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MENSURATION,

FOR

THE USE OF SCHOOLS.


GLASGOW AND LONDON: WILLIAM COLLINS, SONS, \& COMPANY.

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## PREFACE TO SECOND EDITION.

To this Edition there is an Appendix, printed in a separate form, for the use of Teachers, containing the leading properties of the Conic Sections, and the Demonstrations of the Rules of Mensuration. These were in the First Edition interspersed through the work, partly interwoven with the text, and partly in the shape of notes. It is hoped that the present arrangement will better suit the convenience of both Teachers and Pupils. Several other alterations have been made, which, it is hoped, will be found to be improvements.

Teachers should direct their Pupils to learn only such portions of the work as may be necessary for their intended occupations ; for most pupils. the first and second sections, and a few problems in the fourth and sixth, will be quite sufficient,
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## MENSURATION.

## SECTION I.

## PRACTICAL GEOMETRY.

## DEFINITIONS.

1. Geometry teaches and demonstrates the properties of all kinds of magnitudes, or extension ; as solids, surfaces, lines, and angles.
2. Geometry is divided into two parts, theoretical and practical. Theoretical Geometry treats of the various properties of extension abstractedly ; and Prastical Geometry applies these theoretical properties to the various purposes of life. When length and breadth only are considered, the science which treats of them is called Plane Geometry ; but when length, breadth, and thickness are considered, the science which treats of them is called Solid Geometry.
3. A Solid is a figure, or a body, having three dimensions, viz., length, breadth, and thickness ; as A.


The boundaries of a solid are surfaces or superficies.
4. A Superficies, or surface, has length and breadth only ; as B.

## B

The boundaries of a superficies are lines.
5. A Line is length without breadth, and is formed by the motion of a point ; as C B.

The extremities of a line are points.
6. A Straight or Right Line is the shortest distance between two points, and lies evenly between these two points.
7. A Point is that which has no parts or magnitude ; it is indivisible ; it has no length, breadth, or thickness. If it had length, it would then be a line; were it possessed of length and breadth, it would be a superficies; and had it length, breadth, and thickness, it would be a solid. Hence a point is void of length, breadth, and thickness, and only marks the position of their origin or termination in every instance, or of the direction of a line.
8. A Plane rectilineal Angle is the inclination of two right lines, which meet in a point, but are not in the same direction; as S .
9. One angle is said to be less than another, when the lines which form that angle are nearer to each other than those which form the other, measuring at equal distances from the points in which the lines meet. Take $\mathbf{B} n, \mathrm{~B} m, \mathrm{E} x$, and $\mathrm{E} n$, equal to one another ; then if $m n$ be greater than $x n$, the angle ABC is greater than the E angle FED. By conceiving the point A to move towards C, till $m n$ becomes equal to $x n$, the
 angles at $B$ and $E$ would then be equal ; or by conceiving the point F to recede from D , till $x n$ becomes equal to $m n$, then the angles at $B$ and $E$ would be equal.

Hence it appears that the nearer the extremities of the lines forming an angle approach each other, while the point at which they meet remains fixed, the less the angle; and the farther the extreme points recede from each other, the vertical point remaining fixed, as before, the greater the angle.
10. A Circle is a plane figure contained by one line $A \rightarrow B$ called the circumference, which is everywhere equally distant from a point within it, called its centre, as o: and an arc of a circle is any part of its circumference; as AB.
11. The magnitude of an angle does not consist in the length of the lines which form it: the angle CBG is less than the angle ABE, though the lines CB, GB are longer than AB, EB.

12. When an angle is expressed by three letters, as A B E, the middle letter always stands at the angular point, and the other two anywhere along the sides; thus the angle ABE is formed by AB and BE. The angle A B G by A B and GB.
13. In equal circles, angles have the same ratio to each other as the ares on which they stand (33. vi.) Hence also, in the same, or equal circles, the angles vary as the ares on which they stand; and, therefore, the arcs may be assumed as proper measures of angles. Every angle then is measured by an arc of a circle, described about the angular point as a centre; thus the angle A B E is measured by the arc AE; the angle ABG by the arc AF.
14. The circumference of every circle is generally divided into 360 equal parts, called degrees; and every degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds. The angles are measured by the number of degrees contained in the arcs which subtend them; thus, if the are AE contain 40 -degrees, or the ninth part of the circumference, the angle ABE is said to measure 40 degrees.
15. When a straight line H 0 , standing on another AB, makes the angle HOA equal to the angle HOB ; each of these angles is called a right angle; and the line $\mathrm{H} O$ is said to be a perpendicular to AB.


The measure of the angle HOA is 90 degrees, or the fourth part of 360 degrees. Hence a right angle is 90 degrees.
16. An acute angle is less than a right angle; as $\mathrm{A} O \mathrm{G}$, or GOH .
17. An obtuse angle is greater than a right angle; as GOB.
18. A plane Triangle is the space enclosed by three straight lines, and has three angles ; as A.

19. A right angled Triangle is that which has one of its angles right; as A BC. The side BC, opposite the right angle, is called the hypothenuse; the side AC is called the perpendicular; and the side A B is called the base.

20. An obtuse angled Triangle has one of its angles obtuse; as the triangle B , which has the obtuse angle A.

21. An acute angled Triangle has all its three angles acute, as in figure A, annexed to definition 18.
22. An equilateral Triangle has its three sides equal, and also its three angles; as C.

23. An isosceles Triangle is that which has two of its sides equal ; as D.

24. A scalene Triangle is that which bas all its sides unequal ; as $\mathbf{E}$.

25. A quadrilateral figure is a space included by four straight lines. . If its four angles be right, it is called a rectangular parallelograme
26. A Parallelogram is a plane figure bounded by four straight lines, the opposite ones being parallel; that is, if produced ever so far, would never meet.
27. A Square is a fonr sided-figure, having all its sides equal, and all its angles right angles; as H .
28. An Oblong, or rectangle, is a right angled parallelogram, whose length exceeds its breadth; as I.
29. A Rhombus is a parallelogram having all its sides equal, but its angles not right angles; as K .

30. A Rhomboid is a parallelogram having its opposite sides equal, but its angles are not right angles, and its length exceuds its breadth; as M .
31. A Trapezum is a figure included by four straight lines, no two of which are parallel to each other; as N.


A line connecting any two of its opposite angles, is called a diagonal.
32. A Trapezoid is a four-sided figure having two of its opposite sides parallel; as F.

33. Multilateral Figures, or Polygons, are those which have more than four sides. They receive particular names from the number of their sides. Thus, a Pentagon has five sides; a Hexagon has six sides; a Heptagon, seven ; an Octagon, eight; a Nonagon, nine; a Decagon, ten; an Undecagon, eleven; and a Dodecagon has twelve sides.

If all the sides and angles of each figure be equal, it is called a regular polygon; but if either or both be unequal, an irregular polygon.
34. The Diameter of a circle is a straight line passing through the centre, and terminated both ways by the circumference; thus A B is the diameter of the circle. The diameter divides the circle into two equal parts, each of which is called a semicircle; the diameter also divides the circumference into two equal parts, each containing 180 degrees. Any line drawn from the centre to the circumference is called the radius, as A $0,0 \mathrm{~B}$, or 0 S . If 0 S be drawn from the centre perpendicular to AB, it divides the semicircle into two
 equal parts, A OS and BOS, each of which is called a quadrant, or one-fourth of the circle; and the arcs AS and BS contain each 90 degrees, and they are said to be the measure of the angles A 0 S and BOS.
35. A Sector of a circle is a part of the curcle comprchended under
twn Radii, not forming one line, and the part of the circumference between them. From this definition it appears that a sector may be either greater or less than a semicircle; thus AOB is a sector, and is less than a semicircle; and the remaining part of the circle is a sector also, but is greater than a semicircle.

36. A Chord of an arc is a straight line joining its extremities, and is less than the diameter; TS is the chord of the are THS, or of the are TABS.
37. A Segment of a circle is that part of the circle contained between the chord and the circumference, and may be either greater or less than a semicircle; thus TSHT and TABS T are segments, the latter being greater than a semicircle and the former less.
38. Concentric circles are those having the same centre, and the space included between their circumference is called a ring; as F E.

## PROBLEM I.

## To bisect a given straight line AB ; that is, to divide it into twn

 p.qual parts.From the centres A and B , with any radius, greater than half the given line A B, describe two ares intersecting each other at 0 and $S$, then the line joining 0 S will bisect A B.

## PROBLEM II.



Through a given pornt $x$ to draw a straight line CD paralla to a given straight line AB.
In AB take any point $s$, and with the C
$x$ ne with and radius $s x$ describe the are antre and the same ra-
$x$, dius $s x$, describe the are $s y$. Lay the extent $o x$ taken with the compasses from $s$ to $y$; through $x y$ draw CD, which will be parallel to A B.

## PROBLEM III.

To draw a straight line C D parallel to A B, and at a given distance F from it.
In AB take any two points $x, f$; and from the two points as centres with the extent F taken with the compasses, describe two arcs $s, r$; then draw a line CD touching these ares at $r$ and $s$, and it will be at
 the given distance from A B, and parallel to it.

PROBLEM IV.
To divide a straight line A B into any number of equal parts.
Draw AK making any angle with A B; and through B draw B T parallel to AK; take any part AE and repeat it as often as there are parts to be in A B, and from the point $B$ on the line BT, take BI, IS, SV, and VT equal to the parts taken on the line AK; then join AT, EV, GS, HI, and K B , which will divide the line A B into the number of equal parts required, as A C, CD, DF, FB.

## PROBLEM V

From a given point P in a straight line A B to erect a perpendicular.

1. When the given point is in, or near the middle of the line.

On each side of the point P take equal portions, $\mathrm{P} x, \mathrm{P} f$; and from the centres, $x, f$, with any radius greater than $\mathrm{P} x$, describe two arcs, cutting each other at $D$; then the line joining D P will be perpendicular to A B.

Or thus:
From the centre $\mathbf{P}$, with any radius $\mathbf{P} n$, describe an are $n x y$; set off the distance, $\mathrm{P} n$ from $n$ to $x$, and from $x$ to $y$; then from the points $x$ and $y$ with the same or any other radius, describe two arcs intersecting each other at D ; then the line joining the points D and P will be perpendicular to AB .
2. When the point is at the end of the line.

From any centre $q$ out of the line, and with the distance $q \mathrm{~B}$ as radius, describe a circle, cutting AB in $p$; draw $p q 0$; and the line joining the points $O, B$, will be perpendicular to AB.


## Or thus:

Set one leg of the compasses on B, and with any extent $\overline{\mathrm{B}} p$ describe an are $p x$; set off the same extent from $p$ to $q$; join $p q$; from $q$ as a centre, with the extent $p q$ as radius, describe an are $r$; produce $p q$ to $r$, and the line joining $r \mathbf{B}$ will be perpendicular to A B.


## PROBLEM VI.

From a given point D to let fall a perpendicular upon a given line A B.

1. When the point is nearly opposite the middle of the given lene.

From the centre D, with any radius, describe an are $x y$, cutting AB in $x$ and $y$; from $x$ and $y$ as centres, and with the same distance as radius, describe two ares cutting each other at $S$; then the line joining $\mathbf{D}$ and S will be perpendicular to $\mathbf{A B}$.

2. When the point is nearly opposite the end of the given line, and when the given line cannot be conveniently produced.

Draw any line $\mathrm{D} x$, which bisect in 0 ; from $o$ as a centre with the radius $o x$ describe an arc cutting A B in $y$; then the line joining $\mathrm{D} y$ will be perpendicular to AB.

## PROBLEM VII.

To draw a perpendicular, from any angle of a triangle A B C, to its opposite side.

Bisect either of the sides containing the angle from which the perpendicular is to be drawn, as BC in the point $r$; then with the radius $r \mathrm{C}$, and from the centre $r$, describe an arc catting AB (or AB produced if necessary, as in the second figure), in the point $P$; the line joining $C P$ will be per- $\triangle$ pendicular to AB, or to AB produced.


## PROBLEM VIII.

Upon a given right line A B to describe an equilateral triangle.
From the centres $A$ and $B$, with the given line A B as radius, describe two arcs cutting each other at $C$; then the lines drawn from the point $C$ to the points $\mathbf{A}$ and $\mathbf{B}$ will form, with the given line A B, an equilateral triangle, as A B C.


## PROBLEM IX.

To make a triangle whose sides shall be equal to three given right lines A B, AD, and B D, any two of which are greater than the third.
From the centre A with the extent A D, describe an are, and from the centre $\mathbf{B}$ with the radius BD describe another arc cutting the former at D; then join $\mathrm{D} A, \mathrm{D} \mathrm{B}$, and the sides of the triangle $\mathrm{A} B \mathrm{D}$ will be respectively equal to the three given right lines.

## PROBLEM X.



Two sides A B and BC of a right angled triangle being given, to find the hypothenuse.

Place B C at right angles to A B; draw A C, and it will be the hypothenuse required.

## PROBLEM XI.

The hypothenuse A B, and one side A C, of a right angled triangle being given, to find the other side.
Bisect A B in $x$; with the centre $x$, and $x \mathrm{~A}$ as radius, describe an are; and with $A$ as a centre, and AC as radius, describe another are cutting the former at C ; then join AC and CB ; and ABC will be a right angled triangle, and BC the re- A
 quired side.

## PROBLEM XII.

To bisect a given angle; that is, to divide it into two equal parte
Let A CB be the angle to be bisected.
From C as a centre, with any radius $\mathrm{C} x$, describe the $\operatorname{arc} x y$; from the points $x$ and $y$ ascentres, with the same radius, describe two arcs cutting each other at 0 ; join $\cup C$, and it will $A$ bisect the angle ACB.


## PROBLEM XIII.

At a given point A in a given right line A B, to make an angle equal to the given angle C .

From the centre $\mathbf{C}$ with any radius $\mathbf{C} y$, describe an are $x y$; and from the centre A , with
 the same radius, describe another are, on which take the distance $m n$ equal to $x y$; then a line drawn from A through $m$ will make the angleC $m$ A $n$ equal to the angle $x \mathrm{C} y$.


## PROBLEM XIV.

To make an angle containing any proposed number of degrees.

1. Dhen the required angle is less than a quadrant, as 40 degrees.

Take in the compasses the extent of 60 degrees from the line of chords, marked cho. on the scale; and with this chord of 60 degrees as radius, and tre centre A , describe an are $x y$; take from the line of chords 40 degrees, which set off from $n$ to $m$; from A draw a line through $m$; and the angle $m \mathrm{~A} n$ will contain 40 degrees.
2. When the required angle is greater than a quadrant, as 120 degrees.
From the centre 0 , with the chord of 60 degrees as radius, describe the semicircle $y x n \mathrm{~B}$; set off the chord of 90 degrees from $B$ to $n$, and the remaining 30 degrees from $n$ to $x$; join $o x$; and the angle $\mathrm{B} o x$ will contain 120 A
 degrees; or subtract 120 from 180 degrees, and set off the remainder ( 60 degrees) taken from the line of chords from $y$ to $x$ : then join $x$, , and B ox will contain 120 degrees as before.

## PROBLEM XV.

An angle being given, to find, by a scale of chords, how many degrees it contains.
From the vertex A as centre, with the chord of 60 degrees as radius, describe an are $x y$; take the extent $x y$ with the compasses, and setting one foot at the beginning of the line of chords,
 the otherleg will reach to the number of degrees which the angle contains: but if the extent $x y$ should reach beyond the scale, find the number of degrees in $x y$, which deducted from 180, will leave the degrees in the angle Box. See figure to the second case of the last Problem.

## PROBLEM XVI.

## Upon a given right line A B, to construct a square.

With the distance AB as radius, and A as a centre, describe the are EDB; and with the distance AB as radius, and B as a centre, describe the are AFC, cutting the former in $x$; make $x \mathrm{E}$ equal to $x \mathrm{~B}$; join EB ; make $x \mathrm{C}$ and $x \mathrm{D}$ each equal to AF, or $\mathrm{F} x$; then join AD, DC, CB, and ADCB will be the required square.

Or thus.
Draw $\mathrm{B} C$ at right angles to AB , and equal to it; then from the centres $A$ and $C$, with the radius AB and CB , describe two arcs cutting each other at D; join D A and D C, which will complete the square.

## PROBLEM XVII.

To make a rectangular parallelogram of a given length and breadth.

Let $A B$ be the length, and $B C$ the breadth.
Erect B C at right angles to AB; through C and A draw CD and AD , parallel to AB and
 BC.

## PROBLEM XVIII.

To find the centre of a given circle.
Draw any two chords A C, C B; from the points, $\mathrm{A}, \mathrm{C}, \mathrm{B}$, as centres, with any radius greater than half the lines, describe four arcs cutting in $r x$ and $y v$, draw $r x$ and $y v$, and produce them till they meet in 0 , which will be the centre.

## PROBLEM XIX.

L'pon a given right line A B, to describe a rhombus having an angle equal to a given angle A.


Make the angle CAB equal to the angle at A; make A C equal to AB; then from C and B as centres, with the radius AB describe two arcs crossing each other at D; join D C and D B, which will complete the rhombus.

## PROBLEM XX.

To find a mean proportional between two given right lines A B and BC .
Place A B and B C in one straight line; bisect A C in $o$; fron. o as a centre, with Ao or $o \mathrm{C}$ as radius, describe a semicircle ASC; erect the perpendicular B S, and it will be a mean proportional between AB and BC; that is, A B : BS : : B S : B C.


## PROBLEM XXI.

To divide a given right line A B into two such parts, as shall be to each other as $x$ o to of.

From the point A draw AS equal to $x o$, and produce it till FS becomes equal to of; join FB, and draw S T parallel to F B ; then will AT:TB::x0:of.


PROBLEM XXII.
To find a third proportional to two given right lines A B, A S.
Place $A B$ and $A S$ so as to make any angle at A: from the centre A, with the distance AS , describe the are SD ; then draw $\mathrm{D} x$ parallel to $B S$, and $A x$ will be the third proportional required; that is, $\mathrm{AB}: \mathrm{AS}:$ : AS:A $x$.


## PROBLEM XXIII.

To find a fourth proportional to three given right lines, $\mathrm{A} \mathrm{B}, \mathrm{A} \mathrm{C}$, and A D.
Place the right lines AB and AC so as to make any angle at $A$; on $A B$ set off $A D$; join BC; and draw DS parallel to it ; then AS will be the fourth proportional required, viz. AB:AC::AD:AS.

## PROBLEM XXIV.



## In a given circle to inscribe a square.

Draw any two diameters A C, D B at right angles to each other; then join their extremities, and the figure ABCD will be a square inscribed in the given circle.
If a line be drawn from the centre $o$ to the middle of AB , and produced to $f$; the line juining $f^{\mathrm{B}} \mathrm{B}$ will be the side of an octagon inscribed in the circle.


## PROBLEM XXV.

T'o make a regular polygon on a given right line, A B.
Divide 360 degrees by the number of sides contained in the polygon; deduct the quotient from 180 degrees, and the remainder will be the number of degrees in each angle of the polygon. At the points $A$ and $B$ make the angles $o \mathrm{AB}$ and $o \mathrm{BA}$ each equal to half the angle of the polygon; then from $o$ as a centre, and with $o$ A or o B as radius, describe a circle, in which place AB continually.*


## Or thus:

Take the given line A B from the scale of equal parts, and multiply the number of equal parts in it by the number in the third column of the following table, answering to the given number of sides; the product will give the number of equal parts in the radius $\mathbf{A} 0$, or o B , which taken from the scale of equal parts in the compasses, will give

[^0]the radius, with which describe a circle, and place in it the line A B continually, as shewn in the first method.*

> TABLE I.

When the side of the polygon is 1 .

| No. of <br> sides. | Name of the <br> Polygon. | Radius of the circumscrib- <br> ing circle. | Angle OA B, or <br> OB A. |
| :---: | :--- | :---: | :---: |
| 3 | Trigon | .5773503 | 30 |
| 4 | Tetragon | -7071068 | 45 |
| 5 | Pentagon | -5506508 | 54 |
| 6 | Hexagon | 1, Side radius. | 60 |
| 7 | Heptagon | 1.1523825 | $64 \frac{3}{7}$ |
| 8 | Octagon | 1.065630 | $67 \frac{1}{2}$ |
| 9 | Nonagon | 1.4619022 | 70 |
| 10 | Decagon | 1.6186340 | 72 |
| 11 | Undecagon | 1.7747329 | $73 \frac{7}{15}$ |
| 12 | Dodecagon | 1.9318516 | 75 |

## PROBLEM XXVI.

In a given circle to inscribe any regular polygon; or, to dividt the circumference of a given circle into any number of equal parts.

Divide the diameter A B into as many equal parts as the figure has sides; erect the perpendicular $0 x$, from the centre 0 ; divide the radius oy into four equal parts, and set off three of these parts from $y$ to $x$; draw a line from $x$ to the second division $\boldsymbol{z}$, of the diameter AB, and produce it to cut the circumference at C; join A C, and it will be the side of the required polygon. $\dagger$


## PROBLEM XXVII.

To draw a struight line equal to any given arc, of a circle, A B,
Divide the chord AB into four equal parts; and set off one of these parts from $B$ to $D$; then join D C, and it will be equal to the length of half the given are nearly. $\ddagger$

## Or thus:

From the extremity of the arc AB, whose length is required to be found, draw A o m, passing through the centre; divide o $n$ into four equal parts, and set off three of these parts from $n$ to $m$; draw $m \mathrm{~B}$, and produce it to meet AC drawn at right angles to A $m$; then will AC be nearly equal in length to the $\operatorname{arc}$ AB.*


## PROBLEM XXVIII.

To make a square equal in area to a given circle.

First divide the diameter AB into fourteen equal parts, and set off eleven of them from A to $S$; from $S$ erect the perpendicular $S C$, and join A C, the square of which will be very nearly equal to the area of the given circle. $\dagger$


## PROBLEM XXIX.

## To construct a diagonal scale.

Draw an indefinite straight line; set off any distance A E according to the intended length of the scale; repeat A E any number of times, E G, G B, \&c.; draw C D parallel to AB at any convenient distance; then draw the perpendiculars A C, E F, G H, B D, \&c. Divide A E and AC each into ten equal parts; through $1,2,3, \& \mathrm{c}$., draw lines parallel to A B, and through $x y$, \&c., draw $x$ F, $y$ Z, \&c., as in the annexed figure.


The principal use of this scale is, to lay down any line from a given measure; or to measure any line and compare it with others.-Whatever number C F represents, F Z will be the tenth of it, and the subdivisions in the vertical direction FE will be each one-hundredtb part. Thus, if CF be a unit, the small divisions in CF, viz. F \&c., will be 10 ths, and the divisions in the altitude will be the 100 th
parts of a unit. If CF be ten, the small divisions F Z, \&c., will be units, and those in the vertical line, tenths; if C F be a hundred, the others will be tens and units.*

To take any number off the scale, as suppose $2 \frac{38}{100}$, that is, $2 \cdot 38$ : place one foot of the compasses at D , and extend the other to the division marked 3 ; then move the compasses upward, keeping one foot on the line D B , and the other on the line $3 s$, till you arrive at the eighth interval, marked 88, and the extent on the compasses will be that required. This, however, may express $2 \cdot 38,23 \cdot 8$, or 238 , according to the magnitude of the assumed unit.

Note.-If CF were divided into 12 equal parts, each division would be 1 inch, and each vertical division 1-10th of an inch, by making C F one foot.

## PROBLEM XXX.

To reduce a rectilinear figure to a similar one upon etther a smaller or a larger scale.
Take any point P in the figure $\triangle B C D E$, and from this assumed point draw lines to all the angles of the figure; upon one of which P A take $\mathrm{P} a$ agreeably to the proposed scale; then draw $a b$ parallel to A B, bc to BC, \&c., then shall the figure $a b c d e$ :.e similar to the original one, and upon the required scale.
 Or , measure all the sides and diagonals of the figure by a scale, and lay down the same measures respectively from another scale, in the required proportion.

When the figure is complex, the reduction to a different scale is best accomplished by means of the Eidograph, an instrument invented by Professor Wallace, or by means of the improved Pentograph.

## PROBLEM XXXI.

To divide a circle into any number of equal parts, having their perimeters equal also.
Divide the diameter AB into the required number of equal parts, at the points C, D, E, \&c.; then on one side describe the semicircles $1,2,3,4, \& c$., and on the other side of the diameter describe the semicircles $7,8,9,10, \& c$. on the diameters B F, B E, B D, B C, \&c.; so shall the parts $111,210,39,48$, \&c. be equal both in area and perimeter. - LesLie's
 Geometry.

[^1]
## MENSURATION OF SUPERFICIES.

## SECTION II.

The area of any plane figure is the space contained within its boundaries, and is estimated by the number of square miles, square yards, square feet, \&c. which it contains.


In Ireland 21 feet make 1 pole or perch, and 7 yards therefore will make a pole or perch. There are other measures used, for which see Arithmetical Tables.

Land is generally measured by a Chain, of 4 poles, or 22 yards; it consists of 100 links, each link being 22 of a yard. See Section XI. Surveying.

Duodecimals are calculations by feet, inches, and parts, which decrease by twelves: hence they take their name.

Multiplication of feet, inches, and parts, is sometimes called Cross Multiplication, from the factors being multiplied crosswise. It is used in finding the contents of work done by artificers, where the dimensions are taken in feet, inches, and parts.

## Rule.

I. Write the multiplier under the multiplicand in such a manner a $_{\text {}}$ that feet shall be under feet, inches under inches, \&c.
II. Multiply each term of the multiplicand by the number of feet in the multiplier, proceeding from right to left ; carry 1 for every 12, in each product, and set down the remainder under the term multiplied.
III. Next multiply the terms of the multiplicand by the number under the denomination inches, in the multiplier; carry 1 for every 12, as before, but set down each remainder one place farther to the right than if multiplying by a number under the denomination feet.
IV. In like manner proceed with the number in the multipliez
under the denomination parts or lines, remembering to set down each remainder one place farther to the right than if multiplying by a number under the denomination inches. And so on with numbers of inferior denominations.
V. Add the partial products thus placed, and their sum will be the whole product.
in Cross multiplication it is usual to say,
Feet multiplied by feet, give feet.
Feet by inches, give inches.
Feet by parts, give parts.
Inches by inches, give parts.
Inches by parts, give thirds.
Inches by thirds, give fourths.
Parts by parts, give fourths.
Parts by thirds, give fifths.
Parts by fourths, give sixths, \&c.*

1. Multiply $\mathbf{7}$ feet 9 inches by $\mathbf{3}$ feet $\mathbf{6}$ inches.

 9. 4 . 6

|  |
| ---: | :--- |
| $2168 \cdot 0 \cdot 0$ |
| $80 \cdot 3 \cdot 6 \cdot 8$ |
| $10 \cdot 0 \cdot 5 \cdot 4$ |
| $2258 \cdot 4 \cdot 0 \cdot 0$ Ans. |



[^2]F. I. P. F. I. P. F. I. P. ${ }^{\prime \prime \prime n}{ }^{n}$
12. Multiply 321 . 7. 3 by 9 . 3. 6. Ans. 2988 . 2 . 10.4.6.
13. Multiply $4 \cdot 7 \cdot 8$ by $9.6 \cdot \quad-\quad 44 \cdot 0 \cdot 10$.
14. Multiply 39.10 . 7 by 18 . 8 . 4.__ 745 . 4. 10.2.4.

Note.-All these can be solved by the method of aliquot parts, thus:-

$$
\begin{aligned}
& \text { 15. Multiply } \begin{array}{c}
\text { F. } \\
368
\end{array} \quad 7.5 \text { by } 137.8 .4 \\
& \text { 137.8.4 } \\
& 2576 \\
& 1104 \\
& 368
\end{aligned}
$$

$$
\begin{aligned}
& \text { To find the area of a square. }
\end{aligned}
$$

Rule. Multiply the length of the side by itself, and the product will be the area.*

1. Let the side of the square ABCD be 6: what is its area?

Ans. $6 \times 6=36$, the area.
2. What is the area of a square whose side is 15 chains? Ans. 225.
3. What is the area of a square whose side is 7 feet 9 inches? Ans. $60 \frac{1}{16}$.

4. What is the area of a square whose side is 4769 links?

## PROBLEM II.

To find the area of a rectangle.

Rule. Multiply the length of the rectangle by its breadth, and the product will be the area.*

1. Let the sides of the rectangle ABCD be 12 and 9 , what is its grea? Ans. $12 \times 9=108$, the area.
2. What is the superficial content of a plank, whose length is 5 feet 6 inches, and breadth 7 feet 8 inches? Ans. 42 feet 2 inches.
3. What is the area of a field whose boundaries form a rectangle, its length being 176 links and
 preadth 154 links? Ans. 27104 of an acre.

[^3]4. What is the superficial content of a floor, whose length is 40 feet 6 inches, and breadth 28 feet 9 inches?

Ans. 1164 feet, 4 inches, 6 parts.

## PROBLEM III.

To find the area of a rhombus.
Rule. Multiply the length by the perpendicular breadth, and the product will be the area.*

1. What is the area of a rhombus, whose side is 16 feet, and perpendicular breadth 10 feet?

Ans. $16 \times 10=160$ feet, the area.
2. What is the content of a field in the
 form of a rhombus, whose length is $7 \cdot 6$ chains, and perpendicular height 5.7 chains? Ans. 43.32 chains.

- 3. What is the area of a rhombus whose side is 7 feet 6 inches, and perpendicular height 3 feet 4 inches? Ans. 25 feet.

4. What is the area of a rhombus whose length is 3 yards, and perpendicular height 2 feet 3 inches? Ans. 20 feet 3 inches.

## PROBLEM IV.

## To find the area of a triangle.

Rule. Multiply the base by the perpendicular height, and divide the product by two for the area. $\dagger$

1. The base of a triangle is 76.5 feet, and perpendicular 92.2 feet; what is its area?

Ans. $76.5 \times 92.2 \div 2=3526.65$ square feet, the area.
2. The base of a triangle is $72 \cdot 7$ yards, and the perpendicular Leight of 36.5 yards; what is its area? Ans. 1326.775 yards.
3. The base of a triangular field is 1276 links; and perpendicular 976 links; how many acres in it? Ans. 6 acres 36.3008 perches.
4. The base of a triangle measures 15 feet 6 inches, and the perpendicular 12 feet 7 inches; what is its area?

Ans. 97 feet $6 \frac{1}{4}$ inches.

[^4]
## PROBLEM V .

Having the three sides of any triangle given, to find its area.
Rule I. From half the sum of the three sides subtract each side separately, then multiply the half sum and the three remainders together, and the square root of the last product will be the area of the triangle.*

Rule II. Divide the difference between the squares of two sides of the triangle by the third side; to half this third side add half the quotient, and deduct the square of this sum from the square of the greater side, the remainder will be the square of the perpendicular, the square root of which, multiplied by half the base, will give the area of the triangle. $\dagger$


1. Given the side $\mathrm{AB}=9 \cdot 2, \mathrm{BC}=7 \cdot 5$, and $A C=5.5$; required the area of the triangle?

$$
\begin{aligned}
& 9 \cdot 2 \\
& 7 \cdot 5 \\
& 5 \cdot 5
\end{aligned}
$$

Sum $22 \cdot 2$

$$
\left.\begin{array}{rl}
\frac{1}{2} \text { Sum } 11 \cdot 1-9 \cdot 2 & =1 \cdot 9 \\
11 \cdot 1-7 \cdot 5=3.6 \\
11 \cdot 1-5.5 & =5.6
\end{array}\right\} \begin{aligned}
& \text { i then } 425.1744=20.619 \text { the area by }(11 \cdot 1 \times 1.9 \times 3.6 \times 5 \cdot 6)= \\
& \text { Rule I. }
\end{aligned}
$$

Again, $9 \cdot 2^{2}-7 \cdot 5^{2}=84 \cdot 64-56 \cdot 25=28 \cdot 39$; then $28 \cdot 39 \div 5 \cdot 5=$ ©-161818, quotient.

Now $(5 \cdot 161818 \div 2)+(5 \cdot 5 \div 2)=2 \cdot 580909+2 \cdot 75=5 \cdot 3309=$ half quot. plus half third side: then $84 \cdot 64-28 \cdot 41849481=$ $56 \cdot 22150519$, and $\sqrt{ } 56 \cdot 22150519=7 \cdot 498=$ perpendicular; ther $7.498 \times 2.75=20.619$ the area as before.
2. What is the area of a triangle whose sides are 50,40 , and 30 ? Ans. 600.
3. The sides of a triangular field are 4900,5025 , and 2569 links; how many acres does it contain?

Ars. 61 acres, 1 rood, $39 \cdot 68$ perches.
4. What is the area of an isosceles triangle, whose base is 20 , and each of its equal sides 15 ?

Ans. 111-803.
5. How many acres are there in a triangle, whose three sides are 380, 420 , and 765 yards? Ans. 9 acres 38 poles.
6. How many square yards in a triangle, whose three sides are 13, 14 , and 15 feet?

[^5]$\dagger$ Ibid. 12.
7. How many acres, \&c., in a triangle, whose three sides are 49 50.25, and $25 \cdot 69$ chains? Ans. 61 acres, 1 rood, $39 \cdot 68$ perches.

## PROBLEM VI.

## To find the area of an equilateral triangle.

Rule. Square the side, and from this square deduct its fourth part ; then multiply the remainder by the fourth part of the square of the side, and the square root of the product will give the area.* Or multiply $\frac{\mathrm{AB}^{2}}{4}$ by $\sqrt{ } 3$ for the area. $\dagger$

1. Each side of a triangular field, ABC , measures 4 perches, what is its area?
$4^{2}=16$, then $16 \div 4=4$, and $16-4=12:$ then $12 \times \frac{18}{4}=$ $12 \times 4=48$, and $\sqrt{48}=6.928$, the area.
2. How many acres in a field of a triangular form, each of whose sides measures 70 perches? Ans. 13 acres, 1 rood, 1 perch.
3. The perimeter of an equilateral triangle is 27 yards, what is its area? Ans. $35 \cdot 074$.
Note.-When the triangle is isosceles, the perpendicular is equal to the square root of the difference between the squares of either of the equal sides, avd half the base.

## PROBLEM VII.

Given the area and altitude of a triangle, to find the base.
Rule. Divide the area by the altitude or perpendicular, and double the quotient will give the base. $\ddagger$

1. Given the area of a triangle $=12$ yards, and altitude $=4$; what is its base?

Ans. $12 \div 4=3$; then $3 \times 2=6$ yards, the base, A B.

2. A surveyor having lost his field book, and requiring the base of a triangular field, whose content he knew from recollection was 14 acres, and altitude 7 yards, how much is the base?

Ans. 19360 yards.

## PROBLEM VIII.

Given the area of a triangle, and its base, to find its altitude.
Rule. Divide the area by the given base, and double the quotient will give the perpendicular.

The reason of this rule is manifest, from the last.

1. Given the area of a triangle $=12$, and its base $=6$; what is its perpendicular height?

Ans. $12 \div 6=2$; then $2 \times 2=4$ the altitude.

* Swo Appendix, Demonstration 13.
† Ibid. 14.
\$ Ibid. 15


## PROBLEM IX.

Given any two sides of a right angled triangle, to find the third side, and thence its area.

## Rule.

I. To the square of the perpendicular add the square of the base, and the square root of the sum will give the hypothenuse.
II. The square root of the difference of the squares of the hypothenuse and either side, will give the other.
III. Or multiply the sum of the hypothenuse, and either side, by their difference; and the square root of the product will give the other.*

1. Given the base AC3, the perpendicular CB4; required the hypothenusé A B ?
$3^{2}+4^{2}=25$; then $\sqrt{ } 25=5$, the hypothenuse AB.
2. Given A B 5, A C 3 ; required C B ?
$5^{2}-3^{2}=16$; then $\sqrt{ } 16=4$ the side $B C$; or, $(5+3) \times(5-3)=8 \times 2=16$; then $\sqrt{ } 16=4$, as before.
3. Given A B 5, B C 4 ; required A C ?
$5^{2}-4^{2}=9$; then $\sqrt{ } 9=3$ the side AC; or $(5+4) \times(5-4)$ $=9 \times 1=9$; then $\sqrt{ } 9=3$, as before. And $3 \times 4 \div 2=6$ the area of the triangle.
4. The wall of a building on the brink of a river is 120 feet, and the breadth of the river is 70 yards; what is the length of the chord in feet that will reach from the top of the building across the river?

$$
\text { Ans. } 241 \cdot 86 \text { feet. }
$$

5. A ladder 60 feet long, will reach to a window 40 feet from the dags on one side of a street, and by turning the ladder over to the other side of the street, it will reach a window 50 feet from the flags; required the breadth of the street? Ans. $77 \cdot 8875$ feet.
6. The roof of a house, the side walls of which are the same height, forms a right angle at the top, the length of one rafter being 10 feet, and its opposite one 14 feet; what is the breadth of the house?

Ans. $17 \cdot 204$.

## PROBLEM X.

Given the base and perpendicular of a right angled triangle, to find the perpendicular let fall on the hypothenuse from the right angle; and also the segments into which the hypothenuse is divided by this perpendicular.
Rule. Find the hypothenase by Problem IX. Then divide the square of the greater side by the hypothenuse, and the quotient will give the greater segment, which deducted from the entire will give the less. Having found the segments, multiply them together, and the square root of the product will give the perpendicular. $\dagger$

[^6]$\dagger$ Ibid. 17.

1. Given A C 3 yards, and CB 4 yards; required the segments $\mathrm{BD}, \mathrm{D} \mathrm{A}$, and the perpendicular D C.
$3^{2}+4^{2}=25$; then $\sqrt{ } 25=5=\mathrm{AB}$.
$4^{2} \div 5=16 \div 5=3 \cdot 2=\mathrm{BD}$; then $5-3 \cdot 2=1 \cdot 8=\mathrm{AD}$.
Again, $3.2 \times 1.8=5.76$; then $\sqrt{ } 5 \cdot 76=2.4=\mathrm{D}$ C.
2. The roof of a house whose side walls are each 30 feet high, forms a right angle at the top; now if one of the rafters be 10 feet long, and its opposite yoke-fellow 12, required the breadth of the building, the length of the prop set upright to support the ridge of the roof, and the part of the floor at which it must be placed?

Ans. Breadth of the building $15 \cdot 6204$ feet, greater segment 9.2186 feet, lesser segment 6.4018 feet, and length of the prop 37.68 feet.

## PROBLEM XI.

## To find the area of a trapezium.

Rule. Divide the trapezium into two triangles, by joining two of its opposite angles; find the area of each triangle, and the sum 0 ! both areas will give the area of the trapezium.

Or,
Draw two perpendiculars from the opposite angles to the diagonal ; then multiply the sum of these perpendiculars by the diagonal, and half the product will give the area.*

1. In the trapezium ABCD, the diagonal AC is 100 yards, the perpendicular D E 35, and B F 30; what is its area?

| $\begin{aligned} & \text { D E }=35 \\ & \text { B F }=30 \end{aligned}$ |
| :---: |
| 65 |
| 100 |
| 2)6500 |



## 3250 the area.

2. What is the area of a field, whose south side is 2740 links, easi side 3575 links, north side 3755 links, west side 4105 links, and the diagonal from south-west to north-east 4835 links?

Ans. 123 acres 11.8633 perches.
3. In the trapezium ABCD, the side AD is $15, \mathrm{D}$ C 13, C B 14 , and AB 12 ; also the diagonal AC 16; what is its area?

Ans. $172 \cdot 5247$.
4. In the trapezium A B C D, there are given A B 220 yards, D C 265 yards, and AC 378 yards; also A F 100 yards, and EC 70 yards; what is its area?

Ans. $85342 \cdot 2885$ yards $=17$ acres, 2 roods, 21 perches.
5. In the trapezium ABCD, there are given AB 220 yards, D C 265 yards, B F 195.959 yards, D E 255.5875 yards; also FE 208 yards; required the area of the trapezium? Ans. 85342.2885 yards.
6. Suppose in the trapezium ABCD , on account of obstacles, I can only measure AB, D C, B F, DE, and FD, which are respectively 22 yards, 26 yards, 19 yards, 25 yards, and 32 yards; required the area? Ans. 840.5 s̆ square yards.

[^7]
## PROBLEM XII.

To find the arca of a trapezium inscribed in a circle, or of any one whose opposite angles are together equal to two right angles.
Rule. Add the four sides together, and take half the sum, from this half sum deduct each side separately; and the square root of the - product of the four remainders will give the area of the trapezium.*

1. What is the area of a four-sided field, whose opposite angles are together equal to two right angles, the length of the four sides being as follows, viz. A B 12.5, A D 17, D C $17 \cdot 5$, and B C 8 yards?
 $30712 \cdot 50=175 \cdot 25$, the area in yards.
2. There is a trapezium whose opposite angles are together equal to two right angles; the sides are as follows, viz. A B 25, A D 34, DC 35, and BC 16 ; required its area?

Ans. 700.99.

## PROBLEM XIII.

## To find the area of a trapezoid.

Rule. Multiply half the sum of the two parallel sides by the perpendicular distance between them, and the product will give the area.t

1. Let A B CD be a trapezoid, the side $\mathrm{AB}=40, \mathrm{D} C=25, \mathrm{CP}=18$; required the area?

$$
\frac{40}{65} \div 2=32.5 \times 18=585, \text { area }
$$


2. What is the area of a trapezoid, whose parallel sides are 750 and 1225 links, and the perpendicular height 1540 links?

Ans. 15 acres 33.2 perches.
3. What is the area of a trapezoid whose parallel sides are 4 feet 6 inches, and 8 feet 3 inches; and the perpendicular height 5 feet 8 inches?

Ans. 36 feet $1 \frac{1}{2}$ inches.
4. What is the area of a trapezoid whose parallel sides are 1476 and 2073 yards, and perpendicular height 976 yards?

Ans. 220 acres, 3 roods, 25 perches, 7 yards Irish.

## PROBLEM XIV.

## To find the area of an irregular polygon.

Rule. Divide the figure into triangles and trapeziums, and find the area of each separately, by Problem IV. or XI. Add these areas together, and the sum will be the area of the polygon.*

1. What is the area of the irregular polygon ABCDEFGA, the following lines being given?

$$
\begin{aligned}
& \text { A } O=9 \\
& \text { G B }=29 \\
& \text { C } n=11 \\
& \text { G C }=28 \cdot 4 \\
& \text { F } x=14 \cdot 5 \\
& \text { C } y=15 \\
& \text { FD }=35 \\
& \text { E } z=7 \cdot 4
\end{aligned}
$$



$$
\begin{aligned}
& \mathrm{A} 0=9 \\
& \text { C } n=11 \\
& \text { 2) } 20 \mathrm{sum} \\
& 10 \text { half } \\
& 29 \text { diag. G B } \\
& \overline{290}=\text { area of A B C C A } \\
& \text { C } y=13 \\
& \mathrm{E} z=7.4 \\
& \text { 2) } 20.4 \text { sum. } \\
& 10 \cdot 2 \\
& 35 \text { F D } \\
& \overline{3570} \text { area of F C D E F. } \\
& \text { F } x=14.5 \\
& \text { 直 G C }=14 \cdot 2 \\
& 205 \cdot 9 \text { area of G F C. } \\
& 290=\text { area of A B C G A } \\
& 357=\text { area of F C D E F } \\
& 205 \cdot 9=\text { area of G F C }
\end{aligned}
$$

$$
\text { Ans. } \overline{8529}=\text { area of A B C D E F G A. }
$$

2. In a five-sided field G C D E F G there is G C=28 perches, $\mathrm{F} \boldsymbol{x}$ $=14$ perches, $\mathrm{C} y=13$ perches, $z \mathrm{E}=7$ perches, and $\mathrm{F} \mathrm{D}=35$ perches; required its area? Ans. 3 acres, 1 rood, 26 perches.

[^8]3. In the annexed figure, there are given in perches,
\[

$$
\begin{aligned}
& \mathrm{AX}=15 \\
& \mathrm{XR}=8 \\
& \mathrm{RT}=14 \\
& \mathrm{TD}=6 \\
& \mathrm{AP}=17 \\
& \mathrm{PS}=14 \\
& \mathrm{SD}=12 \\
& \mathrm{GX}=5 \\
& \mathrm{FR}=10 \\
& \mathrm{ET}=12 \\
& \mathrm{BP}=20 \\
& \mathrm{CS}=14
\end{aligned}
$$
\]

Required the area?


Ans. 4 acres, 3 roods, $19 \frac{1}{2}$ perches.

## PROBLEM XV.

## To find the area of a regular polygon.

Rule I. Add all the sides together, and multiply half the sum by the perpendicular drawn from the centre of the polygon to the middle of one of the sides, and the product will give the area. This perpendicular is the radius of the inscribed circle.

Rule II. Multiply the square of the side of the polygon by the number standing opposite to its name in the following table, under the word area, and the product will give the area of the polygon.

Rule 1II. Multiply the side of the polygon by the number standing opposite to its name in the column of the following table, headed "Radius of Inscribed Circle," and the product will be the perpendictlar from the centre of the polygon to the middle of one of its sides; then multiply half the sum of the sides by this perpendicular, and the product will give the area.*

TABLE II.
When the side of the polygon is 1 .


Note.-The radius of the circumscribed circle, when the slde of the polygon is 1, may be seen in Table I.

The expressions in the fourth column may be seen in Trigonometry, to Fhich the pupil is referred for a full investigation of them. The tangents of the angle $0 a C$ in the heptagon, nonagon, and undecagon, are extremely difficult to be found without a table of tangents.

1. The side of a pentagon is 20 yards, and the perpendicular from the centre to the middle of one of the sides is 13.76382 ; required the area?

By Rule I. $20 \times 5 \times 13 \cdot 76382 \div 2=1376 \cdot 382 \div 2=688 \cdot 191$. Ans.
By Rule II. $20 \times 20 \times 1 \cdot 720477=688 \cdot 19$, the area as before.
2. The side of a hexagon is 14 , and the perpendicular from the centre $12 \cdot 1243556$; required the area? Ans. 509•2229352.
3. The side of an octagon is 5.7 , required its area?

Ans. 156.875596479.
4. The side of a heptagon is 19.38 yards, what is its area ?

$$
\text { Ans. } 1364 \cdot 84 .
$$

5. The side of an octagon is 10 feet, what is its area ?

Ans. $482 \cdot 84271$.
6. The side of a nonagon is 50 inches, what is its area ?

Ans. $15454 \cdot 5605$.
7. The side of an undecagon is 20 , what is its area?

Ans. $3746 \cdot 25616$.
8. The side of a dodecagon is 40 yards, what is its area? Ans. 17913•84384.

## PROBLEM XVI.

Given the diameter of a circle, to find the circumference; or, the circumference to find the diameter, and thence the area.

Rule.*
I. Say as $7: 22::$ the given diameter : circumference.

Or, as $113: 355::$ the diameter : the circumference.

Or, as $1: 3 \cdot 1416::$ the diameter : the circumference.
II. Say as $22: 7::$ the given circumference : the diameter.

Or, as $355: 113::$ the circumference : the diameter.

Or, as $3 \cdot 1416: 1::$ the circumference : the diameter.

1. The diameter of a circle is 15 , what is its circumference?
$7: 22:: 15: 22 \times 15 \div 7=330 \div 7=47 \cdot 142857$.
Or, $113: 355:: 15: 355 \times 15 \div 113=5325 \div 113=47 \cdot 124$.
Or, $1: 3 \cdot 1416:: 15: \cdot 3 \cdot 1416 \times 15=47 \cdot 124$.
2. The circumference of a circle is 80 , what is its diameter?
$22: 7:: 80: 7 \times 80 \div 22=2 \breve{5} \cdot 45$.
$355: 113:: 80: 113 \times 80 \div 355=25 \cdot 4647$.
$3 \cdot 1416: 1:: 80: 80 \div 3 \cdot 1416=25 \cdot 4647$.

[^9]3. What is the circumference of a circle whose diameter is 10 ?

Ans. $31 \cdot 4285$.
4. What is the diameter of a circle whose circumference is 50 ?

Ans. 15.909.
5. The diameter of the earth is 7958 miles, what is its circumference? Ans. $25000 \cdot 8528$ miles.
6. The circumference of the earth being 25000.8528 miles, what is its diameter?

Ans. 7958 miles.

## PROBLEM XVII.

## To find the length of an arc of a circle.

Rule I. Multiply the radius of the circle by the number of degrees in the given arc, and that product by 01745329 , and the last product will be the length of the arc.*

Rule II. From eight times the chord of half the arc, subtract the chord of the whole arc, one-third of the remainder will give the length of the are, nearly. $\dagger$

1. If the are AB contain 30 degrees, the radius being 9 feet, what is the length of the are?
$30 \times 9=270$, and $270 \times \cdot 01745329=4 \cdot 7124$. Ans.
2. If the chord $A D$ of half the arc $A D B$ be 20 feet, and the chord AB of the whole arc 38 ; what is the length of the are?

$20 \times 8-38=122$; then $122 \div 3=40 \frac{2}{3}$ feet. Ans.
3. The chord of an are is 6 feet, and the chord of half the are is $3 \frac{1}{2}$; required the length of the whole arc? Ans. 71
4. The chord of the whole are is 40 , and the versed sine $\ddagger$ or height of the segment 15 ; what is the length of the are?

$$
\text { Ans. } 53 \frac{1}{\mathrm{~s}} .
$$

5. The chord AB of the whole arc is $48 \cdot 74$, and the chord AD of half the are 30.25 ; required the length of the arc?
6. $\mathrm{A} \mathrm{B}=30, \mathrm{D} P=8$; required the length of the arc?

Ans. $35 \frac{1}{3}$.

## PROBLEM XVIII.

## To find the area of a circle.

Rule I. Multiply half the circumference by half the diameter, for the area.§

Rule II. Multiply the square of the diameter by 7854 , for the area.|l

RULE III. Multiply the square of the circumference by 07958 . ${ }^{T}$
RUs.E IV. As 14 to 11 , so is the square of the diameter to the area.
Rule V. As 88 to 7 , so is the square of the circumference to the area.

[^10]1. To find the area of a circle whose diameter is 100 and circurr ference $314 \cdot 16$.

By Rule I.
$314 \cdot 16$
100
4) 31416

Area 7854
By Rule IV. 11

| $\text { 2) } \longdiv { 7 1 0 0 0 0 }$ |
| :---: |
|  |  |
|  |

$$
100^{2}=10000
$$

$$
\text { Area } 7857
$$

By Rule II.

- 7854
$100^{2}=10000$
Area 7854

By Rule III. 98696.5 sq. cir. -07958
7854. Area.
2. What is the area of a circle whose diameter is 7 ?

Ans. $38 \frac{1}{2}$ nearly.
3. How many square yards are in a circle whose diameter is $1 \frac{1}{6}$ yard?
4. The surveying wheel turns twice in the length of $16 \frac{1}{2}$ feet; in going round a circular bowling-green it turns exactly 200 times ; how many acres, roods, and perches in it?

Ans. 4 acres, 3 roods, 35.8 perches.
5. The circumference of a fish-pond is 56 chains, what is its areal Ans. $249 \cdot 56288$.
6. What is the area of a quadrant, the radius being 100 ? Ans. 7854.
7. Required the length of a chord fastened to a stake at one end, and to a cow's horns at the other, so as to allow her to feed on an acre of grass and no more?
8. The circumference of a circle is 91 , what is its area?

Ans. $659 \cdot 00198$.
9. The diameter of a circle is 15 perches, what is its area?

Ans. 176.715.
10. What is the area of the semicircle of which 20 is the radius?

Ans. 628.32.

## PROBLEM XIX.

Given the diameter of a circle, to find the side of a square equal in area to the circle.
Rule. Multiply the diameter by 8862269 , and the product will be the side of a square equal in area to the circle.*

1. If the diameter of a circle be 100 , what is the side of a square equal in area to the circle ?

Ans. $88 \cdot 62269$.
2. The diameter of a circular fish pond is 200 feet, what is the side of a square fish-pond equal in area to the circular one?

Ans. $177 \cdot 24538$.

[^11]
## PROBLEM XX.

Given the corcumference of a circle, to find the side of a square equal in area to the circle.
Rule. Multiply the circumference by 2820948 , and the product will be the side of the square.*

1. The circumference of a circle is 100 , what is the side of a square equal in area to the circle? Ans. 28.20948.
2. The circumference of a round fish-pond is 200 yards, what is the side of a square fish-pond equal in area to the round one?

## PROBLEM XXI.

Given the diameter, to find the side of the inscribed square.
Rule. Multiply the diameter by $\cdot 7071068=\frac{1}{2} \sqrt{3}$ and the product will give the side of the inscribed square. $\dagger$

1. The diameter of a circle is 100 , what is the side of the inscribed square?

$$
\text { Ans. } 70 \cdot 71068 .
$$

2. The diameter of a circle is 200 , what is the side of the inscribed square?

$$
\text { Ans. } 141 \cdot 42136 .
$$



## PROBLEM XXII.

Given the area of a circle, to find the side of the inscribed squaie.
Rule. Multiply the area by 6366197 , and extract the square root of the product, which will give the side of the inscribed square. $\ddagger$

1. The area of a circle is 100 , what is the side of the inscribed square? Ans. 7-97884.
2. The area of a circle is 200 , what is the side of the inscribed square?
$2 n 0 \times \cdot 6366197=127 \cdot 3239400 ;$ then $\sqrt{ } 127 \cdot 3239400=11 \cdot 2837$. Ans.

## PROBLEM XXIII.

> Given the side of a square, to find the diameter of the circumscribed circle.

Rule. Multiply the side of the square by 1.4142136 , and the pro * duct will give the diameter of the circumscribed circle.§

1. If the side of a square be 10 , what is the diameter of the circumscribed circle? Ans. 14-142136.
2. If the side of a square be 20 , find the diameter of the circumscribed circle? Ans. 28.284272.

## PROBLEM XXIV.

Given the side of a square, to find the circumference of the circumscribed circle.
Rule Multiply the side of the equare by 4.4428934 , and the product will be the circumscribed circle? \|

[^12]1. If the side of a square be 100 , what is the circumference of the circumscribed circle?

Ans. $444 \cdot 28934$.
2. If the side of the square be 30 , what is the circumference of the circumscribed circle?

Ans. $133 \cdot 286802$.

## PROBLEM XXV.

Given the side of a square, to find the diameter of a circle equal in area to the square.
Rule. Multiply the side of the square by $1 \cdot 1283791$, and the product will be the diameter of a circle equal in area to the square whose side is given.*

1. If the side of a square be 100 , what is the diameter of the circle whose area is equal to the square whose side is 100 ?

Ans. 112-83791.
2. What is the diameter of a circle equal in area to a square whose side is 200 ?

Ans. 225•67582.

## PROBLEM XXVI.

Given the sude of a square, to find the circumference of a circle whose area is equal to the square whose side is given.
Rule. Multiply the side of the square by 3.5449076 , and the product will give the circumference of a circle equal in area to the given square. $\dagger$

1. What is the circumference of a circle, whose area may be equal to a square whose side is 100 ?

Ans. $354 \cdot 49076$.
2. Find the circumference of a circle equal in area to a square whose side is 300 ?

Ans. $1063 \cdot 47228$.

## PROBLEM XXVII.

## To find the area of a sector of a circle.

Rule I. Multiply half the length of the are by the radius of the circle, and the product is the area of the sector. $\ddagger$

Rule II. As 360 is to the degrees in the are of the sector, so is the area of the whole circle to the area of the sector.§

1. Let ACBO be a sector less than a semicircle whose radius A O is 20 feet, and chord A B 30 feet, what is the area?

First, $\left.\sqrt{ }\left(\mathrm{A} \mathrm{O}^{2}-\mathrm{A} \mathrm{D}^{2}\right)=\sqrt{ } 400-225\right)=$ $13 \cdot 228=0 \mathrm{D}$; then $0 \mathrm{C}-0 \mathrm{D}=20-13 \cdot 228$ $=6.772=\mathrm{CD}$.
Again, $\sqrt{ }\left(\mathrm{A} \mathrm{D}^{2}+\mathrm{CD} \mathrm{D}^{2}\right)=\sqrt{ } 225+$ $45.859984)=16.4578=\mathrm{AC}$, the chord of half the arc.


Hence, by Problem XVII. the arc AB is 33.8874 ; then $\frac{33.8874}{2} \times 20=338.874$, the area required.

[^13]2. Let $\mathbb{A}$ E BOA be a sector greater than a semicircle, whose radius A 0 is 20 , the chord E B, 38, and chord B F of half EFB, 23 ; required the area?

## 184

$38=$ chord BE
3) $\overline{146}$


$$
48 \cdot 666 \text { \&c. }=\operatorname{arc} \text { B F E }
$$

3. What is the area of a sector whose are contains 18 degrees, the diameter being 3 feet?
```
Then 360:18::7.0686: the area of the sector;
```

Or, $20: 1:: 7 \cdot 0686: 35343$. Ans.
4. What is the area of a sector whose are contains 147 degrees 29 minutes, and radius 25 ?

Ans. 804-3986.
5. What is the area of a sector whose are contains 18 degrees, the radius being 3 feet?

Ans. 1•41372.

## PROBLEM XXVIII.

To fina the area of the segment of a circle.
Rule I. Find the area of the sector having the same arc with the segment, by the last problem; find also the area of the triangle, formed by the chord of the segment and the two radii of the sector. Then add these two areas together, when the segment is greater than a semicircle, but find their difference when it is less than a semicircle, the result will evidently be the answer.

1. What is the area of the segment ACBDA, its chord A B being 24, aud radius A E or E C 20 ?

$$
\sqrt{ }\left(\mathrm{AE}^{2}-\mathrm{AD}^{2}\right)=\sqrt{ }(400-144)=16
$$ $=\mathrm{DE} ; \mathrm{EC}-\mathrm{ED}=20-16=4=\mathrm{CD}$; $\sqrt{ }\left(\mathrm{AD}^{2}+\mathrm{DC}^{2}\right)=\sqrt{ }(144+16)=$ $12.64911=\mathrm{AC}$; then $\frac{(\mathrm{AC} \times 8)-24}{3}=$ $25 \cdot 7309=\operatorname{arc} A C B$,



And $12 \cdot 8654=$ half are

$$
20=\text { radius }
$$

$$
12=\mathrm{AD}
$$

$\overline{257 \cdot 308}=$ area of sector E B CA.
$\overline{192}=$ area of $\triangle \mathrm{ABE}$
$192=$ area of $\triangle \mathrm{A} \mathrm{B} \mathrm{E}$

$$
65 \cdot 308=\text { area of segment A B C A } .
$$

2. Let AGFBA be a segment greater than a semicircle, there are given the chord AB $20 \cdot 5$, F D $17 \cdot 17$, AF 20 , FG $11 \cdot 5$, and AE $11 \cdot 64$; required the area of the segment?
$\frac{(\mathrm{FG} \times}{} \frac{8)-\mathrm{AF}}{3}=\frac{(11.5 \times 8)-20}{3}=24$ the length of the aro
A GF (Problem XVII.); then $24 \times 11 \cdot 64=279 \cdot 36$, area of sector AEBFGA (Problem XXVII.) Again, FD - EF $=17 \cdot 17$ -

$$
11.64=5.53=\mathrm{ED} ; \quad \text { then } \frac{\mathrm{AB} \times \mathrm{ED}}{2}=\frac{20.5 \times 5.53}{2}=56.6825
$$

the area of the triangle A B E, which being added to the area of the sector before found will give the area of the segment, viz., $279 \cdot 36+$ $56 \cdot 6825=336 \cdot 0425$ the area of the segment A G F B A.

Rule II. To two-thirds of the product of the chord and versed sine of the segment, add the cube of the versed sine divided by twice the chord, and the sum will give the area of the segment, nearly.

When the segment is greater than a semicircle, find the area of the remaining segment, and deduct it from the area of the whole circle, the remainder will give the area of the segment.*
3. What is the area of the segment ACB, less than a semicircle, its chord being 18.9 , and height or versed sine D C 2.4 ?

$$
\mathrm{AB} \times \mathrm{DC}=18.9 \times 2.4=45 \cdot 36, \text { and } \frac{2}{3} \mathrm{AB} \times \mathrm{DC}=\frac{2}{3} \times
$$ $45.36=30.24$; then $\frac{2 \cdot 4^{3}}{2 \times 18.9}=36571$; hence $30 \cdot 24+36571=$ 30.60571 the area.

Note.-If two chords of a circle cut one another, the rectangle contained by the segments of one of them is equal to the rectangle contained by the seg. ments of the other. This is the 35th Proposition of Book III. of Euclid.
4. Required the area of the segment AGFB whose height FD is 20 , and chord A B 20?
$\frac{\mathrm{AB}}{2}=\frac{20}{2}=10=\mathrm{AD}$, and $\mathrm{AD}^{2}=100$; but $\mathrm{AD}^{2}=\mathrm{FD} \times \mathrm{DC}$
$\therefore \mathrm{CD}=\frac{\mathrm{AD}^{2}}{\mathrm{~F}} \frac{100}{\mathrm{D}}=5$.
The area of the segment A CB is, by the last case, 697916 ; and the area of the whole circle, by Problem XVIII., is 490.875 ; then $490 \cdot 875-69 \cdot 7916=421 \cdot 0834=$ area of the segment A G F B.
5. What is the area of the segment A GF B, greater than a semicircle, whose chord A B is 12, and versed sine 18? Ans. $297 \cdot 81034$.

Rule III. Divide the height of the segment by the diameter of the circle, to three places of decimals. Find the quotient in the column Height of the table at the end of the practical part of this treatise, and take out the corresponding Area Seg., which multiply by the square of the diameter, and the product will be the area of the segment required. $\dagger$
Note. I.-If the quotient of the height by the diameter be greater than 5 subtract it from 1, and find the Area Seg. corresponding to the remainder, which subtract from 7854 for the correct Area Seg.

NTite II.- If the quotient of the height by the diameter does not terminato in three figures, find the Area Seg. corresponding to the first three decimal figures of the quetient, subtract it from the next greater Area Seg., multiply the remainder by the fractional part of the quotient, and add the product to tho area segment first, taken out of the table. When great accuracy is not required, the fractional part may be omitted.
6. Let the diameter be 20 , and the versed sine 2 , required the area of the segment?
$\frac{2}{20}=\cdot 1$, to which answers $\cdot 040875$
Square of diameter, 400
$16 \cdot 3 \mathrm{~J}$ area.
7. What is the area of a segment, whose diameter is 52 , and versed sine 2 ?
$\frac{2}{62}=\cdot 038 \frac{6}{13}$ which is the tabular versed sine. Then to 0038 answers 009763 , and the difference between this area and the next is $\cdot 000385$, which multiplied by $\frac{6}{13}$ gives $\cdot 000177$, which added to -009763 gives -009940, which is the area corresponding to the versed sine $038 \frac{6}{13}$. Then $52^{2} \times 009940=26.87776$ is the area required.

## PROBLEM XXIX.

I'o find the area of a zone, or the space included by two parallel chords and the arcs contained between them.
RULE. Join the extremities of the parallel chords towards the same parts, and these connecting lines will cut off two equal segments, the areas of which, added to the area of the trapezoid then formed, will give the area of the zone.

1. Suppose the greater chord $\mathrm{AB}=30$, the less CD 20, and the perpendicular distance D $x=25$, required the area of the zone ABDC?
$\frac{1}{2}(\mathrm{AB}-\mathrm{CD})=x \mathrm{~B}=\frac{1}{2}(30-20)=5$ : then $\sqrt{ }\left(x \mathrm{D}^{2}+x \mathrm{~B}^{2}\right)=\mathrm{DB}=\sqrt{ }\left(25^{2}+5^{2}\right)=$ 25.49 . $\mathrm{AB}-\mathrm{B} x=\mathrm{A} x=30-5=25$, and $(\mathrm{A} x \times \mathrm{B} x) \div \mathrm{D} x=\mathrm{F} x=(25 \times 5) \div 25$

$=5 . \mathrm{D} x+\mathrm{F} x=\mathrm{DF}=25+5=30 ; \frac{1}{2}$ $\sqrt{ }\left(\mathrm{CDD}^{2}+\mathrm{D} \mathrm{F}{ }^{2}\right)=\frac{1}{2} \mathrm{CF}=\mathrm{G} z=\frac{1}{2} \sqrt{ }\left(20^{2}\right.$ $\left.+30^{2}\right)=18.027$, the radius of the circle ; ( $\left.\mathrm{DB} \times \mathrm{A} x\right) \div 2 \mathrm{D} x=$ $\mathrm{G} y^{*}=(25 \cdot 49 \times 25) \div(2 \times 25)=12.745 ; \mathrm{G} z-\mathrm{G} y=z y=$ $18 \cdot 027-12 \cdot 745=5 \cdot 282$, the height of the segment $\mathrm{A} \boldsymbol{z}$ C. $36 \cdot 05) 5 \cdot 28(\cdot 146$, the tabular area segment answering to which is $\cdot 071033$, then $071033 \times(36.05)^{2}=92.315=$ the area of the segment $\mathrm{A} z \mathrm{C}$.
$\frac{1}{2}(\mathrm{AB}+\mathrm{CD}) \times \mathrm{D} x=\frac{1}{2}(30+20) \times 25=625$ the area of the trapezoid ABDC: then $625+92.315 \times 2=809 \cdot 63=$ the area of the zone.
2. Let the chord AB be 48 , the chord CD 30 , the chord AC 15.8114 ; what is the area of the zone A B D C ?

Ans. Tb diameter $\mathrm{CF}=50$, height of the segment $\mathrm{A} z \mathrm{C}=$

[^14]$1 \cdot 2829$, area by the table of segments $=13.595$. Area of the zone $\mathrm{ABDC}=534 \cdot 19$ ?
3. Let $\mathrm{AB}=20, \mathrm{CD}=15$, and their distance $=17 \frac{1}{2}$; required the area?

Ans. $395 \cdot 4369$.
4. Let $\mathrm{AB}=96, \mathrm{CD}=60$, and their distance $=26$; required the area?

Ans. 2136.7527.

## PROBLEM XXX.

To find the area of a circular ring, or of the space included between two concentric circles.
Rule. Multiply the sum of the two diameters by their difference, and the product arising by ${ }^{\circ} 7854$ for the area of the ring.*

1. The diameter A B is 30 , and CD 20; what is the area of the ring XX ?
$\frac{30}{20}$
$\frac{50}{10}$
sum

10 difference
$\overline{500}$
-7854

$\overline{392 \cdot 7000}$ area of the ring $\mathbf{X X}$.
2. What is the area of the circular ring, when the diameters are 40 and 30 ?

Ans. 549•78.
3. What is the area of the circular ring, when the diameters are 50 and 45?

Ans. $373 \cdot 065$.

## PROBLEM XXXI.

To find the area of a part of a ring, or of the segment of a sector
Rule. Multiply half the sum of the bounding ares by their distance asunder, and the product will give the area. $\dagger$

1. Let AB be 50 , and $a b 30$, and the distance $\boldsymbol{a} \mathrm{A} 10$; what is the area of the space $a b \mathrm{BA}$ ?

$$
\text { Ans. } \frac{50+30}{2} \times 10=400
$$

2. Let $\mathrm{AB}=60, a b=40$, and $a \mathrm{~A}=2$; required the area of the space $a b \mathrm{BA}$ ? Ans. 100.
3. Let $\mathrm{AB}=25, a b=15$, and $a \mathrm{~A}=6$; required the area of the segment of the sector?

$$
\text { Ans. } 120
$$



## PROBLEM XXXII.

To find the area of a lune, or the space included between the intersecting arcs of two eccentric circles.
Rule. Find the areas of both segments which form the lune, and deduct the less from the greater ; the remainder will evidently bo the area required.

[^15]+ Ibid. 40.

1. Let the chord $\mathrm{AB}=40, \mathrm{EC}=12$, and $\mathrm{ED}=4$; what is the area of the lune ADBCA?

By note, page $38,\left(\mathrm{~A} \mathrm{E}^{2} \div \mathrm{EC}\right)+\mathrm{EC}$ $=$ diameter of the circle of which A C B
 is an arc; and $\left(\mathrm{AE}^{2} \div \mathrm{ED}\right)+\mathrm{ED}=$ the diameter of the circle of which AD B is an are; hence $\left(20^{2} \div 12\right)$ $+12=45 \cdot 3$; and $\left(20^{2} \div 4\right)+4=104$; are the two diameters.

$$
12 \div 45 \cdot 3=-264 . \quad 4 \div 104=\cdot 038
$$

The Area Seg. answering to 264 is 165780 , and $(45 \cdot 3)^{2} \times$ $\cdot 165780=340 \cdot 1954802=$ area of the segment AEBCA.

The Area Seg. answering to 0038 is $\cdot 009763$, and $(104)^{2} \times \cdot 009763$ $=105 \cdot 596608=$ area of the segment A E B D A ; then $340 \cdot 1954802$ $-105 \cdot 596608=234 \cdot 5988722=$ the area of the lune.
2. Let the chord A B be 40, and the heights of the segments EC and ED 15 and 2; required the area of the lume? Ans. $388 \cdot 5$

## PROBLEM XXXIII.

## TO MEASURE LONG IRREGULAR FIGURES.

When irregular figures, not reducible to any known figure, present themselves, their contents are best found by the method of equi-distant ordinates.
Rule. Take the breadths in several places, at equal distances, and divide the sum of the first and last of them by 2 for the arithmetical mean between those two. Add together this mean and all the other breadths, omitting the first and last, and divide their sum by the number of parts so added, the quotient will give the mean breadth of the whole, which being multiplied by the given length will give the area of the figure, very nearly.

It is not necessary sometimes to take the breadths at equal disrances, but to compute each trapezoid separately, and the sum of all the separate areas thus found will give the area of the entire nearly.

Or, add all the breadths together and divide by the number of them for a mean breadth, which being multiplied by the length, as before, will give the area nearly.

1. Let the ordinate AD be $9 \cdot 2$, bf $7, \operatorname{cg} 9, d h 10, \mathrm{BC} 8.8$ and the length A B 30 ; required the area?
$9 \cdot 2 \mathrm{~A} \mathrm{D}$
$8 \cdot 8 \mathrm{~B} \mathrm{C}$
2) 18

9 mean breadth of first and last.

$7 b f$
9 cg
$10 d h$
4) 35 sum
$8 \cdot 75$ mean breadth of all. 30

[^16]2. The length of an irregular figure is 39 yards, and its bretdths, in five equi-distant places, are $4 \cdot 8,5 \cdot 2,4 \cdot 1,7 \cdot 3$, and $7 \cdot 2$; what is its area?

Ans. 220.35 square yards.
3. The length of an irregular figure is 50 yards, and its breadths, at seven equi-distant places, are $5 \cdot 5,6 \cdot 2,7 \cdot 3,6,7 \cdot 5,7$, and $8 \cdot 8$; what is its area?

Ans. $342 \cdot 9150$ square yards.
4. The length of an irregular figure being $37 \cdot 6$, and the breadths, at nine equi-distant places. $0,4 \cdot 4,6 \cdot 5,7 \cdot 6,5 \cdot 4,8,5 \cdot 2,65,6 \cdot 1$; what is the area?

Ans. 218-315.

## EXERCISES.

1. Find the area of a square whose side is 35.25 chains. Ans. 124 acres, 1 rood, 1 perch.
2. Find the area of a rectangular board, whose length is $12 \frac{1}{2}$ feet, and breadth, 9 inches. Ans. $9 \frac{3}{8}$ feet.
3. The sides of three squares being 4,5 , and 6 feet, what is the length of the side of 2 qure which is equal to all three?

$$
\text { Ans. } 8 \cdot 7749 \text { feet. }
$$

4. Required the area of a Nomboid whose length is tofl chains, and breadth, $4 \cdot 28$ chains? Ans. 4 acres, 1 rood, $39^{\prime}$ perches.
5. There is a triangle whose base is 12.6 chains, and altitude 6.4 chains, what is its area?

Ans. 40.32.
6. Find the area of a triangle whose sides are 30,40 , and 50 yards. Ans. 600 square yards.
7. There is a triangular corn-field whose sides are 150,200 , and 250 yards, determine the number of acres contained in the field, and the expense of reaping the corn at 9 s .6 d. per acre.

Ans. Content of the field, 3 acres 15 perches; expense of reaping, £1, 9s. 5d.
8. What must the base of a triangle be to contain 36 square feet, whose vertex is to be 9 feet from the base? Ans. 8 feet.
9. What must be the altitude of a triangle equal in area to the last, whose base is 12 feet? Ans. 6 feet.
10. The height of a precipice standing close by the side of a river is 103 feet, and a line of 320 feet will reach from the top of it to the opposite bank; required the breadth of the river? Ans. 302.97 feet.
11. A ladder $12 \frac{1}{2}$ feet in length stands upright against a wall, how far must the bottom of it be pulled out from the wall so as to lower the top 6 inches?
12. A person wishing to measure the distance from a point A, at one side of a canal, to an object 0 , at the other, and having no instrument but a book, placed a

corner of it on the point A, and directed an edge of it, as in the figure, in a straight line with the object 0 , and drew the straight lines A B, A C; he then placed the hook so that a corner of it rested on the point $B$, at the distance of eight times its length from the point $A$, and directed an edge of it, as before, to the object 0 , and drew the straight line BC which met AC at the distance of three times the length of the book from A; how many times the length of the book is the object 0 from the points $\mathbf{A}$ and B?

Ans. $21 \frac{1}{3}$ and 22.78 times.
13. What is the area of a trapezium whose diagonal is 70.5 feet, and the two perpendiculars 26.5 and $30 \cdot 2$ feet?

Ans. $1998 \cdot 675$ square feet.
14. What is the area of a trapezium whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches? Ans. 6347 feet 36 inches.
15. What is the area of a trapezoid whose two parallel sides are 75 and 122 links, and the perpendicular distance 154 links?

Ans. 15169 square links.
16. A field in the form of a trapezoid, whose parallel sides are 6340 and 4380 yards, and the perpendicular distance between them 121 yards, lets for $£ 207,14 \mathrm{~s}$. per annum ; what is that per acre? $A n s . £ 1,11 \mathrm{~s}$.
17. Two opposite angles of a four-sided field are together equal to two right angles, and the sides are $24,26,28$, and 30 yards; what is its area? Ans. 723.99 square yards, nearly.
18. Required the area of a figure similar to that annexed to the first question under Problem XIV., whose dimensions are double of those there given? Ans. $3411 \cdot 6$.
19. What is the side of an equilateral triangle equal in area to a square, whose side is 10 feet? Ans. $15 \cdot 196$ feet, nearly.
20. Required the area of a regular nonagon, one of whose sides is 8 feet, and the perpendicular from the centre $=10.99$ feet?

Ans. $395 \cdot 64$ square feet.
21. Required the area of a regular decagon, one of whose sides is 20.5 yards?

Ans. $3233 \cdot 491125$ square yards.
22. A wheel of a car turns round 4400 times in a distance of 10 miles; what is its diameter? Ans. 3.819708 feet.
23. If the diameter of a circle be 9 feet, what is the length of the circumference?

Ans. $28 \frac{2}{7}$ feet, nearly.
24. Required the length of an arc of $60^{\circ}$, the radius of the circle being 14 feet?

Ans. $14 \cdot 660772$ feet.
25 . The chord of an arc is 30 feet and the height is 8 feet, what is the length of the are?

Ans. $35 \frac{1}{3}$ feet, nearly.
26. The diameter of a circle is 200 , what is the area of the quadrant? Ans. 7854.
27. The diameters of two concentric circles are 15 and 10 , what is the area of the ring formed by those circles? Ans. 98•175.
28. The circumference of a circle is 628.32 yards, what is the radius of a concentric circle of half the area?

Ans. 70.71.
29. What is the side of a square equal in area to the circle whose diameter is 3 ?

Ans $2 \cdot 658680 z$
30. The two parallel chords of a zone are 16 and 12 , and thei: perpendicular distance is 2 , what is the area of the zone?

Ans. 28.376.
31. The length of a chord is 15 , and the heights of two segments of circles on the same side of it are 7 and 4 ; what is the area of the lune formed by those segments? Ans. 38, nearly.
32. The base and perpendicular of a right-angled triangle are each 1 , what is the area of a circle having the hypothenuse for its diameter? Ans. $1 \cdot 5708$.
33. If the area of a circle be 100 , what is the area of the inscribed square?

Ans. $63 \cdot 66$.

## CONIC SERTIONS.

## SECTION III.

## \{ OF THE ELLIPSIS.* $\}$

## PROBLEM $I$.

The transverse and conjugate diameters of an ellipsis berng given, to find the area.
Rule. Multiply the transverse and conjugate diameters together, and the product arising by $\cdot 7854$, and the result will be the area. $\uparrow$

1. Let the transverse axis be 35 , and the conjugate axis 25 ; required the area? $35 \times 25 \times 7854=687 \cdot 225$. Ans.
2. The longer diameter of an ellipse is 70 , and the shorter 50 ; what is the area? Ans. 2748.9.
3. What is the area of an ellipse whose longer axis is 80 , and shorter axis is 60 ?

Ans. $3769 \cdot 92$.
4. What is the area of an ellipse, whose diameters are 50 and 45 ? Ans. 1767•15.

## PROBLEM II.

To find the area of an elliptical ring.
Rule. Find the area of each ellipse separately, and their difference will be the area of the ring.

Or, From the product of the two diameters of the greater ellipse deduct the product of the two diameters of the less, and multiply the remainder by 7854 for the area of the ring. $\ddagger$

1. The transverse diameter AB is 70, and the conjugate CD 50 ; and the transverse diameter EF of another ellipse having the same centre 0 , is 35 , and the conjugate GH is 25 ; required the area of the elliptical space between their circumferences?


[^17]$70 \times 50 \times \cdot 7854=2748 \cdot 9$; and $35 \times 25 \times 7854=687 \cdot 225$; then $2748 \cdot 9-687 \cdot 225=2061 \cdot 675=$ area of the elliptical ring.
$$
70 \times 50=3500
$$
$$
35 \times 25=875
$$
$$
\overline{2625} \times \cdot 7854=2061 \cdot 675=\text { area }
$$
2. The transverse and conjugate diameters of an ellipse are 60 and 40 , and of another 30 and 10 ; required the area of the space between their circumferences?

Ans. 1649•34.
3. A gentleman has an elliptical flower garden, whose greater diameter is 30 , and less 24 feet; and has ordered a gravel walk to be made round it of 5 feet 6 inches in width; required the area of the walk?

Ans. 371•4942.

## PROBLEM III.

Given the height of an elliptical segment, whose base is parallel to either of the axes of the ellipse, and the two axes of the ellipse, to find the area.
Role. Divide the height of the segment by that diameter of which it is a part, to three places of decimals, find the quotient in the column Height of the table referred to in page 38, and take out the correspondent Area Seg. Multiply the Area Seg. thus found and both the axes of the ellipsis together, and the result will give the area required.*

1. Required the area of an elliptical segment $R A Q$, whose height A P is 20 ; the transverse axis AB being 70, and the tonjugate axis CD 50 ?
$20 \div 70=-285_{7}^{5}=$ the tabular versed sine, the corresponding segment answering $\therefore$ which is 185166 ; then $\cdot 185166 \times 70 \times$
 $50=648^{\circ} 081$, the area.
2. What is the area of an elliptical segment cut off by a chord parallel to the shorter axis, the height of the segment being 10 , and the two diameters 35 and 25 ? Ans. 162.0202.
3. What is the area of an elliptical segment cut off by a chord parallel to the longer axis, the height of the segment being 10 , and the two diameters 40 and 30 ? Ans. $275 \cdot 0064$.
4. What is the area of an elliptical segment cut off by a chord parallel to the shorter diameter, the height being 10 , and the two diameters 70 and 50 ?

$$
\text { Ans. } 240 \cdot 854
$$

## PROBLEM IV.

## To find the circumference of an ellipse, by having the two diameters given.

Rule. Multiply the sum of the two diameters by $1 \cdot 5708$, and the product will give the circumference nearly; that is, putting $t$ for the transverse, $c$ for the conjugate, and $p$ for $3 \cdot 1416$; the circumference will be $(t+c) \times \frac{1}{2} p$. $\dagger$

[^18]+ Ibid. 44.

1. Let the transverse axis be 24 , and the conjugate 18 ; required the circumference
$(24+18) \times 1.5708=42 \times 1.5708=65.9736$ is the circumference nearly.
2. Required the circumference of an ellipse whose transverse axis Is 30 , and conjugate 20 ?

Ans. 78.54.
3. Required the circumference of an ellipse whose diameters are 60 and 40 ?

Ans. $157 \cdot 08$.
4. What is the circumference of an ellipse whose diameters are 6 and 4?

Ans. $15 \cdot 708$.
5. What is the circumference of an ellipse whose diameters are 3 and 2?

Ans. 7•854.

## PROBLEM V.

 To find the length of any arc of an ellipse.Rule. Find the length of the circular are $x y$, intercepted by 0 C , 0 B , and whose radius is half the sum of $0 \mathrm{C}, \mathrm{OB}$ : and it will be erfual to the elliptical are B C nearly.*

[^19]1. Let the axis AD be $24, \mathrm{CK} 18$, and 0 T 3 ; required the length of the arc BC?

Here we have TD=9, and AT=15; then from the property of the ellipsis, we have $\mathrm{A}^{2}: 0 \mathrm{C}^{2}:: \mathrm{AT} \times \mathrm{T} \mathrm{D}: \mathrm{TB} \mathrm{B}^{2}=$ $\frac{9^{2} \times 9 \times 15}{12 \times 12}=\frac{9 \times 9 \times 15}{16}$, and $0 B=\sqrt{ }\left(0 \mathrm{~T}^{2}+\right.$
$\left.T B^{2}\right)=\sqrt{ }\left(9+\frac{9 \times 9 \times 15}{16}\right)=9 \cdot 21616$, the radius
 of the circle of which $G B$ is an arc; but 0 C is the radius of the circle of which CV is an are ; therefore the radius of the circle of which $x y$ is an arc, is $\frac{1}{2} 0 \mathrm{C}+\frac{1}{2} \mathrm{OB}=9 \cdot 10808$. But by Trigonometry, $\uparrow \mathrm{HB} \div 0 \mathrm{~B}=3 \div 9 \cdot 21616=\cdot 325515$, is the sine of the angle COB, or are $x y$, to the radius 1 , answering to $18 \cdot 9968$ degrees. Therefore, by Problem XVII. Rule I., the length of the are $x y$ is $\cdot 01745 \times 18 \cdot 9968 \times 9 \cdot 10808=3.0192$, which is also equal to the length of the elliptical arc C B, nearly.
2. Given AD 30, CK 20, and OT5; required the length of the are BC? Ans. 5•03917786255.
3. Given AD 40, CD 30, and 0 T 5 ; required the length of the are BC?

Ans. 5.033880786.

[^20]
## PROBLEM VI.

## Given the diameter and abscissas, to find the ordinate.

Rule. Say, as the transverse is to the conjugate, so is the square root of the rectangle of the two abscissas, to the ordinate.*

1. In the ellipse ACD K, the -transverse diameter AD is 100 , the conjugate diameter C K 80, and the abscissa D T 10 ; required the length of the ordinate T B ?
$100: 80:: \sqrt{ }(90 \times 10): T B=24$. (See the last figure.)
2. Let the transverse axis be 35 , the conjugate 25 , and the abscissa 7; required the ordinate. Ans. 10.
3. Given the two diameters 70 and 60 , and the abscissa 10 ; required the ordinate?

Ans. 20.9956.

## PROBLEM VII.

Given the transverse axis, conjugate and ordinate, to find the abscissas.
Rule. As the conjugate is to the transverse diameter, so is the square root of the difference of the squares of the ordinate and semiconjugate, to the distance between the ordinate and centre. Then this distance being added to, and subtracted from, the semi-diameter, will give the two abscissas. $\dagger$

1. Let the diameters be 35 and 25 , and the ordinate 10 ; required the abscissas?

By the Rule $\frac{35}{2} \pm \frac{35}{25} \sqrt{ }\left(\left[\frac{25}{2}\right]^{2}-10^{2}\right)=\frac{35+21}{2}=28$ and 7 , the two abscissas.
2. Let the diameters be 120 and 40 , and the ordinate 16 ; required the abscissas?

Ans. 96 and 24.

## PROBLEM VIII.

Given the conjuyate axis, ordinate, and abscissas, to find the transverse axis.
Rule. Find the square root of the difference of the squares of the semi-conjugate axis and the ordinate, which add to, or subtract from, the semi-conjugate, according as the less abscissa or greater is given.

Then say, as the square of the ordinate is to the rectangle of the conjugate, and the abscissa, so is the sum or difference found above to the transverse required. $\ddagger$

1. Let the ordinate be 10 , and the less abscissa 7 ; what is the diameter, allowing the conjugate to be 25 ?

$$
\sqrt{ }\left(\left[\frac{25}{2}\right]^{2}-10^{2}\right)=7 \cdot 5 ; \text { then } 7 \cdot 5+12 \cdot 5=20 ; \text { then } 10^{2}: 25
$$

$\times 7:: 20: 35$, the transverse required.
2. Let the ordinate be 10 , the greater abscissa 28 , and the conjugate 25 ; required the transverse diameter? Ans. 3 ธ.

[^21]
## PROBLEM IX.

Given the transverse axis, ordinate, and abscissa, to find the conjugate.
Rule. The square root of the product of the two abscissas is to the ordinate, as the transverse axis is to the conjugate."

1. Let the transverse axis be 35 , the ordinate 10 , and dee abscissas . 28 and 7 ; required the conjugate?
$V^{\prime}(28 \times 7): 10:: 35: \frac{35 \times 10}{\sqrt{(28 \times 7)}}=\frac{35 \times 10}{14}=25$, the conjugate.
2. Let the transverse diameter be 120 , the ordinate 16 , and the abscissas 24 and 96 ; required the conjugate? . Ans. 40.

## OF THE PARABOLA.

## PROBLEM X.

Given the base and height of a parabola, to find its area.
Note.-Any double ordinate A B, to the axis of a parabola, may be called. its base, and the abscissa OD, to that ordinate, its height.

Rule. Multiply the base by the height, and $\frac{2}{3}$ of the product will be the area. $\dagger$

1. Required the area of a parabola, whose height is 6 and base 12 ?

$$
6 \times 12 \times \frac{9}{3}=48 \text { the area. }
$$


2. What is the area of a parabola, whose base is 24 , and height 4 ? Ans. 64.
3. What is the area of a parabola, whose base is 12 , and height 2 ? Ans. 16.

## PROBLEM XI.

To find the area of the zone of a parabola, or the space between two parallel double ordinates.
Rule I. When the two double ordinates, their distance, and the altitude of the whole parabola are given; find the area of the whole parabola, and find also the area of the upper segment, their difference will be the area of the zone.
II. When the two double ordinates and their distance are given; to the sum of the squares of the two double ordinates, add their product, divide the sum by the sum of the two double ordinates, multip'y the quotient by $\frac{\%}{3}$ of the altitude of the zone, and the product will be the area of the zone. $\ddagger$

1. Given A B 20 , S T 12, and D $x 8$; what is the area of the zone A S T B, the altitude D 0 being 12.5 ?
*See Appondix, Demonstration 49. $\dagger$ Ibid. 50. \$ Ibid. 51.
$(20 \times 12.5) \times \frac{2}{5}=166 \frac{2}{3}=$ area of the parabola A B O, and $(12.5-8)$ $\times 12=54$, and $54 \times \frac{2}{3}=36$; hence $166 \frac{2}{3}-36=130 \frac{2}{3}$, the area.
III. When the altitude of the whole parabola is not given.
2. Suppose the double ordinate $\mathrm{AB}=10$, the double ordinate $\mathrm{ST}=6$, and their distance $\mathrm{D} x=4$; what is the area of the zone ASTB?
$\frac{10^{2}+6^{2}+10 \times 6}{10+6}=12 \frac{1}{4}$; then $12 \frac{1}{4} \times 4 \times \frac{2}{3}=32 \frac{2}{3}$, the area as before.
3. Let the double ordinate $\mathrm{AB}=30, \mathrm{CP}=25$, and their distance D $\mathrm{G}=6$; required the area of the zone A B P C'? Ans. $165_{1 \mathrm{~s}}^{\frac{5}{15}}$

## PROBLEM XII.

To find the length of the curve, or arc of a parabola, out off by a double ordinate to the axis.

Rule.
I. Divide the double ordinate by the parameter, and call the quotient $\varsigma$.
II. Add 1 to the square of the quotient $q$, and call the square root of the sum $s$.
III. To the product of $q$ and $s$, add the hyperbolic logarithm of their sum, then the last sum multiplied by half the parameter, will give the length of the whole curve on both sides of the axis.

Putting $c$ for the curve, $q$ for the quotient of the double ordinate divided by the parameter, $s$ for $\sqrt{ }\left(1+q^{2}\right)$ and $a$ for half the parameter; then

$$
c-a \times\{q s+\text { hyp. log. of }(q+s .)\}^{*}
$$

Note.-The common logarithm of anv number multiplied by $2 \cdot 302585093$ sives the hyperbolical logarithm of the same number.

1. What is the length of the curve of a parabola, cut off by a double ordinate to the axis, whose length is 12 , the abscissa being 2 ?
$x=2$ and $y=6$; then $a=\frac{y^{2}}{2 x}=\frac{36}{4}=9$, and $q=\frac{y}{a}=\frac{6}{y}=\frac{8}{5}$, हlso $s=\sqrt{ }\left(1+q^{2}\right)=\sqrt{ }\left(1+\frac{4}{8}\right)=\sqrt{ }\left(\frac{13}{8}\right)=\frac{1}{3} \sqrt{ }(13)=$ $1 \cdot 2018504=s$. Then $\frac{z}{2}+1 \cdot 2018504=1 \cdot 868517$, whose common logarithm is 271497 , which being multiplied by $2 \cdot 302585093$, produces 6251449 for its hyperbolic logarithm; and also $\frac{2}{3} \times$ $1 \cdot 2018504=8012336$; the sum of these two is $1 \cdot 4263785$, therefore $9 \times 1.4263785=12 \cdot 8374065$, is the length of the curve required.

Rule II. Put $y$ equal to the ordinate, and $q$ equal the quotient arising from the division of the double ordinate by the parameter, or from the division of double the abscissa by the ordinate; then the length of the double curve will be expressed by the infinite series.

$$
\begin{gathered}
2 y \times\left(1+\frac{q^{2}}{2.3}-\frac{q^{4}}{2.4 .5}+\frac{3 q^{6}}{2.4 .6 .7}, \& \mathrm{c}\right) \\
\quad \text { Sce Appendix, Demons }
\end{gathered}
$$

Note.-This series will converge no longer than till $q=1$. For when $q$ is greater than 1, the series will diverge.

Let the last example be resumed, in which the abscissa is 2 , and the ordinate 6 .

Hence, $2 \times 2 \div 6=\frac{2}{3}=q$; then employing $\frac{2}{3}$ instead of $q$ in the last scries, we get
$12 \times\left(1+\frac{\left(\frac{2}{3}\right)^{2}}{2.3}-\frac{\left(\frac{2}{3}\right)^{4}}{2.4 .5}+3 \times \frac{\left(\frac{2}{8}\right)^{6}}{2.4 .6 .7}\right)=12.837$ the length of the curve as before.

Rule III. To the square of the ordinate, add $\frac{4}{3}$ of the square of the abscissa, and the square root of the sum will be the length of the single curve, the double of which will be the length of the double curve, nearly.*
Note.-The two first rules are not recommended in practice. The practical application of this is much simpler, and is therefore to be employed in preference to either.

Retaining the same example, in which $x=2$, and $y=6$, we shall get $v=\sqrt{ }\left(y^{2}+\frac{4}{3} x^{2}\right)=\sqrt{ }\left(36+\frac{16}{3}\right)=6 \cdot 1291$, and $\mathrm{C}=12 \cdot 8582$, nearly.
2. Required the length of the parabolic curve, whose abscissa is 3. and ordinate 8 ?

Ans. 17•435.

## PROBLEM XIII.

Given any two abscrssas and the ordinate to one of them, to find the corresponding ordinate to the second abscissa.
Rule. Say, as the abscissa, whose ordinate is given, is to the square of the given ordinate, so is the other given abscissa to the square of its corresponding ordinate. $\dagger$

1. If the abscissa $x 0=10$, and the ordinate $x \mathrm{~S}=8$, what is the ordinate AD , whose abscissa D 0 is 20 ?
$x 0: x \mathrm{~S}^{2}:: \mathrm{D} 0: \mathrm{AD}^{2}$, viz. $10: 64:: 20: 128$, the square root of which is $11 \cdot 313, \& \mathrm{c} .,=\mathrm{AD}$.
2. If 6 be the ordinate corresponding to the abscissa 9 , required the ordinate corresponding to the abscissa 16? Ans. 8.

## PROBLEM XIV.

Given two ordinates, and the abscissa corresponding to one of them, to find the abscissa corresponding to the other.
Rule. Say, as the square of the ordinate whose abscissa is given, is to the given abscissa, so is the square of the other ordinate to its corresponding abscissa. $\ddagger$

1. Given $\mathrm{S} x=6, x \mathrm{O}=9$, and $\mathrm{AD}=8$; required the abscissa 0 D ? $\quad 36: 9:: 64: 16=0 \mathrm{D}$.
2. Given $\mathrm{S} x=8, x 0=10$, and $\mathrm{AD}=9$; required OD ? Ans. $12 \cdot 656$.

* See Appendix, Demonstration 53.
- Ibid. 54
$\$$ Ibid. 54.


## PROBLEM XV.

Given two ordinates perpendicular to the axis and their distance,
to find the corresponding abscissas.

Rule. Say, as the difference of the squares of the ordinates is to their distance, so is the square of either of them to the corresponding abscissa.*

1. Given $\mathrm{S} x=6, \mathrm{~A} \mathrm{D}=8$, and $x \mathrm{D}=7$; required the abscissas?

$$
\begin{array}{r}
(64-36): 7:: 64 \\
-\quad 28: 7: 64: 16=0 \mathrm{D}, \text { and } \\
28: 7:: 36: 9=0 x .
\end{array}
$$

2. Given $\mathrm{S} x=3, \mathrm{AD}=4$, and $x \mathrm{D}=2$; required the abscissas? Ans. $4 \frac{4}{7}$ and $2 \frac{4}{7}$.

## OF THE HYPERBOLA.

## PROBLEM XVI.

Given the transverse and conjugate diameters, and any atscissa, to find the corresponding ordinate.
Rule. As the transverse is to the conjugate, $s 0$ is the mean proportional between the abscissas to the ordinate. $\dagger$

1. If the transverse be 24 , the conjugate 21 , and the less abscissa AD 8 ; required the ordinate?
Note.-The less abscissa added to the transverse gives the greater.
$24: 21:: \sqrt{ }(32 \times 8): \frac{21 \sqrt{ }(32 \times 8)}{24}=14$ the
 ordinate.
2. If the transverse axis of an hyperbola be 120, the less abscissa 40, the conjugate 72 ; required the ordinate? Ans. 48.
3. The transverse axis being 60 , the conjugate 36 , and the less abscissa 20 ; what is the ordinate?

Ans. 24.

## PROBLEM XVII.

Given the transverse, conjugate, and ordinate, to find the abscissa.
Rule. To the square of half the conjugate, add the square of the ordinate, and extract the square root of the sum. Then say,

As the conjugate is to the transverse, so is that square root to half the sum of the abscissas.

[^22]Then to this half sum, add half the transverse, for the greater abscissa; and from the half sum take half the transverse for the less abscissa.*

1. If the transverse be 24 , and the conjugate 21 ; required the abscissas to the ordinate 14 ?

| $10 \cdot 5=\frac{1}{2}$ conjugate $14=$ |  |
| :--- | ---: |
| $10 \cdot 5$ |  |
| $110 \cdot 25$ | $\frac{14}{196}$ |

## 196

306.25 , the square root of which is 17.5 ; then $21: 24:: 17 \cdot 5$ : $20=$ half sum, $20+12=32$ the greater abscissa, and $20-12=8$ the less abscissa.
2. The transverse is 120 , the ordinate 48 , and the conjugate 72 ; required the abscissas ?

Ans. 40 and 160.

## PROBLEM XVIII.

Given the conjugate, ordinate, and abscissas, to find the transverse.
Rule. To or from the square root of the sum of the squares of the ordinate and semi-conjugate, add or subtract the semi-conjugate, according as the less or greater abscissa is used; then, as the square of the ordinate is to the product of the abscissa and conjugate, so is the sum or difference, above found, to the transverse. $\dagger$

1. Let the conjugate be 21 , the less abscissa 8 , and its ordinate 24 ; required the transverse?

$$
\frac{21 \times 8 \times \sqrt{ }\left(14^{2}+\frac{21^{2}}{4}\right)+10 \frac{1}{2}}{14^{2}}=
$$

$\left.3 \times \sqrt{ }\left(3^{2}+4^{2}\right)+3\right)=3 \times(5+3)=24$ the transverse.
2. The conjugate axis is 72 , the less abscissa 40 , the ordinate 48 ; -equired the transverse? Ans. 120.
3. The conjugate is 36 , the less abscissa 20 , and its ordinate 24 ; required the transverse? Ans. 60.

## PROBLEM XIX.

Given the abscissa, ordinate, and transverse diameter, to find the conjugate.
Rule. As the mean proportional between the abscissas is to the ordinate, so is the transverse to its conjugate $\ddagger$

1. What is the conjugate to the transverse 24 , the less abcissa being 8 , and its ordinate 14 ?

$$
\frac{24 \times 14}{\sqrt{ }(32 \times 8)}=21 \text { the conjugate. }
$$

2. The transverse diameter is 60 , the ordinate 24 , and the less alscissa 20 ; what is the conjugate?
[^23]
## PROBLEM XX.

Fiven any two abscissas, $X, x$, and their ordinates, $Y, y$, to find the transverse to which they belong.
Rule. Multiply each abscissa by the square of the ordinate belonging to the other; multiply also the square of each abscissa by the square of the other's ordinate; then divide the difference of the latter products by the difference of the former; and the quotient will be the transverse diameter to which the ordinates belong.*

1. If two abscissas be 1 and 8 , and their corresponding ordinates $4 \frac{5}{8}$ and 14 , required the transverse to which they belong?

Here $\frac{8^{2} \times 4 \frac{3}{8} \times 4 \frac{3}{8}-1^{2} \times 14^{2}}{1 \times 14^{2}-8 \times 4 \frac{3}{8} \times 4 \frac{3}{8}}=\frac{35 \times 35-14 \times 14}{14 \times 14-35 \times 4 \frac{3}{8}}=\frac{5 \times 5-2 \times 2}{2 \times 2-5 \times \frac{5}{8}}$ $=\frac{21 \times 8}{7}=24$, the transverse.

## PROBLEM XXI.

To find the area of a space ANOB, bounded on one side by the curve of a hyperbola, by means of equi-distant ordinates.
Let AN be divided into any given number of equal parts, AC , CE, E G, \&c., and let perpendicular ordinates A B, CD, EF, \&c., be erected, and let these ordinates be terminated by any hyperbolio curve BDF , \&c.; and let $\mathrm{A}=\mathrm{AB}$ $+\mathrm{NO}, \mathrm{B}=\mathrm{C} \mathrm{D}+\mathrm{GH}+\mathrm{L} \mathrm{M}$, \&c., and $\mathrm{C}=\mathrm{EF}+\mathrm{IK}, \& c$. ; then the common distance AC, of the ordinates, being multiplied by the sum arising from the addition of $\mathrm{A}, 4 \mathrm{~B}$, and 2 C ,
 and one-third of the product taken will be the area, very nearly. That is, $\frac{A+4 B+2 C}{3} \times D=$ the area, putting $D=A C .+$

1. Given the lengths of 9 equi-distant ordinates, viz., $14,15,16$, $17,18,20,22,23,25$ feet, and the common distance 2 feet; required the area?

Ans. $300 \frac{2}{3}$ feet.
2. Given the lengths of 3 equi-distant ordinates, viz., $A B=5$, $\mathrm{CD}=7$, and $\mathrm{EF}=8$, also the length of the base AE 10 ; what is the area of the figure ABFE?

Ans. $68 \frac{1}{3}$ feet.
3. If the length of the asymptote of a hyperbola be 1 , and there be 11 equi-distant ordinates between it and the curve, the common distance of the ordinates will then be $\frac{1}{10}$, and from the nature of the curve their lengths will be $\frac{10}{10}, \frac{10}{11}, \frac{10}{12}, \frac{10}{13}, \frac{10}{14}, \frac{10}{15}, \frac{10}{10}, \frac{10}{17}, \frac{10}{18}, \frac{10}{18}, \frac{10}{20}$ : what is the area of the curved figure? Ans. 69315021 .

This formula will answer for finding the area of all curves by using the sections perpendicular to the axis. The greater the number of ordinates employed, the more accurate the result; but in real practice three or five are in most cases sufficient.

## PROBLEM XXII.

## To find the length of any arc of an hyperbola beginning at the vertex.

## Rule.

I. To 19 times the square of the transverse, add 21 times the square of the conjugate; also to 9 times the square of the transverse add, as before, 21 times the square of the conjugate, and multiply each of these sums by the abscissa.

1I. To each of these two products, thus found, add 15 times the product of the transverse and the square of the conjugate.
III. Then, as the less of these results is to the greater, so is the ordinate to the length of the curve, nearly.*

1. In the hyperbola BAC , the transverse diameter is 80 , the conjugate 60 , the ordinate B D 10, and the abscissa AD 2 ; required the length of the arc BAC? (Fig. p. 52.)

Here $2\left(19 \times 80^{2}+21 \times 60^{2}\right)=2(121600+75600)=394400$.
And $2\left(9 \times 80^{2}+21 \times 60^{2}\right)=2(57600+75600)=266400$.
Whence $15 \times 80 \times 60^{2}+394400=4320000+314400=4714400$.
And $15 \times 80 \times 60^{2}+266400=4320000+266400=4586400$.
Then $4586400: 4714400:: 10: \frac{4714400 \theta}{458640 \theta}=10 \cdot 279=A B$.
Hence A BC $=10 \cdot 279 \times 2=20.558$.
2. In the hyperbola $\mathbf{B A C}$, the transverse diameter is 80 , the conjugate 60, the ordinate BD 10, and the abscissa AD $2 \cdot 1637$; required the length of the arc A B? Ans. $10 \cdot 3005$.

## PROBLEM XXIII.

Given the transverse axis of a hyperbola, the conjugate, and tho abscissa, to find the area.

## Rule.

I. To the product of the transverse and abscissa, add $\frac{5}{7}$ of the square of the abscissa, and multiply the square root of the sum by 21.
II. Add 4 times the square root of the product of the transverse and abscissa, to the preceding product, and divide the sum by 75.
III. Divide 4 times the product of the conjugate and abscissa by the transverse ; this quotient, multiplied by the former quotient, will give the area of the hyperbola, nearly. $\dagger$

1. In the hyperbola BAC (see figure, page 52), the transverse axis is 30 , the conjugate 18 , and the abscissa $A D$ is 10 ; what is the area?

Here $21 \sqrt{ }\left(30 \times 10+\frac{5}{7} \times 10^{2}\right)=21 \sqrt{ }(300+71 \cdot 42857)=21$ $\sqrt{ }(371 \cdot 42857)=21 \times 19272=404 \cdot 712$;
And $\frac{4 \sqrt{ }(30 \times 10)+404.712}{75}=\frac{4 \times 17.3205+404.712}{75}=$ $\frac{69 \cdot 282+404 \cdot 712}{75}=\frac{473994}{75}=6.3199$.

Whence $\frac{18 \times 10 \times 4}{30} \times 6.3199=24 \times 6.3199=151 \cdot 6776$, the area required.
2. What is the area of an hyperbola whose abscissa is 25 , the transverse and conjugate being 50 and 30 ? Ans. 805.0909.
3. The transverse axis is 100 , the conjugate 60 , and abscissa 50 ; required the ares?

# MENSURATION OF SOLIDS. 

## SECTION IV.

## DEFINITIONS.

1. A solid is that which has length, breadth, and thickness.
2. The solid content of any body is the number of cubic inches, feet, yards, \&c., it contains.
3. A cube is a solid, having six equal sides at right angles to one another.

4. A prism is a solid whose ends are plane figures, which are parallel, equal, and similar. Its sides are parallelograms.


It is called a triangular prism, when its ends wre triangles; a square prism, when its ends are squares; a pentagonal prism, when its ends are pentagons; and so on.

5 A parallelopipedon is a solid having six rectangular sides, every opposite pair of which are equal and parallel.

6. A cylinder is a round solid, having crrcular ends, and may be conceived to be describad by the revolution of a rectangle about one of its sides, which remains fixed.

7. A pyramid is a solid, having a plane figure for its base; and whose sides are triangles meeting in a point, called the vertex.

Pyramids have their names from their bases, like prisms.

When the base is a triangle, the solid is called a triangular pyramid; when the base is a square, it is called a square pyramid; and so on.
8. A cone is a round pyramid, having a circle for its base.

9. A sphere is a round solid, which may be conceived to be formed by the revolution of a semicircle about its diameter, which remains fixed.
10. The axis of a solid is a line joining the middle of both ends.
11. When the axis is perpendicular to the base, the solid is called a right prism or pyramid, otherwise it is oblique.
12. The height or altitude of a solid, is a line drawn from its vertex, perpendicular to its base, and is equal to the axis of a right prism or pyramid ; but in an oblique one, the altitude is the perpendicular of a right-angled triangle, whose hypothenuse is the axis.
13. When the base is a regular figure, it is called a regular prism or pyramid; but when the base is an irregular figure, the solid on it is called irregular.
14. The segment of any solid, is a part cut off from the top by a plane parallel to its base.
15. A frustum is the part remaining at the bottom, after the segment is cut off.
16. A zone of a sphere is a part intercepted between two planes which are parallel to each other.
17. A'circular spindle is a solid generated by the revolution of a segment of a circle about its chord, which remains fixed.

18. A wedge is a solid, having a rectangular base, and two of its opposite sides meeting in an calge.
19. A prismoid is a solid, having for its. ,wo ends two right-angled parallelograms; parallel to each other, and its upright sides are four trapezoids.

20. A spheroid is a solid, generated by the rotation of a semi-ellipsis about one of its axis, which remains fixed.

When the ellipsis revolves round the transverse axis, the figure is called a prolate, or oblong spheroid; but when the ellipsis revolves round the shorter axis, the figure is
 called an oblate spheroid.
21. An elliptical spindle is a solid, generated by the rotation of a segment of an ellipsis about its chord.

22. A parabolic conoid, or paraboloid, is a solid generated by the rotation of a semi-parabola about its axis.

23. An ungula or hoof, is a part cut off a solid by a plane oblique to the base.

## PROBLEM I.

To find the solidity of a cube.
Rule. Multiply the side of a cube by itself, and that product again by the side, for the solidity required.*

1. If the side of a cube be 4 inches, required its solidity?

Here, $4 \times 4=16$, the number of cubes of 1 inch deep in the square E F G D, and as the entire solid consists of four such dimensions, its content is $16 \times 4=64$ cubic inches.
2. What is the solidity of a cubical piece of marble, each side being 5 feet 7 inches?

$$
\text { Ans. } 174 \text { feet, nearly. }
$$

3. A cellar is to be dug, whose length,
 breadth, and depth, are each 12 feet 3 inches; required the number of solid feet in it? Ans. 1838 feet 3 inches, nearly.

## PROBLEM II.

## To find the solidity of a parallelopipedon.

Rule. Multiply continually the length, breadth, and depth together for the solidity. $卜$

1. What is the solidity of the parallelopipedon ABCDEFG , the length A B being 10 feet, the breadth A G 4 fect, and thickness A D 5 feet?
$A B \times A G \times A D=10 \times$ $4 \times 5=200$ feet.
2. A piece of timber is 26
 feet long, 10 inches broad, and 8 inches deep; required its solid content? Ans. $14 \frac{4}{9}$ feet.
3. A piece of timber is 10 inches square at the ends and 40 feet long; required its content?
4. A piece of timber 15 inches square at each end, and 18 feet

- See Appendix. Demonstration 的
$\dagger$ Ibid 64
long, is to be measured; required its content, and how far f.um the end must it be cut across, so that the piece cut off may contain 1 solid foot?

Ans. The solidity is 28.125 feet; and 7.68 in length will make one foot.
5. What length of a piece of square timber will make one solia foot, being 2 feet 9 inches deep, and 1 foot 7 inches broad?

Ans. 2.756 inches in length will make one solid foot.

## PROBLEM III.

## To find the solidity of a prism.

Rule. Multiply the area of the base by the perpendicular height, and the product will be the solidity.*

1. What is the solidity of a prism, ABCFIE, whose base C $\AA$ is a pentagon, each side of which being 3.75 , and height 15 feet?

When the side of a pentagon is 1 , its area is 1.720477 (Table 11.); therefore $1720477 \times 3.75^{2}=24 \cdot 1942=$ the area of the base in square feet; hence $24 \cdot 1942 \times$ $15=362 \cdot 913$ solid feet, the content.
2. What is the solidity of a square prism, whose length is $5 \frac{1}{2}$ feet, and each side of its base $1 \frac{1}{3}$ foot? Ans. $9 \frac{7}{9}$ solid feet.
3. What is the solidity of a prism, whose base is an equilateral triangle, each side being 4 feet, and height 10 feet?


$$
\text { Ans. } 69 \cdot 282 \text { teet. }
$$

4. What quantity of water will a prismatic vessel contain, its base being a square, each side of which is 3 feet, and height 7 feet?

Ans. 63 feet.

## PROBLEM IV.

## To find the solidity of a cylinder.

Rule. Multiply the area of the base by its height, and the product will be the solid content. $\dagger$

1. What is the capacity of a right cylinder ABGC, whose height, and the circumference of its base, are each 20 feet?

First $\frac{20}{3 \cdot 1416}=$ the diameter,
 half of which multiplied by half the circumference will give the area of the base (Prob. XVIII. Sec. II.), that is, $10 \times \frac{10}{3 \cdot 1416}=\frac{25}{.7854}=$ the area of the end; then $\frac{25}{.7854} \times 20=636.61828$, the content. - See Appendix, Deinonstration 64.

2 What is the content of the oblique cylinder A B F E, the circumference of whose base is 20 feet, and altitude AC 20 feet?

As before, the area of the base is $\frac{25}{.7854}$; then $\frac{25}{.7854} \times 20=$ 636.61828 , the solid content as before.
3. The length of a cylindrical piece of timber is 18 feet, and its circumference 96 inches; how many solid feet in it?

Ans. 91.676 feet.
4. Three cubic feet are to be cut off a rolling stone 44 inches in circumference; what distance from the end must the section be made?

Ans. $33 \cdot 64$ inches.

## PROBLEM V.

To find the content of a solid formed by a plane passing parallel to the axis of a cylinder.
Rule. Find by Prob. XXVIII., Sec. II., the area of the base, which, multiplied by the height, will give the solidity.*

1. In the cylinder ABGC, whose diameter is 3 , and height 20 feet; let a plane LN pass parallel to the axis, and 1 foot from it; what is the solidity of each of the two prisms into which the cylinder is divided?-(See the last figure.)
$\frac{\stackrel{1}{\mathrm{GC}}}{\mathrm{GC}}=\left(\frac{3}{2}-1\right) \div 3=\frac{\frac{1}{2}}{3}=\frac{1}{6}=166 \frac{2}{8}$ the tabular versed sine, to which, in the Table of Circular Segments, corresponds the area
-08604117
which taken from . . . . . . . . 78539816
leaves the other segment $\quad$ - 69935699
Then $3^{2}=9$ which $\times \cdot 08604117=7 \cdot 7437053=$ seg. D C N.
Also $9 \times \cdot 69935699=6 \cdot 29421291=$ seg. D G N.
Hence $20 \times 7.7437053=15.4874=$ the slice LKACND; and $20 \times 6 \cdot 29421699=125 \cdot 88434=$ the slice L K B G N D.
2. Suppose the right cylinder, whose length is 20 feet, and diameter 50 feet, is cut by a plane parallel to, and at the distance of, $21 \cdot 75$ feet from its axis; required the solidity of the smaller slice?

Ans. $1082 \cdot 95$ feet.

## PROBLEM VI.

## To find the solidity of a pyramid.

Rule. Multiply the area of the base by the one-third of the height, and the product will be the solidity.t

1. What is the solidity of a square pyramid, each side of its base being 4 feet, and height 12 feet?
$4 \times 4=16$ the area of the base:
Then $16 \times \frac{12}{3}=64$ feet, the solidity.
2. Each side of the base of a triangular pyramid is 3 , and height 30 ; required its solidity?

Ans. 38.97117.
3. The spire of a church is an octagonal pyramid, each side at the base being 5 feet 10 inches, and its perpendicular height 45 feet;
also each side of the cavity, or hollow part, at the base is 4 feet 11 inches, and its perpendicular height 41 feet; it is required to know how may solid yards of stone the spire contains.

Ans. 32-19738 yards.
4. The height of a hexagonal pyramid is 45 feet, each side of the hexagon of the base being 10 ; required its solidity?

Ans. 3897•1143.

## PROBLEM VII.

## To find the solidity of a cone.

Rule. Multiply the area of the base by one-third of the height, and the product will be the solidity.*

1. The diameter of the base of a cone is 10 feet, and its perpendicular height 42 feet; what is its solidity?
$10^{2}=100 \times \cdot 7854=78 \cdot 54$; then $78.54 \times \frac{42}{3}=1099.56$ feet.
2. The diameter of the base of a cone is 12 feet, and its perpendicular height 100 ; required its solidity? Ans. $3769 \cdot 92$ feet.
3. The spire of a church of a conical form measures $37 \cdot 6992$ feet round its base; what is its solidity, its perpendicular height being 100 feet? Ans. $3769 \cdot 92$.
4. How many cubic yards in an upright cone, the circumference of the base being 70 feet, and the slant height 30 ? Ans. 134.09.
5. How many cubic feet in an oblique cone, the greatest slant height being 20 feet, the least 16, and the diameter of the base 8 feet? Ans. $254 \cdot 656588$ feet.

## PROBLEM VIII.

## To find the solidity of the frustum of a pyramid.

Rule. Add the areas of two ends and the mean proportional between them together; then multiply the sum by one-third of the perpendicular height, and the product will give the solidity.t

1. In a square pyramid, let $\mathrm{A} 0=7, \mathrm{PD}=5$, and the height $O Q=6$; the solidity of the frustum is required.
$7 \times 7=49=$ the area of the base.
$5 \times 5=25=$ the area of the section SD
$7 \times 5=35=$ the mean proportional between 49 and 25.

Therefore, $\frac{49+35+25}{3} \times 6=218=$ the content of the frustum.

2. What is the content of a pentagonal frustum, whose height is 5 feet, each side of the base 1 foot 6 inches, and each side of the less end 6 inches? Ans. 9.31925 cubic feet.
3. What is the content of a hexagonal frustum, whose height is 6 feet, and the side of the greater end 18 inches, and of the less 12 inches?

Ans. 24.681724.

* See Appendix, Demonstration, 66
- Ibid. 67.

4 How many cubce feet in a squared piece of timber, the areas of the two ends being 504 and 372 inches, and its length $31 \frac{1}{2}$ feet?

Ans. 95.447 feet.
5. What is the solidity of a squared piece of timber, its length being 18 feet, each side of the greater base 18 inches, and each side of the small end 12 inches? Ans. 28.5.

## PROBLEM IX.

## To find the solidity of the frustum of a cone.

Rule. Add the two ends, and the mean proportional between them together, then multiply one-third of the sum by the perpendicular height, and the product will be the content.*

1. How many solid feet in a tapering round piece of timber, whose length is 26 feet, and the diameters of the ends 22 and 18 inches respectively?

Here $22^{2} \times \cdot 7854=380 \cdot 134$ inches, the area of the greater end, and
$18^{2} \times \cdot 7854=254 \cdot 47$ inches $=$ the area of the less end, $(380 \cdot 134 \times$ $9.4 \cdot 47) \frac{1}{n}=311 \cdot 018=$ the mean proportional between the areas of the ends; then by the rule
$\underline{254 \cdot 47+380 \cdot 134+311 \cdot 018} 33 \times 26 \times 12=98345$ cubic inches $=56 \cdot 9$ cubic feet, the answer.
2. How many cubic feet in a round piece of timber, the diameter of the greater end being 18 inches, and that of the less 9 inches, and length $14 \cdot 25$ feet? Ans. 14:68943 feet.
3. What is the solid content of the frustum of a cone, whose height is 1 foot 8 inches, and the diameters of the ends 2 feet 4 inches, and 1 foot 8 inches?

Ans. 5•284.

## PROBLEM X.

## To find the solidity of a wedge.

Rule I. Add the three parallel edges together, and multiply onethird of the sum by the area of that section of the wedge which is perpendicular to these three edges, and the product will give the content. $\dagger$

Note.- When the quadrangular sides are parallelograms, the wedge is a triangular prism, having for its base the triangle BOC; when the quadrangles are rectangular, A $O$ is the height of the prism, and the area of the triangle BOC multiplied by A O will give its content; when the triangle B OC is isosceles and perpendicular to the plane A C, the wedge is of the common kind; C G is its edge, and ARB O its back.


Rule II. To twice the length of the base, add the length of the edge, multiply the sum by the breadth of the base, and the product by the height of the wedge, and one-sixth of the last product will be the solidity, that is, $(2 \mathrm{~L}+l) \times \frac{1}{6} b h$, by putting $\mathrm{L}=\mathrm{RB}$, the length
of the base, $l=\mathrm{GC}$, the length of the edge, $b=\mathrm{A} \mathrm{R}$, the breadth of the base, $h=$ the perpendicular height of the wedge.*

1. Let $\mathrm{A} 0=4, \mathrm{G} \mathrm{C}=3, \mathrm{R} \mathrm{B}=2 \frac{1}{2}$, the perpendicular $\mathrm{D} \mathrm{T}=12$, and $p$ the perpendicular distance of BR from the plane of the face $\mathrm{AC}=$ $3 \frac{1}{2}$ feet; required the solid content?
$\frac{4+3+2 \frac{1}{2}}{3} \times 12 \times \frac{3 \frac{1}{2}}{2}=66 \frac{1}{2}$ cubic feet.
2. The perpendicular height from the point T to the middle of the back AB is $24 \cdot 8$, the length of the edge C G 110 inches, the base R B 70 inches, and its breadth A R 30 inches ; required the solidity? Ans. 31000 cubic inches.
3. How many cubic inchies in a wedge whose altitude is 14 inches, its edge 21 inches, the length of its base 32 inches, and its breadth $4 \frac{1}{2}$ inches?

Ans. 892.5 cubic inches.

## PROBLEA XI.

To find the solidity of a prismoid, which is the frustum of a wedge.
Rule. By either of the foregoing rules, find the solidity of two wedges whose bases are the two ends of the frustum, and height the distance between them, and the sum of both will be the solidity of the prismoid or frustum. +

1. In the prismoid A B PQ, there is given $R B=18$, $\mathrm{A} 0=27, \mathrm{PD}=21, \mathrm{~S} \mathrm{Q}=24, \mathrm{~B} 0=12, \mathrm{D} \mathrm{Q}=4$, and B $I=30$; what is its solidity?
$\frac{18+27+21}{3} \times \frac{30 \times 12}{2}=3960=$ the content of the greater wedge, and $\frac{24+27+21}{3} \times \frac{30 \times 4}{2}=1440$, the content of the other; then $3960+1440=5400$, the
 content of the frustum.
2. What is the solidity of a piece of wood in the form of a prismoid, whose ends are rectangles, the length and breadth of one being 1 foot 2 inches and 1 foot respectively, and the corresponding sides of the other 6 and 4 inches respectively; the perpendicular height being $30 \frac{1}{2}$ feet?

Ans. 18.074 cubic feet.

[^24]
## PROBLEM XII.

To find the solidity of a cylindroid; or the frustum of an elliptical cone.

Rule.
I. To the longer diameter of the greater end, add half the longer

* Seo Appendix, Demonstration 70.
$\dagger$ Ibid. 71
$\ddagger$ Ibid. ${ }^{7}$ ².
diameter of the less end, and multiply the sum by the shorter diameter of the greater end.
II. To the longer diameter of the less end, add half the longer diameter of the greater end, and multiply the sum by the shorter diameter of the less end.
III. Add the two preceding products together, and multiply the sum by $\cdot 2618$ (one-third of $\cdot 7854$ ) and then by the height ; the last product will be the solidity.*

1. Let ABCD be a cylindroid, the base of which is an ellipsis, whose two diameters are 40 and 20 inches, the top a circle, whose diameter is 30 inches; what is its solidity, allowing the height to be 10
 fect?

$$
\begin{aligned}
& \left(\mathrm{AB}+\frac{1}{2} \mathrm{CD}\right) \times \mathrm{GH}=(40+15) \times 20=1100 \\
& \left(C D+\frac{1}{8} \text { A B }\right) \times m \mathrm{P}=(30+20) \times 30=1500 \\
& \text { sum }=2600
\end{aligned}
$$

Then $(2600 \times 2618 \times 10) 6806 \cdot 8$, which, divided by 144 , gives 47.27 feet, the answer.
2. The transverse diameter of the greater base of a cylindroid is 13, and conjugate 8 ; the transverse diameter of the less base 10 , and conjugate $5 \cdot 2$; what is the solidity of the cylindroid, its height being 12 ?

Ans. $721 \cdot 93968$.
3. The transverse diameter at the top of the cylindroid is- 12 inches, and conjugate 7 ; the longer diameter at the bottom is 14 inches, and shorter 12 , and its height 10 feet; required its solidity?

Ans. 6.78 feet.

## PROBLEM XIII.

## To find the solidity of a sphere.

Rule I. Multiply the cube of the diameter by 5236 , and the product will be the content.

Rule II. Multiply the diameter by the circumference of the sphere, and the product multiplied by one-sixth part of the diameter will be the solidity. $\dagger$

1. Suppose the earth to be a perfect sphere, and its diameter $7957 \frac{3}{4}$ miles, how many solid miles does it contain?
$7957 \frac{3}{4} \times 3 \cdot 1416=$ the circumference of the earth (Prob. XVI., Sec. II.) ; then
$7957 \frac{3}{4} \times 3 \cdot 1416 \times 7957 \frac{3}{4}=198943750=$ the surface of the sphere; then $1 \ni 8943750 \times 7957 \frac{3}{4} \times \frac{1}{6}=263857437760$ miles,
 the solidity by Rule II.

Again, $5236 \times d^{3}=5236 \times\left(7957 \frac{3}{4}\right)^{3}=263858149120$ miles, the solidity by Rule I., which gives the result too great on account of taking - 6236 a little too great.
2. What is the solidity of a sphere, whose diameter is 24 inches? Ans. $7238 \cdot 2464$ cubic inches.
3. What is the solid content of the earth, allowing its circumference to be 25000 miles? Ans. 263858149120 miles.
4. Required the solidity of a globe whose diameter is 30 feet?

Ans. 14137.2.

## PROBLEM XIV.

## To find the solidity of the segment of a sphere.

Rule I. From three times the diameter of the sphere deduct twice the height of the segment; multiply the remainder by the square of the height, and that product by $\cdot 5236$; the last product will be the solidity.*

Rule II. To three times the square of the radius of the segment's base add the square of its height; multiply this sum by the heigest, and the product by $\cdot 5286$; the last result will be the solidity.

1. What is the solidity of each of the frigid zones, the diameter of the carth being 7957 星 miles, and half the breadth, or arc of the meridian intercepted between the polar circle and the pole $23 \frac{1}{2}$ degrees; that is, $\mathrm{AD}=23 \frac{1}{2}$ degrees, supposing AB to represent the polar circle.


By Rule I.
As 1 (=tabular radius) : $3978 \frac{7}{8}$ (=radius of the earth) : : -0829399 ( $=$ tabular versed sine of $23 \frac{1}{2}$ degrees) : $330 \cdot 0074946$, the versed sine, or height of the segment

Then $5236 h^{2}=(3 d-2 h)=\cdot 5236 \times 330 \cdot 0074946^{2} \times 23213 \cdot 2350108$ $=1323679710$, the solid content.

## By Rule II.

As $1: 3978 \frac{7}{3}:: 3987491$ ( $=$ the tabular sine of $23 \frac{1}{2}$ degrees) 1586.57282 ã 26 , the radius of the base.

Then $5236 h \times\left(3 r^{2}+h^{2}\right)=\cdot 5236 \times 330 \cdot 0074946 \times 7660544 \cdot 936=$ $1323680299 \cdot 69$, the solidity.
2. Let ABDO be the segment of the sphere whose solidity is required. The diameter AB of the base is 16 inches, and the height OD 4 inches. Ans. $435 \cdot 6352$ cubic inches.
3. Required the solidity of the segment of a sphere, whose diameter is 20 feet, and the height of the segment 5 feet?

Ans. 654.5 feet.

## PROBLEM XV.

To find the soliclity of the frustum or zone of a sphere.
Rite.
I. To the sum of the squares of the radii of the two ends, add $\frac{1}{3}$ of

[^25]the square of their distance, or of the height of the zone; ttis sum multiplied by the height of the zone, and the product again by 1.5708 , will be the solidity.
II. For the middle zone of a sphere. To the square of the diameter of the end add two-thirds of the square of the height; multiply this sum by the height, and then by 7854 , the last result will be the solidity.

Or, From the square of the diameter of the sphere, deduct onethird of the square of the height of the middle zone; multiply the remainder by the height, and then by $\cdot 7854$, the last result will be the solidity.*

1. Required the solidity of the frustum of a sphere, the diameter of whose greater end is 4 feet, the diameter of the less end 3 feet, and the height $2 \frac{1}{2}$ feet?
$\left(2^{2}+1 \cdot 5^{2}+\frac{1}{3} \times 2 \cdot 5^{2}\right) \times 1 \cdot 5708 \times 2.5=8 \frac{1}{3} \times$ $3 \cdot 927=32 \cdot 725$, the solidity of the frustum.
2. What is the solidity of the temperate zone, its breadth being 43 degrees, the radius of the
 top being 1586.57282526 , and the radius of the base $3648 \cdot 86750538$, and height $2062 \cdot 2655$ ?
$\left(3648 \cdot 86750538^{2}+1586 \cdot 57282526^{2}+\frac{1}{3} \times 2062 \cdot 2655^{2}\right) \times 2062 \cdot 2655$ $\times 1 \cdot 5708=17249136 \times 2062 \cdot 2955 \times 1 \cdot 5708=55877778668$, the solidity of each temperate zone.
3. Required the solidity of the torrid zone, which extends $23 \frac{1}{2}$ degrees on each side of the equator, the diameter being $7957 \frac{3}{4}$ miles, and the height $3173 \cdot 14565052$ ?
$\left(7957.75^{2}-\frac{1}{3} \times 3173 \cdot 14565052^{2}\right) \times 3173 \cdot 14565052 \times \cdot 7854=$ 149455081137, the answer.
4. What is the solidity of the middle zone of a sphere, whose top and bottom diameters are each 3 inches, and height 4 inches? Ans. $61 \cdot 7848$.
5. What is the solid content of a zone, whose greater diameter is 20 feet, less diameter 15 feet, and the height 10 feet?

$$
\text { Ans. } 189 \cdot 58
$$

6. How many solid feet in a zone, whose greater diameter is 12 feet, and less diameter 10 ; the height being 2 ?

Ans. 195•8264.

## PROBLEM XVI.

## To find the solidity of a circular spindle.

Rule. Find the distance of the chord of the generating circular segment from the centre of the circle, and also the area of this segment.

Then, from one-third of the cube of half the length of the spindle or half chord of the segment, subtract the product of the central distance, and half the area of the segment ; the remainder, multiplied by $12 \cdot 5664$, will give the solidity. $\dagger$

[^26]Ibid. 77.

1. Let the axis AC of a circular spindle be 40 inches, and its greater diameter BL 30 inches; what is its solidity?
$20^{2} \div 15=26 \frac{2}{3}$, then $26 \frac{2}{3}+15=41 \frac{2}{3}$, the diameter of the circle. Again, $\frac{41 \frac{2}{3}-30}{2}=5 \frac{5}{6}$, the central distance.
Now $15 \div 41 \frac{2}{\mathrm{z}}=36$, the area segment corresponding to v hich is 254550 , which multiplied
 by the square of $41 \frac{2}{3}$, produces 441.92708 the area of the generating segment A B C, the half of which is 220.96354 .

Lastly, $\left(20^{3} \div 3\right)-\left(5 \frac{5}{6} \times 220.96354\right)=1377.71268$, and this multiplied by 12.5664 produces 17312.88862 cubic inches, the solidity required.
2. The axis of a circular spindle is 48 , and the middle diameter 36 ; required the solidity of the spindle? Ans. 29916.6714.

## PROBLEM XVII.

## To find the solidity of the middle frustum of a circular spindle.

## Rule.

I. Find the distance of the centre of the middle frustum, from the centre of the circle.
II. Find the area of a segment of a circle, the chord of which is equal to the length of the frustum, and height half the difference between its greatest and least diameters; to which add the rectangle of the length of the frustum and half its least diameter; the result will be the generating surface.
III. From the square of the radius subtract the square of the central distance, the square root of the remainder will give half the length of the spindle.
IV. From the square of half the length of the spindle take onethird of the square of half the length of the middle frustum, and multiply the remainder by the said half length.
V. Multiply the central distance by the generating surface, and subtract this product from the preceding; the remainder, multiplied by 6.2832 , will give the solidity.*

1. Required the solidity of the middle frustum of a circular spindle, the length DE being 40, the greatest diameter Q F 32, and the least diameter P S 24?
First, $20^{2} \div 4=100$, and $100+4=104$, the diameter of the circle.
Again, $52-16=36$, the central distance. Also, $\frac{1}{2}(32-24)=4$, and $4 \div 104=\cdot 038 \frac{8}{13}$, the area segment corresponding to which is $\cdot 009940$, which, multiplied by the square of 104 , produces 107.51104 , the area of PLQ; and $40 \times 12=480$ the area of the rectangle PDEL.

Hence $107 \cdot 51104+480=587 \cdot 51104$ the area of the generating surface P D L E.

[^27]Next $\sqrt{ }\left(52^{2}-36^{2}\right)=\sqrt{ }(1408)=8 \sqrt{ }(22)=\mathrm{B} 0$ half the length of the spindle;

$$
\text { And }\left(1408-\frac{400}{3}\right) \times 20=25493 \frac{1}{3}
$$

Then $36 \times 587.51104=21150.39744$, and
$\left(25493 \frac{1}{3}-21150 \cdot 39744\right) \times 6 \cdot 2832=27287 \cdot 5347$, the required so. lidity.
2. What is the solidity of the middle frustum P S R L of a circular spindle, whose middle diameter F Q is 36 , the diameter PS of the end 16, and its length D E 40?

Ans. $29257 \cdot 2904$.

## PROBLEM XVIII.

## To find the solidity of a spheroid.

Rule. Multiply the square of the revolving axis by the fixed axis, and this product again by 5236 for the solidity.*

1. What is the solidity of a prolate spheroid whose longer axis A B is 55 inches, and shorter axis CD 33 ?

Here $33^{2} \times 55 \times \cdot 5236=31361.022$ cubic mches, the answer.
2. What is the solidity of an oblate spheroid,
 whose longer axis is 100 feet, and shorter axis 6 ?

$$
\text { Ans. } 31416 \text { cubic feet. }
$$

3. What is the solidity of a prolate spheroid, whose axes are 40 and 50 ?

Ans. 41888.
4. What is the solidity of an oblate spheroid, whose axes are 20 and 10 ?

Ans. 2094.4.

## PROBLEM XIX.

To find the solidity of the segment of a spheroid, the base of the segment being parallel to the revolving axis of the spheroid.

## CASE I.

Rule. From three times the fixed axis, deduct twice the height of the segment, multiply the remainder by the square of the height, and that product by 5236 .

Then say, as the square of the fixed axis is to the square of the revolving axis, so is the product found above to the solidity of the spheroidal segment. $\dagger$

1. What is the content of the segment of a prolate spheroid, the height 0 C being 5 , the fixed axis 50, and the revolving axis 30 ?Sce last igure.

$$
\begin{aligned}
& 50 \times 3-5 \times 2=150-10=140 \text {; then } \\
& 140 \times 5^{2}=3500, \text { and } 3500 \times 5236=1832 \cdot 6 \text {; then } \\
& 25: 9:: 1832 \cdot 6: 659 \cdot 736, \text { the answer. } \\
& \text { CASE II. }
\end{aligned}
$$

When the base is elliptical, or perpendicular to the revolving axis.
Rule. From three times the revolving axis, take double the

* See Appendix, Demonstration 79. + rbid. 80.
height; multiply that difference by the square of the height, and the product again by 5236 .

Then as the revolving axis is to the fixed axis, so is the last product to the content.*
2. What is the content of the segment of a spheroid, whose fixed axis is 50 , revolving axis 30 , and height 6?

$$
30 \times 3-2 \times 6=90-12=78
$$

Then $78 \times 6^{2}=2808$; and $2808 \times 5236=1470 \cdot 2688$ : Then $30: 50:: 1470 \cdot 2688: 2450 \cdot 448$, the answer.
3. In a prolate spheroid, the transverse or fixed axis is 100 , the conjugate or revolving axis is 60 , and the height of the segment, whose base is parallel
 to the revolving axis, is 10: required the solidity? Ans. 5277.888
4. If the axes of a prolate spheroid be 10 and 6 , required the content of the segment, whose height is 1 , its base being parallel to the revolving axis? Ans. $5 \cdot 2^{\prime \prime} 7888$.

## PROBLEM XX.

To find the solidity of the middle zone of a spheroid, the diameter of the ends being perpendicular to the fixed axis, the middle diameter, and that of either end being given, together with the length of the zone.
Rule. To twice the square of the middle diameter, add the square of the diameter of the end; multiply the sum by the length of tho zone, and the product again by 2618 for the solidity. $\dagger$

1. What is the solidity of the middle zone of an oblate spheroid, the middle diameter being 100 , the diameter of the end 80 , and the length 36 ?
$100^{2} \times 2+80^{2}=26400$; then $26400 \times$ $36=950400$, and $950400 \times 2618=$ 248814.72 , the answer.
2. What is the solidity of the middle frustum of a spheroid, the greater diameter being 30 , the diameter of the end 18, and the length 40? Ans. $22242 \cdot 528$.


## PROBLEM XXI.

## To find the solidity of a parabolic conoid.

Rule. Multiply the square of the diameter of its base by 3927 . and that product by the height; the last product will be the solidity. $\mp$

1. What is the solidity of the parabolic conoid, whose height is 10 feet, and the diameter of its base 10 feet?
$10^{2} \times \cdot 3927=39 \cdot 27$; then $39 \cdot 27 \times 10=392 \cdot 7$, the solidity required.

[^28]2. What is the solidity of a parabolic conoid, whose height is 30 , and the diameter of its base 40 ?

Ans. $18849 \cdot 6$.
3. What is the content of the parabolic conoid, whose altitude is 40 , and the diameter of its base 12 ?

Ans. $2261 \cdot 952$.
4. Required the solidity of a parabolic conoid, whose height is 30 , and the diameter of its base 8 ?


## PROBLEM XXII.

## To find the solidity of the frustum of a parabolic conoid.

Rule. Multiply the sum of the squares of the diameters of the two ends by the height, and that product by 3927 ; the last product will be the solidity.*

1. The greater diameter of the frustum is 10 , and the less diameter 5 ; what is the solidity, the length being 12 ?
$10^{2}=100$
$5^{2}=25$
2. Then $125 \times 12=1500$, and
$1500 \times 3927=589 \cdot 05$, the solidity.

3. The greater diameter of the frustum of a parabolic conoid is 20 , the less 10 , and the height 12 ; what is the solidity? Ans. $2357 \cdot 4$.
4. The greater diameter of the frustum of a parabolic conoid is 30 , the less 10 , and the height 50 ; required the solidity? Ans. 19635 .
5. The greater diameter of the frustum of a parabolic conoid is 15 , the less 12 , and the height 8 ; required the solidity?

Ans. 1159•8408.

## PROBLEM XXIII.

## To find the solidity of a parabolic spindle.

Rule. Multiply the square of the middle diameter by $\cdot 7854$, and that product by the length; then $\frac{8}{16}$ of this product will be the solidity. $\dagger$

1. The middle diameter $C D$, of a parabolic spindle is 10 feet, and the length AB is 40 ; required its solidity?


$$
10^{2} \times \cdot 7854 \times 40=3141 \cdot 6 \text { feet }
$$

Then $\frac{8}{15} \times 3141 \cdot 6=1675.52$ feet, the answer.
2. The middle diameter CD , of a parabolic spindle is 12 feet, and the length AB is 30 ; required the solidity? Ans. $1809 \cdot 5616$.
3. The middle diameter of a parabolic spindle is 3 feet, and the length 9 feet; required its solidity?

[^29]$\dagger$ Ibid. 85.
4. The middle diameter of a parabolic spindle is 6 feet, and the length 10 ; required its solidity? Ans. 150.7968.
5. The middle diameter of a parabolic spindle is 30 feet, and the length 50 ; required its solidity?

Ans. 18849•6.

## PROBLEM XXIV.

To find the solidity of the middle frustum of a parabolic spindle.
Rule. To double the square of the middle diameter, add the square of the diameter of the end; and from the sum subtract $\frac{4}{20}$ of the square of the difference between these diameters; the remainder multiplied by the length, and that product by $\cdot 2618$, will be the solidity.*

1. In a parabolic spindle, the middle diameter of the middle frustum is 16 , the least diameter 12, and the length 20 ; required the solidity of the frustum?

Here $2 \times 16^{2}+12^{2}-\frac{4}{10} \times 4^{2}=512+144-$ $6 \cdot 4=649 \cdot 6$; hence $649 \cdot 6 \times 20 \times \cdot 2618=$
 $3401 \cdot 3056$, the solidity.
2. The bung diameter of a cask is 30 inches, the head diameter 20 inches, and the length 40 inches; required its content in ale gallons, allowing 282 cubic inches to be equal to one gallon?

$$
\text { Ans. } 80 \cdot 211 \text { gallons. }
$$

8. The bung diameter of a cask is 40 inches, the head diameter 30 inches, and the length 60 ; how many wine gallons does it contain, 231 cubic inches being equal to one gallon?

Ans. 276.08 gallons.

## PROBLEM XXV.

## Io find the solidity of a hyperbolic conoid.

Rule. To double the height of the solid add three times the transverse axis, multiply the sum by the square of the radius of the base, and that product by the height, and this last product by $\cdot 5236$; the result divided by the sum of the height and transverse axis, will give the solidity.中

1. Required the solidity of an hyperbolic conoid, whose height $\mathrm{V} m$ is 50 , the diameter A B 103.923048, and the transverse axis VE 100 ?
Here $(2 \times 50+3 \times 100) \times \frac{(103.923048)^{2}}{2}=400 \times$
 $2700=1080000$; and $\frac{1080000 \times 50 \times 5236}{150}=188496$, the solidity.
2. What is the content of an hyperboloid, whose altitude is 10 , the radius of its base 12, and the transverse 30 ?

## PROBLEM XXVI.

## To find the solidity of the frustum of an hyperboloid, or hyperbolic conoid.

Rule. To four times the square of the middle diameter, add the sum of the squares of the greatest and least diameters; multiply the result by the altitude, and that product by $\cdot 1309$, for the solidity.*

1. Required the solidity of the frustum ACEHDB of an hyperbolic conoid, whose greatest diameter A B is 96 , least diameter E H 54 , middle diameter CD $76 \cdot 4264352$, and the altitude $m n 25$ ?

Here $4 \mathrm{CD}^{2}+\mathrm{AB}^{2}+\mathrm{EH}^{2}=(5841 \times 4)+$ $9216+2916=35496$, and $35496 \times 25 \times 1309=$ $116160 \cdot 66$, the answer.

2. What is the solidity of an hyperboloidal cask, its bung diameter being 32 inches, its head diameter 24 , and the diameter in the middle between the bung and head $\frac{8}{5} \sqrt{ } 310$, and its length 40 inches?

Ans. $24998 \cdot 69994216$ inches.

## PROBLEM XXVII.

To find the solidity of a frustum of an elliptical spindle, or any other solid formed by the revolution of a conic section about an axis.

Rule. Add together the squares of the greatest and least diameters, and the square of double the diameter in the middle between the two; multiply the sum by the length, and the last product by • 1309 for the solidity.t

1. What is the content of the middle frustum CDIII of any spindle, the length $O P$ being 40 , the greatest, or middle Niameter E F 32, the least, or diameter at either end C D 24, and the diameter G K $30 \cdot 157568$ ?


Here $32^{2}+(2 \times 30 \cdot 157568)^{2}+24^{2}=5237.89$ sum;

$$
\text { Then } 5237.89 \times 40=209515 \cdot 6 \text {, and }
$$

$$
209515 \cdot 6 \times \cdot 1309=27425 \cdot 7 \text {, the answer. }
$$

2. What is the content of the segment of any spindle, the length being 20 , the greatest diameter 10 , the least diameter at either end 5 , aud the diameter in the middle between these 8 ? Ans. $997 \cdot 458$.

## PROBLEM XXVIII.

## I'o find the solidity of a circular ring.

Rule. Tu the thickness of the ring add the inner diameter; multiply the sum by the square of the thickness, and the product by 2.4674 , for the solidity.*

1. The thickness of a cylindrical ring is 2 inches, and the diameter CD 5 inches; required its solidity?
$(2+5) \times 4=28$; then $28 \times 2.4674=$ 69.0872 cubic inches, the answer.
2. Required the solidity of an iron ring whose axis forms the circumference of a circle; the diameter of a section of the ring 2 inches, and the inner diameter, from side
 to side, 18 inches?

Ans. 197.3925 cubic inches.
3. The thickness of a cylindrical ring is 7 inches, and the inner diameter 20 inches; required its solidity? Ans. $3264-3702$.
4. What is the solidity of a circular ring, whose thickness is 2 inches, and its diameter 12 inches? Ans. $138 \cdot 1744$ cubic inches.

* Sto Apperdix. Domonstration 00 .


# THE FIVE REGULAR BODIES. 

## SECTION V.

## DEFINTTIONS.

A regular body is a solid contained under a certain number of similar and equal plane figures.

Only five regular bodies can possibly be formed. Because it is proved in Solid Geometry that only three kinds of equilateral and equiangular plane figures joined together can make a solid angle.

1. The tetraedron, or equilateral pyramid, is a solid having four triangular faces.*

2. The hexaedron, or cube, is a solid having six square faces.

3. The octaedron is a regular solid having eight triangular faces.


[^30]4. The dodecaedron has twelve pentagonal faces.

5. The icosaedron has twenty triangular faces.


PROBLEM I.
To find the solidity of a tetracdron.
Rule I. Multiply $\frac{1}{12}$ of the cube of the lineal side by the square root of 2 , and the product will be the solidity.

Rule II. Multiply the cube of the length of a side of the body by the tabular solidity, and the product will be the solidity of the body.* This rule is general for all the regular bodies.

1. If the side of each face of a tetraedron be $\mathbf{1}$; required its solidity?

Here $\frac{1}{12} \times 1^{3} \times \sqrt{ } 2=\frac{1}{12} \times \sqrt{ } 2=\cdot 11785113$, the solidity.
2. The side of a tetraedron is 12 ; what is its solidity? Ans. 203•6467.


## PROBLEM II.

To find the solidity of a hexaedron, or a cube.
Rule. Cube the side for its solidity. $\dagger$,

1. If the linear side of a hexaedron be 3 , what is its content? Ans. $3 \times 3 \times 3=27$.

## PROBLEM III.

> To find the solidity of an octaedron.

Rule. Multiply the cube of the side by the square root of 2, and $\$$ of the product will be the content. $\ddagger$

1. What is the solidity of an octaedron, when the linear side is 1 ?

$$
1^{3} \times \sqrt{ } 2 \times \frac{1}{3}=\frac{1}{3} \sqrt{ } 2=\cdot 4714045 .
$$

2. What is the solidity of the octaedron, whose linear side is 2? Ans. 3.7712.


## PROBLEM IV.

## To finu the solidity of a dodecaedron.

Rule. To 21 times the square root of 5 add 47, and divide the sum by 40 ; multiply the root of the quotient by 5 times the cube of the lineal side, and the product will be the solidity.*

1. If the lineal side of the dodecaedron be 1 , what is its solidity?

Here $A=1$, consequently $5 A^{3} \sqrt{ } \frac{47 \times 21 \sqrt{ } 5}{40}$ $=7 \cdot 66311896$, the content.
2. The side of a regular dodecaedron is $\mathbf{1 2}$ inches; how many cubic inches does it contain?

Ans. $13241 \cdot 8694592$.


## PROBLEM V.

To find the solidity of an icosaedron.

Rule. To 7 add three times the square root of 5 , take half the sum, multiply the square root of this half sum by $\frac{5}{6}$ of the cube of the lineal side, and the product will be the solidity. +

1. What is the solidity of an icosaedron, whose lineal side is 1 ?

Let the side be denoted by $\mathbf{A}$. Then $\mathbf{A}=\mathbf{1}$, and consequently
$\frac{5}{6} A^{3} \sqrt{ } \frac{7+3 \sqrt{ } 5}{2}=\frac{5}{6} \sqrt{ } \frac{7+3 \sqrt{ } 5}{2}=2 \cdot 18169499$, the content.

2. What is the solidity of an icosaedron, whose lineal side is 12 fuet? Ans. $3769 \cdot 9689$ feet.

[^31]* Sce Appendix Demonstration 93.
$\dagger$ Ibid. 91


## TABLE III.

Shewing the solidity of the five regular bodies, the length in each being 1.

| No of <br> sides. | Names. |  | Solidity. |
| ---: | :--- | :--- | ---: |
| 4 | Tetraedron | $\ldots$ | -1178511 |
| 6 | Hexaedron | $\ldots$ | 1.0000000 |
| 8 | Octaedron | $\ldots$ | .4714045 |
| 20 | Icosaedron | $\ldots$ | 2.1816950 |
| 12 | Dodecaedron | $\ldots$ | 7.6631189 |

## PROBLEM VI.

## To find the surface* of a tetraedron.

Rule I. Multiply the square of the linear side by the square root of 3 , and the product will be the whole surface. $t$

Rule II. Multiply the square of the length of a side of the body, by the tabular area corresponding to the figure, and the product will be the surface of the body. This is a general rule for finding the surfaces of the regular bodies.

1. If the side of a tetraedron be 1 , what is its surface?

Here, $1^{2} \times \sqrt{ } 3=\sqrt{ } 3=1 \cdot 7320508=$ the whole surface.
2. The side of a tetraedron is 12 ; what is its surface?

$$
\text { Ans. } 249 \cdot 415315
$$

## PROBLEM VII.

To find the surface of a hexaedron, or cube.
Rule. Square the side and multiply it by 6 , and the product will be the surface. $\ddagger$

1. If the side be 1 , what is the surface of a hexaedron?
$1^{2} \times 6=6$ the whole surface.
2. If the side be 4 , what is the surface of a hexaedron?

## PROBLEM VIII.

## To find the surface of an octaedron.

Rule. Multiply the square of the side by the square root of 3 , and double the product will be the surface.§

[^32]1. If the side of an octacdron be 1 , what is its surface? $2 \times 1^{2} \sqrt{ } 3=2 \sqrt{ } 3=3 \cdot 4641016=$ the whole surface.
2. If the side of an octaedron be 12 , what is its superficies?

Ans. 498-8306304.
3. If the side of an octaedron be 4 , what is its surface?

Ans. $55 \cdot 4256256$.

## PROBLEM IX.

## To find the superficies of a dodecaedron.

Rule. To 1 add $\frac{2}{6}$ of the root of 5 ; multiply the root of the sum by 15 times the square of the lineal side, and the product will be the surface.*

1. If the lineal side be 1 , what is the surface of a regular dodecaedron?

Here $1^{2} \times 15 \sqrt{ }\left(1+\frac{2}{6} \sqrt{ } 5\right)=15 \sqrt{ }\left(1+\frac{2}{5} \sqrt{ } 5\right)=20.645728807$, the surace.
2. What is the surface of a dodecaedron, whose lineal side is 2 ?

$$
\text { Ans. } 82 \cdot 58292 .
$$

## PROBLEM X.

## To find the superficies of an icosaedron.

Rule. Multiply five times the square of the lineal side by the equare root of 3 , and the product will be the surface. $\dagger$

1. The side of an icosaedron is 1 , what is its surface?

$$
5 \times 1^{2} \times \sqrt{ } 3=5 \sqrt{3}=8 \cdot 66025403
$$

2. What is the surface of an icosaedron whose side is 2 ?

$$
\text { Ans. } 34 \cdot 641 .
$$

3. What is the surface of an icosaedron whose side is 3 ?

Ans. 77.9423.
Note.-In finding the superficial content of the regular bodies, it is particularly recommended to employ the general rule given in Problem VI. in practice, in preference to any other. The particular rules given for each solid ar6 introduced merely to find the tabular numbers by which the pupil is to work.

From the examples given in the preceding rules, in which the lineal side of each regular solid is 1 , the following tabular numbers may be collected.

> TABLE IV.

Shewing the surfaces of the five regular bodies, when the linear side is 1

| Number <br> of sides. | Names. |  | Surface. |
| :---: | :--- | :--- | ---: |
|  | Tetraedron | $\ldots$ | $1 \cdot 7320508$ |
| 4 | Hexaedron | $\ldots$ | $6 \cdot 0000000$ |
| 6 | Octaedron | $\ldots$ | 3.4641016 |
| 8 | Dodecaedron | $\ldots$ | 20.6457288 |
| 12 | Icosaedron | $\ldots$ | 8.6602540 |
| 20 |  |  |  |

[^33]
## SURFACES OF SOLIDS.

## SECTION VI.

## PROBLEM I.

## To find the surface of a prism.

Rule. Multiply the perimeter of the end of the solid by its length, to the product add the area of the two ends, and the sum will be the surface.*

1. If the side HI of the pentagon be 25 feet, and height I D 10, what is its surface?
$25 \times 5=125$, the perimeter;
Then $125 \times 10=1250=$ the upright surface;
$25^{2} \times 1 \cdot 720477=1075 \cdot 298125=$ the area of one end;
And $1075 \cdot 298125 \times 2=2150 \cdot 596250=$ the area of both ends;

Then $2150 \cdot 596250+1250=3400 \cdot 59625=$ the entire surface.
2. If the side of a cubical piece of timber be 3 feet 6 inches, what is the upright surface and whole superficial content?


Ans. $\left\{\begin{array}{l}49 \text { feet upright surface. } \\ 73 \text { feet } 6 \text { in. whole superficial content. }\end{array}\right.$
3. If a stone in the form of a parallelopipedon be 12 feet 9 inches long, 2 feet 3 inches drep, and 4 feet 8 inches broad, what is the upright surface and whole superficial content?

$$
\text { Ans. }\left\{\begin{array}{l}
176 \text { feet } 4 \mathrm{in.} 6 \mathrm{sec} \text {. upright surface. } \\
197 \text { feet } 4 \mathrm{in.} 6 \text { sec. whole sup. content. }
\end{array}\right.
$$

## PROBLEM II.

To find the surface of a pyramid.
Rule. Multiply the slant height by half the circumference of the base, and the product will be the surface of the sides, to which add the area of the base for the whole surface. $\dagger$

Note. -The slant height of a pyramid is the perpendicular distance from the vertex to the middle of one of the sides, and the perpendicular height is a straight line drawn from the vertex to the middle of the base.

1. The slant height of a triangular pyramid is 10 feet, and each side of the base is 1 ; what is its surface?

Half circumference $=\frac{3}{2}$
$\begin{array}{ll}\text { Slant height } & =10 \\ \text { Upright surface } & =15\end{array}$
Area of the base $=-433013$
The entire surface $=15 \cdot 433013$

2. The perpendicular height of a heptagonal pyramid is 13.5 feet, and each side of the base 15 inches; required its surface? Ans. $65 \cdot 0128$ feet.

## PROBLEM III.

## To find the surface of a cone.

Rule. Multiply the slant height by half the circumference of the base, and the product, with the area of the base, will be the whole surface.*

1. What is the surface of a cone whose side is 20 , and the circumference of its base 9 ?

Here $20 \times \frac{9}{2}=90=$ the convex surface.
$92 \times \cdot 07958=6 \cdot 44598=$ the area of the base.
Then $90+6 \cdot 44598=96 \cdot 44598=$ the whole surface.
2. The perpendicular height of a cone is 10.5 feet, and the circumference of its base is 9 feet; what is its superficies?
 Ans. $54 \cdot 1336$ feet.

## PROBLEM IV.

To find the superficies of the frustum of a right, regular pyramid.
Rule. Add the perimeters of the two ends together, and multiply half the sum by the slant height, the product will be the upright surface; to which add the areas of both ends, and the sum will be the whole surface. $\dagger$

1. What is the superficies of the frustum of a square pyramid, each side of the greater base A B being 10 inches, and each side of the less base CD 4 inches, and slant height 20 inches?

Here $10 \times 4=40$ the perimeter of the greater base.
And $4 \times 4=16$ the perimeter of the less end.
Sum $\overline{56}$, the half of which is 28 .


Then $28 \times 20=560=$ the upright surface.
$10 \times 10=100=$ the area of the greater base.
$4 \times 4=16=$ the area of the less end.
Hence $560+100+16=676=$ the whole surface.
2. What is the superficies of the frustum of an octagonal pyramid, each side of the greater base being 9 inches, each side of the less baso 5 inches, and the height 10.5 feet?

[^34]- Ibid. 103.


## PROBLEM V.

## To find the superficies of the frustum of a cone.

Rule. Add the perimeters of both ends together, and multiply half the sum by the slant height, to which add the areas of both ends, for the whole superficies.*

1. If the diameters of the two ends CD and AB are 7 and 3 , and the slant height D B 9, what is the whole surface of the frustum ABCD ?

$$
\begin{aligned}
& \frac{7+3}{2} \times 3 \cdot 1416 \times 9=141.372, \text { the convex surface. D } \\
& 7 \times 7 \times 7854=38 \cdot 4846, \text { the area of the base C D } \\
& 3 \times 3 \times 7854=7.0686, \text { the area of the end A B. }
\end{aligned}
$$



Then $141 \cdot 372+45 \cdot 5532=186 \cdot 9252=$ the whole surface of the frustum.
2. What is the superficies of the frustum of a cone, whose greater diameter is 18 inches, and less diameter 9 inches, and the slant height 171.0592 inches?

Ans. 7572.981.

## PROBLEM VI.

## To find the superficies of a wedge.

Rule. Find the area of the back, which is a rightoangled paralLelogram; find the areas of both ends, which are triangles; and also of both sides, which are trapezoids; all these areas added together will evidently be the whole surface. $\dagger$

1. The back of a wedge is 10 inches long, and 2 inches broad, each of its faces is 10 inches from the edge to the back; required its whole surface?
$10 \times 2=20=$ the area of the back. $10 \times 10 \times 2=200$ the areas of both faces. $\sqrt{ }\left(\mathrm{AE}^{2}-\mathrm{E} x^{2}\right)=\sqrt{ }(100-1)=9 \cdot 949=\mathrm{A} x$; then $9 \cdot 949 \times 2=19 \cdot 898=$ areas of both ends.

Hence $200+20+19 \cdot 898=239 \cdot 898=$ the whole surface of the wedge.

2. The back of a wedge is 20 inches long, and 2 inches broad; each of its faces is 10 iuches from the back to the edge; what is its whole surface? Ans. $459 \cdot 898$.

## PROBLEM VII.

## To find the area of the frustum of a wedge.

Rule. Find the areas of the back and top sections; of the two faces; and of the two ends; the sums of all the separate results will evidently be the whole surface.

[^35]$\dagger$ Tbid. 105.

1. The length and breadth of the back are 10 and 2 inches, the length and breadth of the upper section are 10 and 1 inches, the length of the edge from the back to the upper section is 10 inches; required the whole surface? $10 \times 2=20=$ the area of the back.
$10 \times 1=10=$ the area of the upper section. $10 \times 10 \times 2=200=$ the areas of both faces.
$\frac{2-1}{2}=\frac{1}{2}=\cdot 5$, and $\sqrt{ }(100-\cdot 25)=9 \cdot 98=\mathrm{B} y$. Then $(2+1) \times 9.98=29.94=$ areas of both ends.
 Hence $20+10+200+29 \cdot 94=259 \cdot 94$ inches, the answer.
2. The length and breadth of the back are 10 and 4 , the length and breadth of the upper section are 5 and 2, and the length of each of the faces is 20 ; required the whole superficies? Ans. $470 \cdot 78$.

## PROBLEM VIII.

## To find the surface of a globe or sphere.

Rule. Multiply the diameter of the sphere by its circumference, and the product will be its convex surface.*

1. What is the surface of a globe, whose diameter is 24 inches? $24 \times 3 \cdot 1416=75^{\cdot} \cdot 3984$, the circumference : $75 \cdot 3984 \times 24=1809 \cdot 5616$ inches, the answer.
2. What is the surface of the earth, its diameter being $7957 \frac{3}{4}$, and circumference 25000 miles ? Ans. 198943750 square miles.

## PROBLEM IX.

To find the convex surface of any segment, or zone of a sphere.
Rule. Multiply the circumference of the whole sphere by the neight of the segment, or zone, and the product will be the convex surface. $\dagger$

1. If the diameter of the earth be 7970 miles, the height of the frigid zone will be 252.361283 miles, what is its surface?

Here $7970 \times 3 \cdot 1416=$ the circumference; then $7970 \times 3 \cdot 1416 \times 252 \cdot 361283=6318761 \cdot 107182216$ miles.
2. If the diameter of the earth be 7970 miles, the height of the temperate zone will be $2143 \cdot 6235535$ miles; what is its surface? Ans. $53673229 \cdot 812734532$ miles.
3. If the diameter of the earth be 7970 miles, the height of the torrid zone will be 3178.030327 miles; what is its surface?

$$
\text { Ans. } 79573277 \cdot 600166504 \text { miles. }
$$

Note.-By adding the surfaces of both frigid zones and both temperate zones, to the surface of the torrid zone, the sum $199557259 \cdot 44$, is the surface of the earth in square miles.
4. The diameter of a sphere is 3 , the height of the segment 1 ; what is its convex surface?

Ans. $9 \cdot 4248$.
5. The circumference of a sphere is 33 , the height of the segment ${ }^{13} 4$; what is its convex surface?

Ans. 132.

## PROBLEM X.

## To find the surface of a cylinder.

Rule. Multiply the circumference by the length, and the product will be the convex surface; to which add the area of the two ends, and the sum will be the surface of the entire solid.*

1. What is the entire surface of a cylinder, whose length is 10 feet, and its diameter 5 feet?

$$
3 \cdot 1416
$$

> 15.7080 , then $15.708 \times 10=157.08$ the convex surface. $5 \times 5 \times \cdot 7854=$ the area of the base; then
> $2 \times 5 \times 5 \times \cdot 7854=50 \times \cdot 7854=39 \cdot 2700=$ the area of both bases; then $157 \cdot 08+39 \cdot 27=196 \cdot 35$, the answer.
2. Required the superficial content of a cylinder, whose diameter is 21.5 inches, and height 16 feet? Ans. $95 \cdot 1$ feet.
3. What is the surface of a cylinder whose diameter is 20.75 inches, and its length 55 inches?

$$
\text { Ans. } 29.595 \text { feet. }
$$

## PROBLEM XI.

## T'o find the superficies of a circular cylinder.

Rule. Add the inner diameter to the thickness of the ring, multiply the sum by the thickness, and that product by 9.8696 for the superficies. $\dagger$

1. The thickness AC of a cylindrical ring is 2 inches, the inner diameter CD 5 inches; required its superficial content?
Here $(2+5) \times 2=14$; then $14 \times 9.8696=138.1744$ square inches.

## PROBLEM XII.

## To find the surface of a parallelopipedon.

Rule. Find the area of the sides and ends, and their sum will be the surface.

1. What is the surface of a parallelopipedon, whose length is 10 feet, breadth 4, and depth 2 ? Ans. 136 feet.
$10 \times 4=40=$ the area of one face.
$10 \times 4=40=$ the area of its opposite face.
$10 \times 2=20=$ the area of one face.
$10 \times 2=20=$ the area of its opposite face.
$4 \times 2=8=$ the area of one end.
$4 \times 2=8=$ the area of its opposite end.

$$
\overline{136}=\text { the surface of the whole solid. }
$$

2. The length of a parallelopipedon is 5 , breadth 4 , and depth 3 ; what is its surface?

# MENSURATION OF TIMBER AND OF ARTIFICERS' WORK. 

## SECTION VII.

## DESCRIPTION OF THE CARPENTERS' RULE.

This instrument is sometimes called the sliding rule, and is usea in measuring timber and artificers' work. By it dimensions are taken, and contents computed.

It consists of two equal pieces of box-wood, each one foot long, connected by a folding joint..

One face of the rule is divided into inches and half-quarters or eighths. On the same side or face are several plane scales divided by diagonal lines into twelfths; these are chiefly used in planning dimensions which are taken in feet and inches. The edge of the rule is divided decimally; that is, each foot is divided into 10 equal parts, and each of these again into 10 equal parts. By means of this last scale, dimensions are taken in feet, tenths, and hundredths; and then multiplied as common decimal numbers.

In one of these equal pieces, there is a slider on which are marked the two letters $\mathrm{B}, \mathrm{C}$; on the same face are marked the letters $\mathrm{A}, \mathrm{D}$. The same numbers serve for both these two middle lines, the one being above the numbers, and the other below.

Three of these lines, viz., A, B, C, are called double lines, as they proceed from 1 to 10 twice over. These three lines are exactly alike both in division and numbers, and are numbered from the left hand towards the right, $1,2,3,4,5,6,7,8,9$ to 1 , which stands in the middle ; the numbers then go on, $2,3,4,5,6,7,8,9$ to 10 , which stands at the right-band end of the rule.

These four lines are logarithmic ones; the lower line D, is a single one, proceeding from 4 to 40 , and is called the girt line, from its use in finding the content of timber.

Upon it are also marked W G at $17 \cdot 15$, A G at $18 \cdot 95$, and I G at 18.8 . These are the wine, ale, and imperial gauge points.

On this face is a table of the value of a load, or 50 cubic feet, of timber, at all prices from 6 pence to 2 shillings per foot.

To ascertain the values of the figures on the rule, which have no determinate value of their own, but depend upon the value set on the unit at the left hand of that part of the rule marked $1,2,3, \& c$. ; if
the first unit be called 1 , the 1 in the middle will be 10 , the other figures that follow will be $20,30,40, \& c$. , and the 10 at the righthand end will be 100 . If the left-hand unit be called 10 , the 1 in the middle will be 100, and the following figures will be 200,300 . $400,500, \& c$. ; and the 10 at the right-hand end will be 1000 . I. the 1 at the left-hand end be called 100 , the middle 1 will be 1000 , and the following figures will be $2000,3000,4000, \& c$. , and the 10 at the right hand will be 10,000 . From this it appears that the values of all the figures depend upon the value set on the first unit.

The use of the double line $\mathrm{A}, \mathrm{B}$, is to find a fourth proportional, and also to find the areas of plane figures.

The use of the several lines described here is best learned in practice.

If the rule be unfolded, and the slider moved out of the grove, the back part of it will be seen divided like the edge of the rule, all measuring 3 feet in length.

Some rules have other scales and tables delineated upon them; such as a table of board measure, one of timber measure, another for shewing what length for any breadth will make a square foot. There is also a line shewing what length for any thickness will make a solid foot.

## THE USE OF THE SLIDING RULE

## PROBLEM 1.

## To multiply numbers together.

Set 1 on B to the multiplier on A ; then against the multiplicand on $B$, stands the product on $A$.

1. Multiply 12 and 18 together.

Set 1 on B , to 12 on A; then against 18 on B , stands the product 216 on A.
2. Multiply 36 by 22.

Set 1 on $B$, to 36 on $A$; then as 22 on $B$ goes beyond the rule, look for 2.2 on B, and against it on A stands 79.2 ; but as the real multiplier was divided by 10 , the product $79 \cdot 2$ must be multiplied by 10 , which is effected by taking away the decimal point, leaving the product 792.

## PROBLEM II.

## To divide one number by another.

Set the divisor on $A$, to 1 on $B$; then against the dividend on $A_{3}$ stands the quotient on $B$.

1. Divide 11 into 330.

Set the divisor 11 on $A$, to 1 on $B$; then against the dividend 330 on A, stands the quotient 30 on B
2. Divide 7680 by 24 .

Set 24 on A, to 1 on B; then because 7680 goes beyond the rule on A, look for 768 (the tenth of 7680 ), on A, and against it stands 32 on B ; but as the tenth of the dividend was taken that the number should fall within the compass of the scale A, the quotient 32 must be multiplied by 10 , which gives 320 for the answer.

## PROBLEM III.

To square any number.
Set 1 upon C, to 10 upon D; then if you call the 10 upon D 1, the 1 on C will be 10 ; if you call the 10 on $\mathrm{D}, 10$, then the 1 on C will be 100 ; if you call the 10 on $\mathrm{D}, 100$, then the 1 on C will be 1000 ; this being understood, you will observe that against every number on D , stands its square on C .

1. What are the squares of $25,30,12$, and 20 ?

Proceeding according to the above direction, 625 stands against 25. 900 against 30,144 against 12, 400 against 20.

## PROBLEM IV.

## To extract the square root of a number.

Set 1 or $100, \& \mathrm{c}$., on C , to 1 or $10, \& \mathrm{Ec}$., on D ; then against every number found on C , stands its root on D .

1. What are the square roots of 529 and 1600 ?

Proceeding according to the above directions, opposite 529 stands 23 ; opposite 1600 stands 40 , and so on.

## PROBLEM V.

To find a mean proportional between two numbers, as 9 and 25.
Set the number 9 on C, to the same 9 on D; then against 25 on $\Sigma$, stands 15 on D , the required mean proportional.
The reason of this may be seen from the proportion, viz., $\mathrm{S}: 15:: 15: 25$.

1. What is the mean proportional between 29 and 430 ?

Set one number 29 on C , to the same on D ; then against the other number 430 on C, stands 112 on D, which is the mean proportional, nearly.

## PROBLEM VI.

To find a third proportional to two numbers, as 21 and 32.
Set the first number 21 , on $\mathrm{B}_{2}$ to the second, 32 , on A ; then against the second, 32 , on $B$, stands 488 on $A$, which is the required third proportional.

## PROBLEM VII.

To find a fourth proportional to three given numbers.
Set the first term on B , to the second on A ; then against the third term on B, stands the fourth on A.

If either of the middle numbers fall beyond the line, take one-tenth part of that number, and increase the fourth number found, ten times.

1. Find a fourth proportional to 12,28 , and 114.

Sct the first term, 12 , on $B$, to the second term, 28 , on $A$; then against the third term, 114 , on $B$, stands 266 on $A$, which is the answer.

## TIMBER MEASURE.

## PROBLEM I.

## To find the superficial content of a board or plank.

Rule. Multiply the length by the breadth, and the product will be the area.
Note. When the plank is broader at one end than at the other, add both ends together, and take half the sum for a mean breadth.

## BY THE CARPENTERS' RULE.

Set 12 on $B$, to the breadth in inches on $A$; then against the length in feet, on $\mathbf{B}$, will be found the superficies on $\mathbf{A}$, in feet.

1. If a board be 12 feet 6 inches long, and 2 feet 3 inches broad, how many feet are contained in it?


$$
\text { As } 12 \text { on } \mathrm{B}: 27 \text { on } \mathrm{A}:: 12 \cdot 5 \text { on } \mathrm{B}: 28 \cdot 125 \text { on } \mathrm{A} \text {. }
$$

2. What is the value of a board whose length is 8 feet 6 inches, and breadth 1 foot 3 inches, at $5 d$. per foot? Ans. $4 s .5 d$.
3. What is the value of a board whose length is 12 feet 9 inches, and breadth 1 foot 3 inches, at $5 d$. per foot?
4. What is the value of a plank whose breadth at one end is 2 feet, and at the other end 4 feet, at 6 d . per foot, the length being 12 feet? Ans. $18 s$.
5. How many square feet in a board, whose breadth at one end is 15 inches, and at the other 17 inches, the length being 6 feet?

$$
\text { Ans. } 8 .
$$

6. How many square feet in a plank, whose length is 20 feet, and mean breadth 3 feet 3 inches?

Ans. 65.

## PROBLEM II.

To find the solid content of squared or four sidel timber.
Rule. Take half the sum of the breadth and depth in the middle, (that is, the quarter girt,) square this half sum, and multiply it by the length for the solid content.*

[^36]BY THE CARPENTERS' RULE.
As 12 on D : length on C :: quarter girt on D : the solid content on C .

1. If a piece of squared timber be 3 feet 9 inches broad, 2 feet 7 inches deep, and 20 feet long; how many solid feet are contained therein?

$$
\begin{aligned}
& 3.9 \\
& 2.7 \\
& \overline{2) 6.4} \\
& \text { 3.2 quarter girt. } \\
& 3.2 \\
& 9.6 \\
& 6.4 \\
& 10.0 .4 \text { square of the quarter girt. } \\
& 20 \text { length of the piece. } \\
& 200 \text {. } 6 \text {. } 8 \text { solid content. }
\end{aligned}
$$

## BY THE CARPENTERS' RULE.

$$
\text { As } 12 \text { on D : } 20 \text { on C }:: 38 \text { on D : } 200 \frac{1}{2} \text { on C. }
$$

2. A squared piece of timber is 15 inches broad, 15 inches deep, and 18 feet long; how many solid feet does it contain?

Ans. $28 \frac{1}{8}$ feet, which is the accurate content, as the breadth and depth are equal.
3. What is the solid content of a piece of timber, whose breadth is 16 inches, depth 12 inches, and length 12 feet? Ans. 16 feet.

Rule II. Multiply the breadth in the middle by the depth in the middle, and that product by the length, for the solidity.*
4. The length of a piece of timber is 18 feet 6 inches ; the breadths at the greater and less end 1 foot 6 inches, and 1 foot 3 inches, and the thickness at the greater and less end 1 foot 3 inches, and 1 foot; what is the solid content?


[^37]
## BY THE SLIDING RULE.

$$
\begin{gathered}
\text { B A A } \\
\text { As } \quad \text { A } \\
\text { A }: 13 \frac{1}{2}:: 16 \frac{1}{2}: 223 \text { the mean square. } \\
\text { C D } \\
\text { As } 1: 1 \\
\text { C }
\end{gathered}
$$

As $18 \frac{1}{2}: 12:: 14 \cdot 9: 28 \cdot 6$ the content.

Note.-When the piece to be measured tapers regularly from one end to the other, either take the mean breadth and depth in the middle, or take the dimensions at both ends, and half their sum for the mean dimension. This, however, though very easy in practice, is but a very imperfect approzimation.

When the piece to be measured does not taper regularly, but is thick in some parts and small in others, in this case take several dimensions; add them all together, and divide their sum by the number of dimensions so taken and use the quotient as the mean dimension.

Rule III. Multiply the sum of the breadths of the two ends by the sum of the depths, to which add the product of the breadth and depth of each end; one-sixth of this sum, multiplied by the length, will give the exact solidity of any piece of squared timber tapering regularly.*
5. How many feet in a tree, whose ends are rectangles, the length and breadth of one being 14 and 12 inches, and the corresponding dimensions of the other 6 and 4 inches; also the length $30 \frac{1}{2}$ feet?

| 14 | 12 | $12 \times 14=168$ |
| ---: | ---: | ---: |
| $\frac{6}{20}$ | $\frac{4}{16}$ | $20 \times 16=320$ |

$$
\overline{512} \text { square inches }=\frac{32}{9} \text { square feet. }
$$

Then $\frac{1}{6} \times \frac{32}{9} \times 30 \frac{1}{2}=18 \frac{2}{27}$ feet, the solidity.
6. How many solid inches in a mahogany plank, the length and breadth of one end being $91 \frac{1}{2}$ and 55 inches, the length and breadth of the other end 41 and $29 \frac{1}{2}$ inches, and the length of the plank $47 \frac{1}{4}$ inches? Ans. 126340.59375 cubic inches.

## PROBLEM III.

Given the breadth of a rectangular plank in inches, to find how much in length will make a foot, or any other required quantity.
Rule. Divide 144, or the area to be cut off, by the breadth in inches, and the quotient will be the length in inches.

The Carpenters' Rule is furnished with a scale which answers the

[^38]purpose of this rule. It is called a Table of Board Measure, and is in the following form:-

| 0 | 0 | 0 | 0 | 5 | 0 | $8 \frac{1}{2}$ | 6 | Inches. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 6 | 4 | 3 | 2 | 2 | 1 | 1 | Feet. |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Breadth. |

If the breadth be 1 inch, the length standing against it is 12 feet; if the breadth be 2 inches, the length standing against it is 6 feet; if the breadth be 5 inches, the length is 2 feet 5 inches, \&c.

When the breadth goes beyond the limits of the table on the rule, it must be shut, and then you are to look for the breadth in the line of board measure, which runs along the rule from the table of board measure, and over against it on the opposite side, in the scale of inches, will be found the length required. For example, if the breadth be 9 inches, you will find the length against it to be 16 inches; if the breadth be 11 inches, the length will be found to be a little above 13 inches.

1. If a board be 6 inches broad, what length of it will make a square foot?

Ans. 2 feet.
2. If a board be 8 inches broad, what length of it will make 4 square feet?
3. If a board be 16 inches broad, what length of it will make 7 square feet?

When the board is broader at one end than at the other, proceed according to the following

Rule. To the square of the product of the length, and narrow end, add twice the continual product of tiese quantities, viz., the length, the difference between the breadths of the ends, and the area of the part required to be cut off; extract the square root of the sum ; from the result deduct the product of the length and narrow end, and divide the remainder by the difference between the breadths of the ends.*

If it were required to cut off 60 square inches from the smaller end of a board, A D being 3 inches, CE 6 inches, and A B 20 inches.

Here $\mathrm{A} x=\frac{1}{2 \mathrm{BC}}\left(\sqrt{ }\left\{(\mathrm{AB} \times \mathrm{A} D)^{2}+\right.\right.$
$4 \mathrm{BC} \times \mathrm{AB} \times 60\}-\mathrm{AB} \times \mathrm{AD})=\frac{1}{3}$
 $\left(\sqrt{ }\left\{(20 \times 3)^{2}+6 \times 20 \times 60\right\}-20 \times 3\right)=$ $14 \cdot 64$, the length required.

## PROBLEM IV.

To find how much in length will make a solid foot, or any other required quantity, of squared timber, of equal dimensions from end to end.
Rule. Divide 1728, the solid inches in a foot or the solidity to be cut off, by the area of the end in inches, and the quotient will be the end in inches.

[^39]1. If a piece of timber be 10 inches square, how much in length will make a solid foot?
$10 \times 10=100$ the area of the end; then $1728 \div 100=17 \cdot 28$. Ans.
2. If a piece of timber be 20 inches broad, and 10 inches deep, how much of it will make a solid foot? Ans. $8 \frac{16}{25}$ inches.
3. If a piece of timber be 9 inches broad, and 6 inches deep, how much of it will make 3 solid feet? Ans. 8 feet.
On some Carpenters' Rules, there is a table to answer the purpose of the last rule; it is called a Table of Timber, and is in the following form:-

| 0 | 0 | 0 | 0 | 0 | 0 | 11 | 3 | 1 | Inches. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 144 | 36 | 16 | 9 | 5 | 4 | 2 | 2 | 1 | Feet. |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Side of square. |

## PROBLEM V.

To find the solidity of round or unsquared timber.
Rule I. Gird the piece of timber to be measured round the middle with a string, take one-fourth part of the girth, and square it, an multiply this square by the length for the solidity.

- BY THE SLIDING RULE.

As the length on $\mathrm{C}: 12$ or 10 on $\mathrm{D}::$ quarter girt; in 12 ths or 10ths on D: content on C.
Note - When the tree is very irregular, divide it into several lengths and find the solidity of each part separately; or add all the girths together, and divide the sum by the number of them.

1. Let the length of a piece of round timber be 9 feet 6 inches, and its mean quarter girt 42 inches; what is its content?

| 3.5 quarter girt. | 3. 6 quarter girt. |
| :---: | :---: |
| $3.5$ | 3.6 |
| $\overline{12 \cdot 25}$ | 10.6 |
| $9 \cdot 5$ length. | 1.9 |
| $\overline{116.375}$ content. | 12.3 |
|  | 9.6 length. |
|  | 110.3 |
|  | 6.1 .6 |
|  | 116.4 .6 content. |

BY THE SLIDING RULE.

$$
\text { As } 9 \cdot 5 \text { on C }: 10 \text { on D }:: 35 \text { on D }: 116 \frac{1}{3} \text { on C ; }
$$

$$
\text { Or } 9 \cdot 5: 12:: 42: 116 \frac{1}{5}
$$

Rule II. Multiply the area corresponding to the quarter girt in inches, by the length of the piece in feet, and the product will be tho solidity.

[^40]A TABLE FOR MEASURING TIMBER.

| Quarter Girt. | Area. | Quarter Girt. | Area. | Quarter Girt. | Area. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Inckes. | Feet. | Inches. | Feet. | Inches. | Feet. |
| 6 | $\cdot 250$ | 12 | $1 \cdot 000$ | 18 | $2 \cdot 250$ |
| 64 | -272 | $12 \frac{1}{4}$ | 1.042 | 181 $\frac{1}{2}$ | $2 \cdot 376$ |
| $6 \frac{1}{2}$ | -294 | 122 | 1.085 | 19 | $2 \cdot 506$ |
| $6 \frac{3}{4}$ | -317 | $12 \frac{3}{4}$ | $1 \cdot 129$ | $19 \frac{1}{2}$ | $2 \cdot 640$ |
| 7 | -340 | 13 | $1 \cdot 174$ | 20 | $2 \cdot 777$ |
| $7 \frac{1}{4}$ | -364 | $13 \frac{1}{4}$ | $1 \cdot 219$ | $20 \frac{1}{2}$ | $2 \cdot 917$ |
| 71 | -390 | 132 | $1 \cdot 265$ | 21 | $3 \cdot 062$ |
| $7 \frac{3}{4}$ | -417 | $13 \frac{3}{4}$ | $1 \cdot 313$ | $21 \frac{1}{2}$ | $3 \cdot 209$ |
| 8 | -444 | 14 | $1 \cdot 361$ | 22 | $3 \cdot 362$ |
| $8 \frac{1}{4}$ | -472 | $14 \frac{1}{4}$ | $1 \cdot 410$ | $22 \frac{1}{2}$ | 3.516 |
| 81 | $\cdot 501$ | 14, | $1 \cdot 460$ | 23 | $3 \cdot 673$ |
| $8 \frac{3}{4}$ | -531 | $14 \frac{3}{4}$ | $1 \cdot 511$ | $23 \frac{1}{2}$ | 3.835 |
| 9 | -562 | 15 | 1-562 | 24 | 4.000 |
| 91 | -594 | 151 | $1 \cdot 615$ | $24 \frac{1}{2}$ | $4 \cdot 168$ |
| $9 \frac{1}{2}$ | -626 | $15 \frac{1}{2}$ | 1-668 | 25 | $4 \cdot 340$ |
| $9 \frac{3}{4}$ | -659 | $15 \frac{3}{4}$ | $1 \cdot 722$ | $25 \frac{1}{2}$ | $4 \cdot 516$ |
| 10 | -694 | 16 | $1 \cdot 777$ | 26 | $4 \cdot 694$ |
| $10 \frac{1}{4}$ | -730 | 161 ${ }^{\frac{1}{4}}$ | 1.833 | $26 \frac{1}{2}$ | $4 \cdot 876$ |
| $10 \frac{1}{2}$ | $\cdot 766$ | $16 \frac{1}{2}$ | 1.890 | 27 | $5 \cdot 062$ |
| $10 \frac{3}{4}$ | -803 | $16 \frac{3}{4}$ | $1 \cdot 948$ | 271 | $5 \cdot 252$ |
| 11 | -840 | 17 | 2.006 | 28 | $5 \cdot 444$ |
| $11 \frac{1}{4}$ | -878 | $17 \frac{1}{4}$ | $2 \cdot 066$ | $28 \frac{1}{2}$ | $5 \cdot 640$ |
| 111 | -918 | $17 \frac{1}{2}$ | $2 \cdot 126$ | 29 | $5 \cdot 840$ |
| $11 \frac{3}{4}$ | -959 | $17 \frac{3}{4}$ | $2 \cdot 187$ | 291 $\frac{1}{2}$ | 6.044 |

2. If a piece of round timber be 10 feet long, and the quarter girt $12 \frac{1}{2}$ inches; required the solidity?

To find the solid content by this table, look for the quarter girt $12 \frac{1}{2}$, in the column marked Quarter Girt, and in adjoining column, marked Area, will be found 1.085 , which multiplied by the length, 10 feet, will give 10.85 feet for the solid content.
3. A piece of round timber is 20 feet long, and the quarter girt $14 \frac{1}{4}$; how many feet are contained therein? Ans. 28.2 feet.
4. How many solid feet are contained in a tree 40 feet long, its quarter girt being 9 inches? Ans. $22 \cdot 48$.
5. How many solid fect in a tree 32 feet long, its quarter girt being 8 inches.

Ans. $14 \cdot 208$.
6. How many solid feet in a tree $8 \frac{1}{2}$ feet long, its quarter girt being $7 \frac{1}{2}$ inches.

Ans. 3.315 feet.
7. Required the content of a tree, whose length is 40 feet, and quarter girt $27 \frac{1}{2}$ inches?

Ans. 210.08 feet.
8. What is the content of a tree, whose length is 30 feet 6 inches, and quarter girt $27 \frac{1}{2}$ inches? Ans. $160 \cdot 186$ feet.
9 Required the content of a piece of timber, whose length is 25 feet 9 inches, and quarter girt $12 \frac{3}{4}$ inches? Ans. 29.071 feet.
10. What is the solid content of a piece of timber, whose length is 12 feet, and quarter girt $13 \frac{1}{2}$ inches?

Ans. $15 \cdot 18$ feet.
11. What is the solid content of a piece of timber, whose quarter girt is $14 \frac{3}{4}$ inches, and length 38 feet? Ans. $57 \cdot 418$ feet.

When the square of the quarter girt is multiplied by the length, the product gives a result nearly one-fourth less than the true quantity in the tree. This rule, however, is invariably practised by timber merchants, and is not likely to be abolished. When the tree is in the form of a cylinder, its content ought to be found by Prob. IV. Sec. IV., which gives the content greater than that found by the last rule, nearly in the proportion of 14 to 11. Notwithstanding that the true content is not found by means of the square of the quarter girt, yet some allowance ought to be made to the purchaser on account of the waste in squaring the wood so as to be fit for use. If the cylindrical tree be reckoned no more than what the inscribed square will amount to, the last rule, which is said to give too little, gives too much. When the tree is not perfectly circular, the quarter girt is always too great, and therefore the content, on that account, will be too great.

Dr Hutton recommends the following rule, which will give the content extremely near the truth:-

Rule. Multiply the square of one-fifth of the girt, or circumference, by twice the length, and the product will be the content.

## BY THE SLIDING RULE.

As double the length on $\mathrm{C}: 12$ or 10 on $\mathrm{D}:: \frac{1}{8}$ of the girt, in 12 ths or 10 ths on D : content on C.
12. Required the content of a tree, its length being 9 feet 6 inches, and its mean girt 14 feet.


19


Dr Gregory recommends the following rules given by Mr Andrews :-
Let $L$ denote the length of the tree in feet and decimals, and $G$ the mean girt in inches.

Rule I. Making no allowance for bark.
$\frac{\mathrm{L} \mathrm{G}^{2}}{2304}=$ cubic feet, customary ; and $\frac{\mathrm{L} \mathrm{G}^{2}}{1807}=$ cubic feet, true content.
Rule II. Allowing $\frac{1}{8}$ for bark.
$\frac{\mathrm{L}^{2}}{3009}=$ cubic feet, customary $; \frac{\mathrm{L} \mathrm{G}^{2}}{2360}=$ cubic feet, true content.
Rule III. Allowing $\frac{1}{10}$ for bark.
$\frac{L G^{2}}{2845}=$ cubic feet, customary; $\frac{L G^{2}}{2231}=$ cubic fect, true content.
Rule IV. Allowing $\frac{1}{12}$ for bark.
$\frac{L \mathrm{G}^{2}}{2742}=$ cubic feet, customary; $\frac{\mathrm{L} \mathrm{G}}{}{ }^{2}=$ cubic feet, true content.
What is the solid content of a tree, whose circumference, or girt, is 60 inches, and length 40 feet?

$$
\begin{aligned}
& \text { By Rule I. } \\
& \frac{40 \times 60^{2}}{2304}=62 \frac{1}{2} \text { cubic feet, customary. } \\
& \frac{40 \times 60^{2}}{1807}=79 \frac{3}{4} \text { cubic feet, customary. } \\
& \text { By Rule II. } \\
& \frac{40 \times 60^{2}}{3009}=47.85 \text { cubic feet, customary. } \\
& \frac{40 \times 60^{2}}{2360}=61 \text { cubic feet, true content. } \\
& \text { By Rule III. } \\
& \frac{40 \times 60^{2}}{2845}=50.61 \text { cubic feet, customary } \\
& \frac{40 \times 60^{2}}{2231}=64.54 \text { cubic feet, true content } \\
& \text { By Rule IV. } \\
& \frac{40 \times 60^{2}}{2742}=52.47 \text { cubic feet, customary. } \\
& \frac{40 \times 60^{2}}{2150}=66.97 \text { cubic feet, true content. }
\end{aligned}
$$

When the two ends are very unequal, calculate its content by the rule given for finding the solidity of the frustum of a cone, and deduct the usual allowance from the result.

When it is required to find the accurate content of an irregular borly, not reducible to any figure of which we have already treated, provide a cylindrical or prismatic vessel, capable of containing the
solid to be measured; put the solid into the vessel, and pour in water to cover it, marking the height to which the water reaches. Then take out the solid, and observe how much the water has descended in consequence of its removal ; calculate the capacity of the part of the vessel thus left dry, and it will evidently be equal to the solidity of the body whose content is required.

## ARTIFICERS' WORK.

Artificers compute their works by several different measures:-
Glazing and masonry by the foot.
Plastering, painting, paving, \&c., by the yard of 9 square feet.
Partitioning, roofing, tiling, flooring, \&e., by the square of 100 spuare feet.

Brick work is computed, either by the yard of 9 square feet, or by the perch, or square rood, containing $272 \frac{1}{4}$ square feet, or $30 \frac{1}{4}$ square yards; $272 \frac{1}{4}$ and $30 \frac{1}{4}$ being the squares of $16 \frac{1}{2}$ feet and $5 \frac{1}{2}$ yards respectively.

## CARPENTERS' AND JOINERS' WORK.

of fLooring.
To measure joists, multiply the breadth, depth, and length together for the content.*

1. If a floor be 50 feet 4 inches long, and 22 feet 6 inches broad; how many squares of flooring are in that room?

| $50 \cdot 333$ |  |
| :---: | ---: |
| 22.5 | 50.4 <br> 251665 <br> 100666 |
| 100666 <br> $100) 1132 \cdot 4925$ | 1107.4 <br> 25.2 |
| 100$) 11 \cdot 32.6$ |  |
| $11 \cdot 325$ |  |

Ans. 11 squares $32 \frac{1}{2}$ feet.
2. If a floor be 51 feet 6 inches long, and 40 fect 9 inches broad; Low many squares are contained in that floor? Ans. $20 \cdot 986$ squares.
3. If a floor be 36 feet 3 inches long, and 16 feet 6 inches broad, how many squares are contained in that floor?

Ans. 5 squares $98 \frac{1}{8}$ feet.
4. If a floor be 86 feet 11 inches long, and 21 feet 2 inches broad; how many squares are contained in it?

Ans. 18-3972.
5. In a naked floor the girder is 1 foot 2 inches deep, 1 foot broad, and 22 feet long; there are 9 bridgings, the scantling of each (viz.,

[^41]breadth and depth) being 3 inches, by 6 inches, and length 22 feet; 9 binding joists, the length of each being 10 feet, and scantlings 8 inches by 4 inches; the ceiling joists are 25 in number, each 7 feet long, and their scantlings 4 inches by 3 inches; what is the solidity of the whole?

Ans. 85 feet.
6. What would the flooring of a house three storeys bigh come to, at $£ 5$ per square; the house measures 30 feet long, and 20 broad; there are seven fire-places,* two of which measure, each 6 feet by 4 feet; two others, each 6 feet by 5 feet 6 inches; two, each of 5 feet 6 inches by 4 feet; and the seventh 5 feet by 4 ; the well-hole for the stairs is 10 feet by 8 ?

Ans. £69, 2s.

## OF PARTITIONING.

Partitions are measured by squares of 100 feet, as flooring; therr dimensions are taken by measuring from wall to wall, and from floor to floor ; then multiply the length and height for the content in feet, which bring to squares by dividing 100, as in flooring. When doors and windows are not included by agreement, deductions must be made for their amount. $\dagger$

1. A partition measures 173 feet 10 inches in length, and 10 feet 7 inches in height; required the number of squares in it?

$$
\text { Ans. } 18.3972 \text { squares. }
$$

2. A partition between two rooms measures 80 feet in length, and 50 feet 6 inches in height ; how many squares in it?

$$
\text { Ans. } 40 \frac{2}{6} \text { squares. }
$$

3. If a partition measure 10 feet 6 inches in length, and 10 feet 9 inches in height; how many squares in it? Ans. 1 square $12 \frac{1}{2}$ feet.
4. What is the number of squares in a partition, whose length is 50 feet 6 inches, and height 12 feet 9 inches?

Ans. 6 squares, 43 feet, $10 \frac{1}{2}$ inches.
In roofing, the length of the rafters is equal to the length of a string stretched from the ridge down the rafter till it meets the top of the wall.

To find the content, multiply this length by the breadth and depth of the rafters, and the result will be the content of one rafter; and that multiplied by the number of them will give the content of all the rafters. $\ddagger$

1. If a house within the walls be 42 feet 6 inches long, and 20 feet 3 inches broad; how many squares of roofing in that house?
[^42]| ft. |
| :--- |
| $42 \cdot 5$ |
| $20 \cdot 25$ |
| 2125 |
| 850 |
| 8500 |
| $860 \cdot 625$ |
| $430 \cdot 3125$ |

$\overline{100) 1290.9375}$
12.91 squares.

2. What cost the roofing of a house at $11 s$. per square ; the length within the walls being 50 feet 9 inches, and the breadth 30 feet; the roof being of a true pitch?
3. What number of squares are contained in a house, whose length within the walls is 40 feet, and breadth 18 feet; the roof being common pitch? Ans. 10 squares and 80 feet.
4. How many squares in the roof of a building, the length of the house being 60 feet, and the length of the rafter 14 feet 6 inches?

$$
\text { Ans. } 17 \text { squares and } 40 \text { feet. }
$$

5. How many squares in a building, whose length is 50 feet, and the length of the rafter 15 feet? Ans. 15 squares.
6. How many squares in the roof of a building, whose length is 87 feet, the length of the rafter being 13 feet?

Ans. 9 squares and 62 feet.
7. How many squares in the roof of a building, whose length is 70 feet 6 inches, the length of the rafter being 14 feet 6 inches?

Ars. 20 squares and $44 \frac{1}{2}$ feet.
8. How many squares in the roof of a building, whose length is 50 feet, and the length of a string reaching across the ridge from eave to eave being 30 feet?

Ans. 15 squares.
Note.-All the timbers employed in roofing are measured like those used in flooring, except where there is a necessity of cutting out parallel pieces equal to, or exceeding $2 \frac{1}{2}$ inches broad and two feet long. In this case the amount of the pieces so cut out must be deducted from the content of the whole picce found from its greatest scantlings. When the pieces cut out do not amount to the above dimensions, they are considered as useless, and therefore no deduction is to be made for them.*
9. Let the tie-beam TB be 36 feet long, 9 inches broad, and 1 foot 2 inches thick; the king-post K 11 feet 6 inches high, 1 foot broad at the bottom, and 5 inches thick; out of this post are sawn two equal pieces from the sides, each 7 feet long and 3 inches broad.
 The braces B B are 7 feet 6 inches

[^43]long, and 5 inches by 5 inches square; the rafters R R are 19 feet long, 5 inches broad, and 10 inches deep; the struts S S are 3 feet 6 inches long, 4 inches broad, and 5 inches deep; what is the measurement for workmanship and also for materials?
ft. in. p
31. 6. 0 solidity of the tie-beam T B.
4.9. 6 solidity of the king-post K .
2. T. 3 solidity of the braces B B.
13.2.4 solidity of the rafters R R.
11. 8 solidity of the struts S S.
53.0 .9
solidity for workmanship.
1.5 .6 solidity cut from the king-post.
$\overline{51 \cdot 7 \cdot 3}$ solidity for materials.
OF WAINSCOTTING.
Wainscotting is measured by the yard square, which is 9 square feet.
In taking the dimensions, the string is made to ply close over the cornice, swelling panels, moulding, \&c. The height of the room from the floor to the ceiling being thus taken, is one dimension, and the compass of the room taken all round the floor is the second dimension.

Doors, windows, shutters, \&c., where both their sides are planed, are considered as work and half; therefore in measuring the room, they need not be deducted; but the superficial content of the whole room found as if there were no door, window, \&c., then the contents of the doors and windows must be found, and half thereof added to the content of the whole room.

When there are no shutters, the content of the windows must be deducted; chimneys, window-seats, check-boards, sopheta-boards, linings, \&c., must be measured by themselves.

Windows are sometimes valued at so much per window, and sometimes by the superficial foot. The dimensions of a window are taken in feet and inches, from the under side of the sill to the upper side of the top-rail; and from the outside to outside of the jambs.

When the doors are pannelled on both sides, take double the measure for the workmanship.

For the surrounding architrave, girt round it and inside the jambs, for one dimension, and add the length of the jambs to the length of the cap-piece (taking the breadth of the opening for the length), for the other dimension.

Weather-boarding is measured by the yard square, and somelimes by the square.

Frame-doors are measured by the foot, or sometimes by the yard square.

Staircases are measured by the foot superficial. The dimensions are taken with a string passing over the riser and tread for one dimension, and the length of the step for the other. By the length of the step is meant the length of the front and the returns at the two ends.

For the balustrade, take the whole length of the upper part of the
hand-rail, and girt it over its end till it meet the top of the newelpost, for one dimension; and twice the length of the baluster upon the landing, with the girt of the hand-rail, for the other dimension.

Modillion cornices, coves, \&c., are generally measured by the foot superficial.
Beads, stops, astragals, copings, fillets, boxings to windows, skirt-ing-boards, and water-trunks, are paid for by lineal measure.

Frontispieces are measured by the foot superficial, and the architrave, frieze, and cornice, are measured separately.*

To find the contents of the foregoing work, multiply the two corresponding dimensions together for the superficial content.

1. A room, or wainscot, being girt downwards over the mouldings, measures 12 ft .6 in . and 130 ft .9 in . in compass; how many yards does that room contain?

2. If the wainscot of a room be 15 ft .6 in . high, and the compass of the room 142 ft .6 in . ; how many yards are contained in it?

Ans. $245 \frac{5}{12}$ yards.
3. If the window shutters about a room be 60 ft .6 in . broad, and 6 ft .4 in . high; how many yards are contained therein, at work and a half?

Ans. $63 \frac{31}{36}$ yards.
4. A rectangular room measures 129 feet 6 inches round, and is to be wainscotted at 3 s .6 d . per square yard; after due allowance for girt of cornice, \&c., it is 16 feet 3 inches high; the door is 7 feet by 3 feet 9 inches; the window shutters, two pair, are 7 feet 3 inches by 4 feet 6 inches; the cheek-boards round them come 15 inchey below the shutters, and are 14 inches in breadth; the lining-boards

[^44]round the doorway are 16 inches broad: the door and windowshutters being worked on both sides, are reckoned as work and half, and paid for accordingly; the chimney 3 feet 9 inches by 3 feet, not being enclosed, is to be deducted from the superficial content of the room. The estimate of the charge is required. Ans. $£ 43,4 s .6 \frac{3}{4} d$.
5. The height of a room, taking in the cornice and mouldings, is 12 feet 6 inches, and the whole compass 83 feet 8 inches; the three window-shutters are each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches; the door and shutter, being worked on both sides, are reckoned work and a half. Required the estimate, at 6s. per square yard? - Ans. £36, $12 s .2 \frac{1}{2} d$.

## OF BRICKLAYERS' WORK

## of tiling or slating.

Tiling and slating are measured by the square of 100 feet. There is no material difference between the method employed for finding the estimate of roofing and tiling; bricklayers sometimes require double measure for hips and valleys.

When gutters are allowed double measure, the usual mode is, to measure the length along the ridge tile, and add it to the contents of the roof: this makes an allowance of one foot in breadth along the hips or valleys. Double measure is usually allowed for the eaves, so much as the projector is over the plate, which is generally 18 or 20 inches.

When sky-lights and chimney-shafts are not large, no allowance is to be made for them; but when they are large, their amount is to be deducted.

1. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves) is 30 feet 6 inches, and the lergth 42 feet; how many squares of tiling are contained therein?

| ft. in. | $\mathrm{ft}$. |
| :--- | ---: |
| 30.6 | $30 \cdot 5$ |
| $\frac{42}{1260}$ | $\frac{42}{610}$ |
| $\frac{21}{100) 1281}$ | $\frac{1220}{12.81}$ |

2. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves), is 40 feet 9 inches, and the length 47 feet 6 inches; required the number of squares contained therein?

Ans. 19 squares $35 \frac{5}{8}$ feet.
3. What will the slating of a house cost at $£ 1,5 s .6 d$. per square; the length being 43 feet 10 inches, and the breadth 27 feet 5 inches, on the flat; the eaves projecting 16 inches on each side-true pitch? Ans. £24, 9s. $5 \frac{1}{2} d$.
4. What is the content of a slated roof, the length being 45 feet $\$$ inches, and the whole girt 34 feet 3 inches? Ans. $174 \cdot 104$ yards.

## OF WALLING.

Brick-work is estimated at the rate of a brick and a half thick ; во that if a wall be more or less than the standard thickness, it must be reduced to it : thus, multiply the superficial content of the wall by the number of half bricks in the thickness, and divide the product by 3.

The superficial content is found by multiplying the length by the height. Bricklayers estimate their work by the rod of $16 \frac{1}{2}$ feet, or $272 \frac{1}{4}$ square feet. Sometimes 18 feet are allowed to the rod, that is, 324 square feet; sometimes the work is measured by the rod of 21 feet long, and 3 feet high, that is, 63 square feet; in this case, no regard is paid to the thickness of the wall in measuring, but the price is regulated according to the thickness.

When a piece of brick-work is to be measured, the first thing to be done is to ascertain which of the above measures is to be employed; then, having multiplied the length and breadth together (the dimensions being feet) the product is to be divided by the proper divisor, namely, $272 \cdot 25,324$, or 63 , according to the measure of the rod, and the quotient will be the measure in square rods of that measure.

To measure any arched way, arched window, or door, \&c., the height of the window or door from the crown or middle of the arch, to the bottom or sill, is to be taken, and likewise from the bottom or sill to the spring of the arch, that is, where the arch begins to turn. Then to the latter height add twice the former, and multiply the sum by the breadth of the window, door, \&c., and one-third of the product will be the area sufficiently near the truth for practice.

1. If a wall be 72 feet 6 inches long, and 19 feet 3 inches high, and 5 bricks and a half thick, how many rods of brick-work ars contained therein, when reduced to the standard?
Note.-The standard means a wall a brick and a half thick; therefore, to reduce any wall to the standard, multiply the superficial content of it by the number of half bricks in its thickness, and divide by 3.

$$
\begin{gathered}
\text { ft. in. } \\
72 \cdot 6 \\
19 \cdot 3 \\
\hline 648 \\
72 \\
18 \cdot 1 \cdot 6 \\
9 \cdot 6 \cdot 0 \\
\hline 1395 \cdot 7 \cdot 6 \\
\hline \frac{11}{3) 15351.10 \cdot 6} \\
\hline 272) 5117(18 \text { rods. } \\
\frac{2397}{68)} \\
\frac{221}{17} \text { feet. }
\end{gathered}
$$

Notc.--That 68.06 is the fourth part of 272.26 , and 68 is one-fourth of 272

In reducing feet into rods, it is usual to divide by 272 , rejecting the decimal 25 . By this method, the answer found above is about $4 \frac{1}{2}$ feet too much.
2. How many rods of standard brick-work are in a wall whose length is 57 feet 3 inches, and height 24 feet 6 inches; the wall being $2 \frac{1}{2}$ bricks thick?

Ans. 8.5866 rods.
3. The end wall of a house is 28 feet 10 inches long, and 55 feet 8 inches high to the eaves; 20 feet high is $2 \frac{1}{2}$ bricks thick, another 2 C feet high is 2 bricks thick, and the remaining 15 feet 8 inches is $1 \frac{1}{2}$ half bricks thick, above which is a triangular gable 1 brick thick, which rises 42 courses of bricks, of which every 4 courses make a foot. What is the whole content in standard measure?

Ans. 253.62 yards.

## OF CHIMNEYS.

When a chimney stands by itself, without any party-wall being adjoined, take the girt in the middle for the length, and the height of the storey for the breadth; the thickness is to be the same as the depth of the jambs; if the chimney be built upright from the mantelpiece to the ceiling, no deduction is to be made for the vacancy between the floor (or hearth) and mantel-tree, on account of the gatherings of the breast and wings, to make room for the hearth in the next storey.

When the chimney-back forms a party-wall, and is measured by itself, then the depth of the two jambs is to be measured, and the length of the breast for a length, and the height of the storey for the breadth; the thickness is the same as the depth of the jambs. That part of the chimney which appears above the roof, called the chimueyshaft, is measured by girding it round the middle for the length, and the height is taken for the breadth.

In consideration of plastering and scaffolding, the thickness is generally reckoned half a brick more than it really is ; and in some piaces double measure is allowed on account of extra trouble.

1. Let the dimensions of a chimney, having a double funnel towards the top, and a double shaft, be as follows, viz., in the partour the breast and two jambs measure 18 feet 9 inches, and the height of the room is 12 feet 6 inches; in the first floor, the breast and two jambs girt 14 feet 6 inches, and the height 9 feet; in the second floor, the breast and the jambs girt 10 feet 3 inches, and the height is 7 feet; above the roof, the compass of the shaft is 13 feet 9 inches, and its height 6 feet 6 inches; lastly, the length of the middle partition, which parts the funnel, is 12 feet, and its thickness 1 foot 3 inches; how many rods of brick-work, standard measure, are contained in the chimney, double measure being allowed, and thickness $1 \frac{1}{2}$ bricks?


> ft. in. p. 5th. 1.3.0 12
> 15.0.0 partition. 234.4 . 6 parlour. 130.6.0 first floor. 71.9 .0 second floor. 89 . 4 . 6 shaft.
> $\overline{541.0 .0}$ sum
> 2
> 272) $\overline{1082 \cdot 0.0}$ double.
> $68) \overline{266(3 \text { rods } 3}$ quarters.
> 62 feet.

Ans. 3 rods, 3 quarters, and 62 feet.

## MASONS' WORK.

To masonry belong all sorts of stone-work. The work is sometimes measured by the foot solid, sometimes by the foot in length, and sometimes by the foot superficial. Masons, in taking dimensions, girt all their mouldings, in the same manner as joiners.

Walls, columns, blocks of stone or marble, \&c., are measured by the solid foot, and pavements, slabs, chimney-pieces, \&c., by the square foot.

In estimating for the workmanship, square measure is generally used, but for the materials, solid measure.

In the solid measure, the length, breadth, and thickness are multiplied together.

In the superficial measure, there must be taken the length and breadth of every part of the projection, which is seen without the general upright face of the building.

1. If a wall be 82 feet 9 inches long, 20 feet 3 inches high, and 2 feet 3 inches thick; how many solid feet are contained in that wall?

2. If a wall be 120 feet 4 inches long, and 30 feet 8 inches high; how many superficial feet are contained therein?

Ans. $3690 \frac{2}{3}$ feet.
3. If a wall be 112 feet 3 inches long, and 16 feet 6 inches high; now many superficial rods of 63 square feet are contained therein? Ans. 29 rods 25 feet.
4. What is the value of a marble slab at 8 s . per foot, the length being 5 feet 7 inches, and breadth 1 foot 10 inches?

$$
\text { Ans. } £ 4,1 s .10 \frac{1}{2} d .
$$

## PLASTERERS' WORK.

Plasterers' work is of two kinds, viz., ceiling, which is plastering apon laths; and rendering, which is plastering upon walls. These are measured separately.

The content is sometimes estimated by the foot, sometimes by the yard, and sometimes by the square of 100 feet. Enriched mouldings are calculated by the running foot or yard.

Deductions are made for chimneys, doors, windows, \&c.
In plastering timber partitions, where several of the large braces and other large timbers project from the plastering, a fifth is usually deducted.

Whitening and colouring are measured in the same manner as plastering. In timbered partitions, one-fourth or one-fifth of the whole area is usually added, to compensate for the trouble of colouring the sides of the quarters and braces.

In arches, the girt round them is multiplied by the length for the superficial content.

1. If a ceiling be 40 feet 3 inches long, and 16 feet 9 mehes broad, how many square yards contained therein?

| $\begin{aligned} & \text { ft. in. } \\ & 40.3 \\ & 16.9 \end{aligned}$ | $\begin{gathered} \mathrm{ft} \\ 40 \cdot 25 \\ 16.75 \end{gathered}$ |
| :---: | :---: |
| 640 | 20125 |
| $6=\frac{1}{2} \quad 20.1 .6$ | 28175 |
| $3=\frac{1}{2} \quad 10.0 .9$ | 24150 |
| $3=\frac{1}{4} \quad 4.0 .0$ | 4025 |
| 9) $\overline{674.2 .3}$ | 9) $674 \cdot 1875$ |

## Ans. 74 yards 8 feet.

Ans. 74.9097 yards.
2. The length of a room is 14 feet 5 inches, breadth 13 feet 2 inches, and height 9 feet 3 inches to the under side of the cornice,* which projects 5 inches from the wall, on the upper part next the ceiling; required the quantity of rendering and plastering, there being no deduction but for one door, which is 7 feet by 4 ?

Ans. 53 yards 5 feet of rendering, 18 yards 5 feet of ceiling.
3. The circular vaulted roof of a church measures 105 feet 6 inches in the arch, and 275 feet 5 inches in length; what will the plastering come to at $1 s$. per yard?

Ans. £161, 8s. $5 \frac{3}{4} d$.
4. The length of a room is 18 feet 6 inches, the breadth 12 feet 3 inches, and height 10 feet 6 inches; to how much amount the ceiling and rendering, the former at $8 d$. and the latter at $3 d$. per yard; allowing for the door of 7 feet by 3 feet 8 , and a fire-place of 5 feet square? Ans. $£ 1,13 s .3 d$.

## PLUMBERS' WORK.

Plumbers' work is rated by the pound, or by the hundredweight of 112 lbs . Sheet lead, used in roofing, guttering, \&c., weighs from 6 to 12 pounds per square foot, according to the thickness; and leaden pipes vary in weight per yard, according to the diameters of the bore.

The following table shews the weight of a square foot of sheet lead, according to its thickness; and the common weight of a yard of leaden pipe according to the diameter of its bore.

| Thickness of lead. | Pounds to a square foot. | $\left\|\begin{array}{c} \text { Bore of } \\ \text { leaden pipes. } \end{array}\right\|$ | Pounds per yard. |
| :---: | :---: | :---: | :---: |
| Inch. $\frac{1}{10}$ | 5•899 | $0 \frac{8}{4}$ |  |
| 10 | $6 \cdot 554$ | 1 | 12 |
| $\frac{1}{8}$ | $7 \cdot 373$ | 11 | $18 \square^{\circ}$ |
| $\frac{1}{7}$ | $8 \cdot 427$ | $1 \frac{1}{2}$ | 18 |
| $\frac{1}{6}$ | $9 \cdot 831$ | $1{ }^{\frac{3}{4}}$ | 21 |
| $\frac{1}{6}$ | 11.797 | 2 | 24 |

[^45]1. A piece of sheet lead measures 20 feet 6 inches in length, and 7 feet 9 inches in breadth; what is its weight at $8 \frac{1}{4}$ pounds to the square foot?

2. What weight of lead $\frac{1}{10}$ of an inch thick will cover a flat, 15 feet 6 inches long, and 10 feet 3 inches broad, the lead weighing 6 pounds to the square foot? Ans. 8 cwt ., 2 qrs., $1 \frac{1}{4}$ pounds.
3. What will be the expense of covering and guttering a roof with lead, at 18s. per cwt. ; the length of the roof being 43 feet, and the girt over it 32 feet; the guttering being 57 feet in length, and 2 feet in breadth, allowing a square foot of lead to weigh $8 \frac{3}{4}$ pounds?

$$
\text { Ans. £104, } 15 s .3 \frac{3}{4} d \text {. }
$$

4. What will be the expense of 130 yards of leaden pipe of an inch and half bore, at 4 d . per pound, admitting each yard to weigh 18 pounds? Ans. £39.

## PAINTERS' WORK.

Painters' work is computed in square yards. Every part is measured where the colour lies, and the measuring line is forced into all the mouldings and corners. Double measure is allowed for carved mouldings, \&c.

Windows are done at so much a-piece; sash-frames at a certain price per dozen; sky-lights, window-bars, casements, \&c., are charged at a certain price per piece.

To measure balustrades, take the length of the hand-rail for one dimension, and twice the height of the baluster upon the landing, added to the girt of the hand-rail, for the other dimension.

No general rule can be given for measuring trellis-work; but,
howerer, double the area of one side is often taken for the measure of both sides.

1. If a room be painted, whose height (being girt over the moulding) is 16 feet 4 inches, and the compass of the room 120 feet 9 inches; how many yards of painting in it?


$$
\text { Ans. } 219 \text { yards } 1 \text { foot. }
$$

2. A gentleman had a room to be painted, its length being 24 feet 6 inches, breadth 16 feet 3 inches, and height 12 feet 9 inches; also the size of the door 7 feet by 3 feet 6 inches, and the size of tho window-shutters to each of the windows, there being two, is 7 feet 9 inches by 3 feet 6 inches; but the breaks of the windows themselves are 8 feet 6 inches high, and 1 foot 3 inches deep; what will be the expense of giving it three coats, at $2 d$. per yard each; the size of the fire-place to be deducted, being 5 feet by 5 feet 6 inches?

Ans. £3, 3s. $10 \frac{1}{2} d$.
3. The length of a room is 20 feet, ins breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of painting in it, deducting a fire-place of 4 feet by 4 feet 4 inches, and two windowshutters each 6 feet by 3 feet 2 inches? Ans. $73 \frac{2}{27}$ yards.

## GLAZIERS' WORK.

Glaziers take their dimensions either in feet, inches, and parts; or feet, tenths, and hundredths. They compute their work in square feet.

Windows are sometimes measured by taking the dimensions of one pane, and multiplying its superficies by the number of panes. But generally they take the length and breadth of the whole frame for the glazing. Circular windows are measured as if they were square, taking for their dimensions their greatest length and breadth.

1. If a pane of glass be 3 feet 6 inches and 9 parts long, and 1 foot 3 inches and 3 parts broad; how many feet of glass in that pane?

2. If there be 10 panes of glass, each 4 feet 8 inches 9 parts long, and 1 foot 4 inches and 3 parts broad; how many feet of glass are contained in the 10 panes? Ans. 64.0407.
3. There are 20 panes of glass, each 3 feet 6 inches 9 parts long, and 1 foot 3 inches and 3 parts broad; how many feet of glass are in the 20 panes?

Ans. 90.9224 feet.
4. If a window be 7 feet 6 inches high, and 3 feet 4 inches broad; how many square feet of glass contained therein?

Ans. 25.
5. How many feet in an elliptical fan-light of 14 feet 6 inches in length, and 4 feet 9 inches in breadth? Ans. 68 feet 10 inches.
6. What will the glazing of a triangular sky-light come to at 20 d .; the base being 12 feet 6 inches, and the perpendicular height 6 feet 9 inches?

Ans. £3, 10 s. $3 \frac{1}{2} d$.

## PAVERS' WORK.

Pavers' work is computed by the square yard; and the content is found by multiplying the length by the breadth.

1. What will be paid for paving a foot-path, at $4 s$. the yard, the length being 40 feet 6 inches, and the breadth 7 feet 3 inches?

| ft. in. | ft. |
| :---: | :---: |
| $40 \cdot 6$ | 40.5 |
| $7 \cdot 3$ | $7 \cdot 25$ |
| $283 \cdot 6$ | 2025 |
| $10 \cdot 1 \cdot 6$ |  |
| Ans. | 810 |
| $293 \cdot 7 \cdot 6$ | Ans. $\frac{2835}{293 \cdot 625}$ feet. |

2. What will be the expense of paving a rectangular court-yard, whose lergth is 62 feet 7 inches, and breadth 44 feet 5 inches; and in which there is a foot-path, whose whole length is 62 feet 7 inches, and breadth 5 feet 6 inches, this at $3 s$. per yard, and the rest at $2 s .6 d$. per yard? Ans. £39, 11s. $3 \frac{1}{4} d$.
3. What is the expense of paving a court, at $3 s .2 d$. per yard; the length being 27 feet 10 inches, and the breadth 14 feet 9 inches? Ans. £7, $4 s .5 \frac{1}{4} d$.
4. What will the paving of a walk round a circular bowling-green come to at $2 s .4 d$. per yard, the diameter of the bowling-green being 40 feet, and the breadth of the walk 5 feet? Ans. $£ 9,3 s .3 \frac{3}{25} d$.
5. How many yards of paving in an elliptical walk 4 feet broad, the longer diameter being 60 feet, and shorter 50?

Ans. $82 \cdot 3797$ yards.

## VAULTED AND ARCHED ROOFS.

Arched roofs are either domes, vaults, saloons, or groins.
Domes are formed of arches springing from a circular, or polygonal base. and meeting in a point directly over the centre of that base.

Saloons are made by arches connecting the side walls of a building to a flat roof or ceiling.

Groins are made by the intersection of vaulted roofs with each other.

Vaulted roofs are sometimes circular, sometimes elliptical, and sometimes Gothic.

Circular roofs are those of which the arch is a part of the circumference of the circle.

Elliptical roofs are those of which the arch is a part of the circumference of an ellipsis.

Gothic roofs are made by the meeting of two equal circular arches, exactly above the span of the arch.

Groins are generally measured like a parallelopipedon, and the content is found by multiplying the length and breadth of the base by the height.

Sometimes one-tenth is deducted from the solidity thus found, and the remainder is reckoned as the solidity of the vacuity.

## PROBLEM I.

## To find the solidity of a circular, elliptical, or Gothic vaulted roof.

Rule. Find the area of one end, by one of the foregoing rules, and multiply the area of the end by the length of the roof, or vault, and the product will be the content.

Note. When the arch is a segment of a circle, the area is found by Prob. XXVIII. Sec. II. When the arch is a segment of an ellipsis, multiply the pan by the height, and that product by 7854 for the area of the end. When it is a Cothic arch, find the area of an isosceles triangle, whose base is equal to the span of the arch, and its sides equal to the two chords of the circular segment of the arch; then add the areas of the two segments to the area of the triangle, and the sum will give the area of the end.

1. What is the content of a concavity of a semi-circular vaulted roof, the span being 30 feet, and the length of the vanlt 150 feet?
$30 \times 30=900$; then $900 \times 7854=706 \cdot 86$, hence
$706 \cdot 86 \div 2=353.43$, the area of the end;
Then $353 \cdot 43 \times 150=53014 \cdot 5$, the content.
2. What is the solid content of the vacuity
 AOEB of a Gothic vault, whose span AB is 60 feet, the chord B O, or A 0 , of each arch 60 feet; the distance of each arch from the middle of the chords as $\mathrm{DE}=12$ feet, and the length of the vault 40 feet?

In this example, the triangle ABO equilateral, and its area is $\frac{1}{4}$ $\mathrm{A}^{2} \sqrt{ } 3=900 \sqrt{ } 3=1557$. Again, $\frac{2}{3}(\mathrm{~B} 0 \times \mathrm{DE})+\frac{\mathrm{DE}^{3}}{2 \mathrm{~B} \mathrm{O}}=\frac{2}{3}(60 \times$ 12) $+\frac{12^{3}}{60 \times 2}=494 \frac{2}{6}=$ area of segment $0 E B$, and $494 \frac{2}{5} \times 2=988 \frac{4}{5}$ the areas of the two segments 0 EB and 0 HA ; then $\left(1557+988 \frac{4}{6}\right) \times 40$ $=101832$, the solidity required.

Let MNKL represent a perpendicular section of a vaulted rool (rothic). The span A B is 60 feet, the thickness of the wall M A, or BL, at the spring of the arch $=4$ feet, the thickness 0 P at the crown of the arch $=3$, and the length of the roof $=40$ feet, the chord A 0 or $0 B=60$ ieet, and the versed sine $D E=12$ feet; required the solidity of the materials of the arch?

First, $\sqrt{ }\left(\mathrm{A} \mathrm{O}^{2}-\mathrm{A} \mathrm{C}^{2}\right)=\sqrt{ }\left(60^{2}-30^{2}\right)=51 \cdot 96=\mathrm{S} 0$ the height of the vacuity of the arch, and $\mathrm{SO}+0 \mathrm{P}=51 \cdot 96+3=54 \cdot 96=\mathrm{SP}$; again, $\mathrm{AB}+\mathrm{MA}+\mathrm{BL}=60+4+4=68=\mathrm{ML}$, and $\mathrm{ML} \times \mathrm{SP}=$ the area of the rectangle MNKL; hence ML×SP×40-101832 (the solidity of the vacuity A O B by the last Problem), gives the solidity of the materials ; that is,
$68 \times 54.96 \times 40-101832=47659.2$ feet, the solidity required.
Note.-When the arch A OB is an elliptical segment, its area multiplied by the length of the roof gives the solidity of the vacuity, and ML multiplied by S P, and the product by the length of the arch, gives the solidity of the cubic figure whose end is MNKL; and the difference of the two solidities is the solidity of the mixed solid whose section is A MNKLBEOHA. The materials of a bridge may be calculated after the same manner, by adding the solidities of T, T, and of the battlements, to the solidity as found in this Problem.
3. Required the capacity of the vacuity of an elliptical vault, whose span is 30 feet, and height 15 feet, the length of the vault being 90 feet.

Ans. $31808 \cdot 7$ fcet.

## PROBLEM II.

I' find the concave or convex surface of a circular, elliptical, or Gothic vaulted roof.
Rule. Multiply the length of the arch by the length of the vault, and the product will be the superficies.
Note.- To find the length of the arch, make a line ply close to it, quite across -rom side to side.

1. What is the surface of a vaulted roof, the length of the arch being 45 feet, and the length of the vault 140 feet?

$$
140 \times 45=6300 \text { square feet. }
$$

2. Required the surface of a vaulted roof, the length of the arch being 40 feet 6 inches, and the length of the vault 100 feet?

Ans. 4050 feet.
3. What is the surface of a vaulted roof, the length of the arch being 40.5 feet, and the length of the vault 60 feet? Ans. 2430 feet.

## PROBLEM III.

## To find the solidity of a dome, having the height and the dimensions of its base given.

Rule. Multiply the area of the base by the height, and two-thirds of the product will give the solid content.*

[^46]1. What is the solid content of a dome, in the form of a hemisphere, the diameter of the circular base being 40 feet?

$$
\begin{aligned}
& 40^{2} \times \cdot 7854=1256 \cdot 64=\text { the area of base. } \\
& \frac{2}{3}(1256 \cdot 64 \times 20)=\frac{2}{3}(25132 \cdot 8)=16755 \cdot 2, \text { answer. }
\end{aligned}
$$

2. What is the solid content of an octagonal dome, each side of its 'ase being 20 feet, and the height 21 feet?

Ans. $27039 \cdot 1917$ cubic feet.
3. Required the solidity of the stone-work of an elliptical dome, the two diameters of its base being 40 and 30 feet, the height $17 \cdot 32$ feet, and the stone-work in every part 4 feet thick?

Ans. $9479 \cdot 086848$ cubic feet.

## PROBLEM IF.

To find the superficial content of a dome, the height and dimensions of its base being given.
Rule. Multiply the square of the diameter of the base by 1.5708 , and the product will be the superficial content.*

For an elliptical dome, multiply the two diameters of its base together, and the product resulting by 1.5708 for the superficial content, sufficiently correct for practical purposes.

1. The diameter of the base of a circular dome is 20 feet, and its height 10 feet; required its concave superficies?

$$
20^{2} \times 1.5708=628.32 \text { feet, the answer. }
$$

2. The two diameters of an elliptical dome are 40 and 30 feet, and its height 17.32 feet; required the concave surface?

$$
\text { Ans. } 1884 \cdot 96 \text { square feet. }
$$

3. What is the superficies of a hexagonal spherical dome, each side of the base being 10 feet?

Ans. 519•6152.

## PROBLEM V.

## To find the solid content of a saloon.

Rule. Multiply the area of a transverse section by the compass or circumference of the solid part of the saloon, taken round the middle part. Subtract this product from the whole vacuity of the room, supposing the walls to go upright from the spring of the arch to the flat ceiling, and the difference will be the answer, as will appear evident from the following example.

1. What is the solid content of a saloon with a circular quadrantal

[^47]arch of 2 feet radius, springing over a rectangular room of 20 feet long and 16 feet wide?
$2^{2} \times \cdot 7854=3 \cdot 1416=$ area of the quadrant CDAF. $2 \times 2 \div 2=2=$ area of the triangle CDF; then $3 \cdot 1416-2=1 \cdot 1416=$ area of the segment D AF. Now, $2 \times 2=4=$ area of the rectangle CDEF; then $4-3 \cdot 1416=\cdot 8584=$ area of the section DEFAD. $\sqrt{ }\left(2^{2}+2^{2}\right)=\sqrt{ } 8=2.8284271$. $2 \times 16+2 \times 20=72=$ the compass within the walls. $\frac{1}{2}(2 \cdot 8284271-2)=\cdot 4142136=\mathrm{E} \mathrm{S}$ and $2 \cdot 8284271:-4142136:: 2:-2928932=\mathrm{E} y$ :
 hence $72=(\cdot 2928932 \times 8)=69 \cdot 6568544=$ the circumference of the middle of the solid part of the saloon; therefore $69 \cdot 6568544 \times \cdot 8584=59 \cdot 79344381696=$ the content of the solid part of the saloon.
$20 \times 16=320$ the area of the roof or floor, and $320 \times 2=640=$ the solidity of the upper part of the room ; then $640-59 \cdot 79344=$ $\$ 80.20656$ feet, the solidity of the saloon.
2. If the height $\mathrm{D} E$ of the saloon be $3 \cdot 2$ feet, the chord $\mathrm{DF}=4 \cdot 5$ feet, and its versed sine $=9$ inches; what is the solid content of the solid part, the mean compass being 50 feet? Ans. $138 \cdot 26489$ feet.

## PROBLEM VI.

## To find the superficies of a saloon.

Rule. Find its breadth by applying a string close to it across the surface ; find also its length by measuring along the middle of it, quite round the room; then multiply these two dimensions together for the superficial content.

1. The girt across the face of the saloon is 5 feet, and its mean compass 100 feet; what is its superficial content?

$$
100 \times 5=500 \text {, the answer. }
$$

2. The girt across the face of the saloon is 12 feet, and its mean rompass 98 ; required its surface?

## SPECIFIC GRAVITY.

## SECTION VIII.

1. The specific gravity of a body is the relation which the weight of a given magnitude of that body has to the weight of an equal magnitude of a body of another kind.

In this sense a body is said to be specifically heavier than another, when under the same bulk it weighs more than that other. On the zontrary, a body is said to be specifically lighter than another, when under the same bulk it weighs less than that other. Thus, if there be two equal spheres, each one foot or one inch in diameter, the one of lead and the other of wood, then since the leaden sphere is found to be heavier than the wooden one, it is said to be specifically, or in specie, heavier, and the wooden sphere specifically lighter.
2. If two bodies be equal in bulk, their specific gravities are to each other as their weights, or as their densities.
3. If two bodies be of the same specific gravity or density, theit qbsolute weights will be as their magnitudes or bulks.
4. If two bodies be of the same weight, the specific gravities will be reciprocally as their bulks.
5. The specific gravities of all bodies are in a ratio compounded of the direct ratio of their weights, and the reciprocal ratio of their magnitude. Hence, again, the specific gravities are as the densities.
6. The absolute weights or gravities of bodies are in the compound ratio of their specific gravities and magnitudes or bulks.
7. The magnitudes of bodies are directly as their weights, and reciprocally as their specific gravities.
8. A body specifically heavier than a fluid, loses as much of it3 weight, when immersed in it, as is equal to the weight of a quantity of the fluid of the same bulk or magnitude; if the body be of equal density with the fluid, it loses all its weight, and requires no force but the fluid to sustain it. If it be heavier, its weight in the fluid will be only the difference between its own weight and the weight of the same bulk of the fluid; and therefore it will require a force equal to this difference to sustain it. But if the body immersed be lighter than the fluid, it will require a force equal to the difference between its own weight and that of the same bulk of the fluid, to keep it from rising in the fluid.
9. In comparing the weights of bodies, it is necessary to consider
some one as the standard with which all other bodics may be compared. Rain water is generally taken as the standard, it being found to be nearly alike in all places.

A cubic foot of rain water is found, by repeated experiments, to weigh $62 \frac{1}{2}$ pounds avoirdupois, or 1000 ounces, and a cubic foot containing 1728 cubic inches, it follows that a cubic inch weighs $\cdot 03616898148$ of a pound. Therefore if the specific gravity of any body be multiplied by 03616898148 , the product will be the weight of a cubic inch of that body in pounds avoirdupois; and if this weight be multiplied by 175 , and the product be divided by 144 , the quotient will be the weight of a cubic inch in pounds troy, 144 pounds avoirdupois being exactly equal to 175 pounds troy.
10. Since the specific gravities of bodies are as their absolute gravities under the same bulk, the specific gravity of a fluid will be to the specific gravity of any body immersed in it, as the part of the weight lost by the solid is to the whole weight. Hence the specific gravities of different fluids are as the weights lost by the same solid immersed in them.

## PROBLEM I.

## To find the specific gravity of a body.

## Case I. When the body is heavier than water.

Weigh the body first in water, and afterwards in the open air ; the difference will give the weight lost in water ; then say, as the weight lost in water is to the absolute weight of the body, so is the specific gravity of water to the specific gravity of the body.

## CASE II. When the body is lighter than water.

Fix another body to it, so heavy as that both may sink in water together as a compound mass. Weigh the compound mass and the heavier body separately, both in the water and open air, and find how much each loses in water, by taking its weight in water from its weight in the open air. Then say, as the difference of these remainders is to the weight of the lighter body in air, so is the specific gravity of water to the specific gravity of the lighter body.

## Case III. For a fluid of any kind.

Weigh a body of known specific gravity both in the fluid and open air, and find the loss of weight, by subtracting the weight in water from the weight out of it. Then say, as the whole, or absolute weight, is to the loss of weight, so is the specific gravity of the solid to the specific gravity of the fluid.

The usual way of finding the specific gravities of bodies is the following, viz. :-

On one arm of a balance suspend a globe of lead by a fine thread, and to the other arm of the balance fasten an equal weight sufficient to balance it in the open air; immerse the globe into the fluid, and observe what weight balances it then, by which the lost weight is ascertained, which is proportional to the specific gravity,

Immerse the globe successively in all the fluids whose proportional specific gravity you require, observing the weight lost in each ; then these weights lost in each will be the proportions of the fluids sought.
Examples.-C'ase I.

1. A piece of platina weighel $83 \cdot 1886$ pounds out of water, and in water only 79.5717 pounds; what is its specific gravity, that of water being 1000 ?
$83 \cdot 1886-79 \cdot 5717=3 \cdot 6169$ pounds, which is the weight lost in water; then $3 \cdot 6169: 83 \cdot 1886:: 1000: 23000$ the specific gravity, or the weight of a cubic foot of metal in ounces.
2. A piece of stone weighed 10 pounds in the open air, but in water only $6 \frac{3}{4}$ pounds; what is its specific gravity? Ans. 3077.

## Examples.-Case II.

3. If a piece of elm weigh 15 pounds in the open air, and that a piece of copper, which weighs 18 pounds in open air, and 16 pounds in water, is affixed to it, and that the compound weighs 6 pounds in water ; required the specific gravity of the elm?

| Copper. | Compound. |
| :--- | ---: |
| 18 in air. | 33 |
| 16 in water. | 6 <br> 2 <br> 2 loss. |
|  | 27 |
|  | 2 |

As $\overline{25}: 15:: 1000: 600$, the specific gravity of the elm.
4. A piece of cork weighs 20 pounds in open air, and a piece of granite being affixed to it, which weighs 120 pounds in air, and only 80 pounds in water, the compound mass weighs $16 \frac{2}{3}$ pounds in water; required the specific gravity of the cork? Ans 240.

## Examples.-Case III.

5. A piece of cast iron weighed $259 \cdot 1$ ounces in a fluid, and $298 \cdot 1$ ounces out of it; required the specific gravity of the fluid, allowing the specific gravity of the cast-iron to be 7645 ?
$298 \cdot 1-259 \cdot 1=39$ loss of weight in the iron; then $298 \cdot 1: 39:$ : $7645: 1000$, the specific gravity of the fluid; shewing the fluid to be water.*
6. A piece of lignum vitæ welghed $42 \frac{3}{4}$ ounces in a fluid, and $166 \frac{5}{8}$ ounces out of it; what is the specific gravity of the fluid, that of the lignum vitæ being 1333 ?

Ans. 991 is the specific gravity of the fluid, which shews it to be liquid turpentine or Burgundy wine.

[^48]
## TABLE OF SPECIFIC GRAVITIES.




I'he specific gravity of a body, and its weight, being given, to
find its solidity.

Rule. Say, as the tabular specific gravity of the body is to its weight, in ounces avoirdupois, so is 1 cubic foot to the content.

1. What is the solidity of a block of marble that weighs 10 tons, its specific gravity being 2742 ?

First, 10 tons $=200$ hundreds $=22400$ pounds $=358400$ ounces; then

2. How many cubic inches in an irregular block of marble which weighs 112 pounds, allowing its specific gravity to be 2520 ?

Ans. $1228 \frac{201}{26} \frac{1}{6}$ cubic inches.
3. How many cubic inches of gunpowder are there in 1 pound weight, its specific gravity being 1745 ?

Ans. 153, nearly.
4. How many cubre feet are there in a ton weight of dry oak, its specific gravity being 925 ?

Ans. $38 \frac{1389}{185}$.

## PROBLEM III.

The linear dimensions, or magnitude of a body, being given, and also its specific gravity, to find its weight.
Rule. One cubic foot is to the solidity of the body, as the tabular spreific gravity of the body is to the weight in avoirdupois ounces.

1. What is the weight of a piece of dry oak, in the form of a parallelopipedon, whose length is 56 inches, breadth 18 inches, and depth 12 ?
$56 \times 18 \times 12=12096$ cubic inches, the solid content.
Then $1728: 12096:: 932: 6524$ ounces $=407 \frac{3}{4}$ pounds, the weight required.
2. What is the weight of a block of dry oak, which measures 10 feet long, 3 feet broad, and $2 \frac{1}{2}$ feet deep, its specific gravity being 925 ?

Ans. $4335 \frac{15}{16}$ pounds.
3. What is the weight of a block of marble, whose length is 53 feet, and its breadth and thickness, each 12 feet?

Ans. $694 \frac{11}{160}$ tons.

## PROBLEM IV.

To find the quantities of two ingredients in a given compound.
Rule. Take the difference of every pair of the three specific gravities, viz., of the compound and each ingredient; and multiply the difference of every two by the third.

Then as the greater product is to the whole weight of the compound, so is each of the other products to the weights of the two ingredients.*

1. A composition of 112 pounds being made of tin and copper, whose specific gravity is found to be 8784 ; what is the quantity of each ingredient, the specific gravity of tin being 7320, and of copper 9000?

2. Hiero, king of Sicily, furnished a goldsmith with a quantity of gold, to make a crown. When it came home, he suspected that the goldsmith had used a greater quantity of silver than was necessary in the composition; and applied to the famous mathematician, Archimedes, a Syracusian, to discover the fraud, without defacing the crown.
[^49]To ascertain the quantity of gold and silver in the crown, ho procured a mass of gold and another of silver, each exactly of the same weight with the crown; justly considering that if the crown were of pure gold, it would be of equal bulk, and therefore displace an equal quantity of water with the golden mass; and if of silver, it would be of equal bulk and displace an equal quantity of water with the silver mass ; but if of a mixture of the two, it would displace an intermediate quantity of water.

Now surpose that each of the three weighed 100 ounces; and that on immersing them severally in water, there were displaced 5 ounces of water by the golden mass, 9 ounces by the silver mass, and 6 ounces by the crown; what quantity of gold and silver did the crown contain?

## Ans. $\left\{\begin{array}{l}75 \text { ounces of gold. } \\ 25 \text { ounces of silver. }\end{array}\right.$

Note.-Questions relating to specific gravities may be wrought by the rules of Alligation in Arithmetic, as well as by any Algebraic process that might bo omployed.

## PROBLEM V.

To find how many inches a floating body will sink in a fluid.
Rule. Find, by Problem III. the weight of the floating body from its solidity and specific gravity, and that will be the weight of the fluid which it will displace.

Then say, as the specific gravity of the fluid is to 1728 cubio inches, so is the weight of the body, in ounces, to the cubic inches immersed. The depth will be found from the given dimensions.

1. Suppose a piece of dry oak, in the form of a parallelopipedon, whose length is 56 inches, breadth 18, and depth 12, is to be floated upon common smooth water, on its broadest side; how many inches will it sink, its specific gravity being 932 ?

By Problem III., the weight of the piece of oak is 6524 ounces, which by the preliminary part of this section, is the weight of water displaced.

Then $1000: 1728:: 6524: 11273 \cdot 472$ cubic inches of oak immersed. Therefore, $11273 \cdot 472 \div(56 \times 18)=11 \cdot 184$ inches, the depth it will sink.

To find how far it will sink, allowing it to float on its narrower side, $11273 \cdot 472 \div(56 \times 12)=16.776$ inches.
2. How many inches will a cubic foot of dry oak sink in common water, allowing the specific gravity of the oak to be 970 ?

Ans. 11•64.

## PROBLEM VI.

To find what weight may be attached to a floating body, so that it may be just covered with a given fluid.
Rule. Multiply the cubic feet in the body by the difference between its specific gravity and that of the fluid, and the product will be the weight in ounces avoirdupois, just sufficient to immerse it in the fluid.

1. What weight must be attached to a piece of dry oak, 56 inches
long, 18 inches hroad, and 12 inches deep, to keep it from rising above the surface of a fresh-water lake; the specific gravity of the water being 1000 , and that of the oak 932 ?

> Here $56 \times 18 \times 12=12096$ cubic inches.
> Then $12096 \div 1728=7$ feet.

Then $(1000-932) \times 7=68 \times 7=476$ ounces $=29$ pounds 12 ounces.
2. What weight fixed to a piece of dry oak, 9 inches long, 6 inches broad, and 3 inches deep, will keep it from rising above the surface of common water, the specific gravity of water being 1000, and that of the oak 970 ? Ans. $2 \frac{13}{16}$ ounces.
3. A sailor had half an anker of brandy, the specific gravity of the liquor was 927 , the cask was oak, and contained 216 cubic inches, and its specific gravity was 932 ; to secure his prize from the customhouse offioers, he fixed just as much lead to the cask as would keep it under water, and then threw it into the sea; what weight of lead was necessary for his purpose?

Ans. The cask of brandy contained 1371 cubic inches, the weight of sea-water of an equal bulk was $817 \cdot 20486$ ounces, the cask weighed 116.5 ounces, the brandy $619 \cdot 609375$, both together weighed $736 \cdot 19375$ ounces. The difference between the specific gravity of lead and sea-water is to this remainder, as the specific gravity of lead to its weight in ounces, which will be found to be 89.09495 ounces, or 5 pounds 9 ounces.

## PROBLEM VII.

To find the solidity of a body, lighter than a fluid, which will be sufficient to prevent a body much heavier than the fluid, from sinking.
Rule. Find the solidity of the body to be floated, from its weight and specific gravity, by Problem II. Find also the weight of an equal bnlk of the fluid by Problem III. Then say, as the difference betweell the specific gravity of the fluid, and that of the body lighter than the fluid, is to the difference between the weight of the body to be floated and the weight of an equal bulk of the fluid, so is 1728 to the solidity of the lighter body in cubic inches.

1. How many solid feet of yellow fir, whose specific gravity is 657 , will be sufficient to keep a brass cannon, weighing 56 cwt., afloat at sea, the specific gravity of brass being 8396, and of sea-water 1030 ?

First, $56 \mathrm{cwt}=100352$ ounces, weight of the body to be floated. Then, $8396: 100352:: 1728: 20653 \cdot 675$ cubic inches in the cannon. And, $1728: 20653 \cdot 675:: 1030: 12310 \cdot 9289$, the weight of sea-water equal in bulk to that of the cannon. Hence, 1030-657: 100352$12310 \cdot 9289:: 1728: 407868.5545$ cubic inches $=236.036$ feet, the answer.
2. The specific gravity of lead is 11325 , of cork 240 , and of seawater 1030 ; now it is required to know how many cubic inches of cork will be sufficient to kecp $49 \frac{3}{8}$ pounds of lead afloat at sea?

# TO FIND THE TONNAGE OF SHIPS. 

1st.-VESSELS AGROUND. By the Parliamentary Rule.

## PROBLEM VIII.

For a ship or vessel, the length is to be measured on a straight line along the rabbet of the keel, from a perpendicular, let fall from the back of the main post, at the height of the wing-transom, to a perpendicular at the height of the upper deck (but the middle deck of three-decked ships), from the fore-part of the stern; then from the length between these perpendiculars subtract three-fifths of the extreme breadth for the rake of the stern, and $2 \frac{1}{2}$ inches for every foot of the height of the wing-transom above the lower part of the rabbet of the keel, for the rake abaft; and the remainder will be the length of the keel for tomnage.

The main breadth is to be taken from the outside of the outside plank, in the broadest part of the ship, either above or below the wales, deducting therefrom all that it exceeds the thickness of the plank of the bottom, which shall be accounted the main breadth; so that the moulding breadth, or the breadth of the frame, will then be less than the main breadth, so found, by double the thickness of the plank of the bottom.

Then multiply the length of the keel for tonnage, by the main breadth, so taken, and the product by half the breadth, then divide the whole by 94 , and the quotient will give the tonnage.

In cutters and brigs, where the rake of the stern-post exceeds $2 \frac{1}{2}$ inches to every foot in height, the actual rake is generally subtracted instead of the $2 \frac{1}{2}$ inches to every foot, as before mentioned.

1. Let us suppose the length from the fore-part of the stern, at the height of the upper deck, to the after-part of the stern-post, at the height of the wing-transom, to be 155 feet 8 inches, the breadth from out to outside 40 feet 6 inches, and the height of the wingtransom 21 feet 10 inches, what is the tonnage?

$$
\begin{aligned}
& \text { ft. } \\
& 40.6 \text { breadth. } \\
& \text { deduct } 3 \\
& 40 \cdot 3 \\
& 3 \\
& 5 \longdiv { 1 2 0 \cdot 9 } \\
& 24 \cdot 1 \frac{4}{6}=24 \cdot 15 \\
& 21 \cdot 10 \text { height of wing-transom. } \\
& \text { 12) } \frac{2 \frac{1}{2}}{54^{\frac{7}{2}}} \text { multiply. } \\
& 4 \cdot 55^{2}+24 \cdot 15=28 \cdot 70 \\
& 155 \cdot 66-28 \cdot 70=126 \cdot 96=\text { length } \text {. } \\
& \frac{126 \cdot 96 \times 40 \cdot 25+20 \cdot 125}{94}=1094 \text {, the answer. }
\end{aligned}
$$

2. Suppose the length of the keel to be 50.5 feet, breadth of the nidship-beam 20 feet; required the tonnage? Ans. 107.4.
3. If the length of the keel be 100 feet, and the breadth of the beam 30 feet; what is the tonnage? Ans. 478.

## 2d.-VESSELS AFLOAT.

Drop a plumb-line over the stern of the ship, and measure the distance between such line and the after-part of the stern-post, at the load water-mark: in a paraHel direction with the water, to a perpendicular point immediately over the load water-mark, at the fore-part of the main-stern, subtracting from such measurement the above distance, the remainder will be the ship's extreme length; from which is to be deducted three inches for every foot of the load draught of water for the rake abaft, and also three-fifths of the ship's breadth for the rake forward, the remainder shall be esteemed the just length of the keel to find the tonnage; and the breadth shall be taken from outside to outside of the plank, in the broadest part of the ship, either above or below the main-wales, exclusive of all manner of sheathing or doubling that may be wrought upon the sides of the ship; then multiply the length of the keel, taken as before directed, by the breadth, as before taught, and that product by half the said breadth, and dividing the product by 94 , the quotient is the tonnage.

## 3d.-steam vessels.

The length shall be taken on a straight line, along the rabbet of the keel, from the back of the main-stern-post to a perpendicular line from the fore-part of the main-stem under the bow-sprit; from which deducting the length of the engine-room, and subtracting three-fifths of the breadth, the remainder shall be esteemed the just length of the keel to find the tonnage; and the breadth shall be taken from the outside of the outside plank in the broadest place of the ship or vessel, be it either above or below the main-wales, exclusively of all mamer of doubling planks that may be wrought upon the sides of the ship or ressel; then multiply the length and breadth so found together, and that product by half the same breadth, and dividing by 94 , the quotient will be the tonnage, according to which all such vessels shall be measured.

Note.-Under certain penalties nothing but the fuel can be stowed in the engine-ruom.

Some divide the last product by 100 , to find the tonnage of king's ships, and by 95 , to find that of merchants' ships.

## FLOATING BODIES.

1. The buoyancy of casks, or the load which they will carry without sinking, may be estimated by reckoning 10 pounds avoirdupois to the ale gallon, or $8 \frac{1}{5}$ pounds to the wine gallon
2. The buoyancy of pontoons may be estimated at about half a hundred weight, or 56 pounds for each cubic foot. Therefore a pontoon which contained 96 cubic feet, would sustain 48 hundredweight before it would sink.
N.B.-This is an approximation, in which the difference between ${ }^{\frac{6}{7} T}$ and $\frac{1}{2}$, viz., $\frac{1}{2}$ of the whole weight, is allowed for that of the pontoon itself.
3. The principles of buoyancy are very ingeniously applied in the self-acting flood-gate, which, in the case of common sluices to a mill-dam, prevents inundation when a sudden flood occurs. By means of the same principle, it is that a hollow ball attached to a metallic lever of about a foot long, is made to rise with the liquid in a water-cask, and thus to close the cock and stop the supply from the pipe, just before the time when the water would otherwise run over the top of the vessel.

The property of buoyancy has also been successfully employed in raising ships which had sunk under water, and in pulling up old piles in a river when the tide ebbs and flows. A large barge is brought over a pile as the water begins to rise; a strong chain which has been previously fixed to the pile by a ring, \&c., is made to gird the barge, and is then firmly fastened; then, as the tide rises, the barge rises also, and by means of its buoyant force draws up the pile with it.

In a case which actually occurred, a barge of 50 feet long, 12 feet wide, 6 deep, and drawing 2 feet water, was employed. Then $50 \times 12 \times(6-2) \times \frac{4}{7}=\frac{50 \times 12 \times 16}{7}=192 \times 7 \frac{1}{T}=1344+27_{T}^{\frac{3}{T}}=1371 \frac{3}{T}$ $\mathrm{cwt} .=66 \frac{1}{2}$ tons, nearly, which is the measure of the force with which the barge acted in pulling up the pile.

# WEIGHT AND DIMENSIONS OF BALLS AND SHELLS. 

## SECTION IX.

The foregoing problems furnish rules for finding the weight and dimensions of balls and shells. But they may be found much easier by means of the experimental weight of a ball of a given size, and from the well-known geometrical property, that similar solids are as the cubes of their diameters.

## PROBLEM I.

To find the weight of an iron ball from its diameter.
Rule. Nine times the cube of the diameter being divided by 64 , will express the required weight in pounds.*

1. The diameter of an iron shot is 5 inches; required its weight? $5 \times 5 \times 5=125=$ cube of the ball's diameter.

$$
\text { Then } 125 \times 9 \div 64=17 \frac{37}{64} \text { pounds, the answer. }
$$

2. The diameter of an iron shot being 3 inches; required its weight? Ans. $3 \cdot 8$ pounds.
3. The diameter of an iron shot is $5 \cdot 54$ inches; what is its weight? Ans. 24 pounds.

## PROBLEM II.

- I'o find the werght of a leaden ball, by having its diameter given.

Rule. Multiply the cube of its diameter by 2, and divide the product by 9 , and the quotient will give the weight in pounds. $\dagger$

1. What is the weight of a leaden ball of 5 inches diameter?

$$
5 \times 5 \times 5=125 \text { cube of ball's diameter. }
$$

Then, $125 \times 2 \div 9=250 \div 9=27 \frac{7}{\theta}$ pounds, answer.
2. What is the weight of a leaden ball, whose diameter is 6.6 inches? Ans. $63 \cdot 888$ pounds.
3. What is the weight of a leaden ball, whose diameter is $3 \cdot 5$ inches?
4. What is the weight of a leaden ball, whose diameter is 6 inches? $A n s .48$ pounds.

[^50]$\dagger$ Ibid. 11s.

## PROBLEM III.

Having the weight of an iron ball, to determine its diameter.
Rule. Multiply the weight by $7 \frac{1}{9}$, then take the cube root of the product for the diameter.*

1. What is the diameter of an iron ball, whose weight is 42 pounds? $42 \times 7 \frac{1}{9}=298$ 롱․
Then, $\sqrt[3]{ } 298=6 \cdot 685$ inches, the answer.
2. Required the diameter of an iron ball, whose weight is 24 pounds? Ans. $5 \cdot 54$ inches.
3. What is the diameter of an iron ball, whose weight is 3.8 pounds?

Ans. 3 inches.

## PROBLEM IV.

Having the weight of a leaden ball, to determine its diameter.
Rule. Multiply the weight by 9 , and divide the product by 2 ; and the cube root of the quotient will express the diameter. +

1. What is the diameter of a leaden ball, whose weight is 64 pounds?

$$
64 \times 9=576
$$

$$
\text { Then, } 576 \div 2=288 \text {. }
$$

Hence, $\sqrt[3]{288}=6 \cdot 6$ inches, the answer.
2. Required the diameter of a leaden ball, whose weight is $27 \frac{7}{8}$ pounds?
3. What is the diameter of a leaden ball, whose weight is $63 \cdot 888$ pounds?

Ans. $6 \cdot 6$ inches.

## PROBLEM V.

Having given the external and internal diameter of an iron shell, to find its weight.
Rule. Find the difference between the cubes of the two diameters, and multiply it by 9 ; divide the product by 64 , and the quotient will express the weight in pounds. $\ddagger$

1. What is the weight of an 18 -inch iron bomb-shell, whose mean thickness is $1 \frac{1}{4}$ inches?
$18-2 \frac{1}{2}=15 \frac{1}{2}=$ internal diameter.
Then, $18^{3}=5832$ the cube of external diameter.
$(15 \cdot 5)^{3}=3723 \cdot 875$ the cube of internal diameter. And, $5832-3723 \cdot 875=2108 \cdot 125=$ difference of cubes. Hence, $2108 \cdot 125 \times 9 \div 64=296 \cdot 45$ pounds, the answer.
2. What is the weight of a 9 -inch iron bomb-shell whose mean thickness is $1 \frac{1}{2}$ inch? Ans. $72 \cdot 14$ pounds.
3 What is the weight of an iron bomb-shell, whose external diameter is 9.8 inches, and internal diameter 7 inches?

Ans. $84 \frac{1}{8}$ pounds.

[^51]
## PROBLEM VI.

T'o find how much powder will fill a shell of given dimensions. Rule. Divide the cube of the internal diameter in inches, by $57 \cdot 3$, and the quotient will express the answer.*

1. What quantity of powder will fill a shell, whose internal diameter is 10 inches?

First, $10 \times 10 \times 10=1000=$ cube of diameter. $57 \cdot 3) 1000(17 \cdot 45$ pounds, answer.

| $\frac{573}{4270}$ |
| :--- |
| $\frac{4011}{2590}$ |
| 2292 |
| 2980 |
| 2865 |

## $115, \& c$.

Note.-In some recent works, the cube of the diameter is divided by $59 \cdot 32$, for the weight of powder in pounds.
2. How many pounds of gunpowder are required to fill a hollow shell, whose internal diameter is 13 inches?

Ans. 37 pounds, according to the note. 2 8. Required the number of pounds of powder that will fill a shell, whose internal diameter is 7 inches?

Ans. 6 pounds, by the rule in the text.

## PROBLEM VII.

To find how much powder will fill a rectangular box of given dimensions.
Rule. Multiply the length, breadth, and depth together in inches, and the last result by 0322 , and the last product will give the weight in pounds. $\dagger$

1. How many pounds of powder will fill a rectangular box, whose length is 16 inches, breadth 12 inches, and depth 6 inches?

$$
\begin{aligned}
& 16 \times 12 \times 6=1152=\text { content of the box. } \\
& \text { Then, } 1152 \times \cdot 0322=37.0944 \text {, the answer. }
\end{aligned}
$$

2. How many pounds of powder will fill a rectangular box, whose length is 10 inches, breadth 5 inches, and depth 2 inches?

Ans. $3 \cdot 22$ pounds.
3. How many pounds of powder will fill a rectangular box, whose length is 5 inches, breadth 2 inches, and depth 10 inches? Ans. $3 \cdot 22$ pounds.

## PROBLEM VIII.

Having the length and diameter of a cylinder, to determine how many pounds of gunpowder will fill it.
Rule. Multiply the square of the diameter by the length, and divide the product by 40 , for the weight in pounds. $\ddagger$

1. The diameter of a hollow cylinder is 10 inches, and the length 14 inches; how many pounds will it hold?

$$
\begin{gathered}
10 \times 10=100=\text { square of diameter. } \\
\text { Then, } 100 \times 14=1400 \text {. }
\end{gathered}
$$

Hence, $1400 \div 40=35$ pounds, the answer.
2. The diameter of a hollow cylinder is 5 inches, and its length 40 inches; how much powder will it hold?

Ans. 25 pounds.
3. The diameter of a hollow cylinder is 5 inches, and the length 12 inches; how many pounds will it hold? Ans. $7 \cdot 5$ pounds.

## PROBLEM IX.

To find what portion of a cylinder will be occupied by a given quantity of powder, the diameter of the cylinder being given.
Rule. Multiply the given weight of powder by 40 , and divide the product by the square of the diameter of the cylinder, and the quotient will be the pounds required.*

1. The diameter of a hollow cylinder is 10 inches; how much of it will hold 50 pounds of powder?

$$
50 \times 40=2000 .
$$

Then, $2000 \div 100=20$ inches, the answer.
2. How much of a cylinder of 14 inches diameter will hold 10 pounds of powder?

Ans. 2.05 inches.
3. How much of a cylinder, 12 inches in diameter, will hold 144 pounds of powder? Ans. 40 inches.

## PILING OF BALLS AND SHELLS.

Iron-shot and shells are usually piled in horizontal courses, either in a pyramidical or in a wedge-like form; the base being either an equilateral triangle, a square, or a rectangle.

Those piles whose bases are triangles or squares, terminate in one ball at the top; but piles whose bases are rectangles terminate in a single row of balls.

In triangular and square piles, the number of horizontal rows or courses, is always equal to the number of balls in one side of the bottom row.

And in rectangular piles the number of rows is equal to the nomber of balls in the breadth of the bottom.

Also the number in the top row or edge, is one more than the difference between the length and breadth of the bottom row.

## PROBLEM I.

## To find the number of balls in a triangular.

Rule. Multiply the number in one side of the bottom row, by that number increased by 1 , and the result by the same number increased by 2 ; then the one-sixth of the last product will give the number of balls required. $\dagger$

* See Appendix. Demonstration 119.
t Ibid. 120.
$x=n^{\circ}$ m berta now
$\therefore$ numberoshol $=n(n+1)(n+2)$

1. Required the number of shot in a complete triangular pile, one if whose sides contains 22 balls?

$$
22=\text { the number in one side of base. }
$$

$$
23=\text { the number }+1 .
$$



$$
\frac{\overline{1012}}{\frac{6) 12144}{2024}} \text { the number of shot in the pile. }
$$

2. Required the number of shot in a complete triangular pile, one side of whose base contains 15 balls? Ans. 680 balls.
3. Required the number of balls in a triangular pike, each side of the base containing 30 balls? Ans. 4960.

## PROBLEM II.

To find the number of balls in a square pile.
Rule. Multiply continually together the number in one side of the bottom course, that number increased by 1 , and double the same number increased by 1 ; then one-sixth of the last product will be the answer.*

1. How many balls are in a square pile of 30 rows?

$$
30=\text { number in one side. }
$$

$31=$ number in one side +1 .
$61=$ twice the number in one side +1.

$$
6) 56730
$$

9455 answer.
2. Required the number of shot in a complete square pile, one side of whose base contains 19? Ans. 2470.
3. How many shot in a finished square pile, when a side of the base contains 21 shot?

Ans. 3311.

## PROBLEM III.

To find the number of shot in a finished rectangular pile.
Rule. Add 1 to three times the number of shot contained in the length of the base, subtract the number of shot in the breadth of the base, multiply the remainder by the said number increased by 1 , and this result again by the number in the breadth; then one-sixth of the last result will give the number of shot in the rectangular pile. $\dagger$


1. Required the number of shot in a finished rectangular pile, the length of the base containing 59, and its breadth containing 20 balls ? $59=$ the number of shot in the length.

## 3

$\overline{177}$; then $177+1=178$, and $178-20=158$.
$158 \times 21=3318$, and $3318 \times 20=66360$. Hence $66360 \div 6=$ 11060, the answer.
2. How many balls are in a rectangular complete pile, the length of the bottom course being 46, and its breadth 15 ? Ans. 4960.

## PROBLEM IV.

To determine the number of balls contained in a pile which is not finished, the highest course being complete, and the number of balls in each side thereof being given.
Rule. Find the number of shot which would be contained in the pile if it were complete. Find also the number in that complete pile, each side of whose base contains one shot fewer than the corresponding side of the uppermost course of the unfinished pile, and the difference between these results will evidently give the number of balls in the unfinished pile.

1. How many shot are there in an unfinished triangular pile, a side of whose base contains 23 , and a side of the uppermost course 7 shot?

$$
\begin{aligned}
& \begin{array}{l}
23=\text { number of balls in the base. } \\
\frac{24}{552} \\
=\text { number of balls in the base }+1 . \\
\text { 6) } \frac{25}{\frac{2500}{2300}}
\end{array}=\text { number of the pile when complets. } \\
& \frac{6}{\frac{7}{42}} \\
& \frac{8}{6) 336} \\
& \text { Therefore, } 2300-56=2244 \text {, the answer. }
\end{aligned}
$$

2. How many balls in an incomplete square pile, the side of the base being 24 , and of the top 8 ?
3. How many balls are there in the incomplete rectangular pile of 12 courses, the length and breadth of the base being 40 and 20 ?

## DETERMINING DISTANCES BY SOUND.

The velocity of sound, or the space through which it is propagated in a given time, has been very differently estimated by philosophers who have written on this subject. We shall, however, take it to be 1142 feet in a second.

From repeated experiments it has been ascertained that sound moves uniformly, or to speak more philosophically, that the pulses of air which excite it move uniformly. The velocity of sound is the same with that of the aërial waves, and does not vary much whether it go with the wind or against it. By the wind, no doubt, a certain quantity of air is carried from one place to another, and the sound is somewhat accelerated while its waves move through that part of the air, if their direction be the same as that of the wind. But as the velocity of sound is vastly swifter than the wind, the acceleration it will thereby receive is but inconsiderable, being at most but $\frac{1}{20}$ of the whole velocity.

The chief effect perceptible from the wind is, that it increases and diminishes the space through which sound is propagated. The utmost distance at which sound has been heard is about 200 miles. It is said that the unassisted human voice has been heard from Old to New Gibraltar, a distance of about 12 miles. Dr Derham, placing cannon at different distances, and causing them to be fired off, observed the intervals between the flash and report, by means of which he found the velocity of sound to be as above stated.

1. Having observed the flash of a cannon, I noticed by my watch that 5 seconds elapsed previous to my hearing the report; determine iny distance from the gun.
2. Being at sea I saw the flash of a cannon, and counted 8 seconds between the flash and the report ; required the distance?

Ans. ${ }_{1} \frac{7}{10}$ mile.

## G A U GING.

## SECTION X.

Gauging is the art of measuring the capacities of ressels, such es casks, vats, \&c.

The business of gauging is generally performed by means of two instruments, namely, the gauging or sliding rule, and the gauging or diagonal rod.

## 1. OF THE GAUGING RULE.-LEADBETTER'S.

By this instrument is computed the contents of casks, \&c., after the dimensions have been taken. It is a square rule, having various logarithmic lines on its four faces, and three sliding pieces capable of being moved through grooves in which they fit, in three of these faces.

On the first face are delineated three lines, namely, two markol AB , on which multiplication and division are performed; and the third marked MD, signifies malt depth, and serves to gauge malt. The middle one $\mathbf{B}$ is on the slider, and is a kind of double line, being marked at both edges of the slider, for applying it to both the lines A and MD. These three lines are all of the same radius, or distance from 1 to 10, each containing twice the length of the radius. A and B are numbered and placed exactly alike, each commencing at 1 , which may be either 1 , or $10,100, \& c$., or $\cdot 1$, or $\cdot 01$, or $\cdot 001$, \&c. Whatever the 1 at the beginning is estimated at, the middle division 10 will be 10 times as much, and the last division 100 times as much. But 1 on the line MD is opposite 2220 , or more exactly $2218 \cdot 2$ on the other lines, which number $2218 \cdot 2$ denotes the cubic inch in an imperial malt bushel; and its divisions numbered retrograde to those of A and B. On these two lines are also several other marks and letters; thus on the line A are MB , or sometimes only B , for malt bushel, at the number $2218 \cdot 2$, and A for ale, at 282, the cubic inches in an old ale gallon; and on the line B is $W$, for wine, at 231, the cubic inches in an old wine gallon.

These marks are now usually omitted upon the rule, since the late new act of parliament for uniformity of weights and measures, and G for gallon is put at $277 \cdot 274$ the inches in an imperial gallon,* whether of ale, wine, or spirits.

[^52]On many sliding rules are also found $s i$, for square inscribel, at -707, the side of a square inscribed in a circle whose diameter is $\mathbf{1 ;}$ $8 e$, for square equal, st 886 , the side of a square which is equal to the same circle; and $c$, for circumference, at $3 \cdot 1416$, the circumference of the same circle.

On the second face, or that opposite the first, are a slider and four lines marked D, C, D, E, at one end, and root square, root cube at the other end ; the lines C and D containing, respectively, the squares and cubes of the opposite numbers on the lines $\mathrm{D}, \mathrm{D}$; the radius of D being double to that of $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and triple to that of E ; therefore whatever the first 1 on D denotes, the first on C is its square, and the first on E its cube; that is, if D begin with 1, C and E will begin with 1 ; but if D begin with $10, \mathrm{C}$ will begin with 100 , and E with 1000 ; and so on.

On the line C are marked oc at 0796 , for the area of the circle whose circumference is 1 ; and od, at 7854 , for the area of the circle whose diameter is 1 .

On the line D are marked G S, for gallon square, at 16.65 , and G R for gallon round at 18.789 ; also M S for malt square at 47.097 , and M R for malt round at $53 \cdot 144$.

These are the respective gauge-points for gallons and bushels. The first 16.65 is the side of a square, which at an inoh depth holds a gallon; the second 18.789 , the diameter of a circle, which at an inch depth holds a gallon; the third 47.097 the side of a square, which at an inch depth holds a bushel; the fourth $53 \cdot 144$, the diameter of a circle, which at an inch depth holds a bushel.

On the third face are three lines: one on a slider marked $N$; and two on the stock, marked S S and S L, for segment standing and segment lying, which serve ullaging, standing and lying casks.

And on the fourth side, or opposite face, are a scale of inches, and three other scales, marked spheroid, or 1st variety, 2d variety, 3d variety; the scale for the 4th or conic variety, being on the inside of the slider in the third face. The use of these lines is, to find the mean diameter of casks. On the inside of the two first sliders, besides all those already described, are two other lines, being continued from one slider to the other.

The one of these is a scale of inches, from $2 \frac{1}{2}$ to 36 , and the other is a scale of ale gallons, between the corresponding number 435 and $3 \cdot 61$, which form a table, to shew, in ale gallons, the contents of all cylinders whose diameters are from $12 \frac{1}{2}$ to 36 inches, their common altitude being 1 inch.

[^53]
## VERIE'S SLIDING RULE.

This rule is in the form of a parallelopipedon, and is generally made of box.

1. The line marked A, on the face of this rule, is called Gunter's line, and is numbered $1,2,3,4,5,6,7,8,9,10$. At $2218 \cdot 192$ is fixed a brass pin, marked IM, B, signifying the cubic inches in an imperial bushel; at $277 \cdot 274$ is fixed another brass pin, marked IM, G, denoting the number of cubic inches in an imperial gallon.
2. The line marked B is on the slide, and is divided exactly like that marked A. There is another slide B, on the opposite side, which is used along with this. The slide on the first face is called the second radius, and that on the opposite face, the first radius. The two brass ends, when placed together, make a double radius, numbered from the left hand towards the right. At $277 \cdot 274$, on the second radius, is a fixed brass pin, marked IM, G, denoting the cubic inches in an imperial gallon; at 314 is fixed another brass pin, marked C, signifying the circumference of a circle whose diameter is 1.0 These lines are used and read exactly as the lines A and B on the Carpenter's Rule, which have been already described.
3. The back of one slide or radius, marked B, has the dimensions for imperial gallons, and bushels, green starch, dry starch, hard soap hot, hard soap cold, green soft soap, white soft soap, flint glass, \&c., \&c., as in Table, page 136.

The back of the other slide or radius, marked B, contains the gauge-points corresponding to these divisors, where $S$ denotes squares, and C circles.
4. The line M D on the rule, denoting malt depth, is a line of numbers commencing at $2218 \cdot 192$, and is numbered from the left to the right hand $2,10,9,8,7,6,5 ; 4,3$. This rule is used in maltganging.
5. The two slides B, just described, are always used together, either with the line $\mathrm{A}, \mathrm{MD}$, or the line D , which is on the opposite face of the rule to that already described. The line D is numbered from the left hand towards the right, $1,2,3,31$, to 32 , which is at the right-hand end; it is then continued from the left-hand end of the other edge of the rule, $32,4,5,6,7,8,9,10$. At $16 \cdot 651$ is a brass pin G S, signifying a gauge square, being the square gaugepoint for imperial gallons. At $18 \cdot 789$ is fixed a brass pin, marked G R, denoting gauge round, or circular gauge-point for imperial gallons. At $47 \cdot 097$, M S signifies malt square, the square gaugepoint for malt bushels. At $53 \cdot 144$, M R denotes malt round, the round or circular gauge-point for malt bushels. The line D on this rule is of the same nature as the line marked D on the Carpenter's Rule, which has been already described. The line A and the two slides $\mathbf{B}$, are used together, for performing multiplication, division, simple proportion, \&c.; and the line D, and the same slides B, are used together for extracting the square and cube roots.
6. The other two slides belonging to this rule are marked C , and are divided in the same manner, and used together, like the slides $\mathbf{B}$.

The back of the first slide or radius, marked C, is divided, next the edge, iuto inches, and numbered from the left hand towards the
right $1,2,3,4,5, \& c$., and these inches are again subdivided into ten equal parts. The second line is marked spheroid, and is numbered from the left hand towards the right $1,2,3,4,5,6,7,8$. The third line is marked second variety, and is numbered $1,2,3,4,5,6$. These lines are used, with the scale of inches, for finding a mean diameter.

The back of the second slide or radius, marked C, has several factors for reducing goods of one denomination to others of equivalent values. Thus X. to VI. 6. | signifies that to reduce strong beer at $8 s$. per barrel, to small beer at $1 s .4 d$. you are to multiply by 6. | VI. to X. 17. | signifies that to reduce small beer at $1 s .4 d$. per barrel to strong beer at $8 s$. per barrel, you are to multiply by $\cdot 17$. |C4 $4^{\mathbb{} 2}$ to X. 27. | signifies that 27 is the multiplier for reducing cider at 4s. per barrel to another at $8 s .$, \&c.
7. The two slides C , just described, are always used together, with the lines on the rule marked Seg. St., or S S, segments standing; and Seg. L $y$ or S L, segments lying; for ullaging casks. The former of these lines is numbered $1,2,3,4,5,6,7,8$, which stands at the right-hand end; it then goes on from the left hand on the other edge $8,9,10, \& c$. , to 100 . The latter is numbered in the same manner $1,2,3,4$, which stands at the right-hand end; it then goes on from the left hand on the other edge, $4,5,6,7, \& c$., to 100.

## PROBLEM I.

$$
\begin{gathered}
\text { To find the several multipliers, divisors, and gauge-points, } \\
\text { belonging to the several measures now used. } \\
\text { MULTIPLIERS FOR SQUARES. }
\end{gathered}
$$

As 277.274 solid inches are contained in one imperial gallon, and $2218 \cdot 192$ solid inches in an imperial bushel; then it is obvious that if 1 be divided by $277 \cdot 274$, and $2218 \cdot 192$, respectively, the quotients will be the multipliers for imperial gallons and bushels respectively.

Hence the method of finding the following multipliers is obvious:$277 \cdot 274) 1 \cdot 00000(\cdot 0036065$ multiplier for imperial gallons. $2218 \cdot 192) 1 \cdot 00000(\cdot 0004508$ multiplier for imperial bushels.
Now it is manifest that if the solid inches contained in any vessel be multiplied by the first of these multipliers, the product will be the imperial gallons that vessel will contain; and if multiplied by the other, the product will be the imperial bushels.

## mULTIPLIERS AND DIVISORS FOR CIRCLES.

It has been shewn that when the diameter of a circle is 1 , the area of that circle is $\cdot 785398$, \&c., or $\cdot 7854$, nearly; then by dividing the solid capacity of any figure by $\cdot 7854$, the quotient will be the proper divisor for the square of the diameter of a circular figure. Then to reduce the area at one inch deep into gallons, divide -7854, or $\cdot 785398$, \&c., by $277 \cdot 274$, and $2218 \cdot 192$, and the quotients will give the multipliers for imperial gallons and bushels respectively; and $\cdot 7854$ divided into $277 \cdot 274$ and $2218 \cdot 192$, will give the divisors for the imperial gallons and bushels.
$277 \cdot 274$ ) $\cdot 785398(.002832$ multiplier for imperial gallons. $2218 \cdot 192$ ) $785398(.00354$ multiplier for imperial bushels. -785398) $277 \cdot 274(353 \cdot 0362$ divisor for imperial gallons.
-785398) $2218 \cdot 192$ (2824.2897 divisor for imperial bushels.

The gauge-points are found by extracting the square root of tho divisors.

> gauge-points for squares.
$\sqrt{ }$ 277.274 $=16 \cdot 651$ imperial gallons. $\sqrt{ } 2218 \cdot 192=47 \cdot 097$ imperial bushels. GAUGE-POINTS FOR CIRCLES.
$\sqrt{ } 353.0362=18 \cdot 789$ imperial gallons.
$\sqrt{ } 2824 \cdot 2897=53 \cdot 144$ imperial bushels.
In this manner the numbers in the following Table were calculated


Note.-It very often happens in the practice of gauging, that when the ons given number is set to the gauge-point on the sliding rule, the other given number will fall off the rule; hence in many cases it will be necessary to find a second, or new gauge-point. The second gauge-points are the square roots of teu times the divisors in the above table. Thus, for squares, the new gaugepoint for imperial gallons is 52.65 , for bushels $148^{\circ} 93$; and for circles, the new garge-point for gallons is $59 \cdot 42$, for malt bushels $168 \cdot 05$.

## PROBLEM II.

## To find the area, in imperial gallons, of any rectulineal plane figure.

Rule. By the rules given in Mensuration of Superficies, find the area of the figure in inches, which being divided by $277 \cdot 274$, or multiplied by 0036065 , will give the area in gallons.*

1. Suppose a back or cooler in the form of a parallelogram to be 100 inches in length, and 40 in breadth; required the area in imperial gallons?
$100 \times 40=4000$ the area in inches, which divided by $277 \cdot 274$, the quotient $14 \cdot 426$ =the number of imperial gallons; or if we multiply 4000 by 0036065 , the product 14.426 is the number of imperial gallons as before.

## BY THE SLIDING RULE.

OnA. On B. On A. On B.

$$
\text { As } 277 \cdot 274: 40:: 100: 14 \cdot 4 \text {, nearly. }
$$

2. If the side of a square be 40 inches, what is the area in imperial gallons? Ans. $5 \cdot 77$ gallons.
3. If the side of a rhombus be 40 inches, and its perpendicular oreadth 37 inches; required its area in wine gallons? Ans. $5 \cdot 41$.
4. What is the area of a square cooler, in imperial gallons, the side being 144 inches? Ans. $74 \cdot 785$.
5. Allowing the side of a hexagon to be 64 inches, and the perpendicular from the centre to the middle of one of the sides 55.42 inches; required its area in imperial gallons and malt bushels?

$$
\text { Ans. }\left\{\begin{array}{c}
38.38 \text { imperial gallons. } \\
4.8 \text { malt bushels. }
\end{array}\right.
$$

## PROBLEM III.

The diameter of a circular vessel being given in inches, to find its area in imperial gallons, \&.c.
Rule. Multiply the square of the diameter by 002832 ; or divide the square of the diameter by 353.036 , the product or quotient will give the area in imperial gallons.

When it is required to find the area in any other denomination than imperial gallons, use the proper multiplier or divisor for the required denomination, as given in the Table, page 136.

[^54]1. The diameter of a circular vessel is 32.6 inches; required the ares in imperial gallons?

$$
\begin{gathered}
(32 \cdot 6)^{2}=1062 \cdot 76 \text {. Then, } \\
1062 \cdot 76 \times \cdot 002832=3 \cdot 01 \text { gallons. } \\
\text { Or, } 1062 \cdot 76 \div 353 \cdot 036=3 \cdot 01 \text {. }
\end{gathered}
$$

bY THE SLIDING RULE.
As $18 \cdot 78$ is the circular gauge-point for imperial gallons, say,
On D. On B. On D. On B.

$$
\text { As } 18 \cdot 78: 1:: 32 \cdot 6: 3
$$

2. If the diameter of a circular vessel be 10 inches, what is the area in imperial gallons?
3. Suppose the diameter of a circular vessel is 30 inches, what is its area in imperial gallons?
4. What is the area in imperial gallons of a round vessel, whose diameter is 24 inches?

Ans. 1'631.

## PROBLEM IV.

Given the transverse and conjugate diameter of an elliptical vessel, to find its area in imperial measure.
Rule. Multiply the product of the two diameters by 002832 ; or divide the product of the two diameters by $353^{\circ} 036$; the product or quotient will give the imperial gallons required.

When any other denomination is required, the proper multiplier or divisor in the table is to be employed.

1. Suppose the longer diameter of an elliptical vessel is 10 , and the shorter diameter 6 , required the area in ale and wine gallons?

$$
\text { Here, } 10 \times 6=60 \text {. }
$$

Then, $60 \times \cdot 002832=\cdot 17$ of a gallon.
2. The transverse or longer diameter of an elliptical vessel is 20 , and the conjugate or shorter diameter 10 inches; what is the area in imperial measure?

On A. On B. On A. On B. As $353: 20:: 10: 566$ of a gallon.
8. Suppose the transverse diameter of an elliptical vessel is 70 inches, conjugate 50 inches; required its area in imperial gallons and malt bushels?

Note.-As vessels are seldom or never made truly elliptical, being generally ovals, the area found by the above rule is not correct, except the vessel be a truly mathematical ellipsis; when the vessel is of an oval form, the area is best found by the method of equidistant ordinates.

Let ABCD be the oval vessel whose area is required, and let AB and CD be the transverse and conjugate diameters, at right angles to each other, the former being $102 \cdot 8$ inches. Divide this transverse ( 102.8 ) by some even number which will leave a small remainder, the quotient will be the distance of the ordinates; which distance may be laid off on both sides of the conjugate diameter
a number of times equal to half the even number by which the transverse was divided ; then with chalk and a parellel ruler, draw the ordinates through the points $1,2,3,4$, \&c. Then, by Problem XXI., Sec. III., the area may be found, which being multiplied or divided by the proper tabular numbers, will give the area in gallons, \&c. Or,


1st, Add together the first and last ordinates.
2 d , Add together the even ordinates, that is, the $2,4,6,8,10$, \&c., and multiply the sum by 4.
$3 d$, Add together the odd ordinates, except the first and last ; that 1 s , add the ordinates, $3,5,7,9, \& \mathrm{c}$., and multiply the sum by 2 .

4th, Multiply the sum of the extreme ordinates by their distance from the curve.

5th, Add the three first found sums together, and multiply the sum by the common distance of the ordinates, and to the product add the fourth found sum, and divide the total by 3 , and the quotient resulting by $277 \cdot 274$, or $2218 \cdot 192$, for the area in imperial gallons, or malt bushels, respectively.

First, $102 \cdot 8 \div 10=10$ the distance of the ordinates asunder, and the remainder 2.8 is double the distance of the extreme ordinates from the curve; that is, $1 \cdot 4=$ A 1 , or $\mathbf{B} 11$.

Now let us suppose the lengths of the ordinates to be $20,40.2$, $57,66 \cdot 6,73,75,73,66 \cdot 6,57,40 \cdot 2,20$, respectively beginning at 1 , and proceeding to 11.

$$
\begin{aligned}
& \text { 1st, }\left\{\begin{array}{l}
\left\{\begin{aligned}
& 1=20 \\
& 11=20 \\
& 40 \\
& \text { inches, sum of the first and lsst. }
\end{aligned}\right. \\
\frac{1 \cdot 4}{56}
\end{array}\right. \\
& \text { 2d, }\left\{\begin{array}{l}
2=40 \cdot 2 \\
4=66 \cdot 6 \\
6=75 \cdot 0 \\
8=66 \cdot 6 \\
10=40 \cdot 2
\end{array}\right. \\
& \frac{288 \cdot 6 \times 4=1154 \cdot 4}{} \\
& \text { 8d, }\left\{\begin{array}{l}
3=57 \\
5=73 \\
7=73 \\
9=57
\end{array}\right. \\
& \frac{260}{262} \times 2=520
\end{aligned}
$$

Then, $40+1154 \cdot 4+520=1714 \cdot 4$ sum of first three sums.
10

## 17144

56
3) 17200
5733.3 ; then,
$5733 \cdot 3 \div 277 \cdot 274=20 \cdot 64$ gallons. $5733 \cdot 3 \div 2218 \cdot 192=2 \cdot 58$ malt bushels.
When the vessel is not circular, or elliptical, it is best to measure the equidistant ordinates, which though ever so unequal, will, by proceeding as above, serve to find the area of the base. Whenever the vessel is an irregular curved figure, the area should be invariably found by the method of equidistant ordinates, as the true result cannot be found by any other method.
4. What is the area, in imperial measure, of an ellipse, whose . transverse axis is 24, and conjugate 18? Ans. 1.2234 gallons.

## PROBLEM V .

## To find the content of a prism, in imperial gallons.

Rule. Find the area of the base, by Problem II. in Gauging, which being multiplied by the depth within, will give the content in gallons.

Or, find the solid content by mensuration, and divide that content by $277 \cdot 274$ for imperial gallons.

A vessel, whose base is a right-angled parallelogram, is 49.3 inches in length, the breadth 36.5 inches, and the depth 42.6 inches; required its content in imperial gallons?

Here, $49 \cdot 3 \times 36.5 \times 42 \cdot 6=76656.57$.
Then, $76656 \cdot 57 \div 277 \cdot 274=276 \cdot 465$ gallons.
And $76656 \cdot 57 \div 2218 \cdot 192=34 \cdot 558$ malt bushels.
BY THE SLIDING RULE.

$$
\begin{aligned}
& \text { On B. On D. On B. } \\
& 49 \cdot 3: 49 \cdot 3:: 36 \cdot 5: 42 \cdot 42 .
\end{aligned}
$$

On D. On B. On D.

$$
\left.\begin{array}{l}
16 \cdot 65 \\
46 \cdot 37
\end{array}\right\}: 42 \cdot 6:: 42 \cdot 42:\left\{\begin{array}{l}
27 \cdot 6 \text { gallons. } \\
34 \cdot 5 \text { malt bushels. }
\end{array}\right.
$$

2. Each side of the square base of a vessel is 20 inches, and its depth 10 inches, what is the content in old ale gallons?

Ans. $14 \cdot 28$ gallons.
3. The side of a vessel in the form of a rhombus is 20 inches, breadth 15 inches, and depth 10 inches; required the content in old ale gallons? Ans. 10.638 gallons.
4. What is the content, in old wine gallons, of a vessel in the form of a rhomboid, whose longest side is 20 inches, breadth from side to side 8 inches, and depth 10 inches? Ans. 6.88 wine gallons.

## PROBLEM VI.

To find the content of any vessel, whose ends are squares or rectangles, of any dimensions.
Rule. Multiply the sum of the lengths of the two ends, by the sum of their breadths, to which add the areas of the two ends; this sum, multiplied by one-sixth of the depth, will give the solidity in cubic inches; then divide by $277 \cdot 274$, or $2218 \cdot 192$ for the content in imperial gallons, or malt bushels.

1. Suppose the top and bottom of a vessel are parallelograms, the length of the top is 40 inches, and its breadth 30 inches; the length of the bottom is 30 inches, and its breadth 20 ; and the depth 60 inches; required the contents in imperial gallons?

$$
\begin{aligned}
& \begin{array}{l}
40+30=70 \text { sum of the lengths. } \\
30+20=50 \text { sum of the breadths. } \\
\hline 3500 \\
\text { product. } \\
30 \times 30=1200 \text { area of the greater base. } \\
30 \times 20=\frac{600}{5300} \text { area of the lesser base. } \\
\frac{10}{53000} \text { solidity in cubic inches. } \\
\text { Then, } 53000 \div 277 \cdot 274=191 \cdot 146 .
\end{array} .
\end{aligned}
$$

## BY THE SLIDING RULE.

Find a mean proportional $(\sqrt{ }(40 \times 30)=34 \cdot 64$, $)$ between the length and breadth at the top, and a mean proportional $(\mathcal{V}(30 \times 20)$ $=24 \cdot 49$, ) between the length and breadth at the bottom; the sum of these is $59 \cdot 13$, twice a mean proportional between the length and breadth in the middle. Then,

On D. On B. On D. On B.
$16 \cdot 65: \frac{60}{6}::\left\{\begin{array}{l}34 \cdot 64: \text { च } \\ 24 \cdot 49 \\ 59 \cdot 13\end{array}\right\}$ —— sum $191 \cdot 146$ imperial gallons.
2. Suppose the top and bottom of a vessel are parallelograms, the length of the top is 100 inches, and its breadth 70 inches; the length of the bottom 80 , and its breadth 56, and the depth 42 inches; what is its content in imperial gallons? Ans. $862 \cdot 59$ imperial gallons.

## THE GAUGING OR DIAGONAL ROD.

The diagonal rod is a square rule, having four faces, and is generally 4 feet long. It folds together by joints. This instrument is employed both for gauging and measuring casks, and computing their contents; and that from one dimension only, namely, the diagonal of the cask, or the length from the middle of the bung-hole to the meeting of the cask with the stave opposite the bung; being
the longest line that can be drawn from the middle of the bung-hole to any part within the cask.

On one face of the rule is a scale of inches for measuring this diagonal ; to which are placed the areas, in ale gallons, of circles to the corresponding diameters, in like manner as the lines on the under sides of the three slides in the Sliding Rule.

On the opposite face, there are two scales of ale and wine gallons, expressing the contents of casks having the correspondent diagonals.

All the other lines on the instrument are similar to those on the Sliding Rule, and are used in the same manner.

Example. The diagonal, or distance between the middle of the bung-hole to the most distant part of the cask, as found by the diagonal rod, is $34 \cdot 4$ inches; what is the content in gallons?

To $34 \cdot 4$ inches correspond, on the rod, $90 \frac{3}{4}$ ale gallons, or 111 wine gallons, $92 \frac{1}{2}$ imperial gallons, the content required?
Note.-The contents shewn by the rod answer to the most common form of casks, and fall in between the 2 d and 3 d varieties following.

## OF CASKS AS DIVIDED INTO VARIETIES.

Casks are usually divided into four varieties, which are easily distinguished by the curvature of their sides.

1. The middle frustum of a spheroid belongs to the first variety.
2. The middle fristum of a parabolic spindle belongs to the second variety.
3. The two equal frustums of a paraboloid belong to the third variety.
4. And the two equal frustums of a cone belong to the fourth variety.

If the content of any of these be found in inches by their proper rules, and this divided by $277 \cdot 274$, or $2218 \cdot 2$, the quotient will bo the content in imperial gallons, or bushels, respectively.

## PROBLEM VII.

To find the content of $a$ vessel in the form of the frustum of a cone.
Rule. To three times the product of the two diameters add the square of their difference; multiply the sum by one-third of the depth, and divide the product by 353.0362 for imperial gallons, and by $2824 \cdot 289$ for malt bushels.

1. What is the content of a cone's frustum, whose greater diameter is 20 inches, least diameter 15 inches, and depth 21 inches?

$$
\begin{aligned}
& \begin{array}{l}
20 \times 15 \times 3 \\
20-15=5 \text { and } 5^{2}
\end{array}=\frac{900}{} \quad \begin{aligned}
925
\end{aligned} \times 7=6475 . \text { Then, } \\
& 353 \cdot 0362) 6475(18 \cdot 34 \text { imperial gallons. } \\
& 294 \cdot 12) 6475(22.01 \text { wine gallons. }
\end{aligned}
$$

2. The greater diameter of a conical frustum is 38 inches, the less diameter 20.2 , and depth 21 inches; what is the content in old ale gallons?

## PROBLEM VIII.

## To find the content of the frustum of a square pyramid.

Rul.f. To three times the product of the top and bottom sides, add the square of their difference, multiply their sum by one-third of the depth, and divide the product by 282 and 231 , for old ale and wine gallons, respectively; and by $277 \cdot 274$, for imperial gallons.

1. Suppose the greater base is 20 inches, the less base 15 inches, and depth 21 inches; required the content in old wine measure?

$$
\begin{aligned}
& 20 \times 15 \times 3=900 \\
& 20-15=5 \\
& \text { Then, } 5 \times 5=\frac{25}{925} \times 7 \div 231=27 \cdot 8 \text { gallons. }
\end{aligned}
$$

Note.-The content of the frustum of a pyramid is found just like that of a onne, with the exception of the tabular divisor, or multiplier, the cone requiring the circular factor, and the pyramid the square one.

## PROBLEM IX.

## To find the content of a globe.

Rule. Multiply the diameter of the globe by its cirsumference, and the resulting product by one-sixth of the diameter; then the last product multiplied or divided by the circular factor, will give the content in gallons.

1. Let the diameter be 34 inches, what is its content?

$$
34 \times 34 \times 34 \times 5236=20579 \cdot 5744
$$

Then, $20579 \cdot 5744 \div 282=72 \cdot 9772$ old ale gallons. And, $20579 \cdot 5744 \div 231=89 \cdot 08$ old wine gallons.
Rule. II. Or cube the diameter of the globe, which multiply by $\cdot 001888$ ( $\frac{2}{3}$ of $\cdot 002832$ ) for the content in imperial gallons.
$34^{3}=39304$; then $39304 \times \cdot 001888=74 \cdot 2$ imperial gallons.
2. What is the content of a globe in old ale and wine measare, the diameter being 20 inches?

$$
\text { Ans. }\left\{\begin{array}{l}
14.848 \text { old ale gallons. } \\
18.128 \text { old wine gallons. }
\end{array}\right.
$$

3. Required the content of a globular vessel, whose diameter is 100 inches?

An's. $1888 \frac{1}{3}$ imperial gallons.

## PROBLEM X.

To find the content of the segment of a sphere, as the rising crowr of a copper still, \&c.
Rule. Measure the diameter, or chord of the segment, and the altitude just in the middle. Multiply the square of half the diameter by 3 ; to the product add the square of the altitude; multiply this sum by the altitude, and the product again by 001856 , or 002266 , for old ale or wine measure, respectively, and by 001888 for imperial gallons.

1. The diameter of the crown of a copper still is $27 \cdot 6$, its depth $9 \cdot 2$; required its content?

Here $27 \cdot 6 \div 2=13 \cdot 8$.
Then $13.8 \times 13.8 \times \quad 3=571.32$
$9.2 \times 9.2=84.64$
$\overline{655 \cdot 96}$ sum.
$9 \cdot 2$ depth.
$6034.832 \times \cdot 001888=13 \cdot 39$ imperial gallons.

## PROBLEM XI.

To gauge a copper having either a concave or convex bottom; or what is called a falling bottom, or rising crown.
Rule. If the side of the vessel be straight, with a falling bottom, find the content of the segment $\mathbf{C} y \mathrm{D}$, by Prob. X.; find also the content of the upper part ABDC, by Prob. VII.; the sum of both will give the content of the зopper.

When the copper has a risang crown, find the content of ABCD, by Prob. VII.; from which deduct the content of the
 segment $\mathrm{C} x \mathrm{D}$, and the remainder will be the content of the vessel ABD $x$ C.

## PROBLEM XII.

To gauge a vessel whose side is curved from top to bottom.
Take the diameters at equal distances of $2,3,4$, or 5 inches, according as the case may require; if the side of the vessel be considerably curved, the number of diameters that will be required will be considerable; the less the curvature of the side, the less the number of diameters that will be required.

To gauge the vessel, or copper, ABDC, fasten a piece of pack-thread at $A$ and $B$, as AFB; then with some convenient instrument find the distance $a \mathrm{C}$ of the deepêst part of the copper, which let us suppose to be 47 inches.

By means of the same in-
 strument measure the distance of from the top of the cruwn to $F$ the middle of AB ; which let us suppose to be 42 inehes, this deducted from $a \mathrm{C}, 47$, will leave $5(=o \mathrm{G})$ the height of the crown.

To find the diameter CD, of the bottom of the crown.
Measure the top diameter AB, which suppose to be 99 inches: then hold a thread, so that a plummet attached to the end thereof may hang just over C , and measure $\mathrm{A} a=\mathrm{BF}$, each of which let us
admit to be 17.5 inches; add these together, and deduct their sum (35) from 99, and the remainder (64) will evidently be equal to C D, the diameter at the bottom of the crown. Measure the diameter $m o n$, which touches the top of the crown, which suppose is 65 inches.

Now, as this copper is not considerably curved, the diameters may be taken in the middle of every 6 inches of the depth, which suppose to be as in the second column of the following table; to each diameter find the area in imperial gallons, by Prob. III., which write in the third column; find also the content of every 6 inches, corresponding to these diameters, which write in the fourth column of the table; lastly, find the content of the crown by Prob. X., and subtract it from the content of ABDGC , the remainder will give the capacity of the copper.

Or thus, CD being 64 inches, the area answering to it is 11.6022 ; this multiplied by half the altitude of the crown, viz., by $2 \cdot 5$, gives .29 .0055 gallons, the content of the crown. The content of the part $m n \mathrm{DC}$ is 58.9222 gallons, from which the content of the crown being deducted, the remainder ( $29 \cdot 9167$ gallons) is the quantity of kquor which covers the crown.

| Parts of <br> the depth. | Diameters. | Areas. | Content of every 6 inches. |
| :---: | :---: | :---: | :---: |
| 6 | $95 \cdot 3$ | 25.7257 | $154 \cdot 3542$ |
| 6 | $90 \cdot 1$ | $22 \cdot 9948$ | $137 \cdot 9688$ |
| 6 | 85. | $20 \cdot 4653$ | $122 \cdot 7918$ |
| 6 | 80. | 18•1284 | 108.7704 |
| 6 | $75 \cdot 2$ | 16.0183 | 96.1098 |
|  | $70 \cdot 5$ | 14.0786 | $84 \cdot 4716$ |
| 6 | 66. | 12.3387 | 74.0322 |
| The sum To cover crown |  |  | $778 \cdot 4988$ |
|  |  |  | $29 \cdot 9167$ |
| The whole content |  |  | $808 \cdot 4155$ |

## PROBLEM XIII.

## To find the content of any close cask.

Whatever be the torm of the cask, the following dimensions must be taken; that is,

$$
\left.\begin{array}{l}
\text { The bung diameter, } \\
\text { The head diameter, } \\
\text { The length of the cask, }
\end{array}\right\} \text { within. }
$$

On account of the difficulty in ascertaining the figure of the cask: it is not, in many cases, easy to find the exact contents of casks.

In taking the dimensions of a cask, it is essential that the bunghole be in the middle of the cask, and also that the bung-stave, and the stave opposite to it, are both regular and even within.

It is likewise essential that the heads of casks are equal and truly circular; and if so, the distance between the inside of the chimb to the outside of the opposite stave will be the head diameter within the cask, nearly.

From the variety in the forms of casks, no general rule could be given to answer every form ; two casks may have equal head diameters, equal bung diameters, and equal lengths, and yet their contents may be very unequal.

## PROBLEM XIV.

## To find the content of a cask of the first variety.

Rule. To the square of the head diameter add double the square of the bung diameter, and multiply the sum by the length of the cask. Then multiply the last product by $\cdot 0009 \frac{4}{9}$, or divide by $1059 \cdot 1$, the product or quotient will be the content in imperial gallons.

1. What is the content of a spheroidal cask, whose length is 40 inches, bung diameter 32 inches, and head diameter 24 inches?
$24 \times 24=576$
$32 \times 32=1024$

by the gauging rule.
Set 40 on C, to the G R 18.79 on D, against
24 on D, stands
64.99 on C,

32 on D, stands
116.2 on C ,
$+1162$
3) $297 \cdot 39$
$99 \cdot 13$ gallona
2. What is the content of a spheroidal cask, whose length is 20 inches, bung diameter 16 inches, and head diameter 12 inches?

$$
\text { Ans. }\left\{\begin{array}{l}
12.36 \text { old ale gallons. } \\
14.869 \text { old wine gallons. }
\end{array}\right.
$$

To find the content of a cask by the mean diameter.
Rule. Multiply the difference of the head and bung diameters by $\cdot 68$ for the first variety; by 62 for the second variety; by ${ }^{5} 55$ for the third; and by 5 for the fourth, when the difference between the head and bung diameters is less than 6 inches; but when the difference between these exceeds 6 inches, multiply that difference by 7 for the first variety; by 64 for the second; by $\cdot 57$ for the third; and by $\cdot 52$ for the fourth. Add this product to the head diameter, and the sum will be a mean diameter. Square this mean diameter, and multiply the square by the length of the cask; this product multiplied, or divided, by the proper multiplier or divisor, will give the content.

By resuming the last example but one, we have
Bung diameter 32
Head diameter 24


Mean diameter $\overline{29 \cdot 6}$
In the same manner the content for the second variety will be 94.46 ale gallons; for the third variety 90.87 ale gallons; and for the fourth variety 83.34 gallons.

## PROBLEM XV.

To find the content of a cask of the second variety.
Rule. To the square of the head diameter add double the square of the bung diameter, and from the sum deduct two-fifths of the square of the difference of the diameters; multiply the remainder by the length, and the product again by $0009 \frac{4}{9}$ for the content in imperial gallons.

1. What is the content of a cask, whose length is 40 inches, bung diameter 32 iuches, and head diameter 24 inches?

$$
\begin{aligned}
& 32-24=8 ; \text { then } 8^{2}=64, \text { and } \frac{2}{5} \text { of } 64=25 \cdot 6 \\
& 24^{2}=576, \text { and } 32^{2}=1024, \text { then } 1024 \times 2=2048 \\
& 2048+576=2624, \text { and } 2624-25 \cdot 6=2598 \cdot 4
\end{aligned}
$$

## PROBLEM XVI.

To find the content of a cask of the third variety.
Rule. To the square of the bung diameter add the square of the head diameter ; multiply the sum by the length, and the last product ly -001416 for the answer in imperial gallons.

Let us resume the last example: thius
\(\left.\begin{array}{rl}32^{2} \& =1024 <br>

24^{2} \& =\frac{576}{1600} \times 40\end{array}\right)=\)| $\frac{601416}{90 \cdot 624}$ imperial gallons. |
| ---: | :--- |



## PROBLEM XVII.

To find the content of a cask of the fourth variety.
Rule. Add the square of the difference of the diameters to 3 times the square of their sum ; multiply the sum by the length, and the last product by -000236 for the content in gallons.

Resuming still the last example, $32+24=$ 56 , and $56^{2} \times 3=3136 \times 3=9408$, and $8^{2}=$ 64 , then $9408+64=9472$; then $9472 \times 40=$ 378880 , and $378880 \times \cdot 000236=89 \cdot 41668$ imperial gallons.


## PROBLEM XVIII.

## To find the content of any cask by Dr Hutton's general rule.

Rule. Add into one sum, 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the two diameters; then multiply the sum by the length, and the product again by $\cdot 00031 \frac{4}{8}$ for the content in gallons.

1. What is the content of a cask, whose length is 40 inches, and the bung and head diameters 32 and 24 ?

| $32^{2}=1024$ | $\begin{array}{r} 24^{2}=576 \\ 25 \end{array}$ | $\begin{array}{r} 32 \times 24=768 \\ 26 \end{array}$ |
| :---: | :---: | :---: |
| $\overline{39936}$ | $\overline{14400}$ | 19968 |
| 14400 |  |  |
| 19968 |  |  |
| $\overline{7430 \frac{1}{2}} \times 40=\begin{array}{r} 2972160 \\ .00031 \frac{4}{9} \end{array}$ |  |  |
|  |  |  |
|  | $93 \cdot 4579$ |  |

## ULLAGING.

## PROBLEM XIX.

## To ullage a lying cask.

This is the finding what quantity of liquor is contained in a cask when partly empty.

To ullage a lying cask, the wet and dry inches must be known, as also the content of the cask and bung diameter.

Rule. Take the wet inches, and divide them by the bung diameter; find the quotient in the column of versed sines, in the Table at the end of the practical part of this book, and take out its corresponding segment; multiply this segment by the whole content of the cask, and the product arising by $1 \frac{1}{4}$ for the ullage required, nearly.

1. Find the ullage for 8 wet inches, the bung diameter being 32 inches, and the
 content 92 ale gallons?
$32) 8(\cdot 25$, whose tabular segment is $\cdot 153546$.
Then, $\cdot 153546 \times 92=14 \cdot 126232$.
And, $14 \cdot 126232 \times 1 \frac{1}{4}=17 \cdot 65779$ gallons.

## PROBLEM XX.

To ullage a standing cask.
Rule. Add together the square of the diameter at the surface of the liquor, the square of the diameter of the nearest end, and the square of double the diameter taken in the middle between the other two; multiply the sum by the length between the surface and nearest end, and the product arising by 000472 for the gallons in the less part of the cask whether empty or filled.

1. What is the ullage for 10 wet inches, the three diameters being 24,27 , and 29 inches?

| $24^{2}=576$ | 43330 |
| ---: | :---: |
| $29^{2}=841$ | 000472 |
| $(2 \times 27)^{2}=\frac{2916}{4333}$ | 36660 |
| $\frac{10}{43330}$ | $\frac{1733200}{20.45176}$ gallons. |

## PROBLEM XXI.

To find the content of an ungula, or hoof, of the frustum of a cone.
Rule. For the less hoof, multiply the product of the less diameter and height, by the product of the greater diameter multiplied by a
mean proportional between both diameters, less the square of the less diameter, and this last divided by three times the circular factor multiplied by the difference of the diameters, gives the content of the less hoof.

1. $\mathrm{CD}=30, \mathrm{AB}=40, \mathrm{C} d=20$; required the content of the less hoof?
$40 \times 30=1200$, and $\sqrt{ } 1200=34 \cdot 6$ mean.
$30 \times 20=600,1$ st product.
$40 \times 34 \cdot 6=1384,2 \mathrm{~d}$ product.
$30 \times 30=900$
"

Rule. For the greater hoof multiply the product of the greater diameter and the height of the frustum, by the square of the greater diameter made less by the product of the less diameter multiplied by a mean proportional between those diameters; this remainder, divided by three times the circular divisor multiplied by the difference of the diameters, gives the content of the greater hoof.

Resuming the last example, we have

$$
\begin{gathered}
40 \times 40=1600 \\
20 \times 40=800,1 \text { st product. }
\end{gathered}
$$

$$
40 \times 30=1200, \text { and } \sqrt{ } 1200=34 \cdot 6
$$

$$
34 \cdot 6 \times 30=1038,2 \mathrm{~d} \text { product. }
$$

$$
40-30=10
$$

Then $1600-1038=562$
800
$3 5 9 \times 3 \times 1 0 = 1 0 7 7 0 \longdiv { 4 4 9 6 0 0 }$, last product.
$41 \cdot 74$ old ale gallons.

## PROBLEM XXII.

## 'I'o gauge a still.

Fill the still with water, and draw it off in another vessel of some regular form, whose content is easily computed. This is by far the most accurate method that can be employed.

Or gauge the shoulder by itself, and gauge the body by taking a greater number of diameters at near and equal distances throughout, first covering the bottom, if there be any cavity, with water, the uantity of which is known,

## LAND-SURVEYING.

## SECTION XI.

Land-surveying is that art which enables us to give a true plan or representation of any field or parcel of land, and to determine the superficial content thereof.

In measuring land, the area or superficial content is always expressed in acres, or in acres, roods, and perches; each acre containing 4 roods, and each rood 40 perches.

Land is measured with a chain, called Gunter's chain, of 4 poles or 22 yards in length, which consists of 100 equal links, each link being $\frac{22}{100}$ of a yard long, or $\frac{68}{100}$ of a foot, or 7.92 inches. 10 square chaius, o: 10 chains in length and 1 in breadth, make an acre; or 4840 square yards, 160 square poles, or 100,000 square links make an acre. The length of lines measured with a chain are generally set down in links as integers ; every chain being 100 links in length. Therefore, after the content is found, it will be in square links, and as 100,000 square links make an acre, it will be necessary to cut off five of the figures on the right-hand for decimals, and the rest will be acres. The decimals are reduced to roods by multiplying by 4 , and cutting off five figures as before for decimals, which decimal part is reduced to perches by multiplying by 43 , and cutting off five figures from the product. As an example:-

Suppose the length of a rectangular piece of ground to be 792 links, and its breadth 385 ; required the number of acres, roods, and perches it contains?

| 792 |  |
| :--- | ---: |
| 385 |  |
| 3960 |  |
| 6336 |  |
| 2376 |  |
| 304920 |  |
| 19680 |  |
| 40 |  |

Ans. 3 acres, 0 roods, 7 perches.
The statute perch is $5 \frac{1}{2}$ yards, but the Irish plantation perch is 7 yards; hence the length of a plantation link is 10.08 inches.

## PROBLEM I.

To measure a line or distance on the ground, two persons are employed; the foremost, for the sake of distinction, is called the leader, and the hindermost, the follower.
Ten small arrows or rods, to stick in the ground at the end of each
chain, are provided; also some station-staves, or long poles with coloured flags, to set up in the direction of the line to be measured, if there do not appear some marks naturally in that direction.

The leader takes the 10 arrows in one hand, and one end of the chain by the ring, in the other; the follower stands at the beginning of the line, holding the ring at the end of the chain in his hand, while the leader drags forward the chain by the other end of it, till it is stretched straight, and the leader directed by the follower, by moving his hand, to the right or left, till the follower see him exactly in a line with the mark or direction to be measured to; then both of them holding the chain level and stretched, the leader sticks an arrow upright in the ground, as a mark for the follower to come to, and advances another chain forward, being directed in his position by the follower standing at the arrow, as before, as also by himself, now and at every succeeding chain's length, by moving himself from side to side, till the follower and back-mark be in a direct line. Having then stretched the chain, and stuck down an arrow, as before, the follower takes up the arrow, and thus they proceed till the 10 arrows are employed, or in the hands of the follower, and the leader, without an arrow, is arrived at the end of the eleventh chain-length. The follower then sends or brings the 10 arrows to the leader, who puts one of them down at the end of his chain, and advances with his chain, as before. And thus the arrows are changed from one to the other at every 10 chains' length, till the whole line is finished, if it exceed 10 chains; and the number of changes shews how many times 10 chains the line contains, to which the follower adds the arrows he holds in his hand, and the number of links of another chain over to the mark or end of the line. Thus, if the whole line measure 36 chains 45 links, or 3645 links, the arrows have been changed three times, the follower will have arrows in his hand, the 5 leader 4 , and it will be 45 links from the last arrow, to be taken up by the follower, to the end of the line.

In works on Surveying, it is usual to describe the various instruments used in the art. The pupil, however, will best learn the use of these instruments when actually engaged in the practice. The chief instruments employed are the chain, the plane table, the theodolite, the cross, the circumferentor, the offset staff, the perambulator, used in measuring roads, and other great distances.

Levels, with telescopic or other sights, are used to find the levels between two or more places, or how much one place is higher or lower than the other.

Besides all these, various scales are used in protracting and measuring on paper ; such as plane scales, line of chords, protractor, compasses, reducing scales, parallel and perpendicular rulers, \&c.

## THE FIELD-BOOZ.

In surveying with the plane table, a field-book is not required, as everything is drawn on the table immediately when it is measured. But when the theodolite, or any other instrument, is used, some sort of a field-book is used in order to register all that is done relative to the survey in hand. This book every one contrives and rules as he
thinks fit. It is, however, usually divided into three columns. The middle column contains the different distances on the chain-line, angles, bearings, \&c., and the columns on the right and left are for the offsets on the right and left, which are set against their corresponding distances in the middle column; as also for such remarks as may occur, and may be proper to note in drawing the plan; such as houses, ponds, castles, churches, rivers, trees, \&c. \&c.

But in smaller surveys, an excellent way of setting down the work is, to draw by the eye, on a piece of paper, a figure resembling that which is to be measured; and then write the dimensions, as they are found, against the corresponding parts of the figure. This method may be practised even in larger surveys, and is far superior to any other at present practised. A specimen of this plan will be seen further on.

FORM OF THE FIELD-BOOK.

| Offsets and remarks on the left. | Stations, Bearings, and Distances. | Offsets and remarks on the right. |
| :---: | :---: | :---: |
| Cross a hedge 24 a brook 30 | $\square 1$ $104^{\circ} 25^{\prime}$ 00 67 120 734 954 736 | Brown's barn. ט. : ! 11 Tree. 67 stile. |
| House corner 61 Foot path 15 | $\begin{gathered} 82 \\ 62^{\circ} 25^{\prime} \\ 00 \\ 40 \\ 67 \\ 84 \\ 95 \\ 467 \\ 976 \end{gathered}$ | 44 <br> 14 Spring. |
| Clayton's hedge 24 | $\begin{gathered} \square 3 \\ 54^{\circ} 17^{\prime} \\ 62 \\ 124 \\ 630 \\ 767 \\ 767 \\ 305 \\ 760 \end{gathered}$ | 20 Pond. <br> 30 Stile. |

In this form of a field-book $\square 1$ is the first station, where the angle or bearing is $104^{\circ} 25^{\prime}$. On the left, at 67 links in the distance or principal line, is an offset of 24 ; and at 120 an offset of 30 to a brook on the right; at 67 Brown's barn is situated; at 954 is an offset of 20 to a tree, and at 736 an offset to a stile.
And so on for the other stations.
A line is drawn under the work, at the end of every station, to prevent confusion.

## PROBLEM II.

## To take angles and bearings.

Let it be required to take the bearings of the two objects B, C, from the station A.

In this problem it is required to measure the angle at A, formed by two lines, passing from the station
 A, through two objects B and C.

> 1. By measurement with the chain, \&c.

Measure with the chain any distance along the two lines AB, AC, as $\mathrm{A} b, \mathrm{~A} c$; then measure the distance $b c$; and this being done, transfer the three sides of the triangle Abc to paper, on which measure the angle $c$ A $b$, as in Problem XV., Practical Geometry.

## 2. With the magnetic needle and compass.

Turn the iustrument, or compass, so that the north end of the needle may point to the flower-de-luce. Then direct the sights to a mark at B, noting the degrees cut by the needle. Next direct the sights to another mark at C, noting the degrees cut by the needle as before. Then their sum or difference, as the case may be, will givo the number of degrees in the angle CAB.

## 3. With the theodolite, \&c.

Direct the fixed sights along the line AB , by turning the instrument about till you see the mark B through these sights, and in that position screw the instrument fast. Then turn the moveable index about till, through its sights, you see the other mark C. Then the degrees cut by the index, on the graduated limb or ring of the instrument, shew the number of degrees in the angle C A B.

## 4. With the plane table.

Having covered the table with paper, and fixed it on its stand, plant it at the station A, and fix a fine pin, or a point of the compass, in a proper point of the paper, to represent the station A. Close by the side of this pin, lay the fiducial edge of the index, and turn it about, still touching the pin, till one object B can be seen through the sights; then by the fiducial edge of the index draw a line. By a similar process draw another line in the direction of the object $C$. And it is dune.

## PROBLEM III.

## To measure the offsets.

Let Abcdefg be a crooked hedge, river, or brook, \&c., and A G a base line.

Begin at the point A, and measure towards $G$; and when yon come opposite any of the corners $b c d, \& c$., which is ascertained by

means of the cross-staff, measure the offsets $\mathrm{B} b, \mathrm{C} c, \mathrm{D} d, \& c$., with the chain, and register the dimension, as in the annexed field-book.

FIELD-BOOK.

| 91 | $785=$ A G. | $=$ |
| :---: | :---: | :---: |
| 57 | 634 |  |
| 98 | 510 | - |
| 70 | 340 | - |
| 84 | 220 | - |
| 62 | $\square 5$ |  |
| - | $\square$ A go North. | - |
| Offsets <br> Left. | Base line A G, or <br> $\square$ Station. | Offsets <br> Right. |

## Lo lay down the plan.

Draw the line AG of an indefinite length; then by a diagonal scale, set off AB equal to 45 links; at B erect the perpendicular B $b$ equal to 62 links taken from the same scale. Next set off A C equal to 220 links, or 2 chains 20 links, and at C erect the perpendicular C $c$, equal to 84 links; in the same way set off AD equal to 340 links, or 3 chains 40 links, and at D erect the perpendicular $\mathrm{D} d$ equal to 70 links. Proceed in a similar manner with the remaining offsets, and straight lines joining the points $\mathrm{A}, b, c, d, e$, \&c., will complete the figure.

> To find the content.

Some authors direct to add up all the perpendiculars $\mathrm{B} b, \mathrm{C} c, \& c$., and divide their sum by the number of them, then multiply the quotient by the length A G. This method, however, should never be used, except when the offsets $\mathrm{B} b, \mathrm{C} c, \& c$., are equally distant from each other.

When the offsets are not equally distant from each other, which indeed is generally the case, this method is crroneous; therefore the following method ought to be employed.

Find the content of the space AB b as a triangle, by Problem V., Section II. Find the contents of the figures BC cb, CD dc,\&c., as trapezoids, by Problem XIII., Section II., the sum of all these separate results will be the content of the figure $\mathrm{A} \mathrm{G}_{g f}$ ed $c b \mathrm{~A}$.

The actual calculation is as follows:-

## calculation.



These respective products are evidently double the true contents of the respective figures $\mathrm{AB} b, \mathrm{BC} c b, \mathrm{CD} d c, \& \mathrm{c}$., that is,
$2790=$ double area of AB $b$.
$25550=$ double area of $\mathrm{BC} \boldsymbol{c} b$.
$18480=$ double area of CD de.
$28560=$ double area of DE ed.
$19220=$ double area of E F $f e$.
$22348=$ double area of $\mathrm{FG} g f$.
2) $\overline{116948}=$ double area of the whole in square links.
$\overline{58474}=$ area in square links.
$\overline{58474}=$ area in acres $=0$ A., 2 R., 13.5584 P .
2. Required the plan and content of part of a field, from the following field-book:-


Ans. 0A., 2R., 12P.


## PROBLEM IV.

To measure a field of a triangular form.-1. By the chain.
Set up marks at the three corners A, B, C, and measure with the chain, the distance $\mathrm{AD}, \mathrm{D}$ being the point at which a perpendicular demitted from C , would meet the line AB ; measure also the distance D B; hence you have the measure of A B. Next measure the perpendicular DC; then from the two dimensions A B and D C, the content may be found by Problem IV., Section II.


$$
\text { Let } \mathrm{AD}=794, \mathrm{~A} \mathrm{~B}=1321, \mathrm{D} \mathrm{C}=826 \text { links. }
$$

$$
1321 \times 826 \div 2=545573 \text { links. }
$$

Then $545573 \div 100000=5.45573$ acres.

$$
45573 \times 4=1 \cdot 82292 \text { rQods. }
$$

$$
82292 \times 40=32 \cdot 91680 \text { perches }
$$

Hence the answer is $5 \mathrm{~A} ., 1 \mathrm{R}$., 33P., nearly.
2. What is the area of a triangular field, whose base is 12.25 ehains, and height 8.5 chains? Ans. 5A., 0R., 33P.

> 2. By taking one or more of the angles.

Measure two sides AB, AC, and the angle A, included between them; then half the continual product of the two sides, and the natural sine of the contained angle will give the area.*

Or, measure the two angles A and B, and the adjacent side A B, from which the figure may be planned, and the perpendicular CD found, which perpendicular being multiplied by half the base AB, will give the area. Or by measuring the three sides of the triangle, its area may be found by Problem V., Section II.

## PROBLEM V.

To survey a four-sided field.-1. By the chain.
Measure the diagonal A C, and, as before directed, measure the perpendiculars DE and BF; then the area of each of the triangles A B C, A D. C may be found, as in the last problem, and both areas being added together, will give the content of the four-sided figure ABCD .


1. Let $\mathrm{A}=592, \mathrm{DE}=210, \mathrm{BF}=306$ links. $592 \times 210=124320$ double area of AD C. $592 \times 306=181152$ double area of A B C.
2) $\overline{305472}$ double area of ABCD . $\overline{1 \cdot 52736}=$ area of ABCD.

$$
\overline{2 \cdot 10944}
$$

40
$4 \cdot 37760$ Hence 1A., 2R., 4P., the answer.

- Ene Appenaux, Demonstration 11.


## 2. Dy taking one or more of the angles.

Measure the diagonal A C, also the sides AD and AB. Next measure the angles DAC and BAC; then the area of each of the triangles ABC and ADC may be found by case 2, last problem.
2. Required the plan and content of a field by the following field-book:-

FIELD BOOK.

3. How many acres are there in a four-sided field, whose diagonal is 4.75 chains, and the two perpendiculars falling on it, from its opposite angles, $2 \cdot 25$ and $3 \cdot 6$ chains, respectively?

Ans. 1A., 1R., 22.3P.

## PROBLEM VI.

To survey a field of many sides by the chain only.
Let ABCDEFG be the field whose content is required. Set up marks at the corners of the field, if there be none there naturally. Consider how the field may be best divided into trapeziums and triangles; measure them separately, as in the two last problems; and the sum of all the separate results will give the area of the whole field.

In this way of measuring with the
 chain, the field should be divided into trapeziums and triangles, by drawing diagonals from corner to corner, so that all the perpendiculars may be within the figure.

The last figure is divided into two trapeziums ABCG, GDEF, and the triangle GCD. In the first trapezium measure the diagonal AC , and the two perpendiculars $\mathrm{G} m$ and $\mathrm{B} n$. In the triangle GCD , measure the base G C, and the perpendicular D $q$. Finally, measure the diagonal F D, and the two perpendiculars Go and E $p$. Having drawn a rough figure resembling the field, set all these measures against the corresponding parts of the figure. Or set them down thus:-

| $\text { A m 135) } 130 \mathrm{mG}$ |
| :---: |
| $\left.\begin{array}{lll} A & n & 415 \\ \mathrm{AC} & 550 \end{array}\right\} \begin{aligned} & 130 m \mathrm{G} \\ & 180 n \mathrm{~B} \end{aligned}$ |
|  |  |
|  |
|  |
|  |  |
|  |  |

CALCULATION.

| $130+180=310$ |
| :--- |
| $550 \div 2=275,275 \times$ |
| $310=$ |
|  |
| $440 \times 230 \div 2=\quad 50650=$ ABCG |
| $120+80=200$, |
| $520 \div 2=260,260 \times$ |
| $200=$ |$\quad 52000=$ D EFG.

$\overline{1 \cdot 87850}=$ A B CDE FG.
$3 \overline{51400}$
40
$\overline{20 \cdot 56000}$
1A., 3R., 20.56 p ., answer.
Other methods will naturally present themselves to an ingenious practitioner who has read the preceding part of this work, or who has been previously acquainted with the principles of Mathematics. Every surveyor ought to be well acquainted with Plane Geometry at least. This, with a knowledge of Trigonometry, would be sufficient for the purpose of most surveyors.

The content of the last figure may be found by measuring the sides $\mathrm{A} \mathbf{B}, \mathrm{BC}, \mathrm{CD}, \mathrm{DE}, \mathrm{EF}, \mathrm{FG}, \mathrm{GA}$; and the diagonals $\mathrm{A} \mathbf{C}, \mathrm{CG}$, G D, DF , by which the figure is divided into triangles, the content of each of which may be found by Problem V., Section II.
2. Required the plan and content of a field of an irregular form from the following
FIELD-BOOK.

| 268 | $\begin{aligned} & 900=\mathrm{EG} \\ & 550 \\ & \mathrm{E}, \text { go S.W. } \end{aligned}$ |  |
| :---: | :---: | :---: |
| 280 | $1100=\mathrm{HE}$ 790 350 $\square \mathrm{H}$, go East. | $\overline{410}$ |
| 140 | $\begin{gathered} 1180=\mathrm{CH} \\ 710 \\ 350 \\ \mathrm{C}, \text { go S.W. } \end{gathered}$ | $280$ |
| 200 | $\begin{aligned} & 900=\mathrm{AC} \\ & 430 \\ & 300 \\ & \mathrm{~A}, \text { go S.E. } \end{aligned}$ | $\overline{450}$ |
| Offsets Left. | Stations, $\square$, or Base Lines. | Offsets Right. |

 A.ns. 10A., 18., 24.64P.

## PROBLEM VII.

## To survey a field with the theodolite, \&c. 1. From one point or station.

When all the angles can be seen from one point, as suppose $C$.
Having placed the instrument at C , turn it about till, through the fixed sights, the mark B may be seen. Fixing the instrument in this position, turn the moveable index about, till the mark A is seen through the sights, and note the degrees on the instrument. In the same manner, turn the index successively to the angles $\mathbf{E}$ and $\mathbf{D}$, taking
 care to note the degrees cut off at each; by which you have all the angles, viz., B C A, B C E, B C D. Now, having obtained the angles, measure the lines CB, CA, CE, CD ; entering the respective measures against the corresponding part of a rough figure, drawn to resemble the figure.

> 2. By goong rourd the field.

Set up marks a: B, C, D, \&c. Place the instrument at the point A , and turn it about till the fixed index be in the direction AB, and then screw it fast: turn the moveable index in the direction A F, and the degrees cut off will be the angle A ; next measure AB, and planting the instrument at B , measure, as before, the angle B; measure the line B C, and the angle C: and so proceed round the figure, always measuring the side as you go along, as also
 the anyles.

The 32 d Proposition of the 1st Book of Euclid affords an easy method of proving the work: thus, add all the internal angles, A, B, C, \&c., of the figure together, and their sum must be equal to twice as many right angles as the figure has sides, wanting four right angles. But when the figure has a re-enterant angle as F , measure the external angle, which is less than two right angles, and deduct it from four right angles, or 360 degrees, the remainder will give the internal angle (if such it may be called), which is greater than 180 degrees.

When the field is surveyed from 'one station, as in the first case shewn above, the content of the figure is found as in the second case of Prob. IV., since we have two sides and the angle included between them in each triangle of the figure.

## PROBLEM VIII.

## To survey a field with crooked hedges.

Measure the lengths and positions of lines running as near the sides of the field as you can; and, in proceeding along these lines, measure the offsets to the different corners, as before taught, and
join the ends of the offsets; these connecting lines will represent the required figure. When the plane table is used, the plan will be truly represented on the paper which covers it. But when the survey is made with the theodolite, or other instrument, the different measures are to be noted in the fiold-book, from which the sides and angles are laid down on a map, after returning from the feld.
In surveying the piece ABCDEFGHIKLM, set up marks at $s$ E F $x$. Begin at the station $s$, and measure the lines $s \mathrm{E}, \mathrm{E} p$, $p x, x s$, as also their positions, or the angles Es $x$, $s \mathrm{EF}, \mathrm{E} p x$, and $p x s$; and in going along the four-sided figure $s$ E $p x$, measure the offsets at $a, b$, $d, g, k, l, m$, as before
 taught. By means of the figure $s \mathrm{E} p x$, and of the offsets, the ground is easily planned.

When the principal lines are taken within the figure, as in the nbove case, the contents of the exterior portions $s$ C B A, C D E, \&c., must be added to the area of the quadralateral $s x$ FE. But when the principal lines are taken outside the figure, the portions included oetween them and the boundaries of the field, are to be deducted from the content of the quadralateral, and the remainder will give the true content of the field.


When there aze obstructions within the figure, such as wood, water, hills, \&e., measure the lengths and positions of the four-sided figure $a b c d$, taking care to measure the offsets from the different corners as jou go along.

## PROBLEM IX.

## To survey any pisce of land by two stations.

Choose two stations, from which all the corners of the ground can be seen, if possible; measure the distance between the stations; at each station take the angles formed by every object, from the station line, or distance. Then, the station line and these different angles being laid down from a regular scale, and the external points of intersection connected, the connecting lines will give the boundary.

The two stations may be taken within the bounds, in one of the sides, or without the bounds of the ground to be surveyed.

Let $m$ and $n$ be two stations, from which all the marks A; B, C
\&c., can be seen, plant the instrument at $m$, and by it measure the angles A $m n, \mathbf{B} m n$, $\mathrm{C} m n$, \&c. Next measure $m n$, and planting the instrument at $n$, measure the angles A $n m$, B $n m, \mathrm{C} n m$, \&c. These observations being planned, the lines joining the points of external intersection will give a true map of the ground. The method of finding the content
 will be shewn further on.

The principal objects on the ground may be delineated on the map, by measuring the angles at each station, which every object makes with the station line $m n$. When all the objects to be surveyed cannot be seen from two stations, then three or four may be used, or as many as may be found necessary; taking care to measure the distance from one station to another; placing the instrument at every station, and observing the angles formed by all the visible objects with the respective station line; then the intersection of the lines forming these respective angles will give the positions of all the remarkable objects thus observed.

In this manner may very extensive surveys be taken; and the tositions of hills, rivers, coasts, \&c., ascertained.

## PROBLEM X.

## To survey a large estate.

The following method of surveying a large estate was first given by Emerson, in his "Surveying," page 47. It has been followed by Hutton and Keith.

When the estate is very large, and contains a great number of fields, it cannot be accurately surveyed and planned by measuring each field separately, and then adding all the separate results together; nor by taking all the angles, and measuring the boundaries that enclose it. For in these cases the small errors will be so multiplied as to render it very much distorted.

1. Walk over the estate two or three times, in order to get a perfect idea of its figure. And to help your memory, make a rough draft of it on paper, inserting the names of the different fields within it, and noting down the principal objects.
2. Choose two or more elevated places in the estate for your stations, from which you can see all the principal parts of it; and let these stations be as far distant from each other as possible, as tho fewer stations you have to command the whole, the more exact the work will be.
In selecting the stations, care should be taken that the iines which connect them may run along the boundaries of the estate, or some of the hedges, to which offsets may be taken when necessary.
3. Take such angles, between the stations, as you think necossary,
and measure the distance from station to station, always in a right line ; these things must be done till you get as many lines and angles as are sufficient for determining all the station points. In measuring any of these station distances, mark accurately where these lines meet with any hedges, ditches, roads, lanes, paths, rivulets, \&c., and where any remarkable object is placed, by measuring its distance from the station line; and where a perpendicular from it cuts that line; and always mind, in any of these observations, that you be in a right line, which you may easily know by taking a back-sight and fore-sight, along the station line. In going along any main station line, take offsets to the ends of all hedges, and to any pond, house, mill, bridge, \&c., omitting nothing that is remarkable. All these things must be noted down; for these are the data by which the places of such objects are to be determined on the plan.

Be careful to set up marks at the intersections of all hedges with the station line, that you may know where to measure from when you come to survey the particular fields that are crossed by this line.

These fields must be measured as soon as you have completed your station-line, whilst they are fresh in your memory. In this manner all the station lines must be measured, and the situations of all adjacent objects determined. It will be proper to lay down the work on paper every night, that you may see how you go on.
4. With respect to the internal parts of the estate, they must be determined by new station lines; for, after the main stations are determined, and everything adjoining to them, then the estate must be subdivided into two or three parts by new station lines: taking the inner stations at proper places, where you can have the best view. Measure these station lines as you did the first, and all their intersections with hedges, ditches, roads, \&c., also take offsets to the bends of hedges, and to such objects as appear near these lines. Then proceed to survey the adjoining fields, by taking the angles which the sides make with the station line at the intersections, and measuring the distances to each corner from these intersections; for every station line will be a basis to all future operations, the situation of every object being entirely dependent on them; and therefore ther: should be taken of as great length as possible; and it is best for them to run along some of the hedges or boundaries of one or mors fields, or to pass through some of their angles.

All things being determined for these stations, you must take more inner stations, and continue to divide and subdivide, till at last you come to single fields; repeating the same work for the inner stations as for the outer ones, till the whole is finished. The oftener you close your work, and the fewer lines you make use of, the less you will be liable to error.
5. An estate may be so situated that the whole cannot be surveyed together, because one part of the estate may not be seen from another. In this case you may divide it into three or four parts, and survey these parts separately, as if they were lands, belonging to different persons, and at last join them together.
6. As it is necessary to protract or lay down the work as you proceed in it, you must have a scale of due length to do it by. Tc
get such a scale, measure the whole length of the estate in chains; then consider how many inches long the map is to be: and from these you will know how many chains you must have in an inch; then make your scale accordingly, or choose one already made.
7. The trees in every hedge-row may we placed in their proper situation, which is soon done by the plane table; but may be done by the eye without an instrument; and being thus taken by guess in a rough draft, they will be exact enough, being only to look at; except it be such as are at any remarkable places, as at the ends of hedges, at stiles, gates, \&c., and these must be measured or taken with the plane table, or some other instrument. But all this need not be done till the draft is finished. And observe, in all hedges, what side the gutter or ditch is on, and to whom the fence belongs.

## PROBLEM XI.

## To survey a town or city.

To survey a town or city, it will be proper to have an instrument for taking angles, such as a theodolite or plane table; the latter is a very convenient instrument, because the minute parts may be drawn upon it on the spot. A chain of 50 feet long, divided into 50 links, will be more convenient than the common surveying chain, and an offset staff of 10 feet long will be very useful. Begin at the meeting of two or more of the principal streets, through which you can have the longest prospects, to get the longest station lines. There having fixed the instruments, draw lines of direction along these streets, using two men as marks, or poles set in wooden pedestals, or perhaps some remarkable places in the houses at the farther ends, as windows, doors, corners, \&c. Measure these lines with the chain, taking offsets with the staff, at all corners of streets, bendings, or windings, and to all remarkable objects, as churches, markets, halls, colleges, eminent buildings, \&c. Then remove the instrument to another station, along one of these lines, and there repeat the same process as before. And so continue until the whole is finished.

Thus, fix the instrument at A, and draw lines in the directions of all the streets meeting there; then measure AC, noting the street at $x$. At the second station C , draw the directions of all the streets meeting there ; measure from C to D , noting the place of the street K , as you pass by it. At the third station D, take the direction of all the streets meeting there, and measure DS, noting the cross street at T. Proceed in like manner through all the principal streets; after which proceed to the smaller and intermediate streets; and last of all to
 the lanes, alleys, courts, yards, and every other place which if may be thought proper to represent in the plan.

## PROBLEM XII.

ro compute the content of any survey.

1. In small and separate pieces, the method generally employed is, to compute their contents from the measures of the lines taken in surveying them, without drawing any correct map of them : rules for this purpose have been given in the preceding part of the work. But in large pieces, and whole estates, consisting of a great number of fields, the usual method is, to make an unfinished but correct plan of the whole, and from this plan, the boundaries of which include the whole estate, compute the contents quite independent of the measure, of the lines and angles that were taken in surveying. Divide the plan of the survey into triangles and trapeziums, by drawing new lines through it: measure all the bases and perpendiculars of all these new figures, by means of the scale from which the plan was drawn, and from these dimensions compute the contents, whether triangles or trapeziums, by the proper rules for finding the areas of such figures.

The chief difficulty in computing consists in finding the contents of land bounded by curved or very irregular lines, or in reducing' sach crooked sides or boundaries to straight lines, that shall enclose an equal area with those crooked sides, and so obtain the area of the curved figure by means of the right-lined one, which in general will be a trapezium.

The reduction of crooked sides to straight ones is easily performed, thus:-

Apply a horse-hair or silk thread across the crooked sides in such a manner, that the small parts cut off from the crooked figure by it, may be equal to those taken in. A little practice will enable you to exclude exactly as much as you include ; then, with a pencil draw a line along the thread, or horse-hair. Do the same by the other sides of the figure, and you will thus have the figure reduced to a straight-sided figure equal to the curved one: the content of which, $l$ eing computed as before directed, will be the content of the curved figure proposed.

The best way of using the thread or horse-hair is, to string a small slender bow with it, either of whalebone or wire, which will keep it stretched.

If it were required to find the contents of the following crookedsided figure; draw the four dotted straight lines A B, B C, C D, and J) A, excluding as much from the survey as is taken in by the straight lines; by which the crooked figure is reduced to a right-lined one, both equal in area. Then draw the diagonal B D, which being measured by a proper scale, and multiplied by half
 the sum. of the perpen-
diculars let fall from A and C upon BD (measured on the samo scale), will cive the area required.

Many other methods might have been given for computing the contents of a survey, but they are omitted, the above being, perhaps, the most expeditiens.

## MISCELLANEOUS PROBLEMS.

1. The three sides of a triangle are 12,20 , and 28 ; what is the area? Ans. $60 \sqrt{ } 3$.
2. Find the difference between the area of a triangle whose sides are 3,4 , and 5 feet, and the area of an equilateral triangle having an equal perimeter.

Ans. 928 of a square foot.
3. There is a segment of a sphere, the diameter of whose base is 24 inches, and its altitude 10 inches; required its solidity?

Ans. $2785 \cdot 552$ inches.
4. There is a bushel in the form of a cylinder, whose depth is 8 a inches, and breadth $18 \frac{1}{2}$ inches; required to determine the breadth of another cylindrical vessel of the same capacity as the former, whose depth is only $7 \frac{1}{2}$ inches? Ans. $19 \cdot 107$ inches.
5. A ladder, 40 feet long, may be so planted that it shall reach a window 33 feet from the ground on one side of the street; and by only turning it over, without moving the foot out of its place, it will do the same by a window 21 feet high on the other side; what is the breadth of the street? $\quad 4 n s .56$ feet $7 \frac{3}{4}$ inches.
6. In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made two turns while the inner made but one; the wheels were both 4 feet high; and supposing them fixed at the statutable distance of 5 feet asunder on the axle-tree, what was the circumference of the track described by the outer wheel?

Ans. 63 feet, nearly.
7. A cable which is 3 feet long, and 9 inches in compass, weighs 22 pounds; what will a fathom of that cable weigh which measures a foot about?

Ans. $78 \frac{2}{2}$ pounds.
8. How many solid cubes, a side of which equals 4 inches, may be cut out of a large cube, whose side is 8 inches? Ans. 8.
9. Determine the areas of an equilateral triangle, a square, a hexagon, the perimeter of each being 40 feet.

$$
\text { Ans. } 76 \cdot 980035-100-115 \cdot 47 .
$$

10. A person wants a cylindrical vessel 3 feet deep, that shall contain twice as much as another cylindrical vessel whose diameter is $3 \frac{1}{2}$ feet, and altitude 5 feet; find the diameter of the required vessel. Ans. 6.39 feet.
i1. Three persons having bought a conical sugar-loaf, wish to divide it into three equal parts by sections parallel to the base; it is required to find the altitude of each person's share, the altitude of the loaf being 20 inches.

Ans. Altitude of the upper part $=13.867$ of the middle part $=$ $3 \cdot 604$, of the lower part $=2 \cdot 528$ inches.
12. There is a frustum of a pyramid, whose bases are regular ostagons; each side of the greater base is 21 inches, and each side of the less base 9 inches, and its perpendicular length 15 feet; how many solid feet are contained in it?

Ans. $119 \cdot 2$ feet.
13. Requiring to find the height of a May-pole, I procured a staff 5 feet in length, and placing it in the sunshine, perpendicular to the horizon, I found its shadow to be $4 \cdot 1$ feet. Next I measured the shadow of the May-pole, which I found to be 65 feet; from this data the height of the pole is required.

Ans. $79 \cdot 26$ feet.
14. Given two sides of an obtuse-angled triangle, which are 20 and 40 poles; required the third side, that the triangle may contain just an acre of land? Ans. 58.876 or 23.099 .
15. A circular fish-pond is to be made in a garden, that shall take up just half an acre; what must be the length of the chord that strikes the circle? Ans. $27 \frac{3}{4}$ yards.
16. A gentleman has a garden 100 feet long, and 80 feet broad; now a gravel walk is to be made of an equal width all round it; what must the breadth of the walk be, to take up just half the ground?

$$
\text { Ans. } 12 \cdot 9846 \text { feet. }
$$

17. A silver cup, in form of a frustum of a cone, whose top diameter is 3 inches, its bottom diameter 4, and its altitude 6 inches, being filled with liquor, a person drank out of it till he could see the middle of the bottom ; it is required to find how much he drank?

$$
\text { Ans. } 152127 \text { ale gallons. }
$$

18. I have a right cone, which cost me $£ 5,13 \mathrm{~s} .7 \mathrm{~d}$., at 10 s . \& cubic foot, the diameter of its base being to its altitude as 5 to 8 ; and would have its convex surface divided in the same ratio, by a plane parallel to the base ; the upper part to be the greater; required the slant height of each part?

Ans. $\left\{\begin{array}{l}3.9506486, \text { the slant height of the upper part. } \\ 1.0854612, \text { the slant height of the under part. }\end{array}\right.$
19. How many acres of the earth's surface may be seen from the top of a steeple whose height is 400 feet, the earth being supposed to be a perfect sphere, whose circumference is 25000 miles?

$$
\text { Ans. } 12120981 \cdot 338267112 \text { acres. }
$$

20. Two boys meeting at a farm-house, had a tankard of milk set down to them; the one being very thirsty drank till he could see the centre of the bottom of the tankard; the other drank the rest. Now, if we suppose that the milk cost $4 \frac{1}{2} d$., and the tankard measured 4 inches diameter at the top and bottom; and 6 inches in depth; it is required to know what each boy had to pay, proportionable to th-? quantitr of milk he drank.

$$
\text { Ans. }\left\{\begin{array}{r}
14 \cdot 1802815 \text { farthings, for the first. } \\
3.8197185 \text { farthings, for the second. }
\end{array}\right.
$$

21. If the linear side of a certain cube be increased one inch, the surface of the cube will be increased 246 square inches: determine the side of the cube. Ans. 20 inches.
22. If from a piece of tin, in the form of a sector of a circle, whose radius is 30 inches, and the length of its arc 36 inches, be cut another sector whose radius is 20 inches; and if then the remaining frustum be rolled up so' as to form the frustum of a cone; it is
required to find its content, supposing one-eighth of an inch to be allowed off its slant height for the bottom, and the same allowance of the circumference, of both top and bottom, for what the sides fold over each other, in order to their being soldered together.

Ans. $685 \cdot 3263$ cubic inches.
23. Three men bought a grinding-stone of 40 inches diameter, which cost 20 s ., of which sum the first man paid 9 s ., the second $6 s$., and the third $5 s$. ; how much of the stone must each man grind down, proportionably to the money he paid?

Ans. The first man must grind down $5 \cdot 167603$ inches of the radius; the second 4.832397 inches, and the third 10 inches.
24. There is a frustum of a cone, whose solid content is 20 feet, and its length 12 feet; the greater diameter is to the less as 5 to 2 ; what are the diameters?

Ans. $\left\{\begin{array}{l}2.02012 \text { feet. }\end{array}\right.$
25. A farmer borrowed of his neighbour part of a hay-rick, which measured 6 feet in length, breadth, and thickness; at the next liagtime he paid back two equal cubical pieces, each side of which was Ifeet. Has the debt been discharged ?
$A n s$. No; 88 cubie feet are due.
26 . There is a bowl in form of the segment of an oblong spheroid, whose axes are to each other in the proportion of 3 to 4 , the depth of the bowl one-fourth of the whole transverse axis, and the diameter of its top 20 inches; it is required to determine what number of glasses a company of 10 persons would have in the contents of it, when filled, using a conical glass, whose depth is 2 inches, and the diameter of its top an inch and a hali.

Ans. $114 \cdot 0444976$ glasses each.
27. If a cubical foot of brass were to be drawn into wire, of $\frac{1}{40}$ of an inch in diameter; it is required to determine the length of the said wire, allowing no loss in the metal. Ans. $55 \frac{5}{9}$ miles.
28. How many shot are there in an unfinished oblong pile, the length and breadth of whose base being 48 and 30 , and the length and breadth of the highest course being 24 and 6 ? Ans. 17356 .
29. How many shot are there in an unfinished oblong pile of 12 courses; length and breadth of the top contain 40 and 10 shot respectively? Ans. 8606 shot.
30. Of what diameter must the bore of a cannon be cast for a ball of 24 pounds weight, so that the diameter of the bore may be $\frac{1}{10}$ of an inch more than that of the ball ? Ans. $5 \cdot 757098$ inches.
31. What is the content of a tree, whose length is $17 \frac{1}{2}$ feet, and which girts in five different places as follows, viz., in the first place 9.43 feet, in the second 7.92 , in the third 6.15 , in the fourth 4.74 , and the fifth $3 \cdot 16$ ?

Ans. 42.5195.
32. What three numbers will express the proportions subsisting between the solidity of a sphere, that of the circumscribing cylinder, and circumscribing equilateral cone? Ans. 4, 6, 9.

- 33. Given the side of an equilateral triangle 10 , it is required to find the radii of its circumscribing circle. Ans. 5•7736.

34. Given the perpendicular of a plane triangle 300 , the sum of
the two sides 1150 , and the difference of the segment of the base 495 ; required the base and the sides? Ans. 945, 375, and 780.
35. A side wall of a house is 30 feet high, and the opposite one 40 , the roof forms a right angle at the top, the lengths of the rafters are 10 feet and 12 ; the end of the shorter is placed on the higher wall, and vice versâ; required the length of the upright which supports the ridge of the roof, and the breadth of the house?

Ans. $41 \cdot 803$, length of upright, and 12 feet the breadth of thr: house.

## A TABLE

## OF THE AREAS OF THE SEGMENTS OF A CIRCLE,

I'hose diameter is 1 , and supposed to be divided into 1000 equal parts.

| Height. | Area Seg. | Height. | Area Seg. | Height. | Area Seg. | Height. | Area Seg. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 001 | -000042 | -044 | - 012142 | -087 | - 033307 | $\cdot 130$ | - 059999 |
| -002 | -000119 | . 045 | -012554 | -088 | -033872 | -131 | -060672 |
| -003 | -000219 | -046 | -012971 | -089 | -034441 | -132 | -061348 |
| -004 | -000337 | -047 | -013392 | -090 | -035011 | -133 | -062026 |
| . 005 | -000470 | -048 | -013818 | -091 | -035585 | $\cdot 134$ | -062707 |
| -006 | -000618 | -049 | -014247 | -092 | -036162 | -135 | -063389 |
| -007 | -000779 | -050 | -014681 | -093 | -036741 | -136 | -064074 |
| -008 | -000951 | -051 | -015119 | -094 | -037323 | $\cdot 137$ | -064760 |
| -009 | -001135 | -052 | -015561 | -095 | -037909 | -138 | -065449 |
| -010 | -001329 | -053 | -016007 | -096 | -038496 | -139 | -066140 |
| -011 | -001533 | -054 | -016457 | -097 | -039087 | -140 | -066833 |
| . 012 | -001746 | -055 | -016911 | -098 | -039680 | -141 | - 067528 |
| -013 | -001968 | -056 | -017369 | -099 | -040276 | -142 | -068225 |
| -014 | -002199 | -057 | -017831 | -100 | -040875 | -143 | -068924 |
| -015 | -002438 | -058 | -018296 | -101 | -041476 | -144 | -069625 |
| -016 | -002685 | -059 | -018766 | -102 | -042080 | $\cdot 145$ | -070323 |
| -017 | -002940 | -060 | -019239 | -103 | -042687 | -146 | -071033 |
| -018 | -003202 | -061 | -019716 | -104 | -043296 | -147 | -071741 |
| -019 | -003471 | -062 | -020196 | -105 | - 043908 | $\cdot 148$ | -072450 |
| -020 | -003748 | -063 | - 020681 | -106 | -044522 | -149 | -073161 |
| -021 | -004031 | -064 | -021168 | -107 | -045139 | -150 | -073874 |
| -022 | -004322 | -065 | -021659 | -108 | -045759 | -151 | -074589 |
| -023 | -004618 | -066 | . 022154 | -109 | -046381 | -152 | -075306 |
| -024 | -004921 | -067 | -022652 | -110 | -047005 | -153 | -076026 |
| - 025 | -005230 | -068 | . 023154 | -111 | -047632 | $\cdot 154$ | -076747 |
| . 026 | -005546 | -069 | -023659 | -112 | -048262 | $\cdot 155$ | -077469 |
| -027 | -005867 | -070 | - 024168 | -113 | -048894 | -156 | -078194 |
| -028 | -006194 | -071 | -024680 | $\cdot 114$ | -049528 | -157 | -078921 |
| -029 | -006527 | -072 | -025195 | -115 | -050165 | -158 | -079649 |
| -030 | -006865 | -073 | - 025714 | $\cdot 116$ | -050804 | -159 | -080380 |
| -031 | -007209 | -074 | -026236 | -117 | -051446 | -160 | -081112 |
| -032 | -007558 | . 075 | - 026761 | -118 | -052090 | -161 | -081846 |
| .033 | -007913 | . 076 | -027289 | -119 | -052736 | -152 | -082582 |
| -034 | -008273 | -077 | -027821 | -120 | -053385 | -163 | -083320 |
| -035 | -008438 | -078 | - 028356 | -121 | -054036 | $\cdot 164$ | -084059 |
| .036 | -009008 | -079 | - 028894 | -122 | -054689 | -165 | -084801 |
| -037 | -009383 | -080 | -029435 | -123 | -055345 | -166 | -085544 |
| -038 | -069763 | -081 | -029979 | -124 | -056003 | $\cdot 167$ | -086289 |
| -039 | -010148 | -082 | -030526 | -125 | -056663 | -168 | -087036 |
| -040 | -010557 | -083 | -031076 | -126 | - 057326 | -169 | -087785 |
| $\cdot 041$ | -010931 | -084 | -031629 | -127 | -057991 | -170 | -088535 |
| -042 | . 011330 | -085 | -032180 | -128 | -058658 | -171 | -089287 |
| . 043 | . 011734 | -086 | -032745 | -129 | -059327 | $\cdot 172$ | -090041 |


| Height. | Area Seg. | Height. | Area Seg. | Height. | Area Seg. | Height. | Area Seg |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -173 | -090797 | -224 | -131438 | - 275 | -175542 | -326 | - 222277 |
| $\cdot 174$ | -091554 | -225 | -132272 | -276 | -176435 | -327 | -223215 |
| $\cdot 175$ | -092313 | -226 | -133103 | -277 | -177330 | -328 | -224154 |
| -176 | -093074 | -227 | -133945 | -278 | -178225 | -329 | -225093 |
| $\cdot 177$ | -093836 | -228 | -134784 | -279 | -179122 | -330 | -226033 |
| -178 | -094601 | -229 | -135624 | -280 | -180019 | -331 | -226974 |
| -179 | -095366 | -230 | -136465 | -281 | -180918 | -332 | -227915 |
| -180 | -096134 | -231 | -137307 | -282 | -18:81\% | -333 | - 228858 |
| -181 | -096903 | -232 | -138150 | -283 | -182718 | -334 | -229801 |
| -182 | -097674 | -233 | -138995 | -284 | -183619 | -335 | -230745 |
| -183 | -098447 | -234 | -139841 | -285 | -184521 | -336 | -231689 |
| -184 | -099221 | -235- | -140688 | -286 | -185425 | -337 | -232634 |
| -185 | -099997 | -236 | -141537 | -287 | -186329 | -339 | -233580 |
| -186 | -100774 | -237 | -142387 | -288 | -187334 | -339 | -23 526 |
| -187 | -101553 | -2:38 | -143238 | -289 | -188140 | -340 | -235473 |
| -188 | -102334 | -239 | -144091 | -290 | -189547 | -341 | 236421 |
| -189 | -103116 | -240 | -144944 | -291 | -189955 | -342 | -237369 |
| -190 | -103900 | -241 | - 145799 | -292 | -190864 | -343 | -238318 |
| -191 | -104685 | -242 | -146655 | -293 | -191775 | -344 | -239268 |
| -192 | -105472 | -243 | -147512 | -294 | -192684 | -345 | - 240218 |
| -193 | -106261 | $\cdot 244$ | -148371 | -295 | -193596 | -346 | -241169 |
| 194 | -107051 | $\cdot 245$ | -149:30 | -296 | -194509 | $\cdot 347$ | -242121 |
| 195 | -107842 | $\cdot 246$ | -150091 | -297 | -195422 | -348 | -243074 |
| -196 | -108636 | -247 | -150953 | -298 | -196337 | -349 | -244026 |
| -197 | -109430 | $\cdot 248$ | -151816 | -299 | -197252 | -350 | -244980 |
| -198 | -110226 | - 249 | -152680 | -300 | -198168 | -351 | -245934 |
| -199 | -111024 | - 250 | -153546 | -301 | -199085 | -354 | -246889 |
| - 200 | -111822 | -251 | -154412 | -302 | - 200003 | $\cdot 353$ | -247845 |
| -201 | -112624 | - 252 | -155280 | -303 | -200922 | $\cdot 354$ | -248801 |
| -202 | - 113426 | -253 | -156149 | $\cdot 304$ | -201841 | - 355 | - 249757 |
| -203 | -114230 | -254 | -157019 | -305 | -202761 | -356 | -250715 |
| -204 | -115035 | -255 | -157890 | -306 | - 203683 | - 357 | -251673 |
| -205 | -115842 | -256 | -158762 | $\cdot 307$ | -204605 | -358 | -252631 |
| -206 | -116650 | -257 | -159636 | -308 | -205527 | -359 | -253590 |
| -207 | -117460 | -258 | -160510 | -309 | -206451 | $\because 60$ | -254550 |
| -208 | -118271 | -259 | -161386 | -310 | 207376 | -361 | - 255510 |
| -209 | -119083 | -260 | -162263 | $\cdot 311$ | - 208301 | -362 | - 256471 |
| -210 | -119897 | -261 | -163140 | -312 | -209227 | -363 | -257433 |
| -211 | -120712 | -262 | -164019 | -313 | - 210154 | -364 | -258395 |
| -212 | -121529 | -263 | -164899 | -314 | r2110.82 | -365 | -259357 |
| - 213 | - 122347 | -264 | -165780 | $\cdot 315$ | - 212011 | -366 | -260320 |
| -214 | - 123167 | 265 | -166663 | -316 | - 212940 | -367 | -261284 |
| $\cdot 215$ | -123988 | -266 | -167546 | -317 | - 213871 | -368 | -262248 |
| -216 | -124810 | -267 | -168430 | -318 | - 214802 | -369 | -263213 |
| $\cdot 217$ | -125634 | -268 | -169315 | -319 | - 215733 | -370 | -264178 |
| -218 | -126459 | -269 | -170202 | -320 | - 216666 | -371 | -265144 |
| -219 | -127285 | -270 | -171089 | -321 | - 217599 | -372 | -266111 |
| $\cdot 220$ | -128113 | -271 | -171978 | -322 | - 218533 | -373 | -267.178 |
| -221 | -128942 | -272 | -172867 | -323 | -219468 | -374 | -268045 |
| -222 | -129773 | -273 | - 173758 | $\cdot 324$ | - 220404 | -375 | -26.013 |
| -223 | -130605 | -274 | - 174649 | -325 | -221340 | $\cdot 376$ | -269982 |




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[^0]:    * See Appendix, Demonstration 1.

[^1]:    - See Appendix, Demonstration 7.

[^2]:    * In multiplication, the multiplier must always be a number of times; to talk of multiplying feet by feet, \&c., is absurd, for what notion can be formed of 7 feet taken 3 times? However, since the above easily suggests the correct meaning, and is a concise method of expressing the rule, it has been thought proper to retain it. See Appendix, Demonsturatiou \&

[^3]:    * See Appendix Demonstration.

[^4]:    * See Appendix, Demonstration 9.
    $\dagger$ Ibid. 10

[^5]:    * See Appendix, Demonstration 11.

[^6]:    * Sce Appendix, Demonstration 16.

[^7]:    * See Appendix, Demonstration 18.

[^8]:    * In finding the area of an irregular figure, draw a line through the extreme angles of the figure, on which let fall perpendiculars from all the other angles of the polygon, which will divide it into triangles and trapezoids; then find the area of these by Froblems IV. and XIII.

[^9]:    - See Appendix, Demonstration 12.

[^10]:    * See Appendix, Demonstration 23.
    $\dagger$ thid. 24.
    \# By "versed sine," in works on mensuration, is not meant the trigono metrical versed sine of the whole are, but of half the are.
    § See Appendix, Demonstration 22.
    || Ibid. 32.
    - I Ibid. 25.

[^11]:    - See Appendix, Demonstration 26.

[^12]:    * See Appendix, Demonstration 27.
    $\dagger$ Ibid. 23.
    Ibid. 31.
    $\ddagger$ Ibid. 29.
    § Ibid. 30 .

[^13]:    *Soe Appendix, Demonstration 22 Ibid. 33. $\ddagger$ Ibid. 34. $\$$ Ibid. 35

[^14]:    * See Appendix, Demonstration 38.

[^15]:    *See Appendix, Demonstration 39.

[^16]:    262.50 area of the whole figure

[^17]:    For definitions of the ellipsis (or, as it is frequently written, ellipse) and Wer Conic Sections, see Appendix, Properties of the Conic Sections.
    t See Appendix, Demonstration 11.

[^18]:    *See Apnendix, Demonstration 43.

[^19]:    Note.-The nearer the axes of the ellipse approach towards equality, the wore exact the result of the operation by this Rule; and the less the elliptical arc, the nearer its exact length will approach the arc $x y$.

[^20]:    * See Appêndix, Demonstration 45.
    $\dagger$ It may be done without Trigonometry, by first finding the length of tho suc B by Rule II. Rrob. XVII. Scc. 2, then OG:OY::GB:Y Z.

[^21]:    * Ses Appendix, Demonstration 46. † Ibid. $47 . \quad$ Ibid. 48

[^22]:    * See Appendix, Demonstration 56.
    ; Ibid. 56.

[^23]:    * See Appendix, Demoustration 57.
    $\dagger$ Ibid. 58.
    $\$$ Ibid. 59.

[^24]:    Note.-The following rule will answer for any prismoid, of whatever figure each end may be.
    Rule.-If the bases be dissimilar rectangles, take two corresponding dimensions, and multiply each by the sum of double the other dimenston of the same end, and the dimension of the other end corresponding to this last dimension; then multiply the sum of the products by the height, and one-sixth of the last product will be the solidity. $\ddagger$

[^25]:    * See Appendix. Demarustration 75.

[^26]:    * See Appendix, Demonstration 76.

[^27]:    * See Appendix, Demonstration 78.

[^28]:    * See Appendix, Demonstratiou 81.
    $\dagger$ lbid. 82.
    $\ddagger$ Ibid. 83。

[^29]:    * See Appendix, Demonstration 84.

[^30]:    * If figtures similar to those annexed to the definitions, be drawn on pastoboard, and out out by cutting through the bounding lives, and if the othes lines be cut balf through, and then the parts be turned up and glued together, the bodies defined will be formed.

[^31]:    Note. - The following table may be collected from the examples given in the foregoing rules, each of which has been demonstrated under its particular head. It has also been demonstrated that the cube of the lineal side of any regular solid multiplied by the tabular number corresponding to the figure, will give its content. It is particularly recommended to the pupil to employ the general rule given in Problem I. whenever the content of any of the five rogular bodies is required.

[^32]:    * Though the next section treats exclusively of the surfaces of solids, and would therefore seem to be the proper place for this problem and the following ones in this section, yet it has been thought more convenient to place together the rules both for finding the solidities and surfaces of those curious bodies.
    + See Appendix, Demonstration 95.
    $\ddagger$ Ibid. 96.
    § lbid. 97

[^33]:    * See Arpendix, Demoustration 88.

[^34]:    - See Appeudix, Demonstration 102.

[^35]:    * See Appendix, Demonstration 104.

[^36]:    * This rule, which is generally employed in practice, is far from boing correct, when the breadth and depth differ materially from each other, $3 n d$ the timber does not taper.

[^37]:    * This rule is correct when the timber does not taper; but when the timber tapers considerably, and the breadth and depth are neariy equal, the rule is very erroneous. The measurer, therefore, ought to consider the shapo of tho timber he is about to measure before he applies either of the above rules.

[^38]:    * This rule is correct, being that given for finding the solidity of the pris-moid-which see.

    Let $B$ and $b$ be the breadths of the two ends, $D$ and $d$ the depths, and L the length : $\frac{1}{6}(\mathrm{BD}+(\mathrm{B}+b) \times(\mathrm{D}+d)+b d) \times \mathrm{L}=$ the true solidity, 8 d in the rule for the prismoid.

[^39]:    *Seo Appendix, Demonstration 111.

[^40]:    Note. - It may sometimes happen that the quarter girt exceeds the limits of the table: in this case, take half of it, and four times the coutent thus found will give the required content.

[^41]:    * Joists receive various names from their position; such as girders, bindingjoists, trimming-joists, common-joists, ceiling-joists, \&c. When girders and oists of flooring are designed to bear considerable weight, they should be lot irto the wall at each end about two-thirds of the thickness of the wall.

[^42]:    * Fire-places, \&c., are of course to be deducted.
    $\dagger$ The best and strongest partitions are those made with framed work. The king-posts are measured as roofing, the rest as flooring.
    $\ddagger$ Workmen generally take the flat and half the flat of any house, taken within the walls, to be the measure of the roof of the same house. This, however, is only when the roof is of a true pitch. The usual pitches are tho common, or true pitches, in which the rafters are three-fourths of the breadth of the building; the Gothic pitch is when the length of the principal rafters is equal to the breadth of the building: the pediment pitch is when the perpendicular height is two-ninths of the breadth.

    When the covering of the building is to be plain tiles or slates, the roof is generally of a true or common pitch; the Gothic pitch is used when the covering is of pantiles; the pediment pitch is used when the roof is to be covered with lead.

[^43]:    * In the above figure K is called the king-post, and in measuring the piecer cut out of it, the shortest length is to be taken. T B is called the tie-beam, which prevents the rafters R R from pressing out the wall. The braces BB serve to strengthen the rafters; the struts S S serve for a similar purpose. Besides strengthening the rafters, the braces and struts serve to bind the rool together. When head-room is required, the rafters are braced simply by B B.

[^44]:    * Baluster is a small column or pillar, used for balustrades.

    Balustrade is a row of balusters, joined by a rail ; serving for a rest to the arms, or as an inclosure to balconies, staircases, altars, \&c.

    Cornice is the third and uppermost part of the entablature of a column, or the uppermost ornament of any wainscotting, \&c.

    Bead is a round moulding carved hise beads in necklaces. There is also a kind of plain bead, often set on the eage of each fascia of an architrave, on the upper edge of skirting-boards, on the lining-board of a door-case, de.

    Architrave is that part of a column that bears immediately on the capital. It is supposed to represent the principal beam in timber buildings, in which it is sometimes called the master-piece or reason-piece. In chimneys it is called the mantel-piece. Architrave doors are those which have an architrave on the jambs and over the doors. Architrave windows of timber are usually raised out of the solid timber, and sometimes the mouldings are struck an? laid on.

    Astragal is a small round moulding, encompassing the top of the shaft of a column, like a ring or bracelet. The shaft terminates at the top with an astragal, and at bottom with a fillet, which in this place is called azia.

[^45]:    *Cornices, festoons, \&cc, are put on after the room is plastered and are not, of course, taken into account by the plasterer.

[^46]:    * This rule is correct only in one case, namely, when the dome is half a sphere, and in this case the height is equal to the radius of the circular base. It is a well-known property that the solidity of a sphere is two-thirds of that of a cylinder having the same base and height. But the solidity of a eylinder is found by multiplying the area of its base by the height. Hence the reason

[^47]:    of the rule when applied to this particular case. No general rule can be given to answer every case, as some domes are circular, some elliptical, some polygonal, \&c.; they are of various heights, and their sides of different curvature. When the height of the dome is equal to the radius of its base, (the curved sides being circular or elliptical quadrants), or to half the mean proportional between the two axes of its elliptical base, the above rule will answer pretty vell; but with any other dimensions it ought not to be used.
    *This rule is correct only when the dome is circular, and its height equal to the radius of the base.-See Appendix. Demonstration 112.

[^48]:    * In this manner may the species of a fluid or solid be ascertained, by means of its specific gravity, and the amnexed table. This table has been taken from Gregory's work for practical men.

[^49]:    *For the reason of this rule, see Alligation Total in the Second Book of Arithmetic, published by the Commissioners.

[^50]:    * See Appeudix, Demonstration 113.

[^51]:    * This rule is obvious from Problem I., being the converse thereof.
    $\dagger$ This rule is manifest from Problem III., being its converse.
    $\ddagger$ See Aypeulix, Demonstration 1 \&

[^52]:    * Until 5 George IV., in which a uniform system of weights and measures was established under the denomination of IMPERIAL WEIGHTS AND MEASURES, there were, amongst other sources of inconvenience, different measures, though

[^53]:    of the same name, for ale and wine. A gallon of ale contained 282 cubic inches, and a gallon of wine 231; a buskel of malt contained 2150.42 cubic inches.

    To reduce old measure into new, say, as the number of cubic inches in the imperial standard is to the number of cubic inches in the old standard, so is the number of gallons or bushels, \&c., old measure, to the number of gallons, \&ce., innperial measure.

    When great accuracy is not required, old wine gallons may be reduced to imperial gallons by dividing by $1 \cdot 2$; and old ale gallons may be reduced to imperial gallons by multiplying by 60 , and dividing the product by 59 ; and old, or Winchester bushels, may be reduced to imperial bushels by multiplying by 31 , and dividing the product by 32 .

[^54]:    * The areas of plane figures, in gauging, are expressed in gallons, or bushels. For there will be as many solid inches in any vessel of one inch deep, as there are superficial inches in 1ts base. What is called in gauging a surface or area, is in reality a solid of one inch deep, which mesltiplied by the height will givo the whole content in gallons or bushels.

