



58  
- 13 182

*The Bancroft Library*

University of California • Berkeley

THE THEODORE P. HILL COLLECTION  
of  
EARLY AMERICAN MATHEMATICS BOOKS

Condition noted!

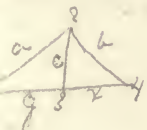


Mary E. Hopkins.  
Ont. You. Soc.  
Canadian.

STATIONARY  
1800 1/2  
STATIONARY



The diff- of the squares of two numbers  
 is equal to the product of the sum & difference  
 of these numbers —



$$a^2 = c^2 + y^2$$

$$b^2 = c^2 + x^2$$


---


$$a^2 - b^2 = y^2 - x^2 =$$

$$(a+b) \times (a-b) = (y+x) \times (y-x)$$

$$(y+x) : (a+b) :: (a-b) : (y-x)$$

Oblique A. G. Theorem 3-

3, 1416-  
 7854

$$\text{area} = \pi r R^2$$

$$\text{Surface} = 4\pi R^2$$

$$\text{Solidity} = \frac{4}{3} \pi R^3$$



Day and Thomson's Series.

---

A

T R E A T I S E

OF

PLANE TRIGONOMETRY,

AND THE

MENSURATION OF HEIGHTS AND DISTANCES.

TO WHICH IS PREFIXED

A SUMMARY VIEW OF THE NATURE AND USE

OF

L O G A R I T H M S .

ADAPTED TO

THE METHOD OF INSTRUCTION IN SCHOOLS AND ACADEMIES

BY JEREMIAH DAY, D.D. LL.D.

LATE PRESIDENT OF YALE COLLEGE.

NEW YORK:

IVISON & PHINNEY, 178 FULTON STREET;

(SUCCESSORS OF NEWMAN & IVISON, AND MARK H. NEWMAN & CO.)

CHICAGO: S. C. GRIGGS & CO., 111 LAKE ST.

BUFFALO: PHINNEY & CO., 188 MAIN STREET

AUBURN: J. C. IVISON & CO. DETROIT: A. M'FARREN.

CINCINNATI: MOORE, ANDERSON & CO.

1855.

ENTERED, according to Act of Congress, in the year 1848, by

JEREMIAH DAY,

In the Clerk's Office of the District Court of the United States for the  
Southern District of New York.

THOMAS B. SMITH, STEREOTYPER,  
216 WILLIAM STREET, N. Y.

J. D. BEDFORD, PRINTER,  
138 FULTON STREET.

## PLANE TRIGONOMETRY.

---

SCARCELY any department of Mathematics is more important, or more extensive in its applications, than Trigonometry. By it the mariner traces his path on the ocean; the geographer determines the latitude and longitude of places, the dimensions and positions of countries, the altitude of mountains, the courses of rivers, &c., and the astronomer calculates the distances and magnitudes of the heavenly bodies, predicts the eclipses of the sun and moon, and measures the progress of light from the stars.

The section on right angled triangles in this treatise, may perhaps be considered as needlessly minute. The solutions might, in all cases, be effected by the theorems which are given for oblique angled triangles. But the applications of rectangular trigonometry are so numerous, in navigation, surveying, astronomy, &c., that it was deemed important, to render familiar the various methods of stating the relations of the sides and angles; and especially to bring distinctly into view the principle on which most trigonometrical calculations are founded, the proportion between the parts of the given triangle, and a similar one formed from the sines, tangents, &c., in the tables.

As this treatise is intended to form a part of Day and Thomson's Course of Mathematics for the use of Schools and Academies, the references to Algebra are made to Thomson's Abridgment; and the references to Geometry, to Thomson's Legendre, as well as to Euclid's Elements.

# PHYSIOLOGICAL OPTICS

BY HERMANN VON HELMHOLTZ

Translated from the German by Charles D. Keyser, M.D., F.R.C.S., F.R.S., and Charles J. Verhulst, M.D., F.R.C.S., F.R.S., with the assistance of Charles D. Keyser, M.D., F.R.C.S., F.R.S., and Charles J. Verhulst, M.D., F.R.C.S., F.R.S.

PHYSIOLOGICAL OPTICS, or the PHYSIOLOGY OF VISION, is a branch of the science of optics, which deals with the physical and physiological processes of vision. It is a branch of the science of optics, which deals with the physical and physiological processes of vision.

The study of the eye and its functions is a branch of the science of optics, which deals with the physical and physiological processes of vision. It is a branch of the science of optics, which deals with the physical and physiological processes of vision.

# CONTENTS.

## LOGARITHMS.

	Page
SECTION I. Nature of Logarithms, . . . . .	7
II. Directions for taking Logarithms and their Numbers from the Tables, . . . . .	14
III. Methods of calculating by Logarithms, . . . . .	
Multiplication, . . . . .	22
Division, . . . . .	25
Involution, . . . . .	27
Evolution, . . . . .	29
Proportion, . . . . .	32
Arithmetical Complement, . . . . .	33
Compound Proportion, . . . . .	35
Compound Interest, . . . . .	37
Increase of Population, . . . . .	40
Exponential Equations, . . . . .	45

## TRIGONOMETRY.

SECTION I. Sines, Tangents, Secants, &c., . . . . .	47
II. Explanation of the Trigonometrical Tables, . . . . .	59
III. Solutions of Right angled Triangles, . . . . .	67
IV. Solutions of Oblique angled Triangles, . . . . .	85
V. Geometrical Construction of Triangles, . . . . .	99
VI. Description and use of Gunter's Scale, . . . . .	107
VII. Trigonometrical Analysis, . . . . .	116
Application of Trigonometry to the mensuration of heights and distances, . . . . .	130
Notes, . . . . .	148

STATE

CHAPTER

7	...
14	...
20	...
25	...
30	...
35	...
40	...
45	...
50	...
55	...
60	...
65	...
70	...
75	...
80	...
85	...
90	...
95	...

100	...
105	...
110	...
115	...
120	...
125	...
130	...
135	...
140	...
145	...
150	...



# LOGARITHMS.

---

## SECTION I.

### NATURE OF LOGARITHMS.

ART. 1. The operations of Multiplication and Division, when they are to be often repeated, become so laborious, that it is an object of importance to substitute, in their stead, more simple methods of calculation, such as Addition and Subtraction. If these can be made to perform, in an expeditious manner, the office of multiplication and division, a great portion of the time and labor which the latter processes require, may be saved.

Now it has been shown, (Algebra, 189, 193,) that *powers* may be multiplied by adding their *exponents*, and divided, by subtracting their exponents. In the same manner, *roots* may be multiplied and divided, by adding and subtracting their fractional exponents. (Alg., 232, 239.) When these exponents are arranged in tables, and applied to the general purposes of calculation, they are called *Logarithms*.

2. LOGARITHMS, THEN, ARE THE EXPONENTS OF A SERIES OF POWERS AND ROOTS.

In forming a system of logarithms, some particular number is fixed upon, as the *base, radix*, or first power, whose logarithm is always 1. From this a series of powers is raised, and the exponents of these are arranged in tables for use. To explain this, let the number which is chosen for the

first power be represented by  $a$ . Then taking a series of powers, both direct and reciprocal, as in Alg. 163 ;

$$a^4, a^3, a^2, a^1, a^0, a^{-1}, a^{-2}, a^{-3}, a^{-4}, \&c.$$

The logarithm of  $a^3$  is 3, and the logarithm of  $a^{-1}$  is—1,

of  $a^1$  is 1,

of  $a^{-2}$  is—2,

of  $a^0$  is 0,

of  $a^{-3}$  is—3, &c.

Universally, the logarithm of  $a^x$  is  $x$ .

3. In the system of logarithms in common use, called *Briggs's* logarithms, the number which is taken for the radix or base is 10. The above series, then, by substituting 10 for  $a$ , becomes

$$10^4, 10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3}, \&c.$$

$$\text{Or } 10000, 1000, 100, 10, 1, \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \&c.$$

Whose logarithms are

$$4, \quad 3, \quad 2, \quad 1, \quad 0, \quad -1, \quad -2, \quad -3, \quad \&c.$$

4. The fractional exponents of *roots*, and of powers of roots, are converted into *decimals*, before they are inserted in the logarithmic tables. See Alg. 208.

The logarithm of  $a^{\frac{1}{3}}$ , or  $a^{0.3333}$ , is 0.3333,

of  $a^{\frac{2}{3}}$ , or  $a^{0.6666}$ , is 0.6666,

of  $a^{\frac{3}{7}}$ , or  $a^{0.4285}$ , is 0.4285,

of  $a^{\frac{11}{3}}$ , or  $a^{3.6666}$ , is 3.6666, &c.

These decimals are carried to a greater or less number of places, according to the degree of accuracy required.

5. In forming a system of logarithms, it is necessary to obtain the logarithm of each of the numbers in the natural series 1, 2, 3, 4, 5, &c. ; so that the logarithm of any number may be found in the tables. For this purpose, the *radix* of the system must first be determined upon ; and then every other number may be considered as some power or root of

this. If the radix is 10, as in the common system, every other number is to be considered as some power of 10.

If the exponent is a fraction, and the *numerator* be increased, the power will be increased; but if the *denominator* be increased, the power will be diminished.

6. To obtain then the logarithm of any number, according to Briggs's system, we have to find a power or root of 10 which shall be equal to the proposed number. The *exponent* of that power or root is the logarithm required. Thus

$$\left. \begin{array}{l} 7=10^{0.8451} \\ 20=10^{1.3010} \\ 30=10^{1.4771} \\ 400=10^{2.6020} \end{array} \right\} \text{therefore the } \left\{ \begin{array}{l} \text{of } 7 \text{ is } 0.8451 \\ \text{of } 20 \text{ is } 1.3010 \\ \text{of } 30 \text{ is } 1.4771 \\ \text{of } 400 \text{ is } 2.6020, \text{ \&c.} \end{array} \right.$$

7. A logarithm generally consists of two parts, an *integer* and a *decimal*. Thus the logarithm 2.60206, or, as it is sometimes written,  $2+.60206$ , consists of the integer 2, and the decimal .60206. The integral part is called the *characteristic* or *index*\* of the logarithm; and is frequently omitted, in the common tables, because it can be easily supplied, whenever the logarithm is to be used in calculation.

By art. 3d, the logarithms of

$$\begin{array}{l} 10000, 1000, 100, 10, 1, .1, .01, .001, \text{ \&c.} \\ \text{are } 4, 3, 2, 1, 0, -1, -2, -3, \text{ \&c.} \end{array}$$

As the logarithms of 1 and of 10 are 0 and 1, it is evident, that, if any given number be between 1 and 10, its logarithm will be between 0 and 1, that is, it will be greater than 0, but less than 1. It will therefore have 0 for its index, with a decimal annexed.

Thus, the logarithm of 5 is 0.69897.

---

\* The term *index*, as it is used here, may possibly lead to some confusion in the mind of the learner. For the logarithm itself is the index or exponent of a power. The characteristic, therefore, is the index of an index.

For the same reason, if the given number be between

10 and 100,	}	the log. {	1 and 2, i.e. 1 + the dec. part.
100 and 1000,			2 and 3, 2 + the dec. part.
1000 and 10000,			3 and 4, 3 + the dec. part.

We have, therefore, when the logarithm of an integer or mixed number is to be found, this general rule :

8. *The index of the logarithm is always one less, than the number of integral figures, in the natural number whose logarithm is sought :* or, the index shows how far the first figure of the natural number is removed from the place of units.

Thus, the logarithm of 37 is 1.56820.

Here, the number of figures being *two*, the index of the logarithm is 1.

The logarithm of 253 is 2.40312.

Here the proposed number 253 consists of *three* figures, the first of which is in the second place from the unit figure. The index of the logarithm is therefore 2.

The logarithm of 62.8 is 1.79796.

Here it is evident that the mixed number 62.8 is between 10 and 100. The index of its logarithm must, therefore, be 1.

9. As the logarithm of 1 is 0, the logarithm of a number less than 1, that is, of any proper fraction, must be *negative*.

Thus, by art. 3d,

The logarithm of $\frac{1}{10}$ or .1	is -1,
of $\frac{1}{100}$ or .01	is -2,
of $\frac{1}{1000}$ or .001	is -3, &c.

10. If the proposed number is between  $\frac{1}{100}$  and  $\frac{1}{1000}$ , its logarithm must be between -2 and -3. To obtain the logarithm, therefore, we must either *subtract* a certain fractional part from -2, or *add* a fractional part to -3; that



is, we must either annex a *negative decimal* to  $-2$ , or a *positive one* to  $-3$ .

Thus, the logarithm

of .008 is either  $-2 - .09691$ , or  $-3 + 90309$ .\*

The latter is generally most convenient in practice, and is more commonly written  $\overline{3}.90309$ . The line over the index denotes, that *that* is negative, while the *decimal* part of the logarithm is positive.

The logarithm  $\left\{ \begin{array}{l} \text{of } 0.3, \text{ is } \overline{1}.47712, \\ \text{of } 0.06, \text{ is } \overline{2}.77815, \\ \text{of } 0.009, \text{ is } \overline{3}.95424, \end{array} \right.$

And universally,

11. *The negative index of a logarithm shows how far the first significant figure of the natural number, is removed from the place of units, on the right; in the same manner as a positive index shows how far the first figure of the natural number is removed from the place of units on the left. (Art. 8.)* Thus, in the examples in the last article,

The decimal 3 is in the *first* place from that of units,  
6 is in the *second* place,  
9 is in the *third* place;

And the indices of the logarithms are  $\overline{1}$ ,  $\overline{2}$ , and  $\overline{3}$ .

12. It is often more convenient, however to make the *index* of the logarithm positive, as well as the decimal part. This is done by adding 10 to the index.

Thus, for  $-1$ , 9 is written, for  $-2$ , 8, &c.  
Because  $-1 + 10 = 9$ ,  $-2 + 10 = 8$ , &c.

---

\* That these two expressions are of the same value will be evident, if we subtract the same quantity,  $+.90309$  from each. The remainders will be equal, and therefore the quantities from which the subtraction is made must be equal.

With this alteration,

The logarithm  $\left\{ \begin{array}{l} \overline{1.90309} \\ \overline{2.90309} \\ \overline{3.90309} \end{array} \right\}$  becomes  $\left\{ \begin{array}{l} 9.90309, \\ 8.90309, \\ 7.90309, \text{ \&c.} \end{array} \right.$

This is making the index of the logarithm 10 too great. But with proper caution, it will lead to no error in practice.

13. The *sum* of the logarithms of two numbers, is the logarithm of the *product* of those numbers; and the *difference* of the logarithms of two numbers, is the logarithm of the *quotient* of one of the numbers divided by the other. (Art. 2.) In Briggs's system, the logarithm of 10 is 1. (Art. 3.) If therefore any number be multiplied or divided by 10, its logarithm will be increased or diminished by 1: and as this is an integer, it will only change the *index* of the logarithm, without affecting the decimal part.

Thus, the logarithm of 4730 is 3.67486

And the logarithm of 10 is 1.

The logarithm of the product  $\underline{47300}$  is 4.67486

And the logarithm of the quotient  $\underline{473}$  is 2.67486

Here the *index* only is altered, while the decimal part remains the same. We have then this important property,

14. The DECIMAL PART of the logarithm of any number is the same, as that of the number multiplied or divided by 10, 100, 1000, &c.

Thus the log. of 45670,	is 4.65963,
4567,	3.65963,
456.7,	2.65963,
45.67,	1.65963,
4.567,	0.65963,
.4567,	$\overline{1.65963}$ , or 9.65963,
.04567,	$\overline{2.65963}$ , 8.65963,
.004567,	$\overline{3.65963}$ , 7.65963

This property, which is peculiar to Briggs's system, is of

great use in abridging the logarithmic tables. For when we have the logarithm of any number, we have only to change the index, to obtain the logarithm of every other number, whether integral, fractional, or mixed, consisting of the same significant figures. The decimal part of the logarithm of a fraction found in this way, is always *positive*. For it is the same as the decimal part of the logarithm of a whole number.

17. *If a series of numbers be in GEOMETRICAL progression, their logarithms will be in ARITHMETICAL progression.* For, in a geometrical series ascending, the quantities increase by a common *multiplier*; (Alg. 359.) That is, each succeeding term is the *product* of the preceding term into the ratio. But the *logarithm* of this product is the *sum* of the logarithms of the preceding term and the ratio; that is, the logarithms increase by a common *addition*, and are, therefore, in arithmetical progression. (Alg. 326.) In a geometrical progression *descending*, the terms decrease by a common *divisor*, and their logarithms, by a common *difference*.\*

Thus, the numbers 1, 10, 100, 1000, 10000, &c., are in geometrical progression.

And their logarithms 0, 1, 2, 3, 4, &c., are in arithmetical progression.

---

\* See Note A.

## SECTION II.

## DIRECTIONS FOR TAKING LOGARITHMS AND THEIR NUMBERS FROM THE TABLES.\*

ART. 24. The purpose which logarithms are intended to answer, is to enable us to perform arithmetical operations with *greater expedition*, than by the common methods. Before any one can avail himself of this advantage, he must become so familiar with the tables, that he can readily find the logarithm of any number; and, on the other hand, the number to which any logarithm belongs.

In the common tables, the *indices* to the logarithms of the first 100 numbers are inserted. But, for all other numbers, the *decimal* part only of the logarithm is given; while the index is left to be supplied, according to the principles in Arts. 8 and 11.

25. *To find the logarithm of any number between 1 and 100:*

Look for the proposed number, on the left; and against it, in the next column, will be the logarithm, with its index  
Thus,

The log. of 18 is 1.25527. The log. of 73 is 1.86332.

26. *To find the logarithm of any number between 100 and 1000; or of any number consisting of not more than three significant figures, with ciphers annexed.*

---

\* The best English Tables are Hutton's in 8vo. and Taylor's in 4to. In these, the logarithms are carried to seven places of decimals, and proportional parts are placed in the margin. The smaller tables are numerous; and, when accurately printed, are sufficient for common calculations.



In the smaller tables, the three first figures of each number, are generally placed in the left hand column ; and the fourth figure is placed at the head of the other columns.

Any number, therefore, between 100 and 1000, may be found on the left hand ; and directly opposite, in the next column, is the decimal part of its logarithm. To this the *index* must be prefixed, according to the rule in Art. 8.

The log. of 458 is 2.66087,    The log. of 935 is 2.97081,  
                   of 796    2.90091,                    of 386    2.58659.

If there are *ciphers* annexed to the significant figures, the logarithm may be found in a similar manner. For, by Art. 14, the *decimal* part of the logarithm of any number is the same, as that of the number multiplied into 10, 100, &c. All the difference will be in the *index* ; and this may be supplied by the same general rule.

The log. of 4580 is 3.66087,    The log. of 326000 is 5.51322,  
                   of 79600    4.90091,                    of 8010000    6.90363.

27. *To find the logarithm of any number consisting of FOUR figures, either with, or without, ciphers annexed.*

Look for the three first figures, on the left hand, and for the fourth figure, at the head of one of the columns. The logarithm will be found, opposite the three first figures, and in the column which, at the head, is marked with the fourth figure.\*

The log. of 6234 is 3.79477,    The log. of 783400 is 5.89398,  
                   of 5231    3.71858,                    of 6281000    6.79803.

28. *To find the logarithm of a number containing MORE than FOUR significant figures.*

By turning to the tables, it will be seen, that if the *differences* between several numbers be small, in comparison with the numbers themselves ; the differences of the *logarithms*

---

\* In Taylor's, Hutton's, and other tables, *four* figures are placed in the left hand column, and the *fifth* at the top of the page.

will be nearly proportioned to the differences of the *numbers*.  
Thus,

The log. of 1000 is 3.00000,	Here the differences in the
of 1001 3.00043,	numbers are, 1, 2, 3, 4, &c.,
of 1002 3.00087,	and the corresponding dif-
of 1003 3.00130,	ferences in the logarithms,
of 1004 3.00173, &c.	are 43, 87, 130, 173, &c.

Now 43 is nearly half of 87, one-third of 130, one-fourth of 173, &c

Upon this principle, we may find the logarithm of a number which is between two other numbers whose logarithms are given by the tables. Thus, the logarithm of 21716 is not to be found in those tables which give the numbers to four places of figures only.

But by the table, the log. of 21720 is 4.33686  
and the log. of 21710 is 4.33666

The difference of the two numbers is 10; and that of the logarithms 20.

Also, the difference between 21710, and the proposed number 21716, is 6.

If, then, a difference of 10 in the numbers  
make a difference of 20 in the logarithms :

A difference of 6 in the numbers will  
make a difference of 12 in the logarithms.

That is,  $10 : 20 :: 6 : 12$ .

If, therefore, 12 be added to 4.33666, the log. of 21710 ;

The sum will be 4.33678, the log. of 21716.

We have, then, this

#### RULE.

To find the logarithm of a number consisting of more than four figures :

Take out the logarithm of two numbers, one greater, and the other less, than the number proposed: Find the difference of the two numbers, and the difference of their logarithms: Take also the difference between the least of the two numbers, and the proposed number. Then say,

As the difference of the two numbers,  
 To the difference of their logarithms;  
 So is the difference between the least of the two numbers, and the proposed number,  
 To the proportional part to be added to  
 the least of the two logarithms.

It will generally be expedient to make the *first four figures*, in the least of the two numbers, the same as in the proposed number, substituting ciphers, for the remaining figures; and to make the greater number the same as the less, with the addition of a unit to the last significant figure. Thus,

For 36843,	take 36840,	and 36850,
For 792674,	792600,	792700,
For 6537825,	6537000,	6538000, &c.

The first term of the proportion will then be 10, or 100, or 1000, &c.

Ex. 1. Required the logarithm of 362572.

The logarithm of 362600 is 5.55943  
 of 362500 5.55931

The differences are 100, and 12.

Then  $100 : 12 :: 72 : 8.64$ , or 9 nearly.

And the log.  $5.55931 + 9 = 5.55940$ , the log. required.

Ex. 2. The log. of 78264 is 4.89356

3. The log. of 143542 is 5.15698

4. The log. of 1129535 is 6.05290.

By a little practice, such a facility in abridging these calculations may be acquired, that the logarithms may be taken

out, in a very short time. When great accuracy is not required, it will be easy to make an allowance sufficiently near, without formally stating a proportion. In the larger tables, the proportional parts which are to be added to the logarithms, are already prepared, and placed in the margin.

29. *To find the logarithm of a DECIMAL FRACTION.*

The logarithm of a decimal is the same as that of a whole number, excepting the *index*. (Art. 14.) To find then the logarithm of a decimal, take out that of a whole number consisting of the same figures; *observing to make the negative index equal to the distance of the first significant figure of the fraction from the place of units.* (Art. 11.)

The log. of 0.07643, is  $\overline{2}.88326$ , or 8.88326, (Art. 12.)

of 0.00259,  $\overline{3}.41330$ , or 7.41330,

of 0.0006278,  $\overline{4}.79782$ , or 6.79782.

30. *To find the logarithm of a MIXED decimal number.*

Find the logarithm, in the same manner as if *all* the figures were integers; and then prefix the index which belongs to the *integral* part, according to Art. 8.

The logarithm of 26.34 is 1.42062.

The index here is 1, because 1 is the index of the logarithm of every number greater than 10, and less than 100. (Art. 7.)

The log. of 2.36 is 0.37291, The log. of 364.2 is 2.56134,

of 27.8 1.44404, of 69.42 1.84148.

31. *To find the logarithm of a VULGAR FRACTION.*

From the nature of a vulgar fraction, the numerator may be considered as a *dividend*, and the denominator as a *divisor*; in other words, the value of the fraction is equal to the quotient of the numerator divided by the denominator. (Alg. 110.) But in logarithms, division is performed by *subtraction*; that is, the *difference* of the logarithms of two numbers, is the logarithm of the *quotient* of those numbers.



(Art. 1.) To find then the logarithm of a vulgar fraction, subtract the logarithm of the denominator from that of the numerator. The difference will be the logarithm of the fraction. Or the logarithm may be found, by first reducing the vulgar fraction to a *decimal*. If the numerator is less than the denominator, the index of the logarithm must be *negative*, because the value of the fraction is less than a unit.

(Art. 9.)

Required the logarithm of  $\frac{34}{87}$ .

The log. of the numerator is 1.53148  
of the denominator 1.93952

of the fraction 1.59196, or 9.59196.

The logarithm of  $\frac{362}{7854}$  is 2.66362, or 8.66362.  
of  $\frac{7}{6329}$  3.04376, or 7.04376.

32. If the logarithm of a *mixed number* is required, reduce it to an improper fraction, and then proceed as before.

The logarithm of  $3\frac{7}{9} = \frac{34}{9}$  is 0.57724.

33. To find the NATURAL NUMBER belonging to any logarithm.

In computing by logarithms, it is necessary, in the first place, to take from the tables the logarithms of the numbers which enter into the calculation; and, on the other hand, at the close of the operation, to find the number belonging to the logarithm obtained in the result. This is evidently done by *reversing* the methods in the preceding articles.

Where great accuracy is not required, look in the tables for the logarithm which is *nearest* to the given one; and directly opposite on the left hand, will be found the *three first* figures, and at the top, over the logarithm, the *fourth* figure of the number required. This number, by pointing off dec-

imals, or by adding ciphers, if necessary, must be made to correspond with the *index* of the given logarithm, according to Arts. 8 and 11.

The natural number belonging

to 3.86493 is 7327,      to 1.62572 is 42.24,  
to 2.90141    796.9,      to 2.89115    0.07783.

In the last example, the index requires that the first significant figure should be in the *second* place from units, and therefore a cipher must be prefixed. In other instances, it is necessary to annex ciphers on *the right*, so as to make the number of figures exceed the index by 1.

The natural number belonging

to 6.71567 is 5196000, to  $\overline{3.65677}$  is 0.004537,  
to 4.67062      46840, to  $\overline{4.59802}$     0.0003963.

34. When great accuracy is required, and the given logarithm is not exactly, or very nearly, found in the tables, it will be necessary to reverse the rule in Art. 28.

Take from the tables two logarithms, one the next greater, the other the next less than the given logarithm. Find the difference of the two logarithms, and the difference of their natural numbers; also the difference between the least of the two logarithms, and the given logarithm. Then say,

As the difference of the two logarithms,  
To the difference of their numbers;  
So is the difference between the given  
logarithm and the least of the other two,  
To the proportional part to be added to  
the least of the two numbers.

Required the number belonging to the logarithm 2.67325.

Next great.log.2.67330. Its numb. 471.3. Given log. 2.67325.

Next less      2.67321. Its numb. 471.2. Next less 2.67321.

Differences                      9                      0.1                      4

Then,  $9 : 0.1 :: 4 : 0.044$ , which is to be added  
to the number  $471.2$

The number required is  $\overline{471.244}$ .

The natural number belonging

to  $4.37627$  is  $23783.45$ , to  $\overline{1.73698}$  is  $54.57357$ ,

to  $3.69479$   $4952.08$ , to  $\overline{1.09214}$   $0.123635$ .

35. *Correction of the Tables.*—The tables of logarithms have been so carefully and so repeatedly calculated, by the ablest computers, that there is no room left to question their general correctness. They are not, however, exempt from the common imperfections of the press. But an error of this kind is easily corrected, by comparing the logarithm with any two others to whose *sum* or *difference* it ought to be equal. (Art. 1.)

Thus  $48 = 24 \times 2 = 16 \times 3 = 12 \times 4 = 8 \times 6$ . Therefore, the logarithm of 48 is equal to the *sum* of the logarithms of 24 and 2, of 16 and 3, &c.

And,  $3 = \frac{6}{2} = \frac{12}{4} = \frac{18}{6} = \frac{24}{8} = \frac{36}{12}$ , &c. Therefore, the logarithm of 3 is equal to the *difference* of the logarithms of 6 and 2, of 12 and 4, &c.

## SECTION III.

## METHODS OF CALCULATING BY LOGARITHMS.

ART. 36. The arithmetical operations for which logarithms were originally contrived, and on which their great utility depends, are chiefly multiplication, division, involution, evolution, and finding the term required in single and compound proportion. The principle on which all these calculations are conducted, is this :

*If the logarithms of two numbers be added, the SUM will be the logarithm of the PRODUCT of the numbers ; and,*

*If the logarithm of one number be subtracted from that of another, the DIFFERENCE will be the logarithm of the QUOTIENT of one of the numbers divided by the other.*

In proof of this, we have only to call to mind, that logarithms are the EXPONENTS of a series of powers and roots. (Arts. 2, 5.) And it has been shown, that powers and roots are multiplied by adding their exponents ; and divided, by subtracting their exponents. (Alg. 189, 193, 232, 239.)

## MULTIPLICATION BY LOGARITHMS.

37. ADD THE LOGARITHMS OF THE FACTORS : THE SUM WILL BE THE LOGARITHM OF THE PRODUCT.

In making the addition, 1 is to be carried for every 10, from the decimal part of the logarithm, to the index. (Art. 7.)

	Numbers.	Logarithms.		Numbers.	Logarithms.
Mult.	36.2 (Art. 30.)	1.55871	Mult.	640	2.80618
Into	7.84	0.89432	Into	2.316	0.36474
Prod.	<u>283.8</u>	<u>2.45303</u>	Prod.	<u>1482</u>	<u>3.17092</u>

The logarithms of the two factors are taken from the



tables. The product is obtained, by finding, in the tables, the natural number belonging to the sum. (Art. 33.)

Mult. 89.24	1.95056	Mult. 134.	2.12710
Into 3.687	0.56667	Into 25.6	1.40824
Prod. <u>329.</u>	<u>2.51723</u>	Prod. <u>3430</u>	<u>3.53534</u>

38. When any or all of the indices of the logarithms are *negative*, they are to be added according to the rules for the addition of positive and negative quantities in algebra. But it must be kept in mind, that the decimal part of the logarithm is *positive*. (Art. 10.) Therefore, that which is carried from the decimal part to the index, must be considered positive also.

Mult. 62.84	1.79824	Mult. 0.0294	<u>2.46835</u>
Into 0.682	1.83378	Into 0.8372	<u>1.92283</u>
Prod. <u>42.86</u>	<u>1.63202</u>	Prod. <u>0.0246</u>	<u>2.39118</u>

In each of these examples, +1 is to be carried from the decimal part of the logarithm. This, added to -1, the lower index, makes it 0; so that there is nothing to be added to the upper index.

If any perplexity is occasioned, by the addition of positive and negative quantities, it may be avoided, by borrowing 10 to the index. (Art. 12.)

Mult. 62.84	1.79824	Mult. 0.0294	8.46835
Into 0.682	9.83378	Into 0.8372	9.92283
Prod. <u>42.86</u>	<u>1.63202</u>	Prod. <u>0.0246</u>	<u>8.39118</u>

Here 10 is added to the negative indices, and afterwards rejected from the index of the sum of the logarithms.

Multiply	26.83	1.42862	1.42862
Into	0.00069	4.83885	or 6.83885
Product	<u>0.01851</u>	<u>2.26747</u>	<u>8.26747</u>

Here +1 carried to -4 makes it -3, which added to the upper index +1, gives -2 for the index of the sum.

Multiply	.00845	<u>3.92686</u>	or	7.92686
Into	1068.	3.02857		3.02857
Product	<u>9.0246</u>	<u>0.95543</u>		<u>0.95543</u>

The product of 0.0362 into 25.38 is 0.9188  
of 0.00467 into 348.1 is 1.626  
of 0.0861 into 0.00843 is 0.0007258

39. *Any number of factors* may be multiplied together, by adding their logarithms. If there are several *positive*, and several *negative* indices, these are to be reduced to one, as in algebra, by taking the difference between the sum of those which are negative, and the sum of those which are positive, increased by what is carried from the decimal part of the logarithms. (Alg. 53.)

Multiply	6832	<u>3.83455</u>		3.83455
Into	0.00863	<u>3.93601</u>	or	7.93601
And	0.651	<u>1.81358</u>		9.81358
And	0.0231	<u>2.36361</u>	or	8.36361
And	62.87	1.79844		1.79844
Prod.	<u>55.74</u>	<u>1.74619</u>		<u>1.74619</u>

**Ex. 2.** The prod. of  $36.4 \times 7.82 \times 68.91 \times 0.3846$  is 7544.

**3.** The prod. of  $0.00629 \times 2.647 \times 0.082 \times 278.8 \times 0.00063$  is 0.0002398.

40. *Negative* quantities are multiplied, by means of logarithms, in the same manner as those which are positive. (Art. 16.) But, after the operation is ended, the proper sign must be applied to the natural number expressing the product, according to the rules for the multiplication of positive and negative quantities in algebra. The negative index of a *logarithm*, must not be confounded with the sign which denotes that the *natural number* is negative. That which the index

of the logarithm is intended to show, is not whether the natural number is *positive or negative*, but whether it is *greater or less than a unit*. (Art. 16.)

Mult.	+36.42	1.56134	Mult.	-2.681	0.42830
Into	-67.31	1.82808	Into	+37.24	1.57101
Prod.	-2451	3.38942	Prod.	-99.84	1.99931

In these examples, the logarithms are taken from the tables, and added, in the same manner, as if both factors were positive. But after the product is found, the negative sign is prefixed to it, because + is multiplied into -. (Alg. 82.)

Mult.	0.263	1.41996	Mult.	0.065	2.81291
Into	0.00894	3.95134	Into	0.693	1.84073
Prod.	0.002351	3.37130	Prod.	0.04504	2.65364

Here the indices of the logarithms are negative, but the product is positive, because the factors are both positive.

Mult.	-62.59	1.79650	Mult.	-68.3	1.83442
Into	-0.00863	3.93601	Into	-0.0096	3.98227
Prod.	+0.5402	1.73251	Prod.	+0.6557	1.81669

DIVISION BY LOGARITHMS.

41. FROM THE LOGARITHM OF THE DIVIDEND, SUBTRACT THE LOGARITHM OF THE DIVISOR; THE DIFFERENCE WILL BE THE LOGARITHM OF THE QUOTIENT. (Art. 36.)

	Numbers.	Logarithms.		Numbers.	Logarithms.
Divide	6238	3.79505	Divide	896.3	2.95245
By	2982	3.47451	By	9.847	0.99330
Quot.	2.092	0.32054	Quot.	91.02	1.95915

42. The *decimal* part of the logarithm may be subtracted

as in common arithmetic. But for the *indices*, when either of them is negative, or the lower one is greater than the upper one, it will be necessary to make use of the general rule for subtraction in algebra; that is, to change the signs of the subtrahend, and then proceed as in addition. (Alg. 60.) When 1 is carried from the decimal part, this is to be considered affirmative, and applied to the index, before the sign is changed.

$$\begin{array}{r}
 \text{Divide } 0.8697 \quad \overline{1.93937} \text{ or } 9.93937 \\
 \text{By } \quad 98.65 \quad \underline{1.99410} \quad \underline{1.99410} \\
 \text{Quot. } \quad \underline{0.008816} \quad \underline{3.94527} \quad \underline{7.94527}
 \end{array}$$

In this example, the upper logarithm being less than the lower one, it is necessary to borrow 10, as in other cases of subtraction; and therefore to carry one to the lower index, which then becomes +2. This changed to -2, and added to -1 above it, makes the index of the difference of the logarithms -3.

$$\begin{array}{r}
 \text{Divide } 29.76 \quad 1.47363 \quad 1.47363 \\
 \text{By } \quad 6254 \quad \underline{3.79616} \quad \underline{3.79616} \\
 \text{Quot. } \quad \underline{0.00476} \quad \underline{3.67747} \text{ or } \underline{7.67747}
 \end{array}$$

Here, 1 carried to the lower index, makes it +4. This changed to -4, and added to 1 above it, gives -3 for the index of the difference of the logarithms.

$$\begin{array}{r}
 \text{Divide } 6.832 \quad \underline{0.83455} \quad \text{Divide } 0.00634 \quad \overline{3.80209} \\
 \text{By } \quad .0362 \quad \underline{2.55871} \quad \text{By } \quad 62.18 \quad \underline{1.79365} \\
 \text{Quot. } \underline{188.73} \quad \underline{2.27584} \quad \text{Quot. } \underline{0.000102} \quad \underline{4.00844}
 \end{array}$$

The quotient of 0.0985 divided by 0.007241, is 13.6

The quotient of 0.0621 divided by 3.68, is 0.01687

43. To divide *negative* quantities, proceed in the same manner as if they were positive, (Art. 40.) and prefix to the



quotient, the sign which is required by the rules for division in algebra.

Divide	+3642	3.56134	Divide	-0.657	<u>1.81757</u>
By	<u>-23.68</u>	<u>1.37438</u>	By	<u>+0.0793</u>	<u>2.89927</u>
Quot.	-153.8	2.18696	Quot.	-8.285	0.91830

In these examples, the sign of the divisor being different from that of the dividend, the sign of the quotient must be negative. (Alg. 100.)

Divide	-0.364	<u>1.56110</u>	Divide	-68.5	<u>1.83569</u>
By	<u>-2.56</u>	<u>0.40824</u>	By	<u>+0.094</u>	<u>2.97313</u>
Quot.	<u>+0.1422</u>	<u>1.15286</u>	Quot.	<u>-728.7</u>	<u>2.86256</u>

## INVOLUTION BY LOGARITHMS.

44. Involving a quantity is multiplying it into itself. By means of logarithms, multiplication is performed by addition. If, then, the logarithm of any quantity be *added to itself*, the logarithm of a *power* of that quantity will be obtained. But adding a logarithm, or any other quantity, to itself, is *multiplication*. The involution of quantities, by means of logarithms, is therefore performed, by multiplying the logarithms.

Thus the logarithm

of 100	is 2	
of $100 \times 100$ , that is, of $\overline{100^2}$	is $2+2$	$=2 \times 2$
of $100 \times 100 \times 100$ ,	$\overline{100^3}$ is $2+2+2$	$=2 \times 3$
of $100 \times 100 \times 100 \times 100$ ,	$\overline{100^4}$ is $2+2+2+2$	$=2 \times 4$

On the same principle, the logarithm of  $\overline{100^n}$  is  $2 \times n$ .

And the logarithm of  $x^n$ , is  $(\log x) \times n$ . Hence,

45. To involve a quantity by logarithms, **MULTIPLY THE LOGARITHM OF THE QUANTITY, BY THE INDEX OF THE POWER REQUIRED.**

The reason of the rule is also evident, from the consideration, that logarithms are the exponents of powers and roots, and a power or root is involved, by *multiplying* its index into the index of the power required. (Alg. 170, 242.)

Ex. 1. What is the cube of	6.296 ?
Root 6.296,	its log. 0.79906
	Index of the power 3
Power 249.6	<u>2.39718</u>

2. Required the 4th power of	21.32
Root 21.32	log. 1.32879
	Index 4
Power 206614	<u>5.31516</u>

3. Required the 6th power of	1.689
Root 1.689	log. 0.22763
	Index 6
Power 23.215	<u>1.36578</u>

4. Required the 144th power of	1.003
Root 1.003	log. 0.00130
	Index 144
Power 1.539	<u>0.18720</u>

46. It must be observed, as in the case of multiplication, (Art. 38.) that what is carried from the *decimal* part of the logarithm is *positive*, whether the index itself is positive or negative. Or, if 10 be added to a negative index, to render it positive, (Art. 12.) this will be multiplied, as well as the other figures, so that the logarithm of the square, will be 20 too great; of the cube, 30 too great, &c.

Ex. 1. Required the cube of	0.0649
Root 0.0649	log. <u>2.81224</u> or 8.81224
	Index 3
Power 0.0002733	<u>4.43672</u> <u>6.43672</u>

2. Required the 4th power of 0.1234  
 Root 0.1234 log.  $\overline{1.09132}$  or 9.09132  
 Index 4 4

Power 0.0002319  $\overline{4.36528}$  6.36528

3. Required the 6th power of 0.9977  
 Root 0.9977 log.  $\overline{1.99900}$  or 9.99900  
 Index 6 6

Power 0.9863  $\overline{1.99400}$  9.99400

4. Required the cube of 0.08762  
 Root 0.08762 log.  $\overline{2.94260}$  or 8.94260  
 Index 3 3

Power 0.0006727  $\overline{4.82780}$  6.82780

5. The 7th power of 0.9061 is 0.5015.

6. The 5th power of 0.9344 is 0.7123.

## EVOLUTION BY LOGARITHMS.

47. Evolution is the opposite of involution. Therefore, as quantities are involved, by the *multiplication* of logarithms, roots are extracted by the *division* of logarithms; that is,

To extract the root of a quantity by logarithms, DIVIDE THE LOGARITHM OF THE QUANTITY, BY THE NUMBER EXPRESSING THE ROOT REQUIRED.

The reason of the rule is evident also, from the fact, that logarithms are the exponents of powers and roots, and evolution is performed, by dividing the exponent, by the number expressing the root required. (Alg. 210.)

1. Required the square root of 648.3

	Numbers.	Logarithms.
Power	648.3	2)2.81178
Root	25.46	1.40589

2. Required the cube root of 897.1

Power	897.1	3)	2.95284
Root	9.645		0.98428

In the first of these examples, the logarithm of the given number is divided by 2; in the other, by 3.

3. Required the 10th root of 6948.

Power	6948	10)	3.84186
Root	2.422		0.38418

4. Required the 100th root of 983.

Power	983	100)	2.99255
Root	1.071		0.02992

The division is performed here, as in other cases of decimals, by removing the decimal point to the left.

5. What is the ten thousandth root of 49680000?

Power	49680000	10000)	7.69618
Root	1.00179		0.00077

We have, here, an example of the great rapidity with which arithmetical operations are performed by logarithms.

48. If the index of the logarithm is *negative*, and is *not divisible* by the given divisor, without a remainder, a difficulty will occur, unless the index be altered.

Suppose the cube root of 0.0000892 is required. The logarithm of this is 5.95036. If we divide the index by 3, the quotient will be  $-1$ , with  $-2$  remainder. This remainder, if it were positive, might, as in other cases of division, be prefixed to the next figure. But the remainder is *negative*, while the decimal part of the logarithm is positive; so that, when the former is prefixed to the latter, it will make neither  $+2.9$  nor  $-2.9$ , but  $-2+.9$ . This embarrassing intermixture of positives and negatives may be avoided, by adding to the index another negative number, to make it ex-



actly divisible by the divisor. Thus, if to the index  $-5$  there be added  $-1$ , the sum  $-6$  will be divisible by 3. But this addition of a negative number must be *compensated*, by the addition of an equal positive number, which may be prefixed to the decimal part of the logarithm. The division may then be continued, without difficulty, through the whole.

Thus, if the logarithm  $\overline{5}.95036$  be altered to  $\overline{6}+1.95036$  it may be divided by 3, and the quotient will be  $\overline{2}.65012$ . We have then this rule,

49. *Add to the index, if necessary, such a negative number as will make it exactly divisible by the divisor, and prefix an equal positive number to the decimal part of the logarithm.*

1. Required the 5th root of	0.009642
Power 0.009642 log.	$\overline{3}.98417$
	or $\overline{5}+2.98417$
Root 0.3952	$\overline{1}.59683$

2. Required the 7th root of	0.0004935.
Power 0.0004935 log.	$\overline{4}.69329$
	or $7\overline{)7}+3.69329$
Root 0.337	$\overline{1}.52761$

50. If, for the sake of performing the division conveniently, the negative index be rendered *positive*, it will be expedient to borrow as many tens, as there are units in the number denoting the root.

What is the fourth root of 0.03698 ?

Power 0.03698	$4\overline{)2}.56797$	or $4\overline{)38}.56797$
Root 0.4385	$\overline{1}.64199$	9.64199

Here the index, by borrowing, is made 40 too great, that is,  $+38$  instead of  $-2$ . When, therefore, it is divided by 4, it is still 10 too great,  $+9$  instead of  $-1$ .

What is the 5th root of 0.008926?

Power	0.008926	5) $\overline{3.95066}$	or 5) $\overline{47.95066}$
Root	0.38916	$\overline{1.59013}$	9.59013

51. A power of a root may be found by first *multiplying* the logarithm of the given quantity into the index of the power, (Art. 45.) and then *dividing* the product by the number expressing the root. (Art. 47.)

1. What is the value of  $(53)^{\frac{6}{7}}$ , that is, the 6th power of the 7th root of 53?

Given number 53	log.	1.72428
Multiplying by		<u>6</u>
Dividing by	7)	$\overline{10.34568}$
Power required	30.06	1.47795

2. What is the 8th power of the 9th root of 654?

#### PROPORTION BY LOGARITHMS.

52. In a proportion, when three terms are given, the fourth is found in common arithmetic, by multiplying together the second and third, and dividing by the first. But when logarithms are used, *addition* takes the place of multiplication, and *subtraction*, of division.

To find, then, by logarithms, the fourth term in a proportion, ADD THE LOGARITHMS OF THE SECOND AND THIRD TERMS, AND *from the sum* SUBTRACT THE LOGARITHM OF THE FIRST TERM. The remainder will be the logarithm of the term required.

Ex. 1. Find a fourth proportional to 7964, 378, and 27960.

	Numbers.	Logarithms.
Second term	378	2.57749
Third term	27960	<u>4.44654</u>
		7.02403
First term	7964	<u>3.90113</u>
Fourth term	1327	3.12290

2. Find a 4th proportional to 768, 381, and 9780.

Second term	381	2.58092
Third term	9780	<u>3.99034</u>
		6.57126
First term	768	<u>2.88536</u>
Fourth term	4852	3.68590

## ARITHMETICAL COMPLEMENT.

53. When one number is to be subtracted from another, it is often convenient, first to subtract it from 10, then to *add the difference* to the other number, and afterwards to reject the 10.

Thus, instead of  $a-b$ , we may put  $10-b+a-10$ .

In the first of these expressions,  $b$  is subtracted from  $a$ . In the other,  $b$  is subtracted from 10, the difference is added to  $a$ , and 10 is afterwards taken from the sum. The two expressions are equivalent, because they consist of the same terms, with the addition, in one of them, of  $10-10=0$ . The alteration is, in fact, nothing more than borrowing 10, for the sake of convenience, and then rejecting it in the result.

Instead of 10, we may borrow, as occasion requires, 100, 1000, &c.

Thus,  $a-b=100-b+a-100=1000-b+a-1000$ , &c.

54. The DIFFERENCE between a given number and 10, or 100, or 1000, &c., is called the ARITHMETICAL COMPLEMENT of that number.

The arithmetical complement of a number consisting of one integral figure, either with or without decimals, is found, by subtracting the number from 10. If there are two integral figures, they are subtracted from 100; if three, from 1000, &c.

Thus, the arithmetical compl't of 3.46 is  $10-3.46=6.54$

of 34.6 is  $100-34.6=65.4$

of 346. is  $1000-346.=654$ . &c.

According to the rule for subtraction in arithmetic, any number is subtracted from 10, 100, 1000, &c. by beginning on the right hand, and taking each figure from 10, after *increasing* all except the first, by *carrying* 1.

Thus, if from	10.00000
We subtract	<u>7.63125</u>

The difference, or arith'l compl't is 2.36875, which is obtained by taking 5 from 10, 3 from 10, 2 from 10, 4 from 10, 7 from 10, and 8 from 10. But, instead of taking each figure, *increased by 1* from 10; we may take it *without being increased*, from 9.

Thus, 2 from 9 is the same as 3 from 10,

3 from 9 the same as 4 from 10, &c. Hence,

55. To obtain the ARITHMETICAL COMPLEMENT of a number, subtract the right hand significant figure from 10, and each of the other figures from 9. If, however, there are ciphers on the right hand of all the significant figures, they are to be set down without alteration.

In taking the arithmetical complement of a logarithm, if the index is *negative*, it must be *added* to 9; for adding a negative quantity is the same as subtracting a positive one. (Alg. 81.) The difference between  $-3$  and  $+9$ , is not 6, but 12.

The arithmetical complement

of 6.24897 is 3.75103	of $\overline{2}$ .70649 is 11.29351
of 2.98643 7.01357	of 3.64200 6.35800
of 0.62430 9.37570	of 9.35001 0.64999

56. The principal use of the arithmetical complement, is in working proportions by logarithms; where some of the terms are to be *added*, and one or more to be *subtracted*. In the Rule of Three or simple proportion, two terms are to be added, and from the sum, the first term is to be subtracted. But if, instead of the logarithm of the first term,



we substitute its arithmetical complement, this may be *added* to the sum of the other two, or more simply all three may be added together, by one operation. After the index is diminished by 10, the result will be the same as by the common method. For subtracting a number is the same, as adding its arithmetical complement, and then rejecting 10, 100, or 1000, from the sum. (Art. 53.)

It will generally be expedient, to place the terms in the same order, in which they are arranged in the statement of the proportion.

<p>1. As 6273 <i>a. c.</i> 6.20252                  Is to 769.4      2.88615                  So is 37.61      <u>1.57530</u>                  To 4.613      <u>0.66397</u></p>	<p>2. As 253 <i>a. c.</i> 7.59688                  Is to 672.5      2.82769                  So is 497      <u>2.69636</u>                  To 1321.1      <u>3.12093</u></p>
<p>3. As 46.34 <i>a. c.</i> 8.33404                  Is to 892.1      2.95041                  So is 7.638      <u>0.88298</u>                  To 147      <u>2.16743</u></p>	<p>4. As 9.85 <i>a. c.</i> 9.00656                  Is to 643      2.80821                  So is 76.3      <u>1.88252</u>                  To 4981      <u>3.69729</u></p>

COMPOUND PROPORTION.

57. In compound, as in single proportion, the term required may be found by logarithms, if we substitute addition for multiplication, and subtraction for division.

Ex. 1. If the interest of \$365, for 3 years and 9 months, be \$82.13; what will be the interest of \$8940, for 2 years and 6 months?

In common arithmetic, the statement of the question is made in this manner.

$$\left. \begin{array}{l} 365 \text{ dollars} \\ 3.75 \text{ years} \end{array} \right\} : \left\{ \begin{array}{l} 8940 \text{ dollars} \\ 2.5 \text{ years} \end{array} \right\} :: 82.13 \text{ dollars} :$$



And the method of calculation is, to *divide* the *product* of the third, fourth, and fifth terms, by the *product* of the first two.\* This, if logarithms are used, will be to *subtract* the *sum* of the logarithms of the first two terms, from the *sum* of the logarithms of the other three.

First two terms	{	365 log.	2.56229
		3.75	0.57403
Sum of the logarithms			3.13632
Third and fourth terms	{	8940	3.95134
		2.5	0.39794
Fifth term		82.13	1.91450
Sum of the logs. of the 3rd, 4th, and 5th,			6.26378
Do.		1st and 2nd,	3.13632
Term required		1341	3.12746

58. The calculation will be more simple, if, instead of *subtracting* the logarithms of the first two terms, we *add* their *arithmetical complements*. But, it must be observed, that *each* arithmetical complement increases the index of the logarithm by 10. If the arithmetical complement be introduced into *two* of the terms, the index of the sum of the logarithms will be 20 too great; if it be in *three* terms, the index will be 30 too great, &c.

First two terms	{	365 a. c.	7.43771
		3.75 a. c.	9.42597
Third and fourth terms	{	8940	3.95134
		2.5	0.39794
Fifth term		82.13	1.91450
Term required		1341	23.12746

The result is the same as before, except that the index of the logarithm is 20 too great.

---

\* See Arithmetic.

Ex. 2. If the wages of 53 men for 42 days be 2200 dollars ; what will be the wages of 87 men for 34 days ?

53 men	}	:	{	87 men	}	::	2200 :
42 days	}		{	34 days	}		
First two terms	{	53. a. c.	8.27572	42. a. c.	8.37675		
Third and fourth terms	{	87	1.93952	34	1.53148		
Fifth term		2200	<u>3.34242</u>				
Term required		2923.5	<u>3.46589</u>				

59. In the same manner, if the product of *any number* of quantities, is to be divided, by the product of several others ; we may add together the logarithms of the quantities to be divided, and the arithmetical complements of the logarithms of the divisors.

Ex. If  $29.67 \times 346.2$  be divided by  $69.24 \times 7.862 \times 497$  ; what will be the quotient ?

Numbers to be divided	{	29.67	1.47232	346.2	2.53933		
Divisors	{	69.24 a. c.	8.15964	7.862 a. c.	9.10447	497 a. c.	<u>7.30364</u>
Quotient		0.03797	<u>8.5794</u>				

In this way, the calculations in *Conjoined Proportion* may be expeditiously performed.

60. In calculating compound interest, the amount for the first year, is made the principal for the second year ; the amount for the second year, the principal for the third

year, &c. Now the amount at the end of each year, must be proportioned to the principal at the beginning of the year. If the principal for the first year be 1 dollar, and if the amount of 1 dollar for 1 year= $a$ ; then, (Alg. 341.)

$$1 : a :: \begin{cases} a : a^2 = \text{the amount for the 2d year, or the principal for the 3d;} \\ a^2 : a^3 = \text{the amount for the third year, or the principal for the 4th;} \\ a^3 : a^4 = \text{the amount for the 4th year, or the principal for the 5th.} \end{cases}$$

That is, the amount of 1 dollar for any number of years is obtained by finding the amount for 1 year, and involving this to a power whose index is equal to the number of years. And the amount of any other principal, for the given time, is found by multiplying the amount of 1 dollar, into the number of dollars, or the fractional part of a dollar.

If logarithms are used, the *multiplication* required here may be performed by *addition*; and the *involution* by *multiplication*. (Art. 45.) Hence,

61. To calculate Compound Interest, *Find the amount of 1 dollar for 1 year; multiply its logarithm by the number of years; and to the product, add the logarithm of the principal.* The sum will be the logarithm of the *amount* for the given time. From the amount subtract the principal, and the remainder will be the *interest*.

If the interest becomes due *half yearly* or *quarterly*; find the amount of one dollar, for the half year or quarter, and multiply the logarithm by the number of half years or quarters in the given time.

If  $P$  = the principal,

$a$  = the amount of 1 dollar for 1 year,

$n$  = any number of years, and

$A$  = the amount of the given principal for  $n$  years; then,

$$A = a^n \times P.$$

Taking the logarithms of both sides of the equation, and reducing it, so as to give the value of each of the four quantities, in terms of the others, we have

$$1. \text{ Log. } A = n \times \text{log. } a + \text{log. } P.$$

$$2. \text{ Log. } P = \text{log. } A - n \times \text{log. } a.$$

$$3. \text{ Log. } a = \frac{\text{log. } A - \text{log. } P}{n}$$

$$4. \quad n = \frac{\text{log. } A - \text{log. } P}{\text{log. } a}.$$

Any three of these quantities being given, the fourth may be found.

Ex. 1. What is the amount of 20 dollars, at 6 per cent. compound interest, for 100 years ?

Amount of 1 dollar for 1 year	1.06	log.	0.0253059
Multiplying by			100
			2.53059
Given principal	20		1.30103
Amount required	\$6786		3.83162

2. What is the amount of 1 cent at 6 per cent. compound interest, in 500 years ?

Amount of 1 dollar for 1 year	1.06	log.	0.0253059
Multiplying by			500
			12.65295
Given principal	0.01		-2.00000
Amount	\$44,973,000,000		10.65295

More exact answers may be obtained, by using logarithms of a greater number of decimal places.

3. What is the amount of 1000 dollars, at 6 per cent. compound interest, for 10 years ? Ans. 1790.80.



4. What principal, at 4 per cent. interest, will amount to 1643 dollars in 21 years?      Ans. 721.

5. What principal, at 6 per cent., will amount to 202 dollars in 4 years?      Ans. 160.

6. At what rate of interest, will 400 dollars amount to  $569\frac{1}{3}$ , in 9 years?      Ans. 4 per cent.

7. In how many years will 500 dollars amount to 900, at 5 per cent. compound interest?      Ans. 12 years.

8. In what time will 10,000 dollars amount to 16,288, at 5 per cent compound interest?      Ans. 10 years.

9. At what rate of interest, will 11,106 dollars amount to 20,000 in 15 years?      Ans. 4 per cent.

10. What principal, at 6 per cent. compound interest, will amount to 3188 dollars in 8 years?      Ans. \$2000.

11. What will be the amount of 1200 dollars, at 6 per cent compound interest, in 10 years, if the interest is converted into principal every *half year*?      Ans. 2167.3 dolls.

12. In what time will a sum of money double, at 6 per cent compound interest?      Ans. 11.9 years.

13. What is the amount of 5000 dollars, at 6 per cent. compound interest, for  $28\frac{1}{4}$  years?      Ans. 25.942 dollars.

INCREASE OF POPULATION.

62. The natural increase of population in a country, may be calculated in the same manner as compound interest; on the supposition, that the yearly rate of increase is regularly proportioned to the actual number of inhabitants. From the population at the beginning of the year, the *rate* of increase being given, may be computed the whole increase during the year. This, *added* to the number at the beginning, will give the amount, on which the increase of the *second* year is to be calculated, in the same manner as the



first year's interest on a sum of money, added to the sum itself, gives the amount on which the interest for the second year is to be calculated.

If  $P$  = the population at the beginning of the year,  
 $a = 1 +$  the fraction which expresses the rate of increase,  
 $n$  = any number of years; and  
 $A$  = the amount of the population at the end of  $n$  years;  
 then, as in the preceding article,

$$A = a^n \times P, \text{ and}$$

$$1. \text{ Log. } A = n \times \text{log. } a + \text{log. } P.$$

$$2. \text{ Log. } P = \text{log. } A - n \times \text{log. } a.$$

$$3. \text{ Log. } a = \frac{\text{log. } A - \text{log. } P}{n}$$

$$4. \quad n = \frac{\text{log. } A - \text{log. } P}{\text{log. } a}$$

Ex. 1. The population of the United States in 1840 was (in round numbers) 17,070,000.\* Supposing the yearly rate

\* For some very interesting views of the progress of population, &c., in the United States, see Prof. George Tucker's elaborate essays, first published in the Merchant's Magazine, 1842—3, and subsequently in a separate volume.

The following tables show the official census of the United States from 1790 to 1840 with the decennial rate of increase.

POPULATION.					
1790.	1800.	1810.	1820.	1830.	1840.
3,929,827	5,305,925	7,239,814	9,638,131	12,866,020	17,069,453

DECENNIAL INCREASE.				
1800.	1810.	1820.	1830.	1840.
35.02	36.45	33.35	33.26	33.67



7. If the population of a country, at a given time, be seven millions; and if the yearly rate of increase be  $\frac{1}{38}$ th; what will be the population at the end of 35 years?

8. The population of the United States in 1800 was 5,306,000. What was it in 1780, supposing the yearly rate of increase to be  $\frac{1}{38}$ ?

9. In what time will the population of a country advance, from four millions to seven millions, if the ratio of increase be  $\frac{3}{100}$ ?

10. What must be the rate of increase, that the population of a place may change from nine thousand to fifteen thousand, in 12 years?

If the population of a country is not affected by immigration or emigration, the rate of increase will be equal to the difference between the ratio of the *births*, and the ratio of the *deaths*, when compared with the whole population.

Ex. 11. If the population of a country, at any given time, be ten millions; and the ratio of the annual number of births to the whole population be  $\frac{1}{4}$ , and the ratio of deaths  $\frac{1}{5}$ , what will be the number of inhabitants, at the end of 60 years?

Here the yearly rate of increase  $= \frac{1}{4} - \frac{1}{5} = \frac{7}{200}$ .

And the population, at the end of 60 years  $= 31,750,000$ .

The rate of increase or decrease from *immigration* or *emigration*, will be equal to the difference between the ratio of immigration and the ratio of emigration; and if this difference added to, or subtracted from, the difference between the ratio of the births and that of the deaths, the whole rate of increase will be obtained.

Ex. 12. If in a country, the ratio of births be  $\frac{1}{3}$ ,  
 the ratio of deaths  $\frac{1}{5}$ ,  
 the ratio of immigration  $\frac{1}{6}$ ,  
 the ratio of emigration  $\frac{1}{8}$ .

and if the population this year be 10 millions, what will it be 20 years hence ?

The rate of the natural increase  $= \frac{1}{30} - \frac{1}{40} = \frac{1}{120}$  ;

That of increase from immigration  $= \frac{1}{50} - \frac{1}{60} = \frac{1}{300}$  ;

The sum of the two is  $\frac{7}{600}$  ;

And the population at the end of 20 years, is 12,611,000.

13. If the ratio of the births be  $\frac{1}{20}$ ,

of the deaths  $\frac{1}{30}$ ,

of immigration  $\frac{1}{40}$ ,

of emigration  $\frac{1}{50}$ ,

in what time will three millions increase to four and a half millions ?

If the period in which the population will *double* be given ; the numbers for several successive periods, will evidently be in a geometrical progression, of which the ratio is 2 ; and as the number of periods will be one less than the number of terms ;

If P = the first term,

A = the last term,

n = the number of periods ;

Then will  $A = P \times 2^n$ , (Alg. 439.)

Or  $\log. A = \log. P + n \times \log. 2$ .

Ex 1. If the descendants of a single pair double once in 25 years, what will be their number at the end of one thousand years ?

The number of periods here is 40.

And  $A = 2 \times 2^{40} = 2,199,200,000,000$ .

2. If the descendants of Noah, beginning with his three sons and their wives, doubled once in 20 years for 300 years, what was their number, at the end of this time ?

Ans. 196,608.

3. The population of the United States in 1820 being



9,638,000; what must it be in the year 2020, supposing it to double once in 25 years?      Ans. 2,467,333,000.

4. Supposing the descendants of the first human pair to double once in 50 years, for 1650 years, to the time of the deluge, what was the population of the world, at that time?

EXPONENTIAL EQUATIONS.

62. An EXPONENTIAL equation is one in which the letter expressing the unknown quantity is an *exponent*.

Thus  $a^x=b$ , and  $x^x=bc$ , are exponential equations. These are most easily solved by logarithms. As the two members of an equation are equal, their logarithms must also be equal. If the logarithm of each side be taken, the equation may then be reduced, by the rules given in algebra.

Ex. What is the value of  $x$  in the equation  $3^x=243$ ?

Taking the logarithms of both sides,  $\log. 3^x=\log. 243$ .

But the logarithm of a *power* is equal to the logarithm of the root, multiplied into the index of the power. (Art. 45.)

Therefore  $(\log. 3) \times x = \log. 243$ ; and dividing by  $\log. 3$ .

$$x = \frac{\log. 243}{\log. 3} = \frac{2.38561}{0.47712} = 5. \quad \text{So that } 3^5=243.$$

64. The exponent of a power may be itself a power, as in the equation

$$a^{m^x}=b;$$

where  $x$  is the exponent of the power  $m^x$ , which is the exponent of the power  $a^{m^x}$ .

Ex. 4. Find the value of  $x$ , in the equation  $9^{3^x}=1000$ .

$$3^x \times (\log. 9) = \log. 1000. \quad \text{Therefore, } 3^x = \frac{\log. 1000}{\log. 9} = 3.14.$$



Then, as  $3^x = 3.14$ .  $x(\log. 3) = \log. 3.14$

$$\text{Therefore, } x = \frac{\log. 3.14}{\log. 3} = \frac{.4969296}{.4771213} = 1.04.$$

In cases like this, where the factors, divisors, &c. are logarithms, the calculation may be facilitated, by taking the *logarithms of the logarithms*. Thus the value of the fraction  $\frac{.4969296}{.4771213}$  is most easily found, by subtracting the logarithm of the logarithm which constitutes the denominator, from the logarithm of that which forms the numerator.

5. Find the value of  $x$ , in the equation  $\frac{ba^x + d}{c} = m$

$$\text{Ans. } x = \frac{\log. (cm - d) - \log. b.}{\log. a.}$$

# TRIGONOMETRY.

---

## SECTION I.

### SINES, TANGENTS, SECANTS, &C.

ART. 71. TRIGONOMETRY treats of the relations of the sides and angles of TRIANGLES. Its first object is to determine the length of the sides, and the quantity of the angles. In addition to this, from its principles are derived many interesting methods of investigation in the higher branches of analysis, particularly in physical astronomy.

72. Trigonometry is either *plane* or *spherical*. The former treats of triangles bounded by *right lines*; the latter, of triangles bounded by *arcs of circles*.

#### *Divisions of the Circle.*

73. In a triangle there are two classes of quantities which are the subjects of inquiry, the *sides* and the *angles*. For the purpose of measuring the latter, a *circle* is introduced.

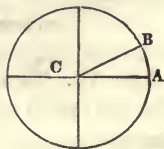
The periphery of every circle, whether great or small, is supposed to be divided into 360 equal parts called *degrees*, each degree into 60 *minutes*, each minute into 60 *seconds*, each second into 60 *thirds*, &c., marked with the characters °, ', ", ''', &c. Thus, 32° 24' 13'' 22''' is 32 degrees, 24 minutes, 13 seconds, 22 thirds.

A degree, then, is not a magnitude of a given *length*; but

a certain *portion* of the whole circumference of any circle. It is evident that the 360th part of a large circle is greater than the same part of a small one. On the other hand, the *number* of degrees in a small circle, is the same as in a large one.

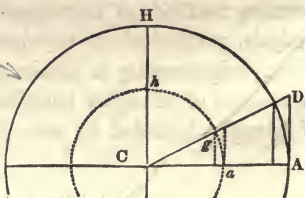
The fourth part of a circle is called a *quadrant*, and contains 90 degrees.

74. To *measure* an angle, a circle is so described that its center shall be the angular point, and its periphery shall cut the two lines which include the angle. The *arc* between the two lines is considered a *measure of the angle*, because, by Euc. 33. 6, angles at the center of a given circle, have the same ratio to each other, as the arcs on which they stand. Thus the arc AB, is a measure of the angle ACB.



It is immaterial what is the size of the circle, provided it cuts the lines which include the angle.

Thus, the angle ACD is measured by either of the arcs AG, *ag*. For ACD is to ACH, as AG to AH, or as *ag* to *ah*. (Euc. 33. 6.)



(Euc. 33. 6.)

75. In the circle ADGH, let the two diameters AG and DH be perpendicular to each other. The angles ACD, DCG, GCH, and HCA, will be right angles; and the periphery of the circle will be divided into four equal parts, each containing 90 degrees. As a right angle is subtended by an arc of 90°, the angle itself is said to contain 90°.



Hence, in two

right angles, there are  $180^\circ$ ; in four right angles,  $360^\circ$ ; and in any other angle, as many degrees, as in the arc by which it is subtended.

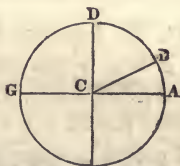
76. The sum of the three angles of any triangle being equal to two right angles, (Euc. 32. 1.\*) is equal to  $180^\circ$ . Hence, there can never be more than one obtuse angle in a triangle. For the sum of two obtuse angles is more than  $180^\circ$ .

77. *The COMPLEMENT of an arc or an angle, is the difference between the arc or angle and 90 degrees.*

The complement of the arc AB is DB; and the complement of the angle ACB is DCB. The complement of the arc BDG is also DB.

The complement of $10^\circ$ is $80^\circ$ ,	of $60^\circ$ is $30^\circ$ ,
of $20^\circ$ is $70^\circ$ ,	of $120^\circ$ is $30^\circ$ ,
of $50^\circ$ is $40^\circ$ ,	of $170^\circ$ is $80^\circ$ , &c.

Hence, an acute angle and its complement are always equal to  $90^\circ$ . The angles ACB and DCB are together equal to a right angle. The two acute angles of a right angled triangle are equal to  $90^\circ$ : therefore each is the complement of the other.



78. *The SUPPLEMENT of an arc or an angle is the difference between the arc or angle and 180 degrees.*

The supplement of the arc BDG is AB; and the supplement of the angle BCG is BCA.

The supplement of $10^\circ$ is $170^\circ$ ,	of $120^\circ$ is $60^\circ$ ,
of $80^\circ$ is $100^\circ$ ,	of $150^\circ$ is $30^\circ$ , &c.

Hence an angle and its supplement are always equal to

\* Thomson's Geometry, 28, 1.



180°. The angles  $BCA$  and  $BCG$  are together equal to two right angles.

79. Cor. As the three angles of a plane triangle are equal to two right angles, that is, to  $180^\circ$  (Euc. 32. 1.) the sum of any two of them is the supplement of the other. So that the third angle may be found, by subtracting the sum of the other two from  $180^\circ$ . Or the sum of any two may be found, by subtracting the third from  $180^\circ$ .

80. A straight line drawn from the centre of a circle to any part of the periphery, is called a *radius* of the circle. In many calculations, it is convenient to consider the *radius*, whatever be its length, as a *unit*. (Alg. 510.) To this must be referred the numbers expressing the lengths of other lines. Thus, 20 will be twenty times the radius, and 0.75, three-fourths of the radius.

*Definitions of Sines, Tangents, Secants, &c.*

81. To facilitate the calculations in Trigonometry, there are drawn, within and about the circle, a number of straight lines, called *Sines, Tangents, Secants, &c.* With these the learner should make himself perfectly familiar. The direct and proper measure of an angle is an arc of a circle. (Art. 74.) But trigonometrical solutions are commonly made with the aid of certain straight lines, which have known relations to the arcs to which they belong.

82. The *SINE* of an arc is a straight line drawn from one end of the arc, perpendicular to a diameter which passes through the other end.

Thus,  $BG$  is the sine of the arc  $AG$ . For  $BG$  is a line drawn from the end  $G$  of the arc, perpendicular to the diameter  $AM$  which passes through the other end  $A$  of the arc.

Cor. The sine is *half the chord of double the arc*. The sine  $BG$  is half  $PG$ , which is the chord of the arc  $PAG$ , double the arc  $AG$ .



83. The **VERSED SINE** of an arc is that part of the diameter which is between the sine and the arc.

Thus, BA is the versed sine of the arc AG.

84. The **TANGENT** of an arc, is a straight line drawn perpendicularly from the extremity of the diameter which passes through one end of the arc, and extended till it meets a line drawn from the centre through the other end.

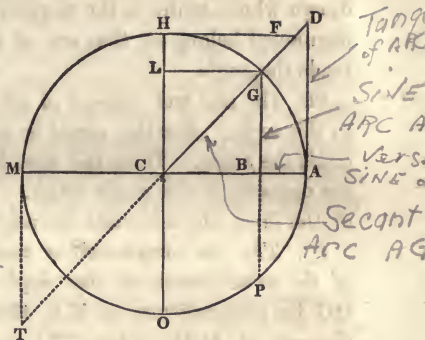
Thus, AD is the tangent of the arc AG.

85. The **SECANT** of an arc is a straight line drawn from the centre, through one end of the arc, and extended to the tangent which is drawn from the other end.

Thus CD is the secant of the arc AG.

86. In Trigonometry, the terms *tangent* and *secant* have a more limited meaning, than in Geometry. In both, indeed, the *tangent touches* the circle, and the *secant cuts* it. But in Geometry, these lines are of no determinate length; whereas, in Trigonometry, they extend from the diameter to the point in which they intersect each other.

87. The lines just defined are sines, tangents, and secants of arcs. BG is the sine of the arc AG. But this arc subtends the angle GCA. BG is then the sine of the arc which subtends the angle GCA. This is more concisely expressed, by saying that BG is the sine of the angle GCA. And universally, the sine, tangent, and secant of an arc, are said to be the sine, tangent, and secant of the angle which stands at the centre of the circle, and is subtended by the arc. Whenever, therefore, the sine, tangent, or secant of an angle is spoken of; we are to suppose a circle to be



drawn whose centre is the angular point; and that the lines mentioned belong to that arc of the periphery which subtends the angle.

88. The *sine* and *tangent* of an acute angle, are *opposite* to the angle. But the *secant* is one of the lines which *include* the angle. Thus, the sine BG, and the tangent AD, are opposite to the angle DCA. But the secant CD is one of the lines which include the angle.

89. *The sine complement or COSINE of an angle, is the sine of the COMPLEMENT of that angle.* Thus, if the diameter HO be perpendicular to MA, the angle HCG is the complement of ACG; (Art. 77.) and LG, or its equal CB, is the sine of HCG. (Art. 82.) It is, therefore, the *cosine* of GCA. On the other hand, GB is the sine of GCA, and the cosine of GCH.

So also the *cotangent* of an angle is the tangent of the *complement* of the angle. Thus, HF is the cotangent of GCA. And the *cosecant* of an angle is the secant of the *complement* of the angle. Thus, CF is the cosecant of GCA.

Hence, as in a right angled triangle, one of the acute angles is the complement of the other; (Art. 77.) the sine, tangent, and secant of one of these angles, are the cosine, cotangent, and cosecant of the other.

90. The sine, tangent, and secant of the *supplement* of an angle, are each equal to the sine, tangent, and secant of the angle itself. It will be seen, by applying the definition. (Art. 82.) to the figure, that the sine of the obtuse angle GCM is BG, which is also the sine of the acute angle GCA. It should be observed, however, that the sine of an acute angle is *opposite* to it; while the sine of an obtuse angle *falls without* the angle, and is opposite to its supplement. Thus BG, the sine of the angle MCG, is not opposite to MCG, but to its supplement ACG.

The *tangent* of the obtuse angle MCG is MT, or its equal

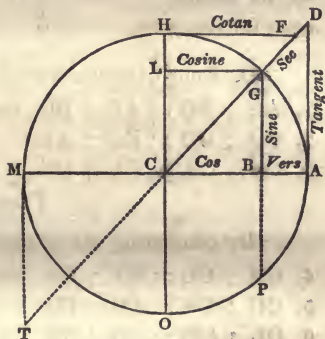
AD, which is also the tangent of ACG. And the *secant* of MCG is CD, which is also the secant of ACG.

91. But the *versed-sine* of an angle is not the same as that of its *supplement*. The versed sine of an *acute* angle is equal to the *difference* between the cosine and radius. But the versed sine of an *obtuse* angle is equal to the *sum* of the cosine and radius. Thus, the versed sine of ACG is  $AB=AC-BC$ . (Art. 83.) But the versed sine of MCG is  $MB=MC+BC$ .

*Relations of Sines, Tangents, Secants, &c., to each other.*

92. The relations of the sine, tangent, secant, cosine, &c., to each other, are easily derived from the proportions of the sides of similar triangles. (Euc. 4. 6.\*) In the quadrant

ACH, these lines form three similar triangles, viz. ACD, BCG or LCG, and HCF. For, in each of these, there is one right angle, because the sines and tangents are, by definition, perpendicular to AC; as the cosine and cotangent are to CH. The lines CH, BG, and AD, are parallel,



because CA makes a right angle with each. (Euc. 27. 1.†) For the same reason, CA, LG, and HF, are parallel. The alternate angles GCL, BGC, and the opposite angle CDA, are equal; (Euc. 29. 1.‡) as are also the angles GCB, LGC, and HFC. The triangles ACD, BCG, and HCF, are therefore similar.

\* Thomson, 18. 4.

† Ibid. 18. 1.

‡ Ibid. 24. 1.





It will not be necessary for the learner to commit these proportions to memory. But he ought to make himself so familiar with the manner of stating them from the figure, as to be able to explain them, whenever they are referred to.

94. Other relations of the sine, tangent, &c., may be derived from the proposition, that the square of the hypotenuse is equal to the sum of the squares of the perpendicular sides. (Euc. 47. 1.—Thomson 11. 4.)

In the right angled triangles CBG, CAD, and CHF,

$$1. \overline{CG}^2 = \overline{CB}^2 + \overline{BG}^2, \text{ that is, } R^2 = \cos^2 + \sin^2, *$$

$$2. \overline{CD}^2 = \overline{CA}^2 + \overline{AD}^2 \quad \sec^2 = R^2 + \tan^2,$$

$$3. \overline{CF}^2 = \overline{CH}^2 + \overline{HF}^2 \quad \operatorname{cosec}^2 = R^2 + \cot^2,$$

And, extracting the root of both sides, (Alg. 296.)

$$R = \sqrt{\cos^2 + \sin^2} = \sqrt{\sec^2 - \tan^2} = \sqrt{\operatorname{cosec}^2 - \cot^2}$$

Hence, if  $R=1$ , (Alg 385.)

$$\operatorname{Sin} = \sqrt{1 - \cos^2}$$

$$\operatorname{Sec} = \sqrt{1 + \tan^2}$$

$$\operatorname{Cos} = \sqrt{1 - \sin^2}$$

$$\operatorname{Cosec} = \sqrt{1 + \cot^2}$$

95.  $\left. \begin{array}{l} \text{The sine of } 90^\circ \\ \text{The chord of } 60^\circ \\ \text{And the tangent of } 45^\circ \end{array} \right\} \text{ are, in any circle, each equal}$

to the radius, and therefore equal to each other.

#### Demonstration.

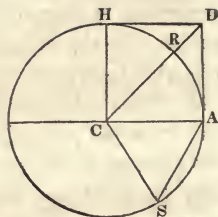
1. In the quadrant ACH, (figure on the next page,) the arc AH is  $90^\circ$ . The sine of this, according to the definition, (Art. 82.) is CH, the radius of the circle.

---

\*  $\operatorname{Sin}^2$  is here put for the square of the sine,  $\operatorname{cos}^2$  for the square of the cosine, &c.



2. Let  $AS$  be an arc of  $60^\circ$ . Then the angle  $ACS$ , being measured by this arc, will also contain  $60^\circ$ ; (Art. 75.) and the triangle  $ACS$  will be equilateral. For the sum of the three angles is equal to  $180^\circ$ . (Art. 76.) From this, taking the angle  $ACS$ , which is  $60^\circ$ , the sum of the remaining two is  $120^\circ$ . But these two are *equal*, because they are subtended by the equal sides,  $CA$  and  $CS$ , both radii of the circle. Each, therefore, is equal to *half*  $120^\circ$ , that is, to  $60^\circ$ . All the angles being equal, the sides are equal, and therefore  $AS$ , the chord of  $60^\circ$ , is equal to  $CS$ , the radius.



3. Let  $AR$  be an arc of  $45^\circ$ .  $AD$  will be its tangent, and the angle  $ACD$  subtended by the arc, will contain  $45^\circ$ . The angle  $CAD$  is a right angle, because the tangent is, by definition, perpendicular to the radius  $AC$ . (Art. 84.) Subtracting  $ACD$ , which is  $45^\circ$ , from  $90^\circ$ , (Art. 77.) the other acute angle  $ADC$  will be  $45^\circ$  also. Therefore the two legs of the triangle  $ACD$  are equal, because they are subtended by equal angles; (Euc. 6. 1.) that is,  $AD$  the tangent of  $45^\circ$ , is equal to  $AC$  the radius.

Cor. The *cotangent* of  $45^\circ$  is also equal to radius. For the complement of  $45^\circ$  is itself  $45^\circ$ . Thus,  $HD$ , the cotangent of  $ACD$ , is equal to  $AC$  the radius.

96. The sine of  $30^\circ$  is equal to *half radius*. For the sine of  $30^\circ$  is equal to half the chord of  $60^\circ$ . (Art. 82. cor.) But by the preceding article, the chord of  $60^\circ$  is equal to radius. Its half, therefore, which is the sine of  $30^\circ$ , is equal to half radius.

Cor. 1. The *cosine* of  $60^\circ$  is equal to half radius. For the cosine of  $60^\circ$  is the sine of  $30^\circ$ . (Art. 89.)

Cor. 2. The cosine of  $30^\circ = \frac{1}{2}\sqrt{3}$ . For

$$\text{Cos}^2 30^\circ = R^2 - \text{sin}^2 30^\circ = 1 - \frac{1}{4} = \frac{3}{4}.$$

Therefore,

$$\text{Cos } 30^\circ = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}.$$

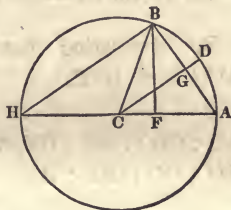
96. b. The sine of  $45^\circ = \frac{1}{\sqrt{2}}$ . For

$$R^2 = 1 = \sin^2 45^\circ + \cos^2 45^\circ = 2 \sin^2 45^\circ$$

Therefore,  $\text{Sin } 45^\circ = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

97. The *chord* of any arc is a *mean proportional*, between the *diameter* of the circle, and the *versed sine* of the arc.

Let ADB, be an arc, of which AB is the chord, BF the sine, and AF the versed sine. The angle ABH is a right angle, (Euc. 31. 3.\*) and the triangles ABH, and ABF, are similar. (Euc. 8. 6.†) Therefore,



$$AH : AB :: AB : AF.$$

That is, the diameter is to the chord, as the chord to the versed sine.

Let the arc  $AD = a$ , and  $ADB = 2a$ . Draw BF perpendicular to AH. This will divide the right angled triangle ABH into two similar triangles. (Euc. 8. 6.) The angles ACD and AHB are equal. (Euc. 20. 3.‡) Therefore the four triangles ACG, AHB, FHB, and FAB are similar; and the line BH is twice CG, because  $BH : CG :: HA : CA$ .

The sides of the four triangles are,

$$\begin{array}{lll} AG = \sin a, & CG = \cos a, & HF = \text{vers. sup. } 2a, \\ AB = 2 \sin a, & BH = 2 \cos a, & AC = \text{the radius,} \\ BF = \sin 2a, & AF = \text{vers } 2a, & AH = \text{the diameter.} \end{array}$$

\* Thomson, 13. 2. Cor. 2.

† Ibid. 22. 4.

‡ Ibid. 13. 2.

A variety of proportions may be stated, between the homologous sides of these triangles: For instance,

By comparing the triangles ACG and ABF,

$$AC : AG :: AB : AF, \text{ that is, } R : \sin a :: 2 \sin a : \text{vers } 2a$$

$$AC : CG :: AB : BF, \quad R : \cos a :: 2 \sin a : \sin 2a$$

$$AG : CG :: AF : BF, \quad \sin a : \cos a :: \text{vers } 2a : \sin 2a$$

Therefore,

$$R \times \text{vers } 2a = 2 \sin^2 a$$

$$R \times \sin 2a = 2 \sin a \times \cos a$$

$$\sin a \times \sin 2a = \text{vers } 2a \times \cos a$$

By comparing the triangles ACG and BFH,

$$AC : CG :: BH : HF, \text{ that is, } R : \cos a :: 2 \cos a : \text{vers. sup. } 2a$$

$$AG : CG :: BF : HF, \quad \sin a : \cos a :: \sin 2a : \text{vers. sup. } 2a$$

Therefore,

$$R \times \text{vers. sup. } 2a = 2 \cos^2 a$$

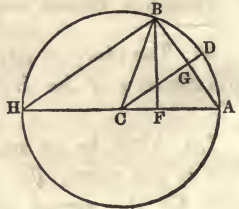
$$\sin a \times \text{vers. sup. } 2a = \cos a \times \sin 2a$$

&c.

&c.

That is, the product of radius into the versed sine of the supplement of twice a given arc, is equal to twice the square of the cosine of the arc.

And the product of the sine of an arc, into the versed sine of the supplement of twice the arc, is equal to the product of the cosine of the arc, into the sine of twice the arc, &c., &c.







number and a decimal. It is evident that all the *secants* also must be greater than radius, as they extend from the centre, to a point without the circle.

102. The numbers in the table here spoken of, are called *natural* sines, tangents, &c. They express the lengths of the several lines which have been defined in Arts. 82, 83, &c. By means of them, the angles and sides of triangles may be accurately determined. But the calculations must be made by the tedious processes of multiplication and division. To avoid this inconvenience, another set of tables has been provided, in which are inserted the *logarithms* of the natural sines, tangents, &c. By the use of these, addition and subtraction are made to perform the office of multiplication and division. On this account, the tables of logarithmic, or as they are sometimes called, *artificial* sines, tangents, &c., are much more valuable, for practical purposes, than the *natural* sines, &c. Still it must be remembered that the former are derived from the latter. The artificial sine of an angle, is the logarithm of the natural sine of that angle. The artificial tangent is the logarithm of the natural tangent, &c.

103. One circumstance, however, is to be attended to, in comparing the two sets of tables. The radius to which the *natural* sines, &c., are calculated, is *unity*. (Art. 100.) The secants, and a part of the tangents are, therefore, *greater* than a unit; while the sines, and another part of the tangents, are *less* than a unit. When the logarithms of these are taken, some of the indices will be *positive*, and others *negative*; (Art. 9.) and the throwing of them together in the same table, if it does not lead to error, will at least be attended with inconvenience. To remedy this, 10 is added to each of the indices. (Art. 12.) They are then all positive. Thus the natural sine of  $20^\circ$  is 0.34202. The logarithm of this is  $\overline{1}.53405$ . But the index, by the addition of 10, be-



comes  $10 - 1 = 9$ . The logarithmic sine in the tables is therefore 9.53405.\*

*Directions for taking Sines, Cosines, &c., from the tables.*

104. The *cosine*, *cotangent*, and *cosecant* of an angle, are the sine, tangent, and secant of the *complement* of the angle. (Art. 89.) As the complement of an angle is the difference between the angle and  $90^\circ$ , and as 45 is the half of 90; if any given angle within the quadrant is greater than  $45^\circ$ , its complement is less; and, on the other hand, if the angle is *less* than  $45^\circ$ , its complement is greater. Hence, every cosine, cotangent, and cosecant of an angle greater than  $45^\circ$ , has its equal among the sines, tangents, and secants of angles less than  $45^\circ$ , and *v. v.*

Now, to bring the trigonometrical tables within a small compass, the same column is made to answer for the *sines* of a number of angles *above*  $45^\circ$ , and for the *cosines* of an equal number *below*  $45^\circ$ .

Thus 9.23967 is the log. *sine* of  $10^\circ$ , and the *cosine* of  $80^\circ$ ,  
 9.53405 the *sine* of  $20^\circ$ , and the *cosine* of  $70^\circ$ , &c.

The tangents and secants are arranged in a similar manner. Hence,

105. *To find the Sine, Cosine, Tangent, &c., of any number of degrees and minutes.*

If the given angle is *less* than  $45^\circ$ , look for the degrees at the *top* of the table, and the minutes on the *left*; then, opposite to the minutes, and under the word *sine* at the head of the column, will be found the sine; under the word *tangent*, will be found the tangent, &c.

---

\* Or the tables may be supposed to be calculated to the radius 10000000000, whose logarithm is 10.

The log. sin of	43° 25'	is 9.83715	The tan of	17° 20'	is 9.49430
	of 17° 20'	9.47411		of 8° 46'	9.18812
The cos	of 17° 20'	9.97982	The cot of	17° 20'	10.50570
	of 8° 46'	9.99490		of 8° 46'	10.81188

The first figure is the index ; and the other figures are the decimal part of the logarithm.

106. If the given angle is between 45° and 90° ; look for the degrees at the *bottom* of the table, and the minutes on the *right* ; then, opposite to the minutes, and *over* the word sine at the foot of the column, will be found the sine ; over the word tangent, will be found the tangent, &c.

Particular care must be taken, when the angle is less than 45°, to look for the title of the column, at the *top*, and for the minutes on the *left* ; but when the angle is between 45° and 90°, to look for the title of the column at the *bottom*, and for the minutes, on the *right*.

The log. sine	of 81° 21'	is 9.99503
The cosine	of 72° 10'	9.48607
The tangent	of 54° 40'	10.14941
The cotangent	of 63° 22'	9.70026

107. If the given angle is *greater* than 90°, look for the sine, tangent, &c., of its *supplement*. (Art. 98, 99.)

The log. sine of	96° 44'	is 9.99699
The cosine of	171° 16'	9.99494
The tangent of	130° 26'	10.06952
The cotangent of	156° 22'	10.35894

108. *To find the sine, cosine, tangent, &c., of any number of degrees, minutes, and SECONDS.*

In the common tables, the sine, tangent, &c., are given only to every *minute* of a degree.\* But they may be found to *seconds*, by taking *proportional parts* of the difference of

---

\* In the very valuable tables of Michael Taylor, the sines and tangents are given to every second.

the numbers as they stand in the tables. For, within a single minute, the variations in the sine, tangent, &c., are nearly proportional to the variations in the angle. Hence,

To find the sine, tangent, &c., to seconds: Take out the number corresponding to the given degree and minute; and also that corresponding to the next greater minute, and find their difference. Then state this proportion;

As 60, to the given number of seconds;

So is the difference found, to the correction for the seconds.

This correction, in the case of sines, tangents, and secants, is to be *added* to the number answering to the given degree and minute; but for cosines, cotangents, and cosecants, the correction is to be *subtracted*;

For, as the sines *increase*, the cosines *decrease*.

Ex. 1. What is the logarithmic sine of  $14^{\circ} 43' 10''$ ?

The sine of $14^{\circ} 43'$ is	9.40490
of $14^{\circ} 44'$	<u>9.40538</u>
Difference	<u>48</u>

Here it is evident that the sine of the required angle is greater than that of  $14^{\circ} 43'$ , but less than that of  $14^{\circ} 44'$ . And as the difference corresponding to a whole minute or  $60''$  is 48; the difference for  $10''$  must be a proportional part of 48. That is,

$$60'' : 10'' :: 48 : 8$$

the correction to be *added* to the sine of  $14^{\circ} 43'$ .

Therefore the sine of  $14^{\circ} 43' 10''$  is 9.40498.

2. What is the logarithmic cosine of  $32^{\circ} 16' 45''$ ?

The cosine of $32^{\circ} 16'$ is	9.92715
of $32^{\circ} 17'$	<u>9.92707</u>
Difference	<u>8</u>

Then,  $60'' : 45'' :: 8 : 6$  the correction to be *subtracted* from the cosine of  $32^{\circ} 16'$ .

Therefore the cosine of  $32^{\circ} 16' 45''$  is 9.92709.

The tangent of  $24^{\circ} 15' 18''$  is 9.65376

The cotangent of  $31^{\circ} 50' 5''$  is 10.20700

The sine of  $58^{\circ} 14' 32''$  is 9.92956

The cosine of  $55^{\circ} 10' 26''$  is 9.75670

If the given number of seconds be any even part of 60, as  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , &c., the correction may be found, by taking a like part of the difference of the numbers in the tables, without stating a proportion in form.

109. *To find the degrees and minutes belonging to any given sine, tangent, &c.*

This is reversing the method of finding the sine, tangent, &c., (Art. 105, 6, 7.)

Look in the column of the same name, for the sine, tangent, &c., which is *nearest* to the given one; and if the title be at the *head* of the column, take the degrees at the *top* of the table, and the minutes on the *left*; but if the title be at the *foot* of the column, take the degrees at the *bottom*, and the minutes on the *right*.

Ex. 1. What is the number of degrees and minutes belonging to the logarithmic sine 9.62863?

The nearest sine in the tables is 9.62865. The title of sine is at the head of the column in which these numbers are found. The degrees at the top of the page are 25, and the minutes on the left are 10. The angle required is, therefore  $25^{\circ} 10'$ .

The angle belonging to

the sine	9.87993	is	$49^{\circ} 20'$	the cos	9.97351	is	$19^{\circ} 48'$
the tan	9.97955		$43^{\circ} 39'$	the cotan	9.75791		$60^{\circ} 12'$
the sec	10.65396		$77^{\circ} 11'$	the cosec	10.49066		$18^{\circ} 51'$

110. *To find the degrees, minutes, and SECONDS, belonging to any given sine, tangent, &c.*



This is reversing the method of finding the sine, tangent, &c., to seconds. (Art. 108.)

First find the difference between the sine, tangent, &c., next greater than the given one, and that which is next less; then the difference between this less number and the given one; then

As the difference first found, is to the other difference;

So are 60 seconds, to the number of seconds, which, in the case of sines, tangents, and secants, are to be *added* to the degrees and minutes belonging to the least of the two numbers taken from the tables; but for cosines, cotangents, and cosecants are to be *subtracted*.

Ex. 1. What are the degrees, minutes, and seconds, belonging to the logarithmic sine 9.40498?

Sine next greater	14° 44'	9.40538		Given sine	9.40498
Next less	14° 43'	9.40490		Next less	9.40490
		48		Difference	8

Then,  $48 : 8 :: 60'' : 10''$ , which added to 14° 43', gives 14° 43' 10'' for the answer.

2. What is the angle belonging to the cosine 9.09773?

Cosine next greater	82° 48'	9.09807		Given cosine	9.09773
Next less	82° 49'	9.09707		Next less	9.09707
		100		Difference	66

Then,  $100 : 66 :: 60'' : 40''$ , which subtracted from 82° 49', gives 82° 48' 20'' for the answer.

It must be observed here, as in all other cases, that of the two angles, the less has the greater cosine.

The angle belonging to

the sin 9.20621 is 9° 15' 6'' the tan 10.43434 is 69° 48' 16''  
 the cos 9.98157 16° 34' 30'' the cot 10.33554 24° 47' 16''



*Method of Supplying the Secants and Cosecants.*

111. In some trigonometrical tables, the secants and cosecants are not inserted. But they may be easily obtained from the sines and cosines. For, by Art. 93, proportion 3d,

$$\cos \times \sec = R^2.$$

That is, the product of the cosine and secant, is equal to the square of radius. But, in logarithms, addition takes the place of multiplication; and, in the tables of logarithmic sines, tangents, &c., the radius is 10. (Art. 103.) Therefore, in these tables,

$$\cos + \sec = 20. \quad \text{Or } \sec = 20 - \cos.$$

Again, by Art 93, proportion 6,

$$\sin \times \text{cosec} = R^2.$$

Therefore, in the tables,

$$\sin + \text{cosec} = 20. \quad \text{Or, } \text{cosec} = 20 - \sin. \quad \text{Hence,}$$

112. To obtain the *secant*, subtract the cosine from 20; and to obtain the *cosecant*, subtract the sine from 20.

These subtractions are most easily performed, by taking the right hand figure from 10, and the others from 9, as in finding the arithmetical complement of a logarithm; (Art. 55.) observing, however, to add 10 to the index of the secant or cosecant. In fact the secant is the arithmetical complement of the cosine, with 10 added to the index.

$$\text{For the secant} \quad = 20 - \cos.$$

$$\text{And the arith. comp. of } \cos = 10 - \cos. \quad (\text{Art. 54.})$$

So also the cosecant is the arithmetical complement of the sine, with 10 added to the index. The tables of secants and cosecants are, therefore, of use, in furnishing the arithmetical complement of the sine and cosine, in the following simple manner:

113. For the arithmetical complement of the *sine*, subtract 10 from the index of the cosecant; and for the arithmetical complement of the *cosine*, subtract 10 from the index of the secant.

By this, we may save the trouble of taking each of the figures from 9.

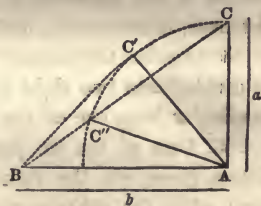
### SECTION III.

#### SOLUTIONS OF RIGHT ANGLED TRIANGLES.

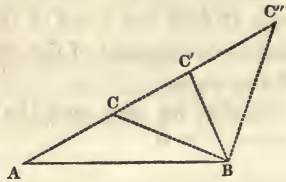
ART. 114. In a triangle there are *six parts*, three sides, and three angles. In every trigonometrical calculation, it is necessary that some of these should be known, to enable us to find the others. *The number of parts which must be given, is THREE, one of which must be a SIDE.*

If only two parts be given, they will be either two sides, a side and an angle, or two angles; neither of which will limit the triangle to a particular form and size.

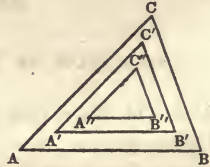
If *two sides* only be given, they may make any angle with each other; and may, therefore, be the sides of a thousand different triangles. Thus, the two lines  $a$  and  $b$  may belong either to the triangle  $ABC$ , or  $ABC'$ , or  $ABC''$ . So that it will be impossible, from knowing two of the sides of a triangle, to determine the other parts.



Or, if a *side and an angle* only be given, the triangle will be indeterminate. Thus, if the side AB and the angle at A be given; they may be parts either of the triangle ABC, or ABC', or ABC''.



Lastly, if two *angles*, or even if *all* the angles be given, they will not determine the length of the sides. For the triangles ABC, A'B'C', A''B''C'', and a hundred others which might be drawn, with sides parallel to these, will all have the same angles. So that one of the parts given must always be a side. If this and any other two parts, either sides or angles, be known, the other three may be found, as will be shown, in this and the following section.

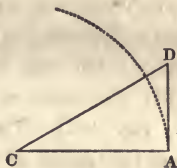


115. Triangles are either *right angled* or *oblique angled*. The calculations of the former are the most simple, and those which we have the most frequent occasion to make. A great portion of the problems in the mensuration of heights and distances, in surveying, navigation and astronomy, are solved by rectangular trigonometry. Any triangle whatever may be divided into two right angled triangles, by drawing a perpendicular from one of the angles to the opposite side.

116. One of the six parts in a right angled triangle, is always given, viz. the right angle. This is a *constant* quantity; while the other angles and the sides are variable. It is also to be observed, that, if one of the *acute* angles is given, the other is known of course. For one is the complement of the other. (Art. 76, 77.) So that, *in a right angled triangle, subtracting one of the acute angles from  $90^\circ$  gives the*

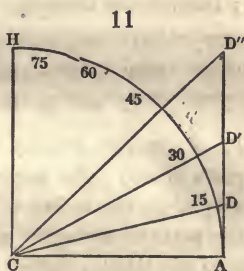
*other.* There remain, then, only *four* parts, one of the acute angles, and the three sides, to be sought by calculation. If any *two* of these be given, with the right angle, the others may be found.

117. To illustrate the method of calculation, let a case be supposed in which a right angled triangle CAD, has one of its sides equal to the radius to which the trigonometrical tables are adapted.



In the first place, let the *base* of the triangle be equal to the tabular radius. Then, if a circle be described, with this radius, about the angle C as a centre, DA will be the *tangent*, and DC the *secant* of that angle. (Art. 84, 85.) So that the radius, the tangent, and the secant of the angle at C, constitute the three sides of the triangle. The *tangent*, taken from the tables of natural sines, tangents, &c., will be the length of the *perpendicular*; and the *secant* will be the length of the *hypotenuse*. If the tables used be logarithmic, they will give the *logarithms* of the lengths of the two sides.

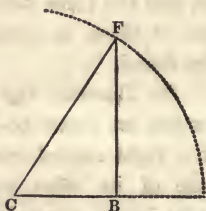
In the same manner, *any* right angled triangle whatever, whose base is equal to the radius of the tables, will have its other two sides found among the tangents and secants. Thus, if the quadrant AH, be divided into portions of  $15^\circ$  each; then, in the triangle



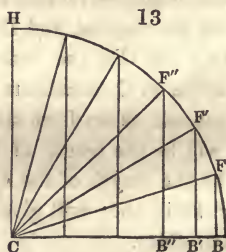
CAD, AD will be the tan, and CD the sec of  $15^\circ$ ,  
 In CAD', AD' will be the tan, and CD' the sec of  $30^\circ$ ,  
 In CAD'', AD'' will be the tan, and CD'' the sec of  $45^\circ$ , &c.



118. In the next place, let the *hypotenuse* of a right angled triangle CBF, be equal to the radius of the tables. Then, if a circle be described, with the given radius, and about the angle C as a centre; BF will be the *sine*, and BC the *cosine* of that angle. (Art. 82, 89.) Therefore the sine of the angle at C, taken from the tables, will be the length of the *perpendicular*, and the cosine will be the length of the *base*.



And any right angled triangle whatever, whose hypotenuse is equal to the tabular radius, will have its other two sides found among the sines and cosines. Thus, if the quadrant AH, be divided into portions of  $15^\circ$  each in the points F, F', F'', &c.; then, in the triangle,



CBF, FB will be the sin, and CB the cos, of  $15^\circ$ ,  
 In CB'F', F'B' will be the sin, and CB' the cos, of  $30^\circ$ ,  
 In CB''F'', F''B'' will be the sin, and CB'' the cos, of  $45^\circ$ , &c.

119. By merely *turning to the tables*, then, we may find the parts of any right angled triangle which has one of its sides equal to the radius of the tables. But for determining the parts of triangles which have *not* any of their sides equal to the tabular radius, the following proportion is used;

*As the radius of one circle,*

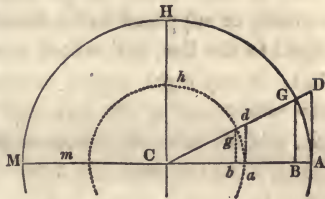
*To the radius of any other ;*

*So is a sine, tangent, or secant, in one,*

*To the sine, tangent, or secant, of the same number  
 of degrees, in the other.*



In the two concentric circles  $AHM$ ,  $ahm$ , the arcs  $AG$  and  $ag$ , contain the same number of degrees. (Art. 74.) The sines of these arcs are  $BG$  and  $bg$ , the tangents  $AD$  and  $ad$ , and the secants  $CD$  and  $Cd$ .



The four triangles,  $CAD$ ,  $CBG$ ,  $Cad$ , and  $Cbg$ , are similar. For each of them, from the nature of sines and tangents, contains one right angle; the angle at  $C$  is common to them all; and the other acute angle in each is the complement of that at  $C$ . (Art. 77.) We have, then, the following proportions. (Euc. 4. 6.\*)

$$1. \quad CG : Cg :: BG : bg.$$

That is, one radius is to the other, as one *sine* to the other.

$$2. \quad CA : Ca :: DA : da.$$

That is, one radius is to the other, as one *tangent* to the other.

$$3. \quad CA : Ca :: CD : Cd.$$

That is, one radius is to the other, as one *secant* to the other.

Cor.  $BG : bg :: DA : da :: CD : Cd.$

That is, as the sine in one circle, to the sine in the other; so is the tangent in one, to the tangent in the other; and so is the secant in one, to the secant in the other.

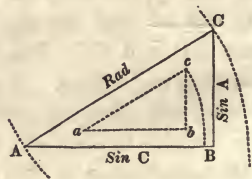
This is a general principle, which may be applied to most trigonometrical calculations. If one of the sides of the proposed triangle be made radius, each of the other sides will be the sine, tangent, or secant, of an arc described by this radius. Proportions are then stated, between these lines, and the *tabular* radius, sine, tangent, &c.

\* Thomson 18. 4.

120. A line is said to be *made radius*, when a circle is described, or supposed to be described, whose semi-diameter is equal to the line, and whose centre is at one end of it.

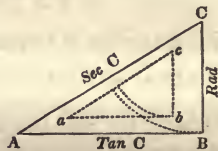
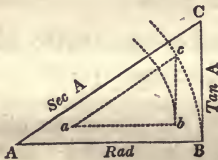
121. In any right angled triangle, if the HYPOTHENUSE be *made radius*, one of the legs will be a SINE of its opposite angle, and the other leg a COSINE of the same angle.

Thus, if to the triangle ABC a circle be applied whose radius is AC, and whose centre is A, then BC will be the *sine*, and BA the *cosine*, of the angle at A. (Art. 82, 89.)



If, while the same line is radius, the other end C be made the centre, then BA will be the *sine*, and BC the *cosine*, of the angle at C.

122. If either of the LEGS be made radius, the other leg will be a TANGENT of its opposite angle, and the hypotenuse will be a SECANT of the same angle; that is, of the angle between the secant and the radius.



Thus, if the base AB (Fig. 15.) be made radius, the centre being at A, BC will be the *tangent*, and AC the *secant*, of the angle at A. (Art. 84. 85.)

But, if the *perpendicular* BC, (Fig. 16.) be made radius, with the centre at C, then AB will be the *tangent*, and AC the *secant*, of the angle at C.

123. As the side which is the sine, tangent, or secant of one of the acute angles, is the cosine, cotangent, or cose-

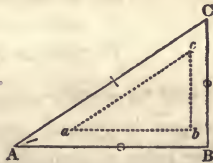
cant of the other ; (Art. 89.) the *perpendicular* BC (Fig. 14.) is the *sine* of the angle A, and the *cosine* of the angle C ; while the *base* AB, is the *sine* of the angle C, and the *cosine* of the angle A.

If the base is made radius, as in Fig 15, the *perpendicular* BC is the *tangent* of the angle A, and the *cotangent* of the angle C ; while the *hypotenuse* is the *secant* of the angle A, and the *cosecant* of the angle C.

If the perpendicular is made radius, as in Fig. 16, the *base* AB is the *tangent* of the angle C, and the *cotangent* of the angle A ; while the *hypotenuse* is the *secant* of the angle C, and the *cosecant* of the angle A.

124. Whenever a right angled triangle is proposed, whose sides or angles are required ; a *similar* triangle may be formed, from the sines, tangents, &c., of the *tables*. (Art. 117, 118.) The parts required are then found, by stating proportions between the similar

sides of the two triangles. If the triangle proposed be ABC, (Fig. 17.) another *abc* may be formed, having the same angles with the first, but differing from it in the length of its sides, so as



to correspond with the numbers in the tables. If similar sides be made radius in both, the remaining similar sides will be lines of *the same name* ; that is, if the perpendicular in one of the triangles be a *sine*, the perpendicular in the other will be a sine ; if the base in one be a *cosine*, the base in the other will be a cosine, &c.

If the *hypotenuse* in each triangle be made radius, as in Fig. 14, the perpendicular *bc*, will be the *tabular sine* of the angle at *a* ; and the perpendicular BC, will be a sine of the equal angle A, in a circle of which AC is radius.

If the *base* in each triangle be made radius, as in Fig. 15, then the perpendicular *bc*, will be the *tabular tangent* of the

angle at  $a$ ; and  $BC$  will be a tangent of the equal angle  $A$ , in a circle of which  $AB$ , is radius, &c.

125. From the relations of the similar sides of these triangles, are derived the two following *theorems*, which are sufficient for calculating the parts of any right angled triangle whatever, when the requisite data are furnished. One is used, when a *side* is to be found; the other, when an *angle* is to be found.

### THEOREM I.

126. When a *side* is required;

AS THE TABULAR SINE, TANGENT, &c., OF THE  
SAME NAME WITH THE GIVEN SIDE,

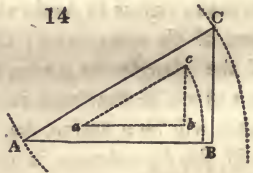
TO THE GIVEN SIDE;

SO IS THE TABULAR SINE, TANGENT, &c., OF THE  
SAME NAME WITH THE REQUIRED SIDE,

TO THE REQUIRED SIDE.

It will be readily seen, that this is nothing more than a statement in general terms, of the proportions between the similar sides of two triangles, one proposed for solution, and the other formed from the numbers in the tables.

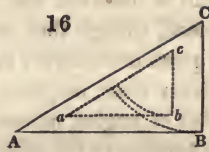
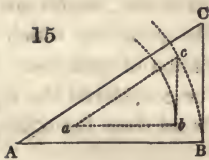
Thus, if the hypotenuse be *given*, and the base or perpendicular be *required*; then in Fig. 14, where  $ac$  is the tabular radius,  $bc$  the tabular sine of  $a$ , or its equal  $A$ , and  $ab$  the tabular sine of  $C$ ; (Art. 124.)



$$ac : AC :: bc : BC, \text{ that is, } R : AC :: \sin A : BC.$$

$$ac : AC :: ab : AB, \quad R : AC :: \sin C : AB.$$





In Fig. 15, where  $ab$  is the tabular radius,  $ac$  the tabular secant of  $A$ , and  $bc$  the tabular tangent of  $A$  ;

$$ac : AC :: bc : BC, \text{ that is, } \sec A : AC :: \tan A : BC.$$

$$ac : AC :: ab : AB, \quad \sec A : AC :: R : AB.$$

In Fig. 16, where  $bc$  is the tabular radius,  $ac$  the tabular secant of  $C$ , and  $ab$  the tabular tangent of  $C$  ;

$$ac : AC :: bc : BC, \text{ that is, } \sec C : AC :: R : BC.$$

$$ac : AC :: ab : AB, \quad \sec C : AC :: \tan C : AB.$$

THEOREM II.

127. When an *angle* is required ;

AS THE GIVEN SIDE MADE RADIUS,

TO THE TABULAR RADIUS ;

SO IS ANOTHER GIVEN SIDE,

TO THE TABULAR SINE, TANGENT, &c., OF THE  
SAME NAME.

Thus, if the side made radius, and one other side be given,  
then, in Fig. 14,

$$AC : ac :: BC : bc, \text{ that is, } AC : R :: BC : \sin A.$$

$$AC : ac :: AB : ab, \quad AC : R :: AB : \sin C.$$

In Fig. 15,

$$AB : ab :: BC : bc, \text{ that is, } AB : R :: BC : \tan A.$$

$$AB : ab :: AC : ac, \quad AB : R :: AC : \sec A.$$

In Fig. 16,

$$BC : bc :: AB : ab, \text{ that is, } BC : R :: AB : \tan C.$$

$$BC : bc :: AC : ac, \quad BC : R :: AC : \sec C.$$



It will be observed that in these theorems, *angles* are not introduced, though they are among the quantities which are either given or required, in the calculation of triangles. But the tabular sines, tangents, &c., may be considered the *representatives* of angles, as one may be found from the other, by merely turning to the tables.

128. In the theorem for finding a *side*, the first term of the proportion is a *tabular number*. But, in the theorem for finding an *angle*, the first term is a *side*. Hence, in applying the proportions to particular cases, this rule is to be observed ;

*To find a SIDE, begin with a tabular number,*

*To find an ANGLE, begin with a side.*

*Radius* is to be reckoned among the tabular numbers.

129. In the theorem for finding an *angle*, the first term is a *side made radius*. As in every proportion, the three first terms must be given to enable us to find the fourth, it is evident, that where this theorem is applied, the side made radius must be a *given* one. But, in the theorem for finding a *side*, it is not necessary that either of the terms should be radius. Hence,

130. *To find a SIDE, ANY side may be made radius,*

*To find an ANGLE, a GIVEN side must be made radius.*

It will generally be expedient, in both cases, to make radius one of the terms in the proportion ; because, in the tables of natural sines, tangents, &c., radius is 1, and in the logarithmic tables it is 10. (Art. 103.)

131. The proportions in Trigonometry are of the same nature as other simple proportions. The fourth term is found, therefore, as in the Rule of Three in Arithmetic, by *multiplying together the second and third terms, and dividing their product by the first term*. This is the mode of calculation, when the tables of *natural* sines, tangents, &c., are used. But the operation by logarithms is so much more expeditious,

that it has almost entirely superseded the other method. In logarithmic calculations, addition takes the place of multiplication; and subtraction the place of division.

The logarithms expressing the lengths of the *sides* of a triangle, are to be taken from the tables of common logarithms. The logarithms of the *sines, tangents, &c.*, are found in the tables of artificial sines, &c. The calculation is then made *by adding the second and third terms, and subtracting the first.* (Art. 52.)

132. The logarithmic radius 10, or, as it is written in the tables, 10.00000, is so easily added and subtracted, that the three terms of which it is one, may be considered as, in effect, reduced to two. Thus, if the tabular radius is in the *first* term, we have only to add the other two terms, and then take 10 from the index; for this is subtracting the first term. If radius occurs in the *second* term, the first is to be subtracted from the third, after its index is increased by 10. In the same manner, if radius is in the *third* term, the first is to be subtracted from the second.

133. Every species of right angled triangles may be solved upon the principle, that the sides of similar triangles are proportional, according to the two theorems mentioned above. There will be some advantages, however, in giving the examples in distinct classes.

There must be given, in a right angled triangle, *two* of the parts, besides the right angle. (Art. 116.) These may be;

1. The hypotenuse and an angle; or
2. The hypotenuse and a leg; or
3. A leg and an angle; or
4. The two legs.

#### CASE I.

134. Given  $\left\{ \begin{array}{l} \text{The hypotenuse,} \\ \text{And an angle,} \end{array} \right\}$  to find  $\left\{ \begin{array}{l} \text{The base and} \\ \text{Perpendicular.} \end{array} \right.$

Ex. 1. If the hypotenuse AC,\* be 45 miles, and the angle at A  $32^{\circ} 20'$ , what is the length of the base AB, and the perpendicular BC?

In this case, as *sides* only are required, *any* side may be made radius. (Art. 130.)

If the hypotenuse be made radius, BC will be the sine of A, and AB the sine of C, or the cosine of A. (Art. 121.) And if *abc* be a similar triangle, whose hypotenuse is equal to the *tabular* radius, *bc* will be the tabular sine of A, and *ab* the tabular sine of C. (Art. 124.)

To find the *perpendicular*, then, by Theorem I, we have this proportion;

$$ac : AC :: bc : BC.$$

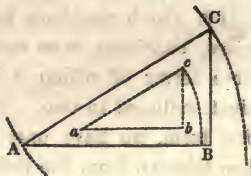
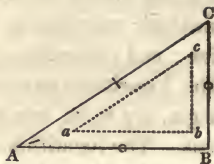
$$\text{Or } R : AC :: \text{Sin } A : BC.$$

Whenever the terms Radius, Sine, Tangent, &c., occur in a proportion like this, the *tabular* Radius, &c., is to be understood, as in Arts. 126, 127.

The numerical calculation, to find the length of BC, may be made, either by *natural* sines, or by *logarithms*. See Art. 131.

*By natural Sines.*

$$1 : 45 :: 0.53484 : 24.068 = BC.$$



\* The parts which are *given* are distinguished by a mark across the line, or at the opening of the angle, and the parts *required* by a cipher.

*Computation by Logarithms.*

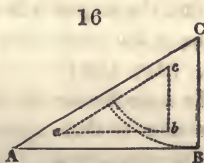
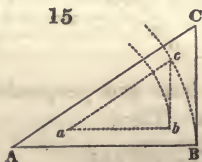
As radius		10.00000
To the hypotenuse	45	1.65321
So is the Sine of A	32° 20'	<u>9.72823</u>
To the perpendicular	24.068	<u>1.38144</u>

Here the logarithms of the second and third terms are added, and from the sum, the first term 10 is subtracted. (Art. 132.) The remainder is the logarithm of 24.068=BC. Subtracting the angle at A from 90°, we have the angle at C=57° 40'. (Art. 116.) Then to find the *base* AB;

$$ac : AC :: ab : AB$$

$$\text{Or } R : AC :: \text{Sin } C : AB=38.023.$$

Both the sides required are now found, by making the hypotenuse radius. The results here obtained may be verified, by making either of the other sides radius.



If the *base* be made radius, as in Fig. 15, the perpendicular will be the *tangent*, and the hypotenuse the *secant* of the angle at A. (Art. 122.) Then,

$$\text{Sec } A : AC :: R : AB$$

$$R : AB :: \text{Tan } A : BC$$

By making the arithmetical calculations, in these two proportions, the values of AB and BC, will be found the same as before.

If the *perpendicular* be made radius, as in Fig. 16, AB will be the *tangent*, and AC the *secant* of the angle at C. Then,



$$\begin{aligned} \text{Sec } C &: AC :: R : BC \\ R &: BC :: \text{Tan } C : AB \end{aligned}$$

Ex. 2. If the hypotenuse of a right angled triangle be 250 rods, and the angle at the base  $46^\circ 30'$ ; what is the length of the base and perpendicular?

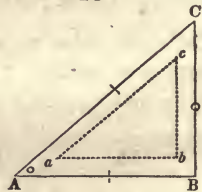
Ans. The base is 172.1 rods, and the perpendic. 181.35.

## CASE II.

135. Given  $\left\{ \begin{array}{l} \text{The hypotenuse,} \\ \text{And one leg.} \end{array} \right\}$  to find  $\left\{ \begin{array}{l} \text{The angles and} \\ \text{The other leg.} \end{array} \right\}$

Ex. 1. If the hypotenuse be 35 leagues, and the base 26; what is the length of the perpendicular, and the quantity of each of the acute angles?

18



To find the angles it is necessary that one of the *given* sides be made radius. (Art. 130.)

If the *hypotenuse* be radius, the base and perpendicular will be sines of their opposite angles. Then,

$$AC : R :: AB : \text{Sin } C = 47^\circ 58\frac{1}{2}'$$

And to find the *perpendicular* by Theorem I;

$$R : AC :: \text{Sin } A : BC = 23.43$$

If the *base* be radius, the perpendicular will be *tangent*, and the hypotenuse *secant* of the angle at A. Then,

$$\begin{aligned} AB &: R :: AC : \text{Sec } A \\ R &: AB :: \text{Tan } A : BC \end{aligned}$$

In this example, where the hypotenuse and base are given, the angles cannot be found by making the *perpendicular* radius. For to find an angle, a *given* side must be made radius. (Art. 130.)



136. Ex. 2. If the hypotenuse be 54 miles, and the perpendicular 48 miles, what are the angles, and the base?

Making the *hypotenuse* radius.

$$\begin{aligned} AC &: R :: BC : \sin A \\ R &: AC :: \sin C : AB \end{aligned}$$



The numerical calculation will give  $A=62^{\circ} 44'$  and  $AB=24.74$ .

Making the *perpendicular* radius.

$$\begin{aligned} BC &: R :: AC : \sec C \\ R &: BC :: \tan C : AB \end{aligned}$$

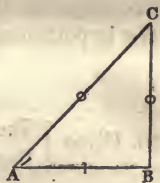
The angles cannot be found by making the *base* radius, when its length is not given.

CASE III.

137. Given  $\left\{ \begin{array}{l} \text{The angles,} \\ \text{and one leg.} \end{array} \right\}$  to find  $\left\{ \begin{array}{l} \text{The hypotenuse,} \\ \text{and the other leg.} \end{array} \right\}$

Ex. 1. If the base be 60, and the angle at the base  $47^{\circ} 12'$ , what is the length of the hypotenuse and the perpendicular?

In this case, as *sides* only are required, *any* side may be radius.



Making the *hypotenuse* radius.

$$\begin{aligned} \sin C &: AB :: R : AC=88.31 \\ R &: AC :: \sin A : BC=64.8 \end{aligned}$$

Making the *base* radius. (Fig. 20.)

$$R : AB :: \text{Sec } A : AC$$

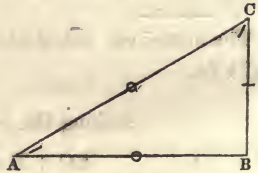
$$R : AB :: \text{Tan } A : BC$$

Making the *perpendicular* radius.

$$\text{Tan } C : AB :: R : BC$$

$$R : BC :: \text{Sec } C : AC$$

138. Ex. 2. If the perpendicular be 74, and the angle C  $61^\circ 27'$ , what is the length of the base and the hypotenuse ?



Making the *hypotenuse* radius.

$$\text{Sin } A : BC :: R : AC$$

$$R : AC :: \text{sin } C : AB$$

Making the *base* radius.

$$\text{Tan } A : BC :: R : AB$$

$$R : AB :: \text{sec } A : AC$$

Making the *perpendicular* radius.

$$R : BC :: \text{sec } C : AC$$

$$R : BC :: \text{tan } C : AB$$

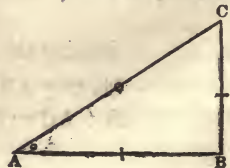
The hypotenuse is 154.83 and the base 136.

#### CASE IV.

139. Given  $\left\{ \begin{array}{l} \text{The base, and} \\ \text{Perpendicular,} \end{array} \right\}$  to find  $\left\{ \begin{array}{l} \text{The hypotenuse,} \\ \text{And the angles.} \end{array} \right\}$

Ex. 1. If the base be 284, and the perpendicular 192, what are the angles, and the hypotenuse ?

In this case, one of the legs



must be made radius, to find an angle; because the hypothenuse is not given.

Making the *base* radius.

$$AB : R :: BC : \tan A = 34^\circ 4'$$

$$R : AB :: \sec A : AC = 342.84$$

Making the *perpendicular* radius.

$$BC : R :: AB : \tan C$$

$$R : BC :: \sec C : AC$$

Ex. 2. If the base be 640, and the perpendicular 480, what are the angles and hypothenuse?

Ans. The hypothenuse is 800, and the angle at the base  $36^\circ 52' 12''$ .

*Examples for Practice.*

1. Given the hypothenuse 68, and the angle at the base  $39^\circ 17'$ ; to find the base and perpendicular. *52.63*
2. Given the hypothenuse 850, and the base 594, to find the angles, and the perpendicular. *52.63*
3. Given the hypothenuse 78, and perpendicular 57, to find the base, and the angles.
4. Given the base 723, and the angle at the base  $64^\circ 18'$ , to find the hypothenuse and perpendicular.
5. Given the perpendicular 632, and the angle at the base  $81^\circ 36'$ , to find the hypothenuse and the base.
6. Given the base 32, and the perpendicular 24, to find the hypothenuse, and the angles.

140. The preceding solutions are all effected, by means of the tabular sines, tangents, and secants. But, when any *two sides* of a right angled triangle are given, the third side may be found, without the aid of the trigonometrical tables, by the proposition, that *the square of the hypothenuse is equal*

to the sum of the squares of the two perpendicular sides. (Euc. 47. 1.)

If the legs be given, extracting the square root of the *sum* of their squares, will give the hypotenuse. Or, if the hypotenuse and one leg be given, extracting the square root of the *difference* of the squares, will give the other leg.

Let  $h$  = the hypotenuse  
 $p$  = the perpendicular  
 $b$  = the base } of a right angled triangle.

Then  $h^2 = b^2 + p^2$ , or (Alg. 248.)  $h = \sqrt{b^2 + p^2}$

By trans.  $b^2 = h^2 - p^2$ , or  $b = \sqrt{h^2 - p^2}$

And  $p^2 = h^2 - b^2$ , or  $p = \sqrt{h^2 - b^2}$

Ex. 1. If the base is 32, and the perpendicular 24, what is the hypotenuse? Ans. 40.

2. If the hypotenuse is 100, and the base 80, what is the perpendicular? Ans. 60.

3. If the hypotenuse is 300, and the perpendicular 220, what is the base?

Ans.  $\sqrt{300^2 - 220^2} = 4160$ , the root of which is 204 nearly.

141. It is generally most convenient to find the difference of the squares by *logarithms*. But this is not to be done by *subtraction*. For subtraction, in logarithms, performs the office of *division*. (Art. 41.) If we subtract the logarithm of  $b^2$  from the logarithm of  $h^2$ , we shall have the logarithm, not of the *difference* of the squares, but of their *quotient*. There is, however, an indirect, though very simple method, by which the difference of the squares may be obtained by logarithms. It depends on the principle, that the *difference of the squares of two quantities is equal to the product of the sum and difference of the quantities*. (Alg. 191.) Thus,

$$h^2 - b^2 = (h + b) \times (h - b)$$

as will be seen at once, by performing the multiplication. The two factors may be multiplied by *adding* their logarithms. Hence,

142. *To obtain the difference of the squares of two quantities, add the logarithm of the sum of the quantities to the logarithm of their difference. After the logarithm of the difference of the squares is found; the square root of this difference is obtained, by dividing the logarithm by 2. (Art. 47.)*

Ex. 1. If the hypotenuse be 75 inches, and the base 45, what is the length of the perpendicular?

Sum of the given sides	120	log.	2.07918
Difference of do.	30		<u>1.47712</u>
	Dividing by	2)	3.55630
Side required	60		<u>1.77815</u>

2. If the hypotenuse is 135, and the perpendicular 108, what is the length of the base? Ans. 81.

## SECTION IV.

### SOLUTIONS OF OBLIQUE ANGLED TRIANGLES.

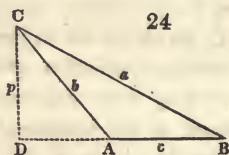
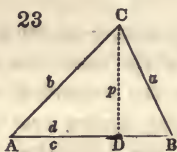
ART. 143. The sides and angles of oblique angled triangles may be calculated by the following theorems.

#### THEOREM I.

In any plane triangle, THE SINES OF THE ANGLES ARE AS THEIR OPPOSITE SIDES.



Let the angles be denoted by the letters A, B, C, and their opposite sides by  $a$ ,  $b$ ,  $c$ , as in Fig. 23 and 24. From one



of the angles, let the line  $p$  be drawn perpendicular to the opposite side. This will fall either within or without the triangle.

1. Let it fall *within* as in Fig. 23. Then, in the right angled triangles ACD, and BCD, according to Art. 126,

$$R : b :: \sin A : p$$

$$R : a :: \sin B : p$$

Here, the two *extremes* are the same in both proportions. The other four terms are, therefore, *reciprocally* proportional:\* that is,

$$a : b :: \sin A : \sin B.$$

2. Let the perpendicular  $p$  fall *without* the triangle, as in Fig. 24. Then in the right angled triangles ACD and BCD;

$$R : b :: \sin A : p$$

$$R : a :: \sin B : p$$

Therefore, as before,

$$a : b :: \sin A : \sin B.$$

Sin A is here put both for the sine of DAC, and for that of BAC. For, as one of these angles is the *supplement* of the other, they have the same sine. (Art. 90.)

The sines which are mentioned here, and which are used

\* Euclid, 23, 5.

in calculation are *tabular* sines. But the proportion will be the same, if the sines be adapted to any other radius. (Art. 119.)

## THEOREM II.

144. In a plane triangle,

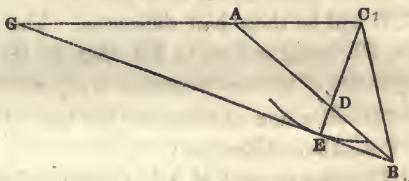
AS THE SUM OF ANY TWO OF THE SIDES,

TO THEIR DIFFERENCE;

SO IS THE TANGENT OF HALF THE SUM OF THE  
OPPOSITE ANGLES;

TO THE TANGENT OF HALF THEIR DIFFERENCE.

Thus, the sum of AB and AC, is to their difference; as the tangent of half the sum of the angles ACB and ABC, to the tangent of half their difference.

*Demonstration.*

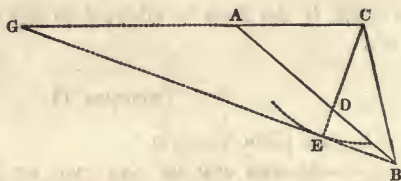
Extend CA to G, making AG equal to AB; then CG is the sum of the two sides AB and AC. On AB, set off AD, equal to AC; then BD is the difference of the sides AB and AC.

The sum of the two angles ACB and ABC, is equal to the sum of ACD and ADC; because each of these sums is the supplement of CAD. (Art. 79.) But as  $AC=AD$  by construction, the angle  $ADC=ACD$  (Euc. 5 1.\*). Therefore ACD is *half the sum* of ACB and ABC. As  $AB=AG$ , the angle  $AGB=ABG$ , or DBE. Also,  $GCE$ , or  $ACD=ADC=BDE$ . (Euc. 15. 1.†) Therefore in the triangles

\* Thomson's Legendre, 11. 1.

† Ibid. 4. 1.

GCE, and DBE, the two remaining angles DEB, and CEG, are equal; (Art. 79.) So that CE is perpendicular to BG. (Euc. Def.



7. 1.\*) If then CE is made radius, GE is the tangent of GCE, (Art. 84.) that is, *the tangent of half the sum of the angles opposite to AB and AC.*

If from the greater of the two angles ACB and ABC, there be taken ACD their half sum; the remaining angle ECB will be their half difference. The tangent of this angle, CE being radius, is EB, that is, *the tangent of half the difference of the angles opposite to AB and AC.* We have then,

CG = the sum of the sides AB and AC ;

DB = their difference ;

GE = the tangent of half the sum of the opposite angles ;

EB = the tangent of half their difference.

But by similar triangles,

$$CG : DB :: GE : EB. \quad \text{Q. E. D.}$$

### THEOREM III.

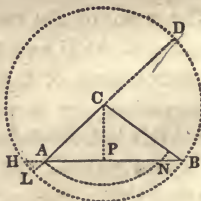
145. If upon the longest side of a triangle, a perpendicular be drawn from the opposite angle ;

AS THE LONGEST SIDE,

TO THE SUM OF THE TWO OTHERS ;

SO IS THE DIFFERENCE OF THE  
LATTER,

TO THE DIFFERENCE OF THE SEG-  
MENTS MADE BY THE PERPEN-  
DICULAR.



In the triangle ABC, if a perpendicular be drawn from C upon AB;

$$AB : CB+CA :: CB-CA : BP-PA.*$$

*Demonstration.*

Describe a circle on the centre C, and with the radius BC. Through A and C, draw the diameter LD, and extend BA to H. Then by (Euc. 35. 3.†)

$$AB \times AH = AL \times AD$$

Therefore,

$$AB : AD :: AL : AH$$

$$\text{But } AD = CD + CA = CB + CA$$

$$\text{And } AL = CL - CA = CB - CA$$

$$\text{And } AH = HP - PA = BP - PA \text{ (Euc. 3. 3.—Thom. 6. 2.)}$$

If, then, for the three last terms in the proportion, we substitute their equals, we have,

$$AB : CB+CA :: CB-CA : PB-PA.$$

146. It is to be observed, that the greater segment is next the greater side. If BC is greater than AC, PB is greater than AP. With the radius AC, describe the arc AN. The segment NP = AP. (Euc. 3. 3.) But BP is greater than NP.

147. The two segments are to each other, as the tangents of the opposite angles, or the cotangents of the adjacent angles. For, in the right angled triangles ACP, and BCP, if CP be made radius, (Art. 126.)

$$R : PC :: \text{Tan } ACP : AP$$

$$R : PC :: \text{Tan } BCP : BP$$

Therefore, by equality of ratios, (Alg. 346.‡)

$$\text{Tan } ACP : AP :: \text{Tan } BCP : BP$$

\* See note B. † Thomson's Legendre, 28. 4. Cor. ; Euc. 11. 5.

That is, the segments are as the tangents of the opposite angles. And the tangents of these are the *cotangents* of the adjacent angles A and B. (Art. 89.)

Cor. The greater segment is opposite to the greater angle. And of the angles at the base, the less is next the greater side. If BP is greater than AP, the angle BCP is greater than ACP; and B is less than A. (Art. 77.)

148. To enable us to find the sides and angles of an oblique angled triangle, *three* of them must be *given*. (Art. 114.)

These may be, either

1. Two angles and a side, or
2. Two sides and an angle *opposite* one of them, or
3. Two sides and the *included* angle, or
4. The three sides.

The two first of these cases are solved by Theorem I, (Art 143.) the third by Theorem II, (Art. 144.) and the fourth by Theorem III. (Art. 145.)

149. In making the calculations, it must be kept in mind, that the greater side is always opposite to the greater angle, (Euc 18, 19. 1.\*) that there can be only one *obtuse* angle in a triangle, (Art. 76.) and therefore, that the angles opposite to the two least sides must be *acute*.

#### CASE I.

150. Given,

Two angles, and	}	to find	{	The remaining angle, and
A side,				The other two sides.

\* Thomson's Legendre, 13. 1.



The third angle is found by merely subtracting the sum of the two which are given from  $180^\circ$ . (Art. 79.)

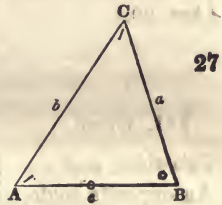
The sides are found, by stating, according to Theorem I, the following proportion ;

As the sine of the angle opposite the *given* side,  
 To the length of the given side ;  
 So is the sine of the angle opposite the *required* side  
 To the length of the required side.

As a *side* is to be found, it is necessary to begin with a *tabular number*.

Ex. 1. In the triangle ABC, the side  $b$  is given 32 rods, the angle A  $56^\circ 20'$ , and the angle C  $49^\circ 10'$ , to find the angle B, and the sides  $a$  and  $c$ .

The sum of the two given angles  $56^\circ 20' + 49^\circ 10' = 105^\circ 30'$ ; which subtracted from  $180^\circ$ , leaves  $74^\circ 30'$  the angle B. Then,



$$\text{Sin B} : b :: \begin{cases} \text{Sin A} : a \\ \text{Sin C} : c \end{cases}$$

Calculation by logarithms.

As the sine of B	$74^\circ 30'$	<i>a. c.</i>	0.01609
To the side $b$	32		1.50515
So is the sine of A	$56^\circ 20'$		<u>9.92027</u>
To the side $a$	27.64		<u>1.44151</u>
As the sine of B	$74^\circ 30'$	<i>a. c.</i>	0.01609
To the side $b$	32		1.50515
So is the sine of C	$49^\circ 10'$		<u>9.87887</u>
To the side $c$	25.13		<u>1.40011</u>

The *arithmetical complement* used in the first term here,

may be found in the usual way, or by taking out the *cosecant* of the given angle, and rejecting 10 from the index. (Art. 113.)

Ex. 2. Given the side  $b$  71, the angle  $A$   $107^{\circ} 6'$ , and the angle  $C$   $27^{\circ} 40'$ ; to find the angle  $B$ , and the sides  $a$  and  $c$ . The angle  $B$  is  $45^{\circ} 14'$ . Then,

$$\text{Sin } B : b :: \begin{cases} \text{Sin } A : a = 95.58 \\ \text{Sin } C : c = 46.43 \end{cases}$$

When one of the given angles is *obtuse*, as in this example, the sine of its *supplement* is to be taken from the tables. (Art. 99.)

### CASE II.

151. Given,

Two sides, and }  
An opposite angle, } to find { The remaining side and  
The other two angles.

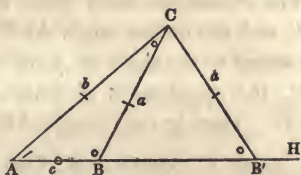
One of the required angles is found, by beginning with a side, and, according to Theorem I, stating the proportion,

As the side opposite the given angle,  
To the sine of that angle;  
So is the side opposite the required angle,  
To the sine of that angle.

The third angle is found, by subtracting the sum of the other two from  $180^{\circ}$ ; and the remaining side is found, by the proportion in the preceding article.

152. In this second case, if the side opposite to the given angle be shorter than the other given side the solution will be *ambiguous*. Two different triangles may be formed, each of which will satisfy the conditions of the problem.

Let the side  $b$ , the angle  $A$ , and the length of the side opposite this angle be given. With the latter for radius, (if it be shorter than  $b$ ,) describe an arc, cutting the line  $AH$  in the points  $B$  and  $B'$ . The lines  $BC$  and  $B'C$ , will be equal. So that, with the same data, there may be formed two different triangles,  $ABC$  and  $AB'C$ .



There will be the same ambiguity in the numerical calculation. The answer found by the proportion will be the *sine* of an angle. But this may be the sine either of the *acute* angle  $AB'C$ , or of the *obtuse* angle  $ABC$ . For,  $BC$  being equal to  $B'C$ , the angle  $CB'B$  is equal to  $CBB'$ . Therefore  $ABC$ , which is the supplement of  $CBB'$ , is also the supplement of  $CB'B$ . But the sine of an angle is the same, as the sine of its supplement. (Art. 90.) The result of the calculation will, therefore, be ambiguous. In practice, however, there will generally be some circumstances which will determine whether the angle required is acute or obtuse.

If the side opposite the given angle be *longer* than the other given side, the angle which is subtended by the latter, will necessarily be acute. For there can be but one obtuse angle in a triangle, and this is always subtended by the longest side. (Art. 149.)

If the *given* angle be obtuse, the other two will, of course, be acute. There can, therefore, be no ambiguity in the solution.

Ex. 1. Given the angle  $A$ ,  $35^\circ 20'$ , the opposite side  $a$  50, and the side  $b$  70; to find the remaining side, and the other two angles.

To find the angle opposite to  $b$ , (Art. 151.)

$$a : \sin A :: b : \sin B$$

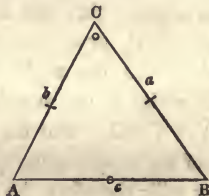
The calculation here gives the acute angle  $AB'C$   $54^\circ 3' 50''$ , and the obtuse angle  $ABC$   $125^\circ 56' 10''$ . If the latter be added to the angle at  $A$   $35^\circ 20'$ , the sum will be  $161^\circ 16' 10''$ , the supplement of which,  $18^\circ 43' 50''$ , is the angle  $ACB$ . Then in the triangle  $ABC$ , to find the side  $c=AB$ ,

$$\sin A : a :: \sin C : c = 27.76$$

If the *acute* angle  $AB'C$   $54^\circ 3' 50''$  be added to the angle at  $A$   $35^\circ 20'$ , the sum will be  $89^\circ 23' 50''$ , the supplement of which,  $90^\circ 36' 10''$ , is the angle  $ACB'$ . Then, in the triangle  $AB'C$ ,

$$\sin A : CB' :: \sin C : AB' = 86.45.$$

Ex. 2. Given the angle at  $A$ ,  $63^\circ 35'$ , the side  $b$   $64$ , and the side  $a$   $72$ ; to find the side  $c$ , and the angles  $B$  and  $C$ .



$$a : \sin A :: b : \sin B = 52^\circ 45' 25''$$

$$\sin A : a :: \sin C : c = 72.05$$

The sum of the angles  $A$  and  $B$ , is  $116^\circ 20' 25''$ , the supplement of which,  $63^\circ 39' 35''$ , is the angle  $C$ .

In this example the solution is *not ambiguous*, because the side opposite the given angle is longer than the other given side.

Ex. 3. In a triangle of which the angles are  $A$ ,  $B$ , and  $C$ , and the opposite sides  $a$ ,  $b$ , and  $c$ , as before; if the angle  $A$  be  $121^\circ 40'$ , the opposite side  $a$   $68$  rods, and the side  $b$   $47$  rods; what are the angles  $B$  and  $C$ , and what is the length of the side  $c$ ? Ans.  $B$  is  $36^\circ 2' 4''$ ,  $C$   $22^\circ 17' 56''$ , and  $c$   $30.3$ .

In this example also, the solution is *not ambiguous*, because the *given* angle is obtuse.



CASE III.

153. Given,

Two sides, and }  
 The included angle, } to find { The remaining side, and  
 { The other two angles.

In this case, the angles are found by Theorem II. (Art. 144.) The required side may be found by Theorem I.

In making the solutions, it will be necessary to observe, that by subtracting the given angle from  $180^\circ$ , the *sum* of the other two angles is found; (Art. 79.) and, that *adding half the difference of two quantities to their half sum gives the greater quantity, and subtracting the half difference from the half sum gives the less.* The latter proposition may be geometrically demonstrated thus;

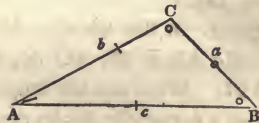
Let AE, be the  
 greater of two \_\_\_\_\_  
 magnitudes, and  
 BE the less. Bisect AB in D, and make AC equal to BE.  
 Then,

- AB is the *sum* of the two magnitudes;
- CE their *difference*;
- DA or DB *half* their *sum*;
- DE or DC *half* their *difference*;

But  $DA + DE = AE$  the *greater* magnitude;

And  $DE - DE = BE$  the *less*.

Ex. 1. In the triangle ABC, the angle A is given  $26^\circ 14'$ , the side  $b$  39, and the side  $c$  53; to find the angles B and C, and the side  $a$ .



The *sum* of the sides  $b$  and  $c$  is  $53 + 39 = 92$

And their *difference*  $53 - 39 = 14$

The *sum* of the angles B and C  $= 180^\circ - 26^\circ 14' = 153^\circ 46'$

And *half the sum* of B and C is  $76^\circ 53'$



Then, by Theorem II, (Fig. 30.)

$$(b+c) : (b-c) :: \tan \frac{1}{2}(B+C) : \tan \frac{1}{2}(B-C)$$

To and from the half sum	76° 53'
Adding and subtracting the half difference	33 8 50
We have the greater angle	<hr style="width: 100%; border: 0.5px solid black;"/> 110 1 50
And the less angle	<hr style="width: 100%; border: 0.5px solid black;"/> 43 44 10

As the greater of the two given sides is  $c$ , the greater angle is  $C$ , and the less angle  $B$ . (Art. 149.)

To find the side  $a$ , by Theorem I.

$$\sin B : b :: \sin A : a = 24.94.$$

Ex. 2. Given the angle  $A$   $101^\circ 30'$ , the side  $b$  76, and the side  $c$  109; to find the angles  $B$  and  $C$ , and the side  $a$ .

$$B \text{ is } 30^\circ 57\frac{1}{2}', C \text{ } 47^\circ 32\frac{1}{2}', \text{ and } a \text{ } 144.8$$

#### CASE IV.

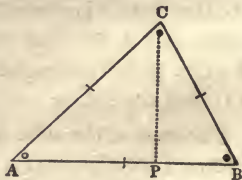
154. Given the three sides, to find the angles.

In this case, the solutions may be made, by drawing a perpendicular to the longest side, from the opposite angle. This will divide the given triangle into two *right angled* triangles. The two segments may be found by Theorem III. (Art. 145.)

There will then be given, in each of the right angled triangles, the hypotenuse and one of the legs, from which the angles may be determined, by rectangular trigonometry. (Art. 135.)

Ex. 1. In the triangle  $ABC$ , the side  $AB$  is 39,  $AC$  35, and  $BC$  27. What are the angles?

Let a perpendicular be drawn from  $C$ , dividing the longest side  $AB$  into the two segments  $AP$  and  $BP$ . Then by Theorem III,



$$AB : AC+BC :: AC-BC . AP-BP.$$

As the longest side	39 a. c.	8.40894
To the sum of the two others	62	1.79239
So is the difference of the latter	8	<u>0.90309</u>
To the difference of the segments	12.72	<u>1.10442</u>

The greater of the two segments is AP, because it is next the side AC, which is greater than BC. (Art. 146.)

To and from half the sum of the segments		19.5
Adding and subtracting half their difference, (Art. 153.)		<u>6.36</u>
We have the greater segment AP		25.86
And the less BP		<u>13.14</u>

Then, in each of the right angled triangles APC and BPC, we have given the hypotenuse and base; and by Art. 135.

$$AC : R :: AP : \cos A = 42^\circ 21' 57''$$

$$BC : R :: BP : \cos B = 60^\circ 52' 42''$$

And subtracting the sum of the angles A and B from  $180^\circ$ , we have the remaining angle  $ACB = 76^\circ 45' 21''$ .

Ex. 2. If the three sides of a triangle are 78, 96, and 104; what are the angles?

Ans.  $45^\circ 41' 48''$ ,  $61^\circ 43' 27''$ , and  $72^\circ 34' 45''$ .

### *Examples for Practice.*

1. Given the angle A  $54^\circ 30'$ , the angle B  $63^\circ 10'$ , and the side  $a$  164 rods; to find the angle C, and the sides  $b$  and  $c$ .
2. Given the angle A  $45^\circ 6'$ , the opposite side  $a$  93, and the side  $b$  108; to find the angles B and C, and the side  $c$ .
3. Given the angle A  $67^\circ 24'$ , the opposite side  $a$  62, and the side  $b$  46; to find the angles B and C, and the side  $c$ .
4. Given the angle A  $127^\circ 42'$ , the opposite side  $a$  381, and the side  $b$  184; to find the angles B and C, and the side  $c$ .

5. Given the side  $b$  58, the side  $c$  67, and the included angle  $A=36^\circ$ ; to find the angles  $B$  and  $C$ , and the side  $a$ .
6. Given the three sides, 631, 268, and 546; to find the angles.

155. The three theorems demonstrated in this section, have been here applied to *oblique angled triangles* only. But they are equally applicable to *right angled triangles*.

Thus, in the triangle  $ABC$ , according to Theorem I, (Art. 143.)

$$\sin B : AC :: \sin A : BC$$

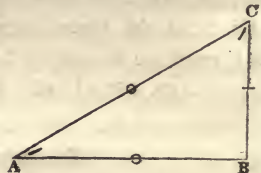
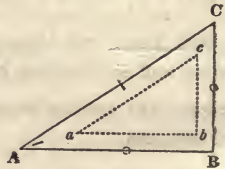
This is the same proportion as one stated in Art 134, except that, in the first term here, the *sine of B* is substituted for *radius*. But, as  $B$  is a right angle, its sine is equal to *radius*. (Art. 95.)

Again, in the triangle  $ABC$ , by the same theorem ;

$$\sin A : BC :: \sin C : AB$$

This is also one of the proportions in rectangular trigonometry, when the hypotenuse is made radius.

The other two theorems might be applied to the solution of right angled triangles. But, when one of the angles is *known* to be a right angle, the methods explained in the preceding section, are much more simple in practice.



## SECTION V.

GEOMETRICAL CONSTRUCTION OF TRIANGLES, BY THE  
PLANE SCALE.

ART. 156. To facilitate the construction of geometrical figures, a number of graduated lines are put upon the common two feet scale; one side of which is called the *Plane Scale*, and the other side, *Gunter's Scale*. The most important of these are the scales of *equal parts*, and the line of *chords*. In forming a given triangle, or any other right lined figure, the parts which must be made to agree with the conditions proposed, are the *lines* and the *angles*. For the former, a scale of equal parts is used; for the latter, a line of chords.

157. The line on the upper side of the plane scale, is divided into *inches* and *tenths* of an inch. Beneath this, on the left hand, are two *diagonal* scales of equal parts,\* divided into inches and half inches, by perpendicular lines. On the larger scale, one of the inches is divided into tenths, by lines which pass *obliquely* across, so as to intersect the parallel lines which run from right to left. The use of the oblique lines is to measure *hundredths* of an inch, by inclining more and more to the right, as they cross each of the parallels.

To take off, for instance, an extent of 3 inches, 4 tenths, and 6 hundredths;

Place one foot of the dividers at the intersection of the perpendicular line marked 3 with the parallel line marked 6,

---

\*These lines are not represented by a figure, as the learner is supposed to have the scale before him.



and the other foot at the intersection of the latter with the oblique line marked 4.

The other diagonal scale is of the same nature. The divisions are smaller, and are numbered from left to right.

158. In geometrical constructions, what is often required, is to make a figure, not *equal* to a given one, but only *similar*. Now figures are similar which have equal angles, and the sides about the equal angles *proportional*. (Euc. Def. 1. 6. \*) Thus a land surveyor, in plotting a field, makes the several lines in his plan to have the same proportion to each other, as the sides of the field. For this purpose a scale of equal parts may be used, of any dimensions whatever. If the sides of the field are 2, 5, 7, and 10 *rods*, and the lines in the plan are 2, 5, 7, and 10 *inches*, and if the angles are the same in each, the figures are similar. One is a copy of the other, upon a smaller scale.

So any two right lined figures are similar, if the angles are the same in both, and if the number of smaller parts in each side of one, is equal to the number of larger parts in the corresponding sides of the other. The several divisions on the scale of equal parts may, therefore, be considered as representing any measures of length, as feet, rods, miles, &c. All that is necessary is, that the scale be not changed, in the construction of the same figure; and that the several divisions and subdivisions be properly proportioned to each other. If the larger divisions, on the diagonal scale, are units, the smaller ones are tenths and *hundredths*. If the larger are tens, the smaller are units and tenths.

159. In laying down an *angle*, of a given number of degrees, it is necessary to *measure* it. Now the proper measure of an angle is an arc of a circle. (Art. 74.) And the measure of an arc, where the radius is given, is its *chord*. For the chord is the distance, in a straight line, from one

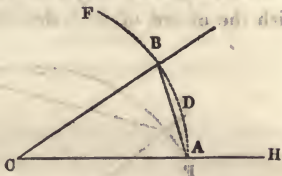
---

\* Thomson's Legendre, Def. 3. 4.



end of the arc to the other. Thus the chord AB, is a measure of the arc ADB, and of the angle ACB.

To form the *line of chords*, a circle is described, and the length of its chords determined for every degree of the quadrant. These measures are put on the plane scale, on the line marked CHO.



160. The chord of  $60^\circ$  is equal to *radius*. (Art. 95.) In laying down or measuring an angle, therefore, an arc must be drawn, with a radius which is equal to the extent from 0 to 60 on the line of chords. There are generally on the scale, two lines of chords. Either of these may be used; but the angle must be measured by the same line from which the radius is taken.

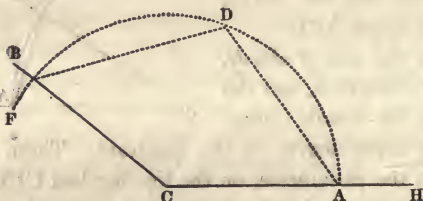
161. To make an angle, then, of a given number of degrees; from one end of a straight line as a centre, and with a radius equal to the chord of  $60^\circ$  on the line of chords, describe an arc of a circle cutting a straight line. From the point of intersection, extend the chord of the given number of degrees, applying the other extremity to the arc; and through the place of meeting, draw the other line from the angular point.

If the given angle is *obtuse*, take from the scale the chord of *half* the number of degrees, and apply it *twice* to the arc. Or make use of the chords of any two arcs whose *sum* is equal to the given number of degrees.

A *right angle* may be constructed, by drawing a perpendicular without using the line of chords.

Ex. 1. To make an angle of 32 degrees. With the point C, in the line CH, for a centre, and with the chord of  $60^\circ$  for radius, describe the arc ADF. Extend the chord of  $32^\circ$  from A to B; and through B, draw the line BC. Then is ACB an angle of 32 degrees.

2. To make an angle of 140 degrees. On the line CH, with the chord of  $60^\circ$ , describe the arc ADF; and extend



the chord of  $70^\circ$  from A to D, and from D to B. The arc  $ADB = 70^\circ \times 2 = 140^\circ$ .

On the other hand :

162. To measure an angle; On the angular point as a centre, and with the chord of  $60^\circ$  for radius, describe an arc to cut the two lines which include the angle. The distance between the points of intersection, applied to the line of chords, will give the measure of the angle in degrees. If the angle be *obtuse*, divide the arc into two parts.

Ex. 1. To measure the angle ACB. (Fig. 33, page 101.) Describe the arc ADF, cutting the lines CH and CB. The distance AB, will extend  $32^\circ$  on the line of chords.

2. To measure the angle ACB. (Fig. 34.) Divide the arc ADB into two parts, either equal or unequal, and measure each part, by applying its chord to the scale. The sum of the two will be  $140^\circ$ .

163. Besides the lines of chords, and of equal parts, on the plane scale; there are also lines of natural *sines*, *tangents*, and *secants*, marked Sin., Tan., and Sec.; of *semitangents*, marked S. T.; of *longitude*, marked Lon. or M. L.; of *rhumbs*, marked Rhu. or Rum., &c. These are not necessary in trigonometrical construction. Some of them are used in Navigation; and some of them in the projections of the Sphere.

164. In Navigation, the quadrant, instead of being graduated in the usual manner, is divided into *eight* portions, called *Rhumbs*. The *Rhumb line*, on the scale, is a line of chords, divided into rhumbs and quarter-rhumbs, instead of degrees.

165. The line of *Longitude* is intended to show the number of geographical miles in a degree of longitude, at different distances from the equator. It is placed over the line of chords, with the numbers in an inverted order: so that the figure above shows the length of a degree of longitude, in any latitude denoted by the figure below.\* Thus, at the equator, where the latitude is 0, a degree of longitude is 60 geographical miles. In latitude 40, it is 46 miles; in latitude 60, 30 miles, &c.

166. The graduation on the line of *secants* begins where the line of sines ends. For the greater sine is only equal to radius; but the secant of the least arc is greater than radius.

167. The *semitangents* are the tangents of *half* the given arcs. Thus, the semitangent of  $20^\circ$  is the tangent of  $10^\circ$ . The line of semitangents is used in one of the projections of the sphere.

168. In the construction of *triangles*, the sides and angles which are *given*, are laid down according to the directions in Arts. 158, 161. The parts *required* are then measured, according to Arts. 158, 162. The following problems correspond with the four cases of oblique angled triangles; (Art. 148.) but are equally adapted to right angled triangles.

169. PROB. I. *The angles and one side* of a triangle being given; to find, by construction, the other two sides.

Draw the given side. From the ends of it, lay off two

\* Sometimes the line of longitude is placed *under* the line of chords.

of the given angles. Extend the other sides till they intersect; and then measure their lengths on a scale of equal parts.

Ex. 1. Given the side  $b$  32 rods, the angle  $A$   $56^{\circ} 20'$ , and the angle  $C$   $49^{\circ} 10'$ ; to construct the triangle, and find the lengths of the sides  $a$  and  $c$ .

Their lengths will be 25 and  $27\frac{1}{2}$ .

2. In a right angled triangle, given the hypotenuse 90, and the angle  $A$   $32^{\circ} 20'$ , to find the base and perpendicular.

The length of  $AB$  will be 76, and of  $BC$  48.

3. Given the side  $AC$  68, the angle  $A$   $124^{\circ}$ , and the angle  $C$   $37^{\circ}$ : to construct the triangle.

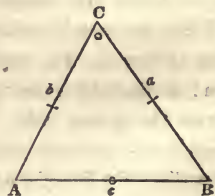
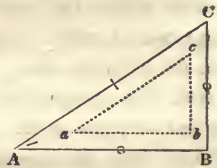
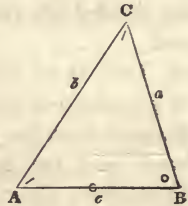
170. PROB. II. *Two sides and an opposite angle* being given, to find the remaining side, and the other two angles.

Draw one of the given sides; from one end of it, lay off the given angle; and extend a line indefinitely for the required side. From the other end of the first side, with the remaining given side for radius, describe an arc cutting the indefinite line. The point of intersection will be the end of the required side.

If the side opposite the given angle be less than the other given side, the case will be *ambiguous*. (Art. 152.)

Ex. 1. Given the angle  $A$   $63^{\circ} 35'$ , the side  $b$  32, and the side  $a$  36.

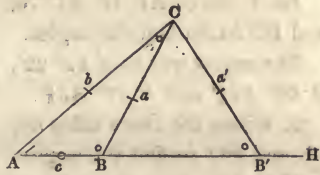
The side  $AB$  will be 36 nearly, the angle  $B$   $52^{\circ} 45\frac{1}{2}'$ , and  $C$   $63^{\circ} 39\frac{1}{2}'$ .





2. Given, the angle  $A$   $35^\circ 20'$ , the opposite side  $a$  25, and the side  $b$  35.

Draw the side  $b$  35, make the angle  $A$   $35^\circ 20'$ , and extend  $AH$  indefinitely. From  $C$  with radius



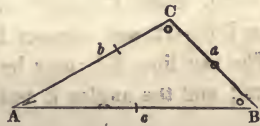
25, describe an arc cutting  $AH$  in  $B$  and  $B'$ . Draw  $CB$  and  $CB'$ , and two triangles will be formed,  $ABC$  and  $AB'C$ , each corresponding with the conditions of the problem.

3. Given the angle  $A$   $116^\circ$ , the opposite side  $a$  38, and the side  $b$  26 ; to construct the triangle.

171. PROB. III. *Two sides and the included angle* being given ; to find the other side and angles.

Draw one of the given sides. From one end of it lay off the given angle, and draw the other given side. Then connect the extremities of this and the first line.

Ex. 1. Given the angle  $A$   $26^\circ 14'$ , the side  $b$  78, and the side  $c$  106 ; to find  $B$ ,  $C$ , and  $a$ .



The side  $a$  will be 50, the angle  $B$   $43^\circ 44'$ , and  $C$   $110^\circ 2'$ .

2. Given  $A$   $86^\circ$ ,  $b$  65, and  $c$  83 ; to find  $B$ ,  $C$ , and  $a$ .

172. PROB. IV. *The three sides* being given ; to find the angles.

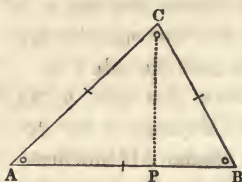
Draw one of the sides, and from one end of it, with an extent equal to the second side, describe an arc. From the other end, with an extent equal to the third side, describe a second arc cutting the first ; and from the point of intersection draw the two sides. (Euc. 22. 1.)



Ex. 1. Given  $AB$  78,  $AC$  70, and  $BC$  54, to find the angles.

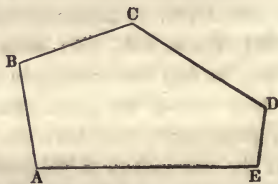
The angles will be  $A$   $42^{\circ} 22'$ ,  $B$   $60^{\circ} 52\frac{2}{3}'$  and  $C$   $76^{\circ} 45\frac{1}{3}'$ .

2. Given the three sides 58, 39, and 46; to find the angles.



173. Any right lined figure whatever, whose sides and angles are given, may be constructed, by laying down the sides from a scale of equal parts, and the angles from a line of chords.

Ex. Given the sides  $AB=20$ ,  $BC=22$ ,  $CD=30$ ,  $DE=12$ ; and the angles  $B=102^{\circ}$ ,  $C=130^{\circ}$ ,  $D=108^{\circ}$ , to construct the figure.



Draw the side  $AB=20$ , make the angle  $B=102^{\circ}$ , draw  $BC=22$ , make  $C=130^{\circ}$ , draw  $CD=30$ , make  $D=108^{\circ}$ , draw  $DE=12$ , and connect  $E$  and  $A$ .

The last line,  $EA$ , may be measured on the scale of equal parts; and the angles  $E$  and  $A$ , by a line of chords.

## SECTION VI.

## DESCRIPTION AND USE OF GUNTER'S SCALE.

ART. 174. An expeditious method of solving the problems in trigonometry, and making other logarithmic calculations, in a mechanical way, has been contrived by Mr. Edmund Gunter. The logarithms of numbers, of sines, tangents, &c., are represented by *lines*. By means of these, multiplication, division, the rule of three, involution, evolution, &c., may be performed much more rapidly, than in the usual method by figures.

The logarithmic lines are generally placed on one side only of the scale in common use. They are,

A line of artificial <i>Sines</i> divided into <i>Rhumbs</i> , and marked,		S. R.
A line of artificial <i>Tangents</i> ,	do	T. R.
A line of the logarithms of <i>Numbers</i> ,		Num.
A line of artificial <i>Sines</i> , to every <i>degree</i> ,		SIN.
A line of artificial <i>Tangents</i> ,	do	TAN.
A line of <i>Versed Sines</i> .		V. S.

To these are added a line of *equal parts*, and a line of *Meridional Parts*, which are not logarithmic. The latter is used in Navigation.

*The Line of Numbers.*

175. Portions of the line of *Numbers*, are intended to represent the *logarithms* of the natural series of numbers 2, 3, 4, 5, &c.

The logarithms of 10, 100, 1000, &c., are 1, 2, 3, &c.  
(Art. 3.)

If, then, the log. of 10 be represented by a line of 1 foot ;  
 the log. of 100 will be repres'd by one of 2 feet ;  
 the log. of 1000 by one of 3 feet ;  
 the lengths of the several lines being proportional to the corresponding logarithms in the tables. *Portions* of a foot will represent the logarithms of numbers between 1 and 10 ; and portions of a line 2 feet long, the logarithms of numbers between 1 and 100.

On Gunter's scale, the line of the logarithms of numbers begins at a brass pin on the left, and the divisions are numbered 1, 2, 3, &c., to another pin near the middle. From this the numbers are repeated, 2, 3, 4, &c., which may be read 20, 30, 40, &c. The logarithms of numbers between 1 and 10, are represented by portions of the first half of the line ; and the logarithms of numbers between 10 and 100, by portions greater than half the line, and less than the whole.

176. The logarithm of 1, which is 0, is denoted, not by any extent of line, but by a *point* under 1, at the commencement of the scale. The distances from this point to different parts of the line, represent other logarithms, of which the *figures* placed over the several divisions are the *natural numbers*. For the intervening logarithms, the intervals between the figures, are divided into tenths, and sometimes into smaller portions. On the right hand half of the scale, as the divisions which are numbered are *tens*, the subdivisions are units.

Ex. 1. To take from the scale the logarithm of 3.6 ; set one foot of the dividers under 1 at the beginning of the scale, and extend the other to the 6th division after the first figure 3.

2. For the logarithm of 47 ; extend from 1 at the beginning, to the 7th subdivision after the second figure 4\*.

---

\* If the dividers will not reach the distance required ; first open them so as to take off *half*, or any part of the distance, and then the remaining part.

177. It will be observed, that the divisions and subdivisions *decrease*, from left to right; as in the tables of *logarithms*, the differences decrease. The difference between the logarithms of 10 and 100, is no greater, than the difference between the logarithms of 1 and 10.

178. The line of numbers, as it has been here explained, furnishes the logarithms of all numbers between 1 and 100.

And if the indices of the logarithms be neglected, the same scale may answer for all numbers whatever. For the *decimal* part of the logarithm of any number is the same, as that of the number multiplied or divided by 10, 100, &c. (Art. 14.) In logarithmic calculations, the use of the indices is to determine the distance of the several figures of the natural numbers from the place of units. (Art. 11.) But in those cases in which the logarithmic line is commonly used, it will not generally be difficult to determine the local value of the figures in the result.

179. We may, therefore, consider the *point* under 1 at the left hand, as representing the logarithm of 1, or 10, or 100; or  $\frac{1}{10}$ , or  $\frac{1}{100}$ , &c., for the decimal part of the logarithm of each of these is 0. But if the first 1 is reckoned 10, all the succeeding numbers must also be increased in a tenfold ratio; so as to read, on the first half of the line, 20, 30, 40, &c., and on the other half, 200, 300, &c.

The whole extent of the logarithmic line,

is from 1	to 100,	or from 0.1	to 10,
or from 10	to 1000,	or from 0.01	to 1,
or from 100	to 10000, &c.	or from 0.001	to 0.1, &c.

Different values may, on different occasions, be assigned to the several numbers and subdivisions marked on this line. But for any one calculation, the value must remain the same.

Ex. Take from the scale 365.

As this number is between 10 and 1000, let the 1 at the



beginning of the scale, be reckoned 10. Then, from this point to the second 3 is 300; to the 6th dividing stroke is 60; and half way from this to the next stroke is 5.

180. Multiplication, division, &c., are performed by the line of numbers, on the same principle, as by common logarithms. Thus,

To *multiply* by this line, *add* the logarithms of the two factors; (Art. 37.) that is, take off, with the dividers, that length of line which represents the logarithm of *one* of the factors, and apply this so as to extend forward from the end of that which represents the logarithm of the *other* factor. The sum of the two will reach to the end of the line representing the logarithm of the product.

Ex. Multiply 9 into 8. The extent from 1 to 8, added to that from 1 to 9, will be equal to the extent from 1 to 72, the product.

181. To *divide* by the logarithmic line, *subtract* the logarithm of the divisor from that of the dividend; (Art. 41.) that is, take off the logarithm of the divisor, and this extent set back from the end of the logarithm of the dividend, will reach to the logarithm of the quotient.

Ex. Divide 42 by 7. The extent from 1 to 7, set back from 42, will reach to 6, the quotient.

182. *Involution* is performed in logarithms, by multiplying the logarithm of the quantity into the index of the power; (Art. 45.) that is, by *repeating* the logarithms as many times as there are units in the index. To involve a quantity on the scale, then, take in the dividers the linear logarithm, and *double it, treble it, &c.*, according to the index of the proposed power.

Ex. 1. Required the square of 9. Extend the dividers from 1 to 9. *Twice* this extent will reach to 81, the square.

2. Required the cube of 4. The extent from 1 to 4 repeated *three times*, will reach to 64 the cube of 4.

183. On the other hand, to perform *evolution* on the scale;



take *half, one-third, &c.*, of the logarithm of the quantity, according to the index of the proposed root.

Ex. 1. Required the square root of 49. *Half* the extent from 1 to 49, will reach from 1 to 7, the root.

2. Required the cube root of 27. *One third* the distance from 1 to 27, will extend from 1 to 3, the root.

184. The *Rule of Three* may be performed on the scale, in the same manner as in logarithms, by adding the two middle terms, and from the sum, subtracting the first term (Art. 52.) But it is more convenient in practice to *begin* by subtracting the first term from one of the others. If four quantities are proportional, the quotient of the first divided by the second, is equal to the quotient of the third divided by the fourth. (Alg. 315.)

Thus, if  $a : b :: c : d$ , then  $\frac{a}{b} = \frac{c}{d}$ , and  $\frac{a}{c} = \frac{b}{d}$ . (Alg. 344.)

But in logarithms, *subtraction* takes the place of division; so that,

$\log. a - \log. b = \log. c - \log. d$ . Or,  $\log. a - \log. c = \log. b - \log. d$ .

Hence,

185. On the scale, *the difference between the first and second terms of a proportion, is equal to the difference between the third and fourth.* Or, the difference between the first and third terms, is equal to the difference between the second and fourth.

The difference between the two terms is taken, by extending the dividers from one to the other. If the second term be greater than the first; the fourth must be greater than the third; if less, less.\* Therefore, if the dividers extend *forward* from *left to right*, that is, from a less number to a greater, from the first term to the second;

---

\* Euclid, 14, 5.

they must also extend forward from the third to the fourth. But if they extend *backward*, from the first term to the second; they must extend the same way, from the third to the fourth.

Ex. 1. In the proportion  $3 : 8 :: 12 : 32$ , the extent from 3 to 8, will reach from 12 to 32; Or, the extent from 3 to 12, will reach from 8 to 32.

2. If 54 yards of cloth cost 48 dollars, what will 18 yds. cost?

$$54 : 48 :: 18 : 16$$

The extent from 54 to 48, will reach *backwards* from 18 to 16.

3. If 63 gallons of wine cost 81 dollars, what will 35 gallons cost?

$$63 : 81 :: 35 : 45$$

The extent from 63 to 81, will reach from 35 to 45.

### *The Line of Sines.*

186. The line on Gunter's scale marked SIN. is a line of logarithmic sines, made to correspond with the line of numbers. The whole extent of the line of numbers, (Art, 179.) is from 1 to 100, whose logs. are 0.00000 and 2.00000, or from 10 to 1000, whose logs. are 1.00000 and 3.00000, or from 100 to 10000, whose logs. are 2.00000 4.00000, the *difference of the indices* of the two extreme logarithms being in each case 2.

Now the logarithmic sine of  $0^{\circ} 34' 22'' 41'''$  is 8.00000  
And the sine of  $90^{\circ}$  (Art. 95.) is 10.00000

Here also the difference of the indices is 2. If then the point directly beneath one extremity of the line of numbers, be marked for the sine of  $0^{\circ} 34' 22'' 41'''$ ; and the point

beneath the other extremity, for the sine of  $90^\circ$ ; the interval may furnish the intermediate sine; the divisions on it being made to correspond with the decimal part of the logarithmic sines in the tables.\*

The first dividing stroke in the line of Sines is generally at  $0^\circ 40'$ , a little farther to the right than the beginning of the line of numbers. The next division is at  $0^\circ 50'$ ; then begins the numbering of the degrees, 1, 2, 3, 4, &c., from left to right.

### *The Line of Tangents.*

187. The first 45 degrees on this line are numbered from left to right, nearly in the same manner as on the line of Sines.

The logarithmic tangent of  $0^\circ 34' 22'' 35'''$  is 8.00000  
And the tangent of  $45^\circ$ , (Art. 95.) is 10.00000

The difference of the indices being 2, 45 degrees will reach to the end of the line. For those above  $45^\circ$  the scale ought to be continued much farther to the right. But as this would be inconvenient, the numbering of the degrees, after reaching 45, is *carried back* from right to left. The same dividing stroke answers for an arc and its *complement*, one above and the other below  $45^\circ$ . For, (Art. 93. Prop. 9.)

$$\tan : R :: R : \cot.$$

In logarithms, therefore, (Art. 184.)

$$\tan - R = R - \cot.$$

That is, the *difference* between the tangent and radius, is equal to the difference between radius and the cotangent: in

---

\* To represent the sines less than  $34' 22'' 41'''$ , the scale must be extended on the left indefinitely. For, as the sine of an arc approaches to 0, its logarithm, which is negative, increases without limit. (Art. 15.)

other words, one is as much *greater* than the tangent of  $45^\circ$ , as the other is *less*. In taking, then, the tangent of an arc greater than  $45^\circ$ , we are to suppose the distance between 45 and the division marked with a given number of degrees, to be added to the whole line, in the same manner as if the line were continued out. In working proportions, extending the dividers *back*, from a less number to a greater, must be considered the same as carrying them *forward* in other cases. See Art. 185.

*Trigonometrical Proportions on the Scale.*

188. In working proportions in trigonometry by the scale; *the extent from the first term to the middle term of the same name, will reach from the other middle term to the fourth term.* (Art. 185.)

In a trigonometrical proportion, two of the terms are the lengths of sides of the given triangle; and the other two are tabular sines, tangents, &c. The former are to be taken from the line of numbers; the latter, from the lines of logarithmic sines and tangents. If one of the terms is a *secant*, the calculation cannot be made on the scale, which has commonly no line of secants. It must be kept in mind that *radius* is equal to the sine of  $90^\circ$ , or to the tangent of  $45^\circ$ . (Art. 95.) Therefore, whenever radius is a term in the proportion, one foot of the dividers must be set on the end of the line of sines or of tangents.

189. The following examples are taken from the proportions which have already been solved by numerical calculation.

Ex. 1. In Case I, of right angled triangles, (Art. 134. ex. 1.)

$$R : 45 :: \sin 32^\circ 20' : 24$$

Here the third term is a *sine*; the first term radius is, therefore, to be considered as the sine of  $90^\circ$ . Then the



extent from  $90^\circ$  to  $32^\circ 20'$  on the line of sines, will reach from 45 to 24 on the line of numbers. As the dividers are set *back* from  $90^\circ$  to  $32^\circ 20'$ ; they must also be set back from 45. (Art. 185.)

2. In the same case, if the base be made radius, (page 60.)

$$R : 38 :: \tan 32^\circ 20' : 24$$

Here, as the third term is a *tangent*, the first term radius is to be considered the tangent of  $45^\circ$ . Then the extent from  $45^\circ$  to  $32^\circ 20'$  on the line of tangents, will reach from 38 to 24 on the line of numbers.

3. If the perpendicular be made radius, (page 62.)

$$R : 24 :: \tan 57^\circ 40' : 38$$

The extent from  $45^\circ$  to  $57^\circ 40'$  on the line of tangents, will reach from 24 to 38 on the line of numbers. For the tangent of  $57^\circ 40'$  on the scale, look for its *complement*  $32^\circ 20'$ . (Art. 187.) In this example, although the dividers extend *back* from  $45^\circ$  to  $57^\circ 40'$ ; yet, as this is from a *less* number to a *greater*, they must extend *forward* on the line of numbers. (Arts. 185, 187.)

4. In Art. 135,

$$35 : R :: 26 : \sin 48^\circ$$

The extent from 35 to 26 will reach from  $90^\circ$  to  $48^\circ$ .

5. In Art. 136,

$$R : 48 :: \tan 27\frac{1}{4}^\circ : 24\frac{3}{4}$$

The extent from  $45^\circ$  to  $27\frac{1}{4}^\circ$ , will reach from 48 to  $24\frac{3}{4}$ .

6. In Art. 150, ex. 1.

$$\sin 74^\circ 30' : 32 :: \sin 56^\circ 20' : 27\frac{1}{2}$$

For other examples, see the several cases in Sections III. and IV.

190. Though the solutions in trigonometry may be ef-

fect by the logarithmic scale, or by geometrical construction, as well as by arithmetical computation; yet the latter method is by far the most accurate. The first is valuable principally for the *expedition* with which the calculations are made by it. The second is of use, in presenting the *form* of the triangle to the eye. But the accuracy which attends arithmetical operations, is not to be expected, in taking lines from a scale with a pair of dividers.\*

---

## SECTION VII.

### THE FIRST PRINCIPLES OF TRIGONOMETRICAL ANALYSIS.

ART. 191. In the preceding sections, sines, tangents, and secants have been employed in calculating the sides and angles of triangles. But the use of these lines is not confined to this object. Important assistance is derived from them, in conducting many of the investigations in the higher branches of analysis, particularly in physical astronomy. It does not belong to an elementary treatise of trigonometry, to prosecute these inquiries to any considerable extent. But this is the proper place for *preparing the formulæ*, the applications of which are to be made elsewhere.

#### *Positive and Negative SIGNS in Trigonometry.*

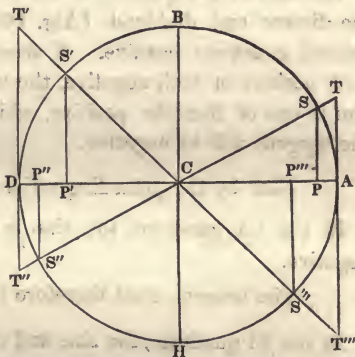
192. Before entering on a particular consideration of the algebraic expressions which are produced by combinations of the several trigonometrical lines, it will be necessary to attend to the positive and negative *signs* in the different

---

\* See note C.

quarters of the circle. The sines, tangents, &c., in the tables, are calculated for a single quadrant only. But these are made to answer for the whole circle. For they are of the same length in each of the four quadrants. (Art. 90.) Some of them however, are *positive*; while others are *negative*. In algebraic processes, this distinction must not be neglected.

193. For the purpose of tracing the changes of the signs, in different parts of the circle, let it be supposed that a straight line  $CT$  is fixed at one end  $C$ , while the other end is carried round, like a rod moving on a pivot; so that the point  $S$  shall describe the circle  $ABDH$ . If the two diameters  $AD$  and  $BH$ , be perpendicular to each other, they will divide the circle into quadrants.



194. In the *first quadrant*  $AB$ , the sine, cosine, tangent, &c., are considered *all positive*. In the *second quadrant*  $BD$ , the sine  $P'S'$  continues *positive*; because it is still on the *upper* side of the diameter  $AD$ , from which it is measured. But the *cosine*, which is measured from  $BH$ , becomes *negative*, as soon as it changes from the *right* to the *left* of this line. (Alg. 382.) In the *third quadrant* the *sine* becomes *negative*, by changing from the upper side to the under side of  $DA$ . The *cosine* continues *negative*, being still on the left of  $BH$ . In the *fourth quadrant*, the *sine* continues *negative*. But the *cosine* becomes *positive*, by passing to the right of  $BH$ .

195. The signs of the *tangents* and *secants* may be derived from those of the sines and cosines. The relations of these several lines to each other must be such, that a uniform method of calculation may extend through the different quadrants.

In the first quadrant, (Art. 93. Propor. 1.)

$$R : \cos :: \tan : \sin, \text{ that is, } \tan = \frac{R \times \sin}{\cos}$$

The sign of the quotient is determined from the signs of the divisor and dividend. (Alg. 100.) The radius is considered as always positive. If then the sine and cosine be both positive or both negative, the tangent will be positive. But if one of these be positive, while the other is negative, the tangent will be negative.

Now by the preceding article,

In the 2d quadrant, the sine is positive, and the cosine negative.

The tangent must therefore be *negative*.

In the 3d quadrant, the sine and cosine are both negative.

The tangent must therefore be *positive*.

In the 4th quadrant, the sine is negative, and the cosine positive.

The tangent must therefore be *negative*.

196. By the 9th, 3d, and 6th proportions in Art. 93.

$$1. \tan : R :: R : \cot, \text{ that is } \cot = \frac{R^2}{\tan}$$

Therefore, as radius is uniformly positive, the *cotangent* must have the same sign as the tangent.

$$2. \cos : R :: R : \sec, \text{ that is, } \sec = \frac{R^2}{\cos}$$



The *secant*, therefore, must have the same sign as the cosine.

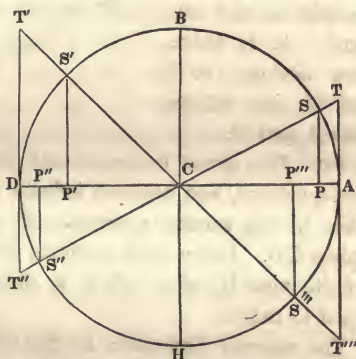
3.  $\text{Sin} : R :: R : \text{cosec}$ , that is,  $\text{Cosec} = \frac{R^2}{\text{sin}}$ .

The *cosecant*, therefore, must have the same sign as the sine.

The *versed sine*, as it is measured from A, in one direction only, is invariably positive.

197. The *tangent*

AT increases, as the arc extends from A towards B. (See also Fig 11. p. 69.) Near B the increase is very rapid; and when the difference between the arc and  $90^\circ$ , is less than any assignable quantity, the tangent is greater than any assignable quantity, and is said



to be *infinite*. (Alg. 447.) If the arc is *exactly*  $90^\circ$ , it has, strictly speaking, *no* tangent. For a tangent is a line drawn perpendicular to the diameter which passes through one end of the arc, and extended till it *meets* a line proceeding from the centre through the other end. (Art. 84.) But if the arc is  $90^\circ$ , as AB, the angle ACB is a right angle, and therefore AT is *parallel* to CB; so that, if these lines be extended ever so far, they never can meet. Still, as an arc infinitely near to  $90^\circ$  has a tangent infinitely great, it is frequently said, in concise terms, that the tangent of  $90^\circ$  is infinite.

In the second quadrant, the tangent is, at first, infinitely

great, and gradually diminishes, till at D it is reduced to nothing. In the third quadrant, it increases again, becomes infinite near H, and is reduced to nothing at A.

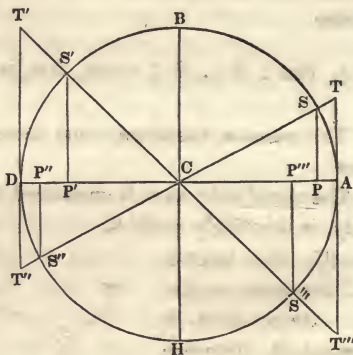
The *cotangent* is inversely as the tangent. It is therefore nothing at B and H, and infinite near A and D.

198. The *secant* increases with the tangent, through the first quadrant, and becomes infinite near B; it then diminishes, in the second quadrant, till at D it is equal to the radius CD. In the third quadrant it increases again, becomes infinite near H, after which it diminishes, till it becomes equal to radius.

The *cosecant* decreases, as the secant increases, and *v. v.* It is therefore equal to radius at B and H, and infinite near A and D.

199. The *sine* increases through the first quadrant, till at B it is equal to radius. (See also Fig. 13. page 70.) It then diminishes, and is reduced to nothing at D. In the third quadrant, it increases again, becomes equal to radius at H, and is reduced to nothing at A.

The *cosine* decreases through the first quadrant, and is reduced to nothing at B. In the second quadrant, it increases, till it becomes equal to radius at D. It then diminishes again, is reduced to nothing at H, and afterwards increases till it becomes equal to radius at A.



In all these cases, the arc is supposed to *begin* at A, and to extend round in the direction of BDH.

200. The *sine* and *cosine* vary from nothing to radius, which they never exceed. The *secant* and *cosecant* are never less than radius, but may be greater than any given length.

The *tangent* and *cotangent* have every value from nothing to infinity. Each of these lines, after reaching its *greatest* limit, begins to *decrease*; and as soon as it arrives at its *least* limit, begins to *increase*. Thus, the sine begins to decrease, after becoming equal to radius, which is its greatest limit. But the secant begins to increase after becoming equal to radius, which is its least limit.

201. The substance of several of the preceding articles is comprised in the following tables. The first shows the *signs* of the trigonometrical lines, in each of the quadrants of the circle. The other gives the *values* of these lines, at the extremity of each quadrant.

	Quadrant	1st	2d	3d	4th
Sine and cosecant		+	+	—	—
Cosine and secant		+	—	—	+
Tangent and cotangent		+	—	+	—
	0°	90°	180°	270°	360°
Sine	0	$r$	0	$r$	0
Cosine	$r$	0	$r$	0	$r$
Tangent	0	$\infty$	0	$\infty$	0
Cotangent	$\infty$	0	$\infty$	0	$\infty$
Secant	$r$	$\infty$	$r$	$\infty$	$r$
Cosecant	$\infty$	$r$	$\infty$	$r$	$\infty$

Here  $r$  is put for radius, and  $\infty$  for infinite.

202. By comparing these two tables, it will be seen, that each of the trigonometrical lines changes from positive to negative, or from negative to positive, in that part of the circle in which the line is either *nothing* or *infinite*. Thus,

the tangent changes from positive to negative, in passing from the first quadrant to the second, through the place where it is infinite. It becomes positive again, in passing from the second quadrant to the third, through the point in which it is nothing.

203. There can be no more than 360 degrees in any circle. But a body may have a number of successive revolutions in the same circle ; as the earth moves round the sun, nearly in the same orbit, year after year. In astronomical calculations, it is frequently necessary to add together parts of different revolutions. The sum may be more than  $360^\circ$ . But a body which has made more than a complete revolution in a circle, is only brought back to a point which it had passed over before. So the sine, tangent, &c., of an arc greater than  $360^\circ$ , is the same as the sine, tangent, &c., of some arc less than  $360^\circ$ . If an entire circumference, or a number of circumferences, be added to any arc, it will terminate in the same point as before. So that, if  $C$  be put for a whole circumference, or  $360^\circ$ , and  $x$  be any arc whatever ;

$$\begin{aligned} \sin x &= \sin (C+x) = \sin (2C+x) = \sin (3C+x), \text{ \&c.} \\ \tan x &= \tan (C+x) = \tan (2C+x) = \tan (3C+x), \text{ \&c.} \end{aligned}$$

204. It is evident also, that, in a number of successive revolutions, in the same circle ;

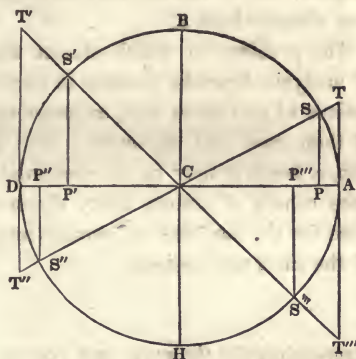
The first quadrant must coincide with the	5th, 9th, 13th, 17th,
The second, with the	6th, 10th, 14th, 18th, &c.
The third, with the	7th, 11th, 15th, 19th, &c.
The fourth, with the	8th, 12th, 16th, 20th, &c.

205. If an arc, extending in a certain direction from a given point, be considered *positive* ; an arc extending from the same point, in an *opposite* direction, is to be considered



*negative.* (Alg. 382.)

Thus, if the arc extending from A to S, be positive; an arc extending from A to S''' will be negative. The latter will not terminate in the *same quadrant* as the other—and the signs of the tabular lines must be accommodated to this circumstance. Thus, the



sine of AS will be positive, while that of AS''' will be negative. (Art. 194.) When a greater arc is subtracted from a less, if the latter be positive, the *remainder* must be negative. (Alg. 40.)

#### TRIGONOMETRICAL FORMULÆ.

206. From the view which has been here taken of the changes in the trigonometrical lines, it will be easy to see, in what parts of the circle each of them increases or decreases. But this does not determine their exact values, except at the extremities of the several quadrants. In the analytical investigations which are carried on by means of these lines, it is necessary to calculate the changes produced in them, by a given increase or diminution of the arcs to which they belong. In this there would be no difficulty, if the sines, tangents, &c., were *proportioned* to their arcs. But this is far from being the case. If an arc is doubled, its sine is *not* exactly doubled. Neither is its tangent or secant. We have to inquire, then, in what manner the sine,

tangent, &c., of one arc may be obtained, from those of other arcs already known.

The problem on which almost the whole of this branch of analysis depends, consists in deriving, from the sines and cosines of two given arcs, expressions for the sine and cosine of their *sum* and *difference*. For, by addition and subtraction, a few arcs may be so combined and varied, as to produce others of almost every dimension. And the expressions for the tangents and secants may be deduced from those of the sines and cosines.

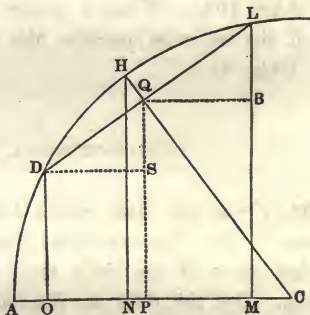
*Expressions for the SINE and COSINE of the SUM and DIFFERENCE of arcs.*

207. Let  $a = AH$ ,  
the greater of the  
given arcs,

And  $b = HL = HD$ ,  
the less.

Then  $a + b = AH + HL = AL$ , the sum of  
the two arcs,

And  $a - b = AH - HD = AD$ , their *difference*.



Draw the chord  $DL$ , and the radius  $CH$ , which may be represented by  $R$ . As  $DH$  is, by construction, equal to  $HL$ ;  $DQ$  is equal to  $QL$ , and therefore  $DL$  is perpendicular to  $CH$ . (Euc. 3. 3.) Draw  $DO$ ,  $HN$ ,  $QP$ , and  $LM$ , each perpendicular to  $AC$ ; and  $DS$  and  $QB$  parallel to  $AC$ .

From the definitions of the sine and cosine, (Arts. 82, 9.) it is evident, that

$$\text{The sine} \begin{cases} \text{of AH, that is, } \sin a = \text{HN,} \\ \text{of HL, } \sin b = \text{QL,} \\ \text{of AL, } \sin (a+b) = \text{LM,} \\ \text{of AD, } \sin (a-b) = \text{DO,} \end{cases}$$

$$\text{The cosine} \begin{cases} \text{of AH, that is, } \cos a = \text{CN,} \\ \text{of HL, } \cos b = \text{CQ,} \\ \text{of AL, } \cos (a+b) = \text{CM,} \\ \text{of AD, } \cos (a-b) = \text{CO.} \end{cases}$$

The triangle CHN is obviously similar to CQP; and it is also similar to BLQ, because the sides of the one are perpendicular to those of the other, each to each. We have, then,

1. CH : CQ :: HN : QP, that is, R : cos b :: sin a : QP,
2. CH : QL :: CN : BL, R : sin b :: cos a : BL,
3. CH : CQ :: CN : CP R : cos b :: cos a : CP,
4. CH : QL :: HN : QB, R : sin b :: sin a : QB,

Converting each of these proportions into an equation;

$$1. \text{QP} = \frac{\sin a \cos b^*}{R} \qquad 3. \text{CP} = \frac{\cos a \cos b}{R}$$

$$2. \text{BL} = \frac{\sin b \cos a}{R} \qquad 4. \text{QB} = \frac{\sin a \sin b}{R}$$

Then adding the first and second,

$$\text{QP} + \text{BL} = \frac{\sin a \cos b + \sin b \cos a}{R}$$

Subtracting the second from the first,

$$\text{QP} - \text{BL} = \frac{\sin a \cos b - \sin b \cos a}{R}$$

---

\* In these formulæ, the sign of multiplication is omitted; sin a cos b being put for sin a × cos b, that is, the product of the sine of a into the cosine of b.

Subtracting the fourth from the third,

$$CP - QB = \frac{\cos a \cos b - \sin a \sin b}{R}$$

Adding the third and fourth,

$$CP + QB = \frac{\cos a \cos b + \sin a \sin b}{R}$$

But it will be seen, from the figure, that

$$QP + BL = BM + BL = LM = \sin(a + b)$$

$$QP - BL = QP - QS = DO = \sin(a - b)$$

$$CP - QB = CP - PM = CM = \cos(a + b)$$

$$CP + QB = CP + SD = CO = \cos(a - b)$$

208. If then, for the first member of each of the four equations above, we substitute its value, we shall have,

$$\text{I. } \sin(a + b) = \frac{\sin a \cos b + \sin b \cos a}{R}$$

$$\text{II. } \sin(a - b) = \frac{\sin a \cos b - \sin b \cos a}{R}$$

$$\text{III. } \cos(a + b) = \frac{\cos a \cos b - \sin a \sin b}{R}$$

$$\text{IV. } \cos(a - b) = \frac{\cos a \cos b + \sin a \sin b}{R}$$

Or multiplying both sides by R,

$$R \sin(a + b) = \sin a \cos b + \sin b \cos a$$

$$R \sin(a - b) = \sin a \cos b - \sin b \cos a$$

$$R \cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$R \cos(a - b) = \cos a \cos b + \sin a \sin b$$

That is, the product of radius and the *sine* of the *sum* of two arcs, is equal to the product of the *sine* of the *first* arc



into the cosine of the second + the product of the sine of the second into the cosine of the first.

The product of radius and the *sine* of the *difference* of two arcs, is equal to the product of the sine of the first arc into the cosine of the second — the product of the sine of the second into the cosine of the first.

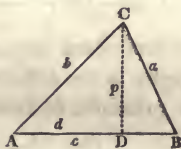
The product of radius and the *cosine* of the *sum* of two arcs, is equal to the product of the cosines of the arcs — the product of their sines.

The product of radius and the *cosine* of the *difference* of two arcs, is equal to the product of the cosines of the arcs + the product of their sines.

These four equations may be considered as fundamental propositions, in what is called the *Arithmetic of Sines and Cosines*, or *Trigonometrical Analysis*.

Expression for the *area* of a triangle, in terms of the sides.

209. Let the sides of the triangle ABC be expressed by  $a$ ,  $b$ , and  $c$ , the perpendicular CD by  $p$ , the segment AD by  $d$ , and the area by  $S$ .



Then  $a^2 = b^2 + c^2 - 2cd$ , (Euc. 13. 2.)

Transposing and dividing by  $2c$  ;

$$d = \frac{b^2 + c^2 - a^2}{2c}. \text{ Therefore } d^2 = \frac{(b^2 + c^2 - a^2)^2}{4c^2}. \text{ (Alg. 171.)}$$

$$\text{By Euc. 47, 1, } p^2 = b^2 - d^2 = b^2 - \frac{(b^2 + c^2 - a^2)^2}{4c^2}$$

Reducing the fraction, (Alg. 120.) and extracting the root of both sides,

$$p = \frac{\sqrt{4b^2c^2 - (b^2 + c^2 - a^2)^2}}{2c}^*$$

This gives the length of the *perpendicular* in terms of the sides of the triangle. But the *area* is equal to the product of the base into half the perpendicular height. (Alg. 393.) That is,

$$S = \frac{1}{2}cp = \frac{1}{4}\sqrt{4b^2c^2 - (b^2 + c^2 - a^2)^2}$$

Here we have an expression for the area, in terms of the sides. But this may be reduced to a form much better adapted to arithmetical computation. It will be seen, that the quantities  $4b^2c^2$ , and  $(b^2 + c^2 - a^2)^2$  are both *squares*; and that the whole expression under the radical sign is the *difference* of these squares. But the difference of two squares is equal to the product of the sum and difference of their roots. (Alg. 191.) Therefore,  $4b^2c^2 - (b^2 + c^2 - a^2)^2$  may be resolved into the two factors,

$$\begin{cases} 2bc + (b^2 + c^2 - a^2) \text{ which is equal to } (b+c)^2 - a^2 \\ 2bc - (b^2 + c^2 - a^2) \text{ which is equal to } a^2 - (b-c)^2 \end{cases}$$

Each of these also, as will be seen in the expressions on the right, is the difference of two squares; and may, on the same principle, be resolved into factors, so that,

$$\begin{cases} (b+c)^2 - a^2 = (b+c+a) \times (b+c-a) \\ a^2 - (b-c)^2 = (a+b-c) \times (a-b+c) \end{cases}$$

\* The expression for the perpendicular is the same, when one of the angles is *obtuse*, as in Fig. 24. page 86. Let  $AD = d$ .

$$\text{Then } a^2 = b^2 + c^2 + 2cd. \text{ (Euc. 12, 2.) And } d = \frac{-b^2 - c^2 + a^2}{2c}$$

$$\text{Therefore, } d^2 = \frac{(-b^2 - c^2 + a^2)^2}{4c^2} = \frac{(b^2 + c^2 - a^2)^2}{4c^2} \text{ (Alg. 169.)}$$

$$\text{And } p = \frac{\sqrt{4b^2c^2 - (b^2 + c^2 - a^2)^2}}{2c} \text{ as above}$$

Substituting, then, these four factors, in the place of the quantity which has been resolved into them, we have,

$$S = \frac{1}{4} \sqrt{(b+c+a) \times (b+c-a) \times (a+b-c) \times (a-b+c)},$$

Here it will be observed, that all the three sides,  $a$ ,  $b$ , and  $c$ , are in each of these factors.

Let  $h = \frac{1}{2}(a+b+c)$  *half the sum* of the sides. Then

$$S = \sqrt{h \times (h-a) \times (h-b) \times (h-c)}$$

210. For finding the area of a triangle, then, when the three sides are given, we have this general rule ;

*From half the sum of the sides, subtract each side severally ; multiply together the half sum and the three remainders ; and extract the square root of the product.*

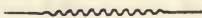
# APPLICATION OF TRIGONOMETRY

TO THE

## MENSURATION

OF

### HEIGHTS AND DISTANCES.



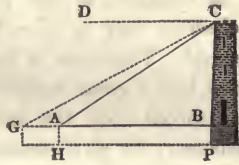
ART. 1. The most direct and obvious method of determining the distance or height of any object, is to apply to it some known measure of length, as a foot, a yard, or a rod. In this manner, the height of a room is found, by a joiner's rule; or the side of a field by a surveyor's chain. But in many instances, the object, or a part, at least, of the line which is to be measured, is *inaccessible*. We may wish to determine the breadth of a river, the height of a cloud, or the distances of the heavenly bodies. In such cases it is necessary to measure some *other* line; from which the required line may be obtained, by geometrical construction, or more exactly, by trigonometrical calculation. The line first measured is frequently called a *base* line.

2. In measuring *angles*, some instrument is used which contains a portion of a graduated circle divided into degrees and minutes. For the proper measure of an angle is an arc of a circle, whose centre is the angular point. (Trig. 74.) The instruments used for this purpose are made in different forms, and with various appendages. The essential parts are a graduated circle, and an index with sight-holes, for taking the directions of the lines which include the angles.

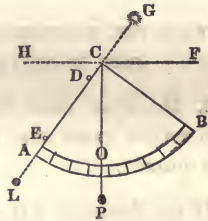
3. Angles of *elevation*, and of *depression* are in a plane



perpendicular to the horizon, which is called a *vertical plane*. An angle of *elevation* is contained between a parallel to the horizon, and an ascending line, as BAC. An angle of *depression* is contained between a parallel to the horizon, and a descending line, as DCA. The *complement* of this is the angle ACB.



4. The instrument by which angles of elevation, and of depression, are commonly measured, is called a *Quadrant*. In its most simple form, it is a portion of a circular board ABC, on which is a graduated arc of 90 degrees, AB, a plumb line CP, suspended from the central point C, and two sight-holes D and E, for taking the direction of the object.



To measure an angle of *elevation* with this, hold the plane of the instrument perpendicular to the horizon, bring the centre C to the angular point, and direct the edge AC in such a manner, that the object G may be seen through the two sight-holes. Then the arc BO measures the angle BCO, which is equal to the angle of elevation FCG. For as the plumb line is perpendicular to the horizon, the angle FCO is a right angle, and therefore equal to BCG. Taking from these the common angle BCF, there will remain the angle  $BCO = FCG$ .

In taking an angle of *depression*, as HCL, the eye is placed at C, so as to view the object at L, through the sight-holes D and E.

5. In treating of the mensuration of heights and dis-

tances, no *new principles* are to be brought into view. We have only to make an application of the rules for the solution of triangles, to the particular circumstances in which the observer may be placed, with respect to the line to be measured. These are so numerous, that the subject may be divided into a great number of distinct cases. But as they are all solved upon the same general principles, it will not be necessary to give examples under each. The following problems may serve as a specimen of those which most frequently occur in practice.

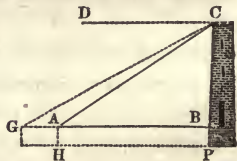
### PROBLEM I.

TO FIND THE PERPENDICULAR HEIGHT OF AN ACCESSIBLE OBJECT STANDING ON A HORIZONTAL PLANE.

6. MEASURE FROM THE OBJECT TO A CONVENIENT STATION, AND THERE TAKE THE ANGLE OF ELEVATION SUBTENDED BY THE OBJECT.

If the distance  $AB$  be measured, and the angle of elevation  $BAC$ ; there will be given in the right angled triangle  $ABC$ , the base and the angles, to find the perpendicular. (Trig. 137.)

As the instrument by which the angle at  $A$  is measured, is commonly raised a few feet above the ground; a point  $B$  must be taken in the object, so that  $AB$  shall be parallel to the horizon. The part  $BP$ , may afterwards be added to the height  $BC$ , found by trigonometrical calculation.



Ex. 1. What is the height of a tower  $BC$ , if the distance  $AB$ , on a horizontal plane, be 93 feet; and the angle  $BAC$   $35\frac{1}{2}$  degrees?

Making the hypotenuse radius (Trig 121.)

Cos. BAC : AB :: Sin. BAC : BC = 69.9 feet.

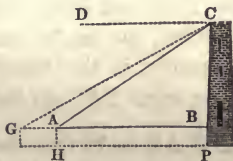
For the *geometrical construction* of the problem, see Trig. 169.

2. What is the height of the perpendicular sheet of water at the falls of Niagara, if it subtends an angle of 40 degrees, at the distance of 163 feet from the bottom, measured on a horizontal plane ?  
 Ans.  $136\frac{3}{4}$  feet.

7. If the height of the object be *known*, its *distance* may be found by the angle of elevation. In this case the angles, and the perpendicular of the triangle are given, to find the base.

Ex. A person on shore, taking an observation of a ship's mast, which is known to be 99 feet high, finds the angle of elevation  $3\frac{1}{2}$  degrees. What is the distance of the ship from the observer ?  
 Ans. 98 rods.

8. If the observer be stationed at the *top* of the perpendicular BC, whose height is known ; he may find the length of the base line AB, by measuring the angle of *depression* ACD, which is equal to BAC.



Ex. A seaman at the top of a mast 66 feet high, looking at another ship, finds the angle of depression 10 degrees. What is the distance of the two vessels from each other ?

Ans.  $22\frac{2}{3}$  rods.

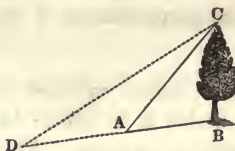
We may find the distance between *two objects* which are in the same vertical plane with the perpendicular, by calculating the distance of each from the perpendicular. Thus AG is equal to the difference between AB and GB.

## PROBLEM II.

TO FIND THE HEIGHT OF AN ACCESSIBLE OBJECT STANDING ON AN INCLINED PLANE.

9. MEASURE THE DISTANCE FROM THE OBJECT TO A CONVENIENT STATION, AND TAKE THE ANGLES WHICH THIS BASE MAKES WITH LINES DRAWN FROM ITS TWO ENDS TO THE TOP OF THE OBJECT.

If the base  $AB$  be measured and the angles  $BAC$  and  $ABC$ ; there will be given, in the oblique angled triangle  $ABC$ , the side  $AB$ , and the angles, to find  $BC$ . (Trig. 150.)



Or the height  $BC$  may be found by measuring the distances  $BA$ ,  $AD$ , and taking the angles,  $BAC$  and  $BDC$ . There will then be given in the triangle  $ADC$ , the angles and the side  $AD$ , to find  $AC$ ; and consequently, in the triangle  $ABC$ , the sides  $AB$  and  $AC$  with the angle  $BAC$ , to find  $BC$ .

Ex. If  $AB$  be 76 feet, the angle  $B$   $101^{\circ} 25'$ , and the angle  $A$   $44^{\circ} 42'$ ; what is the height of the tree  $BC$ ?

$$\text{Sin. } C : AB :: \text{Sin. } A :: BC = 95.9 \text{ feet.}$$

For the *geometrical construction* of the problem, see Trig. 169.

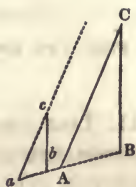
10. The following are some of the methods by which the height of an object may be found, without measuring the angle of elevation.

1. *By shadows.* Let the staff  $bc$  be parallel to an ob-



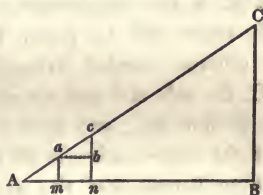
ject BC, whose height is required. If the shadow of BC extend to A, and that of *bc* to *a*; the rays of light CA and *ca* coming from the sun may be considered parallel; and therefore the triangles ABC and *abc* are similar; so that

$$ab : bc :: AB : BC.$$



Ex. If *ab* be 3 feet, *bc* 5 feet, and AB 69 feet, what is the height of BC? Ans. 115 feet.

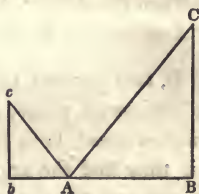
2. *By parallel rods.* If two poles *am* and *cn* be placed parallel to the object BC, and at such distances as to bring the points C, *c*, *a*, in a line, and if *ab* be made parallel to AB; the triangles ABC, and *abc* will be similar; and we shall have



$$ab : bc :: AB : BC.$$

*One pole* will be sufficient, if the observer can place his eye at the point A, so as to bring A, *a*, and C in a line.

3. *By a mirror.* Let the smooth surface of a body of water at A, or any plane mirror parallel to the horizon, be so situated, that the eye of the observer at *c* may view the top of the object C reflected from the mirror. By a law of Optics, the angle BAC is equal to *bAc*; and if *bc* be made parallel to BC, the triangle *bAc* will be similar to BAC; so that



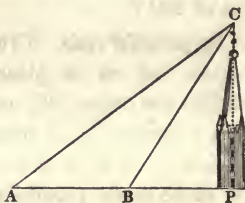
$$Ab : bc :: AB : BC.$$

## PROBLEM III.

TO FIND THE HEIGHT OF AN INACCESSIBLE OBJECT ABOVE A HORIZONTAL PLANE.

11. TAKE TWO STATIONS IN A VERTICAL PLANE PASSING THROUGH THE TOP OF THE OBJECT, MEASURE THE DISTANCE FROM ONE STATION TO THE OTHER, AND THE ANGLE OF ELEVATION AT EACH.

If the base AB be measured, with the angle CBP and CAB; as ABC is the supplement of CBP, there will be given, in the oblique angled triangle ABC, the side AB and the angles, to find BC; and then in the right angled triangle BCP, the hypotenuse and the angles, to find the perpendicular CP.



Ex. 1. If C be the top of a spire, the horizontal base line AB 100 feet, the angle of elevation BAC  $40^\circ$ , and the angle PBC  $60^\circ$ ; what is the perpendicular height of the spire?

The difference between the angles PBC and BAC is equal to ACB. (Euc. 32. 1.)

$$\text{Then } \sin ACB : AB :: \sin BAC : BC = 187.9$$

$$\text{And } R : BC :: \sin PBC : CP = 162\frac{3}{4} \text{ feet.}$$

2. If two persons 120 rods from each other, are standing on a horizontal plane, and also in a vertical plane passing through a *cloud*, both being on the same side of the cloud: and if they find the angles of elevation at the two stations to be  $68^\circ$  and  $76^\circ$ ; what is the height of the cloud?

Ans. 2 miles 135.7 rods.

12. The preceding problems are useful in particular cases.

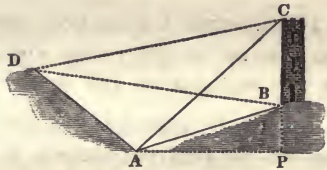
But the following is a *general* rule, which may be used for finding the height of any object whatever, within moderate distances.

PROBLEM IV.

TO FIND THE HEIGHT OF ANY OBJECT, BY OBSERVATIONS AT TWO STATIONS.

13. MEASURE THE BASE LINE BETWEEN THE TWO STATIONS, THE ANGLES BETWEEN THIS BASE AND LINES DRAWN FROM EACH OF THE STATIONS TO EACH END OF THE OBJECT, AND THE ANGLE SUBTENDED BY THE OBJECT, AT ONE OF THE STATIONS.

If BC be the object whose height is required, and if the distance between the stations A and D be measured, with the angles ADC, DAC, ADB, DAB, and BAC;



there will be given in the triangle ADC, the side AD and the angles, to find AC; in the triangle ADB, the side AD and the angles, to find AB; and then, in the triangle BAC, the sides AB and AC with the included angle, to find the required height BC.

If the two stations A and D be in the *same plane* with BC, the angle BAC will be equal to the difference between BAD and CAD. In this case it will not be necessary to measure BAC.

Ex. If AD=83 feet,	{	ADB=33°
{ ADC=51°	{	DAB=121°
{ DAC=95°	{	BAC=26°,

What is the height of the object BC?

$$\sin ACD : AD :: ADC : AC = 115.3 \text{ (Fig. 8.)}$$

$$\sin ABD : AD :: ADB : AB = 103.1.$$

$$(AC + AB) : (AC - AB) :: \tan \frac{1}{2} (ABC + ACB) : \tan \frac{1}{2} (ABC - ACB) = 13^\circ 38'$$

$$\sin ACB : AB :: \sin BAC : BC = 50.57 \text{ feet.}$$

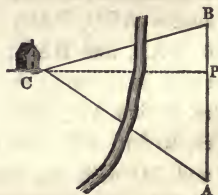
If the object BC be perpendicular to the horizon, its height, after obtaining AB and AC as before, may be found by taking the *angles of elevation* BAP and CAP. The difference of the perpendiculars in the right angled triangles ABP and ACP, will be the height required.

### PROBLEM V.

TO FIND THE DISTANCE OF AN INACCESSIBLE OBJECT.

14. MEASURE A BASE LINE BETWEEN TWO STATIONS, AND THE ANGLES BETWEEN THIS AND LINES DRAWN FROM EACH OF THE STATIONS TO THE OBJECT.

If C be the object, and if the distance between the stations A and B be measured, with the angles at B and A; there will be given, in the oblique angled triangle ABC, the side AB and the angles, to find AC and BC, the distances of the object from the two stations.



For the geometrical construction, see Trig. 169.

Ex. 1. What are the distances of the two stations A and B from the house C, on the opposite side of a river; if AB be 26.6 rods, B  $92^\circ 46'$ , and A  $38^\circ 40'$ ?

The angle C =  $180 - (A + B) = 48^\circ 34'$ . Then

$$\sin C : AB :: \begin{cases} \sin A : BC = 22.17 \\ \sin B : AC = 35.44. \end{cases}$$



2. Two ships in a harbor, wishing to ascertain how far they are from a fort on shore, find that their mutual distance is 90 rods, and that the angles formed between a line from one to the other, and lines drawn from each to the fort are  $45^\circ$  and  $56^\circ 15'$ . What are their respective distances from the fort ?

Ans. 76.3 and 64.9 rods.

15. The *perpendicular* distance of the object from the line joining the two stations may be easily found, after the distance from one of the stations is obtained. The perpendicular distance PC is one of the sides of the right angled triangle BCP. Therefore

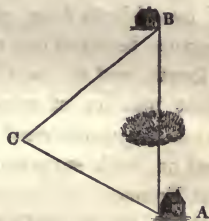
$$R : BC :: \sin B : PC.$$

### PROBLEM VI.

TO FIND THE DISTANCE BETWEEN TWO OBJECTS, WHEN THE PASSAGE FROM ONE TO THE OTHER, IN A STRAIGHT LINE IS OBSTRUCTED.

16. MEASURE THE RIGHT LINES FROM ONE STATION TO EACH OF THE OBJECTS, AND THE ANGLE INCLUDED BETWEEN THESE LINES.

If A and B be the two objects, and if the distances BC and AC be measured, with the angle at C ; there will be given, in the oblique angled triangle ABC, two sides and the included angle, to find the other two angles, and the remaining side. (Trig. 153.)



Ex. The passage between the two objects A and B being obstructed by a morass, the line BC was measured and found to be 109 rods, the line AC 76 rods, and the angle at C  $101^\circ 30'$ . What is the distance AB ?

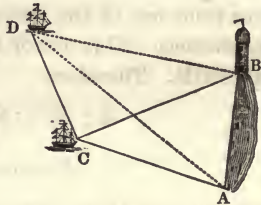
Ans. 144.7 rods.

## PROBLEM VII.

TO FIND THE DISTANCE BETWEEN TWO INACCESSIBLE OBJECTS.

17. MEASURE A BASE LINE BETWEEN TWO STATIONS AND THE ANGLES BETWEEN THIS BASE AND LINES DRAWN FROM EACH OF THE STATIONS TO EACH OF THE OBJECTS.

If A and B be the two objects, and if the distance between the stations C and D be measured, with the angles BDC, BCD, ADC, and ACD; the lines AC and BC may be found as in Problem V, and then the distance AB as in Problem VI.



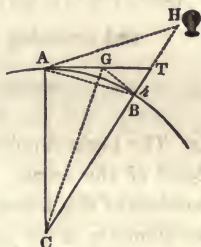
This rule is substantially the same as that in Art. 13. The two stations are supposed to be in the *same plane* with the objects. If they are not, it will be necessary to measure the angle ACB.

18. The same process by which we obtain the distance of *two* objects from each other, will enable us to find the distance between one of these and a third, between that and a fourth, and so on, till a connection is formed between a great number of remote points. This is the plan of the great *Trigonometrical Surveys*, which have been lately carried on, with surprising exactness, particularly in England and France.

19. In the preceding problems for determining altitudes, the objects are supposed to be at such moderate distances, that the observations are not sensibly affected by the *spherical figure of the earth*. The height of an object is measured from an *horizontal plane*, passing through the station at which the angle of elevation is taken. But in an extent

of several miles, the figure of the earth ought to be taken into account.

Let  $AB$  be a portion of the earth's surface,  $H$  an object above it, and  $AT$  a tangent at the point  $A$ , or a horizontal line passing through  $A$ . Then  $HT$ , the oblique height of the object above the horizon of  $A$ , is only a *part* of the height above the surface of the earth, or the level of the ocean.



To obtain the true altitude, it is necessary to add  $BT$  to the height  $HT$  found by observation. The height  $BT$  may be calculated, if the diameter of the earth and the distance  $AT$  be previously known. Or if the height  $BT$  be first determined from observation, with the distance  $AT$ ; the diameter of the earth may be thence deduced.

### PROBLEM VIII.

TO FIND THE DIAMETER OF THE EARTH, FROM THE KNOWN HEIGHT OF A DISTANT MOUNTAIN, WHOSE SUMMIT IS JUST VISIBLE IN THE HORIZON.

20. FROM THE SQUARE OF THE DISTANCE DIVIDED BY THE HEIGHT, SUBTRACT THE HEIGHT.

If  $BT$  (above figure) be a mountain whose height is known, with the distance  $AT$ ; and if the summit  $T$  be just visible in the horizon at  $A$ ; then  $AT$  is a *tangent* at the point  $A$ .

Let  $2BC = D$ , the diameter of the earth,

$AT = d$ , the distance of the mountain,

$BT = h$ , its height.

Then considering AT as a straight line, and the earth as a sphere, we have (Euc. 36. 3.)\*

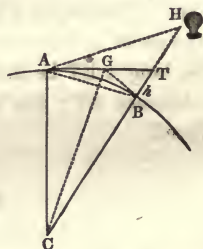
$$(2BC + BT) \times BT = AT^2; \text{ that is, } (D + h) \times h = d^2,$$

and reducing the equation,

$$D = \frac{d^2}{h} - h.$$

Ex. The highest point of the Andes is about 4 miles above the level of the ocean. If a straight line from this touch the surface of the water at the distance of  $178\frac{1}{2}$  miles; what is the diameter of the earth?      Ans. 7940 miles.

21. If the distance AT be *unknown*, it may be found by measuring with a quadrant the angle ATC. Draw BG perpendicular to BC; and join CG. The triangles ACG and BCG are equal, because each has a right angle, the sides AC and BC are equal, and the hypotenuse CG is common. Therefore BG and AG are equal. In the right angled triangle BGT, the angle BTG is given, and the perpendicular BT. From these may be found BG and TG, whose sum is equal to AT, the distance required.†



22. In the common measurement of angles, the light is supposed to come from the object to the eye in a *straight line*. But this is not strictly true. The direction of the light is affected by the *refraction of the atmosphere*. If the object be near, the deviation is very inconsiderable. But in an extent of several miles, and particularly in such nice ob-

\* Thomson's Legendre, 30. 4.

† This method of determining the diameter of the earth is not as accurate as that by measuring a degree of Latitude.



servations as determining the height of distant mountains, and the diameter of the earth, it is necessary to make allowance for the refraction.

PROBLEM IX.

TO FIND THE GREATEST DISTANCE AT WHICH A GIVEN OBJECT CAN BE SEEN ON THE SURFACE OF THE EARTH.

23. TO THE PRODUCT OF THE HEIGHT OF THE OBJECT INTO THE DIAMETER OF THE EARTH, ADD THE SQUARE OF THE HEIGHT; AND EXTRACT THE SQUARE ROOT OF THE SUM.

Let  $2BC = D$ , the diameter of the earth, (Fig. 12.)

$BT = h$ , the height of the object,

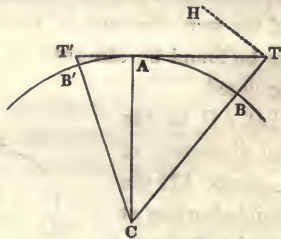
$AT = d$ , the distance required.

$$\text{Then } (D+h) \times h = d^2. \quad \text{And } d = \sqrt{Dh+h^2}.$$

Ex. If the diameter of the earth be 7940 miles, and Mount *Ætna* 2 miles high; how far can its summit be seen at sea? Ans. 126 miles.

The actual distance at which an object can be seen, is increased by the refraction of the air.

24. In this problem the eye is supposed to be placed at the level of the ocean. But if the observer be elevated above the surface, as on the deck of a ship, he can see to a greater distance. If  $BT$  be the height of the object, and  $B'T'$  the height of the eye above the level of the ocean; the distance at which the object can be seen, is evidently equal to the *sum* of the tangents  $AT$  and  $AT'$ .



Ex. The top of a ship's mast 132 feet high is just visible in the horizon, to an observer whose eye is 33 feet above the surface of the water. What is the distance of the ship?

Ans.  $21\frac{1}{8}$  miles.

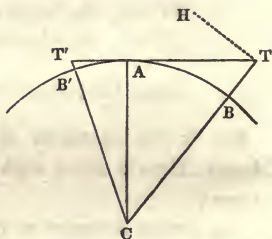
25. The distance to which a person can see the smooth surface of the ocean, if no allowance be made for refraction, is equal to a tangent to the earth drawn from his eye, as T'A. (Fig. 13.)

Ex. If a man standing on the level of the ocean, has his eye raised  $5\frac{1}{2}$  feet above the water: to what distance can he see the surface?

Ans.  $2\frac{7}{8}$  miles.

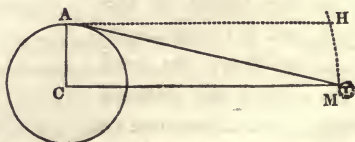
26. If the distance AT, with the diameter of the earth be given, and the *height* BT be required; the equation in Art. 23 gives

$$h = \sqrt{\frac{1}{4}D^2 + a^2} - \frac{1}{2}D$$



27. When the diameter of the earth is ascertained, this may be made a *base line* for determining the distance of the *heavenly bodies*. A right angled triangle may be formed, the perpendicular sides of which shall be the distance required, and the semi-diameter of the earth. If then one of the angles be found by observation, the required side may be easily calculated.

Let AC be the semi-diameter of the earth, AH the sensible horizon at A, and CM the rational horizon, parallel to AH, passing



through the moon M. The angle HAM may be found by astronomical observation. This angle, which is called the *Horizontal Parallax*, is equal to AMC, the angle at the moon subtended by the semi-diameter of the earth. (Euc. 29. 1.)

### PROBLEM X.

TO FIND THE DISTANCE OF ANY HEAVENLY BODY WHOSE HORIZONTAL PARALLAX IS KNOWN.

28. AS RADIUS, TO THE SEMI-DIAMETER OF THE EARTH ; SO IS THE CO-TANGENT OF THE HORIZONTAL PARALLAX, TO THE DISTANCE.

In the right angled triangle ACM, (Fig. 14.) if AC be made radius ;

$$R : AC :: \text{Cot. AMC} : CM.$$

Ex. If the horizontal parallax of the moon be  $0^{\circ} 57'$ , and the diameter of the earth 7940 miles ; what is the distance of the moon from the centre of the earth ?

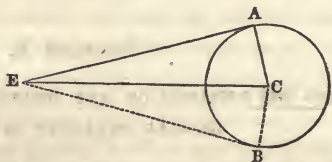
Ans. 239,414 miles.

29. The *fixed stars* are too far distant to have any sensible horizontal parallax. But from late observations it would seem, that some of them are near enough, to suffer a small apparent change of place, from the revolution of the earth round the sun. The distance of the sun, then, which is the semi-diameter of the earth's orbit, may be taken as a *base line*, for finding the distance of the stars.

We thus proceed by degrees from measuring a line on the surface of the earth, to calculate the distances of the heavenly bodies. From a base line on a plane, is determined the height of a mountain ; from the height of a mountain, the diameter of the earth ; from the diameter of the earth, the distance of the sun, and from the distance of the sun the distance of the stars.

30. After finding the distance of a heavenly body, its *magnitude* is easily ascertained; if it have an apparent diameter, sufficiently large to be measured by the instruments which are used for taking angles.

Let AEB be the angle which a heavenly body subtends at the eye. Half this angle, if C be the centre of the body, is AEC; the line EA is a tangent



to the surface, and therefore EAC is a right angle. Then making the distance EC radius,

$$R : EC :: \text{Sin. AEC} : AC.$$

That is, radius is to the distance, as the sine of half the angle which the body subtends, to its semi-diameter.

Ex. If the sun subtends an angle of  $32' 2''$ , and if his distance from the earth be 95 million miles; what is his diameter?  
 Ans. 885 thousand miles.

#### PROMISCUOUS EXAMPLES.

1. On the bank of a river, the angle of elevation of a tree on the opposite side is found to be  $46^\circ$ ; and at another station 100 feet directly back on the same level,  $31^\circ$ . What is the height of the tree?  
 Ans. 143 feet.

2. On a horizontal plane, observations were taken of a tower standing on the top of a hill. At one station the angle of elevation of the top of the tower was found to be  $50^\circ$ ; that at the bottom  $39^\circ$ ; and at another station 150 feet directly back, the angle of elevation of the top of the tower was  $32^\circ$ . What are the heights of the hill and the tower?

Ans. The hill is 134 feet high; the tower 63.



3. What is the altitude of the sun, when the shadow of a tree, cast on a horizontal plane, is to the height of the tree as 4 to 3?  
 Ans.  $36^{\circ} 52' 12''$ .

4. If a straight line from the top of the White Mountains in New Hampshire touch the ocean at the distance of  $103\frac{1}{2}$  miles? what is the height of the mountains?  
 Ans. 7100 feet.

5. From the top of a perpendicular rock 55 yards high, the angle of depression of the nearest bank of a river is found to be  $55^{\circ} 54'$ , that of the opposite bank  $33^{\circ} 20'$ . Required the breadth of the river, and the distance of its nearest bank from the bottom of the rock.

The breadth of the river is 46.4 yards ;

Its distance from the rock 37.2.

6. If the moon subtend an angle of  $31' 14''$ , when her distance is 240,000 miles ; what is her diameter ?

Ans. 2180 miles.

7. Observations are made on the altitude of a balloon, by two persons standing on the same side of the balloon, and in a vertical plane passing through it. The distance of the stations is half a mile. At one, the angle of elevation is  $30^{\circ} 58'$ , at the other  $36^{\circ} 52'$ . What is the height of the balloon above the ground ?  
 Ans.  $1\frac{1}{2}$  miles.

8. The shadow of the top of a mountain, when the altitude of the sun on the meridian is  $32^{\circ}$ , strikes a certain point on a level plain below ; but when the meridian altitude of the sun is  $67^{\circ}$ , the shadow strikes half a mile farther south, on the same plain. What is the height of the mountain above the plain ?  
 Ans. 2245 feet.

## N O T E S .

---

### NOTE A. p. 13.

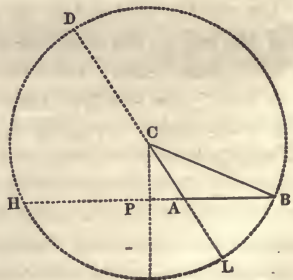
It is common to *define* logarithms to be a series of numbers in arithmetical progression, corresponding with another series in geometrical progression. This is calculated to perplex the learner, when, upon opening the tables, he finds that the natural numbers, as they stand there, instead of being in *geometrical*, are in *arithmetical* progression; and that the logarithms are *not* in arithmetical progression.

It is true, that a geometrical series may be obtained, by taking out, here and there, a few of the natural numbers; and that the logarithms of these will form an arithmetical series. But the definition is not applicable to the whole of the numbers and logarithms, as they stand in the tables.

### NOTE B. p. 89.

If the perpendicular be drawn from the angle opposite the longest side, it will always fall *within* the triangle; because the other two angles must, of course, be acute. But if one of the angles at the base be *obtuse*, the perpendicular will fall *without* the triangle, as CP.

In this case, the side on which the perpendicular



falls, is to the sum of the other two ; as the difference of the latter, to the *sum* of the segments made by the perpendicular.

The demonstration is the same, as in the other case, except that  $AH=BP+PA$ , instead of  $BP-PA$ .

Thus, in the circle BDHL, of which C is the centre,

$$AB \times AH = AL \times AD ; \text{ therefore } AB : AD :: AL :: AH.$$

$$\text{But } AD = CD + CA = CB + CA$$

$$\text{And } AL = CL - CA = CB - CA$$

$$\text{And } AH = HP + PA = BP + PA$$

Therefore,

$$AB : CB + CA :: CB - CA : BP + PA$$

When the three sides are given, it may be known whether one of the angles is obtuse. For any angle of a triangle is obtuse or acute, according as the square of the side subtending the angle is *greater*, or *less*, than the sum of the squares of the sides containing the angle. (Euc. 12, 13. 2.)\*

NOTE C. p. 000.

Gunter's *Sliding Rule*, is constructed upon the same principle as his scale, with the addition of a slider, which is so contrived as to answer the purpose of a pair of dividers, in working proportions, multiplying, dividing, &c. The lines on the *fixed part* are the same as on the scale. The *slider* contains two lines of numbers, a line of logarithmic sines, and a line of logarithmic tangents.

To *multiply* by this, bring 1 on the slider, against one of the factors on the fixed part ; and against the other factor on the slider, will be the product on the fixed part. To *divide*, bring the divisor on the slider, against the dividend on the fixed part ; and against 1 on the slider, will be the quotient

---

\* Thomson's Legendre, 12, 13. 4.

on the fixed part. To work a *proportion*, bring the first term on the slider, against one of the middle terms on the fixed part; and against the other middle term on the slider, will be the fourth term on the fixed part. Or the first term may be taken on the fixed part; and then the fourth term will be found on the slider.

Another instrument frequently used in trigonometrical constructions, is

#### THE SECTOR.

This consists of two equal scales movable about a point as a centre. The lines which are drawn on it are of two kinds, some being parallel to the sides of the instrument, and others diverging from the central point, like the radii of a circle. The latter are called the *double* lines, as each is repeated upon the two scales. The *single* lines are of the same nature, and have the same use, as those which are put upon the common scale; as the lines of equal parts, of chords, of latitude, &c., on one face; and the logarithmic lines of numbers, of sines, and of tangents, on the other.

The *double* lines are

A line of <i>Lines</i> , or equal parts, marked	Lin. or L.
A line of <i>Chords</i> ,	Cho. or C.
A line of natural <i>Sines</i> ,	Sin. or S.
A line of natural <i>Tangents</i> to $45^\circ$ ,	Tan. or T.
A line of tangents, <i>above</i> $45^\circ$ ,	Tan. or T.
A line of natural <i>Secants</i> ,	Sec. or S.
A line of <i>Polygons</i> ,	Pol. or P.

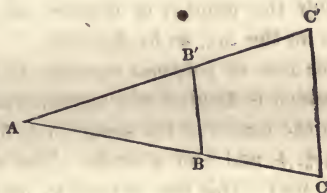
The double lines of *chords*, of *sines*, and of *tangents* to  $45^\circ$ , are all of the same radius; beginning at the central point, and terminating near the other extremity of each scale; the chords at  $60^\circ$ , the sines at  $90^\circ$ , and the tangents at  $45^\circ$ . (See Art. 95.) The line of *lines* is also of the same length, containing ten equal parts which are numbered, and



which are again subdivided. The radius of the lines of secants and of tangents above  $45^\circ$ , is about one-fourth of the length of the other lines. From the end of the radius, which for the secants is at 0, and for the tangents at  $45^\circ$ , these lines extend to between  $70^\circ$  and  $80^\circ$ . The line of polygons is numbered 4, 5, 6, &c., from the extremity of each scale, towards the centre.

The simple principle on which the utility of these several pairs of lines depends is this, that *the sides of similar triangles are proportional*. (Euc. 4. 6.) So that sines, tangents,

&c., are furnished to any radius, within the extent of the opening of the two scales. Let AC and AC' be any pair of lines on the sector, and AB and AB' equal portions of these



lines. As AC and AC' are equal, the triangle ACC' is isosceles, and similar to ABB'. Therefore,

$$AB : AC :: BB' : CC'.$$

Distances measured from the centre on either scale, as AB and AC, are called *lateral distances*. And the distances between corresponding points of the two scales, as BB' and CC', are called *transverse distances*.

Let AC and CC' be radii of two circles. Then if AB be the chord, sine, tangent, or secant, of any number of degrees in one; BB' will be the chord, sine, tangent, or secant of the same number of degrees in the other. (Art. 119.) Thus, to find the *chord* of  $30^\circ$ , to a radius of four inches, open the sector so as to make the transverse distance from 60 to 60, on the lines of chords, four inches; and the distance from 30 to 30, on the same lines, will be the chord required. To find the *sine* of  $28^\circ$ , make the distance from 90 to 90, on the

lines of sines, equal to radius; and the distance from 28 to 28 will be the sine. To find the *tangent* of  $37^\circ$ , make the distance from 45 to 45, on the lines of tangents, equal to radius; and the distance from 37 to 37 will be the tangent. In finding *secants*, the distance from 0 to 0 must be made radius. (Art. 201.)

To lay down an *angle* of  $34^\circ$ , describe a circle, of any convenient radius, open the sector, so that the distance from 60 to 60 on the lines of chords shall be equal to this radius, and to the circle apply a chord equal to the distance from 34 to 34. (Art. 161.) For an angle above  $60^\circ$ , the chord of *half* the number of degrees may be taken, and applied *twice* on the arc, as in Art. 161.

The line of *polygons* contains the chords of arcs of a circle which is divided into equal portions. Thus, the distances from the centre of the sector to 4, 5, 6, and 7, are the chords of  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ , and  $\frac{1}{7}$  of a circle. The distance 6 is the radius. (Art. 95.) This line is used to make a regular polygon, or to inscribe one in a given circle. Thus, to make a *pentagon* with the transverse distance from 6 to 6 for radius, describe a circle, and the distance from 5 to 5 will be the length of one of the sides of a pentagon inscribed in that circle.

The line of *lines* is used to divide a line into equal or proportional parts, to find fourth proportionals, &c. Thus, to divide a line into 7 equal parts, make the length of the given line the transverse distance from 7 to 7, and the distance from 1 to 1 will be one of the parts. To find  $\frac{2}{5}$  of a line, make the transverse distance from 5 to 5 equal to the given line; and the distance from 3 to 3 will be  $\frac{2}{5}$  of it.

In working the *proportions in trigonometry* on the sector, the lengths of the sides of triangles are taken from the line of lines, and the degrees and minutes from the lines of sines, tangents, or secants. Thus, in Art. 135, ex. 1,

$$35 : R :: 26 : \sin 48^\circ.$$

To find the fourth term of this proportion by the sector, make the lateral distance 35 on the line of lines, a transverse distance from 90 to 90 on the lines of sines ; then the lateral distance 26 on the line of lines, will be the transverse distance from 48 to 48 on the lines of sines.

For a more particular account of the construction and uses of the Sector, see Stone's edition of Bion on Mathematical Instruments, Hutton's Dictionary, and Robertson's Treatise on Mathematical Instruments.





Day and Thomson's Series.

---

A

PRACTICAL APPLICATION

OF

THE PRINCIPLES OF GEOMETRY

TO THE

MENSURATION

OF

SUPERFICIES AND SOLIDS.

ADAPTED TO

THE METHOD OF INSTRUCTION IN SCHOOLS AND ACADEMIES.

BY JEREMIAH DAY, D.D. LL.D.

LATE PRESIDENT OF YALE COLLEGE.

NEW YORK:

PUBLISHED BY MARK H. NEWMAN & CO.,  
No. 199 BROADWAY.

1848.

ENTERED, according to Act of Congress, in the year 1848, by  
JEREMIAH DAY,

In the Clerk's Office of the District Court of the United States for the  
Southern District of New York.

THOMAS B. SMITH, STEREOTYPER,  
216 WILLIAM STREET, N. Y.

# CONTENTS.

---

	Page
SECTION I. Areas of figures bounded by right lines, . . . . .	5
II. The Quadrature of the Circle and its parts, . . . . .	19
Promiscuous examples of Areas, . . . . .	34
III. Solids bounded by plane surfaces, . . . . .	37
IV. The Cylinder, Cone, and Sphere, . . . . .	56
Promiscuous examples of Solids, . . . . .	76
V. Isoperimetry, . . . . .	78

## APPENDIX.

Gauging of Casks, . . . . .	92
Notes, . . . . .	99

CONTENTS

1	Introduction
2	Chapter I
3	Chapter II
4	Chapter III
5	Chapter IV
6	Chapter V
7	Chapter VI
8	Chapter VII
9	Chapter VIII
10	Chapter IX
11	Chapter X
12	Chapter XI
13	Chapter XII
14	Chapter XIII
15	Chapter XIV
16	Chapter XV
17	Chapter XVI
18	Chapter XVII
19	Chapter XVIII
20	Chapter XIX
21	Chapter XX
22	Chapter XXI
23	Chapter XXII
24	Chapter XXIII
25	Chapter XXIV
26	Chapter XXV
27	Chapter XXVI
28	Chapter XXVII
29	Chapter XXVIII
30	Chapter XXIX
31	Chapter XXX
32	Chapter XXXI
33	Chapter XXXII
34	Chapter XXXIII
35	Chapter XXXIV
36	Chapter XXXV
37	Chapter XXXVI
38	Chapter XXXVII
39	Chapter XXXVIII
40	Chapter XXXIX
41	Chapter XL
42	Chapter XLI
43	Chapter XLII
44	Chapter XLIII
45	Chapter XLIV
46	Chapter XLV
47	Chapter XLVI
48	Chapter XLVII
49	Chapter XLVIII
50	Chapter XLIX
51	Chapter L
52	Chapter LI
53	Chapter LII
54	Chapter LIII
55	Chapter LIV
56	Chapter LV
57	Chapter LVI
58	Chapter LVII
59	Chapter LVIII
60	Chapter LIX
61	Chapter LX
62	Chapter LXI
63	Chapter LXII
64	Chapter LXIII
65	Chapter LXIV
66	Chapter LXV
67	Chapter LXVI
68	Chapter LXVII
69	Chapter LXVIII
70	Chapter LXIX
71	Chapter LXX
72	Chapter LXXI
73	Chapter LXXII
74	Chapter LXXIII
75	Chapter LXXIV
76	Chapter LXXV
77	Chapter LXXVI
78	Chapter LXXVII
79	Chapter LXXVIII
80	Chapter LXXIX
81	Chapter LXXX
82	Chapter LXXXI
83	Chapter LXXXII
84	Chapter LXXXIII
85	Chapter LXXXIV
86	Chapter LXXXV
87	Chapter LXXXVI
88	Chapter LXXXVII
89	Chapter LXXXVIII
90	Chapter LXXXIX
91	Chapter LXXXX
92	Chapter LXXXXI
93	Chapter LXXXXII
94	Chapter LXXXXIII
95	Chapter LXXXXIV
96	Chapter LXXXXV
97	Chapter LXXXXVI
98	Chapter LXXXXVII
99	Chapter LXXXXVIII
100	Chapter LXXXXIX
101	Chapter LXXXXX

INDEX

1	Introduction
2	Chapter I
3	Chapter II
4	Chapter III
5	Chapter IV
6	Chapter V
7	Chapter VI
8	Chapter VII
9	Chapter VIII
10	Chapter IX
11	Chapter X
12	Chapter XI
13	Chapter XII
14	Chapter XIII
15	Chapter XIV
16	Chapter XV
17	Chapter XVI
18	Chapter XVII
19	Chapter XVIII
20	Chapter XIX
21	Chapter XX
22	Chapter XXI
23	Chapter XXII
24	Chapter XXIII
25	Chapter XXIV
26	Chapter XXV
27	Chapter XXVI
28	Chapter XXVII
29	Chapter XXVIII
30	Chapter XXIX
31	Chapter XXX
32	Chapter XXXI
33	Chapter XXXII
34	Chapter XXXIII
35	Chapter XXXIV
36	Chapter XXXV
37	Chapter XXXVI
38	Chapter XXXVII
39	Chapter XXXVIII
40	Chapter XXXIX
41	Chapter XL
42	Chapter XLI
43	Chapter XLII
44	Chapter XLIII
45	Chapter XLIV
46	Chapter XLV
47	Chapter XLVI
48	Chapter XLVII
49	Chapter XLVIII
50	Chapter XLIX
51	Chapter L
52	Chapter LI
53	Chapter LII
54	Chapter LIII
55	Chapter LIV
56	Chapter LV
57	Chapter LVI
58	Chapter LVII
59	Chapter LVIII
60	Chapter LIX
61	Chapter LX
62	Chapter LXI
63	Chapter LXII
64	Chapter LXIII
65	Chapter LXIV
66	Chapter LXV
67	Chapter LXVI
68	Chapter LXVII
69	Chapter LXVIII
70	Chapter LXIX
71	Chapter LXX
72	Chapter LXXI
73	Chapter LXXII
74	Chapter LXXIII
75	Chapter LXXIV
76	Chapter LXXV
77	Chapter LXXVI
78	Chapter LXXVII
79	Chapter LXXVIII
80	Chapter LXXIX
81	Chapter LXXX
82	Chapter LXXXI
83	Chapter LXXXII
84	Chapter LXXXIII
85	Chapter LXXXIV
86	Chapter LXXXV
87	Chapter LXXXVI
88	Chapter LXXXVII
89	Chapter LXXXVIII
90	Chapter LXXXIX
91	Chapter LXXXX
92	Chapter LXXXXI
93	Chapter LXXXXII
94	Chapter LXXXXIII
95	Chapter LXXXXIV
96	Chapter LXXXXV
97	Chapter LXXXXVI
98	Chapter LXXXXVII
99	Chapter LXXXXVIII
100	Chapter LXXXXIX
101	Chapter LXXXXX

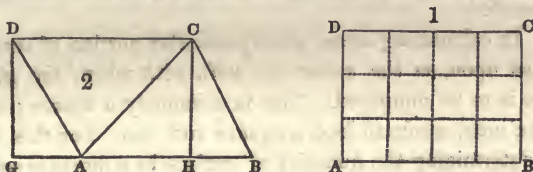


## SECTION I.

### AREAS OF FIGURES BOUNDED BY RIGHT LINES.

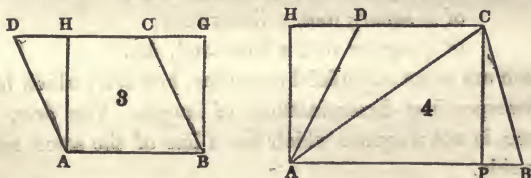
ART. 1. The following definitions, which are nearly the same as in Euclid, are inserted here for the convenience of reference.

I. *Four-sided* figures have different names, according to the relative position and length of the sides. A *parallelogram* has its opposite sides equal and parallel, as ABCD.



(Fig. 2.) A *rectangle*, or *right parallelogram*, has its opposite sides equal, and all its angles right angles; as AC.

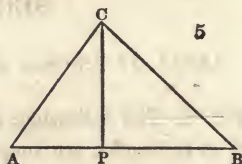
(Fig. 1.) A *square* has all its sides equal, and all its angles right angles; as ABGH. (Fig. 3.) A *rhombus* has all its



sides equal, and its angles oblique; as ABCD. (Fig. 3.) A *rhomboid* has its opposite sides equal, and its angles oblique; as ABCD. (Fig. 2.) A *trapezoid* has only two of its sides parallel; as ABCD. (Fig. 4.) Any other four sided figure is called a *trapezium*.

II. A figure which has more than four sides is called a *polygon*. A *regular polygon* has all its sides equal, and all its angles equal.

III. The *height* of a *triangle* is the length of a perpendicular, drawn from one of the angles to the opposite side; as CP. The *height* of a *four sided* figure is the perpendicular distance between two of its parallel sides; as CP. (Fig. 4.)



IV. The *area* or *superficial contents* of a figure is the *space* contained within the line or lines by which the figure is bounded.

2. In calculating areas, some particular portion of surface is fixed upon, as the *measuring unit*, with which the given figure is to be compared. This is commonly a *square*; as a square inch, a square foot, a square rod, &c. For this reason, determining the quantity of surface in a figure is called *squaring it*, or finding its *quadrature*; that is, finding a square or number of squares to which it is equal.

3. The *superficial* unit has generally the same name, as the *linear* unit which forms the side of the square.

The side of a square inch is a linear inch;

of a square foot, a linear foot;

of a square rod, a linear rod, &c.

There are some superficial measures, however, which have no corresponding denominations of length. The *acre*, for instance, is not a square which has a line of the same name for its side.

The following tables contain the linear measures in common use, with their corresponding square measures.

*Linear Measures.**Square Measures.*

12 inches = 1 foot.	144 inches = 1 foot.
3 feet = 1 yard.	9 feet = 1 yard.
6 feet = 1 fathom.	36 feet = 1 fathom.
$16\frac{1}{2}$ feet = 1 rod.	$272\frac{1}{4}$ feet = 1 rod.
$5\frac{1}{2}$ yards = 1 rod.	$30\frac{1}{4}$ yards = 1 rod.
4 rods = 1 chain.	16 rods = 1 chain.
40 rods = 1 furlong.	1600 rods = 1 furlong.
320 rods = 1 mile.	102400 rods = 1 mile.

An *acre* contains 160 square rods, or 10 square chains.

By reducing the denominations of square measure, it will be seen that

1 sq. mile = 640 acres = 102400 rods = 27878400 feet = 4014489600 inches.

1 acre = 10 chains = 160 rods = 43560 feet = 6272640 inches.

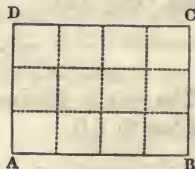
The fundamental problem in the mensuration of superficies is the very simple one of determining the area of a *right parallelogram*. The contents of other figures, particularly those which are rectilinear, may be obtained by finding parallelograms which are equal to them, according to the principles laid down in Euclid.

## PROBLEM I.

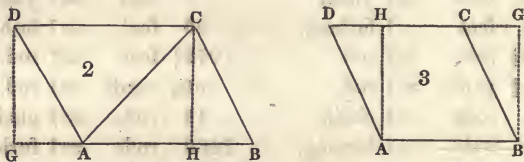
To find the area of a PARALLELOGRAM, square, rhombus, or rhomboid.

4. MULTIPLY THE LENGTH BY THE PERPENDICULAR HEIGHT OR BREADTH.

It is evident that the number of square inches in the parallelogram AC is equal to the number of linear inches in the length AB, repeated as many times as there are inches in the breadth BC. For more particular illustration of this see Alg. 386—389.



The oblique parallelogram or rhomboid ABCD, (Fig. 2.) is equal to the right parallelogram GHCD. (Euc. 36. 1.)\* The



area, therefore, is equal to the length AB multiplied into the perpendicular height HC. And the rhombus ABCD, (Fig. 3.) is equal to the *parallelogram* ABGH. As the sides of a *square* are all equal, its area is found, by *multiplying one of the sides into itself*.

Ex. 1. How many square feet are there in a floor  $23\frac{1}{2}$  feet long, and 18 feet broad?      Ans.  $23\frac{1}{2} \times 18 = 423$ .

2. What are the contents of a piece of ground which is 66 feet square?      Ans. 4356 sq. feet = 16 sq. rods.

3. How many square feet are there in the four sides of a room which is 22 feet long, 17 feet broad, and 11 feet high?      Ans. 858.

ART. 5. If the sides and angles of a parallelogram are given, the perpendicular height may be easily found by trigonometry. Thus, CH (Fig. 2.) is the perpendicular of a right angled triangle, of which BC is the hypotenuse. Then, (Trig. 134.)

$$R : BC :: \sin B : CH.$$

The area is obtained by multiplying CH thus found, into the length AB.

---

\* Thomson's Legendre, 1. 5.



Or, to reduce the two operations to one,

As radius,

To the sine of any angle of a parallelogram ;

So is the product of the sides including that angle,

To the area of the parallelogram.

For *the area* =  $AB \times CH$ , (Fig. 2.) But  $CH = \frac{BC \times \sin B}{R}$

Therefore,

*The area* =  $\frac{AB \times BC \times \sin B}{R}$ . Or,  $R : \sin B :: AB \times BC : \textit{the area}$ .

Ex. If the side AB be 58 rods, BC 42 rods, and the angle B  $63^\circ$ , what is the area of the parallelogram ?

As radius 10.00000

To the sine of B  $63^\circ$  9.94988

{ So is the product of AB 58 1.76343

{ Into BC (Trig. 39.) 42 1.62325

To the area 2170.5 sq. rods 3.33656

2. If the side of a rhombus is 67 feet, and one of the angles  $73^\circ$ , what is the area ? Ans. 4292.7 feet.

6. When the dimensions are given in feet and inches, the multiplication may be conveniently performed by the arithmetical rule of *Duodecimals* ; in which each inferior denomination is one-twelfth of the next higher. Considering a foot as the measuring *unit*, a prime is the twelfth part of a foot ; a second, the twelfth part of a prime, &c. It is to be observed, that, in measures of *length*, inches are *primes* ; but in *superficial* measure they are *seconds*. In both, a prime is  $\frac{1}{12}$  of a foot. But  $\frac{1}{12}$  of a *square* foot is a parallelogram, a foot long and an inch broad. The twelfth part of this is a square inch, which is  $\frac{1}{144}$  of a square foot.



Ex. 1. What is the surface of a board 9 feet 5 inches, by 2 feet 7 inches.

$$\begin{array}{r}
 \text{F} \\
 9 \quad 5' \\
 2 \quad 7 \\
 \hline
 18 \quad 10 \\
 5 \quad 5 \quad 11 \\
 \hline
 24 \quad 3 \quad 11'', \text{ or } 24 \text{ feet } 47 \text{ inches.}
 \end{array}$$

2. How many feet of glass are there in a window 4 feet 11 inches high, and 3 feet 5 inches broad ?

Ans. 16 F. 9' 7'', or 16 feet 115 inches.

7. If the area and one side of a parallelogram be given, the other side may be found by *dividing the area by the given side*. And if the area of a *square* be given, the side may be found by *extracting the square root of the area*. This is merely reversing the rule in Art. 4. See Alg. 520, 521.

Ex. 1. What is the breadth of a piece of cloth which is 36 yds. long, and which contains 63 square yards.

Ans.  $1\frac{3}{4}$  yds.

2. What is the side of a square piece of land containing 289 square rods ?

3. How many yards of carpeting  $1\frac{1}{4}$  yard wide, will cover a floor 30 feet long and  $22\frac{1}{2}$  broad ?

Ans.  $30 \times 22\frac{1}{2}$  feet =  $10 \times 7\frac{1}{2}$  = 75 yds. And  $75 \div 1\frac{1}{4}$  = 60.

4. What is the side of a square which is equal to a parallelogram 936 feet long and 104 broad ?

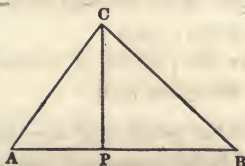
5. How many panes of 8 by 10 glass are there, in a window 5 feet high, and 2 feet 8 inches broad ?

PROBLEM II.

To find the area of a TRIANGLE.

8. RULE I. MULTIPLY ONE SIDE BY HALF THE PERPENDICULAR FROM THE OPPOSITE ANGLE. Or, multiply half the side by the perpendicular, Or, multiply the whole side by the perpendicular and take half the product.

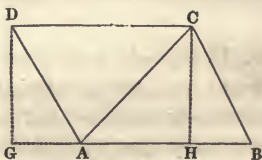
The area of the triangle ABC, is equal to  $\frac{1}{2} PC \times AB$ , because a parallelogram of the same base and height is equal to  $PC \times AB$ , (Art. 4.) and by Euc. 41, 1,\* the triangle is half the parallelogram.



Ex. 1. If AB be 65 feet, and PC 31.2, what is the area of the triangle? Ans. 1014 square feet.

2. What is the surface of a triangular board, whose base is 3 feet 2 inches, and perpendicular height 2 feet 9 inches? Ans. 4 F. 4' 3'', or 4 feet 51 inches.

9. If two sides of a triangle and the included angle, are given, the perpendicular on one of these sides may be easily found by rectangular trigonometry. And the area may be calculated in the same manner as the area of a parallelogram in Art. 5. In the triangle ABC,



$$R : BC :: \sin B : CH$$

And because the triangle is half the parallelogram of the same base and height,

---

\* Thomson's Legendre, 2. 4.

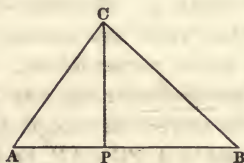
As radius,

To the sine of any angle of a triangle ;

So is the product of the sides including that angle,

To twice the area of the triangle. (Art. 5.)

Ex. If AC be 39 feet, AB 65 feet, and the angle at A  $53^{\circ} 7' 48''$ , what is the area of the triangle? Ans. 1014 square feet.



9. b. If one side and the angles are given ; then

As the product of radius and the sine of the angle opposite the given side,

To the product of the sines of the two other angles ;

So is the square of the given side,

To twice the area of the triangle.

If PC be perpendicular to AB.

$$R : \sin B :: BC : CP$$

$$\sin ACB : \sin A :: AB : BC$$

Therefore, (Alg. 351, 345.)

$$\overline{AB^2} : AB \times CP = \text{twice the area of the triangle.}$$

Ex. If one side of a triangle be 57 feet, and the angles at the ends of this side  $50^{\circ}$  and  $60^{\circ}$ , what is the area?

Ans. 1147 sq. feet.

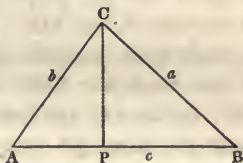
10. If the *sides* only of a triangle are given, an angle may be found, by oblique trigonometry, Case IV, and then the perpendicular and the area may be calculated. But the area may be more directly obtained, by the following method.

**RULE II.** When the three sides are given, *from half their sum subtract each side severally, multiply together the half sum and the three remainders, and extract the square root of the product.*

If the sides of the triangle are  $a$ ,  $b$ , and  $c$ , and if  $h$  = half their sum, then

$$\text{The area} = \sqrt{h \times (h-a) \times (h-b) \times (h-c)}$$

Ex. 1. In the triangle ABC, given the sides  $a$  52 feet,  $b$  39, and  $c$  65; to find the side of a square which has the same area as the triangle.



$$\begin{aligned} \frac{1}{2}(a+b+c) &= h = 78 & h-b &= 39 \\ h-a &= 26 & h-c &= 13 \end{aligned}$$

Then the area =  $\sqrt{78 \times 26 \times 39 \times 13} = 1014$  square feet.

By logarithms.

The half sum	= 78	1.89209
First remainder	= 26	1.41497
Second do.	= 39	1.59106
Third do.	= 13	1.11394
		2)6.01206
The area required	= 1014	2)3.00603
Side of the square	= 31.843 (Trig. 47.)	1.50301

2. If the sides of a triangle are 134, 108, and 80 rods, what is the area? Ans. 4319.

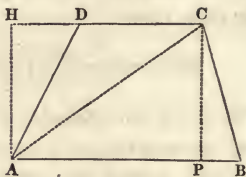
3. What is the area of a triangle whose sides are 371, 264, and 225 feet?

### PROBLEM III.

*To find the area of a TRAPEZOID.*

21. MULTIPLY HALF THE SUM OF THE PARALLEL SIDES INTO THEIR PERPENDICULAR DISTANCE.

The area of the trapezoid ABCD, is equal to half the sum of the sides AB and CD, multiplied into the perpendicular distance PC or AH. For the whole figure is made up of the two triangles ABC and



ADC; the area of the first of which is equal to the product of half the base AB into the perpendicular PC, (Art. 8.) and the area of the other is equal to the product of half the base DC into the perpendicular AH or PC.

Ex. If AB be 46 feet, BC 31, DC 38, and the angle B  $70^\circ$ , what is the area of the trapezoid?

R : BC :: sin B : PC = 29.13. And  $42 \times 29.13 = 1223\frac{1}{2}$ .

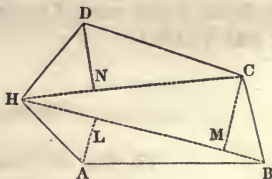
2. What are the contents of a field which has two parallel sides 65 and 38 rods, distant from each other 27 rods?

#### PROBLEM IV.

*To find the area of a TRAPEZIUM, or of an irregular POLYGON.*

13. DIVIDE THE WHOLE FIGURE INTO TRIANGLES, BY DRAWING DIAGONALS, AND FIND THE SUM OF THE AREAS OF THESE TRIANGLES. (Alg. 394.)

If the perpendiculars in two triangles fall upon the *same diagonal*, the area of the trapezium formed of the two triangles, is equal to half the product of the diagonal into the sum of the perpendiculars.



Thus the area of the trapezium ABCH, is

$$\frac{1}{2} BH \times AL + \frac{1}{2} BH \times CM = \frac{1}{2} BH \times (AL + CM.)$$

Ex. In the irregular polygon ABCDH,



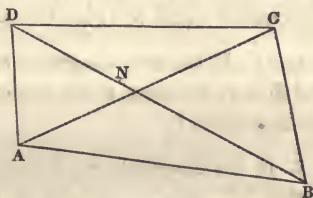
if the diagonals  $\left\{ \begin{array}{l} BH=36, \\ CH=32, \end{array} \right.$  and the perpendiculars  $\left\{ \begin{array}{l} AL=5.3 \\ CM=9.3 \\ DN=7.3 \end{array} \right.$

The area =  $18 \times 14.6 + 16 \times 7.3 = 379.6$ .

14. If the diagonals of a *trapezium* are given, the area may be found, nearly in the same manner as the area of a parallelogram in Art. 5, and the area of a triangle in Art. 9.

In the trapezium ABCD, the sines of the four angles at N, the point of intersection of the diagonals, are

all equal. For the two acute angles are *supplements* of the other two, and therefore have the same sine. (Trig. 90.)



Putting, then,  $\sin N$  for the sine of each of these angles, the areas of the four triangles of which the trapezium is composed, are given by the following proportions; (Art. 9.)

$$R : \sin N :: \left\{ \begin{array}{l} BN \times AN : 2 \text{ area ABN} \\ BN \times CN : 2 \text{ area BCN} \\ DN \times CN : 2 \text{ area CDN} \\ DN \times AN : 2 \text{ area ADN} \end{array} \right.$$

And by addition, (Alg. 349, Cor. 1.)\*

$$R : \sin N :: BN \times AN + BN \times CN + DN \times CN + DN \times AN : 2 \text{ area ABCD.}$$

The 3d term =  $(AN + CN) \times (BN + DN) = AC \times BD$ , by the figure.

Therefore  $R : \sin N :: AC \times BD : 2 \text{ area ABCD}$ . That is,

\* Euclid, 2, 5. Cor.

As Radius,

To the sine of the angle at the intersection of the diagonals of a trapezium;

So is the product of the diagonals,

To twice the area of the trapezium.

It is evident that this rule is applicable to a parallelogram, as well as to a trapezium.

If the diagonals intersect at *right angles*, the sine of  $N$  is equal to radius; (Trig. 95.) and therefore the product of the diagonals is equal to twice the area. (Alg. 356.)\*

Ex. 1. If the two diagonals of a trapezium are 37 and 62, and if they intersect at an angle of  $54^\circ$ , what is the area of the trapezium? Ans. 928.

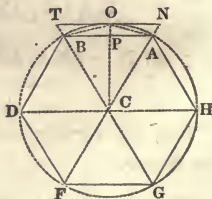
2. If the diagonals are 85 and 93, and the angle of intersection  $74^\circ$ , what is the area of the trapezium?

### PROBLEM V.

*To find the area of a REGULAR POLYGON.*

15. MULTIPLY ONE OF ITS SIDES INTO HALF ITS PERPENDICULAR DISTANCE FROM THE CENTRE, AND THIS PRODUCT INTO THE NUMBER OF SIDES.

A regular polygon contains as many equal triangles as the figure has sides. Thus, the hexagon  $ABDFGH$  contains six triangles, each equal to  $ABC$ . The area of one of them is equal to the product of the side  $AB$ , into half the perpendicular  $CP$ . (Art. 8.) The area of the whole, therefore, is equal to this product multiplied into the *number* of sides.



\* Euclid, 14. 5.

**Ex. 1.** What is the area of a regular octagon, in which the length of a side is 60, and the perpendicular from the centre 72.426 ? Ans. 17382.

2. What is the area of a regular decagon whose sides are 46 each, and the perpendicular 70.7867 ?

16. If only the length and number of sides of a regular polygon be given, the *perpendicular* from the centre may be easily found by trigonometry. The periphery of the circle in which the polygon is inscribed, is divided into as many equal parts as the polygon has sides. (Euc. 16. 4. Schol.)\* The arc, of which one of the sides is a chord, is therefore known ; and of course, the angle at the centre subtended by this arc.

Let AB be one side of a regular polygon inscribed in the circle ABDG. The perpendicular CP bisects the line AB, and the angle ACB. (Euc. 3. 3.)† Therefore, BCP is the same part of  $360^\circ$ , which BP is of the perimeter of the polygon. Then, in the right angled triangle BCP, if BP be radius, (Trig. 122.)

R : BP :: cot BCP : CP. That is,

As Radius,

To half of one of the sides of the polygon ;

So is the cotangent of the opposite angle,

To the perpendicular from the centre.

**Ex. 1.** If the side of a regular hexagon be 38 inches, what is the area ?

The angle BCP =  $\frac{1}{2}$  of  $360^\circ = 30^\circ$ . Then,

R : 19 :: cot  $30^\circ$  : 32.909 = CP, the perpendicular,

And the area =  $19 \times 32.909 \times 6 = 3751.6$

\* Thomson's Legendre, 2. 5. Schol.

† Ibid. 6. 2.

2. What is the area of a regular decagon whose sides are each 62 feet? Ans. 29576.

17. From the proportion in the preceding article, a *table* of perpendiculars and areas may be easily formed, for a series of polygons, of which each side is a unit. Putting  $R=1$ , (Trig. 100.) and  $n$ =the number of sides, the proportion becomes

$$1 : \frac{1}{2} :: \cot \frac{360^\circ}{2n} : \text{the perpendicular}$$

$$\text{So that, the perp.} = \frac{1}{2} \cot \frac{360}{2n}$$

And the *area* is equal to half the product of the perpendicular into the number of sides. (Art. 15.)

Thus, in the trigon, or equilateral triangle, the perpendicular  $= \frac{1}{2} \cot \frac{360^\circ}{6} = \frac{1}{2} \cot 60^\circ = 0.2886752$ .

And the area  $= 0.4330127$ .

In the tetragon, or square, the perpendicular  $= \frac{1}{2} \cot \frac{360^\circ}{8} = \frac{1}{2} \cot 45^\circ = 0.5$ . And the area  $= 1$ .

In this manner, the following table is formed, in which the side of each polygon is supposed to be a unit.

A TABLE OF REGULAR POLYGONS.

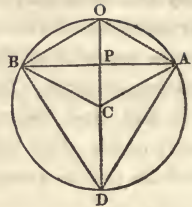
Names.	Sides.	Angles.	Perpendiculars.	Areas.
Trigon,	3	60°	0.2886752	0.4330127
Tetragon,	4	45°	0.5000000	1.0000000
Pentagon,	5	36°	0.6881910	1.7204774
Hexagon,	6	30°	0.8660254	2.5980762
Heptagon,	7	25 $\frac{1}{7}$	1.0382601	3.6339124
Octagon,	8	22 $\frac{1}{2}$	1.2071069	4.8284271
Nonagon,	9	20°	1.3737385	6.1818242
Decagon,	10	18°	1.5388418	7.6942088
Undecagon,	11	16 $\frac{4}{11}$	1.7028439	9.3656399
Dodecagon,	12	15°	1.8660252	11.1961524



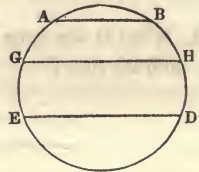


It may be *less* than a semi-circle, as ACBO, or *greater*, as ACBD.

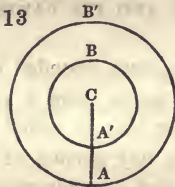
IV. A *Circular Segment* is the space contained between an arc and its chord, as ABO or ABD. The chord is sometimes called the *base* of the segment. The *height* of a segment is the perpendicular from the middle of the base to the arc, as PO.



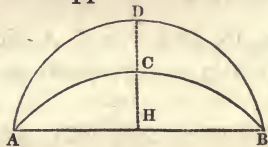
V. A *Circular Zone* is the space between two parallel chords, as AGHB. It is called the *middle zone*, when the two chords are equal, as GHDE.



VI. A *Circular Ring* is the space between the peripheries of two concentric circles, as AA', BB'. (Fig. 13.)



14

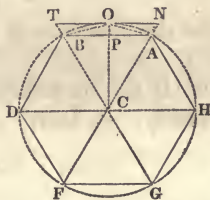


VII. A *Lune* or *Crescent* is the space between two circular arcs which intersect each other, as ACBD. (Fig. 14.)

19. The *Squaring of the Circle* is a problem which has exercised the ingenuity of distinguished mathematicians for many centuries. The result of their efforts has been only an *approximation* to the value of the area. This can be carried to a degree of exactness far beyond what is necessary for practical purposes.

20. If the *circumference* of a circle of given diameter were known, its area could be easily found. For the area is equal to the product of half the circumference into half the diameter. (Sup. Euc. 5, 1.)\*† But the circumference of a circle has never been exactly determined. The method of approximating to it is by inscribing and circumscribing *polygons*, or by some process of calculation which is, in principle, the same. The perimeters of the polygons can be easily and exactly determined. That which is circumscribed is *greater*, and that which is inscribed is *less*, than the periphery of the circle; and by increasing the number of sides, the difference of the two polygons may be made less than any given quantity. (Sup. Euc. 4, 1.)

21. The side of a *hexagon* inscribed in a circle, as AB, is the chord of an arc of  $60^\circ$ , and therefore equal to the radius. (Trig. 95.) The chord of *half* this arc, as BO, is the side of a polygon of 12 equal sides. By repeatedly bisecting the arc, and finding the chord, we may obtain the side of a polygon of an immense number of sides. Or we may calculate the *sine*, which will be half the chord of double the arc, (Trig. 82, cor.) and the *tangent*, which will be half the side of a similar *circumscribed* polygon. Thus the sine AP, is half of AB, a side of the inscribed hexagon; and the tangent NO is half of NT, a side of the circumscribed hexagon. The difference between the sine and the arc AO is less than the difference between the sine and the tangent. In the section on the computation of the canon, (Trig. 223.) by 12 successive bisections, beginning with 60 degrees, an arc is obtained which is the  $\frac{1}{24576}$  of the whole circumference.



\* In this manner, the *Supplement to Playfair's Euclid* is referred to in this work.

† Thomson's Legendre, 11. 5.

The *cosine* of this, if radius be 1, is found to be .99999996732

The *sine* is .00025566346

And the tangent  $= \frac{\text{sine}}{\text{cosine}}$  (Trig. 93.) = .00025566347

The diff. between the sine and tangent is only .00000000001

And the difference between the sine and the *arc* is still less.

Taking then, .000255663465 for the length of the arc, multiplying by 24576, and retaining 8 places of decimals, we have 6.28318531 for the whole circumference, the radius being 1. Half of this,

$$3.14159265$$

is the circumference of a circle whose radius is  $\frac{1}{2}$ , and *diameter* 1.

22. If this be multiplied by 7, the product is 21.99+ or 22 nearly. So that,

$$\text{Diam} : \text{Circum} :: 7 : 22, \text{ nearly.}$$

If 3.14159265 be multiplied by 113, the product is 354.9999+, or 355, very nearly. So that,

$$\text{Diam} : \text{Circum} :: 113 : 355, \text{ very nearly.}$$

The first of these ratios was demonstrated by Archimedes.

There are various methods, principally by infinite series, and fluxions, by which the labor of carrying on the approximation to the periphery of a circle may be very much abridged. The calculation has been extended to nearly 150 places of decimals. But four or five places are sufficient for most practical purposes.

After determining the ratio between the diameter and the circumference of a circle, the following problems are easily solved.

## PROBLEM I.

To find the CIRCUMFERENCE of a circle from its diameter.

23. MULTIPLY THE DIAMETER BY **3.14159**.\*

Or,

*Multiply the diameter by 22 and divide the product by 7. Or, multiply the diameter by 355, and divide the product by 113. (Art. 22.)*

Ex. 1. If the diameter of the earth be 7930 miles, what is the circumference? Ans. 249128 miles.

2. How many miles does the earth move, in revolving round the sun; supposing the orbit to be a circle whose diameter is 190 million miles? Ans. 596,902,100.

3. What is the circumference of a circle whose diameter is 769843 rods?

## PROBLEM II.

To find the DIAMETER of a circle from its circumference.

24. DIVIDE THE CIRCUMFERENCE BY **3.14159**.

Or,

*Multiply the circumference by 7, and divide the product by 22. Or, multiply the circumference by 113, and divide the product by 355. (Art. 22.)*

Ex. 1. If the circumference of the sun be 2,800,000 miles, what is his diameter? Ans. 891,267.

2. What is the diameter of a tree which is  $5\frac{1}{2}$  feet round?

25. As multiplication is more easily performed than division, there will be an advantage in exchanging the *divisor*

---

\* In many cases, 3.1416 will be sufficiently accurate.



3.14159 for a *multiplier* which will give the same result.  
In the proportion

$$3.14159 : 1 :: \text{Circum} : \text{Diam.}$$

to find the fourth term, we may divide the second by the first, and multiply the quotient into the third. Now,  $1 \div 3.14159 = 0.31831$ . If, then, the circumference of a circle be multiplied by .31831, the product will be the diameter.

Ex. 1. If the circumference of the moon be 6850 miles, what is her diameter? Ans. 2180.

2. If the whole extent of the orbit of Saturn be 5650 million miles, how far is he from the sun?

3. If the periphery of a wheel be 4 feet 7 inches, what is its diameter?

### PROBLEM III.

*To find the length of an ARC of a circle.*

26. As  $360^\circ$ , to the number of degrees in the arc ;  
So is the circumference of the circle, to the length of the arc.

The circumference of a circle being divided into  $360^\circ$ , (Trig. 73.) it is evident that the length of an arc of any less number of degrees must be a proportional part of the whole.

Ex. What is the length of an arc of  $16^\circ$ , in a circle whose radius is 50 feet ?

The circumference of the circle is 314.159 feet. (Art. 23.)

$$\text{Then } 360 : 16 :: 314.159 : 13.96 \text{ feet.}$$

2. If we are 95 millions of miles from the sun, and if the earth revolves round it in  $365\frac{1}{4}$  days, how far are we carried in 24 hours? Ans. 1 million 634 thousand miles.

27. The length of an arc may also be found, by multiplying the diameter into the number of degrees in the arc, and



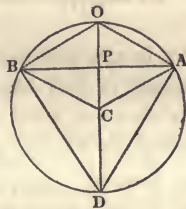
this product into .0087266, which is the length of *one* degree, in a circle whose diameter is 1. For  $3.14159 \div 360 = 0.0087266$ . And in different circles, the circumferences, and of course the degrees, are as the diameters. (Sup. Euc. 8, 1.)\*

Ex. 1. What is the length of an arc of  $10^\circ 15'$  in a circle whose radius is 68 rods?                      Ans. 12.165 rods.

2. If the circumference of the earth be 24913 miles, what is the length of a degree at the equator?

28. The length of an arc is frequently required, when the *number of degrees* is not given. But if the radius of the circle, and either the *chord* or the *height* of the arc, be known; the number of degrees may be easily found.

Let AB be the chord, and PO the height, of the arc AOB. As the angles at P are right angles, and AP is equal to BP; (Art. 18. Def. 4.) AO is equal to BO. (Euc. 4, 1.)† Then,



BP is the *sine*, and CP the *cosine*,  
OP the *versed sine*, and BO the *chord*, } of *half* the arc AOB.

And in the right angled triangle CBP,

$$CB : R :: \begin{cases} BP : \sin BCP \text{ or } BO. \\ CP : \cos BCP \text{ or } BO. \end{cases}$$

Ex. 1. If the radius  $CO = 25$ , and the chord  $AB = 43.3$ ; what is the length of the arc AOB?

$$CB : R :: BP : \sin BCP \text{ or } BO = 60^\circ \text{ very nearly.}$$

The circumference of the circle  $= 3.14159 \times 50 = 157.08$ .  
And  $360^\circ : 60^\circ :: 157.08 : 26.18 = OB$ . Therefore,  $\text{AOB} = 52.36$ .

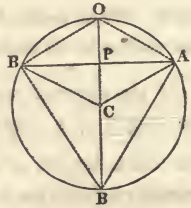
\* Thomson's Legendre, 10. 5.

† Ibid., 5. 1.

2. What is the length of an arc whose chord is  $216\frac{1}{2}$ , in a circle whose radius is 125? Ans. 261.8.

29. If only the *chord* and the *height* of an arc be given, the radius of the circle may be found, and then the length of the arc.

If BA be the chord, and PO the the height of the arc AOB, then (Euc. 35. 3.)\*



$$DP = \frac{\overline{BP}^2}{OP}. \quad \text{And } DO = OP + DP = OP + \frac{\overline{BP}^2}{OP}.$$

That is, the *diameter* is equal to the height of the arc, + the square of half the chord divided by the height.

The diameter being found, the length of the arc may be calculated by the two preceding articles.

Ex. 1. If the chord of an arc be  $173.2$ , and the height  $50$ , what is the length of the arc?

$$\text{The diameter} = 50 + \frac{86.6^2}{50} = 200. \quad \text{The arc contains } 120^\circ,$$

(Art. 28.) and its length is  $209.44$ . (Art. 26.)

2. What is the length of an arc whose chord is  $120$ , and height  $45$ ? Ans. 160.8.

#### PROBLEM IV.

*To find the AREA of a CIRCLE.*

30. MULTIPLY THE SQUARE OF THE DIAMETER BY THE DECIMALS **.7854**.

---

\* Thomson's Legendre, 10. 5.





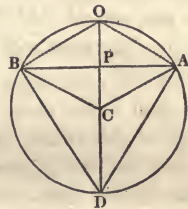
Or,

As 360, TO THE NUMBER OF DEGREES IN THE ARC ;

So IS THE AREA OF THE CIRCLE, TO THE AREA OF THE SECTOR.

It is evident, that the area of the sector has the same ratio to the area of the circle, which the length of the arc has to the length of the whole circumference ; or which the number of *degrees* in the arc has to the number of degrees in the circumference.

Ex. 1. If the arc AOB be  $120^\circ$ , and the diameter of the circle 226 ; what is the area of the sector AOBC ?



The area of the whole circle is 40115. (Art. 30.)

And  $360^\circ : 120^\circ :: 40115 : 13371\frac{2}{3}$ , the area of the sector.

2. What is the area of a quadrant whose radius is 621 ?

3. What is the area of a semi-circle, whose diameter is 328 ?

4. What is the area of a sector which is less than a semi-circle, if the radius be 15, and the chord of its arc 12 ?

Half the chord is the sine of  $23^\circ 34\frac{3}{4}'$  nearly. (Art. 28.)

The whole arc, then, is  $47^\circ 9\frac{1}{2}'$

The area of the circle is 706.86

And  $360^\circ : 47^\circ 9\frac{1}{2}' :: 706.86 : 92.6$  the area of the sector.

5. If the arc ADB be 240 degrees, and the radius of the circle 113, what is the area of the sector ADBC ?

### PROBLEM VI.

*To find the area of a SEGMENT of a circle.*

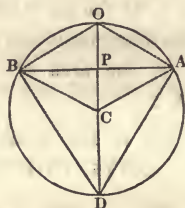
35. FIND THE AREA OF THE SECTOR WHICH HAS THE



SAME ARC, AND ALSO THE AREA OF A TRIANGLE FORMED BY THE CHORD OF THE SEGMENT AND THE RADII OF THE SECTOR.

THEN, IF THE SEGMENT BE LESS THAN A SEMI-CIRCLE; SUBTRACT THE AREA OF THE TRIANGLE FROM THE AREA OF THE SECTOR. BUT, IF THE SEGMENT BE GREATER THAN A SEMI-CIRCLE, ADD THE AREA OF THE TRIANGLE TO THE AREA OF THE SECTOR.

If the triangle  $ABC$ , be taken from the sector  $AOBC$ , it is evident the difference will be the segment  $AOBP$ , less than a semi-circle. And if the same triangle be added to the sector  $ADBC$ , the sum will be the segment  $ADBP$ , greater than a semi-circle.



The area of the triangle (Art. 8.)

is equal to the product of half the chord  $AB$  into  $CP$ , which is the difference between the radius and  $PO$  the height of the segment. Or  $CP$  is the *cosine* of half the arc  $BOA$ . If this cosine and the chord of the segment are not given, they may be found from the arc and the radius.

Ex. 1. If the arc  $AOB$  be  $120^\circ$ , and the radius of the circle be 113 feet, what is the area of the segment  $AOBP$  ?

In the right angled triangle  $BCP$ ,

$R : BC :: \sin BCO : BP = 97.86$ , half the chord. (Art. 28.)

The cosine $PC = \frac{1}{2} CO$ (Trig. 96, Cor.)	= 56.5
The area of the sector $AOBC$ (Art. 34.)	= 13371.67
The area of the triangle $ABC = BP \times PC$	= <u>5528.97</u>
The area of the segment, therefore,	= 7842.7

2. If the base of a segment, less than a semi-circle, be 10

feet, and the radius of the circle 12 feet, what is the area of the segment ?

The arc of the segment contains  $49\frac{1}{4}$  degrees. (Art. 28.)  
 The area of the sector = 61.89 (Art. 34.)  
 The area of the triangle = 54.54  
 And the area of the segment = 7.35 square feet.

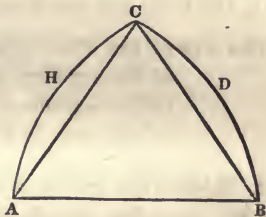
3. What is the area of a circular segment, whose height is 19.2 and base 70 ? Ans. 947.86.

4. What is the area of the segment ADBP, (Fig. 9.) if the base AB be 195.7, and the height PD 169.5 ? Ans. 32272.

36. The area of any figure which is bounded *partly* by arcs of circles, and partly by right lines, may be calculated, by finding the areas of the segments under the arcs, and then the area of the rectilinear space between the chords of the arcs and the other right lines.

Thus, the Gothic arch ACB, contains the two segments ACH, BCD, and the plane triangle ABC.

Ex. If AB be 110, each of the lines AC and BC 100, and the height of each of the segments ACH, BCD 10.435 ; what is the area of the whole figure ?



The areas of the two segments are	1404
The area of the triangle ABC is	<u>4593.4</u>
And the whole figure is	<u>5997.4</u>

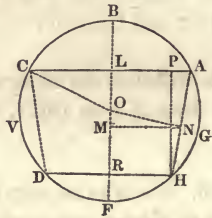
## PROBLEM VII.

To find the area of a circular ZONE.

37. FROM THE AREA OF THE WHOLE CIRCLE, SUBTRACT THE TWO SEGMENTS ON THE SIDES OF THE ZONE.

If from the whole circle there be taken the two segments ABC and DEH, there will remain the zone ACDH.

Or, the area of the zone may be found by subtracting the segment ABC from the segment HBD: Or, by adding the two small segments GAH and VDC, to the trapezoid ACDH. (See Art. 36.)



The latter method is rather the most expeditious in practice, as the two segments at the end of the zone are *equal*.

Ex. 1. What is the area of the zone ACDH, if AC is 7.75, DH 6.93, and the diameter of the circle 8?

The area of the whole circle is	50.26
of the segment ABC	17.32
of the segment DFH	9.82
of the zone ACDH	23.12

2. What is the area of a zone, one side of which is 23.25, and the other side 20.8, in a circle whose diameter is 24?

Ans. 208.

38. If the *diameter* of the circle is not given, it may be found from the sides and the breadth of the zone.

Let the centre of the circle be at O. Draw ON perpendicular to AH, NM perpendicular to LR, and HP perpendicular to AL. Then,

$$AN = \frac{1}{2}AH, \text{ (Euc. 3. 3.)}^* \quad MN = \frac{1}{2}(LA + RH)$$

$$LM = \frac{1}{2}LR, \text{ (Euc. 2. 6.)}^\dagger \quad PA = LA - RH.$$

The triangles APH and OMN are similar, because the sides of one are perpendicular to those of the other, each to each. Therefore,

$$PH : PA :: MN : MO$$

MO being found, we have  $ML - MO = OL$ .

And the radius  $CO = \sqrt{OL^2 + CL^2}$ . (Euc. 47. 1.)<sup>‡</sup>

Ex. If the breadth of the zone ACDH (Fig. 12.) be 6.4, and the sides 6.8 and 6; what is the radius of the circle?

$$PA = 3.4 - 3 = 0.4. \quad \text{And, } MN = \frac{1}{2}(3.4 + 3) = 3.2.$$

Then,  $6.4 : 0.4 :: 3.2 : 0.2 = MO$ . And,  $3.2 - 0.2 = 3 = OL$

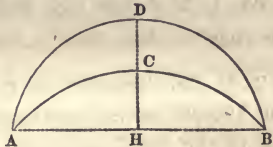
And the radius  $CO = \sqrt{3^2 + (3.4)^2} = 4.534$ .

### PROBLEM VIII.

*To find the area of a LUNE or crescent.*

39. FIND THE DIFFERENCE OF THE TWO SEGMENTS WHICH ARE BETWEEN THE ARCS OF THE CRESCENT AND ITS CHORD.

If the segment ABC, be taken from the segment ABD; there will remain the lune or crescent ACBD.



Ex. If the chord AB be 88, the height CH 20, and the height DH 40; what is the area of the crescent ACBD?

The area of the segment ABD is	2698
of the segment ABC	1220
of the crescent ACBD	1478

---

\* Thomson's Legendre, 6. 2.    † Ibid. 15. 4.    ‡ Ibid. 11. 4.

## PROBLEM IX.

To find the area of a RING, included between the peripheries of two concentric circles.

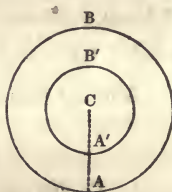
40. FIND THE DIFFERENCE OF THE AREAS OF THE TWO CIRCLES.

Or,

Multiply the product of the sum and difference of the two diameters by .7854.

The area of the ring is evidently equal to the difference between the areas of the two circles AB and A'B'.

But the area of each circle is equal to the square of its diameter multiplied into .7854. (Art. 30.) And the *difference* of these squares is equal to the product of the sum and difference of the diameters. (Alg. 191.)



Therefore the area of the ring is equal to the product of the sum and difference of the two diameters multiplied by .7854.

Ex. 1. If AB be 221, and A'B' 106, what is the area of the ring? Ans.  $(221^2 \times .7854) - (106^2 \times .7854) = 29535$ .

2. If the diameters of Saturn's larger ring be 205,000 and 190,000 miles, how many square miles are there on one side of the ring?

Ans.  $395000 \times 15000 \times .7854 = 4,653,495,000$ .

## PROMISCUOUS EXAMPLES OF AREAS.

Ex. 1. What is the expense of paving a street 20 rods long and 2 rods wide, at 5 cents for a square foot?

Ans.  $544\frac{1}{2}$  dollars.



2. If an equilateral triangle contains as many square feet as there are inches in one of its sides; what is the area of the triangle?

Let  $x$  = the number of square feet in the area.

Then  $\frac{x}{12}$  = the number of linear feet in one of the sides.

$$\text{And, (Art. 11.) } x = \frac{1}{4} \left( \frac{x}{12} \right)^2 \times \sqrt{3} = \frac{x^2}{576} \times \sqrt{3}.$$

Reducing the equation,  $x = \frac{576}{\sqrt{3}} = 332.55$  the area.

3. What is the side of a square whose area is equal to that of a circle 452 feet in diameter?

$$\text{Ans. } \sqrt{(452)^2 \times .7854} = 400.574. \text{ (Arts. 30 and 7.)}$$

4. What is the diameter of a circle which is equal to a square whose side is 36 feet?

$$\text{Ans. } \sqrt{(36)^2 \div 0.7854} = 40.6217. \text{ (Arts. 4 and 32.)}$$

5. What is the area of a square inscribed in a circle whose diameter is 132 feet?

$$\text{Ans. } 8712 \text{ square feet. (Art. 33.)}$$

6. How much carpeting, a yard wide, will be necessary to cover the floor of a room which is a regular octagon, the sides being eight feet each?

$$\text{Ans. } 34\frac{1}{2} \text{ yards.}$$

7. If the diagonal of a square be 16 feet, what is the area?

$$\text{Ans. } 128 \text{ feet. (Art. 14.)}$$

8. If a carriage-wheel four feet in diameter revolve 300 times, in going round a circular green; what is the area of the green?

$$\text{Ans. } 4154\frac{1}{2} \text{ sq. rods, or 25 acres, 3 qrs. and } 34\frac{1}{2} \text{ rods.}$$

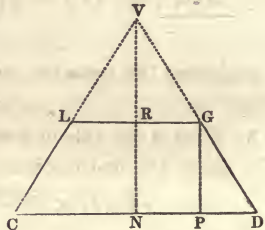
9. What will be the expense of papering the sides of a room, at 10 cents a square yard; if the room be 21 feet long,

18 feet broad, and 12 feet high ; and if there be deducted 3 windows, each 5 feet by 3, two doors 8 feet by  $4\frac{1}{2}$ , and one fire-place 6 feet by  $4\frac{1}{2}$  ?      Ans. 8 dollars 80 cents.

10. If a circular pond of water 10 rods in diameter be surrounded by a gravelled walk  $8\frac{1}{4}$  feet wide ; what is the area of the walk ?      Ans.  $16\frac{1}{2}$  sq. rods. (Art. 40.)

11. If CD, the base of the isosceles triangle VCD, be 60 feet, and the area 1200 feet ; and if there be cut off, by the line LG parallel to CD, the triangle VLG, whose area is 432 feet ; what are the sides of the latter triangle ?

Ans. 30, 30, and 36 feet.



12. What is the area of an equilateral triangle inscribed in a circle whose diameter is 52 feet ?

Ans. 878.15 sq. ft.

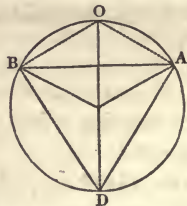
13. If a circular piece of land is inclosed by a fence, in which 10 rails make a rod in length ; and if the field contains as many square rods, as there are rails in the fence ; what is the value of the land at 120 dollars an acre ?

Ans. 942.48 dollars.

14. If the area of the equilateral triangle ABD be 219.5375 feet ; what is the area of the circle OBDA, in which the triangle is inscribed ?

The sides of the triangle are each 22.5167. (Art. 11.)

And the area of the circle is 530.93.



15. If 6 concentric circles are so drawn, that the space between the least or 1st, and the 2d is 21.2058,  
 between the 2d and the 3d is 35.343,  
 between the 3d and the 4th is 49.4802,  
 between the 4th and the 5th is 63.6174,  
 between the 5th and the 6th is 77.7546;

what are the several diameters, supposing the longest to be equal to 6 times the shortest?

Ans. 3, 6, 9, 12, 15, and 18.

16. If the area between two concentric circles be 1202.64 square inches, and the diameter of the lesser circle be 19 inches, what is the diameter of the other?

17. What is the area of a circular segment, whose height is 9, and base 24?

---

## SECTION III.

### SOLIDS BOUNDED BY PLANE SURFACES.

ART. 41. DEFINITION I. A *prism* is a solid bounded by plane figures or faces, two of which are parallel, similar, and equal; and the others are parallelograms.

II. The parallel planes are sometimes called the *bases* or *ends*; and the other figures the *sides* of the prism. The latter taken together constitute the *lateral surface*.

III. A prism is *right* or *oblique*, according as the sides are perpendicular or oblique to the bases.

IV. The *height* of a prism is the perpendicular distance between the planes of the bases. In a right prism, therefore, the height is equal to the length of one of the sides.

V. A *Parallelepiped* is a prism whose bases are parallelograms.

VI. A *Cube* is a solid bounded by six equal squares. It is a right prism whose sides and bases are all equal.

VII. A *Pyramid* is a solid bounded by a plane figure called the base, and several triangular planes, proceeding from the sides of the base, and all terminating in a single point. These triangles taken together constitute the *lateral surface*.

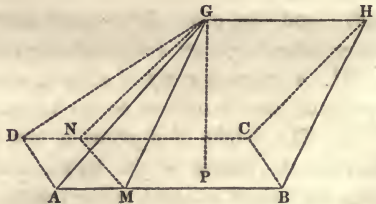
VIII. A pyramid is *regular*, if its base is a regular polygon, and if a line from the centre of the base to the vertex of the pyramid is *perpendicular* to the base. This line is called the *axis* of the pyramid.

IX. The *height* of a pyramid is the perpendicular distance from the summit to the plane of the base. In a *regular pyramid*, it is the length of the *axis*.

X. The *slant-height* of a regular pyramid, is the distance from the summit to the middle of one of the sides of the base.

XI. A *frustum* or *trunk* of a pyramid is a portion of the solid next the base, cut off by a plane parallel to the base. The *height* of the frustum is the perpendicular distance of the two parallel planes. The *slant-height* of a frustum of a *regular pyramid*, is the distance from the middle of one of the sides of the base, to the middle of the corresponding side in the plane above. It is a line passing on the surface of the frustum, through the middle of one of its sides.

XII. A *Wedge* is a solid of five sides, viz. a rectangular base, two rhomboidal sides meeting in an edge, and two triangular ends; as ABHG. The base is ABCD, the sides are ABHG and DCHG, meeting in the edge GH, and the ends are BCH and ADG. The *height* of the wedge is a





perpendicular drawn from any point in the edge, to the plane of the base, as GP.

XIII. A *Prismoid* is a solid whose ends or bases are parallel, but not similar, and whose sides are quadrilateral. It differs from a prism or a frustum of a pyramid, in having its ends dissimilar. It is a *rectangular prismoid*, when its ends are right parallelograms.

XIV. A *linear side* or *edge* of a solid is the line of intersection of two of the planes which form the surface.

42. The common *measuring unit* of solids is a *cube*, whose sides are squares of the same name. The sides of a cubic inch are square inches; of a cubic foot, square feet, &c. Finding the *capacity, solidity,\** or *solid contents* of a body, is finding the number of cubic measures, of some given denomination contained in the body.

*In solid measure.*

1728	cubic inches	= 1 cubic foot,
27	cubic feet	= 1 cubic yard,
4492 $\frac{1}{8}$	cubic feet	= 1 cubic rod,
32768000	cubic rods	= 1 cubic mile,
282	cubic inches	= 1 ale gallon,
231	cubic inches	= 1 wine gallon,
2150.42	cubic inches	= 1 bushel,
1	cubic foot of pure water	weighs 1000 avoirdupois ounces, or 62 $\frac{1}{2}$ pounds.

PROBLEM I.

*To find the SOLIDITY of a PRISM.*

43. MULTIPLY THE AREA OF THE BASE BY THE HEIGHT.

This is a general rule, applicable to parallelopipeds whether right or oblique, cubes, triangular prisms, &c.

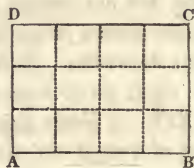
---

\* See note A.



As *surfaces* are measured, by comparing them with a right *parallelogram* (Art. 3.); so *solids* are measured, by comparing them with a right *parallelepiped*.

If ABCD be the base of a right parallelepiped, as a stick of timber standing erect, it is evident that the number of *cubic feet* contained in *one* foot of the height, is equal to the number of *square feet* in the area of the base. And if the solid be of any



other height, instead of one foot, the contents must have the same ratio. For parallelepipeds of the same base are to each other as their heights. (Sup. Euc. 9. 3.)\* The solidity of a right parallelepiped, therefore, is equal to the *product of its length, breadth, and thickness*. See Alg. 397.

And an *oblique* parallelepiped being equal to a right one of the same base and altitude, (Sup. Euc. 7. 3)† is equal to the area of the base multiplied into the perpendicular height. This is true also of *prisms*, whatever be the form of their bases. (Sup. Euc. 2. Cor. to 8, 3. Thomson's Legendre, 12. 7.)

44. As the sides of a *cube* are all *equal*, the solidity is found by *cubing one of its edges*. On the other hand, if the solid contents be given, the length of the edges may be found, by *extracting the cube root*.

45. When solid measure is cast by *Duodecimals*, it is to be observed that *inches* are not *primes* of feet, but *thirds*. If the unit is a cubic foot, a solid which is an inch thick and a foot square is a prime; a parallelepiped a foot long, an inch broad, and an inch thick is a second, or the twelfth part of a prime; and a cubic inch is a third, or the twelfth part of a second. A linear inch is  $\frac{1}{12}$  of a foot, a square inch  $\frac{1}{144}$  of a foot, and a cubic inch  $\frac{1}{1728}$  of a foot.

\* Thomson's Legendre, 9. 7.

† Ibid., 7. 7.

Ex. 1. What are the solid contents of a stick of timber which is 31 feet long, 1 foot 3 inches broad, and 9 inches thick?  
 Ans. 29 feet 9", or 29 feet 108 inches.

2. What is the solidity of a wall which is 22 feet long, 12 feet high, and 2 feet 6 inches thick?

Ans. 660 cubic feet.

3. What is the capacity of a cubical vessel which is 2 feet 3 inches deep?

Ans. 11 F. 4' 8" 3"', or 11 feet 675 inches.

4. If the base of a prism be 108 square inches, and the height 36 feet, what are the solid contents?

Ans. 27 cubic feet.

5. If the height of a square prism be  $2\frac{1}{4}$  feet, and each side of the base  $10\frac{1}{3}$  feet, what is the solidity?

The area of the base =  $10\frac{1}{3} \times 10\frac{1}{3} = 106\frac{7}{9}$  sq. feet.

And the solid contents =  $106\frac{7}{9} \times 2\frac{1}{4} = 240\frac{1}{4}$  cubic feet.

6. If the height of a prism be 23 feet, and its base a regular pentagon, whose perimeter is 18 feet, what is the solidity?  
 Ans. 512.84 cubic feet.

46. The number of *gallons* or *bushels* which a vessel will contain may be found, by calculating the capacity in *inches*, and then dividing by the number of inches in 1 gallon or bushel.

The *weight of water* in a vessel of given dimensions is easily calculated; as it is found by experiment, that a cubic foot of pure water weighs 1000 ounces avoirdupois. For the weight in ounces, then, multiply the cubic feet by 1000; or for the weight in pounds, multiply by  $62\frac{1}{2}$ .

Ex. 1. How many ale gallons are there in a cistern which is 11 feet 9 inches deep, and whose base is 4 feet 2 inches square?

The cistern contains 352500 cubic inches ;  
 And  $352500 \div 282 = 1250$ .

2. How many wine gallons will fill a ditch 3 feet 11 inches wide, 3 feet deep, and 462 feet long?      Ans. 40608.

3. What weight of water can be put into a cubical vessel 4 feet deep?      Ans. 4000 lbs.

## PROBLEM II.

*To find the LATERAL SURFACE of a RIGHT PRISM.*

47. MULTIPLY THE LENGTH INTO THE PERIMETER OF THE BASE.

Each of the sides of the prism is a right parallelogram, whose area is the product of its length and breadth. But the breadth is one side of the base; and therefore, the sum of the breadths is equal to the perimeter of the base.

Ex. 1. If the base of a right prism be a regular hexagon whose sides are each 2 feet 3 inches, and if the height be 16 feet, what is the lateral surface?

Ans. 216 square feet.

If the areas of the two ends be added to the lateral surface, the sum will be the whole surface of the prism. And the superficies of any solid bounded by planes, is evidently equal to the areas of all its sides.

2. If the base of a prism be an equilateral triangle whose perimeter is 6 feet, and if the height be 17 feet, what is the surface?

The area of the triangle is 1.732. (Art. 11.)

And the whole surface is 105.464.

## PROBLEM III.

To find the SOLIDITY of a PYRAMID.

48. MULTIPLY THE AREA OF THE BASE INTO  $\frac{1}{3}$  OF THE HEIGHT.

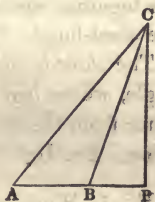
The solidity of a *prism* is equal to the product of the area of the base into the height. (Art. 43.) And a pyramid is  $\frac{1}{3}$  of a prism of the same base and altitude. (Sup. Euc. 15, 3. Cor. 1.)\* Therefore the solidity of a pyramid whether right or oblique, is equal to the product of the base into  $\frac{1}{3}$  of the perpendicular height.

Ex. 1. What is the solidity of a triangular pyramid, whose height is 60, and each side of whose base is 4?

The area of the base is 6.928

And the solidity is 138.56.

2. Let ABC be one side of an oblique pyramid whose base is 6 feet square; let BC be 20 feet, and make an angle of 70 degrees with the plane of the base; and let CP be perpendicular to this plane. What is the solidity of the pyramid?



In the right angled triangle BCP, (Trig. 134.)

$$R : BC :: \sin B :: PC = 18.79.$$

And the solidity of the pyramid is 225.48 feet.

3. What is the solidity of a pyramid whose perpendicular height is 72, and the sides of whose base are 67, 54, and 40? Ans. 25920.

---

\* Thomson's Legendre, 15 and 18. 7.

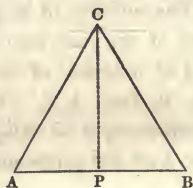


## PROBLEM IV.

To find the LATERAL SURFACE of a REGULAR PYRAMID.

49. MULTIPLY HALF THE SLANT-HEIGHT INTO THE PERIMETER OF THE BASE.

Let the triangle ABC be one of the sides of a regular pyramid. As the sides AC and BC are equal, the angles A and B are equal. Therefore a line drawn from the vertex C to the middle of AB is *perpendicular* to AB. The area of the triangle is equal to



the product of half this perpendicular into AB. (Art. 8.) The perimeter of the base is the sum of its sides, each of which is equal to AB. And the areas of all the equal triangles which constitute the lateral surface of the pyramid, are together equal to the product of the perimeter into half the slant-height CP.

The *slant-height* is the hypotenuse of a right angled triangle, whose legs are the axis of the pyramid, and the distance from the centre of the base to the middle of one of the sides. See Def. 10.

Ex. 1. What is the lateral surface of a regular hexagonal pyramid, whose axis is 20 feet, and the sides of whose base are each 8 feet ?

The square of the distance from the centre of the base to one of the sides. (Art. 16.) = 48.

The slant-height (Euc. 47. 1.)\* =  $\sqrt{48 + (20)^2} = 21.16$

And the lateral surface =  $21.16 \times 4 \times 6 = 507.84$  sq. feet.

2. What is the whole surface of a regular triangular pyr-

\* Thomson's Legendre, 11. 4.



amid whose axis is 8, and the sides of whose base are each 20.78 ?

The lateral surface is	312
The area of the base is	<u>187</u>
And the whole surface is	499

3. What is the lateral surface of a regular pyramid whose axis is 12 feet, and whose base is 18 feet square ?

Ans. 540 square feet.

The lateral surface of an *oblique* pyramid may be found, by taking the sum of the areas of the unequal triangles which form its sides.

PROBLEM V.

*To find the SOLIDITY of a FRUSTUM of a pyramid.*

50. ADD TOGETHER THE AREAS OF THE TWO ENDS, AND THE SQUARE ROOT OF THE PRODUCT OF THESE AREAS; AND MULTIPLY THE SUM BY  $\frac{1}{3}$  OF THE PERPENDICULAR HEIGHT OF THE SOLID.

Let CDGL be a vertical section, through the middle of a frustum of a right pyramid CDV, whose base is a square.

Let  $CD = a$ ,  $LG = b$ ,  $RN = h$ .

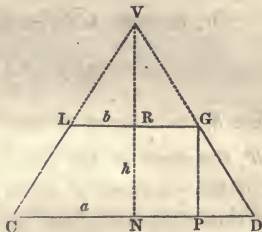
By similar triangles,

$$LG : CD :: RV : NV.$$

Subtracting the antecedents, (Alg. 349.)

$$LG : CD - LG :: RV : NV - RV = RN.$$

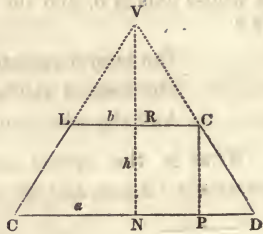
$$\text{Therefore } RV = \frac{RN \times LG}{CD - LG} = \frac{hb}{a - b}$$



The square of CD is the base of the pyramid CDV ;

And the square of LG is the base of the small pyramid LGV.

Therefore, the solidity of the larger pyramid (Art. 48) is



$$\overline{CD}^2 \times \frac{1}{3}(RN + RV) = a^2 \times \frac{1}{3} \left( h + \frac{hb}{a-b} \right) = \frac{ha^3}{3a-3b}$$

And the solidity of the smaller pyramid is equal to

$$\overline{LG}^2 \times \frac{1}{3}RV = b^2 \times \frac{hb}{3a-3b} = \frac{hb^3}{3a-3b}$$

If the smaller pyramid be taken from the larger, there will remain the frustum. CDLG, whose solidity is equal to

$$\frac{ha^3 - hb^3}{3a-3b} = \frac{1}{3}h \times \frac{a^3 - b^3}{a-b} = \frac{1}{3}h \times (a^2 + ab + b^2) \quad (\text{Alg. 194. a.})$$

Or, because  $\sqrt{a^2b^2} = ab$ , (Alg. 210. a.)

$$\frac{1}{3}h \times (a^2 + b^2 + \sqrt{a^2b^2})$$

Here  $h$ , the height of the frustum, is multiplied into  $a$  and  $b^2$ , the areas of the two ends, and into  $\sqrt{a^2b^2}$  the square root of the products of these areas.

In this demonstration the pyramid is supposed to be square. But the rule is equally applicable to a pyramid of any other form. For the solid contents of pyramids are equal, when they have equal heights and bases, whatever be the figure of their bases. (Sup. Euc. 14. 3.)\* And the sec-

\* Thomson's Legendre, 14. 7.

tions parallel to the bases, and at equal distances, are equal to one another. (Sup. Euc. 12. 3. Cor. 2.)\*

Ex. 1. If one end of the frustum of a pyramid be 9 feet square, the other end 6 feet square, and the height 36 feet, what is the solidity ?

The areas of the two ends are 81 and 36.

The square root of their product is 54.

And the solidity of the frustum  $= (81 + 36 + 54) \times 12 = 2052$ .

2. If the height of a frustum of a pyramid be 24, and the areas of the two ends 441 and 121; what is the solidity ?

Ans. 6344.

3. If the height of a frustum of a hexagonal pyramid be 48, each side of one end 26, and each side of the other end 16; what is the solidity ?

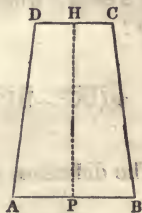
Ans. 56034.

PROBLEM VI.

To find the LATERAL SURFACE of a FRUSTUM of a regular pyramid.

51. MULTIPLY HALF THE SLANT-HEIGHT BY THE SUM OF THE PERIMETERS OF THE TWO ENDS.

Each side of a frustum of a regular pyramid is a *trapezoid*, as ABCD. The slant-height HP, (Def. 11.) though it is oblique to the base of the solid, is perpendicular to the line AB. The area of the trapezoid is equal to the product of half this perpendicular into the sum of the parallel sides AB and DC. (Art. 12.) Therefore the area of all the equal trapezoids which form the lateral surface of the frustum, is equal to the

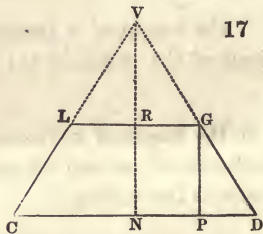


\* Thomson's Legendre, 13. 7. Cor.

product of half the slant-height into the sum of the perimeters of the ends.

Ex. If the slant-height of a frustum of a regular octagonal pyramid be 42 feet, the sides of one end 5 feet each, and the sides of the other end 3 feet each; what is the lateral surface? Ans. 1344 square feet.

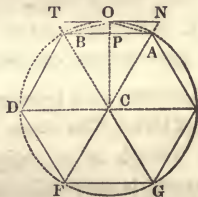
52. If the slant-height be not given, it may be obtained from the perpendicular height and the dimensions of the two ends. Let GD be the slant-height of the frustum CDGL, RN or GP the perpendicular height, ND and RG the radii of the circles inscribed in the perimeters of the two ends.



Then, PD is the difference of the two radii:

And the slant-height  $GD = \sqrt{GP^2 + PD^2}$ .

Ex. If the perpendicular height of a frustum of a regular hexagonal pyramid be 24, the sides of one end 13 each, and the sides of the other end 8 each; what is the whole surface?



$$\sqrt{BC^2 - BP^2} = CP, \text{ that is, } \sqrt{13^2 - 6.5^2} = 11.258$$

$$\text{And } \sqrt{8^2 - 4^2} = 6.928$$

The difference of the two radii is, therefore 4.33

$$\text{The slant-height} = \sqrt{24^2 + 4.33^2} = 24.3875.$$

The lateral surface is 1536.4

And the whole surface, 2141.75.

The height of the *whole pyramid* may be calculated from the dimensions of the frustum. Let VN (Fig. 17.) be the height of the pyramid, RN or GP the height of the frustum, ND and RG the radii of the circles inscribed in the perimeters of the ends of the frustum.

Then, in the similar triangles GPD and VND,

$$DP : GP :: DN : VN.$$

The height of the frustum subtracted from VN, gives VR the height of the small pyramid VLG. The *solidity* and *lateral surface* of the frustum may then be found, by subtracting from the whole pyramid, the part which is above the cutting plane. This method may serve to verify the calculations which are made by the rules in Arts. 50 and 51.

Ex. If one end of the frustum CDGL (Fig. 17.) be 90 feet square, the other end 60 feet square, and the height RN 36 feet; what is the height of the whole pyramid VCD: and what are the solidity and lateral surface of the frustum?

$$DP = DN - GR = 45 - 30 = 15. \quad \text{And, } GP = RN = 36.$$

Then,  $15 : 36 :: 45 : 108 = VN$ , the height of the whole pyramid.

And,  $108 - 36 = 72 = VR$ , the height of the part VLG.

The solidity of the large pyramid is	291600 (Art. 48.)
of the small pyramid	86400
of the frustum CDGL	<u>205200</u>

The lateral surface of the large pyramid is	21060 (Art. 49.)
of the small pyramid	9360
of the frustum	<u>11700</u>





The solidity of the wedge is, therefore,

$$\frac{1}{2}bhl - \frac{1}{6}bh \times (l - L) = \frac{1}{6}bh3l - \frac{1}{6}bh2l + \frac{1}{6}bh2L = \frac{1}{6}bh \times (2L + l)$$

Ex. 1. If the base of a wedge be 35 by 15, the edge 55, and the perpendicular height 12.4; what is the solidity?

$$\text{Ans. } (70 + 55) \times \frac{15 \times 12.4}{6} = 3875.$$

2. If the base of a wedge be 27 by 8, the edge 36, and the perpendicular height 42; what is the solidity?

$$\text{Ans. } 5040.$$

PROBLEM VIII.

To find the SOLIDITY of a rectangular PRISMOID.

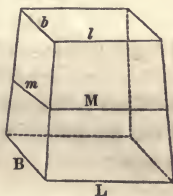
55. TO THE AREAS OF THE TWO ENDS, ADD FOUR TIMES THE AREA OF A PARALLEL SECTION EQUALLY DISTANT FROM THE ENDS, AND MULTIPLY THE SUM BY  $\frac{1}{6}$  OF THE HEIGHT.

Let  $L$  and  $B$  be the length and breadth of one end,

Let  $l$  and  $b$  be the length and breadth of the other end,

Let  $M$  and  $m$  be the length and breadth of the section in the middle.

And  $h$  be the height of the prismoid.



The solid may be divided into two wedges whose bases are the ends of the prismoid, and whose edges are  $L$  and  $l$ . The solidity of the whole, by the preceding article is,

$$\frac{1}{6}Bh \times (2L + l) + \frac{1}{6}bh \times (2l + L) = \frac{1}{6}h(2BL + Bl + 2bl + bL)$$

As  $M$  is equally distant from  $L$  and  $l$ ,

$$2M = L + l, \quad 2m = B + b, \quad \text{and } 4Mm = (L + l)(B + b) = BL + Bl + [bL + lb.$$

Substituting  $4Mm$  for its value, in the preceding expression for the solidity, we have

$$\frac{1}{6}h(BL+bl+4Mm)$$

That is, the solidity of the prismoid is equal to  $\frac{1}{6}$  of the height, multiplied into the areas of the two ends, and 4 times the area of the section in the middle.

This rule may be applied to prismoids of other forms. For, whatever be the figure of the two ends, there may be drawn in each, such a number of small rectangles, that the sum of them shall differ less, than by any given quantity, from the figure in which they are contained. And the solids between these rectangles will be rectangular prismoids.

Ex. 1. If one end of a rectangular prismoid be 44 feet by 23, the other end 36 by 21, and the perpendicular height 72; what is the solidity?

The area of the larger end	= $44 \times 23 = 1012$
of the smaller end	= $36 \times 21 = 756$
of the middle section	= $40 \times 22 = 880$

And the solidity =  $(1012 + 756 + 4 \times 880) \times 12 = 63456$  feet.

2. What is the solidity of a stick of hewn timber, whose ends are 30 inches by 27, and 24 by 18, and whose length is 48 feet?

Ans. 204 feet.

Other solids not treated of in this section, if they be bounded by plane surfaces, may be measured by supposing them to be divided into prisms, pyramids, and wedges. And, indeed, every such solid may be considered as made up of triangular pyramids.

## THE FIVE REGULAR SOLIDS.

56. A SOLID IS SAID TO BE REGULAR, WHEN ALL ITS SOLID ANGLES ARE EQUAL, AND ALL ITS SIDES ARE EQUAL AND REGULAR POLYGONS.

The following figures are of this description ;

- |  |   |                    |   |  |
|--|---|--------------------|---|--|
| <ol style="list-style-type: none"> <li>1. The <i>Tetraedron</i>,</li> <li>2. The <i>Hexaedron or cube</i>,</li> <li>3. The <i>Octaedron</i>,</li> <li>4. The <i>Dodecaedron</i>,</li> <li>5. The <i>Icosaedron</i>,</li> </ol> | } | whose<br>sides are | { | four triangles ;<br>six squares ;<br>eight triangles ;<br>twelve pentagons ;<br>twenty triangles.* |
|--|---|--------------------|---|--|

Besides these five there can be no other regular solids. The only plane figures which can form such solids, are triangles, squares, and pentagons. For the plane angles which contain any solid angle, are together less than four right angles or  $360^\circ$ . (Sup. Euc. 21, 2.) And the least number which can form a solid angle is three. (Sup. Euc. Def. 8, 2.) If they are angles of equilateral *triangles*, each is  $60^\circ$ . The sum of *three* of them is  $180^\circ$ , of *four*  $240^\circ$ , of *five*  $300^\circ$ , and of *six*  $360^\circ$ . The latter number is too great for a solid angle.

The angles of *squares* are  $90^\circ$  each. The sum of *three* of these is  $270^\circ$ , of *four*  $360^\circ$ , and of any other greater number, still more.

The angles of regular *pentagons* are  $108^\circ$  each. The sum of *three* of them is  $324^\circ$  ; of *four*, or any other greater number, more than  $360^\circ$ . The angles of all other regular polygons are still greater.

In a regular solid, then, each solid angle must be contained by three, four, or five equilateral triangles, by three squares, or by three regular pentagons.

---

\* For the geometrical construction of these solids, see Legendre's *Geometry* ; Appendix to Books VI. and VII., or Thomson's *Legendre*, p. 214.

57. As the sides of a regular solid are similar and equal, and the angles are also alike; it is evident that the sides are all equally distant from a central point in the solid. If then, planes be supposed to proceed from the several edges to the centre, they will divide the solid into as many equal *pyramids*, as it has sides. The base of each pyramid will be one of the sides; their common vertex will be the central point; and their height will be a perpendicular from the centre to one of the sides.

### PROBLEM IX.

*To find the SURFACE of a REGULAR SOLID.*

58. MULTIPLY THE AREA OF ONE OF THE SIDES BY THE NUMBER OF SIDES.

Or,

MULTIPLY THE SQUARE OF ONE OF THE EDGES, BY THE SURFACE OF A SIMILAR SOLID WHOSE EDGES ARE 1.

As all the sides are *equal*, it is evident that the area of one of them, multiplied by the number of sides, will give the area of the whole.

Or, if a *table* is prepared, containing the surfaces of the several regular solids whose linear edges are *unity*; this may be used for other regular solids, upon the principle, that the areas of similar polygons are as the squares of their homologous sides. (Euc. 20. 6.)\* Such a table is easily formed, by multiplying the area of one of the sides, as given in Art. 17, by the number of sides. Thus, the area of an equilateral triangle whose side is 1, is 0.4330127. Therefore, the surface

---

\* Thomson's Legendre, 27. 4.



Of a regular tetraedron  $= .4330127 \times 4 = 1.7320508$ .

Of a regular octaedron  $= .4330127 \times 8 = 3.4641016$ .

Of a regular icosaedron  $= .4330127 \times 20 = 8.6602540$ .

See the table in the following article.

Ex. 1. What is the surface of a regular dodecaedron whose edges are each 25 inches ?

The area of one of the sides is 1075.3

And the surface of the whole solid  $= 1075.3 \times 12 = 12903.6$ .

2. What is the surface of a regular icosaedron whose edges are each 102? Ans. 90101.3.

### PROBLEM X.

*To find the SOLIDITY of a REGULAR SOLID.*

59. MULTIPLY THE SURFACE BY  $\frac{1}{3}$  OF THE PERPENDICULAR DISTANCE FROM THE CENTRE TO ONE OF THE SIDES.

Or,

MULTIPLY THE CUBE OF ONE OF THE EDGES, BY THE SOLIDITY OF A SIMILAR SOLID WHOSE EDGES ARE 1.

As the solid is made up of a number of equal pyramids, whose bases are the sides, and whose height is the perpendicular distance of the sides from the centre (Art. 57.); the solidity of the whole must be equal to the areas of all the sides multiplied into  $\frac{1}{3}$  of this perpendicular. (Art. 48.)

If the contents of the several regular solids whose edges are 1, be inserted in a *table*, this may be used to measure other similar solids. For two similar regular solids contain the same number of similar pyramids; and these are to each other as the *cubes* of their linear sides or edges. (Sup. Euc. 15. 3. Cor. 3.)\*

---

\* Thomson's Legendre, 20. 7.

A TABLE OF REGULAR SOLIDS WHOSE EDGES ARE 1.

Names.	No. of sides.	Surfaces.	Solidities.
Tetraedron	4	1.7320508	0.1178513
Hexaedron	6	6.0000000	1.0000000
Octaedron	8	3.4641016	0.4714045
Dodecaedron	12	20.6457288	7.6631189
Icosaedron	20	8.6602540	2.1816950

For the method of calculating the last column of this table, see Hutton's Mensuration, Part. III. Sec. 2.

Ex. What is the solidity of a regular octaedron whose edges are each 32 inches?      Ans. 15447 inches.

## SECTION IV.

### THE CYLINDER, CONE, AND SPHERE.

ART. 61. DEFINITION I. A *right cylinder* is a solid described by the revolution of a rectangle about one of its sides. The *ends or bases* are evidently equal and parallel circles. And the *axis*, which is a line passing through the middle of the cylinder, is perpendicular to the bases.

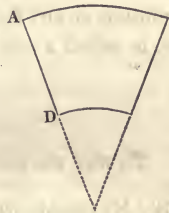
The ends of an *oblique cylinder* are also equal and parallel circles; but they are not perpendicular to the axis. The *height* of a cylinder is the perpendicular distance from one base to the plane of the other. In a right cylinder, it is the length of the axis.

II. A *right cone* is a solid described by the revolution of a right angled triangle about one of the sides which contain the right angle. The *base* is a circle, and is perpendicular to

the *axis*, which proceeds from the middle of the base to the vertex.

The base of an *oblique* cone is also a circle, but is not perpendicular to the axis. The *height* of a cone is the perpendicular distance from the vertex to the plane of the base. In a right cone, it is the length of the axis. The *slant-height* of a right cone is the distance from the vertex to the circumference of the base.

III. A *frustum* of a cone is a portion cut off by a plane parallel to the base. The *height* of the frustum is the perpendicular distance of the two ends. The *slant-height* of a frustum of a right cone, is the distance between the peripheries of the two ends, measured on the outside of the solid; as AD.



IV. A *sphere* or *globe* is a solid which has a centre equally distant from every part of the surface. It may be described by the revolution of a semicircle about a diameter. A *radius* of the sphere is a line drawn from the centre to any part of the surface. A *diameter* is a line passing through the centre, and terminated at both ends by the surface. The *circumference* is the same as the circumference of a circle whose plane passes through the centre of the sphere. Such a circle is called a *great circle*.

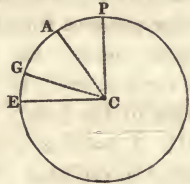
V. A *segment* of a sphere is a part cut off by any plane. The *height* of the segment is a perpendicular from the middle of the base to the convex surface, as LB.



VI. A *spherical zone* or frustum is a part of the sphere included between two parallel planes. It is called the *middle zone*, if the planes are equally distant from the centre.

The *height* of a zone is the distance of the two planes, as LR.\*

VII. A *spherical sector* is a solid produced by a circular sector, revolving in the same manner as the semicircle which describes the whole sphere. Thus a spherical sector is described by the circular sector ACP or GCE revolving on the axis CP.



VIII. A solid described by the revolution of any figure about a fixed axis, is called a *solid of revolution*.

#### PROBLEM I.

To find the CONVEX SURFACE of a RIGHT CYLINDER.

62. MULTIPLY THE LENGTH INTO THE CIRCUMFERENCE OF THE BASE.

If a right cylinder be covered with a thin substance like paper, which can be spread out into a plane; it is evident that the plane will be a *parallelogram*, whose length and breadth will be equal to the length and circumference of the cylinder. The area must, therefore, be equal to the length multiplied into the circumference. (Art. 4.)

Ex. 1. What is the convex surface of a right cylinder which is 42 feet long, and 15 inches in diameter?

$$\text{Ans. } 42 \times 1.25 \times 3.14159 = 164.933 \text{ sq. feet.}$$

2. What is the whole surface of a right cylinder, which is 2 feet in diameter and 36 feet long?

---

\* According to some writers, a *spherical segment* is either a solid which is cut off from the sphere by a single plane, or one which is included between two planes: and a *zone* is the *surface* of either of these. In this sense, the term zone is commonly used in geography.



The convex surface is	226.1945
The area of the two ends (Art. 30.) is	6.2832
The whole surface is	232.4777

3. What is the whole surface of a right cylinder whose axis is 82, and circumference 71?      Ans. 6624.32.

63. It will be observed, that the rules for the *prism* and *pyramid* in the preceding section, are substantially the same, as the rules for the *cylinder* and *cone* in this. There may be some advantage, however, in considering the latter by themselves.

In the base of a *cylinder*, there may be inscribed a polygon, which shall differ from it less than by any given space. (Sup. Euc. 6. 1. Cor.)\* If the polygon be the base of a *prism*, of the same height as the cylinder, the two solids may differ less than by any given quantity. In the same manner, the base of a *pyramid* may be a polygon of so many sides, as to differ less than by any given quantity, from the base of a *cone* in which it is inscribed. A cylinder is therefore considered, by many writers, as a prism of an infinite number of sides; and a cone, as a pyramid of an infinite number of sides. (For the meaning of the term "infinite," when used in the mathematical sense, see Alg. Sec. XV.)

## PROBLEM II.

*To find the SOLIDITY of a CYLINDER.*

64. MULTIPLY THE AREA OF THE BASE BY THE HEIGHT.

The solidity of a *parallelopiped* is equal to the product of the base into the perpendicular altitude. (Art. 43.) And a parallelopiped and a cylinder which have equal bases and altitudes are equal to each other. (Sup. Euc. 17. 3.)†

\* Thomson's Legendre, 9. 5.

† Ibid., 2. 8.



Ex. 1. What is the solidity of a cylinder, whose height is 121, and diameter 45.2?

$$\text{Ans. } 45.2^2 \times .7854 \times 121 = 194156.6.$$

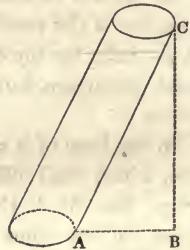
2. What is the solidity of a cylinder, whose height is 424, and circumference 213?

$$\text{Ans. } 1530837.$$

3. If the side AC of an oblique cylinder be 27, and the area of the base 32.61, and if the side make an angle of  $62^\circ 44'$  with the base, what is the solidity?

R : AC :: sin A : BC = 24 the perpendicular height.

And the solidity is 782.64.



4. The Winchester bushel is a hollow cylinder,  $18\frac{1}{2}$  inches in diameter, and 8 inches deep. What is its capacity?

The area of the base =  $(18.5)^2 \times .7853982 = 268.8025$ .

And the capacity is 2150.42 cubic inches. See the table in Art. 42.

### PROBLEM III.

*To find the CONVEX SURFACE of a RIGHT CONE.*

65. MULTIPLY HALF THE SLANT-HEIGHT INTO THE CIRCUMFERENCE OF THE BASE.

If the convex surface of a right cone be spread out into a plane, it will evidently form a *sector* of a circle whose radius is equal to the slant-height of the cone. But the area of the sector is equal to the product of half the radius into the length of the arc. (Art. 34.) Or if the cone be considered as a pyramid of an infinite number of sides, its lateral sur-

face is equal to the product of half the slant-height into the perimeter of the base. (Art. 49.)

Ex. 1. If the slant-height of a right cone be 82, and the diameter of the base 24, what is the convex surface?

Ans.  $41 \times 24 \times 3.14159 = 3091.3$  square feet.

2. If the axis of a right cone be 48, and the diameter of the base 72, what is the whole surface?

The slant-height  $= \sqrt{(36^2 + 48^2)} = 60$ . (Euc. 47. 1.)

The convex surface is 6786

The area of the base 4071.6

And the whole surface 10857.6

3. If the axis of a right cone be 16, and the circumference of the base 75.4; what is the whole surface?

Ans. 1206.4.

#### PROBLEM IV.

*To find the SOLIDITY of a CONE.*

66. MULTIPLY THE AREA OF THE BASE INTO  $\frac{1}{3}$  OF THE HEIGHT.

The solidity of a *cylinder* is equal to the product of the base into the perpendicular height. (Art. 64.) And if a cone and a cylinder have the same base and altitude, the cone is  $\frac{1}{3}$  of the cylinder. (Sup. Euc. 18. 3.)\* Or if a cone be considered as a pyramid of an infinite number of sides, the solidity is equal to the product of the base into  $\frac{1}{3}$  of the height, by Art. 48.

Ex. 1. What is the solidity of a right cone whose height is 663, and the diameter of whose base is 101?

Ans.  $\overline{101^2} \times .7854 \times 221 = 1770622$ .

---

\* Thomson's Legendre, 4. 8. Cor.

2. If the axis of an oblique cone be 738, and make an angle of  $30^\circ$  with the plane of the base; and if the circumference of the base be 355, what is the solidity?

Ans. 1233536.

### PROBLEM V.

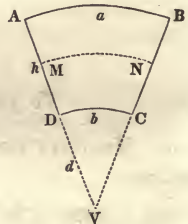
To find the CONVEX SURFACE of a FRUSTUM of a right cone.

67. MULTIPLY HALF THE SLANT-HEIGHT BY THE SUM OF THE PERIPHERIES OF THE TWO ENDS.

This is the rule for a frustum of a *pyramid*; (Art. 51.) and is equally applicable to a frustum of a *cone*, if a cone be considered as a pyramid of an infinite number of sides. (Art. 63.)

Or thus,

Let the sector ABV represent the convex surface of a right cone, (Art. 65.) and DCV the surface of a portion of the cone, cut off by a plane parallel to the base. Then will ABCD be the surface of the frustum.



Let  $AB = a$ ,  $DC = b$ ,  $VD = d$ ,  $AD = h$ .

Then the area  $ABV = \frac{1}{2}a \times (h + d) = \frac{1}{2}ah + \frac{1}{2}ad$ . (Art. 34.)

And the area  $DCV = \frac{1}{2}bd$ .

Subtracting the one from the other,

The area  $ABDC = \frac{1}{2}ah + \frac{1}{2}ad - \frac{1}{2}bd$ .

But  $d : d + h :: b : a$ . (Sup. Euc. 8. 1.)\* Therefore  $\frac{1}{2}ad - \frac{1}{2}bd = \frac{1}{2}bh$ .

The surface of the frustum then, is equal to

$$\frac{1}{2}ah + \frac{1}{2}bh.$$

$$\text{or } \frac{1}{2}h \times (a + b)$$

\* Thomson's Legendre, 10. 5. Cor.

Cor. The surface of the frustum is equal to the product of the slant-height into the circumference of a circle which is *equally distant* from the two ends. Thus, the surface ABCD is equal to the product of AD into MN. For MN is equal to half the sum of AB and DC.

Ex. 1. What is the convex surface of a frustum of a right cone, if the diameters of the two ends be 44 and 33, and the slant-height 84 ?

Ans. 10159.8.

2. If the perpendicular height of a frustum of a right cone be 24, and the diameters of the two ends 80 and 44, what is the whole surface ?

Half the difference of the diameters is 18.

And  $\sqrt{18^2 + 24^2} = 30$ , the slant-height, (Art. 52.)

The convex surface of the frustum is 5843

The sum of the areas of the two ends is 6547

And the whole surface is 12390

#### PROBLEM VI.

*To find the SOLIDITY of a FRUSTUM of a cone.*

68. ADD TOGETHER THE AREAS OF THE TWO ENDS, AND THE SQUARE ROOT OF THE PRODUCT OF THESE AREAS; AND MULTIPLY THE SUM BY  $\frac{1}{3}$  OF THE PERPENDICULAR HEIGHT.

This rule, which was given for the frustum of a *pyramid*, (Art. 50.) is equally applicable to the frustum of a cone; because a cone and a pyramid which have equal bases and altitudes are equal to each other.

Ex. 1. What is the solidity of a mast which is 72 feet long, 2 feet in diameter at one end, and 18 inches at the other ?

Ans. 174.36 cubic feet.





the polygon. The triangles ABH and CGM are similar, because the sides are perpendicular, each to each. Therefore,

$$HB \text{ or } ON : AB :: GM : GC :: \text{circ } GM : \text{circ } GC.$$

So that  $ON \times \text{circ } GC = AB \times \text{circ } GM$ , that is, the surface of the frustum is equal to the product of ON the perpendicular height, into *circ* GC, the perpendicular distance from the centre of the polygon to one of the sides.

In the same manner it may be proved, that the surfaces produced by the revolution of the lines BD and AP about the axis DC, are equal to

$$ND \times \text{circ } GC, \quad \text{and } CO \times \text{circ } GC.$$

The surface of the whole solid, therefore, (Euc. 1. 2.) is equal to

$$CD \times \text{circ } GC.$$

The demonstration is applicable to a solid produced by the revolution of a polygon of *any* number of sides. But a polygon may be supposed which shall differ less than by any given quantity from the circle in which it is inscribed; (Sup. Euc. 4. 1.)\* and in which the perpendicular GC shall differ less than by any given quantity from the radius of the circle. Therefore, the surface of a *hemisphere* is equal to the product of its radius into the circumference of its base; and *the surface of a sphere is equal to the product of its diameter into its circumference.*

Cor. 1. From this demonstration it follows, that the surface of any *segment* or *zone* of a sphere is equal to the product of the height of the segment or zone into the circumference of the sphere. The surface of the zone produced by the revolution of the arc AB about ON, is equal to  $ON \times \text{circ } CP$ . And the surface of the segment pro-

---

\* Thomson's Legendre, 9. 5.



## PROBLEM VIII.

To find the SOLIDITY of a SPHERE.

70. 1. MULTIPLY THE CUBE OF THE DIAMETER BY **.5236**.

Or,

2. MULTIPLY THE SQUARE OF THE DIAMETER BY  $\frac{1}{6}$  OF THE CIRCUMFERENCE.

Or,

3. MULTIPLY THE SURFACE BY  $\frac{1}{6}$  OF THE DIAMETER.

1. A sphere is *two-thirds* of its circumscribing cylinder. (Sup. Euc. 21. 3.)\* The height and diameter of the cylinder are each equal to the diameter of the sphere. The solidity of the cylinder is equal to its height multiplied into the area of its base, (Art. 64.) that is putting D for the diameter,

$$D \times D^2 \times .7854 \text{ or } D^3 \times .7854.$$

And the solidity of the *sphere*, being  $\frac{2}{3}$  of this, is

$$D^3 \times .5236.$$

2. The base of the circumscribing cylinder is equal to half the circumference multiplied into half the diameter; (Art. 30.) that is, if C be put for the circumference,

$$\frac{1}{4}C \times D; \text{ and the solidity is } \frac{1}{4}C \times D^2.$$

Therefore, the solidity of the sphere is

$$\frac{2}{3} \text{ of } \frac{1}{4}C \times D^2 = D^3 \times \frac{1}{6}C.$$

3. In the last expression, which is the same as  $C \times D \times \frac{1}{6}D$ ,

\* Thomson's Legendre, 12. 8.

we may substitute  $S$ , the surface, for  $C \times D$ . (Art. 69.) We then have the solidity of the sphere equal to

$$S \times \frac{1}{3}D.$$

Or, the sphere may be supposed to be filled with small *pyramids*, standing on the surface of the sphere, and having their common vertex in the centre. The number of these may be such, that the difference between their sum and the sphere shall be less than any given quantity. The solidity of each pyramid is equal to the product of its base into  $\frac{1}{3}$  of its height. (Art. 48.) The solidity of the whole, therefore, is equal to the product of the surface of the sphere into  $\frac{1}{3}$  of its radius, or  $\frac{1}{6}$  of its diameter.

71. The numbers 3.14159, .7854, .5236, should be made perfectly familiar. The first expresses the ratio of the *circumference* of a circle to the *diameter*; (Art. 23.) the second, the ratio of the *area* of a circle to the square of the diameter (Art. 30.); and the third, the ratio of the *solidity* of a sphere to the *cube* of the diameter. The second is  $\frac{1}{4}$  of the first, and the third is  $\frac{1}{6}$  of the first.

As these numbers are frequently occurring in mathematical investigations, it is common to represent the first of them by the Greek letter  $\pi$ . According to this notation,

$$\pi = 3.14159, \quad \frac{1}{4}\pi = .7854, \quad \frac{1}{6}\pi = .5236.$$

If  $D$  = the *diameter*, and  $R$  = the *radius* of any circle or sphere;

$$\text{Then, } D = 2R \quad D^2 = 4R^2 \quad D^3 = 8R^3.$$

And  $\pi D$  } = the *periph.*  $\frac{1}{4}\pi D^2$  } = the *area* of  $\frac{1}{6}\pi D^3$  }  
 Or,  $2\pi R$  } or  $\pi R^2$  } the *circ.* or  $\frac{4}{3}\pi R^3$  } = the  
*solidity* of the sphere.

Ex 1. What is the solidity of the earth, if it be a sphere 7930 miles in diameter?

Ans. 261,107,000,000 cubic miles.

2. How many wine gallons will fill a hollow sphere 4 feet in diameter ?

Ans. The capacity is 33.5104 feet =  $250\frac{2}{3}$  gallons.

3. If the diameter of the moon be 2180 miles, what is its solidity ?

Ans. 5,424,600,000 miles.

72. If the solidity of a sphere be *given*, the diameter may be found by reversing the first rule in the preceding article; that is, *dividing by .5236* and *extracting the cube root of the quotient*.

Ex. 1. What is the diameter of a sphere whose solidity is 65.45 cubic feet ?

Ans. 5 feet.

2. What must be the diameter of a globe to contain 16755 pounds of water ?

Ans. 8 feet.

### PROBLEM IX.

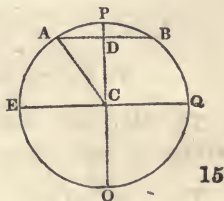
*To find the CONVEX SURFACE of a SEGMENT or ZONE of a sphere.*

73. MULTIPLY THE HEIGHT OF THE SEGMENT OR ZONE INTO THE CIRCUMFERENCE OF THE SPHERE.

For the demonstration of this rule, see Art. 69.

Ex. 1. If the earth be considered a perfect sphere 7930 miles in diameter, and if the polar circle be  $23^{\circ} 28'$  from the pole, how many square miles are there in one of the frigid zones ?

If PQOE be a meridian on the earth, ADB one of the polar circles, and P the pole; then the frigid zone is a spherical segment described by the revolution of the arc APB about PD. The angle ACD subtended by the arc AP is  $23^{\circ} 28'$ . And in the right angled triangle ACD,





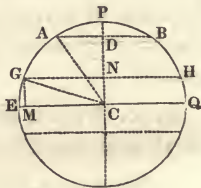
$$R : AC :: \cos ACD : CD = 3637.$$

Then,  $CP - CD = 3965 - 3637 = 328 = PD$  the height of the segment.

And  $328 \times 7930 \times 3.14159 = 8171400$  the surface.

2. If the diameter of the earth be 7930 miles, what is the surface of the torrid zone, extending  $23^\circ 28'$  on each side of the equator?

If  $EQ$  be the equator, and  $GH$  one of the tropics, then the angle  $ECG$  is  $23^\circ 28'$ . And in the right angled triangle  $GCM$ ,



$R : CG :: \sin ECG : GM = CN = 1578.9$  the height of half the zone.

The surface of the whole zone is 78669700.

3. What is the surface of each of the temperate zones?

The height  $DN = CP - CN - PD = 2058.1$

And the surface of the zone is 51273000.

The surface of the two temperate zones is	102,546,000
of the two frigid zones	16,342,800
of the torrid zone	<u>78,669,700</u>
of the whole globe	197,558,500

### PROBLEM X.

*To find the SOLIDITY of a spherical SECTOR.*

74. MULTIPLY THE SPHERICAL SURFACE BY  $\frac{1}{3}$  OF THE RADIUS OF THE SPHERE.

The spherical sector produced by the revolution of  $ACBD$

about CD, may be supposed to be filled with *small pyramids*, standing on the spherical surface ADB, and terminating in the point C. Their number may be so great, that the height of each shall differ less than by any given length from the radius CD, and the sum of their bases shall differ less than by any given quantity from the surface ABD. The



solidity of each is equal to the product of its base into  $\frac{1}{3}$  of the radius CD. (Art. 48.) Therefore, the solidity of all of them, that is, of the sector ADBC, is equal to the product of the spherical surface into  $\frac{1}{3}$  of the radius.

Ex. Supposing the earth to be a sphere 7930 miles in diameter, and the polar circle ADB to be  $23^{\circ} 28'$  from the pole; what is the solidity of the spherical sector ACBP?



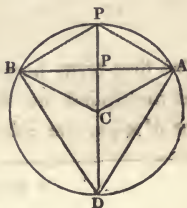
Ans. 10,799,867,000 miles.

PROBLEM XI.

To find the SOLIDITY of a spherical SEGMENT.

75. MULTIPLY HALF THE HEIGHT OF THE SEGMENT INTO THE AREA OF THE BASE, AND THE CUBE OF THE HEIGHT INTO .5236; AND ADD THE TWO PRODUCTS.

As the *circular sector* AOBC consists of two parts, the segment AOBP and the triangle ABC; (Art. 35.) so the *spherical sector* produced by the revolution of AOC about OC consists of two parts, the *segment* produced by the revolution of AOP, and the *cone* produced by the revolution of ACP. If then

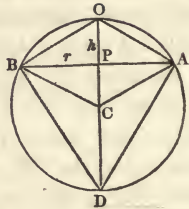


the cone be subtracted from the sector, the remainder will be the segment.

Let  $CO=R$ , the radius of the sphere,  
 $PB=r$ , the radius of the base of  
 the segment.

$PO=h$ , the height of the segment,

Then  $PC=R-h$ , the axis of the cone.



The sector  $= 2\pi R \times h \times \frac{1}{3}R$  (Arts. 71, 73, 74.)  $= \frac{2}{3}\pi hR^2$ .

The cone  $= \pi r^2 \times \frac{1}{3}(R-h)$  (Arts. 71, 66.)  $= \frac{1}{3}\pi r^2 R - \frac{1}{3}\pi hr^2$ .

Subtracting the one from the other,

$$\text{The segment} = \frac{2}{3}\pi hR^2 - \frac{1}{3}\pi r^2 R + \frac{1}{3}\pi hr^2.$$

But  $DO \times PO = \overline{BO}^2$  (Trig. 97.\*)  $= \overline{PO}^2 + \overline{PB}^2$  (Euc. 47. 1.)

That is,  $2Rh = h^2 + r^2$ . So that,  $R = \frac{h^2 + r^2}{2h}$

$$\text{And } R^2 = \left( \frac{h^2 + r^2}{2h} \right)^2 = \frac{h^4 + 2h^2r^2 + r^4}{4h^2}$$

Substituting then, for  $R$  and  $R^2$ , their values, and multiplying the factors,

$$\text{The segment} = \frac{1}{6}\pi h^3 + \frac{1}{3}\pi hr^2 + \frac{1}{6}\frac{\pi r^4}{h} - \frac{1}{6}\pi hr^2 - \frac{1}{6}\frac{\pi r^4}{h} + \frac{1}{3}\pi hr^2$$

Which, by uniting the terms, becomes

$$\frac{1}{2}\pi hr^2 + \frac{1}{6}\pi h^3.$$

The first term here is  $\frac{1}{2}h \times \pi r^2$ , half the height of the segment multiplied into the area of the base; (Art. 71.) and the other  $h^3 \times \frac{1}{6}\pi$ , the cube of the height multiplied into .5236.

\* Euclid 31, 3, and 8, 6. Cor.

If the segment be *greater* than a hemisphere, as ABD ; the cone ABC must be *added* to the sector ACBD.

Let PD= $h$  the height of the segment,  
Then PC= $h-R$  the axis of the cone.

$$\text{The sector ACBD} = \frac{2}{3}\pi hR^2$$

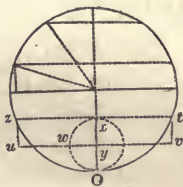
$$\text{The cone} = \pi r^2 \times \frac{1}{3}(h-R) = \frac{1}{3}\pi h r^2 - \frac{1}{3}\pi r^2 R$$

Adding them together, we have as before,

$$\text{The segment} = \frac{2}{3}\pi hR^2 - \frac{1}{3}\pi r^2 R + \frac{1}{3}\pi h r^2.$$

Cor. The solidity of a spherical segment is equal to half a cylinder of the same base and height + a sphere whose diameter is the height of the segment. For a cylinder is equal to its height multiplied into the area of its base ; and a sphere is equal to the cube of its diameter multiplied by .5236.

Thus, if Oy be half Ox, the spherical segment produced by the revolution, of Oxt is equal to the cylinder produced by tvyx + the sphere produced by Oyxz ; supposing each to revolve on the line Ox.



Ex. 1. If the height of a spherical segment be 8 feet, and the diameter of its base 25 feet ; what is the solidity ?

$$\text{Ans. } (25)^2 \times .7854 \times 4 + 8^3 \times .5236 = 2231.58 \text{ feet.}$$

2. If the earth be a sphere 7930 miles in diameter, and the polar circle  $23^\circ 28'$  from the pole, what is the solidity of one of the frigid zones ?      Ans. 1,303,000,000 miles.

## PROBLEM XII.

To find the SOLIDITY of a spherical ZONE or frustum.

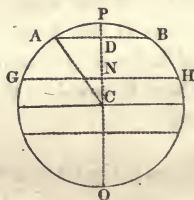
76. FROM THE SOLIDITY OF THE WHOLE SPHERE, SUBTRACT THE TWO SEGMENTS ON THE SIDES OF THE ZONE.

Or,

ADD TOGETHER THE SQUARES OF THE RADII OF THE TWO ENDS, AND  $\frac{1}{3}$  THE SQUARE OF THEIR DISTANCE; AND MULTIPLY THE SUM BY THREE TIMES THIS DISTANCE, AND THE PRODUCT BY .5236.

If from the whole sphere, there be taken the two segments ABP and GHO, there will remain the zone or frustum ABGH.

Or, the zone ABGH is equal to the difference between the segments GHP and ABP.



Let  $NP = H$  } the heights of the two segments.  
 $DP = h$  }  
 $GN = R$  } the radii of their bases.  
 $AD = r$  }

$DN = d = H - h$  the distance of the two bases, or the height of the zone.

Then the larger segment  $= \frac{1}{2}\pi HR^2 + \frac{1}{6}\pi H^3$  } (Art. 75.)  
 And the smaller segment  $= \frac{1}{2}\pi hr^2 + \frac{1}{6}\pi h^3$  }

Therefore the zone  $ABGH = \frac{1}{6}\pi (3HR^2 + H^3 - 3hr^2 - h^3)$

By the properties of the circle, (Euc. 35, 3.)

$ON \times H = R^2$ . Therefore,  $(ON + H) \times H = R^2 + H^2$

$$\text{Or, } OP = \frac{R^2 + H^2}{H}$$



In the same manner,  $OP = \frac{r^2 + h^2}{h}$

Therefore,  $3H \times (r^2 + h^2) = 3h \times (R^2 + H^2)$

Or,  $3Hr^2 + 3Hh^2 - 3hR^2 - 3hH^2 = 0$ . (Alg. 178.)

To reduce the expression for the solidity of the zone to the required form, without altering its value, let these terms be added to it: and it will become

$$\frac{1}{6}\pi(3HR^2 + 3Hr^2 - 3hR^2 - 3hr^2 + H^3 - 3H^2h + 3Hh^2 - h^3)$$

Which is equal to

$$\frac{1}{6}\pi \times 3(H-h) \times (R^2 + r^2 + \frac{1}{3}(H-h)^2)$$

Or, as  $\frac{1}{6}\pi$  equals .5236 (Art. 71.) and  $H-h$  equals  $d$ ,

$$\text{The zone} = .5236 \times 3d \times (R^2 + r^2 + \frac{1}{3}d^2)$$

Ex. 1. If the diameter of one end of a spherical zone is 24 feet, the diameter of the other end 20 feet, and the distance of the two ends, or the height of the zone 4 feet; what is the solidity?                      Ans. 1566.6 feet.

2. If the earth be a sphere 7930 miles in diameter, and the obliquity of the ecliptic  $23^\circ 28'$ ; what is the solidity of one of the temperate zones?                      Ans. 55,390,500,000 miles.

3. What is the solidity of the torrid zone?                      Ans. 147,720,000,000 miles.

The solidity of the two temperate zones is	110,781,000,000
of the two frigid zones	2,606,000,000
of the torrid zone	<u>147,720,000,000</u>
of the whole globe	261,107,000,000

4. What is the convex surface of a spherical zone, whose breadth is 4 feet, on a sphere of 25 feet diameter?

5. What is the solidity of a spherical segment, whose height is 18 feet, and the diameter of its base 40 feet ?

PROMISCUOUS EXAMPLES OF SOLIDS.

Ex. 1. How much water can be put into a cubical vessel three feet deep, which has been previously filled with cannon balls of the same size, 2, 4, 6, or 9 inches in diameter, regularly arranged in tiers, one directly above another ?

Ans.  $96\frac{1}{5}$  wine gallons.

2. If a cone or pyramid, whose height is three feet, be divided into three equal portions, by sections parallel to the base ; what will be the heights of the several parts ?

Ans. 24.961, 6.488, and 4.551 inches.

3. What is the solidity of the greatest square prism which can be cut from a cylindrical stick of timber, 2 feet 6 inches in diameter and 56 feet long ?\*

Ans. 175 cubic feet.

4. How many such globes as the earth are equal in bulk to the sun ; if the former is 7930 miles in diameter, and the latter 890,000 ?

Ans. 1,413,678.

\* The common rule for measuring *round timber* is to multiply the square of the *quarter-girt* by the length. The quarter-girt is one-fourth of the circumference. This method does not give the whole solidity. It makes an allowance of about one-fifth, for waste in hewing, bark, &c. The solidity of a cylinder is equal to the product of the height into the area of the base.

If  $C$  = the circumference, and  $\pi = 3.14159$ , then (Art. 31.)

$$\text{The area of the base} = \frac{C^2}{4\pi} = \left(\frac{C}{\sqrt{4\pi}}\right)^2 = \left(\frac{C}{3.545}\right)^2$$

If then the circumference were divided by 3.545, instead of 4, and the quotient squared, the area of the base would be correctly found. See note B.

5. How many cubic feet of wall are there in a conical tower 66 feet high, if the diameter of the base be 20 feet from outside to outside, and the diameter of the top 8 feet; the thickness of the wall being 4 feet at the bottom, and decreasing regularly, so as to be only two feet at the top?

Ans. 7188.

6. If a metallic globe filled with wine, which cost as much at 5 dollars a gallon, as the globe itself at 20 cents for every square inch of its surface; what is the diameter of the globe?

Ans. 55.44 inches.

7. If the circumference of the earth be 25,000 miles, what must be the diameter of a metallic globe, which, when drawn into a wire  $\frac{1}{8}$  of an inch in diameter, would reach round the earth?

Ans. 15 feet and 1 inch.

8. If a conical cistern be 3 feet deep,  $7\frac{1}{2}$  feet in diameter at the bottom, and 5 feet at the top; what will be the depth of a fluid occupying half its capacity?

Ans. 14.535 inches.

9. If a globe 20 inches in diameter, be perforated by a cylinder 16 inches in diameter, the axis of the latter passing through the centre of the former; what part of the solidity, and the surface of the globe, will be cut away by the cylinder?

Ans. 3284 inches of the solidity, and 502,655 of the surface.

10. What is the solidity of the greatest cube which can be cut from a sphere three feet in diameter?

Ans.  $5\frac{1}{8}$  feet.

11. What is the solidity of a conic frustum, the altitude of which is 36 feet, the greater diameter 16, and the lesser diameter 8?

12. What is the solidity of a spherical segment 4 feet high, cut from a sphere 16 feet in diameter?

## SECTION V.

## ISOPERIMETRY.

ART. 77. It is often necessary to compare a number of different figures or solids, for the purpose of ascertaining which has the *greatest area*, within a given perimeter, or the *greatest capacity* under a given surface. We may have occasion to determine, for instance, what must be the form of a fort, to contain a given number of troops, with the least extent of wall; or what the shape of a metallic pipe to convey a given portion of water, or of a cistern to hold a given quantity of liquor, with the least expense of materials.

78. Figures which have equal perimeters are called *Iso-perimeters*. When a quantity is *greater* than any other of the same class, it is called a *maximum*. A multitude of straight lines, of different lengths, may be drawn within a circle. But among them all, the *diameter* is a *maximum*. Of all *sines* of angles, which can be drawn in a circle, the sine of  $90^\circ$  is a *maximum*.

When a quantity is *less* than any other of the same class, it is called a *minimum*. Thus, of all straight lines drawn from a given point to a given straight line, that which is *perpendicular* to the given line is a *minimum*. Of all straight lines drawn from a given point in a circle, to the circumference, the *maximum* and the *minimum* are the two parts of the diameter which pass through that point. (Euc. 7, 3.)

In isoperimetry, the object is to determine, on the one hand, in what cases the area is a *maximum*, within a given perimeter; or the capacity a *maximum*, within a given surface: and on the other hand, in what cases the perimeter is

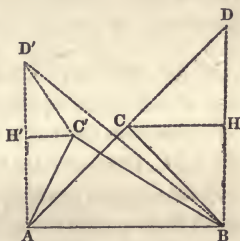
a *minimum* for a given area, or the surface a *minimum*, for a given capacity.

## PROPOSITION I.

79. *An ISOSCELES TRIANGLE has a greater area than any scalene triangle, of equal base and perimeter.*

If  $ABC$  be an isosceles triangle whose equal sides are  $AC$  and  $BC$ ; and if  $ABC'$  be a scalene triangle on the same base  $AB$ , and having  $AC' + BC' = AC + BC$ ; then the area of  $ABC$  is greater than that of  $ABC'$ .

Let perpendiculars be raised from each end of the base, extend  $AC$  to  $D$ , make  $C'D'$  equal to  $AC'$ , join  $BD$ , and draw  $CH$  and  $C'H'$  parallel to  $AB$ .



As the angle  $CAB = ABC$ , (Euc. 5, 1.) and  $ABD$  is a right angle,  $ABC + CBD = CAB + CDB = ABC + CDB$ . Therefore  $CBD = CDB$ , so that  $CD = CB$ ; and by construction,  $C'D' = AC'$ . The perpendiculars of the equal right angled triangles  $CHD$  and  $CHB$  are equal; therefore,  $BH = \frac{1}{2}BD$ . In the same manner,  $AH' = \frac{1}{2}AD'$ . The line  $AD = AC + BC = AC' + BC' = D'C' + BC'$ . But  $D'C' + BC' > BD'$ . (Euc. 20, 1.) Therefore,  $AD > BD'$ ;  $BD > AD'$ , (Euc. 47, 1.) and  $\frac{1}{2}BD > \frac{1}{2}AD'$ . But  $\frac{1}{2}BD$ , or  $BH$ , is the height of the isosceles triangle; (Art. 1.) and  $\frac{1}{2}AD'$  or  $AH'$ , the height of the scalene triangle; and the areas of two triangles which have the same base are as their heights. (Art. 8.) Therefore the area of  $ABC$  is greater than that of  $ABC'$ . Among all triangles, then, of a given perimeter, and upon a given base, the isosceles triangle is a *maximum*.

Cor. The isosceles triangle has a *less perimeter* than any scalene triangle of the same base and area. The triangle



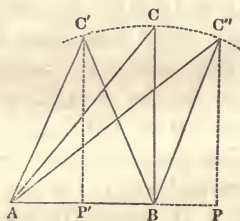
$ABC'$  being less than  $ABC$ , it is evident the perimeter of the former must be enlarged, to make its area equal to the area of the latter.

### PROPOSITION II.

80. *A triangle in which two given sides make a RIGHT ANGLE, has a greater area than any triangle in which the same sides make an oblique angle.*

If  $BC$ ,  $BC'$  and  $BC''$  be equal, and if  $BC$  be perpendicular to  $AB$ ; then the right angled triangle  $ABC$ , has a greater area than the acute angled triangle  $ABC'$ , or the oblique angled triangle  $ABC''$ .

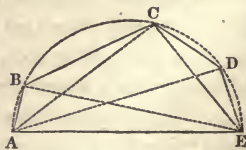
Let  $P'C'$  and  $PC''$  be perpendicular to  $AP$ . Then, as the three triangles have the same base  $AB$ , their areas are as their heights; that is, as the perpendiculars  $BC$ ,  $P'C'$ , and  $PC''$ . But  $BC$  is equal to  $BC'$ , and therefore greater than  $P'C'$ . (Euc. 47. 1.)  $BC$  is also equal to  $BC''$ , and therefore greater than  $PC''$ .



### PROPOSITION III.

81. *If all the sides EXCEPT ONE of a polygon be given, the area will be the greatest, when the given sides are so disposed that the figure may be INSCRIBED IN A SEMICIRCLE, of which the undetermined side is the diameter.*

If the sides  $AB$ ,  $BC$ ,  $CD$ ,  $DE$ , be given, and if their position be such that the area, included between these and another side whose length is not determined, is a *maximum*; the figure may



be inscribed in a semicircle, of which the undetermined side  $AE$  is the diameter.

Draw the lines  $AD$ ,  $AC$ ,  $EB$ ,  $EC$ . By varying the angle at  $D$ , the triangle  $ADE$  may be enlarged or diminished, without affecting the area of the other parts of the figure. The whole area, therefore, cannot be a *maximum*, unless this triangle be a *maximum*, while the sides  $AD$  and  $ED$  are given. But if the triangle  $ADE$  be a *maximum*, under these conditions, the angle  $ADE$  is a right angle; (Art. 80.) and therefore the point  $D$  is in the circumference of a circle, of which  $AE$  is the diameter. (Euc. 31, 3.) In the same manner it may be proved, that the angles  $ACE$  and  $ABE$  are right angles, and therefore that the points  $C$  and  $B$  are in the circumference of the same circle.

The term *polygon* is used in this section to include *triangles*, and *four-sided* figures, as well as other right-lined figures.

82. The area of a polygon, inscribed in a semicircle, in the manner stated above, will not be altered by varying the *order* of the given sides.

The sides  $AB$ ,  $BC$ ,  $CD$ ,  $DE$ , are the *chords* of so many arcs. The sum of these arcs, in whatever order they are arranged, will evidently be equal to the semicircumference. And the *segments* between the given sides and the arcs will be the same in whatever part of the circle they are situated. But the area of the polygon is equal to the area of the semicircle, diminished by the sum of these segments.

83. If a polygon, of which all the sides except one are given, be inscribed in a semicircle whose diameter is the undetermined side; a polygon having the same given sides, cannot be inscribed in any *other* semicircle which is either greater or less than this, and whose diameter is the undetermined side.

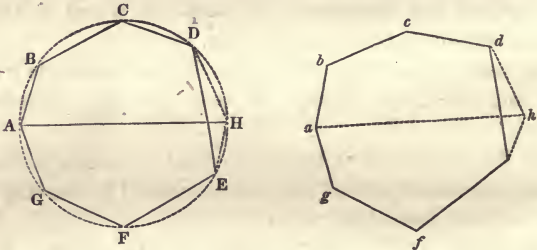
The given sides  $AB$ ,  $BC$ ,  $CD$ ,  $DE$ , are the chords of arcs whose sum is 180 degrees. But in a larger circle, each

would be the chord of a less number of degrees, and therefore the sum of the arcs would be less than  $180^\circ$ : and in a smaller circle, each would be the chord of a greater number of degrees, and the sum of the arcs would be greater than  $180^\circ$ .

#### PROPOSITION IV.

84. *A polygon INSCRIBED IN A CIRCLE has a greater area, than any polygon of equal perimeter, and the same number of sides, which cannot be inscribed in a circle.*

If in the circle ACHF, (Fig. 30.) there be inscribed a



polygon ABCDEFG; and if another polygon *abcdefgh* (Fig. 31.) be formed of sides which are the same in number and length, but which are so disposed, that the figure cannot be inscribed in a circle; the area of the former polygon is greater than that of the latter.

Draw the diameter AH, and the chords DH and EH. Upon *de* make the triangle *deh* equal and similar to DEH, and join *ah*. The line *ah* divides the figure *abcdhefg* into two parts, of which *one at least* cannot, by supposition, be inscribed in a semicircle of which the diameter is AH, nor in any other semicircle of which the diameter is the undetermined side. (Art. 83.) It is therefore less than the corresponding part of the figure ABCDHEFG. (Art. 81.) And the other part of *abcdhefg* is not greater than the correspond-

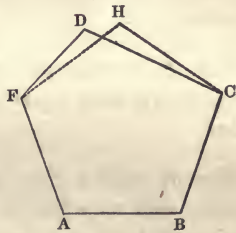
ing part of ABCDHEFG. Therefore, the whole figure ABCDHEFG is greater than the whole figure *abcdhefg*. If from these there be taken the equal triangles DEH and *deh*, there will remain the polygon ABCDEFG greater than the polygon *abcdefg*.

85. A polygon of which all the sides are given in number and length, cannot be inscribed in circles of different diameters. (Art. 83.) And the area of the polygon will not be altered by changing the *order* of the sides. (Art. 82.)

### PROPOSITION V.

86. *When a polygon has a greater area than any other, of the same number of sides, and of equal perimeter, the sides are EQUAL.*

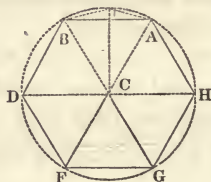
The polygon ABCDF (Fig. 29.) cannot be a *maximum*, among all polygons of the same number of sides, and of equal perimeters, unless it be equilateral. For if any two of the sides, as CD and FD, are unequal, let CH and FH be equal, and their sum the same as the sum of CD and FD. The isosceles triangle CHF is greater than the scalene triangle CDF (Art. 79.); and therefore the polygon ABCHF is greater than the polygon ABCDF; so that the latter is not a *maximum*.



### PROPOSITION VI.

87. *A REGULAR POLYGON has a greater area than any other polygon of equal perimeter, and of the same number of sides.*

For, by the preceding article, the polygon which is a *maximum* among others of equal perimeters, and the same number of sides, is *equilateral*, and by Art. 84, it may be *inscribed in a circle*. But if a polygon inscribed in a circle is equilateral, as ABDFGH, it is also *equiangular*. For the sides of the polygon are the bases of so many isosceles triangles, whose common vertex is the centre C. The angles at these bases are all equal; and two of them, as AHC and GHC, are equal to AHG one of the angles of the polygon. The polygon, then, being equiangular, as well as equilateral, is a *regular polygon*. (Art. 1. Def. 2.)



Thus an *equilateral triangle* has a greater area, than any other triangle of equal perimeter. And a *square* has a greater area than any other four-sided figure of equal perimeter.

Cor. A regular polygon has a *less perimeter* than any other polygon of equal area, and the same number of sides.

For if, with a given perimeter, the regular polygon is greater than one which is not regular; it is evident the perimeter of the former must be diminished, to make its area equal to that of the latter.

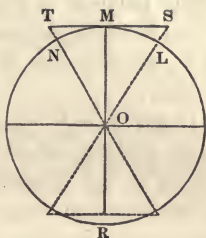
#### PROPOSITION VII.

88. *If a polygon be DESCRIBED ABOUT A CIRCLE, the areas of the two figures are as their perimeters.*

Let ST be one of the sides of a polygon, either regular or



not, which is described about the circle LNR. Join OS and OT, and to the point of contact M draw the radius OM, which will be perpendicular to ST. (Euc. 18, 3.) The triangle OST is equal to half the base ST multiplied into the radius OM. (Art. 8.) And if lines be



drawn, in the same manner, from the centre of the circle, to the extremities of the several sides of the circumscribed polygon, each of the triangles thus formed will be equal to half its base multiplied into the radius of the circle. Therefore the area of the whole polygon is equal to half its perimeter multiplied into the radius: and the area of the circle is equal to half its circumference multiplied into the radius. (Art 30.) So that the two areas are to each other as their perimeters.

Cor. 1. If different polygons are described about the same circle, their areas are to each other as their perimeters. For the area of each is equal to half its perimeter, multiplied into the radius of the inscribed circle.

Cor. 2. The *tangent* of an arc is always greater than the arc itself. The triangle OMT is to OMN, as MT to MN. But OMT is greater than OMN, because the former includes the latter. Therefore, the tangent MT is greater than the arc MN.

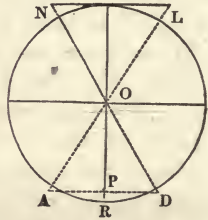
### PROPOSITION VIII.

89. *A CIRCLE has a greater area than any polygon of equal perimeter.*

If a circle and a regular polygon have the same centre, and equal perimeters; each of the sides of the polygon must fall partly *within* the circle. For the area of a *circum-*

*scribing* polygon is greater than the area of the circle, as the one includes the other : and therefore, by the preceding article, the *perimeter* of the former is greater than that of the latter.

Let AD then be one side of a regular polygon, whose perimeter is equal to the circumference of the circle RLN. As this falls partly within the circle, the perpendicular OP is less than the radius OR. But the area of the polygon is equal to half its perimeter multiplied into this perpendicular (Art. 15.); and the area of the circle is equal to half its circumference multiplied into the radius. (Art. 30.) The circle then is greater than the given regular polygon; and therefore greater than any other polygon of equal perimeter. (Art. 87.)



Cor. 1. A circle has a *less perimeter*, than any polygon of equal area.

Cor. 2. Among regular polygons of a given perimeter, that which has the *greatest number of sides*, has also the *greatest area*. For the greater the number of sides, the more nearly does the perimeter of the polygon approach to a coincidence with the circumference of a circle.

### PROPOSITION IX.

90. *A right PRISM whose bases are REGULAR POLYGONS, has a less surface than any other right prism of the same solidity, the same altitude, and the same number of sides.*

If the altitude of a prism is given, the area of the base is as the solidity (Art. 43.); and if the number of sides is

also given, the perimeter is a *minimum* when the **base** is a regular polygon. (Art. 87. Cor.) But the lateral surface is as the perimeter. (Art. 47.) Of two right prisms, then, which have the same altitude, the same solidity, and the same number of sides, that whose bases are regular polygons has the least *lateral* surface, while the areas of the ends are equal.

Cor. A right prism whose bases are regular polygons has a *greater solidity*, than any other right prism of the same surface, the same altitude, and the same number of sides.

#### PROPOSITION X.

91. *A right CYLINDER has a less surface than any right prism of the same altitude and solidity.*

For if the prism and cylinder have the same altitude and solidity, the areas of their bases are equal. (Art. 64.) But the *perimeter* of the cylinder is less, than that of the prism (Art. 89. Cor. 1.); and therefore its lateral surface is less, while the areas of the ends are equal.

Cor. A right cylinder has a *greater solidity*, than any right prism of the same altitude and surface.

#### PROPOSITION XI.

92. *A CUBE has a less surface than any other right parallelopiped of the same solidity.*

A parallelopiped is a prism, any one of whose faces may be considered a base. (Art. 41. Def. I and V.) If these are not all *squares*, let one which is not a square be taken for a base. The perimeter of this may be diminished, without altering its area (Art. 87. Cor.); and therefore the surface

of the solid may be diminished, without altering its altitude or solidity. (Art. 43, 47.) The same may be proved of each of the other faces which are not squares. The surface is therefore a *minimum*, when *all* the faces are squares, that is, when the solid is a *cube*.

Cor. A cube has a *greater solidity* than any other right parallelopiped of the same surface.

### PROPOSITION XII.

93. A CUBE has a *greater solidity* than any other right parallelopiped, the sum of whose length, breadth and depth, is equal to the sum of the corresponding dimensions of the cube.

The solidity is equal to the product of the length, breadth, and depth. If the length and breadth are unequal, the solidity may be increased, without altering the sum of the three dimensions. For the product of two factors whose sum is given, is the greatest when the factors are equal. (Euc. 27. 6.) In the same manner, if the breadth and depth are unequal, the solidity may be increased, without altering the sum of the three dimensions. Therefore, the solid cannot be a *maximum*, unless its length, breadth, and depth are equal.

### PROPOSITION XIII.

94. If a PRISM BE DESCRIBED ABOUT A CYLINDER, the capacities of the two solids are as their surfaces.

The capacities of the solids are as the *areas* of their bases, that is, as the *perimeters* of their bases. (Art. 88.) But the lateral surfaces are also as the perimeters of the bases. Therefore the *whole* surfaces are as the solidities.

Cor. The capacities of different prisms, described about the same right cylinder, are to each other as their surfaces.

## PROPOSITION XIV.

95. *A right cylinder whose height is equal to the diameter of its base has a greater solidity than any other right cylinder of equal surface.*

Let C be a right cylinder whose height is equal to the diameter of its base; and C' another right cylinder having the same surface, but a different altitude. If a square prism P be described about the former, it will be a *cube*. But a square prism P' described about the latter will not be a cube.

Then the surfaces of C and P are as their bases (Art. 47. and 88.); which are as the bases of C' and P', (Sup. Euc. 7, 1.); so that,

$$\text{surf } C : \text{surf } P :: \text{base } C : \text{base } P :: \text{base } C' : \text{base } P' :: \text{surf } C' : \text{surf } P'.$$

But the surface of C is, by supposition, equal to the surface of C'. Therefore, (Alg. 395.) the surface of P is equal to the surface of P'. And by the preceding article,

$$\text{solid } P : \text{solid } C :: \text{surf } P : \text{surf } C :: \text{surf } P' : \text{surf } C' :: \text{solid } P' : \text{solid } C'.$$

But the solidity of P is greater than that of P'. (Art. 92. Cor.) Therefore the solidity of C is greater than that of C'.

Schol. A right cylinder whose height is equal to the diameter of its base, is that which *circumscribes a sphere*. It is also called *Archimedes' cylinder*; as he discovered the ratio of a sphere to its circumscribing cylinder; and these are the figures which were put upon his tomb.

Cor. Archimedes' cylinder has a *less surface*, than any other right cylinder of the same capacity.



## PROPOSITION XV.

96. *If a SPHERE BE CIRCUMSCRIBED by a solid bounded by plane surfaces ; the capacities of the two solids are as their surfaces.*

If planes be supposed to be drawn from the centre of the sphere, to each of the edges of the circumscribing solid, they will divide it into as many pyramids as the solid has faces. The base of each pyramid will be one of the faces ; and the height will be the radius of the sphere. The capacity of the pyramid will be equal, therefore, to its base multiplied into  $\frac{1}{3}$  of the radius (Art. 48.); and the capacity of the whole circumscribing solid, must be equal to its whole surface multiplied into  $\frac{1}{3}$  of the radius. But the capacity of the sphere is also equal to its surface multiplied into  $\frac{1}{3}$  of its radius. (Art. 70.)

Cor. The capacities of different solids circumscribing the same sphere, are as their surfaces.

## PROPOSITION XVI.

97. *A SPHERE has a greater solidity than any regular polyedron of equal surface.*

If a sphere and a regular polyedron have the same centre, and equal surfaces ; each of the faces of the polyedron must fall partly *within* the sphere. For the solidity of a *circumscribing* solid is greater than the solidity of the sphere, as the one includes the other : and therefore, by the preceding article, the *surface* of the former is greater than that of the latter.

But if the faces of the polyedron fall partly within the sphere, their perpendicular distance from the centre must be less than the radius. And therefore, if the surface of the

polyedron be only equal to that of the sphere, its solidity must be less. For the solidity of the polyedron is equal to its surface multiplied into  $\frac{1}{3}$  of the distance from the centre. (Art. 59.) And the solidity of the sphere is equal to its surface multiplied into  $\frac{1}{3}$  of the radius.

Cor. A sphere has a *less surface* than any regular polyedron of the same capacity.

## APPENDIX

---

### GAUGING OF CASKS.

ART. 119. GAUGING is a practical art, which does not admit of being treated in a very scientific manner. Casks are not commonly constructed in exact conformity with any regular mathematical figure. By most writers on the subject, however, they are considered as nearly coinciding with one of the following forms :

- |      |                    |   |                         |
|------|--------------------|---|-------------------------|
| 1. } | The middle frustum | { | of a spheroid,          |
| 2. } |                    |   | of a parabolic spindle. |
| 3. } | The equal frustums | { | of a paraboloid,        |
| 4. } |                    |   | of a cone.              |

The *second* of these varieties agrees more nearly than any of the others, with the forms of casks, as they are commonly made. The first is too much curved, the third too little, and the fourth not at all, from the head to the bung.

120. Rules have already been given, for finding the capacity of each of the four varieties of casks. (Arts. 68, 110, 112, 118.) As the dimensions are taken in *inches*, these rules will give the contents in cubic inches. To abridge the computation, and adapt it to the particular measures used in gauging, the factor .7854 is divided by 282 or 231; and the quotient is used instead of .7854, for finding the capacity in ale gallons or wine gallons.

$$\text{Now } \frac{.7854}{282} = .002785, \text{ or } .0028 \text{ nearly ;}$$

$$\text{And } \frac{.7854}{231} = .0034$$

If then .0028 and .0034 be substituted for .7854, in the rules referred to above; the contents of the cask will be given in ale gallons and wine gallons. These numbers are to each other nearly as 9 to 11.

### PROBLEM I.

*To calculate the contents of a cask, in the form of a middle frustum of a SPHEROID.*

121. Add together the square of the head diameter, and twice the square of the bung diameter : multiply the sum by  $\frac{1}{3}$  of the length, and the product by .0028 for ale gallons, or by .0034 for wine gallons.

If  $D$  and  $d$  = the two diameters, and  $l$  = the length ;

The capacity in inches =  $(2D^2 + d^2) \times \frac{1}{3}l \times .7854$ . (Art. 110.)

And by substituting .0028 or .0034 for .7854, we have the capacity in ale gallons or wine gallons.

Ex. What is the capacity of a cask of the first form, whose length is 30 inches, its head diameter 18, and its bung diameter 24 ?

Ans. 41.3 ale gallons, or 50.2 wine gallons.

### PROBLEM II.

*To calculate the contents of a cask, in the form of the middle frustum of a PARABOLIC SPINDLE.*

122. Add together the square of the head diameter, and twice the square of the bung diameter, and from the sum

subtract  $\frac{2}{3}$  of the square of the difference of the diameters ; multiply the remainder by  $\frac{1}{3}$  of the length, and the product by .0028 for ale gallons, or .0034 for wine gallons.

The capacity in inches  $= (2D^2 + d^2 - \frac{2}{3}(D-d)^2) \times \frac{1}{3}l \times .7854$ . (Art. 118.)

Ex. What is the capacity of a cask of the second form, whose length is 30 inches, its head diameter 18, and its bung diameter 24 ?

Ans. 40.9 ale gallons, or 49.7 wine gallons.

### PROBLEM III.

*To calculate the contents of a cask, in the form of two equal frustums of a PARABOLOID.*

123. Add together the square of the head diameter, and the square of the bung diameter ; multiply the sum by half the length, and the product by .0028 for ale gallons, or .0034 for wine gallons.

The capacity in inches  $= (D^2 + d^2) \times \frac{1}{2}l \times .7854$ . (Art. 112 Cor.)

Ex. What is the capacity of a cask of the third form, whose dimensions are, as before, 30, 18, and 24 ?

Ans. 37.8 ale gallons, or 45.9 wine gallons.

### PROBLEM IV.

*To calculate the contents of a cask, in the form of two equal frustums of a CONE.*

124. Add together the square of the head diameter, the square of the bung diameter, and the product of the two diameters ; multiply the sum by  $\frac{1}{3}$  of the length, and the product by .0028 for ale gallons, or .0034 for wine gallons.

The capacity in inches  $= (D^2 + d^2 + Dd) \times \frac{1}{3}l \times .7854$ . (Art. 68.)



**Ex.** What is the capacity of a cask of the fourth form, whose length is 30, and its diameters 18 and 24?

Ans. 37.3 ale gallons, or 45.3 wine gallons.

125. The preceding rules, though correct in theory, are not very well adapted to practice, as they suppose the form of the cask to be *known*. The two following rules, taken from Hutton's Mensuration, may be used for casks of the usual forms. For the first, *three* dimensions are required, the length, the head diameter, and the bung diameter. It is evident that no allowance is made by this, for different degrees of curvature from the head to the bung. If the cask is more or less curved than usual, the following rule is to be preferred, for which *four* dimensions are required, the head and bung diameters, and a third diameter taken in the middle between the bung and the head. For the demonstration of these rules, see Hutton's Mensuration, Part V. Sec. 2. Ch. 5 and 7.

#### PROBLEM V.

*To calculate the contents of any common cask, from THREE dimensions.*

126. Add together

25 times the square of the head diameter,

39 times the square of the bung diameter, and

26 times the product of the two diameters ;

Multiply the sum by the length, divide the product by 90, and multiply the quotient by .0028 for ale gallons, or .0034 for wine gallons.

The capacity in inches =  $(39 D^2 + 25d^2 + 26Dd) \times \frac{l}{90} \times .7854$ .

**Ex.** What is the capacity of a cask whose length is 30 inches, the head diameter 18, and the bung diameter 24?

Ans. 39 ale gallons, or  $47\frac{1}{2}$  wine gallons.

## PROBLEM VI.

To calculate the contents of a cask from FOUR dimensions, the length, the head and bung diameters, and a diameter taken in the middle between the head and the bung.

127. Add together the squares of the head diameter, of the bung diameter, and of double the middle diameter; multiply the sum by  $\frac{1}{6}$  of the length, and the product by .0028 for ale gallons, or .0034 for wine gallons.

If  $D$  = the bung diameter,  $d$  = the head diameter,  $m$  = the middle diameter, and  $l$  = the length;

The capacity in inches =  $(D^2 + d^2 + 2m^2) \times \frac{1}{6}l \times .7854$ .

Ex. What is the capacity of a cask, whose length is 30 inches, the head diameter 18, the bung diameter 24, and the middle diameter  $22\frac{1}{2}$ ?

Ans. 41 ale gallons, or  $49\frac{2}{3}$  wine gallons.

128. In making the calculations in gauging, according to the preceding rules, the multiplications and divisions are frequently performed by means of a *Sliding Rule*, on which are placed a number of logarithmic lines, similar to those on Gunter's Scale. See Trigonometry, Sec. VI., and Note C. p. 149.

Another instrument commonly used in gauging is the *Diagonal Rod*. By this, the capacity of a cask is very expeditiously found, from a single dimension, the distance from the bung to the intersection of the opposite stave with the head; but this process is not considered sufficiently accurate for casks of a capacity exceeding 40 gallons. The measure is taken by extending the rod through the cask, from the bung to the most distant part of the head. The number of gallons corresponding to the length of the line thus found, is marked on the rod. The *logarithmic* lines on the gauging

rod are to be used in the same manner, as on the sliding rule.

#### ULLAGE OF CASKS.

129. When a cask is *partly* filled, the whole capacity is divided, by the surface of the liquor, into two portions; the *least* of which, whether full or empty, is called the *ullage*. In finding the ullage, the cask is supposed to be in one of two positions; either *standing*, with its axis perpendicular to the horizon; or *lying*, with its axis parallel to the horizon. The rules for ullage which are *exact*, particularly those for lying casks, are too complicated for common use. The following are considered as sufficiently near approximations. See Hutton's Mensuration.

#### PROBLEM VII.

*To calculate the ullage of a STANDING cask.*

130. Add together the squares of the diameter at the surface of the liquor, of the diameter of the nearest end, and of double the diameter in the middle between the other two; multiply the sum by  $\frac{1}{6}$  of the distance between the surface and the nearest end, and the product by .0028 for ale gallons, or .0034 for wine gallons.

If  $D$  = the diameter of the surface of the liquor,

$d$  = the diameter of the nearest end,

$m$  = the middle diameter, and

$l$  = the distance between the surface and the nearest end;

The ullage in inches =  $(D^2 + d^2 + \overline{2m^2}) \times \frac{1}{6}l \times .7854$ .

Ex. If the diameter at the surface of the liquor, in a standing cask, be 32 inches, the diameter of the nearest end 24, the middle diameter 29, and the distance between the sur-

face of the liquor and the nearest end 12 ; what is the ullage?  
 Ans.  $27\frac{4}{5}$  ale gallons, or  $33\frac{3}{4}$  wine gallons.

### PROBLEM VIII.

*To calculate the ullage of a LYING cask.*

131. Divide the distance from the bung to the surface of the liquor, by the whole bung diameter, find the quotient in the column of heights or versed sines in a table of circular segments, take out the corresponding segment, and multiply it by the whole capacity of the cask, and the product by  $1\frac{1}{2}$  for the part which is empty.

If the cask be not half full, divide the depth of the liquor by the whole bung diameter, take out the segment, multiply, &c., for the contents of the part which is full.

Ex. If the whole capacity of a lying cask be 41 ale gallons, or  $49\frac{2}{3}$  wine gallons, the bung diameter 24 inches and the distance from the bung to the surface of the liquor 6 inches ; what is the ullage ?

Ans.  $7\frac{3}{4}$  ale gallons, or  $9\frac{1}{2}$  wine gallons.

## N O T E S .

~~~~~

### NOTE A. p. 39.

THE term *solidity* is used here in the customary sense, to express the magnitude of any geometrical quantity of three dimensions, length, breadth, and thickness; whether it be a solid body, or a fluid, or even a portion of empty space. This use of the word, however, is not altogether free from objection. The same term is applied to one of the general properties of matter; and also to that peculiar quality by which certain substances are distinguished from *fluids*. There seems to be an impropriety in speaking of the *solidity* of a body of *water*, or of a vessel which is *empty*. Some writers have therefore substituted the word *volume* for *solidity*. But the latter term, if it be properly defined, may be retained without danger of leading to mistake.

### NOTE B. p. 76.

The following simple rule for the solidity of round timber, or of any cylinder, is nearly exact:

*Multiply the length into twice the square of  $\frac{1}{5}$  of the circumference.*

If  $C$  = the circumference of a cylinder;

$$\text{The area of the base} = \frac{C^2}{4\pi} = \frac{C^2}{12.566} \text{ But } 2\left(\frac{C}{5}\right)^2 = \frac{C^2}{12.5}$$

It is common to measure *hewn timber*, by multiplying the length into the square of the *quarter-girt*. This gives ex-



actly the solidity of a parallelopiped, if the ends are *squares*. But if the ends are parallelograms, the area of each is *less* than the square of the quarter-girt. (Euc. 27. 6.)

Timber which is *tapering* may be exactly measured by the rule for the frustum of a pyramid or cone (Art. 50, 68.); or, if the ends are not similar figures, by the rule for a prismoid. (Art. 55.) But for common purposes, it will be sufficient to multiply the length by the area of a section *in the middle* between the two ends.

A  
**T A B L E**  
 OF  
**LOGARITHMS OF NUMBERS**  
 FROM  
**1 TO 10,000.**

---

| N. | Log.     | N. | Log.     | N. | Log.     | N.  | Log.     |
|----|----------|----|----------|----|----------|-----|----------|
| 1  | 0.000000 | 26 | 1.414973 | 51 | 1.707570 | 76  | 1.880814 |
| 2  | 0.301030 | 27 | 1.431364 | 52 | 1.716003 | 77  | 1.886491 |
| 3  | 0.477121 | 28 | 1.447158 | 53 | 1.724276 | 78  | 1.892095 |
| 4  | 0.602060 | 29 | 1.462398 | 54 | 1.732394 | 79  | 1.897627 |
| 5  | 0.698970 | 30 | 1.477121 | 55 | 1.740363 | 80  | 1.903090 |
| 6  | 0.778151 | 31 | 1.491362 | 56 | 1.748188 | 81  | 1.908485 |
| 7  | 0.845098 | 32 | 1.505150 | 57 | 1.755875 | 82  | 1.913814 |
| 8  | 0.903090 | 33 | 1.518514 | 58 | 1.763428 | 83  | 1.919078 |
| 9  | 0.954243 | 34 | 1.531479 | 59 | 1.770852 | 84  | 1.924279 |
| 10 | 1.000000 | 35 | 1.544068 | 60 | 1.778151 | 85  | 1.929419 |
| 11 | 1.041393 | 36 | 1.556303 | 61 | 1.785330 | 86  | 1.934498 |
| 12 | 1.079181 | 37 | 1.568202 | 62 | 1.792392 | 87  | 1.939519 |
| 13 | 1.113943 | 38 | 1.579784 | 63 | 1.799341 | 88  | 1.944483 |
| 14 | 1.146128 | 39 | 1.591065 | 64 | 1.806180 | 89  | 1.949390 |
| 15 | 1.176091 | 40 | 1.602060 | 65 | 1.812913 | 90  | 1.954243 |
| 16 | 1.204120 | 41 | 1.612784 | 66 | 1.819544 | 91  | 1.959041 |
| 17 | 1.230449 | 42 | 1.623249 | 67 | 1.826075 | 92  | 1.963788 |
| 18 | 1.255273 | 43 | 1.633468 | 68 | 1.832509 | 93  | 1.968483 |
| 19 | 1.278754 | 44 | 1.643453 | 69 | 1.838849 | 94  | 1.973128 |
| 20 | 1.301030 | 45 | 1.653213 | 70 | 1.845098 | 95  | 1.977724 |
| 21 | 1.322219 | 46 | 1.662758 | 71 | 1.851258 | 96  | 1.982271 |
| 22 | 1.342423 | 47 | 1.672098 | 72 | 1.857333 | 97  | 1.986772 |
| 23 | 1.361728 | 48 | 1.681241 | 73 | 1.863323 | 98  | 1.991226 |
| 24 | 1.380211 | 49 | 1.690196 | 74 | 1.869232 | 99  | 1.995635 |
| 25 | 1.397940 | 50 | 1.698970 | 75 | 1.875061 | 100 | 2.000000 |

N. B. In the following table, in the last nine columns of each page, where the first or leading figures change from 9's to 0's, points or dots are introduced instead of the 0's through the rest of the line, to catch the eye, and to indicate that from thence the annexed first two figures of the Logarithm in the second column stand in the next lower line.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D.  |
|-----|--------|------|------|------|------|------|------|------|------|------|-----|
| 100 | 000000 | 0434 | 0868 | 1301 | 1734 | 2166 | 2598 | 3029 | 3461 | 3891 | 432 |
| 101 | 4321   | 4751 | 5181 | 5609 | 6038 | 6466 | 6894 | 7321 | 7748 | 8174 | 428 |
| 102 | 8600   | 9026 | 9451 | 9876 | .300 | .724 | 1147 | 1570 | 1993 | 2415 | 424 |
| 103 | 012837 | 3259 | 3680 | 4100 | 4521 | 4940 | 5360 | 5779 | 6197 | 6616 | 419 |
| 104 | 7033   | 7451 | 7868 | 8284 | 8700 | 9116 | 9532 | 9947 | .361 | .775 | 416 |
| 105 | 021189 | 1603 | 2016 | 2428 | 2841 | 3252 | 3664 | 4075 | 4486 | 4896 | 412 |
| 106 | 5306   | 5715 | 6125 | 6533 | 6942 | 7350 | 7757 | 8164 | 8571 | 8978 | 408 |
| 107 | 9384   | 9789 | .195 | .600 | 1004 | 1408 | 1812 | 2216 | 2619 | 3021 | 404 |
| 108 | 033424 | 3826 | 4227 | 4628 | 5029 | 5430 | 5830 | 6230 | 6629 | 7028 | 400 |
| 109 | 7426   | 7825 | 8223 | 8620 | 9017 | 9414 | 9811 | .207 | .602 | .998 | 396 |
| 110 | 041393 | 1787 | 2182 | 2576 | 2969 | 3362 | 3755 | 4148 | 4540 | 4932 | 393 |
| 111 | 5323   | 5714 | 6105 | 6495 | 6885 | 7275 | 7664 | 8053 | 8442 | 8830 | 389 |
| 112 | 9218   | 9606 | 9993 | .380 | .766 | 1153 | 1538 | 1924 | 2309 | 2694 | 386 |
| 113 | 053078 | 3463 | 3846 | 4230 | 4613 | 4996 | 5378 | 5760 | 6142 | 6524 | 382 |
| 114 | 6905   | 7286 | 7666 | 8046 | 8426 | 8805 | 9185 | 9563 | 9942 | .320 | 379 |
| 115 | 060698 | 1075 | 1452 | 1829 | 2206 | 2582 | 2958 | 3333 | 3709 | 4083 | 376 |
| 116 | 4458   | 4832 | 5206 | 5580 | 5953 | 6326 | 6699 | 7071 | 7443 | 7815 | 372 |
| 117 | 8186   | 8557 | 8928 | 9298 | 9668 | .38  | .407 | .776 | 1145 | 1514 | 369 |
| 118 | 071882 | 2250 | 2617 | 2985 | 3352 | 3718 | 4085 | 4451 | 4816 | 5182 | 366 |
| 119 | 5547   | 5912 | 6276 | 6640 | 7004 | 7368 | 7731 | 8094 | 8457 | 8819 | 363 |
| 120 | 079181 | 9543 | 9904 | .266 | .626 | .987 | 1347 | 1707 | 2067 | 2426 | 360 |
| 121 | 082785 | 3144 | 3503 | 3861 | 4219 | 4576 | 4934 | 5291 | 5647 | 6004 | 357 |
| 122 | 6360   | 6716 | 7071 | 7426 | 7781 | 8136 | 8490 | 8845 | 9198 | 9552 | 355 |
| 123 | 9905   | .258 | .611 | .963 | 1315 | 1667 | 2018 | 2370 | 2721 | 3071 | 351 |
| 124 | 093422 | 3772 | 4122 | 4471 | 4820 | 5169 | 5518 | 5866 | 6215 | 6562 | 349 |
| 125 | 6910   | 7257 | 7604 | 7951 | 8298 | 8644 | 8990 | 9335 | 9681 | .26  | 346 |
| 126 | 100371 | 0715 | 1059 | 1403 | 1747 | 2091 | 2434 | 2777 | 3119 | 3462 | 343 |
| 127 | 3804   | 4146 | 4487 | 4828 | 5169 | 5510 | 5851 | 6191 | 6531 | 6871 | 340 |
| 128 | 7210   | 7549 | 7888 | 8227 | 8565 | 8903 | 9241 | 9579 | 9916 | .253 | 338 |
| 129 | 110590 | 0926 | 1263 | 1599 | 1934 | 2270 | 2605 | 2940 | 3275 | 3609 | 335 |
| 130 | 113943 | 4277 | 4611 | 4944 | 5278 | 5611 | 5943 | 6276 | 6608 | 6940 | 333 |
| 131 | 7271   | 7603 | 7934 | 8265 | 8595 | 8926 | 9256 | 9586 | 9915 | .245 | 330 |
| 132 | 120574 | 0903 | 1231 | 1560 | 1888 | 2216 | 2544 | 2871 | 3198 | 3525 | 328 |
| 133 | 3852   | 4178 | 4504 | 4830 | 5156 | 5481 | 5806 | 6131 | 6456 | 6781 | 325 |
| 134 | 7105   | 7429 | 7753 | 8076 | 8399 | 8722 | 9045 | 9368 | 9690 | .12  | 323 |
| 135 | 130334 | 0655 | 0977 | 1298 | 1619 | 1939 | 2260 | 2580 | 2900 | 3219 | 321 |
| 136 | 3539   | 3858 | 4177 | 4496 | 4814 | 5133 | 5451 | 5769 | 6086 | 6403 | 318 |
| 137 | 6721   | 7037 | 7354 | 7671 | 7987 | 8303 | 8618 | 8934 | 9249 | 9564 | 315 |
| 138 | 9879   | .194 | .508 | .822 | 1136 | 1450 | 1763 | 2076 | 2389 | 2702 | 314 |
| 139 | 143015 | 3327 | 3639 | 3951 | 4263 | 4574 | 4885 | 5196 | 5507 | 5818 | 311 |
| 140 | 146128 | 6438 | 6748 | 7058 | 7367 | 7676 | 7985 | 8294 | 8603 | 8911 | 309 |
| 141 | 9219   | 9527 | 9835 | .142 | .449 | .756 | 1063 | 1370 | 1676 | 1982 | 307 |
| 142 | 152288 | 2594 | 2900 | 3205 | 3510 | 3815 | 4120 | 4424 | 4728 | 5032 | 305 |
| 143 | 5336   | 5640 | 5943 | 6246 | 6549 | 6852 | 7154 | 7457 | 7759 | 8061 | 303 |
| 144 | 8362   | 8664 | 8965 | 9266 | 9567 | 9868 | .168 | .469 | .769 | 1068 | 301 |
| 145 | 161368 | 1667 | 1967 | 2266 | 2564 | 2863 | 3161 | 3460 | 3758 | 4055 | 299 |
| 146 | 4353   | 4650 | 4947 | 5244 | 5541 | 5838 | 6134 | 6430 | 6726 | 7022 | 297 |
| 147 | 7317   | 7613 | 7908 | 8203 | 8497 | 8792 | 9086 | 9380 | 9674 | 9968 | 295 |
| 148 | 170262 | 0555 | 0848 | 1141 | 1434 | 1726 | 2019 | 2311 | 2603 | 2895 | 293 |
| 149 | 3186   | 3478 | 3769 | 4060 | 4351 | 4641 | 4932 | 5222 | 5512 | 5802 | 291 |
| 150 | 176091 | 6381 | 6670 | 6959 | 7248 | 7536 | 7825 | 8113 | 8401 | 8689 | 289 |
| 151 | 8977   | 9264 | 9552 | 9839 | .126 | .413 | .699 | .985 | 1272 | 1558 | 287 |
| 152 | 181844 | 2129 | 2415 | 2700 | 2985 | 3270 | 3555 | 3839 | 4123 | 4407 | 285 |
| 153 | 4691   | 4975 | 5259 | 5542 | 5825 | 6108 | 6391 | 6674 | 6956 | 7239 | 283 |
| 154 | 7521   | 7803 | 8084 | 8366 | 8647 | 8928 | 9209 | 9490 | 9771 | .51  | 281 |
| 155 | 190332 | 0612 | 0892 | 1171 | 1451 | 1730 | 2010 | 2289 | 2567 | 2846 | 279 |
| 156 | 3125   | 3403 | 3681 | 3959 | 4237 | 4514 | 4792 | 5069 | 5346 | 5623 | 278 |
| 157 | 5899   | 6176 | 6453 | 6729 | 7005 | 7281 | 7556 | 7832 | 8107 | 8382 | 276 |
| 158 | 8657   | 8932 | 9206 | 9481 | 9755 | .29  | .303 | .577 | .850 | 1124 | 274 |
| 159 | 201397 | 1670 | 1943 | 2216 | 2488 | 2761 | 3033 | 3305 | 3577 | 3848 | 272 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D.  |

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D.  |
|-----|--------|------|------|------|------|------|------|------|------|------|-----|
| 160 | 204120 | 4391 | 4663 | 4934 | 5204 | 5475 | 5746 | 6016 | 6286 | 6556 | 271 |
| 161 | 6826   | 7096 | 7365 | 7634 | 7904 | 8173 | 8441 | 8710 | 8979 | 9247 | 269 |
| 162 | 9515   | 9783 | .51  | .319 | .586 | .853 | 1121 | 1388 | 1654 | 1921 | 267 |
| 163 | 212188 | 2454 | 2720 | 2986 | 3252 | 3518 | 3783 | 4049 | 4314 | 4579 | 266 |
| 164 | 4844   | 5109 | 5373 | 5638 | 5902 | 6166 | 6430 | 6694 | 6957 | 7221 | 264 |
| 165 | 7484   | 7747 | 8010 | 8273 | 8536 | 8798 | 9060 | 9323 | 9585 | 9846 | 262 |
| 166 | 220108 | 0370 | 0631 | 0892 | 1153 | 1414 | 1675 | 1936 | 2196 | 2456 | 261 |
| 167 | 2716   | 2976 | 3236 | 3496 | 3755 | 4015 | 4274 | 4533 | 4792 | 5051 | 259 |
| 168 | 5309   | 5568 | 5826 | 6084 | 6342 | 6600 | 6858 | 7115 | 7372 | 7630 | 258 |
| 169 | 7887   | 8144 | 8400 | 8657 | 8913 | 9170 | 9426 | 9682 | 9938 | .193 | 256 |
| 170 | 230449 | 0704 | 0960 | 1215 | 1470 | 1724 | 1979 | 2234 | 2488 | 2742 | 254 |
| 171 | 2996   | 3250 | 3504 | 3757 | 4011 | 4264 | 4517 | 4770 | 5023 | 5276 | 253 |
| 172 | 5528   | 5781 | 6033 | 6285 | 6537 | 6789 | 7041 | 7292 | 7544 | 7795 | 252 |
| 173 | 8046   | 8297 | 8548 | 8799 | 9049 | 9299 | 9550 | 9800 | .50  | .300 | 250 |
| 174 | 240549 | 0799 | 1048 | 1297 | 1546 | 1795 | 2044 | 2293 | 2541 | 2790 | 249 |
| 175 | 3038   | 3286 | 3534 | 3782 | 4030 | 4277 | 4525 | 4772 | 5019 | 5266 | 248 |
| 176 | 5513   | 5759 | 6006 | 6252 | 6499 | 6745 | 6991 | 7237 | 7482 | 7728 | 246 |
| 177 | 7973   | 8219 | 8464 | 8709 | 8954 | 9198 | 9443 | 9687 | 9932 | .176 | 245 |
| 178 | 250420 | 0664 | 0908 | 1151 | 1395 | 1638 | 1881 | 2125 | 2368 | 2610 | 243 |
| 179 | 2853   | 3096 | 3338 | 3580 | 3822 | 4064 | 4306 | 4548 | 4790 | 5031 | 242 |
| 180 | 255273 | 5514 | 5755 | 5996 | 6237 | 6477 | 6718 | 6958 | 7198 | 7439 | 241 |
| 181 | 7679   | 7918 | 8158 | 8398 | 8637 | 8877 | 9116 | 9355 | 9594 | 9833 | 239 |
| 182 | 260071 | 0310 | 0548 | 0787 | 1025 | 1263 | 1501 | 1739 | 1976 | 2214 | 238 |
| 183 | 2451   | 2688 | 2925 | 3162 | 3399 | 3636 | 3873 | 4109 | 4346 | 4582 | 237 |
| 184 | 4818   | 5054 | 5290 | 5525 | 5761 | 5996 | 6232 | 6467 | 6702 | 6937 | 235 |
| 185 | 7517   | 7406 | 7641 | 7875 | 8110 | 8344 | 8578 | 8812 | 9046 | 9279 | 234 |
| 186 | 9513   | 9746 | 9980 | .213 | .446 | .679 | .912 | 1144 | 1377 | 1609 | 233 |
| 187 | 271842 | 2074 | 2306 | 2538 | 2770 | 3001 | 3233 | 3464 | 3696 | 3927 | 232 |
| 188 | 4158   | 4389 | 4620 | 4850 | 5081 | 5311 | 5542 | 5772 | 6002 | 6232 | 230 |
| 189 | 6462   | 6692 | 6921 | 7151 | 7380 | 7609 | 7838 | 8067 | 8296 | 8525 | 229 |
| 190 | 278754 | 8982 | 9211 | 9439 | 9667 | 9895 | .123 | .351 | .578 | .806 | 228 |
| 191 | 281033 | 1261 | 1488 | 1715 | 1942 | 2169 | 2396 | 2622 | 2849 | 3075 | 227 |
| 192 | 3301   | 3527 | 3753 | 3979 | 4205 | 4431 | 4656 | 4882 | 5107 | 5332 | 226 |
| 193 | 5557   | 5782 | 6007 | 6232 | 6456 | 6681 | 6905 | 7130 | 7354 | 7578 | 225 |
| 194 | 7802   | 8026 | 8249 | 8473 | 8696 | 8920 | 9143 | 9366 | 9589 | 9812 | 223 |
| 195 | 290035 | 0257 | 0480 | 0702 | 0925 | 1147 | 1369 | 1591 | 1813 | 2034 | 222 |
| 196 | 2256   | 2478 | 2699 | 2920 | 3141 | 3363 | 3584 | 3804 | 4025 | 4246 | 221 |
| 197 | 4466   | 4687 | 4907 | 5127 | 5347 | 5567 | 5787 | 6007 | 6226 | 6446 | 220 |
| 198 | 6665   | 6884 | 7104 | 7323 | 7542 | 7761 | 7979 | 8198 | 8416 | 8635 | 219 |
| 199 | 8853   | 9071 | 9289 | 9507 | 9725 | 9943 | .161 | .378 | .595 | .813 | 218 |
| 200 | 301030 | 1247 | 1464 | 1681 | 1898 | 2114 | 2331 | 2547 | 2764 | 2980 | 217 |
| 201 | 3196   | 3412 | 3628 | 3844 | 4059 | 4275 | 4491 | 4706 | 4921 | 5136 | 216 |
| 202 | 5351   | 5566 | 5781 | 5996 | 6211 | 6425 | 6639 | 6854 | 7068 | 7282 | 215 |
| 203 | 7496   | 7710 | 7924 | 8137 | 8351 | 8564 | 8778 | 8991 | 9204 | 9417 | 213 |
| 204 | 9630   | 9843 | .56  | .268 | .481 | .693 | .906 | 1118 | 1330 | 1542 | 212 |
| 205 | 311754 | 1966 | 2177 | 2389 | 2600 | 2812 | 3023 | 3234 | 3445 | 3656 | 211 |
| 206 | 3867   | 4078 | 4289 | 4499 | 4710 | 4920 | 5130 | 5340 | 5551 | 5760 | 210 |
| 207 | 5970   | 6180 | 6390 | 6599 | 6809 | 7018 | 7227 | 7436 | 7646 | 7854 | 209 |
| 208 | 8063   | 8272 | 8481 | 8689 | 8898 | 9100 | 9314 | 9522 | 9730 | 9938 | 208 |
| 209 | 320146 | 0354 | 0562 | 0769 | 0977 | 1184 | 1391 | 1598 | 1805 | 2012 | 207 |
| 210 | 322210 | 2426 | 2633 | 2839 | 3046 | 3252 | 3458 | 3665 | 3871 | 4077 | 206 |
| 211 | 4282   | 4488 | 4694 | 4899 | 5105 | 5310 | 5516 | 5721 | 5926 | 6131 | 205 |
| 212 | 6336   | 6541 | 6745 | 6950 | 7155 | 7359 | 7563 | 7767 | 7972 | 8176 | 204 |
| 213 | 8380   | 8583 | 8787 | 8991 | 9194 | 9398 | 9601 | 9805 | .8   | .211 | 203 |
| 214 | 330414 | 0617 | 0819 | 1022 | 1225 | 1427 | 1630 | 1832 | 2034 | 2236 | 202 |
| 215 | 2438   | 2640 | 2842 | 3044 | 3246 | 3447 | 3649 | 3850 | 4051 | 4253 | 202 |
| 216 | 4454   | 4655 | 4856 | 5057 | 5257 | 5458 | 5658 | 5859 | 6059 | 6260 | 201 |
| 217 | 6460   | 6660 | 6860 | 7060 | 7260 | 7459 | 7659 | 7858 | 8058 | 8257 | 200 |
| 218 | 8456   | 8656 | 8855 | 9054 | 9253 | 9451 | 9650 | 9849 | .47  | .246 | 199 |
| 219 | 340444 | 0642 | 0841 | 1039 | 1237 | 1435 | 1632 | 1830 | 2028 | 2225 | 198 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D.  |



| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D.  |
|-----|--------|------|------|------|------|------|------|------|------|------|-----|
| 220 | 342423 | 2620 | 2817 | 3014 | 3212 | 3409 | 3606 | 3802 | 3999 | 4196 | 197 |
| 221 | 4392   | 4589 | 4785 | 4981 | 5178 | 5374 | 5570 | 5766 | 5962 | 6157 | 196 |
| 222 | 6353   | 6549 | 6744 | 6939 | 7135 | 7330 | 7525 | 7720 | 7915 | 8110 | 195 |
| 223 | 8305   | 8500 | 8694 | 8889 | 9083 | 9278 | 9472 | 9666 | 9860 | .54  | 194 |
| 224 | 350248 | 0442 | 0636 | 0829 | 1023 | 1216 | 1410 | 1603 | 1796 | 1989 | 193 |
| 225 | 2183   | 2375 | 2568 | 2761 | 2954 | 3147 | 3339 | 3532 | 3724 | 3916 | 193 |
| 226 | 4108   | 4301 | 4493 | 4685 | 4876 | 5068 | 5260 | 5452 | 5643 | 5834 | 192 |
| 227 | 6026   | 6217 | 6408 | 6599 | 6790 | 6981 | 7172 | 7363 | 7554 | 7744 | 191 |
| 228 | 7935   | 8125 | 8316 | 8506 | 8696 | 8886 | 9076 | 9266 | 9456 | 9646 | 190 |
| 229 | 9835   | .25  | .215 | .404 | .593 | .783 | .972 | 1161 | 1350 | 1539 | 189 |
| 230 | 361728 | 1917 | 2105 | 2294 | 2482 | 2671 | 2859 | 3048 | 3236 | 3424 | 188 |
| 231 | 3612   | 3800 | 3988 | 4176 | 4363 | 4551 | 4739 | 4926 | 5113 | 5301 | 188 |
| 232 | 5488   | 5675 | 5862 | 6049 | 6236 | 6423 | 6610 | 6796 | 6983 | 7169 | 187 |
| 233 | 7356   | 7542 | 7729 | 7915 | 8101 | 8287 | 8473 | 8659 | 8845 | 9030 | 186 |
| 234 | 9216   | 9401 | 9587 | 9772 | 9958 | .143 | .328 | .513 | .698 | .883 | 185 |
| 235 | 371068 | 1253 | 1437 | 1622 | 1806 | 1991 | 2175 | 2360 | 2544 | 2728 | 184 |
| 236 | 2912   | 3096 | 3280 | 3464 | 3647 | 3831 | 4015 | 4198 | 4382 | 4565 | 184 |
| 237 | 4748   | 4932 | 5115 | 5298 | 5481 | 5664 | 5846 | 6029 | 6212 | 6394 | 183 |
| 238 | 6577   | 6759 | 6942 | 7124 | 7306 | 7488 | 7670 | 7852 | 8034 | 8216 | 182 |
| 239 | 8398   | 8580 | 8761 | 8943 | 9124 | 9306 | 9487 | 9668 | 9849 | .30  | 181 |
| 240 | 380211 | 0392 | 0573 | 0754 | 0934 | 1115 | 1296 | 1476 | 1656 | 1837 | 181 |
| 241 | 2017   | 2197 | 2377 | 2557 | 2737 | 2917 | 3097 | 3277 | 3456 | 3636 | 180 |
| 242 | 3815   | 3995 | 4174 | 4353 | 4533 | 4712 | 4891 | 5070 | 5249 | 5428 | 179 |
| 243 | 5606   | 5785 | 5964 | 6142 | 6321 | 6499 | 6677 | 6856 | 7034 | 7212 | 178 |
| 244 | 7390   | 7568 | 7746 | 7923 | 8101 | 8279 | 8456 | 8634 | 8811 | 8989 | 178 |
| 245 | 9166   | 9343 | 9520 | 9698 | 9875 | .51  | .228 | .405 | .582 | .759 | 177 |
| 246 | 390935 | 1112 | 1288 | 1464 | 1641 | 1817 | 1993 | 2169 | 2345 | 2521 | 176 |
| 247 | 2697   | 2873 | 3048 | 3224 | 3400 | 3575 | 3751 | 3926 | 4101 | 4277 | 176 |
| 248 | 4452   | 4627 | 4802 | 4977 | 5152 | 5326 | 5501 | 5676 | 5850 | 6025 | 175 |
| 249 | 6199   | 6374 | 6548 | 6722 | 6896 | 7071 | 7245 | 7419 | 7592 | 7766 | 174 |
| 250 | 397940 | 8114 | 8287 | 8461 | 8634 | 8808 | 8981 | 9154 | 9328 | 9501 | 173 |
| 251 | 9674   | 9847 | .20  | .192 | .365 | .538 | .711 | .883 | 1056 | 1228 | 173 |
| 252 | 401401 | 1573 | 1745 | 1917 | 2089 | 2261 | 2433 | 2605 | 2777 | 2949 | 172 |
| 253 | 3121   | 3292 | 3464 | 3635 | 3807 | 3978 | 4149 | 4320 | 4492 | 4663 | 171 |
| 254 | 4834   | 5005 | 5176 | 5346 | 5517 | 5688 | 5858 | 6029 | 6199 | 6370 | 171 |
| 255 | 6540   | 6710 | 6881 | 7051 | 7221 | 7391 | 7561 | 7731 | 7901 | 8070 | 170 |
| 256 | 8240   | 8410 | 8579 | 8749 | 8918 | 9087 | 9257 | 9426 | 9595 | 9764 | 169 |
| 257 | 9933   | .102 | .271 | .440 | .609 | .777 | .946 | 1114 | 1283 | 1451 | 169 |
| 258 | 411620 | 1788 | 1956 | 2124 | 2293 | 2461 | 2629 | 2796 | 2964 | 3132 | 168 |
| 259 | 3300   | 3467 | 3635 | 3803 | 3970 | 4137 | 4305 | 4472 | 4639 | 4806 | 167 |
| 260 | 414973 | 5140 | 5307 | 5474 | 5641 | 5808 | 5974 | 6141 | 6308 | 6474 | 167 |
| 261 | 6641   | 6807 | 6973 | 7139 | 7306 | 7472 | 7638 | 7804 | 7970 | 8135 | 166 |
| 262 | 8301   | 8467 | 8633 | 8798 | 8964 | 9129 | 9295 | 9460 | 9625 | 9791 | 165 |
| 263 | 9956   | .121 | .286 | .451 | .616 | .781 | .945 | 1110 | 1275 | 1439 | 165 |
| 264 | 421604 | 1768 | 1933 | 2097 | 2261 | 2426 | 2590 | 2754 | 2918 | 3082 | 164 |
| 265 | 3246   | 3410 | 3574 | 3737 | 3901 | 4065 | 4228 | 4392 | 4555 | 4718 | 164 |
| 266 | 4882   | 5045 | 5208 | 5371 | 5534 | 5697 | 5860 | 6023 | 6186 | 6349 | 163 |
| 267 | 6511   | 6674 | 6836 | 6999 | 7161 | 7324 | 7486 | 7648 | 7811 | 7973 | 162 |
| 268 | 8135   | 8297 | 8459 | 8621 | 8783 | 8944 | 9106 | 9268 | 9429 | 9591 | 162 |
| 269 | 9752   | 9914 | .75  | .236 | .398 | .559 | .720 | .881 | 1042 | 1203 | 161 |
| 270 | 431364 | 1525 | 1685 | 1846 | 2007 | 2167 | 2328 | 2488 | 2649 | 2809 | 161 |
| 271 | 2969   | 3130 | 3290 | 3450 | 3610 | 3770 | 3930 | 4090 | 4249 | 4409 | 160 |
| 272 | 4569   | 4729 | 4888 | 5048 | 5207 | 5367 | 5526 | 5685 | 5844 | 6004 | 159 |
| 273 | 6163   | 6322 | 6481 | 6640 | 6798 | 6957 | 7116 | 7275 | 7433 | 7592 | 159 |
| 274 | 7751   | 7909 | 8067 | 8226 | 8384 | 8542 | 8701 | 8859 | 9017 | 9175 | 158 |
| 275 | 9333   | 9491 | 9648 | 9806 | 9964 | .122 | .279 | .437 | .594 | .752 | 158 |
| 276 | 440909 | 1066 | 1224 | 1381 | 1538 | 1695 | 1852 | 2009 | 2166 | 2323 | 157 |
| 277 | 2480   | 2637 | 2793 | 2950 | 3106 | 3263 | 3419 | 3576 | 3732 | 3889 | 157 |
| 278 | 4045   | 4201 | 4357 | 4513 | 4669 | 4825 | 4981 | 5137 | 5293 | 5449 | 156 |
| 279 | 5604   | 5760 | 5915 | 6071 | 6226 | 6382 | 6537 | 6692 | 6848 | 7003 | 155 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D.  |



| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D.  |
|-----|--------|------|------|------|------|------|------|------|------|------|-----|
| 280 | 447158 | 7313 | 7468 | 7623 | 7778 | 7933 | 8088 | 8242 | 8397 | 8552 | 155 |
| 281 | 8706   | 8861 | 9015 | 9170 | 9324 | 9478 | 9633 | 9787 | 9941 | .95  | 154 |
| 282 | 450249 | 0403 | 0557 | 0711 | 0865 | 1018 | 1172 | 1326 | 1479 | 1633 | 154 |
| 283 | 1786   | 1940 | 2093 | 2247 | 2400 | 2553 | 2706 | 2859 | 3012 | 3165 | 153 |
| 284 | 3318   | 3471 | 3624 | 3777 | 3930 | 4082 | 4235 | 4387 | 4540 | 4692 | 153 |
| 285 | 4845   | 4997 | 5150 | 5302 | 5454 | 5606 | 5758 | 5910 | 6062 | 6214 | 152 |
| 286 | 6366   | 6518 | 6670 | 6821 | 6973 | 7125 | 7276 | 7428 | 7579 | 7731 | 152 |
| 287 | 7882   | 8033 | 8184 | 8336 | 8487 | 8638 | 8789 | 8940 | 9091 | 9242 | 151 |
| 288 | 9392   | 9543 | 9694 | 9845 | 9995 | .146 | .296 | .447 | .597 | .748 | 151 |
| 289 | 460898 | 1048 | 1198 | 1348 | 1499 | 1649 | 1799 | 1948 | 2098 | 2248 | 150 |
| 290 | 462398 | 2548 | 2697 | 2847 | 2997 | 3146 | 3296 | 3445 | 3594 | 3744 | 150 |
| 291 | 3893   | 4042 | 4191 | 4340 | 4490 | 4639 | 4788 | 4936 | 5085 | 5234 | 149 |
| 292 | 5383   | 5532 | 5680 | 5829 | 5977 | 6126 | 6274 | 6423 | 6571 | 6719 | 149 |
| 293 | 6868   | 7016 | 7164 | 7312 | 7460 | 7608 | 7756 | 7904 | 8052 | 8200 | 148 |
| 294 | 8347   | 8495 | 8643 | 8790 | 8938 | 9085 | 9233 | 9380 | 9527 | 9675 | 148 |
| 295 | 9822   | 9969 | .116 | .263 | .410 | .557 | .704 | .851 | .998 | 1145 | 147 |
| 296 | 471292 | 1438 | 1585 | 1732 | 1878 | 2025 | 2171 | 2318 | 2464 | 2610 | 146 |
| 297 | 2756   | 2903 | 3049 | 3195 | 3341 | 3487 | 3633 | 3779 | 3925 | 4071 | 146 |
| 298 | 4216   | 4362 | 4508 | 4653 | 4799 | 4944 | 5090 | 5235 | 5381 | 5526 | 146 |
| 299 | 5671   | 5816 | 5962 | 6107 | 6252 | 6397 | 6542 | 6687 | 6832 | 6976 | 145 |
| 300 | 477121 | 7266 | 7411 | 7555 | 7700 | 7844 | 7989 | 8133 | 8278 | 8422 | 145 |
| 301 | 8566   | 8711 | 8855 | 8999 | 9143 | 9287 | 9431 | 9575 | 9719 | 9863 | 144 |
| 302 | 480007 | 0151 | 0294 | 0438 | 0582 | 0725 | 0869 | 1012 | 1156 | 1299 | 144 |
| 303 | 1443   | 1586 | 1729 | 1872 | 2016 | 2159 | 2302 | 2445 | 2588 | 2731 | 143 |
| 304 | 2874   | 3016 | 3159 | 3302 | 3445 | 3587 | 3730 | 3872 | 4015 | 4157 | 143 |
| 305 | 4300   | 4442 | 4585 | 4727 | 4869 | 5011 | 5153 | 5295 | 5437 | 5579 | 142 |
| 306 | 5721   | 5863 | 6005 | 6147 | 6289 | 6430 | 6572 | 6714 | 6855 | 6997 | 142 |
| 307 | 7138   | 7280 | 7421 | 7563 | 7704 | 7845 | 7986 | 8127 | 8269 | 8410 | 141 |
| 308 | 8551   | 8692 | 8833 | 8974 | 9114 | 9255 | 9396 | 9537 | 9677 | 9818 | 141 |
| 309 | 9958   | .99  | .239 | .380 | .520 | .661 | .801 | .941 | 1081 | 1222 | 140 |
| 310 | 491362 | 1502 | 1642 | 1782 | 1922 | 2062 | 2201 | 2341 | 2481 | 2621 | 140 |
| 311 | 2760   | 2900 | 3040 | 3179 | 3319 | 3458 | 3597 | 3737 | 3876 | 4015 | 139 |
| 312 | 4155   | 4294 | 4433 | 4572 | 4711 | 4850 | 4989 | 5128 | 5267 | 5406 | 139 |
| 313 | 5544   | 5683 | 5822 | 5960 | 6099 | 6238 | 6376 | 6515 | 6653 | 6791 | 139 |
| 314 | 6930   | 7068 | 7206 | 7344 | 7483 | 7621 | 7759 | 7897 | 8035 | 8173 | 138 |
| 315 | 8311   | 8448 | 8586 | 8724 | 8862 | 8999 | 9137 | 9275 | 9412 | 9550 | 138 |
| 316 | 9687   | 9824 | 9962 | .99  | .236 | .374 | .511 | .648 | .785 | .922 | 137 |
| 317 | 501059 | 1196 | 1333 | 1470 | 1607 | 1744 | 1880 | 2017 | 2154 | 2291 | 137 |
| 318 | 2427   | 2564 | 2700 | 2837 | 2973 | 3109 | 3246 | 3382 | 3518 | 3655 | 136 |
| 319 | 3791   | 3927 | 4063 | 4199 | 4335 | 4471 | 4607 | 4743 | 4878 | 5014 | 136 |
| 320 | 505150 | 5286 | 5421 | 5557 | 5693 | 5828 | 5964 | 6099 | 6234 | 6370 | 136 |
| 321 | 6505   | 6640 | 6776 | 6911 | 7046 | 7181 | 7316 | 7451 | 7586 | 7721 | 135 |
| 322 | 7856   | 7991 | 8126 | 8260 | 8395 | 8530 | 8664 | 8799 | 8934 | 9068 | 135 |
| 323 | 9203   | 9337 | 9471 | 9606 | 9740 | 9874 | ..9  | .143 | .277 | .411 | 134 |
| 324 | 510545 | 0679 | 0813 | 0947 | 1081 | 1215 | 1349 | 1482 | 1616 | 1750 | 134 |
| 325 | 1883   | 2017 | 2151 | 2284 | 2418 | 2551 | 2684 | 2818 | 2951 | 3084 | 133 |
| 326 | 3218   | 3351 | 3484 | 3617 | 3750 | 3883 | 4016 | 4149 | 4282 | 4414 | 133 |
| 327 | 4548   | 4681 | 4813 | 4946 | 5079 | 5211 | 5344 | 5476 | 5609 | 5741 | 133 |
| 328 | 5874   | 6006 | 6139 | 6271 | 6403 | 6535 | 6668 | 6800 | 6932 | 7064 | 132 |
| 329 | 7196   | 7328 | 7460 | 7592 | 7724 | 7855 | 7987 | 8119 | 8251 | 8382 | 132 |
| 330 | 518514 | 8646 | 8777 | 8909 | 9040 | 9171 | 9303 | 9434 | 9566 | 9697 | 131 |
| 331 | 9828   | 9959 | .90  | .221 | .353 | .484 | .615 | .745 | .876 | 1007 | 131 |
| 332 | 521138 | 1269 | 1400 | 1530 | 1661 | 1792 | 1922 | 2053 | 2183 | 2314 | 131 |
| 333 | 2444   | 2575 | 2705 | 2835 | 2966 | 3096 | 3226 | 3356 | 3486 | 3616 | 130 |
| 334 | 3746   | 3876 | 4006 | 4136 | 4266 | 4396 | 4526 | 4656 | 4785 | 4915 | 130 |
| 335 | 5045   | 5174 | 5304 | 5434 | 5563 | 5693 | 5822 | 5951 | 6081 | 6210 | 129 |
| 336 | 6339   | 6469 | 6598 | 6727 | 6856 | 6985 | 7114 | 7243 | 7372 | 7501 | 129 |
| 337 | 7630   | 7759 | 7888 | 8016 | 8145 | 8274 | 8402 | 8531 | 8660 | 8788 | 129 |
| 338 | 8917   | 9045 | 9174 | 9302 | 9430 | 9559 | 9687 | 9815 | 9943 | .72  | 128 |
| 339 | 530200 | 0328 | 0456 | 0584 | 0712 | 0840 | 0968 | 1096 | 1223 | 1351 | 128 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D.  |

| N.  | 0      | 1     | 2     | 3    | 4    | 5    | 6     | 7    | 8     | 9    | D.  |
|-----|--------|-------|-------|------|------|------|-------|------|-------|------|-----|
| 340 | 531479 | 1607  | 1734  | 1862 | 1990 | 2117 | 2245  | 2372 | 2500  | 2627 | 128 |
| 341 | 2754   | 2882  | 3009  | 3136 | 3264 | 3391 | 3518  | 3645 | 3772  | 3899 | 127 |
| 342 | 4026   | 4153  | 4280  | 4407 | 4534 | 4661 | 4787  | 4914 | 5041  | 5167 | 127 |
| 343 | 5294   | 5421  | 5547  | 5674 | 5800 | 5927 | 6053  | 6180 | 6306  | 6432 | 126 |
| 344 | 6558   | 6685  | 6811  | 6937 | 7063 | 7189 | 7315  | 7441 | 7567  | 7693 | 126 |
| 345 | 7819   | 7945  | 8071  | 8197 | 8322 | 8448 | 8574  | 8699 | 8825  | 8951 | 126 |
| 346 | 9076   | 9202  | 9327  | 9452 | 9578 | 9703 | 9829  | 9954 | . .79 | .204 | 125 |
| 347 | 540329 | 0455  | 0580  | 0705 | 0830 | 0955 | 1080  | 1205 | 1330  | 1454 | 125 |
| 348 | 1579   | 1704  | 1829  | 1953 | 2078 | 2203 | 2327  | 2452 | 2576  | 2701 | 125 |
| 349 | 2825   | 2950  | 3074  | 3199 | 3323 | 3447 | 3571  | 3696 | 3820  | 3944 | 124 |
| 350 | 544068 | 4192  | 4316  | 4440 | 4564 | 4688 | 4812  | 4936 | 5060  | 5183 | 124 |
| 351 | 5307   | 5431  | 5555  | 5678 | 5802 | 5925 | 6049  | 6172 | 6296  | 6419 | 124 |
| 352 | 6543   | 6666  | 6789  | 6913 | 7036 | 7159 | 7282  | 7405 | 7529  | 7652 | 123 |
| 353 | 7775   | 7898  | 8021  | 8144 | 8267 | 8389 | 8512  | 8635 | 8758  | 8881 | 123 |
| 354 | 9003   | 9126  | 9249  | 9371 | 9494 | 9616 | 9739  | 9861 | 9984  | .106 | 123 |
| 355 | 550228 | 0351  | 0473  | 0595 | 0717 | 0840 | 0962  | 1084 | 1206  | 1328 | 122 |
| 356 | 1450   | 1572  | 1694  | 1816 | 1938 | 2060 | 2181  | 2303 | 2425  | 2547 | 122 |
| 357 | 2668   | 2790  | 2911  | 3033 | 3155 | 3276 | 3398  | 3519 | 3640  | 3762 | 121 |
| 358 | 3883   | 4004  | 4126  | 4247 | 4368 | 4489 | 4610  | 4731 | 4852  | 4973 | 121 |
| 359 | 5094   | 5215  | 5336  | 5457 | 5578 | 5699 | 5820  | 5940 | 6061  | 6182 | 121 |
| 360 | 556333 | 6423  | 6544  | 6664 | 6785 | 6905 | 7026  | 7146 | 7267  | 7387 | 120 |
| 361 | 7507   | 7627  | 7748  | 7868 | 7988 | 8108 | 8228  | 8349 | 8469  | 8589 | 120 |
| 362 | 8709   | 8829  | 8948  | 9068 | 9188 | 9308 | 9428  | 9548 | 9667  | 9787 | 120 |
| 363 | 9907   | . .26 | .146  | .265 | .385 | .504 | .624  | .743 | .863  | .982 | 119 |
| 364 | 561101 | 1221  | 1340  | 1459 | 1578 | 1698 | 1817  | 1936 | 2055  | 2174 | 119 |
| 365 | 2293   | 2412  | 2531  | 2650 | 2769 | 2887 | 3006  | 3125 | 3244  | 3362 | 119 |
| 366 | 3481   | 3600  | 3718  | 3837 | 3955 | 4074 | 4192  | 4311 | 4429  | 4548 | 119 |
| 367 | 4666   | 4784  | 4903  | 5021 | 5139 | 5257 | 5376  | 5494 | 5612  | 5730 | 118 |
| 368 | 5848   | 5966  | 6084  | 6202 | 6320 | 6437 | 6555  | 6673 | 6791  | 6909 | 118 |
| 369 | 7026   | 7144  | 7262  | 7379 | 7497 | 7614 | 7732  | 7849 | 7967  | 8084 | 118 |
| 370 | 568202 | 8319  | 8436  | 8554 | 8671 | 8788 | 8905  | 9023 | 9140  | 9257 | 117 |
| 371 | 9374   | 9491  | 9608  | 9725 | 9842 | 9959 | . .76 | .193 | .309  | .426 | 117 |
| 372 | 570543 | 0660  | 0776  | 0893 | 1010 | 1126 | 1243  | 1359 | 1476  | 1592 | 117 |
| 373 | 1709   | 1825  | 1942  | 2058 | 2174 | 2291 | 2407  | 2523 | 2639  | 2755 | 116 |
| 374 | 2872   | 2988  | 3104  | 3220 | 3336 | 3452 | 3568  | 3684 | 3800  | 3915 | 116 |
| 375 | 4031   | 4147  | 4263  | 4379 | 4494 | 4610 | 4726  | 4841 | 4957  | 5072 | 116 |
| 376 | 5188   | 5303  | 5419  | 5534 | 5650 | 5765 | 5880  | 5996 | 6111  | 6226 | 115 |
| 377 | 6341   | 6457  | 6572  | 6687 | 6802 | 6917 | 7032  | 7147 | 7262  | 7377 | 115 |
| 378 | 7492   | 7607  | 7722  | 7836 | 7951 | 8066 | 8181  | 8295 | 8410  | 8525 | 115 |
| 379 | 8639   | 8754  | 8868  | 8983 | 9097 | 9212 | 9326  | 9441 | 9555  | 9669 | 114 |
| 380 | 579784 | 9893  | . .12 | .126 | .241 | .355 | .469  | .583 | .697  | .811 | 114 |
| 381 | 580925 | 1039  | 1153  | 1267 | 1381 | 1495 | 1608  | 1722 | 1836  | 1950 | 114 |
| 382 | 2063   | 2177  | 2291  | 2404 | 2518 | 2631 | 2745  | 2858 | 2972  | 3085 | 114 |
| 383 | 3199   | 3312  | 3426  | 3539 | 3652 | 3765 | 3879  | 3992 | 4105  | 4218 | 113 |
| 384 | 4331   | 4444  | 4557  | 4670 | 4783 | 4896 | 5009  | 5122 | 5235  | 5348 | 113 |
| 385 | 5461   | 5574  | 5686  | 5799 | 5912 | 6024 | 6137  | 6250 | 6362  | 6475 | 113 |
| 386 | 6587   | 6700  | 6812  | 6925 | 7037 | 7149 | 7262  | 7374 | 7486  | 7599 | 112 |
| 387 | 7711   | 7823  | 7935  | 8047 | 8160 | 8272 | 8384  | 8496 | 8608  | 8720 | 112 |
| 388 | 8832   | 8944  | 9056  | 9167 | 9279 | 9391 | 9503  | 9615 | 9726  | 9838 | 112 |
| 389 | 9950   | . .61 | .173  | .284 | .396 | .507 | .619  | .730 | .842  | .953 | 112 |
| 390 | 591065 | 1176  | 1287  | 1399 | 1510 | 1621 | 1732  | 1843 | 1955  | 2066 | 111 |
| 391 | 2177   | 2288  | 2399  | 2510 | 2621 | 2732 | 2843  | 2954 | 3064  | 3175 | 111 |
| 392 | 3286   | 3397  | 3508  | 3618 | 3729 | 3840 | 3950  | 4061 | 4171  | 4282 | 111 |
| 393 | 4393   | 4503  | 4614  | 4724 | 4834 | 4945 | 5055  | 5165 | 5276  | 5386 | 110 |
| 394 | 5496   | 5606  | 5717  | 5827 | 5937 | 6047 | 6157  | 6267 | 6377  | 6487 | 110 |
| 395 | 6597   | 6707  | 6817  | 6927 | 7037 | 7146 | 7256  | 7366 | 7476  | 7586 | 110 |
| 396 | 7695   | 7805  | 7914  | 8024 | 8134 | 8243 | 8353  | 8462 | 8572  | 8681 | 110 |
| 397 | 8791   | 8900  | 9009  | 9119 | 9228 | 9337 | 9446  | 9556 | 9665  | 9774 | 109 |
| 398 | 9833   | 9992  | .101  | .210 | .319 | .428 | .537  | .646 | .755  | .864 | 109 |
| 399 | 600973 | 1082  | 1191  | 1299 | 1408 | 1517 | 1625  | 1734 | 1843  | 1951 | 109 |
| N.  | 0      | 1     | 2     | 3    | 4    | 5    | 6     | 7    | 8     | 9    | D.  |

| N.  | 0      | 1     | 2    | 3    | 4     | 5    | 6     | 7     | 8    | 9     | D.  |
|-----|--------|-------|------|------|-------|------|-------|-------|------|-------|-----|
| 400 | 602060 | 2169  | 2277 | 2386 | 2494  | 2603 | 2711  | 2819  | 2923 | 3036  | 108 |
| 401 | 3144   | 3253  | 3361 | 3469 | 3577  | 3686 | 3794  | 3902  | 4010 | 4118  | 108 |
| 402 | 4226   | 4334  | 4442 | 4550 | 4658  | 4766 | 4874  | 4982  | 5089 | 5197  | 108 |
| 403 | 5305   | 5413  | 5521 | 5623 | 5736  | 5844 | 5951  | 6059  | 6166 | 6274  | 108 |
| 404 | 6381   | 6489  | 6596 | 6704 | 6811  | 6919 | 7026  | 7133  | 7241 | 7348  | 107 |
| 405 | 7455   | 7562  | 7669 | 7777 | 7884  | 7991 | 8098  | 8205  | 8312 | 8419  | 107 |
| 406 | 8526   | 8633  | 8740 | 8847 | 8954  | 9061 | 9167  | 9274  | 9381 | 9488  | 107 |
| 407 | 9594   | 9701  | 9808 | 9914 | . .21 | .128 | .234  | .341  | .447 | .554  | 107 |
| 408 | 610660 | 0767  | 0873 | 0979 | 1086  | 1192 | 1298  | 1405  | 1511 | 1617  | 106 |
| 409 | 1723   | 1829  | 1936 | 2042 | 2148  | 2254 | 2360  | 2466  | 2572 | 2678  | 106 |
| 410 | 612784 | 2890  | 2996 | 3102 | 3207  | 3313 | 3419  | 3525  | 3630 | 3736  | 106 |
| 411 | 3842   | 3947  | 4053 | 4159 | 4264  | 4370 | 4475  | 4581  | 4686 | 4792  | 106 |
| 412 | 4897   | 5003  | 5108 | 5213 | 5319  | 5424 | 5529  | 5634  | 5740 | 5845  | 105 |
| 413 | 5950   | 6055  | 6160 | 6265 | 6370  | 6476 | 6581  | 6686  | 6790 | 6895  | 105 |
| 414 | 7000   | 7105  | 7210 | 7315 | 7420  | 7525 | 7629  | 7734  | 7839 | 7943  | 105 |
| 415 | 8048   | 8153  | 8257 | 8362 | 8466  | 8571 | 8676  | 8780  | 8884 | 8989  | 105 |
| 416 | 9093   | 9198  | 9302 | 9406 | 9511  | 9615 | 9719  | 9824  | 9928 | . .32 | 104 |
| 417 | 620136 | 0240  | 0344 | 0448 | 0552  | 0656 | 0760  | 0864  | 0968 | 1072  | 104 |
| 418 | 1176   | 1280  | 1384 | 1488 | 1592  | 1695 | 1799  | 1903  | 2007 | 2110  | 104 |
| 419 | 2214   | 2318  | 2421 | 2525 | 2628  | 2732 | 2835  | 2939  | 3042 | 3146  | 104 |
| 420 | 623249 | 3353  | 3456 | 3559 | 3663  | 3766 | 3869  | 3973  | 4076 | 4179  | 103 |
| 421 | 4282   | 4385  | 4488 | 4591 | 4695  | 4798 | 4901  | 5004  | 5107 | 5210  | 103 |
| 422 | 5312   | 5415  | 5518 | 5621 | 5724  | 5827 | 5929  | 6032  | 6135 | 6238  | 103 |
| 423 | 6340   | 6443  | 6546 | 6648 | 6751  | 6853 | 6956  | 7058  | 7161 | 7263  | 103 |
| 424 | 7366   | 7468  | 7571 | 7673 | 7775  | 7878 | 7980  | 8082  | 8185 | 8287  | 102 |
| 425 | 8389   | 8491  | 8593 | 8695 | 8797  | 8900 | 9002  | 9104  | 9206 | 9308  | 102 |
| 426 | 9410   | 9512  | 9613 | 9715 | 9817  | 9919 | . .21 | .123  | .224 | .326  | 102 |
| 427 | 630428 | 0530  | 0631 | 0733 | 0835  | 0936 | 1038  | 1139  | 1241 | 1342  | 102 |
| 428 | 1444   | 1545  | 1647 | 1748 | 1849  | 1951 | 2052  | 2153  | 2255 | 2356  | 101 |
| 429 | 2457   | 2559  | 2660 | 2761 | 2862  | 2963 | 3064  | 3165  | 3266 | 3367  | 101 |
| 430 | 633468 | 3569  | 3670 | 3771 | 3872  | 3973 | 4074  | 4175  | 4276 | 4376  | 100 |
| 431 | 4477   | 4578  | 4679 | 4779 | 4880  | 4981 | 5081  | 5182  | 5283 | 5383  | 100 |
| 432 | 5484   | 5584  | 5685 | 5785 | 5886  | 5986 | 6087  | 6187  | 6287 | 6388  | 100 |
| 433 | 6488   | 6588  | 6688 | 6789 | 6889  | 6989 | 7089  | 7189  | 7290 | 7390  | 100 |
| 434 | 7490   | 7590  | 7690 | 7790 | 7890  | 7990 | 8090  | 8190  | 8290 | 8389  | 99  |
| 435 | 8489   | 8589  | 8689 | 8789 | 8888  | 8988 | 9088  | 9188  | 9287 | 9387  | 99  |
| 436 | 9485   | 9586  | 9686 | 9785 | 9885  | 9984 | . .84 | .183  | .283 | .382  | 99  |
| 437 | 640481 | 0581  | 0680 | 0779 | 0879  | 0978 | 1077  | 1177  | 1276 | 1375  | 99  |
| 438 | 1474   | 1573  | 1672 | 1771 | 1871  | 1970 | 2069  | 2168  | 2267 | 2366  | 99  |
| 439 | 2465   | 2563  | 2662 | 2761 | 2860  | 2959 | 3058  | 3156  | 3255 | 3354  | 99  |
| 440 | 643453 | 3551  | 3650 | 3749 | 3847  | 3946 | 4044  | 4143  | 4242 | 4340  | 98  |
| 441 | 4439   | 4537  | 4636 | 4734 | 4832  | 4931 | 5029  | 5127  | 5226 | 5324  | 98  |
| 442 | 5422   | 5521  | 5619 | 5717 | 5815  | 5913 | 6011  | 6110  | 6208 | 6306  | 98  |
| 443 | 6404   | 6502  | 6600 | 6698 | 6796  | 6894 | 6992  | 7089  | 7187 | 7285  | 98  |
| 444 | 7383   | 7481  | 7579 | 7676 | 7774  | 7872 | 7969  | 8067  | 8165 | 8262  | 98  |
| 445 | 8360   | 8458  | 8555 | 8653 | 8750  | 8848 | 8945  | 9043  | 9140 | 9237  | 97  |
| 446 | 9335   | 9432  | 9530 | 9627 | 9724  | 9821 | 9919  | . .16 | .113 | .210  | 97  |
| 447 | 650308 | 0405  | 0502 | 0599 | 0696  | 0793 | 0890  | 0987  | 1084 | 1181  | 97  |
| 448 | 1278   | 1375  | 1472 | 1569 | 1666  | 1762 | 1859  | 1956  | 2053 | 2150  | 97  |
| 449 | 2246   | 2343  | 2440 | 2536 | 2633  | 2730 | 2826  | 2923  | 3019 | 3116  | 97  |
| 450 | 653213 | 3309  | 3405 | 3502 | 3598  | 3695 | 3791  | 3888  | 3984 | 4080  | 96  |
| 451 | 4177   | 4273  | 4369 | 4465 | 4562  | 4658 | 4754  | 4850  | 4946 | 5042  | 96  |
| 452 | 5138   | 5235  | 5331 | 5427 | 5523  | 5619 | 5715  | 5810  | 5906 | 6002  | 96  |
| 453 | 6093   | 6194  | 6290 | 6386 | 6482  | 6577 | 6673  | 6769  | 6864 | 6960  | 96  |
| 454 | 7056   | 7152  | 7247 | 7343 | 7438  | 7534 | 7629  | 7725  | 7820 | 7916  | 96  |
| 455 | 8011   | 8107  | 8202 | 8298 | 8393  | 8488 | 8584  | 8679  | 8774 | 8870  | 95  |
| 456 | 8955   | 9060  | 9155 | 9250 | 9346  | 9441 | 9536  | 9631  | 9726 | 9821  | 95  |
| 457 | 9916   | . .11 | .106 | .201 | .296  | .391 | .486  | .581  | .676 | .771  | 95  |
| 458 | 660865 | 0960  | 1055 | 1150 | 1245  | 1339 | 1434  | 1529  | 1623 | 1718  | 95  |
| 459 | 1813   | 1907  | 2002 | 2096 | 2191  | 2286 | 2380  | 2475  | 2569 | 2663  | 95  |
| N.  | 0      | 1     | 2    | 3    | 4     | 5    | 6     | 7     | 8    | 9     | D.  |



| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D. |
|-----|--------|------|------|------|------|------|------|------|------|------|----|
| 460 | 662758 | 2852 | 2947 | 3041 | 3135 | 3239 | 3324 | 3418 | 3512 | 3607 | 94 |
| 461 | 3701   | 3795 | 3889 | 3983 | 4078 | 4172 | 4266 | 4360 | 4454 | 4548 | 94 |
| 462 | 4642   | 4736 | 4830 | 4924 | 5018 | 5112 | 5206 | 5299 | 5393 | 5487 | 94 |
| 463 | 5581   | 5675 | 5769 | 5862 | 5956 | 6050 | 6143 | 6237 | 6331 | 6424 | 94 |
| 464 | 6518   | 6612 | 6705 | 6799 | 6892 | 6986 | 7079 | 7173 | 7266 | 7360 | 94 |
| 465 | 7453   | 7546 | 7640 | 7733 | 7826 | 7920 | 8013 | 8106 | 8199 | 8293 | 93 |
| 466 | 8386   | 8479 | 8572 | 8665 | 8759 | 8852 | 8945 | 9038 | 9131 | 9224 | 93 |
| 467 | 9317   | 9410 | 9503 | 9596 | 9689 | 9782 | 9875 | 9967 | .60  | .153 | 93 |
| 468 | 670246 | 0339 | 0431 | 0524 | 0617 | 0710 | 0802 | 0895 | 0988 | 1080 | 93 |
| 469 | 1173   | 1265 | 1358 | 1451 | 1543 | 1636 | 1728 | 1821 | 1913 | 2005 | 93 |
| 470 | 672098 | 2190 | 2283 | 2375 | 2467 | 2560 | 2652 | 2744 | 2836 | 2929 | 92 |
| 471 | 3021   | 3113 | 3205 | 3297 | 3390 | 3482 | 3574 | 3666 | 3758 | 3850 | 92 |
| 472 | 3942   | 4034 | 4126 | 4218 | 4310 | 4402 | 4494 | 4586 | 4677 | 4769 | 92 |
| 473 | 4861   | 4953 | 5045 | 5137 | 5228 | 5320 | 5412 | 5503 | 5595 | 5687 | 92 |
| 474 | 5778   | 5870 | 5962 | 6053 | 6145 | 6236 | 6328 | 6419 | 6511 | 6602 | 92 |
| 475 | 6694   | 6785 | 6876 | 6968 | 7059 | 7151 | 7242 | 7333 | 7424 | 7516 | 91 |
| 476 | 7607   | 7698 | 7789 | 7881 | 7972 | 8063 | 8154 | 8245 | 8336 | 8427 | 91 |
| 477 | 8518   | 8609 | 8700 | 8791 | 8882 | 8973 | 9064 | 9155 | 9246 | 9337 | 91 |
| 478 | 9428   | 9519 | 9610 | 9700 | 9791 | 9882 | 9973 | .63  | .154 | .245 | 91 |
| 479 | 680336 | 0426 | 0517 | 0607 | 0698 | 0789 | 0879 | 0970 | 1060 | 1151 | 91 |
| 480 | 681241 | 1332 | 1422 | 1513 | 1603 | 1693 | 1784 | 1874 | 1964 | 2055 | 90 |
| 481 | 2145   | 2235 | 2326 | 2416 | 2506 | 2596 | 2686 | 2777 | 2867 | 2957 | 90 |
| 482 | 3047   | 3137 | 3227 | 3317 | 3407 | 3497 | 3587 | 3677 | 3767 | 3857 | 90 |
| 483 | 3947   | 4037 | 4127 | 4217 | 4307 | 4396 | 4486 | 4576 | 4666 | 4756 | 90 |
| 484 | 4845   | 4935 | 5025 | 5114 | 5204 | 5294 | 5383 | 5473 | 5563 | 5652 | 90 |
| 485 | 5742   | 5831 | 5921 | 6010 | 6100 | 6189 | 6279 | 6368 | 6458 | 6547 | 89 |
| 486 | 6636   | 6726 | 6815 | 6904 | 6994 | 7083 | 7172 | 7261 | 7351 | 7440 | 89 |
| 487 | 7529   | 7618 | 7707 | 7796 | 7886 | 7975 | 8064 | 8153 | 8242 | 8331 | 89 |
| 488 | 8420   | 8509 | 8598 | 8687 | 8776 | 8865 | 8953 | 9042 | 9131 | 9220 | 89 |
| 489 | 9309   | 9398 | 9486 | 9575 | 9664 | 9753 | 9841 | 9930 | .19  | .107 | 89 |
| 490 | 690196 | 0285 | 0373 | 0462 | 0550 | 0639 | 0728 | 0816 | 0905 | 0993 | 89 |
| 491 | 1081   | 1170 | 1258 | 1347 | 1435 | 1524 | 1612 | 1700 | 1789 | 1877 | 88 |
| 492 | 1965   | 2053 | 2142 | 2230 | 2318 | 2406 | 2494 | 2583 | 2671 | 2759 | 88 |
| 493 | 2847   | 2935 | 3023 | 3111 | 3199 | 3287 | 3375 | 3463 | 3551 | 3639 | 88 |
| 494 | 3727   | 3815 | 3903 | 3991 | 4078 | 4166 | 4254 | 4342 | 4430 | 4517 | 88 |
| 495 | 4805   | 4893 | 4981 | 5068 | 5156 | 5244 | 5331 | 5419 | 5507 | 5594 | 88 |
| 496 | 5482   | 5569 | 5657 | 5744 | 5832 | 5919 | 6007 | 6094 | 6182 | 6269 | 87 |
| 497 | 6356   | 6444 | 6531 | 6618 | 6706 | 6793 | 6880 | 6968 | 7055 | 7142 | 87 |
| 498 | 7229   | 7317 | 7404 | 7491 | 7578 | 7665 | 7752 | 7839 | 7926 | 8014 | 87 |
| 499 | 8101   | 8188 | 8275 | 8362 | 8449 | 8535 | 8622 | 8709 | 8796 | 8883 | 87 |
| 500 | 698970 | 9057 | 9144 | 9231 | 9317 | 9404 | 9491 | 9578 | 9664 | 9751 | 87 |
| 501 | 9838   | 9924 | .11  | .98  | .184 | .271 | .358 | .444 | .531 | .617 | 87 |
| 502 | 700704 | 0790 | 0877 | 0963 | 1050 | 1136 | 1222 | 1309 | 1395 | 1482 | 86 |
| 503 | 1568   | 1654 | 1741 | 1827 | 1913 | 1999 | 2086 | 2172 | 2258 | 2344 | 86 |
| 504 | 2431   | 2517 | 2603 | 2689 | 2775 | 2861 | 2947 | 3033 | 3119 | 3205 | 86 |
| 505 | 3291   | 3377 | 3463 | 3549 | 3635 | 3721 | 3807 | 3893 | 3979 | 4065 | 86 |
| 506 | 4151   | 4236 | 4322 | 4408 | 4494 | 4579 | 4665 | 4751 | 4837 | 4922 | 86 |
| 507 | 5098   | 5094 | 5179 | 5265 | 5350 | 5436 | 5522 | 5607 | 5693 | 5778 | 86 |
| 508 | 5864   | 5949 | 6035 | 6120 | 6206 | 6291 | 6376 | 6462 | 6547 | 6632 | 85 |
| 509 | 6718   | 6803 | 6888 | 6974 | 7059 | 7144 | 7229 | 7315 | 7400 | 7485 | 85 |
| 510 | 707570 | 7655 | 7740 | 7826 | 7911 | 7996 | 8081 | 8166 | 8251 | 8336 | 85 |
| 511 | 8421   | 8506 | 8591 | 8676 | 8761 | 8846 | 8931 | 9015 | 9100 | 9185 | 85 |
| 512 | 9270   | 9355 | 9440 | 9524 | 9609 | 9694 | 9779 | 9863 | 9948 | .33  | 85 |
| 513 | 710117 | 0202 | 0287 | 0371 | 0456 | 0540 | 0625 | 0710 | 0794 | 0879 | 85 |
| 514 | 0963   | 1048 | 1132 | 1217 | 1301 | 1385 | 1470 | 1554 | 1639 | 1723 | 84 |
| 515 | 1807   | 1892 | 1976 | 2060 | 2144 | 2229 | 2313 | 2397 | 2481 | 2566 | 84 |
| 516 | 2650   | 2734 | 2818 | 2902 | 2986 | 3070 | 3154 | 3238 | 3323 | 3407 | 84 |
| 517 | 3491   | 3575 | 3659 | 3742 | 3826 | 3910 | 3994 | 4078 | 4162 | 4246 | 84 |
| 518 | 4330   | 4414 | 4497 | 4581 | 4665 | 4749 | 4833 | 4916 | 5000 | 5084 | 84 |
| 519 | 5167   | 5251 | 5335 | 5418 | 5502 | 5586 | 5669 | 5753 | 5836 | 5920 | 84 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D. |

| N.  | 0      | 1      | 2     | 3     | 4      | 5      | 6      | 7     | 8     | 9      | D. |
|-----|--------|--------|-------|-------|--------|--------|--------|-------|-------|--------|----|
| 520 | 716003 | 6087   | 6170  | 6254  | 6337   | 6421   | 6504   | 6588  | 6671  | 6754   | 83 |
| 521 | 6838   | 6921   | 7004  | 7088  | 7171   | 7254   | 7338   | 7421  | 7504  | 7587   | 83 |
| 522 | 7671   | 7754   | 7837  | 7920  | 8003   | 8086   | 8169   | 8253  | 8336  | 8419   | 83 |
| 523 | 8502   | 8585   | 8668  | 8751  | 8834   | 8917   | 9000   | 9083  | 9165  | 9248   | 83 |
| 524 | 9331   | 9414   | 9497  | 9580  | 9663   | 9745   | 9828   | 9911  | 9994  | . . 77 | 83 |
| 525 | 720159 | 0242   | 0325  | 0407  | 0490   | 0573   | 0655   | 0738  | 0821  | 0903   | 83 |
| 526 | 0986   | 1068   | 1151  | 1233  | 1316   | 1398   | 1481   | 1563  | 1646  | 1728   | 82 |
| 527 | 1811   | 1893   | 1975  | 2058  | 2140   | 2222   | 2305   | 2387  | 2469  | 2552   | 82 |
| 528 | 2634   | 2716   | 2798  | 2881  | 2963   | 3045   | 3127   | 3209  | 3291  | 3374   | 82 |
| 529 | 3456   | 3538   | 3620  | 3702  | 3784   | 3866   | 3948   | 4030  | 4112  | 4194   | 82 |
| 530 | 724276 | 4358   | 4440  | 4522  | 4604   | 4685   | 4767   | 4849  | 4931  | 5013   | 82 |
| 531 | 5095   | 5176   | 5258  | 5340  | 5422   | 5503   | 5585   | 5667  | 5748  | 5830   | 82 |
| 532 | 5912   | 5993   | 6075  | 6156  | 6238   | 6320   | 6401   | 6483  | 6564  | 6646   | 82 |
| 533 | 6727   | 6809   | 6890  | 6972  | 7053   | 7134   | 7216   | 7297  | 7379  | 7460   | 81 |
| 534 | 7541   | 7623   | 7704  | 7785  | 7866   | 7948   | 8029   | 8110  | 8191  | 8273   | 81 |
| 535 | 8354   | 8435   | 8516  | 8597  | 8678   | 8759   | 8841   | 8922  | 9003  | 9084   | 81 |
| 536 | 9165   | 9246   | 9327  | 9408  | 9489   | 9570   | 9651   | 9732  | 9813  | 9894   | 81 |
| 537 | 9974   | . . 55 | . 136 | . 217 | . 298  | . 378  | . 459  | . 540 | . 621 | . 702  | 81 |
| 538 | 730782 | 0863   | 0944  | 1024  | 1105   | 1186   | 1266   | 1347  | 1428  | 1508   | 81 |
| 539 | 1589   | 1669   | 1750  | 1830  | 1911   | 1991   | 2072   | 2152  | 2233  | 2313   | 81 |
| 540 | 732304 | 2474   | 2555  | 2635  | 2715   | 2796   | 2876   | 2956  | 3037  | 3117   | 80 |
| 541 | 3197   | 3278   | 3358  | 3438  | 3518   | 3598   | 3679   | 3759  | 3839  | 3919   | 80 |
| 542 | 3999   | 4079   | 4160  | 4240  | 4320   | 4400   | 4480   | 4560  | 4640  | 4720   | 80 |
| 543 | 4800   | 4880   | 4960  | 5040  | 5120   | 5200   | 5279   | 5359  | 5439  | 5519   | 80 |
| 544 | 5599   | 5679   | 5759  | 5838  | 5918   | 5998   | 6078   | 6157  | 6237  | 6317   | 80 |
| 545 | 6397   | 6476   | 6556  | 6635  | 6715   | 6795   | 6874   | 6954  | 7034  | 7113   | 80 |
| 546 | 7193   | 7272   | 7352  | 7431  | 7511   | 7590   | 7670   | 7749  | 7829  | 7908   | 79 |
| 547 | 7987   | 8067   | 8146  | 8225  | 8305   | 8384   | 8463   | 8543  | 8622  | 8701   | 79 |
| 548 | 8781   | 8860   | 8939  | 9018  | 9097   | 9177   | 9256   | 9335  | 9414  | 9493   | 79 |
| 549 | 9572   | 9651   | 9731  | 9810  | 9889   | 9968   | . . 47 | . 126 | . 205 | . 284  | 79 |
| 550 | 740363 | 0442   | 0521  | 0600  | 0678   | 0757   | 0836   | 0915  | 0994  | 1073   | 79 |
| 551 | 1152   | 1230   | 1309  | 1388  | 1467   | 1546   | 1624   | 1703  | 1782  | 1860   | 79 |
| 552 | 1939   | 2018   | 2096  | 2175  | 2254   | 2332   | 2411   | 2489  | 2568  | 2646   | 79 |
| 553 | 2725   | 2804   | 2882  | 2961  | 3039   | 3118   | 3196   | 3275  | 3353  | 3431   | 78 |
| 554 | 3510   | 3588   | 3667  | 3745  | 3823   | 3902   | 3980   | 4058  | 4136  | 4215   | 78 |
| 555 | 4293   | 4371   | 4449  | 4528  | 4606   | 4684   | 4762   | 4840  | 4919  | 4997   | 78 |
| 556 | 5075   | 5153   | 5231  | 5309  | 5387   | 5465   | 5543   | 5621  | 5699  | 5777   | 78 |
| 557 | 5855   | 5933   | 6011  | 6089  | 6167   | 6245   | 6323   | 6401  | 6479  | 6556   | 78 |
| 558 | 6334   | 6412   | 6490  | 6568  | 6645   | 6723   | 6801   | 6879  | 6956  | 7034   | 78 |
| 559 | 7412   | 7489   | 7567  | 7645  | 7722   | 7800   | 7878   | 7955  | 8033  | 8110   | 78 |
| 560 | 748188 | 8266   | 8343  | 8421  | 8498   | 8576   | 8653   | 8731  | 8808  | 8885   | 77 |
| 561 | 8963   | 9040   | 9118  | 9195  | 9272   | 9350   | 9427   | 9504  | 9582  | 9659   | 77 |
| 562 | 9736   | 9814   | 9891  | 9968  | . . 45 | . 123  | . 200  | . 277 | . 354 | . 431  | 77 |
| 563 | 750508 | 0586   | 0663  | 0740  | 0817   | 0894   | 0971   | 1048  | 1125  | 1202   | 77 |
| 564 | 1279   | 1356   | 1433  | 1510  | 1587   | 1664   | 1741   | 1818  | 1895  | 1972   | 77 |
| 565 | 2048   | 2125   | 2202  | 2279  | 2356   | 2433   | 2509   | 2586  | 2663  | 2740   | 77 |
| 566 | 2816   | 2893   | 2970  | 3047  | 3123   | 3200   | 3277   | 3353  | 3430  | 3506   | 77 |
| 567 | 3583   | 3660   | 3736  | 3813  | 3889   | 3966   | 4042   | 4119  | 4195  | 4272   | 77 |
| 568 | 4348   | 4425   | 4501  | 4578  | 4654   | 4730   | 4807   | 4883  | 4960  | 5036   | 76 |
| 569 | 5112   | 5189   | 5265  | 5341  | 5417   | 5494   | 5570   | 5646  | 5722  | 5799   | 76 |
| 570 | 755875 | 5951   | 6027  | 6103  | 6180   | 6256   | 6332   | 6408  | 6484  | 6560   | 76 |
| 571 | 6636   | 6712   | 6788  | 6864  | 6940   | 7016   | 7092   | 7168  | 7244  | 7320   | 76 |
| 572 | 7396   | 7472   | 7548  | 7624  | 7700   | 7775   | 7851   | 7927  | 8003  | 8079   | 76 |
| 573 | 8155   | 8230   | 8306  | 8382  | 8458   | 8533   | 8609   | 8685  | 8761  | 8836   | 76 |
| 574 | 8912   | 8988   | 9063  | 9139  | 9214   | 9290   | 9366   | 9441  | 9517  | 9592   | 76 |
| 575 | 9668   | 9743   | 9819  | 9894  | 9970   | . . 45 | . 121  | . 198 | . 274 | . 347  | 75 |
| 576 | 760422 | 0498   | 0573  | 0649  | 0724   | 0799   | 0875   | 0950  | 1025  | 1101   | 75 |
| 577 | 1176   | 1251   | 1326  | 1402  | 1477   | 1552   | 1627   | 1702  | 1778  | 1853   | 75 |
| 578 | 1928   | 2003   | 2078  | 2153  | 2228   | 2303   | 2378   | 2453  | 2529  | 2604   | 75 |
| 579 | 2679   | 2754   | 2829  | 2904  | 2978   | 3053   | 3128   | 3203  | 3278  | 3353   | 75 |
| N.  | 0      | 1      | 2     | 3     | 4      | 5      | 6      | 7     | 8     | 9      | D. |



| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6      | 7     | 8     | 9      | D. |
|-----|--------|------|------|------|------|------|--------|-------|-------|--------|----|
| 580 | 763428 | 3503 | 3578 | 3653 | 3727 | 3802 | 3377   | 3952  | 4027  | 4101   | 75 |
| 581 | 4176   | 4251 | 4326 | 4400 | 4475 | 4550 | 4624   | 4699  | 4774  | 4848   | 75 |
| 582 | 4923   | 4998 | 5072 | 5147 | 5221 | 5296 | 5370   | 5445  | 5520  | 5594   | 75 |
| 583 | 5669   | 5743 | 5818 | 5892 | 5966 | 6041 | 6115   | 6190  | 6264  | 6338   | 74 |
| 584 | 6413   | 6487 | 6562 | 6636 | 6710 | 6785 | 6859   | 6933  | 7007  | 7082   | 74 |
| 585 | 7156   | 7230 | 7304 | 7379 | 7453 | 7527 | 7601   | 7675  | 7749  | 7823   | 74 |
| 586 | 7898   | 7972 | 8046 | 8120 | 8194 | 8268 | 8342   | 8416  | 8490  | 8564   | 74 |
| 587 | 8638   | 8712 | 8786 | 8860 | 8934 | 9008 | 9082   | 9156  | 9230  | 9303   | 74 |
| 588 | 9377   | 9451 | 9525 | 9599 | 9673 | 9746 | 9820   | 9894  | 9968  | . . 42 | 74 |
| 589 | 770115 | 0189 | 0263 | 0336 | 0410 | 0484 | 0557   | 0631  | 0705  | 0778   | 74 |
| 590 | 770852 | 0926 | 0999 | 1073 | 1146 | 1220 | 1293   | 1367  | 1440  | 1514   | 74 |
| 591 | 1587   | 1661 | 1734 | 1808 | 1881 | 1955 | 2028   | 2102  | 2175  | 2248   | 73 |
| 592 | 2322   | 2395 | 2468 | 2542 | 2615 | 2688 | 2762   | 2835  | 2908  | 2981   | 73 |
| 593 | 3055   | 3128 | 3201 | 3274 | 3348 | 3421 | 3494   | 3567  | 3640  | 3713   | 73 |
| 594 | 3786   | 3860 | 3933 | 4006 | 4079 | 4152 | 4225   | 4298  | 4371  | 4444   | 73 |
| 595 | 4517   | 4590 | 4663 | 4736 | 4809 | 4882 | 4955   | 5028  | 5100  | 5173   | 73 |
| 596 | 5246   | 5319 | 5392 | 5465 | 5538 | 5610 | 5683   | 5756  | 5829  | 5902   | 73 |
| 597 | 5974   | 6047 | 6120 | 6193 | 6265 | 6338 | 6411   | 6483  | 6556  | 6629   | 73 |
| 598 | 6701   | 6774 | 6846 | 6919 | 6992 | 7064 | 7137   | 7209  | 7282  | 7354   | 73 |
| 599 | 7427   | 7499 | 7572 | 7644 | 7717 | 7789 | 7862   | 7934  | 8006  | 8079   | 72 |
| 600 | 778151 | 8224 | 8296 | 8368 | 8441 | 8513 | 8585   | 8658  | 8730  | 8802   | 72 |
| 601 | 8874   | 8947 | 9019 | 9091 | 9163 | 9236 | 9308   | 9380  | 9452  | 9524   | 72 |
| 602 | 9596   | 9669 | 9741 | 9813 | 9885 | 9957 | . . 29 | . 101 | . 173 | . 245  | 72 |
| 603 | 780317 | 0389 | 0461 | 0533 | 0605 | 0677 | 0749   | 0821  | 0893  | 0965   | 72 |
| 604 | 1037   | 1109 | 1181 | 1253 | 1324 | 1396 | 1468   | 1540  | 1612  | 1684   | 72 |
| 605 | 1755   | 1827 | 1899 | 1971 | 2042 | 2114 | 2186   | 2258  | 2329  | 2401   | 72 |
| 606 | 2473   | 2544 | 2616 | 2688 | 2759 | 2831 | 2902   | 2974  | 3046  | 3117   | 72 |
| 607 | 3189   | 3260 | 3332 | 3403 | 3475 | 3546 | 3618   | 3689  | 3761  | 3832   | 71 |
| 608 | 3904   | 3975 | 4046 | 4118 | 4189 | 4261 | 4332   | 4403  | 4475  | 4546   | 71 |
| 609 | 4617   | 4689 | 4760 | 4831 | 4902 | 4974 | 5045   | 5116  | 5187  | 5259   | 71 |
| 610 | 785330 | 5401 | 5472 | 5543 | 5615 | 5686 | 5757   | 5828  | 5899  | 5970   | 71 |
| 611 | 6041   | 6112 | 6183 | 6254 | 6325 | 6396 | 6467   | 6538  | 6609  | 6680   | 71 |
| 612 | 6751   | 6822 | 6893 | 6964 | 7035 | 7106 | 7177   | 7248  | 7319  | 7390   | 71 |
| 613 | 7460   | 7531 | 7602 | 7673 | 7744 | 7815 | 7885   | 7956  | 8027  | 8098   | 71 |
| 614 | 8168   | 8239 | 8310 | 8381 | 8451 | 8522 | 8593   | 8663  | 8734  | 8804   | 71 |
| 615 | 8875   | 8946 | 9016 | 9087 | 9157 | 9228 | 9299   | 9369  | 9440  | 9510   | 71 |
| 616 | 9581   | 9651 | 9722 | 9792 | 9863 | 9933 | . . 4  | . 74  | . 144 | . 215  | 70 |
| 617 | 790285 | 0356 | 0426 | 0496 | 0567 | 0637 | 0707   | 0778  | 0848  | 0918   | 70 |
| 618 | 0988   | 1059 | 1129 | 1199 | 1269 | 1340 | 1410   | 1480  | 1550  | 1620   | 70 |
| 619 | 1691   | 1761 | 1831 | 1901 | 1971 | 2041 | 2111   | 2181  | 2252  | 2322   | 70 |
| 620 | 792392 | 2462 | 2532 | 2602 | 2672 | 2742 | 2812   | 2882  | 2952  | 3022   | 70 |
| 621 | 3092   | 3162 | 3231 | 3301 | 3371 | 3441 | 3511   | 3581  | 3651  | 3721   | 70 |
| 622 | 3790   | 3860 | 3930 | 4000 | 4070 | 4139 | 4209   | 4279  | 4349  | 4418   | 70 |
| 623 | 4488   | 4558 | 4627 | 4697 | 4767 | 4836 | 4906   | 4976  | 5045  | 5115   | 70 |
| 624 | 5185   | 5254 | 5324 | 5393 | 5463 | 5532 | 5602   | 5672  | 5741  | 5811   | 70 |
| 625 | 5880   | 5949 | 6019 | 6088 | 6158 | 6227 | 6297   | 6366  | 6436  | 6505   | 69 |
| 626 | 6574   | 6644 | 6713 | 6782 | 6852 | 6921 | 6990   | 7060  | 7129  | 7198   | 69 |
| 627 | 7268   | 7337 | 7406 | 7475 | 7545 | 7614 | 7683   | 7752  | 7821  | 7890   | 69 |
| 628 | 7960   | 8029 | 8098 | 8167 | 8236 | 8305 | 8374   | 8443  | 8513  | 8582   | 69 |
| 629 | 8651   | 8720 | 8789 | 8858 | 8927 | 8996 | 9065   | 9134  | 9203  | 9272   | 69 |
| 630 | 799341 | 9409 | 9478 | 9547 | 9616 | 9685 | 9754   | 9823  | 9892  | 9961   | 69 |
| 631 | 800029 | 0098 | 0167 | 0236 | 0305 | 0373 | 0442   | 0511  | 0580  | 0648   | 69 |
| 632 | 0717   | 0786 | 0854 | 0923 | 0992 | 1061 | 1129   | 1198  | 1266  | 1335   | 69 |
| 633 | 1404   | 1472 | 1541 | 1609 | 1678 | 1747 | 1815   | 1884  | 1952  | 2021   | 69 |
| 634 | 2089   | 2158 | 2226 | 2295 | 2363 | 2432 | 2500   | 2568  | 2637  | 2705   | 69 |
| 635 | 2774   | 2842 | 2910 | 2979 | 3047 | 3116 | 3184   | 3252  | 3321  | 3389   | 68 |
| 636 | 3457   | 3525 | 3594 | 3662 | 3730 | 3798 | 3867   | 3935  | 4003  | 4071   | 68 |
| 637 | 4139   | 4208 | 4276 | 4344 | 4412 | 4480 | 4548   | 4616  | 4685  | 4753   | 68 |
| 638 | 4821   | 4889 | 4957 | 5025 | 5093 | 5161 | 5229   | 5297  | 5365  | 5433   | 68 |
| 639 | 5501   | 5569 | 5637 | 5705 | 5773 | 5841 | 5908   | 5976  | 6044  | 6112   | 68 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6      | 7     | 8     | 9      | D. |

| N.  | 0      | 1    | 2    | 3     | 4     | 5     | 6     | 7     | 8     | 9     | D. |
|-----|--------|------|------|-------|-------|-------|-------|-------|-------|-------|----|
| 640 | 806180 | 6248 | 6316 | 6384  | 6451  | 6519  | 6587  | 6655  | 6723  | 6790  | 68 |
| 641 | 6858   | 6926 | 6994 | 7061  | 7129  | 7197  | 7264  | 7332  | 7400  | 7467  | 68 |
| 642 | 7535   | 7603 | 7670 | 7738  | 7806  | 7873  | 7941  | 8008  | 8076  | 8143  | 68 |
| 643 | 8211   | 8279 | 8346 | 8414  | 8481  | 8549  | 8616  | 8684  | 8751  | 8818  | 67 |
| 644 | 8886   | 8953 | 9021 | 9088  | 9156  | 9223  | 9290  | 9358  | 9425  | 9492  | 67 |
| 645 | 9560   | 9627 | 9694 | 9762  | 9829  | 9896  | 9964  | ..31  | ..98  | .165  | 67 |
| 646 | 810233 | 0306 | 0367 | 0434  | 0501  | 0569  | 0636  | 0703  | 0770  | 0837  | 67 |
| 647 | 0904   | 0971 | 1039 | 1106  | 1173  | 1240  | 1307  | 1374  | 1441  | 1508  | 67 |
| 648 | 1575   | 1642 | 1709 | 1776  | 1843  | 1910  | 1977  | 2044  | 2111  | 2178  | 67 |
| 649 | 2245   | 2312 | 2379 | 2445  | 2512  | 2579  | 2646  | 2713  | 2780  | 2847  | 67 |
| 650 | 812913 | 2980 | 3047 | 3114  | 3181  | 3247  | 3314  | 3381  | 3448  | 3514  | 67 |
| 651 | 3581   | 3648 | 3714 | 3781  | 3848  | 3914  | 3981  | 4048  | 4114  | 4181  | 67 |
| 652 | 4248   | 4314 | 4381 | 4447  | 4514  | 4581  | 4647  | 4714  | 4780  | 4847  | 67 |
| 653 | 4913   | 4980 | 5046 | 5113  | 5179  | 5246  | 5312  | 5378  | 5445  | 5511  | 66 |
| 654 | 5578   | 5644 | 5711 | 5777  | 5843  | 5910  | 5976  | 6042  | 6109  | 6175  | 66 |
| 655 | 6241   | 6308 | 6374 | 6440  | 6506  | 6573  | 6639  | 6705  | 6771  | 6838  | 66 |
| 656 | 6904   | 6970 | 7036 | 7102  | 7169  | 7235  | 7301  | 7367  | 7433  | 7499  | 66 |
| 657 | 7565   | 7631 | 7698 | 7764  | 7830  | 7896  | 7962  | 8028  | 8094  | 8160  | 66 |
| 658 | 8226   | 8292 | 8358 | 8424  | 8490  | 8556  | 8622  | 8688  | 8754  | 8820  | 66 |
| 659 | 8885   | 8951 | 9017 | 9083  | 9149  | 9215  | 9281  | 9346  | 9412  | 9478  | 66 |
| 660 | 819544 | 9610 | 9676 | 9741  | 9807  | 9873  | 9939  | ..4   | ..70  | .136  | 66 |
| 661 | 820201 | 0267 | 0333 | 0399  | 0464  | 0530  | 0595  | 0661  | 0727  | 0792  | 66 |
| 662 | 0858   | 0924 | 0989 | 1055  | 1120  | 1186  | 1251  | 1317  | 1382  | 1448  | 66 |
| 663 | 1514   | 1579 | 1645 | 1710  | 1775  | 1841  | 1906  | 1972  | 2037  | 2103  | 65 |
| 664 | 2168   | 2233 | 2299 | 2364  | 2430  | 2495  | 2560  | 2626  | 2691  | 2756  | 65 |
| 665 | 2822   | 2887 | 2952 | 3018  | 3083  | 3148  | 3213  | 3279  | 3344  | 3409  | 65 |
| 666 | 3474   | 3539 | 3605 | 3670  | 3735  | 3800  | 3865  | 3930  | 3996  | 4061  | 65 |
| 667 | 4126   | 4191 | 4256 | 4321  | 4386  | 4451  | 4516  | 4581  | 4646  | 4711  | 65 |
| 668 | 4776   | 4841 | 4906 | 4971  | 5036  | 5101  | 5166  | 5231  | 5296  | 5361  | 65 |
| 669 | 5426   | 5491 | 5556 | 5621  | 5686  | 5751  | 5815  | 5880  | 5945  | 6010  | 65 |
| 670 | 826075 | 6110 | 6204 | 6269  | 6334  | 6399  | 6464  | 6528  | 6593  | 6658  | 65 |
| 671 | 6723   | 6787 | 6852 | 6917  | 6981  | 7046  | 7111  | 7175  | 7240  | 7305  | 65 |
| 672 | 7359   | 7424 | 7489 | 7553  | 7628  | 7692  | 7757  | 7821  | 7886  | 7951  | 65 |
| 673 | 8015   | 8080 | 8144 | 8209  | 8273  | 8338  | 8402  | 8467  | 8531  | 8595  | 64 |
| 674 | 8660   | 8724 | 8789 | 8853  | 8918  | 8982  | 9046  | 9111  | 9175  | 9239  | 64 |
| 675 | 9304   | 9368 | 9432 | 9497  | 9561  | 9625  | 9690  | 9754  | 9818  | 9882  | 64 |
| 676 | 9947   | ..11 | ..75 | ..139 | ..204 | ..268 | ..332 | ..396 | ..460 | ..525 | 64 |
| 677 | 830589 | 0653 | 0717 | 0781  | 0845  | 0909  | 0973  | 1037  | 1102  | 1166  | 64 |
| 678 | 1230   | 1294 | 1358 | 1422  | 1486  | 1550  | 1614  | 1678  | 1742  | 1806  | 64 |
| 679 | 1870   | 1934 | 1998 | 2062  | 2126  | 2189  | 2253  | 2317  | 2381  | 2445  | 64 |
| 680 | 832509 | 2573 | 2637 | 2700  | 2764  | 2828  | 2892  | 2956  | 3020  | 3083  | 64 |
| 681 | 3147   | 3211 | 3275 | 3338  | 3402  | 3466  | 3530  | 3593  | 3657  | 3721  | 64 |
| 682 | 3784   | 3848 | 3912 | 3975  | 4039  | 4103  | 4166  | 4229  | 4294  | 4357  | 64 |
| 683 | 4421   | 4484 | 4548 | 4611  | 4675  | 4739  | 4802  | 4866  | 4929  | 4993  | 64 |
| 684 | 5056   | 5120 | 5183 | 5247  | 5310  | 5373  | 5437  | 5500  | 5564  | 5627  | 63 |
| 685 | 5691   | 5754 | 5817 | 5881  | 5944  | 6007  | 6071  | 6134  | 6197  | 6261  | 63 |
| 686 | 6324   | 6387 | 6451 | 6514  | 6577  | 6641  | 6704  | 6767  | 6830  | 6894  | 63 |
| 687 | 6957   | 7020 | 7083 | 7146  | 7210  | 7273  | 7336  | 7399  | 7462  | 7525  | 63 |
| 688 | 7588   | 7652 | 7715 | 7778  | 7841  | 7904  | 7967  | 8030  | 8093  | 8156  | 63 |
| 689 | 8219   | 8282 | 8345 | 8408  | 8471  | 8534  | 8597  | 8660  | 8723  | 8786  | 63 |
| 690 | 838849 | 8912 | 8975 | 9038  | 9101  | 9164  | 9227  | 9289  | 9352  | 9415  | 63 |
| 691 | 9478   | 9541 | 9604 | 9667  | 9729  | 9792  | 9855  | 9918  | 9981  | ..43  | 63 |
| 692 | 840106 | 0169 | 0232 | 0294  | 0357  | 0420  | 0482  | 0545  | 0608  | 0671  | 63 |
| 693 | 0733   | 0796 | 0859 | 0921  | 0984  | 1046  | 1109  | 1172  | 1234  | 1297  | 63 |
| 694 | 1359   | 1422 | 1485 | 1547  | 1610  | 1672  | 1735  | 1797  | 1860  | 1922  | 63 |
| 695 | 1985   | 2047 | 2110 | 2172  | 2235  | 2297  | 2360  | 2422  | 2484  | 2547  | 62 |
| 696 | 2609   | 2672 | 2734 | 2796  | 2859  | 2921  | 2983  | 3046  | 3108  | 3170  | 62 |
| 697 | 3233   | 3295 | 3357 | 3420  | 3482  | 3544  | 3606  | 3669  | 3731  | 3793  | 62 |
| 698 | 3855   | 3918 | 3980 | 4042  | 4104  | 4166  | 4229  | 4291  | 4353  | 4415  | 62 |
| 699 | 4477   | 4539 | 4601 | 4664  | 4726  | 4788  | 4850  | 4912  | 4974  | 5036  | 62 |
| N.  | 0      | 1    | 2    | 3     | 4     | 5     | 6     | 7     | 8     | 9     | D. |

| N.  | 0      | 1    | 2    | 3    | 4    | 5     | 6     | 7     | 8     | 9     | D. |
|-----|--------|------|------|------|------|-------|-------|-------|-------|-------|----|
| 700 | 845093 | 5160 | 5222 | 5284 | 5346 | 5408  | 5470  | 5532  | 5594  | 5656  | 62 |
| 701 | 5718   | 5780 | 5842 | 5904 | 5966 | 6028  | 6090  | 6151  | 6213  | 6275  | 62 |
| 702 | 6337   | 6399 | 6461 | 6523 | 6585 | 6646  | 6708  | 6770  | 6832  | 6894  | 62 |
| 703 | 6955   | 7017 | 7079 | 7141 | 7202 | 7264  | 7326  | 7388  | 7449  | 7511  | 62 |
| 704 | 7573   | 7634 | 7696 | 7758 | 7819 | 7881  | 7943  | 8004  | 8066  | 8128  | 62 |
| 705 | 8189   | 8251 | 8312 | 8374 | 8435 | 8497  | 8559  | 8620  | 8682  | 8743  | 62 |
| 706 | 8805   | 8866 | 8928 | 8989 | 9051 | 9112  | 9174  | 9235  | 9297  | 9358  | 61 |
| 707 | 9419   | 9481 | 9542 | 9604 | 9665 | 9726  | 9788  | 9849  | 9911  | 9972  | 61 |
| 708 | 850033 | 0095 | 0156 | 0217 | 0279 | 0340  | 0401  | 0462  | 0524  | 0585  | 61 |
| 709 | 0646   | 0707 | 0769 | 0830 | 0891 | 0952  | 1014  | 1075  | 1136  | 1197  | 61 |
| 710 | 851258 | 1320 | 1381 | 1442 | 1503 | 1564  | 1625  | 1686  | 1747  | 1809  | 61 |
| 711 | 1870   | 1931 | 1992 | 2053 | 2114 | 2175  | 2236  | 2297  | 2358  | 2419  | 61 |
| 712 | 2480   | 2541 | 2602 | 2663 | 2724 | 2785  | 2846  | 2907  | 2968  | 3029  | 61 |
| 713 | 3090   | 3150 | 3211 | 3272 | 3333 | 3394  | 3455  | 3516  | 3577  | 3637  | 61 |
| 714 | 3638   | 3759 | 3820 | 3881 | 3941 | 4002  | 4063  | 4124  | 4185  | 4245  | 61 |
| 715 | 4306   | 4367 | 4428 | 4488 | 4549 | 4610  | 4670  | 4731  | 4792  | 4852  | 61 |
| 716 | 4913   | 4974 | 5034 | 5095 | 5156 | 5216  | 5277  | 5337  | 5398  | 5459  | 61 |
| 717 | 5519   | 5580 | 5640 | 5701 | 5761 | 5822  | 5882  | 5943  | 6003  | 6064  | 61 |
| 718 | 6124   | 6185 | 6245 | 6306 | 6366 | 6427  | 6487  | 6548  | 6608  | 6668  | 60 |
| 719 | 6729   | 6789 | 6850 | 6910 | 6970 | 7031  | 7091  | 7152  | 7212  | 7272  | 60 |
| 720 | 857332 | 7393 | 7453 | 7513 | 7574 | 7634  | 7694  | 7755  | 7815  | 7875  | 60 |
| 721 | 7935   | 7995 | 8056 | 8116 | 8176 | 8236  | 8297  | 8357  | 8417  | 8477  | 60 |
| 722 | 8537   | 8597 | 8657 | 8718 | 8778 | 8838  | 8898  | 8958  | 9018  | 9078  | 60 |
| 723 | 9138   | 9198 | 9258 | 9318 | 9379 | 9439  | 9499  | 9559  | 9619  | 9679  | 60 |
| 724 | 9739   | 9799 | 9859 | 9918 | 9978 | ..33  | ..98  | ..158 | ..218 | ..278 | 60 |
| 725 | 860338 | 0398 | 0458 | 0518 | 0578 | 0637  | 0697  | 0757  | 0817  | 0877  | 60 |
| 726 | 0937   | 0996 | 1056 | 1116 | 1176 | 1236  | 1295  | 1355  | 1415  | 1475  | 60 |
| 727 | 1534   | 1594 | 1654 | 1714 | 1773 | 1833  | 1893  | 1952  | 2012  | 2072  | 60 |
| 728 | 2131   | 2191 | 2251 | 2310 | 2370 | 2430  | 2489  | 2549  | 2608  | 2668  | 60 |
| 729 | 2728   | 2787 | 2847 | 2906 | 2966 | 3025  | 3085  | 3144  | 3204  | 3263  | 60 |
| 730 | 863323 | 3382 | 3442 | 3501 | 3561 | 3620  | 3680  | 3739  | 3799  | 3858  | 59 |
| 731 | 3917   | 3977 | 4036 | 4096 | 4155 | 4214  | 4274  | 4333  | 4392  | 4452  | 59 |
| 732 | 4511   | 4570 | 4630 | 4689 | 4748 | 4808  | 4867  | 4926  | 4985  | 5045  | 59 |
| 733 | 5104   | 5163 | 5222 | 5282 | 5341 | 5400  | 5459  | 5519  | 5578  | 5637  | 59 |
| 734 | 5696   | 5755 | 5814 | 5874 | 5933 | 5992  | 6051  | 6110  | 6169  | 6228  | 59 |
| 735 | 6287   | 6346 | 6405 | 6465 | 6524 | 6583  | 6642  | 6701  | 6760  | 6819  | 59 |
| 736 | 6878   | 6937 | 6995 | 7055 | 7114 | 7173  | 7232  | 7291  | 7350  | 7409  | 59 |
| 737 | 7467   | 7526 | 7585 | 7644 | 7703 | 7762  | 7821  | 7880  | 7939  | 7998  | 59 |
| 738 | 8056   | 8115 | 8174 | 8233 | 8292 | 8350  | 8409  | 8468  | 8527  | 8586  | 59 |
| 739 | 8644   | 8703 | 8762 | 8821 | 8879 | 8938  | 8997  | 9056  | 9114  | 9173  | 59 |
| 740 | 869232 | 9290 | 9349 | 9408 | 9466 | 5525  | 9584  | 9642  | 9701  | 9760  | 59 |
| 741 | 9818   | 9877 | 9935 | 9994 | ..53 | ..111 | ..170 | ..228 | ..287 | ..345 | 59 |
| 742 | 870404 | 0462 | 0521 | 0579 | 0638 | 0696  | 0755  | 0813  | 0872  | 0930  | 58 |
| 743 | 0989   | 1047 | 1106 | 1164 | 1223 | 1281  | 1339  | 1398  | 1456  | 1515  | 58 |
| 744 | 1573   | 1631 | 1690 | 1748 | 1806 | 1865  | 1923  | 1981  | 2040  | 2098  | 58 |
| 745 | 2156   | 2215 | 2273 | 2331 | 2389 | 2448  | 2506  | 2564  | 2622  | 2681  | 58 |
| 746 | 2739   | 2797 | 2855 | 2913 | 2972 | 3030  | 3088  | 3146  | 3204  | 3262  | 58 |
| 747 | 3321   | 3379 | 3437 | 3495 | 3553 | 3611  | 3669  | 3727  | 3785  | 3844  | 58 |
| 748 | 3902   | 3960 | 4018 | 4076 | 4134 | 4192  | 4250  | 4308  | 4366  | 4424  | 58 |
| 749 | 4482   | 4540 | 4598 | 4656 | 4714 | 4772  | 4830  | 4888  | 4945  | 5003  | 58 |
| 750 | 875061 | 5119 | 5177 | 5235 | 5293 | 5351  | 5409  | 5466  | 5524  | 5582  | 58 |
| 751 | 5640   | 5698 | 5756 | 5813 | 5871 | 5929  | 5987  | 6045  | 6102  | 6160  | 58 |
| 752 | 6218   | 6276 | 6333 | 6391 | 6449 | 6507  | 6564  | 6622  | 6680  | 6737  | 58 |
| 753 | 6795   | 6853 | 6910 | 6968 | 7026 | 7083  | 7141  | 7199  | 7256  | 7314  | 58 |
| 754 | 7371   | 7429 | 7487 | 7544 | 7602 | 7659  | 7717  | 7774  | 7832  | 7889  | 68 |
| 755 | 7947   | 8004 | 8062 | 8119 | 8177 | 8234  | 8292  | 8349  | 8407  | 8464  | 57 |
| 756 | 8522   | 8579 | 8637 | 8694 | 8752 | 8809  | 8866  | 8924  | 8981  | 9039  | 57 |
| 757 | 9096   | 9153 | 9211 | 9268 | 9325 | 9383  | 9440  | 9497  | 9555  | 9612  | 57 |
| 758 | 9669   | 9726 | 9784 | 9841 | 9898 | 9956  | ..13  | ..70  | ..127 | ..185 | 57 |
| 759 | 880242 | 0299 | 0356 | 0413 | 0471 | 0528  | 0585  | 0642  | 0699  | 0756  | 57 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5     | 6     | 7     | 8     | 9     | D. |



| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D. |
|-----|--------|------|------|------|------|------|------|------|------|------|----|
| 760 | 880814 | 0871 | 0928 | 0985 | 1042 | 1099 | 1156 | 1213 | 1271 | 1328 | 57 |
| 761 | 1385   | 1442 | 1499 | 1556 | 1613 | 1670 | 1727 | 1784 | 1841 | 1898 | 57 |
| 762 | 1955   | 2012 | 2069 | 2126 | 2183 | 2240 | 2297 | 2354 | 2411 | 2468 | 57 |
| 763 | 2525   | 2581 | 2638 | 2695 | 2752 | 2809 | 2866 | 2923 | 2980 | 3037 | 57 |
| 764 | 3093   | 3150 | 3207 | 3264 | 3321 | 3377 | 3434 | 3491 | 3548 | 3605 | 57 |
| 765 | 3661   | 3718 | 3775 | 3832 | 3888 | 3945 | 4002 | 4059 | 4115 | 4172 | 57 |
| 766 | 4229   | 4285 | 4342 | 4399 | 4455 | 4512 | 4569 | 4625 | 4682 | 4739 | 57 |
| 767 | 4795   | 4852 | 4909 | 4965 | 5022 | 5078 | 5135 | 5192 | 5248 | 5305 | 57 |
| 768 | 5361   | 5418 | 5474 | 5531 | 5587 | 5644 | 5700 | 5757 | 5813 | 5870 | 57 |
| 769 | 5926   | 5983 | 6039 | 6096 | 6152 | 6209 | 6265 | 6321 | 6378 | 6434 | 56 |
| 770 | 886491 | 6547 | 6604 | 6660 | 6716 | 6773 | 6829 | 6885 | 6942 | 6998 | 56 |
| 771 | 7054   | 7111 | 7167 | 7223 | 7280 | 7336 | 7392 | 7449 | 7505 | 7561 | 56 |
| 772 | 7617   | 7674 | 7730 | 7786 | 7842 | 7898 | 7955 | 8011 | 8067 | 8123 | 56 |
| 773 | 8179   | 8236 | 8292 | 8348 | 8404 | 8460 | 8516 | 8573 | 8629 | 8685 | 56 |
| 774 | 8741   | 8797 | 8853 | 8909 | 8965 | 9021 | 9077 | 9134 | 9190 | 9246 | 56 |
| 775 | 9302   | 9358 | 9414 | 9470 | 9526 | 9582 | 9638 | 9694 | 9750 | 9806 | 56 |
| 776 | 9862   | 9918 | 9974 | .30  | .86  | .141 | .197 | .253 | .309 | .365 | 56 |
| 777 | 890421 | 0477 | 0533 | 0589 | 0645 | 0700 | 0756 | 0812 | 0868 | 0924 | 56 |
| 778 | 0980   | 1035 | 1091 | 1147 | 1203 | 1259 | 1314 | 1370 | 1426 | 1482 | 56 |
| 779 | 1537   | 1593 | 1649 | 1705 | 1760 | 1816 | 1872 | 1928 | 1983 | 2039 | 56 |
| 780 | 892095 | 2150 | 2206 | 2262 | 2317 | 2373 | 2429 | 2484 | 2540 | 2595 | 56 |
| 781 | 2651   | 2707 | 2762 | 2818 | 2873 | 2929 | 2985 | 3040 | 3096 | 3151 | 56 |
| 782 | 3207   | 3262 | 3318 | 3373 | 3429 | 3484 | 3540 | 3595 | 3651 | 3706 | 56 |
| 783 | 3762   | 3817 | 3873 | 3928 | 3984 | 4039 | 4094 | 4150 | 4205 | 4261 | 55 |
| 784 | 4316   | 4371 | 4427 | 4482 | 4538 | 4593 | 4648 | 4704 | 4759 | 4814 | 55 |
| 785 | 4870   | 4925 | 4980 | 5036 | 5091 | 5146 | 5201 | 5257 | 5312 | 5367 | 55 |
| 786 | 5423   | 5478 | 5533 | 5588 | 5644 | 5699 | 5754 | 5809 | 5864 | 5920 | 55 |
| 787 | 5975   | 6030 | 6085 | 6140 | 6195 | 6251 | 6306 | 6361 | 6416 | 6471 | 55 |
| 788 | 6526   | 6581 | 6636 | 6692 | 6747 | 6802 | 6857 | 6912 | 6967 | 7022 | 55 |
| 789 | 7077   | 7132 | 7187 | 7242 | 7297 | 7352 | 7407 | 7462 | 7517 | 7572 | 55 |
| 790 | 807627 | 7682 | 7737 | 7792 | 7847 | 7902 | 7957 | 8012 | 8067 | 8122 | 55 |
| 791 | 8176   | 8231 | 8286 | 8341 | 8396 | 8451 | 8506 | 8561 | 8615 | 8670 | 55 |
| 792 | 8725   | 8780 | 8835 | 8890 | 8944 | 8999 | 9054 | 9109 | 9164 | 9218 | 55 |
| 793 | 9273   | 9328 | 9383 | 9437 | 9492 | 9547 | 9602 | 9656 | 9711 | 9766 | 55 |
| 794 | 9821   | 9875 | 9930 | 9985 | .39  | .94  | .149 | .203 | .258 | .312 | 55 |
| 795 | 900367 | 0422 | 0476 | 0531 | 0586 | 0640 | 0695 | 0749 | 0804 | 0859 | 55 |
| 796 | 0913   | 0968 | 1022 | 1077 | 1131 | 1186 | 1240 | 1295 | 1349 | 1404 | 55 |
| 797 | 1458   | 1513 | 1567 | 1622 | 1676 | 1731 | 1785 | 1840 | 1894 | 1948 | 54 |
| 798 | 2003   | 2057 | 2112 | 2166 | 2221 | 2275 | 2329 | 2384 | 2438 | 2492 | 54 |
| 799 | 2547   | 2601 | 2655 | 2710 | 2764 | 2818 | 2873 | 2927 | 2981 | 3036 | 54 |
| 800 | 903090 | 3144 | 3199 | 3253 | 3307 | 3361 | 3416 | 3470 | 3524 | 3578 | 54 |
| 801 | 3633   | 3687 | 3741 | 3795 | 3849 | 3904 | 3958 | 4012 | 4066 | 4120 | 54 |
| 802 | 4174   | 4229 | 4283 | 4337 | 4391 | 4445 | 4499 | 4553 | 4607 | 4661 | 54 |
| 803 | 4716   | 4770 | 4824 | 4878 | 4932 | 4986 | 5040 | 5094 | 5148 | 5202 | 54 |
| 804 | 5256   | 5310 | 5364 | 5418 | 5472 | 5526 | 5580 | 5634 | 5688 | 5742 | 54 |
| 805 | 5796   | 5850 | 5904 | 5958 | 6012 | 6066 | 6119 | 6173 | 6227 | 6281 | 54 |
| 806 | 6335   | 6389 | 6443 | 6497 | 6551 | 6604 | 6658 | 6712 | 6766 | 6820 | 54 |
| 807 | 6874   | 6927 | 6981 | 7035 | 7089 | 7143 | 7196 | 7250 | 7304 | 7358 | 54 |
| 808 | 7411   | 7465 | 7519 | 7573 | 7626 | 7680 | 7734 | 7787 | 7841 | 7895 | 54 |
| 809 | 7949   | 8002 | 8056 | 8110 | 8163 | 8217 | 8270 | 8324 | 8378 | 8431 | 54 |
| 810 | 908485 | 8539 | 8592 | 8646 | 8699 | 8753 | 8807 | 8860 | 8914 | 8967 | 54 |
| 811 | 9021   | 9074 | 9128 | 9181 | 9235 | 9289 | 9342 | 9396 | 9449 | 9503 | 54 |
| 812 | 9556   | 9610 | 9663 | 9716 | 9770 | 9823 | 9877 | 9930 | 9984 | .37  | 53 |
| 813 | 910091 | 0144 | 0197 | 0251 | 0304 | 0358 | 0411 | 0464 | 0518 | 0571 | 53 |
| 814 | 0624   | 0678 | 0731 | 0784 | 0838 | 0891 | 0944 | 0998 | 1051 | 1104 | 53 |
| 815 | 1158   | 1211 | 1264 | 1317 | 1371 | 1424 | 1477 | 1530 | 1584 | 1637 | 53 |
| 816 | 1690   | 1743 | 1797 | 1850 | 1903 | 1956 | 2009 | 2063 | 2116 | 2169 | 53 |
| 817 | 2222   | 2275 | 2328 | 2381 | 2435 | 2488 | 2541 | 2594 | 2647 | 2700 | 53 |
| 818 | 2753   | 2806 | 2859 | 2913 | 2966 | 3019 | 3072 | 3125 | 3178 | 3231 | 53 |
| 819 | 3284   | 3337 | 3390 | 3443 | 3496 | 3549 | 3602 | 3655 | 3708 | 3761 | 53 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D. |

| N.  | 0      | 1    | 2      | 3      | 4     | 5     | 6     | 7     | 8      | 9      | D. |
|-----|--------|------|--------|--------|-------|-------|-------|-------|--------|--------|----|
| 820 | 913814 | 3867 | 3920   | 3973   | 4026  | 4079  | 4132  | 4184  | 4237   | 4290   | 53 |
| 821 | 4343   | 4396 | 4449   | 4502   | 4555  | 4608  | 4660  | 4713  | 4766   | 4819   | 53 |
| 822 | 4872   | 4925 | 4977   | 5030   | 5083  | 5136  | 5189  | 5241  | 5294   | 5347   | 53 |
| 823 | 5400   | 5453 | 5505   | 5558   | 5611  | 5664  | 5716  | 5769  | 5822   | 5875   | 53 |
| 824 | 5927   | 5980 | 6033   | 6085   | 6138  | 6191  | 6243  | 6296  | 6349   | 6401   | 53 |
| 825 | 6454   | 6507 | 6559   | 6612   | 6664  | 6717  | 6770  | 6822  | 6875   | 6927   | 53 |
| 826 | 6980   | 7033 | 7085   | 7138   | 7190  | 7243  | 7295  | 7348  | 7400   | 7453   | 53 |
| 827 | 7506   | 7558 | 7611   | 7663   | 7716  | 7768  | 7820  | 7873  | 7925   | 7978   | 52 |
| 828 | 8030   | 8083 | 8135   | 8188   | 8240  | 8293  | 8345  | 8397  | 8450   | 8502   | 52 |
| 829 | 8555   | 8607 | 8659   | 8712   | 8764  | 8816  | 8869  | 8921  | 8973   | 9026   | 52 |
| 830 | 919078 | 9130 | 9183   | 9235   | 9287  | 9340  | 9392  | 9444  | 9496   | 9549   | 52 |
| 831 | 9601   | 9653 | 9706   | 9758   | 9810  | 9862  | 9914  | 9967  | . . 19 | . . 71 | 52 |
| 832 | 920123 | 0176 | 0228   | 0280   | 0332  | 0384  | 0436  | 0489  | 0541   | 0593   | 52 |
| 833 | 0645   | 0697 | 0749   | 0801   | 0853  | 0906  | 0958  | 1010  | 1062   | 1114   | 52 |
| 834 | 1166   | 1218 | 1270   | 1322   | 1374  | 1426  | 1478  | 1530  | 1582   | 1634   | 52 |
| 835 | 1686   | 1738 | 1790   | 1842   | 1894  | 1946  | 1998  | 2050  | 2102   | 2154   | 52 |
| 836 | 2206   | 2258 | 2310   | 2362   | 2414  | 2466  | 2518  | 2570  | 2622   | 2674   | 52 |
| 837 | 2725   | 2777 | 2829   | 2881   | 2933  | 2985  | 3037  | 3089  | 3140   | 3192   | 52 |
| 838 | 3244   | 3296 | 3348   | 3399   | 3451  | 3503  | 3555  | 3607  | 3658   | 3710   | 52 |
| 839 | 3762   | 3814 | 3865   | 3917   | 3969  | 4021  | 4072  | 4124  | 4176   | 4228   | 52 |
| 840 | 924279 | 4331 | 4383   | 4434   | 4486  | 4538  | 4589  | 4641  | 4693   | 4744   | 52 |
| 841 | 4796   | 4848 | 4899   | 4951   | 5003  | 5054  | 5106  | 5157  | 5209   | 5261   | 52 |
| 842 | 5312   | 5364 | 5415   | 5467   | 5518  | 5570  | 5621  | 5673  | 5725   | 5776   | 52 |
| 843 | 5828   | 5879 | 5931   | 5982   | 6034  | 6085  | 6137  | 6188  | 6240   | 6291   | 51 |
| 844 | 6342   | 6394 | 6445   | 6497   | 6548  | 6600  | 6651  | 6702  | 6754   | 6805   | 51 |
| 845 | 6857   | 6908 | 6959   | 7011   | 7062  | 7114  | 7165  | 7216  | 7268   | 7319   | 51 |
| 846 | 7370   | 7422 | 7473   | 7524   | 7576  | 7627  | 7678  | 7730  | 7781   | 7832   | 51 |
| 847 | 7883   | 7935 | 7986   | 8037   | 8088  | 8140  | 8191  | 8242  | 8293   | 8345   | 51 |
| 848 | 8396   | 8447 | 8498   | 8549   | 8601  | 8652  | 8703  | 8754  | 8805   | 8857   | 51 |
| 849 | 8908   | 8959 | 9010   | 9061   | 9112  | 9163  | 9215  | 9266  | 9317   | 9368   | 51 |
| 850 | 929419 | 9470 | 9521   | 9572   | 9623  | 9674  | 9725  | 9776  | 9827   | 9879   | 51 |
| 851 | 9930   | 9981 | . . 32 | . . 83 | . 134 | . 185 | . 236 | . 287 | . 338  | . 389  | 51 |
| 852 | 930440 | 0491 | 0542   | 0592   | 0643  | 0694  | 0745  | 0796  | 0847   | 0898   | 51 |
| 853 | 0949   | 1000 | 1051   | 1102   | 1153  | 1204  | 1254  | 1305  | 1356   | 1407   | 51 |
| 854 | 1458   | 1509 | 1560   | 1610   | 1661  | 1712  | 1763  | 1814  | 1865   | 1915   | 51 |
| 855 | 1966   | 2017 | 2068   | 2118   | 2169  | 2220  | 2271  | 2322  | 2372   | 2423   | 51 |
| 856 | 2474   | 2524 | 2575   | 2626   | 2677  | 2727  | 2778  | 2829  | 2879   | 2930   | 51 |
| 857 | 2981   | 3031 | 3082   | 3133   | 3183  | 3234  | 3285  | 3335  | 3386   | 3437   | 51 |
| 858 | 3487   | 3538 | 3589   | 3639   | 3690  | 3740  | 3791  | 3841  | 3892   | 3943   | 51 |
| 859 | 3993   | 4044 | 4094   | 4145   | 4195  | 4246  | 4296  | 4347  | 4397   | 4448   | 51 |
| 860 | 934498 | 4549 | 4599   | 4650   | 4700  | 4751  | 4801  | 4852  | 4902   | 4953   | 50 |
| 861 | 5003   | 5054 | 5104   | 5154   | 5205  | 5255  | 5306  | 5356  | 5406   | 5457   | 50 |
| 862 | 5507   | 5558 | 5608   | 5658   | 5709  | 5759  | 5809  | 5860  | 5910   | 5960   | 50 |
| 863 | 6011   | 6061 | 6111   | 6162   | 6212  | 6262  | 6313  | 6363  | 6413   | 6463   | 50 |
| 864 | 6514   | 6564 | 6614   | 6665   | 6715  | 6765  | 6815  | 6865  | 6916   | 6966   | 50 |
| 865 | 7016   | 7066 | 7117   | 7167   | 7217  | 7267  | 7317  | 7367  | 7418   | 7468   | 50 |
| 866 | 7518   | 7568 | 7618   | 7668   | 7718  | 7769  | 7819  | 7869  | 7919   | 7969   | 50 |
| 867 | 8019   | 8069 | 8119   | 8169   | 8219  | 8269  | 8320  | 8370  | 8420   | 8470   | 50 |
| 868 | 8520   | 8570 | 8620   | 8670   | 8720  | 8770  | 8820  | 8870  | 8920   | 8970   | 50 |
| 869 | 9020   | 9070 | 9120   | 9170   | 9220  | 9270  | 9320  | 9369  | 9419   | 9469   | 50 |
| 870 | 939519 | 9569 | 9619   | 9669   | 9719  | 9769  | 9819  | 9869  | 9918   | 9968   | 50 |
| 871 | 940018 | 0068 | 0118   | 0168   | 0218  | 0267  | 0317  | 0367  | 0417   | 0467   | 50 |
| 872 | 0516   | 0566 | 0616   | 0666   | 0716  | 0765  | 0815  | 0865  | 0915   | 0964   | 50 |
| 873 | 1014   | 1064 | 1114   | 1163   | 1213  | 1263  | 1313  | 1362  | 1412   | 1462   | 50 |
| 874 | 1511   | 1561 | 1611   | 1660   | 1710  | 1760  | 1809  | 1859  | 1909   | 1958   | 50 |
| 875 | 2008   | 2058 | 2107   | 2157   | 2207  | 2256  | 2306  | 2355  | 2405   | 2455   | 50 |
| 876 | 2504   | 2554 | 2603   | 2653   | 2702  | 2752  | 2801  | 2851  | 2901   | 2950   | 50 |
| 877 | 3000   | 3049 | 3099   | 3148   | 3198  | 3247  | 3297  | 3346  | 3396   | 3445   | 49 |
| 878 | 3495   | 3544 | 3593   | 3643   | 3692  | 3742  | 3791  | 3841  | 3890   | 3939   | 49 |
| 879 | 3989   | 4038 | 4088   | 4137   | 4186  | 4236  | 4285  | 4335  | 4384   | 4433   | 49 |
| N.  | 0      | 1    | 2      | 3      | 4     | 5     | 6     | 7     | 8      | 9      | D. |



| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D. |
|-----|--------|------|------|------|------|------|------|------|------|------|----|
| 880 | 944483 | 4532 | 4581 | 4631 | 4680 | 4729 | 4779 | 4828 | 4877 | 4927 | 49 |
| 881 | 4976   | 5025 | 5074 | 5124 | 5173 | 5222 | 5272 | 5321 | 5370 | 5419 | 49 |
| 882 | 5469   | 5518 | 5567 | 5616 | 5665 | 5715 | 5764 | 5813 | 5862 | 5912 | 49 |
| 883 | 5961   | 6010 | 6059 | 6108 | 6157 | 6207 | 6256 | 6305 | 6354 | 6403 | 49 |
| 884 | 6452   | 6501 | 6551 | 6600 | 6649 | 6698 | 6747 | 6796 | 6845 | 6894 | 49 |
| 885 | 6943   | 6992 | 7041 | 7090 | 7140 | 7189 | 7238 | 7287 | 7336 | 7385 | 49 |
| 886 | 7434   | 7483 | 7532 | 7581 | 7630 | 7679 | 7728 | 7777 | 7826 | 7875 | 49 |
| 887 | 7924   | 7973 | 8022 | 8070 | 8119 | 8168 | 8217 | 8266 | 8315 | 8364 | 49 |
| 888 | 8413   | 8462 | 8511 | 8560 | 8609 | 8657 | 8706 | 8755 | 8804 | 8853 | 49 |
| 889 | 8932   | 8981 | 8999 | 9048 | 9097 | 9146 | 9195 | 9244 | 9292 | 9341 | 49 |
| 890 | 949390 | 9439 | 9488 | 9536 | 9585 | 9634 | 9683 | 9731 | 9780 | 9829 | 49 |
| 891 | 9878   | 9926 | 9975 | ..24 | ..73 | .121 | .170 | .219 | .267 | .316 | 49 |
| 892 | 950365 | 0414 | 0462 | 0511 | 0560 | 0608 | 0657 | 0706 | 0754 | 0803 | 49 |
| 893 | 0851   | 0900 | 0949 | 0997 | 1046 | 1095 | 1143 | 1192 | 1240 | 1289 | 49 |
| 894 | 1338   | 1386 | 1435 | 1483 | 1532 | 1580 | 1629 | 1677 | 1726 | 1775 | 49 |
| 895 | 1823   | 1872 | 1920 | 1969 | 2017 | 2066 | 2114 | 2163 | 2211 | 2260 | 48 |
| 896 | 2308   | 2356 | 2405 | 2453 | 2502 | 2550 | 2599 | 2647 | 2696 | 2744 | 48 |
| 897 | 2792   | 2841 | 2889 | 2938 | 2986 | 3034 | 3083 | 3131 | 3180 | 3228 | 48 |
| 898 | 3276   | 3325 | 3373 | 3421 | 3470 | 3518 | 3566 | 3615 | 3663 | 3711 | 48 |
| 899 | 3760   | 3808 | 3856 | 3905 | 3953 | 4001 | 4049 | 4098 | 4146 | 4194 | 48 |
| 900 | 954243 | 4291 | 4339 | 4387 | 4435 | 4484 | 4532 | 4580 | 4628 | 4677 | 48 |
| 901 | 4725   | 4773 | 4821 | 4869 | 4918 | 4966 | 5014 | 5062 | 5110 | 5158 | 48 |
| 902 | 5207   | 5255 | 5303 | 5351 | 5399 | 5447 | 5495 | 5543 | 5592 | 5640 | 48 |
| 903 | 5688   | 5736 | 5784 | 5832 | 5880 | 5928 | 5976 | 6024 | 6072 | 6120 | 48 |
| 904 | 6168   | 6216 | 6265 | 6313 | 6361 | 6409 | 6457 | 6505 | 6553 | 6601 | 48 |
| 905 | 6649   | 6697 | 6745 | 6793 | 6840 | 6888 | 6936 | 6984 | 7032 | 7080 | 48 |
| 906 | 7128   | 7176 | 7224 | 7272 | 7320 | 7368 | 7416 | 7464 | 7512 | 7559 | 48 |
| 907 | 7607   | 7655 | 7703 | 7751 | 7799 | 7847 | 7894 | 7942 | 7990 | 8038 | 48 |
| 908 | 8086   | 8134 | 8181 | 8229 | 8277 | 8325 | 8373 | 8421 | 8468 | 8516 | 48 |
| 909 | 8594   | 8642 | 8689 | 8737 | 8785 | 8833 | 8880 | 8928 | 8976 | 9024 | 48 |
| 910 | 959041 | 9089 | 9137 | 9185 | 9232 | 9280 | 9328 | 9375 | 9423 | 9471 | 48 |
| 911 | 9518   | 9566 | 9614 | 9661 | 9709 | 9757 | 9804 | 9852 | 9900 | 9947 | 48 |
| 912 | 9995   | ..42 | ..90 | .138 | .185 | .233 | .280 | .328 | .376 | .423 | 48 |
| 913 | 930471 | 0518 | 0566 | 0613 | 0661 | 0709 | 0756 | 0804 | 0851 | 0899 | 48 |
| 914 | 0946   | 0994 | 1041 | 1089 | 1136 | 1184 | 1231 | 1279 | 1326 | 1374 | 47 |
| 915 | 1421   | 1469 | 1516 | 1563 | 1611 | 1658 | 1706 | 1753 | 1801 | 1848 | 47 |
| 916 | 1895   | 1943 | 1990 | 2038 | 2085 | 2132 | 2180 | 2227 | 2275 | 2322 | 47 |
| 917 | 2369   | 2417 | 2464 | 2511 | 2559 | 2606 | 2653 | 2701 | 2748 | 2795 | 47 |
| 918 | 2843   | 2890 | 2937 | 2985 | 3032 | 3079 | 3126 | 3174 | 3221 | 3268 | 47 |
| 919 | 3316   | 3363 | 3410 | 3457 | 3504 | 3552 | 3599 | 3646 | 3693 | 3741 | 47 |
| 920 | 963789 | 3835 | 3882 | 3929 | 3977 | 4024 | 4071 | 4118 | 4165 | 4212 | 47 |
| 921 | 4260   | 4307 | 4354 | 4401 | 4448 | 4495 | 4542 | 4590 | 4637 | 4684 | 47 |
| 922 | 4731   | 4778 | 4825 | 4872 | 4919 | 4966 | 5013 | 5061 | 5108 | 5155 | 47 |
| 923 | 5202   | 5249 | 5296 | 5343 | 5390 | 5437 | 5484 | 5531 | 5578 | 5625 | 47 |
| 924 | 5672   | 5719 | 5766 | 5813 | 5860 | 5907 | 5954 | 6001 | 6048 | 6095 | 47 |
| 925 | 6142   | 6189 | 6236 | 6283 | 6329 | 6376 | 6423 | 6470 | 6517 | 6564 | 47 |
| 926 | 6611   | 6658 | 6705 | 6752 | 6799 | 6845 | 6892 | 6939 | 6986 | 7033 | 47 |
| 927 | 7080   | 7127 | 7173 | 7220 | 7267 | 7314 | 7361 | 7408 | 7454 | 7501 | 47 |
| 928 | 7548   | 7595 | 7642 | 7688 | 7735 | 7782 | 7829 | 7875 | 7922 | 7969 | 47 |
| 929 | 8016   | 8062 | 8109 | 8156 | 8203 | 8249 | 8296 | 8343 | 8390 | 8436 | 47 |
| 930 | 968483 | 8530 | 8576 | 8623 | 8670 | 8716 | 8763 | 8810 | 8856 | 8903 | 47 |
| 931 | 8950   | 8996 | 9043 | 9090 | 9136 | 9183 | 9229 | 9276 | 9323 | 9369 | 47 |
| 932 | 9416   | 9463 | 9509 | 9556 | 9602 | 9649 | 9695 | 9742 | 9789 | 9835 | 47 |
| 933 | 9882   | 9928 | 9975 | ..21 | ..68 | .114 | .161 | .207 | .254 | .300 | 47 |
| 934 | 970347 | 0393 | 0440 | 0486 | 0533 | 0579 | 0626 | 0672 | 0719 | 0765 | 46 |
| 935 | 0812   | 0858 | 0904 | 0951 | 0997 | 1044 | 1090 | 1137 | 1183 | 1229 | 46 |
| 936 | 1276   | 1322 | 1369 | 1415 | 1461 | 1508 | 1554 | 1601 | 1647 | 1693 | 46 |
| 937 | 1740   | 1786 | 1832 | 1879 | 1925 | 1971 | 2018 | 2064 | 2110 | 2157 | 46 |
| 938 | 2203   | 2249 | 2295 | 2342 | 2388 | 2434 | 2481 | 2527 | 2573 | 2619 | 46 |
| 939 | 2666   | 2712 | 2758 | 2804 | 2851 | 2897 | 2943 | 2989 | 3035 | 3082 | 46 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | D. |

| N.  | 0      | 1    | 2    | 3      | 4      | 5     | 6     | 7     | 8     | 9     | D. |
|-----|--------|------|------|--------|--------|-------|-------|-------|-------|-------|----|
| 940 | 973123 | 3174 | 3220 | 3266   | 3313   | 3359  | 3405  | 3451  | 3497  | 3543  | 46 |
| 941 | 3590   | 3636 | 3682 | 3728   | 3774   | 3820  | 3866  | 3913  | 3959  | 4005  | 46 |
| 942 | 4051   | 4097 | 4143 | 4189   | 4235   | 4281  | 4327  | 4374  | 4420  | 4466  | 46 |
| 943 | 4512   | 4558 | 4604 | 4650   | 4696   | 4742  | 4788  | 4834  | 4880  | 4926  | 46 |
| 944 | 4972   | 5018 | 5064 | 5110   | 5156   | 5202  | 5248  | 5294  | 5340  | 5386  | 46 |
| 945 | 5432   | 5478 | 5524 | 5570   | 5616   | 5662  | 5707  | 5753  | 5799  | 5845  | 46 |
| 946 | 5891   | 5937 | 5983 | 6029   | 6075   | 6121  | 6167  | 6212  | 6258  | 6304  | 46 |
| 947 | 6350   | 6396 | 6442 | 6488   | 6533   | 6579  | 6625  | 6671  | 6717  | 6763  | 46 |
| 948 | 6808   | 6854 | 6900 | 6946   | 6992   | 7037  | 7083  | 7129  | 7175  | 7220  | 46 |
| 949 | 7266   | 7312 | 7358 | 7403   | 7449   | 7495  | 7541  | 7586  | 7632  | 7678  | 46 |
| 950 | 977724 | 7769 | 7815 | 7861   | 7906   | 7952  | 7998  | 8043  | 8089  | 8135  | 46 |
| 951 | 8181   | 8226 | 8272 | 8317   | 8363   | 8409  | 8454  | 8500  | 8546  | 8591  | 46 |
| 952 | 8637   | 8683 | 8728 | 8774   | 8819   | 8865  | 8911  | 8956  | 9002  | 9047  | 46 |
| 953 | 9093   | 9138 | 9184 | 9230   | 9275   | 9321  | 9366  | 9412  | 9457  | 9503  | 46 |
| 954 | 9548   | 9594 | 9639 | 9685   | 9730   | 9776  | 9821  | 9867  | 9912  | 9958  | 46 |
| 955 | 980003 | 0049 | 0094 | 0140   | 0185   | 0231  | 0276  | 0322  | 0367  | 0412  | 45 |
| 956 | 0458   | 0503 | 0549 | 0594   | 0640   | 0685  | 0730  | 0776  | 0821  | 0867  | 45 |
| 957 | 0912   | 0957 | 1003 | 1048   | 1093   | 1139  | 1184  | 1229  | 1275  | 1320  | 45 |
| 958 | 1366   | 1411 | 1456 | 1501   | 1547   | 1592  | 1637  | 1683  | 1728  | 1773  | 45 |
| 959 | 1819   | 1864 | 1909 | 1954   | 2000   | 2045  | 2090  | 2135  | 2181  | 2226  | 45 |
| 960 | 982271 | 2316 | 2362 | 2407   | 2452   | 2497  | 2543  | 2588  | 2633  | 2678  | 45 |
| 961 | 2723   | 2769 | 2814 | 2859   | 2904   | 2949  | 2994  | 3040  | 3085  | 3130  | 45 |
| 962 | 3175   | 3220 | 3265 | 3310   | 3356   | 3401  | 3446  | 3491  | 3536  | 3581  | 45 |
| 963 | 3626   | 3671 | 3716 | 3762   | 3807   | 3852  | 3897  | 3942  | 3987  | 4032  | 45 |
| 964 | 4077   | 4122 | 4167 | 4212   | 4257   | 4302  | 4347  | 4392  | 4437  | 4482  | 45 |
| 965 | 4527   | 4572 | 4617 | 4662   | 4707   | 4752  | 4797  | 4842  | 4887  | 4932  | 45 |
| 966 | 4977   | 5022 | 5067 | 5112   | 5157   | 5202  | 5247  | 5292  | 5337  | 5382  | 45 |
| 967 | 5426   | 5471 | 5516 | 5561   | 5606   | 5651  | 5696  | 5741  | 5786  | 5830  | 45 |
| 968 | 5875   | 5920 | 5965 | 6010   | 6055   | 6100  | 6144  | 6189  | 6234  | 6279  | 45 |
| 969 | 6324   | 6369 | 6413 | 6458   | 6503   | 6548  | 6593  | 6637  | 6682  | 6727  | 45 |
| 970 | 986772 | 6817 | 6861 | 6906   | 6951   | 6996  | 7040  | 7085  | 7130  | 7175  | 45 |
| 971 | 7219   | 7264 | 7309 | 7353   | 7398   | 7443  | 7488  | 7532  | 7577  | 7622  | 45 |
| 972 | 7666   | 7711 | 7756 | 7800   | 7845   | 7890  | 7934  | 7979  | 8024  | 8068  | 45 |
| 973 | 8113   | 8157 | 8202 | 8247   | 8291   | 8336  | 8381  | 8425  | 8470  | 8514  | 45 |
| 974 | 8559   | 8604 | 8648 | 8693   | 8737   | 8782  | 8826  | 8871  | 8916  | 8960  | 45 |
| 975 | 9005   | 9049 | 9094 | 9138   | 9183   | 9227  | 9272  | 9316  | 9361  | 9405  | 45 |
| 976 | 9450   | 9494 | 9539 | 9583   | 9628   | 9672  | 9717  | 9761  | 9806  | 9850  | 44 |
| 977 | 9895   | 9939 | 9983 | . . 28 | . . 72 | . 117 | . 161 | . 206 | . 250 | . 294 | 44 |
| 978 | 990339 | 0383 | 0428 | 0472   | 0516   | 0561  | 0605  | 0650  | 0694  | 0738  | 44 |
| 979 | 0783   | 0827 | 0871 | 0916   | 0960   | 1004  | 1049  | 1093  | 1137  | 1182  | 44 |
| 980 | 991226 | 1270 | 1315 | 1359   | 1403   | 1448  | 1492  | 1536  | 1580  | 1625  | 44 |
| 981 | 1669   | 1713 | 1758 | 1802   | 1846   | 1890  | 1935  | 1979  | 2023  | 2067  | 44 |
| 982 | 2111   | 2156 | 2200 | 2244   | 2288   | 2333  | 2377  | 2421  | 2465  | 2509  | 44 |
| 983 | 2554   | 2598 | 2642 | 2686   | 2730   | 2774  | 2819  | 2863  | 2907  | 2951  | 44 |
| 984 | 2995   | 3039 | 3083 | 3127   | 3172   | 3216  | 3260  | 3304  | 3348  | 3392  | 44 |
| 985 | 3436   | 3480 | 3524 | 3568   | 3613   | 3657  | 3701  | 3745  | 3789  | 3833  | 44 |
| 986 | 3877   | 3921 | 3965 | 4009   | 4053   | 4097  | 4141  | 4185  | 4229  | 4273  | 44 |
| 987 | 4317   | 4361 | 4405 | 4449   | 4493   | 4537  | 4581  | 4625  | 4669  | 4713  | 44 |
| 988 | 4757   | 4801 | 4845 | 4889   | 4933   | 4977  | 5021  | 5065  | 5108  | 5152  | 44 |
| 989 | 5196   | 5240 | 5284 | 5328   | 5372   | 5416  | 5460  | 5504  | 5547  | 5591  | 44 |
| 990 | 995635 | 5679 | 5723 | 5767   | 5811   | 5854  | 5898  | 5942  | 5986  | 6030  | 44 |
| 991 | 6074   | 6117 | 6161 | 6205   | 6249   | 6293  | 6337  | 6380  | 6424  | 6468  | 44 |
| 992 | 6512   | 6555 | 6599 | 6643   | 6687   | 6731  | 6774  | 6818  | 6862  | 6906  | 44 |
| 993 | 6949   | 6993 | 7037 | 7080   | 7124   | 7168  | 7212  | 7255  | 7299  | 7343  | 44 |
| 994 | 7386   | 7430 | 7474 | 7517   | 7561   | 7605  | 7648  | 7692  | 7736  | 7779  | 44 |
| 995 | 7823   | 7867 | 7910 | 7954   | 7998   | 8041  | 8085  | 8129  | 8172  | 8216  | 44 |
| 996 | 8259   | 8303 | 8347 | 8390   | 8434   | 8477  | 8521  | 8564  | 8608  | 8652  | 44 |
| 997 | 8695   | 8739 | 8782 | 8826   | 8869   | 8913  | 8956  | 9000  | 9043  | 9087  | 44 |
| 998 | 9131   | 9174 | 9218 | 9261   | 9305   | 9348  | 9392  | 9435  | 9479  | 9522  | 44 |
| 999 | 9565   | 9609 | 9652 | 9696   | 9739   | 9783  | 9826  | 9870  | 9913  | 9957  | 43 |
| N.  | 0      | 1    | 2    | 3      | 4      | 5     | 6     | 7     | 8     | 9     | D. |

# SINES AND TANGENTS,

FOR EVERY

## DEGREE AND MINUTE

OF THE QUADRANT.

---

**N.B.** The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top; and those increasing upwards, in the right-hand column, belong to the degrees below.

| M. | Sine     | D      | Cosine.   | D. | Tang.    | D.     | Cotang.   |    |
|----|----------|--------|-----------|----|----------|--------|-----------|----|
| 0  | 0.000000 |        | 10.000000 |    | 0.000000 |        | Infinite. | 60 |
| 1  | 6.463726 | 501717 | 000000    | 00 | 6.463726 | 501717 | 13.536274 | 59 |
| 2  | 764756   | 293485 | 000000    | 00 | 764756   | 293483 | 235244    | 58 |
| 3  | 940847   | 298231 | 000000    | 00 | 940847   | 298231 | 059153    | 57 |
| 4  | 7.065786 | 161517 | 000000    | 00 | 7.065786 | 161517 | 12.934214 | 56 |
| 5  | 162696   | 131968 | 000000    | 00 | 162696   | 131969 | 837304    | 55 |
| 6  | 241877   | 111575 | 9.999999  | 01 | 241878   | 111578 | 758192    | 54 |
| 7  | 308824   | 99653  | 999999    | 01 | 308825   | 99553  | 691175    | 53 |
| 8  | 366816   | 85254  | 999999    | 01 | 366817   | 85254  | 633183    | 52 |
| 9  | 417968   | 76263  | 999999    | 01 | 417970   | 76263  | 582030    | 51 |
| 10 | 463725   | 68988  | 999998    | 01 | 463727   | 68938  | 536273    | 50 |
| 11 | 7.505118 | 62981  | 9.999998  | 01 | 7.505120 | 62981  | 12.494889 | 49 |
| 12 | 542906   | 57936  | 999997    | 01 | 542909   | 57933  | 457091    | 48 |
| 13 | 577668   | 53641  | 999997    | 01 | 577672   | 53642  | 422328    | 47 |
| 14 | 609853   | 49938  | 999996    | 01 | 609857   | 49939  | 390143    | 46 |
| 15 | 639816   | 46714  | 999996    | 01 | 639820   | 46715  | 359180    | 45 |
| 16 | 667845   | 43881  | 999995    | 01 | 667849   | 43882  | 332151    | 44 |
| 17 | 694173   | 41372  | 999995    | 01 | 694179   | 41373  | 305821    | 43 |
| 18 | 718997   | 39135  | 999994    | 01 | 719003   | 39136  | 280997    | 42 |
| 19 | 742477   | 37127  | 999993    | 01 | 742484   | 37128  | 257516    | 41 |
| 20 | 764754   | 35315  | 999993    | 01 | 764761   | 35136  | 235239    | 40 |
| 21 | 7.785943 | 33672  | 9.999992  | 01 | 7.785951 | 33673  | 12.214049 | 39 |
| 22 | 806146   | 32175  | 999991    | 01 | 806155   | 32176  | 193845    | 38 |
| 23 | 825451   | 30895  | 999990    | 01 | 825460   | 30896  | 174540    | 37 |
| 24 | 843934   | 29547  | 999989    | 02 | 843944   | 29549  | 156956    | 36 |
| 25 | 861662   | 28388  | 999988    | 02 | 861674   | 28390  | 138326    | 35 |
| 26 | 878595   | 27317  | 999988    | 02 | 878703   | 27318  | 121292    | 34 |
| 27 | 895085   | 26323  | 999987    | 02 | 895099   | 26325  | 104901    | 33 |
| 28 | 910879   | 25399  | 999985    | 02 | 910894   | 25401  | 089106    | 32 |
| 29 | 926119   | 24538  | 999985    | 02 | 926134   | 24540  | 073866    | 31 |
| 30 | 940842   | 23733  | 999983    | 02 | 940858   | 23735  | 059142    | 30 |
| 31 | 7.955082 | 22980  | 9.999982  | 02 | 7.955100 | 22981  | 12.044909 | 29 |
| 32 | 968870   | 22273  | 999981    | 02 | 968889   | 22275  | 031111    | 28 |
| 33 | 982233   | 21608  | 999980    | 02 | 982253   | 21610  | 017747    | 27 |
| 34 | 995198   | 20981  | 999979    | 02 | 995219   | 20983  | 004781    | 26 |
| 35 | 8.007787 | 20390  | 999977    | 02 | 8.007809 | 20392  | 11.992191 | 25 |
| 36 | 020021   | 19831  | 999976    | 02 | 020045   | 19833  | 979955    | 24 |
| 37 | 031919   | 19302  | 999975    | 02 | 031945   | 19305  | 968555    | 23 |
| 38 | 043501   | 18801  | 999973    | 02 | 043527   | 18803  | 956473    | 22 |
| 39 | 054781   | 18325  | 999972    | 02 | 054809   | 18327  | 945191    | 21 |
| 40 | 065776   | 17872  | 999971    | 02 | 065806   | 17874  | 934194    | 20 |
| 41 | 8.076590 | 17441  | 9.999969  | 02 | 8.076531 | 17444  | 11.923469 | 19 |
| 42 | 086965   | 17031  | 999968    | 02 | 086997   | 17034  | 913903    | 18 |
| 43 | 097183   | 16639  | 999966    | 02 | 097217   | 16642  | 902783    | 17 |
| 44 | 107167   | 16265  | 999964    | 03 | 107202   | 16268  | 892797    | 16 |
| 45 | 116926   | 15908  | 999963    | 03 | 116963   | 15910  | 883937    | 15 |
| 46 | 126471   | 15566  | 999961    | 03 | 126510   | 15568  | 873490    | 14 |
| 47 | 135810   | 15238  | 999959    | 03 | 135851   | 15241  | 861449    | 13 |
| 48 | 144953   | 14924  | 999958    | 03 | 144996   | 14927  | 855091    | 12 |
| 49 | 153997   | 14622  | 999956    | 03 | 153952   | 14627  | 846048    | 11 |
| 50 | 162881   | 14333  | 999954    | 03 | 162727   | 14336  | 837273    | 10 |
| 51 | 8.171280 | 14054  | 9.999952  | 03 | 8.171328 | 14057  | 11.828672 | 9  |
| 52 | 179713   | 13786  | 999950    | 03 | 179763   | 13790  | 820237    | 8  |
| 53 | 187985   | 13529  | 999948    | 03 | 188036   | 13532  | 811954    | 7  |
| 54 | 196102   | 13280  | 999946    | 03 | 196156   | 13284  | 803844    | 6  |
| 55 | 204070   | 13041  | 999944    | 03 | 204126   | 13044  | 795874    | 5  |
| 56 | 211895   | 12810  | 999942    | 04 | 211953   | 12814  | 788047    | 4  |
| 57 | 219581   | 12587  | 999940    | 04 | 219641   | 12590  | 780359    | 3  |
| 58 | 227134   | 12372  | 999938    | 04 | 227195   | 12376  | 772835    | 2  |
| 59 | 234557   | 12164  | 999936    | 04 | 234621   | 12168  | 765379    | 1  |
| 60 | 241855   | 11963  | 999934    | 04 | 241921   | 11967  | 758079    | 0  |
|    | Cosine   |        | Sine      |    | Cotang.  |        | Tang.     | M. |



| M. | Sine     | D.    | Cosine   | D. | Tang.    | D.    | Cotang.   |    |
|----|----------|-------|----------|----|----------|-------|-----------|----|
| 0  | 8.241855 | 11963 | 9.999934 | 04 | 8.241921 | 11967 | 11.758079 | 60 |
| 1  | 249033   | 11768 | 999932   | 04 | 249102   | 11772 | 750898    | 59 |
| 2  | 256094   | 11580 | 999929   | 04 | 256165   | 11584 | 743835    | 58 |
| 3  | 263042   | 11398 | 999927   | 04 | 263115   | 11402 | 736885    | 57 |
| 4  | 269881   | 11221 | 999925   | 04 | 269956   | 11225 | 730044    | 56 |
| 5  | 276614   | 11050 | 999922   | 04 | 276691   | 11054 | 723300    | 55 |
| 6  | 283243   | 10883 | 999920   | 04 | 283323   | 10887 | 716677    | 54 |
| 7  | 289773   | 10721 | 999918   | 04 | 289856   | 10726 | 710144    | 53 |
| 8  | 296207   | 10565 | 999915   | 04 | 296292   | 10570 | 703708    | 52 |
| 9  | 302546   | 10413 | 999913   | 04 | 302634   | 10418 | 697366    | 51 |
| 10 | 308794   | 10266 | 999910   | 04 | 308884   | 10270 | 691116    | 50 |
| 11 | 8.314954 | 10122 | 9.999907 | 04 | 8.315046 | 10126 | 11.684954 | 49 |
| 12 | 321027   | 9982  | 999905   | 04 | 321122   | 9987  | 678878    | 48 |
| 13 | 327016   | 9847  | 999902   | 04 | 327114   | 9851  | 672886    | 47 |
| 14 | 332924   | 9714  | 999899   | 05 | 333025   | 9719  | 666975    | 46 |
| 15 | 338753   | 9586  | 999897   | 05 | 338856   | 9590  | 661144    | 45 |
| 16 | 344504   | 9460  | 999894   | 05 | 344610   | 9465  | 655390    | 44 |
| 17 | 350181   | 9338  | 999891   | 05 | 350289   | 9343  | 649711    | 43 |
| 18 | 355783   | 9219  | 999888   | 05 | 355895   | 9224  | 644105    | 42 |
| 19 | 361315   | 9103  | 999885   | 05 | 361430   | 9108  | 638570    | 41 |
| 20 | 366777   | 8990  | 999882   | 05 | 366895   | 8995  | 633105    | 40 |
| 21 | 8.372171 | 8880  | 9.999879 | 05 | 8.372292 | 8885  | 11.627708 | 39 |
| 22 | 377499   | 8772  | 999876   | 05 | 377622   | 8777  | 622378    | 38 |
| 23 | 382762   | 8667  | 999873   | 05 | 382889   | 8672  | 617111    | 37 |
| 24 | 387962   | 8564  | 999870   | 05 | 388092   | 8570  | 611908    | 36 |
| 25 | 393101   | 8464  | 999867   | 05 | 393234   | 8470  | 606766    | 35 |
| 26 | 398179   | 8366  | 999864   | 05 | 398315   | 8371  | 601685    | 34 |
| 27 | 403199   | 8271  | 999861   | 05 | 403333   | 8276  | 596662    | 33 |
| 28 | 408161   | 8177  | 999858   | 05 | 408304   | 8182  | 591696    | 32 |
| 29 | 413068   | 8086  | 999854   | 05 | 413213   | 8091  | 586787    | 31 |
| 30 | 417919   | 7996  | 999851   | 06 | 418068   | 8002  | 581932    | 30 |
| 31 | 8.422717 | 7909  | 9.999848 | 06 | 8.422869 | 7914  | 11.577131 | 29 |
| 32 | 427462   | 7823  | 999844   | 06 | 427618   | 7830  | 572382    | 28 |
| 33 | 432156   | 7740  | 999841   | 06 | 432315   | 7745  | 567685    | 27 |
| 34 | 436800   | 7657  | 999838   | 06 | 436962   | 7663  | 563038    | 26 |
| 35 | 441394   | 7577  | 999834   | 06 | 441560   | 7583  | 558440    | 25 |
| 36 | 445941   | 7499  | 999831   | 06 | 446110   | 7505  | 553890    | 24 |
| 37 | 450440   | 7422  | 999827   | 06 | 450613   | 7428  | 549387    | 23 |
| 38 | 454893   | 7346  | 999823   | 06 | 455070   | 7352  | 544930    | 22 |
| 39 | 459301   | 7273  | 999820   | 06 | 459481   | 7279  | 540519    | 21 |
| 40 | 463665   | 7200  | 999816   | 06 | 463849   | 7206  | 536151    | 20 |
| 41 | 8.467985 | 7129  | 9.999812 | 06 | 8.468172 | 7135  | 11.531828 | 19 |
| 42 | 472263   | 7060  | 999809   | 06 | 472454   | 7066  | 527546    | 18 |
| 43 | 476498   | 6991  | 999805   | 06 | 476693   | 6998  | 523307    | 17 |
| 44 | 480693   | 6924  | 999801   | 06 | 480892   | 6931  | 519108    | 16 |
| 45 | 484848   | 6859  | 999797   | 07 | 485050   | 6865  | 514950    | 15 |
| 46 | 488963   | 6794  | 999793   | 07 | 489170   | 6801  | 510830    | 14 |
| 47 | 493040   | 6731  | 999790   | 07 | 493250   | 6738  | 506750    | 13 |
| 48 | 497078   | 6669  | 999786   | 07 | 497293   | 6676  | 502707    | 12 |
| 49 | 501080   | 6608  | 999782   | 07 | 501298   | 6615  | 498702    | 11 |
| 50 | 505045   | 6548  | 999778   | 07 | 505267   | 6555  | 494733    | 10 |
| 51 | 8.508974 | 6489  | 9.999774 | 07 | 8.509200 | 6496  | 11.490800 | 9  |
| 52 | 512867   | 6431  | 999769   | 07 | 513098   | 6439  | 426902    | 8  |
| 53 | 516726   | 6375  | 999765   | 07 | 516961   | 6382  | 423039    | 7  |
| 54 | 520551   | 6319  | 999761   | 07 | 520790   | 6326  | 479210    | 6  |
| 55 | 524343   | 6264  | 999757   | 07 | 524586   | 6272  | 475414    | 5  |
| 56 | 528102   | 6211  | 999753   | 07 | 528349   | 6218  | 471651    | 4  |
| 57 | 531828   | 6158  | 999748   | 07 | 532080   | 6165  | 467920    | 3  |
| 58 | 535523   | 6106  | 999744   | 07 | 535779   | 6113  | 464221    | 2  |
| 59 | 539186   | 6055  | 999740   | 07 | 539447   | 6062  | 460553    | 1  |
| 60 | 542819   | 6004  | 999735   | 07 | 543084   | 6012  | 456916    | 0  |
|    | Cosine   |       | Sine     |    | Cotang.  |       | Tang.     | M. |

| M. | Sine     | D.   | Cosine.  | D. | Tang.    | D.   | Cotang.   | M. |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 8.542819 | 6004 | 9.999735 | 07 | 8.543084 | 6012 | 11.456916 | 60 |
| 1  | 546422   | 5955 | 999731   | 07 | 546691   | 5962 | 453309    | 59 |
| 2  | 549995   | 5906 | 999726   | 07 | 550268   | 5914 | 449732    | 58 |
| 3  | 553539   | 5858 | 999722   | 08 | 553817   | 5866 | 446183    | 57 |
| 4  | 557054   | 5811 | 999717   | 08 | 557336   | 5819 | 442664    | 56 |
| 5  | 560540   | 5765 | 999713   | 08 | 560828   | 5773 | 439172    | 55 |
| 6  | 563999   | 5719 | 999708   | 08 | 564291   | 5727 | 435709    | 54 |
| 7  | 567431   | 5674 | 999704   | 08 | 567727   | 5682 | 432273    | 53 |
| 8  | 570836   | 5630 | 999699   | 08 | 571137   | 5638 | 428863    | 52 |
| 9  | 574214   | 5587 | 999694   | 08 | 574520   | 5595 | 425480    | 51 |
| 10 | 577566   | 5544 | 999689   | 08 | 577877   | 5552 | 422123    | 50 |
| 11 | 8.580892 | 5502 | 9.999685 | 08 | 8.581208 | 5510 | 11.418792 | 49 |
| 12 | 584193   | 5460 | 999680   | 08 | 584514   | 5468 | 415486    | 48 |
| 13 | 587469   | 5419 | 999675   | 08 | 587795   | 5427 | 412205    | 47 |
| 14 | 590721   | 5379 | 999670   | 08 | 591051   | 5387 | 408949    | 46 |
| 15 | 593948   | 5339 | 999665   | 08 | 594283   | 5347 | 405717    | 45 |
| 16 | 597152   | 5300 | 999660   | 08 | 597492   | 5308 | 402508    | 44 |
| 17 | 600332   | 5261 | 999655   | 08 | 600677   | 5270 | 399333    | 43 |
| 18 | 603489   | 5223 | 999650   | 08 | 603839   | 5232 | 396161    | 42 |
| 19 | 606623   | 5186 | 999645   | 09 | 606978   | 5194 | 393022    | 41 |
| 20 | 609734   | 5149 | 999640   | 09 | 610094   | 5158 | 389906    | 40 |
| 21 | 8.612823 | 5112 | 9.999635 | 09 | 8.613189 | 5121 | 11.386811 | 39 |
| 22 | 615891   | 5076 | 999629   | 09 | 616262   | 5085 | 383738    | 38 |
| 23 | 618937   | 5041 | 999624   | 09 | 619313   | 5050 | 380687    | 37 |
| 24 | 621962   | 5006 | 999619   | 09 | 622343   | 5015 | 377657    | 36 |
| 25 | 624965   | 4972 | 999614   | 09 | 625352   | 4981 | 374648    | 35 |
| 26 | 627948   | 4938 | 999608   | 09 | 628340   | 4947 | 371660    | 34 |
| 27 | 630911   | 4904 | 999603   | 09 | 631308   | 4913 | 368692    | 33 |
| 28 | 633854   | 4871 | 999597   | 09 | 634256   | 4880 | 365744    | 32 |
| 29 | 636776   | 4839 | 999592   | 09 | 637184   | 4848 | 362816    | 31 |
| 30 | 639680   | 4806 | 999586   | 09 | 640093   | 4816 | 359907    | 30 |
| 31 | 8.642563 | 4775 | 9.999581 | 09 | 8.642982 | 4784 | 11.357018 | 29 |
| 32 | 645428   | 4743 | 999575   | 09 | 645853   | 4753 | 354147    | 28 |
| 33 | 648274   | 4712 | 999570   | 09 | 648704   | 4722 | 351296    | 27 |
| 34 | 651102   | 4682 | 999564   | 09 | 651537   | 4691 | 348463    | 26 |
| 35 | 653911   | 4652 | 999558   | 10 | 654352   | 4661 | 345648    | 25 |
| 36 | 656702   | 4622 | 999553   | 10 | 657149   | 4631 | 342851    | 24 |
| 37 | 659475   | 4592 | 999547   | 10 | 659928   | 4602 | 340072    | 23 |
| 38 | 662230   | 4563 | 999541   | 10 | 662689   | 4573 | 337311    | 22 |
| 39 | 664968   | 4535 | 999535   | 10 | 665433   | 4544 | 334567    | 21 |
| 40 | 667689   | 4506 | 999529   | 10 | 668160   | 4526 | 331840    | 20 |
| 41 | 8.670393 | 4479 | 9.999524 | 10 | 8.670870 | 4488 | 11.329130 | 19 |
| 42 | 673080   | 4451 | 999518   | 10 | 673563   | 4461 | 326437    | 18 |
| 43 | 675751   | 4424 | 999512   | 10 | 676239   | 4434 | 323761    | 17 |
| 44 | 678405   | 4397 | 999506   | 10 | 678900   | 4417 | 321100    | 16 |
| 45 | 681043   | 4370 | 999500   | 10 | 681544   | 4380 | 318456    | 15 |
| 46 | 683665   | 4344 | 999493   | 10 | 684172   | 4354 | 315828    | 14 |
| 47 | 686272   | 4318 | 999487   | 10 | 686784   | 4328 | 313216    | 13 |
| 48 | 688863   | 4292 | 999481   | 10 | 689381   | 4303 | 310619    | 12 |
| 49 | 691438   | 4267 | 999475   | 10 | 691963   | 4277 | 308037    | 11 |
| 50 | 693998   | 4242 | 999469   | 10 | 694529   | 4252 | 305471    | 10 |
| 51 | 8.696543 | 4217 | 9.999463 | 11 | 8.697081 | 4228 | 11.302919 | 9  |
| 52 | 699073   | 4192 | 999456   | 11 | 699517   | 4203 | 300383    | 8  |
| 53 | 701589   | 4168 | 999450   | 11 | 702139   | 4179 | 297861    | 7  |
| 54 | 704090   | 4144 | 999443   | 11 | 704646   | 4155 | 295354    | 6  |
| 55 | 706577   | 4121 | 999437   | 11 | 707140   | 4132 | 292860    | 5  |
| 56 | 709049   | 4097 | 999431   | 11 | 709618   | 4108 | 290382    | 4  |
| 57 | 711507   | 4074 | 999424   | 11 | 712083   | 4085 | 287917    | 3  |
| 58 | 713952   | 4051 | 999418   | 11 | 714534   | 4062 | 285465    | 2  |
| 59 | 716383   | 4029 | 999411   | 11 | 716972   | 4040 | 283028    | 1  |
| 60 | 718800   | 4006 | 999404   | 11 | 719396   | 4017 | 280604    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M  |

| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   | M. |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 8.718800 | 4006 | 9.999404 | 11 | 8.719396 | 4017 | 11.280604 | 60 |
| 1  | 721204   | 3984 | 999398   | 11 | 721806   | 3995 | 278194    | 59 |
| 2  | 723595   | 3962 | 999391   | 11 | 724204   | 3974 | 275796    | 58 |
| 3  | 725972   | 3941 | 999384   | 11 | 726588   | 3952 | 273412    | 57 |
| 4  | 728337   | 3919 | 999378   | 11 | 728959   | 3930 | 271041    | 56 |
| 5  | 730688   | 3898 | 999371   | 11 | 731317   | 3909 | 268683    | 55 |
| 6  | 733027   | 3877 | 999364   | 12 | 733663   | 3889 | 266337    | 54 |
| 7  | 735354   | 3857 | 999357   | 12 | 735996   | 3868 | 264004    | 53 |
| 8  | 737667   | 3836 | 999350   | 12 | 738317   | 3848 | 261683    | 52 |
| 9  | 739969   | 3816 | 999343   | 12 | 740626   | 3827 | 259374    | 51 |
| 10 | 742259   | 3796 | 999336   | 12 | 742922   | 3807 | 257078    | 50 |
| 11 | 8.744536 | 3776 | 9.999329 | 12 | 8.745207 | 3787 | 11.254793 | 49 |
| 12 | 746802   | 3756 | 999322   | 12 | 747479   | 3768 | 252521    | 48 |
| 13 | 749055   | 3737 | 999315   | 12 | 749740   | 3749 | 250260    | 47 |
| 14 | 751297   | 3717 | 999308   | 12 | 751989   | 3729 | 248011    | 46 |
| 15 | 753528   | 3698 | 999301   | 12 | 754227   | 3710 | 245773    | 45 |
| 16 | 755747   | 3679 | 999294   | 12 | 756453   | 3692 | 243547    | 44 |
| 17 | 757955   | 3661 | 999286   | 12 | 758668   | 3673 | 241332    | 43 |
| 18 | 760151   | 3642 | 999279   | 12 | 760872   | 3655 | 239128    | 42 |
| 19 | 762337   | 3624 | 999272   | 12 | 763065   | 3636 | 236935    | 41 |
| 20 | 764511   | 3606 | 999265   | 12 | 765246   | 3618 | 234754    | 40 |
| 21 | 8.766675 | 3588 | 9.999257 | 12 | 8.767417 | 3600 | 11.232583 | 39 |
| 22 | 768828   | 3570 | 999250   | 13 | 769578   | 3583 | 230422    | 38 |
| 23 | 770970   | 3553 | 999242   | 13 | 771727   | 3565 | 228273    | 37 |
| 24 | 773101   | 3535 | 999235   | 13 | 773866   | 3548 | 226134    | 36 |
| 25 | 775223   | 3518 | 999227   | 13 | 775995   | 3531 | 224005    | 35 |
| 26 | 777333   | 3501 | 999220   | 13 | 778114   | 3514 | 221886    | 34 |
| 27 | 779434   | 3484 | 999212   | 13 | 780222   | 3497 | 219778    | 33 |
| 28 | 781524   | 3467 | 999205   | 13 | 782320   | 3480 | 217680    | 32 |
| 29 | 783605   | 3451 | 999197   | 13 | 784408   | 3464 | 215592    | 31 |
| 30 | 785675   | 3431 | 999189   | 13 | 786486   | 3447 | 213514    | 30 |
| 31 | 8.787736 | 3418 | 9.999181 | 13 | 8.788554 | 3431 | 11.211446 | 29 |
| 32 | 789787   | 3402 | 999174   | 13 | 790613   | 3414 | 209387    | 28 |
| 33 | 791828   | 3386 | 999166   | 13 | 792662   | 3399 | 207338    | 27 |
| 34 | 793859   | 3370 | 999158   | 13 | 794701   | 3383 | 205299    | 26 |
| 35 | 795881   | 3354 | 999150   | 13 | 796731   | 3368 | 203269    | 25 |
| 36 | 797894   | 3339 | 999142   | 13 | 798752   | 3352 | 201248    | 24 |
| 37 | 799897   | 3323 | 999134   | 13 | 800763   | 3337 | 199237    | 23 |
| 38 | 801892   | 3308 | 999126   | 13 | 802765   | 3322 | 197235    | 22 |
| 39 | 803876   | 3293 | 999118   | 13 | 804758   | 3307 | 195242    | 21 |
| 40 | 805852   | 3278 | 999110   | 13 | 806742   | 3292 | 193258    | 20 |
| 41 | 8.807819 | 3263 | 9.999102 | 13 | 8.808717 | 3278 | 11.191283 | 19 |
| 42 | 809777   | 3249 | 999094   | 14 | 810683   | 3262 | 189317    | 18 |
| 43 | 811726   | 3234 | 999086   | 14 | 812641   | 3248 | 187359    | 17 |
| 44 | 813667   | 3219 | 999077   | 14 | 814589   | 3233 | 185411    | 16 |
| 45 | 815599   | 3205 | 999069   | 14 | 816529   | 3219 | 183471    | 15 |
| 46 | 817522   | 3191 | 999061   | 14 | 818461   | 3205 | 181539    | 14 |
| 47 | 819436   | 3177 | 999053   | 14 | 820384   | 3191 | 179616    | 13 |
| 48 | 821343   | 3163 | 999044   | 14 | 822298   | 3177 | 177702    | 12 |
| 49 | 823240   | 3149 | 999036   | 14 | 824205   | 3163 | 175795    | 11 |
| 50 | 825130   | 3135 | 999027   | 14 | 826103   | 3150 | 173897    | 10 |
| 51 | 8.827011 | 3122 | 9.999019 | 14 | 8.827992 | 3136 | 11.172008 | 9  |
| 52 | 828884   | 3108 | 999010   | 14 | 829874   | 3123 | 170126    | 8  |
| 53 | 830749   | 3095 | 999002   | 14 | 831748   | 3110 | 168252    | 7  |
| 54 | 832607   | 3082 | 998993   | 14 | 833613   | 3096 | 166387    | 6  |
| 55 | 834456   | 3069 | 998984   | 14 | 835471   | 3083 | 164529    | 5  |
| 56 | 836297   | 3056 | 998976   | 14 | 837321   | 3070 | 162679    | 4  |
| 57 | 838130   | 3043 | 998967   | 15 | 839163   | 3057 | 160837    | 3  |
| 58 | 839956   | 3030 | 998958   | 15 | 840998   | 3045 | 159002    | 2  |
| 59 | 841774   | 3017 | 998950   | 15 | 842825   | 3032 | 157175    | 1  |
| 60 | 843585   | 3000 | 998941   | 15 | 844644   | 3019 | 155356    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M  |



| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   |    |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 8.843585 | 3005 | 9.998941 | 15 | 8.844644 | 3019 | 11.155566 | 69 |
| 1  | 845387   | 2992 | 998932   | 15 | 846455   | 3007 | 153545    | 59 |
| 2  | 847183   | 2980 | 998923   | 15 | 848260   | 2995 | 151740    | 58 |
| 3  | 848971   | 2967 | 998914   | 15 | 850057   | 2982 | 149943    | 57 |
| 4  | 850751   | 2955 | 998905   | 15 | 851846   | 2970 | 148154    | 56 |
| 5  | 852525   | 2943 | 998896   | 15 | 853628   | 2958 | 146372    | 55 |
| 6  | 854291   | 2931 | 998887   | 15 | 855403   | 2946 | 144597    | 54 |
| 7  | 856049   | 2919 | 998878   | 15 | 857171   | 2935 | 142829    | 53 |
| 8  | 857801   | 2907 | 998869   | 15 | 858932   | 2923 | 141068    | 52 |
| 9  | 859546   | 2896 | 998860   | 15 | 860686   | 2911 | 139314    | 51 |
| 10 | 861283   | 2884 | 998851   | 15 | 862433   | 2900 | 137567    | 50 |
| 11 | 8.863014 | 2873 | 9.998841 | 15 | 8.864173 | 2888 | 11.135827 | 49 |
| 12 | 864738   | 2861 | 998832   | 15 | 865906   | 2877 | 134094    | 48 |
| 13 | 866455   | 2850 | 998823   | 16 | 867632   | 2866 | 132368    | 47 |
| 14 | 868165   | 2839 | 998813   | 16 | 869351   | 2854 | 130649    | 46 |
| 15 | 869868   | 2828 | 998804   | 16 | 871064   | 2843 | 128936    | 45 |
| 16 | 871565   | 2817 | 998795   | 16 | 872770   | 2832 | 127230    | 44 |
| 17 | 873255   | 2806 | 998785   | 16 | 874469   | 2821 | 125531    | 43 |
| 18 | 874938   | 2795 | 998776   | 16 | 876162   | 2811 | 123838    | 42 |
| 19 | 876615   | 2785 | 998766   | 16 | 877849   | 2800 | 122151    | 41 |
| 20 | 878285   | 2773 | 998757   | 16 | 879529   | 2789 | 120471    | 40 |
| 21 | 8.879949 | 2763 | 9.998747 | 16 | 8.881202 | 2779 | 11.118798 | 39 |
| 22 | 881607   | 2752 | 998738   | 16 | 882869   | 2768 | 117131    | 38 |
| 23 | 883258   | 2742 | 998728   | 16 | 884530   | 2758 | 115470    | 37 |
| 24 | 884903   | 2731 | 998718   | 16 | 886185   | 2747 | 113815    | 36 |
| 25 | 886542   | 2721 | 998708   | 16 | 887833   | 2737 | 112167    | 35 |
| 26 | 888174   | 2711 | 998699   | 16 | 889476   | 2727 | 110524    | 34 |
| 27 | 889801   | 2700 | 998689   | 16 | 891112   | 2717 | 108888    | 33 |
| 28 | 891421   | 2690 | 998679   | 16 | 892742   | 2707 | 107258    | 32 |
| 29 | 893035   | 2680 | 998669   | 17 | 894366   | 2697 | 105634    | 31 |
| 30 | 894643   | 2670 | 998659   | 17 | 895984   | 2687 | 104016    | 30 |
| 31 | 8.896246 | 2660 | 9.998649 | 17 | 8.897596 | 2677 | 11.102404 | 29 |
| 32 | 897842   | 2651 | 998639   | 17 | 899203   | 2667 | 100797    | 28 |
| 33 | 899432   | 2641 | 998629   | 17 | 900803   | 2658 | 999197    | 27 |
| 34 | 901017   | 2631 | 998619   | 17 | 902398   | 2648 | 990602    | 26 |
| 35 | 902595   | 2622 | 998609   | 17 | 903987   | 2638 | 982213    | 25 |
| 36 | 904169   | 2612 | 998599   | 17 | 905570   | 2629 | 974030    | 24 |
| 37 | 905736   | 2603 | 998589   | 17 | 907147   | 2620 | 966053    | 23 |
| 38 | 907297   | 2593 | 998578   | 17 | 908719   | 2610 | 958281    | 22 |
| 39 | 908853   | 2584 | 998568   | 17 | 910285   | 2601 | 950715    | 21 |
| 40 | 910404   | 2575 | 998558   | 17 | 911846   | 2592 | 943354    | 20 |
| 41 | 8.911949 | 2566 | 9.998548 | 17 | 8.913401 | 2583 | 11.086599 | 19 |
| 42 | 913483   | 2556 | 998537   | 17 | 914951   | 2574 | 935949    | 18 |
| 43 | 915022   | 2547 | 998527   | 17 | 916495   | 2565 | 928505    | 17 |
| 44 | 916555   | 2538 | 998516   | 18 | 918034   | 2556 | 921166    | 16 |
| 45 | 918073   | 2529 | 998506   | 18 | 919568   | 2547 | 913932    | 15 |
| 46 | 919591   | 2520 | 998495   | 18 | 921096   | 2538 | 906804    | 14 |
| 47 | 921103   | 2512 | 998485   | 18 | 922619   | 2530 | 899781    | 13 |
| 48 | 922610   | 2503 | 998474   | 18 | 924136   | 2521 | 892864    | 12 |
| 49 | 924112   | 2494 | 998464   | 18 | 925649   | 2512 | 886051    | 11 |
| 50 | 925609   | 2486 | 998453   | 18 | 927156   | 2503 | 879344    | 10 |
| 51 | 8.927100 | 2477 | 9.998442 | 18 | 8.928658 | 2495 | 11.071342 | 9  |
| 52 | 928587   | 2469 | 998431   | 18 | 930155   | 2486 | 872845    | 8  |
| 53 | 930068   | 2460 | 998421   | 18 | 931647   | 2478 | 866453    | 7  |
| 54 | 931544   | 2452 | 998410   | 18 | 933134   | 2470 | 860166    | 6  |
| 55 | 933015   | 2443 | 998399   | 18 | 934616   | 2461 | 853984    | 5  |
| 56 | 934481   | 2435 | 998388   | 18 | 936093   | 2453 | 847907    | 4  |
| 57 | 935942   | 2427 | 998377   | 18 | 937565   | 2445 | 841935    | 3  |
| 58 | 937398   | 2419 | 998366   | 18 | 939032   | 2437 | 836068    | 2  |
| 59 | 938850   | 2411 | 998355   | 18 | 940494   | 2430 | 830306    | 1  |
| 60 | 940295   | 2403 | 998344   | 18 | 941952   | 2421 | 824648    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M. |



| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   | M. |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 8.940296 | 2403 | 9.998344 | 19 | 8.941952 | 2421 | 11.058048 | 60 |
| 1  | 941738   | 2394 | 998333   | 19 | 943404   | 2413 | 056596    | 59 |
| 2  | 943174   | 2387 | 998322   | 19 | 944852   | 2405 | 055148    | 58 |
| 3  | 944606   | 2379 | 998311   | 19 | 946295   | 2397 | 053705    | 57 |
| 4  | 946034   | 2371 | 998300   | 19 | 947734   | 2390 | 052266    | 56 |
| 5  | 947456   | 2363 | 998289   | 19 | 949168   | 2382 | 050832    | 55 |
| 6  | 948874   | 2355 | 998277   | 19 | 950597   | 2374 | 049403    | 54 |
| 7  | 950287   | 2348 | 998266   | 19 | 952021   | 2366 | 047979    | 53 |
| 8  | 951696   | 2340 | 998255   | 19 | 953441   | 2360 | 046559    | 52 |
| 9  | 953100   | 2332 | 998243   | 19 | 954856   | 2351 | 045144    | 51 |
| 10 | 954499   | 2325 | 998232   | 19 | 956267   | 2344 | 043733    | 50 |
| 11 | 8.955894 | 2317 | 9.998220 | 19 | 8.957674 | 2337 | 11.042326 | 49 |
| 12 | 957284   | 2310 | 998209   | 19 | 959075   | 2329 | 040925    | 48 |
| 13 | 958670   | 2302 | 998197   | 19 | 960473   | 2323 | 039527    | 47 |
| 14 | 960052   | 2295 | 998186   | 19 | 961866   | 2314 | 038134    | 46 |
| 15 | 961429   | 2288 | 998174   | 19 | 963255   | 2307 | 036745    | 45 |
| 16 | 962801   | 2280 | 998163   | 19 | 964639   | 2300 | 035361    | 44 |
| 17 | 964170   | 2273 | 998151   | 19 | 966019   | 2293 | 033981    | 43 |
| 18 | 965534   | 2266 | 998139   | 20 | 967394   | 2286 | 032606    | 42 |
| 19 | 966893   | 2259 | 998128   | 20 | 968766   | 2279 | 031234    | 41 |
| 20 | 968249   | 2252 | 998116   | 20 | 970133   | 2271 | 029867    | 40 |
| 21 | 8.969600 | 2244 | 9.998104 | 20 | 8.971496 | 2265 | 11.028504 | 39 |
| 22 | 970947   | 2238 | 998092   | 20 | 972855   | 2257 | 027145    | 38 |
| 23 | 972289   | 2231 | 998080   | 20 | 974209   | 2251 | 025791    | 37 |
| 24 | 973628   | 2224 | 998068   | 20 | 975560   | 2244 | 024440    | 36 |
| 25 | 974962   | 2217 | 998056   | 20 | 976906   | 2237 | 023094    | 35 |
| 26 | 976293   | 2210 | 998044   | 20 | 978248   | 2230 | 021752    | 34 |
| 27 | 977619   | 2203 | 998032   | 20 | 979586   | 2223 | 020414    | 33 |
| 28 | 978941   | 2197 | 998020   | 20 | 980921   | 2217 | 019079    | 32 |
| 29 | 980259   | 2190 | 998008   | 20 | 982251   | 2210 | 017749    | 31 |
| 30 | 981573   | 2183 | 997996   | 20 | 983577   | 2204 | 016423    | 30 |
| 31 | 8.982883 | 2177 | 9.997984 | 20 | 8.984899 | 2197 | 11.015101 | 29 |
| 32 | 984189   | 2170 | 997972   | 20 | 986217   | 2191 | 013783    | 28 |
| 33 | 985491   | 2163 | 997959   | 20 | 987532   | 2184 | 012468    | 27 |
| 34 | 986789   | 2157 | 997947   | 20 | 988842   | 2178 | 011158    | 26 |
| 35 | 988083   | 2150 | 997935   | 21 | 990149   | 2171 | 009851    | 25 |
| 36 | 989374   | 2144 | 997922   | 21 | 991451   | 2165 | 008549    | 24 |
| 37 | 990660   | 2138 | 997910   | 21 | 992750   | 2158 | 007250    | 23 |
| 38 | 991943   | 2131 | 997897   | 21 | 994045   | 2152 | 005955    | 22 |
| 39 | 993222   | 2125 | 997885   | 21 | 995337   | 2146 | 004663    | 21 |
| 40 | 994497   | 2119 | 997872   | 21 | 996624   | 2140 | 003376    | 20 |
| 41 | 8.995768 | 2112 | 9.997860 | 21 | 8.997908 | 2134 | 11.002092 | 19 |
| 42 | 997036   | 2106 | 997847   | 21 | 999188   | 2127 | 000812    | 18 |
| 43 | 998299   | 2100 | 997835   | 21 | 9.000465 | 2121 | 10.999535 | 17 |
| 44 | 999560   | 2094 | 997822   | 21 | 001738   | 2115 | 998262    | 16 |
| 45 | 9.000816 | 2087 | 997809   | 21 | 003007   | 2109 | 996993    | 15 |
| 46 | 002069   | 2082 | 997797   | 21 | 004272   | 2103 | 995728    | 14 |
| 47 | 003318   | 2076 | 997784   | 21 | 005534   | 2097 | 994466    | 13 |
| 48 | 004563   | 2070 | 997771   | 21 | 006792   | 2091 | 993208    | 12 |
| 49 | 005805   | 2064 | 997758   | 21 | 008047   | 2085 | 991953    | 11 |
| 50 | 007044   | 2058 | 997745   | 21 | 009298   | 2080 | 990702    | 10 |
| 51 | 9.008278 | 2052 | 9.997732 | 21 | 9.010546 | 2074 | 10.989454 | 9  |
| 52 | 009510   | 2046 | 997719   | 21 | 011790   | 2068 | 988210    | 8  |
| 53 | 010737   | 2040 | 997706   | 21 | 013031   | 2062 | 986969    | 7  |
| 54 | 011962   | 2034 | 997693   | 22 | 014268   | 2056 | 985732    | 6  |
| 55 | 013182   | 2029 | 997680   | 22 | 015502   | 2051 | 984498    | 5  |
| 56 | 014400   | 2023 | 997667   | 22 | 016732   | 2045 | 983268    | 4  |
| 57 | 015613   | 2017 | 997654   | 22 | 017959   | 2040 | 982041    | 3  |
| 58 | 016824   | 2012 | 997641   | 22 | 019183   | 2033 | 980817    | 2  |
| 59 | 018031   | 2006 | 997628   | 22 | 020403   | 2028 | 979597    | 1  |
| 60 | 019235   | 2000 | 997614   | 22 | 021620   | 2023 | 978380    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M. |

| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   | M. |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 9.019235 | 2000 | 9.997614 | 22 | 9.021620 | 2023 | 10.978380 | 60 |
| 1  | 020435   | 1995 | 997601   | 22 | 022824   | 2017 | 977166    | 59 |
| 2  | 021632   | 1989 | 997588   | 22 | 024044   | 2011 | 975956    | 58 |
| 3  | 022825   | 1984 | 997574   | 22 | 025251   | 2006 | 974749    | 57 |
| 4  | 024016   | 1978 | 997561   | 22 | 026455   | 2000 | 973545    | 56 |
| 5  | 025203   | 1973 | 997547   | 22 | 027655   | 1995 | 972345    | 55 |
| 6  | 026386   | 1967 | 997534   | 23 | 028852   | 1990 | 971148    | 54 |
| 7  | 027567   | 1962 | 997520   | 23 | 030046   | 1985 | 969954    | 53 |
| 8  | 028744   | 1957 | 997507   | 23 | 031237   | 1979 | 968763    | 52 |
| 9  | 029918   | 1951 | 997493   | 23 | 032425   | 1974 | 967575    | 51 |
| 10 | 031089   | 1947 | 997480   | 23 | 033609   | 1969 | 966391    | 50 |
| 11 | 9.032257 | 1941 | 9.997466 | 23 | 9.034791 | 1964 | 10.965209 | 49 |
| 12 | 033421   | 1936 | 997452   | 23 | 035969   | 1958 | 964031    | 48 |
| 13 | 034582   | 1930 | 997439   | 23 | 037144   | 1953 | 962856    | 47 |
| 14 | 035741   | 1925 | 997425   | 23 | 038316   | 1948 | 961684    | 46 |
| 15 | 036896   | 1920 | 997411   | 23 | 039485   | 1943 | 960515    | 45 |
| 16 | 038048   | 1915 | 997397   | 23 | 040651   | 1938 | 959349    | 44 |
| 17 | 039197   | 1910 | 997383   | 23 | 041813   | 1933 | 958187    | 43 |
| 18 | 040342   | 1905 | 997369   | 23 | 042973   | 1928 | 957027    | 42 |
| 19 | 041485   | 1899 | 997355   | 23 | 044130   | 1923 | 955870    | 41 |
| 20 | 042625   | 1894 | 997341   | 23 | 045284   | 1918 | 954716    | 40 |
| 21 | 9.043762 | 1889 | 9.997327 | 24 | 9.046434 | 1913 | 10.953566 | 39 |
| 22 | 044895   | 1884 | 997313   | 24 | 047582   | 1908 | 952418    | 38 |
| 23 | 046026   | 1879 | 997299   | 24 | 048727   | 1903 | 951273    | 37 |
| 24 | 047154   | 1875 | 997285   | 24 | 049869   | 1898 | 950131    | 36 |
| 25 | 048279   | 1870 | 997271   | 24 | 051008   | 1893 | 948992    | 35 |
| 26 | 049400   | 1865 | 997257   | 24 | 052144   | 1889 | 947856    | 34 |
| 27 | 050519   | 1860 | 997242   | 24 | 053277   | 1884 | 946723    | 33 |
| 28 | 051635   | 1855 | 997228   | 24 | 054407   | 1879 | 945593    | 32 |
| 29 | 052749   | 1850 | 997214   | 24 | 055535   | 1874 | 944465    | 31 |
| 30 | 053859   | 1845 | 997199   | 24 | 056659   | 1870 | 943341    | 30 |
| 31 | 9.054966 | 1841 | 9.997185 | 24 | 9.057781 | 1865 | 10.942219 | 29 |
| 32 | 056071   | 1836 | 997170   | 24 | 058900   | 1869 | 941100    | 28 |
| 33 | 057172   | 1831 | 997156   | 24 | 060016   | 1855 | 939984    | 27 |
| 34 | 058271   | 1827 | 997141   | 24 | 061130   | 1851 | 938870    | 26 |
| 35 | 059367   | 1822 | 997127   | 24 | 062240   | 1846 | 937760    | 25 |
| 36 | 060460   | 1817 | 997112   | 24 | 063348   | 1842 | 936652    | 24 |
| 37 | 061551   | 1813 | 997098   | 24 | 064453   | 1837 | 935547    | 23 |
| 38 | 062639   | 1808 | 997083   | 25 | 065556   | 1833 | 934444    | 22 |
| 39 | 063724   | 1804 | 997068   | 25 | 066655   | 1828 | 933345    | 21 |
| 40 | 064806   | 1799 | 997053   | 25 | 067752   | 1824 | 932248    | 20 |
| 41 | 9.065885 | 1794 | 9.997039 | 25 | 9.068846 | 1819 | 10.931154 | 19 |
| 42 | 066962   | 1790 | 997024   | 25 | 069938   | 1815 | 930062    | 18 |
| 43 | 068036   | 1786 | 997009   | 25 | 071027   | 1810 | 928973    | 17 |
| 44 | 069107   | 1781 | 996994   | 25 | 072113   | 1806 | 927887    | 16 |
| 45 | 070176   | 1777 | 996979   | 25 | 073197   | 1802 | 926803    | 15 |
| 46 | 071242   | 1772 | 996964   | 25 | 074278   | 1797 | 925722    | 14 |
| 47 | 072306   | 1768 | 996949   | 25 | 075356   | 1793 | 924644    | 13 |
| 48 | 073366   | 1763 | 996934   | 25 | 076432   | 1789 | 923568    | 12 |
| 49 | 074424   | 1759 | 996919   | 25 | 077505   | 1784 | 922495    | 11 |
| 50 | 075480   | 1755 | 996904   | 25 | 078576   | 1780 | 921424    | 10 |
| 51 | 9.076533 | 1750 | 9.996889 | 25 | 9.079644 | 1776 | 10.920356 | 9  |
| 52 | 077583   | 1746 | 996874   | 25 | 080710   | 1772 | 919290    | 8  |
| 53 | 078631   | 1742 | 996858   | 25 | 081773   | 1767 | 918227    | 7  |
| 54 | 079676   | 1738 | 996843   | 25 | 082833   | 1763 | 917167    | 6  |
| 55 | 080719   | 1733 | 996828   | 25 | 083891   | 1759 | 916109    | 5  |
| 56 | 081759   | 1729 | 996812   | 26 | 084947   | 1755 | 915053    | 4  |
| 57 | 082797   | 1725 | 996797   | 26 | 086000   | 1751 | 914000    | 3  |
| 58 | 083832   | 1721 | 996782   | 26 | 087050   | 1747 | 912950    | 2  |
| 59 | 084854   | 1717 | 996766   | 26 | 088098   | 1743 | 911902    | 1  |
| 60 | 085894   | 1713 | 996751   | 26 | 089144   | 1738 | 910856    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M. |

| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   | M. |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 9.085894 | 1713 | 9.996751 | 26 | 9.089144 | 1738 | 10.910856 | 60 |
| 1  | 086922   | 1709 | 996735   | 26 | 090187   | 1734 | 909813    | 59 |
| 2  | 087947   | 1704 | 996720   | 26 | 091228   | 1730 | 908772    | 58 |
| 3  | 088970   | 1700 | 996704   | 26 | 092266   | 1727 | 907734    | 57 |
| 4  | 089990   | 1696 | 996688   | 26 | 093302   | 1722 | 906698    | 56 |
| 5  | 091008   | 1692 | 996673   | 26 | 094336   | 1719 | 905664    | 55 |
| 6  | 092024   | 1688 | 996657   | 26 | 095367   | 1715 | 904633    | 54 |
| 7  | 093037   | 1684 | 996641   | 26 | 096395   | 1711 | 903605    | 53 |
| 8  | 094047   | 1680 | 996625   | 26 | 097422   | 1707 | 902578    | 52 |
| 9  | 095056   | 1676 | 996610   | 26 | 098446   | 1703 | 901554    | 51 |
| 10 | 096062   | 1673 | 996594   | 26 | 099468   | 1699 | 900532    | 50 |
| 11 | 9.097065 | 1668 | 9.996578 | 27 | 9.100487 | 1695 | 10.899513 | 49 |
| 12 | 098066   | 1665 | 996562   | 27 | 101504   | 1691 | 898496    | 48 |
| 13 | 099065   | 1661 | 996546   | 27 | 102519   | 1687 | 897481    | 47 |
| 14 | 100062   | 1657 | 996530   | 27 | 103532   | 1684 | 896468    | 46 |
| 15 | 101056   | 1653 | 996514   | 27 | 104542   | 1680 | 895458    | 45 |
| 16 | 102048   | 1649 | 996498   | 27 | 105550   | 1676 | 894450    | 44 |
| 17 | 103037   | 1645 | 996482   | 27 | 106556   | 1672 | 893444    | 43 |
| 18 | 104025   | 1641 | 996465   | 27 | 107559   | 1669 | 892441    | 42 |
| 19 | 105010   | 1638 | 996449   | 27 | 108560   | 1665 | 891440    | 41 |
| 20 | 105992   | 1634 | 996433   | 27 | 109559   | 1661 | 890441    | 40 |
| 21 | 9.106973 | 1630 | 9.996417 | 27 | 9.110556 | 1658 | 10.889444 | 39 |
| 22 | 107951   | 1627 | 996400   | 27 | 111551   | 1654 | 888449    | 38 |
| 23 | 108927   | 1623 | 996384   | 27 | 112543   | 1650 | 887457    | 37 |
| 24 | 109901   | 1619 | 996368   | 27 | 113533   | 1646 | 886467    | 36 |
| 25 | 110873   | 1616 | 996351   | 27 | 114521   | 1643 | 885479    | 35 |
| 26 | 111842   | 1612 | 996335   | 27 | 115507   | 1639 | 884493    | 34 |
| 27 | 112809   | 1608 | 996318   | 27 | 116491   | 1636 | 883509    | 33 |
| 28 | 113774   | 1605 | 996302   | 28 | 117472   | 1632 | 882528    | 32 |
| 29 | 114737   | 1601 | 996285   | 28 | 118452   | 1629 | 881548    | 31 |
| 30 | 115698   | 1597 | 996269   | 28 | 119429   | 1625 | 880571    | 30 |
| 31 | 9.116656 | 1594 | 9.996252 | 28 | 9.120404 | 1622 | 10.879596 | 29 |
| 32 | 117613   | 1590 | 996235   | 28 | 121377   | 1618 | 878623    | 28 |
| 33 | 118567   | 1587 | 996219   | 28 | 122348   | 1615 | 877652    | 27 |
| 34 | 119519   | 1583 | 996202   | 28 | 123317   | 1611 | 876683    | 26 |
| 35 | 120469   | 1580 | 996185   | 28 | 124284   | 1607 | 875716    | 25 |
| 36 | 121417   | 1576 | 996168   | 28 | 125249   | 1604 | 874751    | 24 |
| 37 | 122362   | 1573 | 996151   | 28 | 126211   | 1601 | 873789    | 23 |
| 38 | 123306   | 1569 | 996134   | 28 | 127172   | 1597 | 872828    | 22 |
| 39 | 124248   | 1566 | 996117   | 28 | 128130   | 1594 | 871870    | 21 |
| 40 | 125187   | 1562 | 996100   | 28 | 129087   | 1591 | 870913    | 20 |
| 41 | 9.126125 | 1559 | 9.996083 | 29 | 9.130041 | 1587 | 10.869959 | 19 |
| 42 | 127060   | 1556 | 996066   | 29 | 130994   | 1584 | 869006    | 18 |
| 43 | 127993   | 1552 | 996049   | 29 | 131944   | 1581 | 868056    | 17 |
| 44 | 128925   | 1549 | 996032   | 29 | 132893   | 1577 | 867107    | 16 |
| 45 | 129854   | 1545 | 996015   | 29 | 133839   | 1574 | 866161    | 15 |
| 46 | 130781   | 1542 | 995998   | 29 | 134784   | 1571 | 865216    | 14 |
| 47 | 131706   | 1539 | 995980   | 29 | 135726   | 1567 | 864274    | 13 |
| 48 | 132630   | 1535 | 995963   | 29 | 136667   | 1564 | 863333    | 12 |
| 49 | 133551   | 1532 | 995946   | 29 | 137605   | 1561 | 862395    | 11 |
| 50 | 134470   | 1529 | 995928   | 29 | 138542   | 1558 | 861458    | 10 |
| 51 | 9.135387 | 1525 | 9.995911 | 29 | 9.139476 | 1555 | 10.860524 | 9  |
| 52 | 136303   | 1522 | 995894   | 29 | 140409   | 1551 | 859591    | 8  |
| 53 | 137216   | 1519 | 995876   | 29 | 141340   | 1548 | 858660    | 7  |
| 54 | 138128   | 1516 | 995859   | 29 | 142269   | 1545 | 857731    | 6  |
| 55 | 139037   | 1512 | 995841   | 29 | 143196   | 1542 | 856804    | 5  |
| 56 | 139944   | 1509 | 995823   | 29 | 144121   | 1539 | 855879    | 4  |
| 57 | 140850   | 1506 | 995806   | 29 | 145044   | 1535 | 854956    | 3  |
| 58 | 141754   | 1503 | 995788   | 29 | 145966   | 1532 | 854034    | 2  |
| 59 | 142655   | 1500 | 995771   | 29 | 146885   | 1529 | 853115    | 1  |
| 60 | 143555   | 1496 | 995753   | 29 | 147803   | 1526 | 852197    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M. |



| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   |    |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 9.143555 | 1496 | 9.995753 | 30 | 9.147803 | 1526 | 10.852197 | 60 |
| 1  | 144453   | 1493 | 995735   | 30 | 148718   | 1523 | 851282    | 59 |
| 2  | 145349   | 1490 | 995717   | 30 | 149632   | 1520 | 850368    | 58 |
| 3  | 146243   | 1487 | 995699   | 30 | 150544   | 1517 | 849456    | 57 |
| 4  | 147136   | 1484 | 995681   | 30 | 151454   | 1514 | 848546    | 56 |
| 5  | 148026   | 1481 | 995664   | 30 | 152363   | 1511 | 847637    | 55 |
| 6  | 148915   | 1478 | 995646   | 30 | 153269   | 1508 | 846731    | 54 |
| 7  | 149802   | 1475 | 995628   | 30 | 154174   | 1505 | 845826    | 53 |
| 8  | 150686   | 1472 | 995610   | 30 | 155077   | 1502 | 844923    | 52 |
| 9  | 151569   | 1469 | 995591   | 30 | 155978   | 1499 | 844022    | 51 |
| 10 | 152451   | 1466 | 995573   | 30 | 156877   | 1496 | 843123    | 50 |
| 11 | 9.153330 | 1463 | 9.995555 | 30 | 9.157775 | 1493 | 10.842225 | 49 |
| 12 | 154208   | 1460 | 995537   | 30 | 158671   | 1490 | 841329    | 48 |
| 13 | 155083   | 1457 | 995519   | 30 | 159565   | 1487 | 840435    | 47 |
| 14 | 155957   | 1454 | 995501   | 31 | 160457   | 1484 | 839543    | 46 |
| 15 | 156830   | 1451 | 995482   | 31 | 161347   | 1481 | 838653    | 45 |
| 16 | 157700   | 1448 | 995464   | 31 | 162236   | 1479 | 837764    | 44 |
| 17 | 158569   | 1445 | 995446   | 31 | 163123   | 1476 | 836877    | 43 |
| 18 | 159435   | 1442 | 995427   | 31 | 164008   | 1473 | 835992    | 42 |
| 19 | 160301   | 1439 | 995409   | 31 | 164892   | 1470 | 835108    | 41 |
| 20 | 161164   | 1436 | 995390   | 31 | 165774   | 1467 | 834226    | 40 |
| 21 | 9.162025 | 1433 | 9.995372 | 31 | 9.166654 | 1464 | 10.833346 | 39 |
| 22 | 162885   | 1430 | 995353   | 31 | 167532   | 1461 | 832468    | 38 |
| 23 | 163743   | 1427 | 995334   | 31 | 168409   | 1458 | 831591    | 37 |
| 24 | 164600   | 1424 | 995316   | 31 | 169284   | 1455 | 830716    | 36 |
| 25 | 165454   | 1422 | 995297   | 31 | 170157   | 1453 | 829843    | 35 |
| 26 | 166307   | 1419 | 995278   | 31 | 171029   | 1450 | 828971    | 34 |
| 27 | 167159   | 1416 | 995260   | 31 | 171899   | 1447 | 828101    | 33 |
| 28 | 168008   | 1413 | 995241   | 32 | 172767   | 1444 | 827233    | 32 |
| 29 | 168856   | 1410 | 995222   | 32 | 173634   | 1442 | 826366    | 31 |
| 30 | 169702   | 1407 | 995203   | 32 | 174499   | 1439 | 825501    | 30 |
| 31 | 9.170547 | 1405 | 9.995184 | 32 | 9.175362 | 1436 | 10.824638 | 29 |
| 32 | 171389   | 1402 | 995165   | 32 | 176224   | 1433 | 823776    | 28 |
| 33 | 172230   | 1399 | 995146   | 32 | 177084   | 1431 | 822916    | 27 |
| 34 | 173070   | 1396 | 995127   | 32 | 177942   | 1428 | 822058    | 26 |
| 35 | 173908   | 1394 | 995108   | 32 | 178799   | 1425 | 821201    | 25 |
| 36 | 174744   | 1391 | 995089   | 32 | 179655   | 1423 | 820345    | 24 |
| 37 | 175578   | 1388 | 995070   | 32 | 180508   | 1420 | 819492    | 23 |
| 38 | 176411   | 1386 | 995051   | 32 | 181360   | 1417 | 818640    | 22 |
| 39 | 177242   | 1383 | 995032   | 32 | 182211   | 1415 | 817789    | 21 |
| 40 | 178072   | 1380 | 995013   | 32 | 183059   | 1412 | 816941    | 20 |
| 41 | 9.178900 | 1377 | 9.994993 | 32 | 9.183907 | 1409 | 10.816093 | 19 |
| 42 | 179726   | 1374 | 994974   | 32 | 184752   | 1407 | 815248    | 18 |
| 43 | 180551   | 1372 | 994955   | 32 | 185597   | 1404 | 814403    | 17 |
| 44 | 181374   | 1369 | 994935   | 32 | 186439   | 1402 | 813561    | 16 |
| 45 | 182196   | 1366 | 994916   | 33 | 187280   | 1399 | 812720    | 15 |
| 46 | 183016   | 1364 | 994896   | 33 | 188120   | 1396 | 811880    | 14 |
| 47 | 183834   | 1361 | 994877   | 33 | 188958   | 1393 | 811042    | 13 |
| 48 | 184651   | 1359 | 994857   | 33 | 189794   | 1391 | 810206    | 12 |
| 49 | 185466   | 1356 | 994838   | 33 | 190629   | 1389 | 809371    | 11 |
| 50 | 186280   | 1353 | 994818   | 33 | 191462   | 1386 | 808538    | 10 |
| 51 | 9.187092 | 1351 | 9.994798 | 33 | 9.192294 | 1384 | 10.807706 | 9  |
| 52 | 187903   | 1348 | 994779   | 33 | 193124   | 1381 | 806876    | 8  |
| 53 | 188712   | 1346 | 994759   | 33 | 193953   | 1379 | 806047    | 7  |
| 54 | 189519   | 1343 | 994739   | 33 | 194780   | 1376 | 805220    | 6  |
| 55 | 190325   | 1341 | 994719   | 33 | 195606   | 1374 | 804394    | 5  |
| 56 | 191130   | 1338 | 994700   | 33 | 196430   | 1371 | 803570    | 4  |
| 57 | 191933   | 1336 | 994680   | 33 | 197253   | 1369 | 802747    | 3  |
| 58 | 192734   | 1333 | 994660   | 33 | 198074   | 1366 | 801926    | 2  |
| 59 | 193534   | 1330 | 994640   | 33 | 198894   | 1364 | 801106    | 1  |
| 60 | 194332   | 1328 | 994620   | 33 | 199713   | 1361 | 800287    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M. |



| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   |    |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 9.194332 | 1323 | 9.994620 | 33 | 9.199713 | 1361 | 10.800237 | 60 |
| 1  | 195129   | 1326 | 994600   | 33 | 200529   | 1359 | 799471    | 59 |
| 2  | 195925   | 1323 | 994580   | 33 | 201345   | 1356 | 798555    | 58 |
| 3  | 193719   | 1321 | 994560   | 34 | 202159   | 1354 | 797341    | 57 |
| 4  | 197511   | 1318 | 994540   | 34 | 202971   | 1352 | 797029    | 56 |
| 5  | 198302   | 1316 | 994519   | 34 | 203782   | 1349 | 796218    | 55 |
| 6  | 199091   | 1313 | 994499   | 34 | 204592   | 1347 | 795408    | 54 |
| 7  | 199879   | 1311 | 994479   | 34 | 205400   | 1345 | 794600    | 53 |
| 8  | 200666   | 1308 | 994459   | 34 | 206207   | 1342 | 793793    | 52 |
| 9  | 201451   | 1306 | 994438   | 34 | 207013   | 1340 | 792987    | 51 |
| 10 | 202234   | 1304 | 994418   | 34 | 207817   | 1338 | 792183    | 50 |
| 11 | 9.203017 | 1301 | 9.994397 | 34 | 9.208619 | 1335 | 10.791381 | 49 |
| 12 | 203797   | 1299 | 994377   | 34 | 209420   | 1333 | 791580    | 48 |
| 13 | 204577   | 1296 | 994357   | 34 | 210220   | 1331 | 789780    | 47 |
| 14 | 205354   | 1294 | 994336   | 34 | 211018   | 1328 | 788982    | 46 |
| 15 | 205131   | 1292 | 994316   | 34 | 211815   | 1326 | 788185    | 45 |
| 16 | 206906   | 1289 | 994295   | 34 | 212611   | 1324 | 787389    | 44 |
| 17 | 207679   | 1287 | 994274   | 35 | 213405   | 1321 | 786595    | 43 |
| 18 | 208452   | 1285 | 994254   | 35 | 214198   | 1319 | 785802    | 42 |
| 19 | 209222   | 1282 | 994233   | 35 | 214989   | 1317 | 785011    | 41 |
| 20 | 209992   | 1280 | 994212   | 35 | 215780   | 1315 | 784220    | 40 |
| 21 | 9.210760 | 1278 | 9.994191 | 35 | 9.216568 | 1312 | 10.783432 | 39 |
| 22 | 211526   | 1275 | 994171   | 35 | 217356   | 1310 | 782644    | 38 |
| 23 | 212291   | 1273 | 994150   | 35 | 218142   | 1308 | 781858    | 37 |
| 24 | 213055   | 1271 | 994129   | 35 | 218926   | 1305 | 781074    | 36 |
| 25 | 213818   | 1268 | 994108   | 35 | 219710   | 1303 | 780290    | 35 |
| 26 | 214579   | 1266 | 994087   | 35 | 220492   | 1301 | 779508    | 34 |
| 27 | 215338   | 1264 | 994066   | 35 | 221272   | 1299 | 778728    | 33 |
| 28 | 216097   | 1261 | 994045   | 35 | 222052   | 1297 | 777948    | 32 |
| 29 | 216854   | 1259 | 994024   | 35 | 222830   | 1294 | 777170    | 31 |
| 30 | 217609   | 1257 | 994003   | 35 | 223606   | 1292 | 776394    | 30 |
| 31 | 9.218333 | 1255 | 9.993981 | 35 | 9.224382 | 1290 | 10.775618 | 29 |
| 32 | 219116   | 1253 | 993960   | 35 | 225156   | 1288 | 774844    | 28 |
| 33 | 219868   | 1250 | 993939   | 35 | 225929   | 1286 | 774071    | 27 |
| 34 | 220618   | 1248 | 993918   | 35 | 226700   | 1284 | 773300    | 26 |
| 35 | 221367   | 1246 | 993896   | 36 | 227471   | 1281 | 772529    | 25 |
| 36 | 222115   | 1244 | 993875   | 36 | 228239   | 1279 | 771761    | 24 |
| 37 | 222861   | 1242 | 993854   | 36 | 229007   | 1277 | 770993    | 23 |
| 38 | 223606   | 1239 | 993832   | 36 | 229773   | 1275 | 770227    | 22 |
| 39 | 224349   | 1237 | 993811   | 36 | 230539   | 1273 | 769461    | 21 |
| 40 | 225092   | 1235 | 993789   | 36 | 231302   | 1271 | 768698    | 20 |
| 41 | 9.225833 | 1233 | 9.993768 | 36 | 9.232065 | 1269 | 10.767935 | 19 |
| 42 | 226573   | 1231 | 993746   | 36 | 232826   | 1267 | 767174    | 18 |
| 43 | 227311   | 1228 | 993725   | 36 | 233586   | 1265 | 766414    | 17 |
| 44 | 228048   | 1226 | 993703   | 36 | 234345   | 1262 | 765655    | 16 |
| 45 | 228784   | 1224 | 993681   | 36 | 235103   | 1260 | 764897    | 15 |
| 46 | 229518   | 1222 | 993660   | 36 | 235859   | 1258 | 764141    | 14 |
| 47 | 230252   | 1220 | 993638   | 36 | 236614   | 1256 | 763386    | 13 |
| 48 | 230984   | 1218 | 993616   | 36 | 237368   | 1254 | 762632    | 12 |
| 49 | 231714   | 1216 | 993594   | 37 | 238120   | 1252 | 761880    | 11 |
| 50 | 232444   | 1214 | 993572   | 37 | 238872   | 1250 | 761128    | 10 |
| 51 | 9.233172 | 1212 | 9.993550 | 37 | 9.239622 | 1248 | 10.760378 | 9  |
| 52 | 233899   | 1209 | 993528   | 37 | 240371   | 1246 | 759629    | 8  |
| 53 | 234625   | 1207 | 993506   | 37 | 241118   | 1244 | 758882    | 7  |
| 54 | 235349   | 1205 | 993484   | 37 | 241865   | 1242 | 758135    | 6  |
| 55 | 236073   | 1203 | 993462   | 37 | 242610   | 1240 | 757390    | 5  |
| 56 | 236795   | 1201 | 993440   | 37 | 243354   | 1238 | 756646    | 4  |
| 57 | 237515   | 1199 | 993418   | 37 | 244097   | 1236 | 755903    | 3  |
| 58 | 238235   | 1197 | 993396   | 37 | 244839   | 1234 | 755161    | 2  |
| 59 | 238953   | 1195 | 993374   | 37 | 245579   | 1232 | 754421    | 1  |
| 60 | 239670   | 1193 | 993351   | 37 | 246319   | 1230 | 753681    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M. |

| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   | M. |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 9.239670 | 1193 | 9.993351 | 37 | 9.246319 | 1230 | 10.753681 | 60 |
| 1  | 240336   | 1191 | 993329   | 37 | 247057   | 1228 | 752943    | 59 |
| 2  | 241101   | 1189 | 993307   | 37 | 247794   | 1226 | 752206    | 58 |
| 3  | 241814   | 1187 | 993285   | 37 | 248530   | 1224 | 751470    | 57 |
| 4  | 242526   | 1185 | 993262   | 37 | 249264   | 1222 | 750736    | 56 |
| 5  | 243237   | 1183 | 993240   | 37 | 249998   | 1220 | 750002    | 55 |
| 6  | 243947   | 1181 | 993217   | 38 | 250730   | 1218 | 749270    | 54 |
| 7  | 244656   | 1179 | 993195   | 38 | 251461   | 1217 | 748539    | 53 |
| 8  | 245363   | 1177 | 993172   | 38 | 252191   | 1215 | 747809    | 52 |
| 9  | 246069   | 1175 | 993149   | 38 | 252920   | 1213 | 747080    | 51 |
| 10 | 246775   | 1173 | 993127   | 38 | 253648   | 1211 | 746352    | 50 |
| 11 | 9.247478 | 1171 | 9.993104 | 38 | 9.254374 | 1209 | 10.745626 | 49 |
| 12 | 248181   | 1169 | 993081   | 38 | 255100   | 1207 | 744900    | 48 |
| 13 | 248883   | 1167 | 993059   | 38 | 255824   | 1205 | 744176    | 47 |
| 14 | 249583   | 1165 | 993036   | 38 | 256547   | 1203 | 743453    | 46 |
| 15 | 250282   | 1163 | 993013   | 38 | 257269   | 1201 | 742731    | 45 |
| 16 | 250980   | 1161 | 992990   | 38 | 257990   | 1200 | 742010    | 44 |
| 17 | 251677   | 1159 | 992967   | 38 | 258710   | 1198 | 741290    | 43 |
| 18 | 252373   | 1158 | 992944   | 38 | 259429   | 1196 | 740571    | 42 |
| 19 | 253067   | 1156 | 992921   | 38 | 260146   | 1194 | 739854    | 41 |
| 20 | 253761   | 1154 | 992898   | 38 | 260863   | 1192 | 739137    | 40 |
| 21 | 9.254453 | 1152 | 9.992875 | 38 | 9.261578 | 1190 | 10.738422 | 39 |
| 22 | 255144   | 1150 | 992852   | 38 | 262292   | 1189 | 737708    | 38 |
| 23 | 255834   | 1148 | 992829   | 39 | 263005   | 1187 | 736995    | 37 |
| 24 | 256523   | 1146 | 992806   | 39 | 263717   | 1185 | 736283    | 36 |
| 25 | 257211   | 1144 | 992783   | 39 | 264428   | 1183 | 735572    | 35 |
| 26 | 257898   | 1142 | 992759   | 39 | 265138   | 1181 | 734862    | 34 |
| 27 | 258583   | 1141 | 992736   | 39 | 265847   | 1179 | 734153    | 33 |
| 28 | 259268   | 1139 | 992713   | 39 | 266555   | 1178 | 733445    | 32 |
| 29 | 259951   | 1137 | 992690   | 39 | 267261   | 1176 | 732739    | 31 |
| 30 | 260633   | 1135 | 992666   | 39 | 267967   | 1174 | 732033    | 30 |
| 31 | 9.261314 | 1133 | 9.992643 | 39 | 9.268671 | 1172 | 10.731329 | 29 |
| 32 | 261994   | 1131 | 992619   | 39 | 269375   | 1170 | 730625    | 28 |
| 33 | 262673   | 1130 | 992596   | 39 | 270077   | 1169 | 729923    | 27 |
| 34 | 263351   | 1128 | 992572   | 39 | 270779   | 1167 | 729221    | 26 |
| 35 | 264027   | 1126 | 992549   | 39 | 271479   | 1165 | 728521    | 25 |
| 36 | 264703   | 1124 | 992525   | 39 | 272178   | 1164 | 727822    | 24 |
| 37 | 265377   | 1122 | 992501   | 39 | 272876   | 1162 | 727124    | 23 |
| 38 | 266051   | 1120 | 992478   | 40 | 273573   | 1160 | 726427    | 22 |
| 39 | 266723   | 1119 | 992454   | 40 | 274269   | 1158 | 725731    | 21 |
| 40 | 267395   | 1117 | 992430   | 40 | 274964   | 1157 | 725036    | 20 |
| 41 | 9.268065 | 1115 | 9.992406 | 40 | 9.275658 | 1155 | 10.724342 | 19 |
| 42 | 268734   | 1113 | 992382   | 40 | 276351   | 1153 | 723649    | 18 |
| 43 | 269402   | 1111 | 992359   | 40 | 277043   | 1151 | 722957    | 17 |
| 44 | 270069   | 1110 | 992335   | 40 | 277734   | 1150 | 722266    | 16 |
| 45 | 270735   | 1108 | 992311   | 40 | 278424   | 1148 | 721576    | 15 |
| 46 | 271400   | 1106 | 992287   | 40 | 279113   | 1147 | 720887    | 14 |
| 47 | 272064   | 1105 | 992263   | 40 | 279801   | 1145 | 720199    | 13 |
| 48 | 272726   | 1103 | 992239   | 40 | 280488   | 1143 | 719512    | 12 |
| 49 | 273388   | 1101 | 992214   | 40 | 281174   | 1141 | 718826    | 11 |
| 50 | 274049   | 1099 | 992190   | 40 | 281858   | 1140 | 718142    | 10 |
| 51 | 9.274708 | 1098 | 9.992166 | 40 | 9.282542 | 1138 | 10.717458 | 9  |
| 52 | 275367   | 1096 | 992142   | 40 | 283225   | 1136 | 716775    | 8  |
| 53 | 276024   | 1094 | 992117   | 41 | 283907   | 1135 | 716093    | 7  |
| 54 | 276681   | 1092 | 992093   | 41 | 284588   | 1133 | 715412    | 6  |
| 55 | 277337   | 1091 | 992069   | 41 | 285268   | 1131 | 714732    | 5  |
| 56 | 277991   | 1089 | 992044   | 41 | 285947   | 1130 | 714053    | 4  |
| 57 | 278644   | 1087 | 992020   | 41 | 286624   | 1128 | 713376    | 3  |
| 58 | 279297   | 1086 | 991996   | 41 | 287301   | 1126 | 712699    | 2  |
| 59 | 279948   | 1084 | 991971   | 41 | 287977   | 1125 | 712023    | 1  |
| 60 | 280599   | 1082 | 991947   | 41 | 288652   | 1123 | 711348    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M. |

| M. | Sine     | D.   | Cosine   | D. | Tang.    | D.   | Cotang.   | M. |
|----|----------|------|----------|----|----------|------|-----------|----|
| 0  | 9.280599 | 1082 | 9.991947 | 41 | 9.288652 | 1123 | 10.711348 | 60 |
| 1  | 281248   | 1081 | 991922   | 41 | 289326   | 1122 | 710674    | 59 |
| 2  | 281897   | 1079 | 991897   | 41 | 289999   | 1120 | 710001    | 58 |
| 3  | 282544   | 1077 | 991873   | 41 | 290671   | 1118 | 709329    | 57 |
| 4  | 283190   | 1076 | 991848   | 41 | 291342   | 1117 | 708658    | 56 |
| 5  | 283836   | 1074 | 991823   | 41 | 292013   | 1115 | 707987    | 55 |
| 6  | 284480   | 1072 | 991799   | 41 | 292682   | 1114 | 707318    | 54 |
| 7  | 285124   | 1071 | 991774   | 42 | 293350   | 1112 | 706650    | 53 |
| 8  | 285766   | 1069 | 991749   | 42 | 294017   | 1111 | 705983    | 52 |
| 9  | 286408   | 1067 | 991724   | 42 | 294684   | 1109 | 705316    | 51 |
| 10 | 287048   | 1066 | 991699   | 42 | 295349   | 1107 | 704651    | 50 |
| 11 | 9.287687 | 1064 | 9.991674 | 42 | 9.296013 | 1106 | 10.703987 | 49 |
| 12 | 288326   | 1063 | 991649   | 42 | 296677   | 1104 | 703323    | 48 |
| 12 | 288964   | 1061 | 991624   | 42 | 297339   | 1103 | 702661    | 47 |
| 14 | 289600   | 1059 | 991599   | 42 | 298001   | 1101 | 701999    | 46 |
| 15 | 290236   | 1058 | 991574   | 42 | 298662   | 1100 | 701338    | 45 |
| 16 | 290870   | 1056 | 991549   | 42 | 299322   | 1098 | 700678    | 44 |
| 17 | 291504   | 1054 | 991524   | 42 | 299980   | 1096 | 700020    | 43 |
| 18 | 292137   | 1053 | 991498   | 42 | 300638   | 1095 | 699362    | 42 |
| 19 | 292768   | 1051 | 991473   | 42 | 301295   | 1093 | 698705    | 41 |
| 20 | 293399   | 1050 | 991448   | 42 | 301951   | 1092 | 698049    | 40 |
| 21 | 9.294029 | 1048 | 9.991422 | 42 | 9.302607 | 1090 | 10.697393 | 39 |
| 22 | 294658   | 1046 | 991397   | 42 | 303261   | 1089 | 696739    | 38 |
| 23 | 295286   | 1045 | 991372   | 43 | 303914   | 1087 | 696086    | 37 |
| 24 | 295913   | 1043 | 991346   | 43 | 304567   | 1086 | 695433    | 36 |
| 25 | 296539   | 1042 | 991321   | 43 | 305218   | 1084 | 694782    | 35 |
| 26 | 297164   | 1040 | 991295   | 43 | 305869   | 1083 | 694131    | 34 |
| 27 | 297788   | 1039 | 991270   | 43 | 306519   | 1081 | 693481    | 33 |
| 28 | 298412   | 1037 | 991244   | 43 | 307168   | 1080 | 692832    | 32 |
| 29 | 299034   | 1036 | 991218   | 43 | 307815   | 1078 | 692185    | 31 |
| 30 | 299655   | 1034 | 991193   | 43 | 308463   | 1077 | 691537    | 30 |
| 31 | 9.300276 | 1032 | 9.991167 | 43 | 9.309109 | 1075 | 10.690391 | 29 |
| 32 | 300895   | 1031 | 991141   | 43 | 309754   | 1074 | 690246    | 28 |
| 33 | 301514   | 1029 | 991115   | 43 | 310398   | 1073 | 689602    | 27 |
| 34 | 302132   | 1028 | 991090   | 43 | 311042   | 1071 | 688958    | 26 |
| 35 | 302748   | 1026 | 991064   | 43 | 311685   | 1070 | 688315    | 25 |
| 36 | 303364   | 1025 | 991038   | 43 | 312327   | 1068 | 687673    | 24 |
| 37 | 303979   | 1023 | 991012   | 43 | 312967   | 1067 | 687033    | 23 |
| 38 | 304593   | 1022 | 990986   | 43 | 313608   | 1065 | 686392    | 22 |
| 39 | 305207   | 1020 | 990960   | 43 | 314247   | 1064 | 685753    | 21 |
| 40 | 305819   | 1019 | 990934   | 44 | 314885   | 1062 | 685115    | 20 |
| 41 | 9.306430 | 1017 | 9.990908 | 44 | 9.315523 | 1061 | 10.684477 | 19 |
| 42 | 307041   | 1016 | 990882   | 44 | 316159   | 1060 | 683841    | 18 |
| 43 | 307650   | 1014 | 990855   | 44 | 316795   | 1058 | 683205    | 17 |
| 44 | 308259   | 1013 | 990829   | 44 | 317430   | 1057 | 682570    | 16 |
| 45 | 308867   | 1011 | 990803   | 44 | 318064   | 1055 | 681936    | 15 |
| 46 | 309474   | 1010 | 990777   | 44 | 318697   | 1054 | 681303    | 14 |
| 47 | 310080   | 1008 | 990750   | 44 | 319329   | 1053 | 680671    | 13 |
| 48 | 310685   | 1007 | 990724   | 44 | 319961   | 1051 | 680039    | 12 |
| 49 | 311289   | 1005 | 990697   | 44 | 320592   | 1050 | 679408    | 11 |
| 50 | 311893   | 1004 | 990671   | 44 | 321222   | 1048 | 678778    | 10 |
| 51 | 9.312495 | 1003 | 9.990644 | 44 | 9.321851 | 1047 | 10.678149 | 9  |
| 52 | 313097   | 1001 | 990618   | 44 | 322479   | 1045 | 677521    | 8  |
| 53 | 313698   | 1000 | 990591   | 44 | 323106   | 1044 | 676894    | 7  |
| 54 | 314297   | 998  | 990565   | 44 | 323733   | 1043 | 676267    | 6  |
| 55 | 314897   | 997  | 990538   | 44 | 324358   | 1041 | 675642    | 5  |
| 56 | 315495   | 996  | 990511   | 45 | 324983   | 1040 | 675017    | 4  |
| 57 | 316092   | 994  | 990485   | 45 | 325607   | 1039 | 674393    | 3  |
| 58 | 316689   | 993  | 990458   | 45 | 326231   | 1037 | 673769    | 2  |
| 59 | 317284   | 991  | 990431   | 45 | 326853   | 1036 | 673147    | 1  |
| 60 | 317879   | 990  | 990404   | 45 | 327475   | 1035 | 672525    | 0  |
|    | Cosine   |      | Sine     |    | Cotang.  |      | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.   | Cotang.   | M. |
|----|----------|-----|----------|----|----------|------|-----------|----|
| 0  | 9.317879 | 990 | 9.990404 | 45 | 9.327474 | 1035 | 10.672526 | 60 |
| 1  | 318473   | 988 | 990378   | 45 | 328095   | 1033 | 671905    | 59 |
| 2  | 319066   | 987 | 990351   | 45 | 328715   | 1032 | 671285    | 58 |
| 3  | 319658   | 986 | 990324   | 45 | 329334   | 1030 | 670666    | 57 |
| 4  | 320249   | 984 | 990297   | 45 | 329953   | 1029 | 670047    | 56 |
| 5  | 320840   | 983 | 990270   | 45 | 330570   | 1028 | 669430    | 55 |
| 6  | 321430   | 982 | 990243   | 45 | 331187   | 1026 | 668813    | 54 |
| 7  | 322019   | 980 | 990215   | 45 | 331803   | 1025 | 668197    | 53 |
| 8  | 322607   | 979 | 990188   | 45 | 332418   | 1024 | 667582    | 52 |
| 9  | 323194   | 977 | 990161   | 45 | 333033   | 1023 | 666967    | 51 |
| 10 | 323780   | 976 | 990134   | 45 | 333646   | 1021 | 666354    | 50 |
| 11 | 9.324366 | 975 | 9.990107 | 46 | 9.334250 | 1020 | 10.665741 | 49 |
| 12 | 324950   | 973 | 990079   | 46 | 334871   | 1019 | 665129    | 48 |
| 13 | 325534   | 972 | 990052   | 46 | 335482   | 1017 | 664518    | 47 |
| 14 | 326117   | 970 | 990025   | 46 | 336093   | 1016 | 663907    | 46 |
| 15 | 326700   | 969 | 989997   | 46 | 336702   | 1015 | 663298    | 45 |
| 16 | 327281   | 968 | 989970   | 46 | 337311   | 1013 | 662689    | 44 |
| 17 | 327862   | 966 | 989942   | 46 | 337919   | 1012 | 662081    | 43 |
| 18 | 328442   | 965 | 989915   | 46 | 338527   | 1011 | 661473    | 42 |
| 19 | 329021   | 964 | 989887   | 46 | 339133   | 1010 | 660867    | 41 |
| 20 | 329599   | 962 | 989860   | 46 | 339739   | 1008 | 660261    | 40 |
| 21 | 9.330176 | 961 | 9.989832 | 46 | 9.340344 | 1007 | 10.659656 | 39 |
| 22 | 330753   | 960 | 989804   | 46 | 340948   | 1006 | 659052    | 38 |
| 23 | 331329   | 958 | 989777   | 46 | 341552   | 1004 | 658448    | 37 |
| 24 | 331903   | 957 | 989749   | 47 | 342155   | 1003 | 657845    | 36 |
| 25 | 332478   | 956 | 989721   | 47 | 342757   | 1002 | 657243    | 35 |
| 26 | 333051   | 954 | 989693   | 47 | 343358   | 1000 | 656642    | 34 |
| 27 | 333624   | 953 | 989665   | 47 | 343958   | 999  | 656042    | 33 |
| 28 | 334195   | 952 | 989637   | 47 | 344558   | 998  | 655442    | 32 |
| 29 | 334766   | 950 | 989609   | 47 | 345157   | 997  | 654843    | 31 |
| 30 | 335337   | 949 | 989582   | 47 | 345755   | 996  | 654245    | 30 |
| 31 | 9.335906 | 948 | 9.989553 | 47 | 9.346353 | 994  | 10.653647 | 29 |
| 32 | 336475   | 946 | 989525   | 47 | 346949   | 993  | 653051    | 28 |
| 33 | 337043   | 945 | 989497   | 47 | 347545   | 992  | 652455    | 27 |
| 34 | 337610   | 944 | 989469   | 47 | 348141   | 991  | 651859    | 26 |
| 35 | 338176   | 943 | 989441   | 47 | 348735   | 990  | 651265    | 25 |
| 36 | 338742   | 941 | 989413   | 47 | 349329   | 988  | 650671    | 24 |
| 37 | 339306   | 940 | 989384   | 47 | 349922   | 987  | 650078    | 23 |
| 38 | 339871   | 939 | 989356   | 47 | 350514   | 986  | 649486    | 22 |
| 39 | 340434   | 937 | 989328   | 47 | 351106   | 985  | 648894    | 21 |
| 40 | 340996   | 936 | 989300   | 47 | 351697   | 983  | 648303    | 20 |
| 41 | 9.341558 | 935 | 9.989271 | 47 | 9.352287 | 982  | 10.647713 | 19 |
| 42 | 342119   | 934 | 989243   | 47 | 352876   | 981  | 647124    | 18 |
| 43 | 342679   | 932 | 989214   | 47 | 353465   | 980  | 646535    | 17 |
| 44 | 343239   | 931 | 989186   | 47 | 354053   | 979  | 645947    | 16 |
| 45 | 343797   | 930 | 989157   | 47 | 354640   | 977  | 645360    | 15 |
| 46 | 344355   | 929 | 989128   | 48 | 355227   | 976  | 644773    | 14 |
| 47 | 344912   | 927 | 989100   | 48 | 355813   | 975  | 644187    | 13 |
| 48 | 345469   | 926 | 989071   | 48 | 356398   | 974  | 643602    | 12 |
| 49 | 346024   | 925 | 989042   | 48 | 356982   | 973  | 643018    | 11 |
| 50 | 346579   | 924 | 989014   | 48 | 357566   | 971  | 642434    | 10 |
| 51 | 9.347134 | 922 | 9.988985 | 48 | 9.358149 | 970  | 10.641851 | 9  |
| 52 | 347687   | 921 | 988956   | 48 | 358731   | 969  | 641269    | 8  |
| 53 | 348240   | 920 | 988927   | 48 | 359313   | 968  | 640687    | 7  |
| 54 | 348792   | 919 | 988898   | 48 | 359893   | 967  | 640107    | 6  |
| 55 | 349343   | 917 | 988869   | 48 | 360474   | 966  | 639526    | 5  |
| 56 | 349893   | 916 | 988840   | 48 | 361053   | 965  | 638947    | 4  |
| 57 | 350443   | 915 | 988811   | 49 | 361632   | 963  | 638368    | 3  |
| 58 | 350992   | 914 | 988782   | 49 | 362210   | 962  | 637790    | 2  |
| 59 | 351540   | 913 | 988753   | 49 | 362787   | 961  | 637213    | 1  |
| 60 | 352088   | 911 | 988724   | 49 | 363364   | 960  | 636636    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |      | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.352088 | 911 | 9.988724 | 49 | 9.363364 | 960 | 10.636636 | 60 |
| 1  | 352035   | 910 | 988395   | 49 | 363940   | 959 | 636060    | 59 |
| 2  | 353181   | 909 | 988666   | 49 | 364515   | 958 | 635485    | 58 |
| 3  | 353726   | 908 | 988636   | 49 | 365090   | 957 | 634910    | 57 |
| 4  | 354271   | 907 | 988607   | 49 | 365664   | 955 | 634336    | 56 |
| 5  | 354815   | 905 | 988578   | 49 | 366237   | 954 | 633763    | 55 |
| 6  | 355358   | 904 | 988548   | 49 | 366810   | 953 | 633190    | 54 |
| 7  | 355901   | 903 | 988519   | 49 | 367382   | 952 | 632618    | 53 |
| 8  | 356443   | 902 | 988489   | 49 | 367953   | 951 | 632047    | 52 |
| 9  | 356984   | 901 | 988460   | 49 | 368524   | 950 | 631476    | 51 |
| 10 | 357524   | 899 | 988430   | 49 | 369094   | 949 | 630906    | 50 |
| 11 | 9.358064 | 898 | 9.988401 | 49 | 9.369663 | 948 | 10.630337 | 49 |
| 12 | 358603   | 897 | 988371   | 49 | 370232   | 946 | 629768    | 48 |
| 13 | 359141   | 896 | 988342   | 49 | 370799   | 945 | 629201    | 47 |
| 14 | 359678   | 895 | 988312   | 50 | 371367   | 944 | 628633    | 46 |
| 15 | 360215   | 893 | 988282   | 50 | 371933   | 943 | 628067    | 45 |
| 16 | 360752   | 892 | 988252   | 50 | 372499   | 942 | 627501    | 44 |
| 17 | 361287   | 891 | 988223   | 50 | 373064   | 941 | 626936    | 43 |
| 18 | 361822   | 890 | 988193   | 50 | 373629   | 940 | 626371    | 42 |
| 19 | 362356   | 889 | 988163   | 50 | 374193   | 939 | 625807    | 41 |
| 20 | 362889   | 888 | 988133   | 50 | 374756   | 938 | 625244    | 40 |
| 21 | 9.363422 | 887 | 9.988103 | 50 | 9.375319 | 937 | 10.624681 | 39 |
| 22 | 363954   | 885 | 988073   | 50 | 375881   | 935 | 624119    | 38 |
| 23 | 364485   | 884 | 988043   | 50 | 376442   | 934 | 623558    | 37 |
| 24 | 365016   | 883 | 988013   | 50 | 377003   | 933 | 622997    | 36 |
| 25 | 365546   | 882 | 987983   | 50 | 377563   | 932 | 622437    | 35 |
| 26 | 366075   | 881 | 987953   | 50 | 378122   | 931 | 621878    | 34 |
| 27 | 366604   | 880 | 987922   | 50 | 378681   | 930 | 621319    | 33 |
| 28 | 367131   | 879 | 987892   | 50 | 379239   | 929 | 620761    | 32 |
| 29 | 367659   | 877 | 987862   | 50 | 379797   | 928 | 620203    | 31 |
| 30 | 368185   | 876 | 987832   | 51 | 380354   | 927 | 619646    | 30 |
| 31 | 9.368711 | 875 | 9.987801 | 51 | 9.380910 | 926 | 10.619090 | 29 |
| 32 | 369236   | 874 | 987771   | 51 | 381466   | 925 | 618534    | 28 |
| 33 | 369761   | 873 | 987740   | 51 | 382020   | 924 | 617980    | 27 |
| 34 | 370285   | 872 | 987710   | 51 | 382575   | 923 | 617425    | 26 |
| 35 | 370808   | 871 | 987679   | 51 | 383129   | 922 | 616871    | 25 |
| 36 | 371330   | 870 | 987649   | 51 | 383682   | 921 | 616318    | 24 |
| 37 | 371852   | 869 | 987618   | 51 | 384234   | 920 | 615766    | 23 |
| 38 | 372373   | 867 | 987588   | 51 | 384786   | 919 | 615214    | 22 |
| 39 | 372894   | 866 | 987557   | 51 | 385337   | 918 | 614663    | 21 |
| 40 | 373414   | 865 | 987526   | 51 | 385888   | 917 | 614112    | 20 |
| 41 | 9.373933 | 864 | 9.987496 | 51 | 9.386438 | 915 | 10.613562 | 19 |
| 42 | 374452   | 863 | 987465   | 51 | 386987   | 914 | 613013    | 18 |
| 43 | 374970   | 862 | 987434   | 51 | 387536   | 913 | 612464    | 17 |
| 44 | 375487   | 861 | 987403   | 52 | 388084   | 912 | 611916    | 16 |
| 45 | 376003   | 860 | 987372   | 52 | 388631   | 911 | 611369    | 15 |
| 46 | 376519   | 859 | 987341   | 52 | 389178   | 910 | 610822    | 14 |
| 47 | 377035   | 858 | 987310   | 52 | 389724   | 909 | 610276    | 13 |
| 48 | 377549   | 857 | 987279   | 52 | 390270   | 908 | 609730    | 12 |
| 49 | 378063   | 856 | 987248   | 52 | 390815   | 907 | 609185    | 11 |
| 50 | 378577   | 854 | 987217   | 52 | 391360   | 906 | 608640    | 10 |
| 51 | 9.379089 | 853 | 9.987186 | 52 | 9.391903 | 905 | 10.608097 | 9  |
| 52 | 379601   | 852 | 987155   | 52 | 392447   | 904 | 607553    | 8  |
| 53 | 380113   | 851 | 987124   | 52 | 392989   | 903 | 607011    | 7  |
| 54 | 380624   | 850 | 987092   | 52 | 393531   | 902 | 606469    | 6  |
| 55 | 381134   | 849 | 987061   | 52 | 394073   | 901 | 605927    | 5  |
| 56 | 381643   | 848 | 987030   | 52 | 394614   | 900 | 605386    | 4  |
| 57 | 382152   | 847 | 986998   | 52 | 395154   | 899 | 604846    | 3  |
| 58 | 382661   | 846 | 986967   | 52 | 395694   | 898 | 604306    | 2  |
| 59 | 383168   | 845 | 986936   | 52 | 396233   | 897 | 603767    | 1  |
| 60 | 383675   | 844 | 986904   | 52 | 396771   | 896 | 603229    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D   | Cosine   | D. | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.383675 | 844 | 9.986904 | 52 | 9.396771 | 896 | 10.603229 | 60 |
| 1  | 384182   | 843 | 986873   | 53 | 397309   | 896 | 602691    | 59 |
| 2  | 384687   | 842 | 986841   | 53 | 397846   | 895 | 602154    | 58 |
| 3  | 385192   | 841 | 986809   | 53 | 398383   | 894 | 601617    | 57 |
| 4  | 385697   | 840 | 986778   | 53 | 398919   | 893 | 601081    | 56 |
| 5  | 386201   | 839 | 986746   | 53 | 399455   | 892 | 600545    | 55 |
| 6  | 386704   | 838 | 986714   | 53 | 399990   | 891 | 600010    | 54 |
| 7  | 387207   | 837 | 986683   | 53 | 400524   | 890 | 599476    | 53 |
| 8  | 387709   | 836 | 986651   | 53 | 401058   | 889 | 598942    | 52 |
| 9  | 388210   | 835 | 986619   | 53 | 401591   | 888 | 598409    | 51 |
| 10 | 388711   | 834 | 986587   | 53 | 402124   | 887 | 597876    | 50 |
| 11 | 9.389211 | 833 | 9.986555 | 53 | 9.402656 | 886 | 10.597344 | 49 |
| 12 | 389711   | 832 | 986522   | 53 | 403187   | 885 | 596813    | 48 |
| 13 | 390210   | 831 | 986491   | 53 | 403718   | 884 | 596282    | 47 |
| 14 | 390708   | 830 | 986459   | 53 | 404249   | 883 | 595751    | 46 |
| 15 | 391206   | 828 | 986427   | 53 | 404778   | 882 | 595222    | 45 |
| 16 | 391703   | 827 | 986395   | 53 | 405308   | 881 | 594692    | 44 |
| 17 | 392199   | 826 | 986363   | 54 | 405836   | 880 | 594164    | 43 |
| 18 | 392695   | 825 | 986331   | 54 | 406364   | 879 | 593636    | 42 |
| 19 | 393191   | 824 | 986299   | 54 | 406892   | 878 | 593108    | 41 |
| 20 | 393685   | 823 | 986266   | 54 | 407419   | 877 | 592581    | 40 |
| 21 | 9.394179 | 822 | 9.986234 | 54 | 9.407945 | 876 | 10.592055 | 39 |
| 22 | 394673   | 821 | 986202   | 54 | 408471   | 875 | 591529    | 38 |
| 23 | 395166   | 820 | 986169   | 54 | 408997   | 874 | 591003    | 37 |
| 24 | 395658   | 819 | 986137   | 54 | 409521   | 874 | 590479    | 36 |
| 25 | 396150   | 818 | 986104   | 54 | 410045   | 873 | 589955    | 35 |
| 26 | 396641   | 817 | 986072   | 54 | 410569   | 872 | 589431    | 34 |
| 27 | 397132   | 817 | 986039   | 54 | 411092   | 871 | 588908    | 33 |
| 28 | 397621   | 816 | 986007   | 54 | 411615   | 870 | 588385    | 32 |
| 29 | 398111   | 815 | 985974   | 54 | 412137   | 869 | 587863    | 31 |
| 30 | 398600   | 814 | 985942   | 54 | 412658   | 868 | 587342    | 30 |
| 31 | 9.399088 | 813 | 9.985909 | 55 | 9.413179 | 867 | 10.586821 | 29 |
| 32 | 399575   | 812 | 985876   | 55 | 413699   | 866 | 586301    | 28 |
| 33 | 400062   | 811 | 985843   | 55 | 414219   | 865 | 585781    | 27 |
| 34 | 400549   | 810 | 985811   | 55 | 414738   | 864 | 585262    | 26 |
| 35 | 401035   | 809 | 985778   | 55 | 415257   | 864 | 584743    | 25 |
| 36 | 401520   | 808 | 985745   | 55 | 415775   | 863 | 584225    | 24 |
| 37 | 402005   | 807 | 985712   | 55 | 416293   | 862 | 583707    | 23 |
| 38 | 402489   | 806 | 985679   | 55 | 416810   | 861 | 583190    | 22 |
| 39 | 402972   | 805 | 985646   | 55 | 417326   | 860 | 582674    | 21 |
| 40 | 403455   | 804 | 985613   | 55 | 417842   | 859 | 582158    | 20 |
| 41 | 9.403938 | 803 | 9.985580 | 55 | 9.418358 | 858 | 10.581642 | 19 |
| 42 | 404420   | 802 | 985547   | 55 | 418873   | 857 | 581127    | 18 |
| 43 | 404901   | 801 | 985514   | 55 | 419387   | 856 | 580613    | 17 |
| 44 | 405382   | 800 | 985480   | 55 | 419901   | 855 | 580099    | 16 |
| 45 | 405862   | 799 | 985447   | 55 | 420415   | 855 | 579585    | 15 |
| 46 | 406341   | 798 | 985414   | 56 | 420927   | 854 | 579073    | 14 |
| 47 | 406820   | 797 | 985380   | 56 | 421440   | 853 | 578560    | 13 |
| 48 | 407299   | 796 | 985347   | 56 | 421952   | 852 | 578048    | 12 |
| 49 | 407777   | 795 | 985314   | 56 | 422463   | 851 | 577537    | 11 |
| 50 | 408254   | 794 | 985280   | 56 | 422974   | 850 | 577026    | 10 |
| 51 | 9.408731 | 794 | 9.985247 | 56 | 9.423484 | 849 | 10.576516 | 9  |
| 52 | 409207   | 793 | 985213   | 56 | 423993   | 848 | 576007    | 8  |
| 53 | 409682   | 792 | 985180   | 56 | 424503   | 848 | 575497    | 7  |
| 54 | 410157   | 791 | 985146   | 56 | 425011   | 847 | 574989    | 6  |
| 55 | 410632   | 790 | 985113   | 56 | 425519   | 846 | 574481    | 5  |
| 56 | 411106   | 789 | 985079   | 56 | 426027   | 845 | 573973    | 4  |
| 57 | 411579   | 788 | 985045   | 56 | 426534   | 844 | 573466    | 3  |
| 58 | 412052   | 787 | 985011   | 56 | 427041   | 843 | 572959    | 2  |
| 59 | 412524   | 786 | 984978   | 56 | 427547   | 843 | 572453    | 1  |
| 60 | 412996   | 785 | 984944   | 56 | 428052   | 842 | 571948    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.412996 | 785 | 9.984944 | 57 | 9.428052 | 842 | 10.571948 | 60 |
| 1  | 413467   | 784 | 984910   | 57 | 428557   | 841 | 571443    | 59 |
| 2  | 413938   | 783 | 984876   | 57 | 429062   | 840 | 570938    | 58 |
| 3  | 414408   | 783 | 984842   | 57 | 429566   | 839 | 570434    | 57 |
| 4  | 414878   | 782 | 984808   | 57 | 430070   | 838 | 569930    | 56 |
| 5  | 415347   | 781 | 984774   | 57 | 430573   | 838 | 569427    | 55 |
| 6  | 415815   | 780 | 984740   | 57 | 431075   | 837 | 568925    | 54 |
| 7  | 416283   | 779 | 984706   | 57 | 431577   | 836 | 568423    | 53 |
| 8  | 416751   | 778 | 984672   | 57 | 432079   | 835 | 567921    | 52 |
| 9  | 417217   | 777 | 984637   | 57 | 432580   | 834 | 567420    | 51 |
| 10 | 417684   | 776 | 984603   | 57 | 433080   | 833 | 566920    | 50 |
| 11 | 9.418150 | 775 | 9.984569 | 57 | 9.433580 | 832 | 10.566420 | 49 |
| 12 | 418615   | 774 | 984535   | 57 | 434080   | 832 | 565920    | 48 |
| 13 | 419079   | 773 | 984500   | 57 | 434579   | 831 | 565421    | 47 |
| 14 | 419544   | 773 | 984466   | 57 | 435078   | 830 | 564922    | 46 |
| 15 | 420007   | 772 | 984432   | 58 | 435576   | 829 | 564424    | 45 |
| 16 | 420470   | 771 | 984397   | 58 | 436073   | 828 | 563927    | 44 |
| 17 | 420933   | 770 | 984363   | 58 | 436570   | 828 | 563430    | 43 |
| 18 | 421395   | 769 | 984328   | 58 | 437067   | 827 | 562933    | 42 |
| 19 | 421857   | 768 | 984294   | 58 | 437563   | 826 | 562437    | 41 |
| 20 | 422318   | 767 | 984259   | 58 | 438059   | 825 | 561941    | 40 |
| 21 | 9.422778 | 767 | 9.984224 | 58 | 9.438554 | 824 | 10.561446 | 39 |
| 22 | 423238   | 766 | 984190   | 58 | 439048   | 823 | 560952    | 38 |
| 23 | 423697   | 765 | 984155   | 58 | 439543   | 823 | 560457    | 37 |
| 24 | 424156   | 764 | 984120   | 58 | 440036   | 822 | 559964    | 36 |
| 25 | 424615   | 763 | 984085   | 58 | 440529   | 821 | 559471    | 35 |
| 26 | 425073   | 762 | 984050   | 58 | 441022   | 820 | 558978    | 34 |
| 27 | 425530   | 761 | 984015   | 58 | 441514   | 819 | 558486    | 33 |
| 28 | 425987   | 760 | 983981   | 58 | 442006   | 819 | 557994    | 32 |
| 29 | 426443   | 760 | 983946   | 58 | 442497   | 818 | 557503    | 31 |
| 30 | 426899   | 759 | 983911   | 58 | 442988   | 817 | 557012    | 30 |
| 31 | 9.427354 | 758 | 9.983875 | 58 | 9.443479 | 816 | 10.556521 | 29 |
| 32 | 427809   | 757 | 983840   | 59 | 443968   | 816 | 556032    | 28 |
| 33 | 428263   | 756 | 983805   | 59 | 444458   | 815 | 555542    | 27 |
| 34 | 428717   | 755 | 983770   | 59 | 444947   | 814 | 555053    | 26 |
| 35 | 429170   | 754 | 983735   | 59 | 445435   | 813 | 554565    | 25 |
| 36 | 429623   | 753 | 983700   | 59 | 445923   | 812 | 554077    | 24 |
| 37 | 430075   | 752 | 983664   | 59 | 446411   | 812 | 553589    | 23 |
| 38 | 430527   | 752 | 983629   | 59 | 446898   | 811 | 553102    | 22 |
| 39 | 430978   | 751 | 983594   | 59 | 447384   | 810 | 552616    | 21 |
| 40 | 431429   | 750 | 983558   | 59 | 447870   | 809 | 552130    | 20 |
| 41 | 9.431879 | 749 | 9.983523 | 59 | 9.448356 | 809 | 10.551644 | 19 |
| 42 | 432329   | 749 | 983487   | 59 | 448841   | 808 | 551159    | 18 |
| 43 | 432778   | 748 | 983452   | 59 | 449326   | 807 | 550674    | 17 |
| 44 | 433226   | 747 | 983416   | 59 | 449810   | 806 | 550190    | 16 |
| 45 | 433675   | 746 | 983381   | 59 | 450294   | 806 | 549706    | 15 |
| 46 | 434122   | 745 | 983345   | 59 | 450777   | 805 | 549223    | 14 |
| 47 | 434569   | 744 | 983309   | 59 | 451260   | 804 | 548740    | 13 |
| 48 | 435016   | 744 | 983273   | 60 | 451743   | 803 | 548257    | 12 |
| 49 | 435462   | 743 | 983238   | 60 | 452225   | 802 | 547775    | 11 |
| 50 | 435908   | 742 | 983202   | 60 | 452706   | 802 | 547294    | 10 |
| 51 | 9.436353 | 741 | 9.983166 | 60 | 9.453187 | 801 | 10.546813 | 9  |
| 52 | 436798   | 740 | 983130   | 60 | 453668   | 800 | 546332    | 8  |
| 53 | 437242   | 740 | 983094   | 60 | 454148   | 799 | 545852    | 7  |
| 54 | 437686   | 739 | 983058   | 60 | 454628   | 799 | 545372    | 6  |
| 55 | 438129   | 738 | 983022   | 60 | 455107   | 798 | 544893    | 5  |
| 56 | 438572   | 737 | 982986   | 60 | 455586   | 797 | 544414    | 4  |
| 57 | 439014   | 736 | 982950   | 60 | 456064   | 796 | 543936    | 3  |
| 58 | 439456   | 736 | 982914   | 60 | 456542   | 796 | 543458    | 2  |
| 59 | 439897   | 735 | 982878   | 60 | 457019   | 795 | 542981    | 1  |
| 60 | 440338   | 734 | 982842   | 60 | 457496   | 794 | 542504    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |



| M  | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.440338 | 734 | 9.982842 | 60 | 9.457496 | 794 | 10.542504 | 60 |
| 1  | 440778   | 733 | 982805   | 60 | 457973   | 793 | 542027    | 59 |
| 2  | 441218   | 732 | 982769   | 61 | 458449   | 793 | 541551    | 58 |
| 3  | 441658   | 731 | 982733   | 61 | 458925   | 792 | 541075    | 57 |
| 4  | 442096   | 731 | 982696   | 61 | 459400   | 791 | 540600    | 56 |
| 5  | 442535   | 730 | 982660   | 61 | 459875   | 790 | 540125    | 55 |
| 6  | 442973   | 729 | 982624   | 61 | 460349   | 790 | 539651    | 54 |
| 7  | 443410   | 728 | 982587   | 61 | 460823   | 789 | 539177    | 53 |
| 8  | 443847   | 727 | 982551   | 61 | 461297   | 788 | 538703    | 52 |
| 9  | 444284   | 727 | 982514   | 61 | 461770   | 788 | 538230    | 51 |
| 10 | 444720   | 726 | 982477   | 61 | 462242   | 787 | 537758    | 50 |
| 11 | 9.445155 | 725 | 9.982441 | 61 | 9.462714 | 786 | 10.537286 | 49 |
| 12 | 445590   | 724 | 982404   | 61 | 463186   | 785 | 536814    | 48 |
| 13 | 446025   | 723 | 982367   | 61 | 463658   | 785 | 536342    | 47 |
| 14 | 446459   | 723 | 982331   | 61 | 464129   | 784 | 535871    | 46 |
| 15 | 446893   | 722 | 982294   | 61 | 464599   | 783 | 535401    | 45 |
| 16 | 447326   | 721 | 982257   | 61 | 465069   | 783 | 534931    | 44 |
| 17 | 447759   | 720 | 982220   | 62 | 465539   | 782 | 534461    | 43 |
| 18 | 448191   | 720 | 982183   | 62 | 466008   | 781 | 533992    | 42 |
| 19 | 448623   | 719 | 982146   | 62 | 466476   | 780 | 533524    | 41 |
| 20 | 449054   | 718 | 982109   | 62 | 466945   | 780 | 533055    | 40 |
| 21 | 9.449485 | 717 | 9.982072 | 62 | 9.467413 | 779 | 10.532587 | 39 |
| 22 | 449915   | 716 | 982035   | 62 | 467880   | 778 | 532120    | 38 |
| 23 | 450345   | 716 | 981998   | 62 | 468347   | 778 | 531653    | 37 |
| 24 | 450775   | 715 | 981961   | 62 | 468814   | 777 | 531186    | 36 |
| 25 | 451204   | 714 | 981924   | 62 | 469280   | 776 | 530720    | 35 |
| 26 | 451632   | 713 | 981886   | 62 | 469746   | 775 | 530254    | 34 |
| 27 | 452060   | 713 | 981849   | 62 | 470211   | 775 | 529789    | 33 |
| 28 | 452488   | 712 | 981812   | 62 | 470676   | 774 | 529324    | 32 |
| 29 | 452915   | 711 | 981774   | 62 | 471141   | 773 | 528859    | 31 |
| 30 | 453342   | 710 | 981737   | 62 | 471605   | 773 | 528395    | 30 |
| 31 | 9.453768 | 710 | 9.981699 | 63 | 9.472068 | 772 | 10.527932 | 29 |
| 32 | 454194   | 709 | 981662   | 63 | 472532   | 771 | 527468    | 28 |
| 33 | 454619   | 708 | 981625   | 63 | 472995   | 771 | 527005    | 27 |
| 34 | 455044   | 707 | 981587   | 63 | 473457   | 770 | 526543    | 26 |
| 35 | 455469   | 707 | 981549   | 63 | 473919   | 769 | 526081    | 25 |
| 36 | 455893   | 706 | 981512   | 63 | 474381   | 769 | 525619    | 24 |
| 37 | 456316   | 705 | 981474   | 63 | 474842   | 768 | 525158    | 23 |
| 38 | 456739   | 704 | 981436   | 63 | 475303   | 767 | 524697    | 22 |
| 39 | 457162   | 704 | 981399   | 63 | 475763   | 767 | 524237    | 21 |
| 40 | 457584   | 703 | 981361   | 63 | 476223   | 766 | 523777    | 20 |
| 41 | 9.458006 | 702 | 9.981323 | 63 | 9.476683 | 765 | 10.523317 | 19 |
| 42 | 458427   | 701 | 981285   | 63 | 477142   | 765 | 522858    | 18 |
| 43 | 458848   | 701 | 981247   | 63 | 477601   | 764 | 522399    | 17 |
| 44 | 459268   | 700 | 981209   | 63 | 478059   | 763 | 521941    | 16 |
| 45 | 459688   | 699 | 981171   | 63 | 478517   | 763 | 521483    | 15 |
| 46 | 460108   | 698 | 981133   | 64 | 478975   | 762 | 521025    | 14 |
| 47 | 460527   | 698 | 981095   | 64 | 479432   | 761 | 520568    | 13 |
| 48 | 460946   | 697 | 981057   | 64 | 479889   | 761 | 520111    | 12 |
| 49 | 461364   | 696 | 981019   | 64 | 480345   | 760 | 519655    | 11 |
| 50 | 461782   | 695 | 980981   | 64 | 480801   | 759 | 519199    | 10 |
| 51 | 9.462199 | 695 | 9.980942 | 64 | 9.481257 | 759 | 10.518743 | 9  |
| 52 | 462616   | 694 | 980904   | 64 | 481712   | 758 | 518288    | 8  |
| 53 | 463032   | 693 | 980866   | 64 | 482167   | 757 | 517833    | 7  |
| 54 | 463448   | 693 | 980827   | 64 | 482621   | 757 | 517379    | 6  |
| 55 | 463864   | 692 | 980789   | 64 | 483075   | 756 | 516925    | 5  |
| 56 | 464279   | 691 | 980750   | 64 | 483529   | 755 | 516471    | 4  |
| 57 | 464694   | 690 | 980712   | 64 | 483982   | 755 | 516018    | 3  |
| 58 | 465108   | 690 | 980673   | 64 | 484435   | 754 | 515565    | 2  |
| 59 | 465522   | 689 | 980635   | 64 | 484887   | 753 | 515113    | 1  |
| 60 | 465935   | 688 | 980596   | 64 | 485339   | 753 | 514661    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.465935 | 688 | 9.980596 | 64 | 9.485339 | 555 | 10.514661 | 60 |
| 1  | 466348   | 688 | 980558   | 64 | 485791   | 752 | 514209    | 59 |
| 2  | 466761   | 687 | 980519   | 65 | 486242   | 751 | 513758    | 58 |
| 3  | 467173   | 686 | 980480   | 65 | 486693   | 751 | 513307    | 57 |
| 4  | 467585   | 685 | 980442   | 65 | 487143   | 750 | 512857    | 56 |
| 5  | 467996   | 685 | 980403   | 65 | 487593   | 749 | 512407    | 55 |
| 6  | 468407   | 684 | 980364   | 65 | 488043   | 749 | 511957    | 54 |
| 7  | 468817   | 683 | 980325   | 65 | 488492   | 748 | 511508    | 53 |
| 8  | 469227   | 683 | 980286   | 65 | 488941   | 747 | 511059    | 52 |
| 9  | 469637   | 682 | 980247   | 65 | 489390   | 747 | 510610    | 51 |
| 10 | 470046   | 681 | 980208   | 65 | 489838   | 746 | 510162    | 50 |
| 11 | 9.470455 | 680 | 9.980169 | 65 | 9.490286 | 746 | 10.509714 | 49 |
| 12 | 470863   | 680 | 980130   | 65 | 490733   | 745 | 509267    | 48 |
| 13 | 471271   | 679 | 980091   | 65 | 491180   | 744 | 508820    | 47 |
| 14 | 471679   | 678 | 980052   | 65 | 491627   | 744 | 508373    | 46 |
| 15 | 472086   | 678 | 980012   | 65 | 492073   | 743 | 507927    | 45 |
| 16 | 472492   | 677 | 979973   | 65 | 492519   | 743 | 507481    | 44 |
| 17 | 472898   | 676 | 979934   | 66 | 492965   | 742 | 507035    | 43 |
| 18 | 473304   | 676 | 979895   | 66 | 493410   | 741 | 506590    | 42 |
| 19 | 473710   | 675 | 979855   | 66 | 493854   | 740 | 506146    | 41 |
| 20 | 474115   | 674 | 979816   | 66 | 494299   | 740 | 505701    | 40 |
| 21 | 9.474519 | 674 | 9.979776 | 66 | 9.494743 | 740 | 10.505257 | 39 |
| 22 | 474923   | 673 | 979737   | 66 | 495186   | 739 | 504814    | 38 |
| 23 | 475327   | 672 | 979697   | 66 | 495630   | 738 | 504370    | 37 |
| 24 | 475730   | 672 | 979658   | 66 | 496073   | 737 | 503927    | 36 |
| 25 | 476133   | 671 | 979618   | 66 | 496515   | 737 | 503485    | 35 |
| 26 | 476536   | 670 | 979579   | 66 | 496957   | 736 | 503043    | 34 |
| 27 | 476938   | 669 | 979539   | 66 | 497399   | 736 | 502601    | 33 |
| 28 | 477340   | 669 | 979499   | 66 | 497841   | 735 | 502159    | 32 |
| 29 | 477741   | 668 | 979459   | 66 | 498282   | 734 | 501718    | 31 |
| 30 | 478142   | 667 | 979420   | 66 | 498722   | 734 | 501278    | 30 |
| 31 | 9.478542 | 667 | 9.979380 | 66 | 9.499163 | 733 | 10.500837 | 29 |
| 32 | 478942   | 666 | 979340   | 66 | 499603   | 733 | 500397    | 28 |
| 33 | 479342   | 665 | 979300   | 67 | 500042   | 732 | 499958    | 27 |
| 34 | 479741   | 665 | 979260   | 67 | 500481   | 731 | 499519    | 26 |
| 35 | 480140   | 664 | 979220   | 67 | 500920   | 731 | 499080    | 25 |
| 36 | 480539   | 663 | 979180   | 67 | 501359   | 730 | 498641    | 24 |
| 37 | 480937   | 663 | 979140   | 67 | 501797   | 730 | 498202    | 23 |
| 38 | 481334   | 662 | 979100   | 67 | 502235   | 729 | 497765    | 22 |
| 39 | 481731   | 661 | 979059   | 67 | 502672   | 728 | 497328    | 21 |
| 40 | 482128   | 661 | 979019   | 67 | 503109   | 728 | 496891    | 20 |
| 41 | 9.482525 | 660 | 9.978979 | 67 | 9.503546 | 727 | 10.496454 | 19 |
| 42 | 482921   | 659 | 978939   | 67 | 503982   | 727 | 496018    | 18 |
| 43 | 483316   | 659 | 978898   | 67 | 504418   | 726 | 495582    | 17 |
| 44 | 483712   | 658 | 978858   | 67 | 504854   | 725 | 495146    | 16 |
| 45 | 484107   | 657 | 978817   | 67 | 505289   | 725 | 494711    | 15 |
| 46 | 484501   | 657 | 978777   | 67 | 505724   | 724 | 494276    | 14 |
| 47 | 484895   | 656 | 978736   | 67 | 506159   | 724 | 493841    | 13 |
| 48 | 485289   | 655 | 978696   | 68 | 506593   | 723 | 493407    | 12 |
| 49 | 485682   | 655 | 978655   | 68 | 507027   | 722 | 492973    | 11 |
| 50 | 486075   | 654 | 978615   | 68 | 507460   | 722 | 492540    | 10 |
| 51 | 9.486467 | 653 | 9.978574 | 68 | 9.507893 | 721 | 10.492107 | 9  |
| 52 | 486860   | 653 | 978533   | 68 | 508326   | 721 | 491674    | 8  |
| 53 | 487251   | 652 | 978493   | 68 | 508759   | 720 | 491241    | 7  |
| 54 | 487643   | 651 | 978452   | 68 | 509191   | 719 | 490809    | 6  |
| 55 | 488034   | 651 | 978411   | 68 | 509622   | 719 | 490378    | 5  |
| 56 | 488424   | 650 | 978370   | 68 | 510054   | 718 | 489946    | 4  |
| 57 | 488814   | 650 | 978329   | 68 | 510485   | 718 | 489515    | 3  |
| 58 | 489204   | 649 | 978288   | 68 | 510916   | 717 | 489084    | 2  |
| 59 | 489593   | 648 | 978247   | 68 | 511346   | 716 | 488654    | 1  |
| 60 | 489982   | 648 | 978206   | 68 | 511776   | 716 | 488224    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D   | Cosine   | D. | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.489982 | 648 | 9.978206 | 68 | 9.511776 | 716 | 10.488224 | 60 |
| 1  | 490371   | 648 | 978165   | 68 | 512206   | 716 | 487794    | 59 |
| 2  | 490759   | 647 | 978124   | 68 | 512635   | 715 | 487365    | 58 |
| 3  | 491147   | 646 | 978083   | 69 | 513064   | 714 | 486936    | 57 |
| 4  | 491535   | 646 | 978042   | 69 | 513493   | 714 | 486507    | 56 |
| 5  | 491922   | 645 | 978001   | 69 | 513921   | 713 | 486079    | 55 |
| 6  | 492308   | 644 | 977959   | 69 | 514349   | 713 | 485651    | 54 |
| 7  | 492695   | 644 | 977918   | 69 | 514777   | 712 | 485223    | 53 |
| 8  | 493081   | 643 | 977877   | 69 | 515204   | 712 | 484796    | 52 |
| 9  | 493466   | 642 | 977835   | 69 | 515631   | 711 | 484369    | 51 |
| 10 | 493851   | 642 | 977794   | 69 | 516057   | 710 | 483943    | 50 |
| 11 | 9.494236 | 641 | 9.977752 | 69 | 9.516484 | 710 | 10.483516 | 49 |
| 12 | 494621   | 641 | 977711   | 69 | 516910   | 709 | 483090    | 48 |
| 13 | 495005   | 640 | 977669   | 69 | 517335   | 709 | 482665    | 47 |
| 14 | 495388   | 639 | 977628   | 69 | 517761   | 708 | 482239    | 46 |
| 15 | 495772   | 639 | 977586   | 69 | 518185   | 708 | 481815    | 45 |
| 16 | 496154   | 638 | 977544   | 70 | 518610   | 707 | 481390    | 44 |
| 17 | 496537   | 637 | 977503   | 70 | 519034   | 706 | 480966    | 43 |
| 18 | 496919   | 637 | 977461   | 70 | 519458   | 706 | 480542    | 42 |
| 19 | 497301   | 636 | 977419   | 70 | 519882   | 705 | 480118    | 41 |
| 20 | 497682   | 636 | 977377   | 70 | 520305   | 705 | 479695    | 40 |
| 21 | 9.498064 | 635 | 9.977335 | 70 | 9.520728 | 704 | 10.479272 | 39 |
| 22 | 498444   | 634 | 977293   | 70 | 521151   | 703 | 478849    | 38 |
| 23 | 498825   | 634 | 977251   | 70 | 521573   | 703 | 478427    | 37 |
| 24 | 499204   | 633 | 977209   | 70 | 521995   | 703 | 478005    | 36 |
| 25 | 499584   | 632 | 977167   | 70 | 522417   | 702 | 477583    | 35 |
| 26 | 499963   | 632 | 977125   | 70 | 522838   | 702 | 477162    | 34 |
| 27 | 500342   | 631 | 977083   | 70 | 523259   | 701 | 476741    | 33 |
| 28 | 500721   | 631 | 977041   | 70 | 523680   | 701 | 476320    | 32 |
| 29 | 501099   | 630 | 976999   | 70 | 524100   | 700 | 475900    | 31 |
| 30 | 501476   | 629 | 976957   | 70 | 524520   | 699 | 475480    | 30 |
| 31 | 9.501854 | 629 | 9.976914 | 70 | 9.524939 | 699 | 10.475061 | 29 |
| 32 | 502231   | 628 | 976872   | 71 | 525359   | 698 | 474641    | 28 |
| 33 | 502607   | 628 | 976830   | 71 | 525778   | 698 | 474222    | 27 |
| 34 | 502984   | 627 | 976787   | 71 | 526197   | 697 | 473803    | 26 |
| 35 | 503360   | 626 | 976745   | 71 | 526615   | 697 | 473385    | 25 |
| 36 | 503735   | 626 | 976702   | 71 | 527033   | 696 | 472967    | 24 |
| 37 | 504110   | 625 | 976660   | 71 | 527451   | 696 | 472549    | 23 |
| 38 | 504485   | 625 | 976617   | 71 | 527868   | 695 | 472132    | 22 |
| 39 | 504860   | 624 | 976574   | 71 | 528285   | 695 | 471715    | 21 |
| 40 | 505234   | 623 | 976532   | 71 | 528702   | 694 | 471298    | 20 |
| 41 | 9.505608 | 623 | 9.976489 | 71 | 9.529119 | 693 | 10.470881 | 19 |
| 42 | 505981   | 622 | 976446   | 71 | 529535   | 693 | 470465    | 18 |
| 43 | 506354   | 622 | 976404   | 71 | 529950   | 693 | 470050    | 17 |
| 44 | 506727   | 621 | 976361   | 71 | 530366   | 692 | 469634    | 16 |
| 45 | 507099   | 620 | 976318   | 71 | 530781   | 691 | 469219    | 15 |
| 46 | 507471   | 620 | 976275   | 71 | 531196   | 691 | 468804    | 14 |
| 47 | 507843   | 619 | 976232   | 72 | 531611   | 690 | 468389    | 13 |
| 48 | 508214   | 619 | 976189   | 72 | 532025   | 690 | 467975    | 12 |
| 49 | 508585   | 618 | 976146   | 72 | 532439   | 689 | 467561    | 11 |
| 50 | 508956   | 618 | 976103   | 72 | 532853   | 689 | 467147    | 10 |
| 51 | 9.509326 | 617 | 9.976060 | 72 | 9.533266 | 688 | 10.466734 | 9  |
| 52 | 509696   | 616 | 976017   | 72 | 533679   | 688 | 466321    | 8  |
| 53 | 510065   | 616 | 975974   | 72 | 534092   | 687 | 465908    | 7  |
| 54 | 510434   | 615 | 975930   | 72 | 534504   | 687 | 465496    | 6  |
| 55 | 510803   | 615 | 975887   | 72 | 534916   | 686 | 465084    | 5  |
| 56 | 511172   | 614 | 975844   | 72 | 535328   | 686 | 464672    | 4  |
| 57 | 511540   | 613 | 975800   | 72 | 535739   | 685 | 464261    | 3  |
| 58 | 511907   | 613 | 975757   | 72 | 536150   | 685 | 463850    | 2  |
| 59 | 512275   | 612 | 975714   | 72 | 536561   | 684 | 463439    | 1  |
| 60 | 512642   | 612 | 975670   | 72 | 536972   | 684 | 463028    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.512642 | 612 | 9.975670 | 73 | 9.536972 | 684 | 10.463028 | 60 |
| 1  | 513009   | 611 | 975627   | 73 | 537382   | 683 | 462618    | 59 |
| 2  | 513375   | 611 | 975583   | 73 | 537792   | 683 | 462208    | 58 |
| 3  | 513741   | 610 | 975539   | 73 | 538202   | 682 | 461798    | 57 |
| 4  | 514107   | 609 | 975496   | 73 | 538611   | 682 | 461389    | 56 |
| 5  | 514472   | 609 | 975452   | 73 | 539020   | 681 | 460980    | 55 |
| 6  | 514837   | 608 | 975408   | 73 | 539429   | 681 | 460571    | 54 |
| 7  | 515202   | 608 | 975365   | 73 | 539837   | 680 | 460163    | 53 |
| 8  | 515566   | 607 | 975321   | 73 | 540245   | 680 | 459755    | 52 |
| 9  | 515930   | 607 | 975277   | 73 | 540653   | 679 | 459347    | 51 |
| 10 | 516294   | 606 | 975233   | 73 | 541061   | 679 | 458939    | 50 |
| 11 | 9.516657 | 605 | 9.975189 | 73 | 9.541468 | 678 | 10.458532 | 49 |
| 12 | 517020   | 605 | 975145   | 73 | 541875   | 678 | 458125    | 48 |
| 13 | 517382   | 604 | 975101   | 73 | 542281   | 677 | 457719    | 47 |
| 14 | 517745   | 604 | 975057   | 73 | 542688   | 677 | 457312    | 46 |
| 15 | 518107   | 603 | 975013   | 73 | 543094   | 676 | 456906    | 45 |
| 16 | 518468   | 603 | 974969   | 74 | 543499   | 676 | 456501    | 44 |
| 17 | 518829   | 602 | 974925   | 74 | 543905   | 675 | 456095    | 43 |
| 18 | 519190   | 601 | 974880   | 74 | 544310   | 675 | 455690    | 42 |
| 19 | 519551   | 601 | 974836   | 74 | 544715   | 674 | 455285    | 41 |
| 20 | 519911   | 600 | 974792   | 74 | 545119   | 674 | 454881    | 40 |
| 21 | 9.520271 | 600 | 9.974748 | 74 | 9.545524 | 673 | 10.454476 | 39 |
| 22 | 520631   | 599 | 974703   | 74 | 545928   | 673 | 454072    | 38 |
| 23 | 520990   | 599 | 974659   | 74 | 546331   | 672 | 453669    | 37 |
| 24 | 521349   | 598 | 974614   | 74 | 546735   | 672 | 453265    | 36 |
| 25 | 521707   | 598 | 974570   | 74 | 547138   | 671 | 452862    | 35 |
| 26 | 522066   | 597 | 974525   | 74 | 547540   | 671 | 452460    | 34 |
| 27 | 522424   | 596 | 974481   | 74 | 547943   | 670 | 452057    | 33 |
| 28 | 522781   | 596 | 974436   | 74 | 548345   | 670 | 451655    | 32 |
| 29 | 523138   | 595 | 974391   | 74 | 548747   | 669 | 451253    | 31 |
| 30 | 523495   | 595 | 974347   | 75 | 549149   | 669 | 450851    | 30 |
| 31 | 9.523852 | 594 | 9.974302 | 75 | 9.549550 | 668 | 10.450450 | 29 |
| 32 | 524208   | 594 | 974257   | 75 | 549951   | 668 | 450049    | 28 |
| 33 | 524564   | 593 | 974212   | 75 | 550352   | 667 | 449648    | 27 |
| 34 | 524920   | 593 | 974167   | 75 | 550752   | 667 | 449248    | 26 |
| 35 | 525275   | 592 | 974122   | 75 | 551152   | 666 | 448848    | 25 |
| 36 | 525630   | 591 | 974077   | 75 | 551552   | 666 | 448448    | 24 |
| 37 | 525984   | 591 | 974032   | 75 | 551952   | 665 | 448048    | 23 |
| 38 | 526339   | 590 | 973987   | 75 | 552351   | 665 | 447649    | 22 |
| 39 | 526693   | 590 | 973942   | 75 | 552750   | 665 | 447250    | 21 |
| 40 | 527046   | 589 | 973897   | 75 | 553149   | 664 | 446851    | 20 |
| 41 | 9.527400 | 589 | 9.973852 | 75 | 9.553548 | 664 | 10.446452 | 19 |
| 42 | 527753   | 588 | 973807   | 75 | 553946   | 663 | 446054    | 18 |
| 43 | 528105   | 588 | 973761   | 75 | 554344   | 663 | 445656    | 17 |
| 44 | 528458   | 587 | 973716   | 76 | 554741   | 662 | 445259    | 16 |
| 45 | 528810   | 587 | 973671   | 76 | 555139   | 662 | 444861    | 15 |
| 46 | 529161   | 586 | 973625   | 76 | 555536   | 661 | 444464    | 14 |
| 47 | 529513   | 586 | 973580   | 76 | 555933   | 661 | 444067    | 13 |
| 48 | 529864   | 585 | 973535   | 76 | 556329   | 660 | 443671    | 12 |
| 49 | 530215   | 585 | 973489   | 76 | 556725   | 660 | 443275    | 11 |
| 50 | 530565   | 584 | 973444   | 76 | 557121   | 659 | 442879    | 10 |
| 51 | 9.530915 | 584 | 9.973398 | 76 | 9.557517 | 659 | 10.442483 | 9  |
| 52 | 531265   | 583 | 973352   | 76 | 557913   | 659 | 442087    | 8  |
| 53 | 531614   | 582 | 973307   | 76 | 558308   | 658 | 441692    | 7  |
| 54 | 531963   | 582 | 973261   | 76 | 558702   | 658 | 441298    | 6  |
| 55 | 532312   | 581 | 973215   | 76 | 559097   | 657 | 440903    | 5  |
| 56 | 532661   | 581 | 973169   | 76 | 559491   | 657 | 440509    | 4  |
| 57 | 533009   | 580 | 973124   | 76 | 559885   | 656 | 440115    | 3  |
| 58 | 533357   | 580 | 973078   | 76 | 560279   | 656 | 439721    | 2  |
| 59 | 533704   | 579 | 973032   | 77 | 560673   | 655 | 439327    | 1  |
| 60 | 534052   | 578 | 972986   | 77 | 561066   | 655 | 438934    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.534052 | 578 | 9.972986 | 77 | 9.511066 | 655 | 10.4894   | 60 |
| 1  | 534399   | 577 | 972940   | 77 | 561459   | 654 | 438541    | 59 |
| 2  | 534745   | 577 | 972894   | 77 | 561851   | 654 | 438149    | 58 |
| 3  | 535092   | 577 | 972848   | 77 | 562244   | 653 | 437756    | 57 |
| 4  | 535438   | 576 | 972802   | 77 | 562636   | 653 | 437364    | 56 |
| 5  | 535783   | 576 | 972755   | 77 | 563028   | 653 | 436972    | 55 |
| 6  | 536129   | 575 | 972709   | 77 | 563419   | 652 | 436581    | 54 |
| 7  | 536474   | 574 | 972663   | 77 | 563811   | 652 | 436189    | 53 |
| 8  | 536818   | 574 | 972617   | 77 | 564202   | 651 | 435798    | 52 |
| 9  | 537163   | 573 | 972570   | 77 | 564592   | 651 | 435408    | 51 |
| 10 | 537507   | 573 | 972524   | 77 | 564983   | 650 | 435017    | 50 |
| 11 | 9.537851 | 572 | 9.972478 | 77 | 9.565373 | 650 | 10.434627 | 49 |
| 12 | 538194   | 572 | 972431   | 78 | 565763   | 649 | 434237    | 48 |
| 13 | 838538   | 571 | 972385   | 78 | 566153   | 649 | 433847    | 47 |
| 14 | 538880   | 571 | 972338   | 78 | 566542   | 649 | 433458    | 46 |
| 15 | 539223   | 570 | 972291   | 78 | 566932   | 648 | 433068    | 45 |
| 16 | 539565   | 570 | 972245   | 78 | 567320   | 648 | 432680    | 44 |
| 17 | 539907   | 569 | 972198   | 78 | 567709   | 647 | 432291    | 43 |
| 18 | 540249   | 569 | 972151   | 78 | 568098   | 647 | 431902    | 42 |
| 19 | 540590   | 568 | 972105   | 78 | 568486   | 646 | 431514    | 41 |
| 20 | 540931   | 568 | 972058   | 78 | 568873   | 646 | 431127    | 40 |
| 21 | 9.541272 | 567 | 9.972011 | 78 | 9.569261 | 645 | 10.430739 | 39 |
| 22 | 541613   | 567 | 971964   | 78 | 569648   | 645 | 430352    | 38 |
| 23 | 541953   | 566 | 971917   | 78 | 570035   | 645 | 429965    | 37 |
| 24 | 542293   | 566 | 971870   | 78 | 570422   | 644 | 429578    | 36 |
| 25 | 542632   | 565 | 971823   | 78 | 570809   | 644 | 429191    | 35 |
| 26 | 542971   | 565 | 971776   | 78 | 571195   | 643 | 428805    | 34 |
| 27 | 543310   | 564 | 971729   | 79 | 571581   | 643 | 428419    | 33 |
| 28 | 543649   | 564 | 971682   | 79 | 571967   | 642 | 428033    | 32 |
| 29 | 543987   | 563 | 971635   | 79 | 572352   | 642 | 427648    | 31 |
| 30 | 544325   | 563 | 971588   | 79 | 572738   | 642 | 427262    | 30 |
| 31 | 9.544663 | 562 | 9.971540 | 79 | 9.573123 | 641 | 10.426877 | 29 |
| 32 | 545000   | 562 | 971493   | 79 | 573507   | 641 | 426493    | 28 |
| 33 | 545338   | 561 | 971446   | 79 | 573892   | 640 | 426108    | 27 |
| 34 | 545674   | 561 | 971398   | 79 | 574276   | 640 | 425724    | 26 |
| 35 | 546011   | 560 | 971351   | 79 | 574660   | 639 | 425340    | 25 |
| 36 | 546347   | 560 | 971303   | 79 | 575044   | 639 | 424956    | 24 |
| 37 | 546683   | 559 | 971256   | 79 | 575427   | 639 | 424573    | 23 |
| 38 | 547019   | 559 | 971208   | 79 | 575810   | 638 | 424190    | 22 |
| 39 | 547354   | 558 | 971161   | 79 | 576193   | 638 | 423807    | 21 |
| 40 | 547689   | 558 | 971113   | 79 | 576576   | 637 | 423424    | 20 |
| 41 | 9.548924 | 557 | 9.971066 | 80 | 9.576958 | 637 | 10.423041 | 19 |
| 42 | 548359   | 557 | 971018   | 80 | 577341   | 636 | 422659    | 18 |
| 43 | 548693   | 556 | 970970   | 80 | 577723   | 636 | 422277    | 17 |
| 44 | 549027   | 556 | 970922   | 80 | 578104   | 636 | 421896    | 16 |
| 45 | 549360   | 555 | 970874   | 80 | 578486   | 635 | 421514    | 15 |
| 46 | 549693   | 555 | 970827   | 80 | 578867   | 635 | 421133    | 14 |
| 47 | 550026   | 554 | 970779   | 80 | 579248   | 634 | 420752    | 13 |
| 48 | 550359   | 554 | 970731   | 80 | 579629   | 634 | 420371    | 12 |
| 49 | 550692   | 553 | 970683   | 80 | 580009   | 634 | 419991    | 11 |
| 50 | 551024   | 553 | 970635   | 80 | 580389   | 633 | 419611    | 10 |
| 51 | 9.551356 | 552 | 9.970586 | 80 | 9.580769 | 633 | 10.419231 | 9  |
| 52 | 551687   | 552 | 970538   | 80 | 581149   | 632 | 418851    | 8  |
| 53 | 552018   | 552 | 970490   | 80 | 581528   | 632 | 418472    | 7  |
| 54 | 552349   | 551 | 970442   | 80 | 581907   | 632 | 418093    | 6  |
| 55 | 552680   | 551 | 970394   | 80 | 582286   | 631 | 417714    | 5  |
| 56 | 553010   | 550 | 970345   | 81 | 582665   | 631 | 417335    | 4  |
| 57 | 553341   | 550 | 970297   | 81 | 583043   | 630 | 416957    | 3  |
| 58 | 553670   | 549 | 970249   | 81 | 583422   | 630 | 416578    | 2  |
| 59 | 554000   | 549 | 970200   | 81 | 583800   | 629 | 416200    | 1  |
| 60 | 554329   | 548 | 970152   | 81 | 584177   | 629 | 415823    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.554329 | 548 | 9.970152 | 81 | 9.584177 | 629 | 10.415833 | 60 |
| 1  | 554658   | 548 | 970103   | 81 | 584555   | 629 | 415445    | 59 |
| 2  | 554987   | 547 | 970055   | 81 | 584932   | 628 | 415068    | 58 |
| 3  | 555315   | 547 | 970006   | 81 | 585309   | 628 | 414691    | 57 |
| 4  | 555643   | 546 | 969957   | 81 | 585686   | 627 | 414314    | 56 |
| 5  | 555971   | 546 | 969909   | 81 | 586062   | 627 | 413938    | 55 |
| 6  | 556299   | 545 | 969860   | 81 | 586439   | 627 | 413561    | 54 |
| 7  | 556626   | 545 | 969811   | 81 | 586815   | 626 | 413185    | 53 |
| 8  | 556953   | 544 | 969762   | 81 | 587190   | 626 | 412810    | 52 |
| 9  | 557280   | 544 | 969714   | 81 | 587566   | 625 | 412434    | 51 |
| 10 | 557606   | 543 | 969665   | 81 | 587941   | 625 | 412059    | 50 |
| 11 | 9.557932 | 543 | 9.969616 | 82 | 9.588316 | 625 | 10.411684 | 49 |
| 12 | 558258   | 543 | 969567   | 82 | 588691   | 624 | 411309    | 48 |
| 13 | 558583   | 542 | 969518   | 82 | 589066   | 624 | 410934    | 47 |
| 14 | 558909   | 542 | 969469   | 82 | 589440   | 623 | 410560    | 46 |
| 15 | 559234   | 541 | 969420   | 82 | 589814   | 623 | 410186    | 45 |
| 16 | 559558   | 541 | 969370   | 82 | 590188   | 623 | 409812    | 44 |
| 17 | 559883   | 540 | 969321   | 82 | 590562   | 622 | 409438    | 43 |
| 18 | 560207   | 540 | 969272   | 82 | 590935   | 622 | 409065    | 42 |
| 19 | 560531   | 539 | 969223   | 82 | 591308   | 622 | 408692    | 41 |
| 20 | 560855   | 539 | 969173   | 82 | 591681   | 621 | 408319    | 40 |
| 21 | 9.561178 | 538 | 9.969124 | 82 | 9.592054 | 621 | 10.407946 | 39 |
| 22 | 561501   | 538 | 969075   | 82 | 592426   | 620 | 407574    | 38 |
| 23 | 561824   | 537 | 969025   | 82 | 592798   | 620 | 407202    | 37 |
| 24 | 562146   | 537 | 968976   | 82 | 593170   | 619 | 406829    | 36 |
| 25 | 562468   | 536 | 968926   | 83 | 593542   | 619 | 406458    | 35 |
| 26 | 562790   | 536 | 968877   | 83 | 593914   | 618 | 406086    | 34 |
| 27 | 563112   | 536 | 968827   | 83 | 594285   | 618 | 405715    | 33 |
| 28 | 563433   | 535 | 968777   | 83 | 594656   | 618 | 405344    | 32 |
| 29 | 563755   | 535 | 968728   | 83 | 595027   | 617 | 404973    | 31 |
| 30 | 564075   | 534 | 968678   | 83 | 595398   | 617 | 404602    | 30 |
| 31 | 9.564396 | 534 | 9.968628 | 83 | 9.595768 | 617 | 10.404232 | 29 |
| 32 | 564716   | 533 | 968578   | 83 | 596138   | 616 | 403862    | 28 |
| 33 | 565036   | 533 | 968528   | 83 | 596508   | 616 | 403492    | 27 |
| 34 | 565356   | 532 | 968479   | 83 | 596878   | 616 | 403122    | 26 |
| 35 | 565676   | 532 | 968429   | 83 | 597247   | 615 | 402753    | 25 |
| 36 | 565995   | 531 | 968379   | 83 | 597616   | 615 | 402384    | 24 |
| 37 | 566314   | 531 | 968329   | 83 | 597985   | 615 | 402015    | 23 |
| 38 | 566632   | 531 | 968278   | 83 | 598354   | 614 | 401646    | 22 |
| 39 | 566951   | 530 | 968228   | 84 | 598722   | 614 | 401278    | 21 |
| 40 | 567269   | 530 | 968178   | 84 | 599091   | 613 | 400909    | 20 |
| 41 | 9.567587 | 529 | 9.968128 | 84 | 9.599459 | 613 | 10.400541 | 19 |
| 42 | 567904   | 529 | 968078   | 84 | 599827   | 613 | 400173    | 18 |
| 43 | 568222   | 528 | 968027   | 84 | 600194   | 612 | 399806    | 17 |
| 44 | 568539   | 528 | 967977   | 84 | 600562   | 612 | 399438    | 16 |
| 45 | 568856   | 528 | 967927   | 84 | 600929   | 611 | 399071    | 15 |
| 46 | 569172   | 527 | 967876   | 84 | 601296   | 611 | 398704    | 14 |
| 47 | 569488   | 527 | 967826   | 84 | 601662   | 611 | 398338    | 13 |
| 48 | 569804   | 526 | 967775   | 84 | 602029   | 610 | 397971    | 12 |
| 49 | 570120   | 526 | 967725   | 84 | 602395   | 610 | 397605    | 11 |
| 50 | 570435   | 525 | 967674   | 84 | 602761   | 610 | 397239    | 10 |
| 51 | 9.570751 | 525 | 9.967624 | 84 | 9.603127 | 609 | 10.396873 | 9  |
| 52 | 571066   | 524 | 967573   | 84 | 603493   | 609 | 396507    | 8  |
| 53 | 571380   | 524 | 967522   | 85 | 603858   | 609 | 396142    | 7  |
| 54 | 571695   | 523 | 967471   | 85 | 604223   | 608 | 395777    | 6  |
| 55 | 572009   | 523 | 967421   | 85 | 604588   | 608 | 395412    | 5  |
| 56 | 572323   | 523 | 967370   | 85 | 604953   | 607 | 395047    | 4  |
| 57 | 572636   | 522 | 967319   | 85 | 605317   | 607 | 394683    | 3  |
| 58 | 572950   | 522 | 967268   | 85 | 605682   | 607 | 394318    | 2  |
| 59 | 573263   | 521 | 967217   | 85 | 606046   | 606 | 393954    | 1  |
| 60 | 573575   | 521 | 967166   | 85 | 606410   | 606 | 393590    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang    | M. |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.573575 | 521 | 9.967166 | 85 | 9.606410 | 606 | 10.393590 | 60 |
| 1  | 573888   | 520 | 967115   | 85 | 606773   | 606 | 393227    | 59 |
| 2  | 574200   | 520 | 967064   | 85 | 607137   | 605 | 392863    | 58 |
| 3  | 574512   | 519 | 967013   | 85 | 607500   | 605 | 392500    | 57 |
| 4  | 574824   | 519 | 966961   | 85 | 607863   | 604 | 392137    | 56 |
| 5  | 575136   | 519 | 966910   | 85 | 608225   | 604 | 391775    | 55 |
| 6  | 575447   | 518 | 966859   | 85 | 608588   | 604 | 391412    | 54 |
| 7  | 575758   | 518 | 966808   | 85 | 608950   | 603 | 391050    | 53 |
| 8  | 576069   | 517 | 966756   | 86 | 609312   | 603 | 390688    | 52 |
| 9  | 576379   | 517 | 966705   | 86 | 609674   | 603 | 390326    | 51 |
| 10 | 576689   | 516 | 966653   | 86 | 610036   | 602 | 389964    | 50 |
| 11 | 9.576999 | 516 | 9.966602 | 86 | 9.610397 | 602 | 10.389603 | 49 |
| 12 | 577309   | 516 | 966550   | 86 | 610759   | 602 | 389241    | 48 |
| 13 | 577618   | 515 | 966499   | 86 | 611120   | 601 | 388880    | 47 |
| 14 | 577927   | 515 | 966447   | 86 | 611480   | 601 | 388520    | 46 |
| 15 | 578236   | 514 | 966395   | 86 | 611841   | 601 | 388159    | 45 |
| 16 | 578545   | 514 | 966344   | 86 | 612201   | 600 | 387799    | 44 |
| 17 | 578853   | 513 | 966292   | 86 | 612561   | 600 | 387439    | 43 |
| 18 | 579162   | 513 | 966240   | 86 | 612921   | 600 | 387079    | 42 |
| 19 | 579470   | 513 | 966188   | 86 | 613281   | 599 | 386719    | 41 |
| 20 | 579777   | 512 | 966136   | 86 | 613641   | 599 | 386359    | 40 |
| 21 | 9.580085 | 512 | 9.966085 | 87 | 9.614000 | 598 | 10.386000 | 39 |
| 22 | 580392   | 511 | 966033   | 87 | 614359   | 598 | 385641    | 38 |
| 23 | 580699   | 511 | 965981   | 87 | 614718   | 598 | 385282    | 37 |
| 24 | 581005   | 511 | 965928   | 87 | 615077   | 597 | 384923    | 36 |
| 25 | 581312   | 510 | 965876   | 87 | 615435   | 597 | 384565    | 35 |
| 26 | 581618   | 510 | 965824   | 87 | 615793   | 597 | 384207    | 34 |
| 27 | 581924   | 509 | 965772   | 87 | 616151   | 596 | 383849    | 33 |
| 28 | 582229   | 509 | 965720   | 87 | 616509   | 596 | 383491    | 32 |
| 29 | 582535   | 509 | 965668   | 87 | 616867   | 596 | 383133    | 31 |
| 30 | 582840   | 508 | 965615   | 87 | 617224   | 595 | 382776    | 30 |
| 31 | 9.583145 | 508 | 9.965563 | 87 | 9.617582 | 595 | 10.382418 | 29 |
| 32 | 583449   | 507 | 965511   | 87 | 617939   | 595 | 382061    | 28 |
| 33 | 583754   | 507 | 965458   | 87 | 618295   | 594 | 381705    | 27 |
| 34 | 584058   | 506 | 965406   | 87 | 618652   | 594 | 381348    | 26 |
| 35 | 584361   | 506 | 965353   | 88 | 619008   | 594 | 380992    | 25 |
| 36 | 584665   | 506 | 965301   | 88 | 619364   | 593 | 380636    | 24 |
| 37 | 584968   | 505 | 965248   | 88 | 619721   | 593 | 380279    | 23 |
| 38 | 585272   | 505 | 965195   | 88 | 620076   | 593 | 379924    | 22 |
| 39 | 585574   | 504 | 965143   | 88 | 620432   | 592 | 379568    | 21 |
| 40 | 585877   | 504 | 965090   | 88 | 620787   | 592 | 379213    | 20 |
| 41 | 9.586179 | 503 | 9.965037 | 88 | 9.621142 | 592 | 10.378858 | 19 |
| 42 | 586482   | 503 | 964984   | 88 | 621497   | 591 | 378503    | 18 |
| 43 | 586783   | 503 | 964931   | 88 | 621852   | 591 | 378148    | 17 |
| 44 | 587085   | 502 | 964879   | 88 | 622207   | 590 | 377793    | 16 |
| 45 | 587386   | 502 | 964826   | 88 | 622561   | 590 | 377439    | 15 |
| 46 | 587688   | 501 | 964773   | 88 | 622915   | 590 | 377085    | 14 |
| 47 | 587989   | 501 | 964719   | 88 | 623269   | 589 | 376731    | 13 |
| 48 | 588289   | 501 | 964666   | 89 | 623623   | 589 | 376377    | 12 |
| 49 | 588590   | 500 | 964613   | 89 | 623976   | 589 | 376024    | 11 |
| 50 | 588890   | 500 | 964560   | 89 | 624330   | 588 | 375670    | 10 |
| 51 | 9.589190 | 499 | 9.964507 | 89 | 9.624683 | 588 | 10.375317 | 9  |
| 52 | 589489   | 499 | 964454   | 89 | 625036   | 588 | 374964    | 8  |
| 53 | 589789   | 499 | 964400   | 89 | 625388   | 587 | 374612    | 7  |
| 54 | 590088   | 498 | 964347   | 89 | 625741   | 587 | 374259    | 6  |
| 55 | 590387   | 498 | 964294   | 89 | 626093   | 587 | 373907    | 5  |
| 56 | 590686   | 497 | 964240   | 89 | 626445   | 586 | 373555    | 4  |
| 57 | 590984   | 497 | 964187   | 89 | 626797   | 586 | 373203    | 3  |
| 58 | 591282   | 497 | 964133   | 89 | 627149   | 586 | 372851    | 2  |
| 59 | 591580   | 496 | 964080   | 89 | 627501   | 585 | 372499    | 1  |
| 60 | 591878   | 496 | 964026   | 89 | 627852   | 585 | 372148    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D. | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.591878 | 496 | 9.964026 | 89 | 9.627852 | 585 | 10.372148 | 60 |
| 1  | 592176   | 495 | 963972   | 89 | 628203   | 585 | 371797    | 59 |
| 2  | 592473   | 495 | 963919   | 89 | 628554   | 585 | 371446    | 58 |
| 3  | 592770   | 495 | 963865   | 90 | 628905   | 584 | 371095    | 57 |
| 4  | 593067   | 494 | 963811   | 90 | 629255   | 584 | 370745    | 56 |
| 5  | 593363   | 494 | 963757   | 90 | 629606   | 583 | 370394    | 55 |
| 6  | 593659   | 493 | 963704   | 90 | 629956   | 583 | 370044    | 54 |
| 7  | 593955   | 493 | 963650   | 90 | 630306   | 583 | 369694    | 53 |
| 8  | 594251   | 493 | 963596   | 90 | 630656   | 583 | 369344    | 52 |
| 9  | 594547   | 492 | 963542   | 90 | 631005   | 582 | 368995    | 51 |
| 10 | 594842   | 492 | 963488   | 90 | 631355   | 582 | 368645    | 50 |
| 11 | 9.595137 | 491 | 9.963434 | 90 | 9.631704 | 582 | 10.368296 | 49 |
| 12 | 595432   | 491 | 963379   | 90 | 632053   | 581 | 367947    | 48 |
| 13 | 595727   | 491 | 963325   | 90 | 632401   | 581 | 367597    | 47 |
| 14 | 596021   | 490 | 963271   | 90 | 632750   | 581 | 367250    | 46 |
| 15 | 596315   | 490 | 963217   | 90 | 633098   | 580 | 366902    | 45 |
| 16 | 596609   | 489 | 963163   | 90 | 633447   | 580 | 366553    | 44 |
| 17 | 596903   | 489 | 963108   | 91 | 633795   | 580 | 366205    | 43 |
| 18 | 597196   | 489 | 963054   | 91 | 634143   | 579 | 365857    | 42 |
| 19 | 597490   | 488 | 962999   | 91 | 634490   | 579 | 365510    | 41 |
| 20 | 597783   | 488 | 962945   | 91 | 634838   | 579 | 365162    | 40 |
| 21 | 9.598075 | 487 | 9.962890 | 91 | 9.635185 | 578 | 10.364815 | 39 |
| 22 | 598368   | 487 | 962836   | 91 | 635532   | 578 | 364468    | 38 |
| 23 | 598660   | 487 | 962781   | 91 | 635879   | 578 | 364121    | 37 |
| 24 | 598952   | 486 | 962727   | 91 | 636226   | 577 | 363774    | 36 |
| 25 | 599244   | 486 | 962672   | 91 | 636572   | 577 | 363428    | 35 |
| 26 | 599536   | 485 | 962617   | 91 | 636919   | 577 | 363081    | 34 |
| 27 | 599827   | 485 | 962562   | 91 | 637265   | 577 | 362735    | 33 |
| 28 | 600118   | 485 | 962508   | 91 | 637611   | 576 | 362389    | 32 |
| 29 | 600409   | 484 | 962453   | 91 | 637956   | 576 | 362044    | 31 |
| 30 | 600700   | 484 | 962398   | 92 | 638302   | 576 | 361698    | 30 |
| 31 | 9.600990 | 484 | 9.962343 | 92 | 9.638647 | 575 | 10.361353 | 29 |
| 32 | 601280   | 483 | 962288   | 92 | 638992   | 575 | 361008    | 28 |
| 33 | 601570   | 483 | 962233   | 92 | 639337   | 575 | 360663    | 27 |
| 34 | 601860   | 482 | 962178   | 92 | 639682   | 574 | 360318    | 26 |
| 35 | 602150   | 482 | 962123   | 92 | 640027   | 574 | 359973    | 25 |
| 36 | 602439   | 482 | 962067   | 92 | 640371   | 574 | 359629    | 24 |
| 37 | 602728   | 481 | 962012   | 92 | 640716   | 573 | 359284    | 23 |
| 38 | 603017   | 481 | 961957   | 92 | 641060   | 573 | 358940    | 22 |
| 39 | 603305   | 481 | 961902   | 92 | 641404   | 573 | 358596    | 21 |
| 40 | 603594   | 480 | 961846   | 92 | 641747   | 572 | 358253    | 20 |
| 41 | 9.603882 | 480 | 9.961791 | 92 | 9.642091 | 572 | 10.357909 | 19 |
| 42 | 604170   | 479 | 961735   | 92 | 642434   | 572 | 357566    | 18 |
| 43 | 604457   | 479 | 961680   | 92 | 642777   | 572 | 357223    | 17 |
| 44 | 604745   | 479 | 961624   | 93 | 643120   | 571 | 356880    | 16 |
| 45 | 605032   | 478 | 961569   | 93 | 643463   | 571 | 356537    | 15 |
| 46 | 605319   | 478 | 961513   | 93 | 643806   | 571 | 356194    | 14 |
| 47 | 605606   | 478 | 961458   | 93 | 644148   | 570 | 355852    | 13 |
| 48 | 605892   | 477 | 961402   | 93 | 644490   | 570 | 355510    | 12 |
| 49 | 606179   | 477 | 961346   | 93 | 644832   | 570 | 355168    | 11 |
| 50 | 606465   | 476 | 961290   | 93 | 645174   | 569 | 354826    | 10 |
| 51 | 9.606751 | 476 | 9.961235 | 93 | 9.645516 | 569 | 10.354484 | 9  |
| 52 | 607036   | 476 | 961179   | 93 | 645857   | 569 | 354143    | 8  |
| 53 | 607322   | 475 | 961123   | 93 | 646199   | 569 | 353801    | 7  |
| 54 | 607607   | 475 | 961067   | 93 | 646540   | 568 | 353460    | 6  |
| 55 | 607892   | 474 | 961011   | 93 | 646881   | 568 | 353119    | 5  |
| 56 | 608177   | 474 | 960955   | 93 | 647222   | 568 | 352778    | 4  |
| 57 | 608461   | 474 | 960899   | 93 | 647562   | 567 | 352438    | 3  |
| 58 | 608745   | 473 | 960843   | 94 | 647903   | 567 | 352097    | 2  |
| 59 | 609029   | 473 | 960786   | 94 | 648243   | 567 | 351757    | 1  |
| 60 | 609313   | 473 | 960730   | 94 | 648583   | 566 | 351417    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D. | Tang     | D.  | Cotang.   |    |
|----|----------|-----|----------|----|----------|-----|-----------|----|
| 0  | 9.609313 | 473 | 9.960730 | 94 | 9.648583 | 566 | 10.351417 | 60 |
| 1  | 609597   | 472 | 960674   | 94 | 648923   | 566 | 351077    | 59 |
| 2  | 609880   | 472 | 960618   | 94 | 649263   | 566 | 350737    | 58 |
| 3  | 610164   | 472 | 960561   | 94 | 649602   | 566 | 350398    | 57 |
| 4  | 610447   | 471 | 960505   | 94 | 649942   | 565 | 350058    | 56 |
| 5  | 610729   | 471 | 960448   | 94 | 650281   | 565 | 349719    | 55 |
| 6  | 611012   | 470 | 960392   | 94 | 650620   | 565 | 349380    | 54 |
| 7  | 611294   | 470 | 960335   | 94 | 650959   | 564 | 349041    | 53 |
| 8  | 611576   | 470 | 960279   | 94 | 651297   | 564 | 648703    | 52 |
| 9  | 611858   | 469 | 960222   | 94 | 651636   | 564 | 348364    | 51 |
| 10 | 612140   | 469 | 960165   | 94 | 651974   | 563 | 348026    | 50 |
| 11 | 9.612421 | 469 | 9.960109 | 95 | 9.652312 | 563 | 10.347688 | 49 |
| 12 | 612702   | 468 | 960052   | 95 | 652650   | 563 | 347350    | 48 |
| 13 | 612983   | 468 | 959995   | 95 | 652988   | 563 | 347012    | 47 |
| 14 | 613264   | 467 | 959938   | 95 | 653326   | 562 | 346674    | 46 |
| 15 | 613545   | 467 | 959882   | 95 | 653663   | 562 | 346337    | 45 |
| 16 | 613825   | 467 | 959825   | 95 | 654000   | 562 | 346000    | 44 |
| 17 | 614105   | 466 | 959768   | 95 | 654337   | 561 | 345663    | 43 |
| 18 | 614385   | 466 | 959711   | 95 | 654674   | 561 | 345326    | 42 |
| 19 | 614665   | 466 | 959654   | 95 | 655011   | 561 | 344989    | 41 |
| 20 | 614944   | 465 | 959596   | 95 | 655348   | 561 | 344652    | 40 |
| 21 | 9.615223 | 465 | 9.959539 | 95 | 9.655684 | 560 | 10.344316 | 39 |
| 22 | 615502   | 465 | 959482   | 95 | 656020   | 560 | 343980    | 38 |
| 23 | 615781   | 464 | 959425   | 95 | 656356   | 560 | 343644    | 37 |
| 24 | 616060   | 464 | 959368   | 95 | 656692   | 559 | 343308    | 36 |
| 25 | 616338   | 464 | 959310   | 96 | 657028   | 559 | 342972    | 35 |
| 26 | 616616   | 463 | 959253   | 96 | 657364   | 559 | 342636    | 34 |
| 27 | 616894   | 463 | 959195   | 96 | 657699   | 559 | 342201    | 33 |
| 28 | 617172   | 462 | 959138   | 96 | 658034   | 558 | 341966    | 32 |
| 29 | 617450   | 462 | 959081   | 96 | 658369   | 558 | 341631    | 31 |
| 30 | 617727   | 462 | 959023   | 96 | 658704   | 558 | 341296    | 30 |
| 31 | 9.618004 | 461 | 9.958965 | 96 | 9.659039 | 558 | 10.340961 | 29 |
| 32 | 618281   | 461 | 958908   | 96 | 659373   | 557 | 340627    | 28 |
| 33 | 618558   | 461 | 958850   | 96 | 659708   | 557 | 340292    | 27 |
| 34 | 618834   | 460 | 958792   | 96 | 660042   | 557 | 339958    | 26 |
| 35 | 619110   | 460 | 958734   | 96 | 660376   | 557 | 339624    | 25 |
| 36 | 619386   | 460 | 958677   | 96 | 660710   | 556 | 339290    | 24 |
| 37 | 619662   | 459 | 958619   | 96 | 661043   | 556 | 338957    | 23 |
| 38 | 619938   | 459 | 958561   | 96 | 661377   | 556 | 338623    | 22 |
| 39 | 620213   | 459 | 958503   | 97 | 661710   | 555 | 338290    | 21 |
| 40 | 620488   | 458 | 958445   | 97 | 662043   | 555 | 337957    | 20 |
| 41 | 9.620763 | 458 | 9.958387 | 97 | 9.662376 | 555 | 10.337624 | 19 |
| 42 | 621038   | 457 | 958329   | 97 | 662709   | 554 | 337291    | 18 |
| 43 | 621313   | 457 | 958271   | 97 | 663042   | 554 | 336958    | 17 |
| 44 | 621587   | 457 | 958213   | 97 | 663375   | 554 | 336625    | 16 |
| 45 | 621861   | 456 | 958154   | 97 | 663707   | 554 | 336293    | 15 |
| 46 | 622135   | 456 | 958096   | 97 | 664039   | 553 | 335961    | 14 |
| 47 | 622409   | 456 | 958038   | 97 | 664371   | 553 | 335629    | 13 |
| 48 | 622682   | 455 | 957979   | 97 | 664703   | 553 | 335297    | 12 |
| 49 | 622956   | 455 | 957921   | 97 | 665035   | 553 | 334965    | 11 |
| 50 | 623229   | 455 | 957863   | 97 | 665366   | 552 | 334634    | 10 |
| 51 | 9.623502 | 454 | 9.957804 | 97 | 9.665697 | 552 | 10.334303 | 9  |
| 52 | 623774   | 454 | 957746   | 98 | 666029   | 552 | 333971    | 8  |
| 53 | 624047   | 454 | 957687   | 98 | 666360   | 551 | 333640    | 7  |
| 54 | 624319   | 453 | 957628   | 98 | 666691   | 551 | 333309    | 6  |
| 55 | 624591   | 453 | 957570   | 98 | 667021   | 551 | 332979    | 5  |
| 56 | 624863   | 453 | 957511   | 98 | 667352   | 551 | 332648    | 4  |
| 57 | 625135   | 452 | 957452   | 98 | 667682   | 550 | 332318    | 3  |
| 58 | 625406   | 452 | 957393   | 98 | 668013   | 550 | 331987    | 2  |
| 59 | 625677   | 452 | 957335   | 98 | 668343   | 550 | 331657    | 1  |
| 60 | 625948   | 451 | 957276   | 98 | 668672   | 550 | 331328    | 0  |
|    | Cosine   |     | Sine     |    | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.625948 | 451 | 9.957276 | 98  | 9.668673 | 550 | 10.331327 | 60 |
| 1  | 626219   | 451 | 957217   | 98  | 669002   | 549 | 330998    | 59 |
| 2  | 626490   | 451 | 957158   | 98  | 669332   | 549 | 330668    | 58 |
| 3  | 626760   | 450 | 957099   | 98  | 669661   | 549 | 330339    | 57 |
| 4  | 627030   | 450 | 957040   | 98  | 669991   | 548 | 330009    | 56 |
| 5  | 627300   | 450 | 956981   | 98  | 670320   | 548 | 329680    | 55 |
| 6  | 627570   | 449 | 956921   | 99  | 670649   | 548 | 329351    | 54 |
| 7  | 627840   | 449 | 956862   | 99  | 670977   | 548 | 329023    | 53 |
| 8  | 628109   | 449 | 956803   | 99  | 671306   | 547 | 328694    | 52 |
| 9  | 628378   | 448 | 956744   | 99  | 671634   | 547 | 328366    | 51 |
| 10 | 628647   | 448 | 956684   | 99  | 671963   | 547 | 328037    | 50 |
| 11 | 9.628916 | 447 | 9.956625 | 99  | 9.672291 | 547 | 10.327709 | 49 |
| 12 | 629185   | 447 | 956566   | 99  | 672619   | 546 | 327381    | 48 |
| 13 | 629453   | 447 | 956506   | 99  | 672947   | 546 | 327053    | 47 |
| 14 | 629721   | 446 | 956447   | 99  | 673274   | 546 | 326726    | 46 |
| 15 | 629989   | 446 | 956387   | 99  | 673602   | 546 | 326398    | 45 |
| 16 | 630257   | 446 | 956327   | 99  | 673929   | 545 | 326071    | 44 |
| 17 | 630524   | 446 | 956268   | 99  | 674257   | 545 | 325743    | 43 |
| 18 | 630792   | 445 | 956208   | 100 | 674584   | 545 | 325416    | 42 |
| 19 | 631059   | 445 | 956148   | 100 | 674910   | 544 | 325090    | 41 |
| 20 | 631326   | 445 | 956089   | 100 | 675237   | 544 | 324763    | 40 |
| 21 | 9.631593 | 444 | 9.956029 | 100 | 9.675564 | 544 | 10.324436 | 39 |
| 22 | 631859   | 444 | 955969   | 100 | 675890   | 544 | 324110    | 38 |
| 23 | 632125   | 444 | 955909   | 100 | 676216   | 543 | 323784    | 37 |
| 24 | 632392   | 443 | 955849   | 100 | 676543   | 543 | 323457    | 36 |
| 25 | 632658   | 443 | 955789   | 100 | 676869   | 543 | 323131    | 35 |
| 26 | 632923   | 443 | 955729   | 100 | 677194   | 543 | 322806    | 34 |
| 27 | 633189   | 442 | 955669   | 100 | 677520   | 542 | 322480    | 33 |
| 28 | 633454   | 442 | 955609   | 100 | 677846   | 542 | 322154    | 32 |
| 29 | 633719   | 442 | 955548   | 100 | 678171   | 542 | 321829    | 31 |
| 30 | 633984   | 441 | 955488   | 100 | 678496   | 542 | 321504    | 30 |
| 31 | 9.634240 | 441 | 9.955428 | 101 | 9.678821 | 541 | 10.321179 | 29 |
| 32 | 634514   | 440 | 955368   | 101 | 679146   | 541 | 320854    | 28 |
| 33 | 634778   | 440 | 955307   | 101 | 679471   | 541 | 320529    | 27 |
| 34 | 635042   | 440 | 955247   | 101 | 679795   | 541 | 320205    | 26 |
| 35 | 635306   | 439 | 955186   | 101 | 680120   | 540 | 319880    | 25 |
| 36 | 635570   | 439 | 955126   | 101 | 680444   | 540 | 319556    | 24 |
| 37 | 635834   | 439 | 955065   | 101 | 680768   | 540 | 319232    | 23 |
| 38 | 636097   | 438 | 955005   | 101 | 681092   | 540 | 318908    | 22 |
| 39 | 636360   | 438 | 954944   | 101 | 681416   | 539 | 318584    | 21 |
| 40 | 636623   | 438 | 954883   | 101 | 681740   | 539 | 318260    | 20 |
| 41 | 9.636886 | 437 | 9.954823 | 101 | 9.682063 | 539 | 10.317937 | 19 |
| 42 | 637148   | 437 | 954762   | 101 | 682387   | 539 | 317613    | 18 |
| 43 | 637411   | 437 | 954701   | 101 | 682710   | 538 | 317290    | 17 |
| 44 | 637673   | 437 | 954640   | 101 | 683033   | 538 | 316967    | 16 |
| 45 | 637935   | 436 | 954579   | 101 | 683356   | 538 | 316644    | 15 |
| 46 | 638197   | 436 | 954518   | 102 | 683679   | 538 | 316321    | 14 |
| 47 | 638458   | 436 | 954457   | 102 | 684001   | 537 | 315999    | 13 |
| 48 | 638720   | 435 | 954396   | 102 | 684324   | 537 | 315676    | 12 |
| 49 | 638981   | 435 | 954335   | 102 | 684646   | 537 | 315354    | 11 |
| 50 | 639242   | 435 | 954274   | 102 | 684968   | 537 | 315032    | 10 |
| 51 | 9.639503 | 434 | 9.954213 | 102 | 9.685290 | 536 | 10.314710 | 9  |
| 52 | 639764   | 434 | 954152   | 102 | 685672   | 536 | 314388    | 8  |
| 53 | 640024   | 434 | 954090   | 102 | 685994   | 536 | 314066    | 7  |
| 54 | 640284   | 433 | 954029   | 102 | 686325   | 536 | 313745    | 6  |
| 55 | 640544   | 433 | 953968   | 102 | 686657   | 535 | 313423    | 5  |
| 56 | 640804   | 433 | 953906   | 102 | 686989   | 535 | 313102    | 4  |
| 57 | 641064   | 432 | 953845   | 102 | 687319   | 535 | 312781    | 3  |
| 58 | 641324   | 432 | 953783   | 102 | 687650   | 535 | 312460    | 2  |
| 59 | 641584   | 432 | 953722   | 103 | 687981   | 534 | 312139    | 1  |
| 60 | 641842   | 431 | 953660   | 103 | 688312   | 534 | 311818    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.641842 | 431 | 9.953660 | 103 | 9.688182 | 534 | 10.311818 | 60 |
| 1  | 642101   | 431 | 953599   | 103 | 688502   | 534 | 311498    | 59 |
| 2  | 642360   | 431 | 953537   | 103 | 688823   | 534 | 311177    | 58 |
| 3  | 642618   | 430 | 953475   | 103 | 689143   | 533 | 310857    | 57 |
| 4  | 642877   | 430 | 953413   | 103 | 689463   | 533 | 310537    | 56 |
| 5  | 643135   | 430 | 953352   | 103 | 689783   | 533 | 310217    | 55 |
| 6  | 643393   | 430 | 953290   | 103 | 690103   | 533 | 309897    | 54 |
| 7  | 643650   | 429 | 953228   | 103 | 690423   | 533 | 309577    | 53 |
| 8  | 643908   | 429 | 953166   | 103 | 690742   | 532 | 309258    | 52 |
| 9  | 644165   | 429 | 953104   | 103 | 691062   | 532 | 308938    | 51 |
| 10 | 644423   | 428 | 953042   | 103 | 691381   | 532 | 308619    | 50 |
| 11 | 9.644680 | 428 | 9.952980 | 104 | 9.691700 | 531 | 10.308300 | 49 |
| 12 | 644936   | 428 | 952918   | 104 | 692019   | 531 | 307981    | 48 |
| 13 | 645193   | 427 | 952855   | 104 | 692338   | 531 | 307662    | 47 |
| 14 | 645450   | 427 | 952793   | 104 | 692656   | 531 | 307344    | 46 |
| 15 | 645706   | 427 | 952731   | 104 | 692975   | 531 | 307025    | 45 |
| 16 | 645962   | 426 | 952669   | 104 | 693293   | 530 | 306707    | 44 |
| 17 | 646218   | 426 | 952606   | 104 | 693612   | 530 | 306388    | 43 |
| 18 | 646474   | 426 | 952544   | 104 | 693930   | 530 | 306070    | 42 |
| 19 | 646729   | 425 | 952481   | 104 | 694248   | 530 | 305752    | 41 |
| 20 | 646984   | 425 | 952419   | 104 | 694566   | 529 | 305434    | 40 |
| 21 | 9.647240 | 425 | 9.952356 | 104 | 9.694883 | 529 | 10.305117 | 39 |
| 22 | 647494   | 424 | 952294   | 104 | 695201   | 529 | 304799    | 38 |
| 23 | 647749   | 424 | 952231   | 104 | 695518   | 529 | 304482    | 37 |
| 24 | 648004   | 424 | 952168   | 105 | 695836   | 529 | 304164    | 36 |
| 25 | 648258   | 424 | 952106   | 105 | 696153   | 528 | 303847    | 35 |
| 26 | 648512   | 423 | 952043   | 105 | 696470   | 528 | 303530    | 34 |
| 27 | 648766   | 423 | 951980   | 105 | 696787   | 528 | 303213    | 33 |
| 28 | 649020   | 423 | 951917   | 105 | 697103   | 528 | 302897    | 32 |
| 29 | 649274   | 422 | 951854   | 105 | 697420   | 527 | 302580    | 31 |
| 30 | 649527   | 422 | 951791   | 105 | 697736   | 527 | 302264    | 30 |
| 31 | 9.649781 | 422 | 9.951728 | 105 | 9.698053 | 527 | 10.301947 | 29 |
| 32 | 650034   | 422 | 951665   | 105 | 698369   | 527 | 301631    | 28 |
| 33 | 650287   | 421 | 951602   | 105 | 698685   | 526 | 301315    | 27 |
| 34 | 650539   | 421 | 951539   | 105 | 699001   | 526 | 300999    | 26 |
| 35 | 650792   | 421 | 951476   | 105 | 699316   | 526 | 300684    | 25 |
| 36 | 651044   | 420 | 951412   | 105 | 699632   | 526 | 300368    | 24 |
| 37 | 651297   | 420 | 951349   | 106 | 699947   | 526 | 300053    | 23 |
| 38 | 651549   | 420 | 951286   | 106 | 700263   | 525 | 299737    | 22 |
| 39 | 651800   | 419 | 951222   | 106 | 700578   | 525 | 299422    | 21 |
| 40 | 652052   | 419 | 951159   | 106 | 700893   | 525 | 299107    | 20 |
| 41 | 9.652304 | 419 | 9.951096 | 106 | 9.701208 | 524 | 10.298792 | 19 |
| 42 | 652555   | 418 | 951032   | 106 | 701523   | 524 | 298477    | 18 |
| 43 | 652806   | 418 | 950968   | 106 | 701837   | 524 | 298163    | 17 |
| 44 | 653057   | 418 | 950905   | 106 | 702152   | 524 | 297848    | 16 |
| 45 | 653308   | 418 | 950841   | 106 | 702466   | 524 | 297534    | 15 |
| 46 | 653558   | 417 | 950778   | 106 | 702780   | 523 | 297220    | 14 |
| 47 | 653808   | 417 | 950714   | 106 | 703095   | 523 | 296905    | 13 |
| 48 | 654059   | 417 | 950650   | 106 | 703409   | 523 | 296591    | 12 |
| 49 | 654309   | 416 | 950586   | 106 | 703723   | 523 | 296277    | 11 |
| 50 | 654558   | 416 | 950522   | 107 | 704036   | 522 | 295964    | 10 |
| 51 | 9.654808 | 416 | 9.950458 | 107 | 9.704350 | 522 | 10.295650 | 9  |
| 52 | 655058   | 416 | 950394   | 107 | 704663   | 522 | 295337    | 8  |
| 53 | 655307   | 415 | 950330   | 107 | 704977   | 522 | 295023    | 7  |
| 54 | 655556   | 415 | 950266   | 107 | 705290   | 522 | 294710    | 6  |
| 55 | 655805   | 415 | 950202   | 107 | 705603   | 521 | 294397    | 5  |
| 56 | 656054   | 414 | 950138   | 107 | 705916   | 521 | 294084    | 4  |
| 57 | 656302   | 414 | 950074   | 107 | 706228   | 521 | 293772    | 3  |
| 58 | 656551   | 414 | 950010   | 107 | 706541   | 521 | 293459    | 2  |
| 59 | 656799   | 413 | 949945   | 107 | 706854   | 521 | 293146    | 1  |
| 60 | 657047   | 413 | 949881   | 107 | 707166   | 520 | 292834    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.657047 | 413 | 9.949881 | 107 | 9.707166 | 520 | 10.292834 | 60 |
| 1  | 657295   | 413 | 949816   | 107 | 707478   | 520 | 292522    | 59 |
| 2  | 657542   | 412 | 949752   | 107 | 707790   | 520 | 292210    | 58 |
| 3  | 657790   | 412 | 949688   | 108 | 708102   | 520 | 291898    | 57 |
| 4  | 658037   | 412 | 949623   | 108 | 708414   | 519 | 291586    | 56 |
| 5  | 658284   | 412 | 949558   | 108 | 708726   | 519 | 291274    | 55 |
| 6  | 658531   | 411 | 949494   | 108 | 709037   | 519 | 290963    | 54 |
| 7  | 658778   | 411 | 949429   | 108 | 709349   | 519 | 290651    | 53 |
| 8  | 659025   | 411 | 949364   | 108 | 709660   | 519 | 290340    | 52 |
| 9  | 659271   | 410 | 949300   | 108 | 709971   | 518 | 290029    | 51 |
| 10 | 659517   | 410 | 949235   | 108 | 710282   | 518 | 289718    | 50 |
| 11 | 9.659763 | 410 | 9.949170 | 108 | 9.710593 | 518 | 10.289407 | 49 |
| 12 | 660009   | 409 | 949105   | 108 | 710904   | 518 | 289096    | 48 |
| 13 | 660255   | 409 | 949040   | 108 | 711215   | 818 | 288785    | 47 |
| 14 | 660501   | 409 | 948975   | 108 | 711525   | 517 | 288475    | 46 |
| 15 | 660746   | 409 | 948910   | 108 | 711836   | 517 | 288164    | 45 |
| 16 | 660991   | 408 | 948845   | 108 | 712146   | 517 | 287854    | 44 |
| 17 | 661236   | 408 | 948780   | 109 | 712456   | 517 | 287544    | 43 |
| 18 | 661481   | 408 | 948715   | 109 | 712766   | 516 | 287234    | 42 |
| 19 | 661726   | 407 | 948650   | 109 | 713076   | 516 | 286924    | 41 |
| 20 | 661970   | 407 | 948584   | 109 | 713386   | 516 | 286614    | 40 |
| 21 | 9.662214 | 407 | 9.948519 | 109 | 9.713696 | 516 | 10.286304 | 39 |
| 22 | 662459   | 407 | 948454   | 109 | 714005   | 516 | 285995    | 38 |
| 23 | 662703   | 406 | 948388   | 109 | 714314   | 515 | 285686    | 37 |
| 24 | 662946   | 406 | 948323   | 109 | 714624   | 515 | 285376    | 36 |
| 25 | 663190   | 406 | 948257   | 109 | 714933   | 515 | 285067    | 35 |
| 26 | 663433   | 405 | 948192   | 109 | 715242   | 515 | 284758    | 34 |
| 27 | 663677   | 405 | 948126   | 109 | 715551   | 514 | 284449    | 33 |
| 28 | 663920   | 405 | 948060   | 109 | 715860   | 514 | 284140    | 32 |
| 29 | 664163   | 405 | 947995   | 110 | 716168   | 514 | 283832    | 31 |
| 30 | 664406   | 404 | 947929   | 110 | 716477   | 514 | 283523    | 30 |
| 31 | 9.664648 | 404 | 9.947863 | 110 | 9.716785 | 514 | 10.283215 | 29 |
| 32 | 664891   | 404 | 947797   | 110 | 717093   | 513 | 282907    | 28 |
| 33 | 665133   | 403 | 947731   | 110 | 717401   | 513 | 282599    | 27 |
| 34 | 665375   | 403 | 947665   | 110 | 717709   | 513 | 282291    | 26 |
| 35 | 665617   | 403 | 947600   | 110 | 718017   | 513 | 281983    | 25 |
| 36 | 665859   | 402 | 947533   | 110 | 718325   | 513 | 281675    | 24 |
| 37 | 666100   | 402 | 947467   | 110 | 718633   | 512 | 281367    | 23 |
| 38 | 666342   | 402 | 947401   | 110 | 718940   | 512 | 281060    | 22 |
| 39 | 666583   | 402 | 947335   | 110 | 719248   | 512 | 280752    | 21 |
| 40 | 666824   | 401 | 947269   | 110 | 719555   | 512 | 280445    | 20 |
| 41 | 9.667065 | 401 | 9.947203 | 110 | 9.719862 | 512 | 10.280138 | 19 |
| 42 | 667305   | 401 | 947136   | 111 | 720169   | 511 | 279831    | 18 |
| 43 | 667546   | 401 | 947070   | 111 | 720476   | 511 | 279524    | 17 |
| 44 | 667786   | 400 | 947004   | 111 | 720783   | 511 | 279217    | 16 |
| 45 | 668027   | 400 | 946937   | 111 | 721089   | 511 | 278911    | 15 |
| 46 | 668267   | 400 | 946871   | 111 | 721395   | 511 | 278604    | 14 |
| 47 | 668506   | 399 | 946804   | 111 | 721702   | 510 | 278298    | 13 |
| 48 | 668746   | 399 | 946738   | 111 | 722009   | 510 | 277991    | 12 |
| 49 | 668986   | 399 | 946671   | 111 | 722315   | 510 | 277685    | 11 |
| 50 | 669225   | 399 | 946604   | 111 | 722621   | 510 | 277379    | 10 |
| 51 | 9.669464 | 398 | 9.946538 | 111 | 9.722927 | 510 | 10.277073 | 9  |
| 52 | 669703   | 398 | 946471   | 111 | 723232   | 509 | 276768    | 8  |
| 53 | 669942   | 398 | 946404   | 111 | 723538   | 509 | 276462    | 7  |
| 54 | 670181   | 397 | 946337   | 111 | 723844   | 509 | 276156    | 6  |
| 55 | 670419   | 397 | 946270   | 112 | 724149   | 509 | 275851    | 5  |
| 56 | 670658   | 397 | 946203   | 112 | 724454   | 509 | 275546    | 4  |
| 57 | 670896   | 397 | 946136   | 112 | 724759   | 508 | 275241    | 3  |
| 58 | 671134   | 396 | 946069   | 112 | 725065   | 508 | 274935    | 2  |
| 59 | 671372   | 396 | 946002   | 112 | 725369   | 508 | 274631    | 1  |
| 60 | 671609   | 396 | 945935   | 112 | 725674   | 508 | 274326    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.671609 | 396 | 9.945935 | 112 | 9.725674 | 508 | 10.274326 | 60 |
| 1  | 671847   | 395 | 945868   | 112 | 725979   | 508 | 274021    | 59 |
| 2  | 672084   | 395 | 945800   | 112 | 726264   | 507 | 273716    | 58 |
| 3  | 672321   | 395 | 945733   | 112 | 726588   | 507 | 273412    | 57 |
| 4  | 672558   | 395 | 945666   | 112 | 726892   | 507 | 273108    | 56 |
| 5  | 672795   | 394 | 945598   | 112 | 727197   | 507 | 272803    | 55 |
| 6  | 673032   | 394 | 945531   | 112 | 727501   | 507 | 272499    | 54 |
| 7  | 673268   | 394 | 945464   | 113 | 727805   | 506 | 272195    | 53 |
| 8  | 673505   | 394 | 945396   | 113 | 728109   | 506 | 271891    | 52 |
| 9  | 673741   | 393 | 945328   | 113 | 728412   | 506 | 271588    | 51 |
| 10 | 673977   | 393 | 945261   | 113 | 728716   | 506 | 271284    | 50 |
| 11 | 9.674213 | 393 | 9.945193 | 113 | 9.729020 | 506 | 10.270980 | 49 |
| 12 | 674448   | 392 | 945125   | 113 | 729323   | 505 | 270677    | 48 |
| 13 | 674684   | 392 | 945058   | 113 | 729626   | 505 | 270374    | 47 |
| 14 | 674919   | 392 | 944990   | 113 | 729929   | 505 | 270071    | 46 |
| 15 | 675155   | 392 | 944922   | 113 | 730233   | 505 | 269767    | 45 |
| 16 | 675390   | 391 | 944854   | 113 | 730535   | 505 | 269465    | 44 |
| 17 | 675624   | 391 | 944786   | 113 | 730838   | 504 | 269162    | 43 |
| 18 | 675859   | 391 | 944718   | 113 | 731141   | 504 | 268859    | 42 |
| 19 | 676094   | 391 | 944650   | 113 | 731444   | 504 | 268556    | 41 |
| 20 | 676328   | 390 | 944582   | 114 | 731746   | 504 | 268254    | 40 |
| 21 | 9.676562 | 390 | 9.944514 | 114 | 9.732048 | 504 | 10.267952 | 39 |
| 22 | 676796   | 390 | 944446   | 114 | 732351   | 503 | 267649    | 38 |
| 23 | 677030   | 390 | 944377   | 114 | 732653   | 503 | 267347    | 37 |
| 24 | 677264   | 389 | 944309   | 114 | 732955   | 503 | 267045    | 36 |
| 25 | 677498   | 389 | 944241   | 114 | 733257   | 503 | 266743    | 35 |
| 26 | 677731   | 389 | 944172   | 114 | 733558   | 503 | 266442    | 34 |
| 27 | 677964   | 388 | 944104   | 114 | 733860   | 502 | 266140    | 33 |
| 28 | 678197   | 388 | 944036   | 114 | 734162   | 502 | 265838    | 32 |
| 29 | 678430   | 388 | 943967   | 114 | 734463   | 502 | 265537    | 31 |
| 30 | 678663   | 388 | 943899   | 114 | 734764   | 502 | 265236    | 30 |
| 31 | 9.678895 | 387 | 9.943830 | 114 | 9.735066 | 502 | 10.264934 | 29 |
| 32 | 679128   | 387 | 943761   | 114 | 735367   | 502 | 264633    | 28 |
| 33 | 679360   | 387 | 943693   | 115 | 735668   | 501 | 264332    | 27 |
| 34 | 679592   | 387 | 943624   | 115 | 735969   | 501 | 264031    | 26 |
| 35 | 679824   | 386 | 943555   | 115 | 736269   | 501 | 263731    | 25 |
| 36 | 680056   | 386 | 943486   | 115 | 736570   | 501 | 263430    | 24 |
| 37 | 680288   | 386 | 943417   | 115 | 736871   | 501 | 263129    | 23 |
| 38 | 680519   | 385 | 943348   | 115 | 737171   | 500 | 262829    | 22 |
| 39 | 680750   | 385 | 943279   | 115 | 737471   | 500 | 262529    | 21 |
| 40 | 680982   | 385 | 943210   | 115 | 737771   | 500 | 262229    | 20 |
| 41 | 9.681213 | 385 | 9.943141 | 115 | 9.738071 | 500 | 10.261929 | 19 |
| 42 | 681443   | 384 | 943072   | 115 | 738371   | 500 | 261629    | 18 |
| 43 | 681674   | 384 | 943003   | 115 | 738671   | 499 | 261329    | 17 |
| 44 | 681905   | 384 | 942934   | 115 | 738971   | 499 | 261029    | 16 |
| 45 | 682135   | 384 | 942864   | 115 | 739271   | 499 | 260729    | 15 |
| 46 | 682365   | 383 | 942795   | 116 | 739570   | 499 | 260430    | 14 |
| 47 | 682595   | 383 | 942726   | 116 | 739870   | 499 | 260130    | 13 |
| 48 | 682825   | 383 | 942656   | 116 | 740169   | 499 | 259831    | 12 |
| 49 | 683055   | 383 | 942587   | 116 | 740468   | 498 | 259532    | 11 |
| 50 | 683284   | 382 | 942517   | 116 | 740767   | 498 | 259233    | 10 |
| 51 | 9.683514 | 382 | 9.942448 | 116 | 9.741066 | 498 | 10.258934 | 9  |
| 52 | 683743   | 382 | 942378   | 116 | 741365   | 498 | 258635    | 8  |
| 53 | 683972   | 382 | 942308   | 116 | 741664   | 498 | 258336    | 7  |
| 54 | 684201   | 381 | 942239   | 116 | 741962   | 497 | 258038    | 6  |
| 55 | 684430   | 381 | 942169   | 116 | 742261   | 497 | 257739    | 5  |
| 56 | 684658   | 381 | 942099   | 116 | 742559   | 497 | 257441    | 4  |
| 57 | 684887   | 380 | 942029   | 116 | 742858   | 497 | 257142    | 3  |
| 58 | 685115   | 380 | 941959   | 116 | 743156   | 497 | 256844    | 2  |
| 59 | 685343   | 380 | 941889   | 117 | 743454   | 497 | 256546    | 1  |
| 60 | 685571   | 380 | 941819   | 117 | 743752   | 496 | 256248    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.685571 | 380 | 9.941819 | 117 | 9.743752 | 496 | 10.256248 | 60 |
| 1  | 685799   | 379 | 941749   | 117 | 744050   | 496 | 255950    | 59 |
| 2  | 686027   | 379 | 941679   | 117 | 744348   | 495 | 255652    | 58 |
| 3  | 686254   | 379 | 941609   | 117 | 744645   | 496 | 255355    | 57 |
| 4  | 686482   | 379 | 941539   | 117 | 744943   | 496 | 255057    | 56 |
| 5  | 686709   | 378 | 941469   | 117 | 745240   | 495 | 254760    | 55 |
| 6  | 686936   | 378 | 941398   | 117 | 745538   | 495 | 254462    | 54 |
| 7  | 687163   | 378 | 941328   | 117 | 745835   | 495 | 254165    | 53 |
| 8  | 687389   | 378 | 941258   | 117 | 746132   | 495 | 253868    | 52 |
| 9  | 687616   | 377 | 941187   | 117 | 746429   | 495 | 253571    | 51 |
| 10 | 687843   | 377 | 941117   | 117 | 746726   | 495 | 253274    | 50 |
| 11 | 9.688069 | 377 | 9.941046 | 118 | 9.747023 | 494 | 10.252977 | 49 |
| 12 | 688295   | 377 | 940975   | 118 | 747319   | 494 | 252681    | 48 |
| 13 | 688521   | 376 | 940905   | 118 | 747616   | 494 | 252384    | 47 |
| 14 | 688747   | 376 | 940834   | 118 | 747913   | 494 | 252087    | 46 |
| 15 | 688972   | 376 | 940763   | 118 | 748209   | 494 | 251791    | 45 |
| 16 | 689198   | 376 | 940693   | 118 | 748505   | 493 | 251495    | 44 |
| 17 | 689423   | 375 | 940622   | 118 | 748801   | 493 | 251199    | 43 |
| 18 | 689648   | 375 | 940551   | 118 | 749097   | 493 | 250903    | 42 |
| 19 | 689873   | 375 | 940480   | 118 | 749393   | 493 | 250607    | 41 |
| 20 | 690098   | 375 | 940409   | 118 | 749689   | 493 | 250311    | 40 |
| 21 | 9.690323 | 374 | 9.940338 | 118 | 9.749985 | 493 | 10.250015 | 39 |
| 22 | 690548   | 374 | 940267   | 118 | 750281   | 492 | 249719    | 38 |
| 23 | 690772   | 374 | 940196   | 118 | 750576   | 492 | 249424    | 37 |
| 24 | 690996   | 374 | 940125   | 119 | 750872   | 492 | 249128    | 36 |
| 25 | 691220   | 373 | 940054   | 119 | 751167   | 492 | 248833    | 35 |
| 26 | 691444   | 373 | 939982   | 119 | 751462   | 492 | 248538    | 34 |
| 27 | 691668   | 373 | 939911   | 119 | 751757   | 492 | 248243    | 33 |
| 28 | 691892   | 373 | 939840   | 119 | 752052   | 491 | 247948    | 32 |
| 29 | 692115   | 372 | 939768   | 119 | 752347   | 491 | 247653    | 31 |
| 30 | 692339   | 372 | 939697   | 119 | 752642   | 491 | 247358    | 30 |
| 31 | 9.692562 | 372 | 9.939625 | 119 | 9.752937 | 491 | 10.247063 | 29 |
| 32 | 692725   | 371 | 939554   | 119 | 753231   | 491 | 246769    | 28 |
| 33 | 692906   | 371 | 939482   | 119 | 753526   | 491 | 246474    | 27 |
| 34 | 693087   | 371 | 939410   | 119 | 753820   | 490 | 246180    | 26 |
| 35 | 693268   | 371 | 939339   | 119 | 754115   | 490 | 245885    | 25 |
| 36 | 693448   | 370 | 939267   | 120 | 754409   | 490 | 245591    | 24 |
| 37 | 693628   | 370 | 939195   | 120 | 754703   | 490 | 245297    | 23 |
| 38 | 693808   | 370 | 939123   | 120 | 754997   | 490 | 245003    | 22 |
| 39 | 693987   | 370 | 939052   | 120 | 755291   | 490 | 244709    | 21 |
| 40 | 694166   | 369 | 938980   | 120 | 755585   | 489 | 244415    | 20 |
| 41 | 9.694385 | 369 | 9.938908 | 120 | 9.755878 | 489 | 10.244122 | 19 |
| 42 | 694507   | 369 | 938836   | 120 | 756172   | 489 | 243828    | 18 |
| 43 | 694629   | 369 | 938763   | 120 | 756465   | 489 | 243535    | 17 |
| 44 | 694750   | 368 | 938691   | 120 | 756759   | 489 | 243241    | 16 |
| 45 | 694871   | 368 | 938619   | 120 | 757052   | 489 | 242948    | 15 |
| 46 | 694992   | 368 | 938547   | 120 | 757345   | 488 | 242655    | 14 |
| 47 | 695113   | 368 | 938475   | 120 | 757638   | 488 | 242362    | 13 |
| 48 | 695234   | 367 | 938402   | 121 | 757931   | 488 | 242069    | 12 |
| 49 | 695354   | 367 | 938330   | 121 | 758224   | 488 | 241776    | 11 |
| 50 | 695475   | 367 | 938258   | 121 | 758517   | 488 | 241483    | 10 |
| 51 | 9.695695 | 367 | 9.938185 | 121 | 9.758810 | 488 | 10.241190 | 9  |
| 52 | 6957215  | 366 | 938113   | 121 | 759102   | 487 | 240898    | 8  |
| 53 | 6957435  | 366 | 938040   | 121 | 759395   | 487 | 240605    | 7  |
| 54 | 6957654  | 366 | 937967   | 121 | 759687   | 487 | 240313    | 6  |
| 55 | 6957874  | 366 | 937895   | 121 | 759979   | 487 | 240021    | 5  |
| 56 | 6958094  | 365 | 937822   | 121 | 760272   | 487 | 239728    | 4  |
| 57 | 6958313  | 365 | 937749   | 121 | 760564   | 487 | 239436    | 3  |
| 58 | 6958532  | 365 | 937676   | 121 | 760856   | 486 | 239144    | 2  |
| 59 | 6958751  | 365 | 937604   | 121 | 761148   | 486 | 238852    | 1  |
| 60 | 6958970  | 364 | 937531   | 121 | 761439   | 486 | 238561    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D   | Cosine   | D.  | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.698970 | 364 | 9.937531 | 121 | 9.761439 | 486 | 10.238561 | 60 |
| 1  | 699189   | 364 | 937458   | 122 | 761731   | 486 | 238269    | 59 |
| 2  | 699407   | 364 | 937385   | 122 | 762023   | 486 | 237977    | 58 |
| 3  | 699626   | 364 | 937312   | 122 | 762314   | 486 | 237686    | 57 |
| 4  | 699844   | 363 | 937238   | 122 | 762606   | 485 | 237394    | 56 |
| 5  | 700062   | 363 | 937165   | 122 | 762897   | 485 | 237103    | 55 |
| 6  | 700280   | 363 | 937092   | 122 | 763188   | 485 | 236812    | 54 |
| 7  | 700498   | 363 | 937019   | 122 | 763479   | 485 | 236521    | 53 |
| 8  | 700716   | 363 | 936946   | 122 | 763770   | 485 | 236230    | 52 |
| 9  | 700933   | 362 | 936872   | 122 | 764061   | 485 | 235939    | 51 |
| 10 | 701151   | 362 | 936799   | 122 | 764352   | 484 | 235648    | 50 |
| 11 | 9.701368 | 362 | 9.936725 | 122 | 9.764643 | 484 | 10.235357 | 49 |
| 12 | 701585   | 362 | 936652   | 123 | 764933   | 484 | 235067    | 48 |
| 13 | 701802   | 361 | 936578   | 123 | 765224   | 484 | 234776    | 47 |
| 14 | 702019   | 361 | 936505   | 123 | 765514   | 484 | 234486    | 46 |
| 15 | 702236   | 361 | 936431   | 123 | 765805   | 484 | 234195    | 45 |
| 16 | 702452   | 361 | 936357   | 123 | 766095   | 484 | 233905    | 44 |
| 17 | 702669   | 360 | 936284   | 123 | 766385   | 483 | 233615    | 43 |
| 18 | 702885   | 360 | 936210   | 123 | 766675   | 483 | 233325    | 42 |
| 19 | 703101   | 360 | 936136   | 123 | 766965   | 483 | 233035    | 41 |
| 20 | 703317   | 360 | 936062   | 123 | 767255   | 483 | 232745    | 40 |
| 21 | 9.703533 | 359 | 9.935988 | 123 | 9.767545 | 483 | 10.232455 | 39 |
| 22 | 703749   | 359 | 935914   | 123 | 767834   | 483 | 232166    | 38 |
| 23 | 703964   | 359 | 935840   | 123 | 768124   | 482 | 231876    | 37 |
| 24 | 704179   | 359 | 935766   | 124 | 768413   | 482 | 231587    | 36 |
| 25 | 704395   | 359 | 935692   | 124 | 768703   | 482 | 231297    | 35 |
| 26 | 704610   | 358 | 935618   | 124 | 768992   | 482 | 231008    | 34 |
| 27 | 704825   | 358 | 935543   | 124 | 769281   | 482 | 230719    | 33 |
| 28 | 705040   | 358 | 935469   | 124 | 769570   | 482 | 230430    | 32 |
| 29 | 705254   | 358 | 935395   | 124 | 769860   | 481 | 230140    | 31 |
| 30 | 705469   | 357 | 935320   | 124 | 770148   | 481 | 229852    | 30 |
| 31 | 9.705683 | 357 | 9.935246 | 124 | 9.770437 | 481 | 10.229563 | 29 |
| 32 | 705898   | 357 | 935171   | 124 | 770726   | 481 | 229274    | 28 |
| 33 | 706112   | 357 | 935097   | 124 | 771015   | 481 | 228985    | 27 |
| 34 | 706326   | 356 | 935022   | 124 | 771303   | 481 | 228697    | 26 |
| 35 | 706539   | 356 | 934948   | 124 | 771592   | 481 | 228408    | 25 |
| 36 | 706753   | 356 | 934873   | 124 | 771880   | 480 | 228120    | 24 |
| 37 | 706967   | 356 | 934798   | 125 | 772168   | 480 | 227832    | 23 |
| 38 | 707180   | 355 | 934723   | 125 | 772457   | 480 | 227543    | 22 |
| 39 | 707393   | 355 | 934649   | 125 | 772745   | 480 | 227255    | 21 |
| 40 | 707606   | 355 | 934574   | 125 | 773033   | 480 | 226967    | 20 |
| 41 | 9.707819 | 355 | 9.934499 | 125 | 9.773321 | 480 | 10.226679 | 19 |
| 42 | 708032   | 354 | 934424   | 125 | 773608   | 479 | 226392    | 18 |
| 43 | 708245   | 354 | 934349   | 125 | 773896   | 479 | 226104    | 17 |
| 44 | 708458   | 354 | 934274   | 125 | 774184   | 479 | 225816    | 16 |
| 45 | 708670   | 354 | 934199   | 125 | 774471   | 479 | 225529    | 15 |
| 46 | 708882   | 353 | 934123   | 125 | 774759   | 479 | 225241    | 14 |
| 47 | 709094   | 353 | 934048   | 125 | 775046   | 479 | 224954    | 13 |
| 48 | 709306   | 353 | 933973   | 125 | 775333   | 479 | 224667    | 12 |
| 49 | 709518   | 353 | 933898   | 126 | 775621   | 478 | 224379    | 11 |
| 50 | 709730   | 353 | 933822   | 126 | 775908   | 478 | 224092    | 10 |
| 51 | 9.709941 | 352 | 9.933747 | 126 | 9.776195 | 478 | 10.223805 | 9  |
| 52 | 710153   | 352 | 933671   | 126 | 776482   | 478 | 223518    | 8  |
| 53 | 710364   | 352 | 933596   | 126 | 776769   | 478 | 223231    | 7  |
| 54 | 710575   | 352 | 933520   | 126 | 777055   | 478 | 222945    | 6  |
| 55 | 710786   | 351 | 933445   | 126 | 777342   | 478 | 222658    | 5  |
| 56 | 710997   | 351 | 933369   | 126 | 777628   | 477 | 222372    | 4  |
| 57 | 711208   | 351 | 933293   | 126 | 777915   | 477 | 222085    | 3  |
| 58 | 711419   | 351 | 933217   | 126 | 778201   | 477 | 221799    | 2  |
| 59 | 711629   | 350 | 933141   | 126 | 778487   | 477 | 221512    | 1  |
| 60 | 711839   | 350 | 933066   | 126 | 778774   | 477 | 221226    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.711839 | 350 | 9.933066 | 126 | 9.778774 | 477 | 10.221226 | 60 |
| 1  | 712050   | 350 | 932990   | 127 | 779060   | 477 | 220940    | 59 |
| 2  | 712260   | 350 | 932914   | 127 | 779346   | 476 | 220654    | 58 |
| 3  | 712469   | 349 | 932838   | 127 | 779632   | 476 | 220368    | 57 |
| 4  | 712679   | 349 | 932762   | 127 | 779918   | 476 | 220082    | 56 |
| 5  | 712889   | 349 | 932685   | 127 | 780203   | 476 | 219797    | 55 |
| 6  | 713098   | 349 | 932609   | 127 | 780489   | 476 | 219511    | 54 |
| 7  | 713308   | 349 | 932533   | 127 | 780775   | 476 | 219225    | 53 |
| 8  | 713517   | 348 | 932457   | 127 | 781060   | 476 | 218940    | 52 |
| 9  | 713726   | 348 | 932380   | 127 | 781346   | 475 | 218654    | 51 |
| 10 | 713935   | 348 | 932304   | 127 | 781631   | 475 | 218369    | 50 |
| 11 | 9.714144 | 348 | 9.932228 | 127 | 9.781916 | 475 | 10.218084 | 49 |
| 12 | 714352   | 347 | 932151   | 127 | 782201   | 475 | 217799    | 48 |
| 13 | 714561   | 347 | 932075   | 128 | 782486   | 475 | 217514    | 47 |
| 14 | 714769   | 347 | 931998   | 128 | 782771   | 475 | 217229    | 46 |
| 15 | 714978   | 347 | 931921   | 128 | 783056   | 475 | 216944    | 45 |
| 16 | 715186   | 347 | 931845   | 128 | 783341   | 475 | 216659    | 44 |
| 17 | 715394   | 346 | 931768   | 128 | 783626   | 474 | 216374    | 43 |
| 18 | 715602   | 346 | 931691   | 128 | 783910   | 474 | 216090    | 42 |
| 19 | 715809   | 346 | 931614   | 128 | 784195   | 474 | 215805    | 41 |
| 20 | 716017   | 346 | 931537   | 128 | 784479   | 474 | 215521    | 40 |
| 21 | 9.716224 | 345 | 9.931460 | 128 | 9.784764 | 474 | 10.215236 | 39 |
| 22 | 716432   | 345 | 931383   | 128 | 785048   | 474 | 214952    | 38 |
| 23 | 716639   | 345 | 931306   | 128 | 785332   | 473 | 214668    | 37 |
| 24 | 716846   | 345 | 931229   | 129 | 785616   | 473 | 214384    | 36 |
| 25 | 717053   | 345 | 931152   | 129 | 785900   | 473 | 214100    | 35 |
| 26 | 717259   | 344 | 931075   | 129 | 786184   | 473 | 213816    | 34 |
| 27 | 717466   | 344 | 930998   | 129 | 786468   | 473 | 213532    | 33 |
| 28 | 717673   | 344 | 930921   | 129 | 786752   | 473 | 213248    | 32 |
| 29 | 717879   | 344 | 930843   | 129 | 787036   | 473 | 212964    | 31 |
| 30 | 718085   | 343 | 930766   | 129 | 787319   | 472 | 212681    | 30 |
| 31 | 9.718291 | 343 | 9.930688 | 129 | 9.787603 | 472 | 10.212397 | 29 |
| 32 | 718497   | 343 | 930611   | 129 | 787886   | 472 | 212114    | 28 |
| 33 | 718703   | 343 | 930533   | 129 | 788170   | 472 | 211830    | 27 |
| 34 | 718909   | 343 | 930456   | 129 | 788453   | 472 | 211547    | 26 |
| 35 | 719114   | 342 | 930378   | 129 | 788736   | 472 | 211264    | 25 |
| 36 | 719320   | 342 | 930300   | 130 | 789019   | 472 | 210981    | 24 |
| 37 | 719525   | 342 | 930223   | 130 | 789302   | 471 | 210698    | 23 |
| 38 | 719730   | 342 | 930145   | 130 | 789585   | 471 | 210415    | 22 |
| 39 | 719935   | 341 | 930067   | 130 | 789868   | 471 | 210132    | 21 |
| 40 | 720140   | 341 | 929989   | 130 | 790151   | 471 | 209849    | 20 |
| 41 | 9.720345 | 341 | 9.929911 | 130 | 9.790433 | 471 | 10.209567 | 19 |
| 42 | 720549   | 341 | 929833   | 130 | 790716   | 471 | 209284    | 18 |
| 43 | 720754   | 340 | 929755   | 130 | 790999   | 471 | 209001    | 17 |
| 44 | 720958   | 340 | 929677   | 130 | 791281   | 471 | 208719    | 16 |
| 45 | 721162   | 340 | 929599   | 130 | 791563   | 470 | 208437    | 15 |
| 46 | 721366   | 340 | 929521   | 130 | 791846   | 470 | 208154    | 14 |
| 47 | 721570   | 340 | 929442   | 130 | 792128   | 470 | 207872    | 13 |
| 48 | 721774   | 339 | 929364   | 131 | 792410   | 470 | 207590    | 12 |
| 49 | 721978   | 339 | 929286   | 131 | 792692   | 470 | 207308    | 11 |
| 50 | 722181   | 339 | 929207   | 131 | 792974   | 470 | 207026    | 10 |
| 51 | 9.722385 | 339 | 9.929129 | 131 | 9.793256 | 470 | 10.206744 | 9  |
| 52 | 722588   | 339 | 929050   | 131 | 793538   | 469 | 206462    | 8  |
| 53 | 722791   | 338 | 928972   | 131 | 793819   | 469 | 206181    | 7  |
| 54 | 722994   | 338 | 928893   | 131 | 794101   | 469 | 205899    | 6  |
| 55 | 723197   | 338 | 928815   | 131 | 794383   | 469 | 205617    | 5  |
| 56 | 723400   | 338 | 928736   | 131 | 794664   | 469 | 205336    | 4  |
| 57 | 723603   | 337 | 928657   | 131 | 794945   | 469 | 205055    | 3  |
| 58 | 723805   | 337 | 928578   | 131 | 795227   | 469 | 204773    | 2  |
| 59 | 724007   | 337 | 928499   | 131 | 795508   | 468 | 204492    | 1  |
| 60 | 724210   | 337 | 928420   | 131 | 795789   | 468 | 204211    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D   | Cosine   | D.  | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.724210 | 337 | 9.928420 | 132 | 9.795789 | 468 | 10.204211 | 60 |
| 1  | 724412   | 337 | 928342   | 132 | 796070   | 468 | 203930    | 59 |
| 2  | 724614   | 336 | 928263   | 132 | 796351   | 468 | 203649    | 58 |
| 3  | 724816   | 336 | 928183   | 132 | 796632   | 468 | 203368    | 57 |
| 4  | 725017   | 336 | 928104   | 132 | 796913   | 468 | 203087    | 56 |
| 5  | 725219   | 336 | 928025   | 132 | 797194   | 468 | 202806    | 55 |
| 6  | 725420   | 335 | 927946   | 132 | 797475   | 468 | 202525    | 54 |
| 7  | 725622   | 335 | 927867   | 132 | 797755   | 468 | 202245    | 53 |
| 8  | 725823   | 335 | 927787   | 132 | 798036   | 467 | 201964    | 52 |
| 9  | 726024   | 335 | 927708   | 132 | 798316   | 467 | 201684    | 51 |
| 10 | 726225   | 335 | 927629   | 132 | 798596   | 467 | 201404    | 50 |
| 11 | 9.726426 | 334 | 9.927549 | 132 | 9.798877 | 467 | 10.201123 | 49 |
| 12 | 726626   | 334 | 927470   | 133 | 799157   | 467 | 200843    | 48 |
| 13 | 726827   | 334 | 927390   | 133 | 799437   | 467 | 200563    | 47 |
| 14 | 727027   | 334 | 927310   | 133 | 799717   | 467 | 200283    | 46 |
| 15 | 727228   | 334 | 927231   | 133 | 799997   | 466 | 200003    | 45 |
| 16 | 727428   | 333 | 927151   | 133 | 800277   | 466 | 199723    | 44 |
| 17 | 727628   | 333 | 927071   | 133 | 800557   | 466 | 199443    | 43 |
| 18 | 727828   | 333 | 926991   | 133 | 800836   | 466 | 199164    | 42 |
| 19 | 728027   | 333 | 926911   | 133 | 801116   | 466 | 198884    | 41 |
| 20 | 728227   | 333 | 926831   | 133 | 801396   | 466 | 198604    | 40 |
| 21 | 9.728427 | 332 | 9.926751 | 133 | 9.801675 | 466 | 10.198325 | 39 |
| 22 | 728626   | 332 | 926671   | 133 | 801955   | 466 | 198045    | 38 |
| 23 | 728825   | 332 | 926591   | 133 | 802234   | 465 | 197766    | 37 |
| 24 | 729024   | 332 | 926511   | 134 | 802513   | 465 | 197487    | 36 |
| 25 | 729223   | 331 | 926431   | 134 | 802792   | 465 | 197208    | 35 |
| 26 | 729422   | 331 | 926351   | 134 | 803072   | 465 | 196928    | 34 |
| 27 | 729621   | 331 | 926270   | 134 | 803351   | 465 | 196649    | 33 |
| 28 | 729820   | 331 | 926190   | 134 | 803630   | 465 | 196370    | 32 |
| 29 | 730018   | 330 | 926110   | 134 | 803909   | 465 | 196092    | 31 |
| 30 | 730216   | 330 | 926029   | 134 | 804187   | 465 | 195813    | 30 |
| 31 | 9.730415 | 330 | 9.925949 | 134 | 9.804466 | 464 | 10.195534 | 29 |
| 32 | 730613   | 330 | 925868   | 134 | 804745   | 464 | 195255    | 28 |
| 33 | 730811   | 330 | 925788   | 134 | 805023   | 464 | 194977    | 27 |
| 34 | 731009   | 329 | 925707   | 134 | 805302   | 464 | 194698    | 26 |
| 35 | 731206   | 329 | 925626   | 134 | 805580   | 464 | 194420    | 25 |
| 36 | 731404   | 329 | 925545   | 135 | 805859   | 464 | 194141    | 24 |
| 37 | 731602   | 329 | 925465   | 135 | 806137   | 464 | 193863    | 23 |
| 38 | 731799   | 329 | 925384   | 135 | 806415   | 463 | 193585    | 22 |
| 39 | 731996   | 328 | 925303   | 135 | 806693   | 463 | 193307    | 21 |
| 40 | 732193   | 328 | 925222   | 135 | 806971   | 463 | 193029    | 20 |
| 41 | 9.732390 | 328 | 9.925141 | 135 | 9.807249 | 463 | 10.192751 | 19 |
| 42 | 732587   | 328 | 925060   | 135 | 807527   | 463 | 192473    | 18 |
| 43 | 732784   | 328 | 924979   | 135 | 807805   | 463 | 192195    | 17 |
| 44 | 732980   | 327 | 924897   | 135 | 808083   | 463 | 191917    | 16 |
| 45 | 733177   | 327 | 924816   | 135 | 808361   | 463 | 191639    | 15 |
| 46 | 733373   | 327 | 924735   | 136 | 808638   | 462 | 191362    | 14 |
| 47 | 733569   | 327 | 924654   | 136 | 808916   | 462 | 191084    | 13 |
| 48 | 733765   | 327 | 924572   | 136 | 809193   | 462 | 190807    | 12 |
| 49 | 733961   | 326 | 924491   | 136 | 809471   | 462 | 190529    | 11 |
| 50 | 734157   | 326 | 924409   | 136 | 809748   | 462 | 190252    | 10 |
| 51 | 9.734353 | 326 | 9.924328 | 136 | 9.810025 | 462 | 10.189975 | 9  |
| 52 | 734549   | 326 | 924246   | 136 | 810302   | 462 | 189698    | 8  |
| 53 | 734744   | 325 | 924164   | 136 | 810580   | 462 | 189420    | 7  |
| 54 | 734939   | 325 | 924083   | 136 | 810857   | 462 | 189143    | 6  |
| 55 | 735135   | 325 | 924001   | 136 | 811134   | 461 | 188866    | 5  |
| 56 | 735330   | 325 | 923919   | 136 | 811410   | 461 | 188590    | 4  |
| 57 | 735525   | 325 | 923837   | 136 | 811687   | 461 | 188313    | 3  |
| 58 | 735719   | 324 | 923755   | 137 | 811964   | 461 | 188036    | 2  |
| 59 | 735914   | 324 | 923673   | 137 | 812241   | 461 | 187759    | 1  |
| 60 | 736109   | 324 | 923591   | 137 | 812517   | 461 | 187483    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang    | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.730109 | 324 | 9.923501 | 137 | 9.812517 | 461 | 10.187482 | 60 |
| 1  | 736303   | 324 | 923509   | 137 | 812794   | 461 | 187206    | 59 |
| 2  | 736198   | 324 | 923427   | 137 | 813070   | 461 | 186930    | 58 |
| 3  | 736692   | 323 | 923345   | 137 | 813347   | 460 | 186653    | 57 |
| 4  | 736886   | 323 | 923263   | 137 | 813623   | 460 | 186377    | 56 |
| 5  | 737680   | 323 | 923181   | 137 | 813899   | 460 | 186101    | 55 |
| 6  | 737274   | 323 | 923098   | 137 | 814175   | 460 | 185825    | 54 |
| 7  | 737467   | 323 | 923016   | 137 | 814452   | 460 | 185548    | 53 |
| 8  | 737661   | 322 | 922933   | 137 | 814728   | 460 | 185272    | 52 |
| 9  | 737855   | 322 | 922851   | 137 | 815004   | 460 | 184996    | 51 |
| 10 | 738048   | 322 | 922768   | 138 | 815279   | 460 | 184721    | 50 |
| 11 | 9.738241 | 322 | 9.922686 | 138 | 9.815555 | 459 | 10.184445 | 49 |
| 12 | 738434   | 322 | 922603   | 138 | 815531   | 459 | 184169    | 48 |
| 13 | 738627   | 321 | 922520   | 138 | 816107   | 459 | 183893    | 47 |
| 14 | 738820   | 321 | 922438   | 138 | 816382   | 459 | 183618    | 46 |
| 15 | 739013   | 321 | 922355   | 138 | 816658   | 459 | 183342    | 45 |
| 16 | 739206   | 321 | 922272   | 138 | 816933   | 459 | 183067    | 44 |
| 17 | 739398   | 321 | 922189   | 138 | 817209   | 459 | 182791    | 43 |
| 18 | 739590   | 320 | 922106   | 138 | 817484   | 459 | 182516    | 42 |
| 19 | 739783   | 320 | 922023   | 138 | 817759   | 459 | 182241    | 41 |
| 20 | 739975   | 320 | 921940   | 138 | 818035   | 458 | 181965    | 40 |
| 21 | 9.740167 | 320 | 9.921857 | 139 | 9.818310 | 458 | 10.181690 | 39 |
| 22 | 740359   | 320 | 921774   | 139 | 818585   | 458 | 181415    | 38 |
| 23 | 740550   | 319 | 921691   | 139 | 818860   | 458 | 181140    | 37 |
| 24 | 740742   | 319 | 921607   | 139 | 819135   | 458 | 180865    | 36 |
| 25 | 740934   | 319 | 921524   | 139 | 819410   | 458 | 180590    | 35 |
| 26 | 741125   | 319 | 921441   | 139 | 819684   | 458 | 180316    | 34 |
| 27 | 741316   | 319 | 921357   | 139 | 819959   | 458 | 180041    | 33 |
| 28 | 741508   | 318 | 921274   | 139 | 820234   | 458 | 179766    | 32 |
| 29 | 741699   | 318 | 921190   | 139 | 820508   | 457 | 179492    | 31 |
| 30 | 741889   | 318 | 921107   | 139 | 820783   | 457 | 179217    | 30 |
| 31 | 9.742080 | 318 | 9.921023 | 139 | 9.821057 | 457 | 10.178943 | 29 |
| 32 | 742271   | 318 | 920939   | 140 | 821332   | 457 | 178668    | 28 |
| 33 | 742462   | 317 | 920856   | 140 | 821606   | 457 | 178394    | 27 |
| 34 | 742652   | 317 | 920772   | 140 | 821880   | 457 | 178120    | 26 |
| 35 | 742842   | 317 | 920688   | 140 | 822154   | 357 | 177846    | 25 |
| 36 | 743033   | 317 | 920604   | 140 | 822429   | 457 | 177571    | 24 |
| 37 | 743223   | 317 | 920520   | 140 | 822703   | 457 | 177297    | 23 |
| 38 | 743413   | 316 | 920436   | 140 | 822977   | 457 | 177023    | 22 |
| 39 | 743602   | 316 | 920352   | 140 | 823250   | 456 | 176750    | 21 |
| 40 | 743792   | 316 | 920268   | 140 | 823524   | 456 | 176476    | 20 |
| 41 | 9.743982 | 316 | 9.920184 | 140 | 9.823798 | 456 | 10.176202 | 19 |
| 42 | 744171   | 316 | 920099   | 140 | 824072   | 456 | 175928    | 18 |
| 43 | 744361   | 315 | 920015   | 140 | 824345   | 456 | 175655    | 17 |
| 44 | 744550   | 315 | 919931   | 141 | 824619   | 456 | 175381    | 16 |
| 45 | 744739   | 315 | 919846   | 141 | 824893   | 456 | 175107    | 15 |
| 46 | 744928   | 315 | 919762   | 141 | 825166   | 456 | 174834    | 14 |
| 47 | 745117   | 315 | 919677   | 141 | 825439   | 455 | 174561    | 13 |
| 48 | 745306   | 314 | 919593   | 141 | 825713   | 455 | 174287    | 12 |
| 49 | 745494   | 314 | 919508   | 141 | 825986   | 455 | 174014    | 11 |
| 50 | 745683   | 314 | 919424   | 141 | 826259   | 455 | 173741    | 10 |
| 51 | 9.745871 | 314 | 9.919339 | 141 | 9.826532 | 455 | 10.173468 | 9  |
| 52 | 746059   | 314 | 919254   | 141 | 826805   | 455 | 173195    | 8  |
| 53 | 746248   | 313 | 919169   | 141 | 827078   | 455 | 172922    | 7  |
| 54 | 746436   | 313 | 919085   | 141 | 827351   | 455 | 172649    | 6  |
| 55 | 746624   | 313 | 919000   | 141 | 827624   | 455 | 172376    | 5  |
| 56 | 746812   | 313 | 918915   | 142 | 827897   | 454 | 172103    | 4  |
| 57 | 746999   | 313 | 918830   | 142 | 828170   | 454 | 171830    | 3  |
| 58 | 747187   | 312 | 918745   | 142 | 828442   | 454 | 171558    | 2  |
| 59 | 747374   | 312 | 918659   | 142 | 828715   | 454 | 171285    | 1  |
| 60 | 747562   | 312 | 918574   | 142 | 828987   | 454 | 171013    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.747562 | 312 | 9.918574 | 142 | 9.828987 | 454 | 10.171013 | 60 |
| 1  | 747749   | 312 | 918489   | 142 | 829260   | 454 | 170740    | 59 |
| 2  | 747936   | 312 | 918404   | 142 | 829532   | 454 | 170468    | 58 |
| 3  | 748123   | 311 | 918318   | 142 | 829805   | 454 | 170195    | 57 |
| 4  | 748310   | 311 | 918233   | 142 | 830077   | 454 | 169923    | 56 |
| 5  | 748497   | 311 | 918147   | 142 | 830349   | 453 | 169651    | 55 |
| 6  | 748683   | 311 | 918062   | 142 | 830621   | 453 | 169379    | 54 |
| 7  | 748870   | 311 | 917976   | 143 | 830893   | 453 | 169107    | 53 |
| 8  | 749056   | 310 | 917891   | 143 | 831165   | 453 | 168835    | 52 |
| 9  | 749243   | 310 | 917805   | 143 | 831437   | 453 | 168563    | 51 |
| 10 | 749429   | 310 | 917719   | 143 | 831709   | 453 | 168291    | 50 |
| 11 | 9.749615 | 310 | 9.917634 | 143 | 9.831981 | 453 | 10.168019 | 49 |
| 12 | 749801   | 310 | 917548   | 143 | 832253   | 453 | 167747    | 48 |
| 13 | 749987   | 309 | 917462   | 143 | 832525   | 453 | 167475    | 47 |
| 14 | 750172   | 309 | 917376   | 143 | 832796   | 453 | 167204    | 46 |
| 15 | 750358   | 309 | 917290   | 143 | 833068   | 452 | 166932    | 45 |
| 16 | 750543   | 309 | 917204   | 143 | 833339   | 452 | 166661    | 44 |
| 17 | 750729   | 309 | 917118   | 144 | 833611   | 452 | 166389    | 43 |
| 18 | 750914   | 308 | 917032   | 144 | 833882   | 452 | 166118    | 42 |
| 19 | 751099   | 308 | 916946   | 144 | 834154   | 452 | 165846    | 41 |
| 20 | 751284   | 308 | 916859   | 144 | 834425   | 452 | 165575    | 40 |
| 21 | 9.751469 | 308 | 9.916773 | 144 | 9.834696 | 452 | 10.165304 | 39 |
| 22 | 751654   | 308 | 916687   | 144 | 834967   | 452 | 165033    | 38 |
| 23 | 751839   | 308 | 916600   | 144 | 835238   | 452 | 164762    | 37 |
| 24 | 752023   | 307 | 916514   | 144 | 835509   | 452 | 164491    | 36 |
| 25 | 752208   | 307 | 916427   | 144 | 835780   | 451 | 164220    | 35 |
| 26 | 752392   | 307 | 916341   | 144 | 836051   | 451 | 163949    | 34 |
| 27 | 752576   | 307 | 916254   | 144 | 836322   | 451 | 163678    | 33 |
| 28 | 752760   | 307 | 916167   | 145 | 836593   | 451 | 163407    | 32 |
| 29 | 752944   | 306 | 916081   | 145 | 836864   | 451 | 163136    | 31 |
| 30 | 753128   | 306 | 915994   | 145 | 837134   | 451 | 162866    | 30 |
| 31 | 9.753312 | 306 | 9.915907 | 145 | 9.837405 | 451 | 10.162595 | 29 |
| 32 | 753495   | 306 | 915820   | 145 | 837675   | 451 | 162325    | 28 |
| 33 | 753679   | 306 | 915733   | 145 | 837946   | 451 | 162054    | 27 |
| 34 | 753862   | 305 | 915646   | 145 | 838216   | 451 | 161784    | 26 |
| 35 | 754046   | 305 | 915559   | 145 | 838487   | 450 | 161513    | 25 |
| 36 | 754229   | 305 | 915472   | 145 | 838757   | 450 | 161243    | 24 |
| 37 | 754412   | 305 | 915385   | 145 | 839027   | 450 | 160973    | 23 |
| 38 | 754595   | 305 | 915297   | 145 | 839297   | 450 | 160703    | 22 |
| 39 | 754778   | 304 | 915210   | 145 | 839568   | 450 | 160432    | 21 |
| 40 | 754960   | 304 | 915123   | 146 | 839838   | 450 | 160162    | 20 |
| 41 | 9.755143 | 304 | 9.915035 | 146 | 9.840108 | 450 | 10.159892 | 19 |
| 42 | 755326   | 304 | 914948   | 146 | 840378   | 450 | 159622    | 18 |
| 43 | 755508   | 304 | 914860   | 146 | 840647   | 450 | 159353    | 17 |
| 44 | 755690   | 304 | 914773   | 146 | 840917   | 449 | 159083    | 16 |
| 45 | 755872   | 303 | 914685   | 146 | 841187   | 449 | 158813    | 15 |
| 46 | 756054   | 303 | 914598   | 146 | 841457   | 449 | 158543    | 14 |
| 47 | 756236   | 303 | 914510   | 146 | 841726   | 449 | 158274    | 13 |
| 48 | 756418   | 303 | 914422   | 146 | 841996   | 449 | 158004    | 12 |
| 49 | 756600   | 303 | 914334   | 146 | 842266   | 449 | 157734    | 11 |
| 50 | 756782   | 302 | 914246   | 147 | 842535   | 449 | 157465    | 10 |
| 51 | 9.756963 | 302 | 9.914158 | 147 | 9.842805 | 449 | 10.157195 | 9  |
| 52 | 757144   | 302 | 914070   | 147 | 843074   | 449 | 156926    | 8  |
| 53 | 757326   | 302 | 913982   | 147 | 843343   | 449 | 156657    | 7  |
| 54 | 757507   | 302 | 913894   | 147 | 843612   | 449 | 156388    | 6  |
| 55 | 757688   | 301 | 913806   | 147 | 843882   | 448 | 156118    | 5  |
| 56 | 757869   | 301 | 913718   | 147 | 844151   | 448 | 155849    | 4  |
| 57 | 758050   | 301 | 913630   | 147 | 844420   | 448 | 155580    | 3  |
| 58 | 758230   | 301 | 913541   | 147 | 844689   | 448 | 155311    | 2  |
| 59 | 758411   | 301 | 913453   | 147 | 844958   | 448 | 155042    | 1  |
| 60 | 758591   | 301 | 913365   | 147 | 845227   | 448 | 154773    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.758591 | 301 | 9.913365 | 147 | 9.845227 | 448 | 10.154773 | 60 |
| 1  | 758772   | 300 | 913276   | 147 | 845496   | 448 | 154504    | 59 |
| 2  | 758952   | 300 | 913187   | 148 | 845764   | 448 | 154236    | 58 |
| 3  | 759132   | 300 | 913099   | 148 | 846032   | 448 | 153967    | 57 |
| 4  | 759312   | 300 | 913010   | 148 | 846300   | 448 | 153698    | 56 |
| 5  | 759492   | 300 | 912922   | 148 | 846570   | 447 | 153430    | 55 |
| 6  | 759672   | 299 | 912833   | 148 | 846839   | 447 | 153161    | 54 |
| 7  | 759852   | 299 | 912744   | 148 | 847107   | 447 | 152893    | 53 |
| 8  | 760031   | 299 | 912655   | 148 | 847376   | 447 | 152624    | 52 |
| 9  | 760211   | 299 | 912566   | 148 | 847644   | 447 | 152356    | 51 |
| 10 | 760390   | 289 | 912477   | 148 | 847913   | 447 | 152087    | 50 |
| 11 | 9.760569 | 298 | 9.912388 | 148 | 9.848181 | 417 | 10.151819 | 49 |
| 12 | 760748   | 298 | 912299   | 149 | 848449   | 447 | 151551    | 48 |
| 13 | 760927   | 298 | 912210   | 149 | 848717   | 447 | 151283    | 47 |
| 14 | 761106   | 298 | 912121   | 149 | 848986   | 447 | 151014    | 46 |
| 15 | 761285   | 298 | 912031   | 149 | 849254   | 447 | 150746    | 45 |
| 16 | 761464   | 298 | 911942   | 149 | 849522   | 447 | 150478    | 44 |
| 17 | 761642   | 297 | 911853   | 149 | 849790   | 446 | 150210    | 43 |
| 18 | 761821   | 297 | 911763   | 149 | 850058   | 446 | 149942    | 42 |
| 19 | 761999   | 297 | 911674   | 149 | 850325   | 446 | 149675    | 41 |
| 20 | 762177   | 297 | 911584   | 149 | 850593   | 446 | 149407    | 40 |
| 21 | 9.762356 | 297 | 9.911495 | 149 | 9.850861 | 446 | 10.149139 | 39 |
| 22 | 762534   | 296 | 911405   | 149 | 851129   | 446 | 148871    | 38 |
| 23 | 762712   | 296 | 911315   | 150 | 851396   | 446 | 148604    | 37 |
| 24 | 762889   | 296 | 911226   | 150 | 851664   | 446 | 148336    | 36 |
| 25 | 763067   | 296 | 911136   | 150 | 851931   | 446 | 148069    | 35 |
| 26 | 763245   | 296 | 911046   | 150 | 852199   | 446 | 147801    | 34 |
| 27 | 763422   | 296 | 910956   | 150 | 852466   | 446 | 147534    | 33 |
| 28 | 763600   | 295 | 910866   | 150 | 852733   | 445 | 147267    | 32 |
| 29 | 763777   | 295 | 910776   | 150 | 853001   | 445 | 146999    | 31 |
| 30 | 763954   | 295 | 910686   | 150 | 853268   | 445 | 146732    | 30 |
| 31 | 9.764131 | 295 | 9.910596 | 150 | 9.853535 | 445 | 10.146465 | 29 |
| 32 | 764308   | 295 | 910506   | 150 | 853802   | 445 | 146198    | 28 |
| 33 | 764485   | 294 | 910415   | 150 | 854069   | 445 | 145931    | 27 |
| 34 | 764662   | 294 | 910325   | 151 | 854336   | 445 | 145664    | 26 |
| 35 | 764838   | 294 | 910235   | 151 | 854603   | 445 | 145397    | 25 |
| 36 | 765015   | 294 | 910144   | 151 | 854870   | 445 | 145130    | 24 |
| 37 | 765191   | 294 | 910054   | 151 | 855137   | 445 | 144863    | 23 |
| 38 | 765367   | 294 | 909963   | 151 | 855404   | 445 | 144596    | 22 |
| 39 | 765544   | 293 | 909873   | 151 | 855671   | 444 | 144329    | 21 |
| 40 | 765720   | 293 | 909782   | 151 | 855938   | 444 | 144062    | 20 |
| 41 | 9.765896 | 293 | 9.909691 | 151 | 9.856204 | 444 | 10.143796 | 19 |
| 42 | 766072   | 293 | 909601   | 151 | 856471   | 444 | 143529    | 18 |
| 43 | 766247   | 293 | 909510   | 151 | 856737   | 444 | 143263    | 17 |
| 44 | 766423   | 293 | 909419   | 151 | 857004   | 444 | 142996    | 16 |
| 45 | 766598   | 292 | 909328   | 152 | 857270   | 444 | 142730    | 15 |
| 46 | 766774   | 292 | 909237   | 152 | 857537   | 444 | 142463    | 14 |
| 47 | 766949   | 292 | 909146   | 152 | 857803   | 444 | 142197    | 13 |
| 48 | 767124   | 292 | 909055   | 152 | 858069   | 444 | 141931    | 12 |
| 49 | 767300   | 292 | 908964   | 152 | 858336   | 444 | 141664    | 11 |
| 50 | 767475   | 291 | 908873   | 152 | 858602   | 443 | 141398    | 10 |
| 51 | 9.767649 | 291 | 9.908781 | 152 | 9.858868 | 443 | 10.141132 | 9  |
| 52 | 767824   | 291 | 908690   | 152 | 859134   | 443 | 140866    | 8  |
| 53 | 767999   | 291 | 908599   | 152 | 859400   | 443 | 140600    | 7  |
| 54 | 768173   | 291 | 908507   | 152 | 859666   | 443 | 140334    | 6  |
| 55 | 768348   | 290 | 908416   | 153 | 859932   | 443 | 140068    | 5  |
| 56 | 768522   | 290 | 908324   | 153 | 860198   | 443 | 139802    | 4  |
| 57 | 768697   | 290 | 908233   | 153 | 860464   | 443 | 139536    | 3  |
| 58 | 768871   | 290 | 908141   | 153 | 860730   | 443 | 139270    | 2  |
| 59 | 769045   | 290 | 908049   | 153 | 860995   | 443 | 139005    | 1  |
| 60 | 769219   | 290 | 907958   | 153 | 861261   | 443 | 138739    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.769219 | 290 | 9.907958 | 153 | 9.861261 | 443 | 10.138739 | 60 |
| 1  | 769393   | 289 | 9.97866  | 153 | 861527   | 443 | 138473    | 59 |
| 2  | 769566   | 289 | 907774   | 153 | 861792   | 442 | 138208    | 58 |
| 3  | 769740   | 289 | 907682   | 153 | 862058   | 442 | 137942    | 57 |
| 4  | 769913   | 289 | 907590   | 153 | 862323   | 442 | 137677    | 56 |
| 5  | 770087   | 289 | 907498   | 153 | 862589   | 442 | 137411    | 55 |
| 6  | 770260   | 288 | 907406   | 153 | 862854   | 442 | 137146    | 54 |
| 7  | 770433   | 288 | 907314   | 154 | 863119   | 442 | 136881    | 53 |
| 8  | 770606   | 288 | 907222   | 154 | 863385   | 442 | 136615    | 52 |
| 9  | 770779   | 288 | 907129   | 154 | 863650   | 442 | 136350    | 51 |
| 10 | 770952   | 288 | 907037   | 154 | 863915   | 442 | 136085    | 50 |
| 11 | 9.771125 | 288 | 9.906945 | 154 | 9.864180 | 442 | 10.135820 | 49 |
| 12 | 771298   | 287 | 906852   | 154 | 864445   | 442 | 135555    | 48 |
| 13 | 771470   | 287 | 906760   | 154 | 864710   | 442 | 135290    | 47 |
| 14 | 771643   | 287 | 906667   | 154 | 864975   | 441 | 135025    | 46 |
| 15 | 771815   | 287 | 906575   | 154 | 865240   | 441 | 134760    | 45 |
| 16 | 771987   | 287 | 906482   | 154 | 865505   | 441 | 134495    | 44 |
| 17 | 772159   | 287 | 906389   | 155 | 865770   | 441 | 134230    | 43 |
| 18 | 772331   | 286 | 906296   | 155 | 866035   | 441 | 133965    | 42 |
| 19 | 772503   | 286 | 906204   | 155 | 866300   | 441 | 133700    | 41 |
| 20 | 772675   | 286 | 906111   | 155 | 866564   | 441 | 133436    | 40 |
| 21 | 9.772847 | 286 | 9.906018 | 155 | 9.866829 | 441 | 10.133171 | 39 |
| 22 | 773018   | 286 | 905925   | 155 | 867094   | 441 | 132906    | 38 |
| 23 | 773190   | 286 | 905832   | 155 | 867358   | 441 | 132642    | 37 |
| 24 | 773361   | 285 | 905739   | 155 | 867623   | 441 | 132377    | 36 |
| 25 | 773533   | 285 | 905645   | 155 | 867887   | 441 | 132113    | 35 |
| 26 | 773704   | 285 | 905552   | 155 | 868152   | 440 | 131848    | 34 |
| 27 | 773875   | 285 | 905459   | 155 | 868416   | 440 | 131584    | 33 |
| 28 | 774046   | 285 | 905366   | 156 | 868680   | 440 | 131320    | 32 |
| 29 | 774217   | 285 | 905272   | 156 | 868945   | 440 | 131055    | 31 |
| 30 | 774388   | 284 | 905179   | 156 | 869209   | 440 | 130791    | 30 |
| 31 | 9.774558 | 284 | 9.905085 | 156 | 9.869473 | 440 | 10.130527 | 29 |
| 32 | 774729   | 284 | 904992   | 156 | 869737   | 440 | 130263    | 28 |
| 33 | 774899   | 284 | 904898   | 156 | 870001   | 440 | 129999    | 27 |
| 34 | 775070   | 284 | 904804   | 156 | 870265   | 440 | 129735    | 26 |
| 35 | 775240   | 284 | 904711   | 156 | 870529   | 440 | 129471    | 25 |
| 36 | 775410   | 283 | 904617   | 156 | 870793   | 440 | 129207    | 24 |
| 37 | 775580   | 283 | 904523   | 156 | 871057   | 440 | 128943    | 23 |
| 38 | 775750   | 283 | 904429   | 157 | 871321   | 440 | 128679    | 22 |
| 39 | 775920   | 283 | 904335   | 157 | 871585   | 440 | 128415    | 21 |
| 40 | 776090   | 283 | 904241   | 157 | 871849   | 439 | 128151    | 20 |
| 41 | 9.776259 | 283 | 9.904147 | 157 | 9.872112 | 439 | 10.127888 | 19 |
| 42 | 776429   | 282 | 904053   | 157 | 872376   | 439 | 127624    | 18 |
| 43 | 776598   | 282 | 903959   | 157 | 872640   | 439 | 127360    | 17 |
| 44 | 776768   | 282 | 903864   | 157 | 872903   | 439 | 127097    | 16 |
| 45 | 776937   | 282 | 903770   | 157 | 873167   | 439 | 126833    | 15 |
| 46 | 777106   | 282 | 903676   | 157 | 873430   | 439 | 126570    | 14 |
| 47 | 777275   | 281 | 903581   | 157 | 873694   | 439 | 126306    | 13 |
| 48 | 777444   | 281 | 903487   | 157 | 873957   | 439 | 126043    | 12 |
| 49 | 777613   | 281 | 903392   | 158 | 874220   | 439 | 125780    | 11 |
| 50 | 777781   | 281 | 903298   | 158 | 874484   | 439 | 125516    | 10 |
| 51 | 9.777950 | 281 | 9.903203 | 158 | 9.874747 | 439 | 10.125253 | 9  |
| 52 | 778119   | 281 | 903108   | 158 | 875010   | 439 | 124990    | 8  |
| 53 | 778287   | 280 | 903014   | 158 | 875273   | 438 | 124727    | 7  |
| 54 | 778455   | 280 | 902919   | 158 | 875536   | 438 | 124464    | 6  |
| 55 | 778624   | 280 | 902824   | 158 | 875800   | 438 | 124200    | 5  |
| 56 | 778792   | 280 | 902729   | 158 | 876063   | 438 | 123937    | 4  |
| 57 | 778960   | 280 | 902634   | 158 | 876326   | 438 | 123674    | 3  |
| 58 | 779128   | 280 | 902539   | 159 | 876589   | 438 | 123411    | 2  |
| 59 | 779295   | 279 | 902444   | 159 | 876851   | 438 | 123149    | 1  |
| 60 | 779463   | 279 | 902349   | 159 | 877114   | 438 | 122886    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.779463 | 279 | 9.902349 | 159 | 9.877114 | 438 | 10.122836 | 60 |
| 1  | 779531   | 279 | 902253   | 159 | 877377   | 438 | 122623    | 59 |
| 2  | 779798   | 279 | 902158   | 159 | 877640   | 438 | 122360    | 58 |
| 3  | 779966   | 279 | 902063   | 159 | 877903   | 438 | 122097    | 57 |
| 4  | 780133   | 279 | 901967   | 159 | 878165   | 438 | 121835    | 56 |
| 5  | 780300   | 278 | 901872   | 159 | 878428   | 438 | 121572    | 55 |
| 6  | 780467   | 278 | 901776   | 159 | 878691   | 438 | 121309    | 54 |
| 7  | 780634   | 278 | 901681   | 159 | 878953   | 437 | 121047    | 53 |
| 8  | 780801   | 278 | 901585   | 159 | 879216   | 437 | 120784    | 52 |
| 9  | 780968   | 278 | 901490   | 159 | 879478   | 437 | 120522    | 51 |
| 10 | 781134   | 278 | 901394   | 160 | 879741   | 437 | 120259    | 50 |
| 11 | 9.781301 | 277 | 9.901293 | 160 | 9.880003 | 437 | 10.119997 | 49 |
| 12 | 781468   | 277 | 901202   | 160 | 880265   | 437 | 119735    | 48 |
| 13 | 781634   | 277 | 901105   | 160 | 880528   | 437 | 119472    | 47 |
| 14 | 781800   | 277 | 901010   | 160 | 880790   | 437 | 119210    | 46 |
| 15 | 781966   | 277 | 900914   | 160 | 881052   | 437 | 118948    | 45 |
| 16 | 782132   | 277 | 900818   | 160 | 881314   | 437 | 118686    | 44 |
| 17 | 782298   | 276 | 900722   | 160 | 881576   | 437 | 118424    | 43 |
| 18 | 782464   | 276 | 900626   | 160 | 881839   | 437 | 118161    | 42 |
| 19 | 782630   | 276 | 900529   | 160 | 882101   | 437 | 117899    | 41 |
| 20 | 782796   | 276 | 900433   | 161 | 882363   | 436 | 117637    | 40 |
| 21 | 9.782961 | 276 | 9.900337 | 161 | 9.882625 | 436 | 10.117375 | 39 |
| 22 | 783127   | 276 | 900240   | 161 | 882887   | 436 | 117113    | 38 |
| 23 | 783292   | 275 | 900144   | 161 | 883148   | 436 | 116852    | 37 |
| 24 | 783458   | 275 | 900047   | 161 | 883410   | 436 | 116590    | 36 |
| 25 | 783623   | 275 | 899951   | 161 | 883672   | 436 | 116328    | 35 |
| 26 | 783788   | 275 | 899854   | 161 | 883934   | 436 | 116066    | 34 |
| 27 | 783953   | 275 | 899757   | 161 | 884196   | 436 | 115804    | 33 |
| 28 | 784118   | 275 | 899660   | 161 | 884457   | 436 | 115543    | 32 |
| 29 | 784282   | 274 | 899564   | 161 | 884719   | 436 | 115281    | 31 |
| 30 | 784447   | 274 | 899467   | 162 | 884980   | 436 | 115020    | 30 |
| 31 | 9.784612 | 274 | 9.899370 | 162 | 9.885242 | 436 | 10.114758 | 29 |
| 32 | 784776   | 274 | 899273   | 162 | 885503   | 436 | 114497    | 28 |
| 33 | 784941   | 274 | 899176   | 162 | 885765   | 436 | 114235    | 27 |
| 34 | 785105   | 274 | 899078   | 162 | 886026   | 436 | 113974    | 26 |
| 35 | 785269   | 273 | 898981   | 162 | 886288   | 436 | 113712    | 25 |
| 36 | 785433   | 273 | 898884   | 162 | 886549   | 435 | 113451    | 24 |
| 37 | 785597   | 273 | 898787   | 162 | 886810   | 435 | 113190    | 23 |
| 38 | 785761   | 273 | 898689   | 162 | 887072   | 435 | 112928    | 22 |
| 39 | 785925   | 273 | 898592   | 162 | 887333   | 435 | 112667    | 21 |
| 40 | 786089   | 273 | 898494   | 163 | 887594   | 435 | 112406    | 20 |
| 41 | 9.786252 | 272 | 9.898397 | 163 | 9.887855 | 435 | 10.112145 | 19 |
| 42 | 786416   | 272 | 898299   | 163 | 888116   | 435 | 111884    | 18 |
| 43 | 786579   | 272 | 898202   | 163 | 888377   | 435 | 111623    | 17 |
| 44 | 786742   | 272 | 898104   | 163 | 888639   | 435 | 111361    | 16 |
| 45 | 786906   | 272 | 898006   | 163 | 888900   | 435 | 111100    | 15 |
| 46 | 787069   | 272 | 897908   | 163 | 889160   | 435 | 110840    | 14 |
| 47 | 787232   | 271 | 897810   | 163 | 889421   | 435 | 110579    | 13 |
| 48 | 787395   | 271 | 897712   | 163 | 889682   | 435 | 110318    | 12 |
| 49 | 787557   | 271 | 897614   | 163 | 889943   | 435 | 110057    | 11 |
| 50 | 787720   | 271 | 897516   | 163 | 890204   | 434 | 109796    | 10 |
| 51 | 9.787882 | 271 | 9.897418 | 164 | 9.890465 | 434 | 10.109535 | 9  |
| 52 | 788045   | 271 | 897320   | 164 | 890725   | 434 | 109275    | 8  |
| 53 | 788208   | 271 | 897222   | 164 | 890986   | 434 | 109014    | 7  |
| 54 | 788370   | 270 | 897123   | 164 | 891247   | 434 | 108753    | 6  |
| 55 | 788532   | 270 | 897025   | 164 | 891507   | 434 | 108493    | 5  |
| 56 | 788694   | 270 | 896926   | 164 | 891768   | 434 | 108232    | 4  |
| 57 | 788856   | 270 | 896828   | 164 | 892028   | 434 | 107972    | 3  |
| 58 | 789018   | 270 | 896729   | 164 | 892289   | 434 | 107711    | 2  |
| 59 | 789180   | 270 | 896631   | 164 | 892549   | 434 | 107451    | 1  |
| 60 | 789342   | 269 | 896532   | 164 | 892810   | 434 | 107190    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D   | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.789342 | 269 | 9.896532 | 164 | 9.892810 | 434 | 10.107190 | 60 |
| 1  | 789504   | 269 | 896433   | 165 | 893070   | 434 | 106930    | 59 |
| 2  | 789665   | 269 | 896335   | 165 | 893331   | 434 | 106669    | 58 |
| 3  | 789827   | 269 | 896236   | 165 | 893591   | 434 | 106409    | 57 |
| 4  | 789988   | 269 | 896137   | 165 | 893851   | 434 | 106149    | 56 |
| 5  | 790149   | 269 | 896038   | 165 | 894111   | 434 | 105889    | 55 |
| 6  | 790310   | 268 | 895939   | 165 | 894371   | 434 | 105629    | 54 |
| 7  | 790471   | 268 | 895840   | 165 | 894632   | 433 | 105368    | 53 |
| 8  | 790632   | 268 | 895741   | 165 | 894892   | 433 | 105108    | 52 |
| 9  | 790793   | 268 | 895641   | 165 | 895152   | 433 | 104848    | 51 |
| 10 | 790954   | 268 | 895542   | 165 | 895412   | 433 | 104588    | 50 |
| 11 | 9.791115 | 268 | 9.895443 | 166 | 9.895672 | 433 | 10.104328 | 49 |
| 12 | 791275   | 267 | 895343   | 166 | 895932   | 433 | 104068    | 48 |
| 13 | 791436   | 267 | 895244   | 166 | 896192   | 433 | 103808    | 47 |
| 14 | 791596   | 267 | 895145   | 166 | 896452   | 433 | 103548    | 46 |
| 15 | 791757   | 267 | 895045   | 166 | 896712   | 433 | 103288    | 45 |
| 16 | 791917   | 267 | 894945   | 166 | 896971   | 433 | 103029    | 44 |
| 17 | 792077   | 267 | 894846   | 166 | 897231   | 433 | 102769    | 43 |
| 18 | 792237   | 266 | 894746   | 166 | 897491   | 433 | 102509    | 42 |
| 19 | 792397   | 266 | 894646   | 166 | 897751   | 433 | 102249    | 41 |
| 20 | 792557   | 266 | 894546   | 166 | 898010   | 433 | 101990    | 40 |
| 21 | 9.792716 | 266 | 9.894446 | 167 | 9.898270 | 433 | 10.101730 | 39 |
| 22 | 792876   | 266 | 894346   | 167 | 898530   | 433 | 101470    | 38 |
| 23 | 793035   | 266 | 894246   | 167 | 898789   | 433 | 101211    | 37 |
| 24 | 793195   | 265 | 894146   | 167 | 899049   | 432 | 100951    | 36 |
| 25 | 793354   | 265 | 894046   | 167 | 899308   | 432 | 100692    | 35 |
| 26 | 793514   | 265 | 893946   | 167 | 899568   | 432 | 100432    | 34 |
| 27 | 793673   | 265 | 893846   | 167 | 899827   | 432 | 100173    | 33 |
| 28 | 793832   | 265 | 893745   | 167 | 900086   | 432 | 999914    | 32 |
| 29 | 793991   | 265 | 893645   | 167 | 900346   | 432 | 999654    | 31 |
| 30 | 794150   | 264 | 893544   | 167 | 900605   | 432 | 999395    | 30 |
| 31 | 9.794308 | 264 | 9.893444 | 168 | 9.900864 | 432 | 10.099136 | 29 |
| 32 | 794467   | 264 | 893343   | 168 | 901124   | 432 | 998876    | 28 |
| 33 | 794626   | 264 | 893243   | 168 | 901383   | 432 | 998617    | 27 |
| 34 | 794784   | 264 | 893142   | 168 | 901642   | 432 | 998358    | 26 |
| 35 | 794942   | 264 | 893041   | 168 | 901901   | 432 | 998099    | 25 |
| 36 | 795101   | 264 | 892940   | 168 | 902160   | 432 | 997840    | 24 |
| 37 | 795259   | 263 | 892839   | 168 | 902419   | 432 | 997581    | 23 |
| 38 | 795417   | 263 | 892739   | 168 | 902679   | 432 | 997321    | 22 |
| 39 | 795575   | 263 | 892638   | 168 | 902938   | 432 | 997062    | 21 |
| 40 | 795733   | 263 | 892536   | 168 | 903197   | 431 | 996803    | 20 |
| 41 | 9.795891 | 263 | 9.892435 | 169 | 9.903455 | 431 | 10.096545 | 19 |
| 42 | 796049   | 263 | 892334   | 169 | 903714   | 431 | 996286    | 18 |
| 43 | 796206   | 263 | 892233   | 169 | 903973   | 431 | 996027    | 17 |
| 44 | 796364   | 262 | 892132   | 169 | 904232   | 431 | 995768    | 16 |
| 45 | 796521   | 262 | 892030   | 169 | 904491   | 431 | 995509    | 15 |
| 46 | 796679   | 262 | 891929   | 169 | 904750   | 431 | 995250    | 14 |
| 47 | 796836   | 262 | 891827   | 169 | 905008   | 431 | 994992    | 13 |
| 48 | 796993   | 262 | 891726   | 169 | 905267   | 431 | 994733    | 12 |
| 49 | 797150   | 261 | 891624   | 169 | 905526   | 431 | 994474    | 11 |
| 50 | 797307   | 261 | 891523   | 170 | 905784   | 431 | 994216    | 10 |
| 51 | 9.797464 | 261 | 9.891421 | 170 | 9.906043 | 431 | 10.093957 | 9  |
| 52 | 797621   | 261 | 891319   | 170 | 906302   | 431 | 993698    | 8  |
| 53 | 797777   | 261 | 891217   | 170 | 906560   | 431 | 993440    | 7  |
| 54 | 797934   | 261 | 891115   | 170 | 906819   | 431 | 993181    | 6  |
| 55 | 798091   | 261 | 891013   | 170 | 907077   | 431 | 992923    | 5  |
| 56 | 798247   | 261 | 890911   | 170 | 907336   | 431 | 992664    | 4  |
| 57 | 798403   | 260 | 890809   | 170 | 907594   | 431 | 992406    | 3  |
| 58 | 798560   | 260 | 890707   | 170 | 907852   | 431 | 992148    | 2  |
| 59 | 798716   | 260 | 890605   | 170 | 908111   | 430 | 991889    | 1  |
| 60 | 798872   | 260 | 890503   | 170 | 908369   | 430 | 991631    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang    |    |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.798872 | 260 | 9.890503 | 170 | 9.908369 | 430 | 10.091631 | 60 |
| 1  | 799028   | 260 | 890400   | 171 | 908628   | 430 | 091372    | 59 |
| 2  | 799184   | 260 | 890298   | 171 | 908886   | 430 | 091114    | 58 |
| 3  | 799339   | 259 | 890195   | 171 | 909144   | 430 | 090856    | 57 |
| 4  | 799495   | 259 | 890093   | 171 | 909402   | 430 | 090598    | 56 |
| 5  | 799651   | 259 | 889990   | 171 | 909660   | 430 | 090340    | 55 |
| 6  | 799806   | 259 | 889888   | 171 | 909918   | 430 | 090082    | 54 |
| 7  | 799962   | 259 | 889785   | 171 | 910177   | 430 | 089823    | 53 |
| 8  | 800117   | 259 | 889682   | 171 | 910435   | 430 | 089565    | 52 |
| 9  | 800272   | 258 | 889579   | 171 | 910693   | 430 | 089307    | 51 |
| 10 | 800427   | 258 | 889477   | 171 | 910951   | 430 | 089049    | 50 |
| 11 | 9.800582 | 258 | 9.889374 | 172 | 9.911209 | 430 | 10.088791 | 49 |
| 12 | 800737   | 258 | 889271   | 172 | 911467   | 430 | 088533    | 48 |
| 13 | 800892   | 258 | 889168   | 172 | 911724   | 430 | 088276    | 47 |
| 14 | 801047   | 258 | 889064   | 172 | 911982   | 430 | 088018    | 46 |
| 15 | 801201   | 258 | 888961   | 172 | 912240   | 430 | 087760    | 45 |
| 16 | 801356   | 257 | 888858   | 172 | 912498   | 430 | 087502    | 44 |
| 17 | 801511   | 257 | 888755   | 172 | 912756   | 430 | 087244    | 43 |
| 18 | 801665   | 257 | 888651   | 172 | 913014   | 429 | 086986    | 42 |
| 19 | 801819   | 257 | 888548   | 172 | 913271   | 429 | 086729    | 41 |
| 20 | 801973   | 257 | 888444   | 173 | 913529   | 429 | 086471    | 40 |
| 21 | 9.802128 | 257 | 9.888341 | 173 | 9.913787 | 429 | 10.086213 | 39 |
| 22 | 802282   | 256 | 888237   | 173 | 914044   | 429 | 085956    | 38 |
| 23 | 802436   | 256 | 888134   | 173 | 914302   | 429 | 085698    | 37 |
| 24 | 802589   | 256 | 888030   | 173 | 914560   | 429 | 085440    | 36 |
| 25 | 802743   | 256 | 887926   | 173 | 914817   | 429 | 085183    | 35 |
| 26 | 802897   | 256 | 887822   | 173 | 915075   | 429 | 084925    | 34 |
| 27 | 803050   | 256 | 887718   | 173 | 915332   | 429 | 084668    | 33 |
| 28 | 803204   | 256 | 887614   | 173 | 915590   | 429 | 084410    | 32 |
| 29 | 803357   | 255 | 887510   | 173 | 915847   | 429 | 084153    | 31 |
| 30 | 803511   | 255 | 887406   | 174 | 916104   | 429 | 083896    | 30 |
| 31 | 9.803664 | 255 | 9.887302 | 174 | 9.916362 | 429 | 10.083638 | 29 |
| 32 | 803817   | 255 | 887198   | 174 | 916619   | 429 | 083381    | 28 |
| 33 | 803970   | 255 | 887093   | 174 | 916877   | 429 | 083123    | 27 |
| 34 | 804123   | 255 | 886989   | 174 | 917134   | 429 | 082866    | 26 |
| 35 | 804276   | 254 | 886885   | 174 | 917391   | 429 | 082609    | 25 |
| 36 | 804428   | 254 | 886780   | 174 | 917648   | 429 | 082352    | 24 |
| 37 | 804581   | 254 | 886676   | 174 | 917905   | 429 | 082095    | 23 |
| 38 | 804734   | 254 | 886571   | 174 | 918163   | 428 | 081837    | 22 |
| 39 | 804886   | 254 | 886466   | 174 | 918420   | 428 | 081580    | 21 |
| 40 | 805039   | 254 | 886362   | 175 | 918677   | 428 | 081323    | 20 |
| 41 | 9.805191 | 254 | 9.886257 | 175 | 9.918934 | 428 | 10.081066 | 19 |
| 42 | 805343   | 253 | 886152   | 175 | 919191   | 428 | 080809    | 18 |
| 43 | 805495   | 253 | 886047   | 175 | 919448   | 428 | 080552    | 17 |
| 44 | 805647   | 253 | 885942   | 175 | 919705   | 428 | 080295    | 16 |
| 45 | 805799   | 253 | 885837   | 175 | 919962   | 428 | 080038    | 15 |
| 46 | 805951   | 253 | 885732   | 175 | 920219   | 428 | 079781    | 14 |
| 47 | 806103   | 253 | 885627   | 175 | 920476   | 428 | 079524    | 13 |
| 48 | 806254   | 253 | 885522   | 175 | 920733   | 428 | 079267    | 12 |
| 49 | 806406   | 252 | 885416   | 175 | 920990   | 428 | 079010    | 11 |
| 50 | 806557   | 252 | 885311   | 176 | 921247   | 428 | 078753    | 10 |
| 51 | 9.806709 | 252 | 9.885205 | 176 | 9.921503 | 428 | 10.078497 | 9  |
| 52 | 806860   | 252 | 885100   | 176 | 921760   | 428 | 078240    | 8  |
| 53 | 807011   | 252 | 884994   | 176 | 922017   | 428 | 077983    | 7  |
| 54 | 807163   | 252 | 884889   | 176 | 922274   | 428 | 077726    | 6  |
| 55 | 807314   | 252 | 884783   | 176 | 922530   | 428 | 077470    | 5  |
| 56 | 807465   | 251 | 884677   | 176 | 922787   | 428 | 077213    | 4  |
| 57 | 807615   | 251 | 884572   | 176 | 923044   | 428 | 076956    | 3  |
| 58 | 807766   | 251 | 884466   | 176 | 923300   | 428 | 076700    | 2  |
| 59 | 807917   | 251 | 884360   | 176 | 923557   | 427 | 076443    | 1  |
| 60 | 808067   | 251 | 884254   | 177 | 923813   | 427 | 076187    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.808067 | 251 | 9.884254 | 177 | 9.923813 | 427 | 10.076187 | 60 |
| 1  | 808218   | 251 | 884148   | 177 | 924070   | 427 | 075930    | 59 |
| 2  | 808368   | 251 | 884042   | 177 | 924327   | 427 | 075673    | 58 |
| 3  | 808519   | 250 | 883936   | 177 | 924583   | 427 | 075417    | 57 |
| 4  | 808669   | 250 | 883829   | 177 | 924840   | 427 | 075160    | 56 |
| 5  | 808819   | 250 | 883723   | 177 | 925096   | 427 | 074904    | 55 |
| 6  | 808969   | 250 | 883617   | 177 | 925352   | 427 | 074648    | 54 |
| 7  | 809119   | 250 | 883510   | 177 | 925609   | 427 | 074391    | 53 |
| 8  | 809269   | 250 | 883404   | 177 | 925865   | 427 | 074135    | 52 |
| 9  | 809419   | 249 | 883297   | 178 | 926122   | 427 | 073878    | 51 |
| 10 | 809569   | 249 | 883191   | 178 | 926378   | 427 | 073622    | 50 |
| 11 | 9.809718 | 249 | 9.883084 | 178 | 9.926634 | 427 | 10.073366 | 49 |
| 12 | 809868   | 249 | 882977   | 178 | 926890   | 427 | 073110    | 48 |
| 13 | 810017   | 249 | 882871   | 178 | 927147   | 427 | 072853    | 47 |
| 14 | 810167   | 249 | 882764   | 178 | 927403   | 427 | 072597    | 46 |
| 15 | 810316   | 248 | 882657   | 178 | 927659   | 427 | 072341    | 45 |
| 16 | 810465   | 248 | 882550   | 178 | 927915   | 427 | 072085    | 44 |
| 17 | 810614   | 248 | 882443   | 178 | 928171   | 427 | 071829    | 43 |
| 18 | 810763   | 248 | 882336   | 179 | 928427   | 427 | 071573    | 42 |
| 19 | 810912   | 248 | 882229   | 179 | 928683   | 427 | 071317    | 41 |
| 20 | 811061   | 248 | 882121   | 179 | 928940   | 427 | 071060    | 40 |
| 21 | 9.811210 | 248 | 9.882014 | 179 | 9.929196 | 427 | 10.070804 | 39 |
| 22 | 811358   | 247 | 881907   | 179 | 929452   | 427 | 070548    | 38 |
| 23 | 811507   | 247 | 881799   | 179 | 929708   | 427 | 070292    | 37 |
| 24 | 811655   | 247 | 881692   | 179 | 929964   | 426 | 070036    | 36 |
| 25 | 811804   | 247 | 881584   | 179 | 930220   | 426 | 069780    | 35 |
| 26 | 811952   | 247 | 881477   | 179 | 930475   | 426 | 069525    | 34 |
| 27 | 812100   | 247 | 881369   | 179 | 930731   | 426 | 069269    | 33 |
| 28 | 812248   | 247 | 881261   | 180 | 930987   | 426 | 069013    | 32 |
| 29 | 812396   | 246 | 881153   | 180 | 931243   | 426 | 068757    | 31 |
| 30 | 812544   | 246 | 881046   | 180 | 931499   | 426 | 068501    | 30 |
| 31 | 9.812692 | 246 | 9.880938 | 180 | 9.931755 | 426 | 10.068245 | 29 |
| 32 | 812840   | 246 | 880830   | 180 | 932010   | 426 | 067990    | 28 |
| 33 | 812988   | 246 | 880722   | 180 | 932266   | 426 | 067734    | 27 |
| 34 | 813135   | 246 | 880613   | 180 | 932522   | 426 | 067478    | 26 |
| 35 | 813283   | 246 | 880505   | 180 | 932778   | 426 | 067222    | 25 |
| 36 | 813430   | 245 | 880397   | 180 | 933033   | 426 | 066967    | 24 |
| 37 | 813578   | 245 | 880289   | 181 | 933289   | 426 | 066711    | 23 |
| 38 | 813725   | 245 | 880180   | 181 | 933545   | 426 | 066455    | 22 |
| 39 | 813872   | 245 | 880072   | 181 | 933800   | 426 | 066200    | 21 |
| 40 | 814019   | 245 | 879963   | 181 | 934056   | 426 | 065944    | 20 |
| 41 | 9.814166 | 245 | 9.879855 | 181 | 9.934311 | 426 | 10.065689 | 19 |
| 42 | 814313   | 245 | 879746   | 181 | 934567   | 426 | 065433    | 18 |
| 43 | 814460   | 244 | 879637   | 181 | 934823   | 426 | 065177    | 17 |
| 44 | 814607   | 244 | 879529   | 181 | 935078   | 426 | 064922    | 16 |
| 45 | 814753   | 244 | 879420   | 181 | 935333   | 426 | 064667    | 15 |
| 46 | 814900   | 244 | 879311   | 181 | 935589   | 426 | 064411    | 14 |
| 47 | 815046   | 244 | 879202   | 182 | 935844   | 426 | 064156    | 13 |
| 48 | 815193   | 244 | 879093   | 182 | 936100   | 426 | 063900    | 12 |
| 49 | 815339   | 244 | 878984   | 182 | 936355   | 426 | 063645    | 11 |
| 50 | 815485   | 243 | 878875   | 182 | 936610   | 426 | 063390    | 10 |
| 51 | 9.815631 | 243 | 9.878766 | 182 | 9.936866 | 425 | 10.063134 | 9  |
| 52 | 815778   | 243 | 878666   | 182 | 937121   | 425 | 062879    | 8  |
| 53 | 815924   | 243 | 878547   | 182 | 937376   | 425 | 062624    | 7  |
| 54 | 816069   | 243 | 878438   | 182 | 937632   | 425 | 062368    | 6  |
| 55 | 816215   | 243 | 878328   | 182 | 937887   | 425 | 062113    | 5  |
| 56 | 816361   | 243 | 878219   | 183 | 938142   | 425 | 061858    | 4  |
| 57 | 816507   | 242 | 878109   | 183 | 938398   | 425 | 061602    | 3  |
| 58 | 816652   | 242 | 877999   | 183 | 938653   | 425 | 061347    | 2  |
| 59 | 816798   | 242 | 877890   | 183 | 938908   | 425 | 061092    | 1  |
| 60 | 816943   | 242 | 877780   | 183 | 939163   | 425 | 060837    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang.   | M. |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.816943 | 242 | 9.877780 | 183 | 9.939163 | 425 | 10.060837 | 60 |
| 1  | 817088   | 242 | 877670   | 183 | 939418   | 425 | 060582    | 59 |
| 2  | 817233   | 242 | 877560   | 183 | 939673   | 425 | 060327    | 58 |
| 3  | 817379   | 242 | 877450   | 183 | 939928   | 425 | 060072    | 57 |
| 4  | 817524   | 241 | 877340   | 183 | 940183   | 425 | 059817    | 56 |
| 5  | 817668   | 241 | 877230   | 184 | 940438   | 425 | 059562    | 55 |
| 6  | 817813   | 241 | 877120   | 184 | 940694   | 425 | 059306    | 54 |
| 7  | 817958   | 241 | 877010   | 184 | 940949   | 425 | 059051    | 53 |
| 8  | 818103   | 241 | 876899   | 184 | 941204   | 425 | 058796    | 52 |
| 9  | 818247   | 241 | 876789   | 184 | 941458   | 425 | 058542    | 51 |
| 10 | 818392   | 241 | 876678   | 184 | 941714   | 425 | 058286    | 50 |
| 11 | 9.818536 | 240 | 9.876568 | 184 | 9.941968 | 425 | 10.058032 | 49 |
| 12 | 818681   | 240 | 876457   | 184 | 942223   | 425 | 057777    | 48 |
| 13 | 818825   | 240 | 876347   | 184 | 942478   | 425 | 057522    | 47 |
| 14 | 818969   | 240 | 876236   | 185 | 942733   | 425 | 057267    | 46 |
| 15 | 819113   | 240 | 876125   | 185 | 942988   | 425 | 057012    | 45 |
| 16 | 819257   | 240 | 876014   | 185 | 943243   | 425 | 056757    | 44 |
| 17 | 819401   | 240 | 875904   | 185 | 943498   | 425 | 056502    | 43 |
| 18 | 819545   | 239 | 875793   | 185 | 943752   | 425 | 056248    | 42 |
| 19 | 819689   | 239 | 875682   | 185 | 944007   | 425 | 055993    | 41 |
| 20 | 819832   | 239 | 875571   | 185 | 944262   | 425 | 055738    | 40 |
| 21 | 9.819976 | 239 | 9.875459 | 185 | 9.944517 | 425 | 10.055483 | 39 |
| 22 | 820120   | 239 | 875348   | 185 | 944771   | 424 | 055229    | 38 |
| 23 | 820263   | 239 | 875237   | 185 | 945026   | 424 | 054974    | 37 |
| 24 | 820406   | 239 | 875126   | 186 | 945281   | 424 | 054719    | 36 |
| 25 | 820550   | 238 | 875014   | 186 | 945535   | 424 | 054465    | 35 |
| 26 | 820693   | 238 | 874903   | 186 | 945790   | 424 | 054210    | 34 |
| 27 | 820836   | 238 | 874791   | 186 | 946045   | 424 | 053955    | 33 |
| 28 | 820979   | 238 | 874680   | 186 | 946299   | 424 | 053701    | 32 |
| 29 | 821122   | 238 | 874568   | 186 | 946554   | 424 | 053446    | 31 |
| 30 | 821265   | 238 | 874456   | 186 | 946808   | 424 | 053192    | 30 |
| 31 | 9.821407 | 238 | 9.874344 | 186 | 9.947063 | 424 | 10.052937 | 29 |
| 32 | 821550   | 238 | 874232   | 187 | 947318   | 424 | 052682    | 28 |
| 33 | 821693   | 237 | 874121   | 187 | 947572   | 424 | 052428    | 27 |
| 34 | 821835   | 237 | 874009   | 187 | 947826   | 424 | 052174    | 26 |
| 35 | 821977   | 237 | 873896   | 187 | 948081   | 424 | 051919    | 25 |
| 36 | 822120   | 237 | 873784   | 187 | 948336   | 424 | 051664    | 24 |
| 37 | 822262   | 237 | 873672   | 187 | 948590   | 424 | 051410    | 23 |
| 38 | 822404   | 237 | 873560   | 187 | 948844   | 424 | 051156    | 22 |
| 39 | 822546   | 237 | 873448   | 187 | 949099   | 424 | 050901    | 21 |
| 40 | 822688   | 236 | 873335   | 187 | 949353   | 424 | 050647    | 20 |
| 41 | 9.822830 | 236 | 9.873223 | 187 | 9.949607 | 424 | 10.050393 | 19 |
| 42 | 822972   | 236 | 873110   | 188 | 949862   | 424 | 050138    | 18 |
| 43 | 823114   | 236 | 872998   | 188 | 950116   | 424 | 049884    | 17 |
| 44 | 823255   | 236 | 872885   | 188 | 950370   | 424 | 049630    | 16 |
| 45 | 823397   | 236 | 872772   | 188 | 950625   | 424 | 049375    | 15 |
| 46 | 823539   | 236 | 872659   | 188 | 950879   | 424 | 049121    | 14 |
| 47 | 823680   | 235 | 872547   | 188 | 951133   | 424 | 048867    | 13 |
| 48 | 823821   | 235 | 872434   | 188 | 951388   | 424 | 048612    | 12 |
| 49 | 823963   | 235 | 872321   | 188 | 951642   | 424 | 048358    | 11 |
| 50 | 824104   | 235 | 872208   | 188 | 951896   | 424 | 048104    | 10 |
| 51 | 9.824245 | 235 | 9.872095 | 189 | 9.952150 | 424 | 10.047850 | 9  |
| 52 | 824386   | 235 | 871981   | 189 | 952405   | 424 | 047595    | 8  |
| 53 | 824527   | 235 | 871868   | 189 | 952659   | 424 | 047341    | 7  |
| 54 | 824668   | 234 | 871755   | 189 | 952913   | 424 | 047087    | 6  |
| 55 | 824808   | 234 | 871641   | 189 | 953167   | 423 | 046833    | 5  |
| 56 | 824949   | 234 | 871528   | 189 | 953421   | 423 | 046579    | 4  |
| 57 | 825090   | 234 | 871414   | 189 | 953675   | 423 | 046325    | 3  |
| 58 | 825230   | 234 | 871301   | 189 | 953929   | 423 | 046071    | 2  |
| 59 | 825371   | 234 | 871187   | 189 | 954183   | 423 | 045817    | 1  |
| 60 | 825511   | 234 | 871073   | 190 | 954437   | 423 | 045563    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D.  | Cosine   | D.  | Tang.    | D.  | Cotang    |    |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.825511 | 234 | 9.871073 | 190 | 9.954437 | 423 | 10.045563 | 60 |
| 1  | 825651   | 233 | 870960   | 190 | 954691   | 423 | 045309    | 59 |
| 2  | 825791   | 233 | 870846   | 190 | 954945   | 423 | 045055    | 58 |
| 3  | 825931   | 233 | 870732   | 190 | 955200   | 423 | 044800    | 57 |
| 4  | 826071   | 233 | 870618   | 190 | 955454   | 423 | 044546    | 56 |
| 5  | 826211   | 233 | 870504   | 190 | 955707   | 423 | 044293    | 55 |
| 6  | 826351   | 233 | 870390   | 190 | 955961   | 423 | 044039    | 54 |
| 7  | 826491   | 233 | 870276   | 190 | 956215   | 423 | 043785    | 53 |
| 8  | 826631   | 233 | 870161   | 190 | 956469   | 423 | 043531    | 52 |
| 9  | 826770   | 232 | 870047   | 191 | 956723   | 423 | 043277    | 51 |
| 10 | 826910   | 232 | 869933   | 191 | 956977   | 423 | 043023    | 50 |
| 11 | 9.827049 | 232 | 9.869818 | 191 | 9.957231 | 423 | 10.042769 | 49 |
| 12 | 827189   | 232 | 869704   | 191 | 957485   | 423 | 042515    | 48 |
| 13 | 827328   | 232 | 869589   | 191 | 957739   | 423 | 042261    | 47 |
| 14 | 827467   | 232 | 869474   | 191 | 957993   | 423 | 042007    | 46 |
| 15 | 827606   | 232 | 869360   | 191 | 958246   | 423 | 041754    | 45 |
| 16 | 827745   | 232 | 869245   | 191 | 958500   | 423 | 041500    | 44 |
| 17 | 827884   | 231 | 869130   | 191 | 958754   | 423 | 041246    | 43 |
| 18 | 828023   | 231 | 869015   | 192 | 959008   | 423 | 040992    | 42 |
| 19 | 828162   | 231 | 868900   | 192 | 959262   | 423 | 040738    | 41 |
| 20 | 828301   | 231 | 868785   | 192 | 959516   | 423 | 040484    | 40 |
| 21 | 9.828439 | 231 | 9.868670 | 192 | 9.959769 | 423 | 10.040231 | 39 |
| 22 | 828578   | 231 | 868555   | 192 | 9.960023 | 423 | 039977    | 38 |
| 23 | 828716   | 231 | 868440   | 192 | 960277   | 423 | 039723    | 37 |
| 24 | 828855   | 230 | 868324   | 192 | 960531   | 423 | 039469    | 36 |
| 25 | 828993   | 230 | 868209   | 192 | 960784   | 423 | 039216    | 35 |
| 26 | 829131   | 230 | 868093   | 192 | 961038   | 423 | 038962    | 34 |
| 27 | 829269   | 230 | 867978   | 193 | 961291   | 423 | 038709    | 33 |
| 28 | 829407   | 230 | 867862   | 193 | 961545   | 423 | 038455    | 32 |
| 29 | 829545   | 230 | 867747   | 193 | 961799   | 423 | 038201    | 31 |
| 30 | 829683   | 230 | 867631   | 193 | 962052   | 423 | 037948    | 30 |
| 31 | 9.829821 | 229 | 9.867515 | 193 | 9.962306 | 423 | 10.037694 | 29 |
| 32 | 829959   | 229 | 867399   | 193 | 962560   | 423 | 037440    | 28 |
| 33 | 830097   | 229 | 867283   | 193 | 962813   | 423 | 037187    | 27 |
| 34 | 830234   | 229 | 867167   | 193 | 963067   | 423 | 036933    | 26 |
| 35 | 830372   | 229 | 867051   | 193 | 963320   | 423 | 036680    | 25 |
| 36 | 830509   | 229 | 866935   | 194 | 963574   | 423 | 036426    | 24 |
| 37 | 830646   | 229 | 866819   | 194 | 963827   | 423 | 036173    | 23 |
| 38 | 830784   | 229 | 866703   | 194 | 964081   | 423 | 035919    | 22 |
| 39 | 830921   | 228 | 866586   | 194 | 964335   | 423 | 035665    | 21 |
| 40 | 831058   | 228 | 866470   | 194 | 964588   | 422 | 035412    | 20 |
| 41 | 9.831195 | 228 | 9.866353 | 194 | 9.964842 | 422 | 10.035158 | 19 |
| 42 | 831332   | 228 | 866237   | 194 | 965095   | 422 | 034905    | 18 |
| 43 | 831469   | 228 | 866120   | 194 | 965349   | 422 | 034651    | 17 |
| 44 | 831606   | 228 | 866004   | 195 | 965602   | 422 | 034398    | 16 |
| 45 | 831742   | 228 | 865887   | 195 | 965855   | 422 | 034145    | 15 |
| 46 | 831879   | 228 | 865770   | 195 | 966109   | 422 | 033891    | 14 |
| 47 | 832015   | 227 | 865653   | 195 | 966362   | 422 | 033638    | 13 |
| 48 | 832152   | 227 | 865536   | 195 | 966616   | 422 | 033384    | 12 |
| 49 | 832288   | 227 | 865419   | 195 | 966869   | 422 | 033131    | 11 |
| 50 | 832425   | 227 | 865302   | 195 | 967123   | 422 | 032877    | 10 |
| 51 | 9.832561 | 227 | 9.865185 | 195 | 9.967376 | 422 | 10.032624 | 9  |
| 52 | 832697   | 227 | 865068   | 195 | 967629   | 422 | 032371    | 8  |
| 53 | 832833   | 227 | 864950   | 195 | 967883   | 422 | 032117    | 7  |
| 54 | 832969   | 226 | 864833   | 196 | 968136   | 422 | 031864    | 6  |
| 55 | 833105   | 226 | 864716   | 196 | 968389   | 422 | 031611    | 5  |
| 56 | 833241   | 226 | 864598   | 196 | 968643   | 422 | 031357    | 4  |
| 57 | 833377   | 226 | 864481   | 196 | 968896   | 422 | 031104    | 3  |
| 58 | 833512   | 226 | 864363   | 196 | 969149   | 422 | 030851    | 2  |
| 59 | 833648   | 226 | 864245   | 196 | 969403   | 422 | 030597    | 1  |
| 60 | 833783   | 226 | 864127   | 196 | 969656   | 422 | 030344    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |

| M. | Sine     | D   | Cosine   | D.  | Tang.    | D.  | Cotang.   |    |
|----|----------|-----|----------|-----|----------|-----|-----------|----|
| 0  | 9.833783 | 236 | 9.864127 | 196 | 9.969656 | 422 | 10.030344 | 60 |
| 1  | 833919   | 225 | 864010   | 196 | 969909   | 422 | 030091    | 59 |
| 2  | 834054   | 225 | 863892   | 197 | 970162   | 422 | 029838    | 58 |
| 3  | 834189   | 225 | 863774   | 197 | 970416   | 422 | 029584    | 57 |
| 4  | 834325   | 225 | 863656   | 197 | 970669   | 422 | 029331    | 56 |
| 5  | 834460   | 225 | 863538   | 197 | 970922   | 422 | 029078    | 55 |
| 6  | 834595   | 225 | 863419   | 197 | 971175   | 422 | 028825    | 54 |
| 7  | 834730   | 225 | 863301   | 197 | 971429   | 422 | 028571    | 53 |
| 8  | 834865   | 225 | 863183   | 197 | 971682   | 422 | 028318    | 52 |
| 9  | 834999   | 224 | 863064   | 197 | 971935   | 422 | 028065    | 51 |
| 10 | 835134   | 224 | 862946   | 198 | 972188   | 422 | 027812    | 50 |
| 11 | 9.835269 | 224 | 9.862827 | 198 | 9.972441 | 422 | 10.027559 | 49 |
| 12 | 835403   | 224 | 862709   | 198 | 972694   | 422 | 027306    | 48 |
| 13 | 835538   | 224 | 862590   | 198 | 972948   | 422 | 027052    | 47 |
| 14 | 835672   | 224 | 862471   | 198 | 973201   | 422 | 026799    | 46 |
| 15 | 835807   | 224 | 862353   | 198 | 973454   | 422 | 026546    | 45 |
| 16 | 835941   | 224 | 862234   | 198 | 973707   | 422 | 026293    | 44 |
| 17 | 836075   | 223 | 862115   | 198 | 973960   | 422 | 026040    | 43 |
| 18 | 836209   | 223 | 861996   | 198 | 974213   | 422 | 025787    | 42 |
| 19 | 836343   | 223 | 861877   | 198 | 974466   | 422 | 025534    | 41 |
| 20 | 836477   | 223 | 861758   | 199 | 974719   | 422 | 025281    | 40 |
| 21 | 9.836611 | 223 | 9.861638 | 199 | 9.974973 | 422 | 10.025027 | 39 |
| 22 | 836745   | 223 | 861519   | 199 | 975226   | 422 | 024774    | 38 |
| 23 | 836878   | 223 | 861400   | 199 | 975479   | 422 | 024521    | 37 |
| 24 | 837012   | 222 | 861280   | 199 | 975732   | 422 | 024268    | 36 |
| 25 | 837146   | 222 | 861161   | 199 | 975985   | 422 | 024015    | 35 |
| 26 | 837279   | 222 | 861041   | 199 | 976238   | 422 | 023762    | 34 |
| 27 | 837412   | 222 | 860922   | 199 | 976491   | 422 | 023509    | 33 |
| 28 | 837546   | 222 | 860802   | 199 | 976744   | 422 | 023256    | 32 |
| 29 | 837679   | 222 | 860682   | 200 | 976997   | 422 | 023003    | 31 |
| 30 | 837812   | 222 | 860562   | 200 | 977250   | 422 | 022750    | 30 |
| 31 | 9.837945 | 222 | 9.860442 | 200 | 9.977503 | 422 | 10.022497 | 29 |
| 32 | 838078   | 221 | 860322   | 200 | 977756   | 422 | 022244    | 28 |
| 33 | 838211   | 221 | 860202   | 200 | 978009   | 422 | 021991    | 27 |
| 34 | 838344   | 221 | 860082   | 200 | 978262   | 422 | 021738    | 26 |
| 35 | 838477   | 221 | 859962   | 200 | 978515   | 422 | 021485    | 25 |
| 36 | 838610   | 221 | 859842   | 200 | 978768   | 422 | 021232    | 24 |
| 37 | 838742   | 221 | 859721   | 201 | 979021   | 422 | 020979    | 23 |
| 38 | 838875   | 221 | 859601   | 201 | 979274   | 422 | 020726    | 22 |
| 39 | 839007   | 221 | 859480   | 201 | 979527   | 422 | 020473    | 21 |
| 40 | 839140   | 220 | 859360   | 201 | 979780   | 422 | 020220    | 20 |
| 41 | 9.839272 | 220 | 9.859239 | 201 | 9.980033 | 422 | 10.019967 | 19 |
| 42 | 839404   | 220 | 859119   | 201 | 980286   | 422 | 019714    | 18 |
| 43 | 839536   | 220 | 858998   | 211 | 980538   | 422 | 019462    | 17 |
| 44 | 839668   | 220 | 858877   | 201 | 980791   | 421 | 019209    | 16 |
| 45 | 839800   | 220 | 858756   | 202 | 981044   | 421 | 018956    | 15 |
| 46 | 839932   | 220 | 858635   | 202 | 981297   | 421 | 018703    | 14 |
| 47 | 840064   | 219 | 858514   | 202 | 981550   | 421 | 018450    | 13 |
| 48 | 840196   | 219 | 858393   | 202 | 981803   | 421 | 018197    | 12 |
| 49 | 840328   | 219 | 858272   | 202 | 982056   | 421 | 017944    | 11 |
| 50 | 840459   | 219 | 858151   | 202 | 982309   | 421 | 017691    | 10 |
| 51 | 9.840591 | 219 | 9.858029 | 202 | 9.982562 | 421 | 10.017438 | 9  |
| 52 | 840722   | 219 | 857908   | 202 | 982814   | 421 | 017186    | 8  |
| 53 | 840854   | 219 | 857786   | 202 | 983067   | 421 | 016933    | 7  |
| 54 | 840985   | 219 | 857665   | 203 | 983320   | 421 | 016680    | 6  |
| 55 | 841116   | 218 | 857543   | 203 | 983573   | 421 | 016427    | 5  |
| 56 | 841247   | 218 | 857422   | 203 | 983826   | 421 | 016174    | 4  |
| 57 | 841378   | 218 | 857300   | 203 | 984079   | 421 | 015921    | 3  |
| 58 | 841509   | 218 | 857178   | 203 | 984331   | 421 | 015669    | 2  |
| 59 | 841640   | 218 | 857056   | 203 | 984584   | 421 | 015416    | 1  |
| 60 | 841771   | 218 | 856934   | 203 | 984837   | 421 | 015163    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.  |     | Tang.     | M. |



| M. | Sine     | D.  | Cosine   | D.  | Tang.     | D.  | Cotang.   |    |
|----|----------|-----|----------|-----|-----------|-----|-----------|----|
| 0  | 9.841771 | 218 | 9.856934 | 203 | 9.984837  | 421 | 10.015163 | 60 |
| 1  | 841902   | 218 | 856812   | 203 | 985090    | 421 | 014910    | 59 |
| 2  | 842033   | 218 | 856690   | 204 | 985343    | 421 | 014657    | 58 |
| 3  | 842163   | 217 | 856568   | 204 | 985596    | 421 | 014404    | 57 |
| 4  | 842294   | 217 | 856446   | 204 | 985848    | 421 | 014152    | 56 |
| 5  | 842424   | 217 | 856323   | 204 | 986101    | 421 | 013899    | 55 |
| 6  | 842555   | 217 | 856201   | 204 | 986354    | 421 | 013646    | 54 |
| 7  | 842685   | 217 | 856078   | 204 | 986607    | 421 | 013393    | 53 |
| 8  | 842815   | 217 | 855956   | 204 | 986860    | 421 | 013140    | 52 |
| 9  | 842946   | 217 | 855833   | 204 | 987112    | 421 | 012888    | 51 |
| 10 | 843076   | 217 | 855711   | 205 | 987365    | 421 | 012635    | 50 |
| 11 | 9.843206 | 216 | 9.855588 | 205 | 9.987618  | 421 | 10.012382 | 49 |
| 12 | 843336   | 216 | 855465   | 205 | 987871    | 421 | 012129    | 48 |
| 13 | 843466   | 216 | 855342   | 205 | 988123    | 421 | 011877    | 47 |
| 14 | 843595   | 216 | 855219   | 205 | 988376    | 421 | 011624    | 46 |
| 15 | 843725   | 216 | 855096   | 205 | 988629    | 421 | 011371    | 45 |
| 16 | 843855   | 216 | 854973   | 205 | 988882    | 421 | 011118    | 44 |
| 17 | 843984   | 216 | 854850   | 205 | 989134    | 421 | 010866    | 43 |
| 18 | 844114   | 215 | 854727   | 206 | 989387    | 421 | 010613    | 42 |
| 19 | 844243   | 215 | 854603   | 206 | 989640    | 421 | 010360    | 41 |
| 20 | 844372   | 215 | 854480   | 206 | 989893    | 421 | 010107    | 40 |
| 21 | 9.844502 | 215 | 9.854356 | 206 | 9.990145  | 421 | 10.009855 | 39 |
| 22 | 844631   | 215 | 854323   | 206 | 990398    | 421 | 009602    | 38 |
| 23 | 844760   | 215 | 854109   | 206 | 990651    | 421 | 009349    | 37 |
| 24 | 844889   | 215 | 853986   | 206 | 990903    | 421 | 009097    | 36 |
| 25 | 845018   | 215 | 853862   | 206 | 991156    | 421 | 008844    | 35 |
| 26 | 845147   | 215 | 853738   | 206 | 991409    | 421 | 008591    | 34 |
| 27 | 845276   | 214 | 853614   | 207 | 991662    | 421 | 008338    | 33 |
| 28 | 845405   | 214 | 853490   | 207 | 991914    | 421 | 008086    | 32 |
| 29 | 845533   | 214 | 853366   | 207 | 992167    | 421 | 007833    | 31 |
| 30 | 845662   | 214 | 853242   | 207 | 992420    | 421 | 007580    | 30 |
| 31 | 9.845790 | 214 | 9.853118 | 207 | 9.992672  | 421 | 10.007328 | 29 |
| 32 | 845919   | 214 | 852994   | 207 | 992925    | 421 | 007075    | 28 |
| 33 | 846047   | 214 | 852869   | 207 | 993178    | 421 | 006822    | 27 |
| 34 | 846175   | 214 | 852745   | 207 | 993430    | 421 | 006570    | 26 |
| 35 | 846304   | 214 | 852620   | 207 | 993683    | 421 | 006317    | 25 |
| 36 | 846432   | 213 | 852496   | 208 | 993936    | 421 | 006064    | 24 |
| 37 | 846560   | 213 | 852371   | 208 | 994189    | 421 | 005811    | 23 |
| 38 | 846688   | 213 | 852247   | 208 | 994441    | 421 | 005559    | 22 |
| 39 | 846816   | 213 | 852122   | 208 | 994694    | 421 | 005306    | 21 |
| 40 | 846944   | 213 | 851997   | 208 | 994947    | 421 | 005053    | 20 |
| 41 | 9.847071 | 213 | 9.851872 | 208 | 9.995199  | 421 | 10.004801 | 19 |
| 42 | 847199   | 213 | 851747   | 208 | 995452    | 421 | 004548    | 18 |
| 43 | 847327   | 213 | 851622   | 208 | 995705    | 421 | 004295    | 17 |
| 44 | 847454   | 212 | 851497   | 209 | 995957    | 421 | 004043    | 16 |
| 45 | 847582   | 212 | 851372   | 209 | 996210    | 421 | 003790    | 15 |
| 46 | 847709   | 212 | 851246   | 209 | 996463    | 421 | 003537    | 14 |
| 47 | 847836   | 212 | 851121   | 209 | 996715    | 421 | 003285    | 13 |
| 48 | 847964   | 212 | 850996   | 209 | 996968    | 421 | 003032    | 12 |
| 49 | 848091   | 212 | 850870   | 209 | 997221    | 421 | 002779    | 11 |
| 50 | 848218   | 212 | 850745   | 209 | 997473    | 421 | 002527    | 10 |
| 51 | 9.848345 | 212 | 9.850619 | 209 | 9.997726  | 421 | 10.002274 | 9  |
| 52 | 848472   | 211 | 850493   | 210 | 997979    | 421 | 002021    | 8  |
| 53 | 848599   | 211 | 850368   | 210 | 998231    | 421 | 001769    | 7  |
| 54 | 848726   | 211 | 850242   | 210 | 998484    | 421 | 001516    | 6  |
| 55 | 848852   | 211 | 850116   | 210 | 998737    | 421 | 001263    | 5  |
| 56 | 848979   | 211 | 849990   | 210 | 998989    | 421 | 001011    | 4  |
| 57 | 849106   | 211 | 849864   | 210 | 999242    | 421 | 000758    | 3  |
| 58 | 849232   | 211 | 849738   | 210 | 999495    | 421 | 000505    | 2  |
| 59 | 849359   | 211 | 849611   | 210 | 999748    | 421 | 000253    | 1  |
| 60 | 849485   | 211 | 849485   | 210 | 10.000000 | 421 | 000000    | 0  |
|    | Cosine   |     | Sine     |     | Cotang.   |     | Tang.     | M. |



A  
T A B L E  
OF  
NATURAL SINES AND TANGENTS;  
TO  
EVERY TEN MINUTES OF A DEGREE.

---

If the given angle is less than  $45^\circ$ , look for the title of the column, at the *top* of the page; and for the degrees and minutes, on the *left*. But if the angle is between  $45^\circ$  and  $90^\circ$ , look for the title of the column, at the *bottom*; and for the degrees and minutes on the *right*.

The *Secants and Cosecants*, which are not inserted in this table, may be easily supplied. If 1 be divided by the cosine of an arc, the quotient will be the secant of that arc. (Art. 228.) And if 1 be divided by the sine, the quotient will be the cosecant.

| M  | 0 Deg.    |            | 1 Deg.    |            | 2 Deg.    |            | 3 Deg.    |            | 4 Deg.    |            | M  |
|----|-----------|------------|-----------|------------|-----------|------------|-----------|------------|-----------|------------|----|
|    | Nat. Sine | N. Co-Sine | Nat. Sine | N. Co-Sine | Nat. Sine | N. Co-Sine | Nat. Sine | N. Co-Sine | Nat. Sine | N. Co-Sine |    |
| 0  | 00000     | Unit.      | 01745     | 99985      | 03490     | 99939      | 05234     | 99803      | 06970     | 99756      | 60 |
| 1  | 00029     | 00000      | 01774     | 99984      | 03519     | 99938      | 05263     | 99861      | 07005     | 99754      | 59 |
| 2  | 00053     | 00000      | 01803     | 99984      | 03548     | 99937      | 05292     | 99860      | 07034     | 99752      | 58 |
| 3  | 00087     | 00000      | 01832     | 99983      | 03577     | 99936      | 05321     | 99858      | 07063     | 99750      | 57 |
| 4  | 00116     | 00000      | 01862     | 99983      | 03606     | 99935      | 05350     | 99857      | 07092     | 99748      | 56 |
| 5  | 00145     | 00000      | 01891     | 99982      | 03635     | 99934      | 05379     | 99855      | 07121     | 99746      | 55 |
| 6  | 00175     | 00000      | 01920     | 99982      | 03664     | 99933      | 05408     | 99854      | 07150     | 99744      | 54 |
| 7  | 00204     | 00000      | 01949     | 99981      | 03693     | 99932      | 05437     | 99852      | 07179     | 99742      | 53 |
| 8  | 00233     | 00000      | 01978     | 99980      | 03723     | 99931      | 05466     | 99851      | 07208     | 99740      | 52 |
| 9  | 00262     | 00000      | 02007     | 99980      | 03752     | 99930      | 05495     | 99849      | 07237     | 99738      | 51 |
| 10 | 00291     | 00000      | 02036     | 99979      | 03781     | 99929      | 05524     | 99847      | 07266     | 99736      | 50 |
| 11 | 00320     | 99999      | 02065     | 99979      | 03810     | 99927      | 05553     | 99846      | 07295     | 99734      | 49 |
| 12 | 00349     | 99999      | 02094     | 99978      | 03839     | 99926      | 05582     | 99844      | 07324     | 99731      | 48 |
| 13 | 00378     | 99999      | 02123     | 99977      | 03868     | 99925      | 05611     | 99842      | 07353     | 99729      | 47 |
| 14 | 00407     | 99999      | 02152     | 99977      | 03897     | 99924      | 05640     | 99841      | 07382     | 99727      | 46 |
| 15 | 00436     | 99999      | 02181     | 99976      | 03926     | 99923      | 05669     | 99839      | 07411     | 99725      | 45 |
| 16 | 00465     | 99999      | 02211     | 99976      | 03955     | 99922      | 05698     | 99838      | 07440     | 99723      | 44 |
| 17 | 00495     | 99999      | 02240     | 99975      | 03984     | 99921      | 05727     | 99836      | 07469     | 99721      | 43 |
| 18 | 00524     | 99999      | 02269     | 99974      | 04013     | 99919      | 05756     | 99834      | 07498     | 99719      | 42 |
| 19 | 00553     | 99998      | 02298     | 99974      | 04042     | 99918      | 05785     | 99833      | 07527     | 99716      | 41 |
| 20 | 00582     | 99998      | 02327     | 99973      | 04071     | 99917      | 05814     | 99831      | 07556     | 99714      | 40 |
| 21 | 00611     | 99998      | 02356     | 99972      | 04100     | 99916      | 05844     | 99829      | 07585     | 99712      | 39 |
| 22 | 00640     | 99998      | 02385     | 99972      | 04129     | 99915      | 05873     | 99827      | 07614     | 99710      | 38 |
| 23 | 00669     | 99998      | 02414     | 99971      | 04159     | 99913      | 05902     | 99826      | 07643     | 99708      | 37 |
| 24 | 00698     | 99998      | 02443     | 99970      | 04188     | 99912      | 05931     | 99824      | 07672     | 99705      | 36 |
| 25 | 00727     | 99997      | 02472     | 99969      | 04217     | 99911      | 05960     | 99822      | 07701     | 99703      | 35 |
| 26 | 00756     | 99997      | 02501     | 99969      | 04246     | 99910      | 05989     | 99821      | 07730     | 99701      | 34 |
| 27 | 00785     | 99997      | 02530     | 99968      | 04275     | 99909      | 06018     | 99819      | 07759     | 99699      | 33 |
| 28 | 00814     | 99997      | 02560     | 99967      | 04304     | 99907      | 06047     | 99817      | 07788     | 99696      | 32 |
| 29 | 00844     | 99996      | 02589     | 99966      | 04333     | 99906      | 06076     | 99815      | 07817     | 99694      | 31 |
| 30 | 00873     | 99996      | 02618     | 99966      | 04362     | 99905      | 06105     | 99813      | 07846     | 99692      | 30 |
| 31 | 00902     | 99996      | 02647     | 99965      | 04391     | 99904      | 06134     | 99812      | 07875     | 99689      | 29 |
| 32 | 00931     | 99996      | 02676     | 99964      | 04420     | 99902      | 06163     | 99810      | 07904     | 99687      | 28 |
| 33 | 00960     | 99995      | 02705     | 99963      | 04449     | 99901      | 06192     | 99808      | 07933     | 99685      | 27 |
| 34 | 00989     | 99995      | 02734     | 99963      | 04478     | 99900      | 06221     | 99806      | 07962     | 99683      | 26 |
| 35 | 01018     | 99995      | 02763     | 99962      | 04507     | 99898      | 06250     | 99804      | 07991     | 99680      | 25 |
| 36 | 01047     | 99995      | 02792     | 99961      | 04536     | 99897      | 06279     | 99803      | 08020     | 99678      | 24 |
| 37 | 01076     | 99994      | 02821     | 99960      | 04565     | 99896      | 06308     | 99801      | 08049     | 99676      | 23 |
| 38 | 01105     | 99994      | 02850     | 99959      | 04594     | 99894      | 06337     | 99799      | 08078     | 99673      | 22 |
| 39 | 01134     | 99994      | 02879     | 99959      | 04623     | 99893      | 06366     | 99797      | 08107     | 99671      | 21 |
| 40 | 01164     | 99993      | 02908     | 99958      | 04652     | 99892      | 06395     | 99795      | 08136     | 99668      | 20 |
| 41 | 01193     | 99993      | 02938     | 99957      | 04682     | 99890      | 06424     | 99793      | 08165     | 99666      | 19 |
| 42 | 01222     | 99993      | 02967     | 99956      | 04711     | 99889      | 06453     | 99792      | 08194     | 99664      | 18 |
| 43 | 01251     | 99992      | 02996     | 99955      | 04740     | 99888      | 06482     | 99790      | 08223     | 99661      | 17 |
| 44 | 01280     | 99992      | 03025     | 99954      | 04769     | 99886      | 06511     | 99788      | 08252     | 99659      | 16 |
| 45 | 01309     | 99991      | 03054     | 99953      | 04798     | 99885      | 06540     | 99786      | 08281     | 99657      | 15 |
| 46 | 01338     | 99991      | 03083     | 99952      | 04827     | 99883      | 06569     | 99784      | 08310     | 99654      | 14 |
| 47 | 01367     | 99991      | 03112     | 99952      | 04856     | 99882      | 06598     | 99782      | 08339     | 99652      | 13 |
| 48 | 01396     | 99990      | 03141     | 99951      | 04885     | 99881      | 06627     | 99780      | 08368     | 99649      | 12 |
| 49 | 01425     | 99990      | 03170     | 99950      | 04914     | 99879      | 06656     | 99778      | 08397     | 99647      | 11 |
| 50 | 01454     | 99989      | 03199     | 99949      | 04943     | 99878      | 06685     | 99776      | 08426     | 99644      | 10 |
| 51 | 01483     | 99989      | 03228     | 99948      | 04972     | 99876      | 06714     | 99774      | 08455     | 99642      | 9  |
| 52 | 01513     | 99989      | 03257     | 99947      | 05001     | 99875      | 06743     | 99772      | 08484     | 99639      | 8  |
| 53 | 1542      | 99988      | 03286     | 99946      | 05030     | 99873      | 06773     | 99770      | 08513     | 99637      | 7  |
| 54 | 01571     | 99988      | 03316     | 99945      | 05059     | 99872      | 06802     | 99768      | 08542     | 99635      | 6  |
| 55 | 01600     | 99987      | 03345     | 99944      | 05088     | 99870      | 06831     | 99766      | 08571     | 99632      | 5  |
| 56 | 01629     | 99987      | 03374     | 99943      | 05117     | 99869      | 06860     | 99764      | 08600     | 99630      | 4  |
| 57 | 01658     | 99986      | 03403     | 99942      | 05146     | 99867      | 06889     | 99762      | 08629     | 99627      | 3  |
| 58 | 01687     | 99986      | 03432     | 99941      | 05175     | 99866      | 06918     | 99760      | 08658     | 99625      | 2  |
| 59 | 01716     | 99985      | 03461     | 99940      | 05205     | 99864      | 06947     | 99758      | 08687     | 99622      | 1  |
| M  | N. CS.    | N. S.      | N. CS.    | N. S.      | N. CS.    | N. S.      | N. CS.    | N. S.      | N. CS.    | N. S.      | M  |
|    | 89 Deg.   |            | 88 Deg.   |            | 87 Deg.   |            | 86 Deg.   |            | 85 Deg.   |            |    |

| M  | 5 Deg.  |        | 6 Deg.  |        | 7 Deg.  |        | 8 Deg.  |        | 9 Deg.  |        | M  |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----|
|    | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. |    |
| 0  | 08716   | 99619  | 10453   | 99452  | 12187   | 99255  | 13917   | 99027  | 15643   | 98709  | 60 |
| 1  | 08745   | 99617  | 10482   | 99449  | 12216   | 99251  | 13946   | 99023  | 15672   | 98764  | 59 |
| 2  | 08774   | 99614  | 10511   | 99446  | 12245   | 99248  | 13975   | 99019  | 15701   | 98760  | 58 |
| 3  | 08803   | 99612  | 10540   | 99443  | 12274   | 99244  | 14004   | 99015  | 15730   | 98755  | 57 |
| 4  | 08831   | 99609  | 10569   | 99440  | 12302   | 99240  | 14033   | 99011  | 15758   | 98751  | 56 |
| 5  | 08860   | 99607  | 10597   | 99437  | 12331   | 99237  | 14061   | 99006  | 15787   | 98746  | 55 |
| 6  | 08889   | 99604  | 10626   | 99434  | 12360   | 99233  | 14090   | 99002  | 15816   | 98741  | 54 |
| 7  | 08918   | 99602  | 10655   | 99431  | 12389   | 99230  | 14119   | 98998  | 15845   | 98737  | 53 |
| 8  | 08947   | 99599  | 10684   | 99428  | 12418   | 99226  | 14148   | 98994  | 15873   | 98732  | 52 |
| 9  | 08976   | 99595  | 10713   | 99424  | 12447   | 99222  | 14177   | 98990  | 15902   | 98728  | 51 |
| 10 | 09005   | 99594  | 10742   | 99421  | 12476   | 99219  | 14205   | 98986  | 15931   | 98723  | 50 |
| 11 | 09034   | 99591  | 10771   | 99418  | 12504   | 99215  | 14234   | 98982  | 15959   | 98718  | 49 |
| 12 | 09063   | 99588  | 10800   | 99415  | 12533   | 99211  | 14263   | 98978  | 15988   | 98714  | 48 |
| 13 | 09092   | 99586  | 10829   | 99412  | 12562   | 99208  | 14292   | 98973  | 16017   | 98709  | 47 |
| 14 | 09121   | 99583  | 10858   | 99409  | 12591   | 99204  | 14320   | 98969  | 16046   | 98704  | 46 |
| 15 | 09150   | 99580  | 10887   | 99406  | 12620   | 99200  | 14349   | 98965  | 16074   | 98700  | 45 |
| 16 | 09179   | 99578  | 10916   | 99402  | 12649   | 99197  | 14378   | 98961  | 16103   | 98695  | 44 |
| 17 | 09208   | 99575  | 10945   | 99399  | 12678   | 99193  | 14407   | 98957  | 16132   | 98690  | 43 |
| 18 | 09237   | 99572  | 10973   | 99396  | 12706   | 99189  | 14436   | 98953  | 16160   | 98686  | 42 |
| 19 | 09266   | 99570  | 11002   | 99393  | 12735   | 99186  | 14464   | 98948  | 16189   | 98681  | 41 |
| 20 | 09295   | 99567  | 11031   | 99390  | 12764   | 99182  | 14493   | 98944  | 16218   | 98676  | 40 |
| 21 | 09324   | 99564  | 11060   | 99386  | 12793   | 99178  | 14522   | 98940  | 16246   | 98671  | 39 |
| 22 | 09353   | 99562  | 11089   | 99383  | 12822   | 99175  | 14551   | 98936  | 16275   | 98667  | 38 |
| 23 | 09382   | 99559  | 11118   | 99380  | 12851   | 99171  | 14580   | 98931  | 16304   | 98662  | 37 |
| 24 | 09411   | 99556  | 11147   | 99377  | 12880   | 99167  | 14608   | 98927  | 16333   | 98657  | 36 |
| 25 | 09440   | 99553  | 11176   | 99374  | 12908   | 99162  | 14637   | 98923  | 16361   | 98652  | 35 |
| 26 | 09469   | 99551  | 11205   | 99370  | 12937   | 99158  | 14666   | 98919  | 16390   | 98648  | 34 |
| 27 | 09498   | 99548  | 11234   | 99367  | 12966   | 99156  | 14695   | 98914  | 16419   | 98643  | 33 |
| 28 | 09527   | 99545  | 11263   | 99364  | 12995   | 99152  | 14723   | 98910  | 16447   | 98638  | 32 |
| 29 | 09556   | 99542  | 11291   | 99360  | 13024   | 99148  | 14752   | 98906  | 16476   | 98633  | 31 |
| 30 | 09585   | 99540  | 11320   | 99357  | 13053   | 99144  | 14781   | 98902  | 16505   | 98629  | 30 |
| 31 | 09614   | 99537  | 11349   | 99354  | 13081   | 99141  | 14810   | 98897  | 16533   | 98624  | 29 |
| 32 | 09642   | 99534  | 11378   | 99351  | 13110   | 99137  | 14838   | 98893  | 16562   | 98619  | 28 |
| 33 | 09671   | 99531  | 11407   | 99347  | 13139   | 99133  | 14867   | 98889  | 16591   | 98614  | 27 |
| 34 | 09700   | 99528  | 11436   | 99344  | 13168   | 99129  | 14896   | 98884  | 16620   | 98609  | 26 |
| 35 | 09729   | 99526  | 11465   | 99341  | 13197   | 99125  | 14925   | 98880  | 16648   | 98604  | 25 |
| 36 | 09758   | 99523  | 11494   | 99337  | 13226   | 99122  | 14954   | 98876  | 16677   | 98600  | 24 |
| 37 | 09787   | 99520  | 11523   | 99334  | 13254   | 99118  | 14982   | 98871  | 16706   | 98595  | 23 |
| 38 | 09816   | 99517  | 11552   | 99331  | 13283   | 99114  | 15011   | 98867  | 16734   | 98590  | 22 |
| 39 | 09845   | 99514  | 11580   | 99327  | 13312   | 99110  | 15040   | 98863  | 16763   | 98585  | 21 |
| 40 | 09874   | 99511  | 11609   | 99324  | 13341   | 99106  | 15069   | 98858  | 16792   | 98580  | 20 |
| 41 | 09903   | 99508  | 11638   | 99320  | 13370   | 99102  | 15097   | 98854  | 16820   | 98575  | 19 |
| 42 | 09932   | 99506  | 11667   | 99317  | 13399   | 99098  | 15126   | 98849  | 16849   | 98570  | 18 |
| 43 | 09961   | 99503  | 11696   | 99314  | 13427   | 99094  | 15155   | 98845  | 16878   | 98565  | 17 |
| 44 | 09990   | 99500  | 11725   | 99310  | 13456   | 99091  | 15184   | 98841  | 16906   | 98561  | 16 |
| 45 | 10019   | 99497  | 11754   | 99307  | 13485   | 99087  | 15212   | 98836  | 16935   | 98556  | 15 |
| 46 | 10048   | 99494  | 11783   | 99303  | 13514   | 99083  | 15241   | 98832  | 16964   | 98551  | 14 |
| 47 | 10077   | 99491  | 11812   | 99300  | 13543   | 99079  | 15270   | 98827  | 16992   | 98546  | 13 |
| 48 | 10106   | 99488  | 11840   | 99297  | 13572   | 99075  | 15299   | 98823  | 17021   | 98541  | 12 |
| 49 | 10135   | 99485  | 11869   | 99293  | 13600   | 99071  | 15327   | 98818  | 17050   | 98536  | 11 |
| 50 | 10164   | 99482  | 11898   | 99290  | 13629   | 99067  | 15356   | 98814  | 17078   | 98531  | 10 |
| 51 | 10192   | 99479  | 11927   | 99286  | 13658   | 99063  | 15385   | 98809  | 17107   | 98526  | 9  |
| 52 | 10221   | 99476  | 11956   | 99283  | 13687   | 99059  | 15414   | 98805  | 17136   | 98521  | 8  |
| 53 | 10250   | 99473  | 11985   | 99279  | 13716   | 99055  | 15442   | 98800  | 17164   | 98516  | 7  |
| 54 | 10279   | 99470  | 12014   | 99276  | 13744   | 99051  | 15471   | 98796  | 17193   | 98511  | 6  |
| 55 | 10308   | 99467  | 12043   | 99272  | 13773   | 99047  | 15500   | 98791  | 17222   | 98506  | 5  |
| 56 | 10337   | 99464  | 12071   | 99269  | 13802   | 99043  | 15529   | 98787  | 17250   | 98501  | 4  |
| 57 | 10366   | 99461  | 12100   | 99265  | 13831   | 99039  | 15557   | 98782  | 17279   | 98496  | 3  |
| 58 | 10395   | 99458  | 12129   | 99262  | 13860   | 99035  | 15586   | 98778  | 17308   | 98491  | 2  |
| 59 | 10424   | 99455  | 12158   | 99258  | 13889   | 99031  | 15615   | 98773  | 17336   | 98486  | 1  |
| M  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | M  |
|    | 84 Deg. |        | 83 Deg. |        | 82 Deg. |        | 81 Deg. |        | 80 Deg. |        |    |



| M  | 10 Deg. |        | 11 Deg. |        | 12 Deg. |        | 13 Deg. |        | 14 Deg. |        | M  |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----|
|    | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. |    |
| 0  | 17365   | 98481  | 19081   | 98163  | 20791   | 97815  | 22495   | 97437  | 24192   | 97030  | 60 |
| 1  | 17393   | 98476  | 19109   | 98157  | 20820   | 97809  | 22523   | 97430  | 24220   | 97023  | 59 |
| 2  | 17422   | 98471  | 19138   | 98152  | 20848   | 97803  | 22552   | 97424  | 24249   | 97015  | 58 |
| 3  | 17451   | 98466  | 19167   | 98146  | 20877   | 97797  | 22580   | 97417  | 24277   | 97008  | 57 |
| 4  | 17479   | 98461  | 19195   | 98140  | 20905   | 97791  | 22608   | 97411  | 24305   | 97001  | 56 |
| 5  | 17508   | 98455  | 19224   | 98135  | 20933   | 97784  | 22637   | 97404  | 24333   | 96994  | 55 |
| 6  | 17537   | 98450  | 19252   | 98129  | 20962   | 97778  | 22665   | 97398  | 24362   | 96987  | 54 |
| 7  | 17565   | 98445  | 19281   | 98124  | 20990   | 97772  | 22693   | 97391  | 24390   | 96980  | 53 |
| 8  | 17594   | 98440  | 19309   | 98118  | 21019   | 97766  | 22722   | 97384  | 24418   | 96973  | 52 |
| 9  | 17623   | 98435  | 19338   | 98112  | 21047   | 97760  | 22750   | 97378  | 24446   | 96966  | 51 |
| 10 | 17651   | 98430  | 19366   | 98107  | 21076   | 97754  | 22778   | 97371  | 24474   | 96959  | 50 |
| 11 | 17680   | 98425  | 19395   | 98101  | 21104   | 97748  | 22807   | 97365  | 24503   | 96952  | 49 |
| 12 | 17708   | 98420  | 19423   | 98096  | 21132   | 97742  | 22835   | 97358  | 24531   | 96945  | 48 |
| 13 | 17737   | 98414  | 19452   | 98090  | 21161   | 97735  | 22863   | 97351  | 24559   | 96937  | 47 |
| 14 | 17766   | 98409  | 19481   | 98084  | 21189   | 97729  | 22892   | 97345  | 24587   | 96930  | 46 |
| 15 | 17794   | 98404  | 19509   | 98079  | 21218   | 97723  | 22920   | 97338  | 24615   | 96923  | 45 |
| 16 | 17823   | 98399  | 19538   | 98073  | 21246   | 97717  | 22948   | 97331  | 24644   | 96916  | 44 |
| 17 | 17852   | 98394  | 19566   | 98067  | 21275   | 97711  | 22977   | 97325  | 24672   | 96909  | 43 |
| 18 | 17880   | 98389  | 19595   | 98061  | 21303   | 97705  | 23005   | 97318  | 24700   | 96902  | 42 |
| 19 | 17909   | 98383  | 19623   | 98055  | 21331   | 97699  | 23033   | 97311  | 24728   | 96894  | 41 |
| 20 | 17937   | 98378  | 19652   | 98050  | 21360   | 97692  | 23062   | 97304  | 24756   | 96887  | 40 |
| 21 | 17966   | 98373  | 19680   | 98044  | 21388   | 97686  | 23090   | 97298  | 24784   | 96880  | 39 |
| 22 | 17995   | 98368  | 19709   | 98039  | 21417   | 97680  | 23118   | 97291  | 24813   | 96873  | 38 |
| 23 | 18023   | 98362  | 19737   | 98033  | 21445   | 97673  | 23146   | 97284  | 24841   | 96866  | 37 |
| 24 | 18052   | 98357  | 19766   | 98027  | 21474   | 97667  | 23175   | 97278  | 24869   | 96858  | 36 |
| 25 | 18081   | 98352  | 19794   | 98021  | 21502   | 97661  | 23203   | 97271  | 24897   | 96851  | 35 |
| 26 | 18109   | 98347  | 19823   | 98016  | 21530   | 97655  | 23231   | 97264  | 24925   | 96844  | 34 |
| 27 | 18138   | 98341  | 19851   | 98010  | 21559   | 97648  | 23260   | 97257  | 24953   | 96837  | 33 |
| 28 | 18166   | 98336  | 19880   | 98004  | 21587   | 97642  | 23288   | 97251  | 24982   | 96829  | 32 |
| 29 | 18195   | 98331  | 19908   | 97998  | 21616   | 97636  | 23316   | 97244  | 25010   | 96822  | 31 |
| 30 | 18224   | 98325  | 19937   | 97992  | 21644   | 97630  | 23345   | 97237  | 25038   | 96815  | 30 |
| 31 | 18252   | 98320  | 19965   | 97987  | 21672   | 97623  | 23373   | 97230  | 25066   | 96807  | 29 |
| 32 | 18281   | 98315  | 19994   | 97981  | 21701   | 97617  | 23401   | 97223  | 25094   | 96800  | 28 |
| 33 | 18309   | 98310  | 20022   | 97975  | 21729   | 97611  | 23429   | 97217  | 25122   | 96793  | 27 |
| 34 | 18338   | 98304  | 20051   | 97969  | 21758   | 97604  | 23458   | 97210  | 25151   | 96786  | 26 |
| 35 | 18367   | 98299  | 20079   | 97963  | 21786   | 97598  | 23486   | 97203  | 25179   | 96778  | 25 |
| 36 | 18395   | 98294  | 20108   | 97958  | 21814   | 97592  | 23514   | 97196  | 25207   | 96771  | 24 |
| 37 | 18424   | 98288  | 20136   | 97952  | 21843   | 97585  | 23542   | 97189  | 25235   | 96764  | 23 |
| 38 | 18452   | 98283  | 20165   | 97946  | 21871   | 97579  | 23571   | 97182  | 25263   | 96756  | 22 |
| 39 | 18481   | 98277  | 20193   | 97940  | 21899   | 97573  | 23599   | 97176  | 25291   | 96749  | 21 |
| 40 | 18509   | 98272  | 20222   | 97934  | 21928   | 97566  | 23627   | 97169  | 25320   | 96742  | 20 |
| 41 | 18538   | 98267  | 20250   | 97928  | 21956   | 97560  | 23656   | 97162  | 25348   | 96734  | 19 |
| 42 | 18567   | 98261  | 20279   | 97922  | 21985   | 97553  | 23684   | 97155  | 25376   | 96727  | 18 |
| 43 | 18595   | 98256  | 20307   | 97916  | 22013   | 97547  | 23712   | 97148  | 25404   | 96719  | 17 |
| 44 | 18624   | 98250  | 20336   | 97910  | 22041   | 97541  | 23740   | 97141  | 25432   | 96712  | 16 |
| 45 | 18652   | 98245  | 20364   | 97905  | 22070   | 97534  | 23769   | 97134  | 25460   | 96705  | 15 |
| 46 | 18681   | 98240  | 20393   | 97899  | 22098   | 97528  | 23797   | 97127  | 25488   | 96697  | 14 |
| 47 | 18710   | 98234  | 20421   | 97893  | 22126   | 97521  | 23825   | 97120  | 25516   | 96690  | 13 |
| 48 | 18738   | 98229  | 20450   | 97887  | 22155   | 97515  | 23853   | 97113  | 25545   | 96682  | 12 |
| 49 | 18767   | 98223  | 20478   | 97881  | 22183   | 97508  | 23882   | 97106  | 25573   | 96675  | 11 |
| 50 | 18795   | 98218  | 20507   | 97875  | 22212   | 97502  | 23910   | 97100  | 25601   | 96667  | 10 |
| 51 | 18824   | 98212  | 20535   | 97869  | 22240   | 97496  | 23938   | 97093  | 25629   | 96660  | 9  |
| 52 | 18852   | 98207  | 20563   | 97863  | 22268   | 97489  | 23966   | 97086  | 25657   | 96653  | 8  |
| 53 | 18881   | 98201  | 20592   | 97857  | 22297   | 97483  | 23995   | 97079  | 25685   | 96645  | 7  |
| 54 | 18910   | 98196  | 20620   | 97851  | 22325   | 97476  | 24023   | 97072  | 25713   | 96638  | 6  |
| 55 | 18938   | 98190  | 20649   | 97845  | 22353   | 97470  | 24051   | 97065  | 25741   | 96630  | 5  |
| 56 | 18967   | 98185  | 20677   | 97839  | 22382   | 97463  | 24079   | 97058  | 25769   | 96623  | 4  |
| 57 | 18995   | 98179  | 20706   | 97833  | 22410   | 97457  | 24108   | 97051  | 25798   | 96615  | 3  |
| 58 | 19024   | 98174  | 20734   | 97827  | 22438   | 97450  | 24136   | 97044  | 25826   | 96608  | 2  |
| 59 | 19052   | 98168  | 20763   | 97821  | 22467   | 97444  | 24164   | 97037  | 25854   | 96600  | 1  |
| M  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | M  |
|    | 79 Deg. |        | 78 Deg. |        | 77 Deg. |        | 76 Deg. |        | 75 Deg. |        |    |



| M  | 15 Deg. |        | 16 Deg. |        | 17 Deg. |        | 18 Deg. |        | 19 Deg. |        | M  |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----|
|    | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. |    |
| 0  | 25882   | 96593  | 27564   | 96126  | 29237   | 95630  | 30932   | 95106  | 32557   | 94552  | 60 |
| 1  | 25910   | 96585  | 27592   | 96118  | 29265   | 95622  | 30929   | 95097  | 32584   | 94542  | 59 |
| 2  | 25938   | 96578  | 27620   | 96110  | 29293   | 95613  | 30957   | 95088  | 32612   | 94533  | 58 |
| 3  | 25966   | 96570  | 27648   | 96102  | 29321   | 95605  | 30985   | 95079  | 32639   | 94523  | 57 |
| 4  | 25994   | 96562  | 27676   | 96094  | 29348   | 95596  | 31012   | 95070  | 32667   | 94514  | 56 |
| 5  | 26022   | 96555  | 27704   | 96086  | 29376   | 95588  | 31040   | 95061  | 32694   | 94504  | 55 |
| 6  | 26050   | 96547  | 27731   | 96078  | 29404   | 95579  | 31068   | 95052  | 32722   | 94495  | 54 |
| 7  | 26079   | 96540  | 27759   | 96070  | 29432   | 95571  | 31095   | 95043  | 32749   | 94485  | 53 |
| 8  | 26107   | 96532  | 27787   | 96062  | 29460   | 95562  | 31123   | 95033  | 32777   | 94476  | 52 |
| 9  | 26135   | 96524  | 27815   | 96054  | 29487   | 95554  | 31151   | 95024  | 32804   | 94466  | 51 |
| 10 | 26163   | 96517  | 27843   | 96046  | 29515   | 95545  | 31178   | 95015  | 32832   | 94457  | 50 |
| 11 | 26191   | 96509  | 27871   | 96037  | 29543   | 95536  | 31206   | 95006  | 32859   | 94447  | 49 |
| 12 | 26219   | 96502  | 27899   | 96029  | 29571   | 95528  | 31233   | 94997  | 32887   | 94438  | 48 |
| 13 | 26247   | 96494  | 27927   | 96021  | 29599   | 95519  | 31261   | 94988  | 32914   | 94428  | 47 |
| 14 | 26275   | 96486  | 27955   | 96013  | 29626   | 95511  | 31289   | 94979  | 32942   | 94418  | 46 |
| 15 | 26303   | 96479  | 27983   | 96005  | 29654   | 95502  | 31316   | 94970  | 32969   | 94409  | 45 |
| 16 | 26331   | 96471  | 28011   | 95997  | 29682   | 95493  | 31344   | 94961  | 32997   | 94399  | 44 |
| 17 | 26359   | 96463  | 28039   | 95989  | 29710   | 95485  | 31372   | 94952  | 33024   | 94390  | 43 |
| 18 | 26387   | 96456  | 28067   | 95981  | 29737   | 95476  | 31399   | 94943  | 33051   | 94380  | 42 |
| 19 | 26415   | 96448  | 28095   | 95972  | 29765   | 95467  | 31427   | 94933  | 33079   | 94370  | 41 |
| 20 | 26443   | 96440  | 28123   | 95964  | 29793   | 95459  | 31454   | 94924  | 33106   | 94361  | 40 |
| 21 | 26471   | 96433  | 28150   | 95956  | 29821   | 95450  | 31482   | 94915  | 33134   | 94351  | 39 |
| 22 | 26500   | 96425  | 28178   | 95948  | 29849   | 95441  | 31510   | 94906  | 33161   | 94342  | 38 |
| 23 | 26528   | 96417  | 28206   | 95940  | 29876   | 95433  | 31537   | 94897  | 33189   | 94332  | 37 |
| 24 | 26556   | 96410  | 28234   | 95931  | 29904   | 95424  | 31565   | 94888  | 33216   | 94322  | 36 |
| 25 | 26584   | 96402  | 28262   | 95923  | 29932   | 95415  | 31593   | 94878  | 33244   | 94313  | 35 |
| 26 | 26612   | 96394  | 28290   | 95915  | 29960   | 95407  | 31620   | 94869  | 33271   | 94303  | 34 |
| 27 | 26640   | 96386  | 28318   | 95907  | 29987   | 95398  | 31648   | 94860  | 33298   | 94293  | 33 |
| 28 | 26668   | 96379  | 28346   | 95898  | 30015   | 95389  | 31675   | 94851  | 33326   | 94284  | 32 |
| 29 | 26696   | 96371  | 28374   | 95890  | 30043   | 95380  | 31703   | 94842  | 33353   | 94274  | 31 |
| 30 | 26724   | 96363  | 28402   | 95882  | 30071   | 95372  | 31730   | 94832  | 33381   | 94264  | 30 |
| 31 | 26752   | 96355  | 28429   | 95874  | 30098   | 95363  | 31758   | 94823  | 33408   | 94254  | 29 |
| 32 | 26780   | 96347  | 28457   | 95865  | 30126   | 95354  | 31786   | 94814  | 33436   | 94245  | 28 |
| 33 | 26808   | 96340  | 28485   | 95857  | 30154   | 95345  | 31813   | 94805  | 33463   | 94235  | 27 |
| 34 | 26836   | 96332  | 28513   | 95849  | 30182   | 95337  | 31841   | 94795  | 33490   | 94225  | 26 |
| 35 | 26864   | 96324  | 28541   | 95841  | 30209   | 95328  | 31868   | 94786  | 33518   | 94215  | 25 |
| 36 | 26892   | 96316  | 28569   | 95832  | 30237   | 95319  | 31896   | 94777  | 33545   | 94206  | 24 |
| 37 | 26920   | 96308  | 28597   | 95824  | 30265   | 95310  | 31923   | 94768  | 33573   | 94196  | 23 |
| 38 | 26948   | 96301  | 28625   | 95816  | 30292   | 95301  | 31951   | 94758  | 33600   | 94186  | 22 |
| 39 | 26976   | 96293  | 28652   | 95807  | 30320   | 95292  | 31979   | 94749  | 33627   | 94176  | 21 |
| 40 | 27004   | 96285  | 28680   | 95799  | 30348   | 95284  | 32006   | 94740  | 33655   | 94167  | 20 |
| 41 | 27032   | 96277  | 28708   | 95791  | 30376   | 95275  | 32034   | 94730  | 33682   | 94157  | 19 |
| 42 | 27060   | 96269  | 28736   | 95782  | 30403   | 95266  | 32061   | 94721  | 33710   | 94147  | 18 |
| 43 | 27088   | 96261  | 28764   | 95774  | 30431   | 95257  | 32089   | 94712  | 33737   | 94137  | 17 |
| 44 | 27116   | 96253  | 28792   | 95766  | 30459   | 95248  | 32116   | 94702  | 33764   | 94127  | 16 |
| 45 | 27144   | 96246  | 28820   | 95757  | 30486   | 95240  | 32144   | 94693  | 33792   | 94118  | 15 |
| 46 | 27172   | 96238  | 28847   | 95749  | 30514   | 95231  | 32171   | 94684  | 33819   | 94108  | 14 |
| 47 | 27200   | 96230  | 28875   | 95740  | 30542   | 95222  | 32199   | 94674  | 33846   | 94098  | 13 |
| 48 | 27228   | 96222  | 28903   | 95732  | 30570   | 95213  | 32227   | 94665  | 33874   | 94088  | 12 |
| 49 | 27256   | 96214  | 28931   | 95724  | 30597   | 95204  | 32254   | 94656  | 33901   | 94078  | 11 |
| 50 | 27284   | 96206  | 28959   | 95715  | 30625   | 95195  | 32282   | 94646  | 33929   | 94068  | 10 |
| 51 | 27312   | 96198  | 28987   | 95707  | 30653   | 95186  | 32309   | 94637  | 33956   | 94058  | 9  |
| 52 | 27340   | 96190  | 29015   | 95698  | 30680   | 95177  | 32337   | 94627  | 33983   | 94049  | 8  |
| 53 | 27368   | 96182  | 29042   | 95690  | 30708   | 95168  | 32364   | 94618  | 34011   | 94039  | 7  |
| 54 | 27396   | 96174  | 29070   | 95681  | 30736   | 95159  | 32392   | 94609  | 34038   | 94029  | 6  |
| 55 | 27424   | 96166  | 29098   | 95673  | 30763   | 95150  | 32419   | 94599  | 34065   | 94019  | 5  |
| 56 | 27452   | 96158  | 29126   | 95664  | 30791   | 95142  | 32447   | 94590  | 34093   | 94009  | 4  |
| 57 | 27480   | 96150  | 29154   | 95656  | 30819   | 95133  | 32474   | 94580  | 34120   | 93999  | 3  |
| 58 | 27508   | 96142  | 29182   | 95647  | 30846   | 95124  | 32502   | 94571  | 34147   | 93989  | 2  |
| 59 | 27536   | 96134  | 29209   | 95639  | 30874   | 95115  | 32529   | 94561  | 34175   | 93979  | 1  |
| M  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | M  |
|    | 74 Deg. |        | 73 Deg. |        | 72 Deg. |        | 71 Deg. |        | 70 Deg. |        |    |

| M  | 20 Deg. |        | 21 Deg. |        | 22 Deg. |        | 23 Deg. |        | 24 Deg. |        | M  |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----|
|    | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. |    |
| 0  | 34202   | 93969  | 35837   | 93358  | 37461   | 92718  | 39073   | 92050  | 40674   | 91355  | 60 |
| 1  | 34229   | 93959  | 35864   | 93348  | 37488   | 92707  | 39100   | 92039  | 40700   | 91343  | 59 |
| 2  | 34257   | 93949  | 35891   | 93337  | 37515   | 92697  | 39127   | 92028  | 40727   | 91331  | 58 |
| 3  | 34284   | 93939  | 35918   | 93327  | 37542   | 92686  | 39153   | 92016  | 40753   | 91319  | 57 |
| 4  | 34311   | 93929  | 35945   | 93316  | 37569   | 92675  | 39180   | 92005  | 40780   | 91307  | 56 |
| 5  | 34339   | 93919  | 35973   | 93306  | 37595   | 92664  | 39207   | 91994  | 40806   | 91295  | 55 |
| 6  | 34366   | 93909  | 36000   | 93295  | 37622   | 92653  | 39234   | 91982  | 40833   | 91283  | 54 |
| 7  | 34393   | 93899  | 36027   | 93285  | 37649   | 92642  | 39260   | 91971  | 40860   | 91272  | 53 |
| 8  | 34421   | 93889  | 36054   | 93274  | 37676   | 92631  | 39287   | 91959  | 40886   | 91260  | 52 |
| 9  | 34448   | 93879  | 36081   | 93264  | 37703   | 92620  | 39314   | 91948  | 40913   | 91248  | 51 |
| 10 | 34475   | 93869  | 36108   | 93253  | 37730   | 92609  | 39341   | 91936  | 40939   | 91236  | 50 |
| 11 | 34503   | 93859  | 36135   | 93243  | 37757   | 92598  | 39367   | 91925  | 40966   | 91224  | 49 |
| 12 | 34530   | 93849  | 36162   | 93232  | 37784   | 92587  | 39394   | 91914  | 40992   | 91212  | 48 |
| 13 | 34557   | 93839  | 36190   | 93222  | 37811   | 92576  | 39421   | 91902  | 41019   | 91200  | 47 |
| 14 | 34584   | 93829  | 36217   | 93211  | 37838   | 92565  | 39448   | 91891  | 41045   | 91188  | 46 |
| 15 | 34612   | 93819  | 36244   | 93201  | 37865   | 92554  | 39474   | 91879  | 41072   | 91176  | 45 |
| 16 | 34639   | 93809  | 36271   | 93190  | 37892   | 92543  | 39501   | 91868  | 41098   | 91164  | 44 |
| 17 | 34666   | 93799  | 36298   | 93180  | 37919   | 92532  | 39528   | 91856  | 41125   | 91152  | 43 |
| 18 | 34694   | 93789  | 36325   | 93169  | 37946   | 92521  | 39555   | 91845  | 41151   | 91140  | 42 |
| 19 | 34721   | 93779  | 36352   | 93159  | 37973   | 92510  | 39581   | 91833  | 41178   | 91128  | 41 |
| 20 | 34748   | 93769  | 36379   | 93148  | 37999   | 92499  | 39608   | 91822  | 41204   | 91116  | 40 |
| 21 | 34775   | 93759  | 36406   | 93137  | 38026   | 92488  | 39635   | 91810  | 41231   | 91104  | 39 |
| 22 | 34803   | 93748  | 36434   | 93127  | 38053   | 92477  | 39661   | 91799  | 41257   | 91092  | 38 |
| 23 | 34830   | 93738  | 36461   | 93116  | 38080   | 92466  | 39688   | 91787  | 41284   | 91080  | 37 |
| 24 | 34857   | 93728  | 36488   | 93106  | 38107   | 92455  | 39715   | 91775  | 41310   | 91068  | 36 |
| 25 | 34884   | 93718  | 36515   | 93095  | 38134   | 92444  | 39741   | 91764  | 41337   | 91056  | 35 |
| 26 | 34912   | 93708  | 36542   | 93084  | 38161   | 92432  | 39768   | 91752  | 41363   | 91044  | 34 |
| 27 | 34939   | 93698  | 36569   | 93074  | 38188   | 92421  | 39795   | 91741  | 41390   | 91032  | 33 |
| 28 | 34966   | 93688  | 36596   | 93063  | 38215   | 92410  | 39822   | 91729  | 41416   | 91020  | 32 |
| 29 | 34993   | 93677  | 36623   | 93052  | 38241   | 92399  | 39848   | 91718  | 41443   | 91008  | 31 |
| 30 | 35021   | 93667  | 36650   | 93042  | 38268   | 92388  | 39875   | 91706  | 41469   | 90996  | 30 |
| 31 | 35048   | 93657  | 36677   | 93031  | 38295   | 92377  | 39902   | 91694  | 41496   | 90984  | 29 |
| 32 | 35075   | 93647  | 36704   | 93020  | 38322   | 92366  | 39928   | 91683  | 41522   | 90972  | 28 |
| 33 | 35102   | 93637  | 36731   | 93010  | 38349   | 92355  | 39955   | 91671  | 41549   | 90960  | 27 |
| 34 | 35130   | 93626  | 36758   | 92999  | 38376   | 92343  | 39982   | 91660  | 41575   | 90948  | 26 |
| 35 | 35157   | 93616  | 36785   | 92988  | 38403   | 92332  | 40008   | 91648  | 41602   | 90936  | 25 |
| 36 | 35183   | 93606  | 36812   | 92978  | 38430   | 92321  | 40035   | 91636  | 41628   | 90924  | 24 |
| 37 | 35211   | 93596  | 36839   | 92967  | 38456   | 92310  | 40062   | 91625  | 41655   | 90911  | 23 |
| 38 | 35239   | 93585  | 36867   | 92956  | 38483   | 92299  | 40088   | 91613  | 41681   | 90899  | 22 |
| 39 | 35266   | 93575  | 36894   | 92945  | 38510   | 92287  | 40115   | 91601  | 41707   | 90887  | 21 |
| 40 | 35293   | 93565  | 36921   | 92935  | 38537   | 92276  | 40141   | 91590  | 41734   | 90875  | 20 |
| 41 | 35320   | 93555  | 36948   | 92924  | 38564   | 92265  | 40168   | 91578  | 41760   | 90863  | 19 |
| 42 | 35347   | 93544  | 36975   | 92913  | 38591   | 92254  | 40195   | 91566  | 41787   | 90851  | 18 |
| 43 | 35375   | 93534  | 37002   | 92902  | 38617   | 92243  | 40221   | 91555  | 41813   | 90839  | 17 |
| 44 | 35402   | 93524  | 37029   | 92892  | 38644   | 92231  | 40248   | 91543  | 41840   | 90826  | 16 |
| 45 | 35429   | 93514  | 37056   | 92881  | 38671   | 92220  | 40275   | 91531  | 41866   | 90814  | 15 |
| 46 | 35456   | 93503  | 37083   | 92870  | 38698   | 92209  | 40301   | 91519  | 41892   | 90802  | 14 |
| 47 | 35484   | 93493  | 37110   | 92859  | 38725   | 92198  | 40328   | 91508  | 41919   | 90790  | 13 |
| 48 | 35511   | 93483  | 37137   | 92849  | 38752   | 92186  | 40355   | 91496  | 41945   | 90778  | 12 |
| 49 | 35538   | 93472  | 37164   | 92838  | 38778   | 92175  | 40381   | 91484  | 41972   | 90766  | 11 |
| 50 | 35565   | 93462  | 37191   | 92827  | 38805   | 92164  | 40408   | 91472  | 41998   | 90753  | 10 |
| 51 | 35592   | 93452  | 37218   | 92816  | 38832   | 92152  | 40434   | 91461  | 42024   | 90741  | 9  |
| 52 | 35619   | 93441  | 37245   | 92805  | 38859   | 92141  | 40461   | 91449  | 42051   | 90729  | 8  |
| 53 | 35647   | 93431  | 37272   | 92794  | 38886   | 92130  | 40488   | 91437  | 42077   | 90717  | 7  |
| 54 | 35674   | 93420  | 37299   | 92784  | 38912   | 92119  | 40514   | 91425  | 42104   | 90704  | 6  |
| 55 | 35701   | 93410  | 37326   | 92773  | 38939   | 92107  | 40541   | 91414  | 42130   | 90692  | 5  |
| 56 | 35728   | 93400  | 37353   | 92762  | 38966   | 92096  | 40567   | 91402  | 42156   | 90680  | 4  |
| 57 | 35755   | 93389  | 37380   | 92751  | 38993   | 92085  | 40594   | 91390  | 42183   | 90668  | 3  |
| 58 | 35782   | 93379  | 37407   | 92740  | 39020   | 92073  | 40621   | 91378  | 42209   | 90655  | 2  |
| 59 | 35810   | 93368  | 37434   | 92729  | 39046   | 92062  | 40647   | 91366  | 42235   | 90643  | 1  |
| M  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | M  |
|    | 69 Deg. |        | 68 Deg. |        | 67 Deg. |        | 66 Deg. |        | 65 Deg. |        |    |

| M  | 25 Deg. |        | 26 Deg. |        | 27 Deg. |        | 28 Deg. |        | 29 Deg. |        | M  |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----|
|    | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. |    |
| 0  | 42262   | 90631  | 43837   | 89879  | 45399   | 89101  | 46947   | 88295  | 48481   | 87462  | 60 |
| 1  | 42288   | 90618  | 43863   | 89867  | 45425   | 89087  | 46973   | 88281  | 48506   | 87448  | 59 |
| 2  | 42315   | 90606  | 43889   | 89854  | 45451   | 89074  | 46999   | 88267  | 48532   | 87434  | 58 |
| 3  | 42341   | 90594  | 43916   | 89841  | 45477   | 89061  | 47024   | 88254  | 48557   | 87420  | 57 |
| 4  | 42367   | 90582  | 43942   | 89828  | 45503   | 89048  | 47050   | 88240  | 48583   | 87406  | 56 |
| 5  | 42394   | 90569  | 43968   | 89816  | 45529   | 89035  | 47076   | 88226  | 48608   | 87391  | 55 |
| 6  | 42420   | 90557  | 43994   | 89803  | 45554   | 89021  | 47101   | 88213  | 48634   | 87377  | 54 |
| 7  | 42446   | 90545  | 44020   | 89790  | 45580   | 89008  | 47127   | 88199  | 48659   | 87363  | 53 |
| 8  | 42473   | 90532  | 44046   | 89777  | 45606   | 88995  | 47153   | 88185  | 48684   | 87349  | 52 |
| 9  | 42499   | 90520  | 44072   | 89764  | 45632   | 88981  | 47178   | 88172  | 48710   | 87335  | 51 |
| 10 | 42525   | 90507  | 44098   | 89752  | 45658   | 88968  | 47204   | 88158  | 48735   | 87321  | 50 |
| 11 | 42552   | 90495  | 44124   | 89739  | 45684   | 88955  | 47229   | 88144  | 48761   | 87306  | 49 |
| 12 | 42578   | 90483  | 44151   | 89726  | 45710   | 88942  | 47255   | 88130  | 48786   | 87292  | 48 |
| 13 | 42604   | 90470  | 44177   | 89713  | 45736   | 88928  | 47281   | 88117  | 48811   | 87278  | 47 |
| 14 | 42631   | 90458  | 44203   | 89700  | 45762   | 88915  | 47306   | 88103  | 48837   | 87264  | 46 |
| 15 | 42657   | 90446  | 44229   | 89687  | 45787   | 88902  | 47332   | 88089  | 48862   | 87250  | 45 |
| 16 | 42683   | 90433  | 44255   | 89674  | 45813   | 88888  | 47358   | 88075  | 48888   | 87235  | 44 |
| 17 | 42709   | 90421  | 44281   | 89662  | 45839   | 88875  | 47383   | 88062  | 48913   | 87221  | 43 |
| 18 | 42736   | 90408  | 44307   | 89649  | 45865   | 88862  | 47409   | 88048  | 48938   | 87207  | 42 |
| 19 | 42762   | 90396  | 44333   | 89636  | 45891   | 88848  | 47434   | 88034  | 48964   | 87193  | 41 |
| 20 | 42788   | 90383  | 44359   | 89623  | 45917   | 88835  | 47460   | 88020  | 48989   | 87178  | 40 |
| 21 | 42815   | 90371  | 44385   | 89610  | 45942   | 88822  | 47486   | 88006  | 49014   | 87164  | 39 |
| 22 | 42841   | 90358  | 44411   | 89597  | 45968   | 88808  | 47511   | 87993  | 49040   | 87150  | 38 |
| 23 | 42867   | 90346  | 44437   | 89584  | 45994   | 88795  | 47537   | 87979  | 49065   | 87136  | 37 |
| 24 | 42894   | 90334  | 44464   | 89571  | 46020   | 88782  | 47562   | 87965  | 49090   | 87121  | 36 |
| 25 | 42920   | 90321  | 44490   | 89558  | 46046   | 88768  | 47588   | 87951  | 49116   | 87107  | 35 |
| 26 | 42946   | 90309  | 44516   | 89545  | 46072   | 88755  | 47614   | 87937  | 49141   | 87093  | 34 |
| 27 | 42972   | 90296  | 44542   | 89532  | 46097   | 88741  | 47639   | 87923  | 49166   | 87079  | 33 |
| 28 | 42999   | 90284  | 44568   | 89519  | 46123   | 88728  | 47665   | 87909  | 49192   | 87064  | 32 |
| 29 | 43025   | 90271  | 44594   | 89506  | 46149   | 88715  | 47690   | 87896  | 49217   | 87050  | 31 |
| 30 | 43051   | 90259  | 44620   | 89493  | 46175   | 88701  | 47716   | 87882  | 49242   | 87036  | 30 |
| 31 | 43077   | 90246  | 44646   | 89480  | 46201   | 88688  | 47741   | 87868  | 49268   | 87021  | 29 |
| 32 | 43104   | 90233  | 44672   | 89467  | 46226   | 88674  | 47767   | 87854  | 49293   | 87007  | 28 |
| 33 | 43130   | 90221  | 44698   | 89454  | 46252   | 88661  | 47793   | 87840  | 49318   | 86993  | 27 |
| 34 | 43156   | 90208  | 44724   | 89441  | 46278   | 88647  | 47818   | 87826  | 49344   | 86978  | 26 |
| 35 | 43182   | 90196  | 44750   | 89428  | 46304   | 88634  | 47844   | 87812  | 49369   | 86964  | 25 |
| 36 | 43209   | 90183  | 44776   | 89415  | 46330   | 88620  | 47869   | 87798  | 49394   | 86949  | 24 |
| 37 | 43235   | 90171  | 44802   | 89402  | 46355   | 88607  | 47895   | 87784  | 49419   | 86935  | 23 |
| 38 | 43261   | 90158  | 44828   | 89389  | 46381   | 88593  | 47920   | 87770  | 49445   | 86921  | 22 |
| 39 | 43287   | 90146  | 44854   | 89376  | 46407   | 88580  | 47946   | 87756  | 49470   | 86906  | 21 |
| 40 | 43313   | 90133  | 44880   | 89363  | 46433   | 88566  | 47971   | 87743  | 49495   | 86892  | 20 |
| 41 | 43340   | 90120  | 44906   | 89350  | 46458   | 88553  | 47997   | 87729  | 49521   | 86878  | 19 |
| 42 | 43366   | 90108  | 44932   | 89337  | 46484   | 88539  | 48022   | 87715  | 49546   | 86863  | 18 |
| 43 | 43392   | 90095  | 44958   | 89324  | 46510   | 88526  | 48048   | 87701  | 49571   | 86849  | 17 |
| 44 | 43418   | 90082  | 44984   | 89311  | 46536   | 88512  | 48073   | 87687  | 49596   | 86834  | 16 |
| 45 | 43445   | 90070  | 45010   | 89298  | 46561   | 88499  | 48099   | 87673  | 49622   | 86820  | 15 |
| 46 | 43471   | 90057  | 45036   | 89285  | 46587   | 88485  | 48124   | 87659  | 49647   | 86805  | 14 |
| 47 | 43497   | 90045  | 45062   | 89272  | 46613   | 88472  | 48150   | 87645  | 49672   | 86791  | 13 |
| 48 | 43523   | 90032  | 45088   | 89259  | 46639   | 88458  | 48175   | 87631  | 49697   | 86777  | 12 |
| 49 | 43549   | 90019  | 45114   | 89245  | 46664   | 88445  | 48201   | 87617  | 49723   | 86762  | 11 |
| 50 | 43575   | 90007  | 45140   | 89232  | 46690   | 88431  | 48226   | 87603  | 49748   | 86748  | 10 |
| 51 | 43602   | 89994  | 45166   | 89219  | 46716   | 88417  | 48252   | 87589  | 49773   | 86733  | 9  |
| 52 | 43628   | 89981  | 45192   | 89206  | 46742   | 88404  | 48277   | 87575  | 49798   | 86719  | 8  |
| 53 | 43654   | 89968  | 45218   | 89193  | 46767   | 88390  | 48303   | 87561  | 49824   | 86704  | 7  |
| 54 | 43680   | 89956  | 45243   | 89180  | 46793   | 88377  | 48328   | 87546  | 49849   | 86690  | 6  |
| 55 | 43706   | 89943  | 45269   | 89167  | 46819   | 88363  | 48354   | 87532  | 49874   | 86675  | 5  |
| 56 | 43733   | 89930  | 45295   | 89153  | 46844   | 88349  | 48379   | 87518  | 49899   | 86661  | 4  |
| 57 | 43759   | 89918  | 45321   | 89140  | 46870   | 88336  | 48405   | 87504  | 49924   | 86646  | 3  |
| 58 | 43785   | 89905  | 45347   | 89127  | 46896   | 88322  | 48430   | 87490  | 49950   | 86632  | 2  |
| 59 | 43811   | 89892  | 45373   | 89114  | 46921   | 88308  | 48456   | 87476  | 49975   | 86617  | 1  |
| M  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | M  |
|    | 64 Deg. |        | 63 Deg. |        | 62 Deg. |        | 61 Deg. |        | 60 Deg. |        |    |



| M  | 30 Deg. |        | 31 Deg. |        | 32 Deg. |        | 33 Deg. |        | 34 Deg. |        | M  |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----|
|    | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. |    |
| 0  | 50000   | 866.3  | 51504   | 85717  | 52992   | 84805  | 54464   | 83867  | 55919   | 82904  | 60 |
| 1  | 50025   | 86588  | 51529   | 85702  | 53017   | 84789  | 54488   | 83851  | 55943   | 82887  | 59 |
| 2  | 50050   | 86573  | 51554   | 85687  | 53041   | 84774  | 54513   | 83835  | 55968   | 82871  | 58 |
| 3  | 50076   | 86559  | 51579   | 85672  | 53066   | 84759  | 54537   | 83819  | 55992   | 82855  | 57 |
| 4  | 50101   | 86544  | 51604   | 85657  | 53091   | 84743  | 54561   | 83804  | 56016   | 82839  | 56 |
| 5  | 50126   | 86530  | 51628   | 85642  | 53115   | 84728  | 54586   | 83788  | 56040   | 82822  | 55 |
| 6  | 50151   | 86515  | 51653   | 85627  | 53140   | 84712  | 54610   | 83772  | 56064   | 82806  | 54 |
| 7  | 50176   | 86501  | 51678   | 85612  | 53164   | 84697  | 54635   | 83756  | 56088   | 82790  | 53 |
| 8  | 50201   | 86486  | 51703   | 85597  | 53189   | 84681  | 54659   | 83740  | 56112   | 82773  | 52 |
| 9  | 50227   | 86471  | 51728   | 85582  | 53214   | 84666  | 54683   | 83724  | 56136   | 82757  | 51 |
| 10 | 50252   | 86457  | 51753   | 85567  | 53238   | 84650  | 54708   | 83708  | 56160   | 82741  | 50 |
| 11 | 50277   | 86442  | 51778   | 85551  | 53263   | 84635  | 54732   | 83692  | 56184   | 82724  | 49 |
| 12 | 50302   | 86427  | 51803   | 85536  | 53288   | 84619  | 54756   | 83676  | 56208   | 82708  | 48 |
| 13 | 50327   | 86413  | 51828   | 85521  | 53312   | 84604  | 54781   | 83660  | 56232   | 82692  | 47 |
| 14 | 50352   | 86398  | 51852   | 85506  | 53337   | 84588  | 54805   | 83644  | 56256   | 82675  | 46 |
| 15 | 50377   | 86384  | 51877   | 85491  | 53361   | 84573  | 54829   | 83629  | 56280   | 82659  | 45 |
| 16 | 50403   | 86369  | 51902   | 85476  | 53386   | 84557  | 54854   | 83613  | 56305   | 82643  | 44 |
| 17 | 50428   | 86354  | 51927   | 85461  | 53411   | 84542  | 54878   | 83597  | 56329   | 82626  | 43 |
| 18 | 50453   | 86340  | 51952   | 85446  | 53435   | 84526  | 54902   | 83581  | 56353   | 82610  | 42 |
| 19 | 50478   | 86325  | 51977   | 85431  | 53460   | 84511  | 54927   | 83565  | 56377   | 82593  | 41 |
| 20 | 50503   | 86310  | 52002   | 85416  | 53484   | 84495  | 54951   | 83549  | 56401   | 82577  | 40 |
| 21 | 50528   | 86295  | 52026   | 85401  | 53509   | 84480  | 54975   | 83533  | 56425   | 82561  | 39 |
| 22 | 50553   | 86281  | 52051   | 85385  | 53534   | 84464  | 54999   | 83517  | 56449   | 82544  | 38 |
| 23 | 50578   | 86266  | 52076   | 85370  | 53558   | 84448  | 55024   | 83501  | 56473   | 82528  | 37 |
| 24 | 50603   | 86251  | 52101   | 85355  | 53583   | 84433  | 55048   | 83485  | 56497   | 82511  | 36 |
| 25 | 50628   | 86237  | 52126   | 85340  | 53607   | 84417  | 55072   | 83469  | 56521   | 82495  | 35 |
| 26 | 50654   | 86222  | 52151   | 85325  | 53632   | 84402  | 55097   | 83453  | 56545   | 82478  | 34 |
| 27 | 50679   | 86207  | 52175   | 85310  | 53656   | 84386  | 55121   | 83437  | 56569   | 82462  | 33 |
| 28 | 50704   | 86192  | 52200   | 85294  | 53681   | 84370  | 55145   | 83421  | 56593   | 82446  | 32 |
| 29 | 50729   | 86178  | 52225   | 85279  | 53705   | 84355  | 55169   | 83405  | 56617   | 82429  | 31 |
| 30 | 50754   | 86163  | 52250   | 85264  | 53730   | 84339  | 55194   | 83389  | 56641   | 82413  | 30 |
| 31 | 50779   | 86148  | 52275   | 85249  | 53754   | 84324  | 55218   | 83373  | 56665   | 82396  | 29 |
| 32 | 50804   | 86133  | 52299   | 85234  | 53779   | 84308  | 55242   | 83356  | 56689   | 82380  | 28 |
| 33 | 50829   | 86119  | 52324   | 85218  | 53804   | 84292  | 55266   | 83340  | 56713   | 82363  | 27 |
| 34 | 50854   | 86104  | 52349   | 85203  | 53828   | 84277  | 55291   | 83324  | 56736   | 82347  | 26 |
| 35 | 50879   | 86089  | 52374   | 85188  | 53853   | 84261  | 55315   | 83308  | 56760   | 82330  | 25 |
| 36 | 50904   | 86074  | 52399   | 85173  | 53877   | 84245  | 55339   | 83292  | 56784   | 82314  | 24 |
| 37 | 50929   | 86059  | 52423   | 85157  | 53902   | 84230  | 55363   | 83276  | 56808   | 82297  | 23 |
| 38 | 50954   | 86045  | 52448   | 85142  | 53926   | 84214  | 55388   | 83260  | 56832   | 82281  | 22 |
| 39 | 50979   | 86030  | 52473   | 85127  | 53951   | 84198  | 55412   | 83244  | 56856   | 82264  | 21 |
| 40 | 51004   | 86015  | 52498   | 85112  | 53975   | 84182  | 55436   | 83228  | 56880   | 82248  | 20 |
| 41 | 51029   | 86000  | 52522   | 85096  | 54000   | 84167  | 55460   | 83212  | 56904   | 82231  | 19 |
| 42 | 51054   | 85985  | 52547   | 85081  | 54024   | 84151  | 55484   | 83195  | 56928   | 82214  | 18 |
| 43 | 51079   | 85970  | 52572   | 85066  | 54049   | 84135  | 55509   | 83179  | 56952   | 82198  | 17 |
| 44 | 51104   | 85956  | 52597   | 85051  | 54073   | 84120  | 55533   | 83163  | 56976   | 82181  | 16 |
| 45 | 51129   | 85941  | 52621   | 85035  | 54097   | 84104  | 55557   | 83147  | 57000   | 82165  | 15 |
| 46 | 51154   | 85926  | 52646   | 85020  | 54122   | 84088  | 55581   | 83131  | 57024   | 82148  | 14 |
| 47 | 51179   | 85911  | 52671   | 85005  | 54146   | 84072  | 55605   | 83115  | 57047   | 82132  | 13 |
| 48 | 51204   | 85896  | 52696   | 84989  | 54171   | 84057  | 55630   | 83098  | 57071   | 82115  | 12 |
| 49 | 51229   | 85881  | 52720   | 84974  | 54195   | 84041  | 55654   | 83082  | 57095   | 82098  | 11 |
| 50 | 51254   | 85866  | 52745   | 84959  | 54220   | 84025  | 55678   | 83066  | 57119   | 82082  | 10 |
| 51 | 51279   | 85851  | 52770   | 84943  | 54244   | 84009  | 55702   | 83050  | 57143   | 82065  | 9  |
| 52 | 51304   | 85836  | 52794   | 84928  | 54269   | 83994  | 55726   | 83034  | 57167   | 82048  | 8  |
| 53 | 51329   | 85821  | 52819   | 84913  | 54293   | 83978  | 55750   | 83017  | 57191   | 82032  | 7  |
| 54 | 51354   | 85806  | 52844   | 84897  | 54317   | 83962  | 55775   | 83001  | 57215   | 82015  | 6  |
| 55 | 51379   | 85792  | 52869   | 84882  | 54342   | 83946  | 55799   | 82985  | 57238   | 81999  | 5  |
| 56 | 51404   | 85777  | 52893   | 84866  | 54366   | 83930  | 55823   | 82969  | 57262   | 81982  | 4  |
| 57 | 51429   | 85762  | 52918   | 84851  | 54391   | 83915  | 55847   | 82953  | 57286   | 81965  | 3  |
| 58 | 51454   | 85747  | 52943   | 84836  | 54415   | 83899  | 55871   | 82936  | 57310   | 81949  | 2  |
| 59 | 51479   | 85732  | 52967   | 84820  | 54440   | 83883  | 55895   | 82920  | 57334   | 81932  | 1  |
| M  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | M  |
|    | 59 Deg. |        | 58 Deg. |        | 57 Deg. |        | 56 Deg. |        | 55 Deg. |        |    |



| M  | 35 Deg. |        | 36 Deg. |        | 37 Deg. |        | 38 Deg. |        | 39 Deg. |        | M  |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----|
|    | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. |    |
| 0  | 57358   | 81915  | 58779   | 80302  | 60182   | 79864  | 61566   | 78801  | 62932   | 77715  | 60 |
| 1  | 57381   | 81899  | 58802   | 80885  | 60205   | 79846  | 61589   | 78783  | 62955   | 77696  | 59 |
| 2  | 57405   | 81882  | 58826   | 80867  | 60228   | 79829  | 61612   | 78765  | 62977   | 77678  | 58 |
| 3  | 57429   | 81865  | 58849   | 80850  | 60251   | 79811  | 61635   | 78747  | 63000   | 77660  | 57 |
| 4  | 57453   | 81848  | 58873   | 80833  | 60274   | 79793  | 61658   | 78729  | 63022   | 77641  | 56 |
| 5  | 57477   | 81832  | 58896   | 80816  | 60298   | 79776  | 61681   | 78711  | 63045   | 77623  | 55 |
| 6  | 57501   | 81815  | 58920   | 80799  | 60321   | 79758  | 61704   | 78694  | 63068   | 77605  | 54 |
| 7  | 57524   | 81798  | 58943   | 80782  | 60344   | 79741  | 61726   | 78676  | 63090   | 77586  | 53 |
| 8  | 57548   | 81782  | 58967   | 80765  | 60367   | 79723  | 61749   | 78658  | 63113   | 77568  | 52 |
| 9  | 57572   | 81765  | 58990   | 80748  | 60390   | 79706  | 61772   | 78640  | 63135   | 77550  | 51 |
| 10 | 57596   | 81748  | 59014   | 80730  | 60414   | 79688  | 61795   | 78622  | 63158   | 77531  | 50 |
| 11 | 57619   | 81731  | 59037   | 80713  | 60437   | 79671  | 61818   | 78604  | 63180   | 77513  | 49 |
| 12 | 57643   | 81714  | 59061   | 80696  | 60460   | 79653  | 61841   | 78585  | 63203   | 77494  | 48 |
| 13 | 57667   | 81698  | 59084   | 80679  | 60483   | 79635  | 61864   | 78568  | 63225   | 77476  | 47 |
| 14 | 57691   | 81681  | 59108   | 80662  | 60506   | 79618  | 61887   | 78550  | 63248   | 77458  | 46 |
| 15 | 57715   | 81664  | 59131   | 80644  | 60529   | 79600  | 61909   | 78532  | 63271   | 77439  | 45 |
| 16 | 57738   | 81647  | 59154   | 80627  | 60553   | 79583  | 61932   | 78514  | 63293   | 77421  | 44 |
| 17 | 57762   | 81631  | 59178   | 80610  | 60576   | 79565  | 61955   | 78496  | 63316   | 77402  | 43 |
| 18 | 57786   | 81614  | 59201   | 80593  | 60599   | 79547  | 61978   | 78478  | 63338   | 77384  | 42 |
| 19 | 57810   | 81597  | 59225   | 80576  | 60622   | 79530  | 62001   | 78460  | 63361   | 77366  | 41 |
| 20 | 57833   | 81580  | 59248   | 80558  | 60645   | 79512  | 62024   | 78442  | 63383   | 77347  | 40 |
| 21 | 57857   | 81563  | 59272   | 80541  | 60668   | 79494  | 62046   | 78424  | 63406   | 77329  | 39 |
| 22 | 57881   | 81546  | 59295   | 80524  | 60691   | 79477  | 62069   | 78405  | 63428   | 77310  | 38 |
| 23 | 57904   | 81530  | 59318   | 80507  | 60714   | 79459  | 62092   | 78387  | 63451   | 77292  | 37 |
| 24 | 57928   | 81513  | 59342   | 80489  | 60738   | 79441  | 62115   | 78369  | 63473   | 77273  | 36 |
| 25 | 57952   | 81496  | 59365   | 80472  | 60761   | 79424  | 62138   | 78351  | 63496   | 77255  | 35 |
| 26 | 57976   | 81479  | 59389   | 80455  | 60784   | 79406  | 62160   | 78333  | 63518   | 77236  | 34 |
| 27 | 57999   | 81462  | 59412   | 80438  | 60807   | 79388  | 62183   | 78315  | 63540   | 77218  | 33 |
| 28 | 58023   | 81445  | 59436   | 80420  | 60830   | 79371  | 62206   | 78297  | 63563   | 77199  | 32 |
| 29 | 58047   | 81428  | 59459   | 80403  | 60853   | 79353  | 62229   | 78279  | 63585   | 77181  | 31 |
| 30 | 58070   | 81412  | 59482   | 80386  | 60876   | 79335  | 62251   | 78261  | 63608   | 77162  | 30 |
| 31 | 58094   | 81395  | 59506   | 80368  | 60899   | 79318  | 62274   | 78243  | 63630   | 77144  | 29 |
| 32 | 58118   | 81378  | 59529   | 80351  | 60922   | 79300  | 62297   | 78225  | 63653   | 77125  | 28 |
| 33 | 58141   | 81361  | 59552   | 80334  | 60945   | 79282  | 62320   | 78206  | 63675   | 77107  | 27 |
| 34 | 58165   | 81344  | 59576   | 80316  | 60968   | 79264  | 62342   | 78188  | 63698   | 77088  | 26 |
| 35 | 58189   | 81327  | 59599   | 80299  | 60991   | 79247  | 62365   | 78170  | 63720   | 77070  | 25 |
| 36 | 58212   | 81310  | 59622   | 80282  | 61015   | 79229  | 62388   | 78152  | 63742   | 77051  | 24 |
| 37 | 58236   | 81293  | 59646   | 80264  | 61038   | 79211  | 62411   | 78134  | 63765   | 77033  | 23 |
| 38 | 58260   | 81276  | 59669   | 80247  | 61061   | 79193  | 62433   | 78116  | 63787   | 77014  | 22 |
| 39 | 58283   | 81259  | 59693   | 80230  | 61084   | 79176  | 62456   | 78098  | 63810   | 76996  | 21 |
| 40 | 58307   | 81242  | 59716   | 80212  | 61107   | 79158  | 62479   | 78079  | 63832   | 76977  | 20 |
| 41 | 58330   | 81225  | 59739   | 80195  | 61130   | 79140  | 62502   | 78061  | 63854   | 76959  | 19 |
| 42 | 58354   | 81208  | 59763   | 80178  | 61153   | 79122  | 62524   | 78043  | 63877   | 76940  | 18 |
| 43 | 58378   | 81191  | 59786   | 80160  | 61176   | 79105  | 62547   | 78025  | 63899   | 76921  | 17 |
| 44 | 58401   | 81174  | 59809   | 80143  | 61199   | 79087  | 62570   | 78007  | 63922   | 76903  | 16 |
| 45 | 58425   | 81157  | 59832   | 80125  | 61222   | 79069  | 62592   | 77988  | 63944   | 76884  | 15 |
| 46 | 58449   | 81140  | 59856   | 80108  | 61245   | 79051  | 62615   | 77970  | 63966   | 76866  | 14 |
| 47 | 58472   | 81123  | 59879   | 80091  | 61268   | 79033  | 62638   | 77952  | 63989   | 76847  | 13 |
| 48 | 58496   | 81106  | 59902   | 80073  | 61291   | 79015  | 62660   | 77934  | 64011   | 76828  | 12 |
| 49 | 58519   | 81089  | 59926   | 80056  | 61314   | 78998  | 62683   | 77916  | 64033   | 76810  | 11 |
| 50 | 58543   | 81072  | 59949   | 80038  | 61337   | 78980  | 62706   | 77897  | 64056   | 76791  | 10 |
| 51 | 58567   | 81055  | 59972   | 80021  | 61360   | 78962  | 62728   | 77879  | 64078   | 76772  | 9  |
| 52 | 58590   | 81038  | 59995   | 80003  | 61383   | 78944  | 62751   | 77861  | 64100   | 76754  | 8  |
| 53 | 58614   | 81021  | 60019   | 79986  | 61406   | 78926  | 62774   | 77843  | 64123   | 76735  | 7  |
| 54 | 58637   | 81004  | 60042   | 79968  | 61429   | 78908  | 62796   | 77824  | 64145   | 76717  | 6  |
| 55 | 58661   | 80987  | 60065   | 79951  | 61451   | 78891  | 62819   | 77806  | 64167   | 76698  | 5  |
| 56 | 58684   | 80970  | 60089   | 79934  | 61474   | 78873  | 62842   | 77788  | 64190   | 76679  | 4  |
| 57 | 58708   | 80953  | 60112   | 79916  | 61497   | 78855  | 62864   | 77769  | 64212   | 76661  | 3  |
| 58 | 58731   | 80936  | 60135   | 79899  | 61520   | 78837  | 62887   | 77751  | 64234   | 76642  | 2  |
| 59 | 58755   | 80919  | 60158   | 79881  | 61543   | 78819  | 62909   | 77733  | 64256   | 76623  | 1  |
| M  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | M  |
|    | 54 Deg. |        | 53 Deg. |        | 52 Deg. |        | 51 Deg. |        | 50 Deg. |        |    |

| M  | 40 Deg. |        | 41 Deg. |        | 42 Deg. |        | 43 Deg. |        | 44 Deg. |        | M  |
|----|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|----|
|    | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. | N. S.   | N. CS. |    |
| 0  | 64279   | 76604  | 65646   | 75471  | 66913   | 74314  | 68200   | 73135  | 69466   | 71934  | 60 |
| 1  | 64301   | 76586  | 65628   | 75452  | 66935   | 74295  | 68221   | 73116  | 69487   | 71914  | 59 |
| 2  | 64323   | 76567  | 65650   | 75433  | 66956   | 74276  | 68242   | 73096  | 69508   | 71894  | 58 |
| 3  | 64346   | 76548  | 65672   | 75414  | 66978   | 74256  | 68264   | 73076  | 69529   | 71873  | 57 |
| 4  | 64368   | 76530  | 65694   | 75395  | 66999   | 74237  | 68285   | 73056  | 69549   | 71853  | 56 |
| 5  | 64390   | 76511  | 65716   | 75375  | 67021   | 74217  | 68306   | 73036  | 69570   | 71833  | 55 |
| 6  | 64412   | 76492  | 65738   | 75356  | 67043   | 74198  | 68327   | 73016  | 69591   | 71813  | 54 |
| 7  | 64435   | 76473  | 65759   | 75337  | 67064   | 74178  | 68349   | 72996  | 69612   | 71792  | 53 |
| 8  | 64457   | 76455  | 65781   | 75318  | 67086   | 74159  | 68370   | 72976  | 69633   | 71772  | 52 |
| 9  | 64479   | 76436  | 65803   | 75299  | 67107   | 74139  | 68391   | 72957  | 69654   | 71752  | 51 |
| 10 | 64501   | 76417  | 65825   | 75280  | 67129   | 74120  | 68412   | 72937  | 69675   | 71732  | 50 |
| 11 | 64524   | 76398  | 65847   | 75261  | 67151   | 74100  | 68433   | 72917  | 69696   | 71711  | 49 |
| 12 | 64546   | 76380  | 65869   | 75241  | 67172   | 74080  | 68455   | 72897  | 69717   | 71691  | 48 |
| 13 | 64568   | 76361  | 65891   | 75222  | 67194   | 74061  | 68476   | 72877  | 69737   | 71671  | 47 |
| 14 | 64590   | 76342  | 65913   | 75203  | 67215   | 74041  | 68497   | 72857  | 69758   | 71650  | 46 |
| 15 | 64612   | 76323  | 65935   | 75184  | 67237   | 74022  | 68518   | 72837  | 69779   | 71630  | 45 |
| 16 | 64635   | 76304  | 65956   | 75165  | 67258   | 74002  | 68539   | 72817  | 69800   | 71610  | 44 |
| 17 | 64657   | 76286  | 65978   | 75146  | 67280   | 73983  | 68561   | 72797  | 69821   | 71590  | 43 |
| 18 | 64679   | 76267  | 66000   | 75126  | 67301   | 73963  | 68582   | 72777  | 69842   | 71569  | 42 |
| 19 | 64701   | 76248  | 66022   | 75107  | 67323   | 73944  | 68603   | 72757  | 69862   | 71549  | 41 |
| 20 | 64723   | 76229  | 66044   | 75088  | 67344   | 73924  | 68624   | 72737  | 69883   | 71529  | 40 |
| 21 | 64746   | 76210  | 66066   | 75069  | 67366   | 73904  | 68645   | 72717  | 69904   | 71508  | 39 |
| 22 | 64768   | 76191  | 66088   | 75050  | 67387   | 73885  | 68666   | 72697  | 69925   | 71488  | 38 |
| 23 | 64790   | 76173  | 66109   | 75030  | 67409   | 73865  | 68688   | 72677  | 69946   | 71468  | 37 |
| 24 | 64812   | 76154  | 66131   | 75011  | 67430   | 73846  | 68709   | 72657  | 69966   | 71447  | 36 |
| 25 | 64834   | 76135  | 66153   | 74992  | 67452   | 73826  | 68730   | 72637  | 69987   | 71427  | 35 |
| 26 | 64856   | 76116  | 66175   | 74973  | 67473   | 73806  | 68751   | 72617  | 70008   | 71407  | 34 |
| 27 | 64878   | 76097  | 66197   | 74953  | 67495   | 73787  | 68772   | 72597  | 70029   | 71386  | 33 |
| 28 | 64901   | 76078  | 66218   | 74934  | 67516   | 73767  | 68793   | 72577  | 70049   | 71366  | 32 |
| 29 | 64923   | 76059  | 66240   | 74915  | 67538   | 73747  | 68814   | 72557  | 70070   | 71345  | 31 |
| 30 | 64945   | 76041  | 66262   | 74896  | 67559   | 73728  | 68835   | 72537  | 70091   | 71325  | 30 |
| 31 | 64967   | 76022  | 66284   | 74876  | 67580   | 73708  | 68857   | 72517  | 70112   | 71305  | 29 |
| 32 | 64989   | 76003  | 66306   | 74857  | 67602   | 73688  | 68878   | 72497  | 70132   | 71284  | 28 |
| 33 | 65011   | 75984  | 66327   | 74838  | 67623   | 73669  | 68899   | 72477  | 70153   | 71264  | 27 |
| 34 | 65033   | 75965  | 66349   | 74818  | 67645   | 73649  | 68920   | 72457  | 70174   | 71243  | 26 |
| 35 | 65055   | 75946  | 66371   | 74799  | 67666   | 73629  | 68941   | 72437  | 70195   | 71223  | 25 |
| 36 | 65077   | 75927  | 66393   | 74780  | 67688   | 73610  | 68962   | 72417  | 70215   | 71203  | 24 |
| 37 | 65099   | 75908  | 66414   | 74760  | 67709   | 73590  | 68983   | 72397  | 70236   | 71182  | 23 |
| 38 | 65122   | 75889  | 66435   | 74741  | 67730   | 73570  | 69004   | 72377  | 70257   | 71162  | 22 |
| 39 | 65144   | 75870  | 66458   | 74722  | 67752   | 73551  | 69025   | 72357  | 70277   | 71141  | 21 |
| 40 | 65166   | 75851  | 66480   | 74703  | 67773   | 73531  | 69046   | 72337  | 70298   | 71121  | 20 |
| 41 | 65188   | 75832  | 66501   | 74683  | 67795   | 73511  | 69067   | 72317  | 70319   | 71100  | 19 |
| 42 | 65210   | 75813  | 66523   | 74664  | 67816   | 73491  | 69088   | 72297  | 70339   | 71080  | 18 |
| 43 | 65232   | 75794  | 66545   | 74644  | 67837   | 73472  | 69109   | 72277  | 70360   | 71059  | 17 |
| 44 | 65254   | 75775  | 66566   | 74625  | 67859   | 73452  | 69130   | 72257  | 70381   | 71039  | 16 |
| 45 | 65276   | 75756  | 66588   | 74606  | 67880   | 73432  | 69151   | 72236  | 70401   | 71019  | 15 |
| 46 | 65298   | 75738  | 66610   | 74586  | 67901   | 73412  | 69172   | 72216  | 70422   | 70998  | 14 |
| 47 | 65320   | 75719  | 66632   | 74567  | 67923   | 73393  | 69193   | 72196  | 70443   | 70978  | 13 |
| 48 | 65342   | 75699  | 66653   | 74548  | 67944   | 73373  | 69214   | 72176  | 70463   | 70957  | 12 |
| 49 | 65364   | 75680  | 66675   | 74528  | 67965   | 73353  | 69235   | 72156  | 70484   | 70937  | 11 |
| 50 | 65386   | 75661  | 66697   | 74509  | 67987   | 73333  | 69256   | 72136  | 70505   | 70916  | 10 |
| 51 | 65408   | 75642  | 66718   | 74489  | 68008   | 73314  | 69277   | 72116  | 70525   | 70896  | 9  |
| 52 | 65430   | 75623  | 66740   | 74470  | 68029   | 73294  | 69298   | 72095  | 70546   | 70875  | 8  |
| 53 | 65452   | 75604  | 66762   | 74451  | 68051   | 73274  | 69319   | 72075  | 70567   | 70855  | 7  |
| 54 | 65474   | 75585  | 66783   | 74431  | 68072   | 73254  | 69340   | 72055  | 70587   | 70834  | 6  |
| 55 | 65496   | 75566  | 66805   | 74412  | 68093   | 73234  | 69361   | 72035  | 70608   | 70813  | 5  |
| 56 | 65518   | 75547  | 66827   | 74392  | 68115   | 73215  | 69382   | 72015  | 70628   | 70793  | 4  |
| 57 | 65540   | 75528  | 66848   | 74373  | 68136   | 73195  | 69403   | 71995  | 70649   | 70772  | 3  |
| 58 | 65562   | 75509  | 66870   | 74353  | 68157   | 73175  | 69424   | 71974  | 70670   | 70752  | 2  |
| 59 | 65584   | 75490  | 66891   | 74334  | 68179   | 73155  | 69445   | 71954  | 70690   | 70731  | 1  |
| 60 | 65606   | 75471  | 66913   | 74314  | 68200   | 73135  | 69466   | 71934  | 70711   | 70711  | 0  |
| M  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | N. CS.  | N. S.  | M  |
|    | 49 Deg. |        | 48 Deg. |        | 47 Deg. |        | 46 Deg. |        | 45 Deg. |        |    |

| M  | 0 Degrees.  |         | 1 Degree.   |         | 2 Degrees.  |         | 3 Degrees.  |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 00000       | 0000.00 | 01746       | 57.2000 | 03492       | 28.6333 | 05241       | 19.0811 | 60 |
| 1  | 00329       | 3437.75 | 01775       | 56.3506 | 03521       | 28.3994 | 05270       | 18.9755 | 59 |
| 2  | 00658       | 1718.87 | 01804       | 55.4415 | 03550       | 28.1664 | 05299       | 18.8711 | 58 |
| 3  | 00987       | 1145.92 | 01833       | 54.5613 | 03579       | 27.9372 | 05328       | 18.7678 | 57 |
| 4  | 00116       | 859.436 | 01862       | 53.7086 | 03609       | 27.7117 | 05357       | 18.6656 | 56 |
| 5  | 00145       | 687.549 | 01891       | 52.8821 | 03638       | 27.4899 | 05387       | 18.5645 | 55 |
| 6  | 00175       | 572.957 | 01920       | 52.0807 | 03667       | 27.2715 | 05416       | 18.4645 | 54 |
| 7  | 00204       | 491.106 | 01949       | 51.3032 | 03696       | 27.0566 | 05445       | 18.3655 | 53 |
| 8  | 00233       | 429.718 | 01978       | 50.5485 | 03725       | 26.8450 | 05474       | 18.2677 | 52 |
| 9  | 00262       | 381.971 | 02007       | 49.8157 | 03754       | 26.6367 | 05503       | 18.1708 | 51 |
| 10 | 00291       | 343.774 | 02036       | 49.1039 | 03783       | 26.4316 | 05533       | 18.0750 | 50 |
| 11 | 00320       | 312.521 | 02066       | 48.4121 | 03812       | 26.2295 | 05562       | 17.9801 | 49 |
| 12 | 00349       | 286.478 | 02095       | 47.7395 | 03842       | 26.0307 | 05591       | 17.8863 | 48 |
| 13 | 00378       | 264.441 | 02124       | 47.0853 | 03871       | 25.8348 | 05620       | 17.7934 | 47 |
| 14 | 00407       | 245.552 | 02153       | 46.4489 | 03900       | 25.6418 | 05649       | 17.7015 | 46 |
| 15 | 00436       | 229.182 | 02182       | 45.8294 | 03929       | 25.4517 | 05678       | 17.6106 | 45 |
| 16 | 00465       | 214.858 | 02211       | 45.2261 | 03958       | 25.2644 | 05708       | 17.5205 | 44 |
| 17 | 00495       | 202.219 | 02240       | 44.6386 | 03987       | 25.0798 | 05737       | 17.4314 | 43 |
| 18 | 00524       | 190.984 | 02269       | 44.0661 | 04016       | 24.8978 | 05766       | 17.3432 | 42 |
| 19 | 00553       | 180.932 | 02298       | 43.5081 | 04046       | 24.7185 | 05795       | 17.2558 | 41 |
| 20 | 00582       | 171.885 | 02328       | 42.9641 | 04075       | 24.5418 | 05824       | 17.1693 | 40 |
| 21 | 00611       | 163.700 | 02357       | 42.4335 | 04104       | 24.3675 | 05854       | 17.0837 | 39 |
| 22 | 00640       | 156.259 | 02386       | 41.9158 | 04133       | 24.1957 | 05883       | 16.9990 | 38 |
| 23 | 00669       | 149.465 | 02415       | 41.4106 | 04162       | 24.0263 | 05912       | 16.9150 | 37 |
| 24 | 00698       | 143.237 | 02444       | 40.9174 | 04191       | 23.8593 | 05941       | 16.8319 | 36 |
| 25 | 00727       | 137.507 | 02473       | 40.4358 | 04220       | 23.6945 | 05970       | 16.7496 | 35 |
| 26 | 00756       | 132.219 | 02502       | 39.9655 | 04250       | 23.5321 | 05999       | 16.6681 | 34 |
| 27 | 00785       | 127.321 | 02531       | 39.5059 | 04279       | 23.3718 | 06029       | 16.5874 | 33 |
| 28 | 00814       | 122.774 | 02560       | 39.0568 | 04308       | 23.2137 | 06058       | 16.5075 | 32 |
| 29 | 00844       | 118.540 | 02589       | 38.6177 | 04337       | 23.0577 | 06087       | 16.4283 | 31 |
| 30 | 00873       | 114.539 | 02619       | 38.1885 | 04366       | 22.9037 | 06116       | 16.3499 | 30 |
| 31 | 00902       | 110.892 | 02648       | 37.7686 | 04395       | 22.7518 | 06145       | 16.2722 | 29 |
| 32 | 00931       | 107.426 | 02677       | 37.3579 | 04424       | 22.6020 | 06175       | 16.1952 | 28 |
| 33 | 00960       | 104.171 | 02706       | 36.9560 | 04454       | 22.4541 | 06204       | 16.1190 | 27 |
| 34 | 00989       | 101.107 | 02735       | 36.5627 | 04483       | 22.3081 | 06233       | 16.0435 | 26 |
| 35 | 01018       | 98.2179 | 02764       | 36.1776 | 04512       | 22.1640 | 06262       | 15.9687 | 25 |
| 36 | 01047       | 95.4895 | 02793       | 35.8006 | 04541       | 22.0217 | 06291       | 15.8945 | 24 |
| 37 | 01076       | 92.9085 | 02822       | 35.4313 | 04570       | 21.8813 | 06321       | 15.8211 | 23 |
| 38 | 01105       | 90.4633 | 02851       | 35.0695 | 04599       | 21.7425 | 06350       | 15.7483 | 22 |
| 39 | 01135       | 88.1436 | 02881       | 34.7151 | 04628       | 21.6056 | 06379       | 15.6762 | 21 |
| 40 | 01164       | 85.9398 | 02910       | 34.3678 | 04658       | 21.4704 | 06408       | 15.6048 | 20 |
| 41 | 01193       | 83.8435 | 02939       | 34.0273 | 04687       | 21.3369 | 06437       | 15.5340 | 19 |
| 42 | 01222       | 81.8470 | 02968       | 33.6935 | 04716       | 21.2049 | 06467       | 15.4638 | 18 |
| 43 | 01251       | 79.9434 | 02997       | 33.3662 | 04745       | 21.0747 | 06496       | 15.3943 | 17 |
| 44 | 01280       | 78.1263 | 03026       | 33.0452 | 04774       | 20.9460 | 06525       | 15.3254 | 16 |
| 45 | 01309       | 76.3900 | 03055       | 32.7303 | 04803       | 20.8188 | 06554       | 15.2571 | 15 |
| 46 | 01338       | 74.7292 | 03084       | 32.4213 | 04832       | 20.6932 | 06584       | 15.1893 | 14 |
| 47 | 01367       | 73.1330 | 03114       | 32.1181 | 04862       | 20.5691 | 06613       | 15.1222 | 13 |
| 48 | 01396       | 71.6151 | 03143       | 31.8205 | 04891       | 20.4465 | 06642       | 15.0557 | 12 |
| 49 | 01425       | 70.1533 | 03172       | 31.5284 | 04920       | 20.3253 | 06671       | 14.9898 | 11 |
| 50 | 01455       | 68.7501 | 03201       | 31.2416 | 04949       | 20.2056 | 06700       | 14.9244 | 10 |
| 51 | 01484       | 67.4019 | 03230       | 30.9599 | 04978       | 20.0872 | 06730       | 14.8596 | 9  |
| 52 | 01513       | 66.1055 | 03259       | 30.6833 | 05007       | 19.9702 | 06759       | 14.7954 | 8  |
| 53 | 01542       | 64.8580 | 03288       | 30.4116 | 05037       | 19.8546 | 06788       | 14.7317 | 7  |
| 54 | 01571       | 63.6567 | 03317       | 30.1446 | 05066       | 19.7403 | 06817       | 14.6685 | 6  |
| 55 | 01600       | 62.4992 | 03346       | 29.8823 | 05095       | 19.6273 | 06847       | 14.6059 | 5  |
| 56 | 01629       | 61.3829 | 03376       | 29.6245 | 05124       | 19.5156 | 06876       | 14.5438 | 4  |
| 57 | 01658       | 60.3058 | 03405       | 29.3711 | 05153       | 19.4051 | 06905       | 14.4823 | 3  |
| 58 | 01687       | 59.2659 | 03434       | 29.1220 | 05182       | 19.2959 | 06934       | 14.4212 | 2  |
| 59 | 01716       | 58.2512 | 03463       | 28.8771 | 05212       | 19.1879 | 06963       | 14.3607 | 1  |
| 60 | 01746       | 57.2900 | 03492       | 28.6363 | 05241       | 19.0811 | 06993       | 14.3007 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 89 Degrees. |         | 88 Degrees. |         | 87 Degrees. |         | 86 Degrees. |         |    |



| M  | 4 Degrees.  |         | 5 Degrees.  |         | 6 Degrees.  |         | 7 Degrees.  |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 06993       | 14.3007 | 08749       | 11.4301 | 10510       | 9.51436 | 12278       | 8.14435 | 60 |
| 1  | 07022       | 14.2411 | 08778       | 11.3919 | 10540       | 9.48781 | 12308       | 8.12481 | 59 |
| 2  | 07051       | 14.1821 | 08807       | 11.3540 | 10569       | 9.46141 | 12338       | 8.10536 | 58 |
| 3  | 07080       | 14.1235 | 08837       | 11.3163 | 10599       | 9.43515 | 12367       | 8.08600 | 57 |
| 4  | 07110       | 14.0655 | 08866       | 11.2789 | 10628       | 9.40904 | 12397       | 8.06674 | 56 |
| 5  | 07139       | 14.0079 | 08895       | 11.2417 | 10657       | 9.38307 | 12426       | 8.04756 | 55 |
| 6  | 07168       | 13.9507 | 08925       | 11.2048 | 10687       | 9.35724 | 12456       | 8.02848 | 54 |
| 7  | 07197       | 13.8940 | 08954       | 11.1681 | 10716       | 9.33154 | 12485       | 8.00948 | 53 |
| 8  | 07227       | 13.8378 | 08983       | 11.1316 | 10746       | 9.30599 | 12515       | 7.99058 | 52 |
| 9  | 07256       | 13.7821 | 09013       | 11.0954 | 10775       | 9.28058 | 12544       | 7.97176 | 51 |
| 10 | 07285       | 13.7267 | 09042       | 11.0594 | 10805       | 9.25530 | 12574       | 7.95302 | 50 |
| 11 | 07314       | 13.6719 | 09071       | 11.0237 | 10834       | 9.23016 | 12603       | 7.93438 | 49 |
| 12 | 07344       | 13.6174 | 09101       | 10.9881 | 10863       | 9.20566 | 12633       | 7.91582 | 48 |
| 13 | 07373       | 13.5634 | 09130       | 10.9528 | 10893       | 9.18028 | 12662       | 7.89734 | 47 |
| 14 | 07402       | 13.5098 | 09159       | 10.9178 | 10922       | 9.15554 | 12692       | 7.87895 | 46 |
| 15 | 07431       | 13.4566 | 09189       | 10.8829 | 10952       | 9.13093 | 12722       | 7.86064 | 45 |
| 16 | 07461       | 13.4039 | 09218       | 10.8483 | 10981       | 9.10646 | 12751       | 7.84242 | 44 |
| 17 | 07490       | 13.3515 | 09247       | 10.8139 | 11011       | 9.08211 | 12781       | 7.82428 | 43 |
| 18 | 07519       | 13.2996 | 09277       | 10.7797 | 11040       | 9.05789 | 12810       | 7.80622 | 42 |
| 19 | 07548       | 13.2480 | 09306       | 10.7457 | 11070       | 9.03379 | 12840       | 7.78825 | 41 |
| 20 | 07578       | 13.1969 | 09335       | 10.7119 | 11099       | 9.00983 | 12869       | 7.77035 | 40 |
| 21 | 07607       | 13.1461 | 09365       | 10.6783 | 11128       | 8.98598 | 12899       | 7.75254 | 39 |
| 22 | 07636       | 13.0958 | 09394       | 10.6450 | 11158       | 8.96227 | 12929       | 7.73480 | 38 |
| 23 | 07665       | 13.0458 | 09423       | 10.6118 | 11187       | 8.93867 | 12958       | 7.71715 | 37 |
| 24 | 07695       | 12.9962 | 09453       | 10.5789 | 11217       | 8.91520 | 12988       | 7.69957 | 36 |
| 25 | 07724       | 12.9469 | 09482       | 10.5462 | 11246       | 8.89185 | 13017       | 7.68208 | 35 |
| 26 | 07753       | 12.8981 | 09511       | 10.5136 | 11276       | 8.86862 | 13047       | 7.66466 | 34 |
| 27 | 07782       | 12.8496 | 09541       | 10.4813 | 11305       | 8.84551 | 13076       | 7.64732 | 33 |
| 28 | 07812       | 12.8014 | 09570       | 10.4491 | 11335       | 8.82252 | 13106       | 7.63005 | 32 |
| 29 | 07841       | 12.7536 | 09600       | 10.4172 | 11364       | 8.79964 | 13136       | 7.61287 | 31 |
| 30 | 07870       | 12.7062 | 09629       | 10.3854 | 11394       | 8.77689 | 13165       | 7.59575 | 30 |
| 31 | 07899       | 12.6591 | 09658       | 10.3538 | 11423       | 8.75425 | 13195       | 7.57872 | 29 |
| 32 | 07929       | 12.6124 | 09688       | 10.3224 | 11452       | 8.73172 | 13224       | 7.56176 | 28 |
| 33 | 07958       | 12.5660 | 09717       | 10.2913 | 11482       | 8.70931 | 13254       | 7.54487 | 27 |
| 34 | 07987       | 12.5199 | 09746       | 10.2602 | 11511       | 8.68701 | 13284       | 7.52806 | 26 |
| 35 | 08017       | 12.4742 | 09776       | 10.2294 | 11541       | 8.66482 | 13313       | 7.51132 | 25 |
| 36 | 08046       | 12.4288 | 09805       | 10.1988 | 11570       | 8.64275 | 13343       | 7.49465 | 24 |
| 37 | 08075       | 12.3838 | 09834       | 10.1683 | 11600       | 8.62078 | 13372       | 7.47806 | 23 |
| 38 | 08104       | 12.3390 | 09864       | 10.1381 | 11629       | 8.59893 | 13402       | 7.46154 | 22 |
| 39 | 08134       | 12.2946 | 09893       | 10.1080 | 11659       | 8.57718 | 13432       | 7.44509 | 21 |
| 40 | 08163       | 12.2505 | 09923       | 10.0780 | 11688       | 8.55555 | 13461       | 7.42871 | 20 |
| 41 | 08192       | 12.2067 | 09952       | 10.0483 | 11718       | 8.53402 | 13491       | 7.41240 | 19 |
| 42 | 08221       | 12.1632 | 09981       | 10.0187 | 11747       | 8.51259 | 13521       | 7.39616 | 18 |
| 43 | 08251       | 12.1201 | 10011       | 9.98930 | 11777       | 8.49128 | 13550       | 7.37999 | 17 |
| 44 | 08280       | 12.0772 | 10040       | 9.96007 | 11806       | 8.47007 | 13580       | 7.36389 | 16 |
| 45 | 08309       | 12.0346 | 10069       | 9.93101 | 11836       | 8.44896 | 13609       | 7.34786 | 15 |
| 46 | 08339       | 11.9923 | 10099       | 9.90211 | 11865       | 8.42795 | 13639       | 7.33190 | 14 |
| 47 | 08368       | 11.9504 | 10128       | 9.87338 | 11895       | 8.40705 | 13669       | 7.31600 | 13 |
| 48 | 08397       | 11.9087 | 10158       | 9.84482 | 11924       | 8.38625 | 13698       | 7.30018 | 12 |
| 49 | 08427       | 11.8673 | 10187       | 9.81641 | 11954       | 8.36555 | 13728       | 7.28442 | 11 |
| 50 | 08456       | 11.8262 | 10216       | 9.78817 | 11983       | 8.34496 | 13758       | 7.26873 | 10 |
| 51 | 08485       | 11.7853 | 10246       | 9.76009 | 12013       | 8.32446 | 13787       | 7.25310 | 9  |
| 52 | 08514       | 11.7448 | 10275       | 9.73217 | 12042       | 8.30406 | 13817       | 7.23754 | 8  |
| 53 | 08544       | 11.7045 | 10305       | 9.70441 | 12072       | 8.28376 | 13846       | 7.22204 | 7  |
| 54 | 08573       | 11.6645 | 10334       | 9.67680 | 12101       | 8.26355 | 13876       | 7.20661 | 6  |
| 55 | 08602       | 11.6248 | 10363       | 9.64935 | 12131       | 8.24345 | 13906       | 7.19125 | 5  |
| 56 | 08632       | 11.5853 | 10393       | 9.62205 | 12160       | 8.22344 | 13935       | 7.17594 | 4  |
| 57 | 08661       | 11.5461 | 10422       | 9.59490 | 12190       | 8.20352 | 13965       | 7.16071 | 3  |
| 58 | 08690       | 11.5072 | 10452       | 9.56791 | 12219       | 8.18370 | 13995       | 7.14553 | 2  |
| 59 | 08720       | 11.4685 | 10481       | 9.54106 | 12249       | 8.16398 | 14024       | 7.13042 | 1  |
| 60 | 08749       | 11.4301 | 10510       | 9.51436 | 12278       | 8.14435 | 14054       | 7.11537 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 85 Degrees. |         | 84 Degrees. |         | 83 Degrees. |         | 82 Degrees. |         |    |



| M  | 8 Degrees.  |         | 9 Degrees.  |         | 10 Degrees. |         | 11 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 14054       | 7.11537 | 15838       | 6.31375 | 17633       | 5.67128 | 19438       | 5.14455 | 60 |
| 1  | 14084       | 7.10038 | 15868       | 6.30189 | 17663       | 5.66165 | 19468       | 5.13658 | 59 |
| 2  | 14113       | 7.08546 | 15898       | 6.29007 | 17693       | 5.65205 | 19498       | 5.12862 | 58 |
| 3  | 14143       | 7.07059 | 15928       | 6.27829 | 17723       | 5.64248 | 19529       | 5.12069 | 57 |
| 4  | 14173       | 7.05579 | 15958       | 6.26655 | 17753       | 5.63295 | 19559       | 5.11279 | 56 |
| 5  | 14202       | 7.04105 | 15988       | 6.25486 | 17783       | 5.62344 | 19589       | 5.10490 | 55 |
| 6  | 14232       | 7.02637 | 16017       | 6.24321 | 17813       | 5.61397 | 19619       | 5.09704 | 54 |
| 7  | 14262       | 7.01174 | 16047       | 6.23160 | 17843       | 5.60452 | 19649       | 5.08921 | 53 |
| 8  | 14291       | 6.99718 | 16077       | 6.22003 | 17873       | 5.59511 | 19680       | 5.08139 | 52 |
| 9  | 14321       | 6.98268 | 16107       | 6.20851 | 17903       | 5.58573 | 19710       | 5.07360 | 51 |
| 10 | 14351       | 6.96823 | 16137       | 6.19703 | 17933       | 5.57638 | 19740       | 5.06584 | 50 |
| 11 | 14381       | 6.95385 | 16167       | 6.18559 | 17963       | 5.56706 | 19770       | 5.05809 | 49 |
| 12 | 14410       | 6.93952 | 16196       | 6.17419 | 17993       | 5.55777 | 19801       | 5.05037 | 48 |
| 13 | 14440       | 6.92525 | 16226       | 6.16283 | 18023       | 5.54851 | 19831       | 5.04267 | 47 |
| 14 | 14470       | 6.91104 | 16256       | 6.15151 | 18053       | 5.53927 | 19861       | 5.03499 | 46 |
| 15 | 14499       | 6.89688 | 16286       | 6.14023 | 18083       | 5.53007 | 19891       | 5.02734 | 45 |
| 16 | 14529       | 6.88278 | 16316       | 6.12899 | 18113       | 5.52090 | 19921       | 5.01971 | 44 |
| 17 | 14559       | 6.86874 | 16346       | 6.11779 | 18143       | 5.51176 | 19952       | 5.01210 | 43 |
| 18 | 14588       | 6.85475 | 16376       | 6.10664 | 18173       | 5.50264 | 19982       | 5.00451 | 42 |
| 19 | 14618       | 6.84082 | 16405       | 6.09552 | 18203       | 5.49356 | 20012       | 4.99695 | 41 |
| 20 | 14648       | 6.82694 | 16435       | 6.08444 | 18233       | 5.48451 | 20042       | 4.98940 | 40 |
| 21 | 14678       | 6.81312 | 16465       | 6.07340 | 18263       | 5.47548 | 20073       | 4.98188 | 39 |
| 22 | 14707       | 6.79936 | 16495       | 6.06240 | 18293       | 5.46648 | 20103       | 4.97438 | 38 |
| 23 | 14737       | 6.78564 | 16525       | 6.05143 | 18323       | 5.45751 | 20133       | 4.96690 | 37 |
| 24 | 14767       | 6.77199 | 16555       | 6.04051 | 18353       | 5.44857 | 20164       | 4.95945 | 36 |
| 25 | 14796       | 6.75838 | 16585       | 6.02962 | 18383       | 5.43966 | 20194       | 4.95201 | 35 |
| 26 | 14826       | 6.74483 | 16615       | 6.01878 | 18414       | 5.43077 | 20224       | 4.94460 | 34 |
| 27 | 14856       | 6.73133 | 16645       | 6.00797 | 18444       | 5.42192 | 20254       | 4.93721 | 33 |
| 28 | 14886       | 6.71789 | 16674       | 5.99720 | 18474       | 5.41309 | 20285       | 4.92984 | 32 |
| 29 | 14915       | 6.70450 | 16704       | 5.98646 | 18504       | 5.40429 | 20315       | 4.92249 | 31 |
| 30 | 14945       | 6.69116 | 16734       | 5.97576 | 18534       | 5.39552 | 20345       | 4.91516 | 30 |
| 31 | 14975       | 6.67787 | 16764       | 5.96510 | 18564       | 5.38677 | 20376       | 4.90785 | 29 |
| 32 | 15005       | 6.66463 | 16794       | 5.95448 | 18594       | 5.37805 | 20406       | 4.90056 | 28 |
| 33 | 15034       | 6.65144 | 16824       | 5.94330 | 18624       | 5.36936 | 20436       | 4.89330 | 27 |
| 34 | 15064       | 6.63831 | 16854       | 5.93335 | 18654       | 5.36070 | 20466       | 4.88605 | 26 |
| 35 | 15094       | 6.62523 | 16884       | 5.92283 | 18684       | 5.35206 | 20497       | 4.87882 | 25 |
| 36 | 15124       | 6.61219 | 16914       | 5.91235 | 18714       | 5.34345 | 20527       | 4.87162 | 24 |
| 37 | 15153       | 6.59921 | 16944       | 5.90191 | 18745       | 5.33487 | 20557       | 4.86444 | 23 |
| 38 | 15183       | 6.58627 | 16974       | 5.89151 | 18775       | 5.32631 | 20588       | 4.85727 | 22 |
| 39 | 15213       | 6.57339 | 17004       | 5.88114 | 18805       | 5.31778 | 20618       | 4.85013 | 21 |
| 40 | 15243       | 6.56055 | 17033       | 5.87080 | 18835       | 5.30928 | 20648       | 4.84300 | 20 |
| 41 | 15272       | 6.54777 | 17063       | 5.86051 | 18865       | 5.30080 | 20679       | 4.83590 | 19 |
| 42 | 15302       | 6.53503 | 17093       | 5.85024 | 18895       | 5.29235 | 20709       | 4.82882 | 18 |
| 43 | 15332       | 6.52234 | 17123       | 5.84001 | 18925       | 5.28393 | 20739       | 4.82175 | 17 |
| 44 | 15362       | 6.50970 | 17153       | 5.82982 | 18955       | 5.27553 | 20770       | 4.81471 | 16 |
| 45 | 15391       | 6.49710 | 17183       | 5.81966 | 18986       | 5.26715 | 20800       | 4.80769 | 15 |
| 46 | 15421       | 6.48456 | 17213       | 5.80953 | 19016       | 5.25880 | 20830       | 4.80068 | 14 |
| 47 | 15451       | 6.47206 | 17243       | 5.79944 | 19046       | 5.25048 | 20861       | 4.79370 | 13 |
| 48 | 15481       | 6.45961 | 17273       | 5.78938 | 19076       | 5.24218 | 20891       | 4.78673 | 12 |
| 49 | 15511       | 6.44720 | 17303       | 5.77936 | 19106       | 5.23391 | 20921       | 4.77978 | 11 |
| 50 | 15540       | 6.43484 | 17333       | 5.76937 | 19136       | 5.22566 | 20952       | 4.77286 | 10 |
| 51 | 15570       | 6.42253 | 17363       | 5.75941 | 19166       | 5.21744 | 20982       | 4.76595 | 9  |
| 52 | 15600       | 6.41026 | 17393       | 5.74949 | 19197       | 5.20925 | 21013       | 4.75906 | 8  |
| 53 | 15630       | 6.39804 | 17423       | 5.73960 | 19227       | 5.20107 | 21043       | 4.75219 | 7  |
| 54 | 15660       | 6.38587 | 17453       | 5.72974 | 19257       | 5.19293 | 21073       | 4.74534 | 6  |
| 55 | 15689       | 6.37374 | 17483       | 5.71992 | 19287       | 5.18480 | 21104       | 4.73851 | 5  |
| 56 | 15719       | 6.36165 | 17513       | 5.71013 | 19317       | 5.17671 | 21134       | 4.73170 | 4  |
| 57 | 15749       | 6.34961 | 17543       | 5.70037 | 19347       | 5.16863 | 21164       | 4.72490 | 3  |
| 58 | 15779       | 6.33761 | 17573       | 5.69064 | 19378       | 5.16058 | 21195       | 4.71813 | 2  |
| 59 | 15809       | 6.32566 | 17603       | 5.68094 | 19408       | 5.15256 | 21225       | 4.71137 | 1  |
| 60 | 15838       | 6.31375 | 17633       | 5.67128 | 19438       | 5.14455 | 21256       | 4.70463 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 81 Degrees. |         | 80 Degrees. |         | 79 Degrees. |         | 78 Degrees. |         |    |

| M  | 12 Degrees. |         | 13 Degrees. |         | 14 Degrees. |         | 15 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 21256       | 4.70463 | 23087       | 4.33148 | 24933       | 4.01078 | 26795       | 3.73205 | 60 |
| 1  | 21286       | 4.69791 | 23117       | 4.32573 | 24964       | 4.00582 | 26826       | 3.72771 | 59 |
| 2  | 21316       | 4.69121 | 23148       | 4.32001 | 24995       | 4.00086 | 26857       | 3.72338 | 58 |
| 3  | 21347       | 4.68452 | 23179       | 4.31430 | 25026       | 3.99592 | 26888       | 3.71907 | 57 |
| 4  | 21377       | 4.67786 | 23209       | 4.30860 | 25056       | 3.99099 | 26920       | 3.71477 | 56 |
| 5  | 21408       | 4.67121 | 23240       | 4.30291 | 25087       | 3.98607 | 26951       | 3.71046 | 55 |
| 6  | 21438       | 4.66458 | 23271       | 4.29724 | 25118       | 3.98117 | 26982       | 3.70616 | 54 |
| 7  | 21469       | 4.65797 | 23301       | 4.29159 | 25149       | 3.97627 | 27013       | 3.70188 | 53 |
| 8  | 21499       | 4.65138 | 23332       | 4.28595 | 25180       | 3.97139 | 27044       | 3.69761 | 52 |
| 9  | 21529       | 4.64480 | 23363       | 4.28032 | 25211       | 3.96651 | 27076       | 3.69335 | 51 |
| 10 | 21560       | 4.63825 | 23393       | 4.27471 | 25242       | 3.96165 | 27107       | 3.68909 | 50 |
| 11 | 21590       | 4.63171 | 23424       | 4.26911 | 25273       | 3.95680 | 27138       | 3.68485 | 49 |
| 12 | 21621       | 4.62518 | 23455       | 4.26352 | 25304       | 3.95196 | 27169       | 3.68061 | 48 |
| 13 | 21651       | 4.61868 | 23484       | 4.25795 | 25335       | 3.94713 | 27201       | 3.67638 | 47 |
| 14 | 21682       | 4.61219 | 23516       | 4.25239 | 25366       | 3.94232 | 27232       | 3.67217 | 46 |
| 15 | 21712       | 4.60572 | 23547       | 4.24685 | 25397       | 3.93751 | 27263       | 3.66796 | 45 |
| 16 | 21743       | 4.59927 | 23578       | 4.24132 | 25428       | 3.93271 | 27294       | 3.66376 | 44 |
| 17 | 21773       | 4.59283 | 23608       | 4.23580 | 25459       | 3.92793 | 27326       | 3.65957 | 43 |
| 18 | 21804       | 4.58641 | 23639       | 4.23030 | 25490       | 3.92316 | 27357       | 3.65538 | 42 |
| 19 | 21834       | 4.58001 | 23670       | 4.22481 | 25521       | 3.91839 | 27388       | 3.65121 | 41 |
| 20 | 21864       | 4.57363 | 23700       | 4.21933 | 25552       | 3.91364 | 27419       | 3.64705 | 40 |
| 21 | 21895       | 4.56726 | 23731       | 4.21387 | 25583       | 3.90890 | 27451       | 3.64289 | 39 |
| 22 | 21925       | 4.56091 | 23762       | 4.20842 | 25614       | 3.90417 | 27482       | 3.63874 | 38 |
| 23 | 21956       | 4.55458 | 23793       | 4.20298 | 25645       | 3.89945 | 27513       | 3.63461 | 37 |
| 24 | 21986       | 4.54826 | 23823       | 4.19756 | 25676       | 3.89474 | 27545       | 3.63048 | 36 |
| 25 | 22017       | 4.54196 | 23854       | 4.19215 | 25707       | 3.89004 | 27576       | 3.62636 | 35 |
| 26 | 22047       | 4.53568 | 23885       | 4.18675 | 25738       | 3.88536 | 27607       | 3.62224 | 34 |
| 27 | 22078       | 4.52941 | 23916       | 4.18137 | 25769       | 3.88068 | 27638       | 3.61814 | 33 |
| 28 | 22108       | 4.52316 | 23946       | 4.17600 | 25800       | 3.87601 | 27670       | 3.61405 | 32 |
| 29 | 22139       | 4.51693 | 23977       | 4.17064 | 25831       | 3.87136 | 27701       | 3.60996 | 31 |
| 30 | 22169       | 4.51071 | 24008       | 4.16530 | 25862       | 3.86671 | 27732       | 3.60588 | 30 |
| 31 | 22200       | 4.50451 | 24039       | 4.15997 | 25893       | 3.86208 | 27764       | 3.60181 | 29 |
| 32 | 22231       | 4.49832 | 24069       | 4.15465 | 25924       | 3.85745 | 27795       | 3.59775 | 28 |
| 33 | 22261       | 4.49215 | 24100       | 4.14934 | 25955       | 3.85284 | 27826       | 3.59370 | 27 |
| 34 | 22292       | 4.48600 | 24131       | 4.14405 | 25986       | 3.84824 | 27858       | 3.58966 | 26 |
| 35 | 22322       | 4.47986 | 24162       | 4.13877 | 26017       | 3.84364 | 27889       | 3.58562 | 25 |
| 36 | 22353       | 4.47374 | 24193       | 4.13350 | 26048       | 3.83904 | 27920       | 3.58160 | 24 |
| 37 | 22383       | 4.46764 | 24223       | 4.12825 | 26079       | 3.83449 | 27952       | 3.57758 | 23 |
| 38 | 22414       | 4.46155 | 24254       | 4.12301 | 26110       | 3.82992 | 27983       | 3.57357 | 22 |
| 39 | 22444       | 4.45548 | 24285       | 4.11778 | 26141       | 3.82537 | 28015       | 3.56957 | 21 |
| 40 | 22475       | 4.44942 | 24316       | 4.11256 | 26172       | 3.82083 | 28046       | 3.56557 | 20 |
| 41 | 22505       | 4.44338 | 24347       | 4.10736 | 26203       | 3.81630 | 28077       | 3.56159 | 19 |
| 42 | 22536       | 4.43735 | 24377       | 4.10216 | 26235       | 3.81177 | 28109       | 3.55761 | 18 |
| 43 | 22567       | 4.43134 | 24408       | 4.09699 | 26266       | 3.80726 | 28140       | 3.55364 | 17 |
| 44 | 22597       | 4.42534 | 24439       | 4.09182 | 26297       | 3.80276 | 28172       | 3.54968 | 16 |
| 45 | 22628       | 4.41936 | 24470       | 4.08666 | 26328       | 3.79827 | 28203       | 3.54573 | 15 |
| 46 | 22658       | 4.41340 | 24501       | 4.08152 | 26359       | 3.79378 | 28234       | 3.54179 | 14 |
| 47 | 22689       | 4.40745 | 24532       | 4.07639 | 26390       | 3.78931 | 28266       | 3.53785 | 13 |
| 48 | 22719       | 4.40152 | 24562       | 4.07127 | 26421       | 3.78485 | 28297       | 3.53393 | 12 |
| 49 | 22750       | 4.39560 | 24593       | 4.06616 | 26452       | 3.78040 | 28329       | 3.53001 | 11 |
| 50 | 22781       | 4.38969 | 24624       | 4.06107 | 26483       | 3.77595 | 28360       | 3.52609 | 10 |
| 51 | 22811       | 4.38381 | 24655       | 4.05599 | 26515       | 3.77152 | 28391       | 3.52219 | 9  |
| 52 | 22842       | 4.37793 | 24686       | 4.05092 | 26546       | 3.76709 | 28423       | 3.51829 | 8  |
| 53 | 22872       | 4.37207 | 24717       | 4.04586 | 26577       | 3.76268 | 28454       | 3.51441 | 7  |
| 54 | 22903       | 4.36623 | 24747       | 4.04081 | 26608       | 3.75828 | 28486       | 3.51053 | 6  |
| 55 | 22934       | 4.36040 | 24778       | 4.03578 | 26639       | 3.75388 | 28517       | 3.50666 | 5  |
| 56 | 22964       | 4.35459 | 24809       | 4.03075 | 26670       | 3.74950 | 28549       | 3.50279 | 4  |
| 57 | 22995       | 4.34879 | 24840       | 4.02574 | 26701       | 3.74512 | 28580       | 3.49894 | 3  |
| 58 | 23026       | 4.34300 | 24871       | 4.02074 | 26733       | 3.74075 | 28612       | 3.49509 | 2  |
| 59 | 23056       | 4.33723 | 24902       | 4.01576 | 26764       | 3.73644 | 28643       | 3.49125 | 1  |
| 60 | 23087       | 4.33148 | 24933       | 4.01078 | 26795       | 3.73205 | 28675       | 3.48741 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 77 Degrees. |         | 76 Degrees. |         | 75 Degrees. |         | 74 Degrees. |         |    |

| M  | 16 Degrees. |         | 17 Degrees. |         | 18 Degrees. |         | 19 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 28675       | 3.48741 | 30573       | 3.27085 | 32492       | 3.07768 | 34433       | 2.90421 | 60 |
| 1  | 28706       | 3.48359 | 30605       | 3.26745 | 32524       | 3.07464 | 34465       | 2.90147 | 59 |
| 2  | 28738       | 3.47977 | 30637       | 3.26406 | 32556       | 3.07160 | 34498       | 2.89873 | 58 |
| 3  | 28769       | 3.47595 | 30669       | 3.26067 | 32588       | 3.06857 | 34530       | 2.89600 | 57 |
| 4  | 28800       | 3.47213 | 30700       | 3.25729 | 32621       | 3.06554 | 34563       | 2.89327 | 56 |
| 5  | 28832       | 3.46831 | 30732       | 3.25392 | 32653       | 3.06252 | 34596       | 2.89055 | 55 |
| 6  | 28864       | 3.46448 | 30764       | 3.25055 | 32685       | 3.05950 | 34628       | 2.88783 | 54 |
| 7  | 28895       | 3.46066 | 30796       | 3.24719 | 32717       | 3.05649 | 34661       | 2.88511 | 53 |
| 8  | 28927       | 3.45683 | 30828       | 3.24383 | 32749       | 3.05349 | 34693       | 2.88240 | 52 |
| 9  | 28958       | 3.45301 | 30860       | 3.24049 | 32782       | 3.05049 | 34726       | 2.87970 | 51 |
| 10 | 28990       | 3.44919 | 30891       | 3.23714 | 32814       | 3.04749 | 34758       | 2.87700 | 50 |
| 11 | 29021       | 3.44537 | 30923       | 3.23381 | 32846       | 3.04450 | 34791       | 2.87430 | 49 |
| 12 | 29053       | 3.44154 | 30955       | 3.23048 | 32878       | 3.04152 | 34824       | 2.87161 | 48 |
| 13 | 29084       | 3.43772 | 30987       | 3.22715 | 32911       | 3.03854 | 34856       | 2.86892 | 47 |
| 14 | 29116       | 3.43390 | 31019       | 3.22384 | 32943       | 3.03556 | 34889       | 2.86624 | 46 |
| 15 | 29147       | 3.43008 | 31051       | 3.22053 | 32975       | 3.03260 | 34922       | 2.86356 | 45 |
| 16 | 29179       | 3.42626 | 31083       | 3.21722 | 33007       | 3.02963 | 34954       | 2.86089 | 44 |
| 17 | 29210       | 3.42244 | 31115       | 3.21392 | 33040       | 3.02667 | 34987       | 2.85822 | 43 |
| 18 | 29242       | 3.41862 | 31147       | 3.21063 | 33072       | 3.02372 | 35019       | 2.85555 | 42 |
| 19 | 29274       | 3.41480 | 31178       | 3.20734 | 33104       | 3.02077 | 35052       | 2.85289 | 41 |
| 20 | 29305       | 3.41098 | 31210       | 3.20406 | 33136       | 3.01783 | 35085       | 2.85023 | 40 |
| 21 | 29337       | 3.40716 | 31242       | 3.20079 | 33169       | 3.01489 | 35117       | 2.84758 | 39 |
| 22 | 29368       | 3.40334 | 31274       | 3.19752 | 33201       | 3.01196 | 35150       | 2.84494 | 38 |
| 23 | 29400       | 3.40136 | 31306       | 3.19426 | 33233       | 3.00903 | 35183       | 2.84229 | 37 |
| 24 | 29432       | 3.39771 | 31338       | 3.19100 | 33266       | 3.00611 | 35216       | 2.83965 | 36 |
| 25 | 29463       | 3.39406 | 31370       | 3.18775 | 33298       | 3.00319 | 35248       | 2.83702 | 35 |
| 26 | 29495       | 3.39042 | 31402       | 3.18451 | 33330       | 3.00028 | 35281       | 2.83439 | 34 |
| 27 | 29526       | 3.38677 | 31434       | 3.18127 | 33363       | 2.99738 | 35314       | 2.83176 | 33 |
| 28 | 29558       | 3.38317 | 31466       | 3.17804 | 33395       | 2.99447 | 35346       | 2.82914 | 32 |
| 29 | 29590       | 3.37955 | 31498       | 3.17481 | 33427       | 2.99158 | 35379       | 2.82653 | 31 |
| 30 | 29621       | 3.37594 | 31530       | 3.17159 | 33460       | 2.98868 | 35412       | 2.82391 | 30 |
| 31 | 29653       | 3.37234 | 31562       | 3.16838 | 33492       | 2.98580 | 35445       | 2.82130 | 29 |
| 32 | 29685       | 3.36875 | 31594       | 3.16517 | 33524       | 2.98292 | 35477       | 2.81870 | 28 |
| 33 | 29716       | 3.36516 | 31626       | 3.16197 | 33557       | 2.98004 | 35510       | 2.81610 | 27 |
| 34 | 29748       | 3.36158 | 31658       | 3.15877 | 33589       | 2.97717 | 35543       | 2.81350 | 26 |
| 35 | 29780       | 3.35800 | 31690       | 3.15558 | 33621       | 2.97430 | 35576       | 2.81091 | 25 |
| 36 | 29811       | 3.35443 | 31722       | 3.15240 | 33654       | 2.97144 | 35608       | 2.80833 | 24 |
| 37 | 29843       | 3.35087 | 31754       | 3.14922 | 33686       | 2.96858 | 35641       | 2.80574 | 23 |
| 38 | 29875       | 3.34732 | 31786       | 3.14605 | 33718       | 2.96573 | 35674       | 2.80316 | 22 |
| 39 | 29906       | 3.34377 | 31818       | 3.14288 | 33751       | 2.96288 | 35707       | 2.80059 | 21 |
| 40 | 29938       | 3.34023 | 31850       | 3.13972 | 33783       | 2.96004 | 35740       | 2.79802 | 20 |
| 41 | 29970       | 3.33670 | 31882       | 3.13656 | 33816       | 2.95720 | 35772       | 2.79545 | 19 |
| 42 | 30001       | 3.33317 | 31914       | 3.13341 | 33848       | 2.95437 | 35805       | 2.79289 | 18 |
| 43 | 30033       | 3.32965 | 31946       | 3.13027 | 33881       | 2.95155 | 35838       | 2.79033 | 17 |
| 44 | 30065       | 3.32614 | 31978       | 3.12713 | 33913       | 2.94872 | 35871       | 2.78778 | 16 |
| 45 | 30097       | 3.32264 | 32010       | 3.12400 | 33945       | 2.94590 | 35904       | 2.78523 | 15 |
| 46 | 30128       | 3.31914 | 32042       | 3.12087 | 33978       | 2.94309 | 35937       | 2.78269 | 14 |
| 47 | 30160       | 3.31565 | 32074       | 3.11775 | 34010       | 2.94028 | 35970       | 2.78014 | 13 |
| 48 | 30192       | 3.31216 | 32106       | 3.11464 | 34043       | 2.93748 | 36002       | 2.77761 | 12 |
| 49 | 30224       | 3.30868 | 32139       | 3.11153 | 34075       | 2.93468 | 36035       | 2.77507 | 11 |
| 50 | 30255       | 3.30521 | 32171       | 3.10842 | 34108       | 2.93189 | 36068       | 2.77254 | 10 |
| 51 | 30287       | 3.30174 | 32203       | 3.10532 | 34140       | 2.92910 | 36101       | 2.77002 | 9  |
| 52 | 30319       | 3.29829 | 32235       | 3.10223 | 34173       | 2.92632 | 36134       | 2.76750 | 8  |
| 53 | 30351       | 3.29483 | 32267       | 3.09914 | 34205       | 2.92354 | 36167       | 2.76498 | 7  |
| 54 | 30382       | 3.29139 | 32299       | 3.09606 | 34238       | 2.92076 | 36199       | 2.76247 | 6  |
| 55 | 30414       | 3.28795 | 32331       | 3.09298 | 34270       | 2.91799 | 36232       | 2.75996 | 5  |
| 56 | 30445       | 3.28452 | 32363       | 3.08991 | 34303       | 2.91523 | 36265       | 2.75746 | 4  |
| 57 | 30478       | 3.28109 | 32396       | 3.08685 | 34335       | 2.91246 | 36298       | 2.75496 | 3  |
| 58 | 30509       | 3.27767 | 32428       | 3.08379 | 34368       | 2.90971 | 36331       | 2.75246 | 2  |
| 59 | 30541       | 3.27426 | 32460       | 3.08073 | 34400       | 2.90696 | 36364       | 2.74997 | 1  |
| 60 | 30573       | 3.27085 | 32492       | 3.07768 | 34433       | 2.90421 | 36397       | 2.74748 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 73 Degrees. |         | 72 Degrees. |         | 71 Degrees. |         | 70 Degrees. |         |    |



| M  | 20 Degrees. |         | 21 Degrees. |         | 22 Degrees. |         | 23 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 36397       | 2.74748 | 38386       | 2.60509 | 40403       | 2.47509 | 42447       | 2.35585 | 60 |
| 1  | 36430       | 2.74499 | 38420       | 2.60283 | 40436       | 2.47302 | 42482       | 2.35395 | 59 |
| 2  | 36463       | 2.74251 | 38453       | 2.60057 | 40470       | 2.47095 | 42516       | 2.35205 | 58 |
| 3  | 36496       | 2.74004 | 38487       | 2.59831 | 40504       | 2.46888 | 42551       | 2.35015 | 57 |
| 4  | 36529       | 2.73756 | 38520       | 2.59606 | 40538       | 2.46682 | 42585       | 2.34825 | 56 |
| 5  | 36562       | 2.73509 | 38553       | 2.59381 | 40572       | 2.46476 | 42619       | 2.34636 | 55 |
| 6  | 36595       | 2.73263 | 38587       | 2.59156 | 40606       | 2.46270 | 42654       | 2.34447 | 54 |
| 7  | 36628       | 2.73017 | 38620       | 2.58932 | 40640       | 2.46065 | 42688       | 2.34258 | 53 |
| 8  | 36661       | 2.72771 | 38654       | 2.58707 | 40674       | 2.45860 | 42722       | 2.34069 | 52 |
| 9  | 36694       | 2.72526 | 38687       | 2.58484 | 40707       | 2.45655 | 42757       | 2.33881 | 51 |
| 10 | 36727       | 2.72281 | 38721       | 2.58261 | 40741       | 2.45451 | 42791       | 2.33693 | 50 |
| 11 | 36760       | 2.72036 | 38754       | 2.58038 | 40775       | 2.45246 | 42826       | 2.33505 | 49 |
| 12 | 36793       | 2.71792 | 38787       | 2.57815 | 40809       | 2.45043 | 42860       | 2.33317 | 48 |
| 13 | 36826       | 2.71548 | 38821       | 2.57593 | 40843       | 2.44839 | 42894       | 2.33130 | 47 |
| 14 | 36859       | 2.71305 | 38854       | 2.57371 | 40877       | 2.44636 | 42929       | 2.32943 | 46 |
| 15 | 36892       | 2.71062 | 38888       | 2.57150 | 40911       | 2.44433 | 42963       | 2.32756 | 45 |
| 16 | 36925       | 2.70819 | 38921       | 2.56928 | 40945       | 2.44230 | 42998       | 2.32570 | 44 |
| 17 | 36958       | 2.70577 | 38955       | 2.56707 | 40979       | 2.44027 | 43032       | 2.32383 | 43 |
| 18 | 36991       | 2.70335 | 38988       | 2.56487 | 41013       | 2.43825 | 43067       | 2.32197 | 42 |
| 19 | 37024       | 2.70094 | 39022       | 2.56266 | 41047       | 2.43623 | 43101       | 2.32012 | 41 |
| 20 | 37057       | 2.69853 | 39055       | 2.56046 | 41081       | 2.43422 | 43136       | 2.31826 | 40 |
| 21 | 37090       | 2.69612 | 39089       | 2.55827 | 41115       | 2.43220 | 43170       | 2.31641 | 39 |
| 22 | 37123       | 2.69371 | 39122       | 2.55608 | 41149       | 2.43019 | 43205       | 2.31456 | 38 |
| 23 | 37157       | 2.69131 | 39156       | 2.55389 | 41183       | 2.42819 | 43239       | 2.31271 | 37 |
| 24 | 37190       | 2.68892 | 39190       | 2.55170 | 41217       | 2.42618 | 43274       | 2.31086 | 36 |
| 25 | 37223       | 2.68653 | 39223       | 2.54952 | 41251       | 2.42418 | 43308       | 2.30902 | 35 |
| 26 | 37256       | 2.68414 | 39257       | 2.54734 | 41285       | 2.42218 | 43343       | 2.30718 | 34 |
| 27 | 37289       | 2.68175 | 39290       | 2.54516 | 41319       | 2.42019 | 43378       | 2.30534 | 33 |
| 28 | 37322       | 2.67937 | 39324       | 2.54299 | 41353       | 2.41819 | 43412       | 2.30351 | 32 |
| 29 | 37355       | 2.67700 | 39357       | 2.54082 | 41387       | 2.41620 | 43447       | 2.30167 | 31 |
| 30 | 37388       | 2.67462 | 39391       | 2.53865 | 41421       | 2.41421 | 43481       | 2.29984 | 30 |
| 31 | 37422       | 2.67225 | 39425       | 2.53648 | 41455       | 2.41223 | 43516       | 2.29801 | 29 |
| 32 | 37455       | 2.66989 | 39458       | 2.53432 | 41490       | 2.41025 | 43550       | 2.29619 | 28 |
| 33 | 37488       | 2.66752 | 39492       | 2.53217 | 41524       | 2.40827 | 43585       | 2.29437 | 27 |
| 34 | 37521       | 2.66516 | 39526       | 2.53001 | 41558       | 2.40629 | 43620       | 2.29254 | 26 |
| 35 | 37554       | 2.66281 | 39559       | 2.52786 | 41592       | 2.40432 | 43654       | 2.29073 | 25 |
| 36 | 37588       | 2.66046 | 39593       | 2.52571 | 41626       | 2.40235 | 43689       | 2.28891 | 24 |
| 37 | 37621       | 2.65811 | 39626       | 2.52357 | 41660       | 2.40038 | 43724       | 2.28710 | 23 |
| 38 | 37654       | 2.65576 | 39660       | 2.52142 | 41694       | 2.39841 | 43758       | 2.28528 | 22 |
| 39 | 37687       | 2.65342 | 39694       | 2.51929 | 41728       | 2.39645 | 43793       | 2.28348 | 21 |
| 40 | 37720       | 2.65109 | 39727       | 2.51715 | 41763       | 2.39449 | 43828       | 2.28167 | 20 |
| 41 | 37754       | 2.64875 | 39761       | 2.51502 | 41797       | 2.39253 | 43862       | 2.27987 | 19 |
| 42 | 37787       | 2.64642 | 39795       | 2.51289 | 41831       | 2.39058 | 43897       | 2.27806 | 18 |
| 43 | 37820       | 2.64410 | 39829       | 2.51076 | 41865       | 2.38862 | 43932       | 2.27626 | 17 |
| 44 | 37853       | 2.64177 | 39862       | 2.50864 | 41899       | 2.38668 | 43966       | 2.27447 | 16 |
| 45 | 37887       | 2.63945 | 39896       | 2.50652 | 41933       | 2.38473 | 44001       | 2.27267 | 15 |
| 46 | 37920       | 2.63714 | 39930       | 2.50440 | 41968       | 2.38279 | 44036       | 2.27088 | 14 |
| 47 | 37953       | 2.63483 | 39963       | 2.50229 | 42002       | 2.38084 | 44071       | 2.26909 | 13 |
| 48 | 37986       | 2.63252 | 39997       | 2.50018 | 42036       | 2.37891 | 44105       | 2.26730 | 12 |
| 49 | 38020       | 2.63021 | 40031       | 2.49807 | 42070       | 2.37697 | 44140       | 2.26552 | 11 |
| 50 | 38053       | 2.62791 | 40065       | 2.49597 | 42105       | 2.37504 | 44175       | 2.26374 | 10 |
| 51 | 38086       | 2.62561 | 40098       | 2.49386 | 42139       | 2.37311 | 44210       | 2.26196 | 9  |
| 52 | 38120       | 2.62332 | 40132       | 2.49177 | 42173       | 2.37118 | 44244       | 2.26018 | 8  |
| 53 | 38153       | 2.62103 | 40166       | 2.48967 | 42207       | 2.36925 | 44279       | 2.25840 | 7  |
| 54 | 38186       | 2.61874 | 40200       | 2.48758 | 42242       | 2.36733 | 44314       | 2.25663 | 6  |
| 55 | 38220       | 2.61646 | 40234       | 2.48549 | 42276       | 2.36541 | 44349       | 2.25486 | 5  |
| 56 | 38253       | 2.61418 | 40267       | 2.48340 | 42310       | 2.36349 | 44384       | 2.25309 | 4  |
| 57 | 38286       | 2.61190 | 40301       | 2.48132 | 42345       | 2.36158 | 44418       | 2.25132 | 3  |
| 58 | 38320       | 2.60963 | 40335       | 2.47924 | 42379       | 2.35967 | 44453       | 2.24956 | 2  |
| 59 | 38353       | 2.60736 | 40369       | 2.47716 | 42413       | 2.35776 | 44488       | 2.24780 | 1  |
| 60 | 38386       | 2.60509 | 40403       | 2.47509 | 42447       | 2.35585 | 44523       | 2.24604 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 69 Degrees. |         | 68 Degrees. |         | 67 Degrees. |         | 66 Degrees. |         |    |



| M  | 24 Degrees. |         | 25 Degrees. |         | 26 Degrees. |         | 27 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 44523       | 2.24604 | 46631       | 2.14451 | 48773       | 2.05030 | 50953       | 1.96261 | 60 |
| 1  | 44558       | 2.24428 | 46666       | 2.14288 | 48809       | 2.04879 | 50989       | 1.96120 | 59 |
| 2  | 44593       | 2.24252 | 46702       | 2.14125 | 48845       | 2.04728 | 51026       | 1.95979 | 58 |
| 3  | 44627       | 2.24077 | 46737       | 2.13963 | 48881       | 2.04577 | 51063       | 1.95838 | 57 |
| 4  | 44662       | 2.23902 | 46772       | 2.13801 | 48917       | 2.04426 | 51099       | 1.95698 | 56 |
| 5  | 44697       | 2.23727 | 46808       | 2.13639 | 48953       | 2.04276 | 51136       | 1.95557 | 55 |
| 6  | 44732       | 2.23553 | 46843       | 2.13477 | 48989       | 2.04125 | 51173       | 1.95417 | 54 |
| 7  | 44767       | 2.23378 | 46879       | 2.13316 | 49026       | 2.03975 | 51209       | 1.95277 | 53 |
| 8  | 44802       | 2.23204 | 46914       | 2.13154 | 49062       | 2.03825 | 51246       | 1.95137 | 52 |
| 9  | 44837       | 2.23030 | 46950       | 2.12993 | 49098       | 2.03675 | 51283       | 1.94997 | 51 |
| 10 | 44872       | 2.22857 | 46985       | 2.12832 | 49134       | 2.03526 | 51319       | 1.94858 | 50 |
| 11 | 44907       | 2.22683 | 47021       | 2.12671 | 49170       | 2.03376 | 51356       | 1.94718 | 49 |
| 12 | 44942       | 2.22510 | 47056       | 2.12511 | 49206       | 2.03227 | 51393       | 1.94579 | 48 |
| 13 | 44977       | 2.22337 | 47092       | 2.12350 | 49242       | 2.03078 | 51430       | 1.94440 | 47 |
| 14 | 45012       | 2.22164 | 47128       | 2.12190 | 49278       | 2.02929 | 51467       | 1.94301 | 46 |
| 15 | 45047       | 2.21992 | 47163       | 2.12030 | 49315       | 2.02780 | 51503       | 1.94162 | 45 |
| 16 | 45082       | 2.21819 | 47199       | 2.11871 | 49351       | 2.02631 | 51540       | 1.94023 | 44 |
| 17 | 45117       | 2.21647 | 47234       | 2.11711 | 49387       | 2.02483 | 51577       | 1.93885 | 43 |
| 18 | 45152       | 2.21475 | 47270       | 2.11552 | 49423       | 2.02335 | 51614       | 1.93746 | 42 |
| 19 | 45187       | 2.21304 | 47305       | 2.11392 | 49459       | 2.02187 | 51651       | 1.93608 | 41 |
| 20 | 45222       | 2.21132 | 47341       | 2.11233 | 49495       | 2.02039 | 51688       | 1.93470 | 40 |
| 21 | 45257       | 2.20961 | 47377       | 2.11075 | 49532       | 2.01891 | 51724       | 1.93332 | 39 |
| 22 | 45292       | 2.20790 | 47412       | 2.10916 | 49568       | 2.01743 | 51761       | 1.93197 | 38 |
| 23 | 45327       | 2.20619 | 47448       | 2.10758 | 49604       | 2.01596 | 51798       | 1.93057 | 37 |
| 24 | 45362       | 2.20449 | 47483       | 2.10600 | 49640       | 2.01449 | 51835       | 1.92920 | 36 |
| 25 | 45397       | 2.20278 | 47519       | 2.10441 | 49677       | 2.01302 | 51872       | 1.92782 | 35 |
| 26 | 45432       | 2.20108 | 47555       | 2.10284 | 49713       | 2.01155 | 51909       | 1.92645 | 34 |
| 27 | 45467       | 2.19938 | 47590       | 2.10126 | 49749       | 2.01008 | 51946       | 1.92508 | 33 |
| 28 | 45502       | 2.19769 | 47626       | 2.09969 | 49786       | 2.00862 | 51983       | 1.92371 | 32 |
| 29 | 45537       | 2.19599 | 47662       | 2.09811 | 49822       | 2.00715 | 52020       | 1.92235 | 31 |
| 30 | 45573       | 2.19430 | 47698       | 2.09654 | 49858       | 2.00569 | 52057       | 1.92098 | 30 |
| 31 | 45608       | 2.19261 | 47733       | 2.09498 | 49894       | 2.00423 | 52094       | 1.91962 | 29 |
| 32 | 45643       | 2.19092 | 47769       | 2.09341 | 49931       | 2.00277 | 52131       | 1.91825 | 28 |
| 33 | 45678       | 2.18923 | 47805       | 2.09184 | 49967       | 2.00131 | 52168       | 1.91690 | 27 |
| 34 | 45713       | 2.18755 | 47840       | 2.09028 | 50004       | 1.99986 | 52205       | 1.91554 | 26 |
| 35 | 45748       | 2.18587 | 47876       | 2.08872 | 50040       | 1.99841 | 52242       | 1.91418 | 25 |
| 36 | 45784       | 2.18419 | 47912       | 2.08716 | 50076       | 1.99695 | 52279       | 1.91282 | 24 |
| 37 | 45819       | 2.18251 | 47948       | 2.08560 | 50113       | 1.99550 | 52316       | 1.91148 | 23 |
| 38 | 45854       | 2.18084 | 47984       | 2.08405 | 50149       | 1.99406 | 52353       | 1.91017 | 22 |
| 39 | 45889       | 2.17916 | 48019       | 2.08250 | 50185       | 1.99261 | 52390       | 1.90886 | 21 |
| 40 | 45924       | 2.17749 | 48055       | 2.08094 | 50222       | 1.99116 | 52427       | 1.90741 | 20 |
| 41 | 45960       | 2.17582 | 48091       | 2.07939 | 50258       | 1.98972 | 52464       | 1.90607 | 19 |
| 42 | 45995       | 2.17416 | 48127       | 2.07785 | 50295       | 1.98828 | 52501       | 1.90472 | 18 |
| 43 | 46030       | 2.17249 | 48163       | 2.07630 | 50331       | 1.98684 | 52538       | 1.90337 | 17 |
| 44 | 46065       | 2.17083 | 48198       | 2.07476 | 50368       | 1.98540 | 52575       | 1.90203 | 16 |
| 45 | 46101       | 2.16917 | 48234       | 2.07321 | 50404       | 1.98396 | 52613       | 1.90069 | 15 |
| 46 | 46136       | 2.16751 | 48270       | 2.07167 | 50441       | 1.98253 | 52650       | 1.89935 | 14 |
| 47 | 46171       | 2.16585 | 48306       | 2.07014 | 50477       | 1.98110 | 52687       | 1.89801 | 13 |
| 48 | 46206       | 2.16420 | 48342       | 2.06860 | 50514       | 1.97966 | 52724       | 1.89667 | 12 |
| 49 | 46242       | 2.16255 | 48378       | 2.06706 | 50550       | 1.97823 | 52761       | 1.89533 | 11 |
| 50 | 46277       | 2.16090 | 48414       | 2.06553 | 50587       | 1.97680 | 52798       | 1.89400 | 10 |
| 51 | 46312       | 2.15925 | 48450       | 2.06400 | 50623       | 1.97538 | 52836       | 1.89266 | 9  |
| 52 | 46348       | 2.15760 | 48486       | 2.06247 | 50660       | 1.97395 | 52873       | 1.89133 | 8  |
| 53 | 46383       | 2.15596 | 48521       | 2.06094 | 50696       | 1.97253 | 52910       | 1.89000 | 7  |
| 54 | 46418       | 2.15432 | 48557       | 2.05942 | 50733       | 1.97111 | 52947       | 1.88867 | 6  |
| 55 | 46454       | 2.15268 | 48593       | 2.05789 | 50769       | 1.96969 | 52984       | 1.88734 | 5  |
| 56 | 46489       | 2.15104 | 48629       | 2.05637 | 50806       | 1.96827 | 53024       | 1.88602 | 4  |
| 57 | 46525       | 2.14940 | 48665       | 2.05485 | 50843       | 1.96685 | 53059       | 1.88469 | 3  |
| 58 | 46560       | 2.14777 | 48701       | 2.05333 | 50879       | 1.96544 | 53096       | 1.88337 | 2  |
| 59 | 46595       | 2.14614 | 48737       | 2.05182 | 50916       | 1.96402 | 53134       | 1.88205 | 1  |
| 60 | 46631       | 2.14451 | 48773       | 2.05030 | 50953       | 1.96261 | 53171       | 1.88073 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 65 Degrees. |         | 64 Degrees. |         | 63 Degrees. |         | 62 Degrees. |         |    |

| M  | 28 Degrees. |         | 29 Degrees. |         | 30 Degrees. |         | 31 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 53171       | 1.88073 | 55431       | 1.80405 | 57735       | 1.73205 | 60086       | 1.66428 | 60 |
| 1  | 53208       | 1.87941 | 55469       | 1.80281 | 57774       | 1.73089 | 60126       | 1.66318 | 59 |
| 2  | 53246       | 1.87809 | 55507       | 1.80158 | 57813       | 1.72973 | 60165       | 1.66209 | 58 |
| 3  | 53283       | 1.87677 | 55545       | 1.80034 | 57851       | 1.72857 | 60205       | 1.66099 | 57 |
| 4  | 53320       | 1.87546 | 55583       | 1.79911 | 57890       | 1.72741 | 60245       | 1.65990 | 56 |
| 5  | 53358       | 1.87415 | 55621       | 1.79788 | 57929       | 1.72625 | 60284       | 1.65881 | 55 |
| 6  | 53395       | 1.87283 | 55659       | 1.79665 | 57968       | 1.72509 | 60324       | 1.65772 | 54 |
| 7  | 53432       | 1.87152 | 55697       | 1.79542 | 58007       | 1.72393 | 60364       | 1.65663 | 53 |
| 8  | 53470       | 1.87021 | 55736       | 1.79419 | 58046       | 1.72278 | 60403       | 1.65554 | 52 |
| 9  | 53507       | 1.86891 | 55774       | 1.79296 | 58085       | 1.72163 | 60443       | 1.65445 | 51 |
| 10 | 53545       | 1.86760 | 55812       | 1.79174 | 58124       | 1.72047 | 60483       | 1.65337 | 50 |
| 11 | 53582       | 1.86630 | 55850       | 1.79051 | 58162       | 1.71932 | 60522       | 1.65228 | 49 |
| 12 | 53620       | 1.86499 | 55888       | 1.78929 | 58201       | 1.71817 | 60562       | 1.65120 | 48 |
| 13 | 53657       | 1.86369 | 55926       | 1.78807 | 58240       | 1.71702 | 60602       | 1.65011 | 47 |
| 14 | 53694       | 1.86239 | 55964       | 1.78685 | 58279       | 1.71588 | 60642       | 1.64903 | 46 |
| 15 | 53732       | 1.86109 | 56003       | 1.78563 | 58318       | 1.71473 | 60681       | 1.64795 | 45 |
| 16 | 53769       | 1.85979 | 56041       | 1.78441 | 58357       | 1.71358 | 60721       | 1.64687 | 44 |
| 17 | 53807       | 1.85850 | 56079       | 1.78319 | 58396       | 1.71244 | 60761       | 1.64579 | 43 |
| 18 | 53844       | 1.85720 | 56117       | 1.78198 | 58435       | 1.71129 | 60801       | 1.64471 | 42 |
| 19 | 53882       | 1.85591 | 56156       | 1.78077 | 58474       | 1.71015 | 60841       | 1.64363 | 41 |
| 20 | 53920       | 1.85462 | 56194       | 1.77955 | 58513       | 1.70901 | 60881       | 1.64256 | 40 |
| 21 | 53957       | 1.85333 | 56232       | 1.77834 | 58552       | 1.70787 | 60921       | 1.64148 | 39 |
| 22 | 53995       | 1.85204 | 56270       | 1.77713 | 58591       | 1.70673 | 60960       | 1.64041 | 38 |
| 23 | 54032       | 1.85075 | 56309       | 1.77592 | 58631       | 1.70560 | 61000       | 1.63933 | 37 |
| 24 | 54070       | 1.84946 | 56347       | 1.77471 | 58670       | 1.70446 | 61040       | 1.63826 | 36 |
| 25 | 54107       | 1.84818 | 56385       | 1.77351 | 58709       | 1.70332 | 61080       | 1.63719 | 35 |
| 26 | 54145       | 1.84689 | 56424       | 1.77230 | 58748       | 1.70219 | 61120       | 1.63612 | 34 |
| 27 | 54183       | 1.84561 | 56462       | 1.77110 | 58787       | 1.70106 | 61160       | 1.63505 | 33 |
| 28 | 54220       | 1.84433 | 56500       | 1.76990 | 58826       | 1.69992 | 61200       | 1.63398 | 32 |
| 29 | 54258       | 1.84305 | 56539       | 1.76869 | 58865       | 1.69879 | 61240       | 1.63292 | 31 |
| 30 | 54296       | 1.84177 | 56577       | 1.76749 | 58904       | 1.69766 | 61280       | 1.63185 | 30 |
| 31 | 54333       | 1.84049 | 56616       | 1.76629 | 58944       | 1.69653 | 61320       | 1.63079 | 29 |
| 32 | 54371       | 1.83922 | 56654       | 1.76510 | 58983       | 1.69541 | 61360       | 1.62972 | 28 |
| 33 | 54409       | 1.83794 | 56693       | 1.76390 | 59022       | 1.69428 | 61400       | 1.62866 | 27 |
| 34 | 54446       | 1.83667 | 56731       | 1.76271 | 59061       | 1.69315 | 61440       | 1.62760 | 26 |
| 35 | 54484       | 1.83540 | 56769       | 1.76151 | 59101       | 1.69203 | 61480       | 1.62654 | 25 |
| 36 | 54522       | 1.83413 | 56808       | 1.76032 | 59140       | 1.69091 | 61520       | 1.62548 | 24 |
| 37 | 54560       | 1.83286 | 56846       | 1.75913 | 59179       | 1.68979 | 61561       | 1.62442 | 23 |
| 38 | 54597       | 1.83159 | 56885       | 1.75794 | 59218       | 1.68866 | 61601       | 1.62336 | 22 |
| 39 | 54635       | 1.83033 | 56923       | 1.75675 | 59258       | 1.68754 | 61641       | 1.62230 | 21 |
| 40 | 54673       | 1.82906 | 56962       | 1.75556 | 59297       | 1.68643 | 61681       | 1.62125 | 20 |
| 41 | 54711       | 1.82780 | 57000       | 1.75437 | 59336       | 1.68531 | 61721       | 1.62019 | 19 |
| 42 | 54748       | 1.82654 | 57039       | 1.75319 | 59376       | 1.68419 | 61761       | 1.61914 | 18 |
| 43 | 54786       | 1.82528 | 57078       | 1.75200 | 59415       | 1.68308 | 61801       | 1.61808 | 17 |
| 44 | 54824       | 1.82402 | 57116       | 1.75082 | 59454       | 1.68196 | 61842       | 1.61703 | 16 |
| 45 | 54862       | 1.82276 | 57155       | 1.74964 | 59494       | 1.68085 | 61882       | 1.61598 | 15 |
| 46 | 54900       | 1.82150 | 57193       | 1.74846 | 59533       | 1.67974 | 61922       | 1.61493 | 14 |
| 47 | 54938       | 1.82025 | 57232       | 1.74728 | 59573       | 1.67869 | 61962       | 1.61388 | 13 |
| 48 | 54975       | 1.81899 | 57271       | 1.74610 | 59612       | 1.67752 | 62003       | 1.61283 | 12 |
| 49 | 55013       | 1.81774 | 57309       | 1.74492 | 59651       | 1.67641 | 62043       | 1.61179 | 11 |
| 50 | 55051       | 1.81649 | 57348       | 1.74375 | 59691       | 1.67530 | 62083       | 1.61074 | 10 |
| 51 | 55089       | 1.81524 | 57386       | 1.74257 | 59730       | 1.67419 | 62124       | 1.60970 | 9  |
| 52 | 55127       | 1.81399 | 57425       | 1.74140 | 59770       | 1.67309 | 62164       | 1.60865 | 8  |
| 53 | 55165       | 1.81274 | 57464       | 1.74022 | 59809       | 1.67198 | 62204       | 1.60761 | 7  |
| 54 | 55203       | 1.81150 | 57503       | 1.73905 | 59849       | 1.67088 | 62245       | 1.60657 | 6  |
| 55 | 55241       | 1.81025 | 57541       | 1.73788 | 59888       | 1.66978 | 62285       | 1.60553 | 5  |
| 56 | 55279       | 1.80901 | 57580       | 1.73671 | 59928       | 1.66867 | 62325       | 1.60449 | 4  |
| 57 | 55317       | 1.80777 | 57619       | 1.73555 | 59967       | 1.66757 | 62366       | 1.60345 | 3  |
| 58 | 55355       | 1.80653 | 57657       | 1.73438 | 60007       | 1.66647 | 62406       | 1.60241 | 2  |
| 59 | 55393       | 1.80529 | 57696       | 1.73321 | 60046       | 1.66538 | 62446       | 1.60137 | 1  |
| 60 | 55431       | 1.80405 | 57735       | 1.73205 | 60086       | 1.66428 | 62487       | 1.60033 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 61 Degrees. |         | 60 Degrees. |         | 59 Degrees. |         | 58 Degrees. |         |    |

| M  | 32 Degrees. |         | 33 Degrees. |         | 34 Degrees. |         | 35 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 62487       | 1.60033 | 64941       | 1.53986 | 67451       | 1.48256 | 70021       | 1.42815 | 60 |
| 1  | 62527       | 1.59930 | 64982       | 1.53888 | 67493       | 1.48163 | 70034       | 1.42726 | 59 |
| 2  | 62568       | 1.59826 | 65023       | 1.53791 | 67536       | 1.48070 | 70107       | 1.42638 | 58 |
| 3  | 62608       | 1.59723 | 65065       | 1.53693 | 67578       | 1.47977 | 70151       | 1.42550 | 57 |
| 4  | 62649       | 1.59620 | 65106       | 1.53595 | 67620       | 1.47885 | 70194       | 1.42462 | 56 |
| 5  | 62689       | 1.59517 | 65148       | 1.53497 | 67663       | 1.47792 | 70238       | 1.42374 | 55 |
| 6  | 62730       | 1.59414 | 65189       | 1.53400 | 67705       | 1.47699 | 70281       | 1.42286 | 54 |
| 7  | 62770       | 1.59311 | 65231       | 1.53302 | 67748       | 1.47607 | 70325       | 1.42198 | 53 |
| 8  | 62811       | 1.59208 | 65272       | 1.53205 | 67790       | 1.47514 | 70368       | 1.42110 | 52 |
| 9  | 62852       | 1.59105 | 65314       | 1.53107 | 67832       | 1.47422 | 70412       | 1.42022 | 51 |
| 10 | 62892       | 1.59002 | 65355       | 1.53010 | 67875       | 1.47330 | 70455       | 1.41934 | 50 |
| 11 | 62933       | 1.58900 | 65397       | 1.52913 | 67917       | 1.47238 | 70499       | 1.41847 | 49 |
| 12 | 62973       | 1.58797 | 65438       | 1.52816 | 67960       | 1.47146 | 70542       | 1.41759 | 48 |
| 13 | 63014       | 1.58695 | 65480       | 1.52719 | 68002       | 1.47053 | 70586       | 1.41672 | 47 |
| 14 | 63055       | 1.58593 | 65521       | 1.52622 | 68045       | 1.46962 | 70629       | 1.41584 | 46 |
| 15 | 63095       | 1.58490 | 65563       | 1.52525 | 68088       | 1.46870 | 70673       | 1.41497 | 45 |
| 16 | 63136       | 1.58388 | 65604       | 1.52429 | 68130       | 1.46778 | 70717       | 1.41409 | 44 |
| 17 | 63177       | 1.58286 | 65646       | 1.52332 | 68173       | 1.46686 | 70760       | 1.41322 | 43 |
| 18 | 63217       | 1.58184 | 65688       | 1.52235 | 68215       | 1.46595 | 70804       | 1.41235 | 42 |
| 19 | 63258       | 1.58083 | 65729       | 1.52139 | 68258       | 1.46503 | 70848       | 1.41148 | 41 |
| 20 | 63299       | 1.57981 | 65771       | 1.52043 | 68301       | 1.46411 | 70891       | 1.41061 | 40 |
| 21 | 63340       | 1.57879 | 65813       | 1.51946 | 68343       | 1.46320 | 70935       | 1.40974 | 39 |
| 22 | 63380       | 1.57778 | 65854       | 1.51850 | 68386       | 1.46229 | 70979       | 1.40887 | 38 |
| 23 | 63421       | 1.57676 | 65896       | 1.51754 | 68429       | 1.46137 | 71023       | 1.40800 | 37 |
| 24 | 63462       | 1.57575 | 65938       | 1.51658 | 68471       | 1.46046 | 71066       | 1.40714 | 36 |
| 25 | 63503       | 1.57474 | 65980       | 1.51562 | 68514       | 1.45955 | 71110       | 1.40627 | 35 |
| 26 | 63544       | 1.57372 | 66021       | 1.51466 | 68557       | 1.45864 | 71154       | 1.40540 | 34 |
| 27 | 63584       | 1.57271 | 66063       | 1.51370 | 68600       | 1.45773 | 71198       | 1.40454 | 33 |
| 28 | 63625       | 1.57170 | 66105       | 1.51275 | 68642       | 1.45682 | 71242       | 1.40368 | 32 |
| 29 | 63666       | 1.57069 | 66147       | 1.51179 | 68685       | 1.45592 | 71285       | 1.40281 | 31 |
| 30 | 63707       | 1.56969 | 66189       | 1.51084 | 68728       | 1.45501 | 71329       | 1.40195 | 30 |
| 31 | 63748       | 1.56868 | 66230       | 1.50988 | 68771       | 1.45410 | 71373       | 1.40109 | 29 |
| 32 | 63789       | 1.56767 | 66272       | 1.50893 | 68814       | 1.45320 | 71417       | 1.40022 | 28 |
| 33 | 63830       | 1.56667 | 66314       | 1.50797 | 68857       | 1.45229 | 71461       | 1.39936 | 27 |
| 34 | 63871       | 1.56566 | 66356       | 1.50702 | 68900       | 1.45139 | 71505       | 1.39850 | 26 |
| 35 | 63912       | 1.56466 | 66398       | 1.50607 | 68942       | 1.45048 | 71549       | 1.39764 | 25 |
| 36 | 63953       | 1.56366 | 66440       | 1.50512 | 68985       | 1.44958 | 71593       | 1.39679 | 24 |
| 37 | 63994       | 1.56265 | 66482       | 1.50417 | 69028       | 1.44868 | 71637       | 1.39593 | 23 |
| 38 | 64035       | 1.56165 | 66524       | 1.50322 | 69071       | 1.44778 | 71681       | 1.39507 | 22 |
| 39 | 64075       | 1.56065 | 66566       | 1.50228 | 69114       | 1.44688 | 71725       | 1.39421 | 21 |
| 40 | 64117       | 1.55966 | 66608       | 1.50133 | 69157       | 1.44598 | 71769       | 1.39336 | 20 |
| 41 | 64158       | 1.55866 | 66650       | 1.50038 | 69200       | 1.44508 | 71813       | 1.39250 | 19 |
| 42 | 64199       | 1.55766 | 66692       | 1.49944 | 69243       | 1.44418 | 71857       | 1.39165 | 18 |
| 43 | 64240       | 1.55666 | 66734       | 1.49849 | 69286       | 1.44329 | 71901       | 1.39079 | 17 |
| 44 | 64281       | 1.55567 | 66776       | 1.49755 | 69329       | 1.44239 | 71946       | 1.38994 | 16 |
| 45 | 64322       | 1.55467 | 66818       | 1.49661 | 69372       | 1.44149 | 71990       | 1.38909 | 15 |
| 46 | 64363       | 1.55368 | 66860       | 1.49566 | 69416       | 1.44060 | 72034       | 1.38824 | 14 |
| 47 | 64404       | 1.55269 | 66902       | 1.49472 | 69459       | 1.43970 | 72078       | 1.38738 | 13 |
| 48 | 64446       | 1.55170 | 66944       | 1.49378 | 69502       | 1.43881 | 72122       | 1.38653 | 12 |
| 49 | 64487       | 1.55071 | 66986       | 1.49284 | 69545       | 1.43792 | 72166       | 1.38568 | 11 |
| 50 | 64528       | 1.54972 | 67028       | 1.49190 | 69588       | 1.43703 | 72211       | 1.38484 | 10 |
| 51 | 64569       | 1.54873 | 67071       | 1.49097 | 69631       | 1.43614 | 72255       | 1.38399 | 9  |
| 52 | 64610       | 1.54774 | 67113       | 1.49003 | 69675       | 1.43525 | 72299       | 1.38314 | 8  |
| 53 | 64652       | 1.54675 | 67155       | 1.48909 | 69718       | 1.43436 | 72344       | 1.38229 | 7  |
| 54 | 64693       | 1.54576 | 67197       | 1.48816 | 69761       | 1.43347 | 72388       | 1.38145 | 6  |
| 55 | 64734       | 1.54478 | 67239       | 1.48722 | 69804       | 1.43258 | 72432       | 1.38060 | 5  |
| 56 | 64775       | 1.54379 | 67282       | 1.48629 | 69847       | 1.43169 | 72477       | 1.37976 | 4  |
| 57 | 64817       | 1.54281 | 67324       | 1.48536 | 69891       | 1.43080 | 72521       | 1.37891 | 3  |
| 58 | 64858       | 1.54183 | 67366       | 1.48442 | 69934       | 1.42992 | 72565       | 1.37807 | 2  |
| 59 | 64899       | 1.54085 | 67409       | 1.48349 | 69977       | 1.42903 | 72610       | 1.37722 | 1  |
| 60 | 64941       | 1.53986 | 67451       | 1.48256 | 70021       | 1.42815 | 72654       | 1.37638 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 57 Degrees. |         | 56 Degrees. |         | 55 Degrees. |         | 54 Degrees. |         |    |



| M  | 36 Degrees. |         | 37 Degrees. |         | 38 Degrees. |         | 39 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 72654       | 1.37638 | 75355       | 1.32704 | 78129       | 1.27994 | 80978       | 1.23489 | 60 |
| 1  | 72699       | 1.37554 | 75401       | 1.32624 | 78175       | 1.27917 | 81027       | 1.23416 | 59 |
| 2  | 72743       | 1.37470 | 75447       | 1.32544 | 78222       | 1.27841 | 81075       | 1.23343 | 58 |
| 3  | 72788       | 1.37386 | 75492       | 1.32464 | 78269       | 1.27764 | 81123       | 1.23270 | 57 |
| 4  | 72832       | 1.37302 | 75538       | 1.32384 | 78316       | 1.27688 | 81171       | 1.23196 | 56 |
| 5  | 72877       | 1.37218 | 75584       | 1.32304 | 78363       | 1.27611 | 81220       | 1.23123 | 55 |
| 6  | 72921       | 1.37134 | 75629       | 1.32224 | 78410       | 1.27535 | 81268       | 1.23050 | 54 |
| 7  | 72966       | 1.37050 | 75675       | 1.32144 | 78457       | 1.27458 | 81316       | 1.22977 | 53 |
| 8  | 73010       | 1.36967 | 75721       | 1.32064 | 78504       | 1.27382 | 81364       | 1.22904 | 52 |
| 9  | 73055       | 1.36883 | 75767       | 1.31984 | 78551       | 1.27306 | 81413       | 1.22831 | 51 |
| 10 | 73100       | 1.36800 | 75812       | 1.31904 | 78598       | 1.27230 | 81461       | 1.22758 | 50 |
| 11 | 73144       | 1.36716 | 75858       | 1.31825 | 78645       | 1.27153 | 81510       | 1.22685 | 49 |
| 12 | 73189       | 1.36633 | 75904       | 1.31745 | 78692       | 1.27077 | 81558       | 1.22612 | 48 |
| 13 | 73234       | 1.36549 | 75950       | 1.31666 | 78739       | 1.27001 | 81606       | 1.22539 | 47 |
| 14 | 73278       | 1.36466 | 75996       | 1.31586 | 78786       | 1.26925 | 81655       | 1.22467 | 46 |
| 15 | 73323       | 1.36383 | 76042       | 1.31507 | 78834       | 1.26849 | 81703       | 1.22394 | 45 |
| 16 | 73368       | 1.36300 | 76088       | 1.31427 | 78881       | 1.26774 | 81752       | 1.22321 | 44 |
| 17 | 73413       | 1.36217 | 76134       | 1.31348 | 78928       | 1.26698 | 81801       | 1.22249 | 43 |
| 18 | 73457       | 1.36133 | 76180       | 1.31269 | 78975       | 1.26622 | 81849       | 1.22176 | 42 |
| 19 | 73502       | 1.36051 | 76226       | 1.31190 | 79022       | 1.26546 | 81898       | 1.22104 | 41 |
| 20 | 73547       | 1.35968 | 76272       | 1.31110 | 79070       | 1.26471 | 81946       | 1.22031 | 40 |
| 21 | 73592       | 1.35885 | 76318       | 1.31031 | 79117       | 1.26395 | 81995       | 1.21959 | 39 |
| 22 | 73637       | 1.35802 | 76364       | 1.30952 | 79164       | 1.26319 | 82044       | 1.21886 | 38 |
| 23 | 73681       | 1.35719 | 76410       | 1.30873 | 79212       | 1.26244 | 82092       | 1.21814 | 37 |
| 24 | 73726       | 1.35637 | 76456       | 1.30795 | 79259       | 1.26169 | 82141       | 1.21742 | 36 |
| 25 | 73771       | 1.35554 | 76502       | 1.30716 | 79306       | 1.26093 | 82190       | 1.21670 | 35 |
| 26 | 73816       | 1.35472 | 76548       | 1.30637 | 79354       | 1.26018 | 82238       | 1.21598 | 34 |
| 27 | 73861       | 1.35389 | 76594       | 1.30558 | 79401       | 1.25943 | 82287       | 1.21526 | 33 |
| 28 | 73906       | 1.35307 | 76640       | 1.30480 | 79449       | 1.25867 | 82336       | 1.21454 | 32 |
| 29 | 73951       | 1.35224 | 76686       | 1.30401 | 79496       | 1.25792 | 82385       | 1.21382 | 31 |
| 30 | 73996       | 1.35142 | 76733       | 1.30323 | 79544       | 1.25717 | 82434       | 1.21310 | 30 |
| 31 | 74041       | 1.35060 | 76779       | 1.30244 | 79591       | 1.25642 | 82483       | 1.21238 | 29 |
| 32 | 74086       | 1.34978 | 76825       | 1.30166 | 79639       | 1.25567 | 82531       | 1.21166 | 28 |
| 33 | 74131       | 1.34896 | 76871       | 1.30087 | 79686       | 1.25492 | 82580       | 1.21094 | 27 |
| 34 | 74176       | 1.34814 | 76918       | 1.30009 | 79734       | 1.25417 | 82629       | 1.21023 | 26 |
| 35 | 74221       | 1.34732 | 76964       | 1.29931 | 79781       | 1.25343 | 82678       | 1.20951 | 25 |
| 36 | 74267       | 1.34650 | 77010       | 1.29853 | 79829       | 1.25268 | 82727       | 1.20879 | 24 |
| 37 | 74312       | 1.34568 | 77057       | 1.29775 | 79877       | 1.25193 | 82776       | 1.20808 | 23 |
| 38 | 74357       | 1.34487 | 77103       | 1.29696 | 79924       | 1.25118 | 82825       | 1.20736 | 22 |
| 39 | 74402       | 1.34405 | 77149       | 1.29618 | 79972       | 1.25044 | 82874       | 1.20665 | 21 |
| 40 | 74447       | 1.34323 | 77196       | 1.29541 | 80020       | 1.24969 | 82923       | 1.20593 | 20 |
| 41 | 74492       | 1.34242 | 77242       | 1.29463 | 80067       | 1.24895 | 82972       | 1.20522 | 19 |
| 42 | 74538       | 1.34160 | 77289       | 1.29385 | 80115       | 1.24820 | 83022       | 1.20451 | 18 |
| 43 | 74583       | 1.34079 | 77335       | 1.29307 | 80163       | 1.24746 | 83071       | 1.20379 | 17 |
| 44 | 74628       | 1.33998 | 77382       | 1.29229 | 80211       | 1.24672 | 83120       | 1.20308 | 16 |
| 45 | 74674       | 1.33916 | 77428       | 1.29152 | 80258       | 1.24597 | 83169       | 1.20237 | 15 |
| 46 | 74719       | 1.33835 | 77475       | 1.29074 | 80306       | 1.24523 | 83218       | 1.20166 | 14 |
| 47 | 74764       | 1.33754 | 77521       | 1.28997 | 80354       | 1.24449 | 83268       | 1.20095 | 13 |
| 48 | 74810       | 1.33673 | 77568       | 1.28919 | 80402       | 1.24375 | 83317       | 1.20024 | 12 |
| 49 | 74855       | 1.33592 | 77615       | 1.28842 | 80450       | 1.24301 | 83366       | 1.19953 | 11 |
| 50 | 74900       | 1.33511 | 77661       | 1.28764 | 80498       | 1.24227 | 83415       | 1.19882 | 10 |
| 51 | 74946       | 1.33430 | 77708       | 1.28687 | 80546       | 1.24153 | 83465       | 1.19811 | 9  |
| 52 | 74991       | 1.33349 | 77754       | 1.28610 | 80594       | 1.24079 | 83514       | 1.19740 | 8  |
| 53 | 75037       | 1.33268 | 77801       | 1.28533 | 80642       | 1.24005 | 83564       | 1.19669 | 7  |
| 54 | 75082       | 1.33187 | 77848       | 1.28456 | 80690       | 1.23931 | 83613       | 1.19599 | 6  |
| 55 | 75128       | 1.33107 | 77895       | 1.28379 | 80738       | 1.23858 | 83662       | 1.19528 | 5  |
| 56 | 75174       | 1.33026 | 77941       | 1.28302 | 80786       | 1.23784 | 83712       | 1.19457 | 4  |
| 57 | 75219       | 1.32946 | 77988       | 1.28225 | 80834       | 1.23710 | 83761       | 1.19387 | 3  |
| 58 | 75264       | 1.32865 | 78035       | 1.28148 | 80882       | 1.23637 | 83811       | 1.19316 | 2  |
| 59 | 75310       | 1.32785 | 78082       | 1.28071 | 80930       | 1.23563 | 83860       | 1.19246 | 1  |
| 60 | 75355       | 1.32704 | 78129       | 1.27994 | 80978       | 1.23490 | 83910       | 1.19175 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 53 Degrees. |         | 52 Degrees. |         | 51 Degrees. |         | 50 Degrees. |         |    |



| M  | 40 Degrees. |         | 41 Degrees. |         | 42 Degrees. |         | 43 Degrees. |         | M  |
|----|-------------|---------|-------------|---------|-------------|---------|-------------|---------|----|
|    | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. | N. Tan.     | N. Cot. |    |
| 0  | 83910       | 1.19175 | 86929       | 1.15037 | 90040       | 1.11061 | 93252       | 1.07237 | 60 |
| 1  | 83960       | 1.19105 | 86980       | 1.14969 | 90093       | 1.10996 | 93306       | 1.07174 | 59 |
| 2  | 84009       | 1.19035 | 87031       | 1.14902 | 90146       | 1.10931 | 93360       | 1.07112 | 58 |
| 3  | 84059       | 1.18964 | 87082       | 1.14834 | 90199       | 1.10867 | 93415       | 1.07049 | 57 |
| 4  | 84108       | 1.18894 | 87133       | 1.14767 | 90251       | 1.10802 | 93469       | 1.06987 | 56 |
| 5  | 84158       | 1.18824 | 87184       | 1.14699 | 90304       | 1.10737 | 93524       | 1.06925 | 55 |
| 6  | 84208       | 1.18754 | 87236       | 1.14632 | 90357       | 1.10672 | 93578       | 1.06862 | 54 |
| 7  | 84258       | 1.18684 | 87287       | 1.14565 | 90410       | 1.10607 | 93633       | 1.06800 | 53 |
| 8  | 84307       | 1.18614 | 87338       | 1.14498 | 90463       | 1.10543 | 93688       | 1.06738 | 52 |
| 9  | 84357       | 1.18544 | 87389       | 1.14430 | 90516       | 1.10478 | 93742       | 1.06676 | 51 |
| 10 | 84407       | 1.18474 | 87441       | 1.14363 | 90569       | 1.10414 | 93797       | 1.06613 | 50 |
| 11 | 84457       | 1.18404 | 87492       | 1.14296 | 90621       | 1.10349 | 93852       | 1.06551 | 49 |
| 12 | 84507       | 1.18334 | 87543       | 1.14229 | 90674       | 1.10285 | 93906       | 1.06489 | 48 |
| 13 | 84556       | 1.18264 | 87595       | 1.14162 | 90727       | 1.10220 | 93961       | 1.06427 | 47 |
| 14 | 84606       | 1.18194 | 87646       | 1.14095 | 90781       | 1.10156 | 94016       | 1.06365 | 46 |
| 15 | 84656       | 1.18125 | 87698       | 1.14028 | 90834       | 1.10091 | 94071       | 1.06303 | 45 |
| 16 | 84706       | 1.18055 | 87749       | 1.13961 | 90887       | 1.10027 | 94125       | 1.06241 | 44 |
| 17 | 84756       | 1.17986 | 87801       | 1.13894 | 90940       | 1.09963 | 94180       | 1.06179 | 43 |
| 18 | 84806       | 1.17916 | 87852       | 1.13828 | 90993       | 1.09899 | 94235       | 1.06117 | 42 |
| 19 | 84856       | 1.17846 | 87904       | 1.13761 | 91046       | 1.09834 | 94290       | 1.06056 | 41 |
| 20 | 84906       | 1.17777 | 87955       | 1.13694 | 91099       | 1.09770 | 94345       | 1.05994 | 40 |
| 21 | 84956       | 1.17708 | 88007       | 1.13627 | 91153       | 1.09706 | 94400       | 1.05932 | 39 |
| 22 | 85006       | 1.17638 | 88059       | 1.13561 | 91206       | 1.09642 | 94455       | 1.05870 | 38 |
| 23 | 85057       | 1.17569 | 88110       | 1.13494 | 91259       | 1.09578 | 94510       | 1.05809 | 37 |
| 24 | 85107       | 1.17500 | 88162       | 1.13428 | 91313       | 1.09514 | 94565       | 1.05747 | 36 |
| 25 | 85157       | 1.17430 | 88214       | 1.13361 | 91366       | 1.09450 | 94620       | 1.05685 | 35 |
| 26 | 85207       | 1.17361 | 88265       | 1.13295 | 91419       | 1.09386 | 94676       | 1.05624 | 34 |
| 27 | 85257       | 1.17292 | 88317       | 1.13228 | 91473       | 1.09322 | 94731       | 1.05562 | 33 |
| 28 | 85307       | 1.17223 | 88369       | 1.13162 | 91526       | 1.09258 | 94786       | 1.05501 | 32 |
| 29 | 85358       | 1.17154 | 88421       | 1.13096 | 91580       | 1.09195 | 94841       | 1.05439 | 31 |
| 30 | 85408       | 1.17085 | 88473       | 1.13029 | 91633       | 1.09131 | 94896       | 1.05378 | 30 |
| 31 | 85458       | 1.17016 | 88524       | 1.12963 | 91687       | 1.09067 | 94952       | 1.05317 | 29 |
| 32 | 85509       | 1.16947 | 88576       | 1.12897 | 91740       | 1.09003 | 95007       | 1.05255 | 28 |
| 33 | 85559       | 1.16878 | 88628       | 1.12831 | 91794       | 1.08940 | 95062       | 1.05194 | 27 |
| 34 | 85609       | 1.16809 | 88680       | 1.12765 | 91847       | 1.08876 | 95118       | 1.05133 | 26 |
| 35 | 85660       | 1.16741 | 88732       | 1.12699 | 91901       | 1.08813 | 95173       | 1.05072 | 25 |
| 36 | 85710       | 1.16672 | 88784       | 1.12633 | 91955       | 1.08749 | 95229       | 1.05010 | 24 |
| 37 | 85761       | 1.16603 | 88836       | 1.12567 | 92008       | 1.08686 | 95284       | 1.04949 | 23 |
| 38 | 85811       | 1.16535 | 88888       | 1.12501 | 92062       | 1.08622 | 95340       | 1.04888 | 22 |
| 39 | 85862       | 1.16466 | 88940       | 1.12435 | 92116       | 1.08559 | 95395       | 1.04827 | 21 |
| 40 | 85912       | 1.16398 | 88992       | 1.12369 | 92170       | 1.08496 | 95451       | 1.04766 | 20 |
| 41 | 85963       | 1.16329 | 89045       | 1.12303 | 92223       | 1.08432 | 95506       | 1.04705 | 19 |
| 42 | 86014       | 1.16261 | 89097       | 1.12238 | 92277       | 1.08369 | 95562       | 1.04644 | 18 |
| 43 | 86064       | 1.16192 | 89149       | 1.12172 | 92331       | 1.08306 | 95618       | 1.04583 | 17 |
| 44 | 86115       | 1.16124 | 89201       | 1.12106 | 92385       | 1.08243 | 95673       | 1.04522 | 16 |
| 45 | 86166       | 1.16056 | 89253       | 1.12041 | 92439       | 1.08179 | 95729       | 1.04461 | 15 |
| 46 | 86216       | 1.15987 | 89306       | 1.11975 | 92493       | 1.08116 | 95785       | 1.04401 | 14 |
| 47 | 86267       | 1.15919 | 89358       | 1.11909 | 92547       | 1.08053 | 95841       | 1.04340 | 13 |
| 48 | 86318       | 1.15851 | 89410       | 1.11844 | 92601       | 1.07990 | 95897       | 1.04279 | 12 |
| 49 | 86368       | 1.15783 | 89463       | 1.11778 | 92655       | 1.07927 | 95952       | 1.04218 | 11 |
| 50 | 86419       | 1.15715 | 89515       | 1.11713 | 92709       | 1.07864 | 96008       | 1.04158 | 10 |
| 51 | 86470       | 1.15647 | 89567       | 1.11648 | 92763       | 1.07801 | 96064       | 1.04097 | 9  |
| 52 | 86521       | 1.15579 | 89620       | 1.11582 | 92817       | 1.07738 | 96120       | 1.04036 | 8  |
| 53 | 86572       | 1.15511 | 89672       | 1.11517 | 92872       | 1.07676 | 96176       | 1.03976 | 7  |
| 54 | 86623       | 1.15443 | 89725       | 1.11452 | 92926       | 1.07613 | 96232       | 1.03915 | 6  |
| 55 | 86674       | 1.15375 | 89777       | 1.11387 | 92980       | 1.07550 | 96288       | 1.03855 | 5  |
| 56 | 86725       | 1.15308 | 89830       | 1.11321 | 93034       | 1.07487 | 96344       | 1.03794 | 4  |
| 57 | 86776       | 1.15240 | 89883       | 1.11256 | 93088       | 1.07425 | 96400       | 1.03734 | 3  |
| 58 | 86827       | 1.15172 | 89935       | 1.11191 | 93143       | 1.07362 | 96457       | 1.03674 | 2  |
| 59 | 86878       | 1.15104 | 89988       | 1.11126 | 93197       | 1.07299 | 96513       | 1.03613 | 1  |
| 60 | 86929       | 1.15037 | 90040       | 1.11061 | 93252       | 1.07237 | 96569       | 1.03553 | 0  |
| M  | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | N. Cot.     | N. Tan. | M  |
|    | 49 Degrees. |         | 48 Degrees. |         | 47 Degrees. |         | 46 Degrees. |         |    |

| 44 Degrees. |             |         |    | 44 Degrees. |             |         |    |
|-------------|-------------|---------|----|-------------|-------------|---------|----|
| M           | N. Tan.     | N. Cot. | M  | M           | N. Tan.     | N. Cot. | M  |
| 0           | 96569       | 1.03553 | 60 | 31          | 98327       | 1.01702 | 29 |
| 1           | 96625       | 1.03493 | 59 | 32          | 98384       | 1.01642 | 28 |
| 2           | 96681       | 1.03433 | 58 | 33          | 98441       | 1.01583 | 27 |
| 3           | 96738       | 1.03372 | 57 | 34          | 98499       | 1.01524 | 26 |
| 4           | 96794       | 1.03312 | 56 | 35          | 98556       | 1.01465 | 25 |
| 5           | 96850       | 1.03252 | 55 | 36          | 98613       | 1.01406 | 24 |
| 6           | 96907       | 1.03192 | 54 | 37          | 98671       | 1.01347 | 23 |
| 7           | 96963       | 1.03132 | 53 | 38          | 98728       | 1.01288 | 22 |
| 8           | 97020       | 1.03072 | 52 | 39          | 98786       | 1.01229 | 21 |
| 9           | 97076       | 1.03012 | 51 | 40          | 98843       | 1.01170 | 20 |
| 10          | 97133       | 1.02952 | 50 | 41          | 98901       | 1.01112 | 19 |
| 11          | 97189       | 1.02892 | 49 | 42          | 98958       | 1.01053 | 18 |
| 12          | 97246       | 1.02832 | 48 | 43          | 99016       | 1.00994 | 17 |
| 13          | 97302       | 1.02772 | 47 | 44          | 99073       | 1.00935 | 16 |
| 14          | 97359       | 1.02713 | 46 | 45          | 99131       | 1.00876 | 15 |
| 15          | 97416       | 1.02653 | 45 | 46          | 99189       | 1.00818 | 14 |
| 16          | 97472       | 1.02593 | 44 | 47          | 99247       | 1.00759 | 13 |
| 17          | 97529       | 1.02533 | 43 | 48          | 99304       | 1.00701 | 12 |
| 18          | 97586       | 1.02474 | 42 | 49          | 99362       | 1.00642 | 11 |
| 19          | 97643       | 1.02414 | 41 | 50          | 99420       | 1.00583 | 10 |
| 20          | 97700       | 1.02355 | 40 | 51          | 99478       | 1.00525 | 9  |
| 21          | 97756       | 1.02295 | 39 | 52          | 99536       | 1.00467 | 8  |
| 22          | 97813       | 1.02236 | 38 | 53          | 99594       | 1.00408 | 7  |
| 23          | 97870       | 1.02176 | 37 | 54          | 99652       | 1.00350 | 6  |
| 24          | 97927       | 1.02117 | 36 | 55          | 99710       | 1.00291 | 5  |
| 25          | 97984       | 1.02057 | 35 | 56          | 99768       | 1.00233 | 4  |
| 26          | 98041       | 1.01998 | 34 | 57          | 99826       | 1.00175 | 3  |
| 27          | 98098       | 1.01939 | 33 | 58          | 99884       | 1.00116 | 2  |
| 28          | 98155       | 1.01879 | 32 | 59          | 99942       | 1.00058 | 1  |
| 29          | 98213       | 1.01820 | 31 | 60          | 10000       | 1.00000 | 0  |
| 30          | 98270       | 1.01761 | 30 |             |             |         |    |
| M           | N. Cot.     | N. Tan. | M  | M           | N. Cot.     | N. Tan. | M  |
|             | 45 Degrees. |         |    |             | 45 Degrees. |         |    |

| No. | Name          | Rank     | Regiment   | Company | Service   |
|-----|---------------|----------|------------|---------|-----------|
| 1   | John Smith    | Private  | 1st Regt   | A       | 1862-1864 |
| 2   | James Brown   | Sergeant | 2nd Regt   | B       | 1862-1864 |
| 3   | William Jones | Private  | 3rd Regt   | C       | 1862-1864 |
| 4   | Robert Taylor | Private  | 4th Regt   | D       | 1862-1864 |
| 5   | Thomas White  | Private  | 5th Regt   | E       | 1862-1864 |
| 6   | George Black  | Private  | 6th Regt   | F       | 1862-1864 |
| 7   | Charles Green | Private  | 7th Regt   | G       | 1862-1864 |
| 8   | Henry Lee     | Private  | 8th Regt   | H       | 1862-1864 |
| 9   | Richard King  | Private  | 9th Regt   | I       | 1862-1864 |
| 10  | Edward Clark  | Private  | 10th Regt  | J       | 1862-1864 |
| 11  | Samuel Adams  | Private  | 11th Regt  | K       | 1862-1864 |
| 12  | Joseph Baker  | Private  | 12th Regt  | L       | 1862-1864 |
| 13  | Samuel Adams  | Private  | 13th Regt  | M       | 1862-1864 |
| 14  | John Smith    | Private  | 14th Regt  | N       | 1862-1864 |
| 15  | James Brown   | Private  | 15th Regt  | O       | 1862-1864 |
| 16  | William Jones | Private  | 16th Regt  | P       | 1862-1864 |
| 17  | Robert Taylor | Private  | 17th Regt  | Q       | 1862-1864 |
| 18  | Thomas White  | Private  | 18th Regt  | R       | 1862-1864 |
| 19  | George Black  | Private  | 19th Regt  | S       | 1862-1864 |
| 20  | Charles Green | Private  | 20th Regt  | T       | 1862-1864 |
| 21  | Henry Lee     | Private  | 21st Regt  | U       | 1862-1864 |
| 22  | Richard King  | Private  | 22nd Regt  | V       | 1862-1864 |
| 23  | Edward Clark  | Private  | 23rd Regt  | W       | 1862-1864 |
| 24  | Samuel Adams  | Private  | 24th Regt  | X       | 1862-1864 |
| 25  | Joseph Baker  | Private  | 25th Regt  | Y       | 1862-1864 |
| 26  | Samuel Adams  | Private  | 26th Regt  | Z       | 1862-1864 |
| 27  | John Smith    | Private  | 27th Regt  | AA      | 1862-1864 |
| 28  | James Brown   | Private  | 28th Regt  | BB      | 1862-1864 |
| 29  | William Jones | Private  | 29th Regt  | CC      | 1862-1864 |
| 30  | Robert Taylor | Private  | 30th Regt  | DD      | 1862-1864 |
| 31  | Thomas White  | Private  | 31st Regt  | EE      | 1862-1864 |
| 32  | George Black  | Private  | 32nd Regt  | FF      | 1862-1864 |
| 33  | Charles Green | Private  | 33rd Regt  | GG      | 1862-1864 |
| 34  | Henry Lee     | Private  | 34th Regt  | HH      | 1862-1864 |
| 35  | Richard King  | Private  | 35th Regt  | II      | 1862-1864 |
| 36  | Edward Clark  | Private  | 36th Regt  | JJ      | 1862-1864 |
| 37  | Samuel Adams  | Private  | 37th Regt  | KK      | 1862-1864 |
| 38  | Joseph Baker  | Private  | 38th Regt  | LL      | 1862-1864 |
| 39  | Samuel Adams  | Private  | 39th Regt  | MM      | 1862-1864 |
| 40  | John Smith    | Private  | 40th Regt  | NN      | 1862-1864 |
| 41  | James Brown   | Private  | 41st Regt  | OO      | 1862-1864 |
| 42  | William Jones | Private  | 42nd Regt  | PP      | 1862-1864 |
| 43  | Robert Taylor | Private  | 43rd Regt  | QQ      | 1862-1864 |
| 44  | Thomas White  | Private  | 44th Regt  | RR      | 1862-1864 |
| 45  | George Black  | Private  | 45th Regt  | SS      | 1862-1864 |
| 46  | Charles Green | Private  | 46th Regt  | TT      | 1862-1864 |
| 47  | Henry Lee     | Private  | 47th Regt  | UU      | 1862-1864 |
| 48  | Richard King  | Private  | 48th Regt  | VV      | 1862-1864 |
| 49  | Edward Clark  | Private  | 49th Regt  | WW      | 1862-1864 |
| 50  | Samuel Adams  | Private  | 50th Regt  | XX      | 1862-1864 |
| 51  | Joseph Baker  | Private  | 51st Regt  | YY      | 1862-1864 |
| 52  | Samuel Adams  | Private  | 52nd Regt  | ZZ      | 1862-1864 |
| 53  | John Smith    | Private  | 53rd Regt  | AAA     | 1862-1864 |
| 54  | James Brown   | Private  | 54th Regt  | BBB     | 1862-1864 |
| 55  | William Jones | Private  | 55th Regt  | CCC     | 1862-1864 |
| 56  | Robert Taylor | Private  | 56th Regt  | DDD     | 1862-1864 |
| 57  | Thomas White  | Private  | 57th Regt  | EEE     | 1862-1864 |
| 58  | George Black  | Private  | 58th Regt  | FFF     | 1862-1864 |
| 59  | Charles Green | Private  | 59th Regt  | GGG     | 1862-1864 |
| 60  | Henry Lee     | Private  | 60th Regt  | HHH     | 1862-1864 |
| 61  | Richard King  | Private  | 61st Regt  | III     | 1862-1864 |
| 62  | Edward Clark  | Private  | 62nd Regt  | JJJ     | 1862-1864 |
| 63  | Samuel Adams  | Private  | 63rd Regt  | KKK     | 1862-1864 |
| 64  | Joseph Baker  | Private  | 64th Regt  | LLL     | 1862-1864 |
| 65  | Samuel Adams  | Private  | 65th Regt  | MMM     | 1862-1864 |
| 66  | John Smith    | Private  | 66th Regt  | NNN     | 1862-1864 |
| 67  | James Brown   | Private  | 67th Regt  | OOO     | 1862-1864 |
| 68  | William Jones | Private  | 68th Regt  | PPP     | 1862-1864 |
| 69  | Robert Taylor | Private  | 69th Regt  | QQQ     | 1862-1864 |
| 70  | Thomas White  | Private  | 70th Regt  | RRR     | 1862-1864 |
| 71  | George Black  | Private  | 71st Regt  | SSS     | 1862-1864 |
| 72  | Charles Green | Private  | 72nd Regt  | TTT     | 1862-1864 |
| 73  | Henry Lee     | Private  | 73rd Regt  | UUU     | 1862-1864 |
| 74  | Richard King  | Private  | 74th Regt  | VVV     | 1862-1864 |
| 75  | Edward Clark  | Private  | 75th Regt  | WWW     | 1862-1864 |
| 76  | Samuel Adams  | Private  | 76th Regt  | XXX     | 1862-1864 |
| 77  | Joseph Baker  | Private  | 77th Regt  | YYY     | 1862-1864 |
| 78  | Samuel Adams  | Private  | 78th Regt  | ZZZ     | 1862-1864 |
| 79  | John Smith    | Private  | 79th Regt  | AAA     | 1862-1864 |
| 80  | James Brown   | Private  | 80th Regt  | BBB     | 1862-1864 |
| 81  | William Jones | Private  | 81st Regt  | CCC     | 1862-1864 |
| 82  | Robert Taylor | Private  | 82nd Regt  | DDD     | 1862-1864 |
| 83  | Thomas White  | Private  | 83rd Regt  | EEE     | 1862-1864 |
| 84  | George Black  | Private  | 84th Regt  | FFF     | 1862-1864 |
| 85  | Charles Green | Private  | 85th Regt  | GGG     | 1862-1864 |
| 86  | Henry Lee     | Private  | 86th Regt  | HHH     | 1862-1864 |
| 87  | Richard King  | Private  | 87th Regt  | III     | 1862-1864 |
| 88  | Edward Clark  | Private  | 88th Regt  | JJJ     | 1862-1864 |
| 89  | Samuel Adams  | Private  | 89th Regt  | KKK     | 1862-1864 |
| 90  | Joseph Baker  | Private  | 90th Regt  | LLL     | 1862-1864 |
| 91  | Samuel Adams  | Private  | 91st Regt  | MMM     | 1862-1864 |
| 92  | John Smith    | Private  | 92nd Regt  | NNN     | 1862-1864 |
| 93  | James Brown   | Private  | 93rd Regt  | OOO     | 1862-1864 |
| 94  | William Jones | Private  | 94th Regt  | PPP     | 1862-1864 |
| 95  | Robert Taylor | Private  | 95th Regt  | QQQ     | 1862-1864 |
| 96  | Thomas White  | Private  | 96th Regt  | RRR     | 1862-1864 |
| 97  | George Black  | Private  | 97th Regt  | SSS     | 1862-1864 |
| 98  | Charles Green | Private  | 98th Regt  | TTT     | 1862-1864 |
| 99  | Henry Lee     | Private  | 99th Regt  | UUU     | 1862-1864 |
| 100 | Richard King  | Private  | 100th Regt | VVV     | 1862-1864 |

QA533

D33

1855





- Rad: van: Side adj: Side opp-
- Rad: Sine: Hyp: Side opp-
- Rad: Cos: Hyp: Side adj-

Oblique angled Triangles.

- Side: each: Sin: Sin of an

7

significance.

allie

Rayland

Book - Keeping

otary.

