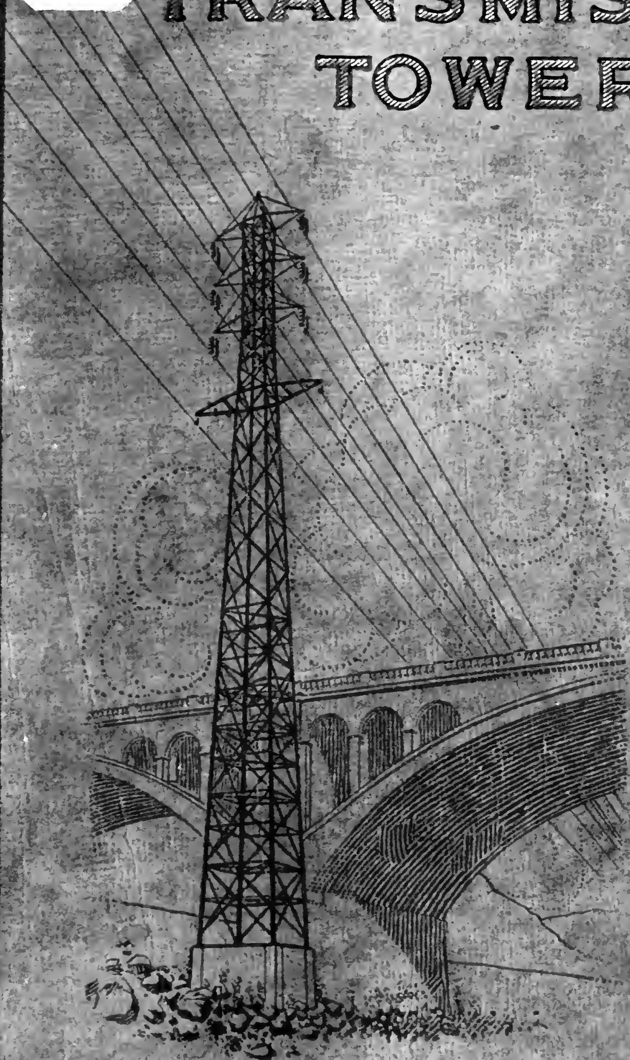
TRANSMISSION TOWERS



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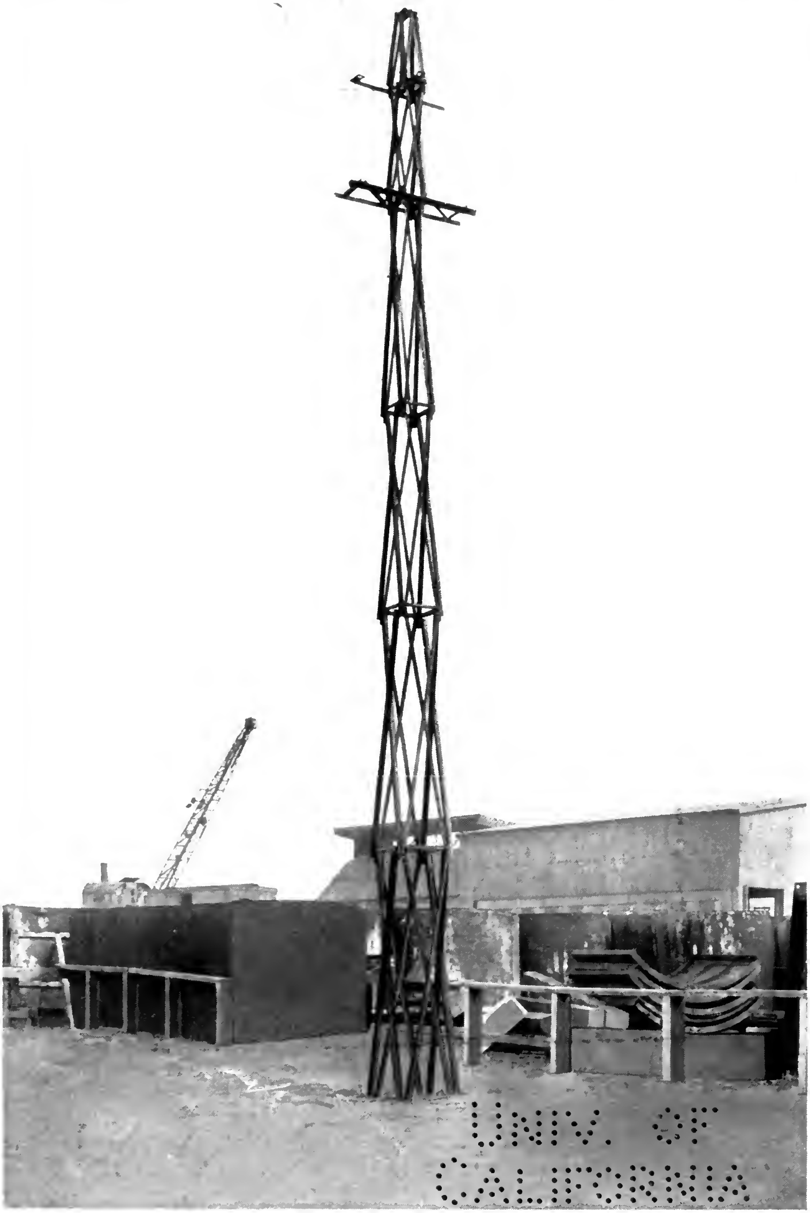

BLAW-KNOX TRANSMISSION POLE
(Patents Applied For)

FORM NO 541

TRANSMISSION TOWERS

Being a reprint of a paper read by E. L. Gemmill, Chief Engineer of the Transmission Tower Department of Blaw-Knox Company, Pittsburgh, before the Engineering Society of the same company.

To which have been added many tables of properties of wires, sags, loads and curves, formulae, etc., to make it a most complete reference book for all interested in the subject.

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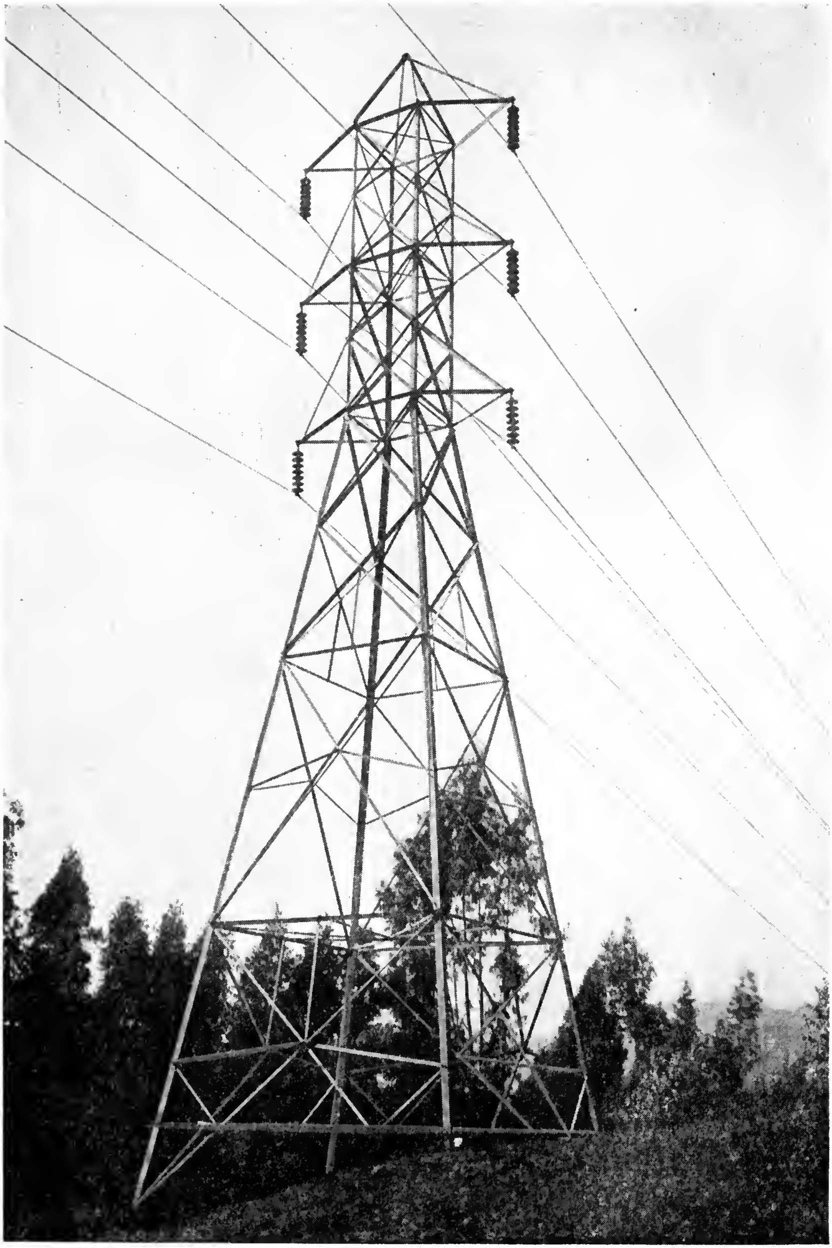


Fig. A—Double Circuit Tower, for 110,000 Volt Line

TRANSMISSION LINE TOWERS

It is only within the last twenty-five to thirty years that it has been considered advisable to carry overhead electric power transmission lines on anything else than wood poles. But with the ever increasing tendency to concentrate power house units, and consequently to make fewer and larger installations, spaced farther apart, it has become necessary to transmit electrical energy over greater distances. This, in turn, has made it advisable to set a higher limit for the voltage at which the electrical energy will be conveyed from one point to another, in order to reduce to the lowest possible minimum the loss in transmission. The using of these higher voltages has, of course, brought in its train the necessity of making more careful provisions for supporting the conductors by means of which the electrical energy is transmitted from one point to another. Naturally, the first change made in the general scheme in vogue was to place the conductors farther apart, which necessitated the use of better cross arms for supporting them. At the same time it was also imperative that, with increased voltage, more clearance be allowed between the ground and the lowest conductor wires under the worst possible conditions of operation. This could best be accomplished by making the supporting structures higher.

So long as the wires were kept only a short distance above the ground, the wood poles made an ideal support for them under ordinary conditions; but when higher supports had to be considered, transmission line engineers began looking about for other supporting structures which would lend themselves more readily to all the varying conditions of service.

The steel structure was immediately suggested as the proper support to take the place of the wood poles, and many arguments were advanced in its favor.

But these supports when built of steel were more expensive than the wood poles had been, and in order to keep the total cost of the line equipment down to a minimum, and to make such an installation compare favorably with a similar line using the wood poles, it became necessary to space the steel supports farther apart, so as to use fewer of them to cover the same length of line.

The steel support, however, had come to stay, and the whole problem resolved itself into a matter of making a careful investigation and study of each installation, in order that there might be used that system which apparently worked out the best in each particular case. From these several projects there have been evolved the different types of structures in use today for transmission line work. They may be roughly divided into three general types, namely:

Poles
Flexible Frames
Rigid Towers

POLES

All supports that are relatively small at the base or ground line are generally classified as Poles. In plan at both ground line and near the top they are made in several different shapes. They may be round, square, rectangular, triangular, or of almost any other section. As a rule, their general outline is continued below the ground line to the extreme bottom of the anchorage. They are usually intended merely to take care of the vertical loads combined with horizontal loads across or at right angles to the direction of the line. They may have greater strength transverse to the line than in the direction of the line, but they are often made of the same strength in each direction. Poles are very rarely designed to take care of any load in the direction of the line when combined with the specified load across the line. They must be spaced closer together than the heavier structures but can be spaced much farther apart than wood poles. A very common spacing for steel poles is about 300 feet apart.

FLEXIBLE FRAMES

Flexible Frames are heavier structures than the poles, and are intended to take care of longer spans. Like the poles, their chief function is to take care primarily of transverse loads with a small margin of safety so that under unusual conditions of service they could also provide a little resistance in the direction of the line; i. e., in a measure, distribute a load coming in this direction over a number of supporting structures, and transfer such a load to the still heavier structures placed at regular intervals in the line. Or the flexible frames may transfer all loads coming on them in the direction of the line to a point where they will be resisted, by a frame of similar construction and strength, but which is made secure against the action of such loads by being anchored in this direction with guy lines.

These flexible frames are almost always rectangular in plan. Generally, the parallel faces in both directions will get smaller as the top is approached, but often the two faces parallel to the direction of the line will be of the same width from the bottom to the top. But the two faces transverse to the line almost always taper from the ground line up, and get smaller toward the top. The two faces parallel to the line are generally extended below the ground line to form the anchorages.

RIGID TOWERS

Rigid Towers are the largest and heaviest structures made for transmission line supports, and, as would be implied by the designation given them, they are intended to have strength to carry loads coming upon them, either in the direction of the line or at right angles to this direction. They are usually designed to take a combination of loads in both directions. These towers are built in triangular, rectangular, and square types, depending upon the particular conditions under which the structure is to be used. When a plan of the tower at the ground line is square in outline, each side of the square will be very much larger than in the case of either poles or flexible frames. The width of one side of a rigid tower, measured at the ground line, will vary somewhere between about one-seventh and one-third of the total height of the structure. This dimension is usually determined by the construction which will give the most economical design, especially when there are a large number of the towers required; but it often happens that the outline of one or more of the structures will be determined by local conditions which are entirely foreign to the matter of economy of design. Then, too, the conditions of loading may be such as to make a special outline the most economical design.

LOADINGS

There are three kinds of loads which come upon transmission line supports:

(1) The dead load of the wires together with any coating on them; also the dead weight of the structure itself.

(2) Wind loads on the wires and the structure transverse to the direction of the line.

(3) Pulls in the direction of the line caused by the dead load and the wind load on the wires.

The dead load on the wires consists of the weight of the wire itself, plus the weight of any insulating covering, plus the weight of any coating of snow or sleet. In most installations the conductors are not covered with any insulating material, and hence at the higher temperatures the dead load will be the weight of the wire only. At the lower temperatures the wires may be coated with a layer of ice, varying up to a thickness of 1″ or more, all around the wire. In some instances ice has been known to accumulate on conductor wires until the thickness of the layer would be as much as 1¼″ all around the wire. But such instances are very rare, especially on wires carrying high voltages because there is generally enough heat in these wires to interfere with the accumulation of much ice on them. But the heaviest coating of ice alone does not often produce the worst conditions of loading for the conductor and the supporting structure. The worst condition of loading is that resulting from the strongest wind blowing against a conductor covered with that coating which offers the greatest area of exposed surface to the direction of the wind under all the several conditions obtaining. This will almost always be true when the wind is blowing horizontally and at right angles to the direction of the line. In this case the total horizontal load on the supporting structure from the wires is the combination of the wind load against the wires and the unbalanced pull in the direction of the line, which is produced by the resultant of the horizontal wind load and the weight of the wire itself and any covering. But it does not follow that this condition will always give the maximum load on the structure. In mountainous districts it may happen that a transmission line will be subjected to a gust of wind blowing almost vertically downward, in which case this pressure, being added directly to the weight of the wire and the ice load, may lead to much more serious results than a wind of equal or even greater intensity blowing horizontally across the line. It may happen in some districts where large sleet deposits are to be encountered, that the vertical load from the dead weight of the wire and its coating of ice will be so great as to produce in the wire a tension large enough to break the wire, even without any added load from the wind. This is especially true if the wire is strung with a very small sag.

Since the design of the transmission line supports is determined very largely by the loads which it is assumed will come upon them, and since the load resulting from the pull in the direction of the line is very often the dominating factor; and, further, since this load is a function of the resultant load on the wire produced by the wind load and the dead load, it naturally follows that the assumptions made regarding the amount of this resultant loading are a matter of prime importance. For this reason some very extensive experimenting has been done to determine the amount of wind pressure against wires, either bare or covered, under extreme conditions of velocity, density of air and temperature. Careful observations have also been made to find out, as near as possible, what is the maximum quantity of ice that will adhere to a wire during and after a heavy storm. It not infrequently happens that the temperature falls and the wind velocity increases immediately after a sleet storm. The falling temperature, of course, tends to make the ice adhere more closely to the wires. On the other hand, a rising wind will tend to remove some of the ice from the wires.

In places where the lower temperatures prevail, the wind velocity rarely gets to be as high as in the warmer districts where sleet cannot form. On the other hand, a moderate wind acting on a wire covered with a coating of ice, will oftentimes put much more stress into the wire than a higher wind acting on the bare wire. This means that the conditions of loading are altogether different for different sections of the country. It is now generally assumed that in those districts where sleet formation is to be met, the worst condition of loading on the wire will be obtained when the wires are covered with a layer of ice ½″ thick, the amount of the wind pressure on them, of course, depending upon the wind velocity and the density of the air.

WIND PRESSURE ON PLANE SURFACES

The wind pressure per unit area on a surface may be obtained by the following formula:

$$P = K \frac{v^2 W}{2\,g} \text{ in which}$$

v = velocity of wind in feet per second;
W = weight of air per unit cube;
g = acceleration of gravity in corresponding units;
K = coefficient for the shape of the surface.

The factor $\frac{v^2 W}{2\,g}$ is called the velocity head.

In considering the pressure on any flat surface normal to the direction of the wind, the pressure may be regarded as composed of two parts:

(1) Front Pressure
(2) Back Pressure

The front pressure is greatest at the center of the figure, where its highest value is equal to that due to the velocity head. It decreases toward the edges. The following conclusions are generally regarded as fair and reliable deductions from the results of many experiments made by several investigators, to determine the amount and distribution of wind pressures on flat surfaces:

(1) The gross front pressure for a circle is about 75% of that due to the velocity head, while for a square it is about 70%, and for a rectangle whose length is very long compared with its width it is somewhere between 83% and 86%.
(2) The back pressure is nearly uniform over the whole area except at the edges.
(3) This back pressure is dependent on the perimeter of the surface and will vary between negative values of 40% and 100% of the velocity head.
(4) The maximum total pressure on an indefinitely long rectangle of measurable width may be taken at 1.83 times the velocity head pressure. For a very small square, the coefficient may be as small as 1.1.

Using the value for W corresponding to a temperature of freezing, or 32° F., and a barometric height of 30 inches, which is 0.08071 pounds per cubic foot, and changing the wind velocity from feet per second to miles per hour, the formula for normal pressure per square foot on a flat surface of rectangular outline becomes:

$$P = 1.83 \times \frac{0.08071}{2 \times 32.2} \times \frac{5280}{60 \times 60} \times \frac{5280}{60 \times 60} \times V^2$$

or

$$P = 0.0049335 \quad V^2$$

WIND PRESSURE ON WIRES

In the case of cylindrical wires the pressure per square foot of projected area is less than on flat surfaces. The coefficient by which the pressure on flat surfaces must be multiplied to obtain the pressure on the projected surface of a smooth cylinder, varies, according to different authorities, from 45% to 79%. Almost all Engineers in this country assume this coefficient to be one-half, and, on this assumption our formula becomes

$$P = 0.00246675 \quad V^2$$

for the pressure per square foot on the projected area of the wire, with any coating it may have on it.

Mr. H. W. Buck has given the results of a series of wind pressure experiments made at Niagara on a 950 ft. span of .58 inch stranded cable, erected so as to be normal to the usual wind. From the data obtained, the following formula was derived:

$$P = 0.0025 \ V^2$$

in which

P = Pressure in pounds per sq. ft. of projected area
V = Wind velocity in miles per hour.

For solid wire previous experimenters had derived the formula

$$P = 0.002 \ V^2$$

It is to be noted that Mr. Buck's formula gives values for pressures 25% in excess of the other formulas, which might be attributed to the fact that for a given diameter, a cable made up of several strands, presents for wind pressure a different kind of surface than a single wire. If we could be sure that this difference exists, then it would be well worth while to take this into consideration when determining the loads for which a tower is to be designed, and to make a careful distinction between towers which are to support solid wires and those which are to carry stranded cables. Almost all Engineers are inclined to accept the formula given by Mr. Buck, and to assume it to be correct for both types of conductors. The fact that this formula agrees so closely with the formula arrived at by assuming that the pressure on the projected area of a cylindrical surface is 50% of the pressure on a rectangular flat surface, would seem to warrant accepting it as being correct.

WIND VELOCITY

In assuming the loadings for which a line of towers are to be designed, the first thing to be determined is the probable wind velocity which will be encountered under the worst conditions. Our calculations, of course, should be based on *actual* velocities. This is mentioned because it is necessary to distinguish between indicated and true wind velocities. The indicated velocities are those determined by the United States Weather Bureau. Their observations are made with the cup anemometer and are taken over five minute intervals. The wind velocities over these short periods of time are calculated on the assumption that the velocity of the cups is one-third of the true velocity of the wind, for both great and small velocities alike. As the result of considerable investigation, it has been found that this assumption is not correct, but that the indicated velocity must be corrected by a logarithmic factor, to convert it into the true velocity. The actual wind velocities corresponding to definite indicated velocities, as given by the United States Weather Reports, are as follows:

Indicated	*Actual*	*Indicated*	*Actual*
10	9.6	60	48.0
20	17.8	70	55.2
30	25.7	80	62.2
40	33.3	90	69.2
50	40.8	100	76.2

It is generally conceded that the wind pressure increases with the height above the ground, and that it is more severe in exposed positions, and where the line runs through wide stretches of open country, than it is in places which are more or less protected by their surroundings.

If we accept the theory advanced by some, to the effect that the ground surface offers a resistance to the wind, which materially lessens its force, then we must conclude that after a certain altitude has been reached the effect of this resistance becomes negligible, and that beyond that altitude the rate of increase in wind pressure must be small. This is especially true, because the density of the air is less in the higher altitudes, which tends to counteract some of the effect of increases in velocity. But experimental data bearing on this matter are very limited, so that the rate of increase in wind pressures for higher elevations above the ground, must in each case be determined by the judgment of the Engineer who is designing the installation.

The curve on Fig. 1 shows the relationship between Indicated velocity and Actual velocity, and the curve on Fig. 2 shows the pressure in pounds per square foot of projected area of wire, corresponding to actual velocities in miles per hour. By placing above the curve given on Fig. 2 a similar curve corresponding to the indicated velocities, a direct comparison between the two different velocities may be made in terms of pressure. This is shown in Fig. 3.

For the general run of transmission line work no special allowance is made for the pressures on towers at different elevations; but pressures are used which are considered to be fair average values for the particular location of the line and for towers of heights which usually prevail. But, of course, there is a distinction made between requirements for a low pole line and for a line on high steel towers. This applies both to the wind pressures, which it is assumed will be encountered, and also to the factor of safety expected in the construction throughout.

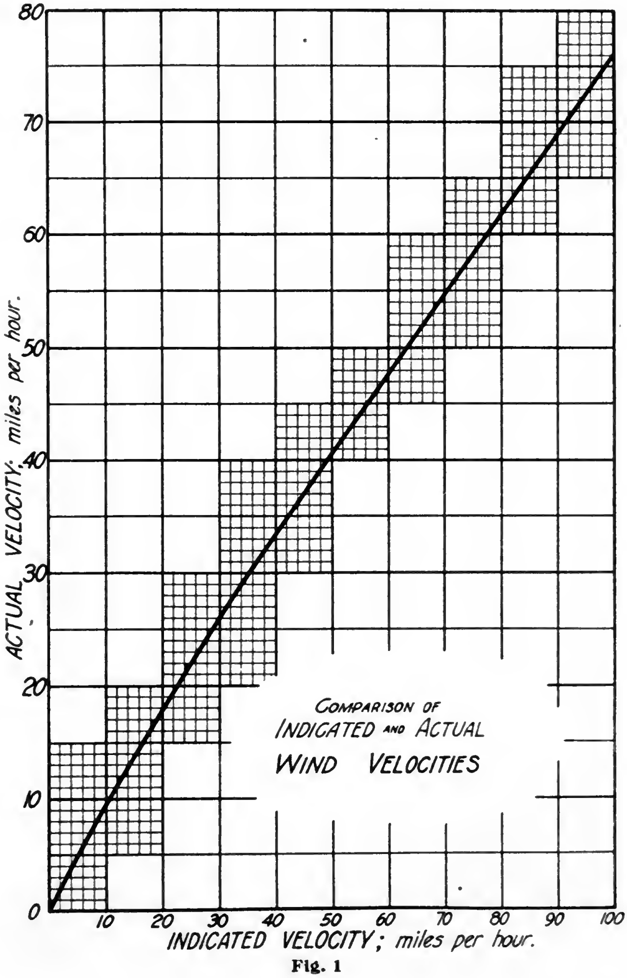


Fig. 1

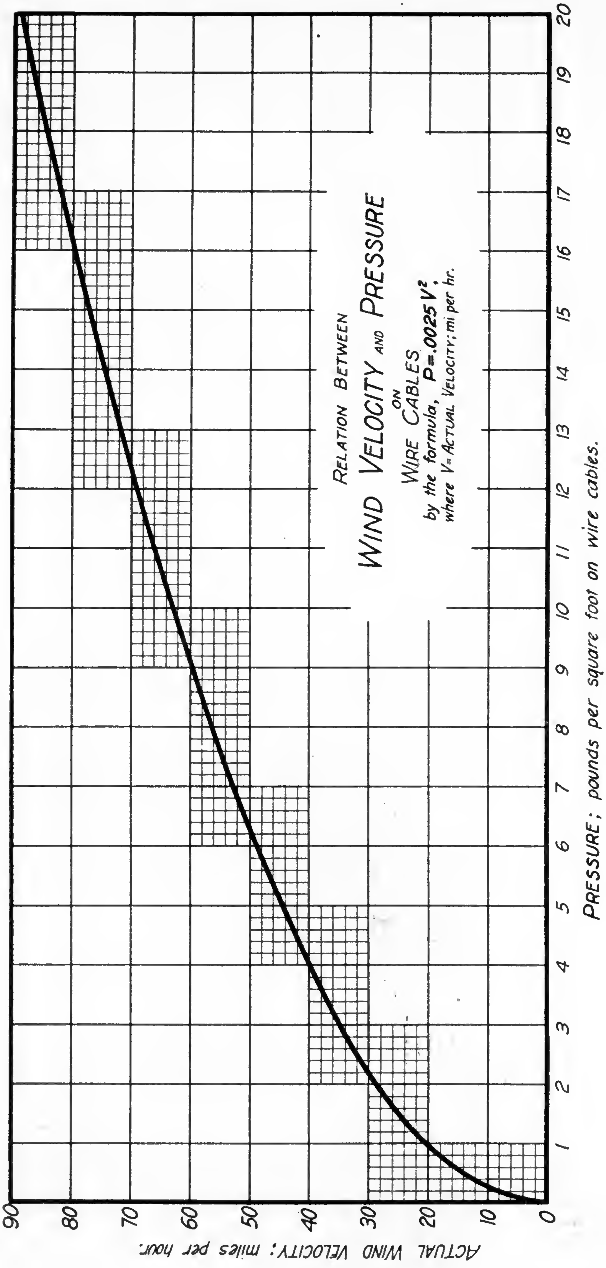

RELATION BETWEEN
WIND VELOCITY AND PRESSURE
ON
WIRE CABLES
by the formula, P=.0025 V²;
where V=ACTUAL VELOCITY; mi per hr.
PRESSURE; pounds per square foot on wire cables.
ACTUAL WIND VELOCITY; miles per hour.
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
0 10 20 30 40 50 60 70 80 90

Fig. 2

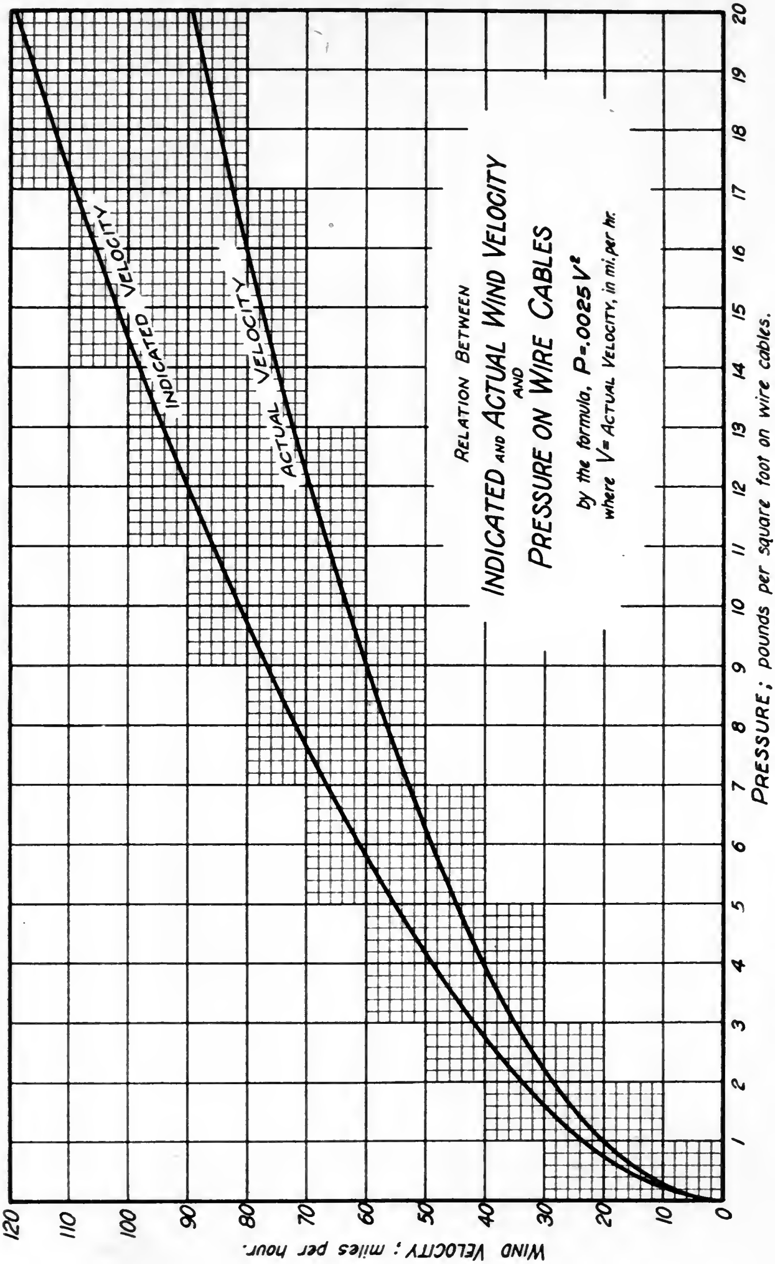

RELATION BETWEEN
INDICATED AND ACTUAL WIND VELOCITY
AND
PRESSURE ON WIRE CABLES
by the formula, $P = .0025V^2$
where V = ACTUAL VELOCITY, in mi. per hr.
INDICATED VELOCITY
ACTUAL VELOCITY
PRESSURE; pounds per square foot on wire cables.
WIND VELOCITY; miles per hour.
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
0 10 20 30 40 50 60 70 80 90 100 110 120

Fig. 3

STANDARD PRACTICE FOR WIND AND ICE LOADS

The Committee on Overhead Line Construction, appointed by the National Electric Light Association of New York, assumes an ice coating ½″ thick all around the wire, for all sizes of conductors, and maximum wind velocities of 50 to 60 miles per hour, as being an average maximum condition of loading. This Committee states that 62 miles per hour is a velocity not likely to be exceeded during the cold months.

Three classes of loading are considered by the Joint Committee on Overhead Crossings, as follows:

Class of Loading:	Vertical Component of Load on Wire:	Horizontal Component of Load on Wire, or Wind Load Across Line:
A	Dead	15 Lbs. per Sq. Ft.
B	Dead + ½″ Ice	8 Lbs. per Sq. Ft.
C	Dead + ¾″ Ice	11 Lbs. per Sq. Ft.

For the Class "B" Loading the ordinary range of temperature is given as—20° to 120° F.

For the calculation of pressures on supporting structures the requirements are 13 lbs. per sq. ft. on the projected area of closed or solid structures, or on 1½ times the projected area of latticed structures The same Joint Committee allows a maximum working stress on copper of 50% of the ultimate breaking stress; in other words, the wires may be stressed to a point very near to the elastic limit.

An analysis of these three classes of loadings would seem to suggest that Class "A" be used for lines in the extreme Southern part of the United States, and that Class "B" be used for all other lines in this country, unless it be for a few lines which might be located in regions where especially cold weather is to be encountered, along with very severe wind storms. For such lines Class "C" would certainly be ample to take care of the most extreme conditions.

Interpreting these loadings in terms of wind velocities, class "A" would allow for an indicated wind velocity of 101.8 miles per hour, or an actual velocity of 77.46 miles per hour, acting against the bare conductor. Class "B" provides for an indicated wind velocity of 71.96 miles per hour, or an actual velocity of 56.57 miles per hour, applied to the projected area of the wire covered with a layer of ice ½″ thick all around. Class "C" assumes an indicated velocity of 85.9 miles per hour, or an actual velocity of 66.33 miles per hour, against the wire covered all around with a layer of ice ¾″ thick.

It has been contended by some Engineers that sleet does not deposit readily on aluminum, owing to the greasy character of the oxide which forms on the surface of aluminum conductors, and that because of this fact the wind loads acting on such lines should not be taken so high as when copper wires are used. But the experience and observation of many other Engineers does not confirm this assumption.

CURVES ASSUMED BY WIRES

When the wires are strung from one structure to another throughout the line, they assume definite curves between each two of the structures, these several curves, of course, depending upon the different conditions attending the stringing.

If a heavy uniform string which is considered to be perfectly flexible, is suspended from two given points, A and B, and is in equilibrium in a vertical plane, the curve in which it hangs will be found to be the common catenary. This is shown in Fig. 4.

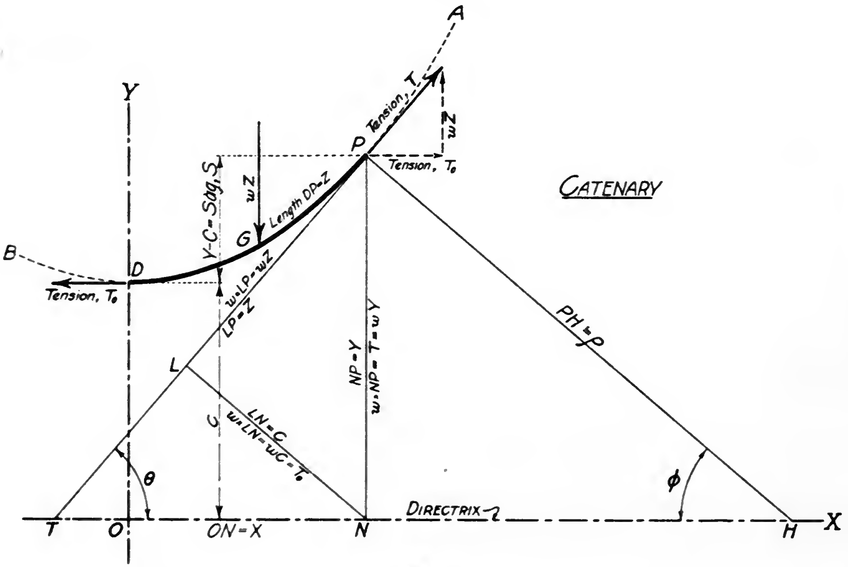


Fig. 4

CATENARY

Let D be the lowest point of the catenary, i. e., the point at which the tangent is horizontal. Take a horizontal straight line O X as the X axis, whose distance from D we may afterwards choose at pleasure. Draw D O perpendicular to this line, and let O be the origin of co-ordinates. Let θ be the angle the tangent at any point P makes with O X. Let T_0 and T be the tensions at D and P respectively, and let the arc D P = Z. The length D P of the string is in equilibrium under three forces, viz: the tensions T_0 and T, acting at D and P in the directions of the arrows, and its weight w Z acting at the center of gravity G of the arc D P.

Resolving horizontally we have

$$T \cos \theta = T_0 \tag{1}$$

Resolving vertically we have

$$T \sin \theta = w Z \tag{2}$$

Dividing equation (2) by equation (1)

$$\frac{dy}{dx} = \tan \theta = \frac{w Z}{T_0} \tag{3}$$

If the string is uniform w is constant, and it is then convenient to write: $T_0 = w C$. To find the curve we must integrate the differential equation (3).

We have,

$$\left(\frac{dz}{dy}\right)^2 = 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{C^2}{Z^2}$$

$$\therefore dy = \pm \frac{z\, dz}{\sqrt{Z^2 + C^2}}$$

$$\therefore y + A = \pm \sqrt{Z^2 + C^2}$$

We must take the upper sign, for it is clear from (3) that, when x and Z increase, y must also increase. When Z = O, y + A = C. Hence, if the axis of X is chosen to be at a distance C below the lowest point D of the string, we shall have A = O. The equation now takes the form,

$$y^2 = Z^2 + C^2 \tag{4}$$

Substituting this value of y in (3), we find,

$$\frac{C\,dz}{\sqrt{Z^2 + C^2}} = dx,$$

where the radical is to have the positive sign. Integrating,

$$C \log \left(Z + \sqrt{Z^2 + C^2}\right) = x + B$$

But x and Z vanish together, hence B = C log C.

From this equation we find,

$$\sqrt{Z^2 + C^2} + Z = C\,e^{\frac{x}{c}}$$

Inverting this and rationalizing the denominator in the usual manner, we have

$$\sqrt{Z^2 + C^2} - Z = C\,e^{-\frac{x}{c}}$$

Adding and subtracting, we deduce by (4)

$$y = \frac{C}{2}\left(e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right),\ Z = \frac{C}{2}\left(e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right) \qquad (5)$$

The first of these is the Cartesian equation of the common catenary. The straight lines which have here been taken as the axes of X and Y are called, respectively, the directrix and the axis of the catenary. The point D is called the vertex.

Adding the squares of (1) and (2), we have by help of (4),

$$T^2 = w^2 (Z^2 + C^2) = w^2y^2;$$

$$\therefore T = w\,y \qquad (6)$$

The equations (1) and (2) give us two important properties of the curve, viz: (1) the horizontal tension at every point of the curve is the same and equal to w C; (2) the vertical tension at any point P is equal to w Z, where Z is the arc measured from the lowest point. To these we join a third result embodied in (6), viz: (3) the resultant tension at any point is equal to w y, where y is the ordinate measured from the directrix.

Referring to Fig. 4, let PN be the ordinate of P, then T = w· PN. Draw NL perpendicular to the tangent at P, then the angle P N L = θ Hence,

$$PL = PN \sin\theta = Z \text{ by } (2)$$

$$NL = PN \cos\theta = C \text{ by } (1)$$

These two geometrical properties of the curve may also be deduced from its cartesian equation (5).

By differentiating (3) we find,

$$\frac{1}{\cos^2\theta} \cdot \frac{d\theta}{dz} = \frac{1}{C} \therefore \rho = \frac{dz}{d\theta} = \frac{C}{\cos^2\theta} \qquad (7)$$

$$\rho \text{ is also } = \frac{y^2}{C}$$

We easily deduce from the right-angled triangle P N H, that the length of the normal, viz: PH, between the curve and the directrix, is equal to the radius of curvature, viz., ρ, at P. At the lowest point of the curve D, the radius of curvature, ρ, $= \frac{C^2}{C} = C$. It will be noticed that these equations contain only one undetermined constant, viz., C; and when this is given, the form of the curve is absolutely determined. Its position in space depends on the positions of the straight lines called its directrix and axis. This constant C is called the parameter of the catenary. Two arcs of catenaries which have their parameters equal are said to be arcs of equal catenaries.

Since $\rho \cos^2 \theta = C$, it is clear that C is large or small according as the curve is flat or much curved near its vertex. Thus, if the string is suspended from two points A and B in the same horizontal line, then C is very large or very small compared with the distance between A and B, according as the string is tight or loose.

The relationship between the quantities y, Z, C, ρ, and θ and T in the common catenary may be easily remembered by referring to the rectilineal figure P L N H. We have $PN = y$, $PL = Z$, $NL = C$, $PH = \rho$, $T = w \cdot PN$ and the angles LNP, NPH are each equal to θ. Thus the important relations (1), (2), (3), (4) and (7) follow from the ordinary properties of a right-angled triangle.

The co-ordinates of the center of curvature for the catenary are:

$$\alpha \text{ (abscissa)} = x - \frac{y}{C}\sqrt{y^2 - C^2}$$

$$\beta \text{ (ordinate)} = 2y$$

When two or more unequal catenaries have similar outlines so that the ratio $\frac{y - C}{x}$ is the same for all of them, the curvature between the points D and P will also be the same for all these catenaries. From this it follows that, at similar points on the different catenaries, the several radii of curvature will vary directly as the values of **x** for the different curves. The radius of curvature at the lowest point D has already been shown to be equal to C, the parameter of the catenary. Since C and y — C both vary directly as the value of **x** for these unequal but similar catenaries, it is evident that y must also vary in the same manner. It will be seen from the triangle PLN, that when C and y both vary in the same manner, LP or Z, which is the length of the arc DP, must also vary in the same manner.

ELASTIC CATENARY

When a heavy elastic string is suspended from two fixed points and is in equilibrium in a vertical plane, its equation may be found as follows:

Using the same figure as for the inelastic string and denoting the unstretched length of arc D P by Z_1, let us consider the equilibrium of the finite part D P;

$$\begin{array}{l} T\cos\theta = T_0 \quad (1) \\ T\sin\theta = wz_1 \quad (2) \end{array} \quad \therefore \frac{dy}{dx} = \tan\theta = \frac{wz_1}{T_0} = \frac{Z_1}{C} \qquad (3)$$

From these equations we may deduce expressions for **x** and **y** in terms of some subsidiary variable. Since $Z_1 = C \tan\theta$ by (3), it will be convenient to choose either Z_1 or θ as this new variable. Adding the squares of (1) and (2), we have,

$$T^2 = w^2 (C^2 + Z_1^2) \qquad (4)$$

$$\text{Since } \frac{dx}{dz} = \cos\theta \text{ and } \frac{dy}{dz} = \sin\theta,$$

we have by (1) and (2)

$$x = \int \frac{T_0}{T}\, dz = \int \frac{wC}{T}\left(1 + \frac{T}{E}\right) dz_1 = \frac{wC}{E} Z_1 + C \log \frac{Z_1 + \sqrt{C^2 + Z_1^2}}{C}$$

$$y = \int \frac{wz_1}{T}\, dz = w\int \frac{Z_1}{T}\left(1 + \frac{T}{E}\right) dz_1 = \frac{w}{2E}\left(C^2 + Z_1^2\right) + \sqrt{C^2 + Z_1^2}$$

where the constants of integration have been chosen to make

$$x = O \text{ and } y = C + \frac{C^2 w}{2E}$$

at the lowest point of the elastic catenary. The axis of X is then the statical directrix.

We have the following geometrical properties of the elastic catenary:

$$w\,y = T + \frac{T^2}{2E} \qquad (1)$$

$$\rho = \frac{C^2 + Z_1^2}{C}\left(1 + \frac{w}{E}\sqrt{C^2 + Z_1^2}\right) \qquad (2)$$

$$Z = Z_1 + \frac{w}{2E}\left\{Z_1\sqrt{C^2 + Z_1^2} + C^2 \log \frac{Z_1 + \sqrt{C^2 + Z_1^2}}{C}\right\} \qquad (3)$$

All of these reduce to known properties of the common catenary when E is made infinite.

These equations have value only from an academic viewpoint. They are too unwieldy to be of any practical value in determining the properties of curves, assumed by transmission line wires under different working conditions. These equations would be still further complicated, if we attempted to make them take care of changes resulting from conditions of loading due to different temperatures.

PARABOLA

If we consider the weight of the wire to be uniformly distributed over its horizontal projection, instead of along its length, its equation will be found to be that of a parabola.

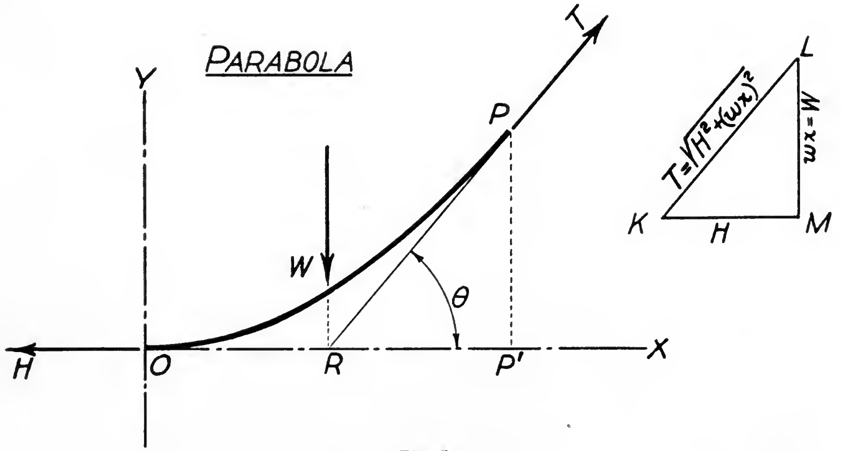


Fig. 5

By referring to Figure 5 and considering the equilibrium of any part OP of the wire, beginning at the lowest point O, the forces acting on this part are seen to be the horizontal tension H at O, the tension T along the tangent at P, and the total weight W of the wire, OP. As this weight is assumed to be uniformly distributed over the horizontal projection $OP^1 = x$ of OP,
the weight is $W = w\,x$, and bisects OP^1.

Resolving the forces in the horizontal and vertical directions, we find as conditions of equilibrium,

$$-H + T\frac{dx}{dz} = O, \qquad -wx + T\frac{dy}{dz} = O,$$

whence, eliminating dz, $\dfrac{dy}{dx} = \dfrac{w}{H}x.$

Integrating and considering that $x = O$ when $y = O$, we get $y = \dfrac{w}{2H}x^2$, which may be put in the form $x^2 = \dfrac{2H}{w}y$. This is the equation to a parabola.

If we substitute $\frac{l}{2}$ for x, and S for y, in the equation for the curve,

it becomes $\left(\dfrac{l}{2}\right)^2 = \dfrac{2H}{w}S$ or $w\,l^2 = 8\,HS$,

from which $H = \dfrac{wl^2}{8S}$,

which is the well known equation for determining the horizontal tension in the wires, when the two points of support are in the same horizontal plane. In that case $\frac{l}{2}$ equals one-half of the span, and S equals the sag or deflection of the wire below the plane of the supports.

The three forces H, T, and W, are in equilibrium; they must intersect in a point R which bisects OP^1, and the force polygon must be similar to the triangle RPP^1.

Drawing such a force diagram K L M, and making L M equal to W or w x, and MK equal to H, KL will be the value of T and equal to $\sqrt{H^2 + (w\,x)^2}$.

Substituting for H and x their values in terms of w, l and S, this

becomes $\sqrt{\left(\dfrac{wl^2}{8S}\right)^2 + \left(w\dfrac{l}{2}\right)^2} = \sqrt{\dfrac{w^2l^4 + 16\,w^2l^2S^2}{64\,S^2}} = \dfrac{wl}{8S}\sqrt{l^2 + 16S^2}.$

A quantity $\sqrt{A^2 + a^2}$, when a is very small relatively to A, may be approximated by using $A + \frac{a^2}{2A}$; hence, an approximation for the above value of T is,

$$\frac{w\,l}{8\,S}\left(l + \frac{16\,S^2}{2\,l}\right), \text{ or, } \frac{w\,l^2}{8\,S} + w\,S.$$

In this form it is very similar to the expression for the tension in the wire at the insulator supports, derived by assuming the curve to be a catenary. It will readily be seen from the above that for very small sags in short spans the maximum tension at the insulator supports is very little more than the tension at the middle of the span.

But it must be noted that in order that the above assumption may be warranted, it is essential that the span considered, be short, and that the length of wire be little more than the span. This, of course, means that the sag in the wire must be rather small.

The equation for the parabola $x^2 = \frac{2\,H}{w}\,y$ has, for the coefficient of y, a constant which is equal to four times the distance between the directrix of the parabola and the vertex O, as shown in Fig. 6.

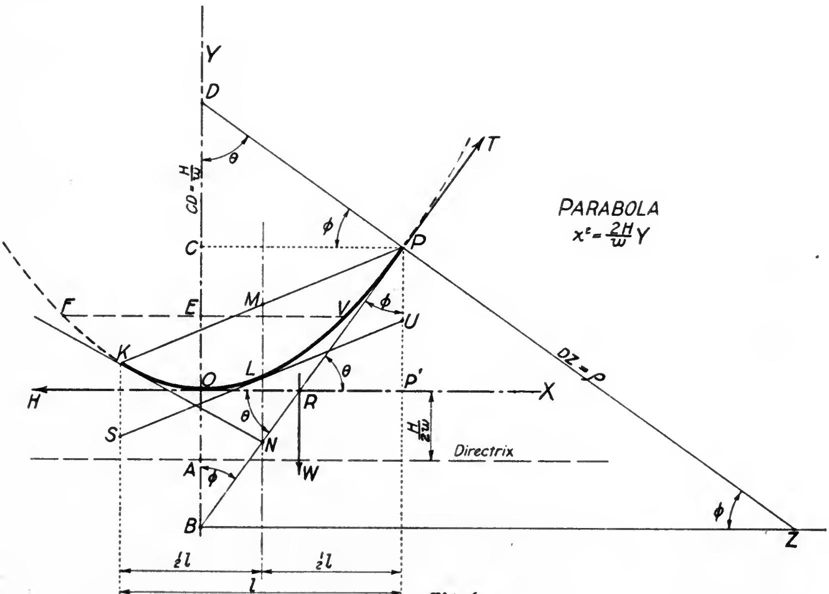


Fig. 6

The directrix is shown passing through the point A, and is parallel to the X axis. The line OY is the axis of the curve. If a line is drawn tangent to the curve at any point P, this tangent will intersect the Y axis at a point B, such that the distance BO will equal the distance OC, where C is the point of intersection of the Y axis with a line drawn through the point P, parallel to the X axis. The length BC is the subtangent and is equal to twice the ordinate of the point of contact. The line PD drawn through the point P and perpendicular to the tangent BP, will intersect the Y axis at point D. The length CD is the subnormal of the curve, and is constant for all points on the curve. It is equal to one-half the co-efficient of y in the original equation, and is therefore equal to $\frac{H}{w}$. The angles TRX and ORB are each equal to the angle PDC or θ.

$$\operatorname{Tan} \theta = \frac{P^1P}{R\,P^1} = \frac{y}{\frac{1}{2}\,x} = \frac{2\,y}{x}$$

PARABOLIC ARC WITH SUPPORTS AT DIFFERENT ELEVATIONS

The curve in which a suspended wire hangs, may be considered to extend indefinitely in both directions, and the suspended wire may be secured to rigid supports at any two points, such as N and U, lying on this curve (Fig. 7), without altering the tension in the wire. The law of this parabola is

$$x^2 = K\,y,$$

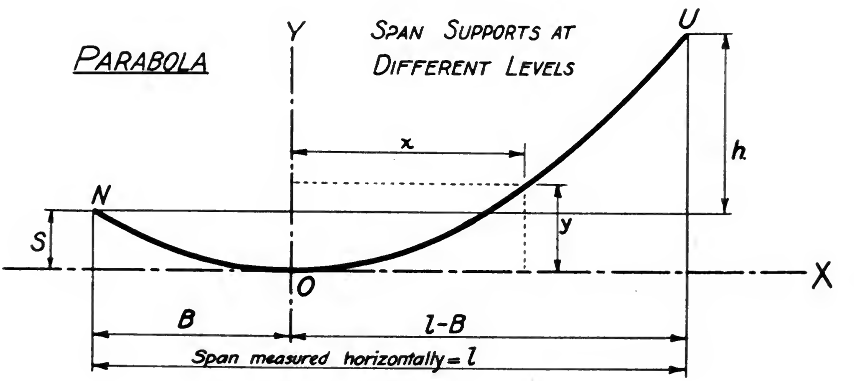


Fig. 7

and in the case of a suspended wire the multiplier K is directly proportional to the tension H, and inversely proportional to the density of the conductor material. The value of K in terms of the horizontal tension and the weight of the conductor has already been found to be $\frac{2\,H}{w}$.

Let S = sag below level of lower support,
B = horizontal distance of lowest point of wire from lower support,
h = difference in level of the two supports,
l = length of span measured horizontally,

all as indicated on Fig. 7; then, by inserting the required values in equation $x^2 = K\,y$, the following equations are derived therefrom:

$$B^2 = K\,S,$$
$$(l - B)^2 = K\,(S + h), \text{ or, } l^2 - 2lB + B^2 = KS + Kh,$$

from which B^2 on one side and its equivalent KS on the other side cancel out, leaving $l^2 - 2lB = Kh$.

Therefore,

$$B = \frac{l^2 - Kh}{2\,l}, \text{ and } S = \frac{B^2}{K}$$

From an inspection of the formula $B = \frac{l^2 - Kh}{2\,l}$, it is seen that if $Kh = l^2$, the lowest point of the wire coincides with the lower support N, while if Kh is greater than l^2, the distance B is negative, and there may be a resultant upward pull on the lower insulator N—a point to bear in mind when considering an abrupt change in the grade of a transmission line.

We may consider the curve of the wire between the two supports N and U as being made up of two distinct parts, NO and OU. The part NO will be equivalent to one-half of a curve whose half span measured horizontally is B, and whose sag is S. Similarly, the part OU will be equivalent to one-half of a curve whose half span measured horizontally is $l - B$, and whose sag is $S + h$.

It is possible, and sometimes convenient, to express the formulas for wires suspended from supports not at the same level, in terms of the equivalent sag (Se) of the same wire, subjected to the same horizontal tension when the horizontal span (l) is unaltered, but the supports are on the same level.

For such a condition, the equation to the curve becomes

$$\left(\frac{l}{2}\right)^2 = \text{K Se},$$

from which

$$\text{K} = \frac{l^2}{4\text{ Se}}$$

If we substitute for K in the above formulas, its equivalent value

$$\text{K} = \frac{l^2}{4\text{ Se}},$$

then we get the following set of formulas, in which B, S, l, and h, are all as indicated in Fig. 7.

$$\text{B} = \frac{l}{2}\left(1 - \frac{\text{h}}{4\text{ Se}}\right), \text{ or, } l - \text{B} = \frac{l}{2}\left(1 + \frac{\text{h}}{4\text{ Se}}\right).$$

$$\text{S} = \text{Se}\left(1 - \frac{\text{h}}{4\text{ Se}}\right)^2, \text{ or, S} + \text{h} = \text{Se}\left(1 + \frac{\text{h}}{4\text{ Se}}\right)^2.$$

COMPARISON OF CATENARY AND PARABOLA

If a straight line is drawn through the point P and any other point K on the parabola, shown in Fig. 6, and this chord KP is bisected at the point M, a line drawn through this point M and parallel to the Y axis will bisect all other chords which are parallel to the chord KP. From this it follows that a line SU drawn tangent to the parabola and parallel to the chord KP, will be tangent to the curve at a point L which lies on a line that is drawn parallel to the Y axis and through the point M. Another property of the parabola is, that the tangents to the curve at the points K and P will intersect at the point N, which also lies on the line that passes through the points L and M. If the horizontal projection of the chord KP be designated by l, then the horizontal projection of KM, MP, KN, and NP will each be equal to $\frac{1}{2}l$.

The total tension T in the wire at any point on the curve equals

$$\sqrt{\text{H}^2 + (\text{w x})^2},$$

or,

$$\text{T} = \text{w}\sqrt{\left(\frac{\text{H}}{\text{w}}\right)^2 + \text{x}^2}.$$

But it is seen from the figure that

$$\sqrt{\left(\frac{\text{H}}{\text{w}}\right)^2 + \text{x}^2} = \text{DP},$$

$$\therefore \text{T} = \text{w} \cdot \text{DP}.$$

In the case of the catenary the total tension in the wire is

$$T = w\,y$$

in which

$$y = \sqrt{C^2 + Z^2}$$

But, when we compare a catenary and a parabola having equivalent horizontal tensions, $C = \frac{H}{w}$, it will be seen that the two formulas for total tension in the wire differ only in that the value Z, which is the true length of the arc, is used in the one case, where x, which is the horizontal projection of the length, is used in the other. But T will always be greater for the catenary except at the lowest point of the curve.

The radius of curvature of the parabola

$$x^2 = \frac{2\,H}{w}\,y \text{ is } \rho = \frac{\frac{H}{w}}{\operatorname{Sin}^3\varphi}$$

in which φ is the angle which the tangent to the curve makes with the Y axis.

$$\rho \sin^3 \varphi = \frac{H}{w} = DP \sin \varphi, \text{ or, } \rho \sin^2 \varphi = DP$$

from which

$$\rho = \frac{DP}{\sin^2\varphi} = DZ.$$

$$\frac{BD}{DZ} = \sin\varphi, \text{ or, } \frac{2y + \frac{H}{w}}{\rho} = \sin\varphi.$$

$$\rho = \frac{2y + \frac{H}{w}}{\sin \varphi} = \frac{2y + \frac{H}{w}}{\cos \theta}$$

If we substitute C for $\frac{H}{w}$, the radius of curvature is,

$$\rho = \frac{C}{\sin^3 \varphi} = \frac{C}{\cos^3 \theta}.$$

In the case of the catenary,

$$\rho = \frac{C}{\cos^2 \theta}.$$

A comparison of these two, shows that the radius of curvature for the parabola, at a point where the tangent makes an angle θ with the horizontal, is $\frac{1}{\cos \theta}$ times that for a similar point on the catenary.

At the lowest point of the curve the radius of curvature is the same for both the parabola and the catenary, when the horizontal tension is the same. It is also true that, when the horizontal tension at the lowest point of the parabola is equivalent to that at the lowest point of the catenary for the same span and loading conditions, the sag at this point below the plane of the supports will be very nearly the same for the two curves.

But the outlines of the two curves differ at all points between the lowest point and the point of support. This difference between the outlines of the two curves becomes greater as the spans and the sags are made larger. It is because of this difference in the outlines of the two curves that the sags will be nearly equal for only one loading condition. Any change in the loading condition will produce different changes in the lengths of the two curves, and hence, will make the sags different.

REACTIONS FOR SPANS ON INCLINES

When wires are strung on towers that are located on steep grades, it is very necessary that we determine carefully the reactions at the points of support and also the deflection of the wire away from a straight line joining the two points of support for any given span. This case is shown in Fig. 8.

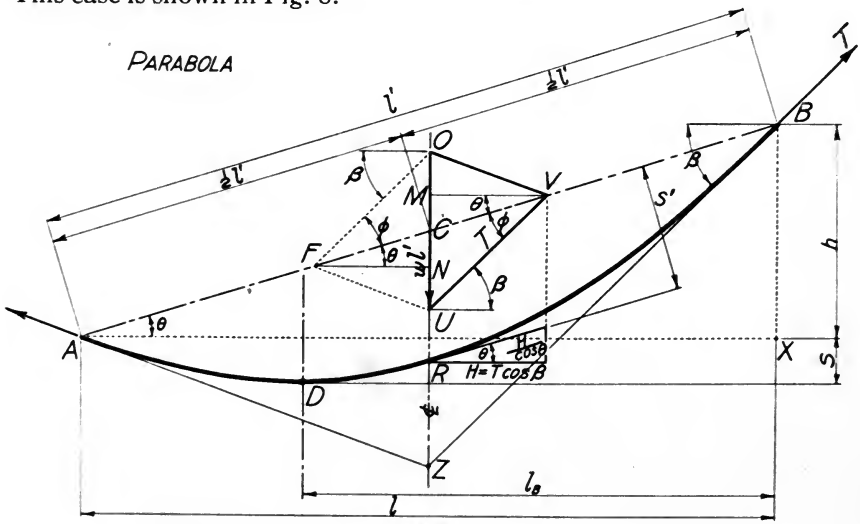


Fig. 8

If we have given the horizontal distance, l, between the supports A and B and also the vertical distance, h, that B is above A, together with the maximum tension, T, in the wire at the point of support B, we can determine the reactions at both of the supports and also the sag in the wire.

The wire ADRB is in equilibrium under three forces; viz., the tensions acting at A and B in the directions of the tangents, and its weight wl^1 acting at its center of gravity. These three forces intersect at the point Z, and the vertical line through this point passes also through the point C on the line AB and midway between A and B. On this vertical line lay off the distance OU equal to W or wl^1, and let it be bisected at the point C so that OC equals CU or $\frac{1}{2} wl^1$. Complete the force diagram by drawing UV and OV parallel respectively to ZB and AZ. UV will then be the tension in the wire at the point B, and OV will be the tension at the point A. The vertical component of the reaction at B is the vertical component of UV and is equal to UM. The vertical component of the reaction at A is the vertical component of OV and is equal to MO or UO — UM.

Complete the parallelogram of forces by drawing OF parallel to UV and FU parallel to OV. The points F and V must lie on the line AB. Let the tension UV in the wire at the point B be denoted by T. Let θ be the angle made by the line AB with the horizontal line AX, and let φ be the angle between the lines FO and AB. Let β be the angle which the tangent at the point B makes with the horizontal plane.

Angle O C F $= \theta + 90°$.

$$\text{Sin } \varphi = \frac{OC}{OF} \cdot \text{Sin } (90° + \theta) = \frac{wl^1}{2T} \cos \theta$$

But

$$l^1 = \frac{l}{\cos \theta} \quad \therefore \sin \varphi = \frac{wl}{2\,T}.$$

$$\beta = \theta + \varphi = \theta + \sin^{-1}\left(\frac{wl}{2\,T}\right)$$

The horizontal component of the stress in the wire at either point of support, is H = T Cos β. This is also the total tension in the wire at the point (if any) where the slope of the wire is zero. The total stress in the wire is greatest at the highest point B, and the vertical component of the reaction at this point is UM = NO = T Sin β.

The weight supported by the lower tower is MO $= wl^1 - T \sin \beta$. In some cases this may be zero, or it may even be a negative quantity, in which case the wire will exert an upward pull on the lower support. In that case the sag S below the point A will be zero. This is not an unusual condition on a steep incline, or on a moderate incline if the spans are short.

The position of the lowest point D in the span is determined by the condition that the vertical component of the force acting at either point of support is the weight of that part of the wire between the point D and the point of support. This is true because the tension in the wire, at the point D, has no vertical component. The horizontal distance of the point D from the support B, is,

$$l_B = l \cdot \frac{ON}{OU} = \frac{l \cdot T \sin \beta}{wl^1} = \frac{T \sin \beta \cdot \cos \theta}{w}$$

It may happen that l_B will be found to be equal to or greater than l, in which case the support A will be the lowest point in the span. If the vertical component of the force at the support B is greater than the total weight of the span (wl^1), it follows that the resultant force at the support A will be in an upward direction.

The total sag is S + h. The value of this sag may be determined by considering l_B to be one-half of a span having supports at the same level, and having the sag,

$$S + h = \frac{w}{\cos \theta} \cdot \frac{(2\, l_B)^2}{8\, H} = \frac{w l_B^2}{2\, H \cos \theta}.$$

STRINGING WIRES IN SPANS ON STEEP GRADES

When transmission lines are carried up steep grades, and are strung on towers in such a manner that the lower support A is the lowest point of the span, it is of considerable advantage in stringing the wires, to know the maximum deflection of the wire from the straight line AB as observed by sighting between the points A and B. Such a condition is shown in Fig. 9.

A line drawn tangent to the curve and parallel to the line AB will be tangent at the point R, which is on a vertical line drawn through the point C at the middle of the line AB. The wire will, therefore, have its maximum deflection from the line AB at this point R. The horizontal projections of AR and RB are equal, and have the value $\frac{1}{2}l$ when the horizontal projection of the span AB is l.

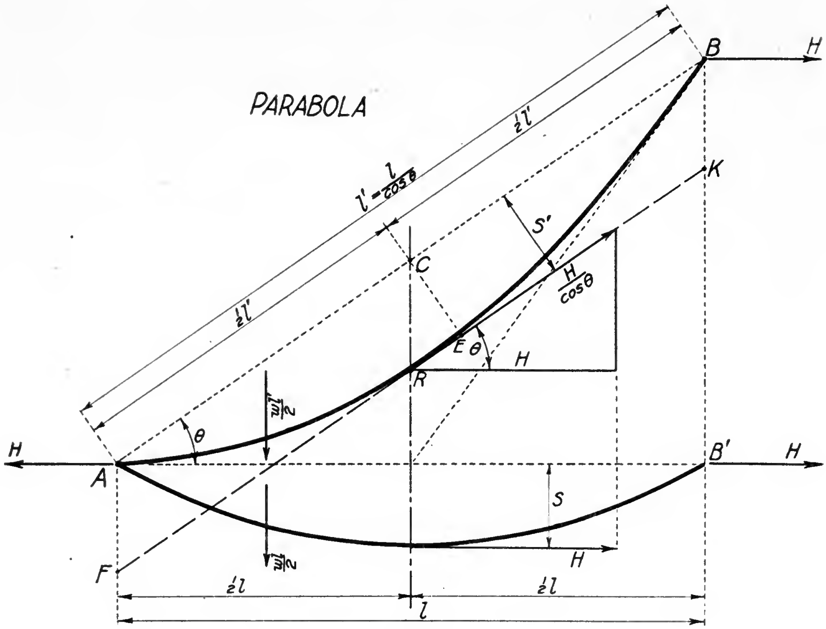


Fig. 9

Let the deflection of the wire, at the point R, from the straight line AB be denoted by S^1. By taking moments about the point A, and putting their sum equal to zero, we may determine the value of this deflection.

$$\frac{H}{\cos\theta}\cdot S^1 = \frac{wl^1}{2}\cdot\frac{l}{4} \text{ or}$$

$$S^1 = \frac{wl^1}{2}\cdot\frac{l}{4}\cdot\frac{\cos\theta}{H} = \frac{wl^2}{8\,H}$$

Comparing this span with a span having the same horizontal projection but supports A and B^1 at the same level, it will be seen that when a wire is strung between supports on a slope, the maximum deflection S^1 of that wire from the straight line joining the two points of support, is exactly the same as the maximum sag S of the same wire when strung between points on the same level; provided the span measured horizontally and the horizontal component H of the tension are the same in both cases.

The above analysis will also show that this relationship between spans having the same horizontal projections will be true even though the lowest point on the wire is not coincident with the lower support.

These formulas for lines carried up steep inclines are all based on the assumption that the total weight of wire is the weight per foot of length of wire multiplied by the length of the line AB (which is $l^1 = \frac{l}{\cos\theta}$). This, of course, is an approximation which is the more nearly correct as the sag is kept small in proportion to the span.

Having determined the value of S^1, and knowing the value of θ, we may determine the value of CR. This distance can then be measured down vertically below the points of support A and B, as shown at F and K, and the wire when strung between these two supports may be drawn up until it becomes tangent to the line FK parallel to AB and at the distance CR vertically below AB. This may be observed by sighting from F to K.

The length of the wire between fixed supports at the same horizontal level is approximately,

$$L = l + \frac{8\,S^2}{3l},$$

in which l is the distance between supports and S the sag at the center, both expressed in feet. In the case of a wire between supports which are not on the same level, the total length may be considered to be made up of two distinct parts of the parabolic curve.

RELATION BETWEEN STRESS, TEMPERATURE AND SAG

All of the formulas so far deduced for determining working conditions for the wires on the basis of using parabolic curves, have been obtained by ignoring the fact that the wires are elastic and will therefore stretch under tension, and that the length will also be affected by changes in temperature.

It is customary to assume that the material of the conductors is perfectly elastic up to a certain critical stress, known as the elastic limit, and that the process of elongation and contraction follows a straight line law. Therefore, the length of the wire will be changed by the amount of elongation or contraction which will be,

$$L_e = L \cdot \frac{T_e}{E},$$

in which L_e is the elongation or contraction, L is the unstressed length of wire, T_e is the stress per square inch in the wire, and E is the modulus of elasticity.

The change in length due to differences of temperature will be,

$$L_t = L_0 (1 + mt),$$

in which L_0 is the original length, t is the number of degrees Fahrenheit change in temperature, and m is the coefficient of expansion for the material in the conductor. These two changes in length of wire are not independent of each other. They act simultaneously and are inter-related, and must be considered together. Any temperature variation causes a change in the length of wire, which, in turn, changes the sag condition, and hence changes the stress, which, in turn, will affect the amount of change in length of wire due to the stress in it.

In the case of long spans it is always necessary to make proper allowances for the changed outline of curve assumed by wires, due both to their elasticity and to the elongation or contraction resulting from changes in temperature. This is also advisable oftentimes in the case of comparatively short spans, especially where a minimum clearance is required under the lowest wires under the worst conditions of loading. This matter has been the subject of quite extensive investigation on the part of several different men, and several different solutions of the problem have been offered. No theoretically correct analytical solution that is easy of application has yet been found, but every one is based on assumptions which involve some approximations. The first assumption generally made is that the parabola approximates near enough to the true curve in which the wire hangs. For short spans this is not much in error, but on the longer spans the difference in results obtained by figuring the curve first as a parabola and then as a catenary, is very considerable, especially when changes due to the elasticity of the wire and to differences in temperature are taken into account.

Some mathematical expressions have been worked out for taking care of these conditions approximately, but they all involve the first and third powers of the unknown quantity. It is a tedious matter to solve such equations, which is another reason why they are not satisfactory, especially when it is known in advance that the result, when obtained, will not be accurate.

THOMAS' SAG CALCULATIONS

After having studied several of these different solutions, the writer is of the opinion that the best one is the semi-graphical method offered by Percy H. Thomas in his article on "Sag Calculations for Suspended Wires," which was presented at the 28th Annual Convention of the American Institute of Electrical Engineers, and which was published in their "Transactions for 1911." Thomas uses the true catenary, and, at the same time, takes care of all changes in the loading conditions. He attacks the given problem by first reducing the span in size, without changing the shape of the curve, until the span is one foot. Having determined all the conditions attending the problem for the similar span of one foot length, it is then an easy matter to convert these into corresponding quantities for the given span.

When the span is reduced in size without changing the shape of the curve, the sag will be reduced in direct proportion to the reduction of span; in other words, the percentage of sag will remain the same. The stress in the wire and the length of wire, also, will be reduced in the same ratio. Again, the stress in the wire for a given span for a definite sag is directly proportional to the total load per foot on the wire.

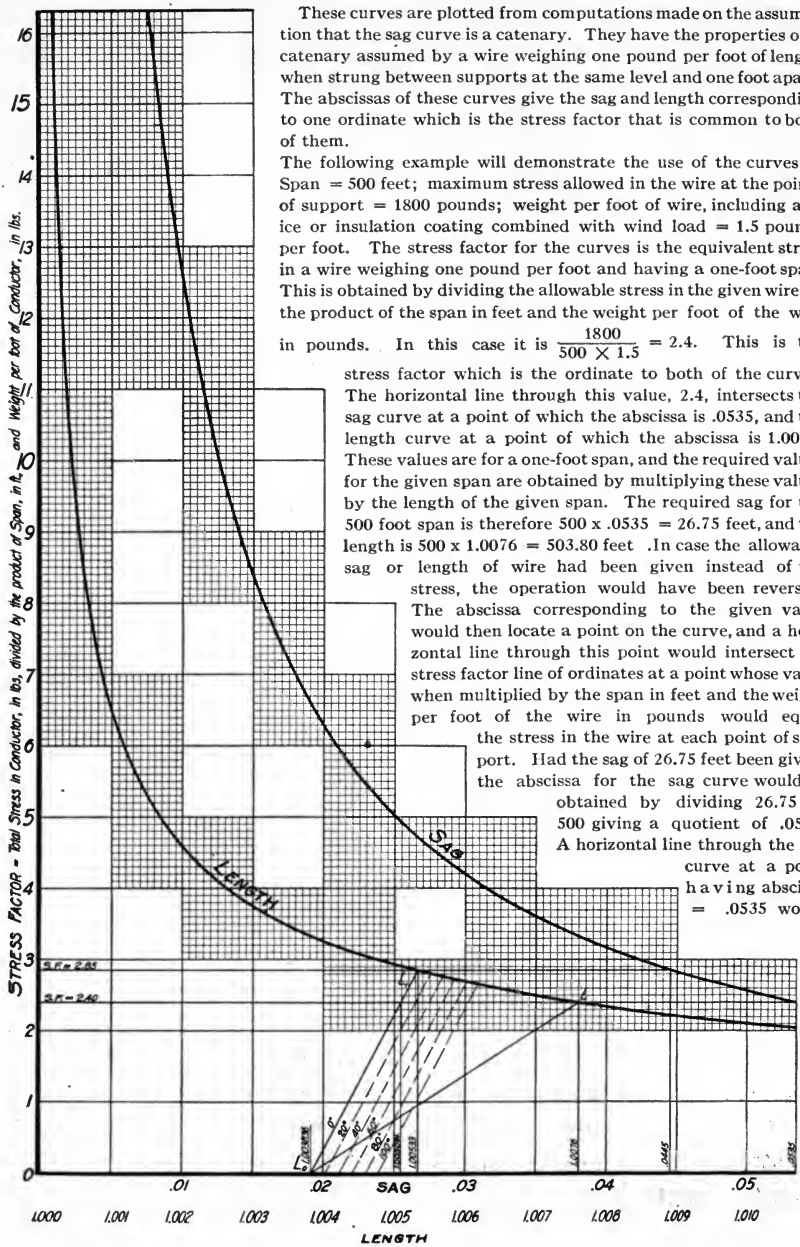
Taking advantage of this fact, two curves can be plotted which will give the sag, stress and length relationships for a wire on which the total load is one pound per foot when strung over a one-foot span, as shown in Fig. 10. These relationships will be directly proportional to those obtaining for the longer spans and for the varied loadings per foot of length on the wire used. Different sets of curves may be plotted for different proportions of sags to spans. A careful study of these curves will show how the stress changes with increase of sag. It will be noted that after the sag has reached 15% to 20% there is little reduction in stress by further increase of sag, and that an actual increase of stress soon results. This is of extreme importance when considering the use of very light wires on long spans.

The sine of the angle made by the wire with the horizontal, at the point of support, is one-half the length of wire in the span divided by the stress with one pound per foot weight of wire, and may be obtained from the length curve.

THOMAS' CURVES FOR SAG AND

These curves are plotted from computations made on the assumption that the sag curve is a catenary. They have the properties of a catenary assumed by a wire weighing one pound per foot of length when strung between supports at the same level and one foot apart. The abscissas of these curves give the sag and length corresponding to one ordinate which is the stress factor that is common to both of them.

The following example will demonstrate the use of the curves. Span = 500 feet; maximum stress allowed in the wire at the points of support = 1800 pounds; weight per foot of wire, including any ice or insulation coating combined with wind load = 1.5 pounds per foot. The stress factor for the curves is the equivalent stress in a wire weighing one pound per foot and having a one-foot span. This is obtained by dividing the allowable stress in the given wire by the product of the span in feet and the weight per foot of the wire in pounds. In this case it is $\frac{1800}{500 \times 1.5} = 2.4$. This is the stress factor which is the ordinate to both of the curves. The horizontal line through this value, 2.4, intersects the sag curve at a point of which the abscissa is .0535, and the length curve at a point of which the abscissa is 1.0076. These values are for a one-foot span, and the required values for the given span are obtained by multiplying these values by the length of the given span. The required sag for the 500 foot span is therefore 500 x .0535 = 26.75 feet, and the length is 500 x 1.0076 = 503.80 feet .In case the allowable sag or length of wire had been given instead of the stress, the operation would have been reversed. The abscissa corresponding to the given value would then locate a point on the curve, and a horizontal line through this point would intersect the stress factor line of ordinates at a point whose value when multiplied by the span in feet and the weight per foot of the wire in pounds would equal the stress in the wire at each point of support. Had the sag of 26.75 feet been given, the abscissa for the sag curve would be obtained by dividing 26.75 by 500 giving a quotient of .0535. A horizontal line through the sag curve at a point having abscissa = .0535 would



STRESS CALCULATIONS (FIG. 10)

intersect the stress factor line of ordinates at 2.4. This value when multiplied by the span in feet and the weight of the wire per foot in pounds would give the desired stress, thus;

$$2.4 \times 500 \times 1.5 = 1800$$

Variations in Loading

Assume the load to decrease from 1.5 to 0.5 lb. per ft., the temperature remaining the same. The resulting conditions of Stress, Sag and Length are determined as follows. If all the load could be removed from the wire it would contract to its "unstressed" length, called L_0, from its full-load length, $L = 1.0076$ ft. per foot of span. If the sectional area of the conductor = .03 sq. in.; the coefficient of elasticity = E = 16,000,000, and the total stress = 1800 lb., then,

$$L = 1.0076 = L_0 + \frac{\frac{1800}{.03}}{16{,}000{,}000} \times L_0,$$

from which,

$$L_0 = \frac{1.0076}{1 + \frac{\frac{1800}{.03}}{16{,}000{,}000}} = \frac{1.0076}{1.00375} = 1.003836 \text{ ft. for one-foot span.}$$

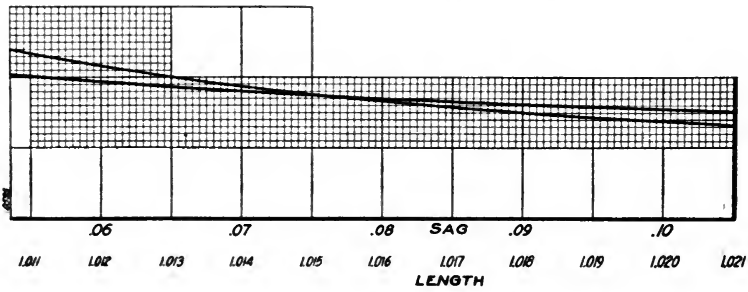
Plot this on the zero Stress Factor line, at L_0. Then the line L_0L is the "stress-stretch" line for this particular span and loading. If the load of 1.5 lb. per ft. stretches the wire for one-foot span from L_0 to L, a load of 0.5 lb. per ft. will stretch it $\frac{0.5}{1.5}(L - L_0) = \frac{1}{3}(1.0076 - 1.003836) = .001255$ ft., which, added to L_0, gives the length of the wire for the lighter load, 1.005091 ft. for one-foot span. Plot this value on the same Stress Factor line as for the preceding load, S. F. = 2.4, and through this point and L_0 draw a line to the Length curve. Its intersection, $L_1 = 1.00533$, is the length of the wire for one-foot span for a load of 0.5 lb. per ft. For this new condition the properties of the 500 ft. span will be: Stress = 500 x 2.85 x 0.5 = 713 lb.; Length = 500 x 1.00533 = 502.665 ft.; and Sag = 500 x .0445 = 22.25 ft. This operation is reversed when working from a light to a heavier load, the principle being the same in all cases.

Temperature Variation

The preceding methods assume a constant temperature, but every change of temperature causes a readjustment of Stress, Sag and Length in any span. To determine the new conditions, first find L_0, the "zero stress" length of wire for one-foot span, as above described. Then compute the change in this length resulting from the change of temperature, and plot this variation from L_0 along the zero Stress Factor line. This gives the "unstressed" length for the new temperature, and through this point draw a line parallel to the "stress-stretch" line for the load then existing. Its intersection with the Length curve gives the new length of wire for one-foot span, and determines the other factors. For example, if the above computation was for a temperature of 0° F., and the properties of the span are required for 20° intervals to 100° F., and the coefficient of heat expanison is .0000096, the length at 20° F. will be,

$$L_{20^\circ} = L_0 (1 + .0000096\, t) = 1.003836 (1 + .0000096 \times 20) = 1.004029 \text{ ft.}$$

Through this new length draw a line parallel to line L_0L_1. It intersects the Length curve at 1.005505, the coincident values being: Stress Factor = 2.8, Sag = .0453. Then for the 500 ft. span, Stress = 500 × 2.80 × .5 = 700 lb., Length = 500 × 1.005505 = 502.7525 ft., Sag = 500 × .0453 = 22.65 ft. Similarly for successive 20° intervals, or for any other temperature changes.

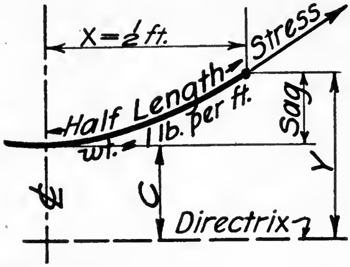


VALUES USED FOR PLOTTING CURVES FOR WIRE WEIGHING ONE POUND PER FOOT OF LENGTH WHEN SUSPENDED IN ONE-FOOT SPAN

$$\text{Stress} = Y = \frac{C}{2}\left(e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right)$$

$$\text{Sag} = Y - C$$

$$\text{Length} = 2 \times \frac{C}{2}\left(e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right)$$



$\frac{X}{C}$	Stress	Sag	Length	$\frac{X}{C}$	Stress	Sag	Length
.0050	100.001 3	.001 250	1.000 004 2	.080	6.270 0	.020 01	1.001 066
.0055	90.910 5	.001 375	1.000 005 1	.085	5.903 6	.021 26	1.001 205
.0060	83.334 8	.001 500	1.000 006 1	.090	5.578 1	.022 52	1.001 351
.0065	76.924 7	.001 625	1.000 007 1	.095	5.286 9	.023 77	1.001 503
.0070	71.430 3	.001 750	1.000 008 2	.100	5.025 0	.025 02	1.001 668
.0075	66.668 5	.001 875	1.000 009 4	.105	4.788 2	.026 27	1.001 839
.0080	62.502 0	.002 000	1.000 010 7	.110	4.573 0	.027 53	1.002 017
.0085	58.825 7	.002 125	1.000 012 0	.115	4.376 6	.028 78	1.002 205
.0090	55.557 8	.002 250	1.000 013 5	.120	4.196 7	.030 04	1.002 402
.0095	52.633 9	.002 375	1.000 015 0	.125	4.031 3	.031 29	1.002 606
.010	50.002 5	.002 50	1.000 017	.130	3.878 7	.032 55	1.002 819
.011	45.457 3	.002 75	1.000 020	.135	3.734 2	.033 80	1.003 040
.012	41.669 7	.003 00	1.000 025	.140	3.606 5	.035 06	1.003 270
.0125	40.003 1	.003 13	1.000 026	.145	3.484 6	.036 31	1.003 508
.013	38.464 8	.003 25	1.000 028	.150	3.370 9	.037 57	1.003 754
.014	35.717 8	.003 50	1.000 033	.170	2.983 8	.042 60	1.004 825
.015	33.337 1	.003 75	1.000 037	.200	2.550 2	.050 17	1.006 680
.016	31.254 0	.004 00	1.000 043	.220	2.328 0	.055 22	1.008 086
.017	29.416 0	.004 25	1.000 048	.25	2.062 8	.062 83	1.010 444
.0175	28.575 8	.004 38	1.000 051	.27	1.919 8	.067 91	1.012 194
.018	27.782 3	.004 50	1.000 054	.30	1.742 2	.075 56	1.015 068
.019	26.320 5	.004 75	1.000 060	.32	1.643 2	.080 68	1.017 154
.020	25.005 0	.005 00	1.000 067	.35	1.517 0	.088 40	1.020 542
.022	22.732 8	.005 50	1.000 081	.37	1.444 9	.093 56	1.022 973
.024	20.839 3	.006 00	1.000 096	.40	1.351 3	.101 34	1.026 881
.025	20.006 3	.006 25	1.000 104	.42	1.297 0	.106 55	1.029 660
.026	19.237 3	.006 50	1.000 113	.45	1.225 5	.114 41	1.034 093
.028	17.864 1	.007 00	1.000 131	.47	1.183 5	.119 68	1.037 224
.030	16.674 2	.007 50	1.000 150	.50	1.127 6	.127 63	1.042 19
.032	15.633 0	.008 00	1.000 171	.55	1.050 1	.141 00	1.051 19
.034	14.714 4	.008 50	1.000 193	.60	.987 9	.154 55	1.061 09
.036	13.897 9	.009 00	1.000 216	.70	.896 5	.182 26	1.083 69
.038	13.167 4	.009 50	1.000 241	.80	.835 8	.210 83	1.110 13
.040	12.510 0	.010 00	1.000 267	.90	.796 2	.240 61	1.140 57
.043	11.638 7	.010 75	1.000 308	1.00	.771 54	.271 54	1.175 20
.047	10.650 1	.011 75	1.000 368	1.10	.758 42	.303 87	1.214 23
.050	10.012 5	.012 50	1.000 417	1.20	.754 44	.337 77	1.257 88
.055	9.104 7	.013 75	1.000 504	1.30	.758 04	.373 43	1.306 45
.060	8.348 3	.015 00	1.000 600	1.40	.768 18	.411 04	1.360 21
.065	7.708 4	.016 26	1.000 704	1.50	.784 14	.450 80	1.419 52
.070	7.160 4	.017 51	1.000 817	1.60	.805 46	.492 96	1.484 73
.075	6.685 4	.018 76	1.000 938				

Considering the case of a span having supports at unequal heights above a given horizontal plane, if the horizontal distance from the higher support to the lowest point of the wire is known, the stress and sag in this part of the span can be determined by considering this part as one-half of a span equal to twice this distance. The smaller stress in the other part can be determined in the same manner.

The following formulas, based upon the catenary, give the horizontal distance from the higher support to the lowest point of the wire,

$$l_u = \frac{l}{2} + \frac{hT}{l} - \frac{h^2}{2l} \tag{A}$$

$$l_u = \frac{dl}{h}\left\{1 - \sqrt{1 - \frac{h}{d}}\right\} = l \cdot \frac{\sqrt{d}}{\sqrt{d-h} + \sqrt{d}} \tag{B}$$

where,

l = the span in feet.

l_u = the horizontal distance in feet from the higher support to the lowest point of the wire.

h = the difference in height of the two supports in feet.

T = the stress in pounds in the wire at the higher support, with *one pound per foot load* on the conductor.

d = the sag in feet measured from the higher point of support.

Formula (A) is useful when the span and the stress to be allowed in the wire are given, and formula (B) when the span and the sag are given.

These formulas are approximate in that the horizontal projection of the wire is substituted for the actual length of it. Formula (A) is correct within from 2% to 4%, when neither sag nor difference in heights of supports exceeds 15% of the span. Formula (B) has an error of less than 1% under these conditions.

SPACING OF TOWERS

The problem of determining the type and the spacing of the towers to be used, is one that requires considerable study of all the foregoing, as the towers are only a part of the complete installation, and a saving on one item may easily be more than offset by an increased cost of some other items affected by the same conditions which made the initial saving. In other words, it is a case of balancing one condition against another, to determine what is the best possible combination. The supporting structures must, of course, be placed as far apart as possible; but an analysis of the various sag conditions for the wires makes it evident that there are definite limits to be observed.

SPACING OF CONDUCTORS

After the spacing of towers has been determined, together with the size of wires to be used and the voltage to be carried by them, the next thing to consider is the spacing of the several wires and the minimum clearance from the ground line to the lowest wire under the worst loading condition. The maximum sag to be allowed must then be determined, and this condition, along with the assumed loading across the line, will determine the pull which may occur in the direction of the line on the wire. The spacing of the wires in both horizontal and vertical directions is dependent upon the voltage carried and upon the length of spans. The minimum spacing, especially in the horizontal direction, will obtain when the wires are supported on pin insulators, or are attached to the cross arms by means of strain insulators. For this condition, it is recommended that, for conductors carrying alternating current, the minimum separation of these conductors, at the points of support, shall be one inch for every twenty feet of span, and one inch additional for each foot of normal sag, but in no case shall the separation be less than:

Line Voltage	*Clearance*
Not exceeding 6600 volts	14½ inches
Exceeding 6600 volts but not exceeding 14000 volts,	24 inches
" 14000 " " " " 27000 "	30 "
" 27000 " " " " 35000 "	36 "
" 35000 " " " " 47000 "	45 "
" 47000 " " " " 70000 "	60 "

For voltages higher than 70000 the minimum separation should be 60 inches plus 0.6 inch for every additional 1000 volts.

When conductors are supported by suspension insulators, the separation of them horizontally must be made greater than when they are supported on pin type insulators. The amount of this increase is empirical, and is more or less a matter of judgment on the part of the Engineer who designs the line. When the conductor wire is supported from the cross arm by strain insulators, it is frequently assumed that the jumper wire will swing to a position, making an angle of thirty degrees with the vertical. It is usually assumed that the maximum swing of a suspension insulator string will be to an angle of forty-five degrees, but this depends upon the size and weight of the conductor, and also upon the assumed maximum loading. It is possible that under unusually severe conditions, two wires suspended from the same cross arm may swing toward each other until each of them will make an angle of about thirty degrees with the vertical. Or even though they do not both swing the same amount, it is a safe assumption that twice the horizontal projection of the length of one insulator string when swung to thirty degrees from the vertical, will be equivalent to the sum of the horizontal projections of the two wires when swinging toward each other under the worst conditions of service. This means that when wires are supported by suspension insulators instead of on pin type insulators, the horizontal separation should be increased by the length of one insulator string.

It is generally recommended that the minimum clearance in any direction between the conductors and the tower, shall not be less than:

Line Voltage	*Clearance*
Not exceeding 14000 volts	9 inches
Exceeding 14000 but not exceeding 27000	15 "
" 27000 " " " 35000	18 "
" 35000 " " " 47000	21 "
" 47000 " " " 70000	24 "

Usually the suspension insulators are made sufficiently long so that when swung out to the assumed position of maximum swing, the vertical distance between the conductor and its supporting cross arm, or any other part of the tower, will meet all the requirements for clearance. The overhead ground wire, or wires, should be, in general, not more than forty-five degrees from the vertical through the adjoining conductor.

The several wires must be spaced far enough apart vertically so that under the worst conditions the wires will not come so close together as to make trouble electrically. This must have careful consideration, especially on the very long spans, because it is entirely possible during storms for the lowest wires to be free from ice loading or to be suddenly relieved of such loading, when they might swing up, close to the wires directly above them, which might be heavily loaded with ice and hence have considerable sag.

The arrangement and spacing of the wires almost always fixes, within certain limits, the general type of the supporting structure to be used. This is at least true of the upper part of it. The outline of the structure below the lowest cross arm will be made that which is the most economical, unless such an outline is prohibitive on account of right-of-way or other limiting conditions.

Where the transmission line consists of three conductor wires, with or without a ground wire, it very often works out to very good advantage to put the three conductor wires in the same horizontal plane, which means that the middle one will pass through the tower. When suspension insulators are used with this arrangement of wires, the tower must be made wide enough to allow ample clearance from the conductor when swung to maximum position either way. But if strain insulators are used, then a much narrower tower may be used by attaching the jumper wire to a pin on the center line of the tower. The narrower tower makes a much more economical construction. When six conductor wires, with or without ground wires, are used, three of the conductors are placed on each side of the tower. These are generally placed so that the three wires in each set are in the same vertical plane, but sometimes the middle one will be put farther from the center line of the tower than the other two wires.

The design of the supporting structures from this point on, consists in determining just what loads are to be considered as coming on the structures, what unit stresses shall be used throughout, and whether a comparatively large or small investment shall be put into them. In other words, it is a matter of first importance whether these structures are to be regarded for a temporary proposition, and hence made as cheaply as possible, or whether they are to be considered as part of a permanent construction and therefore figured a little more liberally.

TEMPORARY STRUCTURES

For a temporary proposition the structures are, of course, made as light as possible and are almost always painted. They are rarely galvanized. In such cases the assumed loadings are kept very low, and are intended to take care of only normal conditions, on the theory that if some of the structures should be subjected to loadings of unusual intensity resulting from specially severe storms, it will be more economical to replace some of them that might be destroyed than to provide additional strength in all the supports. For the same reason the unit stresses are always run as high as possible.

PERMANENT STRUCTURES

On the other hand, where permanency of construction is wanted, the design is made more liberal in every way. To start with, the assumed loadings are such as will be expected to take care of more than ordinary conditions of service. They will be made sufficiently high to be in themselves an insurance against possible interruptions of service due to breakdowns caused by storms. Also, the unit stresses will be kept lower and heavier material will be specified. Generally, but not always, such structures will be required to be galvanized instead of painted, so that the structure will be in service for a longer time.

THICKNESS OF MATERIAL

When the material is required to be galvanized, many specifications will allow web members to be made of material only $\frac{1}{8}''$ thick, but will require a minimum thickness of $\frac{3}{16}''$ or possibly $\frac{1}{4}''$ for the main posts. Almost all specifications require a minimum thickness of material of $\frac{3}{16}''$ for all members when painted; but some specify that no material less than $\frac{1}{4}''$ thick shall be used when painted; while others demand a minimum thickness of $\frac{3}{16}''$ for all material, regardless of whether it is painted or galvanized.

GALVANIZE FOR PERMANENCY

The history of transmission line structures proves that where permanency of construction is desired, they should always be galvanized, not painted. At least all parts of the structure in close proximity to the conductor wires should be galvanized, irrespective of what kind of a protective coating is given to the balance of the structure. This is especially true in those cases where high voltages are used.

SPECIFICATIONS FOR DESIGNS

There is no such thing as a standard practice among Engineers today regarding the method to be pursued in preparing specifications for transmission line towers on which competitive bids are to be received. Usually for a line requiring several towers, the Engineer in charge of the installation will determine the arrangement of all the wires, the limiting dimensions for the structures, and the loadings for them; but the design of the structures will be left generally to the Manufacturer, subject, however, to those provisions of the specifications which are intended to insure that the towers or poles will all be designed to have the same strength. Different Engineers seek to accomplish this result in as many different ways. Some will specify the loadings under which they expect the towers to be used, and will stipulate that the design shall provide sufficient strength to take care of these loadings with a given factor of safety; others will state unit stresses which shall be used in determining the sections in the design, to take care of the stresses resulting from the above loadings. Other Engineers will increase the desired working loads by some factor which they will introduce as a margin of safety, and will then give these increased loadings instead of the working loadings, and will require that the structures be designed to withstand these loadings without failure. Still other Engineers will specify that certain unit stresses shall be used in determining a design, which shall support the specified loadings with a given factor of safety; and, further, that the completed structure must support loads that are twice as large as those specified, but without any restriction regarding unit stresses to be employed.

FACTOR OF SAFETY

The term "Factor of Safety" is in reality a misnomer, and, because of this, it is not always interpreted in the same way by different men. Literally speaking, the structure which is properly designed with a factor of safety of three, should sustain without failure loads three times as great as those which are expected to be the working loads under normal conditions. But the term "Factor of Safety," as it is usually interpreted and applied, means that the unit stresses used throughout shall be one-third the ultimate strength of the material entering into the construction. In actual practice the results of such an interpretation are very disappointing. In a composite structure made up of a large number of different pieces, some of which are undergoing compression while others are in tension, the action of this body as a whole against outside forces will differ radically from what would be expected of any one of its component parts under a similar test. This, of course, is accentuated in the case of transmission towers, because they are always made as light as possible for the work required of them, and, hence when under load, they deflect considerably from their original outlines, and this in turn produces a rearrangement and entirely different distribution of stresses. The net result of all this is, that all such structures will fail when the loading on them reaches the point where some, if not all, of the members making up the construction are stressed to the elastic limit for their material.

Since the elastic limit for steel in either tension or compression is about one-half its ultimate strength, it follows that the structure whose members are determined by using unit stresses equal to one-third of the ultimate strength of the material, will have a total strength only 50% in excess of that required to take care of actual working conditions; so that, instead of having the so-called "Factor of Safety of Three" it has an actual factor of safety of one and one-half.

This fact is recognized by those who first multiply the required working loads by a factor which will provide a margin of safety, and then specify that the towers shall support without failure these increased stipulated loads. It is not often that, under these conditions, the specifications will call for the employment of definite unit stresses in determining the several sections of material to be used. But in all such cases, when unit stresses are specified, it will almost always be found that those recommended are close to the elastic limit for the material.

UNIT STRESSES

The unit stress for a member in either tension or compression is the quotient of the total load divided by the cross sectional area of the member supporting the load. This is given in pounds per square inch. The unit stress for a member in compression is less than that for a member in tension by a quantity which is a function of the ratio between the unsupported length of the member and its least radius of gyration. Usually this unit stress is determined by a straight-line formula, such as

$$S_c = S - C\frac{L}{R}$$ in which

S_c = the desired unit stress in compression,
S = the unit stress allowed in tension,
L = the unsupported length of the member in inches,
R = the least radius of gyration for the member, in inches,
C = a constant determined by experimental investigation.

The elastic limit in tension is about 27000 pounds per square inch of net section. The straight-line formula $27000 - 90\frac{L}{R}$ for unit stresses in compression, gives values which have been proven by actual tests to be approximately the elastic limit for the material.

Where the so-called "Factor of Safety of Three" is wanted, the unit stress generally specified for members in tension is 18000 pounds per square inch of net section, while, for unit stress for members in compression, the formula

$$18000 - 60\frac{L}{R} \text{ is specified.}$$

It will be noted that these values are just two-thirds of those immediately preceding, and, therefore, offer a margin of safety of 50%. It is not often that unit stresses smaller than these are specified for tower work, but occasionally we find specifications which are very severe, considering the infrequency of maximum or even full loads on this type of structure.

It is common practice among Engineers, when specifying that the towers shall safely support certain loads, to refrain from putting any limitations on the design, such as what relationship shall be allowed as a maximum between the length of any compression member and its least radius of gyration. On the other hand, when it is stipulated that the structures shall be figured for carrying certain loads by using given unit stresses, it is almost always also stipulated that the ratio of length of compression members to their least radius of gyration shall be limited to a certain maximum value.

BOLT VALUES

Bolts stressed to 24000 pounds per square inch in shear have values comparable with the strength of members which are figured on the basis of 27000 pounds per square inch of net section in tension, or 27000

$-90\frac{L}{R}$ pounds per square inch of gross section in compression. From this it follows that bolts need not be stressed lower than 16000 pounds per square inch in shear to get values corresponding to those resulting from using 18000 pounds per square inch of net section in tension or $18000 - 60\frac{L}{R}$ pounds per square inch of gross section in compression, for members which are to be connected by means of these bolts. It is evident that smaller values for bolts are unwarranted. Consistent practice in designing requires that the values assumed for bolts shall bear the same ratio to their elastic limit as the ratio obtaining between the working value assumed and the elastic limit for the several members which are connected by the bolts.

LOADINGS

In regard to the specific loadings for which the structures shall be designed, considerable depends upon where they are to be used, as there are several factors entering into this question.

The first thing that should be determined is the kind and maximum value of the vertical load to be taken care of at the end of the cross arm. If the line runs through a comparatively level country, there is no reason why there should ever be any uplift at the end of the cross arm; but if the line runs along steep grades, then there may be times when the vertical load will be upward rather than downward. This is of considerable consequence in the designing of the tower. The vertical load at the end of the cross arm is usually supported by members which run from the end of the cross arm to the main post angles at some point above the cross arm. If the vertical load is downward, these supporting members will act in tension, but if the load can ever be upward instead of downward, then, such members must be capable of taking stress in compression. In cases where the cross arms are long, which is almost always true when suspension insulators are used, these members must be made much heavier to take the stress in compression, rather than tension.

ANGLE TOWERS

The next thing to determine, if possible, is, how many towers will have to take care of angles in the line, and what will be the maximum angle encountered. If this angle should be very large, it will be necessary to provide special structures for such points in the line; but if the angle is very small, provision for it may be made by using one of the straight line towers at this point and shortening the span on each side of it. This shortening of the span reduces the wind load on the wires transverse to the direction of the line, and at the same time reduces the pull in the wires in the direction of the line, if the sag is made a greater percentage of the shortened span than it is in the case of the adjoining spans.

In Fig. 11 there is shown a graphical diagram of the components of the tension in the wire, parallel to the faces of the tower, when its axis parallel to the cross arm bisects the given angle in the line. It will be seen that when the wires leading off in both directions from the end of the cross arm have equal stresses, the component "Y" in one wire balances the corresponding component from the other wire, but that the component "X" is twice what it is when only one wire leads off from the cross arm. This means that in the one case, marked condition "B," the load on the tower is twice the component "X" from one wire, but that for condition "A," the load on the tower is the sum of the components "Y" and "X" from one wire.

It will be noted that for condition "B" the total load on the tower from the pull in the direction of the line will just equal this pull when the tower bisects an angle of sixty degrees in the line, and that this load increases to double the pull on one wire, as a maximum limit, when the angle in the line reaches one hundred eighty degrees. For condition "A" the total load will always be greater than the pull in the wire, no matter how small the angle in the line, and the worst loading will occur when the tower bisects an angle of ninety degrees in the line. When the angle in the line is as large as ninety degrees, it will often be more desirable to construct a special tower, and to set it normal to the direction of the line.

SPECIAL TOWERS

Having determined whether it will be necessary to provide special towers to take care of angles in the line, the next step should be to determine how many, if any, special towers should be provided to take care of such special cases as railroad crossings, and what specifications must govern in the design of these special structures. The Railroad Companies have their own specifications for these structures, and they

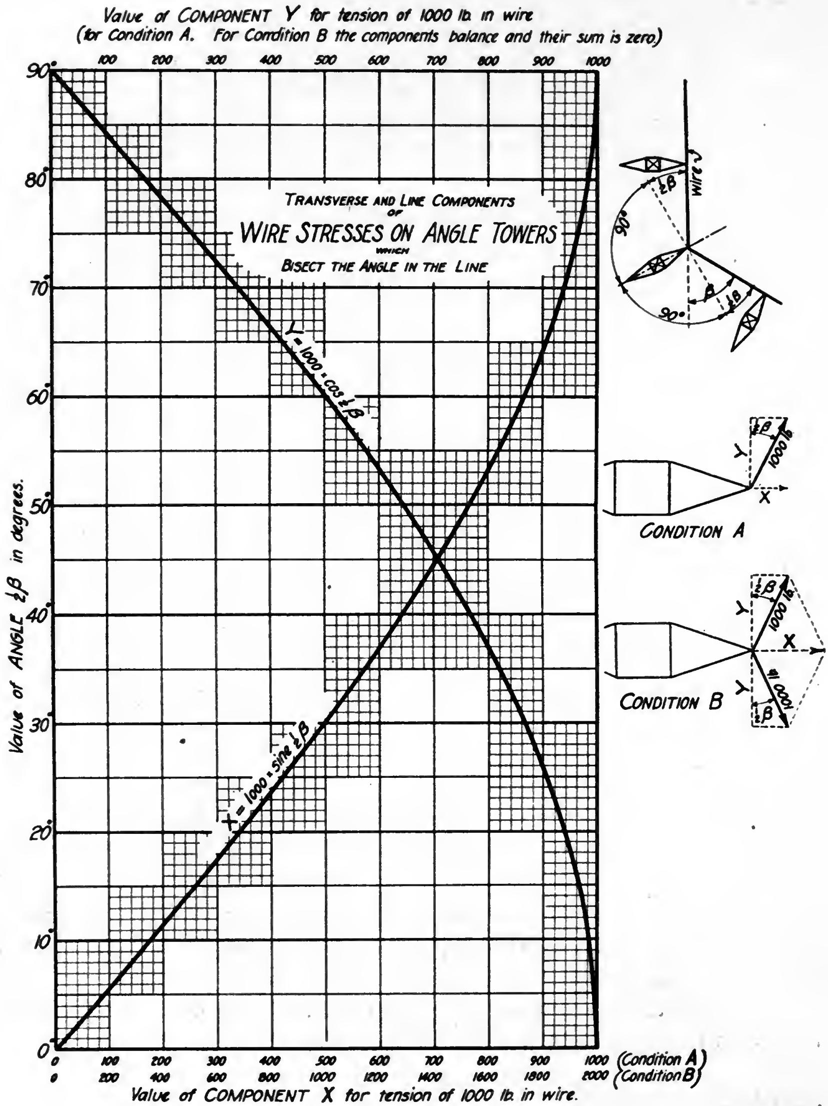

Value of COMPONENT Y for tension of 1000 lb. in wire
(for Condition A. For Condition B the components balance and their sum is zero)
TRANSVERSE AND LINE COMPONENTS OF WIRE STRESSES ON ANGLE TOWERS WHICH BISECT THE ANGLE IN THE LINE
Y = 1000 × cos ½β
X = 1000 × sine ½β
Value of ANGLE ½β in degrees.
Value of COMPONENT X for tension of 1000 lb. in wire.
(Condition A)
(Condition B)
CONDITION A
CONDITION B
Wire

Fig. 11

insist that all wires carried over their crossings shall be supported by structures complying with all their requirements as to loadings and unit stresses to be employed. Their specifications are generally very severe and, hence, special designs almost always are required for those particular points in the line. Of course, one thing always to be kept in mind, is to make as few different designs as circumstances will allow, so that there will be as much duplication as possible in the structures. This is an especial advantage for economical fabrication in the shop, and is also a big advantage when it comes to erecting the towers in the field.

Every line must be carefully studied and designed for its own particular requirements. A line which is taken through a city must be built in a different way from one going through an open country. The working loads might not need to be any heavier, but either the design loads should be heavier or the unit stresses lower, and the towers should be spaced closer together.

REGULAR LINE TOWERS

The average line of any length should have three different types of towers. These may be designated as—Standard or Straight Line, Anchor, and Dead End Towers.

All towers should be designed to take care of the dead weight of the structures and also the vertical loads at the ends of all the cross arms, in addition to and simultaneously with, the horizontal loadings specified below.

STANDARD TOWERS

The Standard, or Straight Line, Towers should predominate, and should be designed to support without failure the required horizontal loads transverse to the direction of the line, combined with a horizontal pull in the direction of the line applied at any one insulator connection equivalent to the value of the wire when stressed to about one-half its ultimate strength. These loads transverse to the line should be large enough to include the wind load across the wires and that against the tower itself, with a little margin of safety.

ANCHOR TOWERS

The Anchor Tower should be designed to support without failure any one of the following horizontal loadings:

(1) The same horizontal loads as those specified for the Standard Tower.

(2) An unbalanced horizontal pull in the direction of the line equivalent to the *working loads* of all the conductor wires and the ground wires, applied at the points of connection of the wires to the tower, combined with the transverse horizontal loads on the wires and the tower.

(3) An unbalanced horizontal pull parallel to the line equivalent to the *working loads* of the wires, applied at one end of each cross arm, all on the same side of the tower and all acting in the same direction, combined with the horizontal transverse loads on the wires and the tower.

DEAD END TOWERS

The Dead End Towers should be designed to support the same loadings as those specified for the Anchor Towers, but the sections should be determined by using smaller unit stresses. Unit stresses of 18000 pounds per sq. in. in tension and $18000 - 60\frac{L}{R}$ for compression, would give these towers approximately 50% more strength than the anchor towers would have when stressed just within the elastic limit.

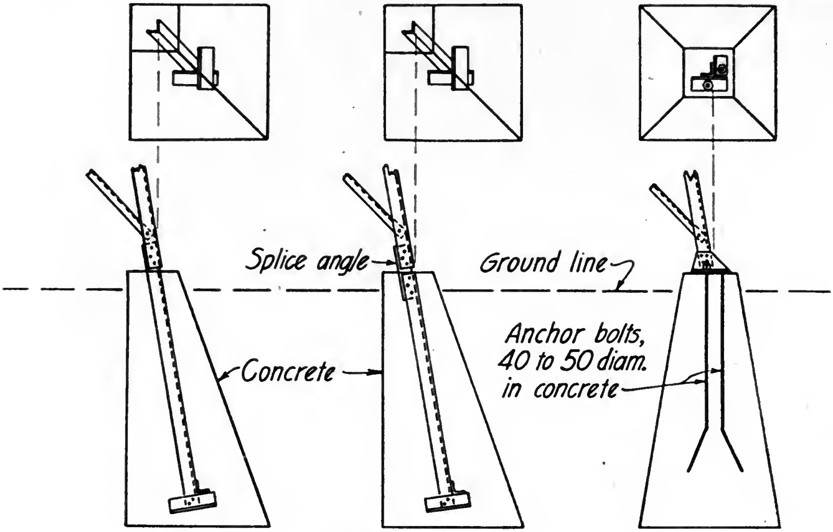
It will be noted that under the above specification, the standard tower will be required to take care of the torque resulting from an unbalanced horizontal pull equivalent to the *allowable tension* (which is one-half the ultimate strength) of one wire, applied at one end of any cross arm and acting parallel to the direction of the line; while, the anchor and the dead end towers are both required to take care of either the torque as given above for the standard tower or the torque resulting from an unbalanced horizontal pull equivalent to the *working loads* (actual tension in the wire under the working conditions) of all the wires on either side of the center line of the tower, applied at one end of each of the cross arms, and all acting parallel to the line and in the same direction. If one anchor tower is placed in the line for every ten or twelve standard towers, all conditions resulting from broken conductor wires should be localized to the territory between two anchor towers. The reason for using lower unit stresses in the dead end towers than in the anchor towers for exactly the same loadings, is that the dead end towers may have to support a large part of this total loading at all times, and all of it very frequently, while the anchor tower may have to support the same loading only once in a great while, and then for only a very brief time.

One of the aims to be kept constantly in mind in designing a transmission structure, is to get a finished tower in which all the stresses can be determined definitely. We usually determine the stresses graphically. The stresses resulting from the horizontal loads applied as so much shear must be determined separately from the stresses resulting from torque. These stress diagrams cannot be combined except in those cases where the slope of the post does not change between the horizontal planes bounding that part of the tower for which the diagrams are wanted. This is true because the horizontal loads which act as so much shear, may be assumed to be acting in a plane containing both posts of the face of the tower, parallel to the direction of the load, in which case the posts may or may not (depending upon the slope of the posts) take up a part of this shear directly. On the other hand, the torque is a moment acting in a horizontal plane and is constant between any two parallel planes, unless it is either increased or decreased by an additional torque of the same or opposite kind.

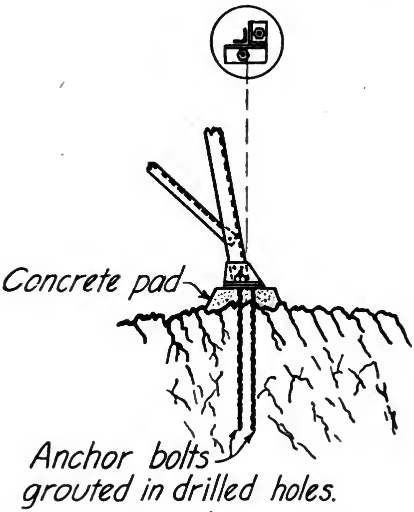
ANCHORAGE DESIGNS

The members for anchoring the structure to the footings are generally the last part of the design to be considered.

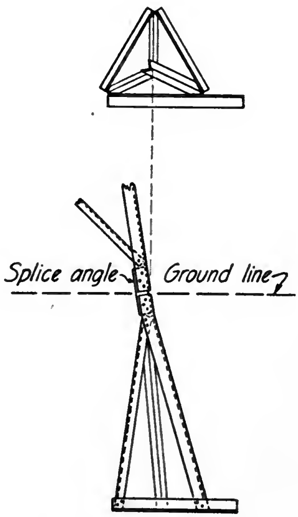
The first question to be determined is whether concrete footings shall be used. These are more simple, and involve much less steel work than any other type of footing used for transmission line structures. The weight of the concrete itself reacts against the tendency of the post to pull away from the base because of the tension in the post on one side of the tower. It also offers more bearing surface against the earth around the footings and introduces the passive resistance of a larger volume of earth against the uplifting tendency of the post on the tension side of the tower. Of course, the saving in the cost of steel in the structure must be balanced against the expense involved in putting the concrete in place, to determine whether or not it is advisable to use concrete footings. This will depend upon many circumstances which must be very carefully considered before reaching a conclusion. It is impossible to overestimate the importance of good anchorages. An otherwise excellent construction may be made inadequate by using footings which are not substantial. If one of the footings should be



Concrete Anchors



Rock Anchor



Earth Anchor

Fig. 12

insufficient to take care of the loads for which the superstructure is intended, it would be very apt to yield under full loading, and, in doing so, would bring about a new distribution of stresses among the members, and would put on some of the members stresses which were not in keeping with those for which the members were designed. Such a rearrangement of stresses may very easily be so vital as to bring about the failure of the superstructure. In view of this fact, it is recommended that, where there is any doubt as to whether concrete footings should be used, the benefit of any small doubt should always be given in favor of such footings. But, it may be that the structures are to be used where such footings would be practically impossible. Under such circumstances, other provisions must, of course, be made.

In the case of poles, the regular outline is generally continued below the surface of the ground whether concrete footings are used or not; but if concrete is not used, then additional steel must almost always be used to get more bearing area against the earth.

In the case of towers there is provided a separate footing for each of the posts. When concrete footings are used the posts are connected to them in one of two ways: In the first method, extensions of the post sections, which are called anchor stubs, may be built in these footings with just sufficient length extending above the concrete so that the lower post sections of the tower may be connected directly to them. These anchor stubs may extend almost to the bottom of the footing, or they may extend into the footings only far enough that the adhesion of the concrete to them will develop their full strength, in which case it will be necessary to add steel reinforcing bars from this point to the bottom of the concrete. This is necessary because provision must be made to bind the concrete together so that it will not break apart under the uplifting force in the post, and thus defeat its purpose. The other method used with the concrete footing is to have a base at the lower end of the post section which will bear directly on the mass of concrete in the footing, and which will at the same time be connected directly to this concrete by means of long bolts or rods extending well into the mass of concrete. These rods, in this case, would be brought into action only when the post is under tension. If these rods are straight for their full length, and fairly large, they should be imbedded in the concrete for a length equal to fifty times their diameter, in order to develop their full breaking strength. But if these rods are bent a little near their lower ends, their breaking strength will be developed by imbedding them in the concrete for a length equal to forty times their diameter. Provision for binding together the concrete in the footing must be made when anchor rods are employed, just the same as when anchor stubs are used.

With any type of footing, there must be provided sufficient bearing surface against the earth to resist the maximum compression in the post, and also an arrangement to lift enough earth to resist the maximum uplifting tendency in the post under the worst condition of loading.

The most positive and direct way to determine the size and outline of a footing for any given loading, is to increase this loading by the desired factor of safety, and then to determine a footing of which the ultimate resisting value will be sufficient to meet the conditions to be imposed. We recommend that the footing be so designed that its ultimate resisting value will be not less than 25% in excess of what is necessary to sustain the loading specified for the pole or tower.

For specially heavy towers which are required to dead-end heavy wires on long spans, it sometimes becomes a troublesome matter to provide adequate footings to take care of the uplift from the posts on the tension side of the tower under the assumed condition of maximum loading. This often happens in the case of River-Crossing Towers. Footings for such cases, if built in the ordinary way, would have to be made very deep and would require a large amount of concrete. It will often be found to be economical to design these footings with special outline and construction.

The following rather unique method has been successfully employed for taking care of cases involving unusually large uplifts, when the footings are built in clay or in mixed clay and sand that is comparatively free of gravel. A square pit is dug deep enough that its bottom will be below the frost line and large enough to afford sufficient bearing area against earth to sustain any possible downward pressure where the tower post may be subjected to either tension or compression. In the center of this pit a hole about twenty inches in diameter is bored with an earth-auger to the depth desired (this depth has been made as much as twenty feet below the bottom of the square pit). Dynamite is then placed in the bottom of this hole and connected with a firing magneto; then the hole is filled with concrete of 1:2:4 mixture, medium wet, and the charge of dynamite is fired immediately. The charge of dynamite that is generally used for this purpose consists of eight one-half pound sticks of 60% dynamite. Reinforcing bars with their ends bent are then pushed down through the concrete to the bottom of the hole and then raised three inches and securely held in this position to prevent them from sinking through the concrete and coming in contact with the earth before the concrete has set. The hole is then refilled with concrete, and the footing in the square pit is also poured and finished. From the moment the dynamite is placed and connected with the firing magneto, it is essential that all the subsequent operations be conducted as rapidly as possible. Not more than five minutes should be allowed between the time when the first pouring of concrete is started and when the dynamite is fired.

The average displacement from such an explosion of dynamite is about one and one-half cubic yards, this, of course, being dependent upon the depth of the hole and the nature of the surrounding earth. Experimental footings placed in this manner show that the enlarged base takes an almost spherical form with its center above the bottom of the excavated hole a distance equal to about one-fourth the horizontal diameter of the enlarged base. This diameter is sometimes almost four feet. It is evident that a footing of this kind can be made to resist a very large uplifting pull.

In the case of light towers it is sometimes considered advisable to put the tower in its erect position above the ground before the anchors are set, and to then bolt these footing members to the lower end of the main tower legs and put concrete or earth back fill around them while the tower is being supported independent of them. But in the case of heavy towers it is generally considered more economical to set the footing members exactly in their position first, and to then erect the towers and connect them to their footings. This latter method of erection requires that the anchor stubs be aligned as accurately as possible, as any inaccuracy in the setting of these anchors will make the subsequent assembling of the tower more difficult and less satisfactory. If the anchor stubs are not set accurately to their true positions, there will be introduced in the tower, additional stresses for which the tower members were not designed. An accurate alignment of the anchors can be accomplished only by using rigid templates that will hold the anchors in their definite positions until they have been secured by either the back fill or concrete.

Almost all towers are built smaller at the top than at the ground line, and the tower leg inclines from the vertical as determined by this outline of the structure. The anchor stub generally follows the direction of the main tower leg, but when it is put in this position and suspended from a template it has a tendency to swing to the vertical position. To obviate this condition the setting template should be trussed as shown in Fig. 13.

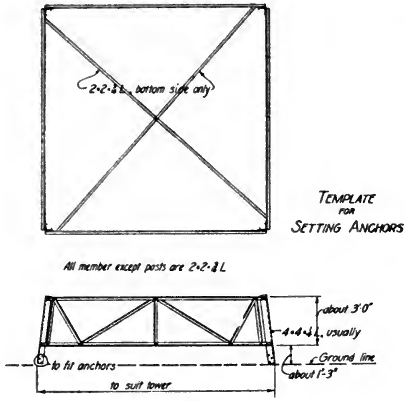


Fig. 13

ERECTION

Transmission towers are erected in one of two ways: they may be erected by assembling the members one at a time in their proper positions in the completed structure, or by assembling the complete structure in a prone position, and raising it to its vertical position by swinging it about two hinge points on or near two anchor stubs.

If the first of these two methods is used, there will generally be required a crew of eight men, including one foreman. The following equipment will generally suffice:

One light gin-pole, about 25 feet long.

One set of two-sheave and three-sheave blocks for ¾" diameter rope.

About 300 feet of ¾" diameter rope; four hand lines, each about 150 feet long; four small gate blocks for the hand lines.

The post members are erected with the gin-pole and tackle, but all the other members are pulled up from the ground with the hand lines. The time required will be about the same whether the tower is light or heavy. The time required will, however, depend upon both the accuracy of the fabrication of the material and the accuracy of the alignment of the anchor stubs.

If the second method is used, the actual work erecting the tower does not consume more than ten or fifteen minutes after all the preparations have been made. These preparations and the erection consist of three distinct operations:

(1) Leveling the ground where required for the erection equipment, and blocking up the tower on rough ground and for side-hill extensions. A crew of seven or nine men including a foreman is required.

(2) Rigging up erection equipment, and bolting erection shoes and struts in place, etc. A crew of about twelve men including a foreman is required.

(3) The actual raising of the tower. Sometimes horses are used for this operation, but it is often found to be more satisfactory to use a caterpillar tractor, especially for raising the heavier towers. One team of horses will generally suffice for this work, but it often requires four and sometimes six horses especially in rough country and for raising towers that are unusually heavy. The Tractor gives a much steadier pull, and will permit of holding the load at any desired point more satisfactorily than when horses are used. A substantial A-frame usually built up of steel pipes is generally employed for raising the tower from the prone to the upright position. A steel cable should also be used in preference to a manilla rope for this purpose in the case of the heavier towers.

When concrete footings are used, and this method of erection is employed, there is an advantage in having the anchor stubs set and concreted in position in advance of the assembling of the tower. When this is done, the tower can be assembled close to the anchor stub and can be raised about hinges fastened to the tops of the anchor stubs; but when the tower is assembled before the concrete is placed around the anchor stubs, it is necessary to assemble the tower a few feet away from the stubs, and then to skid the tower into the position from which it is to be raised. This process of skidding the tower is costly, and is also likely to injure the tower members.

SPACING OF TOWERS

The trend of American practice today in the designing of transmission line installations is to make the spans between supporting structures as great as possible. As the result of considerable study extending over several years of experience with lines having spans some of which were very short while others were exceptionally long, it has been determined that the best and most economical lines, all things considered, are those in which the supporting structures are spaced far apart.

This is true even though the first investment for the original installation is somewhat larger in the case of long spans than where short spans are used. It has been determined from comparative records that the maintenance of lines having the long spans is much less than was the maintenance of the same lines during previous periods when shorter spans were used. This decreased cost of maintenance has been proved to be sufficiently important to warrant making larger initial investments on original projects. The maintenance is not only less expensive with the long spans but it is also less troublesome, because there is less interference with continuous service along the line. This is a matter worthy of careful consideration, as the value of electrical service in almost every case is dependent upon the assurance of its continuity.

By using long spans the number of insulators required is reduced; and, as there is always a chance that a flash-over will occur at the insulator, it is obviously advisable to reduce the number of insulators to a minimum in order to eliminate, as far as possible, this source of trouble for the service.

Another advantage derived from the use of long spans is that the variations of stress in the wires resulting from large changes in temperature will be much less than under similar conditions of loading on short spans. The constant changing of stress in the wires is productive of more trouble than higher stresses which are more uniformly applied.

The long span is especially advantageous for a line carried along a hillside, because it will generally permit of such an arrangement of towers that there will not be any upward pull on any of them. The upward pulls are always a source of trouble, and they should be eliminated wherever conditions will permit an alternative construction. The upward pull causes not only mechanical but also electrical troubles, because, during a rain storm, water will run down along the wire into the insulator, which, of course, immediately produces electrical trouble.

The voltages used on present day high tension lines are such that the suspension-type and strain-type insulators are rapidly displacing the pin-type insulators. This, of course, means longer and heavier cross arms and higher supporting structures. It is also true that the cost of wood is steadily increasing, and will continually increase as the wood becomes less plentiful. These conditions when combined with the tendency for long span construction as described above, mean that the wood pole construction is being rapidly superseded by the better and more permanent steel tower construction.

When the Manufacturer is expected to design the structures for a line of any considerable length, he is generally furnished very definite and complete specifications regarding loadings and unit stresses; but when he is asked for quotations on only a few structures, it is not often that full and complete information regarding working conditions are furnished him. Nor will this information always be forthcoming, even when the customer is requested to give more definite data. As a rule, a part of the necessary information will be furnished by the customer, and it becomes the task of the Manufacturer to complete the design by making his own assumptions regarding the missing data.

The customer will very often profit financially by making as complete as possible the information he gives to the Manufacturer, and it is always much more satisfactory to the designer to know positively what working conditions are to determine the design.

BLAW-KNOX

TABLE 1

Properties of Wire Materials

Material	Ultimate Strength Lbs. per Square In.	Elastic Limit Lbs. per Sq. In.	Mod. Elasticity E Lbs. per Sq. In.	Coef. Expansion 1° Fahr.
Copper, solid, soft-drawn	32–34,000	28,000	12,000,000	0.0000096
Copper, solid, hard-drawn	50–55–57–60,000	30–32–34–35,000	16,000,000	0.0000096
Copper, stranded, soft-drawn	34,000	28,000	12,000,000	0.0000096
Copper, stranded, hard-drawn	60,000	35,000	16,000,000	0.0000096
Aluminum, stranded	23–27,000	14,000	9,000,000	0.0000128
Steel, stranded, Siemens-Martin	75,000	......	29,000,000	0.0000064
Steel, stranded, high-tension	125,000	......	29,000,000	0.0000064
Steel, stranded, ex-high-tension	187,000	......	29,000,000	0.0000064

Loadings Recommended for Wires

Class A loading = Dead Load + 15.0 lbs. per sq. ft. wind pressure.
Class B loading = Dead + ½″ Ice + 8.0 lbs. wind.
Class C loading = Dead + ¾″ Ice + 11.0 lbs. wind.

TABLE 2

Steel Wire—Stranded—Galvanized

Size	Diam. Inches	Area Sq. In.	Siemens-Martin		High Tension		Load per Lin. Foot, Vertical		Load per Lin. Foot Horizontal	Max. Load per Lin. Foot Resultant	Angle of Resultant Load	EA E = 29,000,000
			Ult. Tens.	Allow. Tens.	Ult. Tens.	Allow. Tens.	Dead	Dead ½″ Ice	8.0 lbs. per Sq. Ft. Cond. ½″ Ice			
5/8	.6250	.2356	19,000	9,500	25,000	12,500	.821	1.520	1.083	1.867	35° 28′	6,832,000
9/16	.5625	.1922	14,500	7,250	21,100	10,550	.668	1.329	1.042	1.689	38° 2′	5,574,000
1/2	.5000	.1443	11,000	5,500	18,000	9,000	.510	1.132	.999	1.510	41° 28′	4,185,000
7/16	.4375	.1204	9,000	4,500	15,000	7,500	.415	.998	.958	1.383	43° 25′	3,492,000
3/8	.3750	.0832	6,800	3,400	10,500	5,250	.295	.839	.917	1.243	47° 52′	2,413,000
5/16	.3125	.0606	4,860	2,430	8,100	4,050	.210	.715	.875	1.130	50° 43′	1,757,000

TABLE 3

Copper Wire—Stranded—Bare

Gauge B. & S.	Diam. Ins.	Area Sq. In.	Hard Drawn		Soft Drawn		Load per Lin. Foot Vertical			Load per Lin. Foot Horizontal			Max. Load per Lin. Foot Plane of Resultant			Angle of Resultant Load for Class B Loading	EA	
			Ulti-mate Ten'n Lbs.	Allow. Ten'n	Ulti-mate Ten'n Lbs.	Allow. Ten'n	Dead	Dead + ½" Ice	Dead + ¾" Ice	15.0 lbs. per Sq. Ft.	8.0 lb. per Sq. Ft. ½" Ice	11.0 lbs. per Sq. Ft. ¾" Ice	Class A Load	Class B Load	Class C Load		E 16,000,000	E 12,000,000
500,000	0.819	0.3924	23,540	11,750	13,340	6,650	1.525	2.345	2.989	1.024	1.213	2.126	1.837	2.640	3.668	27° 8′	6,278,400	4,708,800
450,000	0.770	0.3535	21,210	10,600	12,020	6,000	1.373	2.163	2.791	0.963	1.180	2.081	1.677	2.464	3.481	28° 30′	5,636,800	4,242,000
400,000	0.728	0.3141	18,860	9,400	10,680	5,350	1.220	1.984	2.599	0.910	1.152	2.042	1.522	2.294	3.305	29° 52′	5,025,600	3,769,200
350,000	0.679	0.2750	16,500	8,250	9,350	4,650	1.068	1.801	2.401	0.849	1.119	1.997	1.364	2.120	3.123	31° 42′	4,400,000	3,300,000
300,000	0.630	0.2360	14,160	7,100	8,025	4,000	0.915	1.618	2.203	0.788	1.087	1.953	1.208	1.949	2.944	33° 40′	3,776,000	2,832,000
250,000	0.590	0.1965	11,790	5,900	6,680	3,350	0.762	1.440	2.012	0.738	1.060	1.916	1.051	1.788	2.778	36° 3′	3,144,000	2,358,000
0000	0.530	0.1662	9,970	5,000	5,650	2,800	0.645	1.286	1.831	0.663	1.020	1.861	0.925	1.641	2.611	38° 14′	2,659,200	1,994,400
000	0.470	0.1318	7,910	3,950	4,480	2,250	0.513	1.116	1.651	0.588	0.980	1.805	0.780	1.485	2.446	41° 6′	2,108,800	1,581,600
00	0.420	0.1045	6,270	3,150	3,555	1,750	0.406	0.978	1.498	0.525	0.947	1.760	0.664	1.361	2.311	43° 52′	1,672,000	1,254,000
0	0.375	0.0829	4,970	2,500	2,820	1,400	0.322	0.866	1.372	0.469	0.917	1.719	0.569	1.261	2.199	46° 28′	1,326,400	994,800
1	0.330	0.0657	3,940	1,950	2,235	1,100	0.255	0.771	1.263	0.413	0.887	1.678	0.485	1.175	2.100	48° 51′	1,051,200	788,400
2	0.291	0.0521	3,130	1,550	1,770	900	0.203	0.695	1.174	0.364	0.861	1.642	0.417	1.107	2.019	50° 0′	833,600	625,200
3	0.261	0.0413	2,480	1,250	1,405	700	0.160	0.633	1.103	0.326	0.841	1.614	0.363	1.053	1.955	52° 51′	660,800	495,600
4	0.231	0.0328	1,970	1,000	1,115	550	0.127	0.582	1.042	0.289	0.821	1.587	0.316	1.006	1.899	54° 34′	524,800	393,600
5	0.206	0.0260	1,560	800	885	450	0.101	0.540	0.992	0.258	0.804	1.564	0.277	0.970	1.852	55° 57′	416,000	312,000
6	0.184	0.0206	1,235	600	700	350	0.080	0.505	0.951	0.230	0.789	1.543	0.243	0.936	1.813	57° 20′	329,600	247,200

TABLE 4

Copper Wire—Solid—Bare

Gauge B. & S.	Diam. Ins.	Area Sq. In.	Hard Drawn		Soft Drawn		Load per Lin. Foot Vertical			Load per Lin. Foot Horizontal			Max. Load per Lin. Foot Plane of Resultant			Angle of Resultant Load for Class B Loading	EA	
			Ultimate Ten'n Lbs.	Allow. Ten'n	Ultimate Ten'n Lbs.	Allow. Ten'n	Dead	Dead + ½″ Ice	Dead + ¾″ Ice	15.0 lbs. per Sq. Ft.	8.0 lb. per Sq. Ft. ½″ Ice	11.0 lbs. per Sq. Ft. ¾″ Ice	Class A Load	Class B Load	Class C Load		E 16,000,000	E 12,000,000
0000	0.460	0.1662	8,310	4,150	5,650	2,800	0.641	1.238	1.770	0.575	0.973	1.797	0.861	1.575	2.522	38° 10′	2,659,200	1,994,400
000	0.410	0.1318	6,590	3,300	4,480	2,250	0.509	1.074	1.591	0.512	0.940	1.750	0.722	1.427	2.365	41° 18′	2,108,800	1,581,600
00	0.365	0.1045	5,220	2,600	3,555	1,750	0.403	0.940	1.443	0.456	0.910	1.709	0.608	1.309	2.237	44° 3′	1,672,000	1,254,000
0	0.325	0.0829	4,560	2,300	2,820	1,400	0.320	0.833	1.323	0.406	0.883	1.673	0.517	1.214	2.133	46° 44′	1,326,400	994,800
1	0.289	0.0657	3,740	1,850	2,235	1,100	0.253	0.744	1.223	0.362	0.860	1.640	0.442	1.137	2.046	49° 6′	1,051,200	788,400
2	0.258	0.0521	3,120	1,550	1,770	900	0.202	0.673	1.142	0.322	0.838	1.611	0.380	1.075	1.975	51° 15′	833,600	625,200
3	0.229	0.0413	2,480	1,250	1,405	700	0.159	0.613	1.073	0.287	0.820	1.585	0.328	1.024	1.914	53° 10′	660,800	495,600
4	0.204	0.0328	1,960	1,000	1,115	550	0.126	0.564	1.016	0.255	0.803	1.567	0.284	0.981	1.863	54° 52′	524,800	393,600
5	0.182	0.0260	1,560	800	885	450	0.100	0.524	0.969	0.227	0.788	1.542	0.248	0.946	1.821	56° 20′	416,000	312,000
6	0.162	0.0206	1,240	600	700	350	0.079	0.491	0.930	0.203	0.775	1.524	0.218	0.917	1.785	57° 40′	329,600	247,200

TABLE 5

Copper Wire—Stranded—Covered

Gauge B. & S.	Ext. Diam. Inches	Area of Conductor Sq. In.	Hard Drawn		Soft Drawn		Load per Lin. Foot, Vertical		Load per Lin. Foot Horizontal	Max. Load per Lin. Ft. Resultant	Angle of Resultant Load	EA	
			Ult. Tens.	Allow. Tens.	Ult. Tens.	Allow. Tens.	Dead	Dead + ½″ Ice	8.0 lb. per Sq. Ft. Cond. + ½″ Ice			Hard Drawn E = 16,000,000	Soft Drawn E = 12,000,000
500,000	1.108	.3924	23,530	11,770	13,340	6,670	1.894	2.894	1.405	3.217	25° 50′	6,278,000	4,709,000
450,000	1.070	.3535	21,200	10,600	12,020	6,010	1.724	2.701	1.380	3.032	27° 3′	5,656,000	4,242,000
400,000	1.020	.3141	18,840	9,420	10,680	5,340	1.553	2.498	1.347	2.838	28° 18′	5,026,000	3,769,000
350,000	.978	.2750	16,500	8,250	9,370	4,670	1.345	2.264	1.319	2.619	30° 13′	4,400,000	3,300,000
300,000	.930	.2354	14,120	7,060	8,020	4,010	1.174	2.064	1.287	2.432	31° 58′	3,770,000	2,825,000
250,000	.862	.1963	11,780	5,890	6,680	3,340	.985	1.832	1.241	2.217	34° 7′	3,141,000	2,356,000
0,000	.785	.1662	9,980	4,990	5,660	2,830	.800	1.599	1.190	1.993	36° 38′	2,659,000	1,994,000
000	.728	.1318	7,800	3,950	4,480	2,240	.653	1.417	1.152	1.826	39° 6′	2,109,000	1,582,000
00	.662	.1045	6,280	3,140	3,560	1,780	.522	1.245	1.108	1.666	41° 39′	1,672,000	1,254,000
0	.605	.0829	4,980	2,490	2,820	1,410	.424	1.111	1.070	1.542	43° 54′	1,326,000	995,000
1	.518	.0657	3,940	1,970	2,240	1,120	.328	.961	1.012	1.395	47° 27′	1,051,000	788,000
2	.440	.0521	3,120	1,560	1,780	890	.270	.855	.960	1.285	48° 21′	834,000	625,000
3	.408	.0413	2,480	1,240	1,400	700	.206	.771	.939	1.214	50° 37′	661,000	496,000
4	.379	.0328	1,960	980	1,120	560	.170	.717	.919	1.165	52° 5′	525,000	394,000
5	.351	.0260	1,560	780	880	440	.140	.669	.902	1.122	53° 24′	416,000	312,000
6	.327	.0206	1,240	620	700	350	.115	.629	.885	1.086	54° 36′	330,000	247,000

TABLE 6

Copper Wire—Solid, Triple Braid Weather-Proofing

Gauge B. & S.	Ext. Diam. Ins.	Area Sq. In.	Hard Drawn		Soft Drawn		Load per Lin. Foot Vertical			Load per Lin. Foot Horizontal			Max. Load per Lin. Foot Plane of Resultant			Angle of Resultant Load for Class B Loading	EA	
			Ultimate Ten'n Lbs.	Allow. Ten'n	Ultimate Ten'n Lbs.	Allow. Ten'n	Dead	Dead + ½" Ice	Dead + ¾" Ice	15.0 lbs. per Sq. Ft.	8.0 lb. per Sq. Ft. ½" Ice	11.0 lbs. per Sq. Ft. ¾" Ice	Class A Load	Class B Load	Class C Load		E 16,000,000	E 12,000,000
0000	0.640	0.1662	8,310	4,150	5,650	2,800	0.767	1.476	2.064	0.800	1.093	1.951	1.108	1.837	2.847	36° 34′	2,659,200	1,994,400
000	0.593	0.1318	6,590	3,300	4,480	2,250	0.629	1.309	1.882	0.741	1.062	1.918	0.972	1.686	2.687	39° 6′	2,108,800	1,581,600
00	0.515	0.1045	5,220	2,600	3,555	1,750	0.502	1.133	1.682	0.644	1.010	1.847	0.818	1.518	2.498	41° 45′	1,672,000	1,254,000
0	0.500	0.0829	4,560	2,300	2,820	1,400	0.407	1.029	1.573	0.625	1.000	1.833	0.746	1.434	2.415	44° 14′	1,326,400	994,800
1	0.453	0.0657	3,740	1,850	2,235	1,100	0.316	0.909	1.438	0.564	0.968	1.790	0.646	1.328	2.296	46° 50′	1,051,200	788,400
2	0.437	0.0521	3,120	1,550	1,770	900	0.260	0.843	1.367	0.546	0.958	1.775	0.605	1.276	2.240	48° 41′	833,600	625,200
3	0.406	0.0413	2,480	1,250	1,405	700	0.199	0.763	1.278	0.507	0.937	1.747	0.545	1.208	2.164	50° 50′	660,800	495,600
4	0.359	0.0328	1,960	1,000	1,115	550	0.164	0.698	1.199	0.449	0.906	1.704	0.478	1.143	2.083	52° 21′	524,800	393,600
5	0.344	0.0260	1,560	800	885	450	0.135	0.660	1.146	0.430	0.896	1.690	0.451	1.113	2.042	53° 37′	416,000	312,000
6	0.328	0.0206	1,240	600	700	350	0.112	0.627	1.118	0.410	0.885	1.675	0.425	1.084	2.014	54° 41′	329,600	247,200

TABLE 7

Aluminum Wire—Stranded—Bare

Copper Equiv. C. M. or B. & S. Gauge	Size C. M.	Ext. Diam. Inches	Area of Conductor Sq. In.	Ult. Tens.	Allow. Tens.	Load per Lin. Foot Vertical		Load per Lin. Foot Horizontal	Max. Load per Lin. Ft. Resultant	Angle of Resultant Load	EA E =9,000,000
						Dead	Dead + ½″ Ice	8.0 lbs. per Sq. Ft. Cond. + ½″ Ice			
500,000	795,000	1.0255	.6240	14,360	7,180	.732	1.681	1.350	2.156	38° 4′	5,616,000
450,000	715,500	.9737	.5620	12,920	6,460	.658	1.575	1.316	2.052	39° 52′	5,058,000
400,000	636,000	.9184	.5000	11,500	5,750	.585	1.467	1.279	1.946	41° 4′	4,500,000
350,000	556,500	.8555	.4370	10,060	5,030	.512	1.355	1.237	1.835	42° 24′	3,933,000
314,500	500,000	.8115	.3926	9,040	4,520	.460	1.276	1.208	1.757	43° 26′	3,533,000
300,000	477,000	.7925	.3745	8,620	4,310	.439	1.243	1.195	1.724	43° 50′	3,370,000
250,000	397,500	.7235	.3120	7,180	3,590	.365	1.126	1.149	1.608	45° 39′	2,808,000
0,000	336,420	.6570	.2640	6,080	3,040	.310	1.030	1.105	1.511	47° 53′	2,376,000
188,600	300,000	.6280	.2356	5,420	2,710	.276	.978	1.085	1.461	48° 0′	2,120,000
000	266,800	.5859	.2095	4,820	2,410	.246	.921	1.057	1.402	48° 55′	1,885,000
00	211,950	.5220	.1665	3,840	1,920	.195	.831	1.015	1.312	50° 40′	1,498,000
0	167,800	.4644	.1318	3,040	1,520	.155	.755	.976	1.234	52° 19′	1,860,000
1	133,220	.4140	.1045	2,400	1,200	.123	.692	.943	1.170	53° 46′	940,000
2	105,530	.3684	.0829	1,900	950	.097	.637	.912	1.113	55° 6′	746,000
3	83,642	.3279	.0657	1,520	760	.077	.592	.885	1.066	56° 16′	591,000

SAGS

In the following tables are given sags at which conductors shall be strung in order that, when loaded with the specified requirement of one-half inch of ice and a wind load of 8.0 pounds per square foot of projected area at 0 degrees Fahrenheit, the tension in the conductor will not exceed the allowable value of one-half the ultimate strength of the conductor as given in preceding tables. The sags given in the tables for 120 degrees Fahrenheit are greater in every case than the vertical component of the sags at 0 degrees Fahrenheit under the maximum wind and ice load.

Minimum Sags for Stranded Hard-Drawn Bare Copper Wires

No. 4/0 B. & S.

Temp. F.	Span in Feet								
	100 or Less	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*
—20	2	3	5	8	13	20	3.5	6	10
0	2	4	5	9	14	22	3.5	6.5	10.5
20	3	4	6	10	16	24	4	7	11.5
40	3	4	6	11	18	27	4.5	8	12
60	3	5	7	13	20	31	5	8.5	13
80	4	6	8	15	24	35	5.5	9	13.5
100	4	7	10	17	27	40	6	10	14.5
120	5	8	12	20	31	46	7	10.5	15

No. 3/0 B. & S.

Temp. F.	Span in Feet								
	100 or Less	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*
—20	2	3	5	8	13	21	4	7	12
0	2	4	5	9	15	23	4	7.5	12.5
20	3	4	6	10	17	25	4.5	8.5	13.5
40	3	4	6	12	19	29	5	9	14
60	3	5	7	13	22	33	6	9.5	15
80	4	6	8	15	25	38	6.5	10.5	15.5
100	4	7	10	18	29	43	7	11	16
120	5	8	12	21	34	49	7.5	12	17

No. 2/0 B. & S.

Temp. F.	Span in Feet								
	100 or Less	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*
—20	2	3	5	9	14	23	4.5	9	15
0	2	4	5	10	16	26	5	9.5	15.5
20	3	4	6	11	18	29	5.5	10	16
40	3	4	7	12	21	33	6	11	17
60	3	5	7	14	24	37	6.5	11.5	17.5
80	4	6	9	16	28	43	7	12	18
100	5	7	10	19	32	48	8	12.5	18.5
120	6	9	12	23	37	54	8.5	13.5	19.5

No. 0 B. & S.

Temp. F.	Span in Feet								
	100 or Less	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*
—20	2	3	5	9	16	2.5	5.5	11.5	18.5
0	2	4	5	10	18	2.5	6.5	12	19
20	3	4	6	11	21	3	7	12.5	19.5
40	3	5	7	13	24	3.5	7.5	13	20
60	3	5	8	15	27	4	8	14	20.5
80	4	6	9	18	32	4.5	8.5	14.5	21.5
100	5	7	11	21	37	5	9	15	22
120	6	9	13	25	42	5	9.5	15.5	22.5

Minimum Sags for Solid Hard-Drawn Bare Copper Wire

No. 1 B. & S.

Temp. F.	Span in Feet								
	100 or Less	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*
—20	2	4	5	10	19	3	8	14.5	23
0	3	4	6	11	22	3.5	8.5	15	23.5
20	3	4	6	13	25	4	9	16	24
40	3	5	7	15	30	4.5	9.5	16	24.5
60	4	6	8	18	34	5	10	17	25
80	4	7	10	21	39	5.5	10.5	17	25.5
100	5	8	12	25	44	6	11	18	26
120	6	10	16	30	49	6	11.5	18	26.5

No. 2 B. & S.

Temp. F.	Span in Feet								
	100 or Less	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*
—20	2	4	5	12	25	4	10.5	18.5	29
0	3	4	6	14	29	4.5	11	19	29.5
20	3	5	7	16	33	5	11.5	19.5	30
40	3	5	8	19	39	5.5	12	20	30.5
60	4	6	10	23	43	6	12.5	20.5	31
80	4	7	12	27	48	6.5	13	21	31
100	5	9	14	31	53	7	13	21.5	31.5
120	7	11	18	35	58	7.5	13.5	22	32

No. 3 B. & S.

Temp. F.	Span in Feet								
	100 or Less	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
—20	3	4	6	17	3	6	14	24	37.5
0	3	4	7	20	3.5	6.5	14.5	24.5	37.5
20	3	5	8	23	4	7	15	25	38
40	3	6	10	27	4.5	7.5	15	25	38
60	4	7	12	30	5	8	15.5	25.5	38.5
80	5	9	14	35	5.5	8.5	16	26	39
100	6	11	17	39	5.5	8.5	16.5	26	39
120	8	14	22	44	6	9	16.5	26.5	39.5

No. 4 B. & S.

Temp. F.	Span in Feet								
	100 or Less	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
—20	3	4	8	25	5	9	18	31	46
0	3	5	9	29	5.5	9	18.5	31.5	46
20	3	6	11	33	6	9.5	19	31.5	46.5
40	4	7	13	38	6.5	10	19	32	46.5
60	4	9	16	42	6.5	10	19.5	32.5	47
80	5	11	19	46	7	10.5	19.5	32.5	47.5
100	7	13	23	50	7.5	11	20	32.5	47.5
120	9	16	27	54	7.5	11	20.5	33	48

Minimum Sags for Stranded Bare Aluminum Wires

No. 4/0 B. & S.

Temp. F.	Span in Feet: 80 or Less	100	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
—20	1	2	3	5	11	2.5	5	11	19	29
0	1	2	3	6	15	3	5.5	12	19.5	29.5
20	2	3	5	8	21	3.5	6	12.5	20.5	30
40	2	4	7	11	27	4.5	7	13	21	31
60	4	6	11	17	34	5	7.5	13.5	21.5	31.5
80	6	10	16	22	41	5.5	8	14	22	32
100	10	14	20	27	46	6	8.5	14.5	22.5	33
120	13	18	25	32	52	6.5	9	15	23	33.5

No. 3/0 B. & S.

Temp. F.	Span in Feet: 80 or Less	100	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
—20	1	2	3	5	12	3	5.5	13	22	33.5
0	1	2	4	6	17	3.5	6.5	13.5	22.5	34
20	2	3	5	8	24	4.5	7	14	23	34.5
40	2	4	7	12	31	5	7.5	14.5	23.5	35
60	3	5	11	18	38	5.5	8	15	24	35.5
80	6	9	16	23	43	6	8.5	15.5	24.5	36
100	10	13	20	29	49	6.5	9	16	25	36.5
120	13	17	25	33	54	7	9.5	16.5	25.5	37

No. 2/0 B. & S.

Temp. F.	Span in Feet: 80 or Less	100	125	150	200	250	300	400	500	600
	Inches	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
—20	1	2	3	6	2	5	8.5	16.5	28	42
0	2	2	4	8	2.5	5.5	9	17	28.5	42.5
20	2	3	6	12	3	6	9	17.5	29	43
40	2	4	9	18	3.5	6.5	9.5	18	29.5	43
60	4	7	14	24	4	7	10	18.5	29.5	43.5
80	7	12	19	29	4.5	7	10.5	19	30	44
100	10	16	24	33	5	7.5	11	19.5	30.5	44.5
120	14	19	28	38	5.5	8	11.5	20	31	44.5

No. 0 B. & S.

Temp. F.	Span in Feet: 80 or Less	100	125	150	200	250	300	400	500
	Inches	*Inches*	*Inches*	*Inches*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
—20	1	2	4	9	3.5	7	10.5	21	36.5
0	2	3	6	14	4	7	11	21.5	36.5
20	2	4	8	20	4.5	7.5	11.5	22	37
40	3	6	13	26	5	8	12	22	37
60	5	10	18	31	5	8.5	12	22.5	37.5
80	8	14	23	35	5.5	8.5	12.5	23	38
100	12	18	27	39	6	9	13	23	38
120	15	21	31	43	6	9.5	13.5	23.5	38.5

GALVANIZING IRON AND STEEL

We recommend the specifications adopted by the National Electric Light Association, which are as follows:

These specifications give in detail the test to be applied to galvanized material. All specimens shall be capable of withstanding these tests.

a—Coating

The galvanizing shall consist of a continuous coating of pure zinc of uniform thickness, and so applied that it adheres firmly to the surface of the iron or steel. The finished product shall be smooth.

b—Cleaning

The samples shall be cleaned before testing, first with carbona, benzine or turpentine, and cotton waste (not with a brush), and then thoroughly rinsed in clean water and wiped dry with clean cotton waste.

The samples shall be clean and dry before each immersion in the solution.

c—Solution

The standard solution of copper sulphate shall consist of commercial copper sulphate crystals dissolved in cold water, about in the proportion of 36 parts, by weight, of crystals to 100 parts, by weight, of water. The solution shall be neutralized by the addition of an excess of chemically pure cupric oxide (Cu O). The presence of an excess of cupric oxide will be shown by the sediment of this reagent at the bottom of the containing vessel.

The neutralized solution shall be filtered before using by passing through filter paper. The filtered solution shall have a specific gravity of 1.186 at 65 degrees Fahrenheit (reading the scale at the level of the solution) at the beginning of each test. In case the filtered solution is high in specific gravity, clean water shall be added to reduce the specific gravity to 1.186 at 65 degrees F. In case the filtered solution is low in specific gravity, filtered solution of a higher specific gravity shall be added to make the specific gravity 1.186 at 65 degrees Fahrenheit.

As soon as the stronger solution is taken from the vessel containing the unfiltered neutralized stock solution, additional crystals and water must be added to the stock solution. An excess of cupric oxide shall always be kept in the unfiltered stock solution.

d—Quantity of Solution

Wire samples shall be tested in a glass jar of at least two (2) inches inside diameter. The jar without the wire samples shall be filled with standard solution to a depth of at least four (4) inches. Hardware samples shall be tested in a glass or earthenware jar containing at least one-half (½) pint of standard solution for each hardware sample.

Solution shall not be used for more than one series of four immersions.

e—Samples

Not more than seven wires shall be simultaneously immersed, and not more than one sample of galvanized material, other than wire, shall be immersed in the specified quantity of solution.

The samples shall not be grouped or twisted together, but shall be well separated so as to permit the action of the solution to be uniform upon all immersed portions of the samples.

f—Test

Clean and dry samples shall be immersed in the required quantity of standard solution in accordance with the following cycle of immersions.

The temperature of the solution shall be maintained between 62 and 68 degrees Fahrenheit at all times during the following test.

First—Immerse for one minute, wash and wipe dry.

Second—Immerse for one minute, wash and wipe dry.

Third—Immerse for one minute, wash and wipe dry.

Fourth—Immerse for one minute, wash and wipe dry.

After each immersion the samples shall be immediately washed in clean water having a temperature between 62 and 68 degrees Fahrenheit, and wiped dry with cotton waste.

In the case of No. 14 galvanized iron or steel wire, the time of the fourth immersion shall be reduced to one-half minute.

g—Rejection

If after the test described in Section "**f**" there should be a bright metallic copper deposit upon the samples, the lot represented by the samples shall be rejected.

Copper deposits on zinc or within one inch of the cut end shall not be considered causes for rejection.

In the case of a failure of only one wire in a group of seven wires immersed together, or if there is a reasonable doubt as to the copper deposit, two check tests shall be made on these seven wires, and the lot reported in accordance with the majority of the set of tests.

USEFUL DATA

Given, $ax^2 + bx + c = 0$; $X = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

e = Base of Napierian Logarithms = 2.7182818285

$\text{Log}_{10}\, e = 0.4342944819$

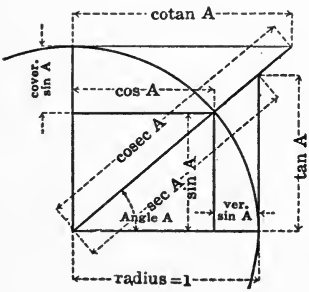
$$\tfrac{1}{2}(e^{v} + e^{-v}) = 1 + \frac{V^2}{\boxed{2}} + \frac{V^4}{\boxed{4}} + \frac{V^6}{\boxed{6}} + \frac{V^8}{\boxed{8}} +$$

$$\tfrac{1}{2}(e^{v} - e^{-v}) = V + \frac{V^3}{\boxed{3}} + \frac{V^5}{\boxed{5}} + \frac{V^7}{\boxed{7}} + \frac{V^9}{\boxed{9}} +$$

One inch = 2.540005 centimeters
One centimeter = 0.3937 inches
One foot = 0.3048006 meter
One meter = 3.2808333 feet
One pound (avoirdupois) = 0.45359 kilograms
One pound per foot = 1.488161 kilograms per meter.
One pound per square inch = 0.0703067 kilograms per square centimeter
One inch-pound = 1.152127 kilogram-centimeters
One kilogram per meter = 0.67197 pounds per foot
One kilogram per square centimeter = 14.2234 pounds per square inch
One kilogram-centimeter = 0.86796 inch-pounds

Trigonometrical Formulae

Radius, $1 = \sin^2 A + \cos^2 A$
$= \sin A \operatorname{cosec} A = \cos A \sec A = \tan A \cot A$



Sine $A = \dfrac{\cos A}{\cot A} = \dfrac{1}{\operatorname{cosec} A} = \cos A \tan A = \sqrt{1 - \cos^2 A}$

Cosine $A = \dfrac{\sin A}{\tan A} = \dfrac{r}{\sec A} = \sin A \cot A = \sqrt{1 - \sin^2 A}$

Tangent $A = \dfrac{\sin A}{\cos A} = \dfrac{1}{\cot A} = \sin A \sec A$

Cotangent $A = \dfrac{\cos A}{\sin A} = \dfrac{1}{\tan A} = \cos A \operatorname{cosec} A$

Secant $A = \dfrac{\tan A}{\sin A} = \dfrac{1}{\cos A}$

Cosecant $A = \dfrac{\cot A}{\cos A} = \dfrac{1}{\sin A}$

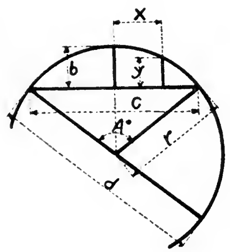
NATURAL TRIGONOMETRIC FUNCTIONS

Degrees	Sines	Cosines	Tangents	Cotangents	Secants	Cosecants	Degrees
0	0.00000	1.00000	0.00000		1.00000		90
1	0.01745	0.99985	0.01746	57.28996	1.00015	57.29869	89
2	0.03490	0.99939	0.03492	28.63625	1.00061	28.65371	88
3	0.05234	0.99863	0.05241	19.08114	1.00137	19.10732	87
4	0.06976	0.99756	0.06993	14.30067	1.00244	14.33559	86
5	0.08716	0.99619	0.08749	11.43005	1.00382	11.47371	85
6	0.10453	0.99452	0.10510	9.51436	1.00551	9.56677	84
7	0.12187	0.99255	0.12278	8.14435	1.00751	8.20551	83
8	0.13917	0.99027	0.14054	7.11537	1.00983	7.18530	82
9	0.15643	0.98769	0.15838	6.31375	1.01247	6.39245	81
10	0.17365	0.98481	0.17633	5.67128	1.01543	5.75877	80
11	0.19081	0.98163	0.19438	5.14455	1.01872	5.24084	79
12	0.20791	0.97815	0.21256	4.70463	1.02234	4.80973	78
13	0.22495	0.97437	0.23087	4.33148	1.02630	4.44541	77
14	0.24192	0.97030	0.24933	4.01078	1.03061	4.13357	76
15	0.25882	0.96593	0.26795	3.73205	1.03528	3.86370	75
16	0.27564	0.96126	0.28675	3.48741	1.04030	3.62796	74
17	0.29237	0.95630	0.30573	3.27085	1.04569	3.42030	73
18	0.30902	0.95106	0.32492	3.07768	1.05146	3.23607	72
19	0.32557	0.94552	0.34433	2.90421	1.05762	3.07155	71
20	0.34202	0.93969	0.36397	2.74748	1.06418	2.92380	70
21	0.35837	0.93358	0.38386	2.60509	1.07115	2.79043	69
22	0.37461	0.92718	0.40403	2.47509	1.07853	2.66947	68
23	0.39073	0.92050	0.42447	2.35585	1.08636	2.55930	67
24	0.40674	0.91355	0.44523	2.24604	1.09464	2.45859	66
25	0.42262	0.90631	0.46631	2.14451	1.10338	2.36620	65
26	0.43837	0.89879	0.48773	2.05030	1.11260	2.28117	64
27	0.45399	0.89101	0.50953	1.96261	1.12233	2.20269	63
28	0.46947	0.88295	0.53171	1.88073	1.13257	2.13005	62
29	0.48481	0.87462	0.55431	1.80405	1.14335	2.06267	61
30	0.50000	0.86603	0.57735	1.73205	1.15470	2.00000	60
31	0.51504	0.85717	0.60086	1.66428	1.16663	1.94160	59
32	0.52992	0.84805	0.62487	1.60033	1.17918	1.88708	58
33	0.54464	0.83867	0.64941	1.53987	1.19236	1.83608	57
34	0.55919	0.82904	0.67451	1.48256	1.20622	1.78829	56
35	0.57358	0.81915	0.70021	1.42815	1.22077	1.74345	55
36	0.58779	0.80902	0.72654	1.37638	1.23607	1.70130	54
37	0.60182	0.79864	0.75355	1.32704	1.25214	1.66164	53
38	0.61566	0.78801	0.78129	1.27994	1.26902	1.62427	52
39	0.62932	0.77715	0.80978	1.23490	1.28676	1.58902	51
40	0.64279	0.76604	0.83910	1.19175	1.30541	1.55572	50
41	0.65606	0.75471	0.86929	1.15037	1.32501	1.52425	49
42	0.66913	0.74314	0.90040	1.11061	1.34563	1.49448	48
43	0.68200	0.73135	0.93252	1.07237	1.36733	1.46628	47
44	0.69466	0.71934	0.96569	1.03553	1.39016	1.43956	46
45	0.70711	0.70711	1.00000	1.00000	1.41421	1.41421	45
Degrees	Cosines	Sines	Cotangents	Tangents	Cosecants	Secants	Degrees

Properties of the Circle

Circumference of Circle of Diameter 1 $= \pi = 3.14159265$
Circumference of Circle $= 2\pi r$
Diameter of Circle = Circumference $\times$ 0.31831

Diameter of Circle of equal periphery as square	= side	$\times$ 1.27324
Side of Square of equal periphery as circle	= diameter	$\times$ 0.78540
Diameter of Circle circumscribed about square	= side	$\times$ 1.41421
Side of Square inscribed in circle	= diameter	$\times$ 0.70711



Arc, $a = \dfrac{\pi r A^\circ}{180} = 0.017453\, r A^\circ$

Angle, $A = \dfrac{180^\circ a}{\pi r} = 57.29578 \dfrac{a}{r}$

Radius $r = \dfrac{4b^2 + c^2}{8b}$ Diameter, $d = \dfrac{4b^2 + c^2}{4b}$

Chord, $c = 2\sqrt{2br - b^2} = 2r \sin \dfrac{A^\circ}{2}$

Rise, $b = r - \tfrac{1}{2}\sqrt{4r^2 - c^2} = \dfrac{c}{2} \tan \dfrac{A^\circ}{4} = 2r \sin^2 \dfrac{A^\circ}{4}$

Rise, $b = r + y - \sqrt{r^2 - x^2}$. $y = b - r + \sqrt{r^2 - x^2}$

$x = \sqrt{r^2 - (r + y - b)^2}$

$\pi = 3.14159265$, log $= 0.4971499$

$\dfrac{1}{\pi} = 0.3183099$, log $= \bar{1}.5028501$

$\pi^2 = 9.8696044$, log $= 0.9942997$

$\dfrac{1}{\pi^2} = 0.1013212$, log $= \bar{1}.0057003$

$\sqrt{\pi} = 1.7724539$, log $= 0.2485749$

$\sqrt{\dfrac{1}{\pi}} = 0.5641896$, log $= \bar{1}.7514251$

$\dfrac{\pi}{180} = 0.0174533$, log $= \bar{2}.2418774$

$\dfrac{180}{\pi} = 57.2957795$, log $= 1.7581226$

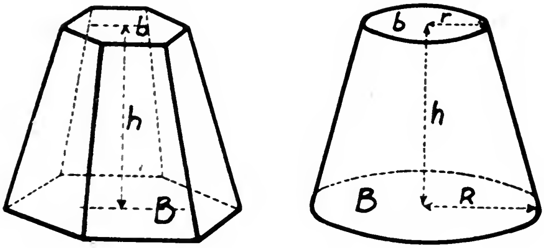
Pyramid and Cone

Volume of any Pyramid or Cone whether regular or irregular equals product of area of base by one-third perpendicular height, or

$$V = \frac{1}{3} Bh$$

in which

V = Volume
B = Area of Base
h = Perpendicular height

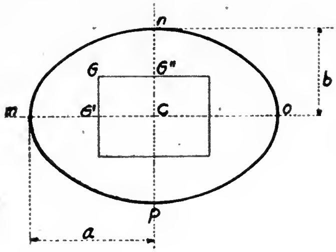


Volume of Frustrum of any Pyramid or Cone with parallel ends equals sum of areas of base and top plus square root of their products, all multiplied by one-third the perpendicular height or distance between the two parallel ends, or

$$V = \frac{1}{3} h (B + \sqrt{Bb} + b)$$

in which

V = volume
h = perpendicular distance between parallel ends
B = area of base
b = area of top



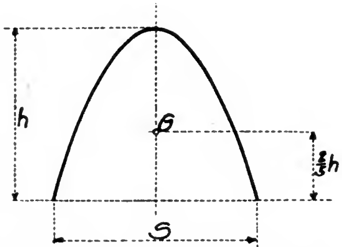
Ellipse

Area $= \pi ab$

Center of Gravity of part mnc is at point G

$$cG^{1} = \tfrac{4}{3} \cdot a \cdot \frac{1}{\pi} = 0.4244 \cdot a = \text{abt. } \tfrac{14}{33}\, a$$

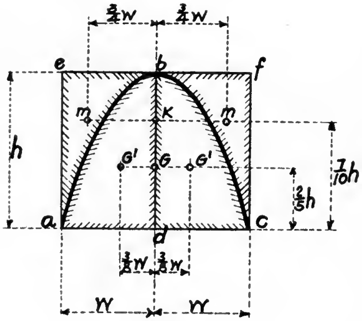
$$cG^{11} = G^{1}G = \tfrac{4}{3} \cdot b \cdot \frac{1}{\pi} = 0.4244 \cdot b = \text{abt. } \tfrac{14}{33}\, b$$



Parabola

Area $= \tfrac{2}{3}\, sh$

Center of Gravity at point G



Semi-Parabola—abd or cbd

Center of Gravity at Point G^1

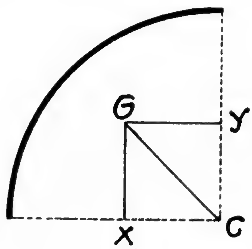
$$dG = \frac{2}{5} h$$

$$GG^1 = \frac{3}{8} W$$

For the area included between the semi-parabola abd and its enclosing rectangle aebd, or between the semi-parabola cbd and its enclosing rectangle cfbd, the center of gravity is at the point m.

$$dk = \frac{7}{10} h$$

$$km = \frac{3}{4} w$$



Circular Quadrant

Center of Gravity at point G

$$CG = \tfrac{4}{3} \text{ Rad.} \times \frac{\sqrt{2}}{\pi} = \text{Rad. } 0.6002$$

$$CX = XG = \tfrac{4}{3} \text{ Rad.} \times \frac{1}{\pi} = \text{Rad} \times 0.4244 \text{ or abt.}$$

$$\text{Rad.} \times \tfrac{14}{33}$$

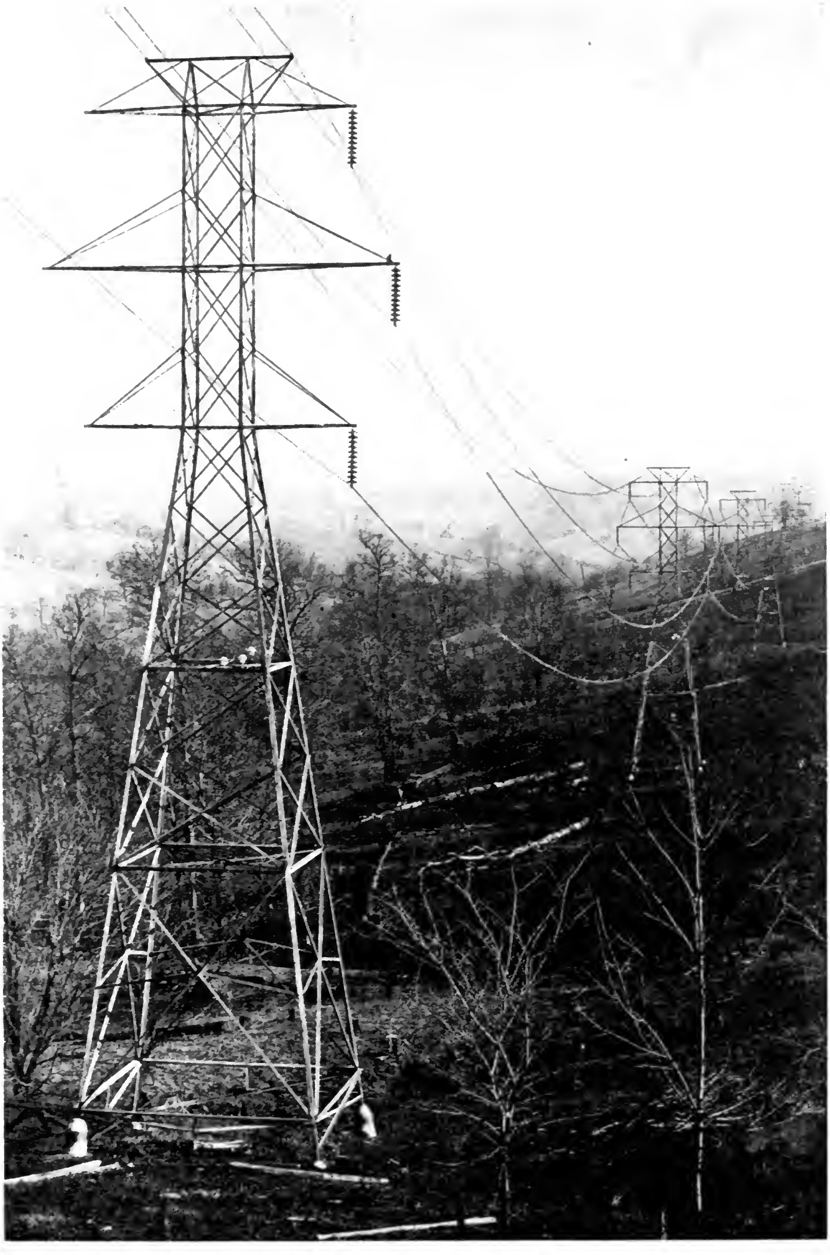


Fig. B—Towers for Double Circuit 130,000 Volt Line

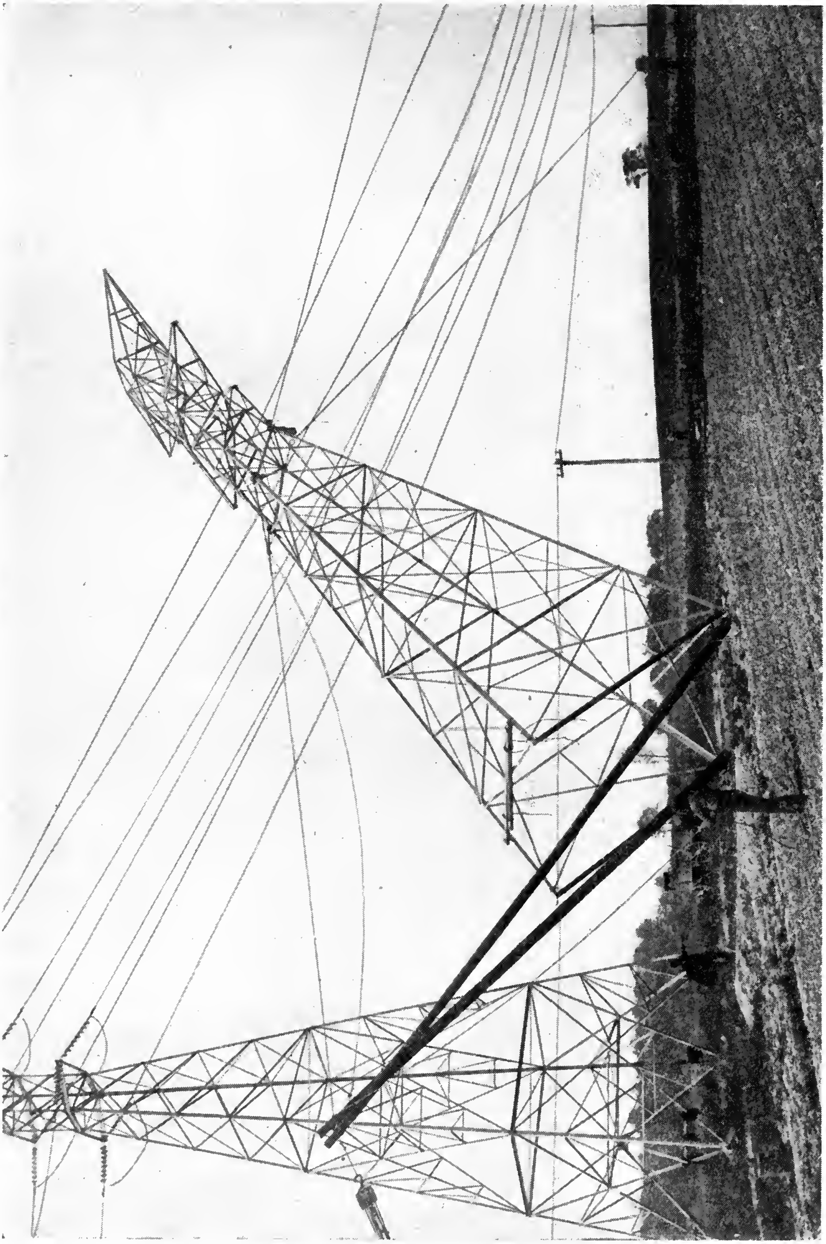


Fig. C—Method of Erecting Towers from Prone Position

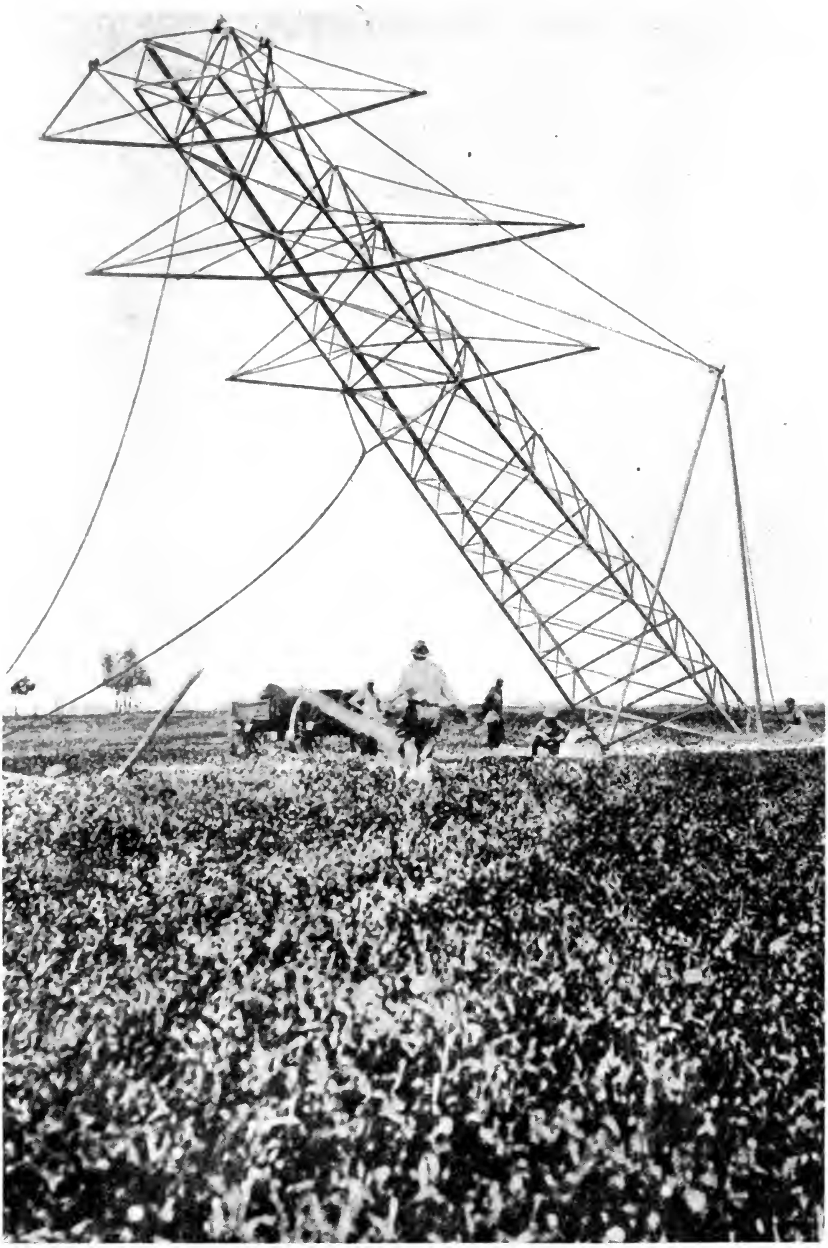


Fig. D—Method of Erecting Flexible A Frames from Prone Position

Fig. E—Method of Erecting Towers in Position

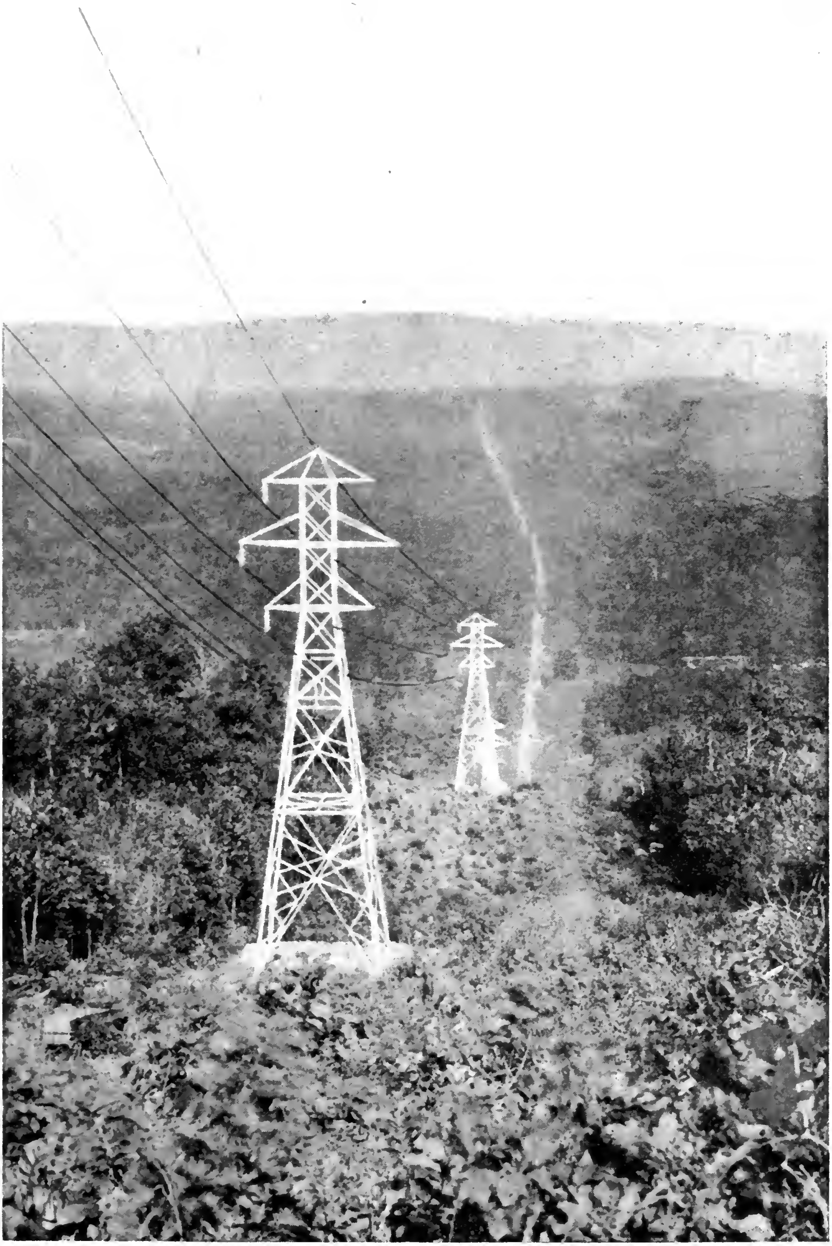


Fig. F—Double Circuit Towers, for 66,000 Volt Line

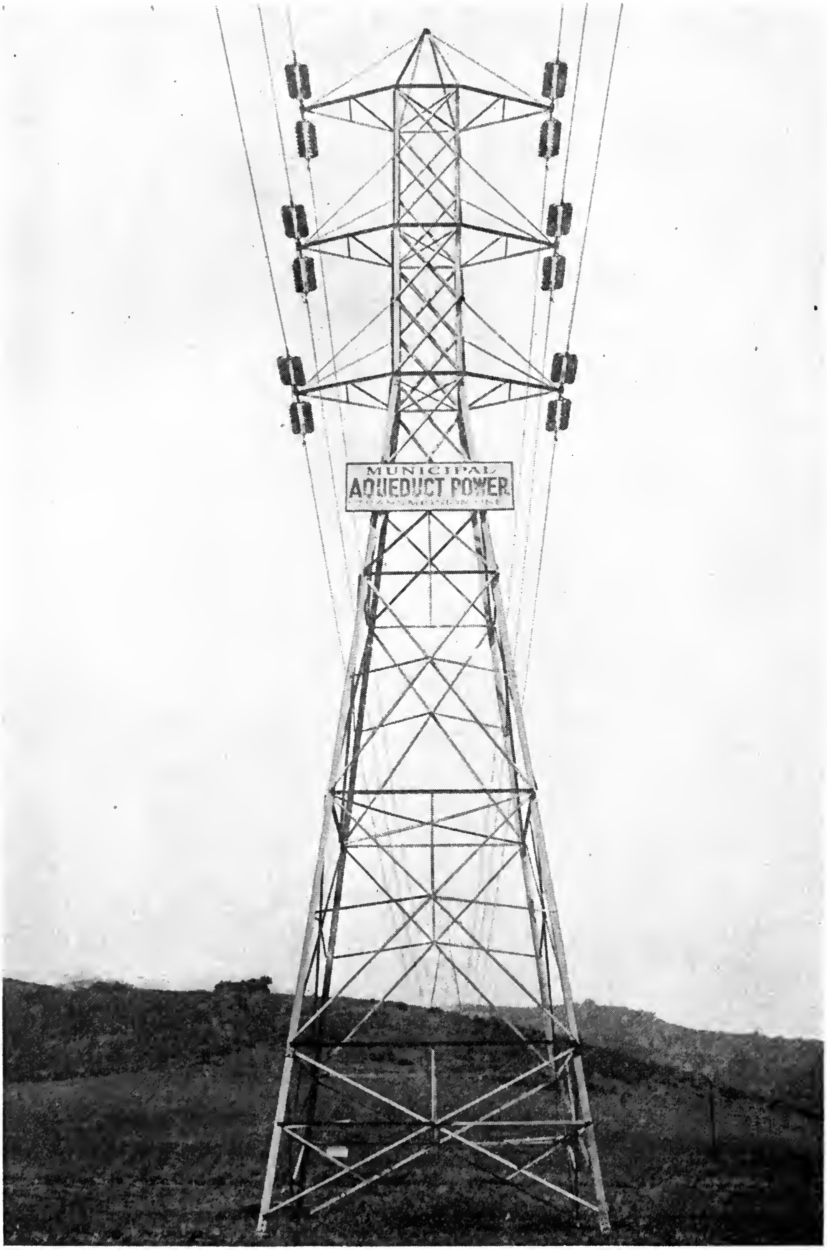


Fig. G—Special Strain Tower, for Double Circuit 110,000 Volt Line

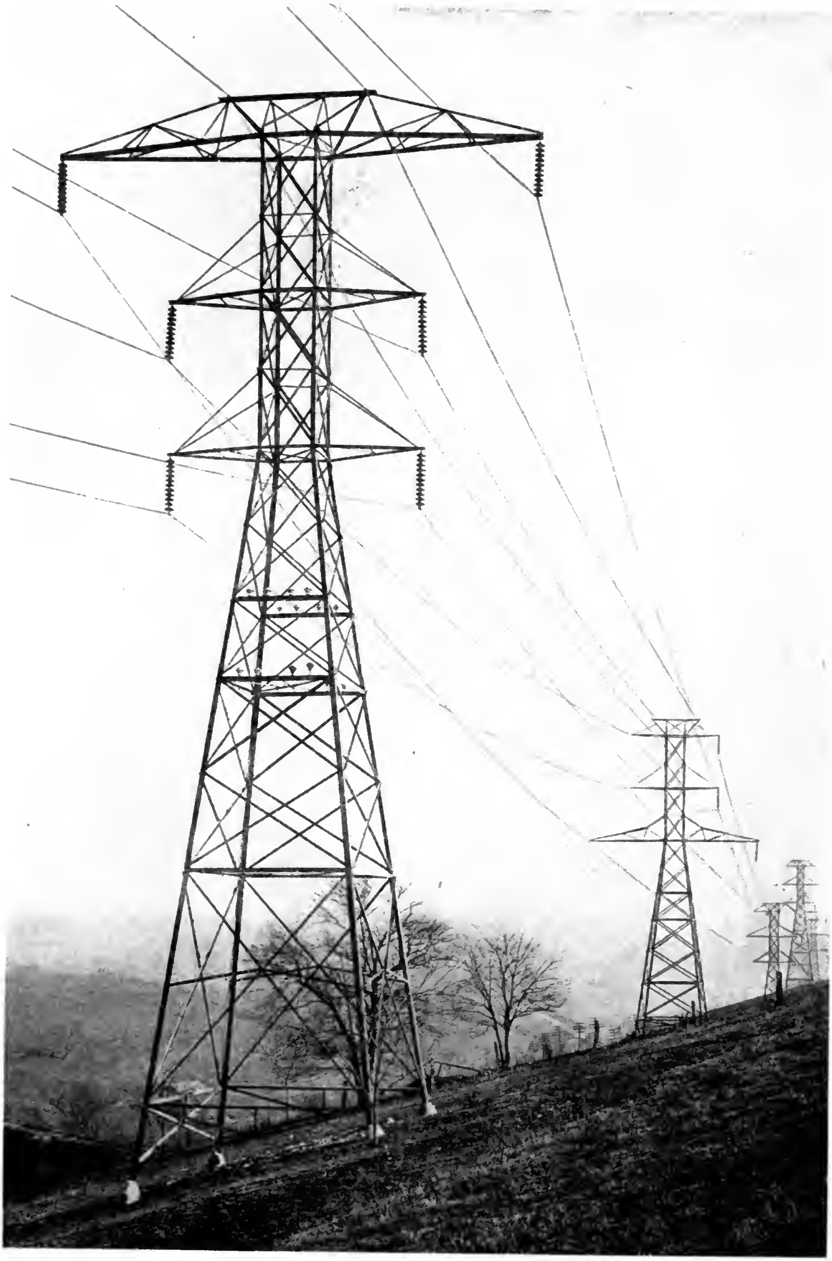


Fig. H—Transposition Tower, for Double Circuit 130,000 Volt Line

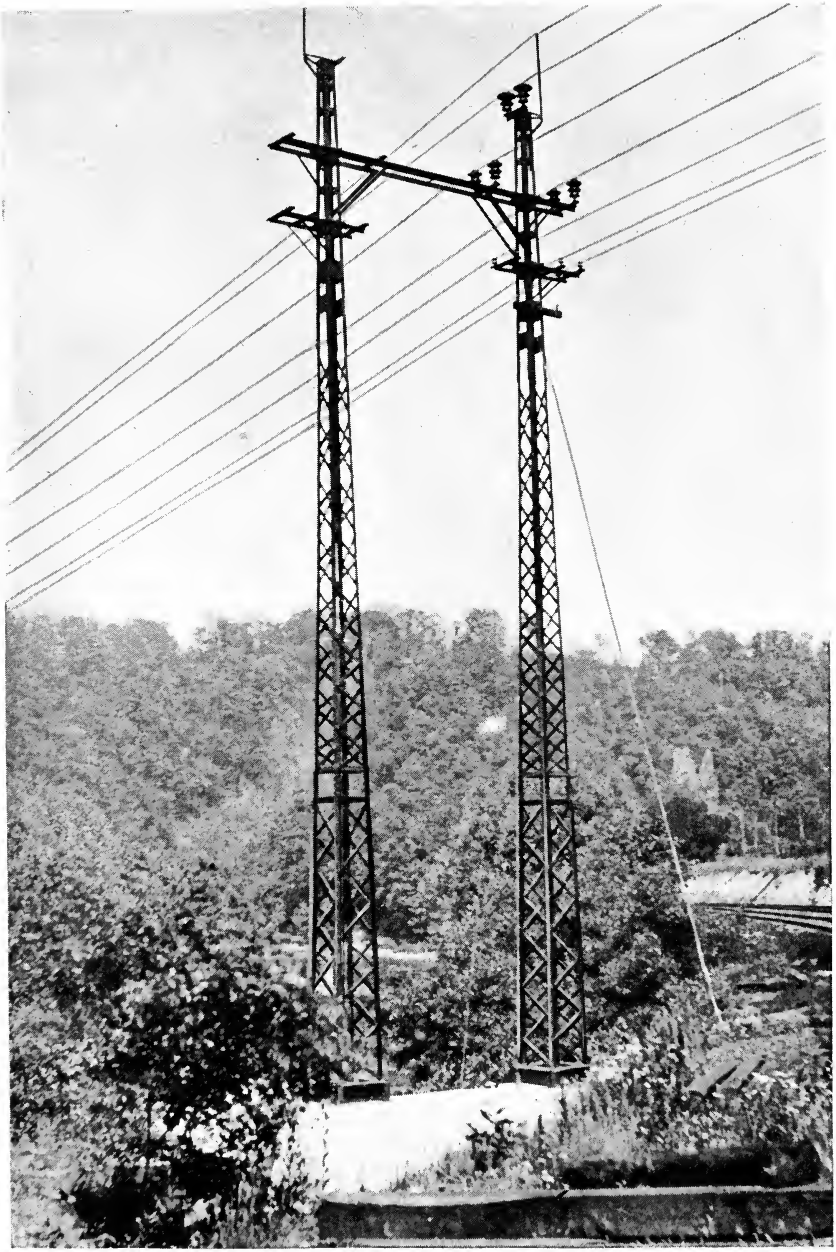


Fig. I—Railroad Crossing Poles, for 6,600 Volt Line

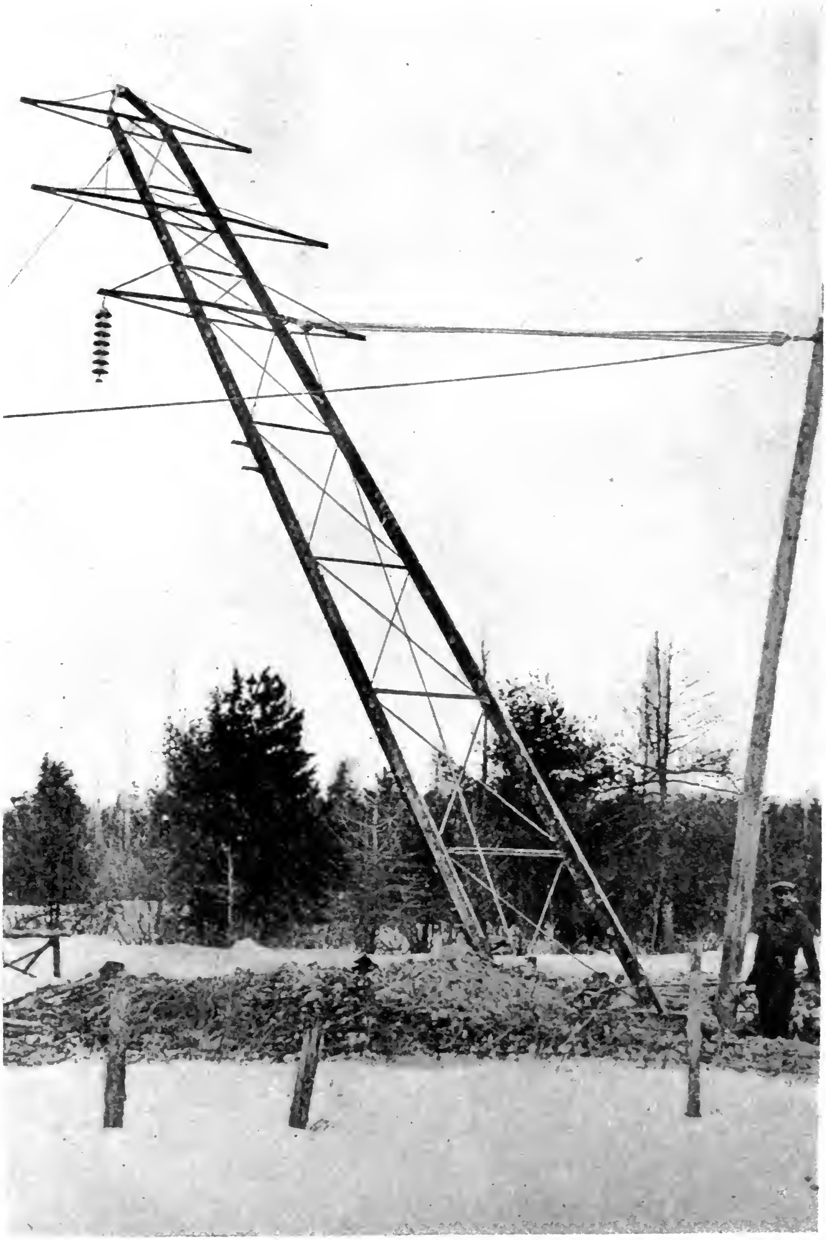


Fig. J—Flexible A Frame, for Double Circuit 66,000 Volt Line

Fig. K—Flexible A Frame, for Single Circuit 66,000 Volt Line

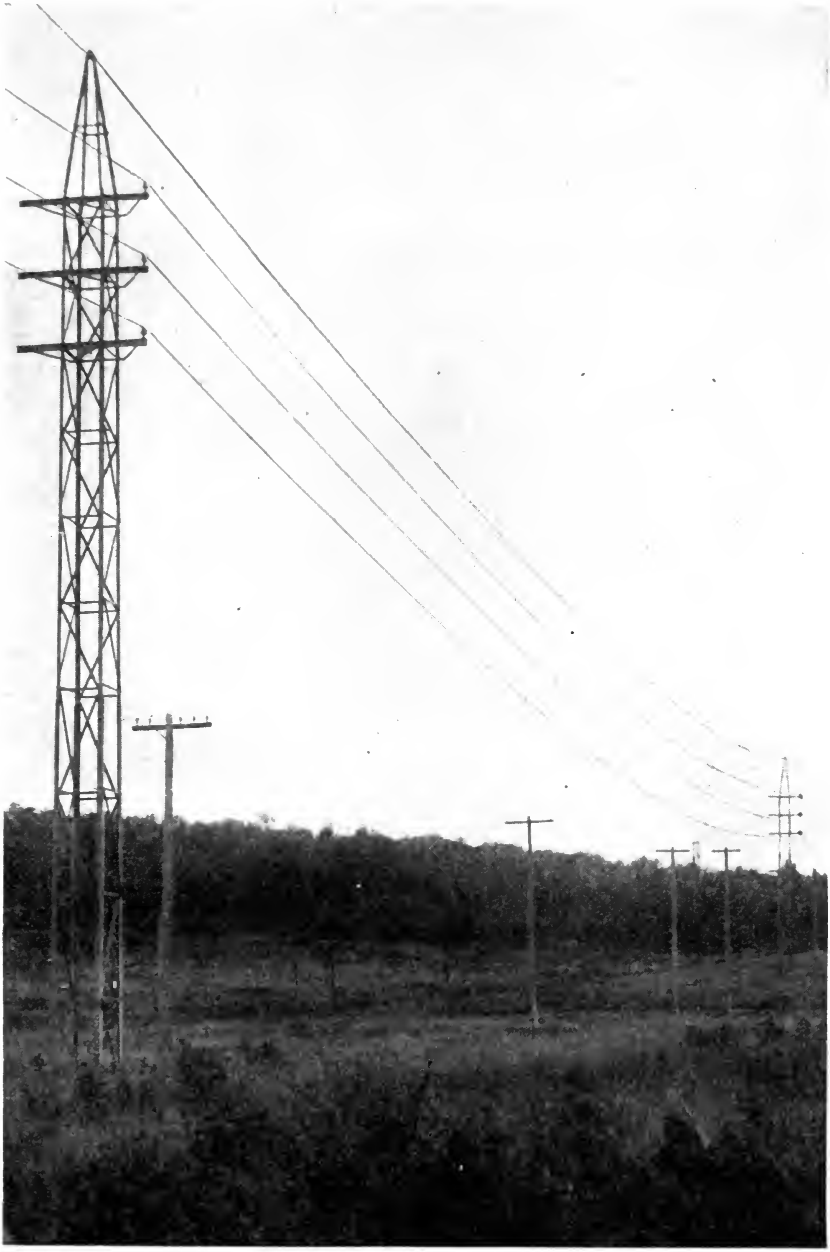


Fig. L—Poles, for Double Circuit 6,600 Volt Line

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PRODUCTS OF THE BLAW-KNOX COMPANY

FABRICATED STEEL

Fabricated steel, one of the principal products of Blaw-Knox Company, includes mill buildings, manufacturing plants, bridges, crane runways, trusses and other construction of a highly fabricated nature.

A corps of highly trained engineers is maintained for consulting and designing services.

TRANSMISSION TOWERS

Four legged straight line or suspension towers, anchor and dead end towers, latticed and channel A-frames, river crossing towers, outdoor sub-stations, switching stations, signal towers, steel poles, derrick towers.

We specialize in the design and fabrication of high tension transmission lines.

PLATE WORK

Riveted, pressed and welded steel plate products of every description, including: accumulators, agitators, water boshes, annealing boxes, containers, digesters, filters, flumes, gear guards, kettles, ladles, pans, penstocks, air receivers, stacks, standpipes, miscellaneous tanks, miscellaneous blast furnace work, etc.

BLAW BUCKETS

Clamshell buckets and automatic cableway plants for digging and rehandling earth, sand, gravel, coal, ore, limestone, tin scrap, slag, cinders, fertilizers, rock products, etc.

For installation on derricks, overhead and locomotive cranes, monorails, dredges, steam shovels, ditchers, cableways, ships for handling cargo and coal, etc.

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Steel forms for every type of concrete construction: aqueducts, bridges, cisterns, columns, culverts, curbs and gutters, dams, factories, floors, foundations, houses, locks, manholes, piers, pipe, reservoirs, roads, sewers, shafts, sidewalks, subways, tanks, tunnels, viaducts, retaining walls, warehouses, etc.

FURNACE APPLIANCES

Knox patented water cooled doors, door frames, front and back wall coolers, ports, bulkheads, reversing valves, etc., for Open Hearth, Glass and Copper Regenerative Furnaces; water-cooled standings, boshes and shields for Sheet and Tin Mills.

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