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FACULTY WORKING PAPER NO. 1174

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August, 1985

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Abstract

In this paper, the issue related to the impact of investment horizon on the CAPM functional form has been re-examined. A generalized translog functional form for CAPM was derived in accordance with the true investment horizon that is finite and unobservable. The securities included in the Dow Jones Industrial Index are used to test whether the risk-return relationship generally follows trans-log type of CAPM. It is found that the translog model improves the precision of estimated parameters the explanatory power of the capital asset pricing model.

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Introduction

The traditional Capital Asset Pricing Model (CAPM) provides a theoretical and empirical foundation for examining risk-return relationship and measuring investment performance. Despite recent criticisms, the CAPM remains as the cornerstone of modern financial theory. Academic scholars and finance practitioners have used the CAPM extensively to estimate systematic risk from realized security returns for various purposes such as market efficiency testing and capital budgeting. However, it is known that the empirical results of the CAPM from ex post data may not be consistent with the ex ante expectation of the model. In particular, a problem of model misspecification can occur in estimating systematic risk when the period length of observed data deviates from the true investment horizon of individual investors. Also, as pointed out by Markowitz [19], the CAPM is based on a simultaneous linear equations constraint set. Extending this constraint into more general linear programming constraint set is non-negative variables will generate an efficient frontier which is very likely nonlinear. These considerations have motivated the investigation of possible nonlinearities in the CAPM.

Jensen [14] first investigated the impact of deviation of observed data period from true investment horizon on the estimation of systematic risk. He proposed a logarithmic linear model to eliminate the impact of time horizon. Kraus and Litzenberger [16] have proposed a quadratic characteristic line for a three-parameter CAPM model. Subsequent studies (see [9], [11], [17] and [18]) have aimed at extending or generalizing Jensen's model to consider the nonlinearity in the CAPM with only limited success. Recently, McDonald [20] provided an extensive analysis of the functional form of the CAPM. He proposed a model of variable elasticity of substitution (VES) to investigate the nonlinearity in the CAPM. His study, based on a very large data set and a sophisticated maximum likelihood method, concluded that the VES model did not significantly improve the estimation of beta coefficients in comparison to Lee's [17] constant elasticity of substitution (CES) model. Further, his study found that the nonlinearity in the CAPM could not solely be attributed to the investment horizon problem.

One important issue, however, was not fully explored in McDonald's study. Although it was understood that the existence of investment horizon problem could produce biased risk estimates, the extent of this bias and the factors affecting the Jensen measure and risk estimations has not been explicitly analyzed. Moreover, the discrepancy between the true and observed investment horizons would likely generate some statistical problems and affect the functional relation of risk and return. As noted by Box and Cox [4], these statistical problems can affect the observed return distribution. More specifically, the moments of the observed return distribution would likely be affected by the data measurement problems, even though the true return distribution remains intact. Yet, the linkage between these statistical issues and the functional relation of risk and return involving higher moments has not been fully investigated.

The purpose of this paper is twofold. First, it introduces a generalized model for the nonlinear risk and return relation .[1] The proposed model includes many familiar asset pricing models as special cases. The paper specifically shows the misspecification of the traditional characteristic lines, and demonstrates how the specification of a more appropriate functional form can help reduce the bias of Jensen measure and risk estimates and improve the explanatory power of the CAPM. Second, this paper provides some empirical evidence obtained from a translog model based on a general nonlinear relation of risk and return. There are several advantages of using this model. First, the translog model provides a generalized functional form that is a local second-order approximation to any nonlinear relationship.[2] Second, the translog model permits greater substitution among variables than many other models. Third, the model can be estimated and tested by relatively straightforward regression methods.

The remainder of this paper is divided into three sections. Section I develops an estimate model for the return and risk relation. This model is then compared to the other risk estimation models often used by academicians and practitioners. Section II discusses data and presents some empirical evidence. The results from several models are compared. Section III summarizes the important findings.

I. The Model

Following Jensen [14] and Lee [17], the risk-return relationship implied by the traditional asset pricing model can be written as

(1) $$E({}_{H}R_{j}) = (1-{}_{H}\beta_{j}){}_{H}R_{f} + {}_{H}\beta_{j}E({}_{H}R_{m})$$

where

$E({}_{H}R_{j})$ = 1 + the expected return on security j over a true investment horizon H.

$E({}_{H}R_{f})$ = 1 + the expected return on a risk-free security over a true investment horizon H.

$E({}_{H}R_{m})$ = 1 + the expected return on the market portfolio over a true investment horizon H.

${}_{H}\beta_{j}$ = the systematic risk of security j in terms of true investment horizon.

Since the observed ex post returns may deviate from the ex ante returns, in general the following relationship holds:

(2) $$E({}_{H}R_{k}) = [E({}_{N}R_{k})]^{\lambda} \quad \text{for } k = j,\ f, \text{ and } m,$$

where $\lambda = \frac{H}{\lambda}$ is a transformation parameter and N is the period when the returns are observed. Substituting (2) into (1) yields

(3) $$E({}_{N}R_{j})^{\lambda} = (1-{}_{H}\beta_{j}){}_{N}R_{f}^{\lambda} + {}_{H}\beta_{j}E({}_{N}R_{m})^{\lambda}.$$

If the values of transformation parameter λ are allowed to vary with each of the security returns, then equation (3) can be rewritten as

(4) $$E({}_{N}R_{j})^{\lambda_{j}} = (1-{}_{H}\beta_{j}){}_{N}R_{f}^{\lambda_{f}} + {}_{H}\beta_{j}E({}_{N}R_{m})^{\lambda_{m}}.$$

Equation (4) is a nonhomogeneous function which permits individual securities to have different transformation parameters. This implies that true investment horizon might be different among individual securities. As Box and Cox [4] indicate, this type of transformation can correct simultaneously for nonadditivity, nonconstant variance and nonnormalities of security returns.

Equation (4) can be approximated by the following translog function (see the Appendix for the derivation):

$$\text{(5)} \qquad \ln E({}_N R_j) = \beta_{1j} \ln {}_N R_f + \beta_{2j} \ln E({}_N R_m) + \beta_{3j} \ln {}_N R_f \ln E({}_N R_m) + \beta_{4j} [\ln {}_N R_f]^2 + \beta_{5j} [\ln E({}_N R_m)]^2$$

As usual, the coefficients can be normalized for convenience in approximating any arbitrary function. A convenient normalization is that $\beta_{1j} + \beta_{2j} = 1$. Given this normalization, equation (5) can be restated in terms of excess returns:

$$\text{(6)} \qquad \ln E({}_N R_j) - \ln {}_N R_f = \beta_{2j} [\ln E({}_N R_m) - \ln {}_N R_f] + \beta_{3j} \ln {}_N R_f \ln E({}_N R_m) + \beta_{4j} [\ln {}_N R_f]^2 + \beta_{5j} [\ln E({}_N R_m)]^2$$

The parameters of equation (6) are related to the original parameters in equation (4) in the following manner:

$$\text{(6a)} \qquad \beta_{2j} = {}_H\beta_j \frac{\lambda_m}{\lambda_j}$$

$$\text{(6b)} \qquad \beta_{3j} = -{}_H\beta_j (1 - {}_H\beta_j) \frac{\lambda_f \lambda_m}{\lambda_j}$$

$$\text{(6c)} \qquad \beta_{4j} = 1/2 \; {}_H\beta_j (1 - {}_H\beta_j) \frac{\lambda_f^2}{\lambda_j}$$

$$\text{(6d)} \qquad \beta_{5j} = 1/2 \; {}_H\beta_j (1 - {}_H\beta_j) \frac{\lambda_m^2}{\lambda_j} \, .$$

Equation (6) states that the excess return of an asset depends not only on the excess return of the market portfolio but also on a multiplicative term involving $\ln {}_N R_f$ and $\ln {}_N R_m$, and the squared terms of the returns on the riskless asset and market portfolio. Inclusion of the multiplicative term permits greater substitution between the riskless asset and market portfolio. The squared terms capture the effect of systematic skewness as Kraus and Litzenberger [16] suggested. The proposed relation in (6) is more general, however, since the co-skewness of individual securities with the riskless asset is also permitted.

To see the above argument more clearly, take the derivation of (6) with respect to $\ln {}_N R_f$ and $\ln E({}_N R_m)$:

$$\text{(6e)} \qquad \frac{\partial \ln E({}_N R_j)}{\partial \ln {}_N R_f} = 1 - \beta_{2j} + \beta_{3j} \ln E({}_N R_m) + 2\beta_{4j} \ln {}_N R_f$$

$$\text{(6f)} \qquad \frac{\partial \ln E({}_N R_j)}{\partial \ln E({}_N R_m)} = \beta_{2j} + \beta_{3j} \ln {}_N R_f + 2\beta_{5j} \ln E({}_N R_m).$$

Equations (6e) and (6f) measure the elasticity of $E({}_N R_j)$ with respect to ${}_N R_f$ and $E({}_N R_m)$ respectively. This contrasts to the log-linear CAPM where the corresponding values are $(1-\beta_{2j})$ and β_{2j}.

An estimate model derived from equation (6) in terms of ex post return can be defined as

$$\text{(7)} \qquad \ln R_{jt} - \ln R_{ft} = \beta_{0j} + \beta_{2j}(\ln R_{mt} - \ln R_{ft}) + \beta_{3j} \ln R_{mt} \ln R_{ft} + \beta_{4j}(\ln R_{ft})^2 + \beta_{5j}(\ln R_{mt})^2 + \varepsilon_{jt}.$$

where, by analogy to the production function, β_{0j} is the efficiency term; $(1-\beta_{2j})$ and β_{2j} are the distribution parameters for $\ln R_{ft}$ and ln R_{mt}; β_{3j}, β_{4j} and β_{5j} are substitution parameters; and ε_{jt} is the stochastic disturbance term. Equation (7) is a nonlinear characteristic line for j^{th} security. Kraus and Litzenberger [16a, 16b] have shown that different types of characteristic line imply different types of CAPM. Therefore, our analyses in this section on different types of characteristic line can be regarded as analyzing different types of CAPM.

Similar to the traditional Jensen's measure, β_{0j} can be used to evaluate the performance of an individual security. Also, if equation (7) is a correct risk-return relation, then omiting the multiplicative and squared term will mean that residuals contain these effects. Therefore, the residuals from the traditional log-linear or linear market model may exhibit hetroscedasticity as shown in Giaccotte and Ali [13].

The assumption on the differential transformation parameters is consistent with the theory of rational choices. For instance, different transformation parameters may be associated with different true investment horizon of individual assets. Investors' determination of optimal investment horizon is generally affected by several crucial factors. The first important factor is transaction cost. The existence of considerable economies of scale implies that large investors are likely to have a shorter investment horizon than small investors. The increasing difference between the transaction costs of large and small investors has been evidenced since the commission rate deregulation in May 1975.

The heterogeneity in investor's expectation may also affect the optimal investment horizon.[3] Investors who possess different forecasts concerning the future prospects of individual assets will likely invest in different securities and have different insights on the market and individual securities timing. Thus, the securities owned by different investor groups possibly will exhibit distinguished investment horizons.

Another factor that plays an important role is the size of the issuing firms. The size of firm is related to the amount of information available to investors. As noted by Zeghal [27] and Barry and Brown [2], more information is produced and disseminated about large firms by external producers of information, such as brokers, financial analysts, institutional investors, and business writers. Also larger firms provide more information than smaller firms due to the economies of scale in the production and dissemination of information. The amount of information is likely to affect the optimal investment horizon. It is not implausible to assume that investors owning the securities of larger firms to have a shorter investment horizon. Given more information available, large firms' securities will generally have higher liquidity and their market values will be closer to the true values. These are the advantages that may help investors shorten the time needed to complete a transaction. In the light of equation (6a), the effect of firm size on the beta estimates can be analyzed. It can be argued that $\frac{\lambda_m}{\lambda_j}$ will be larger than or equal to one for large firms, and will generally be smaller than one for small firms. Therefore, when β is estimated using the traditional market model, it will tend to be overestimated for large firms and underestimated for small firms. In general,

(8) $\beta_{2i} > \beta_{2j}$ if $\frac{\lambda_m}{\lambda_i} > \frac{\lambda_m}{\lambda_j}$,

even though ${}_H\beta_i = {}_H\beta_j$. And, $\frac{\lambda_m}{\lambda_i}$ is an important information for systematic risk estimates. Thus the anomaly of higher average returns to small firm securities unaccounted for by the estimates of systematic risk (e.g. Reinganum [23]) may be attributed to the specification bias of the traditional linear characteristic line.

The translog function provides a generalized model for examining the risk-return relationship. It can be shown that the characteristic lines used in several previous studies to estimate systematic risks are all special cases of the proposed translog relationship. In the following, various restrictions are imposed on the translog function to derive some characteristic functions most familiar in the literature.

(A) $\beta_{3j} = \beta_{4j} = 0$

Equation (5) reduces to the following estimate model:

(9) $\ln R_{jt} = \beta^*_{0j} + \beta_{1j}\ln R_{ft} + \beta_{2j}\ln R_{mt} + \beta_{5j}(\ln R_{mt})^2 + e_i.$

The above estimate model is very similar to the quadratic characteristics line proposed by Kraus and Litzenberger [16a]. If the compounding rates are used in their model, the risk estimates in (9) will be the same as those obtained from the quadratic characteristic line.[4]

(B) $\beta_{3j} = \beta_{4j} = \beta_{5j} = 0$

These restrictions result in the familiar two index model:

(10) $\ln R_{jt} = \beta_{0j} + \beta_{1j} \ln R_{ft} + \beta_{2j} \ln R_{mt} + v_{jt}.$

As noted by Merton [22], the interest rate changing stochastically over time affects the investment opportunity set. Therefore, investors are compensated in terms of expected returns for bearing market systematic risk and for bearing the risk of unfavorable shifts in the investment opportunity set.

(c) $1/2\ \beta_{3j} = \beta_{4j} = \beta_{5j}$

This is equivalent to imposing a homogeneity condition on the risk-return relation; that is, $\lambda_m = \lambda_f$. With these restrictions, the following estimate model is obtained:[5]

(11) $\ln R_{jt} - \ln R_{ft} = \alpha_j + \beta^*_j(\ln R_{mt} - \ln R_{ft}) + \gamma_j(\ln R_{mt} - \ln R_{ft})^2 + w_{jt}.$

This is the CES model proposed in Lee [17]. The γ_j is the systematic skewness coefficient in Lee [17]. The constant investment horizon parameters imply a symmetry restriction on the translog function.

(D) Constant R_f and stationary return distribution

Under this condition, equation (4) can be simplified to generate the following estimate model:

(12) $R_{jt}^{\lambda_j} = \alpha + {}_H\beta_j R_{mt}^{\lambda_m} + v'_{jt}.$

When $\lambda_j = \lambda_m = 1$, the traditional linear market model is obtained. When λ_j and λ_m both approach zero, it can be shown that equation (12) becomes the log-linear market model (see McDonald [20], Spitzer [25]).

(E) Nonstationary return distribution

Kraus and Litzenberger [16a] have demonstrated that under a changing riskless rate, the moments of risky assets return are not intertemporal constants. However, under the assumptions of proportional stochastic growth and either constant relative risk aversion or stationary distribution of per capital end-of-period wealth, they suggested a transformed return variable to resolve the nonstationarity problem.[6] This variable is essentially a deflated excess return defined as

(13a) $r_{jt} = (R_{jt}-R_{ft})/R_{ft}$

(13b) $r_{mt} = (R_{mt}-R_{ft})/R_{ft}$

The transformed variables can be used to estimate the beta systematic risk by the following regression:

(14) $r_{jt} = \alpha_j + \beta_j r_{mt} + u'_{jt}.$

However this function can be shown to be equivalent to the case when $\lambda_j = \lambda_f = \lambda_m = 1$ in equation (4).[7]

In sum, by imposing various restrictions on the transformed function and the translog function, several familiar characteristic lines can be obtained. Thus the proposed risk-return relation as in equation (4) and approximated by equation (6) provides a generalized model to estimate systematic risks when there exists nonstationary return distribution and when investors' utility function involves the higher moment such as skewness.

In the following section, data used to estimate systematic risks are described and some empirical results are reported. In addition, the performance of the translog model is compared with that of the alternative models.

II. Data and Empirical Results

Monthly returns for all the securities included in Dow Jones Industrial Average (DJIA) covering the period 1969-82 were collected from the Compustat tape. One security, American Express, was finally excluded from the sample due to missing observations in the earlier years. The study period was further divided into two subperiods. This resulted in a data base of 29 securities, each with 84 monthly observations. The market rate of return used is the New York Stock Exchange monthly value-weighted index. The monthly treasury bill rate was used as a proxy for the risk-free rate.

The correlation coefficient matrix for the explanatory variables in the translog model is displayed in Table I. In general, the correlation coefficients are fairly stable over time. The sign of the correlation between variables is consistent in two periods with only one exception (the correlation between $\ln R_{ft} \ln R_{mt}$ and $\ln R_{ft}^2$). As shown in the table, the variable $\ln R_{ft} \ln R_{mt}$ is highly correlated with the excess return of the market portfolio (.98 and .94 for the first and second period, respectively). This problem can be attributed to the fact that R_{ft} is smaller and relatively stable over time. The extremely high correlation between these two variables causes a very severe multicollinearity problem. To cope with this problem, the variable $\ln R_{mt} \ln R_{ft}$ is orthogonalized. This procedure involves regressing $\ln R_{mt} \ln R_{ft}$ against the excess market return and obtaining residuals from the regression. The residuals (RES) retain a portion of information in $\ln R_{mt} \ln R_{ft}$ that is not correlated with the excess market return. This residual variable is then used to estimate the coefficient of β_{3j} in equation (7).

The results of the translog regressions are reported in Table II.[8] All the estimates of β_2 coefficients are significant. In addition, the coefficients associated with the quadratic terms are also significant for numerous cases.

In the first period, there are five securities with significant β_3, five securities with significant β_4 and six securities with significant β_5. Out of 29 securities examined, 15 securities have at least one significant coefficient associated with the multiplicative and squared terms. The translog regression results in the second period even perform better. There are 13 securities with significant β_3, five with significant β_4 and five securities with significant β_5. All together, 18 securities have at least one of these three coefficients that are significant. For those securities with significant coefficients associated with the multiplicative and squared terms, the estimation of systematic risks and Jensen performance measures (β_0's) using the traditional CAPM is subjected to specification bias.

The results of the log-linear characteristic line are reported in Table III for comparison. Note that the values of R-square are much larger for the translog regressions. In general, the values of adjusted R-square will increase when the Student's t for the additional parameter introduced is larger than one. For most of the securities included, at least one of the estimated coefficients associated with the higher power terms have the student's t values greater than one. Thus, the proposed translog model improves the explanatory power of the CAPM. Therefore, it will be a more appropriate model for forecasting the security rates of return.

In parallel to McDonald's study, the results of the CES model as approximated by equation (11) are reported in Table IV. As noted earlier, the CES model proposed by Lee [17] is equivalent to the translog model with the restrictions of $\beta_4 = \beta_5 = -1/2\beta_3$. These restrictions are required to satisfy the condition of homogeneity. Table IV shows that most of the β estimates are significant, while the γ coefficients are significant for five and seven cases for the first and second period, respectively. The results are similar to those found in Lee [17] and McDonald [20].

Table V provides the summary statistics of the parameter estimates. For the translog model, the standard deviations of the cross-sectional β_4 and β_5 are relatively higher. The greater dispersion of β_4 and β_5 estimates is attributed to the smaller values of squared terms. Similar to the mean values of β^* and β in the CES and log-linear models, the average of β_2 is close to one as expected. Therefore, the minor differences in the mean values for β_2, β^* and β do not convey significant information for the corresponding risk estimates for individual securities. To provide more details on the differences of systematic risks (β_2, β^* and β) for individual securities, Table VI summarizes the mean, maximum and minimum of the differences and absolute difference in the estimated coefficients. The discrepancy between β

and β^* appear inconsequential. However, the deviations of β_2 from β and β^* are larger. The mean absolute difference, a better measure for bias, indicate that on average the deviations are around .05 to .07, roughly 5 percent to 7.5 percent errors. For individual securities, the maximum difference is around .13 to .79. These figures appear to be not trivial.

The intercept estimates from these alternative model are now compared and analyzed. For the log-linear model, there is only one and three estimated intercepts significant different from zero for the first and second period, respectively. For the CES model, there are one and eight estimated intercepts significantly different from zero and for the translog model, these are four and seven estimated intercepts significantly different from zero. The number of significant intercepts is substantially smaller for the log-linear model. These figures suggest that the Jensen measures estimated from the traditional log-linear market model are likely biased.

To complete the analysis , two likelihood ratio tests are performed. The first test concerns whether the proposed translog model satisfies the homogeneity condition. The second test checks whether the translog model is significantly different from the log-linear market model. Test statistics are reported in Table VII. To perform the first test, the restrictions that $\beta_4 = \beta_5 = -1/2\beta_3$ are imposed. The critical value for F(2, 79, 5%) is equal to 3.15. Out of 29 securities, three and six securities indicate significant differences from homogeneity for the first and second period, respectively. To perform the second test, the denominator and numerator of the ratios of squared residual errors are divided by the associated degree of freedom. The critical F value corresponding to the restriction that the sum of coefficients β_3 to β_5 is zero is equal to 3.92. Out of 29 securities, there are 15 and 19 cases in two respective periods that have F values exceed the critical value. Consequently, for a large proportion of securities studied in this sample, the translog model provides results that are significantly different from the log-linear model. These results tend to support Markowitz's [19] arguments about the nonlinear CAPM.

III. Summary and Concluding Remarks

In this paper, the issue related to the impact of investment horizon on the CAPM functional form has been re-examined. A generalized translog functional form for CAPM was derived in accordance with the true investment horizon that is finite and unobservable. The securities included in the Dow Jones Industrial Index are used to test whether the risk-return relationship generally follows trans-long type of CAPM. It is found that the translog model improves the precision of estimated parameters and the explanatory power of the capital asset pricing model. For a large number of securities, there are systematic risks associated with the multiplicative and squared terms of returns on the riskless asset and market portfolio. The linearity assumption of the CAPM has been rejected for a very large proportion of securities studied in this paper. It is also found that for most of the securities studied the proposed translog model appears to satisfy the condition of homogeneity given symmetry. Implications of the new model derived in this paper to test the capital asset pricing model will be done in the future research.

Footnotes

[1]Jensen [14, 15], Black, Jensen and Scholes [3], Merton [21, 22] have discussed the importance of investment horizon on the capital asset pricing process.

[2]The translog function does not employ additivily and homogeneity as part of the maintain hypothesis. For many production and investment frontiers employed in the econometric studies, the translog frontiers provide accurate global approximations.

[3]Elton and Gruber [10] have recently discussed the effect of heterogeneous expectations on the form of the CAPM. Markowitz [19] also has shown that hetrogeneous expectations is an important issue for testing the CAPM.

[4]Kraus and Litzenberger [16a] proposed the following quadratic characteristic line:

$$R_i - R_f = c_{0i} + c_{1i}(R_m - R_f) + c_{2i}(R_m - \bar{R}_m)^2 + e_i.$$

This function can be simplified as

$$R_i = b_0 + b_1 R_f + b_2 R_m + b_3 R_m^2$$

where $b_0 = c_{0i} + c_{2i}\bar{R}_m^2$

$b_1 = 1 - c_{1i}$

$b_2 = c_{1i} - 2c_{2i}\bar{R}_m$

$b_3 = c_{2i}.$

[5]If $\lambda_j = \lambda_m = \lambda_f = \lambda$, equation (11) can also be obtained by expanding log $E({}_N R_j)$ around $\lambda=0$, and dropping the terms involving powers of $-\lambda$ greater than one.

[6]Rubinstein [24] has also shown that the expected value of transformed variable would be constant over time under the similar conditions.

[7]When λ's are the same, equation (4) can be rewritten as

$$\frac{E({}_H R_j)^{\lambda_j}}{{}_H R_f^{\lambda_f}} = (1-{}_H\beta_j) + \frac{E({}_H R_m)^{\lambda_m}}{{}_H R_f^{\lambda_f}}({}_H\beta_j)$$

or

$$\frac{E({}_N R_j)^{\lambda_j} - {}_H R_f^{\lambda_f}}{{}_H R_f^{\lambda_f}} = {}_H\beta_j \quad \frac{E({}_H R_m)^{\lambda_m} - {}_H R_f^{\lambda_f}}{{}_H R_f^{\lambda}}$$

Setting $\lambda_m = \lambda_f = \lambda_j = 1$ yields:

$$\frac{E({}_H R_j) - {}_H R_f}{{}_H R_f} = \beta\left(\frac{E({}_H R_m) - {}_H R_f}{{}_H R_f}\right)$$

[8]The values included in the parentheses in all the tables are t statistics.

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Appendix

Taking the logarithm of (4) yields the following generalized risk and return relationship:

(A.1) $$\lambda_j \ln E({}_NR_j) = \ln[(1-\beta) \exp(\lambda_f \ln_N R_f) + \beta \exp(\lambda_m \ln E({}_NR_m)].$$

Let

(A.2) $$\phi = \ln E({}_NR_j) = \frac{1}{\lambda j} \ln[(1-\beta) \exp(\lambda_f \ln_N R_f) + \beta \exp(\lambda_m \ln E({}_NR_m)].$$

Then,

(A.3.1) $$\frac{\partial\phi}{\partial \ln_N R_f} = \frac{\lambda_f}{A\lambda_j} [(1-\beta) \exp(\lambda_f \ln_N R_f)]$$

where $A = (1-\beta) \exp(\lambda_f \ln_N R_f) + \beta \exp(\lambda_m \ln E({}_NR_m))$

(A.3.2) $$\frac{\partial\phi}{\partial \ln E({}_NR_m)} = \frac{\lambda_m}{A\lambda_j} [\beta \exp(\lambda_m \ln E({}_NR_m))]$$

(A.3.3) $$\frac{\partial^2\phi}{\partial \ln_N R_f^2} = \frac{\lambda_f^2}{A^2\lambda_j} [(1-\beta) \exp(\lambda_f \ln_N R_f)][\beta \exp(\lambda_m \ln_N R_f)]$$

(A.3.4) $$\frac{\partial^2\phi}{\partial \ln E({}_NR_m)^2} = \frac{\lambda_m^2}{A^2\lambda_j} [(1-\beta) \exp(\lambda_f \ln_N R_f)][\beta \exp(\lambda_m \ln E({}_NR_m))]$$

(A.3.5) $$\frac{\partial^2\phi}{\partial \ln_N R_f \ln E({}_NR_m)} = \frac{\lambda_f\lambda_m}{A^2\lambda_j} [\beta \exp(\lambda_m \ln E({}_NR_m))][(1-\beta) \exp(\lambda_f \ln_N R_f)]$$

Expanding ϕ around $R_f = 1$ and $R_m = 1$ gives the following relation:

(A.4) $$\phi \approx (1-{}_H\beta_j)\frac{\lambda_f}{\lambda_j}\ln {}_NR_f + {}_H\beta_j \frac{\lambda_m}{\lambda_j}\ln E({}_NR_m)$$

$$- {}_H\beta_j(1-{}_H\beta_j)\frac{\lambda_f\lambda_m}{\lambda_j}\ln_N R_f \ln E({}_NR_m)$$

$$+ 1/2\,\frac{\lambda_f^2}{\lambda_j}(1-{}_H\beta_j){}_H\beta_j(\ln_N R_f)^2 + 1/2\,\frac{\lambda_m^2}{\lambda_j}\,{}_H\beta_j(1-{}_H\beta_j)(\ln E({}_NR_m))^2$$

TABLE I

Correlation Matrix of Independent Variables

	$\ln R_{mt}-\ln R_{ft}$	RES	$\ln R^2_{ft}$	$\ln R^2_{mt}$	$\ln R_{ft}\ln R_{mt}$
Period 1					
$\ln R_{mt}-\ln R_{ft}$	1				
RES	.00	1			
$\ln R^2_{ft}$	-.36	.15	1		
$\ln R^2_{mt}$	.05	-.13	.28	1	
$\ln R_{ft}\ln R_{mt}$	.98	.18	.32	.02	1
Period 2					
$\ln R_{mt}-\ln R_{ft}$	1				
RES	.00	1			
$\ln R^2_{ft}$	-.26	.12	1		
$\ln R^2_{mt}$	.25	-.32	.08	1	
$\ln R_{ft}\ln R_{mt}$	.94	.33	-.21	.13	1

TABLE II

Regression Results of the Translog Model

	β_0	β_2	β_3	β_4	β_5	R^2	Adjusted R^2
<u>Period 1</u> (1969-75)							
1.	-0.033** (-2.135)	1.077*** (8.171)	-0.051 (-0.042)	14.976** (2.512)	-2.976* (-1.791)	.463	.436
2.	-0.026 (-1.651)	1.025*** (7.603)	2.899** (2.309)	8.258 (1.354)	1.497 (0.878)	.472	.445
3.	-0.004 (-0.310)	0.718*** (6.302)	0.589 (0.810)	1.486 (0.289)	1.532 (1.063)	.381	.349
4.	-0.014 (-0.869)	1.128*** (8.367)	0.735 (0.586)	10.116* (1.659)	-1.614 (-0.947)	.485	.459
5.	0.012 (0.892)	1.111*** (9.717)	0.400 (0.376)	-3.263 (-0.631)	-0.593 (-0.411)	.600	.580
6.	0.000 (0.040)	1.100*** (6.561)	0.064 (0.042)	0.847 (0.112)	-0.146 (-0.069)	.390	.360
7.	0.026 (1.594)	0.650*** (4.696)	1.278 (0.992)	-9.608 (-1.534)	-1.264 (-0.722)	.323	.289
8.	-0.020 (-1.353)	1.177*** (9.410)	-0.518 (-0.445)	11.029* (1.949)	-0.357 (-0.226)	.541	.518
9.	0.025 (1.595)	0.782*** (5.891)	2.524** (2.042)	-7.985 (-1.330)	0.370 (0.221)	.407	.377
10.	0.015 (1.318)	0.755*** (7.846)	1.038 (1.158)	-2.952 (-0.678)	0.538 (0.442)	.504	.479
11.	0.023* (1.802)	0.636*** (5.908)	1.175 (1.172)	-9.100* (-1.808)	1.859 (1.365)	.423	.393
12.	0.020 (1.169)	0.867*** (5.809)	0.034 (0.025)	-7.839 (-1.162)	0.579 (0.307)	.379	.348
13.	0.004 (0.270)	0.833*** (6.628)	0.656 (0.561)	-3.476 (-0.611)	0.750 (0.472)	.419	.390
14.	-0.017 (-1.056)	1.120*** (8.196)	-0.832 (-0.654)	9.979 (1.614)	-3.423* (-1.980)	.475	.449
15.	-0.041** (-2.505)	1.231*** (8.849)	-0.231 (-0.179)	18.666*** (2.965)	-3.189** (-1.811)	.500	.475

TABLE II (continued)

Regression Results of the Translog Model

	β_0	β_2	β_3	β_4	β_5	R^2	Adjusted R^2
16.	-0.009 (-0.494)	1.039*** (6.490)	-0.286 (-0.192)	5.423 (0.749)	0.235 (0.116)	.371	.339
17.	-0.021 (-1.278)	0.936*** (6.640)	-2.335* (-1.778)	12.371* (1.939)	-0.852 (-0.478)	.373	.341
18.	-0.013 (-0.646)	0.922*** (5.182)	-3.654** (-2.205)	11.020 (1.369)	-6.463*** (-2.871)	.310	.275
19.	-0.014 (-1.008)	0.595*** (4.962)	0.401 (0.359)	3.929 (0.724)	0.056 (0.037)	.255	.217
20.	0.015 (1.235)	0.785*** (7.302)	-0.658 (-0.658)	-7.367 (-1.515)	0.890 (0.654)	.497	.471
21.	0.013 (1.095)	1.027*** (9.888)	0.279 (0.289)	-6.442 (-1.370)	2.071 (1.576)	.630	.611
22.	0.026 (1.244)	0.718*** (4.016)	-1.887 (-1.134)	-15.071* (-1.864)	1.364 (0.603)	.296	.259
23.	-0.001 (-0.139)	0.901*** (8.001)	-0.987 (-0.942)	3.970 (0.779)	-3.315** (-2.326)	.485	.459
24.	-0.012 (-.622)	0.932*** (5.549)	-3.025* (-1.935)	6.619 (0.872)	-2.737 (-1.288)	.315	.280
25.	-0.013 (-0.569)	1.088*** (5.304)	-2.127 (-1.115)	7.374 (0.795)	-2.180 (-0.841)	.282	.246
26.	0.034*** (2.661)	0.834*** (7.659)	0.544 (0.537)	-9.977** (-2.026)	-1.498 (-1.088)	.537	.516
27.	-0.009 (-0.887)	0.678*** (8.001)	-0.479 (-0.607)	3.148 (0.821)	1.278 (1.191)	.487	.460
28.	0.010 (0.839)	1.014*** (9.531)	-0.444 (-0.449)	-1.073 (-0.223)	-2.441* (-1.814)	.587	.567
29.	-0.010 (0.503)	1.082*** (6.284)	-1.216 (0.758)	3.792 (0.487)	-0.159 (-0.073)	.363	.330

TABLE II (continued)

Regression Results of the Translog Model

	β_0	β_2	β_3	β_4	β_5	R^2	Adjusted R^2
Period 2 (1976-82)							
1.	-.002	1.241***	.544	-1.436	-2.555	.288	.252
	(-.444)	(5.037)	(.639)	(-.625)	(-.721)		
2.	-.001	.607***	-.705	.472	-.781	.235	.196
	(-.149)	(4.359)	(-1.464)	(.363)	(-.390)		
3.	.008	.488***	-.517	-.436	.953	.265	.228
	(1.017)	(4.328)	(1.327)	(-.414)	(.587)		
4.	-.023***	1.412***	-.156	1.694	1.594	.669	.652
	(-2.638)	(11.553)	(-.371)	(1.486)	(.907)		
5.	-.011	.658***	-.585	-.076	4.664***	.457	.429
	(-1.355)	(5.767)	(-1.483)	(-.072)	(2.841)		
6.	-.006	1.257***	.185	.007	-.309	.422	.392
	(-.500)	(6.956)	(.297)	(.004)	(-.119)		
7.	-.015	.874***	-.430	.022	3.794**	.482	.456
	(1.599)	(6.871)	(-.978)	(.019)	(2.074)		
8.	-.024**	.880***	-.410	1.732*	3.056*	.537	.503
	(-2.938)	(7.962)	(-1.075)	(1.679)	(1.921)		
9.	-.016	.683***	-1.127**	1.524	.846	.293	.257
	(-1.531)	(4.754)	(-2.267)	(1.136)	(.409)		
10.	-.011	.585***	-.642*	1.172	.663	.317	.283
	(-1.414)	(5.327)	(-1.691)	(1.144)	(.420)		
11.	.006	.759***	.316	-.926	-.100	.478	.451
	(.823)	(7.436)	(.897)	(-.972)	(-.069)		
12.	.010	1.048***	1.441**	-1.462	1.286	.416	.386
	(.812)	(6.082)	(2.419)	(-.909)	(.519)		
13.	.001	.969***	1.399***	-.863	1.945	.463	.436
	(.103)	(6.686)	(2.792)	(-.638)	(.933)		
14.	-.014	.994***	-1.110**	2.481*	-.106	.420	.391
	(-1.392)	(6.901)	(-2.230)	(1.847)	(-.051)		
15.	-.016	1.192***	.462	2.507*	-2.422	.460	.432
	(-1.512)	(7.889)	(.885)	(1.779)	(-1.115)		

TABLE II (continued)

Regression Results of the Translog Model

	β_0	β_2	β_3	β_4	β_5	R^2	Adjusted R^2
16.	-.023*	1.291***	-.436	.470	3.873	.462	.435
	(-1.733)	(6.997)	(-.684)	(.273)	(1.459)		
17.	-.032**	1.123***	-.729	2.492	.896	.351	.319
	(-2.298)	(5.994)	(-1.126)	(1.426)	(.333)		
18.	.004	.903***	-.937*	-1.057	.851	.372	.340
	(.375)	(5.640)	(-1.693)	(-.708)	(.370)		
19.	.004	.689***	.028	-.538	-2.079	.271	.234
	(.437)	(4.976)	(.059)	(-.417)	(-1.043)		
20.	.008	.698***	-.634*	-1.185	.088	.423	.393
	(.998)	(6.312)	(-1.659)	(-1.148)	(.056)		
21.	-.009	.916***	-.906**	.974	2.634*	.597	.576
	(-1.250)	(8.966)	(-2.567)	(1.023)	(1.793)		
22.	.000	1.402***	-.699	.798	.228	.533	.510
	(.003)	(8.697)	(-1.255)	(.531)	(.098)		
23.	.008	.684***	-1.542***	-1.000	-2.008	.343	.310
	(.825)	(4.963)	(-3.237)	(-.778)	(-1.013)		
24.	.036**	.984***	-1.273	-8.102***	-5.091	.376	.344
	(2.081)	(4.126)	(-1.544)	(-3.642)	(-1.484)		
25.	.007	1.167***	-.116	-1.146	1.915	.608	.588
	(.780)	(9.465)	(-.273)	(-.997)	(1.080)		
26.	-.032***	.856***	-1.251***	3.423***	.526	.394	.364
	(-3.141)	(6.183)	(-2.613)	(2.650)	(.264)		
27.	-.001	.260***	-.767***	.287	-.170	.197	.156
	(-.166)	(3.193)	(-2.723)	(.379)	(-.146)		
28.	-.022*	.819**	-1.034**	1.869	2.204	.371	.339
	(-2.097)	(5.634)	(-2.059)	(1.379)	(1.054)		
29.	.003	.919***	-1.436**	.600	-4.909*	.280	.244
	(.281)	(5.079)	(-2.297)	(.356)	(-1.886)		

TABLE III

Regression Results of the Log-Linear Model

	Intercept	β	R^2	Adjusted R^2
Period 1 (1969-75)				
1.	-.002 (-.370)	.939*** (7.571)	.411	.404
2.	-.001 (-.213)	.961*** (7.463)	.404	.397
3.	.003 (.668)	.711*** (6.852)	.364	.356
4.	.008 (1.301)	1.036*** (8.350)	.459	.453
5.	.002 (.425)	1.137*** (10.998)	.596	.591
6.	.002 (.346)	1.093*** (7.252)	.390	.383
7.	-.001 (-.268)	.727*** (5.672)	.281	.273
8.	.007 (1.283)	1.082*** (9.392)	.518	.512
9.	.005 (.884)	.851*** (6.912)	.368	.360
10.	.008** (2.006)	.782*** (8.960)	.494	.488
11.	.004 (.849)	.720*** (7.240)	.390	.382
12.	.001 (.288)	.935*** (6.917)	.360	.368
13.	-.003 (-.535)	.865*** (7.634)	.415	.408
14.	-.000 (-.001)	1.023*** (8.084)	.443	.436
15.	-.001 (-.192)	1.061*** (8.007)	.438	.431

TABLE III (continued)

Regression Results of the Log-Linear Model

	Intercept	β	R^2	Adjusted R^2
16.	.005 (.713)	.994*** (6.879)	.365	.358
17.	.008 (1.263)	.828*** (6.308)	.326	.318
18.	-.001 (-.171)	.805*** (4.714)	.213	.203
19.	-.004 (-.750)	.562*** (5.187)	.247	.237
20.	-.001 (-.200)	.851*** (8.627)	.475	.469
21.	.002 (.439)	1.090*** (11.440)	.614	.610
22.	-.009 (-1.079)	.851*** (5.110)	.241	.232
23.	.000 (.011)	.855*** (8.165)	.448	.441
24.	-.002 (-.282)	.865*** (5.576)	.274	.266
25.	-.000 (-.034)	1.017*** (5.459)	.266	.257
26.	.004 (.952)	.913*** (8.898)	.491	.485
27.	.002 (.609)	.656*** (8.427)	.464	.457
28.	.001 (.346)	1.014*** (10.339)	.565	.560
29.	-.000 (-.117)	1.049*** (6.753)	.357	.349

TABLE III (continued)

Regression Results of the Log-Linear Model

	Intercept	ß	R^2	Adjusted R^2
Period 2 (1976-82)				
1.	-.017 (-1.719)	1.235*** (5.525)	.271	.262
2.	-.000 (-.001)	.580** (4.582)	.203	.194
3.	.007 (1.634)	.518*** (5.017)	.234	.225
4.	-.009* (-1.901)	1.393*** (12.407)	.652	.648
5.	-.002 (-.518)	.745*** (6.665)	.351	.343
6.	-.007 (-1.002)	1.251*** (7.720)	.420	.413
7.	-.007 (-1.367)	.942*** (7.886)	.431	.424
8.	-.006 (-1.386)	.887*** (8.390)	.461	.455
9.	-.004 (-.761)	.656*** (4.860)	.223	.214
10.	-.002 (.545)	.564*** (5.555)	.273	.264
11.	-.000 (-.014)	.783*** (8.467)	.466	.460
12.	.003 (.466)	1.113*** (6.934)	.369	.361
13.	-.000 (-.108)	1.029*** (7.555)	.410	.403
14.	.001 (.203)	.922*** (6.804)	.360	.353
15.	-.005 (-.844)	1.077*** (7.662)	.417	.410

TABLE III (continued)

Regression Results of the Log-Linear Model

	Intercept	β	R^2	Adjusted R^2
16.	-.012*	1.349***	.435	.428
	(-1.690)	(7.955)		
17.	-.013	1.070***	.320	.312
	(-1.784)	(6.223)		
18.	-.000	.949***	.334	.326
	(-.119)	(6.418)		
19.	-.003	.666***	.256	.247
	(-.574)	(5.314)		
20.	.000	.733***	.384	.377
	(.125)	(7.163)		
21.	.002	.936***	.514	.508
	(.496)	(9.319)		
22.	.005	1.384***	.521	.515
	(.876)	(9.455)		
23.	-.002	.675***	.239	.230
	(-.367)	(5.083)		
24.	-.026**	1.118***	.208	.198
	(-2.476)	(4.647)		
25.	.003	1.234***	.596	.591
	(.684)	(11.001)		
26.	-.008	.770***	.284	.275
	(-1.438)	(5.705)		
27.	.001	.249***	.113	.103
	(.159)	(3.245)		
28.	-.005	.806***	.293	.285
	(-.950)	(5.842)		
29.	-.002	.813***	.220	.211
	(-.283)	(4.816)		

*** Significance at 1% level

TABLE IV

Regression Results of the CES Model

	Intercept	β	γ	R^2	Adjusted R^2
Period 1 (1969-75)					
1.	.001 (.234)	.927*** (7.444)	-1.602 (-1.012)	.418	.404
2.	-.005 (-.729)	.973*** (7.528)	1.675 (1.020)	.412	.397
3.	-.000 (0.045)	.722*** (6.939)	1.492 (1.129)	.374	.358
4.	.010 (1.373)	1.030*** (8.227)	-.831 (-.523)	.461	.448
5.	.004 (.750)	1.129*** (10.850)	-.965 (-0.730)	.598	.588
6.	.002 (.312)	1.092*** (7.172)	-.081 (-.042)	.390	.375
7.	.004 (.562)	.710*** (5.548	-2.379 (-1.464)	.300	.283
8.	.005 (.804)	1.088*** (9.356)	.746 (.505)	.519	.507
9.	.007 (1.031)	.845*** (6.801)	-.847 (-.537)	.370	.354
10.	.008 (1.648)	.783*** (8.872)	.066 (.059)	.494	.482
11.	.002 (.376)	.726*** (7.240)	.796 (.625)	.392	.377
12.	.002 (.285)	.934*** (6.837)	-.141 (-.081)	.368	.353
13.	-.003 (-.561)	.868*** (7.575)	.303 (.208)	.415	.401
14.	.005 (.785)	1.005*** (7.967)	-2.339 (-1.459)	.457	.444
15.	.002 (.294)	1.051*** (7.878)	-1.434 (-.846)	.443	.430

TABLE IV (continued)

Regression Results of the CES Model

	Intercept	β	γ	R^2	Adjusted R^2
16.	.003 (.370)	1.000*** (6.854)	.782 (.422)	.367	.351
17.	.006 (.826)	.834*** (6.289)	.729 (.433)	.328	.311
18.	.010 (1.047)	.770*** (4.596)	-4.720** (-2.217)	.258	.239
19.	-.004 (-.757)	.564*** (5.157)	.334 (.240)	.247	.229
20.	-.001 (-.314)	.853*** (8.566)	.345 (.273)	.476	.463
21.	-.001 (-.261)	1.100*** (11.526)	1.422 (1.173)	.621	.611
22.	-.010 (-.993)	.853*** (5.072)	.256 (.167)	.241	.223
23.	.007 (1.139)	.835*** (8.098)	-2.746** (-2.096)	.476	.463
24.	.001 (.179)	.854*** (5.465)	-1.534 (-.772)	.280	.262
25.	.002 (.217)	1.009*** (5.366)	-1.086 (-.455)	.268	.250
26.	.011* (1.863)	.894*** (8.825)	-2.502* (-1.943)	.513	.501
27.	-.001 (-.386)	.669*** (8.640)	1.651* (1.679)	.482	.469
28.	.007 (1.366)	1.000*** (10.290)	-2.441* (-1.984)	.586	.575
29.	-.001 (-.212)	1.052*** (6.703)	.421 (.211)	.357	.341

TABLE IV (continued)

Regression Results of the CES Model

	Intercept	β	γ	R^2	Adjusted R^2
Period 2 (1976-82)					
1.	-.009 (-.834)	1.242*** (5.569)	-3.673 (-1.175)	.283	.265
2.	-.000 (-.148)	.579*** (4.547)	.494 (.277)	.204	.184
3.	.004 (.776)	.514*** (4.993)	1.658 (1.148)	.247	.228
4.	-.013** (-2.359)	1.389*** (12.434)	2.178 (1.391)	.660	.652
5.	-.013** (-2.455)	.734*** (7.063)	5.431*** (3.727)	.446	.432
6.	-.006 (-.705)	1.252*** (7.680)	-.596 (-.261)	.421	.407
7.	-.016** (-2.638)	.933*** (8.106)	4.362*** (2.702)	.478	.465
8.	-.014*** (-2.719)	.879*** (8.655)	4.009*** (2.814)	.509	.497
9.	-.010 (-1.497)	.650*** (4.859)	2.986 (1.592)	.247	.228
10.	-.006 (-1.195)	.560*** (5.545)	1.946 (1.373)	.290	.272
11.	.001 (.323)	.785*** (8.450)	-.820 (-.630)	.469	.455
12.	.006 (.728)	1.116*** (6.923)	-1.421 (-.629)	.372	.357
13.	.000 (.071)	1.030*** (7.518)	-.581 (-.303)	.411	.396
14.	-.003 (.461)	.918*** (6.786)	2.257 (1.191)	.371	.356
15.	-.000 (-.037)	1.982*** (7.725)	-2.515 (-1.280)	.428	.414

TABLE IV (continued)

Regression Results of the CES Model

	Intercept	β^*	γ	R^2	Adjusted R^2
16.	-.022**	1.340***	4.551*	.460	.447
	(-2.491)	(8.032)	(1.947)		
17.	-.019**	1.064***	2.612	.330	.313
	(-2.089)	(6.197)	(1.085)		
18.	-.005	.944***	2.097	.342	.326
	(-.639)	(6.388)	(1.012)		
19.	.001	.671***	-2.176	.270	.252
	(.172)	(5.364)	(-1.241)		
20.	-.001	.731***	.841	.387	.372
	(-.205)	(7.115)	(.584)		
21.	-.006	.928***	4.230***	.567	.557
	(-1.240)	(9.726)	(3.163)		
22.	.002	1.381***	1.532	.524	.513
	(.344)	(9.405)	(.745)		
23.	-.002	.675***	.327	.239	.221
	(-.402)	(5.046)	(.175)		
24.	-.017	1.128***	-4.701	.227	.208
	(-1.363)	(4.711)	(1.401)		
25.	-.000	1.231***	1.776	.602	.592
	(-.021)	(10.984)	(1.131)		
26.	-.015**	.763***	3.310*	.310	.293
	(-2.176)	(5.728)	(1.772)		
27.	-.001	.246***	1.140	.125	.104
	(-.430)	(3.217)	(1.061)		
28.	-.014**	.798***	4.227**	.335	.318
	(-2.014)	(5.919)	(2.236)		
29.	.002	.817***	-2.279	.229	.210
	(.272)	(4.839)	(-.963)		

* Significance at 10% level
** Significance at 5% level
*** Significance at 1% level

TABLE V

Summary Statistics of Parameter Estimates

Translog	Period 1 (1969-75) Mean	Period 1 (1969-75) Standard Deviation	Period 2 (1976-82) Mean	Period 2 (1976-82) Standard Deviation
β_2	.923	.181	.909	.279
β_3	-.202	1.471	-.451	.751
β_4	1.684	8.656	.148	2.087
β_5	-.696	2.012	.396	2.351
R^2	.433		.406	
CES				
$\beta*$	.902	.145	.940	.345
γ	-.504	1.569	1.144	2.629
R^2	.410		.371	
Log-Linear				
β	.906	.146	.912	.283
R^2	.403		.354	

Table VI

Summary Statistics of Differences in Beta Risk Estimates

	Period 1			Period 2		
	Mean	Max	Min	Mean	Max	Min
$\beta_2-\beta^*$	.020	.180	-.135	-.032	.102	-.790
$\lvert\beta_2-\beta^*\rvert$	.069	.180	.004	.069	.790	.001
$\beta_2-\beta$	.017	.170	-.133	-.003	.115	-.134
$\lvert\beta_2-\beta\rvert$	.067	.170	.000	.046	.134	.006
$\beta-\beta^*$	.004	.035	-.013	-.028	.011	-.905
$\lvert\beta-\beta^*\rvert$	.009	.035	.001	.035	.905	.000

TABLE VII

Summary of F Statistics

	Translog vs. CES		Translog vs. Log-Linear	
	Period 1	Period 2	Period 1	Period 2
1.	3.35*	.29	7.67*	1.99
2.	4.74*	1.09	10.41*	2.19
3.	.64	1.12	2.14	3.36
4.	1.99	1.18	3.98*	3.80
5.	.21	1.09	.84	15.91*
6.	.09	1.08	.01	.01
7.	1.44	.21	4.91*	7.46*
8.	2.13	1.45	4.27*	11.03*
9.	2.68	2.57	5.37*	7.55*
10.	1.20	1.76	1.80	5.30*
11.	2.15	.68	4.31*	1.36
12.	.87	2.99	1.50	6.46*
13.	.35	4.06*	.35	8.13*
14.	1.49	3.43*	5.06*	8.24*
15.	4.59*	2.33	9.76*	6.22*
16.	.32	.20	.64	3.96*
17.	2.93	1.41	5.86*	3.84
18.	3.07	1.94	11.23*	4.71*
19.	.38	.18	.77	1.85
20.	1.92	2.61	3.37	5.22*
21.	1.93	2.14	3.61	15.66*
22.	3.05	.82	5.91*	1.91

TABLE VII (continued)

Summary of F Statistics

	Translog vs. CES		Translog vs. Log-Linear	
	Period 1 (1969-75)	Period 2 (1976-82)	Period 1 (1969-75)	Period 2 (1976-82)
23.	.87	6.34*	5.70*	12.73*
24.	2.06	9.49*	4.72*	21.37*
25.	.85	.70	1.84	2.33
26.	2.11	5.56*	7.99*	14.46*
27.	.38	4.32*	3.87	8.65*
28.	.24	2.35	3.92*	9.74*
29.	.37	2.92	.74	6.51*

*Significant at 5 percent level.