TREATISE

ON THE

THEORY OF THE CONSTRUCTION

OF

HELICOIDAL OBLIQUE

ARCHES.

BY

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PREFACE.

I have attempted in this treatise to attain two results:

First.—To supply our American engineering literature with a short, clear treatment of the construction of Helicoidal Oblique Arches; and,

Second.—To render simple all problems connected with their theory or construction. I especially hope that I may make the second plain to all who shall read these pages.

Long since I have been satisfied that much of the confusion and misunderstanding arising from the attempts to understand this subject, have arisen from the fact that authors have failed, either to state the fundamental principle, or to keep it constantly before the student's mind. Hence the general opinion has arisen that helicoidal arches are of the most intricate construction, and too often their consideration has been abandoned with disgust.

The conception of a single principle will clear away all this misunderstanding. It is that of the process of the generation of helicoidal surfaces. It is a simple one, and, if constantly kept in mind, will render all other problems equally simple. No engineer's education is complete without a thorough knowledge of this subject. If the simple propositions of Chapter I. are mastered, there will be no trouble with the remainder of the treatise.

Since the theory of Logarithmic Arches is readily understood, after that of helicoidal arches has once been thoroughly mastered, I have added a short discussion of Logarithmic Oblique Arches.

JOHN L. CULLEY.

CLEVELAND, April, 1886.

Treatise on the Theory of the Construction of Helicoidal Oblique Arches.

CHAPTER I.

ELEMENTARY PRINCIPLES.

1. A *helix* is a cylindrical curve that, in passing over equal portions of the circumference of the cylinder, travels over the same number of equal portions of the length of the cylinder.

2. Thus, in Fig. 1, A B C is the elevation of the semi-cylinder, A C D E, whose diameter is A C, and length is A E. Let A E also be the cylindrical length of the semi-helix, A D, whose elevation is A B C.

Divide A E and the circumference, A B C, each into the same number of parts of equal length, as shown in Fig. 1. Through the points, 1, 2, 3, &c., of A E draw lines parallel to A C, and through the corresponding points, 1′, 2,′ 3′, &c.,

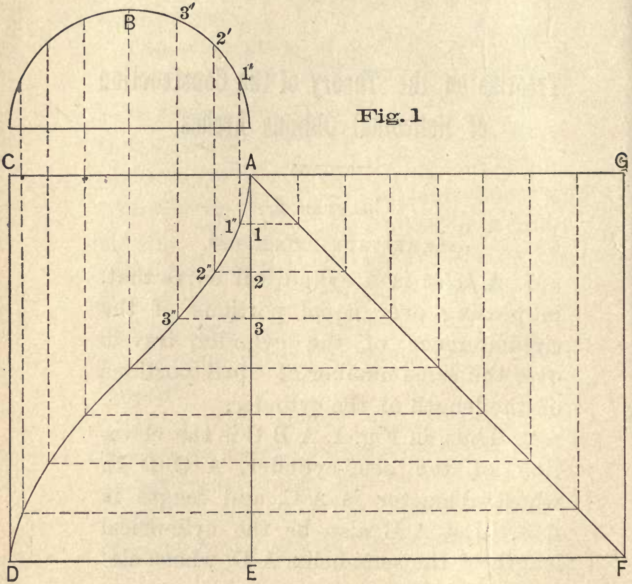


Fig. 1

of A B C, draw lines parallel to A E. The intersections, 1″, 2″, 3″, are points, in the plan, of the semi-helix, A D.

Any number of points may thus be determined, and the curve of the semi-helix readily located, or drawn in the plan, Or, when the relation of the cylindrical length to the semi-circumference is constant, we can readily determine the equation of the curve, A D.

3. If the cylindrical surface, A C D E, be straightened out into a plane surface, its width will evidently be equal to the semi-circumference, A B C, and its length will be equal to A E, whilst the semi-helix, by construction, would become a straight line on this surface. Therefore, in Fig. 1 draw A G equal to the semi-circumference, A B C, perpendicular to A E, and complete the rectangle, A E F G. This rectangle is called the *development* of the semi-cylindrical surface, A C D E, since it is its equivalent in plane surface. If we regard A C D E the inner surface or *soffit* of a right arch, A E F G is the development of the intrados, and the straight line, A F, is the development of the intradosal semi-helix, A D.

4. From this we can determine the location in the plan of a semi-helix I J normal to A D at L (Fig. 2). Its development will evidently be also a straight

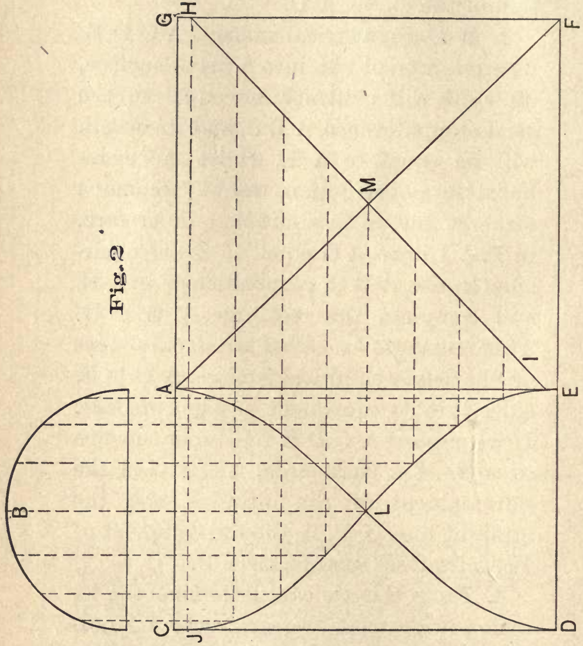


Fig. 2

line perpendicular to A F. Through L draw L M parallel to A G, intersecting A F at M. I M H drawn through M perpendicular to A F, is the development of the semi-helix normal to A D at L. Draw H J parallel to A G, then J and I will be the extremities of the normal semi-helix in plan, whose cylindrical length is I K, and elevation is A B C, whence we can determine the J L I in the same manner as A L D was determined. and as here shown.

5. In Fig. 3 H I J is the elevation, and H J K L the plan of the extrados or outer surface of a right arch of the depth I B, whose extradosal development is H L M N, wherein H N equal to the semi-circumference H I J is drawn perpendicular to H L, and L M and M N are respectively drawn parallel to H N and to H L.

The extradosal and intradosal developments are both made from the axis X X for the convenience of the comparison of these surfaces and their lines. Here H P K is the plan of the semi-extradosal helix H I J,

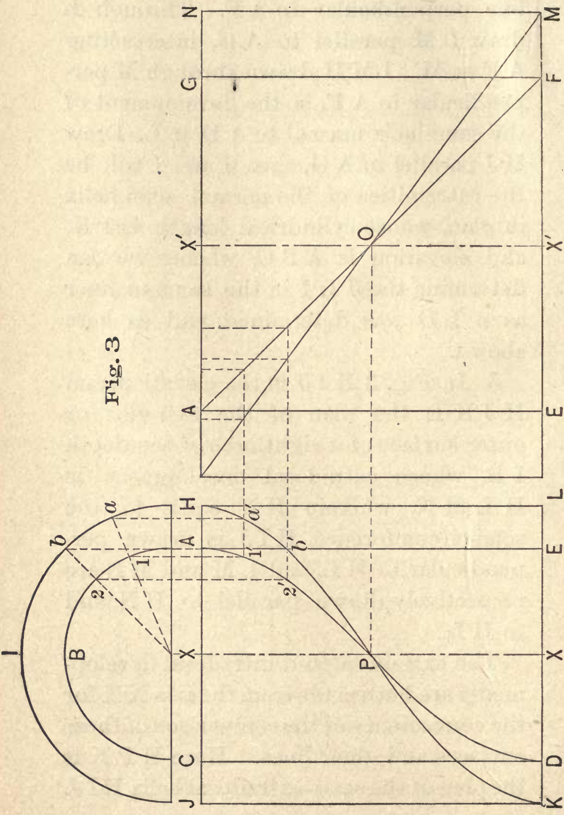


Fig. 3

whose cylindrical length H L is equal to A E of the semi-intradosal helix A P D. The curve H P K is determined from H L and H I J in the same manner as was A P D from A E and A B C. The straight line H O M is the development of H P K.

By construction P and O, the respective points of intersection of the curves H P K and A P D, and of the straight lines H O M and A O F are both on the line X X—X′X′, *i. e.* in plan and in development. They are also on the same line P O drawn at right angles to either X X or to X′ X′. Hence when either point P or O is determined the other point is also determined by the right angled line O P.

6. The surface included between the curves H P K and A P D is called the arch helicoidal surface, or arch helicoid, and a careful examination of Fig. 3 will elucidate to us the fundamental principle of helicoidal arch analyses and construction. This fundamental principle is that of the process of the generation of this helicoidal warped surface.

7. All lines $1\,a-1'a'$, $2\,b-2'b'$, &c., Fig. 3 drawn normal to the axis of the arch and included between the intradosal and extradosal helices are evidently elements of, and the only straight lines lying entirely within this warped surface. $1\,a$, $2\,b$, &c., are the actual lengths of $1\,a-1'a'$ of $2\,b-2'b'$, &c., since $1'a'$, $2'b'$, &c., produced are normal to the axis X X, or parallel to the right section of the arch. They are each equal to I B the depth of the arch as they are the portions of the radii $x\,1\,a : x\,2\,b$, &c., included between A B C and H I J.

Again the radii X $1\,a$, X $2\,b$, &c., at their points of contact in the intrados and in extrados, are perpendicular to these surfaces, and are perpendicular to all lines within these surfaces passing through these points of contact, and are therefore perpendicular to the helices passing through these points. In other words, any radius X $1\,a$ or X $2\,b$ drawn from the axis XX to either intradosal or extradosal helix, is at the point of contact with the helix perpendicular to it. From this it follows that the elements $1\,a - 1'a'$; $2b - 2'b'$, &c., are at the same time perpendicular to both the intradosal and extradosal helices A P D and H P K.

8. Whence, when the right sections of the intrados and the extrados of an arch are circular arcs, we derive the following fundamental principle:

First. *A helicoidal surface is generated by a right line perpendicular to and moving on the axis as one directrix and on a helix as the other directrix, and the arch helicoid is that part of this surface generated by that portion of the generating line, equal to the depth of the arch, and included between the intrados and the extrados. The said generator of the arch helicoid is at all times perpendicular to the intradosal and extradosal helices of such helicoid.*

And as a corollary,

The intradosal and extradosal helices of an arch helicoid are parallel *spiral curves whose perpendicular distances apart equal the depth of the arch.*

9. These deductions are the essence of all problems connected with this subject. Nor should any one pursue it further without a thorough mastery of the principles here stated. A full understanding of the process of the generation of helicoidal surfaces is of the first importance, and such a conception will render all further consideration of the elements of these warped surfaces clear and simple.

CHAPTER II.

HELICOIDAL CURVES AND TEMPLETS—TWIST RULES.

10. In the plan (Fig. 4) of the intradosal helix A P D, make P S′ equal to P R′, and join R′ and S′ by a straight line, R′ S′. R′ S′, by construction, will pass through P. R S will be the elevation of the chord joining the extremities of the helix R′ S′. If, then, we lay off R″ S″

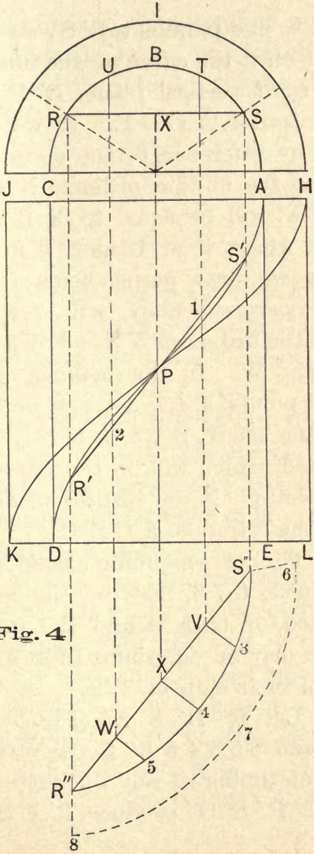


Fig. 4

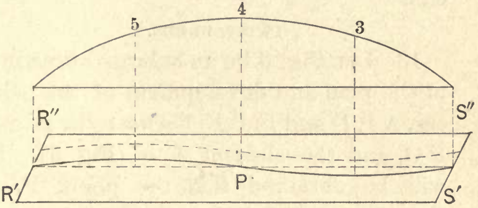
equal to and parallel to $R' \ S'$, we can construct on it the curve of the intersection of a right vertical plane, $R' \ P \ S'$, with the intrados, A C D E. It will be evident, by reference to the elevation, A B C, that the middle ordinate X 4 of R'' S'' at X, will be equal to X B in elevation. Again, if at U and T in the elevation we draw perpendiculars to R S, these perpendiculars will respectively equal the ordinates V 3 and W 5 of $R'' \ S''$ at V and W. In this manner any number of points, 3, 4, 5, &c., may be obtained and the curve, S'', 3, 4, 5, &c., R'' constructed, which will be the curve of the intersection of the vertical plane, $R' \ S'$, with the intrados, A C D E.

11. The corresponding extradosal elliptic curve, 6 7 8, may be similarly constructed, or both it and the intradosal curves may be determined for the equation of their ellipses.

12. Let $R'' \ R' \ S' \ S''$ (Fig. 5) be the plan and S'', 3 4 5 R'' the elevation of a piece of timber of any thickness, $R'' \ R'$. On $R'' \ R' \ S' \ S''$ produce $R' \ P \ S'$ of the

plan of the helix in Fig. 4 and at any convenient distance from, and parallel to, R′ P S′ draw the dotted line here shown.

Fig. 5



The space between R′ P S′ and the dotted parallel line will be the plan, and S″ 3 4 5 R″ the elevation of a templet of the intradosal helix, or the soffit coursing joint, R B S—R′ P S′.

Care should be observed that R′ R″ and S′ S″ in the plan of this templet are made parallel with the axis or spring lines of the arch, and that the spiral R′ P S′ and the dotted curve here shown are made parallel to one another on lines parallel with R′ R″ and S′ S″.

In like manner the templet of the extradosal helix may be constructed. When any considerable portion of a helix is to be treated at one time this is the true way to construct the coursing joint templets.

TWIST RULES.

13. Let Fig. 6 be an enlarged drawing of the plan and development of the helices, A P D and H P K, between the point, P O, and the element, $1'$ a' (Fig. 3). It will be observed that the point, P, is the plan of a vertical element of the arch helicoid, or the element at P is perpendicular to the horizontal plane. The element P and the point, $1'$, of the intradosal helix, locate the vertical plane passing through P and $1'$, whose trace on the horizontal plane is P $1'3'$. Then the line, $a'3'$, drawn from a' in the extradosal helix perpendicular to the vertical plane, P' $1'$ 3, is the measure of the warp of the arch helicoid at $1'$ $a'-1$ a' from the plane, P $1'$ $3'$, for the given depth, I B, of the arch.

14. $1'$ a' is, by construction, equal to I B and a' $3'$ being perpendicular to the

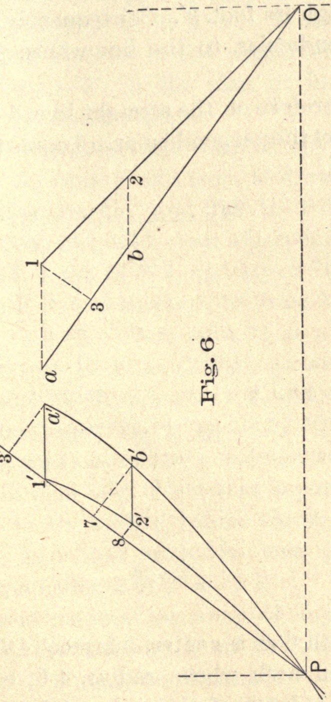


Fig. 6

vertical plane, is perpendicular to any line within the vertical plane passing through its foot, 3′. Consequently a' 3′ is perpendicular to the line whose plan is 1′ 3′.

Therefore on the straight line, 4 5 (Fig. 7) erect the perpendicular, 5 6, equal to a'3′,

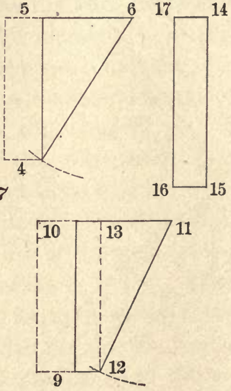


Fig. 7

and with 6 as a center, intersect 4 5 at 4 with an arch whose radius, 4 6, equals I B, the depth of the arch. 4 5 6 is the actual size of the wedge, 1′ 3′ a', in plan that is perpendicular to the vertical plane, P 1′ 3′, and that supports the plane above the helicoid, P 1′ *a*′ at 1′ *a*′. It should be noticed that 4 5 is the actual length of the line, 1′ 3′, in the plan or in the trace of the vertical plane.

15. To determine the warp of any element, 2′ *b*′, from *b*′ and from 2′ draw the perpendiculars, *b*′ 7 and 2′ 8 to P 1′ 3′. Then on any line, 9 10, erect a perpendicular, 10 11, equal to *b*′ 7, and lay of 10 13, equal to 2′ 8, and through 13 draw 12 13 parallel to 9 10. Then, with 11 as a center, intersect 12 13 at 12 with an arc of the radius, 11 12, equal to I B, and draw 9 12 parallel to 10 11. 9 10 11 12 will be the actual size of the truncated wedge shown in plan at 2′ *b*′ 7 8, that is perpendicular to the vertical plane through P and 1′, and that supports this plane above the helicoid at 2′ *b*—2 *b*.

In same manner the warp of any element of the helicoid from the vertical plane, P 1′ 3′, may be obtained.

16. The figures 4 5 6; 9 10 11 12, and a straight line, 14 15, equal to I B, are

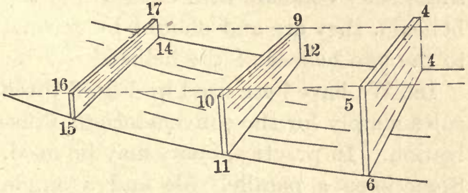
templets of the helicoid respectively at $1'\ a'-1\ a : 2'\ b'-2\ b$, and at P—O, and in applying them care should be exercised that the lines 4 6 and 11 12 are applied to the helicoid with the points 4 and 12 exactly at $1'-1$ and at $2'-2$, and with the points 6 and 11 exactly at $a'-a$ and at $b'-b$. The lines, 4 5 and 9 10 are, of course, to be brought exactly into the plane, P′ 1′ 3′, while the straight line, 14 15, will be applied at the intersection of the plane P 1′ 3′, and the helicoid at P—O. In practice, however, a straight line, 14 15, cannot be used. Therefore, draw 16 17 parallel to 14 15, 14 17 distinct therefrom, and increase the depths of 4 5 6 and of 9 10 11 12 each a distance equal to 14 17 to the dotted lines shown.

Then 16 17 and the dotted lines, 4 5 and 9 10, will be in a plane parallel to the plane P 1′ 3′, and these *parallel* and *twist rules* should, when applied, be always perpendicular to this parallel plane that they may ever be perpendicular to the first plane, P 1′ 3′. It is very important that these rules in application should always be maintained normal to the sight plane. Otherwise it would be impossible to reduce true and correct warped surfaces.

The parallel and twist rules are all known as twist rules, as they are the templets by which the warp of the coursing beds are worked.

17. Fig. 8 is a perspective view showing the manner of applying the twist and

Fig. 8



parallel rules of Fig. 7. 15 11 6, is the extradosal helix whose exact length and that of its parts, 15 11 and 11 6, are taken from the development (Fig. 6), and are respectively equal to O *a*, O *b* and *b a*. 14 12 4 is the intradosal helix equal to 0, 1, and its parts 14, 12 and 12, 4 are equal to 02 and 21 (Fig. 6). The parallel and twist rules are of the same size and characteristics as in Fig. 7. Their upper edges 16 17, 9 10, and 4 5 lie in and coincide with the plane 4 5 16 17, to which these rules are perpendicular. The corners of the rules 6 11 and 15 are in the extradosal helix 6 11 15, while the corners 4 12 and 14 are in the intradosal. It will also be noticed that since the lower edges of these rules coincide with elements of the helicoid, they are and should be normal to the two helices of the helicoid.

18. We have here used in Fig. 8 three rules simply for the convenience of illustration. In practice, many may be used. Sometimes a parallel rule and a single twist rule will be all that are needed, and in fact this will generally be the case. The number of twist rules to be used depends upon the length and warp of the voussoir treated. However, enough of these should always be used to exactly determine the warped helicoids.

19. This method of determining the

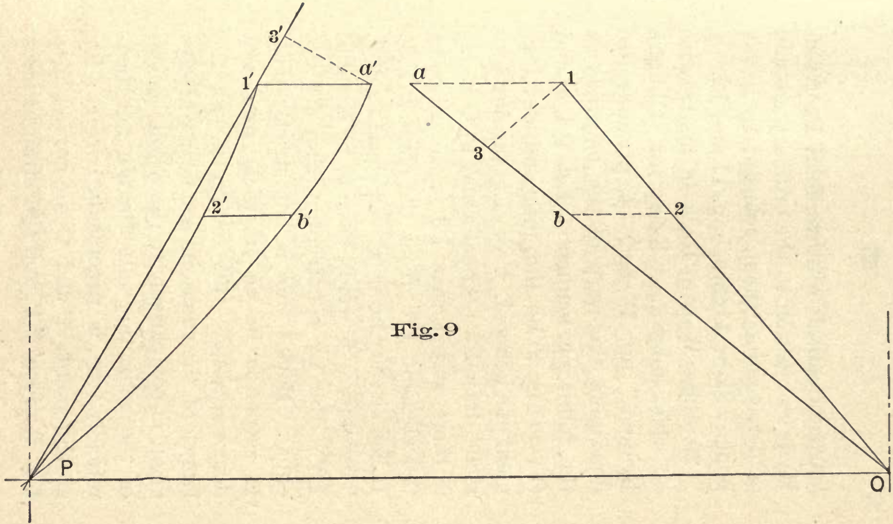


Fig. 9

warped helicoids will overcome Professor Hyde's objection to the ordinary method of determining them, referred to in his book, "Skew Arches," pp. 11 and 12.

20. John Watson Buck in his treatise on this subject, "Essay on Oblique Bridges," pp. 13 and 14, presumes to determine the warp of the helicoids in the following manner: Let P 1′ *a*′ and O 1 *a* (Fig. 9) be the plan and development of a helicoid. From 1 in the development, draw 1 3 perpendicular to O *a*. It is there stated that 1 3 is the warp of the element 1′*a*′ from the plane P 1′ 3′. It has already been proven equal to *a*′ 3′. Therefore *a*′ 3′ would have to be equal to 1 3′. But by construction they are not equal. Buck's rule is not strictly true. It seems to be an empirical rule—not demonstrable. But inasmuch as the length of the voussoirs are nearly always small in comparison to the whole length of the helix, this rule, though not theoretically true, is practically correct; for at the points P and O the curved lines 1′ 2′ P and *a*′ *b*′ P, and the straight lines

1 2 0 and $a\ b\ 0$ make the same included angles.

But if warped helicoids of any magnitude are to be worked at one time, it would not be safe to use Buck's rule. The rule we have given is true for any length of the helicoid, and as easily obtained and applied as Buck's, and there is no reason why it should not be always used. (See art 62.)

21. Produce the lines $1'\ a'$ and $1\ a$ (Fig, 10) to c and d in XX and X′X′

Let the β be *angle of the intrados*, $1\ O\ d'$; β' be the *angle of the extrados* $a\ O\ d'$ (Fig. 10); Φ equal the angle $1'\ P\ c$ $=$angle $3'a'1'$ by construction; r the radius of the intrados and r' the radius of the extrados, $\dfrac{1\ d}{\pi r}=\dfrac{a\ d}{\pi r'}$.

Then

$$Pc=Od=O\ 1 \text{ cosin } \beta=Oa \text{ cosin } \beta' \quad (1)$$

$$1\ d=Od \tan.\ \beta \quad . \quad . \quad (2)$$

and

$$a\ d=Od \tan.\ \beta' \quad . \quad (3)$$

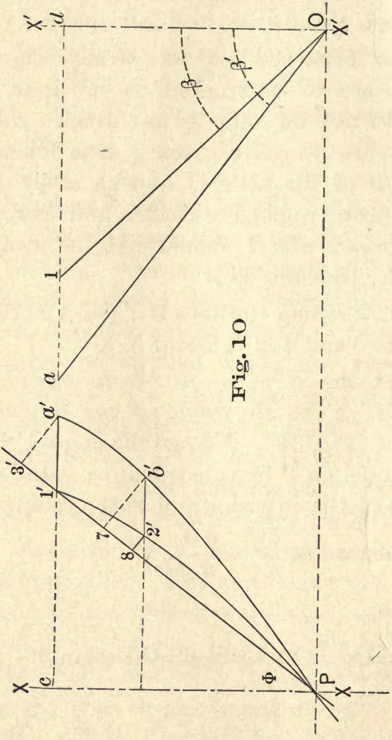


Fig. 10

$$c\,1' = r \sin.\left(\frac{1\,d}{\pi r} \,.\, 180^\circ\right) \quad . \qquad . \quad (4)$$

$$ca' = r' \sin.\left(\frac{a\,d}{\pi\, r'} \,.\, 180^\circ\right) \qquad . \quad (5)$$

Whence we obtain

$$1'a = c\,a' - c1, \text{ and tang. } \Phi = \frac{c\,1'}{\mathrm{P}\,c} \;. \quad (6)$$

$$\therefore \quad 3'a' = 1'a \sin.\ \Phi \qquad . \quad (7)$$

In like manner any warp distances $b'\,7$ and $2'\,8$ may be obtained.

Chapter III.

OBLIQUE ARCHES—HELICOIDAL AND OTHER METHODS.

22. It will often happen in public improvements and in architectural constructions that arches are so located that their parallel ends or faces cannot be placed normal to axes of the arches. Such arches are called *Oblique* or *Skew Arches*, and the acute angle the axis makes with the plan of either face of the arch is the *angle of the obliquity* of the arch.

23. The stability of an arch where the coarse stones or *voussoirs* are constructed in the ordinary manner, with their coursing joints parallel with the axis, is weakened the moment the plane of the arch faces ceases to be normal to the axis. This weakness increases as the angle of obliquity decreases.

24. Thus let Fig. 11 be the plan and elevation of a semi-circular oblique arch, whose coursing joints are parallel to its axis X X. Through H and K draw the perpendiculars H 1 and K 2. The thrust of the arch due to its weight and load would naturally be carried to the abutments in lines parallel to H 1 and K 2. But the triangles H J 1 and L K 2 are each supported by a single abutment. They therefore depend upon the bond and strength of the voussoirs within them and in the body of the arch immediately beyond them, to direct a part of their thrust and stress to the opposite abutments. Moreover, the thrust and stress thus conveyed must necessarily be in lines or planes other than those par-

allel to H 1 and K 2, which would cause an outward movement in these triangles, which would be greatest at or near the

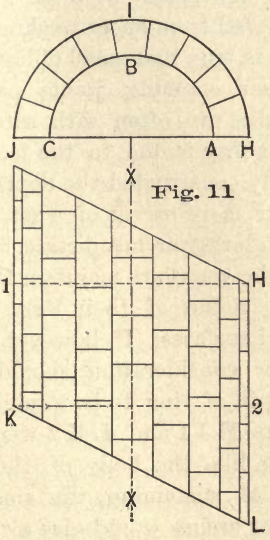


Fig. 11

point J and L. When the angle of obliquity H J 1 or K L 2 is very acute, neither the bond or the strength of the voussoirs will resist the tendency to rupture, nor would the friction of the bedding surfaces produced by the weight and load of the voussoirs prevent the outward movement at J or L, and the arch will fail from sheer weakness.

25. It is true that small oblique arches, with their coursing joints parallel to their axles, are often with safety built. Their stablity is due to the fact that as ordinarily constructed the depth of their voussoirs is in excess of what economy would in larger arches dictate. In other words, the voussoirs maintain their position by virtue of their large bedding frictional surfaces. If, however, economy or other consideration should require their depth of ring to be minimum, or if the parts H J 1 and L K 2 were loaded anything like the body of the arch is capable of sustaining, the smaller like the larger arches would give away.

26. It is evident that to maintain the stability of an oblique arch it is desirable that its stress and thrust should be conveyed to the abutments in planes parallel to the oblique faces of the arch, or as

near so as is possible, and also that this object cannot be attained when the coursing joints are parallel to the axis of the arch. The coursing beds to fully realize this condition should be normal to the oblique faces of the arch wherever they come in contact with them.

27. There are three methods in vogue that closely approximate this requirement. They are known as the *Logarithmic*, the *Corne de Vache*, or *Cow's Horn*, and as the *Helicoidal Methods.*

Without entering into the comparative merits of these three methods, it will be sufficient for the present purposes to state that the main advantages of the Helicoidal Method are:

The successive coursing beds and joints are parallel to one another throughout their entire lengths, rendering all voussoirs of the same warp in their coursing beds, their soffit and in their extrados; also in their bedding surfaces, except in the arch face. Therefore, all voussoirs of the same length are exactly alike, rendering only one set of templets necessary in their execution—a condition not realized in the other two methods, for in both of them each stone is always different from the next one to it in the same course. This consideration materially reduces the number and expense of measurements, drawings and templets. For, in the two other methods each stone has its separate and distinct templets, measurements and drawings, and are the objects of much care and anxiety. Thus it is a helicoidal voussoir will fit into any portion of its course, not requiring to be set at a particular place in its course as is required by the other methods. Again, the ring stones at equal distances from the key or center of the arch face, being of the same dimensions this method preserves the pleasing architectural effect of an arch, without the distressing effect produced by the courses increasing or decreasing in size from one side of the arch to the other as is done by the other methods.

28. Whatever may be said as to the stability of oblique arches by the other

methods, it is true helicoidal arches with very acute angles of obliquity may with safety be constructed. This acuteness can be considerably less than 30°. John Watson Buck, M. Inst. C.E., places its minimum value at 25°. But it is probable that if the skew backs are maintained in their place that this value may be even less.

An oblique arch will, of course, give away from defective construction, as for instance an insecure abutment, a weak foundation, or a lack of proper bond, just as a right arch would under like circumstances. Yet it is quite the custom to condemn the theory of correct construction of oblique arches, when the weakness in the arch was in no way due to the arch proper, either in theory or construction. To be secure an arch in all its parts it must be well and thoroughly built. I have known oblique arches that stood better than could have been expected of right arches under like conditions, simply because the oblique arches were unusually well built.

29. Professor Hyde, in his treatise heretofore referred to, page 101, states that an advantage of this method is owing to the fact that the coursing joints are parallel, that the courses or whole arch, with the exception of the ring stones, may be constructed of bricks, which is very true. Great caution, however, should be exercised in such construction. The crushing strength of ordinary bricks to that of ordinary building stone is as 1 to 7, and arches never should be built of bricks, unless they are of a first-class, hard burnt quality. This is especialiy true in public works exposed to the action of the elements. It would be better and safer to recommend that the arches between the ring stones be made of thin coursed rubble of parallel beds, which material is equally adapted to helicoidal construction.

30. Undoubtedly many oblique arches have been thus constructed of brick, either from a want of knowledge of the principles of oblique arch construction, or from erroneous ideas of the excessive

cost of warped stone voussoirs over that of straight voussoirs for the same arch. This excessive cost is but a small per cent. over right arch construction, as will be proved further on. In oblique arch construction brick work is undoubtedly cheaper than dressed stone work, but is not much cheaper than coursed rubble work, while the latter is far more stable, but neither will compare with dressed stone for strength.

Brick or coursed rubble work can be used to great advantage in the arch between the dressed end ring stones, when the axis and spring lines of the arch are parallel curves. Much less care is necessary in arranging the courses when laid up of brick or of coursed rubble.

HELICOIDAL METHOD.

31. The helicoidal method of construction of oblique arches is one in which the coursing beds of the voussoirs are radial helicoids, whose intradosal and extradosal helices are parallel curves.

32. A helicoid, as stated in article 8,

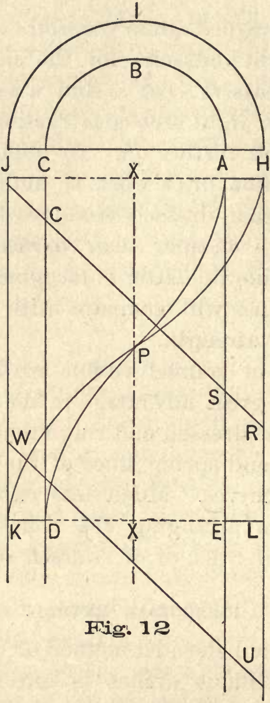


Fig. 12

is generated by a right line perpendicular to, and moving on, the axis of the arch as one directrix, and on a helix as the other directrix, and the arch helicoid is that part of this surface generated by that portion of the generating line that is equal to the depth of the arch, and is included between the intrados and the extrados.

33. J W U H in Fig. 12 is the plan of an oblique arch A B C, and H R J equal to K W U is the angle of obliquity, and the helicoid H P K—A P D represents a coursing bed of one course of voussoirs.

34. The extrados is removed in the plan of Fig. 13 in order to show the intradosal helices of the successive courses and the traces 1 2, 3 4, &c., of the helicoids or coursing beds upon the *imposts* A H and C J of the abutments, while in Fig. 14 these traces 1 2, 3 4, &c., and the corresponding extradosal helices, are shown. The traces 1 2, 3 4, &c., are always normal to the axis of the arch, be the arch segmental or full semi-circular, since the generator of the helicoids is radial and therefore normal to the axis of the arch. These traces are of course

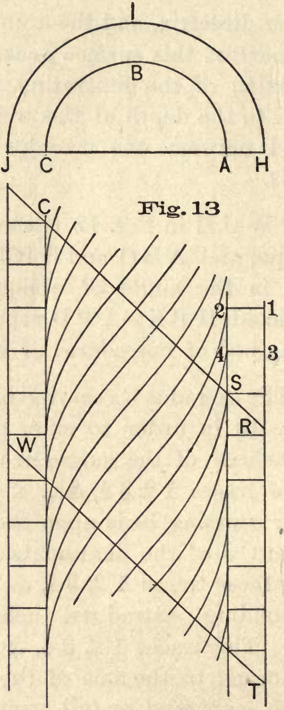


Fig. 13

elements of their several helicoids as well as of the imposts.

Were it not for the confusion of curved lines, both the intradosal and extradosal helices and the traces 1 2, 3 4, &c., might be given in the same plan, and each helicoid or coursing bed would be shown as in Fig. 12. Great confusion would result from such an attempt. Fig. 12, therefore, will suffice for a comparison of the helices of the several helicoids.

Chapter IV.

THE FOREGOING PRINCIPLE APPLIED TO HELICOIDAL OBLIQUE ARCHES AND DEDUCTION OF FORMULÆ.

35. In the application of the foregoing principles to helicoidal oblique arches a constant relation is established for any given case between the right diameter of the arch and the length of either the intradosal or extradosal helix of the arch helicoid. The position of the intradosal helix is then determined by drawing it perpendicular to a straight line, joining

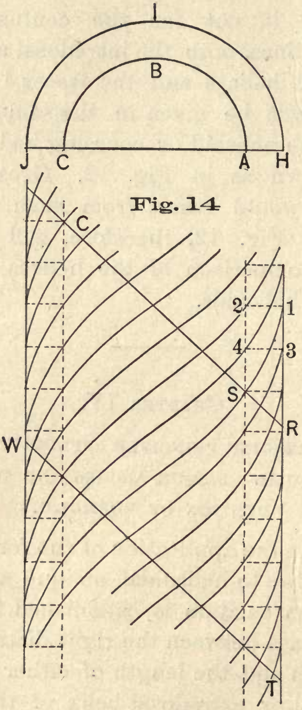


Fig. 14

impost extremities of the arch face end of the developed intrados. Thus in Fig. 16

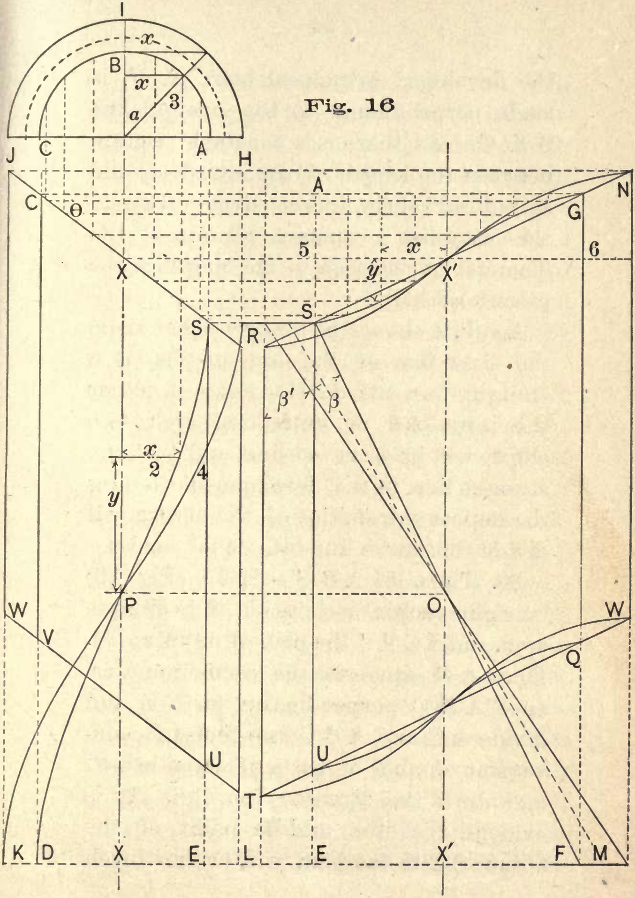


Fig. 16

the developed intradosal helix S. F. is made perpendicular to the straight line S X′ G. As there is a constant relation between the length of the intradosal and extradosal helices, this construction would also establish a constant relation of the diameter of the arch to the length of the extradosal helix.

Again, it should be observed that since the skew face of the arch in plan is a straight line, the development of either the intradosal or extradosal arch face ellipse will be a curved line, and that the straight line in the development joining the impost extremities of the ellipse will divide this curve line into equal curves.

36. Thus, let A B C — H I J (Fig. 16) be right section or elevation of an oblique arch, and S C V U the plan of its intrados. Draw A G equal to the semi-circumference A B C perpendicular to A E and divide A G and A B C into the same convenient number of parts of equal length and draw the straight line G S. It is evident that the middle point of the ellipse S C in the plan will, in the devel-

opment, be the middle point of G S. Then, through the points of division in A B C and A G draw lines parallel with AE, and through the points of intersection with S C draw lines parallel with A G, the points of intersection of these lines the parallel with A G, with the corresponding lines parallel with A E, drawn through the points of division in A G, will be points in the curved line S X′ G.

Through S draw S F perpendicular to the straight line S G. S F will be the development of the helix S D, whose cylindrical length is S E, and from S F and S E the curve S D may be obtained according to article 2. The angle F S E equal to S G A by construction, is the *angle of the intrados* and is designated by β.

37. In Fig. 16 the intrados and extrados of the arch are shown in plan, elevation and in development. The extradosal helix R K is determined from the intradosal helix S D by drawing through S and F, S R and F M perpendicular to R L and to M N and joining R and M by R M. Then R M will be the

development of the helix R P K, whose cylindrical length R L equals S E of the intradosal helix S P D. From R L and R M the curve R P K in the plan can be obtained in the same manner that S P D was. Also the curved end R X′ N is projected in the development similarly for H N and H I J as S X′ G was from A G and A B C. The angle M R L is the *angle of the extrados* and is designated by β'.

38. Let r be the right radius of the intrados; r' the radius of the extrados, b the depth of the arch, and θ the complement of the angle of obliquity. Then $\pi r = \text{A B C} = \text{A G}$, and $\pi r' = \text{H I J} = \text{H N}$. Then the relation of the diameter or radius to the helices will be obtained from the following formulæ:

$$\text{A S} = 2r \text{ tang. } \theta = \pi r \text{ tang. } \beta \qquad (8)$$

$$\therefore \text{ tang. } \beta = \frac{2 \text{ tang. } \theta}{\pi} \quad . \quad . \qquad (9)$$

This eq. is true and β bears this relation to θ so long as S F is normal to the straight-line S G. Sometimes it will be necessary to alter this relation of S F and S G, so that S F will pass through a given point in U Q. This alteration is always slight, and eq. (9) can readily be corrected for it.

$$C S \cos. \theta = 2 r, \text{ or } C S = \frac{2 r}{\cos. \theta} \quad (10)$$

$$G S \cos. \beta = \pi r, \text{ or } G S = \frac{\pi r}{\cos. \beta} (11)$$

Again $(E F = \pi r) = S E \text{ tang. } \beta$,

$$\text{or } S E = \frac{\pi r}{\text{tang. } \beta} = \frac{\pi r'}{\text{tang.} \beta'} = \frac{\pi^2 r}{2 \text{ tang. } \theta} \quad (12)$$

$$S F \sin. \beta = (E F = \pi r) \text{ or } S F = \frac{\pi r}{\sin. \beta} (13)$$

$$R M \sin. \beta' = (L M = \pi r') \text{ or } R M = \frac{\pi r'}{\sin. \beta'} \quad (14)$$

39. In Fig. 16, let X X be the axis of y and P O that of x, with the origin at P, and the equation of the curve S P D with reference to these axes, will be obtained in the following manner:

Let 2 be any point in S P D, whose ordinates are x and y, the angle 3 X B be α, and n be the quotient of the length of the arc B 3, divided by the length of the semi-circumference A B 3 C, then $a = n\,180°$, $x = r \sin. a$, and $y = n\,SE$ $= n\dfrac{\pi^2 r}{2 \text{ tang. } \theta}$ $\therefore r = \dfrac{x}{\sin. a} = \dfrac{2y \tan. \theta}{n\pi^2 r}$ (15)

(15) is the equation of the curve S P D, so that if either x or y is given, n and y or x can be obtained. It is also the equation of the plan of the normal helix to S P D at P, when y equals the nth part of the normal helix's cylindrical length, or n K I in Fig. 2.

40. To determine the equation of R P K let x and y be the ordinates to any point 4 in it, referred to same axes as in article 39. Then, as before,

$$y = n\frac{\pi^2 r}{2 \text{ tang.} \theta}$$

but $$x = r' \sin. a = (r + b) \sin. a$$

$$\therefore r = \frac{2\, y \text{ tang. } \theta}{n\, \pi^2} = \frac{x}{\sin. a} - b \quad (16)$$

41. To determine the equation of the end curve S X′ G of the development with reference axis 5 X′ 6 of x, draw n through X′ parallel to A G and axis of y

X′ X′, let x and y be the ordinates of any point y in S X′ G. Then by reference to Fig. 16 it will appear

$$x = n\,\pi\,r \text{ or } r = \frac{x}{n\,\pi}$$

$$y = r \text{ sin. } a \text{ tang. } \theta \text{ or } r = \frac{y}{\text{sin. } a \text{ tang. } \theta}$$

$$\therefore \frac{x}{n\,\pi} = \frac{y}{\text{sin. } a \text{ tang.} \theta} \qquad (17)$$

42. And for the corresponding extradosal end curve R X′ N we have as in 41

$$x = n\,\pi\,r' \text{ or } r' = \frac{x}{n\,\pi}$$

$$y = r' \text{ sin } a \text{ tang. } \theta \text{ or } r' = \frac{y}{\text{sin. } a \text{ tang. } \theta}$$

$$\therefore \frac{x}{n\,\pi} = \frac{y}{\text{sin. } a \text{ tang. } \theta} \qquad (18)$$

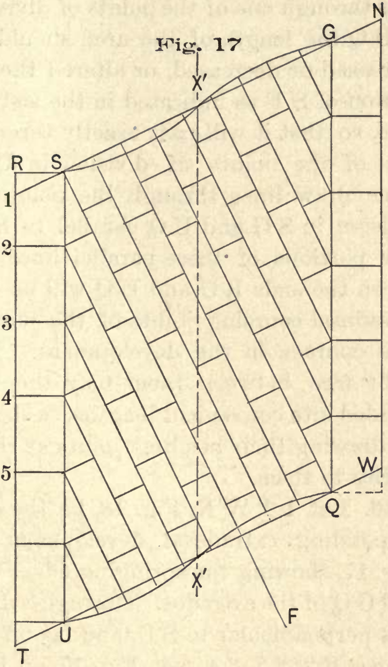
43. In practice it will be impossible to make full-sized drawings of the curves S P D, R P K, S X′ G and R X′ N for their entire lengths. The four equations above will be useful in exactly determining any portion of either one of them, and be of great assistance in laying out the work for construction.

44. In a properly constructed arch the resultant of all faces acting upon it should be confined within the middle third of the depth I B of the arch.

When S F in the development is perpendicular S G, R M cannot be perpendicular to the line joining the impost extremities of arch face ends of the developed intrados. If then for any reason it is desirable to alter the direction of S F it should be so altered whenever practicable, that the dotted line midway between S F and R M (Fig. 16) should approach to or become normal to the straight line, joining the extremities of the development of the middle line of the face of the arch here shown by dotted line in the elevation midway between A B C and H I J.

45. In the intradosal development the coursing joints are laid out in the following manner. Let S U Q G (Fig. 17) be the intradosal development of an arch, and let R T and N W be the outer edges of the extradosal development. Through S draw S F perpendicular to the straight line S G. Divide the straight lines S G

Fig. 17



and U Q each into an odd number of parts of equal length in order to show a key in the arch face. If S F should not pass through one of the points of division in U Q the length of the arch should be increased or decreased, or altered the direction of S F as indicated in the last article, so that it will pass exactly through one of the points of division in U Q. Then draw lines through the points of division in S G and U Q parallel to S F. The portions of these parallel lines between the ends R G and U Q will be the intradosal coursing joints of the successive courses in the development. The soffit face between them may then be divided into convenient lengths, as shown by drawing their heading joints at right angles to them.

46. Let 1 T W N, Fig. 18, be the corresponding extradosal development to Fig. 17, showing the springing edges S U and G Q of the extrados. Through S draw S R perpendicular to S U, and lay off the spaces R 2, 2 3, 3 4, &c., Fig. 17, on R T. Then through R draw R M parallel to R M, Fig. 16, or as altered by Fig. 17, and through 2 3 4, &c., and the corresponding points in N W, draw lines par-

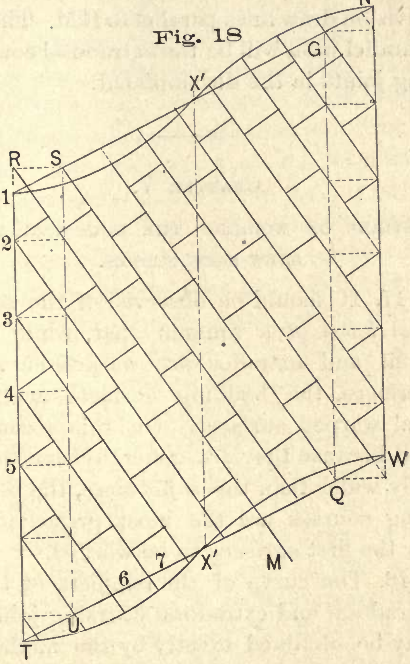


Fig. 18

allel to R M. Then divide the remaining spaces of straight lines 1 N and and T W into equal parts equal to the distance

6 7 on T W, and through the points of division draw lines parallel to R M. These parallel lines will be the extradosal coursing joints in the development.

CHAPTER V.

METHOD OF WORKING THE VOUSSOIRS AND SKEW-BACK STONES.

47. It should be observed of the several faces of a voussoir that, while its soffit and extrados are warped curved surfaces, the bedding surfaces are radial warped surfaces. On this account and because they are generally considerably wider than the soffit faces, the bedding courses are the most presentable for the first surfaces to be worked.

48. The curve of the templets of the intradosal and extradosal coursing joints may be obtained exactly by the method described in Article 12. In practice, however, the elliptical curve, S″3.4.5R″ (Fig. 4.), will be sufficiently exact; for when the length of the voussoir is small

in comparison with the length of the whole semi helix, the curve will vary but slightly from this elliptical curve. Very sharp oblique arches of small diameter have been successfully built, where this curve has been regarded as a circular arct. The method of determining such circular arcts is described further on, in Articles 52 and 53. The elliptical curve is, of course, nearer the true spiral curve than the circular arcs are. But neither of these two approximate curves should be employed unless it is found, after careful examination not to materially depart from the true curve.

49. Let A B C D E (Fig. 19) be the elevation, and E D the plan of a templet of a soffit coursing joint, so that the curve A B C between the center lines of the iron strap, 1 and 3, will be the exact length of a voussoir soffit coursing joint. We will suppose the rules shown in Fig. 7 are the proper ones for the joint A B C at points on it equidistant apart, and that the voussoir bed warps from A H towards C J, the point J being the point in the bed

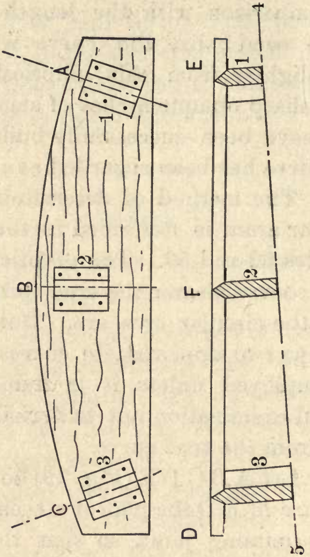


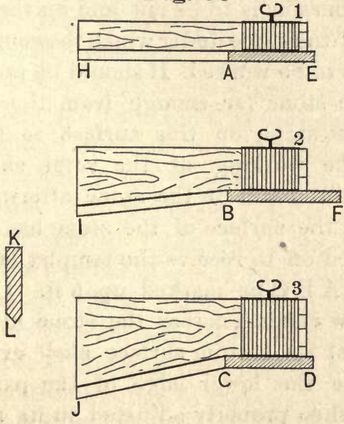
Fig. 19

farthest from a plane passing through the points A C and H. Evidently the parallel rule should be applied at A and the twist rules at B and C, so that their sides will be normal to, and their upper edges shall coincide with the sight plane 4 5.

The iron straps 1, 2 and 3 are fastened

to the templet A B C D E so that their center lines here shown will be normal to the curve A B C, but their sides are normal to the plane 4 5. Their tops, like their rules, coincide with this plane 4 5. Let each of the three rules above referred to, at their intradosal points A, B and C, be extended over the templet A B C D E in an arm that will exactly fit into its respec-

Fig. 20



tive strap 1, 2 or 3 (Fig. 20). Thumb screws in the top of iron straps will prevent the templets moving when they are once adjusted to these straps. The cross section of the parallel and twist rules should be of the form shown at K L in order that its lower edge may occupy the least space when applied to the voussoir bedding surface.

50. Therefore, in working the coursing bed of a voussoir, apply the soffit joint templet (Fig. 19) to the stone from which the voussoir is to be cut and on the surface of the stone under which the coursing bed is to be worked. It should be applied to the stone far enough from the edge of the stone on this surface to allow for the working of the soffit surface of the voussoir in the stone afterwards. When the surface of the stone has been dressed off to receive the templet let the curve A B C be marked upon it. Cut a narrow channel across the stone at A H so that its bottom surface shall exactly receive the lower edge of the parallel rule when properly adjusted in its strap 1, Fig. 20, and the soffit joint templet coincides with its curve already marked

on the dressed surface of the stone. Then cut a narrow channel across the stone at B I, so that its bottom surface will exactly receive the lower edge of its twist rule, when this and the parallel rule, properly adjusted to the soffit joint templet, are applied at B I and A H respectively. In the same manner the third rule is applied at C J and the bottom of its narrow channel reduced to receive the lower edge of this twist rule. Care should be exercised in applying the twist rules that their upper edges are always in the same plane and that the upper edge of the parallel rule is in this plane, all the rules, moreover, should be normal to this plane Any number of twist rules may be thus applied. Ordinarily, a single pair of one parallel and one twist rule will be enough. The balance of the bedding surface may be reduced to the bottom surfaces of the grooves thus cut by applying a straight edge to them on lines parallel to A B and to B C.

Having thus determined the bedding surface the extradosal curve H I J may be drawn upon it parallel to and distant the depth of the arch I B, Fig. 19, from the intradosal curve A B C.

51. The ordinary method of making these bedding surfaces of the voussoirs is: 1st, lay off and sink the soffit course joint A B C. Then the extradosal curve H I J is worked on the rough surface of the stone, and the parallel A H and the twist B I rule are applied normal to the curve A B C, until the upper edges are in the same plane. Then the twist rule C J is similarly applied. Thus used the rules have no arm over the soffit joint templet in the iron straps, as in Figs. 19, 20 and 21, to retain them in correct positions. Obviously this method of reducing these surfaces is attended with uncertain results, and that the method described in the last article is far superior to it. The method there described is true and exact, giving to all these surfaces the same warp or twist, a condition that should always be maintained, and for this reason the method of Article 50 should always be used.

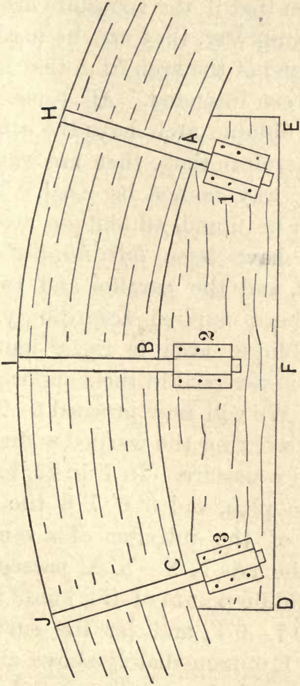


Fig. 21

52. Care should be exercised that the warp is worked in the right direction. Nor should we be deluded by the sup-

position that if the voussoirs are warped the wrong way, they can be used in the other end of the arch from that for which they were intended. All voussoirs in a given oblique arch have the same warp, and therefore those that are warped the wrong way cannot be used. This fact should be noted, all oblique arches here given have been *left handed* oblique arches, and the parallel and twist rules have been applied accordingly. When the oblique arch is right handed the order is reversed in their application.

53. We will now proceed to the method of working the warped soffit surfaces of the voussoirs. In Fig. 22, let 5 6 7 8 be the plan, and 5′ 6′ 7′ 8′ the development of the intrados of a voussoir, so that the axis X X—X′ X′ passes through the middle points of the joints 5 6—5′6′, and 6 7—6′7′, and let the curves 6 B 7 and 5 E 6 immediately above and below 5 6 7 8 each be circular arcs of the right section of the intrados. In the development the heading joint 6′7′ is perpendicular to coursing joint 5′6′ at 6′, there-

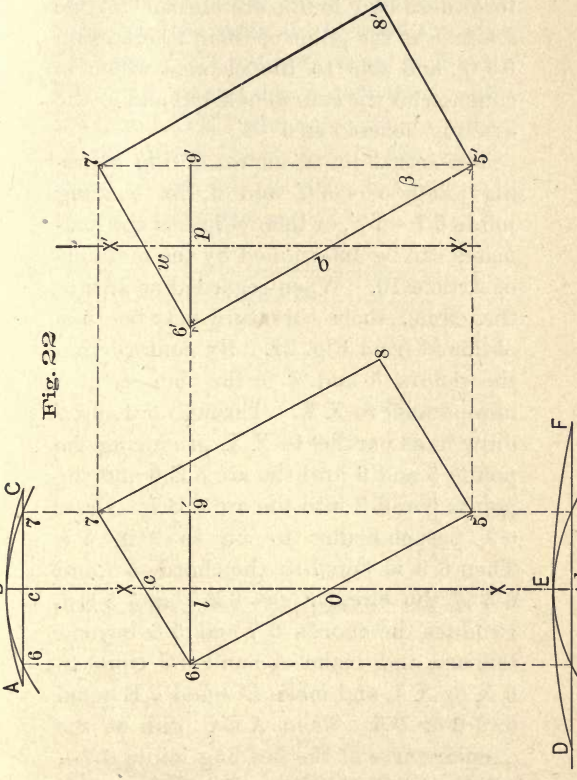


Fig. 22

fore these two joints are normal to one another at the point of their intersection 6—6′, and also to the element which is common to the course helicoid and to the heading helicoid at 6—6′.

The actual lineal curves of the coursing joints 5 6—5′6′ and of the heading joints 6 7—6′7′, or their elliptical approximates can be determined by the methods of Article **10**. When regarded as arcs of the circle, their curvature may be thus obtained from Fig. 22. By construction the points 5 and 7 in the plan are in a line parallel to X X. Through 5 6 and 7 draw lines parallel to X X, producing the points 5 and 6 into the arc 5 E 6 and the points 6 and 7 into the arc 6 B 7. Draw 6 9 perpendicular to *x x* to 9 in 5 7. Then 6 9 is equal to the chords 6 7 and 6 5 of the circular arc 6 B 7 and 5 E 6. Produce the chords 6 7 and 5 6 beyond this arc, and make A *c* and *c* C equal to 6 X or X 7, and make D *d* and *d* E equal to 6 0 or 0 5. Then A B C will be the circular curve of the heading joints 6 7—6′ 7′, and D E F will be that of the coursing joint 5 6—5′ 6′. Their middle ordinates are the same as those of the arcs 6 B 7 and 5 E 6, or both equal to B C or E D. It should be noted that while A B C and D E F are approximate values of, the chords A *c* C, and D *d* E are the actual chords of the true heading and coursing joint curves.

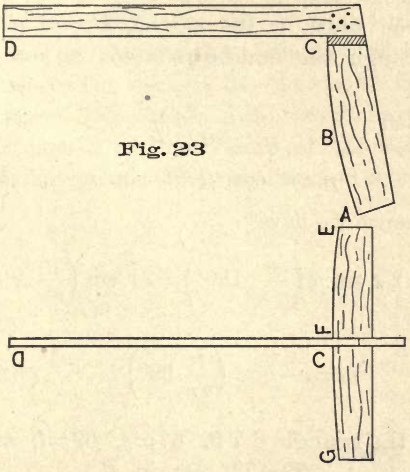


Fig. 23

54. The radii of these circular approx-

imate curves are thus obtained. Let p be the length of each of the arcs 6 B 7 and 6 E 5 or 6 9 in the development; l their chords or 6 9 in the plan; b the breadth 5′ 6′, and w the width 6′ 7′ of the soffit development, c the chord A c C, and c' chord D d F; m the middle ordinate B c or E d; R the radius of A B C, and R′ that of D E F. Then for the given width w the axis X′X′ will pass through the middle points of w and b,

when $$b = \frac{w}{\text{tang. } \beta} \tag{19}$$

or $$p = w \cos. \beta = b \sin. \beta \tag{20}$$

whence we have

$$l = 2\, r \sin. \tfrac{1}{2}\left(\frac{p}{\pi\, r} . 180°\right) = 2\, r \sin.\left(\frac{p}{\pi r} . 90°\right) \tag{21}$$

$$m = r - \cos. \left(\frac{p}{\pi r} . 90°\right) \tag{22}$$

in the triangle 6 7 9, $67 = c$, $69 = l$, and
$79 = 7'9' = w \sin. \beta$,

or $$c^2 = l^2 + (w \sin. \beta)^2 \tag{23}$$

but $\frac{c^2}{4m} = 2\,R - m \quad \therefore \; R = \frac{c^2}{8w} - \frac{1}{2}m$ (24)

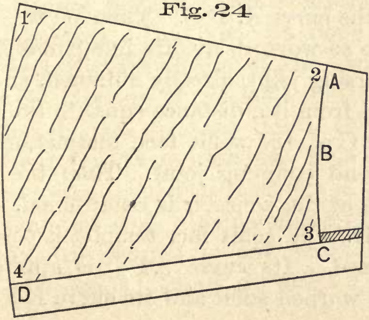
and in the triangle 5 6 9, $56 = c'$, $6\,9 = l$, and $5\,9 = b \cos.\, \beta$,

or $$(c')^2 = l^2 + (b \cos.\, \beta)^2 \tag{25}$$

and $$R' = \frac{c'^2}{8w} - \frac{1}{2}m \tag{26}$$

55. Fig. 23 is the side view and plan of the templet for the soffit face of the voussoir, or simply of the soffit face templet. The blade A B C is of the exact length and curvature of the soffit heading joint of the voussoir, and is per-

Fig. 24



manently attached to the stock, so that its edge C D is normal to the curve ABC at C, and blade E F G is also normal to the stock C D at C, and its curved edge G F E is the curve of the soffit coursing joint.

56. The soffit face templet is thus applied. After the first coursing bed has been worked as described in article 50, apply the stock to it and work off the soffit face of the stone until the curve E F G coincides with the soffit coursing joint line already worked on the coursing bed, at the same time the curve A B C is applied, and the voussoir soffit surface will be worked throughout its entire length to the curve A, B C. Then on the soffit face so worked, lay off a line parallel to its coursing joint, already obtained, and distant from it a distance equal to the curve A B C of the soffit face templet for the second coursing joint. Then the soffit face of the voussoir is determined.

57. The soffit face templet is then reversed. Its curve A B C is applied to the warped soffit and its curve E F G to

the second coursing joint just obtained, and the second coursing bed is worked to the stock C D. Thus let 1 2 3 4 (Fig. 24) be the end view of a voussoir whose upper coursing bed 1 2 and soffit face 2 3 has been worked, the soffit face templet is shown applied to work the second coursing bed 3 4. The extradosal coursing joint on this second bed can be obtained in the same manner it was on the first bed, and then the back of the stone can be worked off, and the whole voussoir will be worked.

ANOTHER METHOD OF WORKING THE VOUSSOIRS.

58. First work in one bed of the stone a convenient portion of the soffit cylindrical surface. This may be done by cutting into it the drafts of the arcs of two right sections of the soffit, and then working the face to these drafts and on lines normal to these sections. This may be done with the templet shown in Fig. 25. The two arms *a a b b* and *d e c f* are permanently secured to one another so that the parallel edges *a a* and *b b* and

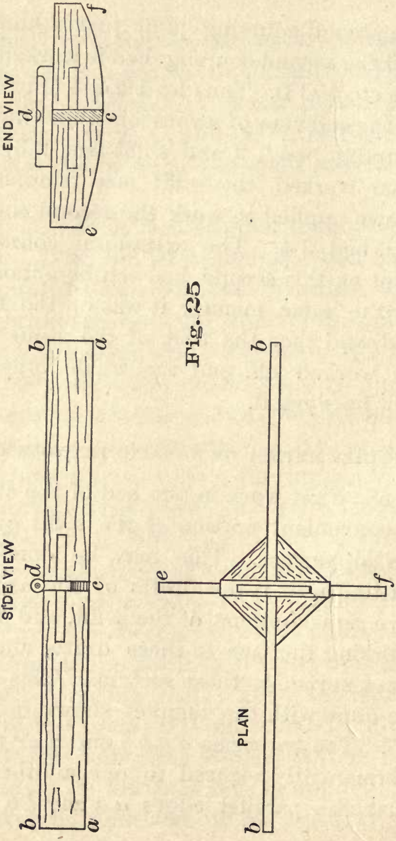


Fig. 25

the plane of the arm *a a b b* are at right angles to the plane of the arm *d c e f*. The lower edges of the two arms coincide at their point of intersection, and the curve *e c f* is a circular arc of the soffit right section.

A line drawn on a cylindrical surface normal to the right section of this surface is a straight line parallel with axis, or in other words, it is an element of the cylindrical surface. If, therefore, two drafts be cut in the face of the stone to receive the lower edges of this templet, the straight draft will be an element of the cylindrical surface of which the curved draft is a right section, and if either arm be moved over its draft without departing from the plane of this arm, and the stone be worked off to receive the sweep of the other arm of the templet, the surface thus obtained will be the soffit cylindrical surface required. If the circular arm extends sufficiently on either side of the straight edge to sweep the entire soffit of a voussoir, the templet may be confined to the single movement of the arm *a a b b* over its straight draft, and this arm may be maintained within a single plane in this movement by a spirit bubble on the upper edge of *d e c f*. In working this cylindrical surface care should be taken that the straight draft should be cut deep enough that the sweep of the circular arm shall be entirely within the body of the stone. Otherwise the voussoir soffit may be deficient, and the process of working the soffit will have to be repeated on new drafts cut deeper into the stone to receive the templet arms.

Through any convenient point A, Fig. 26, of a cylindrical surface thus worked draw an element A E. If 1 2 be the straight draft line over which the arm *a a b b* of the templet (Fig. 25) moved to determine this surface, A E is determined by drawing a line through A parallel to 1 2. A E may be determined by applying the soffit templet to and drawing on the worked soffit the line A E along the edge *a a* of the templet, the edge *a a* passing through the point A in the worked soffit.

Let A′ B′ C′ D′, Fig. 27, be the soffit development of any voussoir with the line A′ E′ drawn on it parallel to the axis or spring lines of the arch. Now cut a templet of card, lead, zinc, sheet iron

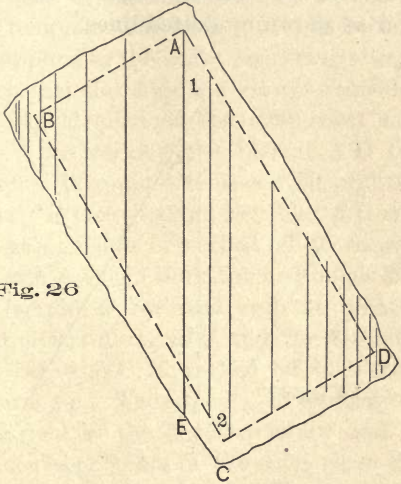


Fig. 26

or other flexible material exactly to its pattern, with any convenient hole *a* cut through it to show the coincidence of the line A E, Fig. 26, with A′ E′ when this templet is applied to the worked soffit. Then apply A′ and line A′ E′ of the templet to A and A E, Fig. 26. Curve the templet exactly to coincide throughout with the worked surface (Fig. 26), and trace on the soffit the edges of the vous. soir as shown by dotted lines.

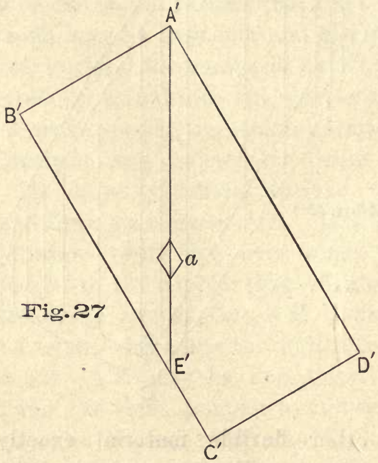


Fig. 27

59. The templet, shown in Fig. 28, is the same as Fig. 25, except the spirit

level is removed and the arm *f g* is added to it. This arm is permanently attached to and is in the same plane as the circular arm *d e c f*, and its inner edge *f g* is normal to the circular arc *e c f*. The coursing beds of the voussoirs are worked to this templet. The arms *a a b b* and *d e c f* are applied to the soffit, care being always taken that the edge *a a* always coincides with an element of the cylindrical surface. In working the bed at A D the templet is applied to the soffit and the point *f* is moved along the joint A D and the stone below is worked off to receive the arm *f g*. To work the bed below B C the templet is reversed, and the point *f* then moves along B C and the coursing bed below B C is worked off to receive the arm *f g*. The edge *f g*, in application, is normal to the heading joints and to the coursing joints of the soffit when the point *f* is at A, B C and D. The line *f g* at such points is therefore at the intersection of the heading surfaces and of the coursing beds when so applied. If, therefore, the templet be applied at A and the direction of fg marked on the coursing bed below A D and then at B and the direction of fg worked on the bed below B C, the heading surface is determined and can be worked to the two lines thus determined and to the heading joint A B. In like manner work off the heading surface at C D.

60. By applying the spirit level, Fig. 25, to the coursing bed templet, Fig. 28, and a thumb screw at f, so that the arm fg can be removed from or attached to the blade $decf$, we would have the soffit and coursing bed templets combined in a single templet. Economy will sometimes render such a templet desirable. It will, however, in the construction of oblique arches of any magnitude, be found best to use the two separate distinct templets here shown.

61. This process has the great advantage of determining all the joints and surfaces of a voussoir with great exactness. It, however, requires great care in application, simple as is its theory. Care should be exercised, 1st, in working the soffit to

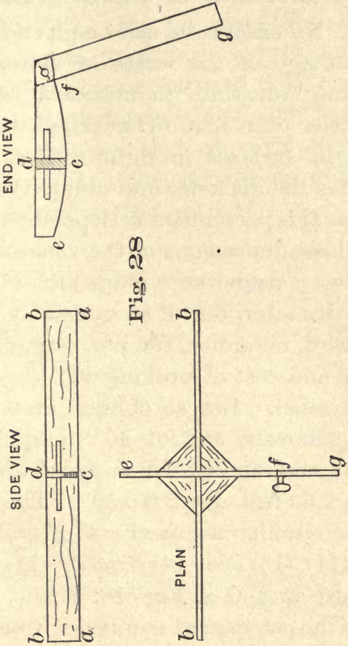


Fig. 28

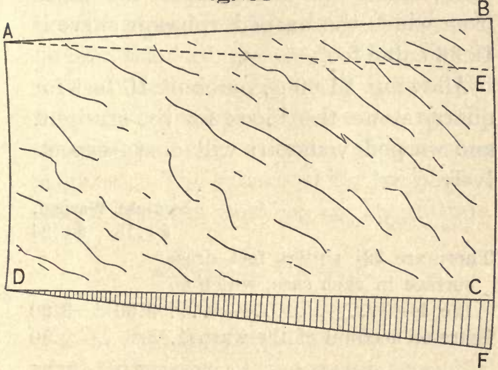
true cylindrical surfaces; and 2d, in working the coursing beds for the soffit, that the templet be so applied that the lower edge of the straight edge arm always coincides with an element of the soffit.

INCREASED COST OF WARPED SURFACES.

62. No exact rules can be given for the percentage of the waste of material in working voussoirs to helicoidal warped surfaces over that of working them to straight surfaces in right arches. For besides the diameter and obliquity of the arches this percentage is dependent upon the three dimensions of the voussoirs.

If we consider an oblique arch of ordinary diameter, but of an unusually sharp angle of obliquity, the per cent. of this waste and cost of working will, of course, be unusual. Let an oblique arch of 30 feet diameter, and of 40° obliquity be such a case, and we will suppose the arch to be 2.50 feet deep. $\theta = 50°$. The right semi-circumference or $\pi r = 47.17$ and from eqs. (11), (13) and (14) $\beta = 37° - 11\frac{1}{3}'$ $\beta' = 41° - 31'$ and G S, Fig. 16, 59.165. Let 45 be the convenient number of voussoirs for the face of the arch, then the length of the bedding joint will be $\frac{59.165}{45} = 1.315$ whence for Eq. (24) R = 23.07. If 3 feet is a convenient soffit length for the voussoirs their extradosal length will, Eq. (25), be 3.61. Eq. (6) gives $\Phi = 37° - 11\frac{1}{3}'$ or equal to β and Eqs. (4) and (5) makes the extreme warp to 0.25 or 3″.* The middle ordinate of the soffit heading joint is .009, and that of soffit coursing joint is .012. The intradosal width is 1.31 and the extradosal 1.45.

Fig. 29



63. Let Fig. 29 be an end view of a

* According to Buck this extreme warp = 3 × tan. $(\beta'-\beta)$ = 0.227, or a full quarter of an inch less than the actual warp. (See art 20).

helicoidal voussoir. The waste of material will be a wedge C D E or B A F whose width is the length of the voussoir, or if the voussoir be of the warp and dimensions given in Article 56. This waste will be 1.03 cubic feet. The contents of the warped voussoir or its equivalent straight voussoir are 11.33 cubic feet, or the waste of material is less than $8\frac{1}{2}$ per cent., since the contents of the block from which the warped voussoir curve is 12.36 cubic feet.

Allowing \$1.60 @ perch of 16 feet for quarry stone the block for the straight, and warped voussoirs will cost respectively,

	Straight.	Warped.
	\$1.13	\$1.24
There are $32\frac{1}{2}$ square feet dress surface in each case, which @ 12c. is........................	3.90	3.90
Extra on account of the warp...		50
	\$5.03	\$5.64

Or the warped voussoirs will cost $12\frac{1}{2}$ per cent. more than the straight ones. This per cent. in practice would be less than 10 per cent. This comparison has been made on the supposition that the block stones were quarried exactly to the given dimensions in both cases. In practice, however, they are quarried considerably larger than needed. The block stone would probably be of the same dimensions in both cases. Again, our illustration is an unusual one. In an ordinary obique arch the obliquity is much less, and the per cent. of cost would be less than 10 per cent. If the workmen are skilled in the use of the templet they will cut warped voussoirs as rapidly as straight ones, and the skill is soon acquired. The extra cost for templets is insignificant, in work of any magnitude.

SKEW-BACK VOUSSOIR.

64. It will be observed by reference to Fig. 17 that the intradosal development of the voussoirs at the spring lines S U and G Q are right angled triangles. Let Fig. 30 be an enlarged cut of one of these triangles. Its spring line length l or C B is obtained by dividing its whole

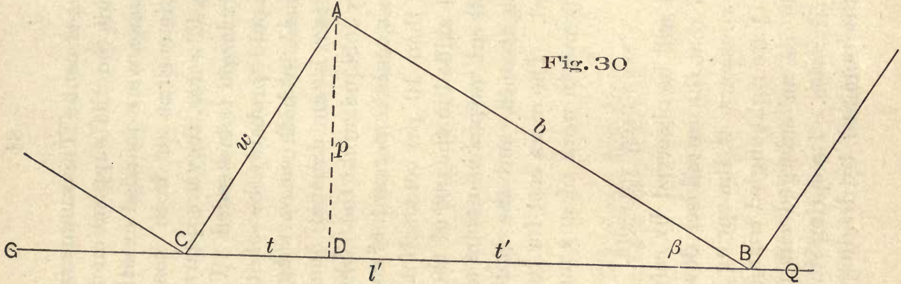


Fig. 30

spring line G Q by the number of these triangles upon it. Draw p or A C perpendicular to l, and let the two parts of l thus divided be t and t'. The angle A B C will be equal to β the angle of the intrados, whence we obtain

$$w = l \sin. \beta \ (27) \qquad b = l \cos. \beta \ (28)$$

$$p = w \cos. \beta \qquad\qquad b \sin. \beta \ (29)$$

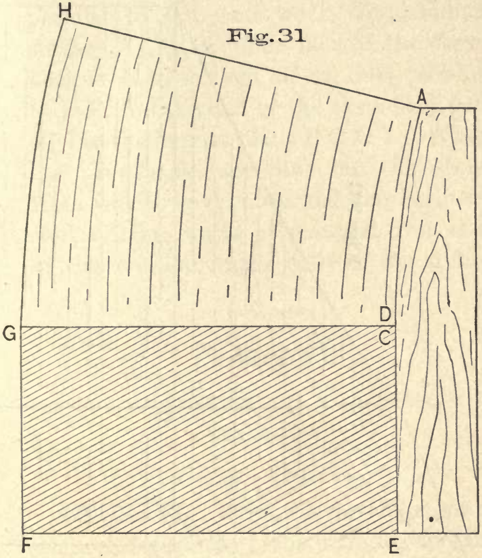
$$t = w \sin. \beta \ (30) \text{ and } t' = b \cos. \beta \ (31)$$

The triangular extradosal development of these voussoirs are shown in Fig. 18. Any of these spring lines, l being an element of the intrados parallel to the axis of the arch is a straight line, and the corresponding spring line of the extrados is equal to and parallel to l. Their impost surfaces are plane surfaces. If they are constructed without being made part of the course immediately below them, they will crack off at B and C, Fig. 30, and the tendency to move over the impost will be great. But if they are made part of the course below this weakness will be overcome, and the tendency to move over the impost will be abutted by the wing walls of the arch.

65. These skew-back voussoirs arranged as suggested by Article 64, are thus constructed. The coursing bed and soffit joint at A B, Fig. 30, is worked for its length A B the same manner as that of any voussoir in the body of the arch is. Let Fig. 31 be an end view of one of these skew-backs, A C G H being that of the heading surface. The line A H is normal to the soffit coursing joint of the bed just worked, and is therefore determined.

Then apply the stock of the soffit face templet to A H and determine C, Figs. 30 and 32, Fig. 32 being a perspective view of the worked skew-back. Work a straight line C B and let the soffit face templet be applied and the soffit between A B and B C worked. Lay off on CB C D $= t$, Eq. (30), and apply the curve A D of the templet A D E, Fig. 31, to the A D, Fig. 32, and work the line D E in the face of the skew-back. In the templet A D E, A D is the curve of the right section

Fig. 31



of the arch and the straight edge D E departs from a normal to this curve at D in conformity to the batter of the face of the abutment. The line B I, Fig. 32, is normal to the curve A B and is determined in working the bed A B I H. Therefore, make C G normal to C B and

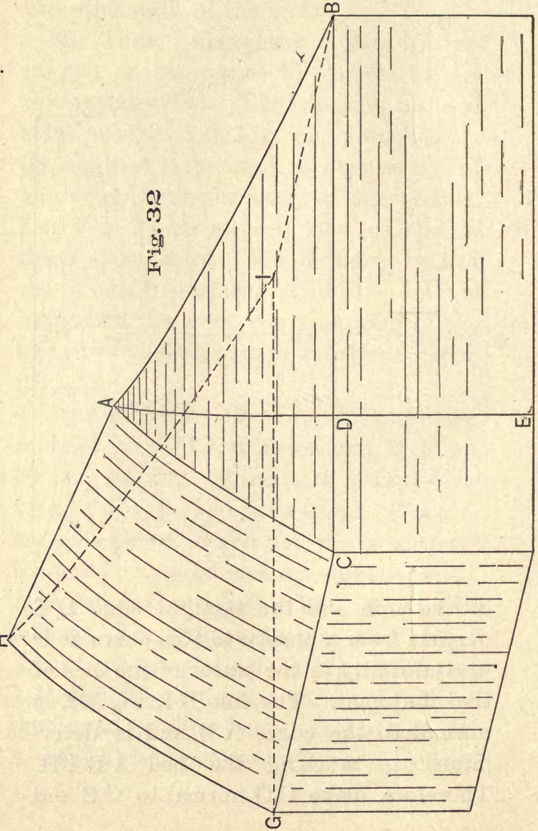


Fig. 32

parallel to I B, and work the heading surface A C G H. The face of the skew-back is then worked off on lines parallel to D E, and the bed of the skew-back parallel to the impost plane B C G I. When the skew-backs are thus cut the stone from whicht hey are worked may be quarried without waste of material over that of the voussoir in the body of the arch.

Chapter VI.

METHOD OF WORKING THE RINGSTONES, CENTERING, &C.

Ring Stones.

66. Let Fig. 32 be the development of an arch face end of the intrados and the extrados between the axis X′X′ and their spring lines S V and R T showing the end curves X′S and X′R and the successive coursing joints of each surface. These end curves should be exactly determined by Eqs. (17) and (18) and the coursing joints by Articles 45 and 46.

Beginning at the axis X′X′ let the corresponding intradosal and extradosal surfaces of the successive courses be numbered 1, 2, 3, 4, &c., and their corresponding coursing joints, 1, 2, 3, 4, &c., as shown.

To determine the angle an end line *c a* of the intrados between any two coursing joints 3 and 4 makes with these joints, through *a*, the point of intersection of the joint 4 with the curve X′ S, draw *a b* perpendicular to the joint 3. Then if A B C D, Fig. 34, be the soffit of a properly worked voussoir, at any convenient point *d* draw *d e* normal to C D or joint 4, which may be readily done with the soffit face templet, Fig. 34. Lay off *e f* on A B or joint 3 equal to *b c*, Fig. 33. If then a flexible straight edge be applied to the soffit and *d f* drawn upon it, *d f* will be the proper location of the end line *ac*, Fig. 34.

All other arch face lines of the soffit may be in a like manner determined.

67. To determine the angle the joint of any coursing bed 5 5, in the face of the arch makes with its soffit coursing

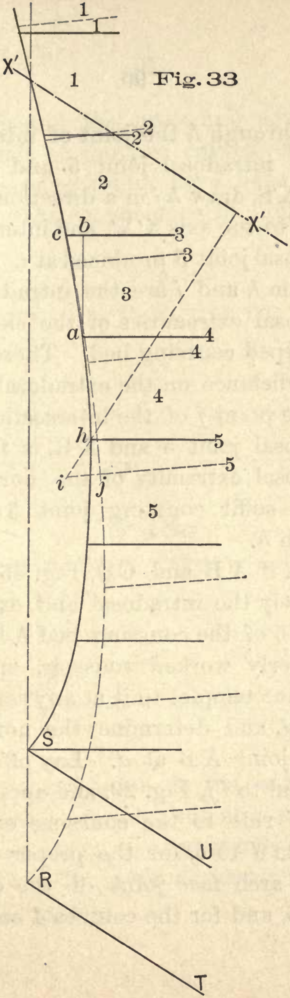


Fig. 33

joint, through h the joint of intersection of the intradosal joint 5 and the end curve X′S, draw $h\,i$ in a direction perpendicular to the axis X′X′, and intersect the extradosal joint 5 produced at i. By construction h and i are the intradosal and extradosal extremities of the element of the warped coursing bed. Therefore $i\,j$ is the distance on the extradosal joint 5 that the point j of the intersection of the extradosal joint 5 and X′R, is from the extradosal extremity of the normal line to the soffit coursing joint 5 passing through h.

Then if A B and C D, Fig. 35, are respectively the intradosal and extradosal joint 5 5 of the coursing bed A B C D of a properly worked voussoir, apply the soffit face templet to it at any convenient point d, and determine the normal $d\,a$ to the joint A B at d. Lay off on C D, $a\,b$ equal to $i\,j$, Fig. 29, and applying the flexible rule to the coursing bed, draw $d\,b$ from d to b, for the proper location of the arch face joint of the coursing bed 5 5, and for the courses 4 and 5.

In this and the preceding article the entire half of the two curved ends and their coursing joints have been considered, for the convenience of illustration.

Fig. 34

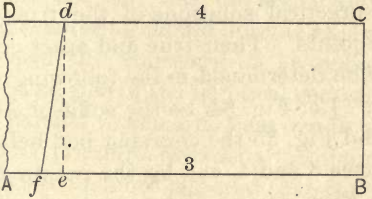
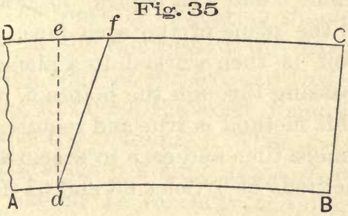


Fig. 35



Such consideration is not always convenient to entertain, but the parts relating to one or more courses may be treated separately by methods that readily suggest themselves.

68. True arch face soffit and bed joints must of course all be within a single plane surface, since the arch face is a plane surface. Hence it is that the methods given in articles 66 and 67 are approximate. Their resultants, however, are practical solutions of the true arch face joints. Their true and exact curves may be determined in the following manner: Let Fig. 34 be the soffit of course 3, and Fig. 35 the coursing bed between courses 3 and 4, and let the points *d* and *d* in Figs. 34 and 35 be identical. Then determine *b* and *f* as before. *b*, *d* and *f* are in the plane of the arch face. The voussoir is then worked to a plane surface passing through the points *b*, *d* and *f*. This method is true and requires but little more time and care to execute than the methods of articles 66 and 67 do.

CENTERING.

69. The centering ribs should be placed in planes parallel to the face of the arch, and, therefore, when so arranged will be elliptical. They are some-

times placed normal to the soffit, or made circular, when, in order to receive the arch under its acute angle, the centering has to be extended beyond the obtuse angle, and there loaded to prevent any movement in the centering when the voussoirs are set near the actual angle. Circular centering should not be employed to receive the voussoirs of an oblique arch. The ribs of the elliptical centering being parallel to the arch face, are in the planes of pressure, are easily maintained and require no more material than is necessary to receive the voussoirs.

70. The sheeting or lagging should be so put on the ribs that it will have an even and smooth surface, and that the centering will be of the exact dimensions to receive the voussoirs. When so prepared the soffit coursing joints of every course should be carefully and permanently marked upon the sheeting, as a guide for the placement of the voussoirs in the arch. The location of these joints on the centering may be determined from the development of the intrados. As the

skew-back stones are generally set before the centering is, no lagging or sheeting will be required on the centering below the intradosal upper courses of the skew-back stones.

SEGMENTAL AND ELLIPTICAL ARCHES.

71. As the same principles are involved in segmental as in full semi-circular right section of a helicoidal oblique arch, no further rules are necessary for their consideration.

As in right segmental arches the axes of oblique segmental arches are in the planes of their imposts or springing surfaces, and their skew-back stones should be constructed accordingly.

Elliptical oblique arches are not recommended, both on account of their structural weakness and the difficulties involved in their construction.

THRUST OF THE ARCH.

72. The thrust of a helicoidal oblique arch being carried to the impost in lines parallel to the face of the arch causes a

tendency in the arch to move outward at the acute angle of the arch, which is resisted by the friction of the coursing beds of the voussoirs. This tendency to move increases with the acuteness of the angle of obliquity, and when very acute this tendency should be resisted by properly constructed wing walls. When so constructed these arches may be constructed to any desired angle. It will, however, be a rare case where the angle of obliquity is less than 25°—the limit calculated by John Watson Buck. If he had considered the arch helicoids constructed as recommended in article 44 the construction would have been more stable, and his limit less than 25°.*

---

* Helicoidal oblique arches are much more stable than they are generally supposed to be. During the summer of 1877 the author superintended the construction of two of these arches, of 66 feet cylindrical length, 16 feet right diameter, and of 40° obliquity, each, and though placed under two tracks over which the heaviest railroad traffic passed, these arches, at this time, March, 1886, do not evince the least weakness. No extra precaution was taken to prevent the skew-back stones moving over their beds. A right arch could not have performed the work better or more satisfactorily.

Logarithmic and Ribbed Oblique Arches.

I.

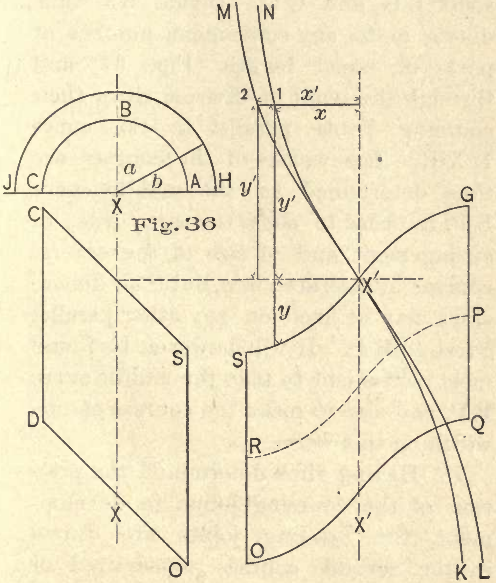
LOGARITHMIC METHOD.

73. This method of constructing oblique arches is so-called because Naperian logarithms are used in their calculations.

The soffit coursing joints by this method are always normal to the plane of the arch face, wherever they come in contact with it, and hence it is these coursing joints are normal to any plane parallel with the arch faces at their points of contact in the parallel planes. The soffit heading joints are elliptical curves in planes parallel with the arch faces, and are, therefore, normal to the coursing joints of the soffit at their intersections. The heading and coursing joints of the soffit being thus normal to one another, they will also be normal to one another in the developed soffit.

74. Fig. 36 shows the plan and devel-

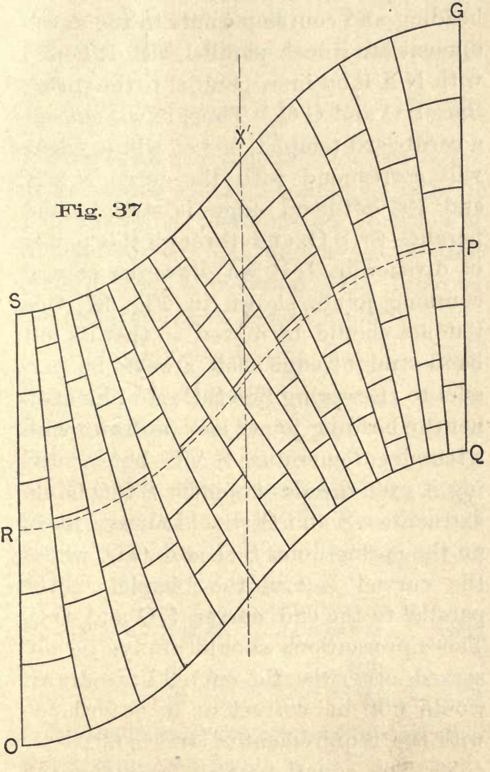
opment of the soffit of a semi-circular oblique arch, whose elevation and right section is shown at A B C and H I J.



The curve N X′K normal to the curve S X′G at its middle joint X′ is the developed soffit coursing joint through that point. Through the middle point R of the spring line S O, draw the dotted line R P parallel to the curved ends of the soffit S G and O Q. Divide R P thus drawn in to any convenient number of parts of equal length (Fig. 37) and through the point of division, draw their coursing joints parallel to the curve N X′R. The widths of the courses are thus determined on the middle curve R P, in order to show the same order of arrangement and of size of the several courses in each arch face, but their dimensions may be fixed on any other parallel curve to R P. It will, however, be found most convenient to take the middle curve R P, and also to make the courses of one width on this curve.

75. Having thus determined the position of the coursing joints in development, the heading joints are drawn in the several courses at desired or convenient points, and their elliptical curves are drawn parallel to R P, or to the curves of the developed arch ends.

Fig. 37



It should be borne in mind that the heading and coursing joints in the development are drawn parallel with R P and with N X′R on lines parallel to the spring lines S O and G Q. Thus, if we cut out a cardboard templet, one of whose edges will correspond with the curve N X′K and its left-hand edge is straight and parallel to S O, and through the points of division in R P we draw the several coursing joints shown in Fig. 37, this templet should be moved so that its left hand straight edge shall always be parallel to the spring line S O. In like manner the heading joints may be drawn with a templet of curvature S X′G, but in moving it over the development S O Q G, its extremities S and G should always move on the spring lines S O and G Q, whilst the curved face of the templet moves parallel to the end curves S G and O Q. These precautions should always be observed, otherwise the curved lines drawn would not be correct or in accordance with the requirement of article 69.

76 The equation of the normal curve

N X′K to S X′G at its middle point X′ will now be determined.

In article 41 will be found the expression

$$y = r \text{ sin. } a \text{ tang. } \theta \qquad (32)$$

Now since x is dependent upon a for its value this equation contains a ratio of x to y, and is therefore an equation of the end curve S X′G, and might be used for it in place of equation (17). Let b be the complement of a, and we have

$$y = r \text{ cos. } b \text{ tang. } \theta \qquad (33)$$

Again a is usually expressed as so many degrees, or as W. 180°. It is in fact the length of an arc of n 180° to radius unity. Thus, the expression tang. 36° means the tangent to an arc of 36° to radius unity, but not the tangent of 36°. a therefore equals $n\,\pi$,

$$\text{but } x = n\,\pi\,r, \quad \therefore\ x = r\,a \qquad (34)$$

$$\text{but } a = \frac{\pi}{2} - b, \quad \therefore\ x = \frac{r\pi}{2} - rb \qquad (35)$$

Differentiating equations (33) and (35) and dividing we have:

$$\frac{dy}{dx} = -\frac{r \text{ tan. } \theta \text{ sin. } b\, d\, b}{r\, d\, b} = -\text{tan. } \theta \text{ sin. } b \quad (36)$$

Now, let y' (Fig. 36) be the ordinate to any point in N X′K that y is for the corresponding point in S X′G for the ordinate x. At an infinitesimal distance from the point of contact X′ the curves N X′ K and S X′G are straight lines. Thus, in Fig. 38, let X′ be the origin of co-ordinates X′Y the axis of y, and X′X that of x. Now, if on X X′ we lay off an infinitesimal distance X′B, and through B draw A C parallel to X′Y, the curves A X′ and C X′ are exactly at right angles to one another within these limits, and the ordinates to A will be:

$-dx$ and dy' and the ordinates to C will be

$-dy$ and $-dx$. The triangles AX′B and BX′C are similar, whence

$$-dy : -dx :: -dx : dy',$$

or
$$\frac{dy}{dx} = -\frac{dx}{dy'} \quad (37)$$

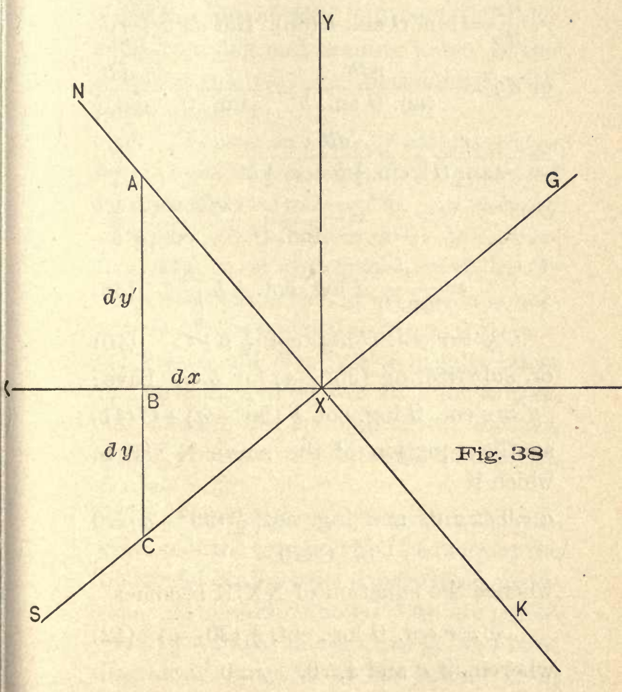


Fig. 38

but $\frac{dy}{dx} = -\text{tang. } \theta \sin. b$ (see eq. 36)

$\therefore \frac{dx}{dy'} = \tan. \theta \sin. b$ (38). But $dx = -rdb$

or $dy' = -\frac{rdb}{\tan. \theta \sin. b} = -\frac{r}{\tan. \theta} \cdot \frac{db}{\sin. b}$

$$= -\frac{r}{\tan. \theta} \frac{db}{2 \sin. \frac{1}{2} b \cos. \frac{1}{2} b}$$

$$= -\frac{r}{\tan. \theta} \cdot \frac{db \cos.^2 \frac{1}{2} b}{\cot. \frac{1}{2} b}$$

$$= \frac{r}{\tan. \theta} d \log. \cot. \tfrac{1}{2} b \qquad (39)$$

$$\therefore y' = r \cot. \theta \log. \cot. \tfrac{1}{2} b + \mathrm{C} \qquad (40)$$

or, substituting $(90° - a)$ for b we have:

$$y' = r \cot. \theta \log. \cot. \tfrac{1}{2} (90° - a) + \mathrm{C} \ (41)$$

as the equation of the curve N X′K in which if

$a = 0$, $x = 0$, and $\log. \cot. \frac{1}{2} (90° - a) = 0$
or $\mathrm{C} = 0$,

whence the equation of NX′K becomes

$$y' = r \cot. \theta \log. \cot. \tfrac{1}{2} (90 - a) \qquad (42)$$

wherein, if a and $x = 0$, $y = 0$

if $a = 90°$, $x = \frac{\pi r}{2}$ and $y = -\infty$

$$\text{if } a = -90^\circ, x = -\frac{\pi r}{2} \text{ and } y = \infty$$

By the aid of Eqs. (42) and (32) the soffit coursing and heading joints in the development may be determined with great precision.

77. *The coursing beds of oblique arches by the logarithmic method are generated by a radial line normal to, and moving along the axis of the arch as one directrix, and on a cylindrical curve as the other directrix, which at all points is normal to planes parallel to the arch faces.*

This second directrix is usually taken in the soffit, and we will so treat it here, but should be in cylindrical surface, midway between the intrados and the extrados.

78. There is great similarity in the generation of the coursing bed surfaces by the helicoidal and by the logarithmic methods. It should be noted both are generated by a radial line normal to, and moving along the axis of the arch. Their difference is in the fact that, in one case the second directrix is a helix, and in the

other, a normal curve to the arch faces. It matters little what this second directrix is so long as its curvature is known. But it is of the greatest importance that we keep in mind that the first directrix is radial in the logarithmic just the same as it is in the helicoidal method; nor should this idea, in the treatment of oblique arches by either of these methods, be ever lost sight of. It is the fundamental principle and renders these two methods quite similar in construction, and for this reason the treatment of logarithmic arches is readily understood when the problems connected with helicoidal arches have been once mastered.

79. Again, it should be noted the only straight line elements in the coursing beds by either methods are radial; that is, they are in lines normal to the axis of the arch, and consequently, are always normal to the coursing joints, both intradosal and extradosal. The only straight lines in the soffit, by either methods, are lines parallel with either the axis or the

spring lines of the arch, nor should this fact be lost sight of.

80. Now, if NX′K (Fig. 36) is the intradosal joint and MX′L the extradosal joint of a coursing bed passing through X′ in the development, and if through any point 1 in NX′K we draw 1 2 in direction perpendicular to S_1O continued, the point 2 in MX′L is the intersection of the radial element through 1 in NX′K. We have already determined the ordinate of 1 to be

$$y' - r \cot. \theta \log. \cot. \tfrac{1}{2}(90 - a) \qquad (42)$$

but y' is also ordinate of the point 2, and Eq. (42) is therefore the equation of MX′L when

$$x' = r'a \qquad (43)$$

II.

METHOD OF WORKING THE VOUSSOIR.

81. Reduce the face of the stone to be worked to a true cylindrical surface by aid of the soffit templet, Fig. 25, in the manner as described in article 58. This reduction may be accomplished in a va-

riety of ways, but the method there described is believed to be the simplest and best.

Let A B C D, Fig. 39, be the soffit thus reduced, and let A′B′C′D′ be the developed soffit of the voussoir to be worked. Through B with the straight blade of the soffit templet draw the element B E, and through B′ in the development draw corresponding line B′E′ parallel to the spring lines, or to the axis of the arch, and through D′ draw D′F′ parallel to B′E′, and through A′ and C′ draw A′G′ and C′F′ perpendicular to B′E′ and to D′F′, respectively. Lay off on B E, B G and G E equal to B′G′ and to G′E′. Then with the curved blade of the soffit templet through G and E draw the circular arcs G A and E C, whose respective lengths shall be equal to A′G′ and E′C′ in the development.

Produce the arc E C to F and make E F equal to E′F′ in the development, and with the straight blade of the soffit templet draw the element F D parallel to B E, and make F D equal to F′D′ in the

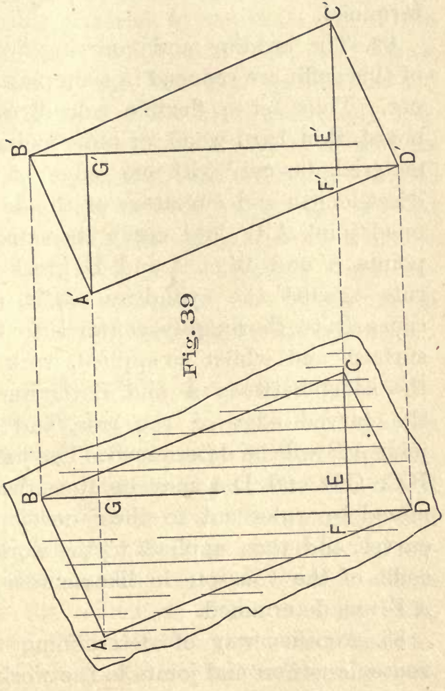


Fig. 39

development. The four corners A, B, C and D of the voussoir soffit are now determined.

82. The heading and coursing joints of this soffit are reduced in a simple manner. Thus let a flexible rule of cardboard, thin hard wood, or other suitable material, be cut, with one edge of the exact length and curvature of the developed joint A′B′, and apply its extreme points A′ and B′ at A and B, press the rule against the cylindrical soffit, and cause it to thoroughly conform to this surface, and whilst so applied, work on the soffit between A and B the line of the curved edge of the rule, and the joint A B will be determined. The joints B C, C D and D A may be thus determined by rules cut to their developed curves, and then applied to the worked soffit of the voussoir in like manner as A B was determined.

83. Another way of determining the voussoir corners and joints in the worked cylindrical soffit is: Let any corner B, Fig. 39, be selected as before, and

through it draw the element B E. Then cut a templet of cardboard, or of other flexible material, to the exact size of the developed soffit A′B′C′D′, and draw across it the line B′E′ parallel to the spring lines of the arch, and apply this templet with its corner B′ at B and its line B′E′ on B E, and cause the templet to conform throughout to the worked cylindrical soffit surfaces, and when so applied, work all the edges of the templet on the stone, and thus determine all the joints of the soffit at one operation.

REDUCTION OF THE COURSING BEDS.

84. When it is remembered that the coursing beds by this method (logarithmic) are generated by a radial line, it will at once appear that the coursing bed templet, Fig. 28, is as applicable to logarithmic arches as to helicoidal arches for the reduction of the coursing beds, when the coursing joints have been located upon the worked soffit of a voussoir.

85. The surfaces below the joints A D

and B C, Fig. 39, are therefore worked to the radial edge of this templet. When so worked, radial lines are. drawn with this radial edge on the coursing beds through the corners A, B, C and D, thus determining the lines of intersections of the heading and of the coursing beds of the voussoir. The heading surfaces are then worked to these radial lines so drawn, causing the surfaces to be normal to the soffit, and therefore normal to the coursing beds.

86. Much has been said as to what should be the character of the heading coursing surfaces, both by the helicoidal and by this method. It has been maintained that these surfaces, by the logarithmic method, should be planes parallel to the arch faces; that the strength and stability of the arch demanded it, &c. It in fact matters little, whether these surfaces are parallel to the arch faces or are warped surfaces, normal to the coursing beds. It is, however, the author's opinion that the latter construction is the more

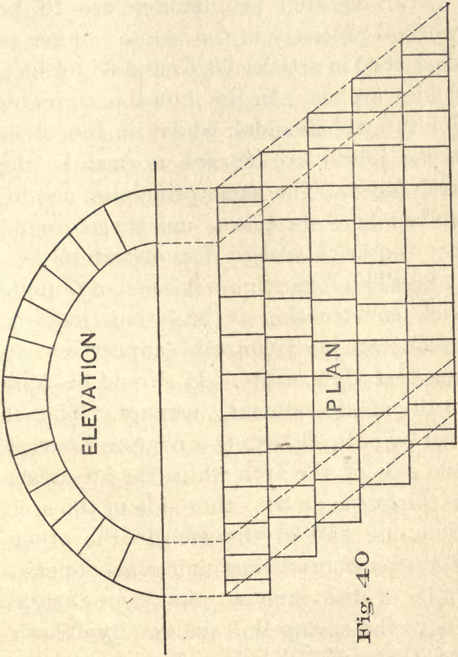


Fig. 40

stable. It has the advantage, also, of simpler construction.

87. The arch face stones are to be worked precisely in the same manner as described in articles 66, 67 and 68 for helicoidal arches. In the one the coursing joints are helicoidal, whilst in the other these joints are curves, normal to the arch faces. The same principles are involved here as there, and therefore do not require a second demonstration.

88. The logarithmic method of oblique arch construction is one that requires great care and constant supervision to successfully execute. It should be done in the most systematic manner. Thus it will be noticed that the courses increase one side of the arch while they decrease in thickness on the other side of the arch from one end of the arch to the other. Yet, two courses beginning at opposite ends of the arch at the same height above the spring line are exactly alike in all their dimensions. Their voussoirs may therefore be worked in duplicate, which, in fact, is the proper way to treat

this class of construction. These two courses may be laid out at once, and all their voussoirs worked. There is no reason why all the voussoirs of the entire arch may not be worked out before any of them are set in position in the arch. All the voussoirs of a course should be kept separate and distinct by themselves. Every voussoir should be marked in plain figures or characteristics, indicating at once its course and position in it.

89. The coursing joints of every course should be marked permanently on the centering, as a guide for the voussoirs as they are set in position. This is best done by transferring to the centering ordinates taken from the development plan of the arch soffit.

RIBBED OBLIQUE ARCHES.

90. Oblique arches are sometimes constructed by placing several narrow elliptical arches or ribs, as they are called, together, as shown in Fig. 40. This method is very faulty, and cannot be too severely condemned. There is no bond between

the several ribs, as each rib is separate and distinct in its construction and its position; the load above the arch is never uniform throughout the whole length of the arch, and on account of this lack of bond in the arch, it will be distorted by its unequal settlement. Again, the outer ribs are constantly being forced outwards by the action of frost upon the material that finds lodgement between their heading surfaces. So serious becomes this weakness that the ribs have to be reconstructed to restore the stability of the arch. True, these ribs are sometimes bonded one to another with iron straps, yet this bungling device is devoid of transverse strength and is susceptible of taking up the longitudinal stress only. Such an arch is not to be compared to an arch of bonded courses. Again, if it is properly bonded with iron straps, a ribbed elliptical arch costs more than an oblique arch constructed in accordance with the rules of any one of the recognized methods of proper construction.

Strength of Oblique Arches.

91. The static problems in masonry arches is to so construct, or so arrange the material composing the arch, that the line of pressure resultant from the weight and load of the arch shall pass through it in planes parallel with the arch faces, to the end that the arch stress shall be uniform throughout the length of the arch, and that the courses shall be relieved, as far as possible, of the tendency to move or slide over one another.

92. Without discussing the problems involved, we will suppose that the bond of the arch is ample to resist all ordinary external forces acting upon the arch, and that the arches now to be considered are so constructed that the line of pressure falls within the middle third of the thickness of the arch, and that it is located on

the center line of the depth of the arch, as for instance on the middle line a a, Fig. 41.

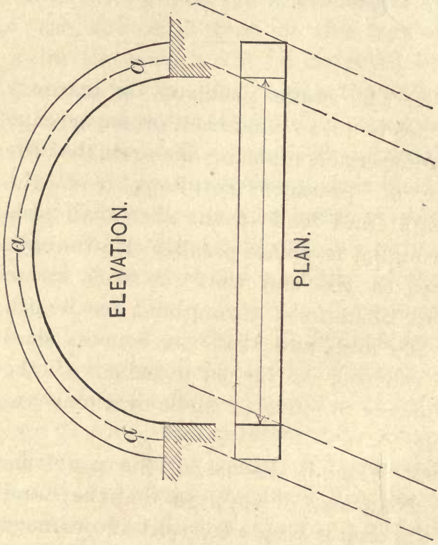


Fig. 41

93. Undoubtedly the disposition of the material composing the arch has much to do in determining the direction of the line of pressures.

In a right arch the coursing beds being parallel with the spring lines or axis of the arch, the line of pressure passes directly to the abutments in lines normal to those beds, and therefore in planes parallel to the arch faces. We have seen (article 24 and Fig. 11) that in an oblique arch with straight beds, there was a tendency for the arch stress to pass to the abutments in lines normal to such straight beds, and it would undoubtedly do so were it not for the distorted strains in the overhanging oblique ends of the arch.

Let Fig. 41 be an elliptical arch. With straight radial beds it is evident that the line of pressure will be parallel with the arch faces, and this is true if the arch is a single narrow rib. Now, an oblique arch may be regarded as composed of a number of elliptical ribs arranged as in Fig. 40. These ribs may be so narrow, that they will not break the continuity of the arch cylindrical intrados. The line of pressure in any single rib will be parallel to the arch face, and therefore the line of pressure for the whole arch must also be parallel with the arch face. The line of pressure would not be altered in this respect if the several ribs were reduced to the exact cylindrical intrados of the arch.

94. Again, in a helicoidal arch there is always a point on either side of the crown, where the coursing beds are normal to the arch face.

Now, if the arch be limited by imposts or abutments at these normal beds, the line of pressure will evidently pass through the arch normal to the impost and in lines parallel with the arch face, the same as in a right segmental arch, provided that no bed between the imposts shall depart from the normal to the arch face, equal to the angle of friction. It will be shown directly that this departure, for arches of the least practicable obliquity never amounts to as much as one-half of the angle of friction. If the abutment be lowered below the normal beds the line of pressure will continue parallel to the arch face until a bed is reached whose departure from the normal to the arch face equals the angle of friction. Even then if the surfaces of this lower bed and those below it resist the tendency to move, the line of pressure will continue parallel with the arch face.

95. We conclude therefore that the line of pressure is parallel to the arch faces when the condition of greatest stability in an oblique arch is to be realized, and that it should be regarded as parallel to the arch faces in discussing the strength of oblique arch.

96. Perfect stability is realized when the coursing beds are exactly normal to the line of pressure. In the logarithmic method the generating line in the coursing beds is exactly normal to the arch face, and, therefore, practically complies with this condition of perfect stability, and needs no further discussion in this particular.

97. Let S X G, Fig. 42, be the end development of the arch for the middle line on the arch face between the intrados and

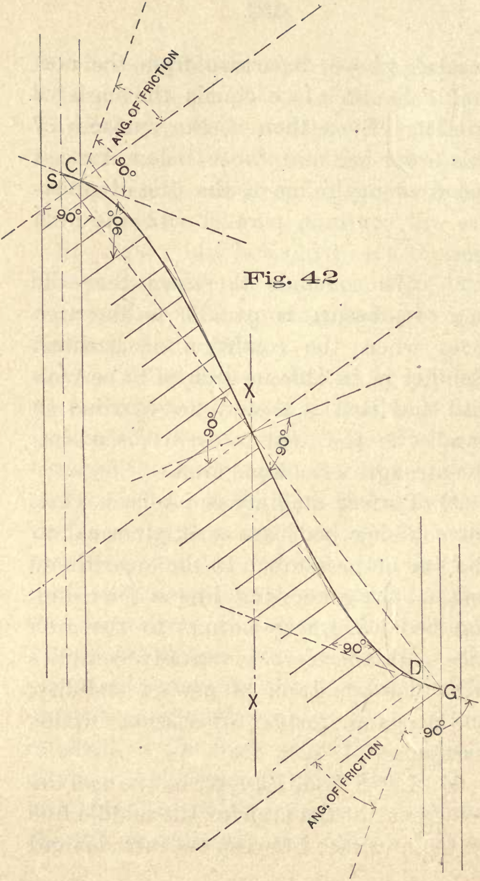


Fig. 42

the extrados. S X G will also represent the direction of the line of pressure, since it is parallel with the arch face. Draw the straight line S X G, joining the extremities of the development, and draw the middle line of the coursing beds normal to this line, as heretofore described. At A and B these lines will be normal also to the line of pressure. X, the middle of the end development, is the point between A and B, of the greatest departure of the middle line of a coursing bed from the normal to the line of pressure. For an oblique arch of 25° obliquity this departure is only 12°, the arch, therefore, is for all practical cases perfectly safe between A and B. At C and D the middle course lines depart from the normal to the line of pressure equal to the angle of friction. And the courses below these points will slide over one another if not otherwise resisted. Between C and S the tendency will be for the courses to move over one another into the arch, and between B and G to move out from the arch. It will not be safe to construct the arch below D, unless precautions be taken to thoroughly resist the outward tendency of the courses between D and G. In an oblique arch of 25° obliquity the point D is 10° above the horizontal on the full semi-circular right section, the angle of friction being taken as 36°. The stability of oblique arches proves that the angle of friction for the courses of the arch must far exceed 36°, especially so for well cemented masonry. The arch of 25° obliquity would unquestionably be safe for 160° on the right section. At 40° above the horizontal the coursing beds in this arch would be normal to the arch face. If the arch below 10° was resisted by the wing walls, or if the courses were doweled to prevent slipping the whole arch could be made stable for a full semicircle on the right section. Such construction is not recommended. Segmental arches are far preferable. It is true, however, that full semi-circular helicoidal arches of slight obliquity have been built, that have given the most satisfactory results.

It should be noted the sliding tendency of the courses immediately back in the arch from its face between D and G, is opposed by the abutment of the arch, and therefore cannot move without disturbing the abutment.

That the courses in the arch face below D do not, in carefully constructed oblique arches, move the least outward, is accounted for by the fact that they are held tight in the arch by the weight of the spandrel or parapet wall above them; that is to say they are so tightly wedged into the arch as to overcome the outward tendency.

Fig. 42 is an exact drawing of the end development, an oblique arch of 25° obliquity for full semi-circular right section.

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