NAVIGATION

A TREATISE

ON

NAVIGATION

FOR THE USE OF STUDENTS

BY

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PREFACE.

Having been engaged for more than twenty years in preparing candidates for the different examinations into which Navigation enters, I have felt the want of a text book which will embrace all that the different examining boards embody under that head. Those engaged in teaching know the difficulty in inducing pupils to study a subject from more than one text book: I have, therefore, hitherto taught the advanced part of Navigation from notes. These I have now enlarged, and have cast them into the form found in the present treatise, hoping thereby to render some slight service to those who wish to obtain a scientific knowledge of the subject.

I have consulted most works extant on Navigation, and have to particularly acknowledge my indebtedness to certain articles in the first volume of 'Naval Science,' to an old work on Navigation by Robertson, to Knox Laughton's 'Marine Surveying,' to the Admiralty Manual of Scientific Inquiry, and others. The examples have been chiefly selected from Examination papers set by the Admiralty and by the Science and Art Department: the latter being marked E, A, or Honours, with the year in which they were set. Others have been taken from Robertson's 'Navigation,' and some are original. They have been solved by logarithms, by construction, and, where practicable, by inspection. The tables used have been the common six-figure logarithms; and for the meridional parts, those computed to a compression of $\frac{1}{321}$, as I am not cognisant of any yet published to the true compression. Results obtained from any of the mathematical tables in general use should approximate very closely to the answers given, except to those in Mercator's Sailing, where a discrepancy must occur unless the same meridional parts be used.

Here I beg to acknowledge with thanks the help I have received from my assistant, Mr. CHARLES MORRIS, F.M.S. and from my son, Mr. W. VENNER MERRIFIELD, B.A. of St. John's College, Cambridge, who have read over the proof sheets, and have tested the accuracy of the answers.

Any suggestions for the improvement of the work, or corrections in the computations, will be gladly received from my fellow teachers and students.

J. M.

PLYMOUTH: *May* 1883.

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CHAPTER I.

Methods of finding a ship's position—Rectangular co-ordinates—Definitions—Latitude—Longitude—Position of places on the earth's surface fixed by their means—Examples—Figure of the earth—Proofs for its shape—Compression—Examination.

MARINERS have recourse to two methods in determining their positions at sea.

I. The direction in which the ship has been sailing and the distance sailed in that direction from some known position are ascertained by means of instruments. This method is called *Dead Reckoning*; and the instruments used are *the Compass*, which shows the directions of the ship's route, and the *Log Ship*, *Log Line*, and *Log Glass*, for measuring the distances in each direction. Then by means of charts, and by rules deduced from trigonometry, both plane and spherical, the position of the ship is fixed. Obtaining the position of the vessel by this means, together with the knowledge requisite for conducting a ship from one part of the world to another and of the laws which govern the instruments, constitute *Theoretical*, or the *Science of Navigation*; whilst the management of the ship, such as making and taking in sail, steering her, selecting the routes to be traversed, &c., make up *Practical*, or the *Art of Navigation.*

II. By means of instruments the positions of heavenly

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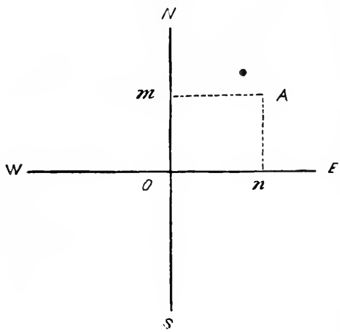
bodies with respect to the horizon are ascertained, and then by rules deduced from spherical trigonometry the ship's place is determined. The instruments used for this purpose are the *Quadrant* or *Sextant*, the *Artificial Horizon* and the *Chronometer.* This method constitutes *Nautical Astronomy*, because a knowledge of astronomy is requisite for the calculations employed.

In this treatise we shall deal exclusively with the science of navigation.

To fix the position of a ship at sea certain lines or points for reference must be agreed on, from which all measurements must be made. These may either be real, as headlands, ports, &c., or a former position of the ship; or they may be imaginary, as the equator and meridians. It is obvious those are most advantageous which can be most generally used; and hence headlands, ports, &c., must be discarded, because a vessel may be situate in any direction and at any distance consistent with the magnitude of the earth from these, and the methods to be used would be cumbersome. But if two lines perpendicular to each other be selected, it is manifest, if the station be on either line, its distance from the other would fix the position of the station; and if not on either line, it must be situate in one of the four right angles made by the lines. Thus, if the position of the point A be required, perpendiculars An, Am are dropped on the lines EW and NS; when these are measured we can say the point A is so many inches, feet, or miles to the right or left of NS; and so many inches, feet, or miles above or below the line EW; and thus the position of the point A would be fully determined. This method is called that of *Rectangular Co-*

FIG. 1.



ordinates; the lines N S and E W are called the *Rectangular Axes*, the point O is called the origin, *n* O the abscissa, and *n* A the ordinate of the point A. As we proceed we shall show that the equator and any meridian have the qualifications for rectangular axes or datum lines.

The earth is spherical or globular in shape, not quite a sphere but somewhat flattened, and is called a spheroid; but for most purposes in navigation it is considered a sphere.

A SPHERE is a solid body every part of whose surface is equally distant from a fixed point which is called its centre; and is generated by the revolution of a semicircle about its diameter, which remains fixed in space.

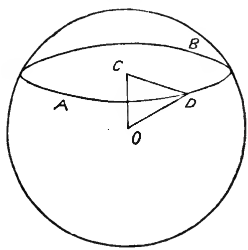
AN OBLATE SPHEROID is a solid body generated by the revolution of a semi-ellipse about its minor diameter; and, therefore, every section through that diameter of the body must be an ellipse.

A DIAMETER of a sphere is any straight line drawn through its centre and terminated both ways by its surface.

Every section of a sphere made by a plane is a circle.

Let A B be the section of the sphere made by any plane, and O the centre of the sphere. Draw O C perpendicular to the cutting plane from O; take any point D in the surface of the sphere where the plane cuts it; join O D, C D. Now since O C is perpendicular to the cutting plane it is perpendicular to C D, a line meeting it in that plane; hence

FIG. 2.



$$CD = \sqrt{OD^2 - OC^2}$$

Now O is a fixed point because it is the centre of the sphere, and C is a fixed point because it is where the perpendicular from C meets the cutting plane; therefore O C is constant and O D is also constant, because it is the radius of the sphere; hence $\sqrt{OD^2 - OC^2}$ or C D is constant. Therefore all points where the plane section meets the surface of the sphere are equally distant from C, and hence the section of a sphere made by a plane is a circle.

A Great Circle of a Sphere is one whose plane passes through the centre of the sphere, such as the circle on the surface made by an even cut through the centre of a cricket ball. It is evident any number of even cuts may be made through the centre, so there may be any number of great circles on the surface of a sphere; and as all of them have the same radius as the sphere, so all great circles of a sphere must be equal.

A Small Circle of a Sphere is one whose plane does not pass through the centre of the sphere.

The Axis of any circle of a sphere is that diameter of the sphere which is perpendicular to the plane of the circle, and *the axis of the earth* is that diameter about which it rotates diurnally; and it will be found the rotation of the earth gives a fixed line on which depends the definitions we shall employ.

The Poles are the ends of the axis, and are thus the extremities of that diameter which is perpendicular to the plane of the circle. The poles of a great circle are equally distant from the circumference of the circle. The poles of a small circle are not equally distant from the circumference of the circle, and to distinguish them they are called respectively the *nearer* and the *further pole*, and for brevity the nearer pole is sometimes called *the* pole.

The Poles of the Earth are those points where the diameter on which it rotates meets the surface.

A few of the great circles on the earth have received particular names, as the Equator and the Meridians.

The Equator is a great circle midway between the two poles It is evident the plane of the equator divides the earth into two equal portions; the one in which England is situate is called the Northern and the other the Southern Hemisphere.

The Celestial Concave is the expanse of the heavens, every part of which appears equally distant from us, and therefore spherical. In the surface of this sphere all the heavenly bodies appear to be situate.

The Equinoctial is a great circle in the heavens formed by producing the plane of the equator to meet the celestial concave.

The Meridians are halves of all great circles extending from pole to pole. Here, again, it is obvious any number of meridians may be drawn, one of which may be conceived to pass through any point on the surface of the sphere. The one which passes through the chief town of a nation, or its national observatory, is usually chosen as *the first meridian* of that nation, and is the one from which all measurements are made. English-speaking people usually select the meridian which passes through Greenwich Observatory as their first meridian; but it would tend to simplify navigation if one meridian could be agreed on as the first meridian for all nations. If we consider any great circle as a *primary one*, all great circles which pass through its poles are called its *secondaries*, and all secondaries cut their primary at right angles. Thus every meridian is a secondary to the equator.

Latitude is the angle at the earth's centre subtended by that portion of a meridian intercepted between the equator and the place; and all places situate to the north of the equator are said to be in north latitude, and all places to the south of the equator in south latitude.

Parallels of Latitude are small circles of the sphere joining all places which have the same latitude. They are therefore parallel to the equator, and four of them have received particular names. They are the two tropics and two polar circles.

The Tropics are parallels of latitude drawn about 23° 28′ on each side of the equator. They are so called because they mark the limits of the sun's apparent path during the earth's annual revolution. The tropic in the northern hemisphere is called *Cancer*; the one in the southern is called *Capricorn*.

The Polar Circles are parallels of latitude drawn about

23° 28′ from the two poles. They mark the limits on the earth where there is no night during one part of the year and no day during another part. The one in the northern hemisphere is called the *Arctic Circle* and the one in the southern hemisphere the *Antarctic Circle.*

Longitude of a place is its angular distance (measured at the earth's axis) E. or W. from the first meridian, or it is the arc of the equator intercepted between the first meridian and that of the place.

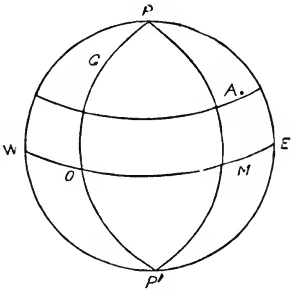
If the earth were a perfect sphere, the meridians would all be equal in length to half the equator, and equal angles at the centre of the earth would be subtended by equal distances measured on the equator or a meridian; hence we can measure latitude on meridians and longitude on the equator; but we cannot measure the latter on parallels of latitude, because they are small circles. We may also measure longitude by the inclination of the plane of any meridian to the plane of the first meridian, and this (Eu. XI. def. 6) is the angle made at the axis of the earth or at the poles; hence we say *latitude* is the angular distance of a place from the plane of the equator measured on a meridian, and *longitude* is the angular distance of a place from the plane of the first meridian measured either on the equator, at the axis of the earth, or at the poles. From this it is evident the greatest latitude a place can have is when the angle it makes at the centre of the earth with the plane of the equator is a right angle or 90°, and the greatest longitude when the angle at the axis is two right angles or 180°.

Difference of Latitude between two places is the angular distance at the centre of the earth measured by the portion of a meridian intercepted between two parallels of latitude, one through each of the places mentioned.

Difference of Longitude between two places is the angular distance between the planes of the meridians which pass through the two places, measured either at the poles or on the equator.

In determining the position of a place on a plane we used two straight lines at right angles to each other, called rectangular axes; on a sphere we shall use two great circles, viz. the equator and the first meridian as rectangular axes; thus great circles on the sphere are analogous to straight lines on the plane. Let P E P′ W be a meridional section of the earth, P P′ its poles, W E the equator, we shall consider the primary: and the meridian P G O P′ through Greenwich as the secondary, from which all distances to the right or left are measured. O is the origin. The position of a point A is fully determined when we know the ordinate A M on the secondary through A, and O M the abscissa on the primary where it is cut by the secondary P A M P′. The former we call latitude, the latter longitude; and as it is agreed that all places to the right of the first meridian P G O P′ shall be called east, and those to the left west, we say A is in north latitude and east longitude.

FIG. 3.



Every circle can be divided into four equal parts, called quadrants, the circumferences of which subtend angles at the centre called right angles; and thus all right angles are equal, and a right angle is used as the unit of angular measurement. In England we subdivide the right angle into ninety equal parts called degrees, each degree into sixty equal parts called minutes, and each minute into sixty equal parts called seconds. Thus we have:—

60 seconds (marked $60''$) = 1 minute,
60 minutes (marked $60'$) = 1 degree,
90 degrees (marked $90°$) = 1 right angle.

In practice we speak of a certain distance on the earth's surface as one, two, three, or more degrees; but it must be understood that what is really meant is, that the distance spoken of subtends one, two, three, or more degrees at the centre of the earth; and also when we speak of a circle being divided into 360° we mean that the 360th part of the circumference subtends an angle at its centre of one degree. Among sailors it is usual to call the minutes of arc miles (nautical); this is evidently correct, because the length of the equator is divided into 360° and each degree into 60 minutes or nautical miles.

Exercise I.

To reduce degrees to nautical miles.—Multiply the degrees by 60, add in the odd number of miles, and reduce the seconds to the decimal of a mile.

Ex. 1. Reduce 57° 31′ 30″ to nautical miles.

$$\begin{array}{r} 57^\circ\ 31'\ 30'' \\ 60 \\ \hline \text{Answer}\quad 3451{\cdot}5 \text{ nautical miles.} \end{array}$$

Ex. 2. (*a*) Reduce 127° 14′ 15″ to nautical miles.

(*b*)	"	1	0	48	"
(*c*)	"	30	15	45	"
(*d*)	"		37	50	"
(*e*)	"	90	0	20	"
(*f*)	"		20	40	"

To reduce nautical miles to degrees.—Divide the miles by 60; the result is the number of degrees: if there be a remainder it is miles. The decimal of a mile must be reduced to seconds.

Ex. 3. Reduce 3451·5 nautical miles to degrees.

$$\begin{array}{r} 6{,}0)345{,}1'{\cdot}5 \\ \hline \text{Answer}\quad 57\ 31{\cdot}5 = 57^\circ\ 31'\ 30'' \end{array}$$

Ex. 4. (*a*) Reduce 2916· nautical miles to degrees, &c.

(*b*)	"	9180·	"	"
(*c*)	"	5400·3	"	"
(*d*)	"	2937·25	"	"
(*e*)	"	50·7	"	"
(*f*)	"	120·45	"	"

Ex. 5. Plymouth is in latitude 50° 22′ 25″ N. How many nautical miles is it from the equator?

Ex. 6. The Naze in Norway is 3477·8 nautical miles from the equator. What is its latitude?

Ex. 7. Quito is close to the equator, in longitude 78° 45′ 10″ west. How many nautical miles is it from the meridian of Greenwich?

To find the difference of latitude between two places:—

(*a*) If both be north or both south, *subtract* the less from the greater and reduce the remainder to nautical miles and decimals.

(*b*) If one place be north and the other south, *add* the two latitudes together and reduce the sum to nautical miles and decimals.

Ex. 8. What is the difference of latitude between London, in latitude 51° 31′ 48″ N., and Rome, in latitude 41° 54′ 6″ N.?

Latitude of London	51° 31′ 48″ N.
„ „ Rome	41 54 6 N.
Difference of latitude	9 37 42 S.
	60
Answer .	577·7 nautical miles.

Ex. 9. Swan River is in latitude 32° 3′ 18″ S., Gilolo in latitude 1° 7′ N. What is the difference of latitude between them?

Latitude Swan River	32° 3′ 18″ S.
„ Gilolo	1 7 0 N.
Difference of latitude	33 10 18 N.
	60
Answer .	1990·3 nautical miles.

To find the middle latitude.—MIDDLE LATITUDE is the latitude of that parallel midway between the given places. It is found thus:—

(*a*) If both places have latitude of the same name, half the sum of the two is the middle latitude.

(*b*) If the latitudes be of different names, half the difference is the middle latitude of the same name as the greater.

Ex. 10. Thurso is in latitude 58° 33′ N. and Plymouth 50° 22′ 25″ N. What is their middle latitude?

Thurso latitude	58° 33′ 0″ N.
Plymouth „	50 22 25 N.
Sum	108 55 25
Mid-latitude	54 27 42·5 N.

Ex. 11. What is the mid-latitude between Pernambuco, in 8° 3′ 36″ S., and Marseilles, in 43° 17′ 42″ N.?

Pernambuco latitude	8° 3′ 36″ S.
Marseilles „	43 17 42 N.
Difference	35 14 6
Mid-latitude	17 37 3 N.

Ex. 12. Find the mid-latitude between the following places :—

(*a*) Stockholm, latitude 59° 20′ 36″ N., and Calais, 50° 57′ 36″ N.

(*b*) Venice, latitude 45° 25′ 54″ N., and Batavia, 6° 8′ 0″ S.

(*c*) New Orleans, latitude 29° 57′ 42″ N., and Colombo, 6° 56′ 6″ N.

(*d*) Chusan, latitude 30° 1′ 0″ N., and South Pole.

(*e*) Wellesley Is., latitude 16° 18′ 15″ S., and Cape St. Mary, 16° 40′ 30″ S.

(*f*) Seven Stones, latitude 50° 3′ 0″ N., and Cape York, 10° 41′ 36″ S.

To find the difference of longitude between two places :—

(*a*) If both be east or both west their difference is the difference of longitude.

(*b*) If one be east and the other west their sum is their difference of longitude. Should their sum exceed 180° subtract it from 360°; the remainder is their difference of longitude.

Ex. 13. Find the difference of longitude between Greenock, in longitude 4° 45′ 12″ W., and Buenos Ayres, in 58° 22′ 0″ W. longitude.

Greenock longitude	4° 45′ 12″ W.
Buenos Ayres „	58 22 0 W.
Difference of longitude . . .	53 36 48 W.

Ex. 14. Wellington, New Zealand, is in longitude 174° 47′ E. and St. Lucia in longitude 60° 57′ W. What is their difference of longitude?

Wellington longitude	174° 47′ E.
St. Lucia „	60 57 W.
	235 44 W.
	360 0
Difference of longitude	124 16 E.

Ex. 15. Find the difference of longitude between—

(*a*) Toulon, longitude 5° 36′ 30″ E., and Valentia, 0° 23′ 0″ W.

(*b*) Madeira, longitude 16° 55′ 0″ W., and Goza, 14° 8′ 0″ E.

(*c*) Cape Agulhas, longitude 20° 0′ 42″ E., and Naze, 7° 2′ 0″ E.

(*d*) Cape Clear, longitude 9° 28′ 36″ W., and Stornoway, 6° 22′ 12″ W.

(*e*) Port Jackson, longitude 151° 18′ 12″ E., and Valparaiso, 71° 41′ 30″ W.

(*f*) San Francisco, longitude 122° 24′ 0″ W., and Sydney, 151° 14′ 0″ E.

The latitude and longitude from are the latitude and longitude of the place left.

The latitude and longitude in are the latitude and longitude arrived at or of the place of destination.

The direction is always reckoned from the latitude and longitude from, to the latitude and longitude in.

Ex. 16. A vessel has sailed from Cape of Good Hope, latitude 34° 22′ S., longitude 18° 29′ E., to Cape Horn, latitude 55° 59′ S., longitude 67° 16′ W.

Find the difference of latitude and difference of longitude she has made and the quadrant she has sailed in.

Cape of Good Hope, latitude	34° 22′ S.	longitude	18° 29′ E.
Cape Horn, latitude	. 55 59 S.	longitude	67 16 W.
	21 37		85 45
	60		60
Diff. lat. . .	1297 S.	Diff. long.	5145 W.

Because the *latitude in* (Cape Horn) is further to the south than the *latitude left* (Cape of Good Hope), therefore the vessel must have sailed to the southward; and because the *longitude in* (Cape Horn) is further to the westward than the *longitude left* (Cape of Good Hope), the vessel must have sailed to the westward. Hence she must have sailed in the south-west quadrant.

Ex. 17. Find the difference of latitude and difference of longitude from Cape Hatteras (35° 15′ N., 75° 30′ W.) to the Lizard (49° 58′ N., 5° 12′ W.); also find the mid-latitude and the quadrant sailed in.

Ex. 18. A vessel left Cape of Good Hope (34° 22′ S., 18° 29′ E.) and made 279·5 miles west longitude and 432·25 miles north latitude. Find her latitude and longitude in, the mid-latitude between the two latitudes, and the quadrant sailed in.

Ex. 19. What is the difference of latitude between two places?

(*a*) One on the tropic of Capricorn and the other on the Arctic Circle.

(*b*) One on the tropic of Cancer, the other at the North Pole.

(*c*) One on the Arctic Circle, the other on the Antarctic Circle.

Ex. 20. A vessel finds by observation she is in latitude 4° 17′ S. and longitude 68° 15′ E., and she has made 637′·8 difference of latitude to the south and 318′·2 difference of longitude to the westward since her former observation. What was the latitude and longitude left?

Figure of the Earth.

We have before stated that the earth is nearly a globe, but a little flattened. This flattening takes place at the poles, so that the equatorial diameter is greater than the polar, and without doubt has been caused by the diurnal rotation of the earth on its axis.

That the earth is not flat is proved—

(*a*) From the fact that when sailing in one general direction, E. or W., in the great southern ocean, the ship returns to the place from which it set out.

(*b*) When standing on the sea-shore and watching vessels leaving land, it is found that the hull, which is the lowest part, is hid first from view, although it is the largest, and therefore should remain longest in sight if the earth were flat; and gradually more and more of the masts and rigging are obscured, the tops of the masts remaining in view the longest. This is seen to occur in whatever direction the vessel sails; and similar phenomena in the reverse order take place when vessels approach land.

(*c*) Travellers, in proceeding north or south, invariably notice that new stars come into view on the northern or southern horizon, and others are obscured.

None of these facts could occur if the earth be flat.

That the earth is sensibly a globe is seen from—

(*a*) An eclipse of the moon, which is known to be caused by the shadow of the earth thrown by the sun on the surface of the moon. If watched, the shadow is always found to be circular, and the only body, which when placed in all positions throws a circular shadow, is a sphere.

(*b*) If an observer in a ship, when in the open ocean, looks around him, the horizon is seen to be circular; and if he ascends, still the horizon, although further away, is of the same form. This occurs in whatever part of the world the observer is situate, and the only body which, when looked at from any external point, always appears circular is a sphere.

(*c*) If parallels of latitude be measured at different distances from the equator, it is found the distances between any two meridians decrease as they approach the pole, but not in the same ratio that they would if the parallels were concentric circles on the plane; but the ratios are very nearly the same as they should be on the sphere.

That the earth is an oblate spheroid is proved by—

(*a*) The actual measurements of an arc of a meridian. It is thus found the part of a meridian subtending a degree of latitude increases in length as the poles are approached. The length of a degree of latitude varies as follows :—

Country	Mean latitude	Length subtending 1° of latitude in English feet
India	12° 32′	362,956
India	16 8	363,044
France	44 15	364,572
England	52 2	364,951
Russia	56 4	365.291
Sweden	66 20	365,744

and hence the further we go from the equator the radius of curvature increases in length and the flatter the earth becomes.

(*b*) *Pendulum Experiments.*—When pendulum clocks are taken from high latitudes towards the equator, it is found they lose time. This is partly owing to the fact that a place near the equator rotates faster than the one in high latitudes, and thus tends to throw away, as it were, bodies from its surface with greater force. But after this has been taken into consideration and allowed for, a portion of the loss still remains, which can only be accounted for by supposing there to be a decrease in the force of gravity. Now this force decreases on the earth's surface as the square of the distance from the earth's centre increases, and thus the equatorial diameter of the earth is proved to be longer than any other, and the shape of the earth is therefore a spheroid.

From calculations of the lengths of the axes by the astronomical methods of Precession, Nutation, and the theory of the moon's motions, Sir George B. Airy has found—

Equatorial diameter = 7925·648 miles
Polar diameter = 7899·114 „

Hence the equatorial diameter exceeds the polar by about $26\frac{1}{2}$ miles, and the earth's compression, which is measured by $\frac{\text{equa. radius} - \text{polar radius}}{\text{equa. radius}}$, is very nearly equal to $\frac{1}{300}$.

Recent investigations have shown that the earth is not a spheroid of revolution, but that the equator is slightly elliptical; its longest diameter meets the surface in 8° 15′ W. and the shortest passes through Ceylon.

Exercise II.

Ex. 21. Describe the methods used by mariners for fixing their positions at sea. By what names are they known? What do you mean by 'Dead Reckoning'?

Ex. 22. What is necessary to be known in fixing positions on the earth's surface? Explain the method called 'Rectangular Co-ordinates' for fixing the position of a point in a plane; and give the modifications necessary for fixing the point on the surface of a sphere. Explain *origin*, *abscissa*, and *ordinate.*

Ex. 23. What are the datum lines used in England for fixing positions on the earth's surface? Why are these chosen? Mention any departure from this usage by other nations.

Ex. 24. For their first meridian the French use the one through Paris. How would you convert longitudes from a French chart into English measurements and *vice versâ*? Find the English measurement of the longitude of a place marked 67° 23′ E. on a French chart (long. Paris = 2° 20′·5 E.)

Ex. 25. It has been said that 'the rotation of the earth gives a fixed line on which the whole system of definitions in navigation depend.' What is that fixed line? Define it, and equator and meridian by reference to that fixed line.

Ex. 26. Prove that every section of a sphere is a circle.

Ex. 27. What is the difference between great and small circles of a sphere? Mention any which have received particular names and define them.

Ex. 28. What do you mean by a primary and a secondary great circle? How are the latter dependent on the former? Where will the origin for the co-ordinates on the sphere be situate?

Ex. 29. Define difference of latitude and difference of longitude. A vessel sails from Cape Clear (51° 25′ N., 9° 29′ W.), making 341′·4 west longitude and 538′·7 north latitude. Find her latitude and longitude in.

Ex. 30. What is the shape of the earth? Define the terms used. Give reasons for believing the earth—

(*a*) Not to be flat.

(*b*) To be spherical.

(*c*) To be an oblate spheroid.

Ex. 31. What do you mean by the earth's compression? What is the value of the terrestrial compression, and how is it found?

Ex. 32. Given the equatorial diameter of Jupiter 87,432 miles, and his polar diameter 82,260 miles, what is the Jovian compression, and what is the ratio between this and the terrestrial compression?

Ex. 33. Give definitions of the earth's axis, poles, meridians, and equator. How is the situation of a place on the earth's surface determined? What are the difference of latitude and difference of longitude between A and B, latitude A 13° 18′ N., longitude A 123° 12′ E., latitude B 33° 17′ S., longitude B 74° 36′ E.? *E.* 1869.

Ex. 34. What is the difference of latitude between two places, one of which is on the tropic of Cancer and the other on the tropic of Capricorn?

Ex. 35. What is the difference of longitude between Ferro Isle (longitude 17° 58′ W.) and St. Petersburg (longitude 30° 19′ E.)? *E.* 1873.

Ex. 36. Show clearly how the position of a place on the surface of the earth is determined, if we know its latitude and longitude.

Ex. 37. How many places answer to the following description: latitude 55° 30′, longitude 36°? These being the numerical values of the latitude and longitude of Moscow, define its position without any ambiguity. What spot on the earth's surface is represented by latitude 0°, longitude 0°? What is the difference of latitude and the difference of longitude between Cape Comorin (8° 5′ N., 77° 44′ E.) and Cape Leewin (34° 24′ S., 115° 28′ E.)? *E.* 1875.

Ex. 38. State the chief reasons for concluding that the earth is a sphere Is it an exact sphere, and if not, how does it differ from one? Mention some of the most important great circles of the earth. *E.* 1878.

Ex. 39. Define latitude, longitude, and middle latitude. Find the latitude in, having given the latitude from, 33° 28′ N., and the true difference of latitude 325′ N. Find also the middle latitude of these two places. What is the difference of longitude between two places—longitude from 51° 20′ E., longitude in 32° 30′ W.? *E.* 1879.

Ex. 40. Define poles of the earth, equator, meridians, and parallels of latitude. What do you understand by the term first meridian? State its special use. What are respectively the greatest latitude and longitude a place can have? Why? *E.* 1881.

Ex. 41. What is the figure of the earth? What is the difference between the equatorial and polar diameters? *Royal Naval College*, 1872.

CHAPTER II.

The compass—Its parts—The needle—The card—Comparison of the card with the horizon—The bowl—Azimuth compass—Adjustments of the compass — Dip — Magnetic induction—Variation—Deviation—How ascertained : by reciprocal bearings, by a distant object, by marks on dock walls, by celestial observations—Exercises—Examination.

The instruments used in navigation for obtaining the position of a ship at sea are the compass, the log-ship, the log-line, and the log-glass.

The mariner's compass is an instrument by which sailors are enabled to determine the direction in which to steer their vessels across the ocean, and to take the bearings of objects within view. It is equally useful to the traveller in directing his route across a new country: and its utility depends on the directive force of the magnet. The principal parts of the compass are—

(*a*) The Needle.
(*b*) The Card.
(*c*) The Bowl.

(*a*) The Needle is a bar of well-hardened homogeneous steel which has been magnetised. It should be well hardened, that it may be retentive of its magnetism, and homogeneous, that no consequent points may be set up. Its shape must be such that its magnetic axis shall coincide with the middle of the bar, and hence is generally made in the form of a lozenge, or of thin bars placed on their edges, and should be strong enough to lift its own weight. A small cap should be formed at the centre of the needle to contain an agate or other hard stone, that the friction may be lessened when the needle is in action.

(*b*) THE CARD is a circular piece of mica or cardboard which represents the plane of the horizon. 'In the open sea a person's vision is bounded by a circle, in the centre of which he is placed. This circle is the *visible horizon*. The plane of the meridian passes through the centre of the horizon and cuts the circumference in the north and south points; a diameter drawn at right angles to the north and south diameter marks the east and west points; and thus the plane of the horizon is divided into four equal parts or quadrants, and each quadrant is subdivided into eight equal parts. The horizon is thus subdivided into thirty-two equal parts, called points.' Then, because the circumference subtends at the centre an angle of 360°—

$$\text{A point} = 360^\circ \div 32 = 11^\circ\ 15'\ 0''$$
$$\tfrac{1}{4}\ \text{,,}\quad \text{,,} \quad = 2\ \ 48\ \ 45$$
$$\tfrac{1}{2}\ \text{,,}\quad \text{,,} \quad = 5\ \ 37\ \ 30$$
$$\tfrac{3}{4}\ \text{,,}\quad \text{,,} \quad = 8\ \ 26\ \ 15$$

The four principal points, North, South, East, and West, are called Cardinal. The intercardinal points bisect the quadrants, and are named north-east (NE.) when between the north and east, south-east (SE.) when between the south and east, south-west (SW.) when between the south and west, and north-west (NW.) when between the north and west. The circumference of the horizon is thus divided into eight equal parts, and the bisectors of these are named from the nearest cardinal point combined with the nearest intercardinal point; e.g. the bisector of the angle between north and north-east is called north-north-east (NNE.), the bisector between east and north-east is called east-north-east (ENE.). The other bisectors are named east-south-east (ESE.), south-south-east (SSE.), &c. The other sixteen points are named from the nearest cardinal or intercardinal point towards the other cardinal point, and has the word *by* between them; thus N. by E. and N. by W. are the points nearest to north towards the east and west respectively, then NE. by N., NE. by E., E. by N., E. by S., and so on. The half and quarter points are named from the nearest cardinal or intercardinal point; e.g. the quarter-point from NE. towards the north is NE$\frac{1}{4}$N., the quarter-point from ESE. towards the south is SE. by E$\frac{3}{4}$E., the half-point from WSW. towards the west is W. by S$\frac{1}{2}$S., &c. On the compass card are represented these subdivisions of the circumference of the horizon, and the north point is distinguished by a fleur-de-lis, a star, or some other ornament. Any straight line drawn from the centre of the card to its

FIG. 4.—MARINERS' COMPASS.

circumference represents a line drawn from the observer to his horizon, and in either case the lines are called *Rhumb Lines.*

Every student should learn the names of the points in order from the card itself, and be able to repeat them, beginning at any point in either direction. This is called *boxing the compass.* To help him to do this a card and table are appended.

A Table of the Angles which every Point and Quarter-point of the Compass makes with the Meridian.

North		Points	Degrees, &c.	Points	South	
			° ′ ″			
		0¼	2 48 45	0¼		
		0½	5 37 30	0½		
		0¾	8 26 15	0¾		
N. by E.	N. by W.	1	11 15 0	1	S. by E.	S. by W.
		1¼	14 3 45	1¼		
		1½	16 52 30	1½		
		1¾	19 41 15	1¾		
NNE.	NNW.	2	22 30 0	2	SSE.	SSW.
		2¼	25 18 45	2¼		
		2½	28 7 30	2½		
		2¾	30 56 15	2¾		
NE. by N.	NW. by N.	3	33 45 0	3	SE. by S.	SW. by S.
		3¼	36 33 45	3¼		
		3½	39 22 30	3½		
		3¾	42 11 15	3¾		
NE.	NW.	4	45 0 0	4	SE.	SW.
		4¼	47 48 45	4¼		
		4½	50 37 30	4½		
		4¾	53 26 15	4¾		
NE. by E.	NW. by W.	5	56 15 0	5	SE. by E.	SW. by W.
		5¼	59 3 45	5¼		
		5½	61 52 30	5½		
		5¾	64 41 15	5¾		
ENE.	WNW.	6	67 30 0	6	ESE.	WSW.
		6¼	70 18 45	6¼		
		6½	73 7 30	6½		
		6¾	75 56 15	6¾		
E. by N.	W. by N.	7	78 45 0	7	E. by S.	W. by S
		7¼	81 33 45	7¼		
		7½	84 22 30	7½		
		7¾	87 11 15	7¾		
East	West	8	90 0 0	8	East	West

The needle with small brass sliding weights (whose use will be explained as we proceed) is attached by means of small brass screws to the card through its centre, so that the north point of the needle shall be exactly below the point marked north on the card, and the south point of the needle below that marked south on the card, and then the card is complete.

(*c*) The Bowl is a hollow hemisphere of brass, heavy at its vertex, which is downwards, and having its open part covered with glass. On the outside of the top of the bowl project two pivots, which should be the ends of a horizontal diameter of the bowl; these fit into a brass ring, from which also project outwards two pivots, whose directions should be at right angles to the direction of the other two. The pivots of the outer ring fit into bearings in a box called *the Binnacle*. This arrangement forms a universal joint called *Gimbals*, and is for the purpose of keeping the open part of the bowl always horizontal when the binnacle is tilted by the heeling of the vessel. From the centre of the bottom of the bowl springs a pivot of hardened steel sharpened to a fine point, on which the needle is placed when in use. On the inside of the bowl are two fine black vertical lines, diametrically opposite to each other; and when the binnacle and its appendages are fixed these vertical lines should lie in a plane cutting the ship amidships from stem to stern, or in a plane parallel to that one. These lines are called *Lubber's Points*. The direction of the ship's head and the course steered are known by noticing at what points the vertical lines meet the card. The course of the ship in this case is the bearing of the ship's head when referred to the compass card, and is called the Compass Course.

The Azimuth Compass is one by which greater accuracy in taking bearings can be attained than by the common steering compass. For this purpose it is fitted with a lever for throwing the needle off the pivot when not in use, and with two sight vanes, to one of which is attached a reflector for taking the bearings of celestial and other elevated objects, and to the other a glass prism for reading off bearings without moving the eye from the slit in the vane. Glass shades are also added for observing the sun, and the whole can be mounted on a tripod for use.

Adjustments of the Compass.—There are certain requisites which a good compass should possess. They are—

(1) *The magnetic axis should lie along the centre of the needle.* This adjustment is made by reversing the needle under the card; then, if the north and south points on the card are also reversed, the adjustment is perfect. Imperfect adjustment in this respect affects all bearings and courses alike. The amount is included in finding the deviation, and is classed under the coefficient A.

(2) *The agate in the centre of the needle on which it revolves must be in the centre of the card.* If not, the difference of bearings between two objects will vary according as two different parts of the card are used, and if this is found to be the case the instrument should be rejected.

(3) *In the azimuth compass the line of sight, or the line joining the slit and thread, should pass directly over the pivot.* If there be an error it is detected by taking the bearing of a distant object; then turn the bowl half round, so as to reverse the vanes, and take the bearing of the same object. If the second observation coincides with the first, the adjustment is perfect, but if not both bearings must be observed, and their mean taken for the correct bearing.

(4) *The sight vanes must be vertical.* To find whether they are or not, the bowl should be taken on shore and tested by a plumb line; if found inaccurate, the compass should be returned to the maker.

Err rs of the Compass.—If a bar of steel be magnetised and freely suspended through its centre of gravity by a thread without torsion, the magnet, as it is then called, will, after many oscillations, come to rest in a definite position, which in England lies in a vertical plane nearly north and south; and if disturbed, the same end of the magnet will always return to the same position when the cause of disturbance is removed. This directive power of the magnet seems to have been known to the Chinese for at least 4,000 years, although it was not made known to Europeans until the twelfth century. The end which in the northern hemisphere points towards the terrestrial north is called, after Sir G. B. Airy, F.R.S., the red-marked end or north pole, the other the blue-marked end or south pole of the magnet. A light steel bar, properly shaped, magnetised and mounted, is what we have spoken of as the needle. It has been found that at all places on the earth, except on the magnetic equator, the needle does not remain horizontal, as it would if not magnetised; but in north magnetic latitudes the red-marked end points downwards, and in south magnetic latitudes the blue-marked end points downwards. This property of the needle is called *Dip or Inclination*, and was discovered by Robert Norman A.D. 1580.

Dip or Inclination is the angle included between the horizontal plane and the axis of the needle when allowed to come freely to rest. This angle varies with the geographical position of the needle, being in London (1882) about 67° 20′. A line joining all places where the needle remains horizontal is the *Magnetic Equator*, and all places having the same dip have the same magnetic latitude. Where the needle takes a vertical direction are the magnetic poles. The north magnetic pole was discovered by Sir James Ross in 1830; it was situate in 70° 14′ N. latitude and 96° 46′ W. longitude. The south magnetic pole is situate about 73°½ S. and 147°½ E. It will be noticed these points are not diametrically opposite to each other. As it is convenient to have the card horizontal, good needles have brass sliding weights attached to them, to be moved nearer or further from the pivot as circumstances require; but as the centre of gravity of one half the needle is thus removed farther from the point of support than the other, one bad result is introduced, viz. unsteadiness; for if a ship be struck by a heavy sea different momenta are acquired by the two halves of the needle, and this causes it to oscillate; and often many vibrations of the card are made before the directive force of the needle can bring it to rest. Sometimes when the disturbance is synchronous with the needle's oscillations it will spin completely around and becomes useless. Many devices have been proposed to prevent unsteadiness; but all are based on a method of retarding the action of the needle, and thus its efficiency is lessened in calm weather.

If we take two bars which have been magnetised and freely suspend each through its centre of gravity, not only will they take up definite positions, but if brought near they will also exert an influence each on the other, and it will be found that *like poles repel and unlike poles attract each other.* This is the fundamental law of magnetism, and has led scientific men to consider the earth a magnet, which has magnetism similar to that in the blue-marked end of a needle residing in the north magnetic pole of the earth and *vice versâ*; and thus the red-marked ends of all magnets are attracted towards the earth's north magnetic pole. If a strong bar magnet be brought near to a long bar of soft iron, the magnetism of the iron undergoes a change, for, whereas before it showed no magnetic properties, it now has become a perfect magnet, and remains so as long as the disturbing bar magnet continues in the same relative position to the iron bar. The end of the iron rod nearest the magnet has a pole of a contrary name to that of the end of the magnet presented to it, and the end farthest from the magnet has a pole of the same name as that presented to the rod; but directly the disturbing magnet is removed the iron bar returns to its former inert condition. This is called *magnetism by induction*, which may be defined as *the development of magnetism by magnetic action exerted at a distance*, and the facts related above are called the laws of induced magnetism. If during the time the iron bar is magnetised by induction it be twisted, hammered, or acted on by mechanical violence, a part of the magnetism is retained by the bar after the disturbing magnet is removed. This has received the name of *subpermanent magnetism*, to distinguish it from the permanent magnetism in steel magnets. Now, as the earth has all the properties of a large magnet, we might expect similar phenomena, and this is justified by experience. If a bar of soft iron be held pointing to the north magnetic pole of the earth, the end nearest the north will be found to have acquired magnetism, the same as resides in the north end of the magnet, and will repel that end, but attract the south. This is magnetism by induction from the earth. In the above experiment we have taken a bar, as most convenient, but similar results will follow if we use a sheet or mass of soft iron instead. What has here been conducted as an experiment is taking place every day in the building of ships. The iron bars, stringers, knees, &c., in wooden ships, the framework of composite ones, and the whole material of iron ships become, whilst building, magnetic by induction from the earth; and the magnetism thus acquired is rendered subpermanent by the hammering, twisting, and riveting which the iron is subject to in constructing the vessel.

VARIATION OR DECLINATION *is the angle contained between a terrestrial and a magnetic meridian.* It is called easterly when the red-marked end points to the right and westerly when it points to the left of terrestrial north, and it is due entirely to the influence the earth exerts on magnetic needles. If a compass be carried to different parts of the earth, it will be found that the needle makes different angles with a terrestrial meridian, that is, it has different variations at different places; and a line joining all places having the same variation is called a *magnetic meridian*, but it will be found that sometimes the terrestrial and magnetic meridians coincide; there we have no variation. The lines joining all places which have no variation are called Agonic lines. At present one passes from King William's Land through Hudson's Bay, Canada, Lake Huron, across the United States to Cape Hatteras, thence a little to the eastward of the West Indian Islands to Cayenne, the mouth of the Amazon, across Brazil to Rio Janeiro to the 20th meridian west, and on to the south magnetic pole. Another line passes through the north of Lapland, the White Sea, through Russia to Astracan on the Caspian, to Hyderabad, through the Laccadives, Sumbawa, and Western Australia to the south magnetic pole. The third line starts from Amoy north, through Mongolia to Lake Baikal, then NE. to the river Tana in Siberia, and southward through the Sea of Okhotsk, the Kurile, and the Philippine Isles back to Amoy, forming on the chart a large ellipse. In proceeding in either direction, east or west, from the agonic lines, the variation of the needle gradually increases and becomes a maximum at some intermediate line between them. Variation charts are constructed, showing the variation of the compass all over the world for the time of printing. Hence if a sailor knows his latitude and longitude he can, by reference, read off the variation. Both the dip and variation are constantly changing, and after long intervals of time the changes are very apparent; thus in London in 1580 the needle pointed to 11° 15′ east of north, whilst in 1819 it pointed to about 24° 30′ west of the same point. At present the variation at Plymouth is about 20° 20′ W. and is decreasing about 7′ annually. The dip in 1723 was 74° 42′, in 1875 was 67° 40′, and is decreasing at the rate of 2′·69 per annum over the British islands. Those changes which recur after long intervals are called *Secular Variations.* Besides those there are *Annual* and *Diurnal* variations, both of which are small; and *irregular changes,* which are called *Perturbations,* but these will be illustrated further on.

Iron is so extensively used in the construction of vessels that, besides their permanent errors, compasses are affected by *Deviation.*

DEVIATION *is the error of the compasses caused by iron in a ship's build, her equipment or cargo.* Another source of error is LOCAL ATTRACTION, which is that caused by volcanoes, the geological formation of coasts, iron cranes, water pipes and pillars, the proximity of other ships, &c. It will be readily seen from the above definitions that deviation is caused by materials in the vessel herself, whilst local attraction is caused by materials external to the ship and confined to the locality. With the latter we have nothing to do, because when a vessel shifts her geographical position the causes of local attraction are left behind, and the effects are no longer felt. Deviation is measured by the angle which a vertical plane through the poles of the needle makes with the magnetic meridian, and may be distinguished from variation and local attraction by its dependence on the position of the ship's head and the angle to which the ship is heeled, whilst the other two are independent of the position of the ship with regard to either the vertical or horizontal planes. A change in the amount of deviation may be expected—

(*a*) With a change in the direction of the ship's head.

(*b*) With the heeling of the vessel.

(*c*) With a change of the geographical position.

(*d*) With the changing or shifting of the cargo.

(*e*) With a heavy blow, as being struck by a sea or by grating on rocks.

(*f*) With the appearance of the aurora.

(*g*) With the ship (if of wood) being struck by lightning. Those marked *a*, *b*, *c*, *d*, *e* are causes of changes which must be especially guarded against by mariners, but no opportunity for testing the accuracy of the compasses should be neglected by them.

Iron and composite ships are supplied with deviation tables in which the amount of this error is registered opposite to every point the ship's head can be placed on; but such tables are only of service when used in about the same magnetic latitudes in which they were made. There are several methods in use for constructing a table of deviations, but in all cases the ship should first be placed beyond all magnetic influence from surrounding objects, and the ship herself be ready for sea. Means must then be taken for *swinging* her, i.e. for bringing her head to every point of the compass. The methods chiefly in use are—

(*a*) *By Reciprocal Bearings.*—An azimuth compass is first compared with the standard compass, whose deviation is required to see what error may arise from difference of centering, mounting the card on the needle, or from other causes. The standard compass is then put in its place and the other set up on shore where it will be free from all disturbing elements, and can be conveniently seen from the standard. The ship is then swung, and at a given signal by those on board the position of the ship's head is observed as well as the bearing of the shore compass from the standard, and at the same instant the bearing of the standard compass is taken from the shore compass and arranged in a table as follows :—

Ship's head by standard compass	Simultaneous bearings		Deviation of standard compass
	From standard compass on board	From shore compass	
North	S. 27° 15′ E.	N. 28° 30′ W.	1° 15′ W.
N. by E.	S. 31 40 E.	N. 30 10 W.	1 30 E.
NNE.	S. 36 10 E.	N. 31 20 W.	4 50 E.
NE. by N.	S. 40 50 E.	N. 31 30 W.	9 20 E.
NE.	S. 46 0 E.	N. 33 10 W.	12 50 E.

A column may be added to insert the time of observation, to serve as a check in case a discrepancy should arise. It is evident that the bearings by the two compasses would be exactly opposite, if nothing but the directive force of the earth influenced either; but, as the vessel disturbs the one on board, *that* one must have deviation, and this is found by reversing the shore bearings, and taking the difference between them so reversed, and the bearings as taken by the standard. This is registered in the table opposite the ship's head, and is marked E. or W. according as the reversed shore bearings are to the right or left of the others.

(*b*) *By taking the Bearings of a Distant Object.*—A prominent object must be selected whose distance must be very great compared with the diameter of the circle described by the standard compass in swinging the ship; generally from six to eight miles will be found sufficient. The bearing of the selected object must be taken and registered as the ship's head is brought to every point of the compass; and its correct magnetic bearing is the mean of the whole so registered; but in practice it is found sufficiently accurate to take the mean from the eight principal points. The differences between the correct magnetic bearing thus obtained and the compass bearings on every point is the table of deviations required. The form used is as follows, and the correct magnetic bearing is known to be N. 63° W :—

Ship's head by standard compass	Bearing of distant object	Deviation	Ship's head by standard compass	Bearing of distant object	Deviation
	° ′	° ′		° ′	° ′
North	N. 59 50 W.	3 10 W.	South	N. 66 10 W.	3 10 E.
N. by E.	N. 65 35 W.	2 35 E.	S. by W.	N. 63 5 W.	0 5 E.
NNE.	N. 71 10 W.	8 10 E.	SSW.	N. 60 0 W.	3 0 W.
NE. by N.	N. 76 10 W.	13 10 E.	SW. by S.	N. 56 30 W.	6 30 W.
NE.	N. 79 50 W.	16 50 E.	SW.	N. 53 20 W.	9 40 W.
NE. by E.	N. 82 30 W.	19 30 E.	SW. by W.	N. 50 0 W.	13 0 W.
ENE.	N. 83 30 W.	20 30 E.	WSW.	N. 46 50 W.	16 10 W.
E. by N.	N. 84 5 W.	21 5 E.	W. by S.	N. 43 45 W.	19 15 W.
East	N. 83 20 W.	20 20 E.	West	N. 41 50 W.	21 10 W.
E. by S.	N. 82 15 W.	19 15 E.	W. by N.	N. 39 40 W.	23 20 W.
ESE.	N. 81 5 W.	18 5 E.	WNW.	N. 39 0 W.	24 0 W.
SE. by E.	N. 79 30 W.	16 30 E.	NW. by W.	N. 39 25 W.	23 35 W.
SE.	N. 77 40 W.	14 40 E.	NW.	N. 41 0 W.	22 0 W.
SE. by S.	N. 75 5 W.	12 5 E.	NW. by N.	N. 44 0 W.	19 0 W.
SSE.	N. 72 40 W.	9 40 E.	NNW.	N. 48 10 W.	14 50 W.
S. by E.	N. 69 0 W.	6 0 E.	N. by W.	N. 53 45 W.	9 15 W.

The mean of the bearings on the eight principal points is N. 62° $52\frac{1}{2}'$ W., and this, therefore, may be assumed to be the correct magnetic bearing of the object. By using this value there will be a difference only of $7\frac{1}{2}'$ from the true with which this table is calculated, and this amount is so small that it may be disregarded in practice.

(*c*) *By Marks on the Dock Walls.*—This is a very convenient method where it can be practised. At Liverpool and Cronstadt there are marks painted on the dock walls showing the magnetic bearings of some conspicuous object inland. As a ship is swung the difference between the correct magnetic bearing and that shown by the standard compass is seen at once, and can be registered without trouble.

(*d*) *By Azimuth and Amplitude Observations.*—The compass errors obtained by observation of celestial objects on board ship always contain variation and deviation, and may also contain local attraction, if the ship be near any disturbing cause. By working an amplitude or azimuth, the true bearing is always found, the difference between which and the bearing by the standard compass at the time the sight was taken gives the whole error of the compass for the particular position the ship's head was in at the time. Declination or variation charts will always give the amount of error due to that cause; and the difference between the variation and the compass error is the deviation, easterly when the compass error is to the right, and westerly when it is to the left of the variation, reckoning from the north. By continuing this process to the eight principal points of the compass, a complete table can be formed by *Napier's Graphic Method*, to be explained hereafter.

Example.—Suppose the compass error obtained by an amplitude on board ship to be 17° 30′ W., and the variation from the chart for the ship's position to be 3° E., what is the deviation?

Compass error	17° 30′ W.
Variation	3 0 E.
Difference = deviation	20 30 W.

Here it must be understood that the difference is the algebraic difference, which may be thus expressed; when of contrary names add, but when of the same name subtract, to get the difference.

Exercise III.

Ex. 42. What instruments are used in navigation? What are the essential parts of the compass? Describe each part.

Ex. 43. What are the qualifications of a good needle? Give reasons for your answer. How is it fastened to the card?

Ex. 44. What does the compass card represent? Compare it with what it represents. On what principle is it divided? What do you mean by boxing the compass?

Ex. 45. What are the lubber's lines? Of what use are they, and how do they answer the ends for which they were designed?

Ex. 46. What are the chief differences between a binnacle compass and an azimuth compass? Give the chief requisites for a good compass, and show how you would test a compass for these requisites.

Ex. 47. Describe the errors of the compass, and show how each is produced. What do you mean by the magnetic equator, by a magnetic meridian, and by agonic lines? Are these great or small circles?

Ex. 48. What is the fundamental law of magnetism? Show how this would act if two compasses be brought near each other. Define and illustrate where you can magnetic induction, terrestrial magnetic induction, and subpermanent magnetism.

Ex. 49. What is variation? Mention what you know about its permanence at any selected place. What is the difference between secular, annual, and diurnal variation and perturbations?

Ex. 50. What do you mean by deviation, and how would you find out whether a compass error was due to deviation, to variation, or to local attraction? With what changes may you expect a change in deviation?

Ex. 51. What do you mean by swinging a ship? What precautions are necessary in swinging a ship, and why? Describe the different methods of forming a deviation table. The standard compass on board marks SW$\frac{1}{2}$W., and that on shore NE. by E$\frac{1}{2}$E.: what is the deviation? *E.* 1872.

Ex. 52. Describe fully the card of the mariner's compass, both the upper surface of it and the under, showing how the needle is placed and the arrangements for balancing it on its point. The indications of two compasses A and B, when together on shore, are the same, the N. point of each being directed to the NNW. point of the horizon. B is taken on board a ship, and its N. point is now directed to the NW. by N. point of the horizon. State in degrees, minutes, &c., the difference of the two, and account for it, and for neither pointing to the true north on shore. *E.* 1874.

Ex. 53. Describe fully the mariner's compass. Explain how the lubber's point bears the same relation to the card that the ship's head does to the horizon. What are the causes which render the indications of the compass uncertain? *E.* 1877.

Ex. 54. State the use of the mariner's compass and explain the principle on which it depends. How is the card divided? Write down the number of points and parts of points between N$\frac{1}{4}$E. and SSE$\frac{1}{2}$S. Express the difference also in degrees, minutes, and seconds. *E.* 1881.

Ex. 55. Not having a table of deviations, I find that in order to sail a course which I know to be true magnetic S$\frac{3}{4}$W. I am obliged to steer her by standard compass S. by W$\frac{1}{4}$S. What is the deviation for this position of the ship's head? *E.* 1869.

Ex. 56. A ship starting on an ENE. course goes round gradually through the N. until she is on a S. by W. course. Through how many points has she turned? State also the number of degrees. *E.* 1869.

Ex. 57. A ship is sailing in the direction NE$\frac{3}{4}$E. Find the number of points and of degrees, &c., between her course and the meridian, reckoning from the S. point. *E.* 1871

Ex. 58. How many degrees, minutes, and seconds are there between two rhumb lines, one NE. by E$\frac{1}{4}$E. and the other E. by S$\frac{1}{2}$S.? *E.* 1873.

Ex. 59. What do you mean by the course? Show how the mariner's compass enables the sailor to keep on a given course, and mention the causes which render its guidance liable to error. Write down the points of the compass between S. by W. and ENE. How many degrees, &c., are there between N$\frac{1}{2}$E. and NE$\frac{1}{4}$N.? *E.* 1876.

Ex. 60. Having been steaming SE. by S$\frac{1}{4}$E., it is desired to change the course so as to proceed in a direction 64° 41′ 15″ to the S. of W. Through how many degrees must the ship's head be turned, and what is the true course? *E.* 1878.

Ex. 61. Define variation and deviation of the compass. Why is it of so much importance to know the variation and deviation? Example: True course NW$\frac{1}{2}$W., compass course WNW$\frac{1}{4}$W., deviation 5° 40′ E. What is the variation?

Ex. 62. State and explain clearly the rule for correcting a compass course for variation and deviation. *Royal Naval College*, 1867.

Ex. 63. Describe the table of deviations of compass used on board ship, and explain fully a method of drawing it up. *A.* 1879.

Ex. 64. Define and explain course, a point, cardinal point, lubber's point. The lubber's point coincides with SSW.: how many points abaft the beam is the north point? Convert the angle into degrees, &c. *E.* 1880.

Ex. 65. Find the deviation of the compass for the several positions of the ship's head in the accompanying table:—

Ship's head	Bearings from ship's compass	Bearings from shore compass
(*a*) North	S. 35° 20′ W.	N. 37° 15′ E.
(*b*) SW.	S. 18 40 E.	N. 31 20 W.
(*c*) ENE.	N. 15 30 W.	S. 7 10 E.
(*d*) N. by W.	S. 22 15 W.	N. 22 15 E.
(*e*) SE. by S.	N. 88 45 E.	N. 83 40 W.
(*f*) West	West	N. 69 20 E.
(*g*) NW. by W.	N. 79 20 E.	S. 62 20 W.
(*h*) South	East	S. 67 50 W.

Ex. 66. The bearings of a distant object were found to be as follows with the ship's head as stated:—

Ship's head	Compass bearing distant object	Ship's head	Compass bearing distant object
North	S. 19° E.	South	S. 5° E.
NE.	S. 8 E.	SW.	S. 24 E.
East	S 6 W.	West	S. 26 E.
SE.	S. 10 W.	NW.	S. 22 E.

Find the correct magnetic bearing of the distant object, also the deviation for each point of the ship's head mentioned.

Ex. 67. The bearings of a distant object were found to be as follows with the ship's head as in the table :—

Ship's head	Compass bearing distant object	Ship's head	Compass bearing distant object
North	N. 84° 10′ E.	South	East
NE.	S. 79 20 E.	SW.	N. 78° 30′ E.
East	S. 73 0 E.	West	N. 77 0 E.
SE.	S. 76 10 E.	NW.	N. 78 50 E.

Find the correct magnetic bearing of the distant object, also the deviation for each point of the ship's head mentioned.

Ex. 68. Describe the method of determining the deviation of the compass by reciprocal bearings. What other methods are there of finding the deviation of the compass? After swinging the ship and determining the bearings on different points of some distant object, how is the correct magnetic bearing found? *Royal Naval College*, 1866.

CHAPTER III.

Leeway—Correction of courses—Exercises—Log-glass—Log-ship—Log-line—Heaving the log—Nautical mile or knot—Why computations by dead reckoning cannot be relied on—Exercises—Variation in length of knot, how found—Patent log—Napier's pressure log—The Dutchman's log—Exercises—Examination.

LEEWAY.

If the wind blow from direct aft it is evident there is no side pressure and the vessel's progress is in the direction of her keel; but if the wind blow on the side of the vessel, if not too far forward, then from her shape she will, if the sails be properly trimmed, make progress in the direction of her head; but a part of the force of the wind is expended in driving the vessel away with the wind, or, as a sailor calls it, to leeward—hence its name leeway. It is obvious this correction must depend on—

(*a*) *The force of the wind.* As the amount of sail a vessel will carry at any time depends on the same thing, it was formerly a custom to estimate the leeway by the amount of sail a vessel was carrying at the time

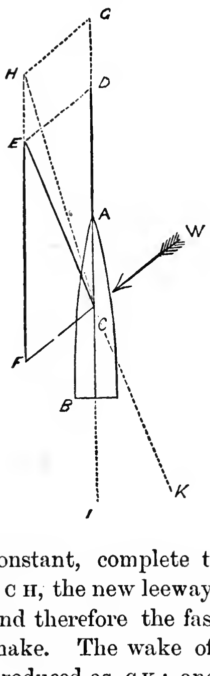
(*b*) *The amount of broadside, rigging, and sails exposed to the wind.*

(*c*) *The depth to which the vessel is laden.* It is evident a light vessel does not meet with so much resistance from the water as she would if deeply laden, and she is therefore more easily driven to leeward.

(*d*) *The rate the vessel is going.* This is not so evident, but it may be seen from the following illustration :—

Let A C B be a vessel sailing in the direction B A G by compass; and let C D represent the headway made in a unit of time, as one minute; W the wind, which (by acting obliquely on the sails, cordage, sides, &c.) drives the vessel in the direction of the arrow a distance represented by C F in the same unit of time. Complete the parallelogram C D E F and join C E, then by the parallelogram of velocities C E is the resultant of C D and C F, the two distances the ship travels; hence the vessel will move in the direction C E, although her head will continue in the direction B A. The angle A C E is called Leeway, and may be thus defined:

FIG. 5.



LEEWAY *is the angle included between the fore and aft line of the ship and the direction in which she is actually moving.* Now it is evident if the vessel makes a greater headway per unit of time, while the effect of the wind in driving her to leeward remains the same—*i.e.* C D is increased to C G, while C F remains constant, complete the parallelogram C G H F, then the angle G C H, the new leeway, will be less than the former leeway D C E, and therefore the faster a vessel sails the less leeway she will make. The wake of the vessel is seen in the direction of E C produced as C K; and if we produce A B to I, it is evident that the angle I C K is equal to the leeway D C E. Hence at sea it is usual to estimate the leeway by noticing the angle which the ship's wake makes with a fore and aft line produced; and this leads us to another definition: LEEWAY is the angle included between a ship's wake and her keel produced.

A vessel is said *to tack* when she alters her course so as to have the wind blowing on the other side of the vessel; and is said to be on the *starboard tack* when the wind blows on the right hand side of the vessel, and on the *port tack* when the wind blows on the left hand side of the vessel; and she is said to be *close-hauled* when she sails as near to the wind as she can lie. Most vessels, when in good trim, can sail within six points of the wind, and some yachts, built entirely for racing purposes, within four points. The wind is said to be on the *beam* when it blows at right angles to the fore and aft line, and on the *quarter* when four points from right aft. Now if the wind blow in a direction not parallel to the ship's keel, it is seen from above a portion of its force must be lost in driving the vessel to leeward; hence, leeway must be allowed from the wind, and this gives rise to the following rules:—

I. When the ship is on the port tack, leeway must be allowed to the right hand.

II. When the ship is on the starboard tack, leeway must be allowed to the left hand.

Right and left are to be thus understood: suppose yourself in the centre of your compass looking towards the vessel's bow, then right and left of this direction will be to your right and left hand respectively.

An easy way of measuring leeway is by screwing a graduated semicircle horizontally on the rail at the stern of the vessel and its diameter athwartships. If where the fore and aft line of the ship cuts the semi-circle be marked zero, and the graduations be made from that point, the angle the ship's wake makes with the fore and aft line is read off without any difficulty.

CORRECTION OF COURSES.

In practice we speak of three different kinds of courses, viz., a *Compass Course*, a *Magnetic Course*, and a *True Course.*

A Compass Course is the angle the ship's track makes with the axis of the compass needle, and contains both deviation and variation.

A Magnetic Course is the angle the ship's track makes with a magnetic meridian, and contains only variation.

A True Course is the angle the ship's track makes with a terrestrial meridian, and contains neither deviation nor variation.

When a course is referred to, which of the three should always be specified.

The courses which are tabulated on board ship must of necessity be compass courses; and, from what has been said under the article Leeway, it is manifest the compass course does not show the direction the ship has actually travelled, but only the compass direction of the ship's head during the time. By correcting this for deviation and variation we obtain the true direction of the ship's head; and, in order to find the track the vessel has gone over, a correction must also be made for leeway. The true course of a ship is necessary to be known in order that her geographical position may be calculated.

The following deviation table was formed for H.M.S. *Jackal*, when swung in Plymouth Sound in 1845:—

Ship's head	Deviation	Ship's head	Deviation
North	2° 15′ W.	South	2° 20′ E.
N. by E.	2 40 E.	S. by W.	0 12 W.
NNE.	8 58 E.	SSW.	2 32 W
NE. by N.	13 7 E.	SW. by S.	4 55 W.
NE.	15 18 E.	SW.	6 30 W.
NE. by E.	17 15 E.	SW. by W.	8 45 W.
ENE.	18 7 E.	WSW.	10 52 W.
E. by N.	17 47 E.	W. by S.	13 50 W.
East	15 35 E.	West	15 45 W.
E. by S.	14 38 E.	W. by N.	17 25 W.
ESE.	13 22 E.	WNW.	18 25 W.
SE. by E.	11 40 E.	NW. by W.	18 25 W.
SE.	9 55 E.	NW.	17 15 W.
SE. by S.	8 5 E.	NW. by N.	15 45 W.
SSE.	5 35 E.	NNW.	11 55 W.
S. by E.	4 12 E.	N. by W.	7 25 W.

To find the True from the Compass Course.

RULE.—Allow westerly deviation and variation to the left, and easterly to the right hand. If the vessel be on the port tack allow leeway to the right, but if on the starboard tack allow leeway to the left hand.

Exercise IV.

Ex. 69. A vessel sails E. by N$\frac{1}{4}$N. by compass, variation 23° 16′ W., deviation according to the table; wind SE. by S., leeway 1$\frac{1}{4}$ points. Required the true course.

Compass course N. 6$\frac{3}{4}$ points E. =	75° 56′ r of N.
Deviation	17 52 r
	93 48 r of N.
Variation	23 16 l
	70 32 r of N.
Starboard tack. Leeway . .	14 4 l
True course	56 28 r of N.

∴ True course is N. 56° 28′ E.

Ex. 70. If the compass course be W. by S., variation 3$\frac{1}{4}$ points E., deviation by table; wind S. by W., leeway 1$\frac{3}{4}$ points; find the true course.

Compass course S. 7 points W. =	78° 45′ r of S.
Deviation	13 50 l
	64 55 r of S.
Variation	36 34 r
	101 29 r of S.
Port tack. Leeway . . .	19 41 r
True course	121 10 r of S.

But as no point on the compass is more than 90° from either N. or S., when the course (as in this example) is more than 90°, the result must be taken from 180° and its name changed thus :—

121° 10′ r of S.
180
58 50 l of N.

∴ True course is N. 58° 50′ W.

Ex. 71. *From the following compass courses, find the true course, using the table of deviations given above.*

Ex.	Compass courses	Variation	Wind	Leeway
a	SSE$\frac{1}{2}$E.	18° 32′ W.	East	9° 20′
b	NE. by E.	1$\frac{1}{2}$ points W.	SE. by E.	8 50
c	SW. by W$\frac{1}{4}$W.	2$\frac{1}{4}$ points E.	S. by E.	6 30
d	E. by S.	24° 30′ E.	S. by E.	1$\frac{1}{2}$ points
e	North	31 50 E.	WNW.	11° 15′
f	W. by N$\frac{1}{2}$N.	3$\frac{1}{4}$ points W.	N$\frac{1}{2}$W.	7 20
g	East	8° 30′ E.	SSE.	2$\frac{1}{4}$ points
h	NE$\frac{3}{4}$N.	25 15 E.	NW. by N.	12° 20′
i	West	$\frac{1}{4}$ point W.	SSW.	9 15
j	SW. by S.	2$\frac{1}{2}$ points E.	SE. by S.	1$\frac{1}{4}$ points
k	South	6° 20′ E.	WSW.	1 point
l	NNW$\frac{3}{4}$W.	3$\frac{1}{2}$ points E.	W$\frac{3}{4}$S.	8° 20′

Given the true course to find the compass course.

After taking celestial observations at sea, and thus finding the true position of the ship, it is the duty of those in charge to give orders how she must be steered to reach her destined port. To do this he must use the converse of the last rule, and to find the compass course he must put into the true course those errors which affect his compass; and therefore they must be allowed in an opposite direction to the former, that is, with a different algebraical sign.

To find the compass course.

Rule.—Apply the leeway *against* the wind, and the variation if west to the right, but if east to the left hand. Then notice what effect the deviation *will have* on your compass *when the work is finished,* and apply it if west to the right, but if east to the left hand.

Ex. 72. If your true course be WNW. to reach the port bound to, variation 2$\frac{1}{2}$ points E., vessel on the port tack, leeway (estimated) $\frac{3}{4}$ point, how must you steer, allowing deviation from the table?

True course N.6W. =	6 points	*l* of N.
Port tack. Leeway	$\frac{3}{4}$	*l*
	6$\frac{3}{4}$	*l* of N.
Variation E. . .	2$\frac{1}{2}$	*l*
	9$\frac{1}{4}$	*l* of N.

The corrections so far give us 9$\frac{1}{4}$ points *l* of N., *i.e.* 6$\frac{3}{4}$ points *r* of S. or W. by S$\frac{1}{4}$S. We must next make a trial correction. The deviation for W. by S. from the table is nearly 1$\frac{1}{4}$ point W., and therefore must be applied to S.6$\frac{3}{4}$W. to the right hand, and this will finally bring the ship's head nearly N. 88° W. Hence we must in our final work use the deviation for the ship's head N. 88° W. Resuming our work:—

Course corrected for leeway and variation = $9\frac{1}{4}$ points or	104° 4′ *l*	of N.
Deviation for west	16 3 *r*	
Compass course	88 1 *l*	of N.

∴ Course to be steered by compass, N. 88° 1′ W.

Ex. 73. My true course to reach the Lizard is ENE$\frac{3}{4}$E., the variation 28° 15′ W., the vessel is on the port tack, estimated leeway 12° 10′; how must I steer to reach the point, deviation to be allowed from the table?

True course N.$6\frac{3}{4}$E. =	75° 56′ *r*	of N.
Port tack. Leeway =	12 10 *l*	
	63 46 *r*	of N.
Variation W. . .	28 15 *r*	
	92 1 *r*	of N.

When the deviation is applied in the trial, it will be found the ship's head is about E. by N$\frac{1}{2}$N.; hence the deviation must be taken for that point, and it will be found to be 17° 55′ E.

Course corrected for leeway and variation =	92° 1′ *r*	of N.
Deviation	17 55 *l*	
Compass course	74 6 *r*	of N.

∴ Course to be steered by compass = N. 74° 6′ E.

Ex. 74. *What must be the compass course to make good the following true courses, using the same deviation table?*

Ex.	True courses	Variation	Tack	Leeway
a	SW. by W.	$1\frac{1}{2}$ points W.	Starboard	$\frac{3}{4}$ point
b	WNW.	27° 30′ E.	Starboard	$1\frac{1}{4}$,,
c	E. by S$\frac{1}{2}$S.	21 50 E.	Port	$\frac{1}{2}$,,
d	SE.	$2\frac{1}{2}$ points W.	Starboard	$1\frac{1}{2}$,,
e	E. by N$\frac{1}{2}$N.	$1\frac{3}{4}$ points W.	Port	$\frac{3}{4}$,,
f	NW$\frac{3}{4}$W.	25° 10′ W.	Port	$\frac{1}{4}$,,
g	NE$\frac{1}{2}$N.	2 points E.	Starboard	$1\frac{3}{4}$,,
h	East	$\frac{3}{4}$ point W.	Starboard	$\frac{1}{4}$,,
i	SW. by S.	37° 20′ W.	Port	$\frac{3}{4}$,,
j	North	22 0 E.	Starboard	$1\frac{1}{4}$,,
k	NW$\frac{1}{2}$N.	31 20 E.	Port	$1\frac{1}{2}$,,
l	West	2 points E.	Port	$\frac{3}{4}$,,

Ex. 75. Define leeway. Is the leeway greater when a ship is sailing free or when she is close-hauled? Explain the reasons for your answer.

The fore and aft line of a ship is in the direction (from stem to stern) SSW.; and the wake she leaves behind is in the direction N. by E. On what tack is she, and what leeway is she making? *E.* 1876.

Ex. 76. A ship is sailing on the apparent course NW$\frac{3}{4}$W. What is the true course; given variation $1\frac{1}{2}$ points E., deviation 8° 10′ W., leeway $1\frac{1}{4}$ points, the direction of the wind N. by E.? *E.* 1868.

Ex. 77. What is meant by the variation and deviation of the compass? What other correction is it sometimes necessary to apply to the compass course to obtain the true course? The compass course is S. by E., the variation is $1\frac{3}{4}$ points E. and deviation 3° W., and the true course is SSW. Explain this. *E.* 1873.

Ex. 78. Explain what you mean by a ship's 'course.' The wind being north, a ship sailing close to the wind on the port tack has her head NE. by E; she now tacks, and is brought close to the wind (which remains steady) on the starboard tack. How many points has her head passed through? Express this in degrees, &c. *E.* 1875.

LOG-GLASS, LOG-SHIP, AND LOG-LINE.

To find the position a ship is in at any moment it is necessary not only to know the directions she has sailed on from her former known position, but also the rate and time or whole distance she has gone on each course. To estimate the rate, time and distance both enter as factors; and the instruments used for getting the ship's rate are—the *log-glass*, for ascertaining the elapsed time, and the *log-ship* and *log-line*, for ascertaining the distance run during that time.

THE LOG-GLASS is an ordinary sand-glass constructed so as generally to run out in half a minute. In the Royal Navy two glasses are used, one to run out in fourteen seconds, called the short-glass, and the other to run out in twenty-eight seconds, called the long-glass. The latter must be used when the rate is less than five knots per hour, and the former when the rate is greater.

THE LOG SHIP is a piece of wood in the form of a sector of a circle, weighted at its circumference, so that when it is thrown overboard it floats with its angular point upwards and the circumference under water. It is connected with the log-line by means of two cords in such a manner that when in use its surface is always at right angles to the ship's direction, and thus presents a maximum resistance to motion through the water, and for all practical purposes it remains stationary.

The Log-line is a line attached to the log-ship, and is marked at equal intervals with pieces of cord let into the strands of the log line, in which are tied knots one for each division from the zero. These are again subdivided into tenths, called fathoms. The line in the Royal Navy is divided for a 28-second glass; but when the 'short-glass' is used the number of knots must be doubled for the rate. The line is wound on a reel, and when the ship's rate is required, the log-ship is thrown overboard, and a quantity of line allowed to run off the reel that the log-ship may float clear of any influence from the ship's motion. This part of the line is called *Stray-line*. It varies in length from ten to twenty fathoms, and its extremity, or zero of the line, is marked with a piece of white bunting.

Heaving the Log.—The rate of the vessel is estimated by heaving the log-ship overboard, and when the white rag passes over the quarter, the officer cries out '*Turn.*' The glass is then turned, and directly the glass is run out the assistant cries '*Stop.*' The line is then checked, and the pressure of the water on the log-ship causes the peg fitted to one of the cords to come out, and so the log-ship is thrown flat on the water to diminish the resistance as it is hauled in. The number of knots and fathoms (tenths) run off the reel between the turning of the glass and checking the line is the rate the vessel is going. When the rate is obtained it is registered on the *log-slate* or *log-board* opposite to the course the vessel is then on; and each day at noon all the courses and distances made during the previous twenty-four hours are copied into the *log-book*, together with every circumstance of note which may have taken place during the day. The place of the vessel is then calculated from these data and entered in the log-book, as well as the place as ascertained by astronomical observations.

Length of the Nautical Mile and Knot.—The log-glass, as we have stated, generally runs out in half a minute, which is the $\frac{1}{120}$th part of an hour; then if the divisions on the line between the knots be the $\frac{1}{120}$th part of a nautical mile, it is at once apparent, if one of these divisions, or the $\frac{1}{120}$th part of a nautical mile, be drawn out in the $\frac{1}{120}$th part of an hour, *one mile* must be drawn out in *one hour*. So, if *two* divisions of the line be drawn out in the same time, then *two miles* must be drawn out in the hour, and so on. Each of these divisions on the line ($\frac{1}{120}$th part of a nautical mile) is called a knot. Hence, if we say a vessel's rate is eight or nine knots per hour, it really means she is going eight or nine nautical miles per hour.

We find the length of a knot thus:—

$$\begin{aligned} \text{Circumference of the earth} &= 360 \text{ degrees.} \\ &= 360 \times 60 \text{ nautical miles.} \\ &= 21600 \quad ,, \quad ,, \quad (a) \end{aligned}$$

Again, using the mean of the earth's radii as given by Sir George Airy, viz., 20888761·5 feet, we get

$$\begin{aligned} \text{Circumference of the earth} &= 2\pi r \\ &= 2 \times 3{\cdot}14159 \cdots \times 20888761{\cdot}5 \text{ ft.} \\ &= 131247848{\cdot}481577 \cdots \text{feet} \quad (b) \end{aligned}$$

Equating (a) and (b)

$$\begin{aligned} 21600 \text{ nautical miles} &= 131247848{\cdot}5 \text{ feet} \\ \therefore \quad 1 \text{ nautical mile} &= \frac{131247848{\cdot}5 \text{ feet}}{21600} \\ &= 6076{\cdot}3 \text{ feet.} \end{aligned}$$

$$\begin{aligned} \text{Hence 1 knot, or } \tfrac{1}{120}\text{th part of a nautical mile} &= \frac{6076{\cdot}3}{120} \\ &= 50{\cdot}64 \text{ feet nearly.} \end{aligned}$$

In practice the nautical mile is usually taken as 6080 feet.

Computations by means of dead reckoning cannot be relied on for reasons which we will now proceed to explain.

First. A fault in steering amounting to a quarter of a point will cause an error in the position of the ship of one mile for every twenty miles of distance made: and few men can steer so closely as that for any length of time.

Secondly. The earth in dead reckoning is supposed to be a plane, and all measurements must on this supposition introduce an element of error.

Thirdly. Wind currents set the ship in directions across their courses, so that, supposing accurate measurements of course and distance to have been made, a mariner can seldom know his position within ten or fifteen miles from this cause alone.

Fourthly. The temperature and the hygrometrical state of the atmosphere materially affect the sand in the glass and the size of the hole through which it runs; so that it takes a much longer time to run out when the weather is cold and damp than when dry and warm.

Fifthly. The length of the knots on the line may from some causes have varied, either becoming stretched or have shrunk from constant wetting.

Hence it is necessary the time the glass takes in running out should often be compared with the chronometer, and the length of the knot should often be re-measured. It is not necessary the time should be constant, but when it varies the line should be re-knotted to compensate for the variation. To do this accurately, *the length of the knot must be made to bear the same proportion to a nautical mile that the time the glass runs does to an hour.*

Thus, let t be the number of seconds the glass runs;
L the length of knot in feet for that time.

Then we must have $$\frac{L}{6080} = \frac{t}{3600}$$

$$\therefore \quad L = t\,\frac{6080}{3600} \text{ feet.}$$

Ex. 79. Find the length of a knot for a 28-second glass.

$$\text{Here } L = t\,\frac{6080}{3600} = 47{\cdot}29 \text{ feet;}$$
$$= 47 \text{ feet } 3\tfrac{1}{2} \text{ inches.}$$

Ex. 80. Find the length of a knot for a 29-second glass.
Ex. 81. Find the length of a knot for a $15\frac{1}{2}$-second glass.
Ex. 82. Find the length of a knot for a $28\frac{1}{2}$-second glass.
Ex. 83. How long must the knot be for a $13\frac{1}{2}$-second glass?

Variation of the Length of the Knot.—The time the glass runs may be constant, but the length of the knot may have varied from water causing the line to shrink, or the line may from some reason have become stretched. Now, if the length of the line should have increased, it is very evident fewer knots must have unwound from the reel in the time than ought to have been; and thus the estimated rate must be too small. On the other hand, if the line has contracted, a greater number of knots must have been unwound in the time than ought to have been, and consequently the estimated rate will be too great; hence, *if the time be constant, the rate estimated by the log-line varies inversely as the length of the knot.* Once more, if the glass runs more seconds than it ought, it is plain more knots must have been drawn off the reel than should have been, and *vice versâ*; hence, *if the knot remains constant, the rate must vary directly as the time.* But should the time the glass runs and the log-line both have become faulty, then *the rate will vary directly as the time, and inversely as the length of the knot.* From the above we see that the registered rate the vessel has sailed may be in error from three causes—

(*a*) The log-glass may have been in error,

(*b*) The log-line may have been in error,

(*c*) Both log-glass and log-line may have been in error.

General Formulæ.—From some causes the length of the knot and the time the glass takes to run out may both have been in error, then the true rate may be computed from the following formulæ obtained from the question thus stated:

If r be the rate of a vessel when the length of a knot be l feet, and the number of seconds the glass takes to run out be t; what is the true rate R for the length of the true knot L feet, and the correct time T seconds?

Rate when	length of	knot is	l	and time	$t = r$
"	"	"	1	"	$t = r.l$
"	"	"	1	"	$1 = \frac{r.l}{t}$
"	"	"	L	"	$1 = \frac{r.l}{t.L}$
"	"	"	L	"	$T = \frac{r.l.T}{t.L}$

and because $\frac{L}{6080} = \frac{T}{3600} \therefore \frac{T}{L} = \frac{3600}{6080} = \frac{45}{76} = \frac{3}{5}$ nearly,

$$\text{Hence, true rate} = \frac{3600}{6080} \frac{r.l}{t}$$

$$= \frac{3}{5} \frac{r.l}{t} \text{ very nearly;}$$

and as the whole distance varies as the rate, the same formula would be obtained by substituting

D for the correct distance gone; and

d for the registered distance.

$$\text{Then } D = \frac{3}{5} \cdot \frac{dl}{t} \text{ nearly.}$$

$$\text{The correct distance is } D = \frac{45}{76} \cdot \frac{dl}{t}.$$

From the above formula we get the rule:—Multiply the estimated distance by the length of the knot used, and divide the product by the number of seconds the glass takes in running out: three-fifths of the result gives the true distance nearly.

Massey's Patent Log.—The log-glass, log and log-line just described, are now almost superseded by *Massey's Patent Log.* This is a brass instrument, with vanes on a spindle, something like a four-bladed screw-propeller. When towed astern of the vessel, the water acts on the vanes, which turn the spindle, and the rate is registered through a train of wheel-work by hands on three faces, the first of which show the number of tens of miles, the second the number of units of miles, and the third the tenths of miles the vessel has run since it was thrown overboard. A distance of 100 miles can thus be registered without taking in the log; but it is obvious *when the course is changed the log must be drawn in and the distance recorded*; and its work is considered very satisfactory if the distance run be measured within 95 per cent. of the true distance.

Napier's Pressure Log.—The late Mr. J. R. Napier, F.R.S., of Glasgow, invented a pressure log, which indicates the rate of a ship by the height a column of water is supported in a tube projecting about six inches through the keel and high enough inside to be read with ease. A hole in the tube near the bottom (which is closed) is placed in the direction the ship is going. If the vessel is stationary the height of water in the tube is the same as that on the outside of the vessel; but when under canvas or steam the pressure exerted through the hole becomes greater as the speed increases: hence the height of the column of water in the tube varies as the rate; and so it is measured either directly by a float, or by an index connected to a string passing over a pulley and fastened to the float.

THE 'DUTCHMAN'S LOG.'—One of the oldest methods for estimating the rate was that called the '*Dutchman's Log.*' This was more exact than that now obtained by the common log, because it was not subject to the variations that the log-line and log-glass are. It was as follows:—Two points as far apart as possible on the ship's rail were marked off and accurately measured; and lines were drawn through these points where vertical planes perpendicular to the fore and aft line of the vessel met the deck. To find the rate, a bottle or piece of wood (hence the name log) was thrown overboard in the direction the ship was going; and when the ship came up to it the exact second that the first mark passed the bottle was called out, and so with the second mark. Then the rate was calculated thus:—

Suppose the marks 185 feet apart, and

Time of passing first mark	3h. 27m. $4\frac{1}{2}$sec.
" " second "	3h. 27m. 17 sec.
Elapsed time	$12\frac{1}{2}$sec.

That is, the ship passed over 185 feet in $12\frac{1}{2}$ seconds.

$$\therefore \text{ In one hour she passed over } \frac{185 \times 3600}{12\frac{1}{2} \times 6080} \text{ knots.}$$

Hence rate of ship = 8·76 knots per hour.

EXERCISE V.

Ex. 84. The apparent distance run at sea is 48 miles, when the glass runs out in 33 seconds. What is the true distance run? *E.* 1868.

Ex. 85. In the Royal Navy the log-glass runs out in 28 seconds, what is the length of the knot on the log-line? It is found on measurement that the length of knot is only 45 feet, and the glass runs out in $28\frac{1}{2}$ seconds, what is the true distance run by the ship which by the log has run 175 miles. *A.* 1869.

Ex. 86. What should be the length of the knot on the log-line when the sand-glass runs out in 27 seconds, a nautical mile being assumed to be 6080 feet. *E.* 1871.

Ex. 87. Suppose the nautical mile to be 6080 feet and the glass to run out in 30 seconds, what is the corresponding length of the knot? *E.* 1872.

Ex. 88. What is the need for stray-line? The distance between two knots on the log-line is $47\frac{1}{3}$ feet, the 'long-glass' being one of 28 seconds, what rate would the ship be going if $4\frac{1}{2}$ knots run out when the short-glass is used? *E.* 1875

Ex. 89. The measured distance run by a ship was 157 miles, the knot on the log-line was 53 feet in length, and the glass runs out in 29 seconds. Find the actual distance run *For Lieutenant, November* 1873.

Ex. 90. The apparent distance run according to the log is 275 miles; but on examination I find the length of the knot is 47 feet, and the glass runs out in 32 seconds instead of 30; what is the real distance run? Prove the formula which you apply. *A.* 1873.

Ex. 91. Using a 29-second glass I ran 300 miles by log; the true distance by the chart was 310 miles; required the error in the log-line (6120 feet = 1 nautical mile.) *Royal Naval College*, 1872.

Ex 92. The apparent rate of a ship was 9·5 knots, but it was found that the log-glass ran out in 13 seconds instead of 14 seconds, and that the length of the knots on the log-line was 5 inches too long. What is the rate of the ship? Why is the short-glass used instead of the long-glass on this occasion. *A.* 1878.

Ex. 93. Taking the nautical mile to be 6080 feet, what is the length of a knot on the log-line when a glass of 30 seconds is used? What error would be introduced in the estimated rate of a ship if the glass were found to have run out in 25 seconds instead of 30 seconds? Describe Massey's patent log. *A.* 1880.

Ex. 94. What do you mean by leeward, windward, port tack, starboard tack, close-hauled, wind on the beam, wind on the quarter? In speaking of right and left on the compass what do you mean?

Ex. 95. Define leeway. On what does its amount depend? Show how to find practically the amount of leeway.

Ex. 96. How many kinds of courses are there? Define each, and show how the true course is found from the compass course and the compass course from the true course.

Ex. 97. What instruments are used in determining the rate of a vessel? Describe each; show to what errors each is liable, and also how the rate is found. What is the stray-line, and what its use?

Ex. 98. Find the length of the nautical mile, and show how it is connected with the length of the knot on the log-line. From what causes may the rate of a vessel be wrongly estimated?

Ex. 99. In estimating the distance run it was found the log-glass and log-line had both become faulty, how is the true distance run to be found? Obtain the formula $D=\frac{3}{5}\cdot\frac{dl}{t}$ and show that a more correct formula is $D=\frac{45}{76}\cdot\frac{dl}{t}$.

Ex. 100. What is meant by the terms log-ship, log-line? How is the log-line attached to the log-ship? What is the object of this mode of attachment? What is the stray-line? Supposing the nautical mile to be 6080 feet, what must be the length of a knot on the log-line when the glass runs out in 28 seconds. *E.* 1869.

Ex. 101. Define rhumb-line. What is the name given to the angle which a rhumb-line makes with a meridian. A ship is sailing NE$\frac{3}{4}$E., and the direction of her head is changed by 118° through the north: what is the new course of the ship in points to the nearest quarter. *E.* 1872.

Ex. 102. What are the several corrections to be applied to the apparent to obtain the true course?

Ex. 103. Correct the following courses:—

Ex.	Apparent course	Variation of the compass	Deviation	Direction of wind	Leeway
a	NW	$1\frac{3}{4}$ points E.	10° W.	NNE.	$2\frac{1}{2}$ points.
b	SW$\frac{1}{2}$W.	2 points W.	$4\frac{1}{2}$° W.	SSE.	$1\frac{3}{4}$ points.
c	N.	$1\frac{1}{4}$ points E.	11° E.	S. by E.	0
d	S. by E.	$1\frac{1}{2}$ points E.	3° E.	WSW.	$1\frac{3}{4}$ points.

Ex. 104. What should be the length of the knot on the log-line with a glass running 28 seconds? Supposing the knot on the line used to be 45 feet, and the distance run estimated by it to be 120 miles; what is the true distance? *A.* 1873.

Ex. 105. How is the distance run by a ship at sea estimated? Compute the length of a knot on the common log-line (1) when the glass runs out in 30 seconds; (2) when it runs out in 14 seconds. The length of a nautical mile may be taken as 6120 feet. *E.* 1881.

Ex. 106. Explain the terms—starboard tack, port tack, close-hauled, windward, and leeward. Define leeway. Under what conditions is the leeway the greatest, and under what circumstances is there no leeway?

Ship on starboard tack going by compass N. by W., wind NE., leeway one point, what is the course by compass made good *E.* 1879.

Ex. 107. Describe the log-line and glass. Show how the estimated distance is affected (1) when the glass runs out too slowly, and (2) when the log-line has shrunk. *A.* 1879.

Ex. 108. A ship makes good a course SW. by W$\frac{1}{2}$W. true, but she steers SSW$\frac{3}{4}$W.; the variation is 2 points E., and the deviation 8° 30′ W. What is the leeway, and from what point of the compass is the wind blowing, supposing the ship to be 5$\frac{1}{2}$ points off the wind? *E.* 1874.

CHAPTER IV.

Plane sailing—Difference of latitude—Departure—Course—Rhumb curve—Nautical distance--Exercises--Traverse sailing—Resolving a traverse—Traverse tables—Exercises—Current sailing—Exercises —Windward sailing—Exercises—Examination.

In Navigation it is usual to consider the place of a ship at sea, either :—

I. With reference to the place left;

II. With reference to the place bound to.

Then five things are involved—viz.: (1) Course; (2) Distance; (3) Difference of Latitude; (4) Departure, and (5) Difference of Longitude; and it is the object of the science to show that these are mutually dependent on each other, so that when some of the above data are given the remainder may be found. In the solution of the two problems presented to us, it is necessary to find :—

(*a*) The position the ship is in at any time, from the courses and distances sailed;

(*b*) The course and distance to sail to reach a port from the given latitudes and longitudes of the ship and port bound to.

The methods of calculation used for these purposes are called THE SAILINGS.

In Plane, Traverse, Current, and Windward Sailings, the earth is considered a plane, and the parallels of latitude and meridians straight lines at right angles to each other, and all questions occurring in either can be solved by the principles laid down in Plane Trigonometry. The earth being so large a sphere no material error is introduced into the calculations by these assumptions if—

(*a*) The distance sailed by the ship be small,
(*b*) The ship's track be near a meridian,
(*c*) When in low latitudes;

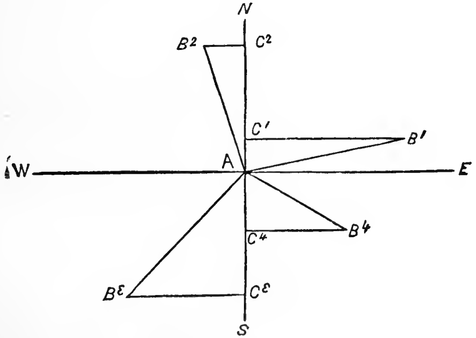
but on long voyages the results obtained by these means must be corrected daily by astronomical observations: because, when we consider the earth a plane, difference of longitude, which depends on the spherical form of the earth, cannot be computed by either of the above sailings.

PLANE SAILING.

The formulæ for Plane Sailing are deduced from the simplest considerations in the solution of right-angled triangles by Plane Trigonometry; and in drawing the figures the student should bear in mind that when not otherwise stated it is usual to consider the top of a map, chart, or book as the north, the bottom the south, the right hand east, and the left hand west.

Thus:—Let A be the position of a ship at starting; draw the vertical line N A S to represent a portion of a meridian, and the horizontal line E A W to represent a portion of a parallel of

FIG. 6.



latitude through A. Now if she sail in any direction A B¹, A B², A B³, or A B⁴, and arrive at either B¹, B², B³, or B⁴, draw lines parallel to E W to meet N S in C¹, C², C³, or C⁴; these will be portions of parallels of latitude through the points arrived at. We have now a right-angled triangle formed in each quadrant, in each of which a portion of a meridian, a portion of a parallel of latitude, and the track of the ship or distance sailed form the sides. Then the line

AB^1, AB^2, AB^3, or AB^4 is called Nautical Distance;
AC^1, AC^2, AC^3, or AC^4 „ Difference of Latitude;
B^1C^1, B^2C^2, B^3C^3, or B^4C^4 „ Departure;

and the angle B^1AC^1, B^2AC^2, B^3AC^3, or B^4AC^4 is called Course.

NAUTICAL DISTANCE is the length of the line in nautical miles intercepted between the place sailed from and the place arrived at.

DIFFERENCE OF LATITUDE is the arc of a meridian intercepted between two parallels of latitude, one through the place sailed from, the other through the place arrived at.

DEPARTURE is the distance made good in a due east or west direction. It is always expressed in nautical miles, and never in degrees, &c., as the difference of longitude is.

COURSE is the angle between a ship's track and a terrestrial meridian, and is always reckoned from the north or south point of the horizon. When it is the subject of calculation, the course is always a true one; and, if obtained, must be reduced to a compass one before it can be used on board in shaping a course to a destined port.

THE RHUMB CURVE, or LOXODROMIC LINE, is that line which cuts every meridian it crosses at equal angles, and is known as the *Equiangular Spiral.* It derives its name from the fact that a vessel in sailing on such a curve keeps always on the same course; that is, with her head on the same rhumb line of the compass. It will be seen there is a distinction between a rhumb-curve and a rhumb line; the latter, as before stated, is a straight line drawn from an observer to any point of his visible horizon; or it is its representative—viz., a straight line drawn from the centre of the compass card to meet the circumference; whereas the former (if not towards one of the cardinal points) is a curve winding round the earth constantly approaching but never reaching the pole.

The accompanying diagram will give the reader an idea of its form. Let A B C D be a stereographic projection of the earth on the plane of the equator, P, its centre, be the North Pole, then the radii will represent meridians. It is a property of the stereographic projection of the sphere that lines intersecting each other on the globe form equal angles on the plane of projection—hence, if Q R S T cut the meridians at equal angles then Q R S T is a part of a rhumb curve or loxodrome. Here it represents one revolution on an ENE. course. That a vessel approaches the pole is evident, because her course is to the northward of east; but that it will never reach it is equally evident, because the pole always bears due north from an observer (in this case) wherever he be situate : hence he can never reach the pole until the course is altered to north, and never whilst sailing in any direction not in a meridian. The explanation here introduced will lead us to another definition of nautical distance, viz. :—

FIG. 7.

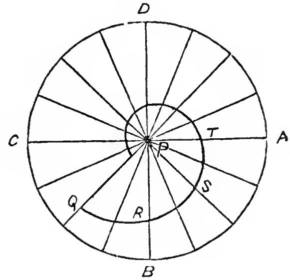
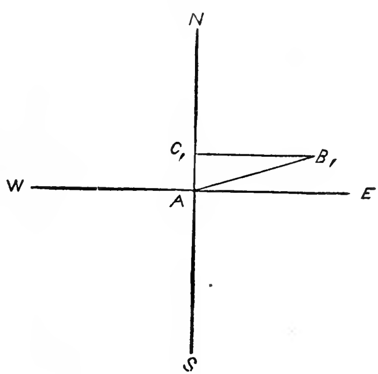


FIG. 8.



NAUTICAL DISTANCE is the arc of the rhumb curve between two places expressed in nautical miles.

To fix the attention of the student we will consider one only of the above triangles, as A B_1 C_1; but what we say concerning this one is equally applicable to all.

I. *With the course and distance given, to find the difference of latitude and departure.*

$$\frac{AC_1}{AB_1} = \cos B_1AC_1 \quad \therefore AC_1 = AB_1 . \cos B_1AC_1 ;$$

i.e. Diff. lat. = Dist. × cos course . . . (*a*)

Again, $\frac{B_1C_1}{AB_1} = \sin B_1AC_1 \quad \therefore B_1C_1 = AB_1 . \sin B_1AC_1 ;$

i.e. Dep. = Dist. × sin course . . . . (*b*)

II. *With the difference of latitude and departure given, to find the course and distance.*

$$\left.\begin{array}{l} \tan B_1AC_1 = \frac{B_1C_1}{AC_1}; \quad i.e. \ \tan \text{ course} = \frac{\text{Dep.}}{\text{Diff. lat.}} \\ \text{or cotan, } B_1AC_1 = \frac{AC_1}{B_1C_1}; \quad i.e. \ \cot \text{ course} = \frac{\text{Diff. lat.}}{\text{Dep.}} \end{array}\right\} \quad (c)$$

Again, $\frac{AB_1}{AC_1} = \sec B_1AC_1 \quad \therefore AB_1 = AC_1 . \sec B_1AC_1 ;$

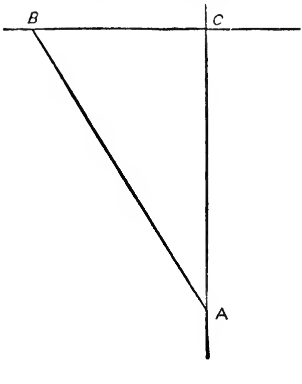
i.e. Dist. = Diff. lat. × sec course. . . . (*d*)

or $\frac{AB_1}{B_1C_1} = \operatorname{cosec} B_1AC_1 \quad \therefore AB_1 = B_1C_1 . \operatorname{cosec} B_1AC_1 ;$

i.e. Dist. = Dep. × cosec course. . . . . (*e*)

FIG. 9.

Scale 20 miles to an inch.



It must be insisted on that no attempt be made at the solution of any question in Navigation until the figure be drawn by the pupil and laid down to scale.

Ex. 109. A yacht sailed from Sercq Island north-westerly 35 miles till the difference of latitude was 30 miles. Find the course on which she sailed and her departure.

Here are given AB = 35 miles
and AC = 30 miles
to find angle BAC and BC.

Now, $\cos BAC = \frac{AC}{AB}$

$$\therefore \cos \text{course} = \frac{\text{diff. lat.}}{\text{distance}}$$

Diff. lat.	30 miles log	= 1·477121
Distance	35 miles log	= 1·544068
Course N. 31° W.	log cos	= 9·933053

Again, $\quad BC = AB. \ \sin BAC$

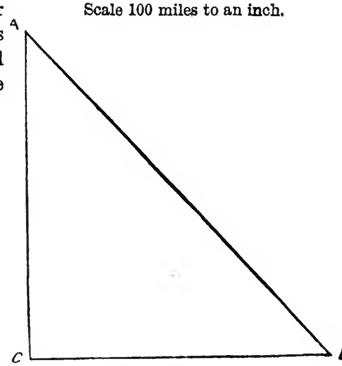
$\therefore$ departure = distance × sin course;

Course	31° log sin	= 9·711839
Distance	35 miles log	= 1·544068
Departure	18 miles log	= 1·255907

Answer { Course N. 31° W.
Departure 18 miles.

Ex. 110. A ship sailed from Cape Race south-easterly until her difference of latitude was 174 miles and her departure 156 miles. Find the course and distance made good.

Fig. 10.
Scale 100 miles to an inch.



Here are given AC = 174 miles
and BC = 156 miles
to find angle BAC and AB.

Now, $\tan BAC = \frac{BC}{AC}$

$$\therefore \tan \text{course} = \frac{\text{dep.}}{\text{diff. lat.}}$$

Departure	156 miles log	= 2·193125
Diff. lat.	174 miles log	= 2·240549
Course S. 41° 53′ E.	log tan	= 9·952576

Again, $\quad AB = BC. \ \operatorname{cosec} BAC$

$\therefore$ distance = departure × cosec course;

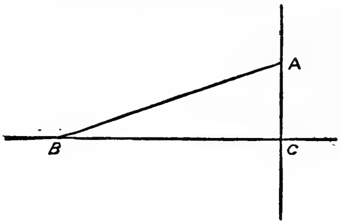
Course S. 41°53′ E.	log cosec	= 10·175473
Departure	156 miles log	= 2·193125
Distance	233·7 miles log	= 2·368598

Answer { Course S. 41° 53′ E.
Distance 233·7 miles.

Ex. 111. From Europa Point (latitude 36° 6′ 42″ N., longitude 5° 20′ W.) a vessel sailed 125 miles S. 70° W. Required her latitude in and departure.

FIG. 11.

Scale 100 miles to an inch.



Here are given A B = 125 miles
and angle B A C = 70°
to find B C and A C.

Now B C = B A. sin B A C
∴ departure = distance × sin course.

Distance 125 miles log	=	2·096910
Course 70° log sin	=	9·972986
Departure 117·5 log	=	2·069896

Again, A C = B A. cos B A C
∴ diff. lat. = distance × cos course.

Distance 125 miles log	=	2·096910
Course 70° log cos	=	9·534052
Diff. lat. 42·75 miles log	=	1·630962

Lat. left	36°	6′	42″	N.
Diff. lat.		42	45	S.
Lat. in	35	23	57	N.

Answer { Lat. in 35° 23′ 57″ N.
Departure 117·4 miles.

EXERCISE VI.

Ex. 112. A ship sails 205 miles ESE., starting from a place in latitude 32° 22′ N. What is the latitude of the place arrived at?

E. 1874.

Ex. 113. Cape Finisterre (latitude 42° 54′ N., longitude 9° 16′ W.) bears from a ship NE½E., distant 120 miles. What is her latitude?

E. 1873.

Ex. 114. A ship sails on a course between south and west for a distance of 109 miles, and finds she is in latitude 1° 13′ higher than that of her place of departure. On what course has she sailed, and how much westing has she made? *E.* 1875.

Ex. 115. Prove:—Departure = T. D. latitude × tan course.
For Lieutenant, December 1874.

Ex. 116. Leaving a place, a ship proceeds SE. by S. and makes 24 miles 'easting.' What distance does she sail, and how much does she alter her latitude? *E.* 1878.

Ex. 117. A ship sails from latitude 38° 4′ N. SE. by S., till her departure is 50 miles. Required the distance she has sailed, and her latitude. Construct a figure. *E.* 1872.

Ex. 118. In plane sailing:—

distance = departure × cosec course.

Required the proof. *Royal Naval College*, 1865.

Ex. 119. A ship sails from Odessa, latitude 46° 30′ N., longitude 30° 45′ E., and after 24 hours is in latitude 43° 18′ N., having made 135 miles of easting. Required her compass course and distance. Variation $1\frac{3}{4}$ points E., deviation 6° 25′ E., leeway $1\frac{1}{4}$ points, wind SSW. *E.* 1869.

Ex. 120. From a place in latitude 35° 20′ S., a ship sails by compass NW. by N. 58 miles; variation of compass 36° 33′ 45″ E., deviation $\frac{1}{4}$ point W. What is the departure and latitude in? *E.* 1878.

Ex. 121. The Lizard (latitude 49° 58′ N., longitude 5° 11′ W) bore by compass NNE$\frac{1}{2}$E., distant 20 miles; the variation of the compass was $2\frac{1}{4}$ points W., the deviation 8° W. Find the departure and the latitude of the ship. *E.* 1880.

Ex. 122. A ship is known to be sailing due south, but the compass course steered is S. by W$\frac{3}{4}$W. The variation of the compass is $1\frac{1}{2}$ points W. What is the deviation? If the latitude left is 5° 18′ N. and the latitude arrived at is 7° 44′ S., what is the distance run? *E.* 1871.

Ex. 123. A ship sails between the north and west until her departure is one-third her difference of latitude. On what compass course has she sailed if the compass error be $2\frac{3}{4}$ points E.

Ex. 124. If the compass error be $3\frac{1}{4}$ points W., and the ship has sailed between the true S. and W. until the distance is double of the difference of latitude she has made, what has been her compass course?

Ex. 125. A steamer left St. Michael's in latitude 37° 48′ 18″ N. on a NNW. course. Her rate was $8\frac{1}{2}$ knots per hour; how long will it take her to get to latitude 40° 41′ 18″ N.

Ex. 126. Ushant is in latitude 48° 28′ 30″ N., longitude 5° 3′ 12″ W. A vessel sailed from there, making 215 miles departure to the westward and 39 miles difference of latitude to the northward; what has been her compass course and distance. Variation 19° 20′ W., deviation 32° 15′ E., wind SW., leeway $1\frac{1}{4}$ points.

TRAVERSE SAILING.

Owing to changes in the direction of the wind, the set and drift of currents and tides, rocks, shoals, intervening land, &c., a mariner can seldom or never go direct from one port to another on the same course; then, in order to determine his position when necessary, he finds it convenient to reduce all the courses and distances sailed on to one resultant course and distance. This he is enabled to do by traverse sailing. The crooked path made by a ship when she sails in several successive directions is called *a traverse,* and the method of finding the single course and distance which would bring a ship to the same place as two or more courses and distances is called *working or resolving a traverse.*

Traverse Sailing *is the method used to reduce any number of successive courses and distances to a resultant one, and to obtain the difference of latitude and departure.* This is done by finding the difference of latitude and departure for each course sailed, and entering them in a table, as in the following example; being careful that the difference of latitude and departure are entered in their proper columns. Thus, if the course be NW. $\frac{3}{4}$ W., the difference of latitude must be entered in the column marked N. and the departure in the one marked W., and so on with all the other courses; but if the vessel has sailed due N., S., E., or W., then the whole distance on the course must be entered under its proper heading. Next add up the columns, and the difference of the sums of those marked N. and S. will be the whole difference of latitude; and the difference of the sums of those marked E. and W. will be the whole departure, in each case of the same name as the greater. Thus, if the sum in the N. column be greater than that in the S. column, the difference of latitude is towards the north; and similarly, if the sum in the column marked W. be greater than that in the E. one, the departure is towards the west. Having thus found the whole difference of latitude and departure the course and distance made good is then found by methods explained in Plane Sailing.

Traverse Table.—To facilitate the working of a traverse, a table called a *Traverse Table* is used. It is thus constructed: All the integral degrees from 1 to 45 are taken as courses, and by plane sailing the differences of latitudes and departures are calculated from the formulæ,

$$\text{Departure} = \text{dist.} \times \sin \text{course},$$
$$\text{Diff. lat.} = \text{dist.} \times \cos \text{course},$$

for all distances from 1 to 300 miles, and tabulated in three parallel columns under the headings, 'Dist.' 'Lat.' and 'Dep., which signify Distance, Difference of Latitude, and Departure. It is necessary to do this only to 45°, because the sine of an angle is equal to the cosine of the complement of that angle, and thus differences of latitude for all degrees of a course will equal the departures for the complements of those degrees, and *vice versâ*; so what is difference of latitude and departure for 1° is departure and difference of latitude for 89°, the difference of latitude and departure for 2° is departure and difference of latitude for 88°, and so on. Hence, because every course must fall in one of the quadrants it cannot exceed 90°, and therefore 45 pages are all that are required for the traverse table. It is marked at the top and bottom of the pages with complementary angles—*e.g.* 36° on the top has $90° - 36° = 54°$ at the bottom; and the column marked 'Lat.' on the top is marked 'Dep.' at the bottom, and *vice versâ*. In practice, such a table is constructed by calculating the difference of latitude and departure for 1° for a distance of one mile correct to five places of decimals; then, for distances of 2, 3, or 4 miles up to 300, this amount is multiplied by 2, 3, 4, up to 300, tabulating only the nearest tenth of a mile. This must be repeated for each degree up to 45; and the traverse table is completed by marking the bottom of the pages in the way spoken of above.

Use of the Traverse Table.—Its use can be seen at once from its construction. The student must first look for the number of degrees in the course, on the top of the page if it be below 45°, and at the bottom if between 45° and 90°. Then opposite the given number in the distance column will be found the difference of latitude and departure in the columns marked 'Lat.' and 'Dep.' Care must be taken if the number of degrees in the course be found at the bottom of the page, then the columns 'Lat.' and 'Dep.' must also be read from the bottom of the page. With a little care all cases of plane sailing can be approximately solved by the aid of the traverse table. We say *approximately*, because the tables are calculated only for the integral degrees and for an integral number of units as distances; hence, if a course be given in degrees and minutes, or the distance in integers and parts, an approximate result only can be deduced. This will be found to be the case especially when the course is a very large or a very small angle, or, what amounts to the same thing, when one side of the triangle is very great in comparison to the other. In all good mathematical tables, another traverse table is inserted, viz. one to the nearest quarter of a point; but its construction is precisely similar to that already explained, and is to be used only when the course is given in points and quarters of a point and not in degrees.

Ex. 127. The last 24 hours we have run the following courses: 1st, SE., 40 miles; 2nd, NE., 28 miles; 3rd, SW. by W., 52 miles; 4th, NW. by W., 30 miles; 5th, SSE., 36 miles; and 6th, SE. by E., 58 miles, Required the difference of latitude, departure, the direct course, and distance made good.

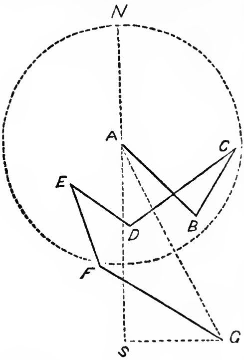
Courses	Distance	Difference of latitude		Departure	
		N.	S.	E.	W.
S. 4 E.	40	—	28·3	28·3	—
N. 4 E.	28	19·8	—	19·8	—
S. 5 W.	52	—	28·9	—	43·2
N. 5 W.	30	16·7	—	—	24·9
S. 2 E.	36	—	33·3	13·8	—
S. 5 E.	58	—	32·2	48·2	—
		36·5	122·7	110·1	68·1
			36·5	68·1	
			86·2	42·0	

Answer { Diff. lat. 86·2 miles; departure 42·0 miles.
Course S. 26° E.; distance 96 miles.

Method of Resolving the Traverse.—Place the courses one under the other, as in the table above, with the distances opposite their own proper courses. Take the first course: SE. is 4 points from south; we therefore enter the Traverse Table (Riddle, page 100), and under 4 points on the top of the page we find 40 in the distance column; directly abreast of the distance we find 28·3 in the difference of latitude column and 28·3 in the departure column, and we enter these amounts in our own table under S. and E. The second course will be similar to the first, but the distance here is 28 miles and the corresponding difference of latitude and departure 19·8; these are entered into our table under N. and E. The third course is SW. by W. or S. 5 points W. We again enter Riddle's Traverse Table, and find the course 5 points at the bottom of page 96, and opposite the distance 52 miles we find 28·9 in the difference of latitude column and 43·2 in the departure column; we therefore enter 28·9 under S. and 43·2 under W. in our own table. This is repeated with all the other courses. The sums of the columns N., S., E., and W. are then taken, and we then see how far the ship has gone in each of these directions, and we find them to be 36·5 to the north, 122·7 to the south, 110·1 to the east, and 68·1 to the west. We then subtract the northing made good from the southing, and have a result 86·2 miles to the south; and similarly we find the difference between the eastings and the westings to leave 42·0 miles to the east. Hence, the whole of the courses sailed on in the 24 hours are now reduced to 86·2 miles to the south and 42 0 miles to the east. The course and distance made good may then be deduced as in plane sailing, or, easier, by still continuing to use the traverse table. Look for 86·2 in the difference of latitude column and 42·0 abreast of it in the departure column, or the nearest that can be taken to these numbers; we find, by searching under 26°, 86·3 under difference of latitude and 42·1 under departure and opposite to 96 in the distance column. We therefore conclude the course to be S. 26° E., and distance 96 miles. The student should prove this by plane sailing; he will then find the course to be S. 25° 59′ E., distance 95·87 miles.

MECHANICAL SOLUTION OF THE PROBLEM.—We will now show how this same question can be solved by the aid of instruments. Describe a circle with any radius (fig. 12), and let A, its centre, be the point of departure. Draw A B between the south and east, making an angle of 4 points or 45° with the meridian through A, and make A B = 40 units in length taken from any scale of equal parts. From B draw the second course B C between north and east, making an angle of 4 points with the meridian through B, and make B C = 28 of same units of length. Through C draw the third course C D between south and west, making an angle of 5 points or 56° 15′ with the meridian through C, and make C D = 52 units in length, and so to E, F, and G; then A B, B C, C D, D E, E F, and F G will represent the courses and distances sailed by the ship in the 24 hours, and G will be the place of the ship at the end of the day. From G draw G S at right angles to the meridian N S through the point A of departure. Then A S will be the number of units the ship is gone to the southward, and by measuring from the same scale we find them 86; and S G, the number of units she has gone to the eastward, these we find to be 42. Join A G; then A G will represent the direct course and distance, or the course and distance equivalent to the six courses she has sailed during the day; by measurement we find A G = 96, and the angle S A G is read off from the protractor 26°; and thus the same result is obtained as by the use of the traverse table.

FIG. 12.
Scale 80 miles to an inch.



As another illustration we will take the following example:

Ex. 128. A ship from latitude 4° 10′ N., wishes to make an island that bears from her NE. by N. 196 miles, and having run the following courses, viz., NE½E., 75 miles; E. by S¾S., 42 miles; NNE¼E., 52 miles; N. by W., 34 miles; and E. by N½N., 49 miles—it is required to know her present latitude, and her direct course and distance to the island.

First, we will find the position of the ship after having made the above courses and distances. This we do by aid of the traverse table as follows:

Courses	Distance	Difference of latitude		Departure	
		N.	S.	E.	W.
N. 4½ E.	75	47·6	—	58·0	—
S. 6¼ E.	42	—	14·1	39·5	—
N. 2¼ E.	52	47·0	—	22·2	—
N. 1 W.	34	33·3	—	—	6·6
N. 6½ E.	49	14·2	—	46·9	—
		142·1	14·1	166·6	6·6
		14·1		6·6	
	Diff. lat.	128·0		160·0 Dep.	

By the question . .	Lat. left =	4° 10′ N.
From traverse . . .	Diff. lat. =	2 8 N.
∴ Latitude in		6 18 N.

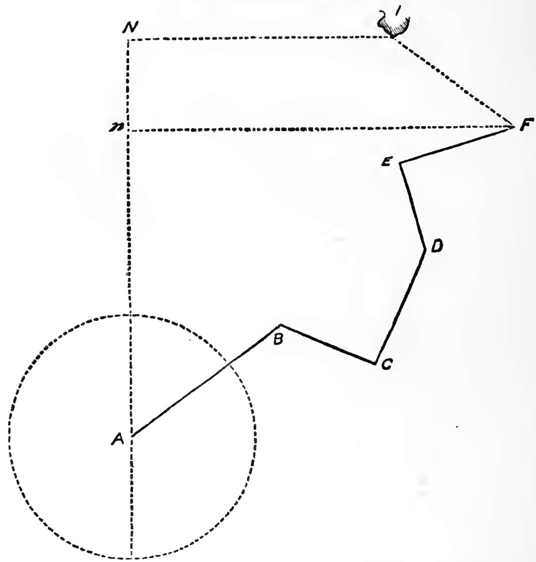
We thus find the ship has made good 128 miles to the north and 160 miles to the east. Our next work will be to find how far north and how far east the island lies from the point of departure; for this purpose we have the bearing N. 3 points E, and distance 196 miles. The traverse tables give the position of the island 163 miles to the north and 108·9 miles to the east of the point of departure. From our resolution of the traverse the ship is 128 miles to the north and 160 miles to the east. Hence, we see

Position of destination	163 miles north	108 9 miles east.
Position of ship	128 „ „	160·0 „ „
∴ Ship must make good	35 miles north and	51·1 miles west.

This again may be solved by plane sailing; or, easier, by the traverse table as follows—Search the table for 35 in difference of latitude column, with 51·1 abreast of it in departure column: this done we find the course to be about N. 55½° W. and distance 62 miles. By plane sailing the answer is, course

N. 55°·36 W., distance 61·93 miles. If the student now lay this question down to scale as the above figure, the lines A B, B C, C D, DE, and E F will show him the courses and distances sailed; F is therefore the place of the ship. He must now lay down I, the place of the island, N. 3 points E., distance A I 196 miles. It is evident the direct course and distance the ship has still to

FIG. 13.
Scale 80 miles to an inch.



make is F I; this measured by the protractor and scale gives N. 55°½ W., distance 62 miles, thus agreeing with the result of resolving the traverse.

EXERCISE VII.

(To be solved by projection as well as by calculation.)

Ex. 129 Since yesterday noon we have run the following courses:—1st, SW. by S., 20 miles; 2nd, W., 16 miles; 3rd, NW. by W., 28 miles; 4th, SSE., 32 miles; 5th, ENE., 14 miles; 6th, SW., 36 miles. What difference of latitude and departure has the ship made, and what is her direct course and distance made good?

Ex. 130. A ship from latitude 28° 32′ N. has run as follows:—1st, NW. by N., 20 miles; 2nd, SW., 40 miles; 3rd, NE. by E., 60 miles; 4th, SE., 55 miles; 5th, W. by S., 41 miles; 6th, ENE., 66 miles. Required her present latitude, with the direct course and distance from the place sailed from to the place arrived at.

Ex. 131. Two ships, A and B, part company in latitude 31° 31′ N., and meet together again at the end of two days, having run as follows:—A: 1st, NNE., 96 miles; 2nd, WSW., 96 miles; 3rd, ESE., 96 miles; 4th, NNW., 96 miles. B: 1st, NNW., 96 miles; 2nd, ESE., 96 miles; 3rd, WSW., 96 miles; 4th, NNE., 96 miles. Required the latitude arrived at and the direct course and distance of each ship.

Ex. 132. Yesterday at noon we were in latitude 0° 15′ S., and bound to a port bearing NE½N. in latitude 2° 15′ N. By our log we have sailed the following courses—viz., NW., 25 miles; NE. by E., 28 miles; E¾S. 32 miles; NNE., 41 miles; E. by S½S., 24 miles; NE. by N., 39 miles; WSW., 24 miles. Required the ship's latitude, with the direct course and distance to the intended port.

Ex. 133. A ship from latitude 18° 14′ N. is bound to a port bearing SW½S. 138 miles, having sailed the following courses:—SSW., 22 miles; S. by E½E., 38 miles; NNE., 30 miles; WSW., 57 miles; NNW., 45 miles; S. by W½W., 50 miles. Find the ship's latitude, and the direct course and distance to the desired port.

Ex. 134. Two ships take their departure from the Lizard in latitude 49° 57′ N.: one bound to St. Michael's, which lies 715 miles to the south and 745 miles to the west; the other bound to Lisbon, lying 661 miles to the south and 215 miles to the west, reckoning from the Lizard; they sail in company SW½W. 610 miles, and then part. What is the direct course and distance of each ship to her port?

Ex. 135. What is meant by resolving a traverse? What is a traverse table? How is it constructed? Traverse tables are usually constructed only for 4 points or 45°; why is it unnecessary to carry it further? With what modifications will the same table serve for points up to 8, and for degrees up to 90? *E.* 1869.

Ex. 136. Explain the construction of the traverse table, and point out some of its uses. *Royal Naval College*, 1865.

Ex. 137. Write down and prove the formulæ used in the construction of the traverse table (in the 'Book of Tables'). Hence show clearly that the calculations need not be carried beyond an angle of 45°. How do you find from the tables the departure and diff. lat. for larger angles? *E.* 1877.

Ex. 138. A port bearing NW. is directly to windward distant 25 miles; a vessel which can sail equally well on either tack within $5\frac{1}{2}$ points of the wind at the rate of 7 miles per hour wishes to reach it. She goes off close-hauled on the starboard tack $2\frac{1}{2}$ hours, and the wind then shifts to W. by S$\frac{1}{2}$S. Find by the traverse table how long after going about she will reach the port if her rate be then increased to 8 miles per hour.

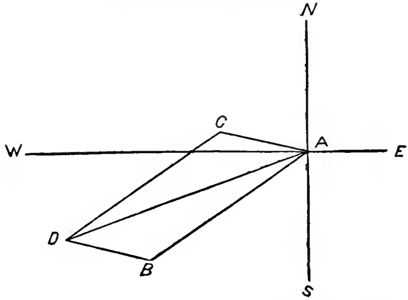
Ex. 139. How is the traverse table constructed? *Ex.* The course being 60° and distance 100 miles, calculate the difference of latitude and departure by arithmetic. *Royal Naval College*, 1865.

CURRENT SAILING.

Currents are streams of water in the ocean; and, unlike the wind, which is named from the quarter from which it blows, currents are named from the point to which they move. Thus, a NE. wind *comes from* the NE., but a NE. current *moves towards* the NE. The direction in which a current moves is called its *set*; the distance it goes in one hour is called its *rate*; and the whole distance travelled in a given time is called its *drift*. Current sailing is the method used to make allowance for the drift of currents and tides. When the course and distance a vessel sails, and also the set and drift of the current, are given for the same time, the question resolves itself into one of simple traverse sailing if we wish to find the course and distance made good. For it is obvious, if a ship sail in any particular direction, as SW. by W. 50 miles, and a current sets her W. by N. 23 miles during the same time, the place reached must be the same as if the vessel had sailed in still water the two courses SW. by W. 50 miles and W. by N. 23 miles; only the

Fig. 14.

Scale 50 miles to an inch.



two directions are made good simultaneously instead of consecutively. To illustrate this, let A be the position of a ship at starting; then she sails from A to B, whilst the current would set her from A to C if she made no headway; hence the ship partakes of the two motions A B and A C simultaneously. Then by the parallelogram of velocities she moves on the diagonal A D of the parallelogram A B D C, and A D is the track of the ship, and therefore represents the course and distance made good in the time. The question is solved thus: Make a traverse table as already directed, entering the set and rate of the current as one course and distance, and solve as a piece of traverse sailing with two courses and distances.

Courses	Distances	Difference of latitude		Departure	
		N.	S.	E.	W.
S. 5 W.	50	—	27·8	—	41·6
N. 7 W.	23	4·5	—	—	22·6
		4·5	27·8		64·2
			4·5		
			23·3		

Answer { Course made good S. 70° W.
Distance „ „ 68¼ miles.

This question is also solved mechanically above and gives the same results.

We will now work the problem trigonometrically.

In the above figure B D = A C = 23 miles

$$\text{the } \angle \text{ CAB} = 4 \text{ points}$$

$$\therefore \angle \text{ ABD} = 12 \;\;,,$$

$$\text{and } \frac{\text{BAD} + \text{BDA}}{2} = 2 \;\;,,$$

We thus have given the two sides A B = 50 miles, B D = 23 miles, and the included angle A B D = 12 points, to find the angles B A D = θ (suppose) and B D A = ϕ, and the side A D the distance.

Now by the usual formula in plane trigonometry—

$\tan \frac{\phi-\theta}{2} = \frac{d-a}{d+a} \cot \frac{B}{2}$	6 pts. log cot 9·617224
$= \frac{27}{73} \cot 6 \text{ pts.}$	27 log 1·431364
	11·048588
$\therefore \frac{\phi-\theta}{2} = 8^\circ 43'$	73 log 1·863323
	8° 43′ log tan 9·185265

From above $\frac{\phi+\theta}{2} = 22^\circ 30'$

Hence θ or BAD $= 13^\circ 47'$

Now the ∠SAD = SAB + BAD

or course = S. 56° 15′ W. + 13° 47′

= S. 70° 2′ W.

To find the distance: $\frac{\text{AD}}{\text{DB}} = \frac{\text{Sin ABD}}{\text{Sin BAD}}$

∴ AD = DB. sin ABD. cosec BAD	23 log = 1·361728
= 23. sin 12 pts. cosec 13° 47′	12 pts. log sin = 9·849485
= 68·26 miles.	13° 47′ log cosec = 10·622965
	68·26 log = 1·834178

Here it will be seen the three methods—viz., mechanically by instruments, by aid of the traverse table, and by plane trigonometry—produce the same results; that is, the course made good S. 70° W. or nearly W. by S$\frac{3}{4}$S., and distance made good 68$\frac{1}{4}$ miles; but the advantage of using the traverse table is clearly seen in the few figures that are necessary for the solution.

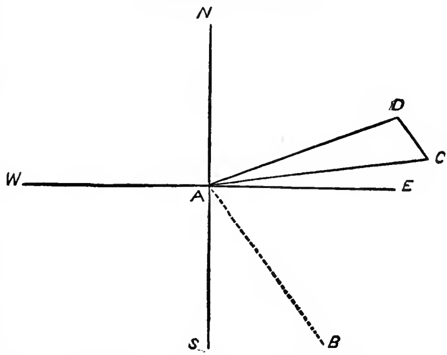
If it be required to shape a course across a known current, or to find the set and drift of the current from known data, the process is not quite so simple as by the traverse table, but the question must be solved as ordinary cases in the solution of triangles by plane trigonometry. These may be better understood by taking examples.

Ex. 140. If a ship is moving through the water at the rate of 8$\frac{1}{2}$ knots per hour, how must I steer across a current, whose set is SE$\frac{3}{4}$S.

$2\frac{1}{4}$ knots per hour, in order to make good an E$\frac{3}{4}$N. course; and what rate am I making good?

Draw the lines N S and E W to represent portions of a meridian and of a parallel of latitude; then draw A B SE$\frac{3}{4}$S. and A C E$\frac{3}{4}$N. The student will see that the angle E A B = $4\frac{3}{4}$ points, and the angle E A C = $\frac{3}{4}$ point; hence the angle B A C = $5\frac{1}{2}$ points. Make the angle A C D equal to the alternate angle C A B—that is, to $5\frac{1}{2}$ points; then, because the current sets the ship towards the south and east, the ship must be kept up against the current—that is, towards the north and west. Draw C D to represent $2\frac{1}{4}$ knots; join A D; this will represent $8\frac{1}{2}$ knots, the rate the vessel is going through the water, and also the line on which the vessel must be steered to counteract the current. It is manifest, if she sail on A D $8\frac{1}{2}$ miles and the current carry her in the direction D C $2\frac{1}{4}$ miles, that at the end of the time she will be at C, which bears E$\frac{3}{4}$N. from A; and, owing to the two directions being made good simultaneously, A C will represent the course and distance made good.

Fig. 15.
Scale 8 miles to an inch.



To find the direction to sail in, we have

$$\frac{\text{Sin CAD}}{\text{Sin ACD}} = \frac{\text{CD}}{\text{AD}}$$

$$\therefore \text{Sin CAD} = \frac{\text{CD}}{\text{AD}} \cdot \text{sin ACD}$$

$$= \frac{2\cdot25}{8\cdot5} \cdot \text{sin } 5\tfrac{1}{2} \text{ points}$$

$$\text{Hence, log sin CAD} = \text{log } 2\cdot25 + \text{log sin } 5\tfrac{1}{2} \text{ points} - \text{log } 8\cdot5$$

$$= 0\cdot352183 + 9\cdot945430 - 0\cdot929419$$

$$= 9\cdot368194$$

$$= \text{log sin } 13^\circ\ 30'\ 1''$$

$$\text{Hence CAD} = 13^\circ\ 30'\ 1''$$

$$\text{and angle NAD} = \text{NAC} - \text{CAD}$$

$$= 81^\circ\ 33'\ 45'' - 13^\circ\ 30'\ 1''$$

$$= 68^\circ\ 3'\ 44''$$

$\therefore$ Course to steer = N. 68° 3′ 44″ E. or ENE. a little easterly.

Again, the angle A D C $= 180° - (\text{A C D} + \text{C A D})$
$= 180° - (61° 52' 30'' + 13° 30' 1'')$
$= 180° - 75° 22' 31''$
$= 104° 37' 29''$

and $\text{A C} = \text{A D}\frac{\sin \text{A D C}}{\sin \text{A C D}}$

$\therefore \log \text{A C} = \log \text{A D} + \log \sin \text{A D C} + \log \text{cosec A C D} - 20$
$= \log 8{\cdot}5 + \log \sin 104°37'29'' + \log \text{cosec } 5\frac{1}{2}\text{pts.} - 20$
$= 0{\cdot}929419 + 9{\cdot}985696 + 10{\cdot}054570 - 20$
$= {\cdot}969685$
$= \log 9{\cdot}326$

$\therefore$ Rate per hour made good $= 9{\cdot}326$ miles.

The same result is obtained by construction.

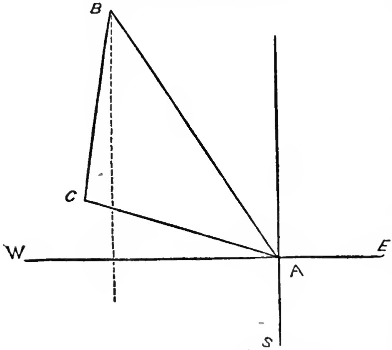
In this example it will be seen the current increases the rate of the vessel towards her destination, and this will be always the case when the angle between the course we wish to make good and the direction of the current is less than 90°; but the rate of the vessel over the ground will be retarded if that angle be greater than 90°

We will take another example, and this time we will find the set, drift, and rate of the current.

Ex. 141. A vessel sails by her log NW. by N. 125 miles in the 24 hours, but by celestial observation the master of her knows he is 83 miles W. by N$\frac{1}{2}$N. of his former position. What is the set, drift, and rate of the current?

FIG. 16.

Scale 80 miles to an inch.



Here, as before, let N S and E W represent portions of the meridian and parallel of latitude through A, the point of departure. Draw A B NW. by N. 125 miles; and A C W. by N$\frac{1}{2}$N., 83 miles. Now the master by his log thinks he is at B, but by his observations he knows he is at C—the current has set him from B to C; hence we must find the direction of B C, and its length will be the drift. We know the angle N A C $= 6\frac{1}{2}$ points and N A B = 3 points, therefore B A C = $3\frac{1}{2}$ points. We have now known B A = 125 miles, C A = 83 miles, and the included angle B A C = $3\frac{1}{2}$ points, to find the other parts of the triangle. This is a well-known case in trigonometry, and we will proceed to do it.

By formula, $\tan \frac{1}{2}(\text{ACB} - \text{ABC}) = \frac{c-b}{c+b} \cot \frac{A}{2}$

$$\therefore \log \tan \tfrac{1}{2}(\text{ACB} - \text{ABC}) = \log (c-b) + \log \cot \frac{A}{2} - \log (c+b)$$
$$= \log 42 + \log \cot 1\tfrac{3}{4} \text{ points} - \log 208$$
$$= 1{\cdot}623249 + 10{\cdot}446353 - 2{\cdot}318063$$
$$= 9{\cdot}751539$$
$$= \log \tan 29^\circ 26' 16''$$
$$\therefore \tfrac{1}{2}(\text{ACB} - \text{ABC}) = 29^\circ 26' 16''$$
$$\text{and } \tfrac{1}{2}(\text{ACB} + \text{ABC}) = \tfrac{1}{2}(180^\circ - \text{BAC})$$
$$= \tfrac{1}{2}(180^\circ - 39^\circ 22' 30'')$$
$$= 70^\circ 18' 45''$$
$$\text{and } \tfrac{1}{2}(\text{ACB} - \text{ABC}) = 29^\circ 26' 16''$$
$$\therefore \text{ACB} = 99^\circ 45' 1'' \text{ and } \text{ABC} = 40^\circ 52' 29''$$

Having found the three angles of the triangle we must now determine the angle which B C makes with the meridian, and this will be the set of the current.

Through B draw a line B D representing a part of a meridian: then A B D = the alternate angle B A N = 33° 45′, and from above A B C = 40° 52′ 29″, therefore the angle D B C = 7° 7′ 29″.

Hence, set of the current = S 7° 7′ 29″ W.

To find the drift $\frac{\text{BC}}{\text{AC}} = \frac{\sin \text{BAC}}{\sin \text{ABC}}$

$$\therefore \text{BC} = \text{AC}.\ \sin \text{BAC}.\ \text{cosec ABC}$$

$$\text{and } \log \text{BC} = \log 83 + \log \sin 3\tfrac{1}{2} \text{ pts} + \log \text{cosec } 40^\circ 52' 29'' - 20$$
$$= 1{\cdot}919078 + 9{\cdot}802359 + 10{\cdot}184152 - 20$$
$$= 1{\cdot}905589$$
$$= \log 80{\cdot}46.$$

Hence, the drift = 80·46 miles

$$\text{and rate} = \frac{80{\cdot}46}{24} = 3{\cdot}35 \text{ miles.}$$

This question may be solved approximately far easier by the traverse table. By dead reckoning the master supposes himself at B; but after observations he knows himself to be at C—

the current has set him from B to C; and the vessel would be brought to the same spot by sailing along B A SE. by S. 125 miles, and then along A C W. by N$\frac{1}{2}$N. 83 miles. This put into the form for resolving a traverse is—

Courses	Distances	Difference of latitude		Departure	
		N.	S.	E.	W.
S. 3 E.	125	—	103·9	69·4	—
N. 6$\frac{1}{2}$ W.	83	24·1	—	—	79·4
		24·1	103·9	69·4	79·4
			24·1		69·4
			79·8		10·0

But as the current produces the same effect as the two courses and distances under consideration, the difference of latitude and departure due to the current are 79·8 miles to the south and 10·0 miles to the west, and these by the traverse table will give 7° as a course and 80$\frac{1}{2}$ miles as a distance. Hence by this method the set is S 7° W. and drift 80·5 miles: a very close approximation to the result found by the longer method. To solve questions of this kind the course and distance made by dead reckoning must always be reversed. By projection with the instruments the same result is obtained.

Exercise VIII.

Ex. 142. A ship leaves port at 8 o'clock in the evening, and runs before the wind, then NNW., at the rate of 3$\frac{1}{2}$ knots an hour, in a tide which runs at the same rate towards the eastward, and next day at noon she found herself 84 miles from her port. Required the ship's true course and the set of the tide.

Ex. 143. Suppose that in 24 hours a ship sails NE. 100 miles by the log, in a current setting E. by S. 1$\frac{3}{4}$ miles per hour. What is the course and distance made good?

Ex. 144. An island bears from a port WSW. 46 miles, but by a strong tide running to the southward a ship, bound to the island, found herself after some time 65 miles from the port and 34 miles from the island. Required the bearing of the port, and the setting of the tide.

Ex. 145. A ship running south at the rate of 5 miles an hour in 10 hours crosses a current which all that time was setting east at the rate of 3 miles an hour. Required the true course and distance the ship sailed.

Ex. 146. A vessel in latitude 41° 18′ N. is bound to a port in latitude 46° 30′ N. lying 258 miles to the west; when she arrived there she found that by dead reckoning she had made 384 miles of northing and 206 miles of westing. Required the set and drift of the current that caused this difference.

Ex. 147. I lay due south of a lighthouse at a distance of $7\frac{1}{2}$ miles; and $2\frac{1}{2}$ hours after I found that a current had set me E. by $S\frac{1}{4}S$. Supposing the rate of the current to be 2 miles an hour, how did I bear in my last position from the lighthouse, which is now invisible, and what is my distance from it? *A.* 1871.

Ex. 148. A ship proceeds at the rate of 8 knots and steers SSE.; she finds at the end of $1\frac{1}{2}$ hours she has made good 14 knots $SE\frac{1}{2}E$. What is the force and direction of the current? *A.* 1873.

Ex. 149. A ship wishes to reach a point the true bearing of which is SW. and distance 15 miles; a current is running SE. by E. (true) 4 knots an hour, and the ship sails at the rate of 7·5 knots an hour; she is on the port tack (wind SSE.), making $\frac{1}{2}$ point leeway, variation $2\frac{1}{2}$ points W., deviation $\frac{1}{4}$ point E. What course must she steer by compass, and how long will she be in reaching the point? *Honours*, 1873.

Ex. 150. What course would a steamer (steaming 6·5 knots an hour) have to steer in order to reach a point 20 miles SE. of her, a current running NE. by E. 3 miles an hour? How long would she be in reaching her destination? *A.* 1874.

Ex. 151. A ship's boat drifted from her when a point of land distant 5 miles bore W. by N.; she picked it up $3\frac{1}{2}$ hours afterwards, when the point of land bore N. by E. distance 12 miles. Find the direction and rate of the current. *A.* 1875.

Ex. 152. A ship in doubling a cape meets a strong tide setting SE.; after running SW. 18 miles by the log the cape bore $N\frac{3}{4}E$. Required the distance of the cape and the drift of the current.

WINDWARD SAILING.

When it is said a vessel can sail within a certain number of points of the wind, it is meant that by a proper management of her sails she can make a course directly towards the wind within the number of points mentioned. Thus, if the wind be SSE. and a vessel can sail within $5\frac{1}{2}$ points of the wind, she can make courses within $5\frac{1}{2}$ points of SSE. on either tack; and she will sail $SW\frac{1}{2}S$. on the port tack and $E\frac{1}{2}S$. on the starboard tack, and she cannot sail in any direction between these courses towards the wind. It frequently happens that when a vessel is nearing her port the wind is directly or partially against her; and then, as most vessels can sail within six points of the wind, by a judicious management in tacking at the right time, a distance may be made good directly to windward, and so the port be reached.

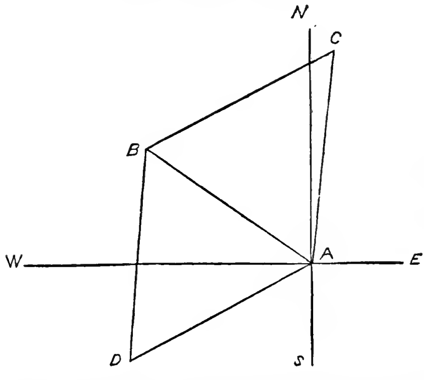
WINDWARD SAILING *is the method by which is found the proper distance to stand on each tack so as to make most headway against a contrary wind.* When a vessel does this she is said to *Sail on a wind* or *Beat to windward*; but if she sails in the direction towards which the wind blows she is said to be *sailing free* or to be *running before the wind.* Problems in windward sailing are solved like the following :—

Ex. 153. What courses and distance must a vessel sail on to reach Cape Hatteras, which bore NW. by W. 42 miles distant, in two tacks, when the wind blows directly from the Cape, supposing she can sail within $5\frac{1}{2}$ points of the wind?

Let A be the position of the ship;
B ,, ,, ,, Cape Hatteras.

The wind is supposed to blow from B towards A. Draw A C $5\frac{1}{2}$ points to the right of A B—that is, N$\frac{1}{2}$E.; and A D $5\frac{1}{2}$ points to the left of A B—that is, SW. by W$\frac{1}{2}$W. Then A C and A D will represent the courses the ship must sail on. From B draw B C parallel to D A to meet A C in C; then the vessel must sail first from A to C N$\frac{1}{2}$E., and then from C to B SW. by W$\frac{1}{2}$W.

FIG. 17.
Scale 10 miles to an inch.



Here the angle B A C$=5\frac{1}{2}$ points, and the angle A B C, which is equal to the alternate angle B A D, also equals $5\frac{1}{2}$ points; hence A B C is an isosceles triangle, and A C is equal to BC—that is, the vessel must make the same distance on each tack, and so one side only of the triangle need to be found. In the triangle ABC all its angles are known, because BCA=16 points−(CAB + CBA)=(16−11) points=5 points; and the side AB is also known, namely 42 miles.

$$\text{Then, } \frac{AC}{AB} = \frac{\sin ABC}{\sin ACB}$$

$$i.e.\ AC = AB \times \sin ABC \times \operatorname{cosec} ACB$$

$$\begin{aligned} \text{and } \log AC &= \log 42 + \log \sin 5\tfrac{1}{2} \text{ points} + \log \operatorname{cosec} 5 \text{ points} - 20 \\ &= 1{\cdot}623249 + 9{\cdot}945430 + 10{\cdot}080154 - 20 \\ &= 1{\cdot}648833 \\ &= \log 44{\cdot}55 \end{aligned}$$

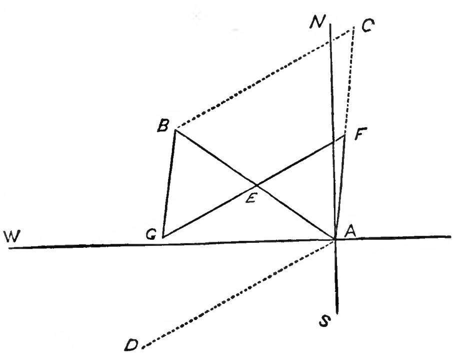
and therefore AC and CB each equal 44·55 miles.

Hence to reach Cape Hatteras the vessel must sail N½E. 44·55 miles, then she must tack and sail SW. by W½W. 44·55 miles.

To vary the question, suppose it be required to know how the vessel must be managed to reach Cape Hatteras *in three tacks.*

Divide AB into two equal parts in E, and through E draw a line FEG parallel to AD, and through B draw BG parallel to AC. Then the vessel must sail first from A to F, next from F to G, and lastly from G to B. It must be noticed that the triangles AFE, BGE are similar and equal in all respects, and that the triangle AFE is similar to the triangle ACB of the last figure; but the side AE is half the side AB, hence AF is half of AC, *i.e.* 22·275 miles. If it be required to solve this question independent of the preceding one, it is to be done thus;

FIG. 18.



We have mentioned that the triangles A F E and B G E are similar and equal in all respects, so if we solve A F E we shall know all that is required. As in the last solution the angles F A E and A E F are each equal to $5\frac{1}{2}$ points, the angle A F E equal to 5 points, and A E the half of 42 miles.

$$\text{Then, } \frac{\text{AF}}{\text{AE}} = \frac{\sin 5\frac{1}{2} \text{ points}}{\sin 5 \text{ points}}$$

$$\begin{aligned} i.e.\ \text{AF} &= \text{AE} \times \sin 5\tfrac{1}{2} \text{ pts.} \times \text{cosec } 5 \text{ pts.} \\ \text{or, } \log \text{AF} &= \log 21 + \log \sin 5\tfrac{1}{2} \text{ pts.} + \log \text{cosec } 5 \text{ pts.} - 20 \\ &= 1{\cdot}322219 + 9{\cdot}945430 + 10\ 080154 - 20 \\ &= 1{\cdot}347803 \\ &= \log 22{\cdot}274 \end{aligned}$$

$$\text{and } \therefore \text{AF} = 22{\cdot}274 \text{ miles} \qquad \text{FE} = 22{\cdot}274 \text{ miles.}$$

$$\text{FG} = 44{\cdot}55 \text{ miles, and GB} = 22{\cdot}274 \text{ miles.}$$

Hence the vessel must be sailed on the port tack N$\frac{1}{2}$E. 22·274 miles, then on the starboard tack SW. by W$\frac{1}{2}$W. 44·55 miles, and again on the port tack N$\frac{1}{2}$E. 22·274 miles. Here we deduce the fact that the vessel must sail the same total distance on each tack, whether she makes two, three, four, or any number of tacks, to reach her destination if direct to windward; and, further, it is seen that no distance is either lost or gained by sailing on two, three, four, or any number of tacks. However, when it is wished to make ground directly to windward, it is advantageous to tack often if the wind be shifty, because any change in its direction must be in the vessel's favour. But, on the other hand, both time and ground are lost every time the vessel is in *stays* (*i.e.* with her head direct to windward). Hence the master of a vessel, with contrary winds, must exercise great judgment in determining how often he will tack to reach the port he is bound to.

Exercise IX.

Ex. 154. A ship bound to a port NNE$\frac{1}{2}$E. must double a cape to the eastward. She having run 45 miles on the port tack, within 6 points of the wind, then at N. by E., the port bore NW. by W$\frac{1}{4}$W. It is required to find how near the wind she must lie on her starboard tack, and what is the distance of the port at each time of setting it.

Ex. 155. Start point bears NW. by W. 14 miles distant; the wind comes exactly over the Start. How must a vessel steer, that can lie within $5\frac{1}{2}$ points of the wind, to reach it in three tacks, and what distance will she sail on each.

Ex. 156. I wish in two tacks to reach an island that bears NE. 21 miles, the wind being NE. My ship can lie within 5 points of the wind, but on the starboard tack she makes $1\frac{3}{4}$ points leeway, and on the port tack 2 points. Find the two courses and distances.

Ex. 157. How long will a master be in taking his ship to a port that lies SSW. 25 miles distant? The wind comes from S. by E., and the ship can lie within $4\frac{1}{2}$ points of the wind, sailing 7 knots per hour.

Ex. 158. A ship bound to a port met a sloop that had sailed from thence SSW$\frac{1}{2}$W. 325 miles, the wind being N. by E.; the ship then went off on the port tack for three days and met a schooner which had sailed a direct course SE. by E$\frac{1}{2}$E. 412 miles since she left the port. Required the ship's course and distance between her meeting those vessels, and how near the wind she lay.

Ex. 159. With the wind at SW., a ship plying to windward, after running 45 miles on the port tack and 45 miles on the starboard tack, finds she has made 30 miles directly to windward. What were her courses, and how near the wind did she lie?

Ex. 160. A pirate gave chase to a sloop 15 miles ahead to the WSW., and both on the starboard tack, the wind being NW.; the sloop, finding she lost way, tacked about for an island in the NW. quarter, 11 miles distant from her and 13 from the pirate. Required how near the wind the sloop must run to fetch the island, and whether the pirate can lie up for it or not?

Ex. 161. A ship sailing close by a point sees two lighthouses, one 18 miles to the NE. by E., the other 8 miles to the SE. by S.; the ship is on the port tack, sailing within $6\frac{1}{4}$ points of the wind, then at SE. by E. Required how far she must run to bring the lighthouses to bear in one line, and what will be the bearing and distances of them from the ship at that time.

Ex. 162. A ship wants to reach a port bearing SW. by S. in two tacks, but must double a point bearing S. by E., the point being 25 miles to the NE. by E. of the port; the ship can lie no nearer the wind, now at SW., than $6\frac{1}{2}$ points. Required her course and distance on each tack.

Ex. 163. A ship which can lie within 5 points of the wind is bound for a port bearing N$\frac{1}{2}$W. 25 miles, the wind being at NW. Required her course and distance on each tack to reach the port, close-hauled in two boards. *A. 1869.*

Ex. 164. A ship wishes to weather an island, which bears from her E. by S$\frac{1}{2}$S., distant 7 miles; the wind is E. by N., and she lies within 6 points of it. How far must she sail on the port tack before going about, so as to weather the island on the other tack? *Honours*, 1875.

Ex. 165. There is a certain port to sail into which you must bring the port and a mill, 4 miles to the east of it, to bear due west; at which time a beacon 5 miles to the NE½N. of the mill must bear N. by E¾E.; you then steer directly for the beacon, and from thence to the port. Required the best wind, in the NW. quarter, for a ship that goes equally well on both tacks, together with the distance she must run on each tack from the said situation to the port.

Ex. 166. What do you understand by the sailings; what by plane sailing; what by traverse sailing; what by current sailing; and what by windward sailing? In all these what assumption is made with regard to the figure of the earth? When may plane and traverse sailing only be used?

Ex. 167. Define: diff. lat., diff. long., departure, course, rhumb curve, and nautical distance. What distinction do you draw between a rhumb line and a rhumb curve?

Ex. 168. What do you mean by 'working or resolving a traverse'? How is the table called a traverse table computed? and why is it necessary to compute one to 45° only?

Ex. 169. When is current sailing used? Why is it necessary? On what principle is it founded?

Ex. 170. What do you mean by sailing on a wind, by running before the wind, by sailing free, by a vessel being in stays, by beating to windward? When a vessel will lie equally well to the wind on both tacks, show that in beating to windward it makes no difference which tack she starts on.

Ex. 171. In the right-angled triangle A B C, $A = 90°$, $C = 67° 30'$, and $CB = 16·8$; find by the traverse table the sides A B and A C. What do you mean by 'resolving a traverse'? Describe how you do it. *E.* 1873.

Ex. 172. Describe fully the construction and use of the traverse table. Show how it is you can apply this table to the solution of such problems as the following:—the height of a ship's mainmast is 180 feet, the angle subtended by it was 4°; required the distance of the observer. Solve this triangle. *E.* 1874.

Ex. 173. A ship is to reach a port 100 miles NE. of her in two tacks. She can lie within 5½ points of the wind, which is NE. by E. What will be her course and distance on each tack? *Royal Naval College*, 1872.

Ex. 174. Define the 'set' and 'drift' of a current. A ship after sailing by dead reckoning NW. by W. 122 miles finds by cross bearings that she has made good 103 miles NW½N. Required the set and drift of the current. *A.* 1877.

Ex. 175. A lighthouse bears from a boat ENE½N. 10·5 miles; find, by the traverse table, how far she must pull to get due south of the lighthouse, and what her distance from it will then be. Explain clearly how it is you can solve this problem by the traverse table. *A.* 1877.

Ex. 176. What do you understand by plane sailing? Write down and prove the formulæ connecting nautical distance, difference of latitude departure, and course. *E.* 1878.

Ex. 177. Prove the following formulæ:

(1) Dep. = dist. × sin course
(2) Diff. lat. = dist. × cos course
(3) Dep. = diff. lat. × tan course.

E. 1879.

Ex. 178. A cutter is distant 7 miles from a ship at anchor, and the ship bears SSW.; the wind is S. by E., and the cutter lies within 6 points of it. How far must she sail on the starboard tack so as to reach the ship on the other tack? *A.* 1879.

Ex. 179. A ship wishes to reach an anchorage in two boards, the anchorage lying NNE. of her; the wind is N. by W., and the ship sails within 6 points of the wind and equally well on either tack. Show that whichsoever tack she sails on she will have to sail over the same distance. *Honours*, 1879.

Ex. 180. Explain the construction of the traverse table. Show clearly why at the top we find 'diff. lat.' and 'dep.' and at the bottom 'dep.' and 'diff. lat.' Are there any other tables where a similar arrangement is adopted? *E.* 1880.

Ex. 181. A ship steams from under a headland in a direction N. by E½E. for 10 miles in 1½ hour; she then finds that the headland bears SW. and is distant 15 miles. Find the direction and strength of the current. *A.* 1880.

Ex. 182. Construct a traverse table for distance 312′ and courses which differ by 15°. *Honours*, 1880.

Ex. 183. A ship is at anchor and bears from my boat W. by N. distant 5 miles; how far must I sail on the port tack before going about so as to speak her when passing on the other tack? My boat sails 6 points to the wind, which is SW. by W.; how much would this distance be reduced if my boat worked in 11 points. *Honours*, 1880.

Ex. 184. A ship wishes to make a harbour lying NE. of her, distant 10 miles, in 3 tacks, the wind being also NE.; she can sail within 6 points of the wind on the starboard and within 7 points on the port tack. How far must she go on each tack (1) supposing her to start on the port, (2) on the starboard tack? *Honours*, 1881

CHAPTER V.

Parallel sailing—Difference of longitude—Meridian distance—Proof of formula—Exercises—Middle-latitude sailing—Proof of formulæ—When middle-latitude sailing should not be used—Exercises—Examination.

In all our calculations up to the present time nothing has been said about any method for finding the difference of longitude or the geographical position of the ship. Because a person's place on the earth is not fully determined by one co-ordinate only, hence he must know his longitude as well as his latitude, and our next care will be to show how it is found. An ordinary observer looking at a terrestrial globe cannot fail to be struck with the fact that all meridians as they recede from the equator approach each other and at last meet in the poles; and thus the arcs of parallels of latitude between any two meridians get less and less as they are in higher latitudes. The sailor says, 'the degrees of longitude get shorter as he nears the pole.' What he means by this is, that the line (the portion of a parallel of latitude) which subtends a degree of longitude gets shorter and shorter as he approaches the poles. On the equator 60 miles subtend a degree of longitude, the same as subtends a degree of latitude all the world over if we consider the earth a sphere; but in latitude 60° a degree of longitude is subtended by 30 miles, in 70° by about 20½ miles, and so on, decreasing as the latitude is increased. We will now show the relation which exists between an arc of the equator and an arc of a parallel of latitude between the same meridians—that is, the meridian distance. This constitutes the fundamental formula in parallel sailing.

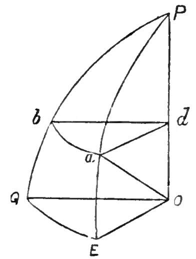
Parallel Sailing *is the method used for determining the difference of longitude, corresponding to a given distance on a parallel of latitude and vice versa.*

Difference of Longitude between two places is the arc of the equator intercepted between their meridians.

Meridian Distance between two places is the arc of the parallel of latitude on which they are situate intercepted between their meridians.

Let E Q P O be a portion of the earth bounded by sections through the meridians P E, P Q, and the equator E Q; then P is the pole, O the centre of the earth, and P O a portion of the axis; also let $a\,b$ be a portion of a parallel of latitude. Draw $a\,b\,d$, the plane of the parallel of latitude, and let it meet the axis in d. Then E Q O and $a\,b\,d$ are parallel planes cutting the axis P O at right angles; hence $a\,d$ is parallel to E O and $b\,d$ to Q O, and therefore Euclid XI. 10:

Fig. 19.



the angle $a\,d\,b =$ the angle E O Q.

Now because the radii of the same sphere are equal,

$$\text{E O} = a\,\text{O};$$

and because the arcs of circles subtending the same angle vary as their radii,

$$\therefore \frac{\text{arc } a\,b}{\text{arc E Q}} = \frac{a\,d}{\text{E O}} = \frac{a\,d}{a\,\text{O}} = \sin a\,\text{O}\,d;$$

$= \cos \text{E O}\,a$, because $a\,\text{O}\,d$ is complementary of E O a.

But E Q is the difference of longitude $=$ L
$a\,b$ is the meridian distance $=$ M
E O a is the latitude $= l$.

Substituting these values in the above equation,

$$\frac{\text{M}}{\text{L}} = \cos l,$$

or meridian distance $=$ diff. long. $\times$ cos lat.

By using and transposing the formula $\text{M} = \text{L} \cos l$, all questions in parallel sailing can be solved.

Ex. 185. A vessel sailed 50 miles due west from Land's End (latitude 50° 4′ N., longitude 5° 44′ 42″ W.). Required the longitude arrived at.

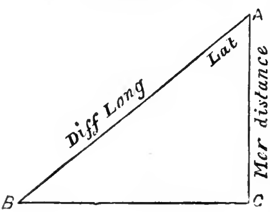
$$L = M \sec l.$$

Meridian distance	M = 50 miles	log =	1·698970
Latitude	l = 50° 4′ N.	log sec =	10·192535
Difference of longitude	L = 77·89	log =	1·891505

Longitude left (Land's End) . .	5° 44′ 42″ W.
Difference of longitude L . . .	1 17 53 W.
Longitude arrived at	7 2 35 W.

All questions in parallel sailing may also be solved by construction. In this instance draw a vertical line A C, and from any scale make A C equal to the meridian distance 50 miles; at the point A make the angle C A B equal to the latitude, in this case 50° 4′, and from C draw C B at right angles to A C to meet A B in B. The hypotheneuse A B will equal the difference of longitude. This is easily seen, for

Fig. 20.
Scale 50 miles to an inch.



$$AB = AC . \sec A$$

and A C was made equal to the mer. dist.
and C A B „ „ latitude;

$$\therefore AB = \text{mer. dist.} \times \sec \text{lat.}$$

but difference of longitude is equal to the same expression.

Hence A B = difference of longitude.

This construction leads to the solution of the question by the traverse table. In the plane triangle, A B represents the distance, A C the difference of latitude, and C A B the course. Hence, if we look in the traverse table for the latitude as the course, viz. 50°, and the meridional distance 50′ in the difference of latitude column, the difference of longitude 78′ is found in the distance column, thus agreeing with the calculation within 7″.

Ex. 186. Suppose a ship to sail 160 miles due west from Gibraltar (longitude 5° 21′ 12″ W.), and finds by observation she is in longitude 8° 39′ 15″ W. Required the latitude.

Longitude in	8° 39′ 15″ W.	
Longitude left	5 21 12 W.	
Difference of longitude L .	3 18 3	= 198·05 miles;

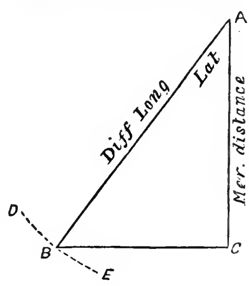
$$\cos l = \frac{M}{L} = \frac{160}{198 \cdot 05}$$

Meridian distance	M =	160	log 2·204120
Difference of longitude	L =	198·05	log 2·296775
Latitude	l =	36° 6′ 40″ N.	log cos 9·907345

To prove this by projection draw A C vertical, as in the last question, to represent the meridian distance = 160 miles, and from C draw C B at right angles to A C. From the centre A, with the radius equal to difference of longitude 198·05 miles, describe an arc D B E meeting C B in B, and join A B; then from the construction the angle B A C is the latitude, and will be found equal to 36°.

Fig. 21.

Scale 133⅓ miles to an inch.



To solve this question by the traverse table we must search until we find 160′ in the difference of latitude column opposite to 198, in the distance column, the course will then be found to be 36°. But in the last question it was shown the course is equal to the latitude; hence the latitude of Gibraltar is 36° nearly.

Ex. 187. At what rate per hour is the Navigation School, Plymouth (latitude 50° 22′ 25″ N.), carried around by the rotation of the earth?

The earth revolves on its axis once in 24 hours—that is, 360° of longitude revolve in that time: this is at the rate of $\frac{360°}{24} = 15°$ per hour. Hence the difference of longitude (L) 15° or 900′, and the latitude (l) 50° 22′ 25″, are given to find the corresponding meridian distance.

$$M = L. \cos l$$

Latitude	l = 50° 22′ 25″	log cos	9·804670
Difference of longitude	L = 900 miles	log	2·954243
Meridian distance	M = 574 miles	log	2·758913

∴ Rate of Navigation School due to rotation of the earth = 574′ per hour.

The student should solve this question for himself, both by projection and by the traverse table.

Exercise X.

Ex. 188. Required the compass course and distance from A to B.

Latitude A .	. 35° 12′ S.	Longitude A .	. 18° 5′ E.
B .	. 35 12 S.	„ B .	. 28 18 E.

Variation $2\frac{1}{2}$ points W.; deviation 11° E. *A.* 1868.

Ex. 189. A ship in latitude 49° 35′ N., longitude 8° 40′ W., sails due south 15 miles, and then alters her course to due east for 25 miles; what are her latitude and longitude in? What courses must she be steered by ship's standard compass, allowing 23° 49′ W. for variation, and 2° W. for deviation in the first position of her head, and 12° 50′ E. in the second? *E.* 1869.

Ex. 190. In sailing on a parallel of latitude, I find the distance actually made good is 38 nautical miles, while I have changed my longitude by one degree. On what parallel am I sailing? *E.* 1872.

Ex. 191. A ship sails due west from Funchal (latitude 32° 57′ N., longitude 16° 58′ W.). How many miles will she go before she arrives at the meridian of Fayal (longitude 28° 43′ W.)? *E.* 1873.

Ex. 192. A ship sails due west from Valparaiso (latitude 33° S., longitude 71° 38′ W.) for a distance of 93 miles. What is the longitude of the place arrived at? *E.* 1875.

Ex. 193. Two ships leave a place in latitude 30° S. and longitude 140° W. together, the one (A) going due south, and the other (B) going due west. If they sail at the same rate, what will be the latitude of A when B is in longitude 141° 10′ W.? *E.* 1875.

Ex. 194. A ship sails along a parallel of latitude towards the west through a distance of 19 miles; her latitude at starting was 51° 20′ N. and her longitude 15° 10′ W. Find her latitude and longitude in. *E.* 1878.

Ex. 195. Two vessels, A and B, bound from St. Helena (latitude 15° 55′ S., longitude 5° 43′ W.) to Ascension (latitude 7° 57′ S., longitude 13° 59′ W.), leave together and sail at the same rate. A sails due west till she arrives in the longitude of Ascension, and then due N.; B sails due N. till she arrives in the latitude of Ascension, and then due W.

Which will arrive first; and by how many miles will she have the advantage? *A.* 1879.

Ex. 196. The Lizard is in latitude 49° 57′ N., longitude 5° 12′ W.; what is the longitude of a place which lies in a direction due east at a distance of 271 miles? *E.* 1876.

Ex. 197. How far must a ship sail due east in latitude l to alter her longitude $a°$? *Royal Naval College*, 1864.

Ex. 198. In what latitude does the meridian distance bear the same ratio to the difference of longitude that 11 does to 12?

Ex. 199. A vessel went due E. x miles, it was then found that the difference of longitude was $\frac{5x}{4}$ miles. In what latitude was the vessel?

Ex. 200. What is the velocity of Rio Janeiro, latitude 22° 54′ 42″ S., due to the earth's rotation on its axis?

Ex. 201. How much faster does a person at Mobile, latitude 30° 14′ N., travel per hour by the rotation of the earth on its axis, than one in New York, latitude 40° 43′ N.?

Ex. 202. Prove that if two places, A and A', in latitude $x°$, be $p°$ of longitude apart; and two places, B and B', in latitude $y°$, be $p°$ of longitude apart; then

$$\frac{\text{Meridian distance between A and A}'}{\text{Meridian distance between B and B}'} = \frac{\cos x°}{\cos y°}$$

Ex. 203. Suppose a vessel sail on the 30th meridian W. from 50° 22′ N. to 44° 37′ S., and then eastward on a parallel of latitude to New Zealand, longitude 165° 30′ E. What distance has she sailed?

Ex. 204. From two ports in latitude 27° 40′ S. distant 319 miles, two ships sail due south at same rate; how far will they be apart when they are in 41° 19′ S.?

Ex. 205. Two ships in latitude 42° 15′ S. are distant from one another 358 miles; if they both sail due north at the same rate until their distance apart is 420 miles, what latitude have they arrived at?

Ex. 206. A ship having run due east for 5 days, at the rate of $5\frac{1}{4}$ miles per hour, finds she has altered her longitude 12° 40′; on what parallel of latitude has she sailed?

Ex. 207. The difference of longitude is 6° 20′, the corresponding meridian distance in north latitude is 247 miles, and in south latitude is 198 miles; what is the distance in nautical miles between the parallels of latitude where the meridian distances are measured?

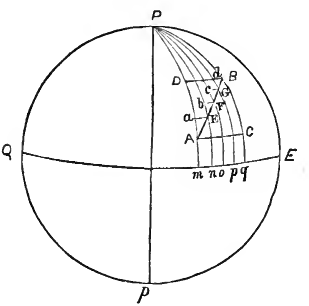
Middle-Latitude Sailing.

When a vessel sails on either the equator or on a parallel of latitude, she changes only her longitude, and when on a meridian she changes only her latitude; but in whatever other direction she sails she must change both her latitude and her longitude.

It is the province of middle-latitude sailing to investigate and to apply formulæ for determining these changes.

Let P E *p* Q represent the earth, E Q the equator, A B the rhumb curve through A the place the ship leaves, and B the place she arrives at. Divide the rhumb curve into a great number of equal parts as in E F G, &c., and through A E F G B, &c., draw meridians P *m*, P *n*, P *o*, P *p*, P *q*, &c., and parallels of latitude A C, E *a*, F *b*, G *c*, B D, &c. Then we have a great number of small right-angled triangles formed, of which *a* E A is the type, in which A *a* is the type of the difference of latitude in each;

Fig. 22.



and when the triangles are infinitely small, and therefore their sides are sensibly straight lines, *a* E is the type of the departure. If now we sum the sides of which A *a* is the type, that is A *a* + E *b* + F *c* + ..., it is evident we get A D, or the difference of latitude between A and B; also if we sum the sides of which *a* E is the type, that is *a* E + *b* F + *c* G + ..., we get the departure; and, similarly, if we sum the sides of which A E is the type, that is A E + E F + F G + ..., we get the nautical distance between A and B. But because *a* E, *b* F, *c* G, &c., get shorter for the same difference of longitude as we approach the poles (*see* Parallel Sailing), the sum of the sides of which *a* E is the type must be less than A C the meridian distance in the lower latitude, and greater than D B the meridian distance in the higher latitude. Therefore, a line representing the sum of the sides of which *a* E is the type—that is, the departure must lie somewhere between A C and B D. In middle-latitude sailing the assumption is made that this is the case in the middle latitude between the place sailed from and the place arrived at. But although nearly correct under certain conditions (for it lies nearer the pole than the middle latitude), it must be remembered *this is a pure assumption*, and is used only because in most cases the error introduced is small.

From the above considerations we get a more exact definition of departure, viz.:

DEPARTURE is the sum of all the meridian distances, corresponding to the infinitely small portions of the rhumb line between the place sailed from and the place arrived at.

All questions in middle-latitude sailing are worked from formulæ obtained by the above assumption—namely, that *the departure is equal to the meridian distance in the middle latitude.* Because of this, when we write the formulæ for parallel sailing,

$$M = L. \cos l$$

we may substitute departure for meridian distance M, and middle latitude for latitude l; the formula then becomes

Departure = diff. long. × cos mid. lat. . . . I.

Referring to our figure we see

$$A\,a = A\,E.\ \cos E\,A\,a$$
$$E\,b = E\,F.\ \cos F\,E\,b$$
$$F\,c = F\,G.\ \cos G\,F\,c$$
$$\&c. = \&c.$$

but E A *a*, F E *b*, G F *c*, &c., are all equal, being the angle made by the ship's track A B with every meridian it crosses (rhumb curve), and is therefore the course.

Hence, A *a* + E *b* + F *c* + ... = (A E + E F + F G + ...) × cos course.

∴ diff. lat. = dist. × cos course.

Again, $a\text{E} = \text{AE}.\ \sin \text{EA}a$

$b\text{F} = \text{EF}.\ \sin \text{FE}b$

$c\text{G} = \text{FG}.\ \sin \text{GF}c$

&c. = &c.

Hence, $a\text{E} + b\text{F} + c\text{G} + \ldots\ldots = (\text{AE} + \text{EF} + \text{FG} + \ldots\ldots) \times \sin \text{course}.$

∴ Departure = dist. × sin course.

Once more, $a\text{E} = \text{A}a.\ \tan \text{EA}a$

$b\text{F} = \text{E}b.\ \tan \text{FE}b$

$c\text{G} = \text{F}c.\ \tan \text{GF}c$

&c. = &c.

Hence, $a\text{E} + b\text{F} + c\text{G} + \ldots\ldots = (\text{A}a + \text{E}b + \text{F}c + \ldots\ldots) \times \tan \text{course}.$

∴ Departure = diff. lat. × tan course

$$\text{and tan course} = \frac{\text{departure}}{\text{diff. lat.}}$$

Substituting from I.

$$\text{tan course} = \frac{\text{diff. long.} \times \text{cos mid. lat.}}{\text{diff. lat.}} \qquad \text{II.}$$

Again, from I. we get

diff. long. = dep. × sec mid. lat.

But from plane sailing

dep. = dist. × sin course.

Hence, diff. long. = dist. × sin course × sec mid. lat. . III.

The formulæ marked I. II. and III. are sufficient to solve all questions which may arise in middle-latitude sailing.

The assumption we have made not being a correct one, it must affect all results obtained from the use of it: and it is now our duty to point out where the errors will be greatest. Middle-latitude sailing should not be used :—

(*a*) *In high latitudes:* because the cosines of all angles change very quickly when the angles are large; and hence the cosine of the mean latitude will not be the mean of the cosines of the two latitudes.

(*b*) *When the difference of latitude is great*: because the greater the difference of latitude, the farther from the middle latitude will the parallel representing the departure be found.

(c) *When the two places under consideration are on different sides of the equator:* because the middle latitude must then be situate much nearer to the equator than the parallel representing the departure would be.

Hence, middle-latitude sailing should only be used when sailing near the equator, when the course is great (more than 45°), when the distance is comparatively small, and when the two places are on the same side of the equator. In all other cases Mercator's sailing should be used.

Ex. 208. Find the course and distance from Rame Head (latitude 50° 19′ N., longitude 4° 13′ W.) to Ushant (latitude 48° 28′ 30″ N., longitude 5° 3′ 12″ W.).

	Diff. lat.	*Mid. lat.*	*Diff. long.*
Rame Head	50° 19′ 0″ N.	50° 19′ 0″ N.	4° 13′ 0″ W.
Ushant .	48 28 30 N.	48 28 30 N.	5 3 12 W.
	1 50 30	2)98 47 30	50 12
	60	49 23 45	50·2 miles.
	110·5 miles.		

To find course.

$$\text{Tan course} = \frac{\text{diff. long} \times \text{cos. mid. lat.}}{\text{diff. lat.}}$$

Difference of longitude .	50·2 miles	log	1·700704
Middle latitude . .	49° 23′ 45″	log cos	9·813468
			11·514172
Difference of latitude .	110·5 miles	log	2·043362
Course . . .	S. 16° 28′ 17″ W.	log tan	9·470810

To find the distance.

$$\text{Dist.} = \text{diff. lat.} \times \text{sec. course.}$$

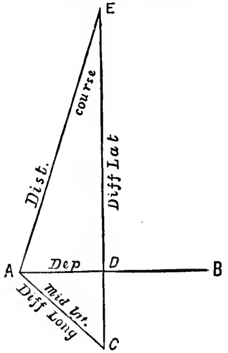
Difference of latitude .	110·5 miles	log	2·043362
Course . . .	16° 28′ 17″	log sec	10·018199
Distance . . .	115·2 miles	log	2·061561

Answer { Course S. 16° 28′ 17″ W.
Distance 115·2 miles.

Middle-latitude sailing can also be solved by construction similar to plane sailing, thus :—

Take a horizontal line A B of indefinite length, and make the angle B A C equal to the middle latitude 42° 24′, and make A C equal to the difference of longitude 50·2′; draw C D a perpendicular to A B, and produce it indefinitely towards E; then A D is the departure, and is equal to 32$\frac{2}{3}$ miles. Next, make D E equal to the difference of latitude, we have then the departure A D and difference of latitude D E known in the plane triangle A D E: join A E: the angle D E A is the course, and measured with a protractor is found equal to 16$\frac{1}{2}$ degrees: and A E is the distance, and is equal to 115 miles.

FIG. 23.
Scale 40 miles to an inch.



This construction suggests the solution by aid of the traverse table alone. Thus, take the middle latitude as a course and the difference of longitude in the distance column; the departure will be found in the difference of latitude column to be equal to 32$\frac{2}{3}$ miles. Again, search the traverse table for 110$\frac{1}{2}$ miles in the difference of latitude column and 32$\frac{2}{3}$ miles in the departure column: these are found under the course 16$\frac{1}{2}$° and distance 115 miles. Hence all three methods of solution give as course S. 16$\frac{1}{2}$W. or S. by W$\frac{1}{2}$W. and distance 115 miles.

Ex. 209. Having given the latitude and longitude of the Rame Head as before, the latitude of Ushant 48° 28′ 30″ N., and the course from the Rame Head thither S. 16° 28′ 17″ W.; find the longitude of Ushant.

Mid. lat.	*Diff. lat.*
49° 23′ 45″ N.	110·5 miles S.

To find difference of longitude.

$$\text{Tan course} = \frac{\text{diff. long} \times \text{cos mid. lat.}}{\text{diff. lat.}}$$

$$\therefore \text{Diff. long.} = \text{diff. lat.} \times \text{tan course} \times \text{sec. mid. la}$$

Difference of latitude .	110·5 miles	log	2·043362
Course . . .	16° 28′ 17″	log tan	9·470810
Middle latitude . .	49 23 45	log sec	10·186532
Difference of longitude	50·2 miles W.	log	1·700704

Longitude, Rame Head	4° 13′ 0″ W.
Difference of longitude, 50·2 miles W. . .	50 12 W.
Longitude, Ushant	5 3 12 W.

The distance is found as before.

The construction for this example is similar to the last (see fig. 23, Ex. 208). Here are given the course, the difference of latitude, and the middle latitude. Draw E C vertical and of indefinite length; make E D equal to $110\frac{1}{2}$ miles, the difference of latitude. At the point E make the angle D E A equal to the course $16\frac{1}{2}°$; from D draw D A at right angles to E C to meet E A in A; then D A is the departure, and is found to be equal to $32\frac{2}{3}$ miles, and E A is the distance 115 miles. Next, at the point A make the angle D A C equal to the middle latitude $49\frac{1}{2}°$, then A C is equal to the difference of longitude 50 miles.

To solve this question by inspection. Turn to the course $16\frac{1}{2}°$ in the traverse table, and find the difference of latitude $110\frac{1}{2}$ in its own proper column; the distance and departure will be found in their respective columns to be 115 and $32\frac{2}{3}$ miles. Next take the middle latitude $49\frac{1}{2}°$ as course, and find the departure $32\frac{2}{3}$ miles in the difference of latitude column, then the difference of longitude, 50 miles, is found in the distance column. This must be applied to the longitude of Rame Head to the westward, which gives the longitude of Ushant as before.

Ex. 210. Given the latitude and longitude of the Rame Head, and the course and distance to Ushant, S. 16° 28′ 17″ W., 115·2 miles. Find the latitude and longitude of Ushant.

To find the latitude of Ushant.

By plane sailing. Diff. lat. = dist. × cos course.

Distance . . .	115·2 miles	log	2·061561
Course	S. 16° 28′ 17″ W.	log cos	9·981801
Difference of latitude .	110·5 S.	log	2·043362

Latitude, Rame Head	50° 19′ 0″ N.
Difference of latitude, 110·5 S. . . .	1 50 30 S.
Latitude, Ushant	48 28 30 N.

To find middle latitude.

Latitude, Ushant	48° 28′ 30″ N.
„ Rame Head	50 19 0 N.
	2)98 47 30 N.
Middle latitude	49 23 45 N.

To find the longitude of Ushant.

Diff. long. = diff. lat. × tan course × sec mid. lat.

Difference of latitude	. 110·5 miles	log	2·043362
Course . .	. S. 16° 28′ 17′ W.	log tan	9·470810
Middle latitude .	. 49 23 45 N.	log sec	10·186532
Difference of longitude	. 50·2 miles	log	1·700704

The student should also solve this question by both construction and inspection.

Longitude left (*Rame Head*)	4° 13′ 0″ W.
Difference of longitude 50·2 miles . . .	50 12 W.
Longitude in (*Ushant*)	5 3 12 W.

Answer	Latitude, Ushant . .	48° 28′ 30 N.
	Longitude, Ushant .	5 3 12 W.

EXERCISE XI.

Ex. 211. Give definitions of the course, distance, true difference of latitude, departure, and middle latitude between two places.

Royal Naval College, 1874.

Ex. 212. A ship sailing from latitude 27° 30′ N., longitude 14° 20′ W., arrives at a place in latitude 29° 45′ N. after making 66 miles easting. What is the longitude arrived at? *E.* 1871.

Ex. 213. A ship in latitude 38° 44′ N., longitude 18° 33′ E., sails by compass ENE. 70 miles; required the latitude and longitude in, given variation of the compass $\frac{3}{4}$ point W., deviation 8° E., leeway 1 point, direction of wind ESE. *E.* 1872.

Ex. 214. The latitudes and longitudes of two places near each other being given, explain the method of finding the course and distance by middle-latitude sailing. *Royal Naval College*, 1867.

Ex. 215. Find the compass course and distance from

Easter Island .	latitude 26° 6′ S;	longitude 109° 17′ W.
Galapagos Island .	latitude 0 0 S;	longitude 92 0 W.

Variation 1 point E.; deviation 3° W. *E.* 1876.

Ex. 216. A ship sails from a place in latitude 22° 20′ S., longitude 90° 40′ W., on a course N. 32° 50′ E., for a distance of 256 miles: required the latitude in and longitude in by middle-latitude method. *E.* 1880

Ex. 217. From the Cape of Good Hope (latitude 34° 29′ S., 1 ngitude

18° 23′ E.) a ship sails N. 48° 25′ W. a distance of 480 miles. Required the latitude and longitude in by middle-latitude sailing. *E.* 1881.

Ex. 218. If the latitude and longitude of Haulbowline Island be 51° 30′ 30″ N. and 8° 18′ 12″ W., and Granville to the south of Haulbowline Island is in longitude 1° 36′ W., whilst the departure between the two places is 256·7 miles, find the latitude of Granville, with the course and distance from Haulbowline Island.

Ex. 219. Since leaving Cape Agulhas (latitude 34° 47′ 42″ S., longitude 20° 0′ 42″ E.) a ship has made good a true S. 65° 30′ W. course and 1650 miles distance. Find her present latitude and longitude, and her course and distance to Cape Horn (latitude 55° 59′ S., longitude 67° 16′ W.), variation 15° 20′ W., deviation 11° 20′ W.

Ex. 220. A ship from latitude 51° 18 N., longitude 22° 6′ W., having sailed between the S. and E. for several days, reckons she has made 564 miles departure and 786 miles difference of longitude: what is the latitude and longitude of the place arrived at, also her direct course and distance.

Ex. 221. From a place in north latitude, a ship sails S. 33° 15′ E. until she has made 564 miles of departure and 786 miles of difference of longitude. What were the latitudes sailed from and arrived at?

Ex. 222. A ship sails from A (latitude 50° 30′ N., longitude 15° 45′ W.) to B as follows: SW½S. 40 miles; N. by W. 15 miles; SSE. 55 miles; W. by N., 70 miles: find the latitude and longitude of B. Also find what course she would have to take from A to B, to arrive at the latter place without altering her direction, and the distance she would have to go. *E.* 1873.

The solution of this question is as follows:—

Courses	Dist.	Difference of latitude		Departure	
		N.	S.	E.	W.
S. 3½ W.	40	—	30·9	—	25·4
N. 1 W.	15	14·7	—	—	2·9
S. 2 E.	55	—	50·8	21·0	—
N. 7 W.	70	13·7	—	—	68·7
		28·4	81·7	21·0	97·0
			28·4		21·0
	Difference of latitude .		53·3	Departure .	76·0

Latitude left	= 50° 30′ N.	Latitude left	50° 30′ N.
Difference of latitude	= 53·3 S.	Latitude in	49 36·7 N.
Latitude in	= 49 36·7 N.		2)100 6·7 N
		Middle latitude	50 3·35 N.

To find the difference of longitude we use the traverse table thus: take the middle latitude to the nearest degree, in this case 50°, as a course, and find the miles departure made, 76·0 in difference of latitude column, the difference of longitude is then found opposite to it in the distance column. In this case the difference of longitude is 118 miles.

Longitude left	15° 45′ W.
Difference of longitude	1 58 W.
Longitude in	17 43 W.

The course and distance must be found as in traverse sailing; with difference of latitude 53·3 and departure 76·0 miles: this gives 55° as a course and 93 miles as a distance.

Answer	Latitude B . .	49° 36′ 42″ N.
	Longitude B .	17 43 W.
	Course . .	S. 55 W.
	Distance . .	93 miles.

The difference of longitude may be found from the formula—

$$L = M \times \text{sec. mid. lat.}$$
$$= 76{\cdot}0 \times \sec\ 50^\circ\ 3'\ 21''$$

Middle latitude . .	50° 3′ 21″	sec	10·192438
Departure	76·0	log	1·880814
Difference of longitude . .	118·4	log	2·073252

The solution must then be completed as before, and differing only 24″ in longitude.

Ex. 223. A ship sails from latitude 20° 10′ N., longitude 30° 30′ W. as follows: NW. by W., 72′; E. by N., 25′; N. by E½E., 240′; SSW., 80′; E. by S¼S. 54′. Required the latitude and longitude in, and the course and distance made good. *E.* 1876.

Ex. 224. A ship sails from latitude 51° 25′ N., longitude 8° 12′ W.; SSE., 30′: E. by S., 18′; SW. by W., 36′; W¾N., 14′; and SE. by E¼E. 46′. Required the equivalent course and distance, and the latitude and longitude of the place arrived at. *E.* 1878.

Ex. 225. January 15, 1881, at noon, was in latitude 28° 13′ N., longitude 63° 14′ W., and sailed during the next 24 hours as under:—

True courses	Distances
NE. by E.	63 miles
N. by W.	48 ,,
NNE.	172 ,,
SSW.	24 ,,
SE¾E	55 ,,

Required the latitude and longitude in on January 16th at noon, and the course and distance made good. *E.* 1881.

Ex. 226. At 9 hours 30 minutes A.M., a steamer steering directly for the Needles observes Portland lighthouse bearing NNW. (true) distant 25 miles. Find her compass course, also the time at which she will have Portland due west: and her distance from it, supposing her to steam 10 knots an hour.

Given lat. Portland lighthouse, 50° 31′ N.; lat. Needles, 50° 40′ N.
long. „ . 2 27 W.; long. „ 1 34 W.
Variation of compass $1\frac{3}{4}$ points W.; deviation 5° E. *A.* 1881.

Ex. 227. Why can I not determine fully the place of my ship by help of the formulæ used in plane sailing only? What additional formula will enable me to do this? Show that I can use a traverse table instead of directly applying this formula. A ship sails from lat. 23° 18′ N., longitude 54° 27′ W., to latitude 27° 19′ N., longitude 51° 59′ W. Draw a diagram showing the course, distance, departure, and difference of latitude, obtaining the departure directly from the traverse table.

E. 1869.

Ex. 228. A ship leaving a place in latitude 23° S., longitude 178° 30′ E., sails on the following courses: NE., 30′ (variation 1 point E., deviation 8° E.; leeway $\frac{1}{2}$ point, wind ENE.); ESE., 50′ (variation $\frac{3}{4}$ point E., deviation 5° W., leeway 1 point, wind N. by E.); E. by N., 30′ (variation $\frac{3}{4}$ point E., deviation 11° E., wind aft). Find the distance and true bearing from the ship in the last position of an island in latitude 22° 50′ S., longitude 180° E. *Honours*, 1874.

Ex. 229. What do you mean by parallel sailing, and by middle-latitude sailing? 'The degrees of longitude get shorter as we near the pole': how do you reconcile this statement with fact?

Ex. 230. What is the assumption made in middle-latitude sailing? Is the departure too great or too little by this assumption? Give the conventional and exact definitions of departure, and illustrate your answer by a figure.

Ex. 231. Show when, and why, middle-latitude sailing should not be used at the times stated. What method should then be adopted?

Prove that
$$\text{cot course} = \frac{\text{mer. diff. lat.}}{\text{diff. long.}}$$
$$\text{diff. long.} = \text{dep.} \times \text{sec. mid. lat.}$$
A. 1868.

Ex. 232. Prove, giving all the steps of the proof, that distance = diff. long. × cos lat. in parallel sailing. *Royal Naval College*, 1867.

Ex. 233. A ship sails from a certain place a miles due E, then a miles due S., and then b miles due W., and arrives at the same longitude as that from which she started. Required the latitude of the place arrived at. Take as an example $a = 100'$, $b = 150'$. *Honours*, 1870.

Ex. 234. Prove the formula :–

$$\text{tan course} = \frac{\text{diff. long.} \times \text{cos mid. lat.}}{\text{true diff. lat.}}$$

Royal Naval College, 1868.

Ex. 235. Prove the formula of middle-latitude sailing.

$$\text{Dep.} = \text{diff. long.} \times \text{cos mid. lat.}$$

Does this give strictly accurate results? Give a reason for your answer. How do you proceed when the two places are situated in different hemispheres? *E.* 1874.

Ex. 236. What do you understand by middle-latitude sailing? Write down its characteristic formulæ. Prove the formula :—

$$\text{Dep} = \text{diff. long.} \times \text{cos lat.}$$ *A.* 1879.

CHAPTER VI.

Taking departure—Methods used—By estimation—By taking two bearings of an object and the run in the interval—By cross bearings—By the dip of the land—By the rate at which sound travels—Exercises—Ship's journal—Log-board and log-book—Dead reckoning—Heaving-to—Day's work—Exercises—Examination.

TAKING DEPARTURES.

WHEN a ship leaves a port, before the master shapes his course for his intended voyage, he must take the bearing and make some estimation of the distance he is from the land he is leaving, and enter this in his journal as the first course and distance he makes. This is called *taking his departure.* The bearing is taken by the compass and is entered into his journal as a compass bearing. There are several methods used for obtaining the distance of the land :—

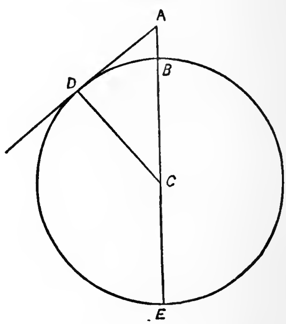
(*a*) *By estimation.*—From constant practice seamen can judge very correctly the distance of objects at sea. This is the method most generally adopted when leaving land.

(*b*) *By taking two bearings of an object and the run in the interval.*—This is accomplished by first taking the bearing of the object, then steering a course for a distance measured by the log, which should be about the same as the distance the object is judged to be, and then taking a second bearing of the same object. More exact results will be obtained if the two bearings make angles between five and six points with the distance run. The bearings and distance run can then be laid down to any scale, and the measurements on the same scale of the other sides of the triangle formed give the distances of the object at the two observations. If laid down on a chart of the place, the scale of the chart must be used. It will also be seen this resolves itself into the solution of a triangle, with two angles and the side included given; and thence the other two sides can be calculated by ordinary methods from $\frac{a}{b} = \frac{\sin A}{\sin B}$.

(*c*) *By cross bearings.*—For this method a chart will facilitate the work and be a check on the calculations. The bearings of two prominent places, such as headlands or lighthouses, must be taken, and corrections applied to bring the bearings to the same name as the chart used, chiefly magnetic; but for *calculation* they must be brought to true bearings. The corrected bearings must then be laid down on the chart, and the point where the two bearings intersect is the position of the ship. The distances from both objects can be measured by the scale on the chart. In practice the best position of the ship is when it and the two selected places form as nearly as possible an equilateral triangle.

(*d*) *By the dip of the land.*—For this method a man ascends the rigging until he can just see the wash of the sea on the shore; or can just catch sight of the light in a lighthouse, or the top of any object whose elevation is known. Then the height of the man's eye above the water must be measured, and the distance of the horizon is then calculated from the following formula:—

FIG. 24.



$$\text{Distance (in nautical miles)} = \frac{26}{23}\sqrt{h \text{ in feet.}}$$

This formula is obtained thus:—

Let the circle represent a section of the earth through its centre C, A B the height of the man above the water; then the wash on the land is seen on the horizon D, where the tangent meets the circle.

Euclid III.
$$\begin{aligned} AD^2 &= AB.\,AE \\ &= AB\,(BE + AB) \\ &= AB.\,BE + AB^2 \end{aligned}$$

In practice AB is always so small a fraction of the earth's diameter that the term AB^2 may be omitted without introducing any appreciable error into the result. Hence we may take

$$AD^2 = AB.\,BE$$

Now BE is very approximately 6875·5 nautical miles; and if h be the number of feet in AB, and AD be required in nautical miles—

$$\begin{aligned} AD^2 &= \frac{h}{6080} \times 6875{\cdot}5 \\ &= \frac{13751}{12160}\,h \end{aligned}$$

$$\text{Hence, } AD = \frac{26}{23}\sqrt{h} \text{ miles very nearly.}$$

When a known light is sighted in the horizon, its height is easily found in books of sailing directions or lists of lighthouses. Its distance from the horizon must be calculated by the foregoing formula as well as the distance of the ship from the horizon by the same formula, then the two results added together will give the required distance.

We must here caution the seaman against implicitly trusting this method; because the heave of the sea, and the varying amount of refraction caused by differences of temperature and the amount of moisture in the atmosphere, render the distance thus ascertained uncertain.

(*e*) *By the rate at which sound travels.*—If the flash of a gun be seen and its report heard after, a fair approximation to the distance may be made. It is known that at the freezing temperature, *i.e.* 32° F., sound travels at the rate of 1,090 feet per second; and for an increase of every degree in temperature an increase in the rate of sound amounting to $1\frac{1}{9}$ feet per second must be made.

Exercise XII.

The following questions have been solved on the Admiralty chart from Dodman Point to Start Point: the student should prove his answers by plane trigonometry.

Ex. 237. Running down Channel, with my compass course W. by N$\frac{1}{4}$N., I took the bearing of Prawle Point, in latitude 50° 12′ N., longitude 3° 43′·4 W., with my standard compass, and found it to be N$\frac{3}{4}$E. After running on my course 8 miles by patent log it bore E. by N. by the same compass. If the variation be 20° 20′ W., and deviation 1$\frac{3}{4}$ points W., what was my departure at the second observation?

Ex. 238. Being bound to Plymouth Harbour or an ENE. compass course, I sighted the Dodman in latitude 50° 13′·1 N, longitude 4° 48′ W., its bearing being by standard compass N. 4° 23′ E. I then ran by my patent log 9$\frac{1}{2}$ miles and the Dodman bore WNW. by the same compass. If the variation be 20° 15′ W., and deviation 1$\frac{1}{2}$ points E., what was my distance at the second sight?

Ex. 239. On passing the Eddystone, steering by compass SSW$\frac{1}{4}$W., it bore W$\frac{1}{4}$S.; after continuing on the same course 8$\frac{1}{4}$ miles by patent log, it then bore NNW$\frac{3}{4}$W. If the variation of the compass be 20° 20 W., and its deviation for the position of the ship's head be $\frac{3}{4}$ point W., what was my departure at the second observation?

Ex. 240. Running down Channel, Start Point (latitude 50° 13′·3 N., longitude 3° 38′·5 W.) bore by compass N. 51° 50′ E., and at the same time Prawle Point (latitude 50° 12′ N., longitude 3° 43′·4 W.) bore N. 2° 50′ E. by the same compass. Find the departure from Start Point, variation 20° 20′ W., deviation 13° 50′ W.

Ex. 241. Wishing to take my departure from the Eddystone (latitude 50° 11′ N., longitude 4° 16′ W.) I took its bearing by the standard compass and found it to be N. by W$\frac{1}{2}$W., and at the same time and with the same compass Bolt Tail (latitude 50° 14′·1 N., longitude 3° 52′ W.) bore E. by N$\frac{1}{4}$N. What was my distance from the Eddystone, variation 20° 20′ W., deviation 19° 41′ W.?

Ex. 242. The lighthouse at Sumburgh Head is in latitude 59° 51′ N., longitude 1° 16′ W., and is 300 feet high. What is my departure from it, if I just see it in the horizon when my eye is 12 feet 3 inches above the water line?

Ex. 243. Wishing to take my departure from the Caskets lighthouse (latitude 49° 43′·4 N., longitude 2° 22′·5 W.) which is 113 feet high, I sent a man up the rigging until he was 40 feet above the water line when he just caught sight of the light. What was my distance?

Ex. 244. The Peak of Teneriffe is $2\frac{1}{2}$ miles high, and the angular depression of the horizon from its summit is $2° 1' 47''$. Find the earth's diameter.

Ex. 245. Between seeing the flash of a gun and hearing the report there elapsed 35 seconds, and the temperature of the air was 57° F. What was the distance?

Ex. 246. What would be the approximate distance between two ships, if the time between seeing the flash and hearing the report from one to the other be $43\frac{1}{2}$ seconds and $46\frac{3}{4}$ seconds respectively, the temperature being 63° F.

Ex. 247. In sailing from the land, the light of a lighthouse is seen in the horizon from the deck 16 feet above the level of the sea; and after 40 minutes have elapsed, a man 81 feet up the rigging sees the same light in the horizon. What is the rate of sailing, and the distance from the light at the second observation? Diameter of the earth $6875\frac{1}{2}$ nautical miles.

Ex. 248. A person standing on the seashore can just see the top of a mountain, the height of which is known to be 1284·8 yards. After ascending vertically to the height of 3 miles in a balloon, he observes the angle of depression of the mountain's summit to be $2° 15'$. Find the earth's radius and the distance of the mountain from the first place of observation.

A SHIP'S JOURNAL.

DAY'S WORK.—Having investigated a method for determining the longitude in, we are now in a position to understand how from day to day the master of a ship fixes his geographical position, and hence how the vessel must be navigated to reach her intended port. The calculations are all brought up to the same time every day—viz., at noon; and because they extend over the twenty-four hours the form used is called *a Day's Work.*

THE LOG-BOARD AND LOG-BOOK.—The log-board is one ruled horizontally for 24 hours, and also into vertical columns, which have the headings, *Hours, Courses, Knots, Tenths, Winds, Leeway, Deviation, Remarks.* It is kept in the steerage or other convenient place for the officer of the watch to enter what courses have been steered, the rate of sailing, &c., opposite to the proper hours, and also an estimation of the set and drift of the current for the whole day, similar to the forms in the following pages. Every day at noon this is copied into the *log-book,* which is ruled like the log-board; and all other matters of importance connected with the working of the ship, such as peculiarities in the set or rate of tides and currents, storms, effect of the swell of the sea, neglect of duty or punishment of the men, damage the ship may have sustained, &c., are also inserted. The position of the vessel is calculated up to that time, and entered, together with her position by observation when obtained, and the course and distance made good during the past twenty-four hours. At the end of the voyage, if properly kept, the journal shows a pretty complete record of all that has occurred on board during the time.

Dead Reckoning *is the method of fixing the position of the ship by means of a day's work.* It depends entirely on terrestrial measurements, and it is extremely difficult to arrive at any certain conclusions from this method, hence celestial observations for fixing the position should be resorted to whenever available. Carelessness or difficulty in steering, errors of the log, sudden squalls, incorrect allowance for rate, leeway, deviation, and currents, are some of the causes which tend to render the ship's position by dead-reckoning only an approximation at the best; but, in the absence of celestial observations, it is the only method available to the mariner, and hence its importance. When the ship's place is fixed by observation, that by dead-reckoning should be discarded.

Heaving-to.—When it is blowing hard with contrary winds and a high sea, no advantage can be gained by sailing: all that can then be done is to use all precaution to avoid damage being done to the ship, and to prevent as far as possible the ship being driven back. This is effected by *lying to* under no more sail than is necessary, to prevent the violent rolling which the ship would otherwise acquire. The tiller is put over to leeward: this brings the vessel's head to the wind, which then has but little effect on her sails, and so she loses way through the water. The rudder then ceases to be acted on, and so its directive power is lost, her head falls off from the wind, the sail which is set fills again, and this gives her fresh way through the water. The rudder then becomes again acted on, the vessel's head is brought again to the wind, and she undergoes a similar series of movements, coming up to the wind and falling off alternately. To make the correction necessary in working a day's work when a ship is *lying-to*, take the middle point between that on which she comes up to the wind, and that on which she falls off for her apparent course, and correct this as you would the other courses, and enter the estimated drift in the time as the distance made good on that course.

THE DEPARTURE COURSE.—On leaving land the first entry in the log-book is the departure course: that is, as has been already explained, the bearing and distance of the land left. It is evident that as the *bearing* of the land is entered, to reduce it to a course—that is, the same as the vessel would have sailed *from* the land—its bearing must be reversed. Hence the rule, *Always reverse the bearing of the land for the departure course.*

WORKING A DAY'S WORK.—(1) Form a table as directed in resolving a traverse.

(2) Correct the departure course by reversing the bearing of the land, then by applying the deviation for the position of the ship's head and the variation. Place the result in the table formed under the head 'Courses,' and opposite it, in the column marked 'Distance,' place the estimated distance of the land.

(3) Correct each of the courses for leeway, deviation, and variation, and enter the results in the table.

(4) Add up the distances for the hours on each course, and place the sums in the distance column, each opposite its proper course.

(5) If there has been a current for the past twenty-four hours, correct its magnetic set for variation, enter the result in the table as a course, with its drift in the distance column.

(6) Work the traverse, get the difference of latitude and departure; thence find the course and distance by inspection, as directed in traverse sailing.

(7) To the latitude left apply the difference of latitude: this will give the latitude the ship is in.

(8) By middle-latitude sailing, get the longitude in, as directed under that heading; and then place down the course, the distance, the latitude in, and the longitude in. At sea it is customary to find the difference of longitude by inspection, thus:—Turn to the traverse table; take the middle latitude as course, then find the departure the ship has made in the difference of latitude column; opposite to this in the distance column will be found the difference of longitude. The reason for this is easily seen from the following considerations:—

From middle latitude sailing $\frac{\text{dep.}}{\text{d. long.}} = \cos \text{mid. lat.}$

„ plane sailing . . $\frac{\text{d. lat.}}{\text{dist.}} = \cos \text{course.}$

Hence, if we make the middle latitude equal to the course, then

$$\frac{\text{dep.}}{\text{d. long.}} = \frac{\text{d. lat.}}{\text{dist.}}$$

In which equation it is evident, if the departure be taken as difference of latitude, the difference of longitude must be equal to the distance.

As the longitude by inspection depends on the middle latitude, the cases in which it should not be used, as explained under middle-latitude sailing, should be attended to; and if the latitude be high, or the distance made good be great on a small course, then correct longitude can only be obtained by finding the position of the ship by Mercator's sailing on every change of course. The reason for this is not far to seek, for in high latitudes the sum of the differences of longitudes made on every course is not equal to that made in the mean of the latitude left and latitude arrived at during the twenty-four hours as is assumed in the day's work: and hence the discrepancies in the answers obtained. The student should compare the results in some typical case selected from the following exercises.

Ex. 249. September 15, 1881. Cape Race (latitude 46° 40′ N., longitude 53° 7′ W.) bore by compass W. by N. 11 miles distant (ship's head NE., deviation 17° E.); the ship then sailed the following courses and distances. Find the course and distance made good, and the latitude and longitude in, on noon September 16.

Hours	Courses	Knots	Tenths	Winds	Lee-way	Devia-tion.	Remarks
1	ESE.	8	3	South	7°	13° E.	
2		8	0				
3		8	0				
4		7	9				
5	SE.	6	0	SSW.	8°	5° E.	Variation of the
6		6	4				compass 30° W.
7		6	3				
8		6	9				
9	SE. by S.	8	7	SW. by S.	6°	$2\frac{3}{4}$° E.	
10		8	4				
11		8	0				
12		8	7				Midnight.
1	S. by W.	6	0	SE. by E.	8°	$8\frac{1}{2}$° W.	
2		6	3				
3		6	5				
4		6	9				
5	NE. by E.	8	0	”	6°	18° E.	
6		9	0				A current set
7		7	0				the ship mag-
8		7	8				netic WSW., $2\frac{3}{4}$
9	SW.	8	7	”	—	$9\frac{1}{4}$ W.	miles per hour
10		8	9				for the last 8
11		8	8				hours.
12		8	4				

Here we notice we have the departure course, six courses during the day, and the current course to correct. This is done as follows :—

Departure course.

W. by N. reversed	78° 45′	*l* of S.
Deviation . . .	17 0	*r*
	61 45	*l* of S.
Variation . .	30 0	*l*
	91 45	*l* of S.
	180 0	
True course . .	88 15	*r* of N.

First course.

ESE. . .	67° 30′	*l* of S.
Deviation . .	13 0	*r*
	54 30	*l* of S.
Variation . .	30 0	*l*
	84 30	*l* of S.
Leeway . .	7 0	*l*
	91 30	*l* of S.
	180	
True course . .	88 30	*r* of N.

Second course.

SE.	45°	0′	*l* of S.
Deviation	5	0	*r*
	40	0	*l* of S.
Variation	30	0	*l*
	70	0	*l* of S.
Leeway	8	0	*l*
True course	78	0	*l* of S.

Third course.

SE. by S.	33°	45′	*l* of S.
Deviation	2	45	*r*
	31	0	*l* of S.
Variation	30	0	*l*
	61	0	*l* of S.
Leeway	6	0	*l*
True course	67	0	*l* of S.

Fourth course.

S. by W.	11°	15′	*r* of S.
Deviation	8	30	*l*
	2	45	*r* of S.
Variation	30	0	*l*
	27	15	*l* of S.
Leeway	8	0	*r*
True course	19	15	*l* of S.

Fifth course.

NE. by E.	56°	15′	*r* of N
Deviation	18	0	*r*
	74	15	*r* of N
Variation	30	0	*l*
	44	15	*r* of N
Leeway	6	0	*l*
True course	38	15	*r* of N

Sixth course.

SW.	45°	0′	*r* of S.
Leeway	0	0	
	45	0	*r* of S.
Deviation	9	15	*l*
	35	45	*r* of S.
Variation	30	0	*l*
True course	5	45	*r* of S.

Current course.

WSW.	67°	30′	*r* of S.
Variation	30	0	*l*
True course	37	30	*r* of S.

The corrected courses are then written down to the nearest degree in the first column of the table: and we must then get the distances sailed on each course. These are obtained by adding up the knots and tenths found opposite each hour the vessel has sailed on each course, thus :—

First course.	Second course.	Third course.	Fourth course.	Fifth course.	Sixth course.	Current course.
8 3	6 0	8 7	6 0	8 0	8 7	2 75
8 0	6 4	8 4	6 3	9 0	8 9	8
8 0	6 3	8 0	6 5	7 0	8 8	22 00
7 9	6 9	8 7	6 9	7 8	8 4	
32 2	25 6	33 8	25 7	31 8	34 8	

The results are tabulated as follows :—

Courses	Dist.	Difference of latitude		Departure	
		N.	S.	E.	W.
N. 88° E.	11′·0	·4	—	11·0	—
N. 89 E.	32·2	·6	—	32·2	—
S. 78 E.	25·6	—	5·3	25·0	—
S. 67 E.	33·8	—	13·2	31·1	—
S. 19 E.	25·7	—	24·3	8·4	—
N. 38 E.	31·8	25·0	—	19·6	—
S. 6 W.	34·8	—	34·6	—	3·7
S. 38 W.	22·0	—	17·3	—	13·5
		26·0	94·7	127·3	17·2
			26·0	17·2	
		Diff. lat.	68·7	110·1	Departure.

The traverse is then resolved as directed under traverse sailing, and the latitude in, longitude in, course, and distance made good, found as directed: thus—

Latitude from . .	46° 40′N.	Latitude from .	46° 40′ N.
Difference of latitude .	1 8·7 S.	Latitude in .	45 31·3 N.
Latitude in	45 31·3 N.		2)92 11·3
		Middle latitude	46 5·65 N.

We will now show how the longitude in is found by inspection. With the middle latitude to the nearest degree, *i.e.* 46°, we turn to the traverse table and find 46° as a course; then seek for the departure made in the question, viz., 110′·1, in the difference of latitude column; then in the distance column, opposite this departure, we find 158′ 5; this is the difference of longitude, and the longitude in is then found as follows :—

Longitude left	53° 7′ W.
Difference of longitude . . .	2 38·5 E.
Longitude in	50 28·5 W.

The difference of latitude and departure must then be sought for in the traverse table in their own respective columns; the course will then be found at either the top or bottom of the page, and the distance opposite to these numbers in the distance column: we thus find the course to be S. 58° E. and distance made good 130 miles.

Answer { Course S. 58° E; distance 130 miles.
Latitude in 45° 31·3′ N.; longitude in 50° 28′·5 W. }

Ex. 250. December 23, 1881, at noon, Sand Island (latitude 28° 25′ N., longitude 178° 28′ W.) bore by compass S½E. distant 23 miles, ship's head NW. by W¾W.; afterwards sailed as by the following log account. Find the latitude in and longitude in on December 24 at noon.

Hours	Courses	Knots	Tenths	Winds	Lee-way	Deviation	Remarks
					pts.	° ′	
1	NW. by W¾W.	5	6	SW¼W.	1¼	19 50 W.	
2		6	0				
3		6	0				
4		6	7				Variation of the compass 1¼ pts. E.
5	W½S.	6	0	S. by W.	1¼	17 0 W.	
6		8	5				
7		9	0				
8		8	7				
9	NW. by N.	8	0	W. by S.	1¾	8 30 W.	
10		9	0				
11		9	2				
12	Hove to.	1	0	Variable.	3	Allow.	Midnight.
1	Head up WNW.	1	0	Between		16 20 W.	
2	„ off SW.	1	0	North			
3		1	0	and			
4		1	0	WNW.			
5	SW½W.	7	0	NW. by W. ½W.	1¼	8 20 W.	
6		5	6				A current set the ship magnetic WNW. at the rate of 3¼ miles per hour for last 22 hours.
7		7	0				
8		6	2				
9	NW¼W.	5	0	WSW.	2	8 40 W.	
10		7	5				
11		7	0				
12		6	8				

The only thing in this day's work which calls for remark is the course from 11 p.m. to 4 a.m., when the ship was *hove-to.* In this course we take the point midway between those on which the ship came up and fell off, that is, between WNW. and SW. The middle point is W. by S.; we then consider this as a course, and correct it precisely the same as others. We now proceed to correct the courses.

Departure course.

$S\frac{1}{2}E$. reversed	$5^\circ\ 38'$	*l* of N.
Deviation	19 50	*l*
	25 28	*l* of N.
Variation	14 4	*r*
True course	11 24	*l* of N.

Course N. $11\frac{1}{2}^\circ$ W.; dist. 23 *m.*

First course.

NW. by $W\frac{3}{4}W$.	$64^\circ\ 41'$	*l* of N.
Leeway	14 4	*r*
	50 37	*l* of N.
Deviation	19 50	*l*
	70 27	*l* of N.
Variation	14 4	*r*
True course	56 23	*l* of N.

Course N. $56\frac{1}{2}^\circ$ W.; dist. 24·3 *m.*

Second course.

$W\frac{1}{2}S$.	$84^\circ\ 23'$	*r* of S.
Leeway	14 4	*r*
	98 27	*r* of S.
Deviation	17 0	*l*
	81 27	*r* of S.
Variation	14 4	*r*
True course	95 31	*r* of S.
	84 29	*l* of N.

Course N. $84\frac{1}{2}^\circ$ W.; dist. 32·2 *m.*

Third course.

NW. by N.	$33^\circ\ 45'$	*l* of N.
Leeway	19 41	*r*
	14 4	*l* of N.
Deviation	8 30	*l*
	22 34	*l* of N.
Variation	14 4	*r*
True course	8 30	*l* of N.

Course N. $8\frac{1}{2}^\circ$ W.; dist. 26·2 *m.*

Fourth course.

Here take the point midway between WNW. and SW.—*i.e.* W. by S.

W. by S.	$78^\circ\ 45'$	*r* of S.
Leeway	33 45	*l*
	45 0	*r* of S.
Deviation	16 20	*l*
	28 40	*r* of S.
Variation	14 4	*r*
True course	42 44	*r* of S

Course S. $42\frac{1}{2}^\circ$ W.; dist. 5 *m.*

Fifth course.

$SW\frac{1}{2}W$.	$50^\circ\ 38'$	*r* of S.
Leeway	14 4	*l*
	36 34	*r* of S.
Deviation	8 20	*l*
	28 14	*r* of S.
Variation	14 4	*r*
True course	42 18	*r* of S.

Course S. $42\frac{1}{2}^\circ$ W.; dist. 25·8 *m.*

Sixth course.

NW. ¼ W. . .	47° 49′ *l* of N.	
Leeway . .	22 30 *r*	
	25 19 *l* of N.	
Deviation . .	8 40 *l*	
	33 59 *l* of N.	
Variation . .	14 4 *r*	
True course . .	19 55 *l* of N.	

Course N. 20° W.; dist. 26·3 *m.*

Current course.

WNW. . .	67° 30′ *l* of N.
Variation . .	14 4 *r*
True course . .	53 26 *l* of N

Course N. 53½° W.; dist. 71·5 *m.*

Courses	Dist.	Difference of latitude		Departure	
		N.	S.	E.	W.
N. 11½° W.	23·0′	22·6	—	—	4·6
N. 56½ W.	24·3	13·4	—	—	20·3
N. 84½ W.	32·2	3·1	—	—	32·1
N. 8½ W.	26·2	25·9	—	—	3·9
S. 42½ W.	5·0	—	3·7	—	3·3
S. 42½ W.	25·8	—	19 0	—	17·4
N. 20 W.	26·3	24·7	—	—	9·0
N. 53½ W.	71·5	42·5	—	—	57·£
		132·2	22·7		148·1
		22·7			
		109·5			

Latitude left . .	28° 25′ N.
Difference of latitude .	1 49·5 N.
Latitude in .	30 14·5 N.

Latitude left .	28° 25′ N.
Latitude in .	30 14·5 N.
	2)58 39·5
Middle latitude .	29 19·8 N.

Longitude left	178° 28′ W.
Difference of longitude by inspection .	2 50 W.
	181 18 W.
	360
Longitude in	178 42 E.

Answer by inspection { Course N. 53½° W.; distance 184 miles.
Lat. in 30° 14′·6 N.; long. 178° 42′ E.

EXERCISE XIII.

Ex. 251. March 25, 1875, at noon, a point of land, in latitude 19° 55′ N., longitude 179° 57′ W., bore by compass NW$\frac{1}{4}$N. distant 14 miles, ship's head being by compass NE$\frac{1}{2}$E.; afterwards sailed as by the following log account. Find the latitude and longitude in on the noon of March 26, together with the course and distance made good.

Hours	Courses	Knots	Tenths	Winds	Lee-way	Deviation	Remarks, &c.
					pts.	° ′	
1	NE$\frac{1}{2}$E.	5	4	ESE.	1$\frac{1}{4}$	24 20 E.	
2		5	9				
3		4	7				
4		4	9				
5		6	1				
6	S$\frac{1}{4}$E.	6	3	WSW.	2$\frac{3}{4}$	2 50 E.	
7		6	0				Variation of the compass 2$\frac{3}{4}$ pts. E.
8		5	1				
9	West.	5	0	SSW.	1$\frac{1}{4}$	14 0 W.	
10		7	2				
11		8	7				
12		7	3				Midnight.
1	NE.	5	0	ESE.	$\frac{1}{2}$	19 40 E.	
2		5	7				
3		5	9				
4		6	4				
5	S. by E.	6	0	SW. by W.	1$\frac{1}{4}$	3 0 E.	
6		5	1				
7		3	9				
8	N. by W$\frac{1}{4}$W	4	7	W. by N.	$\frac{1}{2}$	5 40 W.	A current set the ship magnetic west 2$\frac{3}{4}$ knots per hour for all the day.
9		5	2				
10		5	9				
11		6	8				
12		8	4				

Ex. 252. January 15, 1882, at noon, a point of land (latitude 0° 14′ N., longitude 179° 24′ E.) bore by compass WNW. distant 15 miles, ship's head by compass S. by W.; afterwards sailed as by the following log account. Find the direct course and distance sailed, and the latitude and longitude in, on January 16, 1882, at noon.

Hours	Courses	Knots	Tenths	Winds	Lee-way	Deviation	Remarks
					pts.		
1	S. by W.	9	6	SE. by E.	$1\frac{1}{4}$	$\frac{1}{4}$ pt. W.	
2		9	5				
3	South	9	7	ESE.	$\frac{3}{4}$	None	
4		9	4				
5		9	3				
6	SE. by E.	8	9	NE. by E.	$1\frac{1}{2}$	$\frac{1}{4}$ pt. E.	
7		9	6				
8		9	4				Variation of the compass 2 points W.
9		9	4				
10		8	7				
11		8	9				
12		9	4				Midnight.
1		8	6				
2	E. by N$\frac{1}{4}$N.	8	3	N$\frac{3}{4}$E.	$\frac{1}{2}$	$1\frac{1}{4}$ pts. E.	
3		9	7				
4		8	4				
5		8	2				
6		8	6				
7	SW$\frac{1}{2}$W.	8	7	WNW.	$\frac{3}{4}$	$\frac{1}{2}$ pt. W.	
8		8	4				A current set the ship magnetic ESE $2\frac{3}{4}$ knots per hour for all the day.
9		8	2				
10	SE$\frac{3}{4}$E.	9	6	NNE.	0	$\frac{3}{4}$ pt. E.	
11		9	2				
12		9	4				

Ex. 253. January 20, 1875, at noon, a point of land (latitude 51° 24′ S., longitude 59° 56′ W.) bore by compass NW. distant 10 miles, deviation (ship's head being SE.) 2° 10′ W.; afterwards sailed by the following log account. Find her latitude and longitude in on January 21, at noon, and the course and distance made good.

Hours	Knots	Tenths	Courses	Winds	Lee-way	Remarks
					pts.	
1	5	4	E. by S.	S. by W.	$1\frac{1}{4}$	Deviation 3° E.
2	4	5				
3	3	3				
4	4	5				
5	2	8				
6	5	4				
7	4	2	NW. by N.	SSW.	$\frac{3}{4}$	Deviation 4° 10′ W.
8	5	4				
9	6	3				
10	7	2				
11	6	4				
12	5	5				Variation $1\frac{3}{4}$ points E.
1	4	4	ENE.	SW.	0	Deviation 2° 40′ E.
2	5	1				
3	6	2				
4	4	6				
5	3	7				
6	2	8				
7	5	1				
8	6	2				
9	7	4	SSE.	WSW.	$1\frac{1}{2}$	Deviation 6° W. A current set the ship the last 4 hours N. by E. 4 knots an hour.
10	8	2				
11	7	8				
12	6	5				

A. 1875.

Ex. 254. February 29, 1876, at noon, a point of land (latitude 62° 25′ N., longitude 5° 36′ E.) bore by compass SE¼S., distant 27 miles, ship's head being W. by N¾N.; afterwards sailed as by the following log account. Find the course and distance made good, and the latitude and longitude in, at noon, March 1.

Hours	Courses	Knots	Tenths	Winds	Lee-way	Deviation	Remarks
					pts.	° ′	
1	W. by N¾N.	11	5	SW¼S.	2¼	28 10 W.	
2		11	7				
3		11	9				
4		11	9				
5		11	9				Variation of the compass 23° 40′ W.
6		11	9				
7	North.	10	7	ENE.	2½	16 50 E.	
8		10	2				
9	NNW¼W.	10	9	NE¼N.	1¾	4 20 W.	
10		11	4				
11		11	7				
12	NW¾N.	11	5	NNE¾E.	2¼	11 10 W.	Midnight.
1		11	7				
2		11	9				
3		11	7				
4	NW. by W¾W.	11	9	SW¼W.	2¼	26 30 W.	
5		11	4				
6		11	7				
7		11	2				
8		10	9				A current set N¾E. magnetic 3¼ knots per hour for the last 21 hours.
9	NW¼W.	11	3	NNE.	1½	22 10 W.	
10		10	9				
11		10	7				
12		11	3				

Ex. 255. February 14, 1876, at noon, a point of land (latitude 58° 40′ N., longitude 5° W.) bore by compass SE. distant 15 miles, ship's head WNW., deviation 5° 30′ W.; afterwards sailed as by the following log account. Find the latitude and longitude in on February 15, at noon, and the direct course and distance made good.

Hours	Knots	Tenths	Courses	Winds	Lee-way	Remarks
					pts.	
1	5	2	W. by N.	North	1½	Deviation 8° W.
2	4	5				
3	6	3				
4	5	8				
5	6	6				
6	3	9				
7	2	7	NNE.	NW.	2	Deviation 4° 20′ E.
8	4	0				
9	5	3				
10	5	8				
11	7	2				
12	6	7				Variation 2½ points W.
1	5	6	W.	N. by W.	1	Deviation 7° 20′ W.
2	6	5				
3	7	4				
4	7	3				
5	4	5				
6	4	0	SE.	W.	0	Deviation 9° 10′ E.
7	5	8				
8	4	7				
9	5	0				A current set the ship the last 4 hours 2 knots an hour S. by compass.
10	6	2				
11	7	0				
12	6	0				

A. 1876.

Ex. 256. June 20, 1877, at noon, Cape Leeuwin (latitude 34° 19′ S., longitude 115° 6′ E.) bore by compass E. by N. distant 9 miles (deviation for ship's head 5° 30′ E.); afterwards sailed as by the following log account. Find the course and distance made good, and the latitude and longitude in, on June 21, 1877, at noon.

Hours	Knots	Tenths	Courses	Winds	Lee-way	Remarks
					pts.	
1	5	6	WNW.	N.	½	Deviation 5° 20′ W.
2	6	9				
3	8	0				
4	6	4				
5	4	8				
6	4	4	S½E.	E. by S.	¾	Deviation 1° 10′ W.
7	3	5				
8	5	8				
9	6	9				
10	8	0				
11	8	5				Deviation 7° 0′ E.
12	10	0	NE.	NW. by N.	½	Variation ½ point W.
1	8	5				
2	7	0				
3	8	0				
4	9	5				
5	10	0	NW.	S.	0	Deviation 1° 50′ W.
6	12	0				
7	11	5				
8	8	0				
9	7	5				
10	6	8	E.	S. by E.	½	Deviation 8° E.
11	6	0				A current set the ship the last 3 hours 2 knots an hour SW. by compass.
12	10	0				

A. 1877.

Ex. 257. May 3, 1878, at noon, the flag-staff at Callao (latitude 12° 4′ S., longitude 77° 11′ W.) bore by compass NNE. distant 5 miles, deviation for ship's head 4° 10′ W.; afterwards sailed by the following log account. Find the latitude and longitude in, and the course and distance made good, to noon on May 4.

Hours	Knots	Tenths	Courses	Winds	Lee-way	Remarks
					pts.	
1	4	3	SW.	SSE.	1	Deviation 2° 20′ W.
2	3	8				
3	5	0				
4	4	5				
5	6	0	WNW.	E.	0	Deviation 3° 10′ E.
6	7	5				
7	8	2				
8	9	0				
9	5	5	SW. by S.	ESE.	½	Deviation 4° 30′ W.
10	6	0				
11	6	3				
12	4	5				Variation 1 point E.
1	5	4				
2	5	0				
3	6	0				
4	7	0				
5	6	5	SSE.	SW.	¾	Deviation 1° 30′ W.
6	3	9				
7	5	4				
8	6	3	NE. by N.	SE. by E.	¼	Deviation 5° E.
9	7	4				
10	8	0				
11	9	0				A current set the ship the last three hours 2½ knots an hour NNW.
12	7	5				

A. 1878.

Ex. 258. October 28, 1878, at noon, a point of land (latitude 56° 26 N., longitude 2° 23′ W.) bore by compass W. by N½N. distant 9 miles, ship's head being SE½E.; afterwards sailed as by the following log account. Find the course and distance made good, and the latitude and longitude in, to noon on October 29.

Hours	Courses	Knots	Tenths	Winds	Lee-way	Deviation	Remarks
					pts.		
1	SE½E.	7	9	NE. by E½E.	¼	17° 10′ E.	
2		8	3				Variation of the compass 1¼ pts. W.
3		9	7				
4		10	2				
5	Hove to, head up NE. by N.	1	4	Variable between N. by W. and NE. by N.	2½	Allow 25° 0′ E.	
6		1	5				
7	Head off E. by S.	1	3				
8		1	5				
9	North	9	4	ENE.	½	0 0	
10		8	7				
11		7	6				
12		6	4				Midnight.
1	E. by N¼N.	4	4	N¾E.	¼	25° 20′ E.	
2		3	5				
3		5	4				
4		6	7				
5	E. by S.	8	9	NE. by N.	½	23° 10′ E.	
6		10	2				
7		10	9				
8		9	2				A current set the ship magnetic NE. by E. 17 miles from the time the departure was taken to the end of the day.
9		10	7				
10		11	6				
11		10	4				
12		9	0				

Ex. 259. July 1, 1879, at noon, Cape Horn (latitude 55° 59′ S., longitude 67° 12′ W.) bore by compass W. distant 21 miles, ship's head being E$\frac{3}{4}$S.; afterwards sailed as by the following log account. Find the latitude and longitude in, and the course and distance made good, to July 2, at noon.

Hours	Knots	Tenths	Courses	Winds	Lee-way	Remarks
					pts.	
1	7	8	E$\frac{3}{4}$S.	S. by E$\frac{1}{4}$E.	1$\frac{1}{4}$	Deviation 26° 10′ W.
2	7	7				
3	7	9				
4	6	8	SE$\frac{1}{4}$E.	SSW.	1$\frac{1}{4}$	Deviation 24° 20′ W.
5	7	9				Variation of the compass 23° 20′ E.
6	5	4				
7	9	8	NE. by E$\frac{3}{4}$E.	N$\frac{1}{4}$W.	2$\frac{1}{4}$	Deviation 23° 0′ W.
8	9	4				
9	8	6				
10	8	7	West	SSW.	1$\frac{1}{2}$	Deviation 27° 30′ E.
11	8	6				
12	8	7				
1	9	2				
2	9	8	E. by N$\frac{3}{4}$N.	N$\frac{1}{4}$E.	1$\frac{1}{2}$	Deviation 25° 10′ W.
3	9	7				
4	9	7				
5	9	5				
6	9	7				
7	9	8				
8	9	7				
9	1	3	Hove to	Variable		
10	1	5	Head up SSW.	between	3$\frac{1}{4}$	Allow deviation 3° 20′ E.
11	1	4	Head off SE.	West and SSW		A current set the ship E$\frac{1}{4}$N. magnetic 2$\frac{3}{4}$ knots per
12	1	0				hour for all the day.

Ex. 260. February 23, 1880, at noon, Cape Wilberforce (latitude 23° 30′ S., longitude 35° 50′ E.) bore by compass W. distant 11 miles, ship's head being SW. by S.; afterwards sailed as by the following log account. Find the direct course and distance sailed, and the latitude and longitude in, on February 24, at noon.

Hours	Knots	Tenths	Courses	Winds	Leeway	Remarks
					pts.	
1	5	0	SW. by S.	NNW.	1	Deviation 6° 10′ W.
2	5	8				
3	6	2				
4	7	0				
5	5	0				
6	3	8				
7	5	0				
8	6	0				
9	7	0	E. by S.	North	2	Deviation 7° 10′ E.
10	7	5				
11	8	0				
12	6	8				Variation 2½ points E.
1	8	0	SE.	N. by E.	½	Deviation 2° E.
2	8	5				
3	6	0				
4	4	0	WSW.	East	0	Deviation 8° W.
5	3	5				
6	4	5				
7	5	3				
8	6	0				A current set the ship the last four hours 2 knots an hour ENE. by compass.
9	7	0				
10	5	9				
11	3	5				
12	2	5				

A. 1880.

Ex. 261. August 24, 1881, at noon, North Cape (latitude 71° 10′ N., longitude 26° 1′ E.) bore by compass E. by S¼S., with ship's head N., distant 19 miles; afterwards sailed by following log account. Find the direct course and distance, and the latitude and longitude in, on August 25, at noon.

Hours	Courses	Knots	Tenths	Winds	Leeway	Deviation	Remarks
1	North	9	3	WNW.	2° 20′	2° 10′ W.	
2		9	8				
3		8	2				
4	W¾S.	9	7	S. by W.	2 19	22 10 W.	
5		9	4				
6		9	8				
7		9	9				Variation of the compass 12° 10′ W.
8		10	0				
9		9	7				
10		9	5				
11	N. by W¾W.	8	7	W¼N.	2 18	10 50 W.	
12		7	4				Midnight.
1		7	2				
2	West	10	9	NNW.	2 20	21 50 W.	
3		10	9				
4		10	7				
5		10	7				A current set the ship NE by E¼E. magnetic 3¼ knots per hour for all the day.
6	ESE.	8	7	NE.	1 50	23 20 E.	
7		9	2				
8		9	1				
9		8	3				
10	SW. by W¾W.	8	8	S¼E.	1 40	19 20 W.	
11		9	9				
12		9	5				

Ex. 262. June 17, 1881, at noon, Funchal (latitude 32° 37′ N., longitude 16° 58′ W.) bore by compass NE½E. distant 15 miles, the ship's head being SW. by W.; afterwards sailed as by the following log account. Required the latitude and longitude in on June 18 at noon, and the course and distance made good.

Hours	Knots	Tenths	Courses	Winds	Lee-way	Remarks
					pts.	
1	5	6	SW. by W.	NW.	2	Deviation 6° W.
2	4	7				
3	5	3				
4	4	9				
5	5	2				
6	5	0				
7	4	8				
8	4	6				
9	8	0	ESE.	W.	0	Deviation 6° 20′ E.
10	8	5				
11	9	0				Variation of the compass 2 points W.
12	8	7				
1	6	0	S. by E.	SW.	1½	Deviation 3° 10′ W.
2	5	8				
3	5	9				
4	6	3				
5	6	1				
6	7	5	WSW.	NNW.	1	Deviation 7° 40′ W.
7	8	0				
8	7	4				A current set the ship the last 3 hours 2½ knots an hour E. by N. by compass.
9	6	9				
10	7	5				
11	7	3				
12	8	2				

A. 1881.

Ex. 263. In working a day's work how is the set and rate of the current obtained? What kind of course is the set of the current considered, and why?

Ex. 264. How is the distance of a ship from an elevated point or light-house computed? and how by sound?

Ex. 265. What do you mean by a day's work? Describe how a ship's place is determined from day to day.

Ex. 266. What do you mean by 'heaving-to,' and how is this allowed for in working a day's work?

Ex. 267. How is 'longitude in' found in a day's work by inspection? Prove the rule necessary to be used.

Ex. 268. What do you mean by 'taking a departure'? Give the various methods, both without and with the aid of a chart.

A. 1874.

Ex. 269. Being uncertain of my position by dead reckoning, on running up Channel I sight a known headland; show how I can determine my position by sailing past it. By what observations of this point could I obtain my distance from it without calculation. *Hon.* 1874.

Ex. 270. Supposing a ship, in sailing from A to B, meets with changes of wind which cause her frequently to alter her course, show how her actual position is ascertained at each noon, and explain the construction of the table used in the calculation. *E.* 1875.

Ex. 271. On leaving an anchorage, the top of a cliff, which bears from the ship due N. and is 200 feet above the surface of the sea, has an elevation of 18° 30′. After sailing due S. for a quarter of an hour, the elevation of the cliff is observed on board to be 1° 11′. What distance has the ship passed over?

By log the rate of sailing was computed to be 7 knots, but the glass was discovered to run out in 33 seconds instead of 30 seconds, and the knots on the line were known to be in error. What was the length of the knots on the line used? *A.* 1876.

Ex. 272. What do you mean by 'taking a departure'? Mention the two elements to be determined, and the different methods of doing this.

A. 1880.

Ex. 273. Show how to find the geographical position of a ship: (1) by taking the bearings of the same point of land; (2) by taking the simultaneous bearings of two known points of land.

CHAPTER VII.

MERCATOR'S SAILING—MERIDIONAL PARTS—MERCATOR'S CHARTS.

In the preceding chapter we have proved formulæ for determining the longitude of the ship, and have given reasons why the rules deduced from them are not universally applicable. We now come to a more rigorous method, to one that can be applied in all cases, though it gives neither the shortest nor best route.

From the earliest times charts or ground plans of the parts traversed have been in use. They seem to have been constructed after the simplest fashion—that is, with the meridians and parallels represented by straight lines perpendicular to each other, of equal length, and so enclosing square spaces. Now as distances sailed before the fifteenth century were short and confined chiefly to the coast, this method served the purpose then needed; but with the maritime spirit and the rage for conquest and discovery which sprang up in the latter part of the fifteenth and the sixteenth centuries, the demand for more exact charts came into being. For a long time it had been suggested that future charts would be more correct if the distances between the parallels were lengthened as they receded from the equator; accordingly, Gerard Mercator, a Fleming, published the first of the kind in 1569. The spaces representing the degrees of latitude were gradually lengthened from the equator to the poles; but they were not increased in the proper proportion; neither did Mercator leave any explanation for their construction, hence we have a right to suppose he used only mechanical methods, and that his charts were empirical ones. A writer in 'Naval Science' says, 'It has been conjectured, with probable truth, that it was by careful comparisons of the artificial globe with the plane chart that he obtained his results, constructing his chart by transferring rhumb lines from the surface of the globe to his chart and making them straight lines.' Great good, however, resulted from the introduction of Mercator's charts, for their appearance stimulated thought towards the discovery of the scientific principles on which they should be constructed. The honour of discovering, in 1590, and giving to the world the method now adopted, must be awarded to our own countryman, Edward Wright, of Caius College, Cambridge, and published by him, in 1599, in a work called 'Errors of Navigation Detected and Corrected.' Mr. Wright was lecturer in Navigation to the East India Company, just recently incorporated, and appears to have directed his mathematical skill to the perfection of methods then in use among navigators, as we find he also published 'A Treatise on the Sphere,' 'Tables of Meridional Parts,' 'The Haven-finding Art,' and a 'Treatise on Dialling.'

The principle of the construction by him is derived from the formula in Parallel Sailing

$$\text{L} = \text{M. sec } l.$$

From it we see that if, instead of making all the meridians converge to the poles, we draw them parallel and at right angles to the equator, then we lengthen the meridional distances at any parallel to their difference of longitude between the meridians—that is, in the ratio of the secant of the latitude of that parallel. For example, from the formula

$$\text{M} = \text{L. cos } l$$

we find in latitude 60° that the meridians through two consecutive degrees are thirty nautical miles apart; hence, when we draw them parallel and equal to the distance subtended by a degree of longitude at the equator, that is, to sixty nautical miles, we have increased the meridian distance in 60° to double its proper length, that is, in the proportion of the secant of the latitude, because secant 60° = 2. To compensate for this, on Mercator's chart the space representing a degree of latitude in latitude 60° is also increased in the same proportion—that is, it is doubled; and the same principle is carried out throughout the whole chart: thus, in latitude 45° the line representing a degree of latitude is lengthened to $60 \times \sqrt{2}$, because the secant of $45° = \sqrt{2}$; in latitude 30° it is lengthened to $60 \times \frac{2}{\sqrt{3}}$, because secant $30° = \frac{2}{\sqrt{3}}$, and so on.

From the above considerations a reason is seen why persons who casually look at a chart of the world laid down on Mercator's principle are apt to receive very erroneous impressions of the relative size of countries situate near the equator and those in high latitudes. We have already shown that places in latitude 60° have both their lengths and breadths doubled, consequently their areas are quadrupled; and this is the reason why Scandinavia, extending over but 16° of latitude and possessing but 292,700 square miles of surface, appears on a Mercator's chart at least half as large again as Hindustan, with its 27° of latitude and its 1,200,000 square miles, and also why the coast line of the Arctic shores appears at least three times its real extent.

In applying the principle laid down, we see that on a chart

1′ of the meridian, in latitude				1′,	is represented by		$1' \times \sec 1'$
1′	,,	,,	,,	2′	,,	,,	$1' \times \sec 2'$
1′	,,	,,	,,	3′	,,	,,	$1' \times \sec 3'$
.				.			.
.				.			.
.				.			.
.				.			.
1′	,,	,,	,,	n'	,,	,,	$1' \times \sec n'$

and so on.

Hence, the line on a Mercator's chart which represents a portion of a meridian extending from the equator to n' from it is in length

$=(1' \times \sec 1')+(1' \times \sec 2')+(1' \times \sec 3')+ \ldots + (1' \times \sec n')$
$=(\sec 1' + \sec 2' + \sec 3' + \ldots + \sec n') \times$ length of $1'$ at the equator.

The sum of all these natural secants reckoned in miles is tabulated in nautical tables for every minute of latitude under the title of *Meridional Parts*, and from the construction of the table it is readily seen the meridional parts for any latitude represents the true latitude divided into minutes, each part multiplied by the secant of its own latitude, and then the sum taken of the products. Such was the method adopted by Edward Wright in calculating his table of meridional parts; and so closely does it approximate to the true meridional parts for a sphere, that the error nowhere exceeds half a minute, and no mistake would accrue in practice from using his tables as he left them. By taking one minute as the base of his operations, Wright's supposition was that a section of the earth along a meridian was a polygon, each of whose sides was one minute. A nearer approximation would have been made had he selected a second to make his calculations from; but even then small residual errors would remain, which can only be eliminated by using the more rigorous method supplied by the integral calculus. The formula for calculating the meridional parts by this method will be proved hereafter.

From the above considerations it is evident that the *difference* of latitude on the chart may be obtained by dividing the true difference of latitude into very small portions, and taking the sum of the whole after each has been multiplied by the secant of its own latitude. When the parts are made indefinitely small, and the true difference of latitude thus increased, it is called *meridional difference of latitude*, and in practice is found by taking the difference of the meridional parts for the two given latitudes.

Now because the departure is increased to difference of longitude, that is, in the proportion of the secant of the latitude, and as a compensation the elementary portions of the difference of latitude are increased in the same proportion and then called meridional difference of latitude, we have

$$\frac{\text{diff. long.}}{\text{mer. diff. lat.}} = \frac{\text{dep.}}{\text{true diff. lat.}}$$

$$= \text{tan course} \quad . \quad . \quad . \quad . \quad . \quad \text{I.}$$

$$\therefore \text{diff. long.} = \text{mer. diff. lat.} \times \text{tan course} \quad . \quad \text{II.}$$

Having found the course, the distance is obtained by plane sailing from

$$\text{distance} = \text{true diff. lat.} \times \text{sec course} \quad . \quad . \quad \text{III.}$$

These three formulæ are all that will be required in the solution of questions in Mercator's sailing.

The following questions are solved with meridional parts, computed to a compression of $\frac{1}{321}$ *(Riddle's Tables).*

Ex. 274. Find the course and distance from the Burlings (near Lisbon) to Porto Santo in the Madeiras.

Burlings	lat. 39° 25′ N.	mer. parts 2564	long. 9° 30′·7 W.
Porto Santo	lat. 33 5 N.	mer. parts 2094	long. 16 19·5 W.
	6 20	mer. diff. lat. 470	6 49
	60		60
True diff. lat.	380 S.		diff. long. 409 W.

To find the course.

$$\text{Tan course} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}}$$

Difference of longitude . .	409 miles	log	2·611723
Meridional difference of latitude	470 „	log	2·672098
Course	41° 1′ 49″	log tan	9·939625

To find the distance.

$$\text{Distance} = \text{true diff. lat.} \times \text{sec course.}$$

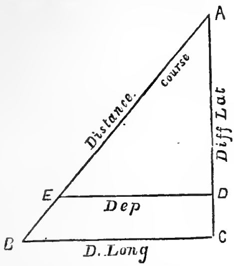
Course	41° 1′ 49″	log sec	10·122418
True difference of latitude	380 miles	log	2·579784
Distance	503·7 „	log	2·702202

∴ Course S. 41° 1′ 49″ W.; distance 503·7 miles.

Questions in Mercator's sailing can also be solved by construction. In this problem we have given the difference of latitude, the meridional difference of latitude, and the difference of longitude, to find the course and distance. Draw two straight lines of indefinite length at right angles to one another, as A C, B C, representing portions of the equator and of a meridian.

From any scale of equal parts take C B, equal to difference of longitude 409 miles, and C A, equal to meridional difference of latitude 470 miles, and join A B. Then the angle B A C is the course, and measured by the protractor is 41°. From A measure A D, equal to the true difference of latitude 380 miles; and from D draw D E parallel to the equator B C, and therefore at right angles to A C; then D E represents the departure, and A E the distance, which is found from the scale to be 504 miles.

Fig. 25.
Scale 400 miles to an inch.



The construction again leads us to the solution of the problem by inspection; thus, turn to the traverse table, and find the difference of longitude and meridional difference of latitude opposite to each other in the departure and difference of latitude columns respectively. If these distances be too great, the better plan will be to divide each by 10; in this we shall then have to look for 40·9 miles in the departure column and 47 miles in the difference of latitude column abreast of one another: we find the nearest to these numbers under the course 41°. Hence we conclude the course to be S. 41° W. The distance is found by looking in the same page for the true difference of latitude 380 miles, or its tenth part, 38 miles, and we find the distance in its proper column to be 50·4 miles. Now, as we divided the true difference of latitude by 10, we have obtained only one-tenth of the distance between the two places. Hence the distance is 504 miles. The results by construction and inspection are the same, or differ only very slightly from those by calculation.

We will now solve the same question by middle-latitude sailing.

Burlings lat.	39° 25′ N.		39° 25′ N.	long.	9° 30′·7 W.
Porto Santo lat.	33 5 N.		33 5 N.	long.	16 19·5 W.
	6 20		2)72 30		6 48·8
	60	Mid. lat.	36 15		60
True diff. lat.	380 S.			Diff. long.	408·8 W.

$$\text{Tan course} = \frac{\text{diff. long.} \times \cos \text{mid. lat.}}{\text{true diff. lat.}}$$

Difference of longitude .	409 miles	log	2·611723
Middle latitude . .	36° 15′	log cos	9·906575
			12·518298
True difference of latitude	380 miles	log	2·579784
Course	S. 40° 57′ 27″ W.	log tan	9·938514

Distance = true diff. lat. × sec course.

Course	40° 57′ 27″	log sec	10·121940
True difference of latitude	380 miles	log	2·579784
Distance	503·2 miles	log	2·701724

Course S. 40° 57′ 27″ W.; distance 503·2 miles.

In this example we see that the course agrees within about $5\frac{1}{2}'$, an amount quite inappreciable in practice, and the distance agrees within half a mile. But here the difference of latitude is not great, neither is either place in a high latitude, hence the near coincidence of results. To show the effects introduced by selecting an example under these conditions, we will work by both methods one involving them.

Ex. 275. Find the course and distance from Jan Mayen Island to point B.

First by Mercator's sailing.

Jan Mayen Island	70° 49′ N.	mer. parts 6092	long. 8° 41′ W.
Point B.	30 49·2 N.	mer. parts 1934	long. 79 52·7 W.
	39 59·8	mer. diff. lat. 4158	71 11·7
	60		60
True diff. lat.	2399·8 S.	Diff. long.	4271·7 W.

To find the course.

$$\text{Tan course} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}}$$

Difference of longitude .	4271·7 miles	log	3·630601
Meridional difference of latitude	4158 miles	log	3·618884
Course	45° 46′ 22″	log tan	10·011717

To find the distance.

Distance = true diff. lat. × sec course.

Course	45° 46′ 22″	sec log	10·156453
True difference of latitude	2399·8 miles	log	3·380175
Distance	3440·55 miles	log	3·536628

∴ Course S. 45° 46′ 22″ W.; distance 3440·55 miles.

By middle-latitude sailing.

Jan Mayen Island	lat. 70° 49′ N.	70° 49′ N.	long. 8° 41′ W.
Point B.	lat. 30 49·2 N.	30 49·2 N.	long. 79 52·7 W.
	39 59·8	2)101 38·2	71 11·7
	60	Mid. lat. 50 49·1	60
True diff. lat.	2399·8 S.		Diff. long. 4271·7 W.

$$\text{Tan course} = \frac{\text{diff. long.} \times \text{cos mid. lat.}}{\text{true diff. lat.}}$$

Middle latitude . .	50° 49′·1	log cos	9·800567
Difference of longitude .	4271·7 miles	log	3·630601
			13·431168
True difference of latitude	2399·8 miles	log	3·380175
Course	48° 21′ 22″	log tan	10·050993

Distance = true diff. lat. × sec course.

Course	48° 21′ 22″	log sec	10·177506
True difference of latitude	2399·8 miles	log	3·380175
Distance	3611·45 miles	log	3·557681

∴ Course S. 48° 21′ 22″ W; distance 3611·45 miles.

In this example we find both a great difference of latitude and one of the places (Jan Mayen Island) in a high latitude; hence we obtain large differences in results, amounting to 2° 35′, or nearly a quarter of a point in the course, and to a difference in the distance of 171 miles; neither of which in practice can be disregarded. The reason for this is, because the parallel in the middle latitude between the two places (as remarked before) does not exactly represent the departure; but a parallel nearer the pole should have been taken: then the middle latitude in the question would have been greater, its cosine therefore smaller, and hence the tangent of the course and the course itself have been less, thus approximating towards that obtained by Mercator's sailing. As the distance sailed varies as the secant of the course, therefore as the course becomes less so does the distance; hence the results obtained by middle-latitude sailing are always too great both in the course and in the distance. The proof that the correct departure between two places is on a parallel of latitude nearer the poles than the parallel of latitude half-way between the latitudes of the places can easily be deduced thus :—

Mercator's sailing $\dfrac{\text{diff. long.}}{\text{mer. diff. lat.}} = \dfrac{\text{dep.}}{\text{diff. lat.}}$

$$\therefore \quad \text{diff. long.} = \frac{\text{mer. diff. lat.}}{\text{diff. lat.}} \times \text{dep.} \qquad \text{I.}$$

Mid.-lat. sailing $\quad \text{diff. long.} = \text{dep} \times \text{sec mid. lat.} \qquad \text{II.}$

Equating I. and II. $\text{sec mid. lat.} = \dfrac{\text{mer. diff. lat.}}{\text{diff. lat.}}$

A table, called after its author 'Workman's Table,' has been formed from the above formula showing the amounts which must be added to the middle latitude in order to obtain the same results as by calculation on Mercator's principle; but as this involves a special table not found in all collections of nautical tables, it is omitted here, and the student is advised to use Mercator's sailing instead.

Now, applying this formula to the problem under discussion, we find the true middle latitude is found thus :—

Mer. diff. latitude .	4158	log	3·618884
True diff. latitude .	2399·8	log	3·380175
True mid. latitude .	54° 44′ 58″	log sec	10·238709
Assumed mid. latitude	50 49 6		
	3 55 52		

In this instance, the parallel, where the departure would be the same as that between the two places, should have been 3° 55′ 52″ nearer the pole. By using this true middle latitude in the formula for middle latitude sailing, we get

True middle latitude . .	54° 44′ 58″	log cos	9·761291
Difference of longitude .	4271·7 miles	log	3·630601
			13·391892
True difference of latitude	2399·8 miles	log	3·380175
Course . .	45° 46 22″	log tan	10·011717

thus agreeing, as it should do, with that obtained in Mercator's sailing.

The solution of this question by both construction and by inspection is precisely similar to the last.

Ex. 276. If a vessel sail SW½S. from Cape Clear (latitude 51° 26′ N., longitude 9° 29′ W.) 950 miles, required the latitude and longitude in.

Diff. lat. = dist. × cos course.

Distance . . .	950 miles	log	2·977724
Course . . .	3½ points	log cos	9·888185
Difference of latitude .	734′·4	log	2·865909

Latitude Cape Clear .	51° 26′ N.	meridional parts	3594
Difference of latitude .	12 14 S.		
Latitude in . .	39 12 N.	meridional parts	2547
		Mer. diff. lat.	1047

Diff. long. = mer. diff. lat. × tan course.

Meridional difference of latitude	1047	log	3·019947
Course	3½ points	log tan	9·914173
Difference of longitude . .	859·3	log	2·934120

Longitude Cape Clear	9° 29 W.
Difference of longitude	14 19 W.
Longitude in	23 48 W.

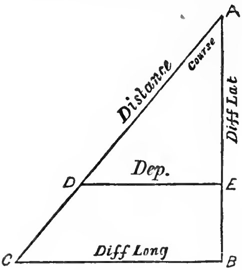
∴ Latitude in 39° 12′ N., longitude in 23° 48′ W.

Solution by construction.—Here we have given the latitude left, and the course and distance, to find the latitude in and longitude in. Draw a vertical line A B to represent a portion of a meridian; at the point A make the angle B A C equal to S. 3½ points W., and from A C cut off A D, equal to the distance 950 miles. From D draw D E perpendicular to A B, then D E = 603 miles represents the departure, and A E = 734 miles the true difference of latitude, which latter amount applied to the latitude left gives the latitude in. Having now the latitude left and latitude in, we find the meridian difference of latitude in the usual way to be 1,047 miles. Make A B equal to 1,047 miles, and from B draw B C perpendicular to A B to meet A C; then A C measured from the scale is 859 miles, and this applied to the longitude left gives the longitude in.

The solution by construction is again the key to the solution by inspection. Turn to the course $3\frac{1}{2}$ points in the traverse table, and opposite the distance 950 we find the departure 603 miles and difference of latitude 734 miles. This applied to the latitude left 51° 26′ N. gives the latitude in, from which we find as before the meridian difference of latitude 1,047 miles. Then under the same course seek for this meridional difference of latitude in the difference of latitude column, and the difference of longitude will be found in the departure column to be 859 miles, and this, as before, must be applied to the longitude left to obtain the longitude in. It will be seen the two latter methods of solution agree very closely with that by calculation. From the examples we have solved in middle-latitude sailing and in Mercator's sailing the student will see how useful the traverse table is in checking the work by calculation, and at sea it should always be resorted to, as every computor is liable to a clerical error.

Fig. 26.

Scale 800 miles to an inch.



Exercise XIV.

Ex. 277. Obtain expressions connecting the *course*, *difference of longitude*, and *meridional difference of latitude*. In what cases would you prefer the rules for Mercator's sailing to the rules for middle-latitude sailing? *A.* 1871.

Ex. 278. Find the true course and distance from Rio Janeiro (latitude 22° 54′ S., longitude 43° 6′ W.) to Sierra Leone (latitude 8° 30′ N., longitude 13° 18′ W.) *H.M.S. Britannia*, 1872.

Ex. 279. Find the course and distance from A to B. Given

Latitude A .	53° 18′ S.	Longitude A .	76° 14′ E.
Latitude B .	56 25 S.	Longitude B .	78 13 E.

Variation $1\frac{1}{4}$ points E.; deviation 8° E.

A. 1871.

Ex. 280. Explain the terms course, distance, true difference of latitude, meridional difference of latitude and middle latitude.

Prove the formula:—

Diff. long. = dep. × sec mid. lat.

H.M.S. Britannia, 1875.

Ex. 281. Required the compass course and distance from A to B:

Latitude A .	37° 44′ N.	Longitude A .	25° 40′ W.
Latitude B .	28 28 N.	Longitude B .	16 15 W.

Variation $2\frac{1}{4}$ points W.; deviation 2° 20′ W.

A. 1873.

Ex. 282. Required the compass course and distance from Cape Finisterre to Terceira.

Cape Finisterre.		*Terceira.*	
Latitude . .	42° 54′ N.	Latitude . .	38° 39′ N.
Longitude . .	9 16 W.	Longitude .	27 14 W.

Variation $2\frac{1}{4}$ points W.; deviation 3° 30′ E.

A. 1874.

Ex. 283. Find by Mercator's sailing the compass course and distance from San Francisco (37° 48′ N., 122° 8′ W.) to Mowee Isle (20° 50′ N., 156° 2′ W.). *A.* 1875.

Ex. 284. Find the compass course and distance from A to B.

Latitude A .	50° 10′ N.	Longitude A .	12° 20′ W.
Latitude B .	38 32 N.	Longitude B .	28 43 W.

Variation $2\frac{1}{2}$ points W.; deviation 5° 40′ E.

A. 1876.

Ex. 285. Find the compass course and distance from Cape Palmas to Cape St. Roque.

Cape Palmas . . latitude 4° 23′ N.; longitude 7° 38′ W.
Cape St. Roque . . latitude 5 10 S.; longitude 35 40 W.

Variation $1\frac{1}{2}$ points W.; deviation 6° 20′ W.

A. 1877.

Ex. 286. Find by Mercator's sailing the compass course and distance from Simon's Bay to St. Helena, having given

Simon's Bay . . latitude 34° 11′ S.; longitude 18° 26′ E.
St. Helena . . latitude 22 16 S.; longitude 5 44 W.

Variation $2\frac{3}{4}$ points W.; deviation 5° 40′ W.

A. 1878.

Ex. 287. Find by Mercator's sailing the compass course and distance between Cape St. Vincent (latitude 27° 3′ N., longitude 9° W.) and Funchal (latitude 32° 37′ N., longitude 16° 58′ W.). Variation 2 points W.; deviation 6° 30′ E. *A.* 1879.

Ex. 288. Find by Mercator's sailing the compass course and distance from St. Paul's Island (latitude 38° 42′ S., longitude 77° 18′ E.) to Mauritius (latitude 20° 10′ S., longitude 57° 28′ E.). Variation $1\frac{3}{4}$ points W.; deviation 3° E. *A.* 1880.

Ex. 289. Find by Mercator's sailing the compass course and distance from Madras (latitude 13° 4′ N., longitude 80° 22′ E.) to Acheen (latitude 5° 36′ N., longitude 95° 30′ E.).

Given variation of the compass $\frac{1}{4}$ point E.; deviation 11° 30′ E.

A. 1881.

Ex. 290. My course from St. Helena (latitude 15° 55 S., longitude 5° 44′ W.) was S. 39° 27′ E., and distance 1,120 miles. Find the latitude and longitude in.

Ex. 291. A vessel from the Eddystone (latitude 50° 10′·9 N., longitude 4° 16′ W.) sailed SW$\frac{1}{4}$W. until her difference of longitude was 620 miles. Required her latitude in and distance sailed

Ex. 292. Since leaving Cape Agulhas (latitude 34° 47′·7 S., longitude 20° 0′·7 E.) a ship has made good true S. 65° 30′ W., distance 1,650 miles. Find her present latitude and longitude, and her compass course and distance to Cape Horn (latitude 55° 59′ S., longitude 67° 16′ W.). Variation 15° 20′ W.; deviation 11° 20′ W.

Ex. 293. A ship, from latitude 37° 0′ N., longitude 22° 56′ W., runs on a course N. 33° 19′ E., until she finds her difference of longitude is 786 miles. What is her present latitude, and distance sailed?

Ex. 294. Give and prove the different rules for finding the difference of longitude. Which is the most accurate method? Explain clearly the reason of your answer. *A.* 1880.

Ex. 295. A ship leaving the south of Vancouver's Island (latitude 48° 23′ N.) finds her course and distance to Owyhee (latitude 20° 17′ N.) by middle-latitude sailing, and also by Mercator's sailing. There is a difference of upwards of 20 miles in the distance, and more than half a degree in the course. Explain clearly the reason of this, and show *à priori* by which method the distance should be the greater and the course the farther from the meridian. *Honours,* 1879.

MERCATOR'S CHARTS.

Mercator's charts possess so many advantages that they are almost universally adopted for sea purposes, and we will now endeavour to show how they are constructed. In an old work on navigation written by Mr. Robertson, he says, 'The difficulty of constructing a true sea-chart seems to have consisted in finding a proper manner of applying the surface of a globe to a plane, which Mr. Wright happily accomplished by a most ingenious conception, the substance of which is as follows:—

'Firstly. Suppose a rectangular plane were rolled about the globe, until the edges of the plane met, and formed a kind of concave cylinder inclosing the globe and touching its equator.

'Secondly. Conceive the surface of this globe to swell (like a bladder while it is blowing up) from the equator towards the poles, *proportionally in latitude as it does in longitude,* until every part of its surface meet that of the concave cylinder, and impress on it the lines that were drawn on the globular surface.

'Thirdly. Then the cylinder, or rectangular plane, being unrolled, will represent a sea-chart, the parts of which bear the same proportion to one another as the corresponding parts of the globe do, and in which all the lines will be right lines.

'For in this formation of the nautical chart, every parallel of latitude on the globe will be increased till it is equal to the equator; and so the distance of the meridians in those parallels will become equal to their distance at the equator; consequently, the meridians on the chart are expressed by parallel right lines.

'Also, the meridians being lengthened as the parallels are increased, every degree of latitude is lengthened in the same proportion as the degrees of longitude are increased; therefore the distances of the parallels of latitude become wider and wider as they approach the poles.

'Again, as the rhumb lines on the globe cut the meridians at equal angles, they will also cut the meridians at equal angles on the chart, and consequently be expressed by right lines, since none but right lines can cut several parallel right lines at equal angles.'

From the above quotation we see the advantages of a Mercator's chart over other plane charts are

(*a*) Because a rhumb line cuts all the meridians it passes on the chart at equal angles, the course between two given places on the chart can be found at once by measuring either angle made.

(*b*) All places of small area retain their proper form on the chart, because the length and breadth of such places are increased in the same proportion, viz., as the secant of the latitude in which they are situate. Thus, a small island in the Arctic Ocean will have a true representation of its form, but will be on a larger scale than a similar one nearer the equator.

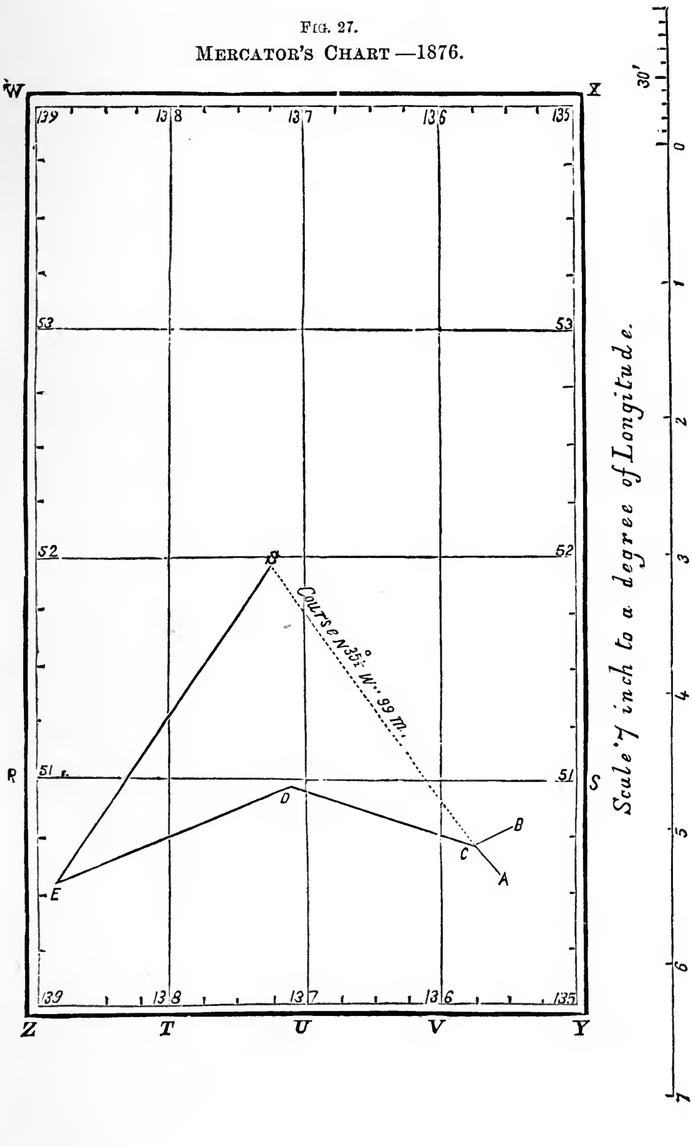
USE OF THE CHART.—*Having given the latitude and longitude of a place, to lay down its position on a chart.*—Find the given latitude on the graduated meridian at the side of the chart, and draw a faint pencil line through it (near the given longitude) parallel to the parallels of latitude. Next find the given longitude on the graduated parallel at the top or bottom of the chart, and draw a similar line through it parallel to the meridians. Where the two lines intersect is the position required. In practice it is found only very short lines need be drawn. If a point, as a headland, a port, &c., be marked on the chart, and it is required to find its latitude and longitude, we have only to reverse the above operations and it is at once known.

To find the course between two given places on the chart.—As it would require a protractor or some other means of measuring the angle between the ship's track and a meridian, figures resembling compass cards are placed in convenient positions about the charts, the true north and south points of which coincide with a meridian, and on which the *magnetic points* are also marked. To find the course, all that is necessary is to lay the edge of a parallel ruler over the two places, and then slide it always parallel to its first position to the centre of the nearest compass, and read from it the course wanted. If it be required to lay down a given course from a given point, we must place the ruler over the given course on the compass and slide it parallel to that position until the given point be reached, and from the given point draw a light pencil line in the proper direction : this will indicate the ship's track for the given course.

To find the distance between two given places on the chart.—Draw a light pencil line between them, and with a pair of compasses bisect the line: this will be the middle latitude of the two places. Take the distance between the middle latitude and either of the points, and from the same middle latitude on

Fig. 27.

Mercator's Chart—1876.



the graduated meridian measure both north and south, counting the number of nautical miles enclosed between the two points of the compasses in each direction. The sum of these two is the distance required. To lay down a distance from a point we must reverse the above process and take in the compasses the distance from the graduated meridian in the middle latitude, and measure this in the given direction from the given point.

To construct a Mercator's Chart.—This will be best understood by working in full the subjoined example given for honours in the South Kensington examination, 1876.

Ex. 296. Construct a Mercator's chart on a scale ·7 inch to a degree of longitude, extending from 50° N. to 54° N., and from 135° W. to 139° W. The position of the ship is fixed by cross bearings of two points of land A and B; A, in latitude 50° 31′ N., longitude 135° 33′ W., bore SE.; and B, in latitude 50° 43′ N., longitude 135° 26′ W., bore ENE. Lay down this position and the following courses and distances—WNW. 55 ; WSW. 70′; NE. by N. 104′. Take off from the chart the latitude and longitude in, and the equivalent course and distance from, taking the cross bearings to the position the ship arrives at.

First.—Construct a scale, making each division equal to ·7 inch: these will represent degrees of longitude; mark them 1, 2, 3, 4, ... Subdivide one of the exterior divisions into 10 equal parts, then each part will represent $\frac{60'}{10}$ or 6 miles. Should great accuracy be required, a diagonal scale should be constructed from which single miles may be taken off.

Secondly.—If the vessel goes towards the north, draw a line near the bottom of the paper on which the chart is to be constructed, as Z Y; and if she goes south, draw the line near the top. At the right hand draw the line Y X at right angles to Z Y. From Y on Y Z mark off divisions V U T Z, &c., each equal to ·7 inch, *i.e.* one division of our scale, and draw from these points lines at right angles to Z Y: these will represent the meridians of our chart, and must be marked 135°, 136°, 137°, 138°, 139°, as the question requires.

Thirdly.—On a separate piece of paper place the degrees of latitude the chart must contain one under the other, and take from the table the 'meridional parts' corresponding to each degree thus:—

Riddle's Tables, Compression $\frac{1}{321}$.

Latitude	50°	meridional parts	= 3458		
,,	51	,,	= 3552	difference	94
,,	52	,,	= 3648	,,	96
,,	53	,,	= 3747	,,	99
,,	54	,,	= 3847	,,	100

and take the difference between the meridional parts for each degree as above. Next with a pair of dividers take 94 miles, *i.e.* in this case one division and nearly $\frac{6}{10}$ of another, from our scale, and lay off northward (because we are in north latitude) from Z and Y towards W and X, as R S; then if Z Y represents the 50th parallel, R S joined will represent the 51st. Similarly take 96 miles, *i. e.* $1\frac{6}{10}$ divisions, in the dividers, and lay off from R and S northward, join these points : this will represent the 52nd parallel; then lay off 99 and 100 miles in the same way to represent the 53rd and 54th parallels, and so on to as many as are required, and we have the skeleton chart on which must be placed the figures representing the degrees. Next subdivide the distances between the degrees of latitude—if divided into four equal parts, each = 15′; if into six, each = 10′; if into twelve, each = 5′, and so on.

Fourthly.—We have now constructed the chart with the meridians and parallels in their proper positions and distances between in their proper proportions. We must now lay down the positions A and B in their respective latitudes and longitudes as directed before, and mark off A C in a NW. direction, thus making A bear SE.; then lay down B C in a WSW. direction, thus making B bear ENE.; and at C, where the two lines intersect, is the position of the ship: we find it in latitude 50° 38′ N., longitude 135° 46′ W. Next take C D on the WNW. rhumb from C and equal to 55′ measured on the graduated meridian as nearly opposite to where C D will fall as possible; then lay down D E on a WSW. rhumb from D and equal to 70′ measured on the graduated meridian in its middle latitude, and so on with all the other courses and distances sailed. Let S be the position of the ship at the end of the operation: its latitude and longitude must be found as already explained. In the present example C is in latitude 50° 38′ N., longitude 135° 46′ W., and the position of the ship S is latitude 51° 59′ N., longitude 137° 18′ W. To find the equivalent course and distance join C S as in the figure, then the course is the angle C S makes with any meridian, and the distance must be measured as directed before on the graduated meridian in the middle latitude. This gives as the course N. $35\frac{1}{2}$° W. and distance 99 miles.

By practice the student can get his figure always in the centre of his paper, and he can add any degree of finish he pleases.

We will now take another example for south latitude given for honours in 1879.

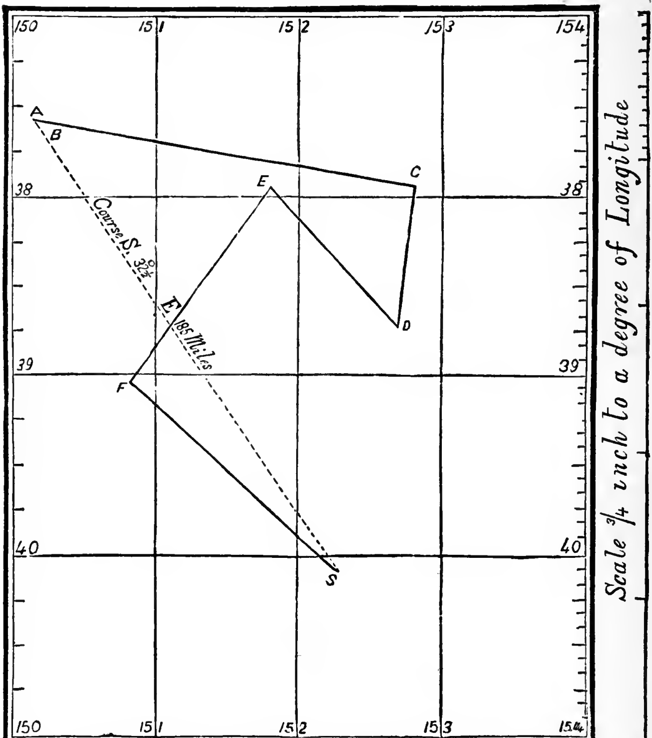
Ex. 297. Construct a Mercator's chart on a scale of $\frac{3}{4}$ of an inch to a degree of longitude, extending from 37° S. to 41° S. latitude, and from 150° E. to 154° E. longitude. A known point of land (latitude

37° 30′ S., longitude 150° 7′ E.) bears from the ship NW. by compass (deviation 7° W.) distant 7 miles; the ship then sails as follows:

Compass courses	Distances	Deviation of compass
E. by S.	130′	10° W.
S.	50	3 W.
NW. by W.	65	6 E.
SSW.	80	5 30′ E.
SE.	90	8 W.

The variation of the compass is 9° E.

Fig. 28.
Mercator's Chart.



Lay down the true courses, and take off the latitude and longitude in, and the direct course and distance from the point of land to the position arrived at.

The bearing of the land and the courses being taken by compass must first be corrected and reduced to true courses before they can be laid down on the chart; this must be done as directed in the chapter on 'Correcting Courses,' and we find the true courses and distances to be :—1st, the ship sailed from the point of land S. 43° E. 7 miles; 2nd, S. 80° E. 130 miles; 3rd, S. 6° W. 50 miles; 4th, N. 41° W. 65 miles; 5th, S. 37° W., 80 miles; and 6th, S. 44° E. 90 miles. We then construct the skeleton chart as described in the last example and finish the question as before. The answer found is—direct course S. $32\frac{1}{2}$° E., direct distance 186 miles, position of the ship latitude 40° 7′ S., longitude 152° 14′ E.

Exercise XV.

Ex. 298. Draw the frame of a Mercator's chart on the scale 1·6 inches to a degree of longitude, extending from latitude 53° S. to latitude 58° S., and from longitude 22° E. to longitude 27° E.; and lay down on it the positions of two rocks, one in latitude 54° 18′ S., longitude 23° 14′ E., and the other in latitude 54° 34′ S., longitude 23° 26′ E.; and lay down the position of a ship bearing from the first SSW$\frac{1}{2}$W., and from the second W$\frac{1}{4}$S., both magnetic. Afterwards sailed by compass NNE. 110′, and S. by E. 38′, and NW. 50′. Lay down the ship's position. Given variation $1\frac{1}{2}$ points E.; deviations 8° 35′ E., 4° 20′ E., and 10° W. respectively. *Honours*, 1871.

Ex. 299. Draw a Mercator's chart on a scale of 1·25 inches to a degree of longitude, extending from 56° to 59° S., and from 29° to 33° W. A headland A (latitude 56° 15′ S., longitude 29° 22′ W.) bore NE. by E$\frac{3}{4}$E., and a rock B (latitude 56° 28′ S., longitude 29° 45′ W.) bore S. by E.; the ship afterwards sailed as under :—

True course	Distance
SW$\frac{3}{4}$W.	85′
SSE.	71
N. by E.	112

Lay down the position of the ship at first, the courses, the latitude and longitude in, and the direct course and distance made good.

Honours, 1874.

Ex. 300. Construct a Mercator's chart on a scale of 1·3 inches to a degree of longitude, extending from 50° N. to 54° N. latitude, and from

10° W. to 14° W. longitude. Lay down the first position of a ship by cross bearings, her subsequent true courses, and find her latitude and longitude in, with the course and distance made good, from the following:—In first position of the ship (ship's head being SW. by S.) a point of land, A (latitude 52° 8′ N., longitude 10° 15′ W.), bore by compass SSE., and another point of land, B (latitude 52° 15 N., longitude 10° 10′ W.), bore E. by N.; afterwards sailed as follows:—

Compass course	Distance	Deviation of compass
SW. by S.	100 miles	6° 10′ W.
WNW.	53 „	6 50 W.
NE.	75 „	10 0 E.
NW. by N.	90 „	3 20 W.
NNE.	80 „	7 30 E.

The variation of the compass was 2 points W. *Honours*, 1875.

Ex. 301. Construct a Mercator's chart on a scale of 1·2 inches to a degree of longitude, extending from latitude 54° to 58° N., and from longitude 135° to 140° W. The position of the ship was fixed by cross bearings of two points, A (latitude 57° 20′ N, longitude 135° 25′ W.) and B (latitude 57° 30′ N., longitude 135° 40′ W.); the true bearing of A being E. by S., and that of B, NE. by N. Afterwards sailed the following magnetic courses (variation 2 points E.) and distances:—SSE., 53′; WNW., 105′; SE., 167′; WSW. 80′. From the final position of the ship a rock bore by compass (deviation 3° 30′ W.) N. 10′. Lay down the rock, and take off its latitude and longitude. *Honours*, 1878.

Ex. 302. Construct a Mercator's chart on a scale of 1·3 inches to a degree of longitude, extending from 67° N to 70° N., and from 60° W. to 65° W. A ship sailed from P (latitude 67° 50′ N., longitude 60° 40′ W.) as follows:

Magnetic courses	Distances	
N. 41° W.	67′	Variation 75° W.
S. 77 E.	100	
N. 15 W.	60	
S. 60 E.	45	
S. 53 W.	22	

Rule true and magnetic meridians through P; protract the true courses; find the latitude and longitude in, and the course and distance made good. *Honours*, 1880.

Ex. 303. On June 25, 1869, at noon, a ship was in latitude 49° 33′ N. and longitude 5° 53′ W., and afterwards sailed as follows:—

Hours	Knots	Tenths	Standard compass course	Leeway points	Winds	Deviation of standard compass	Remarks
1	1	8	SSE.	$\frac{1}{2}$	E. by N.	2° W.	P.M.
2	2	5					
3	2	0					
4	2	3					
5	1	0					
6	5	5	North	0	SSW.	1° E.	
7	6	0					
8	5	2					
9	4	5					
10	3	5	N. by E.	0	SW. by S.	2° E.	
11	3	2					
12	4	0					Midnight
1	3	5					
2	4	5					
3	4	0					
4	3	0					
5	3	0					
6	3	5	NE. by N.	2	NW.	8° E.	
7	2	5					
8	3	6					
9	3	2					Variation allowed 23° W.
10	4	8	NE. by E.	1	NW. by N.	9° E.	
11	6	0					
12	5	5					

Lay down the above day's work on a Mercator's chart on a scale 3·7 inches to the degree of longitude. The chart is to extend from latitude 49° N. to latitude 51° N., and from longitude 4° W. to 6° W. Lay down also a rock in latitude 49° 50′ N., longitude 5° 10′ W., and give (from the chart) the bearing and distance of the rock from the ship at noon, June 26. and the course and distance made good. *Honours*, 1869.

Ex. 304. What is meant by Mercator's projection? How would you find the course and distance between two places on a Mercator's chart? *A.* 1868.

Ex. 305. Explain accurately the principle of Mercator's projection. State its chief value to the navigator. Prove that

$$\tan \text{course} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}}$$ *A.* 1872.

Ex. 306. Explain what you understand by 'Mercator's sailing.' What constitutes the chief value of a Mercator's chart? Prove that

$$\text{diff. long.} = \text{mer. diff. lat.} \times \tan \text{course}.$$ *A* 1877.

MERIDIONAL PARTS.

We will now proceed to investigate the accurate method for calculating a table of meridional parts of the sphere. To do this we shall make use of the integral calculus.

Let $d\,q$ = an infinitesimal portion of the equator.

$d\,p$ = a corresponding portion of a parallel *on the sphere* in latitude m.

$d\,m$ = an infinitesimal portion of the meridian *on the sphere* and adjacent to dp.

$d\,p'$ = a corresponding portion of any parallel *on Mercator's chart* in latitude m.

$d\,m'$ = an infinitesimal portion of a meridian *on Mercator's chart* adjacent to dp'.

Now it has been proved $\text{M} = \text{L}.\cos l$

$$\therefore\ d\,p = d\,q.\cos m \quad . \quad . \quad .. \quad (1)$$

but on Mercator's projection, because the meridians are all parallel,

$$\therefore\ d\,p' = d\,q.$$

Substituting in (1)

$$\therefore\ d\,p = d\,p'.\cos m\,;$$

that is,

$$\frac{d\,p'}{d\,p} = \frac{1}{\cos m} \quad . \quad . \quad . \quad (2)$$

From what has been already said, on Mercator's chart $d\,m'$ exceeds $d\,m$ in the same proportion that $d\,p'$ exceeds $d\,p$.

$$\therefore\ \frac{d\,m'}{d\,m} = \frac{d\,p'}{d\,p}.$$

Substituting in (2) $\quad d\,m' = \dfrac{d\,m}{\cos m}.$

Integrating

$$m' = \int \frac{d\,m}{\cos m} = \int \frac{\cos m}{1 - \sin^2 m}\,d\,m$$

$$= \frac{1}{2}\int \left\{ \frac{\cos m}{1+\sin m} + \frac{\cos m}{1-\sin m} \right\} d\,m$$

$$= \frac{1}{2}\left\{ \log_\varepsilon(1+\sin m) - \log_\varepsilon(1-\sin m \right\}) + \text{C}$$

and because $\log_\epsilon 1 = o$, therefore $c = o$ when the integration is performed between the limits $m = m$ and $m = o$;

$$\therefore\ m' = \tfrac{1}{2}\log_\epsilon \frac{1+\sin m}{1-\sin m} = \tfrac{1}{2}\log\left(\frac{\cos\frac{m}{2}+\sin\frac{m}{2}}{\cos\frac{m}{2}-\sin\frac{m}{2}}\right)^2$$

$$= \log_\epsilon \frac{1+\tan\frac{m}{2}}{1-\tan\frac{m}{2}} = \log_\epsilon \tan\left(45+\frac{m}{2}\right)$$

$$= \log_\epsilon \cot\left(45-\frac{m}{2}\right) = \log_\epsilon \cot \tfrac{1}{2}\text{ co-lat.} \quad . \quad . \ (\text{A})$$

that is, the sum of all the infinitesimal parts dm', or the length of a portion of a meridian on a chart from latitude o to latitude $m = \log_\epsilon \cot \frac{1}{2}$ colat.

We have next to adapt this formula to a form in which we can use the common or Briggs' logarithms. This is done by dividing the Naperian logarithm by the Naperian logarithm of 10 ($= 2{\cdot}30258509+$); and, because we have deduced the formula from the trigonometrical ratios in circular measure, we must adapt it to the terrestrial sphere, and get our results in nautical miles. This is done by multiplying the result so far by the number of nautical miles in the radius of the earth, viz. $3437\frac{3}{4}$ very nearly.

Then $m' = 3437{\cdot}75 \times 2{\cdot}30258509 \times \log\cot\frac{1}{2}$ co-lat.

or $\log m' = \log 3437{\cdot}75 + \log 2{\cdot}30258509 + \log(\log\cot \frac{1}{2}\text{ co-lat.} - 10)$

$$= 3{\cdot}8984895 + \log(\log\cot\tfrac{1}{2}\text{ co-lat.} - 10) \quad . \ (\text{B})$$

Ex. 307. As an example of the application of the above formula (B), we will find the meridional parts for the latitude of the Navigation School, Plymouth.

Latitude $50^\circ\ 22\frac{1}{2}'$ N. $\quad\therefore$ co-latitude $= 39^\circ\ 37\frac{1}{2}'$.

and $\log(\log\cot\frac{1}{2}\text{ co-lat.} - 10) = \log(\log\cot 19^\circ\ 48\frac{3}{4}' - 10)$

$= \log {\cdot}443374$

$\therefore \log m' = 3{\cdot}8984895 + \log {\cdot}443374$

$= 3{\cdot}8984895 + \bar{1}{\cdot}6467709$

$= 3{\cdot}5452604$

$= \log 3509{\cdot}5$

Hence, meridional parts for $50^\circ\ 22\frac{1}{2}' = 3509{\cdot}5$.

It is obvious the same result should be obtained if we multiply ($\log_{10}$ cot $\frac{1}{2}$ co-lat. -10) by $2{\cdot}30258509 \times 3437{\cdot}37$ by ordinary multiplication;

$$\text{i.e.}\ (\log_{10} \cot 19^\circ\ 48\tfrac{3}{4}' - 10) \times 7915{\cdot}712 \ldots.$$
$$= {\cdot}443374 \times 7915{\cdot}712 \ldots$$
$$= 3509{\cdot}66$$

which is correct to two places of decimals. Now these are the same as given by Inman, Raper, Norie, and others, and are the meridional parts for the earth *considering it a sphere.*

In the 'Philosophical Transactions,' No. 219, Dr. Halley, speaking of the correct methods of calculating the meridional parts, says, 'It was discovered by chance, and first published by Mr. Henry Bond in an addition to Norwood's "Epitome of Navigation" about the year 1645, *that the meridian line was analogous to a scale of logarithmic tangents of half the complements of the latitude.*' He adds, 'Nor hath any one, that I know of, yet discovered the rule for computing independently the interval of the meridional parts answering to any two given latitudes.' From considering the stereographic projection of the sphere on the plane of the equator, in which the projection of portions of the meridian representing the co-latitudes of all places on a rhumb curve varies as the tangent of half the co-latitudes, and the differences of longitude were as the logarithms of the ratios of those tangents to unity, he deduced from the properties of the projection and of logarithms that if we divide the log cotan $\frac{1}{2}$ colat. of all places by ·0001263311438, &c., we shall form a table of meridional parts. But dividing by ·0001263311438, &c., is precisely the same as multiplying by 7915·74, &c., and we have shown above that the true multiplier is 7915·712, &c.; the difference occurring from Dr. Halley not having at his command the true radius of the earth since found by Geodic measurements. To illustrate his method we will find the meridional parts for the Navigation School, Plymouth, latitude $50^\circ\ 22\frac{1}{2}'$ N.

$$\text{Co-lat. of school} = 39^\circ\ 37\tfrac{1}{2}'$$
$$\therefore \log \cot \tfrac{1}{2}\ \text{co-lat.} = \log\ (\cot 19^\circ\ 48\tfrac{3}{4}' - 10)$$
$$= {\cdot}443374$$

and meridional parts $= \dfrac{\cdot 443374}{\cdot 0001263311438}$
$= 3509\cdot 62$

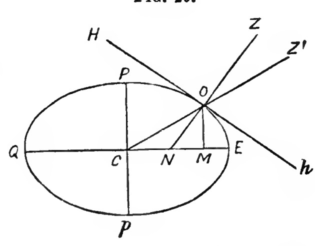
This result agrees very closely with those obtained by the two methods already given.

Formula for the Spheroid.—Those who use Riddle's tables will see there is a discordance in the meridional parts for given latitudes; this is owing to the latter writer making an allowance of $\frac{1}{321}$ for the compression of the earth, and will therefore be more correct than the others mentioned, although the real compression is $\frac{1}{300}$. M. Delambre has shown that the true meridional parts may be computed from the formula $m =$ log cot $\frac{1}{2}$ geocentric colat., but states that the proof was first given by Maclaurin in 1742. Thus for a spheroid of given ellipticity the reduced latitude must be used as the basis for calculation. In order to do this we must explain what is meant by the reduced latitude, and how it is obtained.

Reduced Latitude and Angle of the Vertical.—As the earth is not a perfect sphere, but an oblate spheroid, the normal, *i.e.* the perpendicular to the sensible horizon at any place, except at the equator and poles, will not pass through the centre of the earth.

Let P Q p E be a meridional section of the earth, P p the poles, E Q the points where the plane of the equator cuts the section, C the centre of the earth, O the place of the observer, H O h his sensible horizon, which will be a tangent at O. Through O draw Z O N perpendicular to H O h: O N E measures the *true* or *astronomical* or *latitude by observation* of O; join C O and produce C O to Z′, then O C E measures the *reduced* or *geocentric latitude* of O; Z is the *true zenith*, Z′ the *reduced zenith*, and the angle Z O Z′ is the *angle of the vertical* or *reduction for latitude.*

Fig. 29.



Let $a =$ equatorial radius CE . . . $b =$ polar radius CP
$l =$ true latitude ONE . . . . $l^1 =$ reduced lat. OCE

CM $= x$,—then MN, the subnormal, is by conics $\frac{b^2}{a^2}x$.

Now, $\tan l = \frac{\text{OM}}{\text{MN}}$ and $\tan l^1 = \frac{\text{OM}}{\text{MC}}$

$$\therefore \frac{\tan l^1}{\tan l} = \frac{\text{MN}}{\text{MC}} = \frac{\frac{b^2}{a^2}x}{x} = \frac{b^2}{a^2}$$

Hence, $$\tan l^1 = \frac{b^2}{a^2} \tan l \quad . \quad . \quad . \quad \text{(I.)}$$

Again, $$\tan (l - l') = \frac{\tan l - \tan l^1}{1 + \tan l \, . \, \tan l^1}$$

from (I.) $$= \frac{\tan l \left(1 - \frac{b^2}{a^2}\right)}{1 + \frac{b^2}{a^2} \tan^2 l}$$

$$= \frac{e^2 \tan l}{1 + \frac{b^2}{a^2} \tan^2 l} \quad . \quad . \quad \text{(II.)}$$

Now, $\frac{b^2}{a^2}$ is so nearly equal to unity that no material error will be introduced into the calculation if we let $\frac{b^2}{a^2}$ in the denominator $= 1$. Then

$$\tan (l - l^1) = \frac{e^2 \tan l}{1 + \tan^2 l} \text{ very nearly;}$$

$$= \frac{e^2}{2} \frac{2 \frac{\sin l}{\cos l}}{1 + \frac{\sin^2 l}{\cos^2 l}}$$

multiplying numerator and denominator by $\cos^2 l$

$$\tan (l - l^1) = \frac{e^2}{2} \sin 2l \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

Now, $(l - l^1) =$ ONE $-$ OCE $=$ NOC
$=$ angle of the vertical;

and because $l - l^1$ only amounts (when at its maximum) to a few minutes, the tangent and the arc are very nearly equal.

From III. $\therefore$ the angle of the vertical $= \frac{e^2}{2} \sin 2l$ very nearly.

Because $\frac{e^2}{2}$ is a constant for the earth, therefore *the angle of the vertical varies as sine of twice the latitude*; hence it will be a maximum when $2l$ is so, *i.e.* when the latitude is 45°.

Returning to formula II.: by giving to a and b the values found by the late Astronomer-Royal, viz. $a = 3962{\cdot}824$ and $b = 3949{\cdot}585$ miles, we can find the maximum value of the angle of the vertical thus:

$$\tan(l - l^1) \text{ or tan angle of the vertical} = \frac{e^2 \tan l}{1 + \frac{b^2}{a^2} \tan^2 l};$$

but the angle has just been proved a maximum when $l = 45°$ then $\tan l = 1$; and because $e^2 = \frac{a^2 - b^2}{a^2}$ we get

$$\text{tan angle of the vertical} = \frac{a^2 - b^2}{a^2 + b^2}.$$

Giving a and b their numerical values we find the maximum value of the angle of the vertical $= 11' \, 30''{\cdot}24$.

Ex. 308. Find the reduced latitude and the angle of the vertical for the Plymouth Navigation School, latitude 50° 22′ 25″ N.

$$\tan l^1 = \frac{b^2}{a^2} \tan l;$$

$$\therefore \log \tan l^1 = 2(\log b - \log a) + \log \tan l;$$

$$\begin{aligned} \text{or log tan reduced lat.} &= 2(\log 3949{\cdot}585 - \log 3962{\cdot}824) + \log \tan 50° \, 22' \, 25'' \\ &= 2(3{\cdot}596551 - 3{\cdot}598005) + 10{\cdot}081944 \\ &= \bar{1}{\cdot}997092 + 10{\cdot}081944 \\ &= 10{\cdot}079036 \\ &= \log \tan 50° \, 11' \, 6''{\cdot}1 \text{ nearly.} \end{aligned}$$

$$\begin{aligned} \therefore \text{Angle of the vertical of the school} &= 50° \, 22' \, 25'' - 50° \, 11' \, 6''{\cdot}1 \\ &= 11' \, 18''{\cdot}9 \end{aligned}$$

This gives very nearly the same result for the angle of the vertical as if we had used Equation III., giving e the value $\frac{3}{37}$; but the present result is more correct because in that case $\frac{b^2}{a^2}$ was taken equal to unity.

The log $\frac{b^2}{a^2}$ may be registered (it being equal to $\bar{1}\cdot997092$ as shown above), which would greatly facilitate the calculations; the formula would then be :—

log tan reduced lat. $=\bar{1}\cdot997092+$log tan lat. by observation.

Having now obtained the reduced latitude of the School, we will apply it in finding the meridional parts for its latitude, considering the earth a spheroid whose compression is $\frac{1}{300}$.

Reduced lat. $= 50° \; 11' \; 6''$ ∴ reduced co lat. $= 39° \; 48' \; 54''$ and cotan $\frac{1}{2}$ colat. $- 10 =$ cotan $39° \; 48' \; 54'' - 10$.

$$\begin{aligned}\therefore \text{Log mer. pts.} &= 3\cdot8984895+\log\,(\log \text{cotan } 39° \; 48' \; 54''-10) \\ &= 3\cdot8984895+\log \cdot441120 \\ &= 3\;8984895+\bar{1}\;644557 \\ &= 3\cdot5430465\end{aligned}$$

hence, mer. pts.$=$ 3491·78;

and the meridional parts for any other latitude on the terrestrial spheroid may be computed in the same way.

If we use the geocentric or reduced co-latitude of the School, with compression $\frac{1}{321}$, we shall obtain

mer. pts. $=$ 3493,

which agrees with Riddle's tables.

The true meridional parts for any spheroid are obtained from a formula which we shall now proceed to investigate.

In finding the angle of the vertical we get from Equation I. :

$$\tan l^1 = \frac{b^2}{a^2} \tan l.$$

Using the recognised notation for the co-ordinates of a point in an ellipse this becomes

$$\frac{y}{x} = \frac{b^2}{a^2} \tan l \quad . \quad . \quad . \quad \text{(IV.)}$$

but the equation to the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$;

$$\therefore y^2 = \frac{b^2\,(a^2 - x^2)}{a^2}$$

Squaring IV. and substituting

$$\frac{b^2\,(a^2 - x^2)}{a^2\,x^2} = \frac{b^4}{a^4} \cdot \frac{\sin^2 l}{\cos^2 l}$$

whence $$x^2 = \frac{a^4 \cos^2 l}{a^2 \cos^2 l + b^2 \sin^2 l}$$

$$= \frac{a^2 \cos^2 l}{\cos^2 l + \frac{b^2}{a^2} \sin^2 l}$$

$$= \frac{a^2 \cos^2 l}{\cos^2 l + (1 - e^2) \sin^2 l}$$

$$\therefore x = \frac{a \cos l}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}$$

and $\frac{dx}{dl} =$

$$\frac{a\{-\sin l\,(1-e^2 \sin^2 l)^{\frac{1}{2}} + \frac{1}{2} \cos l\,(1-e^2 \sin^2 l)^{-\frac{1}{2}}.\ 2\,e \,.\sin l \,.\cos l\}}{1 - e^2 \sin^2 l}$$

$$= a\,\frac{-\sin l\,(1 - e^2 \sin^2 l) + e^2 \cos^2 l \,.\sin l}{(1 - e^2 \sin^2 l)^{\frac{3}{2}}}$$

$$= a\,\frac{e^2 \sin l\,(\sin^2 l + \cos^2 l) - \sin l}{(1 - e^2 \sin^2 l)^{\frac{3}{2}}}$$

$$= \frac{-a\,(1 - e^2) \sin l}{(1 - e^2 \sin^2 l)^{\frac{3}{2}}}$$

Hence, $$\left(\frac{dx}{dl}\right)^2 = \frac{a^2\,(1 - e^2)^2 \sin^2 l}{(1 - e^2 \sin^2 l)^3}$$

Similarly, $$y^2 = \frac{b^4 \sin^2 l}{a^2 \cos^2 l + b^2 \sin^2 l}$$

$$= \frac{a^2 \frac{b^4}{a^4} \sin^2 l}{\cos^2 l + \frac{b^2}{a^2} \sin^2 l}$$

$$= \frac{a^2\,(1 - e^2)^2 \sin^2 l}{\cos^2 l + (1 - e^4) \sin^2 l}$$

$$\therefore y = \frac{a\,(1 - e^2) \sin l}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}}$$

nd $\frac{dy}{dl} = a\,(1 - e^2)$

$$\times \frac{\cos l\,(1 - e^2 \sin^2 l)^{\frac{1}{2}} + \frac{1}{2} \sin l\,(1 - e^2 \sin^2 l)^{-\frac{1}{2}}.\ 2\,e \,.\sin l \,.\cos l}{1 - e^2 \sin^2 l}$$

$$= a\,(1-e^2)\,\frac{\cos l\,(1-e^2 \sin{}^2 l) + e^2 \sin{}^2 l \,.\cos l}{(1-e^2 \sin{}^2 l)^{\frac{3}{2}}}$$

$$= \frac{a\;(1-e^2)\cos l}{(1-e^2 \sin{}^2 l\,)^{\frac{3}{2}}}$$

Hence, $\left(\frac{dy}{d\,l}\right)^2 = \frac{a^2\,(1-e^2)^2 \cos{}^2 l}{(1-e^2 \sin{}^2 l)^3}$

Now using the values as before :

a = radius of the equator.

r = radius of the parallel in latitude l.

m = length of an arc of a meridian between the equator and that parallel.

m' = corresponding length on the chart.

$d\,m$ and $d\,m'$ small increments of m and m'; then by the principles of the chart:

$$\frac{d\,m'}{d\,m} = \frac{a}{r}$$

but from the figure, p. 149, it is manifest r is equal to x,

$$\text{then } \frac{dm'}{d\,m} = \frac{(1-e^2 \sin{}^2 l)^{\frac{1}{2}}}{\cos l}$$

and the differential coefficient of the length of the curve with respect to the latitude or $\frac{d\,m}{d\,l}$ is found from

$$\frac{d\,m}{d\,l} = \sqrt{\left(\frac{d\,x}{d\,l}\right)^2 + \left(\frac{d\,y}{d\,l}\right)^2} \qquad \textit{(Tod.'s Diff. Cal., p. 329.)}$$

$$= \frac{a\,(1\;-\;e^2)}{(1\;-\;e^2 \sin{}^2 l)^{\frac{3}{2}}}$$

$$\text{and } \frac{d\,m'}{d\,l} = \frac{d\,m}{d\,l} \times \frac{d\,m'}{d\,m}$$

$$= \frac{a\,(1\;-\;e^2)}{(1\;-\;e^2 \sin{}^2 l)^{\frac{3}{2}}} \times \frac{(1\;-\;e^2 \sin{}^2 l)^{\frac{1}{2}}}{\cos l}$$

$$= \frac{a\;(1\;-\;e^2)}{\cos l\,(1\;-\;e^2 \sin{}^2 l)}$$

$$= \frac{a}{\cos l} - \frac{a\,e^2 \cos l}{1\;-\;e^2 \sin l}$$

Hence $m' = a \int_o^l \sec l \,.\, d\,l - a\,e^2 \int_o^l \frac{\cos l}{1 - e^2 \sin^2 l} \,.\, d\,l.$

The integral of the first term has already been found to be equal to $a \log_\varepsilon \cot \frac{1}{2}$ co-lat; and the integral of the latter term is $\frac{a\,e}{2} \log_\varepsilon \frac{1 + e \sin l}{1 - e \sin l}$, which reduces to $a\,e^2 \sin l$ very nearly;

$\therefore\ m' = a \log_\varepsilon \cot \frac{1}{2}$ co-lat. $- a\,e^2 \sin l$ very nearly

Taking the latter term: $e^2 = \frac{a^2 - b^2}{a^2}$, where a is the major and b the minor radius of the earth.

Then $a\,e^2 \sin l = a \frac{(a + b)\,(a - b)}{a^2} \sin l$

$= (a + b) \times$ compression $\times$ sin lat.

$= 22{\cdot}9 \times$ sin lat. very nearly.

Hence an easy method of finding the meridional parts for any latitude on the terrestrial spheroid is, first find them for a sphere from the formula $3{\cdot}8984895 + \log\,(\log \cot \frac{1}{2}$ co-lat. $-10)$, and then subtract from the result $22{\cdot}9 \times$ sin lat. The reduction $22{\cdot}9 \times$ sin lat. may be found from the traverse table, with the given latitude as course, and the distance 22·9 in the distance column, and the reduction will be found in the departure column.

As an exercise we will apply the last formula for computing the meridional parts for a spheroid in finding those for the School, as before in latitude 50° 22′ 25″.

$\therefore$ co-lat. $= 39° \ 37' \ 35''$

log. mer. pts. $= 3{\cdot}8984895 + \log\,(\log \cot \frac{1}{2}$ co-lat. $- 10)$

$= 3{\cdot}8984895 + \log\,(\log \cot 19°48'47''{\cdot}5 - 10)$

$= 3{\cdot}8984895 + \log {\cdot}443357$

$= 3{\cdot}8984895 + \bar{1}{\cdot}646754$

$= 3{\cdot}5452435$

$\therefore$ mer. pts. for 50° 22′ 25″ on a sphere $= 3509{\cdot}52$

reduction $= 22{\cdot}9 \times$ sin lat.

log reduction $= \log 22{\cdot}9 + \log \sin 50° \ 22' \ 25'' - 10$

$= 1{\cdot}359835 + 9{\cdot}886615 - 10$

$= 1{\cdot}246450$

$\therefore$ reduction $= 17{\cdot}64$

Hence, mer. pts. for 50° 22′ 25″ on the terrestrial spheroid
$= 3509{\cdot}52 - 17{\cdot}64$
$= 3491{\cdot}88$
which agrees very nearly with those deduced from using the reduced latitude.

Exercise XVI.

Ex. 309. Account for the appearance on a Mercator's chart of polar lands and polar seas being so much greater than tropical ones.

Ex. 310. Prove that

$$\text{diff. long.} = \text{mer. diff. lat.} \times \tan \text{course}.$$

For Lieutenant, 1874.

Ex. 311. Explain how the course and distance from one place to another are found on a Mercator's chart.

Prove :—

(1) $\tan \text{course} = \dfrac{\text{diff. long.}}{\text{mer. diff. lat.}} = \dfrac{\text{diff. long} \times \cos \text{mid. lat.}}{\text{true diff. lat.}}$

(2) $\text{dist.} = \text{diff. long.} \times \cos \text{mid. lat.} \times \operatorname{cosec} \text{course}.$ *A.* 1869.

Ex. 312. Explain the construction of the Mercator's chart, and the method of calculating the table of meridional parts.

Beaufort testimonial, 1866.

Ex. 313. Explain clearly what you mean by 'middle-latitude sailing' and 'Mercator's sailing,' and show why the latter is a more perfect method than the former. *A.* 1874.

Ex. 314. Prove the formulæ :—

(1) $\text{Tan course} = \dfrac{\text{dep.}}{\text{true diff. lat.}}$

(2) $\text{Tan course} = \dfrac{\text{diff. long.}}{\text{mer. diff. lat.}}$

(3) $\text{Tan course} = \dfrac{\text{diff. long.} \times \cos \text{mid. lat.}}{\text{true diff. lat.}}$

Define the nautical terms used in the above formulæ. *A.* 1875.

Ex. 315. Explain the construction of Mercator's chart. Point out clearly in what consists its peculiar adaptation for the purposes of the navigator. Describe how you will read off a distance between two places on the chart, and explain the reason of the method. *A.* 1875.

Ex. 316. Explain clearly what you mean by meridional parts. Prove that approximately meridional parts for $l° = \sec 0' + \sec 1' + \sec 2' + \sec 3' + \ldots\ldots + \sec (l° - 1')$, and hence find the meridional parts for 50° 5′, having given the meridional parts for 50° = 3474·47. Give a more accurate formula for the computation of the meridional parts, and verify the preceding result by it. *Honours*, 1877.

Ex. 317 Explain clearly what is meant by a 'sailing.' Write down the characteristic formula of 'Mercator's sailing,' and prove it. Describe the table used in connection with this sailing. *A.* 1878.

Ex. 318. Write down the characteristic formulæ for finding difference of longitude—(1) in middle-latitude sailing; (2) in Mercator's sailing. Which gives the more accurate result if the course and distance are considerable? Why? On what principle is the table of meridional differences of latitude calculated? *A.* 1881.

Ex. 319. Show, on the supposition that the earth is a sphere, that the meridional parts for any latitude may be calculated from the formula,

$$\log \text{mer. parts} = 3{\cdot}8984895 + \log(\log\cot\tfrac{1}{2}\text{ co-lat.} - 10).$$

In what respect does this formula require to be modified if the spheroidal form of the earth is taken into account? Calculate the meridional parts for lat. 51° 50′ N. *Honours*, 1881.

Ex. 320. What do you mean by the angle of the vertical and by the *reduced latitude*? Draw a figure illustrating your answer. What is the reduced latitude for Paris observatory, in latitude 48° 50′ 13″ N.?

CHAPTER VIII.

Great circle sailing—Shortest distance—Definitions—Data employed—Formulæ proved for course—Distance—Position of vertex and succession of points—Additional formulæ for the distance, &c.—Composite sailing—Circular arc sailing—Great circle charts or the Gnomonic Projection—Windward great circle sailing—How a terrestrial globe should be used for the solution of the question—Exercises—Examination.

GREAT-CIRCLE SAILING.

STEAM vessels are so fast supplanting sailing vessels that a very large proportion of the ocean trade is now carried on by their means; and as their method of propulsion renders them in a great measure independent of winds and currents, the masters of them can therefore choose their own routes; and as the shortest possible (all other things being equal) is the one to be desired, great circle sailing is coming into greater use than heretofore when vessels had to depend on the wind for making a passage. The shortest route between any two points on the surface of a sphere is the arc of a great circle. This theorem is a particular case of the general one of finding the shortest distance between two points on any curved surface; but the mathematical proof of it is too advanced for this treatise. The student who wishes to study it is referred to Hall's 'Differential and Integral Calculus,' p. 381, and to Todhunter's 'Integral Calculus,' p. 316. It may be proved experimentally by selecting any two points on a terrestrial globe (say eighteen inches in diameter), bringing them to the wooden horizon, then stretching a piece of thread in the plane of the horizon between the two points, when it will be found this is the least length of thread which will reach from one point to the other. But the wooden horizon is in the plane of a great circle of the globe, therefore the shortest distance is on a great circle. Again, it can be proved, the greater the circle described passing through any two points, the nearer the arc between the two points will approach a straight line between them ; but a great circle on a sphere is the largest which can be drawn on its surface : hence an arc of a great circle drawn between two places on the surface of a sphere must more nearly approach a straight line than the arc of any other circle passing through the two places : and, therefore, an arc of a great circle must be the least distance between two places on the surface of a sphere.

A Great Circle is one whose plane passes through the centre of a sphere, and is therefore the largest which can be drawn on its surface.

A Small Circle is one whose plane does not pass through the centre of a sphere.

A Spherical Angle is the angle on the surface of a sphere formed by the intersection of two great circles, and is measured by the inclination of the planes of the two great circles forming the spherical angle.

A Spherical Triangle is that portion of the surface of a sphere bounded by the arcs of three great circles which do not all intersect in one point. Each of these arcs must be less than a semicircle.

The Course in great circle sailing is the angle included between the plane of the great circle on which the ship sails and the plane of the meridian.

The Angle of Position is the angle *at the place* included between the plane of the great circle and the plane of the meridian; thus it is the first great circle course from the place.

Distance is the length of the arc of a great circle between the two places expressed in nautical miles.

The Vertex is that point in a great circle in the highest latitude. Because all great circles bisect one another, the equator bisects all great circles which can be drawn on the surface of the earth, and hence there must be two points in every great circle farther removed from the equator than any others. These are called *vertices*, and the one in the hemisphere in which is situate the place in the higher latitude is called *the vertex.* The great circle through any two places makes an angle with the equator equal to the latitude of the vertex.

GREAT-CIRCLE SAILING *is the art of finding that route along which a ship must pass in going from one place to another that her track may lie on the arc of a great circle.*

In practice it is most convenient to keep a ship's head always on one course, because in her navigation it produces the least calculation. This is done by sailing on a rhumb curve; but, by so doing, although the direction of the ship's head is the same with every meridian it crosses, yet it is different at every instant in respect to the port bound to. On the contrary, when sailing on a great circle, the ship's head is always the same with regard to the port—that is, *directly towards it, or as 'the crow flies'*; but then it changes its direction with respect to the meridian. By great-circle sailing a vessel is steered as if the port of destination was always in sight, and this is the only method by which it is attained. To do this it is necessary the course of the ship should be continually changed, and to keep her always on a great circle becomes impracticable; but, although she cannot be kept on a great circle, she may frequently be brought on it and kept near it. To participate of the benefits of this method, a succession of points differing but little in longitude is selected, and the course is calculated by Mercator's or middle latitude sailing. Now, it is well known that when the arc is small its chord and tangent will approximate very closely to the length of the arc itself; and, therefore, a difference of 5° in longitude is usually taken as the basis of calculation, except where the great circle between the places lies near a meridian, and then a difference of more than 5° may be selected.

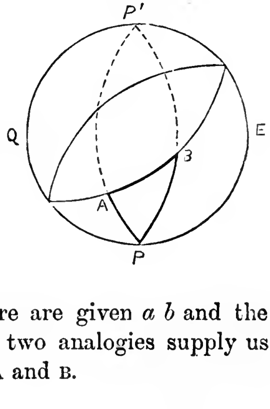
THE DATA EMPLOYED.—The spherical triangle used in solving problems in great circle sailing is formed by the co-latitudes of the places under consideration as sides, the angle at the pole included between the meridians of the two places, i.e. the difference of longitude as the angle included between the two sides already mentioned, and the arc of the great circle through the given places forming the third side. The problem will be simplified by using the pole nearest the place in the higher latitude. In practice it matters little what the actual distance between two ports is, so that the vessel sails on that course which will bring her to her destined port in the quickest time, and the route chosen be practicable. With the data above we shall proceed to find on a great circle :—

I. The course from any one point to another.
II. The distance.
III. The position of the vertex.
IV. A succession of points.

To find the Course.

Let P E P′ Q represent the globe on the plane of a meridian, P P′ the poles, E Q the equator. A B any two points on its curved surface. Join A B by an arc of a great circle, and through A and B draw the meridians P A, P B; then we have a spherical triangle formed by the three arcs P A, P B, and A B, and the difference of longitude is the angle A P B. We will use the recognised notation in trigonometry, viz. the angles of the triangle shall be denoted by capital letters and the sides opposite to each angle shall be denoted by the corresponding small letters.

FIG. 30.



In the spherical triangle P A B there are given a b and the included angle P; then Napier's first two analogies supply us with the means of finding the angles A and B.

$$\left.\begin{aligned}
\text{I.}\quad \tan\frac{A+B}{2} &= \frac{\cos\dfrac{a-b}{2}}{\cos\dfrac{a+b}{2}}\cot\frac{P}{2} = \cos\frac{a-b}{2}\cdot\sec\frac{a+b}{2}\cdot\cot\frac{P}{2}\\
\text{II.}\quad \tan\frac{A-B}{2} &= \frac{\sin\dfrac{a-b}{2}}{\sin\dfrac{a+b}{2}}\cot\frac{P}{2} = \sin\frac{a-b}{2}\cdot\operatorname{cosec}\frac{a+b}{2}\cdot\cot\frac{P}{2}
\end{aligned}\right\}$$

Hence $\frac{A+B}{2}$ and $\frac{A-B}{2}$ are determined,

$$\left.\begin{aligned}
\text{and } \therefore A &= \frac{A+B}{2} + \frac{A-B}{2} \quad . \quad . \quad . \quad (\alpha)\\
\text{and } B &= \frac{A+B}{2} - \frac{A-B}{2} \quad . \quad . \quad . \quad (\beta)
\end{aligned}\right\}\text{ are known.}$$

Now, because $\frac{A+B}{2}$ and $\frac{a+b}{2}$ are of the same affection, that is, are both greater or both less than a right angle, if $\frac{a+b}{2}$ be greater than 90°, the supplement of the angle found by the first analogy is $\frac{1}{2}(A + B)$. If the student *always* select the pole nearest the place in the higher latitude to form his triangle from, then $\frac{a+b}{2}$ will invariably be less than 90°, and $\frac{A+B}{2}$ will be at once found from the first analogy.

From the above we get the following rule :—

Rule.—Reckon the co-latitudes from the pole nearest the place in the higher latitude.

Take half the sum $\left(\frac{a+b}{2}\right)$ and half the difference $\left(\frac{a-b}{2}\right)$ of the co-latitudes of the two places, and half the difference of longitude $\left(\frac{P}{2}\right)$.

Add together—

log cot of half diff. long . . . $\cot\frac{P}{2}$

log secant of half sum of co-lats. . $\sec \frac{a+b}{2}$

log cosine of half diff. of co-lats. . $\cos \frac{a-b}{2}$

The sum is log tan of *half the sum of the two courses* $\left(\log \tan \frac{A+B}{2}\right)$

Again, add together—

log cot of half diff. long. . . . $\cot \frac{P}{2}$

log cosecant of half sum of co-lats. . $\operatorname{cosec} \frac{a+b}{2}$

log sine of half diff. of co-lats. . . $\operatorname{sine} \frac{a-b}{2}$

The sum is log tan of *half the difference of the two courses* $\left(\log \tan \frac{A-B}{2}\right)$

The *sum* of the half sum and half difference of the two courses is the course *from the place in the higher latitude* to the other; and the *difference* of the half sum and half difference of the two courses is the course *from the place in the lower latitude* to the other.

The course is to be reckoned from north in north latitude, and from south in south latitude, and towards the east or west according as the other point lies to the east or west of the one we are reckoning from.

Ex. 321. Find the first course to be sailed on the great circle from Port Otago, New Zealand (latitude 45° 47′ S., longitude 170° 45′ E.), to Callao, Peru (latitude 12° 4′ S., longitude 77° 14′ W.), and *vice versa*.

Port Otago lat. 45° 47′ S. ∴ co-lat.	44° 13′	long.	170° 45′ E.	
Callao lat. 12 4 S. ∴ co-lat.	77 56	long.	77 14 W.	
Sum co-lats. $(a+b)$	122 9	diff. long.	247 59 W.	
Diff. co-lats. $(a-b)$	33 43		360 0	
Diff. long. (P)			112 1 E.	

$\frac{1}{2}$ diff. long.	$\frac{P}{2}$	= 56° 0½′	cot =	9·828851	cot =	9·828851
$\frac{1}{2}$ sum co-lats.	$\frac{a+b}{2}$	= 61 4½	sec =	10·315456	cosec =	10·057866
$\frac{1}{2}$ diff. co-lats.	$\frac{a-b}{2}$	= 16 51½	cos =	9·980923	sin =	9·462407
	$\frac{A+B}{2}$	53 9	tan =	10·125230		
	$\frac{A-B}{2}$	12 35	tan =	. . . . .		9·349124

∴ A = S. 65 44 E. = Course from Port Otago to Callao.
B = S. 40 34 W. = Course from Callao to Port Otago.

To find the Distance.

In the same triangle we have now the three angles and two sides a, b, to find the other side p. Either of Gauss's theorems will now supply us with a ready means of determining p without ambiguity: thus—

$$\cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2} p = \cos \tfrac{1}{2}(a+b) \sin \frac{P}{2}$$

$$\therefore \cos \frac{p}{2} = \cos \frac{a+b}{2} \sec \frac{A+B}{2} \sin \frac{P}{2} \quad . \quad . \quad (\gamma)$$

and we have already obtained $\frac{a+b}{2}$ and $\frac{A+B}{2}$ in finding the course. Hence we have the following rule for the distance:—

Rule.—Add together—

log cosine half the sum of co-lats. .	$\cos \frac{a+b}{2}$
log secant half the sum of two courses	$\sec \frac{A+B}{2}$
log sin half the difference of longitude .	$\sin \frac{P}{2}$

The sum is log cosine of half the distance in degrees and minutes. This must be doubled to find the whole arc, and the result reduced to nautical miles.

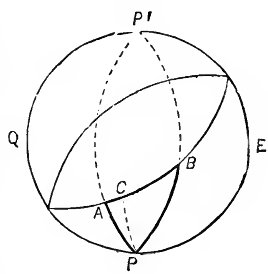
Ex. 322 Find the distance on the great circle from Port Otago to Callao.

We have half the sum of co-latitude	$\frac{a+b}{2}$	$= 61° \; 4\frac{1}{2}'$	cos =	9·684544
half sum of two courses	$\frac{A+B}{2}$	$= 53 \;\; 9$	sec =	10·222050
half difference of longitude	$\frac{P}{2}$	$= 56 \;\; 0\frac{1}{2}$	sin =	9·918617
∴ half distance . . .	$\frac{p}{2}$	$= 48 \;\; 2$	cos =	9·825211
		2		
		96 4		
		60		
Hence distance		5764 nautical miles.		

To find the position of the Vertex.

It is necessary to find the position of the vertex that we may calculate a succession of points on a great circle; and when the vertex falls between the two places, to know the highest latitude into which it will be necessary to take a vessel by sailing on a great circle. By going into too high a latitude the voyage may become dangerous, or at least may occupy more time than by sailing on the rhumb curve.

FIG. 31.



Let C be the position of the vertex; from P draw the meridian P C P′; then because C is the point farthest removed from the equator the tangent at C must be a parallel of latitude, and hence the meridian P C P′ cuts A B at right angles in C.

Then in the right-angled triangle P C A are known P A = co-latitude of A, and angle P A C (from equation α), besides the right angle P C A, to find P C the co-latitude of the vertex, and the angle A P C the difference of longitude between A and the vertex.

From Napier's circular parts

$$\sin PC = \sin PA \,.\, \sin PAC$$

$$\therefore \cos \text{lat. vertex} = \sin b \,.\, \sin A \quad . \quad . \quad . \quad (\delta)$$

Again—

$$\cos \text{PA} = \cot \text{PAC} \,.\, \cot \text{APC};$$
$$\text{or } \cot \text{APC} = \cos \text{PA} \,.\, \tan \text{PAC};$$
$$\therefore \ \cot \text{diff. long. from A} = \cos b \,.\, \tan \text{A} \quad . \quad . \quad . \ (\epsilon)$$

From the formulæ marked (δ) and (ϵ) we derive the following rules.

Rule.—Add together—

log cos latitude of A, which is . .	sin b
and log sin angle A (found in α) . .	sin A

The sum gives log cos lat. of vertex.

Again, add together—

log sin latitude of A, which is . .	cos b
and log tan angle A (found in α) . .	tan A

The sum gives log cot diff. long. of vertex from the place in the highest latitude.

Position of Vertex.—The vertex will not always fall between the two places; but if the angles A and B found in equations (α) and (β) be of the same affection (i.e. both greater or both less than 90°), the vertex will fall between the two places, and the difference of longitude (ϵ) must be applied to the longitude of A *towards* B. But if the angles A and B be not of the same affection, the perpendicular PC will not fall on the arc between A and B, but without it; and the difference of longitude (ϵ) must be then applied to the longitude of A *away from* B.

Ex. 323. Find the position of the vertex of the great circle between Port Otago and Callao.

It will be well, first, to notice the angles A and B are both less than 90°, and, therefore, are of the same affection: hence the difference of longitude must be applied from A towards B.

To find latitude of vertex.			*To find difference of longitude.*	
Lat. A	= 45° 47′	cos 9·843466		sin 9·855342
∠ A	= 65 44	sin 9·959825		tan 10·346000
Lat. vertex	50 31 S.	cos 9·803291	32° 10′	cot 10·201342

Longitude A	= 170° 45′ E.
Difference of longitude	= 32 10 E.
	202 55 E.
	360
Longitude vertex	= 157 5 W.

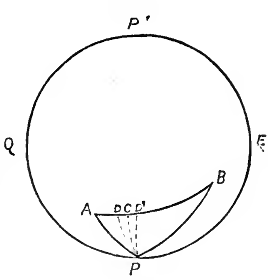
To find a succession of points on a Great Circle.

The distances of the points are always reckoned from the vertex; because, at the vertex, as has been already shown, the meridian cuts the great circle at right angles, and hence a right-angled triangle is formed, and the solution becomes one of extreme simplicity.

It will be found by choosing points at equal distances on each side of the vertex, *when it falls between the two places*; and drawing meridians through the points so selected, we shall form a series of right-angled spherical triangles, of which one on one side of the vertex will be symmetrically (not absolutely) equal to another chosen at an equal distance on the other side of the vertex; and hence the calculations for one can at once be used for the other. *If the vertex does not fall between the two places*, the points should be chosen at equal distances from A toward B.

Select points differing about 5° in longitude, and join each point with the pole by an arc of a meridian. We have then given, besides the right angle, one side, viz. the co-latitude of the vertex, and the angle at the pole formed by this meridian through the point, and the meridian through the vertex to find the co-latitude of the point.

Fig. 32.



PROOF.—Let D and D′ be the points differing about 5° in longitude from the vertex C, then the angle CPD = CPD′ = difference of longitude from the vertex, and PC = co-latitude of vertex; hence from the formula for the solution of right-angled spherical triangles we have

$$\cos \text{CPD} = \tan \text{PC} \cdot \cot \text{PD}$$

$$\therefore \tan \text{PD} = \tan \text{PC} \cdot \sec \text{CPD}$$

i.e. cot lat. points D and D′ = cot lat. vertex × sec diff. long. between D and vertex.

RULE.—Add together—

log cot lat. vertex

log sec diff. long. between D and vertex;

the result gives log cot latitude of the points D and D′; and hence, having the latitude and longitudes of D and D′, their positions are accurately defined.

Ex. 324. Find a succession of points differing 5° in longitude on a great circle, between Port Otago and Cal'ao.

We first take the latitude and longitude of the vertex, and then for our first point we select one differing 5° in longitude from it, and proceed as follows:—

Lat. vertex	50° 31′	cot =	9·915847
Diff. long.	5 0	sec =	10·001656
Lat. 1st point . . .	50 25	cot =	9·917503

This continued with every successive 5° difference of longitude from the vertex will give the results found in the following table:—

Successive points	Diff. long. between vertex and each successive point	Longitude of each point	Latitude of each point	By Mercator's sailing: Courses	By Mercator's sailing: Distances
	° ′	° ′	° ′	° ′	miles
A	32 10	170 45 E.	45 47 S.		
1st intermediate point	30 0	172 55	46 26	S. 67 4 E.	100·1
2nd ” ”	25 0	177 55 E.	47 44	S. 69 2 E.	218·0
3rd ” ”	20 0	177 5 W.	48 45	S. 73 8 E.	210·2
4th ” ”	15 0	172 5	49 32	S. 76 30 E.	201·3
5th ” ”	10 0	167 5	50 5	S. 80 21 E.	196·9
6th ” ”	5 0	162 5	50 25	S. 84 6 E.	194·6
7th ” ”	0 0	157 5	50 31	S 88 17 E.	200·3
8th ” ”	5 0	152 5	50 25	N. 88 17 E.	200·3
9th ” ”	10 0	147 5	50 5	N. 84 6 E.	194·6
10th ” ”	15 0	142 5	49 32	N. 80 21 E.	196·9
11th ” ”	20 0	137 5	48 45	N. 76 30 E.	201·3
12th ” ”	25 0	132 5	47 44	N. 73 8 E.	210·2
13th ” ”	30 0	127 5	46 26	N. 69 2 E.	218·0
14th ” ”	35 0	122 5	44 50	N. 65 37 E.	232·5
15th ” ”	40 0	117 5	42 15	N. 62 5 E.	245·0
16th ” ”	45 0	112 5	40 38	N. 58 37 E.	263·0
17th ” ”	50 0	107 5	37 58	N. 55 31 E.	282·6
18th ” ”	55 0	102 5	34 51	N. 52 17 E.	305·7
19th ” ”	60 0	97 5	31 15	N. 49 25 E.	332·0
20th ” ”	65 0	92 5	27 9	N. 46 58 E.	360·5
21st ” ”	70 0	87 5	22 33	N. 44 43 E.	388·4
22nd ” ”	75 0	82 5	17 26	N. 42 43 E.	417·8
B	79 51	77 14	12 4 S.	N. 41 19 E.	428·7

From the above table it will be seen that the latitudes of points equally distant in longitude from the seventh point—that is, the vertex—are the same; hence, by calculating the latitudes for the first six points, we can by inspection supply those as far as the thirteenth point.

Having now obtained the latitude and longitude of a succession of points on the great circle, the course and distance from one to the other is then calculated by Mercator's or middle-latitude sailing, and it will be found that the distance by this method differs but little from that on the great circle. In practice it is only necessary to calculate for the next point to the one on which the vessel is; so that should she be driven to a point widely separated from the great circle, time and distance would be lost in attempting to regain it, and a new great-circle track should at once be struck off. If the succession of points be laid down on a chart, and a curve be drawn through them, then this line will show the great-circle path.

If the student will now calculate the course and distance by Mercator's sailing from Port Otago to Callao, he will find the former to be N. 70° 40′ 28′·6 E., and the latter 6113 miles; whilst by taking a succession of points as above, differing 5° in longitude from one another, the whole distance is 5799 miles, thus saving 314 miles, and differing only 35 miles from the distance on the great circle.

We will next show how a formula for the distance can be obtained without the aid of the courses, and deduce the courses and position of the vertex by other methods.

Let a, b, and P represent the same data as before, and d be the distance on the great circle. Then by the fundamental formula in spherical trigonometry we have

$$\cos d = \cos a \,.\, \cos b + \sin a \,.\, \sin b \,.\, \cos \text{P}$$

$$= \cos a \,.\, \cos b + \sin a \,.\, \sin b \left(1 - 2 \sin^2 \frac{\text{P}}{2}\right)$$

$$= \cos a \,.\, \cos b + \sin a \,.\, \sin b - 2 \sin a \,.\, \sin b \,.\, \sin^2 \frac{\text{P}}{2}$$

$$= \cos (a - b) - 2 \sin a \,.\, \sin b \,.\, \sin^2 \frac{\text{P}}{2}$$

Add 1 to each side—

$$1+\cos d = 1 + \cos(a-b) - 2\sin a \,.\, \sin b \,.\, \sin^2 \frac{P}{2}$$

$$\therefore\ 2\cos^2\frac{d}{2} = 2\cos^2\frac{a-b}{2} - 2\sin a \,.\, \sin b \,.\, \sin^2\frac{P}{2}$$

and $$\cos^2\frac{d}{2} = \cos^2\frac{a-b}{2} - \sin a \,.\, \sin b \,.\, \sin^2\frac{P}{2}$$

$$\text{Let } \sin a \,.\, \sin b \,.\, \sin^2\frac{P}{2} = \cos^2\theta \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Then } \cos^2\frac{d}{2} = \cos^2\frac{a-b}{2} - \cos^2\theta$$

$$= \cos^2\frac{a-b}{2} - \cos^2\frac{a-b}{2}\cos^2\theta + \cos^2\frac{a-b}{2}\cos^2\theta - \cos^2\theta$$

$$= \cos^2\frac{a-b}{2}(1-\cos^2\theta) - \cos^2\theta\left(1-\cos^2\frac{a-b}{2}\right)$$

$$= \cos^2\frac{a-b}{2} \,.\, \sin^2\theta - \cos^2\theta \,.\, \sin^2\frac{a-b}{2}$$

$$= \sin\left(\theta + \frac{a-b}{2}\right) \,.\, \sin\left(\theta - \frac{a-b}{2}\right) \quad . \quad . \quad . \quad (2)$$

In words the two equations marked (1) and (2) may be thus expressed—

$$\left.\begin{array}{l}\cos^2\theta = \cos(\text{lat. A}) \,.\, \cos(\text{lat. B}) \,.\, \sin^2(\text{half diff. long.}) \\ \cos^2(\text{half dist.}) = \sin(\theta + \tfrac{1}{2}\text{ diff. lat.}) \,.\, \sin(\theta - \tfrac{1}{2}\text{ diff. lat.})\end{array}\right\}$$

We now give another method of procuring the distance by formula we do not remember seeing before; but it possesses the

FIG. 33.

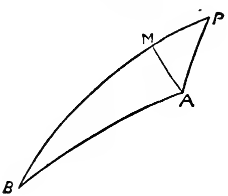
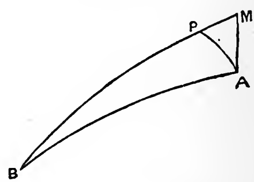


FIG. 34.



advantage of requiring fewer figures in practice than either of the other methods.

Let A be the place in the higher latitude
B " " " lower "
P the pole nearest the place in the higher latitude.

Then A P B = P = difference of longitude between A and B.

From A draw A M perpendicular to B P or to B P produced: the latter will be required when the difference of longitude is greater than 90°.

In the right-angled spherical triangle A M P we have—

$$\pm \cos P = \tan PM \,.\, \cot PA$$
$$\therefore \tan PM = \pm \cos P \,.\, \tan PA$$

If $\theta = PM$

$$\tan \theta = \pm \cos (\text{diff. long.}) \,.\, \cot (\text{lat. } A) \qquad (\alpha)$$

and $BM = BP \mp PM = \text{colat. } B \mp \theta = \phi$ suppose.

Here it must be remarked that the upper sign will correspond with fig. 33—that is, with the difference of longitude less than 90°; and the lower sign with fig. 34—that is, when the difference of longitude is greater than 90°.

In the same triangle

$$\cos AP = \cos AM \,.\, \cos PM$$
$$\therefore \cos AM = \cos AP \,.\, \sec PM \,.\; . \; . \; . \; . \; . \; . \qquad (1)$$

In the right-angled triangle A B M we have—

$$\cos AB = \cos AM \,.\, \cos BM$$

from (1) $\qquad = \cos AP \,.\, \sec PM \,.\, \cos BM$

Hence— $\quad \cos \text{distance} = \sin (\text{lat. } A) \,.\, \sec \theta \,.\, \cos \phi \,.\; . \; . \qquad (\beta)$

The formulæ marked (α) and (β) are all that are required for finding the distance between any two places on the earth's surface; but it must be borne in mind that *when* ϕ or colat. B $\mp \theta$ *is greater than* 90°, *the supplement of the distance just found must be used.*

Having obtained the distance by either of the above methods, the courses may be obtained by using the distance from Gauss's theorems, or without using the distance by Napier's analogies. We shall now use the former, as they are neater, and the latter have been already illustrated. Gauss's theorems are—

$$\left.\begin{aligned}\sin\frac{A+B}{2} &= \cos\frac{P}{2}\,.\,\cos\frac{a-b}{2}\,.\,\sec\frac{d}{2}\\ \sin\frac{A-B}{2} &= \cos\frac{P}{2}\,.\,\sin\frac{a-b}{2}\,.\,\operatorname{cosec}\frac{d}{2}\end{aligned}\right\}$$

These may be expressed in words as follows—

$$\left.\begin{aligned}\sin\frac{A+B}{2} &= \cos\text{ (half d. long.) . cos (half d. lat.). sec (half dist.)}\\ \sin\frac{A-B}{2} &= \cos\text{ (half d. long.) . sin (half d. lat.) cosec (half dist.)}\end{aligned}\right\}$$

$$\text{and}\quad A = \frac{A+B}{2} + \frac{A-B}{2}$$

$$B = \frac{A+B}{2} - \frac{A-B}{2}$$

These give the angles of position from the places A and B.

The latitude and longitude of the vertex may now be found as before, or independently of the courses thus :—

In the spherical triangle P A B (fig. 31)

$$\frac{\sin A}{\sin PB} = \frac{\sin P}{\sin AB}$$

$$\therefore \sin A = \sin P\,.\,\sin PB\,.\,\operatorname{cosec} AB$$
$$= \sin P\,.\,\sin a\,.\,\operatorname{cosec} d.$$

Multiply each side by $\sin b$; then

$$\sin b\,.\,\sin A = \sin P\,.\,\sin a\,.\,\sin b\,.\,\operatorname{cosec} d$$

but $\sin b\,.\,\sin A = \sin PC$; because angles at C are right angles

$$\therefore \sin PC = \sin P\,.\,\sin a\,.\,\sin b\,.\,\operatorname{cosec} d$$

or expressed in words—

cos (lat. vertex) = sin (d. long.) . cos (lat. A). cos (lat. B). cosec (dist.).

The longitude of the vertex is thus found.

In the right angled triangle A P C

$$\cos APC = \tan PC\,.\,\cot PA$$

this expressed in words is—

cos (diff. long. vertex is from A) = cot (lat. vertex). tan (lat. A).

The succession of points must be found as before.

To illustrate these methods we will work the same question as we have already done, viz.—

Ex. 325. Find the distance, angles of position, and the latitude and longitude of the vertex of the great circle, between Port Otago and Callao.

Port Otago	(A) lat.	45° 47′ S.	long.	170° 45′ E.
Callao	(B) lat.	12 4 S.	long.	77 14 W.
	Diff. lat.	33 43	diff. long.	112 1
	Half diff. lat.	16 $51\frac{1}{2}$	half diff. long.	56 $0\frac{1}{2}$

The two formulæ are :—

$$\left.\begin{array}{l}\text{Cos}^2\,\theta = \sin^2(\text{half diff. long.}).\cos(\text{lat. A}).\cos(\text{lat. B}).\\ \text{Cos}^2(\text{half dist.}) = \sin(\theta + \text{half diff. lat.}).\sin(\theta - \text{half diff. lat.})\end{array}\right\}$$

Half diff. long.	56° $0\frac{1}{2}'$	sin	9·918616
			2
			19·837232
Lat. A.	45 47	cos	9·843466
Lat. B.	12 4	cos	9·990297
Half diff. lat.	16 $51\frac{1}{2}$		2)19·670995
θ	46 47	cos	9·835497
θ + half diff. lat.	63 $38\frac{1}{2}$	sin	9·952325
θ − half diff. lat.	29 $55\frac{1}{2}$	sin	9·697984
			2)19 650309
Half distance	48° $2\frac{1}{2}'$	cos	9·825154

∴ Distance . 96 5 = 5765 nautical miles.

By third method for distance.

Port Otago	(A) lat.	45° 47′ S.	long.	170° 45′ E.
Callao	(B) lat.	12 4 S.	long.	77 14 W.
∴ Co-lat.	B.	77 56	diff. long.	112 1

Tan θ = cos (diff. long.). cot (latitude A)

Diff. long.	112° 1′	cos	9·573888
Lat. A	45 47	cot	9·988123
θ	20 $2\frac{1}{2}$	tan	9·562011
Co-lat. B	77 56		
(Co-lat. B + θ)	97 $58\frac{1}{2}$	ϕ	

Here we take (co-lat. B + θ) because the difference of longitude is greater than 90°.

Again, $\cos(\text{dist.}) = \sin(\text{lat. A}) . \sec\theta . \cos\phi$

Lat. A	45° 47′	sin	9·855342
θ	20 $2\frac{1}{2}$	sec	10·027129
ϕ	97 $58\frac{1}{3}$	cos	9·142205
	83 $55\frac{1}{2}$	cos	9·024676
	180		
Distance	96 $4\frac{1}{3}$ = $5764\frac{1}{2}$ nautical miles.		

We have here subtracted the angle taken out from the table from 180° because (co-lat. B + θ) is greater than 90°.

By referring to the three methods by which we have obtained the distance, it will be seen they differ but one mile in nearly 6,000, and would have been exact had we worked to seconds for our arcs.

To find the angles of position at A *and* B.

$$\left.\begin{aligned}\sin\frac{A+B}{2} &= \cos\frac{P}{2}\cos\frac{a-b}{2}\sec\frac{d}{2}\\ \sin\frac{A-B}{2} &= \cos\frac{P}{2}\sin\frac{a-b}{2}\operatorname{cosec}\frac{d}{2}\end{aligned}\right\}$$

$\frac{P}{2}$	56° $0\frac{1}{2}'$	cos	9·747468	cos	9·747468
$\frac{a-b}{2}$	16 $51\frac{1}{2}$	cos	9·980923	sin	9·462408
$\frac{d}{2}$	48 $2\frac{1}{2}$	sec	10·174840	cosec	10·128643
$\frac{A+B}{2}$	53 9	sin	9·903231		
$\frac{A-B}{2}$	12 35			sin	9·338519

A = S. 65 44 E. = angle of position at Port Otago.
B = S. 40 34 W. = angle of position at Callao

To find the position of the Vertex.

Cos (lat. vertex) = sin (d. long.). cos (lat A). cos (lat. B). cosec (dist.)

Diff. long.	112° 1′	sin	9·967115
Lat. A	45 47	cos	9·843466
Lat. B	12 4	cos	9·990297
Distance	96 5	cosec	10·002453
Lat. vertex	50 31 S.	cos	9·803331

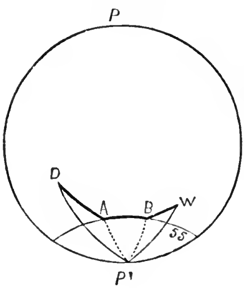
Cos (diff. long. vertex is from A) = cot (lat. vertex). tan (lat. A)

Lat. vertex	50° 31′	cot	9·915847
Lat. A	45 47	tan	10·011877
Diff. long. vertex is from A	32° 9′ E.	cos	9·927724
Long. A	170° 45′ E.		
Long. vertex	202 54 E. = 157° 6′ W.		

Objections.—The chief objection urged against great-circle sailing is that where it would be of the greatest service it generally becomes impossible; this happens when both places have latitude of the same name, with great difference of longitude, and are removed far from the equator. Under such conditions it will be found that the vertex of the great circle will be in so high a latitude that to use this method will be impracticable. Thus, the great circle from the East Cape, New Zealand, to Cape Horn runs as far south as $64\frac{3}{4}$°, and this in southern latitude is very undesirable. Land, ice, currents, &c., interfere with the advantages to be gained by using great circle sailing, and to receive its full benefits great scope is required. The *chief* difficulty, that of being brought into too high a latitude, is obviated by *composite sailing*, which is a combination of great-circle and parallel sailing introduced by the late Mr. J. T. Towson of Liverpool. It is used thus: the highest latitude it is thought prudent to reach is settled on; then a great circle which shall *touch* the parallel and pass through the place of departure is computed, and another great circle which shall pass through the place of destination and touch the same parallel is computed. The vessel is then sailed on the first great circle until she reaches the desired parallel. She is then kept on the parallel of latitude until the longitude of the point where the other great circle meets the parallel is reached, and then the vessel is sailed on the second great circle to the point of destination. The computation of these great circles is exceedingly simple: because they touch the parallel, meridians will pass through the points of contact at right angles to the great circles, and we shall have the co-latitude of the place left as one side, and the co-latitude of the parallel to be reached as another side, of one of the right-angled spherical triangles, to find the angle included between these meridians, which will be the difference of longitude between the place sailed from and the point required in the parallel, where the course must be due east or west. We will work an example in full to show the method used in composite sailing.

Ex. 326. Owing to the season of the year, I wish to limit my latitude to 55° S., and being at Dunedin (latitude 45° 54′ S., longitude 170° 40′ E.) I want to reach Wellington Isle (latitude 49° 7′ S., longitude 75° 34′ W.). In what longitude must I meet and leave the maximum parallel? what are my first and last courses, and the whole distance to be sailed, using composite sailing?

FIG. 35.



First, take the right-angled spherical triangle P′ A D, where D represents Dunedin, and A where the great circle meets the parallel of 55° S.

Cos A P′ D = tan P′A . cot P′ D
= cot lat. A . tan lat. D
= cot 55°. tan 45° 54′

∴ Diff. long. from Dunedin = 43° 44′

Long. left Dunedin =	170° 40′ E.
Diff. long. to meet parallel =	43 44 E.
Long. to meet the parallel =	214 24 E.

= 145° 36′ W. . . . . (*a*)

Again, in the same triangle—

sin P′ A = sin P′ D . sin D
∴ sin D = sin co-lat. A . cosec P′ D
= cos 55°. sec 45° 54′
∴ First course = S. 55° 30½′ E. . . . . (*b*)

Again, in the same triangle—

cos P′ D = cos P′ A . cos A D
cos A D = cos P′ D . sec P′ A
= sin 45° 54′. cosec 55
∴ Distance = 28° 45½′
= 1725·5 miles . . . . (*c*)

Secondly, take the right-angled spherical triangle P′ B W, where W represents Wellington Isle, and B the point where the vessel leaves the parallel to sail on the second great circle.

$$\begin{aligned}
\text{Cos } BP'W &= \tan P'B \,.\, \cot P'W \\
&= \cot \text{lat. } B \,.\, \tan \text{lat. } W. \\
&= \cot 55^\circ \,.\, \tan 49^\circ\, 7'
\end{aligned}$$

$\therefore$ Difference of longitude from Wellington Isle = 36° 1′

Long. destined port	=	75° 34′ W.
Diff. long. to leave parallel	=	36 1 W.
Long. where parallel is left	=	111 35 W. . . . . . (*d*)

Again, in the same triangle—

$$\begin{aligned}
\sin P'B &= \sin P'W \,.\, \sin W \\
\therefore \sin W &= \sin .\text{co-lat. } B \,.\, \text{cosec } P'W \\
&= \cos 55^\circ \,.\, \sec 49^\circ\, 7' \\
\therefore \text{Last course} &= \text{N. } 61^\circ\, 12' \text{ E.} \quad . \quad . \quad . \quad . \ (e)
\end{aligned}$$

Again, in the same triangle—

$$\begin{aligned}
\cos P'W &= \cos P'B \,.\, \cos BW \\
\cos BW &= \cos P'W \,.\, \sec P'B \\
&= \sin 49^\circ\, 7' \,.\, \text{cosec } 55^\circ \\
\therefore \text{Distance} &= 22^\circ\, 38' \\
&= 1358 \text{ miles} \quad . \quad . \quad . \quad . \ (f)
\end{aligned}$$

We have now longitude where first great circle meets the parallel—

	=	145° 36′ W.
Long. where second does	=	111 35 W.
Difference of longitude	=	34 1 = 2041 miles.

This difference of longitude is made good in latitude 55° S.

Lat. . . .	55° S.	cos	9·758591
Diff. long. . .	2041	log	3·309843
Distance on parallel	1170·7	log	3·068434 . . (*g*)

The whole distance travelled by composite sailing is the sum of (*e*) (*f*) and (*g*) or D A + B W = 4254·2 miles.

First course from (*b*)	=	S. 55° 30½′ E.
Last course from (*e*)	=	N. 61 12 E.
Long. to meet parallel 55° S.	=	145 36 W.
„ leave „ „	=	111 35 W.

The course by Mercator's sailing is S. 87° 36½′ E.
The distance „ „ „ 4625 miles
The distance by great-circle sailing is 4136 miles
The latitude of the vertex is 63° 28′ S.
The longitude „ „ „ 130 21 W.

This shows there is a saving of 371 miles in the composite sailing over the rhumb sailing, and a loss of 118 miles from the great-circle track; but the advantage of not going beyond the 55th parallel instead of to 63° 28′ S. more than counterbalances the loss in winter time.

Another method with the same object in view as Mr. Towson had was that suggested by the Rev. Geo. Fisher, M.A., F.R.S., formerly chaplain of Greenwich Schools. It is as follows:—

Join the ship's position on a Mercator's chart with the port bound to by a rhumb curve; bisect this line, and erect a perpendicular at the point of bisection. We have now two given points, viz. the positions of the ship and of the port bound to, and a given straight line, viz. the selected maximum parallel of latitude. An arc of a circle must now be drawn, passing through the two given points and touching the given straight line – a well-known proposition in practical geometry. This will be the shortest distance (limited by the highest latitude it is proposed to reach) between the ship's position and her intended port. A succession of points can then be chosen at will, and the course and distance from one to the other calculated as before either by Mercator's or middle-latitude sailing for the ship's use. Graphic methods for describing the great circle between two places have been proposed by several persons, among whom the late Astronomer Royal, Sir G. B. Airy, F.R.S., is the best known; but in practice the method given above, although not a great circle, is the one mostly used by seamen, as it is so easy of construction. Mr. Hugh Godfray, of St. John's College, Cambridge, has proposed to use a chart on the Gnomonic projection, which ought to be more widely known and more extensively used. Its construction may be gleaned from the following description:—

Let E P Q on the top of the figure be a meridional section of the earth, E Q its equator; and suppose an indefinite tangent plane 15° E., 165° W. to touch the pole of the earth P; the parallels of latitude are dotted in to every 15 degrees; and let

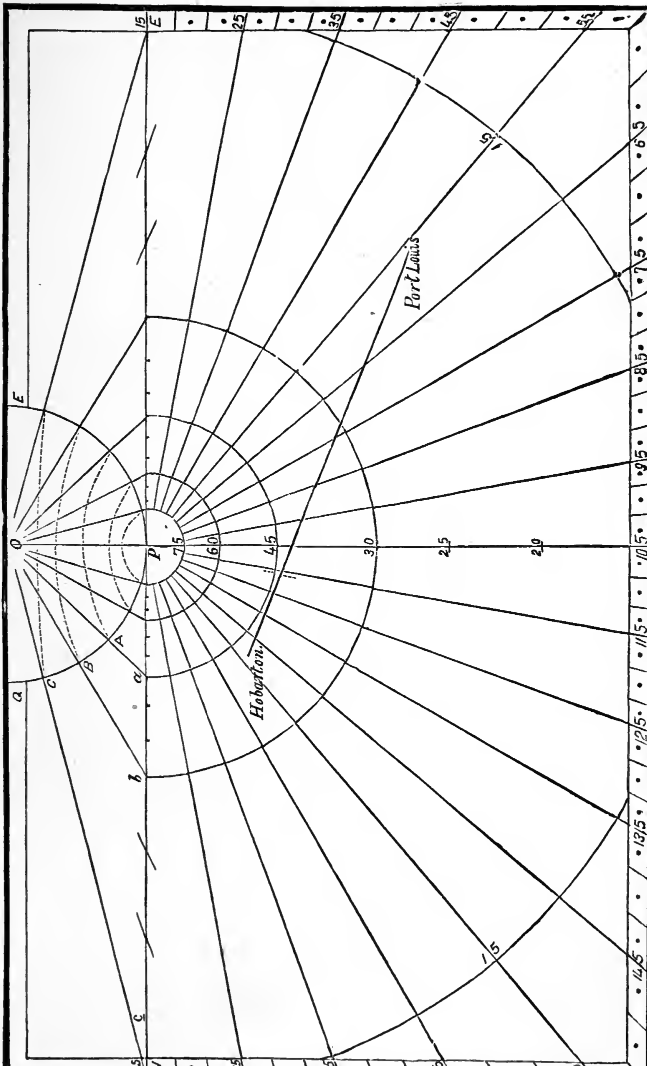


FIG 36.

O be an eye at the centre of the earth; then it will be in the plane of all great circles which can be drawn on the surface of it: hence it will see all its great circles as straight lines, and they will all (except the equator) be projected on the tangent plane as such. The meridians all passing through the pole, will meet each other then on the tangent plane, and make the same angles as they really do on the globe. In the figure they are represented as straight lines making angles of 10° with each other and all radiating from P. The eye O will see the point A in a parallel of latitude projected at *a* on the tangent plane, and the parallel of latitude through A will be seen as a circle whose centre is P and radius P*a*. Similarly B will be seen projected at *b*, C at *c*, and so on; and hence the parallels of latitude will be seen as concentric circles, having the common centre P and radii P*a*, P*b*, P*c*, &c. From the principle on which the chart is constructed it is seen that P*a*, P*b*, P*c*, &c., vary as the tangents of the angles POA, POB, POC, &c., and hence the radii P*a*, P*b*, P*c*, &c., vary as the tangents of the co-latitudes or as the co-tangents of the latitudes. A polar chart constructed on this principle can very well be drawn down to 15° or 20° of latitude, and would then embrace all parts of the earth where great circle sailing is of most advantage. If the place of the ship and the place bound to be on opposite sides of the equator the construction is not so simple. The tangent plane must then be supposed to touch the earth at the equator, on which the meridians would then appear as parallel straight lines at varying distances and the parallels of latitude as equilateral hyperbolas. But it is quite unnecessary to have great-circle charts of low latitudes, because the difference between the rhumb track and the great-circle track is much less than in high latitudes, and the advantages gained are small; hence Mercator's sailing may then be used, especially when we consider that in low latitudes the trade winds and other meteorological phenomena play such important parts in practical navigation.

On the polar chart here shown we have laid down the great-circle track from Port Louis, in latitude 20° 10′ S., longitude 57° 22′ E., to Hobarton, in latitude 42° 54′ S., longitude 147° 21′ E., illustrating Example 328.

The latitude and longitude of the vertex can be seen at once, for it is where the straight line representing the track approaches nearest to the pole (dotted in the figure) at nearly 45° S. latitude and 125° E. longitude. The distance can be estimated on the line 15° E., 165° W., or on either of the meridians if graduated, by taking half the distance and measuring from the middle latitude each way as on a Mercator's chart. The longitude where the great circle cuts any parallel of latitude can be seen by laying a ruler over the pole and reading the longitude from the graduations around the figure, and the course can be taken off the chart by measuring the angle (with a protractor) between the great-circle track and a meridian drawn through the place of the ship; or, if preferred, it could be shaped from point to point by Mercator's sailing. In the Gnomonic projection, if we wish to find the course from a place in one latitude to another in the opposite latitude, we proceed as follows: Lay down the position of either place on the polar chart; then because a great circle which passes through any place also passes through its antipodes, the straight line joining the position of the first place with the antipodes of the second will be the projection of a part of the great circle passing through the two places, and the course can be obtained by measuring the angle between the great circle and the meridian as before.

The advantages of such a chart are:

(*a*) The simplicity of construction. Any master of a vessel can construct one as easily as he can a Mercator's chart.

(*b*) The positions of places can be laid down as easily as on a Mercator's chart, and then a straight line from one to the other shows the great-circle track between them. The latitude in which it cuts any meridian is seen at a glance, and hence the direction in which the ship must be steered from point to point on a great circle is rendered easy and mechanical.

(*c*) The vertex or highest latitude on the great circle is seen by inspection; or, if preferred, it is the point where the perpendicular from the pole meets the great-circle track.

(*d*) If the continents and islands be laid down on such a chart, by simple inspection it is seen if any land prevents sailing on the great circle from one port to another.

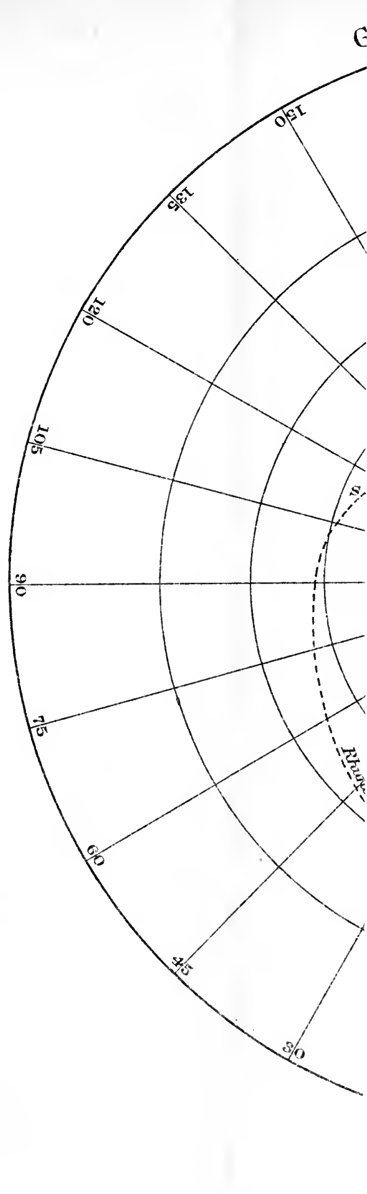
(*e*) If a ship's position be laid down at noon, and a straight line be drawn from that point to the port of destination, any of the above information is obtained by a moment's inspection.

If the student wishes to construct a great-circle chart for use, it is advisable it should be drawn as large as possible, and the whole polar area should be shown as in the accompanying diagram. This chart is especially adapted to *composite sailing*, and to show its use we have laid down the composite track illustrating Example 326, which we have worked from Dunedin to Wellington Isle with the same limitation of maximum latitude. The great circle from Dunedin is first drawn, a straight line touching the circle, which represents the 55° parallel of latitude; the point of contact is the point where the great circle meets the parallel, and is about 145° W.; and a similar tangent to the same circle through Wellington Isle gives the longitude on leaving the parallel $111\frac{1}{2}$° W. The portion of the parallel intercepted between 145° W. and $111\frac{1}{2}$° W. is that part of it sailed over in the composite route. The student can here see how nearly the method by projection agrees with that by calculation.

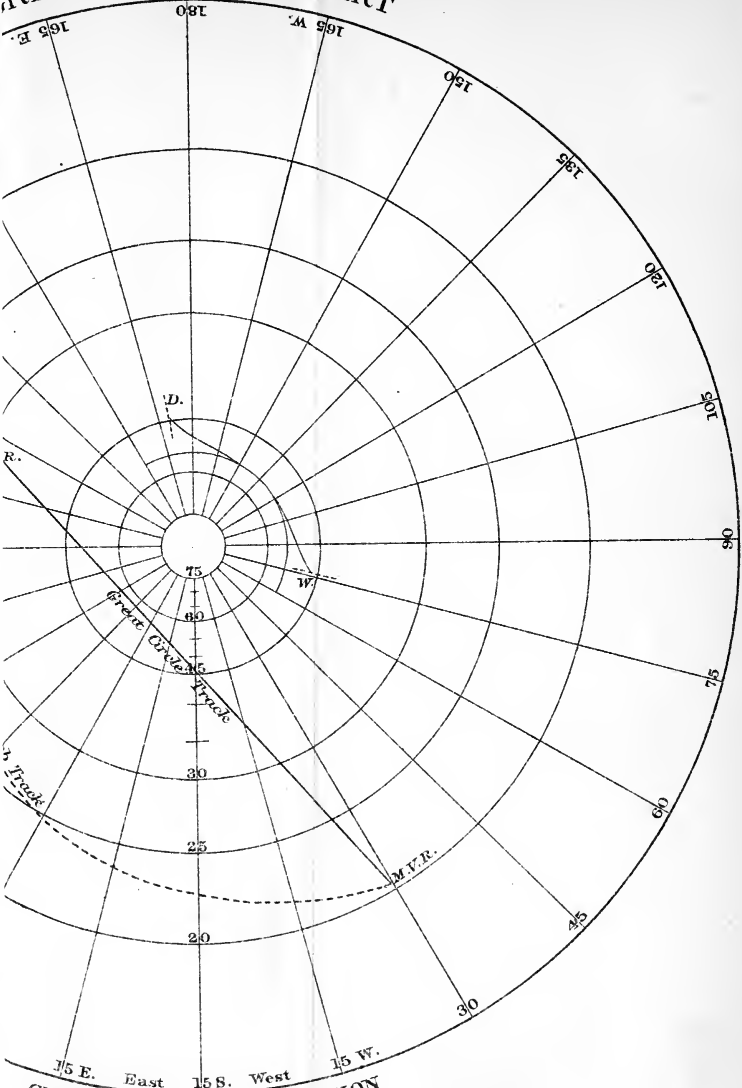
We have also laid down the great-circle track from Martin Vas Rocks (latitude 20° 40′ S., longitude 29° 20′ W.) to Swan River (latitude 32° 3′ S., longitude 115° 45′ E.), where it will be seen the vertex is about 59° S. and about 145° E. To show the difference between this and the rhumb track between the two places (which could not be made at sea because Africa intervenes) we have also dotted in the projection of the rhumb curve.

Windward Great-circle Sailing. — In making long voyages it often happens that a head wind prevails for several

G



REAT CIRCLE CHART



GNOMONIC PROJECTION

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days together; it then becomes of great importance to know on which tack to put the ship so as to save time. It is evident from what has already been said that (all other things being equal) the nearer a vessel can be kept to a great-circle course the less time will be lost. In the example we have worked out, the course on the rhumb curve from Port Otago to Callao is N. 70° 40′ E., or about E. by N$\frac{3}{4}$N.; now suppose the wind to be from the same point (the prevailing wind at the place), and that a vessel can lie within six points of the wind, then by putting her on the starboard tack when leaving Port Otago, she will sail N$\frac{1}{4}$E. and on the port tack SE$\frac{1}{4}$S. To a casual observer it would seem of no importance which course was chosen; but it has been shown that the first course on the great circle is about SE. by E$\frac{3}{4}$E.: hence the vessel should be put on the port tack. In the former case (on the starboard tack), because N$\frac{1}{4}$E. is *ten* points from the first great-circle course, it is clear she is really making two points *away* from her port—that is, in every 10 miles she will be nearly 4 miles further away from her port than when she started; whereas by putting her on the port tack her course will be SE$\frac{1}{4}$S., and thus she will sail within two points of the course that would keep her on the shortest route, or in every 10 miles she sails she will near her port by 9$\frac{1}{4}$ miles. By consulting any chart of winds and currents it will be seen that the rhumb course runs direct against the prevailing winds and currents, hence the great-circle course should here be preferred for that reason; whereas if bound in a contrary direction the master of the vessel could select which method would be most advantageous. Again, in this example the great-circle track cuts the zone of calms more directly than the rhumb course does, and should also be preferred for that reason. It is worthy of notice that if the rhumb differ more than two points from the great-circle course, and a vessel be put on the wrong tack, she will head more than eight points away from her destination, and thus will be increasing her distance from her port.

USE OF A TERRESTRIAL GLOBE IN GREAT-CIRCLE SAILING.—

By carefully adjusting a terrestrial globe, a great circle through any two places may be drawn by the following method :—

Bring both places to coincide with the upper edge of the wooden horizon. This is done by elevating or depressing the pole as necessary. Then with a fine lead pencil draw a line on the globe in the plane of the horizon from one place to the other; this is the required great circle. Take the latitudes and longitudes of as many points on the circle as may be deemed sufficient, and lay them down on a Mercator's chart; then draw a curve through the points. This will be the representation of the great circle on the chart, and the courses and distances from point to point may be calculated by either Mercator's or middle-latitude sailing, or they may be taken from the chart at once. The distance between the two places on the great circle is found by counting the number of degrees on the wooden horizon intercepted between the two places, these, multiplied by 60, give the distance in nautical miles. This is the mechanical solution of great-circle sailing, and will be found by far the shortest method. If a globe of 12 inches diameter be used, the results will be sufficiently accurate for all practical purposes; while the course on the great circle can be laid down each day at noon from the position of the ship to the port of destination without the use of a single figure, and the vessel can be laid on that tack which looks best up for her port. In addition to these advantages, the plane of the wooden horizon passes through all places on the required great circle in both hemispheres, and the latitude and longitude of the vertex is seen at once by inspection. The student should verify the results obtained in each of the following examples by using a terrestrial globe, and also where possible by laying down the places on a chart on the Gnomonic projection.

To show how the work should be arranged we will take another example.

Ex. 327. Find the course and distance on the great circle, and *vice versa*, from Kerguelen's Land (latitude 49° 54′ S., longitude 70° 10′ E.) to Ascension Isle (latitude 7° 56′ S., longitude 14° 26′ W.). Find also the position of the vertex.

Kerguelen's Land	lat. 49° 54′ S.	∴ co-lat. 40° 6′	long. 70° 10′ E.
Ascension Isle .	lat. 7 56 S.	∴ co-lat. 82 4	long. 14 26 W.
Sum co-lats.		122 10	2)84 36
Difference co-lats.		41 58	$\frac{P}{2}$ = 42 18

To find the course.

Half diff. long. $\frac{P}{2}$	= 42° 18′	cot 10·040992	.	cot 10·040992
Half sum co-lats. $\frac{a+b}{2}$	= 61 5	sec 10·315570	.	cosec 10·057831
Half diff. co-lats. $\frac{a-b}{2}$	= 20 59	cos 9·970200	.	sin 9·554000
$\frac{A+B}{2}$	= 64 46	tan 10 326762		
$\frac{A-B}{2}$	= 24 13	. . . . .	.	tan 9·652823

∴ A = S. 88° 59′ W., course from Kerguelen's Land to Ascension.
B = S. 40 33 E., course from Ascension to Kerguelen's Land.

To find the distance.

Half sum co-lats. $\frac{a+b}{2}$	= 61° 5′	. .	cos 9·684430
Half sum course $\frac{A+B}{2}$	= 64 46	. .	sec 10 370279
Half diff. long. $\frac{P}{2}$	= 42 18	. .	sin 9·828023
Half distance . . .	40 14	. .	cos 9·882732
	2		
Distance	80 28 = 4828 nautical miles.		

To find latitude of vertex. *To find longitude of vertex.*

Lat. A	= 49° 54′	cos 9·808969	. .	sin 9·883617
Angle A	= 88 59	sin 9·999932	. .	tan 11·750898
Lat. vertex	= 49 54	cos 9 808901	1° 20′ =	cot 11·634515

Longitude A	70° 10′ E.
Difference of longitude . .	1 20 W.
Longitude vertex	68 50 E.

We will now show how to arrange the work in the second and third methods.

Kerguelen's Land (A) lat.	49° 54′ S.	long.	70° 10′ E.
Ascension Isle (B) lat.	7 56 S.	long.	14 26 W.
Diff. lat.	41 58	diff. long.	84 36
Half diff. lat.	20 59	half diff. long.	42 18

The two formulæ are for the distance :

$$\left.\begin{aligned}\cos^2\theta &= \sin^2(\text{half diff. long.}) \cdot \cos(\text{lat. A}) \cdot \cos(\text{lat. B}) \\ \cos^2(\text{half dist.}) &= \sin(\theta + \text{half diff. lats.}) \sin(\theta - \text{half diff. lats.})\end{aligned}\right\}$$

Half diff. long.	42° 18′	sin	9·828023
			2
		sin ²	19·656046
Lat. A	49 54	cos	9·808969
Lat. B	7 56	cos	9·995823
Half diff. lat.	= 20 59		2)19·460838
θ	= 57 29	cos	9·730419
θ + half diff. lat.	= 78 28	sin	9·991141
θ − half diff. lat.	= 36 30	sin	9·774388
			2)19·765529
Half distance	40 14	cos	9·882765

∴ Distance 80° 28′ = 4828 nautical miles.

To find the distance by third method.

Kerguelen's Land (A) lat.	49° 54′ S.	long.	70° 10′ E.
Ascension Isle (B) lat.	7 56 S.	long.	14 26 W.
Co-lat. B	= 82 4	diff. long.	84 36

Tan θ = cot (lat. A). cos (diff. long.) *Cos (dist.) = sin (lat. A) . sec θ. cos φ*

Lat. A	49° 54′	cot	9·925352	sin	9·883617
Diff. long.	84 36	cos	8·973628		
θ	4 32	tan	8·898980	sec	10·001361
Co-lat. B	82 4				
(Co-lat. B − θ)	77 32		ϕ	cos	9·334195
Distance	80° 28′			cos	9·219173

Hence distance = 4828 nautical miles.

To find the first and last courses.

$$\sin\frac{A+B}{2} = \cos\frac{P}{2} \cdot \cos\frac{a-b}{2} \sec\frac{d}{2}$$

$$\sin\frac{A-B}{2} = \cos\frac{P}{2} \cdot \sin\frac{a-b}{2} \operatorname{cosec}\frac{d}{2}$$

$\frac{P}{2}$	= 42° 18	cos	9·869015	cos	9·869015
$\frac{a-b}{2}$	= 20 59	cos	9·970200	sin	9·554000
$\frac{d}{2}$	= 40 14	sec	10·117236	cosec	10·189833
$\frac{A+B}{2}$	= 64 46	sin	9·956451		
$\frac{A-B}{2}$	= 24 13			sin	9·612848

A = S. 88 59 W., first course from Kerguelen's Land
B = S. 40 33 E., first course from Ascension Isle

To find the position of the vertex.

Cos (lat. vertex) = sin (diff. long.). cos (lat. A). cos (lat. B). cosec (dist.)

Diff. long.	84° 36′	sin	9·998068
Lat. A	49 54	cos	9·808969
Lat. B	7 56	cos	9·995823
Distance	80 28	cosec	10·006040
Lat. vertex	49 54 S.	cos	9·808900

Cos (diff. long. vertex is from A) = cot (lat. vertex) . tan lat. A

Lat. vertex	49° 54′	cot	9·925352
Lat. A	49 54	tan	10·074648
Diff. long. vertex from A	0 0	cos	10·000000

∴ Longitude vertex 70° 10′ E.

It will be observed there is a discrepancy in the longitude of the vertex obtained by the two methods. This arises from not working the angles of position to seconds, as may be seen from the large differences of the logarithmic tangents when the angle is large as A, in this case amounting to 88° 59′; but that the latter is correct is evident, because as the latitude of the vertex is at Kerguelen's Land, so must the longitude be as well. The course and distance when worked by Mercator's sailing is N. 59° 38′ W. and 4981 miles.

Exercise XVII.

Ex. 328. Find the distance on the great circle between Port Louis (latitude 20° 10′ S., longitude 57° 32′ E.) and Hobarton (latitude

42° 54′ S., longitude 147° 21′ E.). Find also the initial and final courses and the latitude and longitude of the vertex.

Compare the projection on the great circle track.

Ex. 329. Find the initial and final courses, the distance and position of the vertex on a great circle between Cape Hatteras (latitude 35° 15′ N., longitude 75° 30′ W.) and the Lizard (latitude 49° 58′ N., longitude 5° 12′ W.).

Ex. 330. Find the initial and final courses, the distance and the position of the vertex on a great circle between Tristan da Cunha (latitude 37° 2′ S., longitude 12° 17′ W.) and Java Head (latitude 6° 47′ S., longitude 105° 13′ E.).

Ex. 331. The latitude of two places is 58°, their difference of longitude is 100°; what is the difference of their distance from each other when measured on their parallel and when measured on the arc of a great circle passing through them. *Royal Naval College*, 1868.

Ex. 332. Prove that the shortest distance between two places on the earth's surface is the arc of the great circle joining them. Find the distance on the arc of a great circle between Pitcairn's Island and Juan Fernandez, and the course on which the ship would start on this passage by great-circle sailing

Juan Fernandez Island .	lat. 33° 20 S.;	long. 79° 0′ W.
Pi cairn's Island . . .	lat. 25 0 S.;	long. 130 10 W.

A. 1876.

Ex. 333. What is the difference in the track between two ships, R and G, in proceeding from a place A (latitude 37° 30′ S., longitude 150° E.) to a place B (latitude 29° 5′ S. longitude 168° E.)? R sails on the rhumb line, and G is supposed to sail on the great circle. *A.* 1878.

Ex. 334. Find the shortest distance between Port Jackson (latitude 33° 52′ S., longitude 151° 16′ E.) and Valparaiso (latitude 33° 1′ S., longitude 71° 52′ W.). Explain the method which is practically used in great-circle sailing. *A.* 1879.

Ex. 335. Find the initial course in sailing on the great circle from the Butt of Lewis (58° 32′ N., 6° 21′ W.) to Cape Race (46° 40′ N; 53° 3′ W.), and the initial course on the return voyage on the great ci·cle. What is the course on the rhumb track in the two cases? Draw a figure showing the two tracks. *Honours*, 1879.

Ex. 336. A ship sails from Cape Cod (latitude 42° 3′ N., longitude 70° 6′ W.) to Cape Clear (latitude 51° 25′ N., longitude 9° 29′ W.) on the great circle, passing through the two places. Find the distance she sails, and the longitude of the point where she attains her highest latitude. *Honours*, 1880.

Ex. 337. Find the distance on the great circle between Cape Horn (latitude 55° 58′ S., longitude 67° 21′ W.) and Ascension (latitude

7° 57′ S., longitude 13° 59′ W.). What advantage is gained in this case by sailing on the great circle over sailing on a direct course? *Honours*, 1881.

Ex. 338. On what projection is the rhumb line joining two places on the earth's surface a straight line? How would you lay down a course on this projection? The rhumb line being a straight line on this projection, is it the shortest distance on the earth's surface? Show by a diagram the form which the shortest distance between the two places 180° of longitude apart would assume on the projection. *A*. 1869.

Ex. 339. Prove the rule for finding the shortest distance between two places on the earth's surface, and the latitude and longitude of the vertex. *Honours*, 1870.

Ex. 340. Investigate a formula for the distance between two places on a great circle. Why is it advantageous to sail on a great circle? State clearly the limitations. *A*. 1872.

Ex. 341. A ship in the southern hemisphere starting on a course NE. followed the great circle for 560 miles, and was then steering N. 38° E.; from what latitude did she start? *Honours*, 1873.

Ex. 342. Why cannot a ship always be kept accurately upon a great-circle track? Is there any particular case in which it can? Investigate a rule for finding the latitude and longitude of the vertex of a great circle between two places. *A*. 1874.

Ex. 343. In sailing from a place A (latitude 51° 25′ N., longitude 9° 29′ W.) to a place B (latitude 46° 40′ N., longitude 53° 3′ W.) on the arc of a great circle, find the initial course from A to B, and on the return voyage from B to A. How much do these differ from the course when sailing on the rhumb between the two places? *Honours*, 1875.

Ex. 344. A ship leaves a place in south latitude, the longitude of which is 40° 30′ E., and sails on the arc of a great circle with an initial course SE. by E.; and after keeping to the great circle for 3,000 miles she is steering ENE. Determine the positions of departure and arrival. *Honours*, 1876.

Ex. 345. A ship sails from A to B, both being places in south latitude, on the arc of the great circle joining them, her initial course is S. 45° W. and her final course N. 60° W., and the latitude of the most southerly point reached was 49° 50′. Required the latitudes of A and B, and their difference of longitudes? *Honours*, 1877.

Ex. 346. A ship in sailing from A to B keeps on the arc of a great circle; her initial course is S. 65° W., and her final course N. 70° W., and the latitude of her most southerly point reached was 42° 30′ S. Required the latitudes of A and B, and the longitude of B, that of A being 90° W. *Honours*, 1878.

Ex. 347. What do you mean by great-circle sailing? How can you show that the arc of a great circle is the shortest distance between any two points on the surface of a sphere?

Ex. 348. Define :—Great circle, small circle, spherical angle, spherical triangle, course in great-circle sailing, angle of position, distance and vertex.

Ex. 349. Why is it necessary in practice to find a succession of points on the great circle, and of what advantage is the knowing of the position of the vertex?

Ex. 350. What objections have been urged to the use of great-circle sailing? Show the advantages of a knowledge of great-circle sailing, especially with wind or current in opposition to the course on a rhumb curve.

Ex. 351. What do you mean by the Gnomonic projection? What are its great advantages? Construct a chart on the Gnomonic principle extending to 20° from the equator, and prove the questions already given in great-circle sailing?

Ex. 352. A ship sails between two points on a great circle. Obtain an expression for the distance. *Royal Naval College*, 1864.

Ex. 353. Explain accurately the difference between the rhumb line (curve) and the great circle which joins two points on the earth's surface. Illustrate by a diagram as they would appear on a Mercator's projection. On what projection would a great circle always be a straight line? Show how great-circle sailing is practically applied Find the difference in distance in sailing on a great circle and on a rhumb line from Cape Clear (latitude 51° 25′ N., longitude 9° 29′ W.) and New Orleans (latitude 29° 58′ N., longitude 90° 11′ W.). *Honours*, 1869.

Ex. 354. Draw a figure and investigate a formula for obtaining courses and distances for sailing in a great circle between two places whose latitudes are l_1, l_2, and longitudes L_1, L_2, about 3,000 miles apart. *Honours*. 1871.

Ex. 355. What do you mean by great-circle sailing, and what do you consider its advantage? Find the distance on the arc of the great circle joining Cape Clear (latitude 51° 25′ N., longitude 9° 29′ W.), and Bermuda (latitude 32° 20′ N., longitude 64° 30′ W.). *A*. 1873.

Ex. 356. Explain what you understand by composite sailing. A ship on the composite track leaves a place A, in latitude 34° 29′ S., and longitude 18° 23′ E., for another place B to the westward, and she is not to go to a higher latitude than 45° S. Find the course at A, and the longitude of the place where she will attain the parallel of maximum latitude. *Honours*, 1873.

Ex. 357. Explain clearly the reason why in sailing from one place to another a ship does not take the shortest route, and show the practical necessity and advantages of sailing on the rhumb line (curve). Prove that the tangent of the course thus shaped from A to B is equal to the difference of longitude of A and B divided by the meridional difference of latitude of A and B. Define and distinguish between 'meridian distance' and 'meridional difference of latitude.' *A.* 1876.

Ex. 358. Define a 'great circle' and 'great-circle sailing.' State the advantages and limitations of great-circle sailing. Why is it necessary to determine a succession of points on the great circle between the places of departure and arrival? Explain the method of finding the latitude and longitude of the vertex. *A.* 1878.

Ex. 359. How does great-circle sailing differ from rhumb sailing? State the characteristic property of each of these two curves which constitutes its special advantage. Write down the formulæ for finding the shortest distance between two points on the earth's surface. *A.* 1880.

CHAPTER IX.

Terrestrial magnetism—Force only directive, not one of translation—How variation, dip, and intensity are found—Semicircular deviation—Quadrantal deviation—Composition of forces—Coefficients A, B, C, D, and E, how produced and how calculated—Heeling error, how produced and how found—Napier's graphic method—Compensations—Exercises and examination.

TERRESTRIAL MAGNETISM AND DEVIATION.

WE will now return to the subject of magnetism and deal more fully with the error called Deviation. We have already stated that the earth's action on a freely suspended needle is the same as if another magnet were brought near enough to influence it. The effects produced by the earth have received the name *terrestrial magnetism.* In order to obtain a complete knowledge of terrestrial magnetism we must know:

(1) The variation or declination of the compass.

(2) The inclination or dip of the earth's magnetic force.

(3) The intensity of that force.

Variation, dip, and *intensity* are called the magnetic elements.

VARIATION *is the angle included between the terrestrial and magnetic meridians.* This error of the compass is due entirely to the action of the earth on the needle, and the force which causes polarity in it is simply directive and not one of translation. This may be proved by floating on water a small magnet attached to a piece of cork. The magnet, after a few oscillations, will assume the position of the magnetic meridian, but will not be translated either towards the north or south. The reason is, the magnetic poles of the earth (in which the controlling power is supposed to lie) are at such distances from the needle, that both poles of the latter may be considered equally distant from them; and although the red-marked end of the needle is attracted to the north by the blue magnetism supposed to be residing there, the south end of the needle is repelled by an equal force. A similar effect results from the south magnetic pole of the earth, and, consequently, the needle moves neither way. From the forces on the needle forming a couple, by the earth's blue magnetism attracting the north end of the needle, and the earth's red magnetism attracting the south end of the needle, the latter is compelled to take up the definite position it does, namely, in the magnetic meridian. Methods for determining the variation of the compass are wholly astronomical. It is found by observing the bearing of a heavenly body with a compass uninfluenced by anything except the earth, and at the same instant taking its altitude. The true bearing for the time is then calculated by one of the following formulæ.

(1) Sin amplitude = sec lat × sin declination.

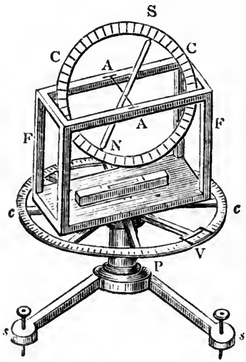
(2) Sin half azimuth = $\sqrt{\text{sec lat . sec alt . cos s. cos } (s \backsim p)}$; but as these methods belong to nautical astronomy we must defer their consideration until we treat of that subject. Having calculated the true bearing by one of the above methods, the variation is found by taking the difference between the true and magnetic bearings of the object. It is easterly if the true be to the right, but westerly if to the left of the magnetic bearing. It is found thus:—

Suppose the calculated true bearing to be	S. 41° 15′ E.
and the observed magnetic bearing	S. 27 40 E.
the variation would be	13 35 W.

Dip or Inclination *is the angle included between the horizontal plane and the axis of the needle when allowed to come freely to rest:* and the straight line through the poles of the needle at that time is called *the line of force.* As a free magnet constantly points to one of the magnetic poles of the earth, it follows that a needle must incline towards that pole which exerts the greatest influence, unless some external force prevents it; or that it be situate at such a distance from the poles that the terrestrial magnetism exerts an equal force in each direction. Such is the case on the magnetic equator. The instrument used for obtaining the dip is called *the Dipping Needle.*

If an ordinary needle be suspended by a horizontal arbor through its centre of gravity, it will hang in any position in which it may be placed, and will show no disposition to depart from that position; but if it be magnetised and placed in the plane of the magnetic meridian, then it will not remain in any but one position, which is inclined to the horizon. On the principle here involved, the dipping needle is constructed. It consists of a graduated circle C C fixed vertically in a frame F F To the frame is attached a Vernier V, which moves with the frame over a horizontal graduated circle *c c*. The whole is supported by a pillar P on a tripod and levelling screws *s s*. Two knife edges A A are fixed at the centre of C C to the frame, and parallel to the plane of the circle. Through the centre of gravity of the needle N S, and at right angles to it, a fine arbor of polished steel is fixed; this supports the needle on the knife edges, and thus friction is almost obliterated. Because the arbor passes through the centre of gravity of the needle, it will, before being magnetised, remain fixed in any position it may be placed; but when magnetised, the instrument levelled, and the circle placed in the plane of the magnetic meridian, the needle will take up the definite position of the dip, which is at once read off the graduated circle C C.

FIG. 37.



As an extemporary method of finding the dip the following may be used. Take a small sensitive pocket compass and when isolated allow it to come to rest, turn the box containing it until the needle coincides exactly with the north and south points of the card. Next take a bar of soft iron, about three feet long, and place it horizontally in the magnetic meridian, so that one end shall be either due east or west of, and level with the centre of the needle, and the other end towards the nearest magnetic pole. It will be found this has caused the needle to deviate from its former position; then gradually raise the remote end of the bar until the needle again coincides with the magnetic meridian. The angle which the bar of iron now makes with the horizontal plane is the complement of the dip. The reason for this is, by raising the end of the bar it gradually reaches the plane parallel to the magnetic equator, and as soon as it comes to that plane it exerts no influence on the needle. The lines of equal dip or isoclinal lines, correspond to parallels of latitude, and the term magnetic latitude is used to denote the position of places with reference to the magnetic dip. In going from the magnetic equator to the magnetic poles, the dip increases more rapidly than geographical latitude does; and the increase is greater near the magnetic equator than near the poles. In the former case it is about 2° for every degree from the equator, whilst near the poles the increase is only about one fourth of this amount. It may be approximately calculated from the formula—

$$\tan \text{dip} = \text{twice} \tan \text{lat.}$$

where the latitude employed is magnetic. This law was discovered by Krafft in 1809.

MAGNETIC INTENSITY *is the force necessary to bring a needle of unit size and of unit strength when freely suspended back to the magnetic meridian when driven from it by any external force.* Magnetic intensity differs in different parts of the earth, and the needle obeys laws similar to those which govern the pendulum: the force which acts on the needle is terrestrial magnetism and on the pendulum is terrestrial gravity; and like gravity terrestrial magnetism is least at the equator and increases towards the poles, where the intensity is between two and three times that at the equator. The following is the method for comparing magnetic intensities. If a needle freely suspended be made to oscillate for a given time at a certain place, and then be removed to some remote place, and again be made to oscillate for an equal interval, it will be found the number of oscillations is different, and the intensities of the earth's magnetism at the two places is respectively proportional to the squares of the number of oscillations. Thus, if at A it made 30 oscillations per minute, and at B 31, we should have—

$$\frac{\text{Intensity of Earth's magnetic force at A}}{\text{Intensity of Earth's magnetic force at B}}=\frac{30^2}{31^2}=\frac{900}{961}=\frac{15}{16}\text{ nearly.}$$

That is, the intensity of the earth's magnetism at the first place is $\frac{15}{16}$ of that at the second place. It should be noted, the needle must be *freely suspended*, and we thus obtain the ratio of the *total* intensities of the earth's magnetism at two places, and these act in the line of force.

The magnetic elements are subject to changes, some of which are regular, others irregular. The regular changes are classed as secular, annual, and diurnal variations.

Secular Variations are those which recur after very long intervals. They take centuries for their completion. The following table, taken mainly from 'On the Deviations of the Compass,' by Capt. E. J. Johnson, R.N., F.R.S., will give some idea of the secular variations in London and its vicinity since accurate observations have been recorded :—

Year	Declination	Year	Dip
1580	11° 15′ E.	1576	71° 50′ N.
1622	5 50 E.	1613	72 30
1657	0 0		
1672	2 30 W.	1676	73 30
1723	4 17 W.	1723	74 42
1745	17 0 W.		
1773	21 9 W.	1772	72 19
1786	23 17 W.	1786	72 8
1790	23 39 W.	1790	71 53
1804	24 8 W.	1805	70 25
1813	24 22 W.		
1815	24 27 W.		
1821	24 23 W.	1821	70 4
1838	23 59 W.	1838	69 17
1846	22 49 W.	1846	68 50
1865	21 6 W.	1865	68 9 N.

From the above table it will be seen that the maximum variation was in 1815, but Colonel Beaufoy placed it about

March 1819, when he states it was in London 24° 41′ W. At the present time it is decreasing at the rate of about 7 minutes annually. The maximum dip occurred in 1723, and is now decreasing at the rate of about 2′·69 per annum. Professor Barlow, in his 'Essay on Magnetic Attraction,' says: 'All these changes may be accounted for by supposing the magnetic poles to revolve round the terrestrial ones.' And thus the changes here described in the whole system of terrestrial magnetism, including all the magnetic elements, are brought about by the revolution backwards of the magnetic pole relatively to the terrestrial pole. The north magnetic pole may be supposed to describe a small circle with 20° radius from the earth's north pole in 960 years. In 1657 it was between London and the true north pole, and since that time it has travelled to the westward to its present position. About the year 2140 A.D. we may expect it again to be due north, but then in longitude 180° or on the other side of the terrestrial pole.

Annual Variations are those changes which take place at stated times of the year. From observations made first by Cassini, and afterwards repeated by him in the cellars under the Observatory at Paris, it appears that from April to July the western declination decreases; for the other nine months, from July to April, the western declination increases. In May and October the positions are nearly the same, and during the winter months the changes are small. According to Hanstein, the dip is about 15′ greater in summer than in winter. Father Secchi says: 'The annual disturbances of the magnetic elements are at a maximum at the Equinoxes, and at a minimum at the Solstices.'

Diurnal Variations were first noticed by Graham in 1722. In the northern hemisphere the mean movement of the red end of the needle, from 8 A.M. to 1 P.M., is from east to west; the other part of the day, from west to east. At the same hours in the southern hemisphere, the same end of the needle moves in opposite directions. The maximum, at 8 A.M., is at Kew about 4′ E. and at 1 P.M. about $6\frac{1}{4}$′ W. of the mean position for the day. The diurnal variation is different in different months, and its greatest range is in May, when it is about 12′, and least in December, when it is about $5\frac{1}{2}'$. The inclination is about 4′ or 5′ greater before noon than after.

Thus it would seem the westerly declination in the Annual and Diurnal variations decreases in amount as the heat received from the sun increases, and *vice versâ*; a coincidence which leads one to believe that the sun's heat must have some influence on terrestrial magnetism, and goes far to support the theory that it is derived from Thermo-Electricity.

Irregular Changes are called perturbations. They take place during thunder-storms, earthquakes, volcanic eruptions, the appearance of the aurora, and when any sudden change occurs in the electrical condition of the atmosphere. When perturbations take place simultaneously over large tracks of the earth, Humboldt called them *magnetic storms*; and General Sabine found that they coincided in frequency with the periods of maximum sun spots—namely, every $11\frac{1}{9}$ years.

How Compasses act under the Influence of Iron Bars.—If a soft iron bar be held vertically, it has been shown that it acquires magnetism at its ends by induction from the earth, and whilst in that position its effects may be regarded as if produced by a permanent magnet. Place a small sensitive compass in the centre of a small circle, and a vertical bar so that its lower end shall be level with the needle, and in the magnetic meridian on the circumference of the circle, which should be of such a size that the directive force of the needle be not wholly overcome by the induced magnetism in the bar. The lower end of the bar will be found to have acquired magnetism of the same name as that in the north end of the needle, and will repel the needle; but as the line of action of its force is in the same direction as that of the needle's, the only effect will be to weaken the action of the needle, as may be proved by making it to oscillate. Now carry the bar at the same level on the circumference of the circle, say towards the east; then, as the directive force of the needle acts at an angle with the bar, it is deflected towards the west, and this will continue until a line from the bar is perpendicular to the needle at its centre, when its deflection will be greatest.

Let the revolution of the bar be continued, then as it comes nearer to the south end of the compass we shall have magnetisms of opposite names in the lower end of the bar and in the south end of the needle contiguous to one another, and attraction is the result, thus keeping the north end of the needle to the west of north. By-and-by, as the bar revolves, it will be seen the needle will again regain its normal position, when it will be found the bar is also in the magnetic meridian, but to the south of the compass, and the directive force of the needle is augmented. Continue carrying the bar around the circle, and the south end of the needle will now follow the bar, causing the north end to deviate towards the east. A similar maximum deviation will also be found on the west side, and when the circle is complete the needle will be found again in the magnetic meridian. Opposite effects, but the same in character, will be seen if the upper end of the rod be used. A superficial observer will notice

(1) The bar is held always vertically.

(2) The deviation is always westerly (with the lower part of the bar) when the bar is in the eastern semicircle, and always easterly when it is in the western semicircle. We should have seen exactly the same effects if the red marked end of a permanent magnet had been used instead of the bar. Hence, because the results are different in the two semicircles, the deviation arising from subpermanent magnetism and from vertical soft iron is called *Semicircular deviation.* It has already been shown that induced magnetism from the earth changes with a change of geographical position; the result from the bar will therefore always be similar, but will vary in magnitude with a change of latitude and longitude, and will even be reversed on opposite sides of the magnetic equator.

Let us vary the experiment and now place the bar horizontally. In this position, too, as already shown, it will be magnetised by terrestrial induction. Beginning as before with the end nearest the compass in the circumference of the circle and its length now in the magnetic meridian, we find the north end of the rod has acquired red magnetism and that nearest the needle has acquired blue magnetism : thus the directive force of the needle will be increased, and its direction not altered. Carry the rod around the circle with its length always pointing to the centre of the compass; then we find that the north end of the needle will follow the bar until the latter comes about NE., after which, as the bar is carried around, the needle will gradually return to its normal position, and then we shall find the bar to be east and west. In that direction the bar is lying parallel to the magnetic equator and has no magnetism imparted to it *in the direction of its length* by terrestrial induction, and hence it has no influence on the needle. As we continue carrying the bar around the circumference of our circle, with its length always pointing to the centre, we find as we get into the south-eastern quadrant that the end of the bar which was formerly pointing southward, and then had acquired blue magnetism, is now pointing northward, and has acquired magnetism of the contrary kind, and will now attract the south end of the needle, causing the north end to deviate towards the west, and the greatest deviation is found when the bar is lying about SE. of the needle. As the experiment is continued, the bar will gradually come into the magnetic meridian again, and then the needle will once more regain its normal position, but its directive force will again be increased. Similar changes will be found as the bar is carried through the south-west quadrant producing easterly deviation, and when carried through the north-west quadrant will produce westerly deviation. Let us now sum up the results of our observation.

(1) The bar has been kept horizontal throughout.

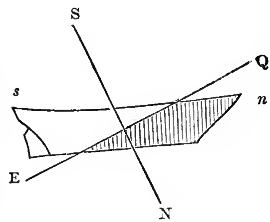
(2) In each quadrant it has produced similar changes; that is, it has produced a maximum deviation in each quadrant at the four intercardinal points, and no deviation at the cardinal points.

(3) The deviations are opposite in kind in contiguous quadrants, and of the same name in opposite quadrants.

Our conclusion must be that horizontal soft iron causes different deviations according to the quadrant it lies in from the compass; and hence the deviation arising from the effects of horizontal soft iron is called *Quadrantal deviation.*

How Compasses act in Iron Ships.—To understand this, it is necessary the student should thoroughly comprehend the effects produced on soft iron by the inductive action of the earth. What has been said about bars is equally applicable to ships. Let *n s* be an iron vessel built with her head magnetic north, and S N the line of force or direction of the dipping needle; then all the part of the vessel below the plane parallel to the magnetic equator E Q will have acquired red or north magnetism (seen by the marking), whilst the part above E Q will have acquired blue magnetism by induction from the earth. The student should draw and trace for himself the changes which take place by altering the direction in which the ship was built. In these, as in the case of iron bars, percussion and vibration by hammering in rivetting render the iron more susceptible to the inductive force of the earth; and a part of the magnetism thus impressed does not again leave the vessel, although, after launching, the major part gradually does. The part retained has received from the Astronomer-Royal the name of *Subpermanent magnetism*, because it is lasting in its character, and to distinguish it from the permanent magnetism in steel magnets. Its effects on compasses will evidently depend on the direction the ship's head was in whilst being built. If built with her head north, the fore part of the ship has acquired red magnetism, and will act like the red pole of a magnet. The line of its action will coincide with that one which was in the magnetic meridian before the ship was launched; hence in our supposed case, in sailing northerly, the red end of the compass needle will be repelled, and the directive power of the needle diminished. On southerly courses the red end of the needle points towards the stern, which has acquired subpermanent blue magnetism, then the directive power of the needle is increased. When sailing easterly or westerly, the effects on the compass are greatest, because the force acts at right angles to the needle;

Fig. 38.



and at all intermediate stations the disturbances due to the position are intermediate. As the ship's head is brought east of north, repulsion of the red end of the needle takes place, and we get westerly deviation, and it reaches its maximum when the ship's fore-and-aft line is at right angles to the needle; beyond that position the fore part of the ship attracts the blue end of the needle, and westerly deviation is still the result. This continues until the ship's head is south, when the line of the ship's action is in the same line as that of the needle, and no disturbance occurs, but the directive power of the needle is greater. When the ship's head is brought round west of south, the blue pole of the needle is still attracted; this causes easterly deviation, and the maximum is again reached when the ship's fore-and-aft line is at right angles to the disturbed needle; this must occur to the north of west. After that point has been reached, we find the fore part of the ship repelling the red end of the needle, and easterly deviation being still the result until the ship's head is again north. Thus we find that in iron ships the disturbance of the compass is nothing when the ship's head is on the same or opposite point to that in which she was built; and that when her head is in one semicircle, easterly deviation is the result, and when in the other, westerly deviation is produced. The deviation resulting from subpermanent magnetism, and from the effects of vertical iron in the ship, has received the name of *Semicircular deviation*, from producing opposite effects when the ship's head is in opposite semicircles of the compass; or easterly deviation is caused when the ship's head is in one-half of the compass, and westerly deviation when the ship's head is in the other half.

Now, because the amount of magnetism induced by the earth in vertical iron changes with a change of geographical position, and is of contrary names on opposite sides of the magnetic equator, hence that part of semicircular deviation which is the result of vertical iron changes as the ship arrives at different latitudes and longitudes, and when she crosses the magnetic equator it becomes of an opposite kind; that is, if westerly deviation be produced on one side, easterly will be produced on the other. The part due to subpermanent magnetism remains constant in kind though different in amount in all latitudes and longitudes, unless the ship be subject to strain or other mechanical violence.

On the Effect of Horizontal Soft Iron on Compasses.—The effects of a horizontal bar of soft iron carried around the compass in a similar way to the magnet, vary as the cosine of the magnetic azimuth, that in the direction of the magnetic meridian they are wholly directive, and therefore produce no disturbance; that at east they vanish because cos 90° = 0, and between these two extremes a maximum is attained. Similarly a disturbance occurs in the SE., SW., and NW. quadrants. Because blue magnetism is induced in the end of the bar nearest the needle, in the NE. and NW. quadrants, attraction of the red end of the needle takes place at those times; and when the ship's head is in either of these quadrants the horizontal iron in her causes a deviation easterly in one quadrant and westerly in the other. Again, in the SE. and SW. quadrants red magnetism is induced in the end nearest the needle, and attraction of the blue pole of the needle is the consequence, thus again causing opposite deviations. Hence, because in the NE. and SW. quadrants the red end of the needle is drawn to the right, easterly deviation is the effect, but in the SE. and NW. quadrants opposite consequences follow. From these results, viz. that in every alternate quadrant different effects are seen from the action of horizontal soft iron, the deviation arising therefrom has been called *Quadrantal deviation.*

In the same way iron ships must play the parts of magnets on all compasses used in them. If a compass be placed in an iron basin of sufficient thickness, it has been found that all the directive power of the earth is completely cut off; with thinner basins part is effaced, and the needle has a sluggish movement: so with iron ships, it has been found that very strong compass needles become inactive when placed on board; and in men-of-war, when placed between the iron decks and sides, its directive power is very feeble indeed. The process recommended in the

Admiralty manual for determining the effect of iron ships on the directive force in the needle is as follows :—Take 'a small flat lenticular needle, $2\frac{3}{4}$ inches long and one-third of an inch broad, fitted with a ruby or sapphire cap, having a pivot of its own made to screw into the socket of the point of the standard compass or into a socket fixed in a small circular box with a glass top. The observer, holding a watch in his hand, causes the needle to deviate through an arc of 45° ; he then marks down the time, counts ten vibrations, and marks down the time ; and so on, marking every tenth vibration till the vibrations become too small to be observed. A well-constructed needle on shore in England should vibrate about 120 times. We thus obtain the times of 12 sets, of 10 vibrations each, the mean of which is taken as the time of 10 vibrations. It will be easily seen from inspection whether the observer has in any of the sets made a mistake by taking eight or twelve vibrations instead of ten.

'If T be the time of ten vibrations on shore, and T' the time of ten vibrations on board, and if H represents the directive force of the needle on shore, H' the directive force on board, then $H' = H \frac{T^2}{T'^2}$. . .'

This observation is very simple, and is completed in a few minutes. If the needle be of good quality the quantity T, or the time of ten vibrations observed on shore, will continue sensibly the same for months, and the shore observations will, therefore, only require to be made occasionally for verification, unless there is a change of geographical position.

Composition of Forces.—To know a force three conditions are necessary.

(1) *Its point of application* ; that is, the point at which it acts.

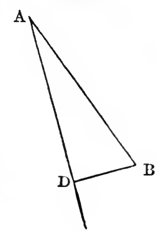
(2) *Its direction* ; that is, the direction in which it tends to move a body.

(3) *Its magnitude* ; that is, the power with which it acts.

A line is taken to represent a force because the point of application can be represented by the point where the line begins ; its direction by the position which the line has with regard to this point; and its magnitude by the length of the line. In all works on mechanics it is proved that *if two forces be represented in direction and magnitude by any two straight lines drawn from the same point, and a parallelogram be described with these two lines as adjacent sides, then the diagonal of the parallelogram drawn from the same point will represent the resultant in direction and magnitude, or the one force which will produce the same effect as the other two.* This proposition is called '*the parallelogram of forces.*' The finding of the resultant of two or more forces is called '*the composition of forces.*' The converse of the above proposition is also true, for if any line represent a force, its component forces will be represented by the adjacent sides of any parallelogram, and, therefore, by the sides of any triangle which may be described on this line. Finding the component forces of any resultant is called '*the resolution of forces.*'

If it be required to resolve any force into two others which act at right angles to each other, all that is necessary is to describe a right-angled triangle on the line representing the given force as hypothenuse. This is the case in most frequent use for our purpose, because the total force of the earth is usually resolved vertically and horizontally; whilst the horizontal force of a ship is resolved in a fore-and-aft line and athwartships.

FIG. 39.



Thus, if a force be represented by A B, and it is required to find the power it exerts in any other given direction as A C; from B draw B D, at right angles to A C, then A D represents the force which A B produces in the direction A C, and

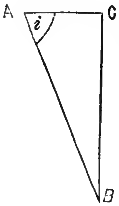
$$\because \text{AD} = \text{AB} \times \cos \text{BAD}$$

$\therefore$ the magnitude of A D is known.

The horizontal and vertical intensities are found by resolving the total intensity in these two directions. Thus, let A B (fig. 40) represent the magnitude and direction of the total intensity. From A draw A C horizontal, and from B draw B D vertical, the angle C A B is the dip: then

the horizontal intensity $AC = AB. \cos i$
and the vertical intensity $BC = AB. \sin i$:

FIG. 40.



that is, the horizontal intensity is found by multiplying the total intensity by the cosine of the dip; and the vertical intensity is found by multiplying the total intensity by the sine of the dip.

In the above method a dipping needle must be used, but the common horizontal needle is much easier of access, and the horizontal intensity may be obtained precisely similar to that above. Having obtained this, we then have—

$$\text{Total intensity } AB = \text{horizontal intensity } AC \times \sec i$$
$$= \text{hor. intensity} \times \text{secant dip.}$$

Then if any station on the earth be settled on as a standard for reference (by the English Greenwich is so selected), all other intensities can be reduced to terms of that for the standard place by either of the above methods. One circumstance must be noticed by the reader: we have *assumed* that the needles used as above retain the same strength during the whole time. If they be well made of hardened steel, and the time occupied be not very long, little or no error will be introduced into the determination.

At the magnetic equator, where there is no dip, the total and horizontal intensities are the same, and there is no vertical intensity. On this line we get the least total intensity, and it increases towards the poles, in the same way as the cosine of an angle decreases as the angle increases. We see from the above formula that the horizontal intensity varies directly as the cosine of the dip; and thus, as the dip increases, a needle which is *kept* horizontal, and therefore is affected only by the horizontal intensity, will oscillate more and more slowly as the magnetic latitude is increased, and at the magnetic pole will stand indifferently in any position it may be placed. For this reason, at Plymouth the compass needle will oscillate slower than at Rome, and at Rome slower than at places nearer the magnetic equator. The earth's horizontal intensity varies from 2·3 in the Bay of Bengal to ·7 in the Gulf of St. Lawrence. The reverse of this takes place with a dipping needle.

If we take the horizontal force at the magnetic equator as unity, the horizontal and vertical forces may be calculated for other places from the formulæ—

Horizontal force = cos magnetic lat.
Vertical force = 2 sin magnetic lat.
Total force = $\sqrt{1 + 3 \sin^2 \text{mag. lat.}}$

For the proof of these the reader is referred to the Admiralty manual.

THE COEFFICIENTS A, B, C, D, and E.—All forces affecting the compasses may be resolved in vertical and horizontal planes passing through the centre of the needle. The former acting vertically can have no effect in turning the card round in a horizontal plane, hence causes no deviation, and it is resisted by the pivot on which the card rests. The resolved part of the total force in the horizontal plane is the only part producing deviation when the ship is upright; and this may be resolved again into two forces at right angles to one another, one acting fore-and-aft and the other athwartships. The late Mr. Archibald Smith, F.R.S., proposed a very convenient notation for expressing the direction of the deviation due to different causes, and classed them under the heads A, B, C, D, and E, and for this purpose the deviation on the eight principal points only are required.

(1) RESIDUAL ERRORS.– By these are meant those that are left after computing for semicircular and quadrantal deviations, and their sum is expressed by the coefficient A. They are caused by errors in the compass itself, such as the index error not being allowed for, errors of prism, lubber lines or card; or it may be from errors in observation, as in taking the bearings or allowing wrong variation from the chart when observations are taken at sea. This coefficient A should never exceed 1° in well compensated compasses, and should it do so, the amount must be sought for among the above causes and rectified.

(2) SEMICIRCULAR DEVIATION.—As has been explained, this error is caused by magnetism induced in vertical iron and sub-permanent acquired in building. Their horizontal resultant,

when again resolved, as directed above, are designated by the coefficients, B and C; + B when the force acts so as to draw the north point of the needle forward, and — B when it acts towards the stern; + C when the force acts to draw the north point of the needle to the starboard side, and — C when the force draws it to the port side. Thus, unless there be large masses of individual iron to interfere, the results of semicircular deviation are—

+ B will be produced when the ship's head is southerly in building, and then the greatest easterly deviation will be produced on all easterly courses:

— B will be produced when the ship's head is northerly in building, and the greatest westerly deviation will be produced on all easterly courses:

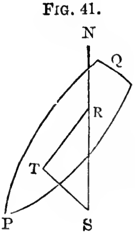
+ C will be produced when the ship is built head east, and easterly deviation is found on all northerly courses, and westerly on all southerly courses:

— C will be produced when the ship is built head west and the results are opposite to those with + C.

There are two points on which the ship's head can be brought where there is no semicircular deviation; these are situate opposite to each other, and are called *neutral points*. Their situation may be calculated when B and C are known, and the deviations for the other positions are found by multiplying the maximum deviation by the sine of the azimuth of the ship's head by the disturbed compass from the neutral points.

If it be required to find the direction in which the ship's head was whilst being built (and masters of vessels should always know in what direction the ship's head was at that time, because it enables them to ascertain under what circumstances they may expect a change in the character and the amount of heeling error), in the figure let P Q be the vessel, N S the magnetic meridian, then the fore part will have acquired blue magnetism, and the horizontal force will act towards the bows, causing A + B, and towards the port side causing A — C. Let R S represent the total horizontal force due to semicircular deviation, and resolve this as explained, then

FIG. 41.



RT is + B and TS is − C, and the angle at T is a right angle. Now if B and C be known,

Euc. I. 47, $T^2 = B^2 + C^2$, or total hor. force $= \sqrt{B^2 + C^2}$,

and tan TRS $= \frac{TS}{TR}$ or tan az. of ship's head at building $= \frac{C}{B}$.

These values are obtained at once from the traverse table thus: reduce B and C to degrees and decimals of a degree, and find B in the difference of latitude and C in the departure columns, then in the distance column is found the total horizontal force due to semicircular deviation, and the corresponding course is the magnetic azimuth the ship's head was on in building. The student should notice we have taken no account of the horizontal force caused by induction in vertical iron, but have supposed the whole of B and C to be caused by subpermanent magnetism; hence the azimuth the ship's head was in whilst being built obtained from the formula, tan TRS $= \frac{C}{B}$ is only approximate.

QUADRANTAL DEVIATION.—The student will remember this error is caused by horizontal soft iron in a ship. As has been shown, its minimum effects are produced when the soft iron is placed pointing to the four cardinal points, and its maximum effects are found at the four intercardinal points. The amount of deviation from this cause varies as the maximum deviation multiplied by the sine of twice the azimuth of the ship's head from north or south measured by the disturbed compass. Quadrantal deviation may be resolved into two forces acting similarly to the resolved parts of semicircular deviation; then + D and + E are used when the forces tend to produce easterly deviation, and − D and − E when they tend to produce westerly deviation. For example, if the maximum quadrantal deviation be 6°, then the deviation from this cause for ship's head NNE.

$$= 6^\circ \times \sin \text{twice } 2 \text{ pts.}$$
$$= 6^\circ \times \sin 4 \text{ pts.}$$

+ D may be regarded as the induced magnetism in iron beams extending athwartships at right angles to a fore-and-aft line.

— D to be produced by iron extending fore-and-aft; but in both cases, if the iron be divided so as to receive a hatchway, skylight, &c., contrary effects are found.

+ E is produced by iron extending from the starboard quarter to the port bow.

— E by iron extending from the port quarter to the starboard bow.

If compasses be well placed, so that the iron in the vessel may be symmetrically arranged around the compass, the coefficients A and E may be neglected.

HOW THE COEFFICIENTS ARE FOUND.—After compensating the compasses, the ship must first be swung, and a table of deviations formed by either of the methods already given; then the coefficients A B C D E are found from the deviations on the eight principal points, remembering that all easterly deviations are considered positive or +, and all westerly ones negative or —.

For A.—Add algebraically the deviations on the four cardinal points, and divide by four. This coefficient is constant for every point of the compass in the same ship; + if easterly deviations be greatest, and — if westerly be greatest.

For B.—Change the sign of the deviation for ship's head west, and take the mean between that (so changed) and the deviation for ship's head east. B is of the same sign as the general coefficient B in the semicircle from north by east to south, and of contrary sign from south to north by west. It is found for every position of the ship's head by multiplying the coefficient B by the sine of the angular distance of the ship's head from north or south.

For C.—Change the sign of the deviation for ship's head south, and take the mean between that result and the deviation for ship's head north. C is of the same sign as the general coefficient C if the ship's head be in the northern semicircle, but of contrary sign if in the southern semicircle. It is found for every position of the ship's head by multiplying the coefficient C by the cosine of the angular distance of the ship's head from north or south.

For D.—Change the signs of the deviations for ship's head NW. and SE., then take the mean between the two so changed, and the deviations for ship's head NE. and SW. D is of the same sign as the general coefficient D in the NE. and SW. quadrants, and of contrary signs in the SE. and NW. quadrants. It is found for every position of the ship's head by multiplying the coefficient D by the sine of twice the angular distance the ship's head is from either of the cardinal points.

For E.—Change the signs of the deviations for ship's head east and west. Then add algebraically those so changed to the deviations at north and south, and divide by four. E is of the same sign as the general co-efficient E in the quadrants from NE. to NW., and in that between SE. and SW.; but in the other two quadrants it has a contrary sign. It is found similarly to D, but the cosine of twice the angular distance the ship's head is from the cardinal points must be used instead of the sine.

EXAMPLE OF FINDING THE COEFFICIENTS.—The following is the table obtained by swinging the ship:—

Ship's Head by the Standard Compass	Deviation of the Standard Compass	Ship's head by the Standard Compass	Deviation of the Standard Compass
North	3° 10′ W.	South	3° 10′ E.
N. by E.	2 35 E.	S. by W.	0 5 E.
NNE.	8 10 E.	SSW.	3 0 W.
NE. by N.	13 10 E.	SW. by S.	6 30 W.
NE.	16 50 E.	SW.	9 40 W.
NE. by E.	19 30 E.	SW. by W.	13 0 W.
ENE.	20 30 E.	WSW.	16 10 W.
E. by N.	21 5 E.	W. by S.	19 15 W.
East	20 20 E.	West	21 10 W.
E. by S.	19 15 E.	W. by N.	23 20 W.
ESE.	18 5 E.	WNW.	24 0 W.
SE. by E.	16 30 E.	NW. by W.	23 35 W.
SE.	14 40 E.	NW.	22 0 W.
SE. by S.	12 5 E.	NW. by N.	19 0 W.
SSE.	9 40 E.	NNW.	14 50 W.
S. by E.	6 0 E.	N. by W.	9 15 W.

We will now show how the coefficients A B C D and E are determined; and we must again remind the student that all easterly deviations are marked + and westerly ones —.

For A — take the mean of the deviations on the four cardinal points.

North	−	3° 10′	East	+	20° 20′
West	−	21 10	South	+	3 10
	−	24 20		+	23 30
	+	23 30			
	4)	0 50			
∴ A =	−	0 12½ always constant.			

For B — change the sign of the deviation for ship's head west, and take the mean between that and the deviation for the ship's head east.

West, with sign changed	+	21° 10′
East	+	20 20
	2)	41 30
∴ B =	+	20 45

For C — change the sign of the deviation for the ship's head south, and take the mean between that and the deviation for the ship's head north.

South, with the sign changed	−	3° 10′
North	−	3 10
2)	−	6 20
∴ C =	−	3 10

For D — change the signs of the deviations for the ship's head NW. and SE., then take the mean between these and the deviations for the ship's head NE. and SW.

NW., with sign changed	+	22° 0′	SE., with sign changed	−	14° 40′
North east	+	16 50	South west	−	9 40
	+	38 50		−	24 20
	−	24 20			
4)	+	14 30			
∴ D =	+	3 37½			

For E — change the signs of the deviations for the ship's head east and west, and take the mean between these and the deviations for the ship's head north and south.

E., with sign changed	− 20° 20′	W., with sign changed	+ 21° 10′
North	− 3 10	South	+ 3 10
	− 23 30		+ 24 20
			− 23 30
		4)	+ 0 50
		∴ E =	+ 0 12½

These are then tabulated as follows :—

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 3
Ship's head by the Standard Compass	Observed Deviation	A = −0° 12½′	B = +20° 45′	C = −3° 10′	D = +3° 37½′	E = +0 12½′	Calculated Deviation
	° ′		° ′	° ′	° ′	° ′	° ′
North.	− 3 10	A constant for every point = − 12½′	+ 0 0	− 3 10	0 0	+ 0 12½	− 3 10
N. by E.	+ 2 35		+ 4 3	− 3 6	+ 1 23	+ 11½	+ 2 20
NNE.	+ 8 10		+ 7 57	− 2 56	+ 2 34	+ 9	+ 7 31½
NE. by N.	+ 13 10		+ 11 32	− 2 38	+ 3 21	+ 4	+ 12 6½
NE.	+ 16 50		+ 14 40	− 2 14	+ 3 37½	0	+ 15 51
NE. by E.	+ 19 30		+ 17 15	− 1 46	+ 3 21	− 4	+ 18 33
ENE.	+ 20 30		+ 19 10	− 1 13	+ 2 34	− 9	+ 20 9½
E. by N.	+ 21 5		+ 20 21	− 0 37	+ 1 23	− 11½	+ 20 43
East.	+ 20 20		+ 20 45	0 0	0 0	− 12½	+ 20 20
E. by S.	- 19 15		+ 20 21	+ 0 37	− 1 23	− 11½	+ 19 11
ESE.	+ 18 5		+ 19 10	+ 1 13	− 2 34	− 9	+ 17 27½
SE. by E.	+ 16 30		+ 17 15	+ 1 46	− 3 21	− 4	+ 15 24½
SE.	+ 14 40		+ 14 40	+ 2 14	− 3 37½	0	+ 13 4
SE. by S.	+ 12 5		+ 11 32	+ 2 38	− 3 21	+ 4	+ 10 40½
SSE.	+ 9 40		+ 7 57	+ 2 56	− 2 34	+ 9	+ 8 15½
S. by E.	+ 6 0		+ 4 3	+ 3 6	− 1 23	+ 11½	+ 5 44
South.	+ 3 10		0 0	+ 3 10	0 0	+ 12½	+ 3 10
S. by W.	+ 0 5		− 4 3	+ 3 6	+ 1 23	+ 11½	+ 0 25
SSW.	− 3 0		− 7 57	+ 2 56	+ 2 34	+ 9	− 2 30½
SW. by S.	− 6 30		− 11 32	+ 2 38	+ 3 21	+ 4	− 5 41½
SW.	− 9 40		− 14 40	+ 2 14	+ 3 37½	0	− 9 1
SW. by W.	− 13 0		− 17 15	+ 1 46	+ 3 21	− 4	− 12 24½
WSW.	− 16 10		− 19 10	+ 1 13	+ 2 34	− 9	− 15 44½
W. by S.	− 19 15		− 20 21	+ 0 37	+ 1 23	− 11½	− 18 45
We-t.	− 21 10		− 20 45	0 0	0 0	− 12½	− 21 10
W. by N.	− 23 20		− 20 21	− 0 37	− 1 23	− 11½	− 22 45
WNW.	− 24 0		− 19 10	− 1 13	− 2 34	− 9	− 23 18½
NW. by W.	− 23 35		− 17 15	− 1 46	− 3 21	− 4	− 22 38
NW.	− 22 0		− 14 40	− 2 14	− 3 37½	0	− 20 44
NW. by N.	− 19 0		− 11 32	− 2 38	− 3 21	+ 4	− 17 39½
NNW.	− 14 50		− 7 57	− 2 56	− 2 34	+ 9	− 13 30
N. by W.	− 9 15		− 4 3	− 3 6	− 1 23	+ 11½	− 8 32

We now proceed to calculate coefficients for each point of the ship's head, from the following formula given by Mr. A. Smith, F.R.S., in the Admiralty Manual.

$$\delta = \text{A} + \text{B} \sin \zeta' + \text{C} \cos \zeta' + \text{D} \sin 2 \zeta' + \text{E} \cos 2 \zeta'.$$

Where δ is the deviation reckoned + when the red end of the needle is drawn to the east, and − when drawn to the west.

ζ' is the compass course, or the azimuth of the ship's head from the direction of the disturbed needle, reckoned + to the eastward, and − to the westward.

In the formula A is the *constant* deviation.

B sin ζ + C cos ζ' is the *semicircular* deviation.

D sin 2 ζ' + E cos 2 ζ' is the *quadrantal* deviation.

The student will see by reference that the practical rules already given are simply the above formula expressed in words.

As the deviations are calculated for every point, and the same number of points is repeated in each quadrant, it will be necessary only to take the natural sines for one, two, three, up to eight points, and remember that the cosine of an angle is the sine of its complement. Thus, the sine of two points is the cosine of six, &c.

Sine of 0 point =	0	= cosine of 8 points
Sine of 1 point =	·195	= cosine of 7 points
Sine of 2 points =	·383	= cosine of 6 points
Sine of 3 points =	·556	= cosine of 5 points
Sine of 4 points =	·707	= cosine of 4 points
Sine of 5 points =	·831	= cosine of 3 points
Sine of 6 points =	·924	= cosine of 2 points
Sine of 7 points =	·981	= cosine of 1 point
Sine of 8 points =	1.	= cosine of 0 point

First, to calculate B *for each point from* B *sin* ζ'. Reduce B to minutes, *i.e.* 20° 45′ = 1245′, multiply the sines for such point above by this amount, the product gives B for every point in minutes.

Thus for North B = 1245′ × 0 = 0
for N. by E. B = 1245 × ·195 = 242·775′ = 4° 3′
for NNE. B = 1245 × ·383 = 476·835 = 7 57

and so on.

Secondly, to calculate C *for each point from* C *cos* ζ'. Reduce C to minutes, *i.e.* 3° 10′ = 190′; and multiply the cosines for each point above by this amount, the product gives C for every point in minutes.

Thus for North	$c = 190' \times \cos 0$ pts. =	$190' =$	$3° 10'$
for N. by E.	$c = 190 \times$	$\cdot 981 = 186\cdot 390 =$	$3 \ . 6$
for NE. by E.	$c = 190 \times$	$\cdot 556 = 105\cdot 640 =$	$1 \ 46$

and so on.

Thirdly, to calculate D *for each point from* D *sin* $2\ \zeta'$. Reduce D to minutes, *i.e.* $3° 37\frac{1}{2}' = 217\cdot 5'$; and multiply this by the sines of twice one point, twice two points, twice three points, and twice four points.

Thus for N. by E.	take sine	2	points	·383
NNE.	„	4	„	·707
NE. by N.	„	6	„	·924
NE.	„	8	„	1·000

and then follow in the reverse order for the other points of the quadrant.

For N. by E.	$D = 217\cdot 5' \times \cdot 383 = 83\ 3025' =$	$1° 2$
NE. by N.	$D = 217\cdot 5 \times \cdot 924 = 200\cdot 9700 =$	$3 \ 21$

and so on.

Fourthly, to calculate E *for each point from* E *cos* $2\ \zeta'$. Multiply the cosines of 0, two, four points, &c. by the minutes in E, thus.

For N. by E.	$E = 12\cdot 5' \times \cdot 924 = 11\cdot 55'$	$= 11\frac{1}{2}'$
NNE.	$E = 12\cdot 5 \times \cdot 707 = 8\cdot 8375$	$= 9$

and so on.

In calculating D and E for each point, it is not necessary to proceed further than four points, for precisely the same reason that we did not carry the calculation of B and C further than eight points.

In the quadrant from east to south the same values of B and C must be copied in a reverse order; that is E. by S. must be the same as E. by N. and so on. The SW. quadrant follows the same order as the SE. one, that is S. by W. is of the same numerical value as S. by E. and so on; and the NW. quadrant follows the same order as the NE. one. It should be noted those points at the same distance from north and south must have the same numerical value. Similarly for the values of D and E those obtained in the half quadrants must be copied, noticing that those equally distant from the four cardinal points have the same numerical value.

We now come to the distinguishing sign for each point. The student must be careful to prefix the sign obtained by multiplying the trigonometrical function by the coefficient B C D or E; thus all the sines that are used are + because the sines of all angles less than sixteen points are +; all cosines under eight points are +, but between eight and sixteen points they are −; and then the product of like signs is + and of unlike ones − determines the sign to be placed before each result in the different columns. Having found the values of B C D and E, as in columns 4, 5, 6, and 7, the deviation table is completed in column 8 by taking the algebraical sum of the five values A B C D and E; thus for E. by S. we have—

$$\begin{array}{rl} A = & -\,0^\circ\ 12\tfrac{1}{2}' \\ D = & -\,1\ \ 23 \\ E = & -\qquad 11\tfrac{1}{2} \\ \hline & -\,1\ \ 47 \end{array} \qquad\qquad \begin{array}{rl} B = & +\,20^\circ\ 21' \\ C = & +\qquad 37 \\ \hline & +\,20^\circ\ 58 \\ & -\,1\quad 47 \\ \hline & +\,19\ 11 \end{array}$$

Hence deviation for E. by S. = 19° 11′ E.

In comparing the observed deviation, column 2, with that calculated column 8, small discrepancies may be observed, but in no case (in this example) do they amount to more than $1\frac{1}{2}$°. This want of agreement arises partly from the fact that the coefficients themselves are only approximations; the correct ones are longer and more tedious, but may be obtained by a process fully described in the Admiralty Manual. The calculation of the deviation as above should never be used when it is greater than 20° on any course, but in all other cases it is sufficiently accurate for practical purposes. Should the deviation exceed 20° on any course the compass should be compensated to reduce it below that amount.

The advantages of using the coefficients are:—

(1) The ship need not be swung to more than the eight principal points.

(2) The amounts produced by different causes, whether from semicircular or quadrantal deviation, can be ascertained; and

the reason for and amount of error being known, compensation is rendered easier and more certain.

(3) In going on a foreign voyage, the kind and amount of changes can in a great measure be anticipated, and an approximate table of deviations can be calculated for any place where the vertical and horizontal forces are known.

THE HEELING ERROR.—All that has hitherto been said refers to vessels only when they are upright, but experience has taught us that ships with iron in their construction affect the compass when heeled over. We have now to consider the effects of heeling. The late Astronomer-Royal says:—'Usually an iron ship, when her head is placed north or south, the ship's inclination through an angle of *n* degrees disturbs the compass through an angle of *n* degrees; but in some particular instances it has been known to disturb the compass as much as 2 *n* degrees. The effect is very serious in those parts of the earth where the wind is steady, and the ship is inclined in the same direction for many days or weeks in succession.' In cases where one degree of heel produces two degrees of change, an alteration of the compass amounting to two points occurs by heeling the ship half a point to starboard and half a point to port, although the ship's head be precisely in the same position with regard to the geographical meridian during the whole time. This accounts for the apparent shifting of the wind observed by sailors when a ship is put on different tacks. When ships are swung, *three* deviation tables should be given to the master, viz. one each for when the ship is upright, when heeling 10° to starboard, and when heeling 10° to port; and corrections for all other degrees of heel can be calculated by simple proportion; thus, if a change in the deviation caused by heeling a vessel 10° be 14° 30′, then a change of $\frac{7}{10}$ of 14° 30′ will be caused by heeling the same vessel 7°, with her head in the same direction.

For if a heel of 10° gives a change of 14° 30′

Then a heel of 1° „ „ $\frac{14° \, 30'}{10}$

And a heel of 7° „ $\frac{7}{10}$ of 14° 30′

$= 10° \, 9'$

How the Amount of Heel is Ascertained.—An instrument called a *clinometer* is used for this purpose, and is fixed athwartships, or at right angles to a fore-and-aft section of the ship. In construction it consists of a brass semicircle graduated to degrees, beginning from the middle of the semicircle and continued both ways. In use the diameter is placed upwards and parallel to the deck, and to the centre of the semicircle is attached a plumb-line; then as the ship heels, because the plumb-line is always vertical, the number of degrees at which the line cuts the graduated edge is the heel of the vessel.

How the Heeling Error is Caused.—Fore-and-aft iron is not disturbed from its horizontal position by a vessel's heeling; but that which runs athwartships as well as vertical iron is so; hence beams, &c., produce the greatest effects. From a little consideration the student will see that the heeling error depends upon

(1) The vertical induction in transverse iron.

(2) The vertical induction in vertical iron.

(3) The vertical force arising from subpermanent magnetism.

The reader must bear in mind that all forces which act on the compass are resolved vertically and horizontally, and as long as the former component acts at right angles to the deck, no effect is produced on the needle: but when the ship heels, this force is removed either to port or starboard, and hence we get a deflection of the needle represented by + c or − c.

When a ship heels, the transverse iron acquires magnetism by induction; and if the iron extends right across the ship, the windward side will, in north latitude, have acquired blue and the leeward side red magnetism; hence from both these causes the red end of the needle must be drawn to windward. But if the iron extends only part of the way across, as when a skylight or hatchway is fitted, the opposite effects must be produced; for then the end of the iron nearest the compass on the weather side must be a red pole, and the end nearest on the lee side must be a blue pole; under these conditions the red end of the needle must be moved to leeward. In vertical iron the force acting on the needle is no longer directly under the needle, but is shifted also to the windward side of the vessel; and thus in north magnetic latitudes, as a general rule, both horizontal and vertical iron tend to draw the north end of the needle to windward. This result is modified by the vertical action arising from subpermanent magnetism, which may act either to increase or decrease the error from the former causes. If a ship has acquired subpermanent magnetism by having been built with her head southerly, the aft part will have acquired red magnetism, and the vertical component of the force will act downward, and its effects will be minus; hence, when the vessel heels, this vertical component is no longer acting vertically through the compass, but is thrust out to the windward side and repels the red end of the needle, thus decreasing the heeling error caused by soft iron. On the contrary, if a vessel be built with her head northerly, the aft part of the ship has acquired blue subpermanent magnetism, and the result is the red end of the needle is attracted in heeling, thereby increasing the change in deviation from the other causes. Thus theory shows why in England the deviation of ships built there, with their heads northerly, are most affected by heeling.

Position of Ship for Greatest and Least Change.—When the ship's head is either magnetic north or south, the disturbing force acts at right angles to the needle; hence the greatest change for a vessel's heeling takes place when her fore-and-aft line is in the magnetic meridian. On the contrary, when the ship's head is either magnetic east or west, the disturbing force, from heeling, still acting at right angles to the fore-and-aft midship line, must tend to bring the needle into the magnetic meridian, and at such times no change in deviation can be produced from heeling; but a greater or less directive force is given to the needle. Hence a change is produced only in that force which acts athwartships; and the different vertical components are found to produce a modification in the coefficients c and e, whilst those which depend on fore-and-aft action (b and d) are unaltered. A slight variation occurs in a, but for all practical purposes we may consider the whole change to take place in c.

The Heeling Coefficent is the amount of heeling error for one degree of heel when the ship's head is either north or south by the disturbed compass, *i.e.* when the heeling error is at its maximum. This is the amount generally used for computing the heeling error for the ship's head on all points of the compass. To obtain this coefficient without observation, the exact ones, 𝔄, 𝔅, ℭ, 𝔇, and 𝔈, are required, instead of the approximate ones, A, B, C, D, and E, as well as two others λ and μ, and these last ones depend on the ratios which the earth's vertical and horizontal forces bear to those of the earth and ship combined; and thus, with a change of geographical position, the heeling error is also changed. The determining of the requisite data is not suited to the pages of such a work as the present; and the student who wishes to study this part of the subject is referred to 'The Admiralty Manual' and Airy's 'Treatise on Magnetism.' The latter authority concludes with the following remark: 'There appears to be no safe way of determining the amount of the effect of heeling, except by making the ship to heel and observing how much the compass is affected.'

To find the Change in Deviation from Heeling.—For the same geographical position this difficulty does not exist if the change in deviation be obtained for any position of the ship's head when not near the east and west points. We have shown that if the change of deviation for heeling any number of degrees be known, the change for all other degrees of heel may be calculated by proportion; and if this be known for the ship's head in any one direction, it may be calculated for the head in any other, because the changes will vary as the cosines of the compass azimuth of the ship's head. The heeling coefficient being known, all that is required is to multiply this by the cosine of the azimuth of the ship's head, and this result again by the number of degrees of heel.

Ex. 360. If the change in deviation when the ship's head is south be 13° 50′ for a heel of 10°, find the change when the ship's head is SE. by E. and heeling 6° in the same direction.

$$\text{Here the heeling coefficient is } \frac{13^\circ\ 50'}{10} = 1^\circ\ 23'$$

∴ Change for 1 degree of heel at SE. by E.
$= 1° 23' \times$ cosine 5 points.
$= 83' \times \cdot 556$
$= 46'$

Hence change for 6 degrees of heel at SE. by S.
$= 46' \times 6$
$= 4° 36'$ Answer.

Or if the change for a certain number of degrees heel be known for any one direction, as for SSE., it may be calculated for any other, as for ESE., thus:—

$$\frac{\text{Change at ESE.}}{\text{Change at SSE.}} = \frac{\text{cosine 6 points}}{\text{cosine 2 points}} = \frac{383}{924}$$

$$\therefore \text{Change at ESE.} = \frac{383}{924} \text{ of change at SSE.}$$

Ex. 361. If the change in deviation when a ship's head is S. by W., and she heels 9°, be 16° 30', what will be the change when the ship's head is WSW. and she heels 6° in the same direction ?

Head S. by W. change in deviation for 9° $= 16° 30'$
∴ " " for 6° $= \frac{6}{9}$ or $\frac{2}{3}$ of $16° 30'$
$= 11°$

$$\frac{\text{Change for WSW.}}{\text{Change for S. by W.}} = \frac{\text{cosine 6 points}}{\text{cosine 1 point}} = \frac{383}{981}$$

$$\text{Hence change for head WSW.} = \frac{383}{981} \text{ of change for S. by W.}$$

$$= \frac{383}{981} \text{ of } 11°$$

$= 4° 18'$ Answer.

Ex. 362. If the ship's head be NNW. and the change of deviation, with 5° of heel, be 8° 30', what will be the change when the ship's head is NW. by W. and the heel 8° in the same direction ?

Ex. 363. If a ship heels 7° and her head be S. by E., thus producing a change of deviation of 4° 20', what will be the change if she heels 5° and her head be SE. ?

Ex. 364. If the change in deviation when a ship's head is NE. by E. be 6° 50' when she heels 9°, what will the change be when her head is north and she heels 5° in the same direction ?

Effects of Heeling.—From what has been already said, if a ship be placed on the opposite tack by a change of wind, even if the ship's head appears the same by compass, owing to a change in the heeling error the vessel will not be moving in the same direction as before, the general rule being that the north point of the needle is drawn to windward. A vessel, if steered for a fixed point on the horizon, will therefore appear to fall off as she heels on northerly courses and to come up on southerly ones: and in north latitude will, if steered on one northerly course, be to the windward of her supposed position if the compass be above the upper deck, and she will in such cases be to leeward on southerly courses. For this reason it is necessary, when steering by compass on either tack, to keep away on all northerly courses, and to keep closer to the wind on either tack on southerly ones.

The reverse of these rules frequently holds good in the Southern Hemisphere; 'but this is a point which can only be ascertained by actual observation for each ship.'

General Conclusions.—The effects of heeling may be summed up thus:—

(1) The amount of error is greatest when sailing N. or S. by compass.

(2) The amount is least when sailing E. or W. by compass.

(3) The amount of error is different on different tacks, and varies as the cosine of the azimuth from N. or S.

(4) The amount of heeling error is proportional to the heel of the vessel.

Napier's Graphic Method.—In almost every branch of mathematical and physical research, a method exists of representing time, force, &c., by distances from certain fixed lines or by curves drawn through certain points; as, for example, the indicator diagram, the barogram, &c. The late Mr. J. R. Napier, F.R.S., has applied the same method to the construction of a deviation table when observations have been made on a few points only and it is not necessary that the deviation should be known on equidistant points of the ship's head. He also applied the method to the correction and shaping of compass courses. The curve may be drawn if the deviation be known on four points only, provided these points be near the intercardinal ones; but it will be found much more accurate if the deviation at the four cardinal points be known in addition.

How a Diagram is Constructed.—Draw a fine vertical line of any length, 18 inches is the most convenient, and divide it into 32 equal parts to represent the points of the compass. Mark N. at the top, then N. by E , NNE., and so on, placing a point of the compass on each division. Also divide the line into 360 equal parts, each will be one-twentieth of an inch long, and will represent a degree; these should be numbered consecutively to the 360°, and also from 0 at north and south to 90° at east and west. Next draw lines through each point N., N. by E., NNE., &c., intersecting each other, and the vertical line at an angle of 60°; and let one be dotted and the other plain, the dotted one inclining downwards on the right hand side of the vertical line. The known deviations must be laid down on the dotted lines, easterly to the right, and westerly to the left hand; the number of degrees being measured from the vertical line, being careful to select that dotted line which passes through the point on which the ship's head was when the observation was made. If no dotted line passes through that point, then lay off the distance parallel to the dotted lines. Continue this for as many deviations as has been calculated, and draw a flowing curve by hand through all the points laid down. This is the curve of deviation.

As an example, we will construct the curve for the deviation table used throughout this chapter, using for that purpose the eight equidistant points of the compass, as shown in the following table :—

Ship's head.	Deviation.	Ship's head.	Deviation.
North	3° 10′ W.	South	3° 10′ E.
NE.	16 50 E.	SW.	9 40 W.
East	20 20 E.	West	21 10 W.
SE.	14 40 E.	NW.	22 0 W.

As was remarked before, it is not necessary that the selected points should be equidistant; neither need they be coincident with even points of the compass, but may be taken at half or quarter points, or at degrees.

How to Draw the Curve.—The vertical line is 18 inches long and is divided into 32 equal parts, each representing a point; and also into 360 equal parts, each representing a degree. The cross lines make an angle of 60° with each other and with the vertical line, and are all drawn through the points marked. The deviations are then laid off, selecting that dotted line which passes through the point representing the direction of the ship's head; thus at north we mark off $3\frac{1}{6}°$ to the left on the dotted line. At NE. we take $16\frac{5}{6}°$ from the vertical line, and lay it off to the right on the dotted line through NE.; at east, $20\frac{1}{3}°$ taken from the vertical line to the right, and so on with all the others. All the points thus obtained are marked in the diagram with a small cross. The curve is then drawn neatly through all the points so laid down; and it will be found that the deviation for any other point taken from the curve corresponds with that taken from the table, and the curve thus drawn can be used instead of the table.

This method will be found particularly useful when the deviations on a few points have been obtained from amplitude and azimuth observations; and if diagrams of the vertical line, with its divisions and the cross lines, be kept on board, a few minutes only will suffice to construct the curve for use.

How the Curve is Used.—(1) If a deviation not obtained by observation be required, as NE$\frac{1}{2}$N., take NE$\frac{1}{2}$N. on the vertical line, and draw a line parallel to the dotted lines until it meets the curve. The length of this line in degrees, taken from the vertical line, gives the deviation. In this example it is 15°.

(2) To find the magnetic course from the compass course. Take the point on the vertical line, representing the given compass course, move a pencil in a direction parallel to the dotted lines, and after meeting the curve, return parallel to the plain lines, until the vertical line be again reached. The point on the vertical line is the magnetic course corresponding to the given compass course.

Ex. 365. If the given compass course be SW$\frac{3}{4}$W., take the point as directed on the vertical line. The diagram shows what lines are necessary, and the result is S. 41° $\frac{3}{4}$ W. for the required magnetic course.

Ex. 366. If the given compass course be ENE., trace the lines as directed; this gives the magnetic course E. 2° N.

(3) To find the compass course when the magnetic course is known from the chart or by other means.

Take the given magnetic course on the vertical line, but in this case move the pencil parallel to the plain lines until the curve be reached, then return to the vertical line parallel to the dotted lines. The course reached on the vertical line will show the required compass course.

Example.—If the given magnetic course be NW½W., find the compass course.

By tracing the lines in the diagram as directed the compass course is found to be N. 32° W.

To assist the memory, the following rhyme is given in the 'Admiralty Manual.'

(1) 'From compass course magnetic course to gain,
Depart by dotted and return by plain;
(2) But if you seek to steer a course allotted,
Take plain from chart and keep her head on dotted.'

We will now work another question in full as set in the Board of Trade Examinations.

In the following table give the correct magnetic bearing of the distant object and thence the deviations.

Ex. 367.

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
North	S. 73° 0′ W.	1° 5′ W.	South	S. 70° 30′ W.	1° 25′ E.
NE.	S. 59 50 W.	12 5 E.	SW.	S. 83 20 W.	11 25 W.
East	S. 54 40 W.	17 15 E.	West	S. 89 30 W.	17 35 W.
SE.	S. 59 40 W.	12 15 E.	NW.	S. 84 50 W.	12 55 W.

In this example we first add together the eight given bearings of the distant object, making 575° 20′, and divide the sum by eight, giving as the result S. 71° 55′ W.; this is the correct magnetic bearing of the distant object. We next take the difference between the magnetic bearing thus obtained and the bearings as given in the table. Thus, for the ship's head North the bearing is S. 73° W., and the difference between this and the magnetic bearing S. 71° 55′ W. is 1° 5′; and because the magnetic is to the left hand of the compass bearing, the deviation is marked 1° 5′ W. Again, take SE. The compass bearing is S. 59° 40′ W., and the difference between this and the magnetic bearing S. 71° 55′ W. is 12° 15′; but now the magnetic is to the right hand of the compass bearing. Hence the deviation is entered opposite to ship's head SE., 12° 15′ E.

The student must now construct a deviation curve on a Napier's diagram thus:—

With a pair of dividers take, on the vertical line from the table formed, the degrees and minutes in the deviation for North, and measure this on the DOTTED line through the North, easterly deviation to the right hand, and westerly to the left hand; mark the point reached. Then take the deviations for NE., E., SE., &c., and proceed in like manner until all eight are laid off, and draw a flowing curve through all the marks made; this line is Napier's curve of deviations.

Ex. 368. With the curve thus constructed, give the courses you would steer by the standard compass to make good the following correct magnetic courses:—

Mag. courses	NE.	West	South	WSW.

Take the given magnetic course on the vertical line, and move the pencil parallel to the plain lines to meet the curve, then return to the vertical line parallel to the dotted lines. The point reached on the vertical line will show the required compass course.

Mag. courses	NE.	West	South	WSW.
Compass courses	N. 35½° E.	N. 73° W.	S. 1½° E.	S. 84½° W.

Ex. 369. Suppose you have steered the following courses by the standard compass:—

SE.	SW.	East	WNW.

Find the correct magnetic courses from the curve drawn.

Take the point on the vertical line, representing the given compass course; move a pencil in a direction parallel to the dotted lines, and after meeting the curve, return parallel to the plain lines, until the vertical line be again reached. The point on the vertical line is the magnetic course corresponding to the given compass course.

Compass courses	SE.	SW.	East	WNW.
Mag. courses	S. $32\frac{1}{2}$° E.	S. $33\frac{1}{2}$° W.	S. 73° E.	N. $83\frac{1}{2}$° W.

Ex. 370. With the ship's head ENE., two distant objects bore by the standard compass N. 39° W. and S. 63° E.; find the deviation from the curve, and thence the correct magnetic bearings.

Take the distance with your dividers on, or parallel to, a dotted line, from the given position of ship's head in the vertical line to the curve; this, measured on the vertical line, is the deviation for the given position. In the case of ship's head ENE., it will be found to be 16° E., and this must be applied to the bearings of the two distant objects, easterly to the right hand and westerly to the left hand.

Comp. bearing, first object	N. 39° W.	Second object	S. 63° E.
Deviation	16 E.	Deviation	16 E.
Mag. bearings, first object	N. 23 W.	Second object	S. 47 E.

Exercise XVIII.

Ex. 371. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
North	S. 14° 30′ W.		South	S. 37° 45′ W.	
NE.	South		SW.	S. 51 45 W.	
East	South		West	S. 53 45 W.	
SE.	S. 13 30 W.		NW.	S. 40 45 W.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses	N$\frac{3}{4}$E.	SW$\frac{1}{2}$W.	S$\frac{1}{2}$E.	W$\frac{1}{4}$S.
Compass courses				

Suppose you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table.

Compass courses	SW$\frac{1}{4}$W.	SSE.	E. by N$\frac{1}{4}$N.	N$\frac{1}{2}$W.
Correct mag. courses				

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at W$\frac{3}{4}$N., find the bearings correct magnetic.

NNE. NW. by W$\frac{3}{4}$W.

Ex. 372. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
	° ′			° ′	
North	N. 83 10 E.		South	N. 79 20 E.	
NE.	N. 69 20 E.		SW.	S. 87 30 E.	
East	N. 61 40 E.		West	S. 80 0 E.	
SE.	N. 69 30 E.		NW.	S. 82 10 E.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses SW. by W. NW$\frac{1}{2}$W. S$\frac{1}{2}$W. E$\frac{1}{2}$S.
Compass courses

Suppose you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table.

Compass courses NE. SE. by E$\frac{1}{2}$E. W$\frac{3}{4}$N. N$\frac{1}{4}$E.
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at E$\frac{3}{4}$S., find the bearings correct magnetic.

N$\frac{3}{4}$W. NE.

Ex. 373. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
				° ′	
North	East		South	S. 87 0 E.	
NE.	S. 67° 0′ E.		SW.	N. 62 45 E.	
East	S. 60 0 E.		West	N. 60 0 E.	
SE.	S. 67 45 E.		NW.	N. 69 0 E.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses SW. by S. N. by E$\frac{3}{4}$E. North E$\frac{1}{4}$N.
Compass courses

Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table.

Compass courses W. by S. NW$\frac{1}{2}$N. S$\frac{1}{2}$W. S. by E$\frac{3}{4}$E.
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at E$\frac{1}{2}$S., find the bearings correct magnetic.

E$\frac{1}{2}$S. W. by S.

Ex. 374. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
	° ′			° ′	
North	S. 88 20 E.		South	S. 84 50 E.	
NE.	S. 74 30 E.		SW.	N. 80 20 E	
East	S. 69 40 E.		West	N. 77 0 E.	
SE.	S. 72 20 E.		NW.	N. 81 40 E.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses SW. W. by N$\frac{3}{4}$N. E. by N$\frac{1}{4}$N. N. by E$\frac{1}{2}$E.
Compass courses

Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table.

Compass courses SSW. SE$\frac{1}{2}$S. NW$\frac{1}{2}$N. W$\frac{3}{4}$S.
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at E. by N.; find the bearings correct magnetic.

E$\frac{3}{4}$S. N. by E.

Ex. 375. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
North	N. 14° 30′ E.		South	N. 8° 15′ W.	
NE.	N. 13 20 W.		SW.	N. 8 45 E.	
East	N. 22 20 W.		West	N. 18 30 E.	
SE.	N. 19 20 W.		NW.	N. 21 30 E.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses NW.byW. SE.byE$\frac{3}{4}$E. S$\frac{1}{4}$W. W.byS$\frac{1}{4}$S.
Compass courses

Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table

Compass courses NE. SW$\frac{1}{2}$W. N$\frac{1}{2}$E. E$\frac{3}{4}$N.
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at S$\frac{3}{4}$W.; find the bearings correct magnetic.

E$\frac{1}{4}$N. South.

Ex. 376. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
North	N. 17° E.		South	N. 25° E.	
NE.	N. 7 W.		SW.	N. 45 E.	
East	N. 9 W.		West	N. 52 E.	
SE.	North		NW.	N. 41 E.	

With the deviations as above, give the courses you wculd steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses E$\frac{3}{4}$S. W. by N$\frac{3}{4}$N. SE. W. by S$\frac{1}{2}$S.
Compass courses

Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table.

Compass courses NE$\frac{1}{2}$N. N$\frac{1}{2}$W. S$\frac{1}{4}$W. W$\frac{1}{2}$N.
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at SW. by W.; find the bearings correct magnetic.

E$\frac{3}{4}$S. S$\frac{1}{2}$W.

Ex. 377. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
North	S. 16° 30′ E.		South	S. 16° 0′ E.	
NE.	S. 5 30 E.		SW.	S. 31 0 E.	
East	South		West	S. 38 0 E.	
SE.	S. 3 0 E.		NW.	S. 34 0 E.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses NW. W$\frac{1}{2}$N. N$\frac{1}{4}$W. E$\frac{1}{2}$N.
Compass courses

Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table.

Compass courses SE. W. by S$\frac{3}{4}$S. NE. by E. South
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at W$\frac{3}{4}$N.; find the bearings correct magnetic.

West North

Ex. 378. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
North	N. 86° 15′ W.		South	S. 86° 15′ W	
NE.	N. 76 30 W.		SW.	S. 73 0 W.	
East	N. 71 45 W.		West	S. 71 0 W.	
SE.	N. 74 0 W.		NW.	S. 78 15 W.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses N$\frac{3}{4}$E. NW$\frac{1}{4}$W. S$\frac{1}{2}$E. N$\frac{1}{4}$W.
Compass courses

Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table.

Compass courses WSW. NE$\frac{1}{2}$E. E$\frac{3}{4}$S. S. by W$\frac{1}{2}$W.
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at E. by S$\frac{1}{2}$S.; find the bearings correct magnetic.

W$\frac{1}{2}$N. S$\frac{1}{4}$W.

Ex. 379. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
North	S. 24° W.		South	S. 26° W.	
NE.	S. 51 W.		SW.	South	
East	S. 58 W.		West	S. 4 E.	
SE.	S. 47 W.		NW.	S. 6 W.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses N$\frac{3}{4}$W. SW. by W. E$\frac{3}{4}$N. N$\frac{1}{4}$E.
Compass courses

Supposing you have steered the following courses by the standard compass, find the correct magnetic courses made from the above deviation table.

Compass courses SE$\frac{1}{2}$E. NNE. South W$\frac{3}{4}$S.
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at NW$\frac{1}{2}$W., find the bearings correct magnetic.

East. W$\frac{3}{4}$S.

Ex. 380. In the following table give the correct magnetic bearing of the distant object, and thence the deviations.

CORRECT MAGNETIC BEARING =

Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required	Ship's head by standard compass	Bearings of distant object by standard compass	Deviation required
North	North		South	North	
NE.	N. 17° 0′ E.		SW.	N. 18° 0′ W.	
East	N. 22 15 E.		West	N. 23 15 W.	
SE.	N. 19 45 E.		NW.	N. 17 45 W.	

With the deviations as above, give the courses you would steer by the standard compass to make the following courses correct magnetic.

Correct mag. courses ENE. North W. by N$\frac{3}{4}$N. NW$\frac{1}{2}$N.
Compass courses

Suppose you have steered the following courses by the standard compass, find the correct magnetic courses made from the above table.

Compass courses S. by E$\frac{3}{4}$E. WSW. South E. by S$\frac{3}{4}$S.
Correct mag. courses

You have taken the following bearings of two distant objects by your standard compass as above, with the ship's head at SW by W$\frac{1}{2}$W.; find the bearings correct magnetic.

W. by S$\frac{1}{4}$S. East.

Compensation *is the employment of means to counteract the effects which magnetism, in surrounding iron, has on the Compasses.* We have seen, as the effects of sub-permanent magnetism, that when an iron vessel sails in the same direction in which she was built, that the directive power is decreased, and a sluggishness of the needle is the result; whilst if she sails in an opposite direction, an undue liveliness is observed in the needle. The directive power can be equalised on all courses by applying a magnet which shall act contrarily to the sub-permanent magnetism of the ship. Again, when the deviations are large, the angle through which the vessel turns is not nearly equal to that recorded by the compass. An example of this was particularly forced on the writer's mind in December, 1865, when swinging the iron ship 'Trevelyan' in Plymouth Harbour, in company with the late Commander Walker, R.N. We then found that when the standard compass showed a change of position of the ship's head from NNE. to NNW., or 4 points *Westerly*, the vessel had actually turned through an angle of 22° to the *Eastward.* Such an occurrence must prove extremely embarrassing to the mariner, and this is lessened by compensating. The late Astronomer-Royal sums up the arguments in favour of compensation thus:—

Non-corrected Compasses. *Using a table of errors.*	Corrected Compasses. *The binnacle being adjustable.*
(1) The directive power on the compass is extremely different on different courses.	(1) The directive power on the compass is sensibly constant.
(2) The principal part of the tabulated errors arises from sub-permanent magnetism, whose effects in producing deviation vary greatly in different parts of the earth.	(2) The magnets which perfectly correct the sub-permanent magnetism in one place will also perfectly correct it in another place.
(3) It is, therefore, absolutely necessary from time to time to make a new table of errors, by observations in numerous positions (not fewer than eight) of the ship's head.	(3) Only when there is suspicion of change in the ship's magnetism are new observations necessary, and then two are sufficient.

(4) In difficult navigation, as in the channels of the Thames or the Mersey, especially with frequent tacks, the use of a table of errors would be attended with great danger.

(4) In any hydrographical difficulty the corrected compass is right on all tacks, and its use is perfectly simple.

'The Astronomer-Royal has no hesitation in giving his own opinion, that the compasses used for directing the ship's course ought to be corrected, and that the efforts of scientific men ought to be directed mainly to the rendering this correction rigorously accurate and easy of application.'

THE DUMB CARD.—To determine the correct angle through which the ship's head moves, which would not be given by a compass affected by deviation, a dumb card is used. This consists of a compass card without a needle, and fitted in every respect, with sight vanes and a prism, like an azimuth compass. If the correct magnetic bearing of any very distant object, celestial or terrestrial, be known, place the card so that it shall point out that direction; the correct magnetic bearing of the ship's head is at once read off by noticing where the fore and aft line cuts the edge of the card. Then if the ship's head be moved, and the magnetic bearing of the object be kept still in the right position of the card, the number of degrees the ship's head has moved through will be shown by the number of degrees passed over by the fore and aft line. In a somewhat similar way the ship's head may be put in any given magnetic direction after taking an amplitude or azimuth.

COMPENSATION OF SEMICIRCULAR DEVIATION.—It is very difficult in practice to separate the semicircular deviation caused by vertical iron from that caused by sub-permanent magnetism. Could this be effected easily, a natural inference would be to compensate the former by vertical soft iron, and the latter by a magnet, each compensator placed so as to produce opposite effects to those of the ship. An arrangement of this kind was adapted to several ships, with good effect, by Mr. Rundell, the Secretary to the Liverpool Compass Committee, who placed a vertical iron bar before the compass, leading from the keel (when possible) to a point at some height above the level of the card. The distance from the compass and height to which it must be carried is determined by experiment; but a difficulty has been experienced in adapting this method so as to ensure success.

The order of proceeding for compensation is as follows:—Make two chalk lines on the deck, intersecting each other at right angles, directly under the centre of the compass. One of these must run direct fore and aft, the other athwartships. Then to compensate + B or — B, the head of the ship must be placed magnetic E. or W., and a magnet of sufficient strength [1] must be placed on the deck, with its centre in the athwartship chalk line, and parallel to the fore and aft line; but in no case must the magnet be placed within twice its length of the compass needle. If the needle be attracted towards the stern, causing — B, the red end of the magnet must be placed aft to neutralise this effect, and *vice versâ*. It must then be moved towards or from the compass, always keeping its centre on the chalk line until the compass shows the ship's head to be east or west, as the case may be. If the ship was built head north, two magnets may be required, one on each side of the compass, and their positions found as before.

To compensate + C or — C, the ship's head must be brought magnetic N. or S., and a magnet placed athwartships, with its centre on the fore and aft chalk line. If by the ship's attraction the red end of the needle be drawn to starboard, causing a + C, this must be compensated by placing the blue end of the magnet to port, thus attracting the red end of the needle, and the magnet must be moved parallel to its first position, with its centre always on the chalk line, until the ship's head bears N. or S. by the compass. If C be large, two magnets may be required here also to be treated exactly as one would be. The compensation should be proved by putting the ship's head on the two cardinal points not already employed. Owing to the

[1] Compensating magnets should be from 10 to 18 inches in length, their breadth one-tenth their length, and their thickness one-fourth their breadth, and should sustain their own weight.

change in the semicircular deviation, this compensation is correct only in the same magnetic latitude where it was first effected; therefore, the magnets should be so placed that the master may be able to move them as required. For this purpose he must bring the ship's head on two adjacent cardinal points by astronomical observation, and slide the magnets along until the compensation be again perfect. When a ship is new, the semicircular deviation is generally diminishing from a loss of magnetism acquired in building; hence it is better to leave a few degrees uncompensated; and if the ship has been swung from right to left, about $1\frac{1}{2}°$ westerly deviation should be left uncompensated, when the ship's head is north or south, and about $\frac{1}{2}°$ when the ship's head is east or west; but if she has been swung from left to right, these amounts should be easterly. The reason for this is, the ship does not at once acquire the amount of induced magnetism due to the position of the ship's head.

Compensation of Quadrantal Deviation.—As quadrantal deviation is caused by the action of horizontal soft iron, a natural inference is, that soft iron should be used for compensating this error, so placed as to cause opposite effects to those of the ship. The compensations for semicircular deviation being complete, the ship's head should be swung to one of the intercardinal points NE., NW., SE., or SW., correct magnetic. The late Astronomer Royal recommends boxes filled with iron chain to be used; but the Liverpool Committee found cast iron correctors, from 9 to 12 inches long and 3 to $3\frac{1}{4}$ inches in diameter, with hemispherical ends, to be preferable when the quadrantal deviation exceeds 2° or 3°. The error from this source is generally of the form $+$ D; $-$ D being caused by divided athwartship iron, is seldom found in any vessel. The chain or correctors for a $+$ D error must be placed one to port, the other to starboard, of the compass, when the ship's head is on one of the intercardinal points; and the compensation is made by moving the correctors towards or from the compass, or by placing in or taking chain from the boxes, until the ship's head shows by compass the intercardinal point chosen. As quadrantal deviation remains the same in every geographical position of the ship, therefore when once this compensation is made, it remains perfect for every place whilst the ship remains under the same conditions, and is subject to but little change by lapse of time. Very often no compensation is made for this error; but it is included in the table of deviations. If the deviation produces — D, the soft iron correctors must be placed fore and aft of the compasses. Between the iron decks of an iron ship the compasses cannot be compensated by soft iron; then Captain Evans, R.N., has recommended that two compasses, equal in every respect, should be used, as is often the case in a double binnacle; the plus quadrantal deviation of the ship is thus compensated by the minus produced by the action of the compasses on each other. This method is not perfect, because the effects produced differ with a change of magnetic latitude.

Professor Airy says, 'The order of operations ought in all cases to be thus:—

'(1) For a compass near the stern, Rundell's vertical bar ought to be fixed.

'(2) The two magnets, or systems of magnets, for effecting the correction with the ship's head N.E.S.W., ought to be applied.

'(3) The masses of iron for correcting the quadrantal deviation ought to be applied, or the modified card ought to be mounted. These will never require alteration, whatever alteration be made in the magnets.

'(4) The ship should, if possible, be sent on a short voyage, or should be exposed to agitation by the sea, and to tremor by her machinery, in different positions of her head for several days.

'(5) The positions of the magnets ought to be readjusted. It will probably be sufficient to place the ship once with her head N. (or S.), and once E. (or W.)'

Captain Evans, R.N., found that if a compass has a single needle, a small sextantal error (or one six times repeated in the circle) is introduced by semicircular compensation, and a small octantal one by quadrantal compensation; but if the compass card be fitted with two needles whose ends are 60° apart, or with four needles 30° apart, these small errors are eliminated.

Compensation for Heeling Error.—This must be effected by fitting a magnet perpendicular to the deck into a groove cut in the pillar which supports the compass, and directly under the centre of the card when the ship is upright. The former compensations being made, the ship is heeled 10° on either side, with her head north when the ship is upright. If the needle be drawn to windward (the general case), the red end of the magnet must be upward, but if to leeward, the blue end must be upward, and it must be slid along the groove until the ship's head appears north by compass. The ship should then be heeled 10° on the other side, and if the head still appears north, the compensation is perfect; if not, one-half must be allowed for heeling in each direction. Mr. Towson says[1] if the compass be uncompensated and the vertical force on board be reduced to nine-fourteenths of that on shore, the compass will not be considerably affected by heeling. This compensation is perfect only in the same magnetic latitude as where it was made; hence arrangements must be made for sliding the magnet along as different latitudes are reached, for removing it altogether, or even for reversing it in high latitudes of opposite name; but it is the general opinion that the error for heel should be left uncompensated except when the voyages the vessel is to make are restricted to the same or to nearly the same magnetic latitudes.

After all the compensations have been accurately performed, there will still remain small residual errors; for these the ship must be swung, and a table of deviations made for use. In the 'Admiralty Manual' the following caution is given: 'Should an arrangement of magnets be employed to neutralise those large deviations occasionally found, and caused by the iron ship's magnetism, the compass so corrected can never be considered as entirely compensated, and the deviation must be

[1] *Practical Information on the Deviation of the Compass*, by J. T. Towson.

expected to change on change of latitude and from other causes. It will thus be seen that the mariner can have no absolutely safe guide, except in the system of actual and unceasing observation.'

Exercise XIX.

Ex. 381. Describe fully the laws to which the effects of the induced magnetism of iron in a ship on the standard compass are subject. What part of the deviation is due to this cause? The compass of an iron ship on first putting to sea is found to have a large amount of deviation, which is much reduced after her first voyage; how do you account for this? *A.* 1869.

Ex. 382. What difference in the sub-permanent magnetism of a ship should I find, according as it is built with its head pointing due N., S., E., or W.? Is it better to build a ship with her head N. or S.? Give reasons. *A.* 1868.

Ex. 383. Explain accurately the cause of that part of the deviation of the compass called the quadrantal deviation, and the changes, both in magnitude and direction, through which it passes while the ship's head turns through 16 points. *Honours*, 1870.

Ex. 384. Explain fully the causes of that part of deviation which is known as quadrantal deviation? Why is this name applied to it? In what direction should a ship's head lie while building? Give your reasons. *A.* 1871.

Ex. 385. Describe the operation of swinging a ship to find the deviation. The bearing of the ship's compass by the shore compass is N. $a°$ E., and the bearing of the shore compass by the ship's compass is S. $b°$ W. Show that the deviation is easterly or westerly, according as a is $>$ or $<$ b. *Honours*, 1871.

Ex. 386. State fully the laws of induced magnetism as affecting the compasses of a ship. What is the name given to that part of the deviation depending on this cause? What is sub-permanent magnetism and the part of deviation due to it? *A.* 1872.

Ex. 387. Distinguish between permanent, sub-permanent, and induced magnetism. Explain what is meant by semicircular deviation and quadrantal deviation, giving the reason of the names. *A.* 1873.

Ex. 388. State the nature of semicircular deviation, and the laws it follows in wood-built ships and in iron-built ships. Explain fully the nature and cause of the heeling deviation. *Honours*, 1873.

Ex. 389. Describe the methods of determining the deviation of a ship's compass. A table of deviation is drawn up for an iron ship soon after she is launched; under what circumstances could this not be depended on? Describe fully the magnetic changes the ship would be subject to. *A.* 1874.

Ex. 390. In an iron ship, show the necessity for the mariner to be acquainted with the circumstances under which she was built. Describe also the means used by the compass-adjuster to compensate for the deviation. Investigate the theory of the heeling error. *Honours*, 1874.

Ex. 391. Describe the effects which the position in which an iron ship is built has upon the action of the compasses on board. How are these effects practically encountered? *A.* 1875.

Ex. 392. Give a full account of the causes of permanent and sub-permanent magnetism in an iron ship, and the effects on the ship's compasses. Describe the effect produced on the deviation of the compass of such an iron ship by heeling, and state the general law for the heeling error. *Honours*, 1875.

Ex. 393. Mention the causes which influence the indications of the compass, and describe the manner in which the several disturbances are practically met and overcome by the mariner. In swinging a ship for deviations the following bearings were obtained: observed on board (1) NNW., (2) SW.; observed on shore (1) SE$\frac{1}{4}$S., (2) ENE. Draw figures for these two cases, and find the deviations. *A.* 1876.

Ex. 394. Distinguish between permanent and sub-permanent magnetism of a ship. State the laws of semicircular and quadrantal deviation. A ship is swung on an even keel: explain clearly why the deviation table so constructed is not always to be depended on at sea. Describe the use of pieces of soft iron and of magnets to reduce and counteract deviation in iron ships. *Honours*, 1876.

Ex. 395. Describe the process of mechanical correction of the several parts of the deviation—viz. (1) the semicircular deviation; (2) the quadrantal deviation; (3) the heeling deviation. *Honours*, 1877.

Ex. 396. The direction of an iron ship's polar magnetism, as affecting her compass, may be inferred from the place at which, and the position in which she was built; explain and illustrate this. Draw the hull of a ship built in north latitude with her head south, showing the parts affected with blue and red magnetism. How are the coefficients A, B, and C found? *Honours*, 1878.

Ex. 397. Distinguish between the correction and compensation of compasses, and describe the methods used in the compensation of compasses. What do you understand by the coefficients A, B, C, D, and E? *Honours*, 1879.

Ex. 398. Describe fully, by diagrams, 'Napier's graphic method,' and illustrate its use. *Honours*, 1880.

Ex. 399. To what causes are the variation and deviation of the compass respectively due? Explain by reference to the laws of induced magnetism that part of the deviation called semicircular. *A.* 1881.

Ex. 400. Describe the effects on a ship's magnetism—(1) by the direction of her head while building; (2) by attraction of the vertical iron on board. How would you compensate for these effects?

Honours, 1881.

Ex. 401. Describe the sub-permanent magnetism of an iron ship, and state when and how it is acquired, and which is the sub-permanent red and which is the blue pole; and why is it called sub-permanent magnetism? *Board of Trade Examination.*

Ex. 402. Describe the coefficients B and C, plus and minus; and why are they said to produce semicircular deviation?

Board of Trade Examination.

Ex. 403. On what points by compass bearing of the ship's head does + B give westerly deviation, and on what does it give easterly? also on what points does − B give westerly, and on what points easterly? *Board of Trade Examination.*

Ex. 404. On what points does + C give westerly, and on what points easterly? also on what points does − C give westerly, and on what points easterly deviation? *Board of Trade Examination.*

Ex. 405. If the value either of coefficient B or C be given, also the magnetic direction of the ship's head while she was being built, how by the traverse tables would you determine the approximate value of the other coefficient C or B? and if the value of both these coefficients be given, how would you determine approximately the direction by compass of the ship's head whilst being built?

Board of Trade Examination.

Ex. 406. Describe quadrantal deviation, and state what coefficients represent it; also on what points of the ship's head by compass each of these coefficients gives the greatest amount of deviation.

Board of Trade Examination.

Ex. 407. On what points of the compass will each of the coefficients D and E + and − give easterly, and on what points westerly deviation? *Board of Trade Examination.*

Ex. 408. Describe the nature of the deviation resulting from + A and − A, and describe the error in the construction of the compass that frequently produces them. *Board of Trade Examination.*

Ex. 409. If an ordinary standard compass placed higher than the iron top sides be compensated whilst the ship is upright, what coefficient will be affected by heeling? *Board of Trade Examination.*

Ex. 410. When generally will this coefficient be plus and when minus? State the exceptions to this general rule. To what extent is the heeling error altered by a change in the magnetic latitude of the ship? *Board of Trade Examination.*

Ex. 411. Given the heel, the direction of the ship's head by compass, and the heeling error observed, to find the approximate heeling error, with a greater or less given heel, and with the ship's head on some other named point of the compass, the ship's magnetic latitude being in both cases the same. Show how this is done.

Board of Trade Examination.

Ex. 412. Write down the formula for the deviation of a ship's compass in terms of the coefficients A, B, C, D, and E. To which parts of the ship's magnetism do the several terms belong? How may these coefficients be determined? *Honours*, 1881.

CHAPTER X.

Soundings—Hand and deep sea leads—Marks and deeps—Sir William Thomson's sounding apparatus—Tides, rise and fall—High and low water—Flood and ebb—Range—Springs and neaps—Establishment of port—Priming and lagging—Admiralty Tide Tables—Calculation of high water and of soundings—Graphic method of finding the rise and fall of the tide—Effects of wind and of atmospheric pressure—Exercises and examination.

SOUNDINGS.

SOUNDING is the method of determining the depth of the sea and of the nature of its bottom. It is usually made by means of a lump of lead hollowed out underneath, attached to a line, divided to fathoms, called the *lead line*. The hollow is filled with wax or tallow, and is then called *an arming*, into which sticks mud, sand, pebbles, shells, &c., when the sounding is taken, thus showing the nature of the bottom. Two leads are in general use at sea :—

(*a*) *The hand lead*, about fourteen pounds weight, attached to a line about 25 fathoms long. The line is divided into *nine* marks and *eleven* deeps. The marks are—leather at two, three, and ten fathoms, the first has two ends; the next three ends, and the last a hole in it;

white linen at five and fifteen fathoms;
red bunting at seven and seventeen fathoms;
blue cloth at thirteen fathoms, and
two knots at twenty fathoms.

These depths are called *marks*, and those not especially marked but judged are called *deeps*. The soundings are taken by the *leadsman* standing in the channels and swinging the lead once or twice around his head; he then throws it as far forward as possible in the direction the ship is going. When he comes perpendicularly over the lead, he stretches the line taut and calls out the depth. If this coincides with either of the marks, he says, 'By the mark,' adding the number of fathoms; if it coincides with either of the unmarked fathoms, he calls out, 'By the deep,' adding the number of fathoms. This method is used to prevent mistakes. The only subdivisions of the fathom in use are its quarter, half, and three quarters.

(*b*) *The deep sea lead* is about twenty-eight pounds weight, attached to a line from 100 to 120 fathoms long. The first 20 fathoms are marked exactly as the hand lead line, and after that with a corresponding knot for every 10 fathoms, and a single knot at each 5 fathoms. In sounding with the deep sea lead it is carried forward to the cathead or fore chains, the line being passed along outside all. The ship's speed is reduced and the lead thrown overboard, the leadsman standing in the after part of the ship; and when he comes perpendicularly over the lead he calls out the depth as before. The error in soundings is almost always in excess, because the line can very seldom be stretched straight from the lead except in shallow water. For deep sea soundings several patents have been taken out, as 'Burt's buoy and nipper,' 'Massey's lead,' something similar to his patent log-line, and Erricson's, which has been improved by Sir William Thomson, F.R.S. This consists of a long glass tube, closed at one end, and coated inside with chromate of silver; then because the volume of the air that is in the tube varies as the pressure (Boyle and Marriotte's law) and the pressure varies as the depth, therefore according to the depth so will the water be forced into the tube and discolour the chromate of silver. When drawn up the depth is calculated or measured on a scale which is graduated according to the pressure the air has sustained. The only error that can arise in this method is from the differences of temperature, and in sounding, a deep sea thermometer passes through strata of water at different temperatures, because of this we have no means of knowing those where the soundings have been made, but, notwithstanding this, this instrument is by far the best yet invented.

Soundings are taken for two reasons :—

(1) In shallow water, to show the pilot or the person in charge of the ship that he has water enough for her safe passage.

(2) In the deep sea, to verify the position of the ship, as nearly every sea and ocean have now the soundings laid down on the charts of the same.

When nearing land, soundings are especially valuable, and are even necessary on entering some bar harbours and docks; and when getting into water not much deeper than the ship's draught, the state of the tide must be taken into account, and for the *calculation* of soundings, the time of high water must first be found.

A superficial observer frequenting the seashore cannot fail to notice that for a certain time every day the water gets deeper and deeper until it attains a maximum depth, and will then get shallower and shallower until it has a minimum depth. We call this *the rise and fall* of the tide; the maximum depth we call *high water*, the minimum depth we call *low water*. He will also most probably notice that two maxima and two minima occur every day. If he were to take an average of the times between two high waters or two low waters extending over a long period, he would find that more than 12 hours would elapse, and if correctly timed it would amount to 12h. 24m.; so that two high waters and two low waters would *not* occur *every* day, but that on those days when the high water came near midnight he would not get a high water at all the next forenoon, but some time soon after noon. When the water is increasing in depth we call it *flood tide*, or say the tide is flowing; when decreasing in depth we call it *ebb tide*, or say the tide is ebbing. The difference in depth from high to low water we call *the daily range*. Again, our observer would notice that on some days the depth at high water is greater than on others, and on those days when we have the greatest depth at high water we get the least depth at low water, and so we should get a greater daily range than at other times. He would see the changes from great daily ranges to small daily ranges come on gradually and periodically, and would take about a fortnight from one maximum range to another, between which he would get a minimum one. The tide of maximum range we call *Spring tide,* and the one of minimum range we call *Neap tide.* If he knew anything about the moon's motion he could not fail being struck with the fact that the daily high waters are connected with the moon's passage across the meridian (called her transit), and on making his calculation, he would find that half a lunar day corresponded to the average of the times occupied from one high water to another, and that half a lunar month corresponded to the average difference of times between two maximum high waters. He would thus get two semi-diurnal maxima and minima depths, called respectively high water and low water, and two semi-menstrual maxima and minima depths, called respectively spring and neap tides. The former he would see was connected with the moon's transit, and the latter with the relative positions of the sun and moon; thus, when they are in conjunction or opposition they would produce spring tides, and when in quadrature they would produce neap tides. If persons at different ports were to take tidal observations, they would find by comparison that the times of high water did not occur at the times of the moon's transit, although they would be governed by it; but that the times would vary very much owing to the configuration of the land, the depth of the sea, and distance from the great Southern Ocean, where the tidal wave takes its origin. The inertia of the water, its friction on its bed, on its shores, and with the atmosphere, as well as among its own particles, all tend to produce a retardation of the time for the tidal wave to travel from port to port, and hence we do not get high water when the moon is on the meridian. But as it is of the utmost importance in navigation to be able to calculate the times of high water as well as the depth of water at any time at any particular place, we must have recourse to some standard tide to reckon from; and it is agreed to take the time and height of that tide which happens next after the full and change of the moon as the standard one, and the time of that high water reckoned from apparent noon is called *the vulgar establishment of the port.*

In calculations nearer results are obtained by using *the mean establishment of the port*, which is found by observing all the lunitidal intervals (that is, all the intervals which elapse between the moon's transits and the times of high water) for a month at least, and taking their mean. The time thus found is the establishment, entered on charts in Roman numerals.

In the Nautical Almanac, we find two tables for facilitating the calculations of the time of high water: the first contains the time of high water at London Bridge for every day in the year; the second one the establishment of the port for nearly two hundred places on the coasts of France, Belgium, Holland, Jutland and the British Isles. By taking the difference of time between the establishment of the port under consideration and that at London Bridge, we obtain a constant to be applied to the time of high water at London Bridge for any particular day. We will work a question, as set in the Board of Trade Examinations.

Ex. 413. Find the times of high water at Portsmouth Dockyard on July 4, 1882, civil time.

We first turn to the table, '*Time of High Water on the Full and Change of the Moon*,' on page 471, and take out the establishments for London Bridge and for the place required, then find the interval elapsed between these establishments. This is the constant for the place—

H. W. at London Bridge	1h.	58m	on full and change.
„ „ Portsmouth .	11	41	„ „ „ „
Constant –	2	17	

The high water at Portsmouth Dockyard is 2h. 17m. before that at London Bridge.

We next turn back a page to the table entitled '*Mean Time of High Water at London Bridge*,' for the given date July 4. It must be noticed that we require the morning and afternoon tides of July 4—that is, the last tide on the astronomical date, July 3, and the first tide on the astronomical date July 4. Then as the constant shows the high water at Portsmouth to be 2h. 17m. before that at London Bridge, we subtract the constant from the times taken out of the Nautical Almanac thus:—

July 4, H. W. London Bridge	3h.	49m.	A.M.	4h.	13m.	P.M.
Constant –	2	17		2	17	
July 4, H. W. Portsmouth .	1	32	A.M.	1	56	P.M.

To work the following questions it will be necessary the student should obtain a Nautical Almanac for 1882.

Exercise XX.

Find the times of high water at the places mentioned on the civil days as follows:—

Ex. 414. 1882, June 13, at London Bridge.

Ex. 415. 1882, February 17, at Newport (Wales).

Ex. 416. 1882, November 15, at Liverpool.

Ex. 417. 1882, October 1, at Leith.

Ex. 418. 1882, April 30, at Yarmouth Roads.

Ex. 419. 1882, September 25, at Wisbeach.

This method is only approximate, because it assumes the difference between the establishment of the port and that at London Bridge to remain constant throughout the half month; whereas it differs from day to day, for many reasons; among which *the priming and lagging of the tide*, or the acceleration and retardation of the times of high water, caused by the shifting of the relative positions of the sun and moon from day to day and the increased pressure and consequent friction of the spring over the neap tides, play a prominent part. From its simplicity this method is frequently used by sailors, as it gives results whose errors affect navigation but very little, seeing that so small a proportion of the whole range rises and falls during the first or last quarter of an hour. Another rough-and-ready way the sailor has of finding his time of high water is to add 48 minutes to the establishment of the port laid down on his chart for every day elapsed since the full or change of the moon, because the lunar day is about 48 minutes longer than the mean solar day. For a nearer approximation for finding the time of high water, but still only an approximation, we must make use of the 'Admiralty Tide Tables.' In that work are given, for at least twelve months in advance, the establishments of the port and range of tide for all the principal places in the world; and for those bordering on British seas, twenty-four ports for reference are added, with the names of the chief places lying near them, together with much very valuable information on the tides around our own shores. By the aid of that work, the time of high water and the rise and fall of the tide can be approximately ascertained for any time at any place which has a port for reference annexed. This method has the advantage of referring ports around the British Isles and France to another port where the ebb and flow are caused by the same tidal wave, and therefore equally affected by the priming and lagging of the tides, which is not the case when London Bridge is selected as the only port for reference; the high water there being caused by the combination of two tidal waves, one of which left Cape Clear twenty-two hours before and travelled round the north of Scotland, the other left the same place ten hours before and travelled up the English Channel.

The method for finding the time of high water is as follows: In the table called 'Tidal Constants,' from page 104 to 108, find the port for which you wish to obtain the time of high water, and opposite it you will see the constant with its proper sign annexed and the port for reference. Then turn back to the month and find the port for reference on the top of the page, and opposite the day required take out the times for the A.M. and P.M. high waters, and apply the constant as directed in the table of Tidal Constants. This will give the times of high water for the day in question at the given port.

Ex. 420. 1878: find the times of high water at Arklow, A.M. and P.M. of January 19.

Constant for Arklow − 2 h. 25 m. from Kingstown.

January 19, H. W. at Kingstown	11h. 14m. A.M.	11h. 37m. P.M.
Constant	− 2 25	− 2 25
January 19, H.W. at Arklow	8 49 A.M.	9 12 P.M.

Exercise XXI.

Ex. 421. 1878, Mar. 14: find the time of H.W. at Berwick A.M. and P.M.
Ex. 422. 1878, Mar. 13: „ „ „ Filey Bay „
Ex. 423. 1878, Jan. 14: „ „ „ Valentia „
Ex. 424. 1878, Jan. 14: „ „ „ Limerick „
Ex. 425. 1878, Jan. 20: „ „ „ Arklow „
Ex. 426. 1878, Jan. 9: „ „ „ Thurso „
Ex. 427. 1878, Mar. 16: „ „ „ Berwick „
Ex. 428. 1878, Mar. 14: „ „ „ Filey Bay „

For the solution of the above questions we have added the necessary data taken from the 'Admiralty Tide Tables,' but it would be better for the student to provide himself with that publication, as then he would become conversant with its use, and much valuable information on the tides could be gleaned therefrom.

Tide Table, 1878.

Standard ports for reference	Date	Morning		Afternoon	
		Time	Height	Time	Height
		h. m.	ft. in.	h. m.	ft. in.
Liverpool . .	January 3	11 0	25 6	11 22	24 10
	„ 4	11 43	26 0	—	—
Thurso . .	„ 7	10 39	11 9	10 57	11 7
	„ 8	11 15	11 4	11 34	11 2
	„ 9	11 53	11 0	—	—
	„ 10	0 12	10 9	0 31	10 5
Galway . .	„ 13	11 6	10 5	11 42	10 5
	„ 14	—	—	0 20	10 6
Queenstown .	„ 14	—	—	0 18	8 10
	„ 15	0 58	8 11	1 38	9 2
Kingstown . .	„ 19	11 14	10 11	11 37	11 1
	„ 20	—	—	noon	11 3
	„ 21	0 23	11 3	0 45	11 4
Brest . . .	February 8	6 56	16 7	7 12	16 1
	„ 9	7 29	15 6	7 49	14 10
	„ 10	8 10	14 2	8 35	13 7
	„ 11	9 4	13 2	9 39	12 10
	„ 12	10 22	12 9	11 14	12 11
	„ 13	—	—	0 3	13 4
	„ 14	0 49	13 11	1 28	14 9
	„ 15	1 59	15 9	2 24	16 10
	„ 16	2 47	17 10	3 10	18 11
	„ 17	3 33	19 8	3 55	20 4
	„ 18	4 17	20 8	4 39	20 11
	„ 19	4 59	21 2	5 19	21 1
	„ 20	5 39	20 11	5 59	20 7
Devonport . .	March 13	11 5	11 3	11 51	11 11
	„ 14	—	—	0 41	11 3
	„ 15	1 32	12 6	2 19	12 1
Sunderland . .	„ 13	9 21	10 4	10 9	10 3
	„ 14	10 58	10 4	11 41	10 8
	„ 15	—	—	0 23	11 3
	„ 16	1 0	11 11	1 29	12 8

Tidal Constants, 1878.

Ports	Constants: Time	Constants: Mean spring range	Constants: Standard port for reference	Constants: Mean spring range
	h. m.			
Valentia Harbour	− 1 19	11 feet	Queenstown	11¾ feet
Limerick . .	+ 1 45	18¾ „	Galway	14¾ „
Warren Point .	0 0	14½ „	Kingstown	11 „
Arklow . .	− 2 25	4 „	„	„
Port Carlisle . .	+ 0 47	20 „	Liverpool	27½ „
Stornoway . .	− 1 42	13½ „	Thurso	13¼ „
Lerwick . .	+ 2 2	6 „	„	„
Berwick . .	− 1 4	15 „	Sunderland	14½ „
Filey Bay . .	+ 0 58	23 „	„	„
Exmouth . .	+ 0 38	12¼ „	Devonport	15½ „
Penzance . .	− 1 13	16¼ „	„	„
Cadiz . . .	− 2 2	9½ „	Brest	19 „
Bordeaux . .	+ 3 3	14 „	„	„
Barfleur . .	+ 5 4	17 „	„	„
Havre . . .	+ 6 4	22 „	„	„

TABLE B.—*For finding the Height of the Tide at any Intermediate Hour between High and Low Water.*

Height above half tide or mean level of the sea	Time from high water												
	h. m. 0 0	h. m. 0 30	h. m. 1 0	h. m 1 30	h. m. 2 0	h. m. 2 30	h. m 3 0	h. m 3 30	h. m. 4 0	h. m. 4 30	h. m. 5 0	h. m. 5 30	h. m. 6 0
	ADD						SUBTRACT						
feet	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.
3	3 0	2 11	2 7	2 1	1 6	0 9	0 0	0 9	1 6	2 1	2 7	2 11	3 0
4	4 0	3 10	3 6	2 10	2 0	1 0	0 0	1 0	2 0	2 10	3 6	3 10	4 0
5	5 0	4 10	4 4	3 6	2 6	1 3	0 0	1 3	2 6	3 6	4 4	4 10	5 0
6	6 0	5 10	5 2	4 3	3 0	1 7	0 0	1 7	3 0	4 3	5 2	5 10	6 0
7	7 0	6 9	6 1	4 11	3 6	1 10	0 0	1 10	3 6	4 11	6 1	6 9	7 0
8	8 0	7 9	6 11	5 8	4 0	2 1	0 0	2 1	4 0	5 8	6 11	7 9	8 0
9	9 0	8 8	7 9	6 4	4 6	2 4	0 0	2 4	4 6	6 4	7 9	8 8	9 0
10	10 0	9 8	8 8	7 1	5 0	2 7	0 0	2 7	5 0	7 1	8 8	9 8	10 0
11	11 0	10 8	9 6	7 9	5 6	2 10	0 0	2 10	5 6	7 9	9 6	10 8	11 0
12	12 0	11 7	10 5	8 6	6 0	3 1	0 0	3 1	6 0	8 6	10 5	11 7	12 0
13	13 0	12 7	11 3	9 2	6 6	3 4	0 0	3 4	6 6	9 2	11 3	12 7	13 0
14	14 0	13 6	12 1	9 11	7 0	3 7	0 0	3 7	7 0	9 11	12 1	13 6	14 0
15	15 0	14 6	13 0	10 7	7 6	3 11	0 0	3 11	7 6	10 7	13 0	14 6	15 0
16	16 0	15 5	13 10	11 4	8 0	4 2	0 0	4 2	8 0	11 4	13 10	15 5	16 0
17	17 0	16 5	14 9	12 0	8 6	4 5	0 0	4 5	8 6	12 0	14 9	16 5	17 0
18	18 0	17 5	15 7	12 9	9 0	4 8	0 0	4 8	9 0	12 9	15 7	17 5	18 0
19	19 0	18 4	16 5	13 5	9 6	4 11	0 0	4 11	9 6	13 5	16 5	18 4	19 0
20	20 0	19 4	17 4	14 2	10 0	5 2	0 0	5 2	10 0	14 2	17 4	19 4	20 0
21	21 0	20 3	18 2	14 10	10 6	5 5	0 0	5 5	10 6	14 10	18 2	20 3	21 0
22	22 0	21 3	19 1	15 7	11 0	5 8	0 0	5 8	11 0	15 7	19 1	21 3	22 0
23	23 0	22 3	19 11	16 3	11 6	5 11	0 0	5 11	11 6	16 3	19 11	22 3	23 0
24	24 0	23 2	20 9	17 0	12 0	6 2	0 0	6 2	12 0	17 0	20 9	23 2	24 0

To find the soundings at any given time at a given place.—The time of the nearest high water to the given time must first be calculated for the given place, and the difference between the given time and the time of high water taken. Turn then to the table at the end of the book and take out the 'Rise of the tide above mean low water level spring tides' for the port for reference and for the given port, and find their difference in rise. Then turn back to the beginning of the book and take out the rise in the column 'Height' for the port for reference opposite the given day of the month, and apply the difference just found. The result gives approximately the height of the high water above the mean low water spring level at the given place for the day required. Subtract half the spring rise at the given place from the height of high water just found; the remainder will give the height above half tide or mean level of the sea which the high water for that day is. This is half the range for the given day. Now place down half the mean spring range at the given place, and to this apply the correction taken from Table B, p. 98, 'Tide Tables' (a copy of which is on p. 252), with the arguments, height above half tide, and time from high water, as directed in that table. The result is the correction to be applied to the soundings by the lead line before it can be compared with the depth marked on the chart.

Ex. 429. On January 4, 1878, at 3h. 55m. A.M., being off Port Carlisle by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead line before comparing it with the depth marked on the chart.

Time constant for Port Carlisle + 0h 47m. to Liverpool.

		ft.	in.	
Mean spring range at Liverpool		27	6	
" " " Port Carlisle		20	0	
Height constant	−	7	6	
		h.	m.	
January 3, high water at Liverpool		11	22	P.M.
Time constant	+		47	
January 4, high water at Port Carlisle		0	9	A.M.
Time of soundings		3	55	A.M.
Soundings taken after high water		3	46	

		ft.	in.
Height of high water at Liverpool		24	10
Height constant	–	7	6
Height of high water at Port Carlisle		17	4
Half spring range at „ „	–	10	0
Height above half tide		7	4

		ft.	in.
Half spring range at Port Carlisle		10	0
Correction from (B) for 3h. 46m. and 7ft. 4in.	–	2	10·1
Correction to be applied	–	7	1·9

Hence I must subtract 7ft. 1·9in. from the soundings taken by the lead line before comparing it with the depth marked on the chart.

Exercise XXII.

Ex. 430. On January 14, 1878, at 9h. 9m. P.M. being off Valentia by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead line before comparing it with the depth marked on the chart.

Ex. 431. On January 14, 1878, at 5h. 27m. A.M. being off Limerick.

Ex. 432. On January 20, 1878, at 7h. 15m. A.M. being off Warren Point.

Ex. 433. On January 21, 1878, at 4h. 50m. P.M. being off Arklow.

Ex. 434. On January 7, 1878, at 2h. P.M. being off Stornoway.

Ex. 435. On February 18, 1878, at 9h. P.M. being off Cadiz.

Ex. 436. On January 4, 1878, at 4h. P.M. being off Liverpool.

Ex. 437. On February 9, 1878, at 1h. 10m. A.M. being off Bordeaux.

A more expeditious, though not so correct a method for determining the rise and fall at any hour of the tide after knowing its daily range was suggested to us by the late Commander W. Walker, R.N., late Queen's Harbourmaster in Devonport dockyard. He found empirically from the tide gauge that—

The rise and fall in 1st hour = $\frac{1}{12}$ of whole range.
„ „ 2nd „ = $\frac{2}{12}$ „
„ „ 3rd „ = $\frac{3}{12}$ „
„ „ 4th „ = $\frac{3}{12}$ „
„ „ 5th „ = $\frac{2}{12}$ „
„ „ 6th „ = $\frac{1}{12}$ „

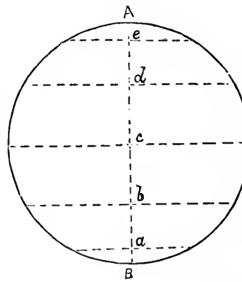
That is, the rise and fall in the individual hours were as the numbers

1 : : 3 : 3 : 2 : 1.

Hence the first hour's rise or fall will be as many inches as there are feet in the daily range, the second and fifth hours twice as many inches, and the third and fourth hours three times as many inches as there are feet in the daily range.

Another method of showing graphically the hourly rise and fall of the tide is as follows, due, we believe, to the late Admiral Beaufort.

FIG. 42.



Draw a circle, whose diameter shall represent the daily range for the given day, and in it place a vertical diameter, as A B. Then, if the circumference of each semicircle be divided into six equal parts and horizontal lines be drawn through these divisions, the vertical diameter will be divided very nearly in the ratio of the hourly rise and fall of the tide. The ratios in this case are found as follows:—

Rise and fall	1st hour	B a =	vers. 30°			= $\frac{1}{8}$ radius nearly	= $\frac{1}{16}$ range.
,,	2nd ,,	$a\,b$ =	,, 60	−	vers. 30°	= $\frac{3}{8}$,,	= $\frac{3}{16}$,,
,,	3rd ,,	$b\,c$ =	,, 90	−	,, 60	= $\frac{1}{2}$,,	= $\frac{4}{16}$,,
,,	4th ,,	$c\,d$ =	,, 120	−	,, 90	= $\frac{1}{2}$,,	= $\frac{4}{16}$,,
,,	5th ,,	$d\,e$ =	,, 150	−	,, 120	= $\frac{3}{8}$,,	= $\frac{3}{16}$,,
,,	6th ,,	e A =	,, 180	−	,, 150	= $\frac{1}{8}$,,	= $\frac{1}{16}$,,

The ratios here are nearly as 1 : 3 : 4 : 4 : 3 : 1.

Hence, if the daily range be divided into sixteen equal parts, one part must be taken for the rise and fall in the first hour, three parts in the second hour, four parts in the third hour, and so on ; a result agreeing very closely with the observations made by Commander Walker, the difference being found only in the first and second hours, and then to the extent only of $\frac{1}{48}$ of the whole range. Finding of soundings by calculation is seldom practised by mariners, because in fine weather it is much easier *to heave the lead* and keep it going than to go through the necessary work ; and in foul weather, calculations take no notice of the action of the wind nor of the height of the barometer, and if they did they would be only guesses. In Plymouth Harbour a strong SW. wind will often drive two feet of water in with the tide, and a strong NE. wind prevent the tide rising to its normal height by the same amount; and in other harbours, other winds affect the rise and fall to the same or greater amounts. Again, the height of the tide is influenced by the same causes which influence the height of the barometer, viz. the pressure of the atmosphere. The specific gravity of mercury is $13\frac{1}{2}$, or it is $13\frac{1}{2}$ times as heavy as fresh water; hence the pressure which will sustain one inch of mercury in the tube of the barometer will sustain $13\frac{1}{2}$ inches of fresh water; and therefore, if the barometer reads one inch lower than the normal pressure, we may expect at least $13\frac{1}{2}$ inches deeper water than that found by calculation. We say *at least* because the removal of the atmosphere representing an inch of mercury from one place to another, must cause an increased pressure at the second place, and a consequent higher level of water at the place where the pressure has been removed from, and the late Professor Whewell says, in 'The Admiralty Manual of Scientific Inquiry,' that a decrease in pressure amounting to 1 inch will cause a rise in the tube of a tide gauge of 20 inches. For these reasons the sailor prefers to take his soundings by the lead to calculating them.

Exercise XXIII.

Ex. 438. What do you understand by taking soundings? Why are they necessary, and what instruments are used in determining the depths? Describe them.

Ex. 439. What is the arming, and how by its aid can you determine the nature of the bottom of the sea when sounding? What errors may affect the soundings by the lead?

Ex. 440. Describe how soundings are taken and the marks on the lead line. What is the difference in the marking of the hand lead and the deep sea lead?

Ex. 441. Describe Sir William Thomson's deep sea lead. How may an error be introduced into soundings taken by this method?

Ex. 442. What do you mean by the 'ebb and flow of the tide,' what by 'the rise and fall of the tide,' what by 'the daily range,' and why is the latter greater in one port than in another?

Ex. 443. What do you mean by the 'priming and lagging' of the tides, and how does this affect the time of high water at different ports? What other causes affect the time of high water?

Ex. 444. Describe any graphic methods for finding the rise and fall at any particular time of tide if the daily range be given.

Ex. 445. How is the depth of the water ascertained—(1) when it is not much greater than the draught of the ship, (2) when the depth is considerable? What is the arming of the lead? Describe the patent log. *E.* 1878.

Ex. 446. What do you mean by the establishment of the port? How does a knowledge of this enable you to find the time of high water.

Ex. 447. Explain all the methods you know for finding the time of high water. Which has most advantages, and why?

Ex. 448. Distinguish between spring and neap tides. Define 'establishment of the port,' and explain its use. Give the establishment of the port of Holyhead. Explain generally how the time of high water may be found on any day. *Honours*, 1880.

Ex. 449. Explain the cause of the tides, and draw a diagram showing the relative position of the earth, moon, and sun at neap tides. Is low-water mark lower at spring tides or neap tides? *Honours*, 1874.

CHAPTER XI.

Surveying, object of—Methods of determining the base line—Triangulation—Tidal observations—How the rise and fall are obtained—On finding the time of high water—Tide gauge—How shoals are discovered—How set and drift of the current are ascertained—Running survey—Examples and examination.

SURVEYING.

SURVEYING is the methodical measurement and observation of a portion or of the whole of a country, so as to enable the surveyor to lay down places on a chart in their relative positions to all others surrounding them. Sir Edward Belcher says in his work on 'Surveying,' this 'subject has never been sufficiently introduced into Epitomes of Navigation and Nautical Astronomy'; but we have not to seek far for the reason. Had examining bodies demanded it, no doubt the subject would have been introduced into standard works on Navigation.

Before commencing; a code of signals must be agreed on, because those making the survey will often be so far apart that they can understand the others engaged by signs only. Several instruments, in addition to those generally found on board, will be required, such as the *theodolite*, *box-sextant*, *prismatic compass*, *station-pointer*, *micrometer*, &c., for a detailed description of, and the method of using which, the student must be referred to some good treatise on mathematical and surveying instruments.

The objects to be obtained in marine surveying should include—

(1) The exact determination of the latitude and longitude of well-defined points.

(2) The horizontal and vertical contour of coasts.

(3) Tidal observations, as
- (*a*) Rise and fall of the tide.
- (*b*) Soundings.
- (*c*) Strength of current, and whether tidal or not.

(4) Position of, and soundings on shoals.

(5) Magnetical and meteorological observations.

(6) Nature of the country.

The subjects included under (2), (3), and (4) will especially engage our attention.

(1) *The exact determination of well-defined points* is useful—first, that all surrounding stations may be referred to them in measurement and direction; and, secondly, that vessels in passing them may have at hand means for testing and correcting their chronometers. The methods to be employed belong to Nautical Astronomy, and must be deferred until we treat of that subject.

(2) *The horizontal and vertical contour of the coast*, and *the position of and soundings on shoals* can best be treated of together. For the survey of any place whatever, the first thing to be done is to take a preliminary view from some elevated position, as the topmast of a vessel, or some neighbouring hill, or by pulling around the coast; and before any actual measurements are made, a rough ground plan should be sketched showing the principal objects to be noted. This will guide the surveyor in the selection of the locality and position of his base line. The base line is so called because it is to be the base of all his future operations, to which all other stations are referred, and from which all his calculations have to be made.

The requisites for a good base line are—

(*a*) To be as level and smooth as practicable, which qualifications are most likely to be found near the seashore.

(*b*) To be, if possible, within sight of all points necessary to be laid down.

(*c*) To be as long as the nature and the extent of the survey demands. From a half to two miles will be found long enough for most practical purposes.

Having selected the position of his base line, the surveyor must then determine the exact latitude and longitude of one end, and the angle its direction makes with the meridian, i.e. its azimuth, both of which must be done by astronomical calculations, as the direction in an unknown country cannot be determined by the compass, because neither the variation nor the local attraction caused by the iron in the rocks of the district has been ascertained. He must next proceed to lay down and measure it. This is not quite so easy as it appears to be, but it is practically one of the most difficult operations to be performed, and the greatest care must be exercised in doing this, or the positions of all other stations deduced from it will be more or less inaccurate. He first places the tripod of his theodolite so that its plumb-bob shall be vertically over the spot he has determined the latitude and longitude of; he then directs its telescope towards the other end of the base line, and his assistants fix up staves or poles in the direct line of the axis of his telescope at about every hundred yards. This being done, if the ground be smooth, two of his assistants next proceed to measure it, in a direct line along the ground, by means of a steel chain 100 feet long, the principal remaining at the theodolite to see that no deviation is made from the straight line. For the purpose of measurement the leader is supplied with ten pickets (pointed irons), and as soon as the chain is stretched, he places one at the end of the chain, and then walks on carrying the end of the chain with him. The follower when he comes to the picket calls to the leader, and another picket is placed in, which the follower takes up as he comes to it. Both persons must keep their eyes on their principal and on each other to see that they are exactly on the base line. This is repeated until 1,000 feet are measured, when the follower returns the ten pickets to the leader, who registers each return in a book that no mistake may be made. This is continued to the length desired, when the principal, after placing a staff to mark the end of the base line, shifts his theodolite to the other end, and a similar measurement is made *backward* on the base line, and should be repeated at least three times from each end, and the mean taken, as it will be found in general no two measurements agree. If the ground be not smooth, then staves are placed in the base line much nearer, and a line stretched taut from staff to staff perfectly level, and the measurement made from end to end of the base line by a long graduated pole, or, better still, by a measured portion of the lead line which has had some use and has therefore been wetted and dried over and over again. Measurements thus made must be repeated, and the mean taken as before. Whichever method is adopted, it is well to verify the result on a calm day by alternately firing a gun several times from each end at given signals, and carefully registering the interval elapsed between the flash and the sound, and also noting the temperature of the air at each observation. The mean of the intervals from each end must be taken, and, if they agree, the distance must be calculated. The velocity of sound in air at the freezing point is 1,090 feet per second; but from the increase of the elasticity of the air with an increase of temperature, the velocity of sound is augmented by $\frac{10}{9}$ of a foot per second for every degree the thermometer rises above 32° F. If the intervals from the two ends do not agree, their mean should be taken, if not too discordant; but if much out, the operation should be performed under more favourable circumstances with a reversion of the positions by guns and watches. Another method of verification, easy of practice, is by means of the micrometer. This is an instrument divided most minutely, and the distances at which a known length subtends each division is registered. All that has to be done then is to fix the pole of known length vertically (by the plumb-line), and read how many divisions of the micrometer is subtended by it, then by reference to the register (which is easily calculated) the distance is at once known. This repeated will give the length of the base line.

If the coast be rocky or rising abruptly, two points in the cliffs, or a point in the cliff and on some rock or buoy in the sea, may of necessity be the two ends of the base line.[1] Sometimes

[1] Sir E. Belcher, p. 68, says lime should be used for marking, and continues, 'In whiting a spot, much time is frequently saved by first wetting the spot well, and then dredging it on by hand. It stands better, and less is lost by *streaming* off; the thickness can be laid on at pleasure, and is *instantly white*.'

it is necessary to have both ends of the base line on the water, then two buoys must be moored, and the distance in each of the latter cases must be ascertained by sound and by the micrometer if possible; but it should be remembered the base line by sound should be as long as practicable, because any error made will bear a smaller ratio to a long base line than to a short one. Where there is sea room the distances can be estimated very correctly by a series of runs to and fro, measuring each run with the patent log and taking the mean. Another method of obtaining the length of the base line is by observing *the masthead angle*. This is done as follows: The vertical height of the masthead is known, and the angle of depression of each end of the base line is observed with a sextant from the masthead. The distance of each end from the observer can easily be calculated from

Vertical height of observer × cosec angle depr. = distance of observer.

All then that is necessary is to take the angle at the eye with the sextant, subtended by the two ends of the base line; and then in the triangle thus formed we shall know the two sides just calculated and the included angle to find the third side or base line. A choice of methods for ascertaining the length of the base line must be left with the surveyor, and he ought to select those which the exigencies of his case demand; but he should never neglect to find it by as many ways as possible for a verification of results. The length by all the methods used should agree very closely if proper precautions and care have been exercised. It will most probably occur to the student that if one end of the base line can be fixed by astronomical observation, the other could also, and we should thus avoid the necessity for such careful and extended operations as has been described to get the length required. But those accustomed to observations know how difficult it is to get an altitude within 10″ or 20″; besides the introduction of the doubtful factor, that of difference of refraction. Now what does 10″ or 20″ difference in altitude mean? In latitude it would mean one-sixth or one-third of a nautical mile, that is, 1,000 or 2,000 feet, and in longitude perhaps three or four times that amount—errors which could not possibly occur in careful measurements.

Having obtained and verified his base line, he must, with either theodolite or sextant, observe the angles included between it and the line of sight to conspicuous places; he will then have two angles and the included side of as many triangles as he has objects. With the observed angles and length of base line he must calculate their bearings relative to it and their distances. He can then (if necessary) use these distances as new base lines for other observations, and so repeat the operations until he has completed his survey. In the measurement of his angles he should, if possible, manage that they fall between 60° and 120°, because greater precision will be thereby attained. Proof lines may be calculated from two sets of observations, and, if they agree, the work so far may be deemed correct; but if not, the mistake must be sought for and found. After all have been calculated the positions may be laid down on the chart, and this of itself, being done to scale, will be a proof of the accuracy of the work. The vertical contour is found precisely in the same way, and to those versed in trigonometrical solutions will present little or no difficulty.

(3) Tidal Observations.—(*a*) *The rise and fall of the tide* should be carefully observed. This may be done afloat in less than seven fathoms of water by ballasting a pole about fifty feet long at its thick end, and graduating it if necessary to inches. This apparatus may be kept vertical near the gangway of the surveying vessel, and the depths of the water read off at stated intervals of time. If in more than seven fathoms of water, the hand lead must be kept constantly going. But where a graduated pole can be set up vertically in a sheltered position near rocks, it will be found the better plan; but the zero division of the pole must be below the lowest low-water springs, and the observations ought to be registered every half hour for a lunar month, and when near high or low water at intervals not exceeding ten minutes. Dr. Whewell says: 'In general the waves will make it difficult to observe the moment of the highest (and lowest) open water with much accuracy. The

following methods may be used to make the observations more accurate:—An upright tube, open below and above, may be placed in the water, reaching above the high water and below the low water. In this tube must be a float (a hollow box or a ball, for example), which must carry an upright rod, or else must have attached to it a string which passes over a pulley and is stretched by a weight, and the part of the rod or of the string which is outside the tube must carry an index, which shall mark on a vertical fixed scale the rise and fall of the float. By making the tube close below, except one or more small openings, the motion of the waves will very little affect the float, and the true rise and fall of the surface may be observed with much accuracy.'

'It may happen that the moment of the highest or lowest water is difficult to determine, either with or without the tube, on account of the water, while near the highest or lowest, stopping or hanging still, without either rising or falling, or else rising and falling irregularly. If there is a considerable time during which the water neither rises nor falls decidedly, note the moment when it ceases to rise and the moment when it begins to fall, and take the time half way between these for the time of high water. Another method is the following:—At certain intervals of time near the time of high water—for example, every ten minutes or every five minutes—let the height of high water be observed, say for half an hour or an hour, and from the height so observed pick out the highest for the high water, and note the height and the time; and in like manner for low water. But the following is a better mode of dealing with observations thus made every five or ten minutes:— Let a number of parallel lines (*ordinates*) be drawn at intervals corresponding to the intervals of observations, and bounded by a line perpendicular to them on one side (the *abscissa*); and on these lines, the ordinates, let the observed heights of the surface be set off from the abscissa, and let a line be drawn through the extremities of the ordinates. This line, if it be tolerably regular, will give the time of high water; and if it be somewhat irregular, it can be smoothed into a curve, and then the time and height of high water read off; and in like manner for low water. It is easy to draw such curves, if we have, ready prepared, *paper ruled* into small squares, the divisions in the horizontal line representing hours and minutes, and the divisions in the vertical line representing feet and inches.' By these means the rise and fall may be very correctly ascertained both at spring and neap tides; and the mean half tide mark (it being invariable under normal circumstances) should be cut in the rock for future guidance.

Tide Gauge.—In many ports where there is a resident observer 'self-registering tide gauges' are erected. In principle they are constructed with the tube and float just explained; but the float carries a pencil, which makes a mark on paper stretched on a revolving drum similar to a self-recording barometer. The drum is driven by clockwork, and is made to turn round once in a day, or longer. If the drum did not revolve, the float moving up and down would simply make a vertical line whose extremities would mark the heights of high and low water: and if the drum revolved but the pencil remained stationary, we should find a horizontal straight line described on the paper when removed from the drum; but if both motions take place simultaneously, we shall find a curve described on the paper, the vertical height showing the rise and fall of the tide, and the horizontal length the 24 hours of the day. The length of paper is therefore divided into 24 equal vertical parts, each representing an hour: and the vertical part into equal horizontal divisions representing feet; and these can be again subdivided into halves and quarters, &c., or into minutes and inches. As it would be inconvenient to have paper representing the actual rise and fall, suitable gear is employed to reduce the height to manageable limits. At a stated period every day the paper on the drum must be changed, unless the gauge be constructed for the drum to revolve in a longer period, as a week, a lunar month, &c. It is evident that the rise and fall of the tide, the times of high and low water, the effects of the wind and atmospheric pressure, &c., will all be recorded by the gauge; and lengthened observations at different places have enabled scientific men to eliminate the effects of the wind, of atmospheric pressure, of freshets in tidal rivers, &c., and thus to obtain the establishment of the port, the rise and fall at springs and neaps, effects of priming and lagging of the tides, and much other valuable information.

Soundings can be ascertained with tolerable accuracy by the hand lead; and much nearer by Sir W. Thomson's sounding apparatus, as differences in temperature are not so likely to be found in harbours to affect the results. If not too deep, depths may be very correctly found by a ballasted graduated pole used by a man in a boat. The series of soundings should all proceed in straight lines at equal distances perpendicular to the coast if straight, or radiating from the ship in harbour, or from a shoal, if found; this may be effected by keeping any two objects on shore in the same line of sight. Shoals may be discovered by ballasting a long pole so as to make it swim vertically to the depth required, and towing this with a boat in every possible direction about the harbour. Or again, by attaching ballast to the required depth to cork, so that the latter may swim just on the surface. If a shoal be met with, the pole will be tilted over, or the cork will be dragged down; then the least depth should be marked by a buoy, and exact soundings and observations for position and extent of the shoal should be made. In all operations for sounding the time of tide must be particularly noted, the height of the barometer should be recorded, as well as the direction and strength of the wind registered, as all these will materially affect the soundings at any particular place. Charts of parts of the world which have been surveyed have laid down on them—

(*a*) The soundings in fathoms at low water ordinary spring tides.

(*b*) The establishment of the port in Roman numerals.

(*c*) The set of currents expressed by an arrow; and

(*d*) The rate of currents distinctly marked.

Tidal charts have also the co-tidal lines drawn.

The set and rate of currents, and whether tidal or not, should be recorded. For this purpose Sir Edward Belcher recommends the following plan:—'Sling two barecas (breakers) and connect them by a line of five fathoms; fill one with water, and add about ten pounds of sand or a hand lead; attach the deep sea line to the centre of the span. It is *convenient* to keep them constantly in the water veered to twenty fathoms, as they then bear their proper strain, and being clear of the ship are ready for immediate use. To try the tide or current, ascertain the interval it takes to run out a hundred fathoms (i.e. one hundred and twenty off the reel, allowing twenty for stray line). This will give a rate free from the influences of *wind* and *surface* current.' Rear-Admiral Beechy, in the 'Admiralty Manual of Scientific Inquiry,' recommends: 'To drop a heavy lead from the quarter, and after it has reached the bottom run out a small quantity of stray line, and then make fast the *nipper* or billet of wood to the line; and at the same time to fasten the end of the log line to it, and veer away both together. Then mark by a watch the time *each knot* is in running out, buoying up the line by a chip of wood; when all the line has run out, take the bearing of the nipper by a compass and haul all in together.'

Running Survey.—For the observations detailed above a more or less protracted stay is necessary; but sometimes a nautical description of a coast is required, which can be obtained in a short time, then 'the heavy boats may be hoisted out and sent in shore of the ship to run in the coast line and the detail, whilst the ship carries on a triangulation and continuation of bases in the distance, making what may be termed a *running survey.* Whenever this can be done, send the boats to a distance of four, five, or six miles at starting, and let them and the ship anchor if possible, to measure a base by sound, and to get astronomical bearings and angles to *the same points.* Fix the ship's position by repeated observations for the latitude and by chronometer; then weigh and put the patent log over, and steer a steady course along the land (sounding, if the depth of water admit of it, without stopping). One boat now runs along the land from point to point, putting in the coast line and its detail, getting astronomical bearings and angles as she proceeds, especially of all transits of points and headlands, and measuring her distance between them by patent log; and sounding, but without stopping. The other boat attends principally to the soundings, fixing herself, as she requires, by angles and bearings between the points determined by the first boat and the ship. At the end of a few miles' run, or at noon, or when necessary to renew the angles and bearings, a signal is to be shown, and the logs are then to be hauled in and read off, but not reset, fresh angles and bearings to be taken, and a new base commenced, the distance between the ship and boats being again measured by sound. The log is then again put over and the course of the vessel resumed. In this manner the day passes, the bearings and observations all being worked out at the moment, the outline run in, views taken, and every particular mapped and booked at the time so as to leave nothing to memory. At the close of the day's operations, anchor in position, measure a base by sound, and repeat operations as at starting, recall the boats, and in the grey of evening get the ship's position by stars and planets, which may at this time be observed with great accuracy before the horizon becomes too obscure. If the ship can remain at anchor she will observe the set of the stream and the rise and fall of water, however roughly it may be done. As early as possible commit the triangulation to paper, that the ship may start in the morning with some points of land well fixed, so as to enable her to continue her triangulation throughout the day without the aid of the boat, although her cooperation as before should be renewed. If there be no anchorage, the ship will maintain her position during the night under canvas, and in the grey of the morning, picking up the place where she left off on the preceding evening, send the boats away, get altitudes of stars for latitude and longitude, measure a base by sound, get astronomical bearings and angles, &c., and, putting over the patent log, continue along the coast as before. Thus far we have considered the observations as being wholly confined to the vessels, but it will add considerably to the accuracy of the survey if landings be occasionally made, and the stations be critically determined by astronomical observation, i.e. by latitudes and chronometers, and the positions connected with the rest of the work.' [1]

[1] *Admiralty Manual of Scientific Inquiry.*

(5) and (6) Magnetical and meteorological observations, as well as those on the nature of the country, can hardly be said to come under the science of Navigation. In the former the magnetic elements, (*a*) dip or inclination, (*b*) variation or declination, and (*c*) total magnetic intensity, are to be found. The first by the dipping needle, the second by astronomical observations, and the third by the oscillations of a horizontal needle; but this, as well as meteorology, belongs to branches of physical science, and the nature of a country can only be ascertained by those well versed in natural science.

Exercise XXIV.

Ex. 450. In surveying a harbour, I place two boats in convenient positions, from each of which the other boat and a house and mill on shore can be seen. The angles at the first boat, between the second boat and the house and mill, respectively, were observed to be 58° 20′ and 95° 20′. The angles at the second boat, between the first boat and the house and mill, respectively, were 98° 45′ and 53° 30′. What is the distance between the house and mill, that between the two boats being 960 yards? *Honours*, 1871.

Ex. 451. Four stations, viz. the ship, a conspicuous hill, an island, and a cliff were used in the survey of a bay. From the island the true bearing of the ship was S. 48° W., the angle observed between the ship's main truck and the water line being 1° 14′, and the height of the main truck above the water line being 165 feet. At the island the angle between the hill and the cliff was found to be 31°, and that between the hill and the ship to be 67°. At the ship the angle between the island and the hill was found to be 61° 30′, and that between the island and the cliff to be 114°. Calculate the base line, and *project* these positions on a scale of 3 inches to a mile of 2,028 yards, and determine by projection the bearing and distance of the hill from the cliff. *Honours*, 1873.

Ex. 452. Having cast anchor in an unknown bay, describe how you would proceed to make a survey of it. Explain the different methods of determining your base line, and the principles you would adopt in the triangulation; also how you would sound, and how you would reduce your soundings before laying them down on your chart. In sounding, you suddenly come upon a shoal; what steps would you take? *Honours*, 1874.

Ex. 453. In nautical surveying, describe the different methods of obtaining a base line. What precautions would you take in determining a base by means of a masthead angle to ensure the required accuracy in the triangulation? In sounding a bay, a shoal cast is suddenly obtained; describe fully how you would proceed to fix the position and examine the locality. *Honours*. 1875.

Ex. 454. An observer at the masthead of a ship, 130 feet above the level of the sea, finds the angle of depression of two floating objects A and B to be 9° 10′ 30″ and 7° 30′; their bearings from the ship are SSE. and E. by S. respectively; find the bearing and distance of A from B. *Honours*, 1875.

Ex. 455. In nautical surveying how do you obtain and lay down the soundings? Mention all the precautions and corrections necessary. Describe a tide gauge, and state what precautions are necessary in using it. *Honours*, 1876.

Ex. 456. In surveying a bay describe fully the method of taking soundings, with all the precautions necessary to be attended to How are soundings expressed on the chart, and what reduction is necessary before they are inserted? *Honours*, 1877.

Ex. 457. Describe the conduct of a running survey. Illustrate by the case of an island which has to be roughly surveyed, but on which landing is impracticable. *Honours*, 1878.

Ex. 458. In marine surveying, after having determined accurately, by astronomical observations, the latitude and longitude of the first principal station, why is not that of the second principal station determined in the same manner? Describe some methods in use for determining the position of the second principal station. *Honours*, 1879.

Ex. 459. Describe a tide gauge, and explain the method of taking tidal observations by means of a tide-pole. What information is given on charts respecting tides? *Honours*, 1880.

Ex. 460. What are the objects of a marine survey, and of what use are they when obtained? How should a surveyor begin his survey?

Ex. 461. Describe all the methods you know for finding a base line. Why is it so called, and why is such precision necessary in obtaining it?

Ex. 462. Having obtained the length and position of the base line, describe the method of triangulation which follows, and illustrate your answer by an imaginary example. What are proof lines? Show how they are obtained in your illustration?

Ex. 463. In tidal observations what is sought after, and how is each found Describe fully how the graphic method may be used for finding the times of high and low water. Illustrate your answer by an example.

Ex. 464. How are shoals found in an unknown harbour? What is the method of procedure after having discovered one?

Ex 465. How is the set and rate of the current ascertained? What is sought in magnetic observations, and how are they determined?

CHAPTER XII.

Cyclones, what they are—Vortex—Axis or line of progression—Where prevalent—Seasons of cyclones—Extent—Rate of progression—Rule for finding the focus—How a ship must act if caught in one—Indications of approach—Examples and examination.

CYCLONES OR REVOLVING STORMS.

THE study of scientific navigation cannot be considered complete without some knowledge of atmospheric phenomena; but the one which most concerns the sailor is that of storms. Piddington, in his 'Sailors' Storm-book,' thus distinguishes between a Gale and a Hurricane.

'*A Gale* means a storm of wind, the direction of which is tolerably steady for a long time; sometimes not only for days, but for weeks, as the common monsoon gales, and the winter gales of the Atlantic, the Channel, and the Bay of Biscay.'

'*A Hurricane* generally means a turning storm of wind blowing with great violence, and often shifting more or less suddenly, so as to blow half or entirely round the compass in a few hours. The words *storm* and *hurricane* are both used to indicate a turning gale: hurricane when it is of excessive violence.'

In *heavy* storms, within the tropics, it is but seldom the direction of the wind remains constant, but to an observer appears to have a circular motion—that is, the direction is gradually changed from point to point of the horizon; at the same time, the storm itself travels onward in a definite direction, and hence the atmosphere in motion must revolve in a spiral curve; but for all practical purposes we may suppose any part of the curve to be circular, the centre of which is *the focus or* *vortex.* Revolving storms have received different names in various parts of the world: they are called *tornadoes, hurricanes,* and *cyclones* in the Atlantic and Indian Oceans, as well as to the East of Australia, and *typhoons* in the Chinese Seas. They have their origin near the hottest parts of the earth, from 10° to 20° of latitude; their cause has been ascribed to the meeting of the polar and equatorial winds, which rush in to fill the partial vacuum formed by the rarefaction of the air vertically under the sun. This alone could not *produce* the revolution of the disc of air which forms a cyclone; but the initial cause may perhaps be sought for in the condensation of the vapour carried up by the ascending rarefied air, and the consequent liberation of an immense amount of heat. This in its turn produces greater rarefaction at the spot where the condensation occurs, hence a greater ascending power is imparted to the air with an intensifying of results; and thus, as in so many other natural phenomena, we find an intermingling of cause and effect Some idea of the quantity of heat liberated in condensation may be gathered from what Professor Tyndall says in his 'Heat as a Mode of Motion.' He says, 'The latent heat of aqueous vapour, at the temperature of its production in the tropics, is about 1,000° F.; for the latent heat augments as the temperature of evaporation descends. A pound of water, then, vaporised at the equator, has absorbed one thousand times the quantity of heat which would raise a pound of the liquid one degree in temperature. But the quantity of heat which would raise a pound of water one degree would raise a pound of cast-iron 10°; hence, simply to convert a pound of the water of the Equatorial Ocean into vapour would require a quantity of heat sufficient to impart to a pound of cast-iron 10,000° of temperature. But the fusing point of cast-iron is 2,000° F.: therefore, for every pound of vapour produced a quantity of heat has been expended by the sun sufficient to raise five pounds of cast-iron to its melting point,' and in condensation the same amount of heat is set free. We can then form some faint idea of the rarefying effects of the heat liberated during the tropical downpours. Irregularities of temperature in the North Atlantic are also produced by the *Gulf Stream* flowing through the colder oceanic waters; and Maury says, 'The excess of heat daily brought into such a region by the waters of the gulf stream would, if suddenly stricken from them, be sufficient to make the column of superincumbent atmosphere hotter than melted iron. With such an element of atmospherical disturbance in its bosom, we might expect storms of the most violent kind to accompany it in its course. Accordingly, the most terrific that rage in the ocean have been known to spend their fury within or near its borders.'

The regions chiefly visited by hurricanes are :—

(*a*) *The North Atlantic Ocean*, to the east of the West Indian Islands, in about 55° W. longitude.

(*b*) *The Indian Ocean*, to the NE. of Madagascar, in about 105° E. longitude.

(*c*) *The Pacific Ocean.* In the tropical regions near the barrier reefs to the east of Australia.

(*d*) *The Chinese Seas*, between the mainland on the west, and Formosa and the Philippine Islands on the east. Cyclones are not confined to these regions, for greater accuracy in, as well as a more extended and systematic course of observation, has led to the general belief that all heavy gales have their origin in cyclonic and anticyclonic action.

At first a revolving storm travels to the westward, probably from the rotation of the earth on its axis from W. to E.; but meeting other air, carried around by the earth, it is soon deflected from its original path, and travels to where it meets the least resistance, i.e. towards the poles, because the velocity of the earth's rotation is less as higher latitudes are reached. The opposition the storm meets with in its course gradually changes the direction in which it travels; so that in the northern hemisphere it moves first westerly, and gradually changes its course to NW., then northerly, and finally NE.; whilst in the southern hemisphere it moves first westerly and changes gradually to SW., then southerly, and finally SE. Typhoons do not recurve in this manner, but travel along the coast of China until they are spent. The paths of hurricanes in the Atlantic and Indian Oceans approximate in form to a parabola, and have been known to travel over 5,000 miles. The vertex of the curve in the Atlantic is generally near the tropic of Cancer, whilst that in the Indian Ocean is near the tropic of Capricorn, but they have been known to recurve in all latitudes from 15° to 30°. The extent of these storms is at first about 100 miles, but after travelling through the West Indies have been observed from 600 to 1,000 miles, in diameter; and their forward rate averages from 16 to 20 miles per hour. In the Indian Ocean they generally range from 1,000 to 1,500 miles in diameter, with a rate of 4 to 7 miles per hour. Those in the Bay of Bengal have been met with from 70 to 350 miles in diameter, and having a progressive rate of from 3 to 15 miles per hour; whilst typhoons have a diameter of from 60 to 250 miles, and travel onward from 7 to 24 miles per hour. The progressive rate of the storm, as a whole, must not be confounded with the actual rate of the wind; the former refers to the distance travelled per hour by the focus in its path, whilst the latter is the distance passed over per hour by any particle of air around the focus. We have seen the rate of the storm bodily may be only a few miles per hour, but the wind itself has been known to blow 120 miles and frequently from 80 to 100 miles per hour. The breadth of the calm area in the centre varies from 5 to 30 miles; and as the storm travels on it increases in area but decreases in violence, except where local causes may intensify its effects. All other things being equal, its greatest force will always be found in the semicircle of the storm farthest from the equator: which may be accounted for, because there the wind has to meet the rotation of the earth. From the fact that hurricanes originate over the hottest parts of the earth, we see at once that they must occur from July to October in the northern hemisphere, and from January to May in the southern. Typhoons are met with from May to October. Cyclones have been occasionally experienced in pairs, which travel in parallel paths, so that vessels have sometimes steered from the centre of one into the other.

In the northern hemisphere the wind *backs*—that is, changes its direction contrary to the apparent motion of the sun or of the hands of a watch; it will therefore shift from north through west to south, thence east to north again. Whilst in the southern hemisphere the wind *veers*—that is, it changes in the direction of the apparent motion of the sun or of the hands of a watch, and hence shifts from north through east to south, thence through west to north again. In consequence of the rotatory motion in a hurricane, the wind blows in opposite directions on each side of the focus of the storm, and it increases in intensity as the centre is approached, where there is usually a dead calm; so that if a cyclone pass directly over a vessel, a terrific wind is experienced from one point of the compass, then a dead calm intervenes for sometimes more than an hour, and afterwards the wind suddenly blows with equal or increased violence from the opposite quarter. The change in the direction of the wind is gradual on the outer edge of the storm, but is quicker as the centre is approached, unless the ship lies on the axis of the storm, when the wind remains stationary until the calm area is entered. During their courses there is a continual intermixture of the lower and warmer strata of the air with the higher and colder, thus giving rise to extremely heavy rains and electrical discharges. Bearing in mind the directions in which the winds rotate, it is easy to deduce the following rules, which are known as Buys Ballot's laws:—

(1) In the *northern hemisphere*, stand with your back to the wind, the *left hand* will point out the direction of the focus or centre of the storm.

(2) In the *southern hemisphere*, stand with your back to the wind, the *right hand* will point out the direction of the focus: and because the focus is the part of the greatest danger, it is at all times to be avoided.

Then by noticing how the wind shifts, it will be easy, by drawing a few concentric circles, to see in what direction the storm itself is travelling, and hence how to sail to escape the focus. The line through the centre of the storm in the direction in which it is moving is called *the axis*, or *line of progression*; and looking in the direction in which it is travelling, the semicircle on either side of the axis is called respectively the right or left-hand semicircle.

As an example, suppose a ship to be in north latitude, the wind blowing heavily, and it has changed from SSE. to SE. and ESE. : state how a person should act. First by standing with his back to the wind, the left hand points out WSW., i.e. eight points to the right of the direction of the wind as the direction of the focus, and the ship is in the NE. quadrant of the storm. The wind has changed to the left, hence the ship is in the left-hand semicircle, or to the left of the line of progression. To prove this, draw three concentric circles and place dots on the circumference of each, where the wind will be found blowing as in the example, beginning with the outermost circle. Join these dots by a straight line—this will give the direction the storm is travelling, in this case easterly; and hence the ship is in the most dangerous quadrant, and should go off to the northward, i.e. keep the wind on the starboard quarter. But if the wind is so heavy as to make it dangerous to sail, the ship must be hove-to, and this should be done on the port tack, because the vessel is in the left-hand semicircle. At first sight this would appear contrary to the law for avoiding the centre; but it must be borne in mind that when hove-to the ship makes but very little progress; and being hove-to on the port tack she always presents her bows to the sea, a necessary precaution when a heavy sea is running. As a second example, suppose a ship to be in the southern hemisphere, the wind freshening first N., then N. by W. and NNW. By standing with the back to the wind, the right hand points out west, i.e. eight points to the left of the direction of the wind as the direction of the focus, and the ship is on the eastern edge of the storm. Now if the wind continued due north, and increased in violence, it is plain the ship would be on the axis of the storm; but as the wind backs to N. by W., then NNW., the ship is again in the left-hand semicircle. This can be proved by drawing three concentric circles, and placing dots as directed: when these are joined by a line, it will be found the storm is travelling to the SE., hence by lying-to or going off on the port tack the centre will be avoided. The rules for determining the direction of the focus and the position of a vessel in a cyclone, and also how to act when caught in one, are :—

I. *For position of the focus.*—In the northern hemisphere allow eight points to the right of the direction of the wind, and in the southern hemisphere allow eight points to the left of the direction of the wind : this will give the direction of the centre of the cyclone.

II. *For the position of the ship.*—If the wind changes to the right the vessel is in the right-hand semicircle; but if it changes to the left she is in the left-hand semicircle of the storm.

III. *For avoiding the greatest danger.*—In the northern hemisphere keep the wind, when sailing, on the starboard side of the vessel, and in the southern hemisphere keep the wind on the port side. But if obliged to heave-to, do so on the starboard tack when in the right-hand semicircle in both hemispheres, and on the port tack when in the left-hand semicircle.

Indications of approaching storms.—(1) Before a storm the barometer usually stands very high, and oscillates; whilst the air is sultry, oppressively hot, and often transparent. The mercury then begins to fall very rapidly : nearly three inches in a few hours has been observed during the progress of a storm. In the hurricane which devastated Guadaloupe on September 6, 1866, the barometer fell 1·693 inches in an hour. The following approximate table is given by Mr. Piddington in his 'Sailors' Horn Book,' p. 252.

Average fall of barometer per hour in inches.	Distance of a ship from the centre of a storm in miles.
From 0·020 to 0·060	From 250 to 150
From 0·060 to 0·080	From 150 to 100
From 0·080 to 0·120	From 100 to 80
From 0·120 to 0·150	From 80 to 50

In north latitude before the storm recurves the fall will be quickest with northerly winds; but after recurving with winds from the east to south-east. In southern latitudes the fall will be quickest with southerly winds before recurving, and with winds from east to north-east after.

(2) A cyclone is often seen advancing as a dense, thick bank or wall of cloud of a heavy leaden colour. The sky takes a wild, threatening aspect, and scud is driven in all directions. The sky has often a deep red colour, with a brickdust haze in the horizon.

(3) A peculiar moaning sound, as if the wind were rising and falling, is frequently heard, accompanied by a distant roar of the elements.

(4) A heavy swell or storm wave is peculiarly characteristic of these tempests, because the disturbance is propagated faster in the dense water than in air.

EXERCISE XXV.

Ex. 466. How would you distinguish a cyclone from an ordinary gale? By what names are revolving storms known in different parts of the world?

Ex. 467. What do you mean by the focus and by the line of progression or axis of a storm? Which is the most dangerous quarter, and why?

Ex. 468. Give any theory to account for cyclones, and show how the theory is borne out by facts. What are the chief hurricane regions, and their tracks?

Ex. 469. Account for the rotation and recurving of cyclones; and can you give any reason why typhoons do not recurve?

Ex. 470. At what periods of the year are cyclones most frequent? Account for this, and show how the times are modified in different places by the sun's declination.

Ex. 471. In what directions do the winds rotate in the northern and southern hemispheres respectively? Can you account for this difference and illustrate your answer by a diagram? What rules have you for finding out the direction of the centre of the storm?

Ex. 472. What are the chief indications of a coming storm? What extent have they in different regions? What difference is there in the rate of progression of the storm and the rate of the wind?

Ex. 473. Describe the characteristics of the circular storms of the North Atlantic and South Indian Oceans. Give the rule for finding the bearing of the centre of a cyclone. In a cyclone in the South Indian Ocean the wind is SE.; between what limits is the probable bearing of the centre of the storm? *Honours*, 1875.

Ex. 474. Give a full account of the class of storms called cyclones, mentioning their distinguishing characteristics, the parts of the world where they are met with, the seasons of the year when they most frequently occur, &c. On finding himself within the range of a cyclone how should a navigator act? *Honours*, 1876.

Ex. 475. Describe fully the hurricanes of the West Indies. State at what time of the year these storms occur, and account for it. *Honours*, 1877.

Ex. 476. Give fully the practical rules for the management of a ship when caught in a cyclone; and state the reasons on which these rules are based. *Honours*, 1878.

Ex. 477. Give the rule for finding the bearing of the centre of a circular storm. Write a full description of the cyclones of the South Indian Ocean. *Honours*, 1879.

Ex. 478. State how the barometer may assist you in estimating the distance of the vortex of a revolving storm. Give the rule for keeping the ship away from the centre of a cyclone. In what parts of the world and at what times of the year are these storms most prevalent? *Honours*, 1880.

Ex. 479. The direction of the wind in a cyclone being ENE., state the probable bearing of its centre from the ship in the southern hemisphere. *Board of Trade Examination.*

Ex. 480. And suppose that the wind during the passage of the same cyclone were found to change towards the eastward, what would be the ship's position with reference to the line of progression of the centre of the cyclone, and what action would you take? *Board of Trade Examination.*

Ex. 481. Under what conditions would the change in the direction of the wind in the cyclone be the reverse of the above? *Board of Trade Examination.*

Ex. 482. What are the usual indications of a ship being on the line of progression of the centre of a cyclone? *Board of Trade Examination.*

Ex. 483. What are the usual indications that a ship is (*a*) approaching the centre of a cyclone, (*b*) receding from it. *Board of Trade Examination.*

Ex. 484. Describe the track usually taken by cyclones in the Bay of Bengal, and state the seasons of the year in which they most frequently occur in that region. *Board of Trade Examination.*

Miscellaneous Exercises.

We shall append a few questions fully worked out, and then several others to exercise the student's ingenuity.

Ex. 485. If the meridional parts for 62° 18′ be 4813·51, find those for 62° 26′.

It has been shown in the text that the meridional parts for any place in latitude n' is equal to

$$\sec 1' + \sec 2' + \sec 3' + \quad . \quad . \quad . + \sec n'$$

In this example we have given—

$$\sec 1' + \sec 2' + \sec 3' + \quad . \quad . + \sec 62^\circ 18' = 4813{\cdot}51$$

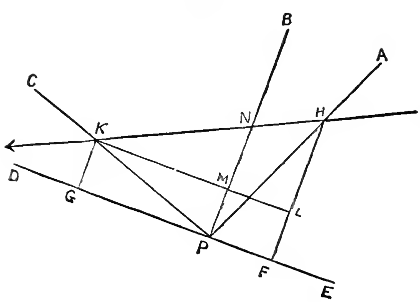
and log sec	62° 19′	=	·332935	∴	sec 62 19	=	2·153
„	62 20	=	·333176	∴	„ 62 20	=	2·154
„	62 21	=	·333417	∴	„ 62 21	=	2·155
„	62 22	=	·333658	∴	„ 62 22	=	2·156
„	62 23	=	333900	∴	„ 62 23	=	2·157
„	62 24	=	·334141	∴	„ 62 24	=	2·158
„	62 25	=	·334383	∴	„ 62 25	=	2·160
„	62 26	=	·334625	∴	„ 62 26	=	2·161

Hence $\sec 1' + \sec 2' + \sec 3' + \quad . \quad . \sec 62^\circ 26' = 4830{\cdot}76$

$\therefore$ Meridional parts for $62^\circ 26' = 4830{\cdot}76$

Ex. 486. Having given three bearings of a vessel steaming in a direct line, with the intervals elapsed between taking the bearings, to find the course of the steamer—

FIG. 43.



Let P be the place of the observer, and the observed bearings be P A, P B, and P C, and the elapsed intervals be m and n minutes respectively.

From point P draw PA, PB, PC on their respective bearings; through P draw D E at right angles to the middle bearing P B; then take on any scale of equal parts—

$$PF : PG :: m : n$$

and draw F H, G K at right angles to D E, cutting P A, P C in H and K respectively.

Then a right line drawn through H K will represent the course of the vessel.

Through K draw K L parallel to D E

Then $m : n :: PF : PG$

$:: ML : MK$

$:: NH : NK$

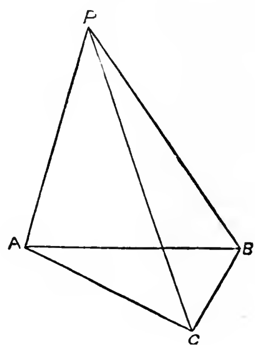
Hence because the points H, N, K are found on the observed bearings, so that the lines N H, N K are in the ratio of the elapsed times, therefore H N K represents the apparent course of the steamer.

The *relative* distances on P A, P B, P C may also be found, as well as the bearing at the least distance from P; but the absolute distance cannot be obtained unless we have further data involving some distance given.

Ex. 487. The lengths of the lines which join three points A, B, C are known; at any point P in the same plane as A, B, C, the angles A P C and B P C are observed; it is required to find the distance of P from each of the points A, B, C. *Todhunter's Plane Trigonometry.*

Let the angle A P C be denoted by α, the angle B P C by β: the angle P A C by x, and the angle P B C by y; then α and β are known, and when x and y are found the required distances P A, P B, P C can be found; for in each of the triangles P A C and P B C two angles and a side will then be known. We will show how x and y may be found.

FIG. 44.



Since the four angles of the quadrilateral P A C B are together equal to four right angles, we have—

$$x + y = 2\pi - \alpha - \beta - C;$$

and the value of C can be deduced from the lengths of the three sides of the triangle A B C: and thus the *sum* of x and y is known.

From the triangle A C P we have—

$$PC = \frac{AC \cdot \sin PAC}{\sin APC} = \frac{b \cdot \sin x}{\sin \alpha}$$

From the triangle B C P we have—

$$PC = \frac{BC \cdot \sin PBC}{\sin BPC} = \frac{a \cdot \sin y}{\sin \beta};$$

therefore
$$\frac{b \cdot \sin x}{\sin \alpha} = \frac{a \cdot \sin y}{\sin \beta}$$

and
$$\frac{\sin x}{\sin y} = \frac{a \cdot \sin \alpha}{b \cdot \sin \beta}$$

Now assume $\tan \phi = \frac{a \cdot \sin \alpha}{b \cdot \sin \beta}$, then the value of ϕ can be found from the trigonometrical tables: thus—

$$\frac{\sin x}{\sin y} = \tan \phi;$$

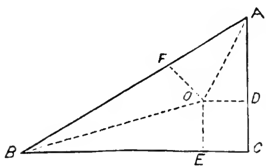
therefore
$$\frac{\sin x - \sin y}{\sin x + \sin y} = \frac{\tan \phi - 1}{\tan \phi + 1} = \tan\left(\phi - \frac{\pi}{4}\right)$$

Hence $$\frac{\tan \frac{1}{2}(x - y)}{\tan \frac{1}{2}(x + y)} = \tan\left(\phi - \frac{\pi}{4}\right)$$

From the last equation we can determine $x - y$, since $x + y$ is known; thus x and y can be found and the problem completed.

Ex. 488. Two ships sail from different ports at the same instant until they meet, on courses bisecting the acute angles of a right angled triangle, showing their position. One sails between the E. and N. a miles, the other between the S. and W. b miles. Required the bearing of one port from the other.

FIG. 45.



Let A B be the points representing the two ports, and O the point where the ships meet. From O draw O D, O E, and O F perpendiculars to the three sides respectively.

Then $$\text{OD} = \text{OA} \,.\, \sin \text{OAD} = b \sin \frac{\text{A}}{2}$$

$$\text{OE} = \text{OB} \,.\, \sin \text{OBE} = a \sin \frac{\text{B}}{2}$$

but Euclid IV. 4 . . . . O D = O E

$$\therefore b . \sin \frac{\text{A}}{2} = a \sin \frac{\text{B}}{2}$$

$$= a \sin\left(45° - \frac{\text{A}}{2}\right)$$

$$= a\left(\sin 45° . \cos \frac{\text{A}}{2} - \cos 45° . \sin \frac{\text{A}}{2}\right)$$

$$= \frac{a}{\sqrt{2}}\left(\cos \frac{\text{A}}{2} - \sin \frac{\text{A}}{2}\right)$$

$$\therefore \frac{a}{\sqrt{2}} . \cos \frac{\text{A}}{2} = \sin \frac{\text{A}}{2}\left(b + \frac{a}{\sqrt{2}}\right)$$

$$\text{and } \cot \frac{\text{A}}{2} = \frac{b}{a}\sqrt{2} + 1$$

Hence $$\text{A} = 2 \cot^{-1}\left(1 + \frac{b}{a}\sqrt{2}\right) \quad \text{Answer.}$$

If $a = 50\sqrt{10}$ and $b = 50\sqrt{5}$ it will be found the bearing of A from B is N. 53° 7′ 48″ E., and distance A B = 250.

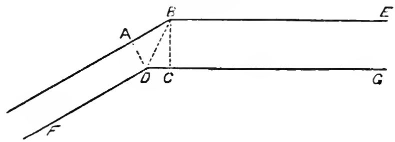
Ex. 489. Two walls have the same height 15 feet, they are inclined one another at an angle of 150°, the breadth of their shadows are 10 and 8 feet respectively. Find the altitude of the sun.

Let A B, B E be a horizontal section of the two walls; F D, D G the extremities of their shadows; B D the shadow of the angle formed by the intersection of the two walls. Let ∠ A B D = ϕ and ∠ C B D = θ

Then $\angle ABE = 150°$ and $\angle EBC = 90°$; hence $\angle ABC = 60°$

$$\left.\begin{array}{l}(1)\ AD = BD \,.\, \sin\phi \\ (2)\ BC = BD \,.\, \cos\theta\end{array}\right\} \text{Dividing (1) by (2) } \frac{AD}{BC} = \frac{\sin\phi}{\cos\theta}$$

FIG. 46.



Now $\frac{AD}{BC} = \frac{8}{10} = \frac{4}{5}$ and $\phi = (60° - \theta)$

$$\therefore \frac{4}{5}\cos\theta = \sin(60° - \theta)$$
$$= \sin 60°.\cos\theta - \cos 60°.\sin\theta$$
$$= \frac{\sqrt{3}}{2}.\cos\theta - \frac{1}{2}\sin\theta$$

and $\sin\theta = \cos\theta\left(\sqrt{3} - \frac{8}{5}\right)$

or $\tan\theta = \cdot 132 \ldots\ldots +$

$BD = BC \,.\, \sec\theta = BC\sqrt{1 + \tan^2\theta}$

$= 10\sqrt{1 + (\cdot 132 \ldots\ldots +)^2}$

Now tan alt. sun $= \frac{\text{height of wall}}{BD} = \frac{15}{10\sqrt{1 + (\cdot 132 \ldots +)}}$

$= \tan 56° 5'$ very nearly.

Hence alt. of sun $= 56° 5'$ very nearly.

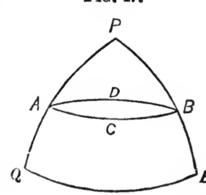
Ex. 490. Two ports are in the same parallel of latitude, their common latitude being l and their difference of longitude $2L$. Show that the saving of distance in sailing from one to the other on the great circle instead of due E. or W. is—

$$2r\{L \,.\, \cos \,.\, l - \sin^{-1}(\sin L \,.\, \cos l)\}$$

L being expressed in circular measure, and r being the radius of the earth.

Todhunter's Spherical Trigonometry.

FIG. 47.



Let P be the po e; EQ part of the equator; ACB the parallel of latitude; ADB the arc of a great circle through the points A and B.

By parallel sailing

Departure $ACB = 2L \,.\, \cos l$ . . . . I.

In the spherical triangle P A D B

$$\cos AB = \cos PA . \cos PB . + \sin PA . \sin PB . \cos P$$
$$= \sin^2 l + \cos^2 l . \cos 2L$$
$$= \sin^2 l + \cos^2 l (1 - 2\sin^2 L)$$
$$= 1 - 2\cos^2 l . \sin^2 L$$
$$\therefore 2\sin^2 L . \cos^2 l = 1 - \cos AB$$
$$= 2\sin^2 \frac{AB}{2}$$

Dividing by 2 and taking the square root of each side—

$$\sin \frac{AB}{2} = \sin L . \cos l$$
$$\therefore AB = 2\sin^{-1}(\sin L . \cos l) \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II.}$$

From I. and II.

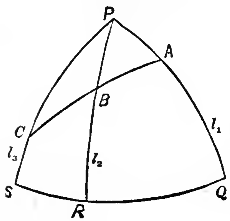
$$ACB - ADB = 2L . \cos l - 2\sin^{-1}(\sin L . \cos l)$$
$$= 2\{L . \cos l - \sin^{-1}(\sin L . \cos l)\}$$

This gives the answer to radius unity, and to adapt it to the sphere with radius r we must multiply the result by r—

$\therefore$ Saving of distance $= 2r\{L . \cos l - \sin^{-1}(\sin L . \cos l)\}$.

Ex. 491. If a ship be proceeding uniformly along a great circle and the observed latitudes be l_1, l_2, l_3 at equal intervals of time, in each of which the distance traversed is S, show that

FIG. 48.



$$S = r . \cos^{-1} \frac{\sin \frac{1}{2}(l_1 + l_3) . \cos \frac{1}{2}(l_1 - l_3)}{\sin l_2}$$

r denoting the earth's radius; and show that the change of longitude may also be found in terms of the three latitudes.

Let the positions be as in the figure Q A $= l_1$, R B $= l_2$, S C $= l_3$, and C B = B A = S

Then $\cos PBC = -\cos PBA$

and
$$\frac{\cos PC - \cos PB . \cos CB}{\sin PB . \sin CB} = \frac{\cos PB . \cos AB - \cos PA}{\sin PB . \sin AB.}$$

or
$$\frac{\sin l_3 - \sin l_2 . \cos S}{\cos l_2 . \sin S} = \frac{\sin l_2 . \cos S - \sin l_1}{\cos l_2 . \sin S}$$

$$\therefore \sin l_3 - \sin l_2 . \cos S = \sin l_2 . \cos S - \sin l_1$$
$$2\sin l_2 . \cos S = \sin l_1 + \sin l_3$$
$$\therefore \cos S = \frac{2\sin \frac{1}{2}(l_1 + l_3) . \cos \frac{1}{2}(l_1 - l_3)}{2\sin l_2}$$

Adapting this to radius r we get

$$S = r\cos^{-1} \frac{\sin \frac{1}{2}(l_1 - l_3) . \cos \frac{1}{2}(l_1 - l_3)}{\sin l_2}$$

In the spherical triangle A P C we have now the three sides given in terms of l_1, l_2, l_3: hence the ∠ A P C which represents the difference of longitude may be found in terms of the three latitudes.

Miscellaneous Questions.

Ex. 492. A ship wishing to sail E. by N. (true), what should be her compass course when the variation is 24° W., deviation 6° W., and leeway $\frac{3}{4}$ point, the wind being NE. by N.? *Britannia*, 1875.

Ex. 493. Explain the theory of the log-line. If with a glass of 25 seconds the line were divided into knots of 50 feet, what would be the error in running 120 miles true? *For Lieutenant*, 1876.

Ex. 494. Using a 31-second glass and a log-line measuring 47 feet, what distance will be shown by it in a true run of 150 miles (6120 feet = 1 knot)? *Royal Naval College*, 1872.

Ex. 495. A ship sails 150 miles (true) with a 29-second glass; her measured distance was 161 miles; required the error in the length of the log-line. *Royal Naval College*, 1872.

Ex. 496. Using a 28-second glass, I measured 128 miles in a true distance of 140 miles. What was the error of my log-line, and what should have been its length to give a correct result? *Royal Naval College*, 1869.

Ex. 497. A supposed half-minute glass ran out in 28 seconds; what will be the real distance run in 100 miles measured by it, if the log-line (measured for a half-minute glass) was afterwards found to be $\frac{1}{51}$ part too long (6120 feet = 1 knot)? *Royal Naval College*, 1868.

Ex. 498. If a vessel sail from the Faroe Isles, latitude 62° 6′ N., longitude 7° 37′ W., 250 miles due N., 250 miles due W., 250 miles due S., and 250 miles due E., how far and in what direction was she from her point of departure?

Ex. 499. If a vessel sail 200 miles due E., then 180 miles due S., and then 290 miles due W., and reach the same meridian, required the latitude arrived at.

Ex. 500. On what course must a ship sail so that

$$\text{difference of latitude} : \text{distance} = \sqrt{3} : 2?$$

Ex. 501. A lighthouse was observed from a ship to bear SW., and after sailing due N. 9 miles it bore S. 30° W. Find the distance from the ship at each observation.

Ex. 502. Explain the construction of the 'traverse table,' and apply it to the solution of the following question:—

Two ships A and B are chasing to windward, the wind being SW. At the end of the given time B is found to bear from A S. by W$\frac{1}{2}$W. distant 2,000 yards. How much has B weathered on A? *For Lieutenant*, 1874.

Ex. 503. Two objects A and B were observed from a ship to be at the same instant in a line bearing NE. After sailing N. 15° W. 7 miles, A bore due E., and B NE. by E. Find the distance between A and B.

Ex. 504. From a ship in latitude 40° N., and longitude 50° E., I sailed between N. and W. 600 miles, having made 418 miles of departure. What was my course and latitude and longitude reached?

Royal Naval College, 1872.

Ex. 505. A ship left a harbour in latitude 50° N., and longitude 20° E.; she sailed W. 600 miles, then N., then E. 400 miles, and then found she was in longitude 20° E. What was her latitude in, and how far N. did she sail? *Royal Naval College*, 1872.

Ex. 506. What is the length of a shroud to reach from the top of a mast 58 feet above the deck to a dead eye placed 8 feet abaft the mast and 7 feet from the deck, the width of the vessel being 27 feet?

Ex. 507. A lighthouse is observed making an angle $x°$ with the direction of the ship's head; after running 14 miles further the angle is doubled: what is now the distance? Give the proof.

Royal Naval College, 1872.

Ex. 508. If a vessel sail between the S. and W. from the Lizard, latitude 49° 57′ 42″ N., longitude 5° 12′ W., until the difference of latitude, departure, and distance are together equal to 144 miles, and the sum of their squares 7.200 miles; find the latitude and longitude of the vessel and the true course she has made if the difference of latitude be greater than the departure.

Ex. 509. Between the flash and the report of a gun fired from a vessel A, bearing N. 29° 50′ E., there elapsed 7 seconds, directly after a gun was fired from another vessel B, bearing N. 7° 25′ W.: the interval between the flash and report of the second was $5\frac{1}{2}$ seconds. Required the bearing and distance of one vessel from the other if sound travels 1,140 feet per second.

Ex. 510. I wished to reach a position fixed by cross bearings S. 35° E. and S. 45° W. of two points lying east and west 5 miles apart. By mistake, the bearings were taken S. 35° E. and S. 35° W.; what was my error in distance? *Royal Naval College*, 1872.

Ex. 511. A ship left a place in latitude 30° S. and longitude 50 E. for a port 300 miles west of her. At the end of the run it was found that the variation 31° E. and the deviation 14° W. had been applied incorrectly; how far was she from the required spot?

Royal Naval College, 1872.

Ex. 512. A ship sailed from a place in latitude 20° N. and longitude 30° W. to reach a port 150 miles NW. of her. After steaming for 15 hours at 10 knots, it was found that the variation (2 points W.) and deviation ($1\frac{1}{2}$ points E.) had been incorrectly applied. What was the error in latitude and longitude? *Royal Naval College*, 1869.

Ex. 513. A ship wishing to regain an anchorage, marked by cross bearings S. 2 points E. and S. 3 points W. from two objects lying W. and E. 2 miles apart, by mistake anchors when they bear N. 3 points W. and N. 2 points E. from her; what are her bearings and distance from her required position? *Royal Naval College*, 1868.

Ex. 514. A ship whose length was 385 feet was sailing ESE. Her bow bore from me NE., and the angle subtended by her head and stern was 1° 15′. Required her distance.

Ex. 515. The steering compasses differing by 5° E. and 6° W. from the standard compass, what will be the *compass* courses by each of them to steer a true S. course? Variation 2 points W., deviation 1 point E., wind SE. by E., leeway $1\frac{1}{2}$ points. *Royal Naval College*, 1868.

Ex. 516. A ship is 245 feet long and her head is SW. by S. The bearing of an object ashore taken from the two ends of the vessel is WNW. and NW$\frac{1}{2}$W. Required the distance of the object from each end of the vessel.

Ex. 517. The *distance, true difference of latitude, departure*, and *course* may be correctly represented by the three sides and one of the angles of a right-angled triangle; also the *meridional difference of latitude* and *difference of longitude* may be represented by two sides of a triangle which is similar to the same right-angled triangle; required the proof. *For Beaufort Testimonial*, 1864.

Ex. 518. The summit of a hill A bore due east of a spectator at B, and ENE. of a spectator at a port C due south of B; the elevation of the point A at B was 20°. Required its elevation at C.

Royal Naval College, 1866.

Ex. 519. A lighthouse bore S$\frac{3}{4}$W. by compass; the vessel stood away ESE. 19 miles, and the light then bore W$\frac{1}{2}$S. Required the distance of the lighthouse at each observation.

Ex. 520. A tower 48 feet high stands on the summit of a slope whose inclination to the horizon is 67° 15′; find the length of a ladder that would reach from its summit to a point on the ground 20 feet from the foot of the tower. *Royal Naval College*, 1868.

Ex. 521. Distinguish between the constant, semicircular, and quadrantal deviations of the compass. How does hard iron differ from soft iron in its effects on the compass needle?

For Beaufort Testimonial, 1874.

Ex. 522. A point of land bore NE. and another NW. by N.: standing away W. by N$\frac{1}{2}$N. 7 miles, we found the first point bore ENE. and the second N$\frac{3}{4}$W. Required the bearing and distance of the second point from the first.

Ex. 523. In swinging ship for deviation of compass the bearings of the shore compass are SW$\frac{1}{4}$W., W$\frac{3}{4}$S., S. by E.; the respective deviations were $\frac{3}{4}$ point E., $1\frac{1}{4}$ points W., $\frac{1}{2}$ point E. Draw figures, and find the bearings of the ship's compass in each case. *For Lieutenant*, 1873.

Ex. 524. Three points, A, B, and C, are on the same parallel of latitude; the distance from A to B is a miles, from B to C is b miles. How far due north from A will the distances subtend equal angles? Which must be the greater, a or b, that the proposition may be possible?

Ex. 525. The bearing of the station B from A is SE. by E. by compass, and that of A from B is by compass NNW$\frac{1}{2}$W. If the deviation of the compass at A be one point E., find the deviation of the compass at B. *Royal Naval College*, 1866.

Ex. 526. After swinging ship, if the deviation be registered as 1 point for ship's head east, is it for a true or compass bearing of the ship's head, and why? *Royal Naval College*, 1869.

Ex. 527. Explain the nature of 'dip' in the compass. Show how the rolling motion of an iron ship affects the compass.

Royal Naval College, 1867.

Ex. 528. Four lights, A, B, C, D, are in a straight line. Their distances apart are AB = 4, BC = 2, and CD = 6 miles respectively. Find a point P out of the line at which they will appear equidistant.

Ex. 529. A ship built on the Clyde leaves a place in latitude 30° N., longitude 10° W., and sails on the starboard tack on the true course SW. by W. for 4 days at the rate of 5·5 knots, during all the time heeling over 5°, each degree of heel introducing 1° of error in the deviation of the compass. Find the latitude and longitude in, and the position which the ship would have had if the heeling error had been disregarded. Give also the bearing and distance of the two positions from one another. *For Beaufort Testimonial*, 1874.

Ex. 530. Two ships, A and B, leave a port at the same time, A steering N. by E. and B steering ESE. After 3 hours A bears NW. from B distant 12 miles; find their rates of sailing. *For Lieutenant*, 1874.

Ex. 531. A ship takes up a position marked by cross bearings N. 45° W., S. 40° W., of two objects lying N. and S. 7 miles apart. How far is she from another ship, from which the objects respectively bear N. 30° W. and S. 60° W.? *For Lieutenant*, 1874.

Ex. 532. A rock was seen from a ship in a current at 8h. 25m. A.M., bearing NE. by E$\frac{1}{2}$E. distant 24 miles. The ship steered NE$\frac{3}{4}$N. 6$\frac{1}{2}$ knots per hour, and was lost on the rock at 2h. 45m. P.M. Required the set and drift of the current.

Ex. 533. By dead reckoning a ship has sailed SW. by W. 27 miles in 5 hours; by cross bearings she is found to have gone W$\frac{3}{4}$N. 29 miles. Find the direction and rate of the current by which she was set.

For Lieutenant, 1873.

Ex. 534. A ship is steered NNW., and steamed at the rate of 9·5 knots an hour; at the end of 2 hours she found that she had made 15 knots NW$\frac{1}{2}$W. What was the force and direction of the current she passed through? *For Lieutenant*, 1873.

Ex. 535. Standing on the paddle-box of a steamer, I observe the direction of a light to be across the *bows* of the steamer, and half an hour afterwards the funnel appears to pass over the light; the vessel is steaming on a straight course at 12 knots per hour; my position on the paddle-box is 50 feet from the bows and 20 feet from the funnel, and the funnel and bows are 45 feet from one another. How far off is the light at the time of each observation?

Ex. 536. To reach a port, a ship's compass course must be NE. by N. when sailing 10 knots; at the end of an hour she has made good 9·16 knots E. (true). What was the force and direction of the current through which she was passing (variation 2 points E.)?

Royal Naval College, 1868.

Ex. 537. An admiral has a look-out frigate 3 points on his port bow, distant 5 miles; the admiral makes a signal to her to look out on the starboard bow, same bearing and distance. Required the frigate's change of course to get into her new station—admiral steaming 8 knots, frigate, to get to her new station, steaming at 11 knots.

Royal Naval College, 1865.

Ex. 538. An admiral has a frigate on detached service, stationed NE. by N. from him distant 71 miles. The admiral has ordered the frigate to meet him on a certain day at 3h. A.M., on a certain bearing from the admiral, namely, ENE. distant 120 miles. How must the frigate steer to meet the admiral, and at what time must she leave her detached station (rate of steaming 10 knots)? *Royal Naval College*, 1865.

Ex. 539. The shadow of two walls, which are at right angles to each other and are 15 and 12 feet high respectively, are observed when the sun is on the meridian to be 17 and 16·5 feet broad. Required the altitude of the sun and the inclination of the first wall to the meridian.

Ex. 540. Two points 3 miles apart have a rock to seaward of them 4 miles from each. What will be the angle subtended by the points from a ship whose distance from each is the same, and which lies one mile to seaward of the rock? *Royal Naval College*, 1872.

Ex. 541. A mill A bears from a lighthouse B NE. by N. 1,500 yards; how will it bear from a rock R 2,500 yards S. by E. from B?

Royal Naval College, 1872.

Ex. 542. An observer a short distance inland notes the time when the funnel of a steamer is just passing a column erected on a beach, and m minutes afterwards the angle subtended by the funnel and pillar is $A°$, and again in n minutes it is $A° + B°$. The bearing of the pillar being known, required the course of the steamer. *For Beaufort Testimonial*, 1864.

Ex. 543. Two headlands A and B and a large tree C form a triangle, whose sides are $AC = 2\frac{1}{2}$, $CB = 3\frac{1}{4}$, and $AB = 4\frac{1}{8}$ miles respectively, the tree appearing between and is further away than the headlands. The angle at the eye on board a vessel subtended by AC observed with a sextant) was 14° 50′, the angle subtended by B C was 12° 40′. Required the distance of the objects from the vessel.

Ex. 544. The altitude of a cloud was observed to be $\alpha°$, at the same time the sun's altitude was $\beta°$, the sun and cloud being in the same plane with the observer, and his distance from the shadow a yards; find the height of the cloud. *For Beaufort Testimonial*, 1864.

Ex. 545. Enumerate the different methods of finding your distance from an object at sea; and show, with figures, the angles taken in each case. *Royal Naval College*, 1872.

Ex. 546. What is meant by the term 'middle latitude'? Is your definition theoretically correct, and in what way is its accuracy dependent on the latitude? *Royal Naval College*, 1872.

Ex. 547. Prove the following rule for finding the distance of the horizon at sea is nearly correct. 'Take the number of feet in the height of the eye above the sea, and increase it by half that number: the square root of this quantity will give the distance of the horizon in miles' (statute). *For Beaufort Testimonial*, 1864.

Ex. 548. A person standing on the seashore can just see the top of a mountain whose height is known to be b miles; after ascending vertically to the height of a miles, he observes the angle of depression of the mountain's summit to be $\alpha°$. Find the earth's radius, and the distance from the first place of observation. *For Beaufort Testimonial*, 1864.

Ex. 549. (*a*) Construct a chart, and lay down the cross bearings and courses; find the latitude and longitude in, from the following:—

Two islands A (latitude 48° 35′ N., longitude 111° 36′ W.) and B latitude 48° 17′ N., longitude 111° 22′ W.) bore respectively NW. by W. and SSW. Afterwards sailed as under:—

True courses.	Distances.
NNW.	62′
ENE.	74
NW. by N.	104
E.	86
SW. by S.	135
ESE.	70

To be drawn on a scale of 1·3 inch to a degree of longitude, and the chart to extend from 48° to 52° N., and from 109° to 113° W.

(*b*) Find the distance and compass bearing of the ship in her last position from a port in latitude 50° 29′ N., and longitude 112° 42′ W.; the variation being 18° W., and deviation 3° E.

(*c*) What is the true bearing of a rock which bears by compass from the ship in her last position NE½N., variation 18° W., deviation 8° E.?

Place the rock on your chart, supposing its distance 15 miles.

Final Examination, Britannia, 1875.

Ex. 550. (*a*) Construct a chart, and, laying down the bearings and courses, find the latitude and longitude in, from the following :—

The latitude of a ship, determined by observation, was known to be 52° 30′ S. ; at the same time the true bearing of a headland, in latitude 52° 20′ S, longitude 73° 10′ E., was N. by W. Afterwards sailed as under :—

True courses.	Distances.
NE.	85′
SSW.	120
ESE.	55
SW. by W.	98
E.	50
N. by E.	105

The chart to be drawn on a scale of 1·25 inches to a degree of longitude, and to extend from latitude 51° S. to latitude 55° S., and from longitude 72° E. to longitude 76° E.

(*b*) Find the distance and compass bearing of the ship in her last position from a port in latitude 53° 27′ S., longitude 75° 48′ E.; the variation being 30° E. and deviation 7° W

(*c*) What is the true bearing of a rock which bears by compass from the ship in her last position WNW., variation 30° E., deviation 12° W. ?

Place the rock on your chart, supposing its distance to be 16 miles.

Final Examination, Britannia, 1875.

Ex. 551. Explain how a table of meridional parts is constructed. Given the meridional parts for 50° 10′ = 3490·06, calculate the meridional parts for 50° 20′. *Honours*, 1882.

Ex. 552. What is meant by great-circle sailing ? and show in what cases it could not be used with advantage. *Royal Naval College*, 1865.

Ex. 553 Two ships A and B sail from one port, their courses making an angle of 50°. A sails between the south and west 120 miles ; B sails between the south and east 150 miles, and is then 45 miles to the southward of A. Required the course each ship has made, and the bearing and distance of one ship from the other.

Ex. 554. Give definitions of the latitude on the sphere and on the spheroid. If δ = difference of latitude on sphere and spheroid, c = compression, and a = semi-major axis, then for any place in latitude l prove that $\delta = \frac{c}{a} \sin 2l$.

Find also in what latitude δ is a maximum. *Royal Naval College*, 1865.

Ex. 555. If a ship sail on a great circle with initial course of W. by N½N. till her difference of longitude is 500 miles, required her latitude and longitude in, if she leaves 47° 20′ N. and 15° 30′ W ?

Ex. 556. Two ships leave a port A, latitude 45° N., longitude 90° E., for a port B, latitude 45° N., longitude 5 ° W. The first follow the parallel of latitude steaming 5·2 knots, the second keeps to the arc of a great circle joining A and B steaming 5 knots. In what time will they respectively accomplish the voyage? *For Lieutenant*, 1873.

Ex. 557. A ship sails to the northwards on a great circle from Bermuda, in latitude 32° 26′ N., longitude 64° 37′ W., for a distance of 895 miles, and her longitude is then 48° 20′ W. Required her first course and latitude in.

Ex. 558. Two ports are on the same parallel of latitude, their common latitude being l. Show that the saving of distance in sailing from one to the other on the great circle instead of sailing E. or W is—

$$2r(\lambda . \cos l - \gamma)$$

when 2λ is the circular measure of the longitude, γ the circular measure of the angle $\sin^{-1}(\sin\lambda . \cot l)$, and r the radius of the earth.

Ex. 559. Explain the principles of triangulation and sounding in the survey of a harbour, and illustrate your description by simple diagrams of any harbour with which you are acquainted. *Honours*, 1882.

Ex. 560. A ship observes two islands, one due N., the other due E.; she sails ENE. 8 miles, and then finds she is equidistant from the two islands; after sailing 3 miles further in the same direction she finds she is in the straight line joining the two islands. Required the bearing and distance of one island from the other.

The following question was given at the Royal Naval College for the rank of Lieutenant, R.N., September, 1874.

Ex. 561. NAUTICAL SURVEYING.

(*a*) Construct a Mercator's chart on a scale of 1·15 inches to a degree of longitude, extending from 64° to 68° S., and from 32° to 37° W.

A ship sailed from latitude 64° 25′ S., longitude 32° 10′ W., as follows:—

Compass courses.	Distances.	
SW½S.	120′	Variation 3½ points W.
NE½N.	100′	
NW. by W½W.	115′	
SSW½W.	155′	
SE½E.	55	
NE.	195	

Lay down the true courses, and find the latitude and longitude in.

(*b*) State why it is necessary to obtain a base in Nautical Surveying.

(*c*) Name the different methods by which a base may be measured.

(*d*) In determining a base by means of the masthead angle, what precautions are necessary to ensure the required accuracy in the triangulation?

(*e*) How is the direction of the meridian line, or true north, obtained?

(*f*) What is a meridian distance? State briefly how it may be determined.

Ex. 562. In the exploration of a coast, a lofty peak, Mount Columbus, known to be in latitude 54° 52′ N., longitude 160° 18′ E., was observed from the ship to be directly in line with a hill near the shore on a *true* bearing of N. 60° W. At the same time the angle between Mount Columbus and a peaked island to the south-west was found to measure 80°.

The vessel then stood to the south west until the peaked island came in line with Mount Columbus on a *true* bearing of N. 36° W.; at the same time the angle between the hill near the shore and the peaked island was 25°, and that between the peaked island and the cliff farther to the south-west, known as Cape Drake, was 101°. The altitude of Mount Columbus was here observed: on 23′ 50″, off 22′ 40″; height of eye 18 feet.

The vessel proceeding to the south-west, when off Cape Drake a landing was effected, and Cape Drake found from observation to be in latitude 54° 16′ 30″ N., the *true* bearing of Mount Columbus being N. 16° W., and the angle between the Mount and the peaked island was found to be 32° 30′.

Project these positions on a scale of quarter of an inch to a mile, and determine *by projection* the longitude of Cape Drake, and the *true* bearing and distance between that Cape and the hill near the shore; also calculate the height of Mount Columbus. *For Beaufort Testimonial*, 1874.

Ex. 563. A ship A, sailing on a known course (N. C° E.), at a known rate, observes another ship B, bearing from her (N. A° E.), at a distance of *d* knots. At the end of two equal intervals of time, the bearing of B from A is observed to be N. A_1° E. and N. A_2° E. respectively; show how to find the course and rate of sailing of B (supposed uniform).

If *a* be the distance sailed by A in each of the two equal intervals of time, show that ϕ, the course of B, may be found from the formula—

$$\tan\left(\phi - \frac{A_1 + A_2}{2}\right) = \tan\frac{A_1 - A_2}{2} \, . \tan(45^\circ + \theta)$$

$$\text{where } \tan\theta = \frac{2\,a \, . \sin(C - A_1) - 2\,d \, . \sin(A - A_1)}{2\,a \sin(C - A_2) - d \, . \sin(A - A_2)}$$

Honours, 1881.

Ex. 564. A cloud is observed bearing S. θ° W. at an elevation of and after a certain interval it is observed to bear S. ϕ° W., at an elevation β°. Assuming it to move in a straight horizontal line, find the direction of the wind by which it is borne.

Ex. $\alpha = 30^\circ$, $\beta = 45^\circ$, $\theta = 22^\circ\ 30'$, and $\phi = 67^\circ\ 30'$.

Ex. 565. Two ships at a given instant are sailing uniformly on different but known courses with given unequal rates towards the same point, the distances of which from the two ships are also known. Find when they will be nearest to each other, and their distance apart at that time.

ANSWERS.

EXERCISE I. (*pages* 8–11).

(2) (*a*) 7634·25 miles.
(*b*) 60·8 „
(*c*) 1815·75 „
(*d*) 37·8$\dot{3}$ „
(*e*) 5400·$\dot{3}$ „
(*f*) 20·$\dot{6}$ „

(4) (*a*) 48° 36′ 0″.
(*b*) 153° 0′ 0″.
(*c*) 90° 0′ 18″.
(*d*) 48° 57′ 15″.
(*e*) 50′ 42″.
(*f*) 2° 0′ 27″.

(5) 3022·41$\dot{6}$ miles.

(6) 57° 57′ 48″ N.

(7) 4725·1$\dot{6}$ miles.

(12) (*a*) 55° 9′ 6″ N.
(*b*) 19° 38′ 57″ N.
(*c*) 18° 26′ 54″ N.
(*d*) 29° 59′ 30″ S.
(*e*) 16° 29′ 22·5″ S.
(*f*) 19° 40′ 42″ N.

(15) (*a*) 5° 59′ 30″ W.
(*b*) 31° 3′ 0″ E.
(*c*) 12° 58′ 42″ W.
(*d*) 3° 6′ 24″ E.
(*e*) 137° 0′ 18″ E.
(*f*) 86° 22′ 0″ W.

(17) Diff. lat. 883 miles N.
Diff. long. 70° 18′ E.
Mid. lat. 42° 36′ 30″ N.
Quadrant NE.

(18) Lat. in 27° 9′ 45″ S.
Long. in 13° 49′ 30″ E.
Mid. lat. 30° 45′ 52·5″ S.
Quadrant NW.

(19) (*a*) 5400 miles N.
(*b*) 3992 „ N.
(*c*) 7984 „ S.

(20) Lat. left 6° 20′ 48″ N.
Long. left 73° 33′ 12″ E.

EXERCISE II. (*pages* 14–15).

(24) Long. 69° 43′ 30″ E.

(29) Lat. in 60° 23′ 42″ N.
Long. in 15° 10′ 24″ W.

(31) $\frac{13267}{3962824} = \frac{1}{298 \cdot 7}$ nearly.

(32) Jovian compression $\frac{431}{7286}$.
Ratio 17·67 : 1.

(33) Diff. lat. 2795 miles S.
Diff. long. 48° 36′ W.

(34) Diff. lat. 2816 miles S.

(35) Diff. long. 48° 17′ E.

(37) Four.
Lat. 55° 30′ N.
Long. 36° 0′ E.
The point where the meri-

dian of Greenwich meets the equator.
Diff. lat. 2549 miles S.
Diff. long. 37° 44′ E.

(39) Lat. in 38° 53′ N.
Mid. lat. 36° 10′ 30″ N.
Diff. long. 83° 50′ W.

Exercise III. (*pages* 29–32).

(51) Deviation 1 point E.
(52) 11° 15′.
(54) $14\frac{1}{4}$ points = 160° 18′ 45″.
(55) No deviation.
(56) 21 points = 236° 25.
(57) $11\frac{1}{4}$ points = 126° 33′ 45″.
(58) 47° 48′ 45″.
(59) 36° 33′ 45″.
(60) 61° 52′ 30″.
True course SSW$\frac{1}{4}$W.
(61) 14° 1′ 15″ E.
(64) 6 points = 67° 30′
(65) (*a*) 1° 55′ E.
(*b*) 12° 40′ W.
(*c*) 8° 20′ E.
(*d*) 0° 0′.
(*e*) 7° 35′ E.
(*f*) 20° 40′ W.
(*g*) 17° 0′ W.
(*h*) 22° 10′ W.
(66) Magnetic bearing S. 11° E.
Deviations 8° E., 3° W., 17° W., 21° W., 6° W., 13° E., 15° E., 11° E.
(67) Magnetic bearing, East.
Deviations 5° 50′ E., 10° 40′ W., 17° 0′ W., 13° 50′ W., 0′ 0°, 11° 30′ E., 13° 0′ E., 11° 10′ E.

Exercise IV. (*pages* 38–40).

(71) (*a*) S. 30° 30′ E.
(*b*) N. 47° 47′ E.
(*c*) S. 81° 36′ W.
(*d*) S. 56° 30′ E.
(*e*) N. 40° 50′ E.
(*f*) S. 45° 3′ W.
(*g*) N. 88° 46′ E.
(*h*) N. 87° 49′ E.
(*i*) S. 80° 41′ W.
(*j*) S. 71° 2′ W.
(*k*) S. 2° 35′ E.
(*l*) N. 2° 0′ E.
(74) (*a*) N. 81° 25′ W.
(*b*) N. 62° 31′ W.
(*c*) N. 61° 45′ E.
(*d*) S. 2° 48′ E.
(*e*) N. 66° 21′ E.
(*f*) N. 20° 7′ W.
(*g*) N. 26° 14′ E.
(*h*) N. 84° 37′ E.
(*i*) S. 75° 40′ W.
(*j*) N. 3° 54′ W.
(*k*) N. 69° 20′ W.
(*l*) S. 70° 48′ W.
(75) Starboard tack, 1 point.
(76) N. 58° 48′ W.
(77) Difference is caused by leeway. Ship on port tack. Leeway 17° 4′.
(78) 10 points = 112° 30′.
(80) 48 ft. 11·73 in.
(81) 26 ft. 2·13 in.
(82) 48 feet 1·6 in.
(83) 22 ft. 9·6 in.

Exercise V. (*pages* 46–49).

(84) $43\frac{7}{11}$ miles.
(85) 47·29 feet; 163·61 miles.
(86) 45·6 feet.
(87) 50 ft. 8 in.
(88) 9·008 miles.
(89) 169·9 miles.
(90) 239·155 miles.
(91) 1 ft. 7·72 in. too long.
(92) 10·41 miles per hour.
(93) 50 ft. 8 in. Rate should be augmented by one-fifth.
(100) 47·29 feet.
(101) NW. by W$\frac{3}{4}$W.
(103) (*a*) N. 63° 26′ W.
(*b*) S. 43° 19′ W.
(*c*) N. 25° 4′ E.
(*d*) S. 11° 4′ E.
(104) 47 ft. $3\frac{1}{2}$ in; 114 19 miles.
(105) 51 feet; 23 8 feet.
(106) NNW.
(108) Wind SSE$\frac{3}{4}$E.
Leeway 16° 57′.

Exercise VI. (*pages* 57–58).

(112) 31° 3′ 33″ N.
(113) 41° 37′ 52″ N.
(114) Course S. 47° 57′ W.
Departure 80·94 miles.
(116) Dist. 43·2 miles;
diff lat. 35·92 miles.
(117) Dist. 90 miles;
lat. 36° 49′ 10″ N.
(119) C. Course S. 47° 9′ E.;
dist. 234·7 miles.
(120) Dep. 0; lat. 34° 22′ S.
(121) Dep. 1·81 miles;
lat. 49° 38′ 5″ N.
(122) Dev. $\frac{1}{4}$ point W.;
dist. 782 miles.
(123) N. 49° 22′ W
(124) N. 83° 26′ W.
(125) 22 03 hours.
(126) S. 73° 18′ W.; 218·5 miles.

Exercise VII. (*pages* 64–66).

(129) Course S. 45° W.;
dist. $71\frac{3}{4}$ miles.
Diff. lat. 50·7 miles;
dep. 50·8 miles.
(130) Course, East; dist. 70·2 miles.
Lat. in 28° 32′ N.
(131) Course, North;
dist. 104 miles.
Lat. in 33° 15′ N.
(132) Course, N. 35° E.;
dist. $82\frac{1}{4}$ miles.
Lat. in 1° 7·7′ N.
(133) Course, S. 19° W.;
dist. 52·6 miles.
Lat. in 17° 16·9′ N.
(134) 1st ship—course, S. 40° W.;
dist. 427 miles.
2nd ship—course, S. 43° E.;
dist., $375\frac{1}{4}$ miles.
(138) 2 h. $52\frac{1}{2}$ m.
(139) Diff. lat. 50 miles;
dep. 86·6 miles.

Exercise VIII. (*pages* 72-73).

(142) S. 63° 54 '35″ E.;
N. 74° 40′ 50″ E.
(143) Course, N. 60° 49′ E.;
dist. 128·2 miles.
(144) N. 37° 37′ E.; S. 4° 46′ E.
(145) Course, S. 30° 58′ E.;
dist. 58·31 miles.
(146) Set. S. 35° 50′ W.;
drift, 88 8 miles.
(147) Bearing, S. 29° 5′ 51·5″ E.;
dist. 9·973 miles.
(148) N. 70½° E.; rate, 4·4 miles per hour.
(149) Course, N. 83° 46′ 23″ W.;
time, 2·673 hours.
(150) Course, S. 18° 5′ E.;
time, 3·134 hours.
(151) Set. S. 33° 52′ 12″ W.;
rate, $3\frac{5}{7}$ miles per hour.
(152) Dist. 22·41 miles;
drift, 13·35 miles.

Exercise IX. (*pages* 76–79).

(154) 6¼ pts. from wind; 30·26 miles; 34·83 miles.
(155) N. ½pt. E.; 7·42 miles.
S. 5½ pts. W.; 14·85 miles.
N. ½ pt. E.; 7·42 miles.
(156) N. 2¾ p's. W.; 48·18 miles.
S. 5 pts. E.; 47·64 miles.
(157) 4·68 hours.
(158) Course, N. 79° 51½′ E.;
dist. 524·8 miles; 68° 36½′ from the wind.
(159) N. 64° 28′ W.; S. 25° 32′ E.;
70° 32′ from the wind.
(160) 54° 56′ from the wind. No.
(161) 23·375 miles; bearing
N. 32° 17′ 15″ E.; dist.
18·95 and 38·65 miles.
(162) SSE½E.; 58·51 miles.
NW. by W½W.; 51·85 miles.
(163) N. by E.; 26·93 miles.
W. by S.; 7·855 miles.
(164) 9·852 miles.
(165) N. 49° 19′ W.; port tack,
N. by E¾E., 4·105 miles;
Starboard tack,
S. 61° 39′ 45″ W.,
8·147 miles.
(171) AB = 15·52; AC = 6·43.
(172) 257·4 feet.
(173) Port, 115·1 miles; starboard
92·97 miles.
(174) Set. N. 72° E.;
drift, 38 miles.
(175) 9·26 miles; 4·95 miles.
(178) 5·5 miles.
(181) Set. N. 82° 19′ 55″ E.;
rate, 5·182 miles.
(183) 6·533 miles; ·7785 miles.
(184) Starboard tack, 8·825; 16·63;
8·825 miles.
Port tack, 8·315; 17·65;
8·315 miles.

Exercise X. (*pages* 83–85).

(188) S. 72° 52′ 30″ E.; 500·91 miles.
(189) 49° 20′ N.; 8° 1′ 38″ W.; S. 25° 49′ W.; S. 79° 1′ E.
(190) 50° 42′ 12″.
(191) 591·6 miles.
(192) 73° 28′ 53″ W.
(193) 31° 0 37″ S.
(194) 51° 20′ N.; 15° 40′ 25″ W.
(195) A arrives first; 14·23 miles.
(196) 1° 49′ 10″ E.
(197) 60 a . cos l miles.
(198) 23° 33′ 23 ′.
(199) 36° 52′ 11″.
(200) 829 miles per hour.
(201) 95·436 miles.
(203) 14048·67 miles.
(204) 270·52 miles.
(205) 29° 43′ 31″ S.
(206) 34° 0′ 33″.
(207) 6183·3 miles.

Exercise XI. (*pages* 92–96).

(212) 13° 4′ 49″ W.
(213) 39° 23′ 20″ N.; 19° 47′ 34″ E.
(215) N. 24° 34′ 33″ E.; 1863·57 miles.
(216) 18° 44′ 54″ S.; 88° 11′ 46″ W.
(217) 29° 10′ 25″ S.; 11° 20′ 25″ E.
(218) 49° 10′ 14″ N. S. 61° 20′ 53″ E.; 292·52 miles.
(219) 46° 11′ 57″ S.; 12° 53′ 44″ W. N. 79° 19″ 14″ W.; 2131·45 miles.
(220) 36° 59′ 38″ N.; 9° 0′ W. S. 33° 18′ 26″ E.; 1027·1 miles.
(221) 51° 18′ 56·3″ N.; 36° 58′ 41·7″ N.
(223) 23° 17·6′ N.; 29° 29·5′ W. N. 16° 40′ E.; 195·8 miles.
(224) S. 19° E.; 76·9 miles. 50° 12·3′ N.; 7° 32′ 26″ W.
(225) 31° 19′ N.; 60° 28′ 15″ W. N. 37° 40′ E.; 235 miles.
(226) N. 51° 33′ 56·5″ E.; 0h. 23·3m. P.M.; 26·9 miles.
(227) 133·8 miles.
(228) 31·42 miles. N. 23° 13′ 53″ W.
(233) 85° 0′ 25″ N.

Exercise XII. (*pages* 100–101).

(237) 8·456 miles.
(238) 8·915 „
(239) 8·25 „
(240) 4·44 „
(241) 10·63 „
(242) 23·537 „
(243) 19·16 „
(244) 7964·5 statute miles.
(245) 6·43 miles.
(246) 8·345 „
(247) 7·9755 knots per hour; 23·3948 miles.
(248) 3993·2 miles; 76 35 miles.

Exercise XIII. (*pages* 111–123).

(251) 20° 26·4′ N.; 179° 34′ E.
N. $40\frac{3}{4}$° W.; 41·5 miles.
(252) S. $61\frac{1}{2}$° E.; 236 miles.
1° 38′ S.; 177° 8′ W.
(253) 51° 20′ $\frac{1}{2}$S.; 57° 44′ $\frac{1}{2}$W.
N. $87\frac{1}{2}$° E.; 82 miles.
(254) N. 78° $\frac{3}{4}$W.; $310\frac{1}{4}$ miles.
63° 26′ N.; 5° 32′ $\frac{1}{2}$W.
(255) 58° 3′ $\frac{1}{2}$N.; 5° 21′ W.
S. 17° W.; 38 miles.
(256) N. 52° $\frac{1}{2}$W.; $30\frac{1}{4}$ miles.
34° 0′ $\frac{1}{2}$S.; 114° 37′ E.
(257) 12° 10′ $\frac{1}{2}$S.; 77° 55 $\frac{3}{4}$W.
S. 81° $\frac{3}{4}$W.; 44 miles.
(258) S. 80° E.; 135 miles.
56° 3′ N.; 1° 36′ E.
(259) 55° 53′ $\frac{3}{4}$S.; 61° 1′ W.
N. 88° $\frac{1}{2}$E.; $207\frac{3}{4}$ miles.
(260) S. 25° W.; $95\frac{1}{2}$ miles.
24° 56′ $\frac{1}{2}$S.; 35° 5′ $\frac{3}{4}$E.
(261) West; 54·1 miles.
71° 10′ N.; 23° 13′ $\frac{1}{2}$E.
(262) 30° 39′ N.; 16° 18′ $\frac{1}{2}$W.
S. 16° E.; $122\frac{3}{4}$ miles.
(271) 9084·7 feet.
Length in feet, 47·586.

Exercise XIV. (*pages* 134–136).

(278) N. 43° 3′ 56″ E.;
2578·8 miles.
(279) S. 42° 13′ 48″ E.; 199·2 miles.
(281) S. 12° 50′ 3″ E.; 731 miles.
(282) N. 85° 27′ 2″ W.; 859·3 miles.
(283) S. 60° 6′ 46″ W.; 2043 miles.
(284) S. 67° 36′ 16″ W.;
989·65 miles.
(285) N. 85° 30′ 44″ W.;
1785·3 miles.
(286) N. 24° 12′ 26″ W.;
1466·1 miles.
(287) N. 35° 13′ 24″ W.;
533·3 miles.
(288) N. 26° 13′ 33″ W.;
1518·3 miles.
(289) S. 77° 50′ 55″ E.;
1005·3 miles.
(290) 30° 19′ 8″ S.; 7° 8·5′ E.
(291) 43° 47′ 6″ N.; 571·5 miles.
(292) 46° 11·9′ S.; 12° 54·6′ W.
N. 79° 21′ 52″ W.; 2126 miles.
(293) 51° 14·9′ N.; 1023 miles.

Exercise XV. (*pages* 143–145).

(298) 53° 14′ S.; 24° 10′ $\frac{1}{2}$E.
(299) 56° 28′ S.; 30° 20′ $\frac{1}{2}$W.
S. 71° W.; $19\frac{1}{2}$ miles.
(300) 53° 32′ $\frac{1}{2}$N.; 12° 40′ $\frac{1}{2}$W.
N. 46° W.; 118·5 miles.
(301) 55° 19′ N.; 138° 24′ $\frac{1}{2}$W.
(302) 69° 0′ $\frac{1}{2}$N.; 62° 6′ W.
N. 24° W.; 77 miles.
(303) Bearing, S. 16° E.;
dist. 45 miles.
Course, N. 14° E.;
dist. 62 miles.

EXERCISE XVI. (*pages* 156–157).

(316) 3482·26.
(319) 3630·98.
(320) 48° 38′ 48″.

EXERCISE XVII. (*pages* 187–191).

(328) 4577 miles, initial course, S. 48° 56′ ½E.; final course. N. 75° 3′ ½E.; latitude vertex, 44° 57′ S.; longitude vertex, 125° 56′ ½E.
(329) Initial course, N. 50° 27′ E.; final course, S. 78° 13′ E.; dist. 3104 miles; lat. vertex, 50° 58′ ½N.; long. vertex, 20° 26′ ½W.
(330) Initial course, S. 67° 11′ ½E.; final course, N. 47° 49′ ½E.; dist. 6429 miles; lat. vertex, 42° 37′ S.; long. vertex, 22° 38′ ½E.
(331) 305·5 miles.
(332) 2700 miles; S. 86° 41′ ½W.
(333) 5½ miles.
(334) 6108 miles.
(335) S. 87° 30′ ½W.; N. 49° 27′ ½E. S. 67° 11′ W.; N. 67° 11′ E.
(336) 2480½ miles; long. 24° 12′ ½W.
(337) 3813 miles; 54 miles.
(341) 51° 58′ S.
(343) N. 82° 11′ W.; N. 64° 12′ E. Course on rhumb, S. 80° 34′ ½W.
(344) Lat. left, 42° 29′ S.; long. left, 40° 30′ E. Lat. arrived at, 48° 25′ S.; long arrived at, 114° 10′ ½E.
(345) Lat. A, 24° 11′ ½S.; lat. B, 41° 51′ ½S.; diff. long., 108° 35′.
(346) Lat. A, 35° 33′ ½S.; lat. B, 38° 19′ S.; long. B, 159° 8′ ¼W.
(353) 163½ miles.
(355) 2636 miles.
(356) S. 59° 4′ ½W.; 28° 14′ W.
(362) 8° 11′.
(363) 2° 14′.
(364) 6° 50′.

EXERCISE XVIII. (*pages* 227–233).

(371) Magnetic bearing, S. 26° 30′ W.
Deviations, 12°0′E.; 26°30′E.; 26° 30′ E.; 13° 0′ E.; 11° 15′ W.; 25° 15′ W.; 27° 15′ W.; 14° 15′ W.
Comp. courses, N. 2° ½W.; S. 78° W.; S. 8° ½W.; N. 70° W.
Magnetic courses, S. 22° W.; S. 21° E.; S. 77° E.; N. 3° ¼E.
Deviation, 25½° W.
Bearings, N. 3° 34′ W. N. 89° W.

(372) Magnetic bearing, N. 81° 40′ E.
Deviations, 1° 30′ W.; 12° 20′ E.; 20° 0′ E.; 12° 10′ E.; 2° 20′ E.; 10° 50′ W.; 18° 20′ W.; 16° 10′ W.
Compass courses, S. 72° W.; N. 37° W.; S. 3° W.; N. 77° E.
Magnetic courses, N. 57$\frac{1}{4}$° E.; S. 46° E; S. 81° W.; N. 3° E.
Deviation, 19° E.
Bearings, N. 10° 34′ E.; N. 64° E.

(373) Magnetic bearing, East.
Deviations, 0° 0′; 23° 0′ W.; 30° 0′ W.; 22° 15′ W.; 3° 0′ W.; 27° 15′ E.; 30° 0′ E.; 21° 0′ E.
Compass courses, S. 21° W; N. 41° E.; North; S. 66° $\frac{1}{4}$E.
Magnetic courses, N. 70° $\frac{1}{2}$W; N. 20° $\frac{1}{4}$W; S. 7° $\frac{1}{2}$W.; S. 32° $\frac{1}{2}$E.
Deviation, 30° 30′ W.
Bearings, N. 66° 8′ E.; S. 48° 15′ W.

(374) Magnetic bearing, S. 86° 20′ E.
Deviations, 2° 0′ E; 11° 50′ W.; 16° 40′ W; 14° 0′ W.; 1° 30′ W.; 13° 20′ E.; 16° 40′ E.; 12° 0′ E.
Compass courses, S. 35° W.; N. 87° $\frac{1}{2}$W.; S. 88° E.; N. 23° $\frac{1}{2}$E.
Magnetic courses, S. 30° $\frac{1}{2}$W.; S. 51° $\frac{1}{2}$E.; N. 28° $\frac{1}{2}$W.; N. 83° W.
Deviation, 15° W.
Bearings, N. 83° 26′ E.; N. 3° 45′ W.

(375) Magnetic bearing, North.
Deviations, 14° 30′ W.; 13° 20′ E.; 22° 20′ E.; 19° 20′ E.; 8° 15′ E.; 8° 45′ W.; 18° 30′ W.; 21° 30′ W.
Compass courses, N. 36° W.; S. 86° E.; S. 9° E.; N. 85° W.
Magnetic courses, N. 59° E.; S. 40° W.; N. 4° $\frac{1}{2}$W.; S. 77° $\frac{3}{4}$E.
Deviation, 6° E.
Bearings, S. 86° 49′ E.; S. 6° W.

(376) Magnetic bearing, N. 20° 30′ E.
Deviations, 3° 30′ E.; 27° 30 E.; 29° 30′ E.; 20° 30′ E.; 4° 30′ W.; 24° 30′ W.; 31° 30′ W.; 20° 30′ W.;
Compass courses, N. 68$\frac{1}{4}$° E.; N. 47° W.; S. 71° E.; N. 77° W.
Magnetic courses, N. 65° E.; N. 6° W.; S. 3° E.; S. 65° W.
Deviation, 28° W.
Bearings, N. 70° 26′ E.; S. 22° 22′ E.

(377) Magnetic bearing, S. 18° E.
Deviations, 1° 30′ W.; 12° 30′ W.; 18° 0′ W.; 15° 0′ W.; 2° 0′ W.; 13° 0′ E.; 20° 0′ E.; 16° 0′ E.
Compass courses, N. 64° W.; S. 76° $\frac{3}{4}$W.; N. 3° W.; S. 78° E.
Magnetic courses, S. 60° E.; S. 89° W.; N. 41° $\frac{3}{4}$E.; S. 2° E.
Deviation, 19$\frac{1}{4}$° E.
Bearings, N. 70° 45′ W.; N. 19° 15′ E.

(378) Magnetic bearing, West.
Deviations, 3° 45′ W.;
13° 30′ W.; 18° 15′ W.;
16° 0′ W.; 3° 45′ E.;
17° 0′ E.; 19° 0′ E.;
11° 45′ E.
Compass courses, N. 15¼° E.;
N. 62¾° W.; S. 6° E.;
N. 2° E.
Magnetic courses, S. 86°½W.;
N. 36¼° E.; N. 80° E.;
S. 28° W.
Deviation, 19° W.
Bearings, N. 76° 37′ W.;
S. 16° 11′ E.

(379) Magnetic bearing, S. 26° W.
Deviations, 2° E.; 25° W.;
32° W.; 21° W.; 0°;
26° E.; 30° E.; 20° E.
Compass courses, N 19° W.;
S. 34° W; S. 70° ¼E.;
N. 2° 49′ E.
Magnetic courses, S. 73° ½E.;
N. 10° E.; South; N. 69° W.
Deviation, 22° E.
Bearings, S. 68° E.;
N. 76° 26′ W.

(380) Magnetic bearing, North.
Deviations, 0°; 17° 0′ W.;
22° 15′ W.; 19° 45′ W.; 0°;
18° 0′ E.; 23° 15′ E.;
17° 45′ E.
Compass courses, East; North;
S. 87° W.; N. 60° ½W.
Magnetic courses, S. 30° E.;
S. 89° W.; South; N. 87° E.
Deviation, 21° E.
Bearings, N. 83° 4′ W.;
S. 69° 0′ E.

EXERCISE XIX. (*pages* 241–243).

(293) 1¾ pts. W

(394) 2 pts. E.

EXERCISE XX. (*page* 249).

(414) 11 h. 48 m. a.m.; no, p.m

(415) 6 h. 10 m. a.m.;
6 h. 34 m. p.m.

(416) 1 h. 37 m. a m.;
1 h. 57 m. p m.

(417) 4 h 36 m. a.m;
4 h. 57 m p m.

(418) 7 h. 30 m. a.m.;
7 h. 52 m. p.m.

(419) 5 h. 8 m. a.m.;
5 h. 40 m. p.m.

EXERCISE XXI. (*page* 250).

(421) 9 h. 54 m. a
10 h. 37 m. p.m.

(422) 10 h. 19 m. a.m.;
11 h. 7 m. p.m.

(423) 10 h. 59 m. a.m.;
11 h. 39 m. p.m.

(424) 1 h. 27 m. a.m.;
2 h. 5 m. p.m.

(425) 9 h. 35 m. a.m.;
9 h. 58 m. p.m.

(426) 11 h. 53 m. a.m.; no, p m.

(427) No. a.m.; 0 h. 25 m. p.m.

(428) 11 h. 56 m. a.m.; no, p.m.

Exercise XXII. (*page* 254).

(430) Subtract 6 ft. 2 in.
(431) Subtract 6 ft. 10 in.
(432) Subtract 1 ft. 4 in.
(433) Add 0 ft. 3 in.
(434) Subtract 2 ft. 2 in.
(435) Add 2 ft. 2 in.
(436) Subtract 6 ft. 2 in.
(437) Subtract 7 ft. 2 in.

Exercise XXIV. (*pages* 278–279).

(450) 1535·3 yards.
(451) 7664 feet.
If ship be northward of hill and cliff:
Bearing, N. 46° E.; distance, 7972 feet.
If ship be southward of hill and cliff:
Bearing, N. 50° E.; distance, 7972 feet.
(454) Bearing, S. 50° 9′ 55″ W.
Distance 860·1 feet.

Miscellaneous Questions.

(492) S. 79° 41′ 15″ E.
(493) $18\frac{2}{3}$ miles too little.
(494) $168\frac{9}{47}$ miles.
(495) $3\frac{2509}{7215}$ feet too short.
(496) $4\frac{13}{30}$ feet too long.
(497) $109\frac{29}{119}$ miles.
(498) 40·65 miles due W.
(499) 80° 23′ 10″ N.
(500) 30°.
(501) 17·39 miles; 24·59 miles.
(502) 1764 yards.
(503) 24·414 miles.
(504) N. 44°9′36′ W.; 47° 10′26″N.; 40° 22′ 54″ E.
(505) 64° 37′ 34″ N.; 877·56 miles.
(506) 53 ft. 4·3 in.
(507) 14 miles.
(508) 49° 9′ 42″ N.; 6° 7′ 30″ W.; S. 36° 52′ 12″ W.
(509) S. 81° 36′ 35″ W.; 4831 feet.
(510) ·768 mile.
(511) 176·68 miles.
(512) 22·7 miles S.; 19·8 miles W.
(513) East, ·4692 miles.
(514) Bows, 16448·6 ft.; stern, 16305·1 ft.
(515) S. 10° 37′ 30″ E.; S. 0° 22′ 30″ W.
(516) 839·9 ft.; 827·8 feet.
(518) 18° 35′ 9·5″.
(519) 9·233 miles; 19 miles
(520) 66·89 feet.
(522) N. 75° 59′ W.; 16·93 miles.
(523) NE. by E.: ENE.; N$\frac{1}{2}$W.
(524) $a\sqrt{\frac{b+a}{b-a}}$.
b must be greater.
(525) $1\frac{1}{2}$ pts. W.
(528) At a perpendicular distance 2·4 miles at 4·8 miles from A.
(529) 25°6′39″·6 N; 18° 15′ 11″·4 W.
25°46′2″·4 N; 18° 43′42″ 6 W.
S. 33° 11′ 4″ E.; 47·05 miles.
(530) 1·56 miles and 3·39 miles per hour.
(531) 6·39 miles.
(532) S. 8° 46′ 22″ W.; 3·475 mile per hour.

(533) N. 17° 56′ 45″ W.; 4·05 miles per hour.

(534) 4·564 miles per hour.; S. 28° 16′ 45″ W.

(535) $6\frac{2}{3}$ miles and $2\frac{2}{3}$ miles.

(536) S. 8° 39′ 5″ E.; 5·62 miles per hour.

(537) 43° 20′ 31″ towards admiral's line of progression.

(538) S. 79° 35′ 48″ E.; 7 h. 44·34 m. p.m.

(539) 29° 18′ 2″; 39° 29′ 41″.

(540) 35° 20′ 40″.

(541) N. 17° 9′ 44″ E.

(543) 8·7197 miles; 6·8375 miles; 9·5546 miles.

(544) $a \,.\, \sin \alpha \,.\, \sin \beta \,.\, \text{cosec}\,(\alpha \backsim \beta)$.

(548) $r = \dfrac{a^2 \cot^2 \alpha - b^2}{2b}$.

dist. $= a \cot. \alpha$

(549) 48° 59′ N; 109° 39′ W.
149 miles; S. 38° E.;
N. 29° 22′ 30″ E.

(550) 52° 53′ S; 74° 43′ E.
$51\frac{1}{2}$ miles; N. 72° W.;
N. 49° 30′ W.

(551) 3505·70.

(553) S. 34°9′22″W.; S. 15°50′38″E.; S. 67° 26′ 29″ E.; 117·3 miles.

(555) 48° 42′ 8″ N.; 23° 50′ W.

(556) 999·386 h.; 1142·25 h.

(557) N. 57° 12′ 28″ E.; 39° 29′ 16″ N.

(560) S. 59° 15′ 16″ E.; 20·05885 miles.

(561) 64° 25′ S.; 32° 9′ W.

(562) 160° 35′ $\frac{1}{2}$E.; N. 11° $\frac{3}{4}$E.; 27 miles; $2006\frac{1}{2}$ feet.

(564) S. 12° 6′ E.

(565) Let u and v be the given rates of sailing, θ the angle between the courses, and a and b the distances from the point of crossing, then time when nearest is,

$$\frac{au + bv - (bu + av)\cos\theta}{u^2 + v^2 - 2uv\cos\theta}$$

and least distance is,

$$\frac{(av - bu)\sin\theta}{\sqrt{u^2 + v^2 - 2uv\cos\theta}}$$

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