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A TREATISE ON HYDRAULICS

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A TREATISE

ON

HYDRAULICS

BY

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PREFACE

In the present treatise the author returns to a subject which occupied his attention at intervals during a long period, and which always seemed attractive. Not only has Hydraulics formed part of his teaching, but he was much engaged in the early sixties in designing water turbines and centrifugal pumps, and has had on many occasions since to consider questions of flow, storage, and measurement of water. In 1876, he wrote the article "Hydraulics" for the ninth edition of the *Encyclopædia Britannica,* which he has reason to think has been useful to many engineers.

Strictly rational hydrodynamics, so far as it has been developed, is concerned mainly with fluids deprived of viscosity, and leads to results flagrantly at variance with the action of actual fluids. Hence in dealing with the practical problems of hydraulics the engineer has recourse to comparatively simple mechanical principles and simplified assumptions which furnish rough formulæ, which can be modified by empirical constants so as to be true to the necessary approximation over any required range of conditions. There now exists an enormous mass of experimental data relating to hydraulic problems, which has been accumulated during a period extending over two centuries, and which is of very varying trustworthiness and importance. It is really on the results of these investigations that the engineer relies in deciding the questions which arise in many branches of professional work, and theoretical formulæ only render partial assistance in reducing to intelligibility and order the mass of empirical

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observations. The difficulty in treating hydraulics appears to the author to lie in the need of giving a sufficient account of experimental investigations to enable a student to realise the limitations of formulæ, and the degree of confidence which can be placed in calculations, without getting involved in a cumbrous and confusing amount of empirical details.

Full references have been given to primary sources of information, in order that students may supplement the necessarily brief statements in the text, by consulting the fuller details in original memoirs.

As to what is special in the present treatise, the author thinks it important that the problems concerning the flow of incompressible fluids, and the closely related problems dealing with compressible fluids, should be treated together. The practical importance of the latter class of problems has increased considerably in recent years.

To most of the chapters numerical examples have been added, selected from those which the author has set for his students during many years past.

July 1907.

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INTRODUCTION

UNITS OF MEASUREMENT

1. In practical hydraulics the most convenient units of measurement are the foot, the pound, and the second. In this treatise these units are used throughout, except in a few cases where other units are specially mentioned.

It happens that a great number of memoirs on hydraulics are in French or German, very important researches having been carried out abroad. In such memoirs metric units are employed. Hence a student of hydraulics finds it necessary to become more or less familiar with formulæ expressed in either English or metric units, and often has to convert formulæ from one system to the other. For that reason some particulars of the conversion factors from metric to English units are given. The convenient units in the metric system are the metre, the kilogram, and the second.

To avoid confusion, the secondary units employed should be the square foot (square metre), cubic foot (cubic metre), foot per second (metre per second), pound per square foot (kilogram per square metre). But in certain cases, especially in dealing with air and steam, the pound per square inch (kilogram per square centimetre) is almost in universal use, though in some respects inconvenient.

The following table gives the relation of the English and metric units, and the logarithms of the factors for conversion :—

CONVERSION OF ENGLISH AND FRENCH MEASURES

	Multiplier.	Logarithm.		Multiplier.	Logarithm.
Feet into metres	0·3048	$\bar{1}$·4840	Metres into feet	3·2808	0·5160
Square feet into square metres	0·0929	$\bar{2}$·9680	Square metres into square feet	10·764	1·0320
Cubic feet into cubic metres	0·02832	$\bar{2}$·4521	Cubic metres into cubic feet	35·315	1·5480
Mass—					
Pounds avoirdupois into kilograms	0·4536	$\bar{1}$·6567	Kilograms into pounds	2·2046	0·3433
Speed—					
Feet per second into metres per second	0·3048	$\bar{1}$·4840	Metres per second into feet per second	3·2808	0·5160
Work—					
Foot pounds into kilogram metres	0·13825	$\bar{1}$·1407	Kilogram metres into foot pounds	7·274	0·8618
Intensity of Load—					
Pounds per foot run into kilograms per metre	1·4882	0·1727	Kilograms per metre into pounds per foot run	0·672	$\bar{1}$·8273
Pressure—					
Pounds per square inch into kilograms per square centimetre	0·0703	$\bar{2}$·8470	Kilograms per square centimetre into pounds per square inch	14·225	1·1530
Pounds per square foot into kilograms per square metre	4·8826	0·6886	Kilograms per square metre into pounds per square foot	0·2048	$\bar{1}$·3114
Heaviness—					
Pounds per cubic foot into kilograms per cubic metre	16·019	1·2046	Kilograms per cubic metre into pounds per cubic foot	0·0624	$\bar{2}$·7954

2. **Units of volume.**—In most hydraulic calculations the convenient unit of volume is the cubic foot (or cubic metre). But in water-supply engineering it has been customary to use the gallon as the volume unit. The imperial gallon is defined to be the volume of 10 lbs. of distilled water at 62° F. Hence, if in general calculations the cubic foot of water is taken to weigh 62·4 lbs., it must also be taken to be equivalent to 6·24 gallons. In the metric system the kilogram is the weight of a cubic decimetre of water at 39°·1 F. Hence a cubic metre of water of maximum density weighs 1000 kilograms, and this value is taken in general calculations on pressure, etc., though at ordinary temperatures the weight is slightly less. In the United States the wine gallon, now disused in England, is the ordinary unit of volume, and is equal to 0·8333 imperial gallon.

Conversion Table

	Multiplier.	Logarithm.
Cubic feet into imperial gallons	6·24	0·7952
Cubic feet into U.S. gallons	7·49	0·8744
Cubic feet into cubic metres	0·02832	$\bar{2}$·4521
Imperial gallons into cubic feet	0·1603	$\bar{1}$·2048
U.S. gallons into cubic feet	0·1336	$\bar{1}$·1256
Cubic metres into cubic feet	35·31	1·5479
U.S. gallons into imperial gallons	0·8333	$\bar{1}$·9208
Imperial gallons into U.S. gallons	1·200	0·0792

To convert imperial gallons per 24 hours into cubic feet per second divide by 539,200.

To convert U.S. gallons per 24 hours into cubic feet per second divide by 647,100.

3. **Heaviness of water.**—In ordinary hydraulic calculations it is usual to disregard the small variations of density of water due to changes of pressure and temperature. In this treatise the weight of a cubic foot of water will be denoted by G and will be taken at 62·4 lbs. In calculations on the metric system the weight of a cubic metre is generally taken at 1000 kilograms from the simplicity which this

introduces into calculation. River and spring water is not sensibly denser than pure water unless in exceptional cases or when carrying mud or sewage. Sea water is usually taken at 64 lbs. per cubic foot, though its density varies somewhat in different localities.

Generally V cubic feet of water weigh GV lbs. in gravitation units. In treatises on theoretical hydromechanics absolute units are employed. Then if M is the mass in poundals, the weight is $W = Mg$ lbs. where g is the acceleration due to gravity in the locality considered. Hence if ρ is the density or mass of unit volume its weight is $g\rho$, and V units of volume weigh $g\rho V$ lbs.

Analyses of Some Typical Waters in Parts per 100,000

	Total Solids in Solution.	Temporary Hardness.	Total Hardness.	
Rain water . .	2·9	...	0·3	...
Loch Katrine .	3·1	...	1·0	From Moorland
Manchester . .	6·2	0·1	3·7	" "
Liverpool . .	8·5	0·1	3·7	" "
London, from .	25·8	...	19·2	Thames and Lea
to .	29·4	...	19·9	" "
London . .	40·3	...	28·7	Chalk wells
Northampton .	57·8	8·6	10·3	Well in Lias
Sea water . .	3898·0	49·0	797·0	...

Change of volume and density of water with change of temperature.—Water expands and decreases in density as the temperature rises, and though in ordinary hydraulic calculations this is disregarded without serious error, it is otherwise when dealing with water raised to steam temperatures. In the following short Table σ is the relative density, that of pure water at 39°·3 F. being taken as unity. G is the weight per cubic foot. Roughly, if the density of pure water is unity, that of river water is on the average 1·0003, that of spring water 1·001, and that of sea water 1·025.

DENSITY OF PURE WATER AT DIFFERENT TEMPERATURES

Temperature Fahr.	Relative Density.	Weight of a cub. ft. in lbs.	Temperature Fahr.	Relative Density.	Weight of a cub. ft. in lbs.
t	σ	G	t	σ	G
32°	·99987	62·416	130	·98608	61·555
39·3	1·00000	62·424	135	·98476	61·473
45	·99992	62·419	140	·98338	61·386
50	·99975	62·408	145	·98193	61·296
55	·99946	62·390	150	·98043	61·203
60	·99907	62·366	155	·97889	61·106
65	·99859	62·336	160	·97729	61·006
70	·99802	62·300	165	·97565	60·904
75	·99739	62·261	170	·97397	60·799
80	·99669	62·217	175	·97228	60·694
85	·99592	62·169	180	·97056	60·586
90	·99510	62·118	185	·96879	60·476
95	·99418	62·061	190	·96701	60·365
100	·99318	61·998	195	·96519	60·251
105	·99214	61·933	200	·96333	60·135
110	·99105	61·865	205	·96141	60·015
115	·98991	61·794	210	·95945	59·893
120	·98870	61·719	212	·95865	59·843
125	·98741	61·638			

For temperatures greater than those in the Table, Rankine's approximate rule may be used:

$$G = \frac{124 \cdot 85}{\frac{t + 461}{500} + \frac{500}{t + 461}}.$$

The following are values at a few temperatures calculated by this rule:—

t	G
50	62·42
100	62·02
200	60·08
250	58·75
300	57·29
350	55·78
400	54·21

It will be seen that in dealing with volumes of water at steam temperatures there would be great error in neglecting the change of density with change of temperature.

4. **Intensity of pressure.**—Very various units of intensity of pressure are adopted in different cases, depending in part on the different methods by which the pressure is measured. The following Table gives equivalent values of various units and the logarithms of the conversion factors :—

Units of Intensity of Pressure

	Multiplier.	Logarithm.
Atmospheres into lbs. per square inch . .	14·7	1·1672
„ „ „ square foot . .	2116·3	3·3256
„ „ kilograms per square centimetre	1·0335	0·0143
Feet of water at 53° into lbs. per square inch .	0·4333	$\bar{1}$·6368
„ „ „ „ square foot .	62·4	1·7952
Pounds per square inch into feet of water .	2·308	0·3632
„ square foot „ „ .	0·01603	$\bar{2}$·2049
Kilograms per square centimetre into lbs. per square inch	14·223	1·1530
Inches of mercury at 32° into lbs. per square inch	0·4912	$\bar{1}$·6912
„ „ „ „ square foot	70·73	1·8496

5. **Atmospheric pressure.**—In most cases a liquid mass has at some point a free surface exposed to atmospheric pressure which is transmitted throughout the mass. In any given case the atmospheric pressure can be deduced from the barometric height at the given place and time. On the average, at sea-level, the atmospheric pressure is 29·92 inches of mercury at 32°, 33·9 feet of water, 14·7 lbs. per sq. inch, or 2116·3 lbs. per sq. foot.

Many forms of pressure gauge indicate only the difference between the pressure at a point and atmospheric pressure. Pressures so observed are termed gauge pressures. The **gauge pressure** plus the atmospheric pressure is termed the **absolute pressure.**

6. **Acceleration due to gravity.**—The acceleration due to gravity, denoted by g, varies with latitude and elevation. In practical calculations it is usual to disregard this variation

in ordinary cases. In this treatise g will be taken at 32·18 ft. per sec. per sec., or at 9·8088 metres per sec. per sec.

English Measures	Logarithm	Metric Measures	Logarithm
$g = 32{\cdot}18$	1·5076	$g = 9{\cdot}8088$	0·9916
$2g = 64{\cdot}36$	1·8086	$2g = 19{\cdot}6176$	1·2927
$\sqrt{g} = 5{\cdot}673$	0·7538	$\sqrt{g} = 3{\cdot}1319$	0·4958
$\sqrt{2g} = 8{\cdot}023$	0·9043	$\sqrt{2g} = 4{\cdot}4292$	0·6463
$\frac{2}{3}\sqrt{2g} = 5{\cdot}349$	0·7283	$\frac{2}{3}\sqrt{2g} = 2{\cdot}9528$	0.4702

The following table gives an idea of the amount of the variation of g with latitude and elevation :—

Values of g and $\sqrt{2g}$

Latitude.	Typical Locality.	Elevation above Sea-Level in Feet.					
		0	2500	5000	0	2500	5000
		Values of g in Feet.			Values of $\sqrt{(2g)}$.		
60°	North Canada .	32·215	32·21	32·20	8·027	8·026	8·025
55°	North Britain .	32·200	32·19	32·18	8·025	8·024	8·023
40°	Mediterranean, Philadelphia	32·154	32·15	32·14	8·019	8·019	8·018
30°	North India, New Orleans	32·124	32·12	32·11	8·016	8·015	8·014
20°	Cuba . . .	32·099	32·09	32·08	8·012	8·011	8·010

At Greenwich $g = 32{\cdot}191$; $\sqrt{2g} = 8{\cdot}024$.
At Paris $g = 32{\cdot}183$; $\sqrt{2g} = 8{\cdot}023$.

7. **Transformation of an equation from one system of units to another.**—Rational homogeneous equations are valid in all systems of units, but a large proportion of hydraulic equations are empirical and require different numerical coefficients for different units. For instance, let

$$M = x\sqrt{\{A(1 + y\sqrt{B})\}}$$

be an equation in which M, A, B are in feet, and x and y are numerical coefficients. It is required to find the values of x and y when M, A, B are in metres. The equivalents of M, A, B metres in feet are 3·28M, 3·28A, 3·28B. Inserting these in the equation,

$$3{\cdot}28M = x\sqrt{\{3{\cdot}28A(1 + y\sqrt{3{\cdot}28B})\}}$$

$$M = x\frac{\sqrt{3{\cdot}28}}{3{\cdot}28}\sqrt{\{A(1 + y\sqrt{3{\cdot}28}\sqrt{B})\}}$$

$$= {\cdot}552x\sqrt{\{A(1 + 1{\cdot}81y\sqrt{B})\}},$$

so that the new constants for a formula in metric measures are $552x$, and $1{\cdot}81y$.

Hydraulic problems are most conveniently solved by the use of tables of four-figure logarithms and antilogarithms. Most hydraulic formulæ are affected by empirical constants which are accurate only to one per cent, or at most in some cases one per thousand. Hence in the answers it is unnecessary and useless to keep more than three, or at most four, significant figures. The short Tables I. and II. in the Appendix will often be useful in obtaining rapidly approximate answers.

Problems

1. A boiler is found to contain 72,000 lbs. of water at a temperature of 55° F. How many pounds will it contain at a temperature of 350° F.? 63,472.
2. How many gallons of water per foot run will a pipe 30 inches in diameter contain? 30·63.
3. Convert ten atmospheres of pressure into pounds per square inch, and into feet of water. 176·4 ; 407·1.
4. On a mountain the barometric pressure is observed to be 24 inches of mercury at 32° F. Find the pressure in pounds per square inch? 11·79.

CHAPTER I

PROPERTIES OF FLUIDS

8. FLUIDS are substances, the parts of which possess an almost unlimited mobility, which oppose almost no resistance to the separation of one part from another, or which offer practically no resistance to distortion of form. A mass of fluid poured into a vessel takes immediately the shape of the vessel and exhibits no rigidity of form.

A **perfect fluid** may be defined as a substance which yields continually to the slightest tangential stress, so that if it is at rest there can be no tangential stress. It is easily deduced from this that the pressure of a perfect fluid is normal to any surface immersed in it, or that the pressure of one part of a fluid on another part is normal to the interface which separates them. The stress at the surface or interface must be a pressure, not a tension, or there would be separation. Further, at any point in a fluid the pressure is the same in all directions, or to put it in another way, the pressure on any small element of surface is independent of its orientation.

Gaseous and liquid fluids.—Fluids are divided into liquids, or incompressible fluids and gases, or compressible fluids. Very great changes of pressure change the volume of liquids only by an extremely small amount, and if the pressure on them is reduced to zero they do not sensibly dilate. On the other hand, in gases or compressible fluids the volume alters sensibly for small changes of pressure, and if the pressure is indefinitely diminished they dilate without limit. In practical hydraulics water is treated as absolutely incompressible, so that its density or weight per cubic foot is considered to be independent of the pressure within the limits of accuracy usually required. In dealing with gases the changes of volume which accompany changes of pressure must always be taken into account, or in other words, the density is always expressed as a function of the pressure.

9. **Compressibility of liquids.**—All liquids are slightly compressible, and up to high pressures the compression is proportional to the pressure. Let $-\Delta V$ be the decrement of volume of V cubic feet for an increment of pressure ΔP in lbs. per square foot. Then $\Delta V/V$ is the compression per unit volume, and

$$k = \frac{\Delta P}{-\frac{\Delta V}{V}} = -V\frac{\Delta P}{\Delta V}$$

is called the co-efficient of elasticity of volume. For water, according to Grassi's observations, k increases from 42,000,000 at 32° F. to 48,000,000 at 128° F. The average value of k may be taken at 44,000,000 in ordinary cases, and then the compression is about 0·00005V for each atmosphere of pressure. Thus one cubic foot of water subjected to a pressure of 1000 lbs. per square inch, or about 64 tons per square foot, would decrease in volume by the amount

$$\Delta V = \frac{\Delta P}{k} = \frac{144{,}000}{44{,}000{,}000} = 0{\cdot}0032 \text{ cubic feet.}$$

One cubic foot weighing 62·4 lbs. uncompressed would become ·9968 cubic foot when compressed. The weight of the compressed water would be 62·4/0·9968 = 62·6 lbs. per cubic foot. It is obvious that the ordinary assumption that water is incompressible involves insignificant errors in ordinary cases.

10. **Viscous fluids.**—Actual fluids do oppose a small resistance to separation of parts and to distortion of form, and there may exist in them temporarily tangential stresses. Such fluids are termed **viscous** fluids.

In an elastic solid a distorting force produces immediately a definite deformation, which is permanent so long as the distorting force acts. In a viscous body the distortion increases as long as the force acts, and an indefinitely large distortion is produced in time by a distorting force however small. Alcohol is less, and oil more viscous than water. Certain substances, such as pitch or sealing-wax, are properly fluids with a very high viscosity. Under the action of gravity a block of pitch will flatten and flow in all directions like water, only the action is very slow. The resistance to distortion of a viscous body is proportional to the velocity of the relative motion of the parts, and becomes zero when the velocity is indefinitely small.

An interesting experiment due to Lord Kelvin illustrates the action of bodies so viscous as to have the appearance of solids. Let a disc of cobbler's wax, about three inches thick, be fixed in a vessel of water below the surface, and let some bullets be placed on the wax, and some corks below it. Under the action of the weight of the bullets and the buoyancy of the corks the wax will slowly yield. After some weeks it will be found that the bullets have sunk through the wax and the corks have risen above it. The disc of wax, however, will be found continuous and unperforated, having closed up during the passage of the solid bodies.

In ordinary fluids the viscosity is small, and in many problems may be neglected without sensible error. On the other hand, when the relative motion of parts of the fluid is rapid, it produces very considerable effects, and in such cases the problems are of so great complexity that usually they have to be dealt with by empirical methods. As water is the most generally diffused liquid, and the one which has generally to be considered in engineering problems, it will be taken as the representative liquid. The great mass of experimental investigation as to the behaviour of liquids under the action of forces has related to water.

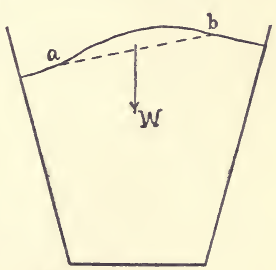


Fig. 1.

11. Free surface of a liquid.—The surface of a liquid at rest is horizontal. For if not, an inclined surface can be taken cutting the water surface in two points *a* and *b*. The weight W of the mass above *ab* will have a component acting down the incline, which could only be resisted by a tangential stress. But as there is no tangential stress in a liquid at rest its surface must be horizontal. In a very large water surface, such as that of a sea, the directions of gravity at distant points are not parallel. In that case the water surface is at all points perpendicular to the direction of gravity. That a liquid surface is a plane appears from the fact that it reflects objects undistorted like a plane mirror; and that it is horizontal appears from the fact that a plumb line and its reflection are in one straight line. If a vessel filled with water is moving with uniform velocity the water surface is still horizontal, for gravity is the only force acting on molecules at the surface. But if the vessel moves with acceleration the particles are subjected to a force equal and opposite to the accelerating force due to their inertia, and the water surface is then perpendicular to the resultant force acting on the molecules. For instance, if a vessel has a constant acceleration p per sec. per sec., the inertia of a molecule of weight W lbs.

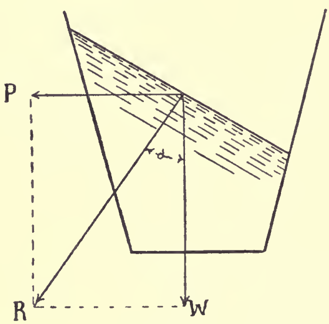


Fig. 2.

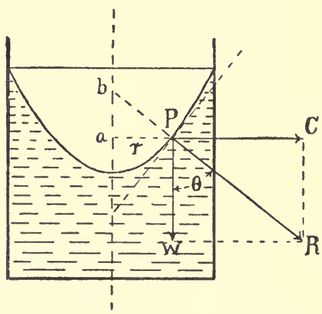


Fig. 3.

is $P = pW/g$. The surface of the water (Fig. 2) is perpendicular to the resultant R of P and W, which makes with the vertical the angle a, such that

$$\tan a = \frac{P}{W} = \frac{p}{g}.$$

If a vessel revolves uniformly about a vertical axis, the friction of the water against the vessel will cause it after a time to revolve with uniform angular velocity also like a solid. Let ω be the angular velocity, W the weight of a particle at the surface at P, where the radius is r. The velocity of the particle at P is ωr, and its radial acceleration is $C = Wr\omega^2/g$.

The resultant R of C and W (Fig. 3) makes with the vertical an angle θ, such that

$$\tan\theta = \frac{C}{W} = \frac{r\omega^2}{g}.$$

Produce R to meet the axis of rotation in b. The subnormal ab is

$$ab = r\cot\theta = \frac{g}{\omega^2},$$

a constant, which is a property of the parabola.

12. **Fluid pressure. Pascal's law.**—Fluid pressure is not only normal to any surface on which it acts, and independent of the orientation of the surface, but it is exerted equally in all directions throughout a fluid mass. Suppose a vessel fitted with pistons of equal area. Any inward force P applied to one of them, A, is instantly transmitted, and acts as an outward force P on all the others.

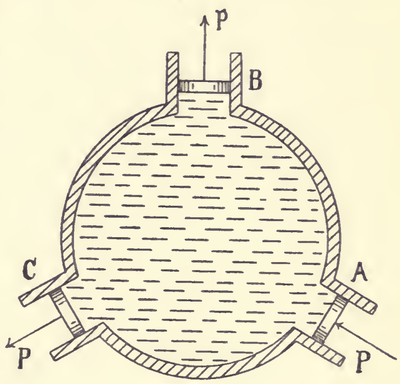


Fig. 4.

In any actual fluid the upper layers press by their weight on the lower layers. Hence, as will be discussed presently, the pressure in a fluid mass varies with the level. But there are light fluids, such as air, in which for a considerable difference of level there is only a small difference of pressure, and in heavier fluids such as water the general pressure may be so great, that the differences of pressure, due to such differences of level as there are in the mass considered, are relatively insignificant. If the pressure at a given level in a mass of water is 100 lbs. per square inch, or 14,400 lbs. per square foot, then for points 10 feet above and below that level the pressures are 13,776 and 15,024 lbs. per square foot, the whole difference being about 4 per cent. In some practical problems this difference can be neglected. With lighter fluids, such as air or steam, the variation of pressure with level is much less, and in a large class of problems is disregarded. In the atmosphere a difference of 1000 feet in level corresponds to less than 4 per cent difference of pressure. Hence it is convenient to divide problems on fluid pressure into two classes, in one of which the pressure is regarded as uniform throughout the mass, as if the fluid were weightless; in the other the variation of pressure with level is taken into account. On a level plane in a fluid the pressure is always uniform.

Fluid pressures are most conveniently measured in hydraulic calculations in lbs. per square foot. But there are other units of intensity of pressure the relations of which have been given in § 4. Pressures are experimentally determined by instruments termed gauges, and usually these measure the excess of the fluid pressure above the atmospheric pressure at the time and place. To find the absolute pressure the barometric pressure must be added to the gauge pressure (see § 5). In hydraulic problems the difference of pressure at two points in the fluid is alone the question, at both of which the atmospheric pressure is the same. Then the atmospheric pressure may be disregarded. But in some cases, for instance the question of the flow of gas in mains, the two points considered may be far apart and different in level, and then the difference of barometric pressure at the two points cannot be disregarded.

13. **Uniform fluid pressure on a plane.**—Consider a plane MN inclined at θ to the vertical and subjected to a uniform pressure p. Let MN be projected on two planes at right angles, for simplicity suppose horizontal and vertical planes. If $A = bl$ is the area of MN, the area of its horizontal projection is $A_h = A \sin \theta$, and that of its vertical projection is $A_v = A \cos \theta$. The resultant normal pressure on MN is $P = pA = pbl$. The vertical component of P is $V = P \sin \theta = pA \sin \theta = pA_h$. Similarly the horizontal component of P is $H = P \cos \theta = pA \cos \theta = pA_v$. Hence the resultant pressure on the plane MN in any given direction is the intensity of pressure p multiplied by the projected area of MN normal to that direction. This is true whatever the shape of the plane.

The pressure being uniformly distributed on the surface MN, the resultant acts through the centre of figure or mass centre of MN. Also its components act through the mass centres of the projections of MN.

Corollary.—On a horizontal plane in a fluid the pressure is always uniform and normal to the surface, and its resultant acts through the mass centre of the surface.

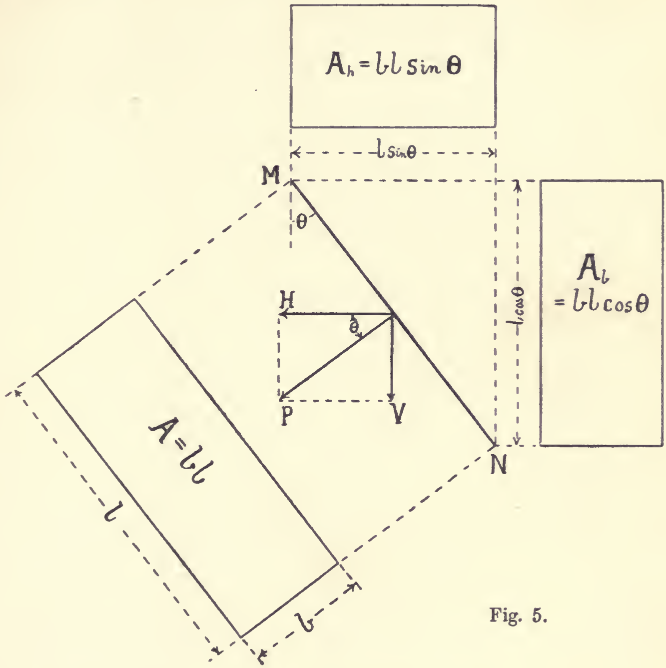


Fig. 5.

14. **Uniform pressure on curved surfaces.**—If the surface of area A on which a uniform fluid pressure p acts is not plane, the total amount of pressure on the surface is pA, but the pressure acts in different directions at different parts of the surface, and the resultant pressure on the surface has a different value.

Let ACB be any curved surface, which for simplicity may be taken to be one foot in length perpendicular to the paper. The uniform internal pressure is p. The total pressure on any small element $\underline{ab}$ is $p \times \overline{ab}$, and the horizontal and vertical components of this are $p \times \overline{be}$ and $p \times \overline{ae}$. But the horizontal component will be exactly balanced by the horizontal pressure on the vertical projection *cf* of the part *cd* of the surface. The only unbalanced part of the pressure on *ab* is $p \times \overline{ae}$, and the resultant vertical pressure on the whole curved surface ACB is $p\ \Sigma\ \overline{ae}$, that is $p \times$ the horizontal projected area of the curved surface—that is, if the ring is one foot in length, *pl* lbs.

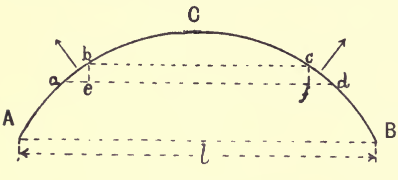


Fig. 6.

Hence the resultant pressure on any curved surface cut off by a plane is normal to the plane, and equal to **the intensity of pressure multiplied by the area of the projection of the surface on the plane.**

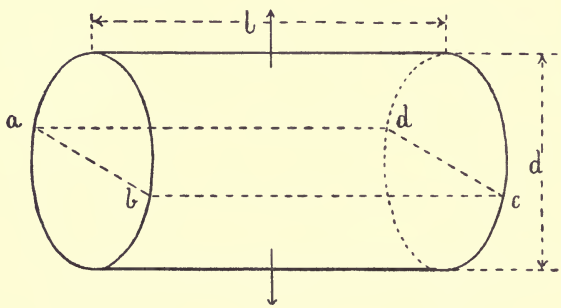


Fig. 7.

Example 1.—Consider a hollow cylinder of diameter *d* feet subjected to a uniform internal pressure *p* lbs. per square foot. Let *abcd* be a diametral plane dividing the cylinder into halves. The resultant pressure P on each half is normal to *abcd*, and equal to

$$P = p \times \text{area } abcd$$
$$= pld \text{ lbs.},$$

because *ld* is the area of the projection *abcd* of the semicylinder.

Example 2.—Some pumps have trunks of half the area of the piston.

Let D be the diameter of the piston *ab* in feet, *d* that of the trunk *cd*, and let p_1, p_2 be the pressures on front and back of the piston in lbs. per square foot. Then $P_1 = p_1 \frac{\pi}{4} D^2$ acts forward on the back of the piston, and $P_2 = p_2 \frac{\pi}{4}(D^2 - d^2)$ acts backwards on the annular face of the piston. The resultant force driving the piston is

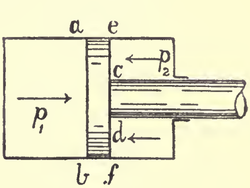


Fig. 8.

$\frac{\pi}{4}\{(p_1-p_2)D^2+p_2d^2\}$ lbs. If the piston faces are recessed or curved in any way the resultant driving pressure is not altered.

15. **Abutment at dead ends or bends of pipes.**—The ends of pipes when blanked off are subject to an endways thrust which, if not resisted by an abutment, would draw the adjacent pipe joints. Let d be the diameter of the pipe in inches, h the greatest statical pressure in the pipe in feet of head, for instance the difference of level of the surface of water in the supply reservoir and the pipe end. Then as, from § 4, the pressure is 0·4333 h lbs. per square inch, the total thrust on the pipe end is

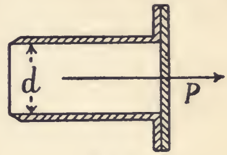


Fig. 9.

$$P=\frac{\pi}{4}\times \cdot 433d^2h=0\cdot 34d^2h \text{ lbs.}$$

This is often a considerable force. In a 36-inch pipe under 200 feet of head the thrust would be 88,180 lbs., or nearly forty tons. Under certain circumstances, such as the sudden shutting of a valve on a branch near the pipe end, an additional thrust due to dynamical action might be produced.

Consider next a pipe bend (Fig. 10), and let $dOb=\theta$, $d=$ the pipe diameter, and h the head in feet. The wedge $abcd$ is acted on by the thrusts $P=P=\frac{\pi}{4}\times 62\cdot 4d^2h=49d^2h$ lbs. along the axis of each pipe. The resultant thrust tending to displace the bend is

$$R=2P\sin\frac{\theta}{2}=98d^2h\sin\frac{\theta}{2} \text{ lbs.}$$

Thus for a 36-inch pipe with a head of 200 feet, bent at an angle of 120°, so that $\theta=60°$,

$$R=98\times 9\times 200\times \sin 30°=88,200 \text{ lbs.}$$

If the water is flowing round the bend there is additional thrust due to the deviation of the water, which will be discussed in a later chapter. It is usual to provide a masonry or concrete block to resist the thrust in such cases.

The result can be arrived at in another way. If we suppose the pipe divided into two troughs of semicircular section by the line *ef*, all the upward-acting pressures act on the upper, and all the downward-acting pressures on the lower trough. The projections of the troughs on a horizontal plane

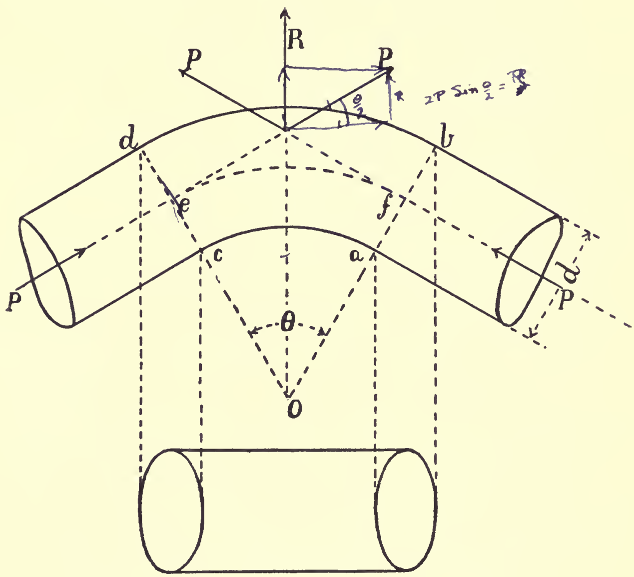


Fig. 10.

are shown below. The difference of their areas is the area of the two ellipses, the major axes of which are d and their minor axes $d \sin \frac{\theta}{2}$. Hence the upward thrust is

$$R = 2\frac{\pi}{4}d^2p \sin \frac{\theta}{2},$$

$$= 2P \sin \frac{\theta}{2} = P\frac{\text{chord } ef}{\text{radius } Oe}.$$

16. **Hydraulic press.**—Suppose a vessel fitted with two pistons of area a and A normal to the direction in which the pistons move. If a downward pressure P_1 is exerted on the smaller piston a much greater upward pressure P_2 will be exerted on the larger. The intensity of pressure in the fluid is P_1/a, and the upward pressure on the large piston is $P_2 = P_1A/a$. This is the principle of the hydraulic press, in

which pressure produced by the plunger of a small pump is transmitted to a very large ram.

Obviously the small piston will move a greater distance

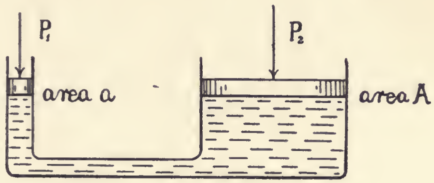


Fig. 11.

than the larger. If v_1, v_2 are the piston velocities, $v_1/v_2 = A/a$. The volume $v_1 a$ displaced by the small piston is equal to the volume $v_2 A$ described by the large piston. Generally the friction of the pistons is not inconsiderable, and this modifies somewhat the ratio of the efforts given above.

Example.—The pump plunger of a large press is $\frac{3}{4}$ inch in diameter, and the press ram is 20 inches in diameter. Then $A/a = 20^2/(\frac{3}{4})^2 = 710$. Suppose a man exerts by a lever a force of $P_1 = 150$ lbs. on the plunger. Then the upward force exerted by the press ram is $710 \times 150 = 106,500$ lbs., or 48 tons. That is neglecting the friction of plunger and ram. To move the press ram one inch the plunger must move through 710 inches. Forging presses have been made on this principle capable of exerting an effort of 10,000 tons.

Problems

1. Treating water as incompressible, find the pressure in tons per square foot on the bed of the Atlantic, the depth being 5 miles, weight of sea water 64 lbs. per cubic foot. 754.
2. With the conditions in the last question, find the weight of a cubic foot of water at the bed of the Atlantic, taking the compression of the water into account. 66·6 lbs. per cubic foot.
3. A pipe 24 inches in diameter has a right-angled bend. The pressure in the pipe is 150 feet of head. Find the force tending to displace the bend. 18·6 tons.
4. Show that the surface of water in the buckets of a water-wheel revolving uniformly are parts of cylindrical surfaces having the same axis.

CHAPTER II

DISTRIBUTION OF PRESSURE IN A LIQUID VARYING WITH THE LEVEL

17. **Pressure column. Free surface level.**—Let a small vertical pipe AB be introduced into a mass of liquid. The liquid will rise in the pipe to some level OO, such that the weight of the column BA balances the pressure on its mouth. This is true whether the liquid is at rest or in motion, provided the mouth of the pipe is parallel to the direction of motion so that the liquid does not impinge on it. The height $AB = h$ measures the pressure at A. Let ω be the area of the cross section of the pipe, p the intensity of pressure at A, and G the weight of a cubic unit of fluid

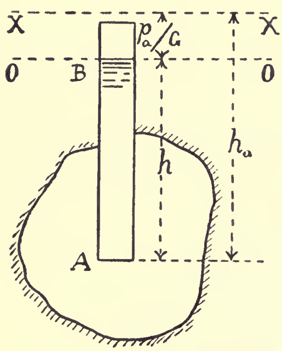


Fig. 12.

$$p\omega = Gh\omega$$

$$p = Gh \text{ or } h = p/G \quad . \quad (1).$$

If h is in feet, p in lbs. per sq. ft., $G = 62{\cdot}4$. For metre-kilogram units $G = 1000$. The result is expressed by saying that h is the height due to the pressure p, or conversely p the pressure due to the height h. The level OO is the free surface level.

In general, atmospheric pressure will be acting on the free surface at OO. Consequently h measures the gauge pressure, not the absolute pressure at A (§ 5). Let p_a be the atmospheric pressure in lbs. per sq. ft. Then p_a/G is the height in feet of water equivalent to atmospheric pressure, and the absolute pressure at A is $p = Gh + p_a$ lbs. per sq. ft., or $h + p_a/G$

feet of water. p_a/G is about 33·9 feet on the average. If a line XX is taken at a height p_a/G above OO, the absolute pressure at A is h_a feet of water, the layer between XX and OO representing a layer of water the weight of which is equivalent to atmospheric pressure. In many hydraulic problems only differences of pressure at two points are concerned, and atmospheric pressure may then be ignored.

18. **Relative level of liquids of different density.**—Suppose two liquids of density G_1, G_2 are placed in a bent tube. At the level of the plane of separation OO the pressure must be the same in both arms. Hence the pressure of the two columns above that level must be the same

$$G_1 h_1 = G_2 h_2$$
$$G_1/G_2 = h_2/h_1 \quad . \quad (2).$$

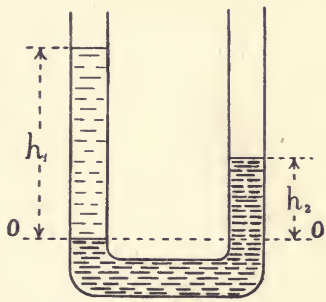


Fig. 13.

As atmospheric pressure is the same on both columns it does not need to be taken into consideration.

Watt's hydrometer.—A bent tube connects two beakers containing fluids of different densities G_1, G_2. If a partial vacuum is formed in the bent tube the liquids will rise to different heights h_1, h_2. Let p_0 be the pressure in the bent tube and p_a the atmospheric pressure on the free surface in the beakers. The pressure due to the weight of the columns in each leg must be equal to the difference of pressure $p_a - p_0$. Hence

$$p_a - p_0 = G_1 h_1 = G_2 h_2$$
$$G_1/G_2 = h_2/h_1 \quad . \quad . \quad (3).$$

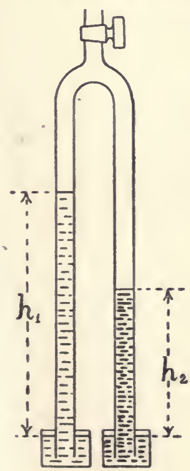


Fig. 14.

If the density of one of the fluids is known, that of the other can be determined by measuring the height of the columns.

Mercury siphon gauge.—Pressure is often measured by a siphon gauge AB containing mercury and open at one end to the atmosphere. Let Fig. 14*a* represent a water main C in which the pressure is to be determined, and let b be the atmospheric pressure in inches of mercury, h the difference of level of the mercury columns in the siphon gauge in inches. The absolute pressure at A is b inches of mercury, that at B is $b+h$ inches of mercury. If the specific gravity of mercury is 13·57, the absolute pressure at B is

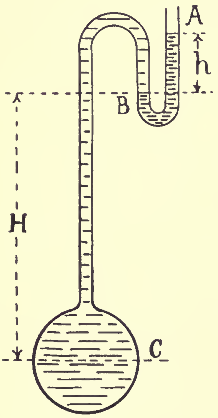


Fig. 14*a*.

$$(b+h)\frac{13 \cdot 57}{12} = 1 \cdot 131(b+h) \text{ feet of water.}$$

If, as is often the case, the siphon gauge is at a considerable height H feet above the centre C of the main, the absolute pressure at C is

$$1 \cdot 131(b+h) + \text{H feet of water,}$$

and the gauge pressure, or pressure in excess of atmospheric pressure, is

$$1 \cdot 131h + \text{H}.$$

19. **Pressure on surfaces varying as the depth from the free surface.**—In any heavy fluid the pressure must increase with the depth reckoned from the actual or virtual free surface.

Let A be a small vertical surface of area ω sq. ft. at a depth h ft. The intensity of pressure at that depth is $p = \text{G}h$ lbs. per sq. ft. The total pressure on the surface is $p\omega = \text{G}h\omega$ lbs. Take a surface B equal and parallel to A at a distance h, and complete the prism AB. Its volume is $h\omega$, and if composed of fluid its weight is $\text{G}h\omega$ lbs.

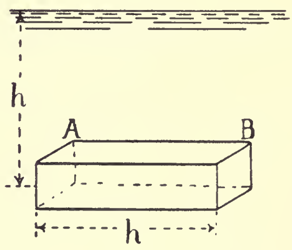


Fig. 15.

Hence the horizontal pressure on a small vertical surface at the depth h is equal to the weight of a prism of fluid of length h and cross section equal to the area of the surface. It will easily be seen that the restriction to a vertical surface is not necessary.

But in any case the resultant pressure thus estimated is normal to the surface.

When the surface is not small it cannot be regarded as all at the same depth. But for each small element of the surface the rule applies. Consider a strip *abcd* of a vertical wall, of width $ab = b$, and height $ad = h$, supporting water pressure. Take $de = h$ and complete the wedge *abcdef*. At any depth the intensity of pressure is proportional to the horizontal thickness of the wedge at that depth. The total

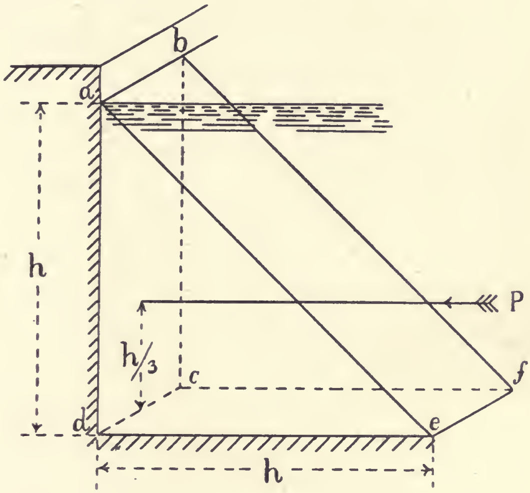


Fig. 16.

pressure on the wall is the weight of a wedge *abcdef* of fluid. The volume of the wedge is $\frac{1}{2}bh^2$ and the pressure on the wall is

$$P = \tfrac{1}{2}Gbh^2 \text{ lbs.} \quad . \quad . \quad . \quad (4).$$

Further, since the distribution of pressure is represented by the wedge, the resultant pressure acts through the mass centre of the wedge, that is at $h/3$ above the base.

As the pressure varies uniformly the mean pressure in this case is

$$p_m = \tfrac{1}{2}Gbh^2/bh = \tfrac{1}{2}Gh \text{ lbs. per sq. ft.},$$

which is the pressure at the mass centre of *abcd*.

The rule is general. The mean pressure on any immersed plane is the pressure at its mass centre due to its depth from the free surface, and the resultant pressure normal to the surface is the mean pressure multiplied by the area of the surface. The point of the surface at which the resultant pressure acts is not in general the mass centre, and this will be determined presently. In the case of a curved surface the total pressure is also the pressure due to the depth of the mass centre multiplied by the area of the surface, but this result has little meaning. As the pressure acts everywhere normal to the surface the total pressure consists of components acting in different directions. The resultant pressure on a curved surface will be found presently.

Example.—A vertical semicircular plate of radius r feet and area $\omega = \frac{1}{2}\pi r^2$, supports water on one side level with its straight edge. The depth of the mass centre of the semicircle is $4r/3\pi$. The mean pressure on the surface is $p_m = 4\mathrm{G}r/3\pi$ lbs. per square foot. The resultant pressure on the surface is

$$\mathrm{P} = p_m \omega = \frac{4\mathrm{G}r}{3\pi} \frac{\pi r^2}{2} = \frac{2}{3}\mathrm{G}r^3 \text{ lbs.}$$

Water at different levels on two sides of a wall.—In cases of this kind it is convenient to consider a strip of the wall one foot in width (Fig. 17).

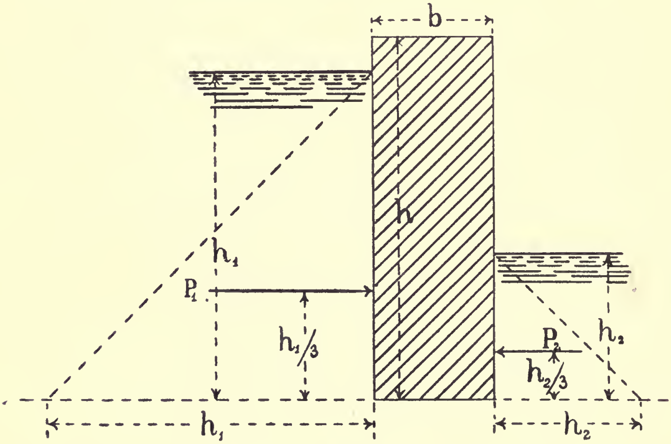


Fig. 17.

Let h_1, h_2 be the depths of water. The distribution of pressure on each side is given by the dotted triangles with bases equal to h_1, h_2, and the total pressures P_1, P_2 are equal to the weight of wedges of water one foot thick of the area of these triangles. Hence the pressures per foot run of wall are $P_1 = \frac{1}{2}Gh_1^2$ and $P_2 = \frac{1}{2}Gh_2^2$ lbs. These pressures act at $h_1/3$ and $h_2/3$ above the base of the wall, and the overturning moment about the toe A of the wall is

$$\tfrac{1}{6}G(h_1^3 - h_2^3) \text{ ft.-lbs.} \quad . \quad . \quad . \qquad (5).$$

Let the wall be h feet high and b feet thick, and let G_m be the weight per cubic foot of masonry. The weight of the wall is $G_m bh$ lbs., and the moment about A resisting overturning is $G_m bh \times \frac{1}{2}b = \frac{1}{2}G_m b^2 h$. If the moment of stability is to be $2\frac{1}{2}$ times the overturning moment

$$\frac{5}{4}G_m b^2 h = \frac{1}{6}G(h_1^3 - h_2^3)$$

$$b = \sqrt{\left\{ \frac{2}{15} \frac{G}{G_m} \frac{h_1^3 - h_2^3}{h} \right\}} \text{ ft.} \quad . \qquad (6).$$

In this case as the total atmospheric pressure is the same on both sides of the wall it is neglected without any error.

20. **Pressure on a flap valve covering the end of a pipe of circular section** (Fig. 18).

Let d be the diameter of the pipe in feet and θ the angle of inclination of the flap to the vertical. The projection of the flap on a vertical plane is a circle of area $A_v = \frac{\pi}{4}d^2$. Its projection on a horizontal plane is an ellipse, the principal axes of which are d and $d \tan \theta$. Hence its area is $A_h = \frac{\pi}{4}d^2 \tan \theta$.

The mean head on the flap is h measured to its centre of figure. The horizontal and vertical components of the pressure on the flap are equal to the mean pressure multiplied by the areas of the vertical and horizontal projections. That is, the vertical component is

$$P_v = Gh \times A_h = \frac{\pi}{4}Ghd^2 \tan \theta \text{ lbs.},$$

and the horizontal component is

$$P_h = Gh \times A_v = \frac{\pi}{4}Ghd^2 \text{ lbs.}$$

The resultant pressure normal to the flap is

$$P = \sqrt{(P_v^2 + P_h^2)} = GhA \text{ lbs.} \quad . \quad . \quad (7)$$

It will be shown presently that the horizontal component acts at a point $h + d^2/16h$ below the water surface, which is more nearly equal to h as h is greater. If the horizontal component

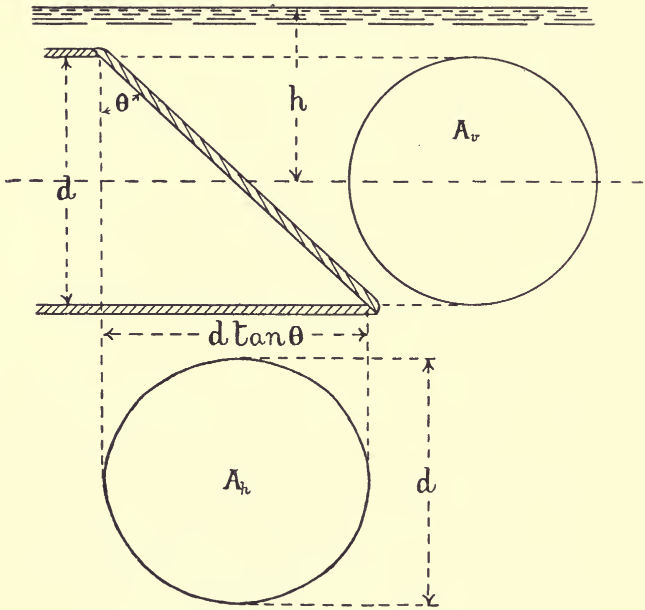


Fig. 18.

is drawn at this depth, the point where it intersects the flap is the centre of pressure at which the resultant pressure on the flap acts.

21. **Centre of pressure on any vertical surface.**—Let AB (Fig. 19) be any surface of area A square feet, the vertical projection of which is given on the right. Let h_1, h_2 be the depths of A and B from the free surface. Let D be the mass centre of the surface at depth h_m and E the centre of pressure at depth z. The resultant pressure on the surface is

$$P = Gh_mA \text{ lbs.}$$

Consider a horizontal strip of the surface between the depths

h and $h + dh$ and of width b. Its area is bdh, and the pressure on it is $\mathrm{G}bhdh$. The moment of this, about a horizontal axis through C, is $\mathrm{G}bh^2dh$. The total moment of the pressure on the surface about C is therefore

$$\mathrm{G}\int_{h_1}^{h_2} bh^2dh = \mathrm{GI},$$

where I is the moment of inertia of the surface about a horizontal axis through C, normal to the plane of the figure.

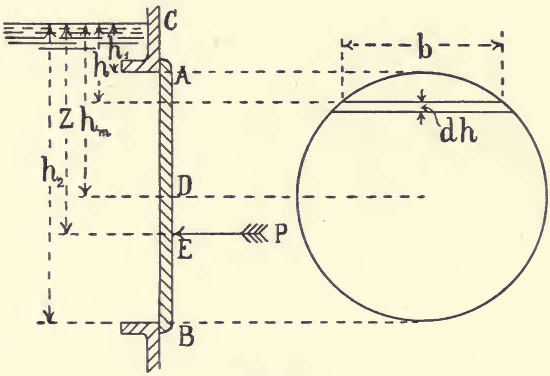


Fig. 19.

But this must be equal to the moment of the resultant pressure about the same axis. Hence

$$\mathrm{P}z = \mathrm{G}h_m\mathrm{A}z = \mathrm{GI}$$

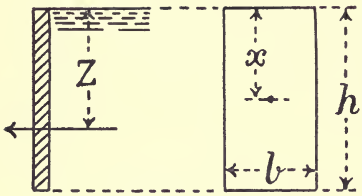
$$z = \frac{\mathrm{I}}{\mathrm{A}h_m} \quad . \quad . \quad . \quad . \quad (8),$$

or if $\mathrm{I} = k^2\mathrm{A}$ where k is the radius of gyration of the surface about the axis through C,

$$z = \frac{k^2}{h_m} \quad . \quad . \quad . \quad . \quad (9).$$

The moment of inertia of a surface about an axis through the mass centre of the surface is known for various surfaces. Let I_0 be the moment of inertia of the surface about an axis through its mass centre and normal to the plane of the figure.

A



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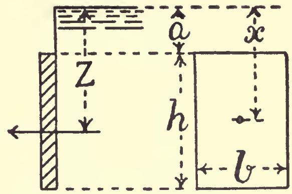

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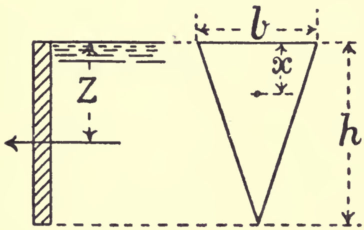

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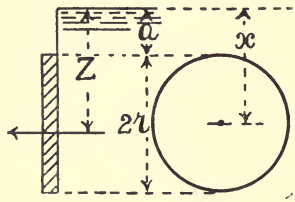

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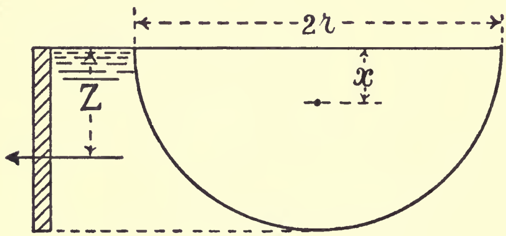

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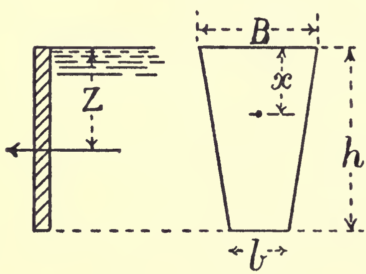

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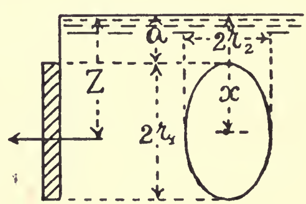

CENTRE OF PRESSURE AND TOTAL PRESSURE

Surface.	Area A.	Depth of Mass Centre from Free Surface x.	Depth of Centre of Pressure from Free Surface z.	Total Pressure on Surface P.
Rectangle A .	bh	$\frac{h}{2}$	$\frac{2}{3}h$	$\frac{1}{2}\mathrm{G}bh^2$
Rectangle B .	bh	$a+\frac{h}{2}$	$\frac{h}{3}\cdot\frac{3a+2h}{2a+h}+a$	$\mathrm{G}\left(a+\frac{h}{2}\right)bh$
Triangle C .	$\frac{1}{2}bh$	$\frac{h}{3}$	$\frac{h}{2}$	$\frac{1}{6}\mathrm{G}bh^2$
Circle D . .	πr^2	$a+r$	$a+r+\frac{r^2}{4(r+a)}$ [$a=0$ $z=5/4r$]	$\mathrm{G}\pi r^2(a+r)$
Semicircle E .	$\frac{1}{2}\pi r^2$	$\frac{4r}{3\pi}$	$\frac{3}{16}\pi r$	$\frac{2}{3}\mathrm{G}r^3$
Ellipse G, one axis vertical	$\pi r_1 r_2$	$a+r_1$	$a+r_1+\frac{r_1^2}{4(a+r_1)}$	$\mathrm{G}\pi r_1 r_2(a+r_1)$
Trapezium F .	$\frac{1}{2}h(\mathrm{B}+b)$	$\frac{h}{3}\frac{\mathrm{B}+2b}{\mathrm{B}+b}$	$\frac{h}{2}\frac{\mathrm{B}+3b}{\mathrm{B}+2b}$	$\frac{1}{6}\mathrm{G}h^2(\mathrm{B}+2b)$

Then by the well-known rule

$$I = I_0 + Ah_m^2$$

$$z = \frac{I_0 + Ah_m^2}{Ah_m} \quad . \quad . \quad . \quad (10).$$

Example.—Let the surface be a circle of diameter d. Then $I_0 = \frac{\pi}{64}d^4$.

$$z = \frac{\frac{\pi}{64}d^4 + \frac{\pi}{4}d^2h_m^2}{\frac{\pi}{4}d^2h_m}$$

$$= h_m + \frac{d^2}{16h_m}.$$

22. **Pressure and centre of pressure on any plane surface.**—Let AB be the surface in a plane normal to the plane of the figure inclined at θ to the horizontal. Take OB for the X axis and an axis through O perpendicular to the plane of the figure for the Y axis. Let A be the area of the surface; R the pressure on it; OA $= x_1$; OB $= x_2$. Let c be the mass centre of the surface and d the centre of pressure, and let $Oc = x_c$ and $Od = x_d$.

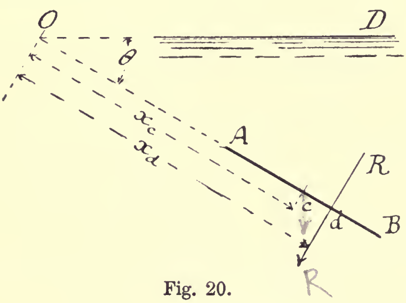

Fig. 20.

Consider a strip of the surface between x and $x + dx$ of breadth y. Its depth below the water surface is $x \sin \theta$, and the total pressure on it is $Gx \sin \theta y dx$. Hence the whole pressure on AB is

$$R = G \sin \theta \int_{x_1}^{x_2} xydx.$$

$$\text{But} \int_{x_1}^{x_2} xydx = Ax_c$$

$$R = GAx_c \sin \theta,$$

where $Gx_c \sin \theta$ is the intensity of pressure at the mass centre of the surface. Taking moments about the Y axis,

$$Rx_d = G \sin \theta \int_{x_1}^{x_2} x^2ydx.$$

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But $\int_{x_1}^{x^2} x^2 y dx$ is the moment of inertia I of the surface about the Y axis,

$$x_d = \frac{\mathrm{GI}\sin\theta}{\mathrm{R}} = \frac{\mathrm{I}}{\mathrm{A}x_c} \quad . \quad . \quad . \quad (11).$$

But $\mathrm{I} = k^2\mathrm{A}$ where k is the radius of gyration of the surface about the Y axis,

$$x_d = \frac{k^2}{x_c} \quad . \quad . \quad . \quad . \quad (12).$$

The lateral position of the centre of pressure is found thus: the mass centre and centre of pressure of the surface are in the same vertical plane, parallel to the plane of the figure.

When surfaces are not vertical it is often convenient to find the component pressures on their horizontal and vertical projections separately and combine them.

The Table on p. 29 gives the pressure and depth of centre of pressure for various vertical surfaces.

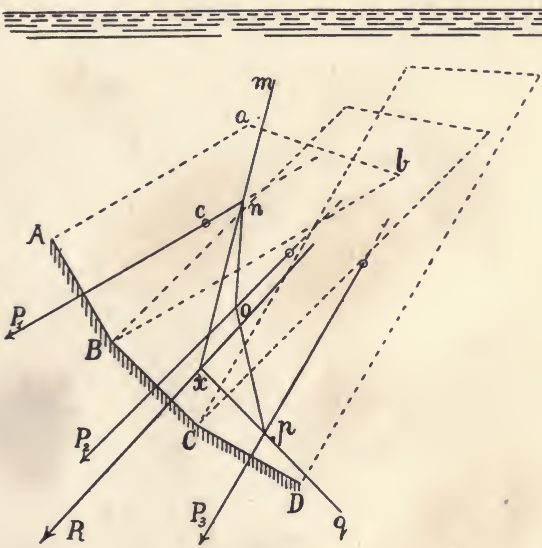

Fig. 20*a*.

23. **Graphic determination of the pressure on surfaces.** —Case of a curved face of a retaining wall or dam. Let Fig 20*a* represent the vertical section of a curved wall,

ABCD, which may be treated as polygonal without serious error if the divisions are taken small enough. It is convenient in such cases to consider one foot length of the wall.

The curved face being divided into lengths AB, BC, CD, each equal to a, the area of these faces will be a also. Let h_1, h_2, h_3, h_4 be the depths of A, B, C, D below the free surface. Take Aa normal to AB and equal to h_1; Bb normal to AB and equal to h_2. Join ab. Then AabB represents in magnitude and distribution the normal pressure on AB. The total pressure on AB is the weight of a prism of water AabB one foot thick. That is $P_1 = \frac{1}{2}Ga(h_1 + h_2)$, and it acts through the mass centre C of AabB normally to AB. Similarly the pressures $P_2 = \frac{1}{2}Ga(h_2 + h_3)$, and $P_3 = \frac{1}{2}Ga(h_3 + h_4)$ can be found in position and direction. Draw the force polygon (Fig. 20b) with sides equal on any scale and parallel to P_1, P_2, P_3. The closing line gives the resultant R in magnitude and direction. Choose a pole O and draw rays to the angles of the force polygon. Next draw the funicular polygon $mnopq$ with sides mn, no, op, pq parallel to the rays, taken in order, and intersecting the pressures P_1, P_2, P_3 at n, o, p. Produce the first and last lines of the funicular polygon to meet in x. Then x is a point through which the resultant R of the pressure acts. R can be drawn through x and parallel to R in the force polygon. The resultant pressure on ABCD is therefore found in magnitude, position, and direction.

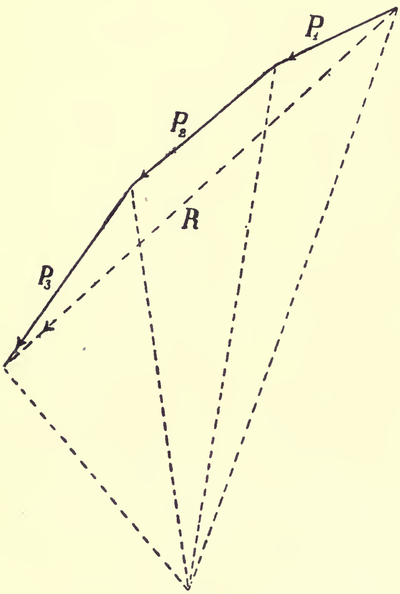

Fig. 20b.

24. **Loss of weight of immersed bodies. Buoyancy. Principle of Archimedes.**—Let Fig. 21 represent a body

immersed in water. Consider a prism *ab* of small cross section at a depth h. Since the vertical projections of the two ends of this prism are equal, and the pressure due to the depth h is the same on each, the horizontal forces on the prism must balance; and since the body can be divided into such prisms the horizontal forces on the whole body must balance also. Next consider a small vertical prism *cd*. If ω is the horizontal cross section, and h_1, h_2 the depths

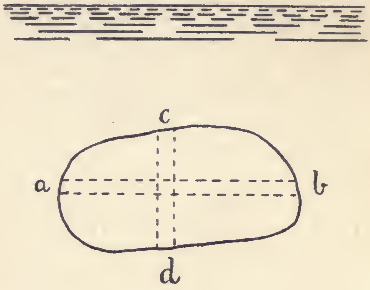

Fig. 21.

of the ends below the free surface, the resultant pressure acting on it is an upward force $G\omega(h_2 - h_1)$. But this is equal and opposite to the weight of a prism *cd* of water. Since the body can be divided into a set of similar vertical prisms, the whole upward pressure on it must be the weight of a volume of water equal to the volume of the body. If W is the weight of the body not immersed, and V its volume, the upward pressure is GV, and the resultant downward force $W - GV$. The body loses, when immersed, a weight equal to the weight of water displaced. The upward pressure GV is termed the buoyancy, and it acts through the mass centre of the water displaced, a point termed the centre of buoyancy. If the body is homogeneous, the centre of buoyancy coincides with the mass centre of the body, provided it is wholly immersed. If the body is not wholly immersed, or is hollow or of varying density, the centre of buoyancy will not generally coincide with the mass centre of the body.

Note that if GV is greater than W the body will float. As part of it rises out of the water, the volume V of water displaced diminishes. The plane of flotation when the body comes to rest is such that $GV = W$ where V is not now the volume of the body, but the volume of the water displaced, the buoyancy then exactly balancing the weight.

25. **Equilibrium of floating bodies.**—If a body floats on water the weight W of the body and the buoyancy B are equal. But W acts at the mass centre *b* of the body, and B

at the mass centre *a* of the displaced water. If these are not on the same vertical there is a couple Wx tending to turn the body, and it must move till *a* is on a vertical through *b*. The line passing through *a* and *b* when the body has taken a position of rest is called the axis of flotation. If the axis of flotation is known, as in the case of various symmetrical bodies, the depth of flotation is easily found. Thus if the body is a prism of section A perpendicular to the axis of flotation, W its weight, and D the depth immersed,

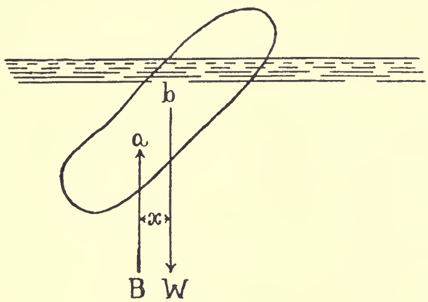

Fig. 22.

$$D = W/GA.$$

Stability of floating bodies. Metacentre.—A body floats in an upright position if a plane through the axis of flotation divides it into symmetrical parts. The body is stable if when slightly displaced it returns to its former position, unstable if a small displacement tends to increase. Let Fig. 23 represent a floating body, and let W be its weight, V its displacement, so that $W = GV$. Let B be the centre of buoyancy when the body floats upright, and G its mass centre. If the body is displaced, the centre of buoyancy moves out to some point B_1. The weight W and buoyancy GV then form a couple tending to rotate the body. Let M be the intersection of GV with the axis of flotation through B and G. This point is termed the **metacentre**. If M is above G the body

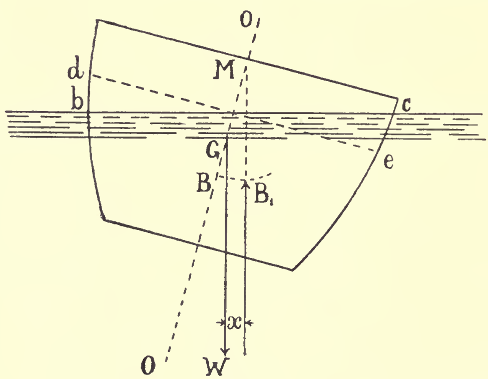

Fig. 23.

will turn so that G sinks and M rises, and the action tends to annul the displacement. If M is below G the body is unstable. If M and G coincide equilibrium is indifferent.

When M is above G the righting couple is Wx, where x is the horizontal distance between the metacentre M and the mass centre G. If MG $= c$ and ϕ is the angle of displacement, the righting couple is W$c \sin \phi$. It increases, therefore, with c. On the other hand the rapidity of rolling increases with c, and therefore there is a limit to the metacentric height which is desirable. But these are questions beyond the scope of the present treatise.

Problems.

1. If mercury is $13\frac{1}{2}$ times heavier than water, find the height in inches of a mercury column corresponding to a pressure of 100 lbs. per square inch. 205·1.
2. A masonry dam vertical on the water side supports water of 100 feet depth. Find the pressure per square foot at 25 and 75 feet from the water surface, and the total pressure on one foot length of dam. 1560 and 4680 lbs. per square foot; 312,000 lbs.
3. Find the resultant pressure on a circular plate 5 feet in diameter, with its top edge 10 feet below the water surface—(1) When the plate is vertical; (2) When the plate is inclined at 30° to the horizontal. Also the position of the centre of pressure when the plate is vertical.
15,310 and 13,790 lbs.; 12·625 feet from surface.
4. A dock entrance is closed by a caisson 50 feet wide at bottom and 60 feet wide at the water surface, 24 feet above the bottom. Find the total pressure on the caisson when the dock is empty.
958,280 lbs.
5. Two lock-gates are each 10 feet wide, and support water 10 feet deep in the head bay, the lock being empty. The gates meet at an angle of 120°. Find the total pressure on each gate, and the thrust at the hollow quoins. 31,200 lbs.; 31,200 lbs.
6. A ship weighs 1000 tons. Find its displacement in sea water.
350,000 cubic feet.
7. If the ship in the last question is vertical-sided near the water-line, and has a section of 1500 square feet at the water-line, by how much would the draught change in passing from sea to fresh water? 6 feet.
8. A homogeneous log is 3 feet wide, 2 feet deep, and 20 feet long. Its density is half that of water. It carries at its centre a load of 2000 lbs. Find its depth of immersion. 18·4 inches.
9. A dam supporting water pressure is vertical for 20 feet below the water surface, slopes at 1 in 5 from 20 feet to 30 feet, and at 1 in 3 from 30 feet to 40 feet. Find, graphically, the magnitude and position of the resultant water pressure.

CHAPTER III

PRINCIPLES OF HYDRAULICS

26. **Hydraulics** is the science of liquids or incompressible fluids in motion, and comprises—

(*a*) The laws of discharge from orifices, and sluices, and over weirs. The application of these is chiefly to the measurement of the flow of water.

(*b*) The laws of flow in pipes, canals, and rivers. The application of these is partly to water measurement, partly to the design of pipes and channels.

(*c*) The laws of impact of water streams on surfaces, the most important applications of which are to the design of some types of water motors.

(*d*) The laws of the resistance of water to the motion of bodies immersed or floating in it. The application of these is to ship design.

Pure theoretical hydrodynamics has proceeded but little beyond the consideration of the action of a perfect fluid without viscosity. The conclusions reached are in no case correct for actual fluids, and in some cases are in startling contradiction with the facts of experience. In practical hydraulics it is impossible to proceed on strictly theoretical lines. There are rational principles which serve for the solution of some elementary problems. In more complex cases dynamical reasoning serves as a basis or guide in generalising the results of experiment. But usually in hydraulics theoretical conclusions have to be checked and modified by the results of observation. In rigid dynamics rational solutions of problems are obtained based on the accurate determination of a few fundamental physical constants. In hydrodynamics the conditions are generally so complex that no such simple rational conclusions can be found. In the strict sense hydraulics is not a science. It is embarrassed by tangles of formulæ, which, initially based on imperfect reasoning, have been modified and adjusted to conform more or less accurately to the results of experiments, themselves affected to some extent by observational errors. On the other hand, it must be recognised that during more than two centuries a very large mass of experimental observation on the motion of water in different circumstances has been accumulated. For the practical purposes of the engineer, the empirical laws of hydraulics used with proper insight into their limitations are sufficient and trustworthy as solutions of practical problems.

27. **The two modes of motion of water.**—The first fundamental difficulty in hydraulics is that water moves in two different and characteristic ways. When water is accelerated or retarded the inertia forces acting on the mass are the same as for any other heavy body. But from the extreme mobility of the parts they readily take relative motions which absorb energy, which is rapidly destroyed by internal retarding forces commonly termed frictional resistances, though they are essentially different from the friction of solids. In certain cases these frictional resistances vary directly as the translational velocity of flow, in others they vary nearly as the square of that velocity. It is clear that in the two cases there must be an essential difference in the character of the motion. Using floating threads, or Professor Osborne Reynolds' method of coloured fluid streams, it is found that in one class of cases the particles follow very direct and constant paths or **stream lines**; in the other the particles eddy about in constantly changing paths of great sinuosity. Professor Reynolds has pointed out that the surface of a slow current of clear water sometimes presents a plate-glass appearance, reflections of objects on the surface being undistorted. That appearance corresponds to non-sinuous or stream-line motion. At other times the surface presents a sheet-glass appearance, reflections being blurred or distorted. That is due to eddy motions slightly disturbing the water surface. In a river in flood the continual breaking up of the surface by eddies is obvious enough.

Now in stream-line motion of the water (Fig. 24, *a*) the resistance is due to the laminæ sliding on each other with very small differences of relative velocity. The relative motion is opposed by the viscosity of the liquid; the resistance is of the nature of a shearing resistance, and is proportional to the velocity of sliding. On the other hand, in eddying or turbulent motion (Fig. 24, *b*) the relative velocities are very much greater, energy is expended in giving motion to the eddies, and this energy is gradually dissipated as the eddies die out in consequence of their mutual friction. The kinetic energy of an eddy is proportional to the square of its velocity, and as this must have a definite relation to the general velocity of translation of the stream it is intelligible that the resistance varies nearly as the square of the velocity. In a stream in turbulent motion there is a continual generation of eddies and stilling of them again by fluid friction, and consequently a continual degradation of mechanical energy into heat throughout the fluid mass. The theory of stream-line motion is much more perfect than the theory of turbulent motion; indeed, in the strict sense there is no rational theory of turbulent motion but only empirical laws deduced from experiment. Unfortunately, almost all cases of practical importance to the engineer are cases of turbulent motion.

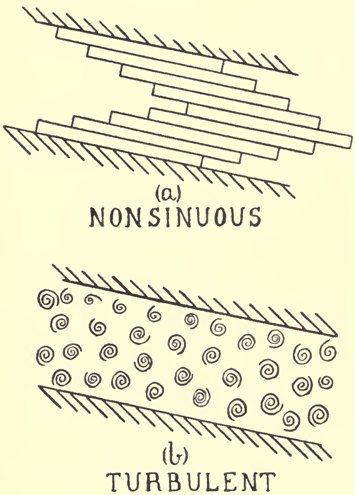

Fig. 24.

In cases of eddying motion, such as that shown in Fig. 24, *b*, the motion may be analysed into two parts: (*a*) a general average motion of translation, and (*b*) an eddying motion superposed which has no resultant motion. It is the former only with which the engineer is in general concerned, and to which the empirical laws of flow apply.

As an instance of how eddying may come in to modify the action of water, an interesting experiment by Mr. Church, at the Cornell University, may be taken. He tried the discharge through two orifices, A and B (Fig. 25). These were exactly of the same size, except that B had a smoothly formed contraction at the inlet; but it was found that B discharged about 10 per cent more than A. Now, why should contracting the section increase the discharge? The reason is simple, viz., that in B the change of section of the water stream is fairly gradual, and there is not much tendency to disturb the stream-line motion and generate eddies. But in A the abrupt inlet angle generates eddies, and so destroys part of the head available for producing the velocity of flow. But if the velocity of discharge is reduced 10 per cent the kinetic energy of the jet is reduced about 20 per cent, or nearly one-fifth of the energy is absorbed by the eddies due to the sharp corner. That is a case where the influence of eddies is comparatively small. In flow through a long pipe it is much greater. Take a pipe of 12 inches diameter with a virtual slope of 1 in 1000. If in such a pipe non-sinuous motion were possible the velocity would be 72 feet per second. But the actual velocity, the motion being turbulent, is only $1\frac{3}{4}$ foot per second. The difference shows the enormous amount of mechanical energy expended in eddy-making.

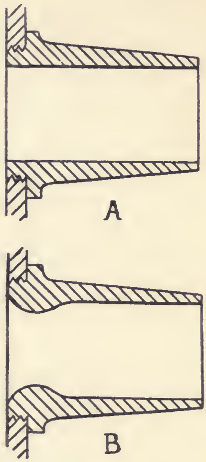

Fig. 25.

28. **Uniform and varying motion.** — Let *ab* (Fig. 26) represent a path along which fluid particles are moving. If the velocity of a particle *a* is constant along the path the motion is *uniform*, if not it is *varying*. In the ordinary cases of turbulent motion it is said to be uniform if the general velocity of translation is constant, and varying if it is not constant. In a canal of constant section the motion along the canal is usually uniform. In a river the section of which varies the motion is varying, that is, it is faster where the section is smaller, and slower where it is greater.

Fig. 26.

Steady and unsteady motion.—This introduces an idea special to hydraulics and of great importance. Consider a definite bounded space (Fig. 27) through which water is flowing along definite stream lines. If in that space the velocity is constant from minute to minute, or from hour to hour, the motion is *steady*. If it changes the motion is said to be unsteady. In turbulent motion, if at a given point the general motion of translation is constant in velocity and direction, the eddies being disregarded, it is said to be steady. If not it is unsteady. At a given point on a river bank in normal conditions the velocity and direction of motion are the same from day to day. But when rising in flood or subsiding afterwards, the velocity varies from minute to minute by some small amount, and the motion is unsteady.

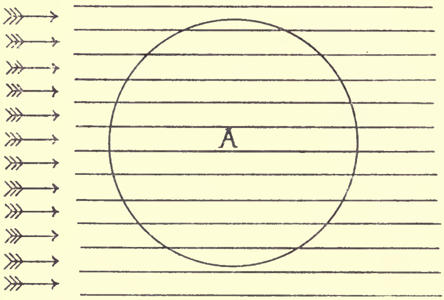

Fig. 27.

In ordinary streams and rivers in which the motion is turbulent, the velocity and direction of motion at any point varies from moment to moment. But if the eddies are disregarded the average velocity over short periods varies very little either in direction or velocity. If the variations are regarded as periodic, then the general motion, apart from the temporary fluctuations, is treated as steady motion. The motion is regarded as equivalent to simple stream-line motion, except that the energy absorbed and dissipated in eddies has to be allowed for by experimental corrections.

29. **Volume of flow.**—Let A (Fig. 28) be any ideal plane surface, of area ω, in a stream, normal to the direction of motion, and let V be the velocity of the fluid. Then the volume flowing through the surface A in unit time is

$$Q = \omega V \quad . \quad . \quad . \quad . \quad (1).$$

Thus, if the motion is rectilinear, all the particles at any instant in the surface A will be found after one second in a similar surface A′, at a distance V, and as each particle is followed by a continuous thread of other particles, the volume of flow is the right prism AA′ having a base ω and length V.

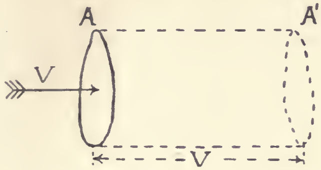

Fig. 28.

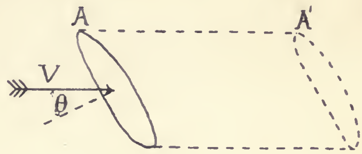

Fig. 29.

If the direction of motion makes an angle θ with the normal to the surface, the volume of flow is represented by an oblique prism AA′ (Fig. 29), and in that case

$$Q = \omega V \cos\theta \quad . \quad . \quad . \quad (2).$$

Mean velocity of flow.—In most practical cases the velocity V will be different at different parts of the cross section A of the stream. In a river, for instance, the velocity is greater towards the middle and top surface, and less towards the bottom and sides. If v is the velocity at some small element $d\omega$ of the section, the volume of flow is

$$Q = \int v d\omega \quad . \quad . \quad . \quad . \quad (3),$$

where the integration extends to the whole surface ω of the cross section. The mean velocity over the section is

$$V_m = \frac{\int v d\omega}{\omega} \quad . \quad . \quad . \quad (4),$$

and in a large number of practical problems it is this mean velocity which is required. Obviously the volume of flow is

$$Q = V_m \omega \quad . \quad . \quad . \quad . \quad (5).$$

If V_m is inclined at θ to the surface,

$$Q = V_m \omega \cos\theta.$$

Principle of continuity.—Consider a fixed bounded space through which liquid is flowing. If for any given time the space is continuously filled with fluid the inflow and outflow in that time must be equal, for the volume in the space is constant. If inflow is reckoned + and outflow −, the volume of flow for all the boundaries is

$$\Sigma Q = 0 \quad . \quad . \quad . \quad . \quad (6).$$

In general the condition that the space should be continuously filled is that the pressure must be a thrust everywhere throughout the space. If water contains air in solution as is ordinarily the case, the air is disengaged, and there is a break in continuity if the thrust falls below a certain value, depending on the amount of air in solution.

Let A_1, A_2 be two cross sections of a stream flowing in rigid boundaries, and V_1, V_2 the normal velocities at those sections. Then from the principle of continuity

$$V_1 A_1 = V_2 A_2$$

$$\frac{V_1}{V_2} = \frac{A_2}{A_1} \quad . \quad . \quad . \quad . \quad (6a);$$

that is, the normal velocities are inversely as the areas of the cross sections. This is true of the mean velocities if at each section the velocity of the stream varies. In a river of varying slope the velocity varies with the slope. It is easy, therefore, to see that in parts of large cross section the slope is smaller than in parts of small cross section.

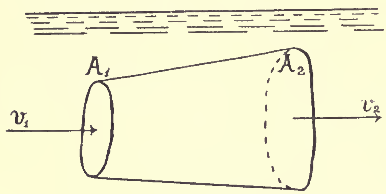

Fig. 30.

If we conceive a space in a liquid bounded by normal sections at A_1, A_2, and between A_1, A_2 by stream lines (Fig. 30), then, as there is no flow across the stream lines,

$$\frac{V_1}{V_2} = \frac{A_2}{A_1} \quad . \quad . \quad . \quad . \quad (7),$$

as in a stream with rigid boundaries.

30. **Application of the principle of the conservation of energy to stream-line motion. Bernoulli's theorem.**

Let AB (Fig. 31) be any one elementary stream in a steadily moving fluid mass. Then from the steadiness of the motion AB is a fixed path in space, and the fluid in it may be regarded as flowing in a tube. Let OO be the free surface level, and XX any horizontal datum plane. Let ω be the area of a normal cross section, v the velocity, p the pressure, and z the elevation above the datum plane at A, and ω_1, v_1, p_1, z_1, the corresponding quantities at B, and let Q be the flow in unit time. Suppose that in a short time t, AB comes to A′B′. Then $AA' = vt$ and $BB' = v_1 t$, and the volumes of fluid AA′, BB′, the equal inflow and outflow $= Qt = \omega v t = \omega_1 v_1 t$. If all frictional or viscous resistances are absent the work of the external forces will be equal to the change of kinetic energy.

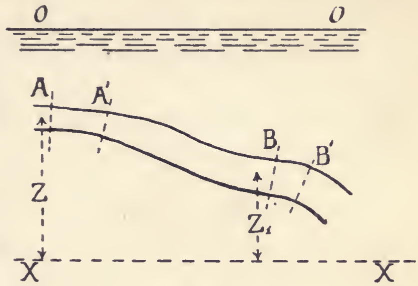

Fig. 31.

The normal pressures on the surface of AB, except at the ends, are everywhere perpendicular to the direction of motion and do no work. Hence the external forces to be reckoned are the pressures on the ends and gravity. The work of gravity when AB falls to A′B′ is the same as if AA′ were transferred to BB′. That is

$$\text{Work of gravity} = GQt(z - z_1) \text{ foot-pounds.}$$

The work of the pressures on the ends, reckoning that at B negative because it opposes motion, is (pressure × volume described)

$$p\omega v t - p_1\omega_1 v_1 t = Qt(p - p_1).$$

The change of kinetic energy in the time t is the difference of the kinetic energy of AA′ and BB′, for in the space A′B the energy is unchanged when the motion is steady.

The mass of AA′ or BB′ is $\frac{G}{g}Qt$, and the change of kinetic energy in t seconds is

$$\frac{G}{g}Qt\left(\frac{v_1^2}{2} - \frac{v^2}{2}\right).$$

Equating work expended and change of kinetic energy,

$$GQt(z - z_1) + Qt(p - p_1) = \frac{G}{g}Qt\left(\frac{v_1^2}{2} - \frac{v^2}{2}\right).$$

Dividing by GQt the weight of fluid and rearranging,

$$\frac{v^2}{2g} + \frac{p}{G} + z = \frac{v_1^2}{2g} + \frac{p_1}{G} + z_1 \qquad . \qquad . \qquad . \qquad (8),$$

or as A and B are any two points,

$$\frac{v^2}{2g} + \frac{p}{G} + z = \text{constant} = \text{H foot-pounds} \qquad . \qquad . \qquad (9),$$

where the quantities are reckoned per pound of fluid. The three terms on the left are quantities of energy, and correspond to the three forms in which energy may exist in a fluid in motion, due to elevation, pressure, and velocity. They are commonly called the heads due to elevation, pressure, and velocity respectively, head being defined as energy per pound of fluid. H is the total energy per pound. If h is the height from the point considered measured up to the free surface,

$$\frac{v^2}{2g} + \frac{p}{G} - h = \text{H foot-pounds} \qquad . \qquad . \qquad (10).$$

The theorem may be expressed thus:—The total head or total energy per pound of fluid, relatively to a given horizontal datum plane, is uniformly distributed along a stream line.

The term head in hydraulics.—The term head is an old millwright's word. A mill was said to have a good head of water if it possessed a waterfall which, from its volume of flow and height, was capable of developing a good amount of power when used on a water-wheel. The term is now scientifically understood as just defined. Since a pound of water falling through a height h acquires h foot-pounds of energy, height and head of elevation are numerically equal. Hence the term head is often used loosely as equivalent to height, but this is misleading. The term head should be restricted to cases in which energy is considered.

Consider water flowing through a frictionless pipe AB (Fig. 32), and that for the present viscosity effects such as the production of eddies are negligible. Let pressure columns be introduced at A and B. Let z, p, v, be the elevation, pressure, and velocity at A, and z_1, p_1, v_1, the same quantities at B. The water will rise in the pressure columns to heights p/G and p_1/G, so that the heights of A′ and B′ above the datum XX are

$z + p/G$ and $z_1 + p_1/G$. Join A′B′ and draw A′D horizontal. A′B′ is called the line of hydraulic gradient, or slope of the pressure-column tops when the liquid is flowing. The fall of the free surface level $DB' = p/G + z - (p_1/G + z_1)$, and this by the theorem above is equal to $(v_1^2 - v^2)/2g$. Consequently if distances $A'A'' = v^2/2g$ and $B'B'' = v_1^2/2g$ are set up, A″B″ is a horizontal line at a height H above the datum XX. The atmospheric pressure is assumed to be the same at both pressure columns; if it is not, the heads due to atmospheric pressure at A and B must be reckoned as part of the pressure

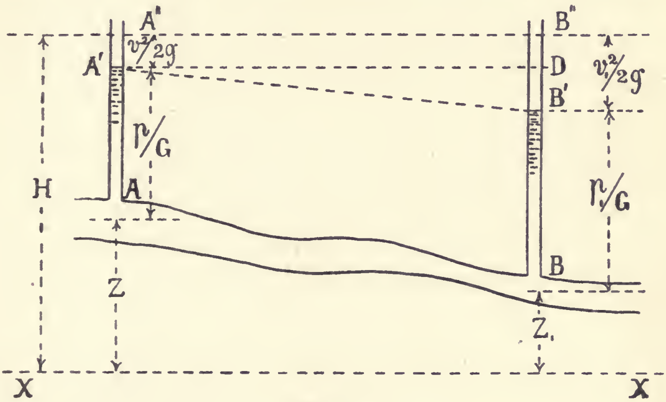

Fig. 32.

heads. The modification of this when friction has to be considered will be given later.

It will be seen from Bernoulli's equation that the three forms of head which make up the total head are convertible. Thus for points on the same level, if the velocity increases the pressure must diminish, and *vice versa.* If the pipe is of uniform section so that the velocity is uniform, then if the elevation increases the pressure diminishes, and *vice versa.*

31. **Illustrations of the theorem of Bernoulli.**—In a lecture to the mechanical section of the British Association in 1875, the late Mr. W. Froude gave some experimental illustrations of the principle of Bernoulli. Mr. Froude remarked that it was a common but erroneous impression that a fluid exercises in a contracting pipe A (Fig. 33) an excess of pressure against the entire converging surface which it meets, and that, con-

versely, as it enters an enlargement B, a relief of pressure is experienced by the entire diverging surface of the pipe. Further, it is commonly assumed that when passing through a

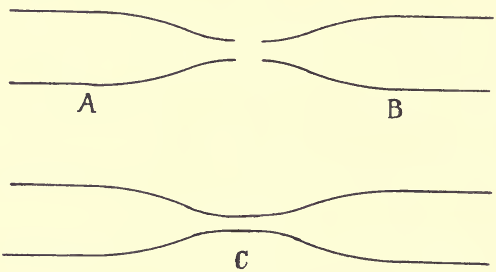

Fig. 33.

contraction C, there is in the narrow neck an excess of pressure due to the squeezing together of the liquid at that point. These impressions are in no respect correct; the pressure is smaller as the section of the pipe is smaller, and conversely.

Fig. 34 shows a pipe so formed that a contraction is

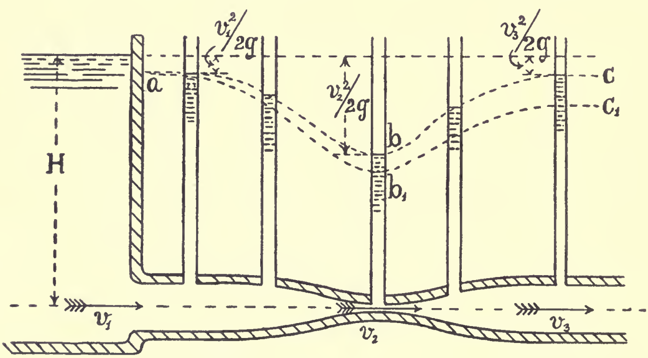

Fig. 34.

followed by an enlargement, and Fig. 35 one in which an enlargement is followed by a contraction. The vertical pressure columns show the decrease of pressure at the contraction, and increase of pressure at the enlargement. The line *abc* in both figures shows the variation of free surface level, supposing the pipe frictionless. In actual pipes, however, work is expended in frictional eddies; the total head diminishes in proceeding along the pipe, and the free surface level is a line such as ab_1c_1, falling below *abc*.

Mr. Froude further points out that, if a pipe contracts

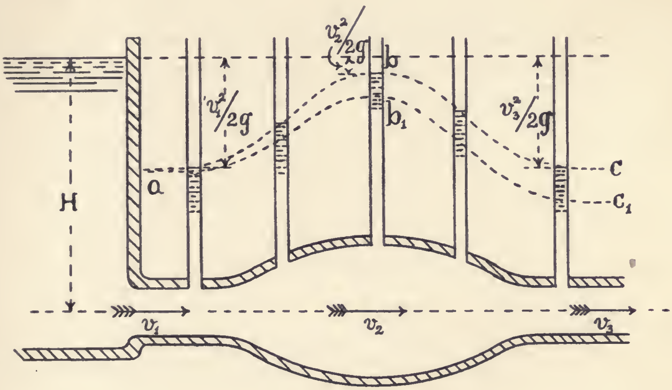

Fig. 35.

and enlarges again to the same size, the resultant pressure on the converging part exactly balances the resultant pressure on the diverging part, so that there is no tendency to move the pipe bodily when water flows through it. Thus the conical part AB (Fig. 36) presents the same projected surface as HI,

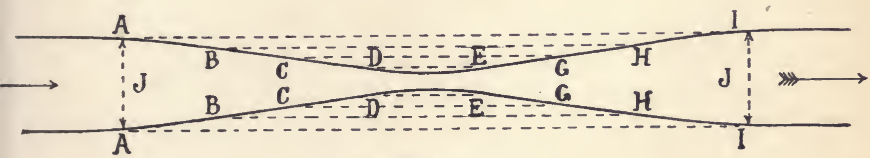

Fig. 36.

and the pressures parallel to the axis of the pipe, normal to these projected surfaces, balance each other. Similarly the pressures on BC, CD, balance those on GH, EG. In the same way, in any combination of enlargements and contractions there is a balance of the pressures parallel to the axis of the pipe, provided the area and direction of the ends are the same. If, however, the eddy loss is taken into account the balance is imperfect, and there is a drag in the direction of the motion of the water.

Let Fig. 37 represent two cisterns A and E provided with a converging pipe B and a diverging pipe D. The water will flow from A, cross the gap C, and fill E, till the level in it is nearly the same as in A. The pressure head h at the datum line XX in A becomes a velocity head $v^2/2g$ at the gap, and is reconverted into a pressure head nearly equal to h in E. There is a small loss due to inexact correspondence of the orifices and to eddy loss. In the jet crossing the gap there is

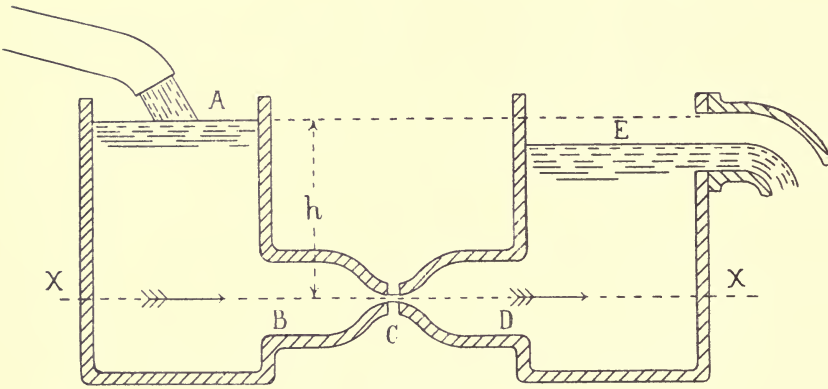

Fig. 37.

no pressure except the atmospheric pressure acting uniformly throughout the system.

31A. **Variation of pressure across the stream lines in two-dimensional motions.**[1]—Let AB, CD be two stream lines in the plane of the figure (Fig. 37a). Along the stream lines the variation of pressure and velocity is determined by Bernoulli's theorem. Normal to the plane of the figure, since the stream lines are parallel, the distribution of pressure is hydrostatic. There remains the direction in the plane of the figure and along the radius of curvature, that is the direction PQ. Let PQ be particles moving along the stream lines at a distance PQ $= ds$, and let z be the elevation above a datum

[1] See Cotterill, "On the Distribution of Energy in a Mass of Fluid in Steady Motion," *Phil. Mag.*, February 1876.

plane, p the pressure, and v the velocity at Q. At Q the total head or energy per pound of fluid is

$$H = z + \frac{p}{G} + \frac{v^2}{2g}.$$

Differentiating, the increment of head between Q and P is

$$dH = dz + \frac{dp}{G} + \frac{vdv}{g}.$$

But $dz = ds \cos \phi$,

$$dH = \frac{dp}{G} + \frac{vdv}{g} + ds \cos \phi \qquad . \qquad . \quad (11),$$

where the last term disappears when the motion is in a horizontal plane.

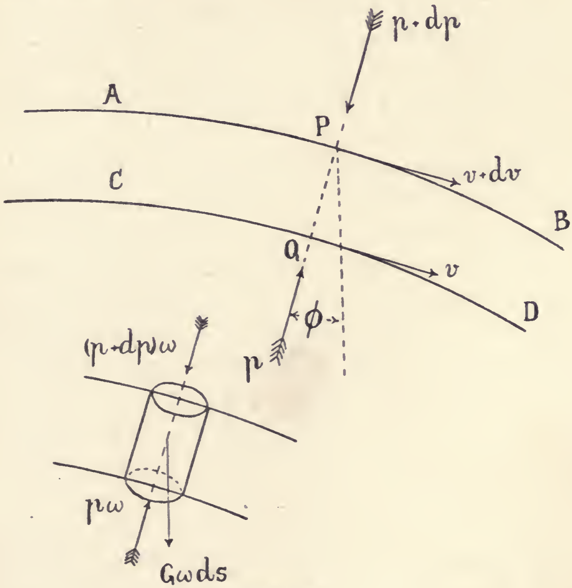

Fig. 37*a*.

Imagine a small cylinder of section ω described round PQ as an axis. This will be in equilibrium under the action of its weight $G\omega ds$; the pressures on its ends $p\omega$ and $(p + dp)\omega$; and its centrifugal force acting along the radius of curvature and equal to $\frac{G\omega ds}{g} \cdot \frac{v^2}{r}$, where r is the radius of curvature at Q. Taking components parallel to PQ,

$$\omega dp = \frac{G}{g} \cdot \frac{v^2}{r} \omega ds - G\omega \cos \phi ds$$

$$\frac{dp}{G} = \left(\frac{v^2}{gr} - \cos \phi\right) ds \quad . \quad . \quad . \quad (12).$$

Introducing this in (11), the increment of head between Q and P is

$$dH = \frac{v^2}{gr} ds + \frac{vdv}{g} \quad . \quad . \quad . \quad (13).$$

Corollary.—If the stream lines are straight and parallel in a horizontal plane, r is infinite and the increment of head across the stream lines is vdv/g. Comparing this with (11), $dp/G = 0$, or the pressure is uniform in a direction normal to the stream lines. If the stream lines are straight and parallel in a vertical plane $dH = vdv/g$, and comparing this with (11), $dp/G = ds \cos \phi = dz$, or $p/G + z =$ constant, that is, the pressure along a vertical varies hydrostatically, or in the same way as in a fluid at rest.

32. **Radiating current.**—Suppose water supplied steadily at the centre and flowing outwards between two parallel plates at a distance d apart (Fig. 38). From the uniformity of conditions the stream lines will be straight and radial. Conceive two cylindric sections of the current at radii r_1 and r_2, where the velocities are v_1 and v_2, and the pressures p_1 and p_2. Since the flow across each section must be the same,

$$Q = 2\pi r_1 d v_1 = 2\pi r_2 d v_2,$$

$$r_1 v_1 = r_2 v_2,$$

$$\frac{r_1}{r_2} = \frac{v_2}{v_1}.$$

The velocity varies inversely as the radius, and would be infinite at the centre if the radial flow could extend so far. The motion being steady,

$$H = \frac{p_1}{G} + \frac{v_1^2}{2g} = \frac{p_2}{G} + \frac{v_2^2}{2g}$$

$$= \frac{p_2}{G} + \frac{r_1^2}{r_2^2} \frac{v_1^2}{2g},$$

$$\frac{p_2 - p_1}{G} = \frac{v_1^2}{2g}\left(1 - \frac{r_1^2}{r_2^2}\right) \quad . \quad . \quad . \quad (14),$$

or in another form

$$\frac{p_2}{G} = H - \frac{r_1^2 v_1^2}{2 g r_2^2} \quad . \quad . \quad . \quad (14a).$$

Hence the pressure increases from the interior outwards in a way indicated by the pressure columns in Fig. 38. In the plane of the figure the curve through the pressure column tops, or curve of the free surface, is a quasi-hyperbola of the form $xy^2 = c^3$. This curve is asymptotic to the vertical axis of the current and to a horizontal line H feet above the plane from which the pressures are measured. It is worth noting that if the discharge is into the air the pressure p_2/G at the circumference is atmospheric pressure. All the pressures at less radii are smaller than atmospheric pressure. Hence the total pressure above the top plate is greater than that below it, and if the top plate is loose it would tend to approach the lower plate and not to recede from it.

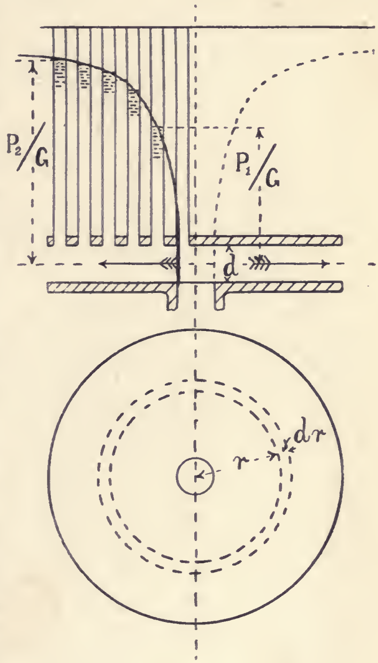

Fig. 38.

Free circular vortex.—A free circular vortex is a revolving mass of water, in which the stream lines are concentric circles, and in which the total head for each stream line is the same. Hence, if by any slow radial motion portions of the water strayed from one stream line to another, they would take freely the velocities proper to their new positions under the action of the existing fluid pressures only.

For such a current, the motion being horizontal, we have for all the circular elementary streams

$$H = \frac{p}{G} + \frac{v^2}{2g} = \text{constant};$$

$$\therefore \ d\mathrm{H} = \frac{dp}{\mathrm{G}} + \frac{vdv}{g} = 0 \quad . \quad . \quad . \quad . \quad (15).$$

Consider two stream lines at radii r and $r + dr$ (Fig. 38). Then in eq. (13) $r = r$ and $ds = dr$,

$$\frac{v^2}{gr}dr + \frac{vdv}{g} = 0,$$

$$\frac{dv}{v} = -\frac{dr}{r},$$

$$v \propto \frac{1}{r} \quad . \quad . \quad . \quad . \quad (16),$$

precisely as in a radiating current; and hence the distribution of pressure is the same, and formulæ 14 and 14a are applicable to this case.

Free spiral vortex.—As in a radiating and circular current the equations of motion are the same, they will also apply to a vortex in which the motion is compounded of these motions in any proportions, provided the radial component of the motion varies inversely as the radius as in a radial current, and the tangential component varies inversely as the radius as in a free vortex. Then the whole velocity at any point will be inversely proportional to the radius of the point, and the fluid will describe stream lines having a constant inclination to the radius drawn to the axis of the current. That is, the stream lines will be logarithmic spirals. When water is delivered from the circumference of a centrifugal pump or turbine into a chamber, it forms a free vortex of this kind. The water flows spirally outwards, its velocity diminishing and its pressure increasing according to the law stated above, and the head along each spiral stream line is constant.

33. **Forced vortex.**—If the law of motion in a rotating current is different from that in a free vortex, some force must be applied to cause the variation of velocity. The simplest case is that of a rotating current in which all the particles have equal angular velocity, as for instance when they are driven round by radiating paddles revolving uniformly. Then in equation (13), considering two circular stream lines of radii r and $r + dr$ (Fig. 39), we have $r = r$, $ds = dr$. If the angular velocity is a, then $v = ar$ and $dv = adr$. Hence

$$dH = \frac{a^2r}{g}dr + \frac{a^2rdr}{g} = \frac{2a^2r}{g}dr.$$

Comparing this with eq. (11), and putting $dz = 0$, because the motion is horizontal,

$$\frac{dp}{G} + \frac{a^2rdr}{g} = \frac{2a^2r}{g}dr,$$

$$\frac{dp}{G} = \frac{a^2r}{g}dr,$$

$$\frac{p}{G} = \frac{a^2r^2}{2g} + \text{constant} \quad . \quad . \quad . \quad (17).$$

Let p_1, r_1, v_1 be the pressure, radius, and velocity at one cylindrical section, p_2, r_2, v_2 those at another; then

$$\frac{p_1}{G} - \frac{a^2{r_1}^2}{2g} = \frac{p_2}{G} - \frac{a^2{r_2}^2}{2g};$$

$$\frac{p_2 - p_1}{G} = \frac{a^2}{2g}({r_2}^2 - {r_1}^2) = \frac{{v_2}^2 - {v_1}^2}{2g} \quad (18).$$

That is, the pressure increases from within outwards in a curve which in radial sections is a parabola, and surfaces of equal pressure are paraboloids of revolution (Fig. 39). This case corresponds to a crude form of centrifugal pump. Apart from a small head producing the radial flow, the lift of the pump is $\frac{(p_2 - p_1)}{G}$ feet, where p_2 and p_1 are the pressures at the outlet and inlet of the pump disc.

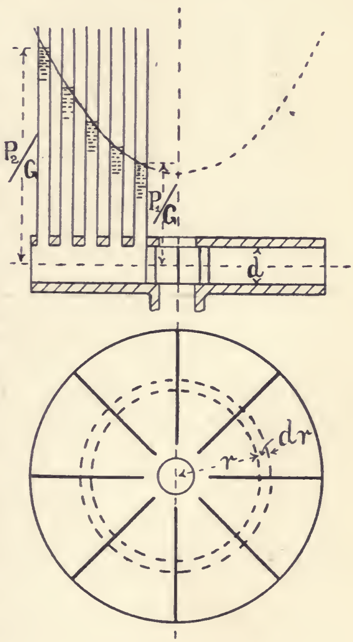

Fig. 39.

34. **Venturi meter.**—An extremely beautiful application of this principle has been made by Mr. Clemens Herschel, in the construction of what he has termed the Venturi meter for measuring water flowing in pipes. Suppose in any water-

main a contraction is made (Fig. 40), the change of section being very gradual to avoid the production of eddies. The ratio ρ of the sections at inlet and throat is in actual meters between 5 to 1 and 20 to 1, and is very carefully determined by the maker of the meter. Then the ratio of the velocity v in the main and the velocity u at the throat is definitely known. Now suppose glass tubes, "piezometer tubes" they are sometimes called, are inserted, in which the water ascends to a height which measures the pressure. Since the velocity is greater at the throat than in the main, the pressure will

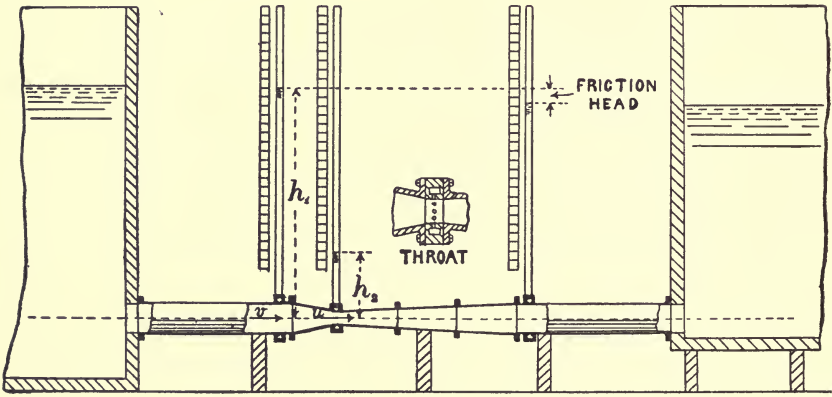

Fig. 40.

be less and the pressure head h_2 will be less than h_1, and this is a quantity easily observed. Using Bernoulli's equation,

$$h_1 + \frac{v^2}{2g} = h_2 + \frac{u^2}{2g},$$

or putting $u = \rho v$, where ρ is the ratio of the cross sections,

$$h_1 + \frac{u^2}{2g\rho^2} = h_2 + \frac{u^2}{2g},$$

$$\frac{u^2}{2g} = (h_1 - h_2)\frac{\rho^2}{\rho^2 - 1} \quad . \quad . \quad . \quad (19),$$

from which the velocity at the throat can be determined if the Venturi head $h_1 - h_2$ is observed, and the ratio ρ of the sections is known. But if u and the area at the throat are known, the discharge of the meter is known. Let Ω be the section of the pipe, then Ω/ρ is the section at the throat. For simplicity let $h_1 - h_2 = h$. Then the discharge is

$$Q = \Omega\sqrt{\left\{\frac{2gh}{\rho^2 - 1}\right\}} \quad . \quad . \quad . \quad (20).$$

Hence, by a simple observation of the piezometric heights, the flow in the main at any moment can be determined. Notice that if a third piezometer is introduced where the water has regained its original section and velocity, the piezometric height will be the same as at first, except for a small loss due to the fact that the motion is not quite non-sinuous, and that some eddies are generated in the meter.

In order to get the pressure head at the throat very exactly, Mr. Herschel surrounds the throat with an annular passage communicating with the throat by small holes, sometimes formed in vulcanite plugs to prevent corrosion.

Although constructed to secure as far as possible non-sinuous motion, the eddy motion cannot be entirely prevented in the Venturi meter. The main effect of this is to cause a loss of head between the two ends of the meter, varying between 1 foot and 5 feet according to the velocity through the meter. But the eddying also affects the difference of head at inlet and throat, from which the discharge through the meter is calculated; consequently, even with this meter, an experimental coefficient must be introduced, determined by tank measurement. However, the range of this coefficient is surprisingly small. Mr. Herschel found coefficients ranging between 0·97 and 1·0 for throat velocities varying between 8 feet per second and 28 feet per second, or inlet velocities varying between 0·9 foot per second and 3·1 feet per second.

Putting eq. (20) in the form

$$Q = c\Omega\sqrt{\left(\frac{2gh}{\rho^2 - 1}\right)} \text{ c. ft. per sec.} \quad . \quad (20a),$$

where c is the coefficient of the meter, the mean value of c is 0·972, and it is rather smaller for small values and greater for large values of the Venturi head h. It is stated to be desirable that the throat velocity should be 15 to 40 feet per second. If the Venturi head is measured by a mercury siphon gauge, let

h_m be the difference of level in the gauge in inches, and let 13·59 be the density of mercury. Then the Venturi head in feet of water is

$$h = \frac{13{\cdot}59 - 1}{12} h_m = 1{\cdot}049\ h_m \qquad . \quad . \quad (21).$$

Mr. Kent of Holborn has constructed two meters for 94-inch mains at the reservoir works at Staines. The coned parts are of riveted steel plates, and have a total length of 84 feet. The throat ratio is 1 to 7, and they can register a flow varying from 400,000 to 6,000,000 gallons per hour. Two still larger meters are being constructed for a pumping station at Divi in the Madras Presidency. The main pipes are 120 inches in diameter. The upstream cones are of steel plate bedded in concrete, and the downstream cones of concrete only. Each meter can register from 1 to 11 million gallons per hour. Various forms of recording apparatus have been used with the meter. In one, a line proportional in length to the discharge is drawn on the recording drum at every quarter hour or other predetermined interval. In another, a line is drawn showing the Venturi head at each instant. An integrating arrangement is also used, the total flow for any given time being shown by a counter.

35. **Principle of the conservation of momentum.**—If a force P acts on a body of weight W, or mass $m = W/g$, moving in the direction of P, the change of velocity from v_1 to v_2 in time t is given by the relation

$$Pt = m(v_2 - v_1) = \frac{W}{g}(v_2 - v_1) \qquad . \quad . \quad (22),$$

where Pt in second-pounds is termed the impulse of the force, and $m(v_2 - v_1)$ the change of momentum. Thus the *impulse of a force is equal to the change of momentum in the direction of the force.* Conversely, if the body suffers a decrease of momentum due to a change of velocity from v_2 to v_1, it must exert an impulse of Pt second-pounds in the direction of the change of momentum. The principle of momentum is of special use in hydraulics, because it can be applied irrespectively of the mutual action of the particles and of their actual motions, only their velocity components in the direction considered being required.

36. **Relation of pressure and velocity in a stream in steady motion when the changes of section of the stream are abrupt.**—When a stream changes section abruptly, rotating eddies are formed which dissipate energy. The energy absorbed in producing rotation is at once abstracted from that effective in causing the flow, and sooner or later it is wasted by frictional resistances due to the rapid relative motion of the eddying parts of the fluid. The energy thus lost is commonly termed energy lost in shock. Suppose Fig. 41 to represent a stream having such an abrupt change of section. Let AB, CD be normal sections at points where ordinary stream-line motion has not been disturbed and where it has been re-established. Let ω, p, v be the area of section, pressure, and velocity at AB, and ω_1, p_1, v_1 corresponding quantities at CD. Then if no work were expended internally, and assuming the stream horizontal,

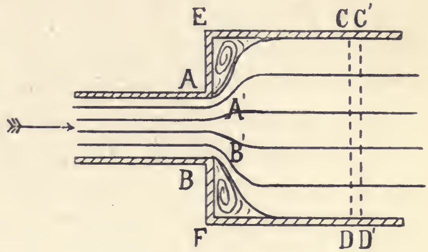

Fig. 41.

$$\frac{p}{G} + \frac{v^2}{2g} = \frac{p_1}{G} + \frac{v_1^2}{2g} \quad . \quad . \quad . \quad (23).$$

But if work is expended in producing irregular eddying motion, the head at the section CD will be diminished.

Suppose the mass ABCD comes in a short time t to A′B′C′D′. The resultant force parallel to the axis of the stream is

$$p\omega + p_0(\omega_1 - \omega) - p_1\omega_1,$$

where p_0 is put for the unknown pressure on the annular space between AB and EF. The impulse of that force is

$$\{p\omega + p_0(\omega_1 - \omega) - p_1\omega_1\}t.$$

The horizontal change of momentum in the same time is the difference of the momenta of CDC′D′ and ABA′B′, because the amount of momentum between A′B′ and CD remains unchanged if the motion is steady. The volume of ABA′B′ or CDC′D′, being the inflow and outflow in the time t, is $Qt = \omega vt = \omega_1 v_1 t$, and the momentum of these masses is $\frac{G}{g}Qvt$ and $\frac{G}{g}Qv_1t$. The change of momentum is therefore $\frac{G}{g}Qt(v_1 - v)$. Equating this to the impulse,

$$\{p\omega + p_0(\omega_1 - \omega) - p_1\omega_1\}t = \frac{G}{g}Qt(v_1 - v).$$

Assume that $p_0 = p$, the pressure at AB extending unchanged through the portions of fluid in contact with AE, BF which lie out of the path of the stream. Then (since $Q = \omega_1 v_1$)

$$(p - p_1) = \frac{G}{g}v_1(v_1 - v);$$

$$\frac{p - p_1}{G} = \frac{v_1(v_1 - v)}{g} \quad . \quad . \quad . \quad (24);$$

$$\frac{p}{G} + \frac{v^2}{2g} = \frac{p_1}{G} + \frac{v_1^2}{2g} + \frac{(v - v_1)^2}{2g} \quad . \quad . \quad (24a).$$

This differs from the expression obtained for cases where no sensible internal work is done, by the last term on the right. That is, $\frac{(v - v_1)^2}{2g}$ has to be added to the total head at CD, which is $\frac{p_1}{G} + \frac{v_1^2}{2g}$, to make it equal to the total head at AB, or $\frac{(v - v_1)^2}{2g}$ is the head lost in shock at the abrupt change of section. But $v - v_1$ is the relative velocity of the two parts of the stream. Hence, when an abrupt change of section occurs, the head due to the relative velocity is lost in shock, or $\frac{(v - v_1)^2}{2g}$ foot-pounds of energy is wasted for each pound of fluid. Experiment verifies this result, so that the assumption that $p_0 = p$ appears to be admissible.

If there is no shock,

$$\frac{p_1}{G} = \frac{p}{G} + \frac{v^2 - v_1^2}{2g}.$$

If there is shock,

$$\frac{p_1}{G} = \frac{p}{G} - \frac{v_1(v_1 - v)}{g}.$$

Hence the pressure head at CD in the second case is less than in the former by the quantity

$$\frac{(v-v_1)^2}{2g},$$

or, putting $\omega_1 v_1 = \omega v$, by the quantity

$$\frac{v^2}{2g}\left(1-\frac{\omega}{\omega_1}\right)^2. \quad . \quad . \quad . \quad (25).$$

The labyrinth piston packing.—Pistons for pumps are sometimes made with a series of circumferential recesses without any other packing. The passage between the cylinder and piston then consists of n wide spaces of cross section A and $n+1$ spaces of smaller cross section a. Let Q be the amount of leakage per second. Then the velocity in the narrow passages is Q/a, and that in the wide passages is Q/A. At each change of velocity in passing from a narrow to a wide passage there will be a loss of head

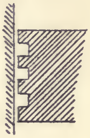

Fig. 42.

$$\frac{Q^2}{2g}\left(\frac{1}{a}-\frac{1}{A}\right)^2.$$

And as the energy in the last narrow passage is also wasted the whole loss of head is

$$\frac{Q^2}{2g}\left[n\left(\frac{1}{a}-\frac{1}{A}\right)^2+\frac{1}{a^2}\right],$$

which when A is large compared with a tends to the limit

$$\frac{Q^2}{2g}\cdot\frac{n+1}{a^2}.$$

As the total difference of head between the two sides of the piston which produces the leakage is a fixed quantity, the greater the head wasted the smaller the leakage. The larger n and the smaller a the less will be the leakage. There are in addition some resistances in the small passages which are not included in this reckoning.

Problems

1. A pipe AB, 100 feet long, has an inclination upwards of 1 in 4. The head due to the pressure at A is 50 feet, the velocity is 4 feet per second, and the section of the pipe is 3 square feet. Find the head due to the pressure at B, where the section is $1\frac{1}{2}$ square feet. 25 feet.

2. The injection orifice of a condenser is at 12 feet below the surface of supply tank. The condenser gauge shows a pressure of 5 inches of mercury. Neglecting frictional resistances, find the velocity at which water will enter the condenser.
50·9 ft. per sec.

3. A Venturi meter has a diameter of 4 feet in the large part and 1·25 feet in the throat. With water flowing through it, the pressure head is 100 feet in the large part and 85 feet at the throat. Find the velocity in the small part and the discharge through the meter. Coefficient of meter taken as unity.
38·3 c. ft. per sec.

4. Ten cubic feet of water are discharged by a pipe per second under a total head of 100 feet. Find h.p. of the stream. 113.

5. Water flows radially outwards between two parallel plates. At 2 feet radius the pressure head is 10 feet and the velocity is 10 feet per second. Find the pressure and velocity at 4 feet radius. 10 ft. per sec.; 14·7 ft.

6. Ten cubic feet of water per second flow through a pipe of 1 square foot area, which suddenly enlarges to 4 square feet area. Taking the pressure at 100 lbs. per square foot in the smaller part of the pipe, find (1) the head lost in shock; (2) the pressure in the larger part; (3) the work expended in forcing the water through the enlargement; (4) the rise of temperature of the water at the enlargement.
0·87 ft.; 136 lbs. per sq. in.; 545 ft.-lbs. per sec.; 0·07° F.

7. A centrifugal pump with radial vanes has diameters of 1 foot inside and 2 feet outside. It revolves 360 times per minute. Find the pressure height produced in the pump. 16·6 ft.

8. A Venturi meter is 3 feet in diameter at each end and 1 foot in diameter at the throat. Find the Venturi head when the inlet velocity is 3 feet per second. Coefficient 0·97. 10·53 ft.

9. Find the energy stored per cubic foot of water in an accumulator loaded to 700 lbs. per square inch. 100,800 ft.-lbs.

10. In a Venturi meter the diameters at inlet and throat are 12 inches and 5 inches. With water flowing through the meter, the Venturi head is observed to be 6 inches of mercury. Find the discharge. 2·9 c. ft. per sec.

CHAPTER IV

DISCHARGE FROM ORIFICES

37. **Experimental observations.**—Some simple laws governing the discharge from orifices are directly indicated by simple observations. Suppose a reservoir arranged as shown in Fig. 43, with a horizontal orifice h feet below the free surface and a vertical jet. That this condition may be permanent, and the flow steady, water must be supplied continuously at the free surface at the rate at which it is discharged by the jet. The jet rises very nearly to the free surface level in the reservoir, and the small difference h_r may reasonably be attributed to small resistances of the air or orifice. Neglecting this small quantity, particles which rise freely to a height h must have issued from the orifice with a velocity given by the relation

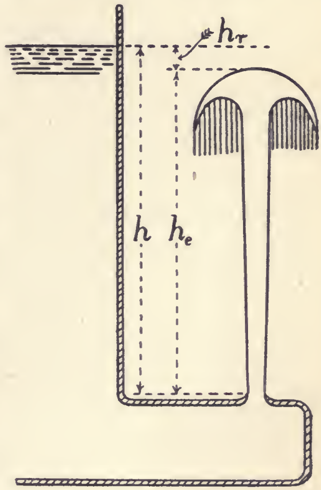

Fig. 43.

$$v = \sqrt{(2gh)} \text{ ft. per sec.} \quad . \quad . \quad . \quad (1).$$

This relation was first discovered by Torricelli and Bernoulli. If the orifice is of a proper conoidal form, the section of the jet at the orifice is equal to the area of the orifice, and the elementary streams forming the jet are normal to the orifice. Let ω be the area of the orifice. Then (§ 29) the discharge must be, neglecting the small resistances,

$$Q = \omega v = \omega \sqrt{(2gh)}$$
$$= 8{\cdot}023\omega \sqrt{h} \text{ c. ft. per sec.} \quad (1a).$$

The actual velocity and discharge will be slightly less than this if the resistances are considered.

In the case of a horizontal orifice the head is the same at all parts of the orifice. But equations (1) and (1a) are used also for the more ordinary case in which the orifice is vertical, and the head varies at different parts of the orifice, and it is necessary to inquire how far this is justifiable. In the case of vertical orifices the head h is taken to be the head measured to the centre of the orifice. Consider a conoidal rectangular orifice such that the section of the jet is identical with the area of the outlet of the orifice (Fig. 44). Let H_1 be the head at the top edge, and H_2 that at the bottom edge of the orifice, and B its breadth. The area is $B(H_2 - H_1)$ and the mean head is $h = \frac{1}{2}(H_2 + H_1)$. Putting these values in eq. (1a),

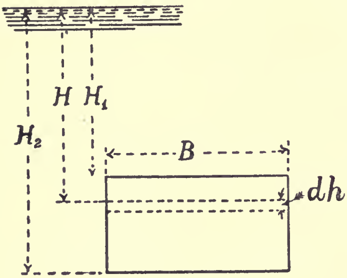

Fig. 44.

$$Q = B(H_2 - H_1) \sqrt{\{g(H_2 + H_1)\}},$$

and the velocity of discharge, the same at all parts of the orifice, on the assumption that the variation of head is negligible, is—

$$v_1 = \sqrt{\{g(H_2 + H_1)\}}.$$

Consider a horizontal lamina issuing between the levels H and H + dH. Its area is BdH, and the discharge is $Bd\text{H}\sqrt{(2gH)}$. The discharge of the whole orifice is

$$Q = B\sqrt{2g}\int_{H_1}^{H_2} H^{\frac{1}{2}}dH$$

$$= \frac{2}{3}B\sqrt{2g}\{H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}\} \quad . \quad . \quad . \quad (2).$$

Hence the mean velocity when the variation of head is taken into the reckoning is

$$v_2 = \frac{2}{3}\sqrt{2g}\,\frac{H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}}{H_2 - H_1} \quad . \quad . \quad . \quad (2a).$$

Comparing this with the velocity found if the variation of head is neglected,

$$\frac{v_2}{v_1} = \frac{2\sqrt{2}}{3}\,\frac{H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}}{(H_2 - H_1)\sqrt{(H_2 + H_1)}}.$$

Let $H_1 = \rho H_2$,

$$\frac{v_2}{v_1} = \frac{2\sqrt{2}}{3}\,\frac{1 - \rho^{\frac{3}{2}}}{(1 - \rho)\sqrt{(1 + \rho)}} \quad . \quad . \quad . \quad (3).$$

$\rho =$	$\frac{v_2}{v_1} =$
0·0	0·9428
0·2	0·9797
0·5	0·9952

It is clear that v_2 is always a little less than v_1, but the difference becomes less as ρ increases and is negligibly small if $\rho > 0{\cdot}5$. Hence, except when the head on the top of the orifice is less than half the head on the bottom, the approximate equation (1) or (1*a*) may be used without sensible error in place of the more complicated equations (2) and (2*a*). The practically important case when $H_1 = 0$ will be dealt with later.

38. **Coefficients of velocity and resistance.**—The approximate formula just given may be made exact for any given conditions by introducing an experimental coefficient. The actual velocity of discharge is

$$v_a = c_v\sqrt{2gh} \quad . \quad . \quad . \quad . \quad (4),$$

where c_v is a coefficient termed the coefficient of velocity, which experiment shows to vary little for a given type of orifice. For well-formed simple orifices c_v is 0·97 to 0·98, and rather greater for very great heads. The velocity of discharge can be expressed in another way. If h_e is the actual height to which the molecules rise, $v_a = \sqrt{2gh_e}$. If the loss of head $h_r = c_r h_e$, where c_r is a coefficient termed the coefficient of resistance,

$$v_a = \sqrt{2gh_e} = \sqrt{\left(2g\frac{h}{1 + c_r}\right)} \quad . \quad . \quad (5)$$

Equating the two expressions for v_a,

$$\left.\begin{aligned} c_v &= \sqrt{\left(\frac{1}{1+c_r}\right)} \\ c_r &= \frac{1}{c_v^2} - 1 \end{aligned}\right\} \quad . \quad . \quad . \quad (6).$$

Thus if $c_v = 0{\cdot}97$, $c_r = 0{\cdot}0628$; and if $c_v = 0{\cdot}98$, $c_r = 0{\cdot}0412$.

The work of gravity on each pound of water descending from the free surface level to the orifice is h ft.-lbs., and if unresisted the water would acquire $v^2/2g$ ft.-lbs. of kinetic energy. The actual energy of the jet is only $v_a^2/2g = h_e$ ft.-lbs. per pound. Hence $h_r = c_r v_a^2/2g$ ft.-lbs. per pound is the energy wasted in overcoming resistances. With the values of c_r given above, from $6\frac{1}{4}$ to $4\frac{1}{8}$ per cent of the head is wasted.

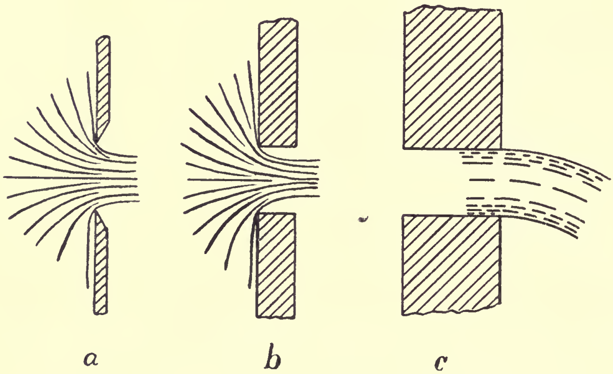

Fig. 45.

Coefficient of contraction.—When a jet issues from an orifice it may either spring clear from the inner edge of the orifice as at a or b (Fig. 45), or it may adhere to the sides of the orifice as at c. The former condition always obtains if the orifice is bevelled to a sharp edge as at a, and generally for cylindrical orifices such as b if the thickness of the plate is not more than the diameter of the orifice. If the plate thickness is $1\frac{1}{2}$ times the diameter of the orifice or more, the condition shown at c obtains, and it is convenient to distinguish orifices of that kind as mouthpieces. At c the jet issues "full bore," or of the same diameter as the orifice, but in the other cases the jet contracts to a diameter smaller than the orifice in consequence of the convergence of the streams which make up the jet.

Let ω be the area of the orifice and $c_c\omega$ the contracted area of the jet. Then c_c is a coefficient to be determined experimentally, called the coefficient of contraction, which is found to be nearly constant for certain types of orifice. For sharp-edged or virtually sharp-edged orifices, such as those shown in *a* and *b*, the average value of c_c is 0·64, but with different kinds of orifice its value may range from 0·5 to 1·0. With $c_c = 0·64$ the diameter of the contracted section of a circular jet is 0·8 of the diameter of the orifice.

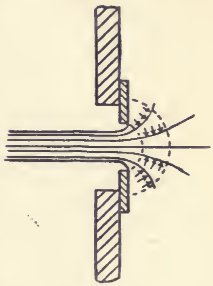

Fig. 46.

It may be noted that as the stream lines are curved when approaching the contracted section there is a centrifugal pressure across the stream lines (Fig. 46). Hence the pressure is greater and the velocity less towards the centre of the converging jet. At the contracted section the stream lines become parallel, the pressure is uniform, and probably the velocity nearly uniform.

Coefficient of discharge.—The discharge $Q = \omega v$ is diminished partly by reduction of velocity, partly by contraction of section. Hence the actual discharge is

$$Q_a = c_v v \times c_c\omega = c_c c_v \omega \sqrt{2gh},$$

or if $c_c c_v = c$, which is termed the coefficient of discharge,

$$Q_a = c\omega\sqrt{2gh} \quad . \quad (7).$$

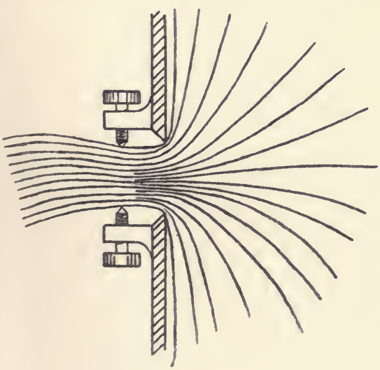

Fig. 47.

For sharp-edged plane orifices c averages about 0·975 × 0·64 = 0·62. But exact values for different cases will be given presently.

39. **Experimental determination of c_v, c_c, and c.**—To determine the coefficient of contraction, the section of the jet must be measured at a distance from the orifice equal to about half its diameter. Fig. 47 shows

an arrangement of set-screws which can be set to touch the jet, and the distance between them afterwards measured. When the orifice is not circular the measurement is difficult, because the section of the jet is not exactly similar to the orifice.

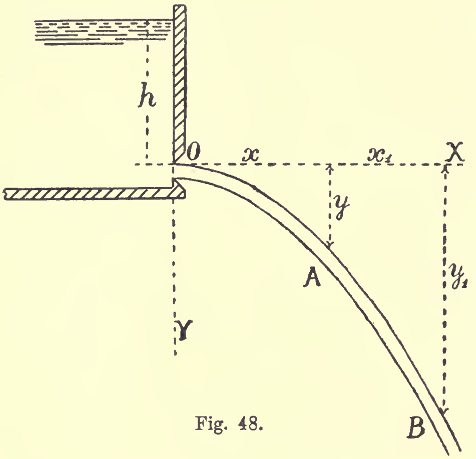

Fig. 48.

The coefficient of velocity is most easily found by measuring the parabolic path of a horizontal jet. Let OAB (Fig. 48) be the path of the jet. Take OX, OY as horizontal and vertical co-ordinate axes. Let h be the head over the centre of the orifice, and x, y the co-ordinates of any point A. If v_a is the horizontal velocity of the jet, and t the time in which a particle falls from O to A,

$$x = v_a t\,; \quad y = \frac{1}{2}gt^2\,; \quad v_a = \sqrt{\left(\frac{gx^2}{2y}\right)}\,;$$

consequently

$$c_v = \frac{v_a}{\sqrt{(2gh)}} = \sqrt{\left(\frac{x^2}{4yh}\right)}.$$

As a check, other co-ordinates, such as x_1, y_1, should be measured. In principle, the coefficient of velocity could be found by measuring the height h_e (Fig. 43) to which a vertical jet rises under a head h. Then

$$c_v = \frac{v_a}{\sqrt{(2gh)}} = \sqrt{\left(\frac{h_e}{h}\right)}\,;$$

but, except for moderately small heads, the measurement is difficult.

In practical hydraulics the coefficient of discharge is much more important than the others, and it can be determined with very great accuracy by tank measurement. In Fig. 49 is shown an arrangement of a measuring tank for gauging the flow from an orifice or notch. The orifice is placed at the end of the reservoir A, and discharges into the waste channel C, and the water flows to waste at F. A trough on rollers B can be slid under the jet, and then delivers the water into the measuring tank D. In the tank is a stilling screen S, and an outlet valve E. Means are provided for very accurately measuring the water-level at the beginning and end of a convenient interval of time, and the area of the tank must be carefully determined. Let the water be discharged into the tank for t seconds, during which the level in the reservoir of

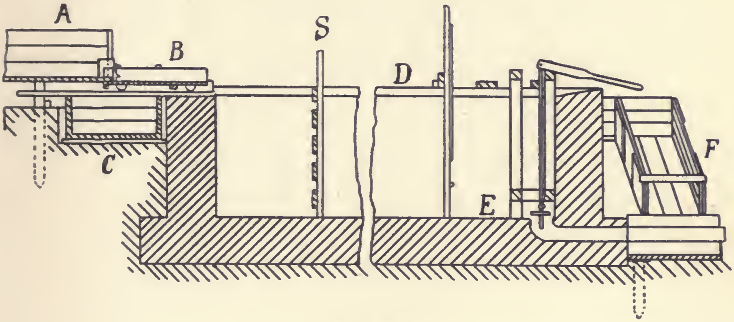

Fig. 49.

area A rises $H_2 - H_1$ feet, and let h be the head at the orifice, and ω its area.

$$Q = \frac{(H_2 - H_1)A}{t} = c\omega\sqrt{2gh} \text{ cubic feet per second,}$$

$$c = \frac{(H_2 - H_1)A}{\sqrt{(2gh)}\omega t} \quad . \quad . \quad . \quad (8).$$

All the required measurements can be made with great accuracy, especially if the tank is large enough to contain the flow during ten or fifteen minutes.

40. **Use of orifices in measuring water.**—The Romans used orifices of bronze to deliver regulated quantities of water from the aqueducts to consumers. The unit of discharge was that from an orifice 0·907 inches diameter, and was termed a quinaria. Fifteen sizes were used, the largest being 8·964 inches diameter, and delivering 97 quinariæ. The discharge was assumed to be proportional to the area of the orifice, and

although it was known that the discharge depended in some way on the head, the arrangements adopted to secure approximate uniformity of head in different cases are not known and appear to have been imperfect (Frontinus, *De Aquis,* translated by Herschel).

In the case of the irrigation works of Northern Italy the water was supplied to estates through orifices, termed modules, for which the height and head were legally fixed, and the width varied according to the amount of water required. This is an almost exact way of delivering a measured quantity of water. The Sardinian unit module was an orifice 0·656 feet square with a head of 0·656 above the top edge, delivering about 2 cubic feet per second.

An old measure of the discharge of the same kind was the so-called water inch, defined by some of the older French hydraulicians as the discharge of an orifice one inch in diameter, with a head of one line above the top edge. In the mining district of California a similar method was used in supplying water to different mines from a supply channel. The unit of discharge was termed the *miner's inch,* and was the discharge through one square inch of orifice with a head of $6\frac{1}{2}$ inches, or about 1·5 cubic feet per minute. But as the form of the orifice and the head were not defined as carefully as in the Italian regulations, the value of the miner's inch varied a good deal in different districts. Later legal definitions of the miner's inch were adopted, varying in different cases from 1·5 to 1·2 cubic feet per minute.

In delivering compensation water from reservoirs to streams in this country an orifice is used, the head on which is regulated so as to be constant. The arrangement is such that any riparian owner interested in the flow in the stream can at any time see whether the proper head, and therefore the proper discharge, is maintained.

41. **Measurement of the head over an orifice.**—The most convenient way of measuring the head over an orifice in a tank is by a gauge-glass, scale, and vernier (Fig. 50). A bar AA is rigidly attached to the tank, having a slot in which the scale BB slides. The scale has at the bottom an adjusting screw by which its zero can be set exactly to the level of the centre of the orifice. A slider C, with a finger

projecting across the gauge-glass, has also a vernier reading on the scale. The scale is most conveniently divided into feet, tenths, and hundredths of a foot. The vernier then reads to 0·001 foot. The zero of the scale can be properly

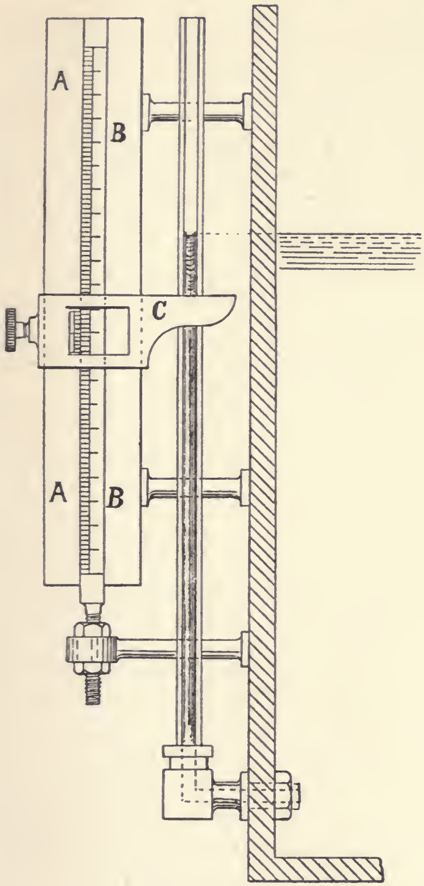

Fig. 50.

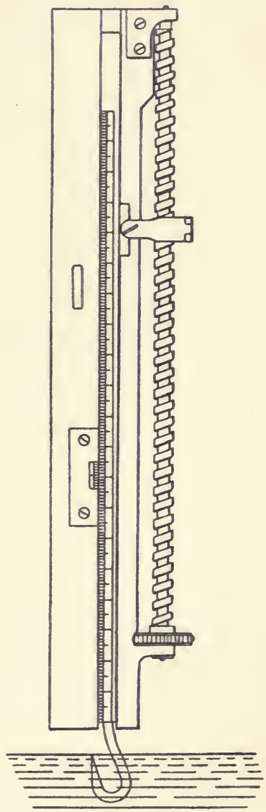

Fig. 51.

fixed by very carefully levelling a surface plate between the orifice and scale, and transferring the centre of the orifice to the scale by a scribing block.

Another method of measuring the head is by using a float. If the float has a cord passing over a pulley, a finger attached to the pulley will give a magnified motion which can be read on a dial. In this case the zero of the scale can be determined by bringing the water-level exactly to the lower edge of the orifice and noting the reading on the finger of the dial.

A more accurate method of determining the exact water-level is the use of the **hook gauge,** invented by Mr. V. Boyden in 1840. It consists of a fixed frame with sliding scale and vernier (Fig. 51). The vernier is fixed to the frame, and the scale slides vertically. The scale carries at its lower end a hook with a fine point, and the scale carrying the hook can be raised or lowered very slowly by a fine-pitched screw. If the hook is depressed below the water surface and then raised gradually by the screw, the moment of its reaching the water surface will be very clearly marked by a sudden reflection from a small capillary elevation of the water surface over the point of the hook. In good light differences of level of 0·0001 of a foot are easily detected by the hook gauge. The gauge is specially useful in measuring the head over weirs which requires to be determined very accurately. The point of the hook should be set by levelling very exactly at the level of the weir crest, and a reading taken. Then the difference of any reading of the water-level and this reading is the head on the weir. It is generally convenient to place the hook gauge in a small cistern, communicating with the stream passing over the weir by a pipe. The water-level in such a cistern fluctuates less than in the stream, and the gauge is more easily read.

42. **Coefficients for bellmouths or conoidal orifices.**—When a bellmouth is formed so as to contract gradually, and finally become cylindric, when in fact it has nearly the form of a contracting jet, the contraction occurs within the mouthpiece and there is no further contraction beyond it. The section of the jet is then equal to the area ω of the smaller end of the mouthpiece. $c_c = 1$, and c_v for moderate heads is about 0·97, which is also the value of c,

$$Q = c_v \omega \sqrt{(2gh)} \quad . \quad . \quad . \quad (9).$$

For such an orifice Weisbach has found the following values of the coefficients with different heads:—

Head over Orifice in Feet = h.	·66.	1·64.	11·48.	55·77.	337·93.
Coefficient of velocity = c_v . .	·959	·967	·975	·994	·994
Coefficient of resistance = c_r . .	·087	·069	·052	·012	·012

Fig. 52 shows a conoidal mouthpiece of approximately correct form.

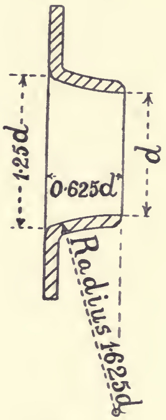

Fig. 52.

43. **Coefficients for sharp-edged orifices with complete contraction.**—Orifices are used in measuring the rate of flow of water. If the flow is discharged through an orifice, and the head at the orifice measured, the rate of flow can be determined, provided the coefficients of the orifice are known. The orifice which can be constructed and measured with most accuracy is a circular hole in a comparatively thin plate. In certain cases a rectangular aperture is more convenient. Sometimes the edges of the hole are bevelled (Fig. 45, *a*); but this is not important, and the edge is more liable to injury. The form *b* is most generally used, the edge being kept as square as possible. A large amount of experimental research has been done in determining the coefficients of orifices of this type of various sizes and under different heads, and the results have been tabulated, so that for most cases the coefficients can be selected. Perhaps the most complete collection of such experiments on the discharge of sharp-edged orifices is to be

found in Hamilton Smith's *Hydraulics* (London, 1886), where the results are discussed and plotted in curves. In cases where great accuracy is important it is desirable that the coefficients for the particular orifice used should be determined by direct experiment. Differences in the condition of the edge and the position of the orifice relatively to the walls of the reservoir cause variations of the coefficient which cannot be indicated in any tables.

Broadly, for large sharp-edged orifices in plane surfaces, and not near lateral boundaries, under moderately large heads, the coefficient of discharge has a fairly constant value not differing much from $c = 0·595$. The value of the coefficient is greater as the head is smaller, and as the area of the orifice is smaller. For small orifices under comparatively small heads it may have the value $c = 0·650$, an increase of 9 per cent. The following tables contain values selected from Hamilton Smith's reductions, modified where necessary to be applicable in the ordinary formula

$$Q = c\omega\sqrt{2gh} \quad . \quad . \quad . \quad (10).$$

For large vertical orifices under small heads there is a decrease of c.

Coefficient of Discharge c of Square Sharp-Edged Orifices in Eq. (10)

Head over Centre in Feet, h.	Length of Side of Square in Feet.					
	0·02.	0·03.	0·05.	0·10.	0·40.	1·00.
0·4	...	...	·637	·621	...	...
0·5	...	·648	·633	·619	·597	...
0·7	·656	·642	·628	·616	·600	·582
1·0	·648	·636	·622	·613	·602	·592
1·5	·641	·629	·618	·610	·604	·599
2·0	·637	·626	·615	·608	·605	·600
3·0	·632	·622	·612	·607	·605	·602
4·0	·628	·619	·610	·606	·605	·602
5·0	·626	·617	·610	·606	·604	·602
7·0.	·621	·615	·608	·605	·604	·602
10·0	·616	·611	·606	·604	·603	·601
20·0	·606	·605	·603	·602	·601	·600

COEFFICIENT OF DISCHARGE c FOR CIRCULAR SHARP-EDGED ORIFICES IN EQ. (10)

Head over Centre in Feet, h.	Diameter of Orifice in Feet.					
	0·02.	0·03.	0·05.	0·10.	0·40.	1·00.
0·4	...	...	·631	·618	...	...
0·5	...	·643	·627	·615	·592	...
0·7	·651	·637	·622	·611	·596	·579
1·0	·644	·631	·617	·608	·597	·586
1·5	·637	·625	·613	·605	·599	·592
2·0	·632	·621	·610	·604	·599	·594
3·0	·627	·617	·606	·603	·599	·597
4·0	·623	·614	·605	·602	·598	·596
5·0	·620	·613	·605	·601	·598	·596
7·0	·616	·609	·603	·600	·598	·596
10·0	·611	·606	·601	·598	·597	·595
20·0	·601	·600	·598	·596	·596	·594

Mr. Mair carried out a series of careful tests on the coefficient of discharge of circular orifices in conditions permitting exceptional accuracy of observation (*Proc. Inst. Civil Engineers,* lxxxiv., 1885-86). The following Table gives a selection of the results:—

VALUES OF c IN EQ. (10)

Head in Feet.	Diameter of Orifice in Inches.				
	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
	Diameter of Orifice in Feet.				
	·083.	·125.	·167.	·208.	·250.
·75	0·616	0·616	0·616	0·607	0·609
1·5	0·610	0·611	0·610	0·603	0·605
2·0	0·609	0·609	0·609	0·604	0·605

The results agree closely with the relation

$$c = 0·6075 + \frac{0·0098}{\sqrt{h}} - 0·0037d,$$

where h is in feet and d in inches. In the case of a 2-inch orifice a minute rounding of the square edge altered the coefficient from 0·612 to 0·622 under the same conditions exactly.

Mr. Ellis measured indirectly by a weir the discharge from a sharp-edged orifice 2 feet square, under heads varying from 2 to $3\frac{1}{2}$ feet. For $h=2$ feet, $c=0{\cdot}611$. For the larger heads c was not sensibly different from 0·60 (*Trans. Am. Soc. Civil Engineers,* 1876).

Rectangular orifices. Experiments of Poncelet and Lesbros.—For rectangular orifices there is a variation of the coefficient of discharge c both with the height a and the width b of the orifice. But for ratios of b/a not exceeding 20, it appears that c depends chiefly on the smaller dimension of the orifice independently of the other. The following are a few values selected from the results obtained by Poncelet and Lesbros: h_2 is the head at the top edge of the orifice, so that the head to the centre of the orifice is $h_2+\frac{a}{2}$. The discharge is therefore

$$Q=cab\sqrt{\left\{2g\left(h_2+\frac{a}{2}\right)\right\}} \quad . \quad . \quad . \quad (11).$$

The sides of the channel of approach were at least $2\frac{3}{4}b$ from the vertical edges, and the bottom at least $2\frac{3}{4}a$ from the lower edge of the orifice. The head was measured not immediately at the orifice, but at some distance back, where the water was nearly at rest.

Coefficients of Discharge c for Rectangular Orifices in Eq. (11)

Head over Top Edge of Orifice, h_2. Feet.	Width, $b=0{\cdot}656$ Feet.				Width, $b=1{\cdot}968$.	
	Height in Feet, $a=$				Height, $a=$	
	·0656.	·164.	·328.	·656.	·0656.	·656.
·066	·659	·615	·596	·572	·643	...
·164	·658	·625	·605	·585	·641	·597
·328	·654	·630	·611	·592	·639	·602
·656	·648	·630	·615	·598	·635	·605
1·64	·640	·628	·617	·603	·630	·607
3·28	·633	·626	·615	·605	·626	·605
4·92	·619	·620	·611	·602	·623	·602
6·56	·612	·613	·607	·601	·620	·602
9·84	·610	·606	·603	·601	·615	·601

44. **Submerged sharp-edged orifices.**—If the orifice is drowned below the tail water the conditions of discharge are in no important way altered, except that the effective head is the difference of level of the free surface of the head and tail water. As there is often some disturbance in the tail water near the orifice the level of the tail water should be taken at a point where the disturbance has subsided. So far as is known, the coefficient of discharge is the same as for an orifice discharging in the air. Some experiments by Hamilton Smith show that this must be very nearly the case.

COEFFICIENT OF DISCHARGE c IN EQ. (10) OF ORIFICES DROWNED TO THE EXTENT OF 0·57 TO 0·73 FEET (HAMILTON SMITH)

Circular, $d=0·05$.		Circular, $d=0·1$.		Square, $0·05 \times 0·05$.		Square, $0·1 \times 0·1$.	
Effective Head, h.	c.	Effective Head, h.	c.	Effective Head, h.	c.	Effective Head, h.	c.
4·08	·602	3·97	·599	4·06	·607	3·95	·605
2·16	·604	2·00	·601	2·21	·609	2·32	·604
·44	·618	·25	·605	·35	·620	·21	·612

45. **Orifice at the end of a channel.**—When the orifice is at the end of a channel the cross section of which Ω is not very large compared with the area ω of the orifice, the velocity of approach to the orifice increases the discharge. In that case the discharge is

$$Q = c\omega\sqrt{\left\{\frac{2gh}{1-\left(\frac{c\omega}{\Omega}\right)^2}\right\}} \quad . \quad . \quad (12);$$

the head h is measured at some distance back from the orifice. The value of c in this case is not well determined.

46. **Self-adjusting orifices for constant discharge. The Spanish module.**—In a number of cases, especially in the case of the distribution of irrigation water, it is required to deliver from a canal or reservoir a constant supply of water, notwithstanding variations of level in the canal or reservoir. A number of devices for this purpose have been

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invented, and the Spanish module used on the canal of Isabella II., which supplies Madrid with water, may be taken as a type. The module, Fig. 53, consists of two chambers, the upper being in free communication with the canal and the lower discharging by a culvert to the fields. In the floor between the chambers there is a sharp-edged orifice in a bronze plate. Hanging in this is a bronze plug of varying

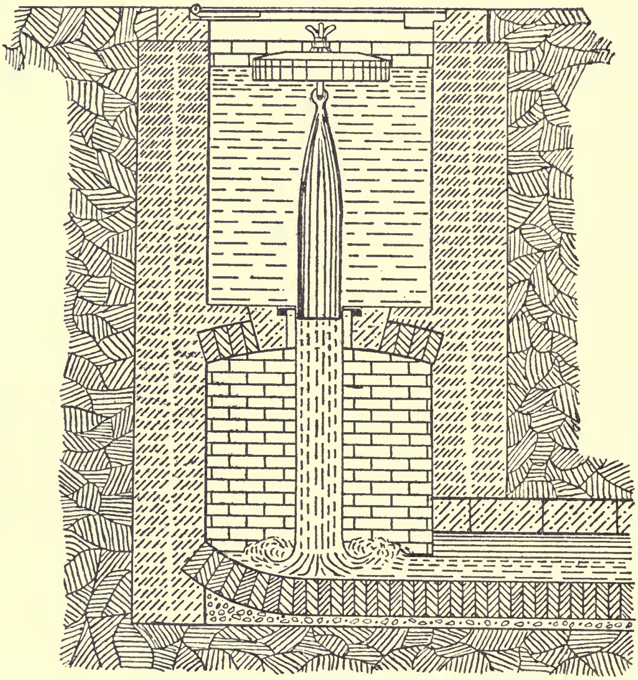


Fig. 53.

diameter suspended from a float. If the water-level falls the plug gives a larger opening, and conversely if the water rises the plug fills a greater part of the orifice. Thus if the plug is properly formed a constant discharge with varying head is obtained. The theory of the module is very simple. Let R (Fig. 54) be the radius of the fixed orifice, r the radius of the plug at a distance h from the plane of flotation of the float, and Q the required constant discharge of the module. Then

$$Q = c\pi(R^2 - r^2)\sqrt{(2gh)}.$$

Taking $c = 0{\cdot}63$,

$$Q = 15{\cdot}88(R^2 - r^2)\sqrt{h},$$

$$r = \sqrt{\left\{R^2 - \frac{Q}{15{\cdot}88\sqrt{h}}\right\}}.$$

A value of R is chosen such that for the lowest head the expression in brackets is not negative, and then values of r can be found for various values of h, and with these the curve of the plug can be drawn. The module in Fig. 53 discharges 1 c. metre per sec. The fixed opening is 0·2 metre diameter, and the greatest head above the orifice is 1 metre.

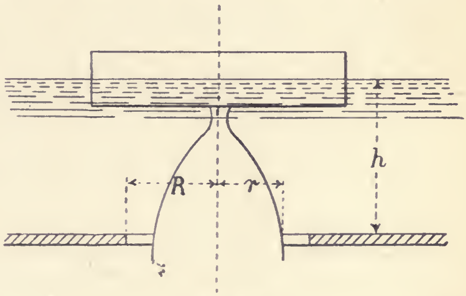


Fig. 54.

47. **Flow from orifices of liquids other than water.**—The same laws apply to all liquids, provided the head is measured in feet of the liquid itself. If a liquid of density G_m lbs. per cubic foot issues under a pressure p lbs. per square foot the corresponding head is p/G_m. Thus if mercury weighs 711 lbs. per cubic foot, a pressure of 50 lbs. per square inch, or 7200 lbs. per square foot, corresponds to a head of $7200/711 = 10{\cdot}12$ feet of mercury, and under this pressure the velocity of issue from an orifice would be $\sqrt{(64{\cdot}4 \times 10{\cdot}12)} = \sqrt{(650{\cdot}4)} = 25{\cdot}5$ feet per second nearly. From a few experiments by Weisbach, the coefficients of velocity and contraction for mercury are not very different from those for water.

Hamilton Smith, with a circular orifice 0·02 feet diameter, found for mercury $c = 0{\cdot}62$ for a head of 0·5 feet; 0·607 for a head of 1 foot; 0·595 for a head of 3 feet. For lubricating oil, with the same orifice, $c = 0{\cdot}75$ for a head of 0·5 feet; 0·735 for a head of 1 foot; 0·72 for a head of 3 feet.

48. **Imperfect contraction.**—If the sides of the channel bounding the stream approaching the orifice are near the edges of the orifice they interfere with the convergence of the elementary streams which causes the contraction. Roughly,

Water Lub Oil
.75 ,75
,735
,72

it may be said that the influence of the lateral boundaries is sensible if their distance from the edge of the orifice is less than $2\frac{3}{4}$ times the corresponding width of the orifice. If a circular orifice of area ω is at the end of a cylindrical pipe of area Ω, then the coefficient of discharge c' to be used in eq. (10) is greater than the coefficient c for the ordinary case in which the contraction is perfect in about the following ratio :—

$\frac{\omega}{\Omega}=$	$\frac{c'}{c}=$
0	1·000
0·1	1·014
0·3	1·059
0·5	1·134
0·75	1·303
0·9	1·470

Partially suppressed contraction.—If an orifice has round part of its edge a rim, or if over part of the edge the orifice touches lateral boundaries, the convergence of the streams at that part is prevented and the coefficient of contraction increased (Fig. 55). If n is the length of the rim measured round the edge of the orifice, and p the whole periphery, then the coefficients of contraction are as follow :—

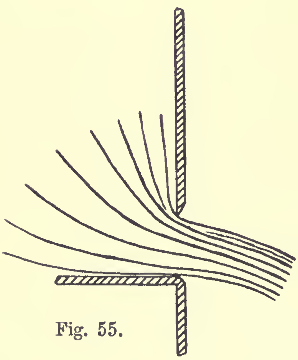


Fig. 55.

	Circular Orifices.	Rectangular Orifices.
$\frac{n}{p}=$	$c_c=$	$c_c=$
0·25	0·640	0·643
0·50	0·660	0·667
0·75	0·680	0·691

But there are few experiments on this point, and the values given can only be taken as a general guide.

49. **Inversion of the jet.**—When an orifice in a vertical wall

has dimensions not small compared with the head, the jet after leaving the orifice passes through remarkable changes of cross section. These were first investigated by Bidone (G. Bidone, *Expériences sur la forme des veines*, Turin, 1829) and Magnus, and later by Rayleigh (*Proc. Roy. Soc.* xxix. 71). Messrs. Strickland and Farmer have also made careful observations in the laboratory at Montreal (*Trans. R. S. Canada*, 1898). The jet from a square orifice (Fig. 56) converges to the vena contracta, where the section is approximately octagonal. Beyond this point sheets spread out perpendicular to the sides of the orifice. The spreading of these sheets reaches a limit in consequence of the action of the surface tension, which then gradually causes the sheets to subside into the central portion of the jet. The distance from the contracted section to this point, which may be considered a wave-length, depends on the head. Beyond this point a second set of sheets is squeezed out, but in directions bisecting the angles between the first sheet, and these are subjected to the same action as the first sheets. Similarly a third or fourth set of sheets may be developed till the jet breaks up into spray. The explanation of these changes of form given by Messrs. Strickland and Farmer is that they are due to the lateral motion of the filaments converging towards the orifice. Hence any filament except the central one has a transverse component of velocity which causes it to press on and displace neighbouring filaments. It is also true that filaments issuing at different heights from the orifice when vertical have different horizontal velocities and tend to describe parabolic paths of different range, and this must cause mutual pressure.

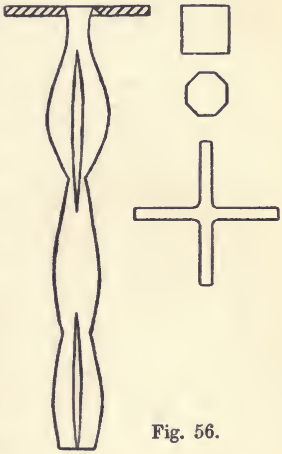


Fig. 56.

50. **Minimum coefficient of contraction.**—In one special case the coefficient of contraction can be determined rationally. Let Fig. 57 represent a vessel with vertical sides, OO being the free surface level. The liquid is discharged by a re-entrant mouthpiece with thin edges. The jet is formed by

filaments converging all round through angles of 180° with the axis of the jet, and as this is the greatest possible convergence, the contraction will be greatest and the coefficient of contraction a minimum. Let Ω be the area of the mouthpiece AB, ω that of the contracted jet *aa*. Suppose that in a short time t, the mass OO*aa* comes to O′O′*a*′*a*′.

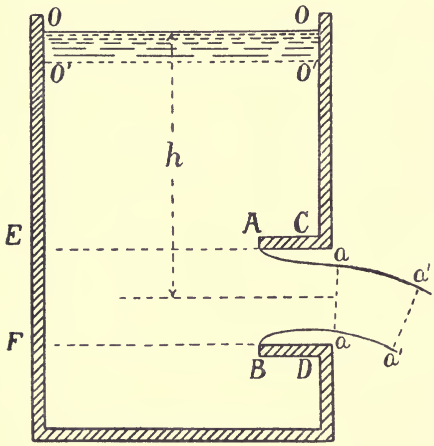


Fig. 57.

The impulse of the external forces estimated horizontally will be equal to the horizontal momentum produced (§ 35).

The pressure on OC will be balanced by that on OE, and so for other parts of the mass except EF and the surface A*aa*B of the jet. Let p_a be the atmospheric pressure and h the depth of the centre of EF from OO. The horizontal pressure exerted by the vessel on the water at EF is $(p_a + Gh)\Omega$. The horizontal pressure of the atmosphere on the surface A*aa*B, which is the pressure on its vertical projection, is $p_a\Omega$. Hence the resultant pressure acting horizontally is $(p_a + Gh)\Omega - p_a\Omega = Gh\Omega$. Since the motion is steady there is no change of horizontal momentum in the time t between OO and *aa*. The momentum generated is the momentum of *aa*′*a*′. If v is the velocity of the jet, the volume *aa*′*a*′ discharged in the time t is ωvt. Its mass is $(G\omega vt)/g$ and its momentum $(G\omega v^2t)/g$. Equating impulse and change of momentum (§ 35),

$$Gh\Omega t = \frac{G}{g}\omega v^2 t,$$

$$\frac{\omega}{\Omega} = \frac{gh}{v^2}.$$

But neglecting the very small resistances,

$$v^2 = 2gh,$$

$$c_c = \frac{\omega}{\Omega} = \frac{1}{2} \quad . \quad . \quad . \quad . \quad (13).$$

Borda found by experiment $c_c = \cdot 5149$; Bidone, $c_c = 0\cdot 5547$; and Weisbach, $c_c = \cdot 5324$, results which do not differ greatly from the theoretical value. The thickness of the edge of the mouthpiece affects the results. The reaction of the jet on the vessel is the pressure $Gh\Omega$. In the case of a simple orifice the velocity of the converging filaments in contact with the vessel in the neighbourhood of C and D reduces the pressure there, and hence the pressure on OE is not balanced by that on OC, and the reaction is greater than $Gh\Omega$. It is easily seen to follow from the equation that the contraction is less, but the exact amount is not calculable.

51. **Application of the principle of Bernoulli to the discharge from orifices.**—A jet is composed of elementary streams, each of which starts into motion at some point in the reservoir where the velocity is zero, and gradually acquires the velocity of the jet. Let Mm (Fig. 58) be such an elementary stream, M being a point where the velocity is insensibly small, and m a point in the contracted section of the jet where the filaments have become parallel and exercise uniform mutual pressure. Take the free surface

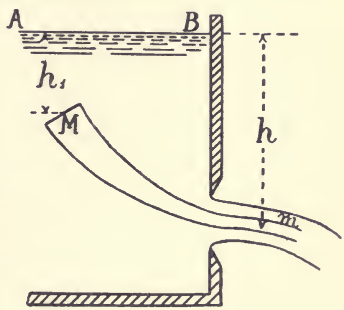


Fig. 58.

AB for datum line, and let p_1, v_1, h_1, be the pressure, velocity, and depth below datum at M; p, v, h, the corresponding quantities at m. Then

$$\frac{v_1^2}{2g} + \frac{p_1}{G} - h_1 = \frac{v^2}{2g} + \frac{p}{G} - h.$$

But at M, since the velocity is insensible, the pressure is the hydrostatic pressure due to the depth; that, is $v_1 = 0$, $p_1 = p_a + Gh_1$. At m, $p = p_a$, the atmospheric pressure round the jet. Hence, inserting these values,

$$0 + \frac{p_a}{G} + h_1 - h_1 = \frac{v^2}{2g} + \frac{p_a}{G} - h;$$

$$\frac{v^2}{2g} = h \quad . \quad . \quad . \quad . \quad (14);$$

or

$$v = \sqrt{2gh} = 8{\cdot}025\sqrt{h} \quad . \quad (14a).$$

That is, neglecting the viscosity of the fluid, the velocity of filaments at the contracted section of the jet is simply the velocity due to the difference of level of the free surface in the reservoir and the orifice. If the orifice is small in dimensions compared with h, the filaments will all have nearly the same velocity, and if h is measured to the centre of the orifice, the equation above gives the mean velocity of the jet.

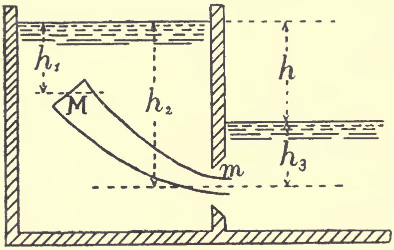


Fig. 59.

Case of a submerged orifice.—Let the orifice discharge below the level of the tail water (Fig. 59). Then at M, $v_1 = 0$, $p_1 = Gh_1 + p_a$; at m, $p = Gh_3 + p_a$.

$$0 + h_1 + \frac{p_a}{G} - h_1 = \frac{v^2}{2g} + h_3 - h_2 + \frac{p_a}{G};$$

$$\frac{v^2}{2g} = h_2 - h_3 = h \quad (15),$$

where h is the difference of level of the head and tail water, and may be termed the *effective head* producing flow.

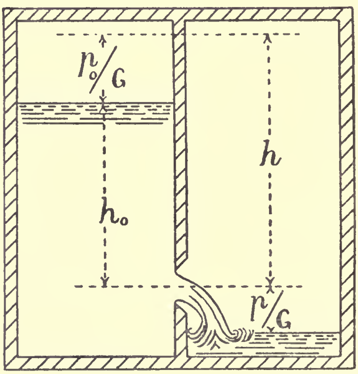


Fig. 60.

Case where the pressures are different on the free surface and at the orifice.—Let the fluid flow from a vessel in which the pressure is p_0 into a vessel in which the pressure is p (Fig. 60). Let h_0 be the height from the centre of the orifice to the free surface in the first vessel. The pressure p_0 will produce the same effect as a layer of fluid of thickness

$\frac{p_0}{G}$ added to the head water; and the pressure p will produce the same effect as a layer of thickness $\frac{p}{G}$ added to the tail water. Hence the effective difference of level, or effective head producing flow, will be

$$h = h_0 + \frac{p_0}{G} - \frac{p}{G};$$

and the velocity of discharge will be

$$v = \sqrt{2g\left\{h_0 + \frac{p_0 - p}{G}\right\}} \quad . \quad . \quad . \quad (16).$$

We may express this result by saying that differences of pressure at the free surface and at the orifice are to be reckoned as part of the effective head.

Hence in all cases thus far treated the velocity of the jet is the velocity due to the effective head, and the discharge, allowing for contraction of the jet, is

$$Q = c\omega v = c\omega\sqrt{2gh} \quad . \quad . \quad . \quad (17),$$

where ω is the area of the orifice, $c\omega$ the area of the contracted section of the jet, and h the effective head measured to the centre of the orifice. If h and ω are taken in feet, Q is in cubic feet per second.

52. **Discharge from a fire nozzle.**—Mr. John R. Freeman has made very accurate tests of the discharge from the nozzles used with hose in delivering water in streams at fires. He has found the coefficients for such nozzles so constant that he suggests their use in measuring the discharge of pumps and in similar cases (*Trans. Am. Soc. of Civil Engineers,* 1891). Fig. 61 shows the arrangement adopted. For three nozzles tried the coefficient of discharge was 0·995, with heads of 12 to 120 feet. The head was corrected for

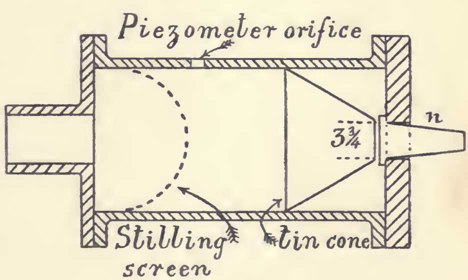


Fig. 61.

velocity of approach, but the correction was very small except for low heads. The nozzles *n* were $1\frac{3}{4}$ to $2\frac{1}{2}$ inches diameter. They were smoothly tapering, with sides converging at 5 to $7\frac{1}{2}$ degrees to the axis, and polished for 3 or 4 diameters back from the outlet. The pressure in the supply chamber was taken at a piezometer orifice made carefully flush with the inside of chamber. With the tin cone removed and a square corner to the brass flange in which the nozzle was screwed, coefficients of 0·985 to 0·990 were obtained.

53. **Flow from a vessel when the effective head varies with the time.**—Various useful problems arise relating to the time of emptying and filling vessels, reservoirs, lock chambers, etc., where the flow is dependent on a head which

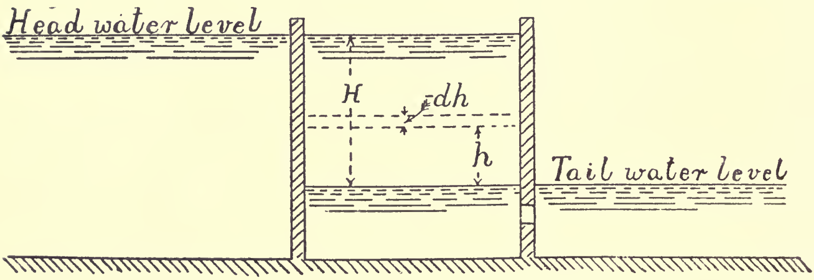


Fig. 62.

increases or diminishes during the operation. The simplest of these problems is the case of filling or emptying a vessel of constant horizontal section, such as a river lock. Suppose the lock chamber, which has a water surface of Ω square feet, is emptied through a sluice in the tail gates, of area ω, placed below the tail-water level. Then the effective head producing flow through the sluice is the difference of level in the lock chamber and tail bay. Let H (Fig. 62) be the initial difference of level, h the difference of level after t seconds. Let $-dh$ be the fall of level in the chamber during an interval dt. Then in the time dt the volume in the chamber is altered by the amount $-\Omega dh$, and the outflow from the sluice in the same time is $c\omega\sqrt{2gh}\,dt$. Hence the differential equation connecting h and t is

$$c\omega\sqrt{2gh}\,dt + \Omega h = 0.$$

For the time t during which the initial head H diminishes to any other value h,

$$-\frac{\Omega}{c\omega\sqrt{2g}}\int_{H}^{h}\frac{dh}{\sqrt{h}}=\int_{0}^{t}dt.$$

$$\therefore\ t=\frac{\Omega}{c\omega\sqrt{2g}}\,2\,(\sqrt{H}-\sqrt{h})$$

$$=\frac{\Omega}{c\omega}\left\{\sqrt{\frac{2H}{g}}-\sqrt{\frac{2h}{g}}\right\}.$$

For the whole time of emptying, during which h diminishes from H to 0,

$$T=\frac{\Omega}{c\omega}\sqrt{\frac{2H}{g}}. \qquad (18).$$

Comparing this with the equation for flow under a constant head, it will be seen that the time is double that required for the discharge of an equal volume under a constant head H.

The time of filling the lock through a sluice in the head

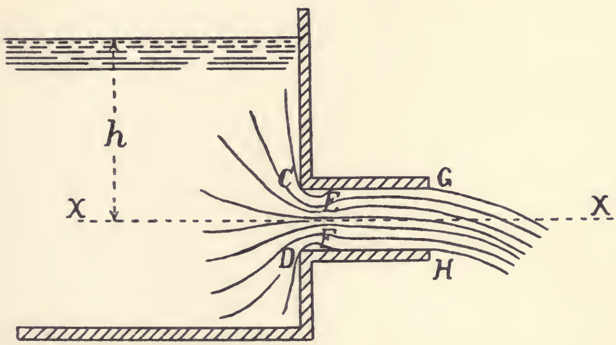


Fig. 63.

gates is exactly the same if the sluice is below the tail-water level. But if the sluice is above the tail-water level, then the head is constant till the level of the sluice is reached, and afterwards it diminishes with the time.

54. **Cylindrical mouthpiece.**—When water is discharged through a short cylindrical mouthpiece, the axis of which is normal to the side of the reservoir (Fig. 63) and its length 2 to 3 times its diameter, there is an internal contraction

at EF due to the convergence of the streams at the inlet, but the jet then expands to fill the mouthpiece and issues full bore. Let Ω be the cross section GH of the mouthpiece and ω the cross section EF of the interior contraction. Then $\omega/\Omega = c_c$ is the coefficient of contraction. Let p and v be the pressure and velocity at GH; p_1, v_1, the pressure and velocity at EF; Q, the discharge per second. Then

$$Q = \omega v_1 = \Omega v$$
$$v_1 = v/c_c.$$

Let h be the head over the axis of the jet, and c the coefficient of discharge of the mouthpiece, which, as there is no external contraction, is also the coefficient of velocity. Then

$$v = c\sqrt{2gh} \quad . \quad . \quad . \quad . \quad (19).$$

Between EF and GH there is the loss of head $(v_1 - v)^2/2g$ due to the change of velocity from v_1 to v (§ 37), and a frictional loss $c_r v^2/2g$ which is negligible for very short mouthpieces. Hence the total head at GH is less than that at EF by these losses.

$$\frac{v^2}{2g} + \frac{p}{G} = \frac{v_1^2}{2g} + \frac{p_1}{G} - \left\{\frac{(v_1 - v)^2}{2g} + c_r \frac{v^2}{2g}\right\}.$$

But $v_1 = v/c_r$ and $v = c\sqrt{(2gh)}$,

$$\frac{p - p_1}{G} = h' = \left[2\left(\frac{1}{c_c} - 1\right) - c_r\right]c^2 h \quad . \quad . \quad (20).$$

Suppose a small vertical pipe dipping into a reservoir at a lower level (Fig. 64) introduced into the mouthpiece at the internal contraction. The pressure p acts on the free surface of the lower reservoir as well as at the outlet of the mouthpiece, and p_1 is the pressure inside the mouthpiece. Hence the water will rise in the tube to a height $KL = h' = (p - p_1)/G$.

If h' is greater than the distance X between the axis of the jet and the surface of the lower reservoir, the water will be continuously pumped up from the lower reservoir and discharged at the level of the mouthpiece. This arrangement is a *jet pump* in its crudest form, in which one body of water descending a distance h pumps up another body of water a height X. Putting for the moment $c = 0{\cdot}82$, $c_c = 0{\cdot}64$, and neglecting the small quantity c_r,

$$h' = 0{\cdot}75h,$$

which is the greatest value of X at which pumping will occur. The values assumed will be seen presently to be about average values of the coefficients.

In order that the continuity of the stream may not be broken, the lowest pressure must not be negative, that is, p_1

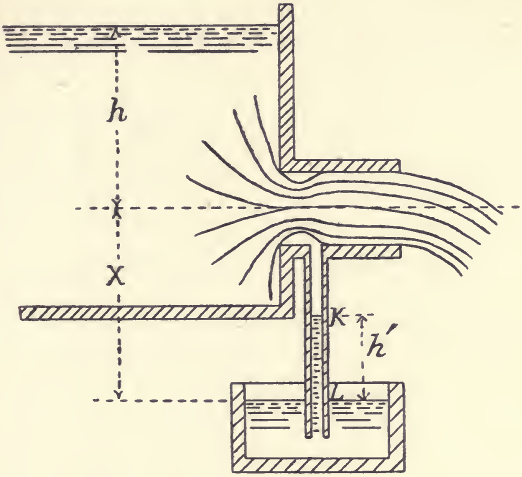


Fig. 64.

must be greater than 0. Let the atmospheric pressure height $p/G = 33{\cdot}9$ feet of water. The condition of flow full bore is—

$$\frac{p_1}{G} = \frac{p}{G} - h' = 33{\cdot}9 - \left\{2\left(\frac{1}{c_c} - 1\right) - c_r\right\}c^2 h > 0$$

$$h < \frac{33{\cdot}9}{\left\{2\left(\frac{1}{c_c} - 1\right) - c_r\right\}c^2} \quad . \quad . \quad . \quad (21).$$

With the values of the coefficients assumed above, h must be less than $33{\cdot}9/0{\cdot}75 = 45$ feet, or the jet will not discharge full bore.

Let c_v be the coefficient of velocity corresponding to the resistances between CD and EF (Fig. 63). Then

$$v_1 = c_v\sqrt{(2gh)},$$

and the head wasted between CD and EF (§ 36) is

$$\left(\frac{1}{c_v^2}-1\right)\frac{v_1^2}{2g}=\frac{1}{c_c^2}\left(\frac{1}{c_v^2}-1\right)\frac{v^2}{2g}.$$

There are therefore three losses of head between CD and GH, two of which have already been given, and the effective head producing the velocity v is h less these three losses.

$$\frac{v^2}{2g}=h-\left[\frac{1}{c_c^2}\left(\frac{1}{c_v^2}-1\right)+\left(\frac{1}{c_c}-1\right)^2+c_r\right]\frac{v^2}{2g}$$

$$=h-\left[\left(\frac{1}{c_c c_v}\right)^2-\frac{2}{c_c}+1+c_r\right]\frac{v^2}{2g}$$

and putting $v=c\sqrt{(2gh)}$,

$$\frac{1}{c^2}=\left(\frac{1}{c_c c_v}\right)^2-\frac{2}{c_c}+2+c_r,$$

and the coefficient of discharge for the mouthpiece is—

$$c=\sqrt{\left\{\frac{1}{\left(\frac{1}{c_c c_v}\right)^2-\frac{2}{c_c}+2+c_r}\right\}} \quad . \quad . \quad (22).$$

Taking $c_c=0·64$, $c_v=0·97$, and neglecting c_r,

$$c=0·824.$$

Weisbach made experiments on some cylindrical mouthpieces of different diameters, and lengths about three diameters, and found the following values of c, which do not differ much from the value just calculated:—

Diameter =	0·032	0·066	0·098	0·131 feet.
c =	·843	·832	·821	·810

The coefficient varies somewhat with the length of the mouthpiece. Its average value may be taken to be as follows:—

$\frac{\text{Length}}{\text{Diameter}}$ =	1	2 to 3	12
c =	0·88	0·82	0·77

55. **Convergent mouthpieces.**—With these there is an external contraction at the outlet as well as the internal contraction. Two cases may be distinguished; the inner

angle may be sharp as at A (Fig. 65), or well rounded as at B.

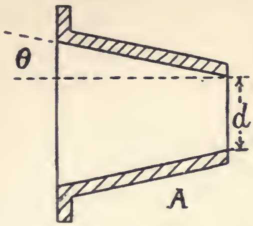


Fig. 65.

In the latter case the loss due to the internal contraction is diminished. The discharge is

$$Q = c_v c_c \Omega \sqrt{2gh} = c\Omega\sqrt{2gh} \quad . \quad . \quad (23),$$

where $\Omega = \frac{\pi}{4}d^2$ is the area at the external end. The length of the mouthpiece is about $3d$.

Angle θ . .	0°	$5\frac{3}{4}$°	$11\frac{1}{4}$°	$22\frac{1}{2}$°	45°	90°
c for case B . .	0·97	0·95	0·92	0·88	0·75	0·63
c for case A . .	0·83	0·94	0·92	0·85	...	...

56. **Divergent conoidal mouthpiece.**—Suppose a mouthpiece with a convergent inlet and divergent outlet so designed that there is nowhere any abrupt change of velocity in the stream passing through it, as in Fig. 66. The inlet may be of the form of a contracted stream from a sharp-edged orifice, and the divergent part should expand very gradually, becoming cylindrical at the end.

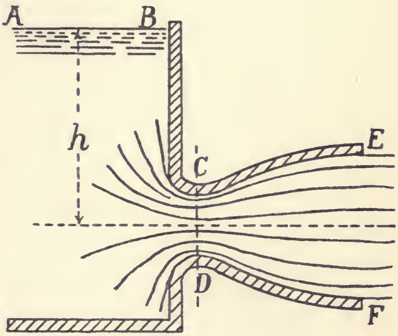


Fig. 66.

Let ω, v, p, be the area of section, velocity, and pressure at CD, and Ω, v_1, p_1, the same quantities at EF. Let the atmospheric pressure be $p_a/G = 33·9$ feet of water, and let h be the head over the mouthpiece.

Then the velocity at EF is

$$v_1 = c_v \sqrt{(2gh)} \quad . \quad . \quad . \quad (24),$$

and the effective head producing this velocity is

$$\frac{v_1^2}{2g} = c_v^2 h. \quad . \quad . \quad . \quad (24a).$$

So that the head wasted in friction and eddies in the mouthpiece is

$$(1 - c_v^2)h.$$

This wasted head may be taken to consist of two parts: z_1 wasted in the converging, and z_2 wasted in the diverging part of the mouthpiece. Then if atmospheric pressure is taken into the reckoning the total head at CD is $h + \frac{p_a}{G} - z_1$, and that at EF is $h + \frac{p_a}{G} - z_1 - z_2$. Consequently if $p_a/G = 33{\cdot}9$,

$$\frac{v^2}{2g} + \frac{p}{G} = h - z_1 + 33{\cdot}9 \quad . \quad . \quad . \quad (24b),$$

$$\frac{v_1^2}{2g} + \frac{p_1}{G} = h - z_1 - z_2 + 33{\cdot}9,$$

or if the jet discharges into the atmosphere $p_1 = p_a$, and

$$\frac{v_1^2}{2g} = h - z_1 - z_2.$$

Then the discharge is

$$Q = \Omega v_1 = \Omega \sqrt{\{2g(h - z_1 - z_2)\}} \quad . \quad . \quad (25),$$

which is independent of the area at the throat CD. But there is one obvious limit to this. As the velocity is greater at CD than EF the pressure must be less, that is, less than atmospheric pressure. If the ratio of the sections $\rho = \Omega/\omega$ is great enough p becomes zero or negative, and flow full bore is impossible.

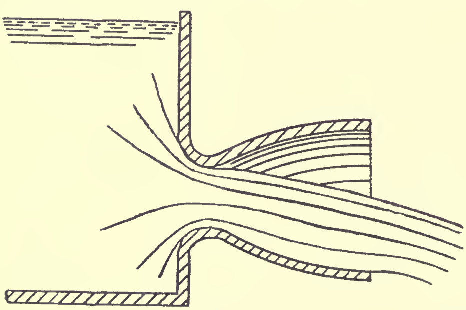


Fig. 67.

The stream breaks away from the mouthpiece as in Fig. 67. But $v = \rho v_1$, and inserting this in eq. (24b),

$$\frac{(\rho v_1)^2}{2g} + \frac{p}{G} = h - z_1 + 33 \cdot 9,$$

$$\frac{p}{G} = (1 - \rho^2)(h - z_1) + \rho^2 z_2 + 33 \cdot 9,$$

p becomes zero if

$$\rho = \sqrt{\frac{h - z_1 + 33 \cdot 9}{h - z_1 - z_2}} \quad . \quad . \quad . \quad (26).$$

From experiments on bellmouths, z_1 may be taken as about $0 \cdot 05h$. The value of z_2 may be considerably greater. In an expanding stream there is great instability and tendency to break up into eddies, which waste energy. If the mouthpiece is short, the stream breaks into eddies; if long, the friction of the surface gives rise to eddies. The following short table is calculated for the limiting cases $z_2 = 0$ and $z_2 = 0 \cdot 9h$.

Limiting Values of ρ

h =	1	5	10	20	50
When $z_2 = 0$	6·06	2·83	2·13	1·66	1·30
When $z_2 = 0 \cdot 9h$	26·4	8·0	4·5	2·8	1·7

Venturi experimented on a mouthpiece of this kind, and concluded that the discharge would be a maximum when the diverging part was of a length equal to nine times its least diameter and the angle of the cone a little more than 5°. Francis (*Lowell Hydraulic Experiments*) obtained results with a similar mouthpiece.

The diameter at CD was 0·102 feet; at EF, 0·321 feet; $\rho = 9 \cdot 9$; the length of the diverging cone 4 feet; the mouthpiece was drowned, and the difference of level of head and tail water was from 0·1 to 1·4 feet. The mean coefficient of velocity (or discharge) was $c_v = 0 \cdot 23$, so that from eq. (24*a*) the effective head was $0 \cdot 23^2 h = \cdot 053h$. Consequently $\cdot 947h$ was the head wasted during the passage of the water through the mouthpiece. This corresponds to the total head lost between inlet and outlet of a Venturi meter, h being the height due to velocity at inlet or outlet.

57. **Influence of temperature on the flow from orifices.**—Experiments were made by the author (*Phil. Mag.*, 1878) with a conoidal mouthpiece 0·394 inches diameter, a head of

1 to $1\frac{1}{2}$ foot. Neglecting the expansion of the reservoir and orifice, the coefficient is—

Temperature F.	Value of *c*.
190°	0·9871
130	0·9740
60	0·9418

With a sharp-edged orifice also 0·394 inches diameter and the same heads, and also neglecting any correction for expansion of the reservoir and orifice—

Temperature F.	Value of *c*.
205°	·5936
140	·5964
62	·5980

The results show that the influence of temperature is very small. The correction for expansion of the reservoir and orifice would be very small.

Mr. Mair repeated these experiments on a much larger scale. With a conoidal orifice $1\frac{1}{2}$ inch in diameter and a head of 1·75 feet, the following values were obtained:—

Temperature F.	Value of *c*.
170°	0·981
110	0·967
55	0·961

With a sharp-edged orifice $2\frac{1}{2}$ inches diameter and 1·75 feet head, the following were the results:—

Temperature F.	Value of *c*.
179°	0·607
110	0·604
57	0·604

In the case of the conoidal orifice the increase of temperature appears to reduce sensibly the frictional loss. In the case of the sharp-edged orifice the influence of temperature is very small.

Problems

1. The pressure in the pump cylinder of a fire-engine is 14,400 lbs. per square foot; assuming the resistances of the valves, hose, and nozzle are such that the coefficient of resistance is 0·7, find the velocity of discharge. 93·5 feet per second.

2. The pressure in the hose of a fire-engine is 13,000 lbs. per square foot; the jet rises to a height of 150 feet. Find the coefficients of velocity and resistance. 0·849 and 1·39.
3. A horizontal jet issues under a head of 9 feet. At 6 feet from the orifice it has fallen vertically 15 inches. Find the coefficient of velocity. 0·89.
4. Required the coefficient of resistance corresponding to a coefficient of velocity = 0·96. State what percentage of the energy due to the head is wasted. 0·085. 7·8 per cent.
5. A fluid of one-quarter the density of water is discharged from a vessel, in which the pressure is 60 lbs. per square inch (absolute), into the atmosphere, where the pressure is 15 lbs. per square inch. Find the velocity due to the head. 163·5 ft. per second.
6. Find the diameter of a circular orifice to discharge 2000 cubic feet per hour under a head of 5 feet. Coefficient 0·62. 3·03 inches.
7. A cylindrical cistern contains water 16 feet deep, and is 1 square foot in cross section. On opening an orifice of 1 square inch in the bottom, the water-level fell 7 feet in one minute. Find the coefficient of discharge. 0·598.
8. A miner's inch is defined to be the discharge through an orifice in a vertical plane of 1 square inch area, under an average head of $6\frac{1}{2}$ inches. Find the supply of water per hour in gallons. Coefficient 0·62. 571.
9. A vessel fitted with a piston of 10 square feet area discharges water under a head of 9 feet. What weight placed on the piston would double the rate of discharge? 270 lbs.
10. Required the discharge from a thin-edged vertical sluice opening 3 feet wide and 1 foot deep. Depth of water to lower edge of orifice = 7 feet, coefficient of discharge = 0·62. 50·7 cubic feet per second.
11. The discharge from an orifice 10 feet below the water surface is 18 cubic feet per minute. What will be the discharge when the head is 25 feet? 28·45 cubic feet per minute.
12. Show that about $\frac{3}{10}$ of the energy due to the head is wasted at a cylindrical mouthpiece. Coefficient 0·83. The loss is 31 per cent.
13. A jet has a diameter of 3 inches when issuing vertically under a head of 9 feet. Find its diameter at 6 feet above the orifice. 3·95 inches.
14. What must be the size of a sluice in a lock gate to empty the lock in ten minutes? Area of water-surface of lock 15 feet by 100 feet. Lift 6 feet. The sluice is below the tail water, and the coefficient of discharge is 0·75. 2·03 square feet.
15. A vessel is of such a form that its horizontal area is $A + Bx + Cx^2$ at x feet above the bottom. Show that if there are h feet initially in the vessel, and it empties through an orifice of area ω, the time of emptying is given by the equation

$$T = \frac{1}{c\omega}\left(2A + \frac{2}{3}Bh + \frac{2}{5}Ch^2\right)\sqrt{\frac{h}{2g}}.$$

16. Coal gas weighs 0·04 lbs. per cubic foot. Treating it as a liquid, find the velocity of discharge from an orifice due to a pressure of 1 inch of water. Coefficient of velocity 0·96.
87·8 feet per second.

17. A tank 1000 square feet in area discharges through an orifice 1 square foot in area. Calculate the time required to lower the level in the tank from 50 feet to 25 feet above the orifice. Coefficient 0·6. 2591 seconds.

18. A vertical-sided lock is 60 feet long and 15 feet wide. Lift 15 feet. Find the area of a sluice below tail water to empty the lock in ten minutes. Coefficient 0·5. 2·895 square feet.

19. A Spanish module has an orifice 18 inches in diameter, and the head in the upper chamber varies from 1·5 to 4 feet. Design the plug so that the discharge shall be 7 cubic feet per second.

CHAPTER V

NOTCHES AND WEIRS

58. **Large vertical rectangular orifices.**—When the head over the top edge of the orifice is less than half the height of the orifice, the variation of head has an influence too great to be neglected (§ 37). If, as in most cases, there is contraction of the jet the theory of flow presents some difficulty. In the plane of the orifice the issuing streams are not normal

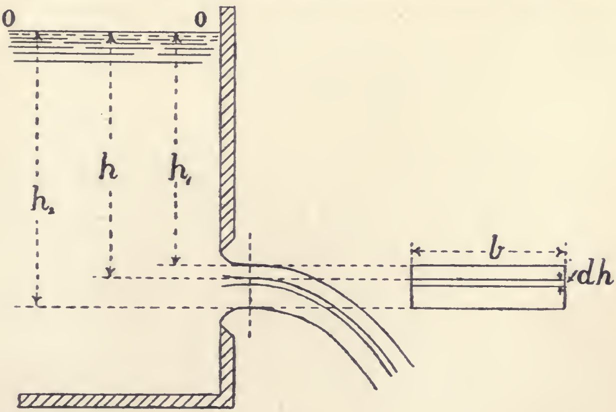


Fig. 68.

to the plane or parallel to each other. At the contracted section the streams are parallel and normal to the section, but the dimensions of the section cannot in general be directly observed. However, let the contracted section, which in the case of a rectangular orifice must itself be very approximately rectangular, be considered. Let h_1, h_2 be the heads over its top and bottom edges and b its width. Consider a lamina between the levels h and $h+dh$. Its cross section is bdh, and neglecting small resistances its velocity is $\sqrt{(2gh)}$, and its discharge $b\sqrt{(2gh)}dh$. Hence the whole discharge of the orifice is

$$Q = b\sqrt{2g}\int_{h_1}^{h_2} h^{\frac{1}{2}}dh$$

$$= \frac{2}{3}b\sqrt{2g}\{h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}\} \quad . \quad . \quad . \quad (1),$$

where the numerical factor on the right is a coefficient depending only on the form of the contracted cross section. Now let H_1, H_2 be the heads at top and bottom edges, and B the width of the orifice itself. Let

$$C = \frac{b(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}})}{B(H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}})}.$$

Then the discharge, in terms of the dimensions of the orifice, is

$$Q = \frac{2}{3}CB\sqrt{2g}\{H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}\} \quad . \quad . \quad . \quad (2),$$

which is commonly given as the theoretical formula for vertical rectangular orifices, and C is often stated to be the coefficient of contraction. But C is clearly not the coefficient of contraction, the value of which must be

$$\frac{b(h_2 - h_1)}{B(H_2 - H_1)}.$$

Equation (2) is only rational if C is understood to be a coefficient the value of which will vary with the proportions of the orifice, and experiment shows this to be the case.

59. **Notches or weirs.**—A practically very important case is that in which $H_1 = 0$ and the jet is discharged from an open notch or orifice extending up to the free surface. Weirs in rivers are cribwork or masonry constructions, primarily intended to raise the surface-level of the river up-stream, while permitting the passage of floods. Notches for measuring purposes are weirs fitted with a plate in which an open notch is formed through which the water passes. The

notch is usually rectangular, but sometimes triangular or trapezoidal. As the water surface falls when approaching the notch, the head h over the bottom of the notch, or over the *crest* of the weir, should be measured some distance back from the weir beyond the origin of the surface curve. The jet or stream passing over a weir may be termed the weir sheet. For an ordinary sharp-edged weir or notch the sheet is of the form shown in Fig. 69, A, B. The weir sheet contracts at the two ends and at its top and bottom surfaces. If the length b of the weir is equal to the width of the channel of approach there are no end contractions, and the weir is termed a weir with suppressed end contractions. If the tail-water level is above the crest of the weir it is termed a drowned weir. If the crest of the weir is broad or rounded,

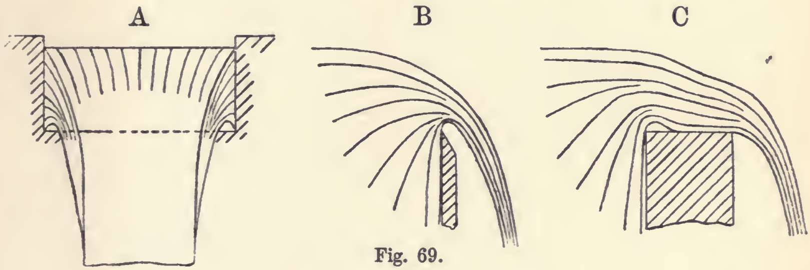


Fig. 69.

or if the upstream or downstream faces of the weir are sloped, the phenomena of discharge are complex, the water sheet in some cases springing clear, and in some cases adhering to the weir (Fig. 69, C).

The equation of discharge for rectangular weirs is found by putting $H_1 = 0$ in eq. (2). Also let h be the head above the crest and l the length of the notch or weir. Then

$$\left.\begin{aligned} Q &= \frac{2}{3}clh\sqrt{(2gh)} \\ &= \frac{2}{3}cl\sqrt{2g}\,h^{\frac{3}{2}} \\ &= 5{\cdot}35clh^{\frac{3}{2}} \end{aligned}\right\} \quad . \quad . \quad . \quad (3),$$

where c is a coefficient of discharge, which varies considerably in different cases. This is the formula which has been most generally used in computing weir discharge, and it is trustworthy for practical purposes if the value of *c* is selected from observations in similar conditions. The following small tables give values selected from those obtained by Hamilton Smith from plottings of various experiments by Francis, Fteley and Stearns, Lesbros, and others. It will be seen that *c* varies more for weirs with end contractions than for weirs with no end contractions.

Coefficients of Discharge for Weirs with Complete Contraction (Hamilton Smith)

Head on Weir Crest in Feet.	Values of *c* when the Length of the Weir is in Feet.						
	1	2	3	5	7	10	19
0·15	·625	·634	·638	·640	·640	·641	·642
0·2	·618	·626	·630	·631	·632	·633	·634
0·3	·608	·616	·619	·621	·623	·624	·625
0·5	·596	·605	·608	·611	·613	·615	·617
0·7	·590	·598	·603	·606	·609	·612	·614
1·0	...	·590	·595	·601	·604	·608	·611
1·5	...	...	·585	·592	·596	·601	·608

Coefficients of Discharge for Weirs with Suppressed End Contractions (Hamilton Smith)

Head on Weir Crest in Feet.	Values of *c* when the length of the Weir is in Feet.					
	3	5	7	10	15	19
0·15	·649	·645	·645	·644	·644	·643
0·2	·642	·638	·637	·627	·636	·635
0·3	·636	·631	·629	·628	·627	·626
0·5	·633	·627	·624	·621	·620	·619
0·7	·635	·628	·624	·620	·619	·618
1·0	·641	·633	·628	·624	·621	·619
1·5	...	·641	·636	·630	·625	·622

60. **Velocity of Approach.**—So far it has been assumed that the stream approaching the weir was of large section compared with the jet over the weir, and that the head *h* was measured where the water was nearly still. In many cases the weir is at the end of a channel of limited section, and

the head must be measured where the water has a velocity too great to be negligible. In that case the observed head has to be corrected for velocity of approach before using it in the weir formula.

Let Fig. 70 represent a vertical rectangular orifice at the end of a channel in which the velocity of approach is u. Let

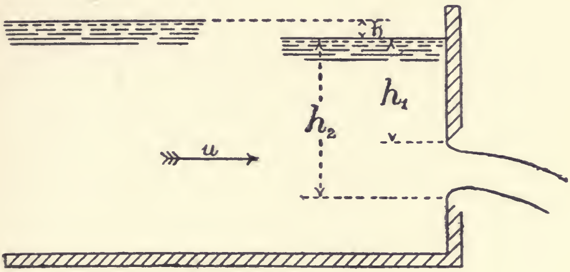


Fig. 70.

b be the width of orifice, and h_1 h_2 be the heads over the top and bottom edges of the orifice measured at a point in the channel where the mean velocity is u. It is obvious that somewhere upstream there must have been a fall of free surface

$$\mathfrak{h} = \frac{u^2}{2g}$$

in producing the velocity u. Hence the true heads over the edges of the orifice, reckoned from still water level, are $h_1 + \mathfrak{h}$ and $h_2 + \mathfrak{h}$. Putting these values in eq. (2),

$$Q = \tfrac{2}{3} cb \sqrt{2g}\{(h_2 + \mathfrak{h})^{\frac{3}{2}} - (h_1 + \mathfrak{h})^{\frac{3}{2}}\} \quad . \quad . \quad (4).$$

In the case of a notch or weir of length l, $h_1 = 0$, and h_2 may be written h,

$$Q = \tfrac{2}{3} cl \sqrt{2g}\{(h + \mathfrak{h})^{\frac{3}{2}} - \mathfrak{h}^{\frac{3}{2}}\} \quad . \quad . \quad (5),$$

which is the equation most generally used for weirs when velocity of approach must be allowed for. It is not from the theoretical point of view entirely satisfactory, because in the section where h is measured the velocity varies, and it is uncertain in what proportion different portions of the stream go to make up the jet over the weir. It is probable that $\mathfrak{h}$ should be affected by an empirical coefficient a to allow for this. In most cases $\mathfrak{h}$ is small compared with h, and the last term in the bracket is very small. Hence for simplicity some writers take

$$Q = \tfrac{2}{3} c l \sqrt{2g}\{(h + a\mathfrak{h})^{\frac{3}{2}}\} \quad . \quad . \quad . \quad (6),$$

which is easier to compute. It appears that $a =$ about 1·5. An analysis of Francis and Fteley and Stearns' experiments led Hamilton Smith to the conclusion that a should be taken 1·33 for weirs with no end contractions, and 1·4 for weirs with end contractions. It will be seen later that new experiments by Bazin have led to a better method of dealing with velocity of approach. The following table will give an idea of the importance of velocity of approach in weir calculations:—

VALUES OF $\mathfrak{h}$

Velocity of Approach u.	$\frac{u^2}{2g}$	$1\frac{1}{3}\frac{u^2}{2g}$	$1{\cdot}4\frac{u^2}{2g}$	Velocity of Approach u.	$\frac{u^2}{2g}$	$1\frac{1}{3}\frac{u^2}{2g}$	$1{\cdot}4\frac{u^2}{2g}$
Feet per second.	Feet.	Feet.	Feet.	Feet per second.	Feet.	Feet.	Feet.
0·2	·0006	·0008	·0009	0·8	·0099	·0133	·0139
0·3	·0014	·0019	·0020	0·85	·0112	·0150	·0157
0·4	·0025	·0033	·0035	0·9	·0126	·0168	·0176
0·5	·0039	·0052	·0054	0·95	·0140	·0187	·0196
0·6	·0056	·0075	·0078	1·0	·0155	·0207	·0218
0·7	·0076	·0102	·0107	1·2	·0224	·0298	·0313
0·75	·0087	·0117	·0122	1·5	·0350	·0466	·0489

When the velocity of approach u is directly measured by a current meter, for instance, eq. (5) or (6) presents no difficulty. More commonly only the cross section Ω of the channel of approach is known. Then if Q is the discharge over the weir,

$$\mathfrak{h} = \frac{Q^2}{2g\Omega^2}.$$

If this value is introduced in eq. (5) or (6) it is very cumbrous. It is better to proceed by approximation. Let Q′ be the discharge if the velocity of approach is neglected, that is, by eq. (3). Then $u' = Q'/\Omega$ is an approximate value of u, and $\mathfrak{h}' = u'^2/2g$ is an approximate value of $\mathfrak{h}$. Putting this in eq. (5) or (6) a second approximation Q'' to Q is obtained. A third approximation can be found, but this is rarely necessary.

61. **Partially submerged orifices. Drowned weirs.**—When the tail-water level is above the lower and below the upper edge of the orifice, it divides the orifice into two parts in which the conditions of flow are different. Let Fig. 71 represent such an orifice, where h_1, h_2, h are the depths below the free surface of the upper and lower edges of the orifice and the tail water, and b is the width of the orifice. An elementary stream M_1m_1 issuing above the tail-water level has the head h', which for different parts of the orifice varies from h_1 to h. An elementary stream M_2m_2 issuing below the tail-water level has a head $h'' - h''' = h$, which is the same for all parts below the tail-water level. If Q_1 Q_2 are the discharges of the upper and lower parts of the orifice,

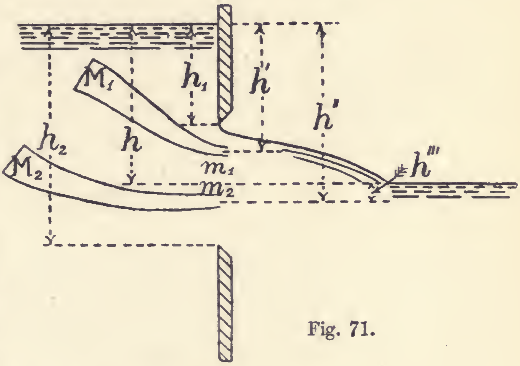


Fig. 71.

$$Q_1 = \tfrac{2}{3}cb\sqrt{2g}\{h^{\frac{3}{2}} - h_1^{\frac{3}{2}}\}$$

$$Q_2 = cb(h_2 - h)\sqrt{(2gh)}.$$

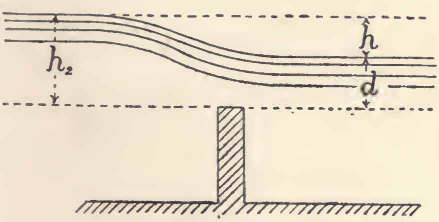


Fig. 72.

The important case is that of a drowned weir, in which the tail-water level is above the weir crest (Fig. 72). Then $h_1 = 0$, and the discharge is

$$Q = Q_1 + Q_2 = cb\sqrt{(2gh)}\{h_2 - \tfrac{1}{3}h\} \quad . \quad . \quad (7),$$

where b is the length of the weir, h_2 the head over the weir measured upstream, and h the difference of level of head and tail water. From some experiments by Fteley and Stearns (*Trans. Am. Soc. of Civil Engineers*, 1883) the following values of c are calculated :—

$\frac{d}{h_2}$	$\frac{h}{h_2}$	c
0·1	0·9	·629
0·2	0·8	·614
0·3	0·7	·600
0·4	0·6	·590
0·5	0·5	·582
0·6	0·4	·578
0·7	0·3	·578
0·8	0·2	·583
0·9	0·1	·596
0·95	0·05	·607
1·0	0·0	·628

The weir was sharp edged, 5 feet in length, with end contractions suppressed. The weir crest was 3·2 feet above the bottom of the channel ; h_2 varied from 0·3 to 0·8 feet.

62. **Broad-crested weirs.**—Broad-crested weirs are unsuitable for water measurement, but it is sometimes necessary to estimate the flow at such weirs. The following is a theory of the flow over broad-crested weirs, which is interesting.

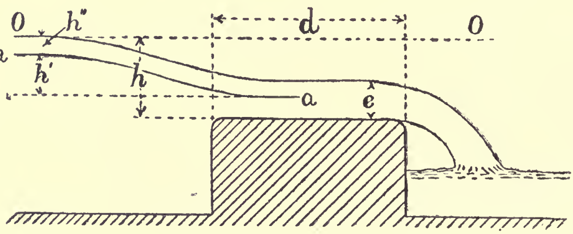


Fig. 73.

Let Fig. 73 represent a weir with a crest of width d such that the stream over it consists of rectilinear and parallel elementary streams. Let the upstream edge be rounded so that there is no contraction there. Consider an elementary stream aa', the point a being so far from the weir that the velocity at that point is negligible. Let OO be the free surface, and let a be h'' below OO and h' above a'. Let a' be z below the free surface at that point. Let h be the head on the weir crest, and e the thickness of the stream on the crest. The pressure at a is Gh'', and at a' is Gz. If v is the velocity at a',

$$\frac{v^2}{2g} = h' + h'' - z = h - e;$$

and if b is the length of the weir,

$$Q = be\sqrt{\{2g(h-e)\}} \quad . \quad . \quad . \quad (8).$$

Now $Q = 0$ for $e = 0$ and for $e = h$. The discharge will be a maximum for a value of e found by putting $dQ/de = 0$. This gives $e = \frac{2}{3}h$. Inserting this value,

$$Q = 0{\cdot}385bh\sqrt{(2gh)} \quad . \quad . \quad . \quad (9).$$

This is equivalent to taking $c = 0{\cdot}577$ in the ordinary weir formula eq. (3). Experiment shows that the discharge of broad-crested weirs approaches and even falls below this value if d is large. The formula is also applicable to large masonry sluice passages with flat floors, over which the water passes with a free surface. With $h > 1{\cdot}5d$ the attachment of the stream to the weir crest is unstable, and with $h > 2d$ the stream springs clear from the upstream edge, and the conditions approximate to those of a sharp-edged weir.

From various experiments the following values are derived. If h is the head at the weir, d the width of crest, and c the coefficient for a sharp-edged weir in the same conditions, then the coefficient of discharge in the formula

$$Q = \tfrac{2}{3}Cbh\sqrt{2gh} \quad . \quad . \quad . \quad (9a)$$

may be taken as follows:—

h/d =	0·25	0·50	0·75	1·00	1·25	1·50
C/c =	0·75	0·78	0·82	0·86	0·90	0·93
If $c = 0{\cdot}63$, C =	0·47	0·50	0·52	0·54	0·57	0·59

The value $c = 0{\cdot}63$ is a mean value for weirs with no end contractions.

The following table gives results of experiments by Mr. Blackwell:—

COEFFICIENTS C, EQ. (9*a*), FOR THE DISCHARGE OVER WEIRS, WITH END CONTRACTIONS (BLACKWELL). WHEN MORE THAN ONE EXPERIMENT WAS MADE WITH THE SAME HEAD, AND THE RESULTS WERE PRETTY UNIFORM, THE RESULTING COEFFICIENTS ARE MARKED WITH AN (*). THE EFFECT OF THE CONVERGING WING-BOARDS IS VERY STRONGLY MARKED.

Heads in Inches, measured from still Water in Reservoir.	Sharp Edge.		Planks Two Inches thick, square on Crest.				Crests Three Feet wide.					
	Three Feet long.	Ten Feet long.	Three Feet long.	Six Feet long.	Ten Feet long.	Ten Feet long, Wing-boards making an Angle of 60°.	Three Feet long, Level.	Three Feet long, Fall 1 in 18.	Three Feet long, Fall 1 in 12.	Six Feet long, Level.	Ten Feet long, Level.	Ten Feet long, Fall 1 in 18.
1	·677	...	·467	·459	...	·754	·452	·545	·467	...	...	·467
2	·675	...	·509*	·561	·585*	·675	·482	·546	·533	...	·479*	·495*
3	·630	·642*	·563*	·597*	·569*	...	·441	·537	·539	·492*	...	...
4	·617	·656	·549	·575	·602*	·656	·419	·431	·455	·497*	...	·515
5	·602	·650*	·588	·601*	·609*	·671	·479	·516	...	...	·518	...
6	·593	...	·593*	·608*	·576*	...	·501*	...	·531	·507	·513	·543
7	...	...	·617*	·608*	·576*	...	·488	·513	·527	·497	...	...
8	...	·581*	·606*	·590*	·548*	...	·470	·491	...	...	·468	·507
9	...	·530	·600	·569*	·558*	...	·476	·492*	·498	·480*	·486	...
10	...	...	·614*	·539	·534*	...	...	...	...	·465*	·455	...
12	...	...	...	·525	·534*	...	...	...	...	·467*	...	...
14	...	...	...	·549*	...	...	...	...	...	...	...	...

63. **Rafter's experiments on broad-crested weirs.**—These experiments were made in 1898 at the Cornell Hydraulic Laboratory (*Trans. Am. Soc. of Civil Engineers*, 1900). The

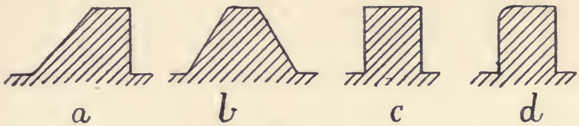


Fig. 74.

height of the weirs varied from $4\frac{1}{2}$ to 5 feet, and the length of crest was 8·58 feet. The forms used are shown in Fig. 74.

In the form *d* the upstream edge was rounded to a radius of 4 inches.

Form of Weir.	Upstream Slope.	Width of Crest, Feet.	Down-stream Slope.	Values of C for $h=$				
				0·5	1·0	1·5	2·0	5·0
a	1 to 2	0·33	Vert.	·626	·687	·713	·704	·692
„	1 to 2	0·66	„	·602	·642	·670	·683	·692
„	1 to 5	0·66	„	·619	·622	·624	·625	·633
„	1 to 4	0·66	„	...	·642	·646	·650	·650
„	1 to 3	0·66	„	·681	·713	·715	·688	·663
b	1 to 2	0·0	1 to 1	·786	·792	·763	·741	·687
„	1 to 2	0·66	1 to 2	·586	·638	·644	·674	·679
„	1 to 2	0·33	1 to 5	·616	·666	·672	·655	·666
c	Vert.	2·62	Vert.	·486	·498	·513	·530	·633
„	„	6·56	„	·467	·486	·474	·463	·504
d	„	2·62	„	·553	·562	·566	·575	·647
„	„	6·56	„	·506	·528	·530	·530	·549

64. **Triangular notches.**—The triangular notch (Fig. 75) has this peculiarity, that whatever the level in the notch, the section of the stream is similar, that is, its linear dimensions have a fixed ratio. Consider two triangular notches of

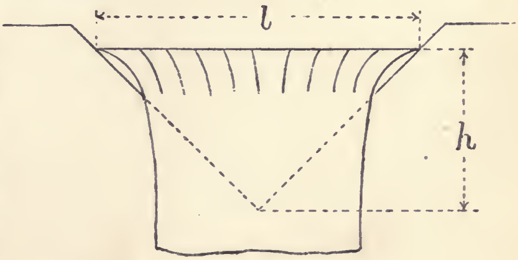


Fig. 75.

the same angle, and in which the ratio of the linear dimensions is 1 to n. The streams through the notches must be made up of similar and similarly situated elementary streams. Taking any pair of corresponding elementary streams, their cross sections must be as 1 to n^2, their depths below the free surface as 1 to n, and their velocities as 1 to $\sqrt{n}$. Consequently the discharge of these two streams must be in the ratio 1 to $n^{\frac{5}{2}}$. As this holds for all pairs of similarly situated elementary streams, the total discharge of the notches must be in the ratio 1 to $n^{\frac{5}{2}}$. But in any one notch, for two different levels of the water the same must hold, and if h_1, h_2 are the heads measured to the vertex of the notch the discharges must be in the ratio $(h_1/h_2)^{\frac{5}{2}}$. Hence, generally, if h is the head at any time the discharge is

$$Q = \beta h^{\frac{5}{2}},$$

and this equation has a more rational basis than the ordinary formula given above for rectangular weirs. It is easy to see that as the surface width l varies directly as h, the equation can be put in the form

$$Q = \tfrac{1}{2} clh \times k\sqrt{(2gh)},$$

where c is a coefficient of contraction, $\frac{1}{2}clh$ is the section of the contracted stream, and k is a constant expressing the ratio of the mean velocity in the contracted stream to the velocity due to the head. The value of k must be about 8/15. Prof. James Thomson first indicated the probability that the coefficient for a triangular notch would be nearly constant. Writing the formula

$$Q = \tfrac{4}{15} clh\sqrt{(2gh)} \quad . \quad . \quad . \quad (10),$$

he found that for a right-angled notch, sharp-edged, $c = 0{\cdot}617$. For a right-angled notch $l = 2h$, and the formula becomes

$$Q = 2{\cdot}64 h^{\frac{5}{2}} \quad . \quad . \quad . \quad . \quad (10a).$$

The notch is convenient for measuring a very variable flow when the quantity is not very large.

65. **Rectangular notch with no end contractions.**— The length of the notch or weir is equal to the distance between the walls of the channel of approach. It is desirable that the side walls should extend a little beyond the crest

of the notch above its level, but provision must be secured for the free access of air below the water stream passing over. As there are no end contractions, and the top and bottom contractions are the same for all vertical slices of the stream, the discharge must be accurately proportional to the length of the weir.

Taking any one vertical slice of the stream of width γh and head h, its discharge must be, as in the case of the triangular notch, proportional to $h^{\frac{5}{2}}$, and as the stream, whatever the head, can be considered as made up of $l/\gamma h$ such slices, the whole discharge must be

$$Q = \frac{l}{\gamma h}\beta h^{\frac{5}{2}},$$

which can be put in the form

$$= clh \times k\sqrt{(2gh)},$$

where c and k have the same meaning, as in the case of the triangular notch, and k must be about 2/3. Then simply

$$Q = \tfrac{2}{3}cl\sqrt{2g}h^{\frac{3}{2}} \quad . \quad . \quad . \quad (11),$$

where c may be expected to be constant for different values of h.

The following are values of c deduced from some very trustworthy experiments on weirs with no end contractions. The values of h have been corrected for velocity of approach, but the correction in all cases was small.

Length of Crest.	Head h.	Discharge Q.	c.		Authority.
5·0	·82	12·61	·6304		Fteley and Stearns
"	·68	9·38	·6276	·6284	"
"	·47	5·37	·6272		"
"	·22	1·747	·6365		"
"	·10	·586	·6852		"
9·995	1·0048	33·49	·6222		Francis
"	·9834	32·56	·6248	·6239	"
"	·7979	23·79	·6246		"
18·996	1·6184	130·12	·6223		Fteley and Stearns
"	·9907	62·02	·6195	·6201	"
"	·4690	20·18	·6186		"

The coefficient increases with very small heads. Excluding these cases, it will be seen that c is very nearly constant.

66. **Sharp-edged weir with end contraction. Francis's formula.**—The influence of the ends in causing contraction extends only for a certain distance, and in a long weir the discharge over the middle part is proportional to the length, as in a weir with no end contractions. Let l be the length of the part where the discharge is proportional to the length, and $\frac{1}{2}mh$ the length of the parts near the ends influenced by end contraction (Fig. 76). Then the whole length $L = l + mh$. The two parts at the ends taken together form a weir of length mh, in which the linear dimensions are in fixed ratio. The discharge of this part must be given by a relation of the form

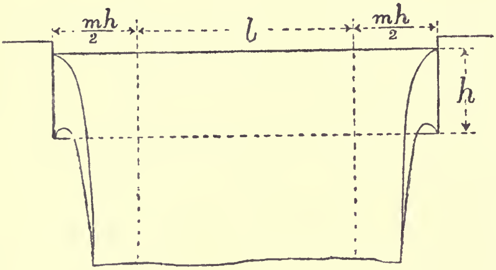


Fig. 76.

$$Q_1 = \delta h^{\frac{5}{2}},$$

as in the case of a triangular weir. The middle part is virtually a weir with no end contractions, and its discharge must be given by a relation of the form

$$Q_2 = \gamma(L - mh)h^{\frac{3}{2}}.$$

Hence the whole discharge is

$$Q = \delta h^{\frac{5}{2}} + \gamma(L - mh)h^{\frac{3}{2}}$$

$$= \gamma\left(L - \frac{\gamma m - \delta}{\gamma}h\right)h^{\frac{3}{2}},$$

which may also be written

$$Q = \frac{2}{3}c\sqrt{(2g)}(L - 2bh)h^{\frac{3}{2}} \quad . \quad . \quad . \quad (12),$$

in which c and b are constants. This is the rational basis of a formula for weirs, arrived at in a purely empirical way by Mr. Francis (*Lowell Hydraulic Experiments*, New York, 1868), which has proved of great service in practical calculations.

For sharp-edged weirs with full end contractions Mr. Francis found for b the value 0·1. The formula is not applicable to short weirs in which L is less than $3h$, nor to cases in which h is very small.

VALUES OF c IN FRANCIS'S FORMULA

Length of Weir.	h	Q	c	Authority.
9·997	1·5598	62·60	·6012	Francis
"	1·0007	32·58	·6089	"
"	·8007	23·43	·6118 mean	"
"	·6246	16·22	·6146 ·606	"
3·999	1·0202	13·14	·5964	"
"	·6830	7·27	·6027	"

It will be seen that for a considerable range of conditions c is very constant in Francis's formula.

67. **Bazin's researches on weirs.**—Some very remarkable researches on flow over various types of weir have been carried out by M. Bazin with exceptional resources, and under conditions which secured the greatest accuracy of measurement. The results are contained in a series of papers in the *Annales des Ponts et Chaussees*, 1888, 1890, 1891, 1894, 1896, and 1898. The first object was to ascertain the coefficients for a standard measuring-weir with no end contractions, especially with reference to the influence of different velocities of approach. This weir was afterwards used in measuring the flow over other weirs of different forms.

The standard weirs had a height of 3·724 feet above the bottom of the channel of approach, a vertical upstream face, and a sharp-edged crest formed by an iron plate $\frac{1}{4}$ inch thick. The length of the weir was equal to the width of the channel of approach, so that end contractions were suppressed. Chambers were formed in the side walls below the weir to admit air below the water sheet and to ensure its free detachment from the crest. The lengths of the weirs of standard type experimented on were 1·64, 3·28, and 6·56 feet.

Taking the rectangular weir formula in its simplest form, and putting $m=\frac{2}{3}c$, the discharge is

$$Q = mlh\sqrt{(2gh)} \quad . \quad . \quad . \quad (13).$$

For the same heads the coefficient m was very approximately the same for the four lengths of weir used. The following table gives a selection of the values obtained from an average of the results on all the weirs. The coefficient for standard weirs will be denoted by m_0:—

STANDARD WEIRS, 3·72 FEET HIGH, WITH NO END CONTRACTIONS

Values of Coefficient m_0 in Eq. (13)

Head. Feet. h	m_0	Head. Feet. h	m_0	Head. Feet. h	m_0	Head. Feet. h	m_0
0·197	·4432	·656	·4262	1·156	·4273	1·575	·4307
·262	·4372	·722	·4259	1·181	·4277	1·640	·4313
·328	·4336	·787	·4258	1·247	·4281	1·706	·4318
·394	·4310	·853	·4260	1·312	·4286	1·772	·4324
·459	·4292	·919	·4263	1·378	·4291	1·837	·4329
·525	·4278	·984	·4266	1·444	·4297	1·903	·4335
·591	·4269	1.050	·4269	1·509	·4302	1·969	·4341

Next, the influence of velocity of approach was examined. For this purpose the height of the weir above the bottom of the approach channel was altered to 2·46, 1·64, 1·15, and 0·787 feet. The following table gives a short selection of the values of m for different heights of weir, and therefore different velocities of approach:—

STANDARD WEIRS OF DIFFERENT HEIGHTS

Values of the Coefficient m_0 for Standard Weirs with no End Contractions

Head. Feet. h	Height of Weir in Feet.				
	3·72	2·46	1·64	1·15	0·79
0·197	·4432	·4438	·4445	·4455	·4468
·394	·4310	·4326	·4349	·4396	·4473
·591	·4269	·4320	·4377	·4463	·4579
·787	·4258	·4345	·4426	·4549	·4699
·984	·4266	·4374	·4184	·4638	·4822
1·181	·4277	·4407	·4544	·4731	·4949
1·378	·4291	·4441	·4605		

To find a general formula accordant with these results, M. Bazin starts from the well-known eq. (6),

$$Q = \mu l \sqrt{2g}\left\{h + a\frac{u^2}{2g}\right\}^{\frac{3}{2}}$$

$$= \mu l h \sqrt{(2gh)}\left\{1 + a\frac{u^2}{2gh}\right\}^{\frac{3}{2}} \quad . \quad . \quad (14),$$

where u is the velocity of approach, and a is a constant having usually a value about 1·5. μ is a coefficient less than m_0, and connected with it by the relation

$$m_0 = \mu\left(1 + a\frac{u^2}{2gh}\right)^{\frac{3}{2}};$$

or since the second term in the bracket is a small fraction,

$$m_0 = \mu\left(1 + 1{\cdot}5a\frac{u^2}{2gh}\right) \text{ nearly} \quad . \quad . \quad (15).$$

If p is the height of the weir, the section of the stream in the channel of approach is $(p+h)l$, and the velocity of approach is $u = Q/l(p+h)$. Replacing Q by its value $m_0 lh\sqrt{(2gh)}$,

$$\frac{u^2}{2gh} = m_0^{\,2}\left(\frac{h}{p+h}\right)^2$$

$$m_0 = \mu\left[1 + K\left(\frac{h}{p+h}\right)^2\right] \quad . \quad . \quad . \quad (16),$$

where K is a new coefficient. With this relation, m in eq. (13) can be found directly from the dimensions of the weir without the need to calculate u. A careful discussion of all the results leads Bazin to adopt the following values of m, and he gives the preference to the second as more convenient:—

$$m_0 = \mu\left(1 + 2{\cdot}5\frac{u^2}{2gh}\right) = \mu\left[1 + 0{\cdot}55\left(\frac{h}{p+h}\right)^2\right] . \quad (17).$$

The coefficient μ varies only with the head, and its average values are:—

Head h	Value of Coefficient μ
0·164	·4481
·328	·4322
·656	·4215
·984	·4174
1·312	·4144
1·640	·4118
1·968	·4092

With these values the coefficient m in eq. (17) can be found, and the discharge over any sharp-edged weir without end contractions calculated, including the influence of the velocity of approach. The formula then supersedes for such weirs the less convenient formulæ (5 or 6) previously given. Further, the values of μ are very approximately given by the relation

$$\mu = 0{\cdot}405 + \frac{1}{100h} \qquad . \quad . \quad . \quad (18).$$

For heads from 0·33 to 1·0 ft. with close approximation

$$m_0 = 0{\cdot}425\left[1 + \frac{1}{2}\left(\frac{h}{p+h}\right)^2\right] \qquad . \quad . \quad (18a),$$

which can be used when a possible error of 2 to 3 per cent can be allowed.

In the case of weirs with vertical faces and flat crests of a width d, such as weirs constructed of horizontal beams of square timber, the weir sheet adheres to the crest if $h < 1{\cdot}5d$; it may adhere or spring clear from the upstream edge if $h > 1{\cdot}5d$ and $< 2d$; and springs clear if $h > 2d$. When the sheet is adherent to the crest the coefficient of discharge depends on the ratio h/d, and is approximately for weirs with no end contractions

$$m = m_0\left[0{\cdot}7 + 0{\cdot}185\frac{h}{d}\right] \qquad . \quad . \quad (19),$$

where m_0 is the coefficient for a standard weir of the same height. Even with a head of 1·48 feet and a width of crest of 6·6 feet, so that $h/d = 0{\cdot}22$, the coefficient of discharge was 0·337, which is little different from the value given by the equation. If $h > 2d$ the coefficient of discharge is the same as for a standard weir of the same height. A rounding of the upstream edge of the crest modifies sensibly the discharge. A rounding to a radius of 4 inches increased the discharge 12 to 14 per cent.

From some experiments on drowned weirs, much too extensive to be described here, Bazin obtained the following expression for the coefficient of discharge:—

$$m = 1{\cdot}05 m_0 \left[1 + \frac{1}{5}\frac{h_1}{p} \right] \sqrt[3]{\frac{z}{h}} \qquad . \qquad . \quad (20),$$

where h is the head above the weir crest on the upstream side, and h_1 that on the downstream side; p is the height of the weir, and z the difference $h - h_1$ of the water-level above and below the weir. The weirs were without end contractions.

Bazin made a very extensive series of researches on weirs

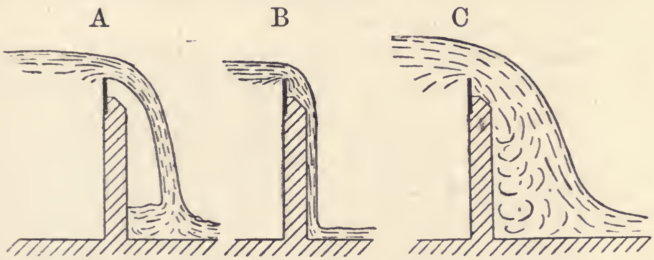


Fig. 77.

with inclined faces, and with crests either sharp, flat, or rounded. A short abstract of these would be of little use; the original account must be referred to. The weir sheet takes the following forms: (1) Free weir sheet, as in the case of a sharp-edged weir, the sheet falling freely in the air. For this condition the coefficient of discharge is best defined. (2) Depressed sheet and sheet drowned underneath. If provision is not made for free access of air below the sheet, and if the head does not exceed a certain limit, the sheet is detached from the weir, and encloses a volume of air at less than atmospheric pressure. The tail water rises in level behind the sheet, and the sheet is depressed by the excess of atmospheric pressure on its outer face (Fig. 77, A). The discharge is somewhat greater than for a free sheet. If the head increases, the whole of the air beneath the sheet is expelled, and the sheet may be said to be drowned underneath (C). It rides over an eddying mass of water in the space which, with a free sheet, is occupied by air. The sheet drowned underneath may or may not be affected by the tail water. If at the foot of the weir there is a rapid followed by a brusque elevation or standing wave, the tail-water level does not influence the discharge. On the other hand, if the tail water covers the foot of the descending sheet, it may influence the discharge, although its level is below the weir crest. (3) Adherent sheets (B). In certain cases with small heads the sheet becomes directly adherent to the downstream face of the weir, without any eddying mass of water behind it. This condition corresponds often to a marked increase of discharge. When the tail water rises above the weir crest, the sheet drowned underneath preserves its general form, until for a certain difference of head and tail water level it breaks into waves.

68. **Measurement of the head at weirs.**—It is assumed in the preceding discussion that the head on the upstream side of the weir is measured at a point above the origin of the curve of surface fall towards the weir. Fteley and Stearns concluded that the distance from the weir should be at least two and a half times the height of the weir above the bottom of the channel of approach, but no doubt this would be an excessive distance if the height of the weir is large compared with h. The exact measurement of the head is very important, and a hook gauge (§ 41) should be used, as accuracy is important. With $h = 0{\cdot}1$ foot, an error of $0{\cdot}001$ foot, or about a hundredth of an inch in the measurement of h, causes an error of $1\frac{1}{2}$ per cent in the calculated discharge. With greater values of h the percentage error is less, but is not unimportant. As the water-level fluctuates, a series of readings at equal intervals of time should be observed and the arithmetical mean taken.

69. **Practical gauging by weirs.**—The most accurate method of gauging the discharge of small streams, as in ascertaining the flow from a catchment basin, is to construct a weir of timber or concrete across the stream. A single reading of the head gives the means of calculating the discharge, and observations are made once or twice a day for as long a period as necessary. For small flows a triangular notch may be

used, but ordinarily the notch is rectangular. An automatic registering apparatus may be used, motion being given to a pencil by a float through the action of a cam designed to allow for the variation of the coefficient of discharge. The reduction of the results is simplified if a weir with no end contractions is used, as the coefficient is nearly constant. The crest of the weir should be a metal plate, flush with the upstream face of the weir, with planed edge accurately levelled.

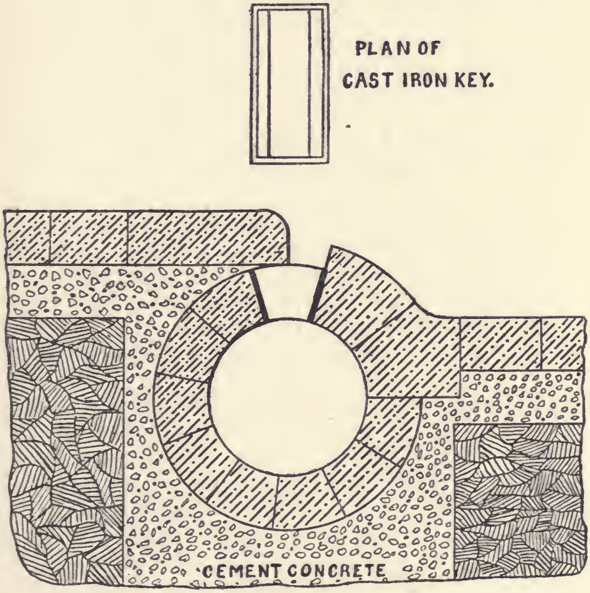


Fig. 78.

70. **Separating weirs.**—When water is collected in reservoirs for towns' supply from moorland districts, it is desirable to separate the clear water of ordinary periods from the discoloured water in periods of flood. The latter is diverted to waste or sent to a reservoir used only to supply compensation water to the streams. This is effected by a separating weir on the stream feeding the reservoir. Fig. 78 shows one form of such a weir. With small or moderate flow the water drops into the circular channel leading to the reservoir. In

flood-time the water springs over the gap, and flows into a channel beyond the weir.

Problems

1. Find the discharge through a rectangular notch, sharp-edged, and with complete contraction. The notch is 3 feet wide, and the head $1\frac{1}{4}$ feet. Velocity of approach negligible.
   13·23 cubic feet per second.
2. What will be the discharge of the same notch if the velocity of approach is 3 feet per second? 16·7 cubic feet per second.
3. Find the discharge over a sharp-edged weir 10 feet wide, with a head of 9 inches. There are no end contractions.
   21·55 cubic feet per second.
4. Find the discharge of the same weir by Bazin's formula, taking the height of the weir to be 2 feet.
   22·96 cubic feet per second.
5. What must be the width of an overfall weir to discharge 24 cubic feet per second, with 8 inches head? Coefficient 0·62.
   13·32 feet.
6. A district of 6500 acres (1 acre = 43,560 square feet) drains into a reservoir. The maximum rate at which rain falls is 2 inches in 24 hours. Supposing this rain to fall when the reservoir is full, it would have to be discharged over the bye-wash weir. Find the length of such a weir under the condition that the head shall not exceed 18 inches. Coefficient of weir 0·66.
   84·16 feet.
7. A sharp-edged weir, with full contraction, is 10 feet long, and has 15 inches of water passing over it. Find the discharge by Francis's formula. 46·27 cubic feet per second.
8. Find the discharge from a triangular right-angled notch with 2 feet head. 14·93 cubic feet per second.
9. A sharp-edged rectangular weir is to discharge daily 30,000,000 gallons of compensation water, with a normal head of 18 inches. The end contractions are suppressed, and the velocity of approach negligible. Find the length of the weir.
   8·99 feet.
10. Draw a curve of discharge from a right-angled triangular notch for different heads. The discharge may be calculated for 2, 4, 8, and 12 inches head. Coefficient 0·6.
11. A lake discharges over a weir 5 feet high above the stream bed and 10 feet wide. The water-level above the weir is 8 feet, and below the weir 6 feet, above the stream bed. Find the discharge, taking the coefficient of the weir $c = 0{\cdot}6$.
    68·1 cubic feet per second.
12. A weir is 30 feet long and has 18 inches head. The height of the weir is 3 feet. The channel of approach is the same width as the weir. Find the discharge.
    198·6 cubic feet per second.

13. If the weir in the last question had end contractions, and the velocity of approach was taken into account, find the discharge.
185·6 cubic feet per second.

14. A weir is 8 feet wide, $2\frac{1}{2}$ feet high, and has a flat crest $1\frac{1}{2}$ feet wide. The head is 15 inches, and there are no end contractions. Find the discharge. 33·7 cubic feet per second.

15. To determine the quantity of water used by a turbine, a sharp-edged weir, with full end contractions, was erected in the tail race. The width of the weir was 12 feet, and the head measured to still water level was 0·75 foot. Find the discharge by Francis's formula. 22·1 cubic feet per second.

CHAPTER VI

STATICS AND DYNAMICS OF COMPRESSIBLE FLUIDS

71. THE present chapter deals with a few problems relating to compressible fluids which are closely related to those discussed in the preceding chapters. In compressible fluids the density varies with ordinary differences of pressure and temperature instead of being nearly constant as in the case of liquids. But some reservations may be made. Gases are so much lighter than water that the variation of pressure with difference of level can often be disregarded. In some cases, as for instance the flow of lighting gas in mains, the difference of pressure causing flow is so small compared with the absolute pressure that the variation of density can be neglected without much error. On the other hand, in a large number of cases the variation of density must be taken into the reckoning, and then the formulæ for compressible fluids are more complicated than those for water.

Heaviness of gases.—The density or weight per cubic unit of volume, G, must be stated with reference to some standard pressure and temperature. The most convenient standards are 32° F., and one atmosphere, or 2116·3 lbs. per square foot. The volume V in cubic feet per pound is the reciprocal of the weight G in pounds per cubic foot. V is often termed the specific volume.

HEAVINESS OF GASES AT 32° F. AND ONE ATMOSPHERE

	Approx. Molecular Weight μ.	Specific Gravity Air=1 s.	Weight in lbs. per cubic feet G_0.	Cubic feet per pound V_0.	P_0V_0.	Gas Constant R.
Hydrogen .	2	0·0693	0·00559	178·30	378819	768·1
Oxygen . .	32	1·106	0·0895	11·17	23710	48·0
Nitrogen . .	28	0·971	0·0786	12·71	26990	54·6
Carbon monoxide	28	0·955	0·0773	12·94	27380	55·5
Carbon dioxide	44	1·529	0·1238	8·08	17145	34·7
Air . . .	29	1·000	0·0810	12·35	26214	53·2
Steam gas .	18	0·622	0·0502	19·91	42141	85·3
Coal gas from	...	0·485	0·0393	25·47	...	109·2
Coal gas to .	...	0·354	0·0287	34·89	...	149·6
Mond gas dry .	...	0·808	0·0654	15·29	...	65·6
Producer gas .	...	0·965	0·0781	12·80	...	55·0

The weight per cubic foot at 32° F. and one atmosphere is $G_0 = \mu/358$, and the corresponding volume per pound is $V_0 = 358/\mu$.

Specific heats of gases.—For the simpler gases the specific heats at constant pressure and volume appear to be nearly independent of the pressure and temperature. For the more complex gases it is now certain that they increase with increase of pressure and temperature. For the calculations in this chapter only the ratio of the specific heats, $\gamma = c_p/c_v$, is required, and it will be sufficient to assume that for air and the so-called permanent gases $\gamma = 1{\cdot}40$; for steam gas and carbon dioxide $\gamma = 1{\cdot}28$.

For air the following values are useful:—

$$\gamma - 1 = 0{\cdot}4; \qquad \frac{\gamma - 1}{\gamma} = 0{\cdot}286; \qquad \frac{1}{\gamma} = 0{\cdot}714;$$

$$\frac{1}{\gamma - 1} = 2{\cdot}5; \qquad \frac{\gamma}{\gamma - 1} = 3{\cdot}5.$$

72. **Gaseous laws. Boyle's law.**—At a constant temperature the pressure of a gaseous mass varies inversely as the volume. If P is the pressure in pounds per square foot, V the volume of a pound in cubic feet, and G the weight of a cubic foot in pounds; then if the temperature is constant,

$$P/G = PV = \text{constant} \quad . \quad . \quad . \quad (1).$$

If P_0, V_0 are the values at 32° F. and one atmosphere, then P_0V_0 is a constant for each gas which has been determined with great precision.

Dalton's law.—In a mixture of gases the pressure is the sum of the pressures which would be exerted by each gas separately if it occupied the space alone. Let $v_1, v_2 \ldots$ be the fractions of a cubic foot of each of the gases in one cubic foot of mixture at a pressure P. Then the pressures due to the different gases are

$$p_1 = Pv_1; \quad p_2 = Pv_2 \ldots$$

Let $w_1, w_2 \ldots$ be the fractions of a pound of each of the gases in one pound of the mixture, and $\mu_1, \mu_2 \ldots$ their molecular weights. Then

$$\left.\begin{aligned} w_1 &= \frac{v_1\mu_1}{\Sigma(v\mu)}; \quad w_2 = \frac{v_2\mu_2}{\Sigma(v\mu)}; \\ v_1 &= \frac{w_1}{\mu_1}\Big/\Sigma\frac{w}{\mu}; \quad v_2 = \frac{w_2}{\mu_2}\Big/\Sigma\frac{w}{\mu}; \end{aligned}\right\} \quad . \quad . \quad (2).$$

Charles's law.—Under constant pressure all gases expand alike. Thus between 32° and 212° F. one cubic foot expands to 1·3654 cubic feet, or, putting it another way, a gas expands 1/493 of its volume at 32° for each degree rise of temperature. Let V_0 be the volume of one pound at 32° and V its volume at t^0, the pressure being the same.

$$V = V_0\left(1 + \frac{t - 32}{493}\right) = V_0\frac{461 + t}{461 + 32} \quad . \quad . \quad (3).$$

If temperatures are reckoned from -461 on the Fahrenheit scale, in which case they are termed *absolute* temperatures, the equation takes a simpler form. Let T, T_0 be the absolute temperatures corresponding to t^0 and 32°.

$$V/V_0 = T/T_0 \quad . \quad . \quad . \quad . \quad (4).$$

The laws of Boyle and Charles can be combined to give the general relation of pressure, volume, and temperature in gases. For, let P_0, V_0, T_0 be the pressure, volume, and temperature of one pound at 32° F., and P, V, T the same quantities under other conditions. By Charles's law, if T_0 changes to T, and V_0 to V′, the pressure remaining constant,

$$V' = V_0T/T_0.$$

But by Boyle's law the product of pressure and volume is constant if the temperature does not change. Let P_0 now change to P and V′ to V at constant temperature. Then

$$PV = P_0V' = P_0V_0T/T_0.$$

Let $P_0V_0/T_0 = R$, which is called the gaseous constant. Then

$$PV = RT \quad . \quad . \quad . \quad . \quad (5)$$

is the general equation connecting pressure, temperature, and volume. Values of R are given in the table on p. 119.

73. **The Mercurial Barometer.**—In the mercurial barometer the pressure due to the height of the column h (Fig. 79) balances the atmospheric pressure. If $G_m = 848{\cdot}8$ is the weight of mercury in pounds per cubic foot, p_a = atmospheric pressure in pounds per square foot, and h is in feet,

$$p_a = G_m h = 848{\cdot}8h \quad . \quad . \quad (6).$$

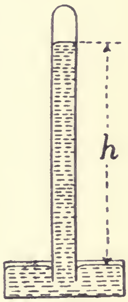


Fig. 79.

If h is given in inches, and p_a is required in pounds per square inch, $p_a = 0{\cdot}4912h$.

As mercury expands with rise of temperature, the actual barometer readings should be corrected to 32° F., the expansion of the brass scale being also allowed for. The correction depends on the height of the barometer at the time, and tables are obtainable giving the correction. If t is the temperature at the time of an observation, the correction for a barometer with brass casing is approximately

$$-h\frac{{\cdot}0001(t-32) - {\cdot}00001(t-62)}{1 + {\cdot}0001(t-32)},$$

the scale being assumed correct at 62° F. The correction is in inches if h is taken in inches.

Let G_a be the weight of air in pounds per cubic foot. If the atmosphere were homogeneous instead of decreasing in density upwards, the height of the atmosphere would be

$$H = \frac{p_a}{G_a} = \frac{G_m h}{G_a} \text{ feet} \quad . \quad . \quad . \quad (7).$$

For mercury $G_m = 848$. The mean barometric pressure at sea-level is 29·92 inches, or 2·493 feet, and the weight of air at 32° and that pressure is $G_a = 0{\cdot}08073$ lbs. per cubic foot. Hence

$$H = \frac{848 \times 2{\cdot}493}{{\cdot}08073} = 26187 \text{ feet.}$$

74. Variation of pressure with elevation. Application of the barometer to determine heights.—Let the atmosphere be at 32° F., and let G be its density at h feet above the point where the pressure is one atmosphere. If p is the pressure at a height h, the pressure at $h + dh$ will be less by the weight of a layer of thickness dh. That is,

$$dp = -G dh.$$

But at constant temperature $p/G = p_0/G_0$, where p_0, G_0 are values at 32° and one atmosphere.

$$G = pG_0/p_0$$
$$dp = -(G_0 p dh)/p_0.$$

Integrating, since $p = p_0$, when $h = 0$,

$$\left.\begin{aligned} \log_e p - \log_e p_0 &= -\frac{G_0 h}{p_0} \\ p &= p_0 e^{-\frac{G_0 h}{p_0}} \end{aligned}\right\} \quad . \quad . \quad . \quad (8).$$

The quantity p_0/G_0 is the height H of a homogeneous atmosphere at 32° F. above a point where there is standard pressure and density.

$$p = p_0 e^{-\frac{h}{H}} \quad . \quad . \quad . \quad . \quad (8a).$$

The height above a point where the height of a homogeneous atmosphere is H is

$$h = H \log_e \frac{p_0}{p},$$

where p, p_0 are the barometric pressures. If p_1, p_2 are the barometric pressures at two stations at heights h_1, h_2 above the point where the pressure is one atmosphere,

$$h_2 - h_1 = H \log_e \frac{p_1}{p_2} \text{ ft.} \quad . \quad . \quad . \quad (8b).$$

As p_1/p_2 is a ratio, the pressures may be taken in any units, for instance inches of mercury. Putting $H = 26190$, and substituting common for natural logarithms,

$$h_2 - h_1 = 60300 \log_{10} \frac{p_1}{p_2} \text{ ft.}$$

Let t_1, t_2 be the temperatures at the two stations. The mean temperature of the air between the stations is approximately $t = \frac{1}{2}(t_1 + t_2)$. But a column of air 1 foot high at 32° expands to

$$k = 1 + \frac{t - 32}{493}$$

at t^0. Hence the true height between the stations corrected for temperature is $k(h_2 - h_1)$.

Example.—The observed barometric heights at two stations were 30 and 27 inches, and the corresponding air temperatures 65° and 50° F.

$$h_2 - h_1 = 60300 \log (30/27) = 6437 \text{ ft.}$$

The mean temperature was 57°·5.

$$k = 1 + (57{\cdot}5 - 32)/493 = 1{\cdot}05.$$

Corrected difference of level $= 6437 \times 1{\cdot}05 = 6759$ ft.

75. **Flow of air through orifices under small differences of pressure.**—In some cases the air is discharged from a vessel in which the pressure is rather more than an atmosphere into the atmosphere. In that case the difference of pressure causing flow is small, and the variation of density of the air is very small also. For instance, if the difference of pressure is one pound per square inch the pressure ratio is 15·7 to 14·7 lbs. per square inch, or 1·07 nearly, and as in the cases under consideration there is no material change of temperature, this is the ratio of variation of density also. In many practical cases the variation of pressure and density is even smaller than this. In such cases the flow may be treated as if the fluid were incompressible.

Let p_1, p_2 be the absolute pressures in pounds per square foot inside and outside the reservoir from which the air flows.

T_1 the absolute initial temperature F.

v the velocity acquired by the air.

V_1 the volume of a pound of air at pressure p_1 and temperature T_1.

G_1 the weight of a cubic foot in the same conditions.

Then neglecting the variation of density, the head producing flow is $(p_1 - p_2)/G_1$.

$$v = \sqrt{\left\{2g\frac{p_1 - p_2}{G_1}\right\}} \text{ ft. per sec.} \quad . \quad . \quad (9).$$

If ω is the area of the orifice in square feet and c the coefficient of discharge, the volume discharged per second is

$$Q = c\omega v = c\omega\sqrt{\left(2g\frac{p_1 - p_2}{G_1}\right)} \text{ c. ft. per sec.} \quad . \quad (10).$$

The weight G_1 of a cubic foot of air at pressure p_1 and temperature T_1 is

$$G_1 = p_1/53{\cdot}2T_1 \text{ lbs. per c. ft.}$$

Hence the weight in pounds discharged per second is

$$\left.\begin{aligned} W = G_1Q &= c\omega\sqrt{\{2gG_1(p_1 - p_2)\}} \text{ lbs.} \\ &= c\omega\sqrt{\left\{2g\frac{p_1(p_1 - p_2)}{53{\cdot}2T_1}\right\}} \text{ lbs.} \\ &= 1{\cdot}1c\omega\sqrt{\frac{p_1(p_1 - p_2)}{T_1}} \text{ lbs.} \end{aligned}\right\} \quad . \quad (11).$$

When dealing with small differences of pressure, it is common to measure the pressures in inches of water column. One inch of water = 5·202 lbs. per square foot. Hence if the pressures are in inches of water,

$$W = 5{\cdot}72c\omega\sqrt{\frac{p_1(p_1 - p_2)}{T_1}} \text{ lbs.} \quad . \quad . \quad (12).$$

Professor Durley has carried out careful experiments on the discharge of sharp-edged orifices, $\frac{5}{16}$ to $4\frac{1}{2}$ inches diameter, with differences of pressure from 1 inch to 6 inches water column. The following table gives the coefficients of discharge obtained :—

VALUES OF c.

Diameter of Orifice in Inches.	Heads in Inches of Water.				
	1	2	3	4	5
$\frac{5}{16}$	0·603	0·606	0·610	0·613	0·616
1	0·601	0·603	0·605	0·606	0·607
2	0·600	0·600	0·600	0·600	0·600
3	0·599	0·598	0·597	0·596	0·596
$4\frac{1}{2}$	0·598	0·596	0·594	0·593	0·592

The channel of approach to the orifice was at least twenty times the area of the orifice, so that the velocity of approach was negligible.

76. **Expansion of compressible fluids.**—Two cases are important. If the expansion takes place without change of temperature, heat must be supplied during expansion; Boyle's law is applicable, and the product PV is constant. Such expansion is termed isothermal or hyperbolic. If no heat is supplied or lost during expansion, it is shown in treatises on thermodynamics that the product PV^{γ} is constant where γ is the ratio of the specific heats at constant pressure and constant volume. The expansion is termed adiabatic, and as external work is done at the expense of the internal energy of the fluid the temperature falls.

Let one pound of air expand from V_1 to V_2, the pressure changing from P_1 to P_2. Then $r = V_2/V_1$ is the volume ratio of expansion, and $\rho = P_2/P_1$ may conveniently be called the pressure ratio of expansion. The relation of pressure and volume during expansion is given graphically by a curve CD. During any

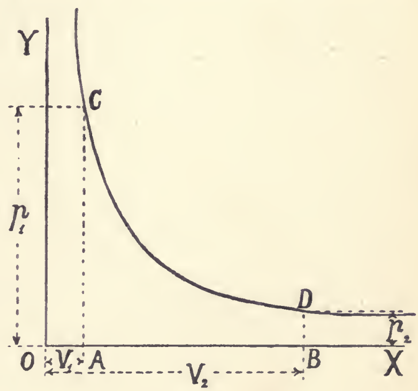


Fig. 80.

small change from V to $V+dV$ the work of expansion is PdV. Hence the whole work of expansion from the state given by P_1V_1 to that given by P_2V_2, reckoned per pound of fluid, is

$$U=\int_{V_1}^{V_2} PdV \text{ ft.-lbs.}$$

Work of isothermal expansion.—Since in this case PV is constant, the expansion curve CD is a hyperbola. $P=P_1V_1/V$. Hence

$$U=P_1V_1\int_{V_1}^{V_2}\frac{dV}{V}=P_1V_1\log_e\frac{V_2}{V_1}$$

$$=P_1V_1\log_e r=P_1V_1\log_e\frac{1}{\rho} \text{ ft.-lbs.} \quad . \quad . \quad (13).$$

Work of adiabatic expansion.—In this case $PV^\gamma=$ constant.

$$U=\int_{V_1}^{V_2} PdV=P_1V_1^\gamma\int_{V_1}^{V_2}\frac{dV}{V^\gamma}$$

$$=\frac{P_1V_1^\gamma}{\gamma-1}\left\{\frac{1}{V_1^{\gamma-1}}-\frac{1}{V_2^{\gamma-1}}\right\}$$

$$\left.\begin{aligned}&=\frac{P_1V_1}{\gamma-1}\left\{1-\left(\frac{1}{r}\right)^{\gamma-1}\right\}\\&=\frac{P_1V_1}{\gamma-1}\left\{1-\rho^{\frac{\gamma-1}{\gamma}}\right\}\\&=\frac{P_1V_1-P_2V_2}{\gamma-1}\\&=\frac{R(T_1-T_2)}{\gamma-1}\end{aligned}\right\}\text{ ft.-lbs.} \quad . \quad (14).$$

It is convenient to remember the following relations in adiabatic expansion:—

$$\left.\begin{aligned}&\rho=\left(\frac{1}{r}\right)^\gamma;\quad r=\left(\frac{1}{\rho}\right)^{\frac{1}{\gamma}}\\&r\rho=\rho^{\frac{\gamma-1}{\gamma}}=r^{1-\gamma}\end{aligned}\right\} \quad . \quad . \quad . \quad (15).$$

It is also useful to state the thermodynamic result that the change of temperature in adiabatic expansion is given by the relation

$$\frac{T_2}{T_1}=\left(\frac{1}{r}\right)^{\gamma-1}=\rho^{\frac{\gamma-1}{\gamma}} \quad . \quad . \quad . \quad . \quad (16).$$

77. **Modification of the theorem of Bernoulli for compressible fluids.**—Suppose that in a short time t the mass AB comes to A′B′. Let P_1, ω_1, v_1, G_1, V_1, T_1, be the pressure, section, velocity, weight per cubic foot, volume per pound, and absolute temperature at A. Let P_2, ω_2, v_2, G_2, V_2, T_2, be the same quantities at B. The motion being steady, the weight of fluid passing A and B in a given time must be the same. If W is the flow in pounds per second,

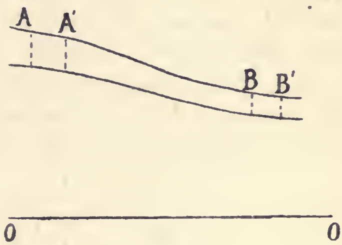


Fig. 81.

$$W = G_1\omega_1 v_1 = G_2\omega_2 v_2.$$

If z_1, z_2 are the heights of A and B above the datum plane, the work of gravity is

$$G_1\omega_1 v_1(z_1 - z_2) = W(z_1 - z_2) \text{ ft.-lbs. per sec.}$$

The work of the pressures on the sections at A and B is

$$P_1\omega_1 v_1 - P_2\omega_2 v_2 = \left(\frac{P_1}{G_1} - \frac{P_2}{G_2}\right)W \text{ ft.-lbs. per sec.}$$

The work of expansion is

$$W\int_{V_1}^{V_2} P dV = WU \text{ ft.-lbs. per sec.}$$

The change of kinetic energy is the difference of the energy of W lbs. entering at A and W lbs. leaving at B. That is,

$$\frac{W}{2g}(v_2^2 - v_1^2) \text{ ft.-lbs. per sec.}$$

Equating the work done to the change of kinetic energy, and for simplicity dividing by W,

$$z_1 - z_2 + \frac{P_1}{G_1} - \frac{P_2}{G_2} + U = \frac{v_2^2 - v_1^2}{2g}$$

$$z_1 + \frac{P_1}{G_1} + \frac{v_1^2}{2g} + U = z_2 + \frac{P_2}{G_2} + \frac{v_2^2}{2g} \quad . \quad . \quad (17).$$

An expression similar to that for liquids, except that the work of expansion U appears. The result may be stated thus: the total head at A, plus the work of expansion between A and B, is equal to the total head at B. Since A and B are any two points, it may be said that the total head along a stream line increases by the work of expansion (or decreases by the work of compression) to that point. If difference of level is neglected and the expansion is adiabatic, eq. (14),

$$\left.\begin{aligned} \frac{v_2^2 - v_1^2}{2g} &= U + \frac{P_1}{G_1} - \frac{P_2}{G_2} = U + P_1V_1 - P_2V_2 \\ &= \frac{\gamma}{\gamma - 1} P_1V_1\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} \\ &= \frac{\gamma}{\gamma - 1}\,\frac{P_1}{G_1}\left\{1 - \rho^{\frac{\gamma-1}{\gamma}}\right\} \end{aligned}\right\} \quad . \quad (18).$$

78. **Flow of compressible fluids from orifices when the variation of density is taken into account.**—When the flow is due to pressure differences which are not small compared with the absolute pressures in the fluid, the work of expansion is not negligible. Suppose the fluid flowing from a point in a reservoir where the pressure is P_1, and where it is sensibly at rest, through an orifice into a space where the pressure is P_2, and where it has acquired the velocity v_2. Neglecting any difference of level, and introducing a coefficient of velocity c_v to allow for the resistance of the orifice, from eq. (18),

$$v_2 = c_v \sqrt{\left[2g\,\frac{\gamma}{\gamma - 1} P_1V_1\left(1 - \rho^{\frac{\gamma-1}{\gamma}}\right)\right]} \quad . \quad (19).$$

Approximate equations.—When the pressure difference is small, let $\delta = (P_1 - P_2)/P_1$, so that $\rho = P_2/P_1 = 1 - \delta$, where δ is a small fraction.

$$1 - \rho^{\frac{\gamma-1}{\gamma}} = \frac{\gamma - 1}{\gamma}\,\delta = \frac{\gamma - 1}{\gamma}\,\frac{P_1 - P_2}{P_1}.$$

Then eq. (19) becomes

$$v_2 = c_v \sqrt{\left\{2g \frac{P_1 - P_2}{G_1}\right\}} \quad . \quad . \quad (20),$$

the approximate equation previously obtained on the assumption that the fluid could be treated as incompressible for small pressure differences. A closer approximation is obtained by taking another term in the expansion of

$$(1-\delta)^{\frac{\gamma-1}{\gamma}},$$

$$1-(1-\delta)^{\frac{\gamma-1}{\gamma}} = \frac{\gamma-1}{\gamma}\delta\left(1+\frac{\delta}{2\gamma}\right),$$

$$v_2 = c_v \sqrt{\left[2g \frac{P_1 - P_2}{G_1}\left(1 + \frac{P_1 - P_2}{2\gamma P_1}\right)\right]} \quad . \quad (21),$$

an equation given by Grashof.

Weight of fluid discharged from an orifice.—Let ω be the area of the orifice, and c_c the coefficient of contraction. Let P_1, V_1 be the pressure and volume per pound in the reservoir; P_2, V_2 the same quantities in the space into which the fluid is discharged. Let r be the volume and ρ the pressure ratio of expansion in the stream issuing from the orifice. The volume discharged per second, reckoned at the lower pressure, is

$$Q_2 = c_c v_2 \omega \text{ cubic feet,}$$

and the weight is

$$W = \frac{c_c v_2 \omega}{V_2} \text{ lbs. per second.}$$

But $V_2 = rV_1$, and putting $c = c_v c_c$, by eq. (18)

$$W = c\omega\sqrt{\left[2g\frac{\gamma}{\gamma-1}\frac{P_1}{V_1}\frac{1-\rho^{\frac{\gamma-1}{\gamma}}}{r^2}\right]}.$$

But $r = 1/\rho^{\frac{1}{\gamma}}$.

$$W = c\omega\sqrt{\left[2g\frac{\gamma}{\gamma-1}\frac{P_1}{V_1}\left(\rho^{\frac{2}{\gamma}} - \rho^{\frac{\gamma+1}{\gamma}}\right)\right]} \quad . \quad (22),$$

and this is a maximum when $P_2/P_1 = \rho$ is

$$\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \phi \quad . \quad . \quad . \quad (23),$$

which may be called the critical pressure ratio. If $\gamma = 1{\cdot}4$, as for air, the discharge is greatest for $\phi = 0{\cdot}528$. The maximum discharge, putting in the value of ρ just found, is

$$W_{max} = c\omega \sqrt{\left[2g \frac{\gamma}{\gamma - 1} \frac{P_1}{V_1} \left\{ \left(\frac{2}{\gamma + 1}\right)^{\frac{2}{\gamma - 1}} - \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}} \right\}\right]} \quad (24),$$

and for $\gamma = 1{\cdot}4$ this becomes

$$W_{max} = 3{\cdot}885 c\omega \sqrt{\frac{P_1}{V_1}} \quad . \quad . \quad . \quad (24a),$$

the external pressure being then a little more than half the pressure in the reservoir. When P_2/P_1 is less than ϕ, the critical value of the pressure ratio, or in other words if P_1 is greater than ϕP_2, the weight of fluid discharged diminishes, a result which is paradoxical and extremely improbable. It must therefore be inquired if there is any defect in the reasoning. There is one assumption which is unverified, namely, that the expansion is completed at the contracted section of the jet, and that the pressure at that section is P_2. Experiments, first made by Mr. R. D. Napier with steam, showed that for P_2/P_1 less than ϕ the pressure at the contracted section was greater than the external pressure P_2, and that the fluid continued to expand after the contracted section was passed. Hence the section at which the pressure is P_2 is a section greater than $c_c\omega$, and may even be greater than the area of the orifice. The jet when P_2/P_1 is less than ϕ takes a form like that shown in Fig. 82.

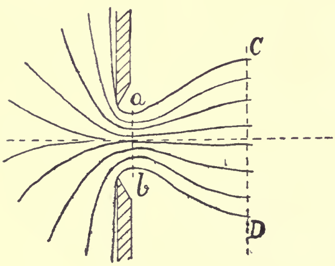


Fig. 82.

The centrifugal force of the curved elementary streams near the contracted section makes the mean pressure there greater than P_2. Experiment shows further that whenever P_2/P_1 is less than ϕ the discharge is found by substituting ϕP_1 for P_2 in the general eq. (22). Hence for such cases the discharge is found by using eq. (24) instead of eq. (22).

Discharge of air from orifices.—For air, $\gamma = 1{\cdot}4$ and $\phi = 0{\cdot}528$. Two cases occur. (*a*) When P_2/P_1 is greater than ϕ, and putting ρ for P_2/P_1,

$$W = c\omega\sqrt{\left[2g\,3{\cdot}5\frac{P_1}{V_1}\left(\rho^{1{\cdot}43} - \rho^{1{\cdot}71}\right)\right]}$$

$$= 15{\cdot}01c\omega\sqrt{\left[\frac{P_1}{V_1}\left(\rho^{1{\cdot}43} - \rho^{1{\cdot}71}\right)\right]}. \quad . \quad (25).$$

(*b*) When P_2/P_1 is less than ϕ,

$$W = 3{\cdot}885c\omega\sqrt{\frac{P_1}{V_1}}. \quad . \quad . \quad . \quad (25a).$$

It appears that for sharp-edged circular orifices, $c = 0{\cdot}64$; for short cylindrical mouthpieces without rounding at the inner edge, $c = 0{\cdot}81$ to $0{\cdot}83$; for short conoidal mouthpieces, $c = 0{\cdot}97$; and for coned blast nozzles, $c = 0{\cdot}86$.

The discharge of steam under great differences of pressure is complicated by variations of wetness in the steam and other circumstances. Careful experiments by Mr. Rosenhain are described in *Proc. Inst. Civil Engineers,* cxl. 199. For dry steam and P_2/P_1 less than ϕ,

$$W = 0{\cdot}1995c_c\omega P_1^{0{\cdot}97} \text{ lbs. nearly} \quad . \quad . \quad (26),$$

or, what is the form of the equation more generally given,

$$W = c\omega\frac{3{\cdot}6P_1}{\sqrt{P_1V_1}} \text{ lbs.} \quad . \quad . \quad . \quad (26a),$$

where V_1 is the specific volume of the steam at the pressure P_1.

CHAPTER VII

FLUID FRICTION

79. When a liquid flows in contact with a solid surface, or when a solid of shipshape form moves in a liquid at rest, there is a resistance to motion which is termed *fluid friction,* though it is wholly different in character from the friction of solids. At very low velocities the motions of the fluid near the solid may be stream-line motions, and the resistance is due to the shearing action of filaments moving with different velocities. Such conditions hardly ever obtain in cases of practical interest to the engineer. Whenever the velocity is not very small, eddies are generated which absorb energy afterwards dissipated in consequence of the viscosity of the fluid. The frictional resistance in this case is measured by the momentum imparted to the water in unit time when a solid moves in still water, or abstracted from the motion of translation and dissipated when a current flows over a surface.

The laws of fluid friction may be stated thus:—

(1) The frictional resistance is independent of the pressure in the fluid.

(2) Under certain restrictions to be stated presently the frictional resistance is proportional to the area of the immersed surface.

(3) At very low velocities the frictional resistance is proportional to the velocity of the fluid relatively to the surface. At all velocities above a certain critical value depending on the general conditions, that is, in all cases in which the motion of the fluid is turbulent, the frictional resistance is nearly proportional to the square of the velocity.

Also in cases where the motion is turbulent:—

(4) The frictional resistance increases very rapidly with the roughness of the solid surface.

(5) The frictional resistance is proportional to the density of the fluid.

These laws can be expressed mathematically for the case of turbulent motion in this way. Suppose a thin board of total area ω, wholly immersed, to move through a fluid at rest with a velocity v. Let f be the frictional resistance reckoned per square foot of the surface at a velocity of one foot per second. Then the total resistance of the board is

$$R = f\omega v^2 \text{ lbs.} \qquad (1),$$

where f is a constant for a given quality of surface and a fluid of given density. It is convenient to express this in another way. Let $\xi = (2gf)/G$, where ξ is termed the coefficient of friction. Then

$$R = \xi G\omega\frac{v^2}{2g} \text{ lbs.} \qquad (2).$$

As the board moves through the fluid the resistance is overcome through a distance of v feet per second. Hence the work expended in overcoming friction is

$$U = f\omega v^3 = \xi G\omega\frac{v^3}{2g} \text{ ft.-lbs. per sec.} \qquad (3).$$

The following are average values of the coefficient of friction for water, obtained from experiments on large plane surfaces moved in an indefinitely large mass of water:—

	Coefficient of Friction ξ	Frictional Resistance in lbs. per square foot f/v^2
New well-painted iron plate	·00489	·00473
Painted and planed plank (Beaufoy) . .	·00350	·00339
Surface of iron ships (Rankine) . . .	·00362	·00351
Varnished surface (Froude). . . .	·00258	·00250
Fine sand surface „	·00418	·00405
Coarser sand surface „	·00503	·00488

80. **Mr. Froude's experiments.**—The most valuable direct

experiments on fluid friction are those carried out by Mr. W. Froude at Torquay.[1] The method adopted was to tow a thin board in a still water canal, the velocity and resistance being simultaneously recorded. The boards were generally 3/16 inch thick and 19 inches deep, with a sharp cutwater, and from 1 to 50 feet in length. The boards were covered with various substances, such as paint, varnish, tinfoil, sand, etc., to determine the influence of different roughnesses of surface. The results obtained by Mr. Froude may be summarised as follows :—

(1) The friction per square foot of surface varies very greatly for different surfaces, being generally greater as the sensible roughness of the surface is greater. Thus, when the surface of the board was covered as mentioned below, the resistance for boards 50 feet long, at 10 feet per second, was :—

Tinfoil or varnish . .	0·25 lb. per square foot.
Calico	0·47 ,, ,, ,,
Fine sand	0·405 ,, ,, ,,
Coarser sand . . .	0·488 ,, ,, ,,

(2) The power of the velocity to which the friction is proportional varies for different surfaces. Thus, with short boards 2 feet long :—

For tinfoil the resistance varied as $v^{2\cdot16}$
For rough surfaces ,, ,, $v^{2\cdot00}$

With boards 50 feet long :—

For varnish or tinfoil the resistance varied as $v^{1\cdot83}$
For sand ,, ,, $v^{2\cdot00}$

(3) The average resistance per square foot of surface was much greater for short than for long boards; or, what is the same thing, the resistance per square foot at the forward part of the board was greater than the friction per square foot of portions more sternward. Thus, at 10 feet per second :—

		Mean Resistance in lbs. per Square Foot.
Varnished surface . .	2 feet long	0·41
,, ,, . .	50 ,,	0·25
Fine sand surface . .	2 ,,	0·81
,, ,, ,, . .	50 ,,	0·405

[1] *British Association Reports*, 1875.

This remarkable result is explained thus by Mr. Froude: "The portion of surface that goes first in the line of motion, in experiencing resistance from the water, must in turn communicate motion to the water in the direction in which it is itself travelling. Consequently, the portion of surface which succeeds the first will be rubbing, not against stationary water, but against water partially moving in its own direction, and cannot therefore experience so much resistance from it."

The following table gives a general statement of the numerical values obtained by Mr. Froude. In all the experiments in this table the boards had a fine cutwater and a fine stern end or run, so that the resistance was entirely due to the surface. The table gives the resistance per square foot in pounds, at the standard speed of 600 feet per minute, and the power of the speed to which the friction is proportional, so that the resistance at other speeds is easily calculated.

	Length of Surface, or Distance from Cutwater, in Feet.											
	Two Feet.			Eight Feet.			Twenty Feet.			Fifty Feet.		
	A	B	C	A	B	C	A	B	C	A	B	C
Varnish .	2·00	·41	·390	1·85	·325	·264	1·85	·278	·240	1·83	·250	·226
Paraffin .	1·95	·38	·370	1·94	·314	·260	1·93	·271	·237	...	...	...
Tinfoil . .	2·16	·30	·295	1·99	·278	·263	1·90	·262	·244	1·83	·246	·232
Calico . .	1·93	·87	·725	1·92	·626	·504	1·89	·531	·447	1·87	·474	·423
Fine sand .	2·00	·81	·690	2·00	·583	·450	2·00	·480	·384	2·06	·405	·337
Medium sand	2·00	·90	·730	2·00	·625	·488	2·00	·534	·465	2·00	·488	·456
Coarse sand .	2·00	1·10	·880	2·00	·714	·520	2·00	·588	·490	...	...	...

Columns A give the power of the speed to which the resistance is approximately proportional.

Columns B give the mean resistance per square foot of the whole surface of a board of the lengths stated in the table.

Columns C give the resistance in pounds of a square foot of surface at the distance sternward from the cutwater stated in the heading.

It may be noticed that although the friction per square foot decreases as the surface is longer in the direction of motion, yet the decrease, which is considerable between 2 feet and 8 feet, is small between 20 feet and 50 feet. Hence for surfaces more than 50 feet long it makes little difference whether the friction is supposed to diminish at the same rate or not to diminish at all. If the decrease of friction sternwards is due to the generation of a current accompanying the moving plane, there is not at first sight any reason why the decrease should not be greater than that shown by the experiments. The current accompanying the board might be assumed to gain in volume and velocity sternwards, till the velocity was nearly the same as that of the moving plane and the friction per square foot nearly zero. That this does not happen appears to be due to the mixing up of the current with the still water surrounding it. Part of the water in contact with the board at any point, and receiving energy of motion from it, passes afterwards to distant regions of still water, and portions of still water are fed in towards the board to take its place. In the forward part of the board more kinetic energy is given to the current than is diffused into surrounding space, and the current gains in velocity. At a greater distance back there is an approximate balance between the energy communicated to the water and that diffused. The velocity of the current accompanying the board becomes constant or nearly constant, and the friction per square foot is therefore nearly constant also.

81. **Friction of discs rotated in water.**—In many hydraulic machines, turbines, and centrifugal pumps, surfaces rotate in water, and the friction is an important cause of loss of energy. A disc rotated in water is virtually a surface of indefinite length in the direction of motion, and experiments carried out in this way by the author, *Proc. Inst. Civil Eng.* lxxx. 1885, permitted considerable variation of the conditions. Fig. 83 shows a section of the apparatus. It consisted of a wooden frame on which was placed a cast-iron cistern C. A cast-iron bracket B at the top of the frame carried a three-armed crosshead *bb*, from which an inner cistern AA was suspended by three fine wires. The crosshead could be adjusted to any position and clamped by the nut *a*. Adjusting-screws in the arms of the crosshead permitted the cistern AA to be levelled. The discs which were to be rotated in water were 10, 15, and 20 inches diameter; one is shown in position at DD keyed on a vertical shaft SS. This shaft was centred on conical ends and driven by a catgut band running on

pulleys P. The rotating disc is contained in the submerged copper cylinder AA. The flat bottom of this is fitted with

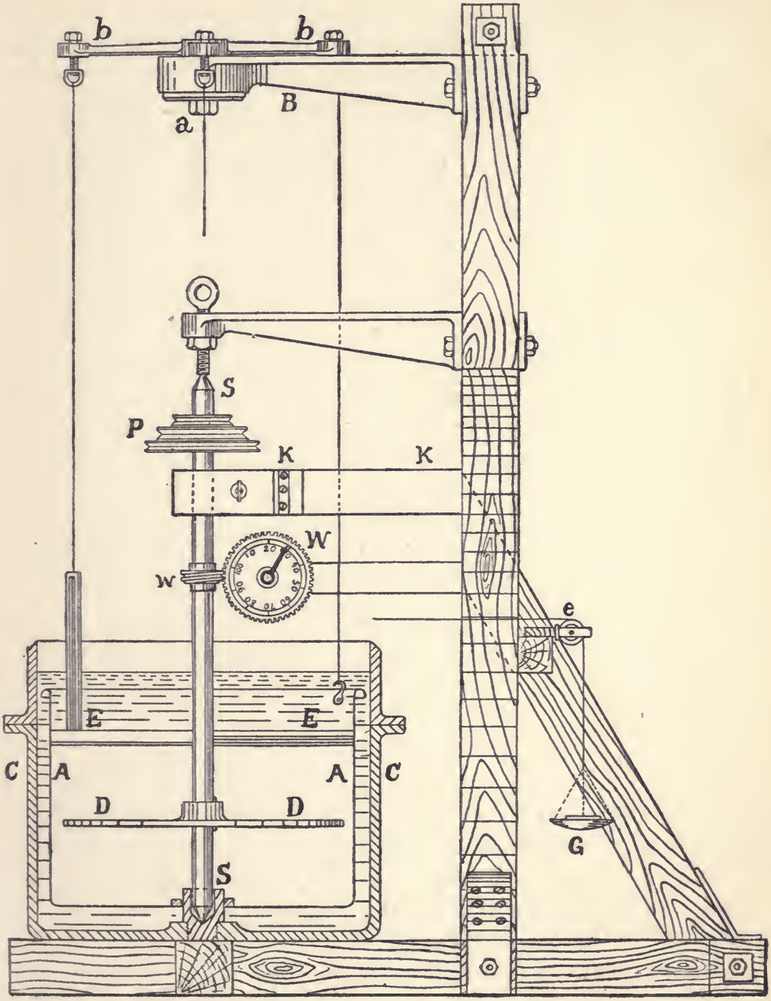


Fig. 83.

very little play round the gun-metal support of the spindle. Above the disc was a flat cover EE parallel to the flat bottom of the cistern. The height of the chamber in which the disc revolved could be varied, the disc being always placed in the centre of the chamber. A thick india-rubber ring bolted round the cover EE made a water-tight connection with the cylinder.

To measure the friction of the disc, the reaction tending to turn the cistern AA was measured, for the reaction on the chamber must be equal and opposite to the effort required to turn the disc. To the suspended cylinder was attached an index-finger moving over a graduated scale. This was adjusted to zero when the apparatus was at rest. When the disc

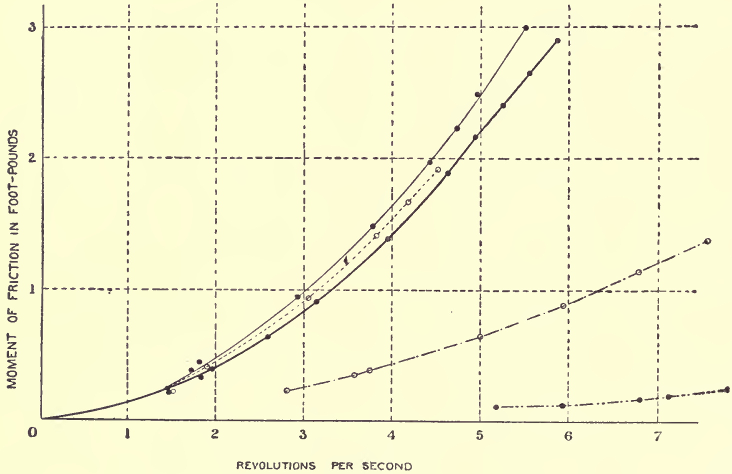


Fig. 84.

rotates, the copper cylinder tends to rotate in the same direction. To measure the effort to rotate which is equal to the effort turning the disc, a fine silk cord attached to an arc on the cistern was carried over the pulley *e* to a scale-pan G. Weights in the scale-pan balanced the friction and kept the index at zero. The rotations were observed by timing the rotations of the worm-wheel W by a chronograph. A clip brake K on the shaft was useful in adjusting the speed.

Fig. 84 shows a plotting of one set of results on brass discs of three sizes. It will be seen that the observations plot in quite regular curves. The three upper curves are for a 20-inch disc of polished brass with $1\frac{1}{2}$, 3, and 6-inch spaces between the disc and the flat ends of the cistern. The resistance diminishes a little as the spaces are narrower. The other curves are for a 15-inch and 10-inch disc of brass.

82. **Theoretical expression for the friction of a disc rotating in liquid.**—Let it be supposed that the general law of fluid friction which applies to large plane surfaces moved uniformly in water may be used to determine the friction of a disc. That is, supposing ω to be the area of any small portion of the disc moving with the velocity v, let it be assumed that the friction of that portion of the surface is $f\omega v^n$; where f is a constant differing for different surfaces, and n a constant which at the velocities used in these experiments does not differ greatly from 2.

Let a be the angular velocity of rotation, R the radius of the disc. Consider a ring of the surface between the radii r and $r+dr$. Its area is $2\pi r dr$, its velocity is ar, and the friction of this portion of the surface is therefore, on the assumption above,

$$f \times 2\pi r dr \times a^n r^n.$$

The moment of the friction of the ring about the axis of rotation is then

$$2\pi a^n f r^{n+2} dr,$$

and the total moment of friction for the two sides of the disc is then

$$M = 4\pi a^n f \int_0^R r^{n+2} dr$$

$$= \frac{4\pi a^n}{n+3} f R^{n+3}.$$

If N is the number of rotations per second, since $a = 2\pi N$,

$$M = \frac{2^{n+2} \pi^{n+1} N^n}{n+3} f R^{n+3} \quad . \quad . \quad . \quad (4).$$

The work expended in rotating the disc is in ft.-lbs. per sec.[1]

[1] If $n=2$, from which it never differs much, this formula becomes

Work expended in friction $= 623 f N^3 R^5$ ft.-lbs. per sec.,

where f varies from 0·002 to 0·003 for ordinarily rough surfaces, and increases to 0·007 for the rough surface of a metal disc covered with coarse sand.

RESULTS OF EXPERIMENTS

Number of Experiment.	Nature of Disc and Surface.	Virtual Radius of Disc.	Thickness of Water Space on each side of Disc.	Temperature Fahr.	Lowest Speed in Rotations per Second.
		Foot.	Inches.	°	
1	Clean polished brass	0·8488	1½	55·0	1·425
2	,, ,, ...	,,	3	53·0	1·459
3	,, ,, ...	,,	6	55·0	1·415
4	Painted cast iron	,,	1½	60·5	1·380
5	,, ,, ...	,,	3	61·0	1·385
6	,, ,, ...	,,	6	59·0	1·787
7	Painted and varnished cast iron .	,,	3	59·0	1·449
8	,, ,, ,, .	,,	6	63·0	1·469
9	Tallowed brass	,,	3	64·5	1·958
10	,, ,, ...	,,	3	67·0	1·950
11	Cast iron	,,	1½	55·0	1·440
12	,, ...	,,	3	54·0	1·419
13	,, ...	,,	6	55·0	1·409
14	,, covered with fine sand .	,,	3	56·5	1·541
15	,, ,, ,, coarse sand .	,,	1½	62·5	1·146
16	,, ,, ,, ,, .	,,	3	62·0	1·113
17	,, ,, ,, ,, .	,,	6	62·0	1·387
2	Clean polished brass	0·8488	3	53·0	1·459
18	,, ,, ...	,,	3	52·0	1·785
16	Cast iron covered with coarse sand .	,,	3	62·0	1·113
19	,, ,, ,, ,, .	,,	3½	53·0	1·086
2	Clean polished brass	0·8488	3	53·0	1·459
20	,, ,, ...	0·6353	3	62·0	2·816
21	,, ,, ...	0·4320	3	54·0	5·230
22	Clean polished brass	0·8488	3	41·2	1·935
2	,, ,, ...	,,	3	53·0	1·459
23	,, ,, ...	,,	3	70·4	1·984
24	,, ,, ...	,,	3	130·5	2·840
2	Clean polished brass	0·8488	3	53·0	1·459
25	,, ,, ...	,,	3	59·5	1·383

Remarks

2. Water a little coloured. 4. Water not quite clear. 5. Water a little coloured.

9. The surface of the tallow on the disc seemed to alter a little during immersion.

11, 12, 13. Cast iron a little rusty.

14. Disc coated with white lead and varnish, and covered with fine sand. Surface about as rough as ashlar stone.

15, 16, 17. Sand-coated cast-iron disc, the sand very coarse, and mixed with small gravel pebbles.

ON ROTATING DISCS

Highest Speed in Rotations per Second.	Mean Value of n for each kind of Surface.	Mean Value of c.	Mean Value of c corrected to 60° Fahr.	Friction per Square Foot at 10 Feet per Second $=f\,10^n$.	——
5·875	1·85	0·1102	0·1089	0·2018	
4·501	,,	0·1149	0·1130	0·2093	
5·531	,,	0·1256	0·1241	0·2299	
4·686	1·86	0·1169	0·1170	0·2182	
5·382	,,	0·1242	0·1245	0·2321	
5·112	,,	0·1329	0·1326	0·2473	
4·892	1·94	0·1106	0·1103	0·2200	
5·470	,,	0·1160	0·1169	0·2331	
4·237	2·06	0·0975	0·0986	0·2167	
5·160	1·86	...	...	...	
5·010	2·00	0·1029	0·1017	0·2129	
5·324	,,	0·1101	0·1085	0·2273	
4·990	,,	0·1176	0·1162	0·2432	
4·456	2·05	0·1572	0·1557	0·3395	
3·300	1·91	0·3004	0·3019	0·5874	
3·604	,,	0·3261	0·3277	0·6376	
3·655	,,	0·3658	0·3676	0·7153	
4·501	1·85	0·1149	0·1130	0·2093	Chamber clean.
4·975	1·95	0·1235	0·1212	0·2436	Chamber coated with rough sand.
3·604	1·91	0·3261	0·3277	0·6376	Chamber clean.
2·735	2·17	0·3381	0·3325	0·7986	Chamber covered with coarse sand.
4·501	1·85	0·1149	0·1130	0·2093	Diameter varied.
7·598	,,	0·0324	0·0326	...	
7·849	,,	0·0048	0·0048	...	
5·668	1·85	0·1215	...	0·2251	Temperature varied. Friction per square foot uncorrected for temperature.
4·501	,,	0·1149	...	0·2128	
5·630	,,	0·1112	...	0·2061	
5·133	,,	0·1003	...	0·1859	
4·501	1·85	0·1149	0·1130	0·2093	In water.
4·708	1·93	0·1195	...	0·2364	In syrup, sp. gr. 1·061.

Remarks

18, 19. The top and bottom of the chamber were coated with coarse sand, like the disc in experiments 15, 16, 17.
20. The disc was slightly greasy.
22. About two pailfuls of ice placed in water outside the copper chamber.
24. Water taken from an engine boiler. It was rather dirty from sediment produced by boiling.
25. Half a hundredweight of sugar dissolved in water in the cistern.

$$Ma = \frac{2^{n+3}\pi^{n+2}N^{n+1}}{n+3} fR^{n+3} \quad . \quad . \quad (5).$$

The experiments give directly the moment of friction M corresponding to any speed N for each disc. But for any given disc

$$M = cN^n \quad . \quad . \quad . \quad . \quad (6),$$

where c is a constant. Hence for any pairs of values of M and N obtained in the experiments on a given disc,

$$n = \frac{\log M_1 - \log M_2}{\log N_1 - \log N_2} \quad . \quad . \quad . \quad (7).$$

The mean value of n thus obtained is given for each of the surfaces tried. When the mean value of n has been obtained from pairs of results in which the speed was different, values of c for each speed were obtained by the formula

$$\log c = \log M - n \log N,$$

and the mean values of c thus found are given in the table, page 140. The values of n for different pairs of speeds never varied very greatly for any given disc in like conditions, nor did the values of c vary greatly for different speeds. Further, the variations from the mean value followed no regular law, so that they may be attributed to errors of observation, or to unavoidable small fluctuations of speed during the observations.

In the formulas above, f is the friction per square foot at unit velocity, but for any given kind of surface in like conditions

$$f = \frac{M(n+3)}{2^{n+2}\pi^{n+1}R^{n+3}N^n} \quad . \quad . \quad . \quad (8).$$

Variation of resistance with diameter of disc.—Three sets of experiments with discs 0·8488, 0·6353, and 0·4320 foot virtual radius, rotating in the same chamber of fixed size, gave moments of resistance in the ratios

$$1 : 0{\cdot}2887 : 0{\cdot}0425,$$

or for discs of different diameters in a chamber of constant size the resistance varies as the $(n + 2{\cdot}82)$th power of the radius. The theoretical formula above (4) is strictly applicable to discs in chambers the linear dimensions of which are proportional to the diameter of the disc, in which case the resistances are as the $(n+3)$th power of the radius. The difference of the two cases is not very great, and is consistent with the experimental result that the resistance with a given disc is greater as the chamber is larger.

Influence of temperature on the resistance.—The four results with a bright brass disc, experiments 2, 22, 23, and 24, show that the friction diminishes with unexpected rapidity as the temperature increases. The diminution is sensible even for a few degrees difference of temperature, and hence it appears that a correction for temperature ought to be introduced in experiments on the flow of water in pipes and channels. The diminution between 41° and 130° Fahr. is about 18 per cent, or 1 per cent for 5° increase of temperature.

The experiments were not numerous enough to determine exactly the law of variation of friction with temperature, and the apparatus was not adapted for securing a constant temperature during a prolonged experiment. The results agree fairly with the empirical formula

$$c_t = 0{\cdot}1328(1 - 0{\cdot}0021t) \quad . \quad . \quad . \quad (9),$$

where c_t is the value of c for a bright brass disc at the temperature $t°$.

In the experiments 1 to 17 the temperature varied in different instances from 53° to 62°. The factor

$$\frac{1 - 0{\cdot}0021 \times 60°}{1 - 0{\cdot}0021t}$$

has been used to reduce the values of c to a standard temperature of 60°. The correction is in any case small, and does not affect the conclusions drawn from the results.

Influence of roughness of surface.—The results of the experiments are altogether in accord with those of Mr. Froude as to the influence of the roughness of the surface. Even the numerical values of the frictional resistance obtained in these experiments differ very little from those obtained by him for long surfaces. Taking Mr. Froude's results for planks 50 feet long, and comparing them with those obtained in the present experiments, the resistances in pounds per square foot at 10 feet per second are:—

Mr. Froude's Experiments.		Present Experiments.	
Tinfoil surface	0·232	Bright brass	0·202 to 0·229
Varnish	0·226	Varnish	0·220 „ 0·233
Fine sand	0·337	Fine sand	0·339
Medium sand	0·456	Very coarse sand	0·587 „ 0·715

Power of the velocity to which resistance is proportional.—There is in this also a remarkable agreement between the present experiments and those of Mr. Froude. For the smoother surfaces the resistance varies as the 1·85th power of the velocity; for the rougher surfaces as a power of the velocity ranging from 1·9 to 2·1. Mr. Froude's results are precisely the same.

Influence of the size of chamber on the resistance.—In all these experiments, without a single exception, the friction of the disc increased when the chamber in which it rotated was made larger. The author is disposed to attribute this to the stilling of the eddies by the surface of the stationary chamber. The stilled water is fed back to the surface of the disc, and hence the friction depends not only on its own surface, but on that of the open chamber in which it rotates. The discs were rotated in chambers 3, 6, and 12 inches deep, and the surfaces of these chambers would be about 1000, 1200, and 1600 square inches. In the larger chambers the kinetic energy of the water may be supposed to be more rapidly destroyed than in the smaller, in consequence of the larger area of stationary surface. The water being more rapidly stilled, and the stilled water fed back to the disc in greater quantity, the resistance of the disc is increased.

Effect of roughening the surface of the chamber.—In experiments 18 and 19 the upper and lower surfaces of the chamber were covered with coarse sand. Roughening the surface of the chamber materially increased the friction of the disc. This may be explained in precisely the same way as increase of friction due to increasing the size of the chamber.

PROBLEMS

1. The resistance of a ship is 1 lb. per square foot of immersed surface at 10 knots. Find the H.P. required to drive a ship having 8000 square feet of immersed surface at 15 knots. One knot $=6086$ feet per hour. 829·9.
2. The disc-shaped covers of a centrifugal pump are 2 feet diameter outside and 1 foot diameter inside. Find the work expended in friction in rotating the pump at 360 revolutions per minute $f=0{\cdot}0025$, and $n=2$. 326 ft.-lbs. per second.

CHAPTER VIII

FLOW IN PIPES

83. **Non-sinuous motion of water.**—When water from a reservoir which has been at rest long enough for eddies to die out issues from a sharp-edged orifice, the stream is perfectly clear and smooth on the surface even at high velocities. Any disturbance of the water in the reservoir shows itself in striation of the jet due to the presence of eddies disturbing the stream-line motion in the jet. The jet from a cylindrical mouthpiece is always troubled from the formation of eddies at the inner edge. In capillary tubes, which have been experimented on by Poisseuille and others, the motion is generally non-sinuous and free from eddies up to considerable velocities. But in ordinary water mains the motion is generally sinuous and turbulent.

Professor Osborne Reynolds investigated the conditions in which sinuous and non-sinuous motion occurred in pipes (*Trans. Roy. Soc.* 1884). A steady stream of water was set up through a glass tube with a flared mouth so that there was no inlet disturbance. Into the stream a small jet of coloured liquid was introduced.

So long as the velocity was low enough the coloured water showed as a straight undisturbed stream line flowing through the tube with the other water. If the velocity was raised there came a point at which the coloured liquid suddenly mingled with the rest of the water, and on viewing the water by an electric spark it was seen that the water contained a mass of more or less distinct coloured curls or eddies. With water at constant temperature and the tank as still as possible the critical velocity at which the stream lines broke up and eddies were formed varied almost exactly inversely as the diameter of the pipe and directly as the viscosity. Very small disturbing causes, such as a disturbance of the water in the tank or fine sediment in the water, caused the break-up to occur at lower velocities. Hence the critical velocity determined in this way is the higher limit of stable stream-line flow in pipes. The coefficient of viscosity for water decreases as the temperature rises, and is given by the equation

$$\eta = \frac{0{\cdot}017}{1 + 0{\cdot}034t + 0{\cdot}00023t^2} \quad . \quad . \quad (1),$$

where t is the temperature centigrade. The denominator of this fraction may be termed the relative fluidity, and will be denoted by f.

The higher critical velocity as determined by Osborne Reynolds by the colour-band method is given by the equation

$$v_c = 0{\cdot}2458 \frac{1}{fd} \text{ ft. per sec.} \quad . \quad . \quad (2),$$

where d is the diameter of the pipe in feet.

HIGHER CRITICAL VELOCITY

$d =$	$\frac{1}{2}$	1	$\cdot 1\frac{1}{2}$	2	inches
$d =$	·0417	·0833	·1250	·1667	feet
v_c at 0° C $=$	5·90	2·95	1·97	1·47	ft. per sec.

Later experiments by Professor Coker, Mr. Clement, and Mr. Barnes have shown that under certain favourable conditions stream-line flow may subsist to considerably higher velocities than those observed by Reynolds, and throw a little doubt on the law that the higher critical velocity varies inversely as the diameter.[1]

In another series of experiments Osborne Reynolds allowed water initially disturbed to flow through a long smooth pipe. It was found that if the velocity was below a certain limit the disturbances died out in a short length of the pipe, and the motion then became non-sinuous. Measuring the resistance to flow in a length of the pipe beyond the disturbed part, it was found that when the motion was non-sinuous the resistance varied very exactly as the velocity, but

[1] *Trans. Royal Society*, 1903. *Proceedings Royal Society*, vol. lxxiv.

that when the motion was turbulent it varied as the 1·72th power of the velocity, or nearly as the square of the velocity. If the velocity in the pipe is slowly increased, the point at which the eddies cease to die out and there is a deviation from the law that the resistance varies as the velocity can be observed, and this velocity may be termed the lower critical velocity. This also was found to vary inversely as the diameter of the pipe and directly as the viscosity. The lower limit of critical velocity found by Osborne Reynolds is given by the equation

$$v_c = 0{\cdot}0387\,\frac{1}{fd}\text{ ft. per sec.} \qquad (2a).$$

LOWER CRITICAL VELOCITY

$d=$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	inches
$d=$	0·0417	0·0833	0·1250	0·1667	feet
v_c at 0°C =	·928	·465	·310	·232	ft. per sec.

Later experiments by Professor Coker and Mr. Clement gave the relation

$$v_c = 0{\cdot}0199\,\frac{1}{fd}\text{ ft. per sec.} \qquad (2b),$$

or about half the values obtained by Osborne Reynolds. The reason of the difference has not been explained.

It will be seen that in somewhat wide limits for small pipes the motion may be sinuous or non-sinuous, but that above the lower limit very small causes of disturbance render the motion turbulent. Practically, for the larger pipes and the velocities with which an engineer has to deal, the motion is always turbulent.

Let d be the diameter of a horizontal pipe, and p the difference of pressure in a length l; the velocity of flow when the motion is in rectilinear stream lines is given by the relation

$$v = \frac{gpd^2}{32\eta l} \qquad (3),$$

where p is in grams per square centimetre and the units are C.G.S. units. A more convenient form is this. Let h be the difference of pressure in a horizontal pipe in a distance l measured in feet of liquid of density ρ.

$$v = 1711 \frac{f\rho h d^2}{l} \text{ centimetre units,}$$

$$= 52150 \frac{f\rho h d^2}{l} \text{ foot units.}$$

Taking for water $\rho = 0{\cdot}999$ and for mercury $\rho = 13{\cdot}6$, then for h in feet of water

$$v = 52100 \frac{fhd^2}{l}$$

and for h in inches of mercury

$$v = 709250 \frac{fhd^2}{l} \qquad \text{(4).}$$

84. **Practical theory of flow in pipes when the motion is turbulent.**—In all ordinary cases with which the engineer has to deal, the water has in addition to its forward motion of translation a distributed eddying motion. It is beyond hope to have a theory which will give rationally the velocity of flow and discharge of pipes in such conditions. It is not only that the eddying motion of the water is so complicated that in the strict sense there is no exact theory, but in addition one of the factors in any formula of flow must express the exact roughness of the surface of the pipe on which the production of eddies depends. There is no scientific measure of roughness, and very small apparent differences in the quality of the pipe surface cause considerable differences in the resistance.

Permissible velocities in pipes.—Theoretically any given discharge can be obtained either by varying the pipe diameter or the head producing velocity of flow, but practically the range of discharge for a given pipe is much limited. If the velocity in the pipe is small it must be of large size and expensive. If great, it is difficult to obtain sufficient pressure in the distant parts of a district supplied, in hours of large consumption, and the risk to the mains from sudden variations of flow, causing what is termed hydraulic shock, is great. A fair rough rule for pipes used in town's supply is the following. Let v be the velocity in a pipe of diameter d (foot units), then

$$v = 1{\cdot}45d + 2$$

$d = 6$	9	12	18	24	36	inches
$= 0{\cdot}5$	0·75	1·0	1·5	2	3	feet
$v = 2{\cdot}7$	3·1	3·4	4·2	4·9	6·3	feet per second.

Of course, cases occur where higher velocities can be permitted. In short supply pipes to turbines, velocities of 7 to 10 feet per second are not unusual. The reason for adopting somewhat lower velocities in small mains is that otherwise the rate of fall of pressure would be excessive.

85. **Steady flow in pipes of uniform diameter.**—If a long pipe connects two reservoirs at different levels, water will flow from the upper to the lower, and the conditions being constant the velocity and rate of discharge will be constant also. Steady flow being established, since the water starts from rest and comes back to rest, the work of gravity on the descending water is exactly balanced by the work of the resistances, of which much the largest is fluid friction. Let Q be the discharge in cubic feet per second, Ω the cross section and d the diameter of the pipe, v the mean forward velocity of the water.

$$Q = \Omega v = \frac{\pi}{4} d^2 v \text{ cubic feet per second} \quad . \quad . \quad (5).$$

As the same quantity of water passes every section in unit time the velocity must be the same, that is if we understand by v the mean velocity of translation along the pipe. In fact, the velocity is greater at the centre of the cross section and less towards the sides of the pipe, and on this general condition eddying motions are superposed. But the mean velocity along the pipe is constant, and for simplicity the complications must be disregarded.

The Chezy formula for flow in pipes.—A very simple theory furnishes an approximate formula which has been of very great service in hydraulics, and which with tabulated values of experimental coefficients is still employed more generally than any other in hydraulic calculations. Let Fig. 85 represent a short portion of a long pipe through which water is steadily flowing. The water enters and leaves at the same velocity, and consequently the work of external forces must be equal to the work in overcoming friction. Let dl be the length of the portion of pipe considered, z and $z + dz$ the elevations of the end sections above any horizontal datum XX, p and $p + dp$ the pressures at the ends, Ω the area of cross section, χ the circumference, and Q the discharge per second. Then, in passing through the length dl, GQ lbs. of water descend a distance $-dz$ feet, and the work of gravity is

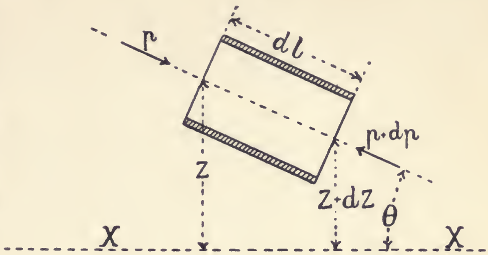


Fig. 85.

$$-\mathrm{GQ}dz,$$

a positive quantity if dz is negative, and *vice versa*. The resultant pressure on the two ends in the direction of motion is $-dp$, and the work of this pressure is

$$-\mathrm{Q}dp,$$

also positive if the pressure is decreasing along the pipe and dp is negative. The only remaining force doing work on the water is the frictional resistance. The area of the pipe surface is χdl, and using the expression obtained above [§ 79, eq. (2)] and putting v for the velocity of the water the frictional work is

$$-\zeta \mathrm{G}\chi dl\frac{v^3}{2g},$$

or, since $\mathrm{Q} = \Omega v$,

$$-\zeta \mathrm{G}\frac{\chi}{\Omega}\mathrm{Q}\frac{v^2}{2g}dl,$$

a quantity always negative because it is work done against a resistance. Adding these portions of work together and dividing by GQ,

$$dz + \frac{dp}{\mathrm{G}} + \zeta\frac{\chi}{\Omega}\frac{v^2}{2g}dl = 0.$$

Integrating,

$$z + \frac{p}{\mathrm{G}} + \zeta\frac{\chi}{\Omega}\frac{v^2}{2g}l = \text{constant} \qquad . \qquad . \quad (6).$$

Let A and B (Fig. 86) be two sections at distances l_1, l_2

from any given point, so that the length of pipe now considered is $L = l_2 - l_1$, and let p_1, z_1 be the pressure and elevation at A_1, p_2, z_2, the same quantities at B. Then, if v is the mean velocity along the pipe,

$$z_1 + \frac{p_1}{G} + \zeta\frac{\chi}{\Omega}\frac{v_2}{2g}l_1 = z_2 + \frac{p_2}{G} + \zeta\frac{\chi}{\Omega}\frac{v^2}{2g}l_2$$

$$\zeta\frac{v^2}{2g} = \frac{1}{L}\left\{\left(z_1 + \frac{p_1}{G}\right) - \left(z_2 + \frac{p_2}{G}\right)\right\}\frac{\Omega}{\chi} \quad . \quad . \quad (7).$$

If pressure columns are introduced at A and B, the water

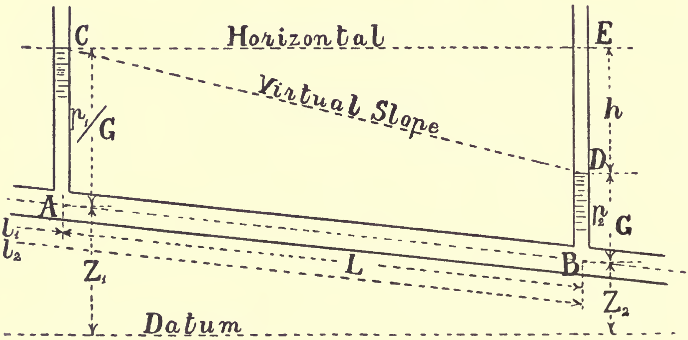


Fig. 86.

will rise to the levels C and D, such that $AC = \frac{p_1}{G}$ and $BD = \frac{p_2}{G}$. It is assumed that the atmospheric pressure is the same at C and D. In a very long pipe this might not be the case. Consequently

$$DE = h = \left(z_1 + \frac{p_1}{G}\right) - \left(z_2 + \frac{p_2}{G}\right) \quad . \quad . \quad (8).$$

The quantity h is the difference of free surface-level at the two points of the pipe considered, and is termed the **virtual fall** of the pipe. The quantity h/L is termed the **virtual slope** of the pipe, and this will be denoted by i. The line CD passing through the pressure-column tops is called the **hydraulic gradient**. The quantity Ω/χ which appears in this and some other equations is termed the **hydraulic mean radius** of the pipe, and will be denoted by m.

The general equation for flow in pipes can now be written more simply

$$\zeta\frac{v^2}{2g}=\frac{\Omega}{\chi}\frac{h}{\mathrm{L}}=mi.$$

For pipes of circular section and diameter d, $m=\Omega/\chi=d/4$. For such pipes the general equation of flow is

$$\zeta\frac{v^2}{2g}=\frac{d}{4}\cdot\frac{h}{\mathrm{L}}=\frac{di}{4} \quad . \quad . \quad . \quad (9).$$

This equation, with a constant value for ζ, is the well-known Chezy formula. It is still extremely useful if values of ζ, varying with certain conditions, are used instead of a constant value.

The following forms of this equation are useful in practical applications. The virtual fall or head lost in the length L is

$$h=\zeta\frac{4\mathrm{L}}{d}\frac{v^2}{2g}=0{\cdot}0622\frac{\zeta\mathrm{L}v^2}{d}\text{ feet} \quad . \quad . \quad (9a).$$

The velocity of flow is

$$v=\sqrt{\left\{2g\frac{d}{4\zeta}\frac{h}{\mathrm{L}}\right\}}=4{\cdot}012\sqrt{\left(\frac{d}{\zeta}\frac{h}{\mathrm{L}}\right)}\text{ feet per sec.} \quad (9b).$$

The discharge is

$$\mathrm{Q}=\frac{\pi}{4}d^2v=3{\cdot}15\sqrt{\left(\frac{d^5h}{\zeta\mathrm{L}}\right)}\text{ cubic feet per sec.} \; . \quad (9c).$$

The diameter for a given discharge is

$$d=0{\cdot}632\sqrt[5]{\left(\frac{\zeta\mathrm{Q}^2\mathrm{L}}{h}\right)}\text{ feet} \quad . \quad . \quad (9d).$$

The head lost for a given discharge is

$$h=0{\cdot}1008\frac{\zeta\mathrm{Q}^2\mathrm{L}}{d^5}\text{ feet} \quad . \quad . \quad . \quad (9e).$$

A form of the equation which is in common use is this:

$$v=c\sqrt{\left(\frac{1}{4}di\right)} \quad . \quad . \quad . \quad (9f),$$

and by some writers this form only is termed the Chezy equation. The constant c is given by the relation

$$c=\sqrt{\frac{2g}{\zeta}}.$$

86. **Case of a pipe connecting two reservoirs. Inlet resistance taken into account.**—Let Fig. 87 represent a pipe connecting two reservoirs at different levels. If the reservoir levels are constant the velocity in the pipe and the rate of discharge are constant. The total head causing flow is the difference of level H, and this is expended in three ways. (1) To give the initial energy to the water corresponding to the velocity v there must be expended a head $v^2/2g$. At the outlet of the pipe this kinetic energy is wasted in shock and eddies, so that this is part of the head lost. (2) There is some resistance due to the form of the inlet, which may be written $\zeta_0 v^2/2g$, where ζ_0 = about 0·5 for a cylindrical inlet, and

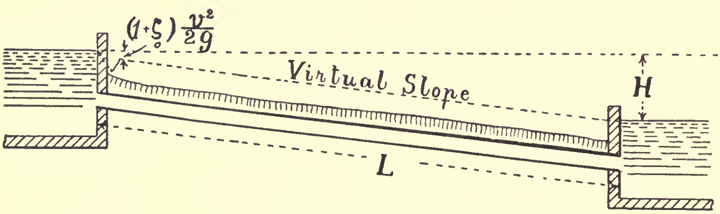


Fig. 87.

about 0·05 if the inlet is bell-mouthed. (3) The friction in the length L has been found to be $\zeta\frac{4L}{d} \cdot \frac{2g}{v^2}$ feet of head, eq. (9a). Adding these together,

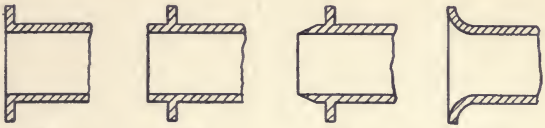
$$H = \left\{\left(1 + \zeta_0\right) + \zeta\frac{4L}{d}\right\}\frac{v^2}{2g},$$

$$v = 8{\cdot}025\sqrt{\left\{\frac{Hd}{(1 + \zeta_0)d + 4\zeta L}\right\}} \quad . \quad . \quad (10),$$

an equation which should always be used for short pipes.

As a matter of fact, water mains are not straight but curved, to follow the variations of level of the ground. Hence their length is really greater than the horizontal projection, and the hydraulic gradient is not strictly a straight line. But in most practical cases the differences of level of the pipe are so small compared with its length that there is no error of practical importance in taking L to be the length of the horizontal projection of the pipe, or in assuming the hydraulic gradient to be straight.

87. **Inlet Resistance.**—The inlet to a pipe may be flush with the reservoir wall, as at A, Fig. 88; re-entrant and with square edges, B; re-entrant with sharp edges, C; or bell-



A B C D

Fig. 88.

mouthed, D. Values of the coefficient of resistance ζ_0 and $1+\zeta_0$ are given in the following table:—

Form of Inlet.	ζ_0.	$1+\zeta_0$.
A	0·5	1·5
B	0·56	1·56
C	1·30	2·30
D	0·02 to 0·05	1·02 to 1·05

The inlet resistance is equivalent to the frictional resistance of a length of pipe given by the equation

$$l_0 = \frac{(1+\zeta_0)d}{4\zeta} \quad . \quad . \quad . \quad (11).$$

VALUES OF l_0/d.

ζ	$1+\zeta_0=$			
	1·05.	1·5.	1·56.	2·3.
·005	53	75	78	115
·0075	35	50	52	77
·010	26	38	39	58

If this length is added to the actual length of the pipe the inlet resistance will be allowed for.

In practical calculations about water mains the length L is usually very large, and $(1+\zeta_0)d$ is small enough compared with $4\zeta L$ to be neglected. Thus let $L=1000$ ft.; $d=1·5$ ft.; $\zeta=·0075$; $\zeta_0=0·5$; $H=10$ ft. The velocity, by eq. (10), is 5·47 ft. per sec., but if the inlet resistance is neglected the velocity is 5·67. The error is here not immaterial, but if the length of the pipe is 10,000 feet and $H = 100$, the velocity, from eq. (10), is 5·65, and if the inlet resistance is neglected the velocity is 5·67, where the difference is in practical cases negligible.

88. **Pressure in the pipe when the water is flowing.**—The vertical from the pipe to the hydraulic gradient is the pressure in the pipe at that point in feet of water, in excess of atmospheric pressure. If h is the height to the gradient, $h + 34$ feet is the pressure, including atmospheric pressure. Hence there could not be negative pressure in the pipe unless it rose more than 34 feet above the hydraulic gradient. With negative pressure the flow would of course be interrupted. But all ordinary water contains air, which would be disengaged, and would interfere with flow if the pressure fell much below atmospheric pressure. Hence, as a practical rule, pipes are not laid so as to rise above the hydraulic gradient. Further, at all anticlinal bends air valves are placed so that the air in the pipe when it is being filled may escape, and also any air carried into the pipe afterwards, which would accumulate at the top of vertical bends and interrupt the flow. Unless the pipe is below the hydraulic gradient these valves cannot act.

89. **Darcy's experimental investigation of the resistance to flow in pipes.**[1]—An extremely important series of measurements of the flow in pipes with different heads was carried out by M. H. Darcy, then Engineer of the Paris Water Supply, under the auspices of the French Government. The general bearing of the results may be stated thus:—

(1) The frictional resistance varies considerably with the nature and degree of roughness of the surface of the pipes. This is in accordance with Froude's results already described, § 80.

(2) The greater part of the experiments were made on new and clean pipes, some of them asphalted. A few were made on old and somewhat incrusted pipes. It was found that the resistance of old and incrusted pipes was double that of new and clean pipes.

(3) The simple Chezy formula

[1] *Recherches expérimentales relatives au mouvement de l'eau dans les tuyaux.* Paris, 1857.

$$\zeta \frac{v^2}{2g} = \frac{di}{4} \quad . \quad . \quad . \quad . \quad (12)$$

very well expressed the results of the tests, if special varying values were given to the coefficient ζ.

(4) The coefficient ζ varies with the velocity of flow, with the diameter of the pipe, and with the roughness of the surface of the pipe. As, for practical reasons, there is not a wide variation of velocity in water mains, the dependence of ζ on the velocity may be disregarded in most practical calculations. On the other hand, the diameters of pipes range from 2 inches to 60 inches, and the variation of ζ with the diameter is very important.

Generally, at ordinary velocities and with cast iron or steel pipes laid in the ordinary way,

$$\zeta = \alpha\left(1 + \frac{\beta}{d}\right) \quad . \quad . \quad . \quad (13),$$

where the constants have the following values :—

	α	β
Drawn wrought-iron or clean cast-iron pipes	·00497	·084
Pipes altered by light incrustations	·0100	·084

Or, in an easily remembered form,

Clean and smooth pipes,

$$\zeta = \cdot 005\left(1 + \frac{1}{12d}\right);$$

Incrusted pipes,

$$\zeta = \cdot 01\left(1 + \frac{1}{12d}\right).$$

[TABLE.

Values of ζ deduced from Darcy's Formula

Diameter of Pipe.		Values of ζ.	
In Inches.	In Feet.	New Pipes.	Incrusted Pipes.
4	0·333	·00622	·01252
5	0·417	·00597	·01202
6	0·500	·00580	·01168
7	0·583	·00568	·01144
8	0·667	·00560	·01126
9	0·75	·00553	·01112
12	1·00	·00539	·01084
15	1·25	·00530	·01067
18	1·50	·00525	·01056
21	1·75	·00521	·01048
24	2·00	·00518	·01042
27	2·25	·00515	·01037
30	2·50	·00514	·01034
33	2·75	·00512	·01031
36	3·00	·00511	·01028
42	3·50	·00509	·01024
48	4·00	·00507	·01021
60	5·00	·00505	·01017

It may be noted that, except for pipes less than about 12 inches in diameter, the variation of ζ is not very great, and in many approximate calculations a constant value of ζ may be assumed without very large error.

(5) There is a variation of ζ with the velocity, and for cases where the velocities were large Darcy proposed the expression

$$\zeta = \alpha + \frac{\alpha_1}{d} + \frac{\beta + \beta_1/d^2}{v} \quad . \quad . \quad . \quad (14),$$

and gave the following values for the constants (foot units), for clean pipes:—

$$\alpha = 0·004346$$
$$\alpha_1 = 0·0003992$$
$$\beta = 0·0010182$$
$$\beta_1 = 0·000005205$$

No doubt Darcy underrated the importance of the influence of velocity on the frictional resistance, and his formula taking account of it is extremely inconvenient. It can be taken into account in a simpler way, which will be given later.

90. **Maurice Levy's formula for pipes.**—Darcy's experiments were made on pipes not more than 20 inches in diameter, and within that limit his formula has considerable authority. M. Maurice Levy came to the conclusion, from experience, that in the case of large pipes Darcy's formula makes the resistance greater than it really is, and leads to the use of pipes unnecessarily large. M. Levy, on partially theoretical grounds, obtained the following formulæ for metric measures:—

For new and clean cast-iron pipes,

$$v = 36{\cdot}4\sqrt{\{ri(1+\sqrt{r})\}}$$

For pipes incrusted,

$$v = 20{\cdot}5\sqrt{\{ri(1+3\sqrt{r})\}} \qquad (15),$$

where r is the radius of the pipe. Reducing to English foot units and substituting the diameter for the radius, these equations become:

For new and clean pipes,

$$\frac{v^2}{2g} = 135(1+0{\cdot}4\sqrt{d})\frac{di}{4}$$

For incrusted pipes,

$$\frac{v^2}{2g} = 42{\cdot}8(1+1{\cdot}17\sqrt{d})\frac{di}{4} \qquad (15a).$$

Where in the Chezy formula, eq. (12), the value of ζ is:

For new and clean pipes,

$$\zeta = \frac{0{\cdot}007408}{1+0{\cdot}4\sqrt{d}}$$

For incrusted pipes,

$$\zeta = \frac{0{\cdot}02335}{1+1{\cdot}17\sqrt{d}} \qquad (16).$$

The following table gives values of ζ calculated by Levy's rule for comparison with those of Darcy:—

Values of ζ from Levy's Formula

Diameter of Pipe.		Values of ζ.	
Inches.	Feet.	New Pipes.	Incrusted Pipes.
4	0 333	·00602	·0139
5	0·417	·00589	·0133
6	0·500	·00577	·0128
7	0·583	·00567	·0123
8	0 667	·00558	·0119
9	0·75	·00550	·0116
12	1·00	·00529	·0108
15	1·25	·00512	·0101
18	1·50	·00497	·0096
21	1·75	·00485	·0092
24	2·00	·00474	·0089
27	2·25	00463	·0085
30	2·50	·00454	·0082
33	2·75	·00445	·0079
36	3·00	·00438	·0077
42	3 50	·00424	·0073
48	4·00	·00412	·0070
60	5·00	·00391	·0065

91. **Later determinations of the values of** ζ.—Imperfect as is the theory on which the Chezy formula is based, it is so convenient that it will continue to be used in engineering calculations. The difficulty in using it is the uncertainty in choosing the proper value of ζ in different cases. In a wide range of cases in which the flow in pipes has been measured by competent observers, ζ has varied from 0·003 to 0·016. Even in cases in many respects identical there is considerable variation. Mr. Gale and Mr. Stearns both measured the flow in asphalted cast-iron pipes 4 feet in diameter, and found $\zeta = 0{\cdot}0031$ and 0·0051 respectively.

In 1886 the author examined all the more carefully made experiments on flow in pipes, including those of Darcy. By classifying pipes according to the quality and condition of their surfaces the range of variation of ζ can be greatly limited. Using a relation between v, d, and i which allows for the influence both of diameter and velocity, which will be explained in Chapter X., it was possible to tabulate values of ζ for most of the conditions which arise in practice.

The following tables give the values of the coefficient ζ in the Chezy formula

$$\zeta\frac{v^2}{2g} = \frac{di}{4}$$

for different kinds of pipe, of different diameters, and with different velocities of flow, deduced in this way.

VALUES OF ζ

When d in Feet is	For Velocities in Feet per Second.			
	1-2.	2-3.	3-4.	4-5.
	Clean Wrought-Iron Pipes.			
0·5-0·75	·0057	·0050	·0046	·0043
0·75-1·0	·0054	·0047	·0043	·0040
1·0-1·5	·0050	·0043	·0040	·0037
1·5-2·0	·0046	·0040	·0037	·0035
2·0-3·0	·0043	·0038	·0034	·0032
3·0-4·0	·0040	·0035	·0032	·0030
	Asphalted Cast-Iron Pipes.			
0·5-0·75	·0064	·0059	·0056	·0054
0·75-1·0	·0062	·0057	·0054	·0052
1·0-1·5	·0059	·0054	·0052	·0050
1·5-2·0	·0056	·0052	·0049	·0047
2·0-3·0	·0054	·0050	·0047	·0045
3·0-4·0	·0052	·0048	·0045	·0043
	New Cast-Iron Uncoated Pipes.			
0·5-0·75	0058	·0056	·0055	·0054
0·75-1·0	·0054	·0053	·0052	·0051
1·0-1·5	·0051	·0050	·0049	·0048
1·5-2·0	·0048	·0047	·0046	·0046
2·0-3·0	·0046	·0044	·0043	·0043
3·0-4·0	·0043	·0042	·0041	·0041
	Incrusted Cast-Iron Pipes. For all Velocities.			
0·5-0·75	·0119			
0·75-1·0	·0113			
1·0-1·5	·0107			
1·5-2·0	·0101			
2·0-3·0	·0095			
3·0-4·0	·0090			

If an expression of the form adopted by Darcy is used, then the results given above agree fairly closely with the following values,

$$\zeta = \alpha\left(1 + \frac{\beta}{d}\right).$$

VALUES OF ζ

Kind of Pipe.	Values of α for Velocities in Feet per Second.				Values of β.
	1-2.	2-3.	3-4.	4-5.	
Drawn wrought iron .	·00375	·00322	·00297	·00275	0·37
Asphalted cast iron .	·00492	·00455	·00432	·00415	0·20
Clean cast iron . .	·00405	·00395	·00387	·00382	0·28
Incrusted cast iron .	At all velocities $\alpha = 0{\cdot}00855$				0·26

These values show that, as was generally believed from practical experience, the influence both of diameter and velocity is greater than Darcy supposed.

92. **Herschel's gaugings of flow in riveted steel pipes.**—Mr. Clemens Herschel, between 1892 and 1896, made numerous gaugings of flow in riveted mains of exceptionally large diameter. The volume of flow was measured by the Venturi meter, a method which may be regarded as very satisfactory. The pipes were asphalted, and some were made with taper lengths and others with cylinder lengths alternately large and small. No very clear difference was found between the two as regards resistance. Mr. Herschel has plotted his results, taking velocities for ordinates, and values of c in the equation $v = c\sqrt{mi}$, where m is the hydraulic mean radius, as abscissae. From the curves drawn through the plotted points he has deduced values of c for various velocities. From these, for comparison with the values of ζ in the tables above, the following values for steel riveted pipes have been deduced :—

Values of ζ for new Steel Riveted Pipes

Diameter in Inches.	For Velocities in Feet per Second.					
	1.	2.	3.	4.	5.	6.
48	·0063	·0055	·0051	·0050	·0051	·0052
48	·0068	·0064	·0062	·0059	·0058	·0058
42	·0070	·0056	·0051	·0051	·0052	·0053
42	·0063	·0059	·0057	·0056	·0055	·0055
36	·0087	·0071	·0060	·0053	·0047	·0042

Broadly, these results confirm the general law given above. The value of ζ diminishes as the velocity increases, and increases as the diameter diminishes. But there are anomalies. There are several cases where ζ is greater at 6 feet per second than at 4 feet per second. What is more anomalous still is that the 48-inch pipe at 6 feet per second has a greater coefficient than the 36-inch pipe. These anomalies must be due to errors of observation. Further, as a whole, the coefficients are somewhat larger than they might be expected to be. There is a series by Darcy and one by Hamilton Smith on riveted pipes which give smaller coefficients if the difference of diameter is allowed for. However, of course, in comparing these with the results on cast-iron pipes, the roughness due to the rivet-heads and joints must be considered, and the resistance can only be determined by direct experiment on riveted pipes.

After some of these pipes had been in use four years some further gaugings were made, and the discharge was found to have diminished considerably. The following are coefficients for the 48-inch main, one set corresponding to the upper part of the main near the supply reservoir, the other to the lower part.

Values of ζ for Old Riveted Steel Pipes

Diameter in Inches.	At Velocities in Feet per Second.					
	1.	2.	3.	4.	5.	6.
48[1]	·0106	·0080	·0075	·0073	·0072	·0072
48[2]	·0068	·0060	·0058	·0060	·0060	·0060

[1] Supply reservoir to Pompton Notch.
[2] Pompton Notch to service reservoir.

It is clear, the author thinks, that during the four years slimy deposits had accumulated in the main and increased the resistance to flow. As would be expected, these were almost entirely in the first length of main from the supply reservoir to Pompton Notch. In the remainder of the main the coefficients are not sensibly different from those obtained in the previous gaugings.

Messrs. Marx, Wing, and Hoskins made gaugings in 1897 and in 1899, by a calibrated Venturi meter, of a remarkable supply pipe 6 feet in diameter, part of which was riveted steel and part of wood staves, at the Pioneer Electric Power Company, Ogden, Utah (*Trans. Am. Soc. of Civil Engineers*, xl. 471, and xliv. 34). The results on the steel part of the pipe plotted in curves furnish the following values for ζ.

COEFFICIENT ζ FOR SIX-FOOT RIVETED STEEL PIPE

$v=$	1·0	1·5	2·0	2·5	3·0	4·0	5·0	5·5
1897 gauging—								
$\zeta=$	·0053	·0052	·0053	·0055	·0055	·0052	—	—
1899 gauging—								
$\zeta=$	·0097	·0076	·0067	·0063	·0061	·0060	·0058	·0058

The increase of resistance with time is very marked at the low velocities if the measurements at these can be trusted. It seems probable, however, that in the earlier gauging the resistance at low velocities was under-estimated, or the resistance at high velocities over-estimated.

93. **Timber stave pipes.**—In the western part of the United States remarkable pipe lines have been constructed of wood staves hooped with steel bands. The wood used is redwood or sequoia, which when wet appears to have great durability. The staves break joint, and at their ends a thin piece of steel is jammed in a saw-cut. By slightly humouring the staves bends of large radius are easily obtained. The staves are usually $1\frac{3}{4}$ inch thick, accurately shaped by machinery. The steel hoops are spaced at different distances according to the pressure, and are drawn tight by a screwed end and nut. These pipes can be put together in difficult country where transport of metal pipes would be very costly.

The results of the gaugings, by Messrs. Marx, Wing, and Hoskins, of the part of the pipe at Ogden constructed of wood staves and 6 feet in diameter (§ 92), gave the following values for the coefficient ζ:—

VALUES OF ζ FOR SIX-FOOT WOOD STAVE PIPE

$v=$	1·0	1·5	2·0	3·0	4·0	5·0	5·5
1897 gauging— $\zeta=$	·0064	·0053	·0048	·0043	·0041	—	—
1899 gauging— $\zeta=$	·0048	·0046	·0045	·0044	·0043	·0043	·0043

In these, as in the results on riveted pipe, there seems some doubt as to the accuracy of the observations at the lowest velocity. The variation of ζ with velocity would be expected to be greater than in the 1899 gaugings for frictional surface as smooth as that of a wood pipe.

94. **Fire hose pipes.**—Very careful experiments on the discharge through fire hose have been made by Freeman (*Trans. Am. Soc. of Civil Engineers*, xxi. 303). For $2\frac{1}{2}$-inch hose pipes of different makes the following were the values of the coefficient ζ obtained:—

VALUES OF ζ

	Velocity in Feet per Second.				
	4.	6.	10.	15.	20.
Unlined canvas . . .	·0095	·0095	·0093	·0088	·0085
Rough rubber-lined cotton .	·0078	·0078	·0078	·0075	·0073
Smooth " " .	·0060	·0058	·0055	·0048	·0045

Let a hose pipe of diameter d connect with a nozzle of diameter δ; let l be the length of hose pipe, H the head of water at inlet, v the velocity in hose pipe, V the velocity of jet from nozzle, and Q the discharge.

The head expended at the nozzle is $h = \mathrm{V}^2/(2gc_v^2)$, where c_v is the coefficient of velocity for the nozzle, which may vary from 0·95 to 0·98. But $\mathrm{V} = vd^2/\delta^2$. Hence

$$h = \frac{v^2 d^4}{2g\delta^4 c_v^2}.$$

The head producing flow in the hose pipe is $H-h$, and therefore

$$v=\sqrt{\frac{2g(H-h)d}{4\zeta l}}$$

$$=\sqrt{\left[\frac{2gdH}{4\zeta l}\right]\Big/\left\{1+\frac{d^5}{4\zeta l\delta^4 c_v^2}\right\}} \quad . \quad . \quad (17).$$

95. **Practical calculations of flow in pipes.**—In the following calculations it is assumed that there are no special obstructions due to valves, bends, etc., and that the pipe is so long that only the frictional resistance requires to be taken into account. In long mains the resistance of ordinary bends is negligible. The fundamental equations are:—

$$\zeta=\alpha\left(1+\frac{\beta}{d}\right) \quad . \quad . \quad . \quad . \quad (1),$$

$$\zeta\frac{v^2}{2g}=\frac{d}{4}\cdot\frac{h}{L}=\frac{di}{4} \quad . \quad . \quad . \quad . \quad (2),$$

$$Q=\frac{\pi}{4}d^2v \quad . \quad . \quad . \quad . \quad . \quad (3).$$

From these equations the following are easily derived, and for convenience are repeated here from § 85:—

$$v=4{\cdot}012\sqrt{\left(\frac{d}{\zeta}\frac{h}{L}\right)} \quad . \quad . \quad . \quad (2a),$$

$$d=0{\cdot}0622\frac{\zeta vL}{h} \quad . \quad . \quad . \quad . \quad (2b),$$

$$v=1{\cdot}273\frac{Q}{d^2} \quad . \quad . \quad . \quad . \quad (3a),$$

$$d=1{\cdot}128\sqrt{\frac{Q}{v}} \quad . \quad . \quad . \quad . \quad (3b),$$

$$d=0{\cdot}632\sqrt[5]{\frac{\zeta Q^2L}{h}} \quad . \quad . \quad . \quad (4),$$

$$Q=3{\cdot}149\sqrt{\frac{hd^5}{\zeta L}} \quad . \quad . \quad . \quad . \quad (4a),$$

$$h=0{\cdot}1008\frac{\zeta Q^2L}{d^5} \quad . \quad . \quad . \quad . \quad (4b).$$

Rough preliminary calculations can be made by the following approximate formulæ obtained by taking a fixed value of ζ. They are least accurate for small pipes:—

For new and clean pipes,

$$\left.\begin{aligned} v &= 56\sqrt{(di)}, \\ Q &= 44\sqrt{(d^5 i)}, \\ d &= 0{\cdot}22\sqrt[5]{\frac{Q^2}{i}}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (5).$$

For old and incrusted pipes,

$$\left.\begin{aligned} v &= 40\sqrt{(di)}, \\ Q &= 31{\cdot}4\sqrt{(d^5 i)}, \\ d &= 0{\cdot}252\sqrt[5]{\frac{Q^2}{i}}, \end{aligned}\right\} \quad . \quad . \quad . \quad (6).$$

When the dimensions of a pipe are given and the velocity and discharge are required there is no great difficulty. If Darcy's value of ζ is used it can be found from eq. (1), and the calculations are straightforward. If a value of ζ depending both on the diameter and velocity is to be used, an approximate value of v can be obtained from eq. (5) or (6), and then the value of ζ can be selected from the tables and v and Q re-calculated. There is rather more difficulty when the discharge is given and the diameter is required. Sometimes from past experience an engineer can assign probable values for d and v, or they can be found approximately by eq. (5) or (6). Then ζ can be found from Darcy's formula or from the tables, and a new value of the diameter calculated by eq. (4). The engineer has to consider whether he will allow for an increase of resistance as the pipe becomes old and incrusted. The rate at which a pipe becomes rougher from corrosion depends on the quality of the water. In some cases the interior of the pipe remains clean for a long time. In some other cases the corrosion is rapid. A common rule of thumb to provide for corrosion is to calculate the diameter of pipe required when clean, to add one inch, and choose the nearest larger commercial size.

96. **Tables of flow in pipes.**—Tables are published giving the velocity and discharge of pipes of different diameters with different heads. Generally these are calculated on a fixed

value of ζ, and the results are therefore only approximate. They are of assistance, however, in settling pipe proportions. The following may be mentioned:—

(1) *Hydraulic and other Tables*, by Thomas Hennell, Spon, 1884. This is based on the Chezy formula.

(2) *Tables for Calculating the Discharge of Water in Pipes*, by A. E. Silk, Spon, 1899. Based on a modified Darcy formula.

(3) *Diagrams of Pipe Discharge*, by E. B. and G. M. Taylor, Batsford. These are based on Kutter's formula with a roughness coefficient 0·013.

(4) *Tables for the Solution of Ganguillet and Kutter's Formula*, by Col. E. C. S. Moore, R.E., Batsford.

(5) Mr. R. O. W. Roberts has designed a very convenient small circular slide-rule for facilitating calculations on flow in pipes. The graduations are based on Kutter's formula. The slide-rule is made by Mr. G. Kent of Holborn.

97. **Secondary losses of head in pipes.**—In very long mains the so-called skin friction or resistance of the pipe surface, which is determined by the equations in § 95, is so large compared with any other losses that the latter are disregarded in ordinary practical calculations or covered by assuming a rather larger value of ζ. It is, however, sometimes necessary to consider these smaller losses, especially in the case of comparatively short mains. The inlet loss has already been considered (§ 87), and can be taken into the reckoning without difficulty. The other losses due to changes of diameter of the pipe, changes of direction, valves, etc., are generally of the nature of losses by shock. All losses are properly ultimately due to fluid friction, but it is rather convenient to speak of these losses as shock losses, as distinguished from the skin friction losses previously discussed.

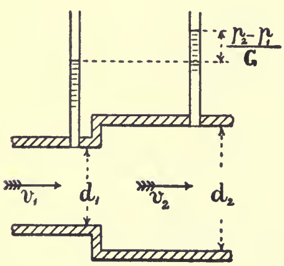


Fig. 89.

Abrupt enlargement of section.—Let d_1, ω_1, v_1 be the diameter, section, and velocity in the narrower, and d_2, ω_2, v_2 the same quantities in the wider part of the pipe, Fig. 89. The head lost in shock (§ 36) is

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$$h_e = \frac{(v_1 - v_2)^2}{2g}.$$

But

$$v_1/v_2 = \omega_2/\omega_1 = d_2^2/d_1^2,$$

$$h_e = \frac{v_2^2}{2g}\left(\frac{\omega_2}{\omega_1} - 1\right)^2 = \frac{v_2^2}{2g}\left(\frac{d_2^2}{d_1^2} - 1\right)^2$$

$$= \zeta_e \frac{v_2^2}{2g} \quad . \quad . \quad . \quad . \quad (18),$$

where ζ_e is a coefficient depending only on the ratio of the sections or diameters at the enlargement.

$\frac{\omega_2}{\omega_1} =$	1·2	1·5	1·7	2·0	3·0
$\frac{d_2}{d_1} =$	1·1	1·22	1·30	1·41	1·73
$\zeta_e =$	0·04	0·25	0·49	1·00	4·00

If p_1, p_2 are the pressures in the two parts of the pipe,

$$\frac{p_2 - p_1}{G} = \frac{v_1^2}{2g} - \frac{v_2^2}{2g}(1 + \zeta_e) = 2\frac{v_2^2}{2g}\left\{\left(\frac{d_2}{d_1}\right)^2 - 1\right\}.$$

Abrupt contraction of section.—When a stream passes from a larger to a smaller section abruptly a contraction is formed at *aa* (Fig. 90), and the stream then enlarges to fill the pipe, eddies being formed as at an abrupt enlargement. Let ω be the section, and v the velocity, where regular steady motion is re-established. At the contraction *aa* the section of the stream is $c_c\omega$ and the velocity is v/c_c, where c_c is a coefficient of contraction. Then the head lost in turbulent motion is

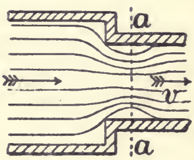

Fig. 90.

$$h_c = \frac{\left(\frac{v}{c_c} - v\right)^2}{2g} = \left(\frac{1}{c_c} - 1\right)^2 \frac{v^2}{2g}$$

$$= \zeta_c \frac{v^2}{2g} \quad . \quad . \quad . \quad (19),$$

where ζ_c is a coefficient. If $c_c = 0·63$, as in a free jet, $\zeta_c = 0·345$.

The value of the coefficient is not well ascertained. Weisbach obtained as the result of experiments the empirical relation

$$\zeta_c = \frac{0 \cdot 077}{c_c} + \left(\frac{1}{c_c} - 1\right)^2 \quad . \quad . \quad . \quad (20).$$

For a quite sharp edge at the change of section $c_c = 0 \cdot 62$ to $0 \cdot 64$. For a rounded edge $c_c = 0 \cdot 7$ to $0 \cdot 8$.

Gradual enlargement.—The resistance in this case can only be ascertained by experiment. Fliegner found the head lost to be (Fig. 91)

$$h_g = \frac{(v_1 - v_2)^2}{2g} \sin \theta \quad . \quad . \quad . \quad (21).$$

Elbows.—The loss of head at elbows appears to be due

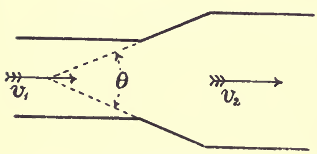

Fig. 91.

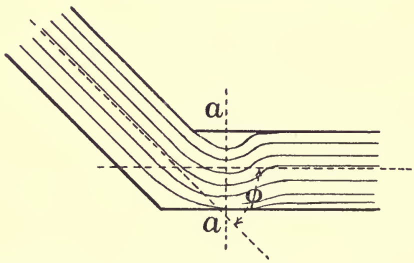

Fig. 92.

to the formation of a contraction and abrupt increase of section (Fig. 92). Weisbach, from experiments on a very small pipe, obtained the expression

$$h_e = \zeta_e \frac{v^2}{2g} \quad . \quad . \quad . \quad (22).$$

$$\zeta_e = 0 \cdot 95 \sin^2 \phi/2 + 2 \cdot 05 \sin^4 \phi/2.$$

$\phi =$	20°	40°	60°	80°	90°	100°	120°
$\zeta_e =$	0·03	0·14	0·37	0·75	1·0	1·27	1·87

This is a loss additional to the pipe friction in the parts constituting the elbow.

98. **Resistance at bends.**—Till lately the resistance at bends has been supposed to be a shock loss due to contraction and abrupt enlargement of the stream at the bend. On this hypothesis, and using the results of some experiments on small

bends, Weisbach found the following empirical expression for the head lost at a bend (*Die experimental Hydraulik*, p. 156).

Let θ be the angle subtended by the bend at the centre of curvature in degrees, v the velocity, r the radius, and d the diameter of the pipe, and R the radius of curvature measured to the centre line of the bend. Then $\rho = r/R$ is the curvature. The head lost is

$$\left.\begin{aligned} h_b &= \zeta_b \frac{v^2}{2g} \frac{\theta}{90} \text{ feet} \\ \zeta_b &= 0{\cdot}131 + 1{\cdot}847\rho^{3{\cdot}5} \end{aligned}\right\} \quad . \quad . \quad . \quad (23).$$

ρ = 0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
R/d = 5	2·5	1·67	1·25	1·0	·83	·71	·62	·55	·5
ζ_b = ·13	·14	·16	·21	·29	·44	·66	·98	1·41	1·98

No great confidence has been placed in these results, as they are based on very limited and small experiments. Recently Mr. Alexander (*Proc. Inst. Civil Engineers*, clix. 341) has made some very careful experiments on small varnished wood bends ($d = 1\frac{1}{4}$ inch) with considerable variation of radius of curvature and velocity of flow. In spite of the small scale of these experiments they throw some light on the nature of the resistance at bends. The most important point is this, that the total resistance at a bend is made up of the skin resistance of a straight length of pipe of the same length as the bend, and an additional resistance due to the curvature which is not a shock resistance but merely an augmentation of the skin friction. Hence the total resistance at a bend can be expressed by the relation

$$h_b = \zeta_b \frac{v^2}{2g} \frac{4l}{d} \text{ feet} \quad . \quad . \quad . \quad (24),$$

where l is the length of the bend measured along its centre line, and d the diameter of the pipe. It appeared in the experiments that the resistance per foot length of bend did not regularly decrease with the curvature, but was a minimum when $\rho = 5$, or when the radius of curvature was $2\frac{1}{2}$ times the diameter of the pipe. Mr. Alexander has given some empirical expressions for loss of head at bends, but they are inconvenient, and it is sufficient for practical purposes to proceed in a simpler way. Assuming the result that the bend resistance is merely an augmented skin friction resistance, so that it can be expressed by the equation (24), the value of ζ_b may be found from such experiments as are available. The most valuable experiments are some by Messrs. Williams, Hubbell and Fenkel, on large bends of asphalted cast iron, and of these the best are on bends in pipes of 30 inches in diameter (*Proc. Am. Soc. of Civil Engineers*, xxvii. 314). The coefficients are deduced for right-angled bends in which $l = \pi R/2$. For any other bends the resistance will be proportional to the angle subtended at the centre of curvature, so that if l_1 is the length of such a bend the coefficient will be greater or less than those given below in the ratio l_1/l.

VALUES OF BEND COEFFICIENT ζ_b FOR RIGHT-ANGLED BENDS

Weisbach, small pipes.

ρ =	·025	·05	·1	·17	·25	·33	0·5
R/d =	20	10	5	3	2	1·5	1·0
ζ_b =	·001	·002	·004	·008	·012	·018	·046

Williams, Hubbell and Fenkel, 30-inch main.

ρ =	·021	·031	·050	·083	·125	·21
R/d =	24	16	10	6	4	2·4
ζ_b =	·009	·0092	·0118	·015	·0155	·018

For small values of the curvature the coefficient of resistance of Weisbach's small pipes is much less than that of the 30-inch pipe, but for large values of the curvature it is not very different. It may be suspected that for the small pipes with small curvature the motion of the water was possibly approximately non-sinuous.

The results may be put in another way. Let l_1 be the length of a straight pipe the resistance of which is equal to that of a right-angled bend of length l along the centre line. Then if ζ is the proper coefficient corresponding to the diameter, velocity, and roughness in the ordinary formula for pipe friction,

$$\zeta\frac{v^2}{2g}\frac{4l_1}{d} = \zeta_b\frac{v^2}{2g}\frac{4l}{d},$$

$$l_1 = \zeta_b l/\zeta \quad . \quad . \quad . \quad . \quad (25).$$

Taking $\zeta = 0·005$ for a 30-inch asphalted pipe, the lengths equivalent to a right-angled bend are as follows :—

$\rho=$	·021	·031	·050	·083	·125	·21	
$R/d=$	24	16	10	6	4	2·4	
$l_1/l=$	1·8	1·84	2·36	3·0	3·1	3·6	
$l=$	94	63	39	24	16	9	feet.
$l_1=$	169	115	92	72	49	32	„
$l_1-l=$	75	52	53	48	33	33	„

It cannot be said that knowledge of the resistance at bends is satisfactory; more experiments on an adequate scale are necessary. But it is fairly certain that the additional resistance at a bend over that of a straight pipe of equal length is not, for practical calculations, a very large or serious quantity when the resistance of long mains is in question.

99. **Valves, cocks, and sluices.**—These contract the section of the pipe, and there is a further contraction of the stream passing the sluice, and an abrupt enlargement of the section of the stream causing loss of head by shock. The loss of head may be expressed by the relation

$$h_s = \zeta_s \frac{v^2}{2g} \quad . \quad . \quad . \quad (26),$$

where v is the velocity in the pipe beyond the sluice where regular motion is re-established.

Pipe of rectangular section.—Section at the sluice, ω_1; in pipe beyond the sluice, ω.

$\frac{\omega_1}{\omega}=$	1·0	0·9	0·8	0·7	0·6	0·5
$\zeta_s=$	0·0	0·09	0·4	0·95	2·08	4·02

$\frac{\omega_1}{\omega}=$	0·4	0·3	0·2	0·1
$\zeta_s=$	8·12	17·8	44·5	193·0

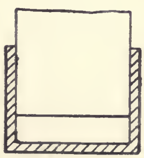

Fig. 93.

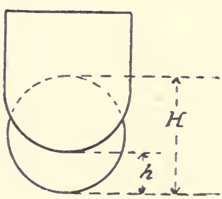

Fig. 94.

Sluice in cylindrical pipe.—Let $\rho=h/H$ be the ratio of height of opening to the diameter of the pipe.

$\rho = 1{\cdot}0$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{\omega_1}{\omega} = 1{\cdot}0$	0·95	0·86	0·74	0·61	·47	·32	·16
$\zeta_s = 0{\cdot}0$	0·07	0·26	0·81	2·1	5·5	17·0	97·8

Some experiments by Kuichling on a 24-inch sluice in a cast-iron main gave the following results:—

$\rho =$	0·66	0·60	0·50	0·37	0·25	0·18
$\zeta_s =$	0·8	1·6	3·3	8·6	22·7	41·2

It will be seen how very largely the pressure beyond the sluice is reduced when the valve is much closed. The form of the valve casing has a good deal of influence on the resistance. With various forms of casing the resistance when the valve or sluice is full open may amount to from two to sixteen times $v^2/2g$.

100. **Flow in a main in which there are secondary resistances.**—The equation for the velocity of flow becomes too cumbrous if expressions for the secondary resistances are inserted. It is best to proceed by approximation. Let H be the total head in the length l. Then taking account only of the inlet resistance and skin friction an approximate value of the velocity v can be found from the equation

$$v = 8{\cdot}025 \sqrt{\left\{\frac{\mathrm{H}d}{(1+\zeta_0)d + 4\zeta l}\right\}} \quad . \quad . \quad (27).$$

Knowing this approximate velocity, the losses of head due to the secondary resistances can be calculated. Let $h =$ the sum of these losses. Then a more approximate value of v can be found from the equation

$$v = 8{\cdot}025 \sqrt{\left\{\frac{(\mathrm{H}-h)d}{(1+\zeta_0)d + 4\zeta l}\right\}} \quad . \quad . \quad (28).$$

Problems.

1. Find an expression for the relative discharge of a square and a circular pipe of the same section and slope. 1·062 to 1.
2. A pipe is 6 inches in diameter, and is laid for a quarter of a mile at a slope of 1 in 50; for another quarter of a mile at a slope of 1 in 100; and for a third quarter of a mile is level. The surface of the supply reservoir is 20 feet above the inlet, and that of the lower reservoir 9 feet above the outlet. Using Darcy's coefficient for clean pipes, find the discharge. Also

draw the hydraulic gradient and mark the pressure in the pipe at each quarter mile. 0·824 cubic foot per second.

3. A pipe, 2000 feet long, discharges Q cubic feet per second. Find how much the discharge would be increased, if for the last 1000 feet a second equal pipe was laid alongside the first, and the water allowed to flow equally through both. Show by a sketch how the hydraulic gradient would be altered.

Ratio of discharge $\sqrt{8}$ to $\sqrt{5}$.

4. A reservoir, the level of which is 50 feet above datum, discharges into a reservoir 30 feet above datum through a 12-inch pipe 5000 feet in length. Using Darcy's coefficient for clean pipes, find the discharge. 2·710 cubic feet per second.

5. The levels of the pipe in the last question are: at the upper reservoir, 40 feet; at 1000 feet, 25 feet; at 2000 feet, 12 feet; at 3000 feet, 12 feet; at 4000 feet, 10 feet; at the lower reservoir, 15 feet above datum. Sketch the line of hydraulic gradient, and write down the pressure in the pipe at each of these points.

6. A pipe, 9 inches in diameter, connects two reservoirs one mile apart, the water-surfaces being 100 feet and 47 feet above datum. Using Darcy's coefficient for incrusted pipes, find the velocity and discharge.

3·3 feet per second; 1·46 cubic feet per second.

7. A pipe is 1500 feet long and 6 inches in diameter. It is to discharge 50 cubic feet of water per minute. Find the loss of head in friction and virtual slope. Use Darcy's coefficient for clean pipes. 19·6 feet; 0·013.

8. What is the head lost per mile in a pipe 2 feet diameter discharging 6,000,000 gallons in 24 hours: (*a*) when new; (*b*) when incrusted. 10·7 feet; 21·5 feet.

9. A pipe is to supply 30,000 gallons per hour. The available head is 80 feet per mile. Find the velocity and diameter (*a*) from the approximate formula; (*b*) from the tabular value of ζ corresponding to the approximate velocity and diameter.

·571 foot and 5·2 feet per second.
·577 foot and 5·10 feet per second.

10. A water main has a virtual slope of 1 in 850, and discharges 35 cubic feet per second. Find the velocity and diameter (*a*) approximately by assuming $\zeta = 0{\cdot}0064$; (*b*) more accurately by selecting a coefficient from the tables for asphalted cast iron.

3·68 feet and 3·3 feet per second.
3·426 feet and 3·80 feet per second.

11. It is required to discharge water from a reservoir through a horizontal pipe 6 inches diameter and 50 feet long. Head over inlet 20 feet. Find the discharge, taking into account the inlet resistance. $\zeta = 0{\cdot}0075$. The inlet is flush with the reservoir. 3·32 cubic feet per second.

12. Find the diameter of a new cast-iron pipe, having a fall of 10 feet per mile, capable of delivering water with a velocity of 3 feet per second. 0·1359 foot.

13. A pipe 12 inches in diameter and 1 mile in length delivers water from one reservoir to another with a difference of level of 60 feet. The surface area of the lower reservoir is 10,000 square feet, and the water-level is observed to be rising at the rate of 11 inches per hour. Find the coefficient of friction ζ of the pipe. 0·0174.

14. A hydraulic main is 6 inches diameter; the velocity is 3 feet per second; and the pressure is 700 lbs. per square inch. What is the gross horse-power transmitted. 108 H.P.

15. Supposing the hydraulic main in the last question to be clean cast iron, find the loss of pressure in pounds per square inch per mile, and the percentage of the energy transmitted wasted in friction. 14·1 lbs. per square inch; 2·01 per cent.

16. A horizontal pipe is in three sections, each of 1000 feet in length, and of diameters 10 inches, 12 inches, and 15 inches respectively. The discharge is 5 cubic feet per second. Taking the coefficient $\zeta = 0·01$, find the loss of head in friction in each length, and the change of pressure at each abrupt change of diameter. Friction = 6·90, 3·96, and 2·03 feet.
Pressure change, 0·555 and 0·288 foot.

17. Taking the pressure at the inlet of the pipe in the last question to be 25 feet, draw the hydraulic gradient with a vertical scale fifty times the horizontal.

18. A pipe connects two reservoirs 1000 feet apart with a difference of surface-level of 20 feet. If a sluice at the outlet into the lower reservoir is partially closed so that the discharge is reduced to one-half, what will be the change in the hydraulic gradient?

CHAPTER IX

DISTRIBUTION OF WATER BY PIPES

101. **Town supply.**—The amount of water supplied per head in different towns varies very greatly. For ordinary domestic purposes 12 gallons per head per day is a small supply, and 18 to 20 gallons an ample supply. For trade and manufacturing purposes 6 to 12 gallons per head per day is generally sufficient. But in a great many towns the supply is larger, and in some cases this is due to waste of water by leakage from the mains. In some towns in the United States the supply reaches 100 to 150 gallons per head per day. The demand for water varies, being small at night and greatest at certain hours in the day. In designing water-mains it is usual to assume the maximum rate of flow to be double the mean rate. In laying new mains a further allowance is made for the prospective increase of population.

The greatest statical pressure in the mains is in ordinary cases 200 to 300 feet of water, and with commercial fittings a higher pressure is undesirable. The lowest pressure which should be provided at points of delivery to consumers is 80 to 100 feet. If a district varies considerably in level it is divided into zones, in each of which the difference of level does not exceed 80 to 100 feet. An independent supply from a service reservoir at least 200 feet above the lowest point in the zone is provided. Such service reservoirs are fed by a trunk main from the source of supply, and usually contain three or more days' supply in case of accident to the main. The distributing mains are calculated so that when losses of head are allowed for there is adequate pressure at all points of delivery during the hours of maximum demand.

The zones are divided into subdistricts, each with an independent supply, and these districts vary in area with the population. One reason for this is the desirability of controlling waste of water by waste-water meters, through which the supply to limited districts can be passed and measured. The smallest mains used are 3 inches in diameter, but generally mains are not less than 4 or 6 inches in diameter.

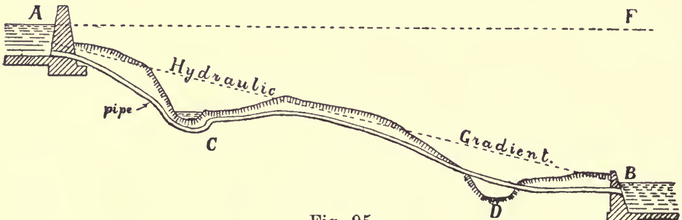

Fig. 95.

102. **Water-supply main.**—Fig. 95 shows the general arrangement of a water-supply main connecting a storage reservoir A and a service reservoir B. The line of hydraulic gradient is drawn from the lowest level in A to the highest in B, the condition in which the rate of flow will be least. The pipe line follows generally the contour of the ground, but

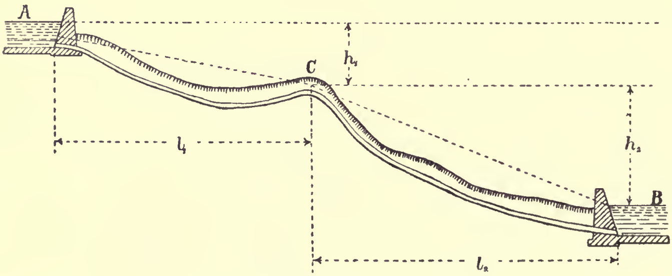

Fig. 96.

is everywhere below the hydraulic gradient. At C is a stream, where the pipe line may be carried under the stream by a specially constructed steel pipe, termed a siphon, or over it on a bridge aqueduct. At D is a valley, which may be crossed by a siphon, or the pipe may be carried on piers. If high ground occurs on the route it may be necessary to place the pipes in a tunnel to avoid rising above the gradient. Another way of dealing with rising ground between the inlet and outlet is to adopt a main with pipes of two diameters. Thus, in Fig. 96, the rising ground at C prevents the adoption of a uniform hydraulic gradient from A to B. Then a larger pipe must be used from A to C, giving the required discharge on the flatter gradient; and a smaller pipe may be used from C to B, giving the same discharge on the steeper gradient.

As to the pressure in the main when the outlet is full open, the pressure in feet of water at any point is the vertical intercept between the pipe line and the hydraulic gradient. But if a valve at the outlet is closed and the water is stationary in the main, the pressure is the vertical intercept between the pipe line and the horizontal AF. Hence generally the strength of the pipe has to be calculated for this latter pressure, if under any circumstances the outlet can be

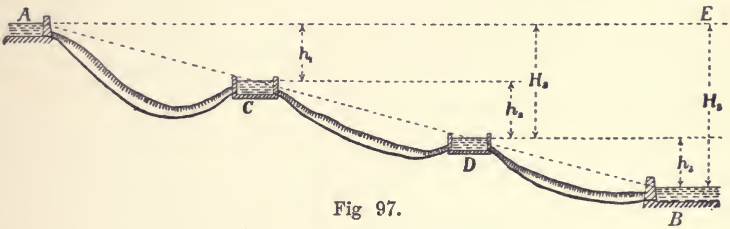

Fig 97.

closed. Any regulation of the flow at the outlet increases the pressure in the main. In certain cases, to reduce the cost of the main, there is no valve at the outlet, and regulation of flow is effected entirely by a valve at the inlet. In that case the pressure at any point is never greater than the height to the hydraulic gradient.

103. **Break-pressure reservoirs.**—When a water-main is of great length, and when there is a large fall H_3 between the supply reservoir at A and the final service reservoir at B, it is often necessary to introduce intermediate balancing or break-pressure reservoirs, such as those shown in Fig. 97 at C and D. The general hydraulic gradient is the line AB from the surface-level in A to the surface-level in B, and for this gradient and the required discharge Q the diameter of the pipes must be calculated. Now, if there are no intermediate reservoirs, the pressures in the main at any point

when the pipe is delivering the full discharge will be the height from the pipe to the hydraulic gradient AB. So far as this condition of things is concerned, intermediate reservoirs are not necessary. But in the working of the main there must be times when the delivery of the main is decreased, and the pressure in the main will then be greater; there must be times when the delivery is stopped, and then the pressure at any point in the main will be the hydrostatic pressure due to the depth of the point below the surface-level in the supply reservoir, or, what is the same thing, the height from the pipe to the horizontal AE. Thus at D the hydrostatic pressure would be H_2, and at B, H_3. Hence, as respects strength, the pipe must be calculated for the hydrostatic pressure in the main when the delivery is stopped, and this may involve inconvenient thicknesses of pipe and unnecessary cost. By taking the pipe line so as to reach at C and D the level of the hydraulic gradient, and introducing balancing reservoirs there, into which one length of main discharges and from which another receives its supply, the pressure conditions are ameliorated. With full delivery the hydraulic gradient is AB as before. But when the delivery is stopped, the hydrostatic pressure in each length can never exceed that due to the nearest higher reservoir. Thus at C the pressure cannot exceed h_1; at D it cannot exceed h_2; and at B it cannot exceed h_3.

104. **Loss of head in a main consisting of sections of different diameters.**—Two cases may be considered. (*a*) The discharge may be taken to be constant throughout the main. (*b*) The velocity may be taken to be constant throughout, portions of the flow being abstracted by branch mains at each change of diameter.

(*a*) *Constant discharge.*—Let Q be the discharge, d_1, d_2, d_3 the diameters, and l_1, l_2, l_3 the lengths of the sections of the main. Then the velocities are

$$v_1 = Q \Big/ \frac{\pi}{4} d_1^2; \quad v_2 = Q \Big/ \frac{\pi}{4} d_2^2; \quad v_3 = Q \Big/ \frac{\pi}{4} d_3^2.$$

The losses of head due to friction are

$$h_1 = \zeta \frac{v_1^2}{2g} \cdot \frac{4l_1}{d_1}; \quad h_2 = \zeta \frac{v_2^2}{2g} \cdot \frac{4l_2}{d_2}; \quad h_3 = \zeta \frac{v_3^2}{2g} \cdot \frac{4l_3}{d_3},$$

where, in approximate calculations, a common mean value can be selected for ζ. The total loss of head due to friction is [§ 95, eq. (4*b*)]

$$H = h_1 + h_2 + h_3$$
$$= 0\cdot1008\zeta Q^2\left\{\frac{l_1}{d_1^{\,5}} + \frac{l_2}{d_2^{\,5}} + \frac{l_3}{d_3^{\,5}}\right\} \quad . \quad . \quad (1).$$

(*b*) *Constant velocity in the main, the discharge diminishing from section to section.*—Let Q_1, Q_2, Q_3 be the discharges in the successive sections, d_1, d_2, d_3 the diameters, and l_1, l_2, l_3 the lengths of the sections, and let v be the common velocity throughout the main. Then the diameters must be fixed by the relations

$$d_1 = \sqrt{\frac{4Q_1}{\pi v}}; \quad d_2 = \sqrt{\frac{4Q_2}{\pi v}}; \quad d_3 = \sqrt{\frac{4Q_3}{\pi v}}.$$

Introducing these quantities into the ordinary equation for loss of head in friction, the total loss is [§ 95, eq. (2*b*)]

$$H = h_1 + h_2 + h_3$$
$$= 0\cdot0622\zeta v^2\left\{\frac{l_1}{d_1} + \frac{l_2}{d_2} + \frac{l_3}{d_3}\right\}$$
$$= 0\cdot0551\zeta v^{\frac{5}{2}}\left\{\frac{l_1}{\sqrt{Q_1}} + \frac{l_2}{\sqrt{Q_2}} + \frac{l_3}{\sqrt{Q_3}}\right\} \quad . \quad . \quad (2).$$

The secondary losses of head are neglected in these equations, and usually have to be allowed for by an addition to H, determined by experience in similar cases.

105. **Equivalent main of uniform diameter.**—It sometimes facilitates calculations of loss of head to substitute for a main

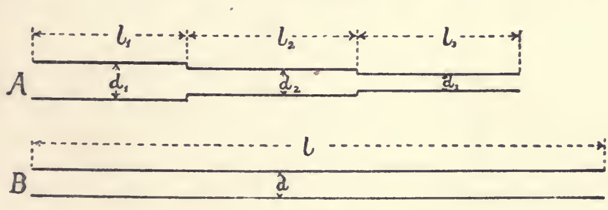

Fig. 98.

in sections of different diameter an equivalent uniform main having the same discharge with the same loss of head. Let A (Fig. 98) be a main of varying diameter having lengths

l_1, l_2, l_3... of diameters d_1, d_2, d_3.... It is required to find the length l of an equivalent main B of diameter d. Let v_1, v_2, v_3... be the velocities in A, and v the velocity in B, with any discharge Q. Since the loss of head in B is to be the same as that in A, from § 95, eq. (2b),

$$\zeta\frac{v^2 l}{d} = \zeta\frac{v_1{}^2 l_1}{d_1} + \zeta\frac{v_2{}^2 l_2}{d_2} + \zeta\frac{v_3{}^2 l_3}{d_3}\ldots\ldots,$$

where a common mean value can be selected for ζ. But

$$Q = \frac{\pi}{4}d^2 v = \frac{\pi}{4}d_1{}^2 v_1 = \frac{\pi}{4}d_2{}^2 v_2\ldots\ldots,$$

$$v_1 = v\frac{d^2}{d_1{}^2}\ ;\ v_2 = v\frac{d^2}{d_2{}^2}\ ;\ v_3 = v\frac{d^2}{d_3{}^2}\ldots\ldots$$

Consequently

$$\frac{l}{d} = \frac{d^4 l_1}{d_1{}^5} + \frac{d^4 l_2}{d_2{}^5} + \frac{d^4 l_3}{d_3{}^5} + \ldots\ldots,$$

$$l = d^5\left\{\frac{l_1}{d_1{}^5} + \frac{l_2}{d_2{}^5} + \frac{l_3}{d_3{}^5} + \ldots\ldots\right\} \quad . \quad . \quad (3),$$

which is the length of the equivalent main.

106. **Main in which the discharge decreases uniformly along the length.**—In street mains water is delivered into branch mains or service pipes, so that the discharge pro-

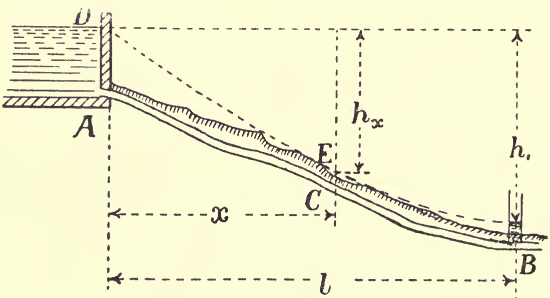

Fig. 99.

gressively decreases. It is useful to consider a limiting case in which the volume of flow in a main of uniform diameter decreases proportionately to the length. Let AB (Fig. 99) be a pipe supplied from a reservoir, and DE its hydraulic gradient. Let Q cubic feet per second be supplied at A, and discharged into service pipes uniformly along the route, so that the pipe loses $q = Q/l$ cubic feet per second per foot run. Let C be any point, $AC = x$, $AB = l$, $h_x =$ the virtual fall from A to C, $h_1 =$ the virtual fall from A to B, and $d =$ the diameter of the pipe. The volume of flow at C is $Q_x = Q - qx$. In a short length dx at C the head lost is [§ 95, eq. (4b)]

$$dh = 0\cdot1008\frac{\zeta}{d^5}(Q - qx)^2 dx.$$

Hence between A and C the head lost is

$$h_x = 0\cdot1008\frac{\zeta}{d^5}\int_0^x (Q - qx)^2 dx.$$

But

$$\int(Q - qx)^2 dx = Q^2\int dx - 2Qq\int x dx + q^2\int x^2 dx,$$

$$\int_0^x (Q - qx)^2 dx = Q^2 x - Qqx^2 + \tfrac{1}{3}q^2 x^3.$$

$$h_x = 0\cdot1008\frac{\zeta}{d^5}\left\{Q^2 x - Qqx^2 + \tfrac{1}{3}q^2 x^3\right\} \quad . \quad . \quad (4).$$

But

$$Q = Q_x + qx.$$

$$h_x = 0\cdot1008\frac{\zeta}{d^5}\left\{Q_x^2 x + Q_x qx^2 + \tfrac{1}{3}q^2 x^3\right\}.$$

At B,

$$Q_x = 0,\ h_x = h_1,\ x = l;\ qx = Q.$$

$$h_1 = 0\cdot1008\frac{\zeta}{d^5}\frac{q^2 l^3}{3} = 0\cdot1008\frac{\zeta}{d^5}\frac{Q^2 l}{3} \quad . \quad . \quad (5).$$

In other words, the total loss of head is precisely one-third of what it would be if the flow was uniform along the pipe instead of uniformly decreasing. The line of hydraulic gradient in this case is a cubic parabola; that is, assuming as usual that lengths measured along the pipe do not sensibly differ from their horizontal projections.

Determination of diameter of pipe which delivers water uniformly en route.—Suppose a pipe of uniform diameter d receives Q cubic feet of water per second at the inlet and delivers Q_x cubic feet at x feet from the inlet, having distributed qx cubic feet uniformly in that distance. From the equation above, the loss of head in the distance x is

$$h_x = 0{\cdot}1008\frac{\zeta}{d^5}\left\{Q_x^2 x + Q_x q x^2 + \tfrac{1}{3}q^2 x^3\right\}.$$

Now let

$$Q'^2 = Q_x^2 + Q_x q x + \tfrac{1}{3}q^2 x^2.$$

Then in a simple form, similar to that for pipes in which the discharge is uniform along the length,

$$h_x = 0{\cdot}1008\frac{\zeta x}{d^5}Q'^2 \quad . \quad . \quad . \quad (6).$$

But Q' is greater than $Q_x + \frac{1}{2}qx$, and is less than $Q_x + \frac{1}{\sqrt{3}}qx$; that is, Q' lies between $Q_x + 0{\cdot}5qx$ and $Q_x + 0{\cdot}57qx$. As an approximation, let $Q' = Q_x + 0{\cdot}55qx$;

$$h_x = 0{\cdot}1008\frac{\zeta x}{d^5}(Q_x + 0{\cdot}55qx)^2 \quad . \quad . \quad (7).$$

So that if the pipe is calculated for the discharge Q_x at the outlet end plus 0·55 of the delivery qx *en route*, like a pipe of uniform discharge, it will satisfy the conditions.

107. **Pipe connecting a supply and a service reservoir, and delivering water en route.**—Let l be the length of the pipe, and h the difference of surface-level in the reservoirs. During the night, when the consumption of water *en route* is zero, the pipe delivers from A to B (Fig. 100) a quantity of water given by the relation [§ 95, eq. (4a)]

$$Q = 3{\cdot}149\sqrt{\frac{hd^5}{\zeta l}}.$$

The hydraulic gradient is then the straight line AB.

When the consumption *en route* reaches the value ql, Q' is received at A, and $Q_x = Q' - ql$ is delivered at B. From the equation above,

$$Q_x = 3{\cdot}149\sqrt{\frac{hd^5}{\zeta l}} - 0{\cdot}55ql \quad . \quad . \quad . \quad (8).$$

If ql increases Q_x diminishes till, when

$$ql = \frac{3{\cdot}149}{0{\cdot}55}\sqrt{\frac{hd^5}{\zeta l}} = 5{\cdot}73\sqrt{\frac{hd^5}{\zeta l}},$$

the discharge into the reservoir B ceases. The line of hydraulic gradient is then a cubic parabola with a horizontal tangent at B. When the service *en route* increases still more, the pipe is

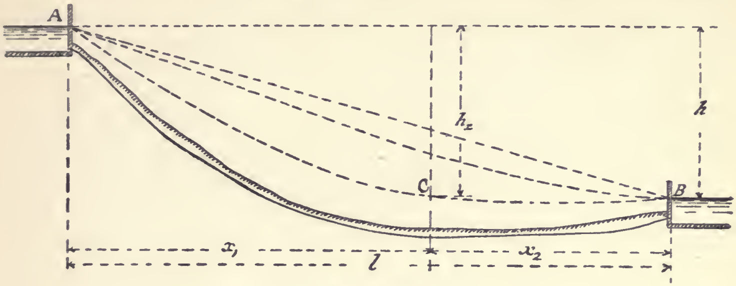

Fig. 100.

fed at one end by the reservoir A and at the other end by the reservoir B. The line of hydraulic gradient remains parabolic, but its horizontal tangent is at some point C.

Let x_1 be the horizontal distance from A to C, and x_2 from C to B, and let h_x be the virtual fall from A to C. From § 106, eq. (5),

$$h_x = 0{\cdot}1008 \frac{\zeta}{d^5} \frac{q^2 x_1^3}{3};$$

and considering the section CB,

$$h_x - h = 0{\cdot}1008 \frac{\zeta}{d^5} \frac{q^2 x_2^3}{3},$$

also $l = x_1 + x_2$. These three relations determine any three of the quantities h, h_x, d, q, x_1, x_2. It may be noticed that

$$\frac{x_1}{x_2} = \frac{\sqrt[3]{h_x}}{\sqrt[3]{(h_x - h)}},$$

$$x_1 = l \frac{\sqrt[3]{h_x}}{\sqrt[3]{h_x} + \sqrt[3]{(h_x - h)}} \quad . \quad . \quad . \quad (9),$$

$$x_2 = l - x.$$

108. **Branched pipe connecting reservoirs at different levels.**—Let A, B, C (Fig. 101) be three reservoirs connected by pipes as shown. Let l_1, d_1, Q_1, v_1 be the length, diameter, discharge, and velocity in the pipe AX; l_2, d_2, Q_2, v_2 the same quantities for BX, and l_3, d_3, Q_3, v_3 for XC. Suppose the

dimensions and positions of the pipes known and the discharges required. If a pressure column is introduced at the junction X the water will rise to a height XR, and *a*R, *b*R, *c*R will be the hydraulic gradients of the pipes. If the surface-level at R is above *b*, the reservoir A supplies B and C. If the surface-level at R is below *b*, the reservoirs A and B supply C. Con-

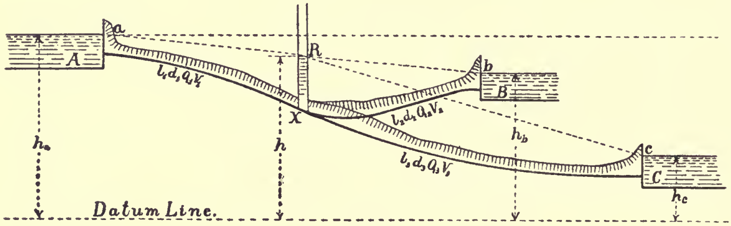

Fig. 101.

sequently there are three cases—(*a*) R above *b*, $Q_1 = Q_2 + Q_3$; (*b*) R level with *b*, $Q_1 = Q_3$ and $Q_2 = 0$; (*c*) R below *b*, $Q_1 + Q_2 = Q_3$. To determine which case has to be dealt with, suppose XB closed by a sluice. Then there is a simple main of two diameters. Let h_a, h_b, h_c be the heights of the surface-level in A, B, and C above datum, and h' the height of R, on the assumption that XB is closed. Then by § 95, eq. (4*b*),

$$h_a - h' = 0{\cdot}1008\frac{\zeta Q_1{}^2 l_1}{d_1{}^5},$$

$$h' - h_c = 0{\cdot}1008\frac{\zeta Q_3{}^2 l_3}{d_3{}^5}.$$

But in the condition assumed $Q_1 = Q_3$.

$$\therefore \frac{h_a - h'}{h' - h_c} = \frac{l_1 d_3{}^5}{l_3 d_1{}^5} \quad . \quad . \quad . \quad (10),$$

from which h' is easily calculated. If then h' is greater than h_b, opening the sluice in XB will allow water to flow into reservoir B, and the case is (*a*). But if $h' = h_b$, the case is (*b*); and if h' is less than h_b, opening the sluice will admit water from B to C, and the case is (*c*). Having distinguished the case, the problem can be solved by approximation, choosing a new value of *h* between h' and h_b, and recalculating Q_1, Q_2, and Q_3. The problem is solved when, with the assumed value of h, the relations of the discharges are those stated above. The approximation seems cumbrous, but is really easy.

109. **Compound main.**—It is sometimes necessary to supplement part of a main by one or more mains laid near it, or between two points there may be several mains through which water can flow. Such a system may be termed a compound main. Suppose the points A and B are connected by mains m, n, and p. Let Q_1, Q_2, Q_3 be the discharges of the mains, d_1, d_2, d_3 their diameters, l_1, l_2, l_3 their lengths, and h the virtual fall or difference of level of the hydraulic gradient between A and B. The total discharge of the mains, from § 95, eq. (4a), is

$$Q_1 + Q_2 + Q_3 = 3{\cdot}149\sqrt{\frac{h}{\zeta}}\left\{\sqrt{\frac{d_1^5}{l_1}} + \sqrt{\frac{d_2^5}{l_2}} + \sqrt{\frac{d_3^5}{l_3}}\right\}.$$

It is sometimes convenient to calculate the diameter of a single equivalent main having the same discharge as m, n, and p with the same virtual fall. Let d be its diameter and l its length. Then

$$3{\cdot}149\sqrt{\frac{hd^5}{\zeta l}} = 3{\cdot}149\sqrt{\frac{h}{\zeta}}\left\{\sqrt{\frac{d_1^5}{l_1}} + \sqrt{\frac{d_2^5}{l_2}} + \sqrt{\frac{d_3^5}{l_3}}\right\},$$

$$d = l^{\frac{1}{5}}\left\{\sqrt{\frac{d_1^5}{l_1}} + \sqrt{\frac{d_2^5}{l_2}} + \sqrt{\frac{d_3^5}{l_3}}\right\}^{\frac{2}{5}} \quad . \quad . \quad (11).$$

If $l = l_1 = l_2 = l_3$,

$$d = \{\sqrt{d_1^5} + \sqrt{d_2^5} + \sqrt{d_3^5}\}^{\frac{2}{5}} \quad . \quad . \quad (12).$$

110. **Hydraulic gradient of a pipe of variable diameter.**—At a change of diameter, where the velocity changes from

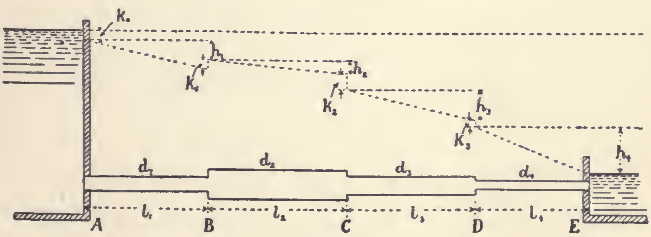

Fig. 102.

v_1 to v_2, there is a change of pressure head $(p_2 - p_1)/G = (v_1^2 - v_2^2)/2g$, and also usually a loss of head in shock, the amount of which for different cases is discussed in § 97. Suppose for simplicity the shock losses neglected and that a mean value is selected for the pipe friction coefficient ζ. Let Fig. 102 represent a main, the sections of which have diameters $d_1, d_2, d_3 \ldots$, and lengths $l_1, l_2, l_3 \ldots$; and let Q be the discharge. The losses of head due to pipe friction are [§ 95, eq. (4*b*)],

$$h_1 = 0{\cdot}1008 Q^2 \zeta \frac{l_1}{d_1{}^5},$$

$$h_2 = 0{\cdot}1008 Q^2 \zeta \frac{l_2}{d_2{}^5}.$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

At B there will be a gain of pressure head due to decrease of velocity from v_1 to v_2; at C and D there will be loss of pressure head due to increase of velocity from v_2 to v_3 and from v_3 to v_4. The velocities can be calculated from the diameters and the discharge, and the changes of head are

$$k_1 = \frac{v_1{}^2 - v_2{}^2}{2g} = 0{\cdot}0252 Q^2 \left(\frac{1}{d_1{}^4} - \frac{1}{d_2{}^4}\right),$$

$$k_2 = \frac{v_2{}^2 - v_3{}^2}{2g} = 0{\cdot}0252 Q^2 \left(\frac{1}{d_2{}^4} - \frac{1}{d_3{}^4}\right)$$

$$k_3 = \frac{v_3{}^2 - v_4{}^2}{2g} = 0{\cdot}0252 Q^2 \left(\frac{1}{d_3{}^4} - \frac{1}{d_4{}^4}\right).$$

The pressure head lost in giving velocity at the inlet is

$$k_0 = \frac{v_1{}^2}{2g} = 0{\cdot}0252 \frac{Q^2}{d_1{}^4}.$$

With these quantities the hydraulic gradient can be drawn, and the total head lost, or virtual fall of the pipe, is

$$H = h_1 + h_2 + \ldots + k_0 + k_1 + k_2 + \ldots.$$

111. **Cost of water-mains.**—The cost of water-mains per foot run laid in the ground, with the ordinarily necessary appendages, is nearly proportional to the diameter, and is about

$$C = 5d \text{ to } 7d \quad . \quad . \quad . \quad (13),$$

where C is in shillings and d in feet. It can be deduced from this that it is more economical to deliver water from one point to another by a single pipe than by several. Hence more than one pipe should be used only if the limit of size for a single pipe is reached. The cost of the pipes to convey a given quantity of water from one point to another is less as the total quantity to be conveyed is greater. The whole cost of a distributing system between given points increases about as the $\frac{3}{8}$th power of the volume of water distributed.

112. **Corrosion and incrustation.**—With some qualities of water, corrosion of iron mains occurs. The corrosion takes the form of nodular or limpet-shaped masses, which in time become confluent and reduce the discharging capacity of the main, partly by reducing its cross section and partly by increasing the roughness. With some other qualities of water incrustations of matter derived from the water, such as carbonate of lime, form on the pipe and have a similar effect.

In the case of some mains the discharge decreases rather rapidly for some time after they are laid, in consequence of corrosion and incrustation. The first case in which this was noticed was at Torquay, where the main had not been coated with asphalt, the idea being that the pure surface-water from the Dartmoor hills would have little action on the pipes. But in eight years the discharge had decreased 51 per cent. At that time Mr. Appold suggested scraping the internal surface of the main by scrapers driven through by the water pressure. This plan was adopted, and after scraping, the delivery increased 28 per cent. The plan has since been adopted in many cases, and the discharge has been increased by scraping by from 28 to 82 per cent in different cases. If scraping is adopted, however, it requires to be repeated, for the protective coating of rust and incrustation is removed, and thus, though slowly, the pipe is worn away. At Torquay the nodules of rust are $\frac{1}{8}$ to $\frac{3}{16}$ inch in height after twelve months (Ingham, *Proc. Inst. Mech. Engineers,* 1873, 1899). In the case of Torquay the water from a granitic district has a serious action on iron, possibly from containing an acid derived from peat. The matter removed by scraping contains about 38 per cent of oxide of iron, 43 per cent of sandy matter deposited from the water, and 18 per cent of organic matter. At Southampton, where the water is obtained from chalk wells, the incrustation consists of 98 per cent of carbonate of lime, and a little sulphate of lime and iron oxide. Well waters from the Old Red Sandstone do not cause much corrosion or incrustation. Soft water appears to have greater action than hard water.[1]

The best protection against corrosion is to coat the pipes with what is known as Dr. Angus Smith's composition. The pipes are heated in a cylindrical stove to about 600° F. and then dipped in a bath of pitch and oil of such a consistency as to produce a tough coating. Natural asphalt is preferred by some, with enough creosote oil to give a tough coat. In the case of steel pipes they should be cleaned in a sulphuric acid bath followed by one of lime water to neutralise the acid, and then dipped in the asphaltic composition kept at nearly boiling temperature.

Slime deposits in pipes carrying unfiltered water.— A serious decrease of discharge occurred in the first length of main of the Vyrnwy aqueduct, which has been traced to the growth of an organic deposit, and no doubt the same cause has operated in other cases. The organisms are brought into the pipe with the water and attach themselves to the pipe. Thread-like organisms with a gelatinous sheath develop, and iron oxide is deposited in the sheaths, which continue to thicken. Solid particles in the water are caught by the gelatinous threads. Acidity other than carbonic acid always characterises water which produces this slime, and an appreciable quantity of iron in solution. Mr. G. F. Deacon has succeeded in removing the slime deposit by a kind of scraper with whalebone brushes which does not injure the pipe (I. C. Brown, *Proc. Inst. Mech. Engineers*, 1903-4).

113. **Pipe aqueducts.**—These are usually of cast iron, sometimes of steel, and in Western America of wood. Cast-iron pipes do not exceed 48 inches diameter, are cast in lengths of 9 or 12 feet, and have spigot and socket joints, the joints being filled with lead. Sometimes the pipe lengths have plain ends, and the joint is made by a collar forming a double socket in which lead is run. The pipes are almost always placed in a trench and covered to protect them from

[1] Figures of various types of pipe-scrapers are given in *Proc. Inst. C.E.* cxvi. p. 307.

frost. As a protection against corrosion they are heated and dipped vertically in a bath of pitch and oil, which forms a smooth hard coating and reduces the frictional resistance to the flow of water. Steel pipes are much thinner, and therefore if corroded lose proportionately more strength and are more liable to deformation by earth pressure. But in some cases they cost less than cast iron, and can be made of larger size. They are made from plates riveted, welded, or made with a special locking-bar joint which is as strong as the solid plate. They usually have collar joints run with lead.

A pipe aqueduct is carried up hill and down dale necessarily below the line of hydraulic gradient, but otherwise at any inclination adapted to the contour of the country, and in some cases a greater velocity may be permitted in a pipe than would be suitable for an open conduit. Changes of direction are effected by special bend pipes, or short straight lengths (about 3 feet) are jointed by double-socketed bevel collars about 12 inches long, the sockets being inclined to each other.

The appurtenances of a pipe line are:—(1) *Air valves*, which are placed at every summit in the pipe line to permit the escape of air when the main is filled, and afterwards if any air is carried into the main. They are also placed on long stretches of nearly level main. They are generally ball-valves lighter than water, which close the air vent so long as they are immersed, but which drop and open the air vent if air accumulates. (2) *Scour valves* are placed at the bottom of all depressions for emptying the main or letting out sediment. (3) *Reflux valves* on ascending parts of the main are flap valves which open in the direction of flow, but which automatically close if a burst occurs and the water flows back. They diminish the damage done by escape of water at a burst. (4) *Momentum valves* are also intended to limit the escape of water at a burst. A disc is placed in the pipe on an arm, counterweighted so that it is not moved by the ordinary flow of water. If a burst occurs the accelerated flow presses back the disc, and the arm releases a catch, and another set of weights cause a disc throttle-valve in the pipe to close gradually and arrest the flow of the water. (5) Sluice stop-valves worked by hand or by a hydraulic cylinder for closing the main or regulating the flow. In the case of large mains the pressure on a large sluice-valve is very great, and the force required to move the sluice when starting from the closed position is very great. Thus on a 36-inch valve, under 250 feet of head the pressure would be nearly 50 tons, and the frictional resistance to moving the valve perhaps 7 tons. To facilitate opening, the valve is sometimes divided into three parts which can be opened separately. In other cases the valve is made about one-third the area of the pipe. The pipe is gradually contracted to the area of the valve and gradually enlarged again. Then, though there is some loss of head at the valve it is not very serious.

In a long main the flow is usually controlled by a sluice at the lower end. In that case, although the pressure in the main when water is flowing is only the pressure due to the depth below the hydraulic gradient, yet when the sluice is closed and the water at rest, the pressure is that due to the depth below the supply reservoir. The strength of the pipes must therefore be sufficient to sustain at all parts the statical pressure due to the depth below top water-level in the reservoir. In the case of the East Jersey main, Mr. Herschel has placed the controlling sluice at the inlet to the main, directions for regulating it being transmitted from the outlet end by telephone. In that case the pressure in the main cannot exceed at any point the pressure due to the depth below the hydraulic gradient. The adoption of this plan permits a material saving of thickness and cost in the pipes.

114. **Examples of pipe aqueducts.**—(1) **The Vyrnwy aqueduct.**—This aqueduct carries 40 million gallons per day from the reservoir at Vyrnwy to a service reservoir at Liverpool, a distance of 68 miles. The water first passes through the Hirnant tunnel of 7 feet diameter and 3900 yards long, and for nearly the whole of the rest of the distance through three lines of cast-iron pipes, each 42 to 39 inches in diameter. As the statical head on the main would be excessive if the pipe line was continuous, the total fall from Vyrnwy to Prescot being 550 feet, balancing reservoirs have been constructed at five points, breaking the pipe line into stretches each having its own hydraulic gradient and a maximum statical pressure due to the level in the reservoir

feeding it. The greatest pressure at any point is 317 feet of head. One of the 42-inch pipe lines, after being laid twelve years, with an hydraulic gradient of 4·5 feet per mile, discharged 15 million gallons per day. This gives a velocity of 2·892 feet per second, and a coefficient $\zeta = 0·00574$.

(2) **East Jersey steel aqueduct**, for the supply of Newark and other towns in New Jersey, U.S.A.—This consists of a steel riveted main, 48 inches in diameter and 21 miles long, with a maximum pressure of 340 feet of head. It delivers 50 million U.S. gallons per day, the velocity in the main being about 6 feet per second. The chief peculiarity of this main is that the cross-joints are riveted, so that the pipe is a continuous riveted structure without provision for expansion. It is calculated that the cross-joints are strong enough to resist the stresses due to 45° F. change of temperature without allowing for any assistance from the friction of the ground.

(3) **The Coolgardie pipe line.**—The longest pipe line is that through which water is pumped from a reservoir at Perth to Coolgardie and Kalgoorlie, Western Australia. Coolgardie is on a tableland which is one of the driest places in the world. A daily supply of 5,600,000 gallons is pumped through a 30-inch steel pipe of the locking-bar construction with collar joints run with lead. There are eight pumping stations. The distance from the storage reservoir to the service reservoir at Coolgardie is 308 miles, and there is a rise of 1290 feet in that distance. From the service reservoir the water gravitates, the total length from Perth being $351\frac{1}{2}$ miles. Most of the pipes are $\frac{1}{4}$ inch thick, which is sufficient for heads up to 250 feet. They were coated with a mixture of one part asphalt and one part coal-tar, and sprinkled on the outside with sand while hot. In a test the following results were obtained, the pipes being new and clean :—

Hydraulic Gradient. Feet per Mile.	Velocity. Feet per Second.	Delivery. Gallons per Day.	Value of ζ.
2·25	1·889	5,000,000	·00480
2·80	2·115	5,600,000	·00476

In arranging the pumping plant a loss of head of 3·76 feet per mile has been allowed for, to provide against contingencies.

115. **Pumping main.**—It is a common case that water has to be raised by pumping from a river to a reservoir, from which it gravitates to the town supplied. In that case the lift of the pumps H is known, the length of the pumping main l, and the volume Q which must be pumped per second. In deciding on the diameter of the rising main, it must be considered that while the smaller the main the less its cost, on the other hand the greater will be the cost of the pumping engines and the annual cost of pumping, because the frictional head to be overcome will be increased. Usually, for various reasons, the velocity in the pumping main is restricted to from $1\frac{1}{2}$ to 4 feet per second, but within these limits a diameter of main can be found which is the most economical.

Let

l = length of main in feet.
Q = volume pumped in cubic feet per second.
d = diameter of main in feet.
H = total lift from river to reservoir.
h = frictional loss of head in main.
p = cost per I.H.P. of pumping engines, including the capitalised cost of maintenance and working.
q = the cost of the main per foot of diameter and per foot of length, including cost of laying.
N = total I.H.P. of the pumping engines.
η = the mechanical efficiency of the engines.

The total cost of the installation of engines and main is

$$C = pN + qdl.$$

The frictional loss of head in the main is

$$h = 0{\cdot}1008\frac{\zeta Q^2 l}{d^5}.$$

Consequently

$$N = \frac{GQ(H+h)}{550\eta} = \frac{0{\cdot}113Q}{\eta}\left(H + 0{\cdot}1008\frac{\zeta Q^2 l}{d^5}\right).$$

Inserting this value,

$$C = {\cdot}113\frac{pQ}{\eta}\left(H + 0{\cdot}1008\frac{\zeta Q^2 l}{d^5}\right) + qdl,$$

where d is the only variable. Differentiating and equating to zero,

$$d = \sqrt[6]{\frac{\cdot 057\zeta}{\eta}}\sqrt[6]{\frac{p}{q}}\sqrt{Q} \quad . \quad . \quad (14).$$

In practice, $d/\sqrt{Q}$ is from 0·75 to 1·0. For instance, in the Coolgardie main

$$d/\sqrt{Q} = 0{\cdot}78.$$

116. **Suction pipe of pumps.**—Let b be the height of the water barometer or atmospheric pressure in feet of water, and h the height from the water-level in the suction well to the bucket of the pump. h must be less than b in any case, or pumping is impossible. Let Ω be the area of the pump bucket and ω the area of the suction pipe, r the radius of the crank and n the number of revolutions per minute. The average speed of the crank pin is $u = 2\pi rn/60$ feet per second, and the connecting rod being supposed long the velocity of the pump bucket is $v = u \sin a$, where a is the crank angle from the lower dead point. The acceleration of the pump bucket at the beginning of its stroke is $f = u^2/r$. The corresponding acceleration of the water in the suction pipe is

$$p = \frac{\Omega}{\omega}\frac{u^2}{r}.$$

Let l be the length of suction pipe. The weight of the water which must be accelerated is $G\omega l$. The pressure acting on the water to make it follow the piston is $G(b-h)\omega$, and this will produce an acceleration

$$\frac{G(b-h)\omega g}{G\omega l} = \frac{(b-h)g}{l}.$$

In order that the water may follow the pump bucket,

$$\frac{(b-h)g}{l} \geqq \frac{\Omega}{\omega}\frac{u^2}{r}.$$

Substituting for u its value above, the greatest speed of the pump is given by the relation

$$u = 9{\cdot}55\sqrt{\left\{\frac{b-h}{l}\frac{g\omega r}{\Omega}\right\}} \quad . \quad . \quad . \quad (15).$$

If the speed exceeds this the water will separate from the bucket at the beginning of the stroke and overtake it afterwards with a shock. This may be prevented by increasing the area ω of the suction pipe, or to a great extent by placing an air vessel on the suction pipe near the pump.

117. **Water hammer.**—When a valve in a long water-main is rapidly closed, the velocity of the column of water behind the valve is retarded and its momentum is destroyed. To change the momentum of the water, a backward force must be exerted by the valve on the water, or conversely a forward pressure is exerted by the water on the valve and pipe, which, if the action is rapid enough, produces a shock termed water hammer. This action is dangerous, and causes in many cases

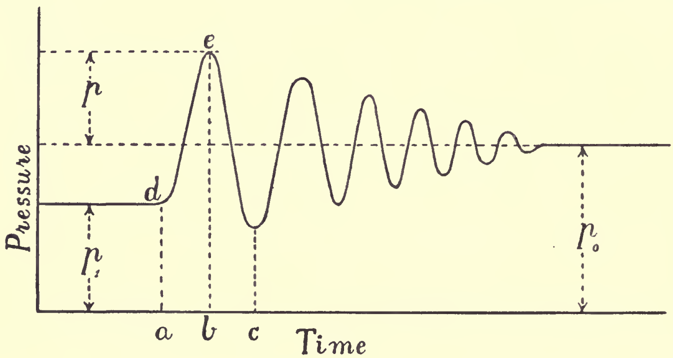

Fig. 103.

fracture of the pipe. It is provided against by arrangements which prevent a rapid closing of important valves.

If a steam-engine indicator is fitted to the pipe, with an arrangement for moving the recording barrel uniformly, a diagram such as is shown in Fig. 103 is produced, the abscissae being time and the ordinates pressure. p_0 is the statical pressure in the pipe when the valve is closed; p_1 is the initial pressure with the water flowing before the valve begins to close. If the valve begins to close at d, the pressure rises to a maximum at e which is in excess of the statical pressure p_0 by an amount p. ab is the time of closing the valve. Waves of pressure follow, gradually diminishing till the water in the pipe comes to rest.

Professor Carpenter made some experiments on a pipe $1\frac{1}{2}$

inches in diameter, with a $\frac{1}{2}$-inch bib-cock at the end. The following were the pressures registered when the cock was suddenly closed. There was a small air chamber near the valve, which in one set of tests was filled with air and in another with water.

GAUGE PRESSURES IN $1\frac{1}{2}$-INCH PIPE

	Air Chamber.	Water Chamber.
Static pressure, lbs. per sq. in.	29	28
Number of impacts	8	9
Maximum pressure	67	76
Minimum pressure	6	9

Consider a column of water in the pipe of unit cross section, extending back from the valve a distance l feet. The weight of this column is Gl lbs. If the initial velocity of the water is v, the momentum of the column is Glv/g second-pounds. If p_m is the mean pressure per unit area exerted in stopping the momentum, and t the time of closing the valve,

$$p_m t = Glv/g.$$

The maximum excess pressure exerted is $p_0 + p - p_1$; and if the mean pressure is taken to be half this,

$$(p_0 + p - p_1)t = 2Glv/g,$$

$$p = 2Glv/gt - p_0 + p_1 \quad . \quad . \quad . \quad (16).$$

The rate u at which a pressure wave is transmitted through water is about 4500 feet per second. Hence l/u seconds must be occupied before the effect of closing the valve reaches the distance l from the valve, and a further time l/u for the pressure due to changing momentum at a distance l is transmitted back to the valve. Hence if the time of closing the valve is less than $2l/u$, that time must be substituted for t in the equation, and then

$$p = Gvu/g - p_0 + p_1 \quad . \quad . \quad . \quad (17).$$

Putting in the numerical quantities, and taking the pressures in pounds per square inch and the velocities in feet per second, the equations become

$$p = 0{\cdot}027lv/t - p_0 + p_1,$$

$$p = 60v - p_0 + p_1.$$

The first equation is to be used if t is greater than $l/2250$ seconds. This equation gives $p = 0$, if

$$t = \text{or} > \frac{{\cdot}027lv}{p_0 - p_1} \quad . \quad . \quad . \quad (18),$$

which is the condition to be satisfied in closing the valve if there is to be no water hammer. The theory involves some assumptions, and must be taken only as a general guide.

Some very elaborate experiments on water hammer in pipes were made by Joukowsky at Moscow (*Stoss in Wasserleitungsröhren*, St. Petersburg, 1900). He used pipes 2, 4, and 6 inches in diameter, and 2494, 1050, and 1066 feet in length. The valve was closed in 0·03 second. Ten recording gauges placed along the pipes showed that the maximum pressures were substantially the same at all points.

The following table gives some of the results :—

VALUES OF $p + p_0 - p_1$ LBS. PER SQUARE INCH

4-inch Pipe.		6-inch Pipe.	
Velocity v.	$p+p_0-p_1$.	Velocity v.	$p+p_0-p_1$.
0·5	31	0·6	43
1·9	115	1·9	106
2·9	168	3·0	173
4·1	232	5·6	369
9·2	519	7·5	426

CHAPTER X

LATER INVESTIGATIONS OF FLOW IN PIPES

118. The different elementary streams which go to form the flow through a pipe have different velocities parallel to the axis of the pipe; those near the sides are retarded by what is often termed skin friction, and these in turn retard those adjacent to them, and so on till the central elementary stream is reached, which has the greatest velocity. It has not been found possible to construct a rational theory of flow which takes account of this distribution of velocity, except at very low velocities. But experiment shows that the resistance to flow involves a loss of energy or head which is proportional to the area of the surface of the pipe and to some function of the mean velocity parallel to the axis of the pipe. The assumption on which the Chezy formula is based is that

$$\frac{d}{4} \cdot \frac{h}{l} = bv^2 \quad . \quad . \quad . \quad . \quad (1),$$

the resistance varying directly as the square of the velocity. In a memoir by Prony in 1804, discussing all the experiments then made, that engineer suggested the expression

$$\frac{d}{4} \cdot \frac{h}{l} = av + bv^2 \quad . \quad . \quad . \quad (2),$$

in which, for metric measures,

$$a = 0{\cdot}0000173, \quad b = 0{\cdot}000343,$$

and for English measures,

$$a = 0{\cdot}0000173, \quad b = 0{\cdot}000105,$$

corresponding to $\zeta = {\cdot}00713$ at 3 feet per second. This binomial expression is exceedingly inconvenient for calculation. It meets the condition that at low velocities the resistance varies as the velocity, and that at high velocities it varies nearly as the square of the velocity; but it makes the transition gradual, whereas it is now known to be abrupt.

119. **Kutter's formula for pipes.**—Messrs. Ganguillet and Kutter, in a laborious investigation on the results of the gauging of streams, arrived at the following complicated empirical formula. Let n be a coefficient of roughness, depending on the character of the surface of the pipe, and m its hydraulic mean radius, i the virtual slope, and v the mean velocity; then, for English measures,

$$v = \left[\frac{41{\cdot}6 + \frac{1{\cdot}811}{n} + \frac{{\cdot}00281}{i}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{i}\right)\frac{n}{\sqrt{m}}} \right] \sqrt{mi} \quad . \quad . \quad (3).$$

There is no good reason for thinking that this formula is specially accurate for flow in pipes. Indeed, it is known not to accord with experiment for small values of i or for small diameters of pipe. But it has been adopted by some engineers, and therefore requires to be mentioned. The usual value of n assumed for clean pipes is 0·013. If in the term $0{\cdot}00281/i$, which is usually relatively small, i is taken as 0·001, and n is taken at 0·013, the formula reduces to the simpler form

$$v = \frac{183{\cdot}72}{1 + \frac{0{\cdot}5773}{\sqrt{m}}} \sqrt{(mi)},$$

or, to put it in a form comparable with the more usual equations,

$$\frac{\left(1 + \frac{1{\cdot}1546}{\sqrt{d}}\right)^2}{524{\cdot}6} \times \frac{v^2}{2g} = \frac{di}{4} \quad . \quad . \quad . \quad (3a),$$

where the first term on the left corresponds to ζ in the Chezy formula.

$d=$ Inches.	Feet.	Kutter's value of ζ.
3	·25	·0209
6	·5	·0132
12	1·0	·0089
24	2·0	·0063
36	3·0	·0053
48	4·0	·0047

120. **Defects of the Chezy formula.**[1]— The Chezy formula is extremely convenient, but involves, if reasonable accuracy is required, the selection of the coefficient ζ amongst a wide range of values. The variation of ζ depends on the following conditions:—

(1) In the case of most pipes the loss of head h does not increase so fast as the square of the velocity v. Consequently ζ must have values which decrease as the velocity is greater.

For instance, in a glass pipe on which Darcy experimented, ζ changed from 0·010 for a velocity of half a foot per second, to 0·0062 for a velocity of 7 feet per second, a decrease of 38 per cent. In a new cast-iron pipe ζ decreased from 0·0114 at half a foot per second to 0·0064 at 10 feet per second, or a decrease of 50 per cent.

(2) Darcy showed that ζ decreases as the size of the pipe is larger. Thus, taking Darcy's experiments on new cast-iron pipes:—

Velocities. Feet per Second.	Values of ζ for Diameters of		
	0·27 feet.	0·45 feet.	0·62 feet.
0·6	·0114	·0073	·0059
10·0	·0064	·0049	·0054

The results are not quite consistent, but they show a considerable decrease in ζ as d increases.

(3) The value of ζ changes with the condition of the inside of the pipe. For asphalted, new, and corroded pipes the values of ζ were proportional to 1, $1\frac{1}{2}$, and 3 in some of Darcy's experiments.

[1] The discussion given here in abbreviated form was published by the author in *Industries* in 1886.

(4) The experiments of Mr. Mair, agreeing with the author's own experiments on discs, show that the resistance decreases as the temperature increases. Thus, for a clean brass pipe, $1\frac{1}{2}$ inches diameter, Mr. Mair obtained the following values :—

At Velocities in Feet per Second of	Values of ζ for Temperatures of		
	56°	90°	160°
$6\frac{1}{2}$	·0047	·0042	·0035
$4\frac{1}{2}$	·0052	·0044	·0038

Alterations of at least 25 per cent for 100° F.

A coefficient which has four independent causes of variation, all of them so large, is not very useful for practical purposes. To get over the difficulty, a formula must be found with more than one constant derived from experiment, and which expresses more nearly the true law of resistance.

It is many years since M. Barré de St. Venant proposed a formula with two arbitrary constants. This is of the form

$$\frac{d}{4} \cdot \frac{h}{l} = mv^n \quad . \quad . \quad . \quad . \quad (4),$$

where m and n are constants derived from the experiments. M. de St. Venant deduced the values $n = \frac{12}{7}$ and $m = 0·0002955$ for metric and 0·0001265 for English measures. When this is written in logarithmic form,

$$\log m + n \log v = \log \left(\frac{d}{4} \cdot \frac{h}{l}\right) \quad . \quad . \quad (5),$$

we have, as St. Venant pointed out, the equation to a straight line, of which m is the ordinate at the origin and n the ratio of the slope. Hence, if the logarithms of a series of experimental values of h and v are plotted, the determination of the constants is reduced to finding the straight line which most nearly passes through the plotted points.

In a remarkable memoir on the influence of temperature on the movement of water in pipes by Hagen (Berlin, 1854), another modification of the St. Venant formula was given; this is

$$\frac{h}{l}=\frac{mv^n}{d^x} \quad . \quad . \quad . \quad . \quad (6).$$

This involves three coefficients, derived from experiment. In the experiments examined by Hagen, he found

$$n=1{\cdot}75,\ x=1{\cdot}25;$$

so that

$$\frac{h}{l}=m\frac{v^{1{\cdot}75}}{d^{1{\cdot}25}} \quad . \quad . \quad . \quad . \quad (6a),$$

in which m was nearly independent of variations both of v and of d. But the range of values of d examined was small.

It is obvious that this form of the equation of flow is very advantageous, even regarded as an empirical formula, for the three constants, n, x, and m, can be taken so as separately to allow for the three principal causes of variation of resistance: the variation of velocity, of diameter, and of roughness of surface.

In a very interesting paper in the *Transactions of the Royal Society*, 1883, Professor Reynolds has made clearer the causes of the change in the character of the motion of water, from the regular stream-line motion at low velocities to the eddying motion which occurs in almost all the cases with which the engineer has to deal. Further, partly by reasoning, partly by induction from the form of the curves of experiments when plotted, he has suggested the general equation

$$A\frac{d^3}{P^2}\cdot\frac{h}{l}=\left(B\frac{dv}{P}\right)^n$$

as applicable both to the case of undisturbed motion and of eddying motion. The constant n having the value 1 for low velocities and undisturbed motion, and a value ranging from 1·7 to 2 for greater velocities. Professor Reynolds's formula reduces to the form

$$\frac{h}{l}=c\frac{v^n}{d^{3-n}}P^{2-n} \quad . \quad . \quad . \quad . \quad (7),$$

where P is a function of the temperature. Neglecting variations of temperature, Professor Reynolds's formula is identical, for velocities not very small, with Hagen's formula; with the exception only that in Reynolds's formula the indices of d and of v are related, so that there are only two independent constants instead of three. For the purpose of obtaining the coefficients from experiment, Hagen's formula is the more convenient.

121. **The experimental data available.**—The earliest experiments on flow in pipes were made by Couplet in 1732, and since that time a considerable number of experiments have been made. In selecting from these it must be borne in mind that it is extremely desirable to exclude from investigation any experiments that are really untrustworthy. No good result can be got by averaging accurate and erroneous results. On the other hand, it would be absolutely wrong in principle to exclude results from examination merely because they did not appear to fit in well with some empirical law.

All experiments may be at once excluded in which the means of measuring the loss of the head or the quantity discharged were unsatisfactory. All experiments may also be excluded in which the condition of the surface of the pipe was not noted. With these exclusions, the number of experiments remaining to be examined is greatly reduced.

Of these experiments, by far the most complete and valuable is the series of experiments on 17 pipes by Henry Darcy. The care and insight with which these experiments were made, and the skilful variation of the conditions of the experiment, are worthy of the highest praise. Of all the conditions to be noted in experimenting, there is only one the importance of which did not occur to Darcy. In many cases he neglected to observe the temperature of the water.

There is, however, one anomaly in Darcy's experiments which cannot now be fully explained, and the nature of which can perhaps best be seen in the plottings of some of his results. Darcy measured the loss of head in two successive 50-metre lengths of his pipes. Now, in almost all cases his results show a rather greater loss in the second 50-metre length than in the first, and this is really not intelligible. On the whole, the author is inclined to think that the measurements in the first 50-metre length are more reliable than those in the second, and only the measurements of head lost in the first 50-metre length are used in these reductions.

From Darcy's experiments have been taken the results on new, cleaned, and incrusted cast-iron pipes, those on wrought-iron gas-pipes, and those on lead pipes. These pipes ranged in diameter from 0·0122 to 0·5 metre, or as 40 to 1. For each pipe the experiments began with a very small loss of head, often only 0·02 metre in 50 metres. The author has excluded the observations in which the loss of head was less than 0·1 metre, partly because some of the experiments with these very small heads correspond to conditions of undisturbed motion, for which the law is different, and partly because the errors in observing very small heads are likely to be relatively large. Up to 6 metres of head the heights were measured by a water column, and beyond that by a mercury column. Now, as the observations with the water gauge give ample range of velocities for the purpose in hand, and as the observations with the mercury gauge at high velocities were, as Darcy mentions, carried out with great difficulty, the former only have been used in these reductions. With a loss of head varying from 0·1 metre to 6 metres, the velocities ranged in different cases from 0·1 metre per second to 5 metres per second, a very ample range for examination.

Of other experiments available, the early (1771) experiments of Bossut on the flow in tin pipes seem very trustworthy, and give values of the constants for a very clean and smooth surface. These extend over a considerable range of velocity.

Dr. Lampe's experiments on the Dantzig main are extremely useful, from the care with which they were carried out, and the fact that they are on a large scale.

Of other experiments, the most valuable are the American data collected in Mr. Hamilton Smith's *Hydraulics.* Of these, there is a very valuable experiment by Mr. Stearns on a cast-iron asphalted pipe, $1\frac{1}{4}$ metres in diameter. Mr. Hamilton Smith's own experiments are also very useful, as filling up and extending the series of results from other sources. This makes the range of diameters of new pipes, on which experiments are available, to extend from 0·266 metre to 1·219 metres.

Then there are some experiments on small wrought-iron gas-pipes, which are useful for comparison with Darcy's, and some experiments on large wrought-iron riveted water-mains.

122. **Method of dealing with the experimental data.**—The greater part of the experimental results are found originally in metric measures. Hence it was convenient to plot the results in metric measures, and to obtain constants for a formula in metric measures. These constants were finally converted to English measures.

Taking Hagen's formula (6), and writing it logarithmically,

$$\log h = n \log v + \log \frac{m}{d^x} + \log \frac{l}{2g} \quad . \quad . \quad (8),$$

in which for any given pipe the second and third terms on the right are constants. This is an equation to a straight line having $\log \{(ml)/(2gd^x)\}$ for the ordinate at the origin, and a slope of n to 1. For all the experimental data, arranged in groups according to the type of pipe, values of $\log h$ were plotted as abscissæ and values of $\log v$ as ordinates, h and v being taken in metric units. One of these plottings is given on a reduced scale in Fig. 104. The values of n corresponding to the average slope of the lines are given in the following table[1] :—

[1] For each of the Darcy pipes two lines are plotted, the full line corresponding to observations in the first, and the dotted to those in the second 50-metre length.

[TABLE.

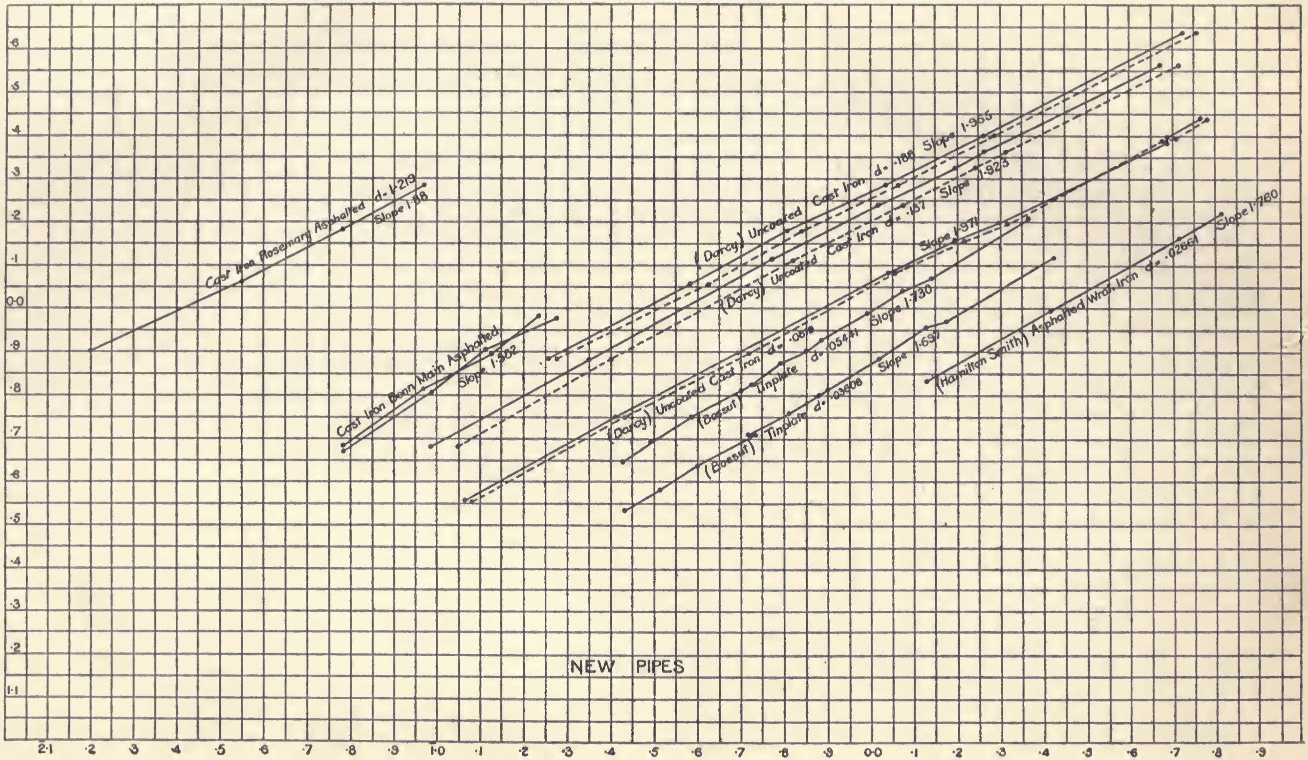
NEW PIPES
Cast Iron Rosemary Asphalted d = 1·213
Slope 1·88
Cast Iron Bonn Main Asphalted
Slope 1·802
(Darcy) Uncoated Cast Iron d = ·188 Slope 1·965
(Darcy) Uncoated Cast Iron d = ·137 Slope 1·923
(Darcy) Uncoated Cast Iron d = ·0678 Slope 1·971
(Bossut) Tinplate d = ·05441 Slope 1·730
(Bossut) Tinplate d = ·03608 Slope 1·697
(Hamilton Smith) Asphalted Wrot. Iron d = ·02861 Slope 1·760

Fig. 104.

VALUES OF THE INDEX OF VELOCITY

Surface of Pipe.	Authority.	Diameter of Pipe in Metres.	Values of n.	
Tin-plate . . .	Bossut	·036	1·697	1·72
	"	·054	1·730	
Wrought iron (gas-pipe)	Hamilton Smith	·0159	1·756	1·75
	"	·0267	1·770	
Lead	Darcy	·014	1·866	1·77
	"	·027	1·755	
	"	·041	1·760	
Clean brass . . .	Mair	·036	1·795	1·795
Asphalted . . .	Hamilton Smith	·0266	1·760	1·85
	Lampe	·4185	1·850	
	Bonn	·306	1·582	
	Stearns	1·219	1·880	
Riveted wrought iron .	Hamilton Smith	·2776	1·804	1·87
	"	·3219	1·892	
	"	·3749	1·852	
Wrought iron (gas-pipe)	Darcy	·0122	1·900	1·87
	"	·0266	1·899	
	"	·0395	1·838	
New cast iron . .	"	·0819	1·950	1·95
	"	·137	1·923	
	"	·188	1·957	
	"	·50	1·950	
Cleaned cast iron .	"	·0364	1·835	2·00
	"	·0801	2·000	
	"	·2447	2·000	
	"	·397	2·07	
Incrusted cast iron .	"	·0359	1·980	2·00
	"	·0795	1·990	
	"	·2432	1·990	

It will be seen that the values of the index n range from 1·72 for the smoothest and cleanest surface to 2·00 for the roughest. The numbers after the brackets are rounded off numbers, not exactly means, but numbers based partly on judgment of the value of the different experiments, which have been adopted in the following reductions.

Taking the values of n thus determined, the value of m/d^x, which should be a constant for any given pipe, is then deduced. For each pipe the values of m/d^x are averaged. It is then possible to see how far the formula fits the experiments, at least so far as the variation of velocity with variation of head is concerned. The following tables give the mean values of m/d^x found for each pipe, and the result of using the formula

$$\frac{h}{l} = \left(\frac{m}{d^x}\right)\frac{v^n}{2g} \quad . \quad . \quad . \quad . \quad (9)$$

to re-calculate h from the observed velocities. It will be seen that, at least so far as variation of the velocity and head are concerned, the formula fits the experiments with extraordinary closeness. The re-calculated values of h approximate to the observed values throughout the whole range of the experiments, with differences which do not exceed the probable experimental errors of observation. Metric measures are used in these tables.

Asphalted Cast Iron

Hamilton Smith. Diam. = 0·02661 m. $\frac{m}{d^x}$ = 1·041 n = 1·85			Bonn Waterworks. Diam. = 0·306 m. $\frac{m}{d^x}$ = 0·0823 n = 1·85 t = 41° F.			Bonn Waterworks. Diam. = 0·306 m. $\frac{m}{d^x}$ = 0·0842 n = 1·85 t = 41° F.		
v	h		v	h		v	h	
	Obs.	Calc.		Obs.	Calc.		Obs.	Calc.
·6767	1·346	1·289	·4779	·0605	·0535	·4734	·0605	·054
·9826	2·609	2·569	·6449	·0930	·0932	·6412	·0975	·094
1·451	5·169	5·285	·7909	·1345	·1358	·7986	·1300	·142
1·659	6·532	6·770	·9523	·1715	·1916	·9436	·1810	·193

Lampe. Diam. = 0·4185 m. $\frac{m}{d^x}$ = 0·0453 n = 1·85 t = 48° F.			Stearns. Diam. = 1·219 m. $\frac{m}{d^x}$ = 0·0111 n = 1·85 t = 38° F.			Loch Katrine. Diam. = 1·219 m. $\frac{m}{d^x}$ = 0·0169 n = 1·85 t = (unknown)		
v	h		v	h		v	h	
	Obs.	Calc.		Obs.	Calc.		Obs.	Calc.
·4806	·0296	·0298	·797	·0159	·0186	1·054	·0474	·0474
·7565	·0688	·0688	1·140	·0356	·0360			
·8256	·0815	·0810	1·513	·0610	·0609			
·9419	·0975	1·034	1·888	·0925	·0916			

Clean Tinplate Pipes

Bossut. Diam. = 0·03608 m. $\frac{m}{d^x}=0·6557$ $n=1·72$ $t=50°$ F.			Bossut. Diam. = ·05441 m. $\frac{m}{d^x}=0·412$ $n=1·72$ $t=67°$ F.		
v	h		v	h	
	Observ.	Calc.		Observ.	Calc.
·3401	·2700	·2615	·4435	·2650	·2594
·3807	·3220	·3174	·4956	·3140	·3139
·4364	·3980	·4015	·5608	·3855	·3884
·5114	·5380	·5273	·6431	·5005	·4916
·5126	·5210	·529	·6692	·526	·5269
·5694	·6410	·634	·7439	·623	·631
·6324	·7540	·760	·7912	·710	·702
·6498	·7915	·796	·8366	·764	·773
·7598	1·0345	1·042	·9685	·987	·994
·8978	1·3470	1·389	1·091	1·194	1·222
·9333	1·494	1·484	1·164	1·398	1·365
1·314	2·649	2·672	1·595	2·324	2·345

Riveted Wrought Iron

Hamilton Smith. Diam. = 0·2776 m. $\frac{m}{d^x}=0·0822$ $n=1·87$ $t=55°$ F.			Hamilton Smith. Diam. = 0·3219 m. $\frac{m}{d^x}=0·0704$ $n=1·87$ $t=55°$ F.		
v	h		v	h	
	Observ.	Calc.		Observ.	Calc.
1·436	·425	·413	1·401	·334	·337
1·858	·667	·667	2·121	·714	·732
2·111	·847	·848	2·635	1·109	1·098
2·639	1·279	1·287	3·262	1·659	1·638
3·054	1·654	1·691			

Riveted Wrought Iron—*Continued*

Hamilton Smith. Diam. = 0·3749 m. $\frac{m}{d^x}=0·549$ $n=1·87$ $t=55°$ F.			Hamilton Smith. Diam. = 0·6566 m. $\frac{m}{d^x}=0·0260$ $n=1·87$ t ?			Hamilton Smith. Diam. = 0·4316 m. $\frac{m}{d^x}=0·0440$ $n=1·87$ t ?		
v	h		v	h		v	h	
	Observ.	Calc.		Observ.	Calc.		Observ.	Calc.
1·336	·251	·241	3·841	·821	·821	6·139	3·336	3·336
2·084	·549	·553						
2 229	·613	·627						
2 579	·823	·824						
3·228	1·235	1·254						
3·684	1·616	1·605						

New Cast-Iron Pipes (Uncoated)

Darcy. Diam. = 0·0819 m. $\frac{m}{d^x}=0·3205$ $n=1·95$ $t=60°$ F.			Darcy. Diam. = 0·137 m. $\frac{m}{d^x}=0·1454$ $n=1·95$ $t=60°$ F.		
v	h		v	h	
	Observ.	Calc.		Observ.	Calc.
·358	·115	·110	·488	·097	·091
·561	·258	·265	·763	·224	·219
·791	·500	·517	1·279	·590	·599
1·185	1·10	1·138	1·714	1·045	1·059
1·418	1·58	1·61	2·098	1·560	1·571
1·571	1·99	1·97	2·281	1·840	1·850
2·453	4·826	4·66	3·640	4·690	4·604
2·487	4·870	4·83			
2·720	5·872	5·75			

New Cast-Iron Pipes (Uncoated)—*Continued*

Darcy. Diam. = 0·188 m. $\frac{m}{d^x}=0·1192$ $n=1·95$ t probably 70° F.			Darcy. Diam. = 0·50 m. $\frac{m}{d^x}=0·0382$ $n=1·95$ t probably 70° F.		
v	h		v	h	
	Observ.	Calc.		Observ.	Calc.
·497	·090	·078	0·7932	0·120	·124
·758	·180	·177	·7951	·125	·125
1·128	·385	·384	1·0412	·210	·211
1·488	·640	·660	1·1135	·230	·240
1·933	1·090	1·098	1·1197	·260	·243
2·506	1·855	1·822	1·1278	·250	·247
4·323	5·276	5·274			

Wrought-Iron (Gas) Pipe

Hamilton Smith. Diam. = ·01594 m. $\frac{m}{d^x}=1·948$ $n=1·75$ t about 60° F.			Hamilton Smith. Diam. = ·02676 m. $\frac{m}{d^x}=1·055$ $n=1·75$ t about 60° F.		
v	h		v	h	
	Observ.	Calc.		Observ.	Calc.
·314	·656	·653	·292	·375	·312
·481	1·375	1·379	·433	·614	·622
·700	2·650	2·657	·656	1·288	1·286
·873	3·892	3·911	·968	2·516	2·537
1·031	5·265	5·245	1·203	3·715	3·716
1·182	6·661	6·653	1·424	5·035	4·991
			1·623	6·350	6·272

CLEANED CAST IRON

Darcy. Diam. = 0·0801 m. $\frac{m}{d^x}=0·3912$ $n=2·0$ t probably 45° F.			Darcy. Diam. = 0·2447 m. $\frac{m}{d^x}=0·1082$ $n=2·0$ t probably 60° F.			Darcy. Diam. = 0·297 m. $\frac{m}{d^x}=0·0770$ $n=2·0$ $t=70°$ F.		
v	h		v	h		v	h	
	Obs.	Calc.		Obs.	Calc.		Obs.	Calc.
·385	·148	·148	·949	·245	·248	·823	·125	·133
·614	·370	·376	1·420	·565	·556	1·155	·255	·262
·624	·375	·389	1·904	1·000	1·000	1·652	·535	·536
·864	·795	·745	2·206	1·350	1·343	2·390	1·170	1·122
1·248	1·51	1·553	2·572	1·840	1·825	2·799	1·570	1·539
1·526	2·30	2·324	4·497	5·505	5·581	3·160	2·022	1·962

INCRUSTED CAST IRON

Darcy. Diam. = 0·359 m. $\frac{m}{d^x}=1·8154$ $n=2·0$ $t=45°$ F.			Darcy. Diam. = 0·0795 m. $\frac{m}{d^x}=0·6898$ $n=2·0$ t probably 45° F.			Darcy. Diam. = 0·2432 m. $\frac{m}{d^x}=0·1873$ $n=2·0$ t probably 68° F.		
v	h		v	h		v	h	
	Obs.	Calc.		Obs.	Calc.		Obs.	Calc.
·130	·081	·078	·251	·111	·111	·452	·098	·098
·253	·300	·296	·446	·355	·350	·707	·235	·239
·381	·665	·672	·678	·800	·808	1·106	·565	·584
·551	1·405	1·406	·931	1·535	1·524	1·547	1·130	1·142
·633	1·85	1·855	1·142	2·265	2·294	1·833	1·580	1·604
						2·073	2·020	2·050

123. **The correction for temperature.**—The correction for temperature is at present imperfectly known. No experiments on the resistance at different temperatures, with very rough surfaces, have been made; but, in the absence of information, it has been thought better to correct all the values of m/d^x to a common temperature of 60°, in accordance

with Mr. Mair's results. Variation of temperature in the different experiments examined ranges from 38° F. to 75° F. In most of the experiments the temperature was between 50° and 70°. For 10° difference from 60°, the temperature correction is under 3 per cent, so that it does not make a great difference whether the temperature correction is applied or not. In some of Darcy's experiments the temperatures are not given, but they can be inferred with some degree of approximation from the dates given.

124. **Variation of resistance with the diameter of the pipe.**—From the values of m/d^x which have been obtained, the value of x, the index of the diameter in the expression for the head lost in the pipe, can be found. If m and x for any given kind of pipe are strictly constant, and if we plot logarithmic values of d as ordinates, and m/d^x as abscissæ, then the points found should lie on a straight line, the slope of which is the required value of x. Broadly speaking, the points corresponding to each set of experiments fell pretty closely on a straight line, those for the pipes with rougher surfaces lying higher than those for the pipes with smoother surfaces. It is not surprising that the lines are more irregular than those previously plotted, for this reason. The points in these lines correspond, not to a series of experiments on one pipe, but to different series of experiments on different pipes. Small differences of roughness in these pipes would quite account for such discrepancies as were found.

On examining the lines, it was found that in all cases the slope is greater than 1 to 1, so that the index x of d, in the formula of loss of head, must be greater than unity, a result in accordance with Darcy's deductions from his experiments. The slope is lowest (1·10 to 1) for the tin-plate pipes of Bossut, which were very smooth, and in which, probably, the joints did not affect the flow so much as in other pipes. Generally, the slope does not exceed 1·2 to 1; but there are one or two exceptions.

The riveted wrought-iron pipes of Hamilton Smith give a slope of 1·39 to 1, which may possibly be due to the different relative effect of the obstruction of the rivet-heads and joints in pipes of different diameters of this kind. Putting aside exceptional values of the index x, the fact that all the other results give values of x lying between 1·10 and 1·21 shows a very satisfactory constancy in the coefficient.

According to Professor Reynolds's formula, the head lost should vary as

$$\frac{v^n}{d^{3-n}}.$$

That is, x should have the value $3-n$. The following table shows how far this reduction of the most trustworthy experiments confirms this law:—

Kind of Pipe.	n	$3-n$	x
Tinplate	1·72	1·28	1·100
Wrought iron (Smith) . .	1·75	1·25	1·210
Asphalted pipes . . .	1·85	1·15	1·127
Riveted wrought iron . .	1·87	1·13	1·390
New cast iron . . .	1·95	1·05	1·168
Cleaned cast iron . .	2·00	1·00	1·168
Incrusted cast iron . .	2·00	1·00	1·160

It will be seen that there is no great discrepancy between the values of x and $3-n$, but there is no appearance of relation in the two quantities. For the present, at least it must be assumed that the value of x is independent to the value of n.

125. **Values of the coefficient** m.—It is now possible to determine the values of the coefficient m from the different series of experiments, using the values of d^x, calculated from the values of x now assigned. It will be a general check on the whole of the preceding reductions, if the values of m for each particular kind of pipe prove to be nearly constant. Hence the values of m for each of the twenty-eight series of experiments which have been discussed are here given. They are placed generally in the order of the index n, and each set of pipes of one general character is placed in the order of the diameters.

Kind of Pipe.	Diam. in Metres.	Value of m.	Mean Value of m.	Authority.
Tinplate . .	0·036	·01697	·01686	Bossut.
	0·054	·01676		
Wrought iron . .	0·016	·01302	·01310	Hamilton Smith.
	0·027	·01319		
Asphalted pipes .	0·027	·01749	·01831	Hamilton Smith.
	0·306	·02058		Bonn, W. W.
	0·306	·02107		Bonn, W. W.
	0·419	·01650		Lampe.
	1·219	·01317		Stearns.
	1·219	·02107		Gale.
Riveted wrought iron	0·278	·01370	·01403	Hamilton Smith.
	0·322	·01440		
	0·375	·01390		
	0·432	·01368		
	0·657	·01448		
New cast iron . .	0·082	·01725	·01658	Darcy.
	0·137	·01427		
	0·188	·01734		
	0·500	·01745		
Cleaned cast iron .	0·080	·01979	·01994	Darcy.
	0·245	·02091		
	0·297	·01913		
Incrusted cast iron .	0·036	·03693	·03643	Darcy.
	0·080	·03530		
	0·243	·03706		

Here, considering the great range of diameters and velocities in the experiments, the constancy of m is very satisfactorily close. The asphalted pipes give rather variable values; but, as some of these were new and some old, the variation is, perhaps, not surprising. The incrusted pipes give a value of m quite double that for new pipes, but that is perfectly consistent with what is known of fluid friction in other cases.

126. **General mean values of constants.**—The general formula

$$\frac{h}{l} = \frac{m}{d^x} \cdot \frac{v^n}{2g} \quad . \quad . \quad . \quad . \quad (10)$$

will be found to agree with the results with convenient closeness, if the following mean values of the coefficients are taken, the unit being a metre:—

Kind of Pipe.	m	x	n
Tinplate	·0169	1·10	1·72
Wrought iron	·0131	1·21	1·75
Asphalted iron	·0183	1·127	1·85
Riveted wrought iron . .	·0140	1·390	1·87
New cast iron	·0166	1·168	1·95
Cleaned cast iron . .	·0199	1·168	2·0
Incrusted cast iron . .	·0364	1·160	2·0

The variation of each of these coefficients is within a comparatively narrow range, and the selection of the proper coefficient for any given case presents no difficulty, if the character of the surface of the pipe is known.

It only remains to give the values of these coefficients when the quantities are expressed in English feet. For English measures the following are the values of the coefficients:—

Kind of Pipe.	m	x	n
Tinplate	·0265	1·10	1·72
Wrought iron	·0226	1·21	1·75
Asphalted iron	·0254	1·127	1·85
Riveted wrought iron . .	·0260	1·390	1·87
New cast iron	·0215	1·168	1·95
Cleaned cast iron . .	·0243	1·168	2·0
Incrusted cast iron . .	·0440	1·160	2·0

If formula (10) is put in the form

$$\frac{mv^{n-2}}{4d^{x-1}} \cdot \frac{v^2}{2g} = \frac{di}{4},$$

it is seen that the coefficient ζ in the Chezy formula can be deduced from these results by taking

$$\zeta = \frac{mv^{n-2}}{4d^{x-1}}.$$

Values of ζ thus obtained have been given in Chapter VIII. § 91. Using these values in the Chezy formula the results are nearly as accurate as if eq. (10) is used.

127. Distribution of velocity in the cross section of a pipe.—The mean velocity of translation along a pipe is necessarily

$$v_m = Q/\left(\frac{1}{4}\pi d^2\right).$$

Strictly, in consequence of the turbulence of the motion, the velocity and direction of motion vary from moment to moment at every point of the cross section. But at each point the variations are temporary fluctuations about a fixed mean value. The mean direction must be parallel to the axis of the pipe, and at each point there must be a constant mean velocity in that direction. Observation shows that these mean velocities at different points are greater near the centre of the cross section and less towards its boundary. Messrs. Williams, Hubbel, and Fenkel found the mean velocity v_m of the whole cross section to be 0·84 of the central mean velocity, and the mean velocity near the boundary to be 0·5 of the central mean velocity. At a radius 0·75 of the radius of the pipe the velocity was equal to the mean velocity v_m of the whole cross section.

The most exact research on the distribution of velocity in pipes is one made by Bazin on a cement pipe 0·8 metre diameter and 80 metres long ("Expériences nouvelles," *Mém. de l'Académie des Sciences*, xxxii., 1897). Let R be the radius of the pipe, and r the radius at which the velocity is observed; let V be the maximum velocity at the centre, v the velocity at radius r, and v_m the mean velocity for the whole cross section. Bazin obtained the following results :—

$\frac{r}{R}$	$\frac{v}{v_m}$	$\frac{V-v}{v_m}$
0	1·1675	0
0·125	1·1605	·0070
0·250	1·1475	·0200
0·375	1·1258	·0417
0·500	1·0923	·0752
0·625	1·0473	·1202
0·750	1·0008	·1667
0·875	0·9220	·2455
0·937	0·8465	·3210
1·000	0·7415	·4260

Let i be the virtual slope of the pipe. Then

$$v = \mathrm{V} - k\left(\frac{r}{\mathrm{R}}\right)^3 \sqrt{\mathrm{R}i},$$

where k varies from 33 to 42, and is on the average 38. At the sides, where $r = \mathrm{R}$, the velocity is $w = \mathrm{V} - 38\sqrt{(\mathrm{R}i)}$. The mean velocity of the whole cross section is

$$v_m = \mathrm{V} - 4{\cdot}64\sqrt{(\mathrm{R}i)}.$$

On the average $\mathrm{V}/v_m = 1{\cdot}24$; $v_m/\mathrm{V} = 0{\cdot}8$; $w/v_m = 0{\cdot}64$, and $w/\mathrm{V} = 0{\cdot}51$. At radius $0{\cdot}74\mathrm{R}$ the velocity is equal to v_m.

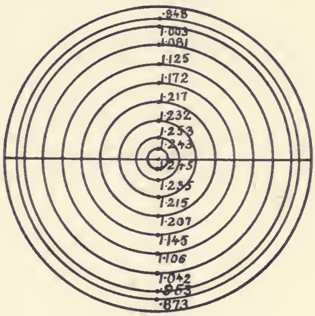

Fig. 105.

Fig. 105 shows the velocities at different radii found by Bazin.

128. **Influence of temperature on the resistance in pipes.**—In the experiments on discs, § 82, it appeared that the frictional resistance diminished as the temperature increased. Froude found a similar result for boards towed in water. Some experiments on flow of water at different temperatures in a brass pipe $1\frac{1}{2}$ inch diameter and 25 feet long were made by Mr. J. G. Mair (*Proc. Inst. of Civil Engineers*, lxxxiv.). The head at inlet was taken at 12 inches from the end of the pipe, to exclude loss at entry. The results agree extremely closely with the equation

$$\frac{h}{l} = \frac{m}{d^{1{\cdot}2}} \cdot \frac{v^{1{\cdot}795}}{2g}.$$

The values of m were as follows:—

Temperature F.	m
57°	0·0178
70	169
80	166
90	161
100	157
110	151
120	147
130	145
160	133

The resistance is therefore 25 per cent less at 160° than at 57°. The resistance varies directly as m, and m is given very closely by the empirical relation

$$m = 0{\cdot}02(1 - 0{\cdot}00215t).$$

CHAPTER XI

FLOW OF COMPRESSIBLE FLUIDS IN PIPES

129. **Notation.**—Let

P = absolute pressure in lbs. per square foot.

T = absolute temperature F.°

G = weight of one cubic foot of fluid in lbs.

V = volume of one pound of fluid in cubic feet.

$u, v,$ = velocities in feet per second.

W = weight of fluid per second in lbs.

Ω = area of cross section of pipe in square feet.

d = diameter of pipe in feet.

$L, l,$ = length of pipe in feet.

R = constant in gaseous equation.

When air flows along a pipe there is necessarily a fall of pressure due to the resistance of the pipe, and consequently the volume and velocity of the air increase going along the pipe in the direction of motion. The effect of the resistance is to create eddying motions which, as they subside, give back to the air the heat equivalent of the work expended in producing them. The result is that, apart from conduction through the walls of the pipe, the flow is isothermal.[1]

130. **Flow in pipes under small differences of pressure.**—In a large number of cases the pressure in a fluid is one atmosphere or more, but the difference of pressure causing flow is only a few inches of water. This is the case in the distribution of lighting gas and in some cases of compressed air transmission. Let P_1, P_2 be the absolute

[1] This was pointed out by the author in a discussion on Pneumatic Transmission in 1875 (*Proc. Inst. C. E.* xliii.). The formula for air-flow in this chapter was first given by the author in 1875 in a paper on the "Motion of Light Carriers in Pneumatic Tubes" in the same volume.

pressures at the inlet and outlet of a pipe. Then when $P_1 - P_2$ is small compared with P_1, the variation of density during flow may be neglected without great error and the hydraulic formulæ are applicable.

Let d be the diameter, l the length of the pipe in feet, v the velocity, $P_1 - P_2$ the pressure difference causing flow in lbs. per square foot, and h the same pressure difference in feet of the fluid itself. If G is the weight of the fluid in lbs. per cubic foot, $P_1 - P_2 = Gh$. Then, as in § 85,

$$\left.\begin{aligned} h &= \frac{\zeta v^2}{2g}\frac{4l}{d} \text{ feet} \\ v &= \sqrt{\left\{\frac{2gdh}{4\zeta l}\right\}} = \sqrt{\left\{\frac{2gd}{4\zeta l}\frac{(P_1 - P_2)}{G}\right\}} \text{feet per second} \\ Q &= \frac{\pi}{4}d^2 v \text{ cubic feet per second} \end{aligned}\right\} \quad (1).$$

If T is the absolute temperature F., then, by the gaseous equation § 72,

$$G = P_1/(RT).$$

If h_w is the pressure difference measured in inches of water, then

$$P_1 - P_2 = (62{\cdot}4h_w)/12 = 5{\cdot}2h_w.$$

Example.—Air initially at one atmosphere and 60°F. (521° absolute) flows through a 12-inch pipe one mile long under a pressure difference of 10 inches of water. $G = 2116{\cdot}3/(53{\cdot}2 \times 521) = 0{\cdot}0764$ lbs. per cubic foot. $P_1 - P_2 = 5{\cdot}2 \times 10 = 52$ lbs. per square foot. The value of ζ may be taken at 0·004. Then

$$v = \sqrt{\left\{\frac{2g \times 1}{{\cdot}016 \times 5280}\frac{52}{{\cdot}0764}\right\}} = 22{\cdot}77 \text{ feet per second.}$$

The discharge is $0{\cdot}7854 \times 22{\cdot}77 = 17{\cdot}88$ cubic feet per second, or $17{\cdot}88 \times 0{\cdot}0764 = 1{\cdot}367$ lbs. of air per second.

131. **Flow of lighting gas in mains.**—Lighting gas is distributed in cast-iron mains under pressure differences of about 2 inches of water column per mile of main, or $2 \times 5{\cdot}2 = 10{\cdot}4$ lbs. per square foot. The velocity is generally not more than about 15 feet per second. In such conditions the hydraulic formulæ are applicable with very little error.

Pressures in gas mains are usually measured by water siphon gauges open to the atmosphere. They indicate therefore the excess of pressure in the main over atmospheric pressure. If h_w is the gauge pressure in inches of water, and the atmospheric pressure is 34 feet of water, then the absolute pressure in the main is $34+\frac{1}{12}h_w$ feet of water, or $62{\cdot}4\,(34+\frac{1}{12}h_w)=2121+5{\cdot}2h_w$ lbs. per square foot.

Head lost in a horizontal main.—Let Fig. 106 represent a length l of horizontal main through which gas of density s (air = 1) is flowing. The difference y_1-y_2 of the water columns in the siphon gauges is the head lost in the length l.

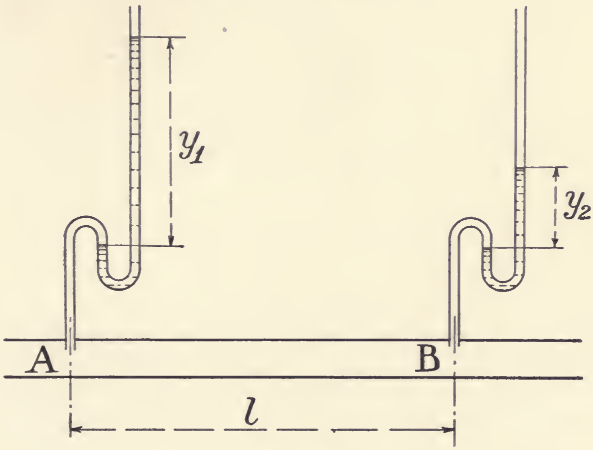

Fig. 106.

Let G_w, G_a, G_g be the weights in lbs. per cubic foot of water, air, and gas respectively. Then $G_g=sG_a$, where for ordinary conditions of pressure and temperature $G_a=0{\cdot}08$ nearly, and $G_w=62{\cdot}4$. Then if y_1, y_2 are measured in inches of water, the height of a column of gas equivalent to y_1-y_2 is

$$h=\frac{1}{12}\frac{G_w}{sG_a}(y_1-y_2)=65\frac{y_1-y_2}{s}\text{ feet} \quad . \quad . \quad (2),$$

and this introduced in the hydraulic equations (1) will give the velocity of flow and discharge.

Head lost in an inclined gas main.—In a falling main (Fig. 107) the atmospheric pressure is greater at B than at A by the amount $G_a\,(z_1-z_2)$ lbs. per square foot, or

$$\frac{G_a}{G_w}(z_1 - z_2) \text{ feet of water ;}$$

a quantity which is negative for a rising main. Hence, taking $y_1 - y_2$ in feet, the head causing flow in feet of gas is

$$h = \frac{1}{sG_a}\left\{G_w(y_1 - y_2) - G_a(z_1 - z_2)\right\} + z_1 - z_2$$

$$= \frac{G_w}{sG_a}(y_1 - y_2) + (z_1 - z_2)\left(1 - \frac{1}{s}\right) \quad . \quad . \quad . \quad (3).$$

Taking the values given above, and now supposing y_1 and y_2 given in inches of water,

$$h = 65\,\frac{y_1 - y_2}{s} + (z_1 - z_2)\left(1 - \frac{1}{s}\right) \quad . \quad . \quad (3a).$$

This is the value of h to be used in the hydraulic equations (1). When there is much difference of level of A and B, the last

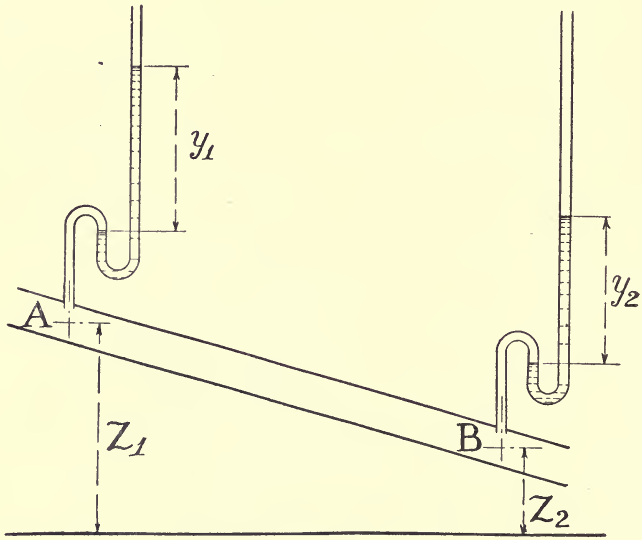

Fig. 107.

term is too large to be neglected. In some rising mains the difference shown by the siphons is negative.

The coefficient of friction in gas mains.—Unfortunately there are very few experiments on the friction in gas mains, and even those which are available are not very satisfactory. A discussion by the author of such results as are available (*Proc. Inst. of Gas Engineers,* 1904) led him to adopt provisionally the following value:—

$$\zeta = 0{\cdot}0044\left(1 + \frac{1}{7d}\right) \quad . \quad . \quad . \quad (4).$$

This gives higher values of ζ than those deduced from tests of air mains, but on the other hand gas mains are rather more roughly jointed, and there were probably in the mains tested some special resistances due to bends, etc.

VALUES OF ζ FOR GAS MAINS

Diameter of Pipe.	ζ
2 inches	0·0082
6 ,,	·0057
12 ,,	·0050
18 ,,	·0048
24 ,,	·0047

Examples.—Let 50,000 cubic feet of gas per hour, or 13·9 cubic feet per second, of density $s = 0{\cdot}4$, be conveyed in a horizontal main, and let it be required to find the pressure head lost in friction per mile of main.

(*a*) Let the main be 8 inches or 0·666 foot in diameter. Then $\zeta = 0{\cdot}0044(1 + 0{\cdot}214) = 0{\cdot}0053$. The cross section of the main is 0·349 square foot, and the velocity is $13{\cdot}9/0{\cdot}349 = 40$ feet per second. Using eq. (1),

$$h = 0{\cdot}0053 \frac{40^2}{64{\cdot}4} \frac{4 \times 5280}{0{\cdot}666} = 4178 \text{ feet}$$

of gas. Taking air in these conditions to weigh 0·08 lb. per cubic foot, the gas weighs $0{\cdot}4 \times 0{\cdot}08 = 0{\cdot}032$ lb. per cubic foot. Hence the pressure difference required per mile of main is $0{\cdot}032 \times 4178 = 133{\cdot}8$ lbs. per square foot, or $133{\cdot}8/5{\cdot}2 = 25{\cdot}7$ inches of water.

(*b*) If the main is sixteen inches or 1·333 feet in diameter, the other conditions being the same, $\zeta = 0{\cdot}0044(1 + 0{\cdot}107) = 0{\cdot}00487$. The cross section of main is 1·395 square feet. The velocity is $13{\cdot}9/1{\cdot}395 = 10$ feet per second.

$$h = 0{\cdot}00487 \frac{10^2}{64{\cdot}4} \frac{4 \times 5280}{1{\cdot}333} = 119{\cdot}9 \text{ feet}$$

of gas, equivalent to 3·84 lbs. per square foot, or 0·74 inch of water per mile of main.

(*c*) If in the case of the 8-inch main in (*a*), the outlet end is 150 feet above the inlet, the frictional loss is the same, but there is a difference of the pressures at the siphon gauges. Using eq. (3*a*),

15

$$h = 4178 = 65\frac{y_1 - y_2}{0{\cdot}4} - 150\left(1 - \frac{1}{0{\cdot}4}\right)$$
$$= 162{\cdot}5\,(y_1 - y_2) + 225$$
$$y_1 - y_2 = 24{\cdot}33 \text{ inches of water.}$$

(*d*) Similarly, if in the case of the 16-inch main in (*b*), the outlet end is 150 feet above the inlet,

$$h = 119{\cdot}9 = 65\frac{y_1 - y_2}{0{\cdot}4} + 225$$
$$y_1 - y_2 = -0{\cdot}647 \text{ inch of water.}$$

That is, the upper siphon-gauge pressure would be greater than the lower.

132. **Flow of air in a long uniform pipe, when the variation of density is taken into account.**—In this case the velocity increases along the pipe as the density diminishes. The work of expansion of the fluid is not negligible. The expansion will be taken to be isothermal.

For air, $P/G = 53{\cdot}2T$ (§ 72), and if the temperature is 60° F., so that $T = 521$, then $P/G = 27710$.

In steady flow the same weight of air must pass every section in any given time. Let W be the weight of air flowing per second, u the velocity, and Ω the area of cross section.

$$W = G\Omega u = \frac{\Omega u P}{RT} \quad . \quad . \quad . \quad (5).$$

Consider a short length dl of the pipe, Fig. 108, between transverse sections A_0A_1. Let d be the diameter, Ω the cross section, m the hydraulic mean radius. Let P and u be the pressure and velocity at A_0, $P + dP$, and $u + du$ the corresponding quantities at A_1. Let W be the weight of air flowing per second—units feet and pounds.

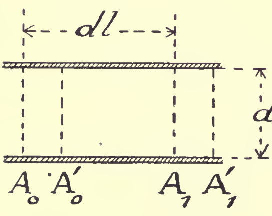

Fig. 108.

If in a short time dt the mass A_0A_1 comes to $A'_0A'_1$, then $A_0A'_0 = udt$ and $A_1A'_1 = (u + du)dt$. Since in a short length the change of density is small the head lost in feet of fluid is

$$\zeta\frac{u^2}{2g}\frac{dl}{m};$$

or if $H = u^2/2g$, the head lost in friction is

$$\zeta H dl/m \text{ feet} \quad . \quad . \quad . \quad (6).$$

And since Wdt lbs. flow in the time dt, the work expended in friction is

$$-\zeta\frac{H}{m}W\,dl\,dt \text{ ft.-lbs.} \qquad (7).$$

The change of kinetic energy in the time dt is the difference of the kinetic energy of $A_1A'_1$, and $A_0A'_0$, that is

$$\frac{Wdt}{2g}\left\{(u+du)^2-u^2\right\}$$

$$=\frac{W}{g}udu\,dt = W\,dH\,dt \text{ ft.-lbs.} \qquad (8).$$

The work of expansion of $\Omega u dt$ cubic feet of air to $\Omega(u+du)dt$ at a pressure initially P is $\Omega P du dt$. But from (5)

$$u=\frac{RTW}{\Omega P}$$

$$\frac{du}{dP}=-\frac{RTW}{\Omega P^2}$$

and the work of expansion is

$$-\frac{RTW}{P}dP\,dt \text{ ft.-lbs.} \qquad (9).$$

The work of gravity is zero if the pipe is horizontal, and in many other cases is negligible.

The work of the pressures on the ends of the mass is

$$P\Omega u dt-(P+dP)\Omega(u+du)dt$$
$$=-(Pdu+udP)\Omega dt.$$

But if the temperature is constant, Pu is constant, and $Pdu+udP=0$. Hence the work of the pressures is zero. Adding the quantities of work and equating them to the change of kinetic energy,

$$WdHdt=-\frac{RTW}{P}dPdt-\zeta\frac{H}{m}Wdldt$$

$$dH+\frac{RT}{P}dP+\zeta\frac{H}{m}dl=0$$

$$\frac{dH}{H}+\frac{RT}{HP}dP+\zeta\frac{dl}{m}=0 \qquad (10).$$

But

$$u = \frac{\mathrm{RTW}}{\Omega \mathrm{P}}$$

$$\mathrm{H} = \frac{u^2}{2g} = \frac{\mathrm{R}^2\mathrm{T}^2\mathrm{W}^2}{2g\Omega^2\mathrm{P}^2}$$

$$\frac{d\mathrm{H}}{\mathrm{H}} + \frac{2g\Omega^2\mathrm{P}}{\mathrm{RTW}^2}d\mathrm{P} + \zeta\frac{dl}{m} = 0$$

For pipes of uniform section, Ω and m are constant, for steady motion W is constant, and for isothermal flow T is constant. Integrating,

$$\log \mathrm{H} + \frac{g\Omega^2\mathrm{P}^2}{\mathrm{W}^2\mathrm{RT}} + \zeta\frac{l}{m} = \text{constant.}$$

For

$l = 0$, let $\mathrm{H} = \mathrm{H}_1$ and $\mathrm{P} = \mathrm{P}_1$

$l = \mathrm{L}$, let $\mathrm{H} = \mathrm{H}_2$ and $\mathrm{P} = \mathrm{P}_2$

$$\log\frac{\mathrm{H}_2}{\mathrm{H}_1} + \frac{g\Omega^2}{\mathrm{W}^2\mathrm{RT}}(\mathrm{P}_2{}^2 - \mathrm{P}_1{}^2) + \zeta\frac{\mathrm{L}}{m} = 0 \quad . \quad (11),$$

where P_1 is the greater and P_2 the less pressure. By replacing H_1, H_2, and W,

$$\log\frac{\mathrm{P}_1}{\mathrm{P}_2} + \frac{g\mathrm{RT}}{u_1{}^2\mathrm{P}_1{}^2}(\mathrm{P}_2{}^2 - \mathrm{P}_1{}^2) + \zeta\frac{\mathrm{L}}{m} = 0 \quad . \quad (12).$$

Hence the initial velocity in the pipe is

$$u_1 = \sqrt{\left\{\frac{g\mathrm{RT}(\mathrm{P}_1{}^2 - \mathrm{P}_2{}^2)}{\mathrm{P}_1{}^2\left(\zeta\frac{\mathrm{L}}{m} + \log\frac{\mathrm{P}_1}{\mathrm{P}_2}\right)}\right\}} \quad . \quad . \quad (13).$$

When L is great, $\log \mathrm{P}_1/\mathrm{P}_2$ is small compared with the other term in the bracket. Then

$$u_1 = \sqrt{\left\{\frac{g\mathrm{RT}m}{\zeta\mathrm{L}} \cdot \frac{\mathrm{P}_1{}^2 - \mathrm{P}_2{}^2}{\mathrm{P}_1{}^2}\right\}} \quad . \quad . \quad (13a).$$

For pipes of circular section and diameter d in feet, $m = d/4$. Let $\mathrm{T} = 521$, then for air $\mathrm{RT} = 27710$, and let p_1, p_2 be the pressures in lbs. per square inch. Then

$$u_1 = \sqrt{\left\{222900\frac{d}{\zeta\mathrm{L}}\frac{p_1{}^2 - p_2{}^2}{p_1{}^2}\right\}} \quad . \quad . \quad (13b).$$

This equation is easily used. In some cases the approximate equation

$$u_1 = \left(1{\cdot}132 - 0{\cdot}726\frac{p_2}{p_1}\right)\sqrt{\left(222900\frac{d}{\zeta L}\right)} \quad . \quad (13c)$$

may be more convenient.

If the terminal pressure p_2 is required in terms of the initial pressure p_1, then

$$p_2 = p_1\sqrt{\left\{1 - \frac{\zeta u_1^2 L}{222900d}\right\}} \quad . \quad . \quad (14).$$

133. **Variation of pressure and velocity in long air mains.**—The following cases have been calculated to give an idea of the way in which pressure and velocity vary in long mains conveying air. The main is assumed to be 12 inches in diameter, and the coefficient of friction to be $\zeta = 0{\cdot}003$.

AIR MAINS

	At distances along main in miles.										
	0	1	2	3	4	5	6	7	8	9	10
CASE I.											
Pressure (absolute) in lbs. per sq. in.	115	112	110	107	104	101	99	96	92	89	86
Velocity in main in ft. per sec.	25	25·6	26·2	26·9	27·6	28·4	29·2	29·9	31·2	32·3	33·6
CASE II.											
Pressure (absolute) in lbs. per sq. in.	115	104	92	79	62	38	0	...	...	...	...
Velocity in main in ft. per sec.	50	55·1	62·3	73·2	93·1	149·0	∞	...	...	...	...

With an initial velocity of 25 feet per second the pressures decrease and the velocities increase slowly. With an initial velocity of 50 feet per second the variation of pressure and velocity is much more rapid. Beyond 5 miles the pressure is very small and the velocity enormous.

134. **Coefficient of friction.**—The author obtained values of the coefficient of friction from experiments made by Professors Riedler and Gutermuth on the mains conveying compressed air in Paris.[1] The mains were $11\frac{3}{4}$ inches in

[1] The details are given in Unwin, *Development and Transmission of Power*. London, 1894.

diameter, and in some tests the length of main tested was 10 miles. Experiments also were made by Mr. Stockalper on the compressed air mains at the St. Gothard tunnel, which were 0·492 and 0·656 feet in diameter.[1]

	$\zeta =$
Mean for 0·492 foot pipe . . .	0·00449
„ 0·656 „ . . .	·00377
„ 0·980 „ . . .	·00290

These results agree with the relation

$$\zeta = 0{\cdot}0027\left(1 + \frac{3}{10d}\right) \quad . \quad . \quad (15).$$

Mr. Batcheller, who has developed and carried out the remarkable systems of pneumatic transmission of parcels in the United States, has also made careful experiments on the resistance to the flow of air in mains. The pipes used were cast-iron pipes bored smooth.

Air is supplied at a pressure of 6 lbs. per square inch, and a carrier weighing 1 lb. 7 oz. passed through with the air. For a main $6\frac{1}{8}$ inches or 0·51 foot diameter the mean value of the coefficient of friction was 0·00435. By the formula above it would be 0·00429.

The coefficient is applicable to gases of other densities.

135. **Distribution of velocity in an air main.**—Threlfall has made experiments on the distribution of velocity in air mains by means of a Pitot tube (*Proc. Inst. of Electr. Eng.* 1903; *Proc. Inst. Mech. Eng.* 1904). The average ratio of mean to maximum central velocity was 0·873 constant at different velocities. The velocity at 0·775 of the radius from the centre was equal to the mean velocity. The highest velocity tried was 60 feet per second. The velocity curve on a diameter approximates to an ellipse.

[1] *Min. Proc. Inst. Civil. Eng.* lxiii. 29.

CHAPTER XII

UNIFORM FLOW OF WATER IN CANALS AND CONDUITS

136. In flow through pipes the section of the stream of water is determined by the cross section of the pipe, and the velocity depends not on the actual slope of the pipe but on that of the hydraulic gradient. When water flows along open channels, its surface is parallel to the bed of the stream, or nearly so, and the velocity depends on the actual slope of the surface of the water. If the slope of the stream-bed varies, the velocity of the stream varies also, being greater where the slope is greater, and *vice versa*. Since in steady motion the same quantity of water must pass every cross section in a given time, the cross sections of the stream must vary inversely as the velocity, being less where the slope is greater and greater where the slope is less.

In artificial canals and conduits for conveying water the slope is constant, and the cross sections of the channel are all similar. In such cases the velocity is uniform, the cross sections of the water stream normal to the direction of flow are equal and similar, and the water surface is parallel to the bed.

137. **Steady flow of water in channels of constant slope and section.**—Let $aa'bb'$ (Fig. 109) be two normal cross sections at a distance dl. Since $aa'bb'$ moves uniformly, the forces acting on it are in equilibrium. Let Ω be the area of cross section, χ the wetted perimeter $pq+qr+rs$, and $m=\Omega/\chi$ the hydraulic mean depth. Let v be the mean velocity, i the slope bc/ab in feet per foot, $W=G\Omega dl$ the weight of $aa'bb'$.

The external forces acting on $aa'bb'$ parallel to the direction of motion are—(*a*) the pressures on aa' and bb', which are equal and opposite since the sections are equal and similar; (*b*) the component of W parallel to ab, that is $G\Omega dl \times$ the cosine of the angle between W and ab, or $G\Omega dl \cos abc = G\Omega i dl$; (*c*) the friction on the surface of the channel.

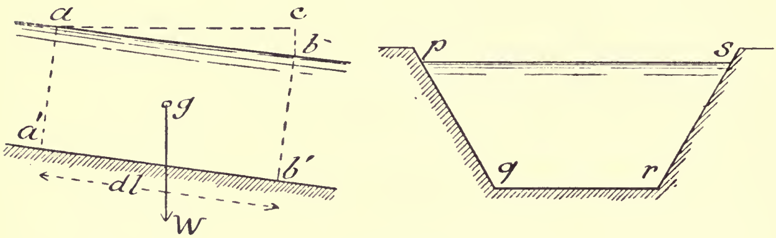

Fig. 109.

This is proportional to the wetted area χdl and to a function of the velocity which may be written $f(v)$, where $f(v)$ is the friction per square foot at the velocity v. Hence the frictional resistance is $\chi dl f(v)$. Equating the sum of the forces to zero—

$$G\Omega i dl - \chi dl f(v) = 0.$$

$$\frac{f(v)}{G} = \frac{\Omega}{\chi} i = mi.$$

But it has been shown in § 79 that $f(v) = \zeta G \frac{v^2}{2g}$, and hence

$$\zeta \frac{v^2}{2g} = mi \quad . \quad . \quad . \quad . \quad (1);$$

or if $\sqrt{\{2g/\zeta\}} = c$,

$$v = c\sqrt{(mi)} \quad . \quad . \quad . \quad . \quad (2);$$

where ζ and c are coefficients depending on the size of the channel and its roughness, and to a smaller extent on the velocity. This is the Chezy formula previously found for flow in pipes (§ 85).

In the case of open channels there is a much greater variation of size and of roughness than in the case of pipes, and consequently a wide variation of values of ζ and c must be expected in different cases. Imperfect as the theory above is, as a theory of flow, the formula is very convenient in practical calculations, and it can be made to give accurate results if the values of ζ and c are those found by experiment in similar cases. Hence the practically useful problem is to find means of selecting values of ζ and c in any given case.

138. **Darcy's research on the value of ζ for open channels.**—M. Darcy carried out an extremely important series of gaugings of the flow in artificial channels of very varied character, and M. Bazin, his successor, continued the investigation after his death. The conclusion arrived at was that the value of ζ depended chiefly on the roughness of the channel and its size, being less for large channels and greater for small ones. It appeared that the influence of size could be provided for by taking for ζ the expression

$$\zeta = \alpha\left(1 + \frac{\beta}{m}\right) \quad . \quad . \quad . \quad . \quad (3),$$

an expression similar to that previously found for pipes. To take account of the roughness of the channels, of which there is no definite measure, Darcy adopted a classification of channels according to their roughness. The following table gives the values of α and β for the different categories in which channels were classed:—

Kind of Channel.	α	β
I. Very smooth channels, sides of smooth cement or planed timber.	0·00294	0·10
II. Smooth channels, sides of ashlar, brickwork, planks.	0·00373	0·23
III. Rough channels, sides of rubble masonry or pitched with stone.	0·00471	0·82
IV. Very rough canals in earth	0·00549	4·10
V. Torrential streams encumbered with detritus .	0·00785	5·74

The last values (Class V.) are not Darcy's, but are taken from experiments by Ganguillet and Kutter on Swiss streams.

The following tables give the values of ζ calculated from Darcy's eq. (3) for use in eq. (1), and the corresponding values of c for use in eq. (2):—

Darcy's Values of ζ

Hydraulic Mean Depth m in Feet.	Values of ζ for Categories				
	I.	II.	III.	IV.	V.
0·5	·00353	·00545	·01243	·0505	·0981
1	·00323	·00458	·00857	·0279	·0529
2	·00308	·00414	·00664	·0167	·0304
5	·00300	·00389	·00546	0100	·0169
10	·00297	·00380	·00508	·0077	·0123
20	·00295	·00376	·00489	·0065	·0101
50	00294	00374	·00477	·0059	·0087
∞	·00294	·00373	·00471	·0055	·0079

Values of c in the Equation $v = c\sqrt{mi}$ deduced from Darcy's Values

Hydraulic Mean Depth m in Feet.	Values of c for Categories				
	I.	II.	III.	IV.	V.
0·5	135	109	72	36	26
1·0	141	119	87	50	35
2·0	145	125	98	62	46
5·0	146	129	109	80	62
10·0	147	130	113	91	72
20·0	148	131	114	100	80
50·0	148	131	116	104	86
∞	148	131	117	108	90

139. **Ganguillet and Kutter's Formula.**—In 1869, the Swiss engineers Messrs. Ganguillet and Kutter undertook a careful analysis of the results of gaugings of open channels then available. Proceeding in a purely empirical way to fit a formula to the results of gaugings, they arrived at the following cumbrous formula :—

$$v = \left[\frac{41\cdot6 + \frac{1\cdot811}{n} + \frac{0\cdot00281}{i}}{1 + \left(41\cdot6 + \frac{0\cdot00281}{i}\right)\frac{n}{\sqrt{m}}}\right]\sqrt{mi}. \quad . \quad (4),$$

in which n is a "coefficient of roughness," and the other symbols have the same signification as above. They adopted Darcy's method of classifying channels according to roughness, and arrived at the values of n given in the following table:—

KUTTER'S CONSTANT n.

$n = 0·009$. Well-planed timber.
$= 0·010$. Pure cement plaster, coated clean pipes.
$= 0·011$. Plaster in cement, iron pipes in best order.
$= 0·012$. Channels of unplaned timber.
$= 0·013$. Ashlar and good brickwork, iron pipes in ordinary condition.
$= 0·015$. Rough brickwork, incrusted iron.
$= 0·017$. Brickwork, ashlar, in bad condition, rubble in cement in good order.
$= 0·020$. Rough rubble in cement, stone pitching.
$= 0·025$. Rivers and canals in perfect order, free from stones or weeds, stone pitching in bad condition.
$= 0·030$. Rivers and canals in good order.
$= 0·035$. Rivers and canals in bad order.
$= 0·050$. Torrential streams encumbered with detritus.

In spite of its complication, Ganguillet and Kutter's formula has been widely adopted, especially in India, where its use has been facilitated by the publication of extensive tables.

A formula with so many arbitrary constants can of course be made to agree with any selected set of results of gauging more closely than a simpler formula. But the formula has only the authority of the results used in obtaining it. If some of these are untrustworthy, the formula must be untrustworthy also. Now the term $0·00281/i$ was introduced chiefly to force the formula into agreement with certain gaugings of the Mississippi, with very large values of m and small values of i. Those gaugings were made by the method of double floats, and it is now known that the velocities so obtained are probably greater than the true velocities.

Let
$$41·6 + \frac{·00281}{i} = k.$$

Then, as Bazin has shown, the formula can be put in the form—

$$\frac{\sqrt{mi}}{nv} - \frac{1}{1·811} = \frac{kn}{kn + 1·811}\left(\frac{1}{\sqrt{m}} - \frac{1}{1·811}\right);$$

and if $\sqrt{m}=1\cdot811$, or $m=3\cdot28$ feet or one metre exactly, then $\sqrt{mi}/v$ is equal to $n/1\cdot811$ for all classes of channels. That is, at this arbitrary limit $\sqrt{mi}/v$ is independent of the term involving the slope in all cases, and the influence of the term in brackets is + or − according as m is > or < one metre. This result is improbable. Further, the comparison which Bazin has made of the formula, with a more extensive list of gaugings than were available when it was deduced, shows that it departs widely in some cases from the results of experiment.

Calculation by Kutter's formula is a little facilitated if the equation is put in the form

$$\mathrm{M}=n\left(41\cdot6+\frac{0\cdot00281}{i}\right)$$

$$v=\frac{\sqrt{m}}{n}\left(\frac{\mathrm{M}+1\cdot811}{\mathrm{M}+\sqrt{m}}\right)\sqrt{mi}. \qquad (5).$$

$i=$	Values of M for $n=$						
	0·010.	0·012.	0·015.	0·017.	0·020.	0·025.	0·030.
·00001	3·2260	3·8712	4·8390	5·4842	6·4520	8·0650	9·6780
·00002	1·8210	2·1852	2·7315	3·0957	3·6420	4·5525	5·4630
·00004	1·1185	1·3422	1·6777	1·9014	2·2370	2·7962	3·3555
·00006	0·8843	1·0612	1·3264	1·5033	1·7686	2·2107	2·6529
·00008	0·7672	0·9206	1·1508	1·3042	1·5344	1·9180	2·3016
·00010	0·6970	0·8364	1·0455	1·1849	1·3940	1·7425	2·0910
·00025	0·5284	0·6341	0·7926	0·8983	1·0568	1·3210	1·5852
·00050	0·4722	0·5666	0·7083	0·8027	0·9444	1·1805	1·4166
·00075	0·4535	0·5442	0·6802	0·7709	0·9070	1·1337	1·3605
·00100	0·4441	0·5329	0·6661	0·7550	0·8882	1·1102	1·3323
·00200	0·4300	0·5160	0·6450	0·7310	0·8600	1·0750	1·2900
·00300	0·4254	0·5105	0·6381	0·7232	0·8508	1·0635	1·2762

The formula can, however, hardly be used in practical work without the aid of extensive tables.

140. **Bazin's later investigation of the results of experiments on flow in channels.**—M. Bazin has lately returned to the study of the results of gaugings on flow in channels, and has examined a more extensive series than has previously been available (*Annales des Ponts et Chaussées*, 1897). He remarks that in the Darcy relation

$$\frac{mi}{v^2} = \alpha\left(1 + \frac{\beta}{m}\right) . \quad . \quad . \quad . \quad (6)$$

which proved to be suitable for pipes, the constants α and β have no very wide range of values so long as the experiments on pipes only are considered. But in the case of open channels, with their great diversity of size and character of surface, the constants α and β have so wide a range of values that the expression ceases to be sufficiently useful as a guide. In addition, the form of the expression is defective. For if m increases indefinitely, $mi/v^2 = \alpha$, and this has a different value for each class of channels. But it is reasonable to suppose that in indefinitely large channels the influence of the roughness of the stream bed must indefinitely diminish, so that in very large channels mi/v^2 should tend to a value common to all classes of channels.

After many trials, M. Bazin has adopted the following relation, which obviates the difficulty just stated :—

$$\sqrt{\left(\frac{mi}{v^2}\right)} = \alpha + \frac{\beta}{\sqrt{m}} \quad . \quad . \quad . \quad (7),$$

in which the constant α has the same value, 0·00635 (English measures), for all classes of channels, and β varies with the character of the surface of the bed.

If the results of gauging are plotted so that the ordinates $y = \sqrt{mi}/v$, and the abscissæ $x = 1/\sqrt{m}$, the expression may be written

$$y = 0{\cdot}00635 + \beta x,$$

or if $\gamma = \beta/\alpha$

$$y = 0{\cdot}00635(1 + \gamma x) \quad . \quad . \quad . \quad (8),$$

the equation to a straight line. Results of this equation plotted give a pencil of rays starting from $x = 0$, $y = 0{\cdot}00635$. The inclination measured by the angular coefficient $0{\cdot}00635\gamma$ increases as the roughness of the bed increases. Fig. 110 shows a plotting.

The following are Bazin's values of the roughness coefficient γ in eq. (8) :—

BAZIN'S VALUES OF γ

I.	*Very smooth.*—Smooth cement, planed timber . .	$\gamma=0{\cdot}109$
II.	*Smooth.*—Planks, ashlar, brick	$\gamma=0{\cdot}290$
III.	*Rough.*—Rubble masonry	$\gamma=0{\cdot}83$
III. *bis.*	*Rough.*—Earth newly dressed, or pitched in whole or part with stone	$\gamma=1{\cdot}54$
IV.	*Very rough.*—Ordinary earth canals	$\gamma=2{\cdot}36$
V.	*Excessively rough.*—Canals encumbered with weeds or boulders	$\gamma=3{\cdot}17$

For practical calculations Bazin's new formula can be put in the form—

$$v=\frac{157{\cdot}6\sqrt{mi}}{1+\dfrac{\gamma}{\sqrt{m}}} \quad . \quad . \quad . \quad . \quad (9).$$

In this form the equation is extremely convenient for calculation. If m is known and v is to be found, the equation

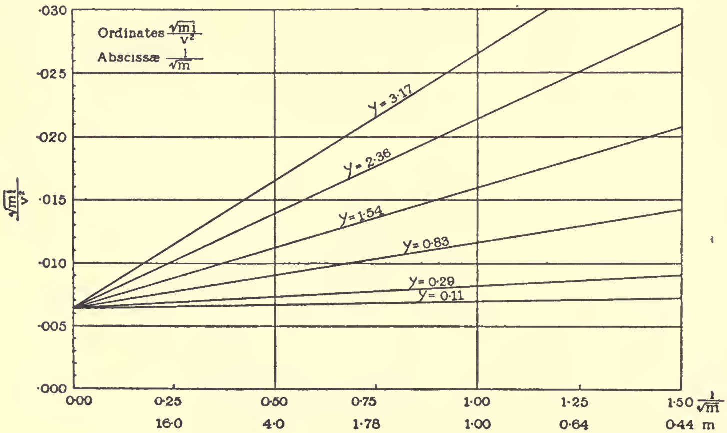

Fig. 110.

can be used quite straightforwardly. If v is given and the dimensions of the channel are to be found, it is best to proceed by approximation. Choose from tables or experience any roughly probable value of m. With this calculate $1+\gamma/\sqrt{m}$, and with this find a new value of m by eq. (9). With this new value recalculate $1+\gamma/\sqrt{m}$, and then find a more approximate value of m by eq. (9). These two steps of approximation are generally sufficient.

It will be seen that the Chezy form of equation

$$\left.\begin{aligned} \zeta\frac{v^2}{2g} &= mi, \\ \text{or} \quad v &= c\sqrt{(mi)} \end{aligned}\right\} \quad . \quad (10)$$

is identical with Bazin's, if

$$\left.\begin{aligned} \zeta &= \frac{2g\left(1+\frac{\gamma}{\sqrt{m}}\right)^2}{157{\cdot}6^2} = 0{\cdot}00259\left(1+\frac{\gamma}{\sqrt{m}}\right)^2 \\ \text{or} \quad c &= \frac{157{\cdot}6}{1+\frac{\gamma}{\sqrt{m}}} \end{aligned}\right\} \quad . \quad (10a)$$

The following tables give values of $1+\frac{\gamma}{\sqrt{m}}$ calculated with Bazin's values of γ, for a series of values of m and for all classes of channels. Also the corresponding values of ζ and c in eq. (10).

In selecting values of ζ or c it should be remembered that the roughness is often increased by organic growths after the channel has been some time in use. Fitzgerald has given some interesting observations on a large aqueduct at Sudbury. The culvert is circular, 9 feet diameter with an invert of 13·2 feet radius; it is lined with brick, with cement joints. It has been found that if the surface of the brickwork is not cleaned it accumulates in the course of a year so much organic slime that the discharge flowing full is diminished 10 per cent (*Trans. Amer. Soc. Civil Engineers*, xliv. 87).

[TABLE.

Values of $1 + \frac{\gamma}{\sqrt{m}}$ in Bazin's Equation

Hydraulic Mean Depth in Feet.	Values of $1+\frac{\gamma}{\sqrt{m}}$ for $\gamma=$					
	·109	·290	·83	1·54	2·36	3·17
	for Categories					
	I.	II.	III.	III. *bis*	IV.	V.
1	1·109	1·290	1·830	2·540	3·360	4·170
2	1·077	1·205	1·587	2·088	2·668	3·241
3	1·063	1·167	1·479	1·888	2·361	2·829
4	1·055	1·145	1·415	1·770	2·180	2·585
5	1·049	1·129	1·371	1·688	2·054	2·416
6	1·044	1·118	1·338	1·628	1·062	2·293
7	1·041	1·110	1·313	1·582	1·892	2·198
8	1·039	1·102	1·294	1·545	1·835	2·122
9	1·036	1·097	1·276	1·512	1·785	2·055
10	1·034	1·092	1·262	1·486	1·745	2·001
11	1·033	1·087	1·249	1·463	1·710	1·954
12	1·032	1·084	1·240	1·445	1·682	1·916
13	1·030	1·080	1·229	1·426	1·653	1·878
14	1·029	1·077	1·221	1·411	1·630	1·846
15	1·028	1·075	1·214	1·397	1·608	1·817
16	1·027	1·073	1·207	1·385	1·590	1·791
17	1·026	1·071	1·202	1·374	1·573	1·770
18	1·026	1·068	1·195	1·363	1·556	1·748
19	1·025	1·066	1·190	1·352	1·540	1·725
20	1·024	1·065	1·185	1·344	1·528	1·710
25	1·022	1·058	1·166	1·308	1·472	1·634
36	1·018	1·048	1·138	1·257	1·393	1·528

BAZIN'S VALUES OF ζ IN EQUATION (10)

Hydraulic Mean Depth in Feet.	Values of ζ for $\gamma=$					
	·109	·290	·830	1·54	2·36	3·17
	for Categories					
	I.	II.	III.	III. *bis*	IV.	V.
1	·00319	·00429	·00867	·01670	·02923	·04506
2	·00300	·00375	·00652	·01129	·01844	·02719
3	·00293	·00352	·00567	·00922	·01442	·02072
4	·00287	·00339	·00518	·00810	·01230	·01730
5	·00285	·00329	·00486	·00738	·01092	·01512
6	·00282	·00323	·00463	·00686	·00997	·01362
7	·00279	·00318	·00445	·00647	·00927	·01250
8	·00279	·00313	·00432	·00619	·00872	·01170
9	·00277	·00311	·00422	·00592	·00826	·01092
10	·00277	·00308	·00411	·00572	·00789	·01036
11	·00277	·00305	·00404	·00554	·00756	·00989
12	·00276	·00304	·00398	·00541	·00732	·00950
13	·00274	·00302	·00391	·00525	·00707	·00914
14	·00274	·00300	·00385	·00515	·00688	·00883
15	·00274	·00298	·00380	·00505	·00670	·00855
16	·00273	·00297	·00377	·00497	·00655	·00831
17	·00271	·00296	·00373	·00488	·00639	·00810
18	·00271	·00295	·00370	·00481	·00626	·00791
19	·00271	·00294	·00366	·00473	·00613	·00771
20	·00270	·00292	·00363	·00467	·00605	·00756
25	·00269	·00290	·00352	·00442	·00562	·00691
36	·00267	·00285	·00336	·00409	·00502	·00606

BAZIN'S VALUES OF c IN THE EQUATION $v=c\sqrt{mi}$

Hydraulic Mean Depth in Feet.	Values of c for $\gamma=$					
	I. 0·109	II. 0·290	III. 0·83	III. *bis* 1·54	IV. 2·36	V. 3·17
1	142·0	122·5	86·2	62·1	47·0	37·8
2	146·4	131·0	99·3	75·5	59·1	48·6
3	148·2	135·2	106·5	83·6	66·8	55·8
4	149·8	137·8	111·4	89·1	72·3	61·0
5	150·2	139·8	115·0	93·4	76·8	65·3
6	151·0	141·2	117·9	96·8	80·4	68·8
7	151·8	142·2	120·2	99·7	83·3	71·7
8	152·1	143·4	122·0	101·9	85·9	74·1
9	152·4	143·9	123·4	104·3	88·2	76·8
10	„	144·5	125·1	106·0	90·3	79·0
11	„	145·3	126·2	107·8	92·3	80·7
12	152·7	145·5	127·1	109·1	93·8	82·3
13	153·3	146·0	128·2	110·7	95·4	83·9
14	„	146·5	129·3	111·7	96·8	85·4
15	„	147·0	130·1	112·9	98·0	86·7
16	153·5	147·2	130·7	113·7	99·2	88·0
17	154·1	147·4	131·3	114·8	100·3	89·1
18	„	147·7	131·9	115·7	101·4	90·2
19	„	147·9	132·6	116·6	102·4	91·4
20	154·4	148·4	133·2	117·3	103·1	92·3
25	154·7	149·0	135·2	120·6	107·0	96·5
36	155·2	149·4	138·4	125·5	113·2	103·0

As examples of the great variation of the coefficients ζ and c in cases of great variation of roughness some results of gauging the Loch Katrine aqueducts may be given. These aqueducts are largely tunnelled in rock, and are only partly lined with cement mortar. In the case of the older Loch Katrine aqueduct, which was largely unlined and in parts very rough, very low values of c were found. Thus, Mr. Gale[1] states that for the Mugdock tunnel, $1\frac{1}{2}$ miles long and exceptionally rough, $c=56·9$. The Loch Katrine tunnel, with about 11 per cent lined, gave $c=67·8$. At the northerly end of the aqueduct, with $21\frac{1}{2}$ per cent lined, the average value of c was 72. In the case of the newer aqueduct the whole of the

[1] "Loch Katrine Waterworks," *Proc. Inst. of Engineers and Shipbuilders in Scotland*, 1895.

invert was concreted, and about 50 per cent completely lined. The area when running full was 64·8 sq. feet in the lined part, and 78·3 sq. feet in the unlined part. With water flowing 7 feet deep, $m = 3{\cdot}1$ in the unlined and 2·87 in the lined part. The gradient is 1 in 5500. The following are some results obtained by Mr. Bruce (*Proc. Inst. Civil Engineers*, cxxiii.) :—

LOCH KATRINE CONDUIT

Depth of water . . .	1·72	2·42	2·73	2·94
Cross section of water . .	14·2	20·8	23·7	25·6
Discharge cubic feet per sec. Q .	26·6	45·8	53·5	53·5
Mean velocity	1·87	2·21	2·26	2·08
Hydraulic mean depth . .	1·23	1·60	1·74	1·81
Value of *c*	125·4	129·3	126·9	135·6

141. **Channels of circular section.**—Aqueducts and sewers are sometimes of circular section, and concrete open channels have been made of semicircular section. For calculations of the discharge of such channels running partly full the following table is useful. Let r be the radius of the channel and d the depth of water :—

$\frac{d}{r}$	$\frac{m}{r}$	$\frac{\Omega}{r^2}$	$\sqrt{\frac{m}{r}}$	$\frac{\Omega\sqrt{m}}{r\sqrt{r}}$
·05	·032	·021	·179	·0037
·10	·052	·060	·229	·0137
·15	·096	·107	·310	·0332
·20	·128	·165	·357	·0589
·30	·185	·294	·430	·1264
·40	·242	·450	·492	·2214
·50	·293	·614	·541	·3321
·60	·343	·795	·586	·4658
·70	·387	·979	·622	·6089
·80	·429	1·175	·655	·772
·90	·466	1·371	·683	·935
1·00	·500	1·571	·707	1·109
1·2	·556	1·968	·746	1·469
1·4	·592	2·349	·769	1·807
1·6	·608	2·694	·780	2·098
1·8	·596	2·978	·772	2·300
2·0	·500	3·141	·707	2·219

$\sqrt{m}/\sqrt{r}$ and $(\Omega\sqrt{m})/(r\sqrt{r})$ are the relative velocities and discharges with different depths in the channel. The greatest velocity is when the depth is $1\cdot 6r$, and the greatest discharge when the depth is $1\cdot 8r$ approximately.

142. **Egg-shaped channels or sewers.**—In sewers for discharging storm water and house drainage the volume of flow is extremely variable; and there is a great liability for deposits to be left when the flow is small, which are not removed during the short periods when the flow is large. The sewer in consequence becomes choked. To obtain uniform scouring action the velocity of flow should be constant or nearly so; a complete uniformity of velocity cannot be obtained with any form of section suitable for sewers, but an approximation to uniform velocity is obtained by making the sewers of oval section. Various forms of oval have been suggested, the simplest being one in which the radius of the crown is double the radius of the invert, and the greatest width is two-thirds the height. The section of such a sewer is shown in Fig. 111, the numbers marked on the figure being proportional numbers.

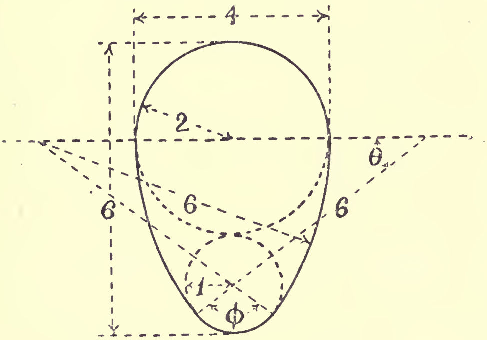

Fig. 111.

The following results facilitate calculations on sewers flowing partly filled. Let d be the greatest width (that is four units in the figure). Then—

Depth of Stream.	Area of Section.	Wetted Perimeter.	Hydraulic Mean Depth.	Relative Discharge.
Full . .	$1\cdot 1485d^2$	$3\cdot 965d$	$0\cdot 2897d$	1·000
$\frac{2}{3}$ full .	$0\cdot 7558d^2$	$2\cdot 394d$	$0\cdot 3157d$	0·687
$\frac{1}{3}$ full .	$0\cdot 2840d^2$	$1\cdot 375d$	$0\cdot 2066d$	0·209

The last column gives the relative discharge neglecting the variation of the coefficient c.

143. **Trapezoidal channels.**—Artificial channels are commonly trapezoidal in section, the side slopes being determined by the stability of the banks and the kind of protection against degradation adopted.

Angle of Side Slopes.	Ratio of Side Slopes.	Character of Bank.
ϕ		
90°	0 to 1	Planks or masonry.
63°20	0·5 to 1	Masonry or brick walls.
45°	1 to 1	Stone pitching.
33°40	$1\frac{1}{2}$ to 1	Firm earth.
26°30	2 to 1	Loose earth.
21°48	$2\frac{1}{2}$ to 1	Loose earth.
18°20	3 to 1	Loose earth.

Let B be the top and b the bottom width, d the depth, ϕ the slope angle, and n the slope ratio, so that $\tan \phi = 1/n$.

$$\text{Top width} = B = b + 2nd.$$
$$\text{Area of section} = \Omega = (b + nd)d.$$
$$\text{Wetted perimeter} = \chi = b + 2d\sqrt{n^2 + 1}.$$

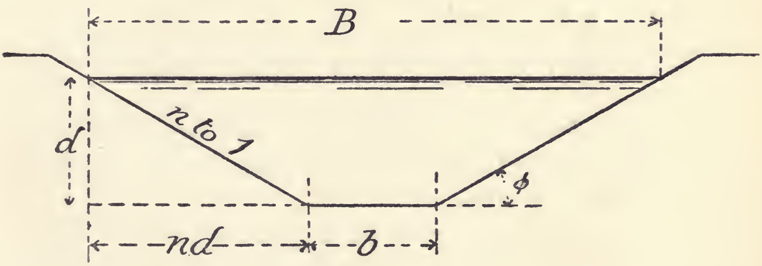

Fig. 112.

144. **Trapezoidal channel of minimum section for given side slopes.**—Various practical considerations determine the general form of the section of a channel. In a navigation canal the depth is fixed by the draught of the boats. In large irrigation canals the depth is limited so as to avoid interference with subsoil drainage, and the canals are of a

width equal to ten or twenty times the depth. In valuable ground the width is restricted and a rectangular section is used. The longitudinal slope i is determined by the slope of the country and the limiting velocity which can be permitted consistently with the stability of the canal bed. The side slopes are fixed by the character of the banks.

If a channel is constructed for a given discharge and given longitudinal and given side slopes, then there is a proportion of breadth to depth which makes the area of cross section, and therefore the amount of excavation, a minimum. The resistance to flow depends on the wetted perimeter, and the velocity will be greatest and the section least for that form for which the wetted perimeter is least.

Differentiating the expressions for Ω and χ given above, and equating to zero,

$$\left(\frac{db}{dd} + n\right)d + b + nd = 0,$$

$$\frac{db}{dd} + 2\sqrt{(n^2 + 1)} = 0.$$

Eliminating db/dd,

$$\frac{b}{d} = 2\{\sqrt{(n^2 + 1)} - n\}.$$

$n = 0$	0·5	1·0	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$\frac{b}{d} = 2$	1·24	0·82	0·60	0·48	0·38	0·32.

If this value of b is inserted in the expressions for Ω and χ, we get a very convenient characteristic of channels of the most economical section—

$$m = \frac{\Omega}{\chi} = \frac{\{2d\sqrt{(n^2 + 1)} - nd\}d}{4d\sqrt{(n^2 + 1)} - 2nd} = \frac{d}{2} \quad . \quad . \quad (11).$$

That is, in channels of the most economical form, with given side slopes, the hydraulic mean depth is half the actual depth. It will easily be seen that this is a characteristic of the semicircle, the half square, and the half hexagon. A simple geometrical construction shows that for all such channels the sides and bottom are tangents to a semicircle having its centre on the water-surface.

Let Fig. 113 represent a trapezoidal channel of minimum section, for side slopes of n to 1. Let E be the centre of the

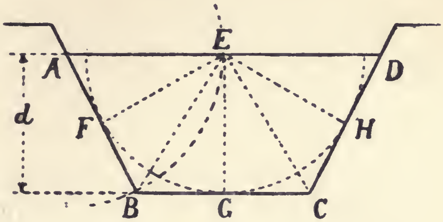

Fig. 113.

water-surface, and drop perpendiculars EF, EG, EH on the sides. Let $AB = CD = a$; $BC = b$; $EF = EH = c$; $EG = d$.

$$\Omega = AEB + EBC + ECD$$

$$= ac + \frac{1}{2}bd,$$

$$\chi = 2a + b.$$

Since the hydraulic mean depth is half the actual depth, $\Omega/\chi = d/2$,

$$ac + \frac{1}{2}bd = \frac{1}{2}(2a + b)d,$$

$$\therefore\ c = d.$$

That is, EF, EG, EH are all equal, and a semicircle with centre at E touches AB, BC, CD. A circle struck from A with radius AE will pass through B.

Proportions of Channels of the most Economical Section

Side Slope Angle.	Ratio of Side Slopes.	$\frac{\Omega}{d^2}$	$\frac{m}{d}$	$\frac{b}{d}$	$\frac{\chi}{d}$	$\frac{\chi}{\sqrt{\Omega}} = k$	$\frac{1}{\sqrt{k}}$	$\frac{B}{d}$
90°	0 to 1	2·000	0·5	2·000	4·00	2·83	·594	2·000
63° 20′	1 „ 2	1·736	„	1·236	3·47	2·63	·616	2·236
45°	1 „ 1	1·828	„	0·828	3·66	2·71	·607	2·828
33° 40′	$1\frac{1}{2}$ „ 1	2·106	„	0·606	4·21	2·90	·587	3·606
26° 30′	2 „ 1	2·472	„	0·472	4·94	3·14	·564	4·472
21° 48′	$2\frac{1}{2}$ „ 1	2·885	„	0·385	5·77	3·40	·543	5·385
18° 20′	3 „ 1	3·325	„	0·325	6·65	3·64	·524	6·325

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The velocity in a channel is

$$v = \sqrt{\frac{2gi}{\zeta}} \sqrt{\frac{\Omega}{\chi}}.$$

Let $k = \chi / \sqrt{\Omega}$

$$v = \sqrt{\frac{2gi}{\zeta}} \sqrt{\frac{\sqrt{\Omega}}{k}} \quad . \quad . \quad . \quad (12).$$

For a given section of channel the velocity and therefore the discharge will be greatest if $1/\sqrt{k}$ is greatest, so that this can be taken as a value figure for channels of various forms.

It is not generally convenient to adopt exactly the form of a channel of minimum section, but the theorem indicates the form towards which actual channel sections should tend if practicable. For other forms of section $m > d/2$, and the mean velocity for a given longitudinal slope is less. The other limit to the value of m is d. For in a channel of great width b, and small depth d, $\Omega = bd$ and $\chi = b$ nearly, so that $m = d$ nearly.

The mean velocity varies as $\sqrt{m}$. Hence, taking the extreme cases of $m = d/2$ and $m = d$, the corresponding mean velocities will have the ratio

$$\frac{v_1}{v_2} = \sqrt{\frac{d}{2}} \Big/ \sqrt{d} = 0{\cdot}709.$$

For a given discharge the areas of the channels would be in the inverse proportion.

145. **Discharge of a channel with different depths of water flowing.**—Consider a rectangular channel with a stream of water of width b and depth d. The area is $\Omega = bd$, the hydraulic mean depth is $m = bd/(b + 2d)$. The discharge is

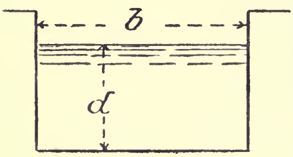


Fig. 114.

$$Q = \Omega v = c\Omega \sqrt{mi} = cbd \sqrt{\frac{bdi}{b + 2d}},$$

that is, as i is constant for a given channel, and c will only vary a little with the variation of size,

$$Q \text{ varies as } \frac{d^{\frac{3}{2}}}{\sqrt{(b+2d)}}.$$

If Q_1 is the discharge determined by gauging for a depth d_1, then the discharge for any other depth is

$$Q = Q_1\left(\frac{d}{d_1}\right)^{\frac{3}{2}}\sqrt{\frac{b+2d_1}{b+2d}} \quad . \quad . \quad . \quad (13).$$

Example.—A rectangular channel draining an area of 572,000 acres is 60 feet wide, with a depth of water of 3 feet it is found to discharge 400 cubic feet per second. Then equation (13) becomes

$$Q = 442\frac{d^{\frac{3}{2}}}{(30+d)^{\frac{1}{2}}}.$$

The following table gives the mean monthly depth of water deduced from daily observations and the discharge calculated by this formula. From this the total discharge of the stream in each month can be found, and this divided by the drainage area, 24,910 million square feet, gives the depth of rainfall in each month equivalent to the stream discharge. The observed mean rainfall is also given. The ratio of the stream discharge or off-flow from the ground to the rainfall varies with the season, and is an important datum in certain problems of water storage.

DISCHARGE AND RAINFALL ON A DRAINAGE AREA

Month.	Mean Depth of Water in Feet, d.	Mean Discharge, Cubic Feet per Second, Q.	Total Discharge per Month in Million Cubic Feet.	Equivalent Depth on Drainage Area in Inches.	Mean Rainfall in Inches.	Ratio of Discharge to Rainfall Per Cent.
January .	4·40	693	1856	·894	2·15	41·6
February .	4·50	720	1742	·840	1·78	47·2
March .	4·25	663	1775	·855	1·70	50·3
April . .	3·90	583	1511	·728	1·87	39·0
May . .	3·25	450	1205	·581	1·55	37·5
June . .	2·80	361	936	·451	1·73	26·1
July . .	3·30	459	1229	·592	1·48	40·0
August .	3·00	400	1071	·516	1·29	40·0
September .	2·85	371	962	·463	1·48	31·3
October .	3·05	410	1097	·529	1·44	36·8
November .	3·10	419	1086	·523	2·00	26·1
December .	3·80	565	1513	·729	1·95	37·4

The mean depth of water and mean rainfall are the average of five years' observations. The smaller the intervals of time for which the means are taken, the more approximate would be the result.

146. **Parabola of discharge.**—In a rectangular channel

of width b and depth d, if b is large compared with d, $\Omega = bd$ and $m = d$ nearly. Then

$$Q = cbd\sqrt{di},$$

that is, for a given channel Q varies as $d^{\frac{3}{2}}$. In a triangular channel the width b is proportional to d, so that $\Omega = \mu d^2$ and $m = \nu d$, where μ and ν are constants depending on the inclination of the sides of the channel. Then

$$Q = c\mu d^2\sqrt{\nu di},$$

or Q varies as $d^{\frac{5}{2}}$. Ordinary channels are of a form between these two, so that at least for a limited variation of d in a given channel the discharge may be taken to vary approximately as d^2. In that case, if the depths of water are taken as ordinates and the discharges as abscissæ the curve of discharge is a parabola. It often happens that an approximate estimate of the total discharge of a stream is required when the only continuous records available are readings on a gauge of the surface-level of the stream. In such cases it may be assumed that Q varies as $(d + \delta)^2$ for the range of variation of level which occurs in such cases, where d is the actual depth of water and δ a quantity to be determined. Suppose that, by gauging, the discharges Q_1, Q_2 for two depths d_1, d_2 of water in the stream have been ascertained.

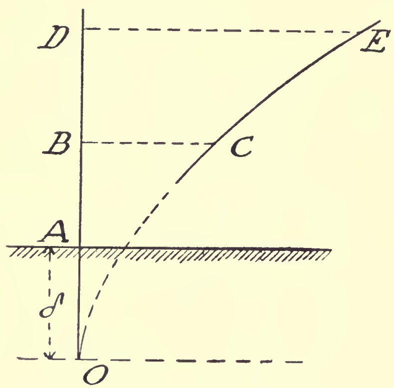


Fig. 115.

Take $AB = d_1$, $BC = Q_1$, $AD = d_2$, $DE = Q_2$ (Fig. 115). Then C and E are points on the discharge curve, which is assumed to be approximately a parabola with its vertex at some point O at δ below A. From the properties of the parabola

$$BC = Q_1 = 4a(d_1 + \delta)^2,$$
$$DE = Q_2 = 4a(d_2 + \delta)^2,$$

where a is the parameter of the parabola. Hence

$$\left.\begin{aligned}\delta &= \frac{d_1 - d_2\sqrt{\dfrac{Q_1}{Q_2}}}{\sqrt{\dfrac{Q_1}{Q_2}} - 1} \\ a &= Q_1/\{4(d_1 + \delta)^2\}\end{aligned}\right\} \quad . \quad . \quad . \quad (14).$$

When δ and a have been determined, the discharge for any value of d is easily calculated to an approximation sufficient in many cases where comparisons of stream discharge and rainfall have to be made.

147. **General distribution of velocity at different points in the cross section of a channel.**—Even a cursory observation of flow in an open channel shows that the velocity of translation along the channel is greater towards the centre and surface and less towards the bottom and sides. A more careful investigation indicates some marked peculiarities, and a knowledge of these is of practical importance in considering various methods of gauging the volume of flow in streams.

By means to be described presently, the mean forward velocity at a number of points in the cross section of a stream can be determined. This was first accomplished in a quite satisfactory way by Darcy, and an example from his work will be taken as an illustration.

Fig. 116 shows the cross section of a rectangular channel, 0·25 metre deep and 0·8 metre wide, in which the velocity was observed at 36 points at the intersection of the verticals *ee*, *ff*, . . ., and the transversals *aa*, *bb*, . . . The velocities at each point on a transversal set up from the transversal vertically give points on a transverse velocity curve. Thus *aaa* is the transverse velocity curve along *aa*, *bbb* that along *bb*, and so on. Similarly, the velocities at each point on a vertical set off from the vertical horizontally give points on a vertical velocity curve. Thus *ee* is the vertical velocity curve for the vertical *ee*, *ff* that for *ff*, and so on. The vertical curves show that the greatest velocity is not at the surface, but somewhat below it. From the level of greatest velocity at any vertical the velocity decreases upwards and downwards. There is another way of representing the distribution of velocity. If at points on the vertical curves where the velocities are 1·2, 1·1, 1·0, 0·9, and 0·8 metres per second, horizontals are drawn to the corresponding verticals, points are found in the section on curves of equal velocity. These curves correspond to the contours of a solid whose base is the cross section of the stream, whose height at any point is the velocity at that point, and whose volume is proportional to the discharge of the stream per second. The maximum velocity is on the centre vertical below the surface, and from that point the velocity decreases in all directions.

Messrs. Fteley and Stearns made very careful gaugings of the brick conduit at Sudbury with different depths of water flowing. The conduit is 9 feet diameter, with an invert of 13·2 feet in radius, the height of the conduit being 7·7 feet (*Trans. Amer. Soc. Civil Engineers*, 1883). With the greatest flow the velocity was measured at 167 points in the cross section. The following are some of the results obtained :—

Sudbury Conduit

Depth of water . .	4·54	4·01	3·00	2·03	1·51
Hydraulic mean depth .	2·33	2·19	1·84	1·38	1·07
Mean velocity . .	2·97	2·90	2·62	2·18	1·90
Maximum velocity . .	3·37	3·32	3·06	2·47	2·14
Bottom velocity (about) .	2·20	2·15	2·10	1·75	1·60
Ratio mean/maximum .	·88	·87	·86	·88	·89
,, mean/bottom .	1·35	1·35	1·25	1·25	1·19
Discharge per sec. Q .	111·5	94·4	62·4	33·3	20·1
Value of c . . .	140·0	139·5	137·5	133·2	129·5

148. **Depression of the point of greatest velocity.**—In calm weather the maximum velocity is below the surface, and this is not due, as has been sometimes supposed, to a resistance of the air similar to that of the stream bed, for it is the case with a wind down stream which should accelerate the surface layer. In a rectangular channel the velocity is highest at the centre and falls to about half depth at the sides. In channels with sloping sides it rises from the centre outwards, and may be at the surface at the edges of the stream. The cause of the depression has been much discussed. Eddies of water stilled by contact with the bed are thrown off, and wander through all parts of the stream, but accumulate and spread out at the surface. In the Mississippi gaugings it was found that

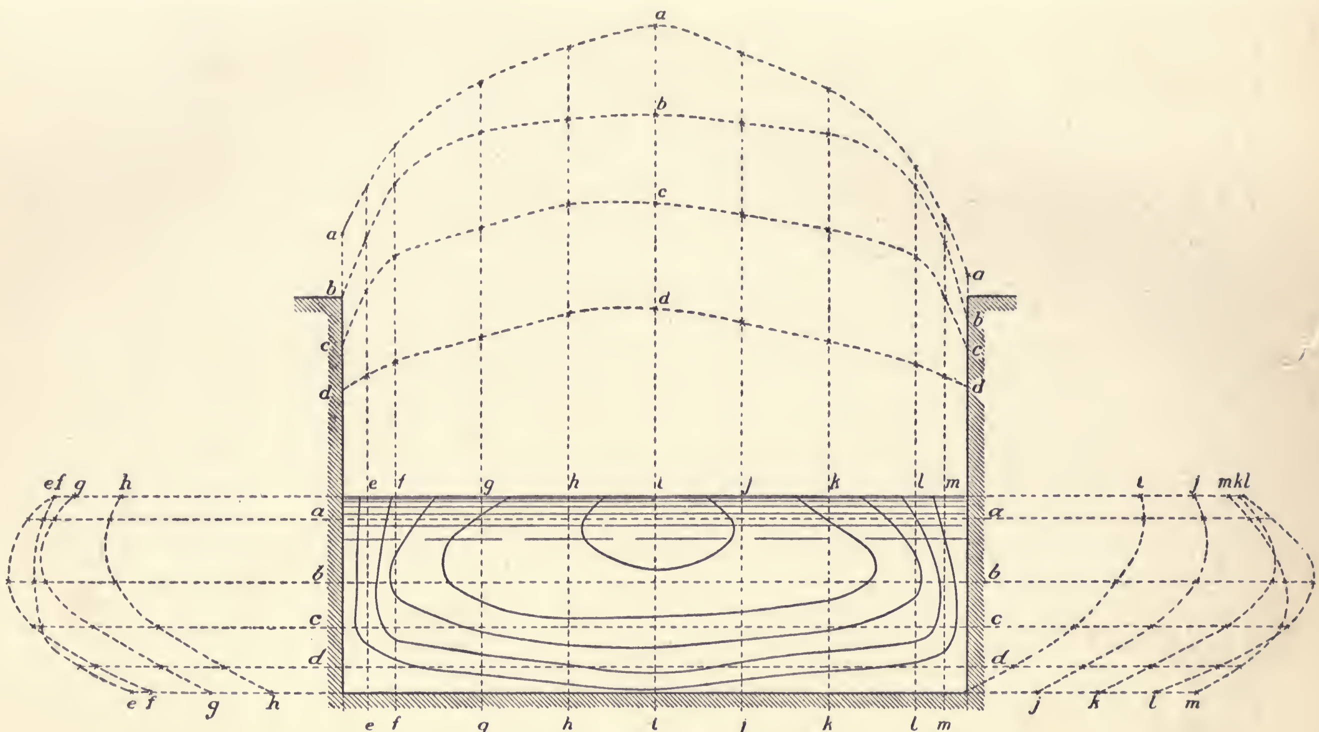


Fig. 116.

the depression of the line of maximum velocity increased with an upstream and decreased with a downstream wind, but this result has not been found in some other cases. Perhaps it depends on the presence or absence of waves or ripples on which the wind can act.

149. **Vertical velocity curve.**—In purely viscous stream-line motion the vertical velocity curve would be a parabola with a horizontal axis at the free surface. In ordinary turbulent motion in streams the vertical velocity curve agrees fairly well with a parabola having a horizontal axis at the level of maximum velocity. Without assuming this to be more than a convenient approximation, it is a result useful in discussing the relations of the velocities at different depths in a stream.

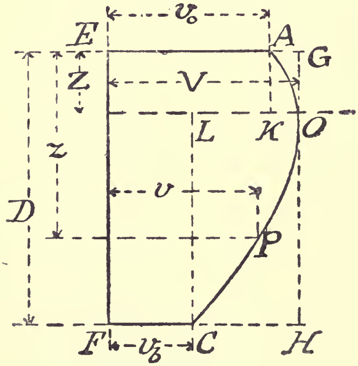


Fig. 117.

Let AOC (Fig. 117) be a parabolic velocity curve, the axis being a horizontal through O. Let V be the maximum, v_o the surface, v_b the bed velocity, and v the velocity at any point P. Let Z be the depth of the filament of greatest velocity, z the depth of P, and D the whole depth of the stream. Then from the properties of the parabola

$$(z - Z)^2 = K(V - v),$$

where K is the parameter of the parabola. Hence

$$v = V - \frac{(z - Z)^2}{K} \quad . \quad . \quad . \quad (15).$$

Mean velocity at a vertical.—If a fairly large number of velocities at equal distances on a vertical are observed, the arithmetic mean is very approximately the mean velocity at the vertical. If the number is small the arithmetic mean is less than the true mean velocity. If through observed points a fair vertical velocity curve can be drawn, the mean velocity at the vertical is the area of the curve divided by the depth of the stream.

Assuming that the vertical velocity curve is a parabola such as is shown in Fig. 117, the mean velocity is the mean ordinate of AOC, that is—

$$U = \frac{1}{D} \{\text{area EAOCF}\}$$

$$= \frac{1}{D}\left\{\text{area EGHF} - \frac{1}{3}(\text{area AGOK} + \text{OLCH})\right\}$$

$$= V - \frac{1}{3}(V - v_o)\frac{Z}{D} - \frac{1}{3}(V - v_b)\left(1 - \frac{Z}{D}\right)$$

$$= \frac{2}{3}V + \frac{1}{3}\left\{\frac{v_o Z}{D} + v_b\left(1 - \frac{Z}{D}\right)\right\}.$$

But by the equation above, when

$$z = O,\ v_o = V - \frac{Z^2}{K}$$

$$z = D,\ v_b = V - \frac{(D - Z)^2}{K}$$

$$U = V - \frac{1}{3KD}\{Z^3 + (D - Z)^3\}$$

$$= V - \frac{D^2}{3K} + \frac{DZ}{K} - \frac{Z^2}{K}$$

$$= v_o + \frac{DZ}{K} - \frac{1}{3}\frac{D^2}{K} \quad . \quad . \quad . \quad . \quad (16).$$

If $v_{\frac{1}{2}D}$ is the velocity at half depth, putting $z = \frac{D}{2}$ in the equation above,

$$v_{\frac{1}{2}D} = v_o + \frac{DZ}{K} - \frac{D^2}{4K},$$

so that the half-depth velocity is greater than the mean velocity at the vertical only by the small quantity $D^2/(12K)$, a result which depends on the assumption of a parabolic curve, but which cannot be much wrong, and this is useful in practical gauging. In Cunningham's Roorkee gaugings with floats, much attention was paid to this point, and the mid-depth velocity was found a little greater than the mean velocity at the vertical in forty-two cases out of forty-six. The average of a large number of results gave $U/v_{\frac{1}{2}D} = 0{\cdot}94$ to $0{\cdot}98$.

If two velocities can be observed on a vertical, then a better approximation to the mean velocity U can be found. Thus the parabolic law shows that if the velocity at the surface and $\frac{2}{3}$ depth is observed,

$$U = \frac{1}{4}(v_0 + 3v_{\frac{2}{3}D}).$$

150. **Transverse velocity curves.**—In a channel symmetrical about its centre line, the transverse velocity curve at any level shows a maximum velocity at the centre, a slow decrease of velocity towards the sides, more rapid as the banks are approached, and very rapid near the banks. In an

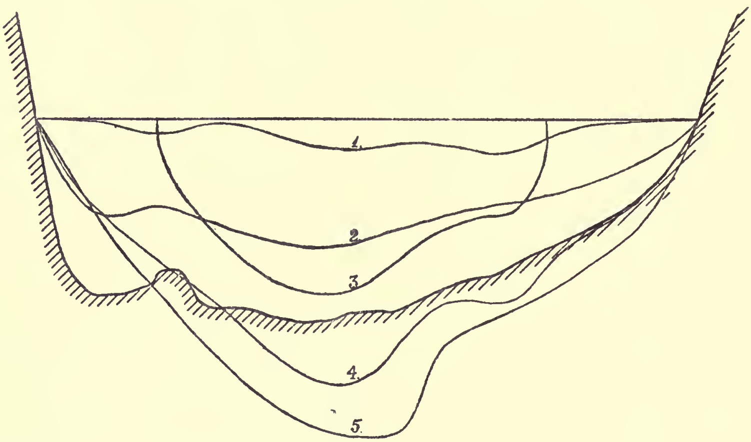


Fig. 118.

unsymmetric channel the greatest velocity is over the deepest part of the stream.

Fig. 118 shows the results of a very careful current-meter gauging of the Eger at Falkenau by Wilhelm Plenkner of Prague. The river is 321·6 feet wide. The vertical scale is exaggerated ten times. The curve 1 passes through the points of maximum velocity, which throughout is somewhat below the surface. Curve 2 passes through the points of mean velocity on each vertical a little below half depth. Curve 3 passes through points where the velocity is equal to the mean velocity of the whole section. Curve 4 is the transverse mean velocity curve, that is, its ordinates are the mean velocities on each vertical. Curve 5 is the transverse surface velocity curve.

151. **Ratio of mean and surface velocities.**—In a gauging of the Rhine at Basel the velocity at 0·58 of the depth was found to be equal to the mean velocity on the same vertical. The ratio of the mean to the surface velocity on one vertical varied from 0·77 to 0·85, the average being 0·82. The ratio of the mean velocity for the whole cross section to the greatest surface velocity was on the average 0·73. Harlacher found the same ratio in gauging the Elbe. The following table gives some values:—

	Mean Velocity of Whole Section. v_m.	Mean Surface Velocity. u_m.	Greatest Surface Velocity. u_o.	$\frac{v_m}{u_m}$.	$\frac{v_m}{u_o}$.
Elbe (high water) .	3·61	4·17	4·66	·86	·77
" (average water) .	3·12	3·61	4·17	·86	·75
" (low water) .	2·49	2·79	3·64	·89	·68
Eger at Warta . .	1·75	1·75	3·21	1·00	·55
" at Falkenau .	2·54	2·77	4·43	·92	·57
" " " .	1·31	1·48	2·26	·89	·58
Sazawa at Poric .	1·61	1·60	2·72	1·00	·59
" " .	·82	·84	1·15	·98	·71
" " .	1·90	1·67	2·61	1·14	·73
Moldau at Budweis .	2·55	2·67	3·53	·96	·72
" " .	5·71	6·51	8·02	·88	·71
" " .	3·07	3·57	4·28	·86	·72

152. **Aqueducts.**—Any work by which water is conveyed may be termed an aqueduct, but the term is usually applied to important works in which water flows by gravitation, and specially to those conveying the water-supply of towns. Where the fall of the country is suitable the water may be conveyed in a channel contoured to the slope of the hydraulic gradient. The channel may be an open channel, such as the conduit which brings water from Staines to London. More commonly it is covered to protect the water from deterioration, but the water flows precisely as in an open channel. Generally,

such an aqueduct is of a composite character—part in tunnel where the ground is above the hydraulic gradient; part in cut and cover, that is, built in an open trench and then covered in. Across valleys the aqueduct must be carried on piers, or more commonly the water is conveyed in one or more pipes, termed inverted siphons, falling from the hydraulic gradient at one end and rising to it again at the other end.

Roman aqueducts.—Amongst the most striking engineering works of antiquity, of which parts still exist, are the aqueducts constructed for the water-supply of Rome and other cities of the Roman empire. The Appian Aqueduct at Rome was constructed in 313 B.C., and conveyed water from springs ten miles distant from the city, in a channel $2\frac{1}{2}$ feet wide by 5 feet deep. Others were subsequently constructed, till there were fourteen aqueducts, of lengths varying from 11 to 59 miles, and aggregating 359 miles. Of the total length, 55 miles were on arches, and the remainder chiefly underground. The channels were lined with cement and roofed with slabs, and the gradients varied from perhaps 1 in 500 to 1 in 3000. Herschel estimates the total supply to the city of Rome at 50 million gallons daily, with an additional supply to districts outside the city. The water was often distributed by lead pipes, and lead siphons of 12 to 18 inches diameter have been found.

Types of aqueducts.—Fig. 119 shows cross sections of some important aqueducts. A, B, C are sections of the new Loch Katrine aqueduct.

153. **Examples of aqueducts.**—(1) **Loch Katrine aqueduct.**—This was designed to convey 50 million gallons per day from Loch Katrine to Glasgow, but the roughness of the channel was not fully allowed for, and it probably carries only about 40 million gallons. The top water surface in Loch Katrine is 367 feet above mean sea-level, and the water is delivered into a service reservoir at Mugdock, 26 miles distant, where the top water-level is 317 feet above mean sea-level. Of the 26 miles of aqueduct, $3\frac{3}{4}$ are cast-iron pipes across valleys, $11\frac{3}{4}$ miles are in tunnel, and $10\frac{1}{4}$ miles are bridges and masonry in cut and cover. The tunnels are 8 feet in diameter, with a fall of 10 inches per mile. The channel in cut and cover has the same gradient as the tunnels.

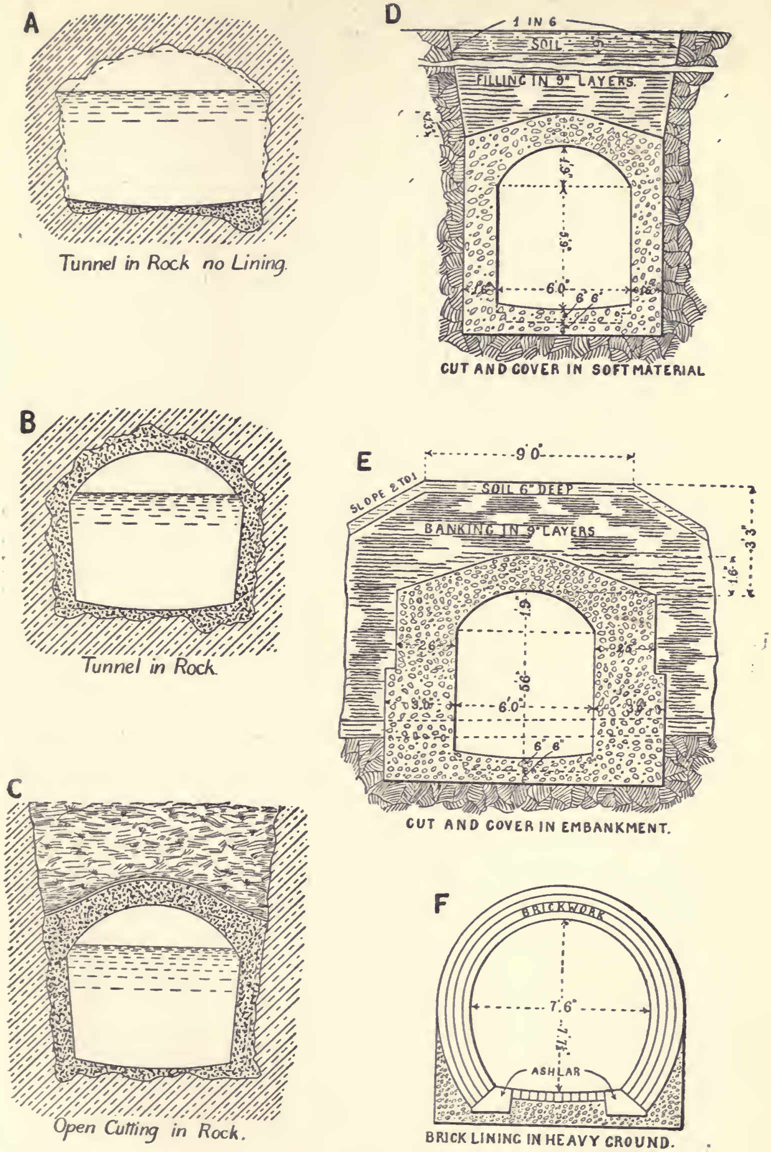

A
Tunnel in Rock no Lining.
D
1 IN 6
SOIL
FILLING IN 9" LAYERS.
CUT AND COVER IN SOFT MATERIAL
B
Tunnel in Rock.
E
SLOPE 2 TO 1
SOIL 6" DEEP
BANKING IN 9" LAYERS
CUT AND COVER IN EMBANKMENT.
C
Open Cutting in Rock.
F
BRICKWORK
ASHLAR
BRICK LINING IN HEAVY GROUND.

Fig. 119.

Portions of the pipe line consist of two 48-inch and one 36-inch pipe, or of four 36-inch pipes, the general hydraulic gradient being 5 feet per mile. An additional aqueduct has now been constructed following generally the line of the old aqueduct, with the object of ultimately maintaining a supply to the city of 100 million gallons per day. In the new aqueduct, with water flowing 7 feet deep, the area of section is 78·3 square feet. The wetted perimeter 24·9 feet. The hydraulic mean depth 3·1 feet. The slope 1 in 5500. The estimated discharge is nearly 72 million gallons per day (*Proc. Inst. Civil Engineers*, 1883).

(2) **Thirlmere aqueduct,** for the supply of water to Manchester.—This is designed to convey 50 million gallons per day from Lake Thirlmere to a service reservoir at Prestwich, a distance of 96 miles. There are 14 miles of tunnel, 37 miles of cut and cover, and 45 miles of cast-iron pipes. The tunnels are 7 feet 1 inch wide, the side walls 5 feet high, and the arch rises 2 feet. They are for the most part lined with concrete, but in parts only the floor is lined. The thickness of floor lining is $4\frac{1}{2}$ inches in close rock to 18 inches in bad ground. Walls 12 inches to 18 inches thick. Arch ring 15 inches thick. Where the tunnels are unlined their width is increased to 8 feet 6 inches, to allow for the greater friction due to irregularities of the rough rock surface. The cut-and-cover channels are also of concrete. At full supply the water in the conduit will be 5 feet 6 inches deep. The pipe line was designed to have three parallel 48-inch pipes in the first part, and five parallel lines of 40-inch pipe in the later part, the pipes varying in thickness from 1 to $1\frac{3}{4}$ inches, with socket joints run with lead. The second pipe laid has been increased in diameter from 40 to 45 inches. The surface of the lake when full is at 584 feet above O.D. The aqueduct starts at 527 feet above O.D. and ends at Prestwich at 353 above O.D. The ruling gradient is 20 inches per mile, but extra fall is given to the pipe line. Along the aqueduct there are manholes at every quarter mile.

New Croton aqueduct, New York, U.S.A.—In this aqueduct there are 30 miles of tunnel, 1 mile of cut and cover, and $2\frac{1}{2}$ miles of pipe. About 7 miles of the tunnel is of circular form $12\frac{1}{4}$ feet in diameter, and is under pressure, amounting at one point to 120 feet of head. The remainder of the tunnel is horseshoe-shaped, 13 feet 7 inches in width and height. For 25 miles the gradient is 0·7 feet per mile. The tunnel is lined with brickwork 12 to 24 inches thick. The discharge is about 300 cubic feet per second.

154. **River bends.**—In rivers flowing in alluvial plains the windings which already exist tend to increase in curvature by the scouring away of material from the outer bank and the deposition of detritus along the inner bank. The sinuosities sometimes increase till a loop is formed with only a narrow strip of land between the two encroaching branches of the river. Finally a "cut off" may occur, a waterway being opened through the strip of land and the loop left separated from the stream, forming a horseshoe-shaped lagoon or marsh. Professor James Thomson has pointed out (*Proc. Royal Soc.* 1877, p. 356; *Proc. Inst. of Mech. Engineers*, 1879, p. 456) that the usual supposition is that the water, tending to go forwards in a straight line, rushes against the outer bank and scours it, at the same time creating deposits at the inner bank. That view is very far from a complete account of the matter, and Professor James Thomson has given a much more ingenious account of the action at the bend, which he has completely confirmed by experiment.

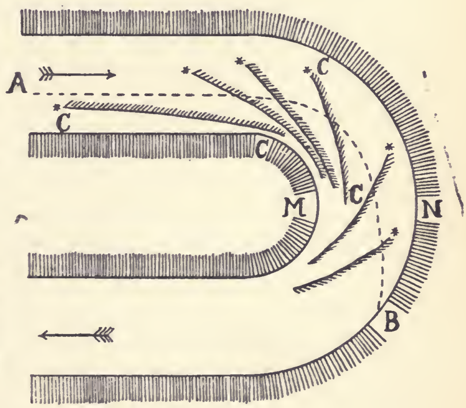


Fig. 120.

When water moves round a circular curve under the action of gravity only, it takes a motion like that in a free vortex. Its velocity is greater parallel to the axis of the stream at the inner than at the outer side of the bend. Hence the scouring at the outer side and the deposit at the inner side of the bend are not due to mere difference of velocity of flow in the general direction of the stream; but, in virtue of the centrifugal force, the water passing round the bend presses outwards, and the free surface in a radial cross section has a slope from the inner side upwards to the outer side (Fig. 121). For the greater part of the water flowing in curved paths, this difference of pressure produces no tendency to transverse motion. But the water immediately in contact with the rough bottom and sides of the channel is retarded, and its centrifugal force is insufficient to balance the pressure due to the greater depth at the outside of the bend. It therefore flows inwards towards the inner side of the bend, carrying with it detritus which is deposited at the inner bank. Conjointly with this flow inwards along the bottom and sides, the general mass of water must flow outwards to take its place. Fig. 120 shows the directions of flow as observed in a small artificial stream, by means of light seeds and specks of aniline dye. The lines CC show the directions of flow immediately in contact with the sides and bottom. The dotted line AB shows the direction of motion of floating particles on the surface of the stream.

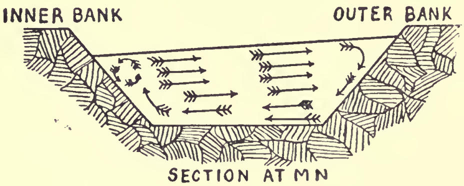


Fig. 121.

Problems.[1]

1. A river has the following section: bottom width, 300 feet; depth of water, 20 feet; side slopes, 1 to 1; fall, 1 foot per mile. Find the discharge, using Darcy's coefficient for earth channels.
   Darcy, $c = 100$; $Q = 37{,}340$ cubic feet per second.
2. A canal is to be constructed for a discharge of 2000 cubic feet per second. The fall is 1·5 feet per mile; side slopes, 1 to 1; bottom width, ten times the depth; $c = 120$. Find the dimensions of the canal.
   Depth, 6·23 feet; bottom width, 62·3 feet.
3. Required the dimensions of a trapezoidal channel of the most economical section to convey 600 cubic feet per second, with a fall of 2 feet per mile, and side slopes $1\frac{1}{2}$ to 1. $\zeta = \cdot 0035$.
   Depth, 7·48 feet; bottom width, 4·49 feet.

[1] When not otherwise stated, Bazin's values of the coefficients for channels have been used.

4. Recalculate the discharge of the channel determined in (3), taking Bazin's coefficient for sides covered with stone pitching.

388 cubic feet per second.

5. An irrigation canal in earth with side slopes $1\frac{1}{2}$ to 1 conveys 600 cubic feet per second at a velocity of $2\frac{1}{2}$ feet per second. Design a suitable canal section with a depth of 3 feet.

Area of section, 240 square feet; $m = 2{\cdot}687$; $c = 64{\cdot}6$; $i = {\cdot}000557$ or 2·94 feet per mile.

6. A brick culvert, 5 feet 6 inches in diameter and 4000 feet long, conveys 150 cubic feet per second when running full. Find the fall in feet necessary. 7·3 feet.

7. An oval brick sewer, flowing two-thirds full, is 4 feet wide and 6 feet high. Find the fall in feet per mile to give a velocity of 3 feet per second, and the discharge.

2·4 feet; 36·3 cubic feet per second.

8. A canal is to be cut in earth with side slopes 2 to 1, and a fall of 9 inches per mile. The discharge is to be 6000 cubic feet per minute, and the depth 3 feet. Find the dimensions of canal. (Solve by approximation.)

Assuming $m = 3$, $b = 18{\cdot}2$ feet; then $m = 2{\cdot}29$, and $b = 24$ feet.

9. A semicircular channel of smooth cement is 5 feet deep and slopes at 1 in 1000. Find the discharge.

115·7 cubic feet per second.

10. A trapezoidal channel of the most economical form, with sides of rubble masonry, has a depth of 10 feet and side slopes of 1 to 1. Find the discharge when the fall is 18 inches per mile.

$b = 8{\cdot}2$; $v = 3{\cdot}28$; $\Omega = 182$; $Q = 597$.

11. A rectangular ashlar masonry channel is 12 feet wide and 4 feet deep, and has a slope of 1 in 5000. Find the velocity and discharge. 2·91 feet per second; 139·6 cubic feet per second.

12. The water section in the aqueduct at Dijon is 2 feet wide and 1 foot deep, and the sides are smooth cement. The slope is 1 in 1000. Find the velocity and discharge.

3·05 feet per second; 6·1 cubic feet per second.

13. Find the equation to the discharge parabola of the Sudbury aqueduct from the data in § 147, and draw the curve.

$Q = 4(d + 0{\cdot}738)^2$.

14. A channel has an hydraulic mean depth of 5 feet. Compare the discharges if the sides are of smooth cement, and of rubble masonry. 1·30 to 1.

15. The top width of an irrigation canal is 200 feet, the depth 10 feet, and the side slopes 3 to 1. The slope is 15 inches per mile. Find the discharge. $v = 3{\cdot}86$; $Q = 6567$.

CHAPTER XIII

GAUGING OF STREAMS

155. For various purposes the engineer needs to gauge the flow of streams. For instance, in determining the value of a fall as a source of water power the volume of flow throughout the year must be ascertained. The flood discharge is of little value unless storage reservoirs can be constructed. The ordinary summer flow and the minimum flow are factors of greater importance generally. Then again, the water-supply of many towns is derived from the drainage of large gathering grounds, flowing off by a stream. In considering the sufficiency of the supply, the flow must be determined partly by rainfall observations, partly by gauging the stream so as to establish a relation between the rainfall and flow from the catchment basin. Usually gauging operations are carried on for a considerable period, as accurate statistics are required in the settlement of difficult questions such as the apportionment of compensation water. Lastly, in the management of irrigation works it is frequently necessary to gauge the flow in canals and distribution channels.

156. **Water-level gauge.**—Wherever stream discharge measurements are carried on, water-level gauges should be established, on which readings of the varying water-level can be taken simultaneously with the velocity observations. The zero of the gauge should be connected by levelling with a permanent bench mark, and the zero should be below the lowest water-level to avoid minus readings. The scale of the gauge should be in feet and tenths. The scale may be fixed to a pile driven into the stream bed or fixed to a masonry structure. Sometimes a scale attached to a float

is convenient, the reading being taken against a fixed mark. Automatic gauges are used in important investigations. A cord attached to a float gives motion through reducing-gear to a pencil which records the water-level on a drum driven by clockwork.

157. **Mean velocity calculated from the longitudinal slope.**—If the longitudinal surface slope of a stream is determined in a part where the channel is of fairly regular section, then the discharge can be ascertained by the formulæ of flow, subject, however, to the difficulty of selecting a coefficient suitable to the character of the stream. In most cases, however, the surface slope is an extremely small quantity, generally less than 1 in 5000, and the oscillations of the water surface render its determination difficult. The slope in natural streams often differs to some extent on the two sides as the current sets to one bank or the other. In Cunningham's experiments on the Ganges Canal twelve measurements of slope on symmetrical 2000 and 4000 feet lengths differed by 25 per cent, but the site was probably a specially difficult one. Usually the mean of the slope determined at the two banks is taken as the virtual slope of the stream.

158. **Gauging by observation of the velocity of flow.**—In streams of moderate size the most accurate method of gauging is by a weir constructed for the purpose across the stream. But often it is impracticable to erect a weir, and the operation of gauging is then effected by determining the cross section Ω and the mean velocity v_m of the stream. The discharge is $Q = \Omega v_m$. For gauging purposes a straight and unobstructed reach of the stream should be selected, where the cross section is fairly uniform in area and form. Then two series of observations are required: (1) a survey of one or more cross sections of the stream; (2) observations of the velocity at one or more points of the cross section.

159. **Measurement of transverse sections.**—The depth of the stream is ascertained at a series of points, equidistant if possible, along the line of the required cross section. For small streams a wire may be stretched across, with equal distances of about 10 feet or less marked on it by tags. If the wire is first set up on land and stretched with a given

weight, the position of the tags can be fixed so that their horizontal distances are equal. The wire is then stretched across the stream with the same tension. The depth at each tag can be taken with a light graduated and loaded rod. Care should be taken that the wire is perpendicular to the thread of the stream.

For large rivers the position of soundings is fixed by angular measurement. A base line AB (Fig. 122), parallel to the stream, is first laid out and measured. Next staves are set up at CA and D along the line of the required section and at right angles to AB. Observers are placed at C and B; a boat drops down stream, and at the moment it crosses the section at E the observer C signals, the sounding is taken in the boat, and B with a box sextant takes the angle ABE. This is repeated till soundings at a sufficient number of points have been ascertained from which to plot the cross section. The soundings may be taken by a graduated rod if the depth is less than 15 or 18 feet, or by a weighted cord or lead-line or chain. If the velocity of the stream is considerable, the weight should be disc-shaped or lenticular, so as to expose as little surface normal to the current as possible. A simple winch and wire are convenient for lowering the weight, and the winch may have a counter which shows the depth. From the observations the section is plotted, and the area Ω and wetted perimeter χ are calculated.

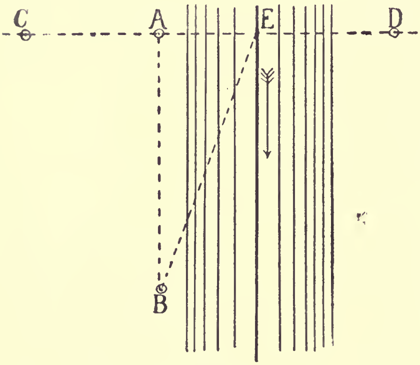


Fig. 122.

The area of a plotted cross section may be obtained by a planimeter, or by dividing the width of the stream into n equal spaces and measuring the $n+1$ vertical ordinates at the dividing points. Let b be the width of a division, and $h_0, h_1, \ldots h_n$ be the measured ordinates. Then by the trapezoidal rule the area is

$$\Omega = \frac{b}{2}\{(h_0 + h_n) + 2(h_1 + h_2 + \ldots + h_{n-1})\}.$$

If the end ordinates are zero,

$$\Omega = b(h_1 + h_2 + \ldots + h_{n-1}) \quad . \quad . \quad (1).$$

If there are ten spaces, Simpson's rule may be used with somewhat greater accuracy—

$$\Omega = \frac{b}{3}\{(h_0 + h_{10}) + 4(h_1 + h_3 + \ldots + h_9) + 2(h_2 + h_4 + \ldots + h_8)\} \quad . \quad . \quad . \quad (2).$$

As the level of a stream varies from time to time, a level gauge should be fixed before operations are begun. The water-level should be noted on this gauge when taking the cross sections, and afterwards when the velocity observations are made.

If velocity observations are to be taken, at least two cross sections should be measured and the average values of χ and Ω computed for use in calculations.

160. **Float gauging.**—The velocity in a stream may be directly observed by taking the time of transit of a float over a measured length of stream. **Surface floats** are used to determine surface velocities. They may be balls, or discs of wood or cork. A tuft of oily cotton-wool, which does not get wet, is a useful means of rendering them visible. Captain Cunningham at Roorkee[1] used thin deal discs 3 inches diameter and 1 inch thick. **Sub-surface floats.**—To observe velocities below the surface, a large relatively heavy float (Fig. 123), connected by a thin wire (about 0·015 inch thick) to a small, light surface float, has been used. It is assumed that the motion of the combination is practically that of the sub-surface float, the influence of the surface float and connector being negligible. But if the large float is made nearly of the density of water, so that the surface float may be small, the eddies prevent the large float from keeping its depth. If the

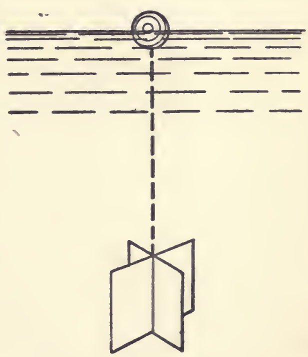


Fig. 123.

[1] *Roorkee Hydraulic Experiments*, by Captain Allan Cunningham, R.E. (Thomason College Press).

lower float is heavy, the upper float must be large, and then its influence on the motion of the combination is not negligible and the velocity observed is not the true sub-surface velocity. Fig. 124 shows the form of sub-surface float used by Captain Cunningham at Roorkee. It consists of a hollow metal ball connected to a disc of cork. The influence of the connecting wire on the motion increases as the depth of the sub-surface

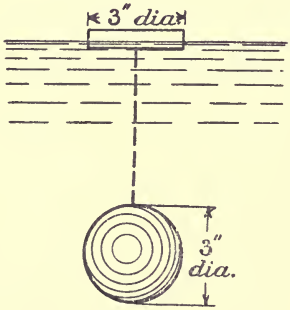


Fig. 124.

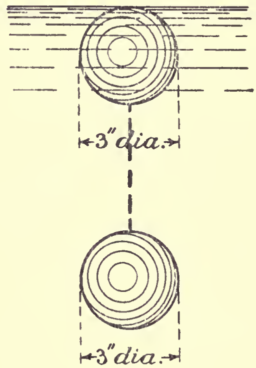


Fig. 125.

float increases, and the observations become less trustworthy the greater the depth. **Twin floats.**—Fig. 125 shows two equal balls connected by a wire, the lower being loaded so that the combination just floats. The motion of the twin float must be nearly the mean of the surface velocity and the velocity at the depth at which the lower float swims. Thus if v_s is the surface velocity, and v_d the velocity at the depth d, the velocity of the twin float is $v = \frac{1}{2}(v_s + v_d)$. If v_s is ascertained by means of a surface float,

$$v_d = 2v - v_s \quad . \quad . \quad . \quad . \quad (3).$$

Captain Cunningham found the twin float more satisfactory than the sub-surface float, but the influence of the connector increases with the depth, and also the uncertainty as to whether the lower float keeps its depth or is tossed about by eddies in the water.

161. **Rod floats.**—Fig. 126 shows another form of float

used in some early researches. Its use has been revived by Captain Cunningham in India. In its simplest form it consists of a wooden rod with a cap at the lower end in which shot can be placed, so that the rod floats nearly upright, and with little projection above the water-surface. Wood rods may be made in lengths which can be screwed together. Cunningham used sets consisting of lengths 0·1, 0·2, 0·3 . . . up to 1 foot, and 1, 2, 3 . . . up to 12 feet; but tube rods of tinplate about 1 inch in diameter made of graduated lengths, adjusted to float at definite depths in still water and marked, were found more convenient. He found that the velocity of a rod, the immersed length of which was nearly equal to the depth of the stream, is a close approximation to the mean velocity on the vertical corresponding to its path, and he considered it the most accurate means of float gauging in suitable conditions. At any rate the gaugings showed that though the rod necessarily was shorter than the full depth of the stream, its velocity was very approximately the mean velocity at the vertical corresponding to its path. The rod float is certainly free from the chief objections to the sub-surface or twin float.

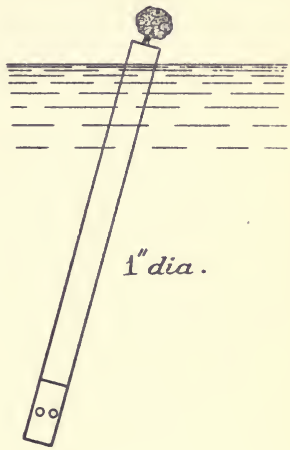


Fig. 126.

162. **Float paths and time of transit.**—In the part of the stream selected for gauging two cross sections are fixed at a measured distance apart, and the time of transit of the floats between these sections is observed. The floats are thrown in above the upper section at various points in the width of the stream. In careful gauging the exact float paths should be observed. The two end sections may be marked by cords stretched across the stream, and if these have coloured tags at equal distances it is possible to note approximately the distance from the bank at which each float crosses each section. If l is the distance between the cross sections, and t the

time of transit, then $v = l/t$ is the velocity of the stream at the position of the float path normal to the cross sections.

In large streams the float paths must be observed by box sextants or theodolites. A base line AB (Fig. 127) is set out parallel to the thread of the stream. Ranging rods are set up at A_1, B_1, on lines at right angles to the base, usually on the lines of surveyed transverse sections. Observers are stationed at A and B with sextants. Floats are dropped into the stream from a boat upstream of AA_1. As the float crosses AA_1 at C, the observer at A signals, and B takes the angle ABC. When the float crosses BB_1 at D, B signals and A takes the angle BAD. An observer also notes with a chronograph the time between the signals. All the data are so obtained for calculating the velocity and plotting the float path CD.

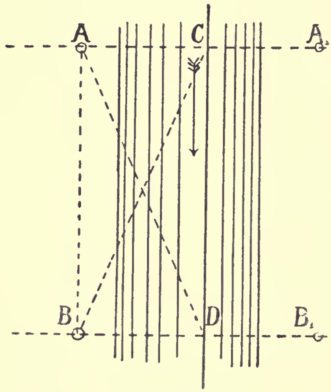


Fig. 127.

The best length of the float path depends on the velocity and regularity of the stream; lengths of 50 to 250 feet have been used. The longer the base the less the error of the time observation. But, on the other hand, the longer the base the more the floats stray about into regions of differing velocity. In the Ganges Canal researches Captain Cunningham found a run of 50 feet best for the central parts of the stream, but near the banks this had to be shortened to $12\frac{1}{2}$ feet. With any longer run the floats strayed to the banks.

163. **The screw current meter.**—This was termed by early hydraulicians the Woltmann Mill. In improved form it is the most generally useful, and, if properly calibrated, the most accurate apparatus for measuring velocity in streams. A screw propeller, like that in Fig. 128, delicately supported, drives a counter by a worm. The counter can be put in or out of gear by a cord. The meter is fixed on a rod or length of gas-pipe, and held in the water in the desired position. A rudder keeps the propeller facing the stream. The counter is put in gear for one minute or more, and from the difference

of the counter readings divided by the duration of run the velocity is calculated. In its ordinary form the meter must be lifted from the water to read the counter, and cannot be conveniently used at greater depths than about seven feet.

Harlacher screw current meter.—This is a current meter with an electrically actuated indicator showing the revolutions. The meter is on a sleeve which slides on a substantial hollow cast-iron rod, and can be moved up and down the rod by a cord passing down inside it. The rod is long enough to be firmly fixed in the bottom of the river. The cord is wound

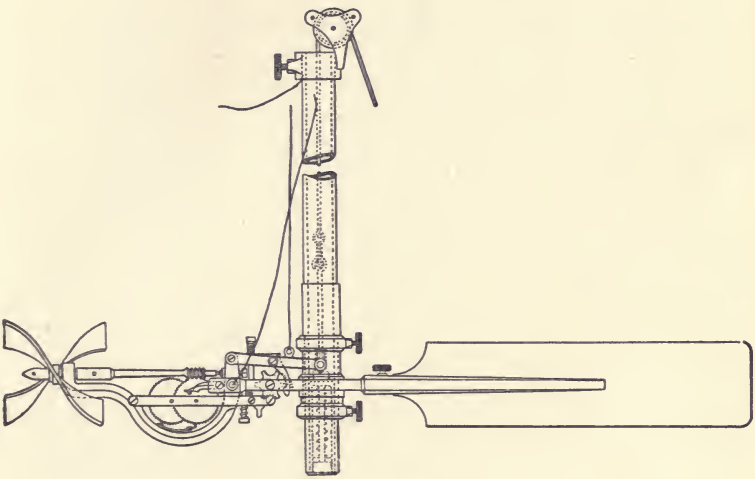


Fig. 128.

on a barrel fixed to the rod, and this has an indicator showing the depth of the meter from the surface. The whole apparatus is fixed on a raft which can be moved across the stream, and anchored at each vertical at which the velocities are to be taken. A current from a small primary battery passes down an insulated wire and back by the rod. A contact-piece on the shaft of the screw closes the circuit every revolution. The current drives a kind of electrical clock with two dials, one showing revolutions and the other hundreds of revolutions. The apparatus being fixed at a vertical in the cross section of the stream, the meter is dropped by the cord to points equidistant on the vertical, and at each the revolutions in one minute or more are observed. The meter is then moved to the next vertical, and similar observations made. The mean velocity on each vertical is calculated from the observations. Otherwise, the mean velocity on a vertical may be found directly by moving the meter slowly and regularly down the vertical, and noting the revolutions and time of transit. It will be seen that all the observations at each vertical can be made rapidly without removing the apparatus from the water. Harlacher used this meter on the Danube in water 26 feet deep running at 10 feet per second (*Proc. Inst. Civil Engineers,* lxvii., 1881).

Current meter of J. Amsler Laffon (Fig. 128).—This can be used on a rod like the primitive meter, and then its chief peculiarity is an improved method of putting the counter in or out of gear. There is a double ratchet, and alternate pulls on a cord throw the counter into gear and out of gear.

But there is a wholly different way in which this meter can be used, the meter M being hung in gimbals, permitting freedom of motion in all directions, and suspended in the water by a wire (Fig. 129). A conical rudder keeps the meter facing the current. The suspending wire is coiled on a small winch A, and this has an index which can be set to show the precise depth at which the meter is suspended. Below the meter, to keep the suspension wire vertical, is a lenticular weight W, of 85 lbs., presenting little resistance to the water, so that the wire is practically vertical. For indicating the revolutions of the meter there is an electric circuit formed by an insulated wire from a battery B, and return through the suspension wire. This circuit is closed, by a contact on one of the counting wheels shown in Fig. 128, at every hundred revolutions of the screw, and a bell is rung. It is only necessary, therefore, to note the time by a stop-watch for 100, 200, or 500 revolutions. A subsidiary arrangement is that, when the foot of the lenticular weight touches ground, a contact is made and the circuit closed, so that the bell rings continuously. The meter is then one foot above the ground. This gives warning, and has the further advantage that the apparatus can be used as a satisfactory sounding instrument in any depth of water.

The suspended meter is generally used thus:—The boat is anchored at a vertical, its position being fixed by angular measurement. The meter is then lowered till its axis is at the water surface and the depth index on the winch is set to zero. The meter is then lowered till the foot touches bottom.

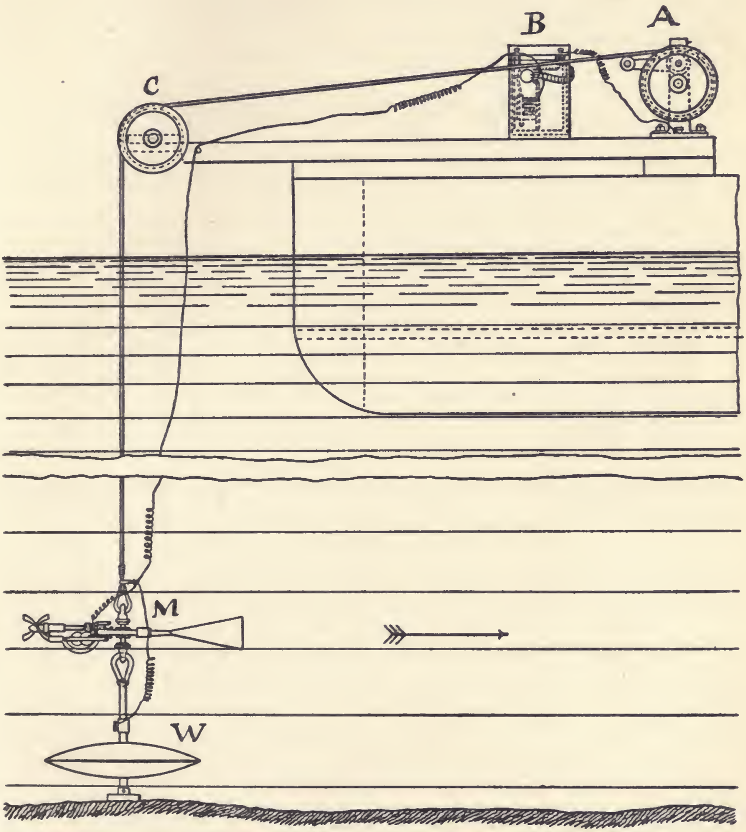


Fig. 129.

If h_1 is the reading, the whole depth of the stream is $H = h_1 + 1$. Then velocities are observed. Let v_1 be the velocity at h_1; v_2 at $h_2 = h_1 - d$; v_3 at $h_3 = h_1 - 2d$; . . . v_n at $h_n = h_1 - (n-1)d$. The mean velocity on the vertical is very nearly

$$v_m = \frac{1}{\mathrm{H}}\left\{d\left(\frac{v_1}{2} + v_2 + v_3 + \ . \ . \ . \ + v_{n-1} + \frac{v_n}{2}\right)\right.$$
$$\left. + h_n v_n + \frac{2}{3}\left(\mathrm{H} - h_1\right)v_1\right\} \quad . \quad . \quad (4).$$

Or the vertical velocity curve may be plotted, and its mean ordinate found. The meter can be used with great facility in rivers even in flood.

Example of Current Meter Observations on a Vertical

Vertical No. 3.
Depth at vertical, 2·6 feet. 2 h. 50 m. p.m.
Distance from zero of transverse section, 32 feet.
Water-level on gauge, 1·65 feet.

Depth.	No. of Revolutions.	Time.	Revolutions per Second.	Mean Revolutions per Second.	Velocity. Feet per Second.
0·3	296	75	3·946	3·973	3·325
0·3	240	60	4·000		
0·63	237	60	3·950	3·958	3·309
0·63	238	60	3·966		
0·96	217	60	3·616	3·637	3·050
0·96	240	60	4·000		
0·96	198	60	3·300		
0·96	218	60	3·633		
1·29	234	65	3·600	3·419	2·870
1·29	211	65	3·246		
1·29	256	75	3·413		
1·62	192	60	3·200	3·088	2·600
1·62	179	60	2·976		
1·95	168	60	2·800	2·753	2·325
1·95	165	61	2·705		
2·60	Bed of	stream.			

Here the mean velocity on the vertical by eq. (4) is

$$v_m = \frac{1}{2{\cdot}6}\left\{{\cdot}33\left(1{\cdot}162 + 2{\cdot}6 + 2{\cdot}87 + 3{\cdot}05 + 3{\cdot}309 + 1{\cdot}662\right)\right.$$
$$\left. + \left(0{\cdot}3 \times 3{\cdot}325\right) + \frac{2}{3}\left({\cdot}65 \times 2{\cdot}325\right)\right\} = 2{\cdot}631 \text{ feet per second.}$$

In connection with this it may be mentioned that in gauging the Severn at Worcester, in 1880, a Deacon electric current meter was used, fixed in a frame suspended from a No. 12 steel wire stretched across the river. The river was 180 feet wide and about 25 feet deep. Velocity measurements were made at every foot of depth on verticals 10 or 20 feet apart in the cross section. The frame carrying the meter was suspended from a small carriage on two 3-inch pulleys, and traversed by an endless wire passing over pulleys on the end supports of the carrying wire. Other wires from the frame, carried over a pulley on the carriage, served for raising and lowering the frame. Lastly, a wire with a cast-iron anchor-plate of 70 lbs. passed through the frame and over the carriage, and served to keep the frame vertically in position during the observations. Insulated wires from the meter, through which a current passed when contact was made at the meter, indicated on shore the revolutions of the meter (Turner, *Proc. Inst. Civil Engineers,* lxxx., 1884). In some cases the meter has been used by observers on a travelling platform suspended from a wire rope stretched across the stream. In a gauging of the Rhine by Baum (*Proc. Inst. Civil Engineers,* lxxi. 456) the current meter was used on a platform between two coupled boats, sliding on a T-iron $4'' \times 2\frac{3}{8}''$.

164. **Calibrating the screw current meter.**—The accuracy of velocity observations by current meter depends entirely on the care and skill used in determining the constants of the instrument. If the screw propeller were of uniform pitch p, and if it were frictionless, then it would make one revolution for p feet of water passing it. The relation of velocity v and revolutions per second n would be $v = pn$. In any actual instrument these conditions are not satisfied. At some velocity v_0 (about 4 inches per second or less) the meter ceases to revolve, being held by friction. Also the pitch cannot be accurately measured. Hence the relation of v and n must be determined by experiment. It is generally assumed that the form of the relation is linear, so that

$$v = an + \beta \quad . \quad . \quad . \quad . \quad (5),$$

where a and β are constants, and β is the velocity at which rotation ceases. Exner has shown that the following equation, on theoretical grounds, is more exact and better agrees with experiment:

$$v = \sqrt{(a^2n^2 + \beta^2)} \quad . \quad . \quad . \quad (6).$$

But when the lowest velocity is not less than 1 foot per second, eq. (5) is practically accurate and more convenient.

Suppose a current meter towed over a length l feet in still water, and that it makes N revolutions, the time of transit being t seconds. The speed of towing or velocity of the water relatively to the meter is $l/t=v$ feet per second, and the speed of the meter is $N/t=n$ revolutions per second. Let a number of observations be taken in this way at different speeds, and let $n_1, n_2, n_3 \ldots$ be the meter speeds corresponding to the velocities $v_1, v_2, v_3 \ldots$ Let m be the number of observations. Then, assuming the relation $v=\alpha n+\beta$, the values of α and β may be found by the method of least squares.

$$\left.\begin{aligned}\alpha&=\frac{m\Sigma(nv)-\Sigma(n)\Sigma(v)}{m\Sigma(n^2)-[\Sigma(n)]^2}\\ \beta&=\frac{\Sigma(v)\Sigma(n^2)-\Sigma(n)\Sigma(nv)}{m\Sigma(n^2)-[\Sigma(n)]^2}\end{aligned}\right\} \quad . \quad . \quad (7).$$

Example.—For instance, the following table contains the results of a series of tests on a meter and the summation of the quantities required in determining the constants. The length of run was 336 feet.

No. of Run.	Time of Transit, t Seconds.	Velocity, Feet per Second, v.	No. of Revolutions per Second, n.	n^2.	nv.
1	115	2·921	2·043	4·174	5·969
2	116	2·896	2·000	4·000	5·792
3	113	2·973	2·053	4·215	6·103
4	130	2·584	1·776	3·154	4·590
5	113	2·973	2·088	4·360	6·209
6	121	2·776	1·892	3·580	5·253
7	125	2·687	1·824	3·327	4·903
Sums . . .		19·810	13·676	26·810	38·819

$$[\Sigma(n)]^2=187{\cdot}03$$

$$m=7$$

$$\alpha=\frac{7\times 38{\cdot}819-13{\cdot}676\times 19{\cdot}810}{7\times 26{\cdot}810-187{\cdot}03}=\frac{0{\cdot}81}{0{\cdot}64}=1{\cdot}266$$

$$\beta=\frac{19{\cdot}810\times 26{\cdot}810-13{\cdot}676\times 38{\cdot}819}{7\times 26{\cdot}810-187{\cdot}03}=\frac{0{\cdot}22}{0{\cdot}64}=0{\cdot}344.$$

Recalculating the velocities from the revolutions, using these values of the constants, and comparing the results with the observed velocities, the following table is obtained :—

Velocity observed	2·921	2·896	2·973	2·584	2·973	2·776	2·687
,, calculated	2·931	2·876	2·943	2·592	2·987	2·739	2·653
	+·01	−·02	−·03	+·008	+·014	−·037	−·034

A different formula of reduction is used by some American engineers. If in the equation $v = an + \beta$ observed values of v and n are inserted, then for m observations a series of m equations can be formed—

$$\left.\begin{array}{l} an_1 + \beta - v_1 = \epsilon_1 \\ an_2 + \beta - v_2 = \epsilon_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{array}\right\} \quad . \quad . \quad . \quad (8),$$

where ϵ_1, ϵ_2 are small errors of individual observations. Since β enters in the same way into all the equations, its most probable value is the arithmetical mean. Let $n_m = (\Sigma n)/m$ be the mean value of n, and $v_m = (\Sigma v)/m$ the mean value of v. Then, as the errors cancel,

$$an_m + \beta - v_m = 0,$$

$$\beta = v_m - an_m \quad . \quad . \quad . \quad . \quad (9).$$

Inserting this value in eq. (8),

$$a(n_1 - n_m) - (v_1 - v_m) = \epsilon_1,$$
$$a(n_2 - n_m) - (v_2 - v_m) = \epsilon_2,$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

To weight these equations multiply each by the coefficient of a. Then

$$a(n_1 - n_m)^2 - (v_1 - v_m)(n_1 - n_m) = \epsilon_1(n_1 - n_m),$$
$$a(n_2 - n_m)^2 - (v_2 - v_m)(n_2 - n_m) = \epsilon_2(n_2 - n_m),$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

Adding these equations,

$$a\Sigma(n - n_m)^2 - \Sigma[(v - v_m)(n - n_m)] = 0.$$

$$a = \frac{\Sigma[(v - v_m)(n - n_m)]}{\Sigma(n - n_m)^2} \quad . \quad . \quad . \quad (10),$$

and β can then be found from eq. (9).

Example.—Taking the data in the table above, the following are the quantities required to determine a and β :—

$$n_m = 13{\cdot}676/7 = 1{\cdot}954 \,.\; v_m = 19{\cdot}310/7 = 2{\cdot}830.$$

	$t.$	$n.$	$v.$	$n-n_m.$	$v-v_m.$	$(n-n_m)^2.$	$(n-n_m)(v-v_m).$
1	115	2·043	2·921	·089	·091	·00792	·00810
2	116	2·000	2·896	·046	·066	·00211	·00304
3	113	2·053	2·973	·099	·143	·00980	·01416
4	130	1·776	2·584	− ·178	− ·246	·03168	·04379
5	113	2·088	2·973	·134	·143	·01796	·01916
6	121	1·892	2·776	− ·062	− ·054	·00384	·00335
7	125	1·824	2·687	− ·130	− ·143	·01690	·01859
Sums	...	13·676	19·810	...	...	·09021	·11019

$$\alpha=\frac{0·11019}{0·09021}=1·2215,$$

$$\beta=2·830-(1·2215\times1·954)=0·444.$$

Recalculating v from the revolutions—

v observed .	2·921	2·896	2·973	2·584	2·973	2·776	2·687
v calculated .	2·940	2·887	2·952	2·613	2·995	2·755	2·672
Difference .	+ ·019	− ·009	− ·021	+ ·029	+ ·022	− ·021	− ·015

When the constants of a meter are determined, a diagram (Fig. 130) may be drawn from which the velocity corresponding to any number of revolutions per second can be read off.

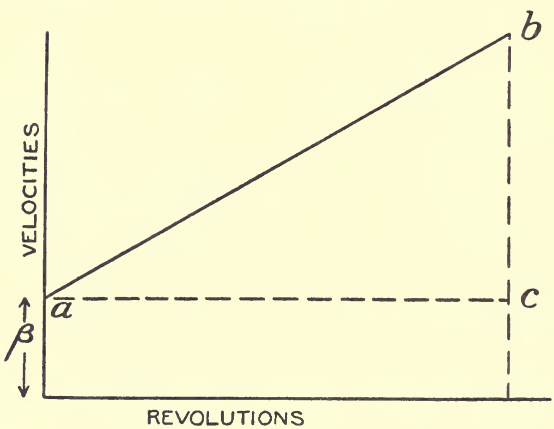
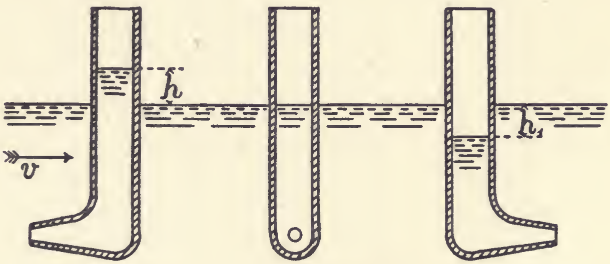


Fig. 130.

The relation is linear, and the line *ab* starts from an ordinate β on the axis of velocities, and has a slope $bc/ac=\alpha$.

The calibration of a meter by fixing it on a boat towed over a measured base-line at different speeds is an operation requiring a good deal of care. It should be repeated many times to eliminate errors. A better plan is to fix the meter on a truck running on rails alongside a quay wall. Slow velocities are best obtained by towing the meter by a winch. Sometimes one current meter can be calibrated by comparing it with another previously calibrated. It is not very satisfactory to obtain the constants by placing the meter in a stream the velocity of which has been determined by floats, but perhaps good results would be obtained if the speed of a stream was determined by a Pitot tube and the current meter used in the same stream at the same place. A check on the calibration of a current meter has sometimes been obtained by using it to measure the volume of flow in a channel the



A B C

Fig. 131.

discharge of which was also measured by a weir. In a few cases the constants of meters have been ascertained by towing them in the Admiralty tank at Torquay, in which ship models are tested. The means of registering time and speed are so perfect in this case that the results are very trustworthy (see Gordon, *Proc. Inst. Mech. Engineers*, 1884).

165. **Pitot tube and Darcy gauge.**—A very early instrument invented by Pitot in 1730, employed in a modified form by Darcy and Bazin in their classical researches, has again come into use in determining the velocity of currents of water and air. Suppose a bent tube, such as that shown in Fig. 131, immersed in a stream of water. When the mouth of the tube points upstream as at A, the impact of the fluid produces a pressure which raises the water in the tube to a height h above the surface outside. If, as at B, the mouth is

parallel to the stream, there is no impact, and the water inside and outside are at the same level. If, as at C, the mouth points downstream there is a certain amount of suction, and the level in the tube is depressed by some distance h_1. Pitot used two tubes arranged as at A and B, and found that the difference of level was very nearly $v^2/2g$. Hence the special advantage of this instrument is that, if properly constructed, it is almost independent of the need of calibration.

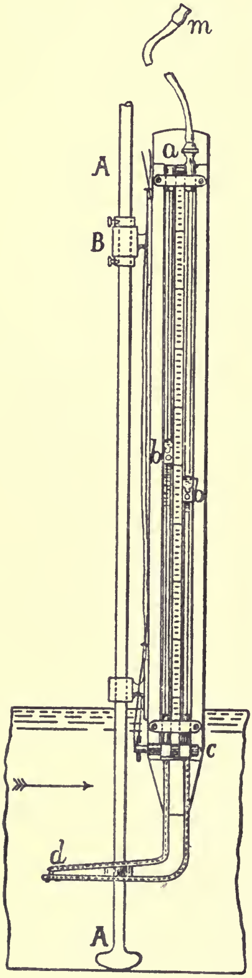


Fig. 132.

An objection to the original Pitot gauge was the difficulty of reading the height h when the gauge was in the water. This is overcome in the modified Darcy gauge shown in Fig. 132. The gauge is shown clamped at B on a rod AA resting on the stream bed. The tubes corresponding to A and B in Fig. 131 are at d, being made very small to avoid disturbing the flow. The mouth of the statical tube opens downwards. The tubes d communicate with the glass tubes b, b, which can be shut off by a two-way cock c actuated by cords. In order to bring the water columns in b, b into a convenient position for reading, a partial vacuum is made above them by sucking out a little air by the tube m and then closing a cock at a. The difference of height of the columns is not altered by raising them. The columns having come to rest, the cock c is closed, and the readings taken by verniers. For a velocity of one foot per second $h = 0{\cdot}186$ inch, which is rather small, but h increases as the square of the velocity, so that at 4 feet per second $h = 3$ inches nearly.

If v is the velocity of the stream and h the difference of level of the columns,

$$v = k\sqrt{(2gh)} \quad . \quad . \quad . \quad (11),$$

where k is a constant depending on the form of the mouths of the instrument and the way they are placed. But if the tubes and orifices are small so as not to create eddies, k differs hardly at all from unity. Darcy calibrated his gauge with great care in three ways. Towing the gauge in still water he found $k = 1{\cdot}034$; observing velocities simultaneously in a stream by floats and by the gauge he found $k = 1{\cdot}006$; and by taking a number of readings in the cross section of a channel the flow in which was known, he found $k = 0{\cdot}993$. He concluded

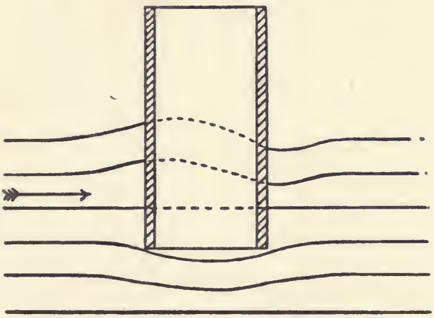


Fig. 133.

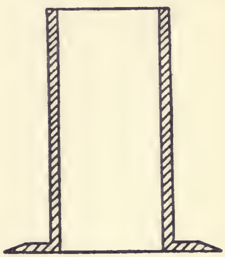


Fig. 134.

that the true value of k did not sensibly differ from unity. White (*Journ. Am. Assoc. of Eng. Soc.*, 1901), and Williams, Hubbell, and Fenkell (*Trans. Am. Soc. of Civil Engineers*, 1902), found that if the tubes were well formed the coefficient was unity. Threlfall (*Proc. Inst. Mech. Engineers*), using Pitot tubes in a current of air, found $k = 0{\cdot}974$; and Stanton, in extremely accurate experiments on the flow of air, found $k = 1{\cdot}03$ (*Proc. Inst. Civil Engineers*, 1903).

The chief cause of variation of the coefficient seems to be the action on the mouth of the statical pressure tube. If this is at all large, the stream lines are bent concave to the mouth (Fig. 133), and there is a slight sucking action which increases h. This may be obviated by a plane disc fitted to the tube, as in Fig. 134. A good arrangement is to form the two tubes concentric, as in Fig. 135, and to place the statical pressure opening on the cylindrical part of the outer tube.

In the case of air of density G lbs. per cubic foot, the head

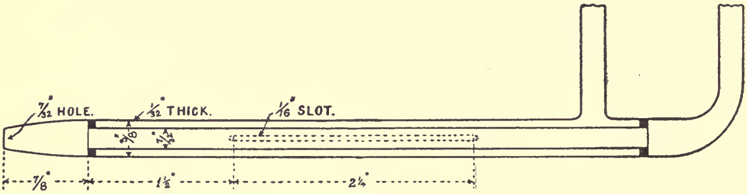


Fig. 135.

corresponding to P lbs. per square foot is P/G. Or if the pressure is measured in inches of water h_w, the head is $5{\cdot}2h_w/G$ in feet of air. Then

$$v = k\sqrt{(2gP/G)} = k\sqrt{(10{\cdot}4gh_w/G)}.$$

If the air is at ordinary pressure and temperature, and $k =$ unity,

$$v = 64{\cdot}6\sqrt{h_w} \quad . \quad . \quad . \quad (12).$$

166. **Ratio of different velocities in a stream. Surface and mean velocities.**—In reducing gauging observations it is necessary to know the relation of the velocities at different parts of a stream. Thus a rough gauging may be made by observing the greatest surface velocity only, if the relation of the mean to the greatest surface velocity is known.

Let V be the mean velocity of the whole cross section, and v_0 the greatest surface velocity, which may be found by using a surface float or current meter. If Ω is the area of cross section the discharge is $Q = \Omega V$. Darcy and Bazin deduced from their researches on small regular channels that

$$V = v_0 - 25{\cdot}4\sqrt{mi} \quad . \quad . \quad . \quad (13).$$

But $V = c\sqrt{mi}$, where c is a constant for a given type of channel (§ 137). Hence

$$V = \frac{c}{c + 25{\cdot}4} v_0 \quad . \quad . \quad . \quad (13a).$$

The following table gives values of V/v_0 for the values of c in § 138:—

Hydraulic Mean Depth m in Feet.	Values of V/v_0 for Darcy's Classes of Channels.				
	I.	II.	III.	IV.	V.
0·5	·84	·81	·74	·59	·51
1·0	·85	·83	·78	·66	·58
2·0	·85	·83	·80	·71	·65
5·0	·85	·84	·81	·76	·71
10·0	·86	·84	·82	·79	·74
20·0	·86	·84	·82	·80	·76
50·0	·86	·84	·82	·81	·78
∞	·86	·84	·82	·81	·78

The ratio decreases as the size of channel decreases, and still more considerably as the roughness of the bed increases. In small wooden channels, probably fairly smooth, Prony found $V/v_0 = 0{\cdot}82$. In the smooth brick conduit at Sudbury, with a depth of 3 feet, the mean velocity was 0·85 of the maximum velocity observed, and about 0·9 of the central surface velocity. In the Vyrnwy stream gauged by Mr. Deacon, the bed width was 33 feet, with side slopes 2 to 1, the bed and sides being pitched with stone and the gauging section lined with concrete. Here in extreme cases the ratio varied from 0·78 to 0·94, the mean of all observations being 0·834.

In rivers with greater roughness and less well-proportioned sections the ratio falls to much lower values.

[TABLE.

RATIO OF MEAN TO GREATEST SURFACE VELOCITY

	Surface Width.	Hydraulic Mean Depth.	$\frac{V}{v_0}$.	Authority.
Weser . . .	276	7·5	·69	Wagner.
Elbe . . .	361	4·6	·78	"
Rhine . . .	705	8·9	·72	Wagner and Grebenau.
Elbe . . .	397	6·6	·77	Harlacher.
" . . .	394	5·3	·75	"
" . . .	344	3·9	·69	"
Rhine . . .	718	5·9	·70	Grebenau.
" . . .	728	8·1	·70	"
" . . .	741	9·2	·74	"
" . . .	778	13·0	·72	"
" . . .	902	19·3	·74	"
Eger at Falkenau .	...	0·62	·57	Plenkner.
" " .	...	1·09	·58	"
Sazawa at Poric .	...	1·09	·59	"
" " .	...	0·90	·71	"
" " .	...	0·38	·73	"
Moldau at Budweis	...	1·12	·73	"
" "	...	0·91	·71	"
" "	...	1·58	·72	"

Wagner deduces for rivers the relation

$$V = 0{\cdot}705 v_0 + 0{\cdot}003 v_0^2 \quad . \quad . \quad . \quad (14),$$

which agrees with some other cases, and is useful in rough gauging.

The central surface velocity is somewhat variable, being affected by wind and other accidents. The mean surface velocity, which can be obtained by a series of surface float observations, has probably a more constant relation to the mean velocity of the cross section. In the gaugings of the Eger and Moldau the ratio (mean velocity)/(mean surface velocity) was 0·90. Wagner found the value 0·88.

The ratio of the mean velocity for the cross section to the mean velocity on the central vertical was 0·95 to 0·98 in Cunningham's float gaugings on the Ganges Canal, 0·93 in Deacon's gaugings of the Vyrnwy stream, and 0·67 on the Eger.

It seems probable that the ratio of the mean velocity for the cross section to the velocity at the centre of figure of the cross section is a fairly constant ratio. The latter could be easily determined by a current meter. In the Vyrnwy stream this ratio was 0·888.

If v_c is the velocity at the centre of figure and V the mean velocity for the cross section, Wagner found in rivers

$$V = 0{\cdot}727 v_c + 0{\cdot}19 v_c^2 \quad . \quad . \quad . \quad (15).$$

This ratio does not differ much from the ratio of the mean velocity of cross section to central mid-depth velocity, which was 0·876 at the Vyrnwy stream.

167. **Velocities on one vertical.**—The following table contains averages from the large mass of float gaugings made by Captain Cunningham on the Ganges Canal.[1] The aqueducts were 85 feet wide with 10 feet depth and less. The main embankment site was 170 feet wide with 11 feet depth and less. The averages are fairly consistent. The individual results vary a good deal.

	Central Surface Velocity, v_0.	Maximum Velocity, v_{max}.	Bed Velocity, v_b.	$\frac{v_0}{v_{max}}$.	$\frac{v_b}{v_{max}}$.
Solani left aqueduct .	3·97	4·01	3·35	0·99	0·84
Solani right aqueduct .	4·06	4·18	3·70	0·97	0·88
Embankment main site	3·56	3·65	3·08	0·98	0·84

The following averages are from the same large mass of observations :—

	Mean Velocity at Central Vertical, V_v.	Half-depth Velocity, $v_{d/2}$.	Rod Float Velocity, v_r.	$\frac{v_{d/2}}{V_v}$.	$\frac{v_r}{V_v}$.	$\frac{v_{d/2}}{v_r}$.
Solani left aqueduct .	3·65	3·69	3·55	1·010	0·972	1·039
Solani right aqueduct .	4·02	4·07	3·85	1·012	0·958	1·057
Embankment main site	3·44	3·48	3·31	1·011	0·963	1·051

[1] *Roorkee Hydraulic Experiments*, Cunningham. Roorkee, 1881.

The half-depth velocity was 1 per cent greater than the mean velocity at a vertical. The rod float velocity was about 4 per cent less than the mean. The mean velocity was computed from double float observations.

Wagner found the mean velocity at a vertical to be 0·8 of the surface velocity at the vertical when the surface velocity was not greater than 2 feet per second. The ratio was 0·85 for velocities from 2 to 4 feet per second, and 0·9 for velocities from 4 to 10 feet per second.

The depth at which the maximum velocity is found at

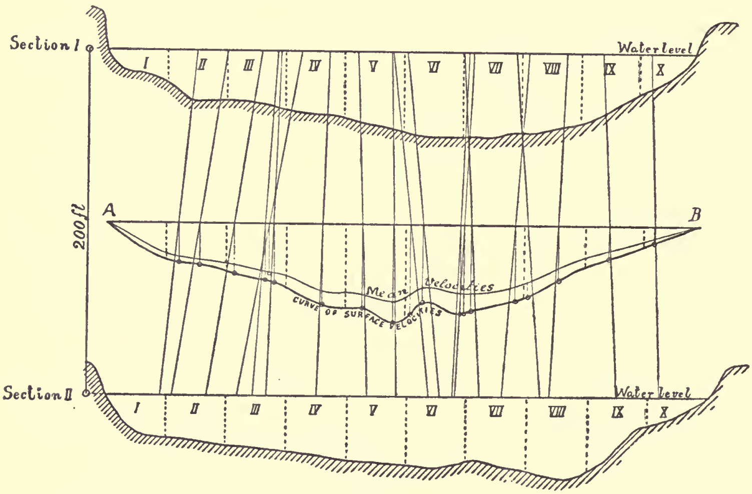


Fig. 136.

the central vertical is from 0 to 0·3 of the whole depth. On other verticals it varies a good deal according to the form of the channel section. The position on a vertical at which the velocity is equal to the mean velocity is fairly constant, and equal to 0·58 to 0·6 of the whole depth. The mid-depth velocity is very slightly greater than the mean velocity.

168. **Surface or rod float gauging.**—Fig. 136 shows a gauging of the Thames by surface floats. Two sections, I. and II., were surveyed at the ends of a 200-foot base-line. These sections are divided into ten compartments of equal width. Between the sections the float paths are plotted. A base-line AB is taken midway between the sections, and at

the points where the float paths cross the line AB the observed velocities are set up as ordinates. Through the points so found the surface velocity curve is drawn. The curve of mean velocities on verticals can be found from this by taking ordinates 0·85 to 0·95 of those of the surface velocity curve, according to the character of the stream. Let Ω_1, Ω_2 . . . be the mean areas of the ten pairs of compartments in the two end sections in square feet, and v_1, v_2 . . . the mean ordinate of the curve of mean velocities corresponding to each compartment in feet per second. Then the discharge of the stream is

$$Q = \Omega_1 v_1 + \Omega_2 v_2 + \ldots + \Omega_{10} v_{10} \text{ cubic feet per second} \quad . \quad (16).$$

The mean velocities might have been observed directly by using rod floats or sub-surface mid-depth floats. In that case the uncertainty due to the selection of the ratio of surface to mean velocity is obviated. The following table gives the results of the gauging shown in Fig. 136. The mean velocities on the verticals are taken at 0·93 of the surface velocities.

River Gauging, October 1877

Compartment.	Mean Area of Section, Square Feet.	Mean Surface Velocity, Feet per Second.	Mean Velocity, Feet per Second.	Discharge, Cubic Feet per Second.
I. . .	59·2	·409	·380	22·5
II. . .	93·5	·659	·613	57·0
III. . .	111·8	·905	·842	93·9
IV. . .	128·1	1·206	1·120	143·5
V. . .	138·2	1·710	1·590	219·7
VI. . .	153·3	1·798	1·670	256·1
VII. . .	157·3	1·631	1·520	239·1
VIII. . .	144·1	1·421	1·339	190·2
IX. . .	116·4	1·115	1·037	121·0
X. . .	44·2	·579	·538	25·7
			Total . .	1368·7

169. **Discharge curve.**—A very convenient method of deducing the discharge from a curve of mean velocities on verticals is to construct a curve with the stream width as base,

and ordinates proportional at each point to the discharge at that point.

Let aeB (Fig. 137) be the stream section, AfB the curve of mean velocities on verticals. Take $ab = af = v$; $ac = k =$ any convenient unit. Join ce, and draw bd parallel to it. Then d is a point on the discharge curve.

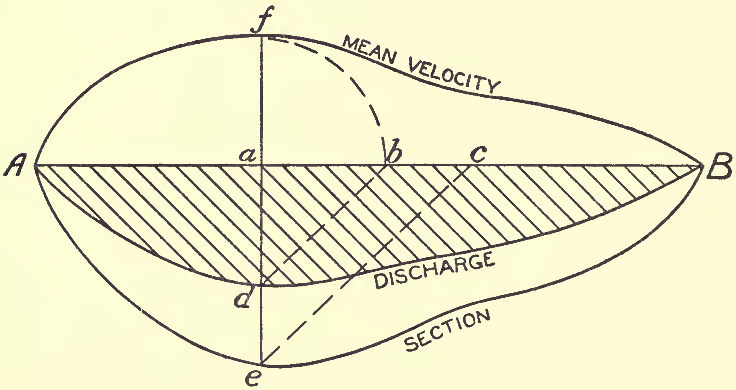


Fig. 137.

If $D = ae$ is the depth, and $v = af = ab$ is the mean velocity at a, the discharge for any small portion dx of the width of the stream at a is $Dvdx$, and the whole discharge of the stream is

$$Q = \int Dvdx.$$

But $ad = (ae \times ab)/(ac)$, that is $ad = (Dv)/k$. Let $y = ad$, then

$$Q = k\int ydx \quad . \quad . \quad . \quad . \quad (17);$$

that is, the whole discharge is proportional to the area of the curve AdB.

If the area of the curve is measured in square inches, and the scales are m feet per second, and n feet to one inch, and k is set off in inches, then the area of the curve must be multiplied by mn^2k to give cubic feet per second.

170. **Calculation of discharge from the vertical velocity curves.**—If the vertical velocity curves have been drawn from current meter observations at different depths, the discharge

between each pair of verticals can be regarded as the volume of a truncated pyramid having the velocity curves as bases. Let b_1, b_2 . . . (Fig. 138) be the distances between the

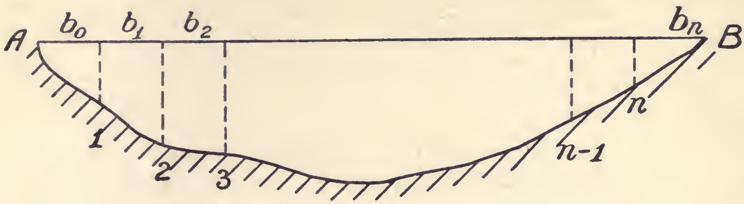


Fig. 138.

verticals; a_1, a_2 . . . the areas of the vertical velocity curves. Then the discharge between the verticals $m-1$ and m is

$$\Delta Q = \frac{b_{m-1}}{3}\left(a_{m-1} + \sqrt{a_{m-1}a_m} + a_m\right).$$

The discharge of the two end sections may be taken as the volumes of pyramids on the bases a_1 and a_n. Hence the whole discharge is

$$Q = \Sigma \frac{b_{m-1}}{3}\left\{a_{m-1} + \sqrt{(a_{m-1}a_m)} + a_m\right\} + \frac{1}{3}\left(a_1 b_0 + a_n b_n\right) \quad (18).$$

If the vertical velocity curve is plotted so that m feet per second = one inch, and n feet of depth = one inch, then one square inch of area represents mn square feet of water passing the vertical per second. The areas of the curves measured in square inches should be multiplied by mn, and the widths taken in feet in the equation, to get the result in cubic feet per second.

171. **Calculation of discharge from contours of equal velocity.**—If contours of equal velocity have been plotted, as in Fig. 116, § 147, a method due to Culmann may be used. Let Ω_0 be the area of cross section of the stream, and Ω_1, Ω_2 . . . the areas included in the successive contours; these should be reckoned in square feet, so that if the scale is m feet to an inch the areas measured in square inches must be multiplied by m^2. Let d be the intervals of velocity for which the contours are plotted in feet per second. Then the discharge of any one layer of thickness d is $\frac{1}{2}(\Omega_{m-1} + \Omega_m)d$.

The top layer of small volume will usually have a thickness δ less than d, and its volume may be reckoned with accuracy enough as $\frac{2}{3}\Omega_n\delta$. Hence the whole discharge is

$$Q = \frac{\Omega_0 + \Omega_1}{2}d + \frac{\Omega_1 + \Omega_2}{2}d \ . \ . \ . + \frac{\Omega_{n-1} + \Omega_n}{2}d + \frac{2}{3}\Omega_n\delta$$

$$= d\left\{\Sigma\Omega - \frac{\Omega_0 + \Omega_n}{2}\right\} + \frac{2}{3}\Omega_n\delta \quad . \quad . \quad . \quad . \quad (19).$$

172. **Gauging streams by chemical means.**—Mr. C. E. Stromeyer has experimented with a chemical gauging method (*Proc. Inst. Civil Engineers*, clx. 349). A fairly concentrated solution of a chemical for which a sensitive reagent is known is discharged at a uniform rate into the stream to be gauged. Analyses are made of the water before the chemical is added, and after it has become well mixed with the stream. Let x be the percentage of chemical in the solution, y the percentage found in the water, a the volume of solution added per second, and Q the discharge of the stream.

$$\frac{x}{y} = \frac{Q}{a}.$$

Chloride of calcium, of magnesium, or of sodium and other chemicals may be used.

CHAPTER XIV

IMPACT AND REACTION OF FLUIDS

173. WHEN a stream of fluid impinges on a solid surface, it exerts a pressure on the surface which is equal and opposite to the force exerted by the surface on the fluid in changing its momentum.

If a fluid glides over a solid also moving, the motion of the former can be resolved into two components—one a motion which the fluid and solid have in common, the other a motion of the fluid relatively to the solid. The motion which the fluid has in common with the solid cannot be affected by their contact. The relative component can be altered in direction, but not in magnitude, for the relative motion must be tangential to the surface, while the pressure between the fluid and solid (friction being neglected) must be normal to the surface. The pressure can deviate the fluid, but cannot alter the magnitude of the relative motion. The absolute velocity of the fluid, after contact with the surface, is found by combining the deviated but otherwise unchanged relative motion, tangential to the solid at the point where the fluid leaves it, with the common velocity of fluid and solid.

The principle of the conservation of momentum has already been explained in § 35. The impulse of the mass of fluid impinging in a given time is equal to the change of momentum, the impulse and change of momentum being estimated in the same direction. If Q cubic feet or GQ/g units of mass impinge in one second with a velocity v_1 in a given direction, and v_2 is the velocity in the same direction after impact, then the pressure exerted, also in the same direction, is

$$P = \frac{GQ}{g}(v_1 - v_2) \text{ lbs.} \quad . \quad . \quad . \quad (1).$$

174. **Jet deviated wholly in one direction.**—Let a jet of water (Fig. 139) impinge on a curved trough-shaped vane

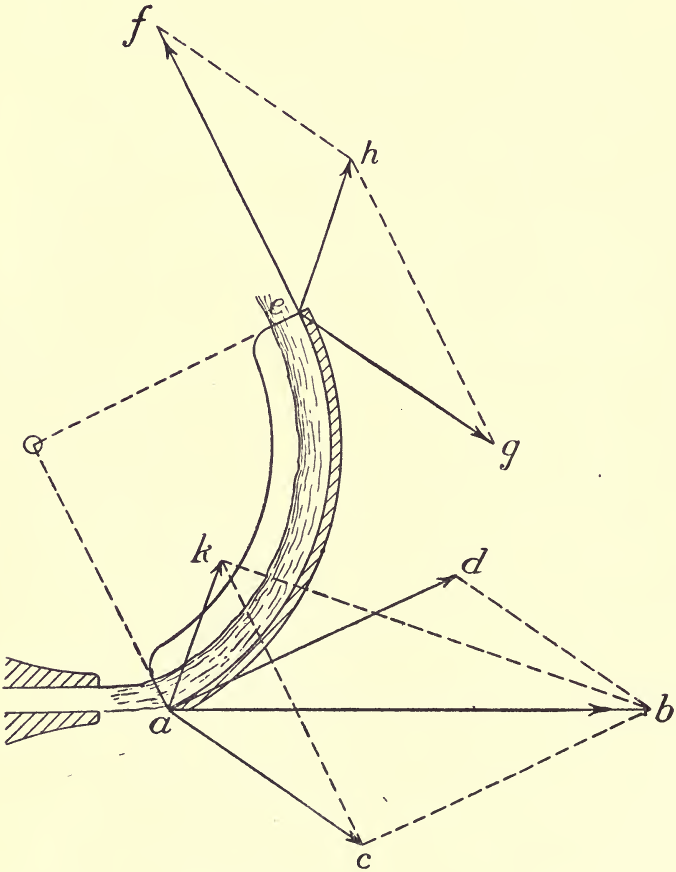


Fig. 139.

ae, so that it is deviated in the plane of the figure. Let *ab* represent in magnitude and direction the velocity v of the jet, and $ac = u$ that of the vane. Completing the parallelogram, $ab = v$ may be resolved into two components—a velocity in common with the vane $ac = u$, and a velocity relative to the vane $ad = v_r$. In order that there may be no shock or disturbance of the water at a, the tangent to the lip of the vane must be parallel to ad. The water glides up the vane with the velocity v_r, and leaves it tangentially with this relative velocity unchanged. Take ef tangential to the vane and equal to v_r, and eg equal and parallel to the common velocity $ac = u$. Completing the parallelogram, eh is the absolute velocity and direction of motion of the water leaving the vane. Take ak equal and parallel to eh, and join kb, kc. Then the initial velocity and direction of motion ab are changed during impact to ak, and $kb = w$ is the change of motion. If Q cubic feet of water impinge per second the pressure on the vane is in the direction kb and equal to

$$P = \frac{GQ}{g} w \text{ lbs.}$$

Since ak is equal and parallel to eh and ac to eg, kc is equal and parallel to hg, and therefore to ef. Hence ck, cb are each equal to v_r and parallel to the initial and final directions of relative motion. It is unnecessary to consider the common velocity in treating the problem. The change of motion kb is represented in magnitude and direction by the third side of an isosceles triangle ckb, the other sides of which are equal to the relative velocity and parallel to the initial and final directions of relative motion.

175. **A jet of water impinges axially on a solid of revolution,** which is moving in the same direction.

The section of the jet (Fig. 140) is supposed much smaller than the solid. The water is deviated symmetrically in all directions and flows away at an angle θ with the axis, each elementary stream being deviated through the same angle. From the symmetry of the conditions the resultant pressure on the solid will be axial. Let v be the velocity of the water, u that of the solid. Since the common velocity is the same before and after impact, it may be disregarded. Parallel to the axis the relative velocity is $v - u$ before impact, and after impact its component in the same direction is $(v - u)\cos\theta$. If ω is the section of the jet, the quantity of water impinging per second is $\omega(v - u)$, and its mass is $G\omega(v - u)/g$. The resultant pressure on the surface, which is equal to the change of momentum per second, estimated in the same direction, is

$$P = \frac{G}{g}\omega(v-u)\{(v-u)-(v-u)\cos\theta\}$$

$$= \frac{G}{g}\omega(v-u)^2(1-\cos\theta) \text{ lbs.} \quad . \quad . \quad . \quad (2).$$

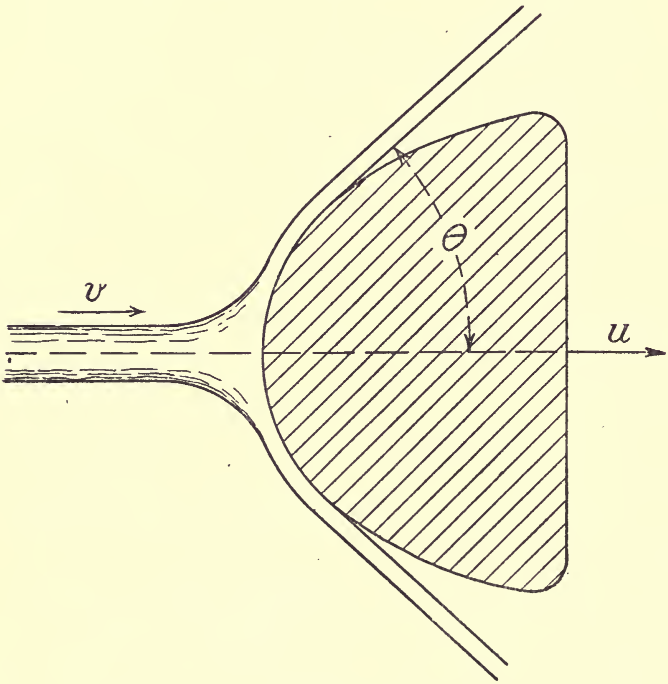


Fig. 140.

The work done by the water in driving the solid is

$$Pu = \frac{G}{g}\omega u(v-u)^2(1-\cos\theta) \text{ ft.-lbs. per second.} \quad . \quad (3).$$

If the solid is at rest, $u = o$, and then

$$P = \frac{G}{g}\omega v^2(1-\cos\theta),$$

and no work is done. The work done will also be zero if $u = v$. Hence there must be an intermediate ratio of u to v, for which the work is a maximum. The total energy issuing from a fixed nozzle would be

$$\frac{G}{g}\omega v\frac{v^2}{2}=\frac{G}{2g}\omega v^3,$$

and the efficiency of the arrangement, considered as a means of utilising the energy of the jet, is

$$\eta=\frac{2u(v-u)^2(1-\cos\theta)}{v^3} \quad . \quad . \quad . \quad (4).$$

Differentiating and equating to zero,

$$\frac{d\eta}{du}=v^2-4vu+u^2=0,$$

whence η is a maximum if $u=v/3$. Inserting this value,

$$\eta_{max}=\frac{8}{27}(1-\cos\theta) \quad . \quad . \quad . \quad (5).$$

In a number of hydraulic machines, a jet acts on a series of vanes which succeed one another in the same position at very short intervals of time. Such vanes are attached to a wheel and therefore have a circular path. But the path of each during the action of the jet is very short, and if the radius of the wheel is large, the curvature of the path may be neglected. Then the quantity of water per second which acts on the series of vanes is ωv, and the equations become

$$P=\frac{G}{g}\omega v(v-u)(1-\cos\theta)\text{ lbs.} \quad . \quad . \quad . \quad (6),$$

$$Pu=\frac{G}{g}\omega vu(v-u)(1-\cos\theta)\text{ ft.-lbs. per second} \quad (7),$$

$$\eta=\frac{2u(v-u)(1-\cos\theta)}{v^2}.$$

The efficiency is greatest if $u=v/2$, and then

$$\eta_{max}=\tfrac{1}{2}(1-\cos\theta) \quad . \quad . \quad . \quad (8).$$

176. **Special Cases.—Case I. A jet impinges normally on a plane moving in the same direction.**—Let v (Fig. 141)

be the velocity of the jet, and u that of the plane. The relative velocity is $v-u$. If ω is the section of the jet, the quantity of water which reaches the plane is $\omega(v-u)$ cubic feet per second. In the direction of the jet the initial velocity of the water is v, and its final velocity after impact is u. The pressure on the plane, which is equal to the change of momentum per second, is

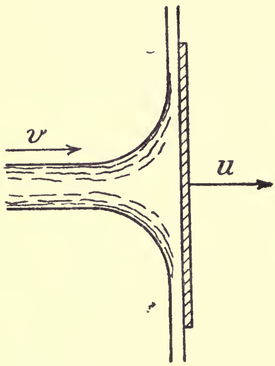


Fig. 141.

$$P = \frac{G}{g}\omega(v-u)(v-u)$$
$$= \frac{G}{g}\omega(v-u)^2 \text{ lbs.},$$

and the work done in driving the plane is

$$Pu = \frac{G}{g}\omega(v-u)^2 u \text{ ft.-lbs. per second.}$$

This is a maximum for $u = v/3$, and then

$$Pu = \frac{4}{27}\frac{G}{g}\omega v^3 \text{ ft.-lbs. per second.}$$

These results can be obtained by putting $\theta = 90°$ in eqs. (2) and (3). If the plane is at rest, $u = 0$, and then

$$P = \frac{G}{g}\omega v^2 \text{ lbs.}$$

It appears that if the area of the plane is less than 16 times the area of the jet, the effective deviation is less than 90°, and the pressure is less.

Case II. A series of plane vanes are interposed in front of the jet in succession.—The other conditions are supposed the same as in the last case. This arrangement is roughly identical with that of an undershot wheel with plane floats which enter in succession in front of a stream issuing with the velocity due to the head driving the wheel. The quantity of water acting per second on the vanes is ωv cubic feet. The pressure on the series of vanes is

$$P = \frac{G}{g} \omega v(v - u) \text{ lbs.}$$

The work done in driving the vanes is

$$Pu = \frac{G}{g} \omega vu(v - u) \text{ ft.-lbs. per second.}$$

This is a maximum if $u = v/2$, and then

$$Pu = \frac{1}{4} \frac{G}{g} \omega v^3.$$

These results can be obtained by putting $\theta = 90°$ in eqs. (6) and (7).

Case III. A jet of water impinges on a series of hemispherical cups moving in the same direction (Fig. 142).—Here the water is deviated through 180°. The initial relative velocity is $v - u$, and the final $-(v - u) = u - v$, both parallel to the direction of the jet. The quantity of water impinging per second is ωv cubic feet.

$$P = \frac{G}{g} \omega v\{(v - u) - (u - v)\}$$

$$= 2 \frac{G}{g} \omega v(v - u) \text{ lbs.}$$

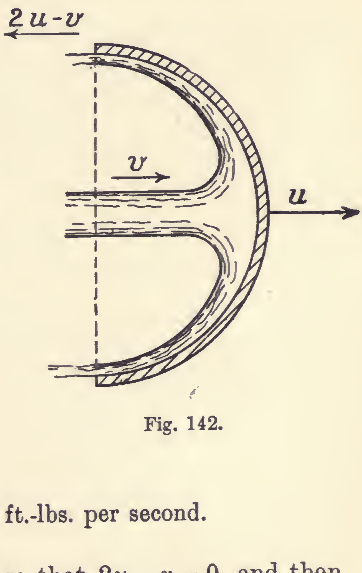


Fig. 142.

The work done is

$$Pu = 2 \frac{G}{g} \omega vu(v - u) \text{ ft.-lbs. per second.}$$

This is greatest when $u = v/2$, so that $2u - v = 0$, and then

$$Pu_{max} = \frac{G}{2g} \omega v^3 \text{ ft.-lbs. per second,}$$

or equal to the whole kinetic energy of the jet. This roughly corresponds to the case of the Pelton wheel, which on high falls reaches an efficiency of 0·8 or more, the loss being due to friction and imperfect deviation of the water as the buckets pass in front of and away from the jet.

177. Pressure of a steady stream of limited section on a plane normal to the direction of motion.—Let CD (Fig. 143) be a thin plate normal to the axis of a pipe through which water is flowing, which for simplicity is taken horizontal. The elementary streams, parallel at A_0, are deviated in front of the plate, form a contraction at A_1, and then converge, leaving a mass of eddies at the back of the plate, and at some section A_2 become parallel again. It may be inferred from the convexity of the stream lines in front and the concavity behind the plate that there is an excess pressure

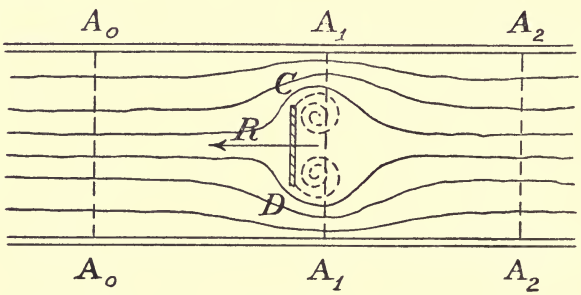


Fig. 143.

in front and a negative pressure behind the plate, the sum of which forms the reaction R causing changes of momentum in the water, and which is equal and opposite to the total pressure of the water on the plate. Since the same amount of water at the same velocity passes the sections A_0, A_2 in a given time, the kinetic energy flowing in and out is the same, and the external forces acting on the mass between A_0 and A_2 must be balanced. Let Ω be the section of the stream at A_0 or A_2, and ω the area of the plate CD. The area of the contracted section of the stream at A_1 is $c_c(\Omega - \omega)$, where c_c is a coefficient of contraction. For simplicity let $\Omega/\omega = \rho$ and $\Omega/\{c_c(\Omega - \omega)\} = r$. Then $r = \rho/\{c_c(\rho - 1)\}$. Let v be the velocity at A_0 and A_2, and v_1 the velocity at A_1.

$$v\,\Omega = c_c(\Omega - \omega)v_1,$$

$$v_1 = \frac{\Omega}{c_c(\Omega - \omega)}v = rv.$$

Let p_0, p_1, p_2 be the pressures at A_0, A_1, A_2 respectively. Applying Bernoulli's theorem to A_0 and A_1,

$$\frac{p_0}{G}+\frac{v^2}{2g}=\frac{p_1}{G}+\frac{v_1{}^2}{2g};$$

and similarly for A_1 and A_2, allowing for the loss in shock due to the relative velocity $v_1 - v$ (§ 36),

$$\frac{p_1}{G}+\frac{v_1{}^2}{2g}=\frac{p_2}{G}+\frac{v^2}{2g}+\frac{(v_1-v)^2}{2g},$$

$$\frac{p_1}{G}=\frac{p_2}{G}-\frac{v(v_1-v)}{g},$$

$$p_0-p_2=G\frac{(v_1-v)^2}{2g},$$

or replacing v_1 by its value above,

$$p_0-p_2=G(r-1)^2\frac{v^2}{2g}.$$

The external horizontal forces acting on the mass between A_0 and A_2 are the difference of the pressures on the sections A_0 and A_2 and the reaction of the plate CD, and these are in equilibrium, there being no resultant change of momentum. Hence

$$(p_0-p_2)\Omega-R=0,$$

and the total resultant pressure on CD is

$$R=G\Omega(r-1)^2\frac{v^2}{2g}=G\rho\omega(r-1)^2\frac{v^2}{2g}$$

$$=KG\omega\frac{v^2}{2g},$$

where K is a coefficient depending only on ρ and c_c. Thus if $c_c = 0{\cdot}85$,

$\rho=$	K=
2	3·6
3	1·8
4	1·3
10	·9
50	2·0

As ρ increases, K diminishes to a minimum and then increases. This is not intelligible, and therefore c_c cannot have a constant value, or, what probably is the same thing, the influence of the plate in deviating the stream lines extends only to a limited distance.

From the equation above,

$$p_1 = p_2 - \frac{G}{g}(v_1 - v)v$$

$$= p_2 - 2G(r-1)\frac{v^2}{2g}.$$

Now in the eddying mass behind the plate the pressure must be practically identical with p_1, and hence the defect of pressure forming part of the reaction R is

$$P_b = (p_1 - p_2)\omega = 2G\omega(r-1)\frac{v^2}{2g} = K_b G\omega\frac{v^2}{2g}.$$

Consequently the front pressure must be

$$P_f = R - P_b = \left\{G\rho\omega(r-1)^2 - 2G\omega(r-1)\right\}\frac{v^2}{2g}$$

$$= G\omega\left\{\rho(r-1)^2 - 2(r-1)\right\}\frac{v^2}{2g} = K_f G\omega\frac{v^2}{2g}.$$

The following values have been calculated, using values of c_c selected by Zeuner on the basis of some experiments of Weisbach.

$\rho =$	$\frac{9}{4}$	4	$\frac{25}{4}$	9
$c_c =$	·824	·852	·873	·892
$r =$	2·19	1·56	1·36	1·26
$K =$	3·18	1·26	·81	·68
$K_b =$	2·38	1·12	·72	·52
$K_f =$	·80	·13	·09	·09

178. **Distribution of pressure on a plane struck normally by a jet.**—Mr. J. S. Beresford made some experiments on the distribution of pressure on a plane struck by a jet. A small hole in the plane communicated by a flexible tube with a pressure column. This aperture was moved across the area struck by the jet. In the following abstract, columns A give the ratio (distance from axis of jet)/(diameter of jet) and the columns B the ratio (pressure head)/(velocity head of jet).

Jet 0·475 Inch Diameter. Velocity Head 43 Inches.		Jet 0·988 Inch Diameter. Velocity Head 42 Inches.		Jet 1·95 Inches Diameter. Velocity Head 27 Inches.	
A	B	A	B	A	B
0	·965	0	·998	0	·993
·21	·917	·10	·988	·17	·967
·42	·839	·20	·958	·37	·808
·63	·672	·30	·891	·48	·669
·84	·334	·51	·546	·61	·463
1·05	·083	·71	·118	·71	·261
1·26	·012	·86	·038	·97	·074

179. **Pressure of an unlimited stream of water on a solid at rest.**—The theorem in § 177, although it elucidates the general action of a stream on a solid immersed in it, does not furnish a numerical solution for the case of a very large stream acting on a small solid. But the general expressions

$$P = (K_f + K_b)G\omega \frac{v^2}{2g} = KG\omega \frac{v^2}{2g},$$

where ω is the projected area of the solid normal to the direction of motion, and K_f, K_b, K are experimental coefficients, have been generally adopted, and appear to agree with the results of experiments so far as they have been carried for unshipshape bodies, the resistance of which is due to the creation of eddies at sharp changes of section in the stream. For quite shipshape bodies, in which the surfaces over which the water slides are of gradual and continuous curvature, and which are wholly immersed, the resistance is due to skin friction, and depends on the total surface of the body, not its projected area.

From some experiments by Dubuat and Duchemin on prisms of cross section $a \times a$ and length l, immersed in a stream of water, the following results were deduced :—

$\frac{l}{a} = 0·03$	1	2	3	6
$K_f = 1·19$	1·19	1·19	1·19	1·19
$K_b =$ ·67	·27	·16	·14	·27
$K = 1·86$	1·46	1·35	1·33	1·46

It is difficult to believe that K_f can be greater than unity. The shortest prism corresponds with a thin plate. In the case of the longest prism it would seem that the increase of resistance is due to skin friction. For a plane one foot square moved in still water Dubuat found $K_f = 1$, $K_b = 0{\cdot}433$, $K = 1{\cdot}433$. Morin, Piobert, and Didion found $K = 1{\cdot}36$ for planes moved normally through air, and Thibault obtained a mean value $K = 1{\cdot}83$.

180. **Stanton's experiments.**[1]—A very careful research has been carried out by Dr. Stanton at the National Physical Laboratory. The solids were placed in a cylindrical trunk 2 feet in diameter and 4 feet 6 inches long, through which a steady current of air was drawn by a fan. It was found that if the area of a plane placed in this trunk was more than 1-144th of the cross section of the trunk, there was a perceptible increase of resistance due to the action of the sides of the trunk which caused an increase of the negative back pressure. Hence the experiments were limited to very small planes. The maximum intensity of front pressure at the centre of a circular or square plane, normal to the current, was always very approximately

$$G \frac{v^2}{2g} \text{ lbs. per square foot,}$$

and the intensity of pressure diminished towards the edges. At the back of the plate there was a negative pressure nearly

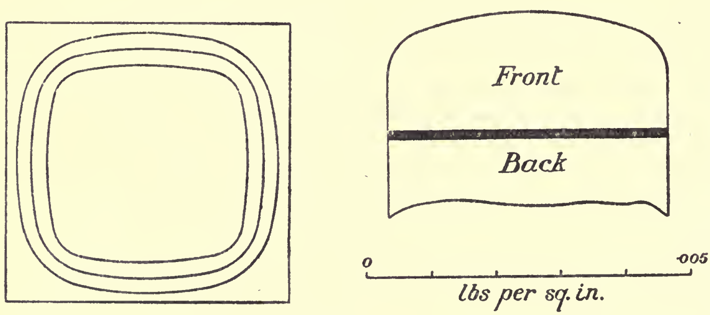


Fig. 144.

uniformly distributed. Fig. 144 shows the distribution of pressure on a square plate and some lines of equal pressure.

[1] *Proc. Inst. Civil Engineers*, clvi., 1903-4.

The average value of K_b was 0·48 for a circular and 0·67 for a square plate. So far as the tests went, the total resistance of similar plates when normal to the stream was directly proportional to the area. The total resistance of square or circular plates, normal to the stream, the velocity of which was v feet per second or V miles per hour, was

$$P = 0{\cdot}00126v^2 = 0{\cdot}0027V^2 \text{ lbs. per square foot,}$$

which is nearly in agreement with the result obtained by Mr. Dines, namely,

$$P = 0{\cdot}0029V^2.$$

If the weight of a cubic foot of air at 60° and 1 atm. is taken at 0·0764 lb., Stanton's result can be put in the form

$$P = 1{\cdot}061G\frac{v^2}{2g} \text{ lbs. per square foot,}$$

and using the result as to negative pressure stated above, this gives

	Coefficient of		
	Front Pressure, K_f.	Back Pressure, K_b.	Total Pressure, K.
Circular Plate . .	0·581	0·48	1·061
Square Plate . .	0·391	0·67	1·061

Stanton's results give somewhat lower pressures than those obtained by earlier observers. He has since carried out experiments on larger planes and solids acted on by wind pressure, and has found that almost uniformly the pressures in these conditions are 18 per cent greater than in the previous experiments on small planes and solids tested in the air trunk. It would appear, therefore, that for planes in an indefinitely large stream

$$P = 1{\cdot}252G\frac{v^2}{2g} \text{ lbs. per square foot.}$$

For rectangular plates, the total resistance was found to increase with the ratio of length to width of plate. The following are some examples deduced from Dr. Stanton's results:—

Dimensions. Inches.	Ratio of Length to Width.	Total Pressure in Lbs.
3 × 1	3	$P = \cdot 00134 v^2$
3·75 × ·75	5	$\cdot 00135 v^2$
5·0 × ·5	10	$\cdot 00151 v^2$
7·5 × ·15	50	$\cdot 00201 v^2$

181. **Pressure on solids of various forms.**—When a solid body is presented to a stream the front pressure is modified if the face of the body is not plane, and the back pressure if the form of the body interferes with or facilitates the convergence in the wake. If k is the ratio of the total pressure on the solid to the pressure on a thin plate normal to the stream and of area equal to the projected area of the solid normal to the stream, then

	$k=$	Direction of stream.
Sphere	0·31	......
Cube	0·80	Normal to face.
,,	0·66	Parallel to diagonal of face.
Cylinder (height = diameter)	0·47	Normal to axis.
Cone (height = diameter of base)	0·38	Parallel to base.

182. **Pressure on planes oblique to the direction of the stream.**—Let Fig. 145 represent a plane moving in a fluid at rest in the direction R, making an angle θ with the normal to the plane, or conversely a plane at rest in a stream moving in the direction R. The resultant pressure on the plane will be a normal pressure N, with a component R in the direction of motion and a lateral component L resisted by the supports of the plane. Obviously

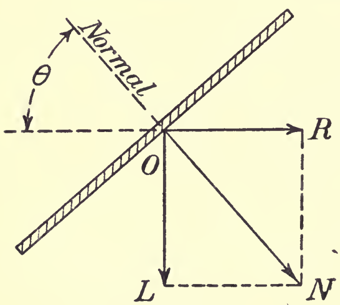


Fig. 145.

$$R = N \cos \theta,$$
$$L = N \sin \theta.$$

The simplest expression for the pressure on the plane in the direction of motion is that of Duchemin,

$$R = P \frac{2 \cos^2 \theta}{1 + \cos^2 \theta} \text{ lbs. per square foot,}$$

where P is the pressure per square foot on a plane in similar conditions normal to the direction of the stream. Consequently the normal pressure on the plane is

$$N = P \frac{2 \cos \theta}{1 + \cos^2 \theta} = \frac{2P}{\sec \theta + \cos \theta}.$$

The following table contains some results calculated by this rule. Dr. Stanton experimented on a small plane 3 inches by 1 inch, with a velocity of stream of 21 feet per second. He found the remarkable result that the normal pressure was different according as the short or the long axis of the rectangle was normal to the current. Further, in the case of the long axis normal to the current, the normal pressure for an inclination of about 45° was considerably greater than when the plane was normal to the stream.

NORMAL PRESSURE ON THIN PLANES

Angle θ.	Values of N/P.		
	By Duchemin's Rule.	Stanton.	
		Long Axis Normal.	Short Axis Normal.
0	1·00	1·00	1·00
15	1·00	1·00	·97
30	·99	1·01	·87
45	·94	1·11	·79
60	·80	·88	·71
75	·49	·30	·64
80	·34	·16	·56
85	·17	·08	·34
90	0	0	0

In 1872 some experiments were made for the Aeronautical Society on the pressure of air on oblique planes. These plates, of 1 to 2 feet square, were balanced by ingenious mechanism designed by Mr. Wenham and Mr. Spencer Browning, in such a manner that both the pressure in the direction of the air current and the lateral force were separately measured. These planes were placed opposite a blast from a fan issuing from a wooden pipe 18 inches square. The pressure of the blast varied from $\frac{6}{10}$ to 1 inch of water pressure. The following are the results given in pounds per square foot of the plane, and a comparison of the experimental results with the pressures given by Duchemin's rule. These last values are obtained by taking P = 3·31, the observed pressure on a normal surface:—

	$\theta =$			
	75°	70°.	30°.	0°.
Horizontal pressure R	0·4	0·61	2·73	3·31
Lateral pressure L	1·6	1·96	1·26	0
Normal pressure $\sqrt{L^2 + R^2}$ . . .	1·65	2·05	3·01	3·31
Normal pressure by Duchemin's rule .	1·605	2·027	3·276	3·31

Lord Rayleigh obtained theoretically the expression

$$N_f = P\frac{(4+\pi)\sin\theta}{4+\pi\sin\theta},$$

but this gives the normal component of the front pressure only. Dr. Stanton found the variation of total normal pressure with inclination to be very different in the case of rectangular plates according as the longer or shorter side was perpendicular to the stream.

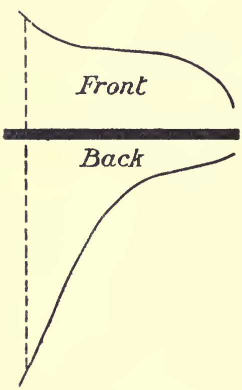


Fig. 146.

183. **Distribution of pressure on an inclined plane.**—In the case of a plane inclined to a stream there is an excess of pressure at the forward part and less pressure sternwards. Fig. 146, from Dr. Stanton's results, shows generally the distribution of positive pressure on the windward and negative pressure on the leeward side of a plane at 45° to the direction of an air current. Clearly the resultant pressure does not act through the centre of the plane. Conversely, if a plane is pivoted about an axis eccentric to its centre line and placed in a stream,

it will assume a position inclined to the stream such that the resultant normal pressure passes through the axis about which it can turn. If, therefore, planes pivoted so that the ratio $\frac{a}{b}$ (Fig. 147) is varied are placed in water, and the angle they make with the direction of the stream is

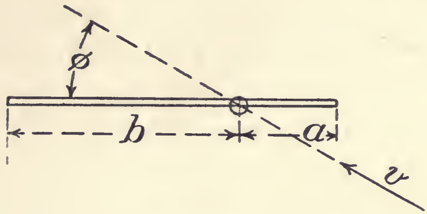


Fig. 147.

observed, the position of the resultant of the pressures on the plane is determined for different angular positions. Experiments of this kind have been made by Hagen. Some of his results are given in the following table :—

$\frac{a}{b}$.	$\frac{a}{a+b}$.	Values of ϕ.		
		Larger Plane.	Smaller Plane.	Calculation.
1·0	·500	...	90°	90°
0·9	·474	75°	72½	66½
0·8	·445	60	57	55
0·7	·412	48	43	45
0·6	·375	25	29	35½
0·5	·333	13	13	26½
0·4	·286	8	6½	16½
0·3	·231	6½	...	6
0·2	·167	4	...	...

Joëssel has given the formula

$$\frac{a}{a+b} = 0{\cdot}2 + 0{\cdot}3 \sin \phi.$$

The last column in the table above gives angles calculated by this rule.

184. **Wind Pressure.**—One of the most important cases to the engineer, in which the pressure of a fluid stream on bodies immersed in it has to be considered, is that of the pressure of wind on structures. Unfortunately the action of the wind is so complex and variable that there is not general agreement as to the allowance to be made for it.

Storm winds are generally rotating eddies generated between two oppositely flowing air currents not of themselves of violent character. Once put in motion, the energy of such

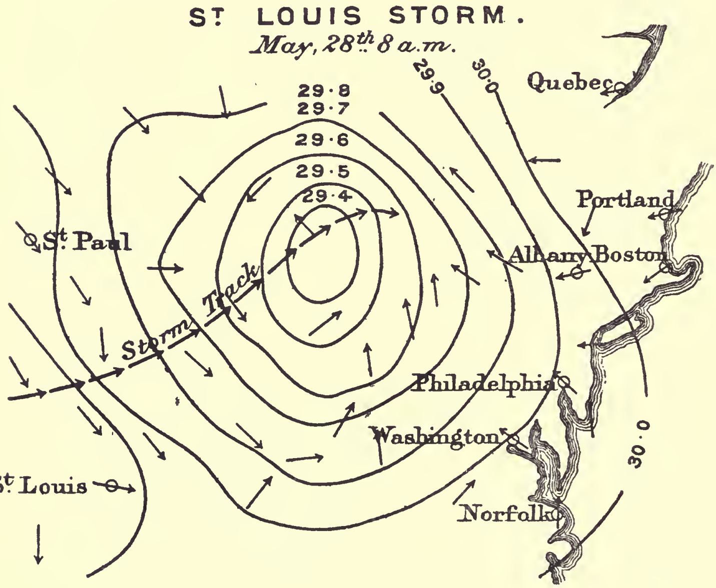


Fig. 148.

an eddy accumulates and the distribution of the energy is a purely mechanical problem. Conditions of dynamical stability involve this, that the pressure diminishes and the velocity increases from the circumference to the centre of the eddy (§ 33). Fig. 148 is a diagram of the St. Louis storm of 1896, which shows that the isobars formed closed curves round the storm centre, the barometric pressure decreasing from 30 inches at the outside to 29·4 inches at the centre. On the other hand, the velocity and violence of the wind increase towards the centre. A storm of this kind is not fixed in position.

Its centre travels along a track generally in the northern hemisphere eastwards or north-eastwards. At any given place, as the storm passes, the wind veers round contrary to the hands of a watch. The storm centre may travel 20 or 30 miles per hour, but the wind velocity near the centre of the storm may be 80 or 100 miles per hour. The area of a storm is extremely variable. It may be 600 or 1200 miles in diameter. In other cases the width of the track over which the wind is violent enough to cause destruction may be only 60 to 1000 feet. Some whirlwinds cut down the trees in a forest along a track as narrow as a road, leaving trees on either side undamaged.

Wind pressures are measured on anemometers of two types, pressure and velocity anemometers. In the former the pressure is measured on a thin vertical plate exposed normally to the wind. It is rare for pressures on such a plate to exceed 30 lbs. per square foot. But at Bidston Observatory near Liverpool pressures of 50 to 80 lbs. per square foot have been registered. There the anemometer is 56 feet above the ground and 251 feet above sea-level. The exposure of the anemometer is complete and severe, but the Board of Trade Committee on the Tay Bridge disaster found no reason to doubt the records. Baier came to the conclusion, after examining some cases of destruction, that the wind pressure in the tornado at St. Louis in 1896 must have ranged from 45 to 90 lbs. per square foot.

A large number of records have been obtained with velocity anemometers of the Robison type, in which hemispherical cups are rotated by the wind, the velocity of the cups being about one-third that of the wind. These records give the average velocity over a more or less considerable period of time. The Board of Trade Committee found that if v_m is the mean velocity during an hour, then the highest pressure during the hour would be approximately

$$P = 0{\cdot}01 v^2_m \text{ lbs. per square foot.}$$

Now observations at Aberdeen show a wind travel of 69 miles an hour, corresponding to a maximum pressure of 48 lbs. per square foot; at Falmouth a travel of 71 miles per hour, corresponding to 50 lbs. per square foot; at Holyhead a travel of 80 miles an hour, corresponding to 64 lbs. per square foot. The velocity anemometer is free of inertia errors, and its indications are not consistent with the supposition that gusts during which the pressure is excessive are necessarily of short duration.

185. **Increase of pressure with elevation.**—Numerous experiments show that the wind velocity and pressure is greater the greater the height from the ground. In some experiments by Mr. Thomas Stevenson in 1878, six velocity anemometers were fixed on a vertical pole 50 feet in height, and observations were taken at various dates when strong winds were blowing. For a height of 15 feet from the ground the velocities were low and irregular even when strong winds were blowing. For heights above 20 feet the velocities increased in a fairly regular way with increase of elevation.

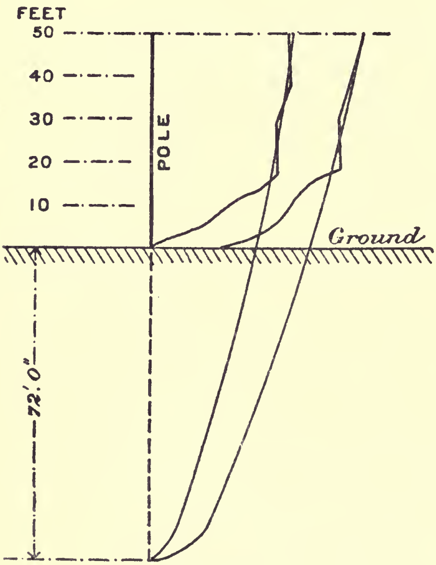


Fig. 149.

Plotted horizontally the wind velocities gave the irregular curves in Fig. 149. For heights above 20 feet the velocity curves agreed fairly with parabolas having their vertices 72 feet below ground-level. If V and v are velocities, and P and p pressures at heights of H and h feet,

$$V = v\sqrt{\frac{H + 72}{h + 72}},$$

$$P = p\,\frac{H + 72}{h + 72}.$$

Suppose that at 25 feet above ground the mean hourly velocity is 30 miles per hour, corresponding to a maximum pressure during the hour of 9 lbs. per square foot. Then at higher elevations the velocities and pressures by Stevenson's rule would be as follows:—

Elevation.	Mean Velocity. Miles per hour.	Maximum Pressure. Lbs. per sq. ft.
25	30·0	9·0
50	33·6	11·3
100	39·9	16·0
200	50·4	25·3
300	58·8	34·6

These results apply only to the case of a flat and nearly unobstructed ground surface.

186. **Evidence of high wind pressure in storms.**—It may be shown that a pressure of 25 to 35 lbs. per square foot distributed over the area of a railway carriage is necessary to overturn it, and this must be chiefly front pressure, as in the case of such a body it cannot be supposed that the negative pressure due to a wake is as completely established as in the case of a thin plate. Now Mr. Seyrig has described the overturning of five carriages of a passenger train at Salces, in France, in 1860. On the same day five waggons of a freight train were overturned at Rivesaltes, and three others thrown off the track. On the same railway, in 1867, a passenger train was almost completely overturned. In 1867 a brake-van and post-office tender were blown over between Chester and Holyhead. In 1864 carriages in two trains on the Eastern Bengal Railway were overturned by wind. In 1870 two spans of a bridge at Decatur, U.S.A., were blown over; and in 1880, one 150-foot span of a bridge at Meredocia. On September 10, 1897, in Paris, a cyclone uprooted every tree from the Quai St. Michel to the Pont Neuf, some barges were sunk, an omnibus overturned, and at the Palais de Justice not a pane was left in the windows.

On the other hand, those who have carefully examined cases of damage by wind have found that structures such as windows, chimneys, roofs, etc., of weak construction, and incapable of standing any considerable lateral pressure, have stood for long periods unharmed. Whether any adequate explanation of the paradox thus presented can be given is doubtful, but certain considerations may be noted: (*a*) At any one place the occurrence of high wind pressure must be very exceptional; (*b*) A structure must be still more rarely struck normally; (*c*) Its form may prevent the creation of a negative pressure; (*d*) Neighbouring obstructions may have the effect of shielding a structure. In this connection the great decrease of wind velocity near the ground is instructive.

187. **The Forth Bridge experiments.**—During the construction of the Forth Bridge some important experiments were carried out by Sir B. Baker. A very large pressure-plate anemometer was erected on Inchgarvie, 20 feet long by 15 feet high, facing east and west. Beside it were erected two small pressure plates, one facing east and west, the other revolving to face the wind. Between 1883 and 1890, on fourteen occasions of storm, pressures ranging from 25 to 65 lbs. per square foot were registered by the revolving pressure plate. In the same period the pressure on the small fixed pressure plate ranged from 16 to 41 lbs. per square foot. Also, during the same period, pressures were registered by the large plate of 300 square feet area ranging from 7 to 35 lbs. per square foot.

For experiments on bodies of complex form, Sir B. Baker adopted a very ingenious device. Experiments in wind storms would have been difficult and inconvenient. Instead of this a light wooden rod was suspended by a cord. At one end, the complex form the resistance of which was required was fixed; at the other, a small cardboard plane. Setting the apparatus swinging, it was obvious at once at which end of the rod the resistance was greatest. Then the area of the cardboard plane was altered until its resistance just balanced that of the body to be tested. In this way the areas of plane having resistance equivalent to that of various bodies of complex form was determined.

For bodies of comparatively simple form, such as cubes and cylinders, the relative resistances were found to be the same as those directly determined by earlier observers. The most interesting point to determine next was the influence of one surface in sheltering another. With discs placed at from one to four diameters apart, there was complete shelter when the distance was one diameter, the resistance being the same as for a simple disc. The resistance was increased by 25 per cent when the discs were $1\frac{1}{2}$ diameters apart; by 40 per cent at 2 diameters; by 60 per cent for 3 diameters; and by 80 per cent for 4 diameters. Intermediate discs did not much increase the resistance. Four discs in series behind each other, with a total distance between first and fourth of $3\frac{1}{2}$ diameters, had no more resistance than two discs at 4 diameters.

Perforated discs were then tried to imitate the effect of shelter of one lattice girder on another. With openings in the discs equal to one-fourth the whole area, the discs being 1 diameter apart, the resistance of the sheltered disc was only 8 per cent of that of the front disc. But with openings half the whole area, the resistance of the sheltered disc was 30 per cent of that of the front disc. At 2 diameters apart, the resistances of the sheltered disc were 40 per cent to 66 per cent of that of the front disc, and at 4 diameters apart, with openings half the total area, the resistance of the sheltered disc was 94 per cent of that of the front disc.

The top members of the Forth Bridge consist each of a pair of box-lattice girders, that is, they are nearly equivalent to four single lattice girders in series. Models of single-web girders made to imitate these were tested in pairs. With distances apart equal to once, twice, and three times the depth of the girders, the resistance of the sheltered girder was 20 per cent, 50 per cent, and 70 per cent of the resistance of the front girder. With additional girders placed between the others the increase of resistance was small. With a complete model of a bay of one top member of the bridge, that is, with the equivalent of two single-lattice girders, the total resistance was 1·75 times the resistance of a plate equal in area to the projection of one lattice girder, that is, to the projection of the solid surfaces excluding the openings.

The bottom member of the Forth Bridge consists of two tubes of circular section braced together by lattice girders. A complete model of one bay was tested. It had a resistance 10 per cent greater than the resistance of a plane surface of the projected area of one tube.

Examples

1. A jet 3 inches in diameter under a head of 400 feet strikes normally a plane at rest. Find the pressure on the plane. 2452 lbs.
2. A jet of water delivers 160 cubic feet per minute at a velocity of 20 feet per second, and strikes a plane normally. Find the pressure on the plane: (1) when the plane is at rest; (2) when it is moving at 7 feet per second in the direction of the jet. In the latter case find the rate at which work is done in driving the plane.
   103·4 lbs.; 43·7 lbs.; 305·8 ft.-lbs. per second.
3. Water impinges on a Poncelet float at 10° with the tangent to the circumference of the wheel. The velocity of the water is double that of the float. Find by construction the angle of the float to receive the water without shock. A slope of 10° is nearly 1 in 6.
4. A cylindrical chimney shaft 100 feet high and 75 feet in diameter is exposed to a wind pressure of 30 lbs. per square foot. Find the overturning moment. 105,750 lbs.
5. A fixed curved vane has a receiving edge making an angle of 45° and a delivering edge an angle of 20° with a line AB. A jet delivers 10 cubic feet per second at a velocity of 30 feet per second, without shock, so that it is deviated along the vane. Find the resultant pressure on the vane, the angle it makes with AB, and the components of the pressure along and at right angles to AB. 970 lbs.; $12\frac{1}{2}$°; 946 lbs.; 210 lbs.
6. Suppose the vane in the previous question is moving in the direction AB at 10 feet per second, and the jet at 45° with AB at 30 feet per second. Find the angle the receiving edge of the vane must make with AB that there may be no shock. Also the relative velocity. 63°; 24 ft. per second.

APPENDIX

TABLE I.—Functions of Numbers from 0·1 to 10·0

n.	$\sqrt{n}$.	$\sqrt[3]{n}$.	$\sqrt{n^3}$.	Natural log. n.	n.	$\sqrt{n}$.	$\sqrt[3]{n}$.	$\sqrt{n^3}$.	Natural log. n.
·1	·3162	·4642	·0316	...	5·1	2·258	1·721	11·52	1·6292
·2	·4472	·5848	·0894	...	5·2	2·280	1·732	11·86	·6487
·3	·5477	·6694	·164	...	5·3	2·302	1·744	12·20	·6677
·4	·6325	·7368	·253	...	5·4	2·324	1·754	12·55	·6864
·5	·7071	·7937	·354	...	5·5	2·345	1·765	12·90	1·7047
·6	·7746	·8434	·465	...	5·6	2·366	1·776	13·25	·7228
·7	·8367	·8879	·586	...	5·7	2·387	1·786	13·61	·7405
·8	·8944	·9283	·716	...	5·8	2·408	1·797	13·97	·7579
·9	·9487	·9655	·854	...	5·9	2·429	1·807	14·33	·7750
1·0	1·0000	1·0000	1·000	0·0000	6·0	2·449	1·817	14·70	1·7918
1·1	1·0488	1·0323	1·153	·0953	6·1	2·470	1·827	15·07	·8083
1·2	1·0954	1·0627	1·315	·1823	6·2	2·490	1·837	15·44	·8245
1·3	1·1402	1·0914	1·483	·2624	6·3	2·510	1·847	15·81	·8405
1·4	1·1832	1·1187	1·655	·3365	6·4	2·530	1·857	16·19	·8563
1·5	1·2247	1·1447	1·837	0·4055	6·5	2·550	1·866	16·57	1·8718
1·6	1·2649	1·1696	2·024	·4700	6·6	2·569	1·876	16·96	·8871
1·7	1·3038	1·1935	2·216	·5306	6·7	2·588	1·885	17·34	·9021
1·8	1·3416	1·2164	2·414	·5878	6·8	2·608	1·895	17·73	·9169
1·9	1·3784	1·2386	2·62	·6419	6·9	2·627	1·904	18·13	·9315
2·0	1·4142	1·2599	2·83	0·6931	7·0	2·646	1·913	18·52	1·9459
2·1	1·4491	1·2806	3·04	·7419	7·1	2·665	1·922	18·92	·9601
2·2	1·4832	1·3006	3·26	·7885	7·2	2·683	1·931	19·32	·9741
2·3	1·5166	1·3200	3·49	·8329	7·3	2·702	1·940	19·72	·9879
2·4	1·5492	1·3389	3·72	·8755	7·4	2·720	1·949	20·13	2·0015
2·5	1·5811	1·3572	3·95	0·9163	7·5	2·739	1·957	20·54	2·0149
2·6	1·6125	1·3751	4·19	·9555	7·6	2·757	1·966	20·95	·0281
2·7	1·6432	1·3925	4·44	·9933	7·7	2·775	1·975	21·36	·0412
2·8	1·6733	1·4095	4·69	1·0296	7·8	2·793	1·983	21·79	·0541
2·9	1·7029	1·4260	4·94	·0647	7·9	2·811	1·992	22·20	·0669
3·0	1·7321	1·4422	5·20	1·0986	8·0	2·828	2·000	22·63	2·0794
3·1	1·7607	1·4581	5·46	·1314	8·1	2·846	2·008	23·05	·0919
3·2	1·7889	1·4736	5·73	·1632	8·2	2·864	2·017	23·48	·1041
3·3	1·8166	1·4888	6·00	·1939	8·3	2·881	2·025	23·91	·1163
3·4	1·8439	1·5037	6·27	·2238	8·4	2·898	2·033	24·34	·1282
3·5	1·871	1·518	6·55	1·2528	8·5	2·915	2·041	24·78	2·1401
3·6	1·897	1·533	6·83	·2809	8·6	2·933	2·049	25·22	·1518
3·7	1·924	1·547	7·12	·3083	8·7	2·950	2·057	25·66	·1633
3·8	1·949	1·560	7·41	·3350	8·8	2·966	2·065	26·10	·1748
3·9	1·975	1·574	7·70	·3610	8·9	2·983	2·072	26·55	·1861
4·0	2·000	1·587	8·00	1·3863	9·0	3·000	2·080	27·00	2·1972
4·1	2·025	1·601	8·30	·4110	9·1	3·017	2·088	27·45	·2083
4·2	2·049	1·613	8·61	·4351	9·2	3·033	2·095	27·91	·2192
4·3	2·074	1·626	8·92	·4586	9·3	3·050	2·103	28·36	·2300
4·4	2·098	1·639	9·23	·4816	9·4	3·066	2·110	28·82	·2407
4·5	2·121	1·651	9·55	1·5041	9·5	3·082	2·118	29·28	2·2513
4·6	2·145	1·663	9·87	·5261	9·6	3·098	2·125	29·74	·2618
4·7	2·168	1·675	10·19	·5476	9·7	3·114	2·133	30·21	·2721
4·8	2·191	1·687	10·51	·5686	9·8	3·130	2·140	30·68	·2824
4·9	2·214	1·698	10·85	·5892	9·9	3·146	2·147	31·15	·2925
5·0	2·236	1·710	11·18	1·6094	10·0	3·162	2·154	31·62	2·3026

TABLE II.—Velocity and Head

n.	Height due to Velocity, $\frac{n^2}{2g}$.		Velocity due to Height, $\sqrt{2gn}$.		n.	Height due to Velocity, $\frac{n^2}{2g}$.		Velocity due to Height, $\sqrt{2gn}$.	
	Metres.	Feet.	Metres.	Feet.		Metres.	Feet.	Metres.	Feet.
·1	·00051	·000155	1·401	2·537	5·1	1·326	·4041	10·00	18·12
·2	·00203	·000622	1·981	3·588	5·2	·378	·4201	·10	·29
·3	·00459	·001398	2·426	4·394	5·3	·432	·4365	·20	·47
·4	·00816	·002486	2·801	5·074	5·4	·486	·4531	·29	·64
·5	·01274	·003885	3·132	5·673	5·5	1·542	·4700	10·39	18·81
·6	·01835	·005593	3·431	6·214	5·6	·599	·4873	·48	·98
·7	·02498	·007613	3·706	6·712	5·7	·656	·5048	·57	19·15
·8	·03262	·009943	3·962	7·176	5·8	·715	·5227	·67	·32
·9	·04129	·01259	4·202	7·611	5·9	·774	·5408	·76	·48
1·0	0·0510	·01554	4·429	8·022	6·0	1·835	·5593	10·85	19·65
1·1	·0617	·01880	4·645	8·414	6·1	·897	·5782	·94	·81
1·2	·0734	·02237	4·852	8·788	6·2	·959	·5973	11·03	·97
1·3	·0861	·02626	5·050	9·147	6·3	2·023	·6167	·12	20·13
1·4	·0999	·03045	5·241	9·492	6·4	·088	·6364	·21	·29
1·5	0·1147	·03496	5·425	9·826	6·5	2·154	·6564	11·29	20·45
1·6	·1305	·03978	·603	·148	6·6	·220	·6768	·38	·61
1·7	·1473	·04490	·775	·460	6·7	·288	·6975	·46	·76
1·8	·1652	·05034	·942	·764	6·8	·357	·7185	·55	·92
1·9	·1840	·05609	6·105	11·059	6·9	·427	·7397	·63	21·07
2·0	0·2039	·06215	6·264	11·346	7·0	2·498	·7613	11·72	21·23
2·1	·2248	·06852	·418	·626	7·1	·570	·7832	·80	·38
2·2	·2467	·07520	·570	·899	7·2	·643	·8055	·88	·53
2·3	·2697	·08219	·717	12·167	7·3	·716	·8280	·97	·67
2·4	·2936	·08950	·862	·429	7·4	·791	·8508	12·05	·82
2·5	0·3186	·09711	7·003	12·685	7·5	2·867	·8740	12·13	21·97
2·6	·3446	·10503	·142	·936	7·6	·944	·8974	·21	22·11
2·7	·3716	·11326	·278	13·182	7·7	3·022	·9212	·29	·26
2·8	·3996	·12182	·411	·424	7·8	·101	·9475	·37	·40
2·9	·4287	·13067	·543	·662	7·9	·181	·9697	·45	·55
3·0	0·4588	·1398	7·672	13·90	8·0	3·262	·9944	12·53	22·69
3·1	·4899	·1493	·798	14·13	8·1	·344	1·0194	·61	·83
3·2	·5220	·1591	·923	·35	8·2	·428	1·0447	·68	·97
3·3	·5551	·1692	8·046	·57	8·3	·512	1·0704	·76	23·11
3·4	·5893	·1796	·167	·79	8·4	·597	1·0963	·84	·25
3·5	0·6244	·1904	8·286	15·01	8·5	3·683	1·1226	12·91	23·39
3·6	·6606	·2014	·404	·21	8·6	·770	1·1492	·99	·53
3·7	·6978	·2127	·520	·42	8·7	·858	1·1761	13·06	·66
3·8	·7361	·2244	·634	·63	8·8	·947	1·2032	·14	·80
3·9	·7753	·2363	·747	·84	8·9	4·038	1·2307	·21	·93
4·0	0·8156	·2486	8·858	16·05	9·0	4·129	1·259	13·29	24·07
4·1	·8569	·2612	·968	·24	9·1	·221	1·287	·36	·20
4·2	·8992	·2741	9·077	·44	9·2	·314	1·315	·43	·33
4·3	·9425	·2873	·184	·63	9·3	·409	1·344	·51	·47
4·4	·9869	·3008	·291	·83	9·4	·504	1·373	·58	·60
4·5	1·0322	·3146	9·396	17·02	9·5	4·600	1·402	13·65	24·73
4·6	·0786	·3288	·500	·20	9·6	·698	1·432	·72	·86
4·7	·1260	·3432	·602	·39	9·7	·796	1·462	·79	·99
4·8	·1745	·3580	·704	·57	9·8	·896	1·492	·87	25·11
4·9	·2239	·3731	·804	·76	9·9	·996	1·523	·94	·24
5·0	1·2744	·3884	9·904	17·94	10·0	5·097	1·554	14·01	25·37

TABLE III.—Slope Table

Fall in Feet per Mile. $f=$	Slope 1 in n. $n=$	Slope Foot per Foot. $i=$	Slope 1 in n. $n=$	Slope Foot per Foot. $i=$	Fall in Feet per Mile. $f=$
0·5	10560	·000095	6000	·000088	·88
0·75	7040	·000142	5000	·0002	1·06
1·0	5280	·000189	4500	·000222	1·17
1·25	4224	·000237	4000	·00025	1·32
1·5	3520	·000284	3500	·000286	1·51
1·75	3017	·000331	3000	·000333	1·76
2·0	2640	·000379	2500	·0004	2·11
3·0	1760	·000568	2000	·0005	2·64
4·0	1320	·000758	1500	·000667	3·52
5·0	1056	·000947	1250	·0008	4·23
6·0	880	·001136	1000	·001	5·28
7·0	754	·001326	750	·00133	7·03
8·0	660	·001515	500	·002	10·56
9·0	587	·001704	400	·0025	13·2
10·0	528	·001894	300	·00333	17·6
11·0	444	·002083	250	·004	21·1
12·0	440	·002273	200	·005	26·4
13·0	406	·002462	175	·00571	30·2
14·0	377	·002651	150	·00667	35·2
15·0	352	·002841	125	·008	42·3
17·5	302	·003311	100	·01	52·8
20·0	264	·003788	75	·0133	70·3
22·5	235	·004255	50	·02	105·6
25·0	211	·004735	40	·025	132
30·0	176	·005682	30	·0333	176
35·0	151	·006629	25	·04	211
40·0	132	·007576	20	·05	264
45·0	117	·008523	15	·0667	352
50·0	105·6	·009470	10	·1	528
60·0	88·0	·011364			
70·0	75·4	·01326			
80·0	66·0	·01515			
90·0	58·7	·01705			
100·0	52·8	·01894			
120·0	44·0	·02273			
140·0	37·7	·02652			
160·0	33·0	·03030			
180·0	29·3	·03409			
200·0	26·4	·03788			
300·0	17·6	·05682			
400·0	13·2	·07576			
500·0	10·6	·09470			

TABLE IV.—TABLE TO FACILITATE CALCULATIONS ON PIPES

Diameter.		Area of Section in Square Feet.	Hydraulic Mean Radius in Feet.	$\sqrt{m}$.	$\Omega\sqrt{m}$.
Inches.	Feet. d.	Ω.	$m=\frac{d}{4}$.		
3	0·250	0·0491	0·0625	·250	·0122
4	0·333	0·0873	·0833	·289	·0252
5	0·417	0·136	·104	·322	·0437
6	0·500	0·196	·125	·354	·0693
7	0·583	0·267	·146	·382	·1019
8	0·666	0·349	·166	·407	·1420
9	0·750	0·442	·188	·434	·1918
10	0·833	0·545	·208	·456	·2485
12	1·000	0·785	·250	·500	·3925
14	1·167	1·069	·292	·540	·577
15	1·250	1·227	·312	·559	·687
16	1·333	1·396	·333	·577	·807
18	1·500	1·767	·375	·612	1·083
20	1·666	2·182	·417	·646	1·408
21	1·750	2·405	·438	·662	1·588
24	2·000	3·142	·500	·707	2·219
27	2·250	3·976	·563	·750	2·985
30	2·500	4·909	·625	·791	3·88
33	2·750	5·939	·688	·829	4·92
36	3·000	7·068	·750	·866	6·14
40	3·333	8·725	·833	·913	7·96
42	3·500	9·621	·875	·935	8·99
45	3·750	11·050	·938	·968	10·65
48	4·000	12·566	1·000	1·000	12·56

TABLE V.—Discharge of Pipes at Different Velocities in Gallons per Hour

Diameter.		Velocities in Feet per Second.											
Inches.	Feet.	1	1½	2	2½	3	3½	4	4½	5	5½	6	6½
5	·4167	3064	4596	6128	7660	9192	10720	12250	13790	15320	16850	18390	19920
6	·5	4412	6618	8823	11030	13230	15440	17650	19860	22060	24270	26460	28670
7	·5833	6005	9007	12010	15010	18020	21010	24020	27020	30020	33030	36030	39030
8	·6667	7843	11770	15690	19610	23530	27460	31370	35300	39210	43140	47060	50980
10	·8333	12250	18380	24500	30630	36760	42880	49010	55130	61260	67380	73500	79640
12	1·0	17640	26460	35290	44110	52930	61760	70580	79390	88220	97040	105900	114700
15	1·25	27560	41340	55120	68900	82680	96470	110300	124000	137800	151600	165400	179200
18	1·5	39690	59540	79380	99220	119100	138900	158800	178600	198400	218300	238100	258000
20	1·667	49020	73530	98050	122600	147100	171600	196100	220600	245100	269600	294100	318600
24	2·0	70580	105900	141200	176400	211700	247000	282300	317700	352900	388200	423500	458800
27	2·25	89310	134000	178600	223300	268000	312600	357300	401900	446600	491200	535900	580600
30	2·5	110300	165400	220600	275700	330900	386000	441100	496300	551400	606500	661600	716900
33	2·75	133500	200250	267000	333750	400500	467250	534000	600750	667500	734250	801000	867750
36	3·0	158800	238200	317600	397000	476400	555800	635200	714600	794000	873400	952800	1032200
42	3·5	216100	324150	432200	540250	648300	756350	864400	972450	1080500	1188600	1296600	1404700
48	4·0	282300	423450	564600	705750	846900	988050	1129200	1270350	1411500	1552650	1693800	1835000

TABLE VI.—DISCHARGE OF PIPES AT DIFFERENT VELOCITIES IN CUBIC FEET PER SECOND

Diameter.		Velocities in Feet per Second.											
Inches.	Feet.	1	1½	2	2½	3	3½	4	4½	5	5½	6	6½
5	·4167	·1364	·2046	·2728	·3410	·4092	·4774	·5456	·6138	·6820	·7502	·8184	·8865
6	·5	·1963	·2946	·3928	·4909	·5891	·6873	·7855	·8837	·9819	1·080	1·178	1·276
7	·5833	·2673	·4010	·5346	·6682	·8019	·9356	1·069	1·203	1·337	1·470	1·603	1·738
8	·6667	·3491	·5237	·6983	·8728	1·047	1·222	1·396	1·571	1·746	1·920	2·094	2·270
10	·8333	·5454	·8181	1·091	1·363	1·636	1·909	2·182	2·454	2·727	3·000	3·272	3·545
12	1·0	·7854	1·178	1·571	1·963	2·356	2·749	3·142	3·534	3·927	4·320	4·713	5·105
15	1·25	1·227	1·841	2·453	3·067	3·681	4·294	4·908	5·521	6·134	6·748	7·361	7·975
18	1·5	1·767	2·650	3·534	4·417	5·301	6·184	7·068	7·950	8·834	9·718	10·60	11·48
20	1·667	2·182	3·273	4·364	5·456	6·547	7·638	8·730	9·819	10·91	12·00	13·09	14·19
24	2·0	3·142	4·713	6·284	7·854	9·427	11·00	12·57	14·14	15·71	17·28	18·85	20·42
27	2·25	3·976	5·964	7·950	9·938	11·93	13·92	15·90	17·89	19·88	21·87	23·85	25·84
30	2·5	4·909	7·364	9·818	12·28	14·73	17·18	19·63	22·09	24·55	27·00	29·45	31·91
33	2·75	5·940	8·910	11·88	14·85	17·82	20·79	23·76	26·73	29·70	32·67	35·64	38·61
36	3·0	7·069	10·60	14·14	17·67	21·21	24·74	28·28	31·81	35·35	38·88	42·41	45·95
42	3·5	9·621	14·43	19·24	24·05	28·86	33·67	38·48	43·30	48·11	52·92	57·73	62·54
48	4·0	12·57	18·86	25·14	31·43	37·72	44·01	50·28	56·57	62·85	69·14	75·42	81·71

TABLE VII.—Loss of Head in New Cast-Iron Pipes

Diameter.		Velocity in Feet per Second.										
		1	1½	2	2½	3	3½	4	4½	5	5½	6
Inches.	Feet.	Loss of Head i in Fractions of a Foot per Foot.										
5	·4167	·0009290	·002047	·003588	·005544	·007912	·01069	·01387	·01746	·02142	·02581	·03058
6	·5	·0007506	·001655	·002899	·004480	·006395	·008638	·01120	·01410	·01732	·02086	·02471
7	·5833	·0006269	·001382	·002422	·003742	·005340	·007214	·00936	·01179	·01446	·01743	·02064
8	·6667	·0005365	·001183	·002072	·003203	·004571	·006175	·008009	·01008	·01238	·01491	·01766
10	·8333	·0004133	·000911	·001597	·002467	·003522	·004756	·006170	·00777	·00954	·01149	·01361
12	1·0	·0003340	·000737	·001290	·001994	·002845	·003844	·004986	·00628	·00771	·00929	·01100
15	1·25	·0002574	·000568	·000994	·001536	·002193	·002962	·003842	·00484	·00594	·00716	·00847
18	1·5	·0002080	·000459	·000804	·001242	·001772	·002394	·003106	·00391	·00480	·00578	·00685
20	1·667	·0001840	·000406	·000710	·001097	·001567	·002116	·002746	·00346	·00424	·00511	·00605
24	2·0	·0001487	·000328	·000574	·000887	·001267	·001711	·002219	·00279	·00343	·00413	·00490
27	2·25	·0001295	·000286	·000500	·000773	·001104	·001490	·001934	·00243	·00299	·00360	·00427
30	2·5	·0001146	·000253	·000443	·000684	·000976	·001318	·001710	·00215	·00264	·00319	·00377
33	2·75	·0001025	·000226	·000396	·000612	·000873	·001180	·001530	·00193	·00236	·00285	·00338
36	3·0	·0000926	·000204	·000358	·000553	·000789	·001065	·001381	·00174	·00214	·00257	·00305
42	3·5	·0000773	·000171	·000299	·000462	·000659	·000890	·001154	·00145	·00178	·00215	·00255
48	4·0	·0000661	·000146	·000256	·000395	·000564	·000761	·000987	·00124	·00153	·00184	·00218

This table is calculated from the equation—

$$i=\frac{h}{l}=\frac{0{\cdot}0215}{d^{1\cdot168}}\frac{v^{1\cdot95}}{2g}.$$

In clean but not quite new pipes the loss of head is 10 to 15 per cent greater.

TABLE VIII.—Loss of Head in Incrusted Cast-Iron Pipes

Diameter.		Velocity in Feet per Second.										
		1	1½	2	2½	3	3½	4	4½	5	5½	6
Inches.	Feet.	Loss of Head i in Fractions of a Foot per Foot.										
5	·4167	·001887	·00425	·00755	·01180	·01698	·02312	·0302	·0382	·0472	·0571	·0680
6	·5	·001527	·00344	·00611	·00954	·01374	·01871	·0244	·0309	·0382	·0462	·0550
7	·5833	·001277	·00287	·00511	·00798	·01149	·01565	·0204	·0259	·0319	·0387	·0460
8	·6667	·001091	·00216	·00437	·00682	·00982	·01338	·0175	·0221	·0273	·0330	·0393
10	·8333	·000845	·001901	·00338	·00528	·00760	·01035	·0135	·0171	·0211	·0256	·0304
12	1·0	·000684	·001538	·00273	·00427	·00615	·00838	·01094	·0139	·0171	·0207	·0246
15	1·25	·000528	·001188	·00211	·00330	·00475	·00647	·00845	·01069	·01319	·01597	·0190
18	1·5	·000427	·000961	·001708	·00267	·00384	·00523	·00683	·00865	·01068	·01292	·01538
20	1·667	·000378	·000850	·001511	·00236	·00340	·00463	·00604	·00765	·00945	·01144	·01361
24	2·0	·000306	·000688	·001224	·00191	·00275	·00375	·00490	·00619	·00765	·00926	·01102
27	2·25	·000267	·000600	·001067	·00167	·00240	·00327	·00427	·00540	·00667	·00807	·00960
30	2·5	·000236	·000531	·000945	·00148	·00213	·00289	·00378	·00478	·00591	·00715	·00850
33	2·75	·000211	·000476	·000846	·001321	·001903	·00259	·00338	·00428	·00529	·00640	·00761
36	3·0	·0001911	·000430	·000764	·001194	·001720	·00234	·00306	·00387	·00478	·00578	·00688
42	3·5	·0001598	·000360	·000639	·000999	·001438	·00196	·00256	·00324	·00400	·00484	·00575
48	4·0	·0001369	·000308	·000547	·000855	·001231	·00168	·00219	·00277	·00342	·00414	·00493

This table is calculated from the equation—

$$i=\frac{h}{l}=\frac{\cdot 044}{d^{1\cdot 16}}\,\frac{v^2}{2g}.$$

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