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FACULTY WORKING
PAPER NO. 1174

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Asset Pricing Model: Theory and Implications

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Theory and Implications

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## Abstract

In this paper, the issue related to the impact of investment horizon on the CAPM functional form has been re-examined. A generalized translog functional form for CAPM was derived in accordance with the true investment horizon that is finite and unobservable. The securities included in the Dow Jones Industrial Index are used to test whether the risk-return relationship generally follows trans-log type of CAPM. It is found that the translog model improves the precision of estimated parameters the explanatory power of the capital asset pricing model.

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## Introduction

The traditional Capital Asset Pricing Model (CAPM) provides a
theoretical and empirical foundation for examining risk-return rela-
tionship and measuring investment performance. Despite recent criti-
cisms, the CAPM remains as the cornerstone of modern financial theory.
Academic scholars and finance practitioners have used the CAPM exten-
sively to estimate systematic risk from realized security returns for
various purposes such as market efficiency testing and capital
budgeting. However, it is known that the empirical results of the
CAPM from ex post data may not be consistent with the ex ante expec-
tation of the model. In particular, a problem of model misspecifica-
tion can occur in estimating systematic risk when the period length of
observed data deviates from the true investment horizon of individual
investors. Also, as pointed out by Markowitz [19], the CAPM is based
on a simultaneous linear equations constraint set. Extending this
constraint into more general linear programming constraint set is non-
negative variables will generate an efficient frontier which is very
likely nonlinear. These considerations have motivated the investiga-
tion of possible nonlinearities in the CAPM.

Jensen [14] first investigated the impact of deviation of observed
data period from true investment horizon on the estimation of system-
atic risk. He proposed a logarithmic linear model to eliminate the
impact of time horizon. Kraus and Litzenberger [16] have proposed a

quadratic characteristic line for a three-parameter CAPM model. Sub-
sequent studies (see [9], [11], [17] and [18]) have aimed at extending
or generalizing Jensen's model to consider the nonlinearity in the
CAPM with only limited success. Recently, McDonald [20] provided an
extensive analysis of the functional form of the CAPM. He proposed a
model of variable elasticity of substitution (VES) to investigate the
nonlinearity in the CAPM. His study, based on a very large data set
and a sophisticated maximum likelihood method, concluded that the VES
model did not significantly improve the estimation of beta coef-
ficients in comparison to Lee's [17] constant elasticity of substitu-
tion (CES) model. Further, his study found that the nonlinearity in
the CAPM could not solely be attributed to the investment horizon
problem.

One important issue, however, was not fully explored in McDonald's
study. Although it was understood that the existence of investment
horizon problem could produce biased risk estimates, the extent of
this bias and the factors affecting the Jensen measure and risk estima-
tions has not been explicitly analyzed. Moreover, the discrepancy be-
tween the true and observed investment horizons would likely generate
some statistical problems and affect the functional relation of risk
and return. As noted by Box and Cox [4], these statistical problems
can affect the observed return distribution. More specifically, the
moments of the observed return distribution would likely be affected by
the data measurement problems, even though the true return distribution
remains intact. Yet, the linkage between these statistical issues and

the functional relation of risk and return involving higher moments has not been fully investigated.

The purpose of this paper is twofold. First, it introduces a generalized model for the nonlinear risk and return relation .[1] The proposed model includes many familiar asset pricing models as special cases. The paper specifically shows the misspecification of the traditional characteristic lines, and demonstrates how the specification of a more appropriate functional form can help reduce the bias of Jensen measure and risk estimates and improve the explanatory power of the CAPM. Second, this paper provides some empirical evidence obtained from a translog model based on a general nonlinear relation of risk and return. There are several advantages of using this model. First, the translog model provides a generalized functional form that is a local second-order approximation to any nonlinear relationship.[2] Second, the translog model permits greater substitution among variables than many other models. Third, the model can be estimated and tested by relatively straightforward regression methods.

The remainder of this paper is divided into three sections. Section I develops an estimate model for the return and risk relation. This model is then compared to the other risk estimation models often used by academicians and practitioners. Section II discusses data and presents some empirical evidence. The results from several models are compared. Section III summarizes the important findings.

## I.  The Model

Following Jensen [14] and Lee [17], the risk-return relationship implied by the traditional asset pricing model can be written as

(1) $\qquad E({}_H R_j) = (1 - {}_H \beta_j) {}_H R_f + {}_H \beta_j E({}_H R_m)$

where

$\quad E({}_H R_j)$ = 1 + the expected return on security j over a true invest-
ment horizon H.

$\quad E({}_H R_f)$ = 1 + the expected return on a risk-free security over a
true investment horizon H.

$\quad E({}_H R_m)$ = 1 + the expected return on the market portfolio over a
true investment horizon H.

$\quad {}_H \beta_j$ = the systematic risk of security j in terms of true
investment horizon.

Since the observed ex post returns may deviate from the ex ante returns,
in general the following relationship holds:

(2) $\qquad E({}_H R_k) = [E({}_N R_k)]^\lambda \qquad$ for k = j, f, and m,

where $\lambda = \dfrac{H}{\lambda}$ is a transformation parameter and N is the period when the
returns are observed. Substituting (2) into (1) yields

(3) $\qquad E({}_N R_j)^\lambda = (1 - {}_H \beta_j) {}_N R_f^\lambda + {}_H \beta_j E({}_N R_m)^\lambda.$

If the values of transformation parameter $\lambda$ are allowed to vary with
each of the security returns, then equation (3) can be rewritten as

(4) $\qquad E({}_N R_j)^{\lambda_j} = (1 - {}_H \beta_j) {}_N R_f^{\lambda_f} + {}_H \beta_j E({}_N R_m)^{\lambda_m}.$

Equation (4) is a nonhomogeneous function which permits individual
securities to have different transformation parameters. This implies
that true investment horizon might be different among individual
securities. As Box and Cox [4] indicate, this type of transformation
can correct simultaneously for nonadditivity, nonconstant variance and
nonnormalities of security returns.

Equation (4) can be approximated by the following translog function (see the Appendix for the derivation):

$$(5) \quad \ln E(_N R_j) = \beta_{1j} \ln {}_N R_f + \beta_{2j} \ln E(_N R_m) + \beta_{3j} \ln {}_N R_f \ln E(_N R_m)$$

$$+ \beta_{4j} [\ln {}_N R_f]^2 + \beta_{5j} [\ln E(_N R_m)]^2$$

As usual, the coefficients can be normalized for convenience in approximating any arbitrary function. A convenient normalization is that $\beta_{1j} + \beta_{2j} = 1$. Given this normalization, equation (5) can be restated in terms of excess returns:

$$(6) \quad \ln E(_N R_j) - \ln {}_N R_f = \beta_{2j} [\ln E(_N R_m) - \ln {}_N R_f] + \beta_{3j} \ln {}_N R_f \ln E(_N R_m)$$

$$+ \beta_{4j} [\ln {}_N R_f]^2 + \beta_{5j} [\ln E(_N R_m)]^2$$

The parameters of equation (6) are related to the original parameters in equation (4) in the following manner:

$$(6a) \quad \beta_{2j} = {}_H \beta_j \frac{\lambda_m}{\lambda_j}$$

$$(6b) \quad \beta_{3j} = - {}_H \beta_j (1 - {}_H \beta_j) \frac{\lambda_f \lambda_m}{\lambda_j}$$

$$(6c) \quad \beta_{4j} = 1/2 \; {}_H \beta_j (1 - {}_H \beta_j) \frac{\lambda_f^2}{\lambda_j}$$

$$(6d) \quad \beta_{5j} = 1/2 \; {}_H \beta_j (1 - {}_H \beta_j) \frac{\lambda_m^2}{\lambda_j} \; .$$

Equation (6) states that the excess return of an asset depends not only on the excess return of the market portfolio but also on a multiplicative term involving $\ln {}_N R_f$ and $\ln {}_N R_m$, and the squared terms of

the returns on the riskless asset and market portfolio. Inclusion of the multiplicative term permits greater substitution between the riskless asset and market portfolio. The squared terms capture the effect of systematic skewness as Kraus and Litzenberger [16] suggested. The proposed relation in (6) is more general, however, since the co-skewness of individual securities with the riskless asset is also permitted.

To see the above argument more clearly, take the derivation of (6) with respect to $\ln {}_N R_f$ and $\ln E({}_N R_m)$:

(6e)     $\dfrac{\partial \ln E({}_N R_j)}{\partial \ln {}_N R_f} = 1 - \beta_{2j} + \beta_{3j} \ln E({}_N R_m) + 2\beta_{4j} \ln {}_N R_f$

(6f)     $\dfrac{\partial \ln E({}_N R_j)}{\partial \ln E({}_N R_m)} = \beta_{2j} + \beta_{3j} \ln {}_N R_f + 2\beta_{5j} \ln E({}_N R_m).$

Equations (6e) and (6f) measure the elasticity of $E({}_N R_j)$ with respect to ${}_N R_f$ and $E({}_N R_m)$ respectively. This contrasts to the log-linear CAPM where the corresponding values are $(1-\beta_{2j})$ and $\beta_{2j}$.

An estimate model derived from equation (6) in terms of ex post return can be defined as

(7)     $\ln R_{jt} - \ln R_{ft} = \beta_{0j} + \beta_{2j}(\ln R_{mt} - \ln R_{ft})$

$+ \beta_{3j} \ln R_{mt} \ln R_{ft} + \beta_{4j}(\ln R_{ft})^2$

$+ \beta_{5j}(\ln R_{mt})^2 + \varepsilon_{jt}.$

where, by analogy to the production function, $\beta_{0j}$ is the efficiency term; $(1-\beta_{2j})$ and $\beta_{2j}$ are the distribution parameters for $\ln R_{ft}$ and

ln $R_{mt}$; $\beta_{3j}$, $\beta_{4j}$ and $\beta_{5j}$ are substitution parameters; and $\varepsilon_{jt}$ is the stochastic disturbance term. Equation (7) is a nonlinear characteristic line for $j^{th}$ security. Kraus and Litzenberger [16a, 16b] have shown that different types of characteristic line imply different types of CAPM. Therefore, our analyses in this section on different types of characteristic line can be regarded as analyzing different types of CAPM.

Similar to the traditional Jensen's measure, $\beta_{0j}$ can be used to evaluate the performance of an individual security. Also, if equation (7) is a correct risk-return relation, then omiting the multiplicative and squared term will mean that residuals contain these effects. Therefore, the residuals from the traditional log-linear or linear market model may exhibit hetroscedasticity as shown in Giaccotte and Ali [13].

The assumption on the differential transformation parameters is consistent with the theory of rational choices. For instance, different transformation parameters may be associated with different true investment horizon of individual assets. Investors' determination of optimal investment horizon is generally affected by several crucial factors. The first important factor is transaction cost. The existence of considerable economies of scale implies that large investors are likely to have a shorter investment horizon than small investors. The increasing difference between the transaction costs of large and small investors has been evidenced since the commission rate deregulation in May 1975.

The heterogeneity in investor's expectation may also affect the optimal investment horizon.[3] Investors who possess different forecasts concerning the future prospects of individual assets will likely

invest in different securities and have different insights on the
market and individual securities timing. Thus, the securities owned
by different investor groups possibly will exhibit distinguished
investment horizons.

Another factor that plays an important role is the size of the
issuing firms. The size of firm is related to the amount of infor-
mation available to investors. As noted by Zeghal [27] and Barry and
Brown [2], more information is produced and disseminated about large
firms by external producers of information, such as brokers, financial
analysts, institutional investors, and business writers. Also larger
firms provide more information than smaller firms due to the economies
of scale in the production and dissemination of information. The
amount of information is likely to affect the optimal investment hori-
zon. It is not implausible to assume that investors owning the
securities of larger firms to have a shorter investment horizon.
Given more information available, large firms' securities will
generally have higher liquidity and their market values will be closer
to the true values. These are the advantages that may help investors
shorten the time needed to complete a transaction. In the light of
equation (6a), the effect of firm size on the beta estimates can be
analyzed. It can be argued that $\frac{\lambda_m}{\lambda_j}$ will be larger than or equal to one
for large firms, and will generally be smaller than one for small
firms. Therefore, when $\beta$ is estimated using the traditional market
model, it will tend to be overestimated for large firms and underesti-
mated for small firms. In general,

(8) $\qquad \beta_{2i} > \beta_{2j} \quad \text{if} \quad \dfrac{\lambda_m}{\lambda_i} > \dfrac{\lambda_m}{\lambda_j},$

even though $_H\beta_i = {_H}\beta_j$. And, $\dfrac{\lambda_m}{\lambda_i}$ is an important information for syste-

matic risk estimates. Thus the anomaly of higher average returns to

small firm securities unaccounted for by the estimates of systematic

risk (e.g. Reinganum [23]) may be attributed to the specification bias

of the traditional linear characteristic line.

The translog function provides a generalized model for examining

the risk-return relationship. It can be shown that the characteristic

lines used in several previous studies to estimate systematic risks

are all special cases of the proposed translog relationship. In the

following, various restrictions are imposed on the translog function

to derive some characteristic functions most familiar in the

literature.

(A) $\qquad \beta_{3j} = \beta_{4j} = 0$

Equation (5) reduces to the following estimate model:

(9) $\qquad \ln R_{jt} = \beta_{0j}^* + \beta_{1j} \ln R_{ft} + \beta_{2j} \ln R_{mt} + \beta_{5j} (\ln R_{mt})^2 + e_i.$

The above estimate model is very similar to the quadratic character-

istics line proposed by Kraus and Litzenberger [16a]. If the com-

pounding rates are used in their model, the risk estimates in (9) will

be the same as those obtained from the quadratic characteristic line.[4]

(B) $\qquad \beta_{3j} = \beta_{4j} = \beta_{5j} = 0$

These restrictions result in the familiar two index model:

(10)     $\ln R_{jt} = \beta_{0j} + \beta_{1j} \ln R_{ft} + \beta_{2j} \ln R_{mt} + v_{jt}.$

As noted by Merton [22], the interest rate changing stochastically over time affects the investment opportunity set. Therefore, investors are compensated in terms of expected returns for bearing market systematic risk and for bearing the risk of unfavorable shifts in the investment opportunity set..

(c)     $1/2\ \beta_{3j} = \beta_{4j} = \beta_{5j}$

This is equivalent to imposing a homogeneity condition on the risk-return relation; that is, $\lambda_m = \lambda_f$. With these restrictions, the following estimate model is obtained:[5]

(11)     $\ln R_{jt} - \ln R_{ft} = \alpha_j + \beta_j^*(\ln R_{mt} - \ln R_{ft}) + \gamma_j(\ln R_{mt} - \ln R_{ft})^2 + w_{jt}.$

This is the CES model proposed in Lee [17]. The $\gamma_j$ is the systematic skewness coefficient in Lee [17]. The constant investment horizon parameters imply a symmetry restriction on the translog function.

(D)     Constant $R_f$ and stationary return distribution

Under this condition, equation (4) can be simplified to generate the following estimate model:

(12)     $R_{jt}^{\lambda_j} = \alpha + {}_H\beta_j R_{mt}^{\lambda_m} + v_{jt}'.$

When $\lambda_j = \lambda_m = 1$, the traditional linear market model is obtained. When $\lambda_j$ and $\lambda_m$ both approach zero, it can be shown that equation (12) becomes the log-linear market model (see McDonald [20], Spitzer [25]).

(E)        Nonstationary return distribution

Kraus and Litzenberger [16a] have demonstrated that under a changing riskless rate, the moments of risky assets return are not intertemporal constants. However, under the assumptions of proportional stochastic growth and either constant relative risk aversion or stationary distribution of per capital end-of-period wealth, they suggested a transformed return variable to resolve the nonstationarity problem.[6] This variable is essentially a deflated excess return defined as

(13a)        $r_{jt} = (R_{jt} - R_{ft})/R_{ft}$

(13b)        $r_{mt} = (R_{mt} - R_{ft})/R_{ft}$

The transformed variables can be used to estimate the beta systematic risk by the following regression:

(14)        $r_{jt} = \alpha_j + \beta_j r_{mt} + u'_{jt}.$

However this function can be shown to be equivalent to the case when $\lambda_j = \lambda_f = \lambda_m = 1$ in equation (4).[7]

In sum, by imposing various restrictions on the transformed function and the translog function, several familiar characteristic lines can be obtained. Thus the proposed risk-return relation as in equation (4) and approximated by equation (6) provides a generalized model to estimate systematic risks when there exists nonstationary return distribution and when investors' utility function involves the higher moment such as skewness.

In the following section, data used to estimate systematic risks are described and some empirical results are reported. In addition, the performance of the translog model is compared with that of the alternative models.

## II. Data and Empirical Results

Monthly returns for all the securities included in Dow Jones Industrial Average (DJIA) covering the period 1969-82 were collected from the Compustat tape. One security, American Express, was finally excluded from the sample due to missing observations in the earlier years. The study period was further divided into two subperiods. This resulted in a data base of 29 securities, each with 84 monthly observations. The market rate of return used is the New York Stock Exchange monthly value-weighted index. The monthly treasury bill rate was used as a proxy for the risk-free rate.

The correlation coefficient matrix for the explanatory variables in the translog model is displayed in Table I. In general, the correlation coefficients are fairly stable over time. The sign of the correlation between variables is consistent in two periods with only one exception (the correlation between $\ln R_{ft} \ln R_{mt}$ and $\ln R_{ft}^2$). As shown in the table, the variable $\ln R_{ft} \ln R_{mt}$ is highly correlated with the excess return of the market portfolio (.98 and .94 for the first and second period, respectively). This problem can be attributed to the fact that $R_{ft}$ is smaller and relatively stable over time. The extremely high correlation between these two variables causes a very severe multicollinearity problem. To cope with this problem, the

variable $\ln R_{mt} \ln R_{ft}$ is orthogonalized. This procedure involves

regressing $\ln R_{mt} \ln R_{ft}$ against the excess market return and obtaining

residuals from the regression. The residuals (RES) retain a portion

of information in $\ln R_{mt} \ln R_{ft}$ that is not correlated with the excess

market return. This residual variable is then used to estimate the

coefficient of $\beta_{3j}$ in equation (7).

The results of the translog regressions are reported in Table II.[8]

All the estimates of $\beta_2$ coefficients are significant. In addition,

the coefficients associated with the quadratic terms are also signifi-

cant for numerous cases.

In the first period, there are five securities with significant

$\beta_3$, five securities with significant $\beta_4$ and six securities with signif-

icant $\beta_5$. Out of 29 securities examined, 15 securities have at least

one significant coefficient associated with the multiplicative and

squared terms. The translog regression results in the second period

even perform better. There are 13 securities with significant $\beta_3$, five

with significant $\beta_4$ and five securities with significant $\beta_5$. All

together, 18 securities have at least one of these three coefficients

that are significant. For those securities with significant coef-

ficients associated with the multiplicative and squared terms, the

estimation of systematic risks and Jensen performance measures ($\beta_0$'s)

using the traditional CAPM is subjected to specification bias.

The results of the log-linear characteristic line are reported in

Table III for comparison. Note that the values of R-square are much

larger for the translog regressions. In general, the values of

adjusted R-square will increase when the Student's t for the additional

parameter introduced is larger than one. For most of the securities

included, at least one of the estimated coefficients associated with

the higher power terms have the student's t values greater than one.

Thus, the proposed translog model improves the explanatory power of

the CAPM. Therefore, it will be a more appropriate model for fore-

casting the security rates of return.

In parallel to McDonald's study, the results of the CES model as

approximated by equation (11) are reported in Table IV. As noted

earlier, the CES model proposed by Lee [17] is equivalent to the

translog model with the restrictions of $\beta_4 = \beta_5 = -1/2\beta_3$. These

restrictions are required to satisfy the condition of homogeneity.

Table IV shows that most of the $\beta$ estimates are significant, while the

$\gamma$ coefficients are significant for five and seven cases for the first

and second period, respectively. The results are similar to those

found in Lee [17] and McDonald [20].

Table V provides the summary statistics of the parameter esti-

mates. For the translog model, the standard deviations of the cross-

sectional $\beta_4$ and $\beta_5$ are relatively higher. The greater dispersion of

$\beta_4$ and $\beta_5$ estimates is attributed to the smaller values of squared

terms. Similar to the mean values of $\beta*$ and $\beta$ in the CES and log-

linear models, the average of $\beta_2$ is close to one as expected. There-

fore, the minor differences in the mean values for $\beta_2$, $\beta*$ and $\beta$ do not

convey significant information for the corresponding risk estimates

for individual securities. To provide more details on the differences

of systematic risks ($\beta_2$, $\beta*$ and $\beta$) for individual securities, Table VI

summarizes the mean, maximum and minimum of the differences and absolute

difference in the estimated coefficients. The discrepancy between $\beta$

and $\beta*$ appear inconsequential. However, the deviations of $\beta_2$ from $\beta$ and $\beta*$ are larger. The mean absolute difference, a better measure for bias, indicate that on average the deviations are around .05 to .07, roughly 5 percent to 7.5 percent errors. For individual securities, the maximum difference is around .13 to .79. These figures appear to be not trivial.

The intercept estimates from these alternative model are now com-. pared and analyzed. For the log-linear model, there is only one and three estimated intercepts significant different from zero for the first and second period, respectively. For the CES model, there are one and eight estimated intercepts significantly different from zero and for the translog model, these are four and seven estimated intercepts significantly different from zero. The number of significant intercepts is substantially smaller for the log-linear model. These figures suggest that the Jensen measures estimated from the traditional log-linear market model are likely biased.

To complete the analysis , two likelihood ratio tests are performed. The first test concerns whether the proposed translog model satisfies the homogeneity condition. The second test checks whether the translog model is significantly different from the log-linear market model. Test statistics are reported in Table VII. To perform the first test, the restrictions that $\beta_4 = \beta_5 = -1/2\beta_3$ are imposed. The critical value for $F(2, 79, 5\%)$ is equal to 3.15. Out of 29 securities, three and six securities indicate significant differences from homogeneity for the first and second period, respectively. To perform the second test, the denominator and numerator of the ratios of squared residual errors are divided by the associated degree of

freedom. The critical F value corresponding to the restriction that the sum of coefficients $\beta_3$ to $\beta_5$ is zero is equal to 3.92. Out of 29 securities, there are 15 and 19 cases in two respective periods that have F values exceed the critical value. Consequently, for a large proportion of securities studied in this sample, the translog model provides results that are significantly different from the log-linear model. These results tend to support Markowitz's [19] arguments about the nonlinear CAPM.

### III. Summary and Concluding Remarks

In this paper, the issue related to the impact of investment horizon on the CAPM functional form has been re-examined. A generalized translog functional form for CAPM was derived in accordance with the true investment horizon that is finite and unobservable. The securities included in the Dow Jones Industrial Index are used to test whether the risk-return relationship generally follows trans-long type of CAPM. It is found that the translog model improves the precision of estimated parameters and the explanatory power of the capital asset pricing model. For a large number of securities, there are systematic risks associated with the multiplicative and squared terms of returns on the riskless asset and market portfolio. The linearity assumption of the CAPM has been rejected for a very large proportion of securities studied in this paper. It is also found that for most of the securities studied the proposed translog model appears to satisfy the condition of homogeneity given symmetry. Implications of the new model derived in this paper to test the capital asset pricing model will be done in the future research.

Footnotes

[1] Jensen [14, 15], Black, Jensen and Scholes [3], Merton [21, 22] have discussed the importance of investment horizon on the capital asset pricing process.

[2] The translog function does not employ additivily and homogeneity as part of the maintain hypothesis. For many production and investment frontiers employed in the econometric studies, the translog frontiers provide accurate global approximations.

[3] Elton and Gruber [10] have recently discussed the effect of heterogeneous expectations on the form of the CAPM. Markowitz [19] also has shown that hetrogeneous expectations is an important issue for testing the CAPM.

[4] Kraus and Litzenberger [16a] proposed the following quadratic characteristic line:

$$R_i - R_f = c_{0i} + c_{1i}(R_m - R_f) + c_{2i}(R_m - \bar{R}_m)^2 + e_i.$$

This function can be simplified as

$$R_i = b_0 + b_1 R_f + b_2 R_m + b_3 R_m^2$$

where $b_0 = c_{0i} + c_{2i}\bar{R}_m^2$

$b_1 = 1 - c_{1i}$

$b_2 = c_{1i} - 2c_{2i}\bar{R}_m$

$b_3 = c_{2i}.$

[5]If $\lambda_j = \lambda_m = \lambda_f = \lambda$, equation (11) can also be obtained by expanding log $E(_NR_j)$ around $\lambda=0$, and dropping the terms involving powers of $-\lambda$ greater than one.

[6]Rubinstein [24] has also shown that the expected value of transformed variable would be constant over time under the similar conditions.

[7]When $\lambda$'s are the same, equation (4) can be rewritten as

$$\frac{E(_HR_j)^{\lambda_j}}{_HR_f{}^{\lambda_f}} = (1-_H\beta_j) + \frac{E(_HR_m)^{\lambda_m}}{_HR_f{}^{\lambda_f}}(_H\beta_j)$$

or

$$\frac{E(_NR_j)^{\lambda_j} - _HR_f{}^{\lambda_f}}{_HR_f{}^{\lambda_f}} = _H\beta_j \frac{E(_HR_m)^{\lambda_m} - _HR_f{}^{\lambda_f}}{_HR_f{}^{\lambda}}$$

Setting $\lambda_m = \lambda_f = \lambda_j = 1$ yields:

$$\frac{E(_HR_j) - _HR_f}{_HR_f} = \beta(\frac{E(_HR_m) - _HR_f}{_HR_f})$$

[8]The values included in the parentheses in all the tables are t statistics.

## References

1.  Aivazian, V. A., J. L. Callen, I. Krinsky and C. C. Y. Kwan. "Mean-Variance Utility Functions and the Demand for Risky Assets: An Empirical Analysis Using Flexible Functional Forms." _Journal of Financial and Quantitative Analysis_, Vol. 18 (December 1983), pp. 411-24.

2.  Barry, C. B. and S. J. Brown. "Differential Information and the Small Firm Effect." _Journal of Financial Economics_, Vol. 13 (June 1984), pp. 283-294.

3.  Black, F., M. C. Jensen, and M. Scholes. "The Capital Asset Pricing Model: Some Empirical Tests." in M. C. Jensen, ed., _Studies in the Theory of Capital Markets_, New York: Praeger Publishers, 1972.

4.  Box, G. E. P. and D. R. Cox. "An Analysis of Transformation." _Journal of the Royal Statistical Society_, Series B, Vol. 26 (1964), pp. 211-243.

5.  Caves, D. W. and L. R. Christensen. "Global Properties of Flexible Functional Forms." _American Economic Review_, Vol. 70 (June 1980), pp. 422-32.

6.  Christensen, L. R., D. W. Jorgenson and L. J. Lau. "Conjugate Duality and the Transcendental Logarithms Production Function." _Econometrica_, Vol. 39 (July 1971), pp. 255-56.

7.  Christensen, L. R., D. W. Jorgenson, and L. J. Lau. "Transcendental Logarithmic Production Functions." _Review of Economics and Statistics_, Vol. 55 (1973), pp. 28-45.

8.  Christensen, L. R., D. W. Jorgenson and L. J. Lau. "Transcendental Logarithmic Utility Functions." _American Economic Review_, Vol. 65 (June 1975), pp. 367-83.

9.  Cheng, P. L. and M. K. Deets. "Systematic Risk and the Horizontal Problem." _Journal of Financial and Quantitative Analysis_, Vol. 8 (March 1973), pp. 299-316.

10. Elton, E. J. and M. J. Gruber. "Non-Standard CAPM's and the Market Portfolio." _Journal of Finance_, Vol. 39 (July 1984), pp. 911-24.

11. Fabozzi, F., J. Francis, and C. F. Lee. "Generalized Functional Form for Mutual Fund Returns." _Journal of Financial and Quantitative Analysis_, Vol. 15 (December 1980), pp. 1107-19.

12. Friend, I. and R. Westerfield. "Co-Skewness and Capital Asset Pricing." _Journal of Finance_, Vol. 25 (September 1980), pp. 897-913.

13. Giaccotto, C. and M. M. Ali. "Optimum Distribution-Free Tests and Further Evidence of Heteroscedasticity in the Market Model." <u>Journal of Finance</u>, Vol. 37 (December 1982), pp. 1247-57.

14. Jensen, M. C. "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios." <u>Journal of Business</u>, Vol. 42 (April 1969), pp. 167-247.

15. Jensen, M. C. "Capital Markets: Theory and Evidence." <u>The Bell Journal of Economics and Management Science</u>, Vol. 3 (Autumn 1972), pp. 357-398.

16a. Kraus, A. and R. H. Litzenberger. "Skewness Preference and the Evaluations of Risk Assets." <u>Journal of Finance</u> (September 1976), pp. 1085-1100.

16b. _____. "On the Distributional Conditions For A Consumption-oriented Three Moment CAPM." <u>Journal of Finance</u>, (December 1983), pp. 1381-1391.

17. Lee, C. F. "Investment Horizon and the Functional Form of the Capital Asset Pricing Model." <u>The Review of Economics and Statistics</u>, Vol. 58 (August 1976), pp. 356-63.

18. Levhari, D. and H. Levy. "The Capital Asset Pricing Model and the Investment Horizon." <u>The Review of Economics and Statistics</u>, Vol. 59 (February 1977), pp. 92-104.

19. Markowitz, H. M. "Nonnegative or Not Nonnegative: A Question about CAPMS." <u>Journal of Finance</u>, Vol. 38 (May 1983), pp. 283-295.

20. McDonald, B. "Functional Forms and the Capital Asset Pricing Model." <u>Journal of Financial and Quantitative Analysis</u>, Vol. 18 (September 1983), pp. 319-29.

21. Merton, R. C. "A Dynamic General Equilibrium Model of the Asset Market and its Application to the Pricing of the Capital Structure of the Firm," Working Paper No. 499-70, Sloan School of Management, Massachusetts Institute of Technology (December 1970).

22. Merton, R. C. "An Intertemporal Capital Asset Pricing Model," <u>Econometrica</u>, Vol. 41 (September 1973), pp. 867-887.

23. Reinganum, M. R. "A Direct Test of Roll's Conjecture on the Firm Size Effect." <u>Journal of Finance</u>, Vol. 37 (March 1982), pp. 27-35.

24. Rubinstein, M. "A Comparative Statistics Analysis of Risk Premiums," <u>Journal of Business</u>, Vol. 46 (October 1973), pp. 605-15.

25.  Spitzer, J.  "The Demand for Money, the Liquidity Trap, and Func-
         tional Forms."  International Economic Review, Vol. 17
         (February 1976), pp. 220-227.

26.  Tobin, J.  "The Theory of Portfolio Selection."  In The Theory
         of Interest Rates, F. Hahn and F. Breechling, eds.  London:
         Macmillan (1965).

27.  Zeghal, D.  "Firm Size and the Information Content of Financial
         Statements."  Journal of Financial and Quantitative Analysis,
         Vol. 19 (September 1984), pp. 299-310.

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Appendix

Taking the logarithm of (4) yields the following generalized risk and return relationship:

(A.1) $\quad \lambda_j \ln E(_N R_j) = \ln[(1-\beta) \exp (\lambda_f \ln_N R_f) + \beta \exp (\lambda_m \ln E(_N R_m)]$.

Let

(A.2) $\quad \phi = \ln E(_N R_j) = \frac{1}{\lambda j} \ln[(1-\beta) \exp (\lambda_f \ln_N R_f) + \beta \exp (\lambda_m \ln E(_N R_m)]$.

Then,

(A.3.1) $\quad \dfrac{\partial \phi}{\partial \ln_N R_f} = \dfrac{\lambda_f}{A \lambda_j} [(1-\beta) \exp (\lambda_f \ln_N R_f)]$

where $A = (1-\beta) \exp (\lambda_f \ln_N R_f) + \beta \exp (\lambda_m \ln E(_N R_m))$

(A.3.2) $\quad \dfrac{\partial \phi}{\partial \ln E(_N R_m)} = \dfrac{\lambda_m}{A \lambda_j} [\beta \exp (\lambda_m \ln E(_N R_m))]$

(A.3.3) $\quad \dfrac{\partial^2 \phi}{\partial \ln_N R_f^2} = \dfrac{\lambda_f^2}{A^2 \lambda_j} [(1-\beta) \exp (\lambda_f \ln_N R_f)][\beta \exp (\lambda_m \ln_N R_f)]$

(A.3.4) $\quad \dfrac{\partial^2 \phi}{\partial \ln E(_N R_m)^2} = \dfrac{\lambda_m^2}{A^2 \lambda_j} [(1-\beta) \exp (\lambda_f \ln_N R_f)][\beta \exp (\lambda_m \ln E(_N R_m))]$

(A.3.5) $\quad \dfrac{\partial^2 \phi}{\partial \ln_N R_f \ln E(_N R_m)} = \dfrac{\lambda_f \lambda_m}{A^2 \lambda_j} [\beta \exp (\lambda_m \ln E(_N R_m))][(1-\beta) \exp (\lambda_f \ln_N R_f)]$

Expanding $\phi$ around $R_f = 1$ and $R_m = 1$ gives the following relation:

$$(A.4) \quad \phi \approx (1-{}_H\beta_j) \frac{\lambda_f}{\lambda_j} \ln {}_NR_f + {}_H\beta_j \frac{\lambda_m}{\lambda_j} \ln E({}_NR_m)$$

$$- {}_H\beta_j(1-{}_H\beta_j) \frac{\lambda_f \lambda_m}{\lambda_j} \ln{}_NR_f \ln E({}_NR_m)$$

$$+ 1/2 \frac{\lambda_f^2}{\lambda_j} (1-{}_H\beta_j){}_H\beta_j (\ln{}_NR_f)^2 + 1/2 \frac{\lambda_m^2}{\lambda_j} {}_H\beta_j(1-{}_H\beta_j)(\ln E({}_NR_m))^2$$

TABLE I

Correlation Matrix of Independent Variables

| | $\ln R_{mt}-\ln R_{ft}$ | RES | $\ln R_{ft}^2$ | $\ln R_{mt}^2$ | $\ln R_{ft}\ln R_{mt}$ |
|---|---|---|---|---|---|
| **Period 1** | | | | | |
| $\ln R_{mt}-\ln R_{ft}$ | 1 | | | | |
| RES | .00 | 1 | | | |
| $\ln R_{ft}^2$ | -.36 | .15 | 1 | | |
| $\ln R_{mt}^2$ | .05 | -.13 | .28 | 1 | |
| $\ln R_{ft}\ln R_{mt}$ | .98 | .18 | .32 | .02 | 1 |
| **Period 2** | | | | | |
| $\ln R_{mt}-\ln R_{ft}$ | 1 | | | | |
| RES | .00 | 1 | | | |
| $\ln R_{ft}^2$ | -.26 | .12 | 1 | | |
| $\ln R_{mt}^2$ | .25 | -.32 | .08 | 1 | |
| $\ln R_{ft}\ln R_{mt}$ | .94 | .33 | -.21 | .13 | 1 |

TABLE II

Regression Results of the Translog Model

| | $\beta_0$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|---|---|

Period 1 (1969-75)

| | $\beta_0$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|---|---|
| 1. | -0.033** | 1.077*** | -0.051 | 14.976** | -2.976* | .463 | .436 |
| | (-2.135) | (8.171) | (-0.042) | (2.512) | (-1.791) | | |
| 2. | -0.026 | 1.025*** | 2.899** | 8.258 | 1.497 | .472 | .445 |
| | (-1.651) | (7.603) | (2.309) | (1.354) | (0.878) | | |
| 3. | -0.004 | 0.718*** | 0.589 | 1.486 | 1.532 | .381 | .349 |
| | (-0.310) | (6.302) | (0.810) | (0.289) | (1.063) | | |
| 4. | -0.014 | 1.128*** | 0.735 | 10.116* | -1.614 | .485 | .459 |
| | (-0.869) | (8.367) | (0.586) | (1.659) | (-0.947) | | |
| 5. | 0.012 | 1.111*** | 0.400 | -3.263 | -0.593 | .600 | .580 |
| | (0.892) | (9.717) | (0.376) | (-0.631) | (-0.411) | | |
| 6. | 0.000 | 1.100*** | 0.064 | 0.847 | -0.146 | .390 | .360 |
| | (0.040) | (6.561) | (0.042) | (0.112) | (-0.069) | | |
| 7. | 0.026 | 0.650*** | 1.278 | -9.608 | -1.264 | .323 | .289 |
| | (1.594) | (4.696) | (0.992) | (-1.534) | (-0.722) | | |
| 8. | -0.020 | 1.177*** | -0.518 | 11.029* | -0.357 | .541 | .518 |
| | (-1.353) | (9.410) | (-0.445) | (1.949) | (-0.226) | | |
| 9. | 0.025 | 0.782*** | 2.524** | -7.985 | 0.370 | .407 | .377 |
| | (1.595) | (5.891) | (2.042) | (-1.330) | (0.221) | | |
| 10. | 0.015 | 0.755*** | 1.038 | -2.952 | 0.538 | .504 | .479 |
| | (1.318) | (7.846) | (1.158) | (-0.678) | (0.442) | | |
| 11. | 0.023* | 0.636*** | 1.175 | -9.100* | 1.859 | .423 | .393 |
| | (1.802) | (5.908) | (1.172) | (-1.808) | (1.365) | | |
| 12. | 0.020 | 0.867*** | 0.034 | -7.839 | 0.579 | .379 | .348 |
| | (1.169) | (5.809) | (0.025) | (-1.162) | (0.307) | | |
| 13. | 0.004 | 0.833*** | 0.656 | -3.476 | 0.750 | .419 | .390 |
| | (0.270) | (6.628) | (0.561) | (-0.611) | (0.472) | | |
| 14. | -0.017 | 1.120*** | -0.832 | 9.979 | -3.423* | .475 | .449 |
| | (-1.056) | (8.196) | (-0.654) | (1.614) | (-1.980) | | |
| 15. | -0.041** | 1.231*** | -0.231 | 18.666*** | -3.189** | .500 | .475 |
| | (-2.505) | (8.849) | (-0.179) | (2.965) | (-1.811) | | |

TABLE II (continued)

Regression Results of the Translog Model

| | $\beta_0$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|---|---|
| 16. | -0.009 | 1.039*** | -0.286 | 5.423 | 0.235 | .371 | .339 |
| | (-0.494) | (6.490) | (-0.192) | (0.749) | (0.116) | | |
| 17. | -0.021 | 0.936*** | -2.335* | 12.371* | -0.852 | .373 | .341 |
| | (-1.278) | (6.640) | (-1.778) | (1.939) | (-0.478) | | |
| 18. | -0.013 | 0.922*** | -3.654** | 11.020 | -6.463*** | .310 | .275 |
| | (-0.646) | (5.182) | (-2.205) | (1.369) | (-2.871) | | |
| 19. | -0.014 | 0.595*** | 0.401 | 3.929 | 0.056 | .255 | .217 |
| | (-1.008) | (4.962) | (0.359) | (0.724) | (0.037) | | |
| 20. | 0.015 | 0.785*** | -0.658 | -7.367 | 0.890 | .497 | .471 |
| | (1.235) | (7.302) | (-0.658) | (-1.515) | (0.654) | | |
| 21. | 0.013 | 1.027*** | 0.279 | -6.442 | 2.071 | .630 | .611 |
| | (1.095) | (9.888) | (0.289) | (-1.370) | (1.576) | | |
| 22. | 0.026 | 0.718*** | -1.887 | -15.071* | 1.364 | .296 | .259 |
| | (1.244) | (4.016) | (-1.134) | (-1.864) | (0.603) | | |
| 23. | -0.001 | 0.901*** | -0.987 | 3.970 | -3.315** | .485 | .459 |
| | (-0.139) | (8.001) | (-0.942) | (0.779) | (-2.326) | | |
| 24. | -0.012 | 0.932*** | -3.025* | 6.619 | -2.737 | .315 | .280 |
| | (-.622) | (5.549) | (-1.935) | (0.872) | (-1.288) | | |
| 25. | -0.013 | 1.088*** | -2.127 | 7.374 | -2.180 | .282 | .246 |
| | (-0.569) | (5.304) | (-1.115) | (0.795) | (-0.841) | | |
| 26. | 0.034*** | 0.834*** | 0.544 | -9.977** | -1.498 | .537 | .516 |
| | (2.661) | (7.659) | (0.537) | (-2.026) | (-1.088) | | |
| 27. | -0.009 | 0.678*** | -0.479 | 3.148 | 1.278 | .487 | .460 |
| | (-0.887) | (8.001) | (-0.607) | (0.821) | (1.191) | | |
| 28. | 0.010 | 1.014*** | -0.444 | -1.073 | -2.441* | .587 | .567 |
| | (0.839) | (9.531) | (-0.449) | (-0.223) | (-1.814) | | |
| 29. | -0.010 | 1.082*** | -1.216 | 3.792 | -0.159 | .363 | .330 |
| | (0.503) | (6.284) | (0.758) | (0.487) | (-0.073) | | |

TABLE II (continued)

Regression Results of the Translog Model

| | $\beta_0$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|---|---|
| **Period 2 (1976-82)** | | | | | | | |
| 1. | -.002 (-.444) | 1.241*** (5.037) | .544 (.639) | -1.436 (-.625) | -2.555 (-.721) | .288 | .252 |
| 2. | -.001 (-.149) | .607*** (4.359) | -.705 (-1.464) | .472 (.363) | -.781 (-.390) | .235 | .196 |
| 3. | .008 (1.017) | .488*** (4.328) | -.517 (1.327) | -.436 (-.414) | .953 (.587) | .265 | .228 |
| 4. | -.023*** (-2.638) | 1.412*** (11.553) | -.156 (-.371) | 1.694 (1.486) | 1.594 (.907) | .669 | .652 |
| 5. | -.011 (-1.355) | .658*** (5.767) | -.585 (-1.483) | -.076 (-.072) | 4.664*** (2.841) | .457 | .429 |
| 6. | -.006 (-.500) | 1.257*** (6.956) | .185 (.297) | .007 (.004) | -.309 (-.119) | .422 | .392 |
| 7. | -.015 (1.599) | .874*** (6.871) | -.430 (-.978) | .022 (.019) | 3.794** (2.074) | .482 | .456 |
| 8. | -.024** (-2.938) | .880*** (7.962) | -.410 (-1.075) | 1.732* (1.679) | 3.056* (1.921) | .537 | .503 |
| 9. | -.016 (-1.531) | .683*** (4.754) | -1.127** (-2.267) | 1.524 (1.136) | .846 (.409) | .293 | .257 |
| 10. | -.011 (-1.414) | .585*** (5.327) | -.642* (-1.691) | 1.172 (1.144) | .663 (.420) | .317 | .283 |
| 11. | .006 (.823) | .759*** (7.436) | .316 (.897) | -.926 (-.972) | -.100 (-.069) | .478 | .451 |
| 12. | .010 (.812) | 1.048*** (6.082) | 1.441** (2.419) | -1.462 (-.909) | 1.286 (.519) | .416 | .386 |
| 13. | .001 (.103) | .969*** (6.686) | 1.399*** (2.792) | -.863 (-.638) | 1.945 (.933) | .463 | .436 |
| 14. | -.014 (-1.392) | .994*** (6.901) | -1.110** (-2.230) | 2.481* (1.847) | -.106 (-.051) | .420 | .391 |
| 15. | -.016 (-1.512) | 1.192*** (7.889) | .462 (.885) | 2.507* (1.779) | -2.422 (-1.115) | .460 | .432 |

TABLE II (continued)

Regression Results of the Translog Model

| | $\beta_0$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|---|---|
| 16. | -.023* | 1.291*** | -.436 | .470 | 3.873 | .462 | .435 |
| | (-1.733) | (6.997) | (-.684) | (.273) | (1.459) | | |
| 17. | -.032** | 1.123*** | -.729 | 2.492 | .896 | .351 | .319 |
| | (-2.298) | (5.994) | (-1.126) | (1.426) | (.333) | | |
| 18. | .004 | .903*** | -.937* | -1.057 | .851 | .372 | .340 |
| | (.375) | (5.640) | (-1.693) | (-.708) | (.370) | | |
| 19. | .004 | .689*** | .028 | -.538 | -2.079 | .271 | .234 |
| | (.437) | (4.976) | (.059) | (-.417) | (-1.043) | | |
| 20. | .008 | .698*** | -.634* | -1.185 | .088 | .423 | .393 |
| | (.998) | (6.312) | (-1.659) | (-1.148) | (.056) | | |
| 21. | -.009 | .916*** | -.906** | .974 | 2.634* | .597 | .576 |
| | (-1.250) | (8.966) | (-2.567) | (1.023) | (1.793) | | |
| 22. | .000 | 1.402*** | -.699 | .798 | .228 | .533 | .510 |
| | (.003) | (8.697) | (-1.255) | (.531) | (.098) | | |
| 23. | .008 | .684*** | -1.542*** | -1.000 | -2.008 | .343 | .310 |
| | (.825) | (4.963) | (-3.237) | (-.778) | (-1.013) | | |
| 24. | .036** | .984*** | -1.273 | -8.102*** | -5.091 | .376 | .344 |
| | (2.081) | (4.126) | (-1.544) | (-3.642) | (-1.484) | | |
| 25. | .007 | 1.167*** | -.116 | -1.146 | 1.915 | .608 | .588 |
| | (.780) | (9.465) | (-.273) | (-.997) | (1.080) | | |
| 26. | -.032*** | .856*** | -1.251*** | 3.423*** | .526 | .394 | .364 |
| | (-3.141) | (6.183) | (-2.613) | (2.650) | (.264) | | |
| 27. | -.001 | .260*** | -.767*** | .287 | -.170 | .197 | .156 |
| | (-.166) | (3.193) | (-2.723) | (.379) | (-.146) | | |
| 28. | -.022* | .819** | -1.034** | 1.869 | 2.204 | .371 | .339 |
| | (-2.097) | (5.634) | (-2.059) | (1.379) | (1.054) | | |
| 29. | .003 | .919*** | -1.436** | .600 | -4.909* | .280 | .244 |
| | (.281) | (5.079) | (-2.297) | (.356) | (-1.886) | | |

TABLE III

Regression Results of the Log-Linear Model

| | Intercept | $\beta$ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|
| Period 1 (1969-75) | | | | |
| 1. | -.002 (-.370) | .939*** (7.571) | .411 | .404 |
| 2. | -.001 (-.213) | .961*** (7.463) | .404 | .397 |
| 3. | .003 (.668) | .711*** (6.852) | .364 | .356 |
| 4. | .008 (1.301) | 1.036*** (8.350) | .459 | .453 |
| 5. | .002 (.425) | 1.137*** (10.998) | .596 | .591 |
| 6. | .002 (.346) | 1.093*** (7.252) | .390 | .383 |
| 7. | -.001 (-.268) | .727*** (5.672) | .281 | .273 |
| 8. | .007 (1.283) | 1.082*** (9.392) | .518 | .512 |
| 9. | .005 (.884) | .851*** (6.912) | .368 | .360 |
| 10. | .008** (2.006) | .782*** (8.960) | .494 | .488 |
| 11. | .004 (.849) | .720*** (7.240) | .390 | .382 |
| 12. | .001 (.288) | .935*** (6.917) | .360 | .368 |
| 13. | -.003 (-.535) | .865*** (7.634) | .415 | .408 |
| 14. | -.000 (-.001) | 1.023*** (8.084) | .443 | .436 |
| 15. | -.001 (-.192) | 1.061*** (8.007) | .438 | .431 |

TABLE III (continued)

Regression Results of the Log-Linear Model

|     | Intercept | β | $R^2$ | Adjusted $R^2$ |
|-----|-----------|---|-------|----------------|
| 16. | .005 (.713) | .994*** (6.879) | .365 | .358 |
| 17. | .008 (1.263) | .828*** (6.308) | .326 | .318 |
| 18. | -.001 (-.171) | .805*** (4.714) | .213 | .203 |
| 19. | -.004 (-.750) | .562*** (5.187) | .247 | .237 |
| 20. | -.001 (-.200) | .851*** (8.627) | .475 | .469 |
| 21. | .002 (.439) | 1.090*** (11.440) | .614 | .610 |
| 22. | -.009 (-1.079) | .851*** (5.110) | .241 | .232 |
| 23. | .000 (.011) | .855*** (8.165) | .448 | .441 |
| 24. | -.002 (-.282) | .865*** (5.576) | .274 | .266 |
| 25. | -.000 (-.034) | 1.017*** (5.459) | .266 | .257 |
| 26. | .004 (.952) | .913*** (8.898) | .491 | .485 |
| 27. | .002 (.609) | .656*** (8.427) | .464 | .457 |
| 28. | .001 (.346) | 1.014*** (10.339) | .565 | .560 |
| 29. | -.000 (-.117) | 1.049*** (6.753) | .357 | .349 |

TABLE III (continued)

Regression Results of the Log-Linear Model

|  | Intercept | β | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|

Period 2 (1976-82)

|  | Intercept | β | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|
| 1. | -.017 (-1.719) | 1.235*** (5.525) | .271 | .262 |
| 2. | -.000 (-.001) | .580** (4.582) | .203 | .194 |
| 3. | .007 (1.634) | .518*** (5.017) | .234 | .225 |
| 4. | -.009* (-1.901) | 1.393*** (12.407) | .652 | .648 |
| 5. | -.002 (-.518) | .745*** (6.665) | .351 | .343 |
| 6. | -.007 (-1.002) | 1.251*** (7.720) | .420 | .413 |
| 7. | -.007 (-1.367) | .942*** (7.886) | .431 | .424 |
| 8. | -.006 (-1.386) | .887*** (8.390) | .461 | .455 |
| 9. | -.004 (-.761) | .656*** (4.860) | .223 | .214 |
| 10. | -.002 (.545) | .564*** (5.555) | .273 | .264 |
| 11. | -.000 (-.014) | .783*** (8.467) | .466 | .460 |
| 12. | .003 (.466) | 1.113*** (6.934) | .369 | .361 |
| 13. | -.000 (-.108) | 1.029*** (7.555) | .410 | .403 |
| 14. | .001 (.203) | .922*** (6.804) | .360 | .353 |
| 15. | -.005 (-.844) | 1.077*** (7.662) | .417 | .410 |

TABLE III (continued)

Regression Results of the Log-Linear Model

|  | Intercept | β | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|
| 16. | -.012* (-1.690) | 1.349*** (7.955) | .435 | .428 |
| 17. | -.013 (-1.784) | 1.070*** (6.223) | .320 | .312 |
| 18. | -.000 (-.119) | .949*** (6.418) | .334 | .326 |
| 19. | -.003 (-.574) | .666*** (5.314) | .256 | .247 |
| 20. | .000 (.125) | .733*** (7.163) | .384 | .377 |
| 21. | .002 (.496) | .936*** (9.319) | .514 | .508 |
| 22. | .005 (.876) | 1.384*** (9.455) | .521 | .515 |
| 23. | -.002 (-.367) | .675*** (5.083) | .239 | .230 |
| 24. | -.026** (-2.476) | 1.118*** (4.647) | .208 | .198 |
| 25. | .003 (.684) | 1.234*** (11.001) | .596 | .591 |
| 26. | -.008 (-1.438) | .770*** (5.705) | .284 | .275 |
| 27. | .001 (.159) | .249*** (3.245) | .113 | .103 |
| 28. | -.005 (-.950) | .806*** (5.842) | .293 | .285 |
| 29. | -.002 (-.283) | .813*** (4.816) | .220 | .211 |

*** Significance at 1% level

TABLE IV

Regression Results of the CES Model

|  | Intercept | β | γ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|

Period 1 (1969–75)

| | Intercept | β | γ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|
| 1. | .001 (.234) | .927*** (7.444) | −1.602 (−1.012) | .418 | .404 |
| 2. | −.005 (−.729) | .973*** (7.528) | 1.675 (1.020) | .412 | .397 |
| 3. | −.000 (0.045) | .722*** (6.939) | 1.492 (1.129) | .374 | .358 |
| 4. | .010 (1.373) | 1.030*** (8.227) | −.831 (−.523) | .461 | .448 |
| 5. | .004 (.750) | 1.129*** (10.850) | −.965 (−0.730) | .598 | .588 |
| 6. | .002 (.312) | 1.092*** (7.172) | −.081 (−.042) | .390 | .375 |
| 7. | .004 (.562) | .710*** (5.548 | −2.379 (−1.464) | .300 | .283 |
| 8. | .005 (.804) | 1.088*** (9.356) | .746 (.505) | .519 | .507 |
| 9. | .007 (1.031) | .845*** (6.801) | −.847 (−.537) | .370 | .354 |
| 10. | .008 (1.648) | .783*** (8.872) | .066 (.059) | .494 | .482 |
| 11. | .002 (.376) | .726*** (7.240) | .796 (.625) | .392 | .377 |
| 12. | .002 (.285) | .934*** (6.837) | −.141 (−.081) | .368 | .353 |
| 13. | −.003 (−.561) | .868*** (7.575) | .303 (.208) | .415 | .401 |
| 14. | .005 (.785) | 1.005*** (7.967) | −2.339 (−1.459) | .457 | .444 |
| 15. | .002 (.294) | 1.051*** (7.878) | −1.434 (−.846) | .443 | .430 |

TABLE IV (continued)

Regression Results of the CES Model

|  | Intercept | β | γ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|
| 16. | .003 (.370) | 1.000*** (6.854) | .782 (.422) | .367 | .351 |
| 17. | .006 (.826) | .834*** (6.289) | .729 (.433) | .328 | .311 |
| 18. | .010 (1.047) | .770*** (4.596) | -4.720** (-2.217) | .258 | .239 |
| 19. | -.004 (-.757) | .564*** (5.157) | .334 (.240) | .247 | .229 |
| 20. | -.001 (-.314) | .853*** (8.566) | .345 (.273) | .476 | .463 |
| 21. | -.001 (-.261) | 1.100*** (11.526) | 1.422 (1.173) | .621 | .611 |
| 22. | -.010 (-.993) | .853*** (5.072) | .256 (.167) | .241 | .223 |
| 23. | .007 (1.139) | .835*** (8.098) | -2.746** (-2.096) | .476 | .463 |
| 24. | .001 (.179) | .854*** (5.465) | -1.534 (-.772) | .280 | .262 |
| 25. | .002 (.217) | 1.009*** (5.366) | -1.086 (-.455) | .268 | .250 |
| 26. | .011* (1.863) | .894*** (8.825) | -2.502* (-1.943) | .513 | .501 |
| 27. | -.001 (-.386) | .669*** (8.640) | 1.651* (1.679) | .482 | .469 |
| 28. | .007 (1.366) | 1.000*** (10.290) | -2.441* (-1.984) | .586 | .575 |
| 29. | -.001 (-.212) | 1.052*** (6.703) | .421 (.211) | .357 | .341 |

TABLE IV (continued)

Regression Results of the CES Model

|  | Intercept | β | γ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|
| Period 2 (1976-82) | | | | | |
| 1. | -.009 (-.834) | 1.242*** (5.569) | -3.673 (-1.175) | .283 | .265 |
| 2. | -.000 (-.148) | .579*** (4.547) | .494 (.277) | .204 | .184 |
| 3. | .004 (.776) | .514*** (4.993) | 1.658 (1.148) | .247 | .228 |
| 4. | -.013** (-2.359) | 1.389*** (12.434) | 2.178 (1.391) | .660 | .652 |
| 5. | -.013** (-2.455) | .734*** (7.063) | 5.431*** (3.727) | .446 | .432 |
| 6. | -.006 (-.705) | 1.252*** (7.680) | -.596 (-.261) | .421 | .407 |
| 7. | -.016** (-2.638) | .933*** (8.106) | 4.362*** (2.702) | .478 | .465 |
| 8. | -.014*** (-2.719) | .879*** (8.655) | 4.009*** (2.814) | .509 | .497 |
| 9. | -.010 (-1.497) | .650*** (4.859) | 2.986 (1.592) | .247 | .228 |
| 10. | -.006 (-1.195) | .560*** (5.545) | 1.946 (1.373) | .290 | .272 |
| 11. | .001 (.323) | .785*** (8.450) | -.820 (-.630) | .469 | .455 |
| 12. | .006 (.728) | 1.116*** (6.923) | -1.421 (-.629) | .372 | .357 |
| 13. | .000 (.071) | 1.030*** (7.518) | -.581 (-.303) | .411 | .396 |
| 14. | -.003 (.461) | .918*** (6.786) | 2.257 (1.191) | .371 | .356 |
| 15. | -.000 (-.037) | 1.982*** (7.725) | -2.515 (-1.280) | .428 | .414 |

TABLE IV (continued)

Regression Results of the CES Model

|  | Intercept | β * | γ | $R^2$ | Adjusted $R^2$ |
|---|---|---|---|---|---|
| 16. | -.022**<br>(-2.491) | 1.340***<br>(8.032) | 4.551*<br>(1.947) | .460 | .447 |
| 17. | -.019**<br>(-2.089) | 1.064***<br>(6.197) | 2.612<br>(1.085) | .330 | .313 |
| 18. | -.005<br>(-.639) | .944***<br>(6.388)· | 2.097<br>(1.012) | .342 | .326 |
| 19. | .001<br>(.172) | .671***<br>(5.364) | -2.176<br>(-1.241) | .270 | .252 |
| 20. | -.001<br>(-.205) | .731***<br>(7.115) | .841<br>(.584) | .387 | .372 |
| 21. | -.006<br>(-1.240) | .928***<br>(9.726) | 4.230***<br>(3.163) | .567 | .557 |
| 22. | .002<br>(.344) | 1.381***<br>(9.405) | 1.532<br>(.745) | .524 | .513 |
| 23. | -.002<br>(-.402) | .675***<br>(5.046) | .327<br>(.175) | .239 | .221 |
| 24. | -.017<br>(-1.363) | 1.128***<br>(4.711) | -4.701<br>(1.401) | .227 | .208 |
| 25. | -.000<br>(-.021) | 1.231***<br>(10.984) | 1.776<br>(1.131) | .602 | .592 |
| 26. | -.015**<br>(-2.176) | .763***<br>(5.728) | 3.310*<br>(1.772) | .310 | .293 |
| 27. | -.001<br>(-.430) | .246***<br>(3.217) | 1.140<br>(1.061) | .125 | .104 |
| 28. | -.014**<br>(-2.014) | .798***<br>(5.919) | 4.227**<br>(2.236) | .335 | .318 |
| 29. | .002<br>(.272) | .817***<br>(4.839) | -2.279<br>(-.963) | .229 | .210 |

*   Significance at 10% level
**  Significance at  5% level
*** Significance at  1% level

## TABLE V

### Summary Statistics of Parameter Estimates

| Translog | Period 1 (1969-75) | | Period 2 (1976-82) | |
|---|---|---|---|---|
| | Mean | Standard Deviation | Mean | Standard Deviation |
| $\beta_2$ | .923 | .181 | .909 | .279 |
| $\beta_3$ | -.202 | 1.471 | -.451 | .751 |
| $\beta_4$ | 1.684 | 8.656 | .148 | 2.087 |
| $\beta_5$ | -.696 | 2.012 | .396 | 2.351 |
| $R^2$ | .433 | | .406 | |
| | | | | |
| **CES** | | | | |
| $\beta*$ | .902 | .145 | .940 | .345 |
| $\gamma$ | -.504 | 1.569 | 1.144 | 2.629 |
| $R^2$ | .410 | | .371 | |
| | | | | |
| **Log-Linear** | | | | |
| $\beta$ | .906 | .146 | .912 | .283 |
| $R^2$ | .403 | | .354 | |

Table VI

Summary Statistics of Differences in Beta Risk Estimates

| | Period 1 | | | Period 2 | | |
|---|---|---|---|---|---|---|
| | Mean | Max | Min | Mean | Max | Min |
| $\beta_2 - \beta*$ | .020 | .180 | -.135 | -.032 | .102 | -.790 |
| $\lvert \beta_2 - \beta* \rvert$ | .069 | .180 | .004 | .069 | .790 | .001 |
| $\beta_2 - \beta$ | .017 | .170 | -.133 | -.003 | .115 | -.134 |
| $\lvert \beta_2 - \beta \rvert$ | .067 | .170 | .000 | .046 | .134 | .006 |
| $\beta - \beta*$ | .004 | .035 | -.013 | -.028 | .011 | -.905 |
| $\lvert \beta - \beta* \rvert$ | .009 | .035 | .001 | .035 | .905 | .000 |

TABLE VII

Summary of F Statistics

|  | Translog vs. CES | | Translog vs. Log-Linear | |
|---|---|---|---|---|
|  | Period 1 | Period 2 | Period 1 | Period 2 |
| 1. | 3.35* | .29 | 7.67* | 1.99 |
| 2. | 4.74* | 1.09 | 10.41* | 2.19 |
| 3. | .64 | 1.12 | 2.14 | 3.36 |
| 4. | 1.99 | 1.18 | 3.98* | 3.80 |
| 5. | .21 | 1.09 | .84 | 15.91* |
| 6. | .09 | 1.08 | .01 | .01 |
| 7. | 1.44 | .21 | 4.91* | 7.46* |
| 8. | 2.13 | 1.45 | 4.27* | 11.03* |
| 9. | 2.68 | 2.57 | 5.37* | 7.55* |
| 10. | 1.20 | 1.76 | 1.80 | 5.30* |
| 11. | 2.15 | .68 | 4.31* | 1.36 |
| 12. | .87 | 2.99 | 1.50 | 6.46* |
| 13. | .35 | 4.06* | .35 | 8.13* |
| 14. | 1.49 | 3.43* | 5.06* | 8.24* |
| 15. | 4.59* | 2.33 | 9.76* | 6.22* |
| 16. | .32 | .20 | .64 | 3.96* |
| 17. | 2.93 | 1.41 | 5.86* | 3.84 |
| 18. | 3.07 | 1.94 | 11.23* | 4.71* |
| 19. | .38 | .18 | .77 | 1.85 |
| 20. | 1.92 | 2.61 | 3.37 | 5.22* |
| 21. | 1.93 | 2.14 | 3.61 | 15.66* |
| 22. | 3.05 | .82 | 5.91* | 1.91 |

TABLE VII (continued)

Summary of F Statistics

| | Translog vs. CES | | Translog vs. Log-Linear | |
|---|---|---|---|---|
| | Period 1 (1969-75) | Period 2 (1976-82) | Period 1 (1969-75) | Period 2 (1976-82) |
| 23. | .87 | 6.34* | 5.70* | 12.73* |
| 24. | 2.06 | 9.49* | 4.72* | 21.37* |
| 25. | .85 | .70 | 1.84 | 2.33 |
| 26. | 2.11 | 5.56* | 7.99* | 14.46* |
| 27. | .38 | 4.32* | 3.87 | 8.65* |
| 28. | .24 | 2.35 | 3.92* | 9.74* |
| 29. | .37 | 2.92 | .74 | 6.51* |

*Significant at 5 percent level.