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U.S. Army
Coastal Engineering
Research Center

TRANSPORTATION OF BED MATERIAL DUE TO WAVE ACTION

TECHNICAL MEMORANDUM NO. 2

DEPARTMENT OF THE ARMY
CORPS OF ENGINEERS

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by

George Kalkanis
University of California

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FOREWORD

A better understanding of the basic mechanisms of sediment transport by wave action is being sought by the Coastal Engineering Research Center (formerly the Beach Erosion Board) through several different approaches. One of these which has been pursued under contract with the University of California at Berkeley, is reported on herein. This report discusses the mechanisms of sediment transport in a layer immediately adjacent to the ocean floor for long waves of small amplitude and relatively deep water. Proceeding on the premise that a bed-load function exists for the assumed type of flow, a bed-load transport equation was developed. Effects of both the unsteady mean flow velocity and the turbulent fluctuations are taken into account. From this equation the rate per unit width at which sediment in the bed layer is shifted by the oscillatory flow is calculated.

This report was prepared at the Hydraulic Engineering Laboratory of the University of California at Berkeley in pursuance of contracts DA-49-055-eng-17 and DA-49-055-CE-63-4 with the Beach Erosion Board (now the Coastal Engineering Research Center). These contracts provided in part for the study of sand movement by wave action. The author of the report, George Kalkanis, was at that time a graduate student and research assistant at the University.

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$a = \frac{H}{2} \operatorname{cosech}(kd)$	semi-amplitude of displacement at the bottom from irrotational theory.
A_1, A_2, A_3, A_4, A_5	constants of proportionality.
$A_L = \frac{\ell}{t}$	constant of proportionality
$A_* = \frac{A_1 A_3}{A_2 A_L}$	constant determined experimentally
$B_* = \frac{2A_2}{C_L \eta_o A_1}$	constant determined experimentally
$c = -0.65$	exponent of $f_1(y)$ from original smooth plate measurement.
c_1, c_2	coefficients
$c\ (y)$	concentration distribution of solid particles in a state of motion within the bed layer
c_o	average value of $c\ (y)$
C_L	coefficient of lift
d	depth of water
D	bed material grain diameter
e	base of natural logarithms
$f_1(y), f_2\ (y)$	empirical functions describing the velocity distribution and the phase shift in the boundary layer respectively
g	gravitational acceleration
$h = 2D$	thickness of bed layer
H	wave height (from crest to trough)
$k = \frac{2\pi}{\lambda}$	wave number
K_1, K_2	empirical constants
ℓ	distance traveled by a solid particle in one realization of motion.
$L = \bar{L} + L'$	lift force - total

$\bar{L}$	lift force due to unsteady mean flow
L'	lift force due to turbulence
m, n	subscripts
N_1	number of particles of size D per unit area of bed surface
p	probability of the lift force exceeding submerged weight
P	pressure
q_B	oscillatory bed-load rate
Q_B	steady mean bed-load rate
r	cylinder radius (Jeffreys model)
R	resultant of all hydrodynamic forces exerted on a solid particle
$t_1 = \frac{\ell}{V_1}$	exchange time
$T = \frac{2\pi}{\omega}$	period of oscillation
T^o	temperature degrees Fahrenheit
u, v	instantaneous boundary layer velocity components
$\bar{u}$, $\bar{v}$	unsteady mean velocity components in the boundary layer
u', v'	turbulent velocity components in the boundary layer
u_1, v_1	unsteady mean velocity components at the boundary from irrotational theory
v_s	settling velocity
$U_\infty = u_o \sin t$	free stream velocity at the outer edge of the boundary layer
U_o	critical free stream velocity
$\bar{U}$	mass-transport velocity
$V_1 = \ell/t_1$	speed of particle propagation

$W_1 = A_2 D^3 \gamma_s$	dry weight of the solid particle
$W' = (\rho_s - \rho_f) g\, A_2 D^3$	submerged weight of the solid particle
X, Y	semi-amplitudes of displacement from irrotational theory
z	dummy variable of integration
$Z = \frac{L'}{L \eta_o}$	standard normal variable
$\alpha = \left(c(1-c)\right)^{1/2}$	coefficient of $f_2(y)$ from original smooth plate measurement
$\beta = \left(\frac{\omega}{2\nu}\right)^{1/2}$	scale parameter for characteristic length associated with $f_1(y)$ and $f_2(y)$
γ_s	dry unit weight of bed material
δ	thickness of the boundary layer
ϵ	roughness diameter
η_o	normalized standard deviation of turbulent lift force
$\theta = \tan^{-1} \frac{f_1(y) \sin f_2(y)}{1 - f_1(y) \cos f_2(y)}$	phase angle
λ	wave length
$\mu = \beta y$	Longuet-Higgins notation
ν	kinematic viscosity of the water
ξ	dummy variable of integration
ρ_s, ρ_f	density of the solid particles and of the water respectively
σ	standard deviation of turbulent lift force

LIST OF SYMBOLS (cont.)

τ	shear stress
ϕ_i	phase increment used in numerical integration
Φ	dimensionless parameter
Ψ	flow intensity function
ω	angular velocity of oscillation

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ABSTRACT

A practical method has been developed which can be used to determine the rate of sediment transportation in a layer adjacent to the ocean floor. The method is applicable only when the flow in this layer caused by surface waves is unstable. The waves in question should be of small amplitude and great length, permitting the linearization of the equations of motion. The fundamental principle of the supporting theory is that at equilibrium the submerged weight of the solid particle resting on the ocean floor is balanced by the vertical component of the resultant of all the hydrodynamic forces exerted on the particle by the flow above. Both effects of the unsteady mean flow velocity as well as of the turbulent fluctuations are taken into account. The distribution of the lift forces associated with the former was determined experimentally while a statistical approach based on the experience with the same phase of the problem in a steady mean flow was used to determine the latter. Proceeding on the premise that a bed-load function exists for the type of flow we are dealing with here the bed-load equation was developed. From this equation it is possible to determine the rate at which sediment in the bed-layer is shifted by the oscillatory flow across a section of unit width. The concentration of bed material in the layer that at any instance is at a state of oscillatory motion can be determined from the above rate. Finally, this concentration in combination with the velocity distribution in the bed-layer associated with any incidental secondary flow is used to calculate the rate of transportation of bed material in the direction of this flow.

1. INTRODUCTION

The purpose of this study is to develop a method by means of which low rates of sediment transport due to the action of surface waves may be predicted with sufficient accuracy. More specifically the waves considered are long waves of small amplitudes in relatively deep water. The problem is of great importance to engineers and scientists as can be witnessed from the considerable amount of research devoted to it especially since the beginning of the century. An extensive survey of the literature, however, revealed that so far the subject has been treated only qualitatively.

The numerous publications are mostly restricted in presenting observations made in the field or the laboratory. With exception of a few purely empirical equations, which describe only particular measurements, no attempt has been made, at least to the writer's knowledge, to derive basic relationships which will constitute the basis of a quantitative treatment. In this paper we will try to analyze separately the individual mechanisms that constitute the overall phenomenon of sediment transport by ocean waves. After each phase of the problem is well-described and understood, the desired relationships will be developed based on all available theoretical concepts and empirical evidence. A more detailed description of our objective is given in the following section.

2. FORMULATION OF THE PROBLEM

It has been observed that loose sediment is moving in considerable quantities near and along the ocean floor even in relatively deep water. This motion cannot be attributed to the action of ocean currents because the flow intensity of these currents is usually very low and the hydrodynamic forces associated with it are not sufficient to overcome the forces resisting motion. It is evident, therefore, that the mechanism mainly responsible for this motion has its origin in the oscillatory flow near the bed which is caused by the surface waves. It is obvious that in general there is no net transport associated with this motion since the particles oscillate more or less about their mean position. The claim set forth is that the hydrodynamic effect of this flow is to relieve the particles of all or part of their weight so as to bring them to a state of incipient equilibrium. At this state any incidental secondary flow or current, no matter how weak, will be able to set the particle in motion. The problem now may be defined in a more precise form. An oscillatory motion is induced by the surface waves on the boundary layer. It is desired

a. To develop an expression that describes the flow field in the boundary layer.

b. To determine the dynamic effect of this field on the solid particles forming the bed.

The individual results of these two phases of the problem will be combined in a logical fashion to obtain a relationship by means of which it will be possible to predict the pattern of the motion of solid particles near the bed for a given set of wave characteristics and bed composition. We proceed first with the study of the flow in the boundary layer.

3. OSCILLATORY BOUNDARY-LAYER FLOW

a. Theoretical Considerations

(i) The Laminar Case. The problem of the boundary-layer flow can be treated in the ordinary fashion. According to the fundamental principle the flow field outside the boundary layer can be described by means of the irrotational theory while the complete equation of motion within the layer is simplified with the help of dimensional arguments. The boundary conditions of the simplified equation are set so as to satisfy the nonslip requirement at the solid boundary and the continuity of the velocity components at the outer edge of the boundary layer. Lin (1957)* has presented a solution to the more general problem in which the flow both inside and outside the boundary layer has a mean steady component as well as an oscillatory one. In the present case the two flows have oscillatory components only. The equations of motion will be

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial P}{\partial x} \quad \nu \frac{\partial^2 u}{\partial y^2} \tag{3-1}$$

and

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} = \frac{1}{\rho}\frac{\partial P}{\partial x} \tag{3-2}$$

inside and outside the boundary layer respectively. Continuity is assumed to be satisfied individually

so that $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3-3}$$

and $$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \tag{3-4}$$

Equations (3-2) and (3-4) describe a two-dimensional inviscid unsteady and incompressible flow. The solution of these two equations under a specific set of boundary conditions will define the velocity field in the fluid as a function of time and space, provided of course, that the effect of viscosity is negligible. In the problem at hand the first boundary condition is a sinusoidal progressive wave at the surface ($y = 0$) and the second, zero vertical velocity component at the bottom ($y = -d$). To the first approximation equation (3-2) may be linearized by omitting the quadratic terms; it can be written then as

$$\frac{\partial u_1}{\partial t} = \frac{1}{\rho}\frac{\partial P}{\partial x} \tag{3-5}$$

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* Numbers in parentheses denote date of reference listed on page 29.

This simplification is justifiable only under the assumption that the slope of the surface waves is very small; this is equivalent to $k \frac{H}{2} << 1$ where k is the wave number and H the amplitude of the surface wave. When the water depth is not very large ($d < \pi/k$) the solution of the simplified equation indicates that the water particles describe elliptical orbits around their mean position with component displacements in the x and y directions given by the expressions: (Lamb, 1932 sec. 229).

$$X = \frac{H \cosh k (y + d)}{2 \sinh kd} \cos (kx - \omega t) \tag{3-6}$$

$$Y = \frac{H \sinh k (y + d)}{2 \sinh kd} \sin (kx - \omega t) \tag{3-7}$$

It is evident that the vertical component of the displacement becomes smaller as the distance from the surface increases and that right at the bottom ($y = -d$) the motion degenerates into a simple harmonic oscillation along the x direction. The corresponding velocity components are obtained by differentiation of equations (3-5) and (3-6); so that

$$u_1 (x,y,t) = \frac{\partial X}{\partial t} = \frac{H}{2} \omega \frac{\cosh k (y + d)}{\sinh kd} \sin (kx - \omega t) \tag{3-8}$$

$$v_1 (x,y,t) = \frac{\partial Y}{\partial t} = -\frac{H}{2} \omega \frac{\sinh k (y + d)}{\sinh kd} \cos (kx - \omega t) \tag{3-9}$$

As $y \rightarrow -d$ $\quad v_1 (-d) \rightarrow 0$, and

$$u_1 (x,-d,t) = \frac{H}{2} \omega \operatorname{cosech} kd \sin (kx - \omega t) \tag{3-10}$$

or $u_1 (x,-d,t) = a\omega \sin (kx - \omega t) = u_o \sin (kx - \omega t)$

where obviously $a = \frac{H}{2} \operatorname{cosech} kd$ and $u_o = a\omega$

On the basis of dimensional considerations it is reasonable to postulate that the thickness of the boundary layer is very small $\left\{ \delta \doteq \sqrt{\frac{\nu}{\omega}} \right\}$ compared to $1/k$ so that for all practical purposes $u_1 (x,t)$ may be assumed constant within the boundary layer and approximately equal to $u_1(x,-d,t)$; as it is customary we will use the notation U_∞ for the free stream velocity at the outer edge, so that

$$U_\infty \doteq u_1 (x,-d,t) = a\omega \sin (kx - \omega t) \tag{3-11}$$

with surface waves of large wave length ($ka << 1$) equation (3-11) becomes

$$U_\infty \doteq a\omega \sin \omega t = u_o \sin \omega t \tag{3-12}$$

Therefore at the edge of the boundary layer equation (3-5) will take the form

$$\frac{\partial U_\infty}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} \tag{3-12a}$$

Subtracting now (3-12a) from the complete boundary layer equation (3-1) we obtain

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} = \frac{\partial U_\infty}{\partial t} + \nu\frac{\partial^2 u}{\partial y^2} \tag{3-13}$$

The value of u and v is zero at the wall and equal to U_∞ and V_∞ respectively at distance δ from it, where $\delta = \sqrt{\frac{\nu}{\omega}}$; within the boundary layer, therefore to the first approximation $|u| \doteq |U_\infty|$. Moreover it is reasonable to asume that in the same region

$$\frac{\partial}{\partial y} \doteq \frac{1}{\delta} \quad , \quad \frac{\partial}{\partial x} \doteq k$$

From continuity then $V \doteq \delta k\, U_\infty$

Let us now examine the order of magnitude of the terms in equation (3-13)

$$\frac{\partial u}{\partial t} \doteq \frac{\partial U_\infty}{\partial t} = u_o\omega = a\omega^2$$

$$u\frac{\partial u}{\partial x} \doteq U_\infty\, k\, U_\infty = k\, a^2\omega^2 = ka\,(a\omega^2)$$

$$V\frac{\partial u}{\partial y} \doteq \delta k\, U_\infty U_\infty/\delta = k\, U_\infty^2 = ka\,(a\omega^2)$$

$$\nu\frac{\partial^2 u}{\partial y^2} \doteq \nu\frac{U_\infty}{\delta^2} = \nu\frac{U_\infty\omega}{\nu} = U_\infty\omega = a\omega^2$$

So we see that the quadratic terms are smaller than the rest by a factor ka and since according to our basic assumption ka << 1 these terms can be omitted.

The boundary layer equation (3-1) to a first approximation may be written now as

$$\frac{\partial u}{\partial t} = \frac{\partial U_\infty}{\partial t} + \nu\frac{\partial^2 u}{\partial y^2} \tag{3-13a}$$

with boundary conditions

$$\left.\begin{aligned} u\,(0,\ t) &= 0 \\ u\,(\infty,\ t) &= U = u_o \sin\omega t \end{aligned}\right\} \tag{3-14}$$

The solution of (3-13) satisfying (3-14) is of the form

$$u\,(y,\ t) = u_o \left\{ \sin\omega t - e^{-\beta y} \sin\,(\omega t - \beta y) \right\} \tag{3-15}$$

where $\beta = \sqrt{\dfrac{\omega}{2\nu}}$

We recognize the second term in the parentheses of (3-15) as the solution of the diffusion equation

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \tag{3-16}$$

with boundary conditions

$$\left.\begin{aligned} u\,(o,t) &= u_o \sin\omega t \\ u\,(\infty,t) &= 0 \end{aligned}\right\} \tag{3-17}$$

which describes the flow near an oscillating flat plate (Schlichting, 1955). The structure of the solution (3-15) suggests that it is possible to determine the velocity component in the x direction within the boundary layer of an oscillating body of water by means of simple superposition of the irrotational component at the bottom and of the solution of the oscillating flat plate. This property has significant importance especially in connection with the experiment. The solution thus obtained is a good approximation of the actual case provided that the flow within the boundary layer is laminar.

However, even when the surface of the wall is hydraulically smooth one would expect that at a certain value of the Reynolds number defined as

$$N_R = \frac{u_o \delta}{\nu}$$

the flow will become unstable. The knowledge of the critical value of Reynolds number is very important to our problem, because the dynamic effects on the particles under laminar flow conditions are quite different than in turbulent flow. It would be very helpful, therefore, to establish criteria of flow stability covering a wide range of wave characteristics and bed roughness. This is rather an involved case in the class of problems of hydrodynamic stability. If we recall the lengthy calculations required to solve the relatively simple case of the Blasius profile on a smooth flat plate we may conclude without much deliberation that the attempt to seek a theoretical solution for our case, of a mean unsteady flow and a rough plate, does not offer much hope for success. Besides this phase of the problem is beyond the scope of the present study. In a practical application it is not as important to know the exact value of the critical Reynolds number as to be able to predict with sufficient confidence that under the existing conditions the flow regime in the boundary layer is not laminar and consequently that the theoretical laminar solution is no more applicable. This type of information can be obtained by experimental methods. The studies of Li (1954) and Manohar (1955) are two outstanding sources of such information. The procedure used by these two investigators as well as their results will be discussed briefly in Appendix A.

(ii) The Turbulent Case. After we have established the fact that the flow in the boundary layer is unstable we proceed with our main task which is the description of the velocity distribution within this layer. We define the instantaneous velocity components in the two directions as

$$u = \bar{u} + u' \tag{3-18}$$

and $$v = v'$$

where $\bar{u}$ is a simple harmonic function of time and space while u' and v' are the turbulence components. We claim that the amplitude of $\bar{u}$ is a function of y only and that its frequency is the same as the frequency of the wave. A reasonable expression for $\bar{u}$ will be then of the form (a form similar to equation (3-15) of the laminar case)

$$\bar{u}(y,t) = u_o \left\{ \sin\omega t - f_1(y) \sin\left[\omega t - f_2(y)\right] \right\} \tag{3-19}$$

where $f_1(y)$ and $f_2(y)$ are functions of y only. These functions can be determined by experimental methods.

This, of course, implies actual measurement of the velocity at points very close to the boundary. The effective thickness, however, of the boundary layer in the actual case does not exceed a few millimeters and even if it were possible to make velocity measurement within such a thin layer the experimental flume must have practically prototype dimensions. The experimental work can be simplified considerably by making use of the principle of superposition mentioned above. Therefore, setting

$$\bar{u} = \bar{u}_1 - \bar{u}_2 \tag{3-20}$$

where $\bar{u}_1 = U_\infty = u_o \sin\omega t$

and $$\bar{u}_2 = u_o f_1(y) \sin\left[\omega t - f_2(y)\right] \tag{3-21}$$

the velocity component $\bar{u}$ in the boundary layer for given values of a and ω can be obtained from measurement of $f_1(y)$ and $f_2(y)$ in a flume in which the water surface is maintained at rest while the bottom is oscillating with a simple harmonic motion in the x direction prescribed by a and ω. The size of the flume can then be kept within reasonable limits since $f_1(y)$ is a rapidly decaying function of y.

b. Experimental Work

Velocity measurements were made in an experimental flume in which a horizontal plate near the bottom was oscillated in its own plane with a simple harmonic motion with respect to the water above. A detailed description of the apparatus and the measuring device has been given elsewhere (Kalkanis, 1957). In that previous study experimenting with a smooth plate it has been found that the expression describing the vertical distribution of the horizontal velocity is of the form

$$\bar{u}_2 = u_o k_1 (\beta y)^c \sin(\omega t - \alpha \ln k_2 \beta y) \tag{3-22}$$

where $\beta = (\frac{\omega}{2\nu})^{\frac{1}{2}}$; the values of the exponent c and of the constants as the results of these experiments indicated were the following

$$c = -0.65, \quad k_1 = 0.342, \quad \alpha = 1.05, \quad k_2 = 2.883$$

The above expression describes the velocity distribution relative to a fixed boundary; the distribution relative to the oscillatory boundary according to equation (3-20) is given by the expression

$$\bar{u} = u_o \left\{ \sin \omega t - k_1 (\beta y)^c \sin(\omega t - \alpha \ln k_2 \beta y) \right\} \tag{3-23}$$

At the time of the conclusion of the experimental work of this first phase of the study the lack of sufficient data led to the adoption of the parameter $1/\beta$ as the proper length scale. The data collected in subsequent tests, however, indicated that more suitable characteristic lengths were the amplitude of the oscillation a for the case of the hydraulically smooth wall and the parameter $a\beta D$ for the rough wall, where D is the roughness diameter (either two or three dimensional).

The analysis of the new experimental data which can be found in Appendix B led to the following expression:

$$\frac{u}{u_o} = \left[1 + f_1^2(y) - 2f_1(y) \cos f_2(y) \right]^{\frac{1}{2}} \sin(\omega t + \theta) \tag{3-24}$$

where $$\theta = \tan^{-1} \frac{f_1(y) \sin f_2(y)}{1 - f_1(y) \cos f_2(y)}$$

The form of the functions $f_1(y)$ and $f_2(y)$ that seemed to best fit the experimental data is the following

(i) Smooth plate $$f_1(y) = 0.3\, e^{-75\frac{y}{a}}$$
$$f_2(y) = 1.55 (\beta y)^{1/3}$$

(ii) Two-dimensional roughness $f_1(y) = e^{-\frac{y}{a\beta D}x10^3}$

$$f_2(y) = 0.5\ (\beta y)^{2/3}$$

(iii) Three-dimensional roughness $f_1(y) = 0.5\ e^{-133\frac{y}{a\beta D}}$

$$f_2(y) = 0.5\ (\beta y)^{2/3}$$

We observe that these equations do not satisfy the boundary condition at the wall which is not a very serious limitation since it can be easily circumvented by assuming the formation of a laminar sublayer as in a steady mean flow.

We want to emphasize, however, that the above expressions are nothing more than a convenient representation of actual measurements. It may be that the true distribution follows some other law which conceivably will be described by an equation derived by a more rigorous analytical process. Nevertheless, one will expect that close to the boundary, which is the region we are mostly concerned with, the results by the two methods will be closely the same since the experimental work was conducted within the range of conditions which most likely will materialize in an actual case. We claim therefore in concluding this phase of the study that by means of the expressions given above it is possible to describe the mean velocity distribution near the ocean bed in terms of the surface wave characteristics, the depth of the water and the grain size of the uniform material forming the bed.

4. HYDRODYNAMIC EFFECTS OF THE FLOW ON THE BOUNDARY

The strong interaction between the flow and the loose material forming the bed of a stream has been studied at great length by numerous investigators. The results of these studies led to the development of theories describing the phenomenon of sediment transport in rivers. It has been reported some time ago, (Einstein, 1948), that a basic similarity exists between the motion of sediment in a river and in a large body of water in which the main motion is oscillatory. This, of course, does not imply that the laws governing the two phenomena are exactly the same, nor that the theories describing the motion in one case are directly applicable to the other. It is, however, reasonable to assume that some fundamental concepts used in the derivation of one theory may be applied in the derivation of the other. Since we intend to use in our study some of the basic principles associated with the motion of sediment by steady streams of water we consider it appropriate to give a brief outline of the historical development of theories related to this problem.

a. Theories of Sediment Transport in Rivers

The older equations which were used to determine rates of sediment transport in rivers have been derived mostly by means of empirical methods.

The basic concept regarding the pattern of motion was that the loose bed is sliding in layers under the action of the flow above. The top layer of the bed is set into motion by the "tractive force" or shear, which in flows where energy is dissipated mainly to overcome friction, is equal per unit area of the bed to the product of the unit weight of the water, of the depth of flow and of the energy gradient. When this force becomes larger than the force resisting motion per unit area, which is proportional to the submerged weight of the particles forming the bed, the latter begin to move. The rate of transport determined experimentally was found to be a function of the difference between the two forces. No effort has been made in developing this theory to explain the actual mechanism of interaction between the solid particles and the flow field. General information therefore cannot be deduced from this theory and its application is necessarily confined to the narrow range of conditions used in its derivation. A major weakness is that it deals only with an average value of the shear which is assumed in each case to be constant in time and space. The implication is that all particles of a certain size will start moving simultaneously over the entire bed, whenever the average shear is larger than critical. This, of course, is contrary to the well-established fact that motion near the bed takes place in the form of sudden jumps by individual particles alternating with rather long periods of rest.

Jeffreys (1929) perhaps, was the first investigator to base a theory on the stability of the individual solid particle. He suggested the application of the solution from classical hydrodynamics regarding the stability of a long circular cylinder of radius r resting on the flat bed of a deep stream with its axis perpendicular to the flow. The expression of the complex potential is then of the form

$$W = \pi r \, U \coth \pi r/z \tag{4-1}$$

and the upward thrust transmitted to the solid can be shown to be equal to

$$L = \pi_{\rho f} \left(\frac{1}{3} + \frac{1}{9} \pi^2\right) U^2 r^2 \tag{4-2}$$

It follows that the condition of motion is given by the inequality

$$\left(\frac{1}{3} + \frac{1}{9} \pi^2\right) U_c^{\,2} > \frac{\rho_s - \rho_f}{\rho_f} \, r \tag{4-3}$$

where U_c is the critical value of the stream velocity. He postulated that for three-dimensional elements the values would be slightly larger. The numerical values given by Jeffreys for water and sand were $U_c = 4.32$ cm/sec for $r = 0.01$ cm and $U_c = 13.6$ cm/sec for $r = 0.1$ cm. This model, of course, gives the correct answer as long as there is sufficient justification for describing dynamic effects in flows of real fluids by means of the irrotational theory. Jeffreys answer to this question is affirmative; he claims that during the initial stage, when a particle is just dropped on the bed of a steady stream, the theory is still applicable because the flow around the particle has not yet been modified by viscosity. This may be true, but even if we accepted the validity of the argument, still we are faced with a phase of our problem which Jeffreys' model does not seem to recognize; this is the erratic fashion by which particles move on the bed. According to his theory the flow which is responsible for the forces induced on the particle is uniform and steady everywhere on the bed, resulting in a uniform force field very similar in character to the one associated with the tractive force theory described above. Therefore a particle that starts moving at some point of the bed will never have a chance to come back to rest at some other point on the bed, a mode which is inconsistent with the actually observed form of motion.

The more modern theories go a step further; they concern themselves again with the stability of the individual particle, but they recognize the fact that the hydrodynamic forces acting upon it vary rapidly with time, a phenomenon strongly associated with turbulence. Moreover, they use some elementary concepts concerning the structure of turbulence to explain the mechanism of suspension. Accordingly, the particles are being moved upwards from lower layers of high concentration to higher ones of lower concentration by the vertical velocity fluctuations. Equations have been derived based on the concept of momentum transfer by turbulence which describe the distribution of concentration of sediment in suspension relative to a specified value at a reference level. If there were a way of predicting this value the rate of sediment moving in suspension would be readily obtained by multiplying corresponding values of local mean velocity and concentration and integrating over the entire depth. One method to predict the concentration at some point in the flow has been proposed by Lane and Kalinske (1939). These authors claimed that the bed itself may be taken as the reference level. The corresponding concentration can be obtained by making use of the statistical properties of turbulence close to the bed. This implies direct exchange of particles between the suspension load and the bed.

Another method proposed by Einstein (1950) interposes a thin layer between the suspension load and the fixed bed. The reasoning behind this model is that very close to the bed the scale of turbulence is so small that the eddies are of about the same order of magnitude as the particles on which they act and, consequently, they are unable to move them away from the bed. Within this thin layer adjacent to the bed and called the "bed layer" the particles move by sliding and rolling, in a fashion much different than the particles in suspension above. The bed layer has been assumed to have a thickness of about two grain diameters, an estimate based on observation. There is a continuous exchange of grains between the bed layer and the bed and between the suspension load and the bed layer. The basic concept of the theory of the bed-load function as it applies in a river flow is that at equilibrium all these exchanges occur at the same rate. The fraction of the total load that is carried within the bed layer is called the bed load. The rate of deposition from the bed layer to the bed is found to be a function of the bed-load rate, while the rate of removal of grains from the bed is a function of the local flow intensity. The functional relationship between the "bed-load rate" in a stream and the flow intensity constitutes the "bed-load function" while the equation expressing this relationship is defined as the "bed-load equation". With the help of this equation it is possible to calculate the bed-load rate for given flow conditions and bed composition. Finally, the concentration in the bed layer which can be easily calculated from the bed-load rate is assumed to be equal to the concentration of the suspension load at the reference level. This helps to find the complete solution to the problem which expresses the total rate of transport as the summation of the suspension rate and the bed-load rate. The significance of the concepts of the bed layer, the bed-load function and the bed-load equation is evident. In the absence of substantial evidence to the contrary it is legitimate to assume that similar concepts hold true in the case of an oscillatory mean flow. Accepting a priori the existence of a bed layer and of a bed-load function, our objective will be to develop the bed-load equation associated with this type of flow. In our effort along this line the valuable experience from the steady mean flow will be used as a guide. The procedure leading to the derivation of the bed-load equation will be described in the following section.

b. The Bed-load Equation

It is a well-established fact that the distortion of the flow field around a solid particle resting on the bed of a stream generates a lift force acting on the particle, even if the latter is well-sheltered within the sublayer. Naturally the larger the size of the particle relative to the thickness of the undisturbed laminar sublayer, the more pronounced is the distortion and the greater the intensity of the lift force. This force will tend to dislocate the particle and move it away from the solid bed. As long, though, as the particle is still in contact with the bed, the lift force is acting only vertically upwards and it can be expressed as

$$L = C_L \rho_f \frac{u^2}{2} A_1 D^2 \tag{4-4}$$

C_L is the coefficient of lift, A_1 a shape factor and u the instantaneous velocity acting at a distance y_o from the theoretical bed. In the case of a steady mean stream the location of the theoretical bed and the level y_o relative to it at which the velocity must be taken have been established experimentally by Einstein and El-Samni (1949). They conducted flume experiments with plastic spherical balls 0.225 feet in diameter placed in a steady stream of water. The theoretical bed has been determined as the reference level from which distances should be taken so that the measured values of the mean velocity would give the best fit to a logarithmic distribution. It has been found that the matching of this profile was the most satisfactory when the theoretical bed was taken at a distance 0.20 D below the top of the spherical particle. The distance y_o at which the velocity should be measured in calculating the lift force has been obtained simultaneously with the determination of the lift coefficient C_L. The method used to this effect consisted of measuring the differential pressure in the fluid between two layers; one at the top of the spheres and the other near their base. This pressure difference which is a measure of the lift force can be expressed as

$$\Delta P = C_L \rho_f \frac{u^2}{2} \tag{4-5}$$

The analysis of the experimental results revealed that C_L in the above expression had a constant value $C_L = 0.178$ for a wide range of flow conditions provided that the average velocity was measured at a distance 0.35 D from the theoretical bed. Since it is practically impossible to determine the value C_L in an unsteady mean flow, it would be reasonable to assume that it is about the same as in a steady stream. As a matter of fact, as it will be shown shortly, the only assumption that is really necessary is that C_L is constant throughout the entire cycle of oscillation. Before we proceed any further we wish to summarize the statements adopted in this study which have their origin in El-Samni's experiments with steady mean flows.

a. The theoretical bed lies at a distance 0.2 D below the top of the grains resting on the fixed bed.

b. The lift force associated with the mean flow can be calculated by using the velocity at a distance 0.35 D from the theoretical bed.

c. The lift coefficient associated with the mean flow has a constant value independent of the Reynolds number.

If it were not for turbulence it would be rather easy to establish a criterion of stability similar to Jeffreys' in the form of the inequality

$$L > W' \tag{4-6}$$

(W' is the submerged weight of the particle)

or
$$C_L \rho_f \frac{\bar{u}^2}{2} A_1 D^2 > (\rho_s - \rho_f)\, g A_2 D^3 \tag{4-7}$$

where by $\bar{u}$ we mean the amplitude of the velocity calculated from equation (3-24) with y = 0.35 D.

In a turbulent stream, however, all the local flow parameters, and consequently the local life as well, vary rapidly with time. An accurate description of the temporal variation of the local life by analytical methods would be possible only if sufficient information regarding the structure of turbulence in the boundary layer were available. Considerable amount of effort is made now by numerous investigators toward developing new theories or toward improving existing ones on the subject. The fact remains that even if we could wait until the theoretical work had been sufficiently advanced to permit practical applications, still some approximations would be required since most of the studies are concerned with steady mean flows and smooth boundaries. In the meantime and due to the lack of more reliable information we are forced to depend again on experimental evidence. A possible criterion of stability in a turbulent stream could be set as

$$L = L' + \bar{L} > W' \tag{4-7a}$$

where L' is the turbulent component of the lift force. The study of the variation of L' with time constituted the last phase of the experimental work conducted by El-Samni. By measuring the instantaneous values of the lift force exerted by a steady stream of water on the plastic spheres mentioned above, he was able to show that L' behaves like a random variable having a normal distribution with mean zero and standard deviation $\sigma = \bar{L}\,\eta_o = \frac{\bar{L}}{2.5}$. Assuming that the behavior of L' in an oscillatory mean flow will be similar although with a different numerical value of σ we could proceed as follows: Let p be defined as the probability that a particle resting at a certain location in the bed becomes just ready to move; this implies that

$$p = Pr\left\{ L' + \bar{L} > W' \right\} = Pr\left\{ \frac{L'}{\bar{L}\,\eta_o} > \frac{W'}{\bar{L}\,\eta_o} - \frac{1}{\eta_o} \right\} \tag{4-8}$$

or $$p = Pr\left\{ Z > \frac{2g(\rho_s - \rho_f)\, A_2 D^3}{C_L \rho_f D^2\, \bar{u}^2\, A_1 \eta_o} - \frac{1}{\eta_o} \right\} \tag{4-9}$$

Let $$\Psi = \frac{\rho_s - \rho_f}{\rho_f} \; \frac{Dg}{\bar{u}^2} \tag{4-10}$$

and $$B_* = \frac{2\,A_2}{C_L\,\eta_o A_1} \tag{4-11}$$

Then $$p = Pr\left\{ Z > \Psi B_* - \frac{1}{\eta_o} \right\} \tag{4-12}$$

and since $Z = \frac{L'}{\bar{L}\eta_o}$ has a normal distribution with mean zero and standard deviation $\sigma = 1$

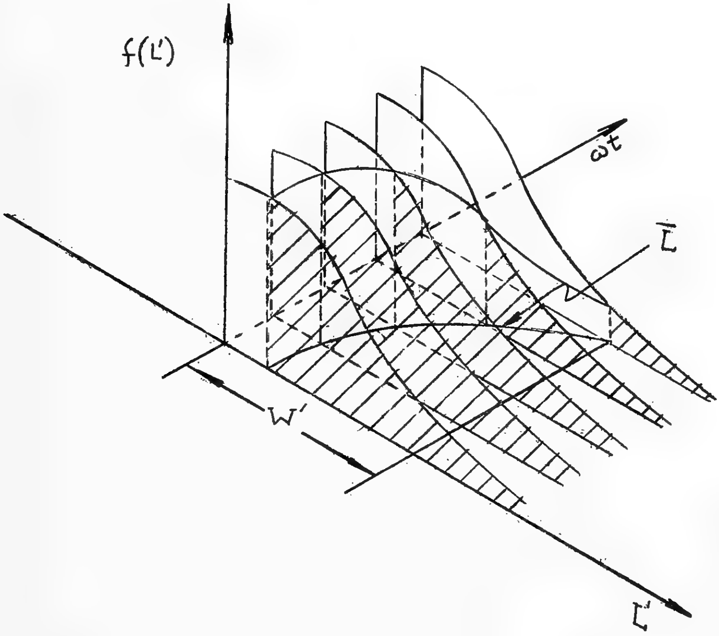
$$p = \frac{1}{\sqrt{2\pi}} \int_{B_* \Psi - \frac{1}{\eta_o}}^{\infty} e^{-z^2/2}\, dz \tag{4-13}$$

where z of course is a dummy variable.

The values of η_o and B_* can only be determined experimentally. It is evident now why it is not necessary to determine the exact value of C_L. As long as C_L is constant its effect will be reflected in the value of B_*. We may recall now that $\bar{u}$ is a periodic function of time which as the experiments indicated can be approximated by the right hand side of equation (3-24). Consequently ψ is a periodic function too which means that a more appropriate form of equation (4-13) would be

$$p = \frac{2}{\pi\sqrt{2\pi}} \int_o^{\pi/2} \int_{B_*\psi - \frac{1}{\eta_o}}^{\infty} e^{-\frac{z^2}{2}} \, dz \; d\omega t \qquad (4\text{-}13a)$$

The geometric representation of the probability p as expressed by equation (4-13a) is shown in the sketch below.



It is evident from the form of equation (4-13a) that the turbulent field was assumed constant during the entire cycle of the oscillatory mean flow.

The number N_1 of particles of size D per unit area of the bed surface that at any instance become free to move and indeed do move is proportional to this probability as well as to the total population of similar particles per unit bed surface. It is evident that

$$N_1 = \frac{p}{A_1 D^2} \tag{4-14}$$

where A_1 is a constant.

The rate of transportation q_B which is defined as the rate in dry weight at which solid particles move across a section of unit width oriented perpendicular to the flow can be expressed as

$$q_B = N_1 W_1 V_1 = \frac{p}{A_1 D^2} W_1 V_1 \tag{4-15}$$

where $W_1 = A_2 D^3 \gamma_s$, γ_s being the dry unit weight of the particles and V_1 the speed of the particle propagation. It should be remembered that the motion of the particles is not continuous, but that it consists of a sequence of discrete steps. It will be reasonable, therefore, to express this speed of propagation as the ratio of the average distance covered by a particle in a step and of the total time required for the completion of a full cycle of motion. If we denoted the former by ℓ and the latter by t equation (4-15) after the substitution for W_1 becomes

$$q_B = \frac{p}{A_1 D^2} A_2 D^3 \gamma_s \frac{\ell}{t} = \frac{A_2}{A_1} pD\gamma_s \frac{\ell}{t} \tag{4-16}$$

The distance ℓ as in the case of a steady mean flow may be considered as a charactsristic length proportional to the grain diameter, so that $\ell = A_L D$. Moreover by definition t can be written as

$$t = t_1 + t_2 \tag{4-17}$$

where t_1 is the part of the cycle during which the particle is at rest (between two consecutive excursions) and t_2 is the part during which the particle is in motion. Experiments with light-weight coarse material and steady mean flows in flumes have demonstrated that in general t_2 is much smaller than t_1 which led to the conclusion that for all practical purposes we may assume $t = t_1$. Einstein (1950) has defined t_1, which he called "the exchange time" as a measure of the time required for the replacement of a particle that is just being picked up by the flow at a certain spot of the bed by a similar particle that is being brought to rest at the same spot. Therefore, one may think of t_1 as a parameter which depends on the properties of the particle and the surrounding fluid only and which, consequently, is independent of the local flow conditions.

The simplest such parameter having the right dimensions is the time required for a particle to settle through a distance equal to its diameter in the fluid at rest. If the settling velocity is denoted by V_s then we can write

$$t_1 = c_1 \frac{D}{V_s} \tag{4-18}$$

c_1 being a constant of proportionality

A basic characteristic of our model is that the criterion of equilibrium is governed mainly by the lift force exerted by the flow on the particle. Since the coefficient of this lift force assumes a constant value even for very small values of the Reynolds number the settling velocity at equilibrium may be written as

$$V_s = c_2 \sqrt{\frac{g(\rho_s - \rho_f)\, D}{\rho_f}} \tag{4-19}$$

Combining now (4-18) and (4-19) and making the proper substitutions for ℓ and t equation (4-16) becomes

$$q_B = \frac{A_2}{A_1} \frac{A_L}{A_3} p\gamma_s D^2 \sqrt{\frac{g(\rho_s - \rho_f)}{D\,\rho_f}} \tag{4-20}$$

which is the "bed-load" equation; after being rearranged (4-20) becomes

$$\frac{q_B}{\gamma_s A_2 A_L D^4} = \frac{p}{A_3 A_1 D^2} \sqrt{\frac{g(\rho_s - \rho_f)}{D\,\rho_f}} \tag{4-21}$$

This is identical with Einstein's (1950) equation (38) provided that the material forming the bed is uniform. The significance of equation (4-21) is that it describes the equality between the rate of deposition (left hand side term) and the rate of erosion (right hand side). The remarkable characteristic of either equation (4-20) or (4-21) is that they indicate that the bed-load rate is only indirectly related to the flow intensity through the probability p. Eliminating this probability between equations (4-13a) and (4-20) we obtain the fundamental relationship between flow intensity and bed-load rate.

Before we conclude this section, it would be necessary to define a little better the average distance of travel $\ell = A_L D$. A rather small value of p implies that only on a small fraction of the bed surface the lift force is strong enough to remove a grain of a given size at any instance. Therefore a particle that has been lifted by the flow will probably come to rest immediately upon completion of the first step since it is very probable that the conditions locally will favor deposition. In this case $\ell = A'_L D$ where A'_L is the true constant of proportionality between distance of travel in a single step and particle diameter. If, on the other hand p is rather large, there is a good chance that the conditions around the spot on the bed where the particle is coming to rest after completing a step of length $A'_L D$ are not favorable to deposition; the result is that the particle will be forced to take an additional step of length $A'_L D$. This can be repeated a number of times until the particle finally finds a point on the bed where it is permitted to rest. This model suggests that the actual distance travelled will be proportional to the length of each step ($A'_L D$) times the number of consecutive steps made in each realization. This number of steps may be thought of as a discrete random variable having a binomial distribution with parameter p. The probability that the particle will travel a distance $A'_L D$ is (1-p) which is the probability of failure in the first trial. The probability of covering a distance $2A'_L D$ is p(1-p), the probability that failure will follow one success. In general the probability of covering a distance $(n + 1) A'_L D$ is $p^n (1-p)$ which is the probability of the first failure occurring after n consecutive successes. The expected value of the distance covered by the particle in a single realization can be expressed then by $A_L D$ where

$$A_L D = \sum_{n=o}^{\infty} p^n (1-p) (1+n) A'_L D = \frac{A'_L D}{1-p} \tag{4-22}$$

We introduce now this expression for $A_L D$ in equation (4-20) to obtain

$$q_B = \frac{A_2 A'_L}{A_1 A_3} \frac{p}{1-p} \gamma_s D^2 \sqrt{\frac{g(\rho_s - \rho_f)}{D \rho_f}} \tag{4-23}$$

Solving (4-23) for p we get

$$\frac{A_* \Phi}{1+A_* \Phi} \tag{4-24}$$

where

$$A_* = \frac{A_1 A_3}{A_2 A'_L} \tag{4-25}$$

and

$$\Phi = \frac{q_B}{\gamma_s} \sqrt{\frac{\rho f}{g(\rho_s - \rho_f)}} D^{-3/2} \tag{4-26}$$

We equate finally the right hand sides of equations (4-13a) and (4-24) to obtain the very important rélationship between flow intensity and bed-load rate in the form of equation (4-27)

$$\frac{2}{\pi \sqrt{2\pi}} \int_o^{\pi/2} \int_{B_* \Psi - \frac{1}{\eta_o}}^{\infty} e^{-\frac{z^2}{2}} dz d\omega t = \frac{A_* \Phi}{1+A_* \Phi} \tag{4-27}$$

The practical application of equation (4-27) necessitates the determination of the constants A_* and B_* and of the standard deviation η_o of the turbulent lift force. The procedure used toward this end will be described in the following section

5. DETERMINATION OF A_*, B_* and η_o

Suppose that a rather large set of corresponding values of the parameters Ψ and Φ is available. These values were calculated from equations (4-10) and (4-26) respectively. The data used in these calculations may have been obtained in the field or in the laboratory but in either case we are confident that they are reasonably accurate. Our task then consists in selecting a set of values of the parameters to be determined so that Ψ and Φ calculated from equations (4-10) and (4-26) will satisfy as closely as possible equation (4-27). The determination of the constants in the present study was based on values of Ψ and Φ calculated from experimental data. The description of the equipment and the procedure followed in this phase of the study is given in Appendix C. In these experiments three different sizes of sand were used and for each size nine runs were made with different amplitudes of oscillation and frequencies. The values of q were directly obtained from the experiment and they were next introduced in equation (4-26) to obtain Φ while equation (4-10) was used to calculate the corresponding Ψ. The latter was then plotted against the former on a log-log paper as shown in Figure 8. On the same paper a family of curves was plotted also representing graphical solutions of equation (4-27) with η_o as parameter. The four curves shown on the figure were calculated with $1/\eta_o$ equal to 1.0, 1.5, 2.0, and 2.5. Both A_* and B_* in these calculations were taken equal to unity. The left hand side of equation (4-27) was integrated numerically with the simultaneous use of normal error tables. The three steps of the procedure used to determine the points of the theoretical curve were the following. First an arbitrary value of Ψ was chosen which was divided by $(\cos \phi_i)^2$ where ϕ_i is the midpoint of one of the nine equal intervals dividing the quarter of the cycle, so that $\phi_1 = 5^o$, $\phi_2 = 15^o$ etc. The probability p which is approximately equal to the average value of p_i where

$$p_i = \frac{1}{\sqrt{2\pi}} \int_{\frac{\Psi}{(\cos \phi_i)^2} - \frac{1}{\eta_o}}^{\infty} e^{-z^2/2} \, dz \tag{5-1}$$

was next obtained. The rearranged form of equation (4-25) was finally used to calculate the corresponding Φ; so that

$$\Phi = \frac{p}{1-p} \tag{5-2}$$

By examining the shape and relative position of the graphs on Figure 8, one may conclude that the translation of the experimental Φ, Ψ curve along the two axes would bring it to a close approximation with the theoretical one constructed on the basis of $1/\eta_o = 1.5$. The coefficients by which the coordinates of the experimental Φ, Ψ curve should be multiplied in order to achieve this approximation express the sought-for values of the parameters A_* and B_*. It is evident from the foregoing that

$$
\begin{aligned}
A_* &= 30 \\
B_* &= 4 \\
\frac{1}{\eta_o} &= 1.5
\end{aligned}
\qquad (5\text{-}3)
$$

We may conclude, therefore, that for a given set of wave characteristics and grain size of bed material it is possible to determine the bed-load rate q_B through equations (4-10), (4-27) and (4-26).

This is an important result, but by no means the final answer to our problem. Because of symmetry the rate in one direction during the first half of the cycle will be exactly equal to the rate in the opposite direction during the second half. The net effect of course will be zero steady movement. The question is now as to how can we make a practical use of the results obtained so far. The argument advanced is that although the calculated value of q_B does not give a direct measure of the amount of sediment that systematically moves in some direction, it can be used to determine the number of solid particles per unit area of bed surface that at any time are exposed to the transportive effort of any incidental flow no matter how weak; this flow is not strong enough to dislocate the particles from their state of rest, but once it finds them in a state of motion produced by the surface wave it is able to move them forward. The rate of sediment transport per unit width of the bed associated with such a secondary flow will be

$$Q_B = \int_o^{2D} U(y)c(y)dy \qquad (5\text{-}4)$$

where h is the thickness of the layer within which motion occurs, U(y) expresses the velocity distribution of the flow and c(y) the concentration distribution of the solid particles in motion. The magnitude of h may be taken equal to 2D, a customary approximation for flows of this type. Another approximation very common with steady mean flows is that c(y) within the layer may be considered constant. The velocity distribution U(y) of course remains undefined, but for any particular case is presumed known. Equation (5-4) then becomes

$$Q_B = c_o \int_o^{2D} U(y)\, dy \qquad (5\text{-}5)$$

which will provice the desired answer to our problem provided of course that c_o is known. This is the phase of the problem where the results obtained in this investigation find a direct and rather important application.

We have shown already that q_B can be determined for a given set of conditions. In the following section a method will be described by means of which c_o could be calculated from known values of q_B.

6. DETERMINATION OF c_o

The rate of flow through a cross section of unit width and height $h = 2D$ is

$$Q_B = \int_o^{2D} U(y)dy \tag{6-1}$$

while the rate of sediment transport through the same section at any instance is

$$Q_B = c_o \int_o^{2D} U(y)dy \tag{6-2}$$

c_o is a measure of the concentration of the sediment which at any time is at a state of motion within the bed layer. In this layer the rate of transport due to the oscillatory flow is by definition equal to q_B. Therefore in a way similar to (6-2) we can write

$$q_B = c_o \int_o^{2D} \bar{u}\, dy \tag{6-3}$$

The integral $\frac{1}{2D}\int_o^{2D} \bar{u}\, dy$ is nothing else but the expression for the mean value of $\bar{u}$ which we will denote by $\bar{u}_m$. We can write then

$$c_o = \frac{q_B}{\int_o^{2D} \bar{u}\, dy} = \frac{q_B}{2D\, \bar{u}_m} \tag{6-4}$$

$\bar{u}$ is obtained from equation (3-24) and naturally is a function of both y and t. Hence the expression for the mean value will be of the form

$$\bar{u}_m = \frac{1}{4\ \pi\ D} \int_o^{2\pi} \int_o^{2D} \bar{u}(y,t)dyd\omega t \tag{6-5}$$

The calculation of $\bar{u}_m$ from (6-5) is a lengthy and tedious operation which has to be performed in each particular case. This renders the method proposed here practically inapplicable.

A simpler approach could be based on the assumption that $\bar{u}_m$ is proportional to the amplitude of the velocity at some arbitrary distance from the wall within the bed-layer. In other words we postulate that

$$\bar{u}_m = A_5 \mid \bar{u}_B \mid \tag{6-6}$$

with A_5 a constant of proportionality and $\overline{u}_B$ the velocity from equation (3-24) at an arbitrary distance, say y = D, from the theoretical bed. A_5 now can be calculated from the equation

$$A_5 = \frac{\int_0^{2\pi}\int_0^{2D} \overline{u}(y,t)dyd\omega t}{4\pi D \left|\overline{u}\right|_{y=D}} \tag{6-7}$$

A large number of values could be obtained from (6-7) for a wide range of variation of the independent variables, and then averaged out. This average value may be considered as a universal constant and be used to calculate $\overline{u}_m$ from (6-6) with $\overline{u}_B = \left|\overline{u}\right|_{y=D}$.

The work could be substantially reduced by the use of a computer; otherwise the operation is not much easier than the more accurate one mentioned previously. Its main advantage over the latter is that it has to be performed only once. Because computer time was not available when the study reached this point, a simpler but less accurate method was used to calculate $\overline{u}_m$. This method is adequate at least in establishing an order of magnitude.

The method consisted of a numerical integration of equation (3-19) in which the two functions f_1 (y) and f_2 (y), as we have seen in section 3, were of the form

$$f_1 (y) = .5e^{-\frac{133y}{a\beta D}}$$

$$f_2 (y) = .5 (\beta y)^{2/3}$$

One may observe that for given values of the parameters $a\beta$ and βD both $f_1(y)$ and $f_2(y)$ are functions of the ratio y/D only. As a characteristic set of these parameters, the mean values from the twenty-seven runs mentioned in a previous section were used. The ratio y/D was made to vary in increments of .4, beginning with y/D = .2 and ending with y/D = 1.8. The angle ωt was varying between 0 and 2π in increments of $\pi/12$ beginning with $\omega t = \pi/24$ and ending with $\omega t = 47\pi/24$. Typical velocity profiles constructed this way and corresponding to different values of the angle ωt are shown of Figure 10. The value of the constant A_5 obtained from the integration of these profiles was found to be equal to .618. Equation (6-4) can be written now as

$$c_o = .618 \frac{q_B}{2D \left|\overline{u}\right|_{y=D}} \tag{6-8}$$

This is the final result of our study. When c_o from (6-8) is introduced in equation (6-2) the desired value of Q_B is obtained provided, of course, that the velocity distribution U(y) of the secondary flow is known. In actual cases it is hard to predict the character of such secondary motions because in general they depend upon local conditions. It is, however, possible to deduce reasonable estimates of their behavior by statistical methods based on long time records. Since the wave characteristics themselves are usually evaluated by similar methods it becomes evident that the error in the estimated values of the parameters entering our problem as independent variables has a bivariate distribution. This, of course, reduces the accuracy of the results. There is at least one case, though, in which the steady mean motion U(y) in a body of water is induced by the surface wave itself. This particular case will be described in the following section.

7. A PARTICULAR CASE OF THE SECONDARY DRIFT

This type of a second-order drift which is generated by the surface waves in a direction parallel to the wave propagation has been originally studied by Stokes (1851) and more recently by numerous investigators. The works of Bagnold (1947) and Longuet-Higgins(1953) wil be singled out because in addition to their scientific merits offer a rather convenient application to the problem at hand. Associated with this secondary motion is a steady mean water particle velocity which usually is called "mass-transport velocity". Stokes' expression for this velocity, which we will denote by $\overline{U}$, is of the form

$$\overline{U} = \frac{a^2 \omega k \cosh 2k(y_1+d)}{2 \sinh^2 kd} - \frac{a^2 \omega \coth kd}{2d} \qquad (7\text{-}1)$$

where y_1 is measured from the mean free surface negative downwards. The only necessary and sufficient condition to be satisfied for the derivation of this expression is that the flow is irrotational. Longuet-Higgins pointed out that the requirement of small amplitude surface wave, as was suggested by Stokes, is not necessary. According to Stokes' theory, the velocity near the bottom is negative (in opposite direction to the wave propagation) and for typical values of the product kd it increases with distance from the bottom attaining its maximum positive value at the free surface. Bagnold (1947) on the other hand has shown experimentally that the actual behavior is quite different. He observed a strong forward velocity near the bottom and a weaker backward velocity at higher levels. This confirmed the belief that Stokes' theory was not accurate, as it has been evidenced from older experiments in which forward velocities were observed both near the bottom and the free surface and backward motion in the interior. A more reliable theory which confirms the experimental results has been developed by Longuet-Higgins (1953). The two fundamental assumptions are that the mean motion is periodic in time and that it can be expressed as a perturbation of a state of rest. The theory recognizes the existence of three distinct regions, the first near the free surface, the second in the interior and the third near the fixed boundary. Associated with each of these regions is a particular mode of secondary motion.

In the present case our interest is concentrated on the boundary layer near the bottom only. Omitting the lengthy procedure of Longuet-Higgins' rigorous derivation it will suffice to present the final results for the case of progressive waves in water of uniform depth. The first-order motion of the fluid is described by means of the irrotational theory so that the horizontal component of the velocity at the boundary may be expressed, as we have seen in section 3, as

$$u_o = a\omega \sin \omega t \tag{7-2}$$

The "mass-transport velocity" can then be obtained from the second-order approximation.

$$\overline{U} = \frac{a^2 \omega k}{4 \sinh^2 kd} f(\mu) \tag{7-3}$$

where

$$f(\mu) = 5 - 8 e^{-\mu} \cos \mu + 3 e^{-2\mu} \tag{7-4}$$

These are Longuet-Higgins' equations (254) and (253) respectively.

Consistent with our notation

$$\mu = \beta y$$

So that the expression for $\overline{U}$ may be written as

$$\overline{U} = \frac{a^2 \omega k}{4 \sinh^2 kd} \left\{ 5 - 8 e^{-\beta y} \cos \beta y + 3 e^{-2\beta y} \right\} \tag{7-5}$$

where y_1 measures the distance upwards from the effective bed. It is evident therefore that the rate of sediment transport of a certain size can be determined by combining the "sediment-transport equation" (6-2) and the "mass-transport velocity equation" (7-5). A given set of surface wave characteristics, temperature and depth of water and grain size of uniform bed material is sufficient for the calculation of this rate.

8. SUMMARY OF RESULTS

The independent variables entering the problem of the sediment transport by wave action are the following:

The amplitude H, the length λ and the angular velocity ω of the surface wave.

The depth d and the temperature T^o of the water (and consequently its kinematic viscosity ν).

The grain diameter D and the density ρ_s of the granular material forming the bed which is assumed to be uniform.

A method has been described in the preceding sections of this study which can be used to predict the rate of sediment transport associated with any given set of values of the independent variables. The procedure to be followed consists of the following seven steps.

a. The necessary condition that has to be satisfied for the method to be applicable is that the flow within the boundary layer caused by the surface wave is unstable. To test whether or not this condition is satisfied in a particular case the graph in Figure 1 may be used. The roughness size ε can be taken equal to D while the velocity u_o is calculated from

$$u_o = a\omega$$

where $$a = \frac{H}{2} \text{ cosech } kd = \frac{H}{2} \text{ cosech } \frac{2\pi d}{\lambda}$$

The point with coordinates D and u_o/ν is plotted in Figure 1 and according to its position relative to the experimental curve for the three-dimensional roughness, a prediction can be made about the character of the flow in the boundary layer.

b. Assuming that the test in step a proved that the condition of instability is satisfied, we proceed with the determination of the flow field in the boundary layer. It has been shown in section 3 that the mean unsteady velocity in the layer is given by the expression

$$\left|\frac{\bar{u}}{u_o}\right| = \left[1 + f_1^2(y) - 2f_1(y) \cos f_2 (y)\right]^{\frac{1}{2}} \sin(\omega t + \theta)$$

where $$\theta = \tan^{-1} \frac{f_1(y) \sin f_2 (y)}{1-f_1(y) \cos f_2(y)}$$

It was also shown that

$$f_1(y) = .5\, e^{-\frac{133y}{a\beta D}}$$

and

$$f_2(y) = .5 (\beta y)^{2/3}$$

where $\beta = \sqrt{\frac{\omega}{2\nu}}$

c. The value of the parameter $|\psi|$ is next calculated from equation (4-10); i.e.

$$|\psi| = \frac{\rho_s - \rho_f}{\rho_f} \frac{gD}{|\bar{u}|^2}$$

The velocity $\bar{u}$ is measured at a distance 0.35D from the theoretical bed, which means that

$$f_1(y) = .5\, e^{-\frac{45.55}{a\beta}}$$

and

$$f_2(y) = .25 (\beta D)^{2/3}$$

d. With $|\Psi|$ known the curve through the experimental points in Figure 8 can be used to obtain Φ. This curve is very closely approximated by the graphical representation of the equation

$$\frac{2}{\pi\sqrt{2\pi}} \int_0^{\pi/2} \int_{\frac{4|\Psi|}{\cos^2\xi} - 1.5}^{\infty} e^{-z^2/2} \, dz d\xi = \frac{30\Phi}{1+30\Phi}$$

e. The oscillatory sediment transport rate q_B in the bed layer which we may call the "oscillatory bed-load rate" is calculated from Φ through equation (4-26), i.e.,

$$\Phi = \frac{q_B}{\gamma_s} \sqrt{\frac{\rho_f}{g(\rho_s - \rho_f)}} \; D^{-3/2}$$

f. The concentration c_o of the solid particles that at any time happen to be in a state of motion within the bed layer (of thickness 2D) is

$$c_o = .618 \; \frac{q_B}{2D\ (\bar{u}_B)}$$

where $\bar{u}_B$ is the value of $\bar{u}$ from (3-24) at $y = D$.

g. The rate of sediment transport per unit width in the direction of any incidental secondary flow described by U(y) will be

$$Q_B = c_o \int_0^{2D} U(y)\, dy$$

In the general case U(y) is an additional independent variable which has to be determined by methods similar to the ones used in the determination of the surface wave characteristics. In the absence, however, of any such flow the only possible steady mean motion in the boundary layer is due to the surface wave itself which is called the "second-order drift flow". This steady flow within the boundary layer is in the direction of the wave propagation. The expression describing the velocity distribution associated with this flow is of the form

$$\bar{U} = \frac{a^2 \omega k}{2 \sinh^2 kd} \left\{ 5 - 8\, e^{-\beta y} \cos\beta y + 3\, e^{-2\beta y} \right\}$$

as proposed by Longuet-Higgins. This expression can be substituted for U(y) in the above integral to calculate Q_B. This is the sought-for value of the rate of transport of bed material in the direction of the wave propagation.

9. DISCUSSION OF THE RESULTS

The procedure outlined in the preceding section leads to a quantitative answer of our problem. In trying to apply the method to an actual case some caution is warranted with regard to its applicability and its accuracy. In developing the various relationships it was necessary to simplify the physical model by making certain assumptions. Unless these assumptions are still valid under the prototype conditions the method may need modification.

The basic assumption which practically governs the entire study was that the amplitude of the surface wave was small ($kH \ll 1$); this permitted the linearization of the equation of motion in the free stream. A second assumption, but of lesser importance was that the depth of the water d was rather large and uniform (small bottom slope.)

We wish to emphasize at this point that the proposed method is only meant to predict the motion of the bed material within the bed layer. Consequently it is not applicable in regions where the mixing is violent and where considerable amount of sediment is being carried in suspension as for instance near the surf zone and onshore.

Another point that calls for attention is that the criterion of instability of the boundary layer is not well defined. As stated in more detail in Appendix A, there is some uncertainty regarding the slope of the empirical curve serving as the criterion of instability in the region $\frac{u_o}{\nu} < 6 \times 10^4 \text{ ft}^{-1}$. (Figure 1). Until this uncertainty is removed, it will be advisable in this region to use the line with the steeper slope proposed by Manohar.

The accuracy of the results, as one would expect, depends entirely upon the quality of the approximations made in the course of developing the various relationships. We shall examine these approximations of the procedure step by step and make suggestions regarding their possible improvement.

a. This step deals only with the applicability of the method and was covered already in the discussion above.

b. The velocity distribution in the boundary layer was determined experimentally in a flume under conditions similar but not identical to those of the prototype. The error introduced this way cannot be very significant and in fact it can be absorbed by the effect of the approximation made in the subsequent step.

c. The value of Ψ was calculated from the velocity profile at a distance y = .35D. This distance was so chosen because it has been proven correct in dealing with steady mean flows; yet there is no proof that it holds true in the case of an oscillatory mean flow also. The accuracy of the method may be improved by determining the distance at which Ψ should be calculated more precisely.

d. The value of Φ was obtained from the curve in Figure 8. Its accuracy depends on how closely the theoretical curve approximates the actual one. The exact shape of this curve can be confirmed only after a considerable amount of reliable data from the field becomes available.

e. No approximation was involved in this step.

f. The determination of the constant in equation (6-8) from the experiment with an oscillating bottom is not quite accurate. The error due to the value of this constant is not serious however, and can easily be reduced as more information from field measurement is obtained.

g. In order to determine the sediment rate in the direction of the flow the concentration was multiplied by the average discharge in the bed layer. This of course is only an approximation of the real mechanism. The concentration in the layer actually remains constant in a statistical sense only. The individual grains are continuously picked up and dropped by the flow. Therefore the solid particles and the surrounding fluid are subjected to random accelerations and decelerations. The effect of the resulting inertia forces was assumed to be reflected by the value of the empirical constant. However the accuracy of the method may be improved considerably by refining the analysis of the force field in the bed layer.

10. CONCLUSIONS

Surface waves are responsible for the formation of an oscillatory boundary layer near the ocean floor. For a certain range of values of the wave parameters the laminar boundary layer becomes unstable. In this case solid particles of bed material are brought to a state of incipient equilibrium, primarily as the result of the instantaneous hydrodynamic lift. The concentration of solid material, in a thin layer adjacent to the bed (the bed layer) which at any instance is thus made free to move can be determined by the method developed here. The rate of the transport of such material in the direction of a secondary flow can be calculated by multiplying this concentration with the average discharge due to the secondary flow in the bed layer. The accuracy of the method should be tested against actual field measurements.

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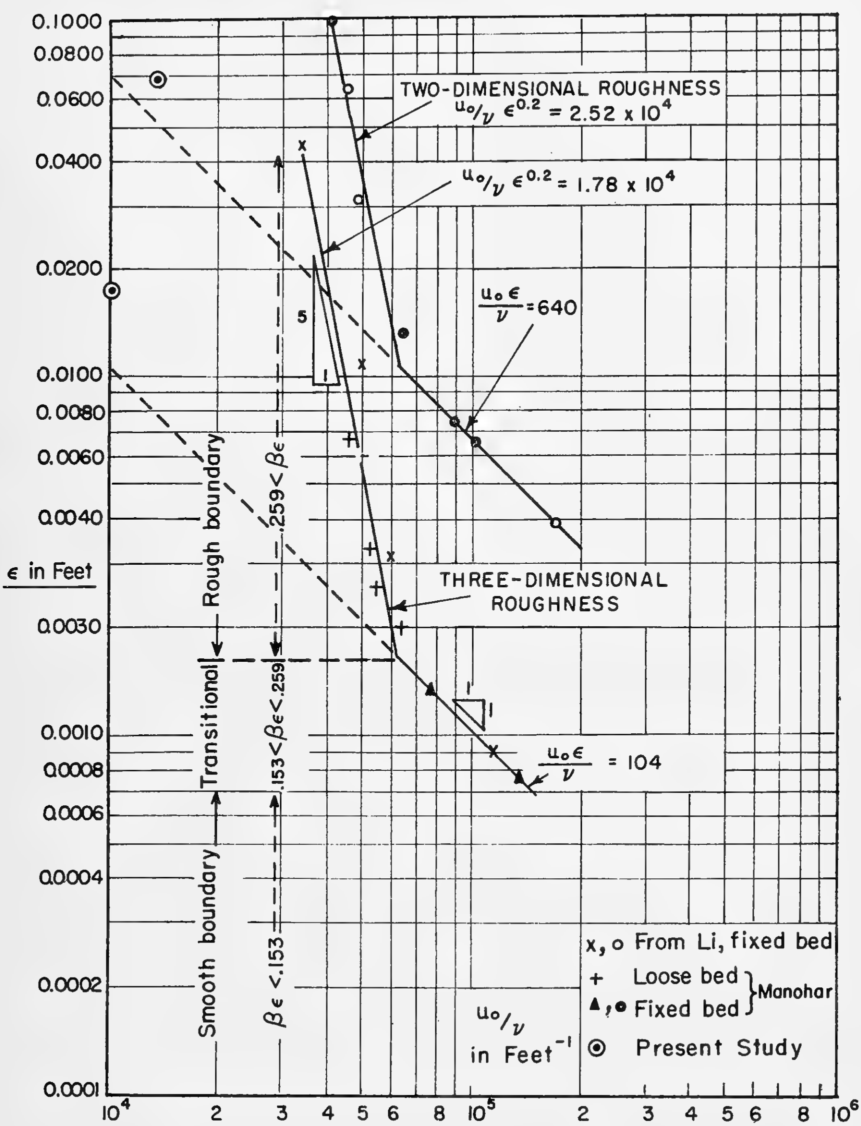


FIGURE I - CRITERION FOR TRANSITION FROM LAMINAR TO TURBULENT FLOW WITH OSCILLATORY MOTION (All grain sizes)

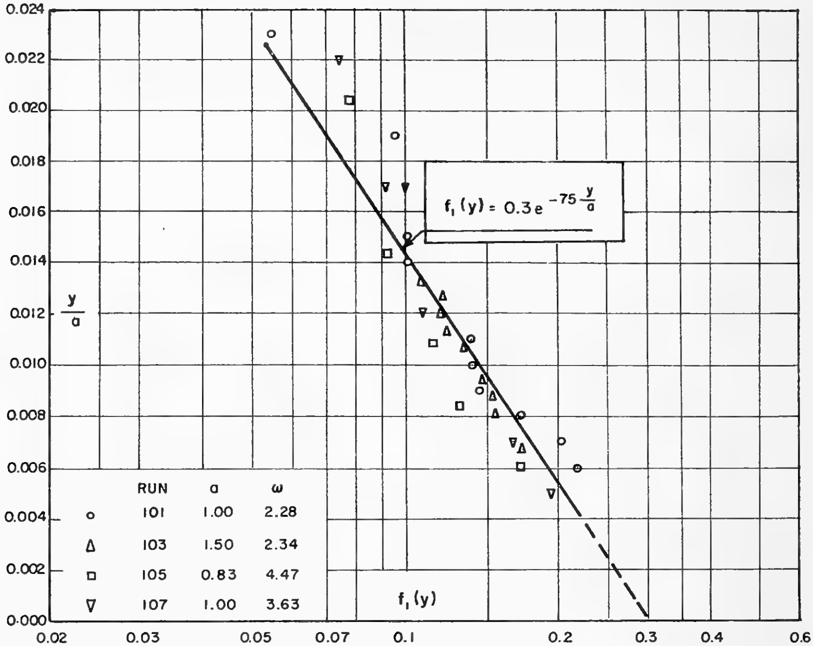


FIGURE 2 - SMOOTH PLATE- MEASURED VELOCITY DISTRIBUTION $f_1(y)$

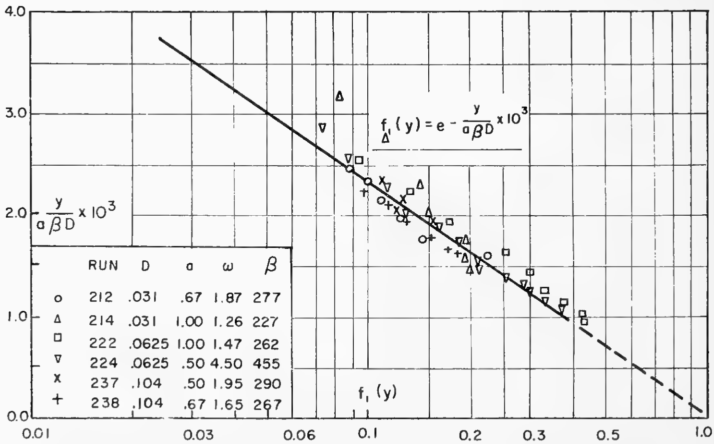


FIGURE 3 - TWO-DIMENSIONAL ROUGHNESS MEASURED VELOCITY DISTRIBUTION $f_1(y)$

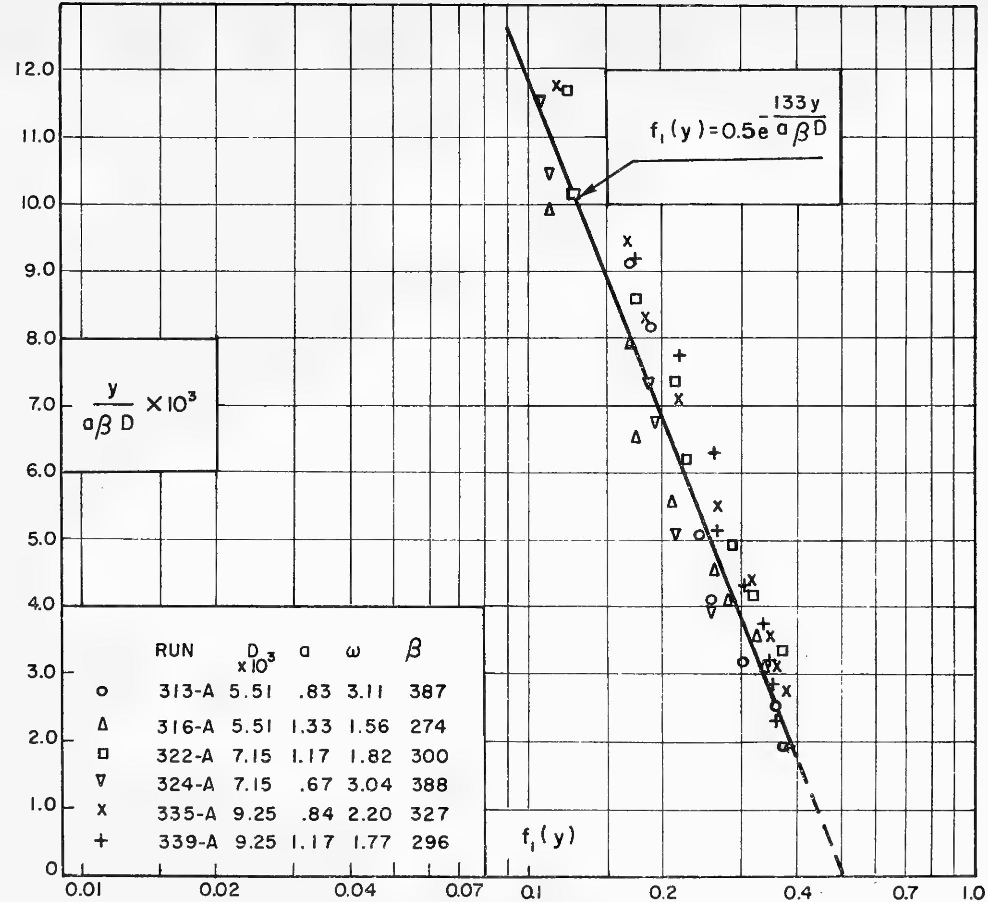


FIGURE 4 - THREE-DIMENSIONAL ROUGHNESS-MEASURED VELOCITY DISTRIBUTION $f_1(y)$

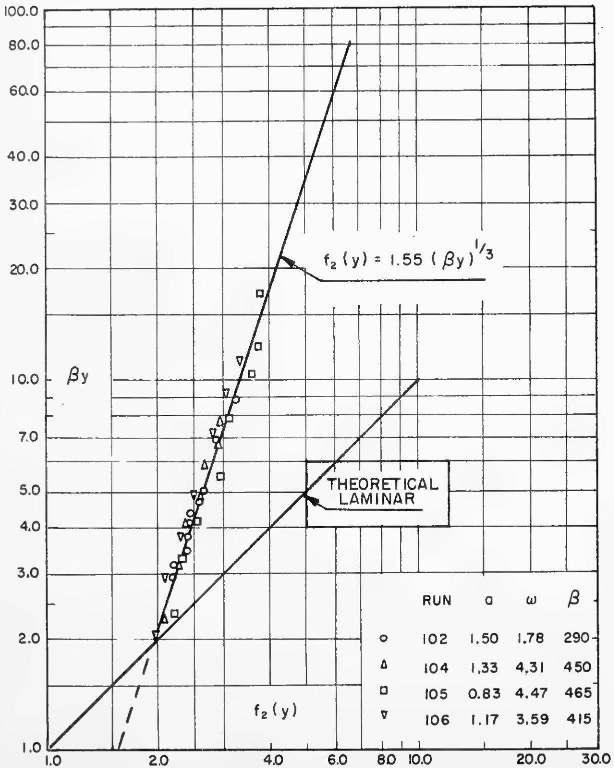


FIGURE 5 - SMOOTH PLATE
MEASURED PHASE SHIFT $f_2(y)$

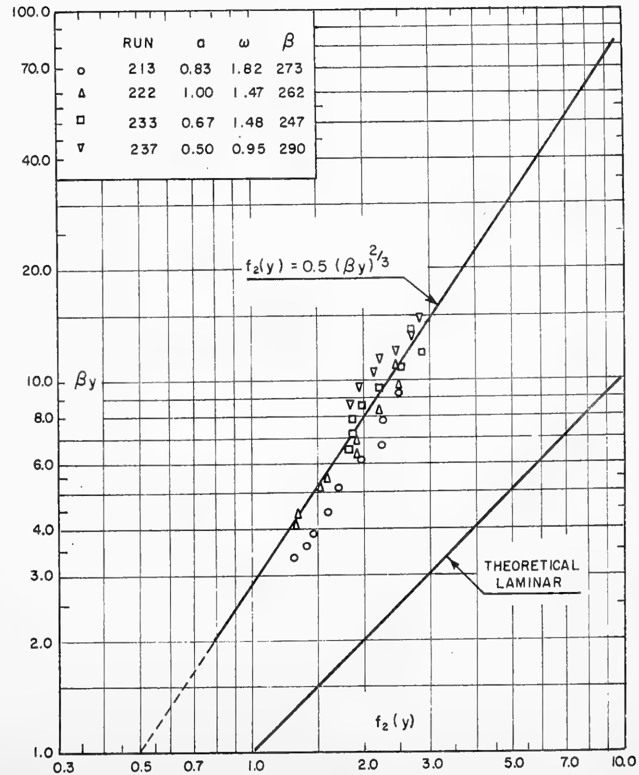


FIGURE 6 - TWO-DIMENSIONAL ROUGHNESS
MEASURED PHASE SHIFT $f_2(y)$

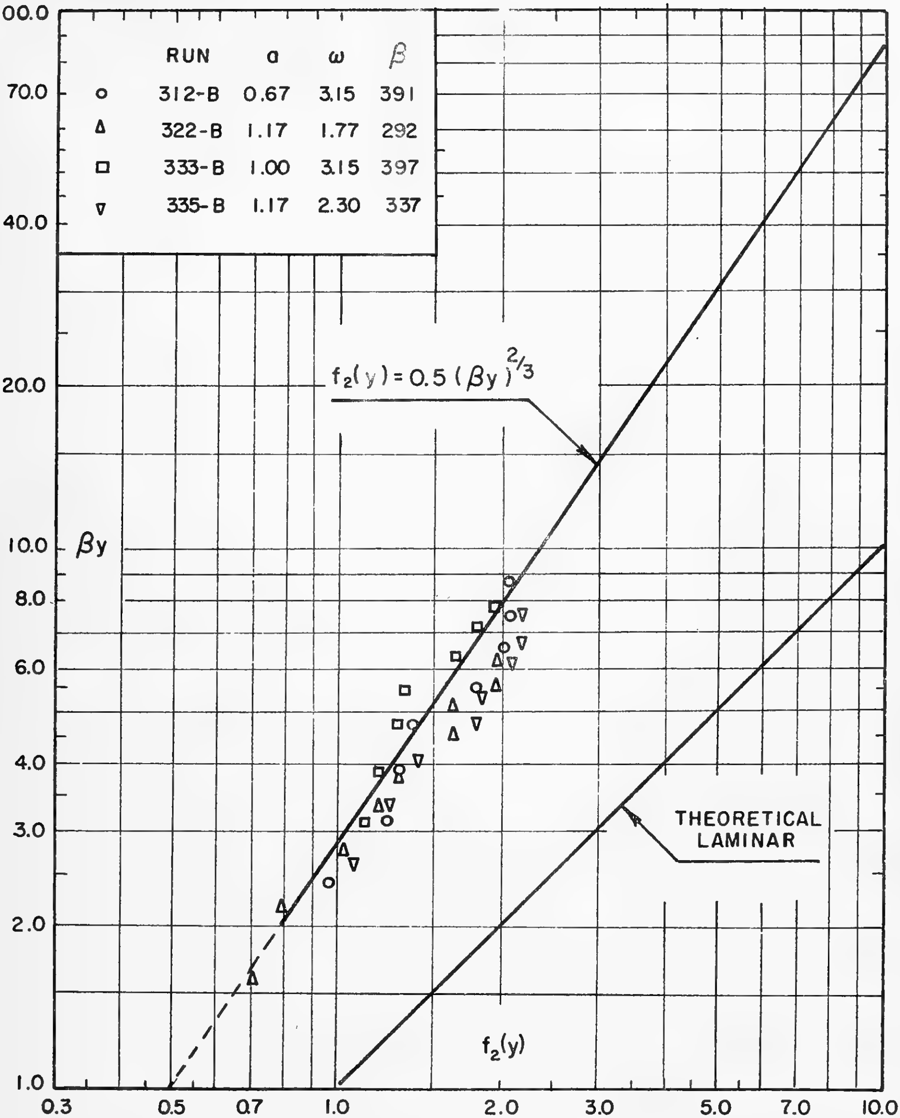


FIGURE 7 - THREE-DIMENSIONAL ROUGHNESS MEASURED PHASE SHIFT $f_2(y)$

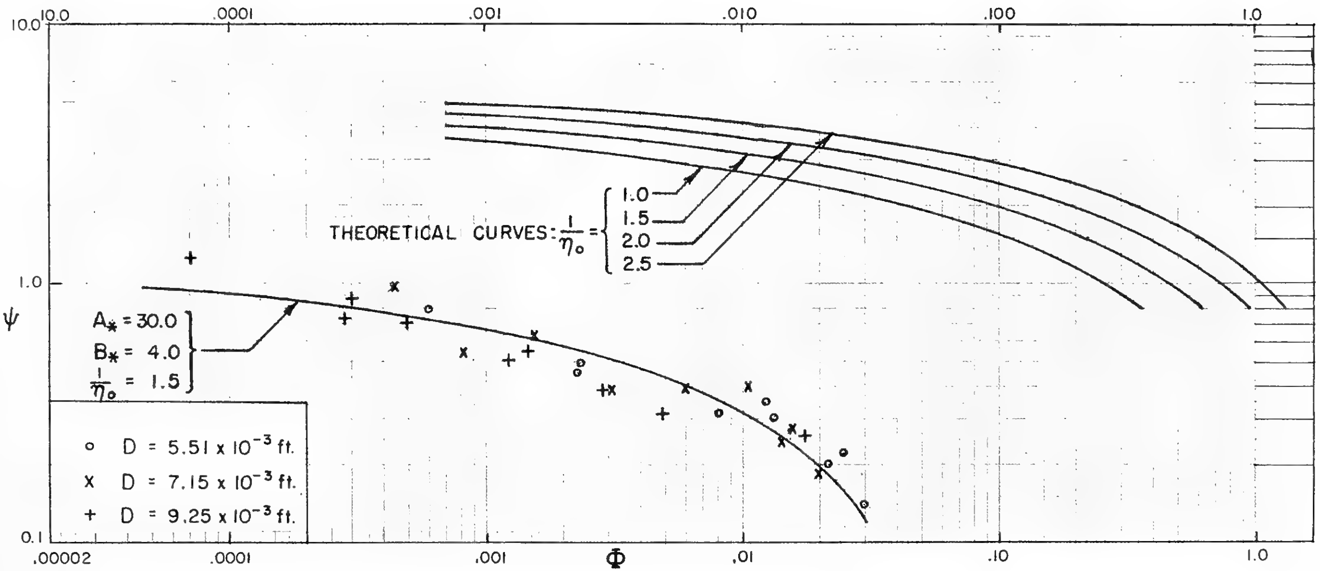


FIGURE 8 - DETERMINATION OF A_*, B_* & η_o

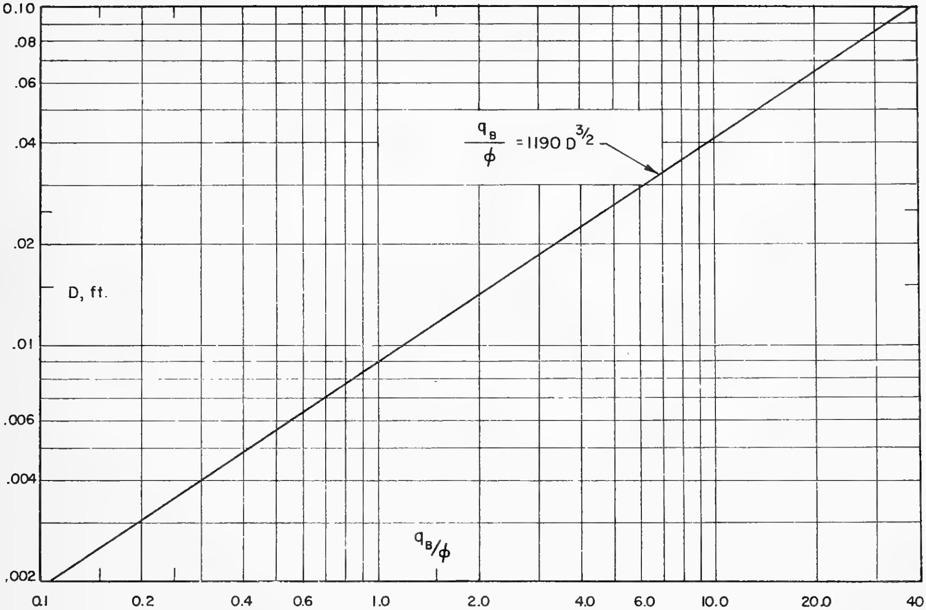


FIGURE 9 - q_B/ϕ VERSUS D CURVE FOR $\frac{\rho_s - \rho_f}{\rho_f} = 1.63$

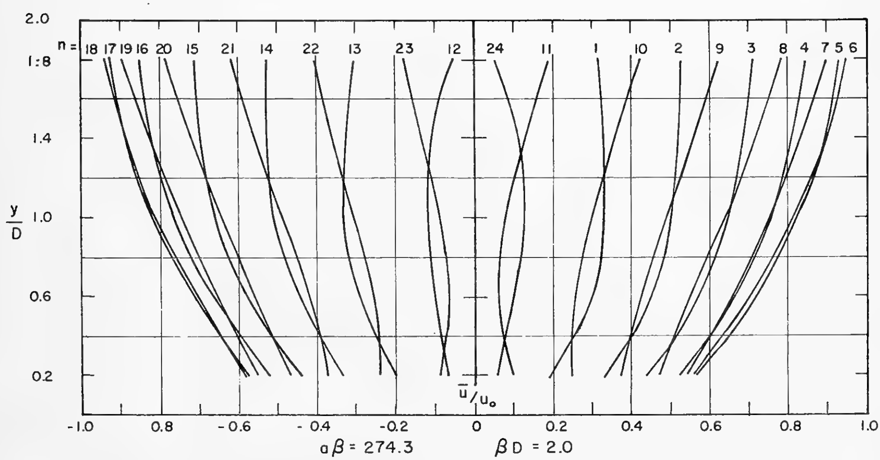


FIGURE 10 - TYPICAL UNSTEADY MEAN VELOCITY PROFILES AT $\omega t = (2n-1)\pi / 24$

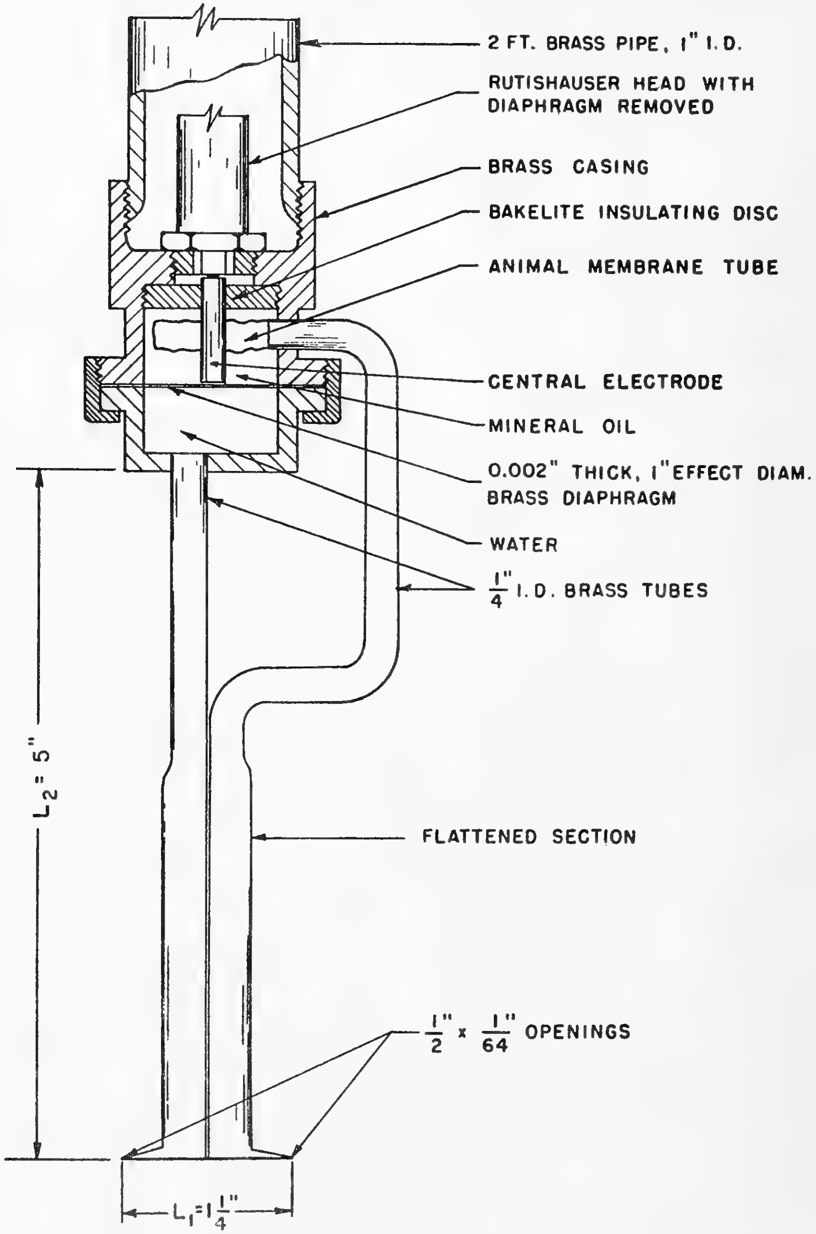


FIGURE 11 - SECTION OF VELOCITY-MEASURING INSTRUMENT

APPENDIX A

STABILITY OF AN OSCILLATORY BOUNDARY LAYER

It has been shown in section 3 that the wave motion at the outer edge of the boundary layer may be approximated by a simple harmonic motion. The stability of this layer and its transition into turbulence plays an important role in the problem associated with the transportation of sediment. In a laminar oscillating stream the predominant force induced by the flow on the particle is a tangential drag. When this force becomes larger than the force resisting motion the particle will begin to move. A criterion of initiation of movement then can be set as

$$\tau_c = K\ (\rho_s - \rho_f)\ g\ \tan\ \theta_1 \tag{A-1}$$

where θ_1 is the angle of repose of the particle. The shear stress τ at any distance from the wall may be determined from equation (3-15). It is evident that the maximum value of τ occurs at the wall so that

$$\tau_{max} = \mu \left.\frac{\partial u}{\partial y}\right|_{y=o} = \mu\beta u_o \tag{A-2}$$

Therefore the criterion of motion in a laminar boundary layer can be set as

$$\omega_c = \left[\frac{K\ (\rho_s - \rho_f)\ g\ D\ \tan\ \theta_1}{a\ \rho_f \nu^{1/2}}\right]^{2/3} \tag{A-3}$$

This is Manohar's (1955) equation (19).

In an unstable boundary layer on the other hand the predominant force, as we have seen in section 4, is the hydrodynamic lift which is associated both with the mean unsteady flow and the turbulent perturbations. The mechanism through which motion is induced is not only different in character for the two cases but also in the unstable case it is more intense by several orders of magnitude.

The purpose of Li's investigation (Li, Huon 1954) was to determine the factors and relationships governing the transition of an oscillatory laminar layer over smooth and rough beds. The mathematical model being so complex, a theoretical approach was ruled out from the outset. The effort therefore was concentrated to obtain empirical results experimentally. The study in the laboratory of the boundary layer created by a surface wave would require equipment of very large size. It was instead considered expedient to investigate the stability of the boundary layer near an oscillating wall in a body of water at rest. It is apparent of course that the patterns of flow on the prototype and in such a model are by no means identical but the reasoning was that the critical values of the governing parameters will be very closely approximated by each other. Following is a brief description of the apparatus and the procedure used in Li's study.

A glass walled flume 12 feet long, 1 foot wide and 3 feet high was used. About 4 inches above the bottom of the flume a plate, 78" x 11-1/2" was mounted on a series of rollers so that it was free to move in a plane parallel to the bottom. Through a series of steel cables and levers the plate was connected to a 1/2-HP AC motor which was mounted external of the flume. By means of an eccentric arm the rotating motion of the motor was converted into a reciprocating one which very nearly approximated simple harmonic motion. The motor was equipped with a speed reducer which allowed the period of the oscillation to vary from 1 to 150 cpm. By varying the eccentricity of the arm the amplitude of the linear motion of the plate could be varied from 2 to 18 inches. The amplitude and the frequency of oscillation in the experiments were made to vary within ranges so as to simulate prototype conditions associated with waves of 0.4 to 60 seconds period and 0.5 to 10 feet height in water about 60 feet deep.

During the testing the water depth was kept at about 2 feet. The shortness of the flume and comparatively shallow depth of water tended to cause the formation of a standing wave. In order to reduce this wave to a minimum a series of vertical baffles together with a set of heavy floats were used.

Three series of tests were made in this flume. The first, designated as the smooth series, was carried out with the surface of the plate waxed. The second, designated as the two-dimensional rough series, was carried out with the use of half-round wooden strips or steel rods as roughness. The third, designated as the three-dimensional rough series, was carried out with the use of sand or gravel as a roughness. In all cases each individual run was carried out with a uniform roughness. That is to say, all roughness elements for any one run were of the same size.

The procedure was to fix a particular roughness to the plate, choose an amplitude of oscillation, and then increase gradually the frequency. The type of the flow regime was determined visually by dropping potassium permanganate crystals from the water surface to the bottom. As the crystals dropped to the bottom they left a trail of dye. Once at the bottom they would oscillate with the motion of the platform. Continuing to dissolve the crystals would leave a clear back and forth trail as long as the flow was stable. The flow was defined as unstable when the trail left by the crystals would break down and mix with layers above. The recorded frequencies and amplitudes at the critical limit together with the size of roughness and the water viscosity were combined to form a critical Reynolds number. The results obtained in these tests as supplemented by Manohar may be summarized as follows:

According to its performance the boundary can be classified as hydraulically smooth, as rough and as a transition from smooth to rough.

The boundary behaves as hydraulically smooth even with two and three-dimensional roughness provided that the parameter β_ϵ where ϵ is the roughness diameter is smaller than a certain value; β of course is equal to $\sqrt{\frac{\omega}{2\nu}}$. The range of β_ϵ for the three classifications determined from the experiments is the following.

	Two-Dimensional Roughness	Three-Dimensional Roughness
Smooth boundary	$\beta_\epsilon < .678$	$\beta_\epsilon < .153$
Transition	$.678 < \beta_\epsilon < 1.77$	$.153 < \beta_\epsilon < .249$
Rough boundary	$1.77 < \beta_\epsilon$	$.249 < \beta_\epsilon$

(i) For the hydraulically smooth boundary it was found that the critical value of the Reynolds number defined as

$$N_R = \frac{\omega^{\frac{1}{2}} a}{\nu^{\frac{1}{2}}}$$

was constant and equal to 400.

(ii) In the transition region the Reynolds number was of the form

$$N_R = \frac{u_o \epsilon}{\nu}$$

with constant critical value for the two-dimensional and three-dimensional roughness; namely

$$N_{R_{cr}} = 640 \quad \text{two-dimensional}$$

$$N_{R_{cr}} = 104 \quad \text{three-dimensional}$$

(iii) In the rough region the governing parameter was not a Reynolds number but the dimensional quantity

$$\frac{u_o \epsilon^{0.2}}{\nu}$$

which was found to have the critical values of

$$\frac{u_o \epsilon^{0.2}}{\nu} = 2.52 \times 10^4 \qquad \text{two-dimensional}$$

$$\frac{u_o \epsilon^{0.2}}{\nu} = 1.78 \times 10^4 \qquad \text{three-dimensional}$$

The summary of the results as reported by Manohar are shown in Figure 1.* The explanation given by these two investigators to the rather singular behavior at the rough region was that the transition from laminar to turbulent is mainly due to the instability of the flow along the wake formed between the individual particles. The characteristic length then is the size of the wake which, although a function of the roughness, diameter, is not necessarily proportional to it. This explanation, although reasonable, is not quite convincing. The writer's reservations were confirmed by some observations made during a more recent phase of the investigation. These observations were made while measuring the phase shift on the vertical in the same experimental flume. The method used involved again the use of dye which this time was injected from above through a thin brass tube. It was observed that the flow was definitely unstable at a value of $\frac{u_o}{\nu}$ equal to 1.3×10^4 for a two-dimensional roughness with $\epsilon = .0625'$. Moreover unstable flow conditions were observed while experimenting with three-dimensional roughness of $\epsilon = .017'$ when the value of $\frac{u_o}{\nu}$ was about equal to 10^4. These points when plotted in Figure 1 demonstrate that the constant value of the critical Reynolds number as defined for the transition regime may well be extended to cover the rough case too. This implies that in Li's and Manohar's experiments the flow in this region was already unstable before it could be established as such from observations. There is a possibility that the crystals and the high-density dye solution were confined within the layer of the dead water under the theoretical bed. The fluid in this layer oscillated back and forth with the plate and as long as the velocities were small it never had a chance to spill over the crest of the roughness elements and mix with the flow above. So even if the flow were unstable there was no way of detecting it. On the other hand, one may argue that the more recent observations do not describe the behavior of the real model either, because the observed premature transition might have been triggered by the tube itself. We want to make clear, however, that the tip of the tube was maintained at a level several millimeters above the bed at a distance where the amplitude of the fluid velocity was negligible while the first indication of instability was observed near the bed. In concluding this Appendix we would like to point out that this phase of the problem needs further investigation so that more reliable information will become available.

* Figure number refers to figures following main text.

APPENDIX B

VELOCITY DISTRIBUTION IN OSCILLATORY BOUNDARY LAYER

This section contains the description of the experimental method and the results obtained in the phase of the study which had as its objective the derivation of an expression describing the velocity distribution in an oscillatory boundary layer. The linearized equation of motion from classical hydrodynamics may be used to describe the flow field in the interior. Within the boundary layer as long as the flow is laminar the velocity field can be described to the first approximation by an expression of the form

$$u = u_o \left\{ \sin\omega t - e^{-\beta y} \sin(\omega t - \beta y) \right\} \qquad \text{(B-1)}$$

where u_o is defined as the amplitude of the velocity at the wall from irrotational theory.

When the flow in the boundary layer becomes unstable the only way the field can be described is by empirical methods. The experimental work involved may be extremely difficult; however, it becomes much simpler if one postulated that the unsteady mean flow in this case can be described by an expression of the form

$$\bar{u} = u_o \left\{ \sin\omega t - f_1(y) \sin\left[\omega t - f_2(y)\right] \right\} \qquad \text{(B-2)}$$

in analogy with the stable solution.

Equation (B-2) may be written as

$$\frac{\bar{u}}{u_o} = \left| \frac{\bar{u}_1}{u_o} \right| \sin\omega t - \left| \frac{\bar{u}_2}{u_o} \right| \sin\left[\omega t - f_2(y)\right] \qquad \text{(B-3)}$$

where $\left| \frac{\bar{u}_1}{u_o} \right| = 1$ and $\left| \frac{\bar{u}_2}{u_o} \right| = f_1(y)$

The purpose therefore of the tests was to determine the functions $f_1(y)$ and $f_2(y)$. The structure of equation (B-2) suggested the possibility of determining these functions by the convenient method of measuring the velocity amplitude and the phase shift in a flume where the bed oscillates with an harmonic motion of the form $u_1 = u_o \sin\omega t$ relative to the water at rest.

The flume and the rest of the equipment used in these tests has been described in detail elsewhere, Li (1954), Manohar (1955), Kalkanis (1957). In Appendix A we gave a brief description of the main apparatus and in this section we will give the necessary information regarding the operation of the additional equipment used in the tests.

The two functions $f_1(y)$ and $f_2(y)$ can be determined independently and indeed in some cases they were so. However, the device used in the measurement of the velocity amplitude at some distance from the oscillating wall was simultaneously measuring the phase angle of the velocity at this level relative to the wall. In other words the same record could be used to determine both $f_1(y)$ and $f_2(y)$. This instrument is essentially a modified pitot tube shown diagrammatically in Figure 11. The basic principle of its operation is that the differential pressure at the two tips causes the diaphragm to deflect; in doing so it modulates a high-frequency signal by altering the capacitance of a Rutishauser pressure pickup. The modulated signal is rectified by a discriminator into a voltage which is proportional to the differential pressure at the tips of the instrument and consequently a function of the incidental velocity. Therefore the excursions of the needle of an oscillograph which is activated by this voltage will give a measure of the local velocity. The instrument was calibrated dynamically by oscillating it with a prescribed simple harmonic motion in water at rest and correlating the amplitude of the needle excursions with the amplitude of the velocity of oscillation.

The tests were made on a smooth plate of a high-gloss finish as well as on rough plates with two and three-dimensional roughness fixed on them. The vertical distances recorded during the tests were measured from the crest of the roughness elements. These values were subsequently corrected to account for the distance between this level and the theoretical bed. The correction in all cases was equal to 0.2D. The values of the function $f_1(y)$ resulting from the analysis of the experimental records are listed on Tables I, II, and III. The graphical representation of the results is shown in Figures 2, 3 and 4. The exponential dependence of the velocity on the distance seems to describe the data better than any other simple relationship. A small number of points only, obtained in some characteristic runs, were plotted in order to make the graphs readable. However, for the selection of the most representative relationship the complete data was used.

The experiments with smooth wall indicated that the proper characteristic length for the normalization of the argument of $f_1(y)$ should be the amplitude of the oscillation. In a previous report (Kalkanis 1957), it was suggested that the parameter $\frac{1}{\beta}$ should be used for this purpose in analogy to the laminar case. The original work on which this suggestion was based consisted of three runs only reported here as runs 111, 112, 113, a very inadequate number. As part of the present investigation, which has as its main objective the study of the rough case, four more runs were made with a smooth wall (114, 115, 116, 117) in order to test the reliability of the instrument. The second set of measurements was made more than a year after the first and considering the fact that the equipment in the meantime underwent certain modifications, the agreement of the results between the two sets was remarkable. In any case the adoption of the amplitude as the characteristic length was strongly supported by the experiments with a rough wall performed more recently.

By adopting a similar purely empirical approach the analysis of the data obtained in these tests revealed that the more suitable length scale to be used with $f_1(y)$ was the parameter $a\beta D$. The forms that most closely approximated the experimental data were

$$f_1(y) = e^{-\frac{y}{a\beta D} \times 10^3} \qquad \text{for two-dimensional roughness}$$

and

$$f_1(y) = .5 \; e^{-\frac{133y}{a\beta D}} \qquad \text{for three-dimensional roughness.}$$

It is understood that we do not claim that these expressions describe the real physical model; on the other hand we believe that the flow field described by these equations cannot be very much different than the actual one since a considerable amount of data was used in their derivation. The implication is that the calculated values by means of these functions will be as close to reality as the form of equation (3-15) permits.

In the determination of the function $f_2(y)$ the characteristic length for all conditions of roughness seemed to be the parameter $\frac{1}{\beta}$ of the laminar case. The phase angle seemed to increase as a power function with distance. The empirical relationships obtained as the results of these tests are the following:

$$f_2(y) = 1.55 \; (\beta y)^{1/3} \qquad \text{Smooth wall}$$

and

$$f_2(y) = .5 \; (\beta y)^{2/3} \qquad \text{Two or three-dimensional rough wall.}$$

The phase angle was measured by two different methods; in the first it was obtained directly from the velocity measurement record. The time at which the flow was changing direction was registered on the time scale of the velocity record while the time of change in direction of the plate motion was recorded by another needle of the oscillograph. The time interval between these two changes of direction after having been averaged out for a number of periods gave the desired phase angle. In the second method the record of the change of flow direction was obtained visually; through a thin brass tube a dye streak was introduced into the flow; the change of direction of the streak at any level was observed and it was recorded by means of a push button arrangement; the motion of the plate was recorded as above. This method is more accurate because the equipment it employs is very simple; but on the other hand it is subject more to personal bias. The two methods were checked against each other in tests with smooth wall and the good agreement of the results was a proof that either one may be used with equal confidence. The first method has been used to measure the phase shift in the smooth case and with a wall having a two-dimensional roughness, while with sand as roughness the phase shift was measured by the dye method.

With both $f_1(y)$ and $f_2(y)$ determined equation (B-3) may now be written as

$$\frac{\bar{u}}{u_o} = \left[1 + f_1^{\,2}(y) - 2f_1(y) \cos f_2(y)\right]^{1/2} \sin(\omega t-\theta) \qquad \text{(B-5)}$$

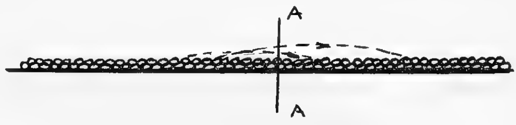
where $\theta = \tan^{-1} \left[\dfrac{f_1(y) \sin f_2(y)}{1 - f_1(y) \cos f_2(y)} \right]$

APPENDIX C

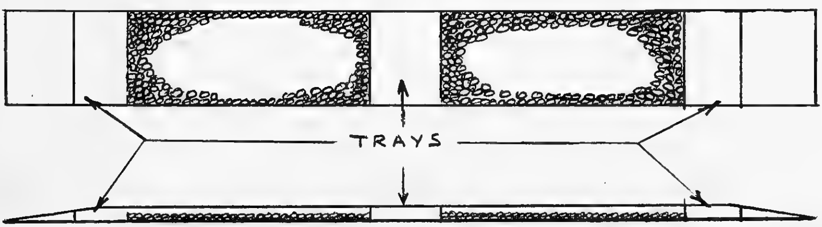
DETERMINATION OF A_*, B_* AND η_o

The method used for the determination of A_*, B_* and η_o has been outlined in section 5 of the text. What we intend to do here is to describe the experimental procedure used in connection with this method.

The same flume which has already been described in previous sections was used, but with some slight modifications. The method called for actual measurement of the quantity q_B. By definition q_B is the rate at which sediment near the bed crosses a section of unit width positioned perpendicular to the wave propagation.



Our experimental procedure was based on the proposition that the flow field and consequently the magnitude of q_B when the bed is fixed and the boundary-layer flow is caused by a surface wave are the same as when the bed is oscillating with a simple harmonic motion while the water surface is at rest. The validity of this argument will be discussed later but for the time being let us assume that it is correct. The oscillating plate had to be modified so that measurement of q_B could be made possible. The plan view and longitudinal section of the modified plate is shown in the sketch below



The procedure is self-explanatory. The space between the trays was filled with loose sand of a given size. The plate then was set into motion and after the completion of a number of periods it was stopped, and the amount of sand collected in the middle tray was removed and measured. A wire screen was placed on top of the trays so as to be flush with the rest of the sandy bed to achieve uniform roughness conditions. Three sizes of sand were used each with three different amplitudes of oscillation. Thus 27 runs were made altogether. The size range used was

$$\#14 < D < \#10 \text{ mesh}$$

$$\#10 < D < \#8 \text{ mesh}$$

$$\#8 < D < \#6 \text{ mesh}$$

The average value of D for each of these ranges were 5.51×10^{-3} feet, 7.15×10^{-3} feet and 9.25×10^{-3} feet, respectively. As a matter of fact, these were exactly the same sizes used in the measurement of the velocity and phase shift distribution. The size of the sample in each run was measured volumetrically. No distinction was made as to whether the material in the tray came over the right or left edge. Each run was repeated a number of times and the measured quantities were averaged out. The bulk volume of the sample was converted into dry weight by multiplication through a coefficient determined experimentally.

Since the width of the flume was equal to 1 foot, the dry weight of the sample when divided by the period and the number of oscillations completed during the testing time gave the value of q_B. Next equation (4-26) was used to determine Φ. This equation is of the form

$$\Phi = \frac{q_B}{\gamma_s} \sqrt{\frac{\rho_f}{g(\rho_s-\rho_f)}} \; D^{-3/2}$$

For given values of $\frac{\rho_s}{\rho_f}$ the ratio $\frac{q_B}{\Phi}$ is proportional to $D^{3/2}$. Herein $\frac{\rho_s}{\rho_f}$ was equal to 2.63 which means that $\frac{q_B}{\Phi} = 1190\ D^{3/2}$. The graphical representation of this equation shown in Figure 9 was used to calculate Φ from measured values of q_B.

On the other hand Ψ was calculated from equation (4-10) which is

$$\Psi = \frac{\rho_s-\rho_f}{\rho_f} \; \frac{Dg}{\bar{u}^2}$$

In fact, the amplitude $|\bar{u}|^2$ was used instead of $\bar{u}^2$ because what we were really interested in was the value of $|\bar{\Psi}|$. A pair of values of the parameters Φ and Ψ were thus obtained for each run. These values were next plotted against each other as shown in Figure 8. Although there is considerable scatter, it seems that the experimental points can be reasonably represented by the empirical curve drawn through them. We may claim therefore that this curve expresses the functional relationship between Φ and Ψ. On the same figure a family of theoretical curves was drawn as it has been explained in section 5 of the text. These curves are the graphical representation of equation (4-27) in which the values A_* and B_* were equal to unity. Each curve corresponds to a particular value of $\frac{1}{\eta_o}$. One may observe that the theoretical curve with $\frac{1}{\eta_o} = 1.5$ is very similar to the experimental one but offset both in the vertical and the horizontal. A parallel translation of the latter by a factor 30 along the horizontal and by a factor 4 along the vertical makes it practically to coincide with the former. This means that the values of Φ and Ψ, satisfying equation (4-27) with $A_* = 30$, $B_* = 4$ and $\frac{1}{\eta_o} = 1.5$ under any set of experimental conditions will be very close to the values of the same parameters calculated directly from measured quantities. In conclusion we set forth the claim that a theoretical curve constructed from equation (4-27) with the constants as determined here could be used to describe the relationship between the two functions Φ and Ψ.

Several reviewers in the past and more recently P. Lhermitte (1961) are rather critical of Li's and Manohar's experiments. More specifically they express some reservations about the applicability of the results in an actual case on the ground that in the experimental flume an inertia force is induced on the particle which in the prototype is absent. Their contention is that this force is significant and consequently cannot be ignored. Since the same flume was used in the present study one may anticipate similar criticism especially in connection with the determination of q_B from direct measurement.

We believe, however, that the criticism is not fully justified because this inertia force under the average experimental conditions is indeed small compared to other forces acting simultaneously on the particle. The maximum value of the angular velocity in the set of runs with a = 1.25 feet was $\omega = 1.86$ rad/sec. corresponding to a maximum tangential acceleration of

$$a\omega^2 = 1.25 \times 1.86^2 = 4.32 \text{ ft/sec}^2$$

which is much smaller than g. The tangential force could have some effect in setting the particle into motion if it were in phase with the lift in which case the combined effect could not be ignored. Since the two forces are 90 degrees out of phase the instant one reaches its maximum value the other is practically equal to zero. Therefore it is justifiable to base the condition of equilibrium on the balance of the vertical forces whose absolute value is relatively large and ignore the effect of a much smaller horizontal component which is fully out of phase.

APPENDIX D

TABLES

TABLE I SMOOTH WALL

RUN: 101 — $a = 1.00$ ft, $\omega = 2.28$ rad/sec, $u_o = 2.28$ ft/sec; $T = 67^oF$, $\nu = 1.06 \times 10^{-5}$ ft^2/sec, $\beta = 328$ ft^{-1}

y ft x 10^3	$\frac{y}{a}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
6.0	6.0	.220	1.97	1.74
7.0	7.0	.205	2.30	2.02
8.0	8.0	.174	2.62	2.09
9.0	9.0	.142	2.95	2.31
11.0	11.0	.136	3.61	2.43
14.0	14.0	.102	4.59	2.60
15.0	15.0	.101	4.92	2.63
19.0	19.0	.096	6.23	2.91
23.0	23.0	.055	7.54	3.08

RUN: 102 — $a = 1.50$ ft, $\omega = 1.78$ rad/sec, $u_o = 2.67$ ft/sec; $T = 67^oF$, $\nu = 1.06 \times 10^{-5}$ ft^2/sec, $\beta = 290$ ft^{-1}

y ft x 10^3	$\frac{y}{a}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
10.0	6.7	.179	2.90	2.16
11.0	7.3	.163	3.19	2.18
12.0	8.0	.162	3.48	2.39
13.0	8.7	.143	3.77	2.39
14.0	9.3	.133	4.06	2.41
15.0	10.0	.132	4.35	2.43
16.0	10.7	.130	4.64	2.62
17.0	11.3	.126	4.93	2.65
18.0	12.0	.120	5.22	--
20.0	13.3	.111	5.80	--
23.0	--	--	6.67	2.89
31.0	--	--	8.99	3.27

TABLE I SMOOTH WALL

RUN: 103 — $a = 1.50$ ft, $\omega = 2.34$ rad/sec, $u_o = 3.51$ ft/sec; $T = 67^oF$, $\nu = 1.06 \times 10^{-5}$ ft^2/sec, $\beta = 333$ ft^{-1}

y ft x 10^3	$\frac{y}{a}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
10.0	6.7	.171	--	--
11.0	--	--	3.66	2.24
12.0	8.0	.150	--	--
13.0	8.7	.149	4.33	2.47
14.0	9.3	.143	--	--
15.0	--	--	5.00	2.65
16.0	10.7	.132	--	--
17.0	11.3	.121	--	--
18.0	12.0	.118	--	--
19.0	12.7	.117	6.33	2.75
20.0	13.3	.108	--	--
23.0	--	--	7.66	3.06
31.0	--	--	10.32	3.48
40.0	--	--	13.32	3.85

RUN: 104 — $a = 1.33$ ft, $\omega = 4.31$ rad/sec, $u_o = 5.73$ ft/sec; $T = 67^oF$, $\nu = 1.06 \times 10^{-5}$ ft^2/sec, $\beta = 450$ ft^{-1}

y ft x 10^3	$\frac{y}{a}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
5.0	3.7	.192	2.25	2.08
7.0	5.3	.161	3.15	2.22
9.0	6.7	.142	4.05	2.35
11.0	8.3	.136	4.95	2.60
13.0	9.8	.121	5.85	2.64
15.0	11.3	.112	6.75	2.76
17.0	12.8	.096	7.65	2.85
19.0	14.3	.091	--	--
21.0	15.8	.082	--	--

TABLE I SMOOTH WALL

RUN: 105

$a = 0.83$ ft
$\omega = 4.47$ rad/sec
$u_o = 3.71$ ft/sec

$T = 72^oF$
$\nu = 1.04 \times 10^{-5}$ ft^2/sec
$\beta = 465$ ft^{-1}

y ft x 10^3	$\frac{y}{a}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
5.0	6.0	.169	2.33	2.25
7.0	8.4	.129	3.26	2.32
9.0	10.8	.113	4.19	2.55
12.0	14.4	.092	5.58	2.87
17.0	20.4	.078	7.91	3.08
22.0	26.4	.067	10.23	3.57
27.0	32.4	.056	12.56	3.66
37.0	44.4	.040	17.20	3.79

RUN: 106

$a = 1.17$ ft
$\omega = 3.59$ rad/sec
$u_o = 4.20$ ft/sec

$T = 72^oF$
$\nu = 1.04 \times 10^{-5}$ ft^2/sec
$\beta = 415$ ft^{-1}

y ft x 10^3	$\frac{y}{a}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
5.0	4.3	.212	2.08	1.98
7.0	6.0	.185	2.91	2.16
9.0	7.7	.157	3.74	2.31
12.0	10.3	.126	4.98	2.50
17.0	14.5	.090	7.06	2.76
22.0	18.8	.081	9.13	3.03
22.0	18.8	.079	9.13	3.01
27.0	--	--	11.21	3.32

TABLE I SMOOTH WALL

RUN: 107

$a = 1.00$ ft
$\omega = 3.63$ rad/sec
$u_o = 3.63$ ft/sec

$T = 72^oF$
$\nu = 1.04 \times 10^{-5}$ ft^2/sec
$\beta = 416$ ft^{-1}

y ft x 10^3	$\frac{y}{a}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
5.0	5.0	.193	2.08	2.08
7.0	7.0	.164	2.91	2.09
9.0	9.0	.143	3.74	2.22
12.0	12.0	.109	4.99	2.51
17.0	17.0	.091	7.07	2.76
17.0	17.0	.099	7.07	2.81
22.0	22.0	.075	9.15	3.24
27.0	--	--	11.23	3.32

TABLE II TWO-DIMENSIONAL ROUGHNESS
(i) D = .031 ft.

RUN: 211

$a = 0.50$ ft, $\omega = 3.15$ rad/sec, $u_o = 1.58$ ft/sec

$T = 60^oF$, $\nu = 1.22 \times 10^{-5}$ ft^2/sec, $\beta = 360$ ft^{-1}, $a\beta D = 5.58$ ft

y_m^* ft x 10^3	y^{**} ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
5.0	11.2	2.00	.140	--	--
6.0	12.2	2.20	.122	4.41	1.51
7.0	13.2	2.38	.109	4.77	1.68
8.0	14.2	2.56	.099	5.31	1.73
10.0	16.2	2.92	.069	6.21	1.90
13.0	19.2	3.46	--	6.93	1.97
16.0	22.2	3.99	--	8.01	2.02
18.0	24.2	4.35	--	8.73	1.99
23.0	29.2	5.25	--	10.53	2.41
28.0	34.2	6.15	--	12.33	--

RUN: 212

$a = 0.67$ ft, $\omega = 1.87$ rad/sec, $u_o = 1.26$ ft/sec

$T = 60^oF$, $\nu = 1.22 \times 10^{-5}$ ft^2/sec, $\beta = 277$ ft^{-1}, $a\beta D = 5.75$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
3.0	9.2	1.60	.221	--	--
4.0	10.2	1.76	.142	--	--
5.0	11.2	1.95	.123	--	--
6.0	12.2	2.12	.108	3.38	1.32
7.0	13.2	2.30	.100	3.66	1.43
8.0	14.2	2.47	.088	3.93	1.52
10.0	16.2	2.82	--	4.49	1.60
13.0	19.2	3.34	--	5.32	1.72
16.0	22.2	3.86	--	6.15	1.93
18.0	24.2	4.21	--	6.70	1.95
23.0	29.2	5.08	--	8.09	2.17
28.0	34.2	5.95	--	9.47	2.13

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(i) D = .031 ft.

RUN: 213

$a = 0.83$ ft, $\omega = 1.82$ rad/sec, $u_o = 1.51$ ft/sec

$T = 60^oF$, $\nu = 1.22 \times 10^{-5}$ ft^2/sec, $\beta = 273$ ft^{-1}, $a\beta D = 7.02$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
4.0	10.2	1.45	.222	--	--
5.0	11.2	1.60	.183	--	--
6.0	12.2	1.74	.181	3.33	1.28
7.0	13.2	1.88	.170	3.60	1.38
8.0	14.2	2.02	.157	3.88	1.45
10.0	16.2	2.31	.125	4.42	1.59
13.0	19.2	2.74	.087	5.24	1.66
16.0	22.2	3.16	--	6.06	1.95
18.0	24.2	3.45	--	6.61	2.24
23.0	29.2	4.16	--	7.97	2.24
28.0	34.2	4.87	--	9.34	2.43

RUN: 214

$a = 1.00$ ft, $\omega = 1.26$ rad/sec, $u_o = 1.26$ ft/sec

$T = 60^oF$, $\nu = 1.22 \times 10^{-5}$ ft^2/sec, $\beta = 227$ ft^{-1}, $a\beta D = 7.04$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
4.0	10.2	1.45	.198	--	--
5.0	11.2	1.59	.193	--	--
6.0	12.2	1.73	.192	2.77	1.17
8.0	14.2	2.02	.148	3.22	1.32
10.0	16.2	2.30	.141	3.68	1.49
13.0	19.2	2.73	.110	4.36	1.62
16.0	22.2	3.15	.082	5.04	1.73
18.0	24.2	3.44	--	5.49	1.87
23.0	29.2	4.15	--	6.63	2.05
28.0	34.2	4.86	--	7.76	2.27
33.0	39.2	5.57	--	8.90	2.35

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(i) D = .031 ft.

RUN: 215 — a = 0.83 ft; ω = 1.28 rad/sec; u_o = 1.06 ft/sec; T = 61°F; ν = 1.205 x 10^{-5} ft^2/sec; β = 230 ft^{-1}; $a\beta D$ = 5.91 ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
4.0	10.2	1.73	.157	--	--
5.0	11.2	1.90	.134	--	--
6.0	12.2	2.06	.130	2.81	1.26
8.0	14.2	2.40	.098	3.27	1.23
10.0	16.2	2.74	.069	3.73	1.19
13.0	19.2	3.24	--	4.42	1.38
18.0	24.2	4.09	--	5.57	1.38
23.0	29.2	4.93	--	6.72	1.86
28.0	34.2	5.78	--	7.87	2.14

RUN: 216 — a = 0.67 ft; ω = 1.83 rad/sec; u_o = 1.23 ft/sec; T = 61°F; ν = 1.205 x 10^{-5} ft^2/sec; β = 276 ft^{-1}; $a\beta D$ = 5.74 ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
4.0	10.2	1.78	.132	--	--
6.0	12.2	2.12	.108	3.37	1.34
7.0	13.2	2.30	.107	3.64	1.43
8.0	14.2	2.47	.095	3.92	1.44
10.0	16.2	2.82	.072	4.47	1.53
13.0	19.2	3.34	--	5.30	1.72
16.0	22.2	3.86	--	6.13	--
18.0	24.2	4.21	--	6.68	1.94
23.0	29.2	5.08	--	8.06	--
28.0	34.2	5.95	--	9.44	2.46

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(ii) D = 0.0625 ft.

RUN: 221 — a = 1.00 ft; ω = 2.33 rad/sec; u_o = 2.33 ft/sec; T = 68.5 F; ν = 1.083 x 10^{-5} ft^2/sec; β = 330 ft^{-1}; $a\beta D$ = 20.63 ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
7.0	19.5	.95	.433	6.44	1.57
8.0	20.5	.99	.395	6.77	1.54
9.0	21.5	1.04	.369	7.10	1.58
11.0	23.5	1.14	.322	7.76	1.86
13.0	25.5	1.24	.309	8.42	1.89
16.0	28.5	1.38	.275	9.41	2.13
19.0	31.5	1.53	.232	10.40	2.11
23.0	35.5	1.72	.206	11.72	2.31
28.0	40.5	1.96	.153	13.37	2.53
33.0	45.5	2.21	.117	15.02	2.82

RUN: 222 — a = 1.00 ft; ω = 1.47 rad/sec; u_o = 1.47 ft/sec; T = 69°F; ν = 1.075 x 10^{-5} ft^2/sec; β = 262 ft^{-1}; $a\beta D$ = 16.38 ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
3.0	15.5	.95	.432	4.06	1.30
4.0	16.5	1.01	.421	4.32	1.35
6.0	18.5	1.13	.374	5.11	1.51
8.0	20.5	1.25	.326	5.37	1.57
11.0	23.5	1.43	.297	6.16	1.90
14.0	26.5	1.62	.252	6.94	1.84
19.0	31.5	1.92	.172	8.25	2.21
24.0	36.5	2.23	.131	9.56	2.44
29.0	41.5	2.53	.094	10.87	2.40

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(ii) D = 0.0625 ft.

RUN: 223

$a = 0.50$ ft, $\omega = 3.72$ rad/sec, $u_o = 1.86$ ft/sec

$T = 68^oF$, $\nu = 1.09 \times 10^{-5}$ ft^2/sec, $\beta = 414$ ft^{-1}, $a\beta D = 12.94$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
6.0	18.5	1.43	.278	7.66	2.08
7.0	19.5	1.51	.275	8.07	1.94
9.0	21.5	1.66	.241	8.90	2.03
11.0	23.5	1.82	.196	9.73	2.05
13.0	25.5	1.97	.194	10.56	2.42
14.0	26.5	2.05	.181	10.97	2.62
15.0	27.5	2.13	.129	11.39	2.65
17.0	29.5	2.28	.126	12.21	2.76
20.0	32.5	2.51	.128	13.46	2.95
23.0	35.5	2.74	.100	14.70	3.20
27.0	39.5	3.05	.059	16.35	3.38
32.0	44.5	3.44	--	18.42	3.55
42.0	54.5	4.21	--	22.56	3.20

RUN: 224

$a = 0.50$ ft, $\omega = 4.50$ rad/sec, $u_o = 2.25$ ft/sec

$T = 68^oF$, $\nu = 1.09 \times 10^{-5}$ ft^2/sec, $\beta = 455$ ft^{-1}, $a\beta D = 17.34$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
6.0	18.5	1.07	.368	8.42	2.05
7.0	19.5	1.13	.324	8.87	2.09
9.0	21.5	1.24	.294	9.78	2.32
10.0	22.5	1.30	.284	10.24	2.45
11.0	23.5	1.36	.252	10.69	2.68
13.0	25.5	1.47	.207	12.06	2.61
15.0	27.5	1.59	.204	12.51	2.73
17.0	29.5	1.70	.184	13.42	2.73
20.0	32.5	1.87	.160	14.79	3.06
23.0	35.5	2.05	.128	16.15	3.15
27.0	39.5	2.28	.112	17.97	3.62
32.0	44.5	2.57	.086	20.25	3.61
37.0	49.5	2.86	.072	22.52	3.62

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(ii) D = 0.0625 ft.

RUN: 225

$a = 0.75$ ft, $\omega = 3.43$ rad/sec, $u_o = 2.57$ ft/sec

$T = 68^oF$, $\nu = 1.09 \times 10^{-5}$ ft^2/sec, $\beta = 397$ ft^{-1}, $a\beta D = 18.60$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
6.0	18.5	.99	.380	7.34	1.80
7.0	19.5	1.05	.370	7.74	1.82
9.0	21.5	1.16	.340	8.54	1.87
10.0	22.5	1.21	.320	8.93	1.90
11.0	23.5	1.26	.269	9.33	2.06
12.0	24.5	1.32	.241	9.73	2.05
13.0	25.5	1.37	.231	10.12	2.13
15.0	27.5	1.48	.218	10.92	2.33
17.0	29.5	1.59	.170	11.71	2.38
20.0	32.5	1.75	.151	12.90	2.56
23.0	35.5	1.91	.119	14.09	2.74
27.0	39.5	2.12	.112	15.68	2.96
32.0	44.5	2.39	.079	17.67	3.19
37.0	49.5	2.66	--	19.65	3.24

RUN: 226

$a = 0.75$ ft, $\omega = 4.30$ rad/sec, $u_o = 3.23$ ft/sec

$T = 69^oF$, $\nu = 1.075 \times 10^{-5}$ ft^2/sec, $\beta = 448$ ft^{-1}, $a\beta D = 21.00$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$	$\frac{\bar{u}_2}{u_o}$	βy	ωt
11.0	23.5	1.12	.310	10.53	2.08
12.0	24.5	1.17	.274	10.98	2.16
15.0	27.5	1.31	.224	12.32	2.45
17.0	29.5	1.40	.217	13.22	2.55
22.0	34.5	1.64	.177	15.46	2.85
27.0	39.5	1.88	.134	17.70	2.87
32.0	44.5	2.12	.121	19.94	3.00

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(iii) D = .104 ft.

RUN: 231

a = 0.83 ft
ω = 2.05 rad/sec
u_o = 1.70 ft/sec

T = 63°F
ν = 1.17 x 10^{-5} ft^2/sec
β = 295 ft^{-1}
$a\beta D$ = 25.0 ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
5.5	26.5	1.06	.322	7.82	1.66
7.5	28.5	1.14	.291	8.41	1.78
10.5	31.5	1.26	.270	9.29	2.03
12.5	33.5	1.34	.259	9.88	2.17
17.5	38.5	1.54	.219	11.36	2.34
20.5	41.5	1.66	.200	12.24	2.36
23.5	44.5	1.78	.191	13.13	2.67
27.5	48.5	1.94	.157	14.31	2.56
30.5	51.5	2.06	.136	15.19	2.94
33.5	54.5	2.18	.131	16.08	2.90
37.5	58.5	2.34	.122	17.26	3.03
42.5	63.5	2:54	.096	18.73	3.02

RUN: 232

a = 0.83 ft
ω = 1.47 rad/sec
u_o = 1.22 ft/sec

T = 63°F
ν = 1.17 x 10^{-5} ft^2/sec
β = 250 ft^{-1}
$a\beta D$ = 21.2 ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
5.5	26.5	1.25	.315	6.63	1.59
7.5	28.5	1.34	.280	7.13	1.73
10.5	31.5	1.49	.249	7.88	1.79
13.5	34.5	1.63	.241	8.63	1.82
17.5	38.5	1.82	.177	9.63	2.16
20.5	41.5	1.96	.173	10.38	--
23.5	44.5	2.10	.171	11.13	--
27.5	48.5	2.29	.138	12.13	2.52
32.5	53.5	2.52	.107	13.38	2.68
37.5	58.5	2.76	.088	14.63	2.72
42.5	63.5	3.00	--	15.88	2.85

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(iii) D = .104 ft.

RUN: 233

a = 0.67 ft
ω = 1.48 rad/sec
u_o = 0.99 ft/sec

T = 63°F
ν = 1.17 x 10^{-5} ft^2/sec
β = 247 ft^{-1}
$a\beta D$ = 17.3 ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
5.5	26.5	1.53	.238	6.55	1.79
7.5	28.5	1.65	.190	7.04	1.81
10.5	31.5	1.82	.151	7.78	1.78
13.5	34.5	1.99	.122	8.52	1.97
17.5	38.5	2.23	.101	9.51	2.19
22.5	43.5	2.51	.097	10.74	2.44
27.5	48.5	2.80	.082	11.98	2.89
33.5	54.5	3.15	--	13.46	2.63

RUN: 234

a = 0.67 ft
ω = 2.87 rad/sec
u_o = 1.92 ft/sec

T = 63°F
ν = 1.17 x 10^{-5} ft^2/sec
β = 350 ft^{-1}
$a\beta D$ = 24.4 ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
4.5	25.5	1.04	.332	8.93	1.80
7.5	28.5	1.17	.265	9.98	2.04
10.5	31.5	1.29	.240	11.03	2.32
13.5	34.5	1.41	.202	12.08	2.55
17.5	38.5	1.58	.178	13.48	2.61
20.5	41.5	1.70	.163	14.53	2.72
23.5	44.5	1.83	.151	15.58	2.83
27.5	48.5	1.99	.122	16.98	2.96
32.5	53.5	2.19	.094	18.73	3.12

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(iii) D = .104 ft

RUN: 235

$a = 0.67$ ft, $\omega = 1.94$ rad/sec, $u_o = 1.30$ ft/sec

$T = 63^oF$, $\nu = 1.17 \times 10^{-5}$ ft^2/sec, $\beta = 298$ ft^{-1}, $a\beta D = 20.0$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
4.5	25.5	1.28	.330	7.60	1.77
7.5	28.5	1.43	.268	8.49	1.93
10.5	31.5	1.58	.237	9.39	2.12
13.5	34.5	1.73	.251	10.28	2.15
17.5	38.5	1.93	.156	11.47	2.21
20.5	41.5	2.08	.142	12.37	2.35
22.5	43.5	2.18	.106	12.96	2.53
27.5	48.5	2.43	.094	14.45	2.68

RUN: 236

$a = 0.67$ ft, $\omega = 3.14$ rad/sec, $u_o = 2.10$ ft/sec

$T = 63^oF$, $\nu = 1.17 \times 10^{-5}$ ft^2/sec, $\beta = 366$ ft^{-1}, $a\beta D = 25.4$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
4.5	25.5	1.00	.300	9.33	2.04
7.5	28.5	1.12	.251	10.43	2.16
10.5	31.5	1.24	.201	11.53	2.32
13.5	34.5	1.36	.198	12.63	2.28
17.5	38.5	1.52	.163	14.09	2.85
22.5	43.5	1.71	.130	15.92	3.18
27.5	48.5	1.91	.121	17.75	3.42
32.5	53.5	2.11	--	19.58	3.50

y_m: measured
$y = y_m + 0.2D$

TABLE II TWO-DIMENSIONAL ROUGHNESS
(iii) D = .104 ft

RUN: 237

$a = 0.50$ ft, $\omega = 1.95$ rad/sec, $u_o = 0.98$ ft/sec

$T = 63.5^o F$, $\nu = 1.165 \times 10^{-5}$ ft^2/sec, $\beta = 290$ ft^{-1}, $a\beta D = 15.08$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$	βy	ωt
9.0	30.0	1.95	.151	8.70	1.89
10.0	31.0	2.02	.120	8.99	1.94
12.0	33.0	2.15	.124	9.57	1.92
15.0	36.0	2.34	.109	10.44	2.12
18.0	39.0	2.54	.091	11.31	2.20
20.0	41.0	2.67	--	11.89	2.42
25.0	46.0	2.99	--	13.34	2.67
30.0	51.0	3.32	--	14.79	2.82

RUN: 238

$a = 0.67$ ft, $\omega = 1.65$ rad/sec, $u_o = 1.11$ ft/sec

$T = 63.5^o F$, $\nu = 1.165 \times 10^{-5}$ ft^2/sec, $\beta = 267$ ft^{-1}, $a\beta D = 18.6$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$	$\frac{\bar{u}_2}{u_o}$	βy	ωt
9.0	30.0	1.62	.183	8.01	2.10
10.0	31.0	1.67	.170	8.28	2.13
12.0	33.0	1.78	.151	8.81	2.27
15.0	36.0	1.94	.129	9.61	2.35
18.0	39.0	2.10	.112	10.41	2.48
20.0	41.0	2.21	.096	10.95	2.65
25.0	46.0	2.48	--	12.28	2.66
30.0	51.0	2.75	--	13.62	2.87

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(i) $D = 5.51 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 311-A — $a = 1.00$ ft, $\omega = 3.25$ rad/sec, $u_o = 3.25$ ft/sec, $T = 72^oF$, $\nu = 1.04 \times 10^{-5}$ ft^2/sec, $\beta = 394$ ft^{-1}, $a\beta D = 2.17$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.1	2.81	.332
7.0	8.1	3.73	.301
9.0	10.1	4.65	.248
12.0	13.1	6.04	.196
15.0	16.1	7.42	.177
20.0	21.1	9.72	.130
27.0	28.1	12.95	.086

RUN: 312-A — $a = 1.00$ ft, $\omega = 1.53$ rad/sec, $u_o = 1.53$ ft/sec, $T = 71^oF$, $\nu = 1.05 \times 10^{-5}$ ft/sec, $\beta = 269$ ft^{-1}, $a\beta D = 1.482$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.1	4.12	.313
7.0	8.1	5.47	.272
8.0	9.1	6.14	.250
9.0	10.1	6.82	.244
11.0	12.1	8.17	.163
13.0	14.1	9.51	.152
15.0	16.1	10.86	.121

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(i) $D = 5.51 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 313-A — $a = 0.83$ ft, $\omega = 3.11$ rad/sec, $u_o = 2.58$ ft/sec, $T = 73^oF$, $\nu = 1.03 \times 10^{-5}$ ft^2/sec, $\beta = 387$ ft^{-1}, $a\beta D = 3.21$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.1	1.90	.364
7.0	8.1	2.52	.357
9.0	10.1	3.14	.300
12.0	13.1	4.08	.251
15.0	16.1	5.01	.238
20.0	21.1	6.27	.221
25.0	26.1	8.12	.188
28.0	29.1	9.06	.170

RUN: 314-A — $a = 0.83$ ft, $\omega = 1.58$ rad/sec, $u_o = 1.31$ ft/sec, $T = 73^oF$, $\nu = 1.03 \times 10^{-5}$ ft^2/sec, $\beta = 275$ ft^{-1}, $a\beta D = 1.262$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
4.0	5.1	4.04	.272
5.0	6.1	4.83	.260
7.0	8.1	6.42	.223
9.0	10.1	8.00	.147
11.0	12.1	9.59	.133
13.0	14.1	11.18	.096
15.0	16.1	12.76	.077

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(i) $D = 5.51 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 315-A — $a = 1.33$ ft, $\omega = 2.99$ rad/sec, $u_o = 3.98$ ft/sec, $T = 73^oF$, $\nu = 1.03 \times 10^{-5}$ ft^2/sec, $\beta = 377$ ft^{-1}, $a\beta D = 2.7664$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
4.0	5.1	1.84	.393
5.0	6.1	2.20	.377
6.0	7.1	2.57	.378
8.0	9.1	3.29	.349
10.0	11.1	4.01	.342
13.0	14.1	5.10	.312
15.0	16.1	5.82	.278
19.0	20.1	7.27	.200
22.0	23.1	8.35	.180
26.0	27.1	9.80	.135

RUN: 316-A — $a = 1.33$ ft, $\omega = 1.56$ rad/sec, $u_o = 2.07$ ft/sec, $T = 72^oF$, $\nu = 1.04 \times 10^{-5}$ ft^2/sec, $\beta = 274$ ft^{-1}, $a\beta D = 2.02$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.1	3.02	.335
6.0	7.1	3.51	.320
7.0	8.1	4.01	.278
8.0	9.1	4.50	.259
10.0	11.1	5.49	.210
12.0	13.1	6.48	.175
15.0	16.1	7.97	.170
19.0	20.1	9.95	.111

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(i) $D = 5.51 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT BY DYE METHOD

RUN: 311-B — $a = .58$ ft, $\omega = 1.98$ rad/sec, $u_o = 1.15$ ft/sec, $T = 73^oF$, $\nu = 1.03 \times 10^{-5}$ ft^2/sec, $\beta = 310$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
4.0	5.1	1.58	.63
6.0	7.1	2.20	.74
9.0	10.1	3.13	1.04
12.0	13.1	4.06	1.11
15.0	16.1	4.99	1.30
18.0	19.1	5.92	1.70
20.0	21.1	6.54	1.71

RUN: 312-B — $a = 0.67$ ft, $\omega = 3.15$ rad/sec, $u_o = 2.06$ ft/sec, $T = 73^oF$, $\nu = 1.03 \times 10^{-5}$ ft^2/sec, $\beta = 391$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
5.0	6.1	2.39	.92
7.0	8.1	3.17	1.22
9.0	10.1	3.95	1.30
11.0	12.1	4.73	1.36
13.0	14.1	5.51	1.80
15.0	16.1	6.30	2.0
18.0	19.1	7.47	2.03
21.0	22.1	8.64	2.04

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(i) $D = 5.51 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT BY DYE METHOD

RUN: 313-B $a = 1.00$ $T = 74^{o}F$
$\omega = 2.01$ rad/sec $\nu = 1.02 \times 10^{-5}$ ft^2/sec
$u_o = 2.01$ ft/sec $\beta = 314$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
5.0	6.1	1.92	.80
7.0	8.1	2.54	.88
9.0	10.1	3.17	.98
12.0	13.1	4.11	1.04
15.0	16.1	5.06	1.41
18.0	19.1	6.00	1.52
21.0	22.1	6.94	1.61
23.0	24.1	7.57	1.63

RUN: 314-B $a = 1.00$ ft $T = 74^{o}F$
$\omega = 3.22$ rad/sec $\nu = 1.02 \times 10^{-5}$ ft^2/sec
$u_o = 3.22$ ft/sec $\beta = 400$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
4.0	5.1	2.04	0.82
6.0	7.1	2.84	1.07
8.0	9.1	3.64	1.32
11.0	12.1	4.84	1.71
14.0	15.1	6.04	1.80
17.0	18.1	7.24	1.91
20.0	21.1	8.44	1.92
23.0	24.1	9.64	2.02

y_m : measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(i) $D = 5.51 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT BY DYE METHOD

RUN: 315-B $a = 1.25$ ft $T = 74^{o}F$
$\omega = 1.83$ rad/sec $\nu = 1.02 \times 10^{-5}$ ft^2/sec
$u_o = 2.29$ ft/sec $\beta = 300$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
5.0	6.1	1.83	.71
6.0	7.1	2.13	.78
8.0	9.1	2.73	.87
10.0	11.1	3.33	.99
13.0	14.1	4.23	1.22
16.0	17.1	5.13	1.51
20.0	21.1	6.33	1.62

RUN: 316-B $a = 1.25$ ft $T = 74.5$ F
$\omega = 3.11$ rad/sec $\nu = 1.01 \times 10^{-5}$ ft^2/sec
$u_o = 3.89$ ft/sec $\beta = 393$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
5.0	6.1	2.40	.96
6.0	7.1	2.79	1.02
8.0	9.1	3.58	1.22
10.0	11.1	4.36	1.53
12.0	13.1	5.15	1.61
15.0	16.1	6.33	1.90
17.0	18.1	7.11	2.01
20.0	21.1	8.29	2.09

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(ii) $D = 7.15 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 321-A

$a = 1.17$ ft, $\omega = 3.02$ rad/sec, $u_o = 3.53$ ft/sec

$T = 74.5$ F, $\nu = 1.01 \times 10^{-5}$ ft^2/sec, $\beta = 386$ ft^{-1}, $a\beta D = 3.23$ ft

y_m ft $\times 10^3$	y ft $\times 10^3$	$\frac{y}{a\beta D} \times 10^3$	$\frac{\bar{u}_2}{u_o}$
6.0	7.43	2.30	.349
7.0	8.43	2.61	.328
8.0	9.43	2.92	.320
9.0	10.43	3.23	.316
11.0	12.43	3.85	.278
13.0	14.43	4.47	.251
15.0	16.43	5.09	.242
18.0	19.43	6.02	.206
21.0	22.43	6.94	.182
25.0	26.43	8.18	.146

RUN: 322-A

$a = 1.17$ ft, $\omega = 1.82$ rad/sec, $u_o = 2.13$ ft/sec

$T = 74.5$ F, $\nu = 1.01 \times 10^{-5}$ ft^2/sec, $\beta = 300$ ft^{-1}, $A\beta D = 2.51$ ft

y_m ft $\times 10^3$	y ft $\times 10^3$	$\frac{y}{a\beta D} \times 10^3$	$\frac{\bar{u}_2}{u_o}$
7.0	8.43	3.36	.361
9.0	10.43	4.16	.312
11.0	12.43	4.95	.282
14.0	15.43	6.15	.224
17.0	18.43	7.34	.216
20.0	21.43	8.54	.172
24.0	25.43	10.13	.126
28.0	29.43	11.72	.121
33.0	34.43	13.72	.097

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(ii) $D = 7.15 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 323-A

$a = 0.67$ ft, $\omega = 3.48$ rad/sec, $u_o = 2.33$ ft/sec

$T = 74.5$F, $\nu = 1.01 \times 10^{-5}$ ft^2/sec, $\beta = 417$ ft^{-1}, $a\beta D = 2.0$ ft

y_m ft $\times 10^3$	y ft $\times 10^3$	$\frac{y}{a\beta D} \times 10^3$	$\frac{\bar{u}_2}{u_o}$
6.0	7.43	3.72	.271
8.0	9.43	4.72	.230
10.0	11.43	5.71	.223
13.0	14.43	7.21	.166
17.0	18.43	9.21	.153
21.0	22.43	11.21	.101
25.0	26.43	13.01	.086
29.0	30.43	15.01	.067

RUN: 324-A

$a = 0.67$ ft, $\omega = 3.04$ rad/sec, $u_o = 2.03$ ft/sec

$T = 74.5$ F, $\nu = 1.01 \times 10^{-5}$ ft^2/sec, $\beta = 388$ ft^{-1}, $a\beta D = 1.86$ ft

y_m ft $\times 10^3$	y ft $\times 10^3$	$\frac{y}{a\beta D} \times 10^3$	$\frac{\bar{u}_2}{u_o}$
6.0	7.43	3.99	.253
8.0	9.43	5.07	.211
11.0	12.43	6.68	.192
12.0	13.43	7.22	.188
15.0	16.43	8.83	.162
18.0	19.43	10.45	.112
20.0	21.43	11.52	.106

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(ii) $D = 7.15 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 325-A

$a = .92$ ft
$\omega = 3.14$ rad/sec
$u_o = 2.89$ ft/sec

$T = 74\overset{\circ}{.}5$ F
$\nu = 1.01 \times 10^{-5}$ ft^2/sec
$\beta = 396$ ft^{-1}
$a\beta D = 2.60$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.43	2.47	.364
7.0	8.43	3.24	.326
9.0	10.43	4.01	.312
11.0	12.43	4.78	.287
14.0	15.43	5.93	.248
16.0	17.43	6.70	.211
20.0	21.43	8.24	.140
22.0	23.43	9.01	.136

RUN: 326-A

$a = 0.92$ ft
$\omega = 2.43$ rad/sec
$u_o = 2.24$ ft/sec

$T = 74\overset{\circ}{.}5$ F
$\nu = 1.01 \times 10^{-5}$ ft^2/sec
$\beta = 346$ ft^{-1}
$a\beta D = 2.277$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.43	2.82	.301
6.0	7.43	3.26	.270
7.0	8.43	3.70	.259
8.0	9.43	4.14	.222
9.0	10.43	4.58	.221
11.0	12.43	5.46	.218

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(ii) $D = 7.15 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT (DYE METHOD)

RUN: 321-B

$a = 1.17$ ft
$\omega = 3.40$ rad/sec
$u_o = 3.98$ ft/sec

$T = 74\overset{\circ}{.}5$ F
$\nu = 1.01 \times 10^{-5}$ ft^2/sec
$\beta = 410$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
6.0	7.43	3.05	1.01
7.0	8.43	3.46	1.03
8.0	9.43	3.87	1.08
10.0	11.43	4.69	1.30
12.0	13.43	5.51	1.32
14.0	15.43	6.33	1.51
17.0	18.43	7.56	1.70
20.0	21.43	8.79	2.01

RUN: 322-B

$a = 1.17$ ft
$\omega = 1.77$ rad/sec
$u_o = 2.07$ ft/sec

$T = 74\overset{\circ}{.}5$ F
$\nu = 1.01 \times 10^{-5}$ ft^2/sec
$\beta = 292$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
4.0	5.43	1.59	.70
6.0	7.43	2.17	.78
8.0	9.43	2.75	1.02
10.0	11.43	3.34	1.20
12.0	13.43	3.92	1.32
14.0	15.43	4.50	1.62
16.0	17.43	5.09	1.62
18.0	19.43	5.67	1.91
20.0	21.43	6.26	1.93

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(ii) $D = 7.15 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT (DYE METHOD)

RUN: 323-B $a = 1.17$ ft $\omega = 3.42$ rad/sec $u_o = 4.00$ ft/sec $T = 74.5°$ F $\nu = 1.01 \times 10^{-5}$ ft^2/sec $\beta = 410$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
4.0	5.43	2.23	.91
6.0	7.43	3.04	.97
8.0	9.43	3.87	1.15
10.0	11.43	4.69	1.18
12.0	13.43	5.51	1.38
14.0	15.43	6.33	1.62
16.0	17.43	7.15	1.68
18.0	19.43	7.97	1.81
20.0	21.43	8.79	2.30

RUN: 324-B $a = 1.17$ ft $\omega = 2.29$ rad/sec $u_o = 2.68$ ft/sec $T = 74.5°$ F $\nu = 1.0. \times 10^{-5}$ ft^2/sec $\beta = 336$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
4.0	5.43	1.82	.77
6.0	7.43	2.50	.86
8.0	9.43	3.17	.86
10.0	11.43	3.84	.98
12.0	13.43	4.51	1.10
14.0	15.43	5.18	1.12
17.0	18.43	6.19	1.27
20.0	21.43	7.20	1.50

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(ii) $D = 7.15 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT (DYE METHOD)

RUN: 325-B $a = .67$ ft $\omega = 3.74$ rad/sec $u_o = 2.51$ ft/sec $T = 74.5°$ F $\nu = 1.01 \times 10^{-5}$ ft^2/sec $\beta = 430$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
5.0	6.43	2.76	1.18
7.0	8.43	3.62	1.31
9.0	10.43	4.48	1.50
11.0	12.43	5.34	1.80
13.0	14.43	6.20	1.82
15.0	16.43	7.06	1.92
18.0	19.43	8.35	2.13
20.0	21.43	9.21	2.26

RUN: 326-B $a = .67$ ft $\omega = 2.41$ rad/sec $u_o = 1.61$ ft/sec $T = 74.5°$ F $\nu = 1.01 \times 10^{-5}$ ft^2/sec $\beta = 347$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
5.0	6.43	2.23	.87
7.0	8.43	2.93	1.11
9.0	10.43	3.62	1.38
11.0	12.43	4.31	1.60
14.0	15.43	5.35	1.74
16.0	17.43	6.05	1.76
19.0	20.43	7.09	1.86
23.0	24.43	8.48	2.10

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 331-A
$a = 1.00$ ft, $\omega = 3.05$ rad/sec, $u_o = 3.05$ ft/sec
$T = 67^oF$, $\nu = 1.12 \times 10^{-5}$ ft^2/sec, $\beta = 369$ ft^{-1}, $a\beta D = 3.41$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.85	2.01	.371
6.0	7.85	2.30	.380
8.0	9.85	2.89	.360
10.0	11.85	3.47	.330
12.0	13.85	4.06	.302
15.0	16.85	4.94	.293
18.0	19.85	5.82	.242
23.0	24.85	7.28	.210

RUN: 332-A
$a = 1.00$ ft, $\omega = 3.95$ rad/sec, $u_o = 3.95$ ft/sec
$T = 66.5$ F, $\nu = 1.13 \times 10^{-5}$ ft^2/sec, $\beta = 418$ ft^{-1}, $a\beta D = 3.87$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
6.0	7.85	2.03	.412
7.0	8.85	2.29	.390
9.0	10.85	2.81	.320
11.0	12.85	3.32	.295
14.0	15.85	4.10	.310
17.0	18.85	4.87	.275
22.0	23.85	6.17	.201
26.0	27.85	7.20	.176

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 333-A
$a = 1.17$ ft, $\omega = 2.97$ rad/sec, $u_o = 3.46$ ft/sec
$T = 66.5$ F, $\nu = 1.13 \times 10^{-5}$ ft^2/sec, $\beta = 362$ ft^{-1}, $a\beta D = 3.92$

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
6.0	7.85	2.00	.351
7.0	8.85	2.26	.332
9.0	10.85	2.77	.301
12.0	13.85	3.53	.234
15.0	16.85	4.30	.254
18.0	19.85	5.06	.276
21.0	22.85	5.83	.197
25.0	26.85	6.85	.176

RUN: 334-A
$a = 0.75$ ft, $\omega = 4.85$ rad/sec, $u_o = 3.64$ ft/sec
$T = 67^oF$, $\nu = 1.12 \times 10^{-5}$ ft^2/sec, $\beta = 465$ ft^{-1}, $a\beta D = 3.23$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
6.0	7.85	2.43	.402
8.0	9.85	3.05	.361
10.0	11.85	3.67	.353
11.0	12.85	3.98	.331
13.0	14.85	4.60	.235
15.0	16.85	5.22	.190
19.0	20.85	6.46	.163
25.0	26.85	8.32	.181
31.0	32.85	10.18	.148

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 335-A

$a = 0.84$ ft, $\omega = 2.20$ rad/sec, $u_o = 1.83$ ft/sec
$T = 73^oF$, $\nu = 1.03 \times 10^{-5}$ ft^2/sec, $\beta = 327$ ft^{-1}, $a\beta D = 2.54$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.85	2.70	.372
6.0	7.85	3.09	.352
7.0	8.85	3.49	.342
9.0	10.85	4.27	.310
12.0	13.85	5.46	.262
16.0	17.85	7.03	.217
19.0	20.85	8.21	.183
22.0	23.85	9.40	.165
28.0	29.85	11.76	.119

RUN: 336-A

$a = 0.84$ ft, $\omega = 2.78$ rad/sec, $u_o = 2.33$ ft/sec
$T = 73^oF$, $\nu = 1.03 \times 10^{-5}$ ft^2/sec, $\beta = 371$ ft^{-1}, $a\beta D = 2.88$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
5.0	6.85	2.38	.352
6.0	7.85	2.73	.340
7.0	8.85	3.07	.325
9.0	10.85	3.77	.285
11.0	12.85	4.46	.262
14.0	15.85	5.50	.217
17.0	18.85	6.54	.168
23.0	24.85	8.63	.152
26.0	27.85	9.67	.135

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 337-A

$a = 0.84$ ft, $\omega = 1.90$ rad/sec, $u_o = 1.59$ ft/sec
$T = 73^oF$, $\nu = 1.03 \times 10^{-5}$ ft^2/sec, $\beta = 307$ ft^{-1}, $a\beta D = 2.37$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
6.0	7.85	3.31	.302
8.0	9.85	4.16	.261
9.0	10.85	4.58	.249
11.0	12.85	5.42	.250
13.0	14.85	6.27	.235
14.0	15.85	6.69	.226
17.0	18.85	7.95	.193
20.0	21.85	9.22	.167
24.0	25.85	10.91	.138

RUN: 338-A

$a = 1.17$ ft, $\omega = 1.77$ rad/sec, $u_o = 2.06$ ft/sec
$T = 74^oF$, $\nu = 1.02 \times 10^{-5}$ ft^2/sec, $\beta = 296$ ft^{-1}, $a\beta D = 3.19$ ft

y_m ft x 10^3	y ft x 10^3	$\frac{y}{a\beta D}$ x 10^3	$\frac{\bar{u}_2}{u_o}$
6.0	7.85	2.46	.387
7.0	8.85	2.77	.356
8.0	9.85	3.09	.332
10.0	11.85	3.71	.300
12.0	13.85	4.34	.245
14.0	15.85	4.97	.248
17.0	18.85	5.91	.218
20.0	21.85	6.84	.173
24.0	25.85	8.10	.165

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 339-A
$a = 1.17$ ft, $\omega = 1.77$ rad/sec, $u_o = 2.06$ ft/sec
$T = 74^oF$, $\nu = 1.02 \times 10^{-5}$ ft^2/sec, $\beta = 296$ ft^{-1}, $a\beta D = 3.19$ ft

y_m ft $\times 10^3$	y ft $\times 10^3$	$\frac{y}{a\beta D} \times 10^3$	$\frac{\bar{u}_2}{u_o}$
5.0	6.85	1.97	.383
6.0	7.85	2.26	.365
8.0	9.85	2.84	.360
9.0	10.85	3.12	.343
11.0	12.85	3.70	.331
13.0	14.85	4.28	.301
16.0	17.85	5.14	.263
20.0	21.85	6.29	.259
25.0	26.85	7.73	.218
30.0	31.85	9.17	.172

RUN: 340-A
$a = 1.17$ ft, $\omega = 2.52$ rad/sec, $u_o = 2.95$ ft/sec
$T = 74^oF$, $\nu = 1.02 \times 10^{-5}$ ft^2/sec, $\beta = 354$ ft^{-1}, $a\beta D = 3.82$ ft

y_m ft $\times 10^3$	y ft $\times 10^3$	$\frac{y}{a\beta D}$	$\frac{\bar{u}_2}{u_o}$
5.0	6.85	1.80	.372
7.0	8.85	2.32	.343
9.0	10.85	2.84	.306
11.0	12.85	3.37	.280
13.0	14.85	3.89	.252
15.0	16.85	4.42	.226
19.0	20.85	5.46	.221
23.0	24.85	6.51	.183
30.0	31.85	8.35	.149

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES A VELOCITY DISTRIBUTION MEASUREMENT

RUN: 341-A
$a = 1.50$ ft, $\omega = 1.65$ rad/sec, $u_o = 2.48$ ft/sec
$T = 74^oF$, $\nu = 1.02 \times 10^{-5}$ ft^2/sec, $\beta = 258$ ft^{-1}, $a\beta D = 4.00$ ft

y_m ft $\times 10^3$	y ft $\times 10^3$	$\frac{y}{a\beta D} \times 10^3$	$\frac{\bar{u}_2}{u_o}$
6.0	7.85	1.97	.389
8.0	9.85	2.47	.342
10.0	11.85	2.96	.321
12.0	13.85	3.46	.322
15.0	16.85	4.21	.293
19.0	20.85	5.21	.276
24.0	25.85	6.46	.242
29.0	30.85	7.71	.193
33.0	34.85	8.71	.187

RUN: 342-A
$a = 1.50$ ft, $\omega = 2.36$ rad/sec, $u_o = 3.54$ ft/sec
$T = 74^oF$, $\nu = 1.02 \times 10^{-5}$ ft^2/sec, $\beta = 343$ ft^{-1}, $a\beta D = 4.76$ ft

y_m ft $\times 10^3$	y ft $\times 10^3$	$\frac{y}{a\beta D} \times 10^3$	$\frac{\bar{u}_2}{u_o}$
5.0	6.85	1.44	.422
6.0	7.85	1.65	.384
8.0	9.85	2.07	.341
11.0	12.85	2.70	.331
13.0	14.85	3.12	.296
18.0	19.85	4.17	.251
23.0	24.85	5.22	.239
27.0	28.85	6.06	.229
32.0	33.85	7.11	.208

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT (DYE METHOD)

RUN: 331-B

$a = 1.43$ ft, $\omega = 2.58$ rad/sec, $u_o = 3.69$ ft/sec

$T = 74.5°$ F, $\nu = 1.01 \times 10^{-5}$ ft^2/sec, $\beta = 357$ ft^{-1}

y ft x 10^3	y ft x 10^3	βy	ωt
6.0	7.85	2.80	.91
8.0	9.85	3.52	1.10
10.0	11.85	4.23	1.18
12.0	13.85	4.95	1.32
14.0	15.85	5.66	1.68
16.0	17.85	6.37	1.83
18.0	19.85	7.09	2.20
20.0	21.85	7.80	2.31

RUN: 332-B

$a = 1.00$ ft, $\omega = 2.64$ rad/sec, $u_o = 2.64$ ft/sec

$T = 74.5°$ F, $\nu = 1.01 \times 10^{-5}$ ft^2/sec, $\beta = 360$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
6.0	7.85	2.83	1.00
8.0	9.85	3.55	1.05
10.0	11.85	4.27	1.11
12.0	13.85	4.99	1.42
14.0	15.85	5.71	1.51
16.0	17.85	6.43	1.52
18.0	19.85	7.15	1.82
21.0	22.85	8.23	1.91

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT (DYE METHOD)

RUN: 333-B

$a = 1.00$ ft, $\omega = 3.15$ rad/sec, $u_o = 3.15$ ft/sec

$T = 74.5°$ F, $\nu = 1.01 \times 10^{-5}$ ft^2/sec, $\beta = 397$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
6.0	7.85	3.12	1.11
8.0	9.85	3.91	1.19
10.0	11.85	4.70	1.29
12.0	13.85	5.50	1.32
14.0	15.85	6.29	1.64
16.0	17.85	7.09	1.68
18.0	19.85	7.88	1.93
20.0	21.85	8.67	2.02

RUN: 334-B

$a = 1.00$ ft, $\omega = 2.38$ rad/sec, $u_o = 2.38$ ft/sec

$T = 74.5°$ F, $\nu = 1.01 \times 10^{-5}$ ft^2/sec, $\beta = 343$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
5.0	6.85	2.35	.97
7.0	8.85	3.04	1.03
9.0	10.85	3.72	1.40
11.0	12.85	4.41	1.61
13.0	14.85	5.09	1.63
15.0	16.85	5.78	1.92
18.0	19.85	6.81	1.98
20.0	21.85	7.49	2.09

y_m: measured
$y = y_m + 0.2D$

TABLE III THREE-DIMENSIONAL ROUGHNESS
(iii) $D = 9.25 \times 10^{-3}$ ft
SERIES B PHASE SHIFT MEASUREMENT (DYE METHOD)

RUN: 335-B

$a = 1.17$ ft
$\omega = 2.30$ rad/sec
$u_o = 2.69$ ft/sec

$T = 74.5^\circ$ F
$\nu = 1.01 \times 10^{-5}$ ft^2/sec
$\beta = 337$ ft^{-1}

y_m ft x 10^3	y ft x 10^3	βy	ωt
6.0	7.85	2.65	1.06
8.0	9.85	3.32	1.22
10.0	11.85	4.00	1.42
12.0	13.85	4.67	1.79
14.0	15.85	5.34	1.81
16.0	17.85	6.02	2.13
18.0	19.85	6.70	2.17
20.0	21.85	7.36	2.21

y_m: measured
$y = y_m + 0.2D$

TABLE IV DETERMINATION OF A_*, B_* AND η_o

a ft	ω rad/sec	u_o ft/sec	ν ft^2/sec $\times 10^5$	β ft^{-1}	$\lvert\bar{u}\rvert^2$ ft^2/sec^2	Ψ	G/T gms/period	q_B lbs/sec $\times 10^3$	Φ $\times 10^3$
SERIES A $D = 5.51 \times 10^{-3}$ ft.									
.75	1.25	.94	1.03	245	.361	.800	.65	.28	.60
.75	1.60	1.20	1.03	279	.579	.499	1.94	1.09	2.31
.75	1.93	1.45	1.02	308	.834	.347	8.47	5.72	12.15
1.00	1.29	1.29	1.02	252	.629	.459	2.37	1.07	2.27
1.00	1.58	1.58	1.02	279	.936	.309	11.24	6.22	13.20
1.00	1.87	1.87	1.02	303	1.312	.220	17.62	11.53	24.48
1.25	1.23	1.59	1.01	248	.912	.317	8.92	3.84	8.15
1.25	1.59	1.99	1.01	281	1.445	.200	18.36	10.22	21.70
1.25	1.86	2.33	1.01	304	1.984	.146	21.50	14.00	29.72
SERIES B $D = 7.15 \times 10^{-3}$ ft.									
.75	1.26	.95	1.03	248	.382	.982	.69	.30	.43
.75	1.57	1.18	1.02	278	.588	.638	1.97	1.08	1.51
.75	1.96	1.47	1.02	310	.918	.408	10.07	7.35	10.29
1.00	1.31	1.31	1.03	252	.678	.553	1.28	.59	.82
1.00	1.56	1.56	1.03	275	.966	.388	4.07	2.18	3.05
1.00	1.84	1.84	1.02	300	1.358	.276	12.20	11.00	15.40
1.25	1.27	1.59	1.02	250	.961	.390	9.20	4.20	5.88
1.25	1.58	1.98	1.01	280	1.517	.247	18.30	10.05	14.07
1.25	1.83	2.29	1.01	301	2.040	.184	22.00	14.10	19.74

TABLE IV DETERMINATION OF A_*, B_* AND η_o

a ft	ω rad/sec	u_o ft/sec	ν ft^2/sec $\times 10^5$	β ft^{-1}	$\lvert\bar{u}\rvert^2$ ft^2/sec^2	Ψ	G/T gms/period	q_B lbs/sec $\times 10^3$	Φ $\times 10^3$
SERIES C $D = 9.25 \times 10^{-3}$ ft.									
.75	1.23	.92	1.03	244	.377	1.286	.16	.07	.07
.75	1.48	1.11	1.03	268	.551	.880	.61	.32	.30
.75	1.86	1.40	1.02	302	.884	.549	2.37	1.54	1.47
1.00	1.25	1.25	1.02	248	.651	.745	.66	.29	.28
1.00	1.52	1.52	1.02	273	.959	.506	2.41	1.28	1.22
1.00	1.93	1.93	1.02	307	1.529	.317	17.87	10.82	10.30
1.25	1.02	1.28	1.02	224	.680	.713	1.45	.52	.49
1.25	1.39	1.74	1.01	263	1.241	.391	6.24	3.04	2.89
1.25	1.73	2.16	1.01	293	1.890	.257	30.55	18.50	17.62

U.S. ARMY COASTAL ENGRG. RES. CENTER, CE., WASH., D.C.

TRANSPORTATION OF BED MATERIAL DUE TO WAVE ACTION by G. Kalkanis. February 1964, 38 pp., 11 illus., and 4 appendices with tables.

TECHNICAL MEMORANDUM No. 2 UNCLASSIFIED

1. Sediment transport by waves and currents
2. Boundary layer
3.

I Kalkanis, G.
II Title

A method is developed for use in determining rate of sediment transportation in a layer adjacent to the ocean floor. Method is applicable only for conditions of unstable flow in this layer associated with long surface waves of small amplitude where it is assumed sediment particles in a bed are brought to a state of incipient equilibrium. By experimental determination of the distribution of lift forces and statistical analysis of turbulent fluctuations, an equation for the rate at which sediment in the bed layer is oscillated and an expression for concentration of sediment in this oscillatory state are developed. The concentration in combination with velocity distribution in bed layer associated with any incidental secondary flow can be used to calculate transport rate of bed material in direction of that flow.

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