HOPKINS & UNDERWOOD'S NEW ARITHMETIC

ADVANCED BOOK

THE MACMILLAN COMPANY

Florian Cajori

HOPKINS AND UNDERWOOD'S NEW ARITHMETICS

ADVANCED BOOK

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO
ATLANTA · SAN FRANCISCO

MACMILLAN & CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

HOPKINS AND UNDERWOOD'S

NEW ARITHMETICS

ADVANCED BOOK

BY

JOHN W. HOPKINS

SUPERINTENDENT OF THE GALVESTON PUBLIC SCHOOLS

AND

P. H. UNDERWOOD

TEACHER OF MATHEMATICS, BALL HIGH SCHOOL, GALVESTON, TEXAS

New York

THE MACMILLAN COMPANY

1908

Set up and electrotyped. Published August, 1903.
New edition revised. Published December, 1907.

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

Muy

PREFACE

This book assumes a working knowledge of the four fundamental rules as applied to integers and United States Money. It contains the essentials of practical arithmetic arranged by topics in conformity with the courses of study in some of the best school systems, each chapter representing a year's work commencing with the fifth grade.

It aims to teach principles rather than rules. As the unitary method is the one most natural to the young learner, the first two chapters give prominence to this style of reasoning. Chapters III and IV give a thorough review of arithmetic principles and practice. In these chapters the method of ratio is brought into prominence. Science to be of value must be more or less deductive in form.

Characteristic features of the book are the early introduction of Decimals, the large number of problems based on the industrial resources of our country, the clearness and directness with which problems are illustrated, and the omission of complicated problems of doubtful utility.

The aim of teaching arithmetic is culture, accuracy, and rapidity in the computation of problems arising in actual life, and the acquisition of correct methods of reasoning. This aim is always kept in view. However, as students possess the power of learning readily to work processes, and, furthermore, as the practice of arithmetic is of more importance to the majority of people than the theory, attention is paid especially to the art of computation.

The book contains a short introduction to the method of obtaining approximate results correct to any required degree of accuracy. This matter is new, but it is believed that it is well worthy of consideration.

Chapters III and IV contain all the arithmetic and mensuration that is most needful to be known, and, in fact, will be found comprehensive enough to suit the requirements of pupils taking a survey of commercial arithmetic.

JOHN W. HOPKINS,
P. H. UNDERWOOD.

GALVESTON, TEXAS,
September 9, 1907.

CONTENTS

CHAPTER II

CHAPTER III

CHAPTER IV

HOPKINS AND UNDERWOOD'S NEW ARITHMETICS

ADVANCED BOOK

ADVANCED BOOK OF ARITHMETIC

CHAPTER I

Arithmetic is the science of numbers and the art of computation.

The fundamental operation in arithmetic is **counting.**

The result of counting is a **number.**

Any one of the natural numbers, one, two, three, etc., is called an **integer,** or **whole number.**

A **unit** is one thing, or a group of things regarded as a single thing.

NUMERATION

The number names are one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, hundred, thousand, million, billion, trillion, etc. By combining these number names all numbers may be expressed in words.

Ones, tens, hundreds, thousands, ten-thousands, are respectively called units of the first order, units of the second order, units of the third order, units of the fourth order, units of the fifth order.

In our system of naming numbers ten units of any order are equal to one unit of the next higher order. On this account our system is known as the **decimal system.**

Numeration is the naming of numbers.

NOTATION

Every number can be expressed by one or more of the following figures, sometimes called **Arabic numerals**: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The first nine of these figures are called **digits**, or **significant figures.**

Tens are written in the second place; hundreds, in the third place; thousands, in the fourth place; millions, in the seventh place. For example, four thousand seven hundred eighty-nine is written 4,789. This number may be read four thousands seven hundreds eight tens nine ones. The nine ones occupy the first place; the eight tens, the second place; the seven hundreds, the third place; the four thousands, the fourth place. To read a number expressed by more than three figures, begin at the right, that is, with units of the first order, and mark off with commas the figures in groups of three each. The three places, or orders, in which units of the first order occur constitute what is called the **units' period**; the next three places, the **thousands' period**; the next three places, the **millions' period**; the next three, the **billions' period**, etc. As an illustration take the number 1734902309; marking this number off into periods, it becomes 1,734,902,309; this is read one billion seven hundred thirty-four million nine hundred two thousand three hundred nine. Observe that each period has its hundreds, tens, and units. The periods most used are the units, thousands, and millions. The billions period is rarely used. The billionth part of a great circle of the earth is less than 2 inches. The names of a few of the succeeding periods are trillions, quadrillions, quintillions, and sextillions.

Notation is the expression of numbers by means of symbols.

EXERCISE 1

Express in words:

1. 45289.
2. 90208.
3. 75307.
4. 392394.
5. 738211.
6. 328993.
7. 401012.
8. 300287.
9. 200020.
10. 910003.
11. 728000.
12. 400098.
13. 902023.
14. 630006.
15. 1000000.
16. 2227001.
17. 3456000.
18. 9287003.
19. 8307308.
20. 8000401.
21. 7000014.
22. 6100079.
23. 3927173.
24. 5009020.
25. 8000904.
26. 6203003.
27. 9000090.

28. What is the largest whole number expressed by two figures?

29. What is the smallest whole number expressed by two figures?

30. What is the largest whole number expressed by three figures?

31. What is the smallest whole number expressed by three figures?

32. Write the largest number expressed by the figures 0, 4, 5.

33. Write the smallest number expressed by the figures 0, 4, 5.

34. Write three numbers expressed by the figures 2, 3, 4.

35. Write four numbers expressed by the figures 7, 6, 8.

36. What is the largest number expressed by the figures 7, 3, 2, 8?

37. What is the smallest number expressed by the figures 2, 5, 3, 4?

EXERCISE 2

Write in figures:

1. Four thousand, eight hundred twenty-seven.
2. Nine thousand, seven hundred one.
3. Sixty-eight thousand, four hundred fifty-two.
4. Forty-seven thousand, three hundred eight.
5. Ninety thousand, six hundred four.
6. Eighty-seven thousand, one hundred one.
7. Twenty-two thousand, three hundred eleven.
8. Twelve thousand, fifteen.
9. Eighteen thousand, eighteen.
10. Fourteen thousand, thirty-four.
11. Thirteen thousand, five.
12. Ninety thousand, nine.
13. Fifty-four thousand, eleven.
14. Seventy-three thousand, one.
15. Six hundred four thousand, two hundred one.
16. One hundred sixty-three thousand, ten.
17. One hundred one thousand, three hundred.
18. One hundred thousand, seven.
19. Four hundred ten thousand, one hundred twenty-seven.
20. Five hundred four thousand, three hundred eight.
21. Five hundred thousand, eleven.
22. Six hundred thousand, seventeen.
23. Nine hundred ninety thousand, fifteen.
24. Two hundred one thousand, one.
25. Seventy-two thousand, four.

DECIMALS

NOTE. Pupils should draw and measure distances such as 9.3 centimeters, 2.87 inches.

The **law** pervading the decimal system of notation is **the value of a digit in any place is always ten times the value of the same digit written in the next place to the right.** A familiar illustration of this law is the notation of United States money. For example, $5.55. The 5 on the left is ten times the value of the second 5, and the second 5 is ten times the value of the 5 on the right. The period separating dollars and cents is called the **decimal point.**

The first place to the right of the decimal point is called the tenths' place; the second place, the hundredths' place; the third place, the thousandths' place; the fourth place, the ten-thousandths' place; the fifth place, the hundred-thousandths' place; the sixth place, the millionths' place.

READING DECIMALS

Since .47 = 4 tenths 7 hundredths $= \frac{4}{10} + \frac{7}{100} = \frac{47}{100}$. Therefore, .47 is read forty-seven hundredths.

Since .372 = 3 tenths 7 hundredths 2 thousandths $= \frac{3}{10} + \frac{7}{100} + \frac{2}{1000} = \frac{372}{1000}$. Therefore, .372 is read 372 thousandths.

In general a decimal is read by regarding it as a whole number and adding the name of the place the right-hand digit occupies. In reading decimals **and** should not be used except to connect the integral and decimal parts of the number. For example, 500.005 is read five hundred and five thousandths. .505 is read five hundred five thousandths. 8.0379 is read eight and three hundred seventy-nine ten-thousandths. .8379 is read eight thousand three hundred seventy-nine ten-thousandths.

EXERCISE 3

Read:

1. 6.2.	**7.** 6.201.	**13.** 5.0067.
2. 7.9.	**8.** 4.027.	**14.** 7.0123.
3. 8.4.	**9.** 5.029.	**15.** 9.1238.
4. 4.32.	**10.** 9.001.	**16.** .0003.
5. .12.	**11.** 6.034.	**17.** .0054.
6. 5.17.	**12.** 8.295.	**18.** .4008.

Write:

1. 33 hundredths.

2. 2005 thousandths.

3. 329 thousandths.

4. 101 thousandths.

5. Two hundred three thousandths.

6. Seven hundred and four thousandths.

7. Nine hundred three thousandths.

8. Nine hundred and three thousandths.

9. Six thousand seven hundred ten-thousandths.

10. Six thousand seven hundred and one ten-thousandth.

11. Five hundred ninety ten-thousandths.

12. Five hundred and ninety ten-thousandths.

13. Six thousand one ten-thousandths.

14. Seven hundred ten-thousandths.

15. Seven hundred ten thousandths.

16. Five hundred thousandths.

17. Five hundred-thousandths.

18. Two hundred seven hundred-thousandths.

19. Two hundred and seven hundred-thousandths.

20. Five thousand two hundred two hundred-thousandths.

21. Six thousand four hundred-thousandths.

22. Three thousand ten thousandths.

23. Three thousand one ten-thousandths.

24. Five hundred seventeen ten-thousandths.

25. One hundred eleven hundred-thousandths.

26. Seventy-eight ten-thousandths.

ADDITION AND SUBTRACTION

The result of combining two or more numbers into a single number is called the **sum.**

Addition is the process of finding the sum of two or more numbers.

Only numbers of the same kind can be added.

The symbol for addition is $+$, and it is read **plus.**

The symbol $=$ is read **equal,** or **equals**; thus, $5 + 8 = 13$ is read five plus eight equal thirteen.

The difference between two numbers is the excess of one over the other.

Subtraction is the process of finding the difference between two numbers.

The number subtracted is the **subtrahend.** The number from which the subtrahend is taken is the **minuend.**

The result of subtraction is called the **remainder,** or **difference.**

The sign of subtraction is $-$, and is read **minus.** Thus, $8 - 5 = 3$ is read eight minus five equals three.

ILLUSTRATIVE EXAMPLE. From 913 take 537.

$$\begin{array}{r} 913 \\ \underline{537} \\ 376 \end{array} \qquad \begin{array}{r} 800 + 100 + 13 = 913 \\ \underline{500 + \ \ 30 + \ \ 7} = \underline{537} \\ 300 + \ \ 70 + \ \ 6 = 376 \end{array}$$

7 from 13 leaves 6; 3 from 10 leaves 7; 5 from 8 leaves 3. This is the usual explanation given by teachers.

When a figure in the subtrahend cannot be taken from the corresponding figure in the minuend, a unit of the next higher order in the minuend is changed to ten units and then added to the figure in the minuend. A better way of subtracting is: 7 and 6 are 13. Write 6, carry 1. 1 and 3 are 4; 4 and 7 are 11. Write 7, carry 1. 1 and 5 are 6; 6 and 3 are 9. Write 3.

EXERCISE 4

The following table gives the number of children of school age, number enrolled, average daily attendance, and total expenditures for the public schools by states and territories for the school year 1904:

State or Territory	Number of Children	Number Enrolled	Average Daily Attendance	Total Expenditures
North Atlantic Division.				
Maine	163,931	131,176	98,257	$ 2,080,109
New Hampshire	91,847	65,673	48,673	1,376,899
Vermont	81,358	66,535	48,845	1,176,784
Massachusetts	673,690	494,042	391,771	16,436,668
Rhode Island	108,471	70,843	51,692	1,804,762
Connecticut	223,174	163,141	123,317	3,795,260
New York	1,859,824	1,300,065	963,780	43,750,277
New Jersey	514,585	352,203	239,505	8,838,515
Pennsylvania	1,782,740	1,200,230	900,234	26,073,565
South Atlantic Division.				
Delaware	50,695	36,895	25,300	453,670
Maryland	347,594	209,978	130,065	2,755,288
District of Columbia	64,766	49,789	39,300	1,576,354
Virginia	611,555	375,601	224,769	2,137,365
West Virginia	319,874	244,040	158,264	2,531,655
North Carolina	666,782	491,838	318,055	2,075,566
South Carolina	490,214	292,115	214,133	1,191,963
Georgia	789,939	502,014	310,400	2,240,247
Florida	180,501	122,636	83,631	945,848

South Central Division.				
Kentucky	700,272	501,482	309,836	$2,662,863
Tennessee	678,782	502,330	344,882	2,602,141
Alabama	652,518	365,171	240,000	1,252,247
Mississippi	563,019	403,647	233,175	1,868,544
Louisiana	483,967	208,737	155,794	1,551,232
Texas	1,128,934	722,904	461,938	6,200,587
Arkansas	467,821	339,542	212,131	1,729,879
Oklahoma	164,882	152,886	93,495	1,359,624
Indian Territory	162,641	38,422	23,053	643,616
North Central Division.				
Ohio	1,151,007	835,607	618,495	15,802,002
Indiana	732,172	550,732	416,047	9,363,450
Illinois	1,428,613	978,554	783,563	21,792,751
Michigan	684,369	497,299	388,092	9,158,014
Wisconsin	658,474	461,214	288,300	7,885,050
Minnesota	566,397	423,663	272,500	8,073,323
Iowa	672,271	545,940	373,023	10,696,693
Missouri	965,598	731,410	464,706	9,878,198
North Dakota	110,938	95,224	58,442	2,316,346
South Dakota	130,844	106,822	73,700	2,239,135
Nebraska	321,822	278,930	180,771	4,774,146
Kansas	455,943	390,236	270,878	5,684,579
Western Division.				
Montana	63,106	44,881	31,471	1,236,253
Wyoming	24,960	14,512	9,650	253,551
Colorado	145,799	134,260	95,117	3,984,967
New Mexico	64,094	39,704	29,582	353,012
Arizona	35,365	21,088	13,022	438,828
Utah	98,762	75,662	56,183	1,657,234
Nevada	9,013	7,319	5,182	257,501
Idaho	54,700	54,480	39,817	1,001,394
Washington	147,302	161,651	110,774	4,053,468
Oregon	118,977	103,877	72,464	1,803,339
California	363,846	299,038	222,182	9,401,465

Find the totals for each of the above divisions. Also find the total in each instance for the entire country.

EXERCISE 5

The following table gives the gross and net earnings of the principal railroads of Texas for the eleven months ending May 31, 1907:

	Railroad	Gross Earnings	Net Earnings
1.	C. R. I. and G.	$ 3,082,314.24	$ 895,096.73
2.	F. W. and D. C.	4,094,588.28	1,345,055.55
3.	G. H. and S. A.	11,130,030.96	2,536,237.69
4.	F. W. and R. G.	1,075,725.13	342,223.14
5.	H. and T. C.	6,572,660.00	2,242,022.12
6.	G. C. and S. F.	12,510,936.83	3,365,234.82
7.	H. E. and W. T.	1,284,929.31	511,161.15
8.	I. and G. N.	8,204,579.38	2,079,764.06
9.	M. K. and T.	9,989,708.55	2,239,523.41
10.	St. L. S. W. of T.	3,936,613.38	506,646.38
11.	S. A. and A. P.	3,518,565.80	1,517,563.03
12.	T. and N. O.	4,103,849.13	968,031.21
13.	T. and P.	15,456,714.54	5,427,188.11
14.	Texas Central	1,149,069.36	489,109.71

Find the operating expenses (difference between gross and net earnings) of each of the above roads.

15. The increase in net earnings of the G. C. and S. F. road for the eleven months ending May 31, 1907, over that of the corresponding months of the preceding year was $1,361,052.87. Find the net earnings for the eleven months ending May 31, 1906.

16. A like increase for the I. and G. N. road was $ 1,235,847.06. Find its net earnings for the eleven months ending May, 1906.

MULTIPLICATION AND DIVISION

$97 + 97 + 97 + 97 + 97 = ?$ If these numbers are added, the sum will be 485. In examples of this character the usual process is as follows:

$$\begin{array}{r} 97 \\ \underline{5} \\ 485 \end{array}$$

Five 7's are 35. Write 5, carry 3. Five 9's are 45, and 3 are 48. The result is 485.

This short method of adding is called **multiplication.**

Multiplication is a short method of addition when the numbers to be added are all the same. The number to be repeatedly added is the **multiplicand.** The number indicating how often the multiplicand is to be taken as an addend is the **multiplier.** The result of a multiplication exercise is the **product.** The multiplicand and multiplier are **factors** of the product. Since the multiplier denotes number of times, it must always be a pure number, or an **abstract** number. The multiplicand may be either a **concrete** or an **abstract** number. *The product is concrete or abstract according as the multiplicand is concrete or abstract.*

The sign of multiplication is $\times$, and is read **multiplied by,** or **times.** Thus, \$34 × 7 means \$34 is to be multiplied by 7. 7 × \$34 means 7 times \$34.

Is $7 \times 8 = 8 \times 7$? Is $9 \times 4 = 4 \times 9$?

Is $7 \times 3 \times 8 = 3 \times 8 \times 7$? Is $9 \times 6 \times 5 = 6 \times 9 \times 5$?

The order in which numbers are multiplied is immaterial.

(*a*) How many times is \$4 contained in \$24? (*b*) What is the sixth part of \$24?

These two examples illustrate the two kinds of division; the first is to determine the number of times one number or quantity is contained in another number or quantity. This is often called **measuring.** The second is to determine a part of a number or quantity, and is often called **parting,** or **dividing.**

Division is the process of determining how often one number is contained in another, and also of determining any given part of a number. The first number is the **divisor**; the second is the **dividend** ; the result is the **quotient.**

When divisor and dividend are concrete numbers, the quotient is abstract. When the dividend is a concrete number, and the divisor an abstract number, the quotient is a concrete number.

The signs of division are $\div$, and a horizontal stroke, the dividend written above, the divisor below. Thus, $27 \div 3$, and $\frac{27}{3}$ indicate that 27 is dividend and 3, divisor.

Illustrative Examples

(*a*) The area of Massachusetts is 8,040 square miles, and the average number of inhabitants per square mile, for the year 1900, was 348.9. Find its population.

```
    348.9
     8040
   ------
   139560
  27912
  ---------
  2805156.0
```

One square mile averages 348.9 inhabitants; therefore, 8,040 square miles will have 8,040 times 348.9 inhabitants.

(*b*) The area of the United States, exclusive of possessions, is 3,026,000 square miles, and the estimated population for the year ending June 30, 1905, was 83,143,000. Find the average number of inhabitants per square mile.

```
          27.4
      --------
3026)83143.
     6052
     -----
     22623
     21182
     ------
      14410
```

Observe the area is 3,026 thousands of square miles and the population is 83,143 thousands of inhabitants. Hence, the required quotient will be obtained by dividing 83,143 by 3,026.

EXERCISE 6

1. An office desk costs \$25. How much will 3 such desks cost? 8 desks? 36 desks? 49 desks?

2. Eggs sell for 26¢ per dozen. Find the cost of 8 dozen; 18 dozen; 94 dozen.

3. There are 5,280 feet in a mile. How many feet are in 19 miles? in 76 miles?

4. How many days are in 39 weeks?

5. A contractor pays in wages \$78 a day. How much will he pay in 78 days?

6. How many hours are in 89 days?

7. A train travels at the rate of 34 miles an hour. How far will it run in 47 hours?

8. How many acres are in a ranch containing 98 sections of land? (1 section = 640 acres.)

9. A degree on a meridian of the earth is about 69 miles. How many miles are in 17 degrees?

10. A cubic foot of rock weighs 148 pounds. How many pounds do 3,297 cubic feet weigh?

11. The rent of a dwelling is \$28 per month. Find the rent for 3 years.

12. A gallon of water contains 231 cubic inches. How many cubic inches are in 368 gallons?

13. A book has 360 pages, each page has 32 lines, and each line averages 9 words. How many words are in the book?

14. A carpenter earns \$3.20 a day. At this rate, how much wages will he receive in 299 days?

15. A brick mason earns \$5.60 a day. How much will he earn in 310 days?

EXERCISE 7

1. A city block is 100 yards long and 90 yards wide. Find its area.

2. Find the area of a square whose side is 84 yards.

3. Find the area of a square having 320 rods for a side.

4. Find the area of a rectangle, the length being 140 yards and the width 84 yards.

5. Find the area of a rectangle 238 yards long and 96 yards wide.

6. A farm, rectangular in shape, 440 yards long and 380 yards wide, contains how many square yards?

7. Find the area of a rectangle 75 rods long and 63 rods wide.

8. Find the area of a grass plot 240 feet by 84 feet.

9. A sheet of paper 18 inches long and 14 inches wide contains how many square inches?

10. A township is 6 miles long and 6 miles wide. How many square miles does it contain?

11. A county, having the shape of a rectangle, is 24 miles long and 18 miles wide. How many square miles are in its area?

12. A street is 1760 yards long and 23 yards wide. How many square yards does it contain?

13. A garden is 50 yards long and 44 yards wide. How many square yards are in its area?

14. A city lot is 43 feet wide and 124 feet deep. How many square feet are in its area?

15. A window is 60 inches by 48 inches. How many square inches are in its area?

16. A yard is 36 inches. How many square inches are in a square yard?

EXERCISE 8

Find the product:

1. $ 79.94 × 8.	12. $ 79.29 × 7.
2. $ 32.20 × 7.	13. $ 29.97 × 8.
3. $ 79.49 × 6.	14. $179.38 × 6.
4. $128.29 × 7.	15. $373.39 × 5.
5. $399.39 × 9.	16. $799.94 × 8.
6. $454.59 × 12.	17. $822.50 × 9.
7. $729.38 × 11.	18. $998.78 × 11.
8. $237.38 × 9.	19. $778.75 × 12.
9. $720.99 × 7.	20. $732.75 × 9.
10. $285.68 × 8.	21. $928.34 × 7.
11. $ 51.33 × 7.	22. $653.82 × 5.

23. When shoes sell for $3.90 a pair, how much will 24 pairs of shoes cost?

24. If overcoats sell for $7.98, find the price of 20 overcoats.

25. Mackintoshes sell for $6.95 apiece. How much will 27 mackintoshes cost?

26. When wheat is 84¢ per bushel, how much will 384 bushels bring?

27. Find the price of 349 bushels of corn at 56¢ a bushel.

28. Cheese costs 13¢ per pound. Find the price of 54 pounds.

29. Find the cost of 325 pounds of sugar at 6¢ per pound.

30. An acre of land is worth $60.75. Find the value of 100 acres.

EXERCISE 9

On the map of the United States published by the General Land Office, Department of the Interior, 1 inch represents 37 miles. On this map the distances in inches between the cities named are given below:

1. New Orleans to Chicago 22.75.
2. Savannah to Indianapolis 16.6.
3. Mobile to Toledo 21.89.
4. Richmond to St. Louis 19.2.
5. Washington to San Antonio 37.88.
6. Boston to Jackson 34.6.
7. Atlanta to Des Moines 20.4.
8. Newport to St. Louis 27.9.
9. New York to Lincoln 32.2.
10. Chicago to San Francisco 50.8.
11. St. Louis to Portland, Oregon, 47.1.
12. Memphis to Seattle 51.2.

Find the distance in miles between each of the above-named cities.

Find the number of inhabitants in the states named:

	State	Areas in Sq. Mi.	Number of Inhabitants per Sq. Mi.
13.	Georgia	58,980	37.6
14.	Iowa	55,475	40.2
15.	Illinois	56,000	86.1
16.	Louisiana	45,420	30.4
17.	Michigan	57,430	42.2
18.	New Jersey	7,525	250.0
19.	Ohio	40,760	102.0
20.	Pennsylvania	44,985	140.0

EXERCISE 10

Divide and prove your answers correct:

1. 77,354 ÷ 16.	9. 99,392 ÷ 36.	17. 828,374 ÷ 56.
2. 79,358 ÷ 18.	10. 59,738 ÷ 35.	18. 528,739 ÷ 64.
3. 97,854 ÷ 20.	11. 49,399 ÷ 40.	19. 629,394 ÷ 72.
4. 92,738 ÷ 21.	12. 99,988 ÷ 42.	20. 273,579 ÷ 81.
5. 100,000 ÷ 24.	13. 68,698 ÷ 48.	21. 179,246 ÷ 84.
6. 73,948 ÷ 25.	14. 123,456 ÷ 49.	22. 739,264 ÷ 90.
7. 69,593 ÷ 23.	15. 876,543 ÷ 50.	23. 543,293 ÷ 56.
8. 85,376 ÷ 32.	16. 789,295 ÷ 54.	24. 665,670 ÷ 81.

EXERCISE 11

1. When sugar sells for 6 cents per pound, how many pounds can be bought for 84 cents?

2. If a boy walks at the rate of 3 miles per hour, how long will it take him to walk 87 miles?

3. In a peck there are 8 quarts. How many pecks are there in 3000 quarts?

4. If a bicyclist rides 9 miles an hour, how many hours will it take him to go from St. Louis to Indianapolis, a distance of 265 miles? After riding 19 hours, how far from Indianapolis will he be?

5. When coal costs 9 dollars a ton, how many tons can be bought for 3456 dollars?

6. A teacher receives a salary at the rate of 4 dollars a day for every day he teaches. His yearly salary is 716 dollars. How many days are in the school year?

7. A brick mason receives 4 dollars a day for every day he works. How many days must he work to earn 900 dollars?

8. How many feet are there in 2,500 inches?

9. How many weeks are there in 364 days?

10. If a dozen penknives cost 9 dollars, how many dozen penknives can be bought for 126 dollars?

11. Plows cost 12 dollars a piece. How many can be bought for 192 dollars?

12. A box of oranges is worth 3 dollars. How many such boxes can be bought for 111 dollars?

13. When a barrel of pork sells for 12 dollars, how many barrels must be sold to realize 5004 dollars?

14. A section foreman rides on a velocipede at the rate of 11 miles an hour. How long will it take him to go from Cincinnati to Cleveland, a distance of 264 miles?

15. Divide 1000 dollars among 8 persons, giving to each the same sum of money.

16. A flock of sheep sells for 966 dollars. How many sheep are in the flock, if each sheep sells for 6 dollars?

17. A man has 795 dollars in 5-dollar gold pieces. How many coins has he?

18. Hogs sell for 8 dollars apiece. At this price how many can be purchased for 360 dollars?

19. A box of soap is listed at 4 dollars. How many such boxes can be purchased for 980 dollars?

20. How many barrels of flour can be bought for 1002 dollars, when flour sells for 6 dollars a barrel?

21. Oyster crackers cost 5 cents a pound. How many pounds can be bought for 95 cents?

22. By buying horses at 75 dollars each and selling them at 84 dollars each, a jobber makes a profit of 324 dollars. How many does he sell?

EXERCISE 12

1. How many bags of Rio coffee can be bought for 882 dollars, if one bag costs 21 dollars?

2. Currants sell for 14 dollars a barrel; at this price how many barrels can be bought for 546 dollars?

3. Granulated sugar is worth 16 dollars a barrel. How many barrels must be sold to realize 1264 dollars?

4. There are 36 inches in one yard. How many yards are in 100,000 inches?

5. There are 32 quarts in one bushel. How many bushels are in 7712 quarts?

6. How many days are in 3000 hours?

7. A degree on a meridian of the earth's surface is 69 miles long. Two places on the same meridian are 2484 miles apart. How many degrees apart are they?

8. How many square yards are in 3276 square feet?

9. A gallon contains 231 cubic inches. How many gallons are in a barrel containing 8316 cubic inches?

10. Oolong tea costs 15 dollars a chest. How many chests can be purchased for 495 dollars?

11. A barrel of sugar weighs 325 pounds. How many barrels are in 105,625 pounds?

12. How long will it take a train, rate 30 miles an hour, to go from New York to San Francisco, a distance of 3270 miles, if 5 hours are allowed for stops?

13. There are 10 square chains in an acre. How many acres are in 10,000 square chains?

14. Rhode Island contains in round numbers 800,000 acres. Find its area in square miles. (640 acres = 1 square mile.)

EXERCISE 13

1. If land is worth \$68 an acre, how many acres can be bought for \$4,624?

2. A tract of land is sold for \$4,795.50 at the rate of \$69.50 an acre. How many acres are in the tract?

3. When horses sell for \$85.40 apiece, how many can be bought for \$36,465.80?

4. If the price of wheat is 76¢ per bushel, how many bushels can be bought for \$4,043.20?

5. When cans of asparagus sell for 35¢ each, how many can be bought for \$85.75?

6. If a pair of patent leather shoes sells for \$3.85, how many pairs must be sold to bring \$1,482.25?

7. A clothier invests in men's trousers \$392.04. How many does he buy, supposing each pair to cost \$1.98?

8. If a keg of pickles cost \$1.70, how many kegs can be purchased for \$28.90?

9. Chipped beef is bought at 17¢ a pound. At this rate, how many pounds can be bought for \$361.25?

10. A 12-pound sack of flour retails at 45¢. How many sacks can be bought for \$322.65?

11. When a can of sardines retails for 27¢, how many cans will \$218.70 buy?

12. A farmer gets for his apples \$816.35 at the rate of \$1.45 per barrel. How many barrels does he sell?

13. Corn is worth 45¢ per bushel. How many bushels can be bought for \$7,876.35?

14. Oats are worth 36¢ per bushel. How many bushels can be bought for \$142.56?

15. A share of railway stock is quoted at \$78.50. How many shares must be sold to realize \$3,061.50?

EXERCISE 14

The following table gives the area and population of some of the principal countries:

	COUNTRY	AREA IN SQ. MI.	POPULATION
1.	Austria-Hungary	241,300	47,355,000
2.	Belgium	11,370	7,161,000
3.	Denmark	15,360	2,574,000
4.	France	207,050	39,300,000
5.	German Empire	208,800	60,478,000
6.	Italy	110,600	33,604,000
7.	Japan	147,700	47,975,000
8.	Netherlands	12,560	5,592,000
9.	Russia	8,660,000	141,000,000
10.	Spain	194,800	18,618,000
11.	United Kingdom	121,370	43,221,000

Find the population per square mile of each of the above countries.

12. Reduce $\frac{17}{42}$ to a decimal.

$$\begin{array}{r|r} 6 & 17.00000 \\ \cline{2-2} 7 & 2.83333 \\ \cline{2-2} & .40476 \end{array}$$

42 can be resolved into two factors, 6 and 7. The simplest way of dividing by 42 is to divide by the factors 6 and 7.

Reduce to decimals:

13. $\frac{1}{2}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{16}$; $\frac{6}{25}$; $\frac{7}{32}$; $\frac{11}{64}$; $\frac{79}{128}$.

14. $\frac{17}{20}$; $\frac{19}{50}$; $\frac{41}{400}$; $\frac{83}{200}$; $\frac{67}{125}$; $\frac{87}{250}$.

15. $\frac{23}{80}$; $\frac{17}{40}$; $\frac{9}{20}$; $\frac{37}{50}$; $\frac{103}{500}$; $\frac{109}{4000}$.

Reduce to decimals, correct to four figures:

16. $\frac{1}{3}$; $\frac{1}{7}$; $\frac{1}{9}$; $\frac{1}{11}$; $\frac{1}{12}$; $\frac{1}{13}$; $\frac{1}{14}$; $\frac{1}{15}$; $\frac{1}{17}$.

17. $\frac{5}{27}$; $\frac{11}{23}$; $\frac{9}{28}$; $\frac{4}{31}$; $\frac{16}{35}$; $\frac{7}{73}$.

SPECIMEN BILL

GALVESTON, TEXAS, Jan. 31, 1907.

MR. A. B. C.

In account with K. M. & CO.,

DEALERS IN

FURNITURE, CARPETS, RUGS, &C.

Jan.	*2*	*3 Chairs @ $2.25*	*$6*	*75*		
	10	*1 Library Table*	*25*	*00*		
	15	*3 Rugs @ $6.75*	*20*	*25*		
	20	*40 yd. Matting @ 45¢*	*18*	*00*		
	24	*2 Wardrobes @ $17.50*	*35*	*00*		
					$105	*00*
		Paid				
		Feb. 1, 1903.				
		K. M. & CO.				
		Per M.				

DALLAS, TEXAS, Feb. 5, 1907.

MR. P. Q. R.

Bought of M. R. S.,

RETAIL GROCERS.

Jan.	*5*	*3 lb. Tea @ 50¢*	*$1*	*50*		
	"	*28 lb. Sugar @ $5\frac{1}{4}$¢*	*1*	*47*		
	"	*3 pks. Potatoes @ 40¢*	*1*	*20*		
	"	*7 lb. Bacon @ 15¢*	*1*	*05*		
	10	*8 lb. Butter @ 35¢*	*2*	*80*		
	12	*3 cans Salmon @ 17¢*		*51*		
	13	*6 lb. Sausage @ 12¢*		*72*	*$9*	*25*
		Paid				
		Feb. 8.				
		M. R. S.				
		Per X.				

EXERCISE 15

Make out the following bills and receipt them:

1. Mr. John Rye bought of William Merchant,

12 yd. Calico	. . . .	@	9¢
15 yd. Sheeting	. . . . . .	@	7¢
11 yd. Flannel	. . . . . .	@	35¢
2 Hats	. . . .	@	$3.75
18 yd. Carpet	. . . . . .	@	75¢
3 Smyrna Rugs	. . . . . .	@	$10.50

2. Mr. J. Hill bought of F. Warner & Co.,

5 Stoves	. . . .	@	$6.50
3 doz. Knives	. . . . . .	@	$4.80
2 Saws	. . . .	@	$1.50
5 Iron Beds	. . . .	@	$15.75
6 Wrenches	. . . .	@	$1.25

3. H. Van Oppen bought of Hegel & Co.,

2 bu. Potatoes	. . . . . .	@	$1.50
5 lb. Tea	. . . .	@	75¢
2 boxes Herring	. . . . .	@	$1.95
25 lb. Ham	. . . .	@	15¢
45 lb. Sugar	. . . .	@	$4\frac{1}{3}$¢

4. Mr. James Kay bought of Simpson, Perdue & Co.,

50 lb. Sugar	. . . .	@	$4\frac{1}{6}$¢
15 cans Tomatoes	. . . .	@	13¢
27 cans Corn	. . . .	@	16¢
10 packages Breakfast Food	.	@	$12\frac{1}{2}$¢
8 cans Salmon	. . . . . .	@	18¢
5 gal. Maple Sirup	. . . .	@	$1.30
25 lb. Butter	. . . .	@	$37\frac{1}{2}$¢
6 lb. Y. H. Tea	. . . . . .	@	75¢

MEASURES AND MULTIPLES

NOTE. Measures and Multiples in this book have reference only to numbers which are both integral and abstract.

A number is prime, or is said to be a **prime number**, when it is exactly divisible by only itself and unity.

Thus, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc., are prime numbers, or prime factors.

A number which is exactly divisible by other numbers, as well as by itself and unity, is called a **composite number**.

Thus, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, etc., are composite numbers.

An **even number** is one which is exactly divisible by 2.

Thus, 4, 6, 8, 10, 12, etc., are even numbers.

An **odd number** is one which is not exactly divisible by 2.

Thus, 3, 5, 7, 9, 11, etc., are odd numbers.

One number is said to be a **measure** of another number when it is contained an exact number of times in that other number.

Instead of the word "measure," *factor*, *divisor*, and *submultiple* are often used. Example: 5 is a measure, factor, divisor, or submultiple of 10, 15, 20, 25, etc.

By **Greatest Common Measure of two or more numbers** is understood the greatest measure which these numbers have in common.

Thus, 6 is the greatest common measure of 12, 18, 30.

Greatest Common Measure is usually designated by the letters G. C. M. It is called also *Greatest Common Divisor* (G. C. D.).

If a number measures each of two or more numbers, it is said to be a common measure of those numbers.

(factors · divisors)

Thus, 2, 3, and 6 are common measures of 12, 18, 30.

Two or more numbers are **prime to each other** when they have no common measure but 1.

Thus, 8 and 9 are prime to each other; so also are 15 and 28 prime to each other. These numbers, while prime to each other, are not themselves prime numbers.

The result obtained by multiplying a number by an integer is called a **multiple** of the number.

Thus, the multiples of 8 are 8, 16, 24, 32, 40, 48, etc.

The **Least Common Multiple of two or more numbers** is the least number which is a multiple of each of the numbers. In other words, the Least Common Multiple of two or more numbers is the least number which is exactly divisible by each of the two or more numbers.

Least Common Multiple is denoted by L. C. M.

Example **1.** What is the L. C. M. of 8 and 12?

Writing the multiples of 8 and 12, we have:

8	16	**24**	32	40	**48**	64	**72**	80
12	**24**	36	**48**	60	**72**	84		

Notice that **24** is a common multiple of 8 and 12. So also are **48** and **72** common multiples. The L. C. M. is 24.

Example **2.** What is the L. C. M. of 12 and 18? Here the second multiple of 18 contains 12 as a factor. Hence 2 times 18 is the L. C. M.

Example **3.** A man buys two kinds of sugar done up in 4-pound bags and in 5-pound bags. What is the least number of pounds he can buy so as to have the same number of pounds of each kind?

Here the answer is obviously a multiple of 4 and 5. The L. C. M. of 4 and 5 is 20. Hence he buys 20 pounds of each kind.

EXERCISE 16

1. A person has equal sums of money in dimes and in 25-cent pieces. Find the least amount he can have.

2. Fence posts in two fences are respectively 14 feet and 21 feet apart. What is the smallest distance corresponding to an exact number of feet in both fences?

3. A man earns $4 a day. How many days must he work so as to be paid in 10-dollar notes?

4. If a person earns $6 a day, how many days must he work to be paid in $20 bills?

5. A housewife puts her flour into 10-pound and 6-pound sacks, and has the same quantity in the 10-pound sacks as she has in the 6-pound sacks. What is the least quantity of flour she can have?

6. A man buys two grades of sheep at $4 and $6 a head respectively. He spends the same amount in the purchase of the two grades of sheep. What is the smallest amount he can spend? How many sheep can he buy?

7. A boy buys oranges at 3¢, 4¢, 5¢ apiece. He spends the same amount on each kind of oranges. What is the least amount he can spend on each kind? How many oranges does he buy?

8. A person spends the same amount of money on eggs at 15¢ a dozen and at 20¢ a dozen. What is the smallest amount he can spend on each kind?

9. Three bells toll at intervals of 4, 5, and 6 seconds respectively. If they start at the same time, after how many seconds will they toll again at the same instant?

10. Find in feet and inches the least distance that will be measured exactly by a 15-inch and an 18-inch rule.

TESTS OF DIVISIBILITY

A number is exactly divisible by 2 when its units' figure is exactly divisible by 2. Thus, 196 is exactly divisible by 2 since 6 is divisible by 2.

A number is exactly divisible by 5 if its units' figure is 5 or 0.

A number is exactly divisible by 3 when the sum of its digits is exactly divisible by 3. Thus, 735 is exactly divisible by 3 since the sum of its digits, 15, is exactly divisible by 3.

A number is exactly divisible by 6 when it is even and the sum of its digits is divisible by 3. Thus, 624 is exactly divisible by 6 since it is an even number and the sum of its digits, 12, is a multiple of 3.

A number is exactly divisible by 9 when the sum of its digits is exactly divisible by 9. Thus, 765 is exactly divisible by 9 because the sum of its digits, 18, is a multiple of 9.

A number is exactly divisible by 11 when the difference between the sums of its digits in the even and odd places is 0 or a multiple of 11. Thus, 94,853, is exactly divisible by 11 since the difference between the sums $3 + 8 + 9$ and $5 + 4$ is a multiple of 11.

A number is exactly divisible by 25 when the number formed by its two right-hand digits is exactly divisible by 25. Thus, 1,275 is exactly divisible by 25 because 75 is exactly divisible by 25.

A number is exactly divisible by 8 when the number formed by its three right-hand digits is exactly divisible by 8. Thus, 19,256 is exactly divisible by 8 because 256 is divisible by 8.

The same rule holds for 125.

NOTATION

2^2 is a short way of writing 2×2.
2^3 is a short way of writing $2 \times 2 \times 2$.
2^4 is a short way of writing $2 \times 2 \times 2 \times 2$.
2^5 is a short way of writing $2 \times 2 \times 2 \times 2 \times 2$.

The result of taking a number any number of times as factor is called a **power of the number**. Thus, $7^4 = 7 \times 7 \times 7 \times 7 = 2{,}401$.

2,401 is the 4th power of 7.

The 4 written to the right of 7 and slightly above it is called the **index** or **exponent** of the power.

Example **1.** Resolve 1,001 into its prime factors. 1,001 is not divisible by 2 because its units' figure is not exactly divisible by 2. It is not divisible by 3 because the sum of the digits, 1, 1, is not divisible by 3. It is not divisible by 5 because its units' figure is not 0 or 5. 7 is contained in 1,001, 143 times. 143 is divisible by 11 because the sum of the digits, 1, 3, equals 4, the digit in the even place. Hence the prime factors of 1,001 are 7, 11, 13. Hence, $1{,}001 = 7 \times 11 \times 13$.

$$\begin{array}{r} 7\overline{)1001} \\ 11\overline{)143} \\ 13 \end{array}$$

Example **2.** Resolve 5,040 into its prime factors, and express 5,040 as the product of prime numbers.

2	5040
2	2520
2	1260
2	630
5	315
3	63
3	21
	7

Divide by 2 as often as possible. Since 315 ends in 5, 5 is a factor of 315. Divide next by 3 as often as possible.

The prime factors of 5,040 are 2, 2, 2, 2, 3, 3, 5, 7.

$$\begin{aligned} 5040 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \\ &= 2^4 \times 3^2 \times 5 \times 7. \end{aligned}$$

EXERCISE 17

Resolve into prime factors and express each number as the product of its prime factors:

1. 8, 12, 16, 18, 20, 24, 27, 28, 30, 32, 36, 39, 40, 42.

2. 45, 48, 49, 50, 56, 60, 65, 69, 72, 75, 77, 80, 84, 88, 92.

3. 98, 99, 111, 117, 119, 120, 124, 128, 132, 133, 135, 140, 144.

4. 240, 720, 343, 512, 216, 729, 736, 608, 544.

5. 1,331, 11,011, 1,309, 858, 1,274, 891, 3,575.

6. Write all the measures of each of the following numbers: 36, 360, 200, 567, 576, 448.

7. Write all the common measures of: (*a*) 36, 24; (*b*) 18, 27; (*c*) 48, 72; (*d*) 21, 63; (*e*) 32, 96; (*f*) 18, 72.

When several numbers are to be taken as a whole and made the subject of an operation, they are inclosed in a sign, or symbol, known as a *parenthesis*, ( ). Thus, $3 + (9 - 2)$ signifies that 3 is to be added to the difference of 9 and 2. A number written immediately to the left of a parenthesis denotes multiplication. Thus, $7 \times 4 + 3(8 + 5)$ means 7 times 4 is to be added to 3 times the sum of 8 and 5.

A composite number can be resolved into only one set of prime factors. Thus, the prime factors of 36 are 2, 2, 3, 3. $36 = 2^2 \times 3^2$. The product of no other prime numbers will give 36.

If a number is prime to each of two other numbers, it is prime to their product.

Illustration. If 7 is prime to 207 and to 8, then 7 is prime to 8×207. For 7 does not appear among the prime factors of the product.

Example 1. Find the G. C. M. of 48, 120, 168.

Expressing these numbers as products of their prime factors,

$$48 = 2^4 \times 3.$$
$$120 = 2^3 \times 3 \times 5.$$
$$168 = 2^3 \times 3 \times 7.$$

2 is contained 3 times as a factor in 168, 3 times as a factor in 120, and 4 times as a factor in 48; 3 is contained once as a factor in each of the numbers. Hence, the G. C. M. $= 2^3 \times 3 = 24$.

To find the G. C. M. of two or more numbers, express each of the numbers as the product of its prime factors, then take the product of the prime factors common to all the numbers, each factor being taken the least number of times it occurs in any of the numbers.

EXERCISE 18

Find the G. C. M. of:

1. 16, 24.
2. 24, 32.
3. 18, 27.
4. 24, 36.
5. 45, 60.
6. 75, 90.
7. 54, 72.
8. 108, 180.
9. 84, 96.
10. 120, 156.
11. 91, 105.
12. 26, 117.
13. 57, 76.
14. 115, 161.
15. 144, 264.
16. 140, 252.
17. 20, 30, 40.
18. 30, 75, 105.
19. 36, 60, 84.
20. 39, 65, 91.
21. 60, 84, 132.
22. 54, 90, 108.
23. 64, 80, 96.
24. 63, 84, 105.
25. 64, 96, 224.
26. 72, 108, 180.
27. 88, 132, 220.
28. 126, 189, 252.
29. 144, 240, 336.
30. 162, 270, 378.
31. 168, 224, 392.
32. 252, 420, 588.
33. 264, 360, 600.

LEAST COMMON MULTIPLE

Since the L. C. M. of two or more numbers is exactly divisible by each of the numbers, it follows that the **L. C. M. contains all the prime factors of each of the given numbers.**

This fact suggests a method of finding the L. C. M. of two or more numbers.

Example. Find the L. C. M. of 48, 60, 72.

$$48 = 2^4 \times 3.$$
$$60 = 2^2 \times 3 \times 5.$$
$$72 = 2^3 \times 3^2.$$

Any multiple of 48 must contain 2, 4 times as a factor. Any multiple of 72 must contain 3 twice as a factor. Hence, the number $2^4 \times 3^2 \times 5 = 720$ contains all the factors of the three numbers 48, 60, 72. Therefore the L. C. M. of 48, 60, 72, is 720.

To find the L. C. M. of two or more numbers, resolve each of the numbers into its prime factors, then find the product of all the prime factors of the given numbers, taking each factor the greatest number of times it occurs in any of the numbers.

Another Method

Example 1. Find the L. C. M. of 48, 60, 72.

$$\begin{array}{r|rrr} 2 & 48 & 60 & 72 \\ \hline 2 & 24 & 30 & 36 \\ \hline 2 & 12 & 15 & 18 \\ \hline 3 & 6 & 15 & 9 \\ \hline & 2 & 5 & 3 \end{array}$$

L. C. M. $= 3 \times 5 \times 2 \times 3 \times 2 \times 2 \times 2 = 720.$

Step 1. Arrange the numbers in a horizontal row.

Step 2. Divide by a prime factor common to two or more of the numbers. Set down the quotients and the undivided numbers.

Step 3. Treat the second horizontal row in the same manner, and so on until a horizontal row is obtained which contains numbers prime to one another. If, at any stage of the process, a horizontal row contains a number which is a factor of some other number in that row, then strike out such factor.

The continued product of the numbers in the last row and of the divisors will be the L. C. M.

EXERCISE 19

Find the L. C. M. of:

1.	16, 20.	**16.**	60, 96.	**31.**	15, 20, 25.
2.	21, 14.	**17.**	84, 108.	**32.**	12, 18, 20.
3.	18, 60.	**18.**	55, 77.	**33.**	45, 63, 70.
4.	18, 45.	**19.**	54, 90.	**34.**	14, 35, 40.
5.	21, 49.	**20.**	72, 108.	**35.**	12, 16, 18.
6.	28, 70.	**21.**	75, 125.	**36.**	14, 24, 40.
7.	42, 56.	**22.**	36, 54, 72.	**37.**	15, 24, 25.
8.	36, 54.	**23.**	36, 90, 60.	**38.**	77, 143, 22.
9.	34, 51.	**24.**	48, 64, 36.	**39.**	18, 20, 45.
10.	48, 72.	**25.**	12, 15, 18.	**40.**	30, 70, 105.
11.	96, 120.	**26.**	14, 21, 35.	**41.**	30, 40, 48.
12.	28, 30.	**27.**	30, 35, 21.	**42.**	21, 28, 35.
13.	32, 80.	**28.**	28, 42, 70.	**43.**	12, 18, 27.
14.	26, 39.	**29.**	32, 35, 150.	**44.**	12, 15, 16, 18.
15.	48, 84.	**30.**	30, 45, 48.	**45.**	36, 40, 45.

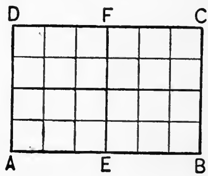
FRACTIONS

If 23 be divided by 4, the process is indicated $\frac{23}{4}$; the quotient is $5\frac{3}{4}$. This means 4 is contained in 23, 5 times and 3 remains to be divided by 4. From questions of this character the term **fraction** (Latin *fractus*, broken in pieces) has arisen. $\frac{23}{4}$, $5\frac{3}{4}$, $\frac{3}{4}$, are all fractions.

A Fraction is an indicated Division.

For purposes of instruction fractions are regarded from another point of view.

If the rectangle $ABCD$ is divided into four equal parts by lines having the same direction as AB, one of these parts is called one fourth of the whole rectangle; two of the parts are called two fourths of the rectangle; three of the parts are called three fourths of the rectangle; and four of the parts are called four fourths of the rectangle. In general, if any one thing is divided into four equal parts, one of the parts is called a fourth; two of the parts are called two fourths; three of the parts, three fourths, etc. Similarly, if any thing is divided into five equal parts, one of the parts is called one fifth; two of the parts are called two fifths; three of the parts, three fifths, etc.



In the above rectangle, if the line EF is drawn so as to divide AB and CD each into two equal parts, the whole figure will be broken up into eight rectangles; one of these rectangles is one eighth of the whole; two of them are two eighths; three of them, three eighths, etc. Divide AE and also EB into three equal parts and draw through the points of division lines parallel to BC. What part of $ABCD$ is one of the small rectangles? two of them? etc.

NOTATION OF FRACTIONS

1 eighth is written $\frac{1}{8}$. 2 eighths is written $\frac{2}{8}$.
3 eighths is written $\frac{3}{8}$. 4 eighths is written $\frac{4}{8}$.
5 eighths is written $\frac{5}{8}$. 6 eighths is written $\frac{6}{8}$, etc.

How many thirds are in 1 thing?
How many fourths are in 1 thing?
How many sevenths are in 1 thing?
How many eighths are in 1 thing?
How many tenths are in 1 thing?

In the notation of fractions, what does the number below the line indicate?

What does the number above the line indicate?

If a unit quantity is divided into any number of equal parts, one of these parts is called a **fractional unit,** or **unit fraction.**

A fraction is one fractional unit, or two or more fractional units of the same denomination.

A fraction is expressed by two numbers, one number being written above a horizontal line and the other number being written below the same horizontal line.

The number above the horizontal line is called the **numerator,** because it *numbers* the parts taken, *i.e.* tells how many fractional units there are.

The number below the line is called the **denominator**: it *names* the fractional unit, and indicates how many fractional units there are in the unit quantity from which the fractional unit is derived.

Thus, $\frac{3}{4}$ signifies the unit quantity is broken into 4 equal parts and 3 of these parts are taken. Here the fractional unit is $\frac{1}{4}$ (one fourth); 3 is the numerator, and 4 is the denominator.

The numerator and denominator are called the **terms of the fraction.**

A **proper fraction** is one whose numerator is less than its denominator. Thus, $\frac{4}{7}$ is a proper fraction because 4 is less than 7.

An **improper fraction** is one whose numerator is greater than or equal to its denominator. Examples: $\frac{14}{3}$, $\frac{7}{7}$.

A **mixed number** is a number, part integral and part fractional. Thus, $4\frac{2}{3}$ is a mixed number.

Read and explain what each represents:

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{3}$, $\frac{3}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, $\frac{7}{3}$, $\frac{9}{3}$, $\frac{7}{4}$, $\frac{11}{4}$, $\frac{13}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{7}{5}$, $\frac{9}{5}$, $\frac{10}{5}$, $\frac{5}{6}$, $\frac{7}{6}$, $\frac{9}{6}$, $\frac{11}{6}$, $\frac{4}{7}$, $\frac{6}{7}$, $\frac{5}{7}$, $\frac{11}{7}$, $\frac{18}{7}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{9}{8}$, $\frac{11}{8}$, $\frac{15}{8}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{19}{10}$, $\frac{29}{10}$.

Is $\frac{2}{7}$ of 1 week equal to $\frac{1}{7}$ of 2 weeks?

Is $\frac{3}{7}$ of 1 week equal to $\frac{1}{7}$ of 3 weeks?

Is $\frac{4}{7}$ of 1 week equal to $\frac{1}{7}$ of 4 weeks?

Is $\frac{2}{5}$ of \$1 equal to $\frac{1}{5}$ of \$2?

Is $\frac{3}{5}$ of \$1 equal to $\frac{1}{5}$ of \$3?

Is $\frac{4}{5}$ of \$1 equal to $\frac{1}{5}$ of \$4?

Is $\frac{3}{4}$ of 1 foot equal to $\frac{1}{4}$ of 3 feet?

The above illustrations show that a fraction may be read in two ways. For example, $\frac{2}{5}$ may be read two fifths, or two divided by five; $\frac{4}{7}$ may be read four sevenths, or four divided by seven.

REDUCTION OF FRACTIONS

Is $1\frac{3}{4}$ feet $= \frac{7}{4}$ of 1 foot? Is $1\frac{3}{4} = \frac{7}{4}$?

Is $\frac{9}{2}$ of 1 hour $= 4\frac{1}{2}$ hours? Is $\frac{9}{2} = 4\frac{1}{2}$?

Is $\frac{1}{2}$ of an apple $= \frac{2}{4}$ of an apple? Is $\frac{1}{2} = \frac{2}{4}$?

Changing the form of fractions without changing their values is called **reduction of fractions.**

EXERCISE 20

Reduce to integers or mixed numbers:

1. $\frac{11}{3}$.	10. $\frac{60}{8}$.	19. $\frac{27}{7}$.	28. $\frac{88}{13}$.
2. $\frac{13}{4}$.	11. $\frac{59}{9}$.	20. $\frac{39}{10}$.	29. $\frac{79}{14}$.
3. $\frac{27}{4}$.	12. $\frac{19}{12}$.	21. $\frac{97}{13}$.	30. $\frac{80}{7}$.
4. $\frac{24}{4}$.	13. $\frac{17}{11}$.	22. $\frac{78}{14}$.	31. $\frac{77}{14}$.
5. $\frac{31}{5}$.	14. $\frac{44}{5}$.	23. $\frac{79}{12}$.	32. $\frac{95}{14}$.
6. $\frac{32}{7}$.	15. $\frac{100}{9}$.	24. $\frac{72}{18}$.	33. $\frac{118}{18}$.
7. $\frac{38}{9}$.	16. $\frac{100}{11}$.	25. $\frac{48}{16}$.	34. $\frac{127}{6}$.
8. $\frac{40}{6}$.	17. $\frac{120}{12}$.	26. $\frac{94}{11}$.	35. $\frac{140}{8}$.
9. $\frac{50}{7}$.	18. $\frac{15}{14}$.	27. $\frac{89}{9}$.	36. $\frac{120}{11}$.

Reduce to an improper fraction $4\frac{5}{6}$.

$$1 = 6 \text{ sixths.}$$
$$4 = 24 \text{ sixths.}$$
$$4\tfrac{5}{6} = 29 \text{ sixths} = \tfrac{29}{6}.$$

Another Method

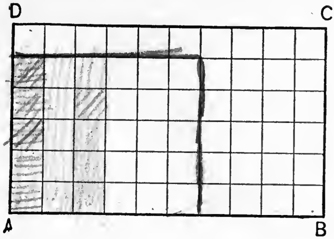
What number divided by 6 gives 4 as a quotient and 5 as a remainder?

The answer is 6 times 4 + the remainder, 5. Therefore, $4\frac{5}{6} = \frac{29}{6}$.

EXERCISE 21

Reduce to improper fractions:

1. $3\frac{1}{2}$.	7. $8\frac{7}{8}$.	13. $19\frac{2}{7}$.	19. $20\frac{10}{11}$.	25. $27\frac{5}{12}$.
2. $3\frac{2}{3}$.	8. $5\frac{5}{6}$.	14. $18\frac{4}{9}$.	20. $16\frac{15}{16}$.	26. $13\frac{12}{13}$.
3. $5\frac{3}{7}$.	9. $9\frac{5}{12}$.	15. $27\frac{5}{11}$.	21. $14\frac{3}{8}$.	27. $14\frac{2}{7}$.
4. $9\frac{3}{8}$.	10. $10\frac{10}{11}$.	16. $38\frac{5}{12}$.	22. $15\frac{2}{7}$.	28. $37\frac{3}{8}$.
5. $9\frac{2}{7}$.	11. $11\frac{1}{10}$.	17. $39\frac{4}{5}$.	23. $19\frac{4}{9}$.	29. $44\frac{4}{11}$.
6. $10\frac{3}{11}$.	12. $10\frac{1}{4}$.	18. $19\frac{2}{3}$.	24. $29\frac{1}{9}$.	30. $30\frac{3}{10}$.



How many squares are in the rectangle *ABCD*?

How many are in $\frac{1}{2}$ of it? in $\frac{1}{3}$ of it?

In $\frac{2}{3}$ of it? in $\frac{1}{4}$ of it? in $\frac{3}{4}$ of it?

In $\frac{1}{5}$ of it? in $\frac{2}{5}$ of it? in $\frac{3}{5}$ of it?

In $\frac{4}{5}$ of it? in $\frac{1}{6}$ of it? in $\frac{5}{6}$ of it?

In $\frac{1}{10}$ of it? in $\frac{3}{10}$ of it? in $\frac{7}{10}$ of it?

In $\frac{9}{10}$ of it? in $\frac{1}{20}$ of it? in $\frac{3}{20}$ of it?

How many squares are in $\frac{6}{10}$ of the rectangle *ABCD*?

How many squares are in $\frac{3}{5}$ of the rectangle *ABCD*?

How do the fractions $\frac{3}{5}$ and $\frac{6}{10}$ compare?

How may the fraction $\frac{6}{10}$ be obtained from $\frac{3}{5}$?

How many squares are in $\frac{2}{5}$ of the rectangle *ABCD*?

How many squares are in $\frac{12}{30}$ of the rectangle *ABCD*?

How do $\frac{12}{30}$ and $\frac{2}{5}$ compare?

How may $\frac{2}{5}$ be obtained from $\frac{12}{30}$?

From the above rectangle can it be shown that:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10} = \frac{10}{20} = \frac{15}{30}?$$

$$\frac{2}{3} = \frac{4}{6} = \frac{10}{15} = \frac{8}{12} = \frac{20}{30}?$$

$$\frac{3}{4} = \frac{9}{12} = \frac{15}{20} = \frac{45}{60}?$$

$$\frac{5}{6} = \frac{10}{12} = \frac{25}{30} = \frac{50}{60}?$$

The terms of a fraction may be multiplied or divided by the same number and the value of the fraction remains unchanged.

Example. Reduce $\frac{3}{7}$ to fourteenths.
$\frac{3}{7}=\frac{6}{14}$. The result is obtained by multiplying the terms of $\frac{3}{7}$ by 2.

EXERCISE 22

Reduce :

1. $\frac{2}{3}$ to 9ths, to 15ths; to 24ths; to 30ths; to 36ths.
2. $\frac{3}{4}$ to 8ths; to 16ths; to 24ths; to 32ds; to 40ths.
3. $\frac{3}{5}$ to 10ths; to 20ths; to 25ths; to 35ths; to 40ths.
4. $\frac{4}{7}$ to 14ths; to 21sts; to 28ths; to 35ths; to 49ths.
5. $\frac{5}{8}$ to 16ths; to 24ths; to 40ths; to 48ths; to 64ths.
6. $\frac{3}{8}$ to 32ds; to 48ths; to 56ths; to 72ds; to 80ths.
7. $\frac{5}{9}$ to 18ths; to 27ths; to 45ths; to 63ds; to 72ds.
8. $\frac{9}{10}$ to 20ths; to 50ths; to 70ths; to 80ths; to 90ths.
9. $\frac{7}{12}$ to 24ths; to 48ths; to 72ds; to 84ths; to 96ths.
10. 3 to 7ths; to 10ths; to 12ths; to 20ths.
11. 5 to 4ths; to 9ths; to 14ths; to 25ths.
12. 7 to 3ds; to 8ths; to 10ths; to 12ths.

Reduce $\frac{3}{4}$ to 28ths. $\frac{3}{4}=\frac{21}{28}$. Do this by multiplying the terms of the fraction $\frac{3}{4}$ by 7. Conversely, reduce $\frac{21}{28}$ to $\frac{3}{4}$ by dividing the terms of the fraction $\frac{21}{28}$ by 7.

A fraction is said to be **in its simplest form** when its terms are integers prime to each other.

A fraction, in its simplest form, is also said to be **in its lowest terms.**

Example. Reduce to lowest terms $\frac{96}{120}$.
$\frac{96}{120}=\frac{48}{60}=\frac{12}{15}=\frac{4}{5}$. Dividing the terms of the fraction by 2, the result is $\frac{48}{60}$. Dividing the terms of this fraction by 4, the result is $\frac{12}{15}$. Dividing the terms of $\frac{12}{15}$ by 3, the result is $\frac{4}{5}$.

EXERCISE 23

Reduce to lowest terms:

1. $\frac{6}{12}$, $\frac{8}{18}$, $\frac{16}{24}$, $\frac{18}{27}$, $\frac{9}{24}$, $\frac{16}{28}$, $\frac{32}{40}$, $\frac{25}{30}$, $\frac{32}{48}$.
2. $\frac{18}{45}$, $\frac{24}{60}$, $\frac{30}{75}$, $\frac{44}{55}$, $\frac{35}{84}$, $\frac{30}{48}$, $\frac{40}{48}$, $\frac{84}{96}$.
3. $\frac{54}{81}$, $\frac{36}{54}$, $\frac{48}{64}$, $\frac{72}{96}$, $\frac{54}{72}$, $\frac{24}{64}$, $\frac{36}{96}$, $\frac{54}{144}$.
4. $\frac{36}{162}$, $\frac{48}{108}$, $\frac{60}{135}$, $\frac{49}{84}$, $\frac{63}{108}$, $\frac{84}{144}$, $\frac{126}{216}$, $\frac{140}{240}$.
5. $\frac{144}{216}$, $\frac{100}{150}$, $\frac{108}{162}$, $\frac{81}{108}$, $\frac{36}{64}$, $\frac{63}{81}$, $\frac{84}{108}$, $\frac{80}{192}$.

ADDITION

Example 1. Add $\frac{1}{2}$, $\frac{1}{3}$.

In adding fractions, select, as a matter of convenience, the lowest denomination common to the fractions.

$$\frac{1}{2} = \frac{3}{6}.$$
$$\frac{1}{3} = \frac{2}{6}.$$

Hence, $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}.$

Example 2. Add $\frac{3}{4}$, $\frac{5}{6}$.

Here the lowest denomination common to both fractions is 12ths. Notice, 12 is L. C. M. of 4, 6.

$$\frac{3}{4} = \frac{9}{12}.$$
$$\frac{5}{6} = \frac{10}{12}.$$

Hence, $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12} = 1\frac{7}{12}.$

Example 3. Add $4\frac{1}{2}$, $2\frac{5}{6}$, $1\frac{5}{12}$.

$$\begin{array}{l} 4\frac{1}{2} = 4\frac{6}{12} \\ 2\frac{5}{6} = 2\frac{10}{12} \\ 1\frac{5}{12} = 1\frac{5}{12} \\ \hline 8\frac{3}{4} \end{array}$$

First add the fractions $\frac{1}{2}$, $\frac{5}{6}$, $\frac{5}{12}$. To do this, find the L. C. M. of the denominators. The L. C. M. of 2, 6, 12, is 12.

$\frac{1}{2} = \frac{6}{12}$, $\frac{5}{6} = \frac{10}{12}$, $\frac{5}{12} = \frac{5}{12}$. Adding $\frac{6}{12}$, $\frac{10}{12}$, $\frac{5}{12}$, the sum is $\frac{21}{12} = \frac{7}{4} = 1\frac{3}{4}$.

Write $\frac{3}{4}$ in the sum and carry 1. Add next the integers 4, 2, 1, and the 1 carried. The sum is $8\frac{3}{4}$.

To add fractions, first reduce them to equivalent fractions having the same denominator. Then add the numerators, and underneath the sum write the common denominator.

If the resulting fraction is not in its simplest form, reduce it to its simplest form.

To add mixed numbers, first add the fractions, and to this sum add the sum of the integers.

EXERCISE 24

Find the sum of:

1. $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$.
2. $\frac{1}{2}, \frac{3}{4}, \frac{2}{3}$.
3. $\frac{1}{2}, \frac{1}{6}, \frac{3}{4}$.
4. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$.
5. $\frac{1}{3}, \frac{5}{6}, \frac{7}{12}$.
6. $\frac{2}{3}, \frac{1}{6}, \frac{5}{12}$.
7. $\frac{1}{6}, \frac{5}{12}, \frac{1}{4}$.
8. $\frac{7}{15}, \frac{2}{5}, \frac{1}{5}$.
9. $\frac{3}{8}, \frac{3}{4}, \frac{1}{2}$.
10. $\frac{2}{5}, \frac{1}{2}, \frac{9}{10}$.
11. $\frac{7}{10}, \frac{1}{2}, \frac{3}{5}$.
12. $\frac{3}{10}, \frac{2}{5}, \frac{1}{2}$.
13. $\frac{2}{3}, \frac{5}{9}$.
14. $\frac{7}{24}, \frac{7}{12}, \frac{5}{6}$.
15. $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{11}{16}$.
16. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{9}{16}$.
17. $\frac{7}{12}, \frac{1}{3}, \frac{1}{6}$.
18. $\frac{11}{18}, \frac{2}{3}, \frac{7}{9}$.
19. $\frac{3}{4}, \frac{7}{12}, \frac{17}{24}$.
20. $\frac{5}{6}, \frac{5}{12}, \frac{13}{24}$.
21. $\frac{3}{4}, \frac{7}{10}, \frac{17}{20}$.
22. $\frac{1}{4}, \frac{4}{5}, \frac{9}{10}$.
23. $\frac{1}{4}, \frac{2}{5}, \frac{7}{10}, \frac{11}{20}$.
24. $\frac{3}{5}, \frac{3}{10}, \frac{19}{20}$.
25. $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{5}{24}$.
26. $\frac{7}{8}, \frac{2}{3}, \frac{3}{4}$.
27. $\frac{5}{8}, \frac{1}{3}, \frac{7}{12}$.
28. $\frac{3}{8}, \frac{2}{3}, \frac{5}{12}$.
29. $\frac{3}{4}, \frac{7}{12}, \frac{5}{6}$.
30. $\frac{2}{3}, \frac{1}{6}, \frac{5}{12}, \frac{13}{24}$.
31. $1\frac{1}{2}, 2\frac{1}{3}, 3\frac{3}{4}$.
32. $2\frac{1}{6}, 5\frac{1}{2}, 7\frac{7}{12}$.
33. $3\frac{1}{4}, 2\frac{1}{2}, 2\frac{7}{8}$.
34. $5\frac{2}{3}, 7\frac{5}{6}, 11\frac{11}{12}$.
35. $9\frac{1}{6}, 7\frac{2}{3}, 10\frac{1}{12}$.
36. $10\frac{5}{12}, 9\frac{3}{4}, 2\frac{2}{3}$.
37. $9\frac{1}{2}, 1\frac{2}{5}, 6\frac{7}{10}$.
38. $2\frac{5}{12}, 3\frac{1}{6}, 7\frac{5}{24}$.
39. $9\frac{1}{3}, 7\frac{3}{8}, 2\frac{7}{12}$.
40. $4\frac{1}{4}, 5\frac{1}{6}, 7\frac{11}{12}$.
41. $8\frac{2}{5}, 3\frac{3}{4}, 12\frac{9}{10}$.
42. $6\frac{7}{8}, 4\frac{13}{32}, 3\frac{5}{16}$.

43. A boy has $\$\frac{1}{2}$ and $\$\frac{1}{5}$. What part of $1 has he?

44. How much is $\frac{1}{2}$ of an hour? $\frac{1}{3}$ of an hour? $\frac{1}{4}$ of an hour?

45. Which is the largest and which the smallest of the three fractions, $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$?

SUBTRACTION

Example 1. From $\frac{4}{5}$ take $\frac{2}{3}$.

$$\frac{4}{5} = \frac{12}{15}.$$
$$\frac{2}{3} = \frac{10}{15}.$$
$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}.$$

Example 2. From $17\frac{3}{10}$ take $12\frac{3}{4}$.

$$17\tfrac{3}{10} = 17\tfrac{6}{20}\ ;\ 17 + \frac{20+6}{20}$$

$$12\tfrac{3}{4} = 12\tfrac{15}{20}\ ;\ \frac{13 + \frac{15}{20}}{4 + \frac{11}{20}}$$

Reduce the fractions to 20ths. $\frac{15}{20}$ cannot be taken from $\frac{6}{20}$. Take it from $1\frac{6}{20}$; that is, from $\frac{20+6}{20}$. Carry 1. 1 and 12 are 13; 13 and 4 are 17. The remainder is $4\frac{11}{20}$.

EXERCISE 25

Find the value of :

1. $\frac{3}{4} - \frac{1}{2}$.
2. $\frac{2}{3} - \frac{1}{2}$.
3. $\frac{2}{3} - \frac{1}{6}$.
4. $\frac{5}{6} - \frac{2}{3}$.
5. $\frac{9}{10} - \frac{1}{2}$.
6. $\frac{7}{8} - \frac{3}{4}$.
7. $\frac{9}{10} - \frac{2}{5}$.
8. $\frac{11}{12} - \frac{3}{4}$.
9. $\frac{7}{12} - \frac{1}{3}$.
10. $\frac{11}{12} - \frac{5}{6}$.
11. $\frac{19}{24} - \frac{2}{3}$.
12. $\frac{4}{5} - \frac{3}{10}$.
13. $3 - 1\frac{1}{2}$.
14. $4 - 1\frac{3}{4}$.
15. $5 - 3\frac{3}{8}$.
16. $6 - 4\frac{7}{10}$.
17. $11 - \frac{4}{5}$.
18. $13 - \frac{7}{8}$.
19. $14 - 5\frac{1}{6}$.
20. $15 - 3\frac{7}{12}$.
21. $28 - 21\frac{9}{10}$.
22. $17\frac{3}{4} - 10\frac{1}{2}$.
23. $18\frac{2}{3} - 11\frac{1}{4}$.
24. $5\frac{4}{5} - 2\frac{3}{10}$.
25. $29\frac{7}{8} - 24\frac{3}{4}$.
26. $33\frac{2}{3} - 17\frac{1}{6}$.
27. $9\frac{5}{12} - 3\frac{1}{4}$.
28. $32\frac{17}{24} - 30\frac{5}{12}$.
29. $31\frac{13}{24} - 20\frac{1}{12}$.
30. $40\frac{2}{3} - 30\frac{11}{24}$.
31. $4\frac{1}{2} - 1\frac{2}{3}$.
32. $7\frac{2}{3} - 3\frac{3}{4}$.
33. $9\frac{1}{2} - 5\frac{7}{12}$.
34. $10\frac{1}{3} - 9\frac{3}{4}$.
35. $16\frac{1}{4} - 9\frac{2}{3}$.
36. $4\frac{2}{5} - 1\frac{7}{10}$.
37. $9\frac{3}{5} - 2\frac{9}{10}$.
38. $8\frac{1}{2} - 3\frac{9}{10}$.
39. $9\frac{3}{4} - 4\frac{11}{12}$.
40. $18\frac{1}{3} - 9\frac{3}{5}$.
41. $28\frac{1}{3} - 8\frac{4}{9}$.
42. $17\frac{1}{4} - 9\frac{7}{20}$.

43. What number must be added to $1\frac{1}{2}$ to make $7\frac{1}{8}$?

44. A man buys a suit of clothes for $\$12\frac{3}{4}$ and gives 3 five-dollar bills in payment. How much change should he receive?

45. From a piece of cloth containing $17\frac{2}{3}$ yards $14\frac{3}{4}$ yards are sold. How many yards are left?

46. A boy buys two books costing $\$\frac{3}{4}$ and $\$\frac{3}{5}$. How much change should he get out of a $\$2\frac{1}{2}$ gold piece?

MULTIPLICATION OF FACTORS. CANCELLATION

Is $2 \times 3 \times 4 \times 9 = (2 \times 3)(4 \times 9) = (2 \times 9) \times (3 \times 4)$?

The product of any number of factors, no matter how the factors are grouped, is the same. This is the **Associative Law.**

Is $5 \times (2 \times 3 \times 4 \times 9) = 10 \times 3 \times 4 \times 9 = 2 \times 15 \times 4 \times 9$
$= 2 \times 3 \times 20 \times 9 = 2 \times 3 \times 4 \times 45$?

Is $(2 \times 3 \times 4 \times 9) \div 2 = 2 \times 3 \times 2 \times 9 = 3 \times 4 \times 9$?

A continued product is multiplied by a number if one of its factors is multiplied by the number.

A continued product is divided by a number if one of its factors is divided by the number.

Cancellation is the shortening of the process of division by dividing dividend and divisor by the same factor or factors.

Find by cancellation the quotient:

$$\frac{18 \times 27 \times 16}{9 \times 8 \times 3} = \frac{2 \times 9 \times 2}{1 \times 1 \times 1}.$$

Dividing 9 into 18, 8 into 16, and 3 into 27, the quotient is $\frac{2 \times 9 \times 2}{1 \times 1 \times 1} = 36$.

EXERCISE 26

Find by cancellation the quotient:

1. $\dfrac{4 \times 18 \times 3 \times 24}{9 \times 4 \times 144}$.

2. $\dfrac{18 \times 90 \times 105}{14 \times 25 \times 3}$.

3. $\dfrac{27 \times 64 \times 8}{108 \times 32}$.

4. $\dfrac{343 \times 125}{35 \times 35}$.

5. $\dfrac{16 \times 16 \times 8 \times 81}{64 \times 32 \times 3}$.

6. $\dfrac{225 \times 216}{75 \times 9 \times 12}$.

7. $\dfrac{108 \times 27 \times 121}{22 \times 33 \times 18}$.

8. $\dfrac{22 \times 88 \times 15}{132 \times 4}$.

9. $\dfrac{1760 \times 99}{4 \times 88 \times 165}$.

10. $\dfrac{5280 \times 14}{176 \times 84}$.

11. $\dfrac{1728 \times 34}{27 \times 136 \times 8}$.

12. $\dfrac{640 \times 5200}{125 \times 512 \times 13}$.

13. $\dfrac{2380 \times 104}{119 \times 8 \times 13}$.

14. $\dfrac{111 \times 39 \times 12}{74 \times 27 \times 13}$.

15. A farmer exchanged 320 acres of land worth \$50 an acre for 25 city lots. Find the price of a lot.

$$(\text{price of a lot}) \times 25 = \$50 \times 320.$$

Hence, price of lot $= \dfrac{\$50 \times 320}{25} = \$640.$

16. How many cows at \$40 a head cost as much as 15 horses at \$64 a head?

17. How many dozen eggs at 35¢ a dozen must be sold to pay for 7 barrels of apples at \$2.10 a barrel?

18. A laborer receives \$3.20 a day. How many days must he work to pay for 6 tons of coal at \$8 per ton?

19. A bicyclist rides at the rate of 9 miles an hour. How long will it take him to travel as far as a train goes in 6 hours at the rate of 33 miles an hour?

20. How many cattle at \$42 a head must be sold to pay for 11,200 bushels of wheat at 75¢ a bushel?

MULTIPLICATION

Multiplication of a fraction by an integer.

Example 1. Multiply $\frac{3}{4}$ by 18.

$\frac{3}{4} \times 18 = 3$ fourths $\times 18 = 54$ fourths $= \frac{54}{4} = 13\frac{2}{4} = 13\frac{1}{2}$.

Example 2. Multiply $3\frac{3}{10}$ by 14.

$$3\tfrac{3}{10} \times 14 = (3 \times 14) + (\tfrac{3}{10} \times 14) = 42 + 4\tfrac{1}{5} = 46\tfrac{1}{5}.$$

To multiply a mixed number by an integer, first multiply the fractional part of the mixed number by the multiplier, next multiply the integral part by the multiplier; add the two results for the final product.

EXERCISE 27

Find the value of:

1. $\frac{2}{3} \times 18$.	**9.** $\frac{3}{14} \times 24$.	**17.** $1\frac{2}{3} \times 10$.	**25.** $7\frac{4}{11} \times 33$.
2. $\frac{3}{4} \times 14$.	**10.** $\frac{5}{16} \times 40$.	**18.** $1\frac{7}{8} \times 12$.	**26.** $8\frac{5}{11} \times 19$.
3. $\frac{4}{5} \times 7$.	**11.** $\frac{9}{16} \times 34$.	**19.** $3\frac{3}{8} \times 10$.	**27.** $9\frac{5}{12} \times 80$.
4. $\frac{9}{10} \times 18$.	**12.** $\frac{11}{12} \times 42$.	**20.** $5\frac{7}{12} \times 54$.	**28.** $6\frac{11}{12} \times 102$.
5. $\frac{7}{10} \times 16$.	**13.** $\frac{13}{16} \times 44$.	**21.** $6\frac{7}{8} \times 36$.	**29.** $7\frac{7}{16} \times 60$.
6. $\frac{3}{8} \times 12$.	**14.** $\frac{19}{20} \times 50$.	**22.** $9\frac{1}{2} \times 16$.	**30.** $11\frac{1}{10} \times 15$.
7. $\frac{5}{8} \times 19$.	**15.** $\frac{17}{20} \times 45$.	**23.** $7\frac{5}{8} \times 44$.	**31.** $12\frac{3}{10} \times 18$.
8. $\frac{7}{8} \times 28$.	**16.** $1\frac{3}{4} \times 9$.	**24.** $10\frac{5}{16} \times 24$.	**32.** $10\frac{2}{9} \times 13$.

33. Find the cost of a dozen cans of baking powder at $37\frac{1}{2}$ cents a can.

34. A pail of mackerel cost $2\frac{1}{8}$ dollars. Find the cost of 20 pails.

35. Find the cost of a barrel of sugar weighing 325 pounds, if the cost per pound is $4\frac{11}{16}$¢.

36. When starch sells for $3\frac{1}{2}$¢ a pound, find the price of 15 pounds of starch.

37. When wheat is $79\frac{1}{8}$ cents a bushel, how much will 164 bushels bring?

38. Find the cost of a 75-pound chest of Hyson tea at $42\frac{1}{2}$ ¢ per pound.

39. If a dozen cakes of yeast cost $42\frac{3}{4}$ ¢, find the cost of 9 dozen cakes of yeast.

40. Pepper sells for $14\frac{3}{4}$ ¢ a pound. Find the cost of 2 bags, each containing 120 pounds.

41. A pound package of chocolate costs $31\frac{1}{4}$ ¢. Find the cost of 25 such packages.

42. A square rod equals $30\frac{1}{4}$ square yards. Reduce 160 square rods to square yards.

43. A link of a surveyor's chain is $7\frac{23}{25}$ inches. If the chain contains 100 links, how many inches long is the chain?

44. When silk sells at $\$\frac{17}{20}$ a yard, what is the cost of 14 yards of silk?

45. A degree on a meridian of the earth's surface is about $69\frac{1}{10}$ miles. How many miles are in 15 degrees? in 40 degrees?

46. A person fails for $9,800. His creditors receive $\$\frac{3}{4}$ on every dollar that he owes. How much in all do they receive?

47. A mass of copper and lead weighs 2,240 pounds; $\frac{4}{5}$ of the mass is copper. How much copper and how much lead is in the mass?

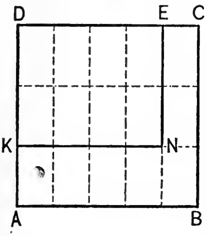
48. A man invests $2,300. At the end of a year his gain is $\frac{7}{20}$ of his investment. Find his gain and the value of the investment at the end of the year.

MULTIPLICATION OF A FRACTION BY A FRACTION

Multiplication of fractions extends the meaning of the term "multiplication."

$\frac{4}{5} \times \frac{2}{3}$, or $\frac{2}{3}$ of $\frac{4}{5}$, means 2 times $\frac{1}{3}$ of $\frac{4}{5}$.
$\frac{7}{8} \times \frac{3}{4}$, or $\frac{3}{4}$ of $\frac{7}{8}$, means 3 times $\frac{1}{4}$ of $\frac{7}{8}$.

Example 1. What is the area of a rectangle whose length is $\frac{4}{5}$ of an inch and width $\frac{2}{3}$ of an inch?



Take AB 1 inch. Let it be divided into five equal parts. Construct upon it a square $ABCD$. Divide AD into three equal parts. Draw lines through the points of division. $KNED$ will have for its dimensions $\frac{4}{5}$ of an inch and $\frac{2}{3}$ of an inch. By counting the small rectangles in $KNED$ the number is found to be eight (4×2), and the number in $ABCD$ is fifteen (5×3). Hence, the area of $KNED$ is $\frac{8}{15}$ of a square inch.

$\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$. Therefore, 2 times $\frac{1}{3}$ of $\frac{4}{5} = \frac{8}{15}$. (Show by figure.)

PROCESS. $\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$.

Example 2. Find the area of a rectangle $3\frac{7}{10}$ by $2\frac{3}{10}$ feet.

$$3\tfrac{7}{10} \times 2\tfrac{3}{10} = \tfrac{37}{10} \times \tfrac{23}{10} = \frac{37 \times 23}{10 \times 10} = \tfrac{851}{100} = 8\tfrac{51}{100}.$$

To multiply a fraction by a fraction, take the product of the numerators for the numerator of the product, and the product of the denominators for the denominator of the product.

To multiply mixed numbers, reduce them to improper fractions, and then apply the rule for multiplication of fractions.

EXERCISE 28

Multiply:

1. $\frac{1}{2}$, $\frac{2}{3}$.
2. $\frac{1}{2}$, $\frac{3}{4}$.
3. $\frac{4}{5}$, $\frac{10}{11}$.
4. $\frac{7}{8}$, $\frac{9}{10}$.
5. $\frac{3}{8}$, $\frac{17}{25}$.
6. $\frac{7}{12}$, $\frac{19}{28}$.
7. $\frac{16}{19}$, $\frac{57}{74}$.
8. $\frac{17}{31}$, $\frac{93}{119}$.
9. $\frac{14}{25}$, $\frac{75}{112}$.
10. $\frac{12}{27}$, $\frac{45}{64}$.
11. $\frac{15}{29}$, $\frac{87}{105}$.
12. $\frac{16}{35}$, $\frac{105}{112}$.
13. $\frac{3}{7}$, $\frac{11}{16}$, $\frac{8}{33}$.
14. $\frac{4}{9}$, $\frac{15}{18}$, $\frac{27}{40}$.
15. $\frac{4}{13}$, $\frac{26}{45}$, $\frac{9}{16}$.
16. $\frac{9}{20}$, $\frac{70}{81}$, $\frac{27}{28}$.
17. $\frac{11}{20}$, $\frac{90}{121}$, $\frac{33}{45}$.
18. $\frac{16}{55}$, $\frac{11}{48}$, $\frac{15}{17}$.
19. $\frac{1}{4}$, $\frac{7}{9}$, $\frac{72}{101}$.
20. $\frac{3}{5}$, $\frac{3}{9}$, $\frac{135}{243}$.
21. $1\frac{2}{3}$, $1\frac{2}{3}$.
22. $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$.
23. $2\frac{1}{7}$, $\frac{7}{15}$, $\frac{3}{4}$.
24. $3\frac{3}{4}$, $\frac{4}{15}$, $\frac{4}{5}$.
25. $2\frac{2}{3}$, $2\frac{2}{3}$, $\frac{5}{64}$.
26. $7\frac{1}{4}$, $1\frac{3}{4}$, $\frac{16}{29}$.
27. $2\frac{5}{8}$, $3\frac{3}{5}$, $\frac{20}{21}$.
28. $4\frac{1}{6}$, $2\frac{3}{5}$, $\frac{64}{65}$.
29. $7\frac{1}{2}$, $7\frac{1}{2}$, $\frac{4}{15}$.
30. $3\frac{1}{3}$, $3\frac{1}{3}$, $\frac{7}{25}$.
31. $5\frac{1}{2}$, $5\frac{1}{2}$, $\frac{8}{11}$.
32. $7\frac{3}{4}$, $3\frac{1}{2}$, $\frac{61}{62}$.
33. $1\frac{4}{5}$, $\frac{5}{18}$, $\frac{2}{3}$.
34. $7\frac{2}{3}$, $1\frac{7}{8}$, $\frac{45}{46}$.
35. $4\frac{1}{4}$, $4\frac{1}{4}$, $3\frac{5}{9}$.
36. $1\frac{1}{5}$, $1\frac{1}{6}$, $\frac{5}{7}$.
37. $20\frac{1}{2}$, $20\frac{1}{2}$.
38. $4\frac{1}{2}$, $5\frac{1}{2}$, $1\frac{3}{11}$.
39. $6\frac{3}{4}$, $1\frac{7}{9}$, $\frac{7}{12}$.
40. $2\frac{2}{5}$, $1\frac{5}{12}$, $\frac{10}{17}$.
41. $2\frac{5}{7}$, $8\frac{3}{5}$, $11\frac{9}{13}$.
42. $6\frac{7}{8}$, $4\frac{2}{9}$, $13\frac{11}{15}$.

43. Find the value of $63\frac{1}{2}$ acres of land at \$$55\frac{1}{2}$ an acre.

44. Find the cost of $12\frac{1}{2}$ pounds of meat at $15\frac{1}{2}$ cents per pound.

45. Find the price of $4\frac{1}{2}$ bushels of wheat at $81\frac{1}{4}$ cents per bushel.

46. A speculator buys 10,000 bushels of wheat at $79\frac{5}{8}$ cents per bushel and sells it when wheat is $81\frac{1}{4}$ cents per bushel. Find his profit.

47. Coal cost \$$9\frac{1}{2}$ a ton. Find the price of $5\frac{1}{2}$ tons.

DIVISION AND RATIO

In division the product of two numbers and one of the numbers are given, and the other number is sought.

Example. Divide $1\frac{7}{9}$ by $\frac{2}{3}$.

$\frac{2}{3}$ of the quotient $= 1\frac{7}{9} = \frac{16}{9}$.

Therefore, $\frac{1}{3}$ of the quotient $= \frac{1}{2}$ of $\frac{16}{9}$.

Therefore, $\frac{3}{3}$ of the quotient $= \frac{3}{2}$ of $\frac{16}{9}$.

Therefore, the quotient $= \frac{3}{2}$ of $\frac{16}{9} = 2\frac{2}{3}$.

Observe that $\frac{16}{9}$ is divided by $\frac{2}{3}$ by multiplying by $\frac{3}{2}$.

Hence the rule for division: **Invert the terms of the divisor and then proceed as in multiplication.**

If the product of two numbers is unity, either is called the **reciprocal** of the other.

Illustrations

$7 \times \frac{1}{7} = 1$. The reciprocal of 7 is $\frac{1}{7}$, and of $\frac{1}{7}$ is 7.

$\frac{9}{2} \times \frac{2}{9} = 1$. The reciprocal of $4\frac{1}{2}$ is $\frac{2}{9}$, and of $\frac{2}{9}$ is $4\frac{1}{2}$.

$\frac{8}{15} \times \frac{15}{8} = 1$. The reciprocal of $\frac{8}{15}$ is $\frac{15}{8}$, and of $\frac{15}{8}$, or $1\frac{7}{8}$, is $\frac{8}{15}$.

Hence the rule for division may be briefly stated: **Multiply the dividend by the reciprocal of the divisor.**

By the **ratio** of one number to another number is meant the quotient of the first number by the second. Thus the ratio of 9 inches to 12 inches is $\dfrac{9 \text{ inches}}{12 \text{ inches}} = \frac{3}{4}$.

The ratio of 9 inches to 12 inches is briefly indicated 9 in.: 12 in.

Example. Find the value of the ratio 4 days : $7\frac{1}{2}$ hours.

4 days = 4 × 24 hours = 96 hours.

96 hours ÷ $7\frac{1}{2}$ hours = 96 ÷ $7\frac{1}{2}$ = 12.8.

EXERCISE 29

Divide:

1. $\frac{18}{25}$ by 27.
2. 1 by $\frac{9}{82}$.
3. $\frac{2}{5}$ by $\frac{4}{18}$.
4. $\frac{20}{27}$ by 16.
5. $3\frac{3}{4}$ by 10.
6. $\frac{3}{10}$ by $\frac{6}{11}$.
7. $\frac{9}{14}$ by 12.
8. $2\frac{1}{8}$ by 3.
9. $\frac{7}{12}$ by $\frac{14}{15}$.
10. $\frac{12}{13}$ by 16.
11. $3\frac{5}{9}$ by 8.
12. $\frac{8}{9}$ by $\frac{4}{5}$.
13. $\frac{10}{13}$ by 15.
14. $4\frac{5}{11}$ by 14.
15. $\frac{9}{14}$ by $\frac{7}{12}$.
16. $\frac{10}{11}$ by 25.
17. $5\frac{5}{13}$ by 21.
18. $\frac{8}{15}$ by $\frac{4}{25}$.
19. $\frac{14}{15}$ by 21.
20. $9\frac{7}{12}$ by 46.
21. $\frac{9}{28}$ by $\frac{18}{35}$.
22. $\frac{18}{21}$ by 36.
23. $9\frac{9}{10}$ by 77.
24. $22\frac{10}{13}$ by 8.
25. 12 by $\frac{2}{3}$.
26. $9\frac{3}{13}$ by 45.
27. $4\frac{4}{5}$ by 9.
28. 14 by $\frac{2}{7}$.
29. $16\frac{4}{5}$ by 63.
30. $4\frac{2}{3}$ by $1\frac{3}{4}$.
31. 16 by $\frac{4}{7}$.
32. $18\frac{1}{5}$ by 26.
33. $3\frac{3}{4}$ by $2\frac{5}{7}$.
34. 18 by $\frac{6}{7}$.
35. $17\frac{1}{7}$ by 75.
36. $4\frac{3}{8}$ by $9\frac{2}{7}$.
37. 21 by $\frac{3}{4}$.
38. $10\frac{2}{7}$ by 48.
39. $7\frac{1}{5}$ by $3\frac{1}{2}$.
40. 25 by $\frac{5}{6}$.
41. $9\frac{2}{7}$ by 52.
42. $4\frac{1}{17}$ by $17\frac{1}{4}$.
43. 26 by $\frac{4}{5}$.
44. $9\frac{1}{11}$ by 75.
45. $3\frac{1}{3}$ by $\frac{7}{9}$.
46. 26 by $\frac{13}{14}$.
47. $8\frac{1}{3}$ by 15.
48. 1 by $4\frac{3}{4}$.
49. 33 by $\frac{6}{11}$.
50. $7\frac{9}{13}$ by 80.
51. 1 by $7\frac{1}{7}$.
52. 40 by $\frac{5}{12}$.
53. $5\frac{15}{17}$ by 50.
54. 1 by $9\frac{2}{3}$.
55. 60 by $\frac{8}{9}$.
56. $8\frac{5}{8}$ by 46.
57. $10\frac{2}{3}$ by $2\frac{2}{3}$.
58. 11 by $\frac{2}{9}$.
59. $\frac{5}{14}$ by $\frac{3}{7}$.
60. $14\frac{2}{3}$ by $1\frac{11}{12}$.
61. 1 by $\frac{1}{2}$.
62. $\frac{1}{3}$ by $\frac{5}{12}$.
63. $\frac{19}{36}$ by $1\frac{7}{12}$.
64. 1 by $\frac{3}{4}$.
65. $\frac{9}{10}$ by $\frac{7}{12}$.
66. $\frac{14}{15}$ by $1\frac{3}{11}$.
67. 1 by $\frac{2}{11}$.
68. $\frac{7}{8}$ by $\frac{8}{9}$.
69. $9\frac{1}{11}$ by $11\frac{1}{9}$.
70. 1 by $\frac{5}{13}$.
71. $\frac{3}{4}$ by $\frac{4}{3}$.
72. $7\frac{1}{7}$ by $4\frac{1}{6}$.

EXERCISE 30

1. What is the ratio of 6 inches to 12 inches?

2. What is the ratio of 1 foot to 1 yard?

3. What is the ratio of 1 square foot to 1 square yard?

4. There are $30\frac{1}{4}$ square yards in 1 square rod. What is the ratio of 1 square yard to 1 square rod?

5. What is the ratio of 3 weeks to 10 days?

6. What is the ratio of 1 hour to 1 minute?

7. What is the ratio of 4 days to 15 hours? of a minute to an hour?

8. What is the ratio of 325 to 100?

Find the values of the following ratios:

9. $2 : \frac{1}{2}$.	15. $6 : \frac{2}{3}$.	21. $3\frac{1}{7} : 2\frac{4}{9}$.	27. $10\frac{2}{5} : 7\frac{1}{9}$.
10. $3 : \frac{1}{3}$.	16. $2\frac{1}{2} : 3\frac{3}{4}$.	22. $9\frac{1}{7} : 31\frac{1}{2}$.	28. $7\frac{9}{13} : 6\frac{1}{4}$.
11. $4 : \frac{1}{4}$.	17. $7\frac{1}{8} : 21\frac{3}{8}$.	23. $5\frac{1}{6} : 7\frac{3}{4}$.	29. $4\frac{1}{6} : 7\frac{1}{7}$.
12. $4 : \frac{3}{4}$.	18. $5\frac{5}{6} : 4\frac{3}{8}$.	24. $4\frac{7}{9} : 2\frac{1}{6}$.	30. $7\frac{8}{9} : 7\frac{1}{10}$.
13. $5 : \frac{1}{5}$.	19. $9\frac{2}{3} : 7\frac{1}{4}$.	25. $5\frac{1}{4} : 1\frac{1}{6}$.	31. $8\frac{1}{4} : 1\frac{3}{8}$.
14. $7 : \frac{6}{7}$.	20. $3\frac{3}{8} : 5\frac{3}{5}$.	26. $6\frac{1}{8} : 3\frac{1}{9}$.	32. $6\frac{5}{11} : 4\frac{1}{6}$.

Example 1. Find the price of 2,000 pounds of wheat at 84¢ a bushel.

To solve this question there are two steps to take.

Step 1. Find the number of bushels by dividing the number of pounds by the number of pounds in one bushel.

Step 2. Multiply the price of one bushel by the number of bushels.

SOLUTION. Number of bushels $= \dfrac{2000}{60}$.

$$\text{Price of the wheat} = 84¢ \times \frac{2000}{60} = \frac{\overset{14}{\cancel{84}} \times 200\cancel{0}}{\underset{\cancel{6}}{\cancel{6}\cancel{0}}}$$

$$= 2800¢ = \$28.$$

Example **2.** Three fifths of a man's money is $2,437. How much money has he?

$$\tfrac{3}{5} \text{ of his money} = \$2437.$$

$$\tfrac{1}{5} \text{ of his money} = \frac{\$2437}{3} \text{ or } \tfrac{1}{3} \text{ of } \$2437.$$

$$\tfrac{5}{5} \text{ of his money} = \frac{\$2437}{3} \times 5 \text{ or } \tfrac{5}{3} \text{ of } \$2437.$$

$$= \frac{\$12185}{3} = \$4061.66\tfrac{2}{3}.$$

Hence, his money = $4,061.67 (to nearest cent).

This method of solving a problem is known as the **analytical method.** It is called also the **unit method,** because the value of the unit of the quantity under consideration is first sought and from this the value of any number of units is then obtained.

Note that the answer is obtained by multiplying $2437 by $\frac{5}{3}$, the reciprocal of $\frac{3}{5}$. In this problem there are given the product of two factors and one of the factors. The other factor is sought. The problem is therefore one of division.

EXERCISE 31

1. Find the price of 78 acres of land if 25 acres are worth $1,375.

2. When 18 pounds of sugar sell for $1, find the cost of 45 pounds.

3. When 7 bushels of wheat sell for $5.95, how much can a person get for 255 bushels?

4. If 5 bushels of barley sell for $2, how much will 343 bushels sell for?

5. If 6 barrels of flour are sold for $45, at this rate how much will 84 barrels sell for?

6. Seven barrels of pork sell for $80.50. Find the cost of 50 barrels of pork.

7. Nine barrels of salt cost $11.70. Find the cost of 19 barrels of salt.

8. Eleven bushels of oats are sold for $4.51. Find the value of 168 bushels.

9. Six barrels of lard bring $115. How much will 46 barrels bring?

10. When 7 yards of sheeting cost 50¢, find how much must be paid for 98 yards.

11. Six yards of cambric sell for 75¢. How much must be given for 34 yards of cambric?

12. Four yards of flannel cost $1.16. How much will 29 yards of flannel cost?

13. Eight yards of gingham cost 60¢. How much will 103 yards cost?

14. Nine yards of cotton fabric cost 75¢. How much will 69 yards cost?

15. Six yards of cotton cheviot cost $1. How much will 81 yards cost?

16. Five-eighths of a man's money is $75. How much money has he?

17. Three-fourths of the length of a pole is 81 feet. Find the length of the pole.

18. The eighth and the twelfth of a number are 15. What is the number?

19. A dealer sold $\frac{1}{3}$ of his coal and had 170 tons left. How many tons had he at first?

20. The fourth part and the sixth part of a number are 25. What is the number?

DECIMALS

It is well to fix in mind the following facts:

Tenths occupy the first place to the right of the decimal point; hundredths, the second place; thousandths, the third place; ten-thousandths, the fourth place; hundred-thousandths, the fifth place; millionths, the sixth place.

Read 22.234. Twenty-two and two hundred thirty-four thousandths.

Write twenty-four tenths.

Write 24 as if it were an integer. Tenths occupy the first place to the right of the decimal point. Hence, 24 tenths is written 2.4.

Write 2,304 hundredths.

Write 2,304 as if it were an integer. Beginning at the right, point off two places for hundredths. Hence, 2,304 hundredths is written 23.04. If $\frac{2304}{100}$ be reduced to a mixed number, it becomes $23\frac{4}{100}$; that is, 23.04.

Write 11 hundred-thousandths.

Write 11 as if it were an integer. Beginning at the right, point off five places for hundred-thousandths. Hence, 11 hundred-thousandths is written .00011. Observe that places having no digits are filled in with ciphers.

Write five hundred and five thousandths.

First write five hundred, and then write five thousandths. Hence, five hundred and five thousandths is written 500.005.

Write seven hundred eight thousandths.

Here the number of units is 708; the denomination is thousandths. As thousandths occupy the third place to the right of the decimal point, hence 708 thousandths is written .708.

MULTIPLICATION AND DIVISION BY POWERS OF TEN

Consider the two numbers,

(*a*) 320.12,
(*b*) 3,201.2.

Both are expressed by the same figures written in the same order. The number (*b*) can be obtained from the number (*a*) by moving each figure one place to the left. But moving a digit one place to the left makes its value ten times as great, and, hence, moving several digits each one place to the left makes the number they represent ten times as great.

The number (*b*) can also be obtained from (*a*) by moving the decimal point in (*a*) one place to the right. Also (*a*) can be obtained from (*b*) by moving the decimal point in (*b*) one place to the left.

To multiply a number by 10, move the decimal point in the number one place to the right.

To divide a number by 10, move the decimal point in the number one place to the left.

Consider the numbers,

(*a*) 320.12,
(*b*) 32,012.

The number (*b*) is obtained from (*a*) by moving each digit in (*a*) two places to the left. This multiplies each digit by 100.

(*b*) may also be obtained from (*a*) by moving the decimal point in (*a*) two places to the right; also (*a*) from (*b*) by moving the decimal point two places to the left.

To multiply a number by 100, move the decimal point in the number two places to the right.

To divide by 100, move the decimal point in the dividend two places to the left.

Consider the numbers,

(*a*) 320.12,
(*b*) 320,120.

(*b*) is here obtained from (*a*) by moving each digit in (*a*) three places to the left. It can also be obtained from (*a*) by moving the decimal point in (*a*) three places to the right.

To multiply a number by 1,000, move the decimal point in the number three places to the right.

To divide a number by 1,000, move the decimal point in the number three places to the left.

The rules for multiplying by 10,000, 100,000, are left for the reader to determine.

Example 1. Multiply 86.4 by 10,000. Moving the decimal point four places to the right, the number becomes 864,000.

Example 2. Divide 12.3 by 100,000. Moving the decimal point five places to the left, the number becomes .000123.

EXERCISE 32

Multiply by 10:

1. 120, 14.2, .1431, .00012, 1.7320, .01234.

Multiply by 100:

2. 173, 172.8, 19.23, .001237, 8,654, 17.1.

Multiply by 1,000:

3. 1156, 32.5, 7.123, .93891, .01275, .00011.

Multiply by 10,000:

4. 345, 34.25, 5.1739, 6.001, .01793, .12.

5. Divide each of the following numbers by 10; by 100; by 1,000; by 10,000; by 100,000:

32,734	9,285.773
3,745.3	325.298
928.49	127
72,173.5	325
12.792	17
99,999.9	18.326
3,728.3	7.294
12.7564	670
1,201	1,000
3,450	7,100

Find the values of the following ratios:

6. 22.3 : .223.
7. 3.74 : .374.
8. 173.2 : 1.732.
9. 7.3 : .073.
10. 1.25 : .0125.
11. 9.28 : .00928.
12. 11.34 : .01134.
13. 7.04 : .0704.
14. 100 : .01.
15. 1,000 : .001.
16. .012 : .12.
17. 1.24 : 124.
18. 9.53 : 9,530.
19. 7.1 : 7,100.
20. 6.5 : 65,000.
21. 11.79 : 11,790.
22. .001 : 10.
23. .005 : 100.
24. 9.265 : 926.5.
25. 12.325 : 1,232.5.
26. 1.534 : 153.4.
27. 1,001 : .1001.
28. 54 : .054.
29. 792 : .0792.
30. 113 : .0113.
31. 79.28 : .7928.
32. 6.45 : 6,450.
33. 99.29 : 99,290.
34. 7.35 : 73,500.
35. 9.24 : 92,400.
36. 8.123 : .008123.
37. .04567 : 45.670.

ADDITION

Find the sum of 3.4, 2.38, 5.005, 6.2374, 11.1.

3.4	Write the numbers so that units of the same
2.38	denomination stand in the same vertical column.
5.005	Then add as integers are added.
6.2374	Write the decimal point in the sum in the same
11.1	vertical line with the decimal points in the
28.1224	addends.

EXERCISE 33

Add:

1. 2.2, .025, 37.3, 5.284, 6.294, 538.1, 77.77.
2. 3.5, 7.12, .339, 47.35, 39.28, .123, 54.275.
3. 9.28, 11.18, .999, 39.28, 7.451, 94.354, 98.76.
4. 12.49, 1.492, 38.75, 53.41, 98.69, 845.5, 892.9.
5. .009, 5.976, 40.99, 6.385, 9.278, 8.239, 64.271.
6. .098, 9.853, 19.47, 17.392, 28.394, 8.01, 77.47.
7. .285, 11.95, 29.99, 94.931, 1.732, 64.6, 78.75.
8. 11.4, 17.5, 99.37, 15.273, 9.394, 71.3, 92.95.
9. 1.21, 12.1, .121, 8.295, 7.777, 68.7, 78.28.
10. 15.9, 9.158, 91.58, 9.158, 2.293, 84.5, .139.
11. 98.5, 11.667, 66.66, 8.394, 9.928, 76.8, 9.359.
12. 77.8, 88.88, 99.99, 6.325, 7.384, 94.9, 1.798.

SUBTRACTION

Find the difference between 4,001 and 1.7003.

	Arrange the numbers so that units of the
4001.0000	same denomination stand in the same vertical
1.7003	column. Ciphers may be inserted after the
3999.2997	decimal point in the minuend. Proceed next
	as in the subtraction of integers.

EXERCISE 34

Find the remainder and verify your answer in each case:

1. 7.73 – 6.78.
2. 9.29 – 3.47.
3. 6.34 – 1.95.
4. 9.82 – 7.78.
5. 7.45 – 3.59.
6. 10.71 – 7.79.
7. 8.94 – 3.95.
8. 5.012 – 2.9.
9. 10.943 – 7.97.
10. 8.325 – 4.378.
11. 8.924 – 5.938.
12. 7.312 – 2.7.
13. 9.419 – 5.57.
14. 10.1 – 7.325.
15. 9.24 – 5.3481.
16. 8.73 – 4.4444.
17. 12.32 – 5.6741.
18. 19.33 – 6.2734.
19. 9.271 – 4.3847.
20. 3.213 – .9875.
21. 4.321 – .73201.
22. 5.204 – 1.3256.
23. 8.731 – 5.4557.
24. 9.21 – 7.2349.
25. 7.29 – 3.4551.
26. 6.001 – 5.112.

27. From seven hundred four thousandths take two hundred five ten-thousandths.

28. From five hundred ten thousandths take five hundred ten-thousandths.

29. From two thousand take two thousandths.

30. How much does one thousandth exceed one hundred-thousandth?

31. Find the difference between a hundred and a hundredth.

32. From 39 tenths take 39 thousandths.

33. From 100 hundredths take 100 ten-thousandths.

34. How much must be added to one and five-tenths to make ten?

35. By how much does 175 hundredths exceed 175 hundred-thousandths? What is the ratio of the first number to the second?

36. By how much does $1 exceed 1 mill?

37. By how much does $2 exceed 15 mills?

MULTIPLICATION

Example **1.** Multiply 3.23 by 25.

$3.23 = 323$ hundredths.

323 hundredths $\times 25 = 8075$ hundredths $= 80.75$.

Example **2.** Multiply 3.23 by .25.

Since the multiplier is $\frac{1}{100}$ of 25, the product $3.23 \times .25 = \frac{1}{100}$ of 3.23×25. $\frac{1}{100}$ of $80.75 = .8075$.

The mechanical work of multiplying may be performed as follows:

$$\begin{array}{r} 3.23 \\ .25 \\ \hline 1615 \\ 646 \\ \hline .8075 \end{array}$$

Multiply as if both numbers were integers, and point off in the product, commencing at the right, as many places as there are decimal places in both multiplicand and multiplier.

Example **3.** Multiply .32 by .018.

$$\begin{array}{r} .018 \\ .32 \\ \hline 36 \\ 54 \\ \hline .00576 \end{array}$$

Point off five places.

Another Explanation

$$\frac{32}{100} \times \frac{18}{1000} = \frac{576}{100000} = .00576.$$

To square a number means to multiply the number by itself or to take the number twice as a factor.

To cube a number means to take the number three times as a factor.

EXERCISE 35

1. Find .04 of $108; .05 of $274; .06 of $720; .07 of $144.

2. Find .09 of $34.50; .3 of $75.30; .08 of $75.80; .07 of $84.70.

3. Find .4 of $29.75; .5 of $69.48; .6 of $68.32; .1 of $328.50.

4. Find .125 of $80.80; .75 of $54; .6 of $300.50; .25 of $98.84.

5. Find .625 of $688; .875 of $792.80; .375 of $900.80.

6. Find .375 of 84 acres; .0625 of 64 acres; .3125 of 96 acres.

7. Find .1 of .1; .3 of .4; .3 of .3; .01 of .2; .01 of 1.2.

8. Multiply 27.9 by 18.
9. Multiply 1,327 by 1.6.
10. Multiply 3,927 by .46.
11. Multiply 120.01 by 3.6.
12. Multiply 25 by .017.
13. Multiply 37.5 by .07.
14. Multiply 11.9 by 2.4.
15. Multiply 182.54 by 1.49.
16. Multiply .286 by 1.96.
17. Multiply 92.24 by 2.7.
18. $.148 \times 1.15 = ?$
19. $.82 \times .51 = ?$
20. $1.875 \times .32 = ?$
21. $1.78 \times 1.89 = ?$
22. $18.24 \times .95 = ?$
23. $1.18 \times .1695 = ?$
24. $.97 \times .97 = ?$
25. $.68 \times .68 = ?$
26. $.373 \times .373 = ?$
27. $.901 \times .901 = ?$
28. $.803 \times .803 = ?$
29. $.693 \times .693 = ?$
30. $.1 \times .1 \times .1 = ?$
31. $.3 \times .3 \times .3 = ?$
32. $.4 \times .4 \times .4 = ?$
33. $.7 \times .7 \times .7 = ?$
34. $1.04 \times 1.04 \times 1.04 = ?$
35. $1.06 \times 1.06 \times 1.06 = ?$
36. $1.08 \times 1.08 \times 1.08 = ?$
37. $.25 \times .25 \times .25 = ?$

38. .7645 of the asphalt found in West Virginia is composed of carbon, .0783 is hydrogen, .1346 is oxygen, and the remainder is ash. How much of each constituent is in 254 tons of asphalt? Check your answers.

39. .7217 of the asphalt found in Oregon is composed of carbon, .079 of hydrogen, .1461 of oxygen, and the remainder of ash. Find the amount of each in 385 tons of asphalt. Check your answer.

40. Multiply the square of 14 by .7854.

41. The area of the surface of a sphere is obtained by multiplying the square of the diameter by 3.1416. Find the area of the surface of the earth, taking the diameter to be 7,920 miles. Compare your answer with the area given in your geography.

42. The moon is nearly 2,200 miles in diameter. Find the area of its surface in square miles.

43. The velocity of the earth in its orbit is 18.5 miles per second. How far does it go in 1 minute? in 1 hour?

44. A hurricane moves at the rate of 146.6 feet per second. How far does it travel in 1 minute? in 1 hour?

45. One meter = 39.37 inches. Find in inches the difference between 64 meters and 70 yards.

DIVISION

Before undertaking Division, it may be well to lay stress on the fact that numbers in the decimal system of notation may be read in many ways. Thus, 32.25 may be read, (*a*) 32 and 25 hundredths; (*b*) 3,225 hundredths; (*c*) 32,250 thousandths; (*d*) 322,500 ten-thousandths; (*e*) 322.5 tenths; (*f*) 3.225 tens.

Example 1. Divide 1.293 by 8.

8)1.293000
.161625

8 into 12 tenths gives 1 tenth, with a remainder 4 tenths. 4 tenths = 40 hundredths; 40 hundredths and 9 hundredths = 49 hundredths. 8 into 49 hundredths gives 6 hundredths, with a remainder 1 hundredth. Change 1 hundredth to thousandths, and proceed as before.

Example 2. Divide .01234 by 4.

4).012340
.003085

The work calls for no explanation.

EXERCISE 36

Divide:

1. 73.21 by 8.
2. 3.45 by 4.
3. 19.362 by 6.
4. 1.791 by 9.
5. 4.564 by 5.
6. 3.927 by 8.
7. .015 by 5.
8. 8.846 by 6.
9. 8.218 by 7.
10. 3.942 by 6.
11. 6.475 by 7.
12. 9.143 by 8.
13. .1234 by 5.
14. .73206 by 6.
15. 1.1466 by 7.
16. 6.2751 by 8.
17. 5.472 by 6.
18. 8.2548 by 9.
19. .34794 by 9.
20. .67356 by 9.
21. .999999 by 7.
22. 7.3745 by 7.
23. 6.2676 by 6.
24. 1.7346 by 7.

Find the difference between .07858 and .078; also find the difference between .07858 and .079.

.07858	.07900
.078	.07858
.00058	.00042

Hence .07858 is nearer to .079 than it is to .078. If, therefore, one is asked to give the value of .07858 correct to three figures, write for answer .079.

Express .73948 correct to three figures. *Ans.* .739.

Express .25764 correct to three figures. *Ans.* .258.

Whenever asked to give a decimal correct to any number of figures, discard the remaining figures if the first one of them is less than 5; if it is 5 or more than 5, increase the last figure by 1.

Example **1.** Divide .0732 by .8.

Make the divisor an integer by moving the decimal point one place to the right. Make a corresponding change in the dividend. This change is equivalent to multiplying divisor and dividend by 10.

$$\begin{array}{r} 8\underline{).7320} \\ .0915 \end{array}$$

Example **2.** Divide 12 by .125.

Move the decimal point in the divisor and in the dividend three places to the right, *i.e.* multiply each by 1,000.

$$\begin{array}{r} 96 \\ 125\overline{)12000} \\ \underline{1125} \\ 750 \\ \underline{750} \end{array}$$

Example **3.** Divide 3.274 by 6.25.

$$\begin{array}{r} .523^{+} \\ 625\overline{)327.400} \\ \underline{3125} \\ 1490 \\ \underline{1250} \\ 2400 \\ \underline{1875} \\ 525 \end{array}$$

Whenever the divisor is a decimal, make it an integer by moving the decimal point to the right. Make a corresponding change in the dividend. After doing this, proceed in exactly the same manner as in long division of integers. Write the decimal point in the quotient in the same vertical line with the decimal point in the dividend transformed.

EXERCISE 37

Divide:

1. 2.34 by .8.
2. .012 by .5.
3. 3.475 by .4.
4. 1.2348 by .6.
5. .1798 by .5.
6. 3.144 by 1.2.
7. 5.96 by 1.6.
8. 3.2903 by 1.3.
9. .27 by .2.
10. 5.376 by 1.6.
11. 9.4851 by 1.5.
12. 3.2 by 6.4.
13. 20 by .5.
14. 10 by .16.
15. 40 by .32.
16. 56 by 1.12.
17. 84 by 5.6.
18. 392 by 7.84.
19. 100 by .625.
20. 100 by .008.
21. 400 by .05.
22. 144 by .288.
23. 15.4 by .616.
24. .096 by .192.
25. 1 by .001.
26. 5 by .004.
27. .1 by .0001.
28. .04 by .0008.
29. .32 by .00128.
30. .45 by .0018.
31. .078 by .00312.
32. .067 by .0268.
33. .01 by .8.
34. .002 by 1.6.
35. .018 by 45.
36. .54 by 81.
37. .243 by 1.944.
38. .216 by 1.44.
39. 5.12 by .16.
40. 7.29 by 270.
41. 34.7231 by .713.
42. 31.8791 by 3.97.
43. .267584 by 2.96.
44. .348336 by .492.
45. .190256 by .188.
46. 59.4204 by 5,860.
47. 55.9911 by 108.3.
48. .575484 by 54.6.
49. .461071 by 122.3.
50. 4.50775 by 123.5.

EXERCISE 38

The mileage and valuation by counties in Texas of the St. Louis and San Francisco Railway as given by the Texas Railroad Commission for the year 1906 are as follows:

	County	Mileage	Valuation
1.	Collin	19.51	$346,538.13
2.	Dallas	2.7	53,300.16
3.	Denton	9.99	188,311.64
4.	Grayson	27.44	843,427.59
5.	Hardeman	8.68	183,997.77
6.	Tarrant	4.56	191,208.29
7.	Wilbarger	12.77	192,843.01

Find the valuation per mile in each of the above counties.

8. On July 16, 1907, a contract for paving Broadway, Denver, Colorado, was awarded on the following itemized specifications and prices:

3,050 ft. 6″ × 18″ stone curb	@ $ 1.05*
2,750 yd. brick gutter	@ $ 2.25
22,900 yd. street asphalt pavement	@ $ 2.25
704 ft. oak header	@ $ .50
945 ft. 27″ pipe sewer	@ $ 2.40
580 ft. 24″ pipe sewer	@ $ 2.00
580 ft. 21″ pipe sewer	@ $ 1.75
580 ft. 15″ pipe sewer	@ $ 1.10
398 ft. 12″ pipe sewer	@ $ .86
516 ft. 10″ pipe sewer	@ $ .75
12 manholes	@ $45.00
17 catch basins	@ $65.00
10 M ft. lumber	@ $30.00

Find the total cost.

* 6″ × 18″ means 6 inches by 18 inches.

REDUCTION OF FRACTIONS TO DECIMALS AND REDUCTION OF DECIMALS TO FRACTIONS

Example 1. Reduce $\frac{7}{8}$ to a decimal.

$$8\overline{)7.000}$$
$$.875$$

Example 2. Reduce $\frac{7}{11}$ to a decimal.

$$11\overline{)7.00000}$$
$$.63636^{+}$$

Example 3. Reduce $\frac{129}{8000}$ to a decimal.

$$\frac{129}{8000} = \frac{.129}{8} = .016125.$$

Divide numerator and denominator by 1,000 by moving the decimal point three places to the left; then divide the numerator by 8.

A fraction in its lowest terms having for denominator a number whose prime factors are 2's or 5's or both can always be exactly expressed as a decimal.

A fraction in its lowest terms having for denominator a number containing prime factors other than 2's and 5's will give rise to a decimal which never terminates.

EXERCISE 39

Reduce to decimals:

1. $\frac{3}{8}, \frac{5}{8}, \frac{7}{16}, \frac{9}{16}, \frac{11}{16}, \frac{13}{16}, \frac{15}{16}, \frac{3}{16}.$
2. $\frac{4}{15}, \frac{7}{15}, \frac{11}{15}, \frac{13}{15}, \frac{14}{15}, \frac{7}{12}, \frac{11}{12}.$
3. $\frac{3}{10}, \frac{79}{100}, \frac{87}{1000}, \frac{183}{10000}, \frac{2779}{10000}.$
4. $\frac{3}{80}, \frac{11}{80}, \frac{29}{80}, \frac{19}{20}, \frac{37}{40}, \frac{31}{60}, \frac{53}{60}.$
5. $\frac{1}{32}, \frac{3}{32}, \frac{5}{32}, \frac{9}{32}, \frac{13}{32}, \frac{19}{32}, \frac{31}{32}.$
6. $\frac{2}{7}, \frac{5}{7}, \frac{1}{13}, \frac{4}{13}, \frac{3}{14}, \frac{9}{14}, \frac{11}{14}, \frac{13}{14}.$
7. $\frac{5}{9}, \frac{19}{99}, \frac{147}{999}, \frac{49}{900}, \frac{274}{9990}, \frac{569}{9900}.$

Example 1. Reduce .0625 to a common fraction. .0625 is read 625 ten-thousandths; $\frac{625}{10000}$ is read in the same way.

$$.0625 = \frac{625}{10000} = \frac{125}{2000} = \frac{25}{400} = \frac{5}{80} = \frac{1}{16}.$$

EXERCISE 40

Reduce to common fractions:

1. .3, .8, .25, .125, .1875.
2. .07, .0125, .00875, .0625, .0075.
3. .009, .0225, .1125, .0275.
4. .072, .0104, .035, .0119, .0375.
5. .144, .0504, .0768, .162, .0112.
6. .288, .0176, .0325, .0175, .425.
7. .2875, .3375, .5125, .7375.

EXERCISE 41

1. A man walks 3 miles an hour. At this rate, how long will it take him to walk 12 miles?

2. A train goes 25 miles an hour. How long will it take it to go 300 miles at this rate?

3. A bicyclist travels at the rate of 9 miles an hour. How long will it take him to go 60 miles?

4. How would you find the time to go any given distance, if you knew the distance gone in a unit of time?

5. A man walks 3.5 miles an hour. At this rate, how long would it take him to go 49 miles?

6. The distance from London to Glasgow is 401.5 miles. An express train goes this distance in 8 hours. Find its rate per hour.

7. From London to Edinburgh is 393.5 miles. The daily mail train takes 7.75 hours to go this distance. Find its rate per hour.

39
3
117
3

3.5
7.5
245 3
175

8. The Empire State Express goes from New York City to Buffalo, a distance of 440 miles, in 8.25 hours. Find its rate per hour.

9. The mail train from Paris to Bayonne goes 486.25 miles in 8.983 hours. Find its rate per hour.

10. The distance from New York City to Cleveland is 568 miles. A train goes this distance in 19.5 hours. Find its average speed.

11. A steamer goes from New York City to Bremen, a distance of 4235 miles, in 7.75 days. Find its rate per day. Also its rate per hour.

12. The earth moves in its orbit at the rate of 1110 miles a minute. How many times faster does the earth move than a train which goes 54 miles an hour?

13. A city lot is worth $1800. If this sum is .75 of the value of the house on it, what is the value of the house?

14. If .7 of a sum of money is $196, what is the sum of money?

15. Cast iron is 7.2 times as heavy as water. How many cubic feet of cast iron weigh as much as 6120 cubic feet of water?

16. Coal is 1.3 times as heavy as water. How many cubic feet of coal weigh as much as 546 cubic feet of water?

17. There are 231 cubic inches in a gallon. How many gallons are in 1 cubic foot? (1 cu. ft. = 1728 cu. in.)

18. If 2000 pounds of coal cost $8.75, find the price of 8750 pounds of this kind of coal.

19. If 3.5 yards of cloth cost $12.25, find the price of 7.5 yards of this cloth.

20. If 1.6 yards of velvet cost $2.88, find the price of 9.75 yards of velvet.

EXERCISE 42

1. What fraction of a yard is 1 foot? What fraction of a yard is 2 feet?

2. What fraction of 1 foot is 1 inch? 3 inches? 4 inches? 5 inches? 7 inches? 8 inches? 9 inches? 10 inches?

3. What fraction of 1 yard is 1 inch? What fraction of a yard is 2 inches? 3 inches? 4 inches? 5 inches? 6 inches? 9 inches? 12 inches? 16 inches? 17 inches? 19 inches? 24 inches? 27 inches?

4. There are 8 quarts in 1 peck. What fraction of a peck is 1 quart? What fraction of a peck is 2 quarts? 3 quarts? 4 quarts? 5 quarts? 6 quarts?

5. What fraction of a square yard is 2 square feet? 3 square feet? 4 square feet? 5 square feet? 6 square feet? 7 square feet?

6. What fraction of 10 is 2? What fraction of 10 is 7?

7. What fraction of 11 is 4? What fraction of 13 is 9?

8. What fraction of 100 is 80?

9. Which of the four fundamental rules enables us to solve a problem of this character: What fraction of a number is some other number?

10. If 4 men can do a piece of work in 7 days, how long will it take 1 man to do the same work?

11. If a team of horses can plow a 40-acre lot in 16 days, how long will it take 4 teams, working together, to plow the same lot?

12. If a man can do a piece of work in 9 days, what fraction of the work can he do in 1 day? in 2 days? in 3 days? in 4 days? in 6 days?

COMPLEX FRACTIONS

A **complex fraction** is a fraction one or both of whose terms contain one, or more than one, fraction.

Thus, $\dfrac{2\frac{1}{2}}{7}$, $\dfrac{3}{4\frac{1}{5}}$, $\dfrac{1+\frac{1}{2}-\frac{1}{3}}{\frac{2}{5}+\frac{2}{3}}$, are complex fractions.

Example 1. Simplify $\dfrac{2\frac{2}{3}}{1\frac{5}{6}}$.

SOLUTION. $2\frac{2}{3} \div 1\frac{5}{6} = \frac{8}{3} \times \frac{6}{11} = 1\frac{5}{11}$.

Or, multiply numerator and denominator by any number which will make the terms of the fraction integers.

$$\frac{2\frac{2}{3}}{1\frac{5}{6}} = \frac{2\frac{2}{3} \times 6}{1\frac{5}{6} \times 6} = \frac{16}{11} = 1\frac{5}{11}.$$

Example 2. Simplify $\dfrac{3\frac{6}{25} - 2\frac{1}{4}}{1\frac{4}{5} - 1\frac{1}{2}}$.

Step 1. Simplify the numerator.

Step 2. Simplify the denominator.

Step 3. Divide the result of Step 1 by the result of Step 2.

EXERCISE 43

1. $\dfrac{4\frac{1}{2}}{39}$.

2. $\dfrac{3\frac{2}{3}}{22}$.

3. $\dfrac{4\frac{4}{5}}{12}$

4. $\dfrac{3\frac{3}{8}}{18}$.

5. $\dfrac{5\frac{3}{8}}{9\frac{5}{9}}$

6. $\dfrac{7\frac{2}{3}}{9\frac{1}{5}}$.

7. $\dfrac{11\frac{7}{8}}{6\frac{10}{11}}$.

8. $\dfrac{13\frac{5}{7}}{9\frac{3}{5}}$.

9. $\dfrac{14\frac{2}{3}}{26\frac{2}{5}}$.

10. $\dfrac{19\frac{1}{4}}{46\frac{1}{5}}$.

11. $\dfrac{25\frac{1}{4}}{40\frac{2}{5}}$.

12. $\dfrac{1\frac{7}{9} - \frac{49}{64}}{2\frac{5}{24}}$.

13. $\dfrac{2\frac{14}{25} - \frac{4}{9}}{2\frac{4}{15}}$.

14. $\dfrac{1\frac{24}{25} - \frac{1}{9}}{2\frac{2}{15}}$.

15. $\dfrac{\frac{4}{7} - \frac{1}{3} + \frac{3}{4}}{\frac{8}{35} - \frac{2}{15} + \frac{6}{20}}$.

16. $\dfrac{\frac{6}{7} + 1\frac{1}{2} - 1\frac{7}{8}}{1\frac{4}{7} - 1\frac{1}{4}}$.

17. $\dfrac{\frac{5}{8} + \frac{3}{4} \text{ of } 1\frac{1}{5}}{\frac{18}{25} + \frac{1}{2}}$.

18. $\dfrac{1\frac{4}{5} - \frac{2}{3} \text{ of } 1\frac{1}{4} \text{ of } 1\frac{1}{5}}{1\frac{1}{20} - \frac{2}{9} \text{ of } 2\frac{3}{8}}$.

19. $\dfrac{\frac{27}{64} - \frac{8}{27}}{\frac{5}{12}} \div 12\frac{1}{18}$.

20. $\dfrac{\frac{3}{4} \text{ of } \frac{5}{6} \div 1\frac{2}{3}}{2\frac{1}{4} \times \frac{5}{18} \text{ of } \frac{3}{5}} \times 1\frac{1}{4} \times 1\frac{1}{5}$.

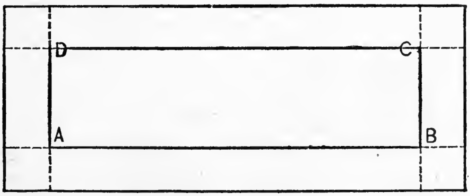
AREAS OF RECTANGULAR FIGURES

EXERCISE 44

1. The dimensions of a room are 40 feet by 30 feet, and 18 feet high. How many square yards are in its walls and ceiling?

2. Find the area, in square yards, of the walls and ceiling of a room 24 feet by 16 feet, and 12 feet high.

3. *ABCD* is a rectangular plot of ground 400 feet by 160 feet. Surrounding it is a road 15 feet wide. Find the area of the road.



4. A rectangular park, 600 feet long by 560 feet wide, has a road surrounding it. Find the area of the road if its width is 24 feet. Suppose the road is fenced in, how many feet of wire will it take to go once round?

5. A rectangular grass plot 252 feet by 180 feet has a walk around it. The width of the walk is 9 feet. How many flags, 9 inches square, will be required to flag the walk?

6. Find the area of each of the following rectangles, in square feet, correct to two decimal figures:

(*a*) 136 feet 8 inches by 115 feet 4 inches.
(*b*) 225 feet by 93 feet 10 inches.
(*c*) 78 feet 5 inches by 56 feet 6 inches.
(*d*) 25 feet 9 inches by 50 feet 2 inches.

(*e*) 104 feet 2 inches by 153 feet 11 inches.
(*f*) 203 feet by 53 feet 9 inches.
(*g*) 223 feet 10 inches by 78 feet.
(*h*) 618 feet 1 inch by 130 feet 7 inches.

HINT. Reduce the inches in each example to the fraction of 1 foot.

7. Find the area of the following rectangles, giving the results in square yards, correct to two decimal figures:

(*a*) 84.5 feet by 76.75 feet.
(*b*) 90.67 feet by 84.33 feet.
(*c*) 96.34 feet by 85.28 feet.
(*d*) 177.33 feet by 82.54 feet.
(*e*) 129.55 feet by 79.63 feet.

8. A cornfield is $213\frac{1}{3}$ rods long and 96 rods wide. How many bushels of corn will it produce at 32 bushels to an acre? Find the value of the crop at \$.$48\frac{3}{4}$ per bushel.

9. A city block is 110 yards long by 90 yards wide. How many acres are in a park which extends 7 blocks one way and 5 blocks the other way?

10. A street is 1,760 yards long and 20 yards wide. How many thousand bricks, 8 inches by 4 inches, will be needed to pave it?

11. How many square tiles, 4 inches on a side, will be required to tile a hall 60 feet by 16 feet?

12. The dimensions of a room are 16 feet by 12 feet, and 10 feet high. How many square yards are in the four walls of the room? How many square yards are in the walls and ceiling?

13. When the pressure per square foot of a hurricane is 19.47 pounds, find in tons the total pressure exerted against the side of a building 50 feet long 45 feet high.

COMPUTATION ON THE BASIS OF 100, 1,000, AND 2,000

Example 1. Find the cost of transporting 5 bales of cotton weighing respectively 510 lb., 515 lb., 508 lb., 496 lb., 487 lb., at 46 ¢ per 100 lb.

$$510 + 515 + 508 + 496 + 487 = 2516$$

$$\frac{2516}{100} \times 46 ¢ = 25.16 \times \$.46 = \$11.5736.$$

Ans. $11.57.

Example 2. Find the cost of shipping 7 head of cattle, average weight 1,089 lb., at 97 ¢ per 100 lb.

$$\frac{7 \times 1089}{100} \times \$.97 = 7 \times 10.89 \times \$.97 = \$73.9431.$$

Ans. $73.94.

$$\begin{array}{r} 10.89 \\ 7 \\ \hline 76.23 \\ 2.2869 = .03 \times 76.23 \\ \hline 73.9431 \end{array}$$

The shortest way to multiply by .97 is to take .03 of the multiplicand from itself.

Example 3. Find the value of a car load of coal weighing 43,275 lb. at $4.80 per ton of 2,000 lb.

$$\frac{43275}{2000} \times \$4.80 = \frac{43.275}{2} \times \$4.80 = 43.275 \times \$2.40$$

$$= \$103.86.$$

Example 4. How much will it cost a man a year to insure his life for $8,750 if the annual premium is $32.80 per $1,000?

$$\frac{8750}{1000} \times \$32.80 = 8.75 \times \$32.80 = \$287.00.$$

NOTE. In the above examples the sign × is to be interpreted as meaning *times*.

EXERCISE 45

The following rates in cents per 100 lb. are taken from the annual Report of the Railroad Commission of the state of Texas for the year 1906.

Find the cost of shipping:

1. 5 bales cotton, average weight 503 lb., @ 45¢.
2. 12 bales cotton, average weight 496 lb., @ 48¢.
3. 15 bales cotton, average weight 490 lb., @ 8¢.
4. 130 bbl. flour, 200 lb. to the barrel, @ 16¢.
5. 124 bbl. flour, 200 lb. to the barrel, @ 17¢.
6. 1 carload grain, weighing 27,500 lb., @ 14¢.
7. 256 sacks flour, 98 lb. to the sack, @ 12¢.
8. 32,800 lb. grain @ $7\frac{1}{2}$¢.
9. 1 carload cotton seed products, weighing 23,800 lb., @ 12¢.
10. 1 carload cotton seed hulls, weighing 28,600 lb., @ $14\frac{1}{2}$¢.
11. 1 carload cotton seed meal, weighing 42,000 lb., @ 16¢.
12. 1 carload cotton seed oil, weighing 43,600 lb., @ 5¢.
13. 1 carload brick, weighing 45,000 lb., @ $5\frac{1}{2}$¢.
14. 1 carload fire brick, weighing 27,000 lb., @ $14\frac{1}{2}$¢.
15. 1 carload common brick, weighing 47,000 lb., @ $5\frac{1}{2}$¢.
16. 1 carload mules, weighing 29,000 lb., @ 23¢.
17. 1 carload cattle, weighing 25,000 lb., @ 14¢.
18. 1 carload sheep, weighing 15,500 lb., @ 15¢.
19. 1 carload crude petroleum, weighing 42,000 lb., @ 9¢.

20. 1 carload asphaltum, weighing 27,000 lb., @ 15¢.

21. 1 carload melons, weighing 20,500 lb., @ 19¢.

22. 5,880 lb. molasses @ $7\frac{1}{2}$¢.

23. 19,200 lb. sugar @ 48¢.

24. The freight rate on coal in cents per ton of 2,000 lb. from Eagle Pass to the points named is :

Weimer 138¢	Flatonia 127¢	Columbus 140¢
Beaumont 217¢	Gonzales 121¢	Schulenburg 134¢

Find the cost of shipping 1 carload of coal, weighing 39,000 lb., from Eagle Pass to each of these points.

25. Find the cost of shipping 105,000 lb. gravel from Austin to San Antonio at 60¢ per ton of 2,000 lb.

26. Find the cost of shipping 116,000 lb. crushed rock from Clay Quarry to Houston at $67\frac{1}{2}$¢ per ton of 2,000 lb.

27. Find the cost of shipping 130,000 lb. crushed rock from Jacksboro to Fort Worth at 50¢ per ton of 2,000 lb.

28. Find the cost of shipping a carload of sand, weighing 50,000 lb., from Kingsbury to San Antonio at 40¢ per ton of 2,000 lb.

29. Find the premium on a $5,500 life insurance policy at $21.50 per $1,000.

30. Find the premium on a life insurance policy for $4,500 at $25.30 per $1,000.

31. What is the premium on a life insurance policy of $6,500 at $19.92 per $1,000 ?

32. Find the premium on a life insurance policy for $10,500 at $29.80 per $1,000.

33. Find the premium on a life insurance policy for $8,500 at $51.20 per $1,000.

34. A man insured his life for $9,450. Find the annual premium at $62.40 per $1,000.

PERCENTAGE

Per cent means by the 100, or on the 100.

Thus, 6 per cent means 6 on 100, or 6 out of 100.

25 per cent means 25 on 100, or 25 out of 100.

6 per cent is written 6%; 25 per cent is written 25%.

If a man invests \$100 and gains on his investment \$100, he makes a profit of 100%. Therefore,

100% of a number = the number.
50 % of a number = $\frac{1}{2}$ of the number.
25 % of a number = $\frac{1}{4}$ of the number.
20 % of a number = $\frac{1}{5}$ of the number.
$16\frac{2}{3}$ % of a number = $\frac{1}{6}$ of the number.
7 % of a number = $\frac{7}{100}$ of the number.

The following per cent equivalents should be remembered:

$100\% = 1$	$50\% = \frac{1}{2}$	$33\frac{1}{3}\% = \frac{1}{3}$
$66\frac{2}{3}\% = \frac{2}{3}$	$25\% = \frac{1}{4}$	$75\% = \frac{3}{4}$
$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$60\% = \frac{3}{5}$
$80\% = \frac{4}{5}$	$16\frac{2}{3}\% = \frac{1}{6}$	$83\frac{1}{3}\% = \frac{5}{6}$
$12\frac{1}{2}\% = \frac{1}{8}$	$37\frac{1}{2}\% = \frac{3}{8}$	$62\frac{1}{2}\% = \frac{5}{8}$
$87\frac{1}{2}\% = \frac{7}{8}$	$150\% = 1\frac{1}{2}$	$175\% = 1\frac{3}{4}$

Example 1. In a city school system there are 5250 children in attendance. If 84 % are promoted, how many are promoted?

$$5250 \times \frac{84}{100} = 5250 \times .84 = 4410.$$

Example 2. Find $58\frac{1}{3}$ % of 3880.

$$\frac{58\frac{1}{3}}{100} = \frac{175}{300} = \frac{7}{12}.$$

$$3880 \times \frac{7}{12} = 2263\frac{1}{3}.$$

EXERCISE 46

Find:

1. 9 % of $84.
2. 8 % of $425.
3. 6 % of $800.
4. 5 % of $2000.
5. 8 % of $3250.
6. 7 % of $4500.
7. 10 % of $2250.
8. 11 % of $4000.
9. 12 % of $7250.
10. 4 % of $3600.
11. 5 % of $983.
12. 8 % of $750.
13. 6 % of $850.
14. 7 % of $1250.
15. 25 % of $4840.
16. 30 % of $3290.
17. 40 % of $4500.
18. 50 % of $3250.
19. 75 % of $4000.
20. 70 % of $3500.
21. 80 % of $2450.
22. 100 % of $7800.
23. 16 % of $3200.
24. 18 % of $9200.
25. 125 % of $4000.
26. 225 % of $5400.

27. A man whose salary is $750 a year saves 35 % of it. How much does he save?

28. In a city school system there are 8250 children; 54 % of this number are girls. How many girls are in these schools? How many boys?

29. A farm of 175 acres has 24 % woodland. How many acres of woodland are in the farm?

30. A house costs $4740. The lot on which it is built cost 32 % of the value of the house. Find the cost of the lot.

31. In a certain year the number of rainy days was 20 % of the number of days in the year. How many rainy days were there? How many fair days?

32. A lawyer charged 6 % for collecting a debt of $3720. Find his fee. How much did he remit to his client?

EXERCISE 47

Find:

1. $33\frac{1}{3}\%$ of \$9600.
2. $66\frac{2}{3}\%$ of \$3240.
3. 25% of \$4920.
4. 20% of \$4500.
5. $16\frac{2}{3}\%$ of \$636.
6. $83\frac{1}{3}\%$ of \$792.
7. $12\frac{1}{2}\%$ of \$3280.
8. $37\frac{1}{2}\%$ of \$4640.
9. $62\frac{1}{2}\%$ of \$5720.
10. $8\frac{1}{3}\%$ of \$5640.
11. $41\frac{1}{3}\%$ of \$9120.
12. $58\frac{1}{3}\%$ of \$7560.
13. $6\frac{2}{3}\%$ of \$4515.
14. $13\frac{1}{3}\%$ of \$4845.
15. $26\frac{2}{3}\%$ of \$3900.
16. $46\frac{2}{3}\%$ of \$2400.
17. $5\frac{5}{9}\%$ of \$3600.
18. $116\frac{2}{3}\%$ of \$672.

19. A man sells his house for \$1800. If he paid for it $83\frac{1}{3}\%$ of the price at which it was sold, what did the house cost?

20. A shoe dealer sold \$720 worth of shoes. The shoes cost him $66\frac{2}{3}\%$ of the selling price. Find the cost price of the shoes.

21. In an apple orchard of 840 trees $58\frac{1}{3}\%$ bore fruit. How many trees were fruit-bearing?

22. A ranchman lost during a blizzard $16\frac{2}{3}\%$ of his sheep. If the number in his flock was originally 960, how many did he lose, and how many were left?

23. If the area of a county is 1230 square miles, and 75% of it arable land, how many square miles are arable land?

24. Piles used in the construction of a railroad bridge are 42 ft. long, and $83\frac{1}{3}\%$ of their length is beneath the water. Find the length in the water.

25. The railroad mileage of the United States in the year 1904 was 212,578. Of this the railroad mileage of Florida was $1\frac{2}{3}\%$. Find the railroad mileage of Florida in 1904.

INTEREST AND PROPERTY INSURANCE

Interest is money paid for the use of money.

The sum loaned is the **principal.**

Interest is always reckoned as a rate per cent of the principal. The rate per cent is for one year unless otherwise stated.

Property insurance is idemnity against loss of property and is reckoned as a rate on the basis of $100 valuation.

The sum paid for insurance is the **premium.**

The written contract between the assured and the insurance company is called the **insurance policy.**

Example. What is the premium on an insurance policy of $15,350 at $1.35 per $100?

$$\frac{15350}{100} \times \$1.35 = 153.5 \times \$1.35 = \$207.225, \text{ or}$$

$$15350 \times \$.0135 = \$207.225.$$

In the first solution, the number of 100's is multiplied by the rate on $100. In the second solution, the number of dollars is multiplied by the rate on $1.00.

EXERCISE 48

Find the interest on:

1. $600 for 1 yr. at 4%; for 1 yr. at 5%; for 1 yr. at 6%; for 1 yr. at 8%.

2. $850 for 1 yr. at 7%; for 1 yr. at 8%; for 1 yr. at 9%.

3. $950 for 1 yr. at 3%; for 1 yr. at 4%; for 1 yr. at 8%.

4. $982 for 2 yr. at 4%; for 3 yr. at 5%; for 1 yr. 6 mo. at 6%.

5. $738 for $\frac{1}{2}$ yr. at 5%; $\frac{1}{2}$ yr. at 6%.
6. $920 for $\frac{1}{4}$ yr. at 6%; $\frac{1}{4}$ yr. at 7%.
7. $1200 for 4 mo. at 5%; 4 mo. at 6%.
8. $1100 for 6 mo. at 7%; 6 mo. at 4%.
9. $1280 for 3 mo. at 8%; 3 mo. at 6%.

Find the premium for insuring dwellings against loss by fire at the rates specified per $100:

10. $2500 at $1.30.
11. $2000 at $1.15.
12. $4500 at $1.50.
13. $3000 at $1.25.
14. $2500 at $1.90.
15. $5500 at $1.70.
16. $6500 at $1.50.
17. $4000 at $1.80.
18. $5400 at $1.70.
19. $3300 at $1.80.
20. $7500 at $1.60.
21. $7250 at $1.40.
22. $10,500 at $1.30.
23. $19,250 at $1.25.
24. $16,450 at $1.60.
25. $7900 at $1.35.
26. $22,500 at $1.10.
27. $18,250 at $1.20.
28. $2400 at $1.80.
29. $9300 at $1.50.
30. $8500 at $1.60.
31. $9450 at $1.50.
32. $6500 at $1.90.
33. $5400 at $1.60.
34. $9500 at $1.90.
35. $12,000 at $1.75.
36. $18,000 at $1.75.
37. $200,000 at $1.25.
38. $15,500 at $1.60.
39. $16,200 at $1.60.
40. $1800 at $1.90.
41. $1750 at $1.75.

42. A man insures his residence, valued at $5000, at $\frac{3}{4}$ of its value at the rate of $1.20 on the $100. Find the premium paid.

43. A jobber insures a quantity of cotton, worth $30,000, at $\frac{2}{3}$ of its value at the rate of 75¢ on the $100. Find his premium.

CHAPTER II

COMPOUND QUANTITIES

CONCRETE quantities of the same kind, but consisting of units of different denominations, are called **compound quantities.**

Seventeen days, 10 hours, and 30 minutes is a compound quantity. Here, we have three units of measurement; namely, a day, an hour, and a minute. These units are of different denominations, but each is of the same kind, inasmuch as it stands for a definite portion of time.

Compound quantities are also called **compound denominate quantities.** Quantities composed of units of one denomination are generally called **simple quantities.**

AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight, or 2000 pounds = 1 ton (T.)
2240 pounds = 1 long ton

Avoirdupois Weight is used in weighing all commercial quantities excepting the precious metals, jewels, and drugs, when sold by retail druggists.

The **unit in Avoirdupois Weight** is the **pound** of 7000 grains. One cubic inch of distilled water weighs in vacuo 252.286 grains, of which 7000 weigh 1 pound.

The long ton is used in the United States custom houses, and in weighing coal and mineral products at the mines.

The process of reducing units of any given denomination to units of higher denomination is called **reduction ascending.**

The process of reducing units of a higher denomination or of higher denominations to units of lower denomination is called **reduction descending.**

Example. Reduce 1,000,201 oz. to higher denominations.

16	1000201
100	62512 lb. 9 oz.
20	625 cwt. 12 lb.
	31 T. 5 cwt.

Divide by 16 to get the number of pounds. Divide by 100 to get the number of hundredweights. Divide by 20 to get the number of tons. *Ans.* 31 T. 5 cwt. 12 lb. 9 oz.

Weights are generally expressed in tons or in pounds.

EXERCISE 49

Reduce to higher denominations :

1. 7800 oz. **3.** 75,497 oz. **5.** 7987 lb.

2. 9763 oz. **4.** 1,000,000 oz. **6.** 32,721 lb.

7. How many ordinary or short tons in 100 long tons?

8. Reduce 10,000 lb. to long tons.

Example. Reduce 7 T. 3 cwt. 12 lb. 10 oz. to ounces.

T.	CWT.	LB.	OZ.
7	3	12	10

20
143 cwt.
100
14312 lb.
16
85882
14312
229002 oz.

Reduce the 7 T. to hundredweights by multiplying by 20. Add 3 cwt. to the product and get 143 cwt. Multiply 143 by 100 and add 12 lb. to the product. This gives 14,312 lb. Multiply this by 16, adding 10 oz., when the first figure is multiplied by 6.

EXERCISE 50

1. Reduce 19 T. to pounds.
2. Reduce 14 T. 4 cwt. to pounds.
3. Reduce 17 T. 3 cwt. to pounds.
4. Reduce 25 T. 2 cwt. to ounces.
5. Reduce 3 T. 15 cwt. 2 lb. to pounds.
6. Reduce 4 T. 11 cwt. 58 lb. to pounds.
7. Reduce 8 T. 2 cwt. 73 lb. to pounds.
8. A dealer buys 50 long tons of coal and sells it by the short ton. How many short tons does he sell?
9. A dealer buys 100 long tons of coal at $6.75 per ton. He sells it by the short ton at $6.75 per ton. How much profit does he make?
10. Convert 784 short tons into long tons.
11. Convert 550 long tons into short tons.
12. Three horses together weigh 2 T. 4 cwt. 91 lb. Find in pounds the average weight of the horses.

LINEAR OR LONG MEASURE

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
6080 feet	= 1 knot, geographical or nautical mile
3 knots	= 1 marine league

1 mi. = 320 rd. = 1760 yd. = 5280 ft.

The **unit of length** is the *yard*.

The yard in the United States is defined as $\frac{3600}{3937}$ of the **meter.**

The standard yard of this country has been adopted since 1893.

EXERCISE 51

1. Reduce 4 yd. 2 ft. to inches.
2. Reduce 110 yd. 1 ft. to inches.
3. Reduce $5\frac{1}{2}$ mi. to yards.
4. Reduce 7 mi. 120 rd. to yards.
5. Reduce 10 mi. 110 rd. 4 yd. to yards.
6. Reduce $445\frac{3}{4}$ mi. to yards.
7. Reduce 7.74 mi. to yards.
8. Reduce 8.35 mi. to rods.
9. Reduce 238 rd. to feet.

SQUARE MEASURE

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
1 acre	= 4840 square yards
1 section	= 1 square mile
36 sections	= 1 township

Square measure is used to measure the areas of surfaces.

A **cube** is a solid bounded by six plane surfaces, each of which is a square.

A solid having the shape of a box or of an ordinary room, *i.e.* one bounded by six plane surfaces, each of which is a rectangle, is called a **rectangular solid.**

The **volume** of a solid means the amount of space it occupies. This is measured by the number of times the solid contains the unit of measurement.

The **unit of volume** is a cube having for an edge the linear unit. The **cubic unit** from which all others are derived is the cubic yard.

CUBIC OR SOLID MEASURE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

MEASURES OF CAPACITY

There are two **measures of capacity** in general use; namely, Liquid Measure and Dry Measure.

LIQUID MEASURE

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

A gallon contains 231 cu. in.

DRY MEASURE

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

One bushel contains 2150.42 cu. in. It is the volume of a cylindrical vessel $18\frac{1}{2}$ in. in diameter and 8 in. deep.

REDUCTION DESCENDING

Example. Reduce 5 gal. 2 qt. 1 pt. 2 gi. to gills.

$$\begin{array}{rl}
\multicolumn{2}{l}{5 \text{ gal. } 2 \text{ qt. } 1 \text{ pt. } 2 \text{ gi.}} \\
\underline{4} & \\
20 & = \text{number of quarts in 5 gal.} \\
\underline{2} & \\
22 & = \text{number of quarts in 5 gal. 2 qt.} \\
\underline{2} & \\
44 & = \text{number of pints in 5 gal. 2 qt.} \\
\underline{1} & \\
45 & = \text{number of pints in 5 gal. 2 qt. 1 pt.} \\
\underline{4} & \\
182 & = \text{number of gills in 5 gal. 2 qt. 1 pt. 2 gi.}
\end{array}$$

EXERCISE 52

Reduce :

1. 1 sq. mi. to sq. rd.
2. $2\frac{1}{2}$ sq. mi. to A.
3. 12 A. to sq. ft.
4. 27 sq. rd. to sq. ft.
5. 3 mi. 50 rd. to ft.
6. 8 mi. 40 rd. to ft.
7. $2\frac{3}{4}$ mi. to yd.
8. 3.75 mi. to yd.
9. 2.125 mi. to ft.
10. 25 cu. yd. to cu. ft.
11. 38 cu. yd. 20 cu. ft. to cu. ft.
12. $17\frac{1}{2}$ cu. yd. to cu. ft.
13. 18.75 cu. yd. to cu. ft.
14. 20.25 cu. yd. to cu. in.
15. 2 gal. 2 qt. to qt.
16. 5 gal. 3 qt. to qt.
17. 3 gal. 1 pt. to pt.
18. 7 gal. 1 pt. to pt.
19. 19.25 gal. to pt.
20. 4 bu. to qt.
21. $3\frac{7}{8}$ bu. to qt.
22. 3.625 bu. to qt.
23. 7 pk. to qt.
24. 7.375 pk. to pt.
25. $18\frac{7}{8}$ bu. to pt.
26. 13 bu. 3 qt. to pt.
27. How many feet are in $\frac{1}{2}$ mi.? in $\frac{1}{4}$ mi.? in $\frac{1}{11}$ mi.?
28. How many yards are in $\frac{1}{4}$ mi.? in $\frac{1}{5}$ mi.? in $\frac{1}{3}$ mi.?
29. What fraction of a mile is 440 yd.? 176 yd.? 88 yd.?
30. How many square yards are in $\frac{1}{4}$ of an A.? in $\frac{1}{2}$ A.?
31. What part of a township is 1 sq. mi.?
32. How many square rods are in $\frac{7}{8}$ A.?
33. How many square rods are in .7 A.? in. .9 A.?
34. How many square feet are in $\frac{3}{4}$ sq. rd.?
35. How many cubic inches are in 1 pt., Dry Measure?
36. How many cubic inches are in 1 pt., Liquid Measure?
37. How many quarts are in $\frac{3}{4}$ pk.?
38. How many gallons are required to fill 10 bu. measures?

REDUCTION ASCENDING

Example 1. Reduce 85 pt. to higher denominations.

2	85 pt.
4	42 qt. 1 pt.
	10 gal. 2 qt. 1 pt.

Example 2. The length of one degree of latitude at 40° north is 364,280 ft. Express this length in miles.

80	364280 ft.
6	4553.500
11	758.916
	68.992

There are 5280 ft. in 1 mi. The factors of 5280 are 80, 6, and 11. (A number is divided by 80 by dividing by 8 and writing each quotient figure one place to the right.) *Ans.* 68.992 mi.

EXERCISE 53

Reduce to higher denominations:

1. 234 pt. (Liquid Measure).
2. 47,385 cu. in.
3. 3456 pt. (Liquid Measure).
4. 10,240 rd.
5. 2000 pt. (Dry Measure).
6. 393,000 cu. in.
7. 20,000,000 A.
8. 15,000 sq. in.

9. The equatorial diameter of the earth is 41,852,404 ft. Express this distance in miles and the decimal of a mile correct to two decimal figures.

10. The polar diameter of the earth is 41,709,790 ft. What is the polar diameter of the earth in miles correct to two decimal figures?

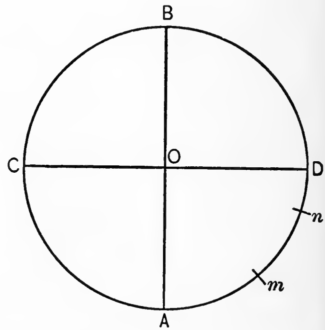
11. By how many miles does the equatorial diameter exceed the polar diameter?

12. Light takes 8 min. 18 sec. to come from the sun to the earth. The mean distance of the sun from the earth is 92,790,000 mi. Find the velocity of light per second.

CIRCULAR ARC MEASURE

A **circle** is a plane figure bounded by a line called the **circumference**, every point of which is equally distant from a point within the figure called the **center.**

A straight line from the center to the circumference is called a **radius.**



A straight line drawn through the center and terminated by the circumference is called a **diameter.**

The lines *AB* and *CD* are diameters.

Any portion of a circumference is called an **arc.** *mn* is an arc.

An arc equal to one half of a circumference is called a **semicircumference.**

An arc equal to one fourth of a circumference is called a **quadrant.**

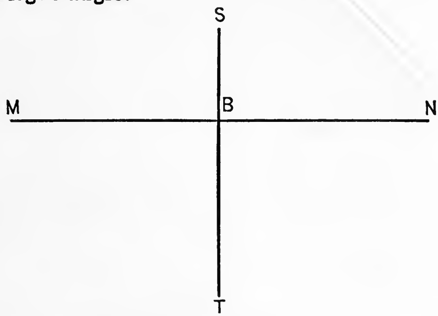
60 seconds (″)	= 1	minute (′)
60 minutes	= 1	degree (°)
360 degrees	= 1	circumference

ANGULAR MEASURE

An **angle** is a figure formed by two straight lines proceeding from a point. Its magnitude depends upon the amount of turning necessary to bring one side into coincidence with the other.

NOTE. Beginners should be provided with a protractor and they should draw and measure angles. To learn things by actual trial and not by mere hearsay is to educate.

If one straight line meets another straight line so as to make the adjacent angles equal to each other, each angle is called a **right angle.**



If the lines *MN*, *ST* meet in *B* so as to make the angles *NBS*, *SBM* equal, then each angle is a right angle.

The **unit of angular measure** is 1 degree.

60 seconds ($''$)	= 1 minute ($'$)
60 minutes	= 1 degree ($^\circ$)
90 degrees	= 1 right angle
2 right angles	= 1 straight angle

EXERCISE 54

1. What part of $1'$ is $1''$?

2. How many seconds are in $\frac{1}{2}$ minute?

3. Reduce $1'\ 30''$ to seconds.

4. What part of a straight angle is a right angle?

5. What part of a right angle is an angle of 45°? 30°? 15°? 18°? 60°? 75°?

6. How many degrees are in 1 straight angle? in $\frac{3}{4}$ of a straight angle?

7. What part of a straight angle is an angle of 15°? 24°? 30°? 45°? 60°? 80°? 100°? 105°? 120°? 135°? 150°?

TIME MEASURE

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 common year (yr.)
366 days	= 1 leap year (yr.)
100 years	= 1 century

There are twelve calendar months in a year.

The following lines will enable one to remember the number of days in each month :

> " Thirty days hath September,
> April, June, and November,
> February twenty-eight alone,
> And all the others thirty-one;
> But leap year, coming once in four,
> Gives February one day more."

A day is the **standard unit of time.** It is of the same duration at all places. It represents the period of time that elapses between two successive passages of the sun across the meridian of any place.

The length of a year is 365 days, 5 hours, 48 minutes, 46 seconds. The common year has 365 days. The difference in length between the common year and the actual, or solar year, gave rise to the introduction of **leap years.** Centennial years are leap years when the number of the year is exactly divisible by 400. Thus, the year 2000 is a leap year because 2000 is divisible by 400. All other years are leap years when their numbers are exactly divisible by 4. The year 1907 is not a leap year, as the number 1907 is not exactly divisible by 4. The year 1828 was a leap year, as 1828 is exactly divisible by 4.

MISCELLANEOUS MEASURE

1 bushel of barley	= 48 lb.
1 bushel of wheat	= 60 lb.
1 bushel of oats	= 32 lb.
1 bushel of rye	= 56 lb.
1 bushel of potatoes (Irish)	= 60 lb.
1 bushel of potatoes (sweet)	= 55 lb.
1 bushel of buckwheat	= 48 lb.
1 bushel of beans	= 60 lb.
1 bushel of shelled corn	= 56 lb.
1 bushel of peas	= 60 lb.
1 bushel of clover seed	= 60 lb.
1 barrel of flour	= 196 lb.
1 barrel of pork or beef	= 200 lb.
1 cental of grain	= 100 lb.

NUMBERS

12 units	= 1 dozen (doz.)
12 dozen	= 1 gross
12 gross	= 1 great gross
20 units	= 1 score

PAPER MEASURE

24 sheets of paper	= 1 quire
20 quires	= 1 ream
2 reams	= 1 bundle
5 bundles	= 1 bale

TROY WEIGHT

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)
1 pound Troy	= 5760 grains

Troy weight is used in weighing precious metals and jewelry.

(The measures in this paragraph are inserted merely for reference.)

EXERCISE 55

Reduce to seconds:

1. $18^\circ\ 20'\ 20''$. **3.** $12^\circ\ 5'\ 10''$. **5.** 120.3°.

2. A quadrant. **4.** $7\frac{3}{4}^\circ$. **6.** $45^\circ\ 30'\ 20''$.

Reduce to minutes:

7. $14\frac{1}{4}^\circ$. **9.** 254.125°. **11.** $18\frac{5}{6}^\circ$.

8. 75.75°. **10.** $4\frac{2}{3}^\circ$. **12.** $13\frac{7}{8}^\circ$.

13. Reduce a common year to minutes.

14. Find the number of minutes in the years 1903, 1904, 1905.

15. Find the number of minutes in February, 1904.

16. Find the number of minutes in the first three months of the year 1903; also in the first three months of the year 1904.

17. Find the number of seconds in a solar year, consisting of 365 da. 5 hr. 48 min. 46 sec.

18. The pulse of a healthy person beats 70 times a minute. At this rate, how many times will it beat in a leap year? How many times will it beat in the four successive years, beginning 1904?

19. Reduce 30 wk. 6 da. 12 hr. to minutes.

20. Reduce $1\frac{3}{5}$ common years to days.

21. Reduce $5\frac{5}{7}$ wk. to hours.

22. Reduce 20.4 yr. to hours, allowing five of them to be leap years.

23. How many days are there between Jan. 30, 1902, and Jan. 30, 1910?

24. Reduce $\frac{2}{15}$ of a circumference to degrees.

25. Reduce $\frac{29}{30}$ of a straight angle to degrees.

ADDITION

In **adding compound quantities**, proceed as follows:

Step 1. Arrange the quantities so that units of the same denomination stand in the same vertical column, the highest denomination being written first, the next to the highest second, and so on.

Step 2. Beginning with the right-hand column, add the numbers in it, divide their sum by the number of units which makes one unit of the next higher denomination. Write the remainder in the right-hand column, and carry the quotient to the next column.

Step 3. Treat the next column to the left in the same manner. The remaining columns are dealt with in the same way.

LINEAR MEASURE

EXERCISE 56

Add:

(1)

YD.	FT.	IN.
9	0	8
11	2	4
6	1	10
5	0	6

(2)

YD.	FT.	IN.
26	1	6
33	2	6
20	1	0
70	1	9

(3)

YD.	FT.	IN.
5	1	11
3	1	2
4	1	3
11	2	5

SQUARE MEASURE

(4)

A.	SQ. RD.
76	144
85	131
37	33
63	99

(5)

A.	SQ. RD.
33	79
173	27
254	28
45	53

(6)

A.	SQ. RD.
127	38
192	99
238	77
413	25

CAPACITY

7. Add: 2 gal. 3 qt. 1 pt., 3 gal. 2 qt. 1 pt., 5 gal. 2 qt. 1 pt., 4 gal. 2 qt. 1 pt.

8. Add: 7 gal. 2 qt. 1 pt., 9 gal. 3 qt., 4 gal. 1 qt. 1 pt., 6 gal. 3 qt. 1 pt., 9 gal. 1 pt., 7 gal. 1 pt.

9. Add: 3 bu. 3 pk. 5 qt., 4 bu. 2 pk. 4 qt., 9 bu. 2 pk. 7 qt., 9 bu. 7 qt., 8 bu. 2 pk. 3 qt., 6 bu. 3 pk. 2 qt.

10. Add: 4 bu. 7 qt., 3 bu. 4 pk. 6 qt., 7 bu. 2 pk. 6 qt., 8 bu. 3 qt., 9 bu. 2 pk. 3 qt., 4 bu. 3 pk. 2 qt.

11. Add: 17 gal. 1 pt., 14 gal. 2 qt. 1 pt., 2 gal. 2 qt. 1 pt., 15 gal. 1 pt., 13 gal. 1 qt. 1 pt., 14 gal. 3 qt. 1 pt.

12. Add: 14 bu. 2 pk. 7 qt., 29 bu. 3 pk. 5 qt., 23 bu. 2 pk. 6 qt., 39 bu. 6 qt., 28 bu. 3 pk., 17 bu. 2 pk. 5 qt.

13. Add: 38 bu. 3 pk. 2 qt., 16 bu. 2 pk. 1 qt., 28 bu. 3 pk. 7 qt., 3 bu. 7 qt., 5 bu. 3 pk., 24 bu. 2 pk. 2 qt.

14. Add: 15 bu. 5 qt., 12 bu. 3 pk., 17 bu. 7 qt., 18 bu. 6 qt., 29 bu. 2 pk. 3 qt., 71 bu. 3 pk. 2 qt., 18 bu. 3 pk.

AVOIRDUPOIS WEIGHT

15. Add: 20 T. 215 lb., 18 T. 425 lb., 17 T. 328 lb., 92 T. 411 lb.

16. Add: 384 lb. 12 oz., 125 lb. 15 oz., 82 lb. 14 oz., 73 lb. 11 oz.

17. Add: 425 lb. 10 oz., 17 lb. 14 oz., 30 lb. 12 oz., 72 lb. 9 oz.

18. Add: 15 T. 290 lb., 17 T. 184 lb., 12 T. 127 lb., 15 T. 9 lb., 18 T. 18 lb.

19. Add: 18 lb. 8 oz., 64 lb. 7 oz., 82 lb. 6 oz., 90 lb. 5 oz., 16 lb. 13 oz.

20. Add: 16 T. 175 lb., 71 T. 29 lb., 28 T. 245 lb., 97 T. 159 lb., 13 T. 1300 lb.

TIME

21. Add: 5 da. 4 hr. 15 min., 17 da. 17 hr. 17 min., 92 da. 14 hr. 14 min., 27 da. 23 hr. 12 min., 29 da. 16 hr. 14 min., 45 da. 15 hr. 18 min.

22. Add: 4 wk. 5 da. 7 hr., 9 wk. 6 da. 11 hr., 18 wk. 5 da. 12 hr., 23 wk. 11 hr., 28 wk. 4 da. 4 hr., 73 wk. 6 da. 19 hr., 82 wk. 5 da. 21 hr.

23. Add: 20 hr. 30 min. 18 sec., 17 hr. 45 min. 37 sec., 14 hr. 18 min. 18 sec., 14 hr. 12 min. 12 sec., 9 hr. 48 min. 48 sec., 8 hr. 39 min. 39 sec.

24. Add: 12 da. 17 hr. 44 min., 15 da. 18 hr. 18 min., 31 da. 19 hr. 19 min., 33 da. 21 hr. 27 min., 12 da. 12 hr. 36 min., 34 da. 20 hr. 23 min.

25. Add: 3 wk. 5 da. 23 hr., 8 wk. 6 da. 16 hr., 9 wk. 5 da. 18 hr., 4 wk. 4 da. 14 hr., 10 wk. 5 da. 13 hr.

26. Add: 14 hr. 14 min. 14 sec., 9 hr. 54 min. 38 sec., 11 hr. 12 min. 19 sec., 4 hr. 31 min. 27 sec., 5 hr. 45 min. 43 sec., 8 hr. 41 min. 42 sec.

VOLUME

27. Add: 4 cu. ft. 1421 cu. in., 9 cu. ft. 294 cu. in., 18 cu. ft. 998 cu. in., 7 cu. ft. 778 cu. in., 9 cu. ft. 499 cu. in., 15 cu. ft. 498 cu. in.

28. Add: 27 cu. yd. 19 cu. ft., 84 cu. yd. 24 cu. ft., 87 cu. yd. 19 cu. ft., 16 cu. yd. 22 cu. ft., 55 cu. yd. 17 cu. ft., 34 cu. yd. 16 cu. ft.

29. Add: 37 cu. yd. 13 cu. ft., 38 cu. yd. 26 cu. ft., 49 cu. yd. 25 cu. ft., 62 cu. yd. 26 cu. ft., 77 cu. yd. 17 cu. ft., 94 cu. yd. 28 cu. ft.

30. Add: 15 cu. ft. 578 cu. in., 18 cu. ft. 902 cu. in., 18 cu. ft. 978 cu. in., 15 cu. ft. 293 cu. in.

SUBTRACTION

From 19 sq. yd. 5 sq. ft. 20 sq. in. take 14 sq. yd. 7 sq. ft. 45 sq. in.

SQ. YD.	SQ. FT.	SQ. IN.
19	5	20
14	7	45
4	6	119

Step 1. Write the quantities so that units of the same denomination are in the same column.

Step 2. Find what concrete quantity added to 45 sq. in. will give 1 sq. ft. 20 sq. in., *i.e.* 164 sq. in. Write the remainder, 119 sq. in., in the column for square inches. Carry 1 sq. ft.

Step 3. Find what concrete quantity added to 8 sq. ft. will give 1 sq. yd. 5 sq. ft., *i.e.* 14 sq. ft. Write the remainder, 6 sq. ft., in the column for square feet. Carry 1 sq. yd.

Step 4. Find the difference between 15 sq. yd. and 19 sq. yd. and write it in its proper place.

EXERCISE 57

CIRCULAR ARC OR ANGULAR MEASURE

1. Subtract 5° 12′ 13″ from 84° 14′ 30″.
2. Subtract 19° 14′ 14″ from 27° 15′ 10″.
3. Subtract 38° 15′ 45″ from 90° 10′ 10″.
4. Subtract 54° 14′ 54″ from 172° 0′ 19″.
5. Subtract 84° 5′ 15″ from 90°.
6. Subtract 113° 13′ 54″ from 180°.
7. Subtract 94° 53′ 50″ from 180°.

8. Subtract 87° 15′ from 133° 12′.

9. Subtract 119° 54′ 17″ from 180°.

10. Subtract 15° 14′ 17″ from 94° 14′ 7″.

11. The tropic of Cancer is 23° 27′ 6″ north of the equator; the Arctic circle is 23° 27′ 6″ south of the north pole. Find the width of the north temperate zone.

CAPACITY

12. From 3 bu. 2 pk. 4 qt. take 1 bu. 2 pk. 5 qt.

13. From 12 gal. 3 qt. 1 pt. take 4 gal. 3 qt.

14. From 11 gal. take 4 gal. 3 qt. 1 pt.

15. From 17 gal. take 11 gal. 1 qt. 1 pt.

16. From 37 bu. 2 pk. 4 qt. take 17 bu. 3 pk. 7 qt.

17. From 29 bu. 1 pk. 2 qt. take 19 bu. 3 pk. 5 qt.

18. From 37 gal. take 17 gal. 1 qt. 1 pt.

19. From 134 gal. take 112 gal. 3 qt. 1 pt.

20. From $1\frac{1}{2}$ gal. take $\frac{3}{4}$ gal. and express the result in quarts.

21. From $1\frac{3}{4}$ bu. take $\frac{7}{8}$ bu. and express the result in quarts.

22. From $5\frac{1}{2}$ bu. take $1\frac{3}{4}$ bu. and express the result in quarts.

TIME

23. From 3 da. 4 hr. 11 min. take 1 da. 7 hr. 14 min.

24. From 11 da. 5 hr. 10 min. take 4 da. 11 hr. 19 min.

25. Almanacs give the time of sunrise in Florida, Louisiana, and Texas on March 5 as 6.22 A.M., and that of sunset as 6.2 P.M. Find the length of the day.

26. On April 1, 1903, the moon rose at 10.28 P.M. On April 4 following, it rose at 12.28 A.M. How many hours and minutes earlier did it rise on April 1 than on April 4?

Time between events happening in two different years.

Example. How many years, months, and days were between Aug. 27, 1880, and Jan. 22, 1901?

YR.	MO.	DA.
1901	1	22
1880	8	27
20	4	25

Since January is the first month of the year and August is the eighth month of the year, we write 1 instead of January and 8 instead of August. In finding the difference, a month is taken as 30 days. The work is then performed as in the subtraction of compound quantities.

EXERCISE 58

1. The battle of New Orleans was fought on Jan. 8, 1815. Find the time from that date to the present day.

2. The first telegraph message was sent by Professor Morse on May 24, 1844. Find the time from that date to the present day.

3. The Spanish fleet under Cervera was destroyed near Santiago on July 3, 1898. Find the time from that date to Feb. 1, 1903.

4. The Mecklenburg Declaration of Independence was signed May 20, 1775. Find the time from this date to the surrender of Cornwallis, Oct. 19, 1781.

5. The following named men were born and died on the dates specified. Find how long each lived.

	BORN	DIED
John Milton	Dec. 9, 1608.	Nov. 8, 1674.
Alexander Pope . .	May 21, 1688.	May 30, 1744.
William Shakespeare .	April 23, 1564.	April 23, 1616.
Edmund Burke . .	Jan. 12, 1730.	July 9, 1797.
Robert E. Lee . . .	Jan. 19, 1807.	Oct. 12, 1870.
U. S. Grant	April 27, 1822.	July 23, 1885.
Oliver Goldsmith . .	Nov. 10, 1728.	April 4, 1774.
Benjamin Franklin .	Jan. 17, 1706.	April 17, 1790.
Alexander Hamilton .	Jan. 11, 1757.	July 12, 1804.
H. W. Longfellow .	Feb. 27, 1807.	March 24, 1882.
J. H. Newman . . .	Feb. 21, 1801.	Aug. 11, 1890.
W. E. Gladstone . .	Dec. 9, 1809.	May 19, 1898.

MULTIPLICATION

Multiply 5 yd. 2 ft. 10 in. by 7.

YD.	FT.	IN.
5	2	10
		7
41	1	10

Multiply 10 in. by 7 and get 70 in. = 5 ft. 10 in. Write 10 in. and carry 5 ft. 7 times 2 ft. are 14 ft. 14 ft. and 5 ft. = 19 ft. = 6 yd. 1 ft. Write 1 ft. Carry 6 yd. 7 times 5 yd. are 35 yd. 35 yd. and 6 yd. = 41 yd.

EXERCISE 59

Multiply:

1. 4 yd. 2 ft. 3 in. by 9.

2. 6 yd. 1 ft. 9 in. by 11.

3. 9 yd. 2 ft. 11 in. by 8.

4. 3 bu. 2 pk. 7 qt. by 7.

5. 4 bu. 1 pk. 6 qt. by 12.

6. 9 gal. 3 qt. 1 pt. by 6.

7. 6 gal. 2 qt. 1 pt. by 12.

8. 3 bu. 3 pk. 7 qt. by 7.

9. 12 T. 400 lb. by 12.
10. 13 T. 387 lb. by 9.
11. 17 T. 254 lb. by 10.
12. 5° 29′ 28″ by 16.
13. 16° 38′ 32″ by 15.
14. 64 A. 150 sq. rd. by 12.
15. 15 A. 27 sq. rd. by 11.
16. 18° 9′ 54″ by 14.
17. Multiply $\frac{5}{8}$ T. by 9 and express the result in pounds.
18. Multiply $\frac{3}{4}$ mile by 19 and give the result in feet.

DIVISION

Divide 97° 10′ 50″ by 8.

$$\begin{array}{r|lll} 8 & 97^\circ & 10' & 50'' \\ \hline & 12^\circ & 8' & 51\tfrac{1}{4}'' \end{array}$$

The eighth part of 97° is 12°, with a remainder of 1°. 1° 10′ = 70′. The eighth part of 70′ is 8′, with a remainder of 6′. 6′ 50″ = 410″. 8 into 410 goes $51\frac{1}{4}$ times.

EXERCISE 60

Divide:

1. 21 yd. 2 ft. 3 in. by 9.
2. 93° 15′ 15″ by 7.
3. 84° 14′ 14″ by 12.
4. 34 yd. 2 ft. 8 in by 6.
5. 77 yd. 2 ft. 4 in. by 7.
6. 13 bu. 3 pk. 4 qt. by 6.

7. How often is 231 cu. in. contained in 1 cu. ft. 582 cu. in.?

8. How often is 7 yd. 1 ft. contained in 1 mi.?

9. A meter is 39.37 inches. How many meters equal 1 mile?

10. The planet Mercury revolves around the sun in 88 days. Find in degrees, minutes, and seconds its daily progress.

11. Civil engineers use a chain 100 feet long. How many of these chains make 5 miles?

Example 1. Reduce .875 yd. to feet and inches.

$$\begin{array}{r} .875 \text{ yd.} \\ 3 \\ \hline 2.625 \text{ ft.} \\ 12 \\ \hline 7.500 \text{ in.} \end{array} \qquad \begin{aligned} .625 \text{ ft.} &= .625 \times 12 \text{ in.} \\ &= 7.5 \text{ in.} \end{aligned}$$

.875 yd. = 2 ft. 7.5 in.

Example 2. Reduce $\frac{7}{12}$ bu. to lower denominations.

$$\frac{7}{12} \text{ bu.} = \frac{7}{12} \text{ of } 4 \text{ pk.} = \frac{7}{3} \text{ pk.} = 2\tfrac{1}{3} \text{ pk.}$$
$$\frac{1}{3} \text{ pk.} = \frac{1}{3} \text{ of } 8 \text{ qt.} = \frac{8}{3} \text{ qt.} = 2\tfrac{2}{3} \text{ qt.}$$
$$\frac{2}{3} \text{ qt.} = \frac{2}{3} \text{ of } 2 \text{ pt.} = \frac{4}{3} \text{ pt.} = 1\tfrac{1}{3} \text{ pt.}$$

Hence, $\frac{7}{12}$ bu. = 2 pk. 2 qt. $1\frac{1}{3}$ pt.

Example 3. Express $\frac{7}{12}$ A. in square yards.

$$\frac{7}{12} \text{ of } 1 \text{ A.} = \frac{7}{12} \text{ of } 4840 \text{ sq. yd.} = \frac{7 \times 4840}{12} \text{ sq. yd.}$$
$$= \frac{7 \times 1210}{3} \text{ sq. yd.} = 2823\tfrac{1}{3} \text{ sq. yd.}$$

EXERCISE 61

Reduce:

1. $\frac{7}{8}$ T. to pounds.
2. .15° to minutes.
3. $\frac{7}{9}$ da. to hours and minutes.
4. .2345 T. to pounds.
5. .95 da. to hours and minutes.
6. .375 bu. to quarts.
7. 18.4 mi. to feet.
8. $\frac{7}{11}$ mi. to yards.
9. .1875 mi. to rods.
10. $15\frac{3}{4}$° to minutes.
11. $\frac{2}{3}$ of a common year to days and hours.
12. .3125 common years to days, hours, and minutes.
13. .45 bu. to a compound quantity.
14. $\frac{2}{3}$ gal. to a compound quantity.
15. .85 A. to square rods.
16. $\frac{39}{44}$ A. to square yards.
17. $\frac{13}{16}$ bu. to quarts.
18. $\frac{7}{16}$ gal. to pints.

EXPRESSION OF ONE QUANTITY AS A FRACTION OF ANOTHER QUANTITY

Example 1. Express 27 rd. 4 yd. 2 ft. as a fraction of 1 mi.

$$\begin{array}{l} 27 \text{ rd. } 4 \text{ yd. } 2 \text{ ft.} \\ \underline{5\tfrac{1}{2}} \\ 13\tfrac{1}{2} \\ \underline{139} \\ 152\tfrac{1}{2} \text{ yd.} \\ \underline{3} \\ 459\tfrac{1}{2} \text{ ft.} \end{array}$$

$$1 \text{ mi} = 5280 \text{ ft.}$$

$$27 \text{ rd. } 4 \text{ yd. } 2 \text{ ft.} = \frac{459\frac{1}{2}}{5280} \text{ of } 1 \text{ mi.}$$

$$= \frac{919}{10560} \text{ of } 1 \text{ mi.}$$

To express one quantity as a fraction of another quantity, reduce both to the same denomination, and divide the first quantity by the second.

Example 2. Express 2 yd. 2 ft. 8 in. as a decimal of a mile.

Step 1. Express 2 yd. 2 ft. 8 in. as a fraction of 1 mi.

Step 2. Reduce this fraction to a decimal.

EXERCISE 62

Reduce:

1. 4400 ft. to the decimal of 1 mi.
2. 293 yd. 1 ft. to the decimal of 1 mi.
3. 117 yd. 1 ft. to the decimal of 1 mi.
4. 1 qt. 1 pt. to the decimal of a gal.
5. 1 pk. 6 qt. to the decimal of a bu.
6. 2 pk. 2 qt. to the decimal of a bu.
7. 3 pk. 1 qt. 1 pt. to the decimal of a bu.
8. $1\frac{1}{8}$ in. to the decimal of 1 ft.
9. 2.34 in. to the decimal of 1 ft.
10. $3° 15'$ to the fraction of a right angle.
11. 1 da. 18 hr. to the decimal of 1 wk.

MEASUREMENTS

EXERCISE 63

1. Find the number of acres in the area of a rectangle whose dimensions are 360 ft. and 121 ft.

2. Find the area of a rectangle 1331 ft. by 720 ft.

3. Find, in acres, the area of a rectangular plot of ground 201 yd. by 10 rd. 5 yd.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Fig. 1.

4. A railway company acquires the right of way through a territory 154 mi. long, and fences in a strip 80 ft. wide. How many acres does it thus inclose, and how much does it pay for the land at $25 an acre?

5. Find the area of a park 396 ft. by 396 ft. Give your answer in acres.

6. How many acres are in a rectangular farm 1.5 mi. long by $1\frac{1}{4}$ mi. wide? Find the value of the farm at $49 an acre.

A **township** is a tract of land 6 mi. square, and it is divided into 36 **sections** each 1 mi. square. The sections are numbered as in Fig. 1.

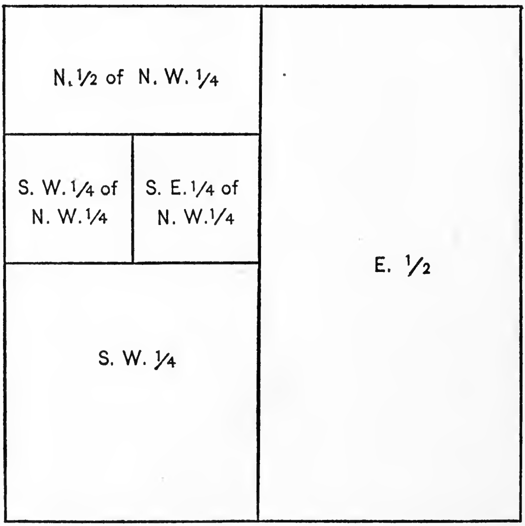


FIG. 2.

A section is subdivided as indicated in Fig. 2. There are two divisions, E. $\frac{1}{2}$ and W. $\frac{1}{2}$. The E. $\frac{1}{2}$ is divided into two equal squares called N.E. $\frac{1}{4}$ and S.E. $\frac{1}{4}$. The W. $\frac{1}{2}$ is divided into two equal squares called N.W. $\frac{1}{4}$ and S.W. $\frac{1}{4}$. These are again subdivided as shown.

EXERCISE 64

1. Draw a figure and locate S.W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$; N. W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$; N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$; S.E. $\frac{1}{4}$ of S. W. $\frac{1}{4}$.

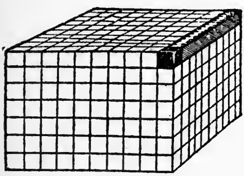
2. Locate N.E. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$; S.W. $\frac{1}{4}$ of N.E. $\frac{1}{4}$; How many acres are in N.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$? How many acres are in N.W. $\frac{1}{4}$? in S.E. $\frac{1}{4}$? in S. $\frac{1}{2}$ of S.W. $\frac{1}{4}$?

3. A man buys $\frac{1}{2}$ of the S. $\frac{1}{2}$ of N.W. $\frac{1}{4}$ and also $\frac{1}{2}$ of the N.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ at the rate of \$20 an acre. Find the cost of his purchase.

VOLUMES OF RECTANGULAR SOLIDS

The **volume of a rectangular solid** is obtained by taking the product of its three dimensions expressed in units of the same denomination.

EXERCISE 65



1. Find the volume of a box 12 ft. by 7 ft. by 6 ft.

2. Find the cubical contents of a room 18 ft. by 12 ft. and 9 ft. high.

3. Find the number of cubic feet in a room 32 ft. by 24 ft. and 12 ft. high.

4. How many cubic yards of earth must be removed for the foundation of a house 75 ft. by 54 ft., if the earth to the depth of 21 ft. is removed?

5. A cistern, in the shape of a rectangular solid, is 22 ft. by 14 ft. and 6 ft. deep. How many gallons of water does it contain?

6. A bin is 8 ft. by 3 ft. and 6 ft. high. How many bushels does it hold?

7. In order to build a concrete wall, earth is removed to the depth of 6 ft. If the wall is 210 ft. long and 12 ft. wide, how many cubic yards of earth must be removed?

8. How many cubic yards of gravel are required to fill, to the depth of 6 in., a street 1 mi. long and 36 ft. wide?

9. How many cubical boxes 2 ft. each way would a storeroom 18 ft. by 12 ft. and 10 ft. high hold?

10. The Sault Ste. Marie Canal is 1.6 mi. long, 160 ft. wide, and 25 ft. deep. Express in cubic yards the volume of water required to fill it.

11. A block of marble is 4 ft. by 3 ft. and 24 ft. long. How many tons does it weigh, if a cubic foot of marble weighs 170 lb.?

12. How many pounds does a cedar beam 14 in. by 10 in. and 40 ft. long weigh, if a cubic foot of cedar wood weighs 38.1 lb.?

13. A cubic foot of clay weighs 75 lb. Find, in tons, the weight of a clay bank 10 ft. by 4 ft. and 80 ft. long.

14. A box 9 in. by 8 in. and 6 in. deep is filled with mercury. Find its weight in pounds if a cubic foot of mercury weighs 13,570 oz.

15. How many 3-in. cubes are required to fill a cubical box each of whose edges is 1 yd.?

16. A pile of 4-ft. wood 8 ft. long and 4 ft. high contains a cord. How many cords of wood are in a pile of 4-ft. wood 120 ft. long and 12 ft. high?

17. Find the weight of the water covering an acre to the depth of 4 inches. 1 cu. ft. of water weighs 1000 oz.

A **triangle** is a portion of a plane bounded by three straight lines. *ABC* is a triangle.

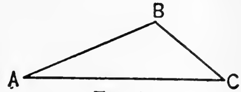


FIG. 1.

Two straight lines are **parallel** if they can never meet no matter how far they may be produced.

A **quadrilateral** is a portion of a plane bounded by four straight lines. Figure 2 represents a quadrilateral.

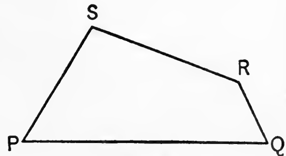


FIG. 2.

A quadrilateral having its opposite sides parallel is called a **parallelogram.** Figure 3 is a parallelogram.

The sides *AB*, *CD* are parallel. Also the sides *AD*, *BC* are parallel.

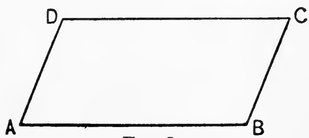


FIG. 3.

Consider next the parallelogram *ABCD* (Fig. 4) and the rectangle *ABKM.* By actual trial the triangle *BKC* is equal to the triangle *AMD.* Take the triangle *AMD* from the figure *ABCM*, the parallelogram remains. If the triangle *BKC* is taken from the figure *ABCM*, the rectangle remains. Hence, the rectangle equals the parallelogram in area. But the area of the rectangle is obtained by taking the product of its two dimensions, *i.e.* *AB* and *BK.* Therefore, the area of the parallelogram is equal to $AB \times BK$. *AB* is called the *base* of the parallelogram; *BK*, *i.e.* the distance between the parallel sides, is called the *altitude*, or *height*, of the parallelogram.

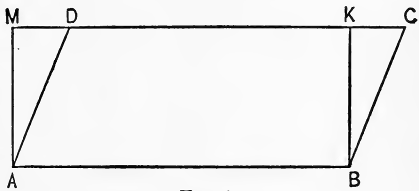


FIG. 4.

Consequently, the *area of a parallelogram* equals the product of its base by its altitude.

A quadrilateral having two sides parallel is called a **trapezoid.** $ABCD$ (Fig. 5) is a trapezoid, having AB parallel to DC. AB and CD are respectively the lower and upper bases of the trapezoid.

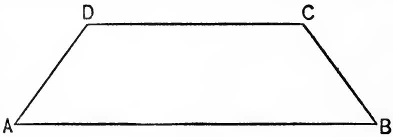


Fig. 5.

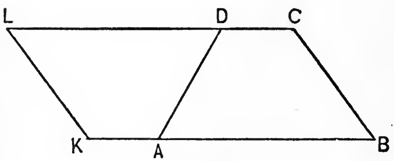


Fig. 6.

Take a piece of paper and make a trapezoid. Make another trapezoid just equal to it.

Place them as shown in Fig. 6. You then have a parallelogram $KBCL$. Its area is equal to $(KA + AB) \times$ height of the parallelogram, *i.e.* equal to $(DC + AB) \times$ height of the parallelogram.

The trapezoid is $\frac{1}{2}$ of $KBCL$.

$\therefore$ the area of the trapezoid $ABCD$ equals $\frac{1}{2}$ of $(DC + AB) \times$ the height of the trapezoid.

Consequently, the *area of a trapezoid* equals one half the sum of its parallel sides multiplied by the distance between them.

If one of the bases, *e.g.* DC, of a trapezoid were to become smaller and smaller, the figure would ultimately be a triangle. Hence, the *area of a triangle* equals one half of its base multiplied by its height.

This may also be seen readily from Fig. 7.

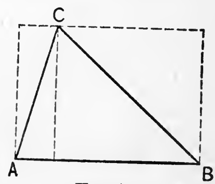


Fig. 7.

EXERCISE 66

Find the areas of the following parallelograms:

1. Base 10′ 6″, altitude 6′ 4″.*
2. Base 17′ 3″, altitude 9′ 8″.
3. Base 12′ 6″, altitude 8′ 3″.
4. Base 15′ 5″, altitude 9′ 4″.
5. Base 36′ 9″, altitude 8′ 4″.
6. Base 40′ 3″, altitude 7′ 6″.
7. Base 27′ 9″, altitude 8′ 7″.
8. Base 28′ 4″, altitude 6′ 3″.

Find the areas of the following triangles:

9. Base 50′ 3″, altitude 23′ 9″.
10. Base 60′ 4″, altitude 42′ 8″.
11. Base 75′ 6″, altitude 35′ 8″.
12. Base 48′ 4″, altitude 29′ 4″.
13. Base 56′ 9″, altitude 27′ 4″.
14. Base 82′ 6″, altitude 64′ 2″.

15. Find the area of the upper surface of a board in the form of a trapezoid whose parallel sides are 12′ 6″ and 5′ 6″ and height 8′ 6″.

16. Find the area of a trapezoid whose parallel sides are 18′ 4″ by 12′ 8″ and 24′ apart.

17. Find the area of a field whose parallel sides are 20 rd. and 36 rd. and width 18 rd.

18. Find the area of a lot whose shape is a trapezoid having for parallel sides 60 ft. and 40 ft. and length 84 ft.

* 6 ft. 4 in., a notation used by engineers, architects, and mechanics.

19. Find the area of a right triangle whose base is 26 ft. and altitude 19 ft. (A right triangle is a triangle having a right angle.)

20. Find the area of a right triangle whose base is 96 ft. 6 in. and altitude 84 ft.

21. Find the area of a right triangle having for base 72 ft. 6 in. and altitude 63 ft. 9 in.

BOARD MEASURE

Lumber is measured by the board foot. A **board foot** is a rectangular solid 12 in. by 12 in. and 1 in. in height.

If a board is 1 in. thick, the number of board feet in it is measured by the number of square feet in its upper or lower surface. Thus, a board 8 ft. long, 8 in. wide, and 1 in. thick contains $8 \times \frac{2}{3}$ board feet. If a board is more than 1 in. thick, the number of board feet in it is measured by the area, in square feet, of its upper or lower surface multiplied by the number denoting the thickness in inches. Thus, a board 9 ft. by 14 in. and $2\frac{1}{2}$ in. thick contains $9 \times \frac{14}{12} \times 2\frac{1}{2}$ board feet.

The number of board feet in a board less than 1 in. in thickness is measured by the number of square feet in its upper or lower surface. Thus, a board 10 ft. by 15 in. and $\frac{2}{3}$ in. thick contains $10 \times \frac{15}{12}$ board feet.

EXERCISE 67

1. How many board feet are in a board 8 ft. by $1\frac{1}{2}$ ft. and 1 in. thick?

2. How many board feet are in a board 9 ft. by 16 in. and $\frac{3}{4}$ in. thick? Find the cost of the board at $3\frac{1}{2}$¢ per board foot.

3. How many board feet are in a plank 24 ft. by 15 in. and 2 in. thick? In a board 20 ft. by 12 in. and 3 in. thick?

4. How many board feet are in a railroad tie 24 ft. by 8 in. and 6 in. thick?

5. Find the number of board feet in each of the following pieces of lumber:

(*a*) 18 ft. by 16 in. and $1\frac{1}{2}$ in. thick.
(*b*) 12 ft. by 8 in. and 6 in. thick.
(*c*) 24 ft. by 9 in. and 3 in. thick.
(*d*) 16 ft. by 8 in. and 4 in thick.
(*e*) 18 ft. by 12 in. and 3 in. thick.
(*f*) 21 ft. by 16 in. and 3 in. thick.
(*g*) 30 ft. by 14 in. and $2\frac{1}{2}$ in. thick.
(*h*) 28 ft. by 15 in. and $3\frac{1}{2}$ in. thick.

6. Find the cost of 480 boards, each $1\frac{1}{2}$ in. by 10 in. and 16 ft. long, @ \$27.50 per M. ("Per M" means "by the 1000" board feet.)

7. Find the cost of 840 boards, each $1\frac{3}{4}$ in. by 10 in. and 12 ft. long @ \$25 per M.

8. How many board feet are in a board 24 ft. long, 12 in. wide at one end and 16 in. wide at the other end, and $2\frac{3}{4}$ in. thick?

9. How many board feet are in a board 18 in. wide at one end, 12 in. wide at the other end, $2\frac{1}{2}$ in. thick, and 28 ft. long?

10. How many board feet are in a cubical block of wood, each of whose dimensions is 2 ft.?

11. How many board feet are in a beam 9 in. by 9 in. and 54 ft. long?

12. How many board feet are in 16 railroad ties $8''$ by $6''$ and $8'\ 6''$ long?

13. How many cubic feet are equivalent to 120 board feet?

14. How many board feet are in a beam 11 in. by 12 in. and 36 ft. long?

15. A piece of lumber contains 2980 cu. in. Express this in board feet, understanding that the piece is more than 1 in. in thickness.

16. Find the number of board feet in a beam 12 in. by 6 in. at one end, 9 in. by 6 in. at the other end, and 27 ft. long.

MASONRY AND BRICKLAYING

Stone work is estimated by the perch, also by the cubic foot.

A perch of masonry is $24\frac{3}{4}$ cu. ft. It is a wall 1 rd. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high. Perch is, in Great Britain, another name for rod. Of the $24\frac{3}{4}$ cu. ft. in a perch, 22 cu. ft. are allowed for stone and $2\frac{3}{4}$ cu. ft. are allowed for mortar, *i.e.* eight ninths for stone and one ninth for mortar.

In estimating the number of perches of masonry in the walls of a building, the outside dimensions of the walls are taken. This method of reckoning counts the corners twice. In estimating the amount of material, the computation should be exact, inside and outside dimensions being reckoned.

Twenty-two common bricks, *i.e.* bricks 8 in. by 4 in. by 2 in., are reckoned as a cubic foot.

EXERCISE 68

1. How many perches of masonry are in a wall 84 ft. long, 16 ft. high, and $1\frac{1}{2}$ ft. thick? How many common bricks are required to build this wall?

2. A cellar is 36 ft. long, 18 ft. wide, and 8 ft. deep. How many cubic feet of masonry are in its walls if they are 2 ft. thick? What is the actual number of cubic feet in the walls?

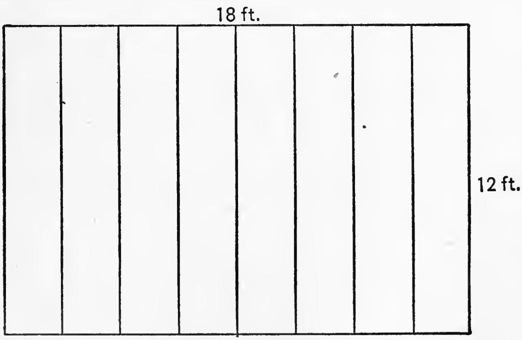
3. How many bricks are necessary to build a wall 25 ft. long, 12 ft. high, and 24 in. in thickness? What is the cost of the bricks at $9 per M?

4. How many cubic feet of mortar are required to build a wall 600 ft. long, 10 ft. high, and 18 in. in thickness? How many common bricks are required to build this wall?

5. At $9.50 per M, what is the cost of the bricks required to build a house 40 ft. by 36 ft. and 40 ft. high to the eaves, the highest point of the gable end being 52 ft. above the ground, and the walls $1\frac{1}{2}$ ft. thick? Allow 270 cu. ft. for openings.

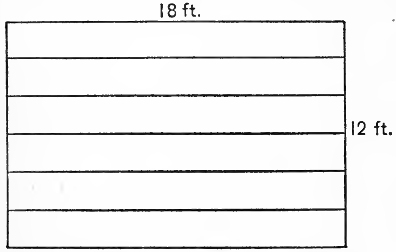
CARPETING

1. How many yards of carpet are needed for a room 18 ft. by 12 ft., if the carpet is 27 in. wide?



SOLUTION. (1) Let the carpet be placed crosswise. Number of strips $= 18$ ft. $\div 27$ in. $= 18$ ft. $\div 2\frac{1}{4}$ ft. $= 8$.
$\therefore$ Number of yards $= 12 \times 8 \div 3 = 32$. *Ans.* 32 yd.

(2) Let the carpet be placed lengthwise.



Number of strips = 12 ft. ÷ $2\frac{1}{4}$ ft. = $5\frac{1}{3}$.

Here 6 strips are needed. The fraction of a strip may be either cut off or turned under.

∴ Number of yards = 6 yd. × 6 = 36 yd. *Ans.* 36 yd.

To find the number of yards of carpet required to carpet a room:

1. Draw a diagram of the room.
2. Find the number of strips.
3. Multiply the number of strips by the number of yards in one strip.

EXERCISE 69

1. How many yards of carpet are required to cover a room 18 ft. by 16 ft. with carpet 30 in. wide, if the strips are laid crosswise?

2. How many yards of carpet $\frac{3}{4}$ of a yard wide are needed to carpet a room 24 ft. by 18 ft., if the carpet runs lengthwise?

3. How many yards of matting 36 in. wide are required to cover a room 16 ft. by 12 ft., the matting being laid crosswise?

4. How many yards of carpet 30 in. wide are needed to carpet a room 24 ft. long, 20 ft. wide, if the carpet is laid crosswise?

5. Find the cost of carpeting a room 16 ft. by 14 ft. with carpet 27 in. wide at 90¢ per yard, the strips being laid lengthwise.

MISCELLANEOUS EXERCISES

EXERCISE 70

1. How long will it take a person to earn \$44.40, if he earns \$1.85 a day?

2. If $\frac{5}{8}$ of a quantity of coal is $71\frac{1}{4}$ tons, how many tons of coal are there in all?

3. A train runs at the rate of $33\frac{1}{3}$ mi. per hour. How many hours will it take the train to run 350 mi.?

4. A man being asked his age replied, "Three elevenths of my age is $13\frac{1}{2}$ years." Find the man's age.

5. A's share is $\frac{3}{5}$ of the joint capital, or \$2540. What is the joint capital?

6. A dealer sells a piano for \$240, thereby gaining $\frac{1}{5}$ of the cost of the piano. What did the piano cost?

7. If a dealer sold a piano for \$240 and lost thereby $\frac{1}{5}$ of the cost price, what did the piano cost?

8. If the dividend is $18\frac{3}{4}$ and the quotient is $10\frac{1}{2}$, what is the divisor?

9. The sum of two numbers is 25, and one of the numbers is $1\frac{1}{2}$ times the other number. Find the numbers.

10. A farmer sells $\frac{2}{9}$ of his cattle to one jobber and $\frac{4}{9}$ to another jobber. If he keeps the remainder, what part does he keep? Suppose the number remaining is 27, how many head of cattle had the farmer originally?

11. A dealer sells $\frac{1}{2}$ of his coal, and then $\frac{1}{3}$ of it, and has $12\frac{1}{2}$ tons left. How many tons had he originally?

12. If the $\frac{1}{5}$ part of a number and the $\frac{1}{6}$ part of the same number together make $60\frac{1}{2}$, what is the number?

13. If .85 of A's money is \$289, how much has he?

14. By gaining .15 of his outlay a man's property amounts to $7245. What was his outlay?

15. Five sevenths of a farm is sold for $1385. What is the value of the farm?

16. How many times will a wheel 12 ft. $4\frac{1}{2}$ in. in circumference revolve in going $3\frac{3}{4}$ mi.?

17. How long will it take a person to go $10\frac{2}{5}$ mi. at the rate of $6\frac{1}{2}$ mi. per hour?

18. A train travels at the rate of $52\frac{1}{2}$ mi. in $1\frac{3}{4}$ hr. Find its rate per hour. How many hours and minutes will it take this train to travel 208 mi.?

19. From the ground to the first floor of a house is $\frac{4}{15}$ of the height of the house; if the first floor is 10 ft. above the ground, how high is the house?

20. What part of $4\frac{5}{6}$ is $\frac{1}{9}$ of $14\frac{1}{2}$?

21. If .625 of a gallon of maple sirup cost $1.25, what will 1 gal. of maple sirup cost?

22. When 3 oranges sell for 5¢, how many oranges can I buy for 80¢?

23. If $8\frac{1}{4}$ yd. of calico cost 66¢, how many yards can I buy for $3.30?

24. A merchant, by selling tea at 69¢ per pound, gains $\frac{3}{20}$ of the cost of the tea. Find the cost per pound of the tea.

25. By selling cloth at 78¢ per yard a clothier loses $\frac{1}{7}$ of the cost of the cloth. Find the cost price per yard of the cloth.

26. The rent of a house for 17 mo. is $225. Find the rent of this house for 12 mo.

27. Lead pencils cost $3\frac{1}{3}$¢ each. At this price, how many can be bought for $1.40?

28. A laborer gets 4¢ per cubit foot for digging a cellar. At this rate how much will he get if the cellar is 15 ft. by 12 ft. and 6 ft. deep?

29. If 3 qt. of oil cost $13\frac{1}{2}$¢, at this rate how much will 1 gal. 2 qt. cost?

30. If 1 lb. 8 oz. of cheese cost 42¢, find the cost of 4 lb.

31. How many tiles 6 in. square will be needed to pave a hearth 5 ft. by 2 ft.?

32. Express $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ as a decimal.

33. Reduce 5.875 hr. to seconds.

34. Find the cost of a car load of coal weighing 35,880 lb. at $5.50 per ton.

35. A man having a salary of $1500 a year spends 25% of it for board, 10% for clothing, and 15% for other things. How much money does he spend?

36. Reduce 2876 in. to yards.

37. Find the value of 32 tubs of butter each weighing 56 lb. at $37\frac{1}{2}$¢ per pound.

38. The wages of a motorman on the Metropolitan West Side Elevated Railway, Chicago, are $30\frac{1}{2}$¢ per hour. How much does he earn in ten weeks, working 8 hr. a day?

39. How many feet are in $87\frac{1}{2}$% of 1 mile?

40. How many square yards are in $62\frac{1}{2}$% of 1 A.?

41. A farmer having 320 bushels of apples sells 75% of them. How many bushels does he sell?

42. How much will a man earn in 10 weeks at $1.50 per day, Sundays excepted?

43. A cubit foot of air weighs .08073 lb. Find the weight of the air in a room 15 by 16 ft. and 9 ft. high?

44. How many grains are in $83\frac{1}{3}$% of 1 lb. Troy?

45. A railroad train runs 1 mi. in 1 min. 15 sec. Find its rate per hour.

46. Find the number of acres in a field 90 rd. by 40 rd.

47. Find the cost of 18,750 bd. ft. of lumber at $30 per M.

48. Find the commission on sales amounting to $4750 at 2%.

49. A man borrows money on April 1, and agrees to pay it in 90 days. On what date should he pay it?

50. How many square inches are in the surface of an 18-in. cube?

51. Find in square rods 15% of 1 A.

52. Out of a class of 64 pupils $12\frac{1}{2}$% failed to be promoted. How many were promoted?

53. A lot is 42 ft. by 120 ft. Find the cost of fencing it at 85¢ per yard.

54. Find the cost of making a concrete sidewalk 63 by 12 ft. at $1.75 per square yard.

55. Express in minutes .075 of 1 day, 16 hours.

56. A railroad train runs 72 ft. in one second. At this rate how far will it run in one hour?

57. Find the cost of plastering the walls of a room 18 ft. by 20 ft. and 10 ft. high at 25¢ per sq. yd.

58. A watch loses 1 min. and 5 sec. in 3 da. At this rate how much will it lose during the month of April?

59. How many pounds avoirdupois are in 420 lb. troy?

60. A tank is 8 ft. by 6 ft. and 7 ft. deep. How many gallons does it hold? (1 cu. ft. = $7\frac{1}{2}$ gal. nearly.)

61. Find the cost of 24 yd. of cloth at $$1.87\frac{1}{2}$ per yd.

62. How many cords of wood are in a pile 50 ft. long, 12 ft. wide, and 8 ft. high?

63. When coal sells for \$7.00 per ton, what is the cost of a sack of coal weighing 200 lb.?

64. A bin is 18 ft. long, 6 ft. wide, and 4 ft. deep. How many bushels will it contain? (1 bu. = $1\frac{1}{4}$ cu. ft.)

65. Change to decimals $\frac{17}{25}$, $\frac{15}{22}$, $\frac{3}{250}$, $\frac{6}{7}$.

66. Change to fractions in lowest terms .875, .00525, $.66\frac{2}{3}$.

67. Express in simplest form

$$7 \times 3.04 \times 10{,}000 - 125{,}000.7.$$

68. A man rents a house for \$550 a year. His rent the previous year was 9% less. What was his rent last year?

69. A grocer having on hand 15 gal. 2 qt. 1 pt. of oil buys 20 gal. 2 qt., and sells 30 gal. 2 qt. 1 pt. How much has he left?

70. Find $8\frac{1}{3}\%$ of 1 cu. ft. Give result in cubic inches.

71. The following is the lowest bid in detail for improving Lick Run Pike, Cincinnati, Ohio, submitted July 19, 1907:

20,000 cu. yd. embankment @ \$$.018\frac{1}{2}$
500 cu. yd. excavation @ \$1.00
1530 cu. yd. stone @ \$2.25
265 cu. yd. screening @ \$2.25
100 cu. yd. cement @ \$8.00
400 ft. 12″ pipe @ \$1.00
10 ft. 12″ slant @ \$.80
30 ft. 4″ box culvert concrete @ \$6.00
60 ft. 5″ box culvert concrete @ \$7.00
2350 ft. 7″ box culvert concrete @ \$16.00
650 ft. 8″ box culvert concrete @ \$18.00
18,400 sq. yd. rolling @ 3¢
6 manholes @ \$40.00

Find the amount of the bid.

REVIEW OF FRACTIONS

EXERCISE 71

Add:

1. $\frac{1}{6}$, $\frac{9}{60}$, $\frac{9}{40}$, $\frac{5}{16}$.
2. $2\frac{8}{9}$, $3\frac{19}{24}$, $4\frac{11}{15}$.
3. $\frac{1}{9}$, $\frac{1}{14}$, $\frac{3}{20}$, $\frac{7}{12}$.
4. $1\frac{7}{9}$, $\frac{11}{27}$, $\frac{7}{24}$, $1\frac{1}{12}$.
5. $\frac{1}{4}$, $\frac{7}{18}$, $\frac{13}{24}$, $\frac{7}{10}$.
6. $2\frac{13}{24}$, $2\frac{29}{36}$, $4\frac{5}{27}$.
7. $\frac{4}{15}$, $\frac{5}{16}$, $\frac{17}{120}$.
8. $\frac{7}{8}$, $3\frac{11}{12}$, $1\frac{11}{16}$, $6\frac{8}{9}$.
9. $\frac{3}{8}$, $\frac{3}{14}$, $\frac{1}{10}$.
10. $\frac{7}{10}$, $3\frac{2}{15}$, $\frac{17}{20}$, $5\frac{23}{45}$.
11. $1\frac{3}{4}$, $1\frac{5}{6}$, $1\frac{8}{9}$, $1\frac{11}{12}$, $1\frac{13}{27}$.
12. $2\frac{1}{12}$, $3\frac{13}{14}$, $2\frac{1}{3}$, $\frac{15}{16}$.

Find the difference between:

13. $4\frac{5}{14}$ and $1\frac{3}{4}$.
14. $1\frac{33}{100}$ and $\frac{78}{100}$.
15. $1\frac{27}{100}$ and $\frac{7}{10}$.
16. $\frac{1}{11}$ and $\frac{1}{12}$.
17. $\frac{1}{9}$ and $\frac{1}{12}$.
18. $\frac{1}{8}$ and $\frac{1}{18}$.
19. $\frac{9}{10}$ and $\frac{8}{9}$.
20. $\frac{5}{6}$ and $\frac{4}{5}$.
21. $\frac{11}{12}$ and $\frac{10}{11}$.
22. $\frac{17}{24}$ and $\frac{13}{20}$.
23. $\frac{17}{18}$ and $\frac{14}{15}$.
24. $7\frac{5}{7}$ and $3\frac{11}{14}$.
25. $8\frac{5}{9}$ and $7\frac{11}{12}$.
26. $9\frac{5}{12}$ and $6\frac{13}{18}$.
27. $5\frac{7}{16}$ and $2\frac{17}{24}$.
28. $6\frac{7}{18}$ and $5\frac{11}{24}$.
29. $8\frac{17}{20}$ and $6\frac{29}{30}$.
30. $12\frac{5}{22}$ and $8\frac{13}{33}$.

31. What number must be added to $3\frac{3}{4}$ to give $5\frac{2}{3}$?

32. A man's capital amounts to \$1727$\frac{1}{2}$. By how much must he increase his capital so that it may amount to \$3000?

33. What number must be taken from $30\frac{7}{8}$ to leave $14\frac{2}{3}$?

34. By how much does $84\frac{1}{6}$ exceed $17\frac{3}{4}$?

35. Of the weight of the earth's atmosphere $\frac{231}{1000}$ is oxygen. What fraction of the weight of the earth's atmosphere do its other constituents aggregate?

EXERCISE 72

Simplify:

1. $\frac{2}{3}$ of $\frac{5}{6}$ of $2\frac{4}{7}$.
2. $\frac{3}{4}$ of $\frac{9}{10}$ of $4\frac{4}{9}$.
3. $\frac{7}{8}$ of $1\frac{2}{5}$ of $\frac{20}{49}$.
4. $\frac{4}{11}$ of $\frac{2}{3}$ of $4\frac{1}{8}$.
5. $\frac{3}{8}\times\frac{3}{11}\times 4\frac{8}{9}$.
6. $\frac{9}{10}\times 1\frac{1}{2}\times 1\frac{1}{18}$.
7. $\frac{3}{13}$ of $4\frac{7}{8}$ of $1\frac{1}{7}$.
8. $\frac{11}{14}\times 5\frac{1}{3}\times 1\frac{10}{11}$.
9. $\frac{7}{16}$ of $1\frac{7}{8}$ of $1\frac{1}{15}$.
10. $2\frac{1}{7}\times 3\frac{1}{7}\times 2\frac{32}{33}$.
11. $5\frac{1}{2}\times 5\frac{1}{2}\times\frac{4}{121}$.
12. $2\frac{3}{4}\times 2\frac{3}{4}\times 1\frac{1}{15}$.
13. $7\frac{1}{8}\times\frac{16}{19}\times\frac{3}{8}$.
14. $9\frac{2}{7}\times 1\frac{8}{13}\times 1\frac{1}{6}$.
15. $1\frac{2}{3}\times 1\frac{2}{3}\times 1\frac{2}{3}\times 1\frac{2}{25}$.
16. $(\frac{3}{4}+\frac{1}{2})\times(\frac{3}{4}-\frac{1}{2})$.
17. $(\frac{5}{6}+\frac{1}{3})\times(\frac{5}{6}-\frac{1}{3})$.
18. $(\frac{9}{10}+\frac{1}{4})\times(\frac{9}{10}-\frac{1}{4})$.
19. $\frac{2}{5}(3\frac{1}{3}+1\frac{1}{4})\times\frac{3}{8}$.
20. $\frac{2}{3}$ of $4\frac{1}{2}-\frac{1}{8}$ of $3\frac{1}{4}\times 3\frac{1}{5}$.
21. $3\frac{9}{11}\times(3\frac{1}{3}+\frac{5}{6}-\frac{20}{21})$.
22. $2\frac{1}{2}\times(1\frac{3}{4}+2\frac{1}{2}+1\frac{7}{8})$.
23. $7\frac{1}{2}\times 1\frac{7}{8}\times\frac{16}{225}\times(\frac{1}{2}+\frac{1}{3}+\frac{1}{4})$.
24. $1\frac{11}{12}\times 1\frac{12}{17}\times\frac{101}{58}\times(\frac{3}{4}+\frac{2}{3}-1\frac{5}{12})$.
25. $2\frac{2}{3}\times 5\frac{1}{4}\times 1\frac{1}{2}\times(\frac{9}{10}+\frac{1}{4}-1\frac{3}{20})$.
26. $3\frac{1}{2}\times 1\frac{1}{14}+7\frac{1}{2}-(1\frac{1}{4}\times 1\frac{1}{2}\times 4\frac{1}{5})$.

EXERCISE 73

Divide:

1. $25\frac{2}{3}$ by $1\frac{5}{6}$.
2. $3\frac{1}{5}$ by $2\frac{2}{5}$.
3. $10\frac{1}{8}$ by $1\frac{35}{64}$.
4. $7\frac{5}{16}$ by $292\frac{1}{2}$.
5. $52\frac{1}{2}$ by $131\frac{1}{4}$.
6. $16\frac{1}{5}$ by $\frac{6}{7}$ of $6\frac{3}{4}$.
7. $\frac{3}{8}$ by $\frac{4}{9}$ of $1\frac{13}{32}$.
8. $\frac{1}{6}$ by $\frac{2}{7}$ of $1\frac{1}{4}$.
9. $\frac{37}{60}$ by $\frac{5}{14}$ of $1\frac{1}{6}$.
10. $2\frac{15}{16}$ by $1\frac{13}{72}$ of $1\frac{3}{5}$.
11. $1\frac{2}{3}$ by $\frac{3}{8}$ of $1\frac{3}{7}$.
12. $3\frac{3}{14}$ by $1\frac{8}{17}$ times $73\frac{4}{15}$.
13. $(\frac{9}{16}-\frac{1}{4})$ by $(\frac{3}{4}+\frac{1}{2})$.
14. $(1\frac{24}{25}-1\frac{21}{100})$ by $(1\frac{2}{5}+1\frac{1}{10})$.
15. $(3\frac{1}{16}-2\frac{1}{4})$ by $3\frac{1}{4}$.
16. $11\frac{1}{9}$ by $(1\frac{5}{6}+1\frac{1}{2})$.

17. If the dividend is $6\frac{1}{2}$ and the quotient $\frac{10}{13}$, find the divisor.

18. What must $10\frac{2}{3}$ be divided by to give as quotient $2\frac{2}{3}$?

PERCENTAGE

The result of taking a rate per cent of a quantity is **percentage.** A percentage of a number is simply a fraction of the number. The central fact in percentage is that 100 % is equivalent to 1, or 1 % is equivalent to $\frac{1}{100}$.

What per cent is $\frac{19}{20}$ equivalent to?

$\frac{19}{20}$ of $1 = \frac{19}{20}$ of $100\% = 95\%$.

Example 1. Find $2\frac{3}{4}\%$ of 1789.

$$
\begin{array}{ll}
17.89 = 1\% \text{ of } 1789. & \\
\underline{\quad 2\frac{3}{4} \quad} & \\
35.78 & 2 \text{ times } 17.89. \\
8.945 & \frac{1}{2} \text{ of } 17.89. \\
\underline{4.4725} & \frac{1}{4} \text{ of } 17.89. \\
49.1975 = 2\frac{3}{4} \text{ times } 17.89. &
\end{array}
$$

Example 2. Find 3.6 % of 2992.

$$2992 \times \frac{3.6}{100} = 2992 \times .036 = 107.712, \text{ or}$$

$$1\% \text{ of } 2992 = 29.92;\ 29.92 \times 3.6 = 107.712.$$

EXERCISE 74

Find:

1. 7 % of 184.
2. 9 % of 275.
3. 6 % of 213.
4. 8 % of 534.
5. 9 % of 3280.
6. 8 % of 3297.
7. 4 % of 615.
8. 7 % of 2630.
9. 9 % of 4280.
10. 10 % of 7850.
11. 11 % of 983.
12. 12 % of 2570.
13. $12\frac{1}{2}\%$ of 928.
14. $23\frac{1}{3}\%$ of 5220.
15. $16\frac{2}{3}\%$ of 733.
16. $37\frac{1}{2}\%$ of 828.
17. $62\frac{1}{2}\%$ of 9200.
18. 60 % of 2855.

19. $4\frac{1}{2}\%$ of 2280.

20. $3\frac{3}{4}\%$ of 3066.

21. $6\frac{1}{2}\%$ of 1820.

22. $5\frac{3}{4}\%$ of 10,656.

23. $\frac{1}{2}\%$ of 1690.

24. $\frac{1}{4}\%$ of 8124.

25. Express the following fractions as per cents:

$$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}, \frac{5}{32}, \frac{7}{16}.$$

26. Express each of the following decimals as a per cent:

.04, .08, $.12\frac{1}{2}$, .0165, .002, .006, .0024.

27. The composition of a piece of coal taken from the Texas and Pacific Coal Company's mine is given as follows: moisture, 5.46%; combustible matter, 35.66%; fixed carbon, 49.17%; ash, 9.71%. Find the amount of each constituent in 2000 lb. of coal. Check your answer.

28. The analysis of a specimen of lignite is given as follows: moisture, 29.07%; combustible matter, 28.96%; fixed carbon, 24.47%; ash, 17.50%. Find the amount of each constituent in a ton of lignite. Check.

29. Distilled water is composed of two gases, $11\frac{1}{9}\%$ by weight being hydrogen, and $88\frac{8}{9}\%$ by weight being oxygen. Find the weight of each gas that can be obtained from 10 lb. of water.

30. A bookkeeper receives a salary of $1800 per annum. If he spends $62\frac{1}{2}\%$ of his salary, and saves the remainder, how much does he spend? How much does he save?

31. An auriferous ore contains .5% of gold. How many pounds avoirdupois of gold would 2240 lb. of this ore yield?

32. A copper ore contains $5\frac{1}{2}\%$ of copper. Find the number of pounds of copper in 500 lb. of this ore.

33. A owns $16\frac{2}{3}\%$ of a boat valued at $12,300. What is the value of A's share of the boat?

34. The estimated value of the exports of the United States for 1899 was $1,275,000,000. The percentages of exports from United States ports for that year are given as follows: New York, 37.4 %; Boston, 10.43 %; Philadelphia, 5.05 %; Baltimore, 8.9 %; New Orleans, 6.4 %; Galveston, 7.17 %. Find the value of the exports from these cities.

Given a number as a per cent of some other number, to find the other number.

Example 1. If 15.5 % of a number equals 22.785, what is the number?

$$15.5\% \text{ of the number} = 22.785$$

$$\therefore \quad 1\% \text{ of the number} = \frac{22.785}{15.5}$$

$$\therefore 100\% \text{ of the number} = \frac{22.785}{15.5} \times 100 = 147.$$

$$\therefore \text{the number is } 147.$$

Example 2. If $3\frac{3}{4}$ % of a number is 2934, what is the number?

$$3\tfrac{3}{4}\% = \frac{3\frac{3}{4}}{100} = \frac{15}{400} = \frac{3}{80}.$$

$$\tfrac{3}{80} \text{ of the number} = 2934.$$

$$\therefore \tfrac{1}{80} \text{ of the number} = 978.$$

$$\therefore \tfrac{80}{80} \text{ of the number} = 78{,}240.$$

$$\therefore \text{the number is } 78{,}240.$$

EXERCISE 75

1. If 5 % of a number equals 185, what is the number?

2. If $12\frac{1}{2}$ % of a man's salary is $156, what is the man's salary?

3. If $37\frac{1}{2}$ % of a man's property is valued at $324, what is the value of his property?

4. The number 360 is equal to 5% of what number? 6% of what number? 8% of what number? 9% of what number?

5. The number 120 is equal to 3% of what number? 4% of what number? 5% of what number? 6% of what number? 8% of what number? 12% of what number?

6. \$750 is 6% of what sum of money? $6\frac{2}{3}$% of what sum of money? $7\frac{1}{2}$% of what sum of money? $12\frac{1}{2}$% of what sum of money?

7. \$108 is 30% of what sum? 25% of what sum? $33\frac{1}{3}$% of what sum? $44\frac{4}{9}$% of what sum?

8. \$450 is $6\frac{1}{4}$% of what sum? $6\frac{2}{3}$% of what sum? $8\frac{1}{3}$% of what sum? $12\frac{1}{2}$% of what sum? $16\frac{2}{3}$% of what sum?

9. \$420 is $37\frac{1}{2}$% of what sum? $62\frac{1}{2}$% of what sum? $87\frac{1}{2}$% of what sum?

10. In 1900 the commercial value of silver was 3% of the value of gold. Find the number of ounces of silver equivalent in value to 126 ounces of gold.

11. Of the population of the United States in 1900, 27,849,760 were married. This number was 36.5% of the population. Find the population in 1900.

12. The census of 1900 gives the number of married men in the United States as 14,003,798. This number was 35.9% of the number of males. Find the male population.

13. The census of the same year gives the number of married women in the United States as 13,845,963. This number was 37.2% of the entire number of females. Find the female population.

14. Of the number of illiterates above 10 years of age in the United States 15.5%, according to the census of 1900, can neither read nor write. This number is 955,840. Find the number of illiterates above 10 years of age in the United States in 1900.

15. Of the average value of the raw cotton exported from the United States in 5 years 50.4% went to England. If the export of raw cotton to England amounts in value to $107,500,000, find the average value of the raw cotton exported.

16. Of the sheep exported from the United States 86% are shipped to England. If the value of the sheep shipped to England in a certain year was $1,685,800, find the total value of the export of sheep for that year.

17. Of the pupils attending school in a certain city .6% are in the senior class of the high school. If the senior class numbers 21, find the number of pupils attending school in that city.

18. In a certain city the number of pupils promoted at the end of the scholastic year was 2765. This number was 79% of the number of pupils in school. Find the number of pupils attending school in that city.

19. The foreign-born population of a city is 10,668. This number is $3\frac{1}{2}$% of the population of the city. Find the population of the city.

20. Of the water of the Dead Sea $22.85\dot{7}$% is saline material. If a quantity of Dead Sea water is evaporated, and the saline material left behind weighs 914.28 lb., what is the weight of the water before evaporation?

To find what per cent one number is of another.

Example 1. What per cent of 12 is 5?

5 is $\frac{5}{12}$ of 12.

$\frac{5}{12} = \frac{5}{12}$ of $100\% = 41\frac{2}{3}\%$.

Example 2. The value of the property of the Fort Worth and Denver City Railroad in Texas, as ascertained by the Railroad Commission of Texas, for the year 1906 was \$5,771,600, and the income from its operation was \$1,178,040. Find the rate per cent of income from operation.

$$\frac{\$1,178,040}{\$5,771,600} \text{ of } 100\% = 20.4\%.$$

To find what per cent one number is of another, find what fraction the first number is of the second and multiply by 100%.

EXERCISE 76

1. What per cent of 15 is 12?
2. What per cent of 20 is 4? is 7? is 11? is 13?
3. What per cent of 40 is 2? is 8? is 12? is 17?
4. What per cent of 90 is 9? is 12? is 27? is $11\frac{1}{4}$?
5. What per cent of 120 is 15? is 18? is 45? is 80?
6. What per cent of 480 is 28.8? is 33.6? is 40?
7. What per cent of 1728 is 345.6? is 155.52? is 288?
8. What per cent of 231 is 21? is 77? is 34.65?
9. What per cent of 5280 is 88? is 440? is 330?
10. What per cent of a bushel is a quart?
11. What per cent of a mile is 8 rd.?
12. What per cent of an acre is a square rod?
13. What per cent of a chain is a yard?
14. What per cent of 1 sq. ch. is 1 sq. rd.?

15. What per cent of 1 gal. is 1 pt.?

16. What per cent of 1 sq. rd. is 7 sq. yd. 5 sq. ft. 9 sq. in.?

17. What per cent of a rod is 1 yd. 2 ft. 6 in.?

18. What per cent of 1 mi. is 1 knot?

19. What per cent of 1 mi. is an arc of 1′ measured on the 40th parallel of latitude? ($1' = 4670$ ft.)

20. What per cent of 1 mi. is 22 yd.? is 176 yd.?

21. What per cent of 1 sq. mi. is the S.W. $\frac{1}{4}$ of N.E. $\frac{1}{4}$ of a section of land?

22. What per cent of a common year is 73 da.? is 219 da.? is 292 da.?

23. A meter is 39.37 in. What per cent of a meter is 1 yd.? What per cent of 1 yd. is 1 meter?

24. A kilometer is 1000 meters. What per cent of 1 mi. is 1 km.?

25. What per cent of 1 lb. avoirdupois is 1 lb. troy?

26. What per cent of 1 oz. avoirdupois is 1 oz. troy?

27. What per cent of the area of each of the following states consists of irrigated land?

	Acres Irrigated	Area in Square Miles
(*a*) California	1,446,114	158,360
(*b*) Colorado	1,611,270	103,925
(*c*) Louisiana	201,685	48,720
(*d*) Montana	951,054	146,080
(*e*) Nevada	501,168	110,700
(*f*) Oregon	388,110	96,030
(*g*) Utah	629,290	84,970
(*h*) Washington	135,470	69,180
(*i*) Wyoming	605,230	97,890

28. Find the increase per cent in the population of each of the following cities for the ten years from 1890 to 1900:

	Population in 1890	Population in 1900
Mobile	31,076	38,469
Little Rock	25,874	38,307
Los Angeles	50,395	102,479
Denver	106,713	133,859
Pensacola	11,750	17,747
Savannah	43,189	54,244
Springfield	24,963	34,159
Evansville	50,756	59,007
Dubuque	30,311	36,297
Kansas City, Kan.	38,316	51,418
Lexington	21,567	26,369
Kansas City, Mo.	132,716	163,752
Minneapolis	164,738	202,718

29. The total production of butter in the United States in 1899 is estimated at 1,430,000,000 lb. Of this quantity 16,002,000 lb. was exported. Find the per cent of the total production exported.

30. For the year 1899 the total production of cheese in the United States is estimated at 300,000,000 lb. Of this quantity 27,203,200 lb. was exported. Find the per cent of cheese exported.

31. The number of farms in the United States June 1, 1900, was 5,739,657. The number of farms operated by owners was 3,713,371. The number of farms operated by tenants was 752,920. The number of farms operated by share tenants was 1,273,366. Find the per cent operated by owners, by tenants, and by share tenants respectively.

COMMERCIAL DISCOUNTS

Commercial or **trade discount** is an allowance made upon the list price of goods, or on the amount of a bill.

Discounts are reckoned upon the basis of 100; in other words, as so many per cent.

Example. If watches are listed at $35 each, and a discount of 10% is allowed, what is the cost of one of the watches?

SOLUTION. 10% of $35 = $3.50 = the discount.
$35 − $3.50 = $31.50.
∴ the cost is $31.50.

EXERCISE 77

Find the cost when the list prices and rates of discount are:

1. $60, 20% off.
2. $75, 30% off.
3. $15, 25% off.
4. $14, 20% off.
5. $24, 10% off.
6. $32, 15% off.
7. $68, $12\frac{1}{2}$% off.
8. $78, $16\frac{2}{3}$% off.
9. $92, $33\frac{1}{3}$% off.
10. $80, $37\frac{1}{2}$% off.
11. $152, $12\frac{1}{2}$% off.
12. $176, 25% off.
13. $88, 5% off.
14. $96, 5% off.
15. $125, 15% off.
16. $175, 2% off.
17. $213, 4% off.
18. $723, 6% off.
19. $3280, 8% off.
20. $2712, 7% off.
21. $5350, $8\frac{1}{3}$% off.
22. $2490, $6\frac{1}{4}$% off.
23. $3778, 9% off.
24. $5062, 12% off.
25. $885, 15% off.
26. $363, 12% off.
27. $689, 8% off.
28. $2034, 7% off.
29. $992, $7\frac{1}{2}$% off.
30. $2572, $4\frac{1}{2}$% off.

COMMERCIAL DISCOUNTS WHEN TWO OR MORE ARE ALLOWED

In some cases several discounts are allowed. If there are two or more discounts, the first is reckoned on the list or catalogue price ; the next is reckoned on the remainder after deducting the first discount ; the third is reckoned on the second remainder ; and so on.

Example 1. What is the cost, if the list price is \$750, and discounts of 20 %, 15 %, and 10 % are allowed ?

SOLUTION. First discount $= 20\%$ of $\$750 = \frac{1}{5}$ of $\$750 = \$150$. $\therefore$ the first remainder $= \$750 - \$150 = \$600$.

The second discount $= 15\%$ of $\$600 = \$90$.

$$\$600 - \$90 = \$510.$$

The third discount $= 10\%$ of $\$510 = \$51$.

$$\$510 - \$51 = \$459. \quad \text{Or}$$

$$100\% - 20\% = 80\% \; ; 100\% - 15\% = 85\% ;$$

$$100\% - 10\% = 90\%.$$

$$90\% \text{ of } 85\% \text{ of } 80\% \text{ of } \$750 = \$459.$$

Example 2. What single discount is equivalent to the three discounts in the above example ?

SOLUTION. $80\% \times 85\% \times 90\% = .8 \times .85 \times .9 = .612 = 61.2\%$.

$$100\% - 61.2\% = 38.8\% \textit{ Ans.}$$

EXERCISE 78

1. A suit of clothes is marked \$70, and is sold with discounts of 25 % and 10 % for cash. Find the selling price.

2. On a bill of \$900 two discounts of 20 % and 15 % are allowed. What is the net amount of the bill?

3. If goods are marked \$175 and sold for \$122.50, what is the discount ?

4. On a bill of $1500 discounts of 25%, 15%, and 6% are allowed. Find the net cash amount of the bill.

5. A piano is listed at $450, with discounts of 20%, $12\frac{1}{2}$%, and 10%. Find the cost price to the purchaser.

6. Find the cash value of a bill of $320 with discounts of 15%, 10%, and 5%.

7. Suppose you were offered a single discount of 45%, or two discounts of 30% and 20%, which would you take? What would be the difference in a bill of $1000?

8. A dealer buys a quantity of goods marked $550, with a discount of 20%. If he sells the goods at 6% above the marked price, what is his gain per cent?

9. If I buy goods listed at $150, with a discount of 10%, and sell them at 12% above the marked price, find my gain per cent.

10. A bookseller buys 100 books marked $1.50 each, at a discount of 20% and sells them at the marked price. What is his gain per cent?

11. A bookseller bought 100 books at $1.25 each. He make a profit of 20% after giving a discount of $\frac{1}{4}$. At what price did he mark each book, and what was his profit?

12. Find the cost price in each case, if the list price and the rates of discount are as follows:

	List Price	Discounts
(*a*)	$480	20%, 30%, 10%
(*b*)	$1000	40%, 5%, 4%
(*c*)	$775	25%, $\frac{1}{5}$
(*d*)	$880	$12\frac{1}{2}$%, 4%
(*e*)	$720	$\frac{1}{3}$, 16%
(*f*)	$960	$12\frac{1}{2}$%, 12%
(*g*)	$1200	10%, 8%, 5%
(*h*)	$1760	25%, $12\frac{1}{2}$%, $16\frac{2}{3}$%

PROFIT AND LOSS

In actual business, gains and losses are reckoned as a per cent of the **cost price.**

Example 1. How much does a person gain by buying 360 yd. of cloth at $1.30 per yd. and selling it at a profit of 15%? What is the selling price per yard?

SOLUTION. 360 × $1.30 × 15% = $70.20, gain.

$$\begin{array}{rl} \$1.30 & \\ .13 & = 10\% \text{ of } \$1.30 \\ .065 & = 5\% \text{ of } \$1.30 \\ \hline \$1.495 & = \text{selling price per yd.} \end{array}$$

Observe, selling price per yd. is 115% of cost price.

Example 2. A dealer buys apples at $1.75 per barrel, and sells them at $2.10 per barrel. Find his gain per cent.

SOLUTION. $2.10 – $1.75 = $.35.

$\frac{\$.35}{\$1.75} = \frac{1}{5}$. The gain is $\frac{1}{5}$ of the cost. $\frac{1}{5} = 20\%$.

EXERCISE 79

1. Find the selling price of articles, the cost prices and rates per cent of profit being given as follows:

	COST PRICE	RATE PER CENT OF PROFIT
(*a*)	$150	12%
(*b*)	$75	25%
(*c*)	$31	$7\frac{1}{2}$%
(*d*)	$215	$16\frac{2}{3}$%
(*e*)	$540	27%
(*f*)	$318	$8\frac{1}{3}$%
(*g*)	$512	$18\frac{3}{4}$%
(*h*)	$234	15%
(*i*)	$457	$12\frac{1}{2}$%

2. Find the rate per cent of profit or loss, if the cost prices and selling prices are given as follows:

	Cost Price	Selling Price
(*a*)	\$ .20	\$ .25
(*b*)	\$ .22	\$ .20
(*c*)	\$ .90	\$1.50
(*d*)	\$2.10	\$1.40
(*e*)	\$125	\$160

Given selling price and rate per cent of profit or loss, to find cost price.

Example **1.** A piano was sold for \$450 at a profit of $12\frac{1}{2}\%$. Find the cost price.

$$(100\% + 12\tfrac{1}{2}\%) \text{ of cost} = 112\tfrac{1}{2}\% \text{ of cost} = \tfrac{9}{8} \text{ of cost.}$$

$$\tfrac{9}{8} \text{ of cost} = \$450.$$

$$\text{Therefore, cost} = \$450 \div \tfrac{9}{8} = \$400.$$

Example **2.** A dealer sells goods for \$200.56 at a loss of 8%. Find the cost price.

$$(100\% - 8\%) \text{ of cost} = 92\% \text{ of cost.}$$

$$92\% \text{ of cost} = \$200.56.$$

$$\therefore \text{cost} = \frac{\$200.56}{92\%} = \$218.00.$$

EXERCISE 80

1. Find the cost price, the selling price and rate per cent of profit being given as follows:

	Selling Price	Rate Per Cent of Profit
(*a*)	\$63	25%
(*b*)	\$143	$8\frac{1}{3}\%$
(*c*)	\$54	8%
(*d*)	\$189	17%
(*e*)	\$205	$2\frac{1}{2}\%$
(*f*)	\$315	5%

2. Find the cost, if the selling price and the rate per cent of loss are given:

	Selling Price	Rate Per Cent of Loss
(*a*)	\$41.30	10 %
(*b*)	\$87.50	20 %
(*c*)	\$90	30 %
(*d*)	\$45.50	50 %
(*e*)	\$59.90	$33\frac{1}{3}$ %
(*f*)	\$33.60	$16\frac{2}{3}$ %
(*g*)	\$55.80	20 %
(*h*)	\$253	$12\frac{1}{2}$ %

Example **1.** A man sold a horse for \$126, thereby losing 20 %. What should have been the selling price to make a profit of 15 % ?

Solution. The selling price is 80 % of cost.

To make a profit of 15 %, the selling price should be 115 % of cost. In this problem 80 % of a number is given and 115 % of it is required. Hence,

$$115 \times \frac{\$126}{80} = \$181.125 = \text{required selling price.}$$

Example **2.** How should goods be marked so that a dealer may give a discount of 20 % off and still make a profit of 15 % ?

Solution. (100 % − 20 %) of marked price = 80 % of marked price.

$$\text{Cost} + 15\,\%\text{ of cost} = 115\,\%\text{ of cost.}$$

$$\therefore\ 80\,\%\text{ of marked price} = 115\,\%\text{ of cost.}$$

$$\therefore\ 1\,\%\text{ of marked price} = \frac{115\,\%}{80}\text{ of cost.}$$

$$\therefore\ 100\,\%\text{ of marked price} = \frac{115\,\%}{80} \times 100 = 143\tfrac{3}{4}\,\%\text{ of cost.}$$

The goods must be marked $43\frac{3}{4}$ % above cost.

EXERCISE 81

1. A sold a lot for \$3835 at a gain of 18%. Find the cost.

2. By selling a piano for \$270, a dealer loses 10%. Find the cost of the piano.

3. By gaining 25% of his capital, a merchant increases his capital to \$4550. What was his original capital?

4. If goods are bought for \$20 and sold for \$22.50, what is the gain per cent?

5. If the cost is \$48 and the selling price is \$52, what is the gain per cent?

6. If the cost is \$190 and the selling price is \$152, what is the loss per cent?

7. If the cost is \$145 and the selling price is \$159.50, what is the gain per cent?

8. If the cost is \$118 and the selling price is \$128.62, what is the gain per cent?

9. If the selling price is \$106.70 and the gain is 10%, what is the cost?

10. If the selling price is \$75.25 and the loss is $12\frac{1}{2}$% what is the cost?

11. If the selling price of a rug is \$90 and the gain is 20%, what is the cost of the rug?

12. If the selling price is \$89.25 and the loss is 15%, what is the cost?

13. If the selling price is \$95 and the gain is $18\frac{3}{4}$%, what is the cost? What is the profit?

14. By selling a horse for \$168, a man gains 40% of the cost. What is the cost of the horse?

15. By selling velvet at \$4.55 a yard, a clothier makes a profit of $8\frac{1}{3}$%. Find the cost per yard of the velvet.

16. At an auction a dealer buys goods at 20 % below the market price. If he sells these goods at the market price, what is his gain per cent?

17. A lawyer collects a debt. He charges 3 % for collection, and remits to his client, after deducting his fee, \$952.54. Find the amount of the debt.

18. If property was sold for \$11,778, at a loss of $2\frac{1}{2}$ %, what was the value of the property?

19. Two horses were sold for \$200 each, one at a gain of 20 % and the other at a loss of 20 %. Did the seller gain or lose by the transaction, and how much?

20. A sells goods to B at a profit of 10 %, and B sells them to C at a profit of 15 %. If C paid \$253 for the goods, find A's cost price.

21. A merchant increases his capital $18\frac{3}{4}$ %, and at the end of a second year he also increases his capital $18\frac{3}{4}$ %. If his capital is then \$14,440, what was it at first?

22. The first year A adds 25 % to his capital, the next year he adds 25 % to the capital of the previous year, the third year he loses 40 % of his capital, and is then worth \$10,800. How much capital did he begin with?

23. If it takes \$81,415 for the running expenses of the schools of a city, and if 95 % of the tax levied for school purposes goes to the support of the schools, what should be the amount levied for school purposes?

24. By selling tea at 65¢ per pound a merchant gained \$118.75. If his gain was $62\frac{1}{2}$ %, how many pounds of tea did he sell?

25. By selling turkeys at \$2.50 each a dealer makes a profit of 60 %. What would have been his gain per cent had he sold the turkeys for \$1.75 each?

26. By selling wine at \$2.10 a gallon, a merchant makes a profit of 20 %. What would be his gain per cent should he sell the wine for \$2 a gallon?

27. A merchant mixes tea which cost him 60¢ per pound with tea which cost him 70¢ per pound in the proportion of 5 lb. of the former tea to 6 lb. of the latter. If he sells the mixture at 80¢ per pound, find his gain per cent.

28. A horse is sold for \$145.50, at a loss of 3 %. At what price should the horse have been sold so as to make a profit of 10 %?

29. If oranges are bought at 5 for 3¢ and sold at 3 for 5¢, what is the gain per cent?

30. If 325 lb. of sugar are bought for \$13, at what price per pound must it be sold to make a profit of 25 %?

31. If 15 yd. of silk are bought for \$26.25, at what price per pard should it be sold to make a profit of 20 %?

32. By selling cloth at the rate of $17\frac{1}{2}$¢ a yard a clothier loses $12\frac{1}{2}$ %. What should the selling price per yard be so as to make a profit of 20 %?

33. A merchant buys butter at 18¢ per pound and sells 120 lb. for \$25.20. What is his gain per cent?

34. A grocer buys cheese at 9¢ per pound and sells it at the rate of 8 lb. for \$1. What is his gain per cent?

35. A merchant mixes two kinds of tea in the ratio 5 : 2. If the teas are worth respectively 68¢ and 75¢ per pound, what should be the selling price per pound so as to make a profit of 20 %?

36. If I buy a horse for \$80 and sell him for \$71, what is the loss per cent?

37. By selling a horse for \$83.30 I lost 15 %. What did the horse cost?

38. Two men, A and B, buy two horses, each paying the same price. A sells his horse for \$90 at a loss of $14\frac{2}{7}\%$. B sells his horse at a loss of 6%. Find the selling price of B's horse.

39. A merchant buys 70 yd. of cloth at 50¢ a yd. and sells it at a gain of 15%. He buys also 70 yd. of silk at 90¢ a yard. On both he gains 10%. At what price per yard does he sell the silk?

40. I buy a quantity of barley and intrust it to an agent to sell. The agent sells it at an advance of 25% on the cost, and after deducting a commission of 2% he remits the balance, \$539. How much did the barley cost me?

41. A fruit dealer buys a crate of oranges for \$2.50. He sells them at 2¢ each, making a profit of 60%. How many oranges are in the crate?

42. If goods are bought at 25% below the retail price and sold at the retail price, what is the gain per cent?

43. A coal dealer buys 120 T. of coal at \$4 a ton. He sells $\frac{1}{2}$ of the coal at an advance of 25%, $\frac{1}{3}$ of it at an advance of 50%, and the remainder at an advance of 10%. Find his entire profit and his gain per cent on the coal.

44. (*a*) How should goods be marked so as to make a profit of 12% after deducting 20% from the marked price?

(*b*) How should they be marked so as to make a profit of 17% after deducting 10% from the marked price?

(*c*) How should they be marked so as to make a profit of 20% after deducting 25% from the marked price?

(*d*) How should they be marked so as to make a profit of 10% after deducting 10% from the marked price?

(*e*) By taking 30% off the marked price a merchant neither gains nor loses. How were the goods marked?

COMMISSION AND BROKERAGE

A very large proportion of the buying and selling of the produce of the country is done through commission merchants and brokers. A commission merchant is usually intrusted with the goods bought or sold. A broker merely buys and sells.

The fee which a commission merchant charges for his services, or which an agent who buys and sells land, or collects rents, charges, is called a **commission.** The fee which a broker charges is called **brokerage.**

Commission and brokerage are usually reckoned as a rate per cent of the buying price when goods or other commodities are bought, or of the selling price when they are sold.

The person who employs another person to buy, to sell, or to collect money is called a **principal.** The person who transacts business for a principal is called an **agent.** Commission merchants and brokers act in the capacity of agents.

Example 1. A commission merchant sold 400 boxes of oranges at $2.75 per box, charging 5% commission. What was his commission, and how much did he remit?

The price of 400 boxes of oranges @ $\$2.75 = \$1100$.
5% of $\$1100 = \$55$, the commission.
$\$1100 - \$55 = \$1045$, amount remitted.

Example 2. A commission merchant charges $94.50 for selling 4500 bushels of wheat. Rate of commission $2\frac{1}{2}\%$. What is the price of wheat per bushel?

$2\frac{1}{2}\% = \frac{1}{40}$. Selling price $= \$94.50 \times 40 = \$3780$.
Selling price of one bushel $= \$\frac{3780}{4500} = \$.84$.

Example 3. A real estate agent remits to his principal \$9788.75, being the amount of the sales of four city lots after deducting a commission of $4\frac{1}{2}\%$. Find the selling price of the four lots and the agent's commission.

SOLUTION. The selling price $-4\frac{1}{2}\%$ of the selling price $= 95\frac{1}{2}\%$ of the selling price.

$95\frac{1}{2}\%$ of the selling price $= \$9788.75$.

$\therefore 1\%$ of the selling price $= \dfrac{\$9788.75}{95\frac{1}{2}}$.

$\therefore 100\%$ of the selling price $= \dfrac{\$9788.75}{95\frac{1}{2}} \times 100 = \$10{,}250$.

$\therefore \$10{,}250 - \$9788.75 = \$461.25$, the commission.

EXERCISE 82

1. What is the commission for selling 200 A. of land at \$40 an acre, the rate of commission being 4 % ?

2. An agent charges 6 % for selling real eatate. If he sells 350 A. of land at \$50 an acre, find his commission and the sum he remits to his principal. What per cent of the selling price does the principal receive?

3. If 200 boxes of oranges are sold at \$3.75 a box by a commission merchant who charges 6 %, what is the commission? How much is remitted to the principal?

4. If 800 bu. of wheat are sold on a commission of $\frac{1}{2}$¢ per bushel, what is the commission?

5. How many bushels of barley at 45¢ per bushel must a commission merchant sell at $2\frac{1}{2}\%$ commission to make an annual salary of \$1080?

6. If 480 bbl. of flour at \$3.90 a barrel are sold on commission at a rate of 3 %, find the commission. What is the net amount realized from the sale?

7. A broker sells 2500 lb. of beef at 12¢ a pound. What is his brokerage at $1\frac{1}{2}\%$?

8. Thirty-six hundred gallons of oil are sold by a broker at 45¢ per gallon. If he charges $3\frac{1}{2}\%$ brokerage, find his brokerage and the amount remitted.

9. A commission merchant sells 1500 T. of hay at \$15 per ton, and charges 5% commission. Find his commission. How much does he remit? What per cent of the selling price does he remit?

10. A coal dealer receives a commission of 90¢ a ton for selling anthracite coal, and 80¢ a ton for selling bituminous coal. If he sells an equal quantity of each kind of coal, and if his entire commission is \$1530, how many tons of each kind does he sell?

11. A broker charges 35¢ for selling a bale of cotton. How many bales must he sell to realize for himself \$295.40? If a bale contains 500 lb., and the brokerage is equivalent to $\frac{7}{8}\%$, find the price of cotton per pound.

12. A commission merchant sells 1200 doz. eggs at 18¢ per dozen. Find his commission at 8%.

13. If a commission of \$70.98 is paid for buying 40 A. of land at \$54.60 an acre, what is the rate per cent of commission?

14. If \$284 is received for selling grain on a commission of $\frac{1}{8}$¢ per bushel, how many bushels of grain were sold?

15. If an agent charges 3% for collecting debts, and in one month his commission from this source is \$126, what is the amount collected?

16. A real estate agent's commission at $4\frac{1}{2}\%$ is \$351. Find the amount of his sales and the amount he remits to his principal.

17. A broker buys 10,000 bu. of wheat at $79\frac{1}{2}$¢, and sells the wheat the next day at $79\frac{7}{8}$¢, charging $\frac{1}{8}$¢ a bushel for buying and $\frac{1}{8}$¢ for selling. Find his commission.

18. An agent remits to his principal $9184.50 as the net proceeds from the sale of 12,000 bu. of wheat. Find the agent's commission if he charges at the rate of $2\frac{1}{2}$%. Find also the selling price of wheat per bushel.

19. After deducting a commission of 3%, an agent remits $1813.90 from the sale of oats at $46\frac{3}{4}$¢ per bushel. How many bushels of oats were sold?

20. When the market price of pork is 13.6¢ per lb., how many pounds can be bought for $520.20, if 2% is charged for brokerage?

21. How many acres of land can be bought for $4635 at $30 an acre, by a real estate agent who charges a commission of 3%?

22. Find the commission and amount remitted to the principal on the sale of the following articles:

(*a*)	Turkeys	1750 lb.	@ $12\frac{3}{4}$¢	8%	commission
(*b*)	Oranges	275 boxes	@ $2.95	5%	commission
(*c*)	Apples	580 bbl.	@ $1.85	8%	commission
(*d*)	Oysters	390 bbl.	@ $1.45	5%	commission
(*e*)	Potatoes	1270 bu.	@ $1.05	$7\frac{1}{2}$%	commission
(*f*)	Celery	560 bunches	@ $ .85	10%	commission
(*g*)	Onions	240 bu.	@ $1.25	8%	commission
(*h*)	Beans	960 bu.	@ $2.15	5%	commission
(*i*)	Rice	3500 lb.	@ $5\frac{3}{4}$¢	5%	commission
(*j*)	Chickens	984 lb.	@ $11\frac{1}{2}$¢	9%	commission
(*k*)	Cotton	287 bales	@ $49.	3%	commission
(*l*)	Peaches	315 boxes	@ $1.35	9%	commission

INTEREST

By **amount** is meant the sum of principal and interest.

To find the simple interest on a sum of money, **Multiply the principal by the rate to get the interest for 1 year, and this product by the time expressed in years.**

Example 1. Find the interest and the amount of $1780 for 5 mo. at 7%.

Solution: $\$1780 \times .07 \times \frac{5}{12} = \$51.92$, interest for 5 mo.
$\$1780 + \$51.92 = \$1831.92$, amount.

A concrete quantity which is contained an exact number of times in another concrete quantity is called an **aliquot part** of that quantity.

Thus, $2\frac{1}{2}$ yd. is an aliquot part of 10 yd.; $2.75 is an aliquot part of $11.

Example. Find the interest on $780 for 1 yr. 2 mo. 10 da. at 7%.

SOLUTION BY ALIQUOT PARTS.

	$780	
	.07	
	$54.60	= int. for 1 yr.
2 mo. = $\frac{1}{6}$ of 1 yr.	9.10	= int. for 2 mo.
10 da. = $\frac{1}{6}$ of 2 mo.	1.52	= int. for 10 da.
	$65.22	= int. for 1 yr. 2 mo. 10 da.

EXERCISE 83

Find the interest on:

1. $700 for 1 yr. at 3%; for 1 yr. at 5%.
2. $278 for 1 yr. at 6%; for 1 yr. at 7%.
3. $598 for 1 yr. at 9%; for 1 yr. at 8%.
4. $289 for 2 yr. at 4%; for 2 yr. at 5%.

5. \$1000 for 1 yr. 3 mo. at 6 %; for 1 yr. 5 mo. at 6 %.
6. \$1200 for 1 yr. 4 mo. at 8 %; for 1 yr. 5 mo. at 7 %.
7. \$1500 for 1 yr. 7 mo. at 9 %; for 1 yr. 8 mo. at 8 %.
8. \$1600 for 2 yr. 3 mo. at 5 %; for 1 yr. 7 mo. at 6 %.
9. \$2000 for 1 yr. 6 mo. at 4 %; for 1 yr. 9 mo. at 5 %.
10. \$3500 for 10 mo. at 7 %; for 7 mo. at 8 %.
11. \$156.40 for 1 yr. at 4½ %; at 5½ %.
12. \$185.50 for 1 yr. at 4½ %; at 7 %.
13. \$375.60 for 5 mo. at 6 %; for 4 mo. at 5 %.
14. \$928.40 for 2 mo. at 8 %; for 3 mo. at 6 %.
15. \$735.60 for 3 mo. at 7 %; for 2 mo. at 8 %.
16. \$1200 for 2 mo. at 8 %; for 5 mo. at 6 %.
17. \$1350.50 for 5 mo. at 5 %; for 4 mo. at 7 %.
18. \$393.80 for 7 mo. at 4 %; for 5 mo. at 3 %.
19. \$385.40 for 8 mo. at 5 %; for 7 mo. at 6 %.
20. \$934.54 for 9 mo. at 8 %; for 8 mo. at 9 %.
21. \$2713.64 for 10 mo. at 9 %; for 7 mo. at 8 %.
22. \$3800 for 11 mo. at 7 %; for 10 mo. at 6 %.
23. \$2825 for 10 mo. at 7 %; for 9 mo. at 5 %.
24. \$2700 for 1 yr. 1 mo. at 6 %; for 14 mo. at 5 %.
25. \$3280 for 1 yr. 3 mo. at 5 %; for 8 mo. at 4 %.
26. \$4500 for 8 mo. 15 da. at 6 %; for 7 mo. at 7 %.
27. \$329.50 for 7 mo. 10 da. at 8 %.
28. \$982 for 9 mo. 18 da. at 6 %.
29. \$545 for 10 mo. 25 da. at 8 %.
30. \$775.24 for 11 mo. 24 da. at 7 %.

Example. Find the interest on \$384.42 from Jan. 11, 1907, to April 30, 1907, at 7 %.

SOLUTION. First, find the difference between the two dates.

YR.	MO.	DA.
1907	4	30
1907	1	11
	3	19

	\$384.42	
	.07	
	\$26.9094	= int. for 1 yr.
3 mo. = $\frac{1}{4}$ of 1 yr.	6.727	= int. for 3 mo.
15 da. = $\frac{1}{6}$ of 3 mo.	1.121	= int. for 15 mo.
3 da. = $\frac{1}{5}$ of 15 da.	.224	= int. for 3 da.
1 da. = $\frac{1}{3}$ of 3 da.	.075	= int. for 1 da.
	\$8.147	= int. for 3 mo. 19 da.

Ans. \$8.15.

EXERCISE 84

Find the interest on:

1. \$450 from Jan. 12 to April 18 at 6 %.
2. \$783 from Feb. 14 to April 24 at 8 %.
3. \$2385 from Mar. 6 to Nov. 11 at 6 %.
4. \$3950 from July 4 to Dec. 6 at 6 %.
5. \$4280 from Aug. 31 to Nov. 18 at 8 %.
6. \$7335 from July 5 to Sept. 14 at 7 %.
7. \$3280 from Jan. 8 to July 5 at 4 %.
8. \$4592.40 from Jan. 10 to May 4 at 3 %.
9. \$384.75 from Mar. 10 to Aug. 14 at 10 %.
10. \$327.50 from April 6 to July 3 at 9 %.
11. \$935 from Jan. 1 to April 30 at $4\frac{1}{2}$ %.
12. \$3540 from Jan. 10 to Oct. 5 at $4\frac{1}{2}$ %.
13. \$1382.60 from Feb. 8 to Nov. 1 at 5 %.

EXERCISE 85

Find the amount of:

1. \$800 for 1 yr. 2 mo. 15 da. at 4%.

   Hint. Find the interest and add it to the principal.

2. \$580 for 3 yr. 3 mo. at 6%.
3. \$750 for 10 mo. 15 da. at 8%.
4. \$327.60 for 11 mo. 12 da. at 9%.
5. \$326.54 for 8 mo. 8 da. at 5%.
6. \$739.90 for 5 mo. 11 da. at 4%.
7. \$843.90 for 6 mo. 14 da. at 8%.
8. \$325 for 9 mo. 12 da. at $4\frac{1}{2}$%.
9. \$982 for 8 mo. 16 da. at 5%.
10. \$375 for 7 mo. 14 da. at 6%.
11. \$1280 from Jan. 10 to July 14 at 8%.
12. \$3580 from Jan. 14 to Aug. 18 at 8%.
13. \$1500 from March 11 to July 19 at 6%.
14. \$4350 from Aug. 14 to Dec. 17 at 7%.
15. \$1200 from Jan. 1 to July 8 at 7%.
16. \$480 for 9 mo. 15 da. at 5%.
17. \$800 for 6 mo. 18 da. at 6%.
18. \$550 for 3 mo. 15 da. at 8%.
19. \$650 for 5 mo. 12 da. at 6%.
20. \$850 for 8 mo. 15 da. at 5%.
21. \$980 for 7 mo. at 6%.
22. \$2329 for 1 yr. 7 mo. 16 da. at 5%.
23. \$3278 for 2 yr. 10 mo. 11 da. at 7%.
24. \$2594 for 1 yr. 5 mo. 10 da. at 3%.
25. \$978 for 1 yr. 1 mo. 20 da. at 6%.
26. \$1857 for 1 yr. 5 mo. 18 da. at 5%.
27. \$903.53 for 1 yr. 3 mo. at 5%.

MEASUREMENTS — SPECIFIC GRAVITY

The ratio of the weight of any given volume of a substance to the weight of an equal volume of another substance taken as a standard is called the **specific gravity** of that substance.

The **standard** taken for solids and liquids is water. Specific gravity, then, simply means how many times as heavy as water a substance is. Thus, cast iron is 7.21 times as heavy as water, and hence its specific gravity is 7.21. Cork is about one fourth as heavy as water, and hence its specific gravity is $\frac{1}{4}$. 1 cu. ft. of water weighs 1,000 oz.

TABLE OF SPECIFIC GRAVITIES

Ash . . .	.84	Ebony .	1.33	Steel . .	7.83	Clay . .	1.2
Beech . .	.85	Glass . .	2.89	Copper .	8.95	Mercury .	13.57
Brass . .	8.40	Gold . .	19.26	Silver .	10.47	Bar iron .	7.79
Butter . .	.94	Granite .	2.78	Lead . .	11.38	Platinum	21.5
		Ice . . .	.92				

Example. Find the weight of 28 cu. in. of mercury.

SOLUTION.

$$\frac{1}{1728} \times 1000 \text{ oz.} = \text{weight of cu. in. of water.}$$

$$\frac{28}{1728} \times 1000 \text{ oz.} = \text{weight of 28 cu. in. of water.}$$

$$13.57 \times \frac{28}{1728} \times 1000 \text{ oz.} = \text{weight of 28 cu. in. of mercury}$$

$$= \frac{13.57 \times 7 \times 1000 \text{ oz.}}{432} = \frac{94990 \text{ oz.}}{432} = 219.88 \text{ oz.}$$

$$\begin{array}{r|l} 12 & 94990 \\ \hline 6 & 7915.833 \\ \hline 6 & 1319.305 \\ \hline 16 & 219.88 \text{ oz.} \\ \hline & 13 \text{ lb., } 11.88 \text{ oz.} \end{array}$$

The factors of 432 are 12, 6, and 6.

EXERCISE 86

1. Find the weight of 1 cu. ft. of steel; 1 cu. ft. of glass; 1 cu. ft. of clay.

2. Find the weight of 1 cu. in. of water. Find the weight of 1 gal. of water.

3. Find the weight of 1 bushel measure filled with water.

4. A cubic foot of marble weighs 2700 oz. Find the specific gravity of marble.

5. A cubic foot of sea water weighs $64\frac{3}{4}$ lb. Find the specific gravity of sea water.

6. A cubic foot of goat's milk weighs 65 lb. Find how many times as heavy as water goat's milk is.

7. The mercury in the barometer exactly counterbalances the pressure of the atmosphere. If the barometer is 30 in. high, find the pressure of atmosphere upon every square inch of surface.

8. A swimming pool of fresh water is 25 ft. by 16 ft. and 5 ft. deep. Find the weight of the water it contains.

9. A block of granite is 6 ft. by 4 ft. and 2 ft. thick. Find its weight in tons.

10. Find the weight of 1 cu. in. of gold; of 1 cu. in. of silver; of 1 cu. in. of platinum; of 1 cu. in. of lead.

11. A block of ice 3 ft. by 2 ft. and 1 ft. thick weighs how many pounds?

12. What is the weight in tons of 1 cu. yd. of clay?

13. Find the weight of the butter required to fill a box 16 in. by 9 in. by 8 in.

14. A cellar is 18 ft. by 12 ft. and 8 ft. deep. Find the weight of the water required to fill it.

15. Find the weight of the air in a hall 27 ft. by 24 ft. and 15 ft. 6 in. high. 1 cu. ft. of air weighs .08073 lb.

16. A cubic foot of coal weighs $81\frac{1}{4}$ lb. Find the specific gravity of coal.

17. How many cubic inches of copper weigh just as much as 1 cu. in. of platinum?

18. Find the weight of a block of ebony 4 ft. long, 9 in. wide, and 8 in. thick.

19. Find the weight of a block of ash 12 in. long, 8 in. wide, and 6 in. thick.

20. How many times as heavy as glass is mercury?

21. What is the weight of a bar of iron 2 in. by 2 in. and 8 ft. long?

22. Find the weight of a beam of beech wood 8 in. by 6 in. and 12 ft. long.

Example **1.** How many sq. yd. are in 1 sq. mile?

1 sq. mi. $= 1760 \times 1760 \times 1$ sq. yd. $= 3,097,600$ sq. yd.

Example **2.** A lot in the form of a rectangle contains 16 A., and is 48 rd. wide. Find its length.

SOLUTION. $48 \times \text{length} = 16 \times 160$. (160 sq. rd. = 1 A.)

Therefore, length $= \dfrac{16 \times 160}{48} = \dfrac{160}{3} = 53\frac{1}{3}$. *Ans.* $53\frac{1}{3}$ rd.

Example **3.** A rectangular plot of ground 55 yd. by 11 yd. produces 2 bu. of buckwheat. How much will 1 A. produce at this rate?

SOLUTION. 55×11 sq. yd. produce 2 bu.

$$1 \text{ sq. yd. produces } \frac{2}{55 \times 11} \text{ bu.}$$

$$4840 \text{ sq. yd. produce } 4840 \times \frac{2}{55 \times 11} \text{ bu} = 16 \text{ bu.}$$

EXERCISE 87

1. The area of a rectangle is 12,500 sq. ft. and one side is 100 ft. Find its other dimension.

2. The area of a rectangle is $30\frac{1}{4}$ sq. yd. and its length is $5\frac{1}{2}$ yd. Find its width.

3. A lot contains 16 A., and its length is 64 rd. Find its width.

4. The area of a room is 252 sq. ft., and its length is 18 ft. Find its width.

5. A rectangular piece of ground 15 yd. by 12 yd. yields 6 bu. of potatoes. Find the yield of an acre.

6. A rectangular tract of land 30 yd. by 11 yd. yields 2 bu. of oats. At this rate how much will 1 A. yield?

7. A rectangular tract of land 50 yd. by 15 yd. yields 3 bu. of corn. At this rate how much will 1 A. yield?

8. The base of a triangle is 84 yd. and its area is 840 sq. yd. Find its height.

9. Find the altitude of a triangle having for base 34 yd. and area 289 sq. yd.

10. Given the area of a triangle 1024 sq. yd. and base 64 yd., find its altitude.

11. The area of the walls of a room is 560 sq. ft. and the dimensions of the room are 16 ft. by 12 ft. Find the height of the room.

12. The width of a rectangle is 3.9 ft. and its area is 17.55 sq. ft. Find the length.

13. The area of a hall is 497 sq. ft., its length is 71 ft. Find its width.

14. A street 1 mile long contains an area of $5\frac{1}{3}$ acres. Find the width of the street in feet.

EXERCISE 88

1. The inside dimensions of a box car are 36 ft. by 8 ft., and 8 ft. 6 in. high. Find the number of cu. ft. in the car, the number of bushels it will hold allowing $1\frac{1}{4}$ cu. ft. to the bushel, and the weight of the carload of corn.

SOLUTION. $36 \times 8 \times 8\frac{1}{2} = 2448$ = number cu. ft. in car.
$2448 \div 1\frac{1}{4} = 2448 \times .8 = 1958.4$ = number bu. in car.
1958.4×56 lb. $= 109670.4$ lb. = weight of corn in car.

2. The dimensions of a box car are 32 ft. by 8 ft. and 7 ft. high. Find the number of cu. ft. in the car, the number of bushels it will hold allowing $1\frac{1}{4}$ cu. ft. to the bushel, and the weight of the carload of oats.

3. A refrigerator car is 28 ft. 9 in. long, 7 ft. 6 in. wide, and 8 ft. high. Find the number of cu. ft. in it. The capacity of this car is 64,000 lb. How many dressed turkeys, averaging $12\frac{1}{2}$ lb. each, will the car hold, allowing 4000 lb. for ice?

4. A car 100,000 lb. capacity, length 40 ft., width 8 ft. 6 in., height 8 ft., inside dimensions, is loaded with wheat. Find the number of cu. ft. in the car. Find the weight of the wheat that will fill it. What fraction of this weight is the capacity of the car?

5. A car 40 ft. long, 8 ft. 6 in. wide, 8 ft. high, is loaded with 4-ft. wood. How many cords does this car contain? What is the value of the wood at $4.75 per cord? What will it cost to ship the wood from Sublime to San Antonio at $1.50 per cord?

6. The electric railroad from Rochester to Avon, New York, is 19 miles long. The rails used in its construction weigh 80 lb. to the yard. Find in tons the weight of the rails of this railroad.

EXERCISE 89

Example **1.** Find the weight in pounds avoirdupois of 5000 silver dollars. A silver dollar weighs $412\frac{1}{2}$ grains.

SOLUTION. 1 dollar weighs $412\frac{1}{2}$ gr.

$$5000 \text{ dollars weigh } 5000 \times 412\tfrac{1}{2} \text{ gr.}$$

$$= \frac{5000 \times 412\frac{1}{2}}{7000} \text{ lb.} = 294\tfrac{9}{14} \text{ lb.}$$

2. Find the weight in pounds troy of 1,000,000 silver dollars. Find also the weight in pounds avoirdupois.

3. A silver dollar is 90 % silver. Find the weight of pure silver in one silver dollar.

4. The commercial value of a silver dollar is the value of the pure silver it contains. What is the commercial value of a silver dollar when pure silver is worth 66¢ per oz. (troy oz.)?

5. A 10-dollar gold piece weighs 258 grains. Find the weight of 10,000 dollars in gold. Find also the weight of 1,000,000 dollars in gold.

6. Gold coins contain 90 % pure gold. Find the weight of pure gold in a 10-dollar gold piece.

7. The alloy in a gold coin neither adds to nor takes from the value of the coin. Compute the value of 480 grains of pure gold. (Remember that 90 % of 258 gr. of gold is worth $10.)

8. The bullion value of the pure silver in a silver dollar at the average price of silver for the year 1905 was $.472. Find the value of 1 oz. troy of pure silver.

9. The value of a fine ounce of silver for the year 1906 was $.6769, and the value of a fine ounce of gold the same year $20.67. Find the commercial ratio of the value of gold to the value of silver for that year.

RATIO

The first number in a ratio is called the **antecedent**, and the second number is called the **consequent**.

Since a ratio is a quotient, we may multiply antecedent and consequent by the same number without affecting the value of the ratio. It follows also that the terms of a ratio must be both abstract, or both concrete, and that the value of a ratio is always an abstract number.

Example 1. Divide 343 in the ratio 4 : 3.

SOLUTION. Divide 343 into 7 equal parts and put 4 of these parts in one group and 3 of them in another.

$$\therefore \text{ first part } = \tfrac{4}{7} \text{ of } 343.$$
$$\therefore \text{ second part } = \tfrac{3}{7} \text{ of } 343.$$

Example 2. Divide 136 into parts proportional to 2, 3, 4.

SOLUTION. Divide into 9 equal parts and group, as in Example 1. $2 + 3 + 4 = 9.$

$$\therefore \text{ first part } = \tfrac{2}{9} \text{ of } 136 = 30\tfrac{2}{9}.$$
$$\therefore \text{ second part } = \tfrac{3}{9} \text{ of } 136 = 45\tfrac{1}{3}.$$
$$\therefore \text{ third part } = \tfrac{4}{9} \text{ of } 136 = 60\tfrac{4}{9}.$$

Example 3. Divide 529 proportional to the numbers $\frac{3}{8}$, $\frac{1}{2}$, $\frac{9}{16}$.

SOLUTION. Multiply each of the fractions by 16, the L. C. M. of 8, 2, and 16. The results are 6, 8, and 9. The number may, therefore, be divided proportional to the numbers 6, 8, and 9.

$$6 + 8 + 9 = 23.$$
$$\therefore \text{ first part } = \tfrac{6}{23} \text{ of } 529 = 138.$$
$$\therefore \text{ second part } = \tfrac{8}{23} \text{ of } 529 = 184.$$
$$\therefore \text{ third part } = \tfrac{9}{23} \text{ of } 529 = 207.$$

CHECK. $138 + 184 + 207 = 529.$

$$138 \div \tfrac{3}{8} = 184 \div \tfrac{1}{2} = 207 \div \tfrac{9}{16}.$$

EXERCISE 90

1. Divide \$130 between two persons in the ratio 8 : 5.
2. Divide \$985 in the ratio 3 : 2.
3. Divide \$3240 in the ratio 4 : 5.
4. Divide \$7128 proportional to 1, 2, 3.
5. Divide \$7225 proportional to 4, 6, 7.
6. Divide 2916 proportional to 7, 9, 11.
7. Divide 4745 proportional to 5, 6, 7, 8.
8. Divide 6728 proportional to 13, 14, 15, 16.
9. Divide 5040 proportional to 21, 22, 23, 24.
10. Divide 2592 proportional to 15, 17, 19, 21.
11. Divide 95 in the ratio $\frac{1}{2} : \frac{1}{3}$.
12. Divide 1736 in the ratio $\frac{1}{3} : \frac{1}{4}$.
13. Divide 365 in the ratio $2\frac{1}{3} : 3\frac{3}{4}$.
14. Divide 1521 in the ratio $7\frac{1}{2} : 4\frac{1}{5}$.
15. Divide 4225 proportional to $1\frac{1}{2}$, $1\frac{2}{3}$, $2\frac{1}{4}$.
16. Divide 2189 proportional to $1\frac{1}{5}$, $2\frac{1}{3}$, $3\frac{1}{10}$.
17. A, B, and C enter into partnership, A contributing \$2450, B, \$3500, and C, \$4000. They gain \$1990. Find each person's share of the profits, supposing each person's gain is proportional to his investment.
18. Divide \$2700 among A, B, and C, so that A shall receive \$4 as often as B receives \$5 and C receives \$6.
19. Three men rent a ranch for \$900. The first man puts in 100 head of cattle, the second 85, and the third 115. How much rent should each pay?
20. Two men buy two lots for \$10,000. One man pays 25% more for his lot than the other man pays. How much does each pay?

PROPORTION

Two equal ratios constitute a proportion.

Thus, $\frac{2}{3} = \frac{6}{9}$ or $2 : 3 = 6 : 9$ is a proportion.

The first and fourth terms of a proportion are called **extremes**; the second and third are called **means.**

The **law of a proportion** is: **The product of the extremes equals the product of the means.**

The simplest method of working examples in proportion is the **analytical** or the **unit** method.

Example **1.** If 17 bu. of wheat cost $15.30, find the cost of 15 bushels.

SOLUTION. 17 bu. cost $15.30.

$$1 \text{ bu. costs } \frac{\$15.30}{17}.$$

$$15 \text{ bu. cost } 15 \times \frac{\$15.30}{17} = \$13.50.$$

Example **2.** If 25 men do a piece of work in 18 days, how long will it take 20 men to do the same work?

SOLUTION. 25 men take 18 da. to do the work.

$\therefore$ 1 man takes 18 da. $\times$ 25 to do the work.

$\therefore$ 20 men take $\frac{1}{20}$ of 18 da. $\times 25 = 22\frac{1}{2}$ days.

EXERCISE 91

Solve analytically:

1. If 19 bu. of wheat cost $14.25, find the price of 11 bu. of wheat.

2. Seven men can dig a trench in 16 da. How long will it take 10 men to do the same work?

3. If 11 sheep cost $71.50, find the cost of 17 sheep at the same rate.

4. If 14 men can do a piece of work in 11 da., how long will it take 21 men to do the same work?

5. The earth revolves on its axis 15° in 1 hour. Through how many degrees does it revolve in 23 minutes?

6. Twelve horses plow a field of 47 acres in 7 days. How many acres will 8 horses plow in the same time?

7. A pole 30 ft. high casts a shadow 24 ft. long. Find the length of the shadow cast by a pole 70 ft. high.

8. How long will it take 20 men to pave a street which 15 men pave in 15 days?

9. A sum of money yields $40.30 interest in 125 days. What interest will the same sum yield in 75 days?

10. A train runs 40 miles in 1 hr. and 20 min. How far will it run in 2 hr.?

11. If $3\frac{1}{2}$ acres of land are worth $259, what is the value of $2\frac{1}{2}$ acres?

12. If $\frac{2}{3}$ of an acre of land is worth $89, how much is a tract measuring $2\frac{1}{2}$ acres worth?

13. A train runs 45 miles an hour. Find how many feet it will go in 1 minute; in 1 second.

14. When the rate of a train is 36 miles an hour, what is its rate in feet per second?

15. Sound travels 1 mile in 5 seconds. How long will it take sound to travel 4400 feet?

16. Light travels from the sun to the earth, a distance of 92,790,000 miles, in 8 minutes and 18 seconds. Find its rate per second.

17. If $\frac{2}{3}$ of a clerk's yearly salary is $900, what is his salary per month?

18. If $11\frac{1}{2}$ yards of carpet can be bought for a certain sum of money, how many yards can be bought for the same sum when the price of carpet falls 8%?

REVIEW

EXERCISE 92

1. Multiply the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, by $1\frac{2}{13}$, and divide the product by $\dfrac{2\frac{1}{2}}{3\frac{3}{4}}$.

2. What number taken from $20\frac{1}{2}$ leaves as a remainder $4\frac{2}{3}$?

3. The difference between two numbers is $1\frac{1}{2}$, and the less is $4\frac{5}{6}$. Find the other number.

4. Which is the greater, $\frac{3}{4}$ of $4\frac{4}{9}$, or $\frac{2}{3}$ of $4\frac{3}{4}$?

5. From $\frac{7}{8}$ of the sum of $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{3}{40}$, take the sum of $\frac{7}{16}$, $\frac{7}{40}$, $\frac{7}{64}$.

6. From $\frac{3}{4}$ of $(\frac{1}{3} - \frac{1}{6} + \frac{8}{9})$ take $(\frac{1}{4} + \frac{2}{3} - \frac{1}{8})$.

7. Express in pounds the difference between .0125 of a ton and $\frac{3}{4}$ cwt.

8. What fraction having 48 for denominator is equivalent to .1875?

9. Find in feet the value of $\frac{15}{16}$ of a mile.

10. Express $\frac{2}{3}$ yd. as the decimal of a rod.

11. Express $\frac{3}{5}$ of 95 lb. as the decimal of $1\frac{1}{4}$ cwt.

12. Express $27\frac{3}{4}$ lb. as the decimal of a ton.

13. Take $\frac{4}{5}$ T. from $1\frac{1}{2}$ T., and express your result in pounds.

14. From $\frac{7}{8}$ of a right angle take $33° \ 45'$.

15. From $\frac{1}{5}$ of a circumference take $\frac{1}{6}$ of the circumference and express your answer in degrees.

16. There are two numbers in the ratio of 3 to 5. What fraction of their sum is their difference?

17. An estate is left to A, B, and C. A gets $\frac{1}{2}$ of the estate, B, $\frac{1}{5}$ of the estate, and C, the remainder. What part of the estate does C get? If C's share is $450, what is the value of the estate? Find A's share and B's share.

18. A man spends $\frac{2}{5}$ of his salary on board, $\frac{1}{3}$ on clothing, $\frac{1}{5}$ on rent. He saves the remainder, amounting to $125. Find his salary.

19. Subtract $\frac{1}{8}$ from 2.1, and divide the remainder by .25.

20. The dividend is 3.562 and the quotient is .3125. Find the divisor.

21. Express $5.24 as a decimal of $100.

22. After giving away $\frac{1}{9}$, $\frac{1}{12}$, and $\frac{17}{36}$ of his money, a man has left $392.95. How much money had he at first?

23. The third part of a number exceeds the fifth part of the same number by 15. What is the number?

24. The sixth part and the eighth part of a number together make $66\frac{1}{2}$. What is the number?

25. If $\frac{1}{3}$ and $\frac{2}{5}$ of a farm are together worth $1650, what is $\frac{1}{4}$ of the remainder worth?

26. Subtract the product of $\frac{1}{2}$ and $\frac{2}{9}$ from their sum.

27. What number divided by $3\frac{3}{5}$ gives $1\frac{1}{12}$ for the quotient?

28. Find the value of $\frac{7}{9}$ of $1\frac{3}{14}$ of $19.80.

29. A man owns $\frac{2}{3}$ of a boat and sells $\frac{3}{4}$ of his share for $750. At this rate, find the value of the boat.

30. If 6 men do a piece of work in 9 days, how long will it take 4 men to do the same work?

31. If 15 men pave a street in 16 days, how long will it take 40 men to pave the same street?

32. If 18 men remove an embankment in 12 da., how long will it take 24 men to remove the embankment?

33. Two trains start at the same time from two stations 840 mi. apart, and travel toward each other, one train going at the rate of 35 mi. an hour, and the other of 25 mi. an hour. In how many hours will they meet?

34. If a man performs $\frac{3}{7}$ of a piece of work in 15 da., in how many days more will he complete the work?

35. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{16}$, and $\frac{7}{20}$. Express the sum as a decimal. Check by reducing to decimals and adding.

36. What is the smallest number which, added to the sum of $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{5}{9}$, will make the final result an integer?

37. If $\frac{3}{5}$ of a barrel of sugar is sold and afterward 40 lb. are sold, how many pounds of sugar were in the barrel originally, supposing it still contains 90 lb.?

38. By selling a piano at $\frac{7}{10}$ of its cost, a dealer loses $98. Find the cost of the piano.

39. If 45 sq. rd. of land cost $18, find the cost of 1 A. Find also the cost of $3\frac{3}{4}$ A.

40. If $\frac{3}{4}$ of a clerk's salary per year is $675, find his salary per month. If his expenses average $48.89 per month, how much will he save per year? How long will it take him to save $652.75?

41. If $3\frac{1}{3}$ A. of land are worth $119, find the value of $2\frac{1}{2}$ A. How much is a rectangular strip of this land $\frac{3}{4}$ of a mile long and 33 feet wide worth?

42. How much is a plot of ground 80 ft. by 60 ft. worth, if an acre is worth $55? If an acre is worth $121?

43. Find the weight of a piece of coal in the shape of a rectangular solid, if its dimensions are $1\frac{1}{2}$ ft. by $1\frac{1}{4}$ ft. by 10 in. A cubic foot of coal weighs $81\frac{1}{4}$ lb.

44. A man's property is assessed at \$4550. If he pays 40¢ on every \$100 for school tax, how many dollars school tax does he pay?

45. How much taxes will be paid on real estate worth \$9580, if the tax is at the rate of \$1.27 on \$100?

46. A man invests \$4800 and gains \$540. How much does he gain on every dollar invested? How much does he gain on every \$100?

47. If an investment of \$9600 produces a gain of \$1056, find the gain on \$1; also on \$100.

48. When \$8400 produces a profit of \$1092, how much does \$1 produce? \$100?

49. The school tax in a city is 2 mills on the dollar. The assessed valuation of the property is \$17,294,000. Find the total tax levied for school purposes. If .95 of this total is collectible, find the amount collected.

50. Find the tax on \$33,254,000 at 4 mills on the dollar.

51. A man insures his dwelling for \$5450. If he pays \$11.50 on every \$1000, how much does he pay altogether?

52. Find the insurance on a house valued at \$7840, if the rate of insurance is $\$1\frac{1}{2}$ on every \$100?

53. If \$47.50 is paid to insure a boat valued at \$9500, how much is paid on \$1? on \$100.

54. The distance from New York City to Plymouth, England, is 2962 knots. The steamship *Deutschland* sailed from Plymouth to New York in July, 1900, in 5 da. 15 hr. 45 min. Find, in knots, its rate per hour. Find also its rate in miles per hour.

55. In March, 1902, the steamship *La Savoie* made the voyage from Havre to New York in 6 da. 10 hr. The distance from Havre to New York is 3170 knots nearly. Find, in knots, the rate per hour. Express the rate also in miles per hour.

56. The steamer *Kronprinz Wilhelm* made, in September, 1902, a voyage from Cherbourg to New York in 5 da. 11 hr. 57 min. Find its rate per hour, the distance from Cherbourg to New York being 3184 knots.

57. In May, 1900, a passenger train ran from Burlington to Chicago, 205.8 mi., in 3 hr. 8 min. 30 sec. Find its rate per hour.

58. The fastest time on record by a passenger train for a distance over 450 miles was made in October, 1895, on the Lake Shore and Michigan Southern Railroad, from Chicago to Buffalo, a distance of 510 mi., in 8 hr. 1 min. Find its rate per hour.

59. The run from London to Edinburgh, $393\frac{1}{2}$ mi., has been made in 7 hr. 45 min. Find the speed per hour.

60. The market quotations, Feb. 19, 1903, were: wheat, $78\frac{3}{4}$¢ per bushel; corn, $45\frac{5}{8}$¢ per bushel; oats, 34¢ per bushel. Find the price of 100 lb. of each of these commodities.

61. Market quotations of live stock sales are in dollars per 100 lb. Find the cost of:

(*a*) 44 cattle, average weight 1121 lb., @ $4.80.
(*b*) 132 cattle, average weight 1018 lb., @ $4.80.
(*c*) 24 cattle, average weight 915 lb., @ $4.20.
(*d*) 23 cattle, average weight 1060 lb., @ $3.90.
(*e*) 133 heifers, average weight 862 lb., @ $4.00.
(*f*) 69 calves, average weight 201 lb., @ $5.00.
(*g*) 82 hogs, average weight 188 lb., @ $6.20.

CHAPTER III

GENERAL REVIEW BY TOPICS

ADDITION

To add numbers is to find a single number equivalent to the numbers jointly. The result is the **sum**.

Only numbers of the same kind can be added. Thus, 5 yd. and 7 yd. may be added; but 5 yd. and $7 cannot be added.

5 ft. and 7 in. may be added provided the 5 ft. is changed to inches, or the 7 in. changed to feet. The sum in one case is 67 inches, in the other case, $5\frac{7}{12}$ feet.

Example 1. Add 279, 514, 928, 763.

$$\begin{array}{r} 279 \\ 514 \\ 928 \\ \underline{763} \\ 2484 \end{array}$$

The process is 3, 11, 15, 24; 24 units = 2 tens and 4 units.

Write the 4 units and carry the 2 tens.

2, 8, 10, 11, 18; 18 tens = 1 hundred and 8 tens.

Write 8 and carry 1.

1, 8, 17, 22, 24; write 24.

Example 2. Add $2.79, $5.14, $9.28, $7.63.

$$\begin{array}{r} \$2.79 \\ 5.14 \\ 9.28 \\ \underline{7.63} \\ \$24.84 \end{array}$$

The process is the same as in Example 1.

Example 3. Add 6 ft. 8 in., 3 ft. 6 in., 5 ft. 4 in.

FT.	IN.
6	8
3	6
5	4
15	6

The process is 4, 10, 18; 18 in. = 1 ft. 6 in. Write 6 in., carry 1 ft. 1, 6, 9, 15. Write 15 ft.

Example 4. Add $6\frac{2}{3}$, $3\frac{1}{2}$, $5\frac{1}{3}$.

$$\begin{array}{r} 6\frac{2}{3} \\ 3\frac{1}{2} \\ 5\frac{1}{3} \\ \hline 15\frac{1}{2} \end{array}$$

Change $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{3}$ to equivalent fractions having 12 for denominator.

$$\frac{2}{3}+\frac{1}{2}+\frac{1}{3}=\frac{8+6+4}{12}=\frac{18}{12}=1\frac{1}{2}.$$

Write $\frac{1}{2}$, carry 1. 1, 6, 9, 15. Write 15.

Observe the same principle pervades the four examples. Units of the same name are placed in the same column, and the columns added separately.

An example in addition may be *checked* by *adding the columns in reverse order.*

EXERCISE 93

Add:

(1)	(2)	(3)	(4)	(5)
4792799	7998992	5998476	5879824	6876894
8399384	8409499	9208503	9473473	9219479
7207383	7294792	8392393	7777888	7328337
9476583	8514798	6555777	6456789	8474494
4728737	9998777	7918924	9875874	9318327
9219777	6666784	5729998	8294295	6438444
6444673	8542728	6329888	7318392	7473478
8299299	5293294	7774673	9299497	9888777
9218288	4445454	6428427	6713729	6666679
7666729	7778898	9444779	9873876	9218289

6. Add vertically and horizontally, and finally sum the vertical and the horizontal totals:

392	965	873	599	222
876	329	888	307	399
543	707	393	448	937
878	538	427	388	542
929	928	925	394	234

7. The mileage of railroads in operation in the several states is given for the years 1903, 1904, and 1905 as follows:

Group and State	1903	1904	1905
New England			
Maine	2,004.79	2,029.89	2,091.12
New Hampshire	1,191.42	1,191.77	1,191.77
Vermont	1,057.84	1,056.96	1,063.20
Massachusetts	2,117.41	2,110.81	2,104.87
Rhode Island	209.84	209.84	209.84
Connecticut	1,025.90	1,020.12	1,020.12
Total			
Middle Atlantic			
New York	8,180.85	8,167.21	8,212.12
New Jersey	2,242.56	2,266.64	2,269.61
Pennsylvania	10,784.54	10,991.97	11,161.45
Delaware	333.63	334.86	333.60
Maryland	1,368.98	1,364.45	1,406.81
District of Columbia	24.70	24.70	24.70
Total			
Central Northern			
Ohio	9,023.61	9,163.97	9,243.26
Michigan	8,459.65	8,467.76	8,521.46
Indiana	6,834.75	6,863.03	7,046.90
Illinois	11,502.38	11,742.10	11,959.09
Wisconsin	6,921.40	7,014.78	7,188.18
Total			

Group and State	1903	1904	1905
South Atlantic			
Virginia	3,833.09	3,823.67	3,862.11
West Virginia	2,565.49	2,820.82	2,966.05
North Carolina	3,790.73	3,913.86	4,015.58
South Carolina	3,112.48	3,146.24	3,184.19
Georgia	6,109.21	6,298.97	6,516.61
Florida	3,469.92	3,585.83	3,635.38
Total			
Gulf and Mississippi Valley			
Alabama	4,442.69	4,590.89	4,758.57
Mississippi	3,156.56	3,367.23	3,541.04
Tennessee	3,355.19	3,484.92	3,606.88
Kentucky	3,193.31	3,261.56	3,355.07
Louisiana	3,419.38	3,592.68	3,764.17
Total			
Southwestern			
Missouri	7,316.62	7,797.18	7,859.57
Arkansas	3,651.28	3,946.54	4,165.72
Texas	11,308.05	11,614.13	11,949.02
Kansas	8,810.50	8,841.09	8,874.58
Colorado	4,852.44	4,989.85	5,093.20
New Mexico	2,450.02	2,441.93	2,596.64
Indian Country	2,320.02	2,585.69	2,686.47
Oklahoma	2,359.52	2,635.64	2,836.19
Total			

Find the total mileage for each group of states as indicated.

Add:

8. $2\frac{1}{2}$, $3\frac{3}{4}$, $5\frac{5}{6}$.

9. $2\frac{3}{10}$, $3\frac{3}{5}$, $5\frac{1}{2}$.

10. $3\frac{1}{4}$, $2\frac{2}{3}$, $5\frac{7}{12}$.

11. $5\frac{5}{8}$, $9\frac{7}{12}$, $2\frac{1}{2}$.

12. $1\frac{3}{4}$, $5\frac{1}{2}$, $2\frac{1}{6}$.

13. $7\frac{2}{5}$, $6\frac{7}{10}$, $9\frac{7}{15}$.

14. $9\frac{9}{16}$, $5\frac{7}{8}$, $7\frac{1}{4}$.

15. $12\frac{2}{9}$, $11\frac{2}{3}$, $6\frac{1}{2}$.

16. 2° 17′ 50″, 7° 24′ 30″, 9° 27′ 37″, 128° 14′ 43″.

17. 2 ft. 9 in., 7 ft. 3 in., 9 ft. 11 in., 15 ft. 7 in.

18. 8 qt. 1 pt., 9 qt. 1 pt., 15 qt., 12 qt. 1 pt.

19. 4 gal. 2 qt., 7 gal. 3 qt., 9 gal. 1 qt., 8 gal. 3 qt.

20. 5 pk. 7 qt., 9 pk. 3 qt., 12 pk. 5 qt., 13 pk. 4 qt.

21. 3 bu. 3 pk., 9 bu. 2 pk., 7 bu. 1 pk., 4 bu. 3 pk.

22. 12 hr. 15 min., 15 hr. 8 min., 17 hr. 42 min., 5 hr. 13 min.

23. 5 da. 12 hr., 18 da. 17 hr., 13 da. 18 hr., 5 da. 3 hr.

24. 15 yd. 2 ft., 25 yd. 1 ft., 32 yd. 2 ft., 9 yd. 1 ft.

25. How many times does a clock strike in 24 hours?

26. If 4 jars contain 3.92 liters, 7.84 liters, 9.57 liters, and 6.3 liters respectively, how many liters are in the four jars?

27. The dimensions of a table are 8 ft. 3 in. by 3 ft. 7 in. How many feet are in its perimeter?

28. The Galveston Sea Wall was constructed by Galveston County and the United States Government; the former built 3.5 miles, and the latter .87 mile. Find the total length of the sea wall.

In its construction there were used 1150 carloads of cement, 6100 carloads of crushed rock, 1400 carloads of round piling, 475 carloads of sheet piling, 4300 carloads of riprap, and 6 carloads of reënforcing rods. How many carloads of material were used in its construction?

How many miles would the cars extend if placed end to end, allowing 39.6 ft. to a car?

29. The decapod locomotives operating between Clarion Junction and Freeman, Ohio, weigh 268,000 lb. each. Express this weight in tons.

SUBTRACTION

Subtraction is the inverse of addition.

To subtract 7 from 16 is to find a number which added to 7 will make 16.

Example 1. Subtract 63 from 92.

$$\begin{array}{r} 92 \\ \underline{63} \\ 29 \end{array}$$

PROCESS. 3 and 9 are 12; write 9, carry 1. 1 and 6 are 7, 7 and 2 are 9; write 2.

Example 2. Subtract 6.3 from 9.2.

$$\begin{array}{r} 9.2 \\ \underline{6.3} \\ 2.9 \end{array}$$

The process is the same as in Example 1.

Example 3. Subtract 6 hr. 3 min. from 9 hr. 2 min.

$$\begin{array}{lr} \text{HR.} & \text{MIN.} \\ 9 & 2 \\ \underline{6} & \underline{3} \\ 2 & 59 \end{array}$$

PROCESS. 3 min. and 59 min. make 1 hr. and 2 min.; write 59 min., carry 1 hr. 1 hr. and 6 hr. are 7 hr. 7 hr. and 2 hr. are 9 hr.; write 2 hr.

Example 4. Subtract $6\frac{3}{5}$ from $9\frac{2}{5}$.

$$\begin{array}{r} 9\frac{2}{5} \\ \underline{6\frac{3}{5}} \\ 2\frac{4}{5} \end{array}$$

PROCESS. $\frac{3}{5}$ and $\frac{4}{5}$ are $1\frac{2}{5}$; write $\frac{4}{5}$, carry 1. 1 and 6 are 7, 7 and 2 are 9; write 2.

Example 5. From 75,218 take the sum of 4799, 3928, 9476, 8873.

$$\begin{array}{r} \underline{75218} \\ 4799 \\ 3928 \\ 9476 \\ \underline{8873} \\ 48142 \end{array}$$

PROCESS. 3, 9, 17, 26; 26 and 2 are 28. Write 2, carry 2. 2, 9, 16, 18, 27; 27 and 4 are 31. Write 4, carry 3. 3, 11, 15, 24, 31; 31 and 1 are 32. Write 1, carry 3. 3, 11, 20, 23, 27; 27 and 8 are 35. Write 8, carry 3. 3 and 4 are 7. Write 4. The remainder is 48,142.

This example shows the practical value of this method of subtraction. (Austrian Method.)

EXERCISE 94

1. Exports of domestic manufactures from the United States for the years ending June 30, 1897, and 1907:

ARTICLE	1897	1907
Iron and steel, manufactures of . . .	$57,497,872	$181,530,871
Copper, manufactures of	31,621,125	88,791,225
Wood, manufactures of	35,679,964	79,704,395
Oils—mineral, refined	56,463,185	78,228,819
Leather and manufactures of	19,161,446	45,476,960
Cotton, manufactures of	21,037,678	32,305,412
Agricultural implements	5,240,686	26,936,456
Naval stores	9,214,958	21,686,752
Carriages, cars, and other vehicles . .	9,952,033	20,513,407
Chemicals, drugs, dyes, and medicines .	8,792,545	18,220,630
Instruments and apparatus	3,054,453	14,661,455
Paper and manufactures of	3,333,163	9,856,733
Paraffin and paraffin wax	4,957,096	9,030,992
Fibers, manufactures of	2,216,184	3,308,112
India rubber, manufactures of	1,926,585	7,428,714
Furs and skins	3,284,349	7,139,221
Books, maps, engravings, etc.	5,647,548	5,813,107
Tobacco, manufactures of	5,025,817	5,735,613
Brass and manufactures of	1,171,431	4,580,455
Gunpowder and other explosives . . .	1,555,318	4,082,402
Paints, pigments, and colors	944,536	3,391,988
Soap	1,136,880	3,806,097
Musical instruments	1,276,717	3,252,063
Nickel and manufactures of	726,789	3,218,862
Clocks, watches	1,770,402	3,160,272
Coke	547,046	3,013,088
Glass and glassware	1,208,187	2,604,717
All other articles	19,799,642	47,295,739

Find the increase in the exports of each of the above articles, or group of articles, and verify your work.

Find the difference between :

2. 200 and .02.
3. 400 and 1.37.
4. \$75 and 73¢.
5. \$700 and \$2.84.
6. \$100 and \$1.75.
7. \$1000 and 5¢.
8. \$324.80 and \$100.99.
9. \$70.73 and \$19.94.
10. \$403.05 and \$92.89.
11. \$60.52 and \$23.87.
12. 100 and .01.
13. 6.29 and 2.9924.
14. 5.001 and 4.0073.
15. 7.2 and 2.77.
16. 11 and 1.5.
17. 17.3 and 11.9.
18. The square of 6.715 and the square of .285.
19. $7\frac{2}{3}$ and $4\frac{11}{12}$.
20. $18\frac{5}{9}$ and $7\frac{5}{6}$.
21. $9\frac{5}{6}$ and $4\frac{3}{4}$.
22. $21\frac{2}{3}$ and $11\frac{13}{18}$.
23. $7\frac{3}{11}$ and $2\frac{1}{2}$.
24. $8\frac{3}{16}$ and $5\frac{9}{10}$.
25. $9\frac{5}{12}$ and $3\frac{7}{8}$.
26. $6\frac{9}{16}$ and $3\frac{2}{3}$.
27. $10\frac{3}{11}$ and $7\frac{1}{4}$.
28. $19\frac{2}{3}$ and $8\frac{5}{8}$.
29. $12\frac{7}{15}$ and $9\frac{9}{10}$.
30. $23\frac{11}{14}$ and $12\frac{16}{21}$.
31. 5 ft. 7 in. and 4 ft. 9 in.
32. 17 ft. 3 in. and 12 ft. 8 in.
33. 19 ft. 1 in. and 9 ft. 4 in.
34. 27 ft. 3 in. and 18 ft. 4 in.
35. 9 lb. 2 oz. and 4 lb. 7 oz.
36. 17 lb. 6 oz. and 5 lb. 11 oz.
37. 33 lb. 2 oz. and 18 lb. 8 oz.
38. 12 hr. 10 min. and 9 hr. 24 min.
39. $90°$ and $34° 14' 15''$.
40. $180°$ and $115° 4' 50''$.
41. $180°$ and the sum of $56° 16'$, and $92° 18'$.
42. 15 pk. 3 qt. and 3 pk. 7 qt.

43. 23 pk. 5 qt. and 13 pk. 6 qt.

44. From 40,000 take the sum of 3211, 4711, 5283, 9438.

45. From 50,580 take the sum of 19,311, 12,218, 1273, 5559.

46. From 18,900 take the sum of 3419, 3428, 4584, 2293.

47. A man owns two houses worth respectively \$2390 and \$4575; he has deposited in the bank \$3280; he owes two notes for \$783 and \$870. How much is he worth?

48. The area of the British Isles is 120,975 square miles; the area of Texas is 265,780 square miles. By how many square miles does the area of Texas exceed the area of the British Isles?

49. The population of the Chinese Empire is 433,553,000; of the British Empire, 363,900,000; of the Russian Empire, 141,000,000; of the United States, exclusive of colonial possessions, 84,150,000; of Germany, 60,478,000. How many more people are in the United States than in Germany? In the British Empire than in Russia, United States, and Germany combined? By how many does the population of China exceed the population of Russia, United States, and Germany together?

50. The areas of Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, and Connecticut in square miles are respectively: 33,040, 9305, 9565, 8315, 1250, 4990. The area of California is 158,360 square miles. By how many square miles does the area of California exceed the area of the six New England states?

51. In going from Galveston to Chicago by rail, a distance of 1410 miles, a man travels the first day 345 miles; the next day, 201 miles; the third day, 290 miles. How far is he from Chicago at the end of the third day?

MULTIPLICATION

If one factor of the product is multiplied by a number, and the other factor divided by the same number, the product will be unchanged.

Thus, $84 \times 20 = 1680.$

$420 \times 4 = 1680.$ Here 84 is multiplied by 5, and 20 is divided by 5.

Example 1. Multiply 3782 by 234.

$$\begin{array}{rl}
3782 & \\
\underline{234} & \\
15128 & = 4 \times 3782 \\
11346 & = 30 \times 3782 \\
\underline{7564} & = \underline{200 \times 3782} \\
884988 & = 234 \times 3782
\end{array}
\qquad \text{or} \qquad
\begin{array}{rl}
3782 & \\
\underline{234} & \\
7564 & = 200 \times 3782 \\
11346 & = 30 \times 3782 \\
\underline{15128} & = \underline{4 \times 3782} \\
884988 & = 234 \times 3782
\end{array}$$

To multiply integers, write multiplicand and multiplier so that units of the same name stand in the same column, then multiply the multiplicand by each digit of the multiplier, placing the first figure of each partial product directly under the digit of the multiplier producing it, and add the partial products.

Example 2. Multiply 17.32 by .47.

$$\begin{array}{r}
17.32 \\
\underline{.47} \\
12\ 124 \\
\underline{69\ 28} \\
8.1404
\end{array}$$

PROCESS. The numbers are multiplied as if both were integers; then beginning at the right of the product four places are pointed off, that is the number of decimal places in multiplicand and multiplier combined. This may be readily seen by multiplying the multiplier by 100 and dividing the multiplicand by 100.

Compare with explanation page 59.

Example 3. Multiply $4\frac{7}{8}$ by $2\frac{2}{3}$.

$$4\tfrac{7}{8} = \tfrac{39}{8}$$
$$2\tfrac{2}{3} = \tfrac{8}{3}$$

Therefore, $4\frac{7}{8} \times 2\frac{2}{3} = \frac{39}{8} \times \frac{8}{3} = \dfrac{\overset{13}{\cancel{39}} \times \cancel{8}}{\cancel{8} \times \cancel{3}} = 13.$

Explanation. If the first factor is multiplied by 8, the result is 39, and if the second factor is multiplied by 3, the result is 8. Hence,

$$8 \times 3 \times \text{required product} = 39 \times 8.$$

$$\text{Therefore, required product} = \frac{39 \times 8}{8 \times 3}.$$

Compare with explanation on page 46.

Example 4. Multiply 5 gal. 2 qt. 1 pt. by 9.

GAL.	QT.	PT.
5	2	1
		9
50	2	1

Process. 9 times 1 pt. = 4 qt. 1 pt.; write 1 pt., carry 4 qt. 9 times 2 qt. = 18 qt. 18 qt. + 4 qt. = 22 qt. = 5 gal. 2 qt.; write 2 qt., carry 5 gal. 9 times 5 gal. = 45 gal. 45 gal. + 5 gal. = 50 gal.

PARTICULAR SHORT METHODS OF MULTIPLICATION

$$5 = \tfrac{1}{2} \text{ of } 10 \qquad 75 = 100 - \tfrac{1}{4} \text{ of } 100$$
$$25 = \tfrac{1}{4} \text{ of } 100 \qquad 875 = 1000 - \tfrac{1}{8} \text{ of } 1000$$
$$125 = \tfrac{1}{8} \text{ of } 1000 \qquad 99 = 100 - 1$$
$$.16\tfrac{2}{3} = \tfrac{1}{6} \qquad 97 = 100 - 3$$

Example 1. Multiply 97.3 by 125.

$125 \times 97.3 = \frac{1}{8}$ of $1000 \times 97.3 = \frac{1}{8}$ of $97300 = 12162.5$

Example 2. Multiply 29.374 by 993.

$$\begin{array}{r@{\;=\;}r} 29374. & 1000 \times 29.374 \\ 205.618 & 7 \times 29.374 \\ \hline 29168.382 & 993 \times 29.374 \end{array}$$

EXERCISE 95

1. Multiply each of the following numbers by 10:
234, 350.2, 25.07, .127, .0788, 1.003.

2. Multiply the following numbers by 100:
505, 67.5, 27.28, 5.347, .07954, .00392.

3. Multiply the following numbers by 1000:
728, 96.4, 12.87, 1.732, .0139, .00782.

4. Multiply the following numbers by 10,000:
318, 25.4, 19.96, 18.832, 27.796, .012.

5. Find in the shortest possible way the following products:

(*a*) 2780×99; 9218×998; 7215×999.
(*b*) 2.79×25; 3.18×125; 243×875.
(*c*) $78 \times .16\frac{2}{3}$; $90 \times .33\frac{1}{3}$; 297×9998.

6. Multiply 5280 by 5280; 1020 by 1020.

7. Multiply 7309 by 256; 9417 by 735.

8. The estimated production and value of the following cereal crops as given in the Annual Report of the Department of Agriculture for the year 1906 are as follows:

Cereals	Yield per Acre	Value per Bushel
	bushels	cents
Corn	30.3	39.9
Wheat	15.5	66.7
Oats	31.2	31.7
Rye	16.7	58.9
Barley	28.3	41.5
Buckwheat	18.6	59.6

Find the value of the yield per acre of each of these cereal crops.

9. The number of bales of cotton produced in Texas in the season 1904–05 was 2,598,949, and in 1903–04, 3,214,133. Allowing 500 lb. to a bale, how many more pounds of cotton were produced in the latter year than in the former?

10. The estimated production and value per ton of the hay crop for the year 1906 are as follows:

State	Yield per Acre	Price per Ton
	tons	
New Hampshire	1.15	$12.50
Massachusetts	1.31	17.00
Connecticut	1.17	15.00
New York	1.28	12.10
New Jersey	1.32	15.95
Pennsylvania	1.30	13.40
Maryland	1.26	13.50
Virginia	1.25	15.50
South Carolina	1.46	15.25
Georgia	1.65	15.75
Alabama	1.95	13.30
Louisiana	1.93	11.50
Tennessee	1.51	13.45
Kentucky	1.35	13.25
Illinois	.98	12.50
Minnesota	1.70	5.50
Kansas	1.28	6.25
Colorado	2.50	9.50
Utah	4.00	7.50
Idaho	2.95	8.00
California	1.85	11.25

Find the value of the yield per acre in each of the above states.

11. A piece of coal taken from the mine at Coos Bay, Oregon, had the following composition by weight:

Moisture	$= .1042$
Combustible matter	$= .4221$
Fixed carbon	$= .4318$
Ash	$= .0419$

Find the amount of each in 87 tons of this coal: in 783 tons. Check your answers.

12. Find to the nearest cent the value of each of the following articles:

(*a*) $25\frac{1}{2}$ bu. corn @ $42\frac{1}{2}$ ¢ per bu.
(*b*) $12\frac{9}{10}$ bu. wheat @ $69\frac{1}{2}$ ¢ per bu.
(*c*) $28\frac{2}{5}$ bu. oats @ $25\frac{1}{2}$ per bu.
(*d*) $16\frac{3}{4}$ bu. rye @ $60\frac{1}{4}$ ¢ per bu.
(*e*) $20\frac{1}{2}$ bu. barley @ $47\frac{3}{4}$ ¢ per bu.
(*f*) $4\frac{3}{4}$ lb. wool @ 61 ¢ per lb.
(*g*) 497 lb. cotton @ $11\frac{3}{8}$ ¢ per lb.
(*h*) 512 lb. cotton @ $10\frac{7}{8}$ ¢ per lb.

13. The inside dimensions of the floor of a box car are 40 ft. $\frac{1}{8}$ in. by 8 ft. 6 in. Find the perimeter of the floor.

14. The inside dimensions of the floor of a refrigerator car are 28 ft. $9\frac{1}{2}$ in. by 8 ft. $1\frac{1}{4}$ in. Find its perimeter.

15. Multiply 5 yd. 2 ft. by 8; 9 ft. 8 in. by 7.

Find the product of:

16. 9 lb. 4 oz. by 5; 16 lb. 11 oz. by 9.

17. 3 hr. 20 min. 30 sec. by 6; 7 hr. 17 min. by 9.

18. $53° 12'$ by 10; $68° 12' 18''$ by 5.

19. 4 pk. 7 qt. by 6; 7 pk. 3 qt. by 8.

20. 5 gal. 2 qt. by 9; 9 gal. 3 qt. by 12.

DIVISION

Division is the inverse of multiplication.

To divide 84 by 7 means to find a number which multiplied by 7 gives 84.

If the divisor and dividend are both multiplied by the same number, the quotient remains unchanged.

Thus,
$$96 \div 8 = 12.$$
$$(96 \times 6) \div (8 \times 6) = 12.$$

Example 1. Divide 2483 by 7.

$$\begin{array}{l} 7\overline{)2483} \\ \quad 354\frac{5}{7} \end{array}$$

PROCESS. 7 is contained in 24 hundreds 3 hundred times, remainder 3 hundreds; 3 hundred = 30 tens. 30 tens and 8 tens = 38 tens. 7 is contained in 38 tens 5 tens times, remainder 3 tens; 3 tens = 30 units. 30 units and 3 units = 33 units. 7 is contained in 33 4 times, with a remainder of 5.

Example 2. Divide .437 by 1.92.

$$\begin{array}{r} .2276^{+} \\ 192\overline{)43.7} \\ 38\,4 \\ \hline 5\,30 \\ 3\,84 \\ \hline 1\,460 \\ 1\,344 \\ \hline 1160 \end{array}$$

PROCESS. Move the decimal point two places to the right in divisor and dividend; this multiplies both by 100. Then write each quotient figure directly above the right-hand figure of the partial dividend which produces it. Write the decimal point in the quotient above the decimal point in the dividend.

Example 3. Divide $3\frac{2}{3}$ by $6\frac{1}{2}$.

$$3\tfrac{2}{3} \div 6\tfrac{1}{2} = (3\tfrac{2}{3} \times 3 \times 2) \div (6\tfrac{1}{2} \times 3 \times 2) =$$
$$(11 \times 2) \div (3 \times 13) = \frac{11 \times 2}{3 \times 13} = \tfrac{22}{39}.$$

Multiply divisor and dividend by 3×2. This makes divisor and dividend both whole numbers.

Example 4. To how many long tons are 3.30693 short tons equivalent?

$$\frac{3.30693 \times \overset{100}{\cancel{2000}}}{\underset{112}{\cancel{2240}}} = \frac{330.693}{112} = 2.9526.$$

To check an example in division, multiply the quotient by the divisor.

EXERCISE 96

1. The estimated acreage, production, and value of the potato crop by states for the year 1905 are as follows:

State	Acreage	Production	Farm Value
	acres	bushels	dollars
New Hampshire	19,700	2,367,000	1,704,000
Rhode Island	6,490	811,200	722,000
Delaware	7,680	714,000	421,200
North Carolina	25,900	1,993,000	1,355,000
Florida	4,110	308,200	369,900
Mississippi	5,860	644,900	548,200
West Virginia	34,400	3,025,100	1,754,500
Michigan	242,000	16,203,000	9,073,700
Illinois	149,000	11,186,000	7,494,600
Missouri	86,100	7,059,000	3,882,614
North Dakota	25,400	2,415,000	917,800
Nevada	2,800	336,700	276,100

Find the number of bushels yielded per acre in each state, and the average price per bushel in cents.

Divide correct to four decimal places:

2. 128.016 by 420.
3. 2.3774 by 7.8.
4. 10.4987 by 3.2.
5. .77087 by .479.
6. .02734 by .044.
7. .035936 by .0888.
8. 1.57899 by .639.
9. 60.247 by 78.8.
10. 5.0748 by 3.88.

Divide:

11. 21 by $1\frac{3}{11}$.

12. 48 by $2\frac{2}{7}$.

13. 42 by $1\frac{5}{16}$.

14. 72 by $3\frac{3}{8}$.

15. $72\frac{1}{2}$ by $4\frac{1}{7}$.

16. $93\frac{3}{7}$ by $62\frac{2}{7}$.

17. $17\frac{3}{4}$ by $9\frac{31}{36}$.

18. $2\frac{2}{5}$ by $3\frac{9}{11}$.

19. $8\frac{1}{10}$ by $4\frac{8}{25}$.

20. $2\frac{11}{20}$ by $2\frac{32}{35}$.

Find the value of:

21. $\frac{2}{3} \times 1\frac{1}{4} \div \frac{3}{4} \div \frac{7}{8}$.

SOLUTION. $\frac{2}{3} \times \frac{5}{4} \times \frac{4}{3} \times \frac{8}{7} = 1\frac{17}{63}$.

22. $\frac{3}{4} \times 1\frac{3}{5} \div 2\frac{2}{5} \times \frac{9}{10}$.

23. $\frac{4}{5} \times 1\frac{2}{3} \div 1\frac{1}{6} \div 3\frac{1}{2}$.

24. $\frac{7}{8} \times 2\frac{2}{3} \times 1\frac{1}{5} \div 1\frac{3}{4}$.

25. $1\frac{9}{10} \div 3\frac{1}{3} \div 3\frac{1}{6} \times \frac{1}{2}$.

26. $1\frac{2}{3} \div (\frac{7}{9} \div 4\frac{1}{2}) \times 4\frac{2}{3}$.

27. $\frac{2}{9}$ of $1\frac{2}{7}$ of $3\frac{1}{2} \div \frac{3}{8}$ of $1\frac{1}{4}$.

HINT. $\frac{2}{9} \times \frac{9}{7} \times \frac{7}{2} \div \frac{15}{32}$.
Observe the divisor is $\frac{3}{8}$ of $1\frac{1}{4}$.

28. $\frac{3}{4}$ of $1\frac{3}{5}$ of $2\frac{1}{2} \div \frac{2}{3}$ of $1\frac{2}{7}$.

29. $\frac{11}{12}$ of $1\frac{5}{7}$ of $\frac{7}{11} \div \frac{6}{13}$ of $\frac{2}{3}$.

30. $\frac{5}{6}$ of $4\frac{4}{5} \times \frac{9}{20} \div \frac{1}{2}$ of $\frac{7}{9}$.

31. $\frac{15}{16}$ of $3\frac{5}{9}$ of $\frac{2}{3} \div \frac{3}{4}$ of $\frac{5}{6}$.

Express in long tons:

32. 3.36 T., 6.6139 T., .00992 T., 4.4092 T.

Express in short tons:

33. 1.9684 long T., 6.8894 long T., .004921 long T.

Express in troy pounds:

34. 5 lb. avoirdupois, 13.228 lb. avoirdupois, 6613.87 lb. avoirdupois.

Express in avoirdupois pounds :

35. 7 lb. troy, 13.396 lb. troy, 5.358 lb. troy.

36. Express 1 ft. as a decimal of 1 mi.

37. Express 1 rd. as a decimal of 1 mi.

38. Express 1 sq. rd. as a decimal of 1 A.

39. Express 1 A. as a decimal of 1 sq. mi.

40. Express 1 sq. yd. as a decimal of 1 A.

41. Express 1 lb. as a decimal of 1 ton.

42. A lot is 40 by 120 feet. How many such lots make 40 acres?

43. How many barrels of $31\frac{1}{2}$ gallons each will a rectangular tank 12 ft. by 8 ft. and 5 ft. deep hold? (Allow $7\frac{1}{2}$ gal. to a cubic foot.)

44. The weight of a half dollar is $12\frac{1}{2}$ grams. How many half dollars can be made out of 7500 grams of standard silver?

45. Find the cost of boring an artesian well 1400 feet deep at \$4 a linear foot for the first 900 feet, \$4.50 per linear foot for the next 200 feet, \$5 per linear foot for the next 100 feet, \$5.50 per linear foot for the next 100 feet, and \$6 per linear foot for the remainder.

46. The rails of the Great Western Railway, England, weigh $97\frac{1}{2}$ lb. per yard. Find in tons the weight of the rails required to construct 1 mile of this railway.

47. Express in the ordinary decimal notation :

3.27×10^6, 17.45×10^9, 9.4×10^3, 7.3×10^6.

48. Given $10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{100}$, $10^{-3} = \frac{1}{1000}$, $10^{-4} = \frac{1}{10000}$, $10^{-5} = \frac{1}{100000}$, $10^{-6} = \frac{1}{1000000}$.

Express in the ordinary decimal notation :

(*a*) 3.2×10^{-3}, 4.71×10^{-6}, 9.83×10^{-3}.

(*b*) 4.98×10^{-5}, 9.371×10^{-6}, 4.329×10^{-4}.

LONGITUDE AND TIME

A **meridian** is an imaginary line running due north and south from pole to pole.

Longitude is the distance, expressed in circular arc measure, east or west from the **prime** or **standard meridian.**

The meridian through any particular place may be used as the prime meridian. The meridians through the observatories of Greenwich, Washington, Paris, Madrid, Rome, Stockholm, Pulkova, and Lisbon have been used as prime meridians by the nations to which these cities belong. The International Geodetic Congress, which met at Washington in 1884, recommended that the meridian passing through the observatory at Greenwich, a suburb of London, be the prime meridian. This recommendation is now generally adopted by the great nations of the world. The meridian of Greenwich is taken as prime meridian in this book.

Longitude is reckoned in either direction halfway around the earth from the prime meridian. The greatest longitude a place can have is 180° E. or 180° W. The meridian 180° E. or 180° W. of the prime meridian is a continuation of the prime meridian on the other side of the earth, and forms with the prime meridian what is called a **great circle** passing through the poles.

The earth rotates on its axis from west to east. Consider two places not on the same meridian; for example, New York City and St. Louis. New York being farther east will come the sooner under the influence of the sun's rays. Therefore, when it is noon in New York City it is before noon in St. Louis. Since the earth's motion is uniform, and furthermore, since

in 24 hr. the earth rotates 360°,
∴ in 1 hr. the earth rotates 15°;
∴ in 1 min. the earth rotates 15′;
∴ in 1 sec. the earth rotates 15″.

A difference of 15° of longitude corresponds to a difference of 1 hr. of time. A difference of 15′ of longitude corresponds to 1 min. of time. A difference of 15″ of longitude corresponds to 1 sec. of time.

Hence, **to convert difference of longitude into difference of time, divide by 15.**

EXERCISE 97

1. When it is noon at London, what is the time at New Orleans, 90° W.?

2. When it is 9 o'clock A.M. on the meridian 75° W., what is the time on the meridian 90° W.?

3. The longitude of Denver is 105° W. When it is 3 o'clock P.M. in Denver, what is the time in London?

4. Two places differ in longitude by 20°. What is their difference in time?

5. A person travels east 15°. What change must he make in the time indicated by his watch so that it may indicate local time? Supposing he goes the same distance west, what change must be made in the time indicated by his watch?

6. When it is noon in London, what is the longitude of the places in which it is 4 o'clock P.M.? 5 o'clock A.M.?

7. When it is 2 o'clock P.M. in Washington, what is the time in places 30° W. of Washington? in places 75° E. of Washington?

8. What is the difference in longitude between places which differ in time by 2 hr. 30 min.? by 4 hr. 10 min.?

9. If a person travels from Denver to New York, will his watch be fast or slow when he reaches New York, and how much?

10. To how many hours does a difference of 80° in longitude correspond?

11. What difference in longitude corresponds to a difference of 4 hr. 20 min. in time?

12. A person living on the 90th meridian W. wishes to send a telegram to a bank in New York City, directing the bank to pay on the same day a sum of money. Up to what hour in the afternoon may he do this, allowing 30 minutes for the transmission of the telegram, taking the longitude of New York as 75° W.? (New York banks close at 3 P.M.)

13. At places on the same parallel of latitude the sun rises at the same instant local time. How many minutes earlier does the sun appear to a person who travels 1° E.?

LONGITUDES OF CITIES REFERRED TO IN THIS CHAPTER

Austin,	97° 44′ W.	Galveston,	94° 47′ W.
Baltimore,	76° 37′ W.	Havana,	82° 21′ 30″ W.
Bangor,	68° 47′ W.	Honolulu,	157° 52′ W.
Bismarck,	100° 47′ W.	Louisville,	85° 46′ W.
Boston,	71° 3′ 50″ W.	Melbourne,	144° 58′ 32″ E.
Brisbane,	153° 2′ E.	Manila,	120° 58′ 3″ E.
Buenos Ayres,	58° 22′ 14″ W.	Mexico City,	99° 6′ 39″ W.
Charleston,	79° 52′ 58″ W.	Montreal,	73° 33′ 4″ W.
Chicago,	87° 40′ W.	New Orleans,	90° 3′ 28″ W.
Cincinnati,	84° 24′ W.	New York,	74° 0′ 24″ W.
Constantinople,	29° 0′ 50″ E.	Norfolk,	76° 17′ 22″ W.
Detroit,	83° 3′ W.	Paris,	2° 20′ 15″ E.
Dublin,	6° 20′ 30″ W.	Pekin,	116° 29′ E.

Pensacola,	87° 16′ 6″ W.	St. Petersburg,	30° 19′ 40″ E.
Philadelphia,	75° 9′ 3″ W.	San Francisco,	122° 24′ 32″ W.
Portland,	122° 40′ W.	Savannah,	81° 5′ 25″ W.
Providence,	71° 24′ 20″ W.	Tientsin,	117° 11′ 44″ E.
Rome,	12° 28′ 40″ E.	Tokyo,	139° 44′ 30″ E.
St. Louis,	90° 16′ W.	Washington,	77° 0′ 36″ W.

In recent years scientific publications often give longitudes in terms of time, the + sign denoting west and the − sign denoting east.

	H.	M.	S.		H.	M.	S.
Harrisburg,	+ 5	7	32	Adelaide,	− 9	14	2
Milwaukee,	+ 5	51	37	Omaha,	+ 6	23	46

Example 1. Find the difference between the longitudes of Austin and Honolulu.

SOLUTION. Honolulu, 157° 52′ W.
Austin, 97° 44′ W.
60° 8′

∴ Honolulu is 60° 8′ farther west than Austin.

Example 2. Find the difference between the longitudes of Galveston and Constantinople.

SOLUTION. Galveston, 94° 47′ W.
Constantinople, 29° 0′ 50″ E.

Here, the places are on opposite sides of the prime meridian. By going east from Galveston 94° 47′, one arrives at the prime meridian, and by going 29° 0′ 50″ still farther east, he arrives at the meridian of Constantinople. Hence, the difference between the longitudes is (94° 47′ + 29° 0′ 50″) = 123° 47′ 50″.

To find the difference in the longitudes of two places: (1) Subtract their longitudes, if the places are on the same side of the prime meridian. (2) Add their longitudes, if the places are on opposite sides of the prime meridian.

EXERCISE 98

Find the difference in longitude between :

1. Baltimore and Bismarck.
2. Bangor and Detroit.
3. Boston and Havana.
4. Buenos Ayres and Chicago.
5. Charleston and Constantinople.
6. Cincinnati and Honolulu.
7. Cincinnati and Melbourne.
8. Havana and Rome.
9. Louisville and St. Petersburg.
10. Constantinople and Tientsin.
11. Paris and Pekin.
12. Norfolk and Paris.
13. Montreal and Mexico City.
14. Pensacola and Portland.
15. St. Louis and St. Petersburg.
16. Savannah and Dublin.
17. San Francisco and Dublin.

Example 1. Find the difference in local time between Boston and Portland, Ore.

Portland	122°	40′		W.
Boston	71°	3′	50″	W.
15)50°		36′	10″	
	3	22	25	*Ans.* 3 hr. 22 min. 25 sec.

Example 2. Find the difference in local time between Washington and Manila.

Washington	77°	0′	36″	W.
Manila	120°	58″	3″	E.
15)197°		58′	39″	
	13	11	54.6	*Ans.* 13 hr. 11 min. 54.6 sec.

EXERCISE 99

Find the difference in the local time of :

1. Mexico City and Montreal.
2. Philadelphia and San Francisco.
3. Philadelphia and Dublin.
4. Norfolk and Tientsin.
5. Chicago and Tokyo.
6. St. Louis and Rome.
7. Austin and St. Petersburg.
8. Savannah and Paris.
9. Washington and Brisbane.
10. Cincinnati and Manila.
11. Havana and Louisville.
12. Rome and Manila.
13. New Orleans and Portland.
14. Providence and St. Petersburg.
15. Montreal and Tokyo.
16. Bangor and Melbourne.
17. Baltimore and Buenos Ayres.

Example **1.** When it is noon, February 22, in St. Louis, it is 15 min. 6 sec. past three o'clock A.M., Feb. 23, in Adelaide, Australia. Find the longitude of Adelaide.

SOLUTION. The time difference between St. Louis and Adelaide is

$$\begin{array}{lrrr} & 15\text{ hr.} & 15\text{ min.} & 6\text{ sec.} \\ \text{Multiply by 15,} & & & 15 \\ \hline & 228^\circ & 46' & 30'' \end{array}$$

∴ Adelaide is 228° 46′ 30″ E. of St. Louis. Longitude of St. Louis is 90° 16′ W. ∴ longitude of Adelaide = (228° 46′ 30″ – 90° 16″) E. = 138° 30′ 30″ E.

EXERCISE 100

Calculate the longitude of each of the following cities, the time difference between New York City and each of them being given :

1. Berlin, 5 hr. 49.5 min.
2. Brussels, 5 hr. 13.4 min.
3. Calcutta, 10 hr. 49.2 min.
4. Edinburgh, 4 hr. 43.2 min.
5. Hamburg, 5 hr. 35.8 min.
6. London, 4 hr. 55.9 min.
7. Madrid, 4 hr. 41.1 min.
8. Vienna, 6 hr. 1.2 min.
9. The time difference between London and Amherst, Mass., is 4 hr. 50 min. 3 sec. Find the longitude of Amherst.
10. Find the difference in the time of sunrise between two points in the same latitude and which differ in longitude by 39° 20′.

STANDARD TIME BELTS.

BORMAY & CO., N.Y.

STANDARD TIME

Standard time is the time of a fixed meridian, generally a multiple of 15°. It was established in the United States in 1883 primarily for the convenience of railroads. It is now adopted generally throughout the civilized world.

STANDARD MERIDIANS AND PLACES USING THEM

0°. Great Britain, Spain, Belgium, Holland.

15° E. Germany, Austria, Italy, Denmark, Norway.

30° E. South Africa, Egypt, Turkey.

$82\frac{1}{2}$° E. British India (since July 1, 1905).

$97\frac{1}{2}$° E. Burma (since July 1, 1905).

120° E. West Australia, eastern coast of China, Philippine Islands.

135° E. Japan.

$142\frac{1}{2}$° E. South Australia.

150° E. Victoria, Queensland, New South Wales.

$172\frac{1}{2}$° E. New Zealand.

60° W. Newfoundland and Eastern Canada.

75° W. Eastern belt of the United States.

90° W. Central belt of the United States.

105° W. Mountain belt of the United States.

120° W. Pacific belt of the United States.

135° W. Alaska.

150° W. Tahiti.

$157\frac{1}{2}$° W. Hawaiian Islands.

France uses Paris time, Ireland uses Dublin time.

EXERCISE 101

1. Mariners carry on board ships chronometers which keep Greenwich time. When it is noon, local time, the chronometer indicates 4 hr. 48 min. P.M. What is the longitude of the ship?

2. When it is 10 o'clock P.M., March 2, in Washington, what is the standard time in Manilla? Melbourne? Berlin?

3. When it is 2 o'clock A.M., standard time, in Denver, what is the standard time of London? Manchester? Glasgow? Tientsin? Constantinople?

4. A telegram is sent from Madrid to Washington at 9 o'clock A.M. Allowing 1 hr. for transmission, when will it reach Washington?

5. At noon, local time, a chronometer indicates 11 o'clock P.M. What is the longitude?

6. A telegram is sent from Galveston to London at 10 o'clock P.M. When will it be received, allowing 2 hr. for transmission?

7. When it is 2 o'clock A.M. in Washington, standard time, what is the time in New Zealand? Tahiti? British India?

8. The San Francisco earthquake occurred April 18, 1906, at 5 A.M. When should the news have reached London? Berlin? Tokyo? Adelaide? (allowing 1 hour for transmission).

9. When it is noon in Paris, France, what is the time in Denver? Natal? Calcutta? Wellington (New Zealand)?

10. When it is 9 o'clock A.M. in Madras, what is the time in St. John's, Newfoundland? Chicago? Sitka?

11. When it is noon in the Hawaiian Islands, what is the time in Cairo (Egypt)? Perth (Western Australia)?

APPROXIMATIONS. CONTRACTED PROCESSES. GENERAL METHODS OF SOLUTION

In business problems results of computation are generally required to be correct to not more than two decimal places. For example: The interest on $79.50 for 4 months at 7% is $1.855. From a business point of view the answer is $1.85.

In all practical measurements of length it requires skill and long practice to get results correct to more than three figures. For example: A surveyor measures the length of a field and finds it to be 3729 feet. It is extremely probable that the last figure in this result is not correct. As the results of measurement are correct to only three or four figures, hence it is useless in computation to give results to more than three decimal places.

Before undertaking to show how results may be obtained correct to any given number of figures, it is well to fix in mind the following facts:

Tenths multiplied by tenths give hundredths.

Tenths multiplied by hundredths give thousandths.

Tenths multiplied by thousandths give ten-thousandths.

Hundredths multiplied by hundredths give ten-thousandths.

Example 1. Multiply .0537928 by 43.27.

```
 .537928
4.327
2.151712
 1613784
  1075856
   3765496
2.327614456
```

Move the decimal point one place to the left in the multiplier.

Move the decimal point one place to the right in the multiplicand.

These changes make no change in the product.

Suppose it be required to get the product correct to two decimal figures. The answer would be 2.33.

Contracted Process
.537982
4.327
2.152
161
11
4
2.328
2.33

Write the units' figure of the multiplier under the third decimal figure of the multiplicand. Multiply 4 by 7 and carry 4 from 4 multiplied by 9 because 36 is nearer 40 than 30. Multiply the remaining figures to the left by 4 in the usual manner.

As tenths multiplied by thousandths give ten-thousandths, multiply 3 by 3 to the left of 7 and carry 2 from 3 times 7. 3 times 5 are 15 and 1 make 16. As hundredths multiplied by hundredths give ten-thousandths, multiply 2 by 5 and carry 1 from 2 times 3. As thousandths multiplied by tenths give ten-thousandths, 7 is multiplied by no figure of the multiplicand, 4 is carried from 7 times 5.

Compare the two processes.

Example **2.** Multiply 253.7 by .079 correct to two decimal figures.

2.537
7.9
17.759
2.283
20.04

Move the decimal point in the multiplier two places to the right. Move the decimal point in the multiplicand two places to the left. These changes make no change in the product. Multiply by 7 in the usual manner. Multiply by 9 beginning with 9 times 3, and adding 6 to the product which is the figure carried from 9 times 7.

Move the decimal point in the multiplier so that it contains one integral figure. Move the decimal point in the multiplicand the same number of places in an opposite direction. Place the units' figure of the multiplier under the third place of the multiplicand, if a product to two decimal figures

is required. If a product to three decimal places is required, place the units' figure of the multiplier under the fourth decimal place of the multiplicand. Then multiply as indicated in the above examples.

Example 3. Multiply .732 by .864 correct to two decimal figures.

.0732	.0732
8.64	468
.586	586
44	44
3	3
.63	.63

Begin multiplying by 8 by taking the product 8×3, carrying 2 from 8×2. Begin the multiplication by 6 with 6×7, carrying 2 from 6×3. Begin the multiplication by 4 with 4×0, carrying 3 from 4×7. The arrangement in the right margin conserves energy, for the multiplication by each figure of the multiplier is begun with the figure directly above it.

Example 4. Divide 120.005 by 17.293 correct to three decimal figures.

$$
\begin{array}{r}
6.939^{+} \\
17293\overline{)120005} \\
103758 \\
\hline
162470 \\
155637 \\
\hline
68330 \\
51879 \\
\hline
164510 \\
155637 \\
\hline
8873
\end{array}
$$

The answer correct to three decimal figures is 6.940 as the next will be 5.

CONTRACTED PROCESS

$$
\begin{array}{r}
6.939^{+} \\
17293\overline{)120005} \\
\underline{103758} \\
16247 \\
\underline{15564} \\
683 \\
\underline{519} \\
164 \\
\underline{155} \\
9
\end{array}
$$

In this example the quotient is required correct to four figures. The divisor contains five figures. *Whenever the divisor contains one or more figures than are required in the quotient, a figure may be struck off the divisor in place of annexing or taking down a figure*, as is usually done in getting each figure of the quotient.

Compare the two processes.

GENERAL METHOD

Example a. If 3 acres of land are worth $129, how much are 5 acres worth at the same rate per acre?

Example b. If 8 masons build a wall in 18 days, how long would it take 9 masons to build the wall?

Example (*a*) The cost of 3 acres = $129.
The cost of 1 acre $= \frac{1}{3}$ of $129.
The cost of 5 acres $= \frac{5}{3}$ of $129 = $205.

Example (*b*) The time 8 masons take = 18 da.
The time 1 mason takes $= 8 \times 18$ da.
The time 9 masons take $= \frac{8}{9} \times 18$ da. = 16 da.

The answer in *Example* (*a*) is a **fraction** of $129.

The answer in *Example* (*b*) is a **fraction** of 18 days.

The solution of examples of this character consists in multiplying the quantity of the same kind as the answer by a fraction.

If the answer is to be greater than the given quantity, form the fraction so that the numerator is greater than the denominator. If the answer is to be less than the given quantity, form the fraction so that the numerator is less than the denominator.

Example 1. If a dealer sells a piano for $425, thereby losing 15 %, what should he have sold it for to make a profit of 15 % ?

In this example 85 % of cost is given, and 115 % of cost is sought. The answer will be obviously more than $425.

Hence, $\frac{115}{85} \times \$425 = \$575$, *Ans.*

Example 2. A kilometer is very nearly equivalent to $\frac{5}{8}$ of a mile. Express a mile in kilometers.

5 eighths of 1 mile is given and 8 eighths is sought.

Hence, $\frac{8}{5}$ of 1 kilometer = 1.6 kilometers.

EXERCISE 102

Solve by the above method.

1. If 6 horses plow a field in 9 days, how long will it take 9 horses to plow the same field?

2. If a train runs in $3\frac{3}{4}$ hours between two stations at the rate of 18 miles an hour, how long will it take a train whose speed is 30 miles an hour to make the same run?

3. If 5 acres of land sell for $423, at this rate what will be the selling price of 7 acres?

4. If 22 yd. of cloth are bought for a sum of money, how many yards may be bought for the same sum when the price falls 12 % ?

5. Eight horses consume a quantity of corn in 24 days. How long should the same quantity of corn last 12 horses?

6. The minute hand of a clock goes 360° in 1 hour. How many degrees does it go in 22 minutes?

7. An arc of 75° is 4 ft. 6 in. How many feet are in the circumference of the circle?

8. If $\frac{11}{200}$ of the number of miles from Paris to Turin is $27\frac{1}{2}$, what is the entire distance separating the cities?

9. If $\frac{11}{15}$ of the number of miles from New York City to Panama is 1727, how far is Panama from New York?

10. Given .9 of the distance from London to Constantinople as 1827 mi., how many miles is it from the former to the latter?

11. If $\frac{13}{55}$ of the distance from Hamburg to Vienna is 143 mi., find the distance between these cities.

12. In the year 1902, $\frac{11}{25}$ of the United States internal revenue receipts from tobacco amounted to $22,852,687. Find the total internal revenue receipts from tobacco for that year.

13. In the year 1902, $\frac{13}{81}$ of the excise tax in the United States on gross receipts under the War Revenue Law of 1898 amounted to $117,221. Find the total tax on gross receipts in 1902.

14. In the year 1902, $\frac{9}{20}$ of the United States internal revenue receipts from the tax on oleomargarine amounted to $1,325,021.40. Find the total receipts from this source.

15. The mark is the unit of money in Germany; $\frac{3}{17}$ of its value in our currency is 42 mills. Express the value of a mark in dollars.

16. The yen is the standard of value in Japan; $\frac{7}{83}$ of its value is equivalent to 4 cents and 2 mills. Express in dollars the value of the yen.

17. In Venezuela, the monetary unit is the Bolivar; $\frac{3}{5}$ of its value is equivalent to $.1158. Find its value in cents.

18. Thirty-two thirty-fifths of a meter is very nearly equivalent to 1 yd. Express the value of a meter in yards.

THE LANGUAGE OF MATHEMATICS, RATIO, PROPORTION, PARTNERSHIP

By **mathematics** is understood those branches of knowledge which deal with quantity. Arithmetic, algebra, geometry, surveying, etc., are included in the term *mathematics.*

Mathematics has a **language** of its own.

The word *eight* conveys a definite idea to the mind; the sign or symbol 8 conveys the same idea. The words *eight squared* convey a definite idea to the mind; the symbol 8^2 conveys the same idea. The words *three fourths of sixteen* convey an idea; the symbols $\frac{3}{4} \times 16$ convey the same idea. Similarly, the words the quotient of *seventy-two divided by eight* convey an idea; the symbol $\frac{72}{8}$ conveys the same idea.

Letters may represent numbers. Thus, a, b, c, x, y, z, etc., may each represent any number whatever. The product of the numbers represented by letters is indicated by writing the letters in succession, one after the other. Thus, abc implies the continued product of a, b, and c. If p stands for principle, r for rate, t for time in years, and i for interest, the rule for computing interest is given by the relation

$$prt = i.$$

In like manner the rule for computing the area of a rectangle may be expressed by the relation

$$F = ba,$$

F stands for area, b for base, and a for altitude. A number written before a letter indicates multiplication. Thus, $5a$ means 5 times a. $5a$ is then a short way of writing $a + a + a + a + a$.

$4b$ is a short way of writing $b + b + b + b$.

a^2 is a short way of writing $a \times a$ or aa.
a^3 is a short way of writing $a \times a \times a$ or aaa.
a^4 is a short way of writing $a \times a \times a \times a$ or $aaaa$.
a^5 is a short way of writing $a \times a \times a \times a \times a$ or $aaaaa$.

The number denoting how many times a number is added is called a **coefficient.** The coefficient of the expression $9b$ is 9.

The expression $a+b$ stands for the sum of any two numbers. The expression $a-b$ stands for the number which when added to b gives a, or in other words, the remainder obtained when b is subtracted from a.

Example 1. If $a=5$, $b=3$, what is the value of $a+b$? $a-b=$? $4a=$? $3b=$? $2a-3b=$?

SOLUTION. $a+b=5+3=8$. $a-b=5-3=2$.
$4a=4\times 5=20$. $3b=3\times 3=9$. $2a-3b=2\times 5-3\times 3=1$.

Example 2. If $a=7$, what is the value of a^2? a^3? $3a^2$? $4a^3$?

$a^2=a\times a=7\times 7=49$. $a^3=a\times a\times a=7\times 7\times 7=343$.
$3a^2=3\times a\times a=3\times 7\times 7=147$. $4a^3=4\times a\times a\times a=4\times 7\times 7\times 7=1372$.

EXERCISE 103

If $a=4$, what is the value of $4a$? $7a$? $11a$? $13a$? $17a$? $19a$? $27a$? $\frac{1}{2}a$? $\frac{1}{4}a$?

If $a=5$, what is the value of a^2? a^3? a^4? $2a^2$? $3a^2$? $2a^3$? $4a^2$? $a+a^2$? a^2+a^3?

If $a=3$, $b=2$, what is the value of $a+b$? $a-b$? $2a+b$? $a+2b$? $2a-b$? $2a-3b$? $3a-2b$?

If $a=6$, $b=3$, what is the value of $6a+3b$? $3a+5b$? $6a-36$? $5a-10b$? $7a-5b$? $3a^2$? a^2+b^2? a^2-b^2?

If $x=5$, $y=6$, what is the value of xy? $2xy$? $3xy$? x^2y? xy? $\frac{1}{3}x^2$? $\frac{1}{4}y^2$?

If $x=9$, $y=4$, what is the value of $2x^2-y^2$? x^2+2y^2? $3x^2+y^2$? $2x^2+3y^2$? y^2-x^2?

If $a=10$, $b=7$, find the value of ab, $5ab$, $ab+a^2$, $2ab+b^2$, $ab+2b^2$.

EXERCISE 104

1. What is the sum of two times a number and three times the same number? What is the sum of $2x$ and $3x$?

2. What is the sum of $4x$ and $3x$? of $8a$ and $3a$? of $5b$ and $2b$? of $6b$ and $4b$?

3. What is the difference between $8x$ and $3x$? $6x$ and $2x$? $11b$ and $7b$? $8a$ and a?

4. Add $5x$ and $7x$; $4x$ and $9x$; $9b$ and $6b$; $10y$ and $6y$; $12x$ and $4x$.

5. Subtract $4x$ from $9x$; $8x$ from $14x$; $9x$ from $16x$; $7x$ from $13x$; $5ab$ from $8ab$.

6. Find the difference between $11y$ and $2y$; $5ab$ and ab; $7ab$ and $4ab$; $12ab$ and $2ab$.

Every sentence conveys a thought. (1) The sum of 3 and 4 is 7. This sentence is expressed in the language of mathematics as follows: $3+4=7$. (2) Write another sentence: The difference between 18 and 7 is 11. This sentence, written in mathematical language, is $18-7=11$. (3) Write a third sentence: Two thirds of 27 is 18. In mathematical language this sentence is written $\frac{2}{3}\times 27=18$. The statements (1), (2), (3), are called equations. An **equation** is a statement in symbols that two expressions are equal to each other.

The part of an equation to the left of the sign of equality is called the **first member** of the equation; the part of an equation to the right of the sign of equality is called the **second member** of the equation.

What is the product of a and a? $a \times a = a^2$.

What is the product of a and a^2? $a \times a^2 = a \times a \times a = a^3$.

What is the product of a^2 and a^3? $a^2 = a \times a$; $a^3 = a \times a \times a$.

$\therefore a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5$.

What is the product of a^4 and a^3?

$$a^4 = a \times a \times a \times a.$$
$$a^3 = a \times a \times a.$$
$$\therefore a^4 \times a^3 = (a \times a \times a \times a) \times (a \times a \times a) = a^7.$$

What is the product of $4a^2$ and $5a^3$?

$$4a^2 = 4 \times a \times a.$$
$$5a^3 = 5 \times a \times a \times a.$$
$$\begin{aligned}\therefore 4a^2 \times 5a^3 &= 4 \times a \times a \times 5 \times a \times a \times a\\ &= 4 \times 5 \times a \times a \times a \times a \times a\\ &= 20a^5. \quad \text{(Associative Law.)}\end{aligned}$$

EXERCISE 105

1. $a \times 2a = ?$
2. $2a \times a^2 = ?$
3. $3a^2 \times a = ?$
4. $3a^2 \times 2a^2 = ?$
5. $4a \times a^3 = ?$
6. $7a^2 \times 2a^2 = ?$
7. $3a \times 5a^3 = ?$
8. $9a^2 \times 2a^3 = ?$
9. $4a^3 \times a^2 = ?$
10. $5x^3 \times 4x = ?$
11. $6x^2 \times x^3 = ?$
12. $5b \times b^2 = ?$
13. $4b^2 \times 3b = ?$
14. $5b^3 \times 4b^3 = ?$
15. $2b^2 \times 5b = ?$
16. $4y^2 \times 3y^3 = ?$
17. $7z^2 \times 5a = ?$
18. $4y \times 8b^3 = ?$

What is the quotient when a^3 is divided by a?

$$a^3 \div a = \frac{a^3}{a} = \frac{a \times a \times a}{a} = a \times a = a^2.$$

Here cancellation is utilized.

What is the quotient of $8a^3$ by $2a$?

$$8a^3 \div 2a = \frac{8a^3}{2a} = \frac{\overset{4}{\cancel{8}} \times a \times a \times a}{\cancel{2} \times a} = 4 \times a \times a = 4a^2.$$

EXERCISE 106

Find the following quotients:

1. $\frac{8a}{2a}$.	6. $\frac{21a^3}{7a^2}$.	11. $\frac{22a^5}{2a^2}$.	16. $\frac{35x^4}{5x^3}$.	21. $\frac{60x^5}{10x^5}$.
2. $\frac{9a}{3}$.	7. $\frac{28a^4}{14a^2}$.	12. $\frac{16a^5}{4a^3}$.	17. $\frac{39x^5}{13x}$.	22. $\frac{26x^5}{13x}$.
3. $\frac{12a}{4}$.	8. $\frac{18a^4}{3a^3}$.	13. $\frac{32a^6}{16a^4}$.	18. $\frac{42x^4}{6x^3}$.	23. $\frac{33b^2}{11b}$.
4. $\frac{16a^2}{4a^2}$.	9. $\frac{24a^4}{8a^2}$.	14. $\frac{24a^5}{8a^3}$.	19. $\frac{45x^4}{9x}$.	24. $\frac{48b^4}{16b}$.
5. $\frac{12a^3}{4a}$.	10. $\frac{11a^4}{a}$.	15. $\frac{25a^4}{5a^2}$.	20. $\frac{50x^5}{10x^4}$.	25. $\frac{46b^5}{23b^2}$.

RATIO

The ratio of one number a to another number b is the quotient obtained by dividing a by b.

The ratio of a to b is written $a : b$. When the quotient $a \div b$ is written $\frac{a}{b}$, the expression $\frac{a}{b}$ is a fraction.

The ratio $b : a$ is called the inverse ratio of a to b.

EXERCISE 107

1. What is the ratio of 2 ft. to 6 ft.?

2. What is the ratio of 4 in. to 1 yd.? of 3 in. to 1 yd.? of 1 yd. to 1 rd.? of $\frac{1}{2}$ rd. to 1 rd.?

3. What is the ratio of 80 A. to 1 sq. mi.? of 120 A. to 1 sq. mi.? of $\frac{1}{2}$ A. to 2 A.?

4. What is the ratio of the distance traveled by two trains in the same time, if the rate of the first train is 20 mi. per hour, and the rate of the second train is 30 mi. per hour?

5. If A walks at the rate of $2\frac{1}{2}$ mi. per hour, and B walks at the rate of 5 mi. per hour, what is the ratio of A's time to B's time in going any given distance?

6. What is the ratio of the time that 8 men take to do a piece of work to the time that 6 men take to do the same piece of work?

7. If you ride in a carriage at the rate of 7 mi. an hour and walk back the same distance at the rate of 3 mi. an hour, what is the ratio of the time in the carriage to the time walking?

8. What is the ratio of the price of 7 lb. of sugar to the price of 10 lb. of sugar of the same kind?

9. What is the ratio of the work done by 6 men to the work done by 9 men?

10. What is the ratio of the time that 9 men take to do a piece of work to the time that 6 men take to do the same work?

11. I can buy two kinds of matting for 40¢ and 50¢ a yard respectively. If I spend the same amount of money in the purchase of the two kinds of matting, what is the ratio of the number of yards of matting of the first kind to the number of yards of the second kind bought?

12. Divide 15 in the ratio $2:3$.

13. Divide 20 in the ratio $3:7$.

14. Divide \$1 in the ratio $18:7$.

15. Divide 1 mi. in the ratio $7:9$.

16. Divide 1 in the ratio $9:11$.

17. Divide 1 gal. in the ratio $1:3$.

18. Divide \$1 in the inverse ratio $9:16$.

19. Divide 22 yd. in the inverse ratio $3:8$.

20. Divide \$1000 in the inverse ratio $3:5$.

PROPORTION

A statement indicating that two ratios are equal is called a **proportion.**

Illustrations, $2:3=4:6.$ (1)

$9:15=12:20.$ (2)

Statement (2) is a proportion because the value of the first ratio is $\frac{3}{5}$, and the value of the second ratio, *i.e.* 12 : 20, is also $\frac{3}{5}$.

Statement (2) may read 9 is as large compared with 15 as 12 is compared with 20.

The first and fourth terms of a proportion are called the **extremes**, and the second and third terms are called the **means**, of the proportion.

In a proportion the product of the extremes is equal to the product of the means.

Let $a:b=c:d$ be any proportion whatever. Then $ad=bc$.

PROOF. $\frac{a}{b}=\frac{c}{d}\cdot$ Multiply each member by bd and get $\frac{abd}{b}=\frac{cbd}{d}\cdot$ $\therefore$ by cancellation $ad=bc$.

This property of a proportion enables us to find any term of a proportion, if three of the terms of the proportion are known.

The proportion $a:b=c:d$ is sometimes written $a:b::c:d$. The double colon used as a sign of equality is now rapidly becoming obsolete.

Example 1. Find x in the proportion $x:4=9:6$.

SOLUTION. The product of the extremes is equal to the product of the means.

$$\therefore 6x=36. \qquad x=6.$$

Example **2.** Find x in the proportion $10 : 35 = x : 42$.

SOLUTION. Since the product of the means is equal to the product of the extremes,

$$35x = 10 \times 42.$$
$$\therefore \quad x = 10 \times \tfrac{42}{35} = 12.$$

Two numbers which vary directly are said to be **directly proportional.** Two numbers which vary inversely are said to be **inversely proportional.**

EXERCISE 108

If x stands for the unknown term in each of the following proportions, find it:

1. $2 : 3 = 6 : x$.

2. $3 : 4 = 6 : x$.

3. $15 : 25 = 12 : x$.

4. $12 : 20 = 18 : x$.

5. $14 : 21 = x : 27$.

6. $21 : 27 = x : 45$.

7. $35 : 84 = x : 72$.

8. $20 : 48 = x : 96$.

9. $16 : 24 = x : 33$.

10. $20 : 32 = x : 72$.

11. $25 : 45 = x : 99$.

12. $45 : 126 = x : 154$.

13. $57 : 133 = x : 126$.

14. $68 : 85 = x : 75$.

15. $36 : x = 52 : 65$.

16. $28 : x = 36 : 63$.

17. $27 : x = 15 : 50$.

18. $15 : x = 21 : 77$.

19. $28 : x = 36 : 81$.

20. $25 : x = 45 : 72$.

21. $35 : x = 30 : 48$.

22. $x : 81 = 16 : 72$.

23. $x : 99 = 26 : 117$.

24. $x : 65 = 24 : 52$.

25. $x : 112 = 45 : 144$.

Example **1.** If 7 bu. of wheat cost \$5.25, find the cost of 11 bu. of wheat at the same rate.

SOLUTION. It is reasonable to assume that the price of 11 bu. of wheat is greater than the price of 7 bu. of wheat. $\therefore$ the price of 11 bu. of wheat $= \frac{11}{7}$ of \$5.25 = \$8.25.

Example 2. If 12 men pave a street in 15 da., how long will it take 9 men to pave a street of the same area?

SOLUTION. It will take 9 men longer than it takes 12 men. $\therefore$ the time 9 men take $= \frac{12}{9}$ of 15 da. $=$ 20 days.

To **solve a problem in proportion**, find first the relation of the answer to the quantity of the same kind as the answer given in the problem. Second, multiply this quantity by a fraction, proper or improper, according as the answer is less or greater than it.

EXERCISE 109

1. If 20 men earn $450 in a given time, how much will 30 men earn in the same time?

2. If 15 bu. of corn cost $7.20, what will 48 bu. of corn cost?

3. If 12 A. of land cost $456.90, what will 16 A. of the same land cost?

4. If 4 men can do a piece of work in 15 da., how long will it take 6 men to do an equal amount of work?

5. If 18 head of cattle cost $1450, what will 27 head of cattle cost at the same rate?

6. If a train goes 400 mi. in 12 hr., how long will it take to go 560 mi.?

7. If 8 masons build a wall in 15 da., how long will it take 6 masons to build a wall of the same size?

8. If 18 horses consume 14 bu. of corn in a week, how much will 24 horses consume in the same time?

9. If 18 horses plow a tract of land in 13 da., how long will it take 26 horses to plow the same tract?

10. How long will it take 126 sheep to eat a quantity of feed which will last 105 sheep 30 da.?

11. A garrison consisting of 1200 men has provisions for 16 da. How many men must be sent away so that the provisions may last 24 da.?

12. A garrison consisting of 1400 men has provisions for 27 da. If the garrison is reënforced by 400 men, in how many days will the provisions be consumed?

13. If I can buy a dozen turkeys for \$20.50, how many turkeys can I buy for \$30.75?

14. If the interest on \$750 for 4 mo. is \$12.50, what is the interest on \$39.60 for the same time?

15. If an arc of $12''$ on the 40th parallel of latitude is 933.92 ft., find the length of $1°$ on the 40th parallel of latitude.

16. If an arc of $30'$ on the circumference of a wheel is $1\frac{1}{4}$ in., find the length of the circumference of the wheel.

17. A fly wheel 63 ft. in circumference makes 150 revolutions per minute. Find the velocity of its rim per second.

18. A train is running at 50 miles an hour. This speed is 25% greater than usual. Find its usual speed.

COMPOUND PROPORTION

If the product of the corresponding terms of two or more ratios are taken, the ratio of the resulting products is called the **ratio compounded** of these ratios. For example, the ratio compounded of the ratios $2:3$, $4:5$, $7:8$, is the ratio $2 \times 4 \times 7 : 3 \times 5 \times 8$, or $56:120$, or $7:15$.

A proportion in which the final result depends upon a ratio compounded of two or more ratios is called a **compound proportion.**

A concrete example may give a clearer conception of compound proportion than any formal definition.

Example 1. If 15 men mow 90 A. in 12 da., how many acres will 12 men mow in 14 da.?

SOLUTION. The 12 men in a given time will mow less than 15 men in the same time. ∴ the 12 men in 12 da. will mow $\frac{12}{15}$ of 90 A. But the 12 men in 14 da. will mow more than this quantity. ∴ 12 men in 14 da. will mow $\frac{14}{12}$ of $\frac{12}{15}$ of 90 A. = $\frac{14}{15}$ of 90 A. = 84 A.

Example 2. If 24 men build a house in 18 da. of 10 hr. each, how many men will it take to build the same house in 30 da. of 8 hr. each?

SOLUTION. *Step.* 1. It will take fewer men to build a house in 30 da. than it will take to build it in 18 da. of the same length.

∴ the number of men it will take to build the house in 30 da. of 10 hr. each = $\frac{18}{30}$ of 24 men.

Step 2. More men are needed when they work 8 hr. a day than when they work 10 hr. a day.

∴ the number of men, in 30 da. of 8 hr. each, required to build the house = $\frac{10}{8}$ of $\frac{18}{30}$ of 24 men = 18 men.

EXERCISE 110

1. If 12 horses plow 84 A. in 6 da., how many acres will 16 horses plow in $4\frac{1}{2}$ da.?

2. If 14 men pave a street 200 ft. long in 8 da., how many feet will 12 men pave in 7 da.?

3. If a man earns \$117 in 3 mo. working 6 hr. a day, how much will he earn in 5 mo. working 8 hr. a day?

4. A garrison of 3650 men consumed in 30 da. 82.3 T. of food. How much food would be required for 7500 men for 1 yr. at the same rate?

5. If 8 masons build in 2 da. a wall 40 ft. long and 6 ft. high, what height of wall 30 ft. long can they build in 5 da.?

6. If 21 men complete a piece of work in 8 da. of $7\frac{1}{2}$ hr. each, in how many days of 10 hr. each can 18 men do the same work?

7. A wall is to be built in 10 da. by 30 men. After 2 da. 10 men are dismissed. In what time will the remaining 20 men finish the work?

8. If 4 men or 6 boys dig a trench in 12 da., in what time can 2 men and 9 boys dig it?

9. If 12 men mow 30 A. in 3 da. of 8 hr. each, how many hours a day must 16 men work to mow 48 A. in 4 da.?

10. If the interest on \$100 for 1 yr. is \$6, find the interest on \$840 for 2 yr. 3 mo.

11. If 12 men working 7 hr. a day earn \$227.50 in 20 da., how much will 15 men earn in 20 da., working 9 hr. each?

12. If 6 men mow $\frac{3}{5}$ of a meadow in $4\frac{1}{2}$ da., how long will it take 8 men to mow the remainder?

13. In 10 da. of 8 hr. each 9 horses can plow $\frac{2}{5}$ of a field. In how many days of 9 hr. each can the remainder of the field be plowed by 15 horses?

14. A marble block 3 ft. by 4 ft. and 5 ft. in length weighs 5.1 T. Find the weight of a marble block 7 ft. by 3 ft. and 10 ft. long.

15. A mason can build 3 yd. of a wall in 15 hr. How long will it take 9 masons to build 24 yd. of a wall which is one-third higher than the other wall?

PARTNERSHIP

NOTE. Partnerships are rapidly becoming a thing of the past. Those partnerships that still survive are conducted on somewhat different principles from the partnerships that existed prior to the introduction of the telegraph, telephone, and modern means of rapid transit.

Example. A, B, and C enter into partnership. A puts in \$840, B puts in \$350, and C puts in \$2000. A withdraws from the concern in 5 mo., C in 7 mo., and at the end of 8 mo. the profits are divided. If the entire profit is \$450, how shall this be divided among A, B, and C?

SOLUTION. A has \$840 in the concern for 5 mo. This is equivalent to \$4200 for 1 mo.

B has \$350 in the concern for 8 mo. This is equivalent to \$2800 for 1 mo.

C has in the concern \$2000 for 7 mo. This is equivalent to \$14,000 for 1 mo.

The profits will be divided in proportion to the numbers 4200, 2800, 14,000, or in proportion to the numbers 3, 2, 10, since 1400 divides each of them.

$$3 + 2 + 10 = 15.$$

$$\therefore \text{ A's share} = \tfrac{3}{15} \text{ of the profits} = \tfrac{3}{15} \text{ of } \$450 = \$90.$$
$$\text{B's share} = \tfrac{2}{15} \text{ of the profits} = \tfrac{2}{15} \text{ of } \$450 = \$60.$$
$$\text{C's share} = \tfrac{10}{15} \text{ of the profits} = \tfrac{10}{15} \text{ of } \$450 = \$300.$$

EXERCISE 111

1. A, B, and C enter into partnership with capitals of \$3000, \$3750, and \$4500 respectively. At the end of the year they divide among themselves a profit of \$3000. Find each person's share.

2. Two partners, A and B, invest \$600 and \$1125. A's money remains in the business 6 mo., and B's 8 mo. If they make a profit of \$2100, find each person's share.

3. Two men rent a pasture for \$171; one puts in the pasture 30 cattle for 30 da., and the other 45 cattle for 18 da. How much rent should each pay?

4. A and B enter into partnership. A's capital is \$200 more than B's. Out of a profit of \$640, B gets \$280. Find A's and B's capital.

5. A and B enter into a partnership, A contributing \$6400 and B \$7200. At the end of 3 mo. A withdraws \$1600, and at the end of 5 mo. B withdraws \$1440. C then enters into the partnership with a capital of \$4800. Seven months later a gain of \$2154 is divided among them. Find each person's share.

6. A, B, and C enter into partnership. A puts in \$1000, B \$1200, and C \$1800. At the end of 3 mo. C withdraws, and at the end of 10 mo. B withdraws. At the end of a year the profits are divided. If C gets \$135, how much do A and B receive?

7. Two men form a partnership. Their capitals are in the ratio 2:3. After 6 mo. the first man increases his capital by $\frac{1}{3}$ of itself, and the second man diminishes his capital by $\frac{1}{3}$ of itself. After 6 mo. more they divide their profits, amounting to \$1450. Find each partner's share.

8. A cistern $66' \times 27'\ 6'' \times 10'$ will hold enough water to irrigate $2\frac{1}{2}$ A. of land to the depth of 2 inches. How many acres will a cistern $77' \times 41'\ 3'' \times 12'\ 6''$ irrigate to the depth of $3\frac{1}{2}$ inches?

9. If A pays $\frac{2}{7}$ of the cost of irrigation when the rate charged is \$4 an acre to the depth of one inch, find his share of the cost.

PERCENTAGE

A **per cent** of a number implies a fraction of the number having 100 for denominator. Thus, 5 per cent, 5 %, $\frac{5}{100}$, and .05 are four ways of expressing the same fact.

The per cent equivalents of the following fractions should be thoroughly fixed in mind:

$$\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{1}{4}, \tfrac{3}{4}, \tfrac{1}{5}, \tfrac{2}{5}, \tfrac{3}{5}, \tfrac{4}{5}, \tfrac{1}{6}, \tfrac{5}{6}, \tfrac{1}{8}, \tfrac{3}{8}, \tfrac{5}{8}, \tfrac{7}{8}, \tfrac{1}{12}.$$

Example 1. The total value of imports into this country through the Atlantic ports for the year 1906 was \$974,562,800; of this 75.35 % came through New York City. Find the value of the imports through this city.

SOLUTION. $\$974{,}562{,}800 \times \dfrac{75.35}{100} = \$9{,}745{,}628 \times 75.35$ $= \$734{,}333{,}069.80$, value of imports through New York.

As 75.35 % is correct to four figures only, the result is not likely to be correct to more than four figures. To get four figures multiply 97.456 millions by 7.535 by the contracted process explained on page 191.

EXERCISE 112

1. Write the equivalent per cents of the following decimals:

   .04, .08, .075, .0525, $.1666\frac{2}{3}$.

2. Express as decimals the following per cents:

   $4\frac{1}{2}$ %, 15 %, $12\frac{1}{2}$ %, $62\frac{1}{2}$ %, $6\frac{1}{4}$ %, $3\frac{3}{5}$ %.

3. Find 5 % of each of the following numbers:

   2151, 366.7, 689.5, 7.188, 12.469.

4. Find 6 % of each of the following numbers:

   5262, 520.7, 2.66, 3.097, 6.41, .783.

5. Find $4\frac{1}{2}$ % of each of the following numbers:

   4150, 1418, 7120, 43.43, 53.17, 2.42.

6. The Engineer's Year Book for the year 1906 gives the cost of railway construction in England as $194,660 per mile. The per cents of cost were as follows:

Land	10	Permanent way	$11\frac{1}{2}$
Fencing	$1\frac{1}{2}$	Sidings	3
Earthworks	24	Junctions	1
Tunnels	12	Stations	$6\frac{1}{2}$
Viaducts and bridges	17	Maintenance	$\frac{1}{2}$
Accommodation works	2	Legal and engineering expenses	6
Culverts	5		

Find the cost of each of the above items of expense.

7. The value of the total imports to the United States for the year 1906 was $1,226,560,000. Of this value 79.45% came through the Atlantic ports, 4.42% through the Gulf ports, 1.38% through the Mexican border ports, 5.41% through the Pacific ports, 7.97% through the northern border ports, 1.37% through the interior ports.

Find the value of the imports through each of these divisions.

8. The value of the total exports of the United States for the year ending June 30, 1906, was $1,743,860,000. The per cents of total value by principal customs districts were as follows:

New York	34.81	Savannah	3.72
Boston	5.66	Puget Sound	2.82
New Orleans	8.63	Detroit	2.02
Galveston	9.54	Buffalo Creek	1.72
		Mobile	1.25
Philadelphia	4.73	Newport News	1.15
Baltimore	6.31	Wilmington	1.06
San Francisco	2.29	Pensacola	1.06

Find the values of the exports through these cities.

Given a quantity, to find its value when decreased by a per cent of itself.

Example 1. In the year 1906 the state of Ohio produced 11,562,500 lb. of wool; this shrunk 50 % from scouring. Find the number of pounds of scoured wool.

SOLUTION. $100\% - 50\% = 50\% = \frac{1}{2}$.

$11{,}562{,}500 \times \frac{1}{2} = 5{,}781{,}250$.

Ans. 5,781,250 lb.

EXERCISE 113

1. The wool production and per cent of shrinkage from scouring for the year 1906, as given by the Bulletin of National Association of Wool Manufacturers, for the states named are as follows:

STATE	NUMBER POUNDS UNWASHED	PER CENT OF SHRINKAGE
Michigan	9,450,000	50
Minnesota	2,450,000	52
Alabama	568,750	40
Montana	35,815,000	65
Wyoming	32,849,750	68
Idaho	16,905,000	67
Oregon	15,300,000	70
California	13,125,000	67
Utah	12,350,000	65
New Mexico	15,950,000	62
Colorado	9,450,000	67
Arizona	4,420,000	66
Texas	9,360,000	66
Washington	4,887,500	70

Find the number of pounds of scoured wool produced in each of the states.

Given a per cent of a number, to find the number.

Example 1. During the month of January the average daily attendance of a school was 414. This number was 92 % of the school enrollment. Find the number enrolled.

SOLUTION. 92 % of enrollment is given.
100 % of enrollment is sought.

$\therefore$ enrollment $= 414 \times \frac{100}{92} = 450$, or
if x stands for enrollment,

$$.9\,x = 414, \text{ therefore } x = \frac{414}{.9} = 450.$$

Example 2. A dealer sells an article for $522 at a gain of 16 %. Find the cost price.

SOLUTION. 116 % of cost price is given.
100 % of cost price is sought.

$$\therefore \text{cost price} = \$522 \times \frac{100}{116} = \$450, \text{ or}$$

$$1.16\,x = \$522.$$

$$\therefore x = \$522 \div 1.16 = \$450.$$

EXERCISE 114

1. Find the number of which 79 is 4 %.

2. In a certain town 60 % of the grown people are married. If there are 2394 married people, how many grown people are in the town?

3. A man spends $320 for board. This sum is 40 % of his income. Find his income.

4. A man spends 83 % of his salary and saves $170. What is his salary?

5. A lot is sold for $3380 at a gain of $12\frac{1}{2}$ %. Find the cost of the lot.

6. After a discount of $16\frac{2}{3}$ % is given, a man pays $84 for a bill of goods. Find the amount of the bill.

7. The total levies of ad valorum taxes and tax rate per cent of assessed valuation are as follows in the states named:

State	Levy	Rate Per Cent
Maine	$ 6,855,776	1.95
Pennsylvania	58,269,455	1.49
South Carolina	3,736,344	1.91
Kansas	14,847,136	4.09
Tennessee	7,626,068	1.88
Washington	9,002,727	3.45
Texas	13,683,526	1.34

Find the assessed valuation of property in each of these states.

To express one number as a percentage of another number.

Example **1.** The foreign population of Danish extraction according to the United Census of 1890 and 1900 was 132,543 and 153,805.

Find the increase per cent during the ten years.

SOLUTION. $153805 - 132543 = 21262$, increase.

$\frac{21262}{132543} =$ fraction the increase is of population in 1890.

$\frac{21262}{132543} \times 100\% = 16.04\%$. Increase per cent.

$$
\begin{array}{r}
16.04 \\
132\cancel{5}\cancel{4}\cancel{3}\overline{)2126200} \\
\underline{132543} \\
80077 \\
\underline{79526} \\
551 \\
\underline{530}
\end{array}
$$

As the divisor contains 6 figures and the quotient is required to 4 figures, for each quotient figure cut off one from the divisor instead of annexing a cipher.

Example 2. A dealer buys goods at a discount of 40% off the list price, and sells them at 16% off the list price. Find his gain per cent.

SOLUTION

Cost price to dealer = 60% of list price (100% − 40%).
Selling price of dealer = 84% of list price (100% − 16%).
Gain = 24% of list price.
$\frac{24}{60} \times 100\%$ = rate per cent of gain. *Ans.* 40%.

EXERCISE 115

1. The railway mileage of the world January 1, 1906, as given by a German statistician was as follows:

COUNTRY	MILES	COUNTRY	MILES
Europe	192,251	North America . .	253,098
Asia	50,593	South America . .	32,859
Africa	16,538	Australasia	17,441

Find the per cent of the total railway mileage in each of the six continents.

2. The foreign-born population of the United States by countries for the years 1890 and 1900 was as follows:

COUNTRY	1890	1900	COUNTRY	1890	1900
Austria . .	123,270	275,910	Germany .	2,785,000	2,663,000
England . .	909,090	840,513	Ireland . .	1,871,500	1,615,500
France . .	113,174	104,197	Scotland .	242,200	233,500

Find the rate per cent of increase or decrease.

3. A dealer buys goods at a discount of 40% off the list price, and sells them at 2% below the list price. What per cent of profit does he make?

4. Eggs are bought at the rate of 5 for 4¢, and sold at the rate of 4 for 5¢. What per cent of profit is made?

5. A lot is sold for $1560 at a profit of $120. Find the rate per cent of profit.

6. Meat is sold at 18¢ per pound at a profit of 20%. Find the cost price per pound.

7. If the butcher has to pay 1¢ per pound more for the meat, how must he sell it to make a profit of 25%?

8. A piano is sold for $470 at a loss of 6%. What would the gain per cent have been if the piano had been sold for $520?

9. A tradesman buys at a discount of 10%, and sells at an advance of 15% on the nominal cost price. Find his rate per cent of profit.

10. A book costs the publisher 60¢ for printing and publishing. At what price should he sell the book in order that he may make a profit of 20%, after paying the author 10% on the selling price?

11. What should be the selling price of an article which costs $15, so that a profit of 20% may be made after giving the dealer a discount of 10%?

12. A tradesman marks his goods at 25% above cost, but allows the customer 6% discount. What per cent of profit does he make?

13. Tea is sold at 60¢ per pound at a profit of $33\frac{1}{3}$%. If the total gain is $15, how much tea is sold?

14. A man buys a house for $4000 which he rents for $40 per month; his taxes are 3% on a valuation of $3000. What per cent does his money yield?

15. A merchant marks his goods 20% above cost. What discount does he give if he sells at cost?

INTEREST

Example 1. Find the interest on $670 at 5% from Jan. 14 to Aug. 10.

SOLUTION.

		MO.	DA.
Aug. 10	=	8	10
Jan. 14	=	1	14
		6	26

	$670	
	.05	
6 mo. = $\frac{1}{2}$ of 1 yr.	2)$33 50	= int. for 1 yr.
	16.75	= int. for 6 mo.
20 da. = $\frac{1}{9}$ of 6 mo.	1.861	= int. for 20 da.
5 da. = $\frac{1}{4}$ of 20 da.	.465	= int. for 5 da.
1 da. = $\frac{1}{5}$ of 5 da.	.093	= int. for 1 da.
	$19.17	= int. for 6 mo. 6 da.

EXERCISE 116

Find the interest and amount of:

1. $728 for 1 yr. 6 mo. at 5%.
2. $670 for 1 yr. 6 mo. at 7%.
3. $1260 for 1 yr. 3 mo. at 8%.
4. $385 for 1 yr. 4 mo. 12 da. at 7%.
5. $2750 for 1 yr. 8 mo. at 3%.
6. $3345 for 1 yr. 4 mo. at 6%.
7. $783 for 1 yr. 1 mo. 10 da. at 4%.
8. $597 for 1 yr. 4 mo. 24 da. at 5%.
9. $3000 for 3 mo. 6 da. at 7%.
10. $940 for 1 yr. 4 mo. at 3%.
11. $1800 for 1 mo. 15 da. at 4%.
12. $2100 for 2 yr. 9 mo. at 4%.
13. $960 for 8 mo. 17 da. at 7%.
14. $2911.25 for 1 yr. 7 mo. 16 da. at 4%.
15. $1857 for 1 yr. 5 mo. 18 da. at 5%.
16. $2775 from May 1 to Dec. 19 at 4%.

17. \$1770 from Jan. 10 to Oct. 5 at $4\frac{1}{2}\%$.
18. \$1975.14 from Feb. 8 to Nov. 1 at $3\frac{1}{2}\%$.
19. \$1218 from March 6 to Nov. 1 at $5\frac{1}{2}\%$.
20. \$1788 from Feb. 14 to Dec. 20 at $7\frac{1}{2}\%$.

EXACT INTEREST

Interest reckoned on the basis of 365 days to the year is called **exact interest.** Exact interest is used by the United States government and sometimes in business transactions.

Example. Find the exact interest on \$2384.50 from Jan. 12 to July 5 at 5%.

SOLUTION. From Jan. 12 to July 5 there are $(19 + 28 + 31 + 30 + 31 + 30 + 5)$ days $= 174$ days.

$$\$2384.50 \times .05 \times \tfrac{174}{365} = \text{exact interest.}$$

$$\frac{\$2384.50 \times .05 \times 174}{365} = \$56.84\text{, nearly.}$$

EXERCISE 117

Find the exact interest on :

1. \$913 from Jan. 4 to Feb. 4 at 5%.
2. \$731.11 from Jan. 14 to Jan. 28 at 7%.
3. \$52.50 from Jan. 1 to April 28 at 7%.
4. \$2745 from Feb. 1 to April 6 at 5%.
5. \$1095.80 from March 6 to June 7 at 5%.
6. \$1911.17 from March 1 to May 11 at 7%.
7. \$1464.98 from Jan. 4 to May 30 at 6%.
8. \$10565.65 from May 13 to June 25 at 4%.
9. \$834 from Feb. 5 to July 12 at 11%.
10. \$3561.50 for 81 da. at 5%.

INVERSE QUESTIONS IN INTEREST

Example 1. What principal will produce \$78.75 interest in 75 days at $7\frac{1}{2}\%$?

Let x denote the principal.

$\therefore$ \$$x$ will produce $\$78.75 \times \frac{360}{75}$ in 1 year.

$$\therefore \frac{7\frac{1}{2}}{100}x = 78.75 \times \frac{360}{75}.$$

$$\therefore x = 78.75 \times \frac{75}{360} \times \frac{100}{7\frac{1}{2}} = \$5040.$$

Example 2. In what time will \$840 produce \$57.40 interest at 5%?

Int. on \$840 for 1 yr. at 5% = \$42.

$$\text{Time} = \frac{\$57.40}{\$42.00}\text{ yr.} = 1\tfrac{11}{30}\text{ yr.}$$

$\frac{11}{30}$ of 1 yr. $= \frac{11}{30}$ of 12 mo. $= 4\frac{2}{5}$ mo.

$\frac{2}{5}$ of a mo. $= \frac{2}{5}$ of 30 da. = 12 da.

The time is 1 yr. 4 mo. 12 da.

Example 3. At what rate per cent will \$720 produce \$42.50 interest in 1 yr. 2 mo. 5 da.?

\$720 will produce $\$42.50 \div (1 + \frac{1}{6} + \frac{5}{360})$ in 1 yr.

\$1 will produce $\frac{\$42.50}{\$720} \div (1 + \frac{1}{6} + \frac{5}{360})$ in 1 yr.

\$100 will produce $100 \times \frac{\$42.50}{720} \div (1 + \frac{1}{6} + \frac{5}{360})$ in 1 yr. = \$5.

$\therefore$ the rate is 5%.

Example 4. What principal will amount to \$136.27 in 1 yr. 3 mo. 15 da. at 5%?

The interest on \$1 for 1 yr. 3 mo. 15 da. is $\$.0645\frac{5}{6}$.

$\therefore$ the amount of \$1 for 1 yr. 3 mo. 15 da. is $\$1.0645\frac{5}{6}$.

$\therefore$ the number of dollars in principal $= \$136.27 \div \$1.0645\frac{5}{6} = \$128$, nearly.

EXERCISE 118

What principal will produce:

1. \$60 in $1\frac{1}{2}$ yr. at 8 %?
2. \$120 in 2 yr. at 5 % ?
3. \$135 in 1 yr. 6 mo. at 9 %?
4. \$36 in 3 yr. at 5 % ?
5. \$144 in 1 yr. 4 mo. at $4\frac{1}{2}$ %?
6. \$12 in 1 yr. at 4 %?
7. \$21 in 1 yr. at $3\frac{1}{2}$ %?
8. \$84 in 3 yr. 6 mo. at 3 %?
9. \$16.90 in 2 yr. 2 mo. at 4 %?
10. \$42 in 2 yr. 4 mo. at 4 %?
11. \$25.50 in 6 mo. at 5 %?
12. \$5.40 in 4 mo. at 5 %?
13. \$6.75 in 9 mo. at 3 %?

EXERCISE 119

In what time will:

1. \$1088.75 produce \$87.10 interest at 8 %?
2. \$144 produce \$21.60 interest at 5 %?
3. \$215 produce \$6.45 interest at 5 %?
4. \$1160 produce \$278.40 interest at 6 %?
5. \$810 produce \$56.70 interest at 7 %?
6. \$312.50 produce \$43.75 interest at 8 %?
7. \$2220 produce \$216.45 interest at 3 %?
8. \$1400 produce \$78.75 interest at 5 %?
9. \$480 produce \$85.50 interest at $9\frac{1}{2}$ %?
10. \$3835 produce \$345.15 interest at 8 %?
11. \$1380 produce \$88.55 interest at $3\frac{1}{2}$ %?
12. \$5400 produce \$267.75 interest at 7 %?
13. \$7630 produce \$1335.25 interest at 6 %?

EXERCISE 120

Find the rate per cent when the interest on —

1. \$750 for 1 yr. is \$45.
2. \$928 for 1 yr. is \$64.96.
3. \$880 for $1\frac{1}{2}$ yr. is \$79.20.
4. \$945 for 6 mo. is \$37.80.
5. \$828 for 8 mo. is \$38.64.
6. \$1200 for 1 yr. 3 mo. is \$90.
7. \$1800 for 9 mo. is \$67.50.
8. \$2400 for 8 mo. is \$64.
9. \$2500 for 9 mo. 18 da. is \$100.
10. \$3000 for 7 mo. 12 da. is \$90.
11. \$3750 for 4 mo. 15 da. is \$112.50
12. \$2754 for 2 mo. 20 da. is \$55.08.
13. \$4846 for 6 mo. 20 da. is \$121.15.
14. \$1440 for 7 mo. 10 da. is \$52.80.

Find the rate per cent when —

15. \$1080 amounts to \$1123.20 in 8 mo.
16. \$1200 amounts to \$1270 in 10 mo.
17. \$1600 amounts to \$1640 in 6 mo.
18. \$2460 amounts to \$2574.80 in 9 mo. 10 da.
19. \$92 amounts to \$102.12 in 2 yr.
20. \$324 amounts to \$333.72 in 8 mo.

EXERCISE 121

What principal will amount to —

1. \$840 in 1 yr. at 5 %?
2. \$749 in 1 yr. at 7 %?
3. \$645 in 1 yr. at $7\frac{1}{2}$ %?
4. \$903 in $1\frac{1}{2}$ yr. at 5 %?
5. \$414 in 3 yr. at 5 %?
6. \$255.30 in 2 yr. at $5\frac{1}{2}$ %?

7. \$12,540.45 in 2 yr. 3 mo. at 4 %?
8. \$168.35 in 7 mo. 6 da. at 6 %?
9. \$618.67 in 1 yr. 4 mo. at 5 %?
10. \$646.80 in 8 mo. at 4 %?
11. \$776.07 in 1 yr. 1 mo. at $4\frac{1}{2}$ %?
12. \$481.50 in 1 yr. at 7 %?
13. \$432.55 in 11 mo. at 8 %?
14. \$282.75 in 2 yr. 4 mo. 15 da. at $7\frac{1}{2}$ %?
15. \$2090.07 in 1 yr. 1 mo. 15 da. at 8 %?
16. \$2067.75 in 1 yr. 5 mo. at $10\frac{1}{2}$%?
17. \$268.28 in 1 yr. 7 mo. at 6%?
18. \$254.25 in 7 mo. 15 da. at $9\frac{1}{2}$%?
19. \$25,346.25 in 1 yr. 3 mo. at 10%?
20. \$843.70 in 8 mo. 3 da. at $7\frac{1}{2}$%?

EXERCISE 122

REVIEW

1. Find the interest on \$4000 for 13 mo. 2 da. at 9%.
2. Find the interest on \$256.30 for 4 mo. 9 da. at 7%.
3. Find the interest on \$30.85 for 11 mo. 6 da. at 5%.
4. Find the interest on \$653 for 2 mo. 16 da. at 4%.
5. Find the interest on \$2105.60 for 84 da. at 5%.
6. Find the amount of \$805 for 10 mo. at 8%.
7. Find the amount of \$507 for 1 yr. 12 da. at 8%.
8. What principal will produce \$20.83 interest in 5 mo. at 5%?
9. What principal will produce \$17.50 interest in 9 da. at 5%?
10. Find the rate of interest when \$500 produces \$2.92 interest in 1 mo.

11. Find the rate of interest when $250 produces $7.30 in 5 mo.

12. How much must I invest at 5% interest to have an annual income of $1200 from my investment?

13. A man buys a house and lot and rents it for $40 a month. Taxes and insurance cost him $120 a year. If his net receipts give him a profit of 6% on his investment, find the cost of the house and lot.

14. For how long a time must $3000 be loaned at 5% to produce $20 interest?

15. If I borrow $2400 at 7% interest and pay in principal and interest $2456, how long did I keep the money?

16. For how long a time must a sum of money be loaned at simple interest at 8% to produce in interest $\frac{1}{5}$ of itself?

17. For how long a time must a sum of money be loaned at simple interest at 6% to produce in interest $\frac{1}{10}$ of itself?

18. Find the exact interest of $1200 for 292 da. at 5%.

19. Find the exact interest of $7300 for 146 da. at 7%.

20. The exact interest of $10,800 at 5% is $324. Find the time.

21. In what time will $260 amount to $262.60 at 5%?

22. What sum must be deposited in a savings bank which pays $3\frac{1}{2}$% interest to produce semiannually $8.75?

23. A man deposits his money in two banks. In one bank he has $572 which pays $3\frac{1}{2}$%. The other bank gives 4% interest. If he receives as interest the same amount from both banks, how much money has he all together?

24. If I invest half my money at 6% and the remainder at 4%, and derive an income of $650 annually, how much money have I invested?

REVIEW QUESTIONS

1. Define *principal*, *rate*, *per cent*, *interest*, *amount*.

2. How does exact interest differ from interest according to the common use of the term? What is the distinction between simple interest and annual interest?

3. How do you find the interest of a sum of money at a given rate and for a given time?

4. If you were given the interest, the rate, and the time, how would you find the principal?

5. Given the principal, interest, and time, how would you find the rate per cent? How would you find the rate?

6. Given the principal, the rate, and the interest, how would you find the time?

7. Given the principal, the amount, and the rate, how would you find the time?

8. Given the principal, the amount, and the time, how would you find the rate?

9. Given the amount, the rate, and the time, how would you find the principal?

10. If you knew the interest of a sum of money for a given time at 6%, how would you find the interest for the same sum for the same time at 5%? at 4%? at 8%?

11. If you knew the interest at 4%, how would you find the interest of the same sum at 7%? at 3%? at 5%? at $3\frac{1}{2}$%?

12. If you were given the interest of a sum of money for a number of days, how would you determine from this the exact interest of the same sum for the same number of days?

PROMISSORY NOTES

A written promise by one person to pay another person on demand, or after a specified time, a sum of money is called a **promissory note.**

The following are promissory notes written in standard form:

Galveston, Texas, March 7, 1907.

$380.75

Sixty days after date I promise to pay to the order of ---------- James Moore ----------

---------- Three hundred eighty ------ $\frac{75}{100}$ Dollars

at ----- the First National Bank ----------

Value received, with interest at 5 %.

John Mosley.

No. 20. May 6/9, 1907.

Dallas, Texas, March 4, 1907.

$474.25

On demand I promise to pay to the order of

---------- John Anderson ----------

---------- Four Hundred seventy-four ----- $\frac{25}{100}$ Dollars

at ---------- the First National Bank ----------

Value received, with interest at 7 %.

James Rowe.

No. 34. Due ------

The first of the above promissory notes is called a **time note**; the second is called a **demand note.**

The person who promises to pay is called the **maker.** John Mosley is the maker of the first note above.

The person to whom the money is to be paid is called the **payee.** The person who has legal possession of a note is called its **holder.**

The sum specified in a note is called its **face.** A time note is legally due on the date indicated. In some states 3 days more than are indicated in the note are allowed before the note is legally due. These days are called **days of grace.** The day on which a note is legally due is called the **day of maturity.**

A note made payable to the order of a person, or a note made payable to the bearer is **negotiable,** *i.e.* it may be transferred from one person to another person.

A note made payable to the payee only is **non-negotiable.**

When a note payable to the order of the payee is transferred, every holder before parting with it must *indorse* it, *i.e.* write his name on the back of it. Every *indorser* thus becomes liable for the payment of the note, if the maker fails to pay it. The holder in whose possession the note is at maturity presents it to the maker for payment. If the maker refuses to pay it, the holder engages a Notary Public to give to the indorser, or indorsers, a written notice of its non-payment. This notice is called a **protest.** A protest must be sent on the date of maturity; otherwise the indorsers are not held responsible for the payment of the note.

An indorser who writes over his signature the words *without recourse* is not held responsible for the payment of the note.

A note made payable to *the bearer* is negotiable without indorsement. In some states a note must contain the words *value received* in order to be legal.

If the words *with interest* are not in a note, no interest is charged. If, however, the note is not paid on the date of maturity, interest at the legal rate may be charged. If a note contains the words *with interest*, and no rate is specified, it is then understood that the note bears the rate of interest usually charged in the state where it is made.

When the time of payment is indicated in months, calendar months are understood.

A note drawn March 6, and payable two months after date, matures on May 6 in states where days of grace are not allowed, and on May 9 in states where days of grace are allowed. About one half of the states and territories allow 3 days of grace.

BANK DISCOUNT

New Orleans, La., Feb. 14, 1903.

$350.00.

Sixty days after date I promise to pay to the order of ---------------*Joseph Coan*---------------

Three hundred fifty------------------------ $\frac{00}{100}$ *Dollars*

Value received.

Alonzo Ryan.

No. 33. Due April 15/18, 1903.

The above time note is negotiable when indorsed. Supposing the payee, Joseph Coan, needs money, he can sell the note to a bank. The sum the bank gives him for the note is called the **proceeds** of the note. The difference between the proceeds and the face of the note is called the **bank discount**.

The **bank discount** is always a rate per cent of the value of the note on its day of maturity, reckoned from the date of the sale of the note to the day of maturity.

The **bank discount** is then the interest on the maturity value of the note computed from the date of discount to the date of maturity. This time is called the **term of discount.**

The maturity value of the note minus the **bank discount** is the **proceeds** of the note.

From the computer's point of view the essential features of a note are the **face, maturity value, date of drawing, date of sale,** or **date of discount, rate of interest** the note bears, **rate of interest charged,** known as **rate of discount,** and **date of maturity.**

BANKERS' INTEREST

Banks charge interest for the exact number of days between dates, allowing 30 days to a month. Banks usually draw notes for 30, 60, or 90 days.

Example. Find the discount and the proceeds of the above note, if it was discounted at 8%, March 1, 1903.

SOLUTION. The bank charges discount from March 1, to April 18.

From March 1 to April 18 is 48 days.

$$\begin{array}{rrl}
 & \$350. & = \text{maturity value.} \\
 & \underline{.08} & \\
 & \underline{28.00} & = \text{int. for 1 yr.} \\
45 \text{ da.} = \frac{1}{8} \text{ of 1 yr.} & 3.50 & = \text{int. for 45 da.} \\
3 \text{ da.} = \frac{1}{15} \text{ of 45 da.} & \underline{.23} & = \text{int. for 3 da.} \\
 & \$3.73 & = \text{int. for 48 da.} \\
 & & = \text{bank discount.}
\end{array}$$

$$\$350 - \$3.73 = \$346.27 = \text{proceeds of the note.}$$

EXERCISE 123

Find the bank discount and the proceeds of the following indicated notes, allowing 3 days of grace in examples 1, 2, 10, 11, 12, 13, and no grace in the remaining: —

	Date	Time	Face	Dis-c'ted	Rate of Disc't
1.	Jan. 12,	60 da.,	$600,	Feb. 13,	8 %.
2.	July 4,	60 da.,	$800,	Aug. 3,	8 %.
3.	Mar. 3,	90 da.,	$500,	Mar. 3,	6 %.
4.	April 5,	60 da.,	$700,	April 5,	6 %.
5.	May 7,	60 da.,	$600,	May 10,	10 %.
6.	May 9,	20 da.,	$900,	May 24,	8 %.
7.	May 30,	90 da.,	$750,	July 1,	6 %.
8.	June 5,	60 da.,	$450,	July 5,	6 %.
9.	Aug. 11,	30 da.,	$800,	Aug. 11,	6 %.
10.	Sept. 9,	30 da.,	$350,	Sept. 12,	10 %.
11.	Oct. 4,	60 da.,	$800,	Oct. 7,	10 %.
12.	Oct. 14,	60 da.,	$500,	Nov. 16,	12 %.
13.	Nov. 10,	90 da.,	$600,	Nov. 25,	8 %.
14.	Dec. 4,	90 da.,	$750,	Feb. 5,	6 %.
15.	Jan. 10,	45 da.,	$650,	Feb. 9,	7 %.
16.	Jan. 5,	60 da.,	$850,	Feb. 5,	9 %.
17.	Jan. 30,	75 da.,	$950,	Feb. 28,	9 %.
18.	July 10,	3 mo.,	$380,	July 12,	9 %.

COMPUTING DISCOUNT ON INTEREST-BEARING NOTES

Find the bank discount and the proceeds of a 90-day note for $250, dated Portland, Me., June 9, 1907, bearing interest at 6 %, and discounted July 8, 1907, at 8 %.

Step 1. Find the maturity value of the note. Maine allows 3 days of grace. Hence, the interest will be computed for 93 days.

$$\begin{array}{rl} \$250 & \\ 3.87 & = \text{interest at } 6\,\% \text{ for 93 days.} \\ \hline \$253.87 & = \text{maturity value of the note.} \end{array}$$

Step 2. Find the term of discount (exact number of days from July 8, to Sept. 10, the date of maturity).

Step 3. Find the bank discount. This is reckoned on the maturity value of the note.

Int. on \$253.87 for 64 days at 8 % = \$3.61.

\$254.87 – \$3.61 = \$250.26, proceeds of note.

EXERCISE 124

Find the discount and the proceeds of the following indicated notes, allowing 3 days of grace in examples 2, 3, 6, 9, 10, 13, and no grace in the remaining: —

	Face	Date	Time	Rate of Int.	Rate of Disc't	Disc't
1.	\$350,	Jan. 1,	45 da.,	10 %,	12 %,	Feb. 1.
2.	\$395,	Jan. 10,	3 mo.,	6 %,	10 %,	Jan. 20.
3.	\$450,	Feb. 1,	30 da.,	6 %,	8 %,	Feb. 18.
4.	\$600,	Mar. 1,	60 da.,	8 %,	12 %,	Mar. 31.
5.	\$500,	Apr. 2,	60 da.,	6 %,	8 %,	Apr. 17.
6.	\$900,	Apr. 10,	30 da.,	6 %,	8 %,	Apr. 10.
7.	\$1000,	Apr. 15,	60 da.,	8 %,	10 %,	Apr. 15.
8.	\$750,	May 4,	60 da.,	6 %,	6 %,	May 5.
9.	\$800,	July 10,	30 da.,	6 %,	8 %,	July 26.
10.	\$400,	Aug. 15,	60 da.,	6 %,	6 %,	Aug. 15.
11.	\$850,	Nov. 11,	90 da.,	7 %,	10 %,	Nov. 12.
12.	\$900,	Dec. 12,	45 da.,	6 %,	6 %,	Jan. 13.
13.	\$1200,	Dec. 5,	3 mo.,	6 %,	10 %,	Jan. 15.

COMMERCIAL DISCOUNTS

Commercial, or **Trade Discount** is a reduction from the list price of goods, or the amount of a bill.

If two or more discounts are allowed, the first is reckoned on the list price, the next on the remainder after deducting the first discount, the third is reckoned on the second remainder, etc.

Example. Find the cost price of a bill of goods, if the list price is \$690 and discounts of 25%, 10%, and 5% are allowed.

SOLUTION.

\$690		
\$172.50	=	25% of \$600.
\$517.50	=	first remainder.
\$ 51.75	=	10% of \$517.50.
\$465.75	=	second remainder.
\$ 23.287	=	5% of \$465.75.
\$442.46	=	cost of the goods.

EXERCISE 125

1. Find the cost if the list price is \$350, and the discount 20%.

2. Find the cost when the list price is \$823, and the discount $12\frac{1}{2}$%.

3. What is the cash value of a bill of goods listed at \$937.50 when a discount of $16\frac{2}{3}$% is given?

4. A suit of clothes is marked \$17.50, and is sold for \$12.00. What is the rate of discount?

5. A bookseller buys 60 books marked \$2.10 each at a discount of $16\frac{2}{3}$%, and sells them at the marked price. What is his gain per cent, and how much profit does he make on the sale of the books?

6. A dealer buys goods at a discount of $16\frac{2}{3}\%$ and sells them at 5% above the list price. What is his gain per cent?

7. Find the cost price in each case, if the list prices and rates of discount are as follows:

	Cost	Discounts
(*a*)	$700	20%, $12\frac{1}{2}\%$, 10%
(*b*)	$7000	10%, 20%, 30%
(*c*)	$3000	15%, 12%
(*d*)	$9690	$\frac{1}{4}\%$, $33\frac{1}{3}\%$
(*e*)	$5000	43%, 10%
(*f*)	$3000	30%, 20%, 20%
(*g*)	$4000	20%, 10%, 5%
(*h*)	$6000	46%, 45%
(*i*)	$2760	30%, 15%

PARTIAL PAYMENTS

When payments are made on a note, these payments are known as **partial payments.** These payments and their dates of payment are written on the back of the note.

There are several methods of computing the amounts due on such notes. The best known and most widely used are the *United States Rule* and the *Merchants' Rule.*

UNITED STATES RULE

Find the amount of the principal until the time of the first payment, or until the sum of two or more payments equals or exceeds the interest.

Subtract the payment, or the sum of the two or more payments from the amount.

Proceed with the remainder as a new principal. Continue in this manner until the date of settlement.

A note for \$600.00 dated Aug. 10, 1902, was indorsed as follows:

1902, Dec. 15, \$100.
1903, Feb. 12, \$150.
1903, March 15, \$150.

Find the amount due April 1, 1903.

SOLUTION.

Dates of Indorsement			Time between Dates			Payments
yr.	mo.	da.	yr.	mo.	da.	
1902	8	10		4	5	\$100
1902	12	15		1	27	\$150
1903	2	12		1	3	\$150
1903	3	15			16	
1903	4	1				

\$600 = first principal. $\$600 \times .06 \times \frac{4\frac{1}{6}}{12} = \$12.50.$
12.50 = int. for 4 mo. 5 da.
\$612.50 = amt. for 4 mo. 5 da.
\$100 = first payment.
\$512.50 = second principal. $\$512.50 \times .06 \times \frac{1.9}{12} = \$4.87.$
4.87 = int. for 1 mo. 27 da.
\$517.37 = amt. for 1 mo. 27 da.
150. = second payment.
\$367.37 = third principal. $\$367.37 \times .06 \times \frac{1.1}{12} = \$2.02.$
2.02 = int. for 1 mo. 3 da.
\$369.39 = amt. for 1 mo. 3 da.
150. = third payment.
\$219.39 = fourth principal. $\$219.39 \times .06 \times \frac{16}{360} = \$.59.$
.59 = int. for 16 da.
\$219.98 = amt. for 16 da. \$219.98. *Ans.*

NOTE. Problems of this type are not as common in business as they were some years ago. Payments are now usually made at equal intervals of time.

EXERCISE 126

1. A man borrows from a loan association $3000 at 8% interest. If he pays $100 at the end of every 3 mo. for 1 yr., find the amount due at the end of the year.

2. A man borrows from a building and loan company $2000 at 8% interest. He pays in monthly installments of $50 each. How much does he owe at the end of 6 mo.?

3. If you borrow $1800 at 6% interest and pay annually $600, how much do you owe at the end of 3 yr.?

4. If you borrow $1500 at 8% interest and pay semiannually $300, how much do you owe at the end of 2 yr.?

5. If $2400 is borrowed at 8% interest, and paid in quarterly installments of $150 each, how much is due at the end of two years?

6. A man borrows $3000 at 8% interest, and pays in semiannual installments of $500 each. How much is due at the end of three years?

7. A man borrows $1800 at 6%, and pays $400 per year. How much is due at the end of five years?

8. If I borrow $2500 at 5% interest, and pay $500 a year, how much do I owe at the end of five years?

9. How much is due at the end of two years on a loan of $2500 at 5% interest when paid in semiannual installments of $600 each?

10. A note dated Jan. 15, 1904, for $4000 had the following indorsements: July 10, 1904, $700; Dec. 24, 1904, $600; June 18, 1905, $800; Nov. 13, 1905, $500; March 17, 1906, $900; Aug. 10, 1906, $400; Feb. 12, 1907, $400. How much was due May 28, 1907, at 6%?

THE MERCHANTS' RULE

The Merchants' Rule for computing interest on partial payments is used when settlement is made within one year from the date of the note. The rule is as follows:

I. Compute the amount of the face of the note until the date of settlement.

II. Compute the amount of each indorsement from its date until the date of settlement, and add these amounts.

III. Subtract the sum of the amounts of the indorsements from the amount of the face of the note. The remainder will be the amount due on the date of settlement.

Example. A note for $500, with interest at 8%, dated Jan. 10, 1902, had the following indorsements: April 1, 1902, $100; May 10, 1902, $200; July 1, 1902, $100. Find the amount due Nov. 1, 1902.

SOLUTION. From Jan. 10 to Nov. 1 is 9 mo. 21 da.
From April 1 to Nov. 1 is 7 mo.
From May 10 to Nov. 1 is 5 mo. 21 da.
From July 1 to Nov. 1 is 4 mo.

$500 for 9 mo. 21 da. amounts to		$532.33
$100 for 7 mo. amounts to	$104.67	
$200 for 5 mo. 21 da. amounts to	207.60	
$100 for 4 mo. amounts to	102.67	
The payments amount to		414.94
Balance due Nov. 1,		$117.39

EXERCISE 127

1. A note for $500, with interest at 6%, dated Jan. 2, 1902, had the following indorsements: March 2, 1902, $100; May 5, 1902, $150; July 10, 1902, $200. Find the amount due Nov. 1, 1902.

2. A note for $600, with interest at 8%, dated Feb. 1, 1902, was indorsed as follows: March 11, 1902, $200; May 15, 1902, $100; July 3, 1902, $100. Find the balance due Aug. 1, 1902.

3. A note for $800, with interest at 8%, dated Feb. 10, 1902, was indorsed as follows; March 15, 1902, $200; April 10, 1902, $100; June 3, 1902, $200. Find the amount due Nov. 1, 1902.

4. A borrows $1500 on Jan. 1, 1907, at 6% interest, and pays $450 each quarter. How much does he owe when three payments are made?

EXCHANGE

The written order of one party to another, requesting the payment of a specified sum of money, is called a **draft.**

The parties to a draft are **drawer**, **drawee**, and **payee**.

The following are forms of drafts:

SIGHT DRAFT

New Orleans, La., March 10, 1903.

$ 600.00

At sight, pay to the order of ------------------
------------------ Jacob Siegel ------------ the sum of
------ Six hundred ---------------------------- $\frac{00}{100}$ Dollars.

Value received and charge to the account of

William Thompson.

To David Meyer,
New York, N. Y.

A time draft payable *after sight* matures as a promissory note.

Bank drafts form one of the most important mediums of exchange. If a merchant owes money in New York, Chicago, or in any other place, he can always purchase a draft from his home bank, and by transmitting this through the mails settle his indebtedness.

TIME DRAFT

> *Louisville, Ky., Aug. 10, 1902.*
>
> *\$1500.00*
>
> *Thirty days after date pay to the order of*
>
> ---------------------------*Self*---------------------------
>
> --------------*Fifteen hundred*-------------- $\frac{00}{100}$ *Dollars.*
>
> *Value received and charge to the account of*
>
> *Richard Weaver.*
>
> *To Henry Morley,*
>
> *No. 8. Minneapolis, Minn. Sept. 9/12.*

BANK DRAFT

> *Galveston, Texas, March 10, 1903.*
>
> *No. 72.*
>
> **Hutchings, Sealy & Co., Bankers,**
>
> *Pay to the order of*------*Frances Newman*-------
>
> *\$300.00*----------*(Three hundred)*---------- $\frac{00}{100}$ *Dollars.*
>
> *H. C. Stein,* Cashier.
>
> *To Seventh National Bank,*
>
> *New York.*

The price of exchange is a matter of supply and demand. If, for instance, the Galveston banks freely offer New York or Chicago exchange, a purchaser will very probably get it for less than its face value. If exchange is scarce, its price is high. When exchange costs more than its face value, it is said to be at a *premium;* when it costs less than its face value, it is said to be at a *discount.* Exchange is *at par* when the cost of a draft is its face value. Exchange is always quoted as a rate per cent, or as so many dollars on a \$1000. The exchange is always reckoned on the face value of the draft. Thus $\frac{1}{8}\%$ premium, or \$1.25 premium, means that the cost of a draft for \$100 is \$100$\frac{1}{8}$, and the cost of a draft for \$1000 is \$1001.25. The rate of exchange is generally a fraction of 1%.

Express companies charge for money orders to any part of this country or Canada not over 30¢ for \$100. It is reasonable to assume that a person will not pay much more than this rate for exchange.

EXERCISE 128

1. What is the cost of exchange on a sight draft for \$400 at $\frac{1}{8}\%$ premium?

2. What is the cost of exchange on a sight draft for \$300 at $\frac{1}{4}\%$ premium?

3. Find the cost of a draft on New York for \$200 at $\frac{1}{8}\%$ premium.

4. Find the cost of a demand draft for \$500 at $\frac{1}{4}\%$ premium.

5. Find the cost of a sight draft for \$600 at $\frac{1}{4}\%$ premium.

6. Find how much must be paid for a sight draft for \$1000 at \$1.25 premium.

7. Find how much must be paid for a sight draft for \$2000 at \$2.50 premium.

8. Find the cost of remitting \$800 from Dallas to Chicago by means of a bank draft when exchange is $\frac{1}{4}\%$ premium.

9. Find the cost of a sight draft on Chicago for \$700, exchange being at $\frac{1}{4}\%$ discount.

10. Find the cost of a sight draft on New York for \$400 when exchange is $\frac{1}{4}\%$ discount.

11. Find the cost of exchange on a draft for \$1000, exchange being $\frac{3}{8}\%$ premium.

12. Find the cost of an express order for \$1200 at the rate of 30¢ per \$100.

13. If you had to remit \$200 to Chicago when exchange is $\frac{1}{4}\%$ premium, which would you prefer, to buy exchange or to buy an express money order at the rate of 30¢ for \$100?

14. What is the cost of an express money order for \$1000?

15. What is the cost of a sight draft for \$3000 at \$1.50 premium?

16. What is the cost of a sight draft for \$3000 at \$2.50 premium?

Example. What is the cost of a draft on Buffalo for \$800, payable in 30 da. at 6% interest, exchange being at the rate of $\frac{1}{4}\%$ premium?

SOLUTION:

The interest on \$800 for 30 da. at 6% = \$4.00.

Cost of exchange = $\frac{1}{4}\%$ of \$800 = \$2.00.

The cost of the draft = \$800 + \$2 − \$4 = \$798.

EXERCISE 129

Find the cost of the following sight drafts:

	Face	Rate of Exchange
1.	$2720,	$\frac{1}{4}\%$ premium.
2.	$3480,	$\frac{1}{2}\%$ premium.
3.	$5080,	$\frac{1}{8}\%$ premium.
4.	$6290,	$2.50 premium.
5.	$8290,	$1.25 discount.
6.	$9980,	$\frac{1}{4}\%$ discount.
7.	$5493,	$1.50 discount.
8.	$5280,	$1.50 premium.
9.	$6040,	$2.00 discount.
10.	$6090,	$1.25 premium.
11.	$5400,	$1.25 discount.
12.	$9870,	1% discount.
13.	$4500,	$\frac{1}{5}\%$ premium.
14.	$6920,	$\frac{1}{10}\%$ discount.
15.	$7780,	$\frac{1}{8}\%$ premium.
16.	$1234,	$\frac{1}{8}\%$ discount.

17. Find the cost of an express money order for $1400 at 30¢ for $100.

18. Find the cost of a draft for $10,000 at $\frac{1}{8}\%$ discount.

19. Find the cost of a draft for $600 payable in 30 da. without grace, the rate of interest being 6%, exchange being $\frac{1}{4}\%$ premium.

20. Find the cost of a draft for $1000 payable in 30 da., exchange being at par and the rate of interest being 8%.

21. Find the cost of a 45 da. draft for $900, exchange being at $\frac{1}{4}\%$ discount, and the rate of interest being 6%.

VALUE OF FOREIGN COINS IN UNITED STATES MONEY

(Proclaimed by the Secretary of the Treasury Oct. 1, 1906)

Country	Standard	Monetary Unit	Value in U. S. Gold Dollar
Argentine Republic	Gold	Peso	$ 0.965
Austria-Hungary	Gold	Crown	.203
Belgium	Gold	Franc	.193
Bolivia	Silver	Boliviano	.485
Brazil	Gold	Milreis	.546
Canada	Gold	Dollar	1.00
Central America	Silver	Peso	.485
Chile	Gold	Peso	.365
China	Silver	Tael { Shanghai	.726
		Haikwan	.808
		Canton	.792
Columbia	Gold	Dollar	1.00
Costa Rica	Gold	Colon	.465
Denmark	Gold	Crown	.268
Ecuador	Gold	Sucre	.487
Egypt	Gold	Pound (100 piasters)	4.943
France	Gold	Franc	.193
Germany	Gold	Mark	.238
Great Britain	Gold	Pound sterling	4.8665
Greece	Gold	Drachma	.193
Haiti	Gold	Gourde	.965
India	Gold	Pound Sterling	4.8665
Italy	Gold	Lira	.193
Japan	Gold	Yen	.498
Mexico	Gold	Peso	.498
Netherlands	Gold	Florin	.402
Newfoundland	Gold	Dollar	1.014
Norway	Gold	Crown	.268
Panama	Gold	Balboa	1.000
Peru	Gold	Libra	4.8665
Portugal	Gold	Milreis	1.08
Russia	Gold	Ruble	.515
Spain	Gold	Peseta	.193
Sweden	Gold	Crown	.268
Switzerland	Gold	Franc	.193
Turkey	Gold	Piaster	.044
Uruguay	Gold	Peso	1.034
Venezuela	Gold	Bolivar	.193

The Circulars of the Secretary of the U. S. Treasury, issued every 3 months, fix the legal equivalent of the monetary units in the principal countries. In countries which have adopted the silver standard monetary units fluctuate in value, owing to changes in the market value of silver.

Example 1. Express \$100 in florins (Netherlands).

$\$.402 = 1$ florin.

$$\therefore \$100 = \frac{100}{.402} \text{ of 1 florin} = 248.76 \text{ florins.}$$

EXERCISE 130

1. Express the value of £1 in French currency; in German currency; in Austrian currency.

2. Change to United States currency (*a*) 450 francs, (*b*) 550 pesetas, (*c*) 280 Norwegian crowns, (*d*) 900 marks, (*e*) 200 colons, (*f*) 720 Austrian crowns, (*g*) 450 Danish crowns.

3. Find in United States currency the difference in value between 1000 francs and \$198.

4. Find the value of \$1000 (*a*) in the money of Costa Rica; (*b*) in Austrian currency; (*c*) in Swedish currency.

5. What is the equivalent in United States money of 200,000 bolivars (Venezuela)?

6. What is the equivalent in United States money of 10,000 Mexican pesos?

7. Express \$1 in Peruvian monetary units.

8. Express \$1 in Norwegian monetary units.

9. Find in United States currency the difference between the value of 10,000 Japanese yen and \$5000.

10. What is the value of 10,000 Portuguese milreis?

ENGLISH MONEY

4 farthings = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound sterling (£)
£1 = 20*s.* = 240*d.* = 960 farthings

The abbreviations £, *s.*, *d.*, are the initial letters of the names of the Roman coins, libra, solidus, denarius.

EXERCISE 131

1. What is the equivalent of $7s.\ 5\frac{3}{4}d.$ in U. S. currency?

$$7s.\ 5\tfrac{3}{4}d. = \left(7 + \frac{5.75}{12}\right)s. = 7.479166s. = £.373958.$$

$$\$4.8665 \times .37396 = \$1.82.$$

2. The average freight rate on wheat per bushel for the year 1906 from Chicago to New York was:

By lake and canal 5.94¢.

By lake and rail 6.48¢.

By all rail delivered to steamer 8.10¢, and from New York to Liverpool $1\frac{7}{16}d.$

Find the cost of shipping 5 carloads of wheat averaging 79,000 lb. from Chicago to Liverpool.

(*a*) By lake and canal to seaboard and thence by steamer.

(*b*) By lake and rail to seaboard and thence by steamer.

(*c*) By rail to seaboard and thence by steamer.

3. Find the cost of shipping from New York to Liverpool 12,000 bu. of wheat at $1\frac{7}{16}d.$ per bu.

4. Find the cost of shipping 50 tons of sacked flour from Chicago to Liverpool at $10.11d.$ per 100 lb.

5. Find the cost of shipping from Galveston to Liverpool: —

(*a*) 10,884,000 lb. of cotton at $1s.\ 6\frac{1}{2}d.$ per 100 lb.

(*b*) 19,490 bu. of wheat at $3d.$ per bu.

(*c*) 37,920 bu. of corn at $5d.$ per bu.

6. Find the cost of shipping from St. Louis to Liverpool *via* New Orleans: —

(*a*) 6475 bbl. of flour at $1s.\ 8\frac{1}{2}d.$

(*b*) 97330 bu. wheat at $7\frac{5}{8}d.$

In examples 5 and 6 take £1 = \$4.80.

FOREIGN EXCHANGE

The settling of outstanding indebtedness between parties in different parts of this country by means of drafts is known as **domestic exchange.** The settling of debts between parties in this country and parties in foreign countries by means of drafts is called **foreign exchange.** Foreign drafts are known as **bills of exchange.** These are issued in duplicate, *i.e.* two are drawn exactly alike. The two bills are sent by different mails. As soon as one is paid, the other is void.

Usually exchange is a little above or a little below the *par of exchange*, *i.e.* intrinsic value of foreign coins.

Exchange on Great Britain is quoted at the number of dollars to £1; exchange on France is quoted at so many cents to the franc, or as so many francs to the dollar; exchange on Germany is quoted as so many cents to 4 marks, or as so many cents to a mark.

BILL OF EXCHANGE

No. 24896 *New York, N.Y., Jan. 4, 1903.*

£1200

At sight of this first of exchange (second of the same date and tenor unpaid)--pay to the order of---------------Williams Phibbs---------------

-----------Twelve hundred pounds sterling-----------

Value received and charge the same to the account of

Bernard Johnson.

To Macmillan & Co., Ltd.,

London, England.

EXERCISE 132

1. Find the cost of a draft on London for £1000, exchange being 4.872.

2. Find the cost of a bill of exchange for £550 15*s.*, exchange being 4.85.

3. Find the cost of a bill of exchange for 5750 francs, exchange being 5.15, *i.e.* 5.15 francs = \$1.

4. Find the cost of a draft on Manchester for £3500 6*s.* 8*d.* when exchange is quoted at $4.87\frac{1}{2}$.

5. Find the cost of a draft on Berlin for 1480 marks when exchange is quoted at $95\frac{1}{4}$, *i.e.* 4 marks = $95\frac{1}{4}$¢.

6. Find the cost of a draft on Vienna for 8500 crowns, rate of exchange being 1 crown = \$.202.

7. Find the cost of a draft for 1500 lira when exchange is quoted at 19.3, *i.e* 19.3¢ = 1 lira.

8. Find the cost of each of the following drafts:

	Face	Where Payable	Rate of Exchange
(*a*)	£928 10*s.*	Liverpool	4.85
(*b*)	£850 6*s.* 8*d.*	Belfast	$4.86\frac{1}{4}$
(*c*)	2500 marks	Berlin	24
(*d*)	9280 francs	Havre	5.16
(*e*)	8500 lira	Naples	$19\frac{1}{4}$
(*f*)	£584 13*s.* 4*d.*	Dublin	$4.85\frac{3}{4}$

Example. What is the face of a New York draft on Liverpool, which costs \$7297.50, when exchange is quoted at $4.86\frac{1}{2}$?

Solution. $\$4.865 = £1.$

$$\$1 = \frac{£1}{4.865}.$$

$$\$7297.50 = \frac{£7297.50}{4.865} = £1500.$$

EXERCISE 133

Find the face of each of the following drafts:

	Cost	Where Payable	Rate of Exchange
1.	$5835	London	$4.86\frac{1}{4}$
2.	$1218.75	Newcastle	$4.87\frac{1}{2}$
3.	$9063.05	Berlin	95.2
4.	$3724.90	Paris	19.3
5.	$718.24	Christiana	26.8
6.	$2366.82	Belfast	$4.86\frac{1}{2}$
7.	$1037	Paris	$5.18\frac{1}{2}$
8.	$5664.80	Hamburg	23.8

STOCKS AND BONDS

A **company** or **corporation** is a number of persons associated under a state law for the purpose of transacting business.

The modern company, or corporation, has supplanted in a great degree the old-time partnership. Nowadays business is conducted on so extensive a scale that a company often numbers several thousand persons.

The money which a company invests in business is called **capital,** or **stock.**

The stock of a company is usually divided into shares of $10, $50, or $100 each. The face value of a share in most large companies is $100.

These shares, as a rule, can be bought and sold. The buying and selling is usually done by agents called **stockbrokers.** The usual fee of a stockbroker is at the rate of $\frac{1}{8}\%$ of the par value for buying a share and $\frac{1}{8}\%$ of the par value for selling a share. A broker's fee is called **brokerage.**

If the business of a company is prosperous, a share may sell for more than its face value. The stock is then said to be at a **premium.** If the business is not prosperous, the value of a share in the market is likely to be less than its face value. The stock is then said to be at a **discount.**

When a share sells in the market for its face value, it is said to be **at par.** A quotation "10 % premium" means the same as 110 % of the face value. A quotation "10 % discount" means the same as 90 % of the face value.

The profits of a company, usually distributed to stockholders annually or semiannually, are called **dividends.** The dividend is generally so many dollars on a share, or a percentage of the face value of the stock. Thus, "4% dividend" means $4 on a share whose face value is $100.

Stock is of two kinds, **common** and **preferred.** The **preferred** stock is guaranteed a specified dividend whether the business is profitable or not. The **common** stock earns what is left over after expenses and the dividends on the preferred stock are paid.

A **bond** is a written instrument made by a government, municipal, state, or national, or by a corporation, for the purpose of borrowing money.

Bonds are of two kinds, **registered** and **coupon.** A **registered bond** is one which is recorded by number and by name of the owner, and it is not transferable except in writing and at the office of the treasurer. **Coupon bonds** are so called because they have interest slips attached to them, which are cut off as the interest falls due. These interest slips are payable to the bearer.

The following is quoted from a daily newspaper giving the New York stock market March 11, 1903.

Stocks

	SALES	HIGHEST	LOWEST	CLOSING BID
Atchison	30,200	82	$80\frac{1}{2}$	$81\frac{7}{8}$
Atchison pf.	1,800	$97\frac{1}{2}$	$97\frac{1}{8}$	$97\frac{3}{8}$
Baltimore & Ohio . .	14,400	93	$91\frac{3}{8}$	$92\frac{7}{8}$
Canadian Pacific . .	13,800	$129\frac{1}{2}$	$127\frac{5}{8}$	$129\frac{1}{2}$
Manhattan L	17,300	$142\frac{3}{4}$	$141\frac{3}{8}$	$142\frac{3}{4}$
Metropolitan St. Ry. .	21,100	$135\frac{1}{8}$	$131\frac{3}{8}$	134
Missouri Pacific . . .	30,800	$108\frac{1}{2}$	$106\frac{5}{8}$	$108\frac{3}{8}$
Pennsylvania	53,400	$144\frac{1}{8}$	$142\frac{1}{4}$	144
St. Paul	58,500	$168\frac{7}{8}$	$167\frac{1}{8}$	$168\frac{1}{4}$
Southern Pacific . . .	54,400	$63\frac{1}{4}$	$61\frac{1}{4}$	$62\frac{7}{8}$

Bonds

U. S. new 4s registered,	135	U. S. old 4s registered,	103
U. S. new 4s coupon,	136	Baltimore & Ohio 4s,	$102\frac{1}{2}$
U. S. old 4s coupon,	$109\frac{1}{2}$	Chicago B. & Q. new 4s,	$93\frac{1}{8}$

Notice that in stock quotations the fractions used are halves, quarters, eighths. Notice further that the variations in the prices of stock in the course of a day are quite considerable. There are instances where stocks have fallen in one day 50% of their face value. Why the prices vary so much would take the proverbial Philadelphia lawyer to explain. In the above quotations the first column shows the number of shares sold; the second column gives the highest prices paid for the shares; the third gives the lowest prices paid; and the fourth gives the closing bids. In the course of the day above referred to there were several other prices than those given.

Example. Find the cost of 10 shares of stock at $107\frac{1}{2}$.

SOLUTION

Market price $= \$107\frac{1}{2}$.
Brokerage $= \$\frac{1}{8}$.
Cost of 1 share $= \$107\frac{5}{8}$ (*i.e.* $\$107\frac{1}{2} + \$\frac{1}{8}$).
Cost of 10 shares $= \$107\frac{5}{8} \times 10 = \$1076.25$.

EXERCISE 134

1. Give the premium or discount of each of the stocks quoted March 11, 1903.

2. What is the cost of 20 shares of Atchison at $80\frac{7}{8}$? at $81\frac{7}{8}$?

3. What is the cost of 40 shares of Baltimore & Ohio at $92\frac{7}{8}$?

4. What is the cost of 200 shares of Canadian Pacific at $145\frac{1}{4}$?

5. What is the cost of 25 shares of stock at $118\frac{1}{2}$?

6. Find the cost of 50 shares of Missouri Pacific at $106\frac{5}{8}$.

7. Find the cost of 100 shares of each of the stocks quoted March 11, 1903, at the prices indicated in the fourth column.

Example **1.** What sum should be received from the sale of 40 shares of Pennsylvania at $144\frac{1}{8}$?

SOLUTION

Market value of 1 share of stock $= \$144\frac{1}{8}$.
Brokerage $= \$\frac{1}{8}$.
Sum received from 1 share $= \$144$.
Sum received from 40 shares $= \$144 \times 40 = \$5760$.

Example 2. What profit is made by purchasing 50 shares of stock at $142\frac{1}{4}$ and selling them at \$144?

SOLUTION. Cost of 1 share $= \$142\frac{1}{4} + \$\frac{1}{8} = \$142\frac{3}{8}$.

Receipts from 1 share $= \$144 - \$\frac{1}{8} = \$143\frac{7}{8}$.

Gain on 1 share $= \$1\frac{1}{2}$.

Gain on 50 shares $= \$1\frac{1}{2} \times 50 = \$75$.

EXERCISE 135

1. What should a person receive from the sale of 100 shares of Manhattan at the closing bid above given? from the sale of 100 shares at the lowest bid?

2. How much should a person receive from the sale of 100 shares of Atchison preferred at $97\frac{1}{2}$?

3. How many shares of Pennsylvania at $142\frac{1}{4}$ can be bought for \$14,237.50?

4. What profit is made by buying 100 shares of Baltimore & Ohio at $91\frac{3}{8}$, and selling them at $92\frac{7}{8}$?

5. If 100 shares of St. Paul are bought at $167\frac{1}{8}$ and sold the same day at $168\frac{5}{8}$, what is the gain?

6. If 200 shares of New York Central are bought at 165 and sold the following day at $167\frac{1}{4}$, what is the gain?

7. A speculator bought Missouri Pacific at the lowest price quoted March 11, 1903, and sold at the closing bid of the same day. If his gain was \$300, how many shares did he buy?

8. If a speculator buys Metropolitan Street Railway at $131\frac{1}{2}$ and sells on the same day at 134, find the number of shares bought to realize a profit of \$225.

9. How many shares of Atchison bought at $80\frac{5}{8}$ and sold at $81\frac{7}{8}$ realize a profit of \$1000?

10. How many Chicago B. & Q. new 4s at $93\frac{1}{8}$ can I buy for \$18,650?

Example 1. By investing money in 4 % stock a man realizes a profit of 5 % on the money invested. Find the price of the stock.

SOLUTION. The gain on \$100 invested is \$5.

The gain on \$20 invested is \$1.

The gain on \$80 invested is \$4.

The stock cost \$80 a share.

The market price of the stock is $\$80 - \$\frac{1}{8} = \$79\frac{7}{8}$.

Example 2. How much money must be invested in United States old 4s registered at 103 to produce an annual income of \$600?

SOLUTION. United States old 4s registered pay a dividend of \$4 a share.

The number of 4s in 600 is 150.

One hundred and fifty shares produce \$600.

The cost of 1 share is $103 + \frac{1}{8}$.

The cost of 150 shares is $103\frac{1}{8} \times 150 = \$15{,}462.50$.

EXERCISE 136

1. If stock paying a dividend of 8 % gives an income at the rate of 5 % on the money invested, what is the market value of the stock?

2. If 6 % stock produces a gain of 4 % on the money invested, find its market value.

3. How much money must I invest in Western Union at $92\frac{5}{8}$, paying a dividend of 5 %, to derive an annual income of \$1000?

4. How much must be invested in Pullman Palace Car at 218, paying a yearly dividend of 8 %, to derive an income of \$1200 a year?

5. If I buy $4\frac{1}{2}$ % stock at $97\frac{7}{8}$, what rate per cent do I get for my money?

6. What must I pay for 5% stock so as to make a profit of 8% on my investment?

7. How much must be invested in Northwestern, paying 6%, at 206, to derive an income of $900?

8. Which is the more profitable investment, 5% stock at $119\frac{7}{8}$ or 8% stock at $219\frac{7}{8}$?

9. How much must be paid for a $1000 bond at $87\frac{1}{2}$?

10. City bonds bought at $89\frac{7}{8}$ pay 5%. What rate of interest do they pay?

11. If I have $5000 stock in United States 4s at 102, what annual dividend do I receive? If I sold my stock, what should it bring?

12. How much should be realized by selling $10,000 of each of the stocks quoted above at the highest prices paid March 11, 1903?

13. Which is the better investment, Illinois Central, paying 6% dividend, at $137\frac{3}{8}$, or United States 4s at $109\frac{7}{8}$?

14. How many shares of stock must a broker sell to make $54.75 brokerage?

CUSTOMS AND DUTIES

Taxes levied on imported goods are called **duties.** Duties are of two kinds, **specific** and **ad valorem.** The latter is a percentage of the foreign value of the goods; the former is a definite amount of money on some standard quantity, such as a yard, gallon, pound, etc. Some articles are subjected to both specific and ad valorem duties. Duties are collected by United States customs officers at places known as **ports of entry.**

The United States revenue from duties for the year ending June 30, 1906, amounted to $300,251,877.79.

EXERCISE 137

1. Find the duty on 3200 lb. of refined sugar at 1.95¢ per pound.

2. Find the duty on 2000 bu. of potatoes at 25¢ per bushel.

3. Find the duty on 1800 lb. of salt at 12¢ per 100 lb.

4. Find the duty on 5 doz. parasols at 4*s.* 6*d.* each at 40% ad valorem.

5. Find the duty on 12 microscopes invoiced at £8 each, the rate of duty being 45% ad valorem.

6. The duty on books printed in English is 25% ad valorem. If I import the following books at the prices specified, how much duty in United States money will I have to pay?

Casey's Elements of Euclid	3*s.* 9*d.*
Casey's Analytical Geometry	10*s.* 0*d.*
Leatham's Spherical Trigonometry . .	4*s.* 0*d.*
Lamb's Infinitesimal Analysis . . .	12*s.* 0*d.*

7. Find the duty on the Clarendon Press edition of Shakespeare's plays, invoiced at 24*s.*, at 25% ad valorem.

8. A hardware merchant imported 80 doz. razors at 2*s.* 6*d.* each, duty $1.75 per dozen and 20% ad valorem. Find the total duty paid.

9. What is the duty on 100 yd. of treble ingrain carpet valued at 90¢ a square yard, the duty being 22¢ a square yard and 40% ad valorem?

10. What is the duty on 40 clocks valued at $4.50 each, at 40% ad valorem? If the clocks are retailed at a profit of 30%, find the selling price of these clocks.

11. Find the duty on 120 doz. linen cuffs at 7*s.* 6*d.* a dozen, the duty being 40¢ a dozen and 20% ad valorem.

12. Find the duty on a fur rug valued at $50, duty 35% ad valorem. At what price should the rug be sold so as to make a profit of 20%?

13. Calculate the duty levied on 5 doz. straw hats valued at $30 a dozen, the duty being $7 a dozen and 20% ad valorem.

14. What is the duty on 800 yd. of silk valued at $1.15 a yard at 60% ad valorem? At what price per yard should the silk be sold to make a profit of 20%?

15. Calculate the duty on 12 horses valued at $250 each at 25% ad valorem.

16. An Axminster carpet, 18 ft. by 12 ft., valued at $1.50 per square yard, is imported. Find the duty at 60¢ per square yard and 40% ad valorem.

17. What is the duty on 5 doz. opera glasses valued at $3.50 each at 45% ad valorem?

18. What is the duty on 500 lb. of glue valued at 50¢ per pound, the rate of duty being 15¢ per pound and 20% ad valorem?

19. A suit of clothes imported from England costs a merchant $38.80. The duty is 60% ad valorem. Taking £1 = $4.85, find the invoice price in pounds sterling.

20. Find the duty on 1200 yd. of linen invoiced at $7\frac{1}{2}$*d.* per yard at 45% ad valorem. At what price per yard should it be marked so that the dealer may give a discount of 10% and make a profit of 20%?

CHAPTER IV

INVOLUTION

In the language of mathematics, $3x$ means three times the number x; *i.e.* $3x$ may stand for three times any number whatever.

$a + b$ stands for the sum of any two numbers, a and b. $a - b$ stands for the difference of two numbers, a and b. In other words, $a - b$ is the number which when added to b gives a for sum.

ab stands for the product of two numbers, a and b. $\frac{a}{b}$ stands for the quotient obtained by dividing a by b. In other words, $\frac{a}{b}$ is that number which when multiplied by b gives a for product.

a^2 stands for the square of a. a^3 stands for the cube of a, $(a + b)^2$ stands for the square of the sum of two numbers, a and b.

$(a - b)^2$ stands for the square of the difference of two numbers, a and b.

$7ab$ stands for seven times the number ab, which is itself the product of two numbers, a and b.

$\frac{a+b}{2} + \frac{a-b}{2}$ stands for one half the sum of two numbers plus one half their difference.

$(a + b)x$ stands for the product of a number, x, by the sum of two numbers, a and b.

$(a-b)x$ stands for the product of a number, x, by the difference of two numbers, a and b.

EXERCISE 138

Add:

1.	2.	3.	4.	5.	6.
$2x$	$7y$	$4a$	$9b$	$4z$	$11a$
$3x$	$8y$	$9a$	$7b$	$7z$	$7a$
$5x$	$9y$	$11a$	$8b$	$9z$	$5a$

7.	8.	9.	10.	11.
$4ab$	$9xy$	$4bc$	$12ab$	$9ab$
$5ab$	$7xy$	$7bc$	$15ab$	$4ab$
$7ab$	$8xy$	$11bc$	$10ab$	$5ab$

12.	13.	14.	15.	16.	17.
$7bc$	ax	$10x$	$11b$	$12m$	$11x$
$8bc$	bx	ax	ab	am	bx
$9bc$					

EXERCISE 139

1. $7x-4x=?$
2. $6x-x=?$
3. $11x-8x=?$
4. $12x-2x=?$
5. $5x-3x=?$
6. $12a-a=?$
7. $15a-11a=?$
8. $17a-12a=?$
9. $18a-13a=?$
10. $12ab-3ab=?$
11. $11ab-4ab=?$
12. $5ab-ab=?$
13. $2xy-xy=?$
14. $14xy-10xy=?$
15. $17xy-9xy=?$
16. $19xy-7xy=?$
17. $ax-bx=?$
18. $cx-dx=?$
19. $mx-nx=?$

Example 1. Multiply $4x$ by $5x$.

SOLUTION. $4x \times 5x = 4 \times x \times 5 \times x = 4 \times 5 \times x \times x = 20x^2$.

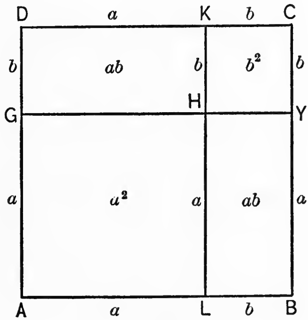
Example 2. Multiply $7x^2y$ by $3xy^2$.

SOLUTION. $7x^2y \times 3xy^2 = (7 \times x \times x \times y) \times (3 \times x \times y \times y) = 7 \times x \times x \times x \times 3 \times y \times y \times y = 21x^3y^3$.

Example 3. What is the square of $a+b$?

$$\begin{array}{l} a+b \\ \underline{a+b} \\ a^2+ab \\ \underline{+ab+b^2} \\ a^2+2ab+b^2 \end{array}$$

Multiply $a+b$ by a, and write the result, a^2+ab. Next multiply $a+b$ by b and write the result, $ab+b^2$. Then add. The following is a graphical representation of the result:



Let $AL=a$,

$LB=b$.

Then $AB=a+b$.

Let $ABCD$ be the square on AB, $ALHG$ be the square on AL. Then $HYCK$ is the square on b, and $GHKD = LBYH$, because their dimensions are equal to each other. Now, $ABCD = ALHG + GHKD + HYCK + LBYH = ALHG + HYCK + 2\,GHKD$.

$\therefore (a+b)^2 = a^2+b^2+2\,ab$. Hence, we have the following important conclusion:

The square of the sum of two numbers equals the square of the first number plus the square of the second number plus twice the product of the two numbers.

The process of finding the power of a number is called **involution.**

Example 1. Square $x + 9$.

SOLUTION. $(x+9)^2 = x^2 + 9^2 + 2 \times 9 \times x = x^2 + 81 + 18\,x$, or $x^2 + 18\,x + 81$.

Example 2. What is the square of 43?

SOLUTION. $43^2 = (40 + 3)^2 = 40^2 + 3^2 + 2 \times 40 \times 3 = 1600 + 9 + 240 = 1849$.

EXERCISE 140

Find by Statement (3) the square of:

1. $10 + x$.	**7.** $70 + b$.	**13.** 29.	**19.** 72.	**25.** 89.
2. $20 + x$.	**8.** $80 + c$.	**14.** 37.	**20.** 76.	**26.** 92.
3. $30 + x$.	**9.** $90 + y$.	**15.** 47.	**21.** 78.	**27.** 93.
4. $40 + x$.	**10.** 14.	**16.** 54.	**22.** 79.	**28.** 67.
5. $50 + x$.	**11.** 15.	**17.** 62.	**23.** 84.	**29.** 98.
6. $60 + a$.	**12.** 24.	**18.** 68.	**24.** 87.	**30.** 55.

EXERCISE 141

1. Find the square of .1, .2, .3, .4, .5, .6, .7, .8, .9.

2. Find the square of the reciprocals of the numbers from 1 to 20 inclusive.

3. Find the squares of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{7}$, $\frac{9}{10}$, $\frac{11}{12}$, $\frac{10}{13}$, $\frac{9}{16}$.

4. Find the squares of $\frac{5}{3}$, $\frac{7}{3}$, $\frac{9}{7}$, $1\frac{1}{4}$, $1\frac{3}{4}$, $1\frac{7}{8}$, $2\frac{1}{5}$, $6\frac{1}{4}$.

5. How does the square of a fraction compare in value with the fraction itself?

6. Find the cubes of the reciprocals of the numbers from 1 to 20 inclusive.

7. Find the value of $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$.

8. Square each of the following numbers and give the product correct to four decimal figures: (*a*) 1.732, (*b*) 9.256, (*c*) 5.401, (*d*) 8.129, (*e*) .6834, (*f*) .7609, (*g*) 9.482, (*h*) .7071, (*i*) .7746, (*j*) .9487.

EVOLUTION — SQUARE ROOT

By the **square root** of a number is meant that number which when squared produces the given number. Thus, 4 is the square root of 16, since $4^2 = 16$. 7 is the square root of 49 for a similar reason.

The square root is defined also as one of the two equal factors of a number. Thus, $7 \times 7 = 49$. One of the factors is the square root of 49.

The symbol for square root, $\sqrt{}$, is called the *radical sign*. It is a degenerate form of the first letter of the word *radix*, the Latin word for root. The exponent $\frac{1}{2}$ is also used as a sign for the square root. $\sqrt{49}$, $49^{\frac{1}{2}}$ are the two ways of indicating the same process, namely, the extraction of the square root of 49. The number written under the radical sign is called the *radicand*.

Students should fix firmly in mind : —

$10^2 = 100$	$40^2 = 1600$	$70^2 = 4900$
$20^2 = 400$	$50^2 = 2500$	$80^2 = 6400$
$30^2 = 900$	$60^2 = 3600$	$90^2 = 8100$

Example 1. What is the square root of 5329?

Solution. By trial the square root of 5329 is more than 70 and less than 80.

$$\therefore \sqrt{5329} = 70 + x.$$
$$\therefore 5339 = 4900 + 140\,x + x^2$$

Subtract 4900, and get,

$$140\,x + x^2 = 439.$$
$$\therefore (140 + x)x = 429.$$

Since 140 is contained in 429 3 times, try 3 as a value of x.

$$(140 + 3)3 = 429.$$
$$\therefore \sqrt{5329} = 70 + 3 = 73.$$

Example **2.** $\sqrt{9025} = ?$

SOLUTION. $\sqrt{9025} = 90 + x.$

$$\therefore 8025 = 8100 + 180\,x + x^2.$$

$$\therefore 180\,x + x^2 = 925.$$

$$\therefore (180 + x)x = 925.$$

Since 180 is contained in 925 5 times, try 5 for the next figure of the root.

$$(180 + 5)5 = 925.$$

$$\therefore \sqrt{9025} = 90 + 5 = 95.$$

In practice, the work is contracted as follows: Beginning at the decimal point, point off the figures of the number in periods of two figures each. By trial find the greatest digit whose square is contained in the number denoted by the period to the left. Write it as the first figure of the root, and write also its square. Subtract the latter from the period to the left and bring down the next period. Double the part of the root just found for trial divisor. Find next the number of times the trial divisor is contained in the number denoted by the remainder and the period brought down. Write this result in the quotient and in the divisor and then multiply.

	9 5
	90.25
	81
	9 25
185	9 25

EXERCISE 142

Find the square root of:

1. 169.
2. 441.
3. 625.
4. 961.
5. 1024.
6. 1849.
7. 2809.
8. 3844.
9. 4225.
10. 5329.
11. 5776.
12. 6724.
13. 7056.
14. 7569.
15. 8464.
16. 9216.
17. 9604.
18. 9801.
19. 7921.
20. 6084.
21. 4761.
22. 3481.
23. 2401.
24. 3364.

Example. $\sqrt{127449} = ?$

$$\begin{array}{r|l}
 & \;\;3\;\;5 \\
 & 12\;74\;49 \\
 & \;\;9 \\
65 & \;\;3\;74 \\
 & \;\;3\;25 \\
\hline
 & \qquad 49\;49
\end{array}$$

SOLUTION. Divide the figures of the number into periods of two as in the previous exercises. Then proceed to extract the square root of the number denoted by the two periods to the left.

The answer is obviously 350 + some number. Let x represent this number; hence

$$350 + x = \sqrt{127449}.$$
$$\therefore (350 + x)^2 = 127449.$$
$$\therefore 122500 + 700\,x + x^2 = 127449.$$
$$\therefore 700\,x + x^2 = 4949.$$
$$\therefore (700 + x)x = 4949.$$

Since 700 is contained in 4900 7 times, try 7 as a value of x.

$$(700 + 7)7 = 4949.$$
$$\therefore \sqrt{127449} = 350 + 7 = 357.$$

Notice the trial divisor is twice the part of the root found. Therefore in a problem in square root where the radicand is an integer consisting of five or six figures, proceed in exactly the same way as has been done in a problem consisting of four figures. Beginning once more, the solution in its contracted form stands as follows:

$$\begin{array}{r|l}
 & \;\;3\;\;5\;\;7 \\
 & 12\;74\;49 \\
 & \;\;9 \\
65 & \;\;3\;74 \\
 & \;\;3\;25 \\
\hline
707 & \quad 49\;49 \\
 & \quad 49\;49 \\
\hline
\end{array}$$

The trial divisor is always twice the part of the root already found.

EXERCISE 143

Extract the square root of:

1. 100,489.	**7.** 229,441.	**13.** 474,721.
2. 110,224.	**8.** 277,729.	**14.** 501,264.
3. 120,409.	**9.** 310,249.	**15.** 654,481.
4. 171,396.	**10.** 354,025.	**16.** 772,641.
5. 190,096.	**11.** 391,876.	**17.** 819,025.
6. 199,809.	**12.** 456,976.	**18.** 826,281.

Memorize : —

$(.1)^2 = .01$	$(.5)^2 = .25$	$(.9)^2 = .81$
$(.2)^2 = .04$	$(.6)^2 = .36$	$(.11)^2 = .0121$
$(.3)^2 = .09$	$(.7)^2 = .49$	$(.12)^2 = .0144$
$(.4)^2 = .16$	$(.8)^2 = .64$	$(.01)^2 = .0001$

To extract the square root of a decimal, begin *at the decimal point*, and proceeding to the right, point off the figures in periods of two ; next proceed as if the number were an integer. Thus, in taking the square root of .0225, first point off in periods of two figures each. This gives .02 25. Next extract the root of the number denoted by the figures 225. The result is 15. Hence, the required root is .15.

To extract the square root of a number part integer and part decimal, begin at the decimal point, and proceeding to the left, point off the integral part in periods of two figures each; next point off the decimal part in periods of two figures each, beginning *at the decimal point.* If there are not enough figures in the decimal part to make an exact number of periods, annex a cipher or as many ciphers as are necessary to make the required number of periods.

Example. Extract the square root of 1.7.

$$\begin{array}{r|l}
\multicolumn{2}{l}{1.\ \ 3\ \ 0\ \ 3\ \ 8\ \ 4} \\
\multicolumn{2}{l}{1.\ 70\ 00\ 00\ 00\ 00} \\
\multicolumn{2}{l}{1.} \\
\hline
23 & 70 \\
 & 69 \\
\hline
2603 & 10000 \\
 & \ \ 7809 \\
\hline
26068 & 219100 \\
 & 208544 \\
\hline
260764 & 1055600 \\
 & 1043056 \\
\hline
\end{array}$$

Double 1 for the first trial divisor. Double 13 for the next trial divisor. Then find the next figure of the root is 0. Write it in the root and in the trial divisor. Then annex two more ciphers, and find that the next figure of the root is 3, and so on.

EXERCISE 144

Extract the square root of:

1. .150932.	**4.** .2909.	**7.** .5319.	**10.** .083.
2. .246016.	**5.** .2632.	**8.** .61575.	**11.** .062.
3. .3448.	**6.** .4616.	**9.** .784.	**12.** .0037.

To extract the square root of a fraction when its numerator and denominator are perfect squares is a simple matter. Thus, the square root of $\frac{25}{49}$ is $\frac{5}{7}$; the square root of $1\frac{17}{64}$, or $\frac{81}{64}$, is $\frac{9}{8}$, or $1\frac{1}{8}$.

To get the square root of a fraction, take the square root of the numerator, and the square root of the denominator, and then write the former result for numerator and the latter result for denominator. The fraction thus found is the required square root.

Example 1. What is the square root of $\frac{17}{36}$?

SOLUTION. $\sqrt{17} = 4.123$; $\sqrt{36} = 6$.

$$\therefore \sqrt{\tfrac{17}{36}} = \frac{4.123}{6} = .687.$$

Example 2. What is the square root of $\frac{23}{39}$?

SOLUTION. $\sqrt{23} = 4.796$; $\sqrt{39} = 6.245$.

$$\therefore \sqrt{\tfrac{23}{39}} = \frac{4.796}{6.245} = .768.$$

This is a roundabout way to take the square root of $\frac{23}{39}$. A shorter and better way is to reduce the fraction to an equivalent decimal, and then to extract the square root of this decimal.

To extract the square root of a fraction whose denominator is not a perfect square, reduce the fraction to an equivalent decimal and then extract the square root of this decimal.

EXERCISE 145

Extract, to three decimal figures, the square root of:

1. 1.2.	4. 5.2.	7. $3\frac{1}{8}$.	10. $4\frac{1}{2}$.	13. $\frac{2}{3}$.
2. 4.25.	5. 3.3.	8. $9\frac{1}{6}$.	11. $2\frac{7}{8}$.	14. $\frac{2}{5}$.
3. 1.1.	6. $5\frac{1}{4}$.	9. $1\frac{3}{8}$.	12. $\frac{7}{12}$.	15. $\frac{7}{11}$.

EXERCISE 146

PROBLEMS INVOLVING SQUARE ROOT

1. The area of a square field is 1 A. Find the length in yards of one of its sides.

2. The area of a square field is 12 A. Find the length in yards of one of its sides.

3. The dimensions of a rectangle are 289 yd. and 196 yd. Find the side of an equivalent square.

4. The dimensions of a rectangle are $1\frac{1}{2}$ mi. and .7 mi. Find, correct to four decimal figures, the side of an equivalent square.

5. Find in rods the perimeter of a square field whose area is $\frac{1}{3}$ of a square mile.

6. The area of a rectangle whose length is twice its breadth is 10 A. Find its dimensions in yards.

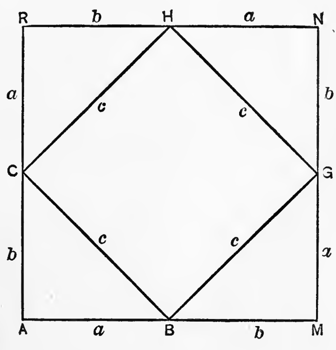
HINT. Draw a diagram; divide it into two equal parts by a line parallel to its width. Notice what each part is.

7. The area of a rectangle whose length is three times its width is 20 A. Find its dimensions in yards.

8. A square and a rectangle have the same area, namely 40 A. If the length of the rectangle is twice its width, find in rods the difference between their perimeters.

The side of a right triangle opposite the right angle is called the **hypothenuse.** The other two sides are called the **legs** of the right triangle. One of the legs is called the **base** of the right triangle, and the other leg is called the **altitude** of the right triangle.

In a right triangle the square on the hypothenuse is equal to the sum of the squares on the two legs. This is the famous Pythagorean Theorem.



Designate the sides of the right triangle ABC by the letters a, b, c. $(a+b)^2 = a^2 + b^2 + 2ab$. But $(a+b)^2 = c^2 + 4$ triangles, each having for its base and altitude a and b. $AMNR = c^2 + 4 \times \frac{1}{2}ab = c^2 + 2ab$.

$$a^2 + b^2 + 2ab = c^2 + 2ab.$$

$$\therefore a^2 + b^2 = c^2.$$

Example. In a right triangle the legs are 7 and 24. Find the hypothenuse.

SOLUTION.
$$a^2 + b^2 = c^2.$$
$$\therefore 7^2 + 24^2 = c^2.$$
$$\therefore 49 + 576 = c^2.$$
$$\therefore c^2 = 625.$$
$$\therefore c = \sqrt{625} = 25.$$

EXERCISE 147

1. In a right triangle, given $a = 6$, $b = 8$, find c.
2. In a right triangle, given $a = 5$, $b = 12$, find c.
3. In a right triangle, given $a = 8$, $b = 15$, find c.
4. In a right triangle, given $a = 20$, $b = 21$, find c.
5. In a right triangle, given $a = 56$, $b = 90$, find c.
6. In a right triangle, given $a = 20$, $b = 99$, find c.
7. In a right triangle, given $a = 17$, $b = 144$, find c.
8. In a right triangle, given $a = 39$, $b = 80$, find c.
9. In a right triangle, given $a = 51$, $b = 140$, find c.
10. In a right triangle, given $a = 44$, $b = 52.5$, find c.
11. In a right triangle, given $a = 87$, $b = 416$, find c.
12. In a right triangle, given $a = 136$, $b = 273$, find c.
13. In a right triangle, given $a = 145$, $b = 408$, find c.
14. In a right triangle, given $a = 207$, $b = 224$, find c.
15. A ladder is placed 14 ft. from a wall 48 ft. high. How long must the ladder be to reach to the top of the wall?
16. Find the length of the diagonal of a square if one side of the square is 10 rods.
17. Find the length of the diagonals of a rectangle, the dimensions of the rectangle being 17 rd. and 25 rd.

Example. If the hypothenuse of a right triangle is 493 and one leg is 468, find the other leg.

SOLUTION. Let the required leg be a.

Then, $$a^2 + 468^2 = 493^2.$$

$$\therefore a^2 + 219,024 = 243,049.$$

Subtract 219,024 from each member of this equation.

$$\therefore a^2 = 24,025.$$

$$\therefore a = 155.$$

EXERCISE 148

1. Hypothenuse = 377, base = 345, find the altitude.
2. Hypothenuse = 545, base = 513, find the altitude.
3. Hypothenuse = 449, base = 351, find the altitude.
4. Hypothenuse = 5.05, base = 4.56, find the altitude.
5. Hypothenuse = .461, base = .38, find the altitude.
6. Hypothenuse = .481, alt. = .36, find the base.
7. Hypothenuse = .641, alt. = .609, find the base.
8. Hypothenuse = .773, alt. = .195, find the base.
9. Hypothenuse = .697, alt. = .528, find the base.

AREAS OF PLANE TRIANGLES

The following rule gives the **area of any triangle**:

(1) Add the three sides and take half the sum.

(2) Subtract each side separately from the half sum.

(3) Find the continued product of the three remainders and the half sum.

(4) The square root of this product is the area.

The proof of this rule is too difficult to be given in an elementary arithmetic. This rule enables one to find the area of a tract of land such as a farm.

Example. Find the area of a triangle whose sides are 34 ch., 65 ch., and 93 ch.

SOLUTION

$$\begin{array}{r} 34 \\ 65 \\ \underline{93} \\ 2)\overline{192} \\ 96 \end{array} \qquad \begin{array}{r} 96 \\ \underline{34} \\ 62 \end{array} \qquad \begin{array}{r} 96 \\ \underline{65} \\ 31 \end{array} \qquad \begin{array}{r} 96 \\ \underline{93} \\ 3 \end{array}$$

$$\text{Area} = \sqrt{96 \times 62 \times 31 \times 3} = \sqrt{553535} = 744.$$

$$\therefore \text{area} = 744 \text{ sq. ch.} = 74.4 \text{ A.}$$

EXERCISE 149

Find the area of each of the following triangles:

1. Given the sides, 13, 20, 21.
2. Given the sides, 13, 30, 37.
3. Given the sides, 33, 34, 65.
4. Given the sides, 35, 52, 73.
5. Given the sides, 29, 60, 85.
6. Given the sides, 140, 143, 157.
7. Given the sides, 507, 603, 721.
8. Given the sides, 46 rd., 75 rd., 109 rd.
9. Given the sides, 40 rd., 51 rd., 77 rd.
10. Given the sides, 3.5 ch., 10 ch., 11.7 ch.
11. Given the sides, 5.6 ch., 6.1 ch., 7.5 ch.
12. Find area of a triangle, each side being 10 rd.
13. Find area of a triangle, each side being 50 rd.
14. Find the area of an isosceles right triangle if the hypothenuse is 27 inches.
15. Find the area of a square whose diagonal is 72 feet.
16. Find the side of a square equivalent to the difference of two squares whose sides are 89 feet and 68 feet.

MENSURATION OF THE CIRCLE, ETC.

Take a string and find the length of the circumference of a circle. Take another string and find the length of the diameter of the circle. Divide the former result by the latter to get the ratio of the circumference of the circle to its diameter.

Let $C =$ area of circle.
$r =$ radius.
$c =$ circumference.
$d =$ diameter.

The ratio of the circumference of a circle to its diameter is approximately 3.14159265. This ratio is denoted by the Greek letter π (Pi). In cases where the numbers involved are not very large, or where extreme accuracy is not demanded, $3\frac{1}{7}$ is a sufficiently accurate approximation of π. The ratio 355 : 113 is a close approximation to the value of π. 3.1416 is generally taken as the value of π. In this chapter we shall consider $\pi = 3.1416$.

Since $\frac{c}{d} = \pi$, $\therefore\ c = \pi d = \pi\,(2\,r) = 2\,\pi r$.

Express in words the relation $c = \pi d$.
Express in words the relation $c = 2\,\pi r$.

EXERCISE 150

Find the circumference when:

1. The diameter is 22.
2. The diameter is 46.
3. The diameter is 150.
4. The diameter is 164.
5. The diameter is 196.
6. The radius is 67.
7. The radius is 86.
8. The radius is 3.6.
9. The radius is 5.9.
10. The radius is 7.3.

11. Find d when $c = 320.44$.
12. Find d when $c = 477.52$.
13. Find d when $c = 24.50$.
14. Find d when $c = 41.7$.
15. Find r when $c = 377$.
16. Find r when $c = 53.41$.
17. Find r when $c = 60.319$.
18. Find r when $c = 42.097$.

19. The diameter of the front wheel of a carriage is 3 ft. 6 in. How many times does the wheel revolve in going 1 mi.? Take $3\frac{1}{7}$ as the value of π.

AREA OF A CIRCLE

A **sector of a circle** is a portion of a circle bounded by two radii and their included arc.

If the arc of a sector is very small, the sector will not sensibly differ from a triangle. Hence, the area of a sector equals $\frac{1}{2}$ the arc multiplied by the radius, and as a circle may be divided into a large number of sectors, hence,

$$C = \tfrac{1}{2}\,cr, \text{ but } c = 2\,\pi r.$$

$$\therefore\ C = \tfrac{1}{2} \times 2\,\pi r \times r = \pi r^2.$$

Also, since $\qquad r^2 = (\frac{1}{2}\,d)^2 = \frac{1}{4}\,d^2,$

$$\therefore\ \pi r^2 = \frac{\pi}{4}\,d^2.$$

Hence, the **three rules for finding the area of a circle:**

(1) Multiply one half the circumference by the radius.

(2) Multiply the square of the radius by π.

(3) Multiply the square of the diameter by $\frac{1}{4}\,\pi$.

There is a **fourth rule** for finding the area of a circle:

$$c = 2\,\pi r. \quad \therefore\ r = \frac{c}{2\,\pi}.$$

$$\therefore\ \pi r^2 = \pi \times \frac{c^2}{4\,\pi^2} = \frac{1}{4\,\pi} \times c^2 = .07958\,c^2.$$

That is, *the area of a circle equals the square of its circumference multiplied by* .07958.

EXERCISE 151

1. Given $r=14$, find C.
2. Given $r=22$, find C.
3. Given $r=36$, find C.
4. Given $r=4.7$, find C.
5. Given $r=6.5$, find C.
6. Given $r=8.6$, find C.
7. Given $r=9.7$, find C.
8. Given $d=78$, find C.
9. Given $d=64$, find C.
10. Given $d=96$, find C.
11. Given $d=74$, find C.
12. Given $d=92$, find C.
13. Given $c=100$, find C.
14. Given $c=78$, find C.
15. Given $c=83$, find C.
16. Given $c=93$, find C.
17. Given $c=8.7$, find C.
18. Given $c=6.9$, find C.
19. Given $c=9.8$, find C.
20. Given $c=10.8$, find C.

Example 1. Given the area of a circle equal to 535.08, find the radius of the circle.

SOLUTION. $\pi r^2 = C.$

$$\therefore\ 3.1416\, r^2 = 535.08.$$

$$\therefore\ r^2 = \frac{535.08}{3.1416} = 170.31.$$

$$\therefore\ r = 13.05, \text{ nearly.}$$

Example 2. Given the area of a circle equal to 658.98, find the circumference of the circle.

SOLUTION. $.07958\, c^2 = C.$

$$.07958\, c^2 = 658.98$$

$$\therefore\ c^2 = \frac{658.98}{.07958} = 8218, \text{ nearly.}$$

$$\therefore\ c = 91, \text{ nearly.}$$

EXERCISE 152

1. Given $C=3019.1$, find r.
2. Given $C=907.9$, find r.
3. Given $C=3421.2$, find r.
4. Given $C=5541.8$, find r.
5. Given $C=21.24$, find r.

6. Given $C = 40.715$, find d.

7. Given $C = 66.476$, find d.

8. Given $C = 75.43$, find d.

9. Given $C = 109.36$, find d.

10. Given $C = 141.03$, find d.

11. Given $C = 4.5964$, find c.

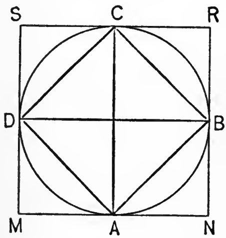
12. Given $C = 6.883$, find c.

13. Given $C = 779.94$, find c.

14. Given $C = 57{,}495$, find c.

15. Given $C = 33{,}621$, find c.

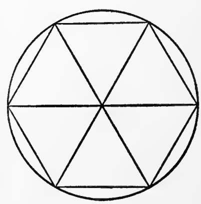
16. How long must a rope be so that by tying one end of it to a stake driven into the ground and fastening the other end to a cow's horn, the cow may graze over 1 A.?



17. *ABCD* is a square described in a circle; *MNRS* is a square described about the circle. If the radius of the circle is 12, find the areas of *ABCD*, *MNRS*, and of the circle.

18. If the diameter of a circle is 34 in., find the difference between the area of the circle and of the square described in the circle.

19. If the diameter of a circle is 88 in., find the difference between the area of the square described about the circle and the area of the circle.



20. A regular hexagon is a figure of six sides each of the same length, and its six angles are equal to one another. A hexagon may be broken up into six equilateral triangles.

If the side of a regular hexagon is 10, find its area.

21. If a regular hexagon is described in a circle, its side equals the radius of the circle. If the radius of a circle is 16 in., find the difference between the areas of the circle and the regular hexagon described in the circle.

22. A regular hexagon and a square have each a perimeter of 60 in. Find their areas.

23. The circumference of a circle equals the perimeter of a square. Find which has the larger area.

Take any angle, A, formed by two radii in a circle, and take another angle, B, formed by two radii of the same circle, then designate the subtended arcs by arc A and arc B, then $A : B =$ arc A : arc B. The reason for this is: If A were twice B, then arc A would be twice arc B. If A were three times B, then arc A would be three times arc B, and so on for any number of times (Fig. 1).

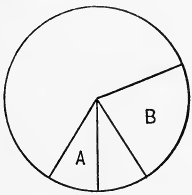


Fig. 1.

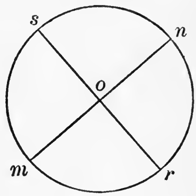
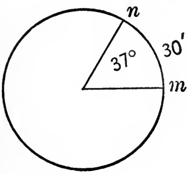


Fig. 2.

$$\frac{A}{B} = \frac{\text{arc } A}{\text{arc } B}.$$

$$\text{or } A : B = \text{arc } A : \text{arc } B. \quad (a)$$

If two diameters, mn, sr (Fig. 2), are drawn at right angles, the four angles at the center are all right angles. If each of the right angles at O is divided into 90 equal parts, each part is equal to $1°$. The arcs these angles intercept on the circumference are all equal, and as the 360th part of the circumference is called a degree on the circumference, hence (a) the number of degrees in an angle at the center is equal to the number of degrees in its arc on the circumference. This fact is generally stated as follows: A **central angle** is measured by its intercepted arc.



Example. The radius of a circle is 15 in. Find the length of an arc of 37° 30′ of the circumference of this circle.

SOLUTION. 360° : 37° 30′ = length of the circumference : length of the arc *mn*.

$$c = 2\pi r.$$

$$\therefore\ c = 2 \times 3.1416 \times 15 \text{ in.} = 94.248 \text{ in.}$$

$$\therefore\ 360 : 37\tfrac{1}{2} = 94.248 \text{ in.} : \text{arc } mn.$$

$$\therefore\ 360 \times \text{arc } mn = 94.248 \times 37\tfrac{1}{2} \text{ in.}$$

$$\therefore\ \text{arc } mn = \frac{94.248 \times 37\frac{1}{2} \text{ in.}}{360} = 9.82 \text{ in., nearly.}$$

EXERCISE 153

1. The radius of a circle is 37 in. Find the length of an arc of 72° of this circle.

2. The radius of a circle is 94 in. Find the length of an arc of 30° of this circle.

3. What is the length of an arc of 1° on a circle whose radius is 58 ft.?

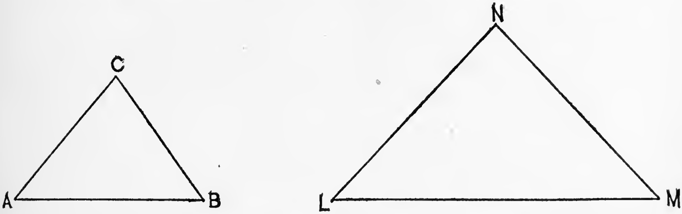
4. What angle does an arc of 40.212 ft. subtend at the center of a circle whose radius is 64 ft.?

5. What angle does an arc of 6.032 ft. subtend at the center of a circle whose radius is 96 ft.?

6. The distance of the moon from the earth is 239,000 mi., and the diameter of the moon is 2170 mi. To an observer on the earth, what angle does the moon's diameter subtend?

SIMILAR FIGURES

Similar figures are figures having the same form.



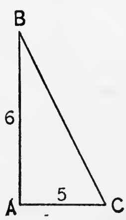
Examples. The triangles ABC, LMN, are similar triangles.

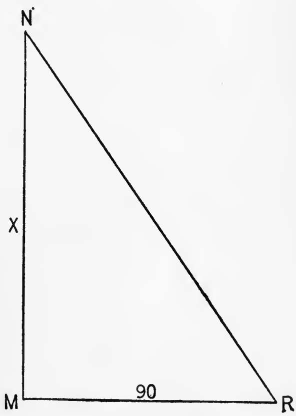
All regular polygons of the same number of sides are similar figures. The drawing which a surveyor makes of a tract of land is similar to the tract of land.

It is shown in geometry that **corresponding dimensions of similar figures have the same ratio**; also that **the areas of similar figures are to each other as the squares of their corresponding dimensions.**

Example 1. When a pole 6 ft. high casts a shadow 5 ft., how high is a steeple whose shadow is 90 ft.?

Solution. Let AB represent the pole, AC its shadow, x the steeple, and MR its shadow. Then $5:90 = 6:x$.
$\therefore\ x = 108$. *Ans.* 108 ft.





Example 2. The area of a triangle, one of whose sides is 5 rd., is 11 sq. rd. Find the corresponding side of a similar triangle whose area is three times as great.

SOLUTION. Let X equal the required side.

The area of the first triangle : the area of the second triangle = square of the side of the first triangle : square of the side of the second triangle.

$$i.e. \quad 11 : 33 = 5^2 : X^2.$$

$$\therefore \quad 11\,X^2 = 33 \times 5^2.$$

$$\therefore \quad X^2 = \frac{33 \times 5^2}{11} = 3 \times 5^2 = 75.$$

$$\therefore \quad X = \sqrt{75} = 8.662. \quad \textit{Ans. } 8.662 \text{ rd.}$$

EXERCISE 154

1. When a tree 90 ft. high casts a shadow 75 ft. long, find the length of the shadow cast by a pole 24 ft. high.

2. How high is an object which casts a shadow 110 ft. when a pole 8 ft. high casts a shadow 5 ft.?

3. A map is drawn to a scale of 40 mi. to 1 in. On this map two cities are $2\frac{3}{4}$ in. apart. How many miles are there between these cities?

4. In a map of a city two public buildings are $9\frac{1}{2}$ in. distant. If the map is drawn to the scale of 1 in. to $\frac{3}{4}$ of a mile, how far is it from one of these buildings to the other?

5. The area of a triangle is 15 sq. ft., and one of its sides is 10 ft. Find the corresponding side of a similar triangle five times as large.

6. The altitude of a triangle is 10 ft. If the triangle is divided into two equal parts by a line parallel to its base, how far from the vertex must this line be drawn?

7. Corresponding sides of two similar quadrilaterals are in the ratio of 4 to 11. Find the ratio of their areas.

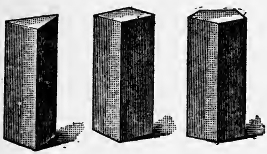
8. The diameters of two circles are 12 and 18 in. Find the ratio of their areas.

9. The distance between two cities is 90 mi., and on a map containing both cities their positions are distant $5\frac{5}{6}$ in. What area is represented by a circle of $\frac{1}{2}$ in. radius on this map?

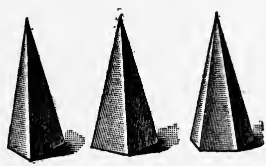
SURFACES OF THE PRISM, PYRAMID, CYLINDER, CONE, AND SPHERE

A **right prism** is a solid, two of whose faces are equal and parallel polygons, and whose other faces are rectangles.

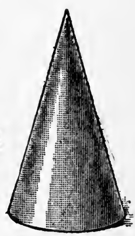
The upper and lower faces are called the *bases*, and the other faces are called *lateral faces*.



PRISMS



PYRAMIDS



CONE

A **right pyramid** is a solid whose base is a regular polygon, and whose other faces are triangles equal in area.

A **cone** is a solid made by the revolution of a right triangle about one of its legs.

A **cylinder** is a solid made by the revolution of a rectangle about one of its sides when that side is fixed.

The following rules may be easily established experimentally by paper cutting or other devices :

The **lateral surface of a right prism** equals the product of the perimeter of its base by the height of the prism.

The **lateral surface of a right pyramid** equals one half the perimeter of its base by the altitude of one of its lateral faces.

The **convex surface of a cone** equals one half the circumference of its base by its slant height.

The **convex surface of a cylinder** equals the circumference of its base by its height.

The **surface of a sphere** equals four times the area of one of its great circles, *i.e.* $4\,\pi r^2$.

EXERCISE 155

1. Find the lateral surface of a quadrangular prism, the dimensions of whose base are 16 ft. by 8 ft., and whose height is 12 ft.

2. Find the area of the walls of a room, having given the dimensions of the floor as 18 ft. by 16 ft., and the height as 10 ft.

3. Find the height of a triangular prism, the sides of its base being 5, 6, and 7 ft., and its lateral area being 190 sq. ft.

4. Find the height of a pentagonal prism whose lateral area is 300 sq. ft., and each side of whose base is 8 ft.

5. The base of a square pyramid is 40 ft. long, and the altitude of each of its triangular faces is 26 ft. Find its lateral area. Find the cost of painting its lateral surface at $2\frac{1}{2}$¢ per square foot.

6. Each side of a hexagonal pyramid is 14 ft. and its slant height is 15 ft. Find the area of its lateral surface.

7. A pyramidal tent whose base is a square 22 ft. on a side has a slant height of 30 ft. Find the cost of the canvas for the tent, at 18¢ per square yard.

8. Find the number of square yards in the lateral surface of a triangular pyramid, each side of the base being 21 ft. and the slant height being 42 ft.

9. The radius of the base of a right cone is 49 in. and the slant height is 50 in. Find its convex surface in

ne,
the ... ant
hei

1 ... e a
co ... and the
slant height 16 ft.?

12. Find the convex surface of a cylinder the diameter of whose base is 19 ft. and whose height is 50 ft.

13. Find the convex surface of a cylinder, the radius of the base being 41 in., and the height, 60 in.

14. A standpipe has a diameter of 30 ft. and is 150 ft. high. Find the cost of painting it at 25¢ per square yard.

15. Find the surface of a sphere whose radius is 98 in.

16. Find the surface of a sphere if its diameter is 42 in.

17. The diameter of the planet Mercury is 3030 mi.; find the area of the planet.

18. The diameter of the planet Venus is 7700 mi.; find the area of the planet.

19. The diameters of the major planets are, respectively, 86,000, 73,000, 32,000, 33,000 mi. Find the number of million square miles in the area of each of these planets.

20. The surface of a sphere is 10,568 sq. in. Calculate its diameter.

VOLUMES OF SOLIDS

The **volume of a rectangular prism** is equal to the area of its base multiplied by its height.

The **volume of a cylinder** is also equal to the product of its base by its height.

The **volume of a pyramid** and that of a cone are each equal to one third the product of the area of the base and height.

The volume of a cylinder having r for radius of base and $2\,r$ for height is $\pi r^2 \times 2\,r = 2\,\pi r^3$.

The volume of a sphere having r for radius is $\frac{2}{3}$ of $2\,\pi r^3 = \frac{4}{3}\,\pi r^3$, and since $r = \frac{1}{2}\,d$, $\therefore\ r^3 = \frac{1}{8}\,d^3$;

$$\therefore\ \tfrac{4}{3}\,\pi r^3 = \tfrac{4}{3}\,\pi \times \tfrac{1}{8}\,d^3 = \frac{\pi}{6}\,d^3 = .5236\,d^3.$$

EXERCISE 156

1. Find the volume of a triangular prism, the sides of the base being 11, 25, 30 in., respectively, and the height of the prism being 40 in.

2. Find the volume of a square pyramid, if the sides of the base are each equal to 10 in., and the height is 21 in.

3. Find the volume of a cone, the radius of its base being 12 in., and its height being 27 in.

4. Find the volume of a cone, if the radius of the base is 25 in., and the height is 24 in.

5. Find the volume of a hexagonal pyramid, each side of its base being 10 in., and its height being 30 in.

6. Find the volume of a sphere whose radius is 20 in.

7. Find the volume of a sphere, the radius being 8 ft.

8. How many gallons does a cylindrical cistern hold, the diameter of its base being 9 ft. 4 in., and its height 8 ft.?

9. How many gallons does a cylindrical cistern contain, if the diameter of its base is 11 ft. and its height is 6 ft. 5 in.?

10. The diameter of the base of a cylinder is 10 in., and its height is 10 in. Find the ratio of the volume of this cylinder to the volume of a sphere 10 in. in diameter.

11. The diameter of the base of a cone is 1 ft. and its height is 1 ft. Find the ratio of the volume of this cone to the volume of a sphere whose diameter is 1 ft.

12. The surface of a cube contains 84 sq. ft. 54 sq. in. Find its volume.

13. The base of a pyramid is a triangle whose sides are 1 ft. 1 in., 3 ft. 1 in., 3 ft. 4 in., and whose volume is 1 cu. ft. 1152 cu. in. Find its height.

14. The surface of a sphere equals 1257 sq. in. Find its volume.

15. The surface of a hemispherical dome is 2513.5 sq. ft. Find its diameter.

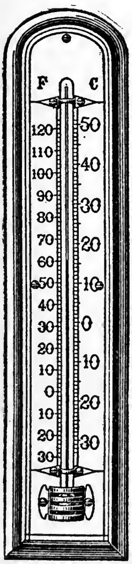
16. The volumes of similar solids are to each other as the cubes of their corresponding dimensions. How many times as large as the earth is the sun? The diameter of the sun is nearly 888,000 mi., and the diameter of the earth is nearly 8000 mi.

17. Find how many times as large as the moon is the earth. The moon's diameter is 2200 mi., nearly.

18. How many times as large as the earth is Saturn? The diameter of Saturn is 73,000 mi.

19. How many times as large as the earth is Jupiter? The diameter of Jupiter is 88,000 mi., nearly.

MEASURE OF TEMPERATURE



A **thermometer** is an instrument for measuring heat. The principle of the thermometer is that substances expand with heat, according to a natural law.

There are two different styles of thermometer in general use,—the Centigrade and the Fahrenheit. The Centigrade thermometer marks the melting point of ice 0°, and the boiling point of water 100°. The interval between these points is divided into 100 parts, or degrees, so that the change in the volume of the mercury between any two consecutive marks is $\frac{1}{100}$ of the change from 0° to 100°.

The Fahrenheit thermometer divides the interval from the melting point of ice to the boiling point of water into 180°. It marks the melting point of ice 32°, and the boiling point of water 212°.

Notation. 92° C. means 92 degrees on the Centigrade thermometer.

45° Fahr. means 45 degrees on Fahrenheit's thermometer.

$+10°$ means 10 degrees above zero.

$-10°$ means 10 degrees below zero.

Verify by counting 32° backward.

$$20° - 32° = -12°.$$
$$10° - 32° = -22°.$$
$$-2° - 32° = -34°, \text{ etc.}$$

(1) **To change from degrees Fahrenheit to degrees Centigrade, subtract 32° and multiply the remainder by $\frac{5}{9}$.**

(2) **To change from degrees Centigrade to degrees Fahrenheit, multiply the number of degrees Centigrade by $\frac{9}{5}$ and add 32 to the product.**

Explanation of the rules:

(1) Suppose the temperature on a Fahrenheit thermometer is n degrees. Subtract 32° to get the number of degrees from 0. A difference of 180° Fahrenheit = a difference of 100° Centigrade. Therefore, a difference of 1° Fahrenheit = a difference of $\frac{5}{9}°$ Centigrade. Therefore, a difference of $(n - 32°)$ Fahrenheit $= \frac{5}{9}(n - 32°)$ Centigrade, which symbolizes the first of the above rules.

(2) A difference of $n°$ C. = a difference of $\frac{9}{5}n°$ Fahrenheit. Hence, $n°$ C. $= (\frac{9}{5}n° + 32°)$ Fahr.

EXERCISE 157

Express the following Fahrenheit temperatures on the C. scale:

1. 86°.	**4.** 248°.	**7.** 38°.	**10.** $-13°$.
2. 77°.	**5.** 68°.	**8.** 23°.	**11.** $-40°$.
3. 203°.	**6.** 54°.	**9.** 15°.	**12.** $-90°$.

Express the following C. temperatures on the Fahr. scale:

13. 35°.	**16.** 20°.	**19.** – 10°.	**22.** – 18°.
14. 55°.	**17.** 18°.	**20.** – 20°.	**23.** – 24°.
15. 25°.	**18.** 8°.	**21.** – 14°.	**24.** – 273°.

TABLE OF MELTING POINTS

Mercury	– 40° C.	Lead	326° C.	Gold	1035° C.
Sulphur	113° C.	Zinc	415° C.	Cast iron,	1100° to 1200° C.

25. Give the above table in the Fahrenheit scale.

26. Water attains its maximum density of 4° C. Express this temperature on Fahrenheit's scale.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES

The metric system is now used by more than forty countries, and it is the only system used in text-books of science. Upward of twenty nations contribute to the support of the International Bureau of Weights and Measures in Paris. For these reasons the Metric System deserves to be called the International System.

The only great nations which have not adopted the Metric System are the United States and Great Britain. In the United States the system is legalized, and none other is used in the Philippines and Porto Rico.

The Metric System is so called because the meter is the basis of the system. The **meter** is the standard unit of linear measure. Its length is the ten-millionth part of the distance from the equator to the north pole measured on the meridian of Paris. Its length in this country is 39.37 inches. In the United Kingdom the legal equivalent of the meter is 39.370113 inches and on the continent of Europe 39.370432 inches.

The standard meter from which all others are derived is a bar made of an alloy of platinum and iridium, kept in the International Bureau of Weights and Measures in Paris. Duplicates of this standard meter have been furnished to all the nations of the world.

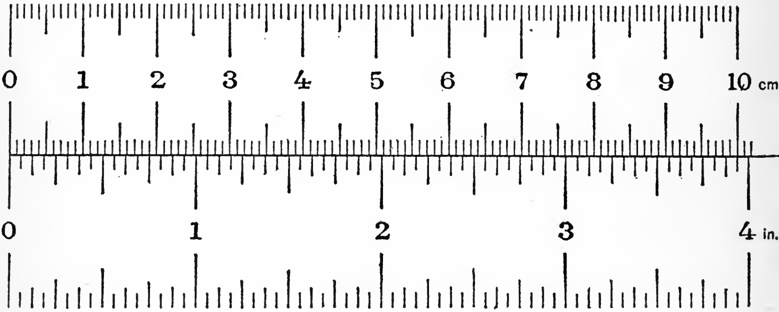
The Metric System is a decimal system. In it there are no compound rules. It is the simplest and the most perfect system ever devised.

The names for the multiples of the standard unit in the Metric System are formed from the names of the standard unit by means of prefixes derived from the Greek words meaning ten, one hundred, one thousand, and ten thousand, *i.e. deka*, *hekaton*, *chilioi*, *murioi*. The names for the submultiples of the standard units are formed in a similar manner from the Latin words meaning ten, one hundred, one thousand, *i.e. decem*, *centum*, *mille*. Thus:

Dekameter means ten meters.
Hektometer means one hundred meters.
Kilometer means one thousand meters.
Myriameter means ten thousand meters.
Decimeter means one tenth of a meter.
Centimeter means one hundredth of a meter.
Millimeter means one thousandth of a meter.

The **standard units** are the *meter*, the *liter*, and the *gram*. The gram being a very small weight, the kilogram is most used in ordinary affairs. In the Metric System the meter, the liter, and the kilogram serve for everyday trade in exactly the same manner as the yard, the quart measure, and the pound Avoirdupois in our system of weights and measures. The international meter and the kilogram are, since 1893, the fundamental standards of length and mass in the United States.

The *gram* is the weight of distilled water at 4° Centigrade, which fills a cubical vessel, one of whose edges is one centimeter.



COMPARISON SCALE: 10 CENTIMETERS AND 4 INCHES. (ACTUAL SIZE.)

LINEAR MEASURE

10 millimeters (mm.)	=	1 centimeter
10 centimeters (cm.)	=	1 decimeter
10 decimeters	=	1 meter
10 meters (m.)	=	1 dekameter
10 dekameters	=	1 hektometer
10 hektometers	=	1 kilometer
10 kilometers (km.)	=	1 myriameter

The units in common use are the centimeter, meter, and kilometer.

SURFACE MEASURE

100 square millimeters (qmm.)	=	1 square centimeter
100 square centimeters (qcm.)	=	1 square decimeter
100 square decimeters	=	1 square meter
100 square meters (qm.)	=	1 ar
100 ars	=	1 hektar
100 hektars (ha.)	=	1 square kilometer (qkm.)

An ar is the area of a square whose side is ten meters. 1 qm. is therefore equal to a centar (ca.). The areas of small tracts of land, such as gardens, are expressed in ars.

The areas of larger tracts, such as farms, are expressed in hektars. The areas of still larger tracts, such as countries, are expressed in square kilometers.

CUBIC MEASURE

1000 cubic millimeters (cmm.)	= 1 cubic centimeter
1000 cubic centimeters (ccm.)	= 1 cubic decimeter, or liter
1000 liters (l.)	= 1 cubic meter (cbm.)

The liter corresponds to our quart. The hektoliter (hl.) corresponds to our bushel. The cubic meter is also called a stere.

WEIGHT

1000 milligrams (mg.)	= 1 gram
1000 grams (g.)	= 1 kilogram, or kilo
1000 kilograms (kg.)	= 1 tonneau (T.), or ton

A gram represents the weight of 1 ccm. of distilled water at 4° C.

A kilogram is the weight of 1 cu. dm. of distilled water at 4° C.

A tonneau, or ton, is the weight of 1 cbm. of distilled water at 4° C.

A quintal is 100 kg.

REDUCTION

Example 1. Reduce 75.623 m. to centimeters, and also to millimeters.

SOLUTION. (*a*) Since there are 100 cm. in 1 m., reduce meters to centimeters by multiplying the number of meters by 100. This is done by moving the decimal point two places to the right.

(*b*) Since there are 1000 mm. in 1 m., therefore multiply the number of meters by 1000. This is done by moving the decimal point three places to the right.

Ans. (*a*) 7562.3 cm.
(*b*) 75623 mm.

Example **2.** Reduce 85679 mm. to meters.

Solution. This is done by dividing by 1000, *i.e.* by moving the decimal point three places to the left.

Ans. 85.679 m.

EXERCISE 158

Add:

(1)	(2)	(3)
96458.23 m.	92435.9 cm.	924357.9 mm.
23458.98 m.	23456.8 cm.	8765.1 mm.
14329.09 m.	98239.2 cm.	986201.8 mm.
920184.60 m.	98209823.9 cm.	87623.7 mm.
1908.02 m.	1980098.76 cm.	987.9 mm.
90327.92 m.	6543097.1 cm.	23456.2 mm.
1872095.72 m.	7609.9 cm.	10098.3 mm.
902376.253 m.	980098.3 cm.	90098.9 mm.
		999999.9 mm.

4. A man walks on four successive days. The first day he walks 11.7 km.; the second day, 984 m.; the third day, 2950 m.; the fourth day, 12.8 km. How far does he travel? Give the answer in meters.

5. From 25.724 km. take 6270 m.

6. Multiply 11.732 m. by 12.

7. How many times is 25 mm. contained in 24 m.?

8. How many times is 7.03 m. contained in .0494209 km.?

9. How many times is 12 ca. contained in 6 ha.?

10. How many farms of 8 ha. each can be made out of a square whose side is 3 km.?

11. A vessel contains 250 ccm. How many such vessels would hold 5 cbm.?

12. How many times is 800 ccm. contained in 50 l.?

13. How many times is 450 ccm. contained in 22.5 hl.?

14. A dime weighs 2.5 g. How many dimes can be coined from 3 kg. standard silver? how many 25¢ pieces, weights being in proportion to values?

15. How often is 1521 mg. contained in 5.9319 kg.?

Example. Find the area of a rectangle, if its length is 425.8 m. and its breadth is .3256 km.

SOLUTION. First, reduce .3256 km. to meters.

$$.3256 \text{ km.} = 325.6 \text{ m.}$$

Second, multiply in the usual manner and get for product 138,640.48 qm. Reduce this to ars by dividing by 100, and reduce the ars to hektars by dividing by 100. Both operations can be performed at once by moving the decimal point four places to the left. *Ans.* 13.864048 ha.

$$\begin{array}{r} 425.8 \\ \underline{325.6} \\ 25548 \\ 21290 \\ 8516 \\ \underline{12774} \\ 138640.48 \text{ qm.} \end{array}$$

EXERCISE 159

Find the area of each of the following rectangles:

	LENGTH	WIDTH		LENGTH	WIDTH
1.	625 m.	125 m.	**4.**	369.4 m.	184.7 m.
2.	305 m.	61 m.	**5.**	488.9 m.	244.45 m.
3.	338 m.	169 m.	**6.**	5767 m.	11.534 m.

7. Find the volume of a rectangular solid whose dimensions are 1.2 m., .9 m., and .75 m.

8. Find the area of a triangle whose sides are 14 m., 48 m., and 50 m.

9. Find the area of a circle whose radius is .78 m.

10. Find the volume of a sphere whose radius is 18 cm.

11. The legs of a right triangle are 56 cm. and 105 cm. Find the hypothenuse.

12. Find the volume of a rectangular prism, the dimensions of the base being 14 m. by 12 m., and height 9.5 m.

13. A boiler has 300 tubes 2.4 m. long, 7.5 cm. diameter. What is the area of the tube heating surface?

14. Find the weight of a spherical cast iron shell 32.5 cm. outside and 27.5 cm. inside diameter.

15. An iron plate 3 mm. thick weighs 1 km. What is its area?

The following approximations should be fixed in mind:

1 meter = 39 inches; 1 kilometer = $\frac{5}{8}$ of 1 mile.

1 centimeter = $\frac{2}{5}$ of an inch; 1 hektar = $2\frac{1}{2}$ acres.

1 liter = 1 quart; 1 kilogram = $2\frac{1}{5}$ pounds Avoirdupois.

EQUIVALENTS OF COMMON UNITS IN METRIC UNITS

1 inch	= 25.4001 mm.	1 sq. foot	= .0929 qm.
1 foot	= .304801 m.	1 sq. yard	= .8361 qm.
1 yard	= .914402 m.	1 A.	= .4047 ha.
1 mile	= 1.60935 km.	1 sq. mi.	= 2.59 qkm.
1 quart	= .94636 l.	1 cu. inch	= 16.3872 ccm.
1 gallon	= 3.78543 l.	1 cu. foot	= .02832 cbm.
1 bushel	= .35239 hl.	1 cu. yard	= .7646 cbm.
1 sq. inch	= .6452 qcm.	1 lb. Avoir.	= .45359 kg.

EQUIVALENTS OF METRIC UNITS IN COMMON UNITS

1 meter	= 39.37 in.	1 sq. centimeter	= .155 sq. in.
1 kilometer	= .62137 mi.	1 sq. meter	= 1.196 sq. yd.
1 hektar	= 2.471 A.		
1 qkm.	= .3861 sq. mi.		
1 cu. centimeter	= .061 cu. in.		
1 cu. meter	= 35.314 cu. ft.		
1 liter	= 1.0567 qt.*	1 kilogram	= 2.20462 lb.
1 hektoliter	= 2.83774 bu.	1 tonneau, or ton	= 2204.6 lb.

* Liquid quarts, or 0.9081 dry quarts.

ANNUAL INTEREST

Lincoln, Neb., Jan. 1, 1899.

$800.00

On demand I promise to pay to ....

................*Jeremiah Hill*................*or order*

Eight hundred..........................$\frac{00}{100}$ *Dollars.*

Value received, with interest annually at 8%.

Wm. Harris.

No. 92. *Due*..........

In some states if a note contains the words "with interest annually," and if the interest remains unpaid for a number of years, then the interest due at the end of each year bears interest until the date of settlement.

The *interest* by this method of reckoning is called **annual interest.** The interest we have up to this time considered is known as **simple interest.**

Example. How much is due on a note for $800, dated Jan. 5, 1898, and bearing interest annually at 8%, if left unpaid, both in principal and interest, until March 10, 1903?

SOLUTION

Interest on $800 for 1 yr. at 8% = $64.

Interest on $800 for 5 yr. 2 mo. 5 da. at 8% = $331.56.

YR.	MO.	DA.
1903	3	10
1898	1	5
5	2	5

The first year's interest is due Jan. 5, 1899, and bears interest until March 10, 1903. The interest falling due at the end of the second year bears interest until March 10, 1903, and so on.

$64 bears interest for	4 yr.	2 mo.	5 da.
$64 bears interest for	3 yr.	2 mo.	5 da.
$64 bears interest for	2 yr.	2 mo.	5 da.
$64 bears interest for	1 yr.	2 mo.	5 da.
$64 bears interest for		2 mo.	5 da.
Adding, $64 bears interest for	10 yr.	10 mo.	25 da.

Interest on $64 for 10 yr. 10 mo. 25 da. at 8% = $55.82.
Interest on $800 for 5 yr. 2 mo. 5 da. at 8% = $331.56.
Amount due = $800 + $331.56 + $55.82 = $1187.38.

EXERCISE 160

1. Find the amount due April 1, 1903, on a note for $900, dated July 15, 1899, and bearing interest annually at 7%.

2. Find interest from March 6, 1898, to Jan. 1, 1903, on note for $1400, interest payable annually at 8%.

3. Find amount due Jan. 1, 1903, on a note for $1200, dated July 1, 1897, interest payable annually at 6%.

4. Find amount due Jan. 1, 1903, on note for $1000, dated Jan. 1, 1898, interest payable semiannually at 5%.

5. Find the amount due at the end of 5 years on a coupon note, interest payable semiannually, if the face of the note is $700, and the rate of interest is 7%.

COMPOUND INTEREST

Savings banks and other banks which give interest on deposits add the interest semiannually or annually to the amount deposited. This interest added bears interest until the next date of balancing the depositor's account book. The next interest is added in the same way.

Suppose a person deposited in a savings bank $250 and allowed it to remain there for 10 yr. The question might be asked, how much will principal and interest amount to at the end of that time? The accruing interest in this case is called **compound interest.**

Designate principal by p, rate of interest by r, time in years by n, and amount by a.

When the rate of interest on \$1 is \$$r$ per year, the amount of \$1 at the end of 1 yr. is \$$(1+r)$.

$\therefore$ the amount of \$1 in 1 yr. = \$$(1+r)$,

$\therefore$ the amount of \$$p$ in 1 yr. = \$$p(1+r)$.

To get the amount of any principal for 1 yr. or for 6 mo., or for any other period at simple interest, multiply the principal by the amount of \$1 for that time.

Take now \$$p(1+r)$ as principal, then the amount of \$$p(1+r)$ in 1 yr. = \$$p(1+r)(1+r)$ = \$$p(1+r)^2$.

The amount of \$$p(1+r)^2$ in 1 yr. = \$$p(1+r)^2(1+r)$ = \$$p(1+r)^3$.

Similarly, the amount of \$$p(1+r)^3$ in 1 yr. = \$$p(1+r)^4$.

At compound interest,

\$$p$ amounts in 2 yr. to \$$p(1+r)^2$.
\$$p$ amounts in 3 yr. to \$$p(1+r)^3$.
\$$p$ amounts in 4 yr. to \$$p(1+r)^4$.
\$$p$ amounts in 5 yr. to \$$p(1+r)^5$.

Generally, \$$p$ amounts in n yr. to \$$p(1+r)^n$.

At compound interest,

The amount of \$1 at 4% for 20 yr. is \$$(1.04)^{20}$.
The amount of \$1 at 5% for 20 yr. is \$$(1.05)^{20}$.
The amount of \$1 at 6% for 20 yr. is \$$(1.06)^{20}$.

The labor of raising 1.06 to the 20th power is considerable.

A student who has a working knowledge of logarithms can do this by the aid of a table of logarithms in less than a minute.

The following table gives the amount of $1 at compound interest:

Yrs.	2%	2½%	3%	4%	5%	6%
1	1.020000	1.025000	1.030000	1.040000	1.050000	1.060000
2	1.040400	1.050625	1.060900	1.081600	1.102500	1.123600
3	1.061208	1.076891	1.092727	1.124864	1.157625	1.191016
4	1.082432	1.103813	1.125509	1.169859	1.215506	1.262477
5	1.104081	1.131408	1.159274	1.216653	1.276282	1.338226
6	1.126162	1.159693	1.194052	1.265319	1.340096	1.418519
7	1.148686	1.188686	1.229874	1.315932	1.407100	1.503630
8	1.171659	1.218403	1.266770	1.368569	1.477455	1.593848
9	1.195093	1.248863	1.304773	1.423312	1.551328	1.689479
10	1.218994	1.280085	1.343916	1.480244	1.628895	1.790848
11	1.243374	1.312087	1.384234	1.539454	1.710339	1.898299
12	1.268242	1.344889	1.425761	1.601032	1.795856	2.012196
13	1.293607	1.378511	1.468534	1.665073	1.885649	2.132928
14	1.319479	1.412974	1.512590	1.731676	1.979932	2.260904
15	1.345868	1.448298	1.557967	1.800943	2.078928	2.396558
16	1.372786	1.484506	1.604706	1.872981	2.182875	2.540352
17	1.400241	1.521618	1.652847	1.947900	2.292018	2.692773
18	1.428246	1.559659	1.702433	2.025817	2.406619	2.854339
19	1.456811	1.598650	1.753506	2.106849	2.526950	3.025599
20	1.485947	1.638616	1.806111	2.191123	2.653298	3.207136

Example 1. Find the amount of $2500 at compound interest for 12 yr. at 5%.

SOLUTION. $a = \$2500\ (1.05)^{12} = \$2500 \times 1.795856 =$ $4489.64.

Example 2. Find the amount of $5000 for 8 yr. at 4% compound interest, the interest being compounded semi-annually.

SOLUTION. $\$5000 \times (1.02)^{16} = \$5000 \times 1.372786 =$ $6863.93.

EXERCISE 161

With the aid of the above table, find the amount at compound interest of:

1. \$2000 for 8 yr. at 4%.
2. \$3000 for 10 yr. at 3%.
3. \$5000 for 12 yr. at 4%.
4. \$6000 for 10 yr. at 5%.
5. \$8000 for 8 yr. at 5%, interest compounded semi-annually.
6. \$2250 for 6 yr. at 4%, interest compounded semi-annually.

MISCELLANEOUS TOPICS

WORK AND TIME

Example 1. A can do a piece of work in 6 da., and B can do the same piece of work in 8 da. In what time can A and B do the work together?

Analytical Solution. A does $\frac{1}{6}$ of the work in 1 da. B does $\frac{1}{8}$ of the work in 1 da.

$\therefore$ A and B together do $(\frac{1}{6}+\frac{1}{8})$ of the work in 1 da.

$\therefore$ A and B do $\frac{7}{24}$ of the work in 1 da.

$\therefore$ A and B do $\frac{1}{24}$ of the work in $\frac{1}{7}$ of 1 da.

$\therefore$ A and B do $\frac{24}{24}$ of the work in $\frac{24}{7}$ of 1 da.

$\therefore$ A and B do the work in $3\frac{3}{7}$ da.

Example 2. A cistern has three pipes. The first pipe fills the cistern in 12 hr., the second, in 15 hr., and the third empties it in 10 hr. In what time will the cistern be filled, if all three pipes run together, and the cistern is empty when the pipes start running?

Analytical Solution. The first pipe fills $\frac{1}{12}$ of the cistern in 1 hr.

The second pipe fills $\frac{1}{15}$ of the cistern in 1 hr.

The third pipe empties $\frac{1}{10}$ of the cistern in 1 hr.

∴ the three pipes fill $(\frac{1}{12} + \frac{1}{15} - \frac{1}{10})$ of the cistern in 1 hr.

∴ the three pipes fill $\frac{1}{20}$ of the cistern in 1 hr.

∴ the three pipes fill $\frac{20}{20}$ of the cistern in 20 hr.

∴ the cistern is filled in 20 hr.

Example **3.** A and B do a piece of work in $3\frac{11}{15}$ hr.; A alone, in 7 hr. In what time does B do the work?

SOLUTION. A and B together do $\dfrac{1}{3\frac{11}{15}}$ of the work in 1 hr.; *i.e.* A and B together do $\frac{11}{56}$ of the work in 1 hr.

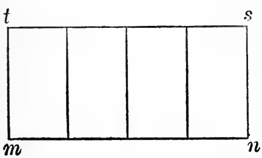
But A alone does $\frac{1}{7}$ of the work in 1 hr.

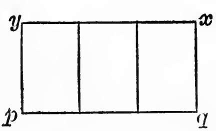
∴ B alone does $(\frac{15}{56} - \frac{1}{7})$ of the work in 1 hr.

∴ B alone does $\frac{1}{8}$ of the work in 1 hr.

∴ B does the work in 8 hr.

Example **4.** A and B can mow a field in 21 hr. A can do $\frac{3}{4}$ as much work as B. Find the time in which each does the work.





SOLUTION. Represent B's work by the rectangle *mnst*.

Represent A's work by the rectangle *pqxy*.

A will do in $\frac{7}{3}$ hr. the work A and B do together in 1 hr.

∴ A will do in $\frac{7}{3}$ hr. × 21 the work A and B do together in 21 hr.

∴ A will do in 49 hr. the work A and B do together in 21 hr.

B will do in $\frac{7}{4}$ hr. the work A and B do together in 1 hr.

∴ B will do in $\frac{7}{4}$ hr. × 21 the work A and B do together in 21 hr.

$\therefore$ B will do in $36\frac{3}{4}$ hr. the work A and B do together in 21 hr.

A's time, 49 hr.; B's time, $36\frac{3}{4}$ hr.

EXERCISE 162

1. If a person can do a piece of work in 7 da., what is his day's work? If he can do the work in $4\frac{1}{2}$ da., what is his day's work?

2. If B can copy a manuscript in $5\frac{3}{4}$ hr., how much can he copy in 1 hr.?

3. John travels $\frac{5}{16}$ of the distance between two cities in 1 hr. How many hours will it take him to travel the remainder of the distance?

4. A can do a piece of work in 3 hr., B in 4 hr., and C in 5 hr. How long will it take the three working together to do the work?

5. A can do a piece of work in 4 hr., B in 5 hr., and A, B, and C together in $1\frac{1}{2}$ hr. How long would it take C alone to do the work?

6. A, B, and C can do a piece of work in 6, 8, and 10 da., respectively. If they begin the work together, what part of the work remains to be done at the end of the second day?

7. A, B, and C can build a fence in 10, 15, and 20 hr. respectively. They work together for 4 hr., when B quits. In what time can A and C finish the work?

8. A cistern has two pipes. One can fill it in 20 min., and the other can empty it in 30 min. If the cistern is empty, in what time can it be filled, if both pipes begin to flow at the same instant?

MOTION IN THE SAME DIRECTION, OR IN OPPOSITE DIRECTIONS

Example 1. A starts to overtake B, who is 100 yd. ahead of him. A travels 11 yd. to B's 9 yd. How far must A travel in order to overtake B?

SOLUTION. A gains on B 2 yd. in every 11 yd. he goes.

$\therefore$ A gains on B 1 yd. in every $5\frac{1}{2}$ yd. he goes.

$\therefore$ A gains on B 100 yd. in every 550 yd. he goes.

Ans. 550 yd.

Example 2. A freight train moving at the rate of 18 mi. an hour is 78 mi. ahead of a passenger train moving in the same direction at the rate of 30 mi. an hour. Find the distance the passenger train must run to overtake the freight train.

SOLUTION. In 1 hr. the passenger train gains on the freight $(30 - 18)$ mi., *i.e.* 12 mi.

$\therefore$ in $(78 \div 12)$ hr. the passenger train will overtake the freight train, *i.e.* in $6\frac{1}{2}$ hr.

In $6\frac{1}{2}$ hr. the passenger train goes 30 mi. $\times 6\frac{1}{2} = 195$ mi.

EXERCISE 163

1. Two ships leave New York for Glasgow, one on Monday morning at 9 o'clock, and the other on the following morning at 9 o'clock. Their rates are 15 and 21 miles an hour respectively. How far from New York City will the second ship overtake the first?

2. Dallas and Galveston are 315 mi. apart. A train leaves Dallas for Galveston at 8 o'clock A.M. at the rate of 30 mi. an hour. At the same time a train leaves Galveston for Dallas at the rate of 33 mi. an hour. How far will the trains be from Dallas when they meet?

3. Paris, Texas, is 584 mi. from St. Louis. A passenger train leaves Paris for St. Louis at 6.50 P.M. Three hours later a freight train leaves St. Louis for Paris. When and where will they meet, the rates being respectively 24 mi. and 16 mi. per hour?

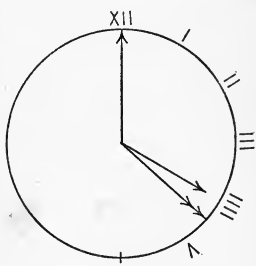
4. A man walking at the rate of 4 mi. an hour is overtaken by a train 88 yd. long, and is passed in 10 sec. Find the rate of the train.

5. A train going at the rate of 40 mi. an hour passes in 6 sec. a man walking in the same direction at the rate of 4 mi. an hour. What is the length of the train?

6. Two trains start from the same station and travel in the same direction. The first train leaves at 7 A.M., and the second train at 9 A.M. How many miles from the station will the second train overtake the first if the rate of the first train is 30 mi. per hour and the rate of the second train is 45 mi. per hour?

CIRCULAR MOTION: CLOCKS

Example 1. At what time between 4 and 5 o'clock are the hands of a clock together?



SOLUTION. At 4 o'clock the hour hand is 20 minute spaces ahead of the minute hand.

In 1 hr. the minute hand goes 60 minute spaces.

In 1 hr. the hour hand goes 5 minute spaces.

∴ ratio of rates of motion of minute hand and of hour hand is 60 : 5, or 12 : 1.

∴ in 12 min. the minute hand gains on the hour hand 11 minute spaces.

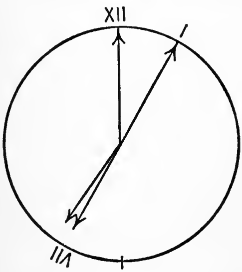
∴ in $1\frac{1}{11}$ min. the minute hand gains on the hour hand 1 minute space.

∴ in $1\frac{1}{11}$ min. $\times$ 20 the minute hand gains on the hour hand 20 minute spaces.

But $1\frac{1}{11}$ min. $\times$ 20 $=$ $21\frac{9}{11}$ min.

∴ the hands are together $21\frac{9}{11}$ min. after 4 o'clock.

Example 2. At what time after 7 o'clock do the hour and minute hands first point in opposite directions?



SOLUTION. They will point in opposite directions whenever they are 30 minute spaces apart.

At 7 o'clock the hour hand is 35 minute spaces ahead of the minute hand.

∴ as soon as the minute hand gains on the hour hand 5 minute spaces, they will point in opposite directions.

But the minute hand gains on the hour hand 5 minute spaces in $1\frac{1}{11}$ min. $\times$ 5, *i.e.* in $5\frac{5}{11}$ min. (See Example 1.) *Ans.* $5\frac{5}{11}$ min. past 7 o'clock.

Example 3. When, between 5 and 6 o'clock, will the hour and minute hands be at right angles?

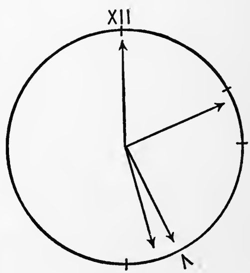


FIG. 1.

SOLUTION. The hour and minute hands will be at right angles when they are 15 minute spaces apart.

∴ they will be at right angles between 5 and 6 o'clock when the minute hand gains on the hour hand (25 − 15) minute spaces, *i.e.* 10 minute spaces.

$1\frac{1}{11}$ min. × 10 = $10\frac{10}{11}$ min. *Ans.* $10\frac{10}{11}$ min. past 5 o'clock.

They will also be at right angles when the minute hand gains on the hour hand (25 + 15) minute spaces, *i.e.* 40 minute spaces. (See Fig. 2.)

$1\frac{1}{11}$ min. × 40 = $43\frac{7}{11}$ min.

the hands will be at right angles to each other at $10\frac{10}{11}$ min. past 5 o'clock and at $43\frac{7}{11}$ min. past 5 o'clock.

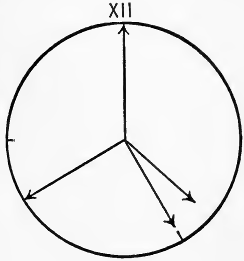


Fig. 2.

To the young learner the following suggestions may be of use:

To solve a question in circular motion: first, draw a diagram showing the things that move; second, find the ratios of the rates of motion of the things moving; third, having found the relative rates of motion of the objects, proceed as in a simple exercise involving motion in a straight line.

EXERCISE 164

1. What angle do the hour and minute hands of a clock make with each other at 1 o'clock? at 2 o'clock? at 3 o'clock? at 4 o'clock? at 5 o'clock? at 6 o'clock? at 12 o'clock?

2. What angle do the hour and minute hands of a clock make when they point to positions 8 minute spaces apart? 12 minute spaces apart? 19 minute spaces apart? 23 minute spaces apart? 27 minute spaces apart? 50 minute spaces apart? At what time between 1 and 2 o'clock do the hands of a clock make an angle of 36°?

3. How many minute spaces apart do the hands of a watch indicate when they make an angle of 36°? 66°? 84°? 114°? 126°? 144°? 162°? 174°?

4. Find at what time between the hours of 2 and 3 o'clock the hands of a clock are together; between 3 and 4 o'clock; between 5 and 6 o'clock; between 7 and 8 o'clock; 9 and 10 o'clock; 10 and 11 o'clock; 11 and 12 o'clock.

5. At what time do the hands of a watch point in opposite directions between

(*a*) 1 and 2 o'clock?
(*b*) 3 and 4 o'clock?
(*c*) 8 and 9 o'clock?
(*d*) 9 and 10 o'clock?
(*e*) 11 and 12 oclock?
(*f*) 12 and 1 o'clock?

6. At what time after 3 o'clock do the hands of a watch first point in opposite directions? When after 6 o'clock do they first point in opposite directions? When after 10 o'clock?

7. At what time or times are the hands of a watch at right angles between (*a*) 2 and 3 o'clock? (*b*) 3 and 4 o'clock? (*c*) 4 and 5 o'clock? (*d*) 6 and 7 o'clock? (*e*) 8 and 9 o'clock? (*f*) 9 and 10 o'clock? (*g*) 11 and 12 o'clock? (*h*) 12 and 1 o'clock?

8. At what time after 4 o'clock do the hands of a watch first point to positions 8 minute spaces apart? 35 minute spaces apart? 24 minute spaces apart? 50 minute spaces apart?

9. Find the time, between 5 and 6 o'clock, when the minute hand is $\frac{1}{6}$ of the circumference of the dial in advance of the hour hand. The time between 9 and 10 o'clock when the minute hand is $\frac{1}{4}$ of the circumference in advance of the hour hand.

MISCELLANEOUS EXAMPLES

(Arranged by Topics)

NOTATION

1. Write in figures ten million ten.

2. Write in figures seven million two hundred five thousand.

3. Write in figures one billion one million one.

4. Write in figures five million fifty thousand.

5. Write in figures ten billion ten million one hundred.

6. Write decimally: Twenty-five tenths. One hundred twenty hundredths. One hundred and fifty-five hundredths. Ten thousand ten hundred thousandths. One million one ten-millionths. Fifty-five thousand two hundred eighteen ten-thousandths. One million two hundred thousand four ten-millionths. Five and five hundredths. Six hundred six hundredths. Seven hundred ten thousandths. Nine hundred and fifteen thousandths. Nine million and nine millionths.

ADDITION AND SUBTRACTION

7. The following table of railroad mileage in the United States is taken from the report of the Interstate Commerce Commission:

Year	Mileage	Year	Mileage
1890	163,597.05	1898	186,396.32
1891	168,402.74	1899	189,294.66
1892	171,563.52	1900	193,345.78
1893	176,461.07	1901	197,237.44
1894	178,708.55	1902	202,471.85
1895	180,657.47	1903	207,977.22
1896	182,776.63	1904	213,904.34
1897	184,428.47	1905	218,101.04

Find the number of miles built during each year, beginning with 1860, up to 1905.

8. From 9000 take .009.

9. From 275 take .000275.

10. Add:

657,987,324,011
119,008,675,987
199,887,564,999
999,555,777,888
987,345,234,876
985,234,678,100
923,524,896,987
987,567,342,959
725,926,846,368
929,935,829,349
929,563,768,968

MULTIPLICATION

11. Multiply 10,500 by 60,600.

12. Multiply 15,010 by 50,080.

13. Multiply 1.01 by 7.07.

14. Multiply 9010.9 by 90.4.

15. A train travels at the rate of 30.25 mi. an hour. How far will it go in $5\frac{1}{2}$ hr.?

16. Find the price of $87\frac{1}{2}$ A. of land at $43.75 an acre.

17. Find the price of 4225 bu. of wheat at $84\frac{1}{2}$¢ per bushel.

18. Find the value of $.1 \times .2 \times .3 \times .4 \times .5 \times .6 \times .7$.

19. Find the value of $(1.03)^4$.

20. The number of bales of cotton produced in Texas in 1901–02 was 2,993,000, and in 1900–01, 3,550,000. Allowing 500 lb. to a bale, how many more pounds of cotton were produced in the latter year than in the former?

DIVISION

21. A steamer's cargo consisted of 120,000 bu. of corn, valued at \$57,000; 13,541 bbl. of flour, valued at \$47,499; 3050 bales of cotton, valued at \$158,224. Find the value of a bushel of corn, a barrel of flour, and a bale of cotton.

22. Divide 26.78508 by .072 (not by long division).

23. The annual consumption of sugar in a certain state was, in 1890, 702,201 T., which was found to be 49.93 lb. per head of population. Find the population.

24. Make a column of eight numbers, the first of which is 73,214, the second is $\frac{2}{3}$ of the first, the third is $\frac{2}{3}$ of the second, and so on for the other numbers.

25. How many miles are in 278,784,000 ft.?

26. Divide $1.125^2 - (.784)^2$ by $1.125 - .784$.

27. Divide $(.75)^3 - (.26)^3$ by $.75 - .26$.

28. Divide 14.302 by 83.92, correct to four places.

29. Divide 24.619 by 56,000.

30. The length of a degree on the earth's surface is approximately 69.15 mi. Two places are on the same meridian and 1000 mi. apart. Find, in degrees, the difference in latitude.

31. Two places on the 60th parallel of latitude are 300 mi. apart. Find the difference of their longitudes. ($1° =$ 183,085 ft.)

32. A bankrupt's liabilities are \$47,875; his assets are \$38,650. How many cents on the dollar can he pay?

33. The product of two numbers is 642,978, and one of the numbers is 5.67. Find the other number.

34. If the quotient is 24,400, the remainder is 15, and the dividend is 6,100,015, find the divisor.

35. The total amount of money in circulation in the United States on March 1, 1903, was $2,353,738,834. The per capita circulation in the United States on the same day was $29.41. Find the population of the United States.

36. Divide the square of 1001 by 77×169.

37. When 450 lb. of sugar cost $20.25, find the price of 84 lb.

38. Find the value of a rectangular plot of ground 726 yd. long and 240 yd. wide, at $50 an acre.

39. Find, in United States currency, the value of £79.

40. When 1.75 yd. of silk cost $3.85, find the cost of 14 yd.

41. Divide 39.328 by .0032.

42. If .6 of a yard of cloth cost 27¢, find the cost of 45 yd.

43. Divide 1 by 1.732.

44. Divide the cube of 11.1 by 27 times 1369.

45. What is the ratio of 25 A. to 640 A.?

G. C. M. AND L. C. M.

46. Find the G. C. M. of 288 and 432.

47. Find all the common measures of 36 and 54.

48. Find the common divisors of 288 and 360.

49. Express 1110, 777, and 1001 as the products of prime numbers. Find their L. C. M.

50. Find the G. C. M. of 208, 572, and 1326.

51. Find the L. C. M. of 26, 28, 48, 70, and 117.

52. Find the G. C. M. of 625 and 2525.

53. Find the G. C. M. and L. C. M. of 209, 304, and 380.

54. Find the prime factors of 80,850.

55. Two numbers have for their G. C. M. 101, and for L. C. M. 27,573. Find the product of these numbers.

56. Resolve 61,776 into its prime factors.

57. Two tracts of land, containing 1225 acres and 1675 acres, are divided into farms each containing the same number of acres. What is the largest possible acreage of each farm?

58. Telephone poles are 231 ft. apart. What is the smallest number of poles which will correspond to an exact number of half miles?

FRACTIONS, DECIMALS, AND DENOMINATE NUMBERS

59. Arrange in order of magnitude $\frac{3}{4}$, $\frac{7}{9}$, $\frac{11}{15}$.

60. Find the difference between the greatest and the least of the fractions $\frac{2}{3}$, $\frac{5}{8}$, $\frac{17}{21}$, and $\frac{19}{24}$.

61. Add: $2\frac{1}{2}$, $3\frac{1}{4}$, $5\frac{5}{8}$, $3\frac{7}{12}$.

62. Reduce to its lowest terms $\frac{999}{1110}$.

63. Express as decimals $\frac{35}{32}$, $\frac{7}{64}$, $\frac{16}{125}$.

64. Reduce to common fractions .0375, .0175, .03125.

65. Simplify $\dfrac{1\frac{3}{4}-\frac{2}{3}}{8\frac{3}{4}-3\frac{1}{3}} \times 15\frac{3}{5}$.

66. Simplify $2\frac{5}{6}$ of $\dfrac{4\frac{1}{5}}{4\frac{2}{3}} - (\frac{1}{7}$ of $17\frac{1}{2}$ of $\frac{2}{5}$ of $1\frac{1}{2})$.

67. Reduce 198 ft. to the decimal of $1\frac{1}{2}$ mi.

68. Reduce $2° \, 30'$ to the decimal of $90°$.

69. Reduce 3 pt. to the decimal of 5 gal.

70. Reduce .375 of 16*s.* 8*d.* + $\frac{2}{3}$ of 15*s.* 6*d.*, to the decimal of £5.

71. Show that if our calendar had 8 leap years in every 33 yr., it would be more correct than it now is.

72. If our calendar were so arranged that 31 leap years would occur in 128 yr., how many years would elapse before the error would amount to 1 da.?

73. The following distances have been run by trains in the times indicated. Find in each case the rate per hour.

Route	Distance in Miles	Time
Jersey City to Oakland . . .	3311	83 hr. 45 min.
New York to Chicago . . .	964	19 hr. 57 min.
Chicago to New York . . .	962	17 hr. 45 min.
London to Aberdeen	539.75	8 hr. 32 min.
Chicago to Buffalo	510.1	8 hr. 1 min. 7 sec.
Albany to Syracuse	147.84	2 hr. 10 min.
Erie to Buffalo Creek	86	1 hr. 10 min. 45 sec.
Camden to Atlantic City . .	58.3	45 min. 45 sec.
Liberty Park to Absecon . .	49.8	37 min. 30 sec.
Berlin to Absecon	35.6	25 min. 45 sec.
New York to Philadelphia . .	90	1 hr. 17 min.

74. Find the value of 25,000 bu. of oats at $46\frac{3}{4}$¢ per bushel.

75. The price of oats in June, 1900, was $26\frac{1}{4}$¢ per bushel, and in August it was 21¢ per bushel. If a speculator lost $1050 by buying oats at the former price and selling at the latter, how many bushels did he buy?

76. A speculator in Chicago bought 10,000 bu. of corn in February, 1901, at $38\frac{3}{8}$¢ per bushel, and sold it in December, 1901, at $69\frac{1}{4}$¢ per bushel. Find his profit.*

77. The total number of bales of cotton exported from the United States for the season of 1901–1902 was 6,715,793, valued at $284,779,190. Find the average price per bale, correct to the cent.

78. The total number of farms in Alabama is 223,220; the total acreage of these is 20,685,427. Find, correct to two decimal places, the average number of acres to a farm.

* Allow $\frac{1}{8}$¢ per bushel brokerage for buying and for selling.

79. The total sugar production of California was, in 1902, 356,500 T., valued at \$15,500,000. Find the average price per 100 lb.

80. According to the census of 1900, the number of persons employed in manufacturing industries in Florida was 1778, and the salaries paid amounted to \$1,295,139. Find the average salary received by each person, correct to the cent.

81. The total enrollment in the elementary and secondary schools in the United States in 1901 was 15,603,451, and the total number of teachers was 430,004. Find the average number of pupils to a teacher.

82. The total expenditure for higher education in Canada in a recent year was \$1,014,254. This expenditure was 19.5¢ per capita of the total population. Find the population of Canada.

83. The total expenditure for higher education in Germany in a recent year was \$7,450,366. The per capita expenditure was 14.3¢. Find the population of Germany.

The cost of higher education in Great Britain and Ireland for a recent year was given as \$8,353,655. The per capita expenditure was 21.7¢. Find the population of Great Britain and Ireland.

84. The total amount of money in circulation in the German Empire is, estimated in our currency, \$1,080,100,000. The per capita circulation is \$19.53. Find the population of Germany.

85. Express in feet .002357 of a mile.

86. Find the value of $\frac{5}{16}$ of a ton + $\frac{7}{8}$ of a hundredweight. Give your answer in pounds.

87. How many times is 12 lb. 8 oz. contained in 2 T.?

88. Light travels at the rate of 185,000 mi. a second. How long does it take a ray of light to pass from the earth to the moon, a distance of 239,000 mi.?

89. How many cubic yards of sand are required to fill a street $1\frac{1}{2}$ mi. long, 40 ft. wide, to the depth of 5 in.?

90. Express $\frac{3}{4}$ of a day as a decimal of a common year.

91. If $\frac{5}{16}$ of an acre of land is worth \$23, find the value of 85 A. of land.

92. If .375 of an acre of land is worth \$22, find the value of 57 A.

93. Multiply 68.4 by .0027, and divide the product by $\frac{3}{4}$ of .96.

94. Find the value of .1875 of a guinea + $\frac{2}{3}$ of £1 + .25 of 7*s.* 8*d.* Give your answer in pounds, shillings, and pence.

95. Reduce 12*s.* 6*d.* to the decimal of £4 sterling.

96. (*a*) Divide \$1293.46 by $.00\frac{7}{8}$. (*b*) Divide \$147.32 by $.00\frac{2}{3}$. (*c*) Divide \$3473.85 by $.00\frac{3}{4}$. (*d*) Divide \$3295 by $.00\frac{2}{5}$.

97. (*a*) Divide \$1456.77 by $.00\frac{7}{8}$. (*b*) Divide \$3947.85 by $.00\frac{3}{4}$.

98. How many acres in a field 160 ch. long, 40 ch. wide?

99. A wheel is $12\frac{1}{2}$ ft. in circumference. How many revolutions will it make in going 6 mi. 80 rd.?

100. How many bushels will a bin 7 ft. by 5 ft. and 4 ft. deep hold?

101. Reduce $\frac{11}{16}$ to a fraction having 12 for denominator.

102. .08 of a boy's money is \$6. How much money has the boy?

103. .875 of a man's property is valued at \$21,700. What is the value of the man's property?

104. How many acres are in a square field whose side is 40 rd.?

105. A and B can mow a field in 7 da. A, B, and C can mow the same field in 5 da. for \$50. What should C receive?

106. Write decimally three-eighths of one hundredth, and reduce it to a simple decimal.

107. Reduce $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{12}$, and $\frac{1}{16}$ to equivalent fractions having 100 for denominator.

108. Find the difference between $\dfrac{.16}{16}$ and $\dfrac{16}{.16}$.

109. Reduce $16\frac{3}{4}$ to an improper fraction having 16 for a denominator.

110. A rectangular field which is 18 rd. wide contains 6 A. How much will it cost to fence it at 75¢ a rod?

111. What decimal of 4 ft. 2 in. is 9 ft. 6 in.?

112. Find the least fraction which added to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ will make the sum an integer.

113. Divide 27.8 of a yard by .00125 of a foot.

114. 8 cwt. 20 lb. of sugar cost $41.42. What will 1 T. cost at the same rate?

115. Find the least length which is a multiple of 1 ft. 3 in., 1 ft. 8 in., 2 ft. 1 in., and 2 ft. 6 in.

116. Twelve tenths of a number equals 42. Find it.

117. Divide 54,218 by 64, using the factors of 64.

118. A train 165 yd. long passes a telegraph pole in 12 sec. Find the rate of the train in miles per hour.

119. A city lot 42 ft. by 120 ft. is sold for $840. At this rate, find the value of 1 A. of land in that city.

120. Find the greatest number which, when divided into 1958 and 2741, will give for remainders 8 and 11 respectively.

121. By buying eggs at 25¢ per dozen and selling them at 60¢ a score, a dealer makes a profit of $10.01. How many eggs does he sell?

122. Reduce $\frac{8568}{10296}$ to its lowest terms.

123. If a sum of money which will pay A's wages for $41\frac{3}{4}$ da. will pay B's wages for $55\frac{2}{3}$ da., for how long will it pay both?

124. If gold weighs 19.3 times as much as water, and copper 8.9 times as much as water, how much heavier than water is an alloy consisting of 16 parts of gold and 3 of copper?

125. A rectangular tank is 18 ft. $8\frac{3}{4}$ in. long, 11 ft. $3\frac{5}{6}$ in. wide, and contains 41 cu. yd., 6 cu. ft., and $34\frac{1}{8}$ cu. in. Find its depth. Find the area of each of its faces.

126. A tennis court is 42 yd. long and 20 yd. wide. It has a walk around it 6 ft. wide. Find the cost of paving the walk at $1.25 per square yard.

127. Telegraph poles along a certain railroad are 132 ft. apart. Find the rate of a train, in miles per hour, which passes 18 poles in 24 sec.

LONGITUDE AND TIME

128. A train leaves New York City at 9 A.M., Apr. 1, 1903, and arrives in Carson City, Nev., in 109 hr. 15 min. Find the hour of the day, and day of the month, Standard time, that it reaches its destination.

129. The time of mail transit between Chicago and Santa Fé, N. M., is 60 hr. 55 min. "The California Limited" leaves Chicago at 10 P.M. At what time, by the clocks in Santa Fé, should "The California Limited" pass Santa Fé?

130. The longitude of Cairo, Egypt, is 31° 21′ E., and the longitude of Savannah, Ga., is 81° 5′ 30″ W. Find the difference in time.

131. The longitude of Toulon is 5° 56′ E. The time difference between Toulon and Halifax, N. S., is 4 hr. 38 min. 4 sec. Find the longitude of Halifax.

132. The time difference between Toulon and Point Barrow, Alaska, is 10 hr. 48 min. 44 sec. Find the longitude of Point Barrow.

133. The time difference between Osaka and Point Barrow is 19 hr. 26 min. 48 sec. Find the longitude of Osaka. (See previous problem.)

134. (*a*) The difference between Standard and local time of Portland, Me., is 19 min. Find the longitude of Portland, Me.

(*b*) The difference between Standard and local time of Fort Wayne, Ind., is 20 min. Find the longitude of Fort Wayne.

(*c*) Cleveland, O., uses Central time; the difference between its local and Standard time is 33 min. Find the longitude of Cleveland, O.

PERCENTAGE

135. The total sugar production of the world for the year 1902 was 9,635,000 T. The amount of sugar consumed in the United States the same year was 2,372,000 T. What per cent of the world's production was the amount consumed in the United States?

136. The foreign-born population of New Orleans, according to the census of 1900, was 30,325; of this number, 1262 came from England, 4428 from France, 8733 from Germany, 5398 from Ireland. What per cent of the foreign-born population of New Orleans came from England? from France? from Germany? from Ireland?

137. The number of Canadians in Detroit, according to the census of 1900, was 25,400; this number was 26.3% of the foreign-born population. Find, correct to 100, the number of foreign-born population of Detroit.

138. A horse is sold, at a loss of 15%, for $127.50. Find the cost of the horse.

139. By selling silk at $1.60 per yard, a dealer makes a profit of 25%. What would the selling price be if he made a profit of $12\frac{1}{2}$%?

140. When cloth is sold for $1.04 per yard, a clothier makes a profit of 30%. What would his profit be if he sold the cloth at 96¢ per yard?

141. A wholesale dealer makes a profit of 10% on canned goods. The retail dealer makes a profit of 25%. Find the original cost of canned goods which cost the consumer $11.

142. A coal merchant buys coal by the long ton at $4.50 a ton, and sells it at the rate of $5 a short ton. Find his gain per cent.

143. How much water must be added to a 25% wine mixture to make it a 20% mixture?

144. A sells goods to B at a profit of 20%; B sells them to C at a profit of 20% on his outlay; C sells them to D for $180, thereby losing $16\frac{2}{3}$%. How much did the goods cost A?

145. A merchant buys goods at 20%, and 10% off list price, and sells them at the list price. Find his per cent of gain.

146. When 20 lb. of tea are sold for what $22\frac{1}{2}$ lb. cost, what is the gain per cent?

147. (*a*) A vessel contains 31 gal. of wine and 17 gal. of water. What per cent of the mixture is wine and what per cent is water? (*b*) How many gallons of water must be added to this mixture to make a mixture containing 60% wine?

148. The following table gives the distances from Atlantic to Pacific ports by the present routes:

New York to San Francisco	13,244 mi., nautical
New York to Sydney	14,560 mi., nautical
Charleston to San Francisco	13,180 mi., nautical
Charleston to Valparaiso	8,296 mi., nautical
New Orleans to San Francisco	13,644 mi., nautical
New Orleans to Melbourne	15,535 mi., nautical
Galveston to San Francisco	13,826 mi., nautical
Galveston to Wellington	14,182 mi., nautical
Liverpool to San Francisco	13,844 mi., nautical
Hamburg to Callao	10,702 mi., nautical
Bordeaux to San Francisco	13,691 mi., nautical

The following table gives the distances from Atlantic to Pacific ports via the Panama Canal route:

New York to San Francisco	5299 mi., nautical
New York to Sydney	9852 mi., nautical
Charleston to San Francisco	4898 mi., nautical

Charleston to Valparaiso	4229 mi., nautical
New Orleans to San Francisco	4698 mi., nautical
New Orleans to Melbourne	9826 mi., nautical
Galveston to San Francisco	4800 mi., nautical
Galveston to Wellington	8392 mi., nautical
Liverpool to San Francisco	8038 mi., nautical
Hamburg to Callao	6527 mi., nautical
Bordeaux to San Francisco	7938 mi., nautical

What per cents of the distances by the old routes are saved by the Panama Canal route?

INTEREST

149. Find the simple interest on $78 for 93 da. at 8%.

150. Find the simple interest on $98 for 63 da. at 7%.

151. Find the amount of $179 for 123 da. at 6%.

152. Find the simple interest on £324 7*s.* 9*d.* from June 12 to Dec. 7 following at 5%.

153. Find the simple interest on £1169 6*s.* 8*d.* from Jan. 25 to June 18 following at 9%.

154. What principal will produce $19.50 in 1 yr. at $6\frac{1}{2}$% ?

155. What principal will produce $180 interest in 3 mo. at 5% ?

156. What principal will amount to $412.50 in $7\frac{1}{2}$ mo. at 5% ?

157. What principal will amount to $1219 in 3 mo. 5 da. at 6% ?

158. What principal will produce $29.17 in 5 mo. at 7% ?

159. At what rate will $1000 produce $23.33 interest in 4 mo.?

160. What principal will produce 75¢ interest in 9 da. at 6% ?

161. Find the exact interest on $73.15 from June 18 to Aug. 1 at 7%.

162. A note for $3500 bearing interest at 8% and dated Jan. 2, 1900, was indorsed as follows: June 7, 1900, $450;

Aug. 2, 1900, \$208; Jan. 2, 1901, \$500; July 7, 1901, \$800; Oct. 4, 1901, \$500; Jan. 11, 1902, \$300; Aug. 4, 1902, \$700. Calculate, by the United States Rule for partial payments, the amount due on this note on Jan. 1, 1903.

163. A demand note dated Jan. 5, 1902, and drawn for \$575 was paid 6 mo. 18 da. later. Find the date of payment and the amount of the note, the rate of interest being 7%.

BANK DISCOUNT

164. A note for 60 da. is drawn on Jan. 10, 1903. Find the proceeds of this note, if its face is \$150, the date of discount Feb. 5, and the rate 6%. (Neglect days of grace.)

165. A note for \$900, dated Mobile, Ala., Jan. 8, 1903, and drawn for 90 da., is discounted March 1. Find the proceeds.

166. A 60-day note bearing interest at 8%, drawn Feb. 1, 1903, for \$1000, is discounted Feb. 27 at 9%. Find the proceeds of this note.

167. A demand note was drawn Oct. 1, 1902, for \$800, and paid 5 mo. 10 da. later. Find the date of payment and the amount of the note; rate of interest, 7%.

168. The proceeds of a note is \$450 when the term of discount is 93 da., and the rate of interest is 8%. What is the maturity value of the note?

MENSURATION

169. Find the area of a parallelogram if its base is 100 yd. and its altitude is 75 yd.

170. Find the area of a trapezoid if its parallel sides are 60 yd. and 80 yd., and its altitude is 50 yd.

171. A tract of land is sold for \$3943.84; the land cost as many dollars per acre as there were acres in the tract. Find the cost per acre.

172. Find the number of square yards in the walls and ceiling of a room 36 by 23, and 16 ft. high.

173. Find the perimeter of a square which contains 40 A.

174. A tract of land in the shape of a rectangle contains 320 A.; its length is twice its width. Find its dimensions.

175. Find the area of an equilateral triangle one side of which is 100 ft.

176. Find the area of a regular hexagon each side of which is 50 ft.

177. Find the circumference of a circle whose radius is 56.5 in.

178. Find the area of a circle if its diameter is 20 in.

179. Find the surface of a sphere whose diameter is 20 in.

180. Find the volume of a cube one of whose dimensions is 1 ft. 3 in.

181. The surface of a cube is 221.0694 sq. in. Find the length of one edge of this cube.

COMPARISON OF PRICES

Express:

182. 6 francs per kilogram as dollars per pound Avoirdupois.

183. 5 francs per meter as dollars per yard.

184. 1.369 francs per liter as dollars per gallon.

185. 9 francs per hektoliter as dollars per bushel.

186. 7 marks per kilogram as dollars per pound Avoidupois.

187. 4 marks per meter as dollars per yard.

188. 2 marks per liter as dollars per U. S. gallon.

189. 9 marks per hektoliter as dollars per bushel.

190. 13.785 marks per meter as dollars per yard.

191. $40 per acre as francs per hektar.

MISCELLANEOUS EXAMPLES (B)

(Taken from Various Examination Papers)

1. What fractional part of $\frac{5}{7}$ of a gallon is $\frac{3}{28}$ of a pint?

2. The difference in time between two places is 2 hr. 33 min. Find the difference in longitude.

3. A bicycle wheel measuring 88 in. in circumference must make how many revolutions a minute to run eighteen miles an hour?

4. A coal bin $16\frac{1}{2}$ ft. long and 8 ft. 9 in. wide must be how deep to contain 10 T. of coal, if one ton of coal occupy 40 cu. ft. of space?

5. Reduce 2 yr. 21 da. to years and decimals of a year.

6. Reduce .09625 bbl. to integers of lower denominations.

7. Find the value of a piece of land 64 ch. by $13\frac{1}{2}$ ch. at $\$48\frac{1}{3}$ an acre.

8. Required the cost of 18 $2\frac{1}{2}$ in. plank 16 ft. long and 10 in. wide, and 33 pieces of scantling 2 in. by 4 in. 16 ft. long, at \$22 per M, board measure.

9. The average yield per bushel of wheat is 14 bu. 1 pk. What will 7 bu. 3 pk. 2 qt. yield?

10. What is the difference in weight, expressed in Avoirdupois pounds, between 100 lbs. Troy and 100 lbs. Avoirdupois?

11. Reduce to simplest form $\dfrac{\frac{2}{3}+\frac{3}{7}}{7-\frac{4}{5}} \div \dfrac{\frac{2}{9}}{2+\frac{1}{2}}$.

12. If it cost \$510 to fence a rectangular field 98 rd. by 72 rd., what will it cost to fence a square field of the same area?

13. Express $\frac{9}{7}$ as a decimal fraction.

14. What is the ratio of 32 ft. to 6 yd.? Express the result decimally.

15. What is the length of a plank $1\frac{1}{2}$ in. thick, 1 ft. 6 in. wide, containing 36 board feet?

16. When it is 12 M. in New York City (74° W.), what is the time in Manila (120° E.)?

17. If the value of $\frac{1}{7}$ of $\frac{3}{8}$ of an estate is \$4500, what is the value of $\frac{1}{14}$ of $\frac{1}{12}$ of it?

18. At \$16 per M, board measure, find the cost of 20 plank 2 in. by 8 in. 18 ft. long, and 30 plank $1\frac{1}{2}$ in. by 6 in. 10 ft. long.

19. A can do a piece of work in 6 da. and B can do the same work in 8 da. How long will it take B to finish after they have worked together two days?

20. $20\frac{3}{4}$ is the product of three factors. Two of these factors are $1\frac{3}{5}$ and $4\frac{5}{8}$. Find the other factor.

21. How many bushels of wheat will a box 6 ft. by $3\frac{1}{2}$ ft. by 2 ft. 8 in. hold?

22. How many yards are in .04675 mi.?

23. If the dividend is 807 and the quotient $34\frac{1}{3}$, what is the divisor?

24. How many rods of fence will inclose a square field whose area is 20 acres?

25. Coal sells at \$5.75 per ton. What will be the cost of 2315 lb. at this rate?

26. How many gallons of water will a tank 5 ft. by 2 ft. by 2 ft. hold?

27. What is the length of one side of a square piece of land whose area is 538,756 sq. rd.?

28. A room is 27 ft. by 22 ft. 6 in. How many yards of carpet 27 in. wide will be required to carpet this room?

29. A man is hired to dig a cellar 20 ft. by 15 ft. by 5 ft. How much money will he receive at 30¢ per cu. yd.?

30. How many days are there between Aug. 14 and Dec. 29?

31. Find the value of a car load of wheat, estimated at 21,643 lb., at 92¢ per bushel.

32. Two persons travel in opposite directions from the same point at the rate of $4\frac{1}{3}$ and $7\frac{3}{4}$ mi. per hour, respectively. How far apart are they after traveling $37\frac{1}{2}$ hrs.?

33. A man was born Nov. 22, 1861. What is his age to-day?

34. Factor the following numbers and from these factors determine the G. C. M.: 42, 112, 140, 308.

35. What will 75 boards 2 in. by 4 in. by 16 ft. long cost at $12 per M board measure?

36. 160 rd. of fence will inclose how many acres in the form of a square?

37. The difference in longitude between two places is 7° 42′ 30″. Find the difference in time.

38. How wide is a rectangular field containing 5 A., the length of the field being 7 ch. 25 l.?

39. A pavement is $5\frac{1}{3}$ rd. long and 8 ft. 6 in. wide. What did it cost at $1.40 per sq. yd.?

40. Three men, A, B, and C do a piece of work; A works 3 da. of 5 hr. each, B, 2 da. of 6 hr. each, and C, 7 da. of 3 hr. each. At the same rate of wages, how should they divide $120, the total amount received for doing the work?

41. A miller charges $\frac{1}{16}$ for toll. How many bushels of wheat must one take to mill to get 12 bbl. of flour, each containing 196 lb., if a bushel of wheat makes 40 lb. of flour?

42. The annual rainfall in a certain locality is 30 in. How many tons of water fall on an acre of land in this locality, if a cubic foot of water weighs 1000 oz.?

43. How much does a man gain or lose on the sale of two houses at $1200 each, if he gains $\frac{1}{3}$ of the cost price on one, and loses $\frac{1}{5}$ of the cost price on the other?

44. What is the ratio of 7 lb. Troy weight to 10 oz. Avoirdupois?

45. The divisor is 357, the quotient is $6\frac{3}{7}$; what is the dividend?

46. A farmer had 28 A. of land left after selling $\frac{1}{4}$ of his farm to one neighbor, $\frac{2}{5}$ of it to another, and $\frac{3}{7}$ of the remainder to another. How large was his farm?

47. Multiply 8.035 by .0035, add 3, and divide the sum by .000625.

48. Divide $459.25 into three parts that shall be to one another as $\frac{3}{4}$, $\frac{5}{6}$, and 3 respectively.

49. When it is two o'clock P.M. in Jerusalem, what is the time in Cincinnati? The longitude of Jerusalem is 35° 12′ E., and of Cincinnati, 84° 26′ W.

50. Find the exact number of days between Dec. 23, 1902, and to-day.

51. A man's farm is mortgaged for $\frac{2}{3}$ of its cost; he sells it for $6000 which is 25% above its cost. How much money will he have after paying the mortgage?

52. A note for $600, dated Oct. 24, 1902, and due in 8 mo., with interest at 6% per annum, is discounted at bank Dec. 20, 1902. Find the proceeds.

53. A man sold two lots each for $600, gaining 20% on one, and losing 20% on the other. What was his gain or loss?

54. A man buys a book the list price of which is $7.20, at a discount of $16\frac{2}{3}$%, and sells it for $7.50. What is his gain per cent?

55. What principal at interest for 1 yr. 3 mo. will amount to $506, the rate of interest being 8% per annum?

56. Twelve per cent of 90 is what per cent of 100?

57. At the following rates per annum of simple interest, what time is required for the accruing interest to equal the principal: 6%, 8%, $9\frac{1}{11}$%?

58. What is the exact interest on $10,000 from Jan. 18, 1903, to May 6, 1904, at $3\frac{1}{2}\%$?

59. A 30-day note, without interest, is discounted at a bank at 8% for $350. What is the face of the note?

60. Bonds bearing 5% interest are bought at 120. What is the rate of income on these bonds?

61. An agent buys sugar at $4\frac{3}{4}$¢ per pound; his commission at $\frac{1}{4}\%$ is $25. How many pounds of sugar does he buy?

62. The discount of a note, discounted at bank, for 3 mo. 18 da. at 5% is $4.20. Find the proceeds.

63. What single discount is equivalent to trade discounts of 25%, 10%, and 5% on the list price of an article?

64. The property in a school district is assessed at $196,000. What rate of taxation would be required to provide about $800 annually for the improved maintenance of the schools? What annual tax would a man pay on this account whose property is assessed at $1200?

65. A man sold two horses at $80 each. On one he gained 20%, on the other he lost 20%. Find the gain or loss.

66. What must I ask for an article worth $36 that, after falling 20%, I may gain 25% on the value?

67. A school district advertised for bids to build a schoolhouse, the lowest bid being $21,049. If it costs 3% to collect the money, how large a levy should be made, supposing 29% of it to be non-collectible?

68. What must a man pay for 4% stock to get 5% on his investment?

69. If you buy United States 3's at 110, what per cent per annum would your investment pay?

70. A merchant's expenses average 10% of his sales. At what per cent advance on cost must he sell his goods to clear 20% profit?

71. A merchant sold goods to the amount of \$760.95, thereby losing 11%. What did he pay for the goods?

72. A ship is insured for half its value for \$374. If the rate is $2\frac{3}{4}$%, what is the value of the ship?

73. A carriage dealer sold 16 buggies at \$200 each; on one half he gained 10%, and on the other half he lost 10%. Find his net gain or loss.

74. What principal will amount to \$1253.86 in 2 yr. 11 mo. 13 da., interest at 5%?

75. How do you find the rate per cent per annum when the principal, interest, and time are given?

76. How do you find the principal when the rate per cent per annum, time, and interest are given?

77. How do you find the time when the principal, rate per cent per annum, and interest are given?

78. The list price of office desks is \$15, but 12 desks are sold for \$126. What rate of discount is allowed?

79. In a certain time \$650 will amount to \$713.05 at 6% simple interest. Find the time.

80. A note for \$500, due in 3 months, is discounted at bank at 6%. Find the proceeds.

81. $236\frac{1}{2}$ is what per cent of $78\frac{5}{6}$?

82. A man insures his life, paying a premium of \$28, which is at the rate of $\frac{7}{8}$% on the amount of his insurance. Find the face of the policy.

83. If 25% of the selling price of an article is profit, what is the per cent of gain on its cost?

84. A man fails in business; his assets amount to \$2100, his liabilities to \$6000. What per cent will his creditors receive?

85. What is the interest on \$475 for 1 yr. 3 mo. 24 da. at 6%?

86. A man bought four loads of hay, each weighing 2750 lb., at $20 per ton; he gave in payment his note, without interest, at 60 da. What are the proceeds of this note, discounted at a bank at 6%?

87. What per cent of $\frac{1}{2}$ is $\frac{1}{8}$?

88. An agent's commissions at 5% amount to $37.65. Find the amount of his sales.

89. The tax on property assessed at $8500 is $48.37. What is the rate on $1000?

90. Find the date of maturity of a note made and dated Sept. 11, 1902, and payable 90 da. after date.

91. Find the cost of 87 shares of stock at $76\frac{1}{4}$, brokerage $\frac{1}{8}$ per cent.

92. A New York sight draft was sold in Atlanta, Ga., for $3542, exchange being at $\frac{3}{4}$% premium. What was the face of the draft?

93. What per cent of 5 lb. is 3 oz. Avoirdupois?

94. An agent's commission for renting a house is $13.25; his rate of commission is $2\frac{1}{2}$%. What is the yearly rent of the house?

95. A man pays a premium of $150 for insuring his house for $\frac{2}{3}$ of its value; the rate of premium is $1\frac{1}{2}$ per cent per annum. What is the value of the house?

96. A building worth $6000 is insured for $\frac{3}{4}$ of its value at 75¢ on the $100. In case of the destruction of the building by fire, what will be the owner's loss, including premium?

97. What per cent of 1 bu. is 3 qt.?

98. A merchant can buy flour on six months' credit at $8 per barrel, or for cash at $7.50 per barrel. He buys 100 bbl., paying cash, but borrows the money at 8% to pay for it. Is this better than to buy on credit, and how much better?

99. A man sells 16 shares of bank stock at $127\frac{3}{4}$, brokerage $\frac{1}{8}$%. How much does he receive for his stock?

ocks at 20 % premium and
discount. What per cent
?

t's books show sales during
the month of March amounting to $1000. One half of his sales are at a profit of 25 % on the cost, and the other half a loss of $16\frac{2}{3}$ % on the cost. Find the cost of the goods sold during the month.

102. A merchant failing in business paid his creditors $3874.75, which was at the rate of 55¢ on every dollar of his indebtedness. Find his indebtedness.

103. The list price of a mower is $38; the retail dealer is allowed discounts of 20 %, 5 %, and 3 %. What does he pay for mowers? If the retailer sells these mowers at a profit of 50 %, what does the farmer pay for these mowers?

104. A certain stock, selling at $121\frac{3}{4}$, pays a semiannual dividend of 4 %. What is the rate per cent per annum on an investment in this stock?

TABLES

Apothecaries' Weight

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (℔)

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
2 barrels	= 1 hogshead (hhd.)

Long Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.), or pole
40 rods	= 1 furlong
8 furlongs	= 1 mile (mi.)

Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Numerical Measure

12 articles	= 1 dozen
12 dozen	= 1 gross
12 gross	= 1 great gross
20 articles	= 1 score

Avoirdupois Weight

16 drams (dr.)	= 1 ounce (oz.)
16 ounces	= 1 pound (lb.)
25 pounds	= 1 quarter
100 pounds	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton (T.)
2240 pounds	= 1 long ton

Circular Measure

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
30 degrees	= 1 sign (S.)
12 signs	= 1 circle (C.) or circumference
360 degrees	= 1 circumference

Nautical Measure

6 feet	= 1 fathom
608 feet	= 1 cable length
10 cable lengths	= 1 nautical mile (6080 feet)

The following denominations are also used:

1.152 statute miles	= 1 geographic mile, or knot
3 geographic miles	= 1 league
60 geographic miles, or 69.1 statute miles	= 1 degree of latitude on a meridian
360 degrees	= the circumference of the earth

4 inches	= 1 hand
9 inches	= 1 span
21.888 inches	= 1 sacred cubit
3 feet	= 1 pace

Time Measure

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
4 weeks	= 1 lunar month
30 days	= 1 commercial month
12 months	= 1 year
365 days	= 1 common year
366 days	= 1 leap year

Surveyors' and Land Measure

7.92 inches	= 1 link (l.)
25 links	= 1 rod
4 rods	= 1 chain (ch.)
10 square chains	= 1 acre
640 acres	= 1 square mile

625 square links (sq. l.)	= 1 pole (P.)
16 poles	= 1 square chain

Apothecaries' Fluid Measure

60 minims (m.)	= 1 fluidrachm (f ʒ)
8 fluidrachms	= 1 fluidounce (f ℥)
16 fluidounces	= 1 pint (O)
8 pints	= 1 gallon (Cong.)

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
16 cubic feet, or 8 cord feet	= 1 cord of wood (cd.)
$24\frac{3}{4}$ cubic feet	= 1 perch of stone or masonry (pch.)

Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod or perch (sq. rd. or sq. pch.)
40 square rods	= 1 square rood (sq. R.)
4 roods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Spanish Land Measure

In Texas, California, New Mexico, and other parts of this country which were formerly parts of the Spanish empire, the vara, the unit of linear measure, is still used in connection with original grants of land. In Texas, the value of the vara is $33\frac{1}{3}$ in. In California and New Mexico it is usually considered 33 in.

1,000,000 square varas = 1 labor = 177.136 acres
25,000,000 square varas = 1 league = 4428.4 acres
3,612,800 square varas = 1 square mile = 640 acres
1,806,400 square varas = $\frac{1}{2}$ square mile = 320 acres
903,200 square varas = $\frac{1}{4}$ square mile = 160 acres
451,600 square varas = $\frac{1}{8}$ square mile = 80 acres
225,800 square varas = $\frac{1}{16}$ square mile = 40 acres
5645 square varas = 1 acre

ANSWERS

Exercise 4. — North Atlantic Division, 5,499,620, 3,843,908, 2,866,074, $105,332,839. South Atlantic Division, 3,521,920, 2,324,906, 1,503,917, $15,907,956. South Central Division, 5,002,836, 3,235,121, 2,074,304, $19,870,733. North Central Division, 7,878,448, 5,895,631, 4,188,517, $107,663,687. Western Division, 1,125,924, 956,472, 685,444, $24,441,012. Totals, 23,028,748, 16,256,038, 11,318,256, $273,216,227.

Exercise 5. — **1.** $2,187,217.51. **2.** $2,749,532.73. **3.** $8,593,793.27. **4.** $733,501.99. **5.** $4,330,637.88. **6.** $9,145,702.01. **7.** $773,768.16. **8.** $6,124,815.32. **9.** $7,750,185.14. **10.** $3,429,967. **11.** $2,001,002.77. **12.** $3,135,817.92. **13.** $10,029,526.43. **14.** $659,959.65. **15.** $2,004,181.95. **16.** $853,917.00.

Exercise 6. — **1.** $75, $200, $900, $1225. **2.** $2.08, $4.68, $24.44. **3.** 100,320 ft.; 401,280 ft. **4.** 273 da. **5.** $6084. **6.** 2136 hr. **7.** 1598 mi. **8.** 62,720 A. **9.** 1173 mi. **10.** 487,956 lb. **11.** $1008. **12.** 85,008 cu. in. **13.** 103,680 pages. **14.** $956.80. **15.** $1736.

Exercise 7. — **1.** 9000 sq. yd. **2.** 7056 sq. yd. **3.** 102,400 sq. rd. **4.** 11,760 sq. yd. **5.** 22,848 sq. yd. **6.** 167,200 sq. yd. **7.** 4725 sq. rd. **8.** 20,160 ft. **9.** 252 sq. in. **10.** 36 sq. mi. **11.** 432 sq. mi. **12.** 40,480 sq. yd. **13.** 2200 sq. yd. **14.** 5332 sq. ft. **15.** 2880 sq. in. **16.** 1296 sq. in.

Exercise 8. — **1.** $639.52. **2.** $225.40. **3.** $476.94. **4.** $898.03. **5.** $3594.51. **6.** $5455.08. **7.** $8023.18. **8.** $2136.42. **9.** $5046.93. **10.** $2285.44. **11.** $359.31. **12.** $555.03. **13.** $239.76. **14.** $1076.28. **15.** $1866.95. **16.** $6399.52. **17.** $7402.50. **18.** $10,986.58. **19.** $9345. **20.** $6594.75. **21.** $6498.38. **22.** $3269.10. **23.** $93.60. **24.** $159.60. **25.** $187.65. **26.** $322.56. **27.** $195.44. **28.** $7.02. **29.** $19.50. **30.** $6075.

Exercise 9. — **1.** 841.75 mi. **2.** 614.2 mi. **3.** 809.93 mi. **4.** 710.4 mi. **5.** 1401.56 mi. **6.** 1280.2 mi. **7.** 754.8 mi. **8.** 1032.3 mi. **9.** 1191.4 mi. **10.** 1,879.6 mi. **11.** 1742.7 mi. **12.** 1894.4 mi. **13.** 2,217,648. **14.** 2,230,095. **15.** 4,821,600. **16.** 1,380,768. **17.** 2,423,546. **18.** 1,881,250. **19.** 4,157,520. **20.** 6,297,900.

Exercise 10. — **1.** 4834, rem. 10. **2.** 4408, rem. 14. **3.** 4892, rem. 14. **4.** 4416, rem. 2. **5.** 4166, rem. 16. **6.** 2957, rem. 23. **7.** 2485, rem. 13. **8.** 2668. **9.** 2760, rem. 32. **10.** 1706, rem. 28. **11.** 1234, rem. 39. **12.** 2380, rem. 28. **13.** 1431, rem. 10. **14.** 2519, rem. 25. **15.** 17,530, rem. 43. **16.** 14,616, rem. 31. **17.** 14,792, rem. 22. **18.** 8261, rem. 35. **19.** 8741, rem. 42. **20.** 3377, rem. 42. **21.** 2133, rem. 74. **22.** 8214, rem. 4. **23.** 9701, rem. 37. **24.** 8217, rem. 3.

Exercise 11. — **1.** 14 lb. **2.** 29 hr. **3.** 375 pk. **4.** 29 hr., 4 mi. rem.; 94 mi. **5.** 384 T. **6.** 179 da. **7.** 225 da. **8.** 2083, 4 in. rem. **9.** 52 wk. **10.** 14 doz. **11.** 16. **12.** 37 boxes. **13.** 417 bbl. **14.** 24 hr. **15.** $125. **16.** 161 sheep. **17.** 159 coins. **18.** 45 hogs. **19.** 245 boxes. **20.** 167 bbl. **21.** 19 lb. **22.** 36 horses.

Exercise 12. — **1.** 42 bags. **2.** 39 bbl. **3.** 79 bbl. **4.** 2777 yd., 28 in. rem. **5.** 241 bu. **6.** 125 da. **7.** 36°. **8.** 364 sq. yd. **9.** 36 gal. **10.** 33 chests. **11.** 325 bbl. **12.** 114 hr. **13.** 1000 A. **14.** 1250 sq. mi.

Exercise 13. — **1.** 68. **2.** 69. **3.** 427. **4.** 5320. **5.** 245. **6.** 385. **7.** 198. **8.** 17. **9.** 2125. **10.** 717 sacks. **11.** 810. **12.** 563 bbl. **13.** 17,503. **14.** 396. **15.** 39.

Exercise 14. — **1.** 196.2. **2.** 629.8. **3.** 167.6. **4.** 189.8. **5.** 289.6. **6.** 303.8. **7.** 324.8. **8.** 445.2. **9.** 16.3. **10.** 95.6. **11.** 356.1. **13.** .5, .75, .8, .3125, .24, .21875, .171875, .6171875. **14.** .85, .38, .1025, .415, .536, .348. **15.** 2875, .425, .45, .74, .206, .02725. **16.** .333, .1429, .111, .0909, .0833, .0769, .0714, .0667, .0588. **17.** .1852, .4783, .3214, .1290, .4571, .0959.

Exercise 15. — **1.** $58.48. **2.** $136.15. **3.** $16.35. **4.** $31.42.

Exercise 16. — **1.** $1. **2.** 42 ft. **3.** 5 da. **4.** 10 da. **5.** 60 lb. **6.** $24, 5 sheep. **7.** 60¢ on each kind, 47 oranges. **8.** 60¢. **9.** 60 sec. **10.** 7 ft. 6 in.

Exercise 18. — **1.** 8. **2.** 8. **3.** 9. **4.** 12. **5.** 15. **6.** 15. **7.** 18. **8.** 36. **9.** 12. **10.** 12. **11.** 7. **12.** 13. **13.** 19. **14.** 23. **15.** 24. **16.** 28. **17.** 10. **18.** 15. **19.** 12. **20.** 13. **21.** 12. **22.** 18. **23.** 16. **24.** 21. **25.** 32. **26.** 36. **27.** 44. **28.** 63. **29.** 48. **30.** 54. **31.** 56. **32.** 84.

Exercise 19. — **1.** 80. **2.** 42. **3.** 180. **4.** 90. **5.** 147. **6.** 140. **7.** 168. **8.** 108. **9.** 102. **10.** 144. **11.** 480. **12.** 420. **13.** 160. **14.** 78. **15.** 336. **16.** 480. **17.** 756. **18.** 385. **19.** 270. **20.** 216. **21.** 375. **22.** 216. **23.** 180. **24.** 576. **25.** 180. **26.** 210. **27.** 210. **28.** 420. **29.** 16,800. **30.** 720. **31.** 300. **32.** 180. **33.** 630. **34.** 280. **35.** 144. **36.** 840. **37.** 600. **38.** 2002. **39.** 180. **40.** 210. **41.** 240. **42.** 420. **43.** 108. **44.** 720. **45.** 360.

Exercise 21. — **1.** $\frac{7}{2}$. **2.** $\frac{11}{3}$. **3.** $\frac{38}{7}$. **4.** $\frac{75}{8}$. **5.** $\frac{65}{7}$. **6.** $\frac{113}{11}$. **7.** $\frac{71}{8}$. **8.** $\frac{35}{6}$. **9.** $\frac{113}{12}$. **10.** $\frac{120}{11}$. **11.** $\frac{111}{10}$. **12.** $\frac{41}{4}$. **13.** $\frac{135}{7}$. **14.** $\frac{166}{9}$. **15.** $\frac{302}{11}$. **16.** $\frac{461}{12}$. **17.** $\frac{199}{5}$. **18.** $\frac{59}{3}$. **19.** $\frac{230}{11}$. **20.** $\frac{271}{16}$. **21.** $\frac{115}{8}$. **22.** $\frac{107}{7}$. **23.** $\frac{175}{9}$. **24.** $\frac{262}{9}$. **25.** $\frac{329}{12}$. **26.** $\frac{181}{13}$. **27.** $\frac{100}{7}$. **28.** $\frac{299}{8}$. **29.** $\frac{488}{11}$. **30.** $\frac{303}{10}$.

Exercise 22. — **1.** $\frac{6}{9}, \frac{10}{15}, \frac{16}{24}, \frac{20}{30}, \frac{24}{36}$. **2.** $\frac{6}{8}, \frac{12}{16}, \frac{18}{24}, \frac{24}{32}, \frac{30}{40}$. **3.** $\frac{6}{10}, \frac{12}{20}, \frac{15}{25}, \frac{21}{35}, \frac{24}{40}$. **4.** $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \frac{20}{35}, \frac{28}{49}$. **5.** $\frac{10}{16}, \frac{15}{24}, \frac{25}{40}, \frac{30}{48}, \frac{40}{64}$. **6.** $\frac{12}{32}, \frac{18}{48}, \frac{21}{56}, \frac{27}{72}, \frac{30}{80}$. **7.** $\frac{10}{18}, \frac{15}{27}, \frac{25}{45}, \frac{35}{63}, \frac{40}{72}$. **8.** $\frac{18}{20}, \frac{45}{50}, \frac{63}{70}, \frac{72}{80}, \frac{81}{90}$. **9.** $\frac{14}{24}, \frac{28}{48}, \frac{42}{72}, \frac{49}{84}, \frac{56}{96}$. **10.** $\frac{21}{7}, \frac{30}{10}, \frac{36}{12}, \frac{60}{20}$. **11.** $\frac{20}{4}, \frac{45}{9}, \frac{70}{14}, \frac{125}{25}$. **12.** $\frac{21}{3}, \frac{56}{8}, \frac{70}{10}, \frac{84}{12}$.

Exercise 23. — **1.** $\frac{1}{2}, \frac{4}{9}, \frac{2}{3}, \frac{2}{3}, \frac{3}{8}, \frac{4}{7}, \frac{4}{5}, \frac{5}{6}, \frac{2}{3}$. **2.** $\frac{2}{5}, \frac{2}{5}, \frac{2}{5}, \frac{4}{5}, \frac{5}{12}, \frac{5}{8}, \frac{5}{6}, \frac{7}{8}$. **3.** $\frac{2}{3}, \frac{2}{3}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{8}, \frac{3}{8}, \frac{3}{8}$. **4.** $\frac{2}{9}, \frac{4}{9}, \frac{4}{9}, \frac{7}{12}, \frac{7}{12}, \frac{7}{12}, \frac{7}{12}, \frac{7}{12}$. **5.** $\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{3}{4}, \frac{9}{16}, \frac{7}{9}, \frac{7}{9}, \frac{5}{12}$.

Exercise 24. — **1.** $1\frac{1}{12}$. **2.** $1\frac{11}{12}$. **3.** $1\frac{5}{12}$. **4.** $1\frac{7}{8}$. **5.** $1\frac{3}{4}$. **6.** $1\frac{1}{4}$. **7.** $\frac{5}{6}$. **8.** $1\frac{1}{15}$. **9.** $1\frac{5}{8}$. **10.** $1\frac{4}{5}$. **11.** $1\frac{4}{5}$. **12.** $1\frac{1}{5}$. **13.** $1\frac{2}{9}$. **14.** $1\frac{17}{24}$. **15.** $1\frac{37}{48}$. **16.** $2\frac{11}{16}$. **17.** $1\frac{1}{12}$. **18.** $2\frac{1}{18}$. **19.** $2\frac{1}{24}$. **20.** $1\frac{19}{24}$. **21.** $2\frac{3}{10}$. **22.** $1\frac{19}{20}$. **23.** $1\frac{9}{10}$. **24.** $1\frac{17}{20}$. **25.** $\frac{23}{24}$. **26.** $2\frac{7}{24}$. **27.** $1\frac{13}{24}$. **28.** $1\frac{11}{24}$. **29.** $2\frac{1}{6}$. **30.** $1\frac{19}{24}$. **31.** $7\frac{7}{12}$. **32.** $15\frac{1}{4}$. **33.** $8\frac{5}{8}$. **34.** $25\frac{5}{12}$. **35.** $26\frac{11}{12}$. **36.** $22\frac{5}{6}$. **37.** $17\frac{3}{5}$. **38.** $12\frac{19}{24}$. **39.** $19\frac{7}{24}$. **40.** $17\frac{1}{3}$. **41.** $25\frac{1}{20}$. **42.** $14\frac{19}{32}$. **43.** $\$\frac{7}{10}$. **44.** $1\frac{1}{12}$ hr. **45.** $\frac{5}{6}$ is largest ; $\frac{2}{3}$ is smallest.

Exercise 25. — **1.** $\frac{1}{4}$. **2.** $\frac{1}{6}$. **3.** $\frac{1}{2}$. **4.** $\frac{1}{6}$. **5.** $\frac{2}{5}$. **6.** $\frac{1}{8}$. **7.** $\frac{1}{2}$. **8.** $\frac{1}{6}$. **9.** $\frac{1}{4}$. **10.** $\frac{1}{12}$. **11.** $\frac{1}{8}$. **12.** $\frac{1}{2}$. **13.** $1\frac{1}{2}$. **14.** $2\frac{1}{4}$. **15.** $1\frac{5}{8}$. **16.** $1\frac{3}{10}$. **17.** $10\frac{1}{5}$. **18.** $12\frac{1}{8}$. **19.** $8\frac{5}{6}$. **20.** $11\frac{5}{12}$. **21.** $6\frac{1}{10}$. **22.** $7\frac{1}{4}$. **23.** $7\frac{5}{12}$. **24.** $3\frac{1}{2}$. **25.** $5\frac{1}{8}$. **26.** $16\frac{1}{2}$. **27.** $6\frac{1}{6}$. **28.** $2\frac{7}{24}$. **29.** $11\frac{11}{24}$. **30.** $10\frac{5}{24}$. **31.** $2\frac{5}{6}$. **32.** $3\frac{11}{12}$. **33.** $3\frac{11}{12}$. **34.** $\frac{7}{12}$ **35.** $6\frac{7}{12}$. **36.** $2\frac{7}{10}$. **37.** $6\frac{7}{10}$. **38.** $4\frac{3}{5}$. **39.** $4\frac{5}{6}$. **40.** $8\frac{11}{15}$. **41.** $19\frac{8}{9}$. **42.** $7\frac{9}{10}$. **43.** $5\frac{5}{8}$. **44.** $\$2\frac{1}{4}$. **45.** $2\frac{11}{12}$. **46.** $1.15.

Exercise 26. — **1.** 1. **2.** 162. **3.** 4. **4.** 35. **5.** 27. **6.** 6. **7.** 27. **8.** 55. **9.** 3. **10.** 5. **11.** 2. **12.** 4. **13.** 20. **14.** 2. **16.** 24 cows. **17.** 42 doz. **18.** 15 da. **19.** 22 hr. **20.** 200.

Exercise 27. — **1.** 12. **2.** $10\frac{1}{2}$. **3.** $5\frac{3}{5}$. **4.** $16\frac{1}{5}$. **5.** $11\frac{1}{5}$. **6.** $4\frac{1}{2}$. **7.** $11\frac{7}{8}$. **8.** $24\frac{1}{2}$. **9.** $5\frac{1}{7}$. **10.** $12\frac{1}{2}$. **11.** $19\frac{1}{8}$. **12.** $38\frac{1}{2}$. **13.** $35\frac{3}{4}$. **14.** $47\frac{1}{2}$. **15.** $38\frac{1}{4}$. **16.** $15\frac{3}{4}$. **17.** $16\frac{2}{3}$. **18.** $22\frac{1}{2}$. **19.** $33\frac{3}{4}$. **20.** $301\frac{1}{2}$. **21.** $247\frac{1}{2}$. **22.** 152. **23.** $335\frac{1}{2}$. **24.** $247\frac{1}{2}$. **25.** 243. **26.** $160\frac{7}{11}$. **27.** $753\frac{1}{3}$. **28.** $705\frac{1}{2}$. **29.** $446\frac{1}{4}$. **30.** $166\frac{1}{2}$. **31.** $221\frac{2}{5}$. **32.** $132\frac{8}{9}$. **33.** $\$4\frac{1}{2}$. **34.** $\$42\frac{1}{2}$. **35.** $\$15.23\frac{7}{16}$. **36.** $52\frac{1}{2}$¢. **37.** $\$129.76\frac{1}{2}$. **38.** $\$31.87\frac{1}{2}$. **39.** $\$3.84\frac{3}{4}$. **40.** $35.40. **41.** $\$7.81\frac{1}{4}$. **42.** 4840 sq. yd. **43.** 792 in.

44. $11.90. 45. $1036\frac{1}{2}$ mi.; 2764 mi. 46. $7350. 47. 1792 lb. copper, 448 lb. lead. 48. Gain $805, investment at end of year $3105.

Exercise 28. — 1. $\frac{1}{3}$. 2. $\frac{3}{8}$. 3. $\frac{8}{11}$. 4. $\frac{63}{80}$. 5. $\frac{51}{200}$. 6. $\frac{19}{48}$. 7. $\frac{24}{37}$. 8. $\frac{3}{7}$. 9. $\frac{3}{8}$. 10. $\frac{5}{16}$. 11. $\frac{3}{7}$. 12. $\frac{3}{7}$. 13. $\frac{1}{14}$. 14. $\frac{1}{4}$. 15. $\frac{1}{10}$. 16. $\frac{3}{8}$. 17. $\frac{3}{10}$. 18. $\frac{1}{17}$. 19. $\frac{14}{101}$. 20. $\frac{1}{9}$. 21. $2\frac{7}{9}$. 22. $3\frac{3}{8}$. 23. $\frac{3}{4}$. 24. $\frac{4}{5}$. 25. $\frac{5}{9}$. 26. 7. 27. 9. 28. $10\frac{2}{3}$. 29. 15. 30. $3\frac{1}{9}$. 31. 22. 32. $26\frac{11}{16}$. 33. $\frac{1}{3}$. 34. $14\frac{1}{16}$. 35. $64\frac{2}{9}$. 36. 1. 37. $420\frac{1}{4}$. 38. $31\frac{1}{2}$. 39. 7. 40. 2. 41. $272\frac{424}{455}$. 42. $\$398\frac{35}{54}$. 43. $\$3524\frac{1}{4}$. 44. $\$1.93\frac{3}{4}$. 45. $\$3.65\frac{5}{8}$. 46. $162.50. 47. $\$52\frac{1}{4}$.

Exercise 29. — 1. $\frac{2}{75}$. 2. $9\frac{1}{9}$. 3. $1\frac{4}{5}$. 4. $\frac{5}{108}$. 5. $\frac{3}{8}$. 6. $\frac{11}{20}$. 7. $\frac{3}{56}$. 8. $\frac{17}{24}$. 9. $\frac{5}{8}$. 10. $\frac{3}{52}$. 11. $\frac{4}{9}$. 12. $1\frac{1}{9}$. 13. $\frac{2}{39}$. 14. $\frac{7}{22}$. 15. $1\frac{5}{49}$. 16. $\frac{2}{55}$. 17. $\frac{10}{39}$. 18. $3\frac{1}{3}$. 19. $\frac{2}{45}$. 20. $\frac{5}{34}$. 21. $\frac{5}{8}$. 22. $\frac{1}{42}$. 23. $\frac{9}{70}$. 24. $2\frac{11}{13}$. 25. 18. 26. $\frac{8}{39}$. 27. $\frac{8}{15}$. 28. 49. 29. $\frac{4}{15}$. 30. $2\frac{2}{3}$. 31. 28. 32. $\frac{7}{10}$. 33. $1\frac{29}{76}$. 34. 21. 35. $\frac{8}{35}$. 36. $\frac{49}{104}$. 37. 28. 38. $\frac{3}{14}$. 39. $2\frac{2}{35}$. 40. 30. 41. $\frac{5}{28}$. 42. $\frac{4}{17}$. 43. $32\frac{1}{2}$. 44. $\frac{4}{33}$. 45. $4\frac{2}{7}$. 46. 28. 47. $\frac{5}{9}$. 48. $\frac{4}{19}$. 49. $60\frac{1}{2}$. 50. $\frac{5}{52}$. 51. $\frac{7}{50}$. 52. 96. 53. $\frac{2}{17}$. 54. $\frac{3}{29}$. 55. $67\frac{1}{2}$. 56. $\frac{3}{16}$. 57. 4. 58. $49\frac{1}{2}$. 59. $\frac{5}{6}$. 60. $7\frac{15}{23}$. 61. 2. 62. $\frac{4}{5}$. 63. $\frac{1}{3}$. 64. $1\frac{1}{3}$. 65. $1\frac{19}{36}$. 66. $\frac{11}{15}$. 67. $5\frac{1}{2}$. 68. $\frac{63}{64}$. 69. $\frac{9}{11}$. 70. $2\frac{3}{5}$. 71. $\frac{9}{16}$. 72. $1\frac{5}{7}$.

Exercise 30. — 9. 4. 10. 9. 11. 16. 12. $5\frac{1}{3}$. 13. 25. 14. $8\frac{1}{6}$. 15. 9. 16. $\frac{2}{3}$. 17. $\frac{1}{3}$. 18. $1\frac{1}{3}$. 19. $1\frac{1}{3}$. 20. $\frac{135}{224}$. 21. $1\frac{2}{7}$. 22. $\frac{128}{441}$. 23. $\frac{2}{3}$. 24. $2\frac{8}{39}$. 25. $4\frac{1}{2}$. 26. $1\frac{31}{32}$. 27. $1\frac{37}{80}$. 28. $1\frac{3}{13}$. 29. $\frac{7}{12}$. 30. $1\frac{1}{9}$. 31. 6. 32. $1\frac{151}{275}$.

Exercise 31. — 1. $4290. 2. $\$2\frac{1}{2}$. 3. $216.75. 4. $\$137\frac{1}{5}$. 5. $630. 6. $575. 7. $24.70. 8. $68.88. 9. $\$881\frac{2}{3}$. 10. $7. 11. $4.25. 12. $8.41. 13. $\$7.72\frac{1}{2}$. 14. $5.75. 15. $\$13\frac{1}{2}$. 16. $120. 17. 108 ft. 18. 72. 19. 255 T. 20. 60.

Exercise 33. — 1. 666.973. 2. 151.987. 3. 261.304. 4. 1943.232. 5. 135.148. 6. 160.687. 7. 282.238. 8. 317.187. 9. 176.483. 10. 212.728. 11. 281.308. 12. 377.077.

Exercise 35. — 1. $4.32, $13.70, $43.20, $10.08. 2. $3.105, $22.59, $6.064, $5.929. 3. $11.90, $34.74, $40.992, $32.85. 4. $10.10, $40.50, $180.30, $24.71. 5. $430, $693.70, $337.80. 6. 31.5 A., 4 A., 30 A. 7. .01, .12, .09, .002, .012. 8. 502.2. 9. 2123.2. 10. 1806.42. 11. 432.036. 12. .425. 13. 2.625. 14. 28.56. 15. 271.9846. 16. .56056. 17. 249.048. 18. .1702. 19. .4182. 20. .6. 21. 3.3642. 22. 17.328. 23. .20001. 24. .9409. 25. .4624. 26. .139129.

27. .811801. **28**. .644809. **29**. .480249. **30**. .001. **31**. .027. **32**. .064. **33**. .343. **34**. 1.124864. **35**. 1.191016. **36**. 1.259712. **37**. .015625. **40**. 153.9384. **41**. 197,061,258. **42**. 15,205,344. **43**. 1110 mi., 66,600 mi. **44**. 8796 ft., 527,760 ft. **45**. .32 in.

Exercise 36. — **1**. 9.15125. **2**. .8625. **3**. 3.227. **4**. .199. **5**. .9128. **6**. .490875. **7**. .003. **8**. 1.47433. **9**. 1.174. **10**. .657. **11**. .925. **12**. 1.142875. **13**. .02468. **14**. .12201. **15**. .1638. **16**. .7843875. **17**. .912. **18**. .9172. **19**. .03866. **20**. .07484. **21**. .142857. **22**. 1.0535. **23**. 1.0446. **24**. .2478.

Exercise 37. — **1**. 2.925. **2**. .024. **3**. 8.6875. **4**. .2058. **5**. .3596. **6**. 2.62. **7**. 3.725. **8**. 2.531. **9**. 1.35. **10**. 3.36. **11**. 6.3234. **12**. .5. **13**. 40. **14**. 62.5. **15**. 125. **16**. 50. **17**. 15. **18**. 50. **19**. 160. **20**. 12,500. **21**. 8000. **22**. 500. **23**. 25. **24**. .5. **25**. 1000. **26**. 1250. **27**. 1000. **28**. 50. **29**. 250. **30**. 250. **31**. 25. **32**. 2.5. **33**. .0125. **34**. .00125. **35**. .0004. **36**. .00666+. **37**. .125. **38**. .15. **39**. 32. **40**. .027. **41**. 48.7. **42**. 8.03. **43**. .0904. **44**. .708. **45**. 1.012. **46**. .01014. **47**. .517. **48**. .01054. **49**. .00377. **50**. .0365.

Exercise 38. — **1**. $17,762.08. **2**. $19,740.80. **3**. $18,850.01. **4**. $30,737.16. **5**. $21,197.90. **6**. $41,931.64. **7**. $15,101.25. **8**. $69,022.28.

Exercise 39. — **1**. .375, .625, .4375, .5625, .6875, .8125, .9375, .1875. **2**. .2667, .4667, .7333, .8667, .9333, .5833, .9167. **3**. .3, .79, .087, .0183, .2779. **4**. .0375, .1375, .3625, .95, .925, .5167, .8833. **5**. .03125, .09375, .15625, .28125, .40625, .59375, .96875. **6**. .2857, .7143, .0769, .3077, .2143, .6429, .7857, .9286. **7**. .5556, .1919, .1471, .0544, .0274, .0569.

Exercise 40. — **1**. $\frac{3}{10}$, $\frac{4}{5}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{16}$. **2**. $\frac{7}{100}$, $\frac{1}{80}$, $\frac{7}{800}$, $\frac{1}{16}$, $\frac{3}{400}$. **3**. $\frac{9}{1000}$, $\frac{9}{400}$, $\frac{9}{80}$, $\frac{11}{400}$. **4**. $\frac{9}{125}$, $\frac{13}{1250}$, $\frac{7}{200}$, $\frac{119}{10000}$, $\frac{3}{80}$. **5**. $\frac{18}{125}$, $\frac{63}{1250}$, $\frac{48}{625}$, $\frac{81}{500}$, $\frac{7}{625}$. **6**. $\frac{36}{125}$, $\frac{11}{625}$, $\frac{13}{400}$, $\frac{7}{400}$, $\frac{17}{40}$. **7**. $\frac{23}{80}$, $\frac{27}{80}$, $\frac{41}{80}$, $\frac{59}{80}$.

Exercise 41. — **1**. 4 hr. **2**. 12 hr. **3**. 6.6667 hr. **5**. 14 hr. **6**. 50.1875 mi. per hr. **8**. 53.33 mi. per hr. **9**. 54.13 mi. per hr. **10**. 29.13 mi. per hr. **11**. 546.45 mi. per da., 22.77 mi. per hr. **12**. 1233.3 times. **13**. $2400. **14**. $280. **15**. 850 cu. ft. **16**. 420 cu. ft. **17**. 7.48 gal. **18**. $38.28. **19**. $26.25. **20**. $17.55.

Exercise 43. — **1**. $\frac{3}{26}$. **2**. $\frac{1}{6}$. **3**. $\frac{2}{5}$. **4**. $\frac{3}{16}$. **5**. $\frac{9}{16}$. **6**. $\frac{5}{6}$. **7**. $1\frac{23}{32}$. **8**. $1\frac{3}{7}$. **9**. $\frac{5}{9}$. **10**. $\frac{5}{12}$. **11**. $\frac{5}{8}$. **12**. $\frac{11}{24}$. **13**. $\frac{14}{15}$. **14**. $\frac{13}{15}$. **15**. $\frac{2}{5}$. **16**. $1\frac{1}{2}$. **17**. $1\frac{1}{4}$. **18**. $1\frac{25}{47}$. **19**. $\frac{1}{40}$. **20**. $1\frac{1}{2}$.

Exercise 44. — **1**. 413.33 sq. yd. **2**. 149.33. sq. yd. **3**. $1966\frac{2}{3}$ sq. yd. **4**. $6442\frac{2}{3}$ sq. yd., 2512 ft. **5**. 14,400. **6**. (*a*) 15,762.22 sq. ft., (*b*) 21,112.5 sq. ft., (*c*) 4430.54 sq. ft., (*d*) 1291.79 sq. ft., (*e*) 16,032.99

sq. ft., (*f*) 10,911.25 sq. ft., (*g*) 17,459 sq. ft., (*h*) 80,711.38 sq. ft. **7.** (*a*) 720.6 sq. yd., (*b*) 849.58 sq. yd., (*c*) 912.88 sq. yd., (*d*) 1626.31 sq. yd., (*e*) 1146.23 sq. yd. **8.** 4096 bu., $1996.80. **9.** 71.6 A. **10.** 1425.6 thousand. **11.** 8640 tiles. **12.** 62.2 sq. yd., 83.55 sq. yd. **13.** 21.9 T.

Exercise 45.—**1.** $11.32. **2.** $28.57. **3.** $5.88. **4.** $41.60. **5.** $42.16. **6.** $38.50. **7.** $30.11. **8.** $24.60. **9.** $28.56. **10.** $41.47. **11.** $67.20. **12.** $21.80. **13.** $24.75. **14.** $39.15. **15.** $25.85. **16.** $66.70. **17.** $35. **18.** $23.25. **19.** $37.80. **20.** $40.50. **21.** $38.95. **22.** $4.41. **23.** $92.16. **24.** Weimer, $26.91; Flatonia, $24.76; Columbus, $27.30; Beaumont, $42.31; Gonzales, $23.59; Schulenburg, $26.13. **25.** $31.50. **26.** $39.15. **27.** $32.50. **28.** $10. **29.** $118.25. **30.** $113.85. **31.** $129.48. **32.** $312.90. **33.** $435.20. **34.** $589.68.

Exercise 46.—**1.** $7.56. **2.** $34. **3.** $48. **4.** $100. **5.** $260. **6.** $315. **7.** $225. **8.** $440. **9.** $870. **10.** $144. **11.** $49.15. **12.** $60. **13.** $51. **14.** $87.50. **15.** $1210. **16.** $987. **17.** $1800. **18.** $1625. **19.** $3000. **20.** $2450. **21.** $1960. **22.** $7800. **23.** $512. **24.** $1656. **25.** $5000. **26.** $12,150. **27.** $262.50. **28.** 4455 girls, 3795 boys. **29.** 42 A. **30.** $1516.80. **31.** 73, 292. **32.** $223.20, $3496.80.

Exercise 47.—**1.** $3200. **2.** $2160. **3.** $1230. **4.** $900. **5.** $106. **6.** $660. **7.** $410. **8.** $1740. **9.** $3575. **10.** $470. **11.** $3769.60. **12.** $4410. **13.** $301. **14.** $646. **15.** $1040. **16.** $1120. **17.** $200. **18.** $784. **19.** $1500. **20.** $480. **21.** 490 trees. **22.** 160, 300. **23.** 922.5 A. **24.** 35 ft. **25.** 3542.97 mi.

Exercise 48.—**1.** $24, $30, $36, $48. **2.** $59.50, $68, $76.50. **3.** $28.50, $38, $76. **4.** $78.56, $147.30, $88.38. **5.** $18.45, $22.14. **6.** $13.80, $16.10. **7.** $20, $24. **8.** $38.50, $22. **9.** $25.60, $19.20. **10.** $32.50. **11.** $23. **12.** $67.50. **13.** $37.50. **14.** $47.50. **15.** $93.50. **16.** $97.50. **17.** $72. **18.** $91.80. **19.** $59.40. **20.** $120. **21.** $101.50. **22.** $136.50. **23.** $240.62. **24.** $263.20. **25.** $106.65. **26.** $247.50. **27.** $219. **28.** $43.20. **29.** $139.50. **30.** $136. **31.** $141.75. **32.** $123.50. **33.** $86.40. **34.** $180.50. **35.** $210. **36.** $315. **37.** $2500. **38.** $248. **39.** $259.20. **40.** $34.20. **41.** $30.62. **42.** $45. **43.** $150.

Exercise 49.—**1.** 4 cwt. 87 lb. 8 oz. **2.** 6 cwt. 10 lb. 3 oz. **3.** 2 T. 7 cwt. 18 lb. 9 oz. **4.** 31 T. 5 cwt. **5.** 3 T. 19 cwt. 87 lb. **6.** 16 T. 721 lb. **7.** 112 T. **8.** $4\frac{13}{28}$ T.

Exercise 50.—**1.** 38,000 lb. **2.** 28,400 lb. **3.** 34,300 lb. **4.** 803,200 oz. **5.** 7502 lb. **6.** 9158 lb. **7.** 16,273 lb. **8.** 56 short tons. **9.** $81. **10.** 700 long tons. **11.** 616 T. **12.** 1497 lb.

Exercise 51.—**1.** 168 in. **2.** 3972 in. **3.** 9680 yd. **4.** 12,980 yd. **5.** 18,209 yd. **6.** 784,520 yd. **7.** 13,622.4 yd. **8.** 2672 rd. **9.** 3927 ft.

Exercise 52.—**1.** 102,400 sq. rd. **2.** 1600 A. **3.** 522,720 sq. ft. **4.** $7350\frac{3}{4}$ sq. ft. **5.** 16,665 rd. **6.** 42,900 ft. **7.** 4840 yd. **8.** 6600 yd. **9.** 11,220 ft. **10.** 675 cu. ft. **11.** 1046 cu. ft. **12.** 816,480 cu. in. **13.** $506\frac{1}{4}$ cu. ft. **14.** 944,784 cu. in. **15.** 10 qt. **16.** 23 qt. **17.** 25 pt. **18.** 57 pt. **19.** 154 pt. **20.** 128 qt. **21.** 124 qt. **22.** 116 qt. **23.** 56 qt. **24.** 59 qt. **25.** 1208 pt. **26.** 838 pt. **27.** 2640 ft., 1320 ft., 480 ft. **28.** 440 yd., 352 yd., $586\frac{2}{3}$ yd. **29.** $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{20}$. **30.** 1210 sq. yd., 2420 sq. yd. **31.** $\frac{1}{36}$. **32.** 140 sq. yd. **33.** 112 sq. rd., 144 sq. rd. **34.** $204\frac{3}{16}$ sq. ft. **35.** 33.6 cu. in. **36.** 28,875 cu. in. **37.** 6 qt. **38.** 93.1 gal.

Exercise 53.—**1.** 29 gal. 1 qt. **2.** 1 cu. yd. 729 cu. in. **3.** 432 gal. **4.** 32 mi. **5.** 31 bu. 1 pk. **6.** 8 cu. yd. 11 cu. ft. 744 cu. in. **7.** 31,250 sq. mi. **8.** 11 sq. yd. 5 sq. ft. 24 sq. in. **9.** 7926.59 mi. **10.** 7899.58 mi. **11.** 27.01 mi. **12.** 186,325 mi.

Exercise 55.—**1.** 66,020″. **2.** 324,000″. **3.** 43,510″. **4.** 27,900″. **5.** 433,080″. **6.** 163,820″. **7.** 855′. **8.** 4545′. **9.** 15,247.5′. **10.** 280′. **11.** 1130′. **12.** $832\frac{1}{2}$′. **13.** 525,600 min. **14.** 1,578,240 min. **15.** 41,760 min. **16.** 129,600 min., 131,040 min. **17.** 31,556,926 sec. **18.** 36,892,800 times. 147,268,800 times. **19.** 311,760 min. **20.** 584 da. **21.** 960 hr. **22.** 178,826.4 hr. **23.** 2922 da. **24.** 48°. **25.** 174°.

Exercise 56.—**1.** 32 yd. 2 ft. 4 in. **2.** 151 yd. 9 in. **3.** 25 yd. 9 in. **4.** 263 A. 87 sq. rd. **5.** 506 A. 27 sq. rd. **6.** 971 A. 79 sq. rd. **7.** 16 gal. 3 qt. **8.** 44 gal. 3 qt. 1 pt. **9.** 42 bu. 3 pk. 4 qt. **10.** 38 bu. 2 pk. 3 qt. **11.** 77 gal. 3 qt. **12.** 153 bu. 3 pk. 5 qt. **13.** 117 bu. 3 pk. 3 qt. **14.** 183 bu. 1 pk. 7 qt. **15.** 147 T. 1379 lb. **16.** 667 lb. 4 oz. **17.** 546 lb. 13 oz. **18.** 77 T. 628 lb. **19.** 272 lb. 7 oz. **20.** 225 T. 1908 lb. **21.** 218 da. 18 hr. 30 min. **22.** 241 wk. 6 da. 13 hr. **23.** 85 hr. 14 min. 52 sec. **24.** 141 da. 13 hr. 47 min. **25.** 38 wk. 0 da. 12 hr. **26.** 54 hr. 20 min. 3 sec. **27.** 64 cu. ft. 1032 cu. in. **28.** 307 cu. yd. 9 cu. ft. **29.** 362 cu. yd. **30.** 67 cu. ft. 1023 cu. in.

Exercise 57.—**1.** 79° 2′ 17″. **2.** 8° 0′ 56″. **3.** 51° 54′ 25″. **4.** 117° 45′ 25″. **5.** 5° 54′ 45″. **6.** 66° 46′ 6″. **7.** 85° 6′ 10″. **8.** 45° 57′. **9.** 60° 5′ 43″. **10.** 78° 59′ 50″. **11.** 43° 5′ 48″.

12. 1 bu. 3 pk. 7 qt. **13.** 8 gal. 0 qt. 1 pt. **14.** 6 gal. 0 qt. 1 pt. **15.** 5 gal. 2 qt. 1 pt. **16.** 19 bu. 2 pk. 5 qt. **17.** 9 bu. 1 pk. 5 qt. **18.** 19 gal. 2 qt. 1 pt. **19.** 21 gal. 0 qt. 1 pt. **20.** 3 qt. **21.** 28 qt. **22.** 120 qt. **23.** 1 da. 20 hr. 57 min. **24.** 6 da. 17 hr. 51 min. **25.** 11 hr. 40 min. **26.** 2 hr.

Exercise 58. — **3.** 4 yr. 6 mo. 28 da. **4.** 6 yr. 4 mo. 29 da. **5.** Milton, 65 yr. 10 mo. 29 da.; Pope, 56 yr. 0 mo. 9 da.; Shakespeare. 52 yr.; Burke, 67 yr. 5 mo. 27 da.; Lee, 63 yr. 8 mo. 23 da.; Grant, 63 yr. 2 mo. 26 da.; Goldsmith, 45 yr. 4 mo. 24 da.; Franklin, 84 yr. 3 mo.; Hamilton, 47 yr. 6 mo. 1 da.; Longfellow, 75 yr. 0 mo. 27 da.; Newman, 89 yr. 5 mo. 20 da.; Gladstone, 88 yr. 5 mo. 10 da.

Exercise 59. — **1.** 42 yd. 2 ft. 3 in. **2.** 72 yd. 1 ft. 3 in. **3.** 79 yd. 2 ft. 4 in. **4.** 26 bu. 1 qt. **5.** 53 bu. 1 pk. **6.** 59 gal. 1 qt. **7.** 79 gal. 2 qt. **8.** 27 bu. 3 pk. 1 qt. **9.** 146 T. 800 lb. **10.** 118 T. 1483 lb. **11.** 171 T. 540 lb. **12.** $87° 51' 28''$. **13.** $249° 38'$. **14.** 779 A. 40 sq. rd. **15.** 166 A. 137 sq. rd. **16.** $254° 18' 36''$. **17.** 11,250 lb. **18.** 75,240 ft.

Exercise 60. — **1.** 2 yd. 1 ft. 3 in. **2.** $13° 19' 19\frac{2}{7}''$. **3.** $7° 1' 11\frac{1}{6}''$. **4.** 5 yd. 2 ft. $5\frac{1}{3}$ in. **5.** 11 yd. 4 in. **6.** 2 bu. 1 pk. 2 qt. **7.** 10 times. **8.** 240 times. **9.** 1609.35. **10.** $4° 5' 27\frac{3}{11}''$. **11.** 264.

Exercise 61. — **1.** 1750 lb. **2.** $9'$. **3.** 18 hr. 40 min. **4.** 469 lb. **5.** 22 hr. 48 min. **6.** 12 qt. **7.** 97,152 ft. **8.** 1120 yd. **9.** 60 rd. **10.** $945'$. **11.** 243 da. 8 hr. **12.** 114 da. 1 hr. 30 min. **13.** 1 pk. 6 qt. 3.2 gi. **14.** 2 qt. 1 pt. $1\frac{1}{3}$ gi. **15.** 136 sq. rd. **16.** 4290 sq. yd. **17.** 26 qt. **18.** $3\frac{1}{2}$ pt.

Exercise 62. — **1.** .8333. **2.** .16667 nearly. **3.** .06667. **4.** .375. **5.** .4375. **6.** .5625. **7.** 796,875. **8.** .09375. **9.** .195. **10.** $\frac{13}{360}$. **11.** .25.

Exercise 63. — **1.** 1 A. **2.** 22 A. **3.** 2.492 A. **4.** $1493\frac{1}{3}$ A. \$$37,333\frac{1}{3}$. **5.** 3.6 A. **6.** 1200 A. \$58,800.

Exercise 64. — **3.** \$1200. **4.** \$5000.

Exercise 65. — **1.** 504 cu. ft. **2.** 72 cu. yd. **3.** 9216 cu. ft. **4.** 3150 cu. yd. **5.** 13,824 gal. **6.** 115.71 bu. **7.** 560 cu. yd. **8.** 3520 cu. yd. **9.** 270 boxes. **10.** $1,251,555\frac{5}{9}$ cu. yd. **11.** 24.48 T. **12.** $1481\frac{2}{3}$ lb. **13.** 120 T. **14.** 212 lb. $\frac{1}{2}$ oz. **15.** 1728 boxes. **16.** 45 cd. **17.** 453.75 T.

Exercise 66. — **1.** $66\frac{1}{2}$ sq. ft. **2.** $166\frac{3}{4}$ sq. ft. **3.** $103\frac{1}{8}$ sq. ft. **4.** $143\frac{8}{9}$ sq. ft. **5.** $306\frac{1}{4}$ sq. ft. **6.** $301\frac{7}{8}$ sq. ft. **7.** $238\frac{3}{16}$ sq. ft.

8. $177\frac{1}{12}$ sq. ft. **9.** $596\frac{23}{32}$ sq. ft. **10.** $1287\frac{1}{9}$ sq. ft. **11.** $1346\frac{5}{12}$ sq. ft. **12.** $708\frac{8}{9}$ sq. ft. **13.** $775\frac{7}{12}$ sq. ft. **14.** $2646\frac{7}{8}$ sq. ft. **15.** $76\frac{1}{2}$ sq. ft. **16.** 372 sq. ft. **17.** 3.15 A. **18.** 4200 sq. ft. **19.** 247 sq. ft. **20.** 4053 sq. ft. **21.** $2310\frac{15}{16}$ sq. ft.

Exercise 67. — **1.** 12. **2.** 12, 42 ¢. **3.** 60, 60. **4.** 96. **5.** (*a*) 36, (*b*) 48, (*c*) 54, (*d*) $42\frac{2}{3}$, (*e*) 54, (*f*) 84, (*g*) $87\frac{1}{2}$, (*h*) $122\frac{1}{2}$. **6.** $264. **7.** $367.50. **8.** 77 bd. ft. **9.** $87\frac{1}{2}$ bd. ft. **10.** 96 bd. ft. **11.** $364\frac{1}{2}$ bd. ft. **12.** 544 bd. ft. **13.** 10 cu. ft. **14.** 396 bd. ft. **15.** $20\frac{25}{36}$ bd. ft. **16.** $141\frac{3}{4}$ bd. ft.

Exercise 68. — **1.** 81.45 perches, 44,352 bricks. **2.** 1984 cu. ft., 1856 cu. ft. **3.** 13,200 bricks, $118.80. **4.** 1000 cu. ft., 198,000 bricks. **5.** $1909.84.

Exercise 69. — **1.** $42\frac{2}{3}$ yd. **2.** 64 yd. **3.** 24 yd. **4.** $66\frac{2}{3}$ yd. **5.** $33.60.

Exercise 70. — **1.** 24 da. **2.** 114 T. **3.** $10\frac{1}{2}$ hr. **4.** $49\frac{1}{2}$ yr. **5.** $$4233\frac{1}{3}$, $$1693\frac{1}{3}$. **6.** $200. **7.** $300. **8.** $1\frac{1}{14}$. **9.** 10 and 15. **10.** $\frac{1}{3}$, 81 cattle. **11.** 75 T. **12.** 165. **13.** $340. **14.** $6300. **15.** $1939. **16.** 1600. **17.** $1\frac{3}{5}$ hr. **18.** 30 mi., 6 hr. 56 min. **19.** $37\frac{1}{2}$ ft. **20.** $\frac{1}{3}$. **21.** $2. **22.** 48 oranges. **23.** $41\frac{1}{4}$ yd. **24.** 60 ¢ per lb. **25.** 91 ¢ per yd. **26.** $158.82. **27.** 42 pencils. **28.** $43.20. **29.** 27 ¢. **30.** $1.12. **31.** 40 tiles. **32.** .783. **33.** 21,150 sec. **34.** $98.67. **35.** $750. **36.** $79\frac{8}{9}$ yd. **37.** $672. **38.** $170.80. **39.** 4620 ft. **40.** 3025 sq. yd. **41.** 240 bu. **42.** $ 90. **43.** 174.3768 lb. **44.** 4800 gr. **45.** 48 mi. **46.** 22.5 A. **47.** $562.50. **48.** $95. **49.** June 30. **50.** 1944 sq. in. **51.** 24 sq. rd. **52.** 56. **53.** $91.80. **54.** $147. **55.** 180 min. **56.** $49\frac{1}{11}$ mi. **57.** $21.11. **58.** 10 min. 50 sec. **59.** 345.6 lb. **60.** 2520 gal. **61.** $45. **62.** $37\frac{1}{2}$ cords. **63.** 70 ¢. **64.** 345.6 bu. **65.** .68, .6818, .012, .8571. **66.** $\frac{7}{8}$, $\frac{21}{4000}$, $\frac{2}{3}$. **67.** 87799.3. **68.** $500.50. **69.** 5 gal. 2 qt. **70.** 144 cu. in. **71.** $60,138.75.

Exercise 71. — **1.** $\frac{41}{48}$. **2.** $11\frac{149}{360}$. **3.** $\frac{577}{630}$. **4.** $3\frac{121}{216}$. **5.** $1\frac{317}{360}$. **6.** $9\frac{115}{216}$. **7.** $\frac{173}{240}$ **8.** $13\frac{53}{144}$. **9.** $\frac{193}{280}$. **10.** $10\frac{7}{36}$. **11.** $8\frac{47}{54}$. **12.** $9\frac{95}{336}$. **13.** $2\frac{17}{28}$. **14.** $\frac{11}{20}$. **15.** $\frac{57}{100}$. **16.** $\frac{1}{132}$. **17.** $\frac{1}{36}$. **18.** $\frac{5}{72}$. **19.** $\frac{1}{90}$. **20.** $\frac{1}{30}$. **21.** $\frac{1}{132}$. **22.** $\frac{7}{120}$. **23.** $\frac{1}{90}$. **24.** $3\frac{13}{14}$. **25.** $\frac{23}{36}$. **26.** $2\frac{25}{36}$. **27.** $2\frac{35}{48}$. **28.** $\frac{67}{72}$. **29.** $1\frac{53}{60}$. **30.** $3\frac{5}{6}$. **31.** $1\frac{11}{12}$. **32.** $$1272\frac{1}{2}$. **33.** $16\frac{5}{24}$. **34.** $66\frac{5}{12}$. **35.** $\frac{769}{1000}$.

Exercise 72. — **1.** $1\frac{3}{7}$. **2.** 3. **3.** $\frac{1}{2}$. **4.** 1. **5.** $\frac{1}{2}$. **6.** $1\frac{17}{40}$. **7.** $1\frac{2}{7}$. **8.** 8. **9.** $\frac{7}{8}$. **10.** 20. **11.** 1. **12.** $8\frac{1}{15}$. **13.** $2\frac{1}{4}$. **14.** $17\frac{1}{2}$. **15.** 5. **16.** $\frac{5}{16}$. **17.** $\frac{7}{12}$. **18.** $\frac{299}{400}$. **19.** $\frac{11}{16}$. **20.** $1\frac{7}{10}$. **21.** $12\frac{3}{11}$. **22.** $15\frac{15}{16}$. **23.** $1\frac{1}{12}$. **24.** 0. **25.** 0. **26.** $3\frac{3}{8}$.

Exercise 73. — **1.** 14. **2.** $1\frac{1}{3}$. **3.** $6\frac{6}{11}$. **4.** $\frac{1}{40}$. **5.** $\frac{2}{5}$. **6.** $2\frac{4}{5}$. **7.** $\frac{3}{5}$. **8.** $\frac{7}{15}$. **9.** $1\frac{12}{25}$. **10.** $1\frac{151}{272}$. **11.** $3\frac{1}{9}$. **12.** $\frac{459}{15386}$. **13.** $\frac{1}{4}$. **14.** $\frac{3}{10}$. **15.** $\frac{1}{4}$. **16.** $3\frac{1}{3}$. **17.** $8\frac{9}{20}$. **18.** 4.

Exercise 74. — **1.** 12.88. **2.** 24.75. **3.** 12.78. **4.** 42.72. **5.** 295.2. **6.** 263.76. **7.** 24.6. **8.** 184.1. **9.** 385.2. **10.** 785. **11.** 108.13. **12.** 308.4. **13.** 116. **14.** 1218. **15.** $122\frac{1}{6}$. **16.** 310.5. **17.** 5750. **18.** 1713. **19.** 102.6. **20.** 114.975. **21.** 118.3. **22.** 612.72. **23.** 8.45. **24.** 20.31. **25.** $33\frac{1}{3}$%, 25%, 20%, $12\frac{1}{2}$%, $6\frac{1}{4}$%, $15\frac{5}{8}$%, $43\frac{3}{4}$%. **26.** 4%, 8%, $12\frac{1}{2}$%, 1.65%, $\frac{1}{5}$%, $\frac{3}{5}$%, .24%. **30.** \$1125. **31.** 11.2 lb. **32.** $27\frac{1}{2}$ lb. **33.** \$2050. **34.** N.Y., \$476,850,000; Boston, \$132,982,500; Phila., \$64,387,500; Baltimore, \$113,475,000; N.O., \$81,600,000; Galveston, \$91,417,500.

Exercise 75. — **1.** 3700. **2.** 1248. **3.** \$864. **4.** 7200, 6000, 4500, 4000. **5.** 4000, 3000, 2400, 2000, 1500, 1000. **6.** \$12,500, \$11,250, \$10,000, \$6000. **7.** \$360, \$432, \$324, \$243. **8.** \$7200, \$6750, \$5400, \$3600, \$2700. **9.** \$1120, \$672, \$480. **10.** 4200 oz. **11.** 76,300,712. **12.** 39,007,793. **13.** 37,220,331. **14.** 6,166,710. **15.** \$213,293,651. **16.** \$1,960,233. **17.** 3500. **18.** 3500. **19.** 304,800. **20.** 4000 lb.

Exercise 76. — **1.** 80%. **2.** 20%, 35%, 55%, 65%. **3.** 5%, 20%, 30%, $42\frac{1}{2}$%. **4.** 10%, $13\frac{1}{3}$%, 30%, $12\frac{1}{2}$%. **5.** $12\frac{1}{2}$%, 15%, $37\frac{1}{2}$%, $66\frac{2}{3}$%. **6.** 6%, 7%, $8\frac{1}{3}$%. **7.** 20%, 9%, $16\frac{2}{3}$%. **8.** $9\frac{1}{11}$%, $33\frac{1}{3}$%, 15%. **9.** $1\frac{2}{3}$%, $8\frac{1}{3}$%. **10.** $3\frac{1}{8}$%. **11.** $2\frac{1}{2}$%. **12.** $\frac{5}{8}$%. **13.** $4\frac{6}{11}$%. **14.** $6\frac{1}{4}$%. **15.** $12\frac{1}{2}$%. **16.** 25%. **17.** $33\frac{1}{3}$%. **18.** 115.15% nearly. **19.** 88.44%. **20.** $1\frac{1}{4}$%, 10%. **21.** $6\frac{1}{4}$%. **22.** 20%, 60%, 80%. **23.** 91.44% nearly, 109.36% nearly. **24.** 62.14% nearly. **25.** $82\frac{2}{7}$%. **26.** $109\frac{5}{7}$%. **27.** (*a*) 1.43%, (*b*) 2.42%, (*c*) .65%, (*d*) 1.02%, (*e*) .71%, (*f*) .63%, (*g*) 1.16%, (*h*) .3%, (*i*) .97%. **28.** Mobile, 23.79%; Little Rock, 48.05%; Los Angeles, 103.35%; Denver, 25.44%; Pensacola, 51.04%; Savannah, 25.6%; Springfield, 36.84%; Evansville, 16.26%; Dubuque, 19.75%; Kansas City, Kan., 34.19%; Lexington, 22.27%; Kansas City, Mo., 23.38%; Minneapolis, 23.05%. **29.** 1.12% nearly. **30.** 9.07% nearly. **31.** 64.69%, 13.12%, 22.19%.

Exercise 77. — **1.** \$48. **2.** \$52.50. **3.** \$11.25. **4.** \$11.20. **5.** \$21.60. **6.** \$27.20. **7.** \$59.50. **8.** \$65. **9.** \$61.33. **10.** \$50. **11.** \$133. **12.** \$132. **13.** \$83.60. **14.** \$91.20. **15.** \$106.25. **16.** \$171.50. **17.** \$204.48. **18.** \$679.62. **19.** \$3017.60. **20.** \$2522.16. **21.** \$4904.17. **22.** \$2334.37. **23.** \$3437.98. **24.** \$4454.56. **25.** \$752.25. **26.** \$319.44. **27.** \$633.88. **28.** \$1891.62. **29.** \$917.60. **30.** \$2456.26.

Exercise 78. — 1. \$47.25. 2. \$612. 3. 30%. 4. \$898.88. 5. \$283.50. 6. \$232.56. 7. \$10. 8. $32\frac{1}{2}$%. 9. $24\frac{4}{9}$%. 10. 25%. 11. \$2, \$25. 12. (*a*) \$241.92, (*b*) \$547.20, (*c*) \$465, (*d*) \$739.20, (*e*) \$403.20, (*f*) \$739.20, (*g*) \$943.92, (*h*) \$962.50.

Exercise 79. — 1. (*a*) \$168, (*b*) \$93.75, (*c*) \$33.32, (*d*) \$250.83, (*e*) \$685.80, (*f*) \$344.50, (*g*) \$608, (*h*) \$269.10, (*i*) \$514.12. 2. (*a*) 25%, (*b*) $9\frac{1}{11}$%, (*c*) $66\frac{2}{3}$%, (*d*) $33\frac{1}{3}$%, (*e*) 28%.

Exercise 80. — 1. (*a*) \$50.40, (*b*) \$132, (*c*) \$50, (*d*) \$161.54, (*e*) \$200, (*f*) \$300. 2. (*a*) \$45.89, (*b*) \$109.37, (*c*) \$128.57, (*d*) \$91, (*e*) \$89.85, (*f*) \$40.32, (*g*) \$69.75, (*h*) \$289.14.

Exercise 81. — 1. \$3250. 2. \$300. 3. \$3640. 4. $12\frac{1}{2}$%. 5. $8\frac{1}{3}$%. 6. 20%. 7. 10%. 8. 9%. 9. \$97. 10. \$86. 11. \$75. 12. \$105. 13. \$80, \$15. 14. \$120. 15. \$4.20. 16. 25%. 17. \$982. 18. 12,080. 19. \$16.67 nearly. 20. \$200. 21. \$10,240. 22. \$11,520. 23. \$85,700. 24. 475 lb. 25. 12%. 26. $14\frac{2}{7}$%. 27. $22\frac{2}{9}$%. 28. \$165. 29. $177\frac{7}{9}$%. 30. 5¢. 31. \$2.10 per yd. 32. 24¢. 33. $16\frac{2}{3}$%. 34. $38\frac{8}{9}$%. 35. 84¢ per lb. 36. $11\frac{1}{4}$%. 37. \$98. 38. \$98.70. 39. $96\frac{1}{2}$¢. 40. 440. 41. 200. 42. $33\frac{1}{3}$%. 43. \$148, $30\frac{5}{6}$%. 44. (*a*) 40% above cost, (*b*) 30% above cost, (*c*) 60% above cost, (*d*) $22\frac{2}{9}$% above cost, (*e*) $42\frac{6}{7}$% above cost.

Exercise 82. — 1. \$320. 2. \$1050, \$16,450. 3. \$45, \$705. 4. \$4. 5. 96,000 bu. 6. \$56.16, \$1815.84. 7. \$4.50. 8. \$56.70, \$1563.30. 9. \$1125, \$21,375, 95%. 10. 900 T. 11. 844 bales, 8¢ per pound. 12. \$17.28. 13. $3\frac{1}{4}$%. 14. 227,200 bu. 15. \$4200. 16. \$7800, \$7449. 17. \$25. 18. \$235.50, $78\frac{1}{2}$¢. 19. 4000 bu. 20. 3750 lb. 21. 150 A. 22. (*a*) \$17.85, \$205.27; (*b*) \$40.56, \$770.69; (*c*) \$85.84, \$987.16; (*d*) \$28.27, \$537.23; (*e*) \$100.01, \$1233.49; (*f*) \$47.60, \$428.40; (*g*) \$24, \$276; (*h*) \$103.20, \$1960.80; (*i*) \$10.06, \$191.19. (*j*) \$10.18, \$101.98. (*k*) \$421.89, \$13,641.11. (*l*) \$38.27, \$386.98.

Exercise 83. — 1. \$21, \$35. 2. \$16.68, \$19.46. 3. \$53.82, \$47.84. 4. \$23.12, \$28.90. 5. \$75, \$85. 6. \$128, \$119. 7. \$213.75, \$200. 8. \$180, \$152. 9. \$120, \$175. 10. \$204.17, \$163.33. 11. \$7.04, \$8.60. 12. \$8.35, \$12.98. 13. \$9.39, \$6.26. 14. \$12.38, \$13.93. 15. \$12.87, \$9.81. 16. \$16, \$30. 17. \$28.14, \$31.51. 18. \$9.19, \$4.92. 19. \$12.85, \$13.49. 20. \$56.07, \$56.07. 21. \$203.52, \$126.64. 22. \$243.83, \$190. 23. \$164.79, \$105.94. 24. \$175.50, \$157.50. 25. \$205, \$87.47. 26. \$191.25, \$183.75. 27. \$16.11. 28. \$47.14. 29. \$39.36. 30. \$53.36.

Exercise 84. — 1. \$7.20. 2. \$12.18. 3. \$97.39. 4. \$100.07. 5. \$74.19. 6. \$98.41. 7. \$64.51. 8. \$43.63. 9. \$16.46. 10. \$7.12. 11. \$13.90. 12. \$117.26. 13. \$51.08. 14. \$8.46. 15. \$87.88. 16. \$42.37.

Exercise 85. — **1.** \$838.67. **2.** \$693.10. **3.** \$802.50. **4.** \$355.61. **5.** \$337.79. **6.** \$753.13. **7.** \$880.28. **8.** \$336.46. **9.** \$1016.92. **10.** \$389. **11.** \$1332.34. **12.** \$3750.25. **13.** \$1532. **14.** \$4454.04. **15.** \$1243.63. **16.** \$499. **17.** \$826.40. **18.** \$562.83. **19.** \$667.55. **20.** \$880.10. **21.** \$1014.30. **22.** \$2518.56. **23.** \$3935.15. **24.** \$2706.41. **25.** \$1044.83. **26.** \$1993.18. **27.** \$960.

Exercise 86. — **1.** 7830 oz., 2890 oz., 1200 oz. **2.** .579 oz., 8.355 lb. **3.** 77.78 lb. nearly. **4.** 2.7. **5.** 1.036. **6.** 1.04. **7.** 14.724 lb. **8.** 62.5 T. **9.** 4.17 T. **10.** 11.146 oz., 6.059 oz., 12.442 oz., 6.586 oz. **11.** 345 lb. **12.** 1.0125 T. **13.** $39\frac{1}{6}$ lb. **14.** 54 T. **15.** 810.85 lb. **16.** 1.3. **17.** 2.402 cu. in. **18.** $166\frac{1}{4}$ lb. **19.** 280 oz. **20.** 4.6955 times. **21.** 108.194 lb. **22.** 212.5 lb.

Exercise 87. — **1.** 125 ft. **2.** $5\frac{1}{2}$ yd. **3.** 40 rd. **4.** 14 ft. **5.** $161\frac{1}{3}$ bu. **6.** $29\frac{1}{3}$ bu. **7.** 19.36 bu. **8.** 20 yd. **9.** 17 yd. **10.** 32 yd. **11.** 10 ft. **12.** 4.5 ft. **13.** 7 ft. **14.** 44 ft.

Exercise 88. — **2.** 1792 cu. ft., 1433.6 bu., 45,875.2 lb. **3.** 1725 cu. ft., 4800 turkeys. **4.** 2720 cu. ft., 130,560 lb., $\frac{625}{816}$. **5.** 20 cords, \$95, \$30. **6.** 2675.2 T.

Exercise 89. — **2.** $71{,}614\frac{7}{12}$ lb. Troy, $58{,}928\frac{4}{7}$ lb. Avoir. **3.** 371.25 grains. **4.** 51.05 ¢. **5.** $36\frac{6}{7}$ lb. Avoir, $3685\frac{5}{7}$ lb. Avoir. **6.** 232.2 grains. **7.** \$20.672. **8.** 61 ¢. **9.** 30.5.

Exercise 90. — **2.** \$80, \$50. **2.** \$591, \$394. **3.** \$1440, \$1800. **4.** \$1188, \$2376, \$3564. **5.** \$1700, \$2550, \$2975. **6.** 756, 972, 1188. **7.** 912.5, 1095, 1277.5, 1460. **8.** 1508, 1624, 1740, 1856. **9.** 1176, 1232, 1288, 1344. **10.** 540, 612, 684, 756. **11.** 57, 38. **12.** 992, 744. **13.** 140, 225. **14.** 975, 546. **15.** 1170, 1300, 1755. **16.** 396, 770, 1023. **17.** \$490, \$700, \$800. **18.** \$720, \$900, \$1080. **19.** \$300, \$255, \$345. **20.** $\$4444\frac{4}{9}$, $\$5555\frac{5}{9}$.

Exercise 91. — **1.** \$8.25. **2.** 11.2 da. **3.** \$110.50. **4.** $7\frac{1}{3}$ da. **5.** $5\frac{3}{4}°$. **6.** $31\frac{1}{3}$ A. **7.** 56 ft. **8.** $11\frac{1}{4}$ da. **9.** \$24.18. **10.** 60 mi. **11.** \$185. **12.** \$333.75. **13.** 3960 ft., 66 ft. **14.** 52.8 ft. **15.** $4\frac{1}{6}$ sec. **16.** 186,000 mi. nearly. **17.** \$112.50. **18.** $12\frac{1}{2}$ yd.

Exercise 92. — **1.** $1\frac{7}{8}$. **2.** $15\frac{5}{6}$. **3.** $6\frac{1}{3}$. **4.** The first. **5.** $\frac{21}{320}$. **6.** 0. **7.** 50 lb. **8.** $\frac{9}{48}$. **9.** 4950 ft. **10.** .1212+ rd. **11.** .456. **12.** .013875 T. **13.** 1400 lb. **14.** 45°. **15.** 12°. **16.** $\frac{1}{4}$. **17.** .3, \$1500, A's \$750, B's \$300. **18.** \$1875. **19.** 7.9. **20.** 11.3984. **21.** .0524. **22.** \$1178.85. **23.** 112.5. **24.** 228. **25.** \$150. **26.** $\frac{11}{18}$. **27.** 3.9. **28.** \$18.70. **29.** \$1500. **30.** $13\frac{1}{2}$ da. **31.** 6 da. **32.** 9 da. **33.** 14 hr. **34.** 20 da. **35.** 1.7875. **36.** $\frac{41}{45}$. **37.** 325 lb. **38.** \$326.67. **39.** \$64, \$240. **40.** \$75. \$313.32, 25 mo. **41.** \$89.25,

$107.10. **42.** $6.06. **43.** 126 lb. $15\frac{1}{4}$ oz. **44.** $18.20. **45.** $121.67. **46.** $11\frac{1}{4}$¢, $11.25. **47.** 11¢, $11. **48.** 13¢, $13. **49.** $34,588, $32,858.60. **50.** $133,016. **51.** $62.675. **52.** $117.60. **53.** 5 mills, 50¢. **54.** 21.82 knots per hr., 25.126 mi. per hr. **55.** 20.584 knots per hr., 23.703 mi. per hr. **56.** 24.13 knots per hr. **57.** 65.5 mi. nearly. **58.** 63.617 mi. per hr. **59.** 50.77 mi. per hr. **60.** Wheat, $$1.31\frac{1}{4}$; corn, $81\frac{1}{2}$¢; oats, $$1.06\frac{1}{4}$ nearly. **61.** (*a*) $2367.55, (*b*) $6450.05, (*c*) $922.32, (*d*) $950.82, (*e*) $4585.84, (*f*) $693.45, (*g*) $955.79.

Exercise 93. — **1.** 75,453,652. **2.** 74,944,016. **3.** 73,781,838. **4.** 80,963,637. **5.** 80,903,198. **7.** New England, 7607.2, 7619.39, 7680.92; Middle Atlantic, 22,930.26, 23,149.83, 23,408.29; Central Northern, 42,741.79, 43,251.64, 43,958.89; South Atlantic, 22,880.92, 23,589.39, 24,179.92; Gulf and Miss. Valley, 17,567.13, 18,297.28, 19,025.73; Southwestern, 43,068.45, 44,852.05, 46,061.39. **8.** $12\frac{1}{12}$. **9.** $11\frac{2}{5}$. **10.** $11\frac{1}{2}$. **11.** $17\frac{17}{24}$. **12.** $9\frac{5}{12}$. **13.** $23\frac{17}{30}$. **14.** $22\frac{11}{16}$. **15.** $30\frac{7}{18}$. **16.** 147° 24′ 40″. **17.** 35 ft. 6 in. **18.** 45 qt. 1 pt. **19.** 30 gal. 1 qt. **20.** 41 pk. 3 qt. **21.** 25 bu. 1 pk. **22.** 50 hr. 18 min. **23.** 43 da. 2 hr. **24.** 83 yd. **25.** 156 times. **26.** 27.63 liters. **27.** 23 ft. 8 in. **28.** 4.37 mi.; 13,431 carloads; 100.7325 mi. **29.** 134 T.

Exercise 94. — **2.** 199.98. **3.** 398.63. **4.** $74.27. **5.** $697.16. **6.** $98.25. **7.** $999.95. **8.** $223.81. **9.** $50.79. **10.** $310.16. **11.** $36.65. **12.** 99.99. **13.** 3.2976. **14.** .9937. **15.** 4.43. **16.** 9.5. **17.** 5.4. **18.** 45.01. **19.** $2\frac{3}{4}$. **20.** $10\frac{13}{18}$. **21.** $5\frac{1}{12}$. **22.** $9\frac{17}{18}$. **23.** $4\frac{17}{22}$. **24.** $2\frac{23}{80}$. **25.** $5\frac{13}{24}$. **26.** $2\frac{43}{48}$. **27.** $3\frac{1}{44}$. **28.** $11\frac{1}{24}$. **29.** $2\frac{17}{30}$. **30.** $11\frac{1}{42}$. **31.** 10 in. **32.** 4 ft. 7 in. **33.** 9 ft. 9 in. **34.** 8 ft. 11 in. **35.** 4 lb. 11 oz. **36.** 11 lb. 11 oz. **37.** 14 lb. 10 oz. **38.** 2 hr. 46 min. **39.** 55° 45′ 45″. **40.** 64° 55′ 10″. **41.** 31° 26′. **42.** 11 pk. 4 qt. **43.** 9 pk. 7 qt. **44.** 17,357. **45.** 12,219. **46.** 5176. **47.** $8592. **48.** 144,805 sq. mi. **49.** 23,672,000, 78,272,000, 147,925,000. **50.** 91,895. **51.** 574 mi.

Exercise 95. — **6.** 27,878,400; 1,040,400. **7.** 1,871,104; 6,921,495. **8.** Corn, $12.09; wheat, $10.34; oats, $9.89; rye, $9.84; barley, $11.74; buckwheat, $11.09. **9.** 30,759,200 lb. **10.** N.H., $14.37; Mass., $22.27; Conn., $17.55; N.Y., $15.49; N.J., $21.05; Penn., $17.42; Md., $17.01; Va., $19.37; S.C., $22.26; Ga., $25.99; Ala., $25.93; La., $22.19; Tenn., $20.31; Ky., $17.89; Ill., $12.25; Minn., $9.35; Kan., $8.00; Col., $23.75; Utah, $30.00; Idaho, $23.60; Cal., $20.81. **12.** (*a*) $10.84, (*b*) $8.97, (*c*) $7.24, (*d*) $10.09, (*e*) $9.79, (*f*) $2.90, (*g*) $56.53, (*h*) $55.68. **13.** 97 ft. $\frac{1}{4}$ in. **14.** 73 ft. $9\frac{1}{2}$ in. **15.** 45 yd. 1 ft.; 67 ft. 8 in. **16.** 46 lb. 4 oz.; 150 lb. 3 oz. **17.** 20 hr. 3 min.; 65 hr. 33 min. **18.** 532°; 341° 1′ 30″. **19.** 29 pk. 2 qt.; 59 pk. **20.** 49 gal. 2 qt.; 117 gal.

Exercise 96.—**1.** N.H., 120.2 bu., 72¢; R.I., 125 bu., 89¢; Del., 93 bu., 59¢; N.C., 77 bu., 68¢; Fla., 75 bu., 120¢; Miss., 110 bu., 85¢; W. Va., 88 bu., 58¢; Mich., 67 bu., 56¢; Ill., 75 bu., 67¢; Mo., 82 bu., 55¢; N.D., 95 bu., 38¢; Nev., 120 bu., 82¢. **2.** .3048. **3.** .3048. **4.** 3.2808. **5.** 1.6093. **6.** .6214. **7.** .4047. **8.** 2.471. **9.** .7646. **10.** 1.3079. **11.** $16\frac{1}{2}$. **12.** 21. **13.** 32. **14.** $21\frac{1}{3}$. **15.** $17\frac{1}{2}$. **16.** $1\frac{1}{2}$. **17.** $1\frac{4}{5}$. **18.** $\frac{22}{35}$. **19.** $1\frac{7}{8}$. **20.** $\frac{7}{8}$. **22.** $\frac{9}{20}$. **23.** $\frac{16}{49}$. **24.** $1\frac{3}{5}$. **25.** $\frac{9}{100}$. **26.** 45. **27.** $2\frac{2}{15}$. **28.** $3\frac{1}{2}$. **29.** $3\frac{1}{4}$. **30.** $4\frac{22}{35}$. **31.** $3\frac{5}{9}$. **32.** 3 long T.; 5.905 long T.; .00886 long T.; 3.9368 long T. **33.** 2.2046 T.; 7.7161 T.; .00551 T. **34.** 6.0764 lb. Troy; 16.076 lb. Troy; 8,037.69 lb. Troy. **35.** 5.76 lb.; 11.023 lb.; 4.409 lb. **36.** .0001894 mi. **37.** .003125 mi. **38.** .00625 A. **39.** .0015625 sq mi. **40.** .0002066 A. **41.** .0005 T. **42.** 363. **43.** $114\frac{2}{7}$ barrels. **44.** 600. **45.** $6150. **46.** 171.6 T.

Exercise 98.—**1.** 24° 10′. **2.** 14° 16′. **3.** 11° 17′ 40″. **4.** 29° 17′ 46″. **5.** 108° 53′ 48″. **6.** 73° 28′. **7.** 229° 22′ 32″. **8.** 94° 50′ 10″. **9.** 116° 5′ 40″. **10.** 88° 10′ 54″. **11.** 114° 8′ 45″. **12.** 78° 37′ 37″. **13.** 25° 33′ 35″. **14.** 35° 23′ 54″. **15.** 120° 35′ 40″. **16.** 74° 44′ 55″. **17.** 116° 4′ 2″.

Exercise 99 (answers correct to the second).—**1.** 1 hr. 42 min. 14 sec. **2.** 3 hr. 9 min. 2 sec. **3.** 4 hr. 35 min. 14 sec. **4.** 12 hr. 53 min. 56 sec. **5.** 15 hr. 9 min. 38 sec. **6.** 6 hr. 50 min. 59 sec. **7.** 8 hr. 32 min. 15 sec. **8.** 5 hr. 33 min. 43 sec. **9.** 15 hr. 20 min. 10 sec. **10.** 13 hr. 41 min. 28 sec. **11.** 13 min. 38 sec. **12.** 7 hr. 13 min. 58 sec. **13.** 2 hr. 10 min. 26 sec. **14.** 6 hr. 46 min. 56 sec. **15.** 14 hr. 13 min. 10 sec. **16.** 14 hr. 15 min. 2 sec. **17.** 1 hr. 12 min. 59 sec.

Exercise 100.—**1.** 13° 22′ 6″ E. **2.** 4° 20′ 36″ E. **3.** 88° 17′ 36″ E. **4.** 3° 12′ 24″ W. **5.** 9° 56′ 36″ E. **6.** 1′ 54″ W. **7.** 3° 43′ 54″ W. **8.** 16° 17′ 36″ E. **9.** 72° 30′ 45″. W. **10.** 2 hr. 37 min. 20 sec.

Exercise 101.—**1.** 72° W. **2.** 11 o'clock A.M. March 3; 1 o'clock P.M. March 3; 4 o'clock A.M. March 3. **3.** 9 o'clock A.M. in London, Manchester, Glasgow; 5 o'clock P.M. in Tien-Tsin; 11 o'clock A.M. in Constantinople. **4.** 5 A.M. **5.** 165° W. **6.** 6 A.M. following day. **7.** 6.30 P.M.; 9 P.M. previous day; 12.30 P.M. **8.** 2 P.M.; 3 P.M.; 11 P.M.; 11.30 P.M. **9.** 4 hr. 50 min. 39 sec. A.M.; 1 hr. 50 min. 39 sec. P.M.; 5 hr. 20 min. 39 sec. P.M.; 11 hr. 20 min. 39 sec. P.M. **10.** 11 hr. 30 min. P.M. previous day; 9 hr. 30 min. P.M. previous day; 6 hr. 30 min. P.M. previous day. **11.** 12.30 A.M. following day; 6.30 A.M. following day.

Exercise 102.—**1.** 6 da. **2.** $2\frac{1}{4}$ hr. **3.** $592.20. **4.** 25 yd. **5.** 16 da. **6.** 132°. **7.** 21.6 ft. **8.** 500 mi. **9.** 2355 mi. **10.** 2030 mi.

11. 605 mi. **12.** $51,937,925. **13.** $730,377. **14.** $2,944,492. **15.** $.238. **16.** $.498. **17.** 19.3¢. **18.** 1.09375.

Exercise 106. — **1.** 4. **2.** $3a$. **3.** $3a$. **4.** 4. **5.** $3a^2$. **6.** $3a$. **7.** $2a^2$. **8.** $6a$. **9.** $3a^2$. **10.** $11a^3$. **11.** $11a^3$. **12.** $4a^2$. **13.** $2a^2$. **14.** $3a^2$. **15.** $5a^2$. **16.** $7x$. **17.** $3x^4$. **18.** $7x$. **19.** $5x^3$. **20.** $5x$. **21.** 6. **22.** $2x^4$. **23.** $3b$. **24.** $3b^3$. **25.** $2b^3$.

Exercise 108. — **1.** 9. **2.** 8. **3.** 20. **4.** 30. **5.** 18. **6.** 35. **7.** 30. **8.** 40. **9.** 22. **10.** 45. **11.** 55. **12.** 55. **13.** 54. **14.** 60. **15.** 45. **16.** 49. **17.** 90. **18.** 55. **19.** 63. **20.** 40. **21.** 56. **22.** 18. **23.** 22. **24.** 30. **25.** 35.

Exercise 109. — **1.** $675. **2.** $23.04. **3.** $609.20. **4.** 10 da. **5.** $2175. **6.** 16 hr. 48 min. **7.** 20 da. **8.** $18\frac{2}{3}$ bu. **9.** 9 da. **10.** 25 da. **11.** 400 men. **12.** 21 da. **13.** 18 turkeys. **14.** 66¢. **15.** 280,176 ft. **16.** 75 ft. **17.** 157.5 ft. **18.** 40 mi. per hr.

Exercise 110. — **1.** 84 A. **2.** 150 ft. **3.** $260. **4.** 2057.5 T. **5.** 20 ft. **6.** 7 da. **7.** 12 da. **8.** 6 da. **9.** 7 hr. 12 min. **10.** $113.40. **11.** $365.625. **12.** $2\frac{1}{4}$ da. **13.** 8 da. **14.** 17.85 T. **15.** $17\frac{7}{9}$ hr.

Exercise 111. — **1.** A's share, $800; B's share, $1000; C's share, $1200. **2.** A's, $600; B's, $1500. **3.** A's, $90; B's, $81. **4.** A's, $900; B's, $700. **5.** A's, $780; B's, $954; C's, $420. **6.** A, $300; B, $300. **7.** $700, $750. **8.** $3\frac{1}{8}$ A. **9.** $12.50.

Exercise 112. — **1.** 4%, 8%, $7\frac{1}{2}$%, $5\frac{1}{4}$%, $16\frac{2}{3}$%. **2.** .045; .15; .125; .625; .0625; .036. **3.** 107.55; 18.335; 34.475; .3594; .62345. **4.** 315.72; 31.242; .1596; .18582; .3846; .04698. **5.** 186.75; 63.81; 320.4; 1.95435; 2.39265; .1089. **6.** Land, $19,466; fencing, $2919.90; earthworks, $46,718.40; tunnels, $23,359.20; viaducts and bridges, $33,092.20; works, $3893.20; culverts, $9733; way, $22,385.90; sidings, $5839.80; junctions, $1946.60; stations, $12,652.90; legal expenses, $11,679.60; maintenance, $973.30. **7.** Ans. correct to one-tenth of one million. $974,500,000; $54,200,000; $16,900,000; $66,300,000; $97,800,000; $16,800,000. **9.** Ans. correct to one-tenth of one million. N.Y., $607,000,000; Savannah, $64,900,000; Boston, $98,700,000; Puget Sound, $49,200,000; New Orleans, $150,000,000; Detroit, $35,200,000; Galveston, $166,400,000; Buffalo Creek, $30,000,000; Philadelphia, $82,500,000; Mobile, $21,800,000; Baltimore, $110,000,000; Newport News, $20,100,000; San Francisco, $39,900,000; Wilmington, $18,500,000.

Exercise 113. — Michigan, 4,725,000 lb.; Minnesota, 1,176,000 lb.; Alabama, 341,250 lb.; Montana, 12,535,250 lb.; Wyoming, 10,511,920 lb.;

Idaho, 5,578,650 lb.; California, 4,331,250 lb.; Utah, 4,322,500 lb.; New Mexico, 6,061,000 lb.; Colorado, 3,118,500 lb.; Arizona, 1,502,800 lb.; Texas, 3,182,400 lb.; Washington, 1,466,250 lb.

Exercise 114. — **1.** 1973. **2.** 3990. **3.** $800. **4.** $1000. **5.** $3004.44. **6.** $100.80. **7.** Maine, $351,577,436. **8.** Pennsylvania, $3,910,701,678; South Carolina, $195,620,105; Kansas, $363,010,660; Tennessee, $405,641,915; Washington, $260,948,609; Texas, $1,021,158,657.

Exercise 115. — **1.** Europe, 34.16%; Asia, 8.99%; Africa, 2.94%; North America, 44.97%; South America, 5.84%; Australasia, 3.10%. **2.** Austria, 123.8%; England, 7.5%; France, 7.9%; Germany, 4.4%; Ireland, 13.7%; Scotland, 3.6%. **3.** $63\frac{1}{3}$%. **4.** $56\frac{1}{4}$%. **5.** $8\frac{1}{3}$%. **6.** 15¢. **7.** 20¢ per lb. **8.** 4%. **9.** $27\frac{7}{9}$%. **10.** 80¢. **11.** $20. **12.** $17\frac{1}{2}$%. **13.** 100 lb. **14.** $9\frac{3}{4}$%. **15.** $16\frac{2}{3}$%.

Exercise 116. — **1.** $54.60; $782.60. **2.** $70.35; $740.35. **3.** $126; $1386. **4.** $36.83; $421.83. **5.** $137.50; $2887.50. **6.** $267.60; $3612.60. **7.** $34.80; $817.80. **8.** $41.79; $638.79. **9.** $56; $3056. **10.** $37.60; $977.60. **11.** $9; $1809. **12.** $231; $2331. **13.** $47.97; $1007.97. **14.** $189.56; $3100.81. **15.** $136.18; $1993.18. **16.** $70.30; $2845.30. **17.** $58.63; $1828.63. **18.** $51.08; $2026.22. **19.** $43.73; $1261.73. **20.** $113.98; $1901.98.

Exercise 117. — **1.** $3.88. **2.** $1.96. **3.** $1.18. **4.** $24.07. **5.** $13.96. **6.** $26.02. **7.** $35.16. **8.** $49.79. **9.** $39.46. **10.** $39.52.

Exercise 118. — **1.** $500. **2.** $1200. **3.** $1000. **4.** $240. **5.** $2400. **6.** $300. **7.** $600. **8.** $800. **9.** $195. **10.** $450. **11.** $1020. **12.** $324. **13.** $300.

Exercise 119. — **1.** 1 yr. **2.** 3 yr. **3.** $\frac{3}{5}$ yr. **4.** 4 yr. **5.** 1 yr. **6.** 1 yr. 9 mo. **7.** 3 yr. 3 mo. **8.** 1 yr. 1 mo. 15 da. **9.** 1 yr. 10 mo. 15 da. **10.** 1 yr. 1 mo. 15 da. **11.** 1 yr. 10 mo. **12.** 8 mo. 15 da. **13.** 2 yr. 11 mo.

Exercise 120. — **1.** 6%. **2.** 7%. **3.** 6%. **4.** 8%. **5.** 7%. **6.** 6%. **7.** 5%. **8.** 4%. **9.** 5%. **10.** $4\frac{32}{37}$%. **11.** 8%. **12.** 9%. **13.** $4\frac{1}{2}$%. **14.** 6%. **15.** 6%. **16.** 7%. **17.** 5%. **18.** 6%. **19.** $5\frac{1}{2}$%. **20.** $4\frac{1}{2}$%.

Exercise 121. — **1.** $800. **2.** $700. **3.** $600. **4.** $840. **5.** $360. **6.** $230. **7.** $11,505. **8.** $162.50. **9.** $580. **10.** $630. **11.** $740. **12.** $450. **13.** $403. **14.** $240. **15.** $1917.50. **16.** $1800. **17.** $245. **18.** $240. **19.** $22,530. **20.** $803.05.

Exercise 122. — **1.** $392. **2.** $6.43. **3.** $1.44. **4.** $5.51. **5.** $24.57. **6.** $858.67. **7.** $548.91. **8.** $999.84. **9.** $14,000. **10.** 7%. **11.** 7%. **12.** $24,000. **13.** $6000. **14.** 1 mo. 18 da. **15.** 4 mo. **16.** 2 yr. 6 mo. **17.** 1 yr. 8 mo. **18.** $48. **19.** $204.40. **20.** 219 da. **21.** 2 mo. 12 da. **22.** $500. **23.** $1072.50. **24.** $13,000.

Exercise 123. — **1.** Discount $4.13. **2.** Discount $5.87. **3.** Discount $7.50. **4.** Discount $7.00. **5.** Discount $9.50. **6.** Discount $1.00. **7.** Discount $7.25. **8.** Discount $2.25. **9.** Discount $4.00. **10.** Discount $2.92. **11.** Discount $13.33. **12.** Discount $5.00. **13.** Discount $10.40. **14.** Discount $3.37. **15.** Discount $1.90. **16.** Discount $6.16. **17.** Discount $10.92. **18.** Discount $8.55.

Exercise 124. — **1.** Discount $1.65, Proceeds $352.72. **2.** Discount $9.25, Proceeds $391.87. **3.** Discount $1.61, Proceeds $450.86. **4.** Discount $6.08, Proceeds $601.92. **5.** Discount $5.05, Proceeds $499.95. **6.** Discount $6.64, Proceeds $898.31. **7.** Discount $16.89, Proceeds $996.44. **8.** Discount $7.45, Proceeds $750.05. **9.** Discount $3.04, Proceeds $801.36. **10.** Discount $4.24, Proceeds $399.96. **11.** Discount $21.38, Proceeds $843.49. **12.** Discount $1.96, Proceeds $904.79. **13.** Discount $17.60, Proceeds $1201.

Exercise 125. — **1.** $280. **2.** $720.13. **3.** $781.25. **4.** $31\frac{3}{7}$%. **5.** 20%, $21. **6.** 26%. **7.** (*a*) $441, (*b*) $3528, (*c*) $2244, (*d*) $4845, (*e*) $2565, (*f*) $1344, (*g*) 2736, (*h*) $1782, (*i*) $1642.20.

Exercise 126. — **1.** $2835.13. **2.** $1776.30. **3.** $233.67. **4.** $480.85. **5.** $1524.53. **6.** $479.46. **7.** $153.97. **8.** $427.89. **9.** $268.02. **10.** $122.08.

Exercise 127. — **1.** $62.84. **2.** $215.47. **3.** $325.31. **4.** $197.25.

Exercise 129. — **1.** $2726.80. **2.** $3497.40. **3.** $5086.35. **4.** $6305.72. **5.** $8279.64. **6.** $9955.05. **7.** $5484.76. **8.** $5287.92. **9.** $6027.92. **10.** $6097.61. **11.** $5393.25. **12.** $9771.30. **13.** $4509. **14.** $6913.08. **15.** $7789.72. **16.** $1232.46. **17.** $4.20. **18.** $9987.50. **19.** $598.50. **20.** $993.33. **21.** $891.

Exercise 130. — **1.** 25.215 francs, 20.45 marks, 23.973 crowns. **2.** (*a*) $86.85, (*b*) $106.15, (*c*) $75.04, (*d*) $214.20, (*e*) $93, (*f*) $146.16, (*g*) $120.60. **3.** $5. **4.** 2150.53 colons, 4926.11 crowns, 3731.34 crowns. **5.** $38,600. **6.** $4980. **7.** .2055 libra. **8.** 3.731 crowns. **9.** $20. **10.** $10,800.

Exercise 131. — **2.** $582.94, $618.49, $725.14. **3.** $349.78. **4.** $205.00. **5.** (*a*) $40,270.80, (*b*) $1169.40, (*c*) $3792. **6.** (*a*) $2654.75, (*b*) $14,842.82.

Exercise 132. — **1.** \$4872. **2.** \$2671.14. **3.** \$1116.50. **4.** \$17,064.12. **5.** \$352.42. **6.** \$1717. **7.** \$289.50. **8.** (*a*) \$4503.22, (*b*) \$4134.75, (*c*) \$600, (*d*) \$1798.45, (*e*) \$1636.25, (*f*) \$2840.02.

Exercise 133. — **1.** £1200. **2.** £250. **3.** 38,080 marks. **4.** 19,300 francs. **5.** 2680 krones. **6.** £486 10*s*. **7.** 5376.85 francs. **8.** 23,801.68 marks.

Exercise 134. — **2.** \$1620, \$1640. **3.** \$3720. **4.** \$29,075. **5.** \$2965.62. **6.** \$5337.50. **7.** \$8200; \$9750; \$9300; \$12,962.50; \$14,287.50; \$13,412.50; \$10,850; \$14,412.50; \$16,837.50; \$6300.

Exercise 135. — **1.** \$14,262.50; \$14,125. **2.** \$9737.50. **3.** 100 shares. **4.** \$125. **5.** \$125. **6.** \$400. **7.** 200 shares. **8.** 100 shares. **9.** 1000 shares. **10.** 200 shares.

Exercise 136. — **1.** $159\frac{7}{8}$. **2.** $149\frac{7}{8}$. **3.** \$18,550. **4.** \$32,718.75. **5.** $4\frac{29}{49}\%$. **6.** $62\frac{3}{8}$. **7.** \$30,918.75. **9.** \$875; no brokerage. **10.** $5\frac{5}{9}\%$. **11.** \$200, \$5093.75. **13.** Central. **14.** 438 shares.

Exercise 137. — **1.** \$62.40. **2.** \$500. **3.** \$2.16. **4.** \$26.28. **5.** £43 4*s*., or \$210.23. **6.** \$1.81. **7.** \$1.46. **8.** \$256.80. **9.** \$58. **10.** \$72, \$8.19 each. **11.** \$91.80. **12.** \$17.50, \$81. **13.** \$65. **14.** \$552, \$2.208. **15.** \$750. **16.** \$28.80. **17.** \$94.50. **18.** \$125. **19.** £5. **20.** \$82.12, 29.4¢.

Exercise 141. — **1.** .01, .04, .09, .16, .25, .36, .49, .64, .81. **2.** 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, $\frac{1}{36}$, $\frac{1}{49}$, $\frac{1}{64}$, $\frac{1}{81}$, $\frac{1}{100}$, $\frac{1}{121}$, $\frac{1}{144}$, $\frac{1}{169}$, $\frac{1}{196}$, $\frac{1}{225}$, $\frac{1}{256}$, $\frac{1}{289}$, $\frac{1}{324}$, $\frac{1}{361}$, $\frac{1}{400}$. **3.** $\frac{4}{9}$, $\frac{9}{16}$, $\frac{25}{36}$, $\frac{4}{49}$, $\frac{81}{100}$, $\frac{121}{144}$, $\frac{100}{169}$, $\frac{81}{256}$. **4.** $2\frac{7}{9}$, $5\frac{4}{9}$, $1\frac{32}{49}$, $1\frac{9}{16}$, $3\frac{1}{16}$, $3\frac{33}{64}$, $4\frac{21}{25}$, $39\frac{1}{16}$. **6.** 1, $\frac{1}{8}$, $\frac{1}{27}$, $\frac{1}{64}$, $\frac{1}{125}$, $\frac{1}{216}$, $\frac{1}{343}$, $\frac{1}{512}$, $\frac{1}{729}$, $\frac{1}{1000}$, $\frac{1}{1331}$, $\frac{1}{1728}$, $\frac{1}{2197}$, $\frac{1}{2744}$, $\frac{1}{3375}$, $\frac{1}{4096}$, $\frac{1}{4913}$, $\frac{1}{5832}$, $\frac{1}{6859}$, $\frac{1}{8000}$. **7.** 3025. **8.** (*a*) 3, (*b*) 85.6735, (*c*) 29.1708, (*d*) 66.0806, (*e*) .4670, (*f*) .5790, (*g*) .8991, (*h*) .5, (*i*) .6, (*j*) .9.

Exercise 142. — **1.** 13. **2.** 21. **3.** 25. **4.** 31. **5.** 32. **6.** 43. **7.** 53. **8.** 62. **9.** 65. **10.** 73. **11.** 76. **12.** 82. **13.** 84. **14.** 87. **15.** 92. **16.** 96. **17.** 98. **18.** 99. **19.** 89. **20.** 78. **21.** 69. **22.** 59. **23.** 49. **24.** 58.

Exercise 143. — **1.** 317. **2.** 332. **3.** 347. **4.** 414. **5.** 436. **6.** 447. **7.** 479. **8.** 527. **9.** 557. **10.** 595. **11.** 626. **12.** 676. **13.** 689. **14.** 708. **15.** 809. **16.** 879. **17.** 905. **18.** 909.

Exercise 144. — **1.** .388. **2.** .496. **3.** .587. **4.** .539. **5.** .513. **6.** .679. **7.** .729. **8.** .785. **9.** .885. **10.** .288. **11.** .249. **12.** .0608.

Exercise 145.—**1.** 1.095. **2.** 2.062. **3.** 1.049. **4.** 2.28. **5.** 1.817. **6.** 2.291. **7.** 1.768. **8.** 3.028. **9.** 1.173. **10.** 2.121. **11.** 1.696. **12.** .764. **13.** .816. **14.** .632. **15.** .7977.

Exercise 146.—**1.** 69.57 yd. **2.** 241 yd. **3.** 238 yd. **4.** 1.025 mi. **5.** 739 rd. **6.** 311.13 yd., 155.56 yd. **7.** 538.89 yd., 179.63 yd. **8.** 19.41 rd.

Exercise 147.—**1.** 10. **2.** 13. **3.** 17. **4.** 29. **5.** 106. **6.** 101. **7.** 145. **8.** 89. **9.** 149. **10.** 68.5. **11.** 425. **12.** 305. **13.** 433. **14.** 305. **15.** 50 ft. **16.** 14.14 rd. **17.** 30.232 rd.

Exercise 148.—**1.** 152. **2.** 184. **3.** 280. **4.** 2.17. **5.** .261. **6.** .319. **7.** .2. **8.** .748. **9.** .455.

Exercise 149.—**1.** 126. **2.** 180. **3.** 264. **4.** 840. **5.** 522. **6.** 9240. **7.** 150,769. **8.** 8.625 A. **9.** 5.775 A. **10.** 1.638 A. **11.** 1.68 A. **12.** 43.301 sq. rd. **13.** 1082.53 sq. rd. **14.** 182.25 sq. in. **15.** 2592 sq. ft. **16.** 57.42 ft.

Exercise 150.—**1.** 69.12. **2.** 144.51. **3.** 471.24. **4.** 515.22. **5.** 615.75. **6.** 420.97. **7.** 540.35. **8.** 22.62. **9.** 37.07. **10.** 45.87 **11.** 102. **12.** 152. **13.** 7.8. **14.** 13.27. **15.** 60. **16.** 8.5. **17.** 9.6. **18.** 6.7. **19.** 480 times.

Exercise 151.—**1.** 615.8. **2.** 1520.5. **3.** 4071.5. **4.** 69.4. **5.** 132.73. **6.** 232.35. **7.** 295.59. **8.** 4778.4. **9.** 3217. **10.** 7238.2. **11.** 4300.8. **12.** 6647.6. **13.** 795.8. **14.** 484.15. **15.** 548.2. **16.** 688.3. **17.** 6.023. **18.** 3.789. **19.** 7.643. **20.** 9.282.

Exercise 152.—**1.** 31. **2.** 17. **3.** 33. **4.** 42. **5.** 2.6. **6.** 7.2. **7.** 9.2. **8.** 9.8. **9.** 11.8. **10.** 13.4. **11.** 7.6. **12.** 9.3. **13.** 99. **14.** 850. **15.** 650. **16.** 39.25 yd. **17.** 288,576,452.4. **18.** 329.9 sq. in. **19.** 1661.9 sq. in. **20.** 259.8. **21.** 139.1 sq. in. **22.** 259.8 sq. in., 225 sq. in. **23.** Circle.

Exercise 153.—**1.** 46.5 in. **2.** 49.22 in. **3.** 1.01 ft. **4.** 36°. **5.** 3° 36′. **6.** 31′.2.

Exercise 154.—**1.** 20 ft. **2.** 176 ft. **3.** 110 mi. **4.** $7\frac{1}{8}$ mi. **5.** 22.36 ft. **6.** 7.07 ft. **7.** 16 : 121. **8.** 4 : 9. **9.** 186.96 sq. mi.

Exercise 155.—**1.** 576 sq. ft. **2.** 680 ft. **3.** $10\frac{5}{9}$ ft. **4.** $7\frac{1}{2}$ ft. **5.** 2080 sq. ft., $52. **6.** 630 sq. ft. **7.** $26.40. **8.** 147 sq. yd. **9.** 53.45 sq. ft. **10.** 4021 sq. in. **11.** 67 yd. **12.** 2984.5 sq. ft. **13.** 15,456.6 sq. in. **14.** $392.70. **15.** 120,687 sq. in. **16.** 5541.7 sq. in. **17.** 28,842,700 sq. mi. **18.** 186,265,000 sq. mi. **19.** (1) Jupiter, 23,235 million sq. mi.; (2) Uranus, 3217 million sq. mi.; (3) Neptune, 3421 million sq. mi.; (4) Saturn, 16,741 million sq. mi. **20.** 58 in.

Exercise 156.—**1.** 5280 cu. in. **2.** 700 cu. in. **3.** 4071.5 cu. in. **4.** 15,708 cu. in. **5.** 2598 cu. in. **6.** 33,510 cu. in. **7.** 2144.7 cu. ft. **8.** 4094 gal. **9.** 4562 gal. **10.** 3 : 2. **11.** 1 : 2. **12.** 52 cu. ft. 1269 cu. in. **13.** 3 ft. **14.** 4189 cu. in. **15.** 40 ft. **16.** 1,367,631. **17.** 48 times. **18.** 760 times. **19.** 1331 times.

Exercise 157.—**1.** 30° C. **2.** 25° C. **3.** 95° C. **4.** 120° C. **5.** 20° C. **6.** $12\frac{2}{9}$° C. **7.** $3\frac{1}{3}$° C. **8.** − 5° C. **9.** − 10° C. **10.** − 25° C. **11.** − 40° C. **12.** $-67\frac{7}{9}$° C. **13.** 95° Fahr. **14.** 131° Fahr. **15.** 77° Fahr. **16.** 68° Fahr. **17.** 64.4° Fahr **18.** 46.4° Fahr. **19.** 14° Fahr. **20.** − 4° Fahr. **21.** 6.8° Fahr. **22.** 0.4° Fahr. **23.** − 11.2° Fahr. **24.** − 459.4° Fahr. **25.** Mercury, − 40°; sulphur, 235.4°; lead, 618.8°; zinc, 779°; gold, 1895°; cast iron, 2012° to 2102°. **26.** 39.2°.

Exercise 158.—**1.** 3,921,138.813 meters. **2.** 107,934,859.86 cm. **3.** 3,131,587.7 mm. **4.** 28.434 km. **5.** 19454 m. **6.** 140.784 m. **7.** 960 times. **8.** 7.03 times. **9.** 5000 times. **10.** $112\frac{1}{2}$. **11.** 20,000. **12.** 62.5 times. **13.** 5000. **14.** 1200, 480.

Exercise 159.—**1.** 7.8125 ha. **2.** 1.8605 ha. **3.** 5.7122 ha. **4.** 6.82 ha. **5.** 11.95 ha. **6.** 6.65 ha. **7.** .81 cbm. **8.** 3.36 a. **9.** 1.91 ca. **10.** 24,429 c.cm. **11.** 119 cm. **12.** 1596 cbm. **13.** 169.65 ca. **14.** 51.08 km. **15.** 462.3 qcm.

Exercise 160.—**1.** \$1156.43. **2.** \$622.91. **3.** \$1650. **4.** \$1278.12. **5.** \$983.59.

Exercise 161.—**1.** \$2737.14. **2.** \$4031.75. **3.** \$8005.16. **4.** \$9773.37. **5.** \$11,876.05. **6.** \$2853.54.

Exercise 162.—**3.** $2\frac{1}{5}$ hr. **4.** $1\frac{13}{47}$ hr. **5.** $4\frac{8}{13}$ hr. **6.** $\frac{13}{60}$. **7.** $\frac{8}{9}$ hr. **8.** 1 hr.

Exercise 163.—**1.** 1260 mi. **2.** 5 hr., 150 mi. **3.** 10.38 A.M. **4.** 22 mi. per hr. **5.** 105.6 yd. **6.** 180 mi.

Exercise 164.—**1.** 48, 72, etc. **2.** 6 min. spaces. **3.** $10\frac{10}{11}$ min. past 2 o'clock, $16\frac{4}{11}$ min. past 3 o'clock, $27\frac{3}{11}$ min. past 5 o'clock, $38\frac{2}{11}$ min. past 7 o'clock, $49\frac{1}{11}$ min. past 9 o'clock, $54\frac{6}{11}$ min. past 10 o'clock. At no time. **5.** (*a*) $38\frac{2}{11}$ min. past 1 o'clock, (*b*) $49\frac{1}{11}$ min. past 3 o'clock, (*c*) $10\frac{10}{11}$ min. past 8 o'clock, (*d*) $16\frac{4}{11}$ min. past 9 o'clock, (*e*) $27\frac{3}{11}$ min. past 11 o'clock, (*f*) $32\frac{8}{11}$ min. past 12 o'clock. **6.** $49\frac{1}{11}$ min. past 3 o'clock, $5\frac{5}{11}$ min. past 7 o'clock. **7.** (*a*) $27\frac{3}{11}$ min. past 2 o'clock, (*b*) $32\frac{8}{11}$ min. past 3 o'clock, (*c*) $5\frac{5}{11}$ min. past 4 o'clock, $38\frac{2}{11}$ min. past 4 o'clock, (*d*) $16\frac{4}{11}$ min. past 6 o'clock, $49\frac{1}{11}$ min. past 6 o'clock, (*e*) $27\frac{3}{11}$ min. past 8 o'clock, (*f*) $32\frac{8}{11}$ min. past 9 o'clock, (*g*) $43\frac{7}{11}$ min. past 11 o'clock, (*h*) $16\frac{4}{11}$ min. past 12 o'clock. **8.** $13\frac{1}{11}$ min. past 4 o'clock, 5 o'clock, 48 min. past 4 o'clock. **9.** $38\frac{2}{11}$ min. past 5 o'clock.

MISCELLANEOUS EXAMPLES (A)

8. 8999.991. **9.** 274.999225. **10.** 8,447,537,940,492. **11.** 636,300,000. **12.** 751,700,800. **13.** 7.1407. **14.** 814,585.36. **15.** 166.375 mi. **16.** $3828.12. **17.** $3570.12. **18.** .000504. **19.** 1.12550881. **20.** 278,500,000. **21.** (1) $.475, (2) $3.508, (3) $51.877. **22.** 372.015. **23.** 28,127,000 nearly. **25.** 52,800 mi. **26.** .341. **27.** .8251. **28.** .1704. **29.** .000439625. **30.** 14.461°. **31.** 8° 39′ 6″. **32.** 80.7 ¢. **33.** 113,400. **34.** 250. **35.** 80,032,000 nearly. **36.** 77. **37.** $3.78. **38.** $1800. **39.** $384.45. **40.** $30.80. **41.** 12,290. **42.** $20.25. **43.** .5774. **44.** .037. **45.** $\frac{5}{128}$, or .0390625. **46.** 144. **47.** 2, 3, 6, 9, 18. **48.** 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. **49.** $1110 = 2 \cdot 3 \cdot 5 \cdot 37$; $777 = 3 \cdot 7 \cdot 37$; $1001 = 7 \cdot 11 \cdot 13$; L.C.M. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 37$. **50.** 26. **51.** 65,520. **52.** 25. **53.** 19; 66,880. **54.** 2, 3, 5^2, 7^2, 11. **55.** 2,784,873. **56.** $2^4 \cdot 3^3 \cdot 11 \cdot 13$. **57.** 25 A. **58.** 80 poles. **59.** $\frac{7}{9}$, $\frac{3}{4}$, $\frac{11}{15}$. **60.** $\frac{31}{168}$. **61.** $14\frac{23}{24}$ **62.** $\frac{9}{10}$. **63.** 1.09375, .109375, .128. **64.** $\frac{3}{80}$, $\frac{7}{400}$, $\frac{1}{32}$. **65.** $3\frac{3}{25}$. **66.** $1\frac{1}{20}$. **67.** .025. **68.** .027. **69.** .075. **70.** .16583. **72.** Upwards of 80,000 yr. **73.** (1) 39.534, (2) 48.321, (3) 54.197, (4) 63.25, (5) 63.615, (6) 68.234, (7) 72.932, (8) 76.459, (9) 79.68, (10) 82.951, (11) 70.13. **74.** $11,687.50. **75.** 20,000 bu. **76.** $3062.50. **77.** $42.40. **78.** $92.67. **79.** $2.174. **80.** $728.42. **81.** 36.3 nearly. **82.** 5,201,300. **83.** 52,100,000 nearly. **84.** 55,310,000. **85.** 12.44496 ft. **86.** 712.5 lb. **87.** 320. **88.** 1.292 sec. **89.** $4888\frac{8}{9}$ cu. yd. **90.** .002055 nearly. **91.** $6256. **92.** $3344. **93.** .2565. **94.** 19*s.* $2\frac{1}{4}$*d.* **95.** .15625. **96.** (*a*) 147,824, (*b*) 22,098, (*c*) 463,180, (*d*) 823,750. **97.** (*a*) 166,488, (*b*) 526,380. **98.** 640 A. **99.** 2640 revolutions. **100.** 112.5 bu. **101.** $\frac{8.25}{12}$. **102.** $75. **103.** $24,800. **104.** 10 A. **105.** $$14\frac{2}{7}$. **106.** (i) $.00\frac{3}{8}$, (ii) .00375. **107.** (*a*) $\frac{12.5}{100}$, (*b*) $\frac{20}{100}$, (*c*) $\frac{8\frac{1}{3}}{100}$, (*d*) $\frac{6.25}{100}$. **108.** 99.99. **109.** $\frac{268}{16}$. **110.** $107. **111.** 2.28. **112.** $\frac{11}{20}$. **113.** 66,720. **114.** $101.02. **115.** 25 ft. **116.** 35. **117.** 847.15625. **118.** $28\frac{1}{8}$. **119.** $7260. **120.** 390. **121.** 1092 eggs. **122.** $\frac{119}{143}$. **123.** $23\frac{6}{7}$. **124.** 17.66. **125.** 5.25 ft., 212.0045 sq. ft., 98.328 sq. ft., 59.427 sq. ft. **126.** $330. **127.** $67\frac{1}{2}$ mi. **128.** 7.15 P.M., Apr. 5. **129.** 9.55 A.M. **130.** 7 hr. 29 min. 46 sec. **131.** 63° 35′ W. **132.** 156° 15′ W. **133.** 135° 27′ E. **134.** (*a*) 70° 15′ W., (*b*) 85° W., (*c*) 81° 45′ W. **135.** 24.62%.

136. (i) 4.16%, (ii) 14.6%, (iii) 28.79%, (iv) 17.8%. **137.** 96,600. **138.** \$150. **139.** \$1.44. **140.** 20%, or 16¢ per yard. **141.** \$8. **142.** $24\frac{4}{9}$%. **143.** $\frac{1}{4}$. **144.** \$150. **145.** $38\frac{8}{9}$%. **146.** $12\frac{1}{2}$%. **147.** (i) $64\frac{7}{12}$%, (ii) $3\frac{2}{3}$ gal. **148.** (*a*) 59.99%, (*b*) 32.34%, (*c*) 62.84%, (*d*) 49.02%, (*e*) 65.57%, (*f*) 36.75%, (*g*) 65.28%, (*h*) 40.83%, (*i*) 41.94%, (*j*) 39.01%, (*k*) 42.02%. **149.** \$1.61. **150.** \$1.20. **151.** \$182.67. **152.** £7 17*s.* 8*d.* **153.** £41 16*s.* 1*d.* **154.** \$300. **155.** \$14,400. **156.** \$400. **157.** \$1200. **158.** \$1000. **159.** 7%. **160.** \$500. **161.** 62¢. **162.** \$546.69. **163.** July 23, \$597.14. **164.** \$149.15. **165.** \$891.80. **166.** \$1004.72. **167.** Mar. 11, 1903, \$824.89. **168.** \$459.50. **169.** 7500 sq. yd. **170.** 3500 sq. yd. **171.** \$62.80. **172.** $301\frac{7}{9}$ sq. yd. **173.** 1 mi. **174.** 1 mi. long, $\frac{1}{2}$ mi. wide. **175.** 4330 sq. ft. **176.** 6495 sq. ft. **177.** 355 in. **178.** 314.16 sq. in. **179.** 1256.64 sq. in. **180.** 1.9531 cu. ft. **181.** 6.07 in. **182.** \$.525. **183.** \$.882. **184.** \$1. **185.** \$.612. **186.** \$.756. **187.** \$.871. **188.** \$1.802. **189.** \$.755. **190.** \$3. **191.** 512.12 francs.

MISCELLANEOUS EXAMPLES (B)

1. $\frac{3}{160}$. **2.** 38° 15′. **3.** 216. **4.** 2.77 ft. **5.** 2.0575 yr. **6.** 3 gal. 1.02 gi. **7.** \$4176. **8.** \$20.94. **9.** 111 bu. 1 pk. $2\frac{1}{2}$ qt. **10.** $17\frac{5}{7}$. lb. **11.** $1\frac{857}{868}$. **12.** \$504. **13.** 1.2857142. **14.** 1.777+. **15.** 16 ft. **16.** 12.56 A.M. **17.** \$500. **18.** \$11.28. **19.** $3\frac{1}{3}$ da. **20.** $2\frac{119}{148}$. **21.** $45\frac{1}{2}$ bu. **22.** 82.28 yr. **23.** $23\frac{52}{103}$. **24.** 226.27+ rd. **25.** \$6.66. **26.** 149.61+ gal. **27.** 734 rd. **28.** 90 yd. **29.** \$16.67. **30.** 137 da. **31.** \$331.86. **32.** $453\frac{1}{8}$ mi. **33.** Answers will depend on date. **34.** 14. **35.** \$9.60. **36.** 10 A. **37.** 30 min. 50 sec. **38.** 6 ch. 89.6+ b. **39.** $\$116.35\frac{5}{9}$. **40.** \$37.50, \$30, \$52.50. **41.** 62.72 bu. **42.** $3403\frac{1}{8}$ T. **43.** Neither gained nor lost. **44.** $9\frac{27}{125}$. **45.** 2295. **46.** 140 A. **47.** 4844.996. **48.** \$75.15, \$83.50, \$300.60. **49.** 6 hr. 1 min. 28 sec. A.M. **50.** Answer will depend on day calculation is made. **51.** \$2800. **52.** \$604.86. **53.** \$50 loss. **54.** 25%. **55.** \$460. **56.** 10.8%. **57.** 16 yr. 8 mo., 12 yr. 6 mo., 11 yr. **58.** \$453.561. **59.** \$352.35. **60.** $4\frac{1}{6}$%. **61.** 210,526.3 lb. **62.** \$275.80. **63.** $35\frac{7}{8}$%. **64.** 41¢ on the \$100, \$4.92. **65.** \$6.67. **66.** \$56.25. **67.** \$30,563.38+. **68.** 80. **69.** $2\frac{8}{11}$%. **70.** $33\frac{1}{3}$%. **71.** \$855. **72.** \$27,200. **73.** \$32.32 loss. **74.** \$1092.56. **78.** 30%. **79.** 1 yr. 7 mo. 12 da. **80.** \$492.50. **81.** 300%. **82.** \$3200. **83.** $33\frac{1}{3}$%. **84.** 35%. **85.** \$37.525. **86.** \$108.90. **87.** 25%. **88.** \$753. **89.** \$5.69. **90.** Dec. 13, 1902. **91.** \$6644.63. **92.** \$3515.63. **93.** $3\frac{3}{4}$%. **94.** \$530. **95.** \$15,000. **96.** \$1533.75. **97.** $9\frac{3}{8}$% **98.** Yes, \$20 better. **99.** \$2042. **100.** 25%. **101.** \$1000. **102.** \$7045. **103.** \$28.01, \$42.02. **104.** $6\frac{278}{487}$.