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URBAN SPATIAL STRUCTURE: AN ANALYSIS WITH A VARYING COEFFICIENT MODEL

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1. Introduction

The density-distance relationship, or more generally the density gradient, has been used in recent years to explain urban spatial structure. The standard functional form assumed for the density gradient is the negative exponential, i.e.,

$$D(u) = D_o e^{-ru} \tag{1}$$

where D(u) is density u distance from the urban center, D_o is the density at the urban center and r, the density gradient, is the percentage by which D(u) falls as distance increases. Previous models of urban economies have focused on explaining the intensity of land use and employment by distance from the urban center with modifications incorporated to include transportation cost, income, past development and selected other socio-economic factors.

This paper proposes an alternative method for analyzing the variable nature of the process of urban growth and change. The varying coefficient model (VCM) depicts urban growth as a dynamic process, allowing for changes in factors reflecting differences in time and urban characteristics. Using the negative exponential density function as a theoretical base, the VCM provides a means for systematically incorporating hypothesized effects of current and past levels of population, income, commuting costs and other factors identified with present urban spatial structures. Thus, the VCM generalizes the simple exponential density function to accommodate more realistic hypotheses about urban structure. Since a number of the structural factors exhibit high secondary relationships with time, the VCM also represents a basis for sharpening existing forecasting tools. Also the VCM can be used with little additional

computational or data collecting effort so it is attractive for exploratory statistical analyses of urban structural and other applied economic problems.

The present study applies the VCM to estimate an urban density function conditioned on factors which vary within and among cities. In Section 2 previous theoretical and empirical results on density gradients are reviewed. Data and the theoretical basis for the hypothesized effects of the conditioning variables to be investigated are discussed in Section 3. The VCM as applied for changing density functions is developed in Section 4. The method for estimating parameters of the VCM using available cross section data is discussed in Section 5. Section 6 contains the results of an application of the VCM to the generalized urban density function problem. Simulated forecasts for selected cities and analyses of structural changes are reported in Section 7. The final section provides a brief summary and some provisional conclusions.

2. Review

Clark (1951) initially employed the negative exponential function to describe the relationship between density and distance. Subsequently, Muth (1969), using Cobb-Douglas supply and demand functions for housing, derived necessary and sufficient conditions for the existence of an exponential function relating density and distance. Since Muth's work a number of theoretical results providing additional justification for the exponential density function have been obtained.[1]

More sophisticated empirical studies have also followed Muth's application of his own model to the analysis of urban density. Kau and Lee (1975d) have derived a stochastic density gradient employing a random coefficient regression model. An index of uncertainty for the density gradient was constructed to determine whether distance is a sufficient variable for measuring the variation of the population density patterns in cities. The deterministic density gradient

developed by Clark and Muth (1969) is, of course, a special case of this formulation. Interestingly for the present study, an index of uncertainty for the density gradient showed that for a number of the cities distance is not sufficient for explaining observed variations in population density patterns. Kau and Lee (1976c) have also applied the Box and Cox (1964) technique to examine the hypothesized functional form for the density gradient. Data for 50 cities indicated that the exponential function is not an appropriate specification in one-half of the cases. The variation in the functional form among cities and the results for the uncertainty index both suggest further investigation of the relationship between the characteristics of a city and the density gradient.

Adding to the uncertainty regarding the simple density gradient, Muth (1975), using a constant-elasticity of substitution (CES) production function and alternative values of the elasticity of substitution between land and structures, demonstrated the inappropriateness of an exponential function derived from the Cobb-Douglas production function in predicting the actual distribution of population densities. More generally, density equations derived from CES production functions, while theoretically more sound, are difficult to estimate because of limited data [See Fallis (1975), Kau and Lee (1976a), Koenker (1972), and Muth (1975)].

Relatedly, Muth (1969) found significant variations from linearity but was unable to draw meaningful conclusions about the role of an included quadratic distance term in a polynomial model explaining urban structure. McDonald and Bowman (1976) studied alternative functional forms and found that the explanatory power of the negative exponential function was improved in some cases by adding a quadratic term. Latham and Yeates (1970) developed the use of a negative quadratic exponential and Mills (1970) has compared linear and log forms of a

distance-density relationship. Kemper and Schmenner (1974) have concluded that the exponential functional form is not completely satisfactory in describing the variation of manufacturing densities with distance. Finally, Fales and Moses (1973) relate density to a variety of variables other than distance. Their results suggest that these other locational characteristics reduce the explanatory power of the distance variable, but represent a means of specializing the results to particular urban structure problems.

In summary the empirical and theoretical work reviewed suggests that distance alone cannot offer an adequate explanation of either population or manufacturing employment densities. The techniques developed in this paper, although conventional in adhering to the theoretically justifiable form for the density function, provide a basis for maintaining the role of distance while allowing for effects of altered economic and institutional factors.

3. Theory and Data

The theoretical foundation for the density gradient provided by Muth (1969) can be used to determine qualitative effects of alternative variables on the intercept and slope of the resulting exponential function. Briefly, housing is produced by using land which surrounds the Central Business District (CBD). Workers residing in these households are assumed to commute to and from jobs in the CBD.[2] The optimum household location for a cost-minimizing worker employed in a CBD occurs when

$$-\partial p/\partial u(q) = \partial T/\partial u, \qquad (2)$$

where p and q are the price and quantity of housing services, respectively; and T represents transport cost. Thus, $-\partial p/\partial u(q)$ is the reduction in expenditure necessary to purchase a given quantity of housing (q) that results from moving

a unit distance (u) away from the CBD. The derivative $\partial T/\partial u$ represents the increase in transport costs (T) incurred by making such a move. It is further assumed that the demand for housing services is given by the expression

$$q = \gamma [I - T(u)]^{\Theta_1} p^{\Theta_2} \tag{3}$$

where I is household money income, and γ, Θ_1 and Θ_2 are parameters. Clearly, Θ_1 is the income elasticity of housing demand and Θ_2 is the money income-constant price elasticity. Using Equation (3) and related formulations of the demand for housing, Muth was able to derive qualitative effects for a number of variables on optimum location. Since the model is well known, this discussion only reviews the qualitative results as specialized for the variables selected for empirical analysis in this study.

Data employed consist of a random sample of 43 census tract densities measured u distance from the CBD for each of 39 United States cities in 1970.[3] Two corresponding sets of additional data were also used. The first of these consists of observations for each of the 43 tracts in the various cities, referred to as tract-specific variables. The tract-specific variables are the percent of commuters using public transportation (X_1) and income (X_2). Percent of public transportation commuters is used to reflect the impact, introduction and continued use of subways or bus systems on urban structures. Relative costs of private versus public transportation are, of course, difficult to determine. Instead of making non-testable statements about relative costs, this study uses observed behavior to establish the importance of the transportation variable. Muth's model shows that an increase in either the fixed or the marginal costs of transport decreases the equilibrium distance from the CBD for any household.

The relation of the optimal household location and income is important because it determines housing consumption patterns in different parts of the

city. For example, consider a general increase in the level of income for the residents of a city. The increase in income would increase housing consumption [(q) in Equation (3)] and, assuming this outweighed effects of increased transport cost and housing prices, the equilibrium distance from the CBD would increase for all households. On the basis of this reasoning, the density gradient is expected to vary inversely with the income level.

The second set of concomitant data is city-wide and designed to explain differences among cities due to variations in past development. Harrison and Kain (1974) have demonstrated the importance of past development on current land use. In fact, they have suggested that the principle differences in urban structures among United States cities are due to differences in the timing of their development. For example, in the Los Angeles metropolitan area dwelling units constructed between 1950 and 1960 accounted for almost 40 percent of the total in 1960, whereas in Boston it was only 16 percent [Harrison and Kain (1974, p. 65)]. Two variables used to capture these effects in the present study are relative age (X_3) of the city and population (X_4). Age, based on the last significant growth spurt, pinpoints the timing of the significant structural changes which occurred in the city.[4] Population levels are used to represent overall scale effects due to past development. Generally, and again based on the Muth results, recent growth spurts and population increases would tend to reduce the density gradient because of technological changes affecting transportation, e.g., freeways and the automobile.

4. The Model

The review of previous work and discussion of the theory and data shows that the density function hypothesis for explaining urban structure has broad empirical support. At the same time it raises a number of questions. These

questions concern the appropriateness of the exponential functional form and relatedly, the possibility that additional specializing arguments may be required to obtain consistency among estimates and improved predictive performance. The present model provides a basis for examining both of these questions using a conventionally specified density function.

Consider the density function represented by the solid line in Figure 1. For convenience, the natural log of the density function has been used, i.e.,

$$\ln D(u) = \ln D_o - ru. \tag{4}$$

Data typical of those used to estimate the parameters of such functions are also plotted in Figure 1. These data points have been selected to suggest some ambiguity in the appropriateness of the log linear functional form; a systematic pattern of errors indicates the possibility of misspecification. Different functional forms and omitted variables are alternatives explanations for this result.

An equally plausible, but slightly altered, interpretation is that the sampled units (cities and/or tracts) each had a different density function. The plotted sample data would then represent points from a collection of density functions. Some density functions conforming to this interpretation are illustrated by the broken lines in Figure 2. The interpretation is consistent with both the partial success in empirically supporting the exponential functional form hypotheses and the inclusion of additional explanatory variables. The latter would, of course, be based on the more complex population density theory discussed in Section 3.

The approach employed in specifying a model consistent with the theory and data presented in Section 3 is to use the exponential density function but introduce systematic parameter changes. That is, the parameters of the density

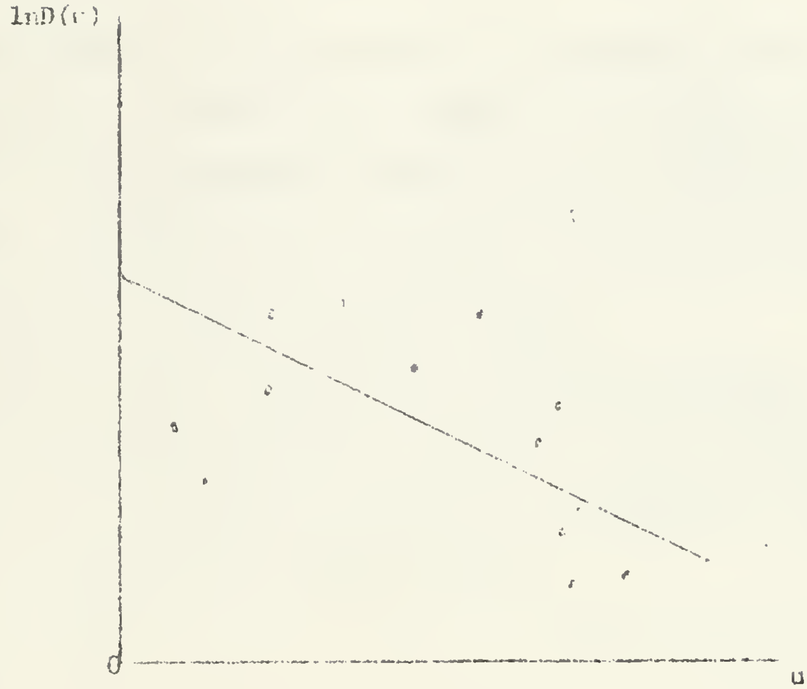


Figure 1. Illustrative Fitted Density Gradient

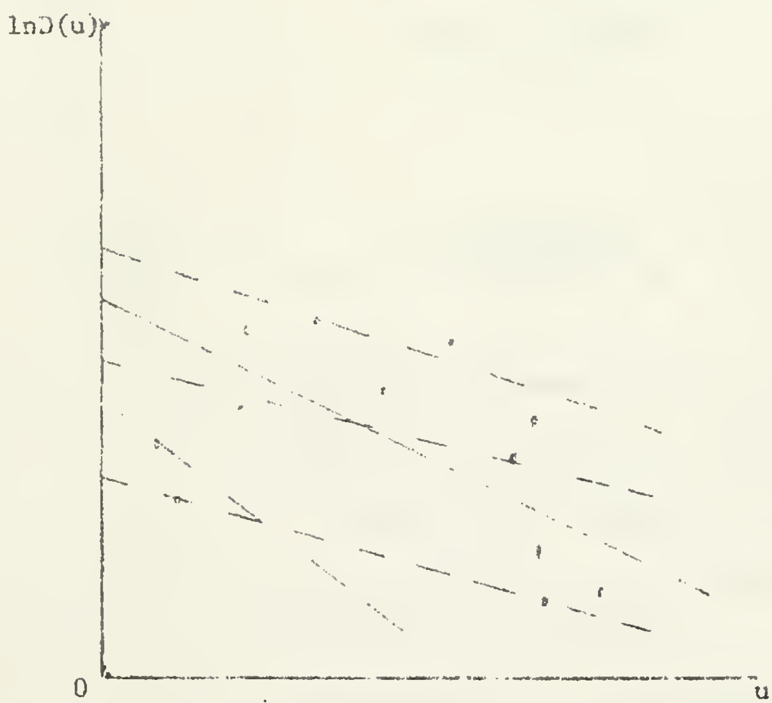


Figure 2. Alternative Functional Forms for Various Data Points

function are hypothesized to vary as a result of the interplay of city and tract-specific variables. As indicated in Section 3, the a priori basis for relating parameters of the exponential density function to city and tract-specific variables is somewhat limited. Generally, the theory only yields conclusions for signs of anticipated parameter changes.

Owing to the limited prior information, a VCM with a polynomial as the structure for possible parameter changes is posited.[5] Since the specification locally approximates more complex relationships it is appealing for exploratory work. To implement the polynomial specification let

$$\ln D_o = \ln D_o(X_1, X_2, X_3, X_4)$$

$$= \sum_{n_1=o}^{q_{o_1}} \sum_{n_2=o}^{q_{o_2}} \sum_{n_3=o}^{q_{o_3}} \sum_{n_4=o}^{q_{o_4}} \beta^o_{n_1,n_2,n_3,n_4} X_1^{n_1} X_2^{n_2} X_3^{n_3} X_4^{n_4}. \quad (5)$$

And similarly for the slope coefficient, r, in model (1), let

$$r = r(X_1, X_2, X_3, X_4)$$

$$= \sum_{n_1=o}^{q_{1_1}} \sum_{n_2=o}^{q_{1_2}} \sum_{n_3=o}^{q_{1_3}} \sum_{n_4=o}^{q_{1_4}} \beta^1_{n_1,n_2,n_3,n_4} X_1^{n_1} X_2^{n_2} X_3^{n_3} X_4^{n_4}. \quad (6)$$

The parameters $\ln D_o$ and r are thus polynomials of orders q_o and q_1, respectively, in the four city and tract-specific variables, X_1, X_2, X_3, X_4. Application of this revised specification to the data represented in Figures 1 and 2 is straightforward. The parameters $\beta^o_{n_1,n_2,n_3,n_4}$ and $\beta^1_{n_1,n_2,n_3,n_4}$ along with values for city and tract-specific variables corresponding to the data points, determine exponential density functions of the type represented by the dotted lines in Figure 2. The special case $n_1=n_2=n_3=n_4=o$ is illustrated by the solid line in Figures 1 and 2, i.e., the constant coefficient, log linear density function.

Advantages of the VCM provided by Equations (5) and (6) combined with the log linear density function hypothesis, should be apparent. The VCM generates city and tract-specific results but within context of a functional form which has theoretical and empirical support. Moreover, the flexibility of the VCM would appear to make the exponential density function more useful for policy analysis and prediction. Since the selected city and tract-specific characteristics may be subject to control by policy action and/or themselves comparatively easily projected on the basis of time, the model can be used for both forecasting and policy analysis, even though estimated from cross section data. While not without statistical limitations, the latter feature should prove especially useful given the data bases available for studying density patterns in urban economies.

5. Estimation Methods

The estimation procedure follows from the error assumptions and additional information restricting the numbers of parameters for the model as expressed in Equations (4), (5), and (6). To begin, the polynomials relating $\ln D_o$ and r to the conditioning variables X_1, X_2, X_3, X_4 are assumed of second order. Even with this assumption, application of the standard formula for permutations shows there are 1320 parameters for each of the hypothesized conditioning structures on the two coefficients, $\ln D_o$ and r. The data, though extensive by comparison to some other studies, obviously cannot support this ambitious specification. Accordingly, the number of parameters required to determine the variable coefficients of the log linear density model was further limited.

The approach used to obtain these restrictions is based on intended model uses and preliminary tests in the sample data. Although there are some obvious

statistical problems with the latter method [Wallace and Ashar (1972)], the situation offered no alternative. First, four versions of density function model were estimated, each with the coefficients a function of only one conditioning variable. For example, in the case of the tract-specific variable, percent of commuters using public transportation (X_1), the assumption was $q_{o_2} = q_{o_3} = q_{o_4} = o$ and $q_{1_1} = q_{1_2} = q_{1_3} = o$, implying structures for the VCM determined on the basis of six parameter estimates. Letting i denote the city and j the tract for this specialized case, the model given in Equations (4), (5), and (6) can be expressed as

$$\ln D(u)_{ij} = \ln D_{o_{ij}} - r_{ij}u + \varepsilon_{ij} \tag{7}$$

for the 43 x 39 observations in the sample. An additive error term ε_{ij} with a subsequently specified structure has been included as well. Applying the specialized assumptions to Equations (5) and (6) yeilds

$$\ln D_o(X_1,X_2,X_3,X_4) = \ln D_o(X_1) = \ln D_{o_{ij}} = \sum_{n_1=o}^{q_{o_1}} \beta^o_{n_1} X_{ij}^{n_1} \tag{8}$$

and

$$r(X_1,X_2,X_3,X_4) = r(X_1) = \ln r_{ij} = \sum_{n_1=o}^{q_{1_1}} \beta^1_{n_1} X_{ij}^{n_1} \tag{9}$$

where the subscripts for β^o and β^1 corresponding to the excluded conditioning variables have been omitted for convenience.

The model specified in Equations (7), (8), and (9) includes coefficient restrictions across tracts and cities. It is clear, therefore, that pooling of the tract and city data is necessary to estimate the required parameters.[6] In addition, plausible assumptions for the distribution of the structural disturbance, ε_{ij}, point to advantages of pooling [Balstra and Nerlove (1966), Wallace and Hassan (1969), and Zellner (1962)]. Although the estimation problem is not

of the classic time series cross section type, it seems reasonable to specify an error structure allowing for different variances between the cities and across city effects. In particular, the error term is assumed normally distributed with mean zero and covariance structure with additive components for identically numbered cities and tracts. Thus, the across city relationship for the errors assumes between tract independence except for those identically numbered. The latter is motivated by the selection procedure for tracts. In as much as possible, tracts were chosen to correspond between cities, relative distance being the major characteristic used in the ordering.

With the assumed error structure and the across tract and city coefficient restrictions, the application of generalized least squares results in estimators which are asymptotically more efficient than those obtained by applying ordinary least squares [Oberhofer and Kamenta (1973)]. For discussing the generalized least squares estimation procedure and tests of homogeneity, a matrix representation is useful. For this representation, let y_i denote the vector of 43 tract observations on the ith city. Similarly define Z_i and ε_i , Z_i being the matrix of observations on newly defined variables obtained by combining Equations (9) and (8) with Equation (7) and ε_i an error vector corresponding to y_i. For the set of observations across cities the vectors y_i are stacked, i.e., $y = (y_1, y_2, \ldots, y_{39})'$. Again the same notational convention carries over to the Z_i's and ε_i's. Specifically, $Z = (Z_1, Z_2, \ldots, Z_{39})'$ and $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{39})'$. Finally, defining $\beta = (\beta_o^o, \beta_1^o, \beta_2^o, \beta_o^1, \beta_1^1, \beta_2^1)'$, the set of 39 x 43 observations on tracts and cities with coefficients varying on the basis of public to private transport, the model can be written as

$$y = Z\beta + \varepsilon. \qquad (10)$$

Estimators of the parameter vector, β, and its sampling variance are straightforwardly obtained, e.g.,

$$b = [Z' (\hat{\Sigma}^{-1} \otimes I)Z]^{-1} Z' (\hat{\Sigma}^{-1} \otimes I)y \qquad (11)$$

and

$$VAR(b) = [Z' (\hat{\Sigma}^{-1} \otimes I)Z]^{-1} \qquad (12)$$

where the Σ is the 39 x 39 covariance matrix, the estimate $\hat{\Sigma}$ is formed using OLS residuals, $\otimes$ is the Krunecker product and I is a 43 x 43 identity matrix.

The four models provided by considering the variables conditioning the coefficients one at a time present the basis for the preliminary tests on which the final VCM was formulated. Comparing Equation (4) and the model given by Equations (7), (8), and (9), it is apparent that abstracting from the error assumptions, they differ by only a set of 4 exclusion restrictions on the structure. These restrictions can be written

$$R\beta = 0 \qquad (13)$$

where R is a 4 x 6 matrix with rows containing only one non-zero element. The restrictions are $\beta_1^o = \beta_2^o = \beta_1^1 = \beta_2^2 = 0$. Two tests of this restriction are made. Both involve a structural norm. The first uses a simple F statistic and evaluates the restrictions on the basis of the improvement in variances of the coefficient estimators [Fisher (1970)]. The second weighs bias and variance -- a reasonable norm given the exploratory nature of the hypothesized varying coefficient structure. This second test involves a weak mean square error norm [Wallace (1972)]. As shown by these authors, a sufficient condition and the lowest bound that will always hold for the restricted estimator to be superior to the unrestricted estimator is that

$$\gamma = \frac{1}{2d_L} \text{ tr. } [S^{-1}R'(RS^{-1}R')^{-1}RS^{-1}] \qquad (14)$$

where $S = [Z'(\hat{\Sigma}^{-1} \otimes I)Z]$ and d_L is the largest given value of the expression under the trace operator (tr.). This inequality can be tested straightforwardly as under the null hypothesis, the statistic

$$\mu = \frac{RSS(b_R) - RSS(b)}{6 - 2} \div \frac{RSS(b)}{43x39-6} \qquad (15)$$

is distributed as a non-central F with (6-2) and (43x39-6) degrees of freedom and non-centrality parameter γ [Wallace (1972)]. μ is just the test statistic for the first norm [Fisher (1970)] with $RSS(b_R)$ and $RSS(b)$ defined as the residual sums of squares under the restricted and unrestricted hypotheses, respectively.

Based on the results from the four simplified VCM's and prior information to be subsequently discussed, a model incorporating effects of all of the coefficient conditioning variables was specified. In terms of Equations (5) and (6) the structure for the density function coefficient variation for this final model is

$$\ln D_o = \beta^o_{oooo} + \beta^o_{1ooo}X_1 + \beta^o_{o1oo}X_2 + \beta^o_{oo1o}X_3 + \beta^o_{ooo1}X_4, \qquad (16)$$

and

$$r = \beta^1_{oooo} + \beta^1_{1ooo}X_1 + \beta^1_{2ooo}X_1^2 + \beta^1_{o1oo}X_2 + \beta^1_{o2oo}X_2^2 + \beta^1_{oo1o}X_3$$
$$+ \beta^1_{oo2o}X_3^2 + \beta^1_{ooo1}X_4 + \beta^1_{ooo2}X_4^2. \qquad (17)$$

As should be apparent final specification concentrates on variation in the density gradient, r. By argument analogous to that made for Equation (7) this variable coefficient structure can be substituted to reparameterize the exponential density function model and generalized least squares methods applied to obtain estimates with desirable asymptotic properties. As well, based on the procedures just described the central and non-central F statistics can be used

to test the null--constant coefficient density function model--hypothesis for appropriateness given the sample data.

6. Empirical Results

Results from an application of the constant coefficient density function model on a city-by-city basis are contained in Table 1. These estimates provide a source of comparison for those from the alternative VCM's subsequently presented. The results in Table 1 demonstrate the aforementioned concern, for the appropriateness of the constant coefficient exponential density hypothesis. Both estimated parameters ($\ln D_o$ and r) are, for most of the 39 cities, statistically significant. There are, however, important differences in their magnitudes, especially for the density gradient r. Also, the estimated density function for the pooled data did not explain a high proportion of the observed variation in the dependent variable. In all cases the explained variation for the city-by-city density function estimates is higher than for the model using pooled data. Although pointing up the limitations of empirical generalizations based on the constant coefficient density function hypothesis, the results are typical of others obtained using data from U.S. cities [See Mills (1970) and Muth (1969)].

Formal statistical tests of the similarity of the density function coefficients presented in Table 1 are equally discouraging regarding the generality of the constant coefficient model. Applications of the F statistic and the test based on the first weak mean square error norm underscore these observed differences. The null hypothesis that the constant coefficient density function, given in Equation (4), is appropriate for all cities is rejected at the 1% level using both norms. Obviously, more elaborate hypotheses are required for explaining population density within and across cities.

TABLE 1

Ordinary Least Squares Estimates of Coefficients for the Exponential Density Function for 39 Cities and for the Pooled City Data

City	Density Function Coefficient Estimates		R^2	City	Density Function Coefficient Estimates		R^2
	$\ln D_o$	r			$\ln D_o$	r	
Akron	9.273	-0.202 (-2.86)	.167	Pittsburgh	9.689	-0.121 (-2.14)	.100
Baltimore	9.767	-0.186 (-12.37)	.783	Portland	9.193	-0.139 (-4.75)	.355
Birmingham	9.017	-0.190 (-6.38)	.498	Providence	9.090	-0.135 (-4.54)	.335
Chicago	9.745	-0.039 (-1.60)	.059	Richmond	8.716	-0.221 (-6.71)	.523
Cincinnati	9.669	-0.162 (-4.78)	.358	Rochester	9.845	-0.327 (-10.32)	.722
Dayton	9.245	-0.179 (-4.62)	.342	Salt Lake City	8.883	-0.128 (-4.17)	.298
Denver	9.624	-0.206 (-5.37)	.413	San Antonio	9.300	-0.212 (-6.44)	.503
Detroit	9.714	-0.075 (-3.86)	.281	San Diego	9.141	-0.065 (-2.79)	.159
Flint	9.482	-0.386 (-6.82)	.532	San Jose	8.990	-0.085 (-2.12)	.099
Fort Worth	8.399	-0.059 (-2.38)	.121	Seattle	9.220	-0.140 (-6.02)	.469
Houston	9.209	-0.153 (-5.17)	.395	St. Louis	10.029	-0.170 (-7.48)	.577
Jacksonville	9.205	-0.343 (-10.34)	.723	Spokane	8.762	-0.256 (-5.24)	.404
Louisville	8.619	-0.139 (-6.12)	.478	Syracuse	9.938	-0.487 (-15.62)	.856
Memphis	9.463	-0.173 (-5.79)	.450	Tacoma	9.078	-0.177 (-4.20)	.284
Milwaukee	10.013	-0.207 (-6.53)	.509	Toledo	9.835	-0.317 (-7.12)	.553
Nashville	9.078	-0.269 (-8.42)	.634	Tucson	8.459	-0.146 (-2.88)	.169
New Haven	9.791	-0.510 (-10.75)	.738	Utica	9.421	-0.374 (-5.78)	.449
Omaha	8.845	-0.114 (-2.41)	.124	Washington, DC	9.980	-0.138 (-3.96)	.277
Philadelphia	10.612	-0.195 (-6.05)	.471	Wichita	9.000	-0.227 (-4.63)	.343
Phoenix	9.089	-0.134 (-4.54)	.335				
Pooled Data	6.41	-.010	.010				

Estimates for the pooled data with the parameters varying according to the scheme given in Equations (8) and (9) are presented in Table 2. Recall that the conditioning variables are public to private transportation (X_1), income (X_2), age (X_3), and population (X_4). The specification is that the coefficients for the density gradient are quadratic functions of these conditioning variables. Examination of the significance levels of the parameters on the linear and quadratic terms for the specifications shown in Table 2 indicates that each of the conditioning variables is important in shifting the density from city to city and between tracts. This general observation is confirmed by comparing the R^2's in Table 2 with that for the constant coefficient model applied to the pooled data and presented in Table 1. Higher R^2's for the VCM's based on each of the four separate conditioning arguments are confirmed as statistically significant by an application of the central and non-central F tests. Both indicate a rejection of the restricted hypothesis at the 1% level.

On a more specific basis, results obtained using the public/private transportation to condition the density function coefficients show that its major effect is on the distance coefficient, r. For the constant term the estimated parameter on the linear term is not statistically significant and the parameter estimate for the quadratic is only marginally so. Estimates on the constant, linear, and quadratic terms for the distance coefficient are -.0867, .456 and -.0517, respectively, and all are statistically significant. The estimates show that the public/private transport variable first increases and then with increase usage decreases density.

More precise interpretations of this and the other results presented in Table 2 require inspection of the sample data. For this purpose, means and standard deviations of the conditioning variables as well as some other variables

TABLE 2

Exponential Density Function Estimated with Coefficients Jointly Conditioned on Selected Income, Public to Private Transportation, Age and Population Variables

Conditioning Variable	Constant Coefficient ($\ln D_o$)[a,b] Constant	Linear (X_k)	Quadratic (X_k^2)	Distance Coefficient (r)[a] Constant	Linear (X_k)	Quadratic (X_k^2)	Coefficient of Determination (R^2)
Public/private transportation (X_1)	8.545 (86.08)[c]	-.179 (.87)	.222 (1.91)	-.0867 (13.92)	.456 (12.76)	-.0517 (2.14)	.32
Income (X_2)	8.895 (87.68)	5.91E-5 (2.72)	-4.714E-9 (4.36)	6.59E-3 (2.81)	-2.17E-5 (12.54)	8.75E-10 (7.33)	.26
Age (X_3)	8.689 (85.88)	.015 (4.81)	-9.96E-5 (5.86)	-.120 (1.42)	-1.19E-3 (2.50)	1.23E-5 (5.03)	.21
Population (X_4)	9.367 (119.59)	-1.33E-6 (8.55)	5.10E-13 (8.51)	-2.90 (22.18)	4.20E-7 (20.92)	-1.14E-13 (16.05)	.28

[a] Constant, linear and quadratic parameters for the indicated conditioning variables on $\ln D_o$ and r--the traditional density function coefficients.

[b] The index k takes on values 1, 2, 3, and 4, indicating public/private transportation, income, age, and population, respectively.

[c] Values in parentheses are estimated students t statistics.

required in the subsequent discussion are presented in Table 3. Using this information, it is apparent that the value for the distance coefficient estimate at the sample mean for the public/private transportation variable is

$$\hat{r} = -.0867 + .456(.2144) - .0517(.2144)^2$$
$$= .01344.$$

What this result shows is that for cities and/or tracts with a low value for the public/private transportation variable the density gradient is lower than in cities for which it has a high value. Thus, other things equal, cities with below average levels for public to private transport and contemplating policy measures designed to increase it should expect a decrease in the absolute value of the density.

The mean for income in the sampled cities and tracts is $9,735. From Table 2 observe that when the density function coefficients are conditioned on income, all are significant. Evaluated at the sample mean the constant term is 9.470 and the distance coefficient is -0.2046. For the constant term, the results show that higher income cities tend to have higher densities at the center. The positive sign on the quadratic term for the distance coefficient indicates that at higher income levels cities and tracts away from the center tend to become less dense.

Age and population are city-specific conditioning variables. Results for the density functions conditioned on age are of interest in that the significant parameter estimates on the quadratic terms show that older cities are less dense at the center and have flatter density gradients. For population, signs on the quadratic terms indicate that larger cities are less dense at the center but have steeper density gradients. These results are somewhat at variance with commonly held views, and possibly due to the highly simplified conditioning

TABLE 3

Mean Values and Standard Deviation for Variables

Used in the Analysis of the Pooled City Data

Variable	Mean	Standard Deviation
Dist	5.625	10.144
Pub/PR	.2144	.1899
Age	55.897	42.228
Pop	567492.	587355.
Inc	9735.6	4026.31

of the coefficients. This observation is supported by the results for the more complex function.

Parameter estimates for the density function specified with coefficients conditioned as hypothesized in Equations (16) and (17) are presented in Table 4. The table is constructed similar to Table 2 except that estimates in the constant columns are repeated for reference. The table shows all parameters statistically significant and the R^2 for the pooled data improved to .49. In general, the parameter estimates are interpreted as were those presented in Table 2.

For the constant coefficient ($\ln D_o$) the estimated parameters on the linear terms show that densities in the CBD increase with increased public and private transportation, income, and age and decrease with population. The significant parameter estimates on the linear and quadratic terms on the distance coefficient show that r increases at higher public/private transport use and income levels and decreases with city age and population. The former two effects would indicate a flatter density gradient in cities with higher average income and greater public transportation usage.

Perhaps the best way to assess the implications of this final version of the VCM is to evaluate the function for each of the cities included in the

within-city sample means. The results are shown in Table 5. Means for income, public/private transportation for each of the cities along with mean, maximum, minimum and variance for distance, μ, are given in Appendix Table 1. With such information, specialized analyses for particular cities can be made using the estimates from Table 4. More generally, on comparing Tables 5 and 1, it is apparent that the VCM produces estimates for the density function which are reasonable. The advantage of the VCM is thus the improved fit, increased realiability of parameter estimates and, most importantly, increases the possibility for

TABLE 4

Exponential Density Function Estimated with Coefficients Jointly Conditioned on Selected Income, Public to Private Transportation, Age and Population Variables

Conditioning Variable	Constant Coefficient ($\ln D_o$)[a,b]			Distance Coefficient (r)[a]			Coefficient of Determination (R^2)
	Constant	Linear (X_k)	Quadratic (X_k^2)	Constant	Linear (X_k)	Quadratic (X_k^2)	
Public/private transportation (X_1)	8.8746 (91.88)[c]	2.3102E-1 (1.71)	--	-2.0146E-1 (-11.53)	2.4260E-1 (6.42)	-3.7874E-3 (-11.21)	0.49
Income (X_2)	8.8746 (91.81)	1.2194E-5 (1.42)	--	-2.0146E-1 (-11.53)	-9.6841E-6 (-7.78)	1.2931E-10 (3.30)	0.49
Age (X_3)	8.8746 (91.88)	5.8489E-4 (0.80)	--	-2.0146E-1 (-11.53)	9.3994E-4 (3.56)	-6.3914E-6 (-4.67)	0.49
Population (X_4)	8.8746 (91.88)	-3.2657E-8 (-0.41)	--	-2.0146E-1 (-11.53)	2.5331E-7 (15.20)	-6.6665E-14 (-14.54)	0.49

[a]Constant, linear and quadratic parameters for the indicated conditioning variables on $\ln D_o$ and r--the traditional density function coefficients.

[b]The index k takes on values 1, 2, 3, and 4, indicating public/private transportation, income, age, and population, respectively.

[c]Values in parentheses are estimated students t statistics.

TABLE 5

Estimates of the Density Function Coefficients

Based on the VCM

City	Density Function Coefficient Estimates $\ln D_o$	Density Function Coefficient Estimates r	City	Density Function Coefficient Estimates $\ln D_o$	Density Function Coefficient Estimates r
Akron	9.0223	-0.1754	Providence	9.0587	-0.19157
Baltimore	9.1219	-0.0865	Richmond	9.0846	-0.12926
Birmingham	9.0055	-0.14101	Rochester	9.0772	-0.15185
Chicago	9.1495	0.0785	Salt Lake City	9.0096	-0.2151
Cincinnati	9.0504	-0.08339	San Antonio	9.0016	-0.10121
Dayton	9.0676	-0.16836	San Diego	8.9008	-0.11302
Denver	9.0399	-0.11955	San Jose	9.0112	-0.19152
Detroit	9.0373	-0.0187	Seattle	9.0423	-0.12774
Flint	9.4459	-0.29751	St. Louis	9.0911	0.08084
Fort Worth	9.0097	-0.1650	Spokane	8.9996	-0.2067
Houston	8.9848	-0.04485	Syracuse	9.0781	-0.16427
Jacksonville	9.0128	-0.10718	Tacoma	9.0094	-0.20905
Louisville	9.081	-0.125008	Toledo	9.0388	-0.15614
Memphis	9.0308	-0.04447	Tuscon	8.9777	-0.2071
Milwaukee	9.0907	-0.05699	Utica	9.0841	-0.2345
Nashville	9.024	-0.1098	Washington, DC	9.1478	-0.000955
New Haven	9.1092	-0.19926	Wichita	9.000	-0.20338
Omaha	9.0436	-0.13125			
Philadelphia	9.1784	0.07524			
Phoenix	8.9984	-0.15133			
Pittsburgh	9.1589	0.03081			
Portland	9.0365	-0.13139			

functional analysis of population density based on the commonly advanced socio-economic conditioning arguments.

7. Specialization of Empirical Results

The results presented in Section 6 have been argued as important for policy and prediction purposes. In this section, two examples are provided to demonstrate how the empirical results can be used in policy and forecasting contexts. One example involves a representative city, obtained by setting the density function coefficient conditioning variables at mean sample values. The second example used in specializing the empirical results is Washington, D.C.

The analysis of impacts of changes in public transportation, income, age and population is made on a partial basis. That is, the value for one of the conditioning variables is changed while others are held at current levels for the two example cities. Initially, three levels are considered for each of the variables assumed to condition the density function coefficients; the current level and 50 and 100 percent increases in it. Results obtained using these assumptions are presented in Table 6. These results show for example, that in the typical city setting public/private transport at the current level increases the constant coefficient, $\ln D_0$, by .0494 and the gradient, r, by .0517. By contrast, increasing the public/private transport variable by 100 percent raises the value of the constant by .0989 and the gradient by .10314. Similar interpretations of the results apply for the second example city, Washington, D.C., and for the other conditioning variables.

What the results in Table 6 show is that the major impact of the conditioning variables is on the density gradient. This is not surprising since the specification of the structure for the varying coefficients featured possible changes in the gradient. What is encouraging is that the results are reasonable for

TABLE 6

Impact of the Explanatory Variables on Central Densities ($\log D_o$)
and the Density Gradient (r) for the Typical City
and Washington, D.C.; Current Values,
50 Percent and 100 Percent Increases
in Levels of the Conditioning Variables

	Conditioning Variables			
	Public Transportation (X_1)	Income (X_2)	Age (X_3)	Population (X_4)
Typical City				
Current				
D_o	.0494	.1187	.3269	.0185
r	.0517	-.0823	.0326	.1223
50% Increase				
D_o	.0742	.1781	.4907	.0278
r	.0775	-.1138	.0343	.1673
100% Increase				
D_o	.0989	.2374	.6539	.0371
r	.10314	-.1395	.0253	.2016
Washington, D.C.				
Current				
D_o	.1003	.1965	.4094	-.0247
r	.1046	-.1030	.0345	.1535
50% Increase				
D_o	.1504	.2347	.6141	-.0371
r	.1563	-.1385	.0283	.2016
100% Increase				
D_o	.2005	.3130	.8189	-.0494
r	.2077	-.1634	.0063	.2306

the changes considered even though some are for values of the conditioning variables far from the sample means. This indicates that the surface being approximated by the polynomial is sufficiently stable so that projections or forecasts based on assumed values of the conditioning variablea can be viewed with some confidence.

To further illustrate the results for the VCM, impacts of changes in the explanatory variables on the gradient, r, are plotted in Figures 3-6, along with representative structural shifts in the density function. The interpretation for the shifted density functions is that they are cross section and thus refer to equilibrium levels. Thus, shifts resulting from changes in the conditioning variables represent density relationships to which the cities would gravitate as a result of policy changes or other possible exogenous effects. Finally, the similarity in the shifting density gradients presented in Figures 3B-6B and Figure 2 shows that the VCM can be consistent with cities and tracts with differing characteristics. In doing so the VCM explains much of what on a simpler hypothesia would be attributed to spurious variation.

Mills (1971), Mohring (1961), Muth (1969), Pendleton (1963), and others have found empirical evidence that improvements in transportation tend to reduce the density gradient. The evidence provided by the VCM indicates that as the percentage of public transit users increase the density gradient (r) decreases; in fact as shown in Figure 3A, r became positive when the number of public transit riders exceeds 30 percent. This occurs in four cities: Chicago, Philadelphia, Pittsburgh and Washington, D.C. Referring to Table 5 the estimates of the gradient, r, based on city specific values for the conditioning variables show that in all cases it was positive except for Washington, D.C., which was essentially zero. Thus, the city specific results based on the VCM, (and as well the ordinary least squares estimates shown in Table 1)

FIGURE 3A

The Impact of Public Transportation on the Density Gradient (r)

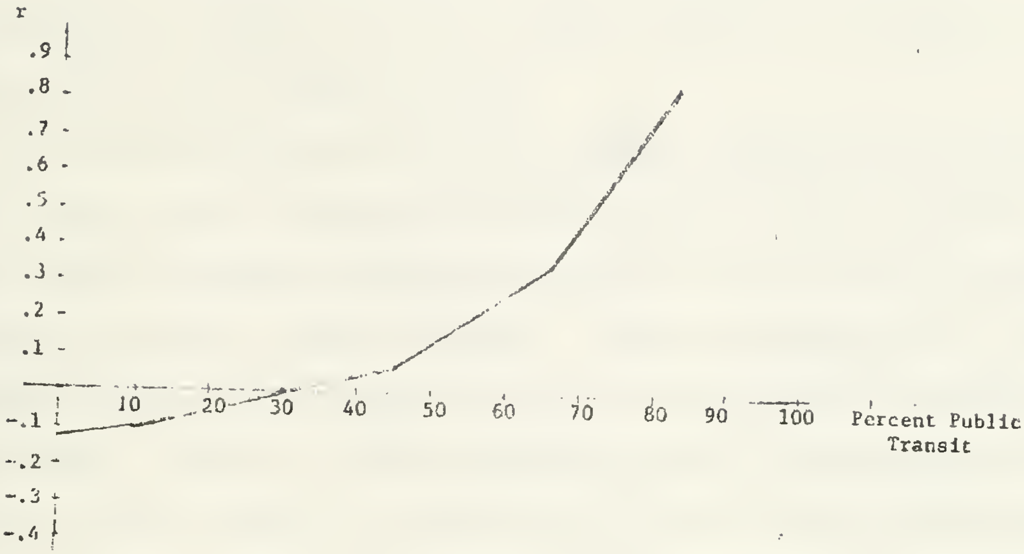


FIGURE 3B

The Impact of Public Transportation on the Density Function.
Examples for 50%, 10% and 0% Public Transit Passengers

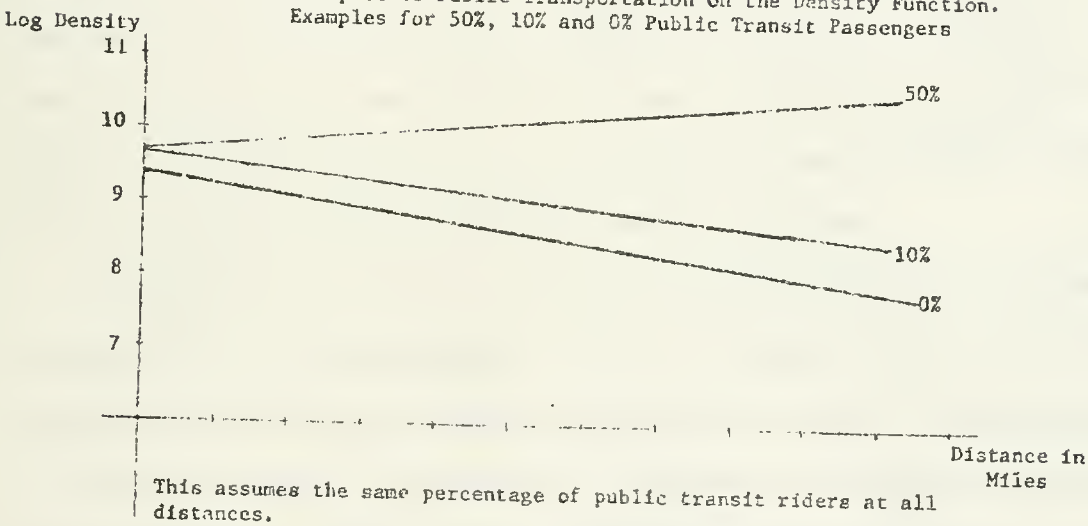
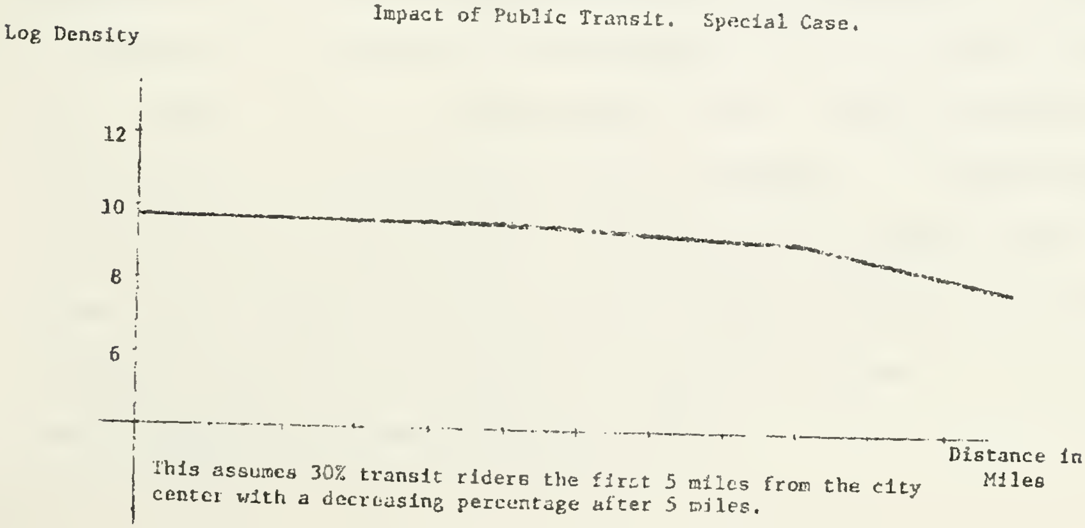


FIGURE 3C

Impact of Public Transit. Special Case.



corroborate the findings of the more general analysis of the impact of transportation on the density gradient.

Additional information for policy analysis is contained in Figure 3C which assumes that a relatively substantial number of riders consistently use public transit for some predetermined distance from the CBD with eventually a decrease in riders at further distances. Since the marginal cost of public transport is mostly time related, this result would apply if identical income groups have a tendency to locate approximately equal distances from the CBD. In general then, subsidies to increase public transit riders would result in decentralization. Since the percentage of public transit is a tract-specific variable, the VCM approach can measure changes in density patterns within a particular area of a city due to a shift in the number of riders. For example, the impact of the new mass-transit system in Washington, D.C. could be approximated for each specific tract. This allows for the development of spatial or more generally three-dimensional density functions.

The other tract-specific variable is income. Again, the analysis is conducted for the representative city and Washington, D.C. The theoretical results as expressed by Equation 2 suggest that higher income households locate at greater distances from the CBD. The empirical results as presented in Table 5 and Figures 4A, B and C, suggest a somewhat different behavior. For incomes between $0 and $37,500, the density gradient (r) decreases; for greater incomes r increases and in all cases it is negative. In all the cities average income fell within the 0 to 38 thousand range. Thus, it would seem that the increase income effect, i.e. increasing housing consumption, on location might be offset by the increased transport costs resulting from the greater value of time. These results combined with the previous analysis on public transportation are consistent with the proposition that changes in transport cost relative to income have dominated the decentralization process.

FIGURE 4A

The Impact of Income on the Density Gradient (r)

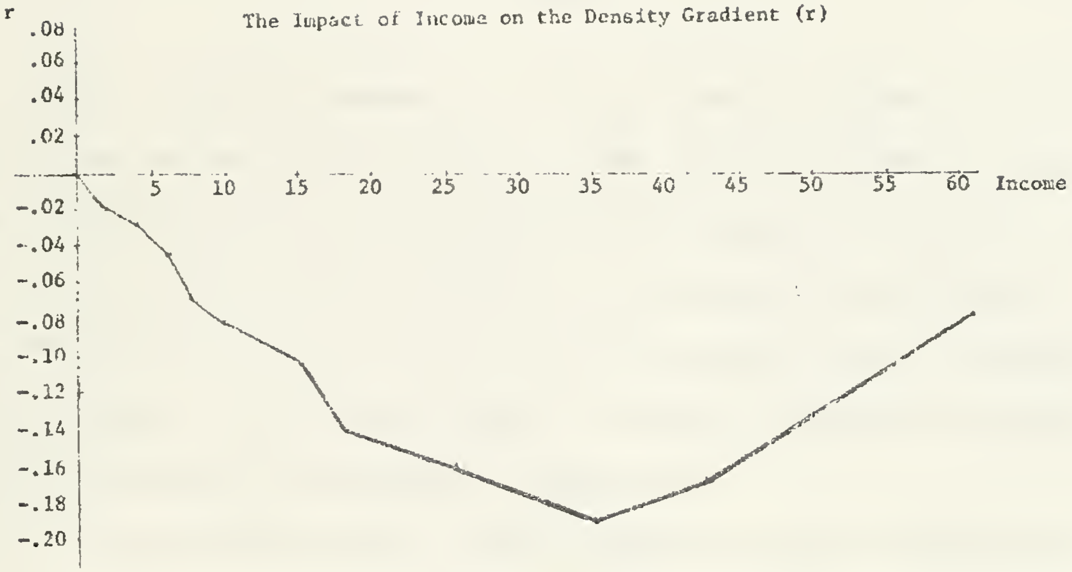


FIGURE 4B

The Impact of Income on the Density Function.
Examples for $6,000, $35,000 and $55,000 Income

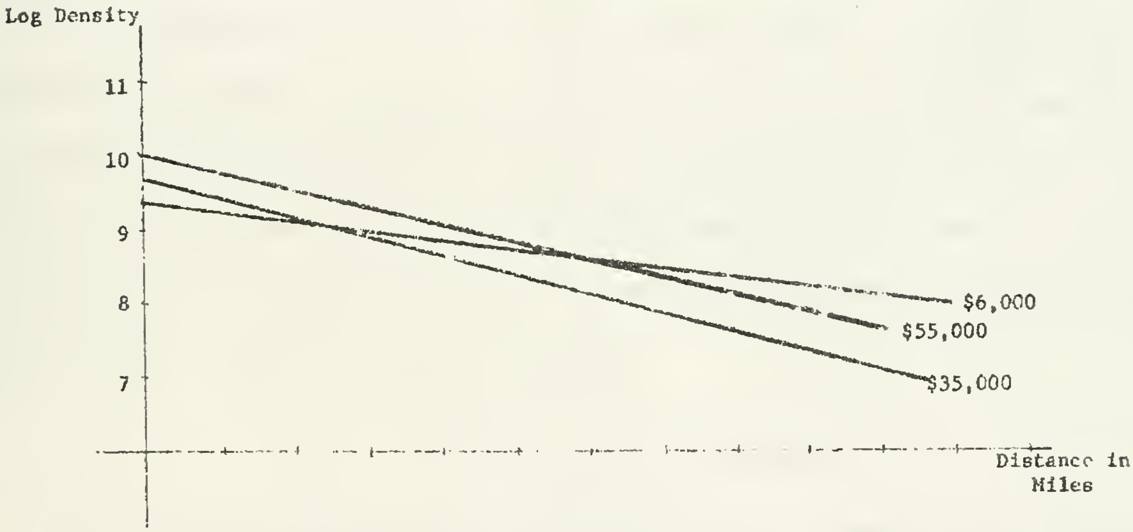
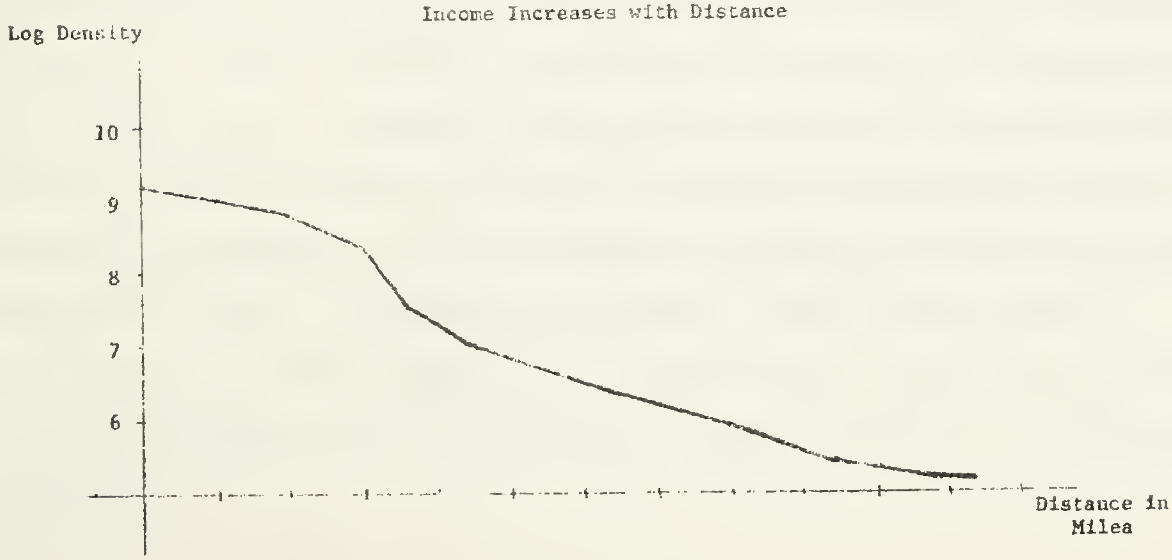


FIGURE 4C

The Impact of Income on the Density Function Assuming
Income Increases with Distance



Age has a definite tendency to reduce the density gradient (r). This was expected because of the rigidity of older cities in adjusting to the technological development of the automobile, (See Figures 5A, B and C). Figure 5C demonstrates that if the assumption of decreasing age with distance is accepted then the effects of age lead to an exponential density function of classical shape. This was approximately the result obtained when assuming increasing income with distance (See Figure 4C).

For the population as with the age variable variation results from comparisons across cities. Within the relevant range for the sample used in this study, population has the effect of increasing r. Associated results are plotted in Figures 6A, B and C. As the figures indicate, population growth at least for smaller cities must result in economics of scale for services (perhaps public transportation) leading to decentralization. At much larger levels of population (over 1,900,000) diseconomies of scale seem to set in making a city inflexible and possibly not responsive to technological changes of the type brought on by the automobile.

8. Summary and Conclusions

The VCM has been proposed as a method for introducing city and tract-specific variables into the exponential density functions used to study urban structure. A major advantage of the VCM is that it permits the introduction of such variables while retaining an interpretation which can be reconciled with the body of theory justifying the use of the exponential functional form. This facilitates comparisons of results obtained by applying the VCM with the massive empirical literature on urban density functions. Most estimated density functions are but special cases of the general VCM with a polynomial structure relating the density function coefficients to the socio-economic conditioning variables.

FIGURE 5A

The Impact of Age on the Density Gradient (r)

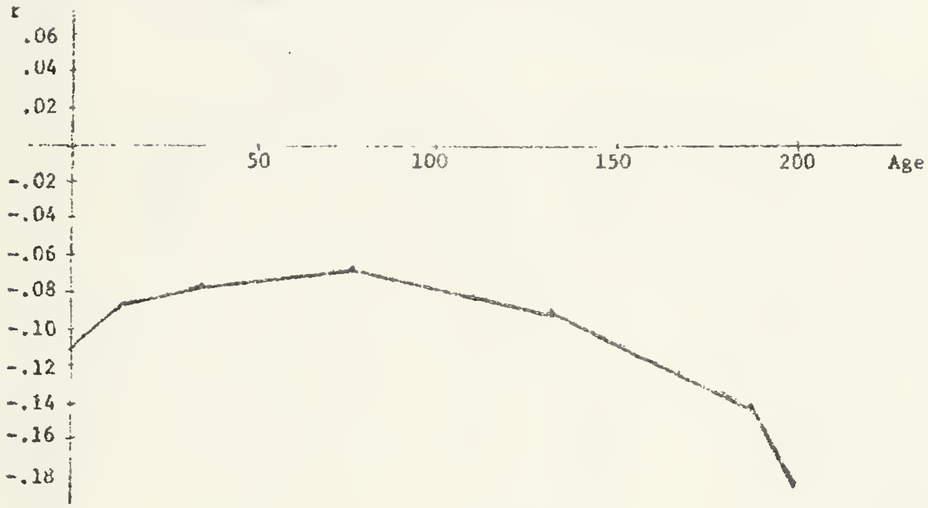


FIGURE 5B

The Impact of Age on the Density Function, Examples for 90, 150 and 200 Years

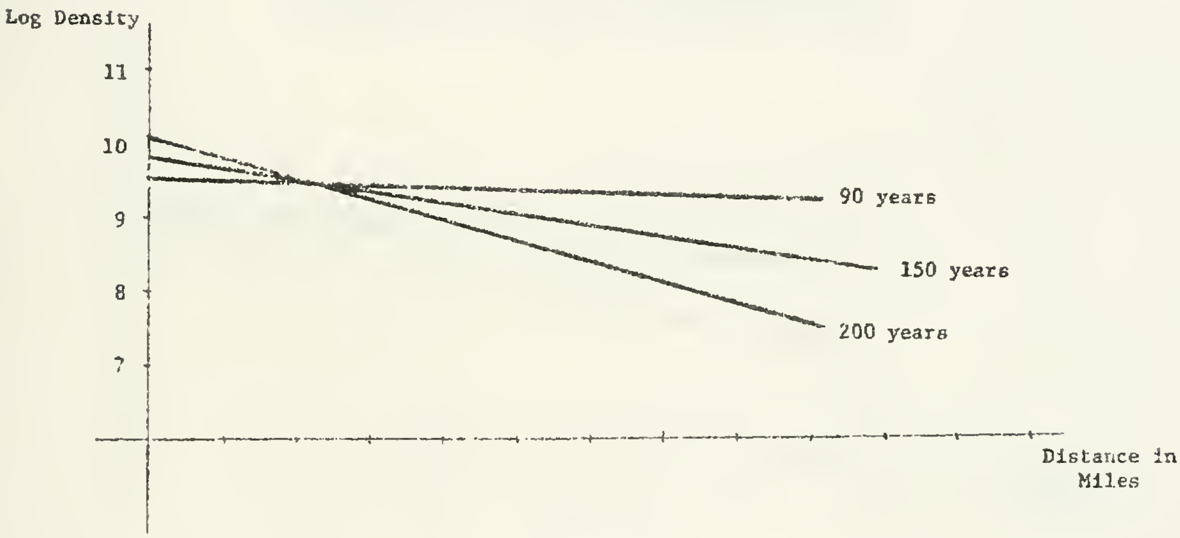


FIGURE 5C

The Impact of Age on the Density Function Assuming Age Decreases with Distance

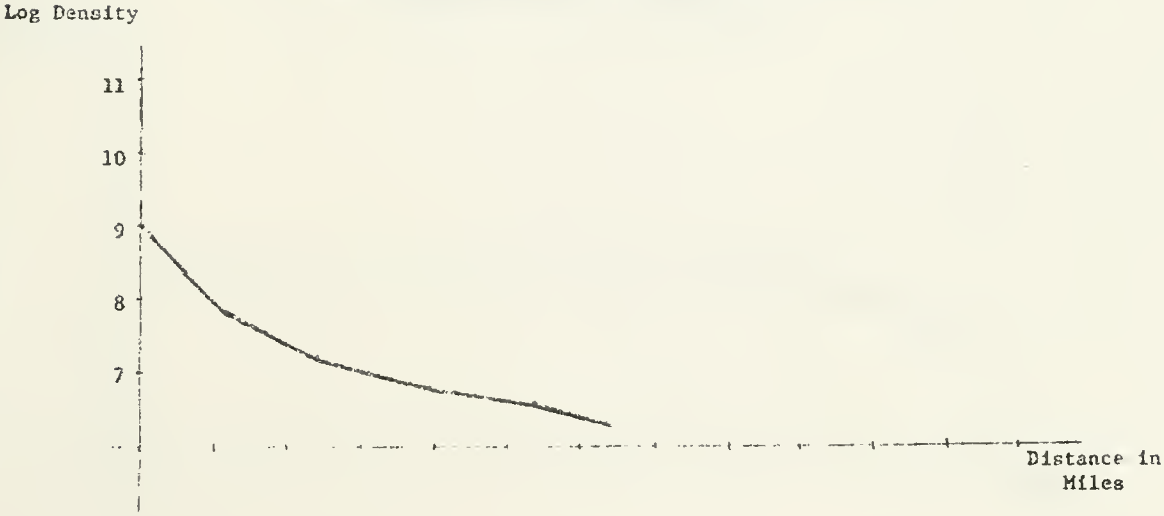


FIGURE 6A

The Impact of Population on the Density Gradient (r)

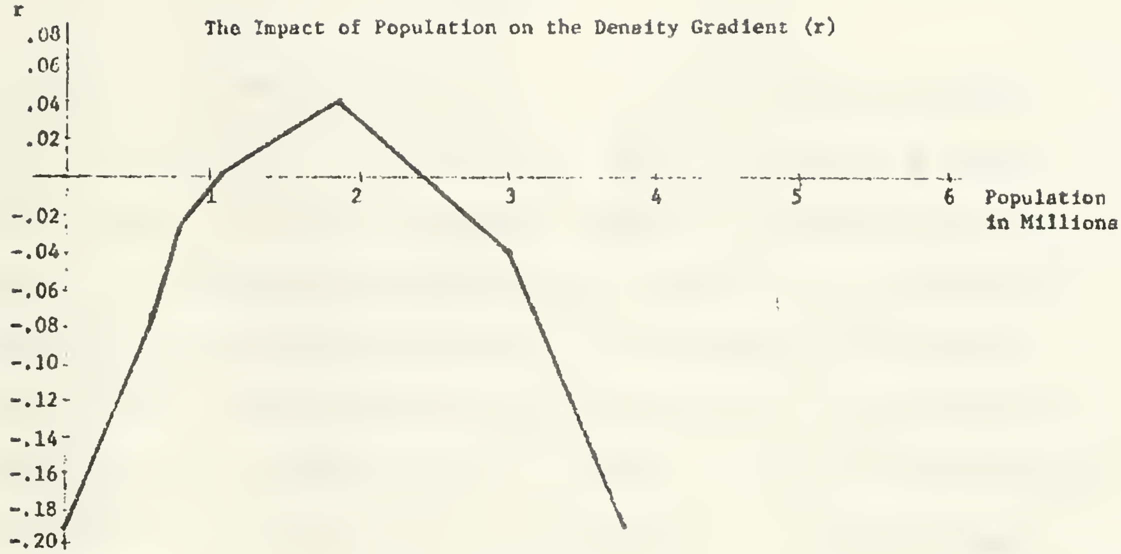


FIGURE 6B

The Impact of Population on the Density Function. Examples for 1/2, 1 and 3 Million Population

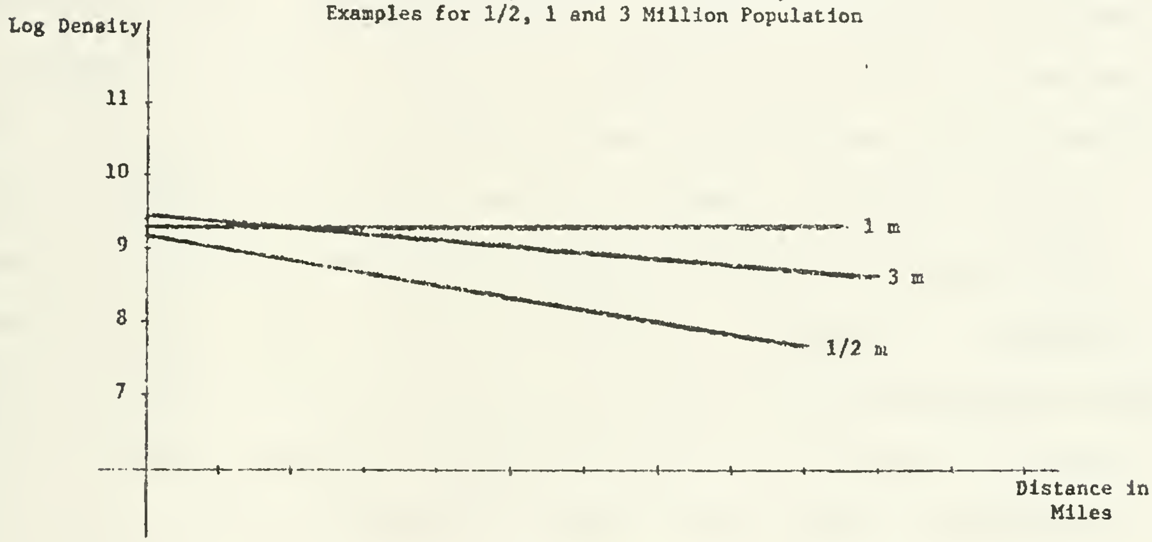
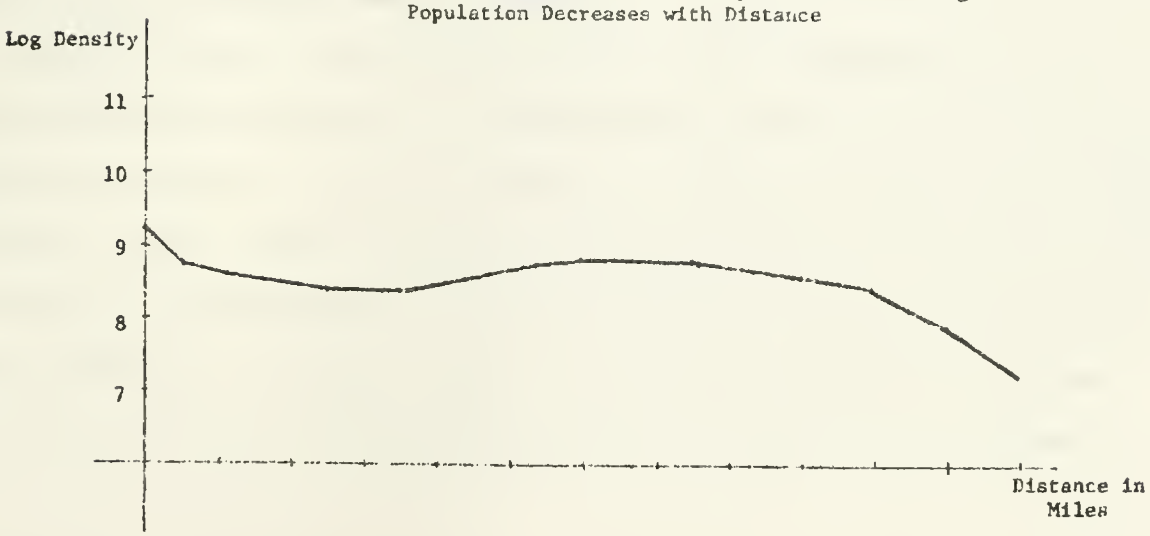


FIGURE 6C

The Impact of Population on the Density Function Assuming Population Decreases with Distance



Application of the VCM specification to data from 43 randomly selected census tracts in each of 39 U.S. cities for the year 1970 provided a number of interesting results. Firstly, the results point to the resolution of a problem raised by recent applied density function studies. It is shown that apparent questions about the appropriateness of the exponential functional form and specification errors associated with the omission of city and tract-specific variables can be handled within the context of applied density function studies using the VCM framework. In the present study the explanatory power of the density function and the significance levels of the structural parameters were greatly enhanced by the application of the VCM in studying the 1970 data.

Secondly, the results showed that the conditioning variables reflecting transport mode, age of city, household income and population could be used to provide reasonable explanations of apparent structural differences between cities and tracts. Of these results, perhaps the most interesting relates income and transport mode to density trade-offs. Analysis of the polynomial structure relating these tract-specific variables to the density gradient gave results which have a natural interpretation based on the opportunity cost of travel time as incomes increase. Other results while perhaps less novel are consistent with the hypotheses which emerge from the more elaborate theories supporting the exponential density function.

The most important results which come out of the application and VCM specification concern the use of the urban density function as a tool for policy analysis and projection. Until the present, the empirical work on urban density functions has been largely descriptive; including tests of the density function form and exploratory analyses of possible additional variables for explaining density patterns. The present study by introducing a method for including possible policy control variables and additional uncontrollable variables

directly related to time, offers an expanded area of application for the density function hypothesis. As shown in the specialized analysis of the typical city and Washington, D.C., effects of policies designed to influence transport mode and income can be directly examined in the context of an estimated density function. Provided that density is a target for urban planning, estimated VCM's of the type presented in this study can assume an important role in the structure of planning models. Regarding projection, the relationship between age and population and time provides an illustration of how the model can be used in forecasting. Since these uncontrollable variables can be accurately projected on the basis of simple expressions in time, the cross sectionally estimated density function can be used for forecasting changes in urban structure. Although such forecasts can yield little information about the adjustment to new equilibrium levels, they should provide urban economists with a tool of some value. As well, the void in the information on rates of adjustment from the cross section data, indicates an area of high potential for further research.

FOOTNOTES

[1]Neidercorn, using a more general model, established the negative exponential as appropriate for population and employment (1969). More recent theoretical work has been rooted in Wilson's (1976) entropy spatial systems. Following a different approach, Beckmann and Wallace (1969) and Golob and Beckmann (1971) have modeled individual trip preferences using interrelationships between opportunity interactions and trips. In these studies net utility for the individual is derived from potential utility of interaction for each spatial opportunity minus the reduction in utility due to traveling time. Smith (1975), following a similar line of argument, presents a theory of travel preferences leading to distance-dependent utility functions. Trip-makers are assumed to discount anticipated opportunity interactions for the distance. Smith (1974) also demonstrates the possibility of exponential spatial discounting behavior within an axiomatic framework. Finally, Isard (1975) in an associated development, provides a rationale for travel behavior consistent with both gravity model trip patterns and exponential spatial discounting.

[2]The VCM does not require that all employment be concentrated in the CBD. The CBD is used as the convenient reference point established in previous theoretical and empirical studies.

[3]The tract-specific data, the ratio of public to private commuters' income, and population used to compute density for each tract are from the 1970 census tract statistics (1970). City-wide data, population and age were taken from the statistical abstract. Areas in square miles were measured with a polar planimeter using tract maps. Distance in miles was measured with a ruler in the tract maps from the center of the CBD to the center of the tract. Density is in terms of population per square miles.

[4]Urban age was determined by examining the historical profile of each city's decennial population growth rate. Each city was assigned a date which corresponded to the decade in which the city experienced its last growth spurt exceeding the growth rate of the national urban population. This technique was taken from a study by Alfred Watkins (1976). For common cities the age data were taken from Watkins' study. Age data for the additional cities were computed using the Watkins technique. The authors wish to thank Watkins for his help in supplying some data and the computational procedure.

[5]It should be noted that Brown, Durbin and Evans (1975) have proposed a similar scheme for dealing with the problem of regression relationships which may change over time. In this case the potential change is across tracts and cities. The method of parameterizing the change is, however, the same. In a somewhat different context with random coefficients, models for parameter change have been specified with the conditioning variable as time [Rosenberg (1973), Rosenberg and McKibben (1973)]. The problem with random coefficients is estimated with a more complex error structure. Also, the fact that time as an artifical variable is unbounded, restricts the structures which can be used to condition the random coefficients and still maintain consistent parameter estimates.

[6] An alternative approach used by Muth (1961) to estimate the density gradient with coefficients conditioned on these variables would be to form city and tract specific subsamples. A two stage procedure could then be applied. First, least squares estimates of the density gradient would be calculated from the sub-samples. Second, polynomials in the conditioning variables would be estimated with the first stage coefficient estimates as dependent variables. This method has several drawbacks as compared to the one currently employed. First, the additional efficiency gained from the co-variance structure for the pooled data used to estimate the VCM would be lost. Secondly, the gains in efficiency from simultaneous imposition of the restrictions could not in general be obtained. Finally, a part of the variance being explained in the second stage of the process would be due to sample size unless more complex random coefficient procedures were applied in the first stage. If the coefficients are treated as random variables in the second stage of the estimation process they must be correspondingly specified in the first stage. Thus, the present method for estimating the VCM by reparameterizing and pooling the data is in general more efficient and, in fact, more simple.

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APPENDIX

Table 1

Mean Values of Distance, Income and Public Transportation
With Minimum and Maximum Values and Variance for Distance

	Akron	Baltimore	Birmingham	Chicago	Cincinnati
MDIST	2.939	9.756	7.159	8.214	4.974
MINC	9,642	11,371	7,524	9,880	8,584
MPT	0.042	0.191	0.109	0.940	0.244
MIND	0.874	1.002	1.048	2.138	0.699
MAXD	6.50	29.00	16.5	14.966	9.873
VD	2.19	64.99	16.84	10.70	6.99

	Dayton	Denver	Detroit	Flint	Fort Worth
MDIST	4.225	5.301	9.256	3.54	6.523
MINC	10,482	10,404	11,148	10,215	10,100
MPT	0.113	0.086	0.202	0.029	0.056
MIND	1.223	0.786	1.781	0.874	1.000
MAXD	10.922	10.485	29.000	8.38	15.75
VD	5.53	7.11	28.40	3.21	17.02

	Houston	Jacksonville	Louisville	Memphis	Milwaukee
MDIST	8.27	5.544	6.145	5.489	5.738
MINC	10,191	8,844	8,556	7,644	11,424
MPT	0.088	0.130	0.1868	0.259	0.230
MIND	1.625	0.625	1.50	1.311	1.50
MAXD	21.25	12.75	16.875	11.009	16.875
VD	17.25	8.79	11.89	8.28	5.13

Table 1 (Continued)

	Nashville	New Haven	Omaha	Philadelphia	Phoenix
MDIST	4.656	3.487	4.128	7.188	6.360
MINC	8,751	11,548	9,452	10.349	10,836
MPT	0.172	0.145	0.154	0.611	0.020
MIND	1.223	0.601	0.601	0.961	0.750
MAXD	11.000	8.750	10.623	17.823	12.750
VD	7.97	3.74	5.68	16.16	9.16

	Pittsburg	Portland	Providence	Richmond	Rochester
MDIST	3.414	5.259	5.555	7.371	4.792
MINC	8,125	9,281	9,927	10,239	11,602
MPT	0.646	0.112	0.069	0.276	0.179
MIND	1.01	0.699	0.699	1.50	1.136
MAXD	7.827	12.844	13.875	19.50	12.500
VD	2.27	8.10	13.28	15.92	11.27

	Salt Lake City	San Antonio	San Diego	San Jose	Seattle
MDIST	5.051	4.747	5.677	5.016	7.651
MINC	9,884	9,340	9,143	11,901	11,454
MPT	0.037	0.098	0.049	0.026	0.145
MIND	0.699	1.000	1.398	0.961	0.437
MAXD	13.50	9.000	11.009	10.485	22.50
VD	12.54	5.18	7.65	5.67	22.11

Table 1 (Continued)

	St. Louis	Spokane	Syracuse	Tacoma	Toledo
MDIST	8.194	5.196	3.783	3.479	4.238
MINC	10,969	8,809	10,268	9,217	10,907
MPT	0.191	0.049	0.163	0.068	0.012
MIND	0.874	0.869	0.612	0.334	0.454
MAXD	17.50	12.25	13.00	8.485	12.00
VD	18.80	9.41	9.07	4.29	7.19

	Tucson	Utica	Washington, DC	Wichita
MDIST	4.589	3.057	6.641	3.287
MINC	8,708	9,355	12,832	10,092
MPT	0.024	0.069	0.434	0.0234
MIND	0.786	0.534	1.804	0.349
MAXD	10.66	8.125	15.25	6.728
VD	5.93	2.81	11.31	1.94

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