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Choices of Competitive Firms

K. C. Chen
Louis O. Scott

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

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K. C. Chen, Assistant Professor
Department of Finance

Louis O. Scott, Assistant Professor
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ABSTRACT

In recent years, inflation has become an increasingly significant factor in corporate decisions. This paper examines the input and output decisions of competitive firms in the presence of uncertain inflation. We find that most firms operate with higher capital-labor ratios when there is uncertain inflation. The effects on output in the model are ambiguous for most cases, but we do find that uncertain inflation has predictable effects on the input decisions of competitive firms.

Input and Output Choices Under Inflation

I. Introduction

The behavior of the firm under uncertainty and the valuation of risky assets have been intensively studied in both the economic and finance literatures (Sandmo [1971], Leland [1972], Hite [1978], etc.). It is often assumed that decisions regarding output and capital investment are made in order to maximize the value of the firm given the risk-return preferences of investors. As a result, the behavior of the firm in product markets is directly linked to financial markets.

In recent years, inflation has become an increasingly significant factor in corporate decisions. Uncertain inflation will affect not only the firm's investment and financing decisions but also its real production decisions including input and output mixes. A rich body of research has existed in the literature. Rappaport and Taggart [1982] incorporate inflation considerations only in the evaluation of capital spending proposals. Chen and Boness [1975] investigate the effects of uncertain inflation on investment and financing decisions. Conine and Tamarkin [1984] further show the effects of changes in uncertain inflation on a firm's optimal investment policy when there are decreasing returns to scale. Hite [1977] integrates financing, investment and output decisions of a firm under uncertainty in capital markets, but the impact of uncertain inflation is not incorporated in his study. The purpose of this paper is to examine the firm's input and output decisions in the presence of uncertain inflation. In Section II, we derive and discuss the optimal input and output choices under uncertain inflation. We summarize our findings in Section III.

II. Model Structure

Using the assumptions that only a nominal risk-free rate of interest exists in a perfectly competitive capital market and that all investors are risk averse and maximize expected utility of real terminal wealth over a single period, Chen and Boness [1975] and Chen [1979] have derived the following equilibrium valuation model in terms of cash flows for a firm under uncertain inflation:[1]

$$V_j = \frac{1}{1+\gamma}\{E(\tilde{Y}_j) - \lambda[\mathrm{cov}(\tilde{Y}_j,\tilde{Y}_m) - \mathrm{cov}(\tilde{Y}_j,\tilde{Y}_a)]\} \qquad (1)$$

where $E(\tilde{Y}_j)$ = the expected end-of-period cash flows to the shareholders of firm j;

γ = the nominal risk-free rate of interest;

$\mathrm{cov}(\tilde{Y}_j,\tilde{Y}_m)$ = the covariance between the cash flows of firm j and the total cash flows of all firms (including firm j);

$\mathrm{cov}(\tilde{Y}_j,\tilde{Y}_a)$ = the covariance between the cash flows of firm j and the total value of random inflation, $\tilde{Y}_a = V_m\tilde{R}_a$, where V_m is the total market value of all firms and $\tilde{R}_a$ is the rate of inflation;

$$\lambda = [E(\tilde{Y}_m) - (1+\gamma)V_m]/[\mathrm{var}(\tilde{Y}_m) - \mathrm{cov}(\tilde{Y}_m,\tilde{Y}_a)].$$

In equilibrium, the value of firm j is the present value of the certainty equivalent of the firm's random cash flows, $\tilde{Y}_j$. In contrast to the traditional CAPM developed by Sharpe [1964], Lintner [1965], and Mossin [1966], this model states that the relevant measure of a firm's riskiness (i.e., the systematic risk) consists of two elements: (1) the variability risk, represented by the covariance between the cash flows of firm j and that of the market portfolio; and (2) the inflation risk, represented by the covariance between the cash flows of firm j and the total value of random inflation. A positive value of $Cov(\tilde{Y}_j, \tilde{Y}_a)$ indicates that firm j is likely to have higher cash flows when actual inflation is greater than expected inflation. Such a firm is defined to be an "inflation-preferred" firm. Likewise, a firm whose cash flows are uncorrelated with inflation is "inflation-neutral" and a firm whose cash flows are negatively correlated with inflation is "inflation-averse." In the following analysis, we drop the j subscript for each firm.

At the beginning of the period, the production process requires a stock of homogeneous capital, K, and labor, L. The capital is purchased with the proceeds of issuing shares to the capital market, and the firm liquidates its stock of capital at the end of the period at an uncertain price. Thus, the depreciation rate designated by $\tilde{\Omega}$ is a random variable:

$$\tilde{\Omega} = \Omega(1 + \tilde{z}), \qquad (2)$$

where Ω is the expected depreciation rate and $\tilde{z}$ is a random variable with mean zero. Note that this depreciation rate is the nominal depreciation rate for capital and it depends on the end-of-period price at which capital is liquidated. Since the firm is assumed to be a price-taker in all factor markets, for simplicity, we choose capital as numeraire and set its price at unity. The wage rate per unit of labor, $\tilde{w}$, which will be paid at the end of the period is independent of the quantity of labor hired.

$$\tilde{w} = w(1 + \tilde{v}), \tag{3}$$

where w is the expected wage rate and $\tilde{v}$ is a random variable with mean zero. The technology for the firm is characterized by a decreasing returns to scale production function, f(K,L).[2] Following the standard analysis for a competitive firm under uncertainty, we assume that the firm is a price taker in its product market and faces a horizontal demand curve:

$$\tilde{P} = P(1 + \tilde{e}), \tag{4}$$

where P is the expected output price and $\tilde{e}$ is a random variable with mean zero.

The firm's uncertain cashflow at the end of the period is

$$\tilde{Y} = \tilde{P}Q - \tilde{w}L - \tau[\tilde{P}Q - \tilde{w}L - \tilde{\Omega}K] + (1 - \tilde{\Omega})K$$

$$= (1 - \tau)[\tilde{P}Q - \tilde{w}L - \tilde{\Omega}K] + K \tag{5}$$

where τ is the corporate income tax rate. Substitute (5) into (1), and the value of the firm is given by

$$V = \frac{1}{1 + \gamma}\{(1 - \tau)\{PQ[1 - \lambda(\sigma_{em} - \sigma_{ea})] - wL[1 - \lambda(\sigma_{vm} - \sigma_{va})]$$

$$- K\Omega[1 - \lambda(\sigma_{zm} - \sigma_{za})]\} + K\} \tag{6}$$

where $\sigma_{em} \equiv Cov(\tilde{e}, \tilde{Y}_m)$; $\sigma_{ea} \equiv Cov(\tilde{e}, \tilde{Y}_a)$; $\sigma_{vm} = \equiv Cov(\tilde{v}, \tilde{Y}_m)$;
$\sigma_{va} \equiv Cov(\tilde{v}, \tilde{Y}_a)$; $\sigma_{zm} \equiv Cov(\tilde{z}, \tilde{Y}_m)$; $\sigma_{za} \equiv Cov(\tilde{z}, \tilde{Y}_a)$.

Using (5), we get the following equation for the covariance of the firm's cashflow with inflation:

$$\mathrm{Cov}(\tilde{Y}, \tilde{Y}_a) = (1 - \tau)[PQ\sigma_{ea} - wL\sigma_{va} - \Omega K\sigma_{za}]. \tag{7}$$

The three covariances with inflation are important for analyzing the changes in firm behavior when uncertain inflation is introduced. Because output prices and wages are generally correlated with uncertain inflation, σ_{ea} and σ_{va} will be positive for most firms. Since the price at which capital can be sold should be positively correlated with uncertain inflation, σ_{za} should be negative for most firms.

The goal of the firm is to maximize the wealth of its shareholders. Therefore, the objective function is to maximize the difference between market value and initial investment, and the decision variables are output (Q), capital, and labor:

$$\max_{Q,K,L} \quad V - K$$

$$\text{s.t.} \quad Q = f(K, L)$$

We use (6) to substitute for V and impose the constraint by substituting the production function for Q in the objective function. The firm's optimization problem becomes

$$\max_{K,L} \; (1 - \tau)[\hat{P}f(K, L) - \hat{w}L - \hat{\Omega}K] - K\gamma$$

where $\hat{P} \equiv P[1 - \lambda(\sigma_{em} - \sigma_{ea})]$, $\hat{w} \equiv w[1 - \lambda(\sigma_{vm} - \sigma_{va})]$, and

$\hat{\Omega} \equiv \Omega[1 - \lambda(\sigma_{zm} - \sigma_{za})]$.

The first-order conditions for this optimization problem are

$$F_1 = \hat{P}f_L(K^*, L^*) - \hat{w} = 0$$

$$F_2 = \hat{P}f_K(K^*, L^*) - \hat{\Omega} - \frac{\gamma}{1 - \tau} = 0, \tag{8}$$

where K* and L* are the optimal levels of capital and labor, respectively. The assumptions for the decreasing returns to scale production function (footnote 2) ensure that the second-order condition is satisfied and that there is a unique solution for K* and L*. For the optimal level of output, we have Q* = f(K*, L*).

By applying the implicit function theorem to F_1 and F_2, we can study the comparative statics for the parameters σ_{ea}, σ_{va}, and σ_{za}. We have placed the mathematical details in the appendix and we present only the results here. For each derivative, we are holding all other exogenous parameters constant and examining the effect of changing a single parameter on the optimal levels of capital and labor. First, we examine the effect of changing the covariance between the firm's output price and uncertain inflation.

$$\frac{dK^*}{d\sigma_{ea}} = \frac{\lambda P f_K(-f_{LL} + f_{LK})}{\hat{P}(f_{LL}f_{KK} - f_{LK}^2)} > 0$$

$$\frac{dL^*}{d\sigma_{ea}} = \frac{\lambda P f_L(-f_{KK} + f_{LK})}{\hat{P}(f_{LL}f_{KK} - f_{LK}^2)} > 0$$

Because f_{LL} and f_{KK} are negative and f_{LK} is positive, both derivatives are positive. As a result both capital and labor increase as σ_{ea} increases and clearly output increases. Next, we examine the effect of changing the covariance between the firm's wage rate and inflation.

$$\frac{dK^*}{d\sigma_{va}} = \frac{f_{LK}\lambda w}{\hat{P}(f_{LL}f_{KK} - f_{LK}^2)} > 0$$

$$\frac{dL^*}{d\sigma_{va}} = \frac{f_{KK}\lambda w}{\hat{P}(f_{LL}f_{KK} - f_{LK}^2)} < 0$$

As σ_{va} increases, the firm decreases labor and increases capital. We obtain similar results for σ_{za}:

$$\frac{dK^*}{d\sigma_{za}} = \frac{f_{LL}\lambda\Omega}{\hat{P}(f_{LL}f_{KK} - f_{LK}^2)} < 0$$

$$\frac{dL^*}{d\sigma_{za}} = \frac{f_{LK}\lambda\Omega}{\hat{P}(f_{LL}f_{KK} - f_{LK}^2)} > 0$$

As σ_{za} increases, the firm decreases capital and increases labor.

The more interesting questions concern the impact of uncertain inflation on the behavior of the competitive firm. What happens to the optimal levels of capital, labor, and output as we move from a world of no uncertain inflation (either no inflation or inflation that is known with certainty) to a world with uncertain inflation? Will the introduction of uncertain inflation alter the optimizing behavior of competitive firms, and if it does, can we predict any of the changes? We approach these questions by studying the first-order conditions in (8); it is here that our definitions for inflation-preferred and inflation-averse firms become useful. When there is no uncertain inflation in our model all of the covariances with inflation are zero. First, we rewrite the first-order conditions by placing the covariance terms on the right hand side:

$$P(1 - \lambda\sigma_{em})f_L - w(1 - \lambda\sigma_{vm}) = -\lambda(P\sigma_{ea}f_L - w\sigma_{va})$$

$$P(1 - \lambda\sigma_{em})f_K - \Omega(1 - \lambda\sigma_{zm}) - \frac{\gamma}{1 - \tau} = -\lambda(P\sigma_{ea}f_K - \Omega\sigma_{za})$$

Note that λ depends on the covariance between the total cashflows of all firms and the total value of random inflation, $\mathrm{Cov}(\tilde{Y}_m, \tilde{Y}_a)$. Let $\bar{\lambda}$ be the value of λ when there is no uncertain inflation:

$$P(1 - \bar{\lambda}\sigma_{em})f_L - w(1 - \bar{\lambda}\sigma_{vm}) = -\lambda(P\sigma_{ea}f_L - w\sigma_{va}) + (\lambda - \bar{\lambda})(P\sigma_{em}f_L - w\sigma_{vm}) \quad (9)$$

$$P(1 - \bar{\lambda}\sigma_{em})f_K - \Omega(1 - \bar{\lambda}\sigma_{zm}) - \frac{v}{1 - \tau} = -\lambda(P\sigma_{ea}f_K - \Omega\sigma_{za}) + (\lambda - \bar{\lambda})(P\sigma_{em}f_K - \Omega\sigma_{zm})$$

The difference in λ, $(\lambda - \bar{\lambda})$, depends on $\mathrm{Cov}(\tilde{Y}_m, \tilde{Y}_a)$, which is the summation of the corresponding covariances for individual firms: $\mathrm{Cov}(\tilde{Y}_m, \tilde{Y}_a) = \sum_{j=1}^{N} \mathrm{Cov}(\tilde{Y}_j, \tilde{Y}_a)$. Some firms will have cashflows that vary negatively with inflation, while others will have cashflows that vary positively with inflation. We argue that much of this covariation washes out across firms so that $\mathrm{Cov}(\tilde{Y}_m, \tilde{Y}_a)$ is very small relative to $\mathrm{Var}(\tilde{Y}_m)$ and that we can treat the difference $(\lambda - \bar{\lambda})$ as being approximately zero. It is not even clear whether $\mathrm{Cov}(\tilde{Y}_m, \tilde{Y}_a)$ is positive or negative. For many years, economists and financial analysts believed that common stocks were a hedge against uncertain inflation, but our experience with inflation during the 1960's and 1970's destroyed much of this belief. Fama and Schwert (1977) have presented empirical evidence which suggests that returns on a large portfolio of common stocks are negatively correlated with unexpected inflation. If $\mathrm{Cov}(\tilde{Y}_m, \tilde{Y}_a)$

is negative, the difference $(\lambda - \overline{\lambda})$ is negative and most of our results below still follow. We argue that this difference, be it positive or negative, is negligible. Given no difference in the λ's we rewrite (9) as follows:

$$P(1 - \lambda\sigma_{em})f_L - w(1 - \lambda\sigma_{vm}) = -\lambda(P\sigma_{ea}f_L - w\sigma_{va}) \qquad (10)$$
$$P(1 - \lambda\sigma_{em})f_K - \Omega(1 - \lambda\sigma_{zm}) - \frac{\gamma}{1 - \tau} = -\lambda(P\sigma_{ea}f_K - \Omega\sigma_{za})$$

When there is no uncertain inflation, the right hand sides are zero for both equations in (10), and competitive firms set labor and capital to satisfy the resulting first-order conditions. To analyze the effects on capital and labor, we rearrange the first-order conditions to yield equations for f_L and f_K:

$$f_L = \frac{w(1 - \lambda\sigma_{vm} + \lambda\sigma_{va})}{P(1 - \lambda\sigma_{em} + \lambda\sigma_{ea})}$$
$$f_K = \frac{\Omega(1 - \lambda\sigma_{zm} + \lambda\sigma_{za}) + \frac{\gamma}{1-\tau}}{P(1 - \lambda\sigma_{em} + \lambda\sigma_{ea})} \qquad (11)$$

When there is no uncertain inflation, the three covariances with inflation are zero. As we introduce uncertain inflation, we consider how the non-zero covariances with inflation alter f_L and f_K and we try to infer the ultimate effects on labor, capital, and output. What happens to the optimal levels of capital and labor as we re-introduce the covariances with uncertain inflation in (10)? We explore three different cases; first we examine two special cases which produce unambiguous results and then we examine the more interesting case where σ_{ea} and σ_{va} are positive and σ_{za} is negative.

i) $\sigma_{ea} > 0$, $\sigma_{va} < 0$, and $\sigma_{za} < 0$

From (7), $Cov(\tilde{Y}, \tilde{Y}_a) > 0$ and the firm is inflation-preferred. The right hand sides for both equations in (10) are negative. In (11), we find that the covariances with inflation lead to decreases in both f_L and f_K. These changes are accomplished by increasing both capital and labor, and as a result output rises. This firm is unambiguously inflation-preferred and K*, L*, and Q* are greater when there is uncertain inflation.

ii) $\sigma_{ea} < 0$, $\sigma_{va} > 0$, and $\sigma_{za} > 0$

From (7), $Cov(\tilde{Y}, \tilde{Y}_a) < 0$ and the firm is inflation-averse. The right hand sides are positive for both equations in (10). As the firm moves from a world without uncertain inflation to a world with uncertain inflation, it increases both f_L and f_K, and this is accomplished by reducing both capital and labor. The firm thus has a lower level of capital, labor, and output when there is uncertain inflation.

iii) $\sigma_{ea} > 0$, $\sigma_{va} > 0$, and $\sigma_{za} < 0$

This case is the one that would apply for most (if not all) firms, but there is some ambiguity regarding the responses of firms to uncertain inflation. Here we make use of $Cov(\tilde{Y}, \tilde{Y}_a)$ to examine the responses of different firms. In the equation for f_K in (11), we decrease the numerator and increase the denominator so that the firm will always lower marginal productivity of capital when we introduce inflation. The effect on f_L is ambiguous because we increase both the numerator and the denominator in the f_L equation in (11). We focus on the quantity $(P\sigma_{ea}f_L - w\sigma_{va})$ in the right hand side of the first equation in (10). If $P\sigma_{ea}f_L - w\sigma_{va} > 0$, then the firm desires a lower f_L when there is uncertain inflation. To reduce both f_K and f_L, the firm will increase labor, capital and hence output. For this firm, it is also the case that

$$L^*(P\sigma_{ea}f_L - w\sigma_{va}) + K^*(P\sigma_{ea}f_K - \Omega\sigma_{za}) > 0.$$

By using a property of the decreasing returns to scale technology $(Q = f(K, L) \geq f_K K + f_L L)$, we have

$$PQ^*\sigma_{ea} - wL^*\sigma_{va} - \Omega K^*\sigma_{za} \geq P\sigma_{ea}(L^*f_L + K^*f_K) - wL^*\sigma_{va} - \Omega K^*\sigma_{za} > 0.$$

This condition implies that the sign of $Cov(\tilde{Y}, \tilde{Y}_a)$ is positive; therefore the firm must be inflation-preferred.

Next we consider inflation-averse and inflation-neutral firms: $Cov(\tilde{Y}, \tilde{Y}_a) \leq 0$. Since $K^*(P\sigma_{ea}f_K - \Omega\sigma_{za})$ must be positive, the condition $Cov(\tilde{Y}, \tilde{Y}_a) \leq 0$ requires $L^*(P\sigma_{ea}f - w\sigma_{va})$ to be less than zero.[3] By examining the right hand side of the first equation in (10), we find that the firm will increase f_L when we introduce uncertain inflation; therefore, the inflation-averse (and inflation-neutral) firm is one that increases f_L. The combination of reducing f_K and increasing f_L can be accomplished by a variety of changes in K^* and L^*, and the ultimate impact on output may be positive or negative. The firm can reduce labor only, raise capital only, or employ a decrease in labor and an increase in capital. The capital-labor ratio will rise, unambiguously, for this firm.

The remaining possibility is a firm that is inflation preferred but has a higher marginal product of labor when there is uncertain inflation. This firm may have a lower level of labor or a higher level of capital or some combination of both. A firm in this category will clearly have a higher capital-labor ratio when there is uncertain inflation. Do all competitive firms increase their capital-labor ratios as we introduce uncertain inflation in (iii)? Our analysis suggests that this may be true for most firms, but the one exception might be those firms which decrease both f_L and f_K. For all firms in (iii), we can easily show that the ratio of f_L to f_K rises as we introduce uncertain inflation. Does this result imply that the capital-labor ratio must also rise? The answer is unambiguously yes if the production function is homothetic, but without additional assumptions we do not have a precise result.

We have also seen that the firm's input-output decisions influence the covariance of its cashflow with inflation. In the model, $\text{Cov}(\tilde{Y}, \tilde{Y}_a)$ has a positive influence on value. A cashflow that is positively correlated with inflation is one that represents a hedge against uncertain inflation and there is value associated with this inflation hedge. Firms that are inflation-averse (or inflation-neutral) are firms that increase their capital-labor ratios and their marginal productivity of labor when we introduce uncertain inflation. These changes decrease the negative covariances of the firms' cashflows with inflation. Firms that are not already inflation hedges will try to move in the direction of becoming inflation hedges.

III. Conclusions

We have employed Chen and Boness' [1975] mean-variance valuation model under uncertain inflation to study the firm's optimal input and output decisions in the presence of uncertain inflation. We find that inflation-averse and inflation-neutral firms operate with higher capital-labor ratios when there is uncertain inflation. These firms adjust labor and capital to decrease the negative covariances of their cashflows with inflation. Some inflation-preferred firms also operate with higher capital-labor ratios and those firms which might be operating with lower capital-labor ratios have higher levels of capital, labor, and output under uncertain inflation. The effects on output in the model are ambiguous for most cases, but we do find that uncertain inflation will have predictable effects on the input decisions of firms.

Footnotes

[1]In addition, Friend, Landskroner and Losq [1976] and Roll [1973] have derived general equilibrium models which incorporate the impact of uncertain inflation on returns in a mean-variance framework. Since all models lead to the same qualitative conclusions regarding the effects of uncertain inflation on returns, we use Chen and Boness' model for the sake of simplicity.

[2]$f_K > 0$, $f_L > 0$, $f(0,L) = f(K,0) = 0$, $f_{LL} < 0$, $f_{KK} < 0$, and $f_{LL} f_{KK} - f_{LK}^2 > 0$. Using the assumptions of decreasing returns to scale (f is a concave function) and $f(0, 0) = 0$, we get the following result: $f(K, L) \geqslant Kf_K + Lf_L$. In addition, we assume that $f_{LK} \geqslant 0$.

[3]Here we reverse the previous argument. $Cov(\tilde{Y}, \tilde{Y}_a) \leq 0$ implies $PQ^*\sigma_{ea} - wL^*\sigma_{va} - \Omega K^*\sigma_{za} \leq 0$.

$$PQ^*\sigma_{ea} - wL^*\sigma_{va} - \Omega K^*\sigma_{za} \geq P\sigma_{ea}(L^*f_L + K^*f_K) - wL^*\sigma_{va} - \Omega K^*\sigma_{za}$$

$$= L^*(P\sigma_{ea} f_L - w\sigma_{va}) + K^*(P\sigma_{ea} f_K - \Omega\sigma_{za})$$

Because $(P\sigma_{ea} f_K - \Omega\sigma_{za}) > 0$, then $(P\sigma_{ea} f_L - w\sigma_{va}) \leq 0$.

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APPENDIX

We set up the two first-order conditions as a system of two equations:

$$F_1 = \hat{P}f_L(K^*, L^*) - \hat{w} = 0$$

$$F_2 = \hat{P}f_K(K^*, L^*) - \hat{\Omega} - \frac{\gamma}{1 - \tau} = 0$$

Next, we totally differentiate both equations and put the exogenous parameters into a vector $\underline{\theta}$:

$$\hat{P}\begin{bmatrix} f_{LL} & f_{LK} \\ f_{LK} & f_{KK} \end{bmatrix}\begin{bmatrix} dL^* \\ dK^* \end{bmatrix} = -\begin{bmatrix} \left(\frac{\partial F_1}{\partial \underline{\theta}}\right)' \\ \left(\frac{\partial F_2}{\partial \underline{\theta}}\right)' \end{bmatrix} d\underline{\theta}$$

Inversion of the matrix on the left side yields:

$$\begin{bmatrix} dL^* \\ dK^* \end{bmatrix} = \frac{-1}{\hat{P}^2(f_{LL}f_{KK} - f_{LK}^2)}\begin{bmatrix} f_{KK} & -f_{LK} \\ -f_{LK} & f_{LL} \end{bmatrix}\begin{bmatrix} \left(\frac{\partial F_1}{\partial \underline{\theta}}\right)' \\ \left(\frac{\partial F_2}{\partial \underline{\theta}}\right)' \end{bmatrix} d\underline{\theta}$$

We examine the effects of changing the parameters σ_{ea}, σ_{va}, and σ_{za}, one at a time. To complete the analysis, we require the following partial derivatives.

$$\frac{\partial F_1}{\partial \sigma_{ea}} = \lambda P f_L \qquad \frac{\partial F_2}{\partial \sigma_{ea}} = \lambda P f_K$$

$$\frac{\partial F_1}{\partial \sigma_{va}} = -\lambda w \qquad \frac{\partial F_2}{\partial \sigma_{va}} = 0$$

$$\frac{\partial F_1}{\partial \sigma_{za}} = 0 \qquad \frac{\partial F_2}{\partial \sigma_{za}} = -\Omega\lambda$$