Sam S. Montague

A TREATISE

ON THE

PRINCIPAL

MATHEMATICAL INSTRUMENTS

EMPLOYED IN

SURVEYING, LEVELLING, AND ASTRONOMY,

&c. &c.

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ON THE

PRINCIPAL

MATHEMATICAL INSTRUMENTS

EMPLOYED IN

SURVEYING, LEVELLING, AND ASTRONOMY;

EXPLAINING THEIR

CONSTRUCTION, ADJUSTMENTS, AND USE:

WITH AN

Appendix and Tables.

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BY

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PREFACE

TO THE FIRST EDITION.

THE want of a work containing a concise and popular description of the principal Instruments used in Practical Astronomy and Surveying has long been felt, as the requsite information with respect to such instruments can only be obtained by consulting various expensive publications, which are not within the reach of many to whom such information is highly interesting and important.

It was the original object of the writer of this little tract to place at the disposal of the young surveyor a description of the instruments which are required in his profession, and such an account of the method of examining and rectifying their adjustments, as would enable him to obtain from them the most accurate results; but he found that, without greatly increasing the size of the book, he might materially add to its utility, by including in his plan the most approved Astronomical Instruments, that amateur astronomers as well as scientific travellers might have at hand a manual of instructions, which would enable them to use their instruments with the utmost advantage.

Usefulness being the author's chief object, he has not scrupled to extract from the works of others whatever he found adapted to his own purpose; and to some kind literary and scientific friends he is under obligations, for which, if he had obtained their permission, he would be glad to thank them by name in this place.

Of Surveying Instruments, those only have been described which are applied in modern practice, no reference being made to those which, having been superseded by better ones, may be said to be out of use.

To the article on Levelling has been added a description of Mr. Troughton's Improved Mountain Barometer, with an easy and accurate method of computing differences of level from barometrical observations. Table II., employed for this purpose, has been carefully recomputed from Mr. Baily's formulæ. The other Tables will, for their several purposes, be found convenient and useful. Tables I. and VIII. are new.

Much attention has been paid to the accuracy of the formulæ given for performing the various computations, and each has been thrown into the form of a practical rule, that persons unacquainted with algebraic notation may be enabled, notwithstanding, to make the requisite calculations.

With respect to such astronomical problems as appertain chiefly to navigation, and require extensive and special tables for their convenient solution, it has been thought better to omit all reference to them in this work, as in Mr. Riddle's Treatise on Navigation, Captain Thompson's Lunar and Horary Tables, and other similar works, all necessary information on the subject may be readily obtained.

The Appendix relates chiefly to the protraction of the work after a survey has been completed, and seems a suitable supplement to the account of Surveying Instruments given in the preceding part of this treatise.

PREFACE

TO THE SECOND EDITION.

In preparing for the press a second edition of my "Treatise on Mathematical Instruments," I have endeavoured to make such additions and improvements as would render it still more acceptable to the reader. To the account of Surveying Instruments I have added a few remarks on the use of the Land Chain, and given some additional particulars, with an engraving, of Captain Everest's Theodolite, which has hitherto been extensively used in India, and is now frequently employed in this country. To the Levelling Instruments I have added a representation and description of Mr. Gravatt's modification of the Spirit Level, and also of the new Levelling Staves; the article on Levelling has also been remodelled, and I hope will be found by the young beginner to contain some useful practical information.

Through the kind friendship of Edward Riddle, Esq., I have been enabled to insert his latest improvements in the practical solution of the problem for determining the *Longitude* by the Moon and Moon-culminating Stars; which will also be contained in the third edition of his valuable Treatise on Navigation, now in the press. Some additional examples and formulæ have likewise been given in the account of the Portable Transit Instrument, which it is hoped will not be without their use.

To the Appendix is added an account of the various methods of copying and reducing or enlarging Plans, &c., including a description of the Pentagraph; also an account of the method of executing a Survey, for the purposes of a Railway or Turnpike Road.

Table VII. has been improved in its arrangement, and the last four Tables have been added to this edition.

GREENWICH, F. W. S.
Feb. 17*th*, 1836.

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SURVEYING INSTRUMENTS.

THE LAND CHAIN.

Gunter's Chain is the one now commonly used in taking the dimensions of land; it is sixty-six feet, or four poles, in length, and is divided into 100 links, each of which is joined to the next by three rings: the length of each link, including the connecting rings, is 7,92 inches, and at the end of every tenth link is attached a piece of brass (each of a different shape), for more readily counting the odd links.

"The English acre contains 4840 square yards, and Gunter's chain is 22 yards in length, and the square chain, or 22 multiplied by 22, gives 484, exactly the tenth part of an acre; and ten square chains are equal to one acre: consequently, as the chain is divided into 100 links, every superficial chain contains 100 multiplied by 100, that is 10,000 square links; and 10 superficial chains, or one acre, contain 100,000 square links.

"If therefore the content of a field, cast up in square links, be divided by 100,000, or (which is the same thing), if from the content we cut off the last five figures, the remaining figures towards the left hand give the content in acres, and consequently the number of acres at first sight; the remaining decimal fraction, multiplied by 4, gives the roods, and the decimal part of this last product, multiplied by 40, gives the poles or perches."

Short distances, or off-sets from the chain-line, are usually measured with a rod, called an off-set staff, the most convenient length for which is 6 feet 7,2 inches, being equal to 10 links of the chain; and it should be divided accordingly.

With the chain must be provided ten arrows, which may be made of strong iron wire, about 12 or 15 inches long, pointed at one end for piercing the ground, and turned up at the other, in the form of a ring, to serve as a handle: their use is to fix in the ground at each extremity of the chain whilst measuring, and to point out the number of chains measured.

The operation of measuring with the chain requires at least two persons, one to lead and the other to follow and direct. The first or leader (taking the ring at one end of the chain upon two fingers of his right hand with one arrow, and the remaining nine in his left), lays his end of the chain, by direction of the follower, in a straight line with the station to be measured to, and there fixes an arrow, while the latter holds the other end of the chain at the starting point; the leader now proceeds onwards until the follower comes to the arrow first laid down, to which he places his end of the chain, and again directs the leader to place a second arrow in line with the forward station: the leader will now have an opportunity of checking the directions of the follower at every succeeding chain's length, by observing if the latter be also in a line with the back station, at the time he directs him to place one of his arrows in the direction of the forward station. They proceed in this manner till the whole line is measured, or the leader has spent all his ten arrows, which, upon counting, he will find in the possession of the follower, (unless some error has been committed,) who must restore them to the leader, and remark in the field-book that they had made one change, or measured 1000 links: they then proceed onwards as before, the leader taking all the ten arrows, until they are again spent, when a second change must be made and entered in the book, and if the line be measured out before a third change takes place, the follower will have in his hand as many arrows as there have been chains laid out upon the last measured part to the distance; which, together with the odd links and the former two changes (or 2000), will make up the entire length of the line.

For the purposes of plotting, &c., it will be necessary to reduce the measurement of the lines which alternately ascend and descend to the correct horizontal measure, for it is evident that the distance between two points, if measured over uneven ground, will be greater than if measured perfectly straight in a horizontal plane. Some surveyors attempt this correction as they proceed, by holding the lower end of the chain above the ground, as nearly horizontal as they can estimate, and, if they aim at considerable accuracy, will have a plumb, which they allow to hang from the hand that holds the chain, over the arrow or mark in the ground. In passing over very steep ground, they frequently take half, or even a quarter, of a chain's length to accomplish their measurements with, as the whole length would be too great to be held horizontal, when the inclination is considerable. But the most correct method is to take the vertical angles along the undulations of the line after it has been measured, and compute the horizontal distances (by a rule in plain trigonometry), as the whole line is then supposed to be divided into a number of right-angled triangles, the measured portion being the hypothenuse, and the horizontal line the base; or it may be more expeditiously accomplished by our Table IX., which shews the quantity to be subtracted from each chain's length for various angles of inclination of the ground, which at once reduces the oblique or hypothenusal measure to the horizontal.

THE SURVEYING CROSS AND OPTICAL SQUARE.

The instrument formerly employed for laying out perpendicular lines was the cross-staff, of which there were various constructions; but that in most general use consisted of four sights, fixed at right angles upon a brass cross, and adapted to the top of a staff, which, being thrust into the ground with two of the sights placed in any given direction, the other two pointed out the perpendicular required. But this instrument has been almost superseded by the optical square, which is much superior to it both for convenience and expedition; and it has also the advantage of greater portability, not being larger than a shallow circular snuff-box, which it resembles in shape. It is made of brass, and contains the two principal glasses of the sextant, *viz.*, the index and horizon glasses, fixed at an angle of 45°; hence, while viewing an object by direct vision, any other, forming a right angle with it, at the place of the observer, will be referred by reflection, so as to coincide with the object viewed. Thus a line may be laid out perpendicular to a station-line, and from any point on it, by simply standing with the instrument over the given point, and looking through it along the line, having a person to go with a mark or station-staff in the direction the perpendicular is required, and signing to him by hand to move to the right or the left, until his staff is seen by reflection to coincide with some object on the line along which the observer is looking, when the place of the staff will be in a perpendicular to the station-line at the place of the observer.

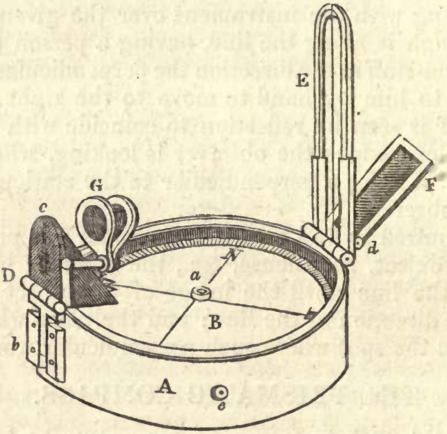
If it be required to find on a line the place of a perpendicular from a fixed object, as a house, &c., the observer himself must move along the line until the image of the object appears, as before, in the direction of the line; and the place where he then stands will be the spot where such perpendicular would fall.

THE PRISMATIC COMPASS.

The use of this little instrument is to measure horizontal angles only, and from its portability is particularly adapted for military surveying, or where but little more than a *sketch map* of the country is required. It is also very useful in filling in the detail of a map, where all the principal points have been correctly fixed by means of the theodolite, and for this purpose has been extensively employed by the gentlemen engaged on the Ordnance

Survey. It may likewise be used for determining approximately the direction of the true meridian, the variation being determined by comparing the observed azimuth of a celestial object with its true azimuth, deduced from an observation made for the purpose.

In the following figure, A represents the compass-box, and B the card, which, being attached to the magnetic needle, moves, as *it* moves, round the agate centre, *a*, on which it is suspended. The circumference of the card is usually divided to 15′ of a degree, but it is doubtful whether an angle can be measured by it even to that degree of accuracy: *c* is a prism, which the observer looks through in observing with the instrument. The perpendicular thread of the sight-vane, E, and the divisions on the card appear *together* on looking through the prism; and the division with which the thread coincides, when the needle is at rest, is the magnetic azimuth of whatever object the thread may bisect. The prism is mounted with a hinge joint, D, by which it can be turned over to the side of the compass-box, that being its position when put into the case. The sight-vane has a fine thread stretched along its opening, in the direction of its length, which is brought to bisect any object, by turning the box round horizontally; the vane also turns upon a hinge joint, and can be laid flat upon the box, for the convenience of carriage. F is a mirror, made to slide on or off the sight-vane, E; and it may be reversed at pleasure, that is, turned face downwards: it can also be inclined at any angle, by means of its joint *d*; and it will remain stationary on any part of the vane, by the friction of its slides. Its use is to reflect the image of an object to the eye of the observer when the object is much above or below the horizontal plane. When the instrument is employed in observing the azimuth of the sun, a dark glass must be interposed; and the coloured glasses, represented at G, are intended for that pur-



pose, the joint upon which they act allowing them to be turned down over the sloping side of the prism-box.

At *e* is shewn a spring, which, being pressed by the finger at the time of observation, and then released, checks the vibrations of the card, and brings it more speedily to rest. A stop is likewise fixed at the other side of the box, by which the needle may be thrown off its centre; which should always be done when the instrument is not in use, as the constant playing of the needle would wear the point upon which it is balanced, and upon the fineness of the point much of the accuracy of the instrument depends. A cover is adapted to the box, and the whole is packed in a leather case, which may be carried in the pocket without inconvenience.

The method of using this instrument is very simple. First raise the prism in its socket, *b*, until you obtain distinct vision of the divisions on the card, and, standing at the place where the angles are to be taken, hold the instrument to the eye, and looking through the slit, *c*, turn round till the thread in the sight-vane bisects one of the objects whose azimuth, or angular distance from any other object, is required; then, by touching the spring, *e*, bring the needle to rest, and the division on the card which coincides with the thread on the vane, will be the azimuth or bearing of the object from the north or south points of the magnetic meridian. Then turn to any other object and repeat the operation; the difference between the bearing of this object and that of the former, will be the angular distance of the objects in question. Suppose the former bearing to be 40° 30′ and the latter 10° 15′, both east, or both west, from the north or south, the angle will be 30° 15′. The divisions are generally numbered 5°, 10°, 15°, &c. round the circle, to 360°. A tripod stand similar to those of the theodolite, described at page 15, can be had with the instrument, if required, on which to place it when observing, instead of holding it in the hand.

THE VERNIER.

This is a contrivance for measuring parts of the space between the equidistant divisions of a graduated scale. It is a scale whose length is equal to a certain number of parts of that to be subdivided, depending on the degree of minuteness to which the subdivision is intended to be carried; but it is divided into parts, which in number are one more or one less than those of the primary scale taken for the length of the vernier: in modern practice the parts on the vernier are generally one more than are contained in the same space on the primary scale.

If it be required to measure to hundredths of an inch the parts of a scale which is graduated to tenths, it may be done by means of a scale whose length is nine-tenths of an inch, and divided into ten equal parts; or by one whose length is eleventenths of an inch, and divided into ten equal parts: for in either case the difference between the divisions of the scale so made and those on the primary scale is the hundredth of an inch. Such a scale, made to move along the edge of that to be subdivided, is called a vernier; and we shall explain how by its application, either to straight lines or arcs of circles, the subdivisions of graduated instruments are read off. For this purpose let us take as a general example the method of reading the sextant, as a person acquainted with the graduations upon this instrument will find no difficulty in becoming familiar with those on any other.

It will be observed* that some of the divisional lines on the limb of the instrument are longer than others, and that they are numbered at every fifth, thus, 0, 5, 10, 15, &c., the 0 being the starting point, or zero. The spaces between these lines represent degrees; and they are again subdivided by shorter lines, each smaller space representing a certain number of minutes. For instance, if the spaces be subdivided into four parts, then there will be three short lines, each of which will indicate the termination of a space of 15 minutes; if there be six parts, there will be five short lines, and each will be at the end of a space of 10 minutes, reckoned from the commencement of the divisions. Likewise it will be observed that some of the divisions on the vernier are longer than others: these indicate in the same manner single minutes, and they are numbered from right to left; the extreme right one is the zero, or commencement of the index divisions, and it is marked 0 or ◊; the shorter divisions show fractions of minutes. If the spaces between each minute (or long division) contain three lines, each space will be 15 seconds, and if five, 10 seconds; the number of subdivisions between the minutes of the vernier is usually, but not necessarily, the same as between the degrees on the limb, so that if the limb be divided into 20′ the vernier is divided into 20″; if the former be divided to 10′ the latter is divided to 10″, &c.

The limb of the instrument now before us is divided to 10′, and the vernier reads to 10″; and, by shewing the manner of reading it off, we shall explain sufficiently the method of reading verniers in general. If the zero division of the vernier coincide (or form a straight line) with any line on the limb, then that line indicates the required angle; thus, if it coincide with the line marked 60, then sixty degrees is the angle; if with the next long division, then 61 degrees will be the angle; but if it coincide with one of the shorter lines between 60 and 61, then the angle will be 60 degrees and a certain number of minutes, according to which of the short lines it coincides with. If it be the first, (of the instrument before us) the angle will be 60° 10′; but if it

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* The reader is supposed to have an instrument before him while perusing these instructions.

coincide with the second, it will be 60° 20′, if with the third, 60° 30′, &c. But when it happens that the zero division of the index does not coincide with any division upon the limb, but stands between two of them, we must observe how many degrees and minutes are denoted by the division it has last passed, and look for a line on the vernier that does coincide with one on the limb; and the number of minutes and seconds from that line to the zero of the index, added to the number read off upon the limb, gives the angle required. Thus, supposing the index to stand between 10′ and 20′ beyond 60°, and the line on the vernier denoting 6′ 10″ (which is the line next beyond the one marked 6) coincides with any one on the limb, then this quantity, added to 60° 10′, gives 60° 16′ 10″, the angle required.

When the arc of excess on the limb of the sextant (the nature of which will be explained hereafter) is required to be read off, observe what quantity is passed to the right of zero by the zero division of the vernier, and find the remaining minutes and seconds to be added to it, by reading the vernier backwards; that is, consider the last numbered division to the left hand as the zero: thus, suppose that (on our instrument) the index stood beyond the third short division on the arc of excess, this would be 30′, and if the third long division from the last numbered one on the left hand (marked 10,) coincided with a line on the limb, this would denote 3′, to be added to the former, making 33′ for the reading on the arc of excess.

On the limbs of small theodolites the spaces between the degrees are generally divided into two parts, consequently the short division represents 30′, and the divisions on the vernier are single minutes; a smaller subdivision must be estimated by the eye, which by a person accustomed to the instrument can be done to 15″.

The subdivision of a straight line, as the scale of a mountain barometer, is likewise effected by a vernier, and is read off in the following manner. The scale is divided into inches, which are subdivided into 10 parts; these tenths are again divided into two, by a shorter division, which will be 5 hundredths of an inch. The long divisions upon the vernier show each of them one hundredth of an inch, and they are numbered at every fifth; these are again subdivided by shorter lines, representing thousandths. Now, to read it off, observe where the zero division of the vernier stands on the scale,—suppose a little above 30 inches and 4 tenths; and as it does not reach the short line denoting 5 hundredths, observe what line on the vernier coincides with one on the scale: if it be a long division, then it is so many hundredths to be added, and if a short division, it will be so many hundredths and thousandths to be added, to make up the measurement, and the readings are written decimally thus, 30·435 inches.

In the subjoined figures, which are given for the purpose of illustration, A B represents a portion of the graduated limb of an instrument, and C D a portion of the vernier scale, the zero point being at C.

Fig. 1.

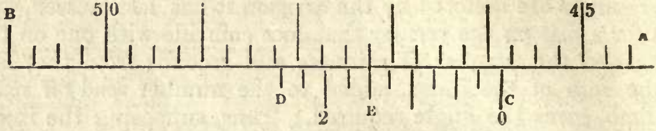


Fig. 2.

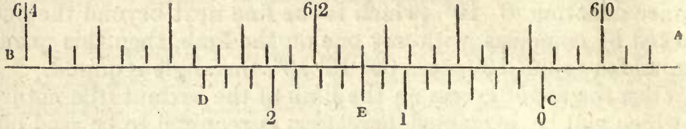


Fig. 3.

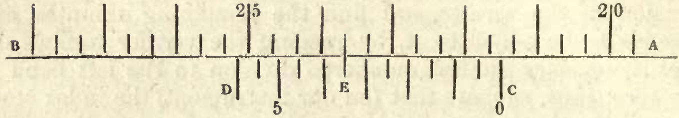
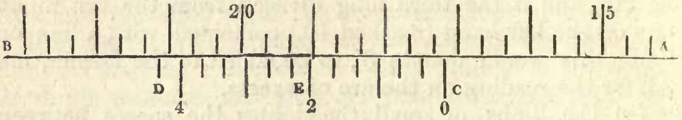


Fig. 4.



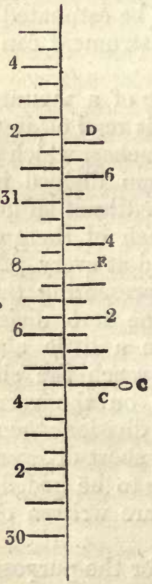
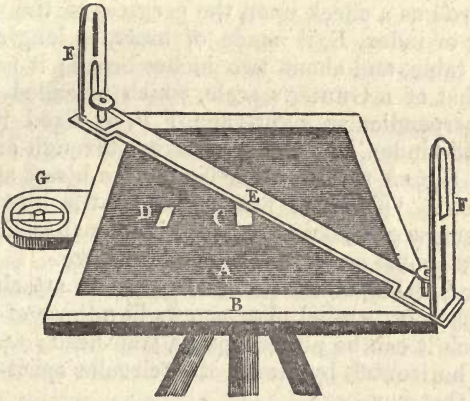


Fig. 5.

In the first figure the limb is divided to 15′, and these divisions are subdivided by the vernier to 15″. In the second figure, the limb is divided to 10′, and subdivided by the vernier to 10″. In the third, the limb is divided to 20′, and subdivided by the vernier to 30″; and in the fourth, the limb is divided to 20′, and subdivided by the vernier to 20″. E, on each figure, is placed where a division on the vernier coincides with one on the limb. In the first, the reading is 45° 46′ 30″; in the second, 60° 21′ 20″; in the third, 21° 23′ 30″; and in the fourth, it is 17° 2′, and between 0′ and 20″, and as the 2′ line is about as much in advance of the one on the limb near to it as the 20″ line is behind the one near to it, the reading may be taken as 17° 2′ 10″. The fifth figure represents the scale of a barometer, reading 30·435 inches, and is drawn much larger than the reality, to render it more intelligible.

THE PLANE TABLE.

Before the theodolite came into general use, the Plane-table was extensively employed in the practice of surveying: it is still sometimes, though seldom, used in surveying small plots of ground, or (where great accuracy is not required) in forming a sketch-map, or laying down the details of a country where the relative situations of the principal conspicuous objects have been previously fixed by triangulation. The expedition with which such work may be performed, by a person who is expert in the use of this instrument, is its chief recommendation.



The construction and size of the plane table has been varied at different times, to suit both the convenience and intentions of the surveyor; but the annexed figure is a representation of that which is now in most general use. It is a board, as A, about sixteen inches square, having its upper edge rabetted, to receive a box-wood frame, B, which being accurately fitted can be placed on the board in any position, with either face upwards. This frame is intended both to stretch and retain the drawing paper upon the board, which it does by being simply pressed down into its place upon the paper, which for this purpose must be cut a little larger than the board.

One face of the frame is divided to 360 degrees, from a centre, C, fixed in the middle of the board, and these are subdivided as minutely as the size of the table will admit. The divisions are frequently numbered each way, to show at sight both an angle and its compliment to 360°. There is sometimes a second centre piece, D, fixed on the table, at about a quarter of its width from one of the sides, and at *exactly* half its length in the other direction. From this centre, and on the other side of the frame, there is graduated 180°: each of these degrees is subdivided to 30 minutes, and numbered, 10, 20, 30, &c., both ways to 180. The object of these graduations is, to make the plane-table supply the place of the theodolite, and an instrument formerly in use called a semicircle. The reverse face of the frame is usually divided into equal parts, as inches and tenths, for the purpose of ruling parallel lines or squares, and for shifting the paper, when the work requires more than one sheet. G is a compass-box, let into one side of the table, with a dove-tail joint, and fastened with a milled-headed screw, that it may be applied or removed at pleasure. The compass, beside rendering the plane-table capable of answering the purpose of a circumferenter, is principally useful in setting the instrument up at a new station parallel to any position that it may have had at a former station, as well as a check upon the progress of the work.

The ruler or index, E, is made of brass, as long as the diagonal of the table, and about two inches broad; it has a sloping edge, like that of a Gunter's scale, which is called the fiducial edge. A perpendicular sight-vane, F F, is fixed to each extremity of the index, and the eye looking through one of them, the vertical thread in the other is made to bisect any required distant object. Upon the flat surface of the index there are frequently engraved scales of various kinds, such as lines of equal parts, with diagonal scales, a line of chords, &c.

To the under side of the table a centre is attached, with a ball and socket, or parallel plate-screws like those of the theodolite, by which it can be placed upon a staff-head; and the table may be set horizontal, by means of a circular spirit-level placed upon it for that purpose.

In preparing the plane-table for use, the first thing to be done is to cover it with drawing paper; the usual method of doing which is the same as that of covering a common drawing board, by damping the under side of the paper, and laying it on the board in an expanded state: press the frame into its place, so that the paper may be squeezed in between the frame and the edge of the table; and the paper, shrinking as it dries, assumes a flat surface for the work to be performed upon. There is one great objection however to this mode of putting on the paper, as when it has once been damped and strained it is easily acted upon by any change in the hygrometrical state of the atmosphere. We therefore prefer putting the paper on dry, taking care to keep it straight and smooth whilst pressing the frame into its place; but it must be acknowledged that this cannot be done so nicely as when it is damped. We have been informed that, if the under side of the paper be covered with the white of an egg well beat up, it may be laid on the board with the greatest nicety, and that when so prepared it is not easily affected by atmospheric changes.

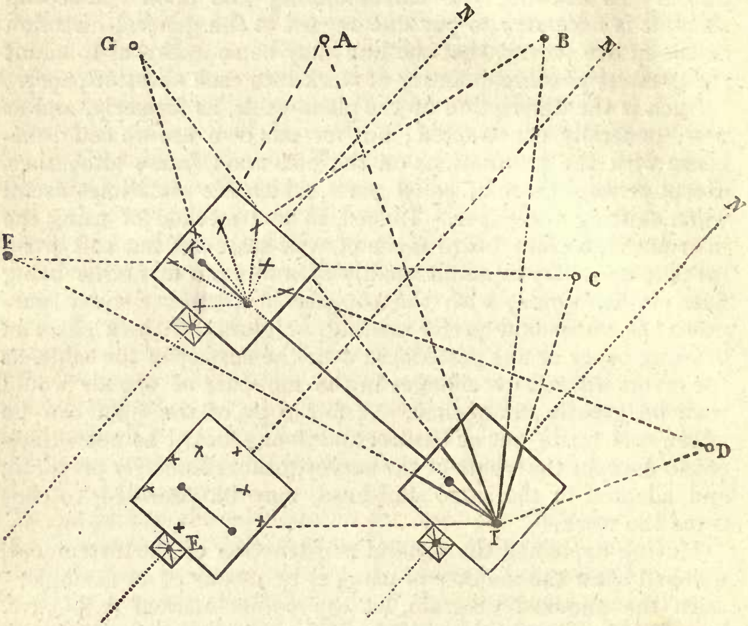
When the survey has been carried to the edge of the paper on the table, and there is occasion to extend the operation further, another sheet must be substituted; but, before removing the old one, a line should be drawn on it, through some particular stations or points of the survey that can be made common to both sheets of paper: then, by drawing a similar line upon the new sheet, and transferring to this line the points or stations that are upon the line in the former sheet, as well as the direction of the last station lines, the survey may be renewed and continued in the same manner, from sheet to sheet, till the whole is completed. In drawing the corresponding line upon the second sheet, it is necessary to pay due regard to the general direction of the future survey, that the line may be so drawn as to admit the greatest possible quantity of work into each sheet of paper.

Such is the description of the plane-table, as formerly, and as now, generally constructed; but for our own use we could dispense with the graduations on the box-wood frame altogether, except *perhaps* those of equal parts, which are sometimes useful when shifting the paper. Indeed, in our method of using the instrument, a plain board made of well-seasoned but soft wood (as pine or cedar) to admit readily of a fine pin or needle being fixed in it, would, with the compass-box, answer every purpose; as we should prefer pasting, or gluing, a thick sheet of drawing paper or fine pasteboard over the surface of the table, as the errors caused by changes in the moisture of the air would then be greatly diminished. A fair copy of the plan can be afterwards made out at leisure, and if one board be not sufficient to contain the whole of the survey, others similarly prepared, and adapted to the same staff-head, may be provided, to continue the work.

Having explained the general construction of the instrument, we shall show the manner of using it by means of an example.

In the annexed diagram, let the points marked A B C, &c. be a few of an extensive series of stations, either fixed or temporary, the relative situations of which are required to be laid down upon the plan. Select two stations, as I and K, (considerably distant from each other,) as the extremities of a base line, from which the greatest number of objects are visible: then, if the scale to which the plan is to be drawn be fixed, the distance, I K, must be accurately measured, and laid off upon the board to the required scale; otherwise a line may be assumed to represent that distance, and at some subsequent part of the work the value of the scale thus assumed must be determined, by measuring a line for that purpose, and comparing the measurement, with its length, as represented on the plan.

Set up the instrument at one extremity of the base, suppose at I, and fix a needle in the table at the point on the paper representing that station, and press the fiducial edge of the index gently against the needle. Turn the table about until the meridian line of the compass-card coincides with the direction of the magnetic needle, and in that position clamp the table firm. Then, always keeping the fiducial edge of the index against the needle, direct the sights to the other station, K, and by the side of the index draw a line upon the paper to represent the base, I K; when, if the scale be fixed, the exact length must be laid off, otherwise the point K may be assumed at pleasure on the line so drawn.



But it is sometimes necessary to draw the base line first, when required, on some particular part of the board, so as to admit of the insertion of a greater portion of the survey. When this is the case, the index must be laid along the line thus drawn, and the table moved till the further end of the base line is seen through both the sights; then fix the table in that position, and observe what reading on the compass-card (or bearing) the needle points to, for the purpose of checking the future operations, and also for setting the table parallel to its first position, wherever it may afterwards be set up. It should be observed that, in placing it over any station, that spot on the table representing such station, and not the centre of the table, should be over the station on the ground: it may be so placed by dropping a plumb-line from the corresponding point on the under side of the table.

Having fixed the instrument and drawn the base line, move the index round the point I, as a centre; direct the sights to the station A, and, keeping it there, draw the line I A along the fiducial edge of the index. Then direct in the same manner to B, and draw the line I B; and so proceed with whatever objects are visible from the station, drawing lines successively in the direction of C D E, &c., taking care that the table remains steady during the operation.

This done, remove the instrument to the station K, and placing the edge of the index along the line I K, turn the table about till the sights are directed to the station I, which if correctly done, the compass-needle will point to the same bearing as it did at the former station (in our example it was set to the meridian). Now remove the needle from I, and fix it in the point K; lay the edge of the index against the needle, and direct the sights in succession to the points A B C, &c., drawing lines from the point K, in their several directions, and the intersection of these lines, with those drawn from the point I, will be their respective situations on the plan.

To check the accuracy of the work, as well as for extending the survey beyond the limits of vision at I and K, the table may be set up at any one or more of the stations thus determined, as at E. The needle being now fixed in the point E on the board, and the edge of the index placed over E and I (or K,) the table may be moved round till the station I is seen through both the sights, and then clamped firm: the compass will now again, (if all be correct) point out its former bearing, and any lines drawn from E, in the direction of A B C, &c. in succession, will pass through the intersection of the former lines, denoting the relative places of those objects on the board; but, should this not be the case with all or any of the lines, it is evident that some error must exist which can be detected only by setting the instrument up and performing similar operations at other stations.

Having a number of objects laid down upon the plan, the situation of any particular spot, as the bend of a road, &c., may at once be determined, by setting the instrument up at the place, and turning the table about till the compass has the same bearing as at any one of the stations. Clamp the table firm, and it will now be parallel to its former position, if no local attraction prevents the magnetic needle from assuming its natural position at the different stations. Fix a needle in the point representing one of the stations, and, resting the edge of the index against it, move the index till the station itself is seen through both the sights, and then draw a line on that part of the paper where the point is likely to fall. Remove the needle to another point or station on the board, and, resting the index against it, direct the sights to the corresponding station on the ground, and draw a line along the edge of the index: the point where this line intersects the last will be the situation on the paper of the place of the observer. But, as a check upon the accuracy of the work, a third or even a fourth line should be drawn in a similar manner in the direction of other fixed points, and they ought also to intersect in the same point.

In this manner the plane-table may be employed for filling in the details of a map: setting it up at the most remarkable spots, and sketching by the eye what is not necessary should be more particularly determined, the paper will gradually become a representation of the country to be surveyed.

THE THEODOLITE.

As an angular instrument, the theodolite has from time to time received such improvements that it may now be considered as the most important one employed in surveying. Instruments of this kind, of the best construction, may to a certain extent be used as altitude and azimuth instruments; and several astronomical operations, such as those required for determining the time, the latitude of place, &c., may be performed by them, and to a degree of accuracy sufficient for most of the purposes that occur in the ordinary practice of a surveyor.

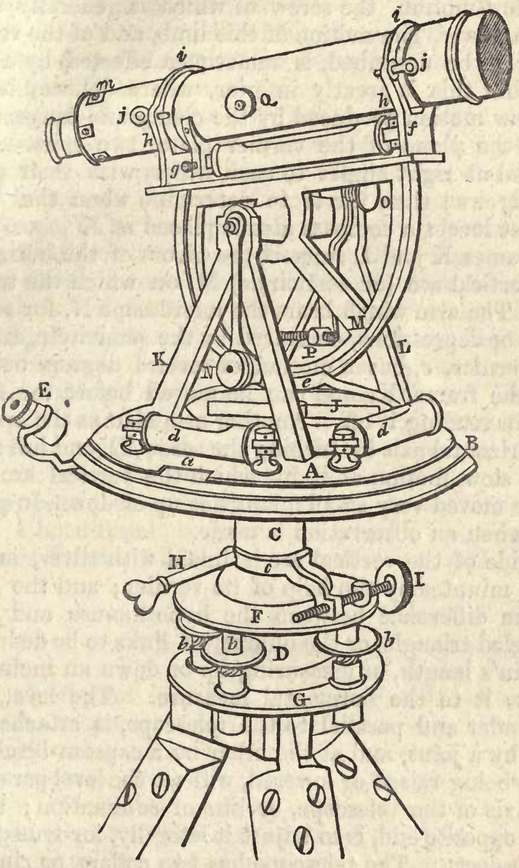
There are various modes of constructing theodolites to suit the convenience or the views of purchasers; but we shall confine ourselves to a description of one of the most perfect, as a person acquainted with the details of its adjustments and use will find no difficulty in comprehending those of others.

Description of the Theodolite.

This instrument (as represented in the next page) consists of two circular plates, A and B, called the horizontal limb, the upper or vernier plate, A, turning freely upon the lower, both having a horizontal motion by means of the vertical axis, C. This axis consists of two parts, external and internal, the former secured to the graduated limb, B, and the latter to the vernier plate, A. Their form is conical, nicely fitted and ground into each other, having an easy and a very steady motion; the external centre also fits into a ball at D, and the parts are held together by a screw at the lower end of the internal axis.

The diameter of the lower plate is greater than that of the upper one, and its edge is chamfered off and covered with silver, to receive the graduations: on opposite parts of the edge of the upper plate, or 180° apart, a short space, *a*, is also chamfered, forming with the edge of the lower plate a continued inclined plane: these spaces are likewise covered with silver, and form the vernier. The lower limb is usually graduated to thirty minutes of a degree, and it is subdivided by the vernier to single minutes, which, being read off by the microscope, E, half or even quarter minutes can easily be estimated.

The parallel plates, F and G, are held together by a ball and socket at D, and are set firm and parallel to each other by four milled-headed screws, three of which, *b b b*, are shown in the figure: these turn in sockets fixed to the lower plate, while their heads press against the under side of the upper plate, and being set in pairs, opposite each other, they act in contrary directions; the instrument by this means is set up level for observation.



Beneath the parallel plates is a female screw adapted to the staff head, which is connected by brass joints to three mahogany legs, so constructed that when shut up they form one round staff, secured in that form for carriage by rings put on them; and, when opened out, they make a very firm stand, be the ground ever so uneven.

The lower horizontal limb can be fixed in any position by tightening the clamping screw, H, which causes the collar, *c*, to embrace the axis, C, and prevent its moving; but, it being requisite that it should be fixed in some precise position more exactly than can be done by the hand alone, the whole instrument, when thus clamped, can be moved any small quantity by means of the slow-motion screw, I, which is attached to the upper parallel plate. In like manner the upper or vernier plate can be fixed to the lower, in any position, by a clamp, (in the plate this clamp is concealed from view,) which is also furnished with a slow-motion, the screw of which is generally called the tangent-screw. The motion of this limb, and of the vertical arc, hereafter to be described, is sometimes effected by a rack and pinion; but this is greatly inferior, where delicacy is required, to the slow motion produced by the clamp and tangent-screw.

Upon the plane of the vernier plate, two spirit-levels, *d d*, are placed at right angles to each other, with their proper adjusting screws; their use is to determine when the horizontal limb is set level: a compass also is placed at J.

The frames K and L support the pivots of the horizontal axis of the vertical arc (or semicircle) M, on which the telescope is placed. The arm which bears the microscope N, for reading the altitudes or depressions, measured by the semicircle, and denoted by the vernier, *e*, has a motion of several degrees between the bars of the frame, K, and can be moved before the face of the vernier for reading it off. Another arm clamps the opposite end of the horizontal axis by turning the screw, O, and has a tangent-screw of slow motion at P, by which the vertical arc and telescope are moved very small quantities up or down, to perfect the contact when an observation is made.

One side of the vertical arc is inlaid with silver, and divided to single minutes by the help of its vernier; and the other side shows the difference between the hypothenuse and base of a right-angled triangle, or the number of links to be deducted from each chain's length, in measuring up or down an inclined plane, to reduce it to the horizontal measure. The level, which is shewn under and parallel to the telescope, is attached to it at one end by a joint, and at the other by a capstan-headed screw, *f*, which, being raised or lowered, will set the level parallel to the optical axis of the telescope, or line of collimation; the screw, *g*, at the opposite end, is to adjust it laterally, for true parallelism in this respect. The telescope has two collars, or rings, of bell metal, ground truly cylindrical, on which it rests in its supports, *h h*, called Y's, from their resemblance to that letter; and it is confined in its place by the clips, *i i*, which may be opened by removing the pins, *j j*, for the purpose of reversing the telescope, or allowing it a circular motion round its axis, during the adjustment.

In the focus of the eye-glass are placed three lines, formed of spider's web, one horizontal, and two crossing it, so as to include a small angle between them,—a method of fixing the wires, which is better than having one perpendicular wire, because an object at a distance can be made to bisect the said small angle with more certainty than it can be bisected by a vertical wire. The screws adjusting the cross wires are shewn at *m:* there are four of these screws, two of which are placed opposite each other, and at right angles to the other two, so that, by easing one and tightening the opposite one of each pair, the intersection of the cross wires may be placed in adjustment.

The object-glass is thrust outwards by turning the milled head, Q, on the side of the telescope, that being the means of adjusting it to show an object distinctly.

A brass plummet and line are packed in the box with the theodolite, to suspend from a hook under its centre, by which it can be placed exactly over the station from whence the observations are to be taken; likewise, if required, two extra eye-pieces for the telescope, to be used for astronomical observations: the one inverts the object, and has a greater magnifying power, but, having fewer glasses, possesses more light; the other is a diagonal eye-piece, which will be found extremely convenient when observing an object that has a considerable altitude,—the observer avoiding the unpleasant and painful position he must assume in order to look through the telescope when either of the other eye-pieces is applied. A small cap, containing a dark coloured glass, is made to apply to the eye-end of the telescope, to screen the eye of the observer from the intensity of the sun's rays, when that is the object under observation. A magnifying glass mounted in a horn frame, a screw-driver, and a pin to turn the capstan-screws for the adjustments, are also furnished with the instrument.

The Adjustments.

The first adjustment is that of the line of collimation,—that is, to make the intersection of the cross wires coincide with the axis of the cylindrical rings on which the telescope turns: it is known to be correct, when an eye looking through the telescope observes their intersection continue on the same point of a distant object during an entire revolution of the telescope. The usual method of making this adjustment is as follows:

First, make the centre of the horizontal wire coincide with some well-defined part of a distant object; then turn the telescope half round in its Y's till the level lies above it, and observe if the same point be again cut by the centre of the wire,—if not, move the wire one half the quantity of deviation, by turning two of the screws at *m*, (releasing one, before tightening the other,) and correct the other half by elevating or depressing the telescope: now, if the coincidence of the wire and object remain perfect in both positions of the telescope, the line of collimation in altitude or depression is correct, but if not, the operation must be repeated carefully, until the adjustment is satisfactory. A similar proceeding will also put the vertical line correct, or, rather, the point of intersection, when there are two oblique lines instead of a vertical one.

The second adjustment is that which puts the level attached to the telescope parallel to the rectified line of collimation. The clips, *i i*, being open, and the vertical arc clamped, bring the air-bubble of the level to the centre of its glass tube, by turning the tangent-screw, P; which done, reverse the telescope in its Y's (that is, turn it end for end), which must be done carefully that it may not disturb the vertical arc, and if the bubble resume its former situation in the middle of the tube, all is right; but if it retire to one end, bring it back one half, by the screw *f*, which elevates or depresses that end of the level, and the other half by the tangent-screw, P: this process must be repeated until the adjustment is perfect; but to make it completely so, the level should be adjusted laterally, that it may remain in the middle of the tube when inclined a little on either side from its usual position immediately under the telescope, which is effected by giving the level such an inclination, and, if necessary, turning the two lateral screws at *g*. If making the latter adjustment should derange the former, the whole operation must be carefully repeated.

The third adjustment is that which makes the azimuthal axis, or axis of the horizontal limb, truly vertical.

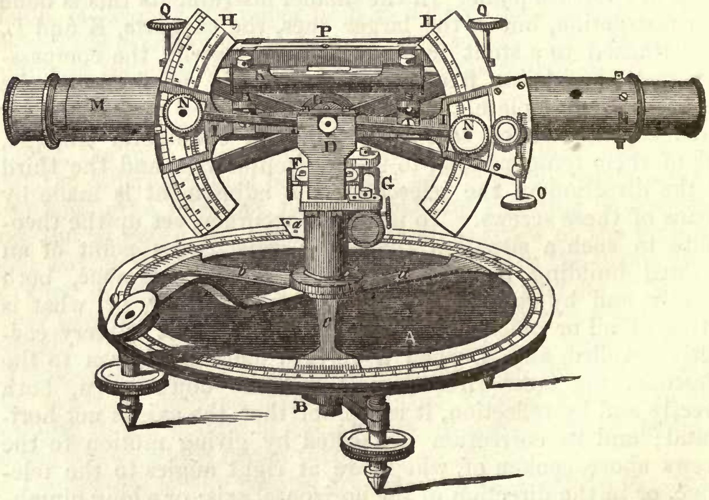
Set the instrument as nearly level as can be done by the eye, fasten the centre of the lower horizontal limb by the staff-head clamp, H, leaving the upper limb at liberty, but move it till the telescope is over two of the parallel plate-screws; then bring the bubble of the level under the telescope to the middle of the tube, by the screw, P: now turn the upper limb half round, that is 180°, from its former position; then, if the bubble return to the middle, the limb is horizontal in that direction, but, if otherwise, half the difference must be corrected by the parallel plate-screws over which the telescope lies, and half by elevating or depressing the telescope, by turning the tangent-screw of the vertical arc; having done which, it only remains to turn the upper limb forward or backward 90°, that the telescope may lie over the other two parallel plate-screws, and by their motion set it horizontal. Having now levelled the limb-plates by means of the telescope level, which is the most sensible upon the instrument, the other air-bubbles fixed upon the vernier plate may be brought to the middle of their tubes by merely giving motion to the screws which fasten them in their places.

The vernier of the vertical arc may now be attended to; it is correct if it point to zero when all the foregoing adjustments are perfect; and any deviation in it is easily rectified by releasing the screws by which it is held, and tightening them again after having made the adjustment,—or, what is perhaps better, note the quantity of deviation as an index error, and apply it, plus or minus, to each vertical angle observed. This deviation is best determined by repeating the observation of an altitude or depression in the reversed positions, both of the telescope and the vernier plate: the two readings will have equal and opposite errors, one half of their difference being the index error. Such a method of observing angles is decidedly the best, since the mean of any equal number of observations taken with the telescope reversed in its Y's must be free from the effects of any error that may exist in the adjustment of the vernier, or zero of altitude.

The theodolite, as constructed in the manner we have described, is not inconveniently heavy, as the diameter of the horizontal limb seldom exceeds five inches; but when the diameter is increased, the other parts must be made proportionably large and strong, and the instrument becomes too weighty and cumbersome to be easily carried from station to station. The object of increasing the dimensions is, to enable the instrument to furnish more accurate results, by applying a telescope of greater power, and by a more minute subdivision of the graduated arcs. With the increase of size, a small variation takes place in the construction, principally consisting in the addition of a second telescope, and in the manner of attaching the supports, K and L (page 15), to the horizontal limb, to afford the means of adjusting the horizontal axis, and, of course, making the telescope and vertical arc move in a vertical plane. In the smaller instruments this is done by construction, but in the larger ones, the supports, K and L, are attached to a stout frame, which also carries the compass-box, instead of being fixed, as represented in our figure, to the upper horizontal plate. The frame is attached to the limb by three capstan-headed screws, forming an equilateral triangle, two of them lying parallel to the horizontal axis, and the third in the direction of the telescope; the adjustment is made by means of these screws. To prove its accuracy, set up the theodolite in such a situation that some conspicuous point of an elevated building may be seen through the telescope, both directly and by reflection, from a basin of water, or, what is better, of oil or quicksilver. Let the instrument be very correctly levelled, and if, when a vertical motion is given to the telescope, the cross-wires do not cut the object seen, both directly and by reflection, it is a proof that the axis is not horizontal; and its correction is effected by giving motion to the screws above spoken of, which are at right angles to the telescope, or in the direction of the horizontal axis: or a long plumb-line may be suspended, and if the cross-wires of the telescope, when it is elevated and depressed, pass exactly along the line, it will be a proof of the horizontality of the axis. The third screw, or that which is under the telescope, serves for adjusting the zero of altitude, or vernier of the vertical arc.

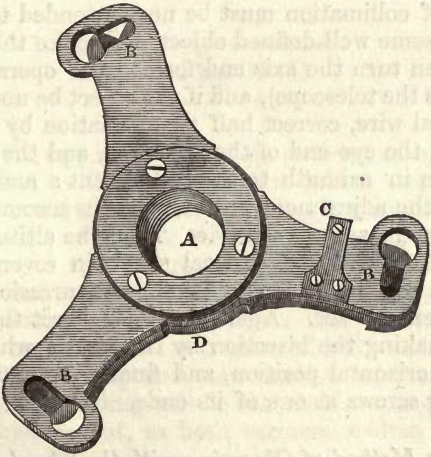
A second telescope is sometimes attached to the instrument beneath the horizontal limb; it admits of being moved, both in a vertical and horizontal plane, and has a tangent-screw attached for slow motion: its use is to detect any accidental derangement that may occur to the instrument whilst observing, which may be done by it in the following manner. After levelling the instrument, bisect some very remote object with the cross-wires of this second telescope, and clamp it firm; if the instrument be steady, the bisection will remain permanent whilst any number of angles are measured, and by examining the bisection from time to time during the operation, at the place where the instrument is set up, any error arising from this cause may be detected and rectified.

At the suggestion of Captain Everest, surveyor-general of India, several small theodolites, differing considerably in construction from that which we have been describing, have lately been made by Messrs. Troughton and Simms, for the great Indian Survey. In principle they are similar to the theodolites of much larger dimensions, and consequently the whole of their essential adjustments are made in the same manner. We shall here give a description and engraving of this kind of instrument, with the particulars of its adjustments, which must be understood as equally applicable to the larger theodolites usually employed in extensive trigonometrical operations.



The horizontal circle (or limb), A, of this instrument consists of one plate only, which, as usual, is graduated at its circumference. The index is formed with four radiating bars, *a, b, c, d,* having verniers at the extremities of three of them, for reading the horizontal angles, and the fourth carries a clamp to fasten the index to the edge of the horizontal limb, and a tangent-screw for slow motion. These are connected with the upper works which carry the telescope, and, turning upon the same centre, show any angle through which the telescope has been moved. The instrument has also the power of repeating the measurement of an angle; for the horizontal limb being firmly fixed to a centre, moveable within the tripod support, B, and governed by a clamp and tangent-screw, C, can be moved with the same delicacy, and secured with as much firmness, as the index above it. Large theodolites, when required, have the power of repeating given them, by means of a particular kind of stand, called a repeating table.

The tripod support, which forms the stand of the instrument, has a foot-screw at each extremity of the *arms* which form the tripod; the heads of the foot-screws are turned downwards, and have a flange (or shoulder) upon them, so that, when they rest upon a triangular plate fixed upon the staff-head, another plate locks over the flange, and, being acted upon by a spring, retains the whole instrument firmly upon the top of the staff, which is similar to that of the theodolite represented at page 15. The great advantage of the tripod stand is, that it can easily be disengaged from the top of the staff, and placed upon a parapet or other support, in situations where the staff cannot be used.



The telescope is mounted in the manner of a transit instrument, that is, the horizontal axis, L, and the telescope, M, form one piece, the axis crossing the telescope about its middle, and terminating at each extremity in a cylindrical pivot. The pivots rest upon low supports, (only one of them, D, being visible in the figure), carried out from the centre, on each side, by a flat horizontal bar, F, to which a spirit-level, G, is attached for adjusting the axis to the horizontal plane. The vertical angles are read off on two arcs of circles, H H, which have the horizontal axis as their centre, and, being attached to the telescope, move with it in a vertical plane. An index, upon the same centre, carries two verniers, I I, and it has a spirit-level, K, attached to it, by which the index can be set in a horizontal position, so that whatever position the telescope, and consequently the graduated arcs, may have, when an observation is made, the mean of the two readings will denote the elevation or depression of the object observed from the horizontal plane.

The following are the adjustments of this instrument. First, to set the instrument level: to accomplish this, bring the spirit-bubble, G, attached to the horizontal bar, in a direction parallel to two of the foot-screws, and by their motion cause the air-bubble to assume a central position in the glass-tube; then turn the telescope, level, &c., half round, and, if the bubble be not central, correct half the deviation by raising or lowering one end of the level itself, and the other half by the foot-screws, which in this instrument perform an office similar to that of the parallel plate-screws of the theodolite already described. Having perfected this part, turn the telescope a quarter round, and the level will be over the third foot-screw, which must be moved to set the level correct; and this part of the adjustment will be complete.

The line of collimation must be next attended to. Direct the telescope to some well-defined object, and make the vertical wire bisect it; then turn the axis end for end (an operation which of course inverts the telescope), and if the object be not now bisected by the vertical wire, correct half the deviation by the collimating screws at the eye-end of the telescope, and the other half by giving motion in azimuth to the instrument: and this must be repeated till the adjustment is satisfactorily accomplished.

Finally, for the zero of altitude. Take the altitude or depression of an object with the vertical sector in reversed positions; half the sum will be its true altitude, or depression, and to this let the verniers be set. Again carefully direct the telescope to the object, making the bisection by the screws which retain the index in a horizontal position, and finally correct the level by the adjusting screws at one of its ends.

The Method of Observing with the Theodolite.

In describing the use of the theodolite, it is not our intention to enter upon an account of the different ways in which it is applied to the purposes of land-surveying, since we do not profess to write a treatise upon that subject; but, in addition to what we here insert, some further particulars will be found in the Appendix, where we purpose explaining the manner of surveying roads, boundaries, &c., in connection with the method of using a circular protractor. Confining ourselves therefore to the manner of measuring angles by its assistance, we observe that, the instrument being placed exactly over the station from whence the angles are to be taken, by means of the plumb-line suspended from its centre, must be set level by the parallel plate-screws, *b b*, &c. (page 15), bringing the telescope over each pair alternately: one must be unscrewed while its opposite one is screwed up, until the two spirit-levels on the vernier plate steadily keep their position in the middle of their tubes, while the instrument is turned quite round upon its staff-head, when it will be ready for commencing operations. (We are now supposing that the adjustments before described have been carefully examined and rectified, otherwise the observations will be good for nothing.) First, clamp the lower horizontal limb firmly in any position, and direct the telescope to one of the objects to be observed, moving it till the cross-wires and object coincide; then clamp the upper limb, and by its tangent-screw make the intersection of the wires nicely bisect the object: now read off the two verniers, the degrees, minutes, and seconds of (either) one, which call A,* and the minutes and seconds only of the other, which call B, and take the mean of the readings thus:—

$$\begin{array}{rrrr} A = & 142^\circ & 36' & 30'' \\ B = & & 37 & 0 \\ \hline \text{Mean} = & 142 & 36 & 45 \\ \hline \end{array}$$

Next release the upper plate, and move it round until the telescope is directed to the second object (whose angular distance from the first is required), and, clamping it, make the cross-wires bisect this object, as was done by the first; again read off the two verniers, and the difference between their mean, and the mean of the first reading, will be the angle required.

Some persons prefer making their first reading = zero, by clamping the upper to the lower plate of 360°, and bisecting the first object by the clamp and slow motion of the lower limb; then their second reading will be the absolute angle subtended by the two objects: but, as both verniers seldom read exactly

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* It would be better to have the letters A, B, &c., engraved over the verniers, making it a rule always to read the degrees from the one called A, which would prevent confusion, and the possibility of a mistake, when observing a number of objects from one station. This is always done (by the makers) upon the verniers of large instruments.

alike,* the mean of them should still be taken, unless one vernier alone is used, which should never be the case; therefore it matters not at what part of the lower, the upper limb is clamped, provided the angle be read off every time an object is bisected, for the difference between any two readings will be the angle subtended by the objects observed.

It would appear, from the above statement, that it is not necessary for the lower horizontal plate to have any motion at all, which is certainly the case when angles are simply to be measured; but its use is important, as it gives us the means of repeating the measure of any angle we may wish to determine with great accuracy, it being evident that a mean of a number of observations will give a more correct result than a single one. To repeat an angle, therefore, after making the second bisection as above directed, leave the upper plate clamped to the lower, and release the clamp of the latter: now move the whole instrument (bodily) round towards the first object, till the cross-wires are in contact with it; then clamp the lower plate firm, and make the bisection with the lower tangent-screw. Leaving it thus, release the upper-plate, and turn the telescope towards the second object, and again bisect it by the clamp and slow motion of the upper plate. This will complete one repetition, and if read off, the difference between this and the first reading will be double the real angle. It is, however, best to repeat an angle four or five times; then the difference between the first and last readings (which are all that it is necessary to note), divided by the number of repetitions, will be the angle required.

The magnetic bearing of an object is taken, by simply reading the angle pointed out by the compass-needle, when the object is bisected; but it may be obtained a little more accurately by moving the upper plate (the lower one being clamped) till the needle reads zero, at the same time reading off the horizontal limb: then, turning the upper plate about, bisect the object and read again; the difference between this reading and the former will be the bearing required.

In taking angles of elevation or depression, it is scarcely necessary to add that, the object must be bisected by the horizontal wire, or rather by the intersection of the wires, and that, after observing the angle with the telescope in its natural position, it should be repeated with the telescope turned half round in its Y's, that is, with the level uppermost; the mean of the two measures will neutralize the effect of any error that may exist in the line of collimation.

The proof of the accuracy of a number of horizontal angles,

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* The reason of their reading differently arises from the errors of eccentricity or of graduation, and perhaps of both: the object of having two readings is to diminish the effect of these errors, which is more effectually done by three verniers; but this being inconvenient in small instruments, two only are applied.

if they quite surround the station from whence they are taken, is to add them altogether, and their sums, if correct, will be 360°. If they be taken at several stations, consider them as the internal angles of a geometrical figure, and the lines connecting the stations as the sides of such figure; then, if the figure have three sides, their sum will = 180°, if four sides, = 360°: if more than four, multiply 90° by double the number of sides, and subtract 360° from the product; the remainder will be the sum of the internal angles.

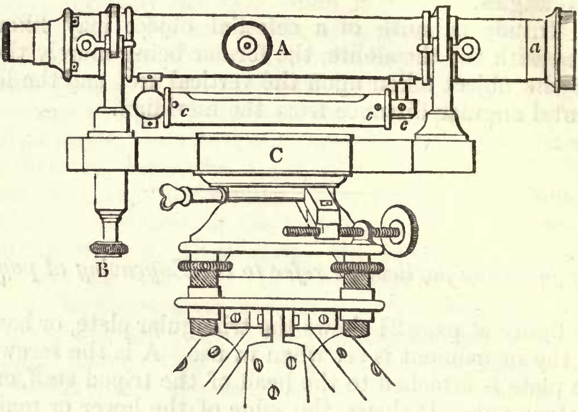
The altitude azimuth of a celestial object may likewise be observed with the theodolite, the former being merely the elevation of the object taken upon the vertical arc, and the latter its horizontal angular distance from the meridian.

The following particulars refer to the Engraving at page 21.

The figure at page 21 shews the triangular plate, or base, upon which the instrument is set when in use. A is the screw whereby the plate is attached to the head of the tripod staff, or legs of the instrument. D shews the edge of the lower or main plate, that is so screwed on to the staff head. B, B, B, is the upper plate, which slides on the surface of the lower one, D: each angle of the upper plate is perforated to admit of the passing of the flanged heads of the foot-screws of the tripod to cells made in the lower plate for their reception; and when the instrument is thus dropped into its place on the stand, it is there secured by sliding the upper plate into the position shewn in the above figure, whereby the narrow part of the perforations are brought over the heads or flanges of the foot-screws, and they are then retained in their places. The two plates are thus kept in the position now described by the catch, C, which acts with a spring, and prevents their having any lateral motion. By the above contrivance the theodolite is readily attached to the stand, or *vice versâ.*

LEVELLING INSTRUMENTS.

THE Y SPIRIT-LEVEL.



The above figure represents this instrument: it has an achromatic telescope, mounted in Y's, like those of the theodolite, and is furnished with a similar system of cross wires for determining the axis of the tube, or line of collimation. By turning the milled-headed screw, A, on the side of the telescope, the internal tube, *a*, will be thrust outwards, which, carrying the object-glass, is by this means adjusted to its focal distance, so as to show a distant object distinctly.

The tube, *c c*, carrying the spirit-bubble, is fixed to the under side of the telescope by a joint at one end and a capstan-headed screw at the other, which sets it parallel to the optical axis of the telescope; at the opposite end is another screw, *e*, to make it parallel in the direction sidewise. One of the Y's is supported in a socket, and can be raised or lowered by the screw, B, to make the telescope perpendicular to the vertical axis. Between the two supports is a compass-box, C, (having a contrivance to throw the magnetic needle off its centre when not in use): it is convenient for taking bearings, and is not necessarily connected with the operations of levelling, but extends the use of the instrument, making it a circumferentor. The whole is mounted on parallel plates and three legs, the same as the theodolite.

It is evident, from the nature of this instrument, that three adjustments are necessary. First, to place the intersection of the wires in the telescope, so that it shall coincide with the axis of the cylindrical rings on which the telescope turns; secondly, to render the level parallel to this axis; and lastly, to set the telescope perpendicular to the vertical axis, that the level may preserve its position while the instrument is turned quite round upon the staves.

To Adjust the Line of Collimation.

The eye-piece being drawn out, to see the wires distinctly, direct the telescope to any distant object, and, by the screw, A, adjust to distinct vision;* bring the intersection of the cross wires to coincide with some well-defined part of the object, then turn the telescope round on its axis as it lies in the Y's, and observe whether the coincidence remains perfect during its revolution: if it does, the adjustment is correct, if not, the wires must be moved one-half the quantity of error, by turning the little screws near the eye-end of the telescope, one of which must be loosened before the opposite one is tightened, which, if correctly done, will perfect this adjustment.

To set the Level parallel to the Line of Collimation.

Move the telescope till it lies in the direction of two of the parallel plate-screws, (the clips which confine the telescope in the Y's being laid open,) and, by giving motion to the screws, bring the air-bubble to the middle of the tube, shewn by the two scratches on the glass. Now reverse the telescope carefully in its Y's, that is, turn it end for end; and should the bubble not return to the centre of the level as before, it shows that it is not parallel to the optical axis, and requires correcting. The end to which the bubble retires must be noticed, and the bubble made to return one-half the distance by the parallel plate-screws, and the other half by the capstan-headed screw at the end of the level, when, if the halves have been correctly estimated, the air-bubble will settle in the middle in both positions of the telescope. This, and the adjustment for the collimation, generally requires repeated trial before they are completed, on account of the difficulty in estimating exactly half the quantity of deviation.

To set the Telescope perpendicular to the Vertical Axis.

Place the telescope over two of the parallel plate-screws, and move them (unscrewing one while screwing up the other) until

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* The eye-piece must first be drawn out, until the cross wires are perfectly well defined, then the object-glass moved till distinct vision is obtained without parallax, which will be the case if, on looking through the telescope at some distant object, and moving the eye sidewise before the eye-glass, the object and the wires remain steadily in contact; but if the wires have any parallax, the object will appear flitting to and from them.

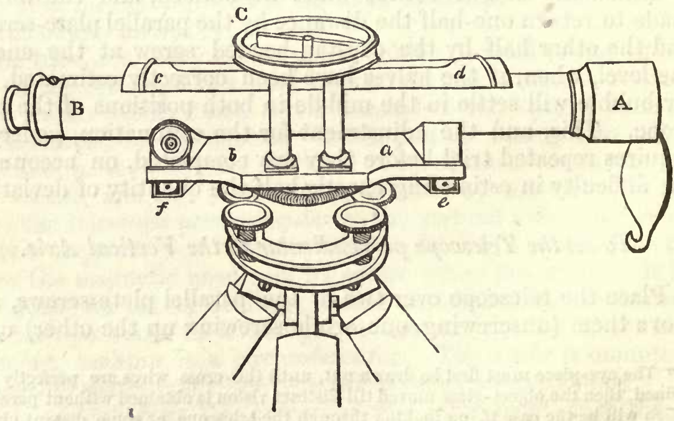
the air-bubble of the level settles in the middle of its tube: then turn the instrument half round upon the vertical axis, so that the contrary ends of the telescope may be over the same two screws, and if the bubble again settles in the middle all is right in that position; if not, half the error must be corrected by turning the screw, B, and the other half by the two parallel plate-screws over which the telescope is placed. Next, turn the telescope a quarter round, that it may lie over the other two screws, and make it level by moving them; and the adjustment will be complete.

Before making observations with this instrument, the adjustments should be carefully examined and rectified, after which the screw, B, should never be touched; the parallel plate-screws alone must be used for setting the instrument level at each station, and this is done by placing the telescope over each pair alternately, and moving them until the air-bubble settles in the middle. This must be repeated till the telescope can be moved quite round upon the staff-head, without any material change taking place in the bubble.

A short tube, adapted to the object-end of the telescope, will occasionally be found useful in protecting the glass from the intensity of the sun's rays, and from damp in wet weather.

TROUGHTON'S IMPROVED LEVEL.

This modification of the instrument has a very decided advantage over the Y level, inasmuch as in its construction it is more compact, and the adjustments when once made are less liable to be deranged; although, to a person unused to the instrument, they will at first appear more tedious to accomplish.



The telescope, A B, rests upon the horizontal bar, *a b*, which turns upon the staff-head (similar to the one employed in the

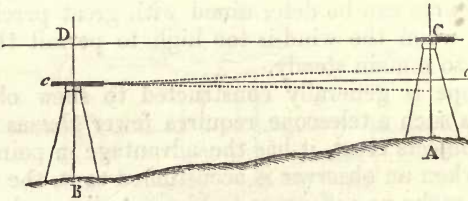
Y level and the theodolite). On the top of the telescope, and partly imbedded within its tube, is the spirit-level, *c d,* over which is supported the compass-box, C, by four small pillars; thus admitting the telescope to be placed so close to the horizontal bar, *a b,* that it is much more firm than in the former instrument. The bubble of the level is sufficiently long for its ends to appear on both sides of the compass-box; and it is shewn to be in the middle by its coinciding with scratches made on the glass tube as usual.

The wire plate (or diaphragm) is generally furnished with three threads, two of them vertical, between which the station-staff may be seen; and the third, by which the observation is made, is placed horizontally. Sometimes a pearl micrometer-scale is fixed perpendicularly on the diaphragm instead of wires. This consists of a fine slip of pearl, with straight edges, one of which is divided into a number of parts, generally hundredths or two-hundredths of an inch; and it is so fixed, that the divided edge intersects the line of collimation, the central division indicating the point upon the staff where the observed level falls. The scale itself may be employed in approximately determining distances, as will be shewn hereafter. It is also very useful in roughly estimating equal distances from the instrument in any direction. Thus, if a man in attendance hold up a staff at any distance, and the observer, looking at it through the telescope, notices how many divisions of the micrometer-scale the staff appears to subtend, then, if the man move in any other direction, retiring until the same staff appears to cover an equal number of divisions, he will be at the same distance from the instrument as before. We have seen a successful application of a delicate wire micrometer to a levelling telescope precisely similar to those applied to astronomical instruments, by means of which distances can be determined with great precision, and will fail only when the wind is too high to permit the instrument or staff to remain steady.

The telescope is generally constructed to shew objects inverted, and as such a telescope requires fewer glasses than one which shews objects erect, it has the advantage in point of brilliancy, and when an observer is accustomed to it, the apparent inversion will make no difference to him. A diagonal eye-piece, however, generally accompanies the instrument, and by it objects can be seen in their natural position. A cap is adapted to the object-end of the telescope, to screen the glass from the rays of the sun, or from the rain: when the cap is used, it should be drawn forwards as much as possible.

The requisite adjustments for this instrument are the same as those of the Y level, *viz.*, that the line of collimation and the level be parallel to each other, and that the telescope be exactly perpendicular to the vertical axis; or, in other words, that the spirit-bubble preserve its position while it is turned round horizontally on the staff-head. The adjustment of the level is effected by correcting half the observed error by the capstan-screws, *e*, *f*, which attach the telescope to the horizontal bar, and the other half by the parallel plate-screws: the capstan-screws, *e*, *f*, have brass covers to defend them from injury or accidental disturbance, but admit their adjustment when necessary.

The spirit-level itself has no adjustment, being firmly fixed in its cell by the maker, and therefore the line of collimation must be adjusted to it, by means of two screws near the eye-end of the telescope: the manner of doing this is as follows. Set up the instrument on some tolerably level spot of ground, and, after levelling the telescope by the parallel plate-screws, direct it to a staff held by an assistant at some distance (from ten to twenty chains); direct him by signals to raise or depress the vane, until its wire coincides with the horizontal wire of the telescope (or central division of the micrometer-scale): now measure the height of the centre of the telescope above the ground, and also note the height of the vane on the staff; let, for example, the former be four feet and the latter six, their difference shews that the ground over which the instrument stood is two feet higher than where the staff is placed. Next, make the instrument and staff change places, and observe in the same manner as before, and if it give the same difference of level the instrument is correct; if otherwise, take half the difference between the results, and elevate or depress the vane that quantity, according as the last observation gives a greater or less difference than the first. Again direct the telescope to the staff, and make the coincidence of the horizontal wire and that on the vane perfect, by turning the collimation-screws.



Suppose the instrument to be set up at A, and the staff at B. C D will be the line of sight. A C, the height of the instrument = 4 feet, B D, the height of vane = 6 feet; their difference = 2 feet. On removing the instrument to B, and the staff to A, *c d* will be the line of sight, giving for the difference of height between B *c* and A *d*=2 feet, as before, if the adjustment be correct; but if it be incorrect, the direction of the line of sight will be either above or below *c d*, as is shewn by the dotted lines. If above it, the difference will be greater than two feet, and the vane must be lowered half that quantity, and the collimation-screws moved to correct the other half; if below the line *c d*, the difference will be less than two feet, and the vane must be raised half that quantity, &c.

Another method of proving the adjustment of the line of collimation is as follows:—Let there be two staves held upright at any convenient distance from each other; call one staff A, and the other B; then place the instrument nearly in a line with the staves, at about one or two chains' length beyond that called A, and, having set it level by the parallel plate-screws, read off both the staves; having done this, remove the instrument to about the same distance from the staff B, set it level, and again read the staves: now, if half the sum of the readings upon the staff A, and also of those upon the staff B, be taken, they will give two points upon the staves that are truly horizontal, by which, or by any other points equidistant therefrom, as may best suit the height at which the instrument is set up, so as to be seen in the field of the telescope, the horizontal wire may be adjusted, that is, moved by its proper screws, so as to coincide with both those points (or readings on the staves).

A third method of adjustment is by means of a sheet of water, and, when practicable, is both convenient and accurate; thus, at the distance of a few chains, drive two stakes close to the water's edge, so that their upper ends may be *even* with the surface of the water: let the level be set up over one of the stakes, and a staff held perpendicular upon the other. Now, having measured the height of the centre of the telescope above the stake over which it is placed, it remains but to move the horizontal wire either up or down, till it points out exactly the same height on the staff, if it does not already do so.

The adjustment of TROUGHTON's level may also be effected by employing as a *collimator* the telescope of a theodolite, or Y level, in the following manner:—First ascertain that the adjustment of the collimating telescope is perfect; then set both instruments up with their telescopes nearly at the same height, and their object glasses opposed to each other, so that, upon placing the eye to either instrument, you may be able to look through both telescopes at once; or, to speak more correctly, you must see the image of the field of the further telescope with its cross-wires distinctly. Both instruments must be carefully levelled, and the telescopes adjusted to about the focus for distinct vision of a remote object: this done, look through the telescope of TROUGHTON's level, and by the rack motion obtain distinct vision of the cross-wires in the collimating instrument; and if the horizontal lines of them both *exactly coincide*, the adjustment is perfect, if not, they must be made to do so by means of the screws that act upon the wire plate. It should be remarked that the level of the instrument employed as the collimator should be at least as sensible as that of the instrument under adjustment, otherwise this method will be very uncertain.

It would be advisable, when the instrument is in perfect adjustment, to fix a level mark on some permanent spot, as a wall, &c., to which the level may be from time to time referred, by simply setting it up at a certain height from the ground, and looking through the telescope at the mark; any error in collimation will be immediately detected, and may be corrected by the collimation-screws only.

The Method of approximately determining Distances by the Micrometer Scale.

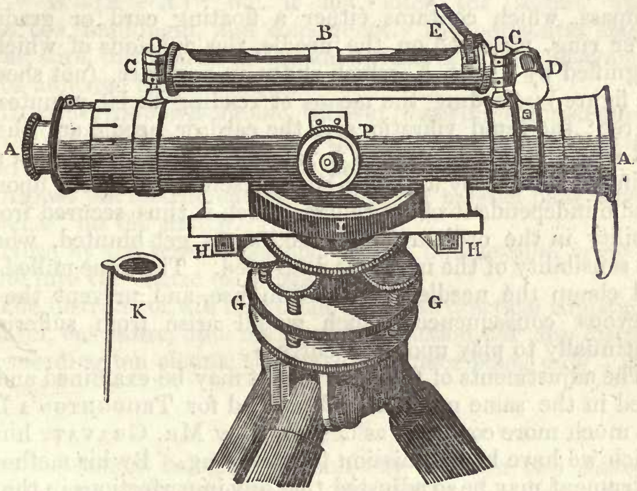
First ascertain the value of the divisions on the scale, and arrange them in a tabular form; to do which, measure off one chain's length from the object end of the telescope, and, having set up a staff there, observe how many divisions and tenths of a division on the scale are occupied by the whole lengh of the staff, or any part of it. Do the same when it is placed at 2, 3, 4, &c., chains, as far as 10, and place the results in a table.

Now, to determine any distance, set up the same staff, or one of equal length, at the distant spot; observe how many divisions and tenths on the scale its whole length subtends, and take from your table the nearest number of divisions and parts which make the first term of an *inverse proportion:* the second term is the number of chains corresponding thereto, the third the observed divisions and parts, and the fourth will be your answer, *viz.*, the distance required.

In making the observations, great care is required in estimating the number of divisions, &c., subtended on the scale by the distant staff, as an error of half a division would occasion a considerable error in the final result.

MR. GRAVATT'S LEVEL.

The following engraving represents Mr. Gravatt's modification of the spirit-level, whereby he obtains advantages both optical and mechanical—the former, by adapting an object-glass of large aperture and short focal length to the telescope, for the purpose of obtaining the light and power of a large instrument without the inconvenience of its length; and the latter, by various contrivances, described as follows:—A A is the telescope, having a diaphragm with cross-wires placed in the usual manner; the internal tube or slide which carries the eye-piece, &c., is nearly equal in length to the external or telescope tube, which, being sprung at its aperture, (as shewn in the cut,) secures to the slide and the eye-piece a steady and parallel motion when adjusting for distinct vision of a distant object by the milled-head, P. The spirit-level is represented at B, placed above the telescope, and attached to two rings passing round it by the capstan-headed screws, C C, which are the means of adjusting the air-bubble of the level for parallelism with the line of collimation. D represents a small level placed across the telescope at right angles to the principal level, C C: it is very convenient in setting the instrument up approximately level by means of the legs only, which saves time, and also the wear of the parallel plate-screws. (Practical men are aware of the uncertainty in judging by the eye alone when the instrument is set nearly level, especially on the side of sloping ground, and duly appreciate the application of the cross-level, by which their valuable time and the wear of their instruments are saved.) Having directed the sight to the staff, and adjusted for distinct vision, the two levels at once shew which of the screws require touching, to perfect the level, before noting the observation.



A mirror, mounted by a hinge-joint on a spring piece of brass, is placed on the telescope, as represented at E; its use is to reflect the image of one end of the air-bubble (in the principal level) to the eye, so that the observer (after having carefully adjusted his level) can, at the same time that he is reading the staff, see that the instrument retains its position, by noticing if the reflected end of the air-bubble coincide with the proper division of the small scale fixed on the bubble-tube:* this is particularly useful in windy weather, or when levelling over soft

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* The small scale spoken of in the text is not applied by the maker unless particularly ordered, but in all cases the requisite marks or divisions are made on the bubble-tube.

or boggy ground, where the least movement of the observer will materially alter the level of the instrument; as by keeping both eyes open, with a little practice, the cross-wires, the bubble, and the staff can be all three seen at the same time, and, by a slight pressure of the hand upon one of the level-legs, any displacement of the bubble may be corrected. The parallel plates and screws, G G, are similar in every respect to those of the former-described instruments. We may remark, that it is convenient to have one of the screws resting in a notch, or Y, fixed on the lower plate exactly over one of the legs; then, by giving motion to that leg only, after the other two are fixed in the ground, the instrument can be set up so nearly level, that a very small motion of the parallel plate-screws will be required to perfect it.

H H represent two capstan-screws, the same as in Troughton's Level, and for a similar purpose, *viz.*, to make the spirit-bubble maintain a central position in its tube, while the instrument is turned completely round on the staff-head. I is the compass, which contains either a floating card or graduated silver ring, mounted on the needle, the divisions of which are magnified by a lens, K, which slides in a socket, (not shewn in the figure,) affording the means of reading to 10 minutes of a degree: the rapid vibrations of the card or needle are checked and speedily brought to rest by a contrivance, in which a spiral spring is moved by a milled-headed screw; this acts upon the needle independent of its centre, which is thus secured from its liability in the ordinary construction to get blunted, whereby the sensibility of the needle is destroyed. The same milled-head will clamp the needle when not in use, and prevent the mischievous consequences which would arise from suffering it continually to play upon its centre.

The adjustments of this instrument may be examined and rectified in the same manner as described for Troughton's Level, but much more correctly as described by Mr. Gravatt himself, which we have his permission for inserting. By his method the instrument may be so adjusted that any imperfections in the slide or tube of the telescope, arising from their not being straight, may not in the least cause the intersection of the cross-wires to deviate from the optical axis of the telescope in its motion, during adjustment for distinct vision.

To examine and correct the Collimation.

"On a tolerably level piece of ground drive in three stakes, at intervals of about four or five chains, calling the first stake *a*,—the second, *b*,—and the third, *c*.

"Place the instrument half-way between the stakes *a* and *b*, and read the staff A, placed on the stake *a*, and also the staff

B, placed on the stake *b;* call the two readings, A' and B': then, although the instrument be out of adjustment, yet the points read off will be equidistant from the earth's centre, and consequently level.

"Now remove the instrument to a point half-way between *b* and *c*. Again read off the staff B, and read also a staff placed on the stake *c*, which call staff C (the one before, called A, being removed into that situation). Now, by adding the difference of the readings on B (with its proper sign) to the reading on C, we get three points, say A', B', and C', equidistant from the earth's centre, or in the same true level.

"Place the instrument at any short distance, say half a chain beyond A, and, using the bubble merely to see that you do not disturb the instrument, read all three staffs, or, to speak more correctly, get a reading from each of the stakes, *a, b, c:* call these three readings, A'' B'' C''. Now, if the stake *b* be half way between *a* and *c*, then ought $C''-C'-(A''-A')$ be equal $2\ [B''-B'-(A''-A')]$; but if not, alter the screws which adjust the diaphragm, and consequently the horizontal spider-line or wire, until such be the case; and then the instrument will be adjusted for collimation.

"To adjust the spirit-bubble, without removing the instrument, read the staff A; say it reads A''': then, adding $(A'''-A')$ with its proper sign to B', we get a value, say B'''.

"Adjust the instrument by means of the parallel plate-screws, to read B''' on the staff B.

"Now, by the screws attached to the bubble-tube, bring the bubble into the centre of its run.

"The instrument will now be in complete practical adjustment for level, curvature, and horizontal refraction, for any distance not exceeding ten chains, the maximum error being only $\frac{1}{1000}$th of a foot."

EXAMPLE.

The instrument being placed half-way between two stakes, *a* and *b* (at one chain from each,)—the staff on *a* or A' read 6·53, and staff on *b* or B' read 3·34; placing the instrument half-way between the stakes *b* and *c*,—(three chains from each) the staff on *b* read 4·01, and the staff on *c* read 5·31.

Hence, taking stake *a* as the datum, we have

Stake.		Above Datum.
a or A'	=	0.00
b or B'	=	3·19
c or C'	=	1·89

The instrument being now placed at *d*, (say five feet from *a*, but the closer the better,) the staff on *a* or A'' read 4·01,—on *b* or B'', 1·03—and on *c* or C'', 3·07. Now, had the instrument been in complete adjustment (under which term curvature and refraction are included), when the reading on staff *a* was 4·01, the readings on *b* and *c* should have been respectively 0·82 and 2·12.

The instrument therefore points upwards, the error at *b* being 0·21, and the error at *c*, 0·95 : now, were the bubble only in error, (as is supposed in all other methods of adjustment,) the error at *c* ought to be four times as great as at *b*,—but 4 × 0·21 = 0·84 only; there is an error, therefore, of 0·95 − 0·84 = 0·11 not due to the bubble.

For the purpose of correcting this error, (and be it remembered, contrary to former practice, for this purpose only,) we must use the capstan-headed screws at the eye-end of the telescope, and, neglecting the actual error of level, we are only to make the error at *b* one-fourth that of *c*.

After a few trials, whilst the reading at *a* continued 4·01, the reading on *b* became 0·75, and that on *c*, 1·84.

Now 0·82−0·75=0·07, and 2·12−1·84=0·28.

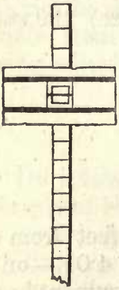
And as 4×0·07=0·28, the telescope is now adjusted for collimation.

All that remains to be done is to raise the object-end of the telescope by means of the parallel plate-screws, until the staff at *c* reads 2·12, and then, by means of the nuts which adjust the bubble-tube, to bring the bubble into the centre of its run.

The operation of collimating, when once performed upon levels on Mr. Gravatt's construction, will scarcely ever need being repeated.

OF THE LEVELLING STAVES.

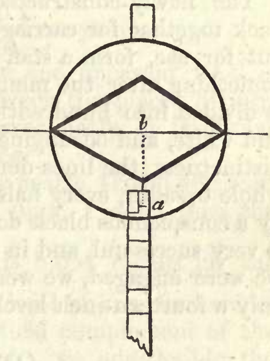
Two mahogany station-staves generally accompany the spirit-level; they consist of two parts, capable of being drawn out when considerable length is required. They are divided into feet and hundredths, or feet, inches, and tenths, and have a sliding vane, with a wire placed across a square hole in the centre, as shewn in the annexed figure: this vane being raised or lowered by the assistant, until the cross-wire corresponds with the horizontal wire of the telescope, the height of the wire in the vane, noted on the staff, is the height of the apparent level above the ground at that place.



When both the staves are used, they should be set up at equal distances on each side of the spirit-level: the difference of the heights of their vanes will be the absolute difference of level between the two stations. But when one staff only is employed, the difference between the height of the vane and the height of the centre of the telescope of the instrument will be the apparent difference of level, which, if the distance between the staff and instrument be great, requires to be corrected for the curvature of the earth. The method of computing this correction will be presently shewn.

TROUGHTON'S LEVELLING STAVES.

These consist of three sliding rods of mahogany, each about four feet long, and they are divided into feet, &c., as those which have just been described. The sliding vane is circular, having at the lower edge a square aperture, one side of which is bevelled; and a line on the bevelled side denotes the reading of the staff. The face of the vane is made of white holly, with an inlaid lozenge of ebony, forming at once a conspicuous object, and one easy of bisection. A circular spirit-level is attached to the top of the hindermost rod, to guide the assistant in holding it perpendicular.



In levelling, the vane must be moved up or down, until the horizontal wire of the telescope bisects the acute angles of the lozenge, or in other words passes through its horizontal extremities, as shewn in the figure.

The line on the bevelled edge, at *a* (as before stated), denotes the reading of the staff; therefore, a piece equal in length to the distance, *a b*, is cut off from the bottom of the staff, or rather the divisions commence at that number of inches above 0.

When the observation requires that the vane be raised to a greater height than four feet, the object is effected by leaving it at the summit of the rod in front, and then sliding this rod up upon the one which is immediately behind it: this will carry the vane up to eight feet; and from that to twelve may be obtained by similarly sliding the second upon the third rod. In the latter steps, the reading is at the side of the staff, the index division remaining stationary, and at four feet from the ground, a circumstance which affords greater facility in reading off.

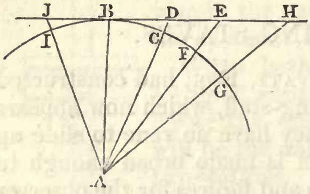
THE NEW LEVELLING-STAVES.

Several years ago, William Gravatt, Esq., had constructed for his own use a new kind of levelling-staff, which now appears likely to come into general use. They have no vane to slide up and down, but the face of each staff is made broad enough to contain sufficiently large graduations and figures for the observer to read with certainty to the one-hundredth part of a foot, at the distance of twelve chains or more, which is sufficient for most practical purposes, thus securing greater certainty and expedition in the work: for it not unfrequently happened, in using the old staves, that when, by a succession of signals, the staff-holder had nearly brought the wire of the vane to coincide with that of the telescope, he would, in his attempt to perfect it, remove the vane further from coincidence than at first; and we have been informed that, on one occasion, the man held the staff upside down, which introduced an error of several feet. To obviate these difficulties, Mr. Gravatt proposed that the observer should read the staff himself, which is now successfully practised.

The newly-constructed staff consists of three parts, which pack together for carriage in a neat manner, and, when opened out for use, form a staff seventeen feet long, jointed together, something after the manner of a fishing-rod: the whole length is divided into hundredths of a foot, alternately coloured black and white, and occupying half the breadth of the staff; but for distinctness, the lines denoting tenths of feet are continued the whole breadth, every half foot or five tenths being distinguished by a conspicuous black dot on each side. The whole contrivance is very successful, and in some late levelling operations in which we were engaged, we were able perfectly to read the staff, with only a fourteen-inch level, at the distance of twelve chains.

ON LEVELLING.

"Levelling is the art of finding a line parallel to the horizon at one or more stations, to determine the height or depth of one place with respect to another. Two or more places are on a true level, when they are equally distant from the centre of the earth. Also, one place is higher than another, or above the level of it, when it is further from the centre of the earth; and a line equally distant from that centre in all its parts is called a line of true level. Hence, because the earth is round, that line must be a curve, and make a part of the earth's circumference, or at least be parallel to it, as the line I B C F G, which has all its points equally distant from A, the centre of the earth—considering it as a perfect sphere.



"But the line of sight, B, D, E, &c., given by the operation of levels, called the apparent line of level, is a tangent, or a right line perpendicular to the semi-diameter of the earth at the point of contact, B, rising always higher above the true line of level the further the distance is. Thus, C D is the height of the apparent level above the true level, a the distance B C or B D; also F E is the excess of height a F; G H, that at G, &c.

The difference, it is evident, is always equal to the excess of the secant of the arc of distance above the radius of the earth.

"Now the difference C D, between the true and apparent level at any distance B C or B D, may be found thus: by a well-known property of the circle, 2 A C + C D : B D : : B D : C D. But, because the diameter of the earth is so great with respect to the line C D, at all distances to which an operation of levelling commonly extends, 2 A C may be taken for 2 A C + C D in this proportion without sensible error. The proportion then will be 2 A C : B D : : B D : C D;

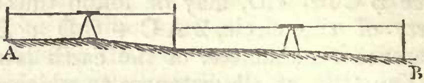
$$\text{whence D C is } = \frac{\text{B D}^2}{2\text{ A C}} \text{ or } \frac{\text{B C}^2}{2\text{ A C}} \text{ nearly:}$$

that is, the difference between the true and apparent level is equal to the square of the distance between the places divided by the diameter of the earth; and, consequently, it is always proportional to the square of the distance."

Now, the diameter of the earth being nearly 41,796,480 feet, or 7916 miles,—if we first take B C equal 1 mile, then the excess $\frac{\text{B C}^2}{2\text{ A C}}$ is $\frac{1}{7916}$ of a mile, which is 8,004 inches for the height of the apparent above the true level at the distance of one mile. Otherwise, if to the arithmetical complement of the logarithm of the diameter, or 2,3788603, we add double the logarithm of the distance in feet, we shall obtain the logarithm of the difference of the true and apparent level in decimals of the same, to be subtracted from the height given by the instrument to reduce it to the true level. In this manner the corrections have been computed, contained in Table I., which shews the difference in decimals of a foot between the true and apparent level, corresponding to any distance from 20 to 5000 feet.

The usual method of obtaining the difference of level between any two places is by a tangent, whose point of contact is exactly in the middle of the level line: this method may be practised without regarding the difference between the apparent and true level; for it is clear that, if from the same station two points of sight be observed equally distant from the eye of the observer, they will be also equidistant from the centre of the earth. Thus, let the instrument be placed at B, (see the last figure,) equally distant from the station staves at C and I, the two points of sight, D and J, marked upon them by the tangent, J D (or J H,) will be level points, and the difference in height between C D and I J will shew how much the one place is higher than the other.

Suppose it were required to determine the difference of level between the two places A and B. First set up your instrument at any convenient distance from A in a line towards B, then, having a staff set up perpendicular at A, measure the distance, and erect another staff beyond you, in the same line and at the same distance as the first, that the instrument may be equally distant from each staff; then direct the telescope towards the first staff,



and sign to a person holding it to move the vane higher or lower, until the wire placed across it coincides with the intersection of the cross-wires in the telescope: he is then to note the height marked by the wire on the staff, which suppose to be 4·69 feet. Now turn the telescope about, and point it towards the second staff, and direct that its vane be raised or lowered, as the former was, until the cross-wire is intersected by the wires of the telescope: it must then be likewise read off; and suppose the reading to be 9·93 feet.

Having completed the first level, let the first staff take the place of the second, and the second to be set up further on in the required direction, as at B. Then, midway between them, set up your level, and direct it to the first staff, and then to the second, making the necessary observations, as before, when, the staves being read off, the operation is completed. Let us suppose the first to read 0·64 feet, and the second, 11·88 feet; the work will then stand thus:—

Reading of the first staff (or back station)	*Feet.*	Reading of the second staff (or forward station)	*Feet.*
First reading . . .	4,69	First reading . . .	9,93
Second reading . .	0,64	Second reading . .	11,88
Sum . . .	5,33	Sum . . .	21,81

The difference of these sums shews that the ground at B is 16·48 feet, or 16 feet 5·76 inches lower than at A.

By continuing the above process, the operation of levelling may be carried on for many miles, the relative height of every station being determined. Also, if the height from the ground of the centre of the levelling telescope be taken at each place at which it is set up, the relative height of that spot will also be determined.

In common levelling operations it is not usual to be particular about placing the instrument in the centre between the staves, but to observe each way, as far as the inclination of the ground will admit. It is also found most eligible to employ staves, such as those described at page 37, in preference to those with sliding vanes, as the observer himself can note and register the various readings, the assistant having nothing more to do than to hold the staff upright. This is a far preferable mode of procedure, for it not unfrequently happens that sufficiently intelligent persons cannot be procured in obscure country places to hold and read off the staff. Besides the greater facility and less liability to error, the observer, with scarcely any loss of time, can make his calculations as he proceeds, so that, following each level, the book contains the absolute differcnce of level of any station above or below a horizontal line drawn through a point assumed as the standard level or point of comparison. This saves a deal of after-trouble in the office, as every requisite is prepared in the field for laying down the section.

The following example shews the form of a levelling book, in which the first staff is called the back station, and the second staff the forward station; when the first forward station will become the second back station, the second forward, the third back, &c.

No. of Station.	Back Station.			Forward Station.			Reduced Levels.
	Bearing.	Distance.	Staff.	Staff.	Distance.	Bearing.	
	o		Ft.	Ft.		o	
1	300·20	140	2·15	14·97	358	120·10	100·00 14·97 85·03 2·15
2	300·40	89	0·50	15·14	420	120·12	87·18 15·14 72·04 0·50
3	300·15	106	0·54	14·12	275	120·0	72·54 14·12 58·42 0·54
4	300·10	109	0·83	15·31	337	120·0	58·96 15·31 43·65 0·83
5	300·0	128	1·49	12·15	609	120·0	44·48 12·15 32·33 1·49
6	300·0	592	5·96	10·50	Bottom	of River.	33·82 10·50 23·32 5·96
			10·50	3·78	215	120·0	29·28 3·78 25·50 10·50
7	300·30	221	8·84	0·90	128	119·40	36·00 0·90 35·10 8·84
							43·94

The contents of each column may be known by the various headings. At the commencement of the operation, the first back station is assumed to be 100 feet above the horizontal line, which is done to avoid the introduction of plus and minus signs in the calculation: this being placed at the top of the column to contain the reduced levels, the reading of the forward station must be subtracted from it, and to the remainder must be added the reading of the back station, which completes the first level,—the result being the height of the forward station above the assumed horizontal line; thus, in the first level, from 100 subtract 14·97, and it leaves 85·03, to which add 2·15, and the result is 87·18, which is the height of the second station, assuming the first to have been 100 feet above the horizontal line. The operation for each of the succeeding levels is precisely similar. The difference of level between any two points in the section may be obtained by simply taking the difference of the heights: thus, to obtain the difference between the first and fourth level, from 87·18 subtract 44·48=42·70, the difference required; and the difference between 100 feet (the assumed height of the starting point) and any other level in the book will be the difference of altitude between those points. Thus, in our example, to find how much the bed of the river is below the point of commencement, subtract 29·28 from 100, and the remainder, 70·72, is the quantity required.

As a check upon the accuracy of the computation, it is necessary to add up the contents of the two columns on each page containing the back and fore observations; and, subtracting the less from the greater, the difference shews the whole amount of the rise or fall of the ground (as in the example given at page 40); and if both computations have been correctly made, it will be identical with the difference as shewn in the column of reduced levels: thus, in our example, page 41, the sum of the back sights is 30·81, and the sum of the fore sights is 86·87; their difference is 56·06, which, taken from 100, (because the first station was assumed to be that height) leaves 43·94,—the same as given by the reduced levels.

There is also another mode of reducing levels, by having two columns, one headed "rise," and the other "fall," in one of which the difference between the back and fore sight must be entered, according as the ground is rising or falling; and then, by the continual adding or subtracting of these quantities, (in a separate column,) the reduced levels are obtained, without assuming the first station to be 100, or any other number of feet high.

In the practice of levelling it is usual to leave, at convenient intervals, what are called bench-marks: these mostly consist of permanent objects, such as gate-posts, stumps of trees, &c., on which it is usual to cut a distinguishing mark, that it may be known hereafter. Their use is chiefly for future reference, in the event of its being necessary either to check the levels by repetition, to change the direction of the line of levels from any point, or to take up and continue the levels at the commencement of a day's work—a bench mark having been left at the close of the day preceding: in the latter case it is more common to leave a peg driven into the ground to renew the work at. When the staff is placed on a bench-mark, the bed of a river, or on any object out of the direct line of levels, the same method of entry and computation must be adopted as shewn in our example, where the bed of the river was taken, no bearing or distance was noticed, but the name of the object entered instead. The computation is precisely the same as before; but when the next forward station was about to be observed, the reading taken to the bed of the river, *viz.*, 10·50, was entered in the column as a back observation, against which the forward reading, with its bearing and distance, was placed: but as both observations were taken from the same spot, they are considered as belonging to the sixth station, as also would any number of intermediate levels; the seventh station being that which has the next back observation taken in the actual line of levelling, this distinction is sufficiently conspicuous to prevent the draftsman plotting the wrong levels in the section, as common bench-marks are not usually noticed in it. In making a section, it is of importance to take the level of all considerable hollows in the ground, the bed of any river, as near the centre as can be obtained, and also of public and private roads.

It is not common to apply the correction for the curvature of the earth, except where extreme accuracy is required; but, by way of illustrating the use of Table I., we have annexed the following example.

No. of Back Station.	Back Station.	Dist. of Instrument from Station.	Correct for Curvat.	Height of Instrument.	Forward Station.	Dist. of Instrument from Station.	Correct for Curvat.	Remarks.
	Ft. In Dec.	Feet.	In. Dec.	Ft. In.	Ft. In. D.	Feet.	In. Dec.	
1	3 1,7	1200	0,413	4 4	11 2,6	800	0,184	
2	6 1,6	480	0,066	4 6	8 1,7	960	0,264	
3	1 7,3	1479	0,629	4 3	6 2,4	1220	0,427	
4	2 4,8	984	0,276	4 7	10 8,3	2160	1,339	
5	4 8,3	764	0,166	4 0	9 3,8	1190	0,406	
6	0 10,2	280	0,022	4 1	11 7,3	340	0,033	
7	7 8,7	1640	0,772	4 5	8 2,1	3100	2,759	
8	2 5,4	660	0,125	4 4	4 3,4	1700	0,829	
Sum	29 0,0	7487	2,469		69 7,6	11470	6,241	
Cor.	2,47				6,24			
	28 9,53				69 1,36			

	Ft.	In.
Sum of forward stations, corrected for curvature . . . . =	69	1,36
Sum of back stations . . . . =	28	9,53
Difference of level between extreme stations =	40	3,83

	Feet.
Sum of distances from the instrument to the back stations	7487
Sum of distance to forward stations	11470
Whole distance levelled . . . . =	18957

In this example, the difference of level being taken in feet and inches, the corrections from Table I. have been multiplied by 12, in order to render them decimals of inches, they being, as contained in the Table, decimals of feet; consequently, if the levels had been taken in feet and decimals of feet, the corrections would have been applied at once, as taken from the Table.

LEVELLING WITH THE THEODOLITE.

The use of the theodolite as a levelling instrument consists in taking a series of angles of elevation and depression along the line, the section of which is required. This must be done at every point where the inclination of the line changes, and the distance measured between the instrument and the station-staff. This distance, it will be evident, is the hypothenuse of a right-angled triangle; the perpendicular of which is the difference of level. To insure accuracy, the angles should be observed both forwards and backwards, by making the instrument and staff change places; and a mean of the two measures should be taken as the correct angle. The instrument should be set up (as nearly as possible) at a constant height from the ground, and the staff used for the observations should have a fixed vane, or conspicuous mark on it, at exactly the same height from the ground as the centre of the telescope, which mark must be bisected by the cross-wires in observing. Great care should be taken that the adjustments of the instrument are correct, more particularly that of the line of collimation, and the level attached to the telescope.

With the measured distance, and the observed angle, the difference of level may be computed, by adding to the logarithm of the measured distance the log. sine of the vertical angle; and their sum, rejecting 10 from the index, will be the log. of the difference of level (in feet or links, as the distance was measured in.*) Having a series of elevations and depressions, the final

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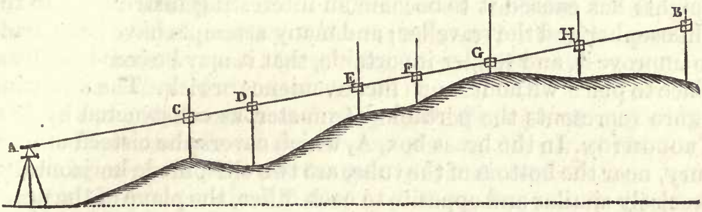
* When the distance has been measured in links of Gunter's chain, and the difference of level is required in feet, it may be obtained by adding to the above logarithm the constant log. 9·8195439, when the sum, rejecting 10 from the index, will be the log. of the difference of level, in feet.

difference of level between the extreme or any two stations, may be found by simply taking the difference of the sums of the intervening elevations and depressions.

A theodolite of larger dimensions than those we have described at page 15, &c., and capable of measuring vertical angles with great accuracy, may be advantageously applied to take the levels of a continually rising line of section: thus, suppose the theodolite set up at A, and the telescope elevated so that the line of sight, A B, may coincide with the vane on a staff exactly at the same height from the ground as the instrument; suppose the staff placed at B, the angle of elevation being carefully noted, the instrument must remain perfectly steady whilst the observer is watching an assistant passing along the line with a staff, which he successively holds up at every change of inclination, as at C D E, &c., the staff-man raising or lowering the vane until the observer perceives the cross-wires of the telescope (or line of sight, A B,) to coincide with that on the vane; the height on the staff is then read off, and noted, which gives the depression of that spot of ground below the line A B, which being done along the whole distance, and a mark made on the ground at each spot, that the distances may likewise be measured, unless determined by a micrometer, as explained at page 29, the undulations of the surface below the line A B is determined; and the inclination of the line of sight being likewise obtained with the theodolite, the requisite data for drawing the section is obtained. This method has been successfully practised by John Macneill, Esq., the engineer of the London and Holyhead roads, with an instrument purposely constructed by Messrs. Troughton and Simms, which is not exactly a theodolite, but rather a large spirit-level, capable of measuring vertical angles with great precision, and has a delicate wire micrometer attached to the eye-end of the telescope, by which the distances of the staff from the instrument are accurately determined.

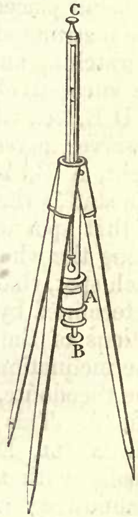


After having obtained the difference of level from station to station, either with a spirit-level or theodolite, the rates of inclination of the surface may be found by dividing the distance by the difference of height; thus, if the distance be 760 feet and the height 38 feet, 760 divided by 38 gives 20,—shewing the rate ofin clination to be 1 in 20.

The rate of inclination may likewise be found by observing the angle of elevation or depression; and Table X., at the end of the volume, shews the corresponding inclination.

LEVELLING WITH THE MOUNTAIN BAROMETER.

The employment of the barometer for the determination of heights has caused it to become an interesting instrument to the philosopher and the traveller; and many attempts have been made to improve it, and render it portable, that it may be conveyed from place to place without much inconvenience or risk. The following figure represents the portable barometer as constructed by Mr. Troughton. In the brass box, A, which covers the cistern of mercury, near the bottom of the tube, are two slits, made horizontally, precisely similar and opposite to each other, the plane of the upper edges of which represent the beginning of the scale of inches, or zero of the barometer. The screw, B, at the bottom, performs a double office; first, it is the means of adjusting the surface of the mercury in the glass cistern to zero, by just shutting out the light from passing between it and the upper edges of the above-named slits; and, secondly, by screwing it up, it forces the quicksilver upwards, and, by filling every part of the tube, renders the instrument portable.



The divided scale on the upper part, is subdivided, by the help of a vernier, to the five-hundredth part of an inch. The screw, C, at the top, moves a sliding-piece on which the vernier scale is divided, the zero of which is at the lower end of the piece. In taking the height of the mercury, this sliding-piece is brought down and set nearly by the hand, and the contact of the zero of the vernier with the top of the mercurial column is then perfected by the screw, C, which moves the vernier the small quantity that may be required just to exclude the light from passing between the lower edges of the sliding-piece and the spherical surface of the mercury.

The barometer is attached to the stand by a ring, in which it turns round with a smooth and steady motion, for the purpose of placing it in the best light for reading off, &c.; and the tripod stand, when closed, forms a safe and convenient packing-case for the instrument.

A thermometer is always attached to the lower part of the barometer, to indicate its temperature; while another, detached from the instrument, is employed at the same time, to shew the temperature of the surrounding air.

The barometrical method of determining differences of level is founded upon the principle that the strata of air decrease in density in a geometrical proportion, when the elevations above the surface of the earth increase in an arithmetical one. Therefore, from the known relation between the densities and the elevations, we can discover the elevations by observations made on the densities by means of the barometer.

Observe at the same time the height of the mercurial columns at both the stations whose difference of elevation is required, and also the temperature of the instrument by the thermometer attached thereto; and that of the surrounding air by another, called the detached thermometer.*

The computations for deducing the difference of height from these observations is rendered very easy by means of Table II., which is computed by the formula given by Mr. Baily, in his volume of Astronomical Tables and Formulæ, and is similar to Table XXXVI. in the same volume, but more extended.

The following is the method of using the Table.

Find in the column headed "S" the sum of the degrees read on the detached thermometers at the two stations, and take out the corresponding number from the adjoining column, headed "A"; next, in the column D, find the difference of the degrees read on the attached thermometers, and take out the opposite number in the column B; lastly, from the column C, take out the number opposite the latitude of the place of observation found in the column L; then,

When the upper thermometer reads less than the lower one,

To the number called B, add the log. of the height of the barometer at the upper station, and subtract their sum from the log. of the height of the barometer at the lower station, and call the remainder R; then take out the log. of R, and add it to the numbers A and C, and the sum, rejecting the tens from the index, will be the log. of the difference of the altitudes of the two stations, in feet.

When the upper thermometer reads more than the lower one,

To the log. of the height of the barometer at the lower station add the number called B, and from their sum subtract the log. of the height of the barometer at the upper station, and call the remainder R; then take out the log of R, and add it to the numbers A and C, and the sum, rejecting the tens from the index, will be the log. of the difference of the altitudes of the two stations, in feet.

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* The mean result of several observations should be taken as that to be used for computation.

EXAMPLE.

The following observations were made in the transit-room of the Royal Observatory, and at the base of the statue of George II. in Greenwich Hospital, latitude 51° 28′, to determine the difference of altitude.

	Upper Station.	Lower Station.
Detached thermometer . . .	71° 5	71° 5
Attached ditto . . .	70 0	70 0
Barometer, mean of 5 obs. .	29 870 inches	30 014 in.

A = 4·81719
B = ... 0·00000
C = 9·99976
log. of bar., upper station, 1·47524
1·47524
log. of bar., lower station, 1·47732
7·31806 log. . . . . R = 0·00208

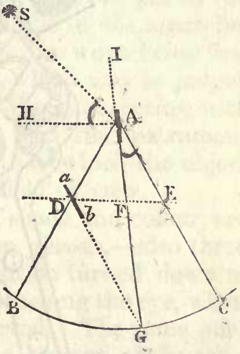
Sum 2·13501 log. of 136·46 feet, the diff. of altitude.

The difference of altitude, as obtained by levelling with the spirit-level, (Phil. Trans., 1831, Part I.) = 135·57 feet, differing only 0·89 feet from that obtained above. The observations should be made simultaneously at both stations, but, to do this, two observers and two barometers are required. When there is only one observer, he should, after making his first observations, lose no time in hastening to his second station, to make his observations there; which, if done quickly, and the atmosphere is undergoing no change at the time, will answer nearly as well as if simultaneous observations were made by a barometer at each station.

ASTRONOMICAL INSTRUMENTS.

THE SEXTANT.

It was our intention, before describing the Sextant, to devote some space to an account of Hadley's Quadrant; but, as the construction of both instruments is essentially the same, we shall confine ourselves to a description of the sextant and its uses only, as comprehending the other instrument, and performing with greater correctness all the operations to which the quadrant can be applied. The principle of its construction may be understood from the following demonstration.

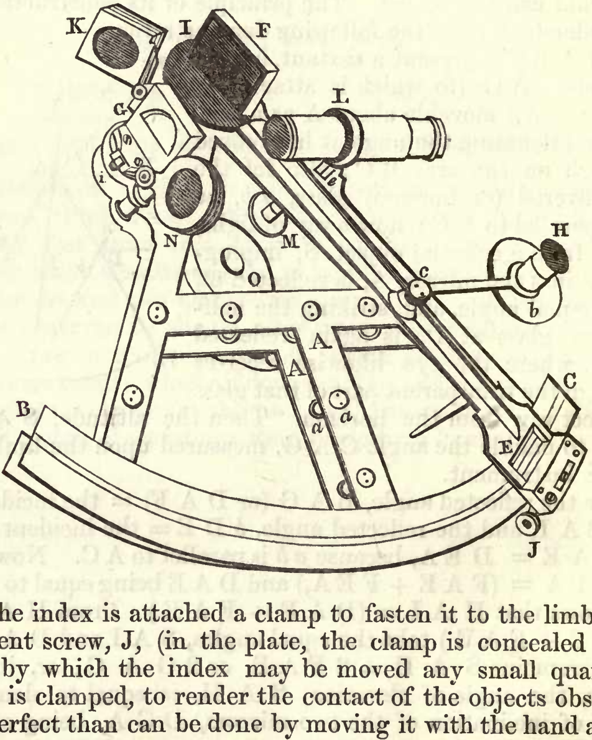


Let A B C represent a sextant, having an index, A G, (to which is attached a mirror at A,) movable about A as a centre, and denoting the angle it has moved through on the arc, B C; also let the half-silvered (or horizon) glass, *a b*, be fixed parallel to A C: now a ray of light, S, A, from a celestial object, S, impinging against the mirror, A, is reflected off at an equal angle, and, striking the half-silvered glass at D, is again reflected to E, where the eye likewise receives through the transparent part of that glass a direct ray from the horizon. Then the altitude, S A H, is equal to double the angle C A G, measured upon the limb, B C, of the instrument.

For the reflected angle, B A G (or D A F) = the incident angle, S A I, and the reflected angle, *b* D E = the incident *a* D A = D A E = D E A, because *a b* is parallel to A C. Now H A I = D F A = (F A E + F E A,) and D A E being equal to D E A, it follows that H A I = (D A E + F A E.) From H A I and (D A E + F A E) take the equal angles, S A I and D A F, and there remains S A H = 2 F A E, or 2 G A C; or, in other words, the angle of elevation, S A H, is equal to double the angle of inclination of the two mirrors, D G A, being equal to G A C.

Hence, the arc on the limb, B C, although only the sixth part of a circle, is divided as if it were 120°, on account of its double being required as the measure of C A B; and it is generally extended to 140°.

The annexed figure represents a sextant of Troughton's construction, having a double frame, A A, connected by pillars, *a a*, &c., thus uniting strength with lightness. The arc, B C, is generally graduated to 10′ of a degree, commencing near the end, C, and it is numbered towards B. The divisions are also continued on the other side of zero, towards C, forming what is called the arc of excess, which is useful in determining the index error of the instrument, as will be explained hereafter. The limb is subdivided by the vernier, E, into 10″, the half of which (or 5″) can be easily estimated: this small quantity is easily distinguishable by the aid of the microscope, H, and its reflector, *b*, which are connected by an arm with the index, I E, at the point *c*, round which it turns as a centre, affording the means of examining the whole vernier, the connecting arm being long enough to allow the microscope to pass over the whole length of it.



To the index is attached a clamp to fasten it to the limb, and a tangent screw, J, (in the plate, the clamp is concealed from view,) by which the index may be moved any small quantity, after it is clamped, to render the contact of the objects observed more perfect than can be done by moving it with the hand alone. The upper end, I, terminates in a circle, across which is fixed the silvered index-glass, F, over the centre of motion, and perpendicular to the plane of the instrument. To the frame, at G, is attached a second glass, called the horizon-glass, the lower half of which only is silvered: this must likewise be perpendicular to the plane of the instrument, and in such a position that its plane shall be parallel to the plane of the index-glass, F, when the vernier is set to 0° (or zero) on the limb, B C. A deviation from this position constitutes the index error before spoken of.

The telescope is carried by a ring, L, attached to a stem, *e*, called the up-and-down piece, which can be raised or lowered by turning the milled screw, M: its use is to place the telescope so that the field of view may be bisected by the line on the horizon-glass that separates the silvered from the unsilvered part. This is important, as it renders the object seen by reflection: and that by direct vision equally bright;* two telescopes and a plain tube, all adapted to the ring, L, are packed with the sextant, one shewing the objects erect, and the other inverting them; the last has a greater magnifying power, shewing the contact of the images much better. The adjustment for distinct vision is obtained by sliding the tube at the eye-end of the telescope in the inside of the other; this also is the means of adapting the focus to suit different eyes. In the inverting telescope are placed two wires, parallel to each other, and in the middle of the space between them the observations are to be made, the wires being first brought parallel to the plane of the sextant, which may be judged of with sufficient exactness by the eye. When observing with this telescope, it must be borne in mind that the instrument must be moved in a contrary direction to that which the object appears to take, in order to keep it in the field of view.

Four dark glasses, of different depths of shade and colour, are placed at K, between the index and horizon glasses,—also three more at N; any one or more of which can be turned down to moderate the intensity of the light, before reaching the eye, when a very luminous object (as the sun) is observed. The same purpose is effected by fixing a dark glass to the eye-end of the telescope: one or more dark glasses for this purpose generally accompany the instrument. They, however, are chiefly used when the sun's altitude is observed with an artificial horizon, or for ascertaining the index error, as employing the shades attached to the instrument for such purposes would involve in the result any error which they might possess. The handle, which is shewn at O, is fixed at the back of the instrument. The hole in the middle is for fixing it to a stand, which is useful when an observer is desirous of great steadiness.

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* This is not the case when one object is much brighter than the other, as the sun and moon; in taking the distance between which, the screw, M, should be moved more than above stated, until they are both nearly of the same brightness, as an observation can be made better when this is the case than when otherwise.

Of the Adjustments.

The requisite adjustments are the following: the index and horizon glasses must be perpendicular to the plane of the instrument, and their planes parallel to each other when the index division of the vernier is at 0° on the arc; and the optical axis of the telescope must be parallel to the plane of the instrument. We shall speak separately of each of these adjustments.

To examine the Adjustment of the Index-glass.

Move the index forward to about the middle of the limb: then, holding the instrument horizontally with the divided limb from the observer, and the index-glass to the eye, look obliquely down the glass, so as to see the circular arc, by direct view and by reflection, in the glass at the same time; and if they appear as one continued arc of a circle, the index-glass is in adjustment. If it require correcting, the arc will appear broken where the reflected and direct parts of the limb meet. This, in a well-made instrument, is seldom the case, unless the sextant has been exposed to rough treatment. As the glass is in the first instance set right by the maker, and firmly fixed in its place, its position is not liable to alter; therefore no direct means are supplied for its adjustment.

To examine the Horizon-glass, and set it perpendicular to the Plane of the Sextant.

The position of this glass is known to be right, when, by a sweep with the index, the reflected image of any object passes exactly over or covers its image as seen directly; and any error is easily rectified by turning the small screw, *i*, at the lower end of the frame of the glass.

To examine the Parallelism of the Planes of the two Glasses, when the Index is set to Zero.

This is easily ascertained; for, after setting the zero on the index to zero on the limb, if you direct your view to some object, the sun for instance, you will see that the two images (one seen by direct vision through the unsilvered part of the horizon-glass, and the other reflected from the silvered part) coincide or appear as one, if the glasses be correctly parallel to each other: but if the two images do not coincide, the quantity of their deviation constitutes what is called the index error. The effect of this error on an angle measured by the instrument is exactly equal to the error itself; therefore, in modern instruments, there are seldom any means applied for its correction, it being considered preferable to determine its amount previous to observing, or immediately after, and apply it with its proper sign to each observation. The amount of the index error may be found in the following manner: clamp the index at about 30 minutes to the left of zero, and looking towards the sun, the two images will appear either nearly in contact or overlapping each other; then perfect the contact, by moving the tangent-screw, and call the minutes and seconds denoted by the vernier the reading on the arc. Next, place the index about the same quantity to the right of zero, or on the arc of excess, and make the contact of the two images perfect as before, and call the minutes and seconds on the arc of excess* the reading off the arc; and half the difference of these numbers is the index error,—additive when the reading on the arc of excess is greater than that on the limb, and subtractive when the contrary is the case.

EXAMPLE.

	′	″
Reading on the arc . . .	31	56
„ off the arc . . .	31	22
Difference	0	34
Index error . . .	= —0	17

In this case, the reading on the arc being greater than that on the arc of excess, the index error, = 17 seconds, must be subtracted from all observations taken with the instrument, until it be found, by a similar process, that the index error has altered. One observation on each side of zero is seldom considered enough to give the index error with sufficient exactness for particular purposes: it is usual to take several measures each way; "and half the difference of their means will give a result more to be depended on than one deduced from a single observation only on each side of zero." A proof of the correctness of observations for index error is obtained by adding the above numbers together, and taking one-fourth of their sum, which should be equal to the sun's semidiameter, as given in the Nautical Almanac. When the sun's altitude is low, not exceeding 20° or 30°, his horizontal instead of his perpendicular diameter should be measured, (if the observer intends to compare with the Nautical Almanac, otherwise there is no necessity); because the refraction at such an altitude affects the lower border (or limb) more than the upper

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* When reading off the arc of excess, the vernier must be read backwards, or from its contrary end as explained at page 7.

so as to make his perpendicular diameter appear less than his horizontal one, which is that given in the Nautical Almanac: in this case the sextant must be held horizontally.

To make the Line of Collimation of the Telescope parallel to the Plane of the Sextant.

This is known to be correct, when the sun and moon, having a distance of 90° or more, are brought into contact just at the wire of the telescope which is nearest the plane of the sextant, fixing the index, and altering the position of the instrument to make the objects appear on the other wire: if the contact still remain perfect, the axis of the telescope is in proper adjustment; if not, it must be altered by moving the two screws which fasten to the up-and-down piece the collar into which the telescope screws. This adjustment is not very liable to be deranged.

Having now gone through the principle and construction of the sextant, it remains to give some instructions as to the manner of using it.

It is evident that the plane of the instrument must be held in the plane of the two objects, the angular distance of which is required; in a vertical plane, therefore, when altitudes are measured,—in a horizontal or oblique plane when horizontal or oblique angles are to be taken. As this adjustment of the plane of the instrument is rather difficult and troublesome to the beginner, he need not be surprised nor discouraged although his first attempts may not answer his expectations. The sextant must be held in the right hand, and as slack as is consistent with its safety, for in grasping too hard the hand is apt to be rendered unsteady.

When the altitude of an object—the sun, for instance—is to be observed, the observer, having the sea horizon before him, must turn down one or more of the dark glasses, or shades, according to the brilliancy of the object; and, directing his sight to that part of the horizon immediately beneath the sun, and holding the instrument vertically, he must with the left hand lightly slide the index forward, until the image of the sun, reflected from the index glass, appears in contact with the horizon, seen through the unsilvered part of the horizon glass. Then clamp it firm, and gently turn the tangent-screw, to make the contact of the upper or lower limb of the sun and the horizon perfect, when it will appear a tangent to his circular disc.* If an artificial

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* If the observer knows his latitude approximately, he may find the meridional altitude nearly, to which he may previously set his instrument; when he will not only find his object more easily, but have only a small quantity to move the index to perfect the observation.

Take from the Nautical Almanac the declination of the object, and if it be of the same name with the latitude, add it to the co-latitude; if of a different name, subtract it: the sum or difference will be the meridian altitude.

horizon be employed, the two images of the sun must be brought into contact with each other; but this will be explained when speaking of that instrument. To the angle read off apply the index error, and then add or subtract the sun's semidiameter, as given in the Nautical Almanac, according as the lower or upper limb is observed, to obtain the apparent altitude of the sun's centre. Before we can use this observation for determining the time, the latitude, &c., it must be further corrected for refraction and parallax, to obtain the true altitude,—subtracting the former and adding the latter; and when the sea horizon is employed, a quantity must also be subtracted for the dip, which is unnecessary when the altitude is taken by means of an artificial horizon.

Tables for obtaining the above corrections may be found in Mr. Baily's Astronomical Tables, &c., in the Requisite Tables, or in any modern work on navigation.

Example.

	″		°	′	″
Obs. alt. of the sun's lower limb . .		=	61	13	5
Index error		=	—		17
Apparent altitude		=	61	12	48,0
* Sun's semidiameter		=	+	15	46,9
,, parallax		=	+	0	4,0
			61	28	38,9
Refraction	— 34,4				
Dip of the horizon, for an elevation of 18 feet . .	—4 3,0	=	—	4	37,4
True altitude of the sun's centre		=	61	24	1,5

If the observer be ignorant of the precise moment of the object being on the meridian, he should, by a slow and gradual motion of the tangent-screw, keep the observed limb in contact with the horizon as long as it continues to rise, and, immediately on the altitude appearing to diminish, cease from observing; and the angle then read on the instrument will be the meridian altitude.

After what has been advanced, little need be said about observing lunar distances, whether of the moon and the sun, or the moon and a fixed star or planet, except that the instrument must be held in the plane of the two objects; and it is generally preferable to direct the telescope to the fainter object, particularly if a star, as it can be more easily kept in view when seen directly

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* An observation of a star requires no correction for either parallax or semidiameter.

than it can when seen by reflection. If the brighter object be to the left, the sextant must be held with the face downwards.

The enlightened limb of the moon is always to be brought into contact with the sun or star, even though the moon's image is made to pass beyond the sun or star before the desired contact can be obtained.

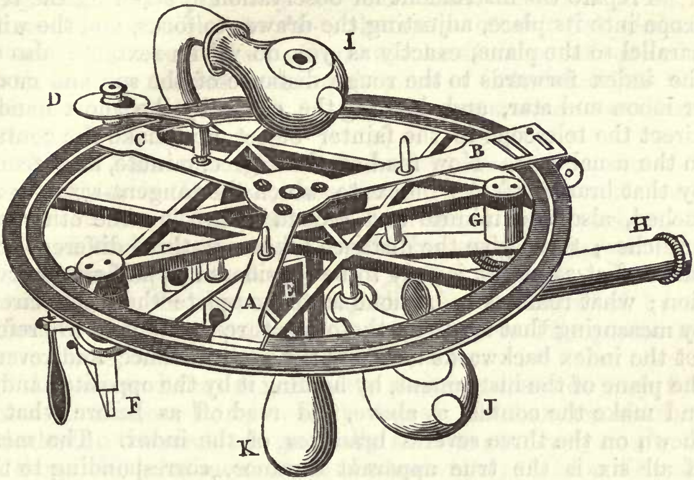
Perhaps the best method of taking a lunar distance is, not to attempt to make the contact perfect by the tangent-screw, but, when the nearest limbs are observed, make the objects overlap each other a little when they are receding; or leave a small space between them when they are approaching, and wait till the contact is perfect, and the reverse, when the furthest limbs are observed.

The altitudes of the two objects should be observed at the same instant as the distance, and the time noted by a chronometer or watch: this would require several observers; but one person may take them all, by having recourse to the following method. " First, observe the altitude of the sun or star—secondly, the altitude of the moon—then any number of distances—next the altitude of the moon—and lastly the altitude of the sun or star, noting the times of each by a watch. Now add together the distances and times when they were observed, and take the mean of each; and, in order to reduce the altitudes to the mean time, making the following proportion: As the difference of times between the observations is to the difference of their altitudes, so is the difference between the time that the first altitude was taken and the mean of the times at which the distances were observed to a fourth number; which, added to or subtracted from the first altitude, according as it is increasing or decreasing, will give the altitude reduced to the mean time."

The angular distances of terrestrial objects are measured by the sextant in the same manner as those of celestial ones; but if the objects be not in the same horizontal plane, a reflecting instrument will not give their horizontal angular distance. But this may be obtained nearly by measuring their angular distances from an object in or near the horizon which subtends a great angle with both, and the sum, or the difference of the angles so measured, will be nearly the required horizontal angle.

Of the sextant it has been said, that it is in itself a portable observatory; and it is doubtless one of the most generally useful instruments that has ever been contrived, being capable of furnishing data to a considerable degree of accuracy for the solution of a numerous class of the most useful astronomical problems; affording the means of determining the time, the latitude and longitude of a place, &c., for which, and many other purposes, it is invaluable to the land surveyor as well as the navigator.

TROUGHTON'S REFLECTING CIRCLE.



The above figure represents this instrument, which in principle and use is the same as the sextant. It has three vernier readings, A B C, moving round the same centre as the index-glass, E, which is upon the opposite face of the instrument. One of the verniers, B, carries the clamp and tangent screw. D represents the microscope for reading the verniers: it is similar to the one used in reading the sextant, and is adapted to each index-bar, by slipping it on a pin placed for that purpose, as shewn in the figure. The horizon-glass is shewn at F. The barrel, G, contains the screws for giving the up-and-down motion to the telescope; it is put in action by turning the milled-head under the barrel. H is the telescope, adapted to the instrument in a manner similar to that of the sextant. I and J are two handles fixed parallel to the plane of the circle, and a third handle, K, is screwed on at right angles to that plane, and can be transferred to the opposite face of the instrument by screwing it into the handle I: the use of this extra handle is for convenience in reading, and in holding the instrument, when observing angles that are nearly horizontal; it can be shifted, according as the face of the instrument is held upwards or downwards. The requisite, dark glasses are attached to the frame-work of the circle, to be used in the same manner and for the same purposes as those of the sextant. With respect to the adjustments and application of this instrument, we cannot do better than use the words of the inventor, Mr. Troughton, contained in a paper which he calls

"*Directions for observing with Trought on's Reflecting Circle.*

"Prepare the instrument for observation by screwing the telescope into its place, adjusting the drawer to focus, and the wires parallel to the plane, exactly as you do with a sextant: also set the index forwards to the rough distance of the sun and moon, or moon and star, and, holding the circle by the short handle, direct the telescope to the fainter object, and make the contact in the usual way. Now read off the degree, minute, and second, by that branch of the index to which the tangent-screw is attached, also the minute and second shewn by the other two branches; these give the distance taken on three different sextants: but as yet, it is only to be considered as half an observation; what remains to be done is, to complete the whole circle, by measuring that angle on the other three sextants. Therefore set the index backwards nearly to the same distance, and reverse the plane of the instrument, by holding it by the opposite handle, and make the contact as above, and read off as before what is shewn on the three several branches of the index. The mean of all six is the true apparent distance, corresponding to the mean of the two times at which the observations were made.

"When the objects are seen very distinctly, so that no doubt whatever remains about the contact in both sights being perfect, the above may safely be relied on as a complete set; but if, from the haziness of the air, too much motion, or any other cause, the observations have been rendered doubtful, it will be advisable to make more: and if, at such times, so many readings should be deemed troublesome, six observations, and six readings may be conducted in the manner following,—Take three successive sights forwards, exactly as is done with a sextant, only take care to read them off on different branches of the index; also make three observations backwards, using the same caution: a mean of these will be the distance required. When the number of sights taken forwards and backwards are unequal, a mean between the means of these taken backwards and those taken forwards will be the true angle.

"It need hardly be mentioned, that the shades, or dark glasses, apply like those of a sextant, for making the objects nearly of the same brightness; but it must be insisted on, that the telescopes should, on every occasion, be raised or lowered, by its proper screw, for making them perfectly so.

"The foregoing instructions for taking distances apply equally for taking altitudes by the sea or artificial horizon, they being no more than distances taken in a vertical plane. Meridian altitudes cannot, however, be taken both backwards and forwards the same day, because there is not time: all therefore that can be done is, to observe the altitude one way, and use the index error; but even here you have a mean of that altitude and this error, taken on three different sextants. Both at sea and land, where the observer is stationary, the meridian altitude should be observed forwards one day and backwards the next, and so on alternately from day to day: the mean of altitudes, deduced severally from such observations, will be the true latitude: but in these there should be no application of index error, for that being constant, the result would in some measure be vitiated thereby.

"When both the reflected and direct images require to be darkened, as is the case when the sun's diameter is measured and when his altitude is taken with an artificial horizon, the attached dark glasses ought not to be used; instead of them, those which apply to the eye-end of the telescope will answer much better: the former, having their errors magnified by the power of the telescope, will, in proportion to this power, and those errors, be less distinct than the latter.

"In taking distances, when the position does not vary from the vertical above thirty or forty degrees, the handles which are attached to the circle are generally most conveniently used; but in those which incline more to the horizontal, that handle which screws into a cock on one side, and into the crooked handle on the other, will be found more applicable.

"When the crooked handle happens to be in the way of reading one of the branches of the index, it must be removed, for the time, by taking out the finger-screw, which fastens it to the body of the circle.

"If it should happen that two of the readings agree with each other very well, and the third differs from them, the discordant one must not on any account be omitted, but a fair mean must always be taken.

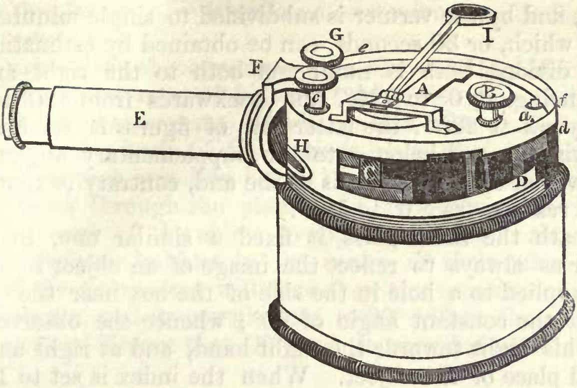
"It should be stated that, when the angle is about thirty degrees, neither the distance of the sun and moon, nor an altitude of the sun, with the sea horizon, can be taken backwards; because the dark glasses at that angle prevent the reflected rays of light from falling on the index-glass: whence it becomes necessary, when the angle to be taken is quite unknown, to observe forwards first, where the whole range is without interruption; whereas in that backwards, you will lose sight of the reflected image about that angle. But in such distances, where the sun is out of the question, and when his altitude is taken with an artificial horizon, (the shade being applied to the end of the telescope,) that angle may be measured nearly as well as any other; for the rays incident on the index-glass will pass through the transparent half of the horizon-glass without much diminution of their brightness.

"The advantages of this instrument, when compared with the sextant, are chiefly these: the observations for finding the index error are rendered useless, all knowledge of that being put out of the question, by observing both forwards and backwards. By the same means the errors of the dark glasses are also corrected; for, if they increase the angle one way, they must diminish it the other way by the same quantity. This also perfectly corrects the errors of the horizon-glass, and those of the index-glass very nearly. But, what is still of more consequence, the error of the centre is perfectly corrected by reading the three branches of the index; while this property, combined with that of observing both ways, probably reduces the errors of dividing to one-sixth part of their simple value. Moreover, angles may be measured as far as one hundred and fifty degrees: consequently, the sun's double altitude may be observed when his distance from the zenith is not less than fifteen degrees,—at which altitude the head of the observer begins to intercept the rays of light incident on the artificial horizon; and, of course, if a greater angle could be measured, it would be of no use in this respect.

"This instrument, in common with the sextant, requires three adjustments. First, the index-glass perpendicular to the plane of the circle. This being done by the maker, and not liable to alter, has no direct means applied to the purpose; it is known to be right when, by looking into the index-glass, you see that part of the limb which is next you reflected in contact with the opposite side of the limb, as one continued arc of a circle: on the contrary, when the arc appears broken, where the reflected and direct parts of the limb meet, it is a proof that it wants to be rectified. The second is, to make the horizon-glass perpendicular. This is performed by a capstan-screw, at the lower end of the frame of that glass, and is known to be right when, by a sweep of the index, the reflected image of any object will pass exactly over or cover the image of that object seen directly. The third adjustment is for making the line of collimation parallel to the plane of the circle. This is performed by two small screws, which also fasten the collar into which the telescope screws to the upright stem on which it is mounted: this is known to be right, when the sun and moon, having a distance of one hundred and thirty degrees, or more, their limbs are brought in contact, just at the outside of that wire which is next to the circle,—and then, examining if it be the same, just at the outside of the other wire. Its being so is the proof of adjustment.

"Should these hints about the adjustments set any over-handy gentleman on tormenting his instrument, it will not be what was intended by them; they were added that, in case of accident, those who are so unfortunate might be enabled thereby to put their own instruments in order."

THE BOX SEXTANT.



This useful little instrument, which is represented in the above figure, might, perhaps with more propriety, have been classed as a surveying instrument, it being chiefly used in that business. The principle of its construction and adjustments is precisely the same as the sextant before described ; a minute description, therefore, would be little more than a recapitulation of what has already been advanced. A is the index, which, instead of being moved along the divided limb, *e f*, by the hand, has a motion given to it by a rack and pinion, concealed within the box, and turned by the milled head, B, which acts as the tangent-screw does to the index of the large sextant. The glasses (shewn at C and D) are within the box, by which they are protected from injury, and their adjustments, when once perfected, kept secure; so much so, that it would require considerable violence to derange them. The horizon-glass, D, alone has a contrivance for adjustment, at *a* and *d*, both to set it perpendicular to the plane of the instrument, and to correct or reduce the index error, which, in this instrument, had better be kept correct, as it is not so likely to get out of order as in the large sextant, which, as we have before observed, seldom admits of its index error being rectified. The key, *c*, is formed to fit both squares, at *a* and *d*, to make the adjustments ; and it is generally tapt into some spare place in the instrument, as at *c*, that it may be always safe and at hand.

It is supplied with a telescope, E, which screws into a shoulder-piece, F, and can be attached to the box by the screw, G: this can be applied or not, at the pleasure of the observer, as there is a contrivance at H to enable him to observe without the telescope, if he prefer plain sights. Two dark glasses are placed within the box, and there is also one adapted to the eye-end of the telescope.

The angle is read off by the help of the glass I, which, being mounted with a joint, can be moved over the vernier on any part of the limb. The instrument is divided to 30 minutes of a degree, and by the vernier is subdivided to single minutes; one-half of which, or 30 seconds, can be obtained by estimation.

The divided limb is numbered both to the right and left, commencing at 0° to 120°, and backwards from 120° to 180°, and beyond to 230°: the latter row of figures is furthest from the divisions, and belongs to the supplementary angles; their zero division of the vernier is at the end, contrary to that of the angles, reading from 0° to 120°.

Beneath the index-glass is fixed a similar one, in such a manner as always to reflect the image of an object to the eye, when applied to a hole in the side of the box near the division 120°, at the constant angle of 90°; whence the observer must direct his sight towards the right hand, and at right angles to the real place of the object. When the index is set to 180°, its glass will also reflect an opposite image to the eye at right angles to the left hand (the two glasses then being exactly across each other); consequently, an eye looking through the hole near the division 120° will (if the adjustments be perfect) perceive objects 180° apart to coincide at right angles to a line connecting them. Thus, a point can be found in line between two stations: the observer, with the instrument set as above, having placed himself as nearly in the line as he can guess, must apply his eye to the hole near 120°, and, looking at right angles to his station line, step backwards or forwards, until he perceives the two distant objects to coincide, when the spot he stands on will be a point in the line joining the objects. To verify this, he should then turn himself half round, and, looking in the opposite direction, see if the two objects still coincide, which they will do if the adjustments of the instrument be correct. If they do not appear in junction, move as before, until you find the spot where they do; then, half-way between the two spots so found will be the true point on the line required.

"The adjustment of this part, as well as the method of observing supplemental angles with it, is performed thus: Choose two objects in the horizon, the further apart the better, but not nearer than 140°; turn your face at right angles to the right-hand object, so as to get sight of its image in the fixed glass, then, by moving the index, bring the image of the other object, seen in the index-glass, exactly to coincide with it on the line of separation of the two glasses: read off the angle, turn yourself half round, and take in like manner the angle which the same objects make the other way. It is evident that the sum of the two angles should be 360°, and, also, that if they exceed that quantity half the excess must be subtracted, and if they fall short of it half the defect must be added, to obtain the true angle. It is, perhaps, better to allow for the errors than to adjust them; but the latter may be done by applying the key, *c*, to a square underneath the box."

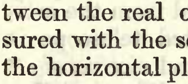
The lid of the box is contrived to screw on the bottom, (as is shewn in the plate,) where it makes a convenient handle for holding the instrument.

Since writing the above we have been shewn by MR. MACNEILL an excellent contrivance of his, for taking altitudes or depressions with the box-sextant, which consists of two small spirit-levels fixed at the back of the horizon-glass, at right angles to each other, so that, standing before the object, you look perpendicularly down through the plane-sight, and, moving the index, bring the image of the object to appear with the levels, which must have their air-bubbles in the centre of their tubes. The reading of the instrument will then shew the supplement of the zenith distance, and its complement to 90° will be the angle required; elevated if more than 90°, and depressed if less than 90°.

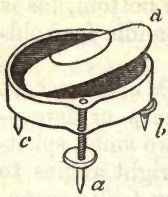
THE ARTIFICIAL HORIZON.

When the altitude of a celestial object is to be taken at sea, the observer has the natural (or sea) horizon as a line of departure; but on shore he is obliged to have recourse to an artificial one, to which his observation may be referred: this consists of a reflecting plane parallel to the natural horizon, on which the rays of the sun or other object falling, are reflected back to an eye placed in a proper position to receive them; the angle between the real object and its reflected image, being then measured with the sextant, is double the altitude of the object above the horizontal plane.

Various natural as well as artificial reflecting surfaces have been made by mechanical arrangements, to afford the means of obtaining double angles: such as pouring water, oil, treacle, or other fluid substances into a shallow vessel; and to prevent the wind giving a tremulous motion to its surface, a piece of thin gauze, talc, or plate-glass, whose surfaces are perfectly plane and parallel, may be placed over it, when used for observation. But the most accurate kind of artificial horizon is that in which fluid quicksilver forms the reflecting surface, the containing vessel being placed on a solid basis, and protected from the influence of the wind. The adjoining figure represents an instrument of this kind. The mercury is contained in an oblong wooden trough, placed under the roof, A, in which are fixed two plates of glass, whose surfaces are plane and parallel to each other. This roof effectually screens the surface of the metal from being agitated by the wind, and when it has its position reversed at a second observation, any error occasioned by undue refraction at either plate of glass will be corrected.



Another and more portable contrivance for an artificial horizon, is represented in the following figure, which consists of a



circular plate of black glass, about two inches diameter, mounted on a brass stand, half an inch deep, with three foot-screws, *a*, *b*, *c*, to set the plane horizontal: the horizontality being determined thus by the aid of a short spirit-level, *d*, having under the tube a face ground plane on which it lies in contact with the reflecting surface,—place the level on the glass in a direction parallel to the line joining two of the three foot-screws, as *a* and *b*, then move one of these screws till the bubble remains in the middle of the tube in both the reversed positions of the level, and the plate will be horizontal in that direction; then place the level at right angles to its former position, and turn the third foot-screw back or forwards till the bubble again settles in the middle of its tube, the former levelling remaining undisturbed, and the plane will then be horizontal. This instrument, from its portability, is extremely convenient for travellers, as, when packed in its case, it can be carried in the pocket without being any incumbrance.

When an artificial horizon is used, the observer must place himself at such a distance that he may see the reflected object as well as the real one; then, having the sextant properly adjusted, the upper or lower limb of the sun's image (supposing that the object) reflected from the index-glass must be brought into contact with the opposite limb of the image reflected from the artificial horizon, observing that, when the inverted telescope is used, the upper limb will appear as the lower, and *vice versâ*: * the angle shewn on the instrument, when corrected for the index error, will be double the altitude of the sun's limb above the horizontal plane; to the half of which, if the semidiameter, refraction, and parallax be applied, the result will be the true altitude of the centre.

EXAMPLE.

	°	′	″
Observed angle	122	25	50,00
Index error	—		17,05
	2) 122	25	32,95
App. alt.	61	12	46,47
Semidiameter	+	15	46,91
Parallax	+		4,00
	61	28	37,38
Refraction	—		34,40
True alt. of sun's centre . . .	61	28	2,98

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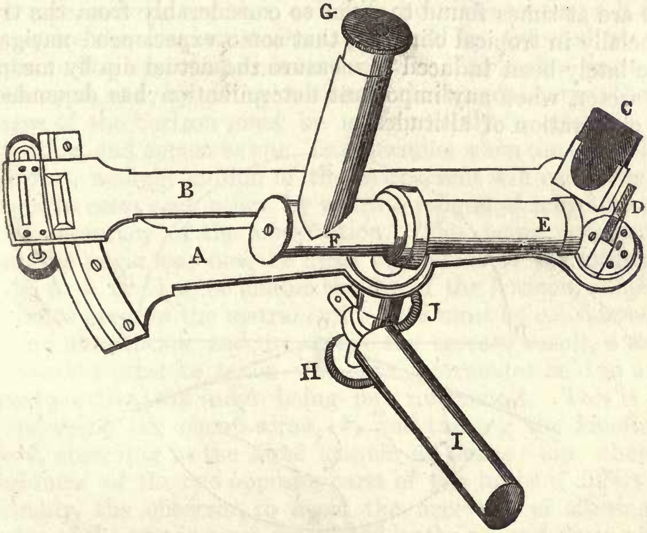
* When the contact is formed at the lower limb, the images will separate shortly after the contact has been made, if the altitude be increasing; but if the altitude be decreasing, they will begin to overlap: but when the contact is formed at the upper limb the reverse takes place. An observer, if in doubt as to which limb he has been observing, should watch the object for a short time after he has made the observation.

THE DIP-SECTOR.

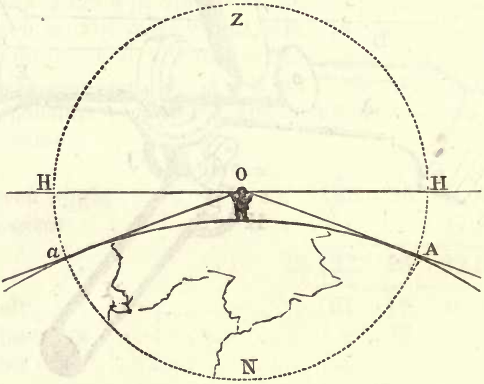
When the late Professor VINCE was engaged in making observations upon extraordinary refraction at Ramsgate, MR. TROUGHTON contrived and constructed for his use an instrument which he called a Refraction-Sector. About five years afterwards, when preparations were making for the first of the late North Polar Expeditions, MR. TROUGHTON was applied to by the late DR. WOOLLASTON, to make him an instrument on the principle of the back observation with the quadrant, to send with the expedition, to measure the dip of the horizon; but, upon MR. TROUGHTON'S producing his Refraction-Sector, which was as well adapted to DR. WOOLLASTON'S purpose as that for which it was devised, the Doctor immediately ordered one to be made for him, and named it a Dip-Sector, proposing at the same time an improvement in the construction of the handle, which, on his suggestion, was made to turn on a centre, to be placed in any position, for convenience in use or packing in its case,—that made for MR. VINCE having two fixed handles, at right angles to the face of the instrument.



The preceding figure represents this instrument: A is the sector, B the index, with its clamp and tangent-screw, exactly similar to that of the sextant: the index-glass, C, and the horizon-glass, D, are fixed at right angles to the plane of the instrument. The telescope, E F, is fitted into a collar, having an up-and-down motion given to it by turning the screw, H; the two images of the horizon can thus be made to appear of the shades most favourable for observation. G represents the eye-piece, fixed at right angles to the telescope; and a diagonal mirror is placed in the telescope, at F, to change the direction of the rays of light, from E F, to F G, in which the observer looks.

The handle, I, turns upon a centre, and is held firmly in any position by tightening the clamp-screw, J. In use it is fixed perpendicular to the length of the instrument, and when wanted it can be turned half-round, and fixed in a similar position on the other side,—a position in which it is required to be when the instrument is reversed for the second observation: it is turned under and parallel to the instrument when packed in its case.

The dip of the horizon, which varies with the height of the observer above the surface of the earth, may always be computed when the height is known: but, as a correction of altitudes observed from the horizon of the sea, it is combined with the effects of refraction upon the apparent place of the horizon, which appears elevated above its true place; and as the effects of refraction are extremely variable, the dip obtained by computation is necessarily very uncertain. Tables containing the dip for various altitudes, allowing for the mean effect of refraction, are to be found in all collections of nautical tables. But these tabular dips are at times found to differ so considerably from the truth, especially in tropical climates, that some experienced navigators have lately been induced to measure the actual dip by means of the sector, when any important determination has depended on the observation of altitudes.

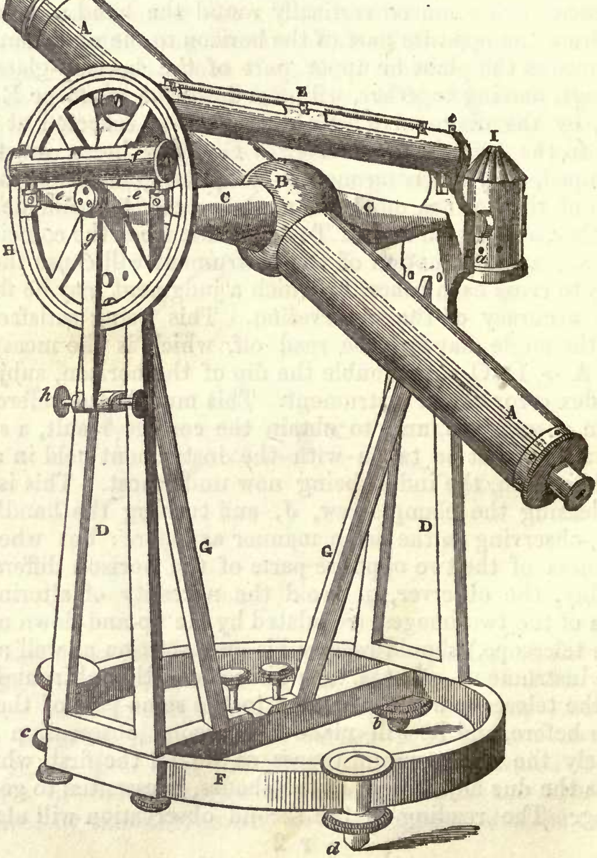


In the above diagram, A *a* represents a portion of the earth's surface, and O the place of an observer; H O H will be his true horizon, O A and O *a* his visible horizon,—these rays being tangents to the earth's surface at A and *a*: the angle, H O A, or H O *a*, is the dip of the horizon, which it is the business of the dip-sector to measure. But the arcs to be measured by this instrument, for the purpose of obtaining the dip, are A Z *a* and A N *a*, the former of which is 180° + double the dip, and the latter 180° — double the dip; therefore the fourth part of the difference is the measure of the dip. But, as the instrument is constructed, only double the dip affected by index error is read from it; and the index error is made so great that the readings are both on the same side of zero: therefore the fourth part of the difference of the readings is the dip angle.

In observing, the face of the instrument must be held in a vertical plane, and lengthwise, in a line with the opposite parts of the horizon whose dip is required; the eye-tube, G F, (page 65) will then be horizontal, and the observer will be looking at right angles to those points of the horizon which he wishes to observe. Suppose the instrument to be held as represented in our engraving, with the index uppermost, the observer will be looking in the direction, G F, when by giving motion to the index B, its glass, C, will receive *a ray* from the visible horizon on the left hand, and reflect it to the silvered part of the horizon-glass, D, and from thence to the telescope; at the same time, the whole instrument being moved vertically round the hand as a centre, *a ray* from the opposite part of the horizon to the right hand will pass through the plain or upper part of the horizon-glass, and both rays, moving together, will pass down the telescope E to F, where, by the diagonal mirror, they will be reflected at right angles to the eye of the observer at G. The index must now be clamped, and, by giving motion to the tangent-screw, the two images of the horizon must be made exactly to coincide with each other, and appear as one. To determine when the coincidence is perfect, a slight motion of the instrument will cause the two images to cross each other, by which a judgment may be formed of the accuracy of the observation. This being satisfactorily done, the angle may now be read off, which is the measure of H O A + H O *a*, or double the dip of the horizon, subject to the index error of the instrument. This must be considered but half an observation, and, to obtain the correct result, a second observation must be taken with the instrument held in an inverted position, the index being now undermost. This is done by releasing the clamp-screw, J, and turning the handle half round, observing in the same manner as before: but when the brightness of the two opposite parts of the horizon differs considerably, the observer, to avoid the necessity of altering the shades of the two images, (regulated by the up-and-down motion of the telescope,) should reverse his own position as well as that of his instrument,—that is, turn himself exactly half round,—for then the telescope will be directed to the same part of the horizon as before, and he will make the second observation under precisely the same circumstances as he did the first, which, as well as the due adjustment of the shades, is essential to good observing. The reading of the second observation will also give double the dip, affected by the same index error as before; and, as both readings are on the same side of zero, *one-fourth* of their difference will give the true result. Several observations should be taken in each position of the instrument, and the mean taken as the final result.

"In using this instrument at sea for the first time, considerable difficulty arises from the constant change in the plane of the instrument, from the perpendicular position in which it is absolutely necessary that it should be held, in order to obtain a correct observation. What at first appears to be a defect, however, is a real advantage, namely, that whenever it is held in the least degree out of the vertical plane, the two horizons (that seen direct and the reflected one) cross each other; and it is only when the plane is vertical that the horizons can appear parallel."

THE PORTABLE TRANSIT-INSTRUMENT.



The Transit is a meridional instrument, employed in conjunction with a clock or chronometer for observing the passage of celestial objects across the meridian, either for obtaining correct time, or determining their difference of right ascension; the latter of which, in the case of the moon and certain stars near her path, that differ but little from her in right ascension, affords the best means of determining the difference of longitude between any two places where corresponding observations may have been made. Such being more especially the use of the *portable* transit-instrument, it forms a valuable accession to the apparatus of the scientific traveller, who, remaining a short time at any station, is enabled thereby to adjust his time-keepers, both with ease and accuracy, and to obtain the best data for finding his longitude. It may also be employed very successfully in determining the latitude.*

The preceding figure represents this instrument as constructed by MR. TROUGHTON, when the telescope does not exceed twenty inches or two feet focal length. The telescope-tube, A A, is in two parts, and connected together by a sphere, B, which also receives the larger ends of two cones, C C, placed at right angles to the direction of the telescope, and forming the horizontal axis. This axis terminates in two cylindrical pivots, which rest in Y's fixed at the upper end of the vertical standards, D D. One of the Y's possesses a small motion in azimuth, communicated by turning the screw, *a:* in these Y's the telescope turns upon its pivots. But, that it may move in a vertical circle, the pivots must be precisely on a level with each other; otherwise the telescope will revolve in a plane oblique (instead of perpendicular) to the horizon. The levelling of the axis, as it is called, is therefore one of the most important adjustments of the instrument, and is effected by the aid of a spirit-level, E, which is made for this purpose to stride across the telescope, and rest on the two pivots.

The standards, D D, are fixed by screws upon a brass circle, F, which rests on three screws, *b c d,* forming the feet of the instrument, by the motion of which the operation of levelling is performed. The two oblique braces, G G, are for the purpose of steadying the supports,—it being essential for the telescope to have not only a free but a steady motion. On the extremity of one of the pivots, which extends beyond its Y, is fixed a circle, H, which turns with the axis while the double vernier, *e e,* remains stationary in a horizontal position, and shews the altitude to which the telescope is elevated. The verniers are set horizontal by means of a spirit-level, *f,* which is attached to them,

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* The transit-instrument is also now much employed by the most eminent civil engineers, in setting out the lines of direction, and the working shafts, in tunnelling; which it is capable of doing with the greatest precision.

and they are fixed in their position by an arm of brass, *g*, clamped to the supports by a screw at *h*: the whole of this apparatus is moveable with the telescope, and, when the axis is reversed, can be attached in the same manner to the opposite standard.

Near the eye-end, and in the principal focus of the telescope, is placed the diaphragm, or wire-plate, which in the theodolite, or levelling telescope, need only carry two cross wires; but in this instrument it has five vertical and two horizontal wires. The centre vertical wire ought to be fixed in the optical axis of the telescope, and perpendicular with respect to the pivots of the axis. It will be evident, upon consideration, that these wires are rendered visible in the daytime by the rays of light passing down the telescope to the eye; but at night, except when a very luminous object, as the moon, is observed, they cannot be seen. Their illumination is therefore effected by piercing one of the pivots, and admitting the light of a lamp fixed on the top of one of the standards, as shewn at I; which light is directed to the wires by a reflector placed diagonally in the sphere B: the reflector, having a large hole in its centre, does not interfere with the rays passing down the telescope from the object, and thus the observer sees distinctly both the wires and the object at the same time. When, however, the object is very faint, (as a small star,) the light from the lamp would overpower its feeble rays: to remedy this inconvenience, the lamp is so constructed, that, by turning a screw at its back, or inclining the opening of the lantern, more or less light may be admitted to the telescope, to suit the circumstances of the case.

The telescope is furnished with a diagonal eye-piece, by which stars near the zenith may be observed without inconvenience.

Of the Adjustments.

Upon setting the instrument up, it should be so placed that the telescope, when turned down to the horizon, should point north and south as near as can possibly be ascertained. This of course can be but approximate, as the correct determination of the meridian can only be obtained by observation, after the other adjustments are completed.

The first adjustment is that of the line of collimation. Direct the telescope to some small, distant, well-defined object, (the more distant the better,) and bisect it with the middle of the central vertical wire; then lift the telescope very carefully out of its angular bearings, or Y's, and replace it with the axis reversed: point the telescope again to the same object, and if it be still bisected, the collimation adjustment is correct; if not, move the wires one half the error, by turning the small screws which hold the diaphragm near the eye-end of the telescope, and the adjustment will be accomplished. But, as half the deviation may not be correctly estimated in moving the wires, it becomes necessary to verify the adjustment by moving the telescope the other half, which is done by turning the screw, *a;* this gives the small azimuthal motion to the Y before spoken of, and, consequently, to the pivot of the axis which it carries. Having thus again bisected the object, reverse the axis as before, and if half the error were correctly estimated, the object will be bisected upon the telescope being directed to it; if not quite correct, the operation of reversing and correcting half the error, in the same manner, must be gone through again, until, by successive approximations, the object is found to be bisected in both positions of the axis: the adjustment will then be perfect. The collimation adjustment may likewise be examined from time to time, by observing the transit of Polaris, or any other close circumpolar star, over the first three wires, which gives the intervals in time from the first to the second, and from the first to the third wire; and then, reversing the axis, observe the same intervals in a reverse order, as the wires which were the three first, in the former position, will now be the three last: if the intervals in the first observations be exactly the same as the intervals in the second, the collimation adjustment is correct; but should the corresponding intervals differ, such difference points out the existence of an error, which must be removed, as before described, one-half by the collimating screws, and the other half by the azimuthal motion of the instrument.

It is desirable that the central or middle wire (as it is usually termed) should be truly vertical; as we should then have the power of observing the transit of a star on any part of it, as well as the centre. It may be ascertained whether it be so, by elevating and depressing the telescope: when directed to a distant object, if it be bisected by every part of the wire, the wire is vertical; if otherwise, it should be adjusted, by turning the inner tube, carrying the wireplate, until the above test of its verticality be obtained, or else care must be taken that the observations are made near the centre only. The other vertical wires are placed by the maker equidistant from each other, and parallel to the middle one; therefore, when the middle one is adjusted, the others are so too: he also places the two transverse wires at right angles to the vertical middle wire. These adjustments are always performed by the maker, and but little liable to derangement. When, however, they happen to get out of order, and the observer wishes to correct them, it is done by loosening the screws which hold the eye-end of the telescope in its place, and turning the end round a small quantity by the hand until the error is removed. But this operation requires very delicate handling, as it is liable to remove the wires from the focus of the object-glass.

The axis on which the telescope turns must next be set horizontal. To do this, apply the level to the pivots; bring the air-bubble to the centre of the glass-tube, by turning the foot-screw, *b*, which raises or lowers that end of the axis, and, consequently, the level resting upon it: then reverse the level, by turning it end for end, and if the air-bubble still remain central, the axis will be horizontal; but if not, half the deviation must be corrected by the foot-screw, *b*, and the other half by turning the small screw, *i*, at one end of the level, which raises or lowers the glass-tube (containing the air bubble) with respect to its supports, which rest upon the pivots. This, like most other adjustments, frequently requires several repetitions before it is accomplished, on account of the difficulty of estimating exactly half the error.

Having set the axis on which the telescope turns parallel to the horizon, and proved the correct position of the central wire or line of collimation, making it describe a great circle perpendicular to that axis, it remains finally to make it move in that vertical circle which is the meridian.

We have supposed the instrument to be nearly in the meridian: the next step is to determine the amount of its deviation, and then, by successive approximations, to bring it exactly into that plane. One of the methods of accomplishing this is, to observe the time of both the upper and lower transits of Polaris, or any other close circumpolar star; and as the middle wire of the instrument, when exactly in the meridian, bisects the circle which the star apparently describes round the polar point in 24 sidereal hours, the time elapsed during its traversing either the eastern or western semicircle will be equal to 12 sidereal hours; but should the interval be greater or less, it is clear that the instrument deviates from the meridian. If the eastern interval be greater than the western, the plane in which the instrument moves from the zenith to the north of the horizon is westward of the true meridian, and *vice versâ* if the western interval be greatest. Having the difference of the interval from 12 hours, the quantity of deviation measured on the horizon may be computed by the following formula,—the latitude of the place and the polar distance of the star being both supposed to be known, at least approximately.

$$\text{Deviation} = \log. \frac{\Delta}{2} + \log. \sec. L + \log. \tan. \pi - 20:$$

in which expression, $\Delta =$ the difference of the intervals from 12^h (reduced to seconds)

$\pi =$ the polar distance of the star

$L =$ the latitude of the place.

This formula, in words, gives the following practical rule: Add together the log. of half the difference of the intervals from 12 hours in seconds, the log. secant of the latitude, and the log. tangent of the polar distance of the star: the sum, rejecting 20 from the index, will be the log. factor of deviation, which may be converted into arc by multiplying it by 15.

The correction of this error may be effected by turning the screw, *a*, if the angular value of one revolution be known, unless the instrument possesses an azimuth circle, by which the telescope may be set exactly that quantity from its present position. But if the quantity of motion to be given to the adjusting-screw, *a*, be not a matter of certainty, the observer, after ascertaining the difference of the intervals, must make the adjustment which he considers sufficient, and again proceed to verify it by observation, until, by continued approximation, he succeeds in fixing his instrument correctly in the meridian.

The above method of determining the instrumental deviation is wholly independent of the tabulated place of the circumpolar star; but it assumes some knowledge of the rate of the time-keeper, and the *perfect stability* of the instrument for twelve hours,—a condition which is rarely to be obtained, except in a regular observatory. The method is still further limited in practice by the uncertainties of the weather and the want of stars sufficiently bright to be observed in the daytime, (Polaris being the only star in the northern hemisphere fit for the purpose, and there is no similar star in the southern.) There are, however, two methods almost as good as the preceding, which depend on the *tabulated* places of the stars only. These will now be explained.

Take two *well-known* circumpolar stars, the nearer the pole the better, differing about twelve hours in right ascension, and observe one above and the other below the pole. Now it is evident that any deviation of the instrument from the meridian will produce *contrary* effects upon the observed times of transit, exactly as in the upper and lower culmination of the same star. Hence, the time which elapses between the two observations will differ, from the time which should elapse according to the catalogue, by the *sum* of the effects of the deviation upon the two stars. Compute what effect a deviation of 15″ will produce on the interval: then, the difference between the observed interval and computed interval divided by the quantity thus computed will be the factor of deviation to be used for correcting transits observed the same night. Or, if the deviation itself be required for altering the position of the instrument, multiply this factor by 15; the result will be the deviation to the east or west of the north in seconds of space.

The effect produced on the interval by a deviation of 15″ is to be computed as follows: let π be the polar distance of the upper star, π' that of the star sub-polo, λ the co-latitude of the place: then the effect in time of a deviation of 15″ is, for the upper star $\frac{\sin.\ (\lambda-\pi)}{\sin.\ \pi}$ and for the star sub-polo $\frac{\sin.\ (\lambda+\pi')}{\sin.\ \pi'}$, acting contrary ways upon the time of transit of each star respectively, and hence affecting the interval by their sum, or by $\frac{\sin.\lambda \sin.(\pi+\pi')}{\sin.\pi \sin.\pi'}$. Hence the factor for instrumental deviation $= \frac{\sin.\pi \sin.\pi'}{\sin.\lambda \sin.(\pi+\pi')} \times$ the difference between the observed and computed intervals. When $\pi=\pi'$, or the same star is observed at the upper and lower culmination, this factor becomes $\frac{\tan.\pi}{2 \sin.\lambda} \times$ the difference.

Practical Rule. To the log. of the difference in seconds between the tabulated and observed interval add the log. sines of the polar distances of the two stars, the log. secant of the latitude, and the log. co-secant of the sum of the two polar distances; reject 40 from the index, and the result will be the log. factor of the deviation (to be used according to the formula, page 86, in correcting the transits of all stars observed the same night). And, as before observed, when it is intended to correct the position of the instrument, this quantity, multiplied by 15, will give the deviation from the meridian in space to the east or west of the north.

In determining the *direction* of the deviation, it must be recollected that, when the deviation is to the east, the star above pole passes too early, and that below pole too late; and, therefore, if the upper star precede, the interval is increased, but if the lower precede, then *vice versâ*. When the deviation is to the west, the star above pole passes too late; while the star below pole passes too early. Hence, if the former precede, the interval is diminished, and *vice versâ*.

EXAMPLE.

At page 79 we have inserted an extract from the Greenwich Observations for 1834, page 10: we shall take for an example the stars Cephei 51 *Hev.* and δ Urs. Min. S. P.

March 18th, 1834.

		Observation. H. M. S.		Naut. Alm. H. M. S.
Cephei 51 *Hev.*	=	6 20 59,00	.	6 20 19,61
δ Urs. Min. S. P. . . .	=	6 26 32,50	.	6 25 51,24
Interval observed . .	=	5 33,50	.	5 31,63
„ tabulated . .	=	5 31,63		
Diff. of Intervals . .	=	1,87	log.	0·27184

		(Brought forward)		
Diff. of Intervals .	=	1,87	log.	0·27184
		° ′ ″		
π the P.D. of Cephei .	=	2 43 50	sine .	8·67796
π′ „ δ Urs. Min. .	=	3 2 4	sine .	8·77536
Latitude . . .	=	51 28 39	secant	0·20564
(π+π′) sum of the Polar distances . .	=	6 8 54	co-sec.	0·97020
		$\overset{s}{0},080$	=	8·90100
Multiply by . . .		15		
Deviation from the meridian in space	=	1,200		

In determining the direction of the above deviation, we must observe, according to our precepts, that the upper star precedes, and the observed interval is greater than the tabulated interval; therefore the deviation is to the east of north.

This method may now be practised very conveniently, as the apparent places of δ Ursæ Minoris and Cephei 51 *Hev.* are given in the Nautical Almanac. In like manner Polaris may be combined, though less advantageously, with the stars of the Great Bear.

Again, Polaris, or any close circumpolar star, the place of which is accurately known, may be combined with any star distant from the pole. The simplest mode of considering this is, that the star which is distant from the pole gives the *time* or error of the time-keeper; and, again, if Polaris give the same error, that the instrument must be in the meridian.* The formula for computation is the same as in the next following method, commonly called that of high and low stars, but is much more accurate.

The last method we shall speak of, for correcting the position of the instrument, is by observing the transit of any two stars differing from each other considerably in declination (at least 40°), and but little in right ascension. The nearer the right

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* Persons desirous of avoiding computation, and who do not want the greatest possible accuracy, may proceed conveniently thus: Get the error of the time-keeper from stars as near the zenith as may be, levelling with the utmost care before each observation, and reversing the instrument once during the series. By taking a mean of the whole, an excellent error of the time-keeper will be found, unaffected by errors of deviation or collimation; and, if the levelling have been performed with all care, of inclination too. With this error, find what time, by the time-keeper, Polaris, δ Ursæ Minoris, or Cephei 51 *Hev.*, should transit, and adjust the azimuthal screws accordingly. If the observer have made out, as he always ought to do, the time between each wire, and the middle wire, as well as the value of the revolutions of his adjusting-screw, he may compute the time for *each* wire, and examine his success at each, as the star passes through the field of the telescope. It is necessary to add that the level should always be examined after touching the azimuth-screws.

ascensions of the stars are to each other the better, as this prevents the possibility of any error arising from a change in the rate of the time-keeper affecting the observations. And, as the apparent places of one hundred principal stars are now given in the Nautical Almanac for every tenth day, it will be better to select a pair from thence, which will save the trouble of computing their apparent right ascensions; and, as many suitable pairs are contained therein, it will seldom happen but that the passage of some of them will occur at a convenient time for observation.

The times of the transits of the two stars being observed (without regard to the *error* of the time-keeper), the deviation of the instrument from the plane of the meridian may be thus determined: Take the difference between the observed passages of the two stars, and also the difference of their computed right ascensions (calling the differences + when the lower star precedes the higher, and *vice versâ)*; and if these differences be exactly equal, the instrument will be correctly in the plane of the meridian; if they be not equal, their difference, that is to say, the difference of the observed times of transit, *minus* the difference of the computed right ascensions, will point out a deviation from that plane, to the eastward of the south when it is +, and west when it is—. As an example let us take the following:

		Observed Time.		Apparent A.R.
		H. M. S.		H. M. S.
Higher star	... ...	5 46 51,91	...	5 46 53,50
Lower „	... ...	6 37 25,66	...	6 37 33,66
Difference	... ... =	– 50 33,75	...	– 50 40,16
Subtract Diff. of A. R. =		– 50 40,16		

+ 6,41 = the difference of time *minus* the difference of right ascension; which, being +, shews that the instrument deviates to the eastward of the south point of the horizon. It is evident that a high star will be less affected by deviation than one in any other situation, and that a star between the pole and zenith will be *differently* affected from a star south of the zenith, it being observed sooner than it ought when the latter is observed later, and *vice versâ.*

The deviation in azimuth may now be computed from the following formula:

Deviation in azimuth = D. sin. π sin. π' co-sec. $(\pi \mp \pi')$ sec. L.

In which D represents the difference of times *minus* the difference of right ascensions, π and π' the polar distances of the higher and lower stars, and L, as before, the latitude of the place of observation.

This formula, in words, gives the following rule: To the log. of the difference of times, *minus* the difference of right ascensions, add the log. sin. of the polar distance of the higher star, the log. sin. of the polar distance of the lower star, the log. cosecant of the difference or sum of the polar distances of both the stars (the difference when they are both above the pole, and the sum when one is above and the other below the pole), and the log. secant of the latitude: the sum will be the log. of the azimuthal deviation, which multiplied by 15 will be the deviation in arc.

As a complete example, let us take a high and low star, from the same day's work at Greenwich that we took our former example from, and see how nearly alike the deviation comes out by the two calculations.

March 18th, 1834. (See page 79.)

	Observed Time.		Apparent A.R. from Naut. Alm.
	H. M. S.		H. M. S.
Higher star, Cephei 51 *Hev.*	6 20 59,00	...	6 20 19,61
Lower ,, Sirius . . .	6 38 30,88	...	6 37 49,76
Difference . . . . =	− 17 31,88	...	− 17 30,15
Subtract diff. of A. R. =	− 17 30,15		
	− 1,73		

− 1,73 = the difference of time *minus* the difference of right ascension, which, being −, shews that the deviation is to the west of south, agreeing with our former determination (page 75), *viz.*, east of north. Let us now compute the deviation in azimuth by our last formula, and see how nearly they agree.

Difference of intervals	$1^s,73$ ..	log. ..	0·23805
π the P. D. of Cephei	2° 43′ 50″ ..	sine ..	8·67796
π' ,, Sirius	106 29 50 ..	sine ..	9·98174
$(\pi'-\pi)$ the diff. of Polar distances	..	co-sec.	0·01266
L = the latitude = 51° 28′ 39″	..	sec. ..	0·20564
		$\overset{s}{0},131$ =	9·11605

Multiply by . . 15

The azimuthal deviation in arc, west of south . . . . } = 1,965

"The time employed in making these observations is supposed to be sidereal time; therefore, if a clock or watch be used which marks mean solar time, the interval between the observations must be corrected accordingly." This correction is made by adding to the difference of the observed times the acceleration of the fixed stars for that interval, (Table IV.,) which will convert that portion of *mean* into an equivalent portion of *sidereal time;* so that, by means of this correction, it will be indifferent whether the clock shews sidereal or mean time.

"If, before or after the passage of the stars, the telescope be pointed to the horizon, and compared with some object there, a meridian mark may be set up, which may be corrected from time to time by subsequent observations on various stars similarly situated; and, when once *correctly* fixed, it will serve to verify both the meridional position of the instrument, and the adjustment of the collimation."

Having, by means of the previous adjustments, made the line of collimation, describe a great circle passing through the zenith of the place and the north and south points of the horizon: the instrument will be in a fit state for making observations. We have said that the telescope contains five vertical and two horizontal wires, placed a short distance from each other; these last are intended to guide the observer in bringing the object to pass across the middle of the field, by moving the telescope until it appears between them: the centre vertical is the meridional wire, and the instant of a star's passing it will be the time of such star's being on the meridian. But as, in noting the time, it will not often happen that an exact second will be shewn by the clock, when the star is bisected by the wire, but it will pass the wire in the interval between two successive seconds, the observer must therefore, whilst watching the star, listen to the beats of his clock, and count the seconds as they elapse; he will then be able to notice the space passed over by the star in every second, and consequently its distance from the wire at the second before it arrives at, and the next second after it has passed it, and with a little practice he will be able to estimate the fraction of a second at which the star was on the wire, to be added to the previous second: thus, suppose the observer counted 4, 5, 6, 7, 8 seconds, whilst watching the passage of a star, which passed the wire between the 7th and 8th, at which times it appeared equally distant on each side of it, the time of the transit would then be $7^s,5$; but if it appeared more distant on one side than the other, it would be $7^s, 3$, or $7^s, 7$, &c., according to its apparent relative distance from the wire.

This kind of observation must be made at each of the five wires, and a mean of the whole taken, which will represent the time of the star's passage over the mean or meridional wire. The utility of having five wires, instead of the central one only, will be readily understood from the consideration that a mean result of several observations is deserving of more confidence than a single one; since the chances are that an error which may have been made at one wire will be compensated by an opposite error at another,—thus destroying each other's effect: the mean result will come out very nearly the same as the observation at the middle wire, if they be made with any tolerable degree of accuracy, and if the intervals of the wires are uniform.

The annexed Table is an example of the Greenwich mode of registering observations made with a transit-instrument.

The heading at the top of the columns sufficiently explains the nature of their contents. The error of the clock from sidereal time is obtained, by taking the difference between the mean of the wires, and the apparent right ascension of the object as given in the Nautical Almanac; and the daily rate is the difference of such errors, divided by the number of days elapsed between the observations. In observing the sun, the times of passing of both the first and second limb over the wires are observed, and set down as distinct observations, the mean of which gives the time of the passage of the centre across the meridian, as is shewn in the annexed example. The wires of the instrument are generally placed by the maker at such a distance from each other, that the first limb of the sun shall have passed all of them before the second limb arrives at the first, and the observer can thus take the observations without hurry or confusion.

One limb only of the moon can be observed, except when her transit happens to be within an hour or two of her opposition: and in observing the larger planets, the first and second limb may be observed alternately over the five wires; that is to say, the first limb over three wires, *viz.*

Example of the Greenwich mode of registering Transit Observations.

Date.	Illuminated End—East.					Mean of Wires.	Clock.		No. of Days.	Object.
	I.	II.	Merid. Wire	IV.	V.		Error.	Rate.		
	s.	s.	M. s.	s.	s.	H. M. s.	M. s.	s.		
1834 Mar. 18	36,1	54,4	50 12,7	31,1	49,5	23 50 12,76	.. ..	..	..	☉ 1 Limb.
	45,2	3,5	52 21,9	40,3	58,6	23 52 21,90	.. ..	..	..	☉ 2 Limb.
	..	..	..	..	..	23 51 17,33	+0 41,32	+0,01	4	☉ Centre.
	14,1	40,5	5 6,6	32,7	59,1	5 5 6,60	+0 41,31	+0,05	3	Capella.
	..	..	20 59,0	..	..	6 20 59,00	.. ..	..	..	Cephei 51 *Hev.*
	..	..	26 32,5	..	..	6 26 32,50	.. ..	..	..	δ Urs. Min. S. P.
	52,6	11,8	38 30,7	50,1	9,2	6 38 30,88	+0 41,13	0,00	3	Sirius.

the first, third, and last, and the second limb over the second and fourth; which, being reduced in the same manner as the observation of the sun, will give the meridional passage of the centre. When an observation at one or more of the wires has been lost, it is impossible to take the mean in the same way as in a perfect observation. If the centre wire be the one that is deficient, the mean of the other four may be taken as the time of the meridional passage, or the mean of any two equally distant on each side of the centre, (supposing the interval of the wires to be equal); but when any of the side wires are lost, and indeed under any circumstance of deficiency in the observation, the most correct method of proceeding is as follows: By a considerable number of careful observations over all the wires, the equatorial interval between each side wire and the centre one is to be deduced and set down for future use. Then, when part of the wires only are observed, each wire is to be reduced to the mean, by adding to, or subtracting from, as the case may be, the time of observation, the equatorial interval between that wire and the centre wire, multiplied by the secant of the declination of the star, as in the following rule:

To the log. of the equatorial interval (from the wire at which the observation was made to the centre) add the log. secant of the star's declination (or co-sec. of its polar distance): the sum, rejecting ten from the index, will be the log. of the interval from the wire at which the transit was taken to the centre wire; which, being added to observations made at the first or second wire, or subtracted from those made at the fourth or fifth, will give the time of the star passing the meridional wire.

The equatorial intervals of the wires may readily be computed by the following rule, from observations made upon any star whose declination is known. To the log. of the interval occupied by the star in passing from any wire to the centre wire add the log. co-sine of the star's declination (or sine of its polar distance); the sum, rejecting ten from the index, will be the log. of the equatorial interval, which being determined for each wire, from observations of a number of stars having different declinations, the mean will be a very correct result. The equatorial intervals of the wires of the transit at the Royal Observatory were found to be—

s.
From the first wire to the third = 36,647
,, second ,, = 18,305
,, fourth ,, = 18,309
,, fifth ,, = 36,606

The middle wire at Greenwich coincides with the mean of the wires, the intervals being very nearly equal; but when this is not the case, the observer must correct the mean of the wires for the difference from the centre wire, to obtain a correct mean. The correction to be applied to the mean of the wires may be computed as follows: divide the difference between the sum of the first two and sum of the last two equatorial intervals by 5, and to the log. of the quotient add the log. co-secant of the polar distance of the star; the sum will be the log. of the correction required, *plus* if the sum of the two first intervals be greater than the second, otherwise *minus*. Such inequality in the intervals should never be allowed to remain, unless circumstances prevented their rectification.

In regular observatories, the transit-instrument is employed, not only for the determination of time, but in forming catalogues of the right ascensions of the fixed stars, and other important operations in astronomy,—purposes for which instruments of a superior class, and fixed in their respective places, are required. But, from the small size and low optical power of the portable transit-instrument, it can be applied with good effect only to the determination of time, and of the longitude by observations of the moon and moon-culminating stars. The Nautical Almanac contains the true apparent right ascension of the sun, and of one hundred of the principal fixed stars; that is, the sidereal time when each of them, respectively, is on the meridian, or on the centre wire of a properly adjusted transit-instrument: and if the instant when a star so passes the central wire be noted by a clock correctly adjusted to sidereal time, the time shewn by the clock will be the right ascension of the star as given in the Almanac. The difference therefore between the time shewn by a clock, and such right ascension, will be the error of the clock from sidereal time, + (or too fast) when the clock time is greater than the right ascension, and − (or too slow) when it is less. Thus, on March 18th, 1834, (page 79,)

	H.	M.	S.
The observed passage of Capella by clock . .	5	5	6,60
Right ascension by Naut. Alm.	5	4	25,29
Clock error	+	0	41,31

In the same manner the error of the clock is deduced from an observed transit of the sun's centre, the time of which, as before shewn, is derived from a mean of the observations of the first and second limbs; but when, from intervening clouds or other circumstances, one limb only can be observed, the passage of the centre may be found, by adding or subtracting *the sidereal time of the sun's semidiameter passing the meridian,* as given in the Nautical Almanac, according as the first or second limb may be observed.

If the clock error be determined in this manner from a number of observations on each day, the mean of the whole will probably be a very accurate determination of the error for the mean of the times at which the observations were made. In like manner the mean daily rate may be found by taking the difference between the errors, as determined by the same object from day to day, and, if more than one day have elapsed between the observations, dividing the change in the error by the number of days elapsed: the rate, when the clock is too fast, will be + (or gaining) when the second error is greater than the first, and − (or losing) when the second error is the least; and *vice versâ*, when the clock is too slow.

When a clock or chronometer, shewing mean solar time, is employed, its error from such time may be found by computing the mean time of the passage of the object over the meridian of the place; and the difference between such mean time, and the observed time of the object's meridian passage will, as before, be the error of the clock from mean time.

The following is the method of computing the mean solar time of the transit of a star across the meridian.

From the right ascension of the star subtract the sidereal time at mean noon for the given day, taken from the Nautical Almanac (adding 24 hours to the former when the latter exceeds it): the remainder is the sidereal interval after noon of that day. From this subtract the acceleration of sidereal upon mean time; and the result is the required mean solar time of the passage. As an example, suppose it were required to find the mean time of the passage of Capella on March 18th, 1834:

	H.	M.	S.
Right ascension of Capella (+ 24 hours)	29	4	25,29
Sidereal time at mean noon	23	42	15,64*
Sidereal interval, past noon . . . . . . =	5	22	9,65
Acceleration of sidereal on mean time for the interval . . . . =		–	52,78
Mean time of passage . . . . =	5	21	16,87

The acceleration of sidereal on mean time is to be taken from Table III.; thus, in the above example:

	M.	S.
Acceleration for 5 hours	0	49,148
" 22 minutes		3,604
" 9 seconds		0,025
" 65 hundredths . . .		0,003
For the whole interval =	0	52,780

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* The sidereal time, as given in the Nautical Almanac, is for mean noon at Greenwich, and therefore must be corrected for any other meridian, as directed in the explanation of the articles, given at the end of the Almanac.

Table III. will not answer for performing the reverse operation, *viz.*, converting a portion of mean solar time into a corresponding portion of sidereal time: Table IV. must be employed for this purpose, adding to the given portion of mean time the quantity taken from the table corresponding thereto; and the sum will be an equivalent portion of sidereal time. As an example we will take the above case of Capella.

	H.	M.	S.
Mean time	5	21	16,87
Table IV.	+		52,78
Sidereal interval	5	22	9,65
Sidereal time at mean noon	23	42	15,64
	29	4	25,29
	− 24		
Sidereal time of the star's passage, or its right ascension	5	4	25,29

The method of taking out the correction from Table IV. is exactly similar to that given in the above example for Table III.

To find the error of a clock or chronometer intended to shew mean time from an observed transit of the sun, nothing more is necessary than to apply the equation of time to 24 hours; and the difference between the result and the time of the sun's transit, as shewn by the chronometer, is the error of the chronometer for mean time, + when the chronometer time is the greater, and − when it is the less.

From the description which has been given of the method of bringing a transit-instrument into a state of perfect adjustment, it might be inferred that it is essential it should be strictly so, to obtain accurate results from the use of it. It is certainly desirable that the adjustments should be examined and rectified as often as possible, as doing so ultimately saves the labour of computing the corrections to be applied to each observation, on account of the errors in the position of the instrument. But in some established observatories, where large instruments are employed, it is not attempted to put them in perfect adjustment, but the amount of the various derangements is ascertained from time to time, and the observations corrected accordingly. The adoption of this method, with so small an instrument as the one which we have been describing, where the adjustments are easily examined and corrected, will give indeed more accurate results, but, on account of the greater trouble, is not perhaps to be generally recommended; we shall, nevertheless, introduce in this place an account of the method of computing these corrections, that persons possessing transit-instruments may adopt which method they think proper.

The first correction is for the deviation of the line of collimation: the amount of the error may be determined by a micrometer attached to the eye-end of the telescope, by which, when the telescope is directed towards any distant object, the angular distance of that object from the central wire is measured in revolutions and parts of the micrometer-screw. The instrument is then reversed, and the distance of the same object from the central wire again measured, when half the difference of the measures is the error of collimation; and the angular value of a revolution of the screw being known, the corresponding value of the error is likewise known. The correction on account of this error to be applied to the time of each observation may be computed from the following formula:—

$$\text{Correction} = \frac{c}{15} \text{ co-sec. } \pi$$

$c =$ the error of collimation, + if the deviation be toward the east.

$\pi =$ (as before) the polar distance of the star.

Hence we have in words this rule. To the log. of the deviation in collimation add the log. co-secant of the polar distance of the star, and the arithmetical complement of the log. of 15: the sum will be the log. of the correction in time required.

The next correction to be considered is that arising from a want of horizontality in the axis. The spirit-level, which we described as striding across the instrument and resting on the pivots, determines the amount of the inclination of the axis, and also, as we have seen, enables the observer to correct it. Above the glass tube, and parallel to its length, is placed a fine graduated scale, the reading of which points out the number of seconds in arc that the pivots deviate from the true level, shewn by the air-bubble receding from the centre towards that pivot which is the highest: but, as it is necessary, when correcting for the adjustment, to remove half the error, by giving motion to the little screw on the level itself, so for the same reason, in finding the measurement of the error, it is necessary to reverse the level on the axis, and read the scale at each extremity of the air-bubble in both its positions,—that is, with the same end of the level on both the east and west pivots alternately; and the difference of the sums of the two readings divided by the number of readings will be the amount of deviation. This may be illustrated by the following example, in which the divisions on the scale represent seconds.*

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* The value of the divisions of the scale may be had from the maker.

Readings of the Scale.

East End.		West End.
″		″
109,0	. . . . .	69,6
109,0	. . . . .	69,8
108,8	. . . . .	69,9
Level Reversed.		
69,0	. . . . .	109,0
68,6	. . . . .	108,9
69,1	. . . . .	109,0
533,5	Sums	536,2
		533,5
Divide by the number of observations, 12)		2,7

$\frac{1}{2}$ difference = 0,23 = the amount of deviation in arc, shewing that the west end of the axis is higher by that quantity than the east end, since the sum of the western readings is greater than the sum of the eastern. This quantity divided by 15 will give the proper factor for inclination. It is more convenient that the scale should be divided into units, each of which is 15″.

Having in this manner determined the inclination of the axis by the level, the correction to be applied to the time of observation of any star made during the existence of that error may be computed from the following formula :—

$$\text{Correction} = b \cos.\ (\pi-\lambda) \text{ co-sec. } \pi$$

b = the factor for the inclination of the axis, + if the west end be too high.

π = the polar distance of the star.

λ = the co-latitude of the place.

This formula in words gives the following practical rule. To the log. of the factor for inclination of the axis add the log. co-secant of the polar distance, and the log. co-sine of the difference between the polar distance and the co-latitude : the sum −20 will be the log. of the correction in time required.

We have already explained the manner of ascertaining the azimuthal deviation of the instrument from the plane of the meridian, page 72, &c. The correction to be added algebraically to the observed time of transit of any star whilst the instrument so deviates may be computed from the following formula :—

$$\text{Correction} = a \sin.\ (\pi-\lambda) \text{ co-sec. } \pi$$

in which a = the factor for azimuthal deviation, + when the instrument deviates to the eastward of the south meridian.

π = the polar distance of the star.
λ = the co-latitude of the place.

This formula in words gives the following

Rule.—To the log. of the factor for azimuthal deviation add the co-secant of the polar distance, and the sine of the difference between the polar distance and co-latitude: the sum will be the log. of the correction required.

As an example, let us take the star ϵ Bootis. (Pearson's Astron. vol. ii. p. 344.)

H. M. S.
Observed time of transit . . . = 14 35 4, 86

Error of collimation . . . 12″ or = + 0, 80
Inclination of the axis . . . = − 1, 75
Deviation of instrument in azimuth . = − 4,737
The errors are in units, each of which = 15″

° ′
Polar distance . . . . = 62 12
Co-latitude . . . . . = 38 27

The Correction for the Collimation.

Deviation = + 0,80 log. . . . . . = + 9·90309
Polar dist. 62° 12′ co-secant . . . = + 0·05326

Correction = + $0^s,904$ . . log. = + 9·95635

The Correction for the Level.

Deviation = −1,75 log. . . . . . = − 0·24304
Polar dist. 62° 12′ co-secant . . . = + 0·05326
Polar dist. minus co-lat. 23° 45′ co-s. . . = + 9·96157

Correction = −$1^s,811$ . . log. = − 0·25787

The Correction in Azimuth.

Deviation = −4,737 log. . . . . . = − 0·67550
Polar dist. 62° 12′ co-secant . . . = + 0·05326
Polar dist. minus co-lat. = 23° 45′ sine . = + 9·60503

Correction = − $2^s,157$ . . log. = − 0·33379

Now apply the sum of these corrections to the observed time of the star's transit, and the actual time of transit will be obtained as correctly as if the instrument had been in a state of perfect adjustment when the observation was made.

	H.	M.	S.
Observed time of transit. . . . . . =	14	35	4,860
Correction for the collimation . . =	+		0,904
„ „ level . . . . . =	−		1,811
„ in azimuth . . . . . . =	−		2,157
Corrected observation . . . . =	14	35	1,796
Computed right ascension . =	14	37	28,910
Clock slow on sidereal time . =		2	27,114

Besides the determination of time, the portable transit-instrument may be successfully employed in determining the longitude. The Nautical Almanac contains, for each lunation, a list of the right ascensions and declinations of the moon-culminating stars, whose meridional transits being observed, together with that of the moon, at any two places, the differences of right ascension thus obtained between the moon's illuminated limb and each of those stars form the data required for computation. "If the moon had no motion, the difference of her right ascension from that of a star would be the same at all meridians, but in the interval of her transit over two different meridians her right ascension varies, and the difference between the two compared differences exhibits the amount of this variation, which, added to the difference of meridians, shews the angle through which the westerly meridian must revolve before it comes up with the moon; hence, knowing the rate of her increase in right ascension, the difference of longitude is easily obtained."

The necessity of having recourse to actual observation of the same stars at the two places, in order to obtain the longitude, may soon be dispensed with, since their apparent right ascensions are given in the Nautical Almanac. At present, however, and until the places of the moon-culminating stars are perfectly well known, corresponding observations are required for the accurate determination of differences of longitude.

The difference of longitude between the stations is supposed to be approximately known, or may be got near enough for an approximation by dividing the difference between the observed and computed right ascension of the moon's bright limb by the hourly motion given in the Nautical Almanac.

The formula for computation, with the necessary explanation, may be found in the Memoirs of the Royal Astronomical Society, vol. ii. p. 1, &c. Availing myself of the kind permission of Mr. Riddle, I am enabled to insert his method of performing the computation, together with Table XXXIII. of his valuable treatise on Navigation.

Practical Rule.

To the estimated longitude in time add the correction from Table XI., and apply the sum to the time of the moon's passing the meridian of Greenwich, as given in the Nautical Almanac, adding if the longitude be *west*, or subtracting it if east; and the sum or the remainder will be the approximate Greenwich date for the moon's passing the given meridian.

Find the moon's right ascension, both for this time and the time of her passing the meridian of Greenwich, and divide the difference of her right ascensions by the hours, &c., in the difference of these times; and the quotient will be the mean hourly change of the moon's right ascension in the interval, which is the argument of Table V.

Take also the declinations roughly for the same two times.

With the mean of these declinations, and the change of the moon's semidiameter, take the correction from Table VI. and apply it to the interval between the transits of the star and the moon's bright limb, as observed at or computed for the more westerly meridian.

Again, with the mean of the declinations take the corrections from Table XII., and, multiplying it by the degrees in the moon's change of declination, apply the product as a second correction to the western interval.

The following formula will shew the signs with which these corrections are to be applied.

Sign of First Correction.

	Moon.	Limb Obsd.	Correct.
Moon's semidiam. increasing	preceding star	W	−
		E	+
	following star	W	+
		E	−

	Moon.	Limb Obsd.	Correct.
Moon's semidiam. decreasing	preceding star	W	+
		E	−
	following star	W	−
		E	+

Sign of Second Correction.

	Moon.	Limb Obsd.	Correct.
Moon's declination increasing	preceding star	W	+
		E	−
	following star	W	−
		E	+

	Moon.	Limb Obsd.	Correct.
Moon's declination decreasing	preceding star	W	−
		E	+
	following star	W	+
		E	−

The change of semidiameter here spoken of is that taken from the Ephemeris, without augmentation for altitude.

The interval at the more westerly meridian being thus corrected, call the seconds of the differences of the intervals, A; or, if more than one star have been observed, call the seconds in the mean of the differences of the corresponding intervals A.

If either of the intervals be in mean time, add to it its 360th part diminished by the 70th part of itself, and the sum will be the corresponding interval in sidereal time. And if both are in mean time, reduce their difference to sidereal time by the same rule. Table IV. may also be used for this purpose.

If the moon precede the star at the easterly and follow it at the westerly meridian, the *sum* of the intervals, instead of the *difference*, will be A.

Then *add* the *logarithm of the seconds in* A, the difference of the *sidereal intervals*, to the *logarithm* from *Table V.*, and the sum will be the *logarithm* of the difference of longitude in *seconds of time.*

Note. The parts for hundredths in Table V. are found in the column of 'parts' opposite the corresponding tenths. Thus, for $1^{m}\,42^{s},57$, the log. for $1^{m}\,42^{s},5$, is 1·534256, and the part for seven hundredths is 304; whence the log. is 1·533952. Striking off the figures on the right, in the column of 'parts,' the remaining figures on the left are parts for thousandths.

Example.

December 8th, 1834.

Star, &c.	Clock Transit observed at Greenwich.			Rate of Clock.	Clock Transit observed at Cambridge.			Rate of Clock.
	H.	M.	S.	s.	H.	M.	S.	s.
96 Aquarii	23	10	58,18	— 0,68	23	10	14,40	— 2,56
n Piscium	23	39	35,32		23	38	51,72	
☽ 's 1st Limb ..	23	47	29,86		23	46	45,52	
s Piscium	23	57	1,18		23	56	17,53	
n Cœti	0	21	45,08		0	21	1,32	

First find the mean intervals between the passage of the stars and the moon at both places, thus:—

Greenwich Intervals.		Cambridge Intervals.	
M.	S.	M.	S.
36	31,68	36	31,12
6	54,54	7	53,80
9	31,32	9	32,01
34	15,22	34	15,80

Intervals corrected for Rate.

36	31,70	36	31,18
7	54,54	7	53,81
9	31,32	9	32,03
34	15,23	34	15,85

Diff. Intervals.
·52
·73
·71
·62
Mean . . . ·65

On December 8th, 1834, the moon passed the meridian of Greenwich at 6^h 40^m, the declination being then about 7°, and it would be about one-thousandth of a degree different at Cambridge; and the 1000th part of ·134 (the number corresponding to 7° of declination in Table XII.) is too small a quantity to be worth attention. This also is the case with the effect of the change in the moon's semidiameter, the change being not more than a thousandth of a second of space; and the effect of that small change on the time of the moon's transit being clearly beyond the reach of notice in ordinary observers. The Nautical Almanac gives the following:—

			M.	S.
Hourly change of ☽'s R.A. from	5 hours to	6	1	50,19
„ „	6 „	7	1	50,03
„ „	7 „	8	1	49,87

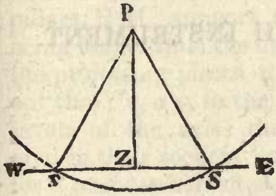
Hence, at 6^h 40^m the hourly rate of change would be about 1^m 50^s,08.

M.	S.		
1	50,08	Table V.	1·502334
	,65	log. . . . . =	9·812913
Longitude of Cambridge in time 21^s,3			1·315247

The longitude of the Cambridge Observatory has been determined by Professor Airy to be 23^s,5. The reader may perhaps be surprised that the above result differs 2^s, 8 from it; but it may be remarked that, by this method of finding longitude, it is absolutely necessary that a great number of results be taken as a satisfactory determination. This arises mostly from the errors made in observing the transit of the moon's limb, which it is well known to practical men is a very difficult observation to make correctly; and a very small error in the observation makes a considerable one in the final result: supposing the transits of the stars to have been observed *perfectly correct*, yet, if an error of only two-tenths of a second be made in that of the moon's limb at either observatory, the longitude deduced from such observation would be incorrect to the amount of 6 seconds in time, at a mean rate of the moon's motion. When both limbs of the moon can be observed at both observatories, which can only be the case when she is near the full at the time of transit, a better result can be obtained.

There is a mode of finding the latitude by the transit-instrument, pointed out by Professor Bessel, and used with great success in the Russian Survey, which we will now explain in some detail, as the method is not so commonly known or practised in this country as it deserves to be.

Place the transit-instrument with its supports north and south, so that the telescope when pointed to the horizon looks due east and west. Observe the passage of a well-known star over the middle wire when the telescope is pointing east, and again observe the passage of the same star over the middle wire when the telescope is pointing west, noting the time carefully. The star should be near the zenith, (such a star as γ Draconis, for instance, in the latitude of London, and for a degree or two to the northwards,) as the observations take less time, and are therefore more independent of the timekeeper employed; the method is also more accurate when the star is near the zenith than when otherwise.



In the accompanying figure, P is the pole, Z the zenith, E Z W the prime vertical passing through the east and west points, the dotted line S *s* the path of the star; all seen as projected on the horizon from a point above Z. Then in the right-angled spherical triangle, P Z S, P S is the north polar distance of the star, P Z the co-latitude, and the angle Z P S half the time elapsed from S to *s*; therefore, tan. P Z = tan. P S × cos. Z P S.

Let Δ'' = half the interval in time reduced to arc between the two transits of the star over the prime vertical (a circle which passes through the zenith, and east and west points of the horizon).

π = the N. P. D. of the star, taken from the Nautical Almanac).

λ = the co-latitude of the place.

then tan. λ = tan. π cos. Δ''

or, in words, to the log. tangent of the star's N. P. D. add the log. co-sine of half the time elapsed; and the sum − 10 will be the log. tangent of the co-latitude required.

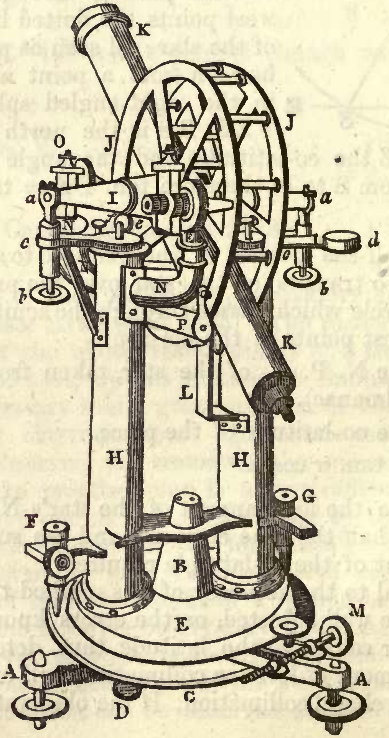
It is essential to the accuracy of this method that the instrument should be well adjusted, or the errors known and allowed for. The error caused in the latitude thus determined, by the want of adjustment of level or collimation, will exactly equal the error of the level and collimation. If the observation be repeated on various nights, the telescope should be reversed. With these precautions, and a level of the best kind, the latitude may be obtained within a second or two, if the place of the star be sufficiently well known, and *differences* of latitude, whether the star be known or not.

To find the time when the star will come to the proper position for observation, *viz.*, the prime vertical: first ascertain when the star will be on the meridian, by the method explained at page 82; then, by the following formula compute the time that would elapse during the passage of the star from the prime vertical to the meridian, or (referring to the preceding diagram) the angle, S P Z; which *time*, subtracted from that of the meridian passage, will give the time of the star being on the prime vertical, or in the required position for making the first observation.

Formula, co-s. S P Z (or Δ'') = tan. P.Z, co-t. P S.

Practical Rule. To the log. tangent of the assumed co-latitude add the log. co-tangent of the star's polar distance; the sum will be the log. co-sine of half the elapsed time in arc, which divided by 15 will give the time required.

THE ALTITUDE AND AZIMUTH INSTRUMENT.



To the centre of the tripod, A A, is fixed the vertical axis of the instrument, of a length equal to about the radius of the circle: it is concealed from view by the exterior cone, B. On the lower part of the axis, and in close contact with the tripod, is centered the azimuth circle, C, which admits of a horizontal circular motion of about three degrees, for the purpose of bringing its zero exactly in the meridian: this is effected by a slow-moving screw, the milled head of which is shewn at D. This motion should, however, be omitted in instruments destined for exact work, as the bringing the zero into the meridian is not requisite either in astronomy or surveying: it is, in fact, purchasing a convenience too dearly, by introducing a source of error not always trivial. Above the azimuth circle, and concentric with it, is placed a strong circular plate, E, which carries the whole of the upper works, and also a pointer, to shew the degree and nearest five minutes to be read off on the azimuth circle; the remaining minutes and seconds being obtained by means of the two reading microscopes, F and G: this plate, by means of the conical part, B, (which is carefully fitted to the axis) rests on the axis, and moves concentrically with it. The conical pillars, H H, support the horizontal or transit axis, I, which being longer than the distance between the centres of the pillars, the projecting pieces, *c c*, fixed to their top, are required to carry out the Y's, *a a*, to the proper distance, for the reception of the pivots of the axis: the Y's are capable of being raised or lowered in their sockets by means of the milled-headed screws, *b b*, for a purpose hereafter to be explained. The weight of the axis, with the load it carries, is prevented from pressing too heavily on its bearing by two friction rollers, on which it rests; one of which is shewn at *e*. A spiral spring, fixed in the body of each pillar, presses the rollers upwards, with a force nearly a counterpoise to the superincumbent weight: the rollers, on receiving the axis, yield to the pressure, and allow the pivots to find their proper bearings in the Y's, relieving them, however, from a great portion of the weight.

The telescope, K, is connected with the horizontal axis, in a manner similar to that of the portable transit-instrument. Upon the axis, as a centre, is fixed the double circle, J J, each circle being close against the telescope, and on each side of it: the circles are fastened together by small brass pillars. By this circle the vertical angles are measured, and the graduations are cut on a narrow ring of silver, inlaid on one of the sides, which is usually termed the *face* of the instrument,—a distinction essential in making observations. The clamp for fixing, and the tangent-screw for giving a slow motion to the vertical circle, are placed beneath it, between the pillars, H H, and attached to them, as shewn at L. A similar contrivance for the azimuth circle is represented at M. The reading microscopes for the vertical circle, are carried by two arms, bent upwards near their extremities, and attached towards the top of one of the pillars. The projecting arms are shewn at N, and the microscopes above, at O.

A diaphragm, or pierced plate, is fixed in the principal focus of the telescope, on which are stretched five vertical and five horizontal wires: the intersection of the two centre ones, denoting the optical axis of the telescope, is the point with which a terrestrial object is bisected, when observing angles for geodetical purposes. The vertical wires are used for the same purpose as those in the transit telescope, and the horizontal ones for taking altitudes of celestial objects. A micrometer, having a moveable wire, is sometimes attached to the eye-end of the telescope, but it is not generally applied to instruments of portable dimensions. The illumination of the wires at night is by a lamp, supported near the top of one of the pillars, as at *d*, and placed opposite the end of one of the pivots of the axis, which, being perforated, admits the rays of light to the centre of the telescope-tube, where, falling on a diagonal reflector, they are reflected to the eye, and illuminate the field of view: the whole of this contrivance is precisely similar to that described as belonging to the transit-instrument.

The vertical circle is usually divided into four quadrants, each numbered, 1°, 2°, 3°, &c., up to 90°, and following one another in the same order of succession: and consequently, in one position of the instrument, altitudes are read off, and with the face of the instrument reversed, zenith distances; and an observation is not to be considered complete, till the object has been observed in both positions. The sum of the two readings will always be 90°, if there be no error in the adjustments, in the circle itself, or in the observations.

It is necessary that the microscopes, O O, and the centre of the circle, should occupy the line of its horizontal diameter; to effect which, the up-and-down motion (before spoken of) by means of the screws, *b b*, is given to the Y's, to raise or lower them until this adjustment is accomplished. A spirit-level, P, is suspended from the arms which carry the microscopes: this shews when the vertical axis is set perpendicular to the horizon. A scale, usually shewing seconds, is placed along the glass-tube of the level, which exhibits the amount, *if any*, of the inclination of the vertical axis. This should be noticed repeatedly whilst making a series of observations, to ascertain if any change have taken place in the position of the instrument after its adjustments have been completed. One of the points of suspension of the level is moveable, up or down, by means of the screw, *f*, for the purpose of adjusting the bubble. A striding-level, similar to the one employed for the transit-instrument, and used for a like purpose, rests upon the pivots of the axis. It must be carefully passed between the radial bars of the vertical circle to set it up in its place, and must be removed as soon as the operation of levelling the horizontal axis is performed. The whole instrument stands upon three foot-screws, placed at the extremities of the three branches which form the tripod;* and brass cups are placed under the spherical ends of the foot-screws. A stone pedestal, set perfectly steady, is the best support for this as well as the portable transit-instrument.

Of the Adjustments.

The first adjustment to be attended to, after setting the instrument up in the place where the observations are to be made, is to set the azimuthal or vertical axis truly perpendicular to the horizon: the method of doing this is to turn the instrument about until the spirit-level, P, is lengthwise in the direction of two of the foot-screws, when by their motion the spirit-bubble must be brought to occupy the middle of the glass-tube, which will be shewn by the divisions on the scale attached to the level. Having done this, turn the instrument half round in azimuth, and if the axis be truly perpendicular the bubble will again settle in the middle of the tube, but if not, the amount of deviation will shew double the quantity by which the axis deviates from the vertical in the direction of the level: this error must be corrected, one half by means of the two foot-screws (in question), and the other half by raising or lowering the spirit-level itself, which is done by the screw represented at *f*. The above process of reversion and levelling should be repeated, to ascertain if the adjustment have been correctly performed; for, as we before observed, when speaking of the transit-instrument, adjustments of every kind can be made perfect only by successive trials and approximations.

Next turn the instrument round in azimuth a quarter of a circle, so that the level, P, shall be at right angles to its former position: it will then be over the third foot-screw, which may be turned until the air-bubble is again central, if not already so; and this adjustment will be completed. If delicately performed,

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* The foot-screws are sometimes made in the following ingenious manner, as described by MR. TROUGHTON, in the Memoirs of the Astronomical Society, vol. i. p. 37. "Each of the three screws is double, that is, a screw within a screw: the exterior one, as usual, has its female in the end of the tripod, and the female of the interior screw is within the exterior; the interior one is longer than the other, its flat end rests on a small cup on the top of the support, and its milled head is a little above the other. Now by this arrangement we gain three distinct motions: for by turning both screws together an effect is produced equal to the natural range of the exterior screw; by turning the interior one alone the effect produced is what is due to this screw; and by turning the exterior one alone (which may be done, because the friction of the interior screw in the cup is greater than that which exists between the two screws,) an effect is produced equal to the difference of the ranges of the two screws. Thus, were the exterior one to have 30 turns in an inch, and the interior 40, the effect last described will be exactly equal to what would be produced by a simple screw of 120 threads in an inch."

the air-bubble will steadily remain in the middle of the level during an entire revolution of the instrument in azimuth. These adjustments should be first performed approximately; for if the third foot-screw be much out of the level it will be impossible to get the other two right. The vertical axis is now adjusted.

The next adjustment is to set the vertical circle at such a height that its two reading microscopes shall be directed to two opposite points in its horizontal diameter, which is done by raising or lowering the Y's which carry the horizontal axis.

The next adjustment is the levelling of the horizontal axis by means of the striding-level, the whole of which operation is in all respects the same as that described for levelling the transit axis, to which therefore the reader is referred. After performing this, the preceding adjustment must be examined, as it will probably be deranged. Indeed, it is better first to set the axis horizontal, and then, by equally raising or depressing the two ends, to bring the microscopes into a diameter, and finally level again.

The adjustment for the line of collimation requires not only that the middle vertical wire shall describe a great circle, but that the middle horizontal wire shall have a definite position with respect to the divisions of the limb. It is usual to rectify the position of one of these at a time, taking the middle vertical wire first.* The error of this wire is ascertained and corrected precisely in the same manner as that of the transit-instrument, with this difference, that, instead of taking the axis out of its bearings and turning it end for end, the whole instrument is turned half round in azimuth, which is an equivalent operation. The middle horizontal wire may be adjusted in the following manner: "Point the telescope to a very distant object, bisect it by the middle horizontal wire (near the intersection of the wires,) and read off by the microscopes the apparent zenith distance; now reverse the instrument in azimuth, and turning the telescope again upon the same object, bisect it as before, and again read off the angle which they show. One of these angles will be an altitude, and the other a zenith distance;" and, if there be no error, the sum of the two readings will be 90°, and half of what it differs from 90° will be the error of collimation, which may be either applied to correct any observation made during its existence, or removed in the following manner. One of the readings being the zenith distance, and the other the altitude of the object, reduce the zenith distance to an altitude, or *vice versâ*, and take the mean: it is evident that "the mean of the two will be the true zenith distance or altitude respectively; and while the telescope bisects the object, the microscopes must be ad-

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* We speak of the middle wire only, as the side wires are supposed to be fixed parallel to it by the maker, and cannot be adjusted by the observer.

justed by their proper screws, so as to shew that mean. This process may be repeated for obtaining a greater degree of accuracy; but its final determination should be deduced from observations upon many heavenly bodies, and the minute error that may remain unadjusted had better be allowed for." This and the preceding operation may be more conveniently performed by a collimating telescope.

The adjustment for setting the cross-wires truly vertical is the same as that described as belonging to the portable transit: the position of the horizontal wires will then depend on the maker, or the horizontal wire may be put right by making it thread an equatorial star at its transit, when the vertical wires will depend upon the maker.

In conclusion, it may be observed that, during a series of observations, if the instrument should be detected to be a small quantity out of level, (having previously gone through the principal adjustments,) it may generally be restored by means of the foot-screws only, when they require but a slight touch to effect it: this is more especially essential when the level of the horizontal axis is the one deranged, as correcting it by moving the Y's would derange the adjustment of the vertical circle with regard to its reading microscopes, the construction and adjustments of which it will next be necessary to describe.

The error of the vertical axis is to be detected by the hanging level, and can very readily be allowed for in computing the observation: as a general rule, when great accuracy is required, it is easier and safer to adjust by computation than by mechanical contrivances.

THE READING MICROSCOPE.

Fig:1.

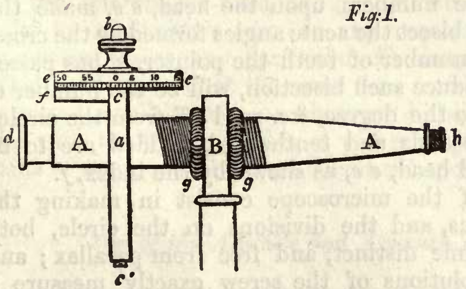
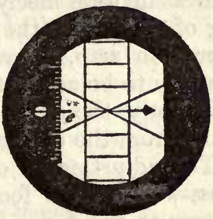


Fig 2.



The divisions on the graduated circle indicate spaces of five minutes each, which are read off along with the *degree*, by means of an index-pointer. The remaining minutes and seconds, if any, are determined by the reading microscope, as was stated when describing the construction of the circle: it now remains to explain the principal parts of the micrometer, the method of adjusting it, and its application to practice. A A, fig. 1, represents the microscope, passing through a collar or support, B, where it is firmly held by the milled nuts, *g g,* acting on a screw cut on the tube of the microscope. These nuts also serve for placing the instrument at the proper distance, for distinct vision, from the divisions it is employed to read. In the body of the microscope, at *a,* the common focus of the object and eye-glasses, are placed two wires, crossing each other diagonally, and they are made to traverse the field of view either up or down, by turning the micrometer-screw, *b,* working in the box, *c c′*. Fig. 2 shews the field of view, with the magnified divisions on the instrument, as seen through the microscope. The shaded part represents the diaphragm, with the cross-wires, the angle made by which may, by turning the micrometer-screw, *b,* be bisected by any line on the circle in the field of view, as is shewn in the figure. On the left hand of the diaphragm appears the comb or scale of minutes, each of the teeth representing one minute. Moveable with the wires along the comb is a small index or pointer, *e,* which in the figure is represented at zero, the centre of the scale, as is shewn by its bisecting the small hole at the back of the comb. Now one revolution of the screw, *b,* moves the wires and the pointer over one tooth of the comb, that is, over a space equal to one minute; and part of a revolution moves them but a fraction of a minute. To determine this fractional quantity, a large cylindrical head, *e e,* is attached to the screw, having its edge divided into 60 equal parts, representing seconds, the index being fixed opposite the eye of the observer, at *f*. In reading off an angle by this instrument, observe first the degree and nearest five minutes shewn by the pointer on the graduated circle; then apply to the microscope, and by turning the screw, *b,* in the order of the numbers upon the head, *e e,* make the nearest division nicely bisect the acute angles formed by the crossing of the wires: the number of teeth the pointer, *e,* has passed over from zero, to produce such bisection, will be the number of minutes to be added to the degree, &c. read off from the circle; and, lastly, the odd seconds and tenths to be added are to be taken from the divided head, *e e,* as shewn by the index, *f*.

The adjustments of the microscope consist in making the cross-wires in its focus, and the divisions on the circle, both appear at the same time distinct, and free from parallax; and also making five revolutions of the screw exactly measure a five minute space on the graduated circle. For the former of these adjustments, draw out the eye-piece, *d,* until distinct vision of the wires is obtained, and observe if the divisions of the instrument be also well defined, and whether any motion of the eye causes the least apparent displacement or parallax of the wires with respect to the graduations. If such a dancing motion be found, the microscope must be moved to or from the circle, by turning the nuts, *g g*, unscrewing one and screwing the other, until the wires and graduations both appear distinct, and no parallax can be detected.

Next, to examine and adjust the *run* (as it is termed) of the screw. If the run have been carefully adjusted by the maker, and no alteration made in the body of the microscope, the image of the space between two of the divisions will be exactly equivalent to five revolutions of the screw, when the wires and divisions are both seen distinctly. Let us, however, suppose that the length of the microscope has been deranged, and that the run is too great; for example, that the space of 5′ on the limb is equal to 5′ 10″ by the micrometer, or that the image is too large. Now the magnitude of the image formed by the object-glass of the microscope depends entirely on the distance of the object-glass from the limb, and is diminished (in the ordinary construction of the microscope) by increasing the distance between the limb and the object-glass, and *vice versâ*. In the case supposed, the image is too large, therefore the object-glass, *h*, must be removed further from the limb. Let this be done by turning the screw at *h* in or towards B. The image now will not be formed at *a*, as it ought to be, but nearer to B; and distinct vision must be gained by bringing the whole body of the microscope nearer to the limb. In this way, by two or three attempts cautiously conducted, we shall make five revolutions of the cross-wires correspond exactly with the image of the space between two divisions; and, for greater accuracy, the 5′ should be read on each side zero, or 10′ on the limb made equal to 10 revolutions of the micrometer.

The screw, *c*′, gives motion to the comb or scale of minutes; and the micrometer-head, being adjustable by friction, can be made to read either zero, or any required second, when the cross-wires bisect any particular division, by holding fast the milled-head, *b*, and at the same time turning the divided head, *e e*, round, until its zero, or whatever division you require, coincides with the index, *f*: this, it will readily be perceived, is the means of accomplishing the adjustment spoken of at page 96.

Use of the Altitude and Azimuth Instrument.

This is the most generally useful of all instruments for measuring angles, being applicable to geodetical as well as astronomical purposes. In the hands of the surveyor it becomes a theodolite of rather large dimensions, measuring with great accuracy both vertical and horizontal angles. It does not possess the power of repetition: but the effect of any error of division on the azimuthal circle may be reduced or destroyed by measuring the same angle upon different parts of the arc; thus,—After each observation turn the whole instrument a small quantity on its stand, and, adjusting it, again measure the required angle. A fresh set of divisions is thus brought into use at every observation, and the same operation being repeated many times, where great accuracy is required, the mean result may be considered as free from any error that may exist in the graduation. A repeating stand has, of late years, been frequently added to this instrument, and is a most powerful and convenient appendage, when great accuracy is required in the measurement of azimuthal angles. The two opposite micrometers being read off at each observation, will always remove the effect of any error in the centring. The vertical angles should, in all cases, be taken twice, reversing the instrument before taking the second observation, when (as before observed) one of the readings will be an altitude, and the other a zenith distance: the sum of the two readings, therefore, if the observation be made with accuracy, and no error exists in the adjustments or the instrument, will be exactly 90°; and whatever the sum differs from this quantity is double the error of the instrument in altitude, and half this double error is the correction to be applied + or — to either of the separate observations, to obtain the true altitude or zenith distance, + when the sum of the two readings is less than 90°, and — when greater.

In applying the instrument to astronomical purposes, it was formerly the custom to clamp it in the direction of the meridian, and, after taking an observation, or series of observations, with the face of the instrument one way, to wait till the next night, or till opportunity permitted, and then take a corresponding series of observations of the same objects, with the face of the instrument in a reversed position. But this method being attended both with uncertainty and inconvenience, it is now usual to complete at once the set of observations, by taking the altitudes in both positions of the instrument as soon as possible after each other. When the meridian altitude is required, several observations may be taken, a short time both before and after the meridional passage, with the face of the instrument in one direction, and with it reversed, noting the time at each observation; and if we have the exact time of the object's transit, its hour angle in time, or its distance from the meridian at the moment of each observation, may be deduced. This, with the latitude of the place (approximately known) and the declination of the object, affords data for computing a quantity called the reduction to the meridian, which, added to the mean of the observed altitudes, when the object is above the pole, and subtracted when the object is below the pole, will give the meridional altitude of the object, and *vice versâ* for zenith distances. The nearer the observations are taken to the meridian the less will the results depend upon an accurate noting or knowledge of the time.

To compute the Reduction to the Meridian.

Practical Rule. Take from Table VII. the natural versed sines of the hour-angles, or times of each observation from the time of transit separately, and take their mean; then, to the log. of this mean add the log. co-sine of the assumed latitude, the co-sine of the declination, the co-secant of the meridian zenith distance, and constant log. 9.31443: the sum, rejecting the tens from the index, will be the log. of the reduction in seconds of space.

The meridional zenith distance employed in the computation need only be approximate; if the latitude of the place and the declination of the body be nearly known, the meridional zenith distance will be equal to the difference between the latitude and the declination when both are north or both south, but equal to their sum when one is north and the other south: and the meridian zenith distance of an object below the pole is equal to the difference between 180° and the sum of the latitude and declination.

As an example, we shall take that given in Woodhouse's Astronomy, vol. i., page 422, of the star Arcturus, as observed at the Dublin Observatory.

	Face of Inst.	Observed Alt.	Hour Angle in Time.	Versed Sine.
		° ′ ″	m. s.	
East of meridian	E.	56 40 5,2	10 35 3	1067
	E.	42 22,9	6 35 3	0413
West of meridian	W.	45 10,0	2 47 7	0074
	W.	43 23,1	7 48 7	0580

Means.......	56 42 45,3	....	533,5	log.	=	2·7271
Reduction.. +	1 52,4					
		Latitude	53° 23′	cos.	=	9·7756
	56 44 37,7	Declination..	20 7	cos.	=	9·9727
Refraction	37,8	Mer. Z. D.=	33 16	cos.	=	0·2608
				Constant log.	=	9·3144
Meridian Alt. .	56 43 59,9					
		Reduction ..	1′ 52″, 4=112″,4	log.		2,0506

If the star be supposed known, the meridian altitude thus determined may be employed in correcting an assumed latitude; or, if the latitude be known, the star's declination may be obtained.

The latitude of a place is its distance from the equator, north or south, and it is equal to the elevation of the celestial pole above the horizon, or to an arc of the meridian contained between the zenith and the celestial equator; which arc can readily be determined by observing the greatest or meridional altitude of a celestial object whose declination at the time is known: for when the declination is greater than the zenith distance, both being of the same denomination (either both north or both south), the latitude will be equal to the declination, *minus* the zenith distance. When the declination and zenith distance are of contrary denominations, then the declination *plus* the zenith distance will be the latitude. And, lastly, when the zenith distance is greater than the declination, then the zenith distance *minus* the declination will be the latitude. And always of the same denomination as the greater of the two.

Another method of determining the latitude is, by observing the meridional zenith distance of a circumpolar star, both at its upper and lower culmination; then, computing the refraction for each observation, the co-latitude will be equal to half the sum of the two zenith distances added to half the sum of the two refractions. The latitude thus obtained does not depend on a previous knowledge of the declination of the object observed.

The method of determining the latitude by an observation of the altitude of the pole-star *at any time of the day,* together with the necessary tables, is given in the Nautical Almanac (as newly arranged).

A very successful and useful application of this instrument is the determination of time and the direction of the true meridian by equal altitudes and azimuths: the method of conducting a series of observations of this kind has been so clearly explained by the late Mr. Wollaston, in his *Fasciculus Astronomicus,* that we shall at once transcribe it nearly in the author's own words.

"In the morning, two or three or more hours before noon, let him (the observer) point the telescope towards the sun, and a little above it, and, clamping the vertical circle, let him follow the sun till its upper limb just touches the first horizontal wire. Then, noting down the exact second of time, as shewn by his chronometer, when that happened, let him follow the sun again till its upper limb just arrives at the second horizontal wire. After setting that down, as before, let him prepare for the third or central wire: by now clamping the instrument in azimuth likewise, and holding its adjusting-screw between his finger and thumb, let him bring the preceding limb of the sun just to touch the third or central perpendicular wire, at the same instant that the upper limb just touches the third or central horizontal one. Noting that instant, and setting it down, let him now read off the azimuth marked on the azimuth circle, and set it down under the other, and then prepare for making the preceding limb to touch the fourth perpendicular wire, at the same instant that the upper limb arrives at the fourth horizontal one: setting that time down again, and reading off the azimuth again, and setting it down, let him do the same by the fifth wire at each way, and record them as before. He will now find the lower limb of the sun, and its second or following limb, ready for observing in the same way, at the first, second, and third wire, making each perpendicular wire a tangent to the sun's last limb, at the instant that its lower limb just leaves the correspondent horizontal wire; and setting down the time, and after reading off the azimuth, setting that down too under the other. After these, the instrument may be released in azimuth, and the lower limb alone be observed, as it quits the fourth and fifth horizontal wires respectively.

"As soon as the sun has thus passed all his wires, he should read off at both the microscopes the zenith distance and altitude at which he had clamped the vertical circle; and if he have a barometer and thermometer, he should set down their station at the same time: for though he probably will have no occasion to regard the precise altitude at which he made these observations, yet if anything should deprive him of the correspondent ones, he may wish to have it in his power to deduce his time or his azimuth from them, and the reading off the microscopes after all is over is attended with very little trouble.

"These things will appear at first hurrying, and till a person becomes a little accustomed to it they certainly will be so. But after a little practice there will be found time enough to go through the whole with ease; for the vertical circle remains clamped the whole time, and all the six azimuths lie much within the limits of their adjusting screw.

"The easiest method of keeping so many observations from confusion is to have a slate, or sheet of paper, ready ruled into five columns, to correspond with the five wires in the telescope, as they occur in succession, in which to write down the observation belonging to each wire, whether that be time or azimuth; for if any cloud or accident should deprive him of any one or more of his observations, he will then at once see afterwards which of them is missing, when he comes to compare the two sets together.

"Leaving the instrument clamped for altitude, and covered entirely from the sun's rays, he must wait till it is at the same distance from noon in the evening to resume his task. For that, he must hold himself ready against the time comes; and previous to it, he will do well to re-examine the adjustment of his instrument, to be certain that no change has happened in the stand or the central cone, so as to throw its axis out of a perpendicular. Let him then observe the same method in this second set of observations as he did in those of the forenoon, considering those wires at first at which the sun's limbs touch first, and setting down the times of their appulse to each respective horizontal wire, and bringing the preceding or subsequent limb to the corresponding perpendicular one, and reading off the azimuths just as he did before. When all are passed, he may release all the clamps, and, replacing his shade, leave the instrument till he has reduced his observations of corresponding altitudes: if he have observed them all, he will have obtained ten pair, and of azimuths six pair, which he must now select from each other, and properly class them, by taking the last in the morning in conjunction with the first in the evening, and so on, till each observation is paired with its opposite corresponding one."

The time of the meridional passage of the sun's centre, as indicated by the time-keeper employed, will be very nearly equal to half the sum of the times at which each pair of the observations were made, and would be exactly so if the declination did not change during the interval elapsed; (similar observations being made upon any star, the result will shew the exact sidereal time of its transit.) The correction to be applied to the time of the sun's transit or apparent noon deduced as above, on account of the change of declination, may be computed by the following formula:*

$$\text{Make } \frac{T}{1440 \text{ sin. } \frac{T}{2}} = -A$$

$$\frac{T}{1440 \text{ tan. } \frac{T}{2}} = B$$

$$\text{correction} = \mp A \,.\, \delta \,.\, \text{tan. } L + B \,.\, \delta \,.\, \text{tan. } D$$

in which L = the latitude of the place (*minus* when south).
δ = the double daily variation in the sun's declination (deduced from the noon of the preceding day, to the noon of the following day; *minus* when the sun is receding from the north).
D = the declination, at the time of noon, on the given day (*minus* when south).
T = the interval of time between the observations expressed in hours.

Note. B is to be considered *plus* when the interval of time is less than 12 hours, otherwise *minus*.

Practical Rule. To the constant log. 3·1584 add the log. sine of half the interval of time between the observations reduced

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* Tables of equation of equal altitudes are contained in MR. BAILY'S volume of Astronomical Tables and Formula, and in Schumacher's Hülfstafeln. The log. of double the sun's daily variation in declination is given in the Berlin Ephemeris as log. μ, in the page relating to *true* noon.

to space, and subtract their sum from the log. of the whole interval, *expressed in hours and decimals;* call the remainder, A, always *minus*.

To the constant log. 3·1584, add the log. tangent of the above half interval, and subtract their sum from the log. of the whole interval as before, and call the remainder, B, *plus*, when the interval is less than twelve hours.

To A add the log. of double the daily variation of the sun's declination, expressed in seconds (*minus* when the sun is *receding* from the north), and the log. tangent of the latitude (*minus* when south); the natural number corresponding to the sum to be considered as seconds of time, &c., *plus* or *minus* as it may result.

To B add the log. of double the daily variation of the sun's declination, as before, and the log. tangent of the sun's declination, *minus* when south, for noon of the given day; the natural number corresponding to the sum must be taken as seconds of time with its proper sign. The algebraic sum of these two quantities will be the correction required, and must be added to, or subtracted from, the half sum of the times of observation, according as it is *plus* or *minus*, to obtain the correct apparent time.

EXAMPLE.

(*From* MR. BAILY'S *Volume of Astronomical Tables, &c.*, p. 227.)

On July 25, 1823, in N. Lat. 54° 20′ at $8^h\ 59^m\ 4^s$ A.M., and at $3^h\ 0^m\ 40^s$ P.M. the sun had equal altitudes. Required the equation or correction to be applied to the mean of those times, in order to find the time of noon. The interval of time is $6^h\ 1^m\ 36^s$, which converted into arc = 90° 24′, and by the Nautical Almanac the declination of the sun at noon on that day was + 19° 48′ 29″, and its double daily variation equal to − 25′ 29″ = − 1529″. The operation will therefore stand thus:—

Constant log. . =	3·1584	. . . . .	3·1584
$\frac{T}{2}$ = 45° 12′ sin. =	9·8510	. . tangent =	0·0030
Sums . . . =	− 3·0094	. . . . . . =	− 3·1614
T = 6·0266 log. =	0·7801	. . . . . .	0·7801
A = (Differences) =	− 7·7707	. . . . . B =	+ 7·6187
δ = − 1529″ log. =	− 3·1844	. . . . . . log.	− 3·1844
L = + 54° 20′ tan.	+ 0·1441	D = 19° 48′ tan.	+ 9·5563
+ $12^s,57$ =	+ 1·0992	. . − $2^s,29$ =	− 0·3594

correction = + $12^s,57$ − $2^s,29$ = + $10^s,28$

This value being added to the mean of the times of the observed altitudes, or $\frac{1}{2}$ ($20^h\ 59^m\ 4^s + 27^h\ 0^m\ 40^s$) = $23^h\ 59^m\ 52^s$, will give $0^h\ 0^m\ 2^s,28$ for the time at apparent noon, to which, if the equation of time be applied, the result will be the time of mean noon.

The equal azimuths may similarly be employed for finding the direction of the true meridian. They must be opposed to each other in pairs, just in the same manner as corresponding altitudes; the first in the morning to the last in the evening, and so of the rest. Then, deducting the one from the other, and applying half the difference between the two to the smallest number in each pair, it will give a number of degrees, minutes, and seconds, in which, if all the observations were perfect, the whole six pair would coincide; and if they do not, the fair mean deduced from among them will approach nearly to the truth, *i.e.*, the error of 180° on the azimuth circle from the true meridian.

To that mean point, deduced from these observations, the instrument must now be turned, and fixed there till the proper correction can be applied to it. Upon the telescope being turned down to the horizon each way, it may be observed what distinct object there may be, either to the north or south, that coincides with one of the perpendicular wires; or, if no such object should occur, a mark may be placed each way, or either way, to which the instrument may be kept, till the correction can be investigated, which is requisite on account of the change of the sun's declination during the interval between the morning and evening observations: for any alteration in his declination will affect the azimuth deduced in this way, as it does the hour. This correction is greatest about the time of the equinoxes, as the change in the sun's declination is then the most rapid: it may be computed from the following formula; but when deduced from a star no such correction is requisite.

$$\text{Correction} = \tfrac{1}{2}\,(D'-D)\ \text{sec. Lat. co-sec.}\frac{(T'-T)}{2}$$

In which expression, $(D'-D)$ = the change in the sun's declination during the interval between the observations, and $(T'-T)$ = the interval itself.

Practical Rule. To the log. of half the change of declination add the log. secant of the latitude, and the log. co-secant of half the interval of time converted into space: the sum − 20 will be the log. of the correction in seconds of space.

When the sun is advancing towards the elevated pole, the middle point, or meridian, as found by equal altitudes, will be too much to the west of the true meridian, by the amount of this correction, and *vice versâ,* when he is receding from the elevated pole; therefore, the telescope, being shifted in azimuth by the quantity thus computed, will be correctly in the meridian.

EXAMPLE.

On February 28, 1834. When the sun had equal altitudes, the azimuth circle read 130° 10′ 15″ and 32° 36′ 15″, therefore the middle point or reading of the approximate meridian was 81° 23′ 15″. The interval of time between the observations was 5 hours, the half of which converted into arc = 37° 30′ 0″. The sun's hourly change of declination = 56″,77, therefore the change for half the interval = 141″,92 (approaching the north pole). The latitude of the place 51° 28′ 39″, required the correction to be applied to the middle point to obtain the direction of the true meridian.

$\frac{1}{2}(D'-D)$ = 141″,92 . . . log. = 2·1520436
Lat. = 51° 28′ 39″ . . . . sec. = 0·2056388
$\frac{(T'-T)}{2}$ = 37° 30′,0 . . . co-sec. = 0·2155529

Correction = 374″,31 . . . log. = 2·5732353
= 6′ 14″,31

° ′ ″
Reading of the middle point . . . . . = 81 23 15
Correction . . . . – 6 14,31

Reading of the instrument when set to the true meridian . . . . } = 81 17 0,69

Another, and an easy method of finding a meridian line, where dependence can be placed upon the time shewn by a chronometer (or watch), is to compute the time of the meridional passage of a star near the pole, either above or below the pole, and, pointing the telescope of the instrument to the star, bisect it at the exact moment; when, if the adjustments of the instrument be perfect, the telescope will be very nearly in the plane of the meridian.

A third method, which admits of great accuracy, when instruments of large dimensions are employed, consists in bisecting a circumpolar star when at its greatest eastern and western elongations; a line bisecting the horizontal interval, contained between the two positions of the telescope, will be the direction of the meridian: this interval being measured on the azimuthal circle, and the telescope moved through half that interval, from either its eastern or western position, will place it in the meridian. But it will often be inconvenient to wait till the star attains its second greatest elongation; and, as one of the observations must be made in the day-time, (except at particular seasons of the year,) a star will not be visible through telescopes of small size. To make a single observation available for the purpose, the azimuth (east or west) of a star, when at its greatest elongation, as well as the time of its attaining such position, must be computed (which may be done by the annexed rules), when the observer must first bisect the star, and follow it in its slow motion, until he is satisfied that it is stationary; or, what is perhaps better, if he be certain of his time, bisect it at the exact moment. The azimuth circle must now be read off, and the position of some fixed object, with respect to the azimuth of the star, should be determined; a lamp may at the time be placed at some distance for reference, and its azimuth being thus obtained, other objects may be referred to it at leisure.

To compute the azimuth of a circumpolar star, when at its greatest elongation.

Rule. From the log. sine of the polar distance, subtract the log. co-sine of the latitude: the remainder will be the log. sine of the azimuth required.

To compute the time (before or after its meridional passage) of a circumpolar star attaining its greatest elongation, either east or west.

Rule. Add together the log. tangent of the polar distance, and the log. tangent of the latitude: their sum, rejecting ten from the index, will be the log. co-sine of the hour-angle (in space); which, divided by fifteen, will be the sidereal time a star attains its greatest elongation before or after it passes the meridian at its upper culmination. Therefore, having the time of the meridional passage (computed, as explained at page 82), the time of its greatest elongation will be known.

The star α Ursæ Minoris, commonly known as the pole-star, is well situated for determining the direction of the meridian by the above method: its apparent motion when near its greatest elongation is so small that it appears stationary at that point for a considerable time, affording us an opportunity of observing it both by direct vision, and also by reflection,—an advantage particularly great, as we need not depend upon the spirit-bubble in levelling the instrument, for the observations expose the slightest deviation, and enable us to correct its position; thus,—

Suppose the polar-star, by previous calculation, is ascertained to be at its greatest elongation at a certain time; having set up the instrument approximately level, place an artificial horizon in a proper position to observe the star by reflection: then direct the telescope to the star, and having bisected it with the intersection of the cross-wires, clamp the horizontal circle. Now depress the telescope till you see the reflected image of the star in the artificial horizon, which, if the instrument be perfectly level, will also appear bisected; if it do not, you must immediately correct *half* the *deviation* by the foot-screws of the instrument, which will set the instrument perfectly level. Bisect the reflected image of the star by giving motion to the horizontal circle; then carefully elevate the telescope to the star itself, which will also be bisected if the estimation of half the amount of deviation have been correctly made, and, if not, it will be a nearer approximation, which must be perfected by a similar process, that is, by removing half the error with the foot-screws, and the other half by the horizontal circle.

Having now set the instrument, so that, upon elevating and depressing the telescope, both the direct and reflected images of the star appear bisected, a satisfactory observation will have been made. This being done at both the eastern and western elongations, and the readings of the azimuth circle noted, the middle point between the two readings will lie in the plane of the true meridian. Or, as before observed, one observation may be made available for the same purpose, by likewise observing a fixed object, as a lamp or church tower, and computing, by the foregoing rule, the azimuth of the star at that time; for the horizontal angle between the star and the fixed object, plus or minus the computed azimuth of the star (plus when the object is on the same side of the meridian and further from it than the star, and minus when it is nearer the meridian than the star, or when they are on opposite sides of the meridian) will give the azimuth of the fixed object from the north, from which the direction of the meridian may be found at any time.

It is only stars whose polar distance is less than the co-latitude of the place of observation that can be used in the two latter methods of determining the direction of the meridian.

The last method which we shall advert to, and which is mostly applied to objects south of the zenith, consists in computing the azimuth of a celestial object from an observation of its altitude, the latitude being known, at the same time observing the horizontal angle contained between it and any fixed object: for the difference or sum of the azimuth of the celestial body, and this observed horizontal angle, will be the angular distance of the fixed object from the meridian,—the sum when the fixed object is on the same side of, and further from the meridian than the celestial object, otherwise, the difference.

Formula for computing the azimuth of a celestial object from its observed altitude, &c.

$$\text{Tang. } \tfrac{1}{2} \text{ azimuth} = \sqrt{\frac{\sin.\frac{s}{2} - z.\ \sin.\frac{s}{2} - \lambda}{\sin.\ \frac{s}{2}.\ \sin.\frac{s}{2} - \pi}}$$

In which $\frac{s}{2}$ = half the sum of the polar distance, the co-latitude and the zenith distance, π and λ = the polar distance and co-latitude, Z = the zenith distance of the object.

Practical Rule. Add together the polar distance, the co-latitude, and the zenith distance, and call their sum S. To the log. sine of half S *minus* the zenith distance add the log. sine of half S *minus* the co-latitude, and increasing the index by 20, call the sum of the two logs. A.

To the log. sine of half S add the log. sine of half S *minus* the polar distance, and call the sum of the two logs. B.

From A subtract B, and divide the remainder by 2; the quotient will be the log. tangent of half the object's azimuth, which doubled will be the whole azimuth, or horizontal angular distance from the south meridian.

EXAMPLE.

On February 20, 1834, in latitude 51° 28′ 39″. The zenith distance of α Geminorum (east of the meridian) corrected for refraction = 56° 20′ 10″, the azimuth circle reading 125° 18′ 24″; after which the clamps of the instrument were released, and a fixed terrestrial object bisected, also to the *east*, but nearer the meridian than the star: the azimuth circle now read 83° 15′ 20″, consequently the horizontal angle between the star and the object = 42° 3′ 4″; required the azimuth of the object from the meridian.

$$
\begin{array}{lrrrll}
 & {}^{\circ} & {}' & {}'' & & \\
\pi\text{ (from the N.A.)} = & 57 & 45 & 16 & & \\
\lambda\ .\ .\ .\ .\ .\ = & 38 & 31 & 21 & & \\
z\ .\ .\ .\ .\ .\ = & 56 & 20 & 10 & & \\
\hline
2)\ & 152 & 36 & 47 & & \\
\hline
\frac{s}{2} = & 76 & 18 & 23 & & \\
\hline
\frac{s}{2} - z\ .\ .\ .\ = & 19 & 58 & 13 & \text{sine} = & 9{\cdot}5334322 \\
\frac{s}{2} - \lambda\ .\ .\ .\ = & 37 & 47 & 2 & \text{sine} = & 9{\cdot}7872371 \\
 & & & & A = & 9{\cdot}3206693 \\
\frac{s}{2}\ .\ .\ .\ .\ = & 76 & 18 & 23 & \text{sine} = & 9{\cdot}9874766 \\
\frac{s}{2} - \pi\ .\ .\ .\ = & 18 & 33 & 7 & \text{sine} = & 9{\cdot}5026514 \\
 & & & & B = & 9{\cdot}4901280 \\
 & & & & A = & 9{\cdot}3206693 \\
 & & & & 2) & 9{\cdot}8305413 = A - B
\end{array}
$$

° ′ ″
39 26 45·5 ½ star's azimuth, tan. = 9·9152706
2

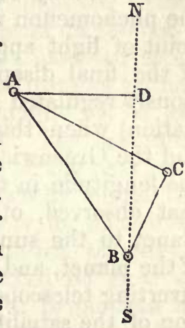
78 53 31·0 = azimuth of star
42 3 4·0 = object near meridian

36 50 27·0 = azimuth of object east of south.

The verification of the meridional position of an instrument by observing the passage of a circumpolar star at both its upper and lower culminations, as well as the method by high and low stars, has been fully explained, when speaking of the transit; and as the altitude and azimuth circle, when firmly clamped in the plane of the meridian, becomes a complete transit instrument, and may be employed precisely in the same manner and for the same purposes, we refer for this use of it to the account which we have given of that instrument.

In addition to the method of determining differences of longitude by the observed transits of the moon and moon-culminating stars, (page 88,) we subjoin the following as applicable to the use of the instrument which we are now describing. The latitudes and longitudes of a great number of the most conspicuous places in this country, as church steeples, &c., having been determined, and published in the account of the Ordnance Survey, they afford a ready means of finding both the latitude and longitude of places adjacent to them, by means of trigonometrical measurement. The process may be understood from the following example.

Let A represent a place, the longitude and latitude of which are known; B the station, the situation of which we wish to determine; C any point to form the triangle; N S the direction of the meridian.



First, the angles at the three points must be observed, and one of the sides measured, when the distance A B must be computed by plane trigonometry. Suppose it to be = 6040·6 feet. Then, the azimuth of A from the meridian, or the angle, A B N, must be determined, which may be done by any of the methods we have described; suppose it = 56° 58′ 40″: now the line A D, perpendicular to the meridian, and B D, the difference of latitude of B and A, may be computed from the right-angled triangle A B D, having A B 6040·6 feet, and the angle A B D = 56° 58′ 40″; A D comes out = 5064·8 feet, and B D = 3292·2 feet.

With the latitude of A, which suppose = 51° 27′ 44″, enter Table VIII. and take out the length of a second, both of latitude and longitude; divide the distances A D and B D by those numbers, and the quotients will be the difference of longitude and latitude (in arc) required. Thus:

$$\frac{\text{A D} = 5064\cdot 8}{\text{Table} = 63\cdot 31} = 80{,}44'' = 1'\ 20{,}00'' = \text{diff. of long. in space.}$$

$$= 5^{s}{,}33 \text{ in time.}$$

$$\frac{\text{B B} = 3292\cdot 2}{\text{Table} = 102\cdot 02} = 32{,}27'' \text{ difference of latitude.}$$

	M.	S.	
Longitude of A =	21	10,40	West.
Difference . .		5,33	East
Longitude of B =	21	5,07	West.

	°	′	″	
Latitude of A =	51	27	44,00	North.
Difference . .		0	32,27	South.
Latitude of B =	51	27	11,73	North.

Lastly, we shall give the method of finding the longitude by observations of the eclipses of Jupiter's satellites.

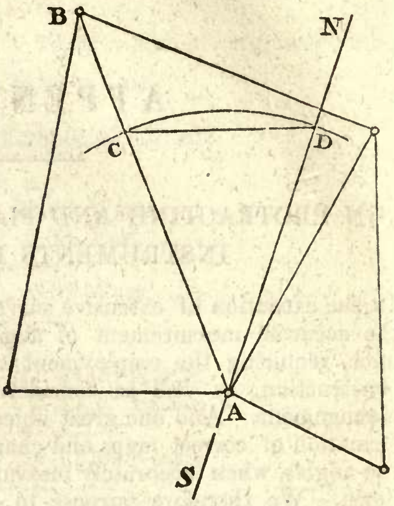
The Nautical Almanac contains the Greenwich mean time when the phenomena happen, consequently the estimated longitude of the place being applied to the time therein given will be the time at which an eclipse may be expected to happen at the station of the observer, who, being at his telescope a few minutes before, should steadily watch the spot near the body of the planet where the phenomenon is expected, till he discovers the first glimpse or point of light appear, if it be an emersion from the shadow, or of the final disappearance, if an immersion; noting, by a previously regulated time-keeper, the exact mean time (at his own station) when this happens. The difference between this time and the Greenwich time given in the Nautical Almanac will be the longitude in time,—east, if the Greenwich time be less than that observed, otherwise west. Before the opposition of the planet to the sun, the eclipses always happen on the west side of the planet, and afterwards on the east. But when using an inverting telescope the appearance will be reversed. The situation of the satellite with respect to the planet where the eclipse takes place is given in the Nautical Almanac.

APPENDIX.

ON PROTRACTING AND PLOTTING, &c., AND THE INSTRUMENTS EMPLOYED.

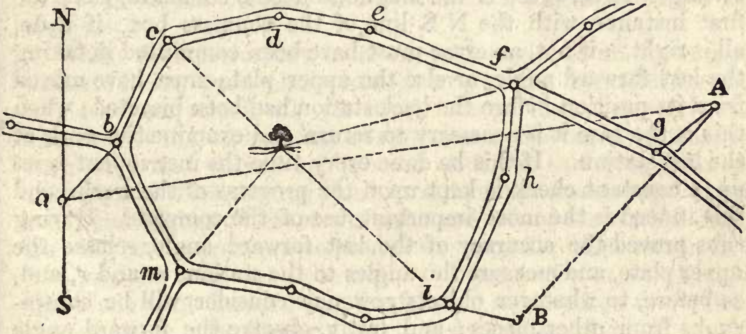
In the execution of extensive surveys upon scientific principles the accurate measurement of angles is of the utmost importance, requiring the employment of instruments of a superior construction, as well as considerable care and skill in their management. And one great object of such surveys being the formation of correct maps and charts, it is no less essential that the angles, when accurately measured, should be accurately laid down. We therefore purpose to describe briefly, in this Appendix, the most approved methods of laying down angles, &c., as supplementary to our account of surveying instruments.

Extensive surveys are best performed by extending a series of triangles over the country to be delineated; and from the length of a side of one triangle measured, or otherwise determined, as a base, and the angles found by means of appropriate instruments, the lengths of the various lines forming the sides of the several triangles throughout the series are computed. The accuracy of the distances thus obtained will depend on the correct measurement of the angles, and the distance assumed as a base,—provided due attention be paid in the first instance to the judicious dispositions of the triangles, which ought to be as nearly equilateral as circumstances will admit. The accurate protracting of the triangles thus determined is of the next importance. They can be more correctly laid down by means of their sides than by their angles; and one side only, for measures of length, can be taken from a scale, and transferred to paper, with more exactness than an angle can be pricked off from a protractor. But it being in most cases requisite, in plotting a survey, to shew the direction of the meridian with regard to the triangulation, it becomes necessary to lay down, from one of the principal stations, the azimuthal angle subtended by some other (remote) station and the meridian: now this angle cannot be laid off from a protractor, even of the most approved construction, so accurately as the plotting of the triangulation may be made from the measured or computed sides of the triangles. To obtain a corresponding degree of exactness, recourse must be had to some other method, and the following is the best that we have seen practised.



Let A and B represent two stations of a trigonometrical survey; and let it be required to lay off the direction of the true meridian, N S, with regard to the line A B, the azimuth of which, *west* of *north*, being 40° 30′ 30″. Take from an accurately divided diagonal scale exactly five inches as a radius, and from A, as a centre, describe an arc C D; now the chord of an arc being equal to twice the sine of half that arc, it follows that the chord C D is equal to twice the natural sine of half the angle C A D or B A D, *viz.*, 20° 15′ 15″; but the radius of the tables of natural sines being =1 or 10, and taking but the half of 10, or 5 inches for our radius, we must take from the table the natural sine of *half* the angle B A D, which will, to radius 5, be equal to C D, the chord of the whole angle: and having taken that distance from the same scale of inches as the radius, place one foot in the point C, and with the other mark the point D on the arc C D, then through D and A draw the line N S, which will represent the meridian. But instead of employing a pair of compasses and a scale for this purpose, it is better to use a beam-compass, graduated to inches, and having a vernier for minute subdivision, as a measure of length can be taken by its means with greater exactness than by a pair of compasses from a scale.

This method of laying off angles may be conveniently employed in dividing a circle to be used as a protractor, when the work is to be laid down to a scale not exceeding six inches to a mile. The protractor may be made either on the same sheet of paper intended to receive the drawing, or on a separate sheet of card-board, when it may be preserved and used on after occasions. During the time which must necessarily be occupied in plotting an extensive and minute survey, the paper which receives the work is often sensibly affected by the changes which take place in the hygrometrical state of the air, causing much annoyance to the draftsman, as the parts laid down from the same scale at different times will not exactly correspond. To remedy in some measure this inconvenience, it has been recommended that the apartments appropriated to the purposes of drawing should be constantly kept in as nearly the same temperature as possible, and also that the intended scale of the plan should be first accurately laid down upon the paper itself; and from this scale all dimensions for the work should invariably be taken, as the scale would always be in the same state of expansion as the plot, though it may no longer retain its original dimensions. The protractor may also be laid down upon the paper; and when a great many angles are to be plotted, as in a road or town survey, made with a theodolite and chain, especially if done by traversing, or what is frequently called surveying by the back angle, this kind of protractor will enable the draftsman to plot the work with great rapidity, and with less chance of error, when the scale is small, than by the method of laying off angles by placing the centre of a metallic protractor at every angular point, and pricking off the angle from its circular edge. The application of the theodolite to surveying by a traverse, as well as the method of protraction, we shall endeavour to explain by means of an example.



Let the above plan represent a survey of roads to be performed with a theodolite and chain. Commencing on a conspicuous spot, *a*, near the place at which two roads meet, the theodolite must be set up and levelled, the upper and lower horizontal plates clamped at zero, and the whole instrument turned about until the magnetic needle steadily points to the N S line of the compass-box, and then fixed in that position by tightening the clamp-screw, H (see page 15). Now release the upper plate, and direct the telescope to any distant conspicuous object within or near the limits of the survey, such as a pole purposely erected in an accessible situation, that it may be measured to, and the instrument placed upon, the same spot at a subsequent part of the operation, as A and B; and after bisecting it with the cross wires, read both the verniers of the horizontal limb, and enter the two readings in the field-book: likewise in the same manner take bearings, or angles, to all such remarkable objects as are likely to be seen from other stations, as the tree situated on a hill; and lastly, take the angle to your forward station, *b,* where an assistant must hold a staff for the purpose, on a picket driven into the ground,* in such a situation as will enable you to take the longest possible sight down each of the roads that meet there. In going through the above process, at this and every subsequent station, great caution must be used to prevent the lower horizontal plate from having the least motion after being clamped in its position by the screw H.

Next measure the distance from *a* to *b,* and set up the instrument at *b;* release the clamp-screw H *only,* not suffering the upper plate to be in the least disturbed from the reading it had when directed at *a* to the forward station *b,* with the instrument reading this forward angle; turn it bodily round, till the telescope is directed to the station *a* (which is now the back station) where an assistant must hold a staff; tighten the clamp-screw H, and by the slow-motion screw, I, (page 15) bisect the staff as near the ground as possible; and having examined the reading, to see that no disturbance has taken place, release the upper plate, and, setting it to zero, see if the magnetic needle coincides, as in the first instance, with the N S line of the compass box: if it do, all is right,—if not, an error must have been committed in taking the last forward angle, or else the upper plate must have moved from its position before the back station had been bisected; when this is the case it is necessary to return and examine the work at the last station. If this be done every time the instrument is set up, a constant check is kept upon the progress of the work; and this indeed is the most important use of the compass. Having thus proved the accuracy of the last forward angle, release the upper plate, and measure the angles to the stations *m* and *r,* and, as before, to whatever objects you may consider will be conspicuous from other places; and, lastly, observe the forward angle to the station *c,* where the theodolite must next be set up, and measure the distance *b c.*

At *c,* and at every succeeding station, a similar operation must be performed, bisecting the back station with the instrument reading the last forward angle: then take bearings to every conspicuous object, as the tree on the hill, the station A, &c., which will fix their relative situations on the plan, and they afterwards serve as fixed points to prove the accuracy of the position of such other stations as may have bearings taken from *them* to the same object; for, if the relative situations of such stations be

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* A picket should always be left in the ground at every station, in order to recognize the precise spot, should it afterwards be found necessary to return to it again.

not correctly determined, these bearings will not all intersect in the same point on the plan. The last operation at each station is to measure the forward angle. In this manner proceed to the stations *d, e, f, g,* &c., and, having arrived at *g,* measure an angle to the pole, A, as to a forward station, and, placing the theodolite upon that spot, direct the telescope to *g,* as a back station, in the usual way; this done, release the upper plate, and direct the telescope to the *first* station *a,* from which A had been observed, and if all the intervening angles have been correctly taken, the reading of the two verniers will be precisely the same as when directed to A from the station *a:* this is called closing the work, and is a test of its accuracy so far as the angles are concerned, independent of the compass-needle. If the relative situation of the conspicuous points, A B, &c., were previously fixed by triangulation, there would be no necessity to have recourse to the magnetic meridian at all, as a line connecting the starting point *a* with any visible *fixed* object may be assumed as a working meridian; and, if it be thought necessary, the reading of the compass-needle may be noted at *a,* when such fixed object is bisected, and upon the theodolite being set to the reading of this assumed meridian, at any subsequent station, the compass-needle will also point to the same reading as it did at first, if the work be all correct, and no local attraction influences the compass.

While the instrument is at A, take angles to all the conspicuous objects, particularly to such as you may hereafter be able to close upon, which will (as in the above instance) verify the accuracy of the intervening observations. Having done this, return to *g* and *f,* &c. and proceed with the survey in the same manner as before, setting the instrument up at each bend in the road, and taking offsets to the right and left of the station lines: arriving at *i,* survey up to, and close upon B; then return to *i,* and proceed from station to station till you arrive at *m,* where, if the whole work be accurate, the forward angle taken to *b* will be the same as was formerly taken from *b* to *m,* which will finish the operation.

The next step is to lay down the lines and angles thus surveyed; and, first, the protractor must be constructed. The great difficulty of dividing a circle accurately is well known, but if the arcs be laid off by means of their chords, the division may be performed with sufficient exactness for the purpose in hand. The length of the chords should be taken from an accurately divided beam-compass, which, to insure success, should be set with the utmost possible exactness.

With a radius of five inches describe a circle, and immediately, without altering the compasses, step round the circle, making a fine but distinct mark at each step: this will divide the circle into six parts of 60° each.

Next set the compasses to the natural sine of 15°, which, to radius *five*, will be equal to the chord of 30°, and this distance will bisect each 60° and divide the circle into arcs of 30° each. A proof may be obtained of the accuracy of the work as it proceeds, by setting the succeeding chords off each way, from those points which they are intended to bisect; for if any inaccuracy exist, the bisection will not be perfect, and if the error prove inconsiderable, the middle point may be assumed as correct.

Each sixty degrees may next be trisected, by setting off the natural sine of 10° (equal the chord of 20° to our radius), which will divide the circle to every ten degrees.

Next, the natural sine of 7° 30′ (equal the chord of 15°), stepped from the points already determined, will divide the circle to every fifth degree.

The natural sine of 3° (equal the chord of 6°), being laid off, divides 30° into five parts, and, set off from the other divisions, divides the circle to single degrees.

Fifteen degrees bisected, or the natural sine of 3° 45′ (equal the chord of 7° 30′) set off from the other divisions, divides the circle into half degrees.

The natural sine of 3° 20′ (equal the chord of 6° 40′) divides 20° into three parts, and, set off from the rest of the divisions, divides the whole circle to every ten minutes, which is as minute a subdivision as such a circle will possibly admit of; smaller quantities must therefore be estimated by the eye. The divisions should be numbered from 0, 10°, 20°, &c. quite round the circle to 360°, the same as the theodolite, which the protractor represents.

It may be considered troublesome to lay down a protractor of this kind upon every sheet of paper to be plotted on, but, having done one, several copies may be obtained from it, by pricking through the divisional points upon paper placed under it for the purpose. Or, if made upon a sheet of card-board, the paper within the graduated circle must be cut out, as the work is plotted within the circle forming the protractor.

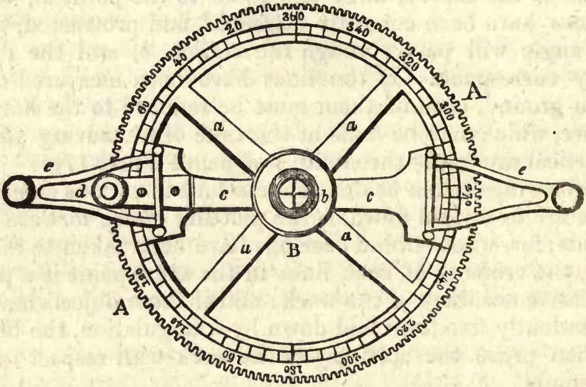
Suppose, with a protractor of the latter kind, we proceed to lay down the work of our survey. First, draw a line through the assumed starting point, *a*, across the paper, to represent the magnetic meridian; or, if the points, A B, &c. have been fixed by previous triangulation, they should be laid down, and a line drawn through *a*, and any one of them (which has been observed from *a*) may be assumed as a working meridian: then, across the protractor, draw a line through the same divisions that were noted on the theodolite for the reading of the meridian, which in our example was zero, or the divisions marked 180° and 360° on the protractor.

Place the protractor upon the paper, so that the line drawn on the former shall coincide with the meridian-line drawn upon the latter, and, to prevent it shifting, lay weights on its corners. Place the edge of a large parallel ruler on the divisions which were read off for the forward angle to *b*, and slide the ruler parallel to itself till its edge passes through the station *a*, and draw a line from *a* in the direction *a b*; then with a pair of compasses, and from the scale of the plot, lay off the distance *a b*, which will determine the point *b*. Next, place the edge of the ruler on the angles taken at *b* to the stations *r* and *c* respectively, and slide it parallel to itself till its edge passes through *b*; then draw the lines *b r* and *b c*, and lay off those distances from the scale of the plot, and the stations *r* and *c* will be fixed. Next, set the ruler to the forward angle take at *c* to the station *d*, and move it till its edge passes through *c*, and draw the line *c d*; lay off the distance *c d*, and the station *d* will be determined. In like manner proceed with the remaining stations of the survey, until you come to the point *m*, when, if the lines have been correctly measured and protracted, the forward angle will pass through the station *b*, and the distance exactly correspond. If the lines have been measured on very uneven ground, each of them must be reduced to the horizontal measure, which may be done at the time of measuring them by the vertical arc of the theodolite (see pages 2 and 17).

The bearings taken at different stations to various conspicuous objects are to be laid down as the plotting of the forward angles proceeds; for, when several bearings have been taken to the same object, the crossing of such lines in the same point is a proof of the relative accuracy of the work: and if these objects have been independently fixed and laid down by triangulation, the bearings will then prove the accuracy of the work with respect to these fixed points.

We have remarked that the plotting must be performed within the circle forming the protractor, which direction is to be understood as applicable only when the protractor is not on the same paper with the plot, for when it is on the same paper, the angles may be transferred by the parallel ruler to any part of the sheet; but care must be taken in numbering the divisions of the protractor, so that the working meridian may be in the best direction for getting into the sheet the greatest portion of the survey. If the protractor be on a separate sheet, and the work has proceeded to its edge, it must be shifted on the paper in the direction of the survey, but must be moved exactly parallel to itself; which may easily be done by drawing more meridian lines parallel to the first meridian, on which to place the protractor, as in the first instance.

When a survey is to be plotted upon a very large scale, it is necessary, to insure the greatest accuracy in laying down the angles, to protract them by their chords, or by means of a circular metallic protractor, as the kind of protractor we have just been describing would not answer the purpose; its chief use being, as has been already described, to plot a traverse upon a moderately small scale. There are several constructions of the protractor adapted to the purpose now under consideration, but the most approved is represented in the subjoined figure. It consists of an entire circle, A A, connected with its centre by four radial bars, *a a*, &c. The centre of the metal is removed, and a circular disk of glass fixed in its place, on which are drawn two lines crossing each other at right angles, and dividing the small circle into four quadrants, the intersection of the lines denoting the centre of the protractor. When the instrument is used for laying down an angle, the protractor must be so placed on the paper that its centre exactly coincides with, or covers, the angular point, which may easily be done, as the paper can be seen through the glass centre-piece.



Round the centre, and concentric with the circle, is fitted a collar, *b*, carrying two arms, *c c*, one of which has a vernier at its extremity, adapted to the divided circle, and the other a milled head, *d*, which turns a pinion, working in a toothed rack round the exterior circle of the instrument: sometimes a third arm is applied at right angles to the other two, to which the pinion is attached, and a vernier can then (if required) be applied to each of the other two, and it also prevents the observer disturbing that part of the instrument with his hand when moving the pinion. The rack and pinion give motion to the arms, which can be thus turned quite round the circle for setting the vernier to any angle that may be required. Upon a joint near the extremity of the two arms (which form a diameter to the circle) turns a branch, *e e*, which for packing may be folded over the face of the instrument, but when in use must be placed in the position shewn in the figure: these branches carry, near each of their extremities, a fine steel pricker, the two points of which, and the centre of the protractor, must (for the instrument to be correct) be in the same straight line. The points are prevented from scratching the paper as the arms are moved round, by steel springs, which lift the branches a small quantity, so that, after setting the centre of the protractor over the angular point, and the vernier in its required position, a slight downward pressure must be given to the branches, and each of the points will make a fine puncture in the paper: a line drawn through one of these punctures and the angular point will be the line required to form the angle.

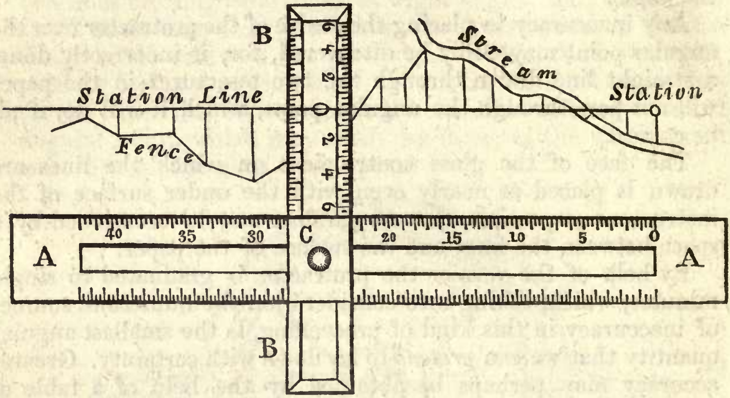
Any inaccuracy in placing the centre of the protractor over the angular point may easily be discovered, for, if incorrectly done, a straight line drawn through the two punctures in the paper will not pass through the angular point, which it will do, if all be correct.

The face of the glass centre-piece on which the lines are drawn is placed as nearly even with the under surface of the instrument as possible, that no parallax may be occasioned by a space between the lines and the surface of the paper.

By help of the vernier the protractor is graduated to single minutes, which, taking into consideration the numerous sources of inaccuracy in this kind of proceeding, is the smallest angular quantity that we can *pretend* to lay down with certainty. Greater accuracy may perhaps be obtained by the help of a table of natural sines and a well-graduated beam-compass, as explained at page 114.

For plotting offsets, measured to the right and left of the station lines, ivory scales with fiducial edges are usually employed. The figure in the following page represents an ingenious contrivance for an offset scale, extensively employed on the Ordnance Survey of Ireland.

The graduated scale, A A, is perforated nearly its whole length by a dove-tail shaped groove, for the reception of a sliding piece, which is fastened to the cross-scale, B B, by the screw, C. It will readily be understood, from an inspection of the figure, that the cross scale, B, slides along the scale, A, the whole length of the groove, and at right angles to it. The graduations on both the scales represent either feet or links, &c., or whatever length may have been assumed as the unit in the operation of measuring. The mode of its application is simply this: place the scale, A A, on the paper, parallel to the line on which the offsets are to be plotted, and at such a distance that the zero division on the cross scale, B, (which is placed about its middle,) may coincide with it as the scale slides along, and also that the zero of the scale, A, may be exactly opposite that end of the line at which the measurement commenced; then, in sliding the scale, B, from the beginning of the line, stop it at every divisional line on A, corresponding to the distance on the station line at which an offset was taken, and lay off the exact length of the offset from the edge of the scale, B, either to the right or left of the station line, to which it will be at right angles as taken in the field; the instrument thus gives both dimensions at the same time. It is perhaps needless to add that, the extremities of the offsets being connected, will represent the curved line, &c., to which they were measured; weights may be placed at the two ends of the scale, A A, to keep it steadily in its position. In our figure, the instrument is represented as in the act of plotting offsets upon a station line.



It very frequently happens that a surveyor requires copies to be made of his plans, and these occasionally on an enlarged or diminished scale. There are various methods of accomplishing this purpose, some of which we shall here enumerate.

When a copy is to be made of the same size as the original, it is a common practice to lay the plan upon the sheet of paper intended for the copy, and press them close together by means of weights; then, with a fine needle, prick through all the corners and leading points on the plan, making corresponding punctures in the paper beneath, which may then be connected by lines to complete the copy. But when the lines on the original are very crooked, this method cannot be successfully applied without the aid of a pair of compasses and tracing paper; when, having pricked off the principal points, the remainder may be found by the compasses, and the curved lines transferred by drawing them on tracing paper, the back of which being rubbed over with powdered black lead, and placed in its correct relative situation on the copy, a blunt point* may be drawn along the lines, which will leave corresponding lines on the copy beneath.

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* The point of a porcupine's quill, or the edge of the eye-end of a fine needle, make good tracing instruments.

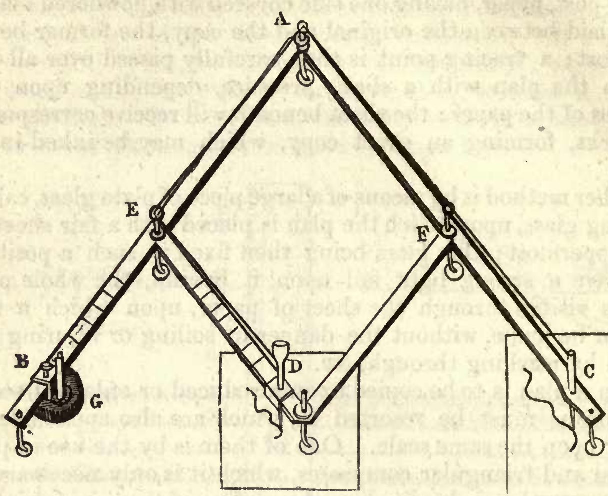
Tracing paper is sometimes thus used for making a copy of the whole plan, but, as this process occupies so much time, it is frequently applied in the following manner: A sheet of tracing, or bank-post, paper, having one side covered with powdered black-lead, is laid between the original and the copy, the former being uppermost; a tracing point is then carefully passed over all the lines on the plan with a slight pressure, depending upon the thickness of the paper: the sheet beneath will receive corresponding marks, forming an exact copy, which may be inked-in at leisure.

Another method is by means of a large piece of plate glass, called a copying glass, upon which the plan is placed with a fair sheet of paper uppermost: the glass being then fixed in such a position as to have a strong light fall upon it behind, the whole plan becomes visible through the sheet of paper, upon which a fair copy can be made, without the danger of soiling or injuring the original by pricking through, &c.

When a plan is to be copied upon a reduced or enlarged scale, other means must be resorted to, which are also applicable to copying upon the same scale. One of them is by the use of proportional and triangular compasses, which it is only necessary to mention; another is by dividing the surface of the original into a great number of small squares, and drawing a similar number upon the copy, which must be formed larger or smaller than those on the original, in the exact proportion of the required difference of the scales: the squares in the latter may then be filled up with the same detail of the plan as is contained in the corresponding squares on the former. When, from the great value of the original, it becomes improper to draw lines upon or otherwise deface it, recourse has been had to a frame of wood or metal, having fine threads stretched across it each way, forming a series of squares: this being laid upon the plan, will, if accurately done, answer the same purpose.

The last method we shall speak of is by means of a well-known instrument called a pentagraph.

The subjoined engraving represents the pentagraph, which consists of four flat rulers, made either of wood or brass: the two outside ones are generally from 15 to 24 inches long, and the others about half that length; the longer ones, A B, and A C, are united together, at A, by a pivot, about which they turn, and the two smaller rulers are similarly attached to each other at D, and to the longer rulers at E and F. A sliding box is placed on each of the arms, A B and E D, which may be fixed by a clamp-screw at any part of the ruler; these slides carry a tube, to contain either a blunt tracing point, a pencil, or pen, or the fulcrum G, which is a heavy weight of lead, having a point on the under side, to pierce the drawing-board and remain immoveable in its proper position, it being the centre upon which the whole instrument turns. Several ivory castors support the surface of the machine parallel to the paper, as well as facilitate its motions.



The arms, E D, and E B, are graduated and marked with the ratios, $\frac{1}{2}$, $\frac{1}{3}$, &c., so that when a copy of a plan is required to be made in any of these proportions, it is only requisite to fix, at the required ratio, the slides carrying the fulcrum, G, and the tube at D, with a pencil or pen, and the instrument will be ready for operation. Thus, suppose it were required to make a copy of a plan exactly one half the size of the original, our engraving represents the pentagraph so employed; the slide carrying the pencil at D, and that working on the fulcrum, G, are each fixed by their respective clamp-screws at the divisions marked $\frac{1}{2}$; the original plan is placed under the tracing point, C, and exactly parallel to it is placed a sheet of paper under the pencil, D,—the pentagraph being first spread out so as to give room for the tracing point to be passed over every line on the plan, whilst the pencil at D is making corresponding marks on the copy, which it is evident will be equal to one half the size of the original. A fine string is attached to the pencil-holder, and passed round by E A, &c. to the tracing point, the pulling at which raises the pencil a small quantity above the paper, to prevent false or improper marks upon the copy. It should also be remarked, that the cup represented on the top of the pencil-holder is intended to receive a weight, to keep the pencil down upon the paper, or when a stronger mark is required.

When the instrument is set for work, the tracing point, the pencil, and the fulcrum, must in all cases be in a straight line, which may be proved by stretching a fine string over them.

When it is required to make an enlarged copy of a plan, the setting of the instrument is precisely the same as above stated, only the tracing point and pencil must change places, the original being placed under D, and the copy under C. But when a copy is to be made of the same size as the original, the fulcrum must be placed in the middle at D, and the pencil at B, under which will be the copy, whilst the original must be placed under the tracing point at C.

When a survey is to be made for the purposes of a line of railway or turnpike road, it is necessary to delineate not only the fields through which it is contemplated the line would pass, but also one or more fields on each side, to the extent of full one hundred yards, for the purpose of admitting hereafter, if necessary, an alteration to that extent at any point on the line. The instrument usually employed on such surveys is the prismatic compass, described at page 3 (or else a circumferentor), together with a land-chain.

To execute a survey of this kind, supposing the line to have been previously chosen, the surveyor must set up his compass at one extremity of the work, and take the bearing of some distant object situated in the direction of the intended line of railway or road; having done which, and entered it in his field-book, he must commence chaining in that direction, taking offsets to the fences of the fields, and every remarkable object within a short distance to the right and left of his line: he must also note the point at which his chain crosses the various fences, and at the same time and place set up his compass, to observe the *bearing* of such fences, or, in other words, the angle their direction makes with the meridian. This angle is at once given by the compass, and furnishes data for laying down their *position* with regard to the main line which crosses them, but does not determine their respective lengths; the surveyor must therefore measure along the side of each fence, both to the right and left of the point at which he crosses it, till he comes to their extremity, or the points where such fence meets the other, or side fences of the field: these now become known or fixed points, from whence the *bearing* of every fence which diverges from this may be taken, giving the means of laying down their several directions on the general plan.

If the surveyor should require to represent the boundaries of the fields which are still more remote from his main line, he must similarly measure the lengths and curves of the fences he has previously taken the bearings of; and then again the bearings, &c. of others, till he possesses sufficient data for his purpose: but he will occasionally find it more convenient to measure secondary or side lines branching from the main line, which, by crossing a number of fences, give so many fixed points to take bearings from, as frequently to reduce his labour materially, both in the field, and afterwards in plotting the work.

Having proceeded onward with the measurement of his first main line, as far as may be convenient for his purpose, and also completed the measurements branching therefrom, the surveyor must again set up his compass at the point where he wishes to change the direction of his course, or commence a *second* main line, when, having taken the bearing of some natural conspicuous object in the required direction, he must proceed to measure such second line, and all its subsidiary dimensions, in the same manner as before, completing as much as possible all the minor measurements depending on each main line before he commences a new one.

Such is the general method of procedure; but as every thing depends upon the experience and tact of the surveyor, it is impossible to give more than a *general description: particular* rules for surveying are useless, as new cases, and sometimes difficult ones, are hourly occurring, which the experience of the surveyor alone will enable him to overcome, and suggest at the time a method which no book, in all probability, could inform him of.

The protraction of a railway survey is the most easily performed, by having a protractor laid down upon the plot itself, from which the angles can be transferred by a parallel ruler to any part of the work, as described at page 118; but, instead of constructing a protractor in the manner there directed, it may be done by laying on the paper a metallic circular protractor, placing the zero divisions (180° and 360°) in the exact position it may be necessary to have the meridian represented; then prick off, and mark *all* the degrees, and, if sufficiently large, the *half degrees* also: thus a protractor may be drawn on the paper, ready for use, in a very few minutes; the plotting is then performed in a manner so precisely similar to that described for the traverse of a road survey, at page 118, that to enter upon the subject here would be merely a recapitulation of what is there stated, to which we accordingly refer.

"Maritime surveying is of a mixed kind: it not only determines the positions of the remarkable headlands and other conspicuous objects that present themselves along the vicinity of a coast, but likewise ascertains the situations of the various inlets, rocks, shallows, and soundings, which occur in approaching the shore. To survey a new or inaccessible coast, two boats are moored at a suitable interval, which is carefully measured or otherwise determined; and from each boat the bearings of all the prominent points of land are taken by means of an azimuth compass, or the angles subtended by these points and the other boat are measured by a sextant. Having now on paper drawn the base to any scale, straight lines radiating from each end at the observed angles will, by their intersections, give the positions of the several points from which the coast may be sketched. But a chart is more accurately constructed by combining a survey made on land with observations taken on the water. A smooth level piece of ground is chosen, on which a base of considerable length is measured, and station staves are affixed at its extremities. If no such place can be found, the mutual distance and position of two points conveniently situated for planting the staves, though divided by a broken surface, are determined from one or more triangles, connected with a shorter and temporary base measured near the beach. A boat then explores the offing, and at every rock, shallow, or remarkable sounding, the bearings of the station staves are noticed. These observations furnish so many triangles, from which the situation of the several points are easily ascertained. When a correct map of the coast can be procured, (or previously constructed) the labour of executing a maritime survey is materially shortened. From each important point on the water the bearings of two known objects on the land are taken, or the intermediate angles subtended by three such objects are observed." The situation of the observer at the time such angles are taken may then be laid down by means of an instrument called a station-pointer, which is represented in the annexed figure, and which we shall now describe.

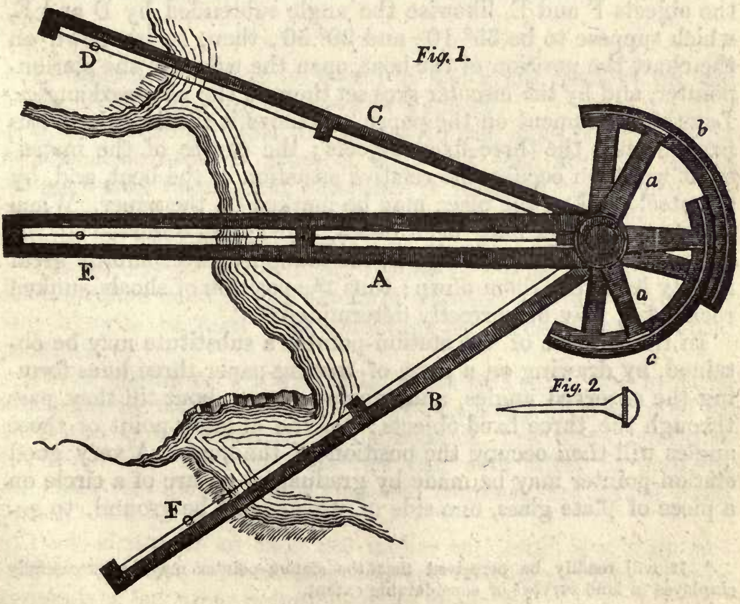


Fig. 1.

Fig. 2

This instrument consists of three rulers, A B C, (fig. 1,) connected together by a common centre upon which they turn, and can be opened to form two angles of any inclination. The ruler, B, is connected with the circular arc, *b*, the ruler, C, with the arc, *c*, and the middle ruler, A, with the two verniers, *a a*, adapted to the two arcs. The middle ruler is double, and has a fine wire or thread stretched along its opening; the other rulers have likewise a fine wire stretched from end to end, and so adjusted by the little projecting pieces which carry them, that all the three wires tend to the centre of the instrument, where they would meet if produced. The graduated circular arcs, *b* and *c*, are for setting the rulers, or rather the fine wires, at whatever angles they may be required to form at the centre of the instrument. Through the centre is an opening sufficiently large to admit a steel pricker (fig. 2) to be gently pressed into the paper, when the instrument is adjusted in its position: the puncture thus made will represent the station required.

That the application of the instrument may the more readily be understood, we have represented it in the act of being used. Suppose the points marked D E F to be three conspicuous objects on the coast, whose relative situations are known and laid down upon the map, and that, on exploring the offing in a boat, a remarkable sounding occurred, which it was necessary should be marked in the chart; the situation of the boat, at the time the sounding was taken, with regard to the shore, must therefore be determined: with a sextant measure the angle subtended by the objects F and E, likewise the angle subtended by D and E, which suppose to be 35° 10′, and 20° 50′, then, to lay down on the chart the position of the boat, open the rulers of the station-pointer, and by the circular arcs set them to the observed angles. Lay the instrument on the paper, and move it till the three wires pass through the three fixed objects; the centre of the instrument will then occupy the relative situation of the boat, and, by the steel pricker, the place may be marked on the paper. When several soundings have been taken, and angles observed at the time to any three fixed objects, the station-pointer affords great facility in laying them down: thus the position of shoals, sunken rocks, &c., may be correctly determined.*

In the absence of the station-pointer a substitute may be obtained, by drawing on a piece of tracing-paper three lines forming the observed angles, and moving them about till they pass through the three fixed objects; and the angular point of these angles will then occupy the position of the boat. A very good station-pointer may be made by graduating an arc of a circle on a piece of plate glass, one side of which must be ground, to re-

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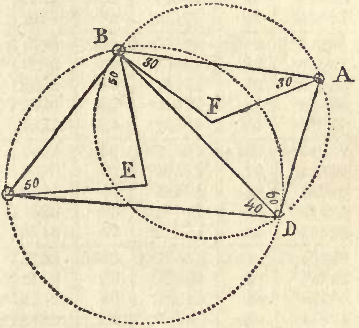
* It will readily be perceived that the station-pointer may be successfully employed in land surveys of considerable extent.

ceive the lines forming the observed angles, and it may be applied to the paper, as above described,—the centre of the graduated arc shewing the situation of the boat on the chart.

The position of the boat may also be determined geometrically, as follows, (but this would be too tedious a process where a great number of stations are to be determined). Let A B C be three fixed objects on shore, and, from the boat at D, suppose the angles C D B and B D A were found = 40° and 60°. Suptract double the angle C D B from 180°, and take half the remainder = 50°, and lay off this angle from C and B: the two lines will meet in E, which will be the centre of a circle passing through B C, and the place of the boat, which will be somewhere on this circle. To find the exact point, take double the angle, B D A, from 180°, and lay off half the remainder = 30° from B and A: these lines will meet in a point, F, which will also be the centre of a circle passing through A, B, and the place of the boat; consequently, where these two circles intersect each other, *viz.* at D, will be the situation of the boat on the plan, with regard to the shore, as required.



In conclusion, it may be useful to add a few remarks on the scales used in plotting the work of a survey.

One chain to an inch (80 inches to one mile) is perhaps the largest scale used in plans of land and road surveys, and is adopted only when great clearness is required, and when the work is of limited extent. It is a very useful scale for plans of building or pleasure grounds.

Two chains to an inch (40 inches to 1 mile) is a very clear scale for land surveys, the extent of which is not very great. It may likewise be used with advantage for gardens and building grounds.

Three chains to an inch ($26\frac{}{3}$ inches to 1 mile) has hitherto not been in very general use, but has lately been adopted by the Tithe Commissioners for the scale of their plans, considering it as " the smallest scale that can with safety be used, in all cases for plans from which the contents are to be computed."

Four chains to an inch (20 inches to 1 mile) is a scale frequently employed in plotting surveys of estates, and is very convenient for either enlargement or reduction.

Smaller scales are usually employed in extensive operations: six inches to 1 mile is a large scale for the survey of a county, and is the one employed in drawing the plans of the Ordnance Survey of Ireland. The English Survey is published on a scale of one inch to a mile.

The plans and sections for projected railways, &c. deposited with the Houses of Parliament, to obtain the sanction of the legislature, are required to be drawn on scales not less than 4 inches to the mile for the plan, and one hundred feet to the inch for the section.

TABLE I.

To reduce the Apparent to the True Level.

Argument = the Distance in Feet.

Dist. in Feet.	Correctⁿ in Decimals of a Foot.	Dist. in Feet.	Correctⁿ in Decimals of a Foot.	Dist. in Feet.	Correctⁿ in Decimals of a Foot.	Dist. in Feet.	Correctⁿ in Decimals of a Foot.	Dist. in Feet.	Correctⁿ in Decimals of a Foot.
20	0·00001	1020	0·02489	2020	0·09762	3020	0·21821	4020	0·38665
40	·00004	40	·02588	40	·09957	40	·22111	40	·39050
60	·00009	60	·02688	60	·10153	60	·22403	60	·39439
80	·00015	80	·02791	80	·10351	80	·22697	80	·39828
100	0·00024	1100	0·02895	2100	0·10551	3100	0·22993	4100	0·40218
20	·00034	20	·03001	20	·10753	20	·23290	20	·40613
40	·00047	40	·03109	40	·10956	40	·23590	40	·41008
60	·00061	60	·03219	60	·11162	60	·23892	60	·41404
80	·00077	80	·03331	80	·11370	80	·24195	80	·41805
200	0·00096	1200	0·03445	2200	0·11580	3200	0·24500	4200	0·42205
20	·00116	20	·03561	20	·11792	20	·24807	20	·42607
40	·00138	40	·03679	40	·12005	40	·25117	40	·43014
60	·00162	60	·03798	60	·12220	60	·25427	60	·43420
80	00187	80	·03920	80	·12437	80	·25740	80	·43827
300	0·00215	1300	0·04043	2300	0·12657	3300	0·26055	4300	0·44239
20	·00245	20	·04169	20	·12878	20	·26372	20	·44650
40	·00276	40	·04296	40	·13101	40	·26691	40	·45066
60	·00310	60	·04425	60	·13326	60	·27011	60	·45483
80	·00345	80	·04556	80	·13553	80	·27334	80	·45899
400	0·00383	1400	0·04689	2400	0·13781	3400	0·27658	4400	0·46320
20	·00422	20	·04824	20	·14012	20	·27985	20	·46742
40	·00462	40	·04961	40	·14244	40	·28313	40	·47166
60	·00506	60	·05100	60	·14480	60	·28643	60	·47591
80	·00551	80	·05241	80	·14715	80	·28975	80	·48020
500	0·00598	1500	0·05383	2500	0·14954	3500	0·29309	4500	0·48449
20	·00647	20	·05528	20	·15194	20	·29644	20	·48881
40	·00697	40	·05674	40	·15436	40	·29982	40	·49316
60	·00750	60	·05822	60	·15680	60	·30323	60	·49749
80	·00805	80	·05972	80	·15926	80	·30664	80	·50189
600	0·00861	1600	0·06125	2600	0·16173	3600	0·31008	4600	0·50627
20	·00920	20	·06279	20	·16423	20	·31353	20	·51067
40	·00980	40	·06435	40	·16675	40	·31700	40	·51511
60	·01042	60	·06593	60	·16929	60	·32050	60	·51957
80	·01106	80	·06753	80	·17184	80	·32401	80	·52404
700	0·01172	1700	0·06914	2700	0·17441	3700	0·32754	4700	0·52852
20	·01240	20	·07078	20	·17701	20	·33110	20	·53302
40	·01310	40	·07244	40	·17962	40	·33466	40	·53755
60	·01382	60	·07411	60	·18225	60	·33825	60	·54211
80	·01456	80	·07581	80	·18490	80	·34186	80	·54667
800	0·01531	1800	0·07752	2800	0·18758	3800	0·34548	4800	0·55124
20	·01609	20	·07925	20	·19026	20	·34913	20	·55586
40	·01688	40	·08100	40	·19298	40	·35280	40	·56048
60	·01769	60	·08277	60	·19571	60	·35650	60	·56512
80	·01853	80	·08456	80	·19844	80	·36018	80	·56978
900	0·01938	1900	0·08637	2900	0·20121	3900	0·36390	4900	0·57447
20	·02025	20	·08820	20	·20400	20	·36766	20	·57917
40	·02114	40	·09005	40	·20681	40	·37142	40	·58388
60	·02205	60	·09191	60	·20962	60	·37520	60	·58860
80	·02298	80	·09380	80	·21247	80	·37899	80	·59337
1000	0·02392	2000	0·09570	3000	0·21533	4000	0·38281	5000	0·59814

The correction to be subtracted from the apparent (or observed) to obtain the true level.

TABLE II.

For determining Altitudes with the Barometer.

Computed by Mr. Baily's Formula XXXVIII.

Thermometers in open Air.						Thermometers to the Barometer.		Latitude of the Place.	
S	A	S	A	S	A	D	B	L	C
40°	4·76891	84°	4·79019	128°	4·81048	0°	0·00000	0°	0·00117
41	·76940	85	·79066	129	·81093	1	·00004	3	·00116
42	·76989	86	·79113	130	·81138	2	·00009	6	·00114
43	·77039	87	·79160	131	·81183	3	·00013	9	·00111
44	·77089	88	·79207	132	·81228	4	·00017	12	·00107
45	4·77138	89	4·79254	133	4·81272	5	0·00022	15	0·00101
46	·77187	90	·79301	134	·81317	6	·00026	18	·00095
47	·77236	91	·79348	135	·81362	7	·00030	21	·00087
48	·77286	92	·79395	136	·81407	8	·00035	24	·00078
49	·77335	93	·79442	137	·81451	9	·00039	27	·00069
50	4·77383	94	4·79488	138	4·81496	10	0·00043	30	0·00059
51	·77433	95	·79535	139	·81541	11	·00048	33	·00048
52	·77482	96	·79582	140	·81585	12	·00052	36	·00036
53	·77531	97	·79629	141	·81630	13	·00056	39	·00024
54	·77579	98	·79675	142	·81675	14	·00061	42	·00012
55	4·77628	99	4.79722	143	4·81719	15	0·00065	45	0·00000
56	·77677	100	·79768	144	·81763	16	·00069	48	9·99988
57	·77726	101	·79814	145	·81807	17	·00074	51	·99976
58	·77774	102	·79860	146	·81851	18	·00078	54	·99964
59	·77823	103	·79907	147	·81896	19	·00083	57	·99952
60	4·77871	104	4·79953	148	4·81940	20	0·00087	60	9·99941
61	·77920	105	·79999	149	·81983	21	·00091	63	·99931
62	·77968	106	·80045	150	·82027	22	·00096	66	·99922
63	·78017	107	·80091	151	·82072	23	·00100	69	·99913
64	·78065	108	·80137	152	·82116	24	·00104	72	·99905
65	4·78113	109	4·80183	153	4·82160	25	0·00109	75	9·99899
66	·78161	110	·80229	154	·82204	26	·00113	78	·99893
67	·78209	111	·80275	155	·82248	27	·00117	81	·99889
68	·78257	112	·80321	156	·82291	28	·00122	84	·99886
69	·78305	113	·80367	157	·82335	29	·00126	87	·99884
70	4·78353	114	4·80412	158	4·82379	30	·00130	90	9·99883
71	·78401	115	·80458	159	·82422	31	0·00134		
72	·78449	116	·80504	160	·82466				
73	·78497	117	·80549	161	·82510				
74	·78544	118	·80595	162	·82553				
75	4·78592	119	4·80641	163	4·82597				
76	·78640	120	·80687	164	·82640				
77	·78688	121	·80732	165	·82683				
78	·78735	122	·80777	166	·82726				
79	·78783	123	·80823	167	·82770				
80	4·78830	124	4·80869	168	4·82813				
81	·78878	125	·80914	169	·82857				
82	·78925	126	·80958	170	·82900				
83	·78972	127	·81003	171	·82943				
84	4·79019	128	4·81048	172	4·82986				

S = the sum of the detached thermometers at the two stations.
D = the difference of the attached thermometers at the two stations.
L = the latitude.
β = height of the barometer at the upper station.
β' = height of the barometer at the lower station.

Make Log R = log. β' − (B + log. β) when upper thermometer reads lowest,
or R = log. β' + B − log. β when upper thermometer reads highest.
Then the log. diff. of altitude in English feet = A + C + log. of R.

TABLE III.

For converting Intervals of Sidereal into corresponding Intervals of Mean Solar Time.

Hours.			Minutes.						Seconds.					
h	m	s	m	s	m	s	m	s	s	s	s	s	s	s
1	0	9,830	1	0,164	21	3,440	41	6,717	1	0,003	21	0,057	41	0,112
2	0	19,659	2	0,328	22	3,604	42	6,881	2	0,005	22	0,060	42	0,115
3	0	29,489	3	0,491	23	3,768	43	7,044	3	0,008	23	0,063	43	0,118
4	0	39,318	4	0,655	24	3,932	44	7,208	4	0,011	24	0,066	44	0,120
5	0	49,148	5	0,819	25	4,096	45	7,372	5	0,014	25	0,068	45	0,123
6	0	58,977	6	0,983	26	4,259	46	7,536	6	0,016	26	0,071	46	0,126
7	1	8,807	7	1,147	27	4,423	47	7,700	7	0,019	27	0,074	47	0,128
8	1	18,636	8	1,311	28	4,587	48	7,864	8	0,022	28	0,076	48	0,131
9	1	28,466	9	1,474	29	4,751	49	8,027	9	0,025	29	0,079	49	0,134
10	1	38,296	10	1,638	30	4,915	50	8,191	10	0,027	30	0,082	50	0,137
11	1	48,125	11	1,802	31	5,079	51	8,355	11	0,030	31	0,085	51	0,140
12	1	57,955	12	1,966	32	5,242	52	8,519	12	0,033	32	0,087	52	0,142
13	2	7,784	13	2,130	33	5,406	53	8,683	13	0,036	33	0,090	53	0,145
14	2	17,614	14	2,294	34	5,570	54	8,847	14	0,038	34	0,093	54	0,148
15	2	27,443	15	2,457	35	5,734	55	9,010	15	0,041	35	0,096	55	0,150
16	2	37,273	16	2,621	36	5,898	56	9,174	16	0,044	36	0,098	56	0,153
17	2	47,103	17	2,785	37	6,062	57	9,338	17	0,047	37	0,101	57	0,156
18	2	56,932	18	2,949	38	6,225	58	9,502	18	0,049	38	0,104	58	0,159
19	3	6,762	19	3,113	39	6,389	59	9,666	19	0,052	39	0,106	59	0,161
20	3	16,591	20	3,277	40	6,553	60	9,830	20	0,055	40	0,109	60	0,164
21	3	26,421												
22	3	36,250												
23	3	46,080												
24	3	55,909												

The quantities taken from this Table must be subtracted from a sidereal interval, to obtain the corresponding interval in mean solar time.

TABLE IV.

For converting Intervals of Mean Solar into corresponding Intervals of Sidereal Time.

Hours.			Minutes.						Seconds.					
h	m	s	m	s	m	s	m	s	s	s	s	s	s	s
1	0	9,856	1	0,164	21	3,450	41	6,735	1	0,003	21	0,057	41	0,112
2	0	19,713	2	0,329	22	3,614	42	6,900	2	0,005	22	0,060	42	0,115
3	0	29,569	3	0,493	23	3,778	43	7,064	3	0,008	23	0,063	43	0,118
4	0	39,426	4	0,657	24	3,943	44	7,228	4	0,011	24	0,066	44	0,120
5	0	49,282	5	0,821	25	4,107	45	7,392	5	0,014	25	0,068	45	0,123
6	0	59,139	6	0,986	26	4,271	46	7,557	6	0,016	26	0,071	46	0,126
7	1	8,995	7	1,150	27	4,436	47	7,721	7	0,019	27	0,074	47	0,128
8	1	18,852	8	1,314	28	4,600	48	7,885	8	0,022	28	0,076	48	0,131
9	1	28,708	9	1,478	29	4,764	49	8,050	9	0,025	29	0,079	49	0,134
10	1	38,565	10	1,643	30	4,928	50	8,214	10	0,027	30	0,082	50	0,137
11	1	48,421	11	1,807	31	5,092	51	8,378	11	0,030	31	0,085	51	0,140
12	1	58,278	12	1,971	32	5,257	52	8,542	12	0,033	32	0,087	52	0,142
13	2	8,134	13	2,136	33	5,421	53	8,707	13	0,036	33	0,090	53	0,145
14	2	17,991	14	2,300	34	5,585	54	8,871	14	0,038	34	0,093	54	0,148
15	2	27,847	15	2,464	35	5,750	55	9,035	15	0,041	35	0,096	55	0,150
16	2	37,704	16	2,628	36	5,914	56	9,199	16	0,044	36	0,098	56	0,153
17	2	47,560	17	2,793	37	6,078	57	9,364	17	0,047	37	0,101	57	0,156
18	2	57,416	18	2,957	38	6,242	58	9,528	18	0,049	38	0,104	58	0,159
19	3	7,273	19	3,121	39	6,407	59	9,692	19	0,052	39	0,106	59	0,161
20	3	17,129	20	3,285	40	6,571	60	9,856	20	0,055	40	0,109	60	0,164
21	3	26,986												
22	3	36,842												
23	3	46,699												
24	3	56,555												

The quantities taken from this Table must be added to a mean interval, to obtain the corresponding interval in sidereal time.

TABLE V.

Logarithms to compute the Longitude from the Difference between the Intervals of the Transit of the Moon's bright Limb and a Star.

1 Min. s	Log.	Parts	1 Min. s	Log.	Parts	1 Min. s	Log.	Parts	2 Min. s	Log.	Parts
42 ·0	1·536442	0	48 ·0	1·510875	0	54 ·0	1·486648	0	0 ·0	1·463627	0
·1	1·536004	43	·1	1·510461	41	·1	1·486255	39	·1	1·463253	37
·2	1·535566	87	·2	1·510047	82	·2	1·485862	78	·2	1·462879	75
·3	1·535130	130	·3	1·509633	123	·3	1·485469	118	·3	1·462505	112
·4	1·534693	174	·4	1·509220	165	·4	1·485077	157	·4	1·462132	149
·5	1·534256	217	·5	1·508808	206	·5	1·484685	196	·5	1·461759	187
·6	1·533821	261	·6	1·508395	247	·6	1·484294	235	·6	1·461386	224
·7	1·533385	304	·7	1·507983	288	·7	1·483903	274	·7	1·461014	261
·8	1·532950	348	·8	1·507571	330	·8	1·483512	313	·8	1·460642	298
·9	1·532515	391	·9	1·507161	371	·9	1·483121	353	·9	1·460270	336
43 ·0	1·532081	0	49 ·0	1·506749	0	55 ·0	1·482731	0	1 ·0	1·459898	0
·1	1·531647	43	·1	1·506337	41	·1	1·482341	39	·1	1·459527	37
·2	1·531214	86	·2	1·505928	82	·2	1·481952	78	·2	1·459156	74
·3	1·530781	130	·3	1·505518	123	·3	1·481562	117	·3	1·458785	111
·4	1·530348	173	·4	1·505108	164	·4	1·481173	155	·4	1·458414	148
·5	1·529916	216	·5	1·504699	205	·5	1·480785	194	·5	1·458044	185
·6	1·529484	259	·6	1·504290	245	·6	1·480397	233	·6	1·457674	222
·7	1·529053	302	·7	1·503881	286	·7	1·480009	272	·7	1·457305	259
·8	1·528622	346	·8	1·503473	327	·8	1·479621	311	·8	1·456936	296
·9	1·528191	389	·9	1·503065	368	·9	1·479234	350	·9	1·456567	333
44 ·0	1·527761	0	50 ·0	1·502658	0	56 ·0	1·478847	0	2 ·0	1·456198	0
·1	1·527331	43	·1	1·502251	41	·1	1·478460	39	·1	1·455830	37
·2	1·526902	86	·2	1·501844	81	·2	1·478074	77	·2	1·455462	73
·3	1·526473	128	·3	1·501438	122	·3	1·477688	116	·3	1·455094	110
·4	1·526044	171	·4	1·501032	162	·4	1·477302	154	·4	1·454727	147
·5	1·525616	214	·5	1·500626	203	·5	1·476917	193	·5	1·454360	184
·6	1·525188	257	·6	1·500221	243	·6	1·476532	231	·6	1·453993	220
·7	1·524761	300	·7	1·499816	284	·7	1·476147	270	·7	1·453627	257
·8	1·524334	342	·8	1·499411	324	·8	1.475763	308	·8	1·453261	294
·9	1·523907	385	·9	1·499007	365	·9	1.475379	347	·9	1·452895	330
45 ·0	1·523481	0	51 ·0	1·498603	0	57 ·0	1·474995	0	3 ·0	1·452529	0
·1	1·523055	42	·1	1·498200	40	·1	1·474612	38	·1	1 452164	36
·2	1·522630	85	·2	1·497797	80	·2	1·474228	76	·2	1·451799	73
·3	1·522205	127	·3	1·497393	121	·3	1·473845	115	·3	1·451434	109
·4	1·521780	170	·4	1·496991	161	·4	1·473462	153	·4	1·451069	146
·5	1·521355	212	·5	1·496589	201	·5	1·473080	191	·5	1·450705	182
·6	1·520932	254	·6	1·496188	241	·6	1·472699	229	·6	1·450341	218
·7	1·520508	297	·7	1·495786	281	·7	1·472317	267	·7	1·449977	255
·8	1·520085	339	·8	1·495385	322	·8	1·471936	306	·8	1·449614	291
·9	1·519662	382	·9	1·494984	362	·9	1·471555	344	·9	1·449251	328
46 ·0	1·519240	0	52 ·0	1·494584	0	58 ·0	1·471174	0	4 ·0	1·448888	0
·1	1·518820	42	·1	1·494184	40	·1	1·470794	38	·1	1·448525	36
·2	1·518400	84	·2	1·493784	80	·2	1·470414	76	·2	1·448163	72
·3	1·517980	126	·3	1·493385	120	·3	1·470034	114	·3	1·447801	108
·4	1·517560	168	·4	1·492986	159	·4	1·469655	152	·4	1·447439	144
·5	1·517140	210	·5	1·492587	199	·5	1·469276	190	·5	1·447078	181
·6	1·516719	252	·6	1·492189	239	·6	1·468897	227	·6	1·446717	217
·7	1·516299	294	·7	1·491791	279	·7	1·468518	265	·7	1·446356	253
·8	1·515879	336	·8	1·491393	319	·8	1·468140	303	·8	1·445995	289
·9	1·515459	378	·9	1·490996	359	·9	1·467762	341	·9	1·445635	325
47 ·0	1·515039	0	53 ·0	1·490599	0	59 ·0	1·467385	0	5 ·0	1·445275	0
·1	1·514621	42	·1	1·490202	39	·1	1·467008	38	·1	1·444915	36
·2	1·514203	83	·2	1·489806	79	·2	1·466631	75	·2	1·444556	71
·3	1·513786	125	·3	1·489410	119	·3	1·466255	113	·3	1·444196	107
·4	1·513369	167	·4	1·489014	158	·4	1·465878	150	·4	1·443837	143
·5	1·512952	208	·5	1·488619	198	·5	1·465502	188	·5	1·443478	179
·6	1·512536	250	·6	1·488224	237	·6	1·465127	226	·6	1·443120	215
·7	1·512120	292	·7	1·487830	277	·7	1·464751	263	·7	1·442762	251
·8	1·511705	333	·8	1·487436	316	·8	1·464376	301	·8	1·442404	286
·9	1·511290	375	·9	1·487042	356	·9	1·464001	338	·9	1·442046	322

TABLE V.

Logarithms to compute the Longitude from the Difference between the Intervals of the Transit of the Moon's bright Limb and a Star.

2 Min. s	Log.	Parts	2 Min. s	Log.	Parts	2 Min. s	Log.	Parts	2 Min. s	Log.	Parts
6 ·0	1 ·441689	0	12 ·0	1 ·420737	0	18 ·0	1 ·400682	0	24 ·0	1 ·381448	0
·1	1 ·441332	36	·1	1 ·420396	34	·1	1 ·400355	33	·1	1 ·381134	31
·2	1 ·440975	71	·2	1 ·420055	68	·2	1 ·400028	65	·2	1 ·380820	63
·3	1 ·440619	107	·3	1 ·419715	102	·3	1 ·399701	98	·3	1 ·380506	94
·4	1 ·440263	142	·4	1 ·419373	136	·4	1 ·399375	130	·4	1 ·380193	125
·5	1 ·439907	178	·5	1 ·419033	170	·5	1 ·399049	163	·5	1 ·379880	156
·6	1 ·439551	214	·6	1 ·418693	204	·6	1 ·398723	196	·6	1 ·379567	188
·7	1 ·439196	249	·7	1 ·418354	238	·7	1 ·398397	228	·7	1 ·379254	219
·8	1 ·438840	285	·8	1 ·418014	272	·8	1 ·398071	261	·8	1 ·378941	250
·9	1 ·438486	320	·9	1 ·417674	306	·9	1 ·397746	293	·9	1 ·378629	282
7 ·0	1 ·438131	0	13 ·0	1 ·417335	0	19 ·0	1 ·397421	0	25 ·0	1 ·378317	0
·1	1 ·437777	35	·1	1 ·416996	34	·1	1 ·397096	32	·1	1 ·378005	31
·2	1 ·437423	71	·2	1 ·416657	67	·2	1 ·396772	65	·2	1 ·377693	62
·3	1 ·437069	106	·3	1 ·416319	101	·3	1 ·396447	97	·3	1 ·377382	93
·4	1 ·436715	141	·4	1 ·415981	135	·4	1 ·396123	129	·4	1 ·377071	124
·5	1 ·436362	177	·5	1 ·415643	168	·5	1 ·395799	161	·5	1 ·376759	155
·6	1 ·436009	212	·6	1 ·415306	202	·6	1 ·395476	194	·6	1 ·376449	186
·7	1 ·435650	247	·7	1 ·414969	236	·7	1 ·395152	226	·7	1 ·376138	218
·8	1 ·435304	282	·8	1 ·414631	276	·8	1 ·394829	258	·8	1 ·375827	249
·9	1 ·434952	318	·9	1 ·414294	303	·9	1 ·394506	291	·9	1 ·375517	280
8 ·0	1 ·434600	0	14 ·0	1 ·413957	0	20 ·0	1 ·394183	0	26 ·0	1 ·375207	0
·1	1 ·434248	35	·1	1 ·413621	33	·1	1 ·393860	32	·1	1 ·374897	31
·2	1 ·433897	70	·2	1 ·413285	67	·2	1 ·393538	64	·2	1 ·374587	62
·3	1 ·433546	105	·3	1 ·412949	100	·3	1 ·393216	96	·3	1 ·374278	92
·4	1 ·433195	140	·4	1 ·412613	134	·4	1 ·392894	128	·4	1 ·373969	123
·5	1 ·432845	175	·5	1 ·412278	167	·5	1 ·392572	160	·5	1 ·373659	154
·6	1 ·432494	210	·6	1 ·411942	201	·6	1 ·392251	193	·6	1 ·373351	185
·7	1 ·432144	245	·7	1 ·411607	234	·7	1 ·391930	225	·7	1 ·373042	216
·8	1 ·431795	280	·8	1 ·411273	268	·8	1 ·391608	257	·8	1 ·372733	246
·9	1 ·431445	315	·9	1 ·410938	301	·9	1 ·391288	289	·9	1 ·372425	277
9 ·0	1 ·431096	0	15 ·0	1 ·410604	0	21 ·0	1 ·390967	0	27 ·0	1 ·372117	0
·1	1 ·430747	35	·1	1 ·410270	33	·1	1 ·390647	32	·1	1 ·371809	31
·2	1 ·430398	70	·2	1 ·409936	67	·2	1 ·390327	64	·2	1 ·371501	61
·3	1 ·430050	104	·3	1 ·409602	100	·3	1 ·390007	96	·3	1 ·371194	92
·4	1 ·429701	139	·4	1 ·409269	133	·4	1 ·389687	128	·4	1 ·370887	122
·5	1 ·429354	174	·5	1 ·408935	166	·5	1 ·389367	159	·5	1 ·370579	153
·6	1 ·429006	209	·6	1 ·408602	200	·6	1 ·389048	191	·6	1 ·370273	184
·7	1 ·428659	244	·7	1 ·408270	233	·7	1 ·388729	223	·7	1 ·369966	214
·8	1 ·428312	278	·8	1 ·407937	266	·8	1 ·388410	255	·8	1 ·369659	245
·9	1 ·427965	313	·9	1 ·407605	300	·9	1 ·388091	287	·9	1 ·369353	275
10 ·0	1 ·427618	0	16 ·0	1 ·407273	0	22 ·0	1 ·387773	0	28 ·0	1 ·369047	0
·1	1 ·427272	35	·1	1 ·406941	33	·1	1 ·387454	32	·1	1 ·368741	30
·2	1 ·426925	69	·2	1 ·406610	66	·2	1 ·387137	63	·2	1 ·368435	61
·3	1 ·426580	103	·3	1 ·406278	99	·3	1 ·386819	95	·3	1 ·368130	91
·4	1 ·426234	138	·4	1 ·405947	132	·4	1 ·386501	127	·4	1 ·367825	122
·5	1 ·425888	172	·5	1 ·405617	165	·5	1 ·386183	158	·5	1 ·367520	152
·6	1 ·425543	207	·6	1 ·405286	198	·6	1 ·385867	190	·6	1 ·367215	183
·7	1 ·425198	242	·7	1 ·404956	231	·7	1 ·385550	222	·7	1 ·366910	213
·8	1 ·424854	276	·8	1 ·404626	264	·8	1 ·385233	254	·8	1 ·366605	244
·9	1 ·424509	310	·9	1 ·404296	297	·9	1 ·384916	285	·9	1 ·366301	274
11 ·0	1 ·424165	0	17 ·0	1 ·403966	0	23 ·0	1 ·384600	0	29 ·0	1 ·365997	0
·1	1 ·423821	34	·1	1 ·403636	33	·1	1 ·384284	31	·1	1 ·365693	30
·2	1 ·423477	69	·2	1 ·403307	66	·2	1 ·383968	63	·2	1 ·365389	61
·3	1 ·423134	103	·3	1 ·402978	98	·3	1 ·383652	94	·3	1 ·365086	91
·4	1 ·422791	137	·4	1 ·402650	131	·4	1 ·383337	126	·4	1 ·364782	121
·5	1 ·422448	171	·5	1 ·402321	164	·5	1 ·383021	157	·5	1 ·364479	151
·6	1 ·422105	206	·6	1 ·401993	197	·6	1 ·382706	189	·6	1 ·364176	182
·7	1 ·421763	240	·7	1 ·401665	230	·7	1 ·382391	220	·7	1 ·363873	212
·8	1 ·421421	274	·8	1 ·401337	262	·8	1 ·382077	252	·8	1 ·363571	242
·9	1 ·421079	309	·9	1 ·401009	295	·9	1 ·381762	283	·9	1 ·363268	273

TABLE V.

Logarithms to compute the Longitude from the Difference between the Intervals of the Transit of the Moon's bright Limb and a Star.

2 Min. s	Log.	Parts	2 Min. s	Log.	Parts	2 Min. s	Log.	Parts	2 Min. s	Log.	Parts
30 ·0	1 ·362966	0	34 ·0	1 ·351035	0	38 ·0	1 ·339396	0	42 ·0	1 ·328034	0
·1	1 ·362664	30	·1	1 ·350740	29	·1	1 ·339109	29	·1	1 ·327753	28
·2	1 ·362362	60	·2	1 ·350446	59	·2	1 ·338821	57	·2	1 ·327473	56
·3	1 ·362061	90	·3	1 ·350152	88	·3	1 ·338534	86	·3	1 ·327192	84
·4	1 ·361759	120	·4	1 ·349858	118	·4	1 ·338248	115	·4	1 ·326912	112
·5	1 ·361458	150	·5	1 ·349564	147	·5	1 ·337961	143	·5	1 ·326632	140
·6	1 ·361157	181	·6	1 ·349271	176	·6	1 ·337675	172	·6	1 ·326352	168
·7	1 ·360856	211	·7	1 ·348977	206	·7	1 ·337388	201	·7	1 ·326073	196
·8	1 ·360556	241	·8	1 ·348684	235	·8	1 ·337102	230	·8	1 ·325793	224
·9	1 ·360255	271	·9	1 ·348391	265	·9	1 ·336816	258	·9	1 ·325514	252
31 ·0	1 ·359955	0	35 ·0	1 ·348098	0	39 ·0	1 ·336530	0	43 ·0	1 ·325235	0
·1	1 ·359655	30	·1	1 ·347805	29	·1	1 ·336244	28	·1	1 ·324956	28
·2	1 ·359355	60	·2	1 ·347513	58	·2	1 ·335959	57	·2	1 ·324677	56
·3	1 ·359055	90	·3	1 ·347220	88	·3	1 ·335674	85	·3	1 ·324399	83
·4	1 ·358756	120	·4	1 ·346928	117	·4	1 ·335389	114	·4	1 ·324120	111
·5	1 ·358457	149	·5	1 ·346636	146	·5	1 ·335104	142	·5	1 ·323842	139
·6	1 358157	179	·6	1 ·346344	175	·6	1 ·334819	171	·6	1 ·323564	167
·7	1 ·357859	209	·7	1 ·346053	204	·7	1 ·334534	199	·7	1 ·323286	195
·8	1 ·357560	239	·8	1 ·345761	234	·8	1 ·334250	228	·8	1 ·323008	222
·9	1 ·357261	269	·9	1 ·345470	263	·9	1 ·333966	256	·9	1 ·322730	250
32 ·0	1 ·356963	0	36 ·0	1 ·345179	0	40 ·0	1 ·333682	0	44 ·0	1 ·322453	0
·1	1 ·356665	30	·1	1 ·344888	29	·1	1 ·333398	28	·1	1 ·322176	28
·2	1 ·356367	59	·2	1 ·344598	58	·2	1 ·333114	57	·2	1 ·321898	55
·3	1 ·356069	89	·3	1 ·344307	87	·3	1 ·332831	85	·3	1 ·321621	83
·4	1 ·355772	119	·4	1 ·344017	116	·4	1 ·332547	113	·4	1 ·321345	111
·5	1 ·355474	148	·5	1 ·343727	145	·5	1 ·332264	141	·5	1 ·321068	138
·6	1 ·355177	178	·6	1 ·343437	174	·6	1 ·331981	170	·6	1 ·320791	166
·7	1 ·354880	208	·7	1 ·343147	203	·7	1 ·331698	198	·7	1 ·320515	194
·8	1 ·354583	238	·8	1 ·342858	232	·8	1 ·331415	226	·8	1 ·320239	222
·9	1 ·354286	267	·9	1 ·342568	261	·9	1 ·331132	255	·9	1 ·319963	249
33 0	1 ·353990	0	37 ·0	1 ·342279	0	41 ·0	1 ·330850	0	45 ·0	1 ·319687	0
·1	1 ·353694	29	·1	1 ·341990	29	·1	1 ·330568	28	·1	1 ·319411	27
·2	1 ·353397	59	·2	1 ·341701	58	·2	1 ·330285	56	·2	1 ·319136	55
·3	1 ·353102	88	·3	1 ·341412	86	·3	1 ·330003	84	·3	1 ·318860	82
·4	1 ·352806	118	·4	1 ·341124	115	·4	1 ·329722	112	·4	1 ·318585	110
·5	1 ·352510	147	·5	1 ·340835	144	·5	1 ·329440	140	·5	1 ·318310	137
·6	1 ·352215	177	·6	1 ·340547	173	·6	1 ·329158	169	·6	1 ·318035	165
·7	1 ·351920	206	·7	1 ·340259	202	·7	1 ·328877	197	·7	1 ·317760	192
·8	1 ·351624	236	·8	1 ·339671	230	·8	1 ·328596	225	·8	1 ·317486	220
·9	1 ·351330	265	·9	1 ·339683	259	·9	1 ·328315	253	·9	1 ·317211	247

TABLE VI.

Effect of a Change in the Moon's Semidiameter on the Time of its passing the Meridian.

Change of ☽'s Semidiam.	Moon's Declination.				
	0°	8°	16°	22°	28°
″	s	s	s	s	s
1	·07	·07	·07	·07	·07
2	·14	·14	·14	·15	·15
3	·21	·21	·22	·23	·23
4	·28	·28	·29	·30	·31
5	·34	·35	·36	·38	·39
6	·41	·42	·43	·45	·47
7	·48	·49	·50	·53	·55
8	·55	·56	·58	·60	·62
9	·62	·63	·65	·68	·70
10	·69	·70	·72	·75	·78

TABLE VII.

Reduction to the Meridian.

Argument = the Hour Angle from the Meridian.

^{s}	0^m	1^m	2^m	3^m	4^m	5^m	6^m	7^m	8^m	9^m	10^m	11^m	12^m	13^m	14^m
0	0	9	38	86	152	238	343	466	609	771	952	1152	1370	1608	1865
1	0	10	39	87	153	240	345	469	612	774	955	1155	1374	1612	1870
2	0	10	39	88	155	241	346	471	614	777	958	1159	1378	1617	1874
3	0	10	40	89	156	243	348	473	617	780	961	1162	1382	1621	1879
4	0	11	41	90	157	244	350	475	619	782	964	1166	1386	1625	1883
5	0	11	41	91	159	246	352	478	622	785	968	1169	1390	1629	1887
6	0	12	42	92	160	248	354	480	624	788	971	1173	1393	1633	1892
7	0	12	43	93	161	249	356	482	627	791	974	1176	1397	1637	1896
8	0	12	43	93	163	251	358	484	630	794	977	1180	1401	1641	1901
9	0	13	44	94	164	253	360	487	632	797	981	1183	1405	1646	1905
10	0	13	45	95	165	254	362	489	635	800	984	1187	1409	1650	1910
11	0	13	45	96	167	256	364	491	637	803	987	1190	1413	1654	1914
12	0	14	46	97	168	257	366	493	640	806	990	1194	1416	1658	1919
13	0	14	47	98	169	259	368	496	643	809	993	1197	1420	1662	1923
14	1	14	47	99	171	261	370	498	645	811	997	1201	1424	1667	1928
15	1	15	48	100	172	262	372	500	648	814	1000	1205	1428	1671	1932
16	1	15	49	102	173	264	374	503	650	817	1003	1208	1432	1675	1937
17	1	16	50	103	175	266	376	505	653	820	1006	1212	1436	1679	1941
18	1	16	50	104	176	267	378	507	656	823	1010	1215	1440	1683	1946
19	1	17	51	105	177	269	380	510	658	826	1013	1219	1444	1688	1950
20	1	17	52	106	179	271	382	512	661	829	1016	1222	1448	1692	1955
21	1	17	53	107	180	272	384	514	664	832	1020	1226	1451	1696	1960
22	1	18	53	108	182	274	386	517	666	835	1023	1230	1455	1700	1964
23	1	18	54	109	183	276	388	519	669	838	1026	1233	1459	1705	1968
24	2	19	55	110	184	278	390	521	672	841	1029	1237	1463	1709	1973
25	2	19	56	111	185	279	392	524	674	844	1033	1241	1467	1713	1978
26	2	20	56	112	187	281	394	526	677	847	1036	1244	1471	1717	1982
27	2	20	57	113	188	283	396	528	680	850	1039	1248	1475	1722	1987
28	2	20	58	114	190	284	398	531	682	853	1043	1251	1479	1726	1992
29	2	21	59	116	191	286	400	533	685	856	1046	1255	1483	1730	1997
30	3	21	59	117	193	288	402	535	688	859	1049	1259	1487	1734	2001
31	3	22	60	118	194	289	404	538	690	862	1053	1262	1491	1739	2005
32	3	22	61	119	196	291	406	540	693	865	1056	1266	1495	1743	2010
33	3	23	62	120	197	293	408	543	696	868	1059	1270	1499	1747	2014
34	3	23	63	121	198	295	410	545	699	871	1062	1273	1503	1751	2019
35	3	24	64	122	200	297	412	547	701	874	1066	1277	1507	1756	2024
36	3	24	64	123	201	299	415	550	704	877	1069	1281	1511	1760	2028
37	4	25	65	124	203	300	417	552	707	880	1073	1284	1515	1764	2033
38	4	25	66	126	204	302	419	555	709	883	1076	1288	1519	1769	2038
39	4	26	67	127	206	304	421	557	712	886	1079	1292	1523	1773	2042
40	4	26	68	128	207	306	423	559	715	889	1083	1295	1527	1777	2047
41	4	27	68	129	209	307	425	562	718	892	1086	1299	1531	1782	2052
42	5	28	69	130	210	309	427	564	720	896	1090	1303	1535	1786	2056
43	5	28	70	131	212	311	429	567	723	899	1093	1307	1539	1790	2061
44	5	29	71	133	213	313	432	569	726	902	1096	1310	1543	1795	2066
45	5	29	72	134	215	315	434	572	729	905	1100	1314	1547	1799	2070
46	6	30	73	135	216	316	436	574	732	908	1103	1318	1551	1804	2075
47	6	30	74	136	218	318	438	577	734	911	1107	1321	1555	1808	2080
48	6	31	75	137	219	320	440	579	737	914	1110	1325	1559	1812	2084
49	6	31	75	139	221	322	442	582	740	917	1114	1329	1563	1817	2089
50	7	32	76	140	222	324	444	584	743	920	1117	1333	1567	1821	2094
51	7	33	77	141	225	326	447	587	745	923	1120	1336	1571	1825	2099
52	7	33	78	142	226	328	449	589	748	927	1124	1340	1575	1830	2103
53	7	34	79	144	227	329	451	592	751	930	1127	1344	1580	1834	2108
54	8	34	80	145	229	331	453	594	754	933	1131	1348	1584	1839	2113
55	8	35	81	146	230	333	455	597	757	936	1134	1352	1588	1843	2117
56	8	36	82	147	232	335	458	599	760	939	1138	1355	1592	1847	2122
57	8	36	83	149	233	337	460	602	763	942	1141	1359	1596	1852	2127
58	9	37	84	150	235	339	462	604	765	945	1145	1363	1600	1856	2132
59	9	37	85	151	236	341	464	607	768	949	1148	1367	1604	1861	2136
60	9	38	86	152	238	343	466	609	771	952	1152	1370	1608	1865	2141

TABLE XI.

Correction of Moon's Meridional Passage.

The application of this and the following Table is explained at page 80.

Long. in Time.	Argument—Daily Change of Mer. Passage.														Long. in Arc.
	40^{m}	42^{m}	44^{m}	46^{m}	48^{m}	50^{m}	52^{m}	54^{m}	56^{m}	58^{m}	60^{m}	62^{m}	64^{m}	66^{m}	
h m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	°
0. 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5
40	1	1	1	1	1	1	1	1	1	2	2	2	2	2	10
1. 0	2	2	2	2	2	2	2	2	2	2	2	2	3	3	15
20	2	2	2	2	3	3	3	3	3	3	3	3	3	4	20
40	3	3	3	3	3	3	3	4	4	4	4	4	4	4	25
2. 0	3	3	4	4	4	4	4	4	4	5	5	5	5	5	30
20	4	4	4	4	5	5	5	5	5	5	6	6	6	6	35
40	4	4	5	5	5	5	6	6	6	6	6	7	7	7	40
3. 0	5	5	5	6	6	6	6	7	7	7	7	7	8	8	45
20	5	6	6	6	6	7	7	7	7	8	8	8	9	9	50
40	6	6	7	7	7	7	8	8	8	9	9	9	9	10	55
4. 0	6	7	7	7	8	8	8	9	9	9	10	10	10	11	60
20	7	7	8	8	8	9	9	9	10	10	10	11	11	11	65
40	7	8	8	9	9	9	10	10	10	11	11	12	12	12	70
5. 0	8	9	9	9	10	10	10	11	11	12	12	12	13	13	75
20	9	9	9	10	10	11	11	12	12	12	13	13	14	14	80
40	9	10	10	11	11	11	12	12	13	13	14	14	14	15	85
6. 0	10	10	11	11	12	12	13	13	13	14	14	15	15	16	90
20	10	11	11	12	12	13	13	14	14	15	15	16	16	17	95
40	11	11	12	12	13	13	14	14	15	15	16	17	17	17	100
7. 0	11	12	12	13	14	14	15	15	16	16	17	17	18	18	105
20	12	12	13	14	14	15	15	16	16	17	18	18	19	19	110
40	12	13	14	14	15	15	16	17	17	18	18	19	20	20	115
8. 0	13	14	14	15	15	16	17	17	18	19	19	20	20	21	120
20	13	14	15	15	16	17	17	18	19	19	20	21	21	22	125
40	14	15	15	16	17	17	18	19	19	20	21	21	22	23	130
9. 0	14	15	16	17	17	18	19	20	20	21	22	22	23	24	135
20	15	16	17	17	18	19	20	20	21	22	22	23	24	25	140
40	15	16	17	18	19	19	20	21	22	22	23	24	25	25	145
10. 0	16	17	18	19	19	20	21	22	22	23	24	25	26	26	150
20	16	18	18	19	20	21	22	22	23	24	25	26	26	27	155
40	17	18	19	20	21	21	22	23	24	25	26	26	27	28	160
11. 0	17	19	20	20	21	22	23	24	25	26	26	27	28	29	165
20	18	19	20	21	22	23	24	25	25	26	27	28	29	30	170
40	18	20	21	22	23	23	24	25	26	27	28	29	30	31	175
12. 0	19	20	21	22	23	24	25	26	27	28	29	30	31	32	180

TABLE XII.

Effect of a Change of 1° in Declination on the Moon's Semidiameter (as given in the Naut. Alm.)

Dec.	Corr.	Dec.	Corr.	Dec.	Corr.	Dec.	Corr.
°	s	°	s	°	s	°	s
0	·000	7	·134	14	·278	21	·445
1	·019	8	·154	15	·300	22	·472
2	·038	9	·173	16	·323	23	·499
3	·057	10	·194	17	·346	24	·527
4	·076	11	·214	18	·368	25	·557
5	·095	12	·235	19	·394	26	·587
6	·114	13	·256	20	·419	27	·619
7	·134	14	·278	21	·445	28	·652

CATALOGUE

OF

INSTRUMENTS,

MADE BY

TROUGHTON AND SIMMS,

OPTICIANS,

AND

MATHEMATICAL INSTRUMENT MAKERS

TO THE

Honourable Board of Ordnance,

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TROUGHTON & SIMMS *beg to caution those who may have occasion to write from abroad, that no reliance can be placed on the genuineness of the Instruments they obtain, unless the application be made* DIRECT, *or through the most respectable channels.*

A CATALOGUE,

&c.

SPECTACLES AND OPERA GLASSES.

		£	s.	d.
Gold Spectacles, Single Joint	from 3*l.* 3*s.* to	5	5	0
Ditto, Double Joint	from 4*l.* 4*s.* to	6	6	0
Ditto, Hand Frames	from 4*l.* 4*s.* to	6	6	0
Silver Spectacles, Single Joint	from 10*s.* to	0	12	0
Ditto, Double Joint	from 13*s.* to	0	18	0
Ditto, Hand Frame		1	1	0
Tortoiseshell Spectacles, Single Joint		0	7	6
Ditto, Double Joint		0	12	6
Ditto, Hand Frame	from 7*s.* 6*d.* to	0	16	0
Fine Blue Steel Spectacles, Single Joint		0	10	6
Ditto, Double Joint		0	12	6
Eye Glasses, Gold Frame	from 1*l.* 1*s.* to	2	2	0
Ditto, Silver Frame	from 6*s.* to	0	8	0
Ditto, Shell Frame	from 4*s.* 6*d.* to	0	6	0
(If with Brazilian Pebbles, 8s. per pair extra.)				
Spectacle Cases	each	0	1	0
Opera Glasses, not Achromatic, in Plated Mountings,	from 6*s.* to	1	11	6
Ditto, Achromatic, with Ivory Bodies and Gilt Mountings	from 2*l.* 2*s.* to	5	5	0

TELESCOPES.

	£	s.	d.
One-foot Achromatic (Camp) Telescope, having one Drawer	1	5	0
Ditto ditto, Portable, with Drawers	1	9	0
Ditto ditto, in Electrum	2	2	0
Ditto ditto, larger aperture in Brass	1	15	0
Eighteen-inch ditto	2	12	6
Two-feet ditto, Reconnoitering	3	13	6

	£	*s.*	*d.*
Two-feet Achromatic, Reconnoitring, in Electrum	5	5	0
Ditto ditto, in Electrum, with Compass, Agate Cap, Needle, &c.	6	6	0
Ditto ditto, ditto, with four Drawers in Brass	3	18	0
Thirty-inch ditto ditto	5	5	0
Three-feet ditto ditto	6	6	0
Military Case and Sling from 10*s.* 6*d.* to	0	12	6
Portable Brass Stand for any of the above Telescopes, from 21*s.* to	2	12	6
Walking Stick Telescope, Portable, with Compass	3	13	6
Ditto, without Compass	3	3	0
Ditto, in one length, with Compass	2	12	6
Ditto, without ditto	2	2	0
Dumpy Navy	4	4	0
Ditto, with Pancratic Eye-piece	4	14	6
Two-feet Navy Telescope	2	12	6
Ditto, ditto, Brass Body, covered with Leather	2	15	0
Ditto, ditto, with Spray Shade	3	3	0
Three-feet ditto	5	5	0
Ditto, ditto, with Spray Shade	5	15	6
Four feet ditto, with Two Powers, in Case	12	12	0
Day or Night Telescope (Deck Glass)	4	4	0
Ditto, ditto, with Spray Shade	4	14	6
Night Glass	3	3	0
Ditto, large size	4	4	0
Ordnance Signal Station Telescope	6	16	6
Thirty-inch Achromatic Telescope, two-and-a-quarter-inch Object Glass, mounted on Pillar-and-claw Stand, with a Terrestrial and an Astronomical Eye-piece, in a Mahogany Case	10	10	0
Ditto, with Vertical Rack Motion	12	12	0
Forty-five-inch Achromatic Telescope, two-and-three quarter-inch Object Glass, on Brass Pillar-and-claw Stand, with a Terrestrial and an Astronomical Eye-piece, in a Mahogany Case	23	2	0
Ditto with Vertical Rack Motion, Finder, and extra Eye-pieces	26	5	0
Ditto, with Horizontal Rack and Steadying Rods, complete	31	10	0
Ditto, three-and-a-quarter-inch Object Glass, with Rack-work Motions, Finder, one Terrestrial and three Astronomical Eye-pieces, in Mahogany Case	42	0	0

	£	s.	d.
Forty-five-inch Achromatic Telescope, three-and-three-quarter-inch Object Glass, mounted as above	68	5	0
Equatorial Stand, instead of Pillar-and-claw to the above Telescopes, constructed to any given latitude, 30 Guineas extra.			
Universal Equatorial, with 30-inch Telescope	80	0	0
Completely mounted Equatorial, with Clock Movement, Micrometers, &c., 5 feet focus, three and three-quarter-inch Object Glass	200	0	0
Ditto ditto, 4-inch Object Glass	230	0	0
Ditto ditto, 8 feet focus, five-and-a-half-inch ditto	400	0	0
Ditto, 10 feet ditto, 6-inch ditto	600	0	0
Varley's Stand, Mahogany, with Brass Fittings, capable of carrying Telescopes from three-and-a-half to seven feet	12	12	0

MICROSCOPES, &c.

	£	s.	d.
Solar Microscopes . . . from 6*l.* 16*s.* 6*d.* to	21	0	0
Botanic ditto, small size	0	15	0
Ditto, ditto	1	5	0
Compound ditto	2	12	6
Ditto ditto	2	15	0
Ditto ditto	3	10	0
Ditto ditto	4	10	0
Ditto ditto	5	10	0
Ditto, ditto, larger, from 10*l.* 10*s.* upwards.			
Cloth Microscopes, Diagonal Print Machines, Black Mirrors, Claude Glasses, Magic Lanterns, &c.			

COMPASSES, SEXTANTS, QUADRANTS, &c.

	£	s.	d.
Binnacle Compasses . . . . . from 12*s.* to	0	18	0
Brass Hanging Compasses for Cabins from 1*l.* 10*s.* to	5	0	0
Common Azimuth Compass	4	14	6
Azimuth (Prismatic) Compass, large size, best construction, with Tripod Stand complete	16	16	0
Ditto ditto, smaller size	10	10	0
Pocket Compasses in Wood, Brass, Metal, Gilt, Silver, and Gold . . . . from 3*s.* 6*d.* to	5	5	0
Ebony Quadrant with Tangent Screw	3	13	6

	£	s.	d.
Ebony Quadrant, with Tangent Screw and Telescope	4	14	6
Ditto ditto, best	5	15	6
Ebony Sextant, with Telescopes	8	8	0
Ditto ditto, with Brass Arch, &c.	10	10	0
Optical Square	1	1	0
Box Sextant, plain	3	13	6
Ditto, with Telescope	4	14	6
Box Sextant, Ordnance Pattern	5	5	0
Ditto, with Supplementary Arc	5	15	6
Ditto, with ditto, and Levels	6	6	0
Leather Case and Strap for Box Sextant	0	9	0
Metal Sextant, 4-inch Radius, divided on Silver to 20 seconds	10	10	0
Ditto, 5-inch ditto, to 20 seconds	13	13	0
Ditto, 6-inch ditto, to 20 seconds	14	14	0
Ditto, 7-inch ditto, to 10 seconds	16	16	0
Ditto, 8-inch ditto, with Double Frames, divided on Silver to 10 seconds	18	18	0
Ditto ditto, divided on Platina	21	0	0
Ditto ditto, divided on Gold	23	2	0
Dip Sector, as described in 'Treatise on Instruments,' by F. W. Simms	12	12	0
Troughton's Reflecting Circle	23	2	0
Six-inch Borda's Repeating Circle by Reflexion	21	0	0
Eight-inch ditto	23	2	0
Ten-inch ditto	25	0	0
Brass Counterpoise Stand for Circle or Sextant, in Mahogany Box	5	15	6
Glass Plane Artificial Horizons, two-and-a-quarter-inch diameter	1	11	6
Ditto, two-and-a-half-inch diameter	2	2	0
Ditto, three-inch diameter	3	3	0
Ditto, three-and-three-quarter inch	4	4	0
Ditto ditto, Ordnance Pattern	4	4	0
Best Mercurial Horizon, with Iron Bottle and Trough, Ordnance Pattern	5	5	0
Ditto ditto, smaller size	4	14	6
Ditto ditto, with Brass Folding-roof	5	5	0
Ditto ditto, with Ebony ditto	4	14	6
Ditto ditto, with Mahogany ditto	4	4	0
Marine Horizon (Captain Becher's)	5	5	0

LEVELS, THEODOLITES, &c.

	£	s.	d.
Four-inch Pocket Level	0	9	6
Six-inch ditto	0	12	6
Eight-inch ditto	0	15	6
Ten-inch ditto	1	1	0
Twelve-inch ditto	1	11	6
Block Level	1	1	0
Level with Sights and Socket, in Box	1	11	6
Portable Levelling Instrument, with Telescope,	8	8	0
Ditto, with Compass, &c.	9	9	0
Fourteen-inch improved Level, with Round Legs	11	11	0
Ditto, with Tripod Stand	12	12	0
Twenty-inch ditto, with Round Legs	13	13	0
Ditto ditto, with Tripod Stand	14	14	0
Y Levels, with nine-inch Telescope	10	10	0
Ditto, with twenty-inch ditto	16	16	0
Ditto, with ditto and Compass	17	17	0
Gravatt's Dumpy Level, without Legs or Compass	12	12	0
Ditto, with Silver Ring Compass and round Legs	15	15	0
Ditto, with Tripod Stand and Silver Ring Compass	16	16	0
Ditto, fourteen-inch, with Round Legs, and Card Compass	15	15	0
Ditto, ditto, with Silver Ring Compass and Tripod Stand	17	17	0
Ditto, large size, complete	22	0	0
Standard Levelling Instrument	42	0	0
Plane Table, with Sights and Round Legs	6	16	6
Ditto, with Telescope, &c.	12	12	0
Cylindrical Cross Staff	0	16	0
Ditto, with Compass and Legs	2	12	6
Circumferenter, in Mahogany, without Legs	2	12	6
Ditto, ditto, larger size	4	4	0
Ditto, in Brass, with Round Legs from 4*l.* 14*s.* 6*d.* to	6	6	0
Ditto, ditto, with Ball and Socket Joint, and Round Leg	6	16	6
Ditto, with Rack Motion, divided to 3 minutes, Ball and Socket Joint and Round Legs	10	10	0
Ditto, ditto, with Levels and Levelling Plates	12	12	0
Best Brass Miners' Compass, with divided Cover, Ball and Socket Joint, and Legs	7	17	6
Ditto, with Vertical Arc, Telescopic and Plain Sights, Levels in Compass, Rack Motion, &c., and Legs, complete	16	16	0
Prismatic Compass, plain	3	3	0

	£	s.	d.
Prismatic Compass, with Azimuth Glasses	3	13	6
Ditto, three-and-a-half inch, plain	3	13	6
Ditto, ditto, with Azimuth Glasses	4	4	0
Prismatic Compass, three-and-a-half inch, with Silver Ring	5	5	0
Stand for Prismatic Compass, with Ball and Socket Joint	1	11	6
Common Theodolite, with Telescope	14	14	0
Four-inch Cradle Theodolite, divided on Silver	16	16	0
Four-inch best Theodolite (Captain Dawson's)	21	0	0
Five-inch Cradle ditto	21	0	0
Five-inch ditto (best construction,) divided on Silver, with Tangent-screw Motions	25	4	0
Five-inch ditto ditto, with two Telescopes	31	10	0
Six-inch ditto, with one Telescope, divided to 20 seconds, complete	31	10	0
Six-inch ditto, with two Telescopes	40	0	0
Six-inch ditto, with Transit Axis and Vertical Circle	36	15	0
Six-inch ditto (Captain J. T. Boileau's construction,) with Axis, Level, &c.	42	0	0
Seven-inch ditto, with one Telescope	35	14	0
Seven-inch ditto, with two Telescopes	45	0	0
Eight-inch ditto, Azimuth and Altitude, with Axis, Level, &c.	52	10	0
Twelve-inch ditto, for Horizontal Angles only	42	0	0
Four-inch ditto (Colonel Everest's construction)	22	0	0
Five-inch ditto, ditto	26	5	0
Seven-inch ditto, ditto	36	15	0
Five-and-a-half-inch Kater's Circle, with Stand, complete	35	0	0
Small Kater's Circle, with Stand	16	0	0
Level Collimator . . . . from 10*l.* 10*s.* to	15	15	0

(Larger Theodolites, &c., made to Order.)

STATION POINTERS, PROTRACTORS, PENTAGRAPHS, &c.

	£	s.	d.
Twelve-inch Station Pointer	6	16	6
Eighteen-inch ditto	7	17	6
Two-feet ditto	9	9	0
Thirty-inch ditto	12	12	0
Three-feet ditto	18	18	0
Wollaston's Goniometer	3	13	6

	£	s.	d.
Eight-inch best Brass Circular Protractor, with Clamp and Tangent-screw and folding Arms	7	7	0
Ditto, ditto, divided upon Silver	8	8	0
Six-inch ditto, with Rack and Pinion	4	14	6
Ditto, ditto, divided upon Silver	5	15	6
Six-inch Semicircular Protractor, with Vernier and Arm	3	3	0
Eight-inch ditto, ditto	3	13	6
Fifteen-inch plain Circular Protractor	3	5	0
Eight-inch ditto . . . . . from 1*l.* 5*s.* to	1	11	6
Six-inch ditto	1	1	0
Semicircular plain Protractors . . . from 16*s.* to	2	2	0
Ivory Protractors . . . . . from 6*s.* to	0	15	0
Ditto, upon Parallel Rollers . . . from 18*s.* to	1	5	0
Eighteen-inch best Brass Pentagraph	5	5	0
Two-feet ditto	6	6	0
Two-and-a-half-feet ditto	7	7	0
Three-feet ditto	8	8	0
Three-and-a-half-feet ditto	9	9	0
Trochiameter, for counting the Revolutions of a Carriage-wheel	2	5	0
Leather Case with Strap for ditto	0	10	6
Plain Perambulators (wood)	9	9	0
Ditto, Brass-mounted	12	12	0
Best ditto, with Metallic Wheel	16	16	0
Common twelve-feet Levelling-Staff	1	11	6
Best ditto	1	15	0
Troughton's Improved Portable ditto, with Level	2	12	6
Sopwith's ditto, for Reading without an Assistant	2	12	6
Ditto, stronger, with painted divisions	3	3	0
Gravatt's Levelling Staff	4	4	0

Tape Measure, 25 feet, links	0	7	0
Ditto, ditto, decimals	0	8	0
Ditto, 33 feet, links	0	8	0
Ditto, ditto, decimals	0	9	0
Ditto, 50 feet, links	0	10	0
Ditto, ditto, decimals	0	12	0
Ditto, 66 feet, links	0	12	0
Ditto, ditto, decimals	0	14	0
Ditto, 100 feet, links	0	16	0
Ditto, ditto, decimals	0	18	0

	£	*s.*	*d.*
Land Chains, 50 feet, and Arrows	0	13	6
Ditto, 100 feet, and ditto . . from 1*l.* 3*s.* 6*d.* to	1	5	0
Ditto, 66 feet, with two Round Rings between each link and Arrows	0	15	6
Ditto, ditto, with three Round Rings, &c.	0	17	6
Ditto, ditto, with two Oval Rings, &c.	0	18	0
Ditto, ditto, with three Oval Rings, &c.	1	1	0
Standard Chain, 50 feet . . . 4*l.* 4*s.* and	5	5	0
Ditto, 66 feet . . . . 5*l.* 5*s.* and	6	16	6
Ditto, 100 feet . . . . 8*l.* 8*s.* and	9	19	6

(*Stronger Chains, &c., made to Order*).

Set of Marquois Scales, in Box	0	12	6
Ditto, in Ivory	2	5	0
Ditto, in Brass	2	12	6
Ditto, in Electrum	4	4	0
Twelve-inch Ivory Plotting Scales . . from 11*s.* to	1	1	0
Twelve-inch Boxwood ditto . . from 4*s.* to	0	7	0
Ivory Offsett and Pocket Scales . from 2*s.* 6*d.* to	0	6	6
Gunter's Scale, Brass, 2 feet	2	2	0
Ditto Boxwood . . . . . from 5*s.* to	0	9	0
Ivory Folding Rules . . . . from 12*s.* to	0	18	0
Boxwood ditto . . . . . from 6*s.* 6*d.* to	0	15	0
Gunner's Rules . . . . . from 3*s.* to	0	10	6
Camera Lucida . . . from 1*l.* 11*s.* 6*d.* to	2	12	6
Stand for ditto . . . . from 1*l.* 1*s.* to	1	11	6

Drawing Instruments, in Skin Cases, (Sappers and Miners)	0	14	0
Ditto, ditto, East India Company's Pattern	1	5	0
Ditto, ditto, Woolwich Pattern	1	15	0
Ditto, ditto, School Pattern	2	2	0
Ditto, ditto, Ordnance Pattern	3	3	0
Ditto, in Mahogany Cases, Addiscombe Pattern	3	3	0
Ditto, ditto, Admiralty Pattern	3	13	6
Ditto, ditto, Sector-jointed Instruments, Parallel Rulers, Sector and Protractor	3	13	6
Ditto, ditto, with Sector double-jointed Dividers	4	4	0
Ditto, ditto, with proportional Compasses	5	15	6
Ditto, ditto, with Spring Bows	7	7	0

	£	s.	d.
Drawing Instruments, in Mahogany Cases, with Road and Wheel Pens, Needle-holder and small Dividers, &c.	9	9	0
Ditto, ditto, in Electrum, packed in Rosewood and Mahogany Cases of the best description, from 5*l.* 5*s.* to	13	13	0
Ditto, ditto, large Magazine Cases	26	5	0
Proportional Compasses	1	11	6
Ditto, ditto, with Adjusting Screw	2	2	0
Plain Beam Compasses	1	15	0
Ditto, with Pen and Pencil Points	2	2	0
Plain Beam Compasses, Ordnance Pattern	2	12	6
Beam Compasses with Double Adjustments and Divided Beam . . . from 4*l.* 4*s.* to	6	6	0
Ditto, ditto, Tubular Beam . . . from 5*l.* 5*s.* to	10	10	0
Plain Ebony Parallel Rulers, 12 inches	0	4	6
Ditto, ditto, 15 inches	0	7	0
Ditto, ditto, 18 inches	0	9	0
Ditto, ditto, 2 feet	0	12	6
Ditto, ditto, with Brass Edges, 18 inches	0	14	0
Ditto, ditto, ditto, 2 feet	1	1	0
Ditto, ditto, ditto, 2 feet 6 inches	1	11	6
Ditto, ditto, ditto, 3 feet	2	2	0
Rolling Ebony Parallel Rulers, with Brass Edges, 12 inches	1	0	0
Ditto, ditto, ditto, 15 inches	1	5	0
Ditto, ditto, ditto, 18 inches	1	11	6
Ditto, ditto, ditto, with Plain Edges, per inch	0	1	0
Ditto, ditto, ditto, with Ivory Edges divided	0	1	6

HORIZONTAL DIALS MADE TO ANY LATITUDE.

	£	s.	d.
Six-inch, to 5 minutes	1	1	0
Nine-inch, to ditto	2	5	0
Twelve-inch, to 2 minutes, and Equation Table	6	16	6
Twelve-inch, with Turned Edge, divided to two minutes, and Equation Table	7	7	0
Fifteen-inch, divided to 1 minute, without Turned Edge	7	17	6
Eighteen-inch, divided to 1 minute, 32 Points Lettered, Equation Table, &c., &c.	18	18	0

(*Larger to Order*).

	£	s	d.
Double Transferrer	3	3	0
Single Transferrer, with Fountain Pipe	1	5	0
Glass Vessel, for Fountain in Vacuo	0	7	0
Six Breaking Squares, Cage and Cap	0	18	0
Apparatus for striking Steel and Flint in Vacuo	0	18	0
Copper Bottle, Beam, and Stand, for weighing Air, and other Experiments	3	3	0
Model of Forcing Pumps for a constant stream, with glass barrels	3	3	0
Gun Lock Experiment	1	1	0
Bacchus ditto	1	14	6
Small-sized Japanned Copper Fountain, with Syringe and 5 Jets	7	0	0
Torricellian Experiments	0	10	6

	£	s	d.
A twelve-inch Electrical Plate Machine, packed with Medical Apparatus	7	10	0
A fifteen-inch ditto packed	10	10	0
An eighteen-inch ditto, packed	12	12	0
A two-feet ditto, packed	18	18	0
A Cylinder Machine, 16 by 10, packed	12	12	0
A ditto ditto, 14 by 8, packed	10	10	0
A ditto ditto, 12 by 7, packed	7	17	6

(*Larger Machines made to Order*).

	£	s	d.
A Universal Discharger and Press	1	16	0
Jointed Discharger, with Glass Handles	0	12	6
Ditto, plain . . . from 4*s*. 6*d*. to	0	8	6
Exhausted Flask for shewing the Aurora Borealis	0	8	6
Electrical Batteries of combined Jars from 2*l*. 12*s*. 6*d*. to	10	10	0
Cuthbertson's Improved Electrometer, with grain Weight	2	12	6
Bennet's Gold Leaf Electrometer	0	18	0
Cavallo's Bottle Electrometer, for Atmospherical purposes . . . from 12*s*. to	4	14	6
Quadrant Electrometer, with divided Arch	0	9	6
Kinnersley's Electrometer	1	1	0
Coulomb's Electrometer	1	16	0
Pith-Ball ditto	0	16	0
Luminous Conductors . . . from 12*s*. to	1	0	0
A Thunder-house, for shewing the use of Conductors	0	8	0

	£	s.	d.
A Thunder-house, with Drawer	0	9	6
A Powder-house for ditto	0	16	0
An Obelisk or Pyramid for ditto	0	10	6
A Magic Picture for giving Shocks from 7*s*. 6*d*. to	0	16	6
Spiral Tubes to illuminate by the Spark from 8*s*. to	0	10	6
A Set of 5 Spiral Tubes on a Stand	1	16	0
Ditto, with a Dome	2	8	0
Luminous Names or Words from 10*s*. 6*d*. to	1	11	6
A Set of 3 Plain Bells	0	10	6
A Set of 8 Bells, containing the Gamut	1	14	0
Diamond or Spotted Jars from 8*s*. to	0	16	0
A Double Jar for explaining the Franklinian Theory, from 18*s*. to	1	8	0
An Electrical Cannon	0	18	0
A Brass Electrical Pistol	0	9	6
Copper Plates and Stand for Dancing Images	0	10	6
A Small Head with Hair	0	8	0
An Artificial Spider	0	1	6
Sportsman and Birds	1	16	0

Atwood's Machine for Demonstrating the Law of Acceleration in Falling Bodies from 20*l*. to	30	0	0
Working Model of Locomotive Engine, 4 wheels	25	0	0
Ditto, 6 wheels	40	0	0
Model of Bramah's Hydrostatic Press	15	15	0
A Syren	2	2	0
A Small Still with Worm Tub and Lamp	2	18	0
Set of Lactometers in Box	0	18	0
Glass Model of a Diving Bell from 2*l*. 12*s*. 6*d*. to	7	7	0
Whirling Table Complete	30	0	0
Hydrostatic Balance from 2*l*. 2*s*., 4*l*. 14*s*. 6*d*. to	8	18	6
Hydrostatic Paradox	7	7	0
Model of the Centrifugal Pump from 4*l*. 10*s*. to	7	7	0
Sectional Model of a Steam Engine	18	0	0

(*Models of Steam Engines, Machinery, &c., made to Order.*)

Bar Magnets for correcting the derangement of the Compass in Iron Vessels, 2 feet each, 2*l*. 2*s*.; 14 inches each, 1*l*. 1*s*.; 8 inches	0	10	6
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