A TREATISE

ON

TOOTHED GEARING.

Containing Complete Instructions

FOR DESIGNING, DRAWING, AND CONSTRUCTING SPUR WHEELS, BEVEL WHEELS, LANTERN GEAR, SCREW GEAR, WORMS, ETC.,

AND

THE PROPER FORMATION OF TOOTH-PROFILES.

FOR THE USE OF

MACHINISTS, PATTERN-MAKERS, DRAUGHTSMEN, DESIGNERS, SCIENTIFIC SCHOOLS, ETC.

BY

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PREFACE.

In presenting to the mechanical public this little work, I am fully aware that I am treading upon well-worn ground, and that I have devoted time and labor to a subject which is well-nigh "old as the hills," and likewise, to many, as familiar. It may also seem to some, who have read more extensively than I have upon the subject of toothed gearing, that this book contains nothing new, or original with its author: had such been my belief, the book would never have been written, much less published.

In my experience as a mechanical engineer I have sought often and earnestly, but always in vain, for a terse, compact, yet complete and comprehensive work on the subject of toothed gearing. Compelled, therefore, by necessity to gain the requisite knowledge from many works, and also from some failures on my own part, and believing, that, in the crowded field of technical literature, room yet remained for such a publication, I decided to write a book on toothed gearing, which should contain all that I had dug out from so many sources, and as much more as my experience and originality had taught me, yet being concise, terse, and simple enough to suit even "the wayfaring man, though a fool." Such were the somewhat exalted intentions of the author in writing this book: whether or not the reality equals the anticipation, is for the reader to judge.

Notwithstanding the apparent tendency to lay aside the old and simple "rules of thumb" for the surer and better methods, involving, to a certain extent, a knowledge of algebra and geometry, there are still many mechanics who continue to look with extreme distrust upon any thing in the shape of a book, because "books are generally too deep and too theoretical." For this reason I have given throughout the following pages simple rules, as well as formulas, for performing each and every operation necessary in designing and laying out the various kinds of gears.

He who possesses the requisite knowledge of algebra and geometry — for which any man will be the better off — may make use of the formulas in designing the gears he may have to construct; while he whose knowledge of mathematics goes not beyond the simple rules of arithmetic may obtain precisely the same results, and do in every way as good work, by using the corresponding rules. Throughout the book I have used a uniform system of notation in order to avoid confusing or burdening the memory of the reader, and the numerous examples will serve to illustrate sufficiently the application of the various rules and formulas. In all cases where the contrary is not stated, forces and weights are taken in pounds, and dimensions in inches. I have also carefully avoided any use of the metric system; because I believe the good old English inch, foot, and pound to be accurate enough for the proper construction of any machine, engine, or thing which can be made by the use of the metric system. In fact, American and English machinery being the best in the world, I see no reason to doubt the efficacy of the English system of weights and measures, from a machinal point of view at least. In writing upon a subject so old, and upon which so much has been written from time to time, it is impossible that I should not, to a certain extent, have copied the thoughts of others, even though in many cases they are also honestly my own. I deem it best, therefore, to say that I have taken the liberty of referring to and quoting such standard writers as Reuleaux, Camus, Unwin, Haswell, and others, but never, I believe, without giving them due credit. In writing the paragraph on "Special Applications of the Principles of Toothed Gearing," I have been greatly assisted by referring to Mr. Henry T. Brown's valuable little book entitled "507 Mechanical Movements," without which the work of collecting the various contrivances explained in this paragraph would have been indeed laborious. I trust, that, while much that is printed in this book may be found in other works on the subject, it also contains much that cannot be found elsewhere, and that my earnest desire to make it a simple, comprehensive, and convenient companion in the shop and scientific school, may be in some measure, if not fully, realized.

J. H. C.

NEW YORK, Feb. 1, 1884.

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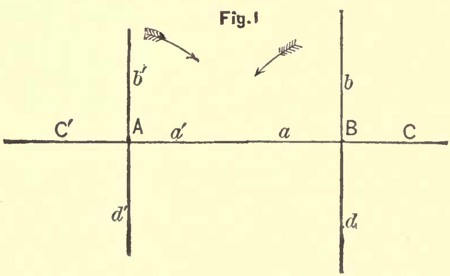
TOOTHED GEARING.

§ I. — *Introduction. — Fundamental Principles.*

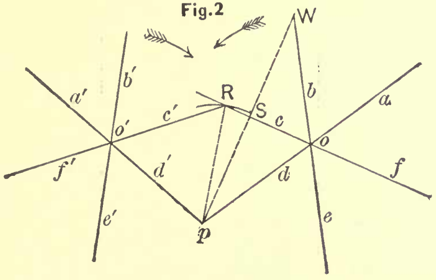
In the Science of Machinery, a science of vast consequence to the world, and vital to the wealth and power of any nation, there is, perhaps, no more important branch than the transmission of power and motion by means of toothed gearing; for in toothed gearing we have practically the only means of the all-necessary transmission. Having been known for thousands of years, and in practical use for centuries, in reviewing this subject we should naturally look for many successive alterations and improvements, even in fundamental principle; but no such result will be found by the most diligent research. Contrary to the natural and seemingly inevitable course of mechanical contrivances, in principle toothed gearing stands as an exception to the well-nigh universally accepted theory of "small beginning and gradual development." Improvement in this branch of machinal science has been slow and retarded; and strangely discordant with the general belief that first principles are always erroneous, or at least faulty ones, is the fact that the fundamental principle of toothed gearing, as it may be expressed to-day, is precisely what it was ten centuries ago. The slow-moving centuries which have witnessed the successive changes in water-motors — from the simple undershot wheel, driven in mid-stream by the impulsive force of the river's current, first to the overshot and Poncelet, then to the turbine and water-engine of the nineteenth century, each involving a different, and, in its turn, an improved, principle — can tell of no such advance in the essential principle of toothed gearing. Throughout the years which have changed the steam-engine from an atmospheric-pressure engine to a high-pressure expansion steam-motor; throughout the years which have produced the locomotive-engine, the ocean steamer, the telegraph, the electric light, the gas-engine, and the telephone, with all their successive alterations in prin-

Fig. 1



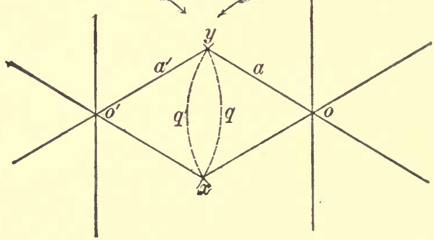
ciple and theory, — the science of toothed gearing almost alone has been able to attest, that in one case at least, if no more, first principles have been sound and perfect, — so perfect as to stand the test of years without change or improvement. This principle, most simple, although the underlying principle of the whole theory and study of toothed gearing, may be succinctly expressed as follows: If two cross-shaped pieces be placed as in Fig. 1, the arms of *A* being somewhat shorter than those of *B*, and the pieces being allowed only the motion of rotation about their fixed axes, or centres, then, if a continuous rotary motion in the direction indicated by the arrow be given to the piece *B*, a similarly continuous rotary motion in the opposite direction will be given to the piece *A*. For the arm *a*, in contact with the arm *a′*, will act as a lever upon it, forcing it downward, and at the same time bringing the arms *b* and *b′* into such relative positions, that a similar action will take place between them. Thus successively each arm of the piece *B* will act upon the corresponding arm of the piece *A*, and a continuous rotary motion will be transmitted from the piece *B* to the piece *A*. Simple

Fig. 2



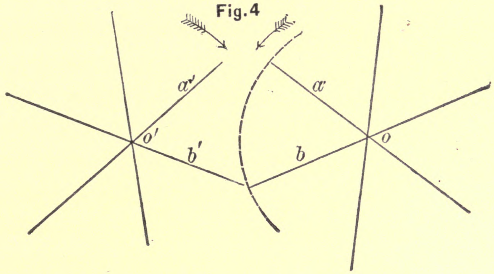
and crude as our sketch may appear, and however childish and primary our statement of this fundamental principle may seem, a most complete analogy exists between them and the most smoothly and accurately running gears of the present day; for each one of the countless scores of accurately profiled teeth, working so industriously and almost noiselessly in our machine-shops and factories, is but the projecting arm of our cross-shaped pieces, modified in accordance with the advance in machine manufacture, and shaped to suit the increased demand for accuracy of transmission. Since, doubtless, the first gear-wheels were similar to those represented in our figure, let us examine a little more minutely their action and the conditions necessary for such action. Let us suppose each wheel to consist of three long, slender pieces, or arms, crossed and fixed in such a manner that their ends divide the circumscribing circles into six equal arcs; that is, they form the diagonals of a regular hexagon (Fig. 2). The arrows indicate the directions in which the wheels revolve. Now, in order that the rotary motion be continuous, it is obvious that contact between the arms

Fig.3



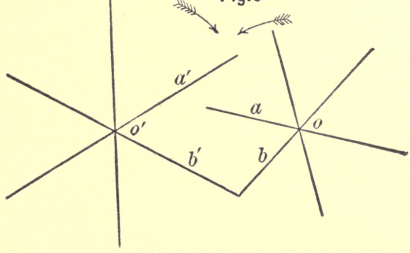
d and *d′* must not cease until contact is begun between the following pair of arms, *c* and *c′*: otherwise the wheel *o* would move some distance without moving the wheel *o′*, and consequently the motion of the wheel *o′* would be intermittent. It is also necessary that the arms of *o′* be somewhat shorter than those of *o*; for if they were equal (Fig. 3), the arcs *xqy* and *xq′y* being also equal, the arms *a* and *a′* would come in contact at their ends, and rotation would be impossible, or, for a greater separation, the arm b would leave the arm b' before the arms a and a' had reached their proper positions, and the wheel o would move on indefinitely without touching the wheel o' (Fig. 4).

Fig. 4



A glance at Fig. 5 is sufficient to show the impossibility of continuous transmission from o to o' when the arms of o' are longer than those of o. Let r be the length of each arm of the wheel o (Fig. 2), and r'

Fig. 5



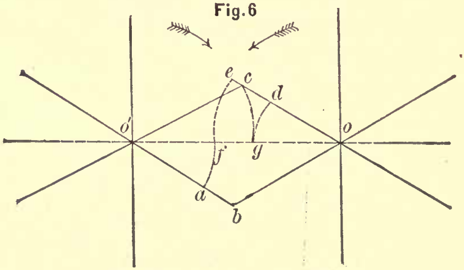
the length of each arm of the wheel o'. Let us suppose that contact between the arms c and c' begins at the moment when contact between d and d' is just about to cease. We have then the distance pR, between the two points of contact equal to r', because

$o'pR$ is an equilateral triangle. But we have seen that r' must be less than r: consequently pR must be less than r. The distance pR must obviously be greater than the distance pS, else there would be no contact at all between the arms c and c'. Since, now, the line pW is perpendicular to and bisects the arm c, we have

$$pS = \sqrt{\overline{po}^2 - \overline{oS}^2} = \sqrt{r^2 - (\tfrac{1}{2}r)^2} = \sqrt{\tfrac{3}{4}r^2} = .866r;$$

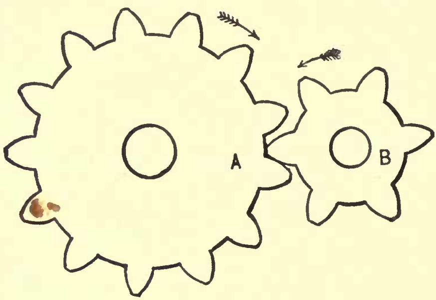
but $pR = r'$ is greater than pS: hence the conditions necessary for uniform transmission from the wheel o to the wheel o' are, that r' be less than r and greater than $.866r$. If there were a wheel of this sort given, to be used as a driver, and we wished to construct a wheel

Fig. 6



which would gear continuously with it, we would proceed as follows: From the point p, with a radius less than r and greater than $.866r$, say $.9r$, we describe an arc cutting the arm c in some point R. Then, with the same radius, we describe a circle passing through the points R and p, and draw the diagonals of the regular inscribed hexagon, of which pR is one side. The end of the arm $o'c$ (Fig. 6) comes in contact with the arm oe at the point c, and slides along its surface until the arms have assumed the positions *o'g* and *of* respectively. Then the end of the arm *oe* (now in the position *of*) comes in contact with the arm *o'c* (now *o'g*) at the point *f*, and slides along its surface until the positions *ob* and *o'b* are reached, after which contact between this pair of arms ceases. That is, during each revolution the end of the arm *o'c* rubs along the surface of the arm *oe* for the distance *cd*, and the end of *oe* rubs along the surface of *o'c* for the greater distance *ab*. The wearing-surfaces being unequal in the two wheels, the wear will be unequal, or, in other words, one wheel will wear out before the other: thus the accuracy of transmission will soon be destroyed, and the wheels rendered useless. Such rude contrivances can, of course, be of no practical use, and are given here, not as practical examples, but because of their natural primitiveness, and because they embody principles from which has been built up the present complete theory of toothed gearing. Whether or not these primitive gear-wheels were ever used for actual transmission, is indeed uncertain; and aside from the natural conclusion that the science of toothed gearing, like all other sciences, must have sprung from a mere germinal conception, and that our simple crossed pieces were most probably the first tangible form, the evidence of their real existence is confined to a few rough old drawings, such as those representing the ancient Greek and Asiatic norias for hoisting water, in which crossed pieces of wood precisely similar to our Fig. 1 are delineated. Certain it is, however, that if these crossed pieces were ever in actual use, time soon effaced the crudeness of their construction, and obliterated the faults which caused their inutility. The number of arms was greatly increased ; the arms themselves changed into pegs, or teeth, projecting at regular intervals from the circumferences of drums or wheels, and formed with curved profiles, in order to distribute the wear evenly over the whole surfaces of the teeth, and, if possible, to diminish the friction between the teeth, and so also the wear itself (Fig. 7). Since, now, the teeth

Fig.7

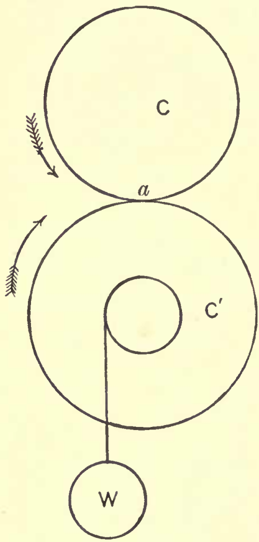


of the wheels *A* and *B* rub or slide against each other when in contact, and thus produce friction and wear, there must be some form of profile, straight or curved, simple or compound, which, when given to the teeth, will reduce the friction between them to a minimum, — some form which will be more advantageous for accurate transmission and uniformity of motion than any other. It is needless here to state of what vast importance is this desired form of tooth-profile ; for the perfection of almost every machine — the most simple and compact, as well as the most complicated and extensive — depends, to a very great degree, upon the action of its gear-wheels, and consequently upon the formation of the tooth-profiles of the gears. The proper formation of the tooth-profile must insure, in the words of another, "a more equable performance of the work in hand, a diminution of the moving-power wasted by friction, and hence the accomplishment of more work with the same amount of power, and a greater durability, and consequently a less cost for repairs in the whole machine." Recognizing, then, the fact that the subject with which we are dealing is of more than ordinary importance, we propose an investigation which aims to present, in as clear and terse a manner as possible, the method of reasoning by which the present development of tooth-profiles has been attained, — an investigation from which have been purposely omitted all the more intricate and tedious mathematical calculations pertaining to the subject, which have been so laboriously worked out by other writers and investigators. Far from thinking, or even wishing, to disparage the labors of men of genius and ability who have devoted their time and energies to the promotion of the purely mathematical and theoretical part of the great study of toothed gearing, on the contrary, believing their investigations and calculations to be the foundation upon which have been built the present more abstruse theories, their investigations have been omitted, because they may be found in almost any comprehensive work on the subject, and because it is thought unnecessary to repeat them here.

§ II. — *Proper Form of Tooth-Profiles.*

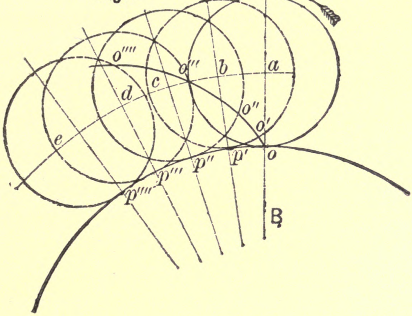
Let *C* and *C′* (Fig. 8) be two circles, in contact at the point *a*. If the circle *C* be made to revolve in the direction indicated by the arrow, the circle *C′* will be made to revolve in an opposite direction by the friction between the two circles, supposing, of course, the friction to be great enough to overcome the resistance. Suppose, now, it is required of the circle C' to perform work, for example, to lift a weight W by means of a string wound around its axle. By varying the pressure of the circle C upon C' at the point a of contact, the friction between the circles may be made just sufficient for the lifting of the weight : the friction between the circles will then be the smallest possible for the given amount of work. Also, if the circle C is driven by a constant and uniform force, since the resistance and motion are constant and uniform, the weight W will be lifted by a constant and uniform force, or, in other words, power and motion will be uniformly transmitted.

Fig. 8



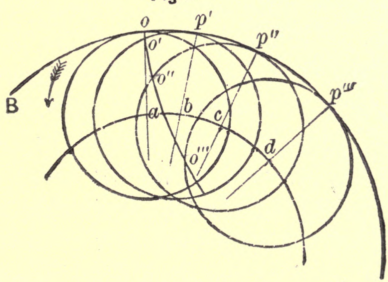
We may therefore conclude, that in order that toothed wheels may work together most uniformly, with the least friction and wasted power, and with the greatest durability, the tooth-profiles must be such that the driving wheel shall cause the driven wheel to revolve as if moved by simple contact. If the circle a roll, in the direction indicated by the arrow, *upon* the circumference of the circle B (Fig. 9), the point o of the circle a will assume successively the positions o', o'', o''', etc., the arc $p'o$ being equal to the arc $p'o'$, the arc $p''o$ being equal to the arc $p''o''$, etc., and the position o' of the point *o* corresponding to the position *b* of the rolling circle, etc. The point of contact, *o*, generates during the rolling the curve *o o′o″o‴o⁗*, obtained by drawing a

Fig. 9



curve through the successive positions of the point *o*. This curve, described by a point of the circumference of circle which rolls *upon* the circumference of another circle, is called an epicycloid. In the same manner, if

Fig. 10



a circle *a* roll, in the direction of the arrow, *within* the circumference of another circle *B* (Fig. 10), the point *o* on the circumference of the rolling circle will generate the curve *o o′o″o‴*; the arcs *p′o′*, *p″o″*, and *p‴o‴* being respectively equal to the arcs $p'o$, $p''o$, and $p'''o$, and the positions o', o'', and o''' of the point o corresponding to the positions b, c, and d of the rolling circle a. This curve, described by a point on the circumference of a circle which rolls *within* the circumference of another circle, is called a hypocycloid. In Fig. 9 the motion of the circle a, relative to the motion of the circle B, is precisely similar to the motion of the circle C, relative to the motion of the circle C' (Fig. 8). For in Fig. 9 equal arcs of the rolling circle are developed, in equal times, upon the circumference of the circle B; and the same is true of the circles C and C' (Fig. 8). Consequently the motion of any point, as o, of the circle a, with reference to the motion of the corresponding point o of the circle B (Fig. 9), must be similar to the motion of the point a of the circle C with reference to the motion of the corresponding point a of the circle C' (Fig. 8). But we have shown that the point o of the circle a (Fig. 9) generates an epicycloid with reference to the motion of the point o of the circle B: hence, also, in Fig. 8, the point a of C generates an epicycloid with reference to the motion of the point a of C'. For the same reasons, the point a of the circle O, revolving about its fixed centre, and thereby causing the circle O' to revolve about its centre (Fig. 11), generates, with reference to the motion of the point a of the circle O', a hypocycloid.

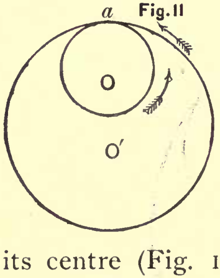
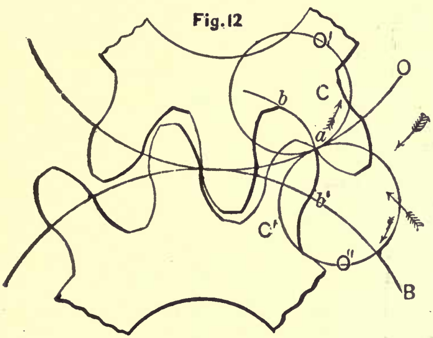


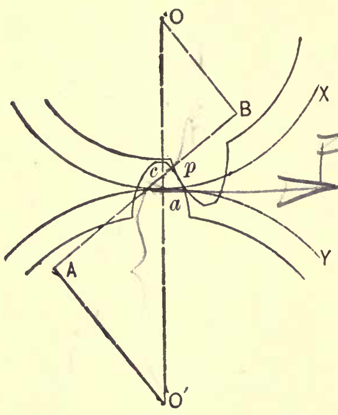
Fig. 11

Let C and C' (Fig. 12) be two teeth, contact between which has just begun, C being the driving, and C' the driven tooth. It is plain, from what has been said, that the motion of the point *a* of the tooth *C′*, with reference to the motion of the point *a* of the tooth *C*, is similar to the path described by the point *a* of a circle *O′*, which rolls within the circumference of the circle *O*. This path, as before explained, is a hypocycloid; and consequently, if we give to the portion *ab* of the tooth *C* (called the *flank* of the tooth) a hypocycloidal form, the profile *ab′* will slide along it with the least possible friction. While the point *a* of the tooth *C′* slides along the profile *ab*, the point *a* of the tooth *C*



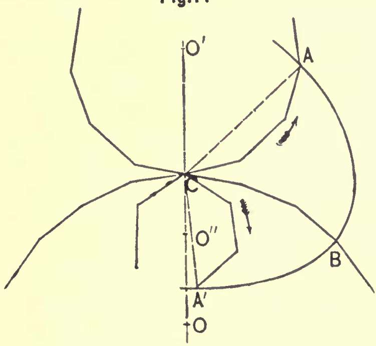
also slides along the profile *ab′*, and generates, with respect to the motion of the point *a* of the tooth *C′*, the epicycloid *ab′*, the path described by the point *a* of a circle *O″*, which rolls upon the circumference of the circle *O*. If, therefore, we give to the portion *ab′* of the tooth *C′* (called the *face* of the tooth) an epicycloidal form, the profile *ab* will slide along it with the least possible friction. Again: let the teeth of the wheels *O* and *O′* be in contact at the point *p* (Fig. 13), and suppose *O′* to be the driver. The driving-force of the wheel *O′* will be transmitted to the wheel *O* through the point *p*, and in the direction of AB, the common normal to the surfaces in contact at the point p. From the centres O' and O draw the lines $O'A$ and OB, each perpendicular to AB. Let F denote the driving-force of the wheel O', or the force exerted by the circumference Y, and F' the force exerted by the point A. From the principles of the simple lever, we have the proportion $F : F' :: O'A : O'a$. Hence $F' = \frac{F \times O'a}{O'A}$. Since

Fig. 13



the lines $O'A$ and OB are parallel, the perpendicular AB will be tangent to two circles drawn with O' and O as centres and $O'A$ and OB as radii, and the force F' of the point A will be directly transmitted to the point B through the line AB. Let P denote the force transmitted to the circumference X. As before, we shall have the proportion $P : F' :: OB : Oa$, or $F' = \frac{P \times Oa}{OB} = \frac{F \times O'a}{O'A}$. From this we obtain $F : P :: Oa \times O'A : O'a \times OB$. From the right-angled triangles cOB and $cO'A$ we may write the proportion $AO' : OB :: cO' : cO$, which, multiplied by $aO : aO' :: aO : aO'$, gives $O'A \times Oa : OB \times O'a :: cO' \times Oa : cO \times O'a$, and consequently we shall have $F : P :: cO' \times Oa : cO \times O'a$. But, for best results, all the force of the wheel O' must be transmitted to the wheel O; also, in order that the wheels may move as simple friction wheels, the velocities at the circumferences must be equal. Hence the forces F and P must be equal; and we will consequently have $cO' \times aO = cO \times aO'$, which can only be true when the points c and a coincide, and form one point. We may conclude from this, that the most advantageous form for the profiles of the teeth is such that the common normal to the profiles at the point of contact will pass through the point of intersection of the line of centres with the *pitch circles X* and *Y.* This point is called the *pitch point.* Suppose, now (Fig. 14), the pitch circle O and the rolling or *generating* circle O' to be regular polygons, having each an infinite number of sides. As the polygon O' rolls in the direction shown by the arrow, the point A generates an epicycloid; and there is, for an instant, a rotation of the polygon O' about the point C. The point A, for that instant, describes an arc of a circle, the centre of which is the point C, and the radius of which is the line CA. But, since the radius of a circle is always normal to the circumference at the point of their intersection, the line CA is a normal to the epicycloid at the point A: it also passes through the pitch point C. These two demonstrations were, we believe, first given by M. Camus in his "Cours de Mathématiques." By a similar course of

Fig. 14



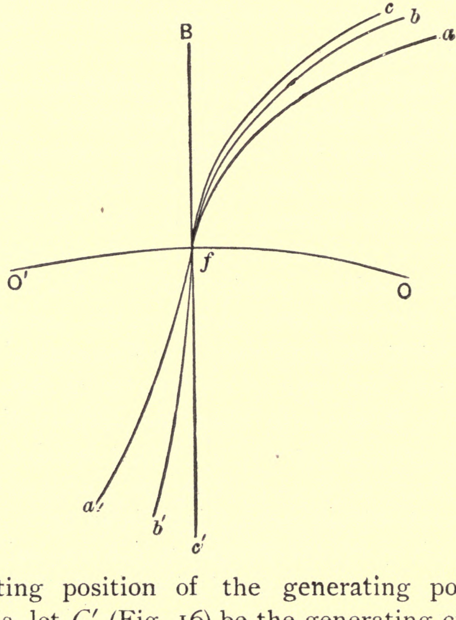
reasoning it may be proved that the normal CA' of the hypocycloid BA' (generated by the point A′ of the polygon O'', which rolls within the polygon O), at the point A' passes through the pitch point C. If, now (Fig. 13), we give to the face of the tooth of O' an epicycloidal form, and to the flank of the tooth of O a hypocycloidal form, the point of contact of the teeth will be the point of contact of two infinitely small circle-arcs, the radii of which are parallel, coincide to form the common normal, and pass through the pitch point *a*. We may now briefly sum up our arguments in order, and the conclusions which must be drawn from them. We have shown (Fig. 8), that, in order that the teeth of wheels work most uniformly together and with the least detrimental friction possible, the action of the driving wheel upon the driven wheel must be such that the wheels shall move as if driven by simple contact. We have also proved (Fig. 12) that this desired action takes place between the teeth when the faces of the teeth are given the epicycloidal and the flanks of the teeth the hypocycloidal form. Further: we have proved (Fig. 13) that the condition necessary for uniform power and velocity is that the common normal to the teeth in contact, at the point of contact, shall pass through the pitch point, and (Fig. 14) that this condition is fulfilled by teeth having epicycloidal faces and hypocycloidal flanks. From these demonstrations but one logical conclusion can be drawn, — that teeth having epicycloidal faces and hypocycloidal flanks fulfil all the conditions required of gear-teeth, and that the desired form of tooth-profile has been determined. Roomer, the celebrated Danish astronomer and inventor, is said to have been the first to demonstrate the advantages of these curves for tooth-profiles. But De la Hire, — who is credited with having first discovered, that, "if the profiles of the teeth of one wheel have an epicycloidal form, the profiles of the teeth of its fellow will properly have the form of a hypocycloid the generating circle of which has the same diameter as that of the epicycloid forming the teeth of the first wheel," * — Brewster, Young, Buchanan, and Reuleaux have been the chief promoters of the application.

Our investigation has now given us the required forms of tooth-profile; but since these curves, like all others, are susceptible of a considerable number of variations, it remains to determine somewhat more specifically the conditions upon which their applicability to wheel-teeth depends. In the first place, then, the amount of curvature, or amount of deviation, of epicycloidal and hypocycloidal curves from the diameter of the *primitive* or pitch circle, which passes through the pitch point, depends upon the diameter of the generating circle and upon the diameter of the primitive circle, or, in other words, upon the ratio of the diameter of the generating circle to that of the primitive circle. Thus, in Fig. 15, the epicycloids a, b, and c were generated by circles having diameters respectively equal to $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{2}$ the diameter of the primitive circle OO'; and the hypocycloids a', b', and c' had for generating circles respectively the same as the epicycloids. If d denote the diameter of the generating circle, and D that of the primitive circle, it is plain from the

* Mr. J. I. Hawkins's translation of Camus on the Teeth of Wheels.

figure, that, as the ratio $\frac{d}{D}$ becomes smaller, the deviation of the curve from the diametral line *Bc′*, passing through the pitch point *f*, becomes greater. If the diameter of the generating circle of a hypocycloid is equal to one-half the diameter of the primitive circle, the curve described will be a straight line coinciding with the diameter of the primitive circle passing through

Fig. 15



the starting position of the generating point. To prove this, let *C′* (Fig. 16) be the generating circle, and *C* the primitive circle. Let *o* be the starting position of the generating point. Since the diameter of *C′* is equal to one-half that of *C*, the circumference of *C′* will be equal to one-half the circumference of *C*, one-half circumference of C' = one-fourth circumference of C, one-fourth circumference of C' = one-eighth circumference of C, etc. Then, when the circle C' rolls sufficiently, the point A will fall upon the point A'; Ao = one-fourth circumference of $C' = oA'$ = one-eighth circumference of C. The diameter AB will then have the position $A'o''$; the diameter oo'' will have the position $A''o'$, at right angles to $A'o''$; and the point o will have the position o' on the diameter oE. Again: when the circle C' rolls sufficiently, the point o'' will fall upon the point d; arc oAo'' = one-half circumference of C' = arc $oA'd$ = one-fourth circumference of C. The diameter $o''o$ will then have the position do'', and the point o will have the position o'', still on the diameter oE. Thus it may be proved, that, for any position of the generating circle C', the point o will fall upon the diameter oE, and consequently that diameter is the path of the point; or the curve generated by the point o will coincide with the diameter of the primitive circle, which passes through the starting position of the point. If, therefore, we use for the generating circle of the tooth-profiles one which has for a diameter one-half that of the primitive circle, the flanks of the teeth will be simply radial straight lines, as is sometimes the case in practice.

Fig. 16

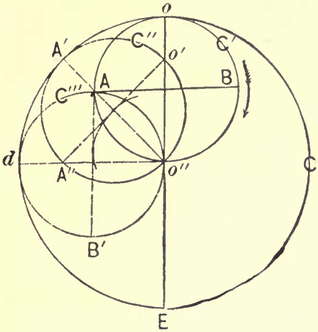
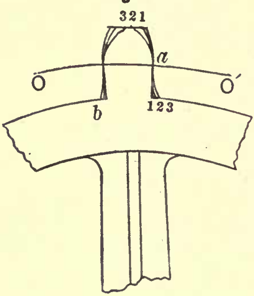


Fig. 17 shows forms of tooth-profile for different generating circles. Thus profile 1*a*1 was generated by a circle having one-half the diameter of the primitive circle OO'; the generating circle of profile $2a2$ had for a diameter three-eighths that of the primitive circle; and the diameter of the generating circle of profile $3a3$ was one-quarter that of OO'. In profile $3a3$ the inclination of the faces is so great, that there may be, by the principles of the inclined plane, a tendency to produce pressure upon the axles of the wheels; while profile $1a1$ is a weak form for teeth, being narrowest at the base $1b$, where it should be widest, because this part of the tooth bears the greatest strain when in action. Profile $1a1$ is also a bad form for wear, because the friction between the face of one tooth of this form and the flank of another is much greater than it would be if the face and flank were more nearly envelopes of each other. Therefore, for greater strength, less friction and wear, and best action between the teeth, we should take for the diameter of our generating circle less than one-half and greater than one-quarter of the diameter of the primitive circle. Generating circles of one-third the diameter of the primitive circle give very good results in practice.

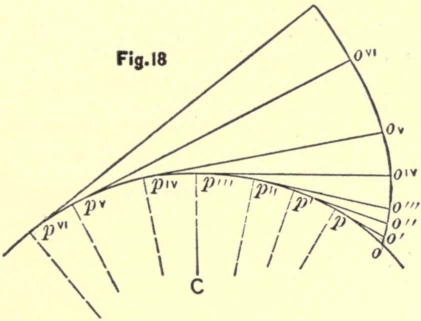
Fig. 17



Let us investigate the subject of profiles further. Let $o\,p\,p' \ldots p^{vi}$ be a string, wound around the circumference of the circle C, and fastened at the point p^{vi} (Fig. 18). If, now, the string be unwound from the point o, and held rigid as it unwinds, the end, or point o, will assume successively the positions o', o'', o''', etc.;

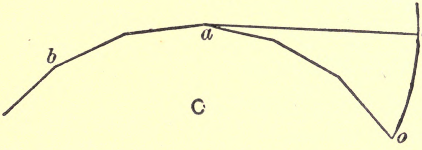
the line po' being equal to the arc po, $p'o'' = p'o$, $p''o''' = p''o$, etc. The curve $o\,o'o''o''' \ldots o^{vi}$, generated by a point of a string as it unwinds from the circumference of a circle, is called an *involute* to the circle, or an *involute* simply. Suppose (Fig. 19) the primi-

Fig. 18



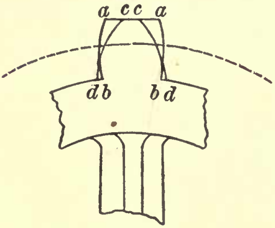
tive circle to be a regular polygon, having an infinite number of sides. As the string bao' unwinds, there will be, for an instant, a revolution about the point a; and the point o' of the string will then generate a circular arc having its centre in the point a, and

Fig. 19



a radius ao'. Therefore, as was shown in Fig. 14 for the epicycloid, the involute also fulfils the condition necessary for uniform power and velocity. For this reason the involute curve has been, and still is, extensively used for tooth-profiles, the curve forming the whole profile, *cd* (Fig. 20); or the teeth having involute faces and radial straight flanks, as in *ab*. We have now two kinds of tooth-profile, cycloidal and involute; each having, it is presumable, its advantages and its disadvantages in practice. A comparison between the two is therefore necessary.

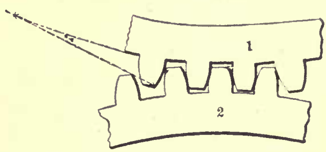
Fig. 20



§ III. — *Comparison. — Advantages and Disadvantages of Cycloidal and Involute Teeth.*

Cycloidal teeth have a great advantage over involute teeth, in that the number of teeth, for wheels of the same diameter gearing together, may be reduced to seven, without in any degree interfering with the uniformity of action. Reuleaux gives the smallest number of involute teeth necessary for proper action, eleven. In cycloidal teeth the loss of power and wear due to friction is not so great as in involute teeth; also the effect upon the action of the teeth by wear is less in cycloidal than in involute teeth, because the wear is evenly distributed in the former, and the teeth, even when considerably worn, present more nearly the original form of profile. Involute teeth, on the other hand, have the advantage of being easier and cheaper to construct than the compound profiles of cycloidal teeth. They are also stronger for the same width on the pitch circle. Again: the axles of wheels having involute teeth may be moved slightly from or toward each other without disturbing the proper action; while a very slight alteration of the distance between the axles of cycloidal gears destroys the accuracy of motion. Straight flanks are acknowledged by all to be poor forms, both on account of their weakness, and loss of work by friction. They should never be used except for large wheels, where the distance of the centres from the pitch circles renders them more nearly parallel, and consequently stronger. The principal objection offered to involute teeth is, that, especially in small wheels, the great obliquity of the profiles tends to produce a pressure upon the journals and bearings, as before noticed. Considerable difference of opinion exists as to the truth of this objection, and of late years actual experiment seems to assert its falsity. The following experiments were tried by Mr. John I. Hawkins, and are taken from his English translation of that portion of M. Camus's "Cours de Mathématiques" relating to the teeth of wheels. Similar experiments tried by the author of this book, with wheels carefully sawed out of black walnut, gave essentially the same results.

Fig. 21



The *approach* noticed by Mr. Hawkins in his Experiment II., however, failed to appear in the experiments of the author. Having constructed the sectors of two wheels, — each of two feet radius, and each containing four teeth of the same curve as those shown in Fig. 21, — one of the sectors (No. 1) was mounted on a fixed axis, and the other on an axis so delicately hung, that a force of even a few grains would cause the axis of the latter to recede from that of the former in a direct line. The following experiments were then made: —

EXPERIMENT I.

The teeth of both sectors being engaged their full depth of an inch and a half, No. 1 was moved forwards and backwards a great number of times, without exhibiting the least tendency to thrust No. 2 to a greater distance, notwithstanding the tangent to the surfaces of the teeth in contact formed an angle of nearly sixteen degrees with the line of centres. The points of contact of the teeth at the line of centres were three-quarters of an inch from the ends of the teeth.

EXPERIMENT II.

The teeth were engaged an inch and a quarter deep: consequently the ends of the teeth were a quarter of an inch free from the bottoms of the spaces; the tangent of contact made an angle of nearly seventeen degrees with the line of centres; and the point of contact at the line of centres was five-eighths of an inch from the ends of the teeth. The sector No. 1, being repeatedly moved forwards and backwards, sometimes caused sector No. 2 to approach, but never to recede. In Experiment I. the approach could not take place, because the teeth were engaged their full depth.

EXPERIMENT III.

The teeth were engaged one inch deep, leaving half an inch between the ends of the teeth and the bottoms of the spaces. The angle of the tangent of contact with the line of centres was eighteen degrees; the points of contact at the line of centres were half an inch from the ends of the teeth. On the sector No. 1 being moved frequently forwards and backwards, no motion of the axle of No. 2 appeared.

EXPERIMENT IV.

The teeth of the sectors were engaged three-quarters of an inch deep: consequently the ends of the teeth were three-quarters of an inch free from the bottoms of the spaces; the points of contact of the teeth at the line of centres were three-eighths of an inch from the ends of the teeth; the angle of the tangent of contact with the line of centres was nineteen degrees. The axle of sector No. 2 neither approached nor receded on numerous trials made by moving No. 1.

EXPERIMENT V.

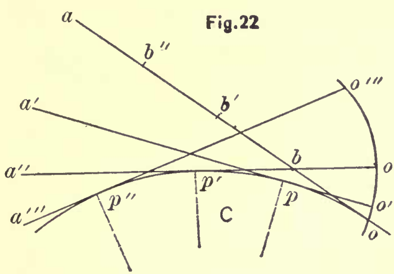
The teeth were engaged half an inch deep; the point of contact was a quarter of an inch from the ends of the teeth at the line of centres; the ends of the teeth were one inch from the bottoms of the spaces; the tangent of contact formed an angle of full twenty degrees with the line of centres. In a great number of repetitions of this experiment, a slight receding of sector No. 2 sometimes appeared.

EXPERIMENT VI.

The teeth were engaged a quarter of an inch: the ends of the teeth, therefore, were one inch and a quarter from the bottoms of the spaces; and the points of contact, one-eighth of an inch from the ends of the teeth at the line of centres; the angle of the tangent of contact with the line of centres was rather more than twenty-one degrees. In this experiment, which was repeated very frequently, a tendency to recede appeared several times, but so slightly as to be of no practical importance. The quiescent state of the axle was much oftener manifest than the receding.

"These experiments," says Mr. Hawkins, "tried with the most scrupulous attention to every circumstance that might affect the results, elicit this important fact, that the teeth of wheels in which the tangent of the surfaces in contact makes a less angle than twenty degrees with the line of centres, possess no tendency to cause a separation of their axes: consequently there can be no strain thrown upon the bearings by such an obliquity of the tooth." Such an obliquity as twenty degrees must, unless counteracted by an opposite force, tend to separate the axes; and, as suggested by Mr. Hawkins, this opposite force is most probably the friction between the teeth, which tends to drag the axes together with as much force as that tending to separate them. Of course the friction between teeth sawed out of wood is greater than in metal teeth; but Mr. Hawkins cites experiments tried by a Mr. Clement, with metal wheels lying loosely upon a work-bench, in which no tendency to separate the axes of the wheels could be noticed. This very serious objection to involute teeth having once been fairly removed, then the relative value of the two kinds of profile must depend upon the action between the teeth in each case, the amount of friction and wasted power, and the relative expense and difficulty of construction. The fact, that, in cycloidal teeth, less power is lost in overcoming friction than in involute teeth, seems to be well established, in theory at least, if, perhaps, not so well in practice; but whether or not the gain in this respect is sufficient to compensate for the additional expense of construction over the involute system, is still a question which must be finally settled by practice and actual experiment. In this practical age, the value of any one mechanism, compared with that of another, is simply a comparison between the relative amounts of work to be obtained from them and the relative costs; and that system of tooth-profiles from which can be obtained "the most work for the least money" must eventually gain the supremacy. In Fig. 18, while generating the involute curve, as fast as any portion of the string is unwound, it is held rigid, and forms a straight line tangent to the circle at the point of contact; as, for instance, the portion $p^{iv}o^{v}$ is tangent to the circle C at the point p^{iv}. Since this portion is that which generates the curve, and upon which alone the curve depends, we may assume the whole string to be rigid and straight, and the result will be the same. Let oa (Fig. 22) be a straight line, which rolls from right to left upon the circumference of the circle C. When the line oa has rolled sufficiently, the point b will fall upon the point p (the arc op being equal to the line ob), and the point o will take the position o'. When the line has rolled sufficiently, the point b' will fall upon the point p' (the arc $p'o$ being equal to the line $b'o$), and the point o will then take the position o''. When the point b'' falls upon the point p'', the point o will take the position o''', etc., and the curve $o\,o'o''o'''$ thus generated will be an involute to the circle C. Thus we have generated an involute by rolling a straight line

Fig. 22



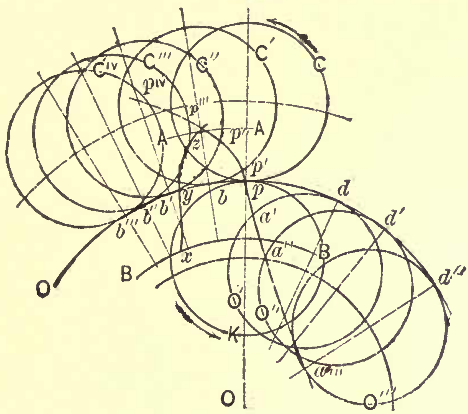
upon the circumference of a circle. But a straight line is the circumference of a circle the radius of which is infinitely long; and the curve generated by a point of a circle which rolls upon the circumference of another circle is an epicycloid: consequently an involute curve is simply an epicycloid the generating circle of which has an infinitely long radius; or, in other words, the involute is but a *limiting case* of the epicycloid. Thus, without coming to any actual decision as to the relative mechanical value of these two curves, or rather two different forms of the same curve, we have, nevertheless, the satisfaction of having verified our former conclusion, and may still assert that the cycloidal form of tooth-profile fulfils all the conditions and requirements, and is therefore the most useful and advantageous.

§ IV. — *Practical Methods for laying out Teeth, Exact and Approximate.*

Because of the difficulty with which exact epicycloidal and hypocycloidal profiles are constructed, approximate methods are very generally used; and they are found to answer the practical purpose very well. Any one of the following approximate methods will give very good results, and will, in ordinary cases, answer as well as the more difficult and tedious exact methods, also given here for use in special cases: —

METHOD I (*exact*). — Let O (Fig. 23) be the primitive or pitch circle. Take the diameters of the rolling circles C and K, each equal to one-third the diameter of the pitch circle. Strike the circles C', C'', C''', etc., which represent the different positions of the rolling circle C, and from the points of tangency, b, b', b'', etc., measure off the following arcs: $bp' = bp$, $b'p'' = b'p$, $b''p''' = b''p$, etc. The points p', p'', p''', etc., thus found, are points of the epicycloid which is to form the face of the tooth; and the curve $pp'p'' \ldots p^{iv}$, drawn through them, is the face-profile. For the hypocycloidal flank, after having struck the circles O', O'', O''', lay off the arcs $da' = dp$, $d'a'' = d'p$, etc., from the points of tangency d, d', etc. The curve $pa'a''a'''$, drawn through the points a', a'', a''', thus found, is the hypocycloid which

Fig.23

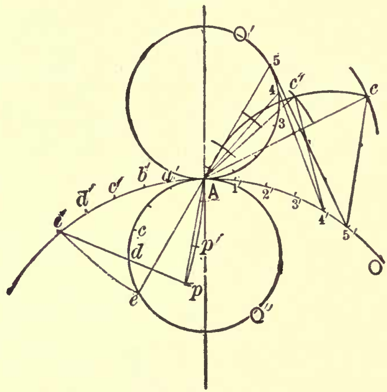


is to form the flank of the tooth. The other profile, xyz, which is similar to the one just found, is found by starting at the point y (py being the given width of the tooth at the pitch circle), and rolling the generating circles in the opposite directions from those just described. We have now but to limit the tooth at the top and bottom by circle-arcs, AA and BB, and the profile is complete.

METHOD 2 (*exact*).—In Fig. 24 O is the pitch circle,

O' and O'' the rolling circles, and A the pitch point. Divide the pitch circle and rolling circles into an equal number of small parts, equal each to each, as shown in the figure. Let the point 5 of O' correspond to point 5′ of O, the point e of O'' correspond to the point e' of O, etc. From A and 5′ as centres, with 5 5′ and the chord $A5$ respectively as radii, describe arcs intersecting in the point c; then from the centres A and 4′, with the radii 4 4′ and $A4$, describe arcs intersecting at c', etc.

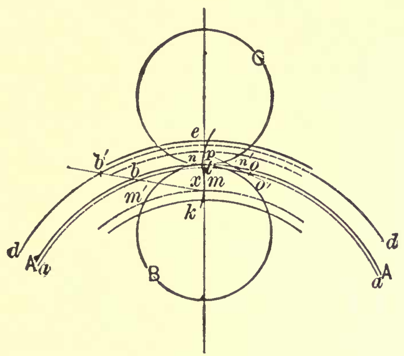
Fig. 24



The points thus found are points of the epicycloid $Ac'c$. Similarly, for the hypocycloid, from the points A and e', with radii $e'e$ and Ae, describe arcs intersecting at the point p, and thus determine the curve $Ap'p$. In these two methods, the closer together the positions of the rolling circles and the points of division of the pitch and rolling circles are taken, the more accurate will be the curves. When either of these methods is used, the work of laying out the teeth may be greatly simplified by accurately working out one entire profile upon a smooth piece of wood, and cutting out this profile for a template with which to trace the profiles around the pitch circle.

METHOD 3 (*approximate*). — From the points 1′, 2′, 3′, etc., a', b', c', etc. (Fig. 24), as centres, and with the corresponding chords of the rolling circles as radii, draw circle-arcs. Thus the radius for centre 5′ is $A5$, for centre 3′ is $A3$, for centre e' is Ae, etc. The envelope of these arcs, or the curve which is tangent to them, is very nearly the correct profile of the tooth.

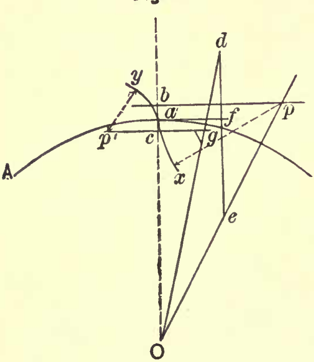
Fig. 25



METHOD 4 (*approximate*). — In Fig. 25 let AA be the pitch circle, and B and C the rolling circles. Let, also, t be the pitch point, and te and tk the heights of the tooth above and below the pitch circle. Take $tn = \frac{2}{3}te$, and strike through n the arc nn' concentric with the pitch circle. Step off on the pitch circle $to = tn'$, and from o as a centre, with the chord $n't$ for a radius, strike an arc cutting nn' in the point p. Draw po. The point p is a point of the epicycloid, and po is the normal to the curve at the point p. Find now, upon the line po, the centre for an arc passing through the points t and p. In the figure, o' is this centre, and $o'p$ is the radius for the faces of the teeth. The centres for all the faces are upon the circle aa drawn through o', and concentric with the pitch circle. Similarly, for the flanks, take $tm = \frac{2}{3} tk$, strike the arc mm', step off $tb = tm'$, and, with the centre b and radius bt, strike an arc cutting mm' in the point x. Draw xb, and find the centre b' for an arc passing through t and x. The radius for the flanks is $b'x$; and the centres are all upon the circle dd, drawn through b', and concentric with the pitch circle.

METHOD 5 (*approximate*). — Let A (Fig. 26) be the pitch circle, and a the pitch point. Draw af tangent to the pitch circle at the pitch point, and make it equal to 0.57 the diameter of the rolling circle, or $1\frac{1}{8}$ times the circular pitch of the teeth. Draw dfe parallel to the diameter aO, make $df = af$, and $ef =$ the diameter of the rolling circle. Draw Od and Oep, and, taking $ab = ac = \frac{1}{8}af$, draw bp and gp' parallel each to af. The point p of the intersection of Op and bp is the centre for the flank ax. Make $p'c = cg$, and p' is the centre for the face ay. As before, all the face centres are upon a circle drawn through p' concentric with the pitch circle, and ali the flank centres are upon a circle drawn through p.

Fig. 26



METHOD 6 (*approximate*). — Let A (Fig. 27) be the pitch circle, C and B the rolling circles, and a the pitch point. Draw $a'c'b$ and $cB'd$ through the centres of the rolling circles, each making angles of 30° with the line of centres. Draw the line *cabf* through the points *c* and *b*, and join *a′* and *d* with the centre *O*. The points *g* and *f* are the centres for the face *bx* and flank *cy* respectively. These approximate methods are from Reuleaux's "Constructeur," and Unwin's "Elements of Machine Design," and are as accurate as any in use at the present time. When a set of wheels is to be constructed so that any wheel of the set will gear with any other, the same generating circles must be taken for all the teeth of the set. Sometimes the generating circle is taken with a diameter equal to the radius of the smallest wheel of the set. The following are some of the simpler and rougher methods of approximation in use: they are convenient and easy, but give poor results, and should only be used in rough work. Fig. 28,

Fig. 27

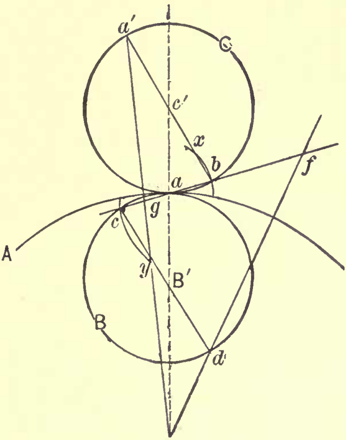
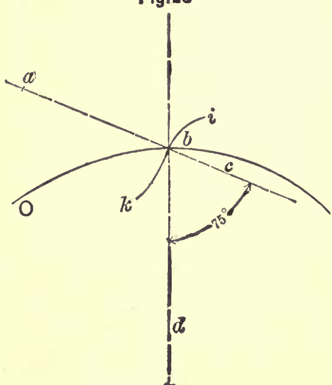
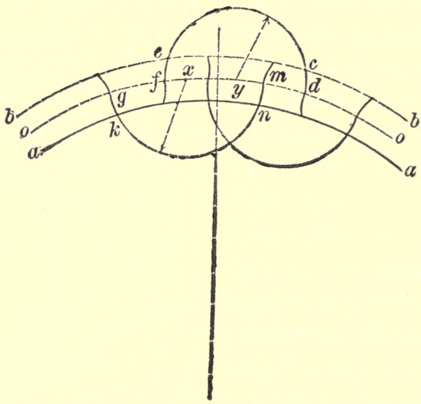


Fig. 28



draw *ac*, making 75° with the line of centres *bd*, and make *bc* equal to one-tenth the pitch of the teeth multiplied by the cube root of the number of teeth. Take $ab = 3bc$: *c* is the centre for the face *bi*, and *a* the centre for the flank *bk*. The following values of *ba* and *bc* give better results: $ba = \frac{p(N+6)}{2N-20}$, and $bc = 0.12p\sqrt[3]{N}$, in which *p* represents the pitch, and *N* the number of teeth. In Fig. 29, *oo* is the pitch circle, and *bb* and *aa* the circles which limit the teeth at top and bottom.

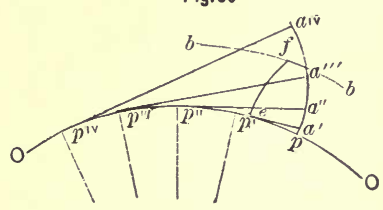
Fig. 29



The centres for both faces and flanks are taken upon the pitch circle; the flank centre for *gk* and *mn* being in the centre of the tooth width at *x*, and the face centre for *cd* and *ef* being in the centre of the space width at *y*. Still another rough rule is to take the centres upon the pitch circle, and take the radius for the faces equal to one and one-fourth times the pitch, making the flanks radial straight lines.

For laying out involute teeth, the exact method is as follows: Fig. 30, O is the circle of the bottoms of the teeth, and p the starting-point of the involute, or the root of the tooth. Lay off the distances pp', $p'p''$, $p''p'''$, etc., along the circle OO; draw the tangents $p'a'$, $p''a''$, etc.; and step off $p'a' =$ arc $p'p$, $p''a'' =$ arc $p''p$, $p'''a''' = p'''p$, etc. The curve $a'a''a'''$, etc., drawn through the points thus found, is the true involute profile. In the same manner, the profile ef is found, and the tooth limited in height by the circle bb. Radial straight flanks are often used in involute teeth; but, for reasons already given, they should never be used except for large wheels, and even then only for rough work. True involute profiles may be easily traced by means of a straight spring arranged to hold a pencil, or other marker, at one end, and fastened at the other end to the circumference of

Fig. 30



a wooden circle-segment of the same radius as the *bottom* or *root circle* of the teeth of the wheel. Because of the comparative ease with which true involute profiles may be traced, approximate or circle arc methods are not much in use. The following methods, however, give very close approximations to the true curve, and are, perhaps, more in use than any others. In Fig. 31 ei is the *working height* of the tooth, i.e., the actual height less the clearance between the end of the tooth of one wheel and the bottom of the corresponding space of the other wheel, and im is the actual height. Make

$ea = \frac{2}{3}ei$, and draw *ad* tangent to the circle *A*; make $pd = \frac{1}{4}ad$, and *p* is the centre for the profile *bak*. A circle through *p*, concentric with the circle *A*, gives the positions of the centres for all the profiles. The part *kc* may be a straight line tangent to *bak* at *k*, since the profile which engages with *bak* does not touch this part at all. It is better, however, to round this part, as in *kf*, for greater strength and better casting. Let *O* (Fig. 32) be the pitch circle, *c* and *d* the circles limiting the tooth at top and bottom (*top circle* and *root circle*), and *a* the pitch point. Draw the straight line *ap* through the pitch point, and making angles of 75° with the line of centres; draw *fp* through the centre *f*, and perpendicular to *ap*; and *p* is the centre for the profile shown in the figure. For small teeth, the centres are often taken on the pitch circle, and the radius taken equal to the pitch of the teeth.

Fig. 31

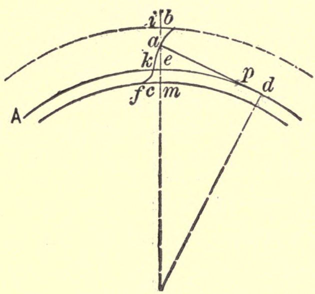
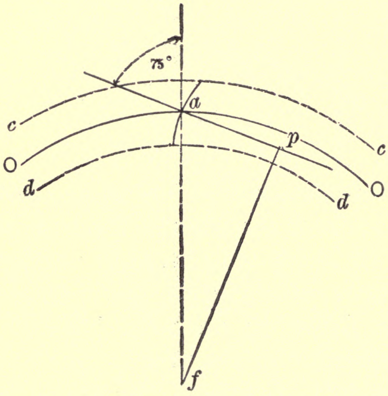


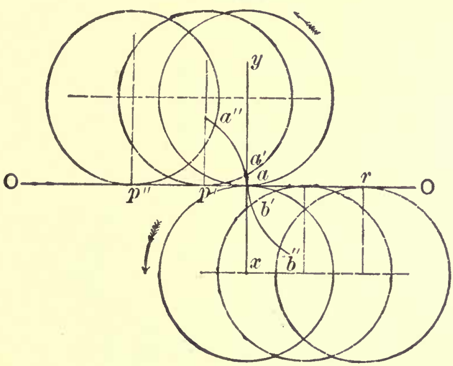
Fig. 32



§ V. — *Rack.* — *Internal Gears.*

If, in a pair of gear-wheels, we assume the radius of one of the pitch circles to be infinitely long, this pitch circle becomes a straight line tangent to the other pitch circle at the pitch point, and the wheel becomes a *rack.* The rolling circles which generate the tooth-profiles for the rack now roll along a straight line instead of upon and within the circumference of a circle, and consequently the faces and flanks of the teeth are no longer epicycloids and hypocycloids, but both are ordinary *cycloids.* Fig. 33 represents one of the exact methods for tracing the teeth. *OO* is the pitch circle,

Fig. 33



and *a* the pitch point. The generating circles roll in the directions indicated by the arrows, and the points a', a'', b', etc., are found as in Fig. 23; the arc $p'a'$ being equal to $p'a$, $p''a'' = p''a$, $rb'' = ra$, etc. The approximate methods for cycloidal teeth, explained in the preceding paragraph, are applicable to the rack. Some of these we give as examples.

Method 4 (*approximate*).—Let A (Fig. 34) be the pitch circle, B and C the rolling circles, and t the pitch point. Let also tl and tk be the heights of the tooth above and below the pitch circle. Take $tn = \frac{2}{3}tl$, and draw nn', cutting the rolling circle C in the point n'. Step off $to =$ arc tn', and from o as a centre, with the chord tn' as a radius, strike an arc cutting nn' in the point p. Draw po, and on it find the centre o' for an arc of a circle passing through the points t and p. It is obvious that the curvature of the flank will be the same as that of the face. Therefore, to find the flank centre, make $o''o''' = o'''o'$, draw $o''b$ parallel to the pitch circle A, and make $ab = xo'$: b is the flank centre. The centres for all the flanks will be on the line $o''b$, and all the face centres will be on the line $o'm$, drawn through the points o' and b, and parallel to the pitch circle A.

Fig.34

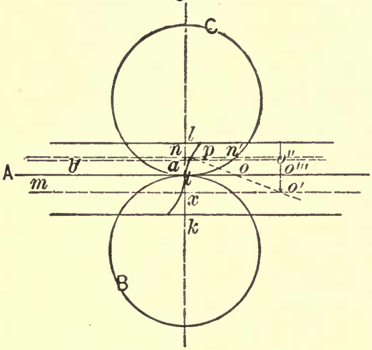
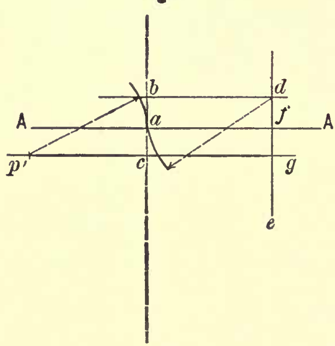
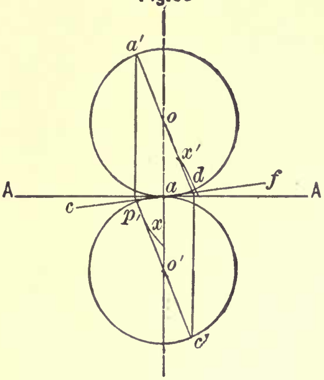


Fig.35



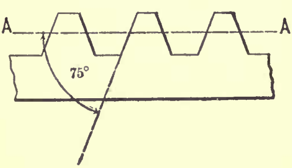
Method 5 (*approximate*).—Fig. 53, A is the pitch circle, and a the pitch point. Take $af = 0.57$ the diameter of the rolling circle, and through f draw dfe parallel to the line of centres. Take $ab = ac = \frac{1}{8}af$, and draw bd and $p'g$ parallel to AA: d is the flank centre. Make $cp' = cg$, and p' is the face centre. Method 6 of the preceding paragraph is greatly simplified when applied to the rack. The lines Odf and Oga' (Fig. 27) become parallel to the line of centres, $c'd$ and $a'p'$ (Fig. 36), and intersect the line cf in the points d and p', where this line meets the 30-degree lines,* giving these points as centres for the profiles. Hence this method, when used for rack teeth, reduces to the following: AA is the pitch circle, a the pitch point, and o and o' the rolling circles. Through the centres o and o' draw the lines $a'od$ and $c'o'p'$, each making angles of 30° with the line of centres. The points d and p', in which these lines meet the circumferences of the rolling circles, are the centres respectively for the flank $p'x$, and face dx'.

Fig. 36



When involute teeth are used for a rack, the profiles reduce to straight lines, making angles of 75° with the pitch circle (Fig. 37). This may be very prettily

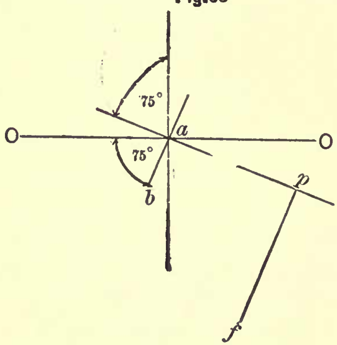
Fig. 37



* This is true only when the rolling circles are equal.

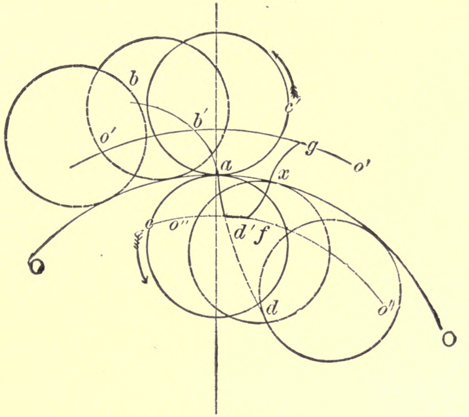
demonstrated by means of the approximate method of Fig. 32 in the preceding paragraph, as follows: Let *OO* (Fig. 38) be the pitch circle, *a* the pitch point, and *ap* the 75-degree line. Since the centre of the pitch circle is infinitely distant from the pitch point, the perpendicular *pf*, which passes through this centre, will also be infinitely distant from the pitch point. The radius *ap* of the profile will therefore be infinitely great, and the profile a straight line perpendicular to this radius, and passing through the pitch point. But since the line *ap* makes angles of 75° with the line of centres, the perpendicular *ab* will make angles of 75° with the pitch circle, which is perpendicular to the line of centres.

Fig. 38



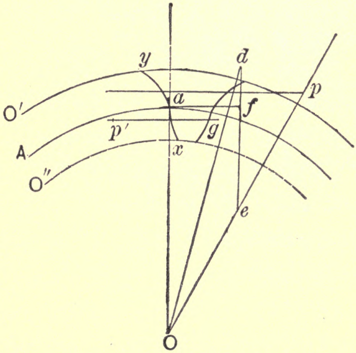
In *internal gears*, the curves forming the faces and flanks of the teeth are reversed as compared with external gears; that is, the faces are hypocycloids, and the flanks epicycloids. The exact method for constructing internal cycloidal teeth is shown in Fig. 39. *O* is the pitch circle, *a* the pitch point, *c'* and *c* the rolling circles, and *o''* and *o'* the top and root circles. Find the profile *bad* by rolling the circles, as in Fig. 23; find, in similar manner, the profile *gxf* (*ax* being the given width of the tooth at the pitch circle), and the tooth *b'ad'fxg* is complete. The approximate methods given for external cycloidal teeth are applicable, without change or difference, to internal gears, remembering, however, that the faces are hypocycloidal, and the flanks epicycloidal curves. The

Fig. 39



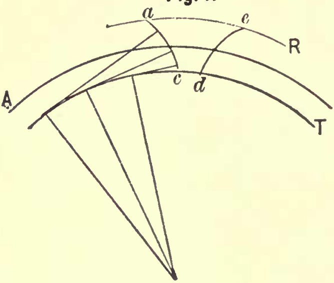
following, for example, is method 5, of the preceding paragraph applied to internal gear-teeth. The centres p and p' (Fig. 40), for the profiles ax and ay respectively, are found as before explained (see Fig. 26), the curves drawn, and the tooth limited by the top and root circles O'' and O'.

Fig. 40



In generating involute teeth for internal gears, the primitive circle, upon which the generating line rolls, or from which the string unwinds, may be taken the same as the top circle of the teeth with very good results. Thus in Fig. 41, for the exact method, *A* is the pitch circle, *T* and *R* the top and root circles. Find the profile *ca*, as before explained. (See the preceding section, Fig. 30.) In a similar manner find the profile *de*, and the tooth is complete. The approximate methods for external involute teeth may be used without change for internal gears. Internal gears were formerly quite extensively used; but of late years they have come to be considered as clumsy contrivances, and are rarely used except in special mechanisms.

Fig. 41

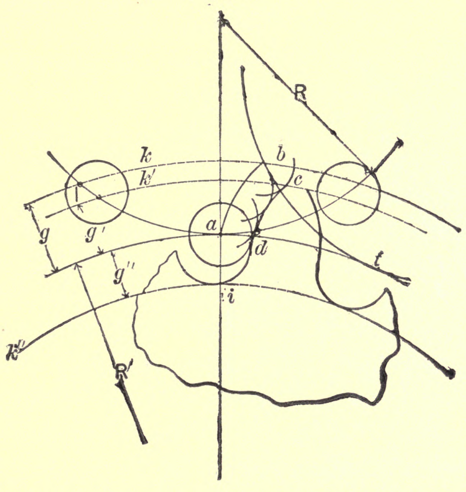


§ VI. — *Special Forms. — Lantern-Gears. — Mixed Gears.*

This paragraph has been translated from the French edition of Professor Reuleaux's valuable work, "Le Constructeur." Straight lines are often used for the profiles of the teeth of gear-wheels, the straight line forming the flank of the tooth, and a curve the face. But teeth obtained thus do not gear together with the necessary exactness, and for this reason ought not to be used in the construction of ordinary machinery. In the teeth of clock-work gears, this kind of profile can be advantageously used; because it permits, at the same time, of the easy cutting-out of the spaces with a file, and of the use of a small number of teeth. If we take the diameter of the generating circle greater than a certain fraction of the radius of the corresponding primitive circle, we obtain teeth which are still of a possible execution, but which, in practice, are admissible only for particular cases. If we take, for the generating circle, the pitch circle of one of the wheels, we obtain, for the profiles of the teeth of the wheel corresponding to the pitch circle upon which it rolls, epicycloidal arcs, while for the other wheel the profiles are reduced to points. It is in this kind of profile that we include *lantern-gears*.

External Lantern Gears (Fig. 42). — From the pitch

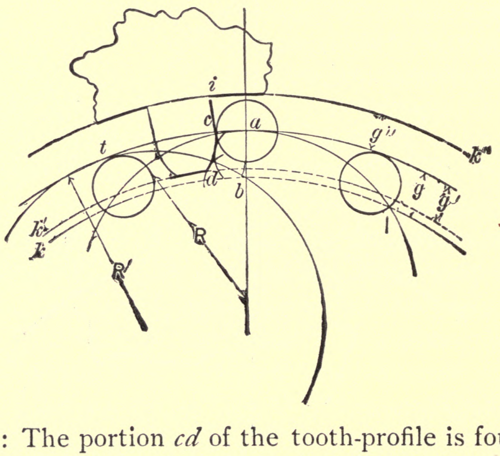
Fig. 42



point a describe a circle having a radius equal to $\frac{19}{80}$ the pitch. This gives the profile of the rung, or spindle, corresponding to the point a. The face of the tooth of the wheel R' is formed by a curve parallel to or equidistant from the epicycloidal arc ab, generated by the point a in the rolling of the circle R upon R' (the arc tb = the arc ta). The envelope of circles described from different points of *ab*, with a radius equal to that of the rung, gives the face profile *cd*: the flank *di* is a circle quadrant. The arc of contact coincides with the circle *R*: its length *al*, of which the limit *l* is determined by the top circle *k*, ought to be greater than the pitch, and hence at least 1.1 times the pitch. This last value serves to determine the height *g* and the real height *g′* of the face.

Internal Lantern-Gears (Fig. 43). — The following manner of proceeding is similar to the one just de-

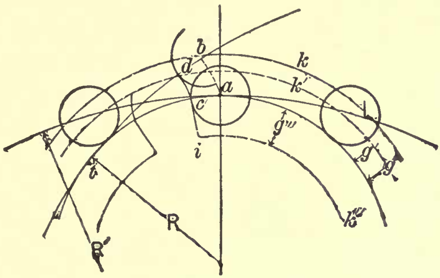
Fig. 43



scribed: The portion *cd* of the tooth-profile is found by a curve parallel to the hypocycloidal arc *ab*, generated by the point *a* in the rolling of the circle *R* within the circle *R′* (the arc *tb* = the arc *ta*). The arc of contact *al* ought to be taken at least equal to 1.1 times the pitch. The flank *ci* is a radial straight line connected with the rim of the wheel by a small circle arc.

In Fig. 44 the hollow wheel is the lantern: the face *cd* is parallel to the pericycloidal arc *ab*, generated by the point *a* in the rolling of the circle R' upon R (the

Fig. 44



arc $tb =$ the arc ta). The arc of contact aI ought to be at least 1.1 times the pitch: the flank *ci* is a radial straight line connected with the rim by a small circle arc.

Fig. 45

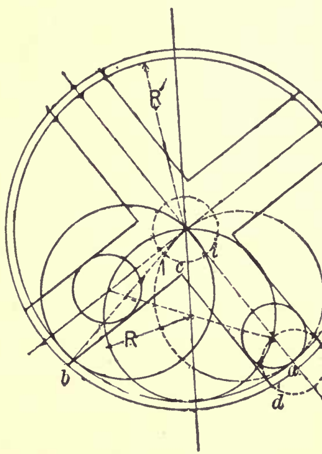
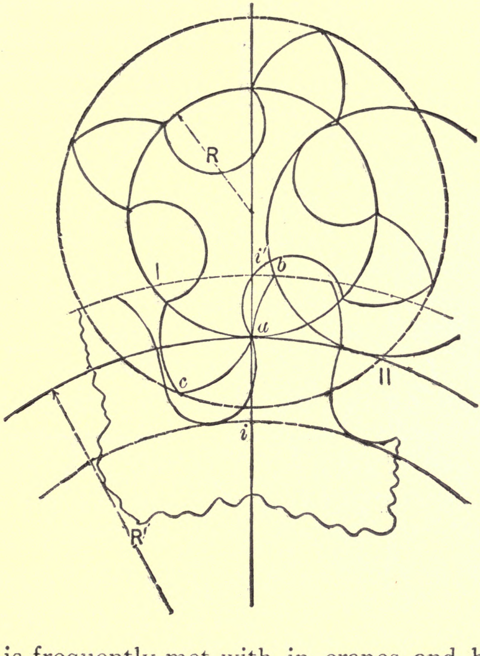


Fig. 45 represents a particular case of Fig. 43. We have $R = \frac{1}{2}R'$, and consequently the number of teeth in R $= \frac{1}{2}$ the number of teeth in R' ($N = \frac{1}{2}N'$). In this case, $N = 2$, and $N' = 4$. The profile *cd* is parallel to the straight line *ai*, to which the hypocycloid reduces (the arc $ab =$ the arc bi): aI is the arc of contact. This arc is here necessarily greater than the pitch: since, however, the straight form of the flanks of the teeth of the wheel R' permits the suppression of all play between the teeth, so that the same rung gears at the same time with two opposite flanks, the arc of contact may be considered equal to twice aI. Many writers regard this kind of gear as a special mechanism, since in actual practice the rungs are movable rollers provided with axles. If in Fig. 43 we consider the radius R' as infinitely long, we obtain the mechanism of the rack, in which the profiles of the teeth upon the rack itself are formed by curves parallel to ordinary cycloids. If, again, in Fig. 44, we consider the radius R' as infinitely long, we obtain a very simple form of rack, which is very often used in preference to the preceding. Upon the pinion the profiles of the teeth are formed by curves parallel to an involute to the pitch circle. Lantern-gears, in cases which require a certain precision and not very frequent use, offer the advantage that the rungs can be easily and exactly described with a pair of compasses. Lantern-racks of wrought iron are very useful in practice for apparatus exposed to cold and wet; such as for lifting gates, drawbridges, etc.

Gear at Two Points (Fig. 46). — If we connect together two gears at a single point, we obtain a new style of gear, which allows us to adopt for one of the wheels a very small number of teeth, and consequently a great difference in the revolutions of the two wheels, even though both wheels are quite small. In the figure the two pitch circles are at the same time the generating circles of the profiles of the teeth: ac is an epicycloidal curve (generated by the rolling of R' upon R), which, for the length of contact aI, gears with the point a of the wheel *R'*; *ab* is a second epicycloidal curve (generated by the rolling of *R* upon *R'*), which, for the length of contact *aII*, gears with the point *a* of the wheel *R*; *ai* and *ai'* are the profiles for the flanks of the teeth for the wheels *R'* and *R*. The small wheel is used frequently for shrouded wheels. This kind

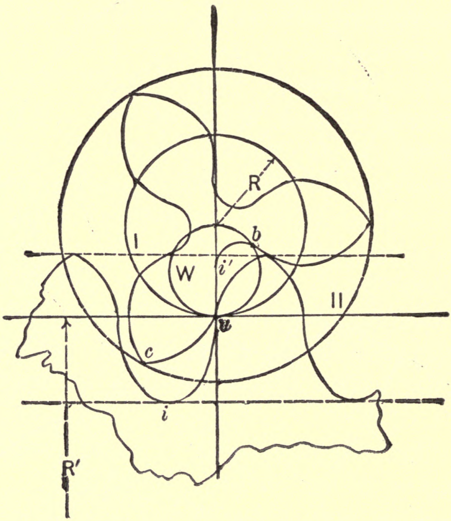
Fig.46



of gear is frequently met with in cranes and hoisting-machines.

Mixed Gear (Fig. 47).—This kind of gear, which is very convenient for the small pinions of hoisting-machines, has the advantage of diminishing the space at the root of the tooth. This result is due to the use of radial straight lines for the flanks of the teeth of the small wheel. In order to obtain a sufficient duration of engagement, it is convenient to use upon both wheels the curves which form the faces of the teeth as far as their points of intersection. In the figure, *ac* is an arc of a cycloid, or involute, generated by the rolling of R′

Fig. 47

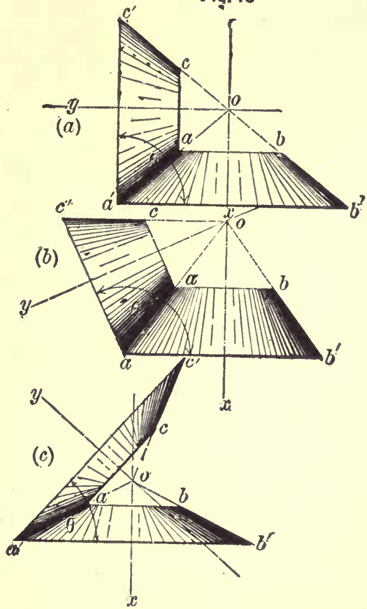


(which here, for a rack, is a straight line) upon *R*: *ai′* is a radial straight line generated by the rolling of the circle *W* upon the inside of *R* (the radius of $W = \frac{1}{2}$ that of *R*). The gearing of the profile *ac* with the point *a* takes place for the length of contact *aII*. The cycloidal arc *ab*, generated by the rolling of *W* upon *R′*, gears with the flank *ai′* for the length of contact *aI*.

§ VII.—*Bevel Gears.*

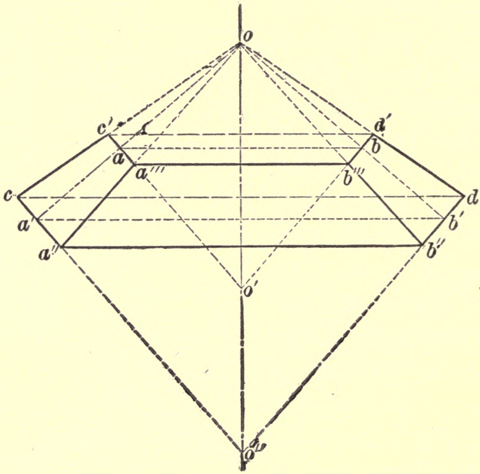
The different gears hitherto described are intended to transmit power from one shaft to another parallel shaft. If we wish to transmit from one shaft to another which is not parallel, or which makes an oblique angle with the first, we must make use of either bevel or screw gears. A bevel or conical gear differs from a cylindrical or spur gear in that its two pitch circles (at the two ends of the teeth) are of different diameters, and consequently the ends of any one tooth are of different heights, widths, etc. The pitch circles of a pair of bevel wheels limit frusta of cones, the apices of which meet at the point of intersection of the axes of the wheels. Thus, in Fig. 48, *o* is the point of intersection of the axes *ox* and *oy*; *a′b′*, *ab*, *a′c′*, and *ac* are the pitch circles; *a′c′ca* and *a′b′ba*, the "pitch frusta;" and *a′c′o* and *a′b′o*, the "pitch cones." The axes may make any angle with each other. It should, however, be remarked that wheels such as are represented in (*c*) are seldom used in practice, since the same angle may be obtained with the wheels shown in (*b*). To lay out

Fig. 48

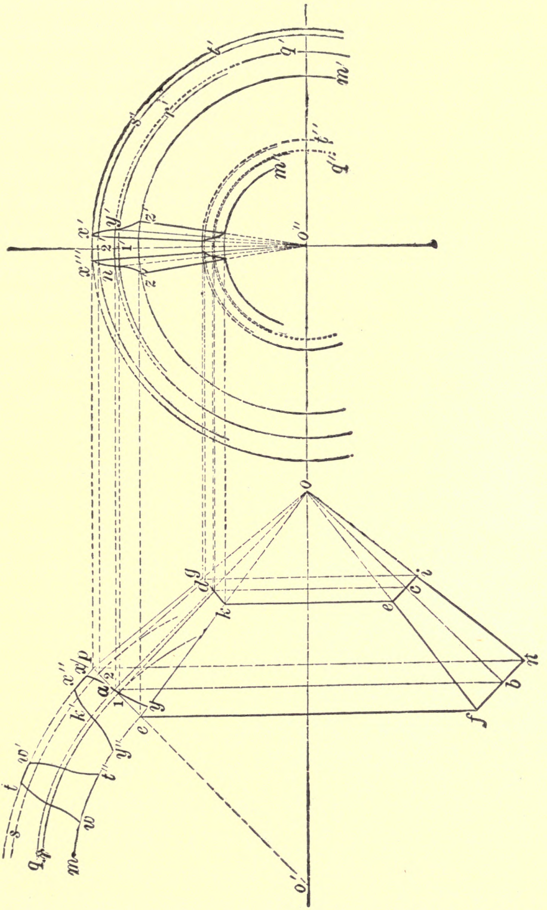


the pitch cones and frusta, we have, then, the following simple rule: Draw *a'c'* and *a'b'* (Fig. 48), making with each other the required angle, and equal respectively to the diameters of the larger pitch circles of the wheels. Draw now the axes *oy* and *ox* perpendicular to their respective pitch circle planes. The point of intersection *o* determines the cones; and the given length *a'a* or *b'b*, the frusta. The ends of bevel teeth lie upon the

Fig. 49



surfaces of cones which are supplementary to the pitch cones, and of which the top and root circles limit frusta. In Fig. 49, *a''cc'a'''* and *b''dd'b'''* represent two teeth; *ab* and *a'b'* are the pitch circles; *a'b'o*, the pitch cone; and *a'b'ba*, the pitch frustum. The root circles are *a''b''* and *a'''b'''*; and the top circles, *cd* and *c'd'*. These circles limit the frusta *cdb''a''* and *c'd'b'''a'''* of the supplementary cones *cdo''* and *c'd'o'*. The larger of the two pitch circles determines the size of the wheel and the tooth dimensions. For instance, a fifteen-inch bevel



of one inch pitch means a bevel of which the larger pitch circle is fifteen inches in diameter, and of which the pitch of the teeth upon this pitch circle is one inch.

The teeth of bevels may be either cycloidal or involute. The latter are, however, more often used, because they are much easier to construct. Fig. 50 shows a convenient method for laying out the teeth of bevels: *abo* is the pitch cone; *pn* and *gi* are the top circles; and *ef* and *kl*, the root circles. Produce *pc* as far as its intersection with the axis of the wheel, and from this point o' as a centre, with $o'a$, $o'p$, and $o'e$ as radii, describe the circle-arcs *q*, *t*, and *m*. These are the *virtual* pitch, top, and root circles. Find by any of the preceding methods the centres for the faces and flanks, regarding *a* as the pitch point: *r* and *s* are the *centre circles* for the faces and flanks respectively. The teeth $xyy''x''$ and $w't''wt$ are correct in size, and are drawn to give the pattern-maker his dimensions. Now project and describe the *actual* pitch, top, and root circles q', t', and m', also the same circles for the small end of the tooth (q'', t'', and m''), and the centre circles r', s', etc. Set off now $x'x''' = xx''$, $y'n = ak'$, and $z'z'' = yy''$, and find upon the centre circles the centres for arcs passing through the points x' and y' (face), and y' and z' (flank). The widths of the small end of the tooth at the pitch, top, and root circles, are determined by the lines $o''y'$, $o''x'$, and $o''z'$, etc., and the faces and flanks drawn as above. Draughtsmen sometimes find the centres for the tooth-profiles upon the actual instead of upon the virtual pitch circle, as we have done; but the height of the teeth upon the actual pitch circle is less than the real height, as a glance at Fig. 50 will show; and consequently the widths upon the top and root circles are respectively too great and too small, thus marring the correctness of the drawing. In Fig. 48 (*a*), the planes of the pitch circles of the two bevels are at right angles with each other (the angle $\theta = 90°$). If, now, we gradually *increase* this angle θ, the wheels take the form of Fig. 48 (*b*); and finally, when the planes become parallel ($\theta = 180°$), become *external* cylindrical or spur gears. If, on the contrary, we gradually *decrease* the angle θ, the bevels take first the form of Fig. 48 (*c*), and finally, when the planes become coincident ($\theta = o$), become *internal* cylindrical gears. Between these latter two cases (Fig. 48 (*c*) and internal cylindrical gears) we have two interesting, if not altogether practicable, cases,—the *internal* bevel and the disk wheel. A pair of bevels, the internal bevel being in section, is represented in Fig. 51, and the "disk wheel" in Fig. 52. The internal bevel, because of the difficulties in the way of its construction, is never used in practice. It may be constructed, however, if desired, by the rules already given for internal cylindrical and bevel gears. The disk wheel is the least

Fig. 51

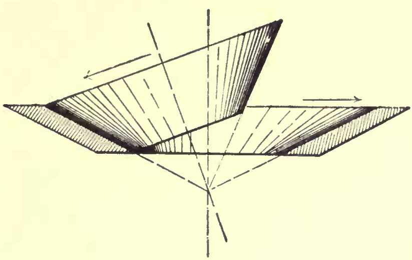
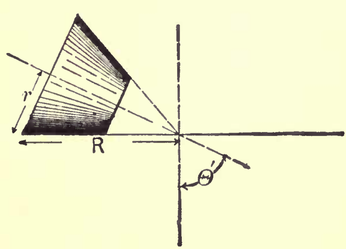


Fig. 52

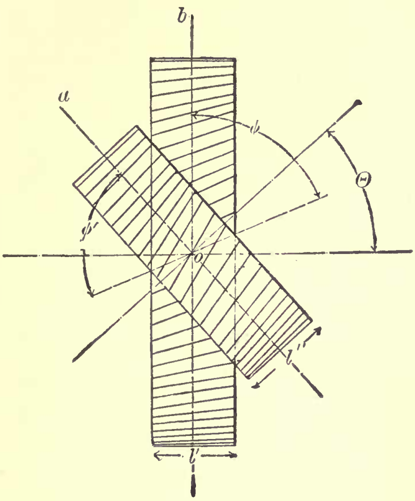


difficult of all bevels to construct, and is, although seldom used, for this reason entitled to a place among practical gears. The disk wheel and pinion possess one peculiarity not found in any other bevel; viz., the ratio of the radius of the pinion to that of the disk wheel depends upon the angle included between the axes of the wheels. If we let r and R be the radii of the pinion and disk wheel, and θ' the angle included between the axes, we shall have the relation $\frac{r}{R} = \cos\ \theta'$. The supplementary cones upon which lie the ends of the teeth become, for the disk wheel, right cylinders having diameters equal to those of the pitch circles, and, when cycloidal teeth are used, the profiles are ordinary cycloids, the disk wheel being regarded as (and is sometimes called) the "bevel rack." In order that a set of bevel gears shall gear together each to each, it is necessary, not only that the pitch and kind of tooth profile be the same, but that the slant height ($b'o$, Fig. 48) of the pitch cones be the same in all the wheels of the set. Practice, however, allows a slight variation from this rule; and, according to Reuleaux, bevels will work sufficiently well together if the difference in the lengths of these slant heights does not exceed five per cent. Such wheels are called *bastard wheels*, and are quite commonly used in cases where there is no necessity for very accurate gear.

§ VIII. — *Screw Gears. — Worm and Wheel.*

Screw gears are cylindrical gears, in which the teeth are not parallel to the axes of the wheels, but make oblique angles with them. All the lines of the teeth, which in spur gears are parallel to the axes of the wheels, are in screw gears parts of helices drawn around the pitch, top, and root circles. Let Fig. 53 represent two screw gears; Θ being the angle included between the axes, and ϕ and ϕ' the angles made by the teeth with the "middle planes" of the wheels, as shown in the figure. It is plain that the angle *aob* is equal to the angle Θ: consequently we have from the figure, $\phi + \phi' + \Theta = 180°$. This condition must be fulfilled, else the wheels will not gear properly together. Another necessary condition in screw gears is, that the pitches of two gears which work together, *taken normal to the directions of the teeth* or the *normal pitches*, must be equal. It is more

Fig. 53



convenient to lay off the pitches on the pitch circles; that is, to lay off the circumferential pitches, instead of the normal. In Fig. 54, *ab* represents the normal pitch, and *ae* the circumferential. The angle *aeb* being equal to ϕ, we have $ae = \frac{ab}{\sin\phi}$, the circumferential pitch equal the normal pitch divided by sin ϕ. In order that the wearing surfaces may be equal, the lengths of the teeth of a pair of screw gears should be equal. The width of face depends upon the length of tooth and the angle ϕ. Thus, in Fig. 54, $l = ec$ being the length of the tooth, and $l' = dc$ the width of face, we have the angle ced = angle ϕ, and consequently $l' = l \sin \phi$. Suppose (Fig. 53), $\Theta = 40°$, and $\phi = 60°$: hence $\phi' + 60° + 40° = 180°$, $\phi' = 80°$. If p and p' represent

Fig. 54

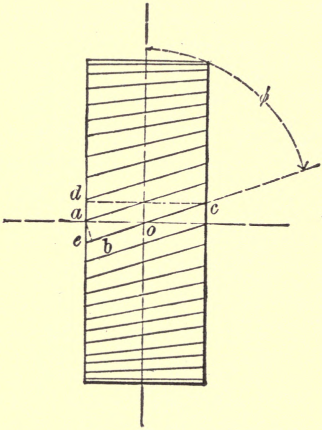
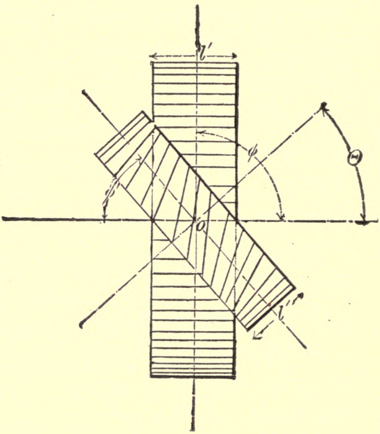


Fig. 55



the circumferential pitches, and n the common normal pitch, we shall have,

$$p = \frac{n}{\sin \phi} = \frac{n}{\sin 60°} = \frac{n}{0.866}$$

and

$$p' = \frac{n}{\sin \phi'} = \frac{n}{\sin 80°} = \frac{n}{0.985}.$$

Also, for the widths of faces of the two wheels, we shall have

$$l' = l \sin \phi = 0.866l, \text{ and } l'' = l \sin \phi' = 0.985l.$$

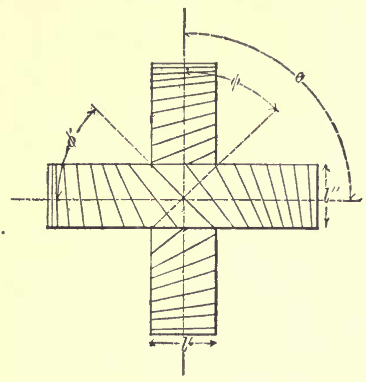
If we make $\phi = 90°$, we will have an ordinary spur-wheel gearing with a screw gear (Fig. 55). In this figure, $\phi = 90°$ and $\Theta = 40°$: hence $\phi' = 180° - (90° + 40°) = 50°$. We therefore have, for the circumferential pitches and widths of faces,

$$p = \frac{n}{\sin 90°} = n, \quad p' = \frac{n}{\sin 50°} = \frac{n}{0.766},$$

$$l' = l \sin 90° = l, \text{ and } l'' = l \sin 50° = 0.766l.$$

Let $\theta = 90°$, that is, the axes are at right angles with each other (Fig. 56): consequently

Fig.56



$$\phi + \phi' = 180° - 90° = 90°.$$

The angles ϕ, ϕ', may be equal or unequal: in the figure they are taken equal.

$$\phi = \phi' = \frac{90°}{2} = 45°.$$

From this,

$$p = p' = \frac{n}{\sin 45°} = \frac{n}{0.707},$$

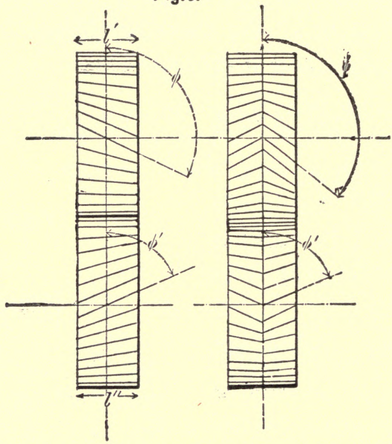
and $l' = l'' = l \sin 45° = 0.707l$.* In Fig. 57 the axes are parallel, or $\theta = o$: hence $\phi + \phi' = 180°$. This signifies that ϕ and ϕ' are supplementary, $\phi = 180° - \phi'$. The inclinations of the teeth across the faces of the wheels are in opposite directions. We have taken $\phi' = 60°$: hence $\phi = 120°$, and we have $p' = \frac{n}{\sin 60°} = \frac{n}{0.866}$.

---

* If we make the angle ϕ less than the angle of repose, which, for cast-iron on cast-iron, is about 10°, only the wheel l'' can be the driver: the wheel l' then restrains motion in the direction opposite to that in which it is driven.

Since the sin. of an angle equals the sin. of its supplement, $p = p'$ and $l' = l'' = l \sin \phi = l \sin \phi' = 0.866$

Fig. 57



Screw Rack and Pinion. — If we make the radius of one of a pair of screw gears infinitely long ($= \infty$), the

Fig. 58

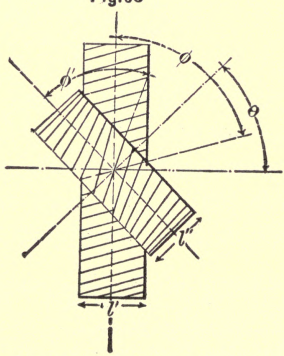
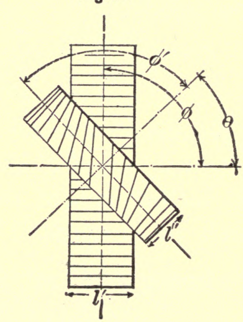


Fig. 59



wheel becomes a *screw rack*, and the pair constitutes a *screw rack and pinion*, shown in Fig. 58. Let $\theta = 45°$, and $\phi = 75°$: hence $\phi' = 180° - (45° + 75°) = 60°$, $p = \frac{n}{\sin\,\phi} = \frac{n}{0.966}$, $p' = \frac{n}{\sin\,\phi'} = \frac{n}{0.866}$, $l' = l \sin\,\phi = 0.966l$, and $l'' = l \sin\,\phi' = 0.866l$.

Fig. 59 represents a spur rack gearing with a screw pinion, $\theta = 45°$, $\phi = 90°$: hence $\phi' = 45°$, $p = \frac{n}{\sin\,\phi} = n$, $p' = \frac{n}{\sin\,\phi'} = \frac{n}{0.707}$, $l' = l \sin\,\phi = l$, and $l'' = l \sin\,\phi' = 0.707l$.

Fig. 60

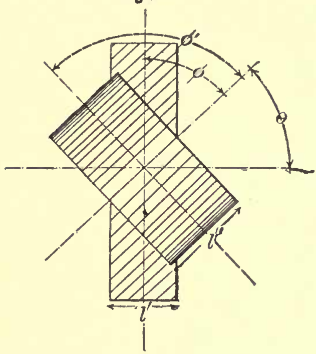
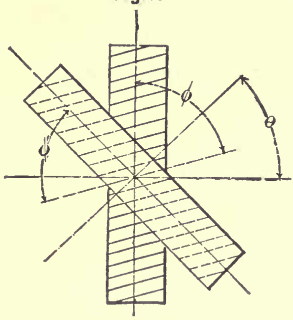


Fig. 61



We may also have a screw rack gearing with a spur pinion, by making $\phi' = 90°$ (Fig. 60). Let $\theta = 45°$, and $\phi' = 90°$: hence $\phi = 45°$, $p = \frac{n}{\sin\,\phi} = \frac{n}{0.707}$, $p' = \frac{n}{\sin\,\phi'} = n$, $l' = l \sin\,\phi = 0.707l$, and $l'' = l \sin\,\phi' = l$.

If we make the radii of both wheels of a pair of screw gears equal to infinity, the pair becomes two screw racks gearing together (Fig. 61); and if we make ϕ or $\phi' = 90°$, we have a spur-rack gearing with a screw rack (Fig. 62).

To draw the tooth profiles for a screw gear we proceed as follows: Having determined the angle ϕ of the teeth, and the length l, draw the horizontal line xy (Fig. 63). Draw db, making the angle ϕ with xy, and make it equal in length to l. Drop the lines dc and bc perpendicular respectively to xy and dc. The line bc is the length of the tooth projected in the plane of the pitch circle P. Strike, now, the pitch, top, and root circles, P, t, and r, and make $ab' = bc$ (a being the pitch point).

Fig. 62

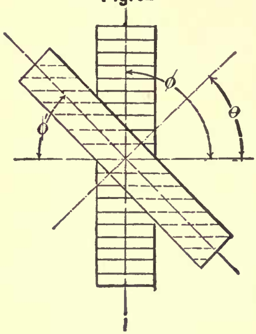
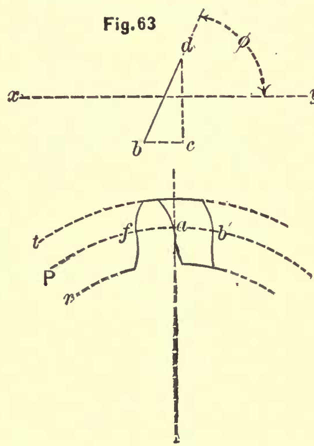


Fig. 63

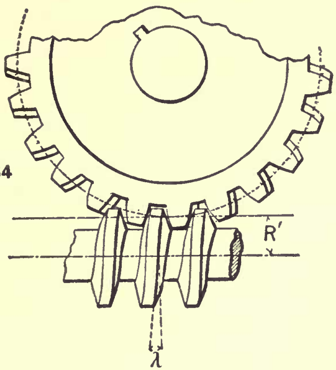


Find the centres for faces and flanks, as in spur gears, and draw the profiles through a, b', and f. In constructing screw gears, it is advantageous to make the angles of the teeth equal ($\phi = \phi'$). The circumferential pitches, and tooth dimensions in the planes of the pitch circles, as also the face widths, will then be equal, thus saving calculation and extra work. The friction between the teeth is also more evenly distributed by this means.

The motion between two well-constructed screw gears is very regular and uniform. They are therefore useful in cases where uniformity of motion is requisite; but, owing to the friction between the teeth, these gears are not very durable, and should be used for the transmission of small powers only, and at comparatively slow motion.

Worm and Wheel.—The mechanism known as the worm and wheel, or the worm and worm-wheel, is a modification of screw gears with axes at right angles, the principal object of which is to obtain conveniently a great difference in the revolutions of two shafts. The worm is an endless screw, and the worm-wheel a screw gear (Fig. 64). It is evident from the figure, that (the worm being the driver), at each revolution of the worm, the wheel will be moved through a distance equal to one tooth. Hence, if the wheel has thirty teeth, the worm will make thirty revolutions while the wheel makes one, or the worm-shaft will revolve thirty times as fast as the wheel-shaft. The common angle λ of the teeth is usually taken such that the worm will drive the wheel, while the wheel will not drive the worm; so that, if at any time the driving-power is taken off, the gearing will remain stationary. For this purpose, the angle λ may be taken from $4\frac{1}{2}°$ to $9°$. If, however, the worm-wheel

Fig. 64



is to be the driver, λ must be taken greater than 10°. The pitch radius R' of the worm may be from one to two times the circumferential pitch.* The tooth-profiles of the worm and wheel may be either cycloidal or involute; and, in either case, those of the worm are drawn as for a rack, and those of the wheel as for a screw gear.

Involute profiles are particularly useful in worms, because the worm is, at best, difficult to construct, and the straight 75° profiles of the involute rack very much facilitate the construction. If we make the radius of the worm-wheel infinitely long, the wheel becomes a screw rack, and the mechanism becomes a worm and screw rack (Fig. 65). We may also have a worm and internal worm-wheel (Fig. 66), or an internal worm and worm-wheel. In either of these cases the profiles are drawn as explained in sections IV. and VIII. As in screw gears, by placing the axes at oblique angles, we may have a worm gearing with an ordinary spur wheel, a

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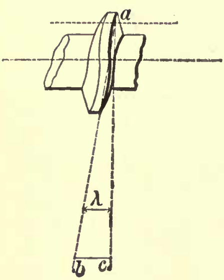
Fig. 64 a



* If we develop in the straight line ac (Fig. 64 a) the circumference of the pitch circle, and in the straight line ab the length of one revolution of the screw, we shall have bc = the pitch $= p$, and ac = the circumference $= 2\pi R'$: hence

$$\tan \lambda = \frac{p}{2\pi R'} = 0.159 \frac{p}{R'}.$$

This condition must be fulfilled: hence, if we make $R' = 2p$, $\frac{p}{R'} = \frac{1}{2}$, $\tan \lambda = 0.159 \times \frac{1}{2} = 0.0795$, $\lambda = 4° 33'$. If $R' = p$, $\frac{p}{R'} = 1$, and $\tan \lambda = 0.159$, $\lambda = 9° 2'$. Inversely, if $\lambda = 12°$,

$$\tan \lambda = 0.213 = 0.159 \frac{p}{R'}, \quad \frac{p}{R'} = 1.34, \quad R' = \frac{3}{4} p.$$

spur rack, or an internal spur wheel. It must, however, not be forgotten that the pitch of the spur gear must be taken equal to the pitch of the worm multiplied by cos λ. In order to obtain more bearing surface between the teeth of the worm and those of the wheel, the bottoms of the spaces in the wheel are sometimes cast in the form of circle-arcs, to fit the threads or teeth of the worm (the radius of curvature equals radius of ends of worm-teeth plus the clearance), and the ends of the wheel-teeth formed to fit the bottoms of the spaces in the worm (radius of curvature equals radius of bottoms of worm-spaces plus the clearance), as shown

Fig. 65

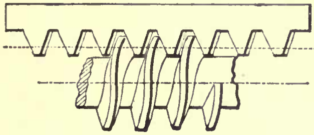


Fig. 66

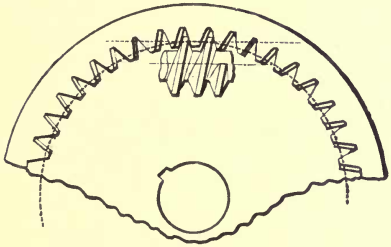
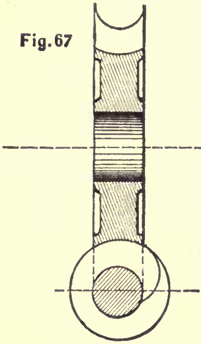


Fig. 67



in Fig. 67. The figure gives a section through the centre of the wheel, showing two teeth entire and an end view of the worm. As with plain screw gears, so with the worm and wheel, the wear is excessive; and, for this reason, only comparatively small powers can be advantageously transmitted by this mechanism. In cases, however, where the gears are not in motion continuously, as in hoisting-machines, cranes, some

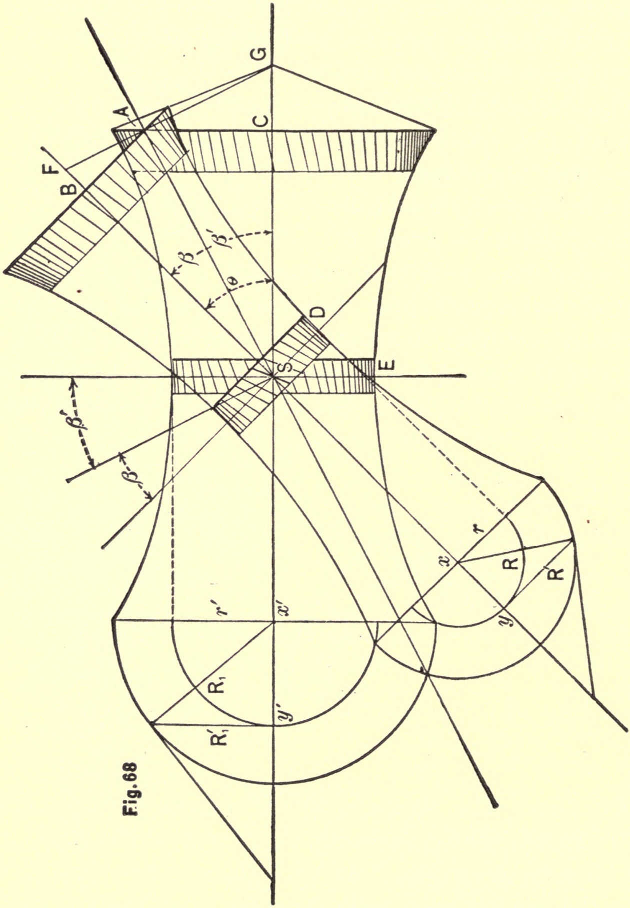


Fig. 68

machine tools, etc., worms may be used for the transmission of considerable powers.

§ IX. — *Hyperbolic Gears.**

Hyperbolic, or, more properly, hyperboloidal, gears are intended to be fixed upon arbors, the axes of which cross, without intersecting each other. Their primitive surfaces (surfaces limited by the primitive or pitch circles) are hyperboloids of revolution, which touch along a common generatrix. This generatrix may be determined as follows: —

In Fig. 68, which is a projection made normally to the shortest distance between the axes, let us divide the angle of inclination θ of the axes into two other angles, β and β', in such a manner that the perpendiculars AB and AC, drawn to some point A of the line of division SA, shall be inversely proportional to the numbers of revolutions of the wheels, i.e., directly proportional to the diameters. SA is, then, the generatrix of contact of the two hyperboloids. $AB = R'$ and $AC = R_1'$ represent the projections of the radii of two normal sections through the point A, and we have,

$$\frac{R'}{R_1'} = \frac{\sin\beta}{\sin\beta'} = \frac{n'}{n} = \frac{N}{N'},$$

n and n' being the numbers of revolutions, and N and N' the numbers of teeth, of the wheels. The real radii, R and R_1, are still to be determined, as also are the radii $SD = r$ and $SE = r'$. Between these last we have the relation,

$$\frac{r}{r'} = \frac{\tan\beta}{\tan\beta'} = \frac{\frac{n'}{n} + \cos\theta}{\frac{n}{n'} + \cos\theta}.$$

---

* From Le Constructeur.

That is, r and r' are in the same relation to each other as are the two segments AF and AG, which are determined by the projections of the axes upon the right line FG, drawn through the point A perpendicular to the generatrix of contact. Representing by a the shortest distance between the axes, we have,

$$\frac{r}{a} = \frac{1 + \frac{n'}{n}\cos\theta}{1 + 2\frac{n}{n'}\cos\theta + \left(\frac{n}{n'}\right)^2} \quad \text{and} \quad \frac{r'}{a} = \frac{1 + \frac{n'}{n}\cos\theta}{1 + 2\frac{n'}{n}\cos\theta + \left(\frac{n'}{n}\right)^2}.$$

The radii R and R_1 are the hypothenuses of right-angled triangles, of which the sides are respectively R' and $xy = r$, R_1' and $x'y' = r'$, and consequently have the values,

$$R = \sqrt{R'^2 + r^2} \quad \text{and} \quad R_1 = \sqrt{R_1'^2 + r'^2}.$$

R' and R_1' are known from what precedes when we have given the length $SA = l$. The angles β and β' are determined by the relation

$$\tan\beta = \frac{\sin\theta}{\frac{n}{n'} + \cos\theta} \quad \text{and} \quad \tan\beta' = \frac{\sin\theta}{\frac{n'}{n} + \cos\theta}.$$

As in bevel gearing, the problem permits of two solutions, according as the line SA is drawn within the angle θ, or within the supplementary angle BSC' (Fig. 69). These two solutions differ from each other in the direction of rotation of the driven arbor. One of these solutions leads to an internal gear, as in bevel gears; but this, to our knowledge, has never been actually constructed, and it cannot possibly have any practical value. When the angle of inclination, θ, is made equal to 90°, we have,

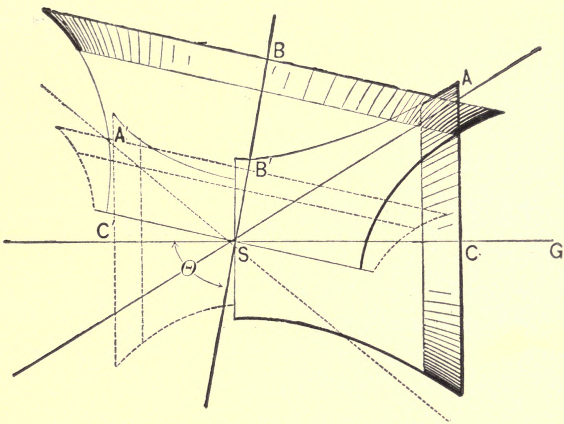
$$\frac{r}{r'} = \tan^2 \beta = \left(\frac{n'}{n}\right)^2$$

and

$$\frac{r}{a} = \frac{n'^2}{n^2 + n'^2}, \quad \frac{r'}{a} = \frac{n^2}{n^2 + n'^2}.$$

It is easily seen, from what precedes, that hyperbolic gears present a more limited number of solutions than

Fig. 69



ordinary screw gears, with which, however, they present many analogies. In the latter, for one value of the angle of inclination of the axes, we can give an arbitrary value to the angle of inclination of the teeth of one of the wheels; while in hyperbolic gears there is only one pair of values admissible for the angles of inclination.

The primitive surfaces of two hyperbolic gears are formed by corresponding zones of two hyperboloids of revolution. When the distance (shortest) between the axes is small, the zones comprising the *circles of the gorge*, of which r and r' (Fig. 68) are radii, cannot be utilized as primitive surfaces, and we must have recourse to zones somewhat removed from these circles. These may ordinarily be replaced by simple frusta of cones, and the construction thus rendered comparatively simple. The following examples will serve to illustrate the preceding formulas and remarks: —

Example 1. $\theta = 40°$, $\frac{n'}{n} = \frac{1}{2}$, $a = 4''$. From the formula $\frac{R'}{R_1'} = \frac{n'}{n}$ we have $\frac{R'}{R_1'} = \frac{1}{2} = 0.5$;

also we have

$$\frac{r}{r'} = \frac{0.5 + \cos 40°}{2 + \cos 40°} = \frac{1.266}{2.766} = 0.4577$$

$$\frac{r}{a} = \frac{1 + 2\cos 40°}{1 + 2 \times 2 \cos 40° + 4} = \frac{2.532}{8.064} = 0.31398$$

$$r = 1.2559'', \quad r' = 2.744''.$$

For the angles β and β' we have

$$\tan \beta = \frac{\sin 40°}{2 + \cos 40°} = \frac{0.6428}{2.766} = 0.232393$$

or $\beta = 13° \ 5'$, and $\beta' = 40° - \beta = 26° \ 55'$. For the distance $SA = l = 8''$ we have

$$R' = l \sin 13° \ 5' = 8 \times 0.226368 = 1.81''$$

$$R_1' = 8 \times \sin 26° \ 55' = 8 \times 0.452634 = 3.62''.$$

Finally,

$$R = \sqrt{1.81^2 + 1.26^2} = 2.20''$$

and

$$R_1 = \sqrt{3.62^2 + 2.74^2} = 4.54''.$$

Example 2. $\theta = 90°$, $\frac{n'}{n} = \frac{9}{5}$ (a value which will be satisfied by the numbers of teeth $N = 36$ and $N' = 20$), and $a = 0.8''$. From the preceding formulas we have

$$\frac{r}{r'} = \left(\frac{9}{5}\right)^2 = \frac{81}{25} = 3.24$$

$$r = a\frac{9^2}{5^2 + 9^2} = \frac{0.8 \times 81}{106} = \frac{64.8}{106} = 0.61''$$

and

$$r' = 0.186''.$$

We have also $\tan \beta = \frac{n'}{n} = 1.80$, or $\beta = 60° \ 57'$, and consequently $\beta' = 29° \ 3'$. For $R = 2''$ we have the formula

$$R' = \sqrt{R^2 - r^2} = \sqrt{2^2 - 0.61^2} = 1.90''$$

and

$$R_1' = \frac{5}{9}R' = \frac{5 \times 1.90}{9} = 1.06''.$$

Also for R_1 we have

$$R_1 = \sqrt{1.06^2 + 0.189^2} = 1.08''.$$

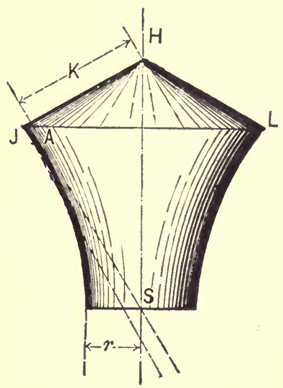
Example 3. $\theta = 90°$, $\frac{n'}{n} = 1$. As before, $\tan\beta = \frac{n'}{n} = 1$, or $\beta = 45°$, $\frac{r}{r'} = \left(\frac{n'}{n}\right)^2 = 1$, or $r = r'$. Also $R = R_1$, and the hyperboloids are congruent.

Example 4. In the particular case where the relation $\frac{n'}{n}$ is numerically equal to $\cos\theta$, and the line of division which determines the angle β is situated within the supplementary angle of θ in such a manner, that, taking into consideration the sign, we have $\frac{n'}{n} = -\cos\theta$, one of the primitive surfaces reduces to a cone, and the other to a hyperboloidal plane. This hyperbolic plane (or disk) wheel corresponds to the disk wheel in bevel gears, and can be made to gear with an ordinary bevel wheel. It offers, however, no practical advantage, since the disk wheel interferes with the prolongation of the arbor of the bevel. For $\theta = 60°$, $\frac{n'}{n} = -\frac{1}{2} = -\cos 60°$, we obtain the disk wheel, and have $\tan\beta = \frac{1}{3}\sqrt{3}$, $\beta = 30°$, $\tan\beta' = \infty$, $\beta' = 90°$, $\frac{R'}{R_1'} = \frac{\sin 30°}{\sin 90°} = \frac{1}{2}$; $r = 0$, $r' = a$, $R = R'$, $R_1 = \sqrt{R_1'^2 + a^2} = \sqrt{4R^2 + a^2}$. If $\frac{n'}{n}$ were negative, and less than $\cos\theta$, we would obtain a hyperbolic internal gear; but gears of this kind are not at all practical.

With hyperbolic gears we may obtain, as a limiting case, the mechanism of a rack and pinion. The rack, in this case, carries oblique teeth; while the pinion is formed by the zone corresponding to the circle of the gorge of a hyperboloid of revolution. But since the construction of this pinion is much more difficult than that of a screw gear, the effect of which is equivalent, it results that the latter should be used in all cases where this effect is to be produced.

Teeth of Hyperbolic Gears. — If we wish to give to the teeth of hyperbolic gears perfectly accurate forms, we meet with very serious difficulties in the execution. We may, however, content ourselves with approximate forms. In this case, to determine the teeth of a hyperbolic gear, we begin by tracing the supplementary cone of the hyperboloidal zone, which is to be used as the primitive surface. The apex H of this cone (Fig. 70) is obtained by drawing a perpendicular AH to the generatrix SA, parallel to the plane of the figure. We then determine the profiles of the teeth for the normal pitch p_n upon the circle of the gorge as if it was acted upon by a screw-wheel having a diameter r, and an inclination of teeth $90° - \beta$; then we continue the profiles thus obtained upon the conical surface HJL, taking care to increase the dimensions parallel to the circle of division in the proportion of p to p_n (p being the circumferential pitch), and the lengths in the proportion of K to r, K representing the length of the generatrix of the supplementary cone. We repeat the same construction

Fig.70



for the supplementary cone corresponding to the other base of the zone, being careful to decrease the values of p and K. Thus we obtain for each tooth two profiles, sufficiently exact, of which the corresponding points must be joined by straight lines to form the body of each tooth.

In certain cases a cone frustum may be substituted for the hyperboloidal zone, upon the condition of properly determining the apex. To this effect, we revolve the generatrix SA about the axis HS until the point A becomes coincident with the point J: the projection of the generatrix, in this position, determines by its intersection with HS the desired apex of the cone.

§ X.—*Relations between Diameter, Circumference, Pitch, Number of Teeth, etc.—Diametral Pitch.—Methods for stepping off the Pitch.*

The circumference of a circle is expressed by the formula

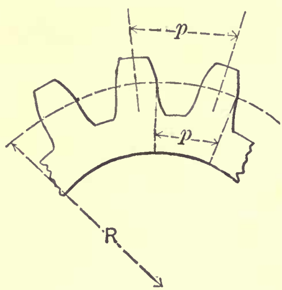
$$C = \pi D, \text{ or } C = 2\pi R \qquad (1)$$

where C is the circumference, D the diameter, R the radius, and π the constant 3.14159. From these formulas we may write,

$$D = \frac{C}{\pi}, \quad R = \frac{C}{2\pi} \qquad (2).$$

Thus, to find the circumference, multiply the diameter by 3.14159, or the radius by $2 \times 3.14159 = 6.28318$. Inversely, to find the diameter, divide the circumference by 3.14159: to find the radius, divide the circumference by 6.28318. The simple, old rule, which says, "To find the circumference of a circle, multiply the diameter by 22, and divide by 7, to find the diameter, multiply the circumference by 7, and divide by 22," ordinarily answers the purpose well enough. The *circumferential pitch* or *circular* pitch (generally called simply the pitch) of a gear of any kind is the distance from the centre of one tooth to the centre of an adjacent tooth, measured on the pitch circle, or, what is the same thing, the distance on the pitch circle, which includes one tooth and one space, p (Fig. 71). This distance, laid off a certain number of times around the pitch circle, divides the pitch circle into a certain number of equal parts, each containing one tooth: consequently the circumference of the pitch circle divided by the pitch will give the number of teeth, and the pitch multiplied by the number of teeth will give the circumference of the pitch circle. In formula, N being the number of teeth, and p the pitch,

Fig. 71



$$N = \frac{C}{p}, \quad C = Np, \quad p = \frac{C}{N} \qquad (3).$$

From formula (1) we may write $\frac{N}{C} = \frac{N}{\pi D}$, and, from the third of formula (3), $\frac{1}{p} = \frac{N}{C}$. Hence $\frac{1}{p} = \frac{N}{\pi D}$, or

$$\frac{N}{D} = \frac{\pi}{p} = p_d \quad \text{and} \quad \frac{\pi}{p_d} = p \qquad (4).$$

This ratio of the constant quantity $\pi = 3.14159$ to the circumferential pitch is called the *diametral* pitch, because it is equal to the ratio of the number of teeth to the diameter of the pitch circle. We represent this diametral pitch by p_d. The diametral pitch gives the number of teeth in a gear wheel per unit (say inch) of length of the pitch-circle diameter. To illustrate, suppose we have a pitch circle of 10″ diameter and a circumferential pitch of 3.14159″. From formula (1) the circumference is $C = \pi \times 10 = 31.4159''$, and from formula (3) the number of teeth is $N = \frac{C}{p} = \frac{31.4159}{3.14159} = 10$. Hence, from formula (4), $p_d = \frac{N}{D} = \frac{10}{10} = 1$; that is, there is one tooth in the gear for each inch of length in the diameter of the pitch circle. In order to distinguish the diametral from the circumferential pitch, the former is often designated as "pitch No. —." Diametral pitch No. 1 = circumferential pitch of $\frac{\pi}{1} = 3.14159''$, diametral pitch No. 2 = circumferential pitch of $\frac{\pi}{2} = 1.57079''$, etc.

Since the circumference of a circle cannot be measured exactly (the quantity π being irrational), it is often tedious work to step off the circumferential pitch around the pitch circle (especially in large gears), a great many trials being necessary before the equal division of the pitch circle is obtained. A formula, by the use of which this work is simplified, may be obtained as follows: Let *bcd* (Fig. 72) be a circle, *bc* a circle chord. In the triangle *abc* we have, from trigonometry, the proportion

$$\sin \text{ angle } bac : bc :: \sin \text{ angle } bca : ab.$$

But $ab = R$, the radius of the circle, and $bc = l''$, the circle chord. Calling the angle bac θ, we have, since $ac = ab = R$, the relation,

$$\text{angle } bca = \frac{180^\circ - \theta}{2}.$$

Substituting these values in the above proportion, we obtain

$$\sin\theta : l'' :: \sin\left(\frac{180^\circ - \theta}{2}\right) : R.$$

Hence

$$l'' = \frac{R \sin\theta}{\sin\left(\frac{180^\circ - \theta}{2}\right)}.$$

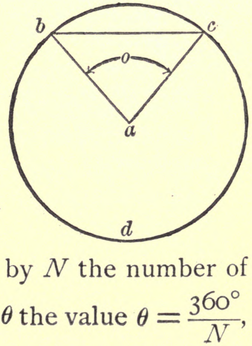
But

$$\sin\left(\frac{180^\circ - \theta}{2}\right) = \sin\left(90^\circ - \frac{\theta}{2}\right) = \cos\tfrac{1}{2}\theta$$

and, from trigonometry, $\sin\theta = 2 \sin\frac{1}{2}\theta \cos\frac{1}{2}\theta$.

These values, substituted in the last expression for l'', give

Fig. 72



$$l'' = \frac{2 R \sin\frac{1}{2}\theta \cos\frac{1}{2}\theta}{\cos\frac{1}{2}\theta}$$

or

$$l'' = 2 R \sin\tfrac{1}{2}\theta \qquad (5).$$

Suppose, now, the arc bc to represent the pitch laid off on the pitch circle of a gear. If we represent by N the number of teeth in the gear, we shall have for θ the value $\theta = \frac{360^\circ}{N}$, and consequently $\frac{1}{2}\theta = \frac{180^\circ}{N}$. From this, by substitution in formula (5), we have for the length of the chord *bc*,

$$l'' = 2R \sin\left(\frac{180^\circ}{N}\right) = D \sin\left(\frac{180^\circ}{N}\right) \qquad (6).$$

Rule. — To find the length of the chord subtended by the pitch arc, multiply the diameter of the pitch circle by the sine of the angle obtained by dividing 180° by the number of teeth.

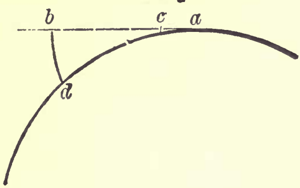
Example 1. — Suppose $D = 24''$ and $N = 80$. Hence $\frac{1}{2}\theta = \frac{180^\circ}{80} = 2^\circ\ 15'$, $\sin 2^\circ\ 15' = 0.03926$, and $l'' = 24 \times 0.03926 = 0.942''$.

Example 2. $D = 39\frac{1}{2}''$ and the pitch $= p = 4''$. From formula (1) the circumference is $C = \pi D = 124''$, and from formula (3) $N = \frac{124}{4} = 31$. Formula (6) therefore gives $l'' = 39\frac{1}{2} \sin\left(\frac{180^\circ}{31}\right) = 39\frac{1}{2} \sin 5^\circ\ 48'\ 23\frac{1}{4}'' = 39\frac{1}{2} \times 0.1011683 = 3.996''$.

Mr. W. C. Unwin, in "Elements of Machine Design," gives the following: —

"*To lay off the Pitch on the Pitch Line.* — The following construction is convenient when the wheel is so large that it is impossible to find the exact pitch by stepping round the pitch line. Let the circle (Fig. 73) be the pitch line. At any point, *a*, draw the tangent *ab*. Make *ab* equal to the pitch. Take *ac* equal to $\frac{1}{4}ab$. With centre *c* and radius *cb*, draw the arc *bd*. Then the arc *ad* is equal to *ab*, and is the pitch laid off on the pitch line. When the wheel has many teeth, the arc *ad* sensibly coincides with its chord; but, if it has few teeth, there is an appreciable error in taking the chord *ad* equal to the pitch."

Unfortunately neither of these rules gives exactly the required distance; for, in the first case, the $\sin\left(\frac{180°}{N}\right)$ is usually a number containing six, eight, or even more decimal places, and consequently the chord *bc* will be such a number, not capable of exact measurement with the compasses; and, in the second case, the pitch (being the circumference — an irrational quantity — divided by the number of teeth) cannot be exactly laid off on the line *ab*. Such simple and easily remembered rules, however, simplify in some degree the work of the draughtsman and mechanic, and are therefore worthy of our notice. An accurately constructed "π-rule" (pi-rule), used in connection with the preceding method, gives very close results. To construct such a rule, have a four-inch circle turned, as accurately as possible, out of wood or metal. Mark a point anywhere upon the circumference, and starting with this point tangent to a straight, true ruler about 14″ long, roll the circle along (taking care not to slip or slide) until the point is again tangent to the ruler. The distance thus developed upon the ruler is equal to the circumference of the 4″ circle, equals 4π. Divide the developed length into four parts: each part is equal to one π (pi), and may be divided into halves, quarters,

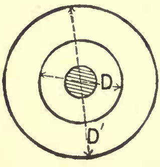
Fig. 73


eighths, etc., or into tenths and hundredths. The total distance now marked off is 4π, and the divisions are equal to π, $\frac{1}{2}\pi$, $\frac{1}{4}\pi$, $\frac{1}{8}\pi$, etc., or $\frac{1}{10}\pi$, and $\frac{1}{100}\pi$. As an example to illustrate the use of the π-rule, suppose the diameter of a gear to be constructed is 10″, and the number of teeth 100. The circumference of the pitch circle is 10π, and the pitch is 10π divided by 100, or $\frac{1}{10}\pi$. This, measured on the π-rule, and laid off on the tangent line *ab* (Fig. 73), will give the arc *ad* (or chord *ad*) as accurately as any method with which we are acquainted.

§ XI. — *Ratios. — Velocity. — Revolution. — Power.*

The velocity ratio of two gear wheels is the velocity at the circumference of one wheel divided by the velocity at the circumference of the other, both velocities being taken in terms of the same unit (generally feet per second), or the ratio of the velocity at the circumference of one to the velocity at the circumference of the other. The velocity ratio of two toothed wheels which gear together is always constant, and equal to unity; that is, the velocity at the circumference of one is equal to the velocity at the circumference of the other.* To prove this, let the circles of Fig. 74 represent the pitch circles of a pair of gear wheels. Suppose R to be the driver,

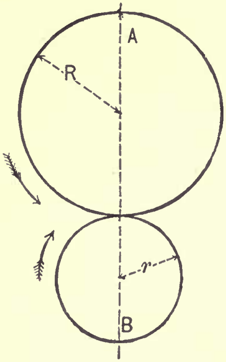
* When two gear wheels are fixed upon the same shaft, their velocities are proportional to their diameters or radii. Thus, let D and D' be the diameters of two such wheels. The velocities at the circumferences of the wheels are $v = Cn$, and $v' = C'n$; v, v', C, and C' being the velocities and circumferences. Hence



$$v = \pi Dn, \quad v' = \pi D'n, \quad \frac{v'}{v} = \frac{\pi D'n}{\pi Dn} = \frac{D'}{D} = \frac{R'}{R}.$$

and r the driven wheel. As the wheels revolve, it is plain, that, as each tooth of R passes the imaginary line AB, it carries with it a tooth of the wheel r. Thus equal numbers of teeth of the two wheels pass the line AB in equal times. But, since the pitches of the wheels are equal, equal numbers of teeth must lie on equal arcs of the two pitch circumferences: therefore, without reference to the relative sizes of the wheels, equal arcs of their pitch circumferences pass the line AB in equal times, or, in other words, the velocities at the circumferences are equal.

Fig. 74



The revolution ratio of two gear wheels which gear together is the greater number of revolutions divided by the less, or the ratio of the greater number of revolutions to the less. For example, if one of a pair of gear wheels makes 100 revolutions per minute and the other 20, the revolution ratio is $\frac{100}{20} = \frac{5}{1}$, and we say the wheels are geared 5 to 1. We have proved in Fig. 74 that equal numbers of teeth of the wheels R and r pass the line AB in equal times. Let us suppose the number of teeth (N) of the wheel R to be 100, and that (N') of r to be 25. When 25 teeth of R have passed the line AB, 25 teeth (all) of r have also passed the line; that is, R has made $\frac{1}{4}$ of a revolution, and r has made 1 entire revolution. When 50 teeth of R have passed the line AB, 50 teeth of r have also passed the line, or R has made $\frac{1}{2}$ of a revolution, and r has made 2 entire revolutions. Thus, when 100 teeth of R have passed AB, or when R has made 1 entire revolution, r has made $\frac{100}{25} = 4$ entire revolutions. The revolution ratio of the pair is therefore $\frac{4}{1}$, the *small* wheel making 4 revolutions while the *large* wheel makes 1. But the ratio of the number of teeth of the small wheel (r) to that of the large wheel (R) is $\frac{25}{100} = \frac{1}{4}$: therefore it is plain that *the revolution ratio of a pair of toothed wheels is inversely equal to the ratio of the numbers of teeth of the wheels.* Letting n, N, R, D, and C represent the number of revolutions, number of teeth, radius, diameter, and circumference respectively, of the smaller wheel, and n', N', R', D', and C' the number of revolutions, etc., of the larger wheel, we have, since the number of teeth is directly proportional to the radius, diameter, or circumference,

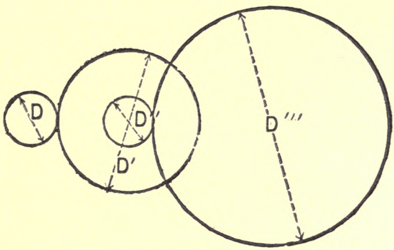
$$\frac{n}{n'} = \frac{N'}{N} = \frac{R'}{R} = \frac{D'}{D} = \frac{C'}{C} \qquad (7).$$

Rule.—The number of revolutions of the smaller wheel is to the number of revolutions of the larger wheel as the number of teeth, radius, etc., of the larger wheel are to the number of teeth, radius, etc., of the smaller wheel.

Example 1.—Two bevel wheels are to gear together so that the revolutions per minute are respectively $n = 160$ and $n' = 40$. The diameter of the smaller wheel is $D = 8''$, and the pitch of the teeth, $p = \frac{1}{2}''$. It is required to find the diameter of the larger wheel (D') and the numbers of teeth (N and N') of each wheel. We have here $\frac{n}{n'} = \frac{160}{40} = \frac{4}{1}$. From formula (7), $\frac{4}{1} = \frac{D'}{8}$, $D' = 32''$. From formula (1), $C = \pi \times 8 = 25.1''$, and, from formula (3), $N = \frac{25.1}{\frac{1}{2}} = 50$. From formula (7), again, $\frac{4}{1} = \frac{N'}{50}$, or $N' = 200$.

Example 2.—A shop shaft makes 120 revolutions per minute. From this shaft it is required to gear down to 8 revolutions per minute. The diameter of the wheel on the first shaft is 12″. Find other diameters and the numbers of teeth of each wheel, supposing the

Fig. 75



pitch $= 1''$. The revolution ratio is $\frac{120}{8} = \frac{15}{1}$. From formula (7), $\frac{15}{1} = \frac{D'}{12}$, $D' = 180'' = 15$ feet. A wheel of this size is out of the question: we therefore must have recourse to a train of wheels such as is represented in Fig. 75. We may take the revolution ratio between D and D' $\frac{3}{1}$, and that between D'' and D''' $\frac{5}{1}$: we then have $\frac{3}{1} \times \frac{5}{1} = \frac{15}{1}$ as the ratio between D and D'''. From formula (7), then, $\frac{n}{n'} = \frac{3}{1} = \frac{D'}{12}$. $D' = 36''$, and $\frac{n''}{n'''} = \frac{5}{1} = \frac{D'''}{D''}$. Taking $D'' = D = 12''$, we have $D''' = 60''$. From formula (1), $C = \pi \times 12 = 37.7$, and, from formula (3), $N = \frac{37 \cdot 7}{1} = 38$. Hence, from formula (7), $N' = 114$, $N'' = 38$, and $N''' = 190$.*

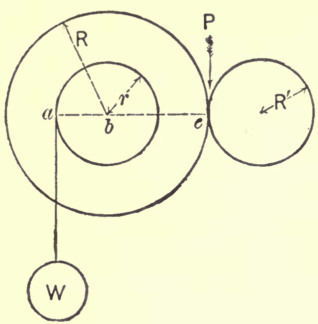
In a pair of gears in which $N = 25$ and $N' = 100$ the revolution ratio is $\frac{n}{n'} = \frac{N'}{N} = \frac{100}{25} = \frac{4}{1}$. The same teeth are therefore in contact once in every revolution of the larger wheel, or once in every 4 revolutions of the smaller wheel. Contact taking place so frequently between the same two teeth, if these teeth happen to be rough and poor, the wear between them must be greater than in any other part of the wheels. If, however, we make $N = 26$, the revolution ratio is $\frac{100}{26} = 3\frac{11}{13}$, practically the same as before, and the two poor teeth are in contact only once in 13 revolutions of the larger, or 50 revolutions of the smaller wheel. By means of this "wear tooth" the wear of the wheels may be more evenly distributed, and the durability of the wheels considerably increased, without seriously interfering with the revolution ratio of the wheels.

Power Ratio. — The power or force of a gear wheel is the force with which the circumference of the wheel turns: it is equal to that force, which, when applied to the circumference in a direction contrary to that of rotation, is just sufficient to stop the rotation of the wheel. The power ratio or force ratio of two gears is the greater power divided by the less, or the ratio of the greater power to the less. The powers of two wheels *which gear together* are equal, the power of the

* The gears D' and D'', being fixed upon the same shaft, of course make the same number of revolutions per minute, regardless of diameters or radii.

driver being transmitted directly to the driven wheel: in this case, therefore, the power ratio, as the velocity ratio, is constant, and equal to unity. Let R and R' (Fig. 76) represent the radii of a pair of gears, and r the radius of a pulley which is fixed upon the axle of R, and arranged to lift a weight W by means of a string passing around its circumference. Let the power or force of the driver R' be denoted by P. This force is transmitted to R in the direction shown by the arrow. We may regard the imaginary line ac as a simple lever, the fulcrum of which is at b, and the arms of which are $ab = r$ and $bc = R$. The force P acts upon the long arm, and the force W upon the short arm. By the principles of the lever, the moments of the forces with reference to the fulcrum must be equal: hence we have

Fig.76



$$Wr = PR, \quad \text{or} \quad \frac{W}{P} = \frac{R}{r} \qquad (8).$$

That is, the forces of the wheels R and r are *inversely proportional to their radii.* Since the radii R and r are directly proportional to the velocities of the circumferences, and the power and velocity of R are equal to the power and velocity of R', we may write,

$$\frac{W}{P} = \frac{V}{v}, \quad W = \frac{PV}{v}, \quad P = \frac{Wv}{V} \qquad (9)$$

where V and v are the circumferential velocities of R and r respectively. From this formula we may write the following: —

Rule. — The relative powers of the wheels of a train of gears are inversely proportional to the circumferential velocities of the wheels. To find the power of any wheel of a train of gears when the power of the next wheel is known, multiply the power of the latter by its own velocity, and divide by the velocity of the former.

Example 1. — In a train of gears such as is represented in Fig. 76, the force of the driver is $P = 50$ pounds, the velocity of the driver is $V = 10$ feet per second; that of the pulley, $v = 5$ feet per second. Required the weight W which can be lifted by the pulley. The force of R is equal to that of the driver, since their velocities are equal. By the rule,

$$\text{force of } R = \frac{\text{force of } R' \times \text{velocity of } R'}{\text{velocity of } R} = \text{force of } R'.$$

From formula (9), or the rule,

$$W = \frac{PV}{v} = \frac{50 \times 10}{5} = 100 \text{ pounds.*}$$

Example 2. — In the gear train represented in Fig. 77 the force of the driver R'' is $P = 500$ pounds, $R = R' = 12''$, $r = r' = 5''$. It is required to find the

* The gain in power is obtained by a sacrifice of time; for the wheel R, having twice the velocity and half the power of the pulley r, can lift twice as far a weight equal to $\frac{1}{2}W$ in the same time, or just as far a weight equal to $\frac{1}{2}W$ in half the time. The *work* inherent in these two wheels is therefore the same: r simply does double work in double time. If, however, we have only 50 pounds of force at our disposal, we can lift 100 pounds at one lift only by means of such a train, or a similar mechanism.

weight, W, which can be lifted by the pulley r, and the distance per minute which W can be lifted, supposing the wheel R' to make 15 revolutions per minute. The power of R' is equal to that of the driver $= P$. From formula (8), P' representing the power of r',

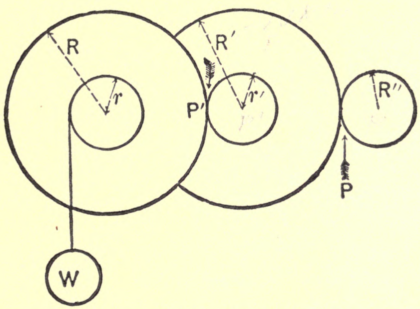
$$\frac{P'}{P} = \frac{R'}{r'}, \quad \frac{P'}{500} = \frac{12}{5}, \quad P' = 1200 \text{ pounds.}$$

This power is transmitted directly to R: hence

$$\frac{W}{P'} = \frac{R}{r}, \quad \frac{W}{1200} = \frac{12}{5}, \quad W = 2880 \text{ pounds.}$$

R' and r' make the same number of revolutions, being

Fig.77



on the same shaft. From formula (7), n' and n being the numbers of revolutions of r' and R, we have

$$\frac{n}{n'} = \frac{r'}{R'}, \quad \frac{n}{15} = \frac{5}{12}, \quad n = 6\tfrac{1}{4}.$$

The circumferential velocity of r, which is the velocity with which the weight W is lifted, is

$$\frac{2\pi rn}{12} = \frac{2\pi \times 5 \times 6\frac{1}{4}}{12} = 16.36 \text{ feet per minute.}$$

Example 3. — Required an expression for the weight which can be lifted by a train similar to that of Fig. 77, containing any number of wheels. From Example 2, $\frac{P'}{P}=\frac{R'}{r'}$ or $P'=\frac{PR'}{r'}$ and $\frac{W}{P'}=\frac{R}{r}$ or $W=\frac{P'R}{r}$. Substituting in this expression the value of P', just written, we obtain $W=\frac{PRR'}{rr'}$. In the same manner, for any number of wheels, R, R', R'', R''', etc., representing the radii of the large wheels, and r, r', r'', r''', etc., those of the pinions, we obtain $W=\frac{PRR'R''R''', \text{ etc.}}{rr'r''r''', \text{ etc.}}$. Inversely, $P=\frac{Wrr'r''r''', \text{ etc.}}{RR'R''R''', \text{ etc.}}$.

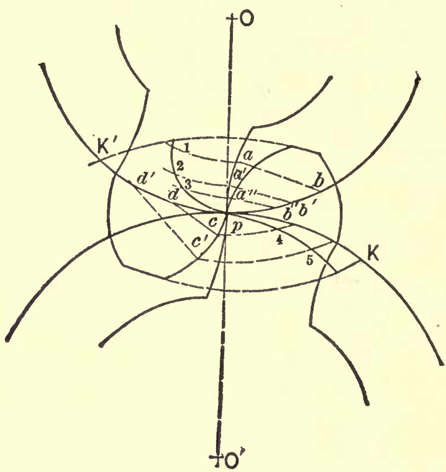
Example 4. — We have a shaft which drives a gear with a force of 250 pounds: we wish with this power to lift a weight of 1,500 pounds. Required the radii of the wheels of the necessary train. We can see at a glance that a simple train, such as Fig. 76, will not be practicable, for in this case $\frac{W}{P}=\frac{R}{r}=\frac{1500}{250}=\frac{6}{1}$; and, if $r=6''$ (as small as is convenient), $R=r\times 6=36''$, or the diameter of our large gear will have to be 6 feet. This is practically out of the question: we must therefore use a train with 4 or more wheels. Let us try 4. From Example 3 we have $W=\frac{PRR'}{rr'}$. Taking $r=r'=6''$, $W=\frac{PRR'}{36}$, $1500=\frac{250RR'}{36}$, $RR'=\frac{1500\times 36}{250}=216$. We can now assume a value for R, and find the corresponding value of R'. Say $R=12''$, then $R'=\frac{216}{12}=18''$.

In the preceding examples no account has been taken of the friction of the gear teeth and axles, since they are given simply to illustrate the use of the rules and formulas which precede them. The detrimental friction is, of course, very considerable, even in the best wheels, and increases rapidly as we increase the number of wheels in a train: therefore the trains spoken of in the examples, if actually made and used, would accomplish considerably less than the examples give them credit for. Were this not the case, we could, with the slightest possible amount of power, by means of a train containing a sufficient number of wheels, perform an infinitely great amount of work — manifestly, from a practical point of view at least, an absurdity.

§ XII. — *Line of Contact. — Arc of Contact.*

In a pair of toothed wheels, each tooth of one wheel is in contact, for a certain, definite length of time or distance of revolution, with a tooth of the other wheel, and there is always at least one pair of teeth in contact. Whether or not the same two teeth come into contact at each revolution depends, as we have already seen, upon the relative numbers of teeth of the two wheels. If, during the contact of a pair of teeth, a curve be drawn through all the successive points of contact, this curve will represent the entire contact of the teeth. Such a curve is called the *line of contact*, and its length represents the duration of the contact. The line of contact may be found by drawing different positions of two teeth while in contact, and drawing a curve through the points of contact thus determined. This operation is, however, often a difficult one, because the effect of the preceding pair of teeth upon the early contact of the pair in question cannot easily be taken into consideration, and this effect is very often too important to be neglected. Reuleaux has pointed out the following method for determining the line of contact: Let O and O' (Fig. 78) be the centres of two toothed wheels which gear together, OpO' the line of centres, and p the pitch

Fig.78



point. From different points along the profile apc' draw normal lines intersecting the pitch circle in the points b, b', $b,''$ d', etc., and from O as a centre strike circle-arcs through the points a, a', a'', c, etc. We have seen, that, for uniform velocity ratio, it is necessary that the common normal to two teeth in contact at the point of contact shall pass through the pitch point. If, therefore, from the pitch point p as a centre, with radii equal to ab, $a'b'$, $a''b''$, dc, etc., we strike arcs intersecting the above-mentioned arcs, the points of intersection will be points of contact of the teeth, and a curve drawn through these points will be the line of contact. The arcs Kp and $K'p$, taken on the pitch circles, and limited by the top circles, are called the *arcs of approach* and *recess*, according to the direction of rotation, and together form the *arc of contact.* The length of the arc of contact depends upon the diameters of the pitch circles of the gears and the height of the teeth between the pitch and top circles; while the length and position of the line of contact depend not only upon these dimensions, but also upon the form of the profiles of the teeth and the number of teeth in contact at one time. In ordinary gearing, where the height of the teeth between the pitch and top circles is the same for both wheels, the arcs of approach and recess are equal, and, in wheels having cycloidal profiles, the lengths of the line and arc of contact are, according to Reuleaux, equal. The length of the arc of contact must be at least equal to the pitch of the teeth, else there would be less than one pair of teeth in contact at one time: in ordinary machine gearing this length varies from $1\frac{1}{2}$ to $2\frac{1}{4}$ times the pitch.

§ XIII.—*Strength of Teeth.—Rules for determining the Pitch, and other Tooth Dimensions.*

Before taking up the subject of strength of wheel teeth, our notation for the calculations under this head must be explained. The total height of the tooth, i.e., the sum of the heights above and below the pitch circle, we denote by h $(= h' + h'')$; the breadth of the tooth on the pitch circle, by b; and the face width of the tooth, by l (see Fig. 79). In calculating the strength of a wheel tooth, the curved profile is disregarded, as is also, in ordinary gearing, the influence of the velocity of the wheel, and the tooth regarded as a simple beam or semi-girder supported at one end, and having a weight or force, W, acting at the other end (Fig. 80). The width b is taken equal to the width of the tooth on the pitch circle. The safe working-load for a beam

Fig. 79

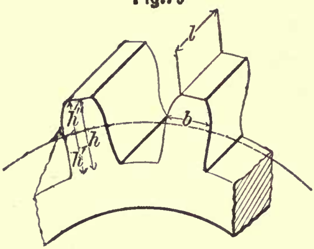
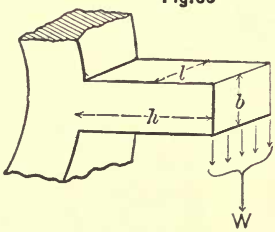


Fig. 80



such as is represented in Fig. 80 is expressed by the formula

$$W = \frac{flb^2}{6h}$$

in which W is the safe working-load, f the greatest safe working-stress in pounds per square inch for the material used, and the other quantities the same as in Fig. 80. It is evident that the width b of the tooth must be less than half the pitch, else the space would not be wide enough to admit the tooth of the mate-wheel; and, in order that the tooth may be sufficiently strong when it becomes worn, we take, at the suggestion of Unwin, $b = 0.36p$; p being the circumferential pitch. Also we may take, as is now generally done,

$h = h' + h'' = 0.4p + 0.3p = 0.7p$. These values, substituted in the above formula, give

$$W = \frac{fl \times 0.1296p^2}{4.2p}.$$

In this expression W is the actual load or strain on one tooth. It is more convenient to use this formula in terms of the total force, P, transmitted by the wheel. Ordinarily, more than one pair of teeth are in gear at once: therefore the whole force transmitted is not sustained by one tooth. The number of teeth in gear at once varies considerably in different wheels; but we may safely say that no tooth bears more than three-fourths of the entire force transmitted. We have, then, $W = \frac{3}{4}P$, and consequently

$$\tfrac{3}{4}P = \frac{fl \times 0.1296p^2}{4.2p}.$$

Reducing this equation, we obtain

$$P = \frac{0.04114fp^2l}{p}.$$

From this, by transposing,

$$p^2 = \frac{P}{0.04114f} \times \frac{p}{l}$$

or

$$p = \sqrt{\frac{P}{0.04114f} \times \frac{p}{l}} = 4.93\sqrt{\frac{P}{f} \times \frac{p}{l}} \qquad (10).$$

This formula may be termed the *general* formula for determining the pitch. It may be used for any material whatever by substituting for the quantity f its proper value. From the formula, therefore, we may write the following: —

General Rule. — To determine the pitch of a gear of any material, divide the total force to be transmitted by the greatest safe working-stress per square inch for the material of the wheel, multiply the quotient thus obtained by the ratio of the pitch to the face width,* extract the square root of this product, and multiply the result by 4.93.

The degree of safety necessary in calculations for strength of gear teeth varies with the work to be done by the gear, in other words, with the amount of danger to be incurred. Thus the degree of safety necessary is greater when the gear is to be subjected to sudden, violent shocks than when no such shocks occur, because the danger of breakage or accident is greater. This degree of safety we obtain conveniently by varying the value of the quantity f, taking small values when the danger is great, and *vice versa.*

For ordinary, good cast-iron we may take $f = 4{,}000$ pounds when there are sudden, violent shocks upon the gear, $f = 5{,}000$ pounds when only moderate shocks occur, and $f = 10{,}000$ pounds when there is little or no shock. By substituting these values of f, in turn, in formula (10), and reducing, we obtain the following formula for determining the pitch of a *cast-iron* gear: —

* Ordinarily this ratio is assumed. For example, we may assume $\frac{p}{l} = \frac{1}{2}$, or $\frac{p}{l} = \frac{1}{2\frac{3}{4}}$, and determine the pitch for the particular value assumed.

$$\left.\begin{array}{lll} \text{For violent shock,} & p = 0.078\sqrt{P \times \frac{p}{l}} & (a) \\ \text{For moderate shock,} & p = 0.07\sqrt{P \times \frac{p}{l}} & (b) \\ \text{For little or no shock,} & p = 0.05\sqrt{P \times \frac{p}{l}} & (c) \end{array}\right\} \quad (11).$$

Rule. — To determine the pitch for a cast-iron gear, multiply the total force to be transmitted by the ratio of the pitch to the face width, extract the square root of the product, and multiply the result by 0.078 for violent shock, 0.07 for moderate shock, or 0.05 for little or no shock.

In ordinary machine-gearing the face width is very often taken equal to twice the pitch $\left(l = 2p,\ \frac{p}{l} = \frac{1}{2}\right)$; because a greater relative face width does not, in the same degree, add strength to the tooth, the principal effect being to increase the stiffness of the tooth. If we make $\frac{p}{l} = \frac{1}{2}$ in each of the formulas (11), we obtain

$$p = 0.078\sqrt{P \times \tfrac{1}{2}},\ p = 0.07\sqrt{P \times \tfrac{1}{2}},\ \text{and}\ p = 0.05\sqrt{P \times \tfrac{1}{2}}.$$

Reducing these, we have, for the three cases given above, the formulas

$$\left.\begin{array}{ll} p = 0.055\sqrt{P} & (a) \\ p = 0.05\sqrt{P} & (b) \\ p = 0.035\sqrt{P} & (c) \end{array}\right\} \quad (12).$$

Rule. — To determine the pitch for cast-iron gears when the face width is equal to twice the pitch, multiply the square root of the total force to be transmitted by 0.055 for violent shock, 0.05 for moderate shock, or 0.035 for little or no shock.

A *horse-power*, as commonly used, is that force which will lift a weight of 33,000 pounds one foot high in one minute, — 33,000 foot-pounds. If we let H represent the horse-power, and v the velocity at the circumference of the wheel in *feet per second*, we shall have the expression,

$$P = \frac{33000H}{60v} = \frac{550H}{v}.$$

This value of P, substituted in formulas (11), gives the following convenient formulas for the pitch when the horse-power and the velocity in feet per second, instead of the force transmitted in pounds, are given : —

$$\left.\begin{array}{lll} \text{For violent shock,} & p = 1.83\sqrt{\frac{H}{v}\frac{p}{l}} & (a) \\ \text{For moderate shock,} & p = 1.64\sqrt{\frac{H}{v}\frac{p}{l}} & (b) \\ \text{For little or no shock,} & p = 1.17\sqrt{\frac{H}{v}\frac{p}{l}} & (c) \end{array}\right\} \quad (13).$$

Rule. — To determine the pitch from the horse-power and velocity in feet per second, multiply the ratio of the horse-power to the velocity by the ratio of the pitch to the face width, extract the square root of the product, and multiply the result by 1.83 for violent shock, 1.64 for moderate shock, or 1.17 for little or no shock.

By substituting the above value of P in formulas (12), we obtain for the pitch, when the face width is not less than twice the pitch, the formulas : —

$$\left.\begin{array}{lll} \text{For violent shock,} & p = 1.29\sqrt{\frac{H}{v}} & (a) \\ \text{For moderate shock,} & p = 1.17\sqrt{\frac{H}{v}} & (b) \\ \text{For little or no shock,} & p = 0.82\sqrt{\frac{H}{v}} & (c) \end{array}\right\} \quad (14).$$

Rule.—To determine the pitch from the horse-power and velocity in feet per second, when the face width is equal to twice the pitch, multiply the square root of the ratio of the horse-power to the velocity by 1.29 for violent shock, 1.17 for moderate shock, or 0.82 for little or no shock.

If we represent by n the number of revolutions per minute, and by D the diameter of the wheel in inches, we may obtain, for the velocity in feet per second, the value,

$$v = \frac{\pi D n}{12 \times 60} = 0.00436Dn.$$

This value, substituted for v in formulas (13), gives

$$\left.\begin{array}{lll} \text{For violent shock,} & p = 27.71\sqrt{\frac{H}{Dn}\frac{p}{l}} & (a) \\ \text{For moderate shock,} & p = 24.84\sqrt{\frac{H}{Dn}\frac{p}{l}} & (b) \\ \text{For little or no shock,} & p = 17.72\sqrt{\frac{H}{Dn}\frac{p}{l}} & (c) \end{array}\right\} \quad (15).$$

Rule.—To determine the pitch from the horse-power, diameter, and number of revolutions, divide the horse-power by the product of the diameter, in inches, into the number of revolutions per minute, multiply the quotient by the ratio of the pitch to the face width, extract the square root of the product, and multiply the result by 27.71 for violent shock, 24.84 for moderate shock, or 17.72 for little or no shock.

By substituting $v = 0.00436Dn$ for v in formulas (14), the following formulas for the pitch, when the face width is equal to twice the pitch, may be obtained:—

$$\left.\begin{array}{lll} \text{For violent shock,} & p = 19.54\sqrt{\frac{H}{Dn}} & (a) \\ \text{For moderate shock,} & p = 17.72\sqrt{\frac{H}{Dn}} & (b) \\ \text{For little or no shock,} & p = 12.42\sqrt{\frac{H}{Dn}} & (c) \end{array}\right\} \quad (16).$$

Rule.—To determine the pitch from the horse-power, diameter, and number of revolutions, when the face width is equal to twice the pitch, divide the horse-power by the product of the diameter into the number of revolutions, extract the square root of the quotient, and multiply the result by 19.54 for violent shock; 17.72 for moderate shock, or 12.42 for little or no shock.

Example 1.—A cast-steel gear wheel is required which will transmit a force of 100,000 pounds. The gear is to be subjected to severe shock. Suppose a specimen of the steel to be used has been tested, and the *breaking-weight* found to be 140,000 pounds per square inch. We may take, for the greatest safe working-stress per square inch, $f = \frac{1}{6} \times 140000 = 23333$ pounds, say, $f = 23000$. If, now, we take $\frac{p}{l} = \frac{1}{4}$, formula (10) gives,

$$p = 4.93\sqrt{\frac{100000}{23000} \times \frac{1}{4}} = 4.93\sqrt{1.087} = 4.93 \times 1.042 = 5.137 \text{ in.}$$

For the other dimensions we have

$$l = 4p = 4 \times 5.137 = 20.548 \text{ inches}$$

$$h = 0.7p = 0.7 \times 5.137 = 3.5959 \text{ inches}$$

and

$$b = 0.46p = 0.46 \times 5.137 = 2.363 \text{ inches.}$$

Taking these values to the nearest eighth, sixteenth, etc., taking care to err only on the safe side, we have $p = 5\frac{9}{64}$ inches, $l = 20\frac{9}{16}$ inches, $h = 3\frac{19}{32}$ inches, and $b = 2\frac{3}{8}$ inches.

Example 2. — Required the tooth dimensions for a wooden cog-wheel to transmit a force of 10,000 pounds, moderate shock. Let us suppose the cogs to be of oak, the breaking-strength of which is 15,000 pounds per square inch. We may safely take

$$f = \frac{1}{6} \times 15000 = 2500 \text{ pounds.}$$

If $\frac{p}{l} = \frac{1}{2}$, formula (10) gives

$$p = 4.93\sqrt{\frac{10000}{2500} \times \frac{1}{2}} = 4.93\sqrt{2} = 4.93 \times 1.414 = 6.971 \text{ in.}$$

$$l = 2p = 13.942 \text{ inches}$$

$$h = 0.7p = 4.8797 \text{ inches}$$

and

$$b = 0.46p = 3.20666 \text{ inches.}$$

In fractions the dimensions are, $p = 6\frac{63}{64}$ inches, $l = 13\frac{61}{64}$ inches, $h = 4\frac{7}{8}$ inches, and $b = 3\frac{7}{32}$ inches.

Example 3. — With the data of Example 2, required the tooth dimensions for a wheel to be subjected to little or no shock. In this case we may take

$$f = \tfrac{1}{4} \times 15000 = 3750.$$

As before, formula (10) gives for the pitch,

$$p = 4.93\sqrt{\frac{10000}{3750} \times \frac{1}{2}} = 4.93\sqrt{1.333} = 4.93 \times 1.154 = 5.689''$$

$$l = 2p = 11.378''$$

$$h = 0.7p = 3.982''$$

$$b = 0.46p = 2.617''.$$

Example 4. — Required the tooth dimensions for a cast-iron gear wheel which is to be worked by hand and crank. Suppose a man can exert by means of the crank a force of 300 pounds on the wheel. From formula (12, *c*) we have, for little or no shock,

$$p = 0.035\sqrt{300} = 0.035 \times 17.32 = 0.606''$$

$$l = 2p = 1.212''$$

$$h = 0.7p = 0.4242''$$

and

$$b = 0.46p = 0.37876''.$$

If, for any reason, it is necessary to make $\frac{p}{l} = \frac{1}{4}$, we must use formula (11, *c*), which gives

$$p = 0.05\sqrt{300 \times \tfrac{1}{4}} = 0.05 \times 8.66 = 0.433''.$$

Hence

$$l = 4p = 1.732'', \text{ etc.}$$

Example 5. — A cast-iron gear is to have a circumferential velocity of 10 feet per second, and is to transmit a force of 15-horse power, moderate shock. Required the dimensions of the teeth when $\frac{p}{l} = \frac{1}{3\frac{1}{2}}$. From formula (13, *b*),

$$p = 1.64\sqrt{\frac{15}{10} \times \frac{1}{3\frac{1}{2}}} = 1.64\sqrt{0.4286} = 1.64 \times 0.6546 = 1.0725''.$$

Hence

$$l = 3\tfrac{1}{2}p = 3.7538'', \text{ etc.}$$

If $\frac{p}{l} = \frac{1}{2}$, formula (14, *b*) gives

$$p = 1.17\sqrt{\frac{15}{10}} = 1.17\sqrt{1.50} = 1.17 \times 1.2247 = 1.4329''$$

and

$$l = 2p = 2.8658'', \text{ etc.}$$

Example 6.—The diameter of a cast-iron gear which is to transmit 25-horse power, excessive shock, is 30″. Required the dimensions, taking $\frac{p}{l} = \frac{1}{2}$, and the number of revolutions per minute $n = 80$. From formula (16, *a*),

$$p = 19.54\sqrt{\frac{25}{30 \times 80}} = 19.54\sqrt{0.0104} = 19.54 \times 0.102 = 2''$$

$$l = 4'', \text{ etc.}$$

Example 7.—Required a cast-steel gear which will safely transmit 100-horse power, excessive shock. The diameter of the gear is to be 36″, and the number of revolutions per minute 20. From the expressions

$v = 0.00436Dn$ and $P = \frac{550H}{v}$ we have, by combining,

$$P = \frac{550H}{0.00436Dn} = \frac{550 \times 100}{0.00436 \times 36 \times 20} = 17520 \text{ pounds.}$$

Taking $f = 23{,}000$ pounds, as in Example 1, and $\frac{p}{l} = \frac{1}{4}$, formula (10) gives

$$p = 4.93\sqrt{\frac{17520}{23000} \times \frac{1}{4}} = 4.93\sqrt{0.19} = 4.93 \times 0.436 = 2.149''$$

$$l = 4p = 8.596'', \text{ etc.}$$

As has already been noticed, the ratio $\frac{p}{l}$ is commonly taken equal to $\frac{1}{2}$; that is, the face width equals twice the circumferential pitch. Formulas (12), (14), and (16), which have been obtained from formulas (11), (13), and (15) by substituting for $\frac{p}{l}$ the value $\frac{1}{2}$, are therefore much the more often used. The following tables, obtained from formulas (12), (14), and (16), will be found convenient:—

TABLE I.

From formula (12, *a*, *b*, and *c*).

p in inches.	*P* for			No.
	Little or no shock.	Moderate shock.	Violent shock.	
1	816	400	331	1
1¼	1,276	625	513	2
1½	1,837	900	743	3
1¾	2,500	1,225	1,012	4
2	3,265	1,600	1,322	5
2¼	4,133	2,025	1,670	6
2½	5,102	2,500	2,066	7
2¾	6,173	3,025	2,500	8
3	7,347	3,600	2,975	9
3¼	8,622	4,225	3,491	10
3½	10,000	4,900	4,050	11
3¾	11,480	5,625	4,649	12
4	13,061	6,400	5,289	13
4¼	14,745	7,225	5,971	14
4½	16,531	8,100	6,694	15
4¾	18,418	9,025	7,459	16
5	20,408	10,000	8,265	17
5¼	22,500	11,025	9,111	18
5½	24,694	12,100	10,000	19
5¾	26,990	13,225	10,930	20
6	29,388	14,400	11,900	21
6½	34,490	16,900	13,967	22
7	40,000	19,600	16,198	23
7½	45,918	22,500	18,595	24
8	52,245	25,600	21,157	25

TABLE II.

From formula (14, *a*, *b*, and *c*.)

p in inches.	$\frac{H}{v}$ for			No.
	Little or no shock.	Moderate shock.	Violent shock.	
1	1.49	0.73	0.60	1
1¼	2.32	1.14	0.94	2
1½	3.35	1.64	1.35	3
1¾	4.55	2.24	1.84	4
2	5.95	2.92	2.40	5
2¼	7.53	3.69	3.04	6
2½	9.30	4.56	3.76	7
2¾	11.25	5.52	4.54	8
3	13.38	6.57	5.41	9
3¼	15.71	7.72	6.35	10
3½	18.22	8.95	7.36	11
3¾	20.91	10.27	8.45	12
4	23.80	11.69	9.61	13
4¼	26.86	13.19	10.85	14
4½	30.11	14.79	12.11	15
4¾	33.56	16.48	13.56	16
5	37.18	18.26	15.02	17
5¼	40.99	20.13	16.56	18
5½	44.99	22.09	18.18	19
5¾	49.17	24.15	19.87	20
6	53.54	26.30	21.63	21
6½	62.83	30.86	25.39	22
7	72.87	35.80	29.45	23
7½	83.66	41.09	33.80	24
8	95.18	46.75	38.46	25

TABLE III.

From formula (16, *a*, *b*, and *c*).

p in inches.	$\frac{H}{Dn}$ for Little or no shock.	$\frac{H}{Dn}$ for Moderate shock.	$\frac{H}{Dn}$ for Violent shock.	No.
1	0.0065	0.0032	0.0026	1
1¼	0.0101	0.0050	0.0041	2
1½	0.0146	0.0072	0.0059	3
1¾	0.0198	0.0098	0.0080	4
2	0.0259	0.0127	0.0105	5
2¼	0.0328	0.0161	0.0133	6
2½	0.0405	0.0199	0.0164	7
2¾	0.0490	0.0241	0.0198	8
3	0.0583	0.0287	0.0236	9
3¼	0.0685	0.0336	0.0277	10
3½	0.0794	0.0390	0.0321	11
3¾	0.0912	0.0448	0.0368	12
4	0.1037	0.0510	0.0419	13
4¼	0.1171	0.0575	0.0473	14
4½	0.1313	0.0645	0.0530	15
4¾	0.1463	0.0719	0.0591	16
5	0.1621	0.0796	0.0655	17
5¼	0.1787	0.0878	0.0722	18
5½	0.1961	0.0963	0.0792	19
5¾	0.2143	0.1053	0.0866	20
6	0.2334	0.1146	0.0943	21
6½	0.2739	0.1346	0.1107	22
7	0.3177	0.1560	0.1283	23
7½	0.3647	0.1790	0.1473	24
8	0.4149	0.2038	0.1676	25

Example 1. — Required the pitch of a cast-iron bevel wheel which will transmit a force of 10,000 pounds, moderate shock. In Table I., column for moderate shock, line 17, we find $P = 10{,}000$ pounds. In the pitch column, and directly opposite this value of P, we find the required pitch, $p = 5''$. Hence $l = 2p = 10''$, etc.

Example 2. — The force transmitted by a cast-iron gear under violent shock is 6,000 pounds. Required the necessary pitch. Table I., column for violent shock, line 14, gives $P = 5{,}971$ pounds; and the corresponding pitch is $p = 4\frac{1}{4}''$. Since this pitch corresponds to a value of P slightly less than the required one, we may take for our required pitch $p = 4\frac{3}{8}''$.

Example 3. — The pitch of a cast-iron gear subjected to little or no shock is $2\frac{1}{4}''$. Required the force in pounds which can be safely transmitted by the gear. In Table I., pitch column, line 6, we find $p = 2\frac{1}{4}''$. The value of P for little or no shock, corresponding to this pitch, is 4,133 pounds.

Example 4. — Required the pitch for a cast-iron gear which will safely transmit 24-horse power, violent shock, at a circumferential velocity of 8 feet per second. In this case $\frac{H}{v} = \frac{24}{8} = 3$. In Table II., column for violent shock, line 6, we find $\frac{H}{v} = 3.04$; and the corresponding pitch (found opposite this value of $\frac{H}{v}$ in the pitch column) is $p = 2\frac{1}{4}''$.

Example 5. — A certain cast-iron gear transmits 75-horse power. The pitch of the gear is $3\frac{1}{2}''$. Required the circumferential velocity safe for the gear at moderate shock. We have from Table II., column for moderate shock, the value $\frac{H}{v} = 8.95$, corresponding to $p = 3\frac{1}{2}$. Hence $\frac{75}{v} = 8.95$, $v = \frac{75}{8.95} = 8.38$ feet per second.

Example 6. — Required the pitch for a cast-iron gear to transmit safely 50-horse power, violent shock, at 100 revolutions per minute; the diameter of the gear being 16″. We have

$$\frac{H}{Dn} = \frac{50}{16 \times 100} = \frac{1}{32} = 0.03125.$$

In Table III., column for violent shock, line 11, we find $\frac{H}{Dn} = 0.0321$. The corresponding pitch is $p = 3\frac{1}{2}''$.

The following table will be found very convenient in converting decimals into fractions: —

TABLE IV.

4ths.	8ths.	16ths.	32ds.	64ths.		4ths.	8ths.	16ths.	32ds.	64ths.	
				1	0.015625					33	0.515625
			1		0.031250				17		0.531250
				3	0.046875					35	0.546875
		1			0.062500			9			0.562500
				5	0.078125					37	0.578125
			3		0.093750				19		0.593750
				7	0.109375					39	0.609375
	1				0.125000		5				0.625000
				9	0.140625					41	0.640625
			5		0.156250				21		0.656250
				11	0.171875					43	0.671875
		3			0.187500			11			0.687500
				13	0.203125					45	0.703125
			7		0.218750				23		0.718750
				15	0.234375					47	0.734375
1					0.250000	3					0.750000
				17	0.265625					49	0.765625
			9		0.281250				25		0.781250
				19	0.296875					51	0.796875
		5			0.312500			13			0.812500
				21	0.328125					53	0.828125
			11		0.343750				27		0.843750
				23	0.359375					55	0.859375
	3				0.375000		7				0.875000
				25	0.390625					57	0.890625
			13		0.406250				29		0.906250
				27	0.421875					59	0.921875
		7			0.437500			15			0.937500
				29	0.453125					61	0.953125
			15		0.468750				31		0.968750
				31	0.484375					63	0.984375
2					0.500000	4					1.000000

For very high speed gears, we may take, at the suggestion of Professor Reuleaux and Mr. W. C. Unwin, $f = \frac{10000}{\sqrt[3]{v}}$, v being the circumferential velocity in feet per second. For example, suppose it was required to determine the pitch of a cast-iron gear, the velocity of which is 35 feet per second, and the force to be transmitted is 5,000 pounds. We have for our safe working-stress

$$f = \frac{10000}{\sqrt[3]{35}} = \frac{10000}{3.27} = 3058.$$

From formula (10), taking $\frac{p}{l} = \frac{1}{4}$, we have

$$p = 4.93\sqrt{\frac{5000}{3058} \times \frac{1}{4}} = 4.93\sqrt{0.4088} = 4.93 \times 0.639$$

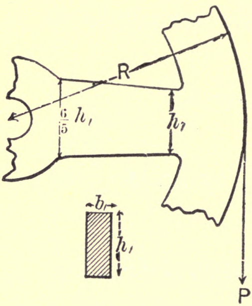
$$= 3.15'' = 3\tfrac{5}{32}'' \text{ nearly.}$$

§ XIV.—*Strength of Arms, Rim, Nave, Shafts, etc.*

The arms of a gear-wheel being symmetrically placed with reference to each other and to the rim of the wheel, we may assume, without demonstration, that each arm bears an equal share of the total strain upon the rim: in other words, the strain upon each arm is equal to the total strain upon the rim divided by the number of arms in the gear. This is a bending-strain acting in a direction perpendicular to the axis of the arm and in the plane of the wheel. The proper strength for the arms may therefore be calculated similarly to that of the teeth.

Fig. 81 represents a portion of a gear showing one arm. R is the radius of the pitch circle in inches; P, the total strain upon the rim (the total force transmitted in pounds); h_1 and b_1 respectively, the width (in the plane of the wheel) and thickness of the arm in inches; and n_1', the number of arms.* Denoting by f the greatest safe working-stress for the material, the equation for equilibrium is

Fig. 81



$$\frac{P}{n_1'} = \frac{fb_1h_1^2}{6R}.$$

We may take, in this case, for cast-iron, $f = 3{,}000$ pounds. Consequently

$$\frac{P}{n_1'} = \frac{3000b_1h_1^2}{6R} = \frac{500b_1h_1^2}{R}.$$

From this, by transposing, we have

$$b_1h_1^2 = \frac{PR}{500n_1'} \qquad (17)$$

which may be easily solved by assuming a value for b_1 in terms of h_1 $\left(\frac{b_1}{h_1} = \frac{1}{2},\ \frac{b_1}{h_1} = \frac{1}{4}, \text{ etc.}\right)$, and finding the corresponding value of h_1.

For convenience, we may write formula (17) in the form of an equation having one unknown quantity, thus: —

$$h_1 = x\sqrt[3]{PR} \qquad (18)$$

---

* The dimensions b_1 and h_1 are taken *at the rim* of the wheel, and tapered, as shown in Fig. 81.

and find values of the co-efficient x for different values of b_1 and n_1'.*

The following table gives values of x for different values of $\frac{b_1}{h_1}$ and n_1':—

TABLE V.

$\frac{b_1}{h_1}$	x for $n_1'=$				
	4	5	6	8	10
$\frac{1}{2}$	0.100	0.093	0.087	0.079	0.074
$\frac{1}{4}$	0.126	0.117	0.110	0.100	0.093
$\frac{1}{6}$	0.144	0.134	0.126	0.114	0.106
$\frac{1}{8}$	0.159	0.147	0.139	0.126	0.117

Rule.—To determine the width of cast-iron gear arms in the plane of the wheel, multiply the force transmitted by the radius of the pitch circle, extract the cube root of this product, and multiply the result by the tabular number corresponding to the given values of $\frac{b_1}{h_1}$ and n_1'.

Example 1.—A cast-iron gear the diameter of which is 48″ transmits a force of 5,000 pounds. Required the width and thickness of the arms, of which there are 5. If we assume $\frac{b_1}{h_1} = \frac{1}{4}$, Table V. gives, for the value of the co-efficient, $x = 0.117$. Hence formula (18) becomes

* This form is given by Unwin in Elements of Machine Design.

$$h_1 = 0.117\sqrt[3]{PR} = 0.117\sqrt[3]{5000 \times 24} = 0.117\sqrt[3]{12000} = 0.117 \times 49.324 = 5.77''$$

or in fractions, from Table IV., $h_1 = 5\frac{49}{64}''$: hence

$$b_1 = \tfrac{1}{4}h_1 = \tfrac{1}{4} \times 5.77 = 1.4425'' = 1\tfrac{7}{16}''.$$

Example 2. — A cast-iron 72″ gear transmits a force of 15,000 pounds. Taking $n_1' = 6$, and $\frac{b_1}{h_1} = \frac{1}{2}$, required the dimensions of the arms. From Table V., $x = 0.087$: hence, from formula (18), we have

$$h_1 = 0.087\sqrt[3]{PR} = 0.087\sqrt[3]{15000 \times 36} = 0.087 \times 81.433 = 7.0847'' = 7\tfrac{5}{64}''.$$

For the thickness we have

$$b_1 = \tfrac{1}{2}h_1 = \tfrac{1}{2} \times 7.0847 = 3.54235'' = 3\tfrac{35}{64}''.$$

If, instead of rectangular, we have circular cross-sections for gear arms, and represent the diameter by d', the equation for equilibrium becomes

$$\frac{P}{n_1'} = \frac{f \times 0.0982d'^3}{R}$$

or, for cast-iron,

$$\frac{P}{n_1'} = \frac{3000 \times 0.0982d'^3}{R}.$$

Reducing and transposing this equation gives

$$d' = 0.15\sqrt[3]{\frac{PR}{n_1'}} \qquad (19).$$

Rule. — To determine the diameter for cast-iron gear arms having circular cross-sections, multiply the force transmitted by the pitch radius, divide this product by the number of arms, extract the cube root of the quotient thus obtained, and multiply the result by 0.15.

Example 3. — A cast-iron gear of 36″ diameter has 5 arms (circular cross-sections), and transmits a force of 600 pounds. Required the diameter for the arms. From formula (19) we have

$$d' = 0.15\sqrt[3]{\frac{18 \times 600}{5}} = 0.15\sqrt[3]{2160} = 0.15 \times 12.927 = 1.939''$$

or, from Table IV., $d' = 1\frac{15}{16}''$.

For elliptical cross-sections, representing by a the major and by b' the minor axis, the equation for equilibrium is

$$\frac{P}{n_1'} = \frac{f \times 0.0982b'a^2}{R}$$

or, for cast-iron,

$$\frac{P}{n_1'} = \frac{3000 \times 0.0982b'a^2}{R}.$$

Hence, by reducing and transposing, we obtain

$$b'a^2 = 0.00339\frac{PR}{n_1'} \qquad (20).$$

Rule. — To determine the dimensions for cast-iron gear arms having elliptical cross-sections, multiply the force transmitted by the pitch radius, multiply the product thus obtained by 0.00339, and divide the result by the number of arms. This gives the product of the minor into the square of the major axis ($b'a^2$) : the axes may then be found as in formula (17).

Example 4. — Required the axes for the cross-sections of the elliptical arms of a cast-iron gear, the diameter of which is 24″. The force transmitted is 800 pounds, and the number of arms 3. Let us assume a relation between the cross-section axes, say $b' = \frac{a}{2}$. Formula (20) then gives

$$\frac{a^3}{2} = 0.00339 \frac{800 \times 12}{3}$$

or

$$a^3 = \frac{0.00339 \times 800 \times 12 \times 2}{3} = 21.696.$$

Hence

$$a = \sqrt[3]{21.696} = 2.789'' = 2\tfrac{51}{64}''.$$

Also

$$b' = \frac{a}{2} = \frac{2.789}{2} = 1.3945'' = 1\tfrac{25}{64}''.$$

For arms having flanged cross-sections, such as is shown in Fig. 82, the equation for equilibrium becomes

$$\frac{P}{n_1'} = \frac{f}{R} \times \frac{b_{\prime\prime}H'^3 + Bh_{\prime\prime}^3}{6H'}.$$

Substituting for f its value of 3,000 pounds, and reducing, we obtain, for cast-iron,

$$\frac{P}{n_1'} = \frac{500(b_{\prime\prime}H'^3 + Bh_{\prime\prime}^3)}{RH'}$$

or

$$\frac{b_{\prime\prime}H'^3 + Bh_{\prime\prime}^3}{H'} = \frac{PR}{500n_1'} \qquad (21).$$

Example 5.—Required the dimensions for the arms of a 36″ cast-iron gear which transmits a force of 800 pounds; the arms to be flanged, as in Fig. 82, and to be 4 in number. Let us assume relations between the

Fig. 82

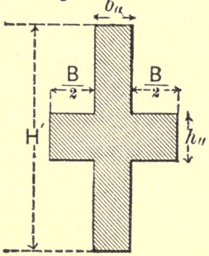
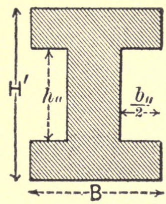


Fig. 83



several unknown quantities in formula (21). Thus, suppose

$$b_{\prime\prime} = h_{\prime\prime} = \frac{B}{2} = \frac{H'}{5}.$$

By substitution the formula becomes

$$\frac{\left(\frac{H'}{5} \times H'^3\right) + \left(\frac{2H'}{5} \times \frac{H'^3}{125}\right)}{H'} = \frac{800 \times 18}{500 \times 4}.$$

Reducing, we have

$$\frac{H'^3}{5} + \frac{2H'^3}{625} = \frac{127H'^3}{625} = \frac{36}{5}.$$

Hence

$$H' = \sqrt[3]{\frac{36 \times 625}{5 \times 127}} = \sqrt[3]{35.433} = 3.2845''.$$

For the other dimensions,

$$b_{\prime\prime} = h_{\prime\prime} = \frac{H'}{5} = \frac{3.2845}{5} = 0.6569''$$

and

$$B = \frac{2H'}{5} = \frac{2 \times 3.2845}{5} = 1.3138''.$$

Converted into fractions by means of Table IV., the arm dimensions are $H' = 3\frac{9}{32}''$, $b_{\prime\prime} = h_{\prime\prime} = \frac{21}{32}''$, and $B = 1\frac{5}{16}''$.

For arms with cross-sections, flanged as in Fig. 83, the equation for equilibrium is

$$\frac{P}{n_1'} = \frac{f}{R} \times \frac{BH'^3 - b_{\prime\prime}h_{\prime\prime}^3}{6H'}.$$

Making $f = 3{,}000$ in this equation, we have, for cast-iron,

$$\frac{P}{n_1'} = \frac{500(BH'^3 - b_{\prime\prime}h_{\prime\prime}^3)}{RH'}$$

or

$$\frac{BH'^3 - b_{\prime\prime}h_{\prime\prime}^3}{H'} = \frac{PR}{500n_1'} \qquad (22).$$

Example 6. — A 48″ cast-iron gear transmits a force of 1,000 pounds, and has 5 arms, the cross-sections being flanged as in Fig. 83. Required the arm dimensions. Let us take $B = \frac{H'}{2}$, $h_{\prime\prime} = \frac{3}{4}H'$, and $\frac{b_{\prime\prime}}{2} = \frac{1}{8}H'$. These values, substituted in formula (22), give

$$\frac{\left(\frac{H'}{2} \times H'^3\right) - \left(\frac{H'}{4} \times \frac{27H'^3}{64}\right)}{H'} = \frac{1000 \times 24}{500 \times 5}.$$

Reducing, we have

$$\frac{H'^3}{2} - \frac{27H'^3}{256} = \frac{48}{5}.$$

$$H' = \sqrt[3]{\frac{48}{5} \times \frac{256}{101}} = \sqrt[3]{24.333} = 2.898'' = 2\tfrac{29}{32}''$$

$$B = \frac{H'}{2} = \frac{2.898}{2} = 1.449'' = 1\tfrac{29}{64}$$

$$h_{,,} = \tfrac{3}{4}H' = 2.1735'' = 2\tfrac{11}{64}''$$

and

$$\frac{b_{,,}}{2} = \tfrac{1}{8}H' = 0.3623'' = \tfrac{23}{64}''.$$

The number of arms in a gear-wheel is often determined, according to the pitch diameter, by the following table:—

For a gear of	1½	to	3¼	feet diameter,	4 arms.
For a gear of	3¼	to	5	feet diameter,	5 arms.
For a gear of	5	to	8½	feet diameter,	6 arms.
For a gear of	8½	to	16	feet diameter,	8 arms.
For a gear of	16	to	25	feet diameter,	10 arms.

Reuleaux gives for the number of arms the formula

$$n_1' = 0.56\sqrt{N}\sqrt[4]{p} \qquad (23)$$

in which n_1' is the number of arms, N the number of teeth in the gear, and p the pitch.*

Rule.—To determine the number of arms for a gear-wheel, extract the square root of the number of teeth and the fourth root of the pitch, multiply the roots together and the product by 0.56.

Example 6 *a.*—A gear-wheel has 100 teeth and a

* Small pinions, and sometimes narrow-faced gears, are made without arms; i.e., having a continuous web cast between the rim and nave.

pitch of 1″. Required the number of arms. From formula (23)

$$n_1' = 0.56\sqrt{100\sqrt[4]{1}} = 0.56 \times 10 \times 1 = 5.6 \text{ or } 6.$$

A convenient formula for the arm dimensions, in terms of the horse-power transmitted and the revolutions, may be obtained as follows. As explained in § XIII., we have the expressions

$$v = 0.00873Rn \quad \text{and} \quad P = \frac{550H}{v}$$

v being the circumferential velocity in feet per second, H the horse-power, and n the revolutions per minute. By combining these we obtain

$$P = \frac{550H}{0.00873Rn} = 63000\frac{H}{Rn}.$$

This value of P substituted in formula (17) gives

$$b_1h_1^2 = \frac{63000H}{Rn} \times \frac{R}{500n_1'}$$

or

$$b_1h_1^2 = 126\frac{H}{nn_1'}, \qquad (24).$$

Rule.—To determine the quantity $b_1h_1^2$ (the thickness multiplied by the square of the width) for cast-iron gear arms, from the horse-power and revolutions, multiply the horse-power by 126, and divide by the product of the number of revolutions into the number of arms.

Example 7.—A 36″ cast-iron gear makes 80 revolutions per minute, and transmits 15-horse power. Required the dimensions of the arms. From the table we have for the number of arms $n_1' = 4$, and from formula (24)

$$b_1h_1^2 = \frac{126 \times 15}{80 \times 4} = 5.906.$$

We may now assume $b_1 = \frac{h_1}{4}$: hence

$$b_1h_1^2 = \frac{h'^3}{4} = 5.906$$

and

$$h_1 = \sqrt[3]{23.624} = 2.869'' = 2\tfrac{7}{8}''$$

$$b_1 = \frac{h_1}{4} = \frac{2.869}{4} = 0.717'' = \tfrac{23}{32}''.$$

For arms having circular cross-sections we have, as above,

$$P = 63000 \frac{H}{Rn}$$

which, substituted in formula (19), gives, for the diameter of the arm cross-section,

$$d' = 0.15\sqrt[3]{63000 \frac{H}{Rn} \times \frac{R}{n_1'}}$$

or

$$d' = 5.969\sqrt[3]{\frac{H}{nn_1'}} \qquad (25).$$

Rule.—To determine the diameter for cast-iron gear arms having circular cross-sections, from the horse-power and revolutions, divide the horse-power by the product of the number of revolutions per minute into the number of arms, extract the cube root of this quotient, and multiply the result by 5.969.

Example 8.—The diameter of a cast-iron gear is 48″, the horse-power transmitted 15, and the number of revolutions per minute 40. Required the diameter for the circular cross-sections of the arms. From the table, the number of arms is 5: hence, from formula (25),

$$d' = 5.969\sqrt[3]{\frac{15}{40 \times 5}} = 5.969 \times 0.4217 = 2.517 = 2\tfrac{33}{64}''.$$

For elliptical cross-sections, of which a and b' are respectively the major and minor axes, we have, by substituting in formula (20), the value

$$P = 63000\frac{H}{Rn},$$

$$b'a^2 = 0.00339 \times 63000\frac{H}{Rn} \times \frac{R}{n_1'}$$

or

$$b'a^2 = 213.57\frac{H}{nn_1'} \qquad (26).$$

Rule. — To determine the quantity $b'a^2$ (the minor axis multiplied by the square of the major), for cast-iron gear arms having elliptical cross-sections, from the horse-power and revolutions, multiply the horse-power by 213.57, and divide by the product of the number of revolutions per minute into the number of arms.

Example 9. — A 48″ cast-iron gear makes 40 revolutions per minute, and transmits 20-horse power. Required the arm dimensions for elliptical cross-sections. In this case, $n_1' = 4$, and hence formula (26) gives

$$b'a^2 = 213.57\frac{20}{40 \times 5} = 21.357.$$

If we take $b' = \frac{1}{2}a$, we shall have

$$b'a^2 = \frac{a^3}{2} = 21.357$$

and

$$a = \sqrt[3]{21.357 \times 2} = 3.496'' = 3\tfrac{1}{2}''$$

$$b' = \frac{a}{2} = \frac{3.496}{2} = 1.748'' = 1\tfrac{3}{4}''.$$

For arms having cross-sections flanged, as shown in Fig. 82, we obtain, by substituting in formula (21) the value of P determined above,

$$\frac{b_{\prime\prime}H'^3 + Bh_{\prime\prime}^3}{H'} = 63000\frac{H}{Rn} \times \frac{R}{500n_1'}$$

or

$$\frac{b_{\prime\prime}H'^3 + Bh_{\prime\prime}^3}{H'} = \frac{126H}{nn_1'} \qquad (27)$$

which may be solved as explained in Example 5 of this section.

Similarly, for arms having cross-sections flanged, as in Fig. 83, we obtain

$$\frac{BH'^3 - b_{\prime\prime}h_{\prime\prime}^3}{H'} = \frac{126H}{nn_1'} \qquad (28).$$

It is often convenient to calculate the dimensions of the arms from the pitch and radius of the gear. Formulas for the arm dimensions, in terms of these quantities, may be obtained as follows: —

From formula (12, b) we may write

$$P = \frac{p^2}{0.0025}$$

which, substituted in formula (17), gives

$$b_1 h_1^2 = \frac{p^2}{0.0025} \times \frac{R}{500 n_1'} = \frac{p^2 R}{12.5 n_1'}$$

or

$$b_1 h_1^2 = \frac{0.8 p^2 R}{n_1'} \qquad (29).$$

Rule. — To determine the quantity $b_1 h_1^2$ (the thickness of the arm multiplied by the square of its width) from the pitch and radius of the gear, divide the continued product of 0.8 into the square of the pitch into the radius, by the number of arms.

Example 10. — Required the dimensions for the arms of a gear-wheel, the diameter of which is 24″, and the pitch 1″. In this case, $n_1' = 4$: hence, from formula (29),

$$b_1 h_1^2 = \frac{0.8 \times 1 \times 12}{4} = 2.4.$$

If we take $b_1 = \frac{h_1}{6}$,

$$b_1 h_1^2 = \frac{h_1^3}{6} = 2.4$$

$$h_1 = \sqrt[3]{2.4 \times 6} = 2.433'' = 2\tfrac{7}{16}''$$

$$b_1 = \frac{h_1}{6} = \tfrac{13}{32}''.$$

By substituting $P = \frac{p^2}{0.0025}$ in formulas (19), (20), (21), and (22), the following formulas may be obtained. For arms having circular cross-sections, of which d' is the diameter,

$$d' = 1.105 \sqrt[3]{\frac{p^2 R}{n_1'}} \qquad (30).$$

For elliptical cross-sections, a and b' being the major and minor axes respectively,

$$b'a^2 = 1.356\frac{p^2R}{n_1'} \qquad (31).$$

For cross-sections, as shown in Fig. 82,

$$\frac{b_{\prime\prime}H'^3 + Bh_{\prime\prime}^3}{H'} = \frac{0.8p^2R}{n_1'} \qquad (32).$$

For cross-sections, as shown in Fig. 83,

$$\frac{BH'^3 - b_{\prime\prime}h_{\prime\prime}^3}{H'} = \frac{0.8p^2R}{n_1'} \qquad (33).$$

Example 11.—Taking the data of Example 10, required the diameter for arms having circular cross-sections. Formula (30) gives, by substituting the numerical data,

$$d' = 1.105\sqrt[3]{\frac{1 \times 12}{4}} = 1.105\sqrt[3]{3} = 1.15937'' = 1\tfrac{19}{32}''.$$

Example 12.—With the same data, required the dimensions for arms having elliptical cross-sections. From formula (31) we have

$$b'a^2 = 1.356\frac{1 \times 12}{4} = 1.356 \times 3 = 4.068.$$

Assuming $b' = \frac{1}{2}a$

$$b'a^2 = \frac{a^3}{2} = 4.068$$

$$a = \sqrt[3]{4.068 \times 2} = 2''$$

$$b' = \tfrac{1}{2}a = 1''.$$

Example 13. — Using the same data, it is required to determine the dimensions for flanged arms having cross-sections, such as shown in Fig. 83.

From formula (33)

$$\frac{BH'^3 - b_{''}h_{''}^3}{H'} = \frac{0.8 \times 1 \times 12}{4} = 2.4.$$

Let us take $B = h_{''} = \frac{1}{2}H'$ and $\frac{b_{''}}{2} = \frac{1}{8}H'$: hence

$$\frac{BH'^3 - b_{''}h_{''}^3}{H'} = \frac{\frac{H'^4}{2} - \left(\frac{H'}{4} \times \frac{H'^3}{8}\right)}{H'} = \frac{H'^3}{2} - \frac{H'^3}{32} = \frac{15H'^3}{32} = 2.4$$

$$H' = \sqrt[3]{2.4 \times \tfrac{32}{15}} = \sqrt[3]{5.12} = 1.724'' = 1\tfrac{47}{64}''$$

$$B = h_{''} = \tfrac{1}{2}H' = 0.862'' = \tfrac{55}{64}''$$

and

$$\frac{b_{''}}{2} = \tfrac{1}{8}H' = 0.215'' = \tfrac{7}{32}''.$$

More often than otherwise, the arms of gear-wheels are made straight, as in Fig. 81: sometimes, however, especially in large gears and in gears subjected to violent shock and strain, curved arms are preferred, as tending to stiffen and support the rim better. Also curved arms, as a general rule, cast better. When single curved arms are used, they may be constructed as follows: —

After having determined the number of arms by one of the foregoing rules, and having marked their centres *A*, *C* (Fig. 84), upon the circumference *ABC*, take the arc $AB = \frac{2}{3}$ arc *AC*, and draw the radial line *OB*. From the centre *O* of the wheel, erect the line *OD* perpendicular to *OB*, and find upon *OD*, by trial, the centre *a* for a circular arc passing through the points *O* and *A*. This arc is the axis of the arm. Lay off, as shown in the figure, *h* (*d'*, *a*, or *H'*, according as the cross-sections are to be rectangular, circular, elliptical, or flanged*) at the rim, and not less than $\frac{6}{5}h$ ($\frac{6}{5}d'$, $\frac{6}{5}a$, or $\frac{6}{5}H'$) at the nave. Find upon *OD*, by trial, the centres *b* and *c* for the arcs *gk* and *df*, which determine the form of the arms.

Fig.84

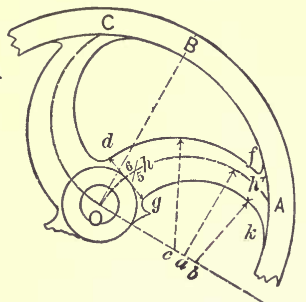
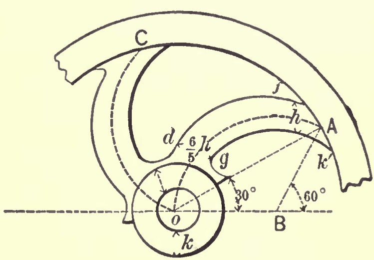


Fig. 85 shows another method for drawing curved arms. Through the centre *o* of the wheel draw the line *oA*, making 30° with the horizontal. Draw also the line *AB*, making 60° with the horizontal. The point *B* is the centre for the axis *oA* of the arm. Lay off, as before, *h* and $\frac{6}{5}h$, and find upon the line *oB* the centres for the arcs *df* and *gk'*.

Fig.85

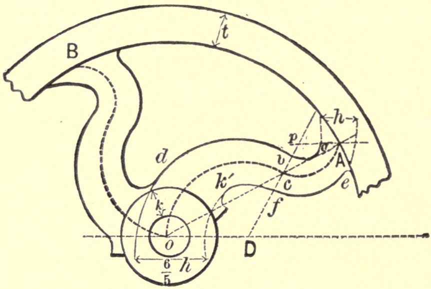


Double curved arms are sometimes used for large gears. Fig. 86 shows a simple method for their construction. Draw the radial line *oA*, making 30° with the horizontal. Take $oc = \frac{2}{3}oA$, and through the point *c* draw the line *pD*, making 60° with the horizontal. Intersect

* The cross-sections of curved arms are generally elliptical, the curved form giving sufficient stiffness to dispense with flanges, etc.

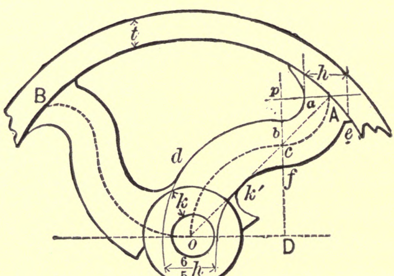
the line pD by a horizontal line through the point A: the points D and p are respectively the centres for the arcs oc and cA, which together form the axis

Fig.86



of the arm. Lay off the arm widths as shown in the figure. From the point p as a centre strike the arcs ab and ef, and find upon the line oD the centres for the remaining arcs bd and fk'.

Fig.87



Another very similar method for laying out double curved arms is shown in Fig. 87. Draw the radial line oA, making 45° with the horizontal. Take $oc = \frac{2}{3}oA$, and through the point c draw the vertical line pD. Intersect the line pD by the horizontal line Ap. The points p and D are the centres for the arcs of the axis. Lay off h and $\frac{6}{5}h$, as shown in the figure, and proceed, as in Fig. 86, to strike the arcs ab, ef, bd, and fk'.

Rim: For the thickness of the rim in the plane of the wheel, t (Fig. 87), Reuleaux gives the formula

$$t = 0.12 + 0.4p \qquad (34)$$

in which t is the rim thickness, and p the pitch.

Rule. — To determine the thickness of the rim of a cast-iron gear-wheel, multiply the pitch by 0.4, and to this product add 0.12″.

Example 14. — Required the thickness of rim for a gear having a pitch of $3\frac{1}{2}''$. From formula (34)

$$t = 0.12 + 0.4 \times 3.5 = 0.12 + 1.40 = 1.52'' = 1\tfrac{1}{2}''.$$

A simple and not very accurate rule in use in the shops is to take the rim thickness equal to $\frac{3}{4}$ the pitch.

Nave: The old formulas for the thickness of the nave — (k, Fig. 85) $k = \frac{3}{4}p$ and $k = \frac{1}{2}d$, in which k is the nave thickness, p the pitch, and d the diameter of the eye of the wheel — are probably nearly correct, notwithstanding their simplicity. Unwin gives the formula

$$k = 0.4\sqrt[3]{p^2R} + \tfrac{1}{2} \qquad (35)$$

in which p is the pitch of the teeth, and R the radius of the wheel.

Rule. — To determine the thickness of the nave of a cast-iron gear wheel, multiply the square of the pitch by the radius of the wheel, extract the cube root of this product, multiply the result by 0.4, and add $\frac{1}{2}''$.

Example 15.—The diameter of a gear is 36″ and the pitch $1\frac{1}{2}''$. Required the thickness of the nave. From formula (35) we have

$$k = 0.4\sqrt[3]{(1\tfrac{1}{2})^2 \times 18} + 0.5 = 0.4\sqrt[3]{40.5} + 0.5$$

$$= 0.4 \times 3.434 + 0.5 = 1.874'' = 1\tfrac{7}{8}''.$$

By the formula $k = \frac{3}{4}p$, we would have

$$k = 0.75 \times 1.5 = 1.125'' = 1\tfrac{1}{8}''.$$

Thus, in this case the difference between the results of the two formulas is $\frac{3}{4}''$.

For the length of the nave we may use the formula

$$l' = l + \frac{D}{30} \qquad (36)$$

in which l' is the length of the nave, l the face width of the teeth, and D the diameter of the gear.

Rule.—To determine the nave length of a gear, divide the diameter of the gear by 30 and to the quotient add the face width of the teeth.

Example 16.—The diameter of a gear is 60″ and the face width of the teeth 8″. Required the length of the nave. Formula (36) gives

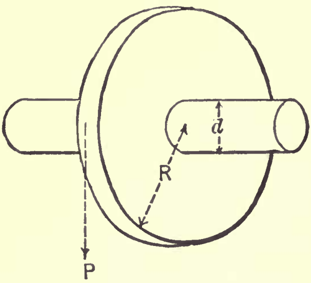
$$l' = 8 + \tfrac{60}{30} = 8 + 2 = 10''.$$

According to Unwin, the length of the nave should never be less than three times its thickness. He gives, for the length, the formula $l' = l + 0.06R$, which agrees very nearly with formula (36).

Shafts: When a shaft is so supported by its bearings as to be subjected to a *torsional* strain only, as is almost invariably the case in gear shafts (the bending-strain, due to the weight of the gear and the pressure between the gears in the direction of a line joining their centres, being ordinarily slight enough to be safely neglected), the calculation of the proper strength for the shaft may be made as follows:—

In Fig. 88, P represents the total force tending to twist the shaft, i.e., the total force transmitted by the gear; R, the distance from the centre of the shaft to the point at which the force acts, i.e., the radius of the gear; and d, the diameter of the shaft. The greatest safe torsional strain which can be sustained by the shaft is given by the expression

Fig. 88



$$P = \frac{\pi f' d^3}{16R} = 0.19635 \frac{f' d^3}{R}$$

in which f' is the greatest safe shearing-stress in pounds per square inch for the material of the shaft. From this,

$$d = \sqrt[3]{\frac{PR}{0.19635 f'}}$$

or

$$d = 1.720 \sqrt[3]{\frac{PR}{f'}} \qquad (37).$$

Rule.—To determine the diameter of a gear shaft of any material, multiply the total force transmitted by the gear by the radius of the gear, divide this product by the greatest safe shearing-stress in pounds per square inch for the material of the shaft, extract the cube root of the quotient thus obtained, and multiply the result by 1.720.

Example 17. — Required the diameter for an oak shaft, upon which is a 60″ gear transmitting a force of 1,000 pounds, taking $f' = 500$ pounds. From formula (37),

$$d = 1.720\sqrt[3]{\frac{1000 \times 30}{500}} = 1.720 \times 3.915 = 6.734'' = 6\tfrac{47}{64}''.$$

We propose to take, for steel, $f' = 12{,}000$ pounds; for wrought-iron, $f' = 8{,}000$ pounds; and, for cast-iron, $f' = 4{,}000$ pounds. These values of f' are nearly mean between those used by Stoney, Haswell, and Unwin, which differ far more than is conducive to any degree of accuracy. Substituting the above values of f' successively in formula (37), and reducing, we obtain;

For steel, $d = 0.075\sqrt[3]{PR}$ (38)

For wrought-iron, $d = 0.086\sqrt[3]{PR}$ (39)

For cast-iron, $d = 0.108\sqrt[3]{PR}$ (40).

Rule. — To determine the diameter for a gear shaft of steel, wrought or cast iron, multiply the total force transmitted by the radius of the gear, extract the cube root of the product, and multiply the result by 0.075 for steel, 0.086 for wrought-iron, and 0.108 for cast-iron.

Example 18. — A 48″ gear transmits a force of 100,000 pounds. Required the diameter for a steel shaft. From formula (38) we have

$$d = 0.075\sqrt[3]{100000 \times 24} = 0.075 \times 62.145 = 4.66'' = 4\tfrac{43}{64}''.$$

Example 19. — Taking the data of Example 18, required the diameter for a shaft of cast-iron. Formula (40) gives

$$d = 0.108\sqrt[3]{100000 \times 24} = 0.108 \times 62.145 = 6.712'' = 6\tfrac{45}{64}''.$$

Formulas for the diameters of gear shafts, in terms of the horse-power transmitted and the revolutions per minute, may be obtained as follows: —

As before explained, we have the expression

$$P = 63000\frac{H}{Rn}$$

H representing the horse-power, R the radius of the gear, and n the number of revolutions per minute. Substituting this value of P in formulas (37), (38), (39), and (40), and reducing, we obtain the following: —

General formula, $d = 68.44\sqrt[3]{\dfrac{H}{nf'}}$ (41)

For steel, $d = 2.984\sqrt[3]{\dfrac{H}{n}}$ (42)

For wrought-iron, $d = 3.422\sqrt[3]{\dfrac{H}{n}}$ (43)

For cast-iron, $d = 4.297\sqrt[3]{\dfrac{H}{n}}$ (44).

Rule. — To determine the diameter for a gear shaft of any material, from the horse-power and number of revolutions per minute, divide the horse-power by the product of the number of revolutions into the greatest safe shearing-stress in pounds per square inch for the material of the shaft, extract the cube root of the quotient thus obtained, and multiply the result by 68.44.

To determine the diameter for a gear shaft of steel, wrought or cast iron, from the horse-power and number of revolutions per minute, divide the horse-power by the number of revolutions, extract the cube root of the quotient, and multiply the result by 2.984 for steel, 3.422 for wrought-iron, and 4.297 for cast-iron.

Example 20. — Required the diameter for an oak gear shaft which transmits a force of 10-horse power, and makes 40 revolutions per minute. If we take for the greatest safe shearing-stress for oak $f' = 500$ pounds per square inch, we shall have, from formula (41),

$$d = 68.44\sqrt[3]{\frac{10}{40 \times 500}} = 68.44\sqrt[3]{\frac{1}{2000}} = 68.44 \times \frac{1}{12.60}$$
$$= 5.432'' = 5\tfrac{7}{16}'' \text{ nearly.}$$

Example 21. — Taking the data of Example 20, required the diameters for shafts of steel and wrought-iron. From formula (42),

$$d = 2.984\sqrt[3]{\tfrac{10}{40}} = 2.984\sqrt[3]{0.25} = 2.984 \times 0.62996 = 1.88'' = 1\tfrac{57}{64}''$$

for steel. From formula (43),

$$d = 3.422\sqrt[3]{\tfrac{10}{40}} = 3.422 \times 0.62996 = 2.1557'' = 2\tfrac{5}{32}''$$

for wrought-iron.

Convenient formulas for gear-shaft diameters in terms of the pitch and radius, may be obtained in the following manner. From formula (12, *b*) we have, as before,

$$P = \frac{p^2}{0.0025} = 400p^2$$

which value, substituted in formulas (37), (38), (39), and (40), gives the following formulas:—

General formula, $d = 12.673\sqrt[3]{\frac{p^2R}{f'}}$ (45)

For steel, $d = 0.553\sqrt[3]{p^2R}$ (46)

For wrought-iron, $d = 0.634\sqrt[3]{p^2R}$ (47)

For cast-iron, $d = 0.796\sqrt[3]{p^2R}$ (48).

Rule. — To determine the diameter of a gear shaft of any material, from the pitch and radius of the gear, multiply the square of the pitch by the radius, divide the product by the greatest safe shearing-stress in pounds per square inch for the material of the shaft, extract the cube root of the quotient thus obtained, and multiply the result by 12.673. To determine the diameter of a gear shaft of steel, wrought or cast iron, from the pitch and radius of the gear, multiply the square of the pitch by the radius, extract the cube root of the product, and multiply the result by 0.553 for steel, 0.634 for wrought-iron, and 0.796 for cast-iron.*

* The expression $P = 400p^2$ is true only for cast-iron gears: hence the value of p^2 in formulas (45), (46), (47), and (48), must be for a cast-iron gear.

Example 22. — A cast-iron gear has a diameter of 12″ and a pitch of $\frac{1}{2}$″. Required the diameter for a brass shaft, supposing $f' = 3{,}000$ pounds for brass. From formula (45)

$$d = 12.673\sqrt[3]{\frac{(\frac{1}{2})^2 \times 6}{3000}} = 12.673\sqrt[3]{0.0005} = 12.673 \times 0.07937 = 1''.$$

Example 23. — The diameter of a cast-iron gear is 60″ and the pitch 2″. Required the diameters for shafts of steel and wrought-iron. From formula (46)

$$d = 0.553\sqrt[3]{4 \times 30} = 0.553\sqrt[3]{120} = 0.553 \times 4.932$$
$$= 2.727'' = 2\tfrac{47}{64}''$$

for steel. From formula (47) we have

$$d = 0.634\sqrt[3]{4 \times 30} = 0.634 \times 4.932 = 3.127'' = 3\tfrac{1}{8}''$$

for wrought-iron.

Gear shafts are most commonly of wrought-iron: when, however, wrought-iron shafts, in order to give the necessary strength, become so large as to be inconvenient, steel shafts are used. Cast-iron shafts are, as a rule, unreliable and treacherous; they are therefore seldom used, except for the transmission of slight powers, and in cheap, inferior machinery. The following tables, calculated from formulas (38), (39), (42), and (43), to the nearest $\frac{1}{64}$″, will be found very convenient in designing gear shafts of steel and wrought-iron: —

TABLE VI.

PR	*d* for steel.	*d* for wrought iron.	*PR*	*d* for steel.	*d* for wrought iron.
250	$\frac{15}{32}''$	$\frac{35}{64}''$	60000	$2\frac{15}{16}''$	$3\frac{23}{64}'$
500	$\frac{19}{32}$	$\frac{11}{16}$	70000	$3\frac{3}{32}$	$3\frac{35}{64}$
1000	$\frac{3}{4}$	$\frac{55}{64}$	80000	$3\frac{15}{64}$	$3\frac{45}{64}$
1500	$\frac{55}{64}$	$\frac{63}{64}$	90000	$3\frac{23}{64}$	$3\frac{55}{64}$
2000	$\frac{61}{64}$	$1\frac{5}{64}$	100000	$3\frac{31}{64}$	4
2500	$1\frac{1}{64}$	$1\frac{11}{64}$	110000	$3\frac{19}{32}$	$4\frac{1}{8}$
3000	$1\frac{5}{64}$	$1\frac{15}{64}$	120000	$3\frac{45}{64}$	$4\frac{15}{64}$
3500	$1\frac{9}{64}$	$1\frac{5}{16}$	130000	$3\frac{54}{64}$	$4\frac{23}{64}$
4000	$1\frac{3}{16}$	$1\frac{23}{64}$	140000	$3\frac{57}{64}$	$4\frac{15}{32}$
4500	$1\frac{15}{64}$	$1\frac{27}{64}$	150000	$3\frac{63}{64}$	$4\frac{9}{16}$
5000	$1\frac{9}{32}$	$1\frac{15}{32}$	175000	$4\frac{13}{64}$	$4\frac{13}{16}$
6000	$1\frac{23}{64}$	$1\frac{9}{16}$	200000	$4\frac{25}{64}$	$5\frac{1}{32}$
7000	$1\frac{7}{16}$	$1\frac{41}{64}$	250000	$4\frac{23}{32}$	$5\frac{27}{64}$
8000	$1\frac{1}{2}$	$1\frac{23}{32}$	500000	$5\frac{61}{64}$	$6\frac{53}{64}$
10000	$1\frac{39}{64}$	$1\frac{55}{64}$	750000	$6\frac{13}{16}$	$7\frac{13}{16}$
12500	$1\frac{3}{4}$	2	1000000	$7\frac{1}{2}$	$8\frac{39}{64}$
15000	$1\frac{27}{32}$	$2\frac{1}{8}$	1500000	$8\frac{37}{64}$	$9\frac{27}{32}$
20000	$2\frac{1}{32}$	$2\frac{21}{64}$	2000000	$9\frac{29}{64}$	$10\frac{53}{64}$
25000	$2\frac{3}{16}$	$2\frac{33}{64}$	2500000	$10\frac{11}{64}$	$11\frac{43}{64}$
30000	$2\frac{21}{64}$	$2\frac{43}{64}$	3000000	$10\frac{13}{64}$	$12\frac{13}{32}$
35000	$2\frac{29}{64}$	$2\frac{13}{16}$	3500000	$11\frac{25}{64}$	$13\frac{1}{16}$
40000	$2\frac{9}{16}$	$2\frac{15}{16}$	4000000	$11\frac{29}{32}$	$13\frac{21}{32}$
45000	$2\frac{43}{64}$	$3\frac{1}{16}$	4500000	$12\frac{3}{8}$	$14\frac{13}{64}$
50000	$2\frac{49}{64}$	$3\frac{11}{64}$	5000000	$12\frac{53}{64}$	$14\frac{45}{64}$

TABLE VII.

$\frac{H}{n}$	d for steel.	d for wrought iron.	$\frac{H}{n}$	d for steel.	d for wrought iron.
0.025	$\frac{7}{8}''$	$1''$	3.75	$4\frac{41}{64}''$	$5\frac{5}{16}''$
0.050	$1\frac{3}{32}$	$1\frac{17}{64}$	4	$4\frac{47}{64}$	$5\frac{7}{16}$
0.075	$1\frac{17}{64}$	$1\frac{7}{16}$	4.25	$4\frac{53}{64}$	$5\frac{35}{64}$
0.100	$1\frac{25}{64}$	$1\frac{19}{32}$	4.50	$4\frac{15}{16}$	$5\frac{21}{32}$
0.150	$1\frac{37}{64}$	$1\frac{13}{16}$	4.75	$5\frac{1}{64}$	$5\frac{3}{4}$
0.200	$1\frac{3}{4}$	2	5	$5\frac{7}{64}$	$5\frac{55}{64}$
0.250	$1\frac{7}{8}$	$2\frac{5}{32}$	5.50	$5\frac{17}{64}$	$6\frac{3}{64}$
0.300	2	$2\frac{19}{64}$	6	$5\frac{27}{64}$	$6\frac{7}{32}$
0.350	$2\frac{7}{64}$	$2\frac{13}{32}$	6.50	$5\frac{9}{16}$	$6\frac{25}{64}$
0.400	$2\frac{13}{64}$	$2\frac{33}{64}$	7	$5\frac{45}{64}$	$6\frac{35}{64}$
0.500	$2\frac{3}{8}$	$2\frac{23}{32}$	8	$5\frac{31}{32}$	$6\frac{27}{32}$
0.600	$2\frac{33}{64}$	$2\frac{57}{64}$	9	$6\frac{13}{64}$	$7\frac{1}{8}$
0.700	$2\frac{21}{32}$	$3\frac{1}{32}$	10	$6\frac{27}{64}$	$7\frac{3}{8}$
0.800	$2\frac{49}{64}$	$3\frac{11}{64}$	11	$6\frac{41}{64}$	$7\frac{39}{64}$
0.900	$2\frac{57}{64}$	$3\frac{5}{16}$	12	$6\frac{53}{64}$	$7\frac{53}{64}$
1	$2\frac{63}{64}$	$3\frac{27}{64}$	14	$7\frac{3}{16}$	$8\frac{1}{4}$
1.25	$3\frac{7}{32}$	$3\frac{11}{16}$	16	$7\frac{33}{64}$	$8\frac{5}{8}$
1.50	$3\frac{27}{64}$	$3\frac{59}{64}$	18	$7\frac{13}{16}$	$8\frac{31}{32}$
1.75	$3\frac{19}{32}$	$4\frac{1}{8}$	20	$8\frac{3}{32}$	$9\frac{9}{32}$
2	$3\frac{49}{64}$	$4\frac{5}{16}$	22	$8\frac{23}{64}$	$9\frac{37}{64}$
2.25	$3\frac{29}{32}$	$4\frac{31}{64}$	25	$8\frac{23}{32}$	10
2.50	$4\frac{3}{64}$	$4\frac{41}{64}$	27	$8\frac{61}{64}$	$10\frac{17}{64}$
2.75	$4\frac{11}{64}$	$4\frac{51}{64}$	30	$9\frac{17}{64}$	$10\frac{5}{8}$
3	$4\frac{5}{16}$	$4\frac{15}{16}$	32	$9\frac{15}{32}$	$10\frac{55}{64}$
3.25	$4\frac{27}{64}$	$5\frac{1}{16}$	35	$9\frac{49}{64}$	$11\frac{3}{16}$
3.50	$4\frac{17}{32}$	$5\frac{3}{16}$	40	$10\frac{13}{64}$	$11\frac{45}{64}$

Example 24. — Required the diameter for a wrought-iron shaft for a 40″ gear which transmits a force of 10,000 pounds. In this case

$$PR = 10000 \times 20 = 200000$$

and, from Table VI., the value of d for wrought-iron corresponding to $PR = 200{,}000$ is $d = 5\frac{1}{32}''$.

Example 25. — The diameter of a wrought-iron gear shaft is $4\frac{1}{8}''$. Required the force which the shaft can safely transmit by means of a 24″ gear. From Table VI. the value of PR corresponding to $d = 4\frac{1}{8}''$ for wrought-iron is 110,000: hence we will have

$$P = \frac{110000}{R} = \frac{110000}{12} = 9167 \text{ pounds nearly.}$$

Example 26. — A gear transmitting a force of 20-horse power makes 200 revolutions per minute. Required the diameter for a shaft of steel. We have

$$\frac{H}{n} = \frac{20}{200} = \frac{1}{10} = 0.100$$

and, from Table VII., the value of d for steel corresponding to $\frac{H}{n} = 0.100$ is $d = 1\frac{25}{64}''$.

Example 27. — A 2″ steel shaft transmits a force of 25-horse power. It is required to determine the proper number of revolutions per minute. From Table VII. the value of $\frac{H}{n}$ which corresponds to $d = 2''$ for steel is $\frac{H}{n} = 0.300$: hence we have

$$\frac{H}{n} = \frac{25}{n} = 0.300$$

or

$$n = 83\tfrac{1}{3} \text{ revolutions per minute.}$$

Keys: We may take for the mean width of the key which fixes the gear upon its shaft $S = 0.28d$, and, for the thickness, $S' = 0.014d$; S and S' being respectively the mean width and thickness of the key, and d the diameter of the shaft. More accurately, according to Reuleaux,

$$S = 0.16 + \frac{d}{5} \qquad (49)$$

and

$$S' = 0.16 + \frac{d}{10} \qquad (50).$$

Rule. — To determine the mean width of the fixing-key, divide the diameter of the shaft by 5, and to the result add 0.16″. To determine the key thickness, divide the diameter of the shaft by 10, and to the result add 0.16″.

Example 28. — Required the mean width and thickness for a fixing-key of sufficient strength for the gear and shaft given in Example 24. From formula (49) we have

$$S = 0.16 + \frac{5.03125}{5} = 0.16 + 1.00625 = 1.16625'' = 1\tfrac{11}{64}''.$$

From formula (50),

$$S' = 0.16 + \frac{5.03125}{10} = 0.16 + 0.503125 = 0.663125'' = \tfrac{43}{64}''.$$

Weight of Gears: The approximate weight of a spur-wheel may be calculated by the following formula, given by Reuleaux. G represents the approximate weight in pounds, N the number of teeth, and p and l respectively the pitch and face width:—

$$G = lp^2(0.215N + 0.0014N^2) \qquad (51).$$

Rule. — To determine the approximate weight of a spur wheel, add 0.215 times the number of teeth to 0.0014 times the square of the number of teeth, and multiply the sum by the product of the square of the pitch into the face width.

Example 29. — Required the approximate weight of a spur wheel having 50 teeth, a pitch of 2″, and a face width of 4½″. From formula (51) we have

$$G = 4\tfrac{1}{2} \times 2^2(0.215 \times 50 + 0.0014 \times 50^2)$$
$$= 18(10.75 + 3.50) = 18 \times 14.25 = 256.50 \text{ pounds.}$$

When the face width is twice the pitch ($l = 2p$), formula (51) becomes

$$G = 2p^3(0.215N + 0.0014N^2)$$

or

$$G = p^3(0.430N + 0.0028N^2) \qquad (52).$$

Rule. — To determine the approximate weight of a spur wheel when the face width is equal to twice the pitch, add 0.430 times the number of teeth to 0.0028 times the square of the number of teeth, and multiply the sum by the cube of the pitch.

Example 30. — Required the approximate weight of a spur wheel having 50 teeth, a pitch of 2″, and a face width of 4″. From formula (52) we have

$$G = 2^3(0.430 \times 50 + 0.0028 \times 50^2) = 8 \times 28.50 = 228 \text{ pounds.}$$

The following table, computed from formula (51), gives values of $\frac{G}{lp^2}$ for different numbers of teeth: —

TABLE VIII.

N	0	2	4	6	8
20	4.86	5.41	5.97	6.54	7.12
30	7.71	8.31	8.93	9.55	10.19
40	10.84	11.50	12.17	12.85	13.55
50	14.25	14.97	15.69	16.43	17.18
60	17.94	18.71	19.49	20.29	21.09
70	21.91	22.74	23.58	24.43	25.29
80	26.16	27.04	27.94	28.84	29.76
90	30.69	31.63	32.58	33.54	34.52
100	35.50	36.50	37.50	38.52	39.55
120	45.96	47.07	48.19	49.32	50.46
140	57.54	58.76	59.99	61.23	62.49
160	70.24	71.57	72.91	74.27	75.63
180	84.06	85.50	86.96	88.42	89.90
200	99.00	100.56	102.12	103.70	105.29
220	115.06	116.73	118.41	120.10	121.80

Example 31.—Required the approximate weight of a spur wheel having 126 teeth, the pitch being 3″ and the face width 7″. From Table VIII. the value of $\frac{G}{lp^2}$, which corresponds to $N = 126$, is 49.32: hence

$$\frac{G}{7 \times 3^2} = 49.32$$

$$G = 49.32 \times 7 \times 9 = 3107.16 \text{ pounds.}$$

To determine the approximate weight of a bevel gear, proceed as explained in the above example for a spur wheel, except that the tabular number must be multiplied by 0.855.

Example 32. — Required to determine the approximate weight of a bevel wheel, for which $N = 48$, $p = 3''$, and $l = 7''$. From the table the value of $\frac{G}{lp^2}$ corresponding to $N = 48$ is 13.55. This multiplied by 0.855 gives 11.585: hence

$$\frac{G}{7 \times 9} = 11.585$$

$$G = 11.585 \times 7 \times 9 = 729.855 \text{ pounds.}$$

§ XV. — *Recapitulation of Formulas and Rules.*

For convenience in designing, the various rules and formulas developed in the foregoing pages have been gathered together in the following recapitulation: —

NOTATION.

R = radius of the pitch circle.
D = diameter of the pitch circle.
C = circumference of the pitch circle.
π = constant 3.14159.
p = circumferential pitch.
p_d = diametral pitch.
N = number of teeth.
l'' = length of chord subtending the pitch.
n = number of revolutions per minute.
P = total force transmitted.
W = total force transmitted.
v = circumferential velocity in feet per second.
V = circumferential velocity in feet per second.
f = greatest safe working-stress in pounds per square inch for the material.
l = face width.

h = total height of teeth.
h' = height of teeth below pitch circle.
h'' = height of teeth above pitch circle.
b = breadth of teeth at pitch circle.
H = horse-power transmitted.
n_1' = number of arms.
h_1 = width of rectangular arms in plane of the pitch circle.
b_1 = thickness of rectangular arms.
x = variable co-efficient.
d' = diameter of circular arms.
H', B, $h_{\prime\prime}$, and $b_{\prime\prime}$ = dimensions for Figs. 82 and 83.
a = major axis for elliptical arms.
b' = minor axis for elliptical arms.
t = thickness of rim.
k = thickness of nave.
l' = length of nave.
d = diameter of shaft.
S = mean width of fixing-key.
S' = thickness of fixing-key.
G = approximate weight of spur wheel.

Dimensions are in inches, forces and weights in pounds, unless otherwise stated.

$$C = \pi D = 2\pi R \qquad (1).$$

Rule.—To find the circumference of the pitch circle, multiply the diameter by 3.14159, or the radius by 6.28318.

$$D = \frac{C}{\pi}, \quad R = \frac{C}{2\pi} \qquad (2).$$

Rule.—To find the diameter of the pitch circle, divide the circumference by 3.14159. To find the radius, divide the circumference by 6.28318.

$$N = \frac{C}{p}, \quad C = Np, \quad p = \frac{C}{N} \qquad (3).$$

Rule. — To find the number of teeth, divide the circumference by the pitch. To find the circumference, multiply the number of teeth by the pitch. To find the pitch, divide the circumference by the number of teeth.

$$p_d = \frac{N}{D} = \frac{\pi}{p}, \quad p = \frac{\pi}{p_d} \qquad (4).$$

Rule. — To find the *diametral* pitch, divide the number of teeth by the diameter, or divide 3.14159 by the pitch. To find the pitch, divide 3.14159 by the diametral pitch.

$$l'' = 2R \sin \tfrac{1}{2}\theta \qquad (5).$$

Rule. — To find the length of the chord which subtends the pitch, multiply twice the radius by the natural sine of half the angle limited by the pitch.

$$l'' = D \sin\left(\frac{180^\circ}{N}\right) \qquad (6).$$

Rule. — To find the length of the chord which subtends the pitch, divide 180° by the number of teeth, take the natural sine of the angle thus obtained, and multiply by the diameter.

$$\frac{n}{n'} = \frac{N'}{N} = \frac{R'}{R} = \frac{D'}{D} = \frac{C'}{C} \qquad (7).$$

Rule. — The ratio of the numbers of revolutions of a pair of gears is inversely proportional to the ratio of their numbers of teeth to the ratio of their radii, diameters, or circumferences.

$$Wr = PR, \quad \frac{W}{P} = \frac{R}{r} \qquad (8).$$

Rule. —The ratio of the powers of two gears on the same shaft is inversely proportional to the ratio of their radii.

$$\frac{W}{P} = \frac{V}{v}, \quad W = \frac{PV}{v}, \quad P = \frac{Wv}{V} \qquad (9).$$

Rule. —The ratio of the powers of two gears on the same shaft is inversely proportional to the ratio of their circumferential velocities.

$$p = 4.93\sqrt{\frac{P}{f} \times \frac{p}{l}} \qquad (10).$$

Rule. —To find the pitch for a gear of any material, divide the force transmitted by the greatest safe working-stress in pounds per square inch for the material, multiply the quotient by the ratio of the pitch to the face width, extract the square root of the product, and multiply the result by 4.93.

*For cast-iron.**

$$\left.\begin{array}{lll} \text{Violent shock,} & p = 0.078\sqrt{P \times \frac{p}{l}} & (a) \\ \text{Moderate shock,} & p = 0.07\sqrt{P \times \frac{p}{l}} & (b) \\ \text{Little or no shock,} & p = 0.05\sqrt{P \times \frac{p}{l}} & (c) \end{array}\right\} \qquad (11).$$

Rule. —To find the pitch for a cast-iron gear, multiply the force transmitted by the ratio of the pitch to the face width, extract the square root of the product, and

* $h = 0.7p$, $h' = 0.4p$, $h'' = 0.3p$, and $b < 0.5p$.

multiply the result by 0.078 for violent shock, 0.07 for moderate shock, or 0.05 for little or no shock.

When $l = 2p$,

$$\left.\begin{array}{lll} \text{Violent shock,} & p = 0.055\sqrt{P} & (a) \\ \text{Moderate shock,} & p = 0.05\sqrt{P} & (b) \\ \text{Little or no shock,} & p = 0.035\sqrt{P} & (c) \end{array}\right\} \quad (12).$$

Rule.—To find the pitch for a cast-iron gear when the face width is twice the pitch, multiply the square root of the force transmitted by 0.055 for violent shock, 0.05 for moderate shock, or 0.35 for little or no shock.

$$\left.\begin{array}{lll} \text{Violent shock,} & p = 1.83\sqrt{\frac{H}{v} \times \frac{p}{l}} & (a) \\ \text{Moderate shock,} & p = 1.64\sqrt{\frac{H}{v} \times \frac{p}{l}} & (b) \\ \text{Little or no shock,} & p = 1.17\sqrt{\frac{H}{v} \times \frac{p}{l}} & (c) \end{array}\right\} \quad (13)$$

Rule.—To find the pitch for a cast-iron gear from the horse-power transmitted and circumferential velocity in feet per second, divide the horse-power by the circumferential velocity, multiply the quotient by the ratio of the pitch to the face width, extract the square root of the product, and multiply the result by 1.83 for violent shock, 1.64 for moderate shock, or 1.17 for little or no shock.

When $l = 2p$,

$$\left.\begin{array}{lll} \text{Violent shock,} & p = 1.29\sqrt{\frac{H}{v}} & (a) \\ \text{Moderate shock,} & p = 1.17\sqrt{\frac{H}{v}} & (b) \\ \text{Little or no shock,} & p = 0.82\sqrt{\frac{H}{v}} & (c) \end{array}\right\} \quad (14)$$

Rule. — To find the pitch for a cast-iron gear, from the horse-power and velocity, when the face width is twice the pitch, divide the horse-power by the velocity, extract the square root of the quotient, and multiply the result by 1.29 for violent shock, 1.17 for moderate shock, or 0.82 for little or no shock.

$$\left.\begin{array}{lll}\text{Violent shock,} & p = 27.71\sqrt{\dfrac{H}{Dn} \times \dfrac{p}{l}} & (a) \\ \text{Moderate shock,} & p = 24.84\sqrt{\dfrac{H}{Dn} \times \dfrac{p}{l}} & (b) \\ \text{Little or no shock,} & p = 17.72\sqrt{\dfrac{H}{Dn} \times \dfrac{p}{l}} & (c)\end{array}\right\} \quad (15)$$

Rule. — To find the pitch for a cast-iron gear from the horse-power and number of revolutions per minute, divide the horse-power by the product of the diameter into the number of revolutions, multiply the quotient by the ratio of the pitch to the face width, extract the square root of the product, and multiply the result by 27.71 for violent shock, 24.84 for moderate shock, or 17.72 for little or no shock.

When $l = 2p$,

$$\left.\begin{array}{lll}\text{Violent shock,} & p = 19.54\sqrt{\dfrac{H}{Dn}} & (a) \\ \text{Moderate shock,} & p = 17.72\sqrt{\dfrac{H}{Dn}} & (b) \\ \text{Little or no shock,} & p = 12.42\sqrt{\dfrac{H}{Dn}} & (c)\end{array}\right\} \quad (16).$$

Rule. — To find the pitch for a cast-iron gear, from the horse-power and number of revolutions per minute, when the face width is twice the pitch, divide the horse-power by the product of the diameter into the number of revolutions, extract the square root of the quotient, and multiply the result by 19.54 for violent shock, 17.72 for moderate shock, or 12.42 for little or no shock.

$$b_1 h_1^2 = \frac{PR}{500 n_1'} \qquad (17).$$

Rule. — To find the quantity $b_1 h_1^2$ (the thickness of the arm multiplied by the square of the width) for cast-iron arms, multiply the force transmitted by the radius of the pitch circle, and divide the product by 500 times the number of arms.

$$h_1 = x\sqrt[3]{PR} \qquad (18).$$

Rule. — To find the width of the arms in the plane of the pitch circle, multiply the force transmitted by the radius of the pitch circle, extract the cube root of the product, and multiply the result by the tabular number (in Table V.) corresponding to the required number of arms and value of $\frac{b_1}{h_1}$.

$$d' = 0.15\sqrt[3]{\frac{PR}{n_1'}} \qquad (19).$$

Rule. — To find the diameter for cast-iron arms having circular cross-sections, multiply the force transmitted by the radius of the pitch circle, divide the product by the number of arms, extract the cube root of the quotient, and multiply the result by 0.15.

$$b'a^2 = 0.00339 \frac{PR}{n_1'} \qquad (20).$$

Rule. — To find the quantity of $b'a^2$ (the minor axis of *elliptical* cross-section multiplied by the square of the major axis) for cast-iron arms, multiply the force transmitted by the radius of the pitch circle, divide the product by the number of arms, and multiply the result by 0.00339.

$$\frac{b_{,,}H'^3 + Bh_{,,}^3}{H'} = \frac{PR}{500n_1'} \qquad (21)\ *$$

$$\frac{BH'^3 - b_{,,}h_{,,}^3}{H'} = \frac{PR}{500n_1'} \qquad (22)\ \dagger$$

$$n_1' = 0.56\sqrt{N}\sqrt[4]{p} \qquad (23).$$

Rule.—To find the number of arms, extract the fourth root of the pitch and the square root of the number of teeth, multiply the two roots together, and the product by 0.56.

$$b_1h_1^2 = \frac{126H}{nn_1'} \qquad (24).$$

Rule.—To find the quantity $b_1h_1^2$ (see formula 17) for cast-iron arms, from the horse-power and number of revolutions per minute, multiply the horse-power by 126, and divide by the product of the number of revolutions into the number of arms.

$$d' = 5.969\sqrt[3]{\frac{H}{nn_1'}} \qquad (25).$$

Rule.—To find the diameter for cast-iron arms having circular cross-sections, from the horse-power and number of revolutions per minute, divide the horse-power by the product of the number of revolutions into

* See Fig. 82. † See Fig. 83.

the number of arms, extract the cube root of the quotient, and multiply the result by 5.969.

$$b'a^2 = 213.57 \frac{H}{nn_1'} \qquad (26).$$

Rule. — To find the quantity $b'a^2$ (see formula 20) for cast-iron arms, from the horse-power and number of revolutions per minute, divide the horse-power by the product of the number of revolutions into the number of arms, and multiply the quotient by 213.57.

$$\frac{b_{\prime\prime}H'^3 + Bh_{\prime\prime}^3}{H'} = \frac{126H}{nn_1'} \qquad (27)\ *$$

$$\frac{BH'^3 - b_{\prime\prime}h_{\prime\prime}^3}{H'} = \frac{126H}{nn_1'} \qquad (28)\ \dagger$$

$$b_1h_1^2 = \frac{0.8p^2R}{n_1'} \qquad (29).$$

Rule. — To find the quantity $b_1h_1^2$ (see formula 17) for cast-iron arms, from the pitch, multiply 0.8 times the square of the pitch by the radius of the pitch circle, and divide the product by the number of arms.

$$d' = 1.105\sqrt[3]{\frac{p^2R}{n_1'}} \qquad (30).$$

Rule. — To find the diameter of cast-iron arms having circular cross-sections, from the pitch, multiply the square of the pitch by the radius of the pitch circle, divide the product by the number of arms, extract the cube root of the quotient, and multiply the result by 1.105.

* See Fig. 82. † See Fig. 83.

$$b'a^2 = 1.356\frac{p^2R}{n_1'} \qquad (31).$$

Rule. — To find the quantity $b'a^2$ (see formula 20) from the pitch, multiply the square of the pitch by the radius of the pitch circle, divide the product by the number of arms, and multiply the result by 1.356.

$$\frac{b_{\prime\prime}H'^3 + Bh_{\prime\prime}^3}{H'} = \frac{0.8p^2R}{n_1'} \qquad (32)\ *$$

$$\frac{BH'^3 - b_{\prime\prime}h_{\prime\prime}^3}{H'} = \frac{0.8p^2R}{n_1'} \qquad (33)\ †$$

$$t = 0.12 + 0.4p \qquad (34).$$

Rule. — To find the thickness of the rim, add 0.12″ to 0.4 times the pitch.

$$k = 0.4\sqrt[3]{p^2R} + \tfrac{1}{2} \qquad (35).$$

Rule. — To find the thickness of the nave, multiply the square of the pitch by the radius of the pitch circle, extract the cube root of the product, multiply the root by 0.4, and to the result add $\frac{1}{2}$″.

$$l' = l + \frac{D}{30} \qquad (36).$$

Rule. — To find the length of the nave, divide the diameter of the pitch circle by 30, and to the result add the face width of the teeth.

$$d = 1.720\sqrt[3]{\frac{PR}{f'}} \qquad (37).$$

Rule. — To find the diameter of a gear shaft of any

* See Fig. 82. † See Fig. 83.

material, multiply the force transmitted by the radius of the pitch circle, divide the product by the greatest safe shearing-stress in pounds per square inch for the material, extract the cube root of the quotient, and multiply the result by 1.720.

For steel, $$d = 0.075\sqrt[3]{PR} \quad (38)$$

For wrought-iron, $$d = 0.086\sqrt[3]{PR} \quad (39)$$

For cast-iron, $$d = 0.108\sqrt[3]{PR} \quad (40).$$

Rule. — To find the diameter of a gear shaft, multiply the force transmitted by the radius of the pitch circle, extract the cube root of the product, and multiply the result by 0.075 for steel, 0.086 for wrought-iron, and 0.108 for cast-iron.

$$d = 68.44\sqrt[3]{\frac{H}{nf'}} \quad (41).$$

Rule. — To find the diameter of a gear shaft of any material from the horse-power and number of revolutions, divide the horse-power by the product of the number of revolutions into the greatest safe shearing-stress in pounds per square inch for the material, extract the cube root of the quotient, and multiply the result by 68.44.

For steel, $$d = 2.984\sqrt[3]{\frac{H}{n}} \quad (42)$$

For wrought-iron, $$d = 3.422\sqrt[3]{\frac{H}{n}} \quad (43)$$

For cast-iron, $$d = 4.297\sqrt[3]{\frac{H}{n}} \quad (44).$$

Rule. — To find the diameter of a gear shaft from the horse-power and number of revolutions, divide the horse-power by the number of revolutions, extract the cube root of the quotient, and multiply the result by 2.984 for steel, 3.422 for wrought-iron, and 4.297 for cast-iron.

$$d = 12.673\sqrt[3]{\frac{p^2R}{f'}} \qquad (45).$$

Rule. — To find the diameter of a gear shaft of any material from the pitch, multiply the square of the pitch by the radius of the pitch circle, divide the product by the greatest safe shearing-stress in pounds per square inch for the material used, extract the cube root of the quotient, and multiply the result by 12.673.

For steel, $d = 0.553\sqrt[3]{p^2R}$ (46)

For wrought-iron, $d = 0.634\sqrt[3]{p^2R}$ (47)

For cast-iron, $d = 0.796\sqrt[3]{p^2R}$ (48)

Rule. — To find the diameter of a gear shaft from the pitch, multiply the square of the pitch by the radius of the pitch circle, extract the cube root of the product, and multiply the result by 0.553 for steel, 0.634 for wrought-iron, and 0.796 for cast-iron.

$$S = 0.16 + \frac{d}{5} \qquad (49)$$

$$S' = 0.16 + \frac{d}{10} \qquad (50).$$

Rule. — To find the mean width of a fixing-key, divide the diameter of the shaft by 5, and to the result add 0.16″. To find the thickness of the key, divide the diameter of the shaft by 10, and to the result add 0.16″.

$$G = lp^2(0.215N + 0.0014N^2) \qquad (51).$$

Rule. — To find the approximate weight of a spur wheel, add 0.215 times the number of teeth to 0.0014, the square of the number of teeth, and multiply the sum by the product of the face width into the square of the pitch.

When $l = 2p$,

$$G = p^3(0.430N + 0.0028N^2) \qquad (52).$$

Rule. — To find the approximate weight of a spur wheel when the face width is twice the pitch, add 0.430 times the number of teeth to 0.0028 times the square of the number of teeth, and multiply the sum by the cube of the pitch.

§ XVI. — *Complete Design of Spur-Wheel, Bevels, Worm, Screw Gear, etc.*

Example 1. — Required to design and make full working drawings for a 36″ cast-iron spur wheel to transmit a force of 5,000 pounds, violent shock.

For the pitch we have, from formula (12, *a*),

$$p = 0.055\sqrt{5000} = 0.055 \times 70.71 = 3.889''$$

for the face width,

$$l = 2p = 2 \times 3.889 = 7.778''.$$

As explained in § XIII., we have for the total height of the teeth, and heights below and above the pitch circle,

$$h = h' + h'' = 0.4p + 0.3p = 0.7 \times 3.889 = 2.7223''$$
$$h' = 0.4 \times 3.889 = 1.5556''$$
$$h'' = 0.3 \times 3.889 = 1.1667''.$$

We may take, for the breadth of the teeth on the pitch circle, $b = 0.45p = 1.75''$. From formulas (1) and (3), for the circumference and number of teeth,

$$C = 3.14159 \times 36 = 113.10$$

and

$$N = \frac{113.10}{3.889} = 29.$$

From formula (23), the number of arms is

$$n_1' = 0.56\sqrt{29}\sqrt[4]{3.889} = 0.56 \times 5.385 \times 1.40 = 4.$$

If we wish to have elliptical cross-sections for the arms, we have, from formula (20),

$$b'a^2 = 0.00339 \times \frac{5000 \times 18}{4} = 0.00339 \times 22500 = 76.275.$$

Taking

$$b' = \frac{a}{2}, \quad b'a^2 = \frac{a^3}{2} = 76.275;$$

or, for the major axis of the cross-section,

$$a = \sqrt[3]{152.55} = 5.343''$$

and, for the minor axis,

$$b' = \frac{5.343}{2} = 2.6715''.$$

For the thickness of the rim, from formula (34),

$$t = 0.12 + 0.4 \times 3.889 = 0.12 + 1.5556 = 1.6756.''$$

Formula (35) gives, for the thickness of the nave,

$$k = 0.4\sqrt[3]{3.889^2 \times 18} + \tfrac{1}{2} = 0.4 \times 6.481 + \tfrac{1}{2} = 3.092''.$$

The length of the nave is, from formula (36),

$$l' = 7.778 + \tfrac{36}{30} = 7.778 + 1.2 = 8.978''.$$

Formula (39) gives, for the diameter of the wrought-iron shaft,

$$d = 0.086\sqrt[3]{5000 \times 18} = 0.086 \times 44.814 = 3.854''.$$

For the mean width and thickness of the fixing-key we have, from formulas (49) and (50),

$$s = 0.16 + \frac{3.854}{5} = 0.16 + 0.7708 = 0.9308''$$

and

$$s_1 = 0.16 + \frac{3.854}{10} = 0.16 + 0.3854 = 0.5454''.$$

We may now recapitulate our dimensions, and by means of Table IV. convert the decimals into convenient fractions: —

Diameter,	$D = 36''$
Pitch,	$p = 3\frac{57}{64}''$
Face width,	$l = 7\frac{25}{32}''$
Total tooth height,	$h = 2\frac{23}{32}''$
Height below pitch circle,	$h' = 1\frac{35}{64}''$
Height above pitch circle,	$h'' = 1\frac{11}{64}''$
Breadth of tooth on pitch circle,	$b = 1\frac{3}{4}''$

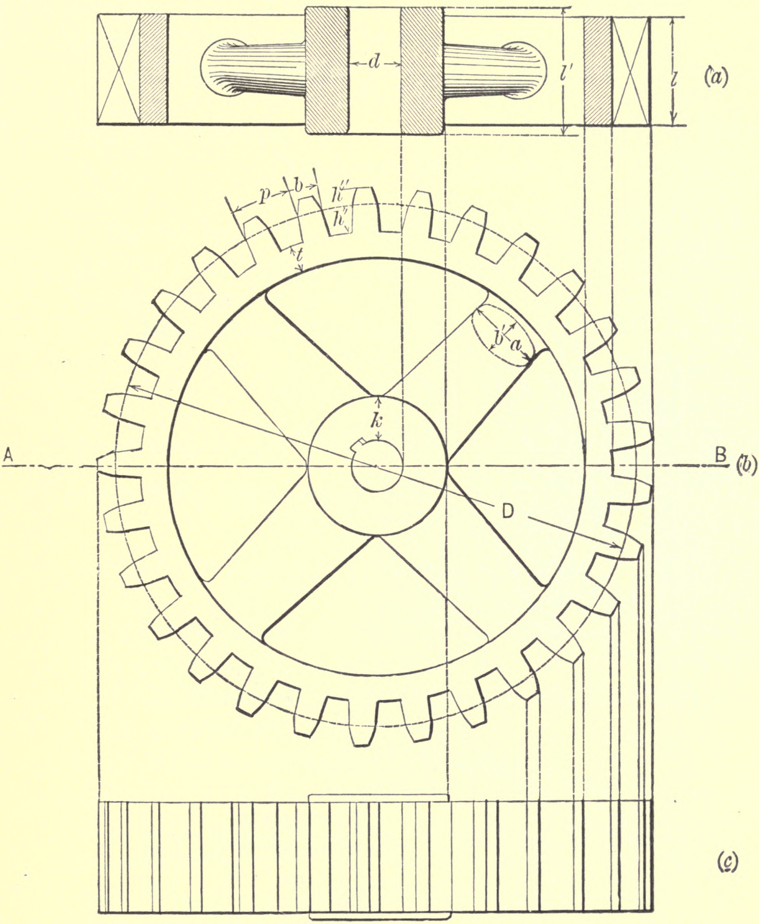
Number of teeth,	$N = 29$
Number of arms,	$n_1' = 4$
Axes of arm cross-sections,	$\begin{cases} a = 5\frac{11}{32}'' \\ b' = 2\frac{43}{64}'' \end{cases}$
Thickness of rim,	$t = 1\frac{43}{64}''$
Nave length,	$l' = 8\frac{63}{64}''$
Nave thickness,	$k = 3\frac{3}{32}''$
Diameter of shaft,	$d = 3\frac{55}{64}''$
Key width,	$s = \frac{15}{16}''$
Key thickness,	$s_1 = \frac{35}{64}''$.

Fig. 89 shows the working drawings for the above spur wheel. Fig. (*b*) is a simple horizontal projection of the gear, showing the pitch, tooth dimensions, thickness of rim and nave, dimensions of arms, number of teeth, arms, etc. Fig. (*c*) is a vertical projection taken from Fig. (*b*), as shown by the dotted lines, and Fig. (*a*) is a sectional, vertical projection taken from Fig. (*b*) on the line *AB*, and showing the face width, nave length, etc. The profiles were drawn by the method of § IV., Fig. 26.

Example 2. — Required to design and make full working drawings for a pair of cast-iron bevel wheels to transmit a force of 10-horse power from a smoothly running turbine wheel (moderate shock), the smaller bevel to be fixed upon the 3″ shaft of the turbine wheel, which makes 30 revolutions per minute, the bevel wheels to be 15″ and 30″ diameters. The circumferential velocity of the smaller bevel (as also that of the larger) is

$$v = \frac{30 \times \pi \times 15}{12 \times 60} = \frac{30 \times 47.124}{12 \times 60} = 2 \text{ feet per second nearly.}$$

Fig. 89



SCALE = $\frac{3}{40}$*

* The scale of all working drawings should be $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, etc. The scale of $\frac{3}{40}$ is taken here in order to bring the drawings of convenient size.

For the smaller bevel, from formula (14, *b*), we have, therefore, for the pitch,

$$p = 1.17\sqrt{\frac{10}{2}} = 1.17 \times 2.236 = 2.616''.$$

For the face width,

$$l = 2 \times 2.616 = 5.232''.$$

For the total height of the teeth,

$$h = 0.7 \times 2.616 = 1.8312''.$$

For the heights below and above the pitch circle,

$$h' = 0.4 \times 2.616 = 1.0464''$$

and

$$h'' = 0.3 \times 2.616 = 0.7848''.$$

Taking, for the breadth of the teeth at the pitch circle, $b = 0.48p$, we have

$$b = 0.48 \times 2.616 = 1.25568''.$$

The bevel, being so small, may be made without rim or arms, i.e., cast solid, as shown in the drawing (Fig. 91, *a*). From formula (3) the number of teeth is

$$N = \frac{47.124}{2.616} = 18.$$

For the thickness of the nave, from formula (35),

$$k = 0.4\sqrt[3]{2.62^2 \times 7\tfrac{1}{2}} + \tfrac{1}{2} = 2''.$$

From formula (36), for the length of the nave, we have

$$l' = 5.232 + \tfrac{15}{30} = 5.732''.$$

The diameter of the shaft is that of the turbine, or $d = 3''$. From formulas (49) and (50) the mean width of the key which fixes the bevel to its shaft is

$$s = 0.16 + \frac{3}{5} = 0.76''$$

and the thickness,

$$s' = 0.16 + \frac{3}{10} = 0.46''.$$

For the larger bevel the pitch and tooth dimensions are the same as for the smaller bevel. From formula (3) the number of teeth is

$$N = \frac{\pi \times 30}{2.616} = \frac{94.25}{2.616} = 36.$$

From formula (34) the thickness of the rim is

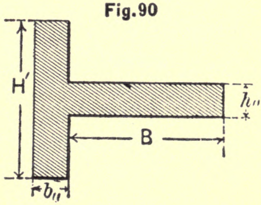
$$t = 0.12 + 0.4 \times 2.616 = 1.1664''.$$

Formula (23) gives for the number of arms,

$$n_1' = 0.56\sqrt{36}\sqrt[4]{2.616} = 4.$$

For the number of revolutions per minute, we have, from formula (7), $n = 15$. For the flanged cross-sections of the arms, such as that represented in Fig. 90, taking $b_{\prime\prime}$ equal to the rim-thickness $= 1.1664''$, $h_{\prime\prime} = 1''$, and $B = H'$, we have, from formula (27),

Fig. 90



$$\frac{1.1664 \times H'^3 + H' \times 1}{H'} = \frac{126 \times 10}{15 \times 4}$$

or

$$1.1664H'^3 + 1 = 21.$$

Hence

$$H' = \sqrt[3]{\frac{20}{1.1664}} = 4.141''$$

and

$$B = H' = 4.141''.$$

For the thickness of the nave, from formula (35) we have

$$k = 0.4\sqrt[3]{2.62^2 \times 15} + \tfrac{1}{2} = 2.36''.$$

Formula (36) gives, for the length of the nave,

$$l' = 5.232 + \tfrac{30}{30} = 6.232''.$$

For the diameter of the wrought-iron shaft we have, from formula (43),

$$d = 3.422\sqrt[3]{\tfrac{10}{15}} = 3''.$$

Formulas (49) and (50) give, for the mean width and thickness of the fixing-key,

$$s = 0.16 + \tfrac{3}{5} = 0.76''$$

and

$$s' = 0.16 + \tfrac{3}{10} = 0.46''.$$

Our dimensions in fractions instead of decimals are as follows : —

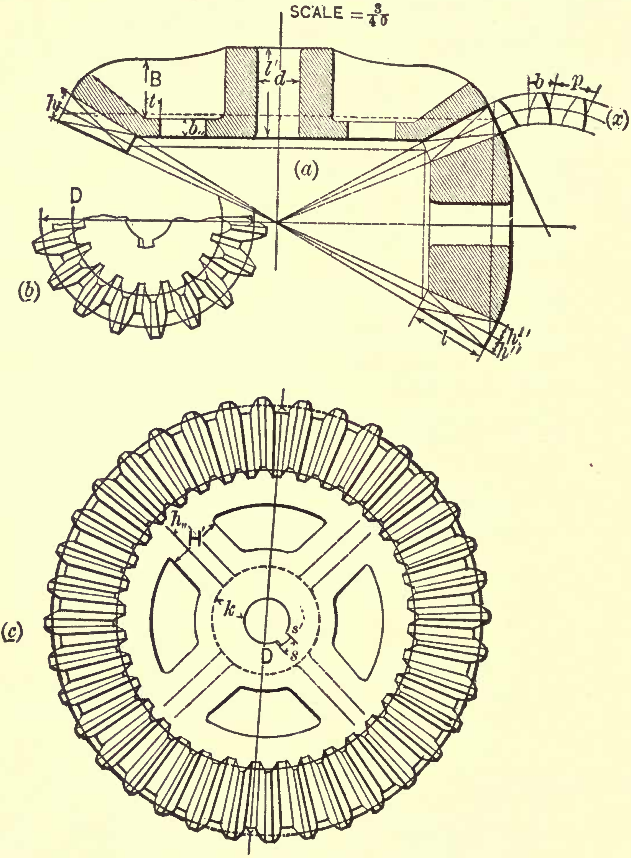
	For smaller bevel.
Diameter,	$D = 15''$
Pitch,	$p = 2\frac{39}{64}''$
Face width,	$l = 5\frac{15}{64}''$
Total height of teeth,	$h = 1\frac{53}{64}''$
Height below pitch circle,	$h' = 1\frac{3}{64}''$
Height above pitch circle,	$h'' = \frac{25}{32}''$

	For smaller bevel.
Breadth on pitch circle,	$b = 1\frac{17}{64}''$
Number of teeth,	$N = 18$
Thickness of nave,	$k = 2''$
Length of nave,	$l' = 5\frac{47}{64}''$
Diameter of shaft,	$d = 3''$
Key width,	$s = \frac{3}{4}''$
Key thickness,	$s' = \frac{15}{32}''$.

	For larger bevel.
Diameter,	$D = 30''$
Pitch,	$p = 2\frac{39}{64}''$
Face width,	$l = 5\frac{15}{64}''$
Total height of teeth,	$h = 1\frac{53}{64}''$
Height below pitch circle,	$h' = 1\frac{3}{64}''$
Height above pitch circle,	$h'' = \frac{25}{32}''$
Breadth of teeth at pitch circle,	$b = 1\frac{17}{64}''$
Number of teeth,	$N = 36$
Rim thickness,	$t = 1\frac{11}{64}''$
Number of arms,	$n_1' = 4$
Arm dimensions. See Fig. 90.	$H' = 4\frac{9}{64}''$
	$B = 4\frac{9}{64}''$
	$b_{\prime\prime} = 1\frac{11}{64}''$
	$h_{\prime\prime} = 1''$
Thickness of nave,	$k = 2\frac{23}{64}''$
Length of nave,	$l' = 6\frac{15}{64}''$
Diameter of shaft,	$d = 3''$
Key width,	$s = \frac{3}{4}''$
Key thickness,	$s' = \frac{15}{32}''$.

Fig. 91 gives the working drawings, drawn to a scale of $\frac{3}{40}$. Fig. (*a*) is a sectional drawing of both bevels in gear, showing teeth, rim, nave thickness, etc., and at x the true form of the profiles and true tooth dimensions. Fig. (*b*) is a partial projection of the smaller bevel; and Fig. (*c*), a projection of the larger bevel, showing the arms, fixing-key, etc.

Fig. 91



Example 3. — Required to design, and make complete working drawings for, a worm and wheel to transmit a force of 850 pounds, little or no shock, the wheel to be 12″ in diameter. From formula (12, *c*) we have, for the pitch,

$$p = 0.035\sqrt{850} = 0.035 \times 29.15 = 1.02'',$$

for the heights of the teeth,

$$h = 0.7 \times 1.02 = 0.714''$$

$$h' = 0.4 \times 1.02 = 0.408''$$

and

$$h'' = 0.3 \times 1.02 = 0.306''.$$

The breadth of the teeth at the pitch circle is

$$b = 0.48 \times 1.02 = 0.4896''.$$

For the number of teeth in the wheel, from formula (3),

$$N = \frac{37 \cdot 7}{1.02} = 37.$$

Face width of wheel,

$$l = 2 \times 1.02 = 2.04''.$$

From formula (23) the number of arms is

$$n_1' = 0.56\sqrt{37}\sqrt[4]{1.02} = 0.56 \times 6.08 \times 1.005 = 4.$$

For the thickness and width of the arms* we have, from formula (17), taking $b_1 = \frac{h_1}{2}$,

$$\frac{h_1^3}{2} = \frac{850 \times 6}{500 \times 4} = \frac{1275}{500}$$

---

* Ordinarily so small a gear would be made without arms. For the purpose of illustrating, however, we use four arms, as given by the formula.

or

$$h_1 = \sqrt[3]{5.1} = 1.72''$$

$$b_1 = \frac{1.72}{2} = 0.86''.$$

Formula (34) gives, for the thickness of the rim,

$$t = 0.12 + 0.408 = 0.528''.$$

From formula (35) the thickness of the nave is

$$k = 0.4\sqrt[3]{1.02^2 \times 6} + \tfrac{1}{2} = 0.4 \times 1.841 + \tfrac{1}{2} = 1.236''.$$

The length of the nave is, from formula (36),

$$l' = 2.04 + \tfrac{12}{30} = 2.44''.$$

Formula (39) gives, for the diameter of the wrought-iron shaft of the wheel,

$$d = 0.086\sqrt[3]{850 \times 6} = 0.086 \times 17.29 = 1.48''.$$

From formulas (49) and (50) the width of the fixing-key is

$$s = 0.16 + \frac{1.48}{5} = 0.456''$$

and the thickness

$$s' = 0.16 + \frac{1.48}{10} = 0.308''.$$

From § VIII., taking the radius of the worm equal to $1\frac{1}{2}$ times the pitch, we have

$$R' = 1\tfrac{1}{2} \times 1.02 = 1.53''$$

and, for the angle (λ) of the teeth,

$$\tan\lambda = 0.159\frac{1.02}{1.53} = 0.159 \times 0.6667 = 0.106$$

or $\lambda = 6° \; 3$. From formula (39) the shaft diameter for the worm is

$$d = 0.086\sqrt[3]{850 \times 1.53} = 0.939''.$$

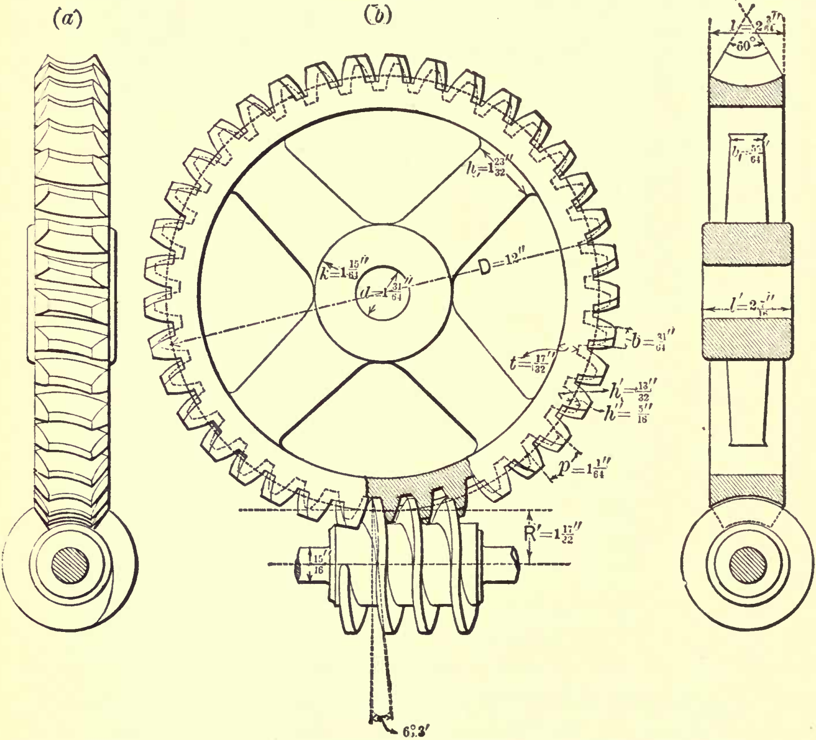
		Dimensions.
Diameter of wheel,	D =	$12''$
Pitch,	p =	$1\frac{1}{64}''$
Total height of teeth,	h =	$\frac{23}{32}''$
Height below pitch circle,	h' =	$\frac{13}{32}''$
Height above pitch circle,	h'' =	$\frac{5}{16}''$
Breadth of teeth on pitch circle,	b =	$\frac{31}{64}''$
Face width,	l =	$2\frac{3}{64}''$
Number of teeth on wheel,	N =	37
Number of arms on wheel,	n_1' =	4
Thickness of rim,	t =	$\frac{17}{32}''$
Width of arms,	h_1 =	$1\frac{23}{32}''$
Thickness of arms,	b_1 =	$\frac{55}{64}''$
Thickness of nave,	k =	$1\frac{15}{64}''$
Length of nave,	l' =	$2\frac{7}{16}''$
Diameter of shaft,	d =	$1\frac{31}{64}''$
Width of key,	s =	$\frac{29}{64}''$
Thickness of key,	s' =	$\frac{19}{64}''$
Radius of worm,	R' =	$1\frac{17}{32}''$
Angle of the teeth,	λ =	$6° \; 3'$
Shaft diameter for worm,	d =	$\frac{15}{16}''$.

The working drawings, with dimensions, are given in Fig. 92, of which Fig. (*b*) is a full projection, showing the arms, rim, nave thickness, tooth dimensions in section, angle (λ) of inclination of the teeth, etc. Fig. (*c*) is a sectional projection of Fig. (*b*), showing the shape of the wheel teeth, arm thickness, nave length, etc.; and Fig. (*a*) is a full projection taken from Fig. (*b*).

Example 4. — Required to design, and make full working drawings for, a pair of screw gears to transmit a force of 2-horse power, little or no shock; the larger

Fig. 92

SCALE $= \frac{3}{16}$



gear to be fixed upon a $1\frac{1}{4}''$ wrought-iron shaft, which makes 20 revolutions per minute, and the smaller gear to make 40 revolutions per minute. The angle included between the two gear shafts to be 60°.

Suppose we take, for the diameter of the smaller gear, 6″: hence, from formula (7), the diameter of the larger gear is 12″.

The circumferential velocity is

$$v = \frac{\pi D n}{12 \times 60} = \frac{37.7 \times 20}{12 \times 60} = 1.047 \text{ feet per second.}$$

From § VIII. we have for the angles of inclination (ϕ and ϕ') of the teeth, the angle (Θ) included between the axes of the shafts being 60°,

$$\phi + \phi' + \Theta = 180, \quad \phi + \phi' = 180° - 60° = 120°.$$

If we assume $\phi = 60°$, we have,

$$\phi' = 120° - 60° = 60°.*$$

For the larger wheel the dimensions are calculated as follows: The pitch, from formula (14, c), is

$$p = 0.82\sqrt{\frac{2}{1.047}} = 0.82\sqrt{1.91} = 0.82 \times 1.382 = 1.133''.$$

The face width is

$$l = 2 \times 1.133 = 2.266''.$$

The heights of the teeth are

$$h = 0.7 \times 1.133 = 0.793''$$

$$h' = 0.4 \times 1.133 = 0.4532''$$

and

$$h'' = 0.3 \times 1.133 = 0.3399''.$$

---

* We can assume $\phi = 90°$, in which case the gear upon which the inclination of the teeth is $\phi = 90°$ is a spur wheel, and then have $\phi' = 120° - 90° = 30°$ for the inclination of the teeth of the other gear.

For the breadth of the teeth at the pitch circle we may take

$$b = 0.48p = 0.48 \times 1.133 = 0.54384''.$$

From formula (3) we have, for the number of teeth,

$$N = \frac{37.7}{1.133} = 33.$$

Formula (34) gives, for the rim thickness,

$$t = 0.12 + (0.4 \times 1.133) = 0.5732''.$$

From formula (35), the thickness of the nave is

$$k = 0.4\sqrt[3]{1.133^2 \times 6} + \tfrac{1}{2} = 0.4\sqrt[3]{7.70} + \tfrac{1}{2}$$
$$= 0.4 \times 1.97 + \tfrac{1}{2} = 1.29''.$$

Formula (36) gives, for the nave length,

$$l' = 2.266 + \tfrac{12}{30} = 2.666''.$$

The fixing-key width and thickness are, from formulas (49) and (50),

$$s = 0.16 + \frac{1.25}{5} = 0.41''$$

and

$$s' = 0.16 + \frac{1.25}{10} = 0.285''.$$

The gear is small enough to be made without arms. The thickness of the web between the nave and rim may be calculated from formula (24), by assuming the gear to have 10 arms, the width of each being one-tenth the outer circumference of the nave. Thus the shaft diameter is 1.25″, and the nave thickness 1.29″: hence the diameter across the nave is

$$1.25 + (2 \times 1.29) = 3.83''$$

and the circumference 12.032″. The width of the assumed arms is therefore $\frac{12.032}{10}$, or $h_1 = 1.203''$. Formula (24) becomes

$$b_1 \times 1.203^2 = \frac{126 \times 2}{20 \times 10}$$

or

$$b_1 = \frac{1.26}{1.447} = 0.87''.$$

For the smaller gear the pitch and tooth dimensions are the same as for the larger gear, as is also the rim thickness. The thickness of the nave is, from formula (35),

$$k = 0.4\sqrt[3]{1.133^2 \times 3} + \tfrac{1}{2} = 0.4\sqrt[3]{3.851} + \tfrac{1}{2}$$
$$= 0.4 \times 1.567 + \tfrac{1}{2} = 1.1268''.$$

From formula (36) we have, for the length of the nave,

$$l' = 2.266 + \tfrac{6}{30} = 2.466''.$$

From formula (3), for the number of teeth, we have

$$N = \frac{18.85}{1.133} = 17.$$

The diameter of the wrought-iron shaft is, from formula (43),

$$d = 3.422\sqrt[3]{\tfrac{2}{40}} = 3.422\sqrt[3]{0.050} = 3.422 \times 0.368 = 1.2593''.$$

Formulas (49) and (50) give, for the mean width and thickness of the fixing-key,

$$s = 0.16 + \frac{1.2593}{5} = 0.41186''$$

and

$$s' = 0.16 + \frac{1.2593}{10} = 0.28593''.$$

For the thickness of the web between the rim and nave, the diameter across the nave is

$$1.2593 + (2 \times 1.1268) = 3.51''$$

and the circumference 11.03″: hence $h_1 = 1.10''$. And formula (24) gives

$$b_1 \times 1.10^2 = \frac{126 \times 2}{40 \times 10}$$

or

$$b_1 = \frac{0.63}{1.21} = 0.521''.$$

Dimensions for larger gear.

Diameter,	d	$= 12''$
Pitch,	p	$= 1\frac{9}{64}''$
Face width,	l	$= 2\frac{17}{64}''$
Total height of teeth,	h	$= \frac{51}{64}''$
Height below pitch circle,	h'	$= \frac{29}{64}''$
Height above pitch circle,	h''	$= \frac{11}{32}''$
Breadth at pitch circle,	b	$= \frac{35}{64}''$
Rim thickness,	t	$= \frac{37}{64}''$
Number of teeth,	N	$= 33$
Thickness of web,	b_1	$= \frac{7}{8}''$
Thickness of nave,	k	$= 1\frac{9}{32}''$
Length of nave,	l'	$= 2\frac{43}{64}''$

	Dimensions for larger gear.
Diameter of shaft,	$d = 1\frac{1}{4}''$
Width of fixing-key,	$s = \frac{13}{32}''$
Thickness of fixing-key,	$s' = \frac{9}{32}''$
Angle of the teeth,	$\phi = 60°$.

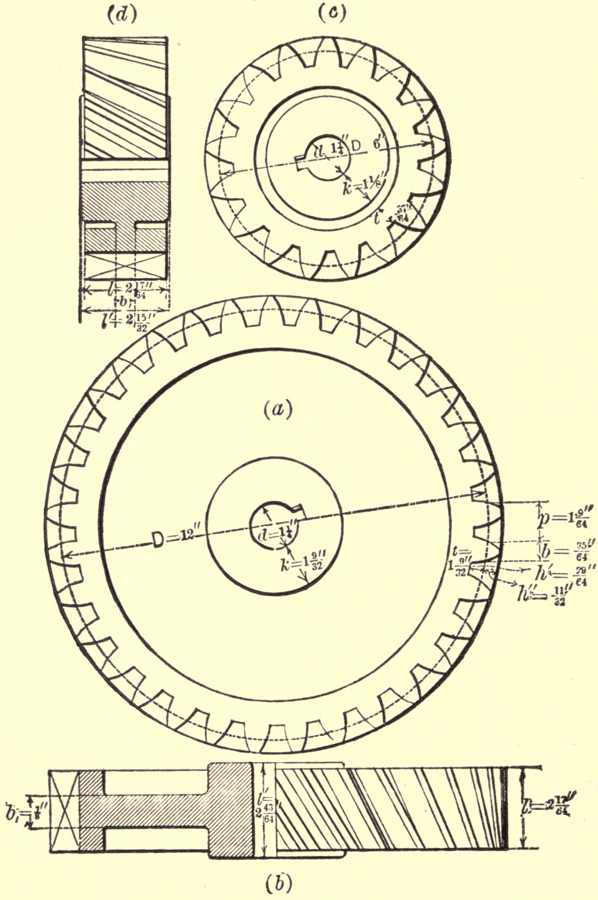
	Dimensions for smaller gear.
Diameter,	$D = 6''$
Pitch,	$p = 1\frac{9}{64}''$
Face width,	$l = 2\frac{17}{64}''$
Total height of teeth,	$h = \frac{51}{64}''$
Height below pitch circle,	$h' = \frac{29}{64}''$
Height above pitch circle,	$h'' = \frac{11}{32}''$
Breadth of teeth,	$b = \frac{35}{64}''$
Rim thickness,	$t = \frac{37}{64}''$
Number of teeth,	$N = 17$
Thickness of web,	$b_1 = \frac{33}{64}''$
Thickness of nave,	$k = 1\frac{1}{8}''$
Length of nave,	$l' = 2\frac{15}{32}''$
Diameter of shaft,	$d = 1\frac{1}{4}''$
Width of fixing-key,	$s = \frac{13}{32}''$
Thickness of fixing-key,	$s' = \frac{9}{32}''$
Angle of the teeth,	$\phi' = 60°$.

The working drawings with marked dimensions are given in Fig. 92 A. Fig. (*a*) is a full projection of the larger gear, showing the pitch, tooth dimensions, rim, etc. The right half of Fig. (*b*) is a full projection of the larger gear, taken from Fig. (*a*); and the left half is a sectional projection taken from Fig. (*a*), showing the web thickness, etc. Similarly, for the smaller gear, Fig. (*c*) is a full projection, and Fig. (*d*) a full and sectional projection taken from Fig. (*c*).

Example 5. — Required to design, and make complete working drawings for, a cast-iron internal spur gear and pinion which will safely transmit a force of 6,197 pounds

Fig. 92 A

SCALE = $\frac{3}{16}$



moderate shock, the pinion to be fixed upon a $3\frac{11}{16}''$ wrought-iron shaft, the face width to be $2\frac{1}{2}$ times the pitch, and the revolution ratio 3 to 1. From formula (11, *b*) we have, for the pitch,

$$p = 0.07\sqrt{6197 \times \frac{1}{2\frac{1}{2}}} = 0.07\sqrt{2478.80} = 0.07 \times 49.8 = 3.486''.$$

For the face width,

$$l = 2\tfrac{1}{2} \times 3.486 = 8.715''.$$

The heights are,

$$h = 0.7 \times 3.486 = 2.44''$$

$$h' = 0.4 \times 3.486 = 1.394''$$

and

$$h'' = 0.3 \times 3.486 = 1.046''.$$

Taking the breadth of the teeth equal to $0.45p$, we have

$$b = 0.45 \times 3.486 = 1.569''.$$

If we take for the diameter of the pinion $25\frac{1}{2}''$, we shall have for the number of teeth, from formula (3),

$$N = \frac{80.10}{3.486} = 23.$$

From formula (7) the diameter of the internal gear is

$$25\tfrac{1}{2} \times 3 = 76\tfrac{1}{2}'',$$

and from formula (3) the number of teeth is

$$N = \frac{240.30}{3.486} = 69.$$

Formula (34) gives, for the rim thickness of the pinion,

$$t = 0.12 + (0.4 \times 3.486) = 1.51''.$$

Since in an internal gear the rim is not supported by the arms, as in an external gear (see Fig. 93, *b*), we may take the rim thickness for the internal gear equal to $2t = 3''$. From formula (35), the thickness of the nave for the pinion is

$$k = 0.4\sqrt[3]{3.486^2 \times 12.75} + \tfrac{1}{2} = 0.4\sqrt[3]{12.15 \times 12.75} + \tfrac{1}{2}$$
$$= 0.4 \times 5.371 + \tfrac{1}{2} = 2.6484'',$$

and, for the internal gear,

$$k = 0.4\sqrt[3]{3.486^2 \times 38.25} + \tfrac{1}{2} = 0.4\sqrt[3]{12.15 \times 38.25} + \tfrac{1}{2}$$
$$= 0.4 \times 7.746 + \tfrac{1}{2} = 3.598''.$$

Formula (36) gives for the nave lengths of the pinion and internal gear respectively,

$$l' = 8.715 + \frac{25.50}{30} = 8.715 + 0.85 = 9.565''$$

and

$$l' = 8.715 + \frac{76.50}{30} = 8.715 + 2.55 = 11.265''.$$

The pinion may be without arms, and the thickness of the web calculated from formula (29) by assuming the pinion to have 10 arms, each having a width of one-tenth the outer circumference of the nave. Thus the diameter of the shaft is 3.6875″, and the nave thickness 2.6484″: hence the diameter across the nave is

$$3.6875 + (2 \times 2.6484) = 9'',$$

and the circumference 28.27″. We therefore have $h_1 = 2.827''$; and formula (29) gives

$$b_1 \times 2.827^2 = \frac{0.80 \times 3.486^2 \times 12.75}{10} = 12.393,$$

or

$$b_1 = \frac{12.393}{7.99} = 1.55''.$$

For the number of arms for the internal gear, formula (23) gives

$$n_1' = 0.56\sqrt{69}\,\sqrt[4]{3.486} = 0.56 \times 8.307 \times 1.366 = 6.35,$$

say $n_1' = 7$. If we wish to have elliptical arm cross-sections, we have from formula (31), taking the minor axis equal to one-half the major,

$$b_1 a^2 = \frac{a^3}{2} = 1.356\,\frac{3.486^2 \times 38.25}{7} = 90.03.$$

Hence

$$a = \sqrt[3]{90.03 \times 2} = 5.647''$$

and

$$b_1 = \frac{5.647}{2} = 2.824''.$$

From formula (39), the diameter of the wrought-iron shaft for the internal gear is

$$d = 0.086\sqrt[3]{6197 \times 38.25} = 0.086 \times 61.888 = 5.322''.$$

Formulas (49) and (50) give, for the mean width and thickness of the fixing-key for the pinion,

$$s = 0.16 + \frac{3.6875}{5} = 0.16 + 0.7375 = 0.8975''$$

$$s' = 0.16 + \frac{3.6875}{10} = 0.16 + 0.36875 = 0.52875''$$

and the same for the internal gear,

$$s = 0.16 + \frac{5.322}{5} = 1.2244''$$

and

$$s' = 0.16 + \frac{5.322}{10} = 0.6922''.$$

	Dimensions for pinion.
Diameter,	$D = 25\frac{1}{2}''$
Pitch,	$p = 3\frac{31}{64}''$
Face width,	$l = 8\frac{23}{32}''$
Total height of teeth,	$h = 2\frac{7}{16}''$
Height below pitch circle,	$h' = 1\frac{25}{64}''$
Height above pitch circle,	$h'' = 1\frac{3}{64}''$
Number of teeth,	$N = 23$
Breadth of teeth,	$b = 1\frac{9}{16}''$
Thickness of rim,	$t = 1\frac{33}{64}''$
Thickness of nave,	$k = 2\frac{41}{64}''$
Length of nave,	$l' = 9\frac{9}{16}''$
Thickness of web,	$b_1 = 1\frac{35}{64}''$
Diameter of shaft,	$d = 3\frac{11}{16}''$
Mean width of fixing-key,	$s = \frac{29}{32}''$
Thickness of fixing-key,	$s' = \frac{3}{8}''$

	Dimensions for internal gear.
Diameter,	$D = 76\frac{1}{2}''$
Pitch,	$p = 3\frac{31}{64}''$
Number of teeth,	$N = 69$
Total height of teeth,	$h = 2\frac{7}{16}''$
Height below pitch circle,	$h' = 1\frac{25}{64}''$
Height above pitch circle,	$h'' = 1\frac{3}{64}''$
Face width,	$l = 8\frac{23}{32}''$
Breadth of teeth,	$b = 1\frac{9}{16}''$
Rim thickness,	$2t = 3''$
Nave thickness,	$k = 3\frac{19}{32}''$

		Dimensions for internal gear
Nave length,	l'	$= 11\frac{17}{64}''$
Number of arms,	n_1'	$= 7$
Major axis of arm cross-sections,	a	$= 5\frac{41}{64}''$
Minor axis of arm cross-sections,	b'	$= 2\frac{53}{64}''$
Diameter of shaft,	d	$= 5\frac{21}{64}''$
Width of fixing-key,	s	$= 1\frac{7}{32}''$
Thickness of fixing-key,	s'	$= \frac{11}{16}''$

Fig. 93 shows the working drawings for the pair of gears, to the scale of $\frac{3}{80}$; Fig. (*a*) being a full projection of both gears in position for action, showing the pitch, tooth dimensions, number of arms, etc., and Fig. (*b*) being a sectional projection of both gears, taken from Fig. (*a*), on the line *xy*, showing the shape of the arms of the larger gear necessary to the proper action of the pair, etc.

Example 6.—Required to design, and make full working drawings for, a cast-iron rack and pinion to transmit a force of 1,000 pounds, moderate shock; the pinion to make 20 revolutions per minute, and the rack (which is to be 9 feet long) to move at the rate of $63\frac{1}{2}$ feet (762″) per minute. From formula (12, *b*), for the pitch we have

$$p = 0.05\sqrt{1000} = 0.05 \times 31.62 = 1.581''.$$

The face width is, consequently,

$$l = 2 \times 1.581 = 3.162''.$$

For the heights of the teeth,

$$h = 0.7 \times 1.581 = 1.1067''$$

$$h' = 0.4 \times 1.581 = 0.6324''$$

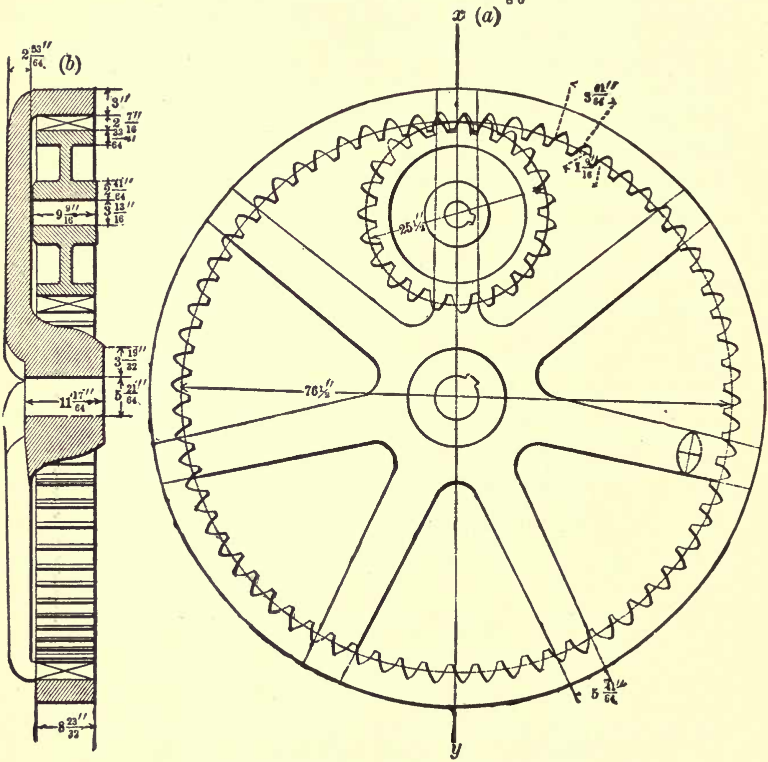
and

$$h'' = 0.3 \times 1.581 = 0.4743''.$$

Taking the breadth of the teeth equal to 0.48 times the pitch gives

$$b = 0.48 \times 1.581 = 0.7589''.$$

Fig. 93
SCALE = $\frac{3}{80}$



The circumferential velocity of the pinion (which is equal to the velocity of the rack) is 762″ per minute: hence the circumference of the pinion must be $\frac{762}{20} = 38.1''$.

From formula (3), the number of teeth is

$$N = \frac{38.1}{1.581} = 24,$$

and from formula (2), the diameter is

$$\frac{38.1}{3.14159} = 12\tfrac{1}{8}''.$$

Formula (34) gives, for the thickness of the rim,

$$t = 0.12 + (0.4 \times 1.581) = 0.7524'',$$

and, from formula (35), the nave thickness is

$$k = 0.4\sqrt[3]{1.581^2 \times 6.0625} + \tfrac{1}{2} = 0.4\sqrt[3]{15.156} + \tfrac{1}{2}$$
$$= 0.4 \times 2.475 + \tfrac{1}{2} = 1.49''.$$

The nave length is, from formula (36),

$$l' = 3.162 + \frac{12.125}{30} = 3.566''.$$

From formula (39) we have, for the diameter of the wrought-iron pinion shaft,

$$d = 0.086\sqrt[3]{1000 \times 6.0625} = 0.086\sqrt[3]{6062.5}$$
$$= 0.086 \times 18.234 = 1.57''.$$

Formulas (49) and (50) give, for the width and thickness of the pinion fixing-key,

$$s = 0.16 + \frac{1.57}{5} = 0.474''$$

and

$$s' = 0.16 + \frac{1.57}{10} = 0.317''.$$

The pinion is small enough to be made without arms. For the thickness of the web we have the following. The diameter across the nave is

$$1.57 + (2 \times 1.49) = 4.55'',$$

and the circumference is

$$4.55 \times 3.14159 = 14.294''.$$

Hence $h_1 = 1.43''$, and formula (17) becomes

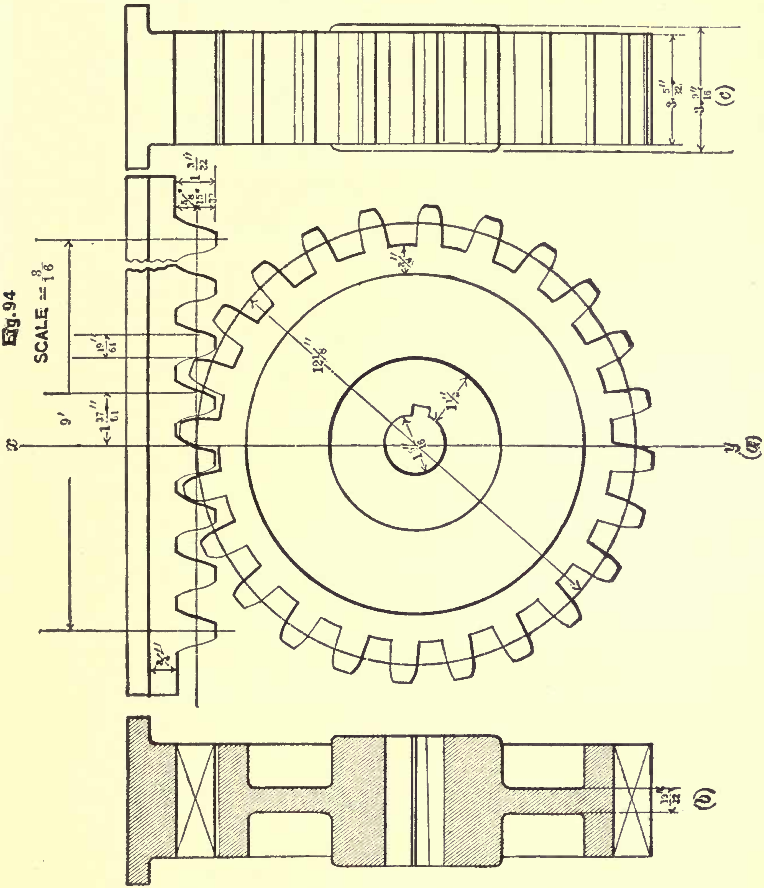
$$b_1 h_1^2 = 2.045 b_1 = \frac{1000 \times 6.0625}{500 \times 10}$$

$$b_1 = \frac{1.2125}{2.045} = 0.599''.$$

The dimensions, converted into fractions, are as follows:—

Diameter of pinion,	$D =$	$12\frac{1}{8}''$
Length of rack,	$9'$	
Pitch,	$p =$	$1\frac{37}{64}''$
Number of teeth,	$N =$	24
Face width,	$l =$	$3\frac{5}{32}''$
Total height of teeth,	$h =$	$1\frac{3}{32}''$
Height below pitch circle,	$h' =$	$\frac{5}{8}''$
Height above pitch circle,	$h'' =$	$\frac{15}{32}''$
Breadth of teeth,	$b =$	$\frac{49}{64}''$
Thickness of rim,	$t =$	$\frac{3}{4}''$
Thickness of nave,	$k =$	$1\frac{1}{2}''$
Length of nave,	$l' =$	$3\frac{9}{16}''$
Diameter of shaft,	$d =$	$1\frac{9}{16}''$
Width of key,	$s =$	$\frac{15}{32}''$
Thickness of key,	$s' =$	$\frac{5}{16}''$
Thickness of web,	$b_1 =$	$\frac{19}{32}''$.

The working drawings are shown in Fig. 94, drawn to a scale of $\frac{3}{16}$, and dimensions marked. Fig. (*a*) is a



full projection of the rack and pinion in gear, the rack being broken in order to save space; Fig. (*c*), a full projection taken from Fig. (*a*); and Fig. (*b*), a sectional projection of the rack and pinion, taken from Fig. (*a*), on the line *xy*. The cycloidal profiles of the teeth were drawn by the method given under Fig. 26 for the pinion, and under Fig. 35 for the rack.

Example 7. — Required to design, and make working drawings for, a cast-iron lantern gear and pinion to transmit a force of 1,600 pounds, moderate shock, the revolution ratio of the lantern to the pinion being $\frac{1}{3}$.

From formula (12, *b*), for the pitch,

$$p = 0.05\sqrt{1600} = 0.05 \times 40 = 2''.$$

The total height of the teeth is

$$h = 0.7 \times 2 = 1.4'',$$

and the breadth may be

$$b = 0.46 \times 2 = 0.92''.$$

The face width is

$$l = 2 \times 2 = 4''.$$

If we take, for the diameter of the lantern, $19\frac{1}{8}''$, we have for the number of teeth, from formula (3),

$$N = \frac{60.08}{2} = 30$$

and, from formula (7), the diameter of the pinion is

$$\frac{19\frac{1}{8}}{3} = 6\frac{3}{8}''.$$

The number of teeth for the pinion is

$$\frac{20.02}{2} = 10.$$

Formula (34) gives, for the rim thickness,

$$t = 0.12 + (0.4 \times 2) = 0.92''.$$

Formula (35) gives for the nave thickness, for the lantern,

$$k = 0.4\sqrt[3]{2^2 \times 9.5625} + \tfrac{1}{2} = 0.4\sqrt[3]{38.25} + \tfrac{1}{2}$$
$$= 0.4 \times 3.369 + \tfrac{1}{2} = 1.8476'',$$

and for the pinion,

$$k = 0.4\sqrt[3]{2^2 \times 3.1875} + \tfrac{1}{2} = 0.4\sqrt[3]{12.75} + \tfrac{1}{2}$$
$$= 0.4 \times 2.336 + \tfrac{1}{2} = 1.4344''.$$

For the nave length of the pinion we have, from formula (36),

$$l' = 4 + \frac{6.375}{30} = 4.2125'',$$

and for the lantern,

$$l' = 4 + \frac{19.125}{30} = 4.6375''.$$

The diameter of the pinion shaft, from formula (39), is

$$d = 0.086\sqrt[3]{1600 \times 3.1875} = 0.086\sqrt[3]{5100}$$
$$= 0.086 \times 17.213 = 1.48'',$$

and the diameter of the lantern shaft,

$$d = 0.086\sqrt[3]{1600 \times 9.5625} = 0.086\sqrt[3]{15300}$$
$$= 0.086 \times 24.826 = 2.135''.$$

For the pinion, the width and thickness of the fixing-key, from formulas (49) and (50), are

$$s = 0.16 + \frac{1.48}{5} = 0.456''$$

and

$$s' = 0.16 + \frac{1.48}{10} = 0.308''.$$

For the lantern,

$$s = 0.16 + \frac{2.135}{5} = 0.587''$$

and

$$s' = 0.16 + \frac{2.135}{10} = 0.3735''.$$

The pinion is small enough to be made without arms. Formula (23) gives, for the number of arms in the lantern,

$$n_1' = 0.56\sqrt{30}\sqrt[4]{2} = 0.56 \times 5.48 \times 1.19 = 4.$$

For arms having circular cross-sections, formula (19) gives a diameter of

$$d' = 0.15\sqrt[3]{\frac{1600 \times 9.5625}{4}} = 0.15\sqrt[3]{3825} = 0.15 \times 15.64 = 2.346''.$$

As explained in § VI., under Fig. 42, the radius for the lantern rungs is $\frac{19}{80} \times 2 = 0.475''$.

Dimensions for the lantern.

Diameter,	D =	$19\frac{1}{8}''$
Pitch,	p =	$2''$
Face width,	l =	$4''$*
Radius of rungs,	=	$\frac{15}{32}''$
Number of rungs,	N =	30
Thickness of rim,	t =	$\frac{59}{64}''$
Number of arms,	n_1' =	4
Diameter of arm cross-section,	d' =	$2\frac{11}{32}''$
Thickness of nave,	k =	$1\frac{27}{32}''$
Length of nave,	l' =	$4\frac{41}{64}''$
Diameter of shaft,	d =	$2\frac{9}{64}''$
Width of fixing-key,	s =	$\frac{19}{32}''$
Thickness of fixing-key,	s' =	$\frac{3}{8}''$

* See Fig. 95 (c).

	Dimensions for the pinion.
Diameter,	$D = 6\frac{3}{8}''$
Pitch,	$p = 2''$
Face width,	$l = 4''$
Total height of teeth,	$h = 1\frac{13}{32}''$
Breadth of teeth,	$b = \frac{59}{64}''$
Number of teeth,	$N = 10$
Thickness of nave,	$k = 1\frac{7}{16}''$
Length of nave,	$l' = 4\frac{7}{32}''$
Diameter of shaft,	$d = 1\frac{31}{64}''$
Width of fixing-key,	$s = \frac{29}{64}''$
Thickness of fixing-key,	$s' = \frac{5}{16}''$.

Fig. 95 gives the working drawings of the lantern and pinion, drawn to a scale of $\frac{3}{32}$. One of the lantern rungs is shown in section in Fig. (*c*) in order to show that the rungs are to be cast on the lantern, instead of being made separately, and driven into holes along the lantern rim, as is ordinarily the case. The arrangement of the rim, etc., is sufficiently explained by the figure. The teeth of the pinion are drawn according to the explanation given in § VI., under Fig. 42.

Example 8. — Given the data and dimensions of the pinion of Example 7, it is required to design an internal lantern, the revolution ratio of which to the pinion shall be $\frac{1}{4}$; the rungs of the lantern to be of wrought-iron, and to be driven into holes along the rim.

The radius for the rungs is the same as in Example 7, as is also the calculated rim thickness. But for an internal gear we take the rim thickness from $1\frac{1}{2}$ times to twice that of an external gear (see Fig. 96, *b*).

From formula (7), the diameter of the lantern is

$$D = 6\tfrac{3}{8} \times 4 = 25\tfrac{1}{2}'',$$

and, from formula (3), the number of rungs is

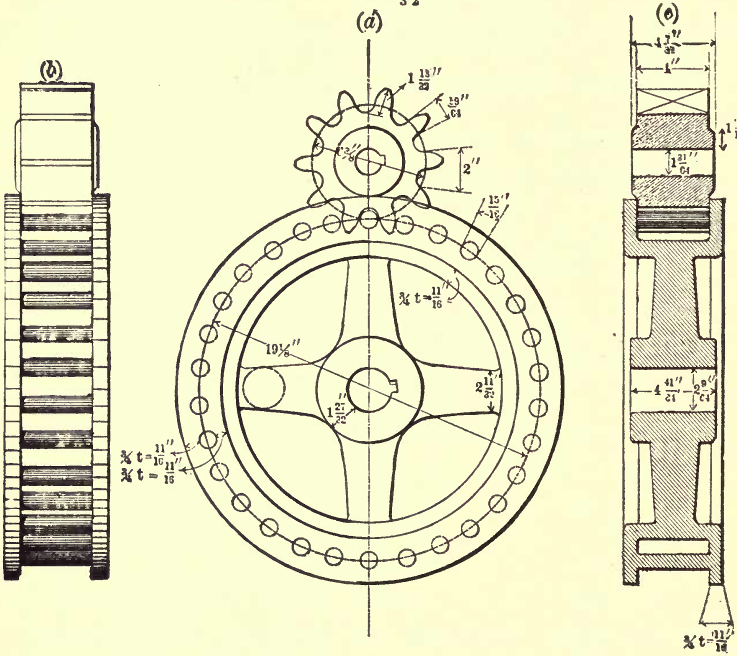
$$N = \frac{80.1}{2} = 40.$$

Formula (23) gives, for the number of arms,

$$n_1' = 0.56\sqrt{40}\sqrt[4]{2} = 0.56 \times 6.32 \times 1.187 = 5.$$

Fig.95

SCALE $= \frac{3}{32}$



From formula (19) the diameter for the circular cross-section of the arms is

$$d' = 0.15\sqrt[3]{\frac{1600 \times 12.75}{5}} = 0.15\sqrt[3]{4080} = 0.15 \times 15.98 = 2.40''.$$

For the nave thickness, formula (35) gives,

$$k = 0.4\sqrt[3]{2^2 \times 12.75} + \tfrac{1}{2} = 0.4\sqrt[3]{51} + \tfrac{1}{2} = 0.4 \times 3.7 + \tfrac{1}{2} = 1.98'',$$

and the nave length is, from formula (36),

$$l' = 4 + \frac{25.5}{30} = 4.85''.$$

Formula (39) gives, for the diameter of the wrought-iron lantern shaft,

$$d = 0.086\sqrt[3]{1600 \times 12.75} = 0.086\sqrt[3]{20400} = 0.086 \times 27.32$$
$$= 2.349''.$$

The width and thickness of the fixing-key are, from formulas (49) and (50),

$$s = 0.16 + \frac{2.349}{5} = 0.53''$$

and

$$s' = 0.16 + \frac{2.349}{10} = 0.395''.$$

	Dimensions for lantern.
Diameter,	$D = 25\frac{1}{2}''$
Pitch,	$p = 2''$
Face width,	$l = 4''$
Radius of rungs,	$\frac{15}{32}''$
Number of rungs,	$N = 40$
Thickness of rim,	$t = \frac{59}{64}''$
Number of arms,	$n_1' = 5$
Diameter of arms,	$d' = 2\frac{13}{32}''$
Thickness of nave,	$k = 1\frac{63}{64}''$
Length of nave,	$l' = 4\frac{27}{32}''$
Diameter of shaft,	$d = 2\frac{23}{64}''$
Width of fixing-key,	$s = \frac{17}{32}''$
Thickness of fixing-key,	$s' = \frac{25}{64}''$

	Dimensions for pinion.
Diameter,	$D = 6\frac{3}{8}''$
Pitch,	$p = 2''$
Face width,	$l = 4''$
Total height of teeth,	$h = 1\frac{13}{32}''$
Breadth of teeth,	$b = \frac{59}{64}''$
Number of teeth,	$N = 10$
Thickness of nave,	$k = 1\frac{7}{16}''$
Length of nave,	$l' = 4\frac{7}{32}''$
Diameter of shaft,	$d = 1\frac{31}{64}''$
Width of fixing-key,	$s = \frac{29}{64}''$
Thickness of fixing-key,	$s' = \frac{5}{16}''$.

The working drawings for Example 8 (shown in Fig. 96, drawn to a scale of $\frac{3}{32}$) need but little explanation. The dimensions are marked on the drawings; and the arrangement of the lantern arms, proportions of the rim, etc., will be sufficiently explained by a glance at Fig. (*b*). The teeth of the pinion were drawn by the method explained in § VI., under Fig. 44.

Example 9. — Required to design a train of cast-iron gears to lift a weight of 8,000 pounds (say, moderate shock) by means of a drum and cord as outlined in Fig. 97. The circumferential force of the driving-gear *r* is 1,000 pounds, and the diameter of the driver 12″. Let us assume that ten per cent of the driving-force is lost in overcoming the friction of the gear teeth, shaft bearings, etc. We have, therefore, an effectual force of $1000 - 1000 \times 0.10 = 900$ pounds, with which to lift the weight of 8,000 pounds. We must gear our power from 900 pounds to 8,000 pounds, or, in other words, we must gear our power up $\frac{8000}{900} = 9$ times. Since the powers of the gears are inversely proportional to their radii (formula 8), we must gear down our radii 9 times. We can gear from R to r' $2\frac{1}{4}$ times, and from R' to

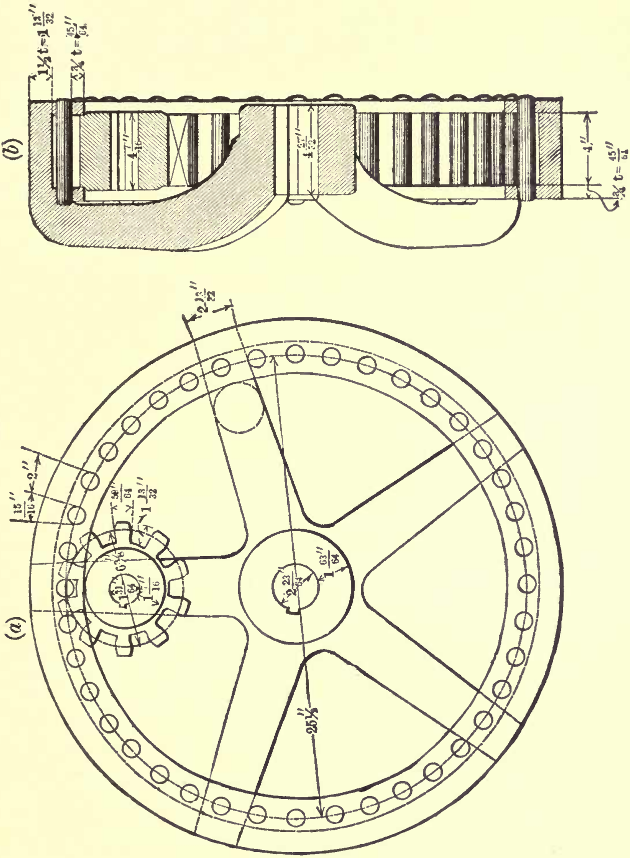


Fig. 96
SCALE = $\frac{3}{32}$

the drum r'' 4 times $(2\frac{1}{4} \times 4 = 9)$. If, therefore, we take $R = 13\frac{1}{2}''$, we have

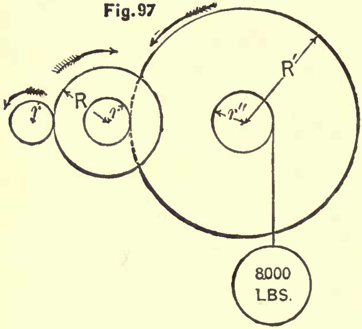
$$r' = \frac{13\frac{1}{2}}{2\frac{1}{4}} = 6'';$$

and, if we take $R' = 28''$, we have, for the radius of the drum,

$$r'' = \frac{28}{4} = 7''.$$

The power (or circumferential force) of the gear R is, of course, that of the driver r, 1,000 pounds;* and from formula (8) the power of the gear r' (and consequently that of the gear R') is $1000 \times 2\frac{1}{4} = 2250$ pounds. The total power of the drum is $2250 \times 4 = 9000$ pounds. Our example is now reduced to two very simple ones; viz., first to design a pair of gears (r and R) to transmit a force of 1,000 pounds (moderate shock), the diameters to be $2r = 12''$, and $2R = 27''$; and, second, to design a pair of gears (r' and R') to transmit a force of 2,250 pounds (moderate shock), the diameters being $2r' = 12''$, and $2R' = 56''$. Let us take them in the order given. From formula (12, b), the pitch for the gears r and R is

Fig. 97



$$p = 0.05\sqrt{1000} = 0.05 \times 31.62 = 1.581''$$

for the face width,

$$l = 2 \times 1.581 = 3.162''.$$

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* We do not take the lost power into account in calculating the strength of the gears.

The heights are,

$$h = 0.7 \times 1.581 = 1.1067''$$

$$h' = 0.4 \times 1.581 = 0.6324''$$

and

$$h'' = 0.3 \times 1.581 = 0.4743''.$$

Taking the breadth of the teeth equal to $0.45p$ gives

$$b = 0.45 \times 1.581 = 0.7115''.$$

From formula (3), the number of teeth for r is

$$N = \frac{37.699}{1.581} = 24$$

and for R,

$$N = \frac{84.823}{1.581} = 54.$$

Formula (34) gives, for the rim thickness,

$$t = 0.12 + (0.4 \times 1.581) = 0.7524''.$$

The gear r is without arms. For the gear R the number of arms, from formula (23), is

$$n_1' = 0.56\sqrt{54}\sqrt[4]{1.581} = 0.56 \times 7.348 \times 1.121 = 5.$$

For elliptical cross-sections, taking $b' = \frac{a}{2}$, formula (20) gives

$$b'a^2 = \frac{a^3}{2} = 0.00339 \frac{1000 \times 13.5}{5} = 9.153$$

or

$$a = \sqrt[3]{18.306} = 2.636''$$

$$b' = \frac{2.636}{2} = 1.318''.$$

Formula (35) gives, for the nave thickness for r,

$$k = 0.4\sqrt[3]{1.581^2 \times 6} + \tfrac{1}{2} = 0.4\sqrt[3]{15} + \tfrac{1}{2}$$
$$= 0.4 \times 2.466 + \tfrac{1}{2} = 1.486''$$

and for R,

$$k = 0.4\sqrt[3]{1.581^2 \times 13.5} + \tfrac{1}{2} = 0.4\sqrt[3]{33.744} + \tfrac{1}{2}$$
$$= 0.4 \times 3.231 + \tfrac{1}{2} = 1.794''.$$

From formula (36), the nave length for r is

$$l' = 3.162 + \tfrac{12}{30} = 3.562'',$$

and for R,

$$l' = 3.162 + \tfrac{27}{30} = 4.062''.$$

The diameter of the shaft for r is, from formula (39),

$$d = 0.086\sqrt[3]{1000 \times 6} = 0.086 \times 18.17 = 1.5626''$$

and for R,

$$d = 0.086\sqrt[3]{1000 \times 13.5} = 0.086 \times 23.81 = 2.0477''.$$

Formulas (49) and (50) give, for the width and thickness of the fixing-key for r,

$$s = 0.16 + \frac{1.5626}{5} = 0.4725''$$

and

$$s' = 0.16 + \frac{1.5626}{10} = 0.3163'',$$

and for R,

$$s = 0.16 + \frac{2.0477}{5} = 0.5695''$$

and

$$s' = 0.16 + \frac{2.0477}{10} = 0.3648''.$$

For the thickness of the web between the nave and rim of the gear r, the calculations are as follows. The diameter across the nave is equal to

$$d + 2k = 1.5626 + (2 \times 1.486) = 4.535''$$

and the circumference is 14.25″. Supposing the gear to have 10 arms, each having a width of one-tenth the nave circumference, we have

$$h_1 = \frac{14.25}{10} = 1.425''.$$

Formula (17) therefore gives, for the web thickness,

$$b_1 h_1^2 = 2.03 b_1 = \frac{1000 \times 6}{500 \times 10}$$

or

$$b_1 = \frac{1000 \times 6}{500 \times 10 \times 2.03} = 0.591''.$$

For the second pair of gears, r' and R', formula (12, b) gives a pitch of

$$p = 0.05\sqrt{2250} = 0.05 \times 47.434 = 2.3717''.$$

The face width is

$$l = 2 \times 2.3717 = 4.7434''.$$

For the heights of the teeth we have

$$h = 0.7 \times 2.3717 = 1.6602''$$

$$h' = 0.4 \times 2.3717 = 0.9487''$$

and

$$h'' = 0.3 \times 2.3717 = 0.7115''.$$

The breadth of the teeth at the pitch circle is

$$b = 0.45 \times 2.3717 = 1.0673''.$$

From formula (3), the number of teeth for r' is

$$N = \frac{37.699}{2.3717} = 16$$

and for R',

$$N = \frac{175.93}{2.3717} = 74.$$

The small gear r' is without arms. From formula (23), the number of arms for R' is

$$n_1' = 0.56\sqrt{74}\sqrt[4]{2.3717} = 0.56 \times 8.60 \times 1.241 = 6.$$

For elliptical cross-sections, taking $b' = \frac{a}{2}$, formula (20) gives

$$b'a^2 = \frac{a^3}{2} = 0.00339\frac{2250 \times 28}{6} = 35.60,$$

or

$$a = \sqrt[3]{71.20} = 4.1447''$$

and

$$b' = \frac{4.1447}{2} = 2.0723''.$$

The thickness of the rim is, from formula (34),

$$t = 0.12 + (0.4 \times 2.3717) = 1.0687''.$$

Formula (35) gives, for the nave thickness for r',

$$k = 0.4\sqrt[3]{2.37^2 \times 6} + \tfrac{1}{2} = 0.4\sqrt[3]{33.701} + \tfrac{1}{2}$$
$$= 0.4 \times 3.23 + \tfrac{1}{2} = 1.792''$$

and for the gear R',

$$k = 0.4\sqrt[3]{2.37^2 \times 28} + \tfrac{1}{2} = 0.4\sqrt[3]{157.273} + \tfrac{1}{2}$$
$$= 0.4 \times 5.398 + \tfrac{1}{2} = 2.6592''.$$

From formula (36), the nave length for r' is

$$l' = 4.7434 + \tfrac{12}{30} = 5.1434'',$$

and for R',

$$l' = 4.7434 + \tfrac{56}{30} = 6.61''.$$

For the shaft diameter for r', formula (39) gives

$$d = 0.086\sqrt[3]{2250 \times 6} = 0.086\sqrt[3]{13500} = 0.086 \times 23.81 = 2.048''$$

and for R',

$$d = 0.086\sqrt[3]{2250 \times 28} = 0.086\sqrt[3]{63000} = 0.086 \times 39.79 = 3.4219''.$$

For the width and thickness of the fixing-key for r', formulas (49) and (50) give

$$s = 0.16 + \frac{2.048}{5} = 0.5696''$$

and

$$s' = 0.16 + \frac{2.048}{10} = 0.3648'',$$

and for R',

$$s = 0.16 + \frac{3.4219}{5} = 0.8444''$$

and

$$s' = 0.16 + \frac{3.4219}{10} = 0.5022''.$$

For the web thickness for r', as before, the nave diameter is

$$d + 2k = 2.048 + (2 \times 1.792) = 5.63'',$$

and the circumference is 17.69″: hence

$$h_1 = \frac{17.69}{10} = 1.769''.$$

From formula (17),

$$b_1h_1^2 = 3.13b_1 = \frac{2250 \times 6}{500 \times 10}$$

or

$$b_1 = \frac{2250 \times 6}{500 \times 10 \times 3.13} = 0.8626''.$$

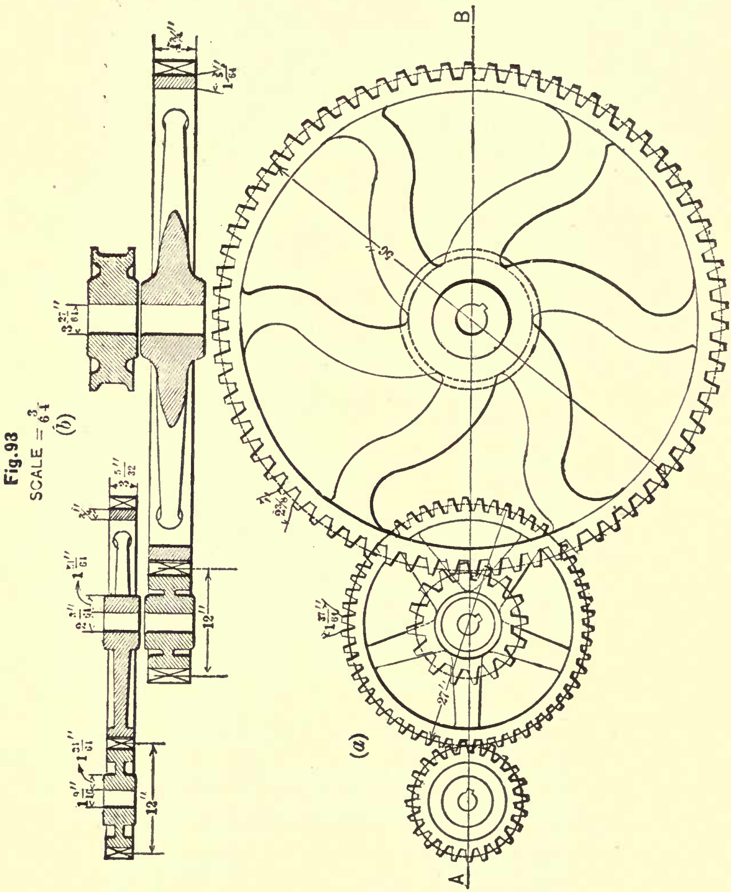
	Dimensions for gear r.
Diameter,	$D = 12''$
Pitch,	$p = 1\frac{37}{64}''$
Face width,	$l = 3\frac{5}{32}''$
Total height of teeth,	$h = 1\frac{7}{64}''$
Height below pitch circle,	$h' = \frac{41}{64}''$
Height above pitch circle,	$h'' = \frac{15}{32}''$
Breadth of teeth on pitch circle,	$b = \frac{23}{32}''$
Number of teeth,	$N = 24$
Rim thickness,	$t = \frac{3}{4}''$
Nave thickness,	$k = 1\frac{31}{64}''$
Nave length,	$l' = 3\frac{9}{16}''$
Shaft diameter,	$d = 1\frac{9}{16}''$
Width of fixing-key,	$s = \frac{15}{32}''$
Thickness of fixing-key,	$s' = \frac{5}{16}''$
Thickness of web,	$b_1 = \frac{19}{32}''$

	Dimensions for gear R.
Diameter,	$D = 27''$
Pitch,	$p = 1\frac{37}{64}''$
Face width,	$l = 3\frac{5}{32}''$
Total height of teeth,	$h = 1\frac{7}{64}''$
Height below pitch circle,	$h' = \frac{41}{64}''$
Height above pitch circle,	$h'' = \frac{15}{32}''$
Breadth of teeth on pitch circle,	$b = \frac{23}{32}''$
Number of teeth,	$N = 54$
Rim thickness,	$t = \frac{3}{4}''$
Number of arms,	$n_1' = 5$
Major axis of cross-sections,	$a = 2\frac{5}{8}''$
Minor axis of cross-sections,	$b' = 1\frac{5}{16}''$
Nave thickness,	$k = 1\frac{51}{64}''$
Nave length,	$l' = 4\frac{1}{16}''$
Shaft diameter,	$d = 2\frac{3}{64}''$
Width of fixing-key,	$s = \frac{37}{64}''$
Thickness of fixing-key,	$s' = \frac{23}{64}''$

	Dimensions for gear r'.
Diameter,	$D = 12''$
Pitch,	$p = 2\frac{3}{8}''$
Face width,	$l = 4\frac{3}{4}''$
Total height of teeth,	$h = 1\frac{21}{32}''$
Height below pitch circle,	$h' = \frac{61}{64}''$
Height above pitch circle,	$h'' = \frac{45}{64}''$
Breadth of teeth on pitch circle,	$b = 1\frac{5}{64}''$
Number of teeth,	$N = 16$
Rim thickness,	$t = 1\frac{5}{64}''$
Nave thickness,	$k = 1\frac{51}{64}''$
Nave length,	$l' = 5\frac{9}{64}''$
Shaft diameter,	$d = 2\frac{3}{64}''$
Width of fixing-key,	$s = \frac{37}{64}''$
Thickness of fixing-key,	$s' = \frac{23}{64}''$
Thickness of web,	$b_1 = \frac{55}{64}''$

	Dimensions for gear R'.
Diameter,	$D = 56''$
Pitch,	$p = 2\frac{3}{8}''$
Face width,	$l = 4\frac{3}{4}''$
Total height of teeth,	$h = 1\frac{21}{32}''$
Height below pitch circle,	$h' = \frac{61}{64}''$
Height above pitch circle,	$h'' = \frac{45}{64}''$
Breadth of teeth on pitch circle,	$b = 1\frac{5}{64}''$
Number of teeth,	$N = 74$
Rim thickness,	$t = 1\frac{5}{64}''$
Number of arms,	$n_1' = 6$
Major axis for cross-sections,	$a = 4\frac{9}{64}''$
Minor axis for cross-sections,	$b' = 2\frac{5}{64}''$
Nave thickness,	$k = 2\frac{21}{32}''$
Nave length,	$l' = 6\frac{39}{64}''$
Shaft diameter,	$d = 3\frac{27}{64}''$
Width of fixing-key,	$s = \frac{27}{32}''$
Thickness of fixing-key,	$s' = \frac{1}{2}''$.

The working drawings for the train are given in Fig. 98, drawn to a scale of $\frac{3}{64}$. Fig. (*a*) is a full projec-

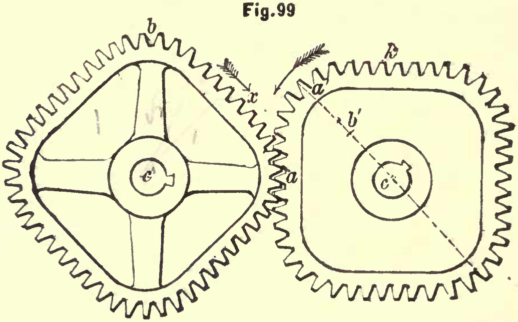


tion of the whole train, showing the pairs in gear; and Fig. (*b*) is a sectional projection of the whole train, taken from Fig. (*a*), on the line *AB*. The double curved arms of the large (56″) gear were drawn by the method explained in § XIV., under Fig. 87. It may be remarked here that very often, perhaps in the majority of cases, in order to save calculation and work, the pitches for all the gears of a train are taken the same. Obviously, when such is the case, the common pitch must be taken equal to that of the gear which transmits the greatest force; in the last example, that of the gear R' (or r' which transmits the same force). Suppose the driving-gear r to make 120 revolutions per minute; then, from formula (7), the number of revolutions per minute made by R is $120 \times \frac{12}{27} = 53\frac{1}{3}$. The gear r', being fixed to the same shaft, makes the same number of revolutions as R; and the number of revolutions per minute of R', and consequently of the drum r'', is $53\frac{1}{3} \times \frac{12}{56} = 11.43$. The diameter of the drum is 14″, and its circumference 43.98″: hence the circumferential velocity of the drum, or the velocity with which the weight will be lifted, is $\frac{43.98 \times 11.43}{12} = 41.89$ feet per minute.

§ XVII. — *Special Applications of the Principles of Toothed Gearing.*

In the foregoing pages the subject of toothed gearing has been treated in so far as it relates to ordinary machinery only. The simple, uniform, rotary motion of the spur wheel, bevel, or screw gear, the continuous rectilinear movement of the rack — these are met with daily in almost every shop and factory. But there are many special cases in which these simple, uniform motions are not sufficient. According to the work which is to be performed, we need, in one case, an intermittent rotary or rectilinear motion; in another, a gradually increasing or decreasing speed; and, in another, a reciprocating movement. These variations must be obtained from the uniformly rotating shop-shaft; and there are few fields in which the ingenuity of man has had wider scope, or produced more variety and beauty of mechanism, than in that of special gear-contrivances. Some of the more useful and common of these many special mechanisms will be found explained in the following pages.

Fig. 99

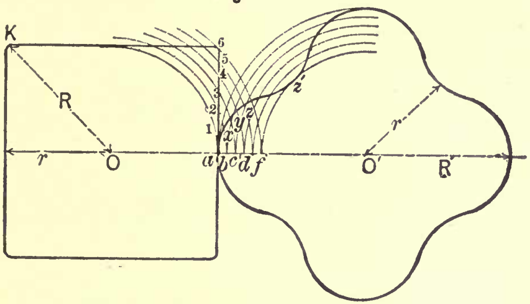


(1) *Spur Gearing.* — Fig. 99 represents a pair of "square" or "rectangular" gears, the object of which is to obtain a varying speed for the driven gear c' from the uniform rotary motion of the driver c. As explained in § XI., we have the expression,

$$\frac{n}{n'} = \frac{R'}{R}, \text{ or } n' = \frac{Rn}{R'},$$

in which R and n are the radius and number of revolutions of the driver, and R' and n' the same for the driven gear. From this last expression it is plain, that, if we increase R', we decrease the value of n'; if we decrease R', we increase n'; if we increase R, we increase n'; and, if we decrease R, we decrease n'. In Fig. 99, while the gears are in the position shown, the greatest radius of the driver gears with the smallest radius of the driven gear: the speed of the driven gear is, therefore, then at its maximum. As the gears revolve in the directions indicated by the arrows, the radius of the driver gradually decreases, and that of the driven gear gradually increases, until the points x and a' are in contact. The speed of the driven gear, therefore, grad-

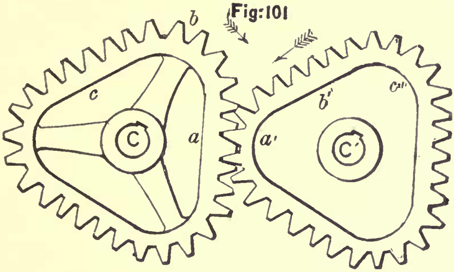
Fig. 100



ually decreases during this eighth of a revolution. From the moment of contact between the points x and a', the reverse action takes place, and the speed of the driven gear gradually increases until the points b and k are in contact. Thus, during the entire revolution, the driven gear continues to alternate from a gradually decreasing to a gradually increasing speed, and *vice versâ*. In order that rectangular gears shall work properly together, it is necessary, first, that the pitch peripheries of the two gears be equal in length, and, second, that the sum of the radii of each pair of points

(points which come into contact with each other) on the two pitch peripheries be equal to the distance between the centres of the gears. These two conditions suggest the method for finding the pitch periphery (or pitch line) of a rectangular gear which shall properly gear with a given driver, shown in Fig. 100, which is as follows. O is the given driver. Since the smallest radius of the driver gears with the greatest radius of the driven gear, and the sum of these two radii is equal to the distance between the centres, make $aO' = R$, and O' is the centre for the required gear. Divide the periphery of O into a number of small parts, $a1$, 12, 23, etc.; and from the point O as a centre, and $O1$, $O2$, $O3$, etc., as radii, strike arcs cutting the line of centres in the points b, c, d, etc. With the centre O' and radii $O'b$, $O'c$, $O'd$, etc., strike circle arcs, and lay off the arcs ax, xy, yz, etc., equal respectively to $a1$, 12, 23, etc., taking care that the points x, y, z., etc., fall upon the corresponding arcs, of which the point O' is the centre; so on, until the entire required pitch line is determined by the points thus found. As may be at once seen by comparing Figs. 99 and 100, the shape of the driven periphery depends upon the amount of curvature of the "corners" of the driver. Thus, for very slightly curved corners, the driven periphery becomes more nearly star shaped, as in Fig. 100. If we take the radius of curvature for the corners ($b'a'$, Fig. 99) equal to $b'c'$, the gear peripheries become equal and similar, and the gears square, with rounded corners.

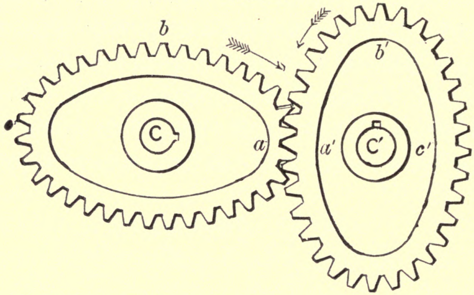
Fig. 101 represents a pair of "triangular" gears, the object of which is to obtain an alternating, varying speed from the uniformly rotating driver, as in rectangular gears. Triangular gears give fewer changes of speed per revolution than rectangular gears. In Fig. 101, *C* being the driver and *C′* the driven gear, the speed of the latter is at its minimum when the gears are in the positions shown in the figure. Since, from these positions, the radius of the driver gradually increases, and that of the driven gear decreases, as far as the points *b* and *b′*, the speed of the driven gear will gradually increase until the points *b* and *b′* are in contact, or for one-sixth of an entire revolution. The



reverse action will then take place until the points *c* and *c′* are in contact, and so on. Thus while, in rectangular gears, each gradually increasing or decreasing period takes place during one-eighth of a revolution, in triangular gears each of these periods occupies one-sixth of a revolution; that is, in rectangular gears there are eight alternately increasing and decreasing periods in one entire revolution of the driven gear, and in triangular gearing there are but six.

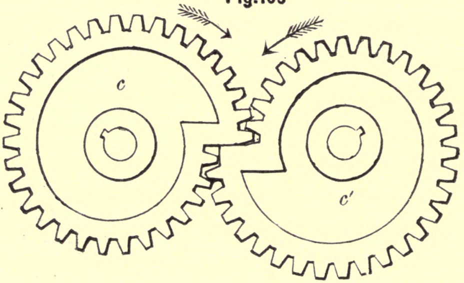
In "elliptical" gears (shown in Fig. 102) we have still another means of obtaining the same result, with the difference, that, in elliptical gears, each period of gradually increasing and decreasing speed takes place during one-fourth of a revolution: in other words, there are but four periods of increasing and decreasing

Fig. 102



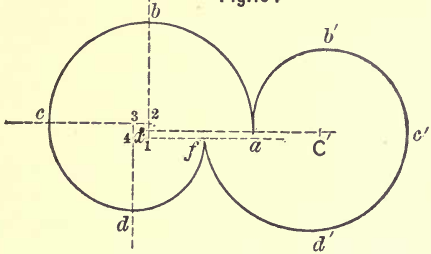
speed during one entire revolution of the driver. To construct the pitch lines of triangular and elliptical gears, we proceed as already explained, under Fig. 100, for rectangular gears. Fig. 103 represents a pair of

Fig. 103



"scroll" gears; *c* being the driver, and *c'* the driven gear. From the positions shown in the figure (in which the greatest radius of the driver gears with the smallest radius of the driven gear), as the gears revolve in the directions indicated by the arrows, the radius of the driver gradually and uniformly decreases, while that of the driven gear gradually and uniformly increases. The speed of the driven gear is therefore at its maximum when the gears are in the positions shown, and gradually and uniformly decreases during the entire revolution. The moment before the positions shown in the figure are reached, the smallest radius of the driver gears with the greatest radius of the driven gear: the speed of the latter is then at its minimum, and suddenly (as the gears assume the positions in the figure)

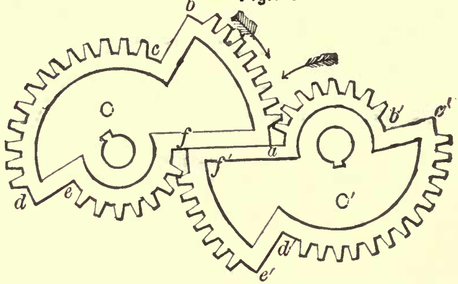
Fig. 104



changes to its maximum. To construct the pitch lines for a pair of scroll gears, proceed as follows. Construct the square 1234 (Fig. 104), each side of which is equal to one-fourth the distance *af*, which determines the rapidity of variation in the speed of the driven gear. Produce the sides of the square, as shown in the figure. From the point 1 as a centre, and a radius 1*a*, strike the arc *ab*; with the point 2 as a centre, and 2*b* as a radius, strike the arc *bc*; with centre 3, and radius 3*c*, strike the arc *cd*; and with centre 4, and radius 4*d*, strike the arc *df*. These four arcs together form the pitch line of the driver, the axis of which is at the centre x of the square 1234. Make $aC' = fx$, and C' is the centre for the driven pitch line; after which proceed to find points, and construct the pitch line $ab'c'd'$ as explained, for rectangular gears, under Fig. 100.

The mechanism represented in Fig. 105 is known as "sector" gears, and the object is to obtain a series of

Fig.105



different uniform speeds. In the figure, C is the driver, and C' the driven gear. As long as the arcs ab and ab' are in gear, the speed of the driven gear is the same. When the arcs cd and $c'd'$ come into gear, the speed of the driven gear becomes slower, but remains the same throughout the gearing of these two arcs. Similarly, when the arcs ef and $e'f'$ come into gear, the speed of the driven gear becomes still slower, but uniform during the gearing of these arcs. Thus, during each revolution, the driven gear has three periods of uniform speed, each differing from the others. In order that sector gears shall work properly together, it is necessary that the arcs which gear together be equal in length ($ab = ab'$, $cd = c'd'$, etc.), and that the sum of the arc lengths upon one gear be equal to the sum of the arc lengths upon the other ($ab + cd + ef = ab' + c'd' + e'f'$). Also the sum of the radii of each two arcs which gear together must be equal to the distance between the centres of the gears. Sector gears are somewhat difficult to construct, because considerable care must be taken that no two sectors of the driver gear at the same time with the driven gear. To illustrate, suppose (Fig. 105) that the arcs *ab* and *ab′* gear together at the same time as do the arcs *ef* and *e′f′*; that is, that the last few teeth of *ab* gear with the driven gear at the moment when the first few teeth of *ef* do the same. The driver will then strive to drive the driven gear at its maximum and minimum speeds at the same time, — an attempt which must obviously result in a fracture. In the figure, the arc *ef* ceases to gear with the driven gear at the moment when the arc *ab* begins to gear. Thus each arc of the driver must escape gear just in time for its successor to begin gear, and yet leave between these events no appreciable interval to disturb the uniformity of motion. Fig. 106 represents a peculiar kind of spur wheel and pinion. The wheel has two sets of teeth, one set being on each side; and the teeth of the two sets alternating in position, as shown in the figure. The pinion consists of two heart-cams, so arranged that each gears, in turn, with one set of teeth of the spur wheel. By this means a very slow motion is obtained for the spur wheel, which is moved through a distance of two teeth at each revolution of the cam-pinion. In the figure, the cam *a* leaves the tooth *a′* some time before the cam *b* comes into contact with the tooth *b′*: during this time, therefore, the spur wheel remains motionless, or, in other words, the motion of the spur wheel is intermittent. The length of time during which the driven gear remains motionless depends upon the shape of the cams. Thus, if we give to the cam *b* the shape indicated by the dotted outline, the cam will engage sooner with the tooth *b'*: consequently the period of rest will be shorter, and the period of motion longer. Also, if the cams differ from each other in shape, the periods of rest pre-

Fig. 106

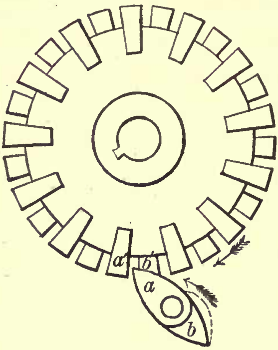
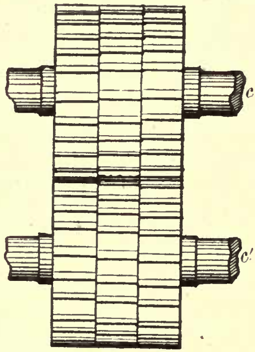
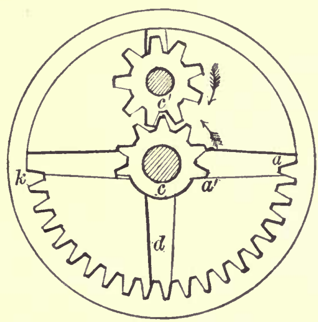


Fig. 107



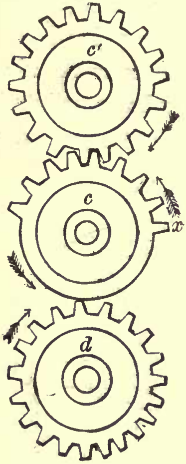
ceding the engagement of the two cams will be of different lengths, and the motion of the driven gear will be rendered still more variable. Fig. 107 represents the device known as "stepped" gears. This arrangement is used when very heavy powers are to be transmitted, and is met with sometimes in large and powerful machine tools. In the figure, each of the shafts *c* and *c'* bears three spur wheels; the pitches, diameters, etc., being equal, and the three gears being keyed firmly to the shaft. The gears are so fixed upon the shaft that their teeth are arranged in steps along the combined face, as shown in the figure; i.e., each gear is turned round upon the shaft slightly farther than the preceding one, so that instead of there being, say, $1\frac{1}{2}$ teeth gearing with the driver at one time, as is the case in a pair of ordinary spur wheels, there are $3 \times 1\frac{1}{2} = 4\frac{1}{2}$. The strain is thus divided among three gears, and the contrivance is capable of transmitting three times the power which can be transmitted by one pair of the gears. The device represented in Fig. 108 consists of a mutilated spur driver *c*, a spur pinion *c'*, and a mutilated internal gear *d;* the gears *c* and *d* are fixed upon the same shaft. The mutilated spur wheel *c* drives the pinion *c'* in the direction shown by the arrow, until the point *a'* is reached, when the gear *c* ceases to be the driver, and the mutilated internal gear takes its place. This drives the pinion *c'* in a contrary direction, until the point *k* is reached, when *c* again becomes the driver, and again reverses the direction of rotation of the pinion. The gear *c* being small in diameter, and the gear *d* large, the former drives the pinion *c'* at a slow speed, and the latter gives to it a high speed. The mechanism is therefore useful where a slow forward motion and a quick return are needed, as in the planer, and other machine tools.

Fig. 108



The arrangement represented in Fig. 109, which consists of two spur wheels and a mutilated spur driver, is intended to give to the spur wheels *c'* and *d* an alternating, intermittent motion. The driver *c*, rotating in the direction indicated by the arrow, drives the spur wheel *c′* in the direction shown, until the tooth *x* comes into gear. From that moment the driver acts upon the spur-wheel *d*, which it drives in the same direction as that given to the gear *c′*. When the tooth *x* comes into contact with the gear *d*, the driver ceases to act upon this gear, and returns to the gear *c′*. Thus the intermittent motions of the two spur wheels are made to alternate; the gear *c′* remaining at rest while the gear *d* is in motion, and contrariwise.

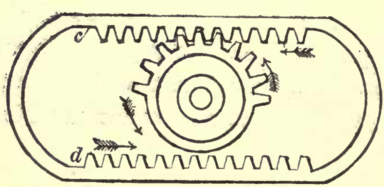
Fig. 109



In Fig. 110 the mutilated driving pinion engages alternately with the racks *c′* and *d*, which it drives, at the same speed, in opposite directions. The two racks being rigidly fixed to one frame, a reciprocating rectilinear motion is given to the frame; the forward and return motions being the same in velocity.

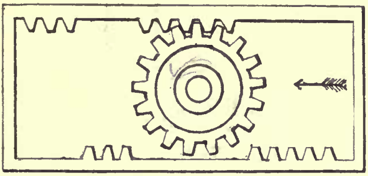
If the two racks are mutilated and the pinion entire, as in Fig. 111, the mutilations being alternately situated on the two racks, a continuous rectilinear motion of the rack frame will give to the pinion an alternating rotary motion; the speeds of advance and return motion being the same. If the rack mutilations are of different lengths, the motion of the pinion will be variable; the pinion moving over a greater distance when

Fig. 110



engaging with a long toothed part, and a less distance when engaging with a part of the rack containing a few teeth only. Fig. 112 represents a device for obtaining a uniform rectilinear motion in one direction, and a sudden return motion. The mutilated pinion, rotating continuously in the direction shown by the arrow, imparts a downward motion to the rack until the toothless part of the pinion is reached. The rack, being then free, is lifted quickly into its original position by means of the weight *W*, cord and pulley *K*. This arrangement is sometimes used on special automatic drills, in which case the rack is fixed upon a frame within which the drill spindle works. The spindle bears a raised ring, which fits into an annular depression within the frame. This allows the spindle to revolve freely, still enables the pinion and rack to give to the spindle the rectilinear motion necessary for the feed; and at the proper time the weight returns the spindle to its original position in readiness to repeat the desired operation.

Fig. 111



In Fig. 113 the mutilated driver *C* acts upon the gear *C′*, driving it uniformly in the direction shown by the arrow, until the toothless parts are opposite each other, when, the gear *C′* being free, the weight *W* falls, and quickly carries the gear *C′* into such a position that the driver again gears with it; and the same action again takes place. Thus a variable rotary motion is imparted to the driven gear, — slow when the driver acts upon it, and fast when it is acted upon by the weight *W*. If we change the numbers of teeth, so that, when the teeth of the driver and driven gear cease contact, the weight has the position *W′* (the directions of rotation being the same as in the figure), the weight, in falling, will carry the gear *C′* in a direction contrary to that imparted by the driver, and the motion of the driven gear will be an alternating or oscillating one, made up of a slow forward and a quick return movement.

Fig. 112

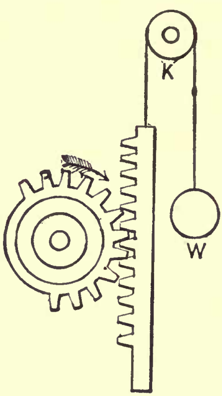
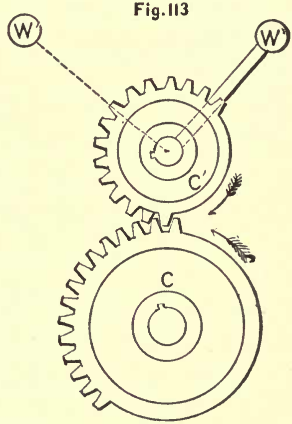


Fig. 113



(2) *Bevel Gearing.* — Under the head of special applications of bevel gearing we propose to include some pairs of gears which are not, strictly speaking, bevel gears, since the teeth are not "bevelled," but which resemble bevels in that their shafts are not parallel, but form either oblique or right angles with each other. An example of such a pair is seen in Fig. 114. The gear *C* is an ordinary spur wheel, and *C′* what is termed a "crown gear." The teeth of the latter gear are made so thin that their sides are practically parallel, and

hence gear with the spur wheel, notwithstanding the fact that their side lines all intersect at the centre of the pitch circle. Because of the necessarily thin teeth of the crown gear, such a pair as is shown in the figure can be used only for the transmission of very slight powers. They are very seldom seen in practice, except in models, mathematical instruments, and such like.

Fig.114

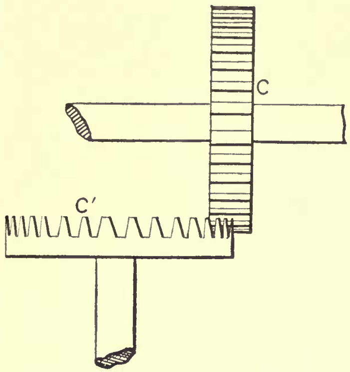
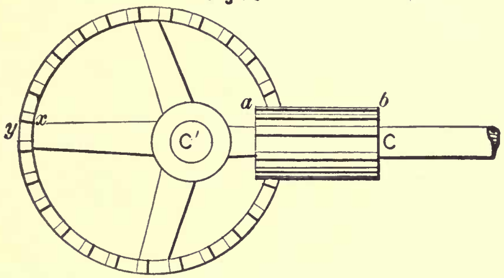


Fig. 115 represents a crown gear, *C′*, which engages with a wide-faced spur driver, *C*. The shaft of the crown gear is set eccentrically, instead of in the centre of the gear: hence a variable motion is given to it by

Fig.115



the uniform rotary motion of the driver. The motion of the crown gear is fast when the gears are in the positions shown in the figure, and gradually decreases until the largest radius comes into gear, when the reverse action takes place for the remaining half-revolution. The face width, *ab*, of the driver, must be at least equal to the difference between the greatest and smallest inner radii of the crown gear plus the thickness of the crown gear teeth, *xy*.

Fig. 116 represents a contrivance for obtaining three different uniform rotary motions for the shaft C' from a uniformly revolving shaft C, the two shafts being at

Fig. 116

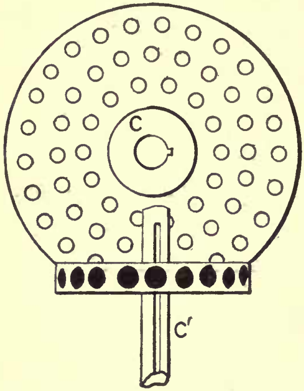
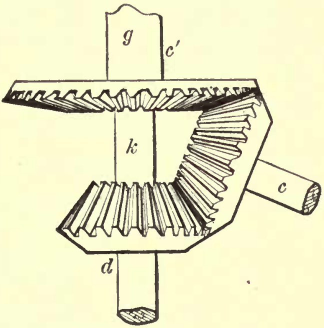


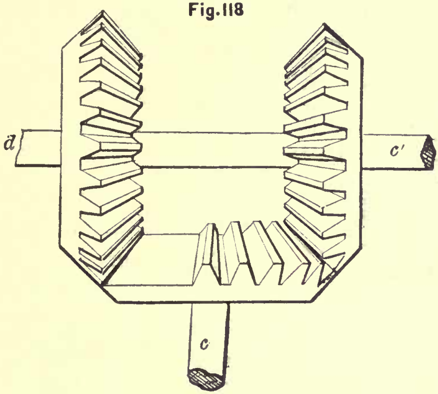
Fig. 117



right angles with each other. The wheel C has three sets of projecting pins, arranged in circles of different diameters, as shown in the figure. The pitches (distance between the centres of two adjacent pins) of all the circles are equal. The gear C' has a slotted face, the slots being slightly larger than the pins of the wheel C, and equally distant from each other. By sliding the gear C' along its shaft, it may be made to engage at will with either of the three circles of the wheel C, thus obtaining a quick or slow motion as may be required.

Fig. 117, which represents a device for giving two different velocities to the same shaft, consists of a driving bevel *c*, and two driven bevels *c′* and *d*, of different diameters, and running on the same shaft. The bevel *d*, being smaller than the bevel *c′*, is driven at a greater speed, and in a direction contrary to that of *c′*. The bevel *c′* is fixed to a collar or hollow shaft, *g*, which fits over the shaft *k*, thus allowing it to revolve in a contrary direction. If the driving bevel *c* is mutilated, and

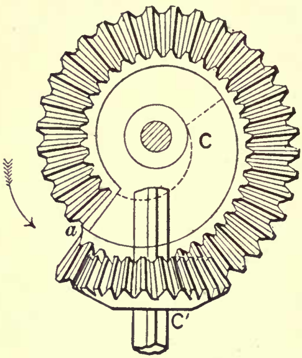
Fig. 118



the bevels *c′* and *d* fixed to the shaft *k*, an alternating rotary motion will be given to the shaft, the alternations being at different speeds. The same result may be obtained for the shaft *c* by mutilating the gears *c′* and *d* so that the toothed part of one is opposite the toothless part of the other, and making the bevel *c* the driven gear.

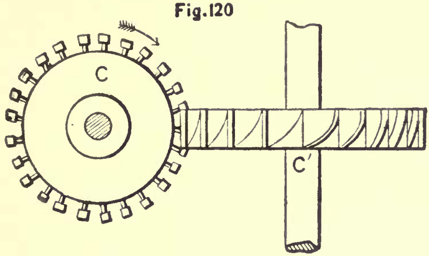
Fig. 118 represents a method of obtaining an alternating rotary motion from a uniformly rotating shaft, the driving and driven shafts being at right angles with each other. The mutilated driving bevel *c* drives the shaft *c'd* alternately in opposite directions, according as it gears with the bevel *c'* or *d*. The speeds of the forward and return motions are the same, since the bevels *c'* and *d* are of the same diameter. This contrivance was once used to give the reciprocating motion to planer-beds; a thread on the shaft *c'd*, which worked in a female thread in the bed, producing the rectilinear motion. The arrangement soon fell into disuse, for the reason that as much time was required for the return as for the forward motion, a waste which is now obviated by the more modern "quick return."

Fig. 119



The device represented in Fig. 119 is intended to transmit a gradually increasing speed to a shaft from the uniform rotary motion of a shaft at right or oblique angles. The scroll bevel *C* is the driver, and the ordinary bevel *C'* the driven gear. Starting with the smallest radius of the scroll bevel (at the point *a*) in gear with the driven bevel, and rotating in the direction indicated by the arrow, the radius gradually and steadily increases until the bevels assume the positions shown in the figure: consequently the speed of the driven bevel gradually and steadily increases during the entire revolution. The toothed part of the scroll bevel may be carried farther than in the figure, as indicated by the dotted lines, and the described action thus made to take place during more than one revolution. The shaft of the driven bevel carries a feather, which allows the bevel to slide along it without interfering with the rotary motion. In the figure, when the described action begins, the driven bevel *C′* is in its highest position on the shaft; and, as during the rotation the radius of the driving bevel increases, the former bevel is forced downward upon its shaft until the positions shown in the figure are reached. If the scroll bevel be made to rotate in a direction opposite to that indicated in the figure, it is plain that the teeth, not being prevented from so

Fig. 120

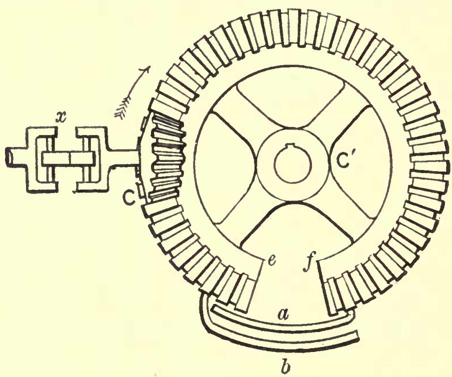


doing by the converging of their lines, will lift out of gear as the radius decreases, and thus destroy the action.

Fig. 120 represents a peculiar kind of bevel gear, more properly a pair of right-angle gears. The driving gear *C* bears upon its circumference small rollers, which gear into curved projections, or grooves, in the face of the driven gear *C′*, and, by rolling down these curves, give to the driven gear a rotary motion at right angles with that of the driver. The motion of the driven gear depends upon the shape of the projections. If these are curved, the curves being more oblique to the vertical at the bottoms than at the tops, as in the figure, the motion of the driven gear will be variable, — slow when each roller of the driver gears with the upper part of a projection, and gradually faster as the roller progresses downward. If, instead of being curved, the profiles of the projections are straight lines, the motion of the driven gear will be nearly uniform.

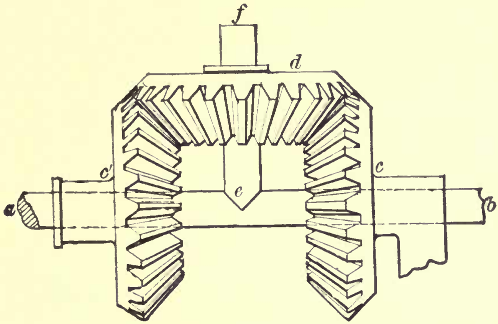
The motions described under Fig. 113 may be transmitted from one shaft to another at right or oblique

Fig. 121



angles, by using mutilated bevels in place of the spur gears shown in that figure. Fig. 121 represents an arrangement of bevels known as the "mangle wheel" and pinion, the object of which is to obtain an alternating rotary motion for the mangle wheel *C′*. This wheel has teeth upon both sides, one side only being shown in the figure. As the driving bevel *C* rotates, it drives the mangle wheel in the direction indicated by the arrow, until the opening *ef* is reached. At this point the guide *a* comes into contact with the shaft of the driver, which it forces downward through the opening, and into such a position, that the driver gears with the teeth on the other side of the mangle wheel. The latter is then driven in an opposite direction, until the opening *cf* is again reached, when the guide *b* lifts the driver up through the opening into gear with the first-mentioned side of the mangle wheel. This operation is repeated indefinitely; the mangle wheel making one entire revolution alternately in each direction. The shaft of the driving bevel carries a universal joint, *x*, which allows

Fig. 122



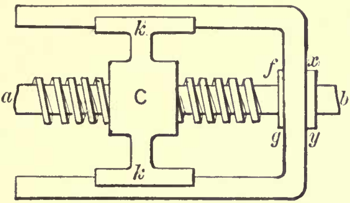
it enough freedom of motion to fall and rise through the opening in the mangle wheel.

Fig. 122 represents an arrangement of bevel gears, the object of which is to produce a double or half speed; the three bevel gears having the same diameter. The bevel *c* is rigidly fixed (so that it cannot rotate) to the bed of the mechanism, and the shaft *ab* runs loosely through it. The bevel *c′* runs loose upon the shaft *ab*, which carries a short, right-angle shaft, *cf*. Upon this right-angle shaft the bevel *d* runs loose. If, now, a rotary motion be given to the shaft *ab*, the right-angle shaft *ef*, and with it the bevel *d*, will be made to revolve in a vertical plane about the axis *ab*. The bevel *d* will also, by its gearing with the fixed bevel *c*, be made to rotate upon its own axis, *ef*. Since the bevels *c* and *d* are of the same diameter, the speeds of these two rotations will be the same: therefore the bevel *d* will transmit to the bevel c' the effect of two speeds, each equal to that of the shaft *ab*. And, since the speeds are in the same direction, the bevel c' will be made to rotate about the shaft *ab* with a speed equal to twice that of the shaft; that is, while the shaft *ab* makes one entire revolution in a given direction, the bevel c' will make two revolutions in the same direction. If the bevel c' be made the driver, its rotary motion will transmit to the bevel *d* a rotary motion about its axis *ef*, and, by means of the fixed bevel *c*, also a revolving motion in a vertical plane about the axis *ab*. The bevels having equal diameters, half the speed of the driver is transmitted in the rotation of the bevel *d* about its axis *ef*, and half in the vertical rotation about the axis *ab*; that is, the shaft *ab* will be made to rotate with a speed equal to one-half that of the driver: while the driver c' makes two entire revolutions in a given direction, the shaft *ab* will make one revolution in the same direction. The relative speeds of the shaft *ab* and the bevel c' may be varied by changing the relative diameters of the bevels.

(3) *Screw Gearing.* — Fig. 123 represents a very common mode of transforming uniform rotary into uniform rectilinear motion. The threaded shaft *ab*, rotating upon its axis, and restrained from other motion by the collars *xy* and *fg*, works in a female thread in the piece

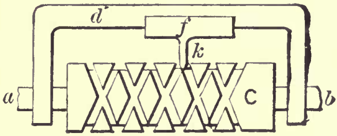
C, thus giving to the latter piece a rectilinear motion upon the slides *k*, *k*. By reversing the direction of rotation of the shaft *ab*, the direction of the motion of the piece *C* will also be changed. This device is seen in the leading-screws of lathes, in the arrangement for feeding the tool holders in planing machines, drills, etc.

Fig. 123



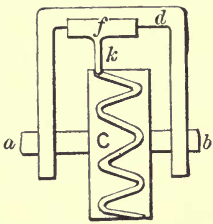
In Fig. 124 the cylinder *C* has right and left spiral grooves cut in its surface, as shown in the figure. The tooth *k* of the slide *f* fits into the grooves. Upon giving to the cylinder a rotary motion about its axis *ab* (supposing the tooth *k* to be working in the right-hand groove), the slide *f* is made to move along the frame *d*, upon which it rests, until the end of the groove is reached, when the tooth runs into the left-hand groove, and the slide *f* returns in the opposite direction. Thus a reciprocating rectilinear motion is obtained from the uniform rotary motion of the cylinder.

Fig. 124



In Fig. 125 a uniform rotary motion of the pulley *C* gives to the slide *f* a reciprocating rectilinear motion along the frame *d*, by means of the zigzag groove upon the pulley surface, in which the tooth *k* of the slide

Fig. 125



works. By giving to the groove in the surface of the pulley the proper shape, the motion of the slide *f* may be made uniform, variable, or intermittent.

Fig. 126 represents a device for transforming uniform rotary motion into two rectilinear motions in opposite directions. The shaft *ab*, which carries the right and left screw-threads shown in the figure, rotates within its bearing *O*. The right and left screws work in female screws within the pieces *C* and *C'*: consequently these pieces are driven in contrary directions, approaching each other, or receding from each other, according to the direction of rotation of the shaft *ab*. This arrangement is used in presses of various kinds, the arms indicated by the dotted lines being drawn together at their tops by the action of the screws, and the point *x* being forced slowly downward with great force and steadiness. The arrangement of worm wheels represented in Fig. 127 is intended to produce two uniform rotary motions in opposite directions. The right and left worms on the shaft *ab* cause the worm wheels *C* and *C'* to rotate in opposite directions when the shaft is given a rotary motion. The

Fig. 126

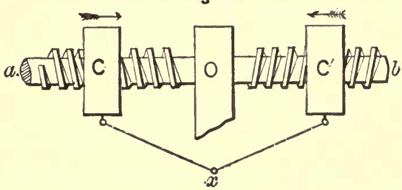
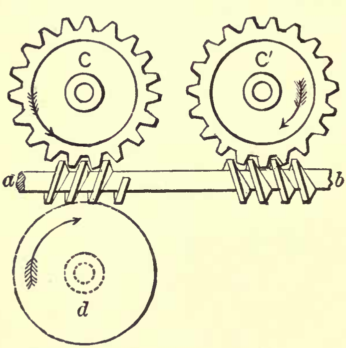


Fig. 127



same effect may be obtained with one worm, by gearing with it two worm wheels, *C* and *d*, on opposite sides of the shaft, as indicated by the dotted circle.

Fig. 128 represents a peculiar example of screw gearing. The disk *C* carries upon its side an elevated spiral, as shown in the figure. This spiral gears with an ordinary spur gear *C'*, the shaft of which is at right angles with that of the disk. At each revolution of the disk *C*, the constantly changing radius of the spiral causes the spur gear to rotate for a distance equal to one tooth; the pitch *p* of the spiral being equal to the pitch of the spur gear. By gearing with the spiral two spur gears (the second is indicated in the figure by the dotted lines), motion may be transmitted from the spiral to two shafts at right angles with each other. In a like manner the spiral may be made to drive several spur gears at once, the shafts making oblique angles with each other.

Fig. 128

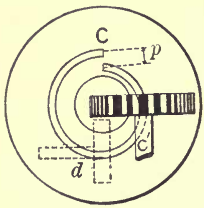
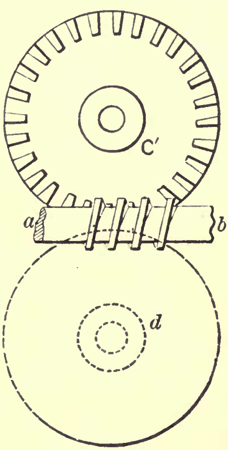


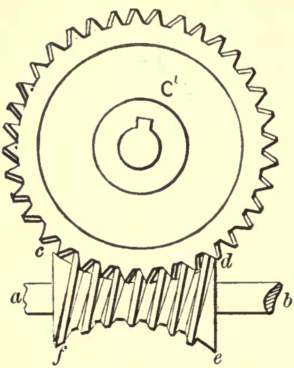
Fig. 129



In Fig. 129 we have represented a "side" worm wheel *C'*, and worm. The former carries upon its side projections or teeth, as shown in the figure; and the worm on the shaft *ab*, gearing with these teeth, causes the wheel *C'* to rotate uniformly, the action being similar to that of an ordinary worm and wheel. By gearing with the worm two side worm wheels (the second being indicated in the figure by the dotted circle *d*), the teeth being on the sides of the wheels which face towards each other, two uniform rotary motions in opposite directions may be obtained, as in Fig. 127. The motions described under Fig. 113 may be obtained for shafts at right angles with each other by substituting for the mutilated spur gears a worm and mutilated worm wheel.

Fig. 130 represents a kind of worm and worm wheel sometimes used to transmit very heavy powers. The primitive surface *cdef*, of the worm, instead of being a right cylinder, as in ordinary worms, is a solid of revolution generated by the revolution of the circle arc *ef* about the axis *ab*. The object of this is to obtain a contact of several teeth at one time. In the figure, seven teeth of the worm are in gear at the same time with the teeth of the worm wheel, and each tooth sustains an equal share of the transmitted strain. In Fig. 127 only two teeth of the worm are in gear at one time with the teeth of the driven wheel *C*. If, therefore, we have to transmit such a force that the strain on the teeth is 10,000 pounds, for example, each tooth of the worm in Fig. 127 will sustain a strain of $\frac{10000}{2} = 5{,}000$ pounds; while under similar circumstances each tooth of the worm in Fig. 130 will sustain a strain of $\frac{10000}{7} = 1{,}430$ pounds: in other words, the latter worm is capable of transmitting $\frac{7}{2} = 3\frac{1}{2}$ times the force of the former worm with the same strain upon each tooth.

Fig. 130



APPENDIX.

The present tendency among mechanical men in favor of the use of the diametral instead of the older and more widely known circumferential pitch, together with the increasing importance of cut gears (in the construction of which the diametral pitch seems to be especially convenient), has induced the author to devote an appendix to the brief discussion of the relative values of the two kinds of pitch, to a brief explanation of the method of constructing cut gears, and to the working-out of simple rules and formulas, by means of which all the necessary calculations may be made without the use of the circumferential pitch. From § X we have the expression $p_d = \frac{\pi}{p}$, in which p_d and p represent respectively the diametral and circumferential pitch, and π the irrational constant 3.14159+. The following table gives values for the diametral pitch, for different circumferential pitches, in inches. A glance at the table will show, that, in the list of most common circumferential pitches, not one corresponds to a diametral pitch of whole numbers, or even exact eighths, sixteenths, thirty-seconds, etc. In fact, the diametral pitch can be a whole number only when the corresponding circumferential pitch is an exact divisor of the irrational constant π, a condition which is not at all likely to be fulfilled.

p	p_d	p	p_d
⅛	25.1327	3½	0.8976
¼	12.5664	4	0.7854
½	6.2832	4½	0.6981
¾	4.1888	5	0.6283
1	3.1416	5½	0.5711
1⅛	2.7926	6	0.5236
1¼	2.5132	6½	0.4833
1½	2.0944	7	0.4488
1¾	1.7952	7½	0.4188
2	1.5708	8	0.3927
2¼	1.3963	9	0.3491
2½	1.2566	10	0.3142
2¾	1.1424	12	0.2618
3	1.0472	14	0.2244

For this reason, in all gears which have to be laid out, — as cast gears, in the construction of which the pitch must be stepped off around the pitch circumference in the drawings and pattern, — the circumferential pitch only can be conveniently used. In such cases, even if we have given the diametral pitch, we must practically find the circumferential pitch before we can properly divide our pitch circumference, and lay out the teeth. At this point of the construction, the important questions are, "How many teeth is the gear to have?" and "How much space on the pitch circle does each tooth need?" We care as little how many teeth there are per inch of diameter as how many teeth there may be per pound of metal. In performing the calculations necessary to the laying-out of gears, the diametral pitch offers no advantages over the circumferential. Thus, to obtain the number of teeth with the latter pitch, we divide the pitch circumference (an irrational quantity) by the pitch; while in using the former pitch the case is no better, for, to find the number of teeth in the gear, we must multiply the pitch diameter by the diametral pitch (itself an irrational quantity). Again: the rules and formulas for the tooth dimensions at present in use in the shops are in terms of the circumferential pitch, — for example, the formulas $l = 2p$, or $l = 2\frac{1}{2}p$, $h = 0.7p$, etc., given in the preceding pages, — and, while using the diametral pitch, we must either obtain the circumferential pitch in order to find our tooth dimensions, or devise and introduce new rules and formulas in terms of the diametral pitch. The author having taken the pains to ask a considerable number (68) of draughtsmen and pattern-makers in the States of New York, Pennsylvania, New Jersey, and Connecticut, their preferences, finds that a very large majority (61 to 7) of those spoken to favor the use of the old circumferential pitch. This would seem to indicate, that, while the same theorists who are striving to force upon the American mechanic the French metric system are clamoring for an absolute discontinuance of the use of the old pitch, the practical mechanic, who does the measuring and constructing, goes steadily on with his work, looking neither to the right for a "centimeter," nor to the left for a "diametral pitch."

But while, according to the opinion and experience of the author, the diametral pitch is of no practical use in cast gears, it cannot be reasonably disputed, that, in the construction of cut gears, this pitch has, indeed, advantages over the circumferential, and for this reason deserves the attention and respect of every intelligent mechanic.

In the construction of cut gears the wheels are first cast without teeth, the entire thickness of the rim being its own thickness when finished plus the height of the teeth $(t + h)$. The spaces between the teeth are then cut out by means of revolving circular cutters, the blades of the cutters being as nearly as possible the shape of the required spaces. In order to properly construct cut gears, a shop must be provided with different sets of cutters, corresponding to the different pitches and diameters of gears. The principle of the gear-cutter series may be illustrated as follows. Suppose we wish to construct a set of cutters for a No. 1 pitch. The extreme variation in the shape of the cutters must obviously be between the cutter for the gear having the greatest diameter (the rack) and that for the gear having the smallest diameter (say the pinion having eleven teeth). Between these two we must have a sufficient number of cutters to cut No. 1 teeth for a gear of any diameter without serious error. Similarly, for each other necessary pitch, we must have a set of cutters composed of a sufficient number to make our errors unimportant. Of course the greater number of cutters we have in each set, the more accurate will be our work. Thus, if we have a cutter of each pitch for a gear of eleven teeth, another for a gear of twelve teeth, another for thirteen teeth, and so on, our gears will be theoretically accurate.

But gear-cutters are expensive tools, and it is therefore important to reduce the number to the minimum which can be used without making the errors so great as to do practical harm. Mr. George B. Grant, in an article published some months ago in the "American Machinist," points out the fact, that, since the extreme variation in the shape of the cutters is less for fine than for coarse pitches, the number of cutters necessary for the same degree of accuracy is less in the former than in the latter. He gives for the proper number of cutters in the different sets the following table: —

For a	16 pitch or finer,	6 cutters
For an	8 to 16 pitch,	12 cutters
For a	4 to 8 pitch,	24 cutters
For a	2 to 4 pitch,	48 cutters.

If we substitute for the circumferential pitch, p in formula (10), its value in terms of the diametral pitch,

$$\left(p = \frac{\pi}{p_d} = \frac{3.14159}{p_d}\right)$$

we shall have

$$\frac{\pi}{p_d} = 4.93\sqrt{\frac{P}{f} \times \frac{p}{l}}$$

or

$$\frac{3.14159}{p_d} = 4.93\sqrt{\frac{P}{f} \times \frac{p}{l}}.$$

From this, by transposing, we have,

$$p_d = \frac{3.14159}{4.93}\sqrt{\frac{f}{P} \times \frac{l}{p}}$$

or

$$p_d = 0.637\sqrt{\frac{f}{P} \times \frac{l}{p}} \qquad (1).$$

Rule.—To find the diametral pitch for a gear of any material, divide the greatest safe working-stress in pounds per square inch for the material used by the force transmitted, multiply the quotient by the assumed ratio of the face width to the circumferential pitch, extract the square root of the product thus obtained, and multiply the result by 0.637.

By substituting

$$p = \frac{\pi}{pd} = \frac{3.14159}{pd}$$

in formulas (12, *a*, *b*, *c*), we obtain,

$$\frac{\pi}{pd} = \frac{3.14159}{pd} = 0.055\sqrt{P}$$

$$\frac{\pi}{pd} = \frac{3.14159}{pd} = 0.05\sqrt{P}$$

and

$$\frac{\pi}{pd} = \frac{3.14159}{pd} = 0.035\sqrt{P}.$$

From these, by transposing, we have,

$$pd = \frac{3.14159}{0.055}\sqrt{\frac{1}{P}}$$

$$pd = \frac{3.14159}{0.05}\sqrt{\frac{1}{P}}$$

and

$$pd = \frac{3.14159}{0.035}\sqrt{\frac{1}{P}}$$

or, reducing the three last found equations. we have,

$$\left.\begin{array}{lll} \text{For violent shock,} & p_d = 57.12\sqrt{\frac{1}{P}} & (a) \\ \text{For moderate shock,} & p_d = 62.83\sqrt{\frac{1}{P}} & (b) \\ \text{For little or no shock,} & p_d = 89.76\sqrt{\frac{1}{P}} & (c) \end{array}\right\} \quad (2).$$

Formulas (12, *a*, *b*, *c*) were determined upon the condition that the face width equals twice the circumferential pitch: hence substituting $p = \frac{\pi}{p_d}$ in the expression $l = 2p$ gives,

$$l = \frac{2\pi}{p_d} = \frac{6.283}{p_d}.$$

After determining the diametral pitch p_d from formula (2), the face width must not be taken less than $\frac{6.283}{p_d}$.

Rule. — To determine the diametral pitch for a cast-iron gear, when $l \geqq \frac{6.283}{p_d}$, extract the square root of the reciprocal of the force transmitted, and multiply the result by 57.12 for violent shock, 62.83 for moderate shock, or 89.76 for little or no shock.

The above value of the circumferential in terms of the diametral pitch, substituted in formulas (14, *a*, *b*, *c*), gives

$$\frac{\pi}{p_d} = \frac{3.14159}{p_d} = 1.29\sqrt{\frac{H}{v}}$$

$$\frac{\pi}{p_d} = \frac{3.14159}{p_d} = 1.17\sqrt{\frac{H}{v}}$$

and

$$\frac{\pi}{p_d} = \frac{3.14159}{p_d} = 0.82\sqrt{\frac{H}{v}}.$$

By transposing,

$$p_d = \frac{3.14159}{1.29}\sqrt{\frac{v}{H}}$$

$$p_d = \frac{3.14159}{1.17}\sqrt{\frac{v}{H}}$$

and

$$p_d = \frac{3.14159}{0.82}\sqrt{\frac{v}{H}}$$

or, reducing, we have,

$$\left.\begin{array}{lll} \text{For violent shock,} & p_d = 2.435\sqrt{\frac{v}{H}} & (a) \\ \text{For moderate shock,} & p_d = 2.685\sqrt{\frac{v}{H}} & (b) \\ \text{For little or no shock,} & p_d = 3.831\sqrt{\frac{v}{H}} & (c) \end{array}\right\} \quad (3).$$

As before, the condition $l \geqq \frac{6.283}{p_d}$ must be fulfilled.

Rule. — To determine the diametral pitch for a cast-iron gear from the horse-power and circumferential velocity in feet per second, when $l \geqq \frac{6.283}{p_d}$, divide the velocity by the horse-power, extract the square root of the quotient, and multiply the result by 2.435 for violent shock, 2.685 for moderate shock, or 3.831 for little or no shock.

In a similar manner, by substituting $p = \frac{\pi}{p_d}$ in formulas (16, *a*, *b*, *c*), we obtain,

$$\frac{3.14159}{p_d} = 19.54\sqrt{\frac{H}{Dn}}$$

$$\frac{3.14159}{p_d} = 17.72\sqrt{\frac{H}{Dn}}$$

and

$$\frac{3.14159}{p_d} = 12.42\sqrt{\frac{H}{Dn}}$$

Transposing and reducing these three equations, as with the preceding, we have

$$\left.\begin{array}{lll} \text{For violent shock,} & p_d = 0.161\sqrt{\dfrac{Dn}{H}} & (a) \\ \text{For moderate shock,} & p_d = 0.177\sqrt{\dfrac{Dn}{H}} & (b) \\ \text{For little or no shock,} & p_d = 0.253\sqrt{\dfrac{Dn}{H}} & (c) \end{array}\right\} \quad (4).$$

Rule. — To determine the diametral pitch for a cast-iron gear from the horse-power and number of revolutions per minute, when $l \geqq \frac{6.283}{p_d}$, multiply the diameter of the gear by the number of revolutions, divide the product by the horse-power, extract the square root of the quotient thus obtained, and multiply the result by 0.161 for violent shock, 0.177 for moderate shock, or 0.253 for little or no shock.

Example 1. — Required the diametral pitch for a steel gear which will transmit a force of 30,000 pounds, assuming $\frac{l}{p} = 3$; the greatest safe working-stress per square inch being 20,000 pounds. From formula (1) we have,

$$p_d = 0.637\sqrt{\frac{20000}{30000} \times 3} = 0.637\sqrt{2} = 0.637 \times 1.41 = 0.898.$$

Example 2. — Required the diametral pitch for a cast-iron gear to transmit a force of 900 pounds, moderate shock. From formula (2, *b*) we have,

$$p_d = 62.83\sqrt{\frac{1}{900}} = \frac{62.83}{30} = 2.09.$$

Hence

$$l = \frac{6.283}{p_d} = \frac{6.283}{2.09} = 3''.$$

Example 3. — The horse-power to be transmitted by a cast-iron gear is 10, moderate shock, and the circumferential velocity 5 feet per second. Required the diametral pitch. Formula (3, *b*) gives

$$p_d = 2.685\sqrt{\frac{5}{10}} = 2.685\sqrt{\frac{1}{2}} = \frac{2.685}{1.41} = 1.90 .$$

$$l = \frac{6.283}{1.90} = 3.31''.$$

Arms: If, in formula (23), we substitute for p its value of $\frac{\pi}{p_d}$, we will have

$$n_1' = 0.56\sqrt{N}\sqrt[4]{\frac{\pi}{p_d}}.$$

Extracting the fourth root of π in this equation gives

$$n_1' = 0.56 \times 1.3313\sqrt{N}\sqrt[4]{\frac{1}{p_d}}$$

or

$$n_1' = 0.746\sqrt{N}\sqrt[4]{\frac{1}{p_d}} \qquad (5).$$

Rule. — To determine the number of arms in a gear, extract the square root of the number of teeth and the fourth root of the reciprocal of the diametral pitch; multiply these two roots together, and their product by 0.746.

From the expression $p = \frac{3.14159}{p_d}$ we may obtain, by squaring both sides, $p^2 = \frac{9.86965}{p_d^2}$. This, substituted in formula (29), gives

$$b_1 h_1^2 = \frac{0.8 \times 9.86965 R}{p_d^2 n_1},$$

or

$$b_1 h_1^2 = 7.896 \frac{R}{p_d^2 n_1}, \qquad (6).$$

Rule. — To determine the quantity $b_1 h_1^2$ (the thickness of the arm multiplied by the square of the width), divide the radius of the gear by the product of the square of the diametral pitch into the number of arms, and multiply the quotient by 7.896.

By substituting $p^2 = \frac{9.86965}{p_d^2}$, in formula (30), we obtain,

$$d' = 1.105 \sqrt[3]{9.86965 \frac{R}{p_d^2 n_1}},$$

which reduces to

$$d' = 2.37 \sqrt[3]{\frac{R}{p_d^2 n_1}}, \qquad (7).$$

Rule. — To determine the diameter for arms having circular cross-sections, divide the radius of the gear by the product of the square of the diametral pitch into the number of arms, extract the cube root of the quotient, and multiply the result by 2.37.

In a similar manner we may obtain from formulas (31), (32), and (33), the expressions,

$$b'a^2 = 1.356 \times 9.86965\frac{R}{p_d^2 n_1},$$

$$\frac{b_{,,}H'^3 + Bh_{,,}^3}{H'} = 0.8 \times 9.86965\frac{R}{p_d^2 n_1},$$

and

$$\frac{BH'^3 - b_{,,}h_{,,}^3}{H'} = 0.8 \times 9.86965\frac{R}{p_d^2 n_1},$$

which reduce respectively to the following : —

$$b'a^2 = 13.383\frac{R}{p_d^2 n_1}, \qquad (8)$$

$$\frac{b_{,,}H'^3 + Bh_{,,}^3}{H'} = 7.896\frac{R}{p_d^2 n_1}, \qquad (9)\ *$$

and

$$\frac{BH'^3 - b_{,,}h_{,,}^3}{H'} = 7.896\frac{R}{p_d^2 n_1}, \qquad (10).\ †$$

Rim, Nave, etc. : The total rim thickness before the spaces between the teeth are cut out is equal to $t + h$. If we add together formula (34) and the expression for the total height of the teeth, $h = 0.7p$, we shall have,

$$t + h = 0.12 + 0.4p + 0.7p$$

and, by substituting for p its value of $\frac{3.14159}{p_d}$,

$$t + h = 0.12 + \frac{0.4 \times 3.14159}{p_d} + \frac{0.7 \times 3.14159}{p_d}.$$

---

* See Fig. 82. † See Fig. 83.

Or, calling the total thickness of the rim t', and reducing, we obtain

$$t' = 0.12 + \frac{3.46}{p_d} \qquad (11).$$

Rule. — To determine the total thickness of the rim (the height of the teeth plus the true rim thickness), divide 3.46 by the diametral pitch, and to the quotient add 0.12″.

The expression $p^2 = \frac{9.86965}{p_d^2}$, substituted in formula (35), gives

$$k = 0.4\sqrt[3]{\frac{9.86965R}{p_d^2}} + \tfrac{1}{2} = 0.4 \times 2.145\sqrt[3]{\frac{R}{p_d^2}} + \tfrac{1}{2}$$

or

$$k = 0.858\sqrt[3]{\frac{R}{p_d^2}} + \tfrac{1}{2} \qquad (12).$$

Rule. — To determine the thickness of the nave, divide the radius of the gear by the square of the diametral pitch, extract the cube root of the quotient, multiply the root by 0.858, and to the result add $\frac{1}{2}$ inch.

For the length of the nave we have formula (36), which is,

$$l' = l + \frac{D}{30} \qquad (13).$$

Formula (45) becomes, on substituting for p^2 its value in terms of the diametral pitch,

$$d = 12.673\sqrt[3]{9.86965\frac{R}{p_d^2 f'}}$$

which reduces to

$$d = 27.184\sqrt[3]{\frac{R}{p_d^2 f'}} \qquad (14).$$

Rule. — To determine the diameter of a gear shaft of any material, divide the radius of the gear by the product of the square of the diametral pitch into the greatest safe shearing-stress in pounds per square inch for the material of the shaft, extract the cube root of the product thus obtained, and multiply the result by 27.184.

Similarly we may obtain from formulas (46), (47), and (48) the equations,

$$d = 0.553\sqrt[3]{9.86965\frac{R}{pd^2}}$$

$$d = 0.634\sqrt[3]{9.86965\frac{R}{pd^2}}$$

and

$$d = 0.796\sqrt[3]{9.86965\frac{R}{pd^2}}.$$

From which, by reducing, we obtain the following: —

For steel, $$d = 1.186\sqrt[3]{\frac{R}{pd^2}} \qquad (15)$$

For wrought-iron, $$d = 1.36\sqrt[3]{\frac{R}{pd^2}} \qquad (16)$$

For cast-iron, $$d = 1.707\sqrt[3]{\frac{R}{pd^2}} \qquad (17).$$

Rule. — To determine the diameter of a gear shaft, divide the radius of the gear by the square of the diametral pitch, extract the cube root of the quotient, and multiply the result by 1.186 for steel, 1.36 for wrought-iron, or 1.707 for cast-iron.

The formulas for the mean width and thickness of the fixing-key are, as before explained for formulas (49) and (50),

$$S = 0.16 + \frac{d}{5} \qquad (18)$$

and

$$S' = 0.16 + \frac{d}{10} \qquad (19).$$

Example 4. — Required to design a 24″ cut gear-wheel (of cast-iron) which will safely transmit a force of 1,000 pounds, moderate shock.

From formula (2, *b*) we have, for the diametral pitch,

$$p_d = 62.83\sqrt{\frac{1}{1000}} = \frac{62.83}{31.62} = 2 \text{ very nearly.}$$

The face width is consequently

$$l = \frac{6.283}{2} = 3.14''.$$

For the number of teeth in the gear we have the expression,

$$N = p_d D = 2 \times 24 = 48.$$

Therefore formula (5) gives, for the number of arms,

$$n_1' = 0.746\sqrt{48}\sqrt[4]{\tfrac{1}{2}} = 0.746 \times 6.928 \times \frac{1}{1.19} = 4.$$

If we wish to have rectangular cross-sections for our arms, and take the thickness equal to one-half the width, formula (6) gives

$$b_1 h_1^2 = \frac{h_1^3}{2} = \frac{7.896 \times 12}{4 \times 4}.$$

Hence

$$h_1 = \sqrt[3]{11.844} = 2.28''$$

and

$$b_1 = \frac{2.28}{2} = 1.14''.$$

From formula (11) we have, for the total thickness of the rim,

$$t' = 0.12 + \frac{3.46}{2} = 0.12 + 1.73 = 1.85''.$$

The thickness of the nave is, from formula (12),

$$k = 0.858\sqrt[3]{\tfrac{12}{4}} + \tfrac{1}{2} = 0.858 \times 1.44 + \tfrac{1}{2} = 1.736''$$

and the length, from formula (13), is

$$l' = 3.14 + \tfrac{24}{30} = 3.94''.$$

Formula (16) gives, for the diameter of the wrought-iron shaft,

$$d = 1.36\sqrt[3]{\tfrac{12}{4}} = 1.36 \times 1.44 = 1.96'', \text{ say } 2''.$$

Formulas (18) and (19) give, for the mean width and thickness of the fixing-key,

$$S = 0.16 + \tfrac{2}{5} = 0.56''$$

and

$$S' = 0.16 + \tfrac{2}{10} = 0.36''.$$

The following table will be found convenient in constructing cut gears of cast-iron. To illustrate its application, suppose we have to construct a cut gear which will transmit a force of 4,000 pounds, moderate shock. We find in the table, column for moderate shock, $P = 3{,}948$ pounds, which corresponds to a No. 1 diametral pitch. We also find in the table the face width of 6.28″, and the total rim thickness of 3.58″.

pd	P Violent shock.	P Moderate shock.	P Little or no shock.	$\frac{v}{H}$ Violent shock.	$\frac{v}{H}$ Moderate shock.	$\frac{v}{H}$ Little or no shock.	t' In inches.	l In inches.
$\frac{1}{4}$	52203	63166	128910	0.0106	0.0087	0.0043	13.96	25.13
$\frac{3}{8}$	23201	28074	57293	0.0237	0.0195	0.0095	9.36	16.76
$\frac{1}{2}$	13051	15791	32227	0.0423	0.0350	0.0170	7.04	12.57
$\frac{5}{8}$	8352	10106	20625	0.066	0.054	0.027	5.66	10.05
$\frac{3}{4}$	5800	7018	14323	0.095	0.078	0.038	4.73	8.38
1	3263	3948	8057	0.169	0.139	0.068	3.58	6.28
$1\frac{1}{4}$	2088	2527	5156	0.265	0.217	0.107	2.89	5.03
$1\frac{1}{2}$	1450	1754	3581	0.380	0.312	0.153	2.43	4.19
$1\frac{3}{4}$	1066	1290	2631	0.519	0.425	0.208	2.10	3.59
2	816	987	2014	0.676	0.555	0.272	1.85	3.14
$2\frac{1}{4}$	644	779	1591	0.858	0.702	0.344	1.66	2.79
$2\frac{1}{2}$	522	632	1289	1.056	0.867	0.425	1.50	2.51
$2\frac{3}{4}$	431	522	1065	1.283	1.049	0.514	1.37	2.28
3	362	439	895	1.521	1.248	0.612	1.27	2.09
4	204	247	504	2.704	2.219	1.088	0.99	1.57
5	131	158	322	4.225	3.467	1.700	0.81	1.26
6	91	110	224	6.084	4.993	2.448	0.70	1.05
7	67	81	164	9.281	6.796	3.332	0.61	0.90
8	51	62	126	10.816	8.877	4.352	0.55	0.79
9	40	49	99	13.689	11.235	5.508	0.50	0.70
10	33	40	81	16.863	13.870	6.812	0.47	0.63
12	23	27	56	24.336	19.973	9.792	0.41	0.52

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