Horizon.—Cementstone series.

Neuropterideæ.

Cardiopteris, Schimper.

Cardiopteris polymorpha, Göpp., sp.

Cardiopteris polymorpha, Göpp., sp. See *ante*, p. 749.

Locality.—Pit heaps, quarter mile N.E. of Chalton Lime Works.

Horizon.—Calcareous series. Coal above Woodend Limestone. Upper part of the Calciferous Sandstone series = Oil Shale group of Midlothian.

Locality.—Warksburn, North Tynedale, Northumberland.

Horizon.—Calciferous Sandstone series. Lower part of Midlothian Oil Shale group = Scremerston series.

Alcicornopterideæ.

Alcicornopteris, Kidston.

Alcicornopteris convoluta, Kidston.

Alcicornopteris convoluta, Kidston, see *ante*, p. 749.

Locality.—River Tweed, 100 yards below Norham Castle, Northumberland.

Horizon.—Cementstone series.

Locality.—Horncliff Dean, near Mill, river Tweed, S. of Horncliff village, Northumberland.

Horizon.—Cementstone series, upper part, a few hundred feet below Fell Sandstones.

Locality.—River Coquet, half mile N.N.E. of Holystone, Northumberland.
Horizon.—Cementstone series.
Locality.—Spithope Burn, Rede Water, Northumberland.
Horizon.—Cementstone series.
Locality.—Hawkburn, near Catcleuch, Redesdale, Northumberland.
Horizon.—Cementstone series.
Locality.—Crawley Dean, one-third mile S. of Powburn, near Ingram, Northumberland.
Horizon.—Cementstone series.
Locality.—Coombsdon Burn, half mile S.W. from its junction with the river Rede, Northumberland.
Horizon.—Cementstone series.
Locality.—Bull Cleuch, Bewcastle, Cumberland.
Horizon.—Cementstone series.

Fern Stem.

Eskdalia, Kidston.

Eskdalia minuta, Kidston, sp.

Eskdalia minuta, Kidston, see *ante*, p. 750.

Locality.—River Coquet, half mile N.N.E. of Holystone, Northumberland.
Horizon.—Cementstone series.

Equisetaceæ.

Asterocalamites, Schimper.

Asterocalamites scrobiculatus, Schlotheim, sp., p. 751.

Locality.—Yate Burn, two miles S.S.W. of High Long House, Northumberland.
Horizon.—Upper part of Calciferous Sandstone series = Oil Shale group of Midlothian.
Locality.—Sandstone quarry, east of Long Crag.
Horizon.—(?)

Lycopodiaceæ.

Lepidodendron, Sternberg.

Lepidodendron Veltheimii, Sternberg.

Lepidodendron Veltheimii, Sternb., see *ante*, p. 754.

Locality.—Yate Burn, two miles S.S.W. of High Long House, Northumberland.

Horizon.—Upper part of Calciferous Sandstone series = Oil Shale group of Midlothian.

Locality.—Chattlehope Burn, two miles S.W. of Chattlehope House, Rede Water, Northumberland.

Horizon.—Cementstone series.

Lepidodendron Volkmannianum, Sternberg.

(Plate II. fig. 19.)

1820. "*Schuppenpflanze*," Rhode, *Beitr. z. Pflanzenkunde d. Vorwelt*, p. 32, pl. vii. figs. 4–5.
1826. *Lepidodendron Volkmannianum*, Sternb., *Essai flore monde prim.*, vol. i. fasc. 4, p. 10, pl. liii. figs. 3*a*, 3*b*, 3*c*.
1877. *Lepidodendron Volkmannianum*, Stur, *Culm Flora*, Heft ii. p. 392, pl. xxxv. fig. 4, pl. xl. figs. 2, 3 (fig. 4 ?) (*non* fig. 5).
1882. *Lepidodendron Volkmannianum*, Weiss, *Aus d. Steink.*, p. 8, pl. iv. fig. 29 (zweiter abdr.).
1899. *Lepidodendron Volkmannianum*, Frech, *Lethæa geog.*, 1 Theil, *Lethæa Palæoz.*, Band ii. Lief. ii., *Die Steinkohlenform.*, pl. xxxvii.*a* figs. 1*a*, 1*b*.
1899. *Lepidodendron Volkmannianum*, Hofmann and Ryba, *Leitpflanzen*, p. 81, pl. xv. figs. 2, 3.
1901. *Lepidodendron Volkmannianum*, Potonié, *Silur- und Culm-Flora d. Harzes u. d. Magdeburgischen*, p. 113, figs. 68–71 (*Abhandl. d. k. preuss. geol. Landesanstalt, Neue Folge*, Heft 36).
1838. *Sagenaria Volkmanniana*, Presl., *in Sternb.*, *Essai flore monde prim.*, vol. ii. pl. 179, p. lxviii. fig. 8.
1854. *Sagenaria Volkmanniana*, Göpp., *in Roemer*, *Beitr. z. Kenntn. d. Nordwest. Harzgebirges. Palæont.*, vol. iii. p. 46, pl. vii. fig. 15.
1838. *Sagenaria affinis*, Presl, *in Sternberg*, *Essai flore monde prim.*, vol. ii. p. 180, pl. lxviii. fig. 9.
1852. *Sagenaria Roemeriana*, Göpp., *Foss. Flora d. Ubergangs*, p. 184.
1862. *Sagenaria concinna*, Roemer, *Palæont.*, vol. ix. p. 10, pl. iv. fig. 8.

Remarks.—The specimen figured here is the only example I have seen from England, and represents that form of the plant to which Presl gave the name of *Sagenaria affinis*.

Lepidodendron Volkmannianum, Sternb., is very rare in Britain.

Locality.—Quarry, one mile S.W. of Glororum farm-house, Bamborough, Northumberland.

Horizon.—Upper part of Calciferous Sandstone series = Oil Shale group of Midlothian.

Lepidodendron Harcourti, Witham.

1832. *Lepidodendron Harcourtii*, Witham, "On the *Lepidodendron Harcourtii*," *Trans. Nat. Hist. Soc. of Northumberland, Durham, and Newcastle-on-Tyne*, vol. ii. p. 236, pl. v. figs. 1–7, pl. vi. figs. 1–7, 1838.
1833. *Lepidodendron Harcourtii*, Witham, *Internal Structure of Fossil Vegetables*, pp. 51, 75, pl. xii. figs. 1–7, pl. xiii. figs. 1–7.
1833. *Lepidodendron Harcourtii*, L. and H., *Fossil Flora*, vol. ii. pl. xcviii., pl. xcix.
1839. *Lepidodendron Harcourtii*, Brongt., "Observ. sur la Struct. intér. du *Sigillaria elegans* comparée à celle des Lepidodendron et des Stigmaria," *Archives d. Mus. d'hist. nat.* (*Paris*), vol. i. p. 417 *et seq.*, pl. xxx. and pl. xxxi.

Remarks.—It has been believed that the museum of the Yorkshire Philosophical Society, York, possessed the original specimen from which the sections had been prepared that are described and figured by LINDLEY and HUTTON in their *Fossil Flora.**

Through the kindness of the Curator, Mr H. M. PLATNAUER, I had an opportunity of examining the reputed types of LINDLEY and HUTTON's descriptions and figures in the York museum. The material consists of part of a stem showing structure and a microscopical section. On comparing the stem with the section, it was evident, owing to the larger size of the latter and its different character of preservation, that it had not formed part of the supposed LINDLEY and HUTTON stem. On this being pointed out, with the object of arriving at a definite conclusion as to the stem being LINDLEY and HUTTON's type, Mr PLATNAUER obtained permission from the Council of the museum for having the stem sliced. This has been done, and it now transpires that the stem is the original block from which BRONGNIART's transverse section, was derived. Apparently BRONGNIART possessed little more than a transverse section, as most of the longitudinal sections he publishes are copies of WITHAM's and LINDLEY and HUTTON's figures.

The transverse section to which I have already referred, and on the glass of which is written with a diamond "Northumberland Limestone, Vernon," is the transverse section figured by WITHAM in the *Trans. Nat. Hist. Soc. of Northumberland, Durham, and Newcastle-on-Tyne*, plate ii. fig. 1, and in his *Fossil Vegetables*, pl. xiii. fig. 1. With these conclusions in regard to the identifications of these two specimens, Dr D. H. SCOTT, to whom I showed the specimen and sections, fully agrees.

On the other hand, I cannot identify either of these with the figures of *Lepidodendron Harcourtii* given by LINDLEY and HUTTON. The specimen from which their sections came may have been all cut up at the time ; but if not, it seems to be lost or buried in some collection, where it is unrecognised as LINDLEY and HUTTON's type. Several examples, however, seem to have been found, for LINDLEY and HUTTON say, "the *fossils* are found partly in the coal and partly in the roof, which in some cases consists of a mass of encrinal remains and shells, such as *Productæ*, *Melaniæ*, etc."†

The type specimens of *Lepidodendron Harcourtii* came from Hesley Heath, near Rothbury, Northumberland, from rocks belonging to the Scremerston group of the Calciferous Sandstone series, but the late Prof. WILLIAMSON, in part xix. of his Memoirs "On the organisation of the Fossil Plants of the Coal Measures,"‡ describes a *Lepidodendron* from the Lower Coal Measures which he identifies as *Lepidodendron Harcourtii* of WITHAM. It is, however, a well ascertained fact that not a single species of *Lepidodendron* which occurs in the Lower Carboniferous has ever been known to pass into the Upper Carboniferous ; it is therefore most improbable that the plant identified by WILLIAMSON as WITHAM's species can really be that plant, notwithstanding the great similarity of stem structure. It is perhaps to be expected that plants so closely related, though undoubtedly specifically distinct, as the Upper and Lower Carboniferous

* WILLIAMSON, *Proc. Roy. Soc. London*, vol. xlii. p. 6.

† LINDLEY and HUTTON, *l.c.*, vol ii. p. 45.

‡ *Phil. Trans.*, 1893, B, p. 1. See also *Proc. Roy. Soc. London*, vol. xlii p. 6, 1886.

Lepidodendra may possess an internal structure so similar that at present we are unable to distinguish any specific difference in their structure. It is, however, much to be desired that fresh material might be procured from the original *Horizon* of WITHAM'S *Lepidodendron Harcourtii*, so that a careful structural comparison might be made between the *Lepidodendron Harcourtii*, Witham, and that described by WILLIAMSON under the same name, and so to ascertain if there are not some anatomical differences.

It has been thought by some botanists that *Lepidodendron Harcourtii* should, on account of the '*corona*' which surrounds the outer margin of the vascular axis, be classed with the *Lepidophloios*, as in *Lepidophloios* stems which have shown structure this '*corona*' seems to be always present.

Locality.—Hesley Heath, near Rothbury, Northumberland.

Horizon.—Carbonaceous series = Scremerston group.

Bothrodendron, L. and H.

Bothrodendron Kidstoni, Weiss.

1889. *Bothrodendron Wükianum*, Kidston (*pars*), *Ann. and Mag. Nat. Hist.*, ser. 6, vol. iv. p. 65, pl. iv. fig. 2.
1889. *Bothrodendron Wükianum*, Kidston (*pars*), *Proc. Roy. Phys. Soc. Edin.*, vol. x. p. 94, pl. iv. fig. 2.
1893. *Sigillaria* (*Bothrodendron*) *Kidstoni*, Weiss, *Die Sigillarien d. preuss. Steink. u. Rothl. Gebiete*, ii. *Gruppe. Die Subsigillarien*, p. 56, pl. xxviii. fig. 110 (*Abhandl. d. Königl. preuss. geol. Landesanstalt, Neue Folge*, Heft 2).

Remarks.—The specimen which WEISS names *Bothrodendron Kidstoni* I previously included with *Bothrodendron Wükianum*, regarding it as possibly a younger condition of that species.

It differs from *Bothrodendron Wükianum* in possessing a smooth bark and closer leaf scars, from which two lateral lines extend downwards. I now believe I was in error in regarding this specimen as a young condition of *Bothrodendron Wükianum*.

The type specimen of *Bothrodendron Kidstoni*, Weiss, was received from the late Mr HUGH MILLER.

Locality.—Little Whickhope Burn, near first branch above Cross Syke, Northumberland.

Horizon.—Well down in the Calciferous Sandstone series—Cementstone series or Fell Sandstone series.

Lepidophyllum, Brongniart.

Lepidophyllum lanceolatum, L. and H.

Lepidophyllum lanceolatum, L. and H. See *ante*, p. 756.

Locality.—Yate Burn, two miles S.S.W. of High Long House, Northumberland.

Horizon.—Calciferous Sandstone series = Oil Shale group of Midlothian.

Locality.—Gunnerston Coal Pit, North Tynedale, Northumberland.
Horizon.—Carbonaceous series = Scremerston group.

Lepidostrobus, Brongniart.

Lepidostrobus variabilis, L. and H.

Lepidostrobus variabilis, L. and H. See *ante*, p. 756.

Locality.—East bank of Lewis Burn, Barney's Cut, a little over quarter mile S.W. of Lewis Burn Bridge, North Tynedale, Northumberland.
Horizon.—Carbonaceous series—Lewis Burn Coal group.

Lepidostrobus fimbriatus, Kidston.

Lepidostrobus fimbriatus, Kidston. See *ante*, p. 756.

Locality.—Black Cleuch, tributary of Chirdon Burn, above Tarset, Northumberland.
Horizon.—Well up in the Calciferous Sandstone series = Scremerston group.
Locality.—River Irthing, five-eighth mile S. of Lampert boundary line between Northumberland and Cumberland.
Horizon.—Carbonaceous series = Scremerston group.
Locality.—Near foot of Bailey Water, Cumberland.
Horizon.—Cementstone series.
Locality.—Black Line, Clattering Ford, Cumberland.
Horizon.—Scremerston group (?).
Locality.—Trout Beck, below King Water, Cumberland.
Horizon.—Near base of Fell Sandstones.
Locality.—King Water, half mile above foot of Trout Beck, Cumberland.
Horizon.—Near base of Fell Sandstones.
Locality.—King Water, half mile S. of Spottey Bank, Cumberland.
Horizon.—Near base of Fell Sandstones.
Locality.—Tributary of Kershope Burn, one mile E. of Kershope Foot, Cumberland.
Horizon.—Scremerston group.
Locality.—Kershope Head, Cumberland.
Horizon.—Scremerston group.
Locality.—River Irthing, three-quarter mile E. of Waterhead, Cumberland.
Horizon.—Carbonaceous series = Scremerston group.
Locality.—Lewis Burn, about 100 yards above the Old Lewis Burn bridge, Northumberland.
Horizon.—Carbonaceous series = Scremerston group.
Locality.—Lewis Burn, half mile N.W. of bridge, Northumberland.
Horizon.—Carbonaceous series = Scremerston group.

Locality.—Lewis Burn, about 200 yards below Lewis Burn Colliery, North Tynedale, Northumberland.

Horizon.—Carbonaceous series = Scremerston group.

Locality.—Lewis Burn, a little over 100 yards below Lewis Burn Colliery, Northumberland.

Horizon.—Carbonaceous series = Scremerston group.

Stigmaria, Brongniart.

Stigmaria ficoides, Sternberg, sp.

Stigmaria ficoides, Sternberg, sp. See *ante*, p. 757.

Locality.—Open coal working near Hawkhope coal working, one and a half miles N.E. of Falstone church, Northumberland.

Horizon.—Scremerston group.

Locality.—Lishaw Burn, Yate Burn, five-eighth mile S. of High Long House, Northumberland.

Horizon.—Scremerston group.

Locality.—Shore section, W. of Budle, Chesterfield Slakes, Northumberland.

Horizon.—Upper part of Calciferous Sandstone series = Oil Shale group of Midlothian.

Locality.—Lewis Burn, over 200 yards below Lewis Burn Colliery, North Tynedale, Northumberland.

Horizon.—Carbonaceous series = Scremerston group.

Locality.—Buck Burn, three-quarter mile N.W. of Willow Bog, Oakenshaw Burn, North Tynedale, Northumberland.

Horizon.—Upper part of Fell Sandstone group.

Stigmaria ficoides, var. **undulata,** Göpp.

1841. *Stigmaria ficoides*, var. *undulata*, Göpp., *Gatt. d. foss. Pflanzen.*, Lief. 1–2, pp 13, 30, pl. ix. figs. 5–8 (? fig. 9).
1852. *Stigmaria ficoides*, var. *undulata*, Göpp., *Foss. Flora d. Ubergangs*, p. 245, pl. xxxii. fig. 2.
1884. *Stigmaria ficoides*, var. *undulata*, Lesqx., *Indiana Dept. of Geol. and Nat. Hist. 13th Ann. Rept.*, part ii., *Palæont.*, p. 96, pl. xix. fig. 3.
1873. *Stigmaria ficoides*, Feistmantel, *Zeitsch. d. deut. geol. Gesell.*, vol. xxv. pp. 535, 540, pl. xvii. fig. 37.
1865. *Stigmaria*, with scars in rhomboidal areoles, Dawson, *Quart. Journ. Geol. Soc.*, vol. xxii. p. 169, pl. xii. fig. 83.

Locality.—Found lying loose, not far from Ox Crags, top of Bewcastle Fells, Northumberland (H. Miller).

Horizon.—(?) Boulders here have travelled from many miles to the west.

Cordaiteæ.

Samaropsis, Göppert.

Samaropsis nervosa, Kidston, sp.

1894. *Cardiocarpus nervosus*, Kidston, *Proc. Roy. Phys. Soc.*, vol. xii. p. 266, pl. v. figs. 3–5.

Locality.—Horncliff Dean, near mill, River Tweed, S. of Horncliff village, Northumberland.

Horizon.—Cementstone series = Ballagan group.

Locality.—River Coquet, half mile N.N.E. of Holystone, Northumberland.

Horizon.—Cementstone series = Ballagan group.

Rhabdocarpus, Göppert and Berger.

Rhabdocarpus curvatus, Kidston, n.sp.

(Plate II. fig. 14 *a, b, c, d.*)

Seeds oblong, blunt at both extremities, slightly curved, and bearing about eight slight ridges on the exposed surface.

Remarks.—All the specimens figured occur on the same block,—*a*, *b*, *c* all shown in their relative position as they occur on the slab. The specimen *d* was some little distance from the others.

The seeds vary in size from 1·30 cm. to 2 cm. long, and from 0·50 cm. to 0·70 cm. wide.

Locality.—Lewis Burn, rather over 200 yards below Lewis Burn Colliery, North Tynedale, Northumberland.

Horizon.—Carbonaceous series = Scremerston group.

Cordaicarpus, Geinitz.

Cordaicarpus planus, Kidston, n.sp.

1883. (?) *Cardiocarpus apiculatus*, Kidston (*non* Göppert), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 545, pl. xxxi. figs. 13, 13*a*.

Small oval or pointed seeds, about 0·70 cm. long, and rather less wide. Surface smooth, and surrounded by a narrow wing or border.

These small seeds vary in form. At plate xxxi. fig. 13 (*l.c.*) is shown a specimen terminating in a point, while that shown at fig. 13*a* is oval. These specimens occur on a slab thickly covered with other examples of the same species.

Locality.—Lewis Burn, North Tynedale, Northumberland.
Horizon.—Carbonaceous series = Scremerston group.

Pitys, Witham, emend.

Pitys Withami, L. and H., sp.

1831. *Observations on Fossil Vegetables*, p. 30, pl. iii. figs. 8–12.
1831. *Pinites Withami*, L. and H., *Fossil Flora*, vol. ii. p. 9, pl. ii.
1833. *Pinites Withami*, Witham, *Internal Structure of Fossil Vegetables*, pp. 27, 72, pl. iv. figs. 8–12, pl. v., pl. vi., figs. 1–4, pl. vii. figs. 1–6.
1841. *Pinites Withami*, Unger, *Chloris Protogæa. Beitr. z. Flora d. Vorwelt*, p. 29.
1845. *Pinites Withami*, Unger, *Synop. Plant. Foss.*, p. 205.
1847. *Dadoxylon Withami*, Eudlicher, *Synop. Coniferarum fossilium*, p. 34.
1850. *Dadoxylon Withami*, Unger, *Genera et Species*, p. 378.
1845. *Araucarites Withami*, Göppert, *Descrip. d. Végét. foss. recueillis par M. P. de Tchihatcheff en Sibérie*, p. 10 (*Voyage scientifique dans l'Altai oriental*, pp. 379–390).
1850. *Araucarites Withami*, Göppert, *Monog. d. foss. Coniferen*, p. 231.
1870. *Araucarioxylon Withami*, Kraus, *in Schimper, Traité d. paléont. végét.*, vol. ii. p. 384.
1880. *Pitys Withami*, Göppert, *Revision meiner Arbeiten über die Stämme der fossilen Coniferen*, p. 18.
1902. *Pitys Withami*, Scott, *Trans. Roy. Soc. Edin.*, vol. xl. p. 354, pl. ii. fig. 10, pl. vi. fig. 21.
1831. Witham (*Pinites medullaris*, L. and H.), *Trans. Nat. Hist. Soc. of Northumberland, Durham, and Newcastle-upon-Tyne*, vol. i. p. 297, pl. xxv. figs. 3–8.
1831. *Pinites medullaris*, L. and H., *Fossil Flora*, vol. i. p. 13, pl. iii.
1833. *Pinites medullaris*, Witham, *Internal Structure of Fossil Vegetables*, pp. 35, 72, pl. vi. figs. 5–8, pl. vii. figs. 7, 8.
1845. *Pinites medullaris*, Unger, *Synop. plant. foss.*, p. 205.
1845. *Araucarites medullaris*, Göppert, *Descriptions d. végét. foss. recueillis par M. P. de. Tchihatcheff en Sibérie*, p. 10.
1850. *Araucarites medullaris*, Göppert, *Monog. d. foss. Coniferen*, p. 231.
1847. *Dadoxylon medullare*, Eudlicher, *Synop. Coniferarum fossilium*, p. 34.
1890. *Dadoxylon medullare*, Knowlton, *Revision of the Genus Araucarioxylon of Kraus, etc.*, *Proc. U. S. Nat. Museum*, vol. xii. p. 610.
1870. *Araucarioxylon medullare*, Kraus, *in Schimper, Traité d. paléont végét.*, vol. ii. p. 385.
1880. *Pitys medullaris*, Göppert, *Revision meiner Arbeiten über die Stämme der fossilen Coniferen*, p. 18.

Locality.—In Syke, west from Eastnook, near Elsdon, Northumberland.
Horizon.—Near top of the Calciferous Sandstone series.

Pitys primæva, Witham, sp.

1833. *Pitus primæva*, Witham, *Internal Structure of Fossil Vegetables*, pp. 37–39, 71, pl. viii. figs. 4–6 pl. xvi. fig. 2.
1841. *Pissadendron primævum*, Unger, *Chloris protogæa. Beitr. z. Flora d. Vorwelt*, p. 29.
1845. *Pissadendron primævum*, Unger, *Synop. plant. foss.*, p. 205.
1847. *Pissadendron primævum*, Eudlicher, *Synop. Coniferum fossilium*, p. 33.
1850. *Pissadendron primævum*, Göppert, *Monog. foss. Coniferen*, p. 230.
1850. *Pissadendron primævum*, Unger, *Genera et species*, p. 377.

1845. *Araucarites primæva*, Göppert, *Descriptions d. végét. foss. recueillis par M. P. de Tchihatcheff en Sibérie*, p. 11.
1870. *Araucarioxylon primævum*, Kraus, *in Schimper, Traité d. paléont. végét.*, vol. ii. p. 385.
1880. *Pitys primæva*, Goppert, *Revision meiner Arbeiten über die Stämme der fossilen Coniferen, etc.*, p. 18.
1902. *Pitys primæva*, Scott, *Trans. Roy. Soc. Edin.*, vol. xl. p. 355, pl. ii. fig. 11, pl. vi. figs. 22–23.

Locality.—River Irthing, below Lampert and Shankend, county boundary between Cumberland and Northumberland.

Horizon.—Near the top of the Calciferous Sandstone series.

Incertæ sedis.

Ptilophyton, Dawson.

Ptilophyton plumula, Dawson.

Ptilophyton plumula, Dawson, sp. See *ante*, p. 761.

Locality.—Head of Black Burn, Humble Burn, two and a quarter miles S.W. of Cranecleuch, Northumberland.

Horizon.—Lower part of Calciferous Sandstone series.

Locality.—King Water, half mile S. of Spottey Bank, Northumberland.

Horizon.—Calciferous Sandstone series, near base of Fell Sandstones.

Sorocladus, Lesquereux.

1879–1880. *Sorocladus*, Lesqx., *Coal Flora*, vol. i. p. 327.

Note.—I employ this genus as originally proposed by Lesquereux, with no further significance than that the fossil included here is a fern fructification.

Sorocladus antecedens, Kidston.

1887. *Sorocladus antecedens*, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. p. 143, pl. viii. fig. 6*b*.

Locality.—Lewis Burn, over 200 yards below Lewis Burn Colliery, Northumberland.

Horizon.—Carbonaceous series = Scremerston group.

The foregoing list of plants from Northumberland and Cumberland contains the species from all the divisions of the Calciferous Sandstone series of that area ; and with the purpose of comparing the fossil plants from the same rocks of Dumfriesshire with those from the North of England a table is given, showing in the first column the fossil plants from the North of England, the second column contains the Dumfriesshire specimens, while the third column shows the further distribution of these plants in the Calciferous Sandstone series of other areas of Scotland.

	Calciferous Sandstone Series.		
	North of England.	Dumfries-shire.	Other areas of Scotland.
Bythotrephis acicularis, Göpp., sp.,	×	×	
,, *plumosa*, Kidston, sp,,	×	×	
,, *simplex*, Kidston, sp.,		×	
,, *Scotica*, Kidston, sp.,		×	
,, *gracilis*, var. *intermedia*, Hall.		× *	
,, ,, var. *crassa*, Hall,	×		
Spirophyton cauda-galli, Vanuxem, sp.,		×	×
Calymmatotheca bifida, L. and H., sp.,	×	×	×
,, *affinis*, L. and H., sp.,	×		×
Sphenopteris elegans, Brongt.,	×		×
,, *crassa*, L. and H.,		×	×
,, *pachyrrhachis*, Göpp.,		×	×
,, *Macconochiei*, Kidston, sp.,		×	
,, *obovata*, L. and H.,		×	×
,, *Hibberti*, L. and H., var.,		×	×
,, *decomposita*, Kidston,	×	×	×
,, *dicksonioides*, Göpp., sp.,	×		
,, sp.,		×	
,, sp.,	×		
Rhodea Machaneki, Ett., sp.,		×	×
,, *patentissima*, Ett., sp.,	×		×
,, *moravica*, Ett., sp.,	×		×
,, *dissecta*, Brongt., sp.,	×		×
Rhacopteris inæquilatera, Göpp., sp.,		×	×
,, *Geikiei*, Kidston, sp.,		×	
,, *flabellata*, Tate, sp.,	×		×
,, *subcuneata*, Kidston,	×		
Adiantites antiquus, Ett., sp.,	×		×
Cardiopteris polymorpha, Göpp., sp.,	×	×	×
Alcicornopteris convoluta, Kidston,	×	×	×
Eskdalia minuta, Kidston,	×	×	
Asterocalamites scrobiculatus, Schl., sp.,	×	×	×
Volkmannia, sp.,		×	
Pinnularia, sp.,		×	×
Lepidodendron Veltheimi, Sternb.,	×	×	×
,, *Volkmannianum*, Sternb.,	×		×
,, *Harcourtii*, Witham,	×		
Bothrodendron Kidstoni, Weiss,	×	×	
,, *Wükianum*, Kidston,		×	×
Lepidophyllum lanceolatum, L. and H.,	×	×	×
Lepidostrobus variabilis, L. and H.,	×	×	×
,, *fimbriatus*, Kidston,	×	×	
Stigmaria ficoides, Sternb., sp.,	×	×	×
,, ,, var. *undulata*, Göpp.,	×	×	×
Cordaites, sp.,		×	×
Samaropsis nervosa, Kidston, sp.,	×		
Rhabdocarpus curvatus, Kidston,	×		
Cordaicarpus planus, Kidston,	×		
Carpolithes, sp.,		×	
Pitys Withami, L. and H., sp.,	×		×
,, *primæva*, Witham, sp.,	×		×
Ptilophyton plumula, Dawson, sp.,	×	×	
Schutzia, sp.,		×	
Sorocladus antecedens, Kidston,	×		

* This specimen comes from Kirkcudbrightshire.

This table shows very clearly the great similarity of the fossil flora of Dumfriesshire and the North of England, and also its agreement with other areas of the Calciferous Sandstone series of Scotland.

I have little doubt that further collecting in Cumberland and Northumberland would augment the lists of Calciferous Sandstone plants from these counties.

INDEX.

EXPLANATION OF PLATES.

Plate I.

Fig. 1. *Bythotrephis gracilis*, Hall, var. *intermedia*, Hall. Arbigland, Kirkcudbrightshire. Collected by Mr J. Linn. Natural size. Reg. No. K/3236.

Figs. 2–3. *Bythotrephis Scotica*, Kidston. Glencartholm, Eskdale. Specimens in the Collection of the Geological Department of the British Museum. Natural size.

Fig. 4. *Eskdalia minuta*, Kidston. Glencartholm, Eskdale. Natural size. Reg. No. K/2740.

Figs. 5–6. *Eskdalia minuta*, Kidston. Scars enlarged from last specimen. Fig. 5 × 2.

Fig. 7. *Eskdalia minuta*, Kidston. Glencartholm, Eskdale. Natural size. Reg. No. K/2739.

Fig. 8. *Eskdalia minuta*, Kidston. Scars from last specimen, enlarged.

Fig. 9. *Pinakodendron Macconochiei*, Kidston, n.sp. River Esk, left bank, about 30 yards below junction of Byre Burn, Canonbie. Portion of cortex showing leaf scars and ornamentation of outer surface. × 3. Reg. No. K/3146.

Fig. 10. *Pinakodendron Macconochiei*, Kidston, n.sp. Same specimen as last. Natural size.

Fig. 11. *Pinakodendron Macconochiei*, Kidston, n.sp. Ornamentation of surface of cortex much enarged. Same specimen as fig. 10.

Fig. 12. *Cordaicarpus Cordai*, Geinitz, sp. A few yards above bridge at foot of Byre Burn, Canonbie. Natural size. Reg. No. K/3167.

Fig. 13. *Cordaicarpus Cordai*, Geinitz, sp. Same specimen enlarged × 3.

Plate II.

Fig. 14. *Rhabdocarpus curvatus*, Kidston, n.sp. Lewis Burn, rather over 200 yards below Lewis Burn Colliery, North Tynedale, Northumberland. Specimen in the Collection of the Geological Survey of Great Britain, Jermyn St., London. Natural size.

Fig. 15. *Stigmaria* (? *Stigmariopsis*) *rimosiformis*, Kidston, n.sp. River Esk, right bank, about 150 yards below Gilnockie Bridge, Eskdale. Natural size. Reg. No. K/3189.

Fig. 16. *Sphenopteris*, sp. Shore west of Budle, Northumberland. Specimen in the Collection of the Geological Survey of Great Britain, Jermyn St., London. Natural size.

Fig. 17. *Lepidodendron fusiforme*, Corda, sp. Near Halifax, Yorkshire. From plaster cast of impression in the Collection of the Geological Department, British Museum. Natural size.

Fig. 18. *Lepidodendron fusiforme*, Corda, sp. Outline sketch of cushion and leaf scar from last specimen. Natural size.

Fig. 19. *Lepidodendron Volkmannianum*, Sternb. Quarry one mile S.W. of Glororum farm-house, Bamborough, Northumberland. Specimen in the Collection of the Geological Survey of Great Britain, Jermyn St., London. Natural size.

Figs. 20–21. *Lepidodendron Glincanum*, Eichwald, sp. River Esk, right bank, about 200 yards above foot of Byre Burn, Canonbie. Natural size. Fig. 20, Reg. No. K/3125. Fig. 21, Reg. No. K/3124.

Plate III.

Fig. 22. *Lepidodendron fusiforme*, Corda, sp. Jockie's Syke, one mile E. by N. of Riddings Junction, Cumberland. Portion of specimen shown at fig. 25. × 2.

Fig. 23. *Lepidodendron fusiforme*, Corda, sp. Outline sketch of cushion and leaf scar from same specimen. × 2.

Fig. 24. *Lepidodendron fusiforme*, Corda, sp. Leaf scar from same specimen. × $3\frac{1}{2}$.

Fig. 25. *Lepidodendron fusiforme*, Corda, sp. Jockie's Syke, one mile E. by N. of Riddings Junction, Cumberland. Natural size. Reg. No. K/3186.

Fig. 26. *Sigillaria Canobiana,* Kidston, n.sp. River Esk, right bank, about 200 yards above foot of Byre Burn, Canonbie. Natural size. Reg. No. K/3118.

Figs. 27–28. *Lepidodendron Glincanum,* Eichwald, sp. River Esk, right bank, about 200 yards above foot of Byre Burn, Canonbie. Natural size. Fig. 27, Reg. No. K/3126. Fig. 28, Reg. No. K/3122.

Plate IV.

Fig. 29. *Sigillaria Canobiana,* Kidston, n.sp. Portion of specimen shown on Pl. III. fig. 26. × 2.

Fig. 30. *Sigillaria Canobiana,* Kidston, n.sp. Outline sketch of leaf scar and portion of rib from same specimen. × 2.

Figs. 31–32. *Sigillaria Canobiana,* Kidston, n.sp. River Esk, right bank, about 200 yards above foot of Byre Burn, Canonbie. Figure × 2. Reg. No. K/3115. Fig. 32, outline leaf scar from same specimen, × 2.

Fig. 33. *Sigillaria Canobiana,* Kidston, n.sp. River Esk, right bank about 200 yards above foot of Byre Burn, Canonbie. Natural size. Reg. No. K/3116.

Figs. 34–35. *Sigillaria Canobiana,* Kidston, n.sp. Portion of last specimen, × 2. Fig. 35, outline sketch of leaf scar of same, × 2.

Fig. 36. *Calamites* (*Calamitina*) *pauciramis,* Weiss. River Esk, right bank, about 30 yards below junction of Byre Burn, Canonbie. Natural size. Reg. No. K/3144.

Figs. 37–38. *Lepidodendron Glincanum,* Eichwald, sp. Portion of specimen shown on Pl. II. fig. 21, × 2. Fig. 38, outline sketch of cushion and leaf scar, × 2.

Figs. 39–40. *Lepidodendron Glincanum,* Eichwald, sp. Portion of specimen shown on Pl. II. fig. 20, × 2. Fig. 40, outline sketch of cushion and leaf scar, × 2.

Plate V.

Fig. 41. *Lepidodendron Glincanum,* Eichwald, sp., var *rimosum,* Schmalhausen. River Esk, right bank, about 200 yards above foot of Byre Burn, Canonbie. Natural size. Reg. No. K/3127.

Figs. 42–43. *Lepidodendron Glincanum,* Eichwald, sp. Portion of last specimen, × 2. Fig. 43, outline sketch of cushion and leaf scar, × 2.

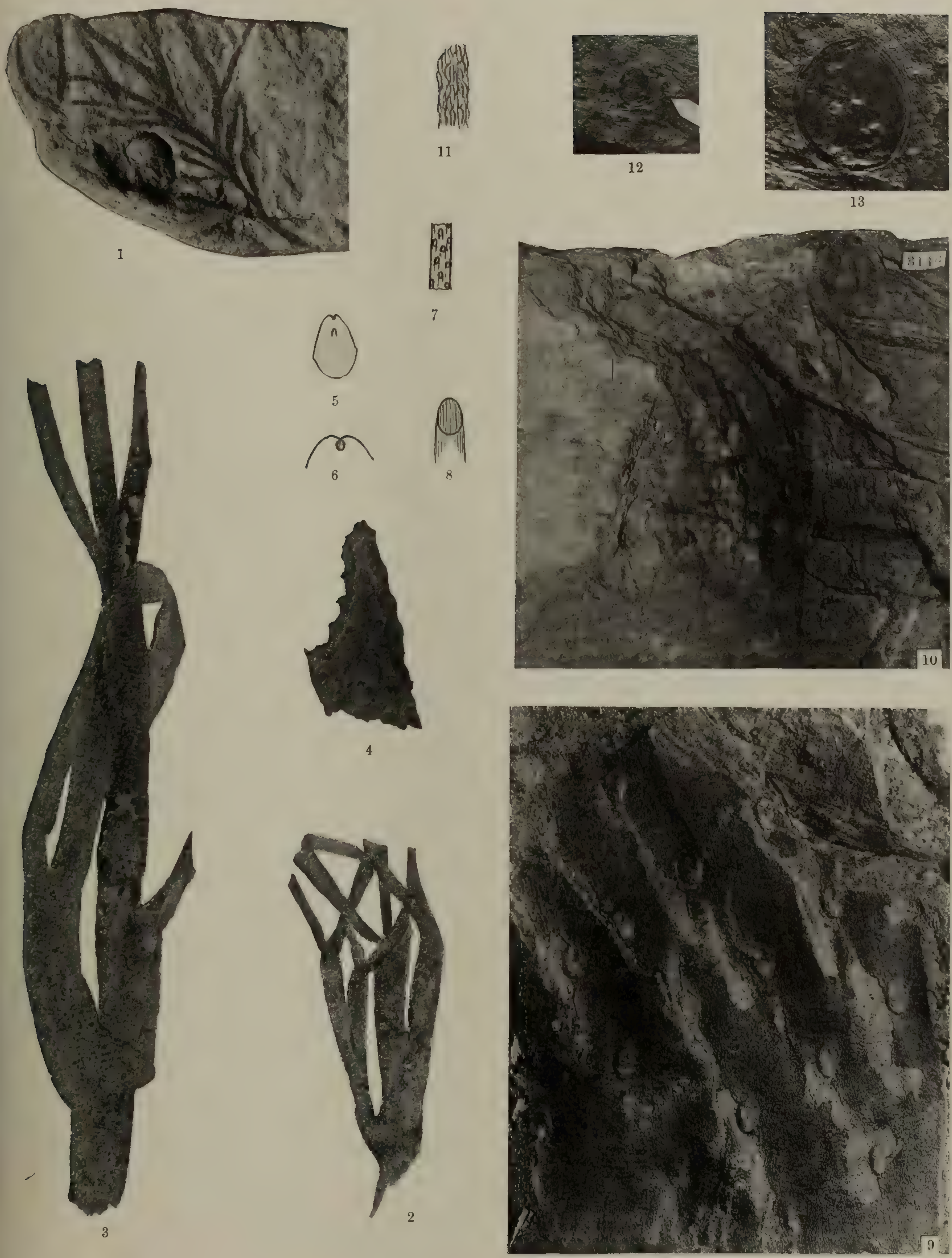
Fig. 44. *Calamites* (*Calamitina*) *pauciramis,* Weiss. River Esk, left bank, about 30 yards below junction of Byre Burn, Canonbie. Natural size. Reg. No. K/3145.

Fig. 45. *Sigillaria Canobiana,* Kidston, n.sp. River Esk, right bank, about 200 yards above foot of Byre Burn, Canonbie. Natural size. Reg. No. K/3120.

Figs. 46–47. *Sigillaria Canobiana,* Kidston, n.sp. Portion of same specimen as last, × 2. Fig. 47, outline sketch of leaf scar and rib, × 2.

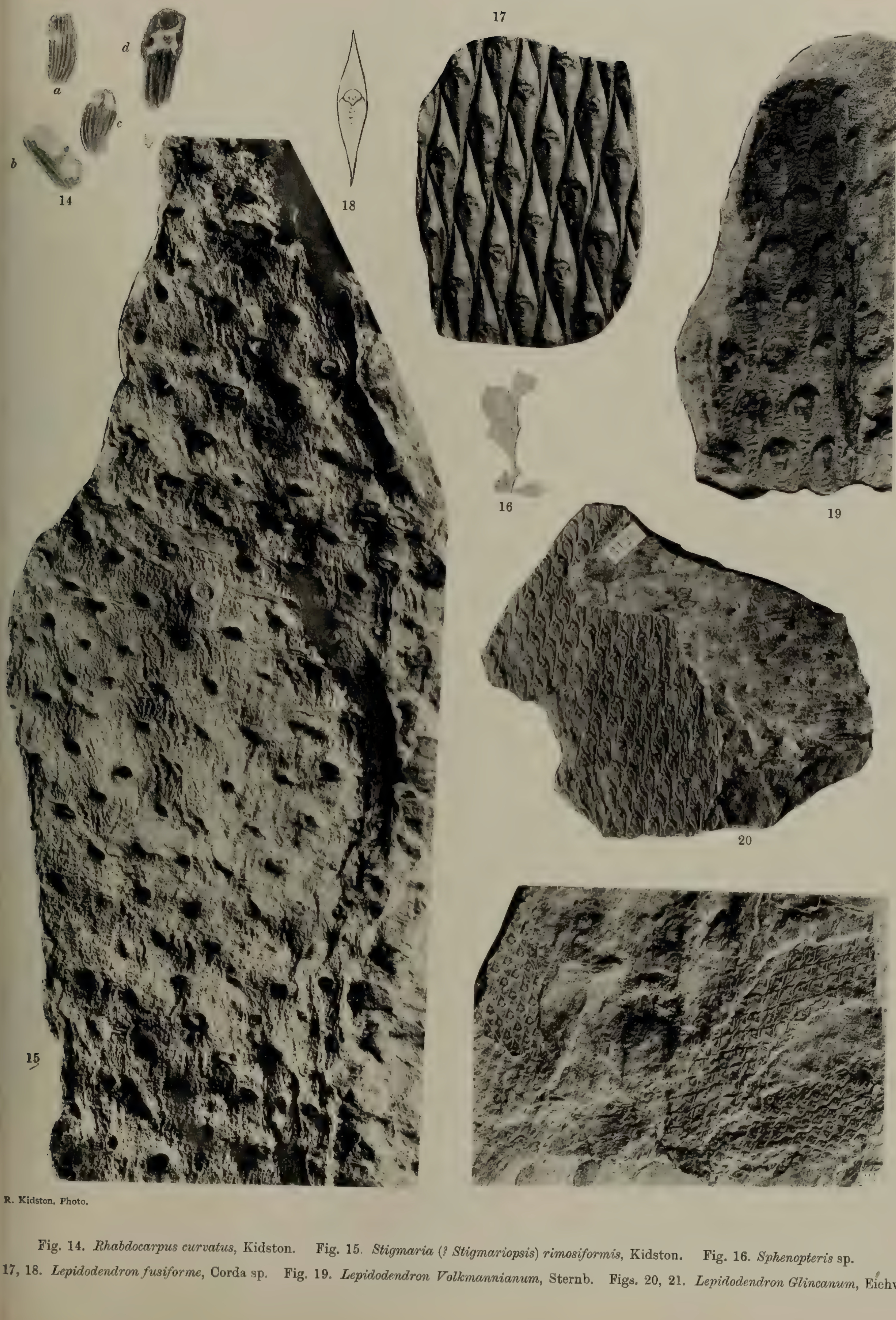
Figs. 48–50. *Rhodea Moravica,* Ettingshausen, sp. Shore west of Budle, Chesterfield Slakes, Northumberland. Specimens in the Collection of the Geological Survey of Great Britain, Jermyn St., London. Figs. 48–49, natural size. Fig. 50, pinnule enlarged.

Note.—The registration numbers distinguished by a "K" refer to specimens in the Author's collection.



Kidston, Photo.

g. 1. *Bythotrephis gracilis*, Hall, var. *intermedia*, Hall. Figs. 2, 3. *Bythotrephis Scotica*, Kidston. Figs. 4-8. *Eskdalia minuta*, Kidston.
Figs. 9-11. *Pinakodendron Macconochiei*, Kidston. Figs. 12, 13. *Cordaicarpus Cordai*, Geinitz sp.



R. Kidston, Photo.

Fig. 14. *Rhabdocarpus curvatus*, Kidston. Fig. 15. *Stigmaria (? Stigmariopsis) rimosiformis*, Kidston. Fig. 16. *Sphenopteris* sp.
Figs. 17, 18. *Lepidodendron fusiforme*, Corda sp. Fig. 19. *Lepidodendron Volkmannianum*, Sternb. Figs. 20, 21. *Lepidodendron Glincanum*, Eichw. sp.

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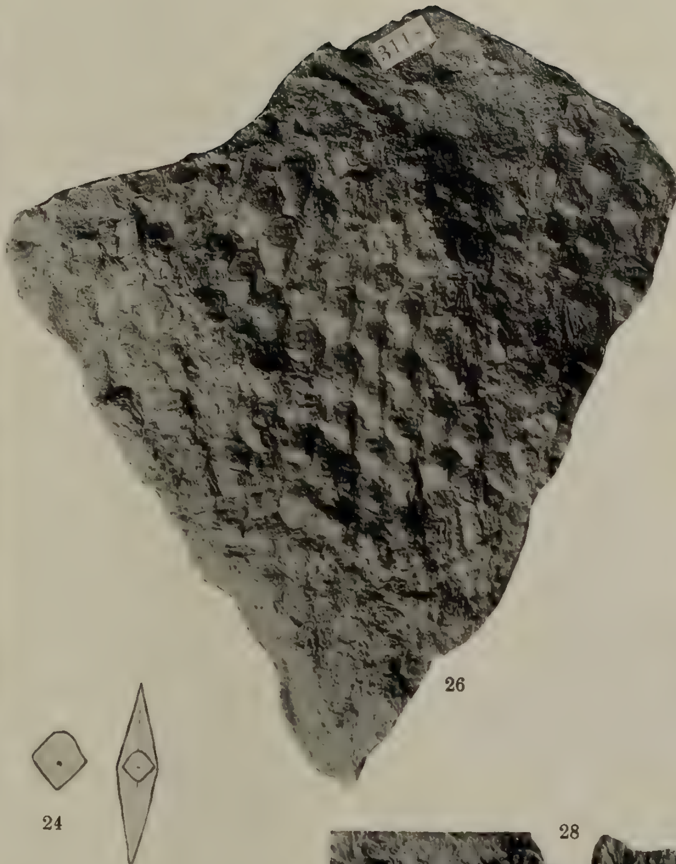

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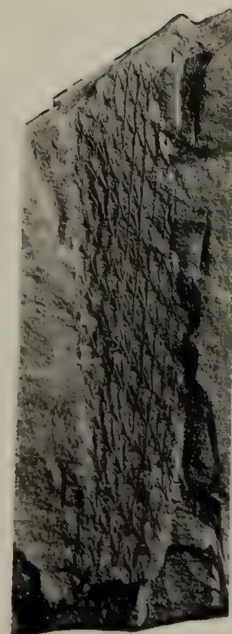

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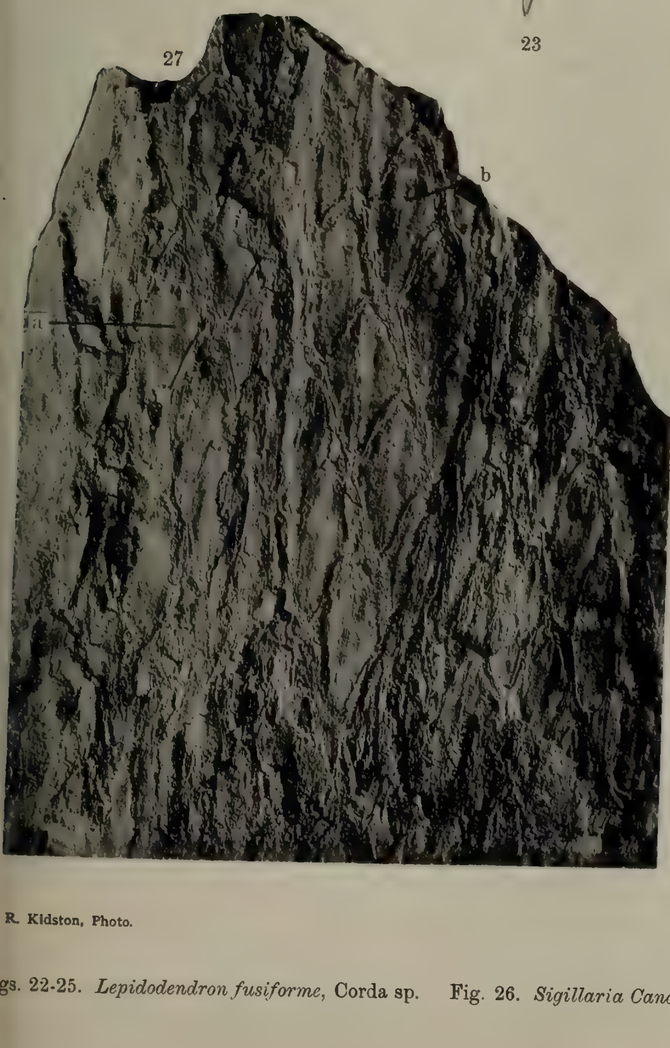

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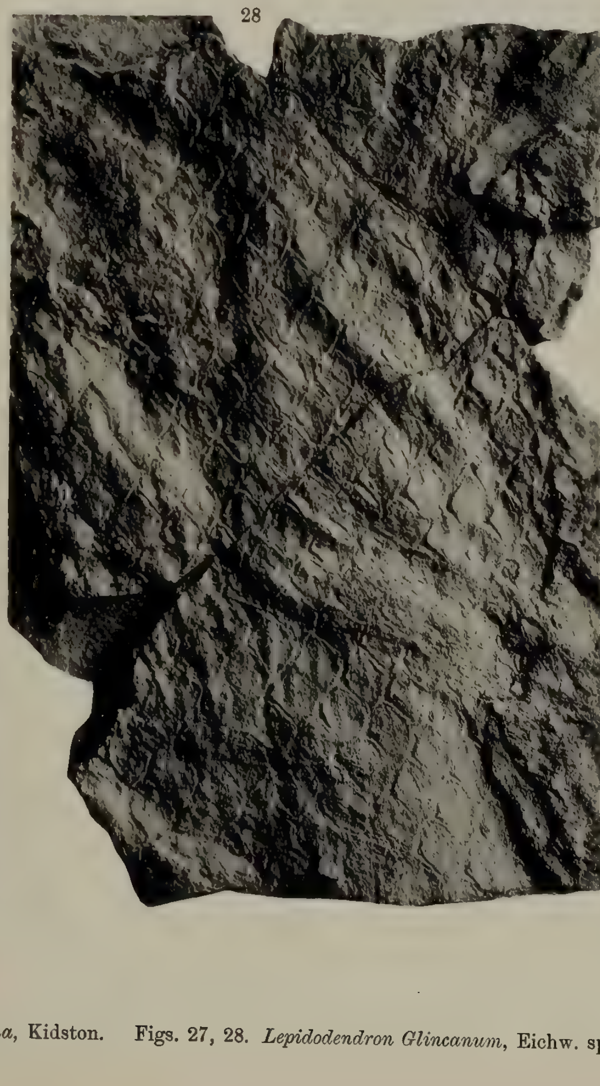

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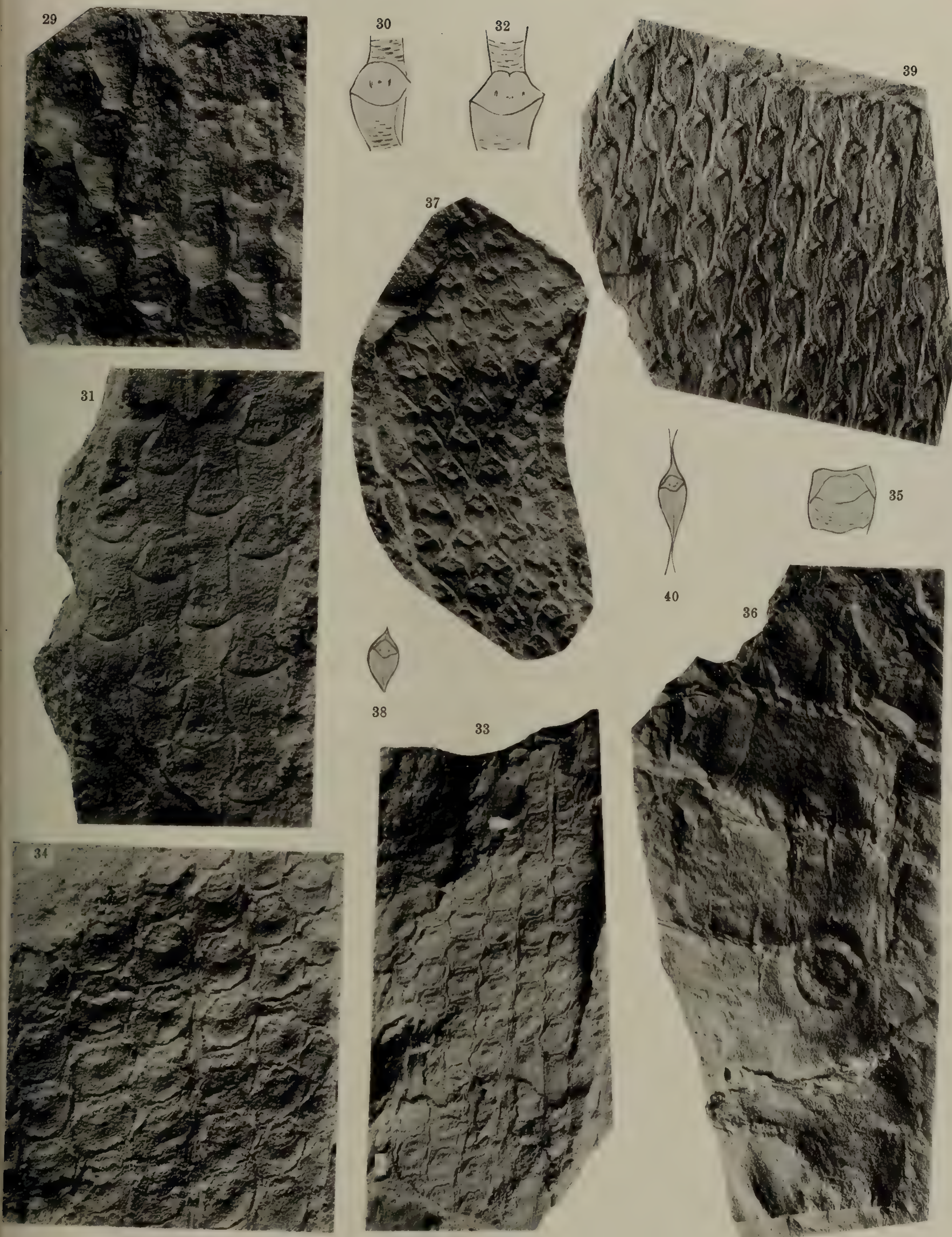
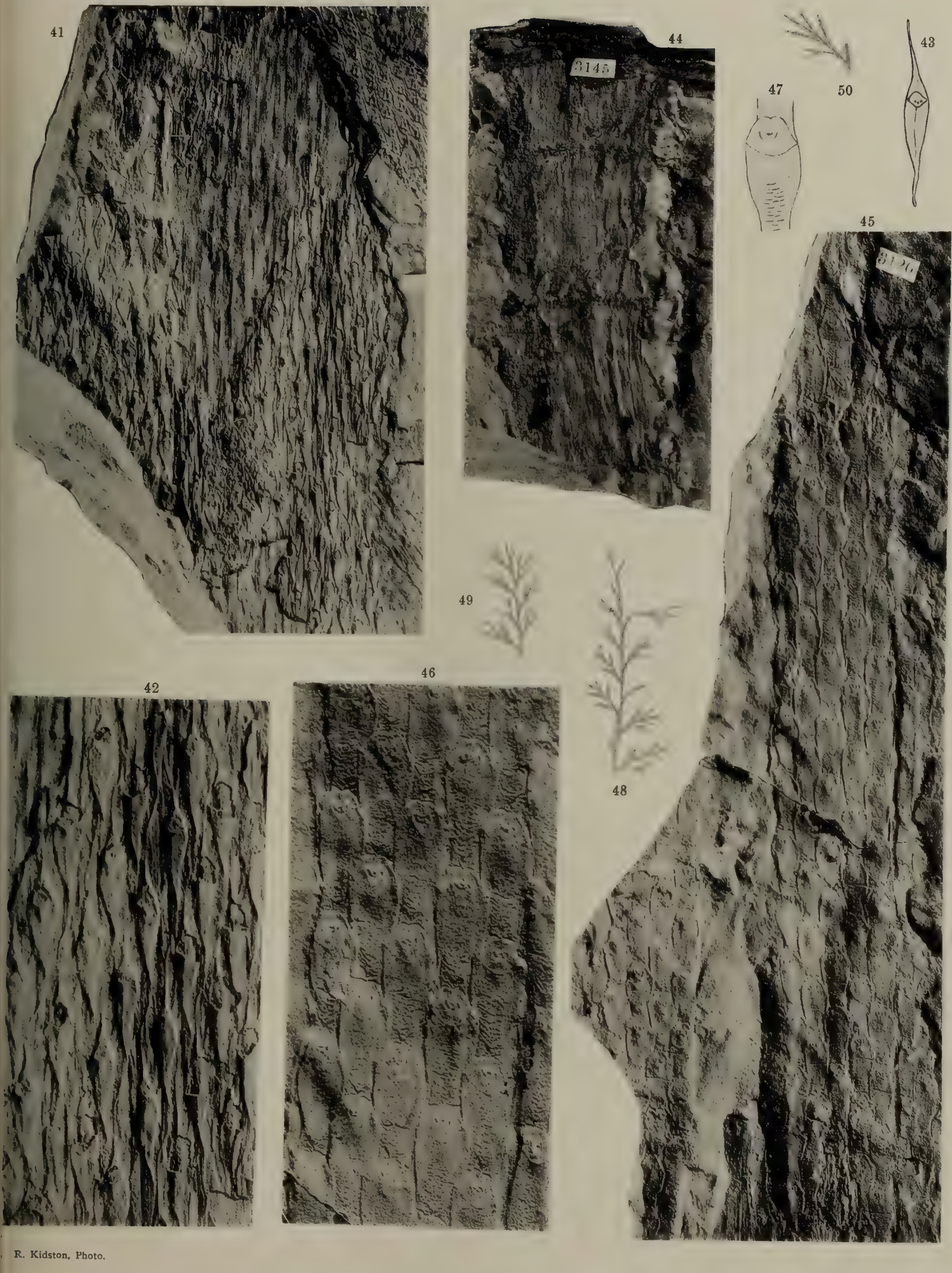
R. Kidston, Photo.

igs. 22-25. *Lepidodendron fusiforme*, Corda sp. Fig. 26. *Sigillaria Canobiana*, Kidston. Figs. 27, 28. *Lepidodendron Glincanum*, Eichw. sp.



R. Kidston, Photo.

Figs. 29-35. *Sigillaria Canobiana*, Kidston. Fig. 36. *Calamites* (*Calamitina*) *pauciramis*, Weiss. Figs. 37-40. *Lepidodendron Glincanum*, Eichw. sp.



R. Kidston, Photo.

Figs. 41-43. *Lepidodendron Glincanum*, Eichw. sp. Fig. 44. *Calamites* (*Calamitina*) *pauciramis*, Weiss. Figs. 45-47. *Sigillaria Canobiana*, Kidston.
Figs. 48-50. *Rhodea Moravica*, Ett. sp.

XXXII.—*The Canonbie Coalfield: its Geological Structure and Relations to the Carboniferous Rocks of the North of England and Central Scotland.* By B. N. Peach, LL.D., F.R.S., and J. Horne, LL.D., F.R.S. (With Four Plates.)*

(Read June 15, 1903. Given in for publication November 11, 1903. Issued separately December 31, 1903.)

CONTENTS.

I. Previous Researches.

The Canonbie Coalfield occupies a small tract of ground between the Liddel Water and the river Esk in the south-east part of the county of Dumfries. Though of limited extent, the coalfield has aroused considerable interest, due partly to the important series of plants obtained from the beds, and partly to the questions bearing on the correlation of the Carboniferous rocks of the Scottish Border with those in the North of England and Central Scotland.

In 1861 an elaborate paper, with numerous sections and a geological map, was communicated by Mr Edmund Gibsone to the North of England Institute of Mining Engineers on "The Border Districts of Dumfriesshire, Cumberland, and Part of Roxburghshire, including the Coal Formation of Canonbie."† The following classification of the Carboniferous rocks was adopted by the author. (1) The Carboniferous Limestone, comprising a lower series of sandstones, shales, and thin limestones, and an upper series consisting of thick limestones (Peterscrook, Harelaw Hill, Springkell, and

* Communicated by permission of the Director of H.M. Geological Survey.

† *Trans. North of England Inst. of Mining Engineers*, vol. xi. p. 65, 1861–2.

Kelhead), sandstones, black and blue shales, with numerous seams of coal, from a few inches to two feet thick. (2) The Coal Formation, including a lower group of sandstones and shales (Millstone Grit) underlying the Byre Burn coal-seams, and an upper group embracing the workable coal-seams of Rowanburn. In the geological map accompanying this paper, and in the descriptive notes, all the red sandstones lying to the south of the foregoing subdivisions are regarded as of Permian age; the boundary line between the two systems being a fault, which is referred to as "the great Permian Fault."

In 1863 an important advance in the classification of the Carboniferous rocks of the Canonbie district was made by Mr E. W. BINNEY, in a paper contributed to the Literary and Philosophical Society of Manchester on "Further Observations on the Carboniferous, Permian, and Triassic Strata of Cumberland and Dumfries.* While accepting Mr GIBSONE'S correlation of the Byre Burn and Rowanburn coal-bearing strata with the Coal-measures, he differed from him in regarding all the red sandstones to the south of the so-called great Permian fault as of Permian age. He contended that some of the red sandstones, as for instance those visible in the Esk north of Canonbie bridge, and in the Liddel south of Penton, belong to Upper Coal-measures. His reasons for this view were "that in their physical characters they are more like Carboniferous than Permian deposits, and that they contain the *Spirorbis* limestone, *Stigmaria ficoides*, and other Coal plants." Immediately to the north of Canonbie bridge, in certain red shales exposed in the Esk, Mr BINNEY found rootlets of *Stigmaria ficoides*, which deposits were regarded by him as "the highest Coal-measures ever yet noticed in Great Britain." Again, further up the river, at the Knotty Holm, he obtained plant-remains from a mottled sandstone, which he referred to *Calamites approximatus* and *Dadoxylon*. Still northwards in this section, but to the south of the great Permian fault defined by Mr GIBSONE, he noted a thin bed of limestone, six inches thick, in red and purple shales and clays, containing *Spirorbis carbonarius* and a *Cypris*? In view of this evidence, and on the assumption that the red sandstones of the Upper Coal-measures and the Middle Coal-measures of Byre Burn and Rowanburn are comformable, Mr BINNEY estimated that a bore sunk at Canonbie bridge would have to pass through from 350 to 400 fathoms of strata before reaching the workable coal-seams of Canonbie. This estimate is of special interest in the light of the bores put down in recent years by His Grace the Duke of Buccleuch, to which reference will be made in the sequel.

At a later date Mr BINNEY revisited the Canonbie district with his friend Mr J. W. KIRKBY, when he obtained further evidence in support of his correlation of some of the red sandstones of the Esk and the Liddel with the Upper Coal-measures.†

In 1876 the Geological Survey began the mapping of the Carboniferous tract of the Scottish border extending from Liddisdale westwards towards Annandale, the operations in the field being carried on by Mr R. LOGAN JACK, Mr SKAE, and Mr WILSON.

* *Memoirs of the Lit. and Phil. Soc. of Manchester*, third series, vol. ii. p. 343 : also an abstract of same paper, *Proc. of the Lit. and Phil. Soc. of Manchester*, vol iii. p. 162.

† "Note on the Upper Coal-measures of Canonbie, Dumfriesshire," by E. W. BINNEY, F.R.S., *Proc. of the Lit. and Phil. Soc. of Manchester*, vol. xvi. p. 192.

Before the survey of that district was completed Mr JACK, who had mapped the greater portion of the area, left for Queensland, and the completion of the work was shared by Mr B. N. PEACH. In the course of the survey great difficulty was experienced in correlating the subdivisions of the Carboniferous system as there developed with those of the Midland valley of Scotland, due partly to the variation in some of the groups from the normal Scottish types, and partly to the fact that the mapping of the Carboniferous rocks of the north of England had not been completed to the Scottish border. Eventually, the view was adopted and expressed in the Geological Survey map of the district (sheet 11—one inch) that the Canonbie coalfield belonged to the Calciferous Sandstone series, which represented part of the Carboniferous Limestone series of England.

The palæontological evidence, however, did not harmonise with this conclusion. After the mapping was completed Mr MACCONOCHIE began the fossil-collecting in that district, and obtained a series of plants from the Canonbie coalfield and from Carboniferous strata occupying a lower geological horizon. These plants were named and described by Mr KIDSTON, the results of his researches being published in the Transactions of this Society.* In his paper a list of the plants from that coalfield was given, but no geological horizon was assigned to them, out of deference to the view then held by the Geological Survey. On the evidence of the plants alone, he was led to the same conclusion as that of Mr GIBSONE and Mr BINNEY, that the coal-bearing strata of Byre Burn and Rowanburn belong to the Coal-measures. In the course of his work Mr MACCONOCHIE incidentally found plants in certain red shales at Jockie's Sike, near Riddings Junction, which suggested to Mr KIDSTON that these red shales and sandstones near the border might be the representatives of the Upper Coal-measures of England. This striking confirmation of Mr BINNEY's sagacious conclusion regarding the age of these sandstones was first announced by Mr KIDSTON in his presidential address to the Royal Physical Society, Edinburgh, in 1893.

It may be further noted that Mr MACCONOCHIE, while collecting the fossils from the massive limestones of Peterscrook, Harelaw Hill, and Gilnockie, was struck with the resemblance of the facies of organic remains to that found in the lower limestones of the Edge Coal series of the Midland valley of Scotland. This opinion was shared by the late Mr BENNIE, who compared the microzoa from the shales of the Gilnockie limestone with those obtained from the horizon of the Hurlet limestone of Fife.

The subsequent completion of the mapping of the Carboniferous rocks in Northumberland, northwards to Berwick and the Cheviots, threw much light on the sequence and peculiar lithological features of the members of that system in Liddisdale and Eskdale.

The revision of the Scottish coalfields, now in progress, furnished an opportunity

* "Report on Fossil Plants collected by the Geological Survey of Scotland in Eskdale and Liddisdale," *Trans. Roy. Soc. Edin.*, vol. xxx. p. 531.

last year of re-examining certain typical sections in Liddisdale, Eskdale, and westwards towards the Annan valley. A large collection of plants was obtained by Mr KIDSTON and Mr MACCONOCHIE, which are described by Mr KIDSTON in his paper now presented to the Society* on "The Fossil Plants of the Carboniferous Rocks of Canonbie, Dumfriesshire, and of Parts of Cumberland and Northumberland." On the evidence of the plants he correlates the Rowanburn coal-bearing group with the Lower Coal-measures, the Byre Burn group with the Middle Coal-measures, and the red sandstones and shales in the Liddel between Penton and Riddings Junction and in the Esk north of Canonbie bridge with the Upper Coal-measures of England.

Important information has been supplied by two deep bores sunk in recent years through the red sandstones and shales (Upper Coal-measures) near Canonbie. By the courtesy of His Grace the Duke of Buccleuch, we have been furnished with copies of the journals of these bores, and have received his permission to publish them. We likewise obtained leave to examine the cores of these deep bores, now stored at the Rowanburn Colliery; and in the course of our work we have been supplied with much information by the mining managers. For such valuable aid, so generously rendered, we desire to express our cordial thanks.

In the sequel, we propose to describe in turn the various subdivisions of the Carboniferous system in the tract extending from Liddisdale to Annandale, illustrating the geological structure of the region by a series of horizontal sections. Thereafter it will be shown by means of comparative vertical sections that the Lower Carboniferous sequence of the Scottish border closely resembles that of Northumberland, and differs in important points from that of Central Scotland.

II. DESCRIPTION OF THE CARBONIFEROUS SUBDIVISIONS IN ESKDALE AND LIDDISDALE.

The order of succession of the strata and the lithological characters of the various subdivisions of the Carboniferous system of the Scottish border is presented in the subjoined table:—

* *Trans. Roy. Soc. Edin*, vol. xl. pp. 741-833.

TABLE OF THE CARBONIFEROUS SYSTEM IN ESKDALE AND LIDDISDALE.

Trias		Brick-red sandstones and marls.
		Unconformability.
Upper Carboniferous	UPPER COAL MEASURES	Red Sandstones and Shales: stained in part.—With plants of Upper Coal-measures.
	MIDDLE COAL MEASURES	Byre Burn Coal group.—Sandstones, shales, coals, and thin ironstones; stained in part.—With plants of Middle Coal-measures.
	LOWER COAL MEASURES	Rowanburn Coal group.—Several workable coals; sandstones, shales, and ironstones.—With plants of Lower Coal-measures.
	MILLSTONE GRIT	Coarse sandstones, shales, and several thin coals; proved in bores.
Lower Carboniferous	UP. LS. GROUP	Marine limestones, sandstones, and shales. About 240 feet thick.
	COAL-BEARING GROUP	A group of five thin coals—Kilnholm coals (Horizon of Lickar and Edge coals)—sandstones and shales. Upwards of 340 feet thick.
	LOW. LS. GROUP	Group of marine limestones, sandstones, shales, and thin coals.
	LAWSTON LINN and MUIR BURN COAL GROUP	Sandstones, shales, marine limestones, thin coals, and ironstones.—Horizon of Lewisburn and Plashetts (Scremerston coals).
	GLENCARTHOLM VOLCANIC GROUP	Basic tuffs and lavas (Olivine-basalts), with interbedded shales and mudstones.
	FELL SANDSTONES	Grey and yellow sandstones, with red marls and thin impure limestones.
	CEMENTSTONE GROUP	Cementstones and impure limestones, clays, sandstones, and a zone of marine limestone near the top—(Larriston and Thorlieshope limestones).
	WHITA SANDSTONE	Grey and yellow sandstones.
	BIRRENSWARK VOLCANIC GROUP	Lavas (Olivine-basalts).
Old Red Sandstone	UPPER OLD RED SANDSTONE	Red sandstones and shales, with cornstone and chert at top.
		Unconformability.
Silurian		Silurian strata.

Before proceeding to the description of the subdivisions of the Carboniferous system, brief allusion may be made to the succession of red sandstones which, though they pass conformably upwards into that system, are grouped with the Old Red Sandstone, in virtue of their fish fauna.

Upper Old Red Sandstone.

Along the southern flanks of the Silurian tableland the members of this system rest unconformably on the folded and denuded edges of the Upper Silurian rocks. Owing to the uneven floor on which they were deposited, their thickness varies in every section where they are exposed. Near Langholm it is about 300 feet, and yet about three miles to the west of that town these rocks almost wholly disappear. Near the base the strata consists of reddish pebbly sandstones, composed mainly of materials derived from the Silurian tableland, but at certain localities, as for instance on the west slope of Whita Hill, there is an admixture of debris of igneous rocks resembling the Lower Old Red Sandstone andesites of the Cheviots. These are overlaid by red carious weathering sandstones, with occasional pebbles; and towards the top the calcareous matter is aggregated in knots and lenticles, evidently representing the horizon of the cornstone which occurs near the top of this formation. Indeed, near Riccarton, in the north-west, this zone does occur, where it is often accompanied by a lenticular red chert band. No fish-remains have been found in these strata near Langholm, but in the sandstones of Dinley Burn, near the Dinley Spout, a tributary of the Hermitage Water, scales of *Holoptychius nobilissimus* have been met with.

Lower Carboniferous Rocks.

i. *The Volcanic Rocks of the Tarras Water and Birrenswark.*

In the district now under consideration the Upper Old Red Sandstone strata are everywhere surmounted by a zone of contemporaneous volcanic rocks that form a well marked horizon in the geological sequence. The lava flows, which are usually slaggy and much decomposed, are of a basic character, ranging from olivine-basalts to andesitic basalts. Hardly any tuff or volcanic agglomerate has been observed in this volcanic zone. As might naturally be expected, few vents filled with volcanic agglomerate pierce the strata of a lower geological horizon to the north of the volcanic platform. But in the area north of the Tarras and Ewes Waters (sheet 17—one-inch) numerous plugs or stocks of intrusive igneous rocks, of intermediate or acid types, resembling those of the Eildons near Melrose, appear within the Silurian area. An excellent example of a plug of andesitic basalt rising through the Upper Old Red Sandstone underlying the volcanic zone is to be found on Arkleton Hill, six miles north of Langholm, in the basin of the Ewes Water.

This volcanic zone forms a narrow fringe round the Carboniferous area, though the outcrop is much interrupted by faults, some of which are of considerable magnitude. It has been traced for several miles along the eastern margin of the Silurian inlier near Riccarton. From Dinley on the Hermitage Water, it has been followed at intervals across the heights to the Whita Hill near Langholm, thence westwards by Waterbeck and Middlebie to Birrenswark, and beyond the Dumfries basin of the New Red Sandstone it reappears in the district of Kirkbean.

ii. *The Whita Sandstone.*

The volcanic zone of Birrenswark and the Tarras Water is overlaid by a group of sandstones which are characteristically developed on the Whita Hill, about half a mile east of Langholm. At the base, the beds consist of pink pebbly sandstones, the pebbles being well rounded and composed of vein quartz. These pass upwards into yellowish or grey mottled gritty sandstones, with marked false-bedding, and containing clay galls and specks of decomposing carbonates. Near the base of this type, and above the pinkish sandstone, there are occasional lenticular beds of ochreous cementstone. The highest members of the group consist usually of coarse, mottled, carious weathering sandstones, with decomposed, rusty carbonates. Occasionally beds of greenish shale and cementstone are intercalated in the series near the top. It is highly probable that the galls of shale and particles of carbonate found in the sandstone have been derived from the erosion of the beds of shale and cementstone that appear to have been laid down during pauses in the deposition of coarser sediment. The thickness of this zone ranges from 600 to 700 feet.

In the Esk south of Langholm, from Longwood to below Broomholm—a distance of about a mile—this arenaceous group may be traced in natural sequence above the volcanic zone of the Tarras Water and Birrenswark. Thence they stretch northwards, forming the tops of the Fells from the Whita Hill to Dinley Fell near the Hermitage Water—a distance of nearly twelve miles. Across the Hermitage Water they may be followed north-eastwards as far as the Whitterhope Edge, where they are truncated by a fault. A second outcrop of this group of sandstones appears in the upper part of the valley of the Liddel and to the east of the Silurian inlier at Riccarton. From Liddel Castle, about three miles above New Castleton, it extends north-eastwards, flanking the rocks of the Birrenswark volcanic zone, across the Riccarton, the Dawston, and the Caddroun Burns, beyond which the sandstones are obliquely truncated by a fault. Along this outcrop the Whita sandstones cannot exceed 300 feet in thickness, and they appear to thin out towards the north-east.

iii. *Cementstone Group.*

This subdivision consists mainly of green, blue, and grey mudstones and sandy shales, with bands of impure muddy limestone and cementstone, with occasional intercalations of coarse grey calcareous sandstone. Near the top there is a zone of true marine limestone. In the Langholm district the thickness of this group varies from 1200 to 1500 feet.

The cementstone group of Eskdale and Liddisdale was evidently deposited along shore under estuarine conditions, the mudflats being suitable for the growth of lamellibranchs, which occur in profusion in certain beds. Indeed, in some bands one species of *Modiola* appears, to the exclusion of every other form. The characteristic shell is *Modiola Macadami* and its varieties, but other bivalves also occur, though less abundantly. A good section of these beds exposed in the cliff of the Liddel Water opposite the manse, about two miles above New Castleton, yielded the following assemblage of lamellibranchs, with which *Spirorbis* is usually associated :—

Spirorbis, sp.
Anthracomya subparallela.
Aviculopecten Geikiei, Eth. jnr. M.S.
Edmondia josepha, De Kon.
Leiopteria, sp.
Modiola Macadami, Portl.
Myalina sublamellosa, Eth. jnr.
„ *Verneuili*, M'Coy.
Naiadites, sp.
Nuculana attenuata, Flem.
„ *stilla*, M'Coy.
Protoschizodus axiniformis, Portl.
„ *nuculoides*, M'Coy.
„ sp.
Sanguinolites roxburgense, Hind.
Schizodus, sp.
Tellinomorpha cuneiformis, De Kon.

The forms given in the foregoing list are often associated with remains of the higher crustacea of the genera *Palæocrangon*, *Anthrapalæmon*, and *Pseudogalathea*, while entomostraca are frequently abundant. Among the Xiphosura, *Prestwichia* and *Cyclus* occur. Occasionally there is evidence of purer marine conditions, indicating incursions of the sea, as we may gather from the presence of *Lingula squamiformis*, *Discina nitida*, and more commonly *Camarophoria crumena*. Some of the beds of thin limestone in this group are composed mainly of the remains of the last of these forms in association with *Athyris ambigua* and *Orthotetes crenistria*. Such marked proofs of more marine conditions are rare in the lower part of the Cementstone group, embracing about 500 feet of strata ; but above that level they are more numerous. Indeed, from the thickness of the marine limestones which have been worked in the upper part of Liddisdale, in the valley of Larriston Burn, Thorlieshope, and near Dead Water, on this horizon, it is clear that marine conditions must have lasted for considerable intervals of time. This zone is characterised by a form of *Syringothyris* (*Spirifera*) *cuspidata*, which, with one exception, has been found in this area only in this zone. The exception referred to occurs on a slightly higher horizon than the Larriston Burn limestone.

A list of fossils is subjoined from the limestones of Thorlieshope and Larriston Burn, which is sufficient to demonstrate that marine conditions prevailed during their deposition. At the same time it may be observed that many forms which appear in the marine limestones overlying the Fell sandstones are absent from this list. The abundance of lamellibranchs and gasteropods in this marine zone below the Fell sandstones seems to imply that a considerable amount of sediment was then present in the sea-water.

List of fossils from the marine limestones of Thorlieshope and Larriston Burn :—

Chætetes tumidus, Phill.
Palæacis cyclostoma, Phill.
Syringopora ramulosa, Goldf.
Lithostrotion junceum, Flem.
Palæchinus globulus.
Poteriocrinus crassus, Miller.
Spirorbis ambiguus, Flem.
Entomostraca.
Athyris ambigua, Sow.
„ *Roysii*, Lév.
Camarophoria crumena, Mart.
Lingula scotica, Dav.
Productus longispinus, Sow.
„ *semireticulatus*, Mart.
Rhynchonella, sp.
Spirifera bisulcata, Sow.
„ *trigonalis*, Mart.
Syringothyris (*Spirifera*) *cuspidata.*
Allorisma sulcata, Flem.
Aviculopecten cœlatus, M'Coy.

Aviculopecten interstitialis, Phill.
„ *macrotis*, M'Coy.
Modiola Macadami, Portl.
„ sp.
Myalina sublamellosa, Eth. jnr.
Nuculana attenuata, Flem.
Nucula stilla, M'Coy.
Protoschizodus axiniformis, Portl.
Sanguinolites costellatus, M'Coy.
„ *striato-lamellosus*, De Kon.
„ sp.
Tellinomorpha cuneiformis, De Kon.
Edmondia josepha, De Kon.
Edmondia pectunculus.
Entolium Sowerbyi, M'Coy.
Loxonema curvilinea, Phill.
„ sp.
Murchisonia Verneuliana, De Kon.
Macrocheilus acutus, Sow.
Naticopsis plicistria, Phill.
Ivania (*Pleurotomaria*) *Ivani*, Lév.
Conularia quadrisulcata, Sow.
Discitoceras (*Discites*) *sulcatus*, Sow.
Orthoceras attenuatum, Flem.
„ *lineale*, De Kon.
„ sp.

Above the horizon of the marine limestones of Thorlieshope the strata indicate a reversion to estuarine conditions. Immediately below the Fell sandstones, however, a well marked and persistent band of limestone appears, charged with a peculiar calcareous organism, named *Mitcheldeania gregaria* by the late Professor ALLEYNE NICHOLSON. It occurs in the well known limestone at Kershopefoot and at Kidds Linn.

Though land plants occur more or less frequently throughout the group, it is only occasionally that a true land surface is indicated by an under-clay with underground rhizomes such as *Stigmaria* in place. One thin coal-seam, however, about ten inches thick, is found near the top of the Cementstone group, under Peel Fell on the Scottish side, which is the lowest known coal-seam on the border. The characteristic ferns of the group are *Calymmatotheca* (*Sphenopteris*) *affinis* and *C. bifida*; at Tarrasfoot *Rachopteris inæquilatera* also occurs.

The fishes found in the Cementstone group are not of common occurrence, and, as might be expected, are almost wholly of an estuarine character. These have been submitted to Dr TRAQUAIR for determination, who has identified the following forms in the collection : *Strepsodus*, sp., *Rhadinichthys Macconochiei*, Traq., and *Styracopterus fulcratus*, Traq. In the marine limestone zone of Thorlieshope and Larriston Burn palatal teeth of marine sharks have been obtained.

Land animals also occur in the form of Scorpions, while Myriapods belonging to more than one genus have been found in rocks of this horizon in the basin of the Tweed near Coldstream.

From the peculiar lithological characters of the Cementstone group, the distribution of the beds is comparatively clear in Eskdale and Liddisdale. In the Esk and in the lower part of the basin of the Tarras Water they form a simple outcrop overlying the Whita sandstone, and dipping generally in a southerly direction at an average angle of about 20°. When followed north-eastwards to the Tinnis Burn they are spread over a wide area, owing to repetition by folding, and occupy a basin three miles in width, which extends up the Hermitage Water towards the northern margin of sheet 11 of the one-inch map of Scotland. On its eastern side this basin is truncated by a powerful north and south fault which brings the Cementstone group successively in contact with the Upper Silurian inlier of Arnton Fell near Riccarton, and with the Upper Old Red sandstone and Birrenswark volcanic zone near the junction of the Liddel and Hermitage Water. (See Plate III. section 1.)

In Upper Liddisdale, to the east of the Silurian inlier just referred to, near Riccarton, the Cementstone group is again repeated, resting in natural sequence on the Whita sandstone and Birrenswark volcanic zone, and dipping towards the south-east. In that district they floor the course of the Liddel Water, and form the lower slopes of the Larriston Fells. Owing to minute folding of the beds it is difficult to give an accurate estimate of their thickness, but it is probably about 1200 feet. The Larriston Burn furnishes a good section, especially of the marine limestone zone, near the top.

iv. *The Fell Sandstones.*

The Cementstone group of Liddisdale and Eskdale is overlaid by a succession of sandstones, with intercalations of red and green marly clays, and occasional impure cementstone bands, varying in thickness from 400 to 600 feet. The sandstones are siliceous and usually fine-grained, but sometimes become coarse and pebbly. At certain localities they contain marine fossils such as *Aviculopecten*, while the impure limestone bands contain cyprids and modioliform shells, but there is no indication of clear water conditions.

In the district of Peel Fell there is evidence of successive land surfaces in the form of dirt beds, and even of thin coal-seams, which accompany the red and green marls and impure fireclays separating the beds of sandstone.

From a stratigraphical point of view this group of sandstones is of great importance, inasmuch as the zone is persistent and easily traceable. They form much of the higher part of the Larriston Fells, where they lie in a synclinal fold, overlaid by the upper volcanic zone at the base of the Lewisburn coal-bearing beds (Scremerston position), to which reference will be made in the sequel. (See Plate III. section 1.) When traced towards the south-west, owing to the fall in the ground, the Fell sandstones appear in the centre of the trough, but on Caerby Hill they are capped by the upper volcanic zone. Crossing the Liddel at Kershopefoot, they extend westwards along the slopes south of the Tinnis Burn, and form the high ground separating that stream from the Tarras Water. They are visible in the Esk at Irvine House, and towards the south-west they are traceable across the moorland to the south of Ecclefechan, where they form the prominent eminences of Brown Moor and Woodcock Air, on either side of the Annan. West of the Nith this zone appears on the shore between Arbigland and Southerness Point.

v. *Glencartholm Volcanic Group.*

Next in order above the Fell sandstones comes the volcanic group of Glencartholm, which, though of no great thickness, has been of service in working out the stratigraphical arrangement of the beds between the Esk and the Liddel. In the Esk section the group consists of fine decomposing basic tuffs and thin basic lava, in the midst of which there is a zone of sediments, comprising black shales, oil shale, and black cherts, followed by fine-grained calcareous shale of unique palæontological importance. From this horizon of calcareous shale a great variety of organic remains has been obtained, including plants, ostracods, brachiopods, lamellibranchs, cephalopods, crustaceans, fishes and land animals (scorpions and eurypterids). The extraordinary feature of the band is the very large number of new genera and species gathered from this single exposure in the Esk, which has made it one of the classic fossil-localities in Scotland. The discovery was made by the skilled fossil-collector of the Geological Survey, Mr A. Macconochie. From the fact that these sediments are both underlaid and overlaid by tuff, it is evident that they are merely an episode in the phase of volcanic activity on this horizon. Indeed, it is worthy of note that this rich palæontological zone, though carefully sought for, has not been found at any other locality.

Though a great variety of forms has been obtained from the Glencartholm shales, it is interesting to observe that they are not equally distributed through the successive layers. For example, the fishes are usually found underneath a band in which *Orthoceras* is a conspicuous fossil. The scorpions and plants usually occur together in a separate bed, while the crustaceans are found in association with the fishes. The ferns are usually represented by separate fronds, and they are often covered with a calcareous incrustation, as if they had floated about in concentrated calcareous solutions before becoming embedded. No coal-seam nor root-bed appears in this zone. The remains of the crustacea seem to have been filled in with orbicular calcite during decomposition, as if they had lain in water highly charged with calcium sulphate. This feature seems to point to lagoon conditions, as if arms of the sea had been temporarily cut off from the open ocean and subjected to desiccation. Strings of *Spirorbis* and of an adherent brachiopod shell are often found, fixed to carbonaceous stems of decomposing plants. Some of the bands of shale are covered with the chitinous tubes of marine worms. Sea-weeds are represented by *Bythotrephis*.

In view of the evidence regarding the conditions of entombment of the organic remains, it is highly probable that the Glencartholm shales may have been deposited in a muddy creek, shut off at intervals from the open ocean.

In his recent valuable paper "On the Distribution of Fossil Fish-remains in the Carboniferous Rocks of the Edinburgh District,* Dr Traquair states that out of the large number of fishes found at Glencartholm, only one (*Tristychius minor*) is found in the Lower Carboniferous rocks of central Scotland. But it is quite possible, when the divisions of the Lower Carboniferous rocks in Berwickshire, the Lothians, Fife, and the West of Scotland have been thoroughly searched, some forms, now restricted to Glencartholm, may be found. The present revision of the coalfields furnishes an opportunity of testing this question. In connection with this point it may be observed that some of the crustaceans which were at first thought by Dr Peach

* *Trans. Roy. Soc. Edin.*, vol. xl. p. 687.

to be peculiar to the Glencartholm shales, have since been proved to possess a wide distribution. Some have been found in the Cementstone group in the Whiteadder and Blackadder sections in Berwickshire and at Belhaven Bay in Haddingtonshire, and some on still higher horizons in the Granton sandstone at Craigleith, and in the Wardie shales on the shores of the Firth of Forth. Indeed, Dr PEACH is confident that further search may extend the present known limits of their distribution.

The subjoined list gives the fossils collected from the Glencartholm shales.*

Bythotrephis acicularis, Göpp., sp.
„ *plumosa*, Kidston, sp.
„ *simplex*, Kidston, sp.
„ *Scotica*, Kidston.
Calymmatotheca bifida, L. and H., sp.
Sphenopteris crassa, L. and H.
„ *pachyrrhachis*, Göpp.
„ *obovata*, L. and H.
„ *Hibberti*, L. and H., *var.*
„ *decomposita*, Kidston.
„ *Macconochiei*, Kidston.
Rhodea Machaneki, Ett., sp.
Rhacopteris inæquilatera, Göpp., sp.
„ *Geikiei*, Kidston, sp.
Cardiopteris polymorpha, Göpp., sp.
Eskdalia minuta, Kidston, sp.
Asterocalamites scrobiculatus, Schl., sp. = *Pothocites Grantoni*, Paterson.
Volkmannia, sp.
Lepidodendron Veltheimi, Sternb.
Lepidophyllum lanceolatum, L. and H.
Lepidostrobus variabilis, L. and H.
„ *fimbriatus*, Kidston.
Carpolithes, sp.
Ptilophyton plumula, Dawson, sp.
Beyrichia gigantea, Jones.
Acanthocaris elongatus, Peach.
„ *attenuatus*, Peach.
„ „ *scorpioides*, Peach.
Anthrapalæmon Etheridgei, Peach.
„ „ var. *latus*, Peach.
„ *formosus*, Peach.
Pseudogalathea Macconochiei, Eth. jnr., sp.
Rostrocaris falcatus, Peach.
„ *Traquairi*, Peach.
Palæosquilla Parki, Peach.
„ sp.
Palæocaris scoticus, Peach.
Palæocrangon elegans, Peach.
„ *eskdalensis*, Peach.
Eoscorpius euglyptus, Peach.
„ *glaber*, Peach.
„ sp.
Glyptoscorpius, sp.
Cyclus testudo, Peach.
Prestwichia rotundata, Peach.
Discina nitida, Phill.
Lingula mytiloides, Sow.
„ *squamiformis*, Phill.
„ sp.
Productus semireticulatus, Martin.
Small adherent brachiopod.
Avicula Hendersoni, Eth.
Aviculopecten Geikiei, Eth., M.S.
„ *eskdalensis*, Hind.
„ *interstitialis*, Phill.
„ *papyracea*, (?) Goldf.
„ *planicostatus*, M'Coy.
„ sp.
Edmondia josepha, De Kon.
„ sp.
Entolium Sowerbyi, M'Coy.
Leiopteria divisa, M'Coy.
Lithodomus carbonarius, Hind.
Modiola Macadami, Portl.
Myalina sublamellosa, Eth. Jnr.
„ sp.
Pinna mutica, M'Coy.
Posidonomya radiata, Hind.
Pteronites angustatus, M'Coy.
Sanguinolites variabilis, M'Coy.
Sedgwickia ovata, Hind.
Euomphalus catillus, Sow.
„ *pentangulatus*, Sow.
Murchisonia sulcata, M'Coy.
Naticopsis plicistria, Phill.
Conularia quadrisculcata, Sow.
Orthoceras, sp.
Acanthodes nitidus, A. S. Woodw.
Acrolepis ortholepis, Traq.
Canobius elegantulus, Traq.
Cheirodopsis Geikiei, Traq.
Chondrenchelys problematica, Traq.
Cladodus, sp.
Cœlacanthus Huxleyi, Traq.
Cycloptychius concentricus, Traq.
Elonichthys pulcherrimus, Traq.
„ *serratus*, Traq.

* The list of fishes from Glencartholm given by Dr TRAQUAIR in his paper already referred to has been embodied in the above list.

Eurynotus, two species.
Mesolepis rhombus, Traq.
„ *tuberculatus*, Traq.
Mesopoma politum, Traq.
„ *pulchellum*, Traq.
„ *Ramsayi*, Traq.
Phanerosteon mirabile, Traq.
Platysomus supberbus, Traq.
Rhadinichthys angustulus, Traq.
Rhadinichthys delicatulus, Traq.
„ *fusiformis*, Traq.
„ *Macconochiei*, Traq.
„ *tuberculatus*, Traq.
Sphenacanthus costellatus, Traq.
Strepsodus, two species.
Tarrasius problematicus, Traq.
Tristychius minor, Portl.

The tuffs of the Glencartholm volcanic group have been traced for five miles towards the E.N.E. to the head of Muir Burn that joins the Liddel Water at Liddel Bank, but the richly fossiliferous shales have not been found in association with them. Chert beds, however, accompany the tuffs and basaltic lavas in Muir Burn. The small outlier of basic lava that caps the hill near Dinwoodie, east of Muir Burn, is probably on this horizon. East of the Liddel we encounter this volcanic zone in the Kershope Burn, about half a mile above its junction with that stream, whence its outcrop sweeps northwards by the top of Caerby Hill, and curves eastwards till it recrosses the Kershope Burn and passes into Cumberland, about a mile and a half north-east of Kershopefoot Station. Owing to the synclinal arrangement of the strata in Larriston Fells, the basaltic lava on this horizon forms in that region a narrow outcrop, encircling sediments at the base of the overlying Lewis Burn coal-bearing group.

West of the Esk, this volcanic zone can be traced up the south bank of the Irvine Burn, and still further to the west it appears in the Palling Burn—a tributary of the Water of Sark—about four miles W.S.W. of Glencartholm.

vi. *Marine Limestone Series with Coal-seams.*

In Eskdale and Liddisdale the Glencartholm volcanic zone is followed in natural sequence by sediments in which marine limestones are a prominent feature with thin coal-seams on two horizons which have been worked at certain localities. The members of this series may be classified as follows :—

	Thickness.
4. Upper Limestone group	300–400 feet
3. Kilnholm Coal group (horizon of Lickar coals) . . .	342 „
2. Lower Limestone group	500–700 „
1. Lawston Linn and Lewis Burn Coal group (horizon of Scremerston coals)	400–500 „

1. *The Lawston and Lewis Burn Coal Group.*—In Eskdale and Liddisdale the members of this subdivision consist of sandstones, shales, thin coals, and thin marine limestones. At Lawston Linn, on the Liddel, a coal from one foot six inches to two feet thick was formerly extensively wrought on this horizon, which is the most prominent seam. Other coals of less thickness, together with an oil shale, also occur at that locality. Again, at Muir Burn, near the head of Archer Beck, similar thin coals appear in this position. In the Esk section thin coals almost immediately succeed the

Glencartholm volcanic tuff, occupying the same relative position as the Lawston seams, but too thin to be of any economic value. They are visible also in tributaries of the Esk, on both sides of Glencartholm, where, as in the Esk, they frequently have limestone roofs.

A limestone, from six to eight feet thick, almost immediately overlies the coal formerly wrought at Lawston Linn, which, from the fossils given in the annexed list, is of undoubted marine origin:—

Clisiophyllum, sp.	*Productus giganteus*, Mart.
Lithostrotion cæspitosum, Mart.	„ *punctatus*, Mart.
Athyris ambigua, Sow.	*Spirifera trigonalis* var. *bisulcata*, Mart.

Calcareous nodules in soft shales from the same locality yielded the following assemblage of organic remains, which indicate less purely marine conditions.

Clisiophyllum, sp.	*Allorisma sulcata*, Flem.
Lithostrotion irregulare, Phill.	*Aviculopecten cœlatus*, M'Coy.
Archæocidaris Urei, Flem.	„ *Geikiei*, Eth. M.S.
Fenestella, sp.	„ *interstitialis*, Phill.
Athyris ambigua, Sow.	*Edmondia sulcata*, Phill.
„ *Roysii*, Lév.	*Nuculana attenuata*, Flem.
Camarophoria crumena, Mart.	*Pteronites angustatus*, M'Coy.
Productus punctatus, Mart.	*Sanguinolites roxburgensis*, Hind.
Spirifera lineata, Mart.	„ *variabilis*, M'Coy.
Syringothyris (*Spirifera*) *cuspidata*, Mart.	

Similar fossil-lists might be supplied from the limestones on this horizon in Archer Beck and in the Esk.

In the Kershope Burn, not far up stream from Kershopefoot, the lowest beds of this group rest on the Glencartholm volcanic zone, where two coal-seams were formerly wrought on the English side of the border. Again, in the upper part of Tweeden Burn and its tributaries, south-east of New Castleton, there are sections showing outcrops of coal-seams, some of which seem to have been formerly wrought, in association with sediments that overlie the upper volcanic zone so well seen on the Fell top, near Tweedenhead. These strata are evidently the continuation of those forming the Lewis Burn Coal group just across the border. The limestones on this horizon in the Tweeden Burn do not indicate such clear water-conditions as those of the Liddel, Muir Burn, or Archerbeck, as shown by the fossils in the subjoined list.

Lingula mytiloides, Sow.	*Leiopteria*, sp.
„ *squamiformis*, Phill.	*Naiadites* (*Myalina*) *crassa*, Flem.
Camaraphoria crumena, Mart.	*Nuculana attenuata*, Flem.
Productus cora, d'Orb.	*Protoschizodus axiniformis*, Portl.
Aviculopecten cœlatus, M'Coy.	*Sedgwickia ovata*, Hind.
„ *dissimilis*, Phill.	*Bellerophon hiulcus*, Sow.
„ *planicostatus*, M'Coy.	„ *decussatus*, Flem.
„ *segregatus*, M'Coy.	„ „ var. *striatus*, Flem.
„ sp.	*Euomphalus Dionysii*, Goldf.
Edmondia unioniformis, Phill.	*Loxonema constrictum*, Sow.
Entolium Sowerbyi, M'Coy.	„ *curvilineum*, Phill.

Narica variata, Phill.
Naticopsis plicistria, Phill.
Ivania (*Pleurotomaria*) *Ivani*, Lév.
Trochus hisingerianus.
Conularia quadrisulcata, Sow.
Orthoceras, sp.
Archichthys Portlocki, Ag.
Cœlacanthus lepturus, Ag.
Eurynotus aprion, Traq.
„ *crenatus*, Ag.
Megalichthys, sp.
Strepsodus sauroides, Ag. M.S.
Wardichthys cyclosoma, Traq.

2. *The Lower Limestone Group.*—As developed in Eskdale and Liddisdale, this subdivision differs in one important aspect from that just described. While the group as a whole consists of a constant alternation of sandstones, shales, fireclays, thin coals, and limestones, its distinctive feature is the massive nature of some of the limestones and their true marine character. The latter are admirably seen in the Liddel Water at Penton Linns, at Harelaw Hill quarry, and in the Esk above Gilnockie bridge. The following sequence is visible in the lower quarry of Harelaw Hill.

8. Thin black shale.
7. Grey solid limestone, 4 feet.
6. Black shales with ironstone nodules, 15–20 feet.
5. Grey solid limestone, 20–22 feet.
4. Fossiliferous black shales, 1 foot 6 inches.
3. Grey limestone, $3\frac{1}{2}$ feet.
2. Black shale.
1. Coal, 1 foot.

The fossils given in the annexed list which indicate true marine conditions have been obtained from the limestone in Harelaw Hill quarry.

Lithostrotion junceum, Flem.
Hydreionocrinus globularis, De Kon.
Dithyrocaris, sp.
Phillipsia seminifera, Phill.
Polyzoa.
Chonetes laguessiana, De Kon.
Productus giganteus, Mart.
„ *longispinus*, Sow.
„ *semireticulatus*, Mart.
„ sp.
Spirifera lineata, Mart.
„ *trigonalis*, Mart.
„ „ var. *bisulcata*, Sow.
Terebratula hastata, Sow.
Aviculopecten cœlatus, M'Coy.
„ *Geikiei*, Eth.
Edmondia sulcata, Phill.
Sanguinolites striato-lamellosus, De Kon.
Euomphalus carbonarius, Sow.
Orthoceras sulcatum, Flem.

The section exposed in the Esk between Glencartholm and Gilnockie bridge furnishes favourable opportunities for studying the members of this subdivision when the river is low. The strata are affected by numerous small folds and faults, but on the whole there is an ascending sequence, with an inclination to the south or south-east. In that part of the section between Gilnockie Tower and Canonbie Mills the limestones are well displayed in the bed and banks of the river, where they are richly charged with corals, brachiopods, cephalopods, and other organic remains. The subjoined list gives the fossils from the limestone and calcareous shales in the Esk near Gilnockie Tower.

Chœtetes septosus, Flem.
Clisiophyllum, sp.
Lithostrotion aranea, M'Coy.
„ *junceum*, Flem.
„ *Portlocki*, M. Edw.
Zaphrentis Enniskilleni, Edw. and Haime
„ *Phillipsi*, Edw. and Haime.
Poteriocrinus crassus, Miller.
Crinoid stems.
Stenopora Howelli, Nich.
Athyris ambigua, Sow.
„ *Roysii*, Lév.
Camarophoria crumena, Mart.
Chonetes comoides, Sow.

Lingula squamiformis, Phill.
Orthis Michelini, Lév.
Productus complectens, Eth. jnr.
„ *giganteus*, Mart.
„ *llangollensis*, Dav.
„ *scabriculus*, Mart.
Spirifera lineata, Mart.
„ *trigonalis*, var. *bisulcata*, Sow.
Aviculopecten, sp.
Lithodomus carbonarius.
Myalina, sp.
Nuculana attenuata, Flem.
Protoschizodus axiniformis, Portl.
Bellerophon Urei, Flem.
Naticopsis, sp.
Pleurotomaria, sp.
Orthoceras, sp.

Perhaps the finest section in the Border region of the limestone group underlying the Kilnholm coals is that visible in the Liddel Water at Penton Linns, about a mile east of Rowanburn Colliery, where the river flows through a gorge carved out of these strata. Along their southern margin the sandstones and shales of the Lower Marine limestone group are truncated by a fault which brings down the red sandstones and shales of the Upper Coal-measures (see Plate III. section 3). North of this fault the members of the marine limestone group have, for a distance of 300 yards, a general dip to the E.S.E. at a moderate angle, where the fossiliferous character of the shales and limestones is admirably seen. Here the limestones are traversed by an east and west fault, with a downthrow to the south, repeating the beds. North of this second fault towards the Penton bridge, the massive limestones of this group are thrown into a well marked anticline, the axis of which appears about a hundred yards west of the latter locality. On the west side of the arch the strata are inclined at high angles, and the successive beds of limestone can there be studied to advantage. From the limestones and calcareous shales at Penton Linns the fossils given in the annexed list have been collected.

Saccammina Carteri, Brady.
Clisiophyllum turbinatum, M'Coy.
Lithostrotion irregulare, Phill.
Forbesocrinus, sp.
Hydreionocrinus globularis, De Kon.
Poteriocrinus crassus, Miller.
Crinoid stems.
Phillipsia seminifera, Phill.
Diastopora megastoma, M'Coy.
Fenestella, sp.
Chonetes buchiana, De Kon,
„ *laguessiana*, De Kon.
Orthis Michelini, Lév.
Productus giganteus, Mart.
„ *longispinus*, Sow.
„ *semireticulatus*, Mart.
Rhynchonella pleurodon, Phill.
Spinifera trigonalis, Mart.
„ „ var. *bisulcata*, Sow.
Ctenodonta pentonensis, Hind.
Edmondia pentonensis, Hind.
Myalina Verneuili, M'Coy.
Nucula brevirostris, Phill.
„ *gibbosa*, Flem.
Pinna flabelliformis, Mart.
Protoschizodus axiniformis Portl.
Sanguinolites variabilis, M'Coy.
Streblopteria, sp.
Bellerophon decussatus, Flem.
„ *hiulcus*, Sow.
„ *Urei*, Flem.
Euomphalus carbonarius, Sow.
Loxonema rugifera, Phill.
„ *scalaroidea*, Phill.
Macrocheilus, sp.
Murchisonia angulata, Phill.
Naticopsis lirata, Phill.
Pleurotomaria canaliculata, M'Coy.
Orthoceras cinctum, Sow.
„ sp.

The fossils obtained from the limestones and shales of this group at Penton Linns in the Liddel, near Gilnockie Tower on the Esk, and at Harelaw Hill quarry, prove beyond doubt that truly marine conditions prevailed during their deposition; and further, they recall the assemblage of organic remains so characteristic of the Lower Limestone group of the Edge Coal series (Hurlet and Hosies) of central Scotland.

The massive character of the limestones towards the top of the Lower Limestone group has been proved in the Catsbit bore (see sketch map, Plate I.), sunk close to the farmhouse of Catsbit, about three-quarters of a mile E.N.E. of Rowanburn Colliery, where seventy-five feet of limestone with few intercalations of sediment were pierced beneath the Kilnholm coals. A copy of the journal of this bore is given below.

Section of Strata in Bore (by Diamond Drill Process) at Catsbit, commenced December 1891.

Description of strata :	Fms.	Ft.	Ins.
Surface soil	0	2	0
Soft red clay	2	0	0
Boulders and clay	4	3	10
Yellow sandstone	2	2	6
Yellow sandstone with coal strains	0	1	6
White sandstone	0	1	6
White sandstone	0	1	11
White sandstone with coal strains	1	2	0
Yellow sandstone	1	2	0
Broken sandstone (cuttry)	0	2	3
Space vacant	0	2	0
Yellow sandstone	0	5	0
Coal, soft and loose	0	0	4
Fine white sandstone	0	4	5
Yellow sandstone	0	5	3
Gray fakes	0	1	6
Blue blaes	0	3	0
Sandstone white	0	5	8
Gray fakes	0	2	6
Blue blaes	0	1	6
Soft blaes and fireclay	0	0	6
Blaes and coal	0	0	4
Coal	0	1	11
Gray fakes	0	0	6
Hard white sandstone	0	3	2
Blue fakes and blaes	0	2	0
Blue blaes and balls	0	3	2
Coal	0	0	5
Light fakes	0	0	6
Fakes and sandstone	0	2	3
Blue blaes	0	2	9
Coarse parrot coal	0	1	3
Dark blaes	0	1	2
Coal and sulphur	0	0	4
Coal	0	0	8
Dark gray fakes	0	2	0
Light fireclay	0	1	2
Light fireclay and ironstone ball	0	4	8
Blue fakes	0	2	9
Soft blaes	0	0	9
Fakes and sandstone	0	1	6
White sandstone cuttry	0	4	0
Carry forward	24	4	5

Description of strata :	Fms.	Ft.	Ins.
Brought forward	24	4	5
Dark gray fakes	0	4	0
Fakes and blaes	0	5	0
White sandstone	6	3	8
White sandstone with coal strains	0	2	0
Dark gray fakes	0	4	0
Fakes and sandstone	1	2	0
Dark blaes	0	0	6
White sandstone	0	4	6
Gray fakes	0	1	0
Yellow sandstone	4	1	6
Coarse sandstone and coal strains	0	3	0
Fakes and blaes	1	2	6
Coarse sandstone	0	1	9
Inferior **limestone**	0	1	4
Gray sandstone	1	0	2
Fakes and blaes	1	0	0
Coal (loose)	0	1	0
Sandstone, white	1	3	2
Fakes and sandstone	0	2	6
Blue blaes	0	5	6
Dark fakes	0	5	8
Coal	0	0	7
Sandstone, white	0	3	0
Gray fakes	0	4	0
Fakes and blaes	0	5	0
Gray fakes	0	5	6
Fakes and coal strains	0	0	4
Gray fakes	0	2	6
Sandstone, gray	1	1	0
Blue blaes	0	4	5
White sandstone	0	5	0
Fakes and sandstone	0	3	3
Gray sandstone	0	3	0
White sandstone	1	2	3
Gray sandstone	1	0	9
White sandstone	3	3	5
Coal (soft)	0	0	4
Fakes and blaes	0	0	8
White sandstone	3	5	6
Dark fakes and coal strains	0	0	4
Coarse white sandstone	1	3	6
Carry forward	67	5	6

Description of strata :	Fms.	Ft.	Ins.
Brought forward	67	5	6
Soft fakes	0	0	2
Coarse sandstone, white . .	2	3	8
Blue blaes and sandstone . .	0	1	0
Coarse sandstone	0	3	7
Blaes	0	0	2
Coarse sandstone	0	1	5
Blaes, blue	0	0	2
White sandstone	0	2	6
Fakes and sandstone . . .	0	4	10
White sandstone	0	0	4
Fakes, blue	0	2	2
Fakes and blaes	1	0	0
Gray fakes	0	1	3
Blaes and **limestone** . . .	0	1	0
Limestone	0	2	0
Gray sandstone	0	3	9
Limey fakes	1	1	2
Blue blaes	1	0	0
Limestone	3	1	6
Coal	0	0	6
Carry forward	81	0	8

Description of strata :	Fms.	Ft.	Ins.
Brought forward	81	0	8
Fakey sandstone	0	2	10
White sandstone	2	3	10
Fakey sandstone	1	2	5
Blue fakes	0	3	6
Blue blaes	1	0	8
Blaes and **limestone** . . .	0	3	0
Limestone	0	2	8
Limey fakes	1	3	6
Blue blaes, limey . . .	0	4	0
Blue blaes	1	0	2
Blue blaes and balls . . .	0	4	0
Blue blaes, limestone ribs . .	0	4	8
Limestone	12	5	9
Coal and sandstone	0	0	8
Sandstone, white	0	3	9
Dark gray fakes	0	5	7
Coarse **coal**	0	0	7
Light fireclay, soft . . .	0	3	0
Limestone, brown and gray .	2	0	0
Total,	110	1	3

3. *Kilnholm Coal Group.*—Above the marine limestones just described there is a group of thin coals which, though of little economic value, are of considerable importance from a stratigraphical point of view. Their relative position to the underlying marine limestones of Penton Linns is defined in the Liddel section above Penton bridge, where several thin coals are visible in the banks of the stream, one of which was formerly wrought at Kilnholm. There the sandstones, shales, and thin coals, varying from a few inches to a foot or more in thickness, have an easterly dip, and follow in natural sequence the Penton limestones. They are also visible in the railway section on the English side near Penton House.

In the Esk section, between Gilnockie bridge and the foot of Byre Burn, on an anticlinal fold of the strata, thin coal-seams also appear which are probably on this horizon.

The relative position of this group of thin coal-seams to the Lower Coal-measures of Rowanburn has also been proved in the Rowanburnhead bore (see Plate II.), where they occur underneath the upper limestones. On this horizon six seams were passed through in this bore, five of them being less than one foot thick, and the sixth measuring two feet four inches (see journal of bore, p. 855). Again, in the Catsbit bore (see Plate II.) several thin coal-seams were pierced above the massive lower limestones, all of which, with one exception, are of no economic importance.

In the sequel, evidence will be adduced pointing to the conclusion that the Kilnholm coals occupy the position of the Lickar coals of Northumberland, which have been correlated with the Edge coals of the Carboniferous Limestone series of central Scotland. It is evident, therefore, that there is a marked difference in the economic value of this coal-bearing group to the north and south of the Silurian tableland in Scotland.

4. *Upper Limestone Group.*—The members of this subdivision were proved in an important bore sunk at Rowanburnhead, near the northern margin of the Rowanburn Colliery, to which reference has already been made. A glance at the journal of this bore (see p. 855), and at the diagram of vertical sections, Plate II., shows that the bore was begun in the pavement of the Seven Feet Coal, the position of which in the Rowanburn coalfield is well known. Deducting the thickness of sand and boulder clay at the surface, the first limestone was pierced at a depth of one hundred and eleven fathoms below the pavement of the Seven Feet Seam. Altogether three beds of limestone were passed through, measuring respectively one foot two inches, ten feet, and the lowest, with some intercalations of shale, about twenty feet. Underneath the limestones, as already indicated, lie the thin seams of the Kilnholm coals.

Further evidence relating to the position of this limestone group was obtained in a bore in the bottom of the Old Furnace Pit, Rowanburn, which showed that they underlie some thin coals below the Seven Feet Seam.

Owing to the extensive faulting of the Coal-measures in the Canonbie district, the infraposition of this limestone group to the Lower Coal-measures of Rowanburn has not been proved in any stream section. In the Esk, about one hundred and fifty yards up stream from the foot of Byre Burn, a limestone about three feet thick and calcareous shales appear, charged with *Productus*, *Orthoceras*, and other marine fossils, which may represent one of the bands in this group.

Upper Carboniferous Rocks.

Proceeding now to the consideration of the subdivisions of the Upper Carboniferous rocks of Eskdale and Liddisdale, we encounter serious difficulties owing to the absence of any stream sections showing the original order of succession of the various groups. Judging from the evidence visible at the surface, the field-geologist is at a loss to decide the true sequence of the various subdivisions. The area occupied by these rocks in the Canonbie district is so much traversed by important faults, which have obscured the order of superposition, that any attempt to construct a geological map on surface evidence alone would be liable to error. It is not surprising that the Byre Burn Coal group was regarded as lying beneath the Rowanburn coals, nor that the deep bore sunk from the pavement of the Seven Feet Seam was put down with the view of finding the workable coals of Byre Burn below. That attempt proved a failure, though the upper limestones of the underlying Marine Limestone group were passed through in that trial bore. The result of this bore in some measure paved the way for the classification to which Mr Kidston has recently been led by the evidence of the plants, viz., that the Rowanburn coals represent the Lower Coal-measures, the Byre Burn coals the Middle Coal-measures, and the stained red sandstones and shales the Upper Coal-measures.

vii. *Millstone Grit.*

Between the workable coal-seams of the Rowanburn group and the upper limestones of the Marine Limestone series, as proved in the Rowanburnhead bore, there is a succession of sandstones, shales, fireclays, and thin coal-seams, about one hundred and eleven fathoms in thickness, which may represent, in part at least, the arenaceous group (Millstone Grit) that intervenes between the Coal-measures above and the Marine Limestone series below. Some of the bands of sandstone revealed in this bore are from eighteen to thirty feet thick. During our recent examination of the district we had an opportunity, through the courtesy of the manager, Mr Bowie, of studying the cores of this bore, and paid special attention to these massive sandstones, some of which are coarse, false-bedded, and pebbly. No plants have been collected from this horizon; and so far as this line of evidence is concerned, it is impossible to say where the boundary should be drawn between the upper and lower divisions of the Carboniferous system.

viii. *Rowanburn Coal Group* (*Lower Coal-measures*).

This important group of strata contains the coal-seams which have so long been wrought in the Canonbie district. The area uncovered by the stained red sandstones of the Upper Coal-measures is about half a square mile. The strata are not exposed in any stream section, and the information regarding the sequence of the beds and geological structure of the field is based solely on mining plans. The following vertical section supplied by Mr Bonar, present manager of the Canonbie Colliery, gives the sequence of the coal-seams in descending order.

	Fm.	Ft.	Ins.		Fms.	Ft.	Ins.
1. Upper coal	0	3	4	5. Three Feet coal . . .	0	3	6
Strata	15	4	11	Strata	7	0	0
2. Main coal	1	0	0	6. Five Feet coal . . .	0	5	0
Strata	12	5	2	Strata	7	0	0
3. Splint or Nine Feet coal .	1	3	0	7. Black top coal . . .	0	4	9
Strata	0	4	0	Strata	4	0	0
4. Coal (good)	0	1	8	8. Seven Feet coal (local name) .	1	0	0
Strata	1	3	0				

On referring to the geological sketch map of the district (Plate I.), it will be seen that the coal-seams crop out to the west and north, being truncated on the south by a powerful east and west fault that brings down the stained sandstones and shales of the Upper Coal-measures. On their north-eastern side the strata are likewise bounded by a fault which brings them in contact with the members of the Marine Limestone series. The beds dip towards the east and south-east, and from the coal-workings it appears that they curve up against the great bounding fault on the south side of the field at Rowanburn (see Plate III. section 4).

Reference has already been made to the prevalent opinion that the coals of the Byre Burn group underlie the Rowanburn coals, and with the object of testing this conclusion a deep bore was sunk at Rowanburnhead, a copy of the journal of which is given below.

Section of Strata in Bore (by Diamond Drill Process) at Rowanburnhead, commenced at Sill or Pavement of "Seven Feet Seam," with the object of finding the Coal-seams of Byre Burn. Boring commenced March 1891.

Description of strata :	Fms.	Ft.	Ins.
Surface and sand	1	1	0
Boulder clay and stones	9	1	0
Fireclay (soft)	3	1	0
Light fakes	0	1	9
Fireclay broken	0	1	8
Coal (loose)	0	0	9
Fireclay	0	0	4
Broken sandstone	0	5	0
Fakes and blaes	0	1	6
Coal (soft)	0	1	5
Fireclay and blaes	0	2	6
Light fakes	0	1	6
Fakes and fireclay	0	2	0
Blue fakes	2	0	0
Blue fakes and blaes	0	2	0
Blue blaes	0	2	5
Coal	0	1	4
Coal (soft)	0	0	3
Fireclay	0	1	6
Fireclay and blaes	0	2	6
Sandstone	0	2	3
Dark blaes	0	1	4
Coal	0	1	3
Dark fakes	0	0	2
Gray fakes	0	0	7
Blue fakes	0	2	9
Sandstone	0	5	0
Fakey sandstone	1	4	6
Fakes and blaes	0	1	6
Coal	0	0	7
Fireclay	0	2	0
Fakes	0	1	0
Sandstone	0	1	9
Fakey sandstone	3	0	0
Fakey fireclay and blaes	0	4	0
Soft fireclay and balls	0	4	6
Sandstone	5	0	0
Sandstone	0	0	5
Fakey sandstone	0	1	8
Sandstone	0	1	7
Blue Fakes	1	2	3
Sandstone	0	2	5
Fakes and sandstone	1	2	0
Sandstone (variegated)	1	5	6
Sandstone	0	5	0
Carry forward	41	1	5

Description of strata :	Fms.	Ft.	Ins.
Brought forward	41	1	5
Fakes and sandstone (variegated)	3	0	8
Sandstone	1	2	6
Blue fakes and blaes	2	1	6
Dark fakes	0	1	3
Gray fakes	0	3	3
Blue fakes and blaes	1	0	0
Fireclay and coal	0	0	8
Light fakes and fireclay	0	4	6
Sandstone	0	2	3
Dark fakes	3	2	11
Dark fakes and sandstone	1	5	0
Blue fakes	0	4	2
Sandstone	0	1	8
Blue fakes	0	0	10
Sandstone	0	0	5
Dark blaes	0	1	7
Coal	0	1	5
Dark fireclay and **coal**	0	0	7
Brown fireclay	0	0	5
Coal	0	1	8
Dark fireclay and **coal**	0	0	4
Light fireclay	1	0	0
Dark fireclay and blaes	0	2	4
Light fireclay	0	4	0
Sandstone	0	3	6
Fakes and sandstone	0	2	0
Fakey blaes	0	3	0
Dark fireclay and **coal**	0	1	9
Fireclay	0	1	3
Fakes and fireclay	0	1	6
Fakes and sandstone	0	2	6
Soft blaes and **coal**	0	0	9
Sandstone	2	2	3
Extra hard sandstone	0	3	10
Fakes and blaes	0	3	2
Extra hard sandstone	2	5	11
Fakey blaes	0	2	3
Light fireclay	0	4	6
Light fakes and sandstone	0	5	0
Hard cuttry sandstone	1	1	0
Fakey sandstone	0	2	6
Blue blaes	0	2	0
Dark blaes and **coal**	0	0	10
Fakes and fireclay	0	1	4
Carry forward	74	0	2

Description of strata:	Fms.	Ft.	Ins.
Brought forward	74	0	2
Sandstone	1	0	3
Extra hard sandstone	0	0	9
Sandstone	0	0	10
Fakes and fireclay	0	3	3
Fakes and sandstone	0	1	6
Fakey blaes	0	2	6
Reddish fireclay	2	2	6
Dark blaes	0	0	4
Blue fakes	0	3	0
Light fakes and sandstone	0	3	0
Sandstone	0	2	0
Reddish fakes	0	1	3
Blue blaes and fireclay	0	2	6
Blue fakes	0	1	2
Light fakes	0	4	2
Gray fakes	0	4	2
Blue fakes and blaes	0	1	9
Light fakes	0	1	9
Sandstone	0	2	9
Light fakes	0	3	0
Blaes and iron ribs	0	2	6
Coal	0	0	9
Sandstone	1	1	6
Blue fakes	0	2	3
Ironstone clay band	0	0	3
Blue fakes and sandstone	0	2	8
Ironstone clay band	0	0	2
Hard sandstone	0	1	8
Blue blaes	0	1	9
Dark blaes, parroty	0	0	9
Light fakes	0	3	7
Sandstone	0	4	6
Fakes and fireclay	0	3	4
Fakes	0	1	6
White sandstone	3	1	6
Cuttry sandstone	7	4	6
Coal and sulphur	0	0	2
Gray sandstone	0	1	10
Hard sandstone	0	3	0
Gray fakes	2	2	0
Dark blaes	0	3	2
Ironstone balls and sulphur	0	0	3
Gray sandstone	4	3	1
Coal strains and sandstone, variegated	1	3	0
Fakes and **coal**	0	0	10
Fireclay and blaes	0	4	0
Blue fakes	0	2	0
Sandstone	2	1	8
Light fireclay	0	1	6
Light fireclay and balls	1	3	9
Sandstone	2	0	1
Fakes and sandstone	0	2	4
Light fireclay	0	2	6
Coal (coarse)	0	0	9
Blue fakes	0	2	0
Light fakes	1	2	2
Light gray fakes and blaes	0	3	6
Blue fakes and blaes, limey	0	4	0
Limestone	0	1	2
Limey fakes	0	1	6
Carry forward	122	0	0

Description of strata:	Fms.	Ft.	Ins.
Brought forward	122	0	0
Gray fakes	0	3	10
Blue fakes and blaes	1	0	10
Dark blaes	0	0	7
Fireclay and **coal**	0	0	9
Light fireclay	0	2	8
Dark fireclay and balls	1	1	0
Fakes	0	0	8
Sandstone	1	3	5
Coarse sandstone	1	3	0
Light fakes	0	1	0
Fakey sandstone	0	1	2
Limey blaes	1	2	8
Light fakes	2	3	0
Blaes and balls	1	2	6
Light fakes	0	1	6
Fakes	1	0	0
Fakes and sandstone	1	1	4
Fakes and iron balls	0	2	8
Sandstone	0	2	0
Fakes and sandstone	2	0	0
Blaes and fakes	0	1	6
Blue blaes	1	2	2
Gray **limestone**	0	2	0
Limey sandstone	0	0	3
Fakes and sandstone	1	1	3
Blue fakes and blaes	0	2	0
Fakey blaes and balls	2	1	6
Dark blaes	1	4	6
Gray **limestone**	0	2	8
Limestone cuttry	0	3	1
Limestone	1	4	10
Blue blaes	0	1	0
Fireclay and **coal**	0	0	6
White sandstone	2	4	0
Light fireclay	0	2	5
White sandstone	0	5	0
Gray fakes	0	2	10
Blue blaes	0	1	1
Gray fakes	0	3	6
White sandstone	0	3	9
Fakes and blaes	0	3	6
Soft dark blaes	0	1	6
Gray sandstone	0	1	8
Fakes and sandstone	0	2	7
Blue blaes	2	5	6
Limestone, gray	0	1	6
Blaes and **limestone**	0	3	0
Gray **limestone**	0	3	6
Blaes and **limestone**	0	5	6
Dark gray **limestone**	1	0	0
Fakey blaes	0	0	3
Gray sandstone	0	3	3
Gray fakes	0	1	3
Blue blaes	0	0	9
Sandstone	1	2	0
Dark fakes and blaes	0	1	0
Blue fakes	0	2	6
Sandstone	2	1	6
Coarse sandstone	1	1	10
Blue blaes and sandstone	1	1	0
Carry forward	171	2	0

Description of strata:	Fms.	Ft.	Ins.
Brought forward	171	2	0
Coarse sandstone	2	0	2
Coarse sandstone	1	3	0
Blue blaes and balls	0	5	2
Blue blaes and musse	0	0	10
Gray fakes	0	3	6
Dark blaes	0	0	5
Coal	0	0	10
Dark blaes and **coal**	0	0	8
Dark blaes	0	1	0
Sandstone	1	3	6
Gray fakes	0	4	6
Blue blaes	0	2	7
Coal, soft	0	0	4
Blue blaes	0	1	0
Dark fireclay	0	0	8
Gray sandstone	1	2	0
Gray fakes	0	3	6
Dark fakes	0	1	6
Dark gray fakes	0	2	0
Coal	0	0	8
Coal and sulphur	0	0	2
Dark fakes	0	0	2
Gray sandstone	0	4	10
Fakey sandstone	0	1	5
Dark fakes and blaes	0	3	6
Dark blaes	0	1	2
Gray sandstone	0	5	5
Gray sandstone	0	3	6
Fakes and blaes	0	2	10
Gray sandstone	2	1	1
Coal	0	0	4
Fakey blaes	0	0	8
White sandstone	0	4	6
Fakes and sandstone	0	2	0
Dark blue blaes	0	0	10
Carry forward	190	0	3

Description of strata:	Fms.	Ft.	Ins.
Brought forward	190	0	3
Dark gray fakes	0	5	0
White sandstone	1	4	3
Dark blaes	0	0	11
Coal splint	0	2	4
Fakey fireclay	0	0	3
White sandstone	1	0	9
Fakes and blaes	0	0	10
Blue fakes	0	3	6
Hard sandstone	0	0	4
Gray fakes	0	2	8
Fakes and blaes	1	5	4
Gray fakes	2	0	10
Fakes and **coal**	0	1	6
White sandstone	0	4	7
Fakes and blaes	0	4	0
Parrot **coal**	0	0	6
Fakey blaes	0	2	0
Fakes and sandstone	2	1	2
Blue blaes	0	1	2
White sandstone	6	0	2
Sandstone and coal strains	0	4	0
White sandstone	4	4	9
Blue fakes	0	2	6
Fakes and blaes	1	1	0
Limestone	0	0	3
Blue blaes	0	1	0
Coarse sandstone	0	5	8
Lime and sandstone	0	3	8
Hard gray coarse sandstone	1	1	6
Dark fakes and **coal**	0	0	7
Soft fireclay and blaes	0	2	0
White sandstone	2	0	0
Hard gray sandstone, limey	0	0	9
White sandstone	1	4	6
Fakey sandstone	0	3	4
Total,	224	5	10

This deep bore furnishes important evidence as to the sequence of beds below the Seven Feet Seam down to the upper limestones at the top of the Marine Limestone series. The Coals of the Byre Burn group (see page 858) were not met with in this intervening group, and it is therefore probable, as already suggested, that the latter represents, in part at least, the Millstone Grit division.

ix. *Byre Burn Coal Group* (*Middle Coal-measures*).

The members of this subdivision are exposed in the Byre Burn that joins the Esk, on the east bank, about a mile north of Canonbie village, where they consist of sandstones, shales, fireclays, and coal-seams. In 1816 a bore was sunk through this group, a copy of the journal of which is subjoined.

	Ft.	Ins.		Ft.	Ins.
			Brought forward	84	4
Till or alluvial clay	15	0	Gray beds	34	0
Blue limestone	1	0	White stone	2	0
Blue metals	15	0	Craw coal	1	0
White freestone	2	0	Fireclay	1	0
Main coal (deficient)*	0	4	Ironstone	1	2
Fireclay	0	3	Gray beds	30	0
White freestone	30	0	White freestone	2	0
Blue metal	1	0	Craw coal	1	6
$\frac{3}{4}$ coal (deficient)†	1	0	Fireclay	1	0
Fireclay	1	6	White freestone	15	0
Gray beds of stone and blaes	15	0	Lime coal: top bed	1	0
Craw coal	0	8	Lime coal: metal	3	0
Fireclay	0	1	Lime coal: bottom bed	3	0
White stone	1	6			
Carry forward	84	4		180	0

In the above section the deficient thicknesses of the Main and Three-Quarter Seams are due to the bore having passed through old workings.

In the lower part of the Byre Burn, close to the Esk, the strata dip to the south-east at angles varying from 20° to 25°, while further up, towards the railway viaduct, they are thrown into an anticline. On their western side they are bounded by a fault that enters the Esk a few yards above the foot of Byre Burn and runs down the river channel for a distançe of 200 yards, thereafter ascending the west bank. By means of this dislocation they are brought in contact with strata referred to the upper part of the Marine Limestone series. The limits of this subdivision are not clearly defined, but the beds can be traced down the east bank of the Esk at Byreburnfoot, till they are abruptly cut off by the east and west fault that brings down the red sandstones and shales of the Upper Coal-measures. The area occupied by the Byre Burn group, at present known, measures about a quarter of a square mile.

x. *Red Sandstone Group* (*Upper Coal-measures*).

The red sandstones and shales that form the highest division of the Carboniferous system in Eskdale and Liddisdale are well seen in the Esk between Byreburnfoot and Canonbie bridge, and in the Liddel above and below the junction of Archerbeck with that river. Along their northern margin, as already indicated, they are bounded by a powerful east and west fault, which brings them successively in contact with the Marine Limestone series in the Liddel Water, with the Lower Coal-measures at Rowanburn, and with the Middle Coal-measures in the Esk (see Plate I.). The fault is admirably seen in the Liddel, where the strata on either side form a high cliff, and also in the Rowanburn close to the colliery. In the Esk the general dip of the beds is to the south or south-east, at angles varying from 15° to 35°. In the Liddel and in Archerbeck, however, there is evidence of folding and faulting of the strata, for they are inclined in various directions, and the lines of dislocation are visible in the stream sections.

* Usually 4 feet 10 inches, with 4 inches metal. † Usually 3 feet thick.

Lithologically, as Mr BINNEY pointed out, the sandstones, marls, and clayey shales of this Carboniferous subdivision are distinguishable from the Triassic rocks to the south. Much of the red and purple colour of the former is due to staining, and they lack the rich brick-red hue of the Triassic sandstones. This contrast is very apparent in the Esk. Reference has already been made to the fact that Mr BINNEY detected in the Esk at Knottyholm a bed of *Spirorbis* limestone, six inches thick, in association with red and purple shales, clays, bands of grit, and two seams of calcareous ironstone.

The plants obtained by Mr MACCONOCHIE from the red and greenish shales in the upper part of this series at Jockie's Syke, near Riddings Junction, have been unhesitatingly referred by Mr KIDSTON to the horizon of the Upper Coal-measures.* In the absence of determinable plants from the members of this group exposed in the Esk section, he is at present unable to determine whether the latter belong to the Upper Transition series of England or to the Upper Coal-measures.

Along their southern margin these Upper Carboniferous sandstones are covered unconformably by the Trias, the boundary between the two being a sinuous line. As shown on the sketch map (Plate I.), they occupy on the Scottish side a narrow belt about three miles in length from the Liddel westwards beyond the Esk. Owing to the thick accumulation of drift (boulder clay and gravel), their limits west of the Esk cannot be definitely fixed, but it is probable that they extend as far as Bulmans Knowe, which is about a mile west of that river. On this assumption the area which they cover in Scotland is two square miles.

A question of great economic importance arises in connection with this highest division of the Carboniferous system in the border region, which relates to the existence of a concealed coalfield underneath the area occupied by these sandstones. There is no evidence at present known to us pointing to any unconformability at their base; indeed, all the available data indicate continuous deposition from the Whita sandstones and cementstones to the top of the system. Under these circumstances it is obvious that both the valuable coal-seams of Rowanburn and those of Byre Burn might naturally be found below these sandstones. With the view of testing this question, the representatives of His Grace the Duke of Buccleuch sank two deep bores. The first was put down at Rowanburn about 200 yards south of the great fault that bounds these sandstones on the north (see Plate I.). A copy of the journal of this bore is given below:—

* *Summary of Progress, H.M. Geological Survey*, 1902, p. 214. *Trans. Roy. Soc. Edin.*, vol. xl. pp. 741–833.

Section of Strata bored by "Diamond Drill" through overlying Red Sandstone Formation into Coal Measures at Rowanburn, near Canonbie Colliery. Boring commenced 7th March 1889, finished 7th March 1890.

	Fms.	Ft.	Ins.
Surface sand	0	4	6
Sand and gravel	0	2	6
Red muddy clay	3	3	0
,, sandy clay (firm) . . .	0	4	0
,, muddy clay	1	0	6
,, fakes, broken (thin beds laminated)	2	4	0
,, soft fireclay	0	3	6
,, soft sandstone	1	2	0
,, blaes	0	2	0
,, fireclay	0	1	6
,, sandstone, soft . . .	1	3	6
,, fireclay and blaes . . .	1	0	6
,, sandstone, soft . . .	0	5	0
,, blaes, soft	1	1	0
,, fakes	0	2	0
,, light fakes and fireclay . .	1	4	6
,, fakey sandstone . . .	0	3	6
,, sandstone	1	1	6
,, fakes	0	1	0
,, fakes and fireclay . . .	0	5	0
,, fakes	0	2	0
,, fireclay	0	2	3
,, sandstone	0	2	6
,, fakes and fireclay . . .	0	3	4
,, fakes	0	2	3
,, fakes and blaes . . .	0	3	9
,, fireclay	0	2	6
,, fakes	0	2	4
Light red fakes	1	2	6
Gray sandstone (solid) . . .	12	2	10
Red blaes	3	0	0
,, fakes and blaes . . .	1	3	2
,, fakes	1	4	0
,, fakes and sandstone . . .	1	3	6
,, soft blaes	0	2	0
,, sandstone	5	2	0
,, blaes *	0	1	6
,, sandstone	1	2	0
,, fakes	0	5	6
,, fakes and sandstone . .	6	0	6
,, fakes	0	1	0
,, fakes and sandstone . .	2	0	0
,, blaes	0	1	6
Gray sandstone	3	0	6
Red irony sandstone . . .	0	0	6
Brown sandstone	0	3	9
Red sandstone (conglomerate) . .	0	3	4
,, blaes (variegated) . . .	0	4	0
Brown irony fakes	0	0	2
Red fakes (variegated) . . .	0	1	10
Brown fakes	1	4	7
Carry forward	70	0	7

	Fms.	Ft.	Ins.
Brought forward	70	0	7
Red fireclay	4	1	8
,, blaes	2	2	0
,, blaes	1	5	0
,, fakes	0	3	9
,, sandstone	0	4	9
,, fakes and sandstone . .	2	3	9
,, blaes (variegated) . . .	1	2	0
Brown irony fakes	0	0	2
Red sandy fakes (variegated) . .	0	2	0
,, sandy fakes (limey) . . .	0	5	0
,, blaes (limey)	1	1	2
,, blaes, hard (limey) . . .	3	2	10
Light red fakes	0	4	0
Light red fakes and iron balls .	0	3	9
Red fakes and fireclay . . .	0	4	0
,, fakes and blaes . . .	1	1	0
,, fakes	0	3	0
,, blaes	2	1	8
,, fakes and blaes . . .	1	0	0
,, and green fakes and blaes .	1	5	0
,, irony blaes	0	0	2
Light grey blaes	0	1	10
,, bluish grey blaes . . .	1	3	6
Reddish fireclay and ochre . .	2	1	8
Red fireclay	2	4	2
Hard white sandstone . . .	1	5	0
Reddish fakes and sandstone . .	1	4	0
Red fakes	0	4	0
Fakes and sandstone . . .	1	2	0
Gray sandstone	0	2	0
Reddish fakes and sandstone . .	3	5	9
Bluish fakes	0	2	0
Red fakes and sandstone . .	0	1	0
,, blaes (variegated) . . .	0	5	0
,, hard blaes	1	5	8
Reddish fakes and blaes . . .	2	0	0
Light red fakes and fireclay . .	1	0	6
Red sandstone	0	5	0
Reddish blaes	0	4	0
Red fakes	0	1	0
Sandstone (hard, white) . . .	0	1	2
Light blaes	1	2	8
Extra hard gray sandstone . .	0	1	0
Gray fakes	0	1	3
Light fireclay and blaes . . .	0	2	0
Red fireclay and blaes . . .	0	5	2
Red and brownish blaes . . .	0	4	7
Red and gray sandstone . . .	1	3	0
Sandstone (conglomerate) . .	0	3	0
Blaes (variegated)	1	2	6
Carry forward	130	3	8

* *Note.*—From 53 fathoms to 83 fathoms thin seams of gypsum were met with, not exceeding ¼ inch thick.

	Fms.	Ft.	Ins.
Brought forward	130	3	8
Red fakes and sandstone	0	1	10
Red blaes	0	5	0
Light gray fireclay	0	5	7
Red fakes and blaes	1	2	1
,, fakes and sandstone	0	2	3
,, blaes	0	5	0
,, sandstone (solid)	1	0	3
Soft fireclay	0	3	0
Blaes (hard)	0	2	5
Fireclay and ochre	0	3	0
Red sandstone and ochre	1	1	0
White(ish) sandstone	0	3	0
Reddish fakes (variegated)	2	3	8
Red and white sandstone	1	2	0
Red blaes (variegated)	0	2	0
,, sandstone	0	3	0
,, fakes	0	2	9
,, sandstone	0	2	0
Extra hard gray sandstone	0	0	9
Red sandstone	0	2	6
,, fakes and blaes	0	3	0
Extra hard gray sandstone	0	1	0
Red sandstone	1	5	0
,, fakes	0	4	0
,, fakes and sandstone	4	1	0
,, blaes, variegated	0	1	6
,, and grayish sandstone	0	4	3
,, fakes and sandstone	1	3	0
,, blaes, soft	0	2	6
,, fakes	0	1	6
Reddish fireclay and blaes	0	2	6
Red sandstone	0	5	0
,, blaes	0	3	0
,, fakes and sandstone	1	2	0
,, and gray sandstone	0	3	8
,, sandstone	0	2	0
Extra hard sandstone	0	1	2
Red and white sandstone	1	1	0
,, blaes	0	5	1
,, and gray fakes and sandstone	0	5	6
,, blaes	0	1	3
,, fakes	0	2	0
Sandstone, hard gray	0	1	2
Red blaes	0	0	11
Sandstone, gray, extra hard	0	1	6
Sandstone (whitish)	0	1	0
Sandstone (extra hard)	0	0	6
Red fakes and sandstone	0	5	3
,, blaes	0	2	0
,, fakes and sandstone	0	5	7
,, blaes	0	1	0
,, and gray sandstone	0	2	6
,, blaes	0	1	9
,, and gray sandstone	1	2	0
,, fakes	0	1	6
,, blaes	0	5	6
,, blaes (variegated)	0	2	9
Light bluish blaes	0	4	3
Red and light blaes	1	1	7
blaes	1	3	9
Carry forward	173	6	2

	Fms.	Ft.	Ins.
Brought forward	173	6	2
Red and light blaes	2	0	6
,, fakes and blaes	1	1	0
Sandstone	0	2	6
Red blaes	0	3	0
Light fakes and blaes	0	1	6
Red fakes and sandstone	2	1	4
Light fakes and sandstone	0	4	0
Red fakes and blaes	1	0	0
Blue blaes	0	0	4
Light blaes	0	4	6
Blaes, variegated	0	3	9
Red blaes	1	3	0
,, and light blaes	0	2	8
Bluish fireclay	0	3	4
Light fakes	0	3	0
Blaes and ochre	0	2	3
Red and light blaes	0	4	0
,, fireclay and blaes	1	3	0
Blue fireclay (*upper bed of coal measures, A. Bowie*)	0	5	0
Sandstone	0	3	0
Red and bluish fakes	0	3	6
Sandstone (extra hard)	0	0	6
Blue fireclay and blaes	0	3	7
Black blaes	0	0	5
Blue fireclay and ironstone balls	0	4	0
Light fireclay and balls	0	0	7
Blaes and **coal**	0	0	2
1st **coal**	0	0	5
Blaes and **coal**	0	0	5
Fireclay	0	0	6
Blue fakes	2	2	6
Fireclay (soft, reddish)	1	0	0
Fakes (gray)	0	5	4
Blue blaes	0	1	6
Fakes and sandstone	1	0	0
Blue blaes and balls	0	4	0
Gray fakes	0	3	7
Dark blaes and balls	1	4	0
Coaly blaes	0	0	5
Black shelly blaes	0	1	0
2nd **coal**	0	0	6
Dark blaes	0	0	2
Light fireclay	0	3	0
Irony fireclay	0	1	0
Light fakes and fireclay	0	5	0
Gray fakes	0	4	6
Blue blaes and balls	0	1	6
Ironstone ball	0	0	1
Blue blaes	0	0	4
3rd **coal**	0	1	0
Dark blaes	0	0	4
Sandstone	0	4	3
Blue blaes	0	1	3
Sandstone	0	0	8
Dark blaes	0	0	10
Ironstone	0	0	3
Dark blaes	0	1	0
Blue fakes	1	0	0
Blue blaes and ironstone balls	1	2	3
Carry forward	208	2	2

	Fms.	Ft.	Ins.
Brought forward	208	2	2
Dark blaes	0	0	2
4th **coal**	0	0	6
Dark fireclay	0	0	7
Light fireclay and ironstone balls	0	2	0
Red and blue fakes and sandstone	0	4	0
Red and white sandstone	5	0	2
Blue blaes and ironstone balls	0	3	0
Blue fakes and blaes	4	5	4
Blue blaes and ironstone balls	1	1	8
Light fireclay	0	1	6
Light blue blaes and ironstone balls	2	2	7
Blue fakes and blaes	0	2	0
Blue blaes	0	3	3
Fakes and sandstone	2	5	7
Dark blaes	0	0	10
5th **coal**	0	0	5
Dark blaes	0	0	2
Light fireclay	0	2	2
Dark fireclay	0	1	5
Blaes and red balls	0	0	7
Red sandstone	0	1	6
Red and white sandstone	2	0	0
Dark fakes	0	2	0
Blaes and balls	1	2	4
6th **coal**	0	1	8
Light fireclay	0	3	3
Dark fakes	1	1	3
Red and gray sandstone	0	3	6
Reddish fakes	0	0	9
Gray and white sandstone	3	1	6
Dark gray fakes	0	2	6
Dark blaes	0	2	4
7th **coal**	0	1	10
Light fireclay	0	1	0
Carry forward	239	5	6

	Fms.	Ft.	Ins.
Brought forward	239	5	6
Light fireclay and ironstone balls	0	1	5
Gray fakes and balls	0	2	0
Fakes and sandstone	1	3	7
White sandstone	1	2	6
Dark blue blaes	0	2	7
Ironstone ball	0	0	2
Dark blaes	2	1	1
8th **coal**, soft	0	1	2
Dark fireclay	0	1	0
Dark blaes	1	1	10
9th **coal**	0	1	6
Light fireclay	0	1	2
Light fireclay	0	4	0
Blaes	2	0	2
Sandstone (white)	0	1	0
Fakes and sandstone (white)	0	3	0
Blaes (blue)	0	2	2
Blaes (black-coaly)	0	0	9
Fireclay (light)	0	1	7
Sandstone (gray)	0	4	6
Fakes (gray)	0	0	9
Sandstone (white and hard)	1	2	9
Fakes (gray)	0	1	3
Sandstone (white)	2	1	10
Blaes (black)	0	0	4
10th **coal** (soft)	0	2	5
Fakes and fireclay	0	0	6
Sandstone (white)	0	1	1
Sandstone, white and hard	0	0	10
Blaes (dark)	0	1	1
Blue blaes and ironstone balls	4	0	0
Mussel band	0	0	10
Blaes, black (with ironstone balls)	0	3	3
Blaes, blue	0	2	0
Total,	263	1	7

In the Rowanburn bore ten coal-seams were pierced: four under one foot thick, five ranging from one to two feet, and one measuring two feet five inches, which was reached at a depth of 257 fathoms.

At a later date the second bore was sunk on the west bank of the Esk, near the forge at Canonbie bridge, about half a mile to the south of the great fault that bounds the Carboniferous Red Sandstones on the north side (see Plate I.). A copy of the journal of this bore is subjoined:—

Section of Strata bored by "Diamond Drill" through overlying Red Sandstone Formation into Coal-measures at West Side of River Esk, near Forge. Commenced June 1892, finished 1893.

	Fms.	Ft.	Ins.
Sand	1	1	3
Coarse gravel	1	0	0
Red fireclay and blaes	1	1	9
Red sandstone	3	0	0
Red sandstone (fakey)	1	0	0
Red sandstone	2	1	0
Red fakes and blaes	0	4	6
Red sandstone	12	2	4
Soft fakes and fireclay	0	2	6
Fakes and blaes	0	2	3
Blaes	3	4	4
Reddish sandstone	3	0	10
Red fakes	0	3	6
Red blaes	0	5	2
Red blaes (hard)	0	5	9
Red blaes (soft)	1	3	7
Fakes and blaes	0	5	2
Red sandstone	4	0	10
Sandstone (conglomerate)	0	2	0
Red sandstone	0	2	2
Red fireclay	0	2	0
Red blaes	1	2	6
Fireclay and blaes	1	0	6
Red blaes	1	0	0
Red fakey sandstone	0	3	0
Fireclay and blaes	1	0	0
Red blaes	0	4	2
Red blaes and fireclay	1	0	5
Blaes	0	3	7
Red sandstone	4	3	6
Red blaes	2	3	3
Red fakes	0	5	10
Gray sandstone	0	1	0
Red blaes	3	0	8
Red blaes (hard)	2	1	11
Red and greenish blaes	0	1	0
Red blaes	0	1	9
Red fakes	1	1	4
Red fakes and sandstone	1	1	2
Red fakes and blaes	1	2	0
Red blaes	1	0	6
Red fakes and blaes	1	0	0
Red fakes	0	4	0
Red fakes and sandstone	2	1	0
Red blaes	0	5	2
Fakes and blaes, red	1	0	0
Red blaes	1	0	0
Fakes (hard, coarse, conglomerate)	0	4	0
Red blaes (hard)	0	5	0
Red fireclay and blaes	0	4	10
Fakes (hard)	1	2	5
Fireclay and blaes	0	5	0
Red blaes (hard)	2	1	6
Red and light blaes	3	2	4
Carry forward	83	4	3

	Fms.	Ft.	Ins.
Brought forward	83	4	3
Red blaes	1	3	0
Blue fakes and blaes	2	5	2
Light reddish fireclay	1	4	6
Gray sandstone	1	0	6
Fakes and fireclay	0	2	6
Fakey sandstone	1	0	0
Gray sandstone	1	5	0
Reddish sandstone	1	3	0
Red sandstone	8	5	2
White sandstone	0	3	0
Red blaes and soft partings	5	2	9
Red fakey blaes	0	2	5
Red fireclay and blaes	0	2	9
Black blaes	0	0	4
Red blaes	0	0	6
Reddish sandstone	0	5	0
Red blaes (fakey)	1	2	0
Red sandstone	0	2	10
Red blaes (fakey)	1	0	9
Red sandstone	0	4	9
Green fireclay	0	3	0
Black blaes	0	0	4
Light fireclay	0	1	4
1st **coal**	0	0	7
Light greenish fireclay	0	2	8
Dark fakes and coal	0	0	5
Dark fakes	0	2	0
Limey sandstone	0	3	6
Limestone (inferior)	1	2	9
Green fireclay and balls of ironstone	0	3	9
Red and greenish fireclay	1	3	0
Greenish blaes	0	4	8
Red blaes	1	2	4
Red and green blaes	1	4	7
Limey blaes	0	5	5
Red and green blaes	2	0	0
Light greenish blaes and balls	1	3	3
Light blaes	1	1	3
2nd **coal**	0	1	2
Light fireclay	0	1	8
Light fakes	0	1	6
Sandstone	1	1	0
Light bluish blaes	0	1	6
3rd **coal**	0	1	4
Coal and sulphur	0	1	1
Dark fakes	0	0	4
Light fakes	1	0	0
Red and greenish fakes	0	4	0
Bluish fakes	0	2	9
Red and green blaes (variegated)	1	5	0
Red and green blaes	1	5	4
Red fakes and blaes	1	2	4
Red and green blaes	4	0	1
Carry forward	145	2	1

	Fms.	Ft.	Ins.
Brought forward	145	2	1
Blue blaes	0	4	4
Bluish blaes (and strains of ochre)	1	0	9
Red blaes	0	3	4
Dark blaes	0	0	4
Gray fakes	0	0	9
Red and greenish fakes	0	3	8
Red and white sandstone	0	2	0
Blaes and ochre	0	1	0
Red blaes	0	2	0
Red blaes and ochre	1	3	11
Red blaes (variegated)	0	4	6
Red blaes	0	5	0
Red blaes, variegated	2	4	0
Sandstone	0	4	6
Red and white blaes (variegated)	3	4	4
Red sandstone	1	4	7
Red and white sandstone	1	2	6
Blaes (variegated)	0	4	0
Red sandstone	1	1	0
Red blaes	0	3	0
Red and white sandstone	0	2	3
Red sandstone	1	2	6
Blaes, variegated	0	1	6
Red sandstone	0	4	10
Blaes (variegated)	0	3	2
Fireclay (variegated)	0	5	0
Fakes (hard)	0	2	0
Red sandstone	1	3	7
Red blaes	0	2	6
Red and white sandstone	0	5	6
Red fireclay	0	5	0
Fireclay and ochre	1	0	0
Red and white blaes	0	5	0
Red fireclay	0	4	0
Red blaes	1	0	0
Red and lightish blaes	1	1	3
Red and white sandstone	1	4	6
Red and white fakes and blaes	1	0	6
Red fireclay blaes	1	1	8
Red and light blaes and ochre	2	1	10
Red and white sandstone	0	2	0
Red and light blaes	1	3	10
Red and white fakes and sandstone	0	3	8
Red and light blaes	0	4	0
Red and white sandstone	1	0	0
Fakey blaes	0	1	3
White sandstone	0	1	0
Fakey blaes	0	2	9
Light blaes	1	4	0
White sandstone	3	0	0
Red and light blaes	1	0	0
Red and light fireclay and blaes	5	0	0
Light blaes	0	1	6
Red blaes	0	3	0
Light fireclay	0	0	6
Dark blaes	0	1	2
Light fireclay	0	5	0
Red fireclay	0	5	6
Red and light fireclay	1	0	6
Carry forward	204	1	10

	Fms.	Ft.	Ins
Brought forward	204	1	10
Bluish fireclay	1	2	6
Light blaes	0	5	6
Light fireclay	0	4	3
Dark blaes	0	1	2
Dark blaes and **coal**	0	0	6
Dark fireclay	0	1	6
Light fireclay	1	3	7
White sandstone	1	2	3
Greenish fakes and sandstone	1	1	8
Greenish fakes and blaes	1	1	7
4th **coal** (soft)	0	0	7
Blue fireclay	0	1	0
Light fireclay	1	0	0
Fakey sandstone	0	5	4
Blue fakes	0	2	0
Light fakey sandstone	0	1	10
Blue fakes and hard ribs	2	2	0
Light fakes	1	0	0
Dark blaes	0	4	0
Light fakes	1	2	0
Light sandstone	1	5	0
Fakes and sandstone	0	5	6
Blue blaes and fireclay	0	2	0
Light fireclay (soft running)	0	5	8
Light fakes	0	1	8
Dark fakes and sandstone	3	2	4
White sandstone	0	0	10
Sandstone and **coal** strains	0	0	6
Fakey sandstone	0	1	8
Blue blaes	0	0	4
White sandstone	0	2	6
Blue blaes	0	0	6
White sandstone	0	3	10
Fireclay and **coal**	0	0	9
Fakes and fireclay	1	0	9
Fakey sandstone	2	0	0
White sandstone	1	5	8
Light fakes	1	1	6
Dark blaes and balls (ironstone)	0	5	6
5th **coal**	0	0	9
Dark blaes	0	3	0
Light fireclay	0	2	3
Light fakes	1	1	0
White sandstone (coarse)	3	2	0
Fakes and sandstone	0	5	6
Gray fakes	0	1	6
Blaes and ironstone ribs	0	3	6
Dark blaes	0	1	6
6th { **coal**	0	1	3
6th { blaes (black)	0	0	3
6th { **coal** (splint)	0	0	9
Dark fireclay	0	0	6
Dark blaes	0	1	6
Dark fakes	0	4	6
Dark blue blaes	1	5	10
7th **coal**	0	1	9
Bluish fireclay	0	4	0
Fakes and sandstone	0	3	7
Total depth,	250	4	6

A glance at the journal of the Forge bore shows that seven coal-seams were passed through, ranging in thickness from seven inches to two feet, the lowest seam being reached at a depth of 249 fathoms.

The evidence obtained from these two bores demonstrates the existence of thin coal-seams underneath the Carboniferous red sandstones of Canonbie. Though they cannot be correlated with the known Canonbie coals, it is not improbable that they may belong to the upper part of the Byre Burn group, the whole sequence of which has not been proved.

In the sheet of vertical sections (Plate II.), illustrating diagrammatically the more important bores put down in the Canonbie district, we have shown what we believe to be the relative stratigraphical position of the strata in each bore in the Carboniferous system. In our opinion, had the bores at the Forge or Rowanburn, within the area of Carboniferous red sandstone, been sunk to a sufficient depth, they would have passed through, in turn, the Middle Coal-measures of Byre Burn, the Lower Coal-measures of Rowanburn, the Upper Limestones and Kilnholm coals in the Rowanburn bore, and eventually the massive Lower Limestones shown in the Catsbit section.*

III. Description of Horizontal Sections.

1. *Buchtknowe to Larriston Fells and Kershope Burn.*—This line of section illustrates the structure of the area in the northern part of sheet 11 one-inch map, embracing portion of the Hermitage Water and Upper Liddisdale. It shows the ascending sequence from the Upper Old Red Sandstone (c^3) resting on the Silurian floor at Dinley Spout, through the Birrenswark volcanic zone (d^{i}) and the Whita sandstone (d^{ii}) to the Cementstone group (d^{iii}). On the west slope of Arnton Fell these subdivisions are faulted against the inlier of Upper Silurian strata on that ridge; while on its eastern side the sequence of the lower groups is again repeated in Upper Liddisdale; the higher part of the Cementstone group, with its marine limestones, being surmounted by the Fell sandstones (d^{iv}) and the Lawston coals (d^{vi}) on the Larriston Fells.

2. *Arkleton Fell to Caerby Hill and Kershope Burn.*—On the heights between the Ewes Water and the basin of the Liddel (Arkleton Fell and Cloak Knowe), the sequence from the Upper Old Red Sandstone to the Whita sandstone is exposed, the strata being there pierced by several necks of volcanic agglomerate and massive igneous rocks. Eastwards the members of the Cementstone group succeed, with which a sheet of basic lava is associated on Bedda Hill, till near Sorbietrees, south of New Castleton, they are overlaid by the Fell sandstones. On Caerby Hill these sandstones are capped by basic lava, probably representing the volcanic zone of Glencartholm, followed in turn by the Lawston Linn coals.

* The lamellibranchs from the rocks of the Canonbie coalfield have been examined by Dr Wheelton Hind, who is of opinion that they confirm the evidence obtained from the plant-remains that the strata in which they occur belong to the Coal-measures,—*Summary of Progress, Geological Survey for* 1902, p. 137.

3. *Ewes Water by Archerbeck to the Liddel Water.*—On the north-west slope of Whita Hill, east of Langholm, the unconformability at the base of the Upper Old Red Sandstone is well seen, as shown in section; the members of this formation being followed by the Birrenswark volcanic zone and the Whita sandstone. South-eastwards, towards the Tarras Water, the Cementstone group succeeds, excellent exposures of which are visible in the latter stream, overlaid in turn by the Fell sandstones (d^{iv}), the basic tuffs of the Glencartholm volcanic zone (d^{v}), and the beds on the horizon of the Lawston coals (d^{vi}). By means of a north and south fault on the moor between Tarras Water and Archerbeck, the two last groups are repeated as shown in section 3, followed by higher beds of the Marine Limestone series which stretch across the moorland to Archerbeck. In the latter stream, at a point about two miles above its junction with the Liddel, the arch of Lower Marine Limestones (d^{vii}) is truncated by a fault with a downthrow to the south (see section). On Harelaw Hill the massive Lower Marine Limestones again appear from underneath the beds on the horizon of the Kilnholm coals (Lickar position), and to the south-east at Penton Linns in the Liddel, as shown in section, the same limestones are exposed in sharp folds, being followed on the English side of the border by the thin Kilnholm coals, which are cut off by the great fault that throws down the red sandstones of the Upper Coal-measures (d^{xiii}).

4. *Wauchope Water by Glencartholm, Byre Burn, and Rowanburn to the Liddel Water.*—This section has been prepared to show the relations of the strata in the Esk below Langholm and in the Rowanburn coalfield, as proved by the mining plans. It is observable that the sequence from Langholm to Glencartholm is extremely clear, each group, from the Whita sandstone to the Glencartholm volcanic zone, with its fossiliferous shale, following each other in natural sequence. Between Glencartholm and the foot of Byre Burn the various subdivisions of the Marine Limestone series are met with, which are affected by numerous small faults and folds, most of which are omitted in the section. The small patch of Middle Coal-measures at Byre Burn is represented (d^{xii}), bounded on both sides by faults; but without the assistance of the fossil plants, the field geologist would hardly realise from the evidence on the ground that the Middle Coal-measures are there faulted, on the north-west side, against the higher part of the Marine Limestone series. The section further shows the gently inclined and faulted coal-seams of Rowanburn curving upwards, along the south-east margin near that colliery.

The remaining portion of this section is of special interest, as it shows the position of the great fault that bounds the Rowanburn coalfield on the south, the belt of Carboniferous red sandstone of the Upper Coal-measures (d^{xiii}) beyond, overlaid unconformably by the Triassic sandstones to the south.

5. *Birrenswark by Ecclefechan to the Kirtle Water.*—This line of section lies about twelve miles to the west of the river Esk, and furnishes important evidence of the great unconformability at the base of the Trias in that part of Dumfriesshire. From the slopes of Birrenswark the basic lavas dip gently southwards, followed by the Whita

sandstone, the Cementstone group, and by the Fell sandstones, which form a conspicuous eminence at Brownmoor Wood (d^{iv}). These, in turn, are overlaid by the members of the Marine Limestone series, which occupy the area southwards to the margin of the Trias. At Kirtlebridge the limestones of the Lower Limestone group (Penton Linns and Gilnockie) are exposed in various quarries, where they have been extensively worked. Their outcrops show that the beds were folded in arches and troughs before the deposition of the Trias, for the latter rests on the upturned and denuded edges of the members of the Marine Limestone series (see section 5). It is obvious, therefore, that in pre-Triassic time there must have been prolonged denudation of the Carboniferous rocks in that region; for all the divisions of the Canonbie Coal-measures were removed, and the Marine limestones underneath the Kilnholm coals were laid bare before the Triassic sandstones were deposited.

Similar evidence is obtained in the Cadgill Burn, a tributary of the river Sark, about half-way between Canonbie and Kirtlebridge. In that stream, within a few yards of the margin of the Trias, there is a band of stained limestone, charged with encrinite stems, *Euomphalus carbonarius*, *Bellerophon*, *Machrochilina*, and indeterminable fragments of brachiopods, with stained sandstone containing modioliform shells. Here again the Triassic sandstone rests on the upturned and denuded edges of the Marine limestones, and the Coal-measures have been removed.

Westwards beyond the Annan, at Kelhead and Clarencefield, the limestones of this horizon appear. Much of that district is thickly covered with drift, but at the latter locality the Marine limestone occurs within a mile of the margin of the New Red Sandstone, which points to transgression of the Trias and the removal by denudation of the Coal-measures.

IV. Estimate of Coal-supply in Concealed Coalfield underlying the Red Sandstones of the Upper Coal-measures, North of the Triassic Rocks at Canonbie.

In the previous sections of this paper descriptive of the subdivisions of the Coal-measures, the evidence has been given which leads to the conclusion that the Middle Coal-measures of Byre Burn and the Lower Coal-measures of Rowanburn lie underneath the Carboniferous red sandstones (d^{xiii}). It was further stated that the area covered by the latter amounts to two square miles.

We have prepared the following estimate of the coal-supply of this concealed coalfield on the basis that a seam of coal one foot thick and one square mile in area contains 900,000 tons of coal:—

Rowanburn Coals, estimated thickness 38 feet, . .	68,400,000 tons.
Byre Burn Coals, estimated thickness 6 feet, . .	10,800,000 tons.

An important economic question arises as to the probability of finding workable coal-seams in the Carboniferous area north of the Triassic rocks between Canonbie and the river Nith below Dumfries. In the previous section (p. 866) evidence has been given to prove that in the tract extending from the river Sark by Kirtlebridge, westwards to Clarencefield, near the mouth of the Nith, the three divisions of the Coal-measures at Canonbie were removed by denudation before the Triassic rocks were deposited. There is therefore no prospect of finding any part of the Coal-measures within that area north of the Trias. The same evidence renders it very improbable that the Coal-measures will be found underneath the Trias extending from the Cadgill Burn near Kirkpatrick south-westwards to Annan, or below the basin of New Red Sandstone at Dumfries.

Bores have been put down near Springkell, east of Kirtlebridge, which have proved the existence there of thin coal-seams, but none workable. This result is what might be expected from a consideration of the evidence in the field, for these thin seams lie there below the Marine limestones of Kirtlebridge (d^{vii}, Plate III.), on the horizon of the Lawston Linn coals of Liddisdale and the Scremerston coals of Northumberland (d^{vi}, Pl. III.). These coals, as developed in Dumfriesshire, are of little or no economic importance. It is probable, however, that representatives of the Kilnholm coals (d^{viii}, Pl. III.) might be found underneath the Trias between Cadgill Burn and Annan, but even these seams, as they appear in Liddisdale, are too thin to be worked at the present time.

V. Comparison of the Carboniferous Subdivisions in Eskdale and Liddisdale with those in Northumberland and Central Scotland.

The evidence relating to the correlation of the Carboniferous subdivisions in Eskdale and Liddisdale with those in Northumberland is fortunately of a conclusive character. To Mr Tate, of Alnwick, belongs the merit of having been the first to establish the sequence of the Carboniferous rocks in north Northumberland, where he worked it out between 1849 and 1868. His classification is given below:—

3. Calcareous group, embracing all the beds from the base of the Millstone Grit down to the Dun Limestone, and containing numerous marine limestones, with alternations of sandstones, shales, and coal-seams.
2. Carbonaceous group, with various workable coal-seams (Scremerston coals) and thin limestones, usually impure.
1. Tuedian group, comprising all the strata between the base of the Carbonaceous division and the Upper Old Red Sandstone, and containing shales, clays, sandstones, and thin beds of argillaceous limestone (Cementstone group).

This classification has been adopted by the Geological Survey with one modification, viz., the insertion of the Fell Sandstone group between the Tuedian and Carbonaceous divisions. The detailed mapping of the border territory has shown that TATE's classification applies not only to north Northumberland, but to north-east Cumberland, Liddisdale, and Eskdale.

The foregoing order of succession is exposed in clear sections in the basin of the Tweed near Berwick, and along the shore to the south-east of that town as far as Cheswick, which we had an opportunity of examining this year.

The conformable passage from the Upper Old Red Sandstone through the contemporaneous volcanic rocks of Kelso, which at Carham are overlain by a prominent band of cornstone, into the overlying Cementstone group, is well seen in various streams, as for instance in the tributary of the Whiteadder near Preston, west of Edrom. Throughout the Merse of Berwickshire there is an extensive development of the Cementstone group, where they consist of green, grey, and red shales and clays, sandstones, and pale argillaceous limestones and cementstones, which, save on certain horizons, rarely yield fossils. Plant remains occur in some of the beds, but there are no coal-seams. No limestones, similar to those at Larriston and Thorlieshope in Liddisdale, with corals, brachiopods, and other organic remains indicating open sea conditions, have yet been found in the Cementstone group in the Merse. The fauna is largely estuarine, the characteristic form being *Modiola Macadami*. In the higher part of the group in the Tweed, near Coldstream, lamellibranch limestones, with *Orthoceras*, *Pleurotomaria*, fish-remains, scorpions, and crustaceans occur. Similar evidence is obtained at the head of Redesdale, where one of the limestone bands near the top of the group is richly charged with lamellibranchs, together with *Orthoceras* and *Rhynchonella*.*

The district south of the Tweed from Norham and Berwick, south by Lowick to beyond Belford, was accurately mapped and described by our late colleague Mr GUNN, where the order of succession is remarkably clear. On the slope overlooking the Tweed between Norham and Berwick, the Cementstone group is surmounted by the Fell sandstones, which in the north-east part of that area reach a thickness of 300 feet, but gradually swell out towards the south-west to 600 feet. Next in order come the members of the Carbonaceous group (Scremerston coals), with several workable coal-seams, the outcrops of which are laid down on the Geological Survey maps (sheets 110, N.W., N.E., old series, England and Wales). The average thickness of this division was estimated by Mr GUNN at 800 feet.†

The Scremerston coals and associated strata are followed in normal sequence by the Calcareous division, which, according to TATE's classification, as already indicated, embraces all the beds from the base of the Dun Limestone to the base of the Millstone Grit. The Calcareous division has been further classified into a Lower Calcareous subgroup, including the beds from the base of the Dun Limestone to the top of the Dryburn Limestone (1480 feet),‡ and an Upper Calcareous subgroup comprising the

* *Mem. Geol. Surv.*,—Geology of the Country round Otterburn and Elsdon, p. 10.

† *Ibid.*, Geology of the Coast south of Berwick-on-Tweed, p. 4.

‡ *Ibid.*, p. 17.

strata from the top of the Dryburn limestone to the base of the Millstone Grit (600 feet). At the base of the Upper Calcareous subgroup lie the Lickar coals (see Pl. IV.).

On the shore from Spittal, south-east to Cheswick, at low water, there are tolerably continuous sections of the Lower Calcareous subgroup, where the individual bands may be studied to advantage. The characteristic feature of the group is the presence of marine limestones, charged with corals, brachiopods, gasteropods, and other organic remains, indicating true marine conditions. In the lower portion there are three marked beds of limestone, the Dun, the Woodend, and the Oxford (see Pl. IV.), with sandstones, shales, thin seams of coal, and a band of oil-shale. In the upper part of this subgroup the marine limestones appear in force, which are here given in descending order, with the local names given to them at Lowick.*

No. 1	Limestone	(Dryburn)	25 feet.
,, 2	,,	(Low Dean)	
,, 3	,,	(Acre)	20 ,,
,, 4	,,	(Eelwell)	20 ,,

Sandstones, shales, and thin coal-seams are associated with these limestones; the highest (No. 1) being eventually succeeded at Cheswick by the Lickar Main coal, which was formerly wrought at that locality.

Our late colleague Mr Gunn states in his valuable paper on "The Correlation of the Lower Carboniferous Rocks of England and Scotland," that these four limestones (Nos. 1 to 4) have been traced almost continuously for nearly 100 miles in the northern counties of England, under various local names, so that there can be no doubt as to the identity of the limestones.†

At Lickar, about one mile north of Lowick, a small group of coals (the Lickar coals) succeeds the Dryburn limestone, embracing three and in some sections four seams, which seem to be inconstant.‡ These, in descending order, are the Limestone coal, Parrot coal, Rough coal, and Main coal.

South of Alnwick, towards Shilbottle and Felton, on the river Coquet, the representative of the Dryburn limestone is followed by the Upper Calcareous subgroup, including several limestones, the highest of which, laid down on the Geological Survey map (sheet 109, S.W., old series), is the Fell Top band. This subdivision is followed towards the east by the Millstone Grit and the Coal-measures.

The evidence now adduced shows clearly the striking resemblance between the sequence of the Lower Carboniferous rocks in Northumberland and that in Eskdale and Liddisdale, which is represented in graphic form in the vertical sections in Pl. IV. Apart from the resemblance in the successive groups, the correlation is further strengthened by the fact that some of the subdivisions have been traced more or less continuously from the one region to the other. For example, if we exclude the area in

* *Mem. Geol. Sur.*,—Geology of Coast south of Berwick-on-Tweed, p. 16.

† *Trans. Edin. Geol. Soc.*, vol. vii. p. 365.

‡ *Mem. Geol. Sur.*,—Geology of Belford, Holy Island, and the Farne Islands, p. 39.

the Cheviots occupied by the Lower Old Red Sandstone volcanic rocks, the Fell sandstones extend from Tweedmouth, by Carter Fell and across Dumfriesshire, to the mouth of the Nith. In like manner the group of Scremerston coals has been traced from the shore to the Old Red volcanic platform of the Cheviots, and reappears at Lewisburn in the basin of the Rede Water, Northumberland, where, as Mr CLOUGH * has shown, they overlie the mass of the sandstones of Peel Fell, and come beneath the coal-seams of the Plashetts. The Lewisburn coals cross over into the upper part of the Kershope Water and into the head of Tweeden Burn, and appear in Scotland as the Muirburn and Lawston Linn coals.

It is obvious, therefore, that the Calcareous division of Northumberland, with its dominant bands of marine limestone, are the equivalents of the Calcareous series which in Liddisdale and Eskdale overlie the Lawston Linn and the Muir Burn coals. We may reasonably proceed one step further and suggest that the massive marine limestones at Lowick, Northumberland, including the Dryburn, Low Dean, Acre and Eelwell bands, may be wholly or partly represented by the limestones of Penton Linns, Harelaw Hill and Gilnockie in Liddisdale. But while there is doubtless a striking general resemblance in lithological and palæontological characters in the Lower Carboniferous rocks of these two areas, pointing to similar terrestrial movements along the margin of the old Silurian tableland, yet there are some specific distinctions worthy of note. It has been shown that even below the Fell sandstones, in the upper part of the Cementstone group in Liddisdale, marine limestones appear charged with crinoids, corals, brachiopods, gasteropods and other organisms which have not been found in the same group in Berwickshire nor in Northumberland—a fauna, indeed, which is characteristic of the Carboniferous Limestone series. It further appears that the Scremerston coals, which comprise several workable seams south of the Tweed, gradually diminish in number and dwindle in importance when traced south-westwards into Dumfriesshire. The same observation applies to the Lickar coals.

Proceeding now to the consideration of the Carboniferous subdivisions in the Lothians and Fife, we meet with certain marked divergencies from the types of sedimentation in the border territory. Nevertheless it is possible to correlate the main divisions. A glance at the vertical sections of the Carboniferous system in Edinburgh and Fife shows how the sequence varies according to the special districts in which they are taken. For our present purpose it will be sufficient if we indicate the general characters of these divisions, and their equivalents in the border territory.

In central Scotland the Lower Carboniferous rocks are grouped in two divisions: (1) The Calciferous Sandstone series, overlain by (2) the Carboniferous Limestone series. The former is subdivided into (*a*) the Cementstone group, consisting of green, gray and red shales and clays, sandstones of various tints with pale argillaceous limestones or cementstones, which, like their equivalents in Berwickshire and Dumfriesshire, are singularly barren of organic remains, save on certain horizons; and (*b*) the oil shale

* *Mem. Geol. Sur.*, "The Geology of Plashetts and Kielder," p. 36.

group, composed of gray, white and yellow sandstones, black and blue shales, oil-shales, occasional thin coal-seams, clay ironstones, and thin limestones. The palæontological researches of Dr Traquair, Mr R. Etheridge, jun., the late Mr Kirkby, and others, have shown that while the fauna points mainly to estuarine or brackish water conditions, there are marine bands particularly in Midlothian and the east of Fife which increase in number near the top of the group as we approach the base of the Carboniferous limestone series. Indeed, Mr Kirkby has shown that the fauna of the Carboniferous limestone is present in the upper part of the Calciferous Sandstone series, so that the boundary line between these two divisions is merely an arbitrary one.

The normal Cementstone group appears to the north of the Silurian tableland, on the shore at Cockburnspath, where it rests comformably on the cornstone zone of the Upper Old Red Sandstone, the latter yielding scales of *Holoptychius nobilissimus*. The Cementstones there, as shown by Mr CLOUGH,* are of no great thickness, being truncated by a fault bringing in a subgroup of shales, sandstones, fireclays and thin coals (the latter under one foot thick), which probably represent the Scremerston division of Northumberland. Though on the whole unfossiliferous, the Cementstones there contain a brecciated lamellibranch limestone with plant remains, which recalls similar types in this group in the border region and in the Randerston beds in Fife, to which attention will be immediately directed. Overlying the Carbonaceous group on the shore at Cove, near Cockburnspath, we find sandstones, shales, two thin marine crinoidal limestones, clays, and a thin oil shale.† The late Mr GUNN suggested that the group of the Dun and Woodend limestones might be represented by the marine limestones in Cove Harbour, and that the oil-shale might be the equivalent of that beneath the Oxford limestone.‡ But whether this be correct or not, there can be no doubt that the Cove oil-shale represents a stage of the oil-shale group of the Lothians. Unfortunately there is no continuous section from the beds just described up to the marine limestones at Longcraig, Skateraw, and Chapel Point, at the base of the Carboniferous Limestone series, east of Dunbar, but the section so far is a connecting link between the Carboniferous subdivisions of the border region and central Scotland.

In Midlothian the normal Cementstones appear underneath the volcanic platform of Arthur's Seat, and overlying the Upper Old Red Sandstone in the southern part of Edinburgh. From a recent exposure in the city of Edinburgh, the remains of plants, ostracods, worms, crustaceans and fishes were obtained. The late Mr KIRKBY stated, as the result of his examination of the ostracods from this section, that had "the lot been found in Fife it would not have been higher than the Billow Ness beds." (See Plate IV.)

Above the volcanic platform of Arthur's Seat comes the great succession of strata ranging from the Granton sandstone and Wardie shales to the Hurlet limestone, the

* *Geol. Survey, Sum. of Progress for* 1902, p. 121.
† *Ibid.*
‡ *Trans. Edin. Geol. Soc.*, vol. vii. p. 366.

total thickness of strata amounting to about 3650 feet. No workable oil-shales occur in the lowest part of this group till we reach the level of 800 feet beneath the Burdiehouse Limestone, which is the position of the Pumpherston band. According to Mr CADELL's computations the oil-shales appear on different horizons in a series of strata whose vertical thickness is about 2750 feet, the highest band being the Raeburn shale, about 450 feet beneath the Hurlet limestone.*

It is obvious, therefore, that the oil-shale group as developed in Midlothian differs in a marked degree from the higher part of the Calciferous Sandstone series found in East Lothian.

In East Fife the Calciferous Sandstone series has an exceptional development, with special lithological and palæontological characters, which have been admirably described by Sir A. GEIKIE in his recent memoir on "The Geology of East Fife." † He calls attention to the fact that in place of the widely separated marine platforms, with comparatively few fossils, to be found in the Lothians and Western Fife, there is a great succession of marine bands crowded with organic remains, and alternating with numerous coal-seams, which distinguishes the group in the East of Fife from any other equivalent strata in Scotland, the total thickness amounting to about 4500 feet.

Below the sandstones of Fife Ness, as Sir A. GEIKIE has pointed out, there emerges a group of shales, clays, and thin seams of cementstone, resembling the Cementstone group in other parts of Scotland.‡ This group passes downwards into a nodular cornstone, which may represent the band at the top of the Upper Old Red Sandstone. Next in order come the Randerstone beds, composed of alternations of *Spirorbis* and lamellibranch limestones, sandstones, shales, with occasional fireclays, root beds and thin coals. No purely marine bands occur in this estuarine subdivision, though such marine organisms as *Bellerophon*, *Rhynchonella* and *Orthoceras* are met with. These are overlaid by the Billowness sandstones, followed by alternations of impure oil-shales, sandstones, cyprid limestones, with nineteen thin coals, and eventually by the "Encrinite bed," which forms a marked horizon in the Calciferous Sandstone of East Fife. This impure limestone is charged with corals, crinoids, polyzoa, brachiopods, including four species of *Productus*, a fauna which is characteristic of the Carboniferous Limestone series. Above the Encrinite limestone comes a succession of sandstones, shales, ironstones and thin coals, near the top of which there are bands of limestone (Abden limestones), which contain a typical marine fauna like that of the Hurlet and Hosies limestones.

It is obvious that in the Calciferous Sandstone of the East of Fife there is a striking departure from the type of strata which in Northumberland intervenes between the Fell sandstones and the Eelwell limestone. It is not improbable that the "Encrinite bed" of Fife may be the equivalent of the crinoidal limestone at Cove, near Cockburnspath.

In central Scotland the Carboniferous Limestone series, as is well known, is represented by a lower group containing marine limestones, sandstones, shales, fireclays and

* *Trans. Geol. Soc. Edin.*, vol. viii., part i., p. 136.

† *Memoirs of the Geol. Survey*, "The Geology of East Fife," 1902, p. 71.

‡ *Ibid.*, p. 121.

occasional coal-seams; a middle group (Edge Coals), consisting of many valuable coal-seams, ironstones, sandstones and shales, with no limestones; and an upper group, composed of limestones, sandstones, shales and coal-seams. This triple classification obtains throughout the midland valley, the lithological and palæontological types being remarkably persistent.

The lower group of limestones (Hurlet and Hosies) has been correlated by the late Mr GUNN with the massive marine limestones at Lowick,* Northumberland (Dryburn, Low Dean, Acre and Eelwell), which, as already indicated, are separated from the upper limestone group by the Lickar Coals. (See Plate IV.) If this correlation, which is highly probable, should ultimately prove to be correct, then it follows that the group of strata which, in Northumberland and Eskdale, intervene between the base of the Fell sandstones and the Eelwell and Gilnockie limestones, represents that part of the Calciferous Sandstone in central Scotland which overlies the Cementstone group. It may be further noted that the Lickar and Kilnholm coals are meagre representatives of the valuable series of Edge Coals in the midland valley. (See Plate IV.)

The triple classification of the Coal-measures at Canonbie adopted by Mr KIDSTON, from the evidence of the plants, does not obtain in central Scotland. In the latter area only the Lower and Middle Coal-measures are represented: the lower group containing a valuable series of coals and ironstones, and the middle consisting of red sandstones, shales, clays, marls, thin limestones, and poor coals, yielding plants, molluscs, crustaceans and fishes. On the evidence of the plants Mr KIDSTON correlates this Red Sandstone group, like that of Byre Burn at Canonbie, with the middle Coal-measures of England.†

LIST OF PAPERS REFERRING TO THE GEOLOGY OF THE DISTRICTS UNDER REVIEW.

1844. MILNE HUME, "Geological Account of Roxburghshire," *Trans. Roy. Soc. Edin.*, vol. xv. p. 433.

1853. TATE, G., "The Fossil Flora of the Mountain Limestone Formation of the Eastern Borders," in *Johnstone's Nat. Hist. of the Eastern Borders*, p. 290.

1857. TATE, G., "Anniversary Address to the Members of the Berwickshire Naturalists' Club," *Proc. Berwickshire Nats. Club*, vol. iii. p. 135. (Read 1853.)

1857. EMBLETON, R., "Address delivered to the Berwickshire Naturalists' Club," *Proc. Berwickshire Nats. Club*, vol. iii. p. 219. (Read 1856.)

1861. HOWELL, H. H., and (Sir) A. GEIKIE, *Memoir of Geological Survey of Great Britain*, "Geology of the Neighbourhood of Edinburgh" (sheet 32).

1862. GIBSONE, E., "The Coal Formation of Canonbie," *Trans. North of England Institute of Mining Engineers*, vol. xi. p. 65.

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* *Trans. Geol. Soc. Edin.*, vol. vii. p. 306.

† We wish to acknowledge the valuable assistance rendered by Mr A. MACCONOCHIE and Mr D. TAIT in the preparation of the fossil lists embodied in this paper and the list of papers referring to the geology of the districts under review.

1863. Geikie, A., *Memoir of Geological Survey of Great Britain*, "The Geology of Eastern Berwickshire" (sheet 34).

1863. Tate, G., "Fauna of the Mountain Limestone on the Berwickshire Coast, with a preliminary notice of the succession of the strata on the Eastern Borders," *Proc. Berwickshire Nats. Club*, vol. iv. p. 151.

1864. Binney, E. W., "Observations on the Carboniferous, Permian, and Triassic Strata of Cumberland and Dumfries," *Proc. Lit. and Phil. Soc. Manchester*, vol. iii. p. 162.

1865. Binney, E. W., "Further Observations on the Carboniferous, Permian, and Triassic Strata of Cumberland and Dumfries," *Mem. Lit. and Phil. Soc. Manchester*, series 3, vol. ii. p. 343. (Read October 1863.)

1866. Geikie, (Sir) A., H. H. Howell., and J. Young., *Memoir Geological Survey of Great Britain*, "The Geology of East Lothian" (sheets 33, 34, 41).

1868. Bailes, G., "Sections of Mountain Limestone Strata at Scremerston, Northumberland, with a 'Note on the Scremerston Sections,' by G. Tate," *Proc. Berwick Nat. Field Club*, vol. v. p. 349.

1868. Tate, G., "Miscellanea Geologica for 1866," *Proc. Berwickshire Nats. Club*, vol. v. p. 283.

1869. Tate, G., "The Geology, Botany, and Zoology of the Neighbourhood of Alnwick," 8vo, Alnwick. (Reprint of chapters from the History.)

1874. Goodchild, J. G., "Note on the Carboniferous Conglomerates of the Eastern Part of the Basin of the Eden," *Quart. Jour. Geol. Soc.*, vol. xxx. p. 394.

1875. Lebour, G. A., "On the Limits of the Yoredale Series in the North of England," *Geol. Mag.*, Dec. 2, vol. ii. p. 539.

1876. Lebour, G. A., "On the Larger Divisions of the Carboniferous System in Northumberland," *Trans. North of England Institute Mining Engineers*, vol. xxv. p. 225.

1877. Binney, E. W., "Note on the Upper Coal Measures, Canonbie," *Proc. Lit. and Phil. Soc. Manchester*, vol. xvi. p. 192.

1878. Etheridge, R., jun., "The Invertebrate Fauna of the Lower Carboniferous or Calciferous Sandstone Series of the Edinburgh Neighbourhood." (Read November 7, 1877.) *Quart. Jour. Geol. Soc.*, vol. xxxiv. pp. 1–26.

1878. Lebour, G. A., Geological Map of Northumberland.

1878. Lebour, G. A., *Outlines of the Geology of Northumberland*, 8vo, Newcastle (2nd ed. in 1886).

1879. Geikie, (Sir) A., J. Geikie, and B. N. Peach., *Mem. Geol. Sur. Scot.* (sheet 31).

1881. Geikie, (Sir) A., "A Recent 'Find' in British Palæontology," *Nature*, vol. xxv. p. 1.

1883. Peach, B. N., "On some New Crustaceans from the Lower Carboniferous Rocks of Eskdale and Liddisdale," *Trans. Roy. Soc. Edin.*, vol. xxx. p. 73. (Read July 1880.)

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DESCRIPTION OF PLATES.

Plate I.

Geological map of the Canonbie District on the scale of two inches to a mile, showing the areas occupied by the Lower, Middle and Upper Coal-measures.

Plate II.

Table of vertical sections prepared from journals of bores sunk in the Canonbie district.

Abbreviations:—Upper C.M = Upper Coal-measures; M.C.M. = Middle Coal-measures; L.C.M. = Lower Coal-measures; M.G. = Millstone Grit; C.L. = Carboniferous Limestone.

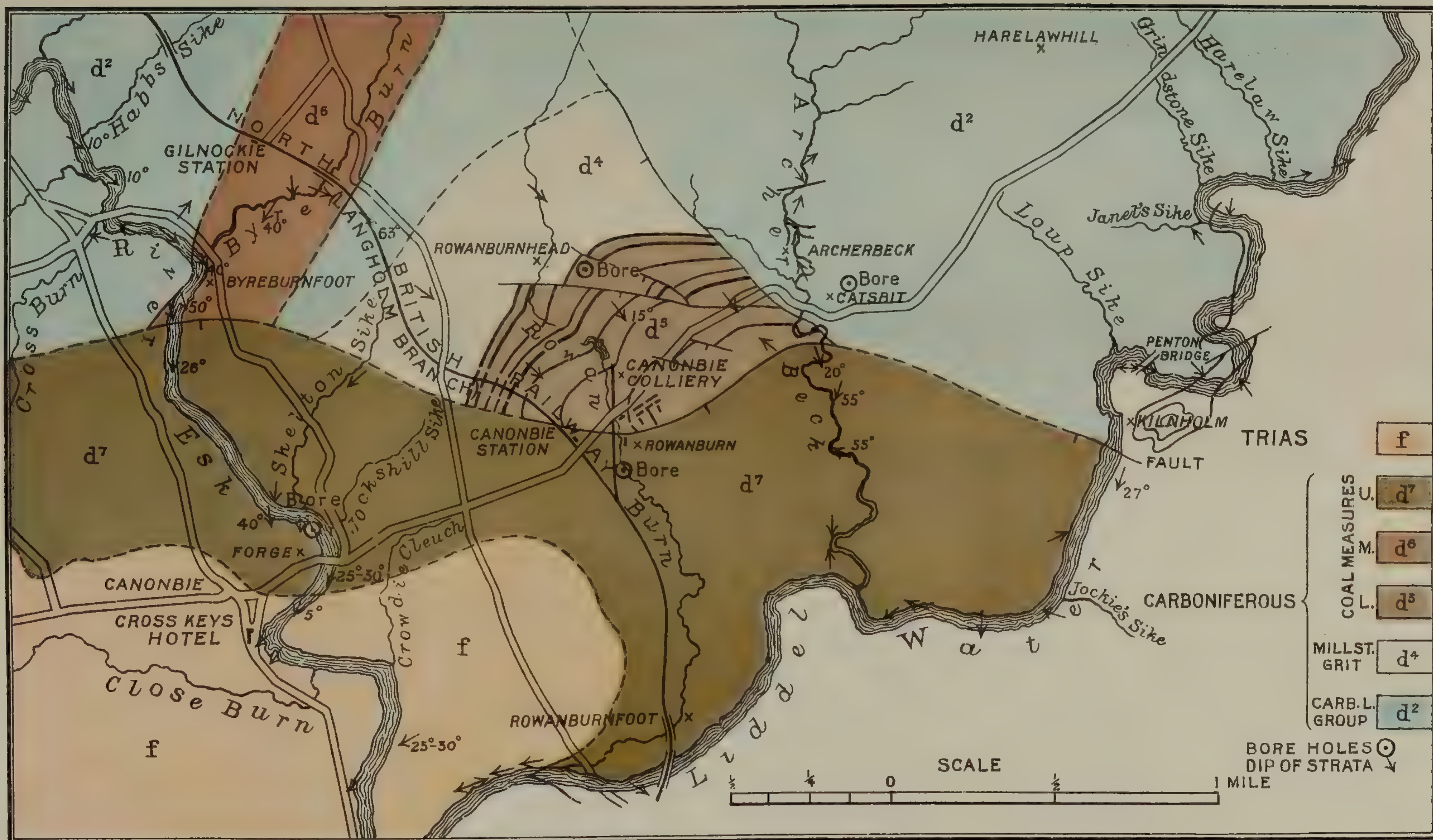
Plate III.

Series of horizontal sections to illustrate the geological structure of the Border region between Liddisdale and Annandale.

Explanation of Geological Signs:—b^5 = Upper Silurian; c^3 = Upper Old Red Sandstone; d^i = Volcanic Zone of Tarras Water and Birrenswark; d^{ii} = Whita Sandstone; d^{iii} = Cementstone Group; d^{iv} = Fell Sandstones; d^v = Glencartholm Volcanic Group; d^{vi} = Lawston Coals; d^{vii} = Marine Limestones; d^{viii} = Kilnholm Coals; d^{ix} = Upper Limestones; d^x = Millstone Grit; d^{xi} = Rowanburn Coals; d^{xii} = Byre Burn Coals; d^{xiii} = Red Sandstones of Canonbie (Upper Coal-measures); f = Trias.

Plate IV.

Comparative series of vertical sections of the Carboniferous system in (1) Eskdale and Liddisdale; (2) Berwick and Northumberland; (3) Fife; (4) Midlothian.



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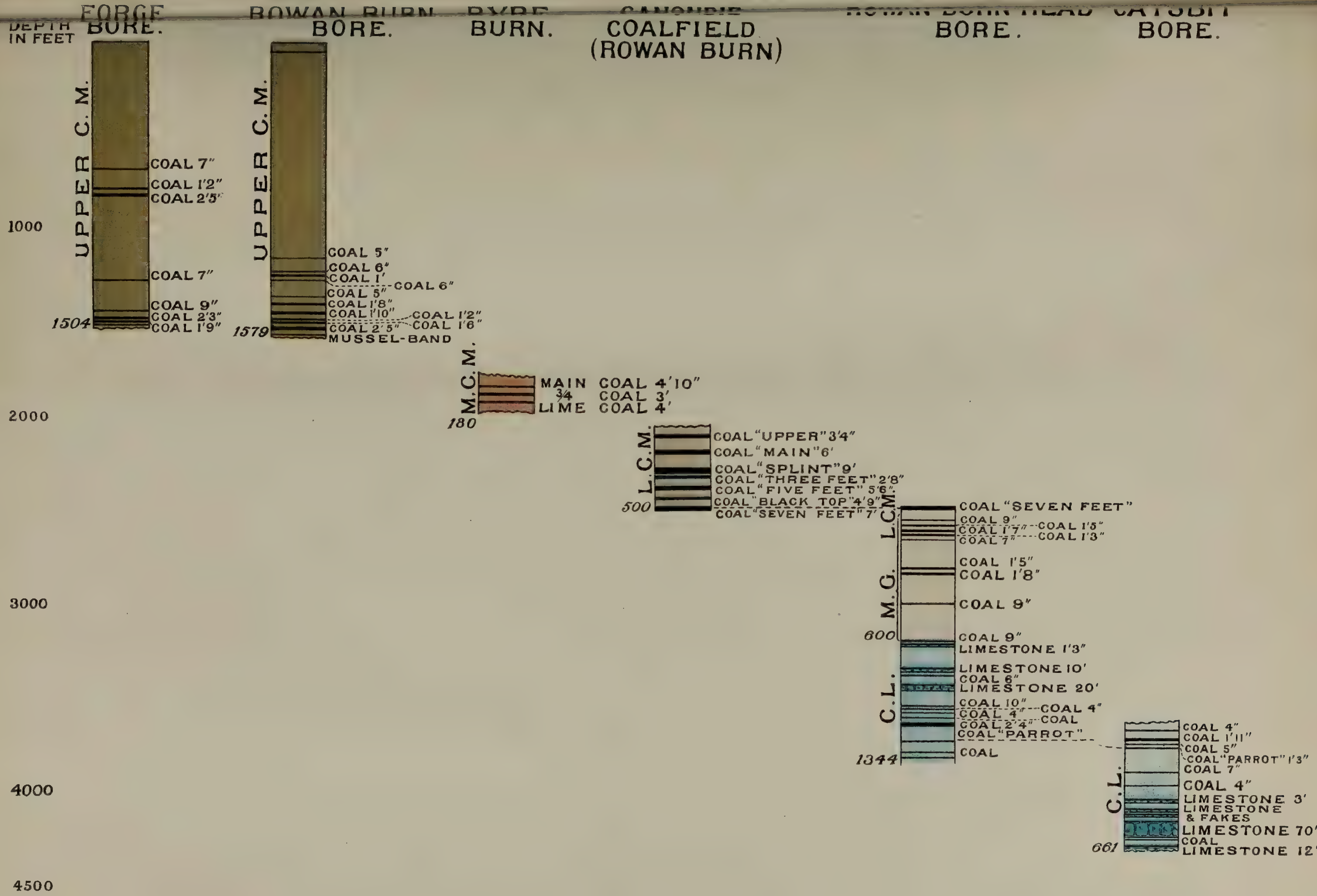

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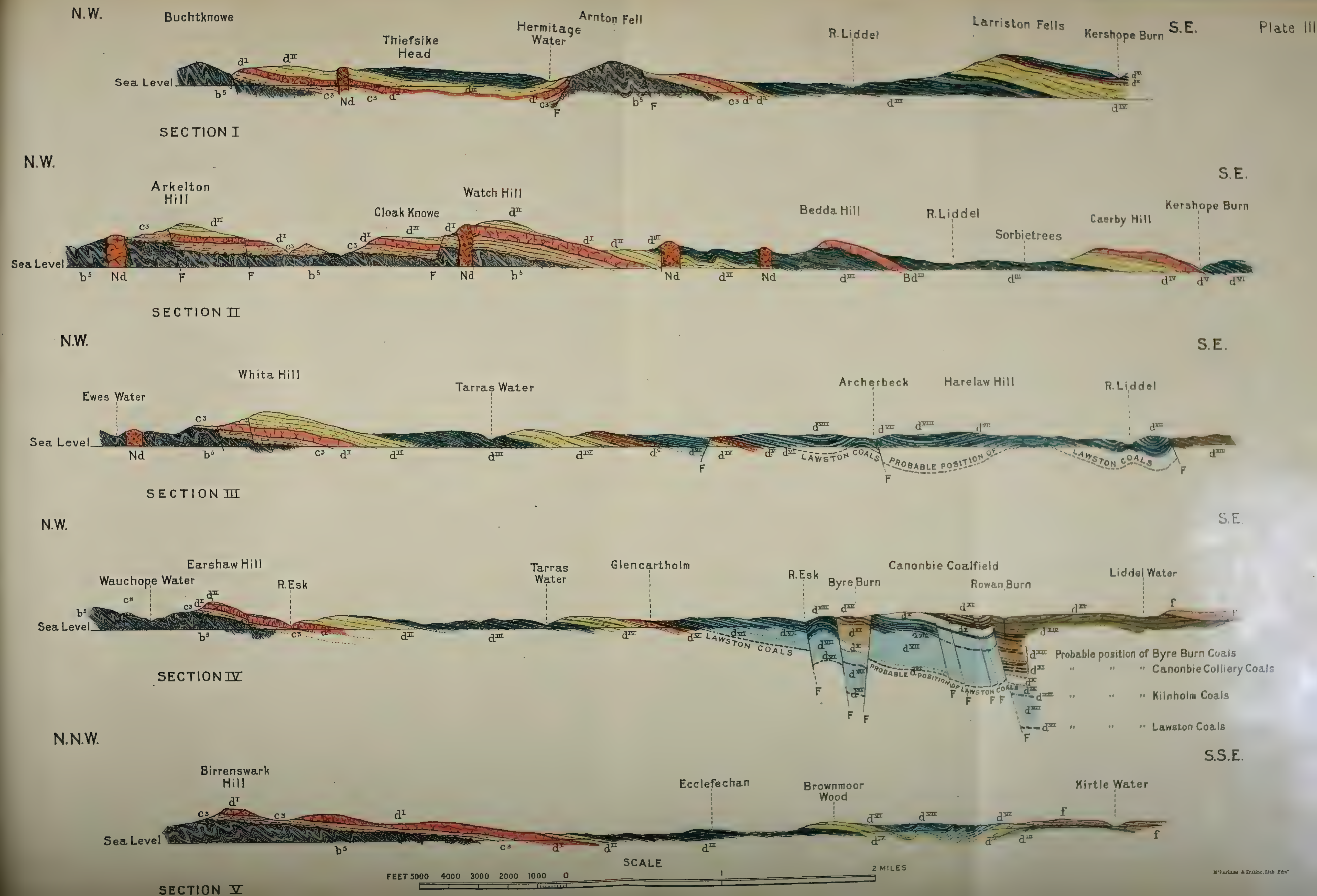
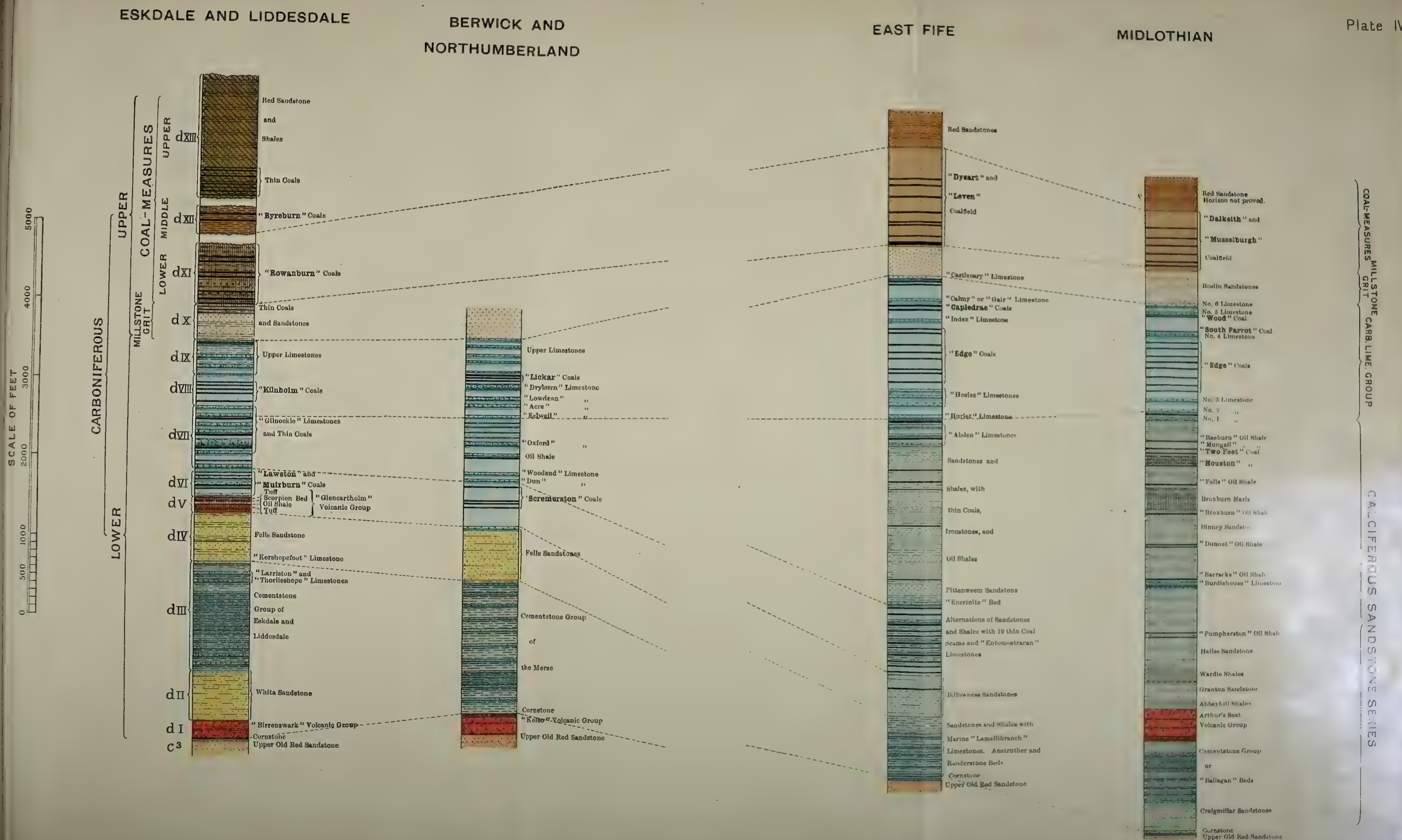


Plate III.
N.W.
Buchtknowe
Thiefsike Head
Hermitage Water
Arnton Fell
R. Liddel
Larriston Fells
Kershope Burn
S.E.
Sea Level
SECTION I
N.W.
Arkelton Hill
Cloak Knowe
Watch Hill
Bedda Hill
R. Liddel
Sorbietrees
Caerby Hill
Kershope Burn
S.E.
Sea Level
SECTION II
N.W.
Ewes Water
Whita Hill
Tarras Water
Archerbeck
Harelaw Hill
R. Liddel
S.E.
Sea Level
LAWSTON COALS
PROBABLE POSITION OF
LAWSTON COALS
SECTION III
N.W.
Earshaw Hill
Wauchope Water
R. Esk
Tarras Water
Glencartholm
R. Esk
Byre Burn
Canonbie Coalfield
Rowan Burn
Liddel Water
S.E.
Sea Level
LAWSTON COALS
PROBABLE POSITION OF LAWSTON COALS
Probable position of Byre Burn Coals
" " " Canonbie Colliery Coals
" " " Kilnholm Coals
" " " Lawston Coals
SECTION IV
N.N.W.
Birrenswark Hill
Ecclefechan
Brownmoor Wood
Kirtle Water
S.S.E.
Sea Level
SCALE
FEET 5000 4000 3000 2000 1000 0
1
2 MILES
SECTION V

Plate IV.


ESKDALE AND LIDDESDALE
BERWICK AND NORTHUMBERLAND
EAST FIFE
MIDLOTHIAN
SCALE OF FEET
0
500
1000
2000
3000
4000
5000
CARBONIFEROUS
UPPER
LOWER
COAL-MEASURES
UPPER
MIDDLE
LOWER
MILLSTONE GRIT
dXIII
dXII
dXI
dX
dIX
dVIII
dVII
dVI
dV
dIV
dIII
dII
dI
c3
Red Sandstone and Shales
Thin Coals
"Byreburn" Coals
"Rowanburn" Coals
Thin Coals and Sandstones
Upper Limestones
"Kilnholm" Coals
"Gilnockie" Limestones and Thin Coals
"Lawston" and "Muirburn" Coals
Tuff
Scorpion Bed
Oil Shale
Tuff
"Glencartholm" Volcanic Group
Fells Sandstone
"Kershopefoot" Limestone
"Larriston" and "Thorlieshope" Limestones
Cementstone Group of Eskdale and Liddesdale
Whita Sandstone
"Birrenswark" Volcanic Group
Cornstone
Upper Old Red Sandstone
Upper Limestones
"Lickar" Coals
"Dryburn" Limestone
"Lowdean"
"Acre"
"Eelwell"
"Oxford"
Oil Shale
"Woodend" Limestone
"Dun"
"Scremerston" Coals
Fells Sandstones
Cementstone Group of the Merse
Cornstone
"Kelso" Volcanic Group
Upper Old Red Sandstone
Red Sandstone
"Dysart" and "Leven" Coalfield
"Castlecary" Limestone
"Calmy" or "Gair" Limestone
"Capledrae" Coals
"Index" Limestone
"Edge" Coals
"Hosies" Limestones
"Hurlet" Limestone
"Ablen" Limestone
Sandstones and Shales, with thin Coals, Ironstones, and Oil Shales
Pittenweem Sandstone
"Encrinite" Bed
Alternations of Sandstones and Shales with 19 thin Coal Seams and "Entomostracan" Limestones
Billowness Sandstones
Sandstones and Shales with Marine "Lamellibranch" Limestones. Anstruther and Randerstone Beds
Cornstone
Upper Old Red Sandstone
Red Sandstone
Horizon not proved.
"Dalkeith" and "Musselburgh" Coalfield
Roslin Sandstones
No. 6 Limestone
No. 5 Limestone
"Wood" Coal
"South Parrot" Coal
No. 4 Limestone
"Edge" Coals
No. 3 Limestone
No. 2
No. 1
"Raeburn" Oil Shale
"Mungall"
"Two Feet" Coal
"Houston"
"Fells" Oil Shale
Broxburn Marls
"Broxburn" Oil Shale
Binney Sandstone
"Dunnet" Oil Shale
"Barracks" Oil Shale
"Burdiehouse" Limestone
"Pumpherston" Oil Shale
Hailes Sandstone
Wardie Shales
Granton Sandstone
Abbeyhill Shales
Arthur's Seat Volcanic Group
Cementstone Group or "Ballagan" Beds
Craigmillar Sandstones
Cornstone
Upper Old Red Sandstone
COAL-MEASURES
MILLSTONE GRIT
CARB. LIME. GROUP
CALCIFEROUS SANDSTONE SERIES

XXXIII.—*Supplementary Report on Fossil Fishes collected by the Geological Survey of Scotland in the Upper Silurian Rocks of Scotland.* By Ramsay H. Traquair, M.D., LL.D., F.R.S., Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh. (With Three Plates.)

(Read July 5, 1901. Given in for publication November 29, 1904. Issued separately January 26, 1905.)

Since the publication of my "Report" in December 1899, Mr Tait, fossil collector to the Geological Survey of Scotland, has been successful in adding a number of specimens of great interest to the valuable series of Scottish Silurian Fishes which supplied the material for that memoir, and with the permission of the Director of the Survey, to whom my best thanks are due, I propose to give descriptions of them in the following Supplement. The search for these remarkable fishes has likewise been taken up by private collectors, and I have therefore pleasure in acknowledging my indebtedness to Messrs J. Young, of Lesmahagow, P. Macnair, of Glasgow, A. Whyte and F. White, of Muirkirk, and D. Nimmo, of Hamilton, for kindly submitting to me the material resulting from their own work in this field.

The present communication is concerned only with the recording of additional facts, further discussion of the theoretical points suggested by the study of these ancient fishes being reserved for another paper.

Historical Note.

The first finder of these fish-remains in the Silurian rocks of the South of Scotland seems to have been the late Dr Slimon, the well-known collector of the fossil *Crustacea* and *Merostomata* of the Lesmahagow district. It was from Mr Macnair that I received information that the Slimon Collection, now in the possession of Miss Slimon, contained examples of *Thelodus* and *Birkenia* from the "Ludlow" beds of Logan Water, and I can confirm the fact from personal examination of the specimens. These are, however, imperfect; and as they have no labels, we have no evidence as to whether or not Dr Slimon recognised them as fish-remains.

To Mr Macnair I am also indebted for the loan of a microscopic slide of scales of *Thelodus Scoticus*, apparently from Logan Water, which he found in the collection of fossils formed by the late Dr John Young, and now in the Glasgow Corporation's Museum at Kelvingrove. In this slide the minute structure of the simple dentine substance of which the scales are composed is very clearly shown.

In my previous memoir I mentioned the fact that Mr James Young had found a fish in the Logan Water beds before the locality had been visited by the collectors of the Geological Survey. I have since that time had the pleasure of seeing the specimen, and find it to be a very fair example of *Thelodus Scoticus*, in which the caudal fin is exceedingly well shown.

Order HETEROSTRACI, Lankester.

Family Cœlolepidæ, Pander.

Thelodus Scoticus, Traquair.

When I wrote my previous memoir, the general configuration of this genus and species was known principally from specimens from the Downtonian beds of Seggholm, Monks Burn, and Birkenhead Burn, those from the Ludlow rocks of Logan Water being more useful for the study of the scales. Since that time, however, specimens have turned up in the last-named horizon and locality which considerably add to our knowledge of the species, and necessitate a correction in the definition of the genus.

In Plate I. fig. 4 is represented a specimen of *Th. Scoticus* collected by Mr Tait in the Logan Water beds, and showing essentially the same contour as those from the Downtonian horizon, as may be seen by comparing figs. 3 and 4, Plate I. of my former "Report." The fish is in front compressed vertically, and the outline of the lateral flaps is well shown, while behind the tail is twisted so as to display a side view of the caudal fin, of which the upper lobe is unfortunately wanting. The specimen looks indeed short and stumpy compared with some others, but I need scarcely remind the reader that in the absence of an exoskeleton of closely fitted plates, or of a well-ossified endoskeleton, the contour of a fossil fish is extremely liable to variation.

Dorsal fin.—The absence of all fins except the caudal (unless the pectoral flaps are to be looked upon as paired members) was in my former memoir and in my address to the Zoological Section of the British Association at Bradford in 1900 given as a character of the entire family of Cœlolepidæ, but clear evidence is now forthcoming that, in *Thelodus* at least, a small dorsal fin was present. It was Mr James Young, of Lesmahagow, who first informed me that specimens of *Thelodus Scoticus* in his possession had more fins than I had credited the genus with, and to his kindness I am indebted for the privilege of figuring three specimens from Logan Water which show the fin in question.

These are represented in Plate I. figs. 1, 2, and 3, the dorsal fin being seen at *d*. This fin is situated not far in front of the caudal, is small in size, rounded in contour, and covered with minute shagreen scales of the same type as those of the rest of the fish behind the head. I may here mention that, subsequent to my receiving these specimens from Mr Young, Mr Tait showed me one of the same species from the Downtonian of Seggholm in which the same fin was distinctly visible.

Eyes.—In Plate I. figs. 5 and 6 are represented two heads of *Thelodus Scoticus*, collected by Mr Tait from the Ludlow beds of Shanks Castle, Logan Water, in which, as already noted in my Bradford "Address,"* certain appearances are visible which are at least strongly suggestive of eyes. These appearances, observed by Mr TAIT himself before the specimens were referred to me, consist of two small dark rounded spots (*o*), one at each antero-external angle of the head, and which come out still more distinctly if the specimen be wetted with a little water. In fig. 5 the dark spots take on a somewhat ring-shaped appearance, the centre being paler than the circumference.

On examination with a strong lens, or with the compound microscope (one inch

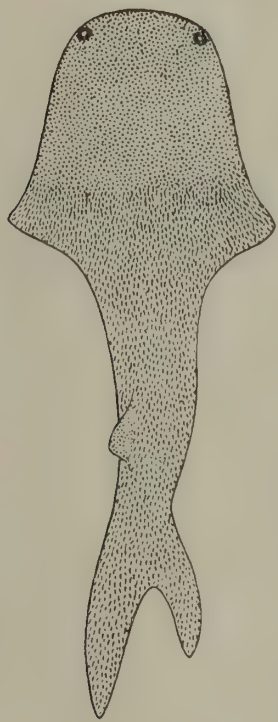


FIG. 1.—Diagrammatic restoration of *Thelodus Scoticus*, Traq., showing the dorsal fin and the position of the eyes. The tail is turned round so as to show the caudal fin in profile.

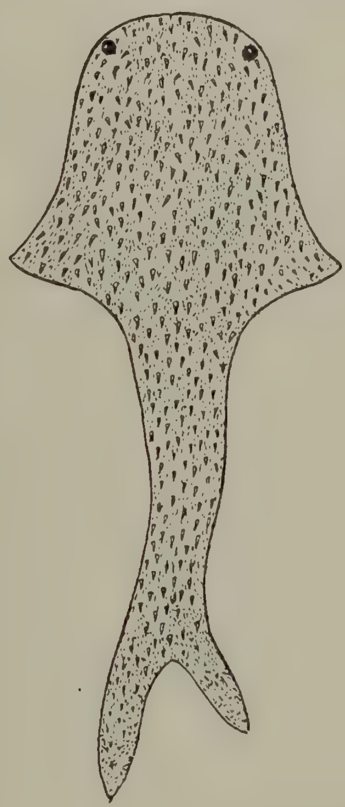


FIG. 2.—A similar outline of *Lanarkia spinosa*, Traq. In this genus (*L. horrida*) the eyes have been found, but not the dorsal fin as yet.

objective), the spaces occupied by these dark spots are seen to be covered with minute scales like the rest of the skin, but which appear as if stained by a darker colour, which, as already said, is intensified the moment the surface is wetted. As the same phenomenon is also observed in a specimen of *Lanarkia horrida*, I shall return to it presently in dealing with that genus and species.

Scales.—A specimen from Logan Water in the Geological Survey Collection shows beautifully the cœlolepid character of the scales, these being split or divided horizontally so as to expose the internal pulp cavity. The external sculpture of the pointed

* Also in the *Summary of Progress of the Geological Survey of the United Kingdom*, for the year 1900, p. 175.

scales of the tail and body behind the head is admirably shown in a specimen lent me by Mr MACNAIR, and from which the drawings of magnified scales given in Plate I. figs. 7 and 8 were taken.

Lanarkia horrida, Traquair.

A specimen of *Lanarkia horrida* from Birkenhead Burn (Geological Survey Collection), represented, natural size, in Plate II. fig 1, distinctly displays, even when dry, two small round spots in the same relative position as those in the two examples of *Thelodus Scoticus* described above, and in like manner, when the fossil is wetted, stand out with all the greater distinctness. On examination with a powerful lens or with the compound microscope (one inch objective), nothing more can be made out than the appearance of a dark bituminous-looking stain at the spots concerned. The white dot seen in the centre of the left dark spot in fig. 1 is due to the carbonate of lime filling up the pulp cavity of a dermal spinelet which has become broken across at this place.

That the above-described dark-stained spots represent the position of the eyes in *Thelodus* and *Lanarkia* will, I think, scarcely be questioned, and indeed the phenomenon does not stand alone in the field of fossil ichthyology. In the case of the little Cyprinodont fish *Prolebias cephalotes* (Ag.), which occurs in shoals in the Lower Oligocene of Aix in France, the pigment of the eye, as well as of the abdominal cavity, is preserved; and on a slab of fissile grey shale from China, and probably of Jurassic age, now before me, numerous specimens of a small *Lycoptera* show a conspicuous round black spot occupying the well-defined interior of the orbit. That no definite orbits are observable in the case of the Cœlolepidæ is inseparable from the non-coherent nature of the elements of the exoskeleton.

It is, of course, interesting to find that these eye-spots in the Cœlolepidæ occupy exactly the same relative position on the head as the orbits, or supposed orbits, of *Pteraspis* and *Drepanaspis*, seeing that, if my view of the interaffinities of these creatures is correct, such a position is the one we would naturally expect. But upholders of the arthropod idea of vertebrate derivation may also point out that the lateral eyes of certain Eurypterids (*Pterygotus*, *Slimonia*) also occupy the same situation in the head. Other members of the same group (*Eurypterus*, *Stylonurus*) have their lateral eyes on the upper surface of the cephalic shield towards the middle, as the Cephalaspidæ,—in either case I fear we have to do with what the Germans call a "Convergenz-Erscheinung."

? *Lanarkia*, sp.

In Plate I. fig. 2 is represented the anterior part of a fish apparently belonging to the Cœlolepidæ, and here provisionally referred to *Lanarkia*, seeing that, so far as the lens can discover, the exoskeleton seems to consist of minute, pointed, hollow spinelets. What exists of the counterpart is shown in fig. 3.

What is of interest in this specimen is, in the first place, a dark line on each side, which runs parallel with and internal to each of the right and left margins respectively, at a distance of about $\frac{1}{8}$ inch. In front these two lines are joined by a cross commissure running behind the front of the head, which is oblique, owing to the deformation which is frequent in these specimens; this line is not distinct enough in the figures, being only clearly seen when the stone is wet. At the sides we observe, between the margin and the intramarginal line referred to, several transverse dark bars, which divide the space concerned into a corresponding number of compartments, how many in all we cannot say, as the fossil is cut off behind; the counterpart, however, shows on one side as many as six.

On examination with strong lens the above-noted dark lines seem to be due to some extent to a greater closeness of the minute dermal spinelets, which are black in colour, with a lesser abundance of them on the paler interspaces. Be that as it may, I have figured this specimen because of the manner in which these lateral markings recall to our minds those on internal casts of the carapace in *Cyathaspis integer* (Kunth) and *Cy. Sturii*, Alth., and which have been supposed to indicate the position of branchial pouches. Geological Survey Collection; collected by Mr TAIT at Monks Water.

Order OSTEOSTRACI, Lankester.

Family CEPHALASPIDÆ, Traquair.

Ateleaspis tessellata, Traquair.

As indicated in my British Association "Address," and in the "Report of Progress" for 1900, material has come to hand enabling us to form a much better conception of the configuration and structure of *Ateleaspis* than was possible from the meagre specimens available for my former memoir. It will therefore be necessary to furnish a new description of the whole animal, from which it will appear that it is more closely allied to *Cephalaspis* than I had previously imagined.

General form.—In Plate III. fig. 1 a nearly entire specimen is represented, only the tip of the tail being wanting. The head, here vertically compressed, while the tail is twisted round so as to be seen in profile, is almost of an ovoid outline, rounded in front, more gently so laterally, and assuming on each side behind a contour which, though evenly rounded and with no tendency to angularity, recalls the "pectoral flap" of the Cœlolepidæ. In fig. 2 the head is wanting, but the hinder extremity is better seen, and shows that the heterocercal caudal fin was not, as I had taken for granted, bilobate, as in *Thelodus* and *Lanarkia*, but unilobate, as in *Cephalaspis*. In both figures, but likewise most distinctly in fig. 2, we see a small dorsal fin of a rounded contour, and occupying a position identical with that of the dorsal of *Cephalaspis*, namely, opposite a space just in front of the caudal.

Head.—As previously described, the head-shield of very numerous small polygonal pieces, ornamented externally by a delicate tuberculation. At the back of the head these pass into the rhombic scales of the body, while on the lateral or "opercular" flaps they become almost minute. In the specimen represented in fig. 1 it is hardly possible to recognise the position of the orbits: their existence is, however, clearly indicated in another piece, the anterior part of a head, which to all appearance belongs to the same species. On carefully washing off the decayed bony matter from the counterpart of this specimen a sharp impression of the central part was procured, which is represented of the natural size in fig. 9 of the same plate.

Fig. 3.—Diagrammatic restored outline of *Ateleaspis tessellata*, Traq., the tail turned round so as to appear in profile.

Here we see the rounded contour of the front of the head-shield, and a due distance back from it are the two rounded orbits, connected with each other by a narrow bridge, as in *Tremataspis*, or indeed as in the *Asterolepidæ*, though there the connection is wider. This bridge is, however, filled up by a narrow plate, which at each lateral extremity joins a rounded piece covering over the orbit like the "eyelid" of the last-named family. Just in front of this bridge, between the orbits, is the impression of an anteorbital fossa (*a. o.*), which shows clear evidence of having been furnished, as in *Tremataspis* and *Cephalaspis*, with a median elevation (frontal organ of Rohon), perforated at the apex by a minute opening. Again, behind the orbital bridge, there are clear traces of the postorbital fossa (parietal organ of Rohon), as found in *Cephalaspis*, but it is presently cut off behind by the edge of the stone, as seen in the figure. Another specimen, not figured, shows, however, that the postorbital fossa was similar in proportions and form to that of *Cephalaspis*, and that the same arrangement of radiating vascular canals existed under the middle layer of the shield.

Scales.—In figs. 1 and 2, Plate III., the scales are seen to be arranged on the flank in nearly vertical rows, which, on approaching the middle line of the back, turn forwards at an obtuse angle,—whether they also change direction at the ventral margin is not seen in any specimen. But their disposition on the sides and dorsally coincides with the arrangement of the body-scales in *Cephalaspis*, save that in this case the long narrow lateral and dorsal scales are represented by rows of separate rhombic scales.

Posteriorly the scales become smaller, and on the caudal body-prolongation are irregularly angular in shape. The dorsal and caudal fin-membranes are also covered by minute scales, which, on the last-named fin at least, tend to a linear arrangement, as already noticed in my previous "Report."

To that "Report," p. 836, I may also refer for an account of the external sculpture and internal microscopic structure of the scales; here I take the opportunity of supplementing that description by a drawing, Pl. II. fig. 10, which is so far diagrammatic that it combines in one view the results obtained from more than one less comprehensive vertical section. The three layers of the scale-substance are here very distinctly shown, and special attention may be directed to the *lacunæ*, abundantly seen in the sections of the ridges and tubercles which make up the outer layer.

Observations.—The question first arises as to whether the specimens from which the above description has been taken are strictly identifiable with those already adopted as types of the genus and species. There can, I think, be no mistake with regard to the originals of figs. 7, 8, 11, and 12 on Plate IV. of my previous memoir: the difficulty is with regard to the original of fig. 6, which, as I have already stated (*op. cit.*, p. 835), shows no trace of the antorbital fossæ, or of the postorbital valley, which are prominent markings on the shield of *Cephalaspis*. As, however, there are clear indications of orbits in that specimen, and as it conforms in other respects and in general appearance to the new examples now figured, I am inclined to believe that the non-appearance of these markings is due to the mode of preservation, namely, to extreme crushing, whereby they have become indistinguishable, and that we are therefore justified in assuming that one and the same animal is represented in all those examples.

We have thus a creature whose general appearance reminds us, in the first place, of the Cœlolepidæ by reason of its broad depressed head, which terminates on each side behind in an obtusely round and apparently flexible "flap." But though there are no cornua, the orbits and associated appearances are of the type characteristic of *Cephalaspis*, and it is therefore clear that these flaps or lappet-like expansions in question are the same with the structures in the last-named genus, originally considered by LANKESTER to be pectoral fins, and afterwards by Dr A. S. WOODWARD as being of the nature of opercula. That they are homologous with the pectoral flaps of *Thelodus* and *Lanarkia* I still believe, but that they are of the nature of pectoral fins is another question, the consideration of which must be left for the present.

Finally, as the general resemblance between *Ateleaspis* and *Cephalaspis* is now seen to be so close, I no longer see the necessity of allotting a distinct family to the former genus, and propose therefore to classify it henceforth as a member of the Cephalaspidæ.

Order ANASPIDA.

Family Birkeniidæ.

Lasanius problematicus, Traquair.

Pl. II. figs. 4–8.

A considerable amount of fresh and interesting information is available, and accordingly more than one-half of Plate II. is occupied with figures illustrating new or little-known details of configuration and structure of this still problematic little fish.

In my previous memoir I showed how that in many specimens of *Lasanius* the general form was indicated in parts by a carbonaceous film, and how that in one

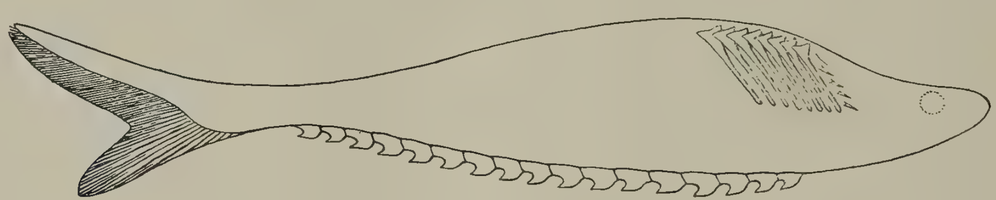


Fig. 4.—Restored outline of *Lasanius problematicus*.

example (*L. armatus*, Pl. V. fig. 12) the rays of the caudal fin were preserved, proving that that fin was heterocercal in structure, and thereby giving a clue to the position of the median row of scales, which accordingly turned out to be ventral, and not dorsal as I had originally supposed.

Since then a number of fresh specimens have been collected at Seggholm by Mr Tait, in which the general outline is still better displayed, as will be seen on inspecting Plate II. figs. 4–7 of the present paper. The constancy in the examples there figured of a feature, which I looked upon as merely an accidental one in the two specimens figured in my previous memoir (Pl. V. figs. 6 and 12), induces me to produce a new restored outline of *Lasanius* (text figure), embodying a modification of the contour of the anterior part of the creature. It will be seen that in all these specimens the greatest depth is at the region of the post-cephalic rods, and that in front of these the dorsal outline suddenly falls or slopes away down to a distance of about half the depth of the body, and then passes forwards again nearly horizontally into the outline of the bluntly pointed head, which is thus only one-half the depth of the body a little way back.

Fig. 4, Plate II., shows one of the most complete specimens as regards the preservation of the external form. The rays of the caudal fin are also to some extent preserved, giving ample corroboration to the demonstration of its heterocercal structure, previously afforded by *L. armatus.* The same condition is still better shown in fig. 6, where the division of this caudal fin goes into two unequal lobes, with fin-rays arranged after the heterocercal manner, is perfectly obvious.

Body muscles.—In fig. 5, a little way behind the post-cephalic rods, and decidedly in the dorsal half of the body, a number of short, dark lines, passing obliquely upwards and backwards, may be distinguished in the carbonaceous film which represents the soft parts. The same phenomenon is also seen to a lesser extent in fig. 6 ; but in fig. 7 we have, in addition, a set of similar lines on the hæmal aspect, which pass obliquely downwards and backwards. In this last specimen both sets are more complete, and extend nearly the whole length of the fish. Of this phenomenon only two explanations are possible. These parallel oblique lines represent either the neural and hæmal spines of the vertebral axis, or the oblique septa of the myotomes of the body. The second interpretation seems to me the most natural.

Skin.—In addition to the median ventral row of aculeate scutes and the gridiron-like arrangement of bony rods just behind the dorsal aspect of the head, there is some evidence of the skin having been furnished with other hard parts, though not of such a nature as to be ordinarily capable of preservation. This is seen in the specimen from Birkenhead Burn, represented in fig. 8, which is a portion of the skin near the front of the fish, in this case an unusually large example,—the position being indicated by the post-cephalic rods, which stand out as prominent oblique black lines. On wetting the specimen and examining it with a strong lens, the portion of skin in question is seen to be obliquely crossed by dark, closely-set linear markings, parallel with each other, and with the direction of the post-cephalic rods, below and behind which they are situated. That these markings represent the remains of structures by which the skin was strengthened cannot be doubted.

Lasanius armatus, Traquair.

This species was described by me in my former memoir from a single specimen, from Seggholm, in the Collection of the Geological Survey of Scotland. Another specimen, corroborating the first in all essential details, has been found since that time in the same locality by Mr Adam Whyte, of Muirkirk.

EXPLANATION OF THE PLATES.

Plate I.

All the specimens figured on this plate are from the "Ludlow" beds of Logan Water, and are represented of the natural size, with the exception of figures 7 and 8.

Fig. 1. *Thelodus Scoticus*, Traq., slightly deficient on the right side of the head. *d*, dorsal fin. From a specimen collected and lent by Mr James Young, Lesmahagow.

Fig. 2. Another specimen, showing the dorsal fin, also lent by Mr Young.

Fig. 3. Another specimen, in Mr Young's collection, and showing the dorsal fin.

Fig. 4. Another specimen, showing the contour of the head and lateral flaps. In the Collection of the Geological Survey of Scotland.

Fig. 5. Head showing the eye-spots. In the Collection of the Geological Survey of Scotland.

Fig. 6. Another head showing the eye-spots, but not so distinctly as the previous specimen. Geological Survey of Scotland.

Fig. 7. Caudal scales, magnified.

Fig. 8. Caudal scales in apposition, magnified. From a specimen lent by Mr Peter Macnair, Glasgow.

Plate II.

All the specimens represented in this plate were collected by Mr Tait from the "Downtonian" horizon, and are in the Collection of the Geological Survey of Scotland, except the original of fig. 4.

Fig. 1. *Lanarkia horrida*, Traq., showing the eye-spots. From Birkenhead Burn. Natural size. From Monks Water.

Fig. 2. Head portion of a *Lanarkia*-like fish, with markings possibly indicative of branchial pouches.

Fig. 3. Counterpart of the same specimen.

Fig. 4. *Lasanius problematicus*, Traq., enlarged by one-half, the contour of the body, including the caudal fin, being indicated by a carbonaceous film. From Birkenhead Burn. In the Royal Scottish Museum, presented by the Geological Survey.

Fig. 5. Another specimen, from Seggholm, twice natural size, showing the contour of the body, but wanting the caudal fin. The oblique lines indicating the dorsal intermuscular septa are seen in the posterior half of the specimen. Geological Survey of Scotland.

Fig. 6. Another specimen, from Seggholm, twice natural size, showing well the heterocercal caudal fin, while a few of the dorsal intermuscular septa are visible about the middle of the body.

Fig. 7. Another specimen, from Monks Water, twice natural size, showing both dorsal and ventral intermuscular septa.

Fig. 8. Portion of the skin of a large specimen of *Lasanius problematicus*, from Birkenhead Burn, natural size, showing the post-cephalic rods, and behind and below these the dermal striations.

Fig. 9. Impression of the outer surface of a head of *Ateleaspis tessellata*, Traq. From Birkenhead Burn.

Fig. 10. Schematised vertical section of a scale of *Ateleaspis tessellata*, highly magnified.

Plate III.

The two specimens figured on this plate are from the "Downtonian" horizon, and are in the Collection of the Geological Survey of Scotland.

Fig. 1. *Ateleaspis tessellata*, Traq., a nearly perfect specimen, showing the contour of the head. From The Mount, three miles south-east of Strathavon. Natural size.

Fig. 2. Another specimen, wanting the head, but showing well the arrangement of the scales on the body, and the contour of the dorsal and caudal fins. Natural size. From Birkenhead Burn.

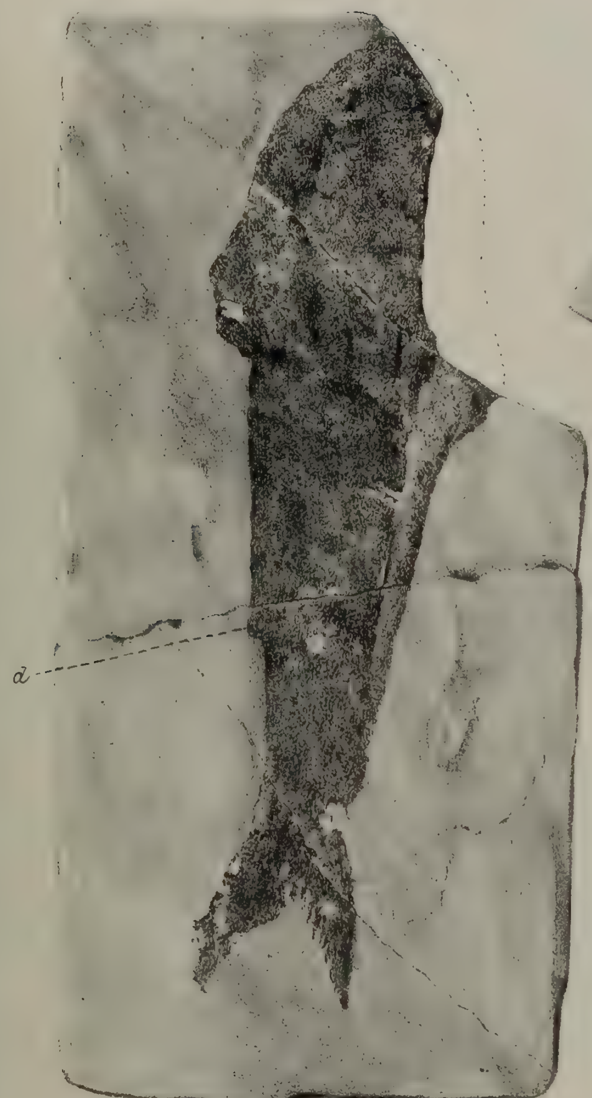


Fig. 1.

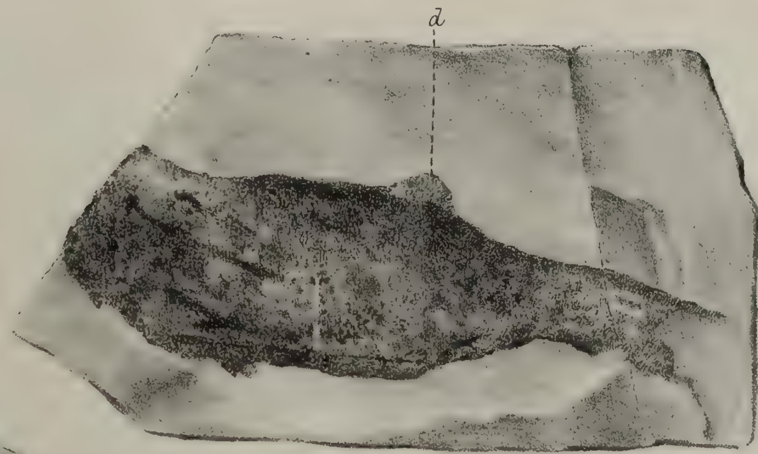


Fig. 2.

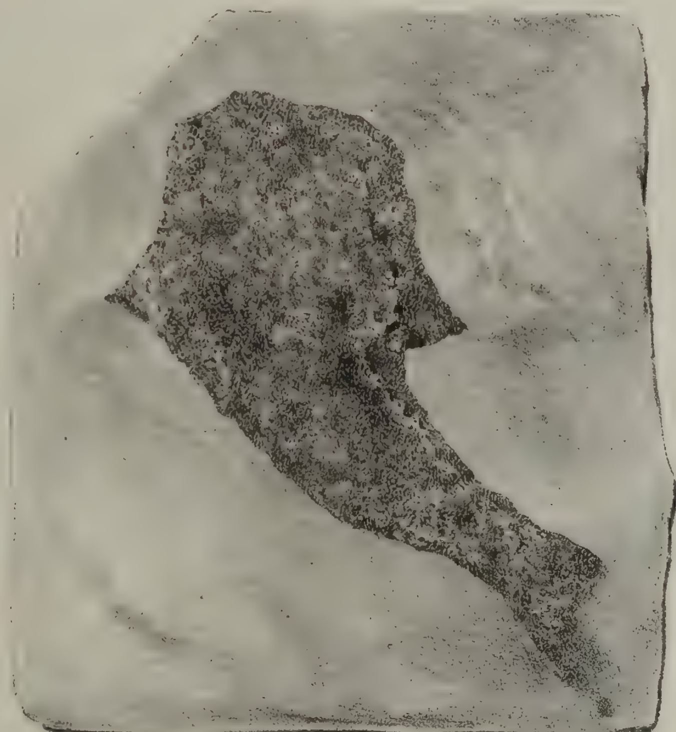


Fig. 4.

Fig. 5.

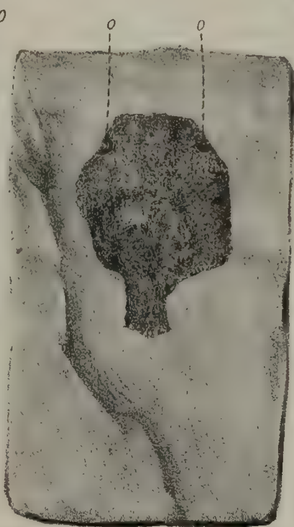


Fig. 6.

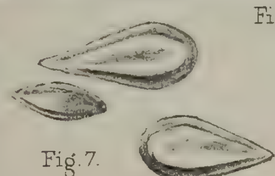


Fig. 7.

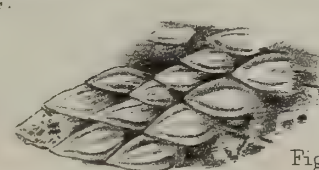


Fig. 8.

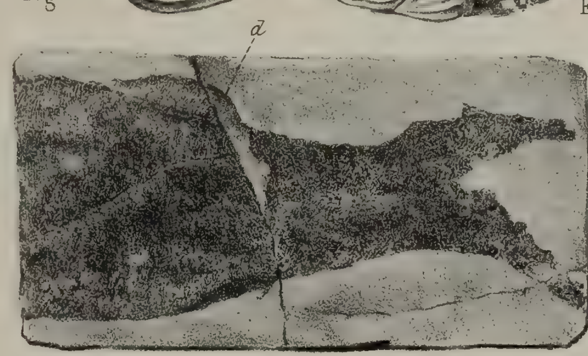


Fig. 3.

Figs. 1-4 J. Green.
Figs. 5-8 R. H. Traquair.

J. Green in lap.
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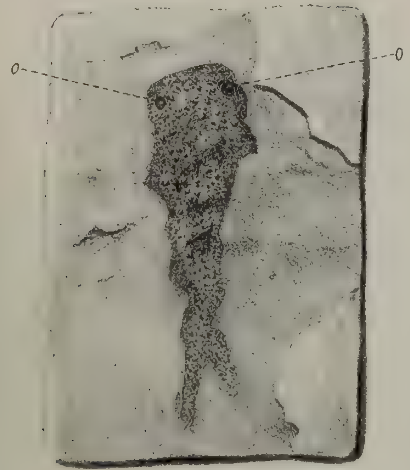

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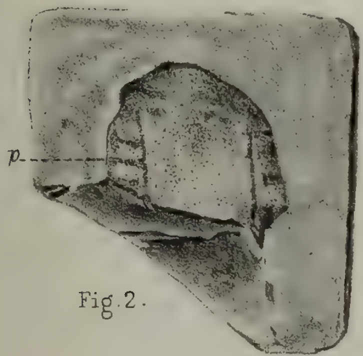
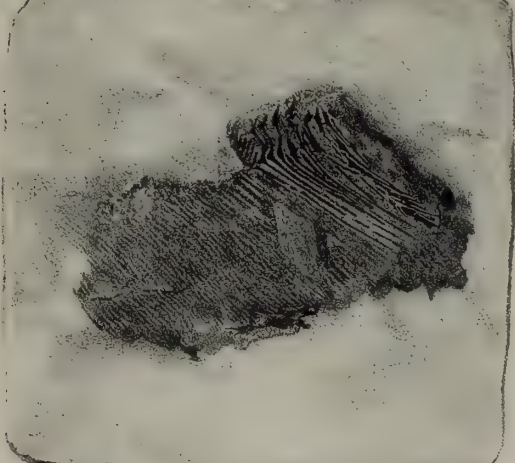


Fig. 2.



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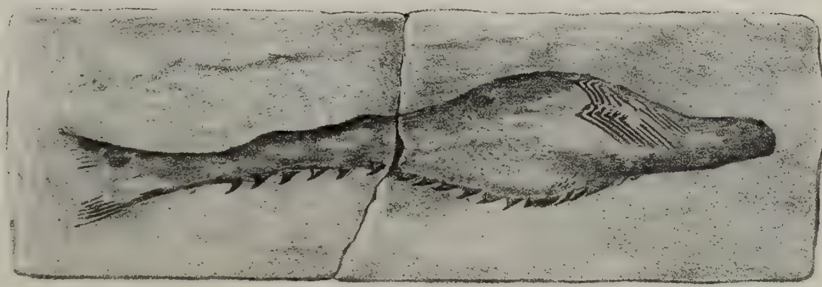

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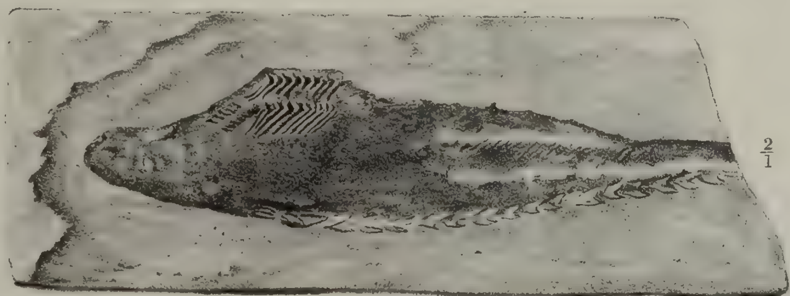

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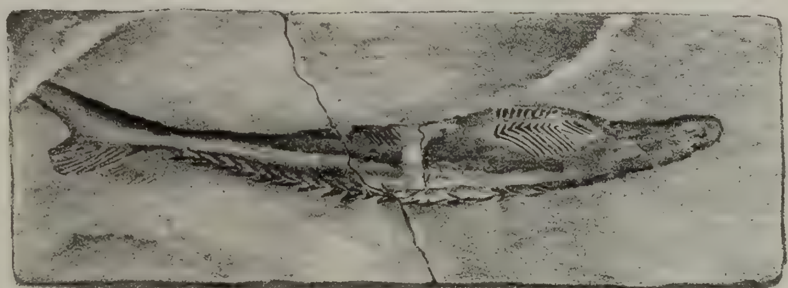

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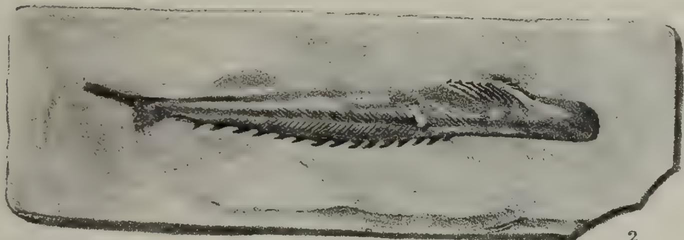

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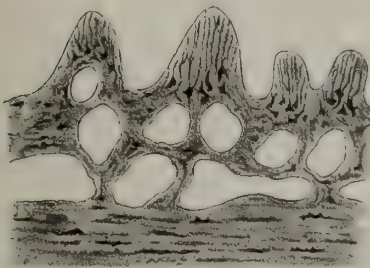
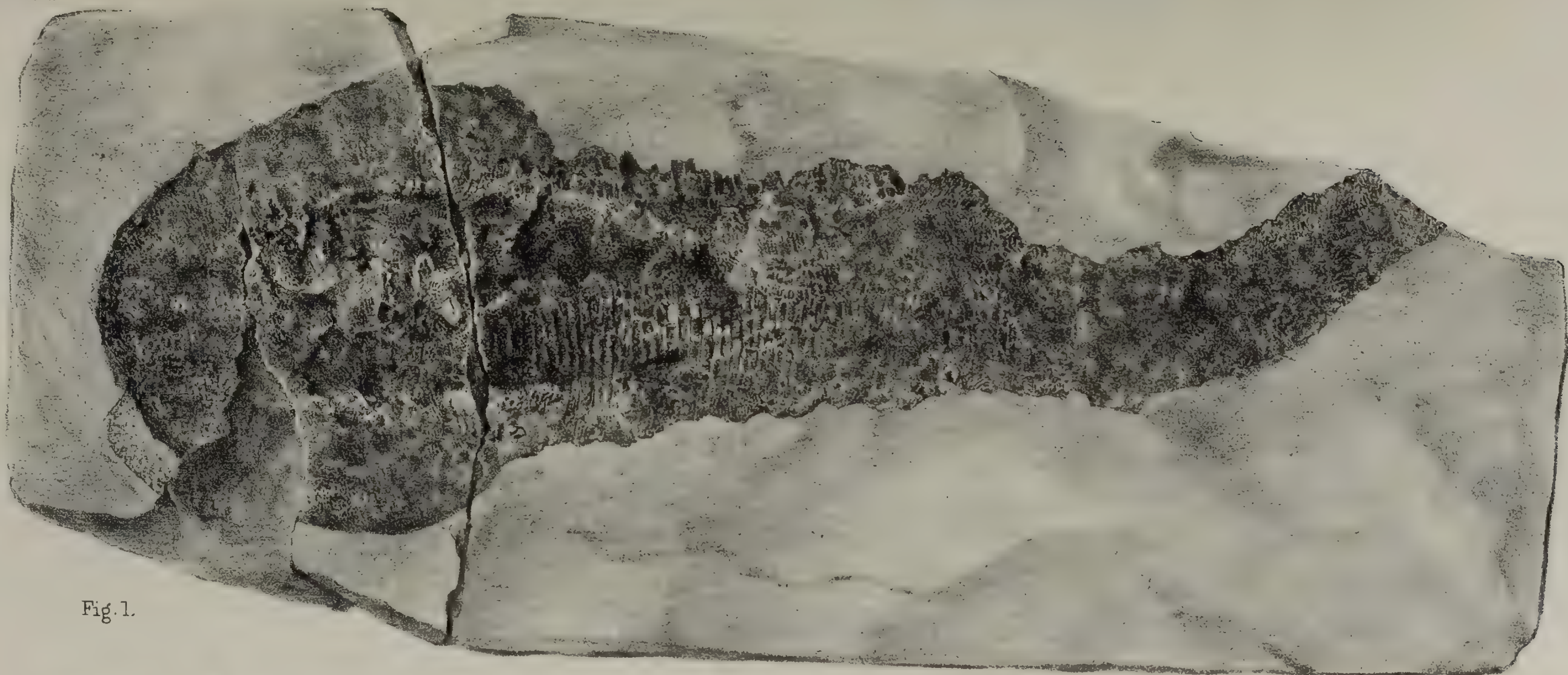


Fig. 10.

Figs. 1, 4–7, 10, R. H. Traquair.
Figs. 2, 3, 8, 9, J. Green.

J. Green in lap.
Mintern Bros. imp. London.



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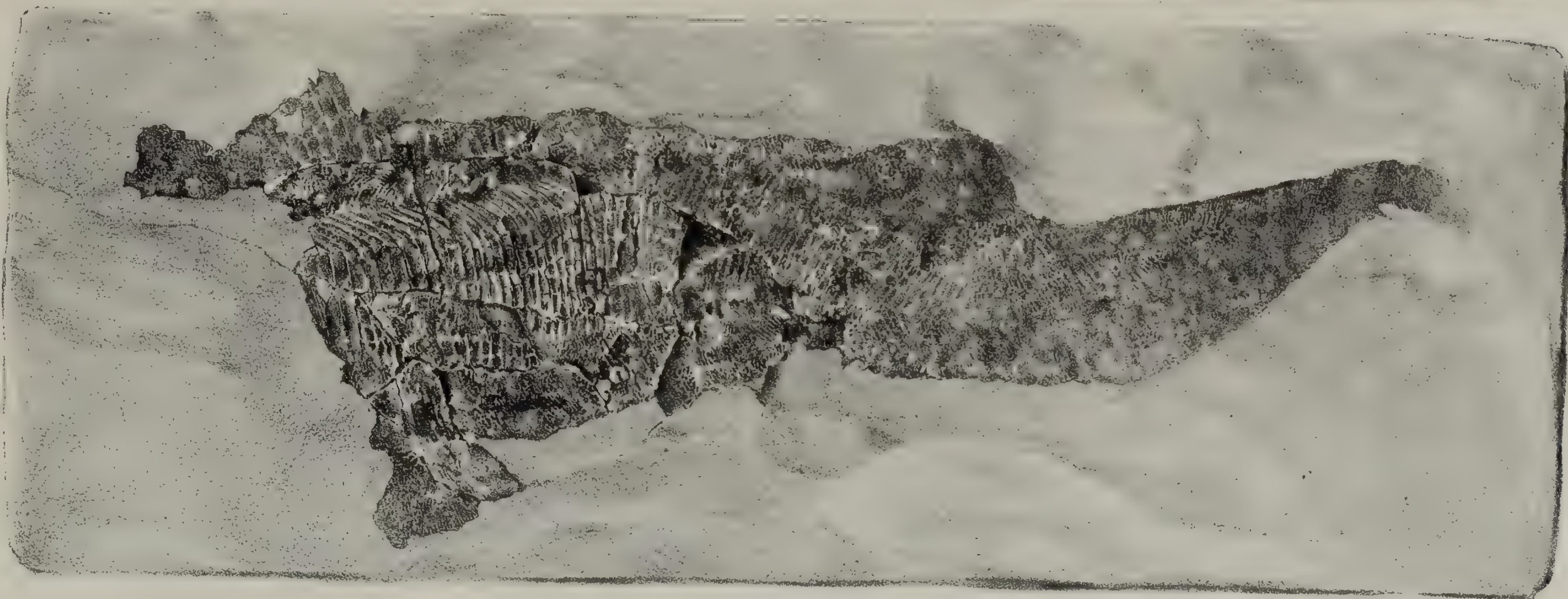

Fig. 2.

J. Green del

J. Green in lap.
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APPENDIX.

TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

CONTENTS.

ROYAL SOCIETY OF EDINBURGH.

LIST OF MEMBERS.

COUNCIL,

ALPHABETICAL LIST OF ORDINARY FELLOWS,

AND LIST OF HONORARY FELLOWS,

At October 1904.

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OF

THE ROYAL SOCIETY OF EDINBURGH.

OCTOBER 1904.

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OF

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CORRECTED TO OCTOBER 1904.

N.B.—*Those marked * are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, Keith Medal.
N. ,, ,, ,, Neill Medal.
V. J. ,, ,, ,, the Gunning Victoria Jubilee Prize.
C. ,, ,, ,, contributed one or more Communications to the Society's TRANSACTIONS OR PROCEEDINGS.

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1893	B. C.	* Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow	
1882	C.	Beddard, Frank E., M.A. Oxon., F.R.S., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London	
1887		* Begg, Ferdinand Faithful, 24 Lansdowne Road, London	
1886		* Bell, A. Beatson, 17 Lansdowne Crescent, Edinburgh	45
1874		Bell, Joseph, M.D., F.R.C.S.E., 2 Melville Crescent	
1900		* Bennett, James Bower, Memb. Inst. C.E., 2 Thorburn Road, Colinton	
1887		* Bernard, J. Mackay, of Dunsinnan, B.Sc., 25 Chester Street	
1875		Bernstein, Ludwik, M.D., Lismore, New South Wales	
1893	C.	* Berry, George A., M.D., C.M., F.R.C.S., 31 Drumsheugh Gardens	50
1897	C.	* Berry, Richard J., M.D., F.R.C.S.E., 4 Howard Place	
1881		* Berry, Walter, of Glenstriven, K.D., Danish Consul-General, 11 Atholl Crescent	
1904		* Beveridge, Erskine, LL.D., St Leonard's Hill, Dunfermline	
1880	C	Birch, De Burgh, M.D., Professor of Physiology, Yorkshire College, Victoria University, 16 De Grey Terrace, Leeds	

Date of Election.			
1900		* Bisset, James, M.A., F.L.S., F.G.S., 9 Greenhill Park	55
1884		* Black, John S., M.A., LL.D., 6 Oxford Terrace	
1850		Blackburn, Hugh, M.A., LL.D., Emeritus Professor of Mathematics in the University of Glasgow, Roshven, Lochailort	
1897		* Blaikie, Walter Biggar, 6 Belgrave Crescent	
1898		* Blyth, Benjamin Hall, M.A., Memb. Inst. C.E., 17 Palmerston Place	
1878	C.	Blyth, James, M.A., LL.D., Prof. of Natural Philosophy in Anderson's College, Glasgow	60
1894		* Bolton, Herbert, Curator of the Bristol Museum, Queen's Road, Bristol	
1884		Bond, Francis T., B.A., M.D., M.R.C.S., Gloucester	
1872	C.	Bottomley, J. Thomson, M.A., D.Sc., F.R.S., F.C.S., Lecturer on Natural Philosophy in the University of Glasgow, 13 University Gardens, Glasgow	
1869	C.	Bow, Robert Henry, C.E., 7 South Gray Street	
1886		* Bower, Frederick O., M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow, 1 St John's Terrace, Hillhead, Glasgow	65
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., Assoc. Inst. C.E., Assoc. Inst. M.E., M.I.E.E., &c., Spinningfield, Deansgate, Manchester	
1901		Bradbury, J. B., M.D., Downing Professor of Medicine, University of Cambridge	
1903	C.	* Bradley, O. Charnock, M.B., Ch.B., Royal Veterinary College, Edinburgh	
1886		* Bramwell, Byrom, M.D., F.R.C.P.E., 23 Drumsheugh Gardens	
1895		* Bright, Charles, Assoc. Memb. Inst. C.E., Memb. Inst. E.E., F.R.A.S., F.G.S., 21 Old Queen Street, Westminster, London	70
1886		Brittle, John Richard, Memb. Inst. C.E., Farad Villa, Vanbrugh Hill, Blackheath, Kent	
1877		Broadrick, George, Memb. Inst. C.E., Broughton House, Broughton Road, Ipswich	
1893		Brock, G. Sandison, M.D., 2 Via Veneto, Rome, Italy	
1892		* Brock, W. J., M.B., D.Sc., 5 Manor Place	
1901	C.	* Brodie, W. Brodie, M.B., 28 Hamilton Park Terrace, Hillhead, Glasgow	75
1887		* Brown, A. B., C.E., Memb. Inst. Mech. E., 19 Douglas Crescent	
1864	C. K. B.	Brown, Alex. Crum, M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S. (SECRETARY), Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent	
1898		* Brown, David, F.C.S., F.I.C., Willowbrae House, Midlothian	
1883	C.	* Brown, J. J. Graham, M.D., F.R.C.P.E., 3 Chester Street	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 2 Frognal, London, N.W.	80
1883	C.	* Bruce, Alexander, M.A., M.D., F.R.C.P.E., 8 Ainslie Place	
1898	C.	* Bryce, T. H., M.A., M.D. (Edin.), 2 Granby Terrace, Glasgow	
1888		* Bryson, William A., Electrical Engineer, 16 Charlotte Street, Leith	
1869	C. B. V. J.	Buchan, Alexander, M.A., LL.D., F.R.S., Secretary to the Scottish Meteorological Society (CURATOR OF LIBRARY AND MUSEUM), 2 Dean Terrace	
1870	C. K.	Buchanan, John Young, M.A., F.R.S., Christ's College, Cambridge	85
1902		* Buchanan, Robert M., M.B., F.F.P.S.G., 2 Northbank Terrace, Glasgow	
1882		* Buchanan, T. R., M.A., 12 South Street, Park Lane, London, W.	
1887	C.	* Buist, J. B., M.D., F.R.C.P.E., 1 Clifton Terrace	
1902		* Burgess, A. G., M.A., Mathematical Master, Edinburgh Ladies College, 2 Craigcrook Terrace, Blackhall	
1894	C.K.	* Burgess, James, C.I.E., LL.D., M.R.A.S., M. Soc. Asiatique de Paris, *H.*A.R.I.B.A., 22 Seton Place	90
1902		* Burn, The Rev. John Henry, B.D., The Parsonage, Ballater	
1887		* Burnet, John James, Architect, 18 University Avenue, Hillhead, Glasgow	

Date of Election.			
1888		* Burns, Rev. T., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent	
1903		* Butler, Rev. Dugald, M.A., Minister of the Tron Parish, 54 Blacket Place	
1896		* Butters, J. W., M.A., B.Sc., Rector of Ardrossan Academy	95
1887	C.	* Cadell, Henry Moubray, of Grange, B.Sc., Bo'ness	
1897		* Caird, Robert, LL.D., Shipbuilder, Greenock	
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, 7 East Castle Road, Merchiston	
1894		* Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn	
1904		* Campbell, Charles Duff, 21 Montague Terrace, Inverleith Row	100
1878		Campbell, John Archibald, M.D., Gothic Villa, St Aubyn's Road, Jersey	
1899	C.	* Carlier, Edmund W. W., M.D., B.Sc., Prof. of Physiology in Mason College, Birmingham	
1902		* Carmichael, Sir Thomas D. Gibson, Bart., Castle Craig, Dolphinton	
1901		Carslaw, H. S., M.A., D.Sc., Professor of Mathematics in the University of Sydney, New South Wales	
1898		* Carter, Wm. Allan, Memb. Inst. C.E., 32 Great King Street	105
1898		Carus-Wilson, Cecil, F.R.G.S., F.G.S., Royal Societies Club, St James Street, London	
1882		* Cay, W. Dyce, Memb. Inst. C.E., 1 Albyn Place	
1890		Charles, John J., M.A., M.D., C.M., Prof. of Anatomy and Physiology, Queen's College, Cork	
1899		* Chatham, James, Actuary, 98 Inverleith Place	
1874		Chiene, John, C.B., M.D., LL.D., F.R.C.S.E., Professor of Surgery in the University of Edinburgh, 26 Charlotte Square	110
1880	C. K.	Chrystal, George, M.A., LL.D., Professor of Mathematics in the University of Edinburgh (General Secretary), 5 Belgrave Crescent	
1891		* Clark, John B., M.A., Mathematical and Physical Master in Heriot's Hospital School, Garleffin, Craiglea Drive	
1903		* Clarke, William Eagle, F.L.S., Natural History Department, Royal Scottish Museum, Edinburgh, 35 Braid Road	
1875		Clouston, T. S., M.D., President of the Royal College of Physicians, Tipperlinn House, Morningside	
1892		* Coates, Henry, Pitcullen House, Perth	115
1887		* Cockburn, John, F.R.A.S., The Abbey, North Berwick	
1904	C.	Coker, Ernest George, M.A., D.Sc., Professor of Mechanical Engineering and Applied Mathematics, City and Guilds Technical College, Finsbury, London	
1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1888	C.	Collie, John Norman, Ph.D., F.R.S., F.C.S., Professor of Organic Chemistry in the University College, Gower Street, London	
1904		* Colquhoun, Walter, M.A., M.B., Muirhead Demonstrator of Physiology, University of Glasgow, 7 Stanley Street, Glasgow, W.	120
1886		Connan, Daniel M., M.A., Education Department, Cape of Good Hope	
1872		Constable, Archibald, LL.D., 11 Thistle Street	
1894		Cook, John, M.A., Principal of the Government Central College, Bangalore, India	
1891		* Cooper, Charles A., LL.D., 41 Drumsheugh Gardens	
1890	C.	* Copeland, Ralph, Ph.D., F.R.A.S., Astronomer-Royal for Scotland, and Professor of Practical Astronomy in the University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh	125
1875		Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons 71 Bruntsfield Place	

Date of Election.			
1898		* Crawford, Francis Chalmers, 19 Royal Terrace	
1903		Crawford, Lawrence, M.A., D.Sc., Professor of Mathematics in the South African College, Cape Town.	
1887		* Crawford, William Caldwell, 1 Lockharton Gardens, Colinton Road, Edinburgh	
1870		Crichton-Browne, Sir Jas., M.D., LL.D., F.R.S., Lord Chancellor's Visitor and Vice-President of the Royal Institution of Great Britain, 61 Carlisle Place Mansions, Victoria Street, and Royal Courts of Justice, Strand, London	130
1886		* Croom, Sir John Halliday, M.D., F.R.C.P.E., President, Royal College of Surgeons, Edinburgh, 25 Charlotte Square	
1898		* Cullen, Alexander, F.S.A. Scot., Millburn House, by Hamilton	
1878		Cunningham, Daniel John, M.D., LL.D., D.C.L., F.R.S., F.Z.S., Professor of Anatomy in the University of Edinburgh (SECRETARY), 18 Grosvenor Crescent	
1898		* Currie, James, M.A. Cantab., Larkfield, Golden Acre	
1904		* Cuthbertson, John, Secretary, West of Scotland Agricultural College, 4 Charles Street, Kilmarnock	135
1889		* Dalrymple, James D. G., F.S.A. Lond. and Scot., Meiklewood, Stirling	
1885		* Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, c/o Messrs Buchan & Buchan, S.S.C., 37 Great King Street	
1897		* Davidson, Hugh, of Braedale, Lanark	
1884		Davy, R., F.R.C.S. Eng., Surgeon to Westminster Hospital, Burstone House, Bow, North Devon	
1894		* Denny, Archibald, Braehead, Dumbarton	140
1895		* Deuchar, David, F.I.A., F.F.A., Actuary, 12 Hope Terrace	
1869	C.	Dewar, Sir James, M.A., LL.D., D.C.L., D.Sc. Dub., F.R.S., F.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London	
1904		Dickinson, Walter George Burnett, F.R.C.V.S., Boston, Lincolnshire	
1884		* Dickson, The Right Hon. Charles Scott, K.C., Lord-Advocate of Scotland, M.P. for the Bridgeton Division of Glasgow, 22 Moray Place	
1888	C.	* Dickson, H. N., B.Sc., 2 St Margaret's Road, Oxford	145
1876	C.	Dickson, J. D. Hamilton, M.A., Fellow and Tutor, St Peter's College, Cambridge	
1885	C.	Dixon, James Main, M.A., President, Columbia College, Milton, Oregon, United States	
1897		* Dobbie, James Bell, F.Z.S., 2 Hailes Street	
1904	C.	* Dobbie, James Johnston, M.A., D.Sc., F.R.S., Director of the Royal Scottish Museum, Edinburgh, 27 Polwarth Terrace	
1881	C.	* Dobbin, Leonard, Ph.D., Lecturer on Chemistry in the University of Edinburgh, 7 Cobden Road	150
1902		Dollar, John A. W., M.R.C.V.S., 56 New Bond Street, London	
1867	C.	Donaldson, J., M.A., LL.D., Principal of the University of St Andrews, St Andrews	
1896		* Donaldson, William, M.A., Viewpark House, Spylaw Road	
1882	C.	* Dott, D. B., Memb. Pharm. Soc., 29 Spring Gardens	
1892		Doyle, Patrick, C.E., M.R.I.A., F.G.S., Editor of *Indian Engineering*, Calcutta	155
1901		* Douglas, Carstairs Cumming, M.D., B.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow, 2 Royal Crescent, Glasgow	
1866		Douglas, David, 22 Drummond Place	
1901		* Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., 25 Blacket Place	
1876		Duncan, James, 52 Shakespeare Street, Hove, Sussex	

Date of Election.			
1878		Duncanson, J. J. Kirk, M.D., F.R.C.P.E., 22 Drumsheugh Gardens	160
1904		* Dunlop, William Brown, M.A., 7 Carlton Street	
1859	C.	Duns, Rev. Professor, D.D., 5 Greenhill Place	
1903		* Dunstan, John, M.R.C.V.S., Professor of Surgery, Royal Veterinary College, Edinburgh	
1892	C.	Dunstan, M. J. R., B.A., F.C.S., Director of Technical Education in Agriculture, The College, Wye, Kent	
1888		* Durham, James, F.G.S., Wingate Place, Newport, Fife	165
1899		* Duthie, George, M.A., Inspector-General of Education, Salisbury, Rhodesia	
1893		Edington, Alexander, M.D., Colonial Bacteriologist, Graham's Town, South Africa	
1904		* Edwards, John, 4 Great Western Terrace, Kelvinside, Glasgow	
1904		* Elder, William, M.D., F.R.C.P.E., 4 John's Place, Leith	
1885		Elgar, Francis, Memb. Inst. C.E., LL.D., F.R.S., 18 Cornwall Terrace, Regent's Park, London	170
1875		Elliot, Daniel G., Curator of Department of Zoology, Field Columbian Museum, Chicago, U.S.	
1897	C.	* Erskine-Murray, James Robert, D.Sc., 39 Watcombe Circus, Nottingham	
1884		* Evans, William, F.F.A., 38 Morningside Park	
1879	C. N.	Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.L.S., Professor of Natural History, University of Edinburgh	
1902		* Ewen, J. T., B.Sc., Memb. Inst. Mech. E., H.M.I.S., Millbank House, Forfar	175
1878	C.	Ewing, James Alfred, M.A., B.Sc., LL.D., Memb. Inst. C.E., F.R.S., Director of Naval Education, Royal Naval College, Greenwich	
1900		Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), The Bacteriological Laboratories, Guy's Hospital, London, 19 Villiers Street, London	
1875		Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds	
1888	C.	* Fawsitt, Charles A., 9 Foremount Terrace, Dowanhill, Glasgow	
1859		Fayrer, Sir Joseph, Bart., K.C.S.I., M.D., F.R.C.P.L., F.R.C.S. L. and E., LL.D., F.R.S., Honorary Physician to the Queen, 16 Devonshire St., Portland Pl., London, W.	180
1883	C.	* Felkin, Robert W., M.D., F.R.G.S., Fellow of the Anthropological Society of Berlin, 12 Oxford Gardens, North Kensington, London, W.	
1899		* Fergus, Andrew Freeland, M.D., 22 Blythswood Square, Glasgow	
1904		* Ferguson, James Haig, M.D., F.R.C.P.E., F.R.C.S.E., 25 Rutland Street	
1888		* Ferguson, John, M.A., LL.D., Professor of Chemistry in the University of Glasgow	
1868	C.	Ferguson, Robert M., Ph.D., LL.D. (Society's Representative on George Heriot's Trust), 5 Douglas Gardens	185
1898		* Findlay, John R., M.A. Oxon., 27 Drumsheugh Gardens	
1899		* Finlay, David W., B.A., M.D., LL.D., F.R.C.P., D.P.H., Professor of Medicine in the University of Aberdeen, 2 Queen's Terrace, Aberdeen	
1900	C.N.	* Flett, John S., M.A., D.Sc., Geological Survey Office, 28 Jermyn Street, London	
1880		Flint, Robert, D.D., Corresponding Member of the Institute of France, Corresponding Member of the Royal Academy of Sciences of Palermo, Emeritus Professor of Divinity in the University of Edinburgh (Vice-President), 1 Mountjoy Terrace, Musselburgh	
1872	C.	Forbes, Professor George, M.A., Memb. Inst. C.E., Memb. Inst. E.E., F.R.S., F.R.A.S., 34 Great George Street, Westminster	190
1904		Forbes, Norman Hay, F.R.C.S.E., Drummiuor, Tunbridge Wells, Kent	
1892		* Ford, John Simpson, F.C.S., 4 Nile Grove	

Date of Election.			
1858		Fraser, A. Campbell, Fellow of the British Academy, Hon. D.C.L. Oxford, LL.D., Litt.D., Emeritus Professor of Logic and Metaphysics in the University of Edinburgh, Gorton House, Hawthornden	
1896		* Fraser, John, M.B., F.R.C.P.E., one of H.M. Commissioners in Lunacy for Scotland, 13 Heriot Row	
1892		* Fraser, Patrick Neill, Rockville, Murrayfield	195
1867	C K. B.	Fraser, Sir Thomas R., M.D., LL.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh, Honorary Physician to the King in Scotland, 13 Drumsheugh Gardens	
1891		* Fullarton, J. H., M.A., D.Sc., Brodick, Arran	
1891		* Fulton, T. Wemyss, M.D., Scientific Superintendent, Scottish Fishery Board, 417 Great Western Road, Aberdeen	
1888	C.	* Galt, Alexander, D.Sc., Keeper of the Technological Department, Royal Scottish Museum, Edinburgh	
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur States, Jaipur, India	200
1899		Gatehouse, T. E., Assoc. Memb. Inst. C.E., Memb. Inst. M.E., Memb. Inst. E.E., Tulse Hill Lodge, 100 Tulse Hill, London	
1867		Gayner, Charles, M.D., F.L.S.	
1900		Gayton, William, M.D., M.R.C.P.E., 11 Redbourne Avenue, North Finchley, London	
1889		* Geddes, George H., Mining Engineer, 8 Douglas Crescent	
1880	C.	Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh	205
1861	C. B.	Geikie, Sir Archibald, LL.D. Oxf., D.Sc. Camb. Dub., F.R.S., F.G.S., Foreign Member of the Reale Accad. Lincei, Rome, of the National Acad. of the United States, Corresponding Member of the Institute of France and of the Academies of Berlin, Vienna, Munich, Göttingen, Turin, Belgium, Stockholm, Christiania, Philadelphia, New York, &c., 10 Chester Terrace, Regent's Park, London	
1871	C. B.	Geikie, James, LL.D., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh (VICE-PRESIDENT), Kilmorie, Colinton Road	
1881	C.	* Gibson, George Alexander, D.Sc., M.D., F.R.C.P.E., 3 Drumsheugh Gardens	
1890		* Gibson, George A., M.A., Professor of Mathematics in the Glasgow and West of Scotland Technical College, 8 Sandyford Place, Glasgow	
1877	C.	Gibson, John, Ph.D., Professor of Chemistry in the Heriot-Watt College, Ringlewood, Colinton	210
1892		Gifford, Herbert James, Assoc. M. Inst. C.E.	
1900		Gilchrist, Douglas A., B.Sc., Professor of Agriculture, Durham College of Science, Newcastle-upon-Tyne	
1897	C.	* Gillespie, A. Lockhart, M.D., F.R.C.P.E., 12 Walker Street	
1887		* Gilmour, William, 9 Inverleith Row	
1880		Gilruth, George Ritchie, Surgeon, 53 Northumberland Street	215
1898		* Glaister, John, M.D., F.F.P.S. Glasgow, D.P.H. Camb., Professor of Forensic Medicine in the University of Glasgow, 3 Newton Place, Glasgow	
1901		Goodwillie, James, M.A., B.Sc., Liberton, Edinburgh	
1899		* Goodwin, Thomas S., F.C.S., Professor of Chemistry, Veterinary College, Glasgow	
1897		Gordon-Munn, John Gordon, M.D., 34 Dover Street, London, W.	
1891		* Graham, Richard D., 11 Strathearn Road	220

Date of Election.		
1898	C.	* Gray, Albert A., M.D., 14 Newton Terrace, Glasgow
1883		* Gray, Andrew, M.A., LL.D., F.R.S., Professor of Natural Philosophy in the University of Glasgow
1880	C.	Gray, Thomas, B.Sc., Professor of Physics, Rose Polytechnic Institute, Terre Haute, Indiana, U.S.
1886		* Greenfield, W. S., M.D., F.R.C.P.E., Professor of General Pathology in the University of Edinburgh, 7 Heriot Row
1897		Greenlees, Thomas Duncan, M.D. Edin., The Residency, Grahamstown, South Africa 225
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Church Hill
1888	C.	Guppy, Henry Brougham, M.B., Rosario, Salcombe, Devon
1899		Hamilton, Allan M'Lane, M.D., 44 East Twenty-ninth Street, New York
1881	C.	* Hamilton, D. J., M.B., F.R.C.S.E., Professor of Pathological Anatomy in the University of Aberdeen, 35 Queen's Road, Aberdeen
1876	C.	Hannay, J. Ballantyne, Cove Castle, Loch Long 230
1902		* Hargreaves, Andrew Fuller, F.C.S., Eskhill House, Roslin
1896		* Harris, David, Fellow of the Statistical Society, Lyncombe Rise, Prior Park Road, Bath
1896	C.	* Harris, David Fraser, B.Sc. (Lond.), M.D., F.S.A. Scot., Lecturer on Physiology in the University of St Andrews
1888		* Hart, D. Berry, M.D., F.R.C.P.E., 29 Charlotte Square
1869		Hartley, Sir Charles A., K.C.M.G., Memb. Inst. C.E., 26 Pall Mall, London 235
1877	C.	Hartley, W. N., D.Sc., F.R.S., F.I.C., Prof. of Chemistry, Royal College of Science for Ireland, Dublin
1881		* Harvie-Brown, J. A., of Quarter, F.Z.S., Dunipace House, Larbert, Stirlingshire
1880	C.	Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff
1892	C.	* Heath, Thomas, B.A., Assistant Astronomer, Royal Observatory, Edinburgh
1862		Hector, Sir J., K.C.M.G., M.D., F.R.S., Director of the Geological Survey, Colonial Laboratory, Meteorological and Weather Departments, and of the New Zealand Institute, Wellington, New Zealand 240
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S.L., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India
1890	C.	Helme, T. Arthur, M.D., M.R.C.P.L., M.R.C.S., 3 St Peter's Square, Manchester
1900		Henderson, John, D.Sc., Assoc. Inst. E.E., Kinnoul, Warwick's Bench Rd., Guildford, Surrey
1890	C.	* Hepburn, David, M.D., Professor of Anatomy in the University College of South Wales and Monmouthshire, Cardiff
1896	C.	* Herbertson, Andrew J., M.A., Ph.D., Lecturer in Regional Geography, and Curator, School of Geography, University of Oxford, 4 Broad Street, Oxford 245
1881	C. N.	* Herdman, W. A., D.Sc., F.R.S., F.L.S., Prof. of Natural History in University College, Liverpool, Croxteth Lodge, Ullet Road, Liverpool
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Valentine Mount, Freshwater Bay, Isle of Wight
1902		* Hinxman, Lionel W., B.A., Geological Survey Office, George IV. Bridge, Edinburgh
1904		Hobday, Frederick T. G., F.R.C.V.S., 6 Berkeley Gardens, Kensington, London
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Prof. of Chem. and Physics at the Royal Military Acad. and Royal Artillery Coll., Woolwich, 18 Glenluce Road, Blackheath, Kent 250
1881	C. N.	* Horne, John, LL.D., F.R.S., F.G.S., Director of the Geological Survey of Scotland, Sheriff-Court Buildings, Edinburgh

Date of Election.			
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley	
1904		* Horsburgh, Ellice Martin, M.A., B.Sc., Lecturer in Technical Mathematics, University of Edinburgh, 3 Eglinton Crescent	
1897		Houston, Alex. Cruikshanks, M.B., C.M., D.Sc., 14 Upper Addison Gardens, Kensington, London	
1893		Howden, Robert, M.A., M.B., C.M., Professor of Anatomy in the University of Durham, 24 Burdon Terrace, Newcastle-on-Tyne	255
1899		Howie, W. Lamond, F.C.S., Hanover Lodge, West Hill, Harrow	
1883	C.	* Hoyle, William Evans, M.A., D.Sc., M.R.C.S., 25 Brunswick Road, Withington, Manchester	
1872		Hughes-Hunter, Colonel Charles, of Pläs Cöch, Llanfairpwll, Anglesea, and Junior United Service Club, London	
1886		Hunt, Rev. H. G. B., Mus. D. Dub., Mus. B. Oxon., F.L.S., La Belle Sauvage, London	
1887	C.	* Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian	260
1887	C.	* Hunter, William, M.D., M.R.C.P. L. and E., M.R.C.S., 54 Harley Street, London	
1882	C.	* Inglis, J. W., Memb. Inst. C.E., c/o National Bank of New Zealand, Paeoroa, Auckland, New Zealand	
1904		Innes, R. T. A., Director, Government Observatory, Johannesburg, Transvaal	
1904		* Ireland, Alexander Scott, S.S.C., 2 Buckingham Terrace	
1875		Jack, William, M.A., LL.D., Professor of Mathematics in the University of Glasgow	265
1894		Jackson, Sir John, LL.D., 10 Holland Park, London	
1889	C.	* James, Alexander, M.D., F.R.C.P.E., 10 Melville Crescent	
1882		* Jamieson, Prof. A., Memb. Inst. C.E., 16 Rosslyn Terrace, Kelvinside, Glasgow	
1880		Japp, A. H., LL.D., Dunrose, Fair Dene Road, Purley R.S.O., Surrey	
1901		* Jardine, Robert, M.D., M.R.C.S. Eng., F.F. P. and S. Glas., 5 Clifton Place, Glasgow	270
1900		Jee, Sir Bhagvat Sinh, G.C.I.E., M.D., LL.D. Edin., H.H. The Thakore Sahib of Gondal, Gondal, Kathiawar, Bombay	
1900		* Jerdan, David Smiles, M.A., D. Sc., Ph.D., Gorgie Mills	
1895		Johnston, Lieutenant-Colonel Henry Halcro, C.B., R.A.M.S., D.Sc., M.D., F.L.S., Orphir House, Kirkwall, Orkney	
1903		* Johnston, Thomas Nicol, M.B., C.M., Corstorphine House, Corstorphine	
1902		Johnstone, George, Lieut. R.N.R., Marine Superintendent, British India Steam Navigation Co., 16 Strand Road, Calcutta, India	275
1874		Jones, Francis, Lecturer on Chemistry, Beaufort House, Alexandra Park, Manchester	
1888		Jones, John Alfred, Memb. Inst. C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras, c/o Messrs Parry & Co., 70 Gracechurch Street, London	
1847	C. K. V. J.	Kelvin, The Right Hon. Lord, G.C.V.O., P.C., LL.D., D.C.L., F.R.S. (PRESIDENT), Grand Officer of the Legion of Honour of France, Member of the Prussian Order *Pour le Mérite*, Foreign Associate of the Institute of France, and Emeritus Professor of Natural Philosophy in the University of Glasgow, Netherhall, Largs, Ayrshire, and 15 Eaton Place, London, S.W.	
1892		* Kerr, Rev. John, M.A., Manse, Dirleton	
1903	C.	* Kerr, John Graham, M.A., Professor of Zoology in the University of Glasgow	280
1891		Kerr, Joshua Law, M.D., Biddenden Hall, Cranbrook, Kent	
1886	C. N.	* Kidston, Robert, F.R.S., F.G.S., 12 Clarendon Place, Stirling	
1877		King, Sir James, of Campsie, Bart., LL.D., 115 Wellington Street, Glasgow	
1880		King, W. F., Lonend, Russell Place, Trinity	
1883		* Kinnear, The Rt. Hon. Lord, one of the Senators of the College of Justice, 2 Moray Pl.	285

Date of Election.			
1878		Kintore, The Right Hon. the Earl of, M.A. Cantab., LL.D. Cambridge, Aberdeen and Adelaide, Keith Hall, Inverurie, Aberdeenshire	
1901		* Knight, The Rev. G. A. Frank, M.A., St Leonard's United Free Church, Perth	
1880	C. K.	Knott, C. G., D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh (late Prof. of Physics, Imperial University, Japan), 42 Upper Gray Street, Edinburgh	
1896	C.	* Kuenen, J. P., Ph.D. (Leiden), Prof. of Natural Philosophy in University College, Dundee	
1886		* Laing, Rev. George P., 17 Buckingham Terrace	290
1878	C.	Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics, University of St Andrews	
1885	C.	* Laurie, A. P., M.A., D.Sc., Principal of the Heriot-Watt College, Edinburgh	
1894	C.	* Laurie, Malcolm, B.A., D.Sc., F.L.S., Royal College of Surgeons, Edinburgh	
1870		Laurie, Simon S., M.A., LL.D., Emeritus Professor of Education in the University of Edinburgh, 22 George Square	
1903		* Leighton, Gerald Rowley, M.D., 17 Hartington Place	295
1874	C. K.	Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast	
1889		* Lindsay, Rev. James, D.D., B.Sc., F.G.S., M.R.A.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Associate of the Philosophical Society of Louvain, Minister of St Andrew's Parish, Springhill Terrace, Kilmarnock	
1870	C. B.	Lister, The Right Hon. Lord, P.C., M.D., F.R.C.S.L., F.R.C.S.E., LL.D., D.C.L., F.R.S., Foreign Associate of the Institute of France, Emeritus-Prof. of Clinical Surgery, King's College, Surgeon Extraordinary to the King, 12 Park Crescent, Portland Pl., London	
1903		Liston, William Glen, M.D., Captain, Indian Medical Service, c/o Grindlay Groom & Co., Bombay, India	
1903		* Littlejohn, Henry Harvey, M.A., M.B., B.Sc., F.R.C.S.E., 1 Atholl Crescent	300
1897	C.	Lloyd, Richard John, M.A., D.Lit., 49A Grove Street, Liverpool	
1898		* Lothian, Alexander Veitch, M.A., B.Sc., 11 Holborn Terrace, Kelvinside, Glasgow	
1884		* Low, George M., Actuary, 11 Moray Place	
1888		* Lowe, D. F., M.A., LL.D., Headmaster of Heriot's Hospital School, Lauriston	
1900		Lusk, Graham, Ph.D., M.A., Prof. of Physiology, Univ. and Bellevue Medical College, N.Y.	305
1894		* Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire	
1887		M'Aldowie, Alexander M., M.D., 6 Brook Street, Stoke-on-Trent	
1891		Macallan, John, F.I.C., 3 Rutland Terrace, Clontarf, Dublin	
1888	C.	M'Arthur, John, F.C.S., 196 Trinity Road, Wandsworth Common, London	
1883		* M'Bride, P., M.D., F.R.C.P.E., 16 Chester Street	310
1903		* M'Cormick, W. S., M.A., LL.D., 13 Douglas Crescent	
1899		* M'Cubbin, James, B.A., Rector of the Burgh Academy, Kilsyth	
1894		* Macdonald, James, Secretary of the Highland and Agricultural Society of Scotland, 2 Garscube Terrace	
1897	C.	* Macdonald, James A., M.A., B.Sc., H.M. Inspector of Schools, Glengarry, Dingwall	
1904		* Macdonald, J. A., M.A., B.Sc., Olive Lodge, Polwarth Terrace	315
1886		* Macdonald, The Rt. Hon. Sir J. H. A., K.C.B., K.C., LL.D., F.R.S., M.I.E.E., Lord Justice-Clerk, and Lord President of the Second Division of the Court of Session, 15 Abercromby Place	
1904		Macdonald, William, B.Sc., M.Sc., Chief of the Division of Publications under the Department of Agriculture, Pretoria Club, Pretoria, Transvaal	
1886		* Macdonald, William J., M.A., Comiston Drive	
1901	C.	* MacDougal, R. Stewart, M.A., D.Sc., 13 Archibald Place	

Date of Election.			
1888	C.	* M'Fadyean, John, M.B., B.Sc., Professor of Pathology and Dean of the Royal Veterinary College, Camden Town, London	320
1878	C.	Macfarlane, Alexander, M.A., D.Sc., LL.D., Lecturer in Physics in Lehigh University, Pennsylvania, Gowrie Grove, Chatham, Ontario, Canada	
1885	C.	* Macfarlane, J. M., D.Sc., Professor of Biology in the University of Pennsylvania, Lansdowne, Delaware Co., Pennsylvania	
1897		* M'Gillivray, Angus, C.M., M.D., South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, Middlesex	
1886		* MacGregor, Rev. James, D.D., 3 Eton Terrace	325
1880	C.	MacGregor, James Gordon, M.A., D.Sc., LL.D., F.R.S., Prof. of Natural Philosophy in the University of Edinburgh, 6 Chalmers Crescent	
1903		* M'Intosh, D. C., M.A., 37 Warrender Park Terrace	
1869	C. N.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, 2 Abbotsford Crescent, St Andrews	
1895	C.	* Macintyre, John, M.D., 179 Bath Street, Glasgow	
1882		* Mackay, John Sturgeon, M.A., LL.D., Mathematical Master in the Edinburgh Academy, 69 Northumberland Street	330
1873	C. B.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S., Professor of Physiology in the University of Glasgow, 2 Buckingham Terrace, Glasgow	
1900	C.	* M'Kendrick, John Souttar, M.D., 2 Florentine Gardens, Hillhead, Glasgow	
1894		* Mackenzie, Robert, M.D., Napier, Nairn	
1898		Mackenzie, W. Cossar, D.Sc., Principal of the College of Agriculture, Gheezeh, Egypt	
1904		* Mackenzie, W. Leslie, M.A., M.D., D.P.H., Medical Inspector to the Local Government Board for Scotland, 1 Stirling Road, Trinity.	335
1904		* Mackintosh, Donald James, M.V.O., M.B., Superintendent of the Western Infirmary, Glasgow	
1894		* Maclagan, Philip R. D., F.F.A. (Treasurer), St Catherine's, Liberton	
1869	C.	Maclagan, R. C., M.D., F.R.C.P.E., 5 Coates Crescent	
1869	C.	M'Laren, The Hon. Lord, LL.D. Edin. & Glasg., F.R.A.S., one of the Senators of the College of Justice (Vice-President), 46 Moray Place	
1899		Maclean, Ewan John, M.D., M.R.C.P. London, 12 Park Place, Cardiff	340
1888	C.	* Maclean, Magnus, M.A., D.Sc., Memb. Inst. E. E., Prof. of Electrical Engineering in the Glasgow and West of Scotland Technical College, 51 Kerrsland Ter., Hillhead, Glasgow	
1876		Macleod, Very Rev. Norman, D.D., Westwood, Inverness	
1876		Macmillan, John, M.A., D.Sc., M.B., C.M., F.R.C.P.E., 48 George Square	
1893		* M'Murtrie, The Rev. John, M.A., D.D., 13 Inverleith Place	
1884		* Macpherson, Rev. J. Gordon, M.A., D.Sc., Mathematical Examiner in the University of St Andrews, Ruthven Manse, Meigle	345
1890		* M'Vail, John C., M.D., 32 Balshagray Avenue, Partick, Glasgow	
1898	C.	Mahalanobis, S. C., B.Sc., Professor of Physiology, Presidency College, Calcutta, India	
1880	C.	Marsden, R. Sydney, M.B., C.M., D.Sc., F.I.C., F.C.S., Rowallan House, Cearns Road, and Town Hall, Birkenhead	
1882	C.	Marshall, D. H., M.A., Professor of Physics in Queen's University and College, Kingston, Ontario, Canada	
1901	C.	* Marshall, F. H. A., B.A., D.Sc., Zoological Department, University of Edinburgh	350
1888	C. K.	* Marshall, Hugh, D.Sc., F.R.S., Lecturer on Chemistry and on Mineralogy and Crystallography in the University of Edinburgh, 12 Lonsdale Terrace	

Date of Election.		
1892		* Martin, Francis John, W.S., 17 Rothesay Place
1903		Martin, Nicholas Henry, F.L.S., F.C.S., Ravenswood, Low Fell, Gateshead
1864		Marwick, Sir James David, LL.D., 19 Woodside Terrace, Glasgow
1866		Masson, David, LL.D., Litt. D. Dub., Emeritus-Professor of Rhetoric and English Literature in the Univ. of Edin., H.M. Historiographer for Scotland, 2 Lockharton Gardens 355
1885	C.	* Masson, Orme, D.Sc., F.R.S., Professor of Chemistry in the University of Melbourne
1898	C. B.	* Masterman, Arthur Thomas, M.A., D.Sc., Inspector of Fisheries, Board of Agriculture, Whitehall, London
1890		* Matheson, The Rev. George, M.A., B.D., D.D., LL.D., 19 St Bernard's Crescent
1902		Matthews, Ernest Romney, C.E., F.G.S., Bridlington, Yorkshire
1901		* Menzies, Alan W. C., M.A., B.Sc., F.C.S., Professor of Chemistry in St Mungo's College, Glasgow 360
1888		* Methven, Cathcart W., Memb. Inst. C.E., F.R.I.B.A., Durban, Natal, S. Africa
1902	C.	Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Professor of Mathematics, Syracuse University, Syracuse, N.Y.
1885	C. B.	* Mill, Hugh Robert, D.Sc., LL.D., 62 Camden Square, London
1904	C.	* Milne, James Robert, B.Sc., 56 Manor Place
1886		* Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen 365
1899		* Milroy, T. H., M.D., B.Sc., Professor of Physiology in Queen's College, Belfast, 14 Ashley Avenue, Belfast
1866		Mitchell, Sir Arthur, K.C.B., M.A., M.D., LL.D., 34 Drummond Place
1889	C.	Mitchell, A. Crichton, D.Sc., Professor of Pure and Applied Mathematics, and Principal of the Maharajah's College, Trivandrum, Travancore, India
1897		* Mitchell, George Arthur, M.A., 2 Lilybank Gardens, Glasgow
1900		* Mitchell, James, M.A., B.Sc., 7 Bath Street, Nairn 370
1899		* Mitchell-Thomson, Sir Mitchell, Bart., 6 Charlotte Square
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., The Poplars, 20 Avenue Road, Regent's Park, London
1887	C.	Moos, N. A. F., L.C.E., B.Sc., Professor of Physics, Elphinstone College, and Director of the Government Observatory, Colaba, Bombay
1901		* More, James, jun., M. Inst. C.E., 74 George Street
1896		* Morgan, Alexander, M.A., D.Sc., Rector, Church of Scotland Training College, 6 Cluny Terrace 375
1892		Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony
1901		Moses, O. St John, M.D., B.Sc., 6 Lansdowne Road, Calcutta, India
1892	C.	* Mossman, Robert C.
1874	C. K.	Muir, Thomas, C.M.G., M.A., LL.D., F.R.S., Superintendent-General of Education for Cape Colony, Education Office, Cape Town, and Mowbray Hall, Rosebank, Cape Colony
1888	C.	* Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers 380
1887		Mukhopâdhyay, Âsûtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta
1894		* Munro, J. M. M., Memb. Inst. E.E., 136 Bothwell Street, Glasgow
1891	C.	* Munro, Robert, M.A., M.D., LL.D., Hon. Memb. R.I.A., Hon. Memb. Royal Soc. of Antiquaries of Ireland (VICE-PRESIDENT), 48 Manor Place and Elmbank, Largs, Ayrshire
1896		* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent

Date of Election.			
1892	C.	* Murray, George Robert Milne, F.R.S., F.L.S., Keeper of the Botanical Department, British Museum (Natural Hist.), Cromwell Road, London	385
1877	C. B. N.	Murray, Sir John, K.C.B., LL.D., D.C.L., Ph.D., D.Sc., F.R.S., Member of the Prussian Order *Pour le Mérite*, Director of the Challenger Expedition Publications (Vice-President). Office, 45 Frederick Street. House, Challenger Lodge, Wardie, and United Service Club	
1887		Muter, John, M.A., F.C.S., South London Central Public Laboratory, 325 Kennington Road, London	
1902		Mylne, The Rev. R. S., M.A., B.C.L., Oxford, F.S.A. Lond., Great Amwell, Herts	
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P.L., General Hospital, Adelaide, S. Australia	
1895		* Napier, James, M.A., Drums, Old Kilpatrick	390
1897		Nash, Alfred George, C.E., B.Sc., Engineer, Department of Public Works, Jamaica, Belretiro, Mandeville, Jamaica, W.I.	
1887		* Nasmyth, T. Goodall, M.D., C.M., D.Sc., Cupar-Fife	
1898		Newman, George, M.D., D.P.H. Cambridge, 2 Woburn Square, London	
1884		* Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park	
1880	C.	Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place	395
1878		Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham	
1902		Nunn, Joshua Arthur, C.I.E., D.S.O., F.R.C.V.S., Barrister-at-Law, Lincoln's Inn; Veterinary Lieut.-Colonel and Deputy Director-General, Army Veterinary Department; Junior United Service Club, London, and Conservative Club, St James Street, London	
1888		* Ogilvie, F. Grant, M.A., B.Sc., Principal Assistant Secretary for Science, Art, and Technology, Board of Education, Whitehall, London	
1888		* Oliphant, James, M.A., 12 Murrayfield Road	
1886	C.	Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 18 Gordon Square, London	400
1895		Oliver, Thomas, M.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	
1884	C. K.	* Omond, R. Traill, Hon. Superintendent of Ben Nevis Observatory, Fort-William, 3 Church Hill, Edinburgh	
1892		Parker, Thomas, Memb. Inst. C.E., 1B Chapel Street, Edgeware Road, London	
1901		* Paterson, David, F.C.S., Lea Bank, Rosslyn, Midlothian	
1886	C.	* Paton, D. Noël, M.D., B.Sc., F.R.C.P.E., 22 Lyndoch Place	405
1889		* Patrick, David, M.A., LL.D., c/o W. & R. Chambers, 339 High Street	
1892		* Paulin, David, Actuary, 6 Forres Street	
1881	C. N.	* Peach, B. N., LL.D., F.R.S., F.G.S., Acting Palæontologist of the Geological Survey of Scotland, 30 Mayfield Road	
1904		* Peck, James Wallace, M.A., H.M. Inspector of Schools, 22 Duke Street	
1889		* Peck, William, F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh	410
1863		Peddie, Alexander, M.D., F.R.C.P.E., 15 Rutland Street	
1887	C.B.	* Peddie, Wm., D.Sc., Lecturer on Natural Philosophy, Edinburgh University, 14 Ramsay Gardens	
1900		Penny, John, M.B., C.M., D.Sc., Great Broughton, near Cockermouth, Cumberland	
1893		Perkin, Arthur George, F.R.S., 8 Montpellier Terrace, Hyde Park, Leeds	
1889		* Philip, R. W., M.A., M.D., F.R.C.P.E., 45 Charlotte Square	415
1883		Phillips, Charles D. F., M.D., LL.D., 10 Henrietta St., Cavendish Sq., London, W.	

Date of Election.			
1886		* Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow	
1852		Powell, Eyre B., C.S.I., M.A., 25 Kirkstall Road, Streatham Hill, London	
1888		Prain, David, Major I.M.S., Superintendent, Royal Botanic Gardens, Shibpur, Calcutta	
1902		* Preller, Charles Du Riche, M.A., Ph.D., Assoc. Memb. Inst. C.E., 61 Melville Street	420
1892		* Pressland, Arthur J., M.A. Camb., Edinburgh Academy	
1875	C.	Prevost, E. W., Ph.D., Weston, Ross, Herefordshire	
1885		* Pullar, J. F., Rosebank, Perth	
1903		* Pullar, Laurence, The Lea, Bridge of Allan	
1880		Pullar, Sir Robert, Tayside, Perth	425
1898		* Purves, John Archibald, D.Sc., 53 York Place	
1897		* Rainy, Harry, M.B., C.M., F.R.C.P. Ed., 16 Gt. Stuart Street	
1899		* Ramage, Alexander G., 8 Western Terrace, Murrayfield	
1884		Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, Curator of Australian Museum, Sydney, N.S.W.	
1891		* Rankine, John, M.A., LL.D., Advocate, Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place	430
1904		Ratcliffe, Joseph Riley, M.B., C.M., Elmdon, Wake Green Road, Morley, Birmingham	
1900		Raw, Nathan, M.D., Mill Road Infirmary, Liverpool	
1883	C.	* Readman, J. B., D.Sc., F.C.S., Mynde Park, Tram Inn, Hereford	
1889		Redwood, Boverton, D.Sc. (Hon.), F.I.C., F.C.S., Assoc. Inst. C.E., Wadham Lodge, Wadham Gardens, London	
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., Barrister-at-Law, National Liberal Club, Whitehall Place, London	435
1902		Reid, George Archdall O'Brien, M.B., C.M., 9 Victoria Road South, Southsea, Hants	
1875		Richardson, Ralph, W.S., 10 Magdala Place	
1872		Ricarde-Seaver, Major F. Ignacio, Athenæum Club, Pall Mall, London	
1898	C.	Roberts, Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa	
1880		Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John Street, Manchester	440
1872		Robertson, D. M. C. L. Argyll, M.D., F.R.C.S.E., LL.D., Surgeon Oculist to the King for Scotland, Mon Plaisir, St Aubins, Jersey	
1900		* Robertson, Joseph M'Gregor, M.B., C.M., 26 Buckingham Terrace, Glasgow	
1896		* Robertson, Robert, M.A., 25 Mansionhouse Road	
1902	C.	* Robertson, Robert A., M.A., B.Sc., Lecturer on Botany in the University of St Andrews	
1896	C.	* Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., 26 Minto Street	445
1881		Rosebery, The Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh	
1880		Rowland, L. L., M.A., M.D., President of the Oregon State Medical Society, and Professor of Physiology and Microscopy in Williamette University, Salem, Oregon	
1902	C.	* Russell, James, 11 Argyll Place	
1880		Russell, Sir James A., M.A., B.Sc., M.B., F.R.C.P.E., LL.D., Woodville, Canaan Lane	
1904		Sachs, Edwin O., Architect, 7 Waterloo Place, London	450
1903		* Samuel, John S., 8 Park Avenue, Glasgow	
1897		* Sanderson, William, Talbot House, Ferry Road	
1864		Sandford, The Right Rev. Bishop D. F., D.D., LL.D., 4 Coates Crescent	
1903		* Sarolea, Charles, Ph.D., D. Litt., Lecturer on French Language, Literature, and Romance Philology, University of Edinburgh, Hermitage, Colinton	

Date of Election.			
1895		Savage, Thomas, M.D., F.R.C.S. England, M.R.C.P. London, Professor of Gynæcology, Mason College, Birmingham, The Ards, Knowle, Warwickshire	455
1891		Sawyer, Sir James, Knt., M.D., F.R.C.P., F.S.A., J.P., Consulting Physician to the Queen's Hospital, 31 Temple Row, Birmingham	
1900	C.	* Schäfer, Edward Albert, M.R.C.S., LL.D., F.R.S., Professor of Physiology in the University of Edinburgh	
1885	C.	Scott, Alexander, M.A., D.Sc., F.R.S., The Davy-Faraday Research Laboratory of the Royal Institution, London	
1880		Scott, J. H., M.B., C.M., M.R.C.S., Prof. of Anatomy in the University of Otago, New Zealand	
1902		Senn, Nicholas, M.D., LL.D., Professor of Surgery, Rush Medical College, Chicago, U.S.A.	460
1872	C.	Seton, George, M.A., Advocate, Ayton House, Abernethy, Perthshire	
1897		* Shepherd, John William, Carrickarden, Bearsden, Glasgow	
1894		* Shield, Wm., Memb. Inst. C.E., 33 Old Queen Street, Westminster, London	
1872		Sibbald, Sir John, M.D., Commissioner in Lunacy (retired), 18 Great King Street	
1870		Sime, James, M.A., Craigmount House, 10 Grange Road	465
1871		Simpson, A. R., M.D., Professor of Midwifery in the University of Edinburgh, 52 Queen Street	
1900	C.	* Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh, 52 Queen Street	
1903		* Skinner, Robert Taylor, M.A., Governor and Headmaster, Donaldson's Hospital, Edinburgh	
1901		* Smart, Edward, B.A., B.Sc., Benview, Craigie, Perth	
1891	C.	* Smith, Alex., B.Sc., Ph.D., Prof. of General Chemistry, University of Chicago, Ills., U.S.	470
1882	C.	Smith, C. Michie, B.Sc., F.R.A.S., Director of the Kodaikánal and Madras Observatories, The Observatory, Kodaikánal, South India	
1885		* Smith, George, F.C.S., Polmont Station	
1871	C.	Smith, John, M.D., F.R.C.S.E., LL.D., 11 Wemyss Place	
1904		* Smith, William Charles, K.C., M.A., LL.B., Advocate, 6 Darnaway Street	
1880		Smith, William Robert, M.D., D.Sc., Barrister-at-Law, Professor of Forensic Medicine in King's College, 74 Great Russell Street, Bloomsbury Square, London	475
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Coventry	
1880		Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., late Fellow of St John's College, Cambridge, and Professor of Geology and Palæontology in the University of Oxford	
1889	C.	Somerville, Wm., M.A., D.Sc., D.Oec., Assistant Secretary, H.M. Board of Agriculture, 4 Whitehall Place, London	
1882		* Sorley, James, F.I.A., C.A., 32 Onslow Square, London	
1896		* Spence, Frank, M.A., B.Sc., 25 Craiglea Drive	480
1874	C.	Sprague, T. B., M.A., LL.D., Actuary, 29 Buckingham Terrace	
1891		* Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College	
1886	C.	* Stevenson, Charles A., B.Sc., Memb. Inst. C.E., 28 Douglas Crescent	
1884		* Stevenson, David Alan, B.Sc., Memb. Inst. C.E., 45 Melville Street	
1868		Stevenson, John J., 4 Porchester Gardens, London	485
1888	C.	* Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, 9 Learmonth Gardens	
1868		Stewart, Major-General J. H. M. Shaw, late R.E., Assoc. Inst. C.E., F.R.G.S., 7 Inverness Terrace, London, W.	

Date of Election.		
1904		* Stewart, Thomas W., M.A., B.Sc., Science Master, Edinburgh Ladies' College, 29 Bruntsfield Gardens
1873		Stewart, Walter, 3 Queensferry Gardens
1877		Stirling, William, D.Sc., M.D., LL.D., Brackenbury Professor of Physiology and Histology in Owens College and Victoria University, Manchester 490
1902		* Stockdale, Herbert Fitton, Clairinch, Milngavie, Dumbartonshire
1889	C.	* Stockman, Ralph, M.D., F.R.C.P.E., Professor of Materia Medica and Therapeutics in the University of Glasgow
1903		Sutherland, David W., M.D., M.R.C.P. Lond., Captain, Indian Medical Service, Professor of Pathology and Materia Medica, Medical College, Lahore, India
1896		* Sutherland, John Francis, M.D., Dep. Com. in Lunacy for Scotland, 3 Moston Terrace
1885	C.	* Symington, Johnson, M.D., F.R.C.S.E., F.R.S., Prof. of Anatomy in Queen's College, Belfast 495
1904		* Tait, John W., B.Sc., Rector of Leith Academy, 18 Netherby Road, Leith
1898		Tait, William Archer, B.Sc., Memb. Inst. C.E., 38 George Square
1895		Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., Professor of Geology, Univ. of Utah, Salt Lake City, Utah
1890	C.	Tanakadate, Aikitu, Prof. of Nat. Phil. in the Imperial University of Japan, Tokyo, Japan
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow 500
1899		* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy, 3 Melgund Terrace
1872		Teape, Rev. Charles R., M.A., Ph.D., Rector of St Andrew's Episcopal Church, 15 Findhorn Place
1892		Thackwell, J. B., M.B., C.M., Ravenswood Hospital, Ravenswood, Queensland
1885	C.	* Thompson, D'Arcy W., C.B., B.A., F.L.S., Professor of Natural History in University College, Dundee
1887		* Thomson, Andrew, M.A., D.Sc., F.I.C., Rector, Perth Academy, Ardenlea, Pitcullen, Perth 505
1896		* Thomson, George Ritchie, M.B., C.M., Cumberland House, Von Brandis Square, Johannesburg, Transvaal
1903		Thomson, George S., F.C.S., Dairy Commissioner for Queensland, Department of Agriculture, Brisbane, Queensland
1887	C.	* Thomson, J. Arthur, M.A., Regius Prof. of Natural History in the University of Aberdeen
1880		Thomson, John Millar, LL.D., F.R.S., Prof. of Chem. in King's College, Lond., 9 Campden Hill Gardens, London
1899		* Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow 510
1902		* Thomson, Robert Stevenson, M.D., D.Sc., F.F.P.S.G., 17 Woodside Crescent, Glasgow
1870		Thomson, Spencer C., Actuary, 10 Eglinton Crescent
1882		Thomson, Wm., M.A., B.Sc., LL.D., Registrar, University of the Cape of Good Hope, University Buildings, Cape Town
1876		Thomson, William, Royal Institution, Manchester
1874	C. B. N.	Traquair, R. H., M.D., LL.D., F.R.S., F.G.S., Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh (VICE-PRESIDENT), 8 Dean Park Crescent 515
1874		Tuke, Sir J. Batty, M.D., D.Sc., LL.D., F.R.C.P.E., M.P. for the Universities of Edinburgh and St Andrews, 20 Charlotte Square
1888		* Turnbull, Andrew H., Actuary, The Elms, Whitehouse Loan
1861	C. N.	Turner, Sir William, K.C.B., M.B., F.R.C.S.E., LL.D., D.C.L., D.Sc. Dub., F.R.S., Principal of the University of Edinburgh, 6 Eton Terrace
1895		Turton, Albert H., A.I.M.M., The Gwyn Mines (Merioneth) Ltd., nr Dolgelly, North Wales

Date of Election.		
1898	C.	* Tweedie, Charles, M.A., B.Sc., Lecturer on Mathematics in the University of Edinburgh, 12 Nelson Street 520
1877		Underhill, Charles E., B.A., M.B., F.R.C.P.E., F.R.C.S.E., 8 Coates Crescent
1889		Underhill, T. Edgar, M.D., F.R.C.S.E., Dunedin, Barnt Green, Worcestershire
1875		Vincent, Charles Wilson, F.I.C., F.C.S., M.R.I., Librarian of the Reform Club, Pall Mall, London, 38 Queen's Road, South Hornsey, Middlesex
1888		Walker, James, Memb. Inst. C.E., Engineer's Office, Tyne Improvement Commission, Newcastle-on-Tyne
1891	C. B.	* Walker, James, D.Sc., Ph.D., F.R.S., Professor of Chemistry in University College, Dundee, 8 Windsor Terrace, Dundee 525
1873	C.	Walker, Robert, M.A., University, Aberdeen
1902		* Wallace, Alexander G., M.A., 25 Belvidere Crescent, Aberdeen
1886	C.	* Wallace, R., F.L.S., Prof. of Agriculture and Rural Economy in the Univ. of Edinburgh
1898		Wallace, Wm., M.A., Principal, Cockburn Science School, Leeds
1891		* Walmsley, R. Mullineux, D.Sc., Prin. of the Northampton Inst., Clerkenwell, London 530
1901	C.	* Waterston, David, M.A., M.D., F.R.C.S.E., Lecturer on Regional Anatomy in the University of Edinburgh, 23 Colinton Road
1904		* Watson, Charles B. Boog, 82 Polwarth Terrace
1866		Watson, Sir Patrick Heron, M.D., F.R.C.S.E., LL.D., Surgeon in Ordinary to the King in Scotland, 16 Charlotte Square
1862	C.	Watson, Rev. Robert Boog, B.A., LL.D., F.L.S., Past President of the Conchological Society, 11 Strathearn Place
1900		* Watson, Thomas P., M.A., B.Sc., Principal, Keighley Institute, Keighley 535
1896		Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynæcology, Rush Medical College, Chicago, 706 Reliance Buildings, 100 State Street, Chicago
1903	C.	* Wedderburn, J. H. Maclagan, M.A., 8 East Fettes Avenue
1896		Wenley, R. M., M.A., D.Sc., D.Phil., LL.D., Prof. of Philosophy in the University of Michigan, U.S.
1896	C.	White, Philip J., M.B., Prof. of Zoology in University College, Bangor, North Wales
1890		White, Sir William Henry, K.C.B., Memb. Inst. C.E., LL.D., F.R.S., late Assistant Controller of the Navy, and Director of Naval Construction, Cedarscroft, Putney Heath, London 540
1881		Whitehead, Walter, F.R.C.S.E., Professor of Clinical Surgery, Owens College and Victoria University, 499 Oxford Road, Manchester
1894		Whymper, Edward, F.R.G.S., 29 Ludgate Hill, London
1879		Will, John Charles Ogilvie, M.D., 379 Union Street, Aberdeen
1897		* Williams, W. Owen, F.R.C.V.S., Professor of Veterinary Medicine and Surgery, University of Liverpool, The Veterinary School, The University, Liverpool
1900		Wilson, Alfred C., F.C.S., Voewood Croft, Stockton-on-Tees 545
1879		Wilson, Andrew, Ph.D., F.L.S., Lecturer on Zoology and Comparative Anatomy, 110 Gilmore Place
1902		* Wilson, Charles T. R., M.A., F.R.S., Glencorse House, Peebles, and Sidney Sussex College, Cambridge
1895		Wilson-Barker, David, F.R.G.S., Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," Greenhithe, Kent
1882		Wilson, George, M.A., M.D., 7 Avon Place, Warwick
1891		* Wilson, John Hardie, D.Sc., University of St Andrews (39 South Street, St Andrews) 550

Date of Election.		
1902		Wilson, William Wright, F.R.C.S.E., M.R.C.S. Eng., Cottesbrook House, Acock's Green, Birmingham
1886	C.	* Woodhead, German Sims, M.D., F.R.C.P.E., Prof. of Pathology in the University of Cambridge
1884		Woods, G. A., M.R.C.S., Eversleigh, 1 Newstead Road, Lee, Kent
1890		* Wright, Johnstone Christie, Northfield, Colinton
1896		* Wright, Robert Patrick, Professor of Agriculture, West of Scotland Agricultural College, 6 Blythswood Square, Glasgow 555
1882		* Young, Frank W., F.C.S., H.M. Inspector of Science and Art Schools, 32 Buckingham Terrace, Botanic Gardens, Glasgow
1892		Young, George, Ph.D., Firth College, Sheffield
1896	C.	* Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton
1900		* Young, J. M'Lauchlan, F.R.C.V.S., Lecturer on Veterinary Hygiene, University of Aberdeen
1904		Young, R. B., M.A., B.Sc., Transvaal Technical Institute, Johannesburg, Transvaal 560

LIST OF HONORARY FELLOWS

AT OCTOBER 1904.

HIS MOST GRACIOUS MAJESTY THE KING.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X.).

Elected		
1897	Alexander Agassiz,	*Cambridge (Mass.).*
1897	E.-H. Amagat,	*Paris.*
1900	Arthur Auwers,	*Berlin.*
1900	Adolf Ritter von Baeyer,	*Munich.*
1889	Marcellin Pierre Eugène Berthelot,	*Paris.*
1895	Ludwig Boltzmann,	*Vienna.*
1897	Stanislao Cannizzaro,	*Rome.*
1902	Jean Gaston Darboux,	*Paris.*
1902	Anton Dohrn,	*Naples.*
1902	Albert Gaudry,	*Paris.*
1888	Ernst Haeckel,	*Jena.*
1883	Julius Hann,	*Graz.*
1879	Jules Janssen,	*Paris.*
1864	Albert von Kölliker,	*Würzburg.*
1902	Samuel Pierpont Langley,	*Washington.*
1897	Gabriel Lippmann,	*Paris.*
1895	Éleuthère-Élie-Nicolas Mascart,	*Paris.*
1888	Demetrius Ivanovich Mendeléef,	*St Petersburg.*
1895	Carl Menger,	*Vienna.*
1897	Fridtjof Nansen,	*Christiania.*
1881	Simon Newcomb,	*Washington.*
1895	Jules Henri Poincaré,	*Paris.*
1889	Georg Hermann Quincke,	*Heidelberg.*
1897	Ferdinand von Richthofen,	*Berlin.*
1897	Giovanni V. Schiaparelli,	*Milan.*
1878	Otto Wilhelm Struve,	*Carlsruhe.*
1886	Tobias Robert Thalén,	*Upsala.*
1897	Ferdinand Zirkel,	*Leipzig.*

Total, 28.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X.).

Elected		
1902	Sir Benjamin Baker, K.C.M.G., Mem.Inst.C.E., F.R.S.,	*London.*
1889	Sir Robert Stawell Ball, Kt., LL.D., F.R.S., M.R.I.A., Lowndean Professor of Astronomy in the University of Cambridge,	*Cambridge.*
1900	Edward Caird, LL.D., Master of Balliol College, Oxford,	*Oxford.*
1892	Colonel Alexander Ross Clarke, C.B., R.E., F.R.S.,	*Redhill, Surrey.*
1897	George Howard Darwin, M.A., LL.D., F.R.S., Plumian Professor of Astronomy in the University of Cambridge,	*Cambridge.*
1892	Sir David Gill, K.C.B., LL.D., F.R.S., His Majesty's Astronomer at the Cape of Good Hope,	*Cape of Good Hope.*
1900	David Ferrier, M.D., LL.D., F.R.S., Prof. of Neuro-pathology, King's College, London,	*London.*
1900	Andrew Russell Forsyth, D.Sc., F.R.S., Sadlerian Professor of Pure Mathematics in the University of Cambridge,	*Cambridge.*
1895	Albert C. L. G. Günther, Ph.D., F.R.S.,	*London.*
1883	Sir Joseph Dalton Hooker, K.C.S.I., M.D., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1902	Sir Richard C. Jebb, Litt. D., D.C.L., M.P., Regius Professor of Greek in the University of Cambridge,	*Cambridge.*
1900	Archibald Liversidge, LL.D., F.R.S., Professor of Chemistry in the University of Sydney,	*Sydney.*
1884	Sir William Huggins, K.C.B., LL.D., D.C.L., P.R.S., Corresp. Mem. Inst. of France,	*London.*
1886	The Lord Rayleigh, D.C.L., LL.D., D.Sc. Dub., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1884	Sir J. S. Burdon Sanderson, Bart., M.D., LL.D., D.Sc. Dub., F.R.S.,	*Oxford.*
1900	Thomas Edward Thorpe, D.Sc., LL.D., F.R.S., Principal of the Government Laboratories, London,	*London.*
1895	Sir Charles Todd, K.C.M.G., F.R.S., Government Astronomer, South Australia,	*Adelaide.*

Total, 17.

ORDINARY FELLOWS ELECTED

During Session 1899–1900.

Arranged according to the Date of their Election.

4th December 1899.

John Penny, M.D.
John Henderson, D.Sc.
Professor Graham Lusk, Ph.D.
Alfred C. Wilson, F.C.S.
John W. H. Eyre, M.D.
James Bisset, M.A.

5th February 1900.

Thomas P. Watson, M.A.
Sir Bhagvat Sinh Jee, G.C.I.E., M.D., LL.D., Thakore Sahib of Gondal.
Douglas A. Gilchrist, B.Sc.

5th March 1900.

David Smiles Jerdan, M.A., Ph.D.
John Flett, M.A., D.Sc.
W. L. Sargant, M.A.
T. Edgecumbe Edwardes, B.A.
Professor E. A. Schäfer.
George A. O'Brien Reid, M.B., C.M.

7th May 1900.

John Souttar M'Kendrick, M.D.
Joseph M'Gregor Robertson, M.B., C.M.

4th June 1900.

J. M'Lauchlan Young, F.R.C.V.S.

2nd July 1900.

James Young Simpson, M.A., B.Sc.
William Gayton, M.D.
James Mitchell, M.A., B.Sc.
James Bower Bennett, Assoc.Memb.Inst.C.E.
Nathan Raw, M.D.

HONORARY FELLOWS ELECTED

During Session 1899–1900.

FOREIGN.

Dr Arthur Auwers, Secretary, Royal Prussian Academy of Sciences.
Professor Wilhelm His, Leipzig.
Professor Adolf Ritter von Baeyer, Munich.

BRITISH.

Edward Caird, LL.D., Master of Balliol College, Oxford.
David Ferrier, LL.D., Prof. of Neuro-pathology, King's College, London.
George Francis Fitzgerald, D.Sc., Professor of Natural and Experimental Philosophy, Trinity College, Dublin.
Andrew Russell Forsyth, D.Sc., Sadlerian Professor of Pure Mathematics in the University of Cambridge.
Archibald Liversidge, LL.D., Professor of Chemistry in the University of Sydney.
Thomas Edward Thorpe, D.Sc., Principal of the Government Laboratories, London.

FELLOWS DECEASED OR RESIGNED

During Session 1899–1900.

ORDINARY FELLOWS DECEASED.

John Anderson, C.M.G., M.D., LL.D.
His Grace The Duke of Argyll, K.G., K.T., *Hon. V.-P.*
C. Leopold Field, F.C.S.
Major-General W. D. Gosset, R.E.
His Excellency Robert Halliday Gunning, LL.D.
W. H. Lowe, M.D., F.R.C.P.S.
John Mackenzie.
Sir Douglas Maclagan, M.D., LL.D., *Hon. V.-P.*
Peter Maclagan of Pumpherston.
D. Bruce Peebles.
Adam Gillies Smith, C.A.
Emeritus Professor Piazzi Smyth, LL D.
Professor Sir Thomas Grainger Stewart, M.D., LL.D.
Henry Hannotte Vernon, M.D.
John Winzer.

RESIGNED.

James Campbell Irons. Peter Fyfe. W. A. Taylor.

HONORARY FELLOWS DECEASED.

Session 1899–1900.

BRITISH.

Sir James Paget, Bart., LL.D., D.C.L.

FOREIGN.

Alphonse Milne-Edwards.

ORDINARY FELLOWS ELECTED

During Session 1900–1901.

Arranged according to the Date of their Election.

3rd December 1900.

Alan W. C. Menzies, M.A., B.Sc.
Professor J. B. Bradbury, M.D.

7th January 1901.

Fred. P. Pullar, F.R.G.S.
Carstairs Cumming Douglas, M.D., B.Sc.
R. Stewart MacDougall, M.A., D.Sc.

4th March 1901.

F. H. A. Marshall, B.A.

6th May 1901.

W. Brodie Brodie, M.B.
H. S. Carslaw, M.A., D.Sc.
Thomas W. Drinkwater, L.R.C.P.E.
Professor Sanjiban Ganguli, M.A.
David Waterston, M.A., M.D.
James More, Jun., M.Inst.C.E.

3rd June 1901.

Robert Jardine, M.D.
Edward Smart, B.A., B.Sc.

1st July 1901.

James Goodwillie, M.A., D.Sc.
The Rev. G. A. Frank Knight, M.A.
O. St John Moses, M.D., B.Sc.
David Paterson, F.G.S.

FELLOWS DECEASED, RESIGNED, OR CANCELLED

During Session 1900–1901.

ORDINARY FELLOWS DECEASED.

Professor George F. Armstrong.
Sir Thomas C. Clark, Bart.
Andrew Fleming, M.D.
John Henderson.
J. Slater Lewis, M.Inst.C.E.
Stevenson Macadam, Ph.D
James M'Lintock, M.D., B.Sc.
The Rev. Canon W. Scott Moncrieff.
Wm. Pole, M.Inst.C.E., F.R.S.
Baden Henry Baden Powell, C.I.E.
Fred. P. Pullar, F.R.G.S.
John Rattray, M.A., B.Sc.
Wm. Skinner, W.S.
Professor P. G. Tait.
Principal W. Williams.
Thomas Graham Young.

RESIGNED.

E. F. de Jong.
W. L. Sargant.
Rt. Hon. Lord Robertson.

CANCELLED.

G. C. Robinson.

HONORARY FELLOWS DECEASED.

Session 1900–1901.

BRITISH.

Professor George F. Fitzgerald, D.Sc., F.R.S.
The Rt. Rev. W. Stubbs, D.D., LL.D., Bishop of Oxford.

FOREIGN.

Charles Hermite.
Max von Pettenkofer.
Henry A. Rowland.

ORDINARY FELLOWS ELECTED

During Session 1901–1902.

Arranged according to the Date of their Election.

4th November 1901.

Ernest Romney Matthews, C.E., F.G.S.

2nd December 1901.

George Johnstone, Lieut. R.N.R.
J. T. Ewen, B.Sc., M.Inst.C.E.

6th January 1902.

The Rev. John Henry Burn, B.D.
Lionel W. Hinxman, B.A.
James Russell.

3rd February 1902.

The Rev. R. S. Mylne, M.A., B.C.L.
Wm. Wright Wilson, F.R.C.S.E.
Lt.-Col. Joshua A. Nunn, C.I.E., D.S.O.

3rd March 1902.

A. G. Burgess, M.A.
Robert M. Buchanan, M.B.
Sir Thomas D. Gibson Carmichael, Bart.
Professor William H. Metzler.
Herbert Fitton Stockdale.
Alexander G. Wallace, M.A.

5th May 1902.

W. B. Bannerman, M.D., Major I.M.S.
Andrew Fuller Hargreaves, F.C.S.
Charles Du Riche Preller, M.A., Ph.D.
Professor Nicholas Senn, M.D., LL.D.

2nd June 1902.

Robert Stevenson Thomson, M.D.
Charles T. R. Wilson, M.A., F.R.S.

7th July 1902.

Robert A. Robertson, M.A., B.Sc.
John A. W. Dollar, M.R.C.V.S.
John Vernon Rees Roberts, M.D., D.Sc.

HONORARY FELLOWS ELECTED

Session 1901–1902.

FOREIGN.

Samuel Pierpont Langley, Secretary of the Smithsonian Institution, Washington.
Albert Gaudry, Member of the Institute, Professor of Palæontology in the Natural History Museum, Paris.
Jean Gaston Darboux, Member of the Institute, Dean of the Faculty of Sciences, Paris.
Anton Dohrn, Director of the Naples Zoological Station.

BRITISH.

Sir Richard C. Jebb, Litt.D., D.C.L., M.P., Regius Professor of Greek in the University of Cambridge.
Sir Benjamin Baker, K.C.M.G., M.Inst.C.E., F.R.S.

FELLOWS DECEASED, RESIGNED, OR CANCELLED

During Session 1901–1902.

ORDINARY FELLOWS DECEASED.

Sir Thomas J. Boyd.
Sheriff R. Vary Campbell.
John Christie.
Col. J. H. B. Hallen.
Major-General Sir John Hills.
Robert Irvine.
Alexander H. Lee.
W. Ivison Macadam.
John M. M'Candlish, W.S.
Dr R. Peel Ritchie.
J. A. Wenley.

RESIGNED.

A. B. Griffiths.
Rev. Robt. Munro.

HONORARY FELLOWS DECEASED.

Session 1901–1902.

FOREIGN.

Rudolf Virchow.

ORDINARY FELLOWS ELECTED

During Session 1902–1903.

Arranged according to the Date of their Election.

3rd November 1902.

John S. Samuel.

David Anderson Berry, M.D., F.S.A.Scot.

1st December 1902.

Henry Harvey Littlejohn, M.A., M.B., B.Sc., F.R.C.S.E.

5th January 1903.

The Rev. Dugald Butler, M.A.

Professor Lawrence Crawford, M.A., D.Sc.

Professor J. Graham Kerr, M.A.

Nicholas Henry Martin, F.L.S., F.C.S.

Laurence Pullar.

David W. Sutherland, M.D., M.R.C.P.L.

2nd February 1903.

George S. Thomson, F.C.S.

William Glen Liston, M.D.

D. C. M'Intosh, M.A.

Robert Taylor Skinner, M.A.

2nd March 1903.

O. Charnock Bradley, M.B., Ch.B.

Wm. Eagle Clarke, F.L.S.

4th May 1903.

Noel Dean Bardswell, M.D., M.R.C.P. (Ed. and Lond.).

W. S. M'Cormick, M.A., LL.D.

John Dunstan, M.R.C.V.S.

Gerald Rowley Leighton, M.D.

1st June 1903.

J. H. Maclagan Wedderburn, M.A.

Thomas Nicol Johnston, M.B., C.M.

6th July 1903.

Charles Sarolea, D.Ph., D.Litt.

FELLOWS DECEASED, RESIGNED, OR CANCELLED

During Session 1902–1903.

ORDINARY FELLOWS DECEASED.

C. M. Aikman, M.A., D.Sc.
Dr George W. Balfour.
W. H. Barlow, M.Inst.C.E.
Wm. Boyd, M.A.
John Calderwood, F.I.C.
Lt.-Col. J. Wilson Johnston, M.D.
Rev. Dr Hugh Macmillan.
James Mactear, F.C.S.
David Munn, M.A.
George A. Panton.
John J. Rogerson, LL.D.
John Scott, C.B.
James Stevenson, LL.D.

RESIGNED.

Professor Butcher.
T. E. Edwards.

HONORARY FELLOWS DECEASED.

Session 1902–1903.

FOREIGN.

Luigi Cremona.
Carl Gegenbaur.
Theodore Mommsen.
Alphonse Renard.
Rudolph Leuckart.

BRITISH.

Sir George Gabriel Stokes, Bart.

ORDINARY FELLOWS ELECTED

During Session 1903–1904.

Arranged according to the Date of their Election.

2nd November 1903.

James Johnstone Dobbie, M.A., D.Sc., F.R.S.

7th December 1903.

Ernest George Coker, M.A., D.Sc.
Alfred Charles Coles, M.D., D.Sc.
Frederick T. G. Hobday, F.R.C.V.S.

4th January 1904.

W. Leslie Mackenzie, M.A., M.D.
Charles B. Boog Watson.
William Charles Smith, K.C., M.A.
Alexander Scott Ireland, S.S.C.
Wm. Gibson Wedderspoon, M.A., LL.D.

1st February 1904.

Erskine Beveridge, LL.D.
Norman Hay Forbes, F.R.C.S.E.
Charles Duff Campbell.
Thomas W. Stewart, M.A., B.Sc.
Wm. Brown Dunlop.
James Wallace Peck, M.A.
Joseph Riley Ratcliffe, M.B., C.M.

7th March 1904.

John Cuthbertson.
James Haig Ferguson, M.D., F.R.C.P.E.
R. B. Young, M.A., B.Sc.
Ellice M. Horsburgh, M.A., B.Sc.

2nd May 1904.

William Muirhead Baxter.
Walter Colquhoun, M.A., M.D.
Walter George Burnett Dickinson, F.R.C.V.S.
R. T. A. Innes.
James Robert Milne, B.Sc.
William Elder, M.D., F.R.C.P.E.

6th June 1904.

Donald James Mackintosh, M.V.O., M.B.
William Macdonald, B.Sc., M.Sc.

3rd July 1904.

John Edwards.
James Barr, M.D., F.R.C.P. (Lond.).
J. A. Macdonald, M.A., B.Sc.
Edwin O. Sachs.
John W. Tait, B.Sc.

FELLOWS DECEASED, RESIGNED, OR CANCELLED

During Session 1903–1904.

ORDINARY FELLOWS DECEASED.

Andrew Peebles Aitken, M.A., D.Sc.
A. Hamilton Bryce, LL.D., D.C.L.
Dr Thomas B. Christie.
Dr R. T. Blair Cunynghame.
Robert Etheridge, F.R.S.
Professor J. D. Everett, M.A., F.R.S.
Maj.-Gen. J. G. Forlong.
Charles Henry Gatty, M.A., LL.D.
John Hislop, LL.D.
Thomas Isherwood, LL.D., D.C.L.
The Hon. George Waldegrave Leslie, LL.D.
Dr Charles M'Bride.
Dr John Ivor Murray (died July 24, 1902).
Dr R. Milne Murray.
The Rev. John Stevenson, LL.D.
The Very Rev. Andrew Tait, D.D., LL.D.
George Hunter Thoms.

HONORARY FELLOWS DECEASED.

Session 1903–1904.

FOREIGN.

Otto Martin Torell.
Professor Wilhelm His.

BRITISH.

The Rev. George Salmon, D.D., LL.D.
Alexander William Williamson, LL.D.

LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH.

AS REVISED 18TH JULY 1904.

LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows. Title.

II.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.* Fellows may compound for these contributions on such terms as the Council may from time to time fix. The fees of Ordinary Fellows residing in Scotland.

III.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from further payment. Payment to cease after 25 years.

IV.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt Fees of Non-Resident Ordinary Fellows.

* A modification of this rule, in certain cases, was agreed to at a Meeting of the Society held on the 3rd January 1831.

At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

Case of Fellows becoming Non-Resident.

from any further payment. In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall be exhibited publicly in the Society's Rooms for one month, after which it shall be considered by the Council. If the Candidate be approved by the Council, notice of the day fixed for the election shall be given in the circulars of at least two Ordinary Meetings of the Society.

* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

X.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states. Honorary Fellows, British and Foreign.

XI.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X. Royal Personages.

XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election. Recommendation of Honorary Fellows. Mode of Election.

XIII.

The election of Ordinary Fellows shall take place only at one Afternoon Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote. Election of Ordinary Fellows.

XIV.

The Ordinary Meetings shall be held on the first and third Mondays of each month from November to March, and from May to July, inclusive; with the exception that when there are five Mondays in January, the Meetings for that month shall be held on its second and fourth Mondays. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make. Ordinary Meetings.

* We hereby recommend ________________ ________________
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

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To the President and Council of the Royal Society of Edinburgh.

XV.

The Transactions.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote.

XVI.

How Published.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council.

That there shall be formed a Council, consisting—First, of such gentlemen as may have filled the office of President; and Secondly, of the following to be annually elected, viz.:—a President, Six Vice-Presidents (two at least of whom shall be resident), Twelve Ordinary Fellows as Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers.

An Extraordinary Meeting for the election of Office-Bearers shall be held annually on the fourth Monday of October, or on such other lawful day in October as the Council may fix, and each Session of the Society shall be held to begin at the date of the said Extraordinary Meeting.

XX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in October, he shall present the accounts for the preceding year, duly audited.

At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor.

At the Extraordinary Meeting in October, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXV.

Curator of Museum and Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXVI.

Use of Museum and Library.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations as the Council from time to time shall appoint.

XXVII.

Register Book.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

XXVIII.

Power of Expulsion.

If, in the opinion of the Council of the Society, the conduct of any Fellow is unbecoming the position of a Member of a learned Society, or is injurious to the character and interests of this Society, the Council may request such Fellow to resign ; and, if he fail to do so within one month of such request being addressed to him, the Council shall call a General Meeting of the Fellows of the Society to consider the matter ; and, if a majority of the Fellows present at such Meeting agree to the expulsion of such Member, he shall be then and there expelled by the declaration of the Chairman of the said Meeting to that effect ; and he shall thereafter cease to be a Fellow of the Society, and his name shall be erased from the Roll of Fellows, and he shall forfeit all right or claim in or to the property of the Society.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The Keith Prize, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1905–1906 for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1903–04 and 1904–05." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1906–1907, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1906.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented to the Society during the Sessions 1904–05, 1905–06, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1907–1908.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 8th July 1907,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. Gunning, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES, FROM 1827 TO 1904.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his Researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–43.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1857–59.—Not awarded.

17TH BIENNIAL PERIOD, 1859–61.—JOHN ALLAN BROUN, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18TH BIENNIAL PERIOD, 1861–63.—Professor WILLIAM THOMSON, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19TH BIENNIAL PERIOD, 1863–65.—Principal FORBES, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20TH BIENNIAL PERIOD, 1865–67.—Professor C. PIAZZI SMYTH, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21ST BIENNIAL PERIOD, 1867–69.—Professor P. G. TAIT, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22ND BIENNIAL PERIOD, 1869–71.—Professor CLERK MAXWELL, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871–73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873–75.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875–77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877–79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II. having appeared in the volume for 1877–78.

27TH BIENNIAL PERIOD, 1879–81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881–83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883–85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885–87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, &c.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887–89.—Professor E. A. LETTS, for his Papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889–91.—R. T. OMOND, Esq., for his Contributions to Meteorological Science, many of which are contained in Vol. XXXIV. of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891–93.—Professor THOMAS R. FRASER, F.R.S., for his Papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in Vols. XXXV., XXXVI., and XXXVII. of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893–95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895–97.—Dr THOMAS MUIR, for his continued Communications on Determinants and Allied Questions.

36TH BIENNIAL PERIOD, 1897–99.—Dr JAMES BURGESS, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$, with extended Tables of Values," printed in Vol. XXXIX. of the Transactions of the Society.

37TH BIENNIAL PERIOD, 1899–1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38TH BIENNIAL PERIOD, 1901–03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., &c., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I., II., likewise published in the Transactions of the Society.

II. MAKDOUGALL-BRISBANE PRIZE.

1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.

2ND BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1864–66.—Not awarded.

5TH BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1868–70.—Not awarded.

7TH BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10TH BIENNIAL PERIOD, 1876–78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1878–80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1880–82.— Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1882–84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculation of Logarithms both of Numbers and Trigonometrical Ratios, —the former communication being published in the Proceedings of the Society.

14TH BIENNIAL PERIOD, 1884–86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15TH BIENNIAL PERIOD, 1886–88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1888–90.—Dr LUDWIG BECKER, for his Paper on "The Solar Spectrum at Medium and Low Altitudes," printed in Vol. XXXVI. Part I. of the Society's Transactions.

17TH BIENNIAL PERIOD, 1890–92.—HUGH ROBERT MILL, Esq., D.Sc., for his Papers on "The Physical Conditions of the Clyde Sea Area," Part I. being already published in Vol. XXXVI. of the Society's Transactions.

18TH BIENNIAL PERIOD, 1892–94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, Vol. XX., pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19TH BIENNIAL PERIOD, 1894–96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound; many of which have appeared in the Society's publications.

20TH BIENNIAL PERIOD, 1896–98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.

21ST BIENNIAL PERIOD, 1898–1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in Vol. XXXIX. of the Transactions of the Society.

22ND BIENNIAL PERIOD, 1900–02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in Vol. XL. of the Transactions of the Society.

23RD BIENNIAL PERIOD, 1902–04.—Mr JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in Vol. XLI. of the Transactions of the Society.

III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856–59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2ND TRIENNIAL PERIOD, 1859–62.—ROBERT KAYE GREVILLE, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862–65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and Memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geographical Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865–68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868–71.—Professor WILLIAM TURNER, for his papers "on the great Finner Whale ; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD, 1871–74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874–77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877–80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880–83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883–86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886–89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889–92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892–95.—ROBERT IRVINE, Esq., for his papers on the action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895–98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898–1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in Vol. XXXIX. of the Transactions of the Society.

16TH TRIENNIAL PERIOD, 1901–04.—Professor J. GRAHAM KERR, M.A., for his researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884–87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887–90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890–93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's Publications.

4TH TRIENNIAL PERIOD, 1893–96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1ST QUADRENNIAL PERIOD, 1896–1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.

2ND QUADRENNIAL PERIOD, 1900–04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., &c., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

PROCEEDINGS

OF THE

STATUTORY GENERAL MEETINGS,

27TH NOVEMBER 1899,

22ND OCTOBER 1900,

28TH OCTOBER 1901,

27TH OCTOBER 1902,

26TH OCTOBER 1903,

AND OF A

SPECIAL GENERAL MEETING,

26TH NOVEMBER 1903.

STATUTORY MEETING.

HUNDRED AND SEVENTEENTH SESSION.

Monday, 27th November 1899.

At the Annual Statutory Meeting,

Sir ARTHUR MITCHELL, K.C.B., in the Chair,

The Minutes of last Annual Statutory Meeting of 28th November 1898 were read, approved, and signed.

On the motion of Dr BUCHAN, Mr WENLEY and Dr MUNRO were appointed Scrutineers, and the Ballot for the New Council commenced.

The TREASURER submitted his Accounts for the year. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following Council had been duly elected :—

The Right Hon. Lord KELVIN, G.C.V.O., LL.D., D.C.L., F.R.S., President.

Professor JOHN G. M'KENDRICK, M.D., LL.D., F.R.S., Professor GEORGE CHRYSTAL, M.A., LL.D., Sir ARTHUR MITCHELL, K.C.B., LL.D., Sir WILLIAM TURNER, M.B., F.R.S., Professor COPELAND, Astronomer-Royal for Scotland, The Rev. Professor DUNS,	Vice-Presidents.

Professor P. G. TAIT, M.A., D.Sc., General Secretary.

Professor CRUM BROWN, F.R.S., Sir JOHN MURRAY, K.C.B., LL.D., F.R.S.,	Secretaries to Ordinary Meetings.

PHILIP R. D. MACLAGAN, F.F.A., Treasurer.

ALEXANDER BUCHAN, M.A., LL.D., F.R.S., Curator of Library and Museum.

COUNCILLORS.

Sir J. Batty Tuke, M.D., D.Sc.
Mr A. Beatson Bell, Advocate.
Professor Shield Nicholson, M.A., D.Sc.
Dr John Gibson.
The Hon. Lord M'Laren, LL.D.
C. G. Knott, D.Sc.
Dr Alex. Bruce, M.A., F.R.C.P.E.
Mr James A. Wenley.
The Rev. Professor Flint, D.D.
Dr James Burgess, C.I.E.
Dr R. M. Ferguson.
Mr Robert Irvine.

On the motion of Dr Buchan, thanks were voted to the Treasurer.

On the motion of Dr Crum Brown, thanks were voted to the Scrutineers.

On the motion of Dr Burgess, thanks were voted to the Auditors, who were reappointed.

On the motion of Professor Tait, thanks were voted to the Chairman.

John M'Laren, *C.*

STATUTORY MEETING.

HUNDRED AND EIGHTEENTH SESSION.

Monday, 22nd October 1900.

At the Annual Statutory Meeting,

The Hon. Lord M'Laren, LL.D., Vice-President, in the Chair,

The Minutes of last Annual Statutory Meeting of 27th November 1899 were read, approved, and signed.

On the motion of Professor Crum Brown, Professor M'Intosh and Colonel Bailey were appointed Scrutineers, and the Ballot for the New Council commenced.

The Treasurer submitted his Accounts for the year. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following New Council had been duly elected:—

The Right Hon. Lord Kelvin, G.C.V.O., LL.D., F.R.S., President.

Professor George Chrystal, M.A., LL.D.,
Sir Arthur Mitchell, K.C.B., LL.D.,
Sir William Turner, M.B., F.R.S.,
Professor Copeland, Astronomer-Royal for Scotland,
The Rev. Professor Duns, D.D.,
Professor James Geikie, LL.D.,
Vice-Presidents.

Professor P. G. Tait, M.A., D.Sc., General Secretary.

Professor Crum Brown, F.R.S.,
Dr R. H. Traquair, F.R.S.,
Secretaries to Ordinary Meetings.

Philip R. D. Maclagan, F.F.A., Treasurer.

Alexander Buchan, M.A., LL.D., F.R.S., Curator of Library and Museum.

COUNCILLORS.

The Hon. Lord M'Laren, LL.D.	R. M. Ferguson, Ph.D., LL.D.
C. G. Knott, D.Sc.	Mr Robert Irvine.
Dr Alex. Bruce, M.A., F.R.C.P.E.	Professor J. G. M'Kendrick, M.D., LL.D., F.R.S.
Mr James A. Wenley.	Professor Schäfer, F.R.S.
The Rev. Professor Flint, D.D.	Dr Robert Munro, M.A.
Dr James Burgess, C.I.E.	J. S. Mackay, LL.D.

On the motion of Professor Tait, thanks were voted to the Treasurer.

On the motion of Professor Crum Brown, thanks were voted to the Scrutineers.

On the motion of Lord M'Laren, thanks were awarded to the Auditors, Messrs Lindsay, Haldane, & Jamieson, who were reappointed.

On the motion of Professor Duns, thanks were voted to the Chairman.

John M'Laren,
Chairman.

STATUTORY MEETING.

HUNDRED AND NINETEENTH SESSION.

Monday, 28th October 1901.

At the Annual Statutory Meeting,

The Hon. Lord M'Laren, LL.D., Vice-President, in the Chair,

The Minutes of last Annual Statutory Meeting of 22nd October 1900 were read, approved, and signed.

On the motion of Dr Buchan and Professor Crum Brown, the Rev. Dr Macmillan and Mr John Scott, C.B., were appointed Scrutineers, and the Ballot for the New Council commenced.

The Treasurer submitted his Accounts for the year. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following New Council had been duly elected:—

The Right Hon. Lord Kelvin, G.C.V.O., LL.D., D.C.L., F.R.S., President.

Sir Arthur Mitchell, K.C.B., LL.D., Sir William Turner, M.B., F.R.S., Professor Ralph Copeland, Ph.D., Astronomer-Royal for Scotland, The Rev. Professor Duns, D.D., Professor James Geikie, LL.D., The Hon. Lord M'Laren, LL.D.,	Vice-Presidents.

Professor George Chrystal, LL.D., General Secretary.

Professor A. Crum Brown, F.R.S., LL.D., Dr R. H. Traquair, F.R.S.,	Secretaries to Ordinary Meetings.

Philip R. D. Maclagan, F.F.A., Treasurer.

Alexander Buchan, M.A., LL.D., F.R.S., Curator of Library and Museum.

COUNCILLORS.

The Rev. Professor FLINT, D.D.	Dr ROBERT MUNRO, M.A.
Dr JAMES BURGESS, C.I.E.	Dr J. S. MACKAY.
Dr R. M. FERGUSON.	Sir JOHN MURRAY, K.C.B., LL.D., F.R.S.
Mr ROBERT IRVINE.	Mr R. TRAILL OMOND.
Prof. J. G. M'KENDRICK, M.D., LL.D., F.R.S.	Mr F. GRANT OGILVIE, M.A., B.Sc.
Professor SCHÄFER, F.R.S.	Dr GEO. A. GIBSON, F.R.C.P.E.

On the motion of Professor CRUM BROWN, thanks were voted to the Treasurer.

On the motion of Dr MUNRO, thanks were voted to the Scrutineers.

On the motion of Lord M'LAREN, thanks were voted to the Auditors, who were reappointed.

On the motion of Sir ARTHUR MITCHELL, thanks were voted to the Chairman.

JOHN M'LAREN, *V.P.*,
Chairman.

STATUTORY MEETING.

HUNDRED AND TWENTIETH SESSION.

Monday, 27th October 1902.

At the Annual Statutory Meeting,

The Hon. Lord M'Laren, Vice-President, in the Chair,

The Minutes of last Annual Statutory Meeting of 28th October 1901 were read, approved, and signed.

On the motion of Dr Crum Brown, Dr James Burgess and the Hon. John Abercromby were appointed Scrutineers, and the Ballot for the New Council commenced.

The Treasurer submitted his Accounts for the year. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following New Council had been duly elected:—

The Right Hon. Lord Kelvin, G.C.V.O., LL.D., D.C.L., F.R.S., President.

Sir William Turner, K.C.B., M.B., F R.S., Professor Copeland, Astronomer-Royal for Scotland, The Rev. Professor Duns, D.D., Professor James Geikie, LL.D., F.R.S., The Hon. Lord M'Laren, LL.D., The Rev. Professor Flint, D.D.,	Vice-Presidents.

Professor George Chrystal, LL.D., General Secretary.

Professor Crum Brown, F.R.S., Ramsay H. Traquair, M.D., LL.D., F.R.S.,	Secretaries to Ordinary Meetings.

Philip R. D. Maclagan, Esq., F.F.A., Treasurer.

Alex. Buchan, Esq., M.A., LL.D., F.R.S., Curator of Library and Museum.

COUNCILLORS.

Professor Schäfer, F.R.S.
Dr Robert Munro, M.A.
J. S. Mackay, LL.D.
Sir John Murray, K.C.B., LL.D, F.R.S.
Mr R. Traill Omond.
Mr F. Grant Ogilvie, M.A., B.Sc.
Dr Geo. A. Gibson, F.R.C.P.E.
Sir Arthur Mitchell, K.C.B., LL.D.
Professor J. G. MacGregor, LL.D., F.R.S.
John Horne, LL.D., F.R.S.
C. G. Knott, D.Sc.
Arthur T. Masterman, M.A., D.Sc.

R. M. Ferguson, Ph.D., LL.D., Representative on George Heriot's Trust.

On the motion of Dr Ferguson, thanks were voted to the Treasurer.

On the motion of Dr Munro, thanks were voted to the General and other Secretaries.

On the motion of Mr Grant Ogilvie, thanks were voted to the Auditors, who were reappointed.

On the motion of Dr Burgess, thanks were voted to the Chairman.

J. Duns, *V.P.*,
Chairman.

STATUTORY MEETING.

HUNDRED AND TWENTY-FIRST SESSION.

Monday, 26th October 1903.

At the Annual Statutory Meeting,

The Rev. Dr DUNS in the Chair,

The Minutes of last Annual Statutory Meeting of 27th October 1902 were read, approved, and signed.

On the motion of Dr BUCHAN, Dr R. M. FERGUSON and Mr WM. SANDERSON were appointed Scrutineers, and the Ballot for the New Council commenced.

The TREASURER submitted his Accounts for the year. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following New Council had been duly elected:—

The Right Hon. Lord KELVIN, G.C.V.O., LL.D., D.C.L., F.R.S., President.

The Rev. Professor DUNS, D.D., Professor JAMES GEIKIE, LL.D., F.R.S., The Hon. Lord M'LAREN, LL.D., The Rev. Professor FLINT, D.D., ROBERT MUNRO, M.A., M.D., LL.D., Sir JOHN MURRAY, K.C.B., LL.D., F.R.S.,	Vice-Presidents.

Professor GEORGE CHRYSTAL, LL.D., General Secretary.

Professor CRUM BROWN, F.R.S., RAMSAY H. TRAQUAIR, M.D., LL.D., F.R.S.,	Secretaries to Ordinary Meetings.

PHILIP R. D. MACLAGAN, F.F.A., Treasurer.

ALEX. BUCHAN, M.A., LL.D., F.R.S., Curator of Library and Museum.

COUNCILLORS.

Mr R. Traill Omond.
Dr Geo. A. Gibson, F.R.C.P.E.
Sir Arthur Mitchell, K.C.B., LL.D.
Professor J. G. MacGregor, LL.D., F.R.S.
John Horne, LL.D., F.R.S.
Cargill G. Knott, D.Sc.
Arthur T. Masterman, M.A., D.Sc.
Professor Ralph Stockman, M.D., F.R.C.P.E.
Professor James Walker, D.Sc., Ph.D., F.R.S.
Professor Andrew Gray, M.A., LL.D., F.R.S.
Robert Kidston, F.R.S., F.G.S.
Professor D. J. Cunningham, M.D., LL.D., F.R.S.

On the motion of Professor Crum Brown, thanks were voted to the Treasurer.

On the motion of Principal W. Owen Williams, thanks were voted to the Scrutineers.

On the motion of Mr A. Beatson Bell, thanks were voted to the Auditors, who were reappointed.

On the motion of Dr Buchan, thanks were voted to the Chairman.

John M'Laren, *V.P.*,
Chairman.

SPECIAL GENERAL MEETING.

Thursday, 26th November 1903.

The Right Hon. Lord KELVIN, President, in the Chair.

A Special Meeting of the Society was held on Thursday, 26th November 1903, in response to a requisition addressed to the Council in the following terms:—

" TO THE SECRETARY OF THE ROYAL SOCIETY.

" We beg respectfully to ask you to call a Special Meeting of the Fellows of the Royal " Society, at 4 p.m. on Thursday, 26th November 1903, in accordance with Law XX. of the " Society.

" The special object of the meeting is to consider the report by the Departmental " Committee to inquire into the administration of the Board of Manufactures, in so far " as that report affects the position of the Royal Society and the other Scottish Scientific " Societies. Those Fellows of the Royal Society who have a special interest in other " Scottish Scientific Societies are particularly requested to attend this Meeting.

" JOHN MURRAY.
" ALEXANDER BUCHAN.
" R. T. OMOND.
" T. N. JOHNSTON.
" ROBERT MUNRO.
" JOHN HORNE.
" ROBERT FLINT."

Sir JOHN MURRAY, called upon by the PRESIDENT, gave an explanation of the objects for which the meeting was summoned; thereafter moving the following resolution, seconded by Dr JOHN HORNE:—

" This meeting of the Fellows of the Royal Society resolves to instruct the Council to " enter into formal communication with the other scientific societies having their head- " quarters in Edinburgh, with the view of concerting measures for obtaining the use of " the Royal Institution building wholly and exclusively for Scottish scientific societies."

Professor COSSAR EWART, Professor CHIENE, Dr MUNRO, Dr BUCHAN, Professor HUDSON BEARE, Sir JAMES RUSSELL, and Professor CHRYSTAL also spoke in support of the motion.

The resolution having been put to the meeting by the PRESIDENT, it was unanimously adopted.

On the motion of the PRESIDENT, thanks were voted to Sir JOHN MURRAY and the other gentlemen who had joined in summoning the meeting; and to Professor CHRYSTAL for so ably placing the claims of the Society before the Departmental Committee.

On the motion of Professor CHRYSTAL, a vote of thanks to the PRESIDENT was cordially adopted.

JOHN M'LAREN, *V.P.*,
Chairman.

The following Public Institutions and Individuals are entitled to receive Copies of the Transactions and Proceedings of the Royal Society of Edinburgh:—

London, British Museum.
... ... (Natural History Department), Cromwell Road.
... Royal Society, Burlington House.
... Anthropological Institute of Great Britain and Ireland, 3 Hanover Square.
... British Association for the Advancement of Science, Burlington House.
... Society of Antiquaries, Burlington House.
... Royal Astronomical Society, Burlington House.
... Royal Asiatic Society, 22 Albemarle St.
... Society of Arts, John Street, Adelphi.
... Athenæum Club.
... Chemical Society, Burlington House.
... Institution of Civil Engineers, 25 Great George Street.
... Royal Geographical Society, Burlington Gardens.
... Geological Society, Burlington House.
... Royal Horticultural Society, South Kensington.
... Hydrographic Office, Admiralty.
... Imperial Institute.
... Royal Institution, Albemarle Street, W.
... Linnean Society, Burlington House.
... Royal Society of Literature, 20 Hanover Square.
... Royal Medical and Chirurgical Society, 20 Hanover Square.
... Royal Microscopical Society, 20 Hanover Square.
... Museum of Economic Geology, Jermyn Street.
... Royal Observatory, Greenwich.
... Pathological Society, 20 Hanover Sq.
... Royal Statistical Society, 9 Adelphi Terrace, Strand, London.
... Royal College of Surgeons of England, 4 Lincoln's Inn Fields.
... United Service Institution, Whitehall Yard.
... University College, Gower Street.
... Zoological Society, 3 Hanover Square.
... The Editor of *Nature*, 29 Bedford Street, Covent Garden.

London, The Editor of the *Electrician*, Salisbury Court, Fleet Street.
... The Editor, *Science Abstracts*.
Cambridge Philosophical Society.
... University Library.
Leeds Philosophical and Literary Society.
Liverpool, University College Library.
Manchester Literary and Philosophical Society.
Oxford, Bodleian Library.
Plymouth, Marine Biological Laboratory, Citadel Hill.
Richmond (Surrey), Kew Observatory.
Sheffield, University College.
Teddington, Middlesex, National Physical Laboratory.
Yorkshire Philosophical Society.

SCOTLAND.

Edinburgh, Advocates' Library.
... University Library.
... Royal College of Physicians.
... Highland and Agricultural Society, 3 George IV. Bridge.
... Royal Medical Society, 7 Melbourne Place.
... Royal Observatory.
... Royal Physical Society, India Buildings.
... Royal Scottish Society of Arts, 117 George Street.
... Royal Botanic Garden, Inverleith Row.
Aberdeen, University Library.
Dundee, University College Library.
Glasgow, University Library.
... Philosophical Society, 207 Bath Street.
St Andrews, University Library.

IRELAND.

Dublin, Royal Dublin Society.
... Royal Irish Academy.
... Library of Trinity College.
... National Library of Ireland.

COLONIES, DEPENDENCIES, ETC.

Bombay, Royal Asiatic Society.
... Elphinstone College.
Calcutta, Asiatic Society of Bengal.

Calcutta, Geological Survey of India.
... Meteorological Department of the Government of India.
Madras, Literary Society.
Canada, Geological and Natural History Survey.
... Queen's University, Kingston.
... Royal Society of Canada, Ottawa.
... Quebec, Literary and Philosophical Society.
... ... The Canadian Institute.
... ... University Library.
Cape of Good Hope, The Observatory.
Mauritius, Royal Alfred Observatory.
Melbourne, University Library.
Sydney, University Library.
... Geological Survey.
... Linnæan Society of New South Wales.
... Royal Society of New South Wales.
Transvaal Meteorological Department, Johannesburg.
Wellington, New Zealand Institute.

CONTINENT OF EUROPE.

Amsterdam, Koninklijke Akademie van Wetenschappen.
... Koninklijk Zoologisch Genootschap.
Athens, University Library.
... Observatoire National.
Basle, Die Schweizerische Naturforschende Gesellschaft.
Bergen, Museum.
Berlin, Königliche Akademie der Wissenschaften.
... Physikalische Gesellschaft.
... Deutsche Geologische Gesellschaft.
Bern, Allgemeine Schweizerische Gesellschaft für die gesammten Naturwissenschaften.
Bologna, Accademia delle Scienze dell' Istituto.
Bordeaux, Société des Sciences Physiques et Naturelles.
Bremen, Naturwissenschaftlicher Verein.
Brussels, Académie Royale des Sciences, des Lettres et des Beaux-arts.
... Musée Royal d'Histoire Naturelle de Belgique.
... Musée du Congo.
.. L'Observatoire Royal de Belgique, Uccle.
... La Société Scientifique.
Bucharest, Academia Romana.
Buda-Pesth, Magyar Tudományos Akadémia—Die Ungarische Akademie der Wissenschaften.
Buda-Pesth, Königliche Ungarische Naturwissenschaftliche Gesellschaft.
Catania, Accademia Gioenia di Scienze Naturali.
Charlottenburg, Physikalisch-Technische Reichsanstalt.
Christiania, University Library.
... Meteorological Institute.
Coimbra, University Library.
Copenhagen, Royal Academy of Sciences.
Cracow, Académie des Sciences.
Danzig, Naturforschende Gesellschaft.
Dorpat, University Library.
Ekatherinebourg, La Société Ouralienne d'Amateurs des Sciences Naturelles.
Erlangen, University Library.
Frankfurt-am-Main, Senckenbergische Naturforschende Gesellschaft.
Gand (Ghent), University Library.
Geneva, Société de Physique et d'Histoire Naturelle.
Genoa, Museo Civico di Storia Naturale.
Giessen, University Library.
Göttingen, Königliche Gesellschaft der Wissenschaften.
Graz, Naturwissenschaftlicher Verein für Steiermark.
Groningen, Holland, University Library.
Haarlem, Société Hollandaise des Sciences Exactes et Naturelles.
... Musée Teyler.
Halle, Kaiserliche Leopoldino - Carolinische Deutsche Akademie der Naturforscher.
... Naturforschende Gesellschaft.
Hamburg, Naturwissenschaftlicher Verein.
... Naturhistorisches Museum.
Hanover, Naturhistorische Gesellschaft.
Helder, Nederlandsche Dierkundige Vereeniging.
Helsingfors, Sällskapet pro Fauna et Flora Fennica.
... Societas Scientiarum Fennica (Société des Sciences de Finlande).
Jena, Medicinisch-Naturwissenschaftliche Gesellschaft
Kasan, University Library.
Kiel, University Library.
... Kommission zur Wissenschaftlichen Untersuchung der Deutschen Meere.
Kiev, University of St Vladimir.
Königsberg, University Library.
Leyden, The University Library.
Leipzig, Königliche Sächsische Akademie.

Leipzig, Editor of the *Annalen der Physik.*
Lille, Société des Sciences.
... Société Géologique du Nord.
Lisbon, Academia Real das Sciencias de Lisboa.
... Sociedade de Geographia, 5 Rua Capello.
Louvain, University Library.
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Lyons, Académie des Sciences, Belles Lettres et Arts.
... Société d'Agriculture.
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Marseilles, Faculté des Sciences.
Milan, Reale Istituto Lombardo di Scienze, Lettere, ed Arti.
Modena, Regia Accademia di Scienze, Lettere, ed Arti.
Montpellier, Académie des Sciences et Lettres.
Moscow, Société Impériale des Naturalistes de Moscou.
... Société Impériale des Amis d'Histoire Naturelle, d'Anthropologie et d'Ethnographie.
... Imperial University.
... Musée Polytechnique.
... L'Observatoire Impérial.
Munich, Königlich-Bayerische Akademie der Wissenschaften (2 copies).
Nantes, Société des Sciences de l'Ouest de la France.
Naples, Zoological Station, Dr Anton Dohrn.
... Società Reale di Napoli—Accademia delle Scienze Fisiche e Matematiche.
... R. Istituto d'Incorragiamento di Napoli.
Neufchatel, Société des Sciences Naturelles.
Nice, L'Observatoire.
Padua, R. Accademia di Scienze, Lettere ed Arti.
Palermo, Società di Scienze Naturali ed Economiche.
Paris, Académie des Sciences de l'Institut.
... Académie des Inscriptions et Belles Lettres de l'Institut.
... Association Française pour l'Avancement des Sciences.
... Bureau International des Poids et Mesures, Sèvres.
... Société d'Agriculture.
... Société Nationale des Antiquaires de France.
Paris, Société de Biologie.
... Société de Géographie.
... Société Géologique de France.
... Société d'Encouragement pour l'Industrie Nationale.
... Bureau des Longitudes.
... Dépôt de la Marine.
... Société Mathématique.
... École des Mines.
... Ministère de l'Instruction Publique.
... Musée Guimet, 30 Avenue du Trocadéro.
... Muséum d'Histoire Naturelle, Jardin des Plantes.
... L'Observatoire.
... École Normale Supérieure, Rue d'Ulm.
... Société Française de Physique.
... École Polytechnique.
... Société Zoologique de France.
Prague, Königliche Sternwarte.
... Königlich-Böhmische Gesellschaft der Wissenschaften.
... Ceska Akademie Cisare Frantiska Josefa pro Vedy, Slovesnost a Umeni.
Rennes, Société Scientifique et Médicale de l'Ouest.
Rome, R. Accademia dei Lincei.
... Accademia Ponteficia dei Lincei.
... Società Italiana delle Scienze (detta dei XL.), S. Pietro in Vincoli.
... Società degli Spettroscopisti Italiani.
... Comitato Geologico, 1 Via Santa Susanna.
Rotterdam, Bataafsch Genootschap der Proefondervindelijke Wijsbegeerte.
St Petersburg, Académie Impériale des Sciences.
... Comité Géologique.
... Imperial University.
... L'Institut Impérial de Médecine Expérimentale.
... L'Observatoire Impérial de Pulkowa.
... Physikalisches Central-Observatorium.
... Physico-Chemical Society of the University of St Petersburg.
... Russian Ministry of Marine.
Stockholm, Kongliga Svenska Vetenskaps-Academien.
Strasbourg, University Library.
Stuttgart, Verein für Vaterländische Naturkunde zu Würtemberg.

Throndhjem, Kongelige Norske Videnskabers Selskab.
Toulouse, Faculté des Sciences.
... L'Observatoire.
Tübingen, University Library.
Turin, Reale Accademia delle Scienze.
Upsala, Kongliga Vetenskaps-Societeten.
... University Library.
Venice, Reale Istituto Veneto di Scienze, Lettere ed Arti.
Vienna, Kaiserliche Akademie der Wissenschaften.
... Oesterreichische Gesellschaft für Meteorologie, Hohe Warte, Wien.
... Geologische Reichsanstalt.
... Zoologisch-Botanische Gesellschaft.
Zurich, University Library.
... Commission Géologique Suisse.
... Naturforschende Gesellschaft.

ASIA.

Java, Bataviaasch Genootschap van Kunsten en Wetenschappen.
... The Observatory.
Japan, The Imperial University of Tokio (Teikoku-Daigaku).

UNITED STATES OF AMERICA.

Albany, New York State Library.
American Association for the Advancement of Science.
Baltimore, Johns Hopkins University.
... Maryland Geological Survey.
Boston, The Bowditch Library.
... American Academy of Arts and Sciences, Beacon Street, Boston.
... Society of Natural History.
California, Academy of Sciences, San Francisco.
Cambridge, Mass., Harvard University.
... Harvard College Observatory.
Chicago Observatory.
Chicago, Field Columbian Museum.
Clinton, Litchfield Observatory, Hamilton College.
Denison, University and Scientific Association.
Iowa Academy of Sciences.
Jefferson City (Missouri), Bureau of Geology and Mines.
Kansas, University of Kansas, Lawrence.
New York, American Mathematical Society.
Philadelphia, American Philosophical Society.
... Academy of Natural Sciences, Logan Square.
... University of Pennsylvania.
... Geological Survey of Pennsylvania.
Cleveland (Ohio), The Geological Society of America.
Salem, The Essex Institute.
St Louis, Academy of Sciences.
Washington, United States National Academy of Sciences.
... Bureau of Ethnology.
... United States Coast Survey.
... United States Fishery Commission.
... United States Naval Observatory.
... United States Geological Survey.
... United States Department of Agriculture, Weather Bureau.
... The Smithsonian Institution.
... Surgeon-General's Office, United States Army.
Wisconsin, University (Washburn Observatory), Madison.
Yale College, Newhaven, Connecticut.
Yerkes Observatory (University of Chicago), Williams Bay, Wisconsin.

MEXICO.

Mexico, Observatorio Meteorologico-Magnetico Central.
... Sociedad Cientifica "Antonio Alzate."

SOUTH AMERICA.

Buenos Ayres, Public Museum.
Cordoba, Argentine Republic, Academia Nacional de Ciencias.
... The Observatory.
La Plata, Argentine Republic, Museo de La Plata.
Rio de Janeiro, The Astronomical Observatory.
Santiago, Société Scientifique du Chili.
Pará, Brazil, Museu Paraense de Historia Natural e Ethnographia.

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... Scottish Fishery Board, 101 George Street.
... Royal Scottish Geographical Society.
... Mathematical Society.
... Scottish Meteorological Society, 122 George Street.
... Pharmaceutical Society, 36 York Pl.
... Royal College of Physicians Laboratory, 8 Lauriston Lane.
Glasgow, Geological Society, 207 Bath Street.
... University Observatory.
... Natural History Society.
... Marine Biological Association of the West of Scotland.
Berwickshire Naturalists' Club, Old Cambus, Cockburnspath.

ENGLAND.

London, Geologists' Association, University College.
... Mathematical Society, 22 Albemarle Street, London, W.
... Institution of Mechanical Engineers, Storey's Gate, St James' Park, Westminster.
... Meteorological Office, 116 Victoria Street.
... Royal Meteorological Society, Princes Mansions, 70 Victoria Street, Westminster.
... *Nautical Almanac* Office, 3 Verulam Buildings, Gray's Inn.
... Pharmaceutical Society, 17 Bloomsbury Square, London.
... The Editor of the *Electrical Engineer*, 139–40 Salisbury Court, Fleet Street.
Birmingham Philosophical Society.
Cardiff, University College of South Wales.
Cornwall, Geological Society.
... Royal Institution of Cornwall, Truro.
Epping Forest and County of Essex Naturalists' Field Club.
Liverpool, Literary and Philosophical Society.
... Biological Society, University College.
... Geological Society.
Manchester, Geological Society, 36 George Street.
... Microscopical Society.
Newcastle, Philosophical Society.
... North of England Institute of Mining and Mechanical Engineers.
Norfolk and Norwich Naturalists' Society, The Museum, Norwich.
Oxford, Ashmolean Society.
... Radcliffe Observatory.
Scarborough, Philosophical Society.
Whitby, Philosophical Society.
Yorkshire, Geological and Polytechnic Society, Hopton, Mirfield.

IRELAND.

Dublin, Dunsink Observatory.
Belfast, Natural History and Philosophical Society.

COLONIES, DEPENDENCIES, ETC.

Adelaide, South Australia, University Library.
... Royal Society.
... Royal Geographical Society (*South Australian Branch*).
Canada, Natural History Society of Montreal.
... Canadian Society of Civil Engineers, 112 Mansfield Street, Montreal.
... Astronomical and Physical Society of Toronto.
Cape Town, South African Philosophical Society.
... Geological Commission.
Geelong, Victoria, Gordon Technical College.
Halifax, Nova Scotian Institute of Science.
Melbourne, Royal Society of Victoria.
Natal, Geological Survey of Natal, Pietermaritzburg.
Perth, Western Australia, Geological Survey Office.
Sydney, The Australian Museum.

Hong Kong, China Branch of the Asiatic Society.
... The Observatory.
Jamaica, The Institute of Jamaica, Kingston.
Madras, Superintendent of Government Farms of Madras Presidency.
Ottawa, Literary Scientific Society.
Queensland, Royal Society, Brisbane.
... Queensland Branch of Geographical Society.
... Government Meteorological Office.
... Water Supply Department.
Tasmania, Royal Society.
Wellington, N.Z., Polynesian Society.

CONTINENT OF EUROPE.

Amsterdam, Genootschap der Mathematische Wetenschappen.
Berlin, Deutsche Meteorologische Gesellschaft.
... Königl. Preussisches Meteorologisches Institut.
... Gesellschaft Naturforschender Freunde.
... K. Technische Hochschule.
Bern, Naturforschende Gesellschaft.
Bonn, Naturhistorischer Verein der Preussischen Rheinlande und Westfalens.
Bordeaux, Société de la Géographie Commerciale.
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Chemnitz, Naturwissenschaftliche Gesellschaft.
Cherbourg, Société Nationale des Sciences Naturelles.
Constantinople, Société de Médecine.
Copenhagen, Naturhistoriske Forening.
... Danske Biologiske Station.
Delft, École Polytechnique.
Dijon, Académie des Sciences.
Erlangen, Physico-Medical Society.
Frankfurt a. Oder, Naturwissenschaftlicher Verein.
Giessen, Oberhessische Gesellschaft für Natur- und Heilkunde.
Gothenburg, Kungl. Vetenskaps-och Vitterhetssamhället.
Gratz, Chemisches Institut der K. K. Universität.
Halle, Verein für Erdkunde.
... Naturwissenschaftlicher Verein für Sächsen und Thüringen.
Hamburg, Verein für Naturwissenschaftliche Unterhaltung, 29 Steindamm, St Georg.
Helsingfors, Société de Géographie Finlandaise.
Iceland, Islenzka Fornleifafelag, Reikjavik, Iceland.
Kasan, Société Physico-Mathématique.
Kiel, Naturwissenschaftlicher Verein für Schleswig-Holstein.
Lausanne, Société Vaudoise des Sciences Naturelles.
Leipzig, Naturforschende Gesellschaft.
... Deutsche Mathematiker Vereinigung.
Lemberg, Société Scientifique de Chevtchenko.
Liége, Institut de Physiologie, Université de Liége.
Lille, University Library.
Lübeck, Geographische Gesellschaft und Naturhistorisches Museum.
Luxembourg, L'Institut Royal Grand-Ducal.
Lyons, Société Botanique.
... Société Linnéenne, Place Sathonay.
Marseilles, Société Scientifique Industrielle.
Milan, Società Crittogamologica Italiana.
Modena, Società dei Naturalisti.
Moscow, Observatoire Magnétique et Météorologique de l'Université Impériale.
Neuchatel, Société Neuchâteloise de Géographie.
Nijmegen, Nederlandsche Botanische Vereeniging.
Oberpfalz und Regensburg, Historischer Verein.
Odessa, Société des Naturalistes de la Nouvelle Russie.
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Osnabrück, Naturwissenschaftlicher Verein.
Paris, Société d'Anthropologie.
... Société Académique Indo-Chinoise de France.
... Société Philomathique.
... École Libre des Sciences Politiques.
... Bureau des Ponts et Chaussées.
... Sociétés des Jeunes Naturalistes et d'Études Scientifiques, 35 Rue Pierre-Charron.
... *Revue Générale des Sciences Pures et Appliquées.*
Pisa, *Il Nuovo Cimento.*
Sarajevo, Governor-General of Bosnia and Herzegovina.
Sassari, Istituto Fisiologico della R. Università.

St Petersburg, Imperatorskoe Russkoe Geographicheskoe Obtshéstvo.
... Russian Society of Naturalists and Physicians.
... Société Impériale Minéralogique.
... Société des Naturalistes (Section de Géologie et de Minéralogie).
... Société Astronomique Russe.
Sofia, Station Centrale Météorologique de Bulgarie.
Stavanger, Museum.
Stockholm, Svenska Sällskapet för Anthropologi och Geografi.
Tiflis, Physical Observatory.
Toulouse, Académie des Sciences.
Trieste, Società Adriatica di Scienze Naturali.
... Museo Civico di Storia Naturale.
... Osservatorio Astronomico-Meteorologico.
Tromsö, The Museum.
Utrecht, Provinciaal Genootschap van Kunsten en Wetenschappen.
Vienna, K. K. Naturhistorisches Hofmuseum.
Vilafranca del Panades (Cataluña), Observatorio Meteorologico.
Zurich, Schweizerische Botanische Gesellschaft.

ASIA.

China, Shanghai, North China Branch of the Royal Asiatic Society.
Japan, Tokio, The Seismological Society.
... Zoological Society of Tokio.
... The Asiatic Society of Japan.
... Deutsche Gesellschaft für Natur- und Völkerkunde Ostasiens.
Java, Koninklijke Natuurkundige Vereeniging, Batavia.

UNITED STATES.

Allegheny Observatory.
Annapolis, Maryland, St John's College.
Buffalo, Society of Natural Sciences.
California, State Mining Bureau, Sacramento.
... The Lick Observatory, Mount Hamilton, *viâ* San José, San Francisco.
... University of California (Berkeley).
Chapel Hill, North Carolina, Elisha Mitchell Scientific Society.
Chicago, University of Chicago.
... Academy of Sciences.
Cincinnati, Observatory.
... Society of Natural History.
... Ohio Mechanics' Institute.
Colorado, Scientific Society.
Connecticut, Academy of Arts and Sciences.
Davenport, Academy of Natural Sciences.
Ithaca, N.Y., The Editor, *Physical Review.*
... N.Y., The Editors, *Journal of Physical Chemistry.*
Indiana, Academy of Sciences, Indianopolis.
Iowa, The State University of Iowa.
... Geological Survey.
Kansas, Academy of Science, Topeka.
Mass., Tuft's College Library.
Meriden, Conn., Meriden Scientific Association.
Minnesota, The Geological and Natural History Survey of Minnesota, Minneapolis, Minnesota.
... State Botanist.
Missouri, Botanical Garden, St Louis.
Nebraska, The University of Nebraska, Lincoln.
New Orleans, Academy of Sciences.
New York, The American Museum of Natural History.
... American Geographical Society.
... American Institute of Electrical Engineers.
Philadelphia, Wagner Free Institute of Science.
... Geographical Club.
... Commercial Museum.
... The Editor, *American Naturalist.*
Portland (Maine), Society of Natural History.
Rochester, N.Y., Academy of Science.
Texas, Academy of Science, Austin.
Trenton, Natural History Society.
Washington, Philosophical Society.
... United States National Museum.
... United States Department of Agriculture (Division of Ornithology and Mammalogy).
... United States Patent Office.
Wisconsin, Academy of Sciences, Arts, and Letters.
... Geological and Natural History Survey.

SOUTH AMERICA.

Montevideo, Museo Nacional de Montevideo.

Quito, Ecuador, Observatorio Astronomico y Meteorologico.

Rio de Janeiro, Museu Nacional.

San Salvador, Observatorio Astronómico y Meteorólogico.

Santiago, Deutscher Wissenschaftlicher Verein.

MEXICO.

Academia Mexicana de Ciencias Exactas Naturales.

Instituto Geológico de México.

Tacubaya, Observatorio Astronómico.

Xalapa, Observatorio Meteorologico Central del Estado Vera Cruz.

Honolulu (Hawaiian Islands), Bernice Pauahi Bishop Museum.

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