VALUATION, DEPRECIATION AND THE RATE-BASE

BY

CARL EWALD GRUNSKY, Eng. D.

Member of the American Society of Civil Engineers

ASSISTED BY

CARL EWALD GRUNSKY, Jr., E. M.

Member of the American Institute of Mining Engineers

FIRST EDITION

FIRST THOUSAND

NEW YORK

JOHN WILEY & SONS, Inc.

London: CHAPMAN & HALL, Limited

1917

Stanhope Press
F. H. GILSON COMPANY
BOSTON, U.S.A.

PREFACE

A thorough discussion of the principles which control the engineer, the economist, the assessor and the business man in making valuations will not be attempted in these pages. This book is the result of personal contact with the valuation problem. No apology need be made for the fact that, in presenting an original analysis of the problem, the arrangement of the material is not always as logical as might have been expected if the present state of the art only had been brought under review. Special consideration has been given to a discussion of the non-agreement of the actual life of articles which have a limited period of usefulness with their probable or normal life. The effect of this non-conformity has been studied by the author and the results have been thought worthy of a special chapter. These results show that there is great advantage in adopting, instead of "present value," a rate-base without deduction of depreciation, which will include but little, if anything, other than legitimate and properly estimated cost as the starting point when rates are to be fixed. He therefore recommends for the thoughtful consideration of the student, the method of procedure which he has named the Unlimited Life Method and which is herein fully explained. The author feels, too, that the owner of a public utility is, generally, entitled to larger earnings than will yield a bare interest return on the invested capital. Volume of business transacted and also the unearned increment should, sometimes, be taken into account when estimates are made of the earnings that may with propriety be allowed. Such matters as these have received attention and it is hoped that some of the ideas that are herein advanced may stimulate further thought along similar lines and may, here and there, prove helpful to those who have appraisals to make or rates for an output or for a service to establish. Careful attention has been given to the fundamental principles which should control when appraisals of public utilities are made for rate-fixing purposes and their practical application is explained.

The tables which are presented in this volume are intended to meet the needs of valuation engineers, but will also be found useful by any one having problems of finance and bonding to solve. They are based throughout on original calculations. Those relating to replacement requirements and to expectancy are of a novel type not to be found elsewhere. Special care has been taken to secure accuracy and convenient arrangement.

Acknowledgment is due to the office staff of the American Engineering Corporation the members of which have assisted in the preparation of the tables.

C. E. GRUNSKY.

San Francisco, Cal.,
February 1st, 1916.

ABBREVIATIONS AND NOTATION

Amort. = Amortization.
Ann. = Annual.
Beg. = Beginning.
Eq. = Equal.
Payt. = Payment.

A = the annual replacement requirement for each dollar of capital invested annually in a growing plant.

A_m = the accrued amortization in m years when the annual amortization installment is a and the interest rate is i.

A' = the amount of $1.00 at compound interest at the end of the nth year at the interest rate i.

A'' = the amount of an annuity of $1.00 paid at the end of each year at compound interest at the interest rate i.

a_n = the amortization installment which must be invested annually, in order to amount at compound interest to $100 in n years.

a_n' = the annual installment which at compound interest at the rate i will amount to $1.00 in n years.

a_n'' = the annuity receivable at the end of each year which $1.00 will buy for n years.

a_m = the current amortization in the mth year, *i.e.*, the amortization increment a plus interest on the amortization fund already accumulated. It is the amortization installment which in the remaining years of life will retire the remaining capital.

C = cost of replacing a group of articles.

c = the annual renewal requirement for a group of articles whose cost of replacement is C.

e = expectation, that is the probable remaining years of usefulness of any article whose probable life new was n years.

e' = relative expectancy of an article whose probable life new is 10 years when compared with an article m years old whose probable life new is n years.

g = the average annual investment in additions to a plant.

i = the rate of interest per year expressed fractionally — thus for 6 per cent; $i = 0.06$.

m = a number of years.

m' = relative age of an article whose probable life new is 10 years, when compared with an article m years old, whose probable life new is n years.

n = the probable life term in years of any article; or the term of amortization.

P = the present value of \$1.00 due at the end of n years.

P' = the present value of an annuity of \$1.00 receivable at the end of each year during n years.

R = amount in the replacement fund expressed in percentage of the original investment.

S = the sum of all annual replacement requirements estimated for a number of articles of various ages.

S_n = the sum of all annual replacement requirements during n years.

CONTENTS

VALUATION, DEPRECIATION AND THE RATE-BASE

CHAPTER I

INTRODUCTION AND GENERAL NOTES

Conflicting Views Relating to Procedure. — The need of harmonizing the conflicting views relating to the best methods of dealing with the establishment of rates that are to be charged by public service corporations for the service which they render or for the commodity which they supply is recognized by economists and engineers. This need is the more pressing in view of the fact that the public, in exercising control over the operation of the utility through properly constituted authority, may and no doubt frequently has established onerous and burdensome regulations, resulting at times in the confiscation of property. At any rate the limitations relating to permissible charges have frequently been such that the question of the reasonableness and sufficiency of these charges has been taken into court and the courts have reached certain conclusions from which there is apparently no appeal but concerning some of which the wisdom of permanent enforcement may well be called in question.

Rapid progress is being made in recognizing the fundamental principles which should control when an appraisal of public service properties is to be made as a basis for the establishment of rates for service rendered. This subject is, however, today still in a controversial stage. The assembling of facts relating to such fundamental principles as will be of service to the appraisers of operating plants of any character, whether used in the public service or not, is the task which was set in undertaking the preparation of this volume. It is hoped that the same will fill the gap for a time until wider experience of the valuation experts shall have eliminated the involved and ofttimes useless procedures which are still in practice and explanations of which have, therefore, been included in the text.

The Public and the Owner. — The question may well be asked whether the reformer, who has been seeking protection against the demands of the trusts and corporations which have frequently been guilty of over-capitalizing, has not in his turn gone too far in the endeavor to reduce the earnings of the corporation-owned public utilities. The public has demanded and, in some measure, has obtained, from various rate-fixing bodies, and from the courts, the introduction of methods of procedure that are not always equitable, but the baleful effects of which are slow in becoming manifest. There has, for example, been no little confusion in the matter of defining depreciation and not a few valuation experts have had difficulty in grasping the fact, which when once recognized seems fundamental, that the repayment of capital (amortization), is one thing and that the making of provisions for the replacement of worn-out or discarded parts at the end of their term is another thing.

It is fundamental that the owner of a public utility should be allowed:

a. To obtain, sooner or later, a reasonable return on his investment.

b. To recover at some time the capital invested for the public good.

c. To reap a suitable reward for establishing and managing the enterprise.

It is the knowledge that these principles have obtained universal recognition which prompts the capitalist to embark upon such ventures as those known as public utilities.

Control of Public Utilities a Recent Innovation. — Until within the last few decades the control by the public of the operations of the public utility concern has generally been so loosely exercised that the margin for a possible profit was large; regulation was more or less perfunctory and, very frequently, due to the imperfect knowledge of the problem, neither the owner of the utility nor the rate-payer knew whether the earnings were deficient, reasonable, or excessive. The recent tendency of the public to insist upon its right to control and the attempt to make a close study of what would in each case be reasonable earnings has stimulated the work of the economist and the valuation engineer and has led to a consideration of many new phases of an interesting subject.

Prof. John H. Gray in an address before the Economic Club of San Francisco on July 28, 1914, in presenting a historical survey of public utilities said:

"We have very recently entered upon the experiment of administrative control through central commissions. But we have failed entirely to grasp the significance of the problem, or the steps necessary to solve it, if we fail for a moment to remember that until the establishment of the Board of Gas Commissioners in Massachusetts, in 1885, no industries, save that of transportation, were classed as utilities, and that the other states have been very slow to follow the lead of Massachusetts in this matter. In fact, it is only within the last decade that the problem may be said to have been taken up seriously by the country as a whole. That is altogether too short a time to adjust the law, the theories, or the practices to the economic needs and conditions of the situation. Much more is it too brief a period in which to have modified to any considerable degree the pioneer philosophy that still dominates the private owners of these industries.

"The scientific theory is that the utilities should render adequate, safe, and universal service, at just, reasonable, and fair prices to all, and that the sovereignty shall be the final judge in every case of these matters. This statement implies, of course, that the total gains or rewards of the owners shall be reasonable under all circumstances, including that of virtually guaranteed monopoly, and that they shall have just compensation in case of expropriation."

Fundamental Principles Apply to Industrial Establishments. — The work which has been done in studying the fundamental principles looking to the fair treatment of both the rate-payer and the owner has also been of benefit in a large way to the factory, and to all the great industrial establishments of the country, because it points the way to a closer approximation of the actual cost of production, thereby establishing a better basis for fixing the sale price of the output.

The discussions in the following pages are necessarily directed mainly to the valuation of public service properties and to a discussion of the procedure to be followed in determining what are reasonable earnings, but the general fundamental principles are equally applicable to the private concern.

Value must be ascertained when the ownership of property is to be transferred from one person to another as in the case of a sale; it must be ascertained when the property is capitalized as a basis for the issuance of securities, and, too, for taxation purposes. When rates or prices to be charged for service or commodity output are to be fixed, it is necessary to know the amount of capital which is properly invested — to know, in other words, what the amount is which should be taken into account as a "rate-base."

Depreciation and Withdrawal of Capital. — The practice is altogether too common of treating every earned depreciation allowance as equivalent to a withdrawal of invested capital and of assuming that any of the various methods of estimating present value are equitable, regardless of the past history of the plant under investigation. So, too, there is sometimes a too rigid construction of the generally recognized requirement that only those parts of a property which are actually in use or which are useful elements are to be considered in valuing the property for rate-fixing purposes. This has occasionally worked a hardship upon the public service corporation and may therefore be regarded as one of the factors which have caused a demand for large allowances for all kinds of intangible and more or less indefinite elements of value to offset losses and unproductive investments.

Every private industrial establishment, too, has its depreciation problem to deal with. Depreciation enters into the cost of production and must be covered in the sale price if there is to be any profit. The cost of the perishable parts of the plant which are consumed, just as fuel is consumed though at a slower rate, must be recovered sooner or later.

Definition of Terms. — The definitions of terms, herein presented, relate to the sense in which these terms are used in this volume. There is not yet any perfect agreement among engineers and accountants relating to all of these terms and, as time goes on, some modification of terms or of the definitions of terms is to be expected. These definitions are therefore offered with due appreciation of the fact that some of them may not survive and that others remain subject to modification.

The Agency Theory. — Every owner of a public service property is in a certain sense the agent of the rate-payer. If regarded as such agent he is under obligation to render the service which he has engaged to perform in a satisfactory manner; he is entitled to protection against unreasonable competition, and he is entitled to protection of his investment, provided, of course, that the investment is a proper and reasonable one. There may be some cases, however, where unwise investments have been made, and others where part compensation for making the investment has been found in the increased value of real estate or other property, perhaps not connected with the utility but owned by the same individuals who own the utility, and in such cases the value of the service rendered to the public may bear an unusual relation to the investment.

Thus an irrigation canal, which was constructed to develop a region of little or no value without water, when considered by itself, apart from the land which it irrigates, may have cost so much that the service charges cannot be made high enough to yield a revenue on its cost. And yet the canal was justified, though perhaps not as a separate venture. In such cases a transfer of the canal to the land owner would be appropriate in order that the question of a reasonable return on invested capital might be eliminated, or as an alternative a bonus charge against the land whether under this name or under the name of "water right," or otherwise, might be agreed to, such that the capital remaining in the canal enterprise, as a proposition apart from the land, would be reduced to a reasonable amount. If neither of these alternatives are adopted the canal venture may be permanently unprofitable, from the standpoint of present owners. This will be the case when the rates that would yield interest on the investment are in excess of the value of the service rendered and cannot be supplemented by a participation of the canal owner in the unearned increment which the construction of the canal has brought to the land owner.

As another illustration a railroad may be considered which was primarily constructed for the development of a tract of land or for the development of mineral or timber resources. There can be no question that when considered from the broad standpoint of the advantage to society the construction of a railroad under such circumstances may be justified. When this railroad begins to serve the public and the freight tariff and passenger rates are to be fixed, these may have to be established on the basis of the fair value of the service rendered without any attempt to make the earnings yield interest on the capital invested in the road, or suitable consideration may have to be given to the fact that a part of the investment was made for the benefit of some special enterprise such as a rock quarry or a lumber business and should not have been charged to the road at all.

These limitations upon the use of an appraisal as an element for consideration in fixing rates are fully recognized, but these and other limitations are not such as to detract from the desirability of referring to and using appraisals under ordinary circumstances.

Basis for Sale Price — Business Enterprises. — Everyone who engages in a business enterprise must know:

The cost of the article in which he deals at the time the article is sold.

The cost of the plant such as buildings, roads and equipment which are necessary to manufacture or to care for the article pending the time of sale.

The operating costs of the business.

The depreciation of the plant or the replacement requirements thereof.

The profit which it is desired to make.

When these elements are known and a proper analysis of operating cost has been made the prudent business man will distribute his fixed elements of cost, such as management, depreciation and the like with a view to making the price of the article or the charge for the service which he renders, attractive to his customers.

In a large machine works where the charges for repair work appeared unusually high, it was found upon investigation that overhead expenses were unequally distributed to new and to the repair work. This was done in order that new work which had to be obtained in competition with other concerns could be taken at lower figures. While this is only an isolated example, it shows the importance of being in a position to analyze costs in order that charges may be fixed low enough to secure business and yet high enough to yield an adequate profit.

Reference to the Tables in this Volume. — As an aid to the appraisers of industrial and public utility properties, this volume would fail in its purpose without tables containing information relating to the probable term of service and the expectancy of the articles in common use in such establishments or plants and also interest and annuity, amortization and depreciation tables.

The amortization and depreciation table not only covers a range from 2-year life to 75-year life, but it has been so arranged that the remaining years of service, instead of age, can be made the starting point when it is desired to enter the table for present value and the so-called current rate of depreciation.

While such tables and explanations as are furnished in this volume are generally applicable they have been prepared, as stated, with special reference to the requirements of appraisers and of the public service commissions and of other rate-fixing bodies. They will meet, too, the requirements of the owners of industrial and other properties, who owing to lack of such information occasionally allow over-capitalization and excessive dividends, coupled with inadequate earnings, to cripple their enterprises.

The Panama Canal and Fundamental Principles. — The necessity for a clear understanding of the fundamental principles to be followed when rates are to be fixed may be illustrated by reference to the case of the Panama Canal. Here is a great work representing an investment by the United States of about \$375,000,000. It is a type of canal which involves large operating expense. In connection with this canal the question arises: What should the earnings be?

The problem of the canal tolls may be broadly considered along the following lines:

First: Who is ultimately to pay for the canal? Is it to be the user of the canal?

Second: Has the canal been built by the United States as a revenue-producing investment or is the canal to be regarded as worth what it has cost for military purposes and as an instrument of general benefit?

Third: What shall be included in the operating expenses?

The ordinary citizen is under the impression that the \$375,000,000 which the canal has cost is an investment similar to those which the United States has made in constructing the Eads jetties at the mouth of the Mississippi river, in constructing the Ambrose channel for access to the New York harbor, and in building the San Pedro breakwater for the protection of the harbor of Los Angeles and the Columbia and Eureka bar jetties for the improvement of other Pacific Coast harbors of more or less local importance. If this is the fact, the investment in the canal has been made as a non-revenue-producing investment for all time and there will be no need of recovering the cost of the canal from those who use it nor even interest on this cost. If the United States plans to collect the cost of the canal in installments from the users of the canal, no matter how small these installments may be, the canal will ultimately be paid for, in part at least, by foreign people. And if interest only is covered by the toll charges, then this Nation will stand in the position of having made a loan to the commercial interests of the world of $375,000,000 for the construction of the canal.

Once the fundamental principle has been adopted relating to the extent to which commerce shall recoup the United States for its outlay and the question has been settled as to whether the users of the canal shall pay interest on the investment, the question of operating expenses will have to be taken up. Salaries, wages, materials and supplies are readily determinable. All expenditures for repairs and upkeep will be included with perhaps no difficulty until the question of depreciation is to be considered.

Shall each element or class of elements be considered separately, according to probable useful life, and some sum be set aside which together with interest thereon will replace the worn-out parts, or shall the amount to be set apart for replacements be determined directly from the relation of probable life to the cost of replacement of each perishable part by setting apart annually such a fraction of the cost that these annual increments, without interest, added together will be equal to the cost, or finally shall provision be made from year to year for the replacements that will probably have to be made in each year? It is hoped that what is presented in this volume may prove of some assistance in the solution of problems of this character.

Unprofitable Expenditures and Early Losses. — Unprofitable and unproductive expenditures which may be the results of mistakes or of accidents and early losses in the business are elements which should not be regarded as adding value to a property. They belong in some measure to the hazards of the business. They are, nevertheless, to be taken into account when rates are to be fixed because they usually represent legitimate expenditures. They represent an outlay which in nearly every case would have been incurred under consent of the ratepayer, if the owner had been acting as the agent for the rate-payer and with his approval. Consequently the rates should be fixed with a view to amortize sooner or later such expenditures.

In thus making provision for unproductive expenditures determined, perhaps, from actual expenditures legitimately incurred in excess of cost of works in use, care must be taken not to go too far. Wasteful expenditure is not to be sanctioned and wise and prudent management is entitled to reward. Experience alone can determine what allowance should be made for hazards of the business and for the cost of establishing the business and bringing it up to a paying basis. If it be found, for example, that a suburban electric road will not be on a paying basis for a number of years, the losses (or deficient earnings) during these years may be added to the cost of the road not as elements of value but as a part of the investment on which an interest return is to be allowed or as the preferable alternative, net earnings in excess of interest on the cost of the road can be allowed in such amount that within a reasonable time the early losses will be amortized. Thereafter a continuation of some excess of earnings above the returns from ordinary safe investments will be the owner's reward for having engaged in the enterprise.

Franchise and Water-right Values. — When the public through properly constituted authority grants a franchise or confers a privilege to enter upon a business which is in the nature of a public service, as, when it grants the use of water for power, for irrigation or for other purposes, the franchise or water-right is valuable only to the extent that the public provides a market for the service rendered or commodity supplied. When the rates to be charged are subject to regulation and are not fixed in the franchise or water-right grant, no basis exists (except in the cases of strategic value) for determining franchise or water-right value. This value depends, as will be explained, upon earnings in excess of a fair return on the investment and if earnings are not protected by franchise terms, it will lie in the power of the rate-fixing body to eliminate franchise and water-right value altogether. This is as it should be. At the same time with due regard to the share in the general prosperity to which the owner of the utility is entitled, he should be allowed to earn more than ordinary interest on his investment and a capitalization of any such excess earnings when these are fairly assured will furnish him with a basis for approximating the sum of all intangible values of whatever nature.

Amortization and Depreciation. — A clear distinction should be made between the amortization of capital that is invested in any enterprise and provision for depreciation or the replacement of perishable elements. The sinking fund for the retirement of bonds, or other debt, should not, in other words, be confounded with the fund usually referred to as the "depreciation" fund which is intended to meet the replacement requirements as they arise.

Valuation for Purchase or Sale. — In appraising for a sale, various matters are to be taken into account which are of minor importance when rates are to be fixed. If the accrued depreciation has actually been allowed in the earnings and has been permitted to accumulate in a fund, and if the accumulated fund properly represents the accrued depreciation, and is a part of the property to be transferred, then the appraisal including this fund should be in substantial agreement with the legitimate investment. If, on the other hand, there is no depreciation fund, if the earned and collected adequate depreciation allowance in excess of the expenditures therefrom for replacements has been absorbed for other purposes by the owner, then a purchaser would make the depreciated value or the legitimate investment less depreciation his starting point in determining the amount which a plant is worth, because in purchasing the property he assumes an obligation to replace perishable parts as they go out of use. If the plan is followed of allowing earnings which will meet all replacement requirements together with interest on the full investment, without deduction for depreciation, then a purchaser in view of the assured return will make the legitimately invested capital without deduction for depreciation his starting point in determining what he can afford to pay.

This last case is in conformity with the requirements herein explained, that the investment in a public utility should remain, at all times, unimpaired. It is at variance with the plan advocated by many and approved in various court decisions, that the appraisal for rate-fixing purposes shall fluctuate from year to year with the age and remaining service value of the perishable elements that go to make up a public service plant.

Present Value as a Starting Point. — Despite prevailing sentiment and rulings of the public service commissions and decisions of the courts in favor of making "present value" the starting point when rates are to be regulated, it will be apparent to the student that what is estimated as "depreciation" which may never have been earned and collected has not necessarily been a repayment of capital and that therefore "present value" is not always the best starting point.

The wisdom of starting with "present value" or with "value" in any form may well be questioned because value is a result of assured earnings and such values as are not covered by the investment do not exist until the earnings create them. In appraising for rate-fixing purposes the aim should be to make the legitimate investment and not the depreciated value the rate-base.

It is sometimes difficult to modify the "fair value" basis of rates as heretofore insisted upon by the courts so that it will fit local conditions as in the following illustrations:

1. Adjacent to the built-up section of a community which is being supplied with water at satisfactory and in every way equitable rates, a large subdivision of acre property is being made. Streets have been laid out and paved, sidewalks have been constructed, an adequate sewer system has been provided and a system of water mains has been constructed. These water mains conform in every respect to the requirements of the owner of the water-works to whom this system of mains is now offered on the sole condition that he supply prospective customers with water.

Acceptance of this system of mains may bring no immediate increase of revenue. The development of the newly subdivided area may be slow. Some years may elapse before the sales of water in the district will be equal to the added burden of maintenance, collection of rates, and depreciation.

Shall part of the system thus acquired be included in the valuation on which the owner is to receive a return? Or shall such inclusion be deferred? And if deferred for how long a time?

The inclusion of such property merely because it has " value " can perhaps be justified on the plea that it makes no difference what a plant has cost the owner and that according to the decision of the courts value alone should be considered. It will be held that it represents a necessary extension of the distributing system that could best be made at the same time that other street improvements were made, and that it would be impracticable to attempt in the case of this considerable addition to the system to fix the time when the extent of the use of such new pipe and the addition of revenue resulting from such use would no longer add to the burden of the remaining, or rather, older rate-payers. But the fact remains that when considered as the agent of the public the water-works owner has not made the investment. The investment has been temporarily made by the owner of the subdivided tract. He will be reimbursed by the purchaser of lots, for the outlay for this and all other improvements made on the tract will appear in some form in the selling price of the lots.

If the municipality owned the water-works, in what situation would it now find itself? The same treatment should be accorded the owner as would be accorded to an agent of the municipality. Viewed from this standpoint, the owner, when he accepts the addition to his distributing pipe system as a gift, assumes at once also the obligation to keep this part of the system in repair and to replace the same when worn out or discarded for other reasons. The owner is therefore entitled (necessity for the extension of the system being admitted) to the necessary allowance for operating expenses including a proper allowance for anticipated future replacements but should not always immediately be allowed interest on the full cost of this extension which, as already stated, together with other improvements such as street grading, paving, sidewalk and sewer construction will be included in the price charged by the owner for the lots and will therefore represent a contribution by this new section of the municipality to the cost of the privately owned water-works.

In some measure a donated extension to a water-works system, as here used for the purpose of illustration, bears a similar relation to the plant, as does property held for future use in excess of the capacity of the plant, with the difference that in the one case the public donates the property to the owner, in the other, the owner secured the same by investing his own capital. In the fixing of rates the increase of capital due to the receipt of gifts and donations should generally be considered separately from the actually invested capital. Individual cases will arise requiring special treatment. The owner in most cases will not be entitled to as large a return on this donated capital as on his actual investment.

2. The above case is not materially different from that of the municipality which needs water-works but cannot afford to make the necessary expenditures. To induce the investment of private capital in a water-works enterprise, the offer of a bonus is made. To avoid legal restrictions this has sometimes been done through the medium of extra large annual payments extending through a number of years for the water required for street sprinkling, for sewer flushing, for extinguishing fires and other municipal uses. Whenever a direct or an indirect contribution is thus made, the owner's invested capital — the rate-base — is not necessarily represented by the cost of the reproduction of the entire water-works.

Fair Value as a Rate-Base. — In concluding this introductory chapter attention may be called to the accepted view of the courts in the matter of valuations for rate-fixing purposes. It appears from the various decisions of the courts which have been cited and from many others that might be referred to, that the majority of the courts are definitely committed to the idea of making the "fair" value of the property used for the convenience of the public, that is to say the "present value" of this property the basis of all calculation as to the reasonableness of the rates to be charged by a public service corporation.

If the general analysis of the fundamental principles, which must control when rates are to be fixed, as presented in this volume, contributes in any degree to the modification of this attitude of the courts, the author will feel repaid for having attempted the analysis. It seems to him illogical to make the earnings dependable on value which itself is the direct result of the earning capacity of the property for whose output rates are to be fixed. The real starting point is the properly invested capital and the volume of the business which is transacted. What consideration should be given to these elements and to other factors affecting a legitimate return to the owner of the public service property will be further discussed.

CHAPTER II

DEFINITIONS

Value. — Value is the worth of anything measured by any standard of purchasing power. It is the exchange power which one commodity or service has in relation to another. The word value is used in this sense throughout this volume. The other meaning of the word relating to adaptability for a certain purpose has not been brought under discussion. It is only value in exchange and not utility which is of interest to the appraiser.

A careful distinction must be drawn between the cost of an article and its value. The cost frequently determines the price at which an article is offered and taken in exchange and there may therefore be at times a close relation between cost and value. But the words are in no sense synonymous and cannot be used interchangeably.

Value in the sense of worth estimated by any standard of purchasing power, is, in the case of such properties as public utilities, a result of the earning capacity. In the case of certain properties such as highly improved residence property a determination of value from earning capacity may not be immediately apparent but the rental value is nevertheless generally there and can be determined. In the case of such properties as works of art, however, which are not ordinarily revenue producing, but which are desired for the pleasure which they give and for educational purposes, the value in exchange cannot thus be measured, and consideration must be given to the more abstract question of supply and demand. The discussion in these pages will be restricted to the valuation of properties which have an earning capacity.

Market Value. — In the sense in which used in this volume "value" is synonymous with market value.

Fair Value. — The term "fair value" is used so frequently in the valuations of public service properties for rate-fixing purposes, that it deserves more than passing notice. As a basis for the fixing of rates it is difficult to define. Apparently it is usually intended to mean market value, — the value which would be ascertained by a prudent purchaser making thorough inquiry relating to all circumstances affecting value. But in reality the value is a result and not a basis of earnings. It should not be made the "rate-base." The term "fair value" has been used in a measure, no doubt, to rule out "book value" or cost which may include items and amounts of doubtful propriety and to rule out estimates of value based on stock and bond issues and the market value of such securities.

The term fair value as used by the courts has not yet been satisfactorily interpreted and no attempt will here be made to reconcile divergent views in relation thereto. But attention may be called to the difficulty which has been experienced by all who have attempted to make appraisals for rate-fixing purposes, in reconciling the value to a purchaser with the "fair value" which the courts wish to have considered when fixing rates. Why should there be one valuation for purchase and another for rate-fixing? The answer that has been given by the courts is practically to the effect that there should be no such difference, and experts have found difficulty in so appraising values that any such difference shall disappear.

The value to an investor is unhesitatingly determined from the net earnings, with due regard to hazards of the business. The value for rate-fixing purposes as the courts say is to be that value on which, with the same regard for the hazards of the business, the owner is to be allowed to earn a fair interest return. Value should be the same whether determined by a rate-fixing body or whether determined for a purchaser.

When the acquisition of the properties of the Maine Water Company by the Kennebec Water District was under consideration, the Supreme Judicial Court of Maine included in its instructions to the appraisers (Dec. 27, 1902) the following:

"The capitalization of income, even at reasonable rates, cannot be adopted as a sufficient or satisfactory test of present value. But while not a test, present and probable future earnings at reasonable rates are properly to be considered in determining the present value of the system."

In this case, "Kennebec Water District *vs.* City of Waterville et al" (97 Maine 185; 54 Atlantic 6), the district was authorized by law to acquire the entire plant property and franchise, rights and privileges of the Maine Water Co. and the instructions referred to were issued by the court upon a joint request.

According to the decisions of the courts, as the matter stands today, that which is to be ascertained and made the starting point when rates are to be fixed, is the present value of the property devoted to the public use.

In this connection the Supreme Court of the United States says in the Minnesota Rate Cases (230 U.S. 352): "The basis of calculation is the fair value of the property used for the convenience of the public." In San Diego Land and Town Co. *vs.* National City the Court says, "What the company is entitled to demand, in order that it may have just compensation, is a fair return upon the reasonable value of the property at the time it is being used for the public." In the Minnesota Rate Cases the Court also quotes with its approval from Smyth *vs.* Ames (169 U.S. 466).

"In order to ascertain that value, the original cost of construction, the amount expended in permanent improvements, the amount and market value of its bonds and stock, the present as compared with the original cost of construction, the probable earning capacity of the property under particular rates prescribed by statutes, and the sum required to meet operating expenses, are all matters for consideration and are to be given such weight as may be just and right in each case. We do not say that there may not be other matters to be regarded in estimating the value of the property. What the company is entitled to ask is a fair return upon the value of that which it employs for the public convenience. On the other hand, what the public is entitled to demand is, that no more be extracted from it for the use of a public highway than the services rendered by it are reasonably worth."

It is to be noted, too, that the U. S. Supreme Court holds to the view that the value as a basis for establishing rates is to be the present value. The Court uses the following language in Willcox *vs.* Consolidated Gas Co. of N. Y. (212 U.S. 19, 52).

"And we concur with the court below in holding that the value of a property is to be determined as of the time when the inquiry is made regarding the rates. If the property, which legally enters into the consideration of the question of rates, has increased in value since it was acquired, the company is entitled to the benefit of such increase. This is at any rate the general rule. We do not say there may not possibly be an exception to it, where the property may have increased so enormously in value as to render a rate permitting a reasonable return upon such increased value unjust to the public. How such facts should be treated is not a question now before us, as this case does not present it. We refer to the matter only for the purpose of stating that the decision herein does not prevent an inquiry into the question when, if ever, it should be necessarily presented."

The Court has apparently taken the view that the unearned increment may generally be allowed to go to the owner of the utility either as an offset in whole or in part to early losses or as a reward for having undertaken the enterprise. The fact should, however, not be lost sight of that any such increase in value is, strictly speaking, a part of the aggregate earnings and due consideration to this fact should be given in making appraisals.

While these rulings of the highest tribunal in the country seem to compel the valuing of a public utility property according to accepted standards at present worth, when value is to be ascertained and made the starting point for fixing rates, we may nevertheless be permitted, and we consider it a duty, to point out the limitations of these rulings and the desirability of securing some modification thereof. This matter will be further discussed in the chapter on depreciation.

Remaining Value. — The term "remaining value" is equivalent in its meaning to present value when applied to an article whose value lessens with age. It is generally dependent upon and to be computed from three elements; (*a*) the probable useful life term of the article new; (*b*) its expectancy or probable remaining term of usefulness; and (*c*) the cost of replacing it with some article of equivalent usefulness, which, when prices are not subject to change, is the original cost less the residual value of the article when it goes out of use.

Residual Value. — The "residual value" is the value which remains in any article after the same has ceased to be useful as an integral part of a public service or other property. The residual value may be only scrap value, or it may be the price at which it can be disposed of for use in connection with some other property. It is usually estimated as the value to an outside purchaser, less the cost of delivery to such purchaser.

Condition Per Cent. — The existing condition of any article can be expressed by comparison with the condition of the same article new. If the article when new is taken at 100 per cent and present condition in comparison therewith is noted in per cent this is called the condition per cent. It is the present or remaining value in comparison with value new, based upon the condition of the article, expressed in percentage.

Accrued Depreciation. — The accrued depreciation is the difference expressed in money between the original cost of an article and its remaining value.

Probable Life. — The probable life term of an article is the time, usually expressed in years, during which it may reasonably be expected to render useful service. The probable life term of any article is to be determined by the experience, the world over, with other articles of the same kind. It depends not alone upon the time required for an article to become valueless by use, by ordinary wear and tear, but also upon the time when by reason of accidental destruction, inadequacy or obsolescence, the article must be replaced by a new one better adapted to fulfill its purpose. If the life of a large number of articles of the same kind, as for example telegraph poles, be recorded it will be found that a few fail almost immediately, being destroyed by storms, by fire or by accidents, that a larger number will fail within a few years due to inherent defects in the timber and other causes in addition to the causes of failure already named, that failures will multiply at some period such as 10 to 15 years according to the character of the timber of which the poles are constructed and that some will survive many years beyond the average life term of all. From such records the probable life term of a new pole is ascertained; but while this probable life, here synonymous with average life, is known or can be ascertained as explained, the actual life of individual poles will depart therefrom more or less. The total number of service years of the poles which fail early will fall short of the expected total number by as many service years as those of the poles which survive the probable term will exceed this number.

Expectancy or Remaining Life. — The expectancy of any article is the probable time, usually expressed in years, during which it may reasonably be expected to render efficient service. For a new article the expectancy is its probable life. When the life of a candle which will burn for a number of hours is under consideration, the expectancy is ascertained by subtracting age from probable life new. But in all ordinary cases the expectancy cannot be ascertained by any such simple mathematical process. Every article which has been in service for some years, and has escaped the accidents which might have put it out of business in its early life, stands a better chance of being among those which will outlive the probable life term fixed for it when it was new, than it had when new to outlive this term. Consequently the expectancy is not to be determined by subtracting age from probable life. It is to be determined from the actual condition of the article and all local circumstances which may affect its continued usefulness.

A high-duty pump, which originally had a probable life of 25 years, when it reaches the end of this term, may be in a condition almost as good as new. It has escaped the possible accidents of the early years of its life and by careful attention and replacements of its wearing parts is still rendering first-class service. The value of this pump is not to be written off the books, neither should it be regarded as good as new. Its value is ascertained by determining its probable additional years of usefulness and the probable cost of replacing it at the end of this term.

An irrigation canal usually improves with age. So far as wear and tear is concerned, it has unlimited life. But under the development of extensive areas, the small original canal may in the course of time be superseded. The probable life of this canal, and therefore, too, the annual replacement increment, is estimated on some assumption relating to the rate of this development. Finally the time comes when the project for a comprehensive canal system has taken shape and it may reasonably be assumed that within a definite period, five or ten, or some other number of years, the original canal will be superseded; its diverting dam and its headworks, and perhaps the canal itself, will then be abandoned. The remaining life or expectancy of the canal is at that time only five or ten or some other number of years, as the case may be, and within this time the remaining investment in the canal is the amount under consideration for replacement.

Composite Life. — The composite life of a complex plant is that term of years within which the accruing depreciation of all items of which the plant is composed, on the assumption of no replacements, would amount to the cost of the plant. As in the case of articles in classes according to their probable life, so in the case of entire plants, the life thus determined for various types of plants is made an aid in estimating the current depreciation. Without recourse to the more laborious method of dealing separately with each item of which the plant is made up, it becomes possible if composite life is known to approximate the current depreciation or replacement requirement.

Composite Age. — The composite age of a complex plant is that age which it would have to acquire, if treated as a depreciating unit, to make its accrued depreciation equal to the aggregate accrued depreciation of the individual items of which the plant is made up.

"Amortization." — In the sense used in this paper the term amortization applies to the retirement of capital. When an article is to be permanently retired from connection with a business and not to be replaced, the annual depreciation increment or the annual replacement increment allowed in the earnings becomes an amortization increment.

Ordinarily a discarded article is replaced by a new one which will render equivalent or better service. The amount to be provided during the life of the article and made available at the time it goes out of use is, for all practical purposes, its replacement cost, being the cost of installing an equivalent new article, less the residual value of the original article. The amount thus provided may be treated as amortization, but there is no need of doing so as will be fully explained.

Depreciation. — Depreciation is the lessening in worth of any perishable article which takes place from use and advancing age. It is not solely due to inherent deterioration or to wear and tear but results, too, from the fact that owing to progress in the arts and sciences and methods of manufacture, an article may become obsolete and that due to growth of communities or other causes, appliances or structures may become inadequate and have to be replaced in the course of time by new ones better adapted to the requirements. Every circumstance tending to limit the term of usefulness of an article should be taken into account when its probable life or expectation and consequent rate of depreciation are to be determined.

Annual Depreciation. — The annual depreciation or the current depreciation is the annual theoretical lessening in worth, expressed in money.

Appreciation. — Appreciation is the increase in worth expressed in terms of money. It applies not alone to real estate but to any article, structure or thing which increases in value due to general prosperity or to the more obvious cause of higher wages and greater cost of materials.

The Current or Annual Replacement Requirement. — The current or annual replacement requirement is that amount which should annually be covered by the earnings, to meet the renewals which must be made from time to time. The amount necessary to accomplish this may be estimated from the known character, number and cost of the articles which must annually be replaced or in the case of large units, from the rate of depreciation.

The Obligation to Replace. — The obligation which every owner of a public utility has assumed to continue in business for either a definite or an indefinite time period carries with it an obligation to replace the articles which are essential for the proper conduct of the business with equivalent new articles whenever the original articles cease to be useful. This obligation to replace grows with the age of the article in service. When expressed in money, it is equivalent to the accrued theoretical depreciation. This obligation to replace, together with deferred maintenance, must be taken into account by the appraiser of property which is to be purchased.

Deferred Maintenance. — "Deferred Maintenance" is the neglect, expressed in dollars, which has resulted from failure to keep an article in good condition and repair. It is the sum which should at once be expended to restore the article to ordinary good service condition, and to protect it against causes of rapid destruction so that its deterioration will not be unduly rapid. Ordinarily there should be no "deferred maintenance." By proper attention to maintenance and repairs the service rendered should be kept at the standard which is expected of the plant. The plant in other words should, practically at all times, be at 100 per cent efficiency.

Wear and Tear. — "Wear and tear" is the term applied to the deterioration of an article from use. The article is kept in serviceable condition by the renewal of its worn-out parts and by suitable attention to its wearing parts. Maintenance and repair are to be included in operating expenses. The expenditures for repairs and maintenance are intended to keep the article at or near 100 per cent efficiency.

It is sometimes difficult to draw a close distinction between repairs and replacements and between deferred maintenance and accrued depreciation. It is generally assumed that deferred maintenance results from neglect to make the lesser repairs and renewals such as are current from year to year and that depreciation comes into consideration when dealing with the more important items which when new have a probable life of at least a number of years.

Replacement Cost. — The replacement cost is the cost of a new article with which a worn-out article is replaced, less the residual value of the original article. It is the cost of effecting a change from the worn-out part of the property to a new part of equivalent service value.

Invested Capital or Investment. — The investment or the capital invested hardly needs a definition. It is the aggregate of the expenditures which have been made and which remain in the business. It is the summation of the cost of the various items which make up the property in question and render the same efficient for the purpose for which it is intended, less such sums as may have been applied out of earnings for the retirement of the capital. In connection with the valuation of public utility properties it may be regarded as the aggregate of the reasonable and proper expenditures which have been incurred to make these properties serviceable. It may and generally does include such items as the cost of establishing the business.

Wearing or Service Value. — When the fact is taken into account that an article after it has ceased to be useful in connection with one enterprise or property may still have value in connection with another, and usually does have some scrap value, it will be plain that the lessening of worth is not to be estimated from the original full cost of the article. It is the difference between the first cost and the residual value which is being consumed during the useful life of the article. This difference is frequently referred to as the wearing or service value of the article.

Remaining Service Value. — The service value of an article which is no longer new is its remaining service value. It is determined from the original service value by deducting the accrued depreciation.

Intangible Values. — The term intangible value is applied to any element of value other than elements of a physical nature. The value of a franchise is an intangible value, so too the "good-will" of a business; "going value;" "rights of access;" "water-rights;" "power-rights," and the like. The cost of organizing and establishing a business is an element for consideration in making an appraisal of value just as legitimate as the cost of the physical elements of a property.

Cost of Business Development or the Cost of Establishing the Business. — The cost of development or the cost of establishing the business is a summation of the expenses not chargeable to construction which have been incurred in building up the business and bringing it upon a paying basis. Preliminary expenses, advertising, commissions and losses due to inadequate revenue in the early years may all be brought under review in making up the cost of development. It is not always a simple matter to know where to stop in the matter of charging up expenses to the cost of development and to the capital account. To a certain extent the losses due to deficient business in early years may be regarded as part of the capital reasonably invested. The European practice is to do this to a far greater extent than is customary in the United States. When feasible, the better practice would be to recognize the early losses as a necessary incident to the business and to make provision for their amortization as circumstances may justify. In general it may be said that any legitimate expense which it may be difficult to assign to construction or operating accounts can be regarded as development expense.

Franchise. — A franchise is a privilege to do business granted by the public. A franchise may be limited to a specified period of time, it may be indeterminate or it may be perpetual. It may be, but is not necessarily, an exclusive privilege. It may be coupled with all sorts of conditions such as division of profits; the fulfilment of all kinds of obligations; the limitation of charges for the service rendered or the commodity furnished. It may be nothing more than a privilege to occupy the public highways with rails for transportation purposes or with pole or pipe lines for the transmission or delivery of electricity, water, oil or gas. It may be a grant without cost or may have to be paid for at a fixed price or subject to competitive bidding. It is one of the intangible elements which may represent value and frequently appears among the items of cost.

Going Concern. — The term "going concern" is applied to such business enterprises as are in actual operation. If in successful operation, the value of the "going concern" should be somewhat greater than the sum of the value of the physical elements of which the property is made up.

Going Value. — The increment of value which is due to the fact that a business has been established and brought upon a paying basis is its "going value." In some degree the cost of developing the business, such as the early losses, can be made the measure of going value. This can be only to the extent that the going concern has an advantage over a similar enterprise subject to ordinary reasonable business development expenditures. If the early losses have been treated in the accounts as operating loss and not as invested capital and if subsequent years of operation have wiped out the early losses, it may be found difficult to justify the inclusion of any cost of developing the business among the intangible elements of value. It may in other words be difficult to determine how much of the aggregate of the intangible value should be classed as "going value."

Good-Will. — The term "good-will" is closely related to "going value." It represents the intangible element of value in an ordinary business and is applied generally in those cases where there is competition with other like business ventures. The courts hold that where there is no competition there can be no good-will.

Overhead Expenses. — Certain expenditures in connection with the development and operation of any revenue-producing property are usually classed as overhead expenses. Economists and engineers are not all agreed as to just what part of the cost of construction or operation should be classed as overhead. Ordinarily the cost of management is treated as the overhead. When work is done by contract, the amount paid to the contractor is the cost. An analysis of cost to the contractor when compared with what he has received may show a profit but this is no part of the overhead expense. The compensation of the general manager, on the other hand, and of the engineer and legal staff and the expense of the general office should be treated as overhead — so, too, interest during construction, taxes and insurance. Whether or not they are taken into account in making valuations for the purpose of establishing a "rate-base" (it being no concern of the public how the funds for establishing an enterprise are raised), the commissions to brokers who sell bonds are a real item of expense to the owner and will be classed by him either among the overhead expense or as a promotion expense.

Promotion Expense. — The term "promotion expense" is used to designate certain expenditures, which cannot well be included in the "overhead" nor yet in the legitimate cost of establishing the business. To this class of expenditures belong a variety of expenditures usually incurred prior to the permanent organization of the business enterprise and may include commissions to real-estate agents, and the various expenditures that have been incurred apart from actual cost in acquiring the properties forming the nucleus of the enterprise. Advertising and discounts and commissions to bond brokers may also be at times properly classed as promotion expense. The cost of bringing together the various fundamental properties into one holding, representing the result of the promoter's activities, except to the extent that the cost of promotion work appears in enhanced value, is generally accepted as being covered by the item "promotion expense."

The amount of the promotion expense may vary within wide limits according to the character of the works, the difficulties to be overcome and the thoroughness with which preliminary investigations are made. Such expenditures are generally much less in connection with the additions to an established enterprise than they would be in connection with an original plant of the same character and magnitude.

Reproduction Cost New. — The ascertainment of what it would cost to construct an exactly equivalent property, identical with that to be valued, is frequently an acceptable aid in determining value. Any article which forms a part of a revenue-producing property has a value in the service which may reasonably be measured by the cost of replacing it. Its value in the service may be estimated from the cost of installing a new article at the end of the expectancy term. Generally the value at some particular time is under consideration and it would be theoretically correct to apply the prices of material and labor which are current at that time in making the estimate of reproduction cost. But construction was a process requiring time and reconstruction would also require time. Furthermore the appraisal may have to serve for some time, perhaps for a term of years. It is reasonable and logical, therefore, to depart from the strictly theoretical requirement and to adopt, in making such an estimate, unit prices which represent average conditions, preferably for a period of about 5 years. The instructions to appraisers by the Supreme Judicial Court of Maine, in the Kennebec Water District Case (1902), (97 Maine 185; 54 Atlantic 6), when the properties of the Maine Water Co. were to be valued contained the following reference to reproduction cost:

"The appraisers may properly consider what the existing system can be reproduced for. But the cost of reproduction will not be conclusive. It will be evidence having some tendency to prove present value. The inquiry along the line of reproduction should be limited to the replacing of the present system by one substantially like it."

Public Utility. — A public utility is an enterprise which renders a service or which supplies a commodity of general necessity or convenience to the public. There may be public ownership of the utility, or the privilege of serving the public may be delegated to a private person. When the private person is a corporation, we have the public service corporation.

Rate-base. — The "rate-base" is the starting point, the basis of the calculation when earnings are to be determined to which the owner of a public utility is entitled. The owner is entitled to a fair return on the legitimate investment which he has made for the benefit of the public. He may also be entitled to more than what would be considered a reasonable return on ordinary investments but this fact is to be determined by the circumstances in each particular case. To know the legitimate investment, whether ascertained from book records or by estimating the cost of reproduction or otherwise, is fundamental.

The rate-base which is the most satisfactory is the actual reasonable and proper investment or original cost new, including a proper allowance for the cost of establishing and developing the business, undiminished by depreciation, though in some cases, perhaps, diminished by the bonus which may have been contributed by the public or by the rate-payer.

The endeavor is made in these pages to show that the value of the service rendered and the earnings are independent of accrued depreciation and that for this reason the present or depreciated value of the physical elements is not the best criterion for a determination of the legitimate amount of the earnings. But the sacrifice which the owner has made, the amount of money which is in the business legitimately as of the date of construction, the original cost, in other words, less the capital which has been contributed to the work as a bonus and not as amortization, is the essential element for consideration and should be made the starting point when rates are to be regulated. It is to be understood, as a matter of course, that special consideration may have to be given to the overbuilt plant and to property held for future use. If the owner has gone too far in the matter of providing capacity or in the matter of securing property not immediately required, the cost of such property may have to be omitted from the rate-base or the rate of return on the rate-base should be lower than would otherwise be allowed.

CHAPTER III

FUNDAMENTAL PRINCIPLES WHICH CONTROL WHEN APPRAISALS OF PUBLIC SERVICE PROPERTIES ARE TO SERVE AS A BASIS FOR FIXING RATES

1. The earnings of the public service property should be such that, within the life of the property, there will be returned to the owner, the capital which he has properly invested in it, and in addition thereto, interest at a reasonable rate upon such amount of capital as from time to time actually and properly remains as an investment in the property.

2. The reasonable cost of each item which goes to make up a property may be returned to the owner during the probable or during the actual life of that item or it may be returned to him in a lump sum when that item ceases to be useful. In the case of items which are of sufficient importance to be individualized the amortization of cost may be progressive during a fixed term which is generally determined by the probable life of the item. In the case of numerous articles the amortization, with equal propriety, may be deferred to the end of the term of usefulness of each article.

3. During the early years of most public utilities the operating costs will exceed the revenue. The business is conducted at a loss. When such losses are legitimate, they may be regarded as the cost of establishing the business and may be added to the invested capital. They may, as an alternative, be regarded as temporary advances which are then to be amortized within a reasonable time.

4. The owner of the public service property is entitled to proper compensation for assuming the hazards of the business and for establishing and operating the utility.

5. The owner of a public utility is entitled to a reasonable share in the general prosperity of the community which he serves. For this reason he should be allowed a share in the increase of value which results from population growth. If present value is made the rate-base, he will get the appreciation of real estate (if such increase be not excessive). If the public utility has but little or no property which appreciates then some other factor may be brought into consideration in determining the fair return. It would be sound doctrine to exclude appreciation from the rate-base thus avoiding uncertain and accidental reward and to allow the unearned increment to appear in the net earnings.

6. Where the invested capital is small and the volume of business large, the owner of the utility is entitled to have the volume of business considered as well as the invested capital when rates are to be fixed. It would generally be good practice to take the volume of business into account as well as the rate-base when rates are to be regulated, but this is not yet an established practice.

7. The depreciation of an item used in the public service does not reduce the value of the service or commodity which the public utility supplies to the consumer. The value of the service is ordinarily unaffected by depreciation. Depreciation is a lessening of worth which may result from any cause. Wear and tear suggest themselves as the prime cause of the depreciation of articles in use, but it is sometimes difficult to establish a connection between the rate at which an article is consumed in service by wear and tear and its lessening worth. The accrued depreciation of any article is ascertainable from the probable life new of the article, the cost of replacing it at the end of its life, its probable remaining term of usefulness, and its original cost. In practice the accrued depreciation is usually computed from the cost of the article or rather from its wearing value, its probable life new and its expectancy. The cost affords a convenient basis for approximation. (This does not apply to certain articles which are readily replaced such as automobiles when

considered apart from a complex property. Treated by itself the automobile shows a marked drop in value as soon as it goes into service and thereafter may be considered as decreasing at a fairly uniform rate per year.)

8. Whenever the rates to be charged for the service rendered by a public utility are to be fixed the current depreciation or the current replacement requirement must be estimated. This is usually done by compound interest annuity methods or by the so-called Straight Line Method. According to the procedure under which the necessary amount of earnings are determined, either the current depreciation or the current replacement requirement are then made a part of the necessary earnings.

9. According to the procedure which is adopted for treating the replacement requirement, depreciation and amortization in the accounting system, the computation of earnings that are necessary from year to year will vary. Any one of a number of methods of procedure may be adopted with due regard to the following:

a. Is it desirable to keep the required earnings in the early years relatively low?

b. Is any portion of the invested capital to be amortized or is there to be any amortization of expenditures for unproductive work?

c. Has there been any amortization of capital in the past?

d. What has been the relation of earnings to operating expenses?

e. What amount of prospective business is to be taken into account?

10. An earned depreciation allowance may be regarded as an amortization increment. The depreciation allowance is then for all practical purposes a repayment to the owner of a part of his invested capital. It will, in that event, make no difference in the ultimate result, at what rate the repayment of capital is made. Ordinarily capital is retired as follows:

a. In equal annual amounts which without interest will in the life of the article whose cost is to be amortized amount to that cost. (The Straight Line Method.)

b. In amounts which increase from year to year, figured on the compound interest basis. (Equal Annual Payment Method.)

c. In equal annual amounts which together with compound interest thereon will in the life of an article amount to its cost. (Sinking Fund Method.)

11. A public service property may be regarded as having an unlimited life. Taken as a whole when rates are to be fixed the accrued depreciation may then be disregarded; but provision must be made for the renewal of those parts which for any reason become useless. In this event there need be no repayment of capital, except only those portions thereof which may be regarded as temporarily invested.

12. In so far as this may be practicable, the earnings of the public utilities should be adjusted to the ability of the rate-payer to pay. Ordinarily, therefore, the aggregate earnings should be kept relatively low in the early years when the number of rate-payers is small.

13. The owner of a public utility has the right to do what he pleases with any portion of his capital which is paid back to him, which is, in other words, eliminated from the rate-base.

14. The owner of a public utility should be held accountable for all sums collected from the rate-payers for the specific purpose of making repairs and renewals. A diversion to other uses of any fund intended for this purpose is equivalent to a repayment of capital.

15. The basis of calculation, the starting point when rates are to be fixed for a public utility, is the amount of capital whose investment was necessary to build the plant and develop its business and on which there should be a proper interest return to the owner. This may be either the legitimate original cost new, or it may be the properly invested capital reduced by the accomplished amortization. This basic amount is for convenience called the " rate-base."

16. As a general proposition the earnings present and prospective should be such that they will give the property a value in excess of the capital actually (and properly) invested — the

excess of value being the amount of the reward (including perhaps some unearned increment) to which the owner is entitled for having established and for managing the business.

17. Strategic value, resulting from a combination of circumstances such as low operating cost and ample market for the output at prices fixed by or for competing concerns which have not the same advantage of low operating cost, is a part of the reward to which the owner of a public utility may be legitimately entitled.

18. The various procedures for estimating the required earnings of public service properties are usually based on the assumption that the same procedure has been used from the beginning of operations. A procedure which may be correct if consistently and continuously applied, may be unfair to either the owner or the rate-payer, if introduced at a later period. Consequently it should be regarded as imperative that consideration be given to past history when the rates of an operating concern are to be fixed.

19. Franchises and water-rights and in general the privilege to do business are not to be made a part of the rate-base except in certain cases in which such rights and privileges have definitely ascertainable strategic value or have had to be purchased as, for example, when a city sells the right to construct and maintain a street car system or when the acquisition of a water-right involves the purchase of adverse riparian or other rights.

20. The net earnings of a public service property should in some measure exceed the return from ordinary safe investments.

21. The prospective business must be taken into account when the rates are to be fixed particularly in the case of a new enterprise, where the rate-payers are too few at the outset to produce, at a reasonable charge for the service, the revenue which would yield a proper current return on the investment.

22. The replacement requirement is to be determined from the expectancy (remaining years of usefulness), the probable life when new of similar articles and the estimated cost of effecting the replacement. It is not dependent on original cost nor age,

although original cost and age may frequently be valuable aids in making an estimate thereof.

23. Whenever the current replacement requirement or depreciation are to be estimated, due consideration must be given to the fact that the actual period of usefulness of perishable articles rarely coincides with their probable term of life.

CHAPTER IV

ESSENTIALS OF VALUE

Cost of Physical Elements

Supply and Demand. — The value of any property is usually determined by the law of supply and demand. This is perhaps best illustrated by reference to perishable property such as articles of food which can be preserved in marketable condition for a limited time only. According to information that may be available relating to the world's supply of wheat and of coffee the value of the wheat or coffee will be high or low. The harvest prospect in the case of such perishable elements has a direct effect upon the market value. Or as an additional illustration take the limited output of a great sculptor or artist. The demand, particularly after the sculptor or artist has ceased to produce, may greatly exceed the supply and in such case the market value may rise far in excess of what would otherwise be the value.

The popular actor, or vocalist, the skillful musician, the poet and the author have something to offer in the world's market which is unique and may be in great demand, and therefore may command a return out of all proportion to the compensation for ordinary individual effort.

Cost of Physical Elements in Relation to Value. — Productions of this type which have value are not here under consideration. It is the revenue producing property which is to be discussed and to which the chapters of this volume are addressed. Nearly every such property is made up of both physical and intangible elements. Taken in the aggregate this combination of physical and intangible elements has "value" based upon earning capacity. This is a fundamental fact which if lost sight of may lead into confusion when the term " value " is used. Both the physical and the intangible elements which make up the entire property may have been acquired by the outlay of money or in exchange for other valuable property. Consequently the question of cost deserves consideration and may at times be found a fair guide in determining value. Usually the physical elements of any property in successful use are worth at least as much as it would cost to reproduce them in the condition in which they are found at the time of the valuation. At any rate a prudent purchaser would desire to know what this cost would be and he would be very apt to make it his starting point in determining what he could afford to pay for the property.

The first step therefore in practically every valuation is the determination of the cost of the physical elements which go to make up the property.

In making the valuation of the physical properties account must be taken:

1. Of the cost of replacing each inventoried item with a new equivalent article.

2. Of the probable remaining term of usefulness of the item.

3. Of the probable term of usefulness of the same or an equivalent new article.

4. Of the cost of replacing the article with an equivalent article at the time when the article will probably be discarded.

5. In the case of valuation for purchase or sale account should also be taken of the accrued depreciation, which will aid in determining the present value.

The Determination of Cost. — The appraiser will therefore begin with an inventory of the property which should be in sufficient detail to facilitate a cost estimate such as an engineer would make who is called upon to construct a duplicate of the property to be valued. He will call to aid in making this inventory the records of the owner and he may find that he need only verify or amplify a list of items already enumerated. He will then estimate the cost of reproduction and here again he should call to aid, if it is available, the record of the cost of construction. The actual cost, intelligent direction and supervision being assumed, is generally a better guide to the cost of accomplishing a result, than the estimate of the expert who, no matter how much he knows about present cost, must make certain assumptions relating to the past or the future which will always brand his conclusions as approximations, even though the approximations are in some cases to be accepted as being quite as dependable and as acceptable as though they were of absolute accuracy.

It is not proposed to introduce into this volume sample sheets enumerating all the various items that may be encountered in valuing the various classes of public utilities and industrial establishments. The blanks in use for these purposes are obtainable, at least for the utilities, from any of the public service commissions and will be found an excellent guide in listing and appraising the physical elements of value.

The Relation of Present Worth to the Cost of Replacement. — After the cost of reproduction has been determined the time must be estimated during which — everything considered — the article to be valued may reasonably be assumed to continue in service. With this time and the probable life new of articles of the same class it will, as elsewhere explained, be possible to estimate the relation which the present worth of the article bears to the cost of replacing it at the time it will probably be discarded. If now the cost of reproduction be so modified that it will represent the cost of replacing the article at the end of its period of usefulness and the present worth or condition factor is applied to this cost of replacement the result will be the present worth.

When the property to be valued includes land, rights-of-way or water-rights, regard should be had to the principles discussed in the chapters on the "Valuation of Land in Eminent Domain Proceedings" and the "Value of a Water-Right and of Reservoir Lands."

Extent of Detail Required. — It has become a common practice in valuing property for rate-fixing purposes to approximate the legitimate investment or the present value of physical properties by means of a close estimate of the cost of reproduction. Such estimates are a check upon the records of actual cost and are valuable even when used for no other purpose than as a check.

The fashion is growing of making the estimates of the cost of reproduction with attention to the minutest detail. These estimates are frequently made with more care and with more careful enumeration of items, of specials, fixtures and incidentals and of labor and overhead, than would satisfy a contractor bidding on the work. And when this is done, when the sleeves and elbows of the pipe lines have all been counted and weighed and fence posts have been pulled to determine their length, unit prices are applied — sometimes as they prevail at the date of the valuation but more frequently as they have prevailed throughout a series of years 2, 3, 5 or 10, in order to give weight in a measure to the conditions under which the plant came into being. An approximation results. Then, perhaps, the accrued depreciation and deferred maintenance are estimated by some method of approximation and are deducted and finally appreciation of value may be allowed and an addition is made for the "cost of establishing the business" for "going value" or for other intangible elements such as "franchise" or "water-rights." To the original approximation other approximations are added with a result not always entirely satisfactory.

Where this is the case, one may well pause to consider how far into detail the appraisal of the physical elements should go, and whether there are not many cases where actual cost, with such checks as may be necessary to determine whether the cost is strictly reasonable and legitimate, would serve as the best starting point when a rate-base is to be determined.

The expense of making an appraisal with strict attention to every possible detail is not always justified and it may, moreover, defeat its own purpose, because by the time that a first valuation is completed, as in the case of the railroad systems of the country, a revaluation may be necessary if the same standard of approximation to reconstruction cost is to be continuously maintained. At any rate, the public utility commissions of the country should see to it that useless work on these lines be not demanded. It is their province to fix the standard in this matter.

When rates are to be fixed, they are based on a forecast of what the earnings should be. The exact result of charging for the service rendered at any fixed rates cannot be known until the rates have been in effect for a time. A deficiency in the earnings is discouraging to the owner and makes him prone to give inferior service. He may be forced into the courts and will then incur legal and other expenses which in the end will be paid by the consumer. An accumulating surplus, on the other hand, should insure high-class service and can be used to safeguard the property against unforeseen accident and to make sure of its proper upkeep without giving to the owner, in the long run, any larger amount in dividends than he is legitimately entitled to receive. If there be error, it should be on the side of the adequate return, provided always that a limit is set to the rate of return on the investment and to the compensation for managing the business, and that the charge for the service rendered be a reasonable charge.

Overhead Expenses

Cost Includes Overhead Expenses. — Overhead expenses have already been defined. They include primarily the cost of management, engineering, interest during construction, taxes and insurance.

In the determination of " cost " whether the actual cost or the estimated cost of reproduction, whether at present day prices or at prices applying to a selected period or to a particular time, it will be essential to have due regard to the overhead expense. It is not enough to be able to state item for item what it would cost to reproduce each in its exact present condition of serviceability, but proper account should be taken of the amount of general expense, of the cost of management, engineering and supervision of the expenditures due to unforeseen conditions, of unprofitable expenditures, and of losses by accident, that would ordinarily be anticipated in the execution of similar work under the same general conditions. The finished plant if constructed under better than average conditions is entitled to a credit due to this favorable circumstance, and if constructed under adverse conditions, may not be entitled to an appraisal at full actual cost.

The determination of the allowance which should be made for overhead expense is therefore a matter of considerable importance as will be seen from the quotations presented here at some length, which will give a fair idea, based upon experience, of the actual overhead expense connected with many public enterprises.

The Special Committee of the Am. Soc. C. E. on the Valuation of Public Utilities is Cited on Overhead. — In reference to engineering as an overhead expense the Special Committee of the American Society of Civil Engineers on the Valuation of Public Utilities in the progress report which it submitted in January, 1914, says:

Am. Soc. C. E. Committee, Engineering. — "Under the head Engineering are usually included not only strictly engineering expenses, but those of other technical employes and of inspectors. It is difficult to draw the line between the preliminary engineering and that during construction, and it is probably best in most cases to include the part of the preliminary engineering expenses connected with the preparation of the final design of the works with those incurred during their construction.

"The percentage of the cost of the work represented by engineering differs with the character of the works and with the amount of care and skill exercised in their design and construction. On railroads it is commonly estimated that the engineering cost will amount to 5 per cent of the physical valuation of the property, exclusive of overhead charges. Statistics of the cost of engineering are available in connection with several municipal and other works as follows:

"*Metropolitan Water-Works,* Massachusetts. This property, costing to the end of 1912, $42,036,000, is to the extent of $15,-300,000 made up of old works purchased from the City of Boston and others, on which the engineering charge is not known, leaving $26,736,000, of which the engineering charge was $2,077,-000, equal to 7.77 per cent. Based on the total cost, exclusive of engineering, the percentage is 8.42. These amounts include both the preliminary engineering and that during construction.

"*New York Water-Works,* now in process of construction. The disbursements upon this work to the end of September, 1913, amounted to $103,178,000. The engineering expense directly attributable to the work under construction amounting to $10,050,000, equal to 9.78 per cent of the total disbursements for this work. In addition, there were engineering expenses relating to investigations of other drainage areas, which are only indirectly attributable to the work under construction, amounting to $394,000 or 0.38 per cent of the total disbursements, making the total for engineering 10.16 per cent. It is to be noted, however, that there is included in this case under the head "engineering" the cost of unusually extensive borings and investigations which were not included as an engineering charge upon the Metropolitan Water-Works. There is not included, however, the cost of expensive investigations by sinking shafts at the Hudson River, which, although originally charged to the engineering account, has been deducted because the shafts afterward became a part of the final construction.

"*Boston Subways.* The following table has been compiled from official reports:

"BOSTON TRANSIT COMMISSION

(1895–1912)

	Total cost.	Percentage of total cost.	Percentage of construction cost.
Subway:			
Engineering	$407,475.48	9.88	11.34
General expense (inc. commission)	131,681.87	3.19	3.68
Construction	3,586,002.33	86.93	100.00
Total	$4,125,159.68	100.00	
East Boston Tunnel:			
Engineering	$191,466.57	5.90	6.62
General expense	161,134.78	4.96	5.57
Construction	2,894,595.01	89.14	100.00
Total	$3,247,196.36	100.00	
Boston Tunnel and Subway:			
Engineering	$417,866.25	5.05	5.48
General expense	226,441.57	2.73	3.58
Construction	7,623,206.56	92.22	100.00
Total	$8,267,514.38	100.00	
Cambridge Connection:			
Engineering	$96,575.46	7.10	8.05
General expense	62,355.20	4.59	5.20
Construction	1,199,904.39	88.31	100.00
Total	$1,358,835.05	100:00	

"CHARLES RIVER BASIN COMMISSION, BOSTON

Dam, Lock, Embankments, Marginal Conduits, Etc., Completed 1909

			Total cost.	Percentage of total cost.	Percentage of construction cost.
Administration			$94,011.97	2.6	3.2
Engineering			446,096.03	12.4	15.5
Construction:					
Preliminary	$10,783.86	0.3	....	......	......
Contracts	2,629,671.95	73.1	....	......	......
Additional	237,433.22	6.6	....	......	......
			2,877,889.03	80.0	100.0
Real estate			179,730.77	5 0	6.2
			$3,597,727.80	100.00	

"*Pennsylvania Railroad Tunnels.* — The cost of engineering for the East River Division of the Pennsylvania Railroad tunnels amounted to 5.8 per cent of the total cost of the work, including excavation, retaining walls around the station area in New York, the tunnels eastward under the streets and East River to the surface in Long Island, terminal yard in Long Island, and engineering. This is equivalent to 6.1 per cent of the cost, exclusive of engineering.

"KENNEBEC WATER DISTRICT, MAINE

New Gravity Water Supply, with Auxiliary Steam Pumping Plant, 1906

	Total cost.	Percentage of total cost.	Percentage of construction cost.
Administration	$6,161.20	2.30	2.53
Engineering	17,872.35	6.69	7.35
Construction:			
Rights of way	926.97	0.36	100.00
Pumping station	20,833.39	7.80	
Steam plant	6,275.59	2.34	
China Lake pipe line	215,302.93	80.51	
Total	$267,408.43	100.00	

"LOUISVILLE, KY., SEWERAGE WORKS, 1906 TO 1912

This Work Consisted of Intercepting Sewers, Trunk Sewers, and a Small Proportion of Lateral Sewers

	Total cost.	Percentage of total expenditures.	Percentage of payments to contractors.
Administration	$78,025.03	2.09	2.37
Engineering	336,544.87	9.00	10.23
Rights of way	12,319.36	0.33	0.37
Castings and other metal work	15,461.29	0.41	0.47
Damage suits (exc. of rights of way)	8,307.52	0.22	0.25
Amounts of payments to contractors	3,289,330.89	87.95	100.00
	$3,739,988.96	100.00	

The above total is exclusive of the cost of preliminary engineering, which involved an expense of approximately $57,000, equivalent to 1.7 per cent of the construction cost ($3,289,330.89) shown above.

"*Watertown.* — Mechanical filters built in 1903; cost, \$97,065; engineering cost, 5.9 per cent.

"*Ogdensburg.* — Sand filters built in 1910; cost of work \$167,694; engineering charges, 7 per cent, excluding cost of preliminary report, which amounted to \$500.

"*Hudson River State Hospital.* — Sand filter plant built in 1904; cost \$36,000; total cost of engineering, 10½ per cent.

"*Peekskill.* — Sand filters built in 1908; total cost, \$63,304; cost of engineering, 7.1 per cent.

"*Yonkers.* — Open sand filters, built 1903; total cost, \$50,165; total cost of engineering, 7.3 per cent. Covered sand filters, built in 1907; total cost, \$106,708; cost of engineering, 8.7 per cent.

"*Ithaca, N. Y.* — Filters built in 1903 on a percentage basis under rush conditions; cost, \$192,114; engineering cost, 7 per cent.

"*Springfield, Mass.* — Water-works built 1910; construction of additional supply from Little River, including diversion works, reservoir, filters, and pipe lines; cost \$1,465,393; cost of engineering, 10 per cent. The basis of computation does not include the sum of \$268,000 paid for land, legal and other expenses.

"*Springfield, Mass.* — Ludlow filters; built in 1906 at a cost of \$43,306, to meet an emergency, and requiring very rigid inspection to secure proper grade of sand and proper sanitary conditions during construction; cost of engineering, including board, livery, cots, bedding, and provisions for inspectors on work, 17 per cent.

"It is obvious that the cost of engineering varies with the character of the work. For instance, the construction of an important dam or aqueduct, built in place and requiring skill in designing and a careful inspection of every part of the work as it is built, requires a larger expenditure for engineering than a large cast-iron pipe line where the cost of laying the pipes in a trench is but a small percentage of the total cost of the line, and the work progresses so rapidly that the inspection cost is small in proportion to the total cost.

"The cost of engineering varies not only with the class of work but with the character of the design and execution. For instance, works may be built with very little inspection, from crude designs prepared by unskilled engineers, with the result that the cost of works may be large although the percentage paid for engineering may be small. Works skilfully designed and efficiently constructed necessarily involve a larger cost for engineering, which should be recognized in any valuation when the works give evidence of such skill and efficiency.

"The cost of engineering for additions and extensions of a property may be as large, or even a larger percentage of the total cost than for the original plant, but under a continuous system of regulation engineering for additions and extensions, if charged to current expense, should not be included in the valuation.

"It is sometimes suggested that no engineering charge should be made in the acquisition of real estate, and there may be some cases where such a view would be substantially correct. It is frequently the case, however, especially where strips of land are to be acquired, that the engineering cost is as great as for other portions of the work.

"The record of the cost of the Wachusett Reservoir, where the engineers had little to do with the acquisition of land other than to prepare the plans of the lands and to make appraisals of the mill property and water-rights, amounted to about 3 per cent of the total cost of the property acquired. The percentage would have been considerably larger if nearly all of the property had not been acquired by purchase and very little by condemnation proceedings.

"The records of the New York Water-Works show that the engineering connected with the acquisition of land to the end of September, 1913, amounted to 1.87 per cent of the total expenditures on land account."

Am. Soc. C. E. Committee, General Expense. — The same committee in referring to general expenses says that these include:

"All administrative, legal and other general expenses during the period of construction. Such expenses include general office rent, the salaries of the officers of the corporation and of the secretary, treasurer, legal advisers, clerks and others. Statistics showing the amount of such expenses in the case of corporations are not available to the Committee. The statistics of municipal public service properties throw some light upon the subject, although generally defective in that the offices are frequently in public buildings where no rent is charged and the financial and legal duties are performed to a large extent by those who receive salaries paid out of general funds.

"On the Massachusetts Metropolitan Water-Works, the administrative and expert services, with a small portion of the legal services, amounted to 1.43 per cent of the cost of the work, an unusually low figure, as there was very little litigation in connection with the work.

"On the New York Water-Works the strictly administrative expenses to the end of September, 1913, were 1.12 per cent of the total disbursements to that date, but in addition, advertising and the fees of special counsel and commissioners of appraisal to the end of 1912, all in connection with the acquisition of land, amounted to $2,966,000, equivalent to 3.39 per cent of the total disbursements to that date, making the total for general expenses, exclusive of police services, 4.51 per cent.

"On the Boston subways the general expenses, as given by accounts, have amounted to 3.42 per cent of the total, but some items usually classed as general expenses were charged directly to the various sections of the work.

"It should be remembered that in all cases above cited, a part of the general expenses have been paid out of general funds and are not included in the above percentages.

"In modern works in populated sections of the country there is a strong tendency toward an increase in general expenses, owing to the greater attention paid to policing and sanitation where large bodies of men are employed.

"On the Metropolitan Water-Works the charge for police services amounted to $211,000 for works costing $26,737,000, equal to 0.79 per cent.

"On the New York Water-Works to the end of 1912, the total disbursements amounted to $87,551,000, of which $1,369,000 was for police, equal to 1.56 per cent of the total.

"The Committee believes that original conditions should be considered in determining the proper percentage for general expenses and that the allowance above suggested for police and sanitation should be included only when similar expenditures were actually made in the creation of the property under consideration."

Am. Soc. C. E. Committee, Contingencies. — "In making an estimate of the cost of a projected undertaking, the experienced engineer adopts a policy of liberality with the intention of reaching the probable actual cost of the proposed work, and even under such circumstances the actual cost is as likely to exceed as to run below the estimate.

"This policy of liberality includes an addition to the computed theoretical or geometrical quantities in all cases where the actual quantities are likely to be greater, the adoption of liberal rather than minimum prices for the various items of work, and a further allowance for contingencies. If the plans are incomplete, so that many minor features are omitted, he properly adds more for omissions and contingencies than where the plans are in greater detail.

" In the valuation of a public service property, the same ideas should be kept in view and the percentage or sum to be allowed for contingencies should be governed, to a considerable extent, by the completeness of the inventory and the amount already allowed for omissions, by the extent to which additions have been made to the computed theoretical quantities and by the degree of liberality of the prices affixed to the various items of the inventory, but in no case should the contingencies be omitted or reduced to a small figure. Large contingent expenses are necessarily incurred in practically all important public works. They may occur from very many causes, among which may be enumerated the failure of contractors and the cost and legal expenses incident thereto; to the delay of certain parts of the work caused by such failure; to injunctions or to the inability to obtain possession of the land in due season, thereby necessitating the execution of such portions of the work under winter conditions or other adverse circumstances; to stringencies in the money market, causing a temporary shortage of funds, and a consequent disorganization of the forces employed on the work; to protracted strikes; to the necessity of rebuilding parts of the work which have failed because of improper design or unforeseen causes; to making alterations found to be necessary or desirable after the work is built; to the slipping of earth or rock; and to making changes in plans which increase the cost of the work.

" Such contingent expenses as those above enumerated, with the exception of the last two, should be included as contingent expenses in any valuation of an existing property because the existence of the property does not give any clew to the amount of most of the contingent expenses involved in its creation."

Am. Soc. C. E. Committee, Insurance and Risk. — "There is another subject closely related to contingencies which may be classed as insurance or risk. For instance, if an owner of property is constructing a building he runs the risk that it may be burned. If a fire occurs when a building is nearly finished and the owner has to rebuild it, the cost of the completed structure, if he has no insurance, will be nearly that of two buildings. In estimating the reproduction cost of the building, only one building would appear on the inventory, but in estimating its value, there should be added to the reproduction cost of the single building as otherwise determined, the sum necessary to insure its whole value against fire, and this sum should be added whether the owner actually paid it to an insurance company for carrying the risk or whether he assumed the risk himself. Similarly any other property which involves risk during its construction and testing should have added a contingent sum representing what the cost of insurance would be were there insurance companies to insure against such risks.

" It is seldom that a large public service property is examined that there is not a disclosure of some large expenditure for works which have been destroyed, reconstructed, or not used because of faulty design or construction. In some cases this is distinctly the result of negligence, but in a majority of cases such expenditures have taken place where the owners were not negligent in that they have taken due care in the selection of engineers to design and contractors to construct the work. They are the result of human fallibility.

" Many examples of failures have been furnished by masonry dams. A considerable portion of the Quebec bridge failed while in process of construction; there was a great slip in the Necaxa Dam, one of the highest earth dams ever built, when it was far advanced toward completion; the Loetschberg tunnel in Switzerland encountered bad ground, which required the abandoning of the tunnel for about a mile and its relocation through another portion of the mountain; the change in plan of the new Croton Dam of the New York Water-Works involved an additional expense of more than $1,000,000 for construction and interest. These are only a few instances of many which might be cited. Such disastrous occurrences are not contemplated by engineers when they make a provision for contingencies in preliminary estimates of the cost of works, but it seems proper in the valuation of an existing property which has been completed and successfully tested to recognize that the owner has been required to assume the risk of accident and failure and should be compensated therefor by at least the amount which insurance companies would charge for taking such risks were they doing this kind of business. This feature may properly be included in the valuation by increasing the amount allowed for contingencies and risks."

Am. Soc. C. E. Committee, Interest and Taxes. — Referring to interest and taxes the Committee says:

"Interest upon the capital invested in the plant up to the time when it is first operated and begins to have earning capacity is an unavoidable expense.

"Under the most favorable conditions it is necessary to raise the money required for the construction of the work months in advance of its expenditure and in many cases the whole amount must be raised before beginning the work in order to insure against a suspension of operations with the large loss necessarily incident thereto.

"The rate allowed for interest during the construction should be the prevailing rate at the time of the valuation, having regard, however, to the variations in rate in different localities and the character of the property. The rate of interest on money required for the original plant will generally be larger than on that required for extensions and betterments as the corporation naturally has a stronger financial standing after it possesses a successful operating plant.

"The amount to be allowed for taxes during construction must be determined largely on the basis of local tax rates and other local conditions and in this case, as in others relating to overhead charges, the original plant and subsequent additions should be treated separately."

Overhead in Railroad Construction. — While as in the case of other items of cost the records of expenditures may be the best guide when overhead charges of an operating property are to be determined, they are not always to be accepted without question; the appraiser must know whether they are reasonable or not. It is for this reason that such data as above presented are of interest to the appraiser. Following this matter further it may be noted that in valuing the railroads of the State of Washington Mr. H. P. Gillette ascertained from their records of cost that their overhead costs were as shown in the following table:

TABLE 1. OVERHEAD COST ITEMS

Railroads in the State of Washington. By H. P. Gillette

	Gt. N. 955 M., per cent.	Gt. N. 488 M., per cent.	W. and Gt. N. 84 M., per cent.	F. & S. Ry. 32 M., per cent.	S. & N. Ry. 132 M., per cent.	N. P. R. R. 1654 M., per cent.	O. R. & N. 501 M., per cent.
Engineering	2.50	3.23	4.40	3.56	3.50	5.51	2.83
General expense	0.28	0.26	0.08	....	1.00	1.22	0.48
Legal expense	....	....	....	3.55	....	0.01	0.02
Insurance	....	....	0.01	0.06	....	....	....
Interest on advances	0.93	1.23	3.25	....	....	....	....
Bond interest during construction	2.44	3.84	....	....	5.25	13.60	2.61
Bond expense	0.10	0.18	....	....	....	....	....
Taxes	0.02	....	0.17	....	....	....	0.05
Undistributed accounts	....	....	....	....	....	0.91	2.64
Totals	6.27	8.74	7.91	7.17	9.75	21.25	8.63

Gt. N. = Great Northern.
W. & Gt. N. = Washington & Great Northern.
F. & S. = Fairhaven & Southern.
S. & N. = Spokane Falls & Northern.
N. P. = Northern Pacific.
O. R. & N. = Oregon Railway & Navigation Co.

In valuing the railroads of Michigan, Prof. M. E. Cooley who had charge of this work adopted the following percentages in estimating the reasonable allowance for overhead costs:

	Per cent.
Engineering	4.00
Legal expense	0.50
Organization expense	1.50
Interest during construction	3.00
	9.00

The Wisconsin Railroad Commission on Overhead Allowances

The following information is found in the records of the Wisconsin Railroad Commission:

In Hill et al *vs.* Antigo Water Co. (decided Aug. 3, 1909). Overhead charges in the case of water companies are usually 10 to 18 per cent. The staff engineers estimated in this case at 10 per cent or the minimum (engineering, 5 per cent; interest during construction, 3 per cent; and contingencies, 2 per cent).

In City of Ripon *vs.* Ripon Light and Water Co. (decided March 28, 1910) the staff allowed 12 per cent (5 per cent being for engineering and superintendence, 4 per cent interest during construction and 3 per cent legal expenses, organization expense and casualties). The respondent claimed 6 per cent interest during construction.

In Dick et al *vs.* Madison Water Commission (decided Nov. 11, 1910) the staff allowed 10 per cent instead of the usual 12 per cent because the plant is a municipal plant.

In State Journal Printing Co. *vs.* Madison Gas and Electric Co. (decided March 8, 1910), engineering was estimated at 5 per cent on the cost of the original physical plant, but not on additions to the plant. The staff of the Commission assumed that construction extended throughout one year and allowed 3 per cent for interest but subsequently changed to 4 per cent as the construction period may have been underestimated. The respondent contended for 6 per cent interest during construction, claiming a 2-year period of construction.

In the tentative valuation the staff had allowed 2 per cent for various items including contingencies, omissions, casualties, legal and organization expenses but on revision allowed 3 per cent. The respondent claimed through various experts 4 to 7 per cent, the larger amount being based on 2 per cent for omissions and contingencies, 3 per cent for legal and organization expenses and 2 per cent for casualty liability.

The Commission in this case says it is fair "that rates charged for the service rendered be fixed at a level that is high enough to cover all reasonable costs."

In City of Racine *vs.* Racine Gas Light Co. (decided Jan. 21, 1911) the staff allowed 12 per cent for interest during construction, engineering contingencies, etc.

In the application of the LaCrosse Gas and Electric Co. the staff allowed 12 per cent while the petitioner claimed 15 per cent for interest during construction, engineering and contingencies.

In City of Milwaukee *vs.* Milwaukee Electric Railway and Lighting Co. (decided Aug. 23, 1912) 3 Cent Fare Case, the respondent contended that 22 per cent for overhead is not excessive and that there are cases in which such additions have been as high as 51 per cent.

Mr. W. D. Pence of the engineering staff of the Commission testifying to an appraisal of the property made in 1907 says the basis was:

Four per cent for engineering and superintendence.

Two per cent for organization and legal expense.

Three per cent for contingencies.

Three per cent for interest during construction.

Mr. Beggs, the manager of the Company, stated that contractor's profit would be at least 15 per cent of cost of labor and materials. (The Commission included contractor's profit in unit prices.)

Mr. Beggs also claimed that interest during construction could not be less than 5 to $7\frac{1}{2}$ per cent and engineering from 5 to $6\frac{1}{2}$ per cent.

Prof. M. E. Cooley in revising the staff's figures allowed 5 per cent for contingencies during construction on all items except land, furniture, etc., besides 5 per cent for contingent omissions on the investment not including land, cars and car equipment; 4 per cent for engineering, $\frac{1}{2}$ per cent for insurance, $2\frac{1}{2}$ per cent for organization and legal expense, and 6 per cent interest; total 22 per cent.

Mr. M. G. Starrett on behalf of the Company claimed 4 per cent for engineering and superintendence, 2 per cent for organization and legal expense, 3 per cent for interest, 5 per cent for contingencies of construction, 9 per cent discount on bonds and 1 per cent for working capital; total 24 per cent.

The Commission as the result of a study of 30 cases of actual expenditures finds a range of 4.5 to 23.43 per cent and an average of 10.13 per cent for engineering, superintendence, organization and legal expense and interest during construction. This does not include contingencies. Some of the individual percentages were low because a portion of the cost of supervising extensions was charged to construction.

In *re* Purchase of the Oshkosh Water-Works plant (Sept. 17, 1913) the City estimated 10 per cent for engineering, superintendence, interest during construction, contingencies, etc., this on the physical plant not including general equipment. The Commission allowed 15 per cent and said:

"It will be noted that the valuation recently placed on the Oshkosh Water-Works property contained an item of 15 per cent for engineering, superintendence, interest, organization, etc. It will also be noted that the report of the previous valuation only contained 12 per cent to cover these items. It should be stated that this change from 12 to 15 per cent represents the results of an extended study of the costs of such work and it is my opinion that where the actual costs are not known for such a property as the Oshkosh Water-Works, 15 per cent of the cost of specific construction represents about a fair amount for these overhead charges."

In *re* Application Manitowoc Gas Co. (Dec. 4, 1913) the Commission allowed 15 per cent exclusive of contractor's profits which should be included in the separate cost items.

In *re* Investigation Ashland Water Co. (July 10, 1914) the Commission allowed 15 per cent.

Promotion Expenses

Promotion Expenses. — There is an additional class of expenditures incurred in connection with many of the public service and some private enterprises which has a more remote connection with the cost of these enterprises than the overhead or the cost of developing the business. Capital is timid and needs guidance. Preceding nearly every investment of capital in the development of an enterprise of any magnitude the same has been in the hands of the promoter whose investigations and efforts are supposed to have been directed to a unification or combination of properties that will make them available as the basis for the enterprise. The reward to the promoter should appear in the enhanced value of the unified property when compared with the cost of the individual items that go to make it up. But this is not all that there is to promotion expense.

Advertising may be necessary to find a market for the securities whose sale is to furnish the money with which construction will be undertaken. A commission may have to be paid to the agent who buys the properties or who markets the securities. The securities may have to be sold at a discount so that the indebtedness on which interest is paid may exceed the sum that was actually required to construct and acquire all the essential property. Promotion expense of this character does not usually appear in any valuation, nor in the estimate of the investment, though it may nevertheless have been an essential element of cost. The necessity which may have compelled the incurrence of a promotion expense may be deplored and yet it must be admitted that, on the whole, society has benefitted by the promoters' activity and by the carrying out of many enterprises which were not at the outset so attractive that bonds based thereon would sell at par or command a premium.

In this connection it may be noted that the Wisconsin R. R. Commission holds that the cost of marketing bonds should be taken into account as a promotion expense but that all discounts do not necessarily become proper additions to physical value.

The appraiser whether of value or of the investment will rarely be required to make a close estimate of the promotion expense, but the fact that under the prevailing conditions there has generally been some legitimate promotion expense which may not appear in the valuation, should not be overlooked when rates are fixed.

Intangible Elements of Value

The Total Intangible Value. — Whenever the net return exceeds a fair interest rate on the capital invested in the physical elements of any enterprise, there will be other elements of value in the same, not represented by the cost of any physical portion thereof. These values being apart from the actual investment in physical properties are usually regarded as intangible values. Ordinarily the sum of all intangible values will be the capitalized net annual earnings, less the investment in the physical properties (if the accounting system throughout has been proper and the enterprise has met with no untoward experiences) or what may generally be a safer approximation, the capitalized net earnings, less the cost of reproducing the physical properties.

A taxicab concern, for example, has invested in the business $100,000. The net annual earnings after allowing for depreciation are $30,000. If it be assumed that 10 per cent per annum is a reasonable return on an investment of this character and permanency of the business may be assumed the capitalization of the concern might be at $300,000. The good-will of this taxicab business appears at $200,000. That is to say, if there were no fear of a reduction of income through competition or other causes, a valuation three times as great as the actual investment might be justified. In such a case the sum of all intangible values connected with the business which may include besides what is strictly good-will, advantageous leases on desirable space for the taxi stands and contracts with railroads or other transportation companies, is the capitalization of the net earnings of $30,000 per year, or $300,000, less the value of the physical properties assumed as above stated to be $100,000.

Ordinarily, however, in a business of this character, the earnings may at any time be cut down by competition. Any intending purchaser taking this into account as a hazard of the business will conclude that, while earnings of 10 per cent a year on the value of the tangible property may be adequate, the return on the value of uncertain intangible elements should be very much greater.

He may find circumstances that will justify him in concluding that the return on any allowance for good-will should be 20 per cent and in this event he will find:

Total annual net earnings	$ 30,000
Ten per cent of value of physical properties	10,000
Net annual return on intangible values	$ 20,000
Capitalization of $20,000 per year at 20 per cent	$100,000

Consequently, the amount which he, as a prudent purchaser, would be willing to pay for the business would be only $200,000 instead of $300,000. In other words, $300,000 would be an over-capitalization in view of the risks of the business, and an adequate annual net return from a business of this character should exceed 10 per cent on the total value thereof.

As another illustration of relatively high intangible value, a newspaper route may be cited. A thousand subscribers, at $4 each, would make the value of this route $4000. The investment in physical elements such as a cart and horse may be insignificant. There are, no doubt, many such routes where the bicycles and hand carts in use by the delivery boys are furnished by these boys and not by the owner of the route. Practically the entire value of such a route is represented by good-will.

Intangible Value Under a Restricted Franchise. — When public utilities are under consideration, the problem may become quite complex and each case will have to be considered by itself. A restricted or limited franchise, for example, may make a monopoly of the enterprise, and its terms may be such that charges for service cannot be reduced by any rate-fixing body during the life of the franchise. Under such circumstances, when the volume of business is determinable, it is possible to forecast with some degree of certainty, on the basis of past experience, the annual net earnings and a capitalization of these at a fair rate of interest leads directly to a determination of value. A comparison between value thus ascertained and the cost of reproduction of the physical elements establishes the aggregate value of all intangible elements of whatever nature connected with the utility. Of course, the capitalization of earnings, as here set forth, may not in all cases be a simple matter, because many factors are to be taken into account:

The allowance for replacements must be correctly determined.

The hazards of the business must not be overlooked.

The value of the properties remaining on hand subject to sale at the termination of the franchise may be a factor of some moment in determining how much of the investment is to be amortized within the life of the franchise.

Nevertheless, the fact remains that the restricted, exclusive franchise usually affords a definite basis for estimating its value. When the rates for service rendered or for commodity furnished are subject to regulation by public service commissions or other similar rate-fixing bodies, this is not so, at least not until rates thus regulated become dependable for a number of years and the policy to be pursued by such bodies in the discharge of their duties can be forecast with confidence.

Over-capitalization. — The cases have been so frequent where exorbitant earnings, real or in some measure fictitious, have been made the basis of enormous over-capitalization of enterprises both private and quasi-public in character, that the general public leans strongly toward extreme restriction of earnings whenever the law and circumstances and the power of the rate-fixing bodies will permit of such restriction. Why allow any public utility to earn more than is ordinarily earned by a safe investment? Why allow anything for unprofitable investments even though made under good expert advice? Why take into account any lean years of the past? If the utility was not profitable or is not profitable, why should not the loss fall upon those whose poor business judgment led them into a losing venture? Such questions and others of similar character deserve serious consideration when rates are to be established, which are to be fair alike to the owner and to the rate-payer.

Kansas City Water-Works Case and Intangible Value.—Justice Brewer, in stating the conclusions of the Court in the Kansas City Water-Works Case (Circuit Court of Appeals, Eighth Circuit, July 2, 1894, 62 Feb. Rep. 853), says:

"A completed system of waterworks, such as the company has, without a single connection between the pipes in the streets and the buildings of the city, would be a property of much less value than that system connected, as it is, with so many buildings, and earning, in consequence thereof, the money which it does earn. The fact that it is a system in operation, not only with a capacity to supply the city, but actually supplying many buildings in the city — not only with a capacity to earn but actually earning — makes it true that 'the fair and equitable value' is something in excess of the cost of reproduction. The fact that the company does not own the connections between the pipes in the streets and the buildings — such connections being the property of the individual property owners — does not militate against the proposition last stated, for who would care to buy, or at least give a large price for a waterworks system without a single connection between the pipes in the streets and the buildings adjacent. . . . It (the city) should pay therefore not merely the value of a system which might be made to earn, but that of a system which does earn."

According to this view of the court there is no question about the inclusion of intangible elements among the properties which have value. The Supreme Judicial Court of Maine in its instructions to the appraisers of the properties of the Maine Water Co. (1902) in the Kennebec Water District Case (97 Maine 185; 54 Atlantic 6) said:

"In addition to structure values, the appraisers should allow just compensation for all the franchises, rights, and privileges to be taken."

Here, too, the court recognizes the fact that franchises and privileges may have value.

Intangibles as a Protection to the Owner. — The courts and the public service commissions must protect the investor whose enterprise is developing the latent resources of the country and who is, therefore, to be encouraged, and they must at the same time prevent as far as may be the robbing of the public by over-capitalization and over-bonding with consequent over-charging for the service rendered. But in protecting the public they should not lose sight of the fact that the owner of the utility is entitled to fair compensation for his time and his business ability, and for the risks which he assumed in embarking upon a venture for general benefit.

To do this adequately, in conformity with the rulings of public service commissions and the decisions of the courts, the appraiser has frequently had recourse to intangible elements of value under such names as "unification of properties," "going value," "solidification of road-bed," "appreciation of land" or of "conduit values due to paving," "cost of establishing business" or under some other designation. Such an addition of intangible value under the name of "going value" based on the losses during the early years of operation has been seriously advocated. If regard should be had in this connection to the exact amount in each case of early operating losses, of unprofitable investments, such as tunnels or wells for water which turned out to be unproductive; destruction by fire or earthquake and the like, then the allowance for unprofitable expenditures might be greatest in the case of the least worthy enterprise, which would be an absurdity. Nevertheless, some allowance for unproductive expenditures and for early losses is frequently justified and these may be taken care of either in the appraisal of the rate-base or they may be taken care of in the rate of return which when fixed somewhat higher than the return on ordinary safe investments will in the course of time amortize a part or all of the expenditures which do not appear in the rate-base. Under such a procedure there is a recognition of the fact that early losses may have been unavoidable, that no blame may attach for having made certain unprofitable expenditures and that the owner who has invested wisely and under more fortunate circumstances is entitled to the reward which will be brought to him either by giving suitable consideration to "going value" or by allowing him to earn more than ordinary interest on his actual investment.

Example of Intangible Value Created by Earnings. — Take as an illustration the case of a property operated at a loss for 5 years and thereafter at a profit. Suppose that the investment in the property was $1,000,000 before operation commenced; that cost of operation exceeded the earnings in the first five years of operation by $100,000 and that money for the entire investment had been borrowed at 6 per cent. During the five years the interest payments, compounded at 6 per cent, amounted to $338,220, which, together with the operating loss, makes $438,220 as the actual outlay by the owner at the close of the fifth year in addition to the original investment of $1,000,000.

No compensation for hazard or management is included in the foregoing figures. The property as assumed will yield more than operating expenses after the first five years. Unless the excess over operating expenses is more than 6 per cent on the total outlay of $1,438,220 the owner will still be conducting business at a loss and unless it is sufficiently in excess of 6 per cent to compensate him adequately for management and risk he will not realize all that he had a right to expect.

By reason of increase of population increased demand for his output, or for the service which is rendered by the utility it may be possible after the first five years to reproduce the utility or to construct a substitutional plant with established business at a less cost than $1,438,220.

The question is how to determine what will be fair earnings. Two procedures are open:

a. The actual cost of developing the business may be added to the cost of reproducing the physical plant and the sum approximating $1,438,220 may be introduced into the calculation as the rate-base.

b. The cost of reproducing the physical plant together with actual cost of franchises, water-rights or rights-of-way or about $1,000,000 is made the rate-base, and the cost of developing the business, in this case approximately $438,220, is estimated and treated as a business loss subject to amortization in a reasonable number of years.

Any allowance less than will result from these procedures would be confiscation of a part of the investment and therefore unfair to the owner, who is in this illustration assumed to have used good judgment in undertaking and developing the enterprise.

Kennebec Water District Case on "Going Concern." — In this connection, too, the instructions issued by the court in the Kennebec Water District Case to the appraisers of the Maine Water Co. (97 Main 185; 54 Altantic 6) may be cited:

"In estimating even the structure value of the plant, allowance should be made for the fact, if proved, that the company's water system is a going concern, with a profitable business established, and with a present income assured and now being earned."

" Going Value " and " the Going Concern." — A distinction is to be made between the " value of the going concern " and " going value." The value of the going concern is the total value of the entire property, its market value. " Going value " is the increment of value due to the fact that the business is established and in successful operation. It is at least equal to the reasonable cost of establishing the business, including a reasonable allowance for losses in early, lean business years. If assured earnings are large when compared with the operating cost it may greatly exceed early losses just as value may exceed cost. In the case of unprofitable business, as when a company's properties are in a receiver's hand, going value will be only nominal.

It sometimes happens that the public is responsible in some measure for inadequate earnings as in the case of competing utilities. By permitting the competition and a duplication of works, the cost of bringing the business up to a profitable basis is increased and may be the cause of increased losses.

Going Value in the Decisions of the Wisconsin R. R. Commission. — On the subject of " going value " the Wisconsin Railroad Commission in the Cashton Light and Power Co. case (Wis. R. C. R., Vol. 3, p. 85) declares this to be an element of value which must be taken into account. The Commission after stating that it is akin to " good-will " refers to Justice Brewer's views in National Waterworks Co. *vs.* Kansas City (62 Fed. Rep. 853) and then cites the opinion in Cedar Rapids Water Co. *vs.* Cedar Rapids (118 Iowa, 234) to the effect that " going value " is " that value which arises from having an established going business . . . and attaches to the business rather than the property employed in such business."

The Commission takes a sound view when it says in the Antigo Water Co. case (Wis. R. C. R., Vol. 3, p. 707) that the question as to whether deficits resulting from operation, or due to other causes, should be regarded as investment, will depend upon the circumstances of each case.

The Commission discusses methods of ascertaining the "cost of establishing the business" and "going value" in Green Bay *vs.* Green Bay Water Co. (Wis. R. C. R., Vol. 11, p. 243–252), and in this case says:

"The method which is generally followed by the Commission aims to determine, as far as possible, what the actual cost of developing the business in question has been, and to what extent, if at all, such losses have been recovered in later years of operation. There are a number of difficulties in determining, by this method, what the cost of building up a business has been, among which may be mentioned:

"1. Entire or partial lack of records covering the development period.

"2. Difficulty of finding original cost of physical plant.

"3. Difficulty of eliminating from reported operating expenses amounts which are the results of extravagance, inefficiency, or other causes which tended to keep the costs above a nominal figure.

". . . This method, where it can be applied to its full extent, enables the investigator to determine what it has actually cost the utility in question to build up its business. This sum, added to the actual investment in the physical plant, gives the total amount which the plant and the business have actually cost. . . . Where it is impracticable to determine what the actual cost of the physical property has been the only method of arriving at the value of that property is to ascertain the cost of reproduction.

"The method of determining going value as followed by Mr. Alvord, and which, for the sake of convenience we will refer to as Alvord's method, is an attempt to fix the amount which it would cost to reconstruct the business of the utility, somewhat as a physical valuation reveals the cost of reconstructing the physical plant. There are two assumptions vital to this method:

"1. A city similar in all respects to the one under consideration, except that there is no public water supply system but in which the people are in a general way cognizant of the advantages of such a water supply.

"2. Capital seeking investment which may be either used to construct a plant and business in the city with no water supply or to purchase the existing plant and business.

"In a computation of going value according to Alvord's method the going value is the present worth of the amounts by which the net earnings of the comparative plant are less than the net earnings of the existing plant during the entire period from the date of the first preliminary work until the earnings of both plants are equal. . . . The loss to capital invested in the comparative plant is not to be measured by the extent to which its earnings during the construction period fall short of the net revenues of the existing plant, but rather by the amount by which they are less than the returns which have been foregone in order to enter the new field of investment. . . . The detailed computation of the cost of developing the amount of business necessary to yield a reasonable return upon the property involves a number of further assumptions which make the accuracy of the result very questionable."

Franchises and Related Matters

The Franchise Value. — Any privilege or right granted by legislative authority to engage in a particular business is a franchise.

Whenever the franchise or privilege to do business is an exclusive privilege and grants some latitude in the matter of the charges which may be made for the service rendered thereunder, it is an element that can be valued. It is well stated by the Supreme Judicial Court of Kennebec County, Maine, in instructions to appraisers that its value "depends upon its net earning power present and prospective, developed and capable of development, at reasonable rates, and the value to be assessed is the value to the seller and not to the buyer."

When the value of a franchise is in question, all circumstances that affect the earning powers of a property must be taken into account and by an analysis thereof the excess of the earnings over a fair return on the capital invested in physical properties must be determined. This excess of earnings when capitalized will give some idea of the total value of all intangible elements and may lead to the determination of the franchise value.

Frequently, the only stipulation in the franchise in the matter of rates is that these shall be fair and subject to regulation. In such cases the franchise value should not be made a part of a rate-base, determined by investment, except only to the extent that its acquisition has actually required legitimate investment, as is the case, for instance, when a city sells to a Street Railway Corporation the right to operate cars on certain streets for a certain period of time. The franchise value together with the value of other intangible elements is independent of the actual cost of these elements; but a franchise value may result from the establishment of rates which produce earnings in excess of reasonable interest return on the capital invested in the physical properties.

The ascertainment of franchise value and value of all other intangible elements connected with public utilities is similar to the ascertainment of the value of the good-will of a private business but with this difference. . . . In the case of the public utility there is usually protection against competition, while in the case of the good-will of a private business, competition or the possibility of competition is to be assumed and such competition may materially affect the value.

A franchise is sought for and operations thereunder are undertaken for profit. Sometimes, of course, the profit to the party who accepts the franchise and operates thereunder is an indirect one, as in the case of the railroads which make large land holdings valuable. While the privilege to do business is granted in order that certain property may be used for the benefit of the public, the person or corporation undertaking the business must be assumed to be doing so in the hope of reaping an adequate reward.

The Franchise Term. — In many of the states perpetual franchises have been granted. Rights to use water-powers, wharf and water-front privileges, the privilege to supply gas or water and the right to occupy streets for various purposes when thus granted without time limit may acquire relatively high value and may prove a serious obstacle to the ultimate rational development of natural resources. The perpetual franchise has, however, one feature in its favor. The operator thereunder can look with some confidence into the future and if circumstances justify, will find it to his interest and may be expected to make suitable provision for the growth and future demands of the community. Knowing that the prospective earnings justify this course, he can build far ahead of the immediate requirements. But this advantage is not of sufficient importance to justify the grant of perpetual privileges and such privileges are now rarely, if at all, granted by any of the states.

The term of a franchise should not be too short. When the evils of the perpetual franchise began to be fully realized, the next step was to the other extreme and terms were in some of the states restricted to 25 years. Usually several years are consumed in the construction of works, 5 to 10 years or more in the development of the business and thereafter there should be ample time left within which to make some profit. A term of 40 to 50 years would seem to be about right, not alone to encourage reasonable development under a franchise, but also to give the public opportunity to renew, at not too great intervals, the conditions subject to which the privilege is granted.

Amortization During the Life of the Franchise. — In any event suitable provision should always be made to amortize the capital invested either by a purchase on an agreed basis at the end of the franchise term or by the amortization of a part or all of the investment during the life of the franchise with a view to an acquisition of the property by the public. If the latter plan is followed, there may be danger of neglect in the last years of service, as it will be to the advantage of the owner to expend as little as possible in upkeep, provided only that he can continue rendering the service. The provision which must be made in such a case to amortize the investment may cover the entire term or only a portion of the franchise term.

The Indeterminate Franchise. — The indeterminate franchise is fast finding favor. No definite term of life is fixed but provision is made that at any time after a specified number of years, the community may take over the property either at an agreed price or at an appraised value. Usually the method of making the valuation is agreed upon in advance in order that there may be as few points of disagreement as possible. The indeterminate franchise may be granted subject to various conditions such, for example, as an allowance of 10 per cent or some other amount on the actual investment if the property is taken over within 10 years; or that after a certain number of years a part of the earnings will be turned over to the community; or that certain requirements relating to the character and quality of the service will be complied with.

The indeterminate franchise has not yet been fully tried out, but in those states in which suitable provision has been made for the regulation of rates, there is good reason to believe that it will prove satisfactory.

Capitalization of the Franchise. — The tendency has been to capitalize the value of the franchise, in other words, to use the franchise as a basis for the issuance of securities. Perhaps there is some reason for this in the case of a perpetual privilege, when thereunder the assured earnings exceed the ordinary fair interest return on other similar investments; but the capitalization of the franchise, except the actual cost thereof, is now quite generally prohibited by the laws which provide for the control and regulation of the public service corporations, and the decisions of the courts are adverse to such capitalization.

The Wisconsin Railroad Commission says on this subject in the Antigo Water Case (Aug. 3, 1909), "That if the municipality required the payment of money or its equivalent, or there was necessary legitimate payment made for the franchise, then the sum which may be reasonably said to have been paid for the franchise may be included in the valuation, the same as money necessarily invested in physical property. But the Commission refuses to consider the claim of some experts and corporations that franchises for which no money was paid may have 'intangible' values which should be considered in the making of rates."

The Public Service Commission Law of New York provides: "The Commission shall have no power to authorize the capitalization of any franchise to be a corporation or to authorize the capitalization of any franchise or the right to own, operate or enjoy any franchise whatsoever in excess of the amount (exclusive of any tax or annual charge) actually paid to the State or to a political subdivision thereof as the consideration for the grant of such franchise."

Appreciation and the Unearned Increment

Earnings Affect Value. — The earning power of a property determines its value. As its net earnings increase, its value increases. The earning power of public utilities, as a general rule, if rates remain undisturbed, increases as population density increases although not in the same ratio. The appreciation or increase of value which results when net earnings, in their relation to the investment, are increasing is the reverse of depreciation, but it follows no definite law and it cannot be forecast with that degree of certainty which can, with some reason, be claimed for depreciation.

The Unearned Increment. — Appreciation or increase of value without increase of investment is the unearned increment which results from the changing conditions of environment. Usefulness in service does not always decrease, but may increase with age. This may be the case with a dam or with a railway embankment, and this increase of usefulness when it can be expressed in terms of money, or value in exchange, represents appreciation.

Increase in the price of labor and materials, or a change in the conditions under which an enterprise was first established, may add to or may take from its value according to whether it would be more or less expensive to construct and establish the same enterprise under the altered conditions.

As a general proposition, it may be stated that values as expressed in terms of money are increasing. It is also true that, as a general rule, public utilities are to be included among the principal factors which are responsible for the growth of the community, and that, when viewed in this light, the owner of the utility is entitled to participate in the unearned increment just as the owner of land participates. He does so, of course, in a measure as his business increases, but if held down to earnings which will barely yield the ordinary interest rates on safe investments, the extent of doing this may fall far short of the advance in property value shared in by practically all the owners of the realty in the community.

Application to Public Utilities. — Such considerations as this, although not thus expressed, have led the U. S. Supreme Court to hold that the owner of a public utility is entitled, in most cases at any rate, to have the present value of his property made the basis of the computation when rates are to be fixed. In the Consolidated Gas Co. case (Wm. R. Willcox et al. *vs*. Consolidated Gas Co. of N. Y., 212 U. S. 19, 29 Sup. Ct. Rep. 192) as already quoted the court says:

> "And we concur with the court below in holding that the value of the property is to be determined as of the time when the inquiry is made regarding the rates. If the property which legally enters into the consideration of the question of rates has increased in value since it was acquired, the company is entitled to the benefit of such increase. This is at any rate the general rule. We do not say that there may not possibly be an exception to it where the property may have increased so enormously in value as to render a rate permitting a reasonable return upon such value unjust to the public."

The court has, perhaps, overlooked the fact that unearned increments are earnings and can be allowed without adding them to the investment. It may have been perfectly fair to make the valuation allowance to the full extent of the unearned increment in the Consolidated Gas Co. case and this allowance is not here made the subject of criticism. It is only the practical application of the ruling which is brought into question. It is believed that as these matters are better understood there will be a general acceptance of the view that appreciation had better not be included in the rate-base.

In the case of farm lands the situation may obtain of a greater supply thereof than can be made use of by the inhabitants of the country. The rental value is then low and the value is fixed rather by the supply of the desirable tracts of land than by the revenue which can be produced by cultivation. The unimproved farm and the unimproved town lot may be a source of expense instead of a source of income. The owner of such property expects appreciation to bring him a reward for having acquired and for holding the property. If he analyses his investment in such property after he has held it for a number of years, he may find that the first cost at compound interest, at savings bank rates, plus annual taxes amounts to more than other equally desirable property can be obtained for. His investment under such circumstances was not a judicious one, provided, of course, that the property while thus held was not income producing. But when real estate is held and is in use by the owner of a public utility, the intent is, whether always realized or not, to allow him to recover in the earnings at least interest on what this real estate has cost him. He is not in the position of the person who owns an unproductive piece of property. Nevertheless, as already stated, in order to share in the unearned increment which he has helped to create for the community he should be allowed, if the prosperity of the community justifies this course, to earn more than ordinary interest on his investment. In the Kennebec Water District Case, already cited, the court instructed the appraisers of the Maine Water Company's properties that " subject to all the foregoing limitations, the owner is entitled to any appreciation due to natural causes."

If, in any measure, appreciation goes to the owner of a public utility which includes among its properties land holdings, then in the case of other utilities which include no appreciating property there should be a like opportunity for profit. In the case of these other utilities the unearned increment cannot be measured by the appreciation of land. Appreciation does not increase the invested capital. It is not essential that appreciation of land or of any other property be added to the rate-base; but the owner of the utility is, nevertheless, entitled to a reasonable share in the general prosperity which he helps to create. This can best be allowed him, not in estimating actual appreciation, which is more or less uncertain and irregular, and, therefore, not always dependable, but in a suitable interest return on the original investment.

Interstate Commerce Commission Comments on U. S. Supreme Court Decision. — The difficulty of conforming to the decision of the United States Supreme Court in the matter of allowing the appreciation of real estate was felt by the Interstate Commerce Commission of the United States which says in its opinion in the Western Advanced Rate Case (20 I. C. C. Rep. 344, decided Feb. 22, 1911):

"Certainly if the Supreme Court may decline to lay down the absolute rule that 'in every case failure to produce some profit to those who have invested their money in the building of a road is conclusive that the tariff is unjust and unreasonable' (Reagan *vs.* Farmer Loan and Trust Co. 154 U. S. 412), it is a conservative statement of the law to hold that a railroad may not increase the rates upon a number of commodities solely because its real estate has risen in value."

"While it is evident, therefore, that each case must be decided upon the facts peculiar to it, the Commission believes it proper in this case to follow the general rule, as stated by Judge Hough of the United States Circuit Court (Consolidated Gas Co. *vs.* City of New York et al., 157 Fed. Rep. 849, 855), 'Upon reason, it seems clear that in solving this equation the plus and minus quantities should be equally considered, and appreciation and depreciation treated alike.' . . . Thus land has been taken at its fair value and not at its original cost, and the annual appreciation of land has been treated as a profit. By this method all property is treated absolutely alike, as Judge Hough suggests. No difference is made, except that as depreciation represents a decrease in assets, it is placed as a debit against operation, while appreciation is placed as a credit because it is an increase in assets."

Treatment of Appreciation and Depreciation. — The real difference between the way in which depreciation and appreciation should be treated has apparently been overlooked by the courts but recognized by the Interstate Commerce Commission. The lessening of worth is offset, or at any rate is intended to be offset, by an increase of the earnings. Depreciation then appears on both sides of the account. The owner of the property if fairly treated is no better nor worse off in consequence of deteriorating articles because they are made good to him as they deteriorate. But appreciation is usually not estimated from year to year and is not therefore entered with the annual revenue. When the occasional appraisal discloses appreciation, it appears as a profit. The Interstate Commerce Commission regards it as income. The United States Supreme Court holds that the owner of the public utility is entitled to the appreciation unless the same is excessive in amount in addition to a fair return upon what would, without the inclusion of appreciation, be the reasonable amount of capital invested. How much simpler it would be to grant to the utility some reasonable share in the general prosperity not measured solely by the increase in the value of the real estate which it happens to own and use in the public service.

The Secretary of the Treasury of the United States in decisions relating to the income tax says:

"Profits realized on the sale of real estate during the year, also increase of value of unsold property, if taken up on the books of the corporation, (are) to be included in income."

Appreciation and the Rate-Base. — In weighing the question whether or not appreciation is to be added to the rate-base, consideration may be given to the alternative of the rental value of equivalent property. Suppose for example that among the properties owned by a public utility there is a large tract of land located in a region in which real estate values are advancing normally. If instead of acquiring this land the owner of the utility had entered into a lease thereof based upon an agreement that the rent from year to year should be commensurate with a proper valuation of the land, the amount of the rent increasing from year to year would be included in the cost of operation and rates would be fixed as though, in the case of actual ownership, the rate-base had included appreciation.

While this circumstance may justify the inclusion of appreciation in the appraisal of the rate-base in some cases, there will be others where the appreciation is out of all proportion to the original investment. Furthermore, a strict analysis will show that such inclusion would deprive the rate-payer of any share in the unearned increment to which he, too, is a contributor. It is, moreover, as already stated, difficult and oftentimes impossible to ascertain the rate of appreciation which might fairly be taken into account and finally, if appreciation is included in the appraisal of land, it should also be included in the appraisal of all other kinds of property. Such inclusion would greatly and unnecessarily complicate every calculation relating to rates and would introduce an element of much uncertainty. It will be much simpler and yet fair to all concerned to exclude appreciation from the rate-base but, as already suggested, to allow the owner of every utility to participate in the general prosperity by allowing the rate of return on the rate-base to be higher than it would otherwise be estimated.

If this practice coupled with the Unlimited Life Method of procedure could be made general there would be no revision of a rate-base, once established, except as made necessary by additions to the utility properties or changes therein, nor would there be any need of determining present value, and yet the unearned increment would not be ignored. It would appear in the earnings and not in the rate-base, and the equitable apportionment of the unearned increment to the utility owner and to the rate-payer would be facilitated.

If the unearned increment (appreciation) is estimated from time to time and is treated as income and the total net return on the rate-base does not exceed ordinary interest rates on safe investments, the estimated appreciation should be added to the rate-base in fairness to the owner of the utility.

If appreciation is estimated from time to time and added to the rate-base but is not treated as income, the rate-payer will be denied any share thereof.

If appreciation is ignored, and a rate-base, once determined, is allowed to remain uninfluenced by appreciation, the owner of the public utility should be allowed a higher rate of return than if appreciation were included in the appraisal of the rate-base.

The endeavor should be to secure a general recognition of the latter principle and thereby secure the same liberal treatment for those utilities which do not include among their properties land holdings, as is now accorded with approval of the courts, to those which do include large amounts of real estate.

If treated in this way the owner will never be in a position to capitalize appreciation, unless the use of the property in the public service be abandoned. This is as it would be if the owner were acting as agent, and is equitable.

In the Minnesota Rate Cases, the United States Supreme Court apparently recognizes the broad principle that the public service property should earn a return upon the increasing value of its properties, provided the appreciation be properly ascertained. The Court in these cases says:

> "Assuming that the company is entitled to a reasonable share in the general prosperity of the communities which it serves, and thus to attribute to its property an increase in value, still the increase so allowed, apart from any improvements it may make, can not properly extend beyond the fair average of the normal market value of the land in the vicinity having a similar character."

It would be much better to let this principle, recognized generally by the courts when they say that present value is to be the basis of the calculations when rates are to be fixed, be applied, as already suggested, without reference to or without being dependent upon and restricted to the present value of land, so that the same share in the general prosperity would come to the owner of the utility which owns no real estate as to the other utility which owns broad acres.

Cost to Reproduce New Includes Appreciation. — When reproduction cost is used as a means of approximating the investment or the "rate-base," the basis of the calculation may be current or comparatively recent prices and the general conditions as they exist at the time of the appraisal. Such reproduction cost will include appreciation whether the same be enhanced value of real estate or increased cost of construction due to higher wages and higher prices of materials, or due to other conditions at variance with those which prevailed when the work was first done. Such appreciation comes up for consideration in the case of a pipe line which was laid before the street was paved but which is to be appraised after the street has been paved. The cost of reproduction in this case includes the cost of cutting through and replacing the pavement. This cost of reproduction may be some indication of present value but it is not a good measure of the investment, and at its best will be more or less unsatisfactory as a method of approximating the amount on which interest should be earned. Or a pipe which is to be valued may have been laid along an ungraded street. It may have been laid in a temporary trench to be deepened when the street was graded or it may have been placed in a deep trench. The cost of reproduction, if estimated after the street has been graded, will again fail to indicate correctly the amount invested in the pipe and is not entirely satisfactory because the first cost of a deep trench or the lowering of the pipe during the work of grading were legitimate expenditures which the owner of the public utility, if he had been an agent of the rate-payer, would have been justified in incurring. Such considerations as these show how unsatisfactory any close adherence to the cost of reproduction may be and they show, too, that a wide latitude is sometimes allowable when the legitimate investment is approximated by estimating the cost of reproduction. Furthermore, this method can hardly be applied satisfactorily without adopting some definite rule in the matter of valuing real estate. Suppose the rule to be that real estate and all other property be appraised on the assumption of present day cost of acquisition and construction, disregarding low original cost, donations and all other factors and conditions that may have prevailed at the time of construction and that may have kept cost down. Under such a rule all relation between the capital legitimately invested and the cost of reproduction may be lost. In a private enterprise the former and not the latter would be used as the guide in determining what the earnings should be. But the cost of reproduction is generally held to deserve special consideration when the "fair value," required by the courts, is to be appraised and it is frequently accepted as the starting point. When the courts shall have accepted the view that "fair value" is not the proper starting point, then the importance of close estimates of the cost of reproduction, which includes both depreciation and appreciation, will fall away.

CHAPTER V

ELEMENTS WHICH REDUCE VALUE

Deductions from Value

The Lessening of Worth. — The value of a property may increase or decrease with age. Having in the preceding chapter called attention to the essentials of value and to certain elements which add value, there will be considered briefly in this chapter some of the factors which reduce value.

In the case of any operating plant there will be parts which deteriorate or wear out under the effect of continued use. By ordinary care the rate at which the perishable parts of a plant are consumed in rendering useful service is held at or near the average rate which experience teaches should be regarded as inevitable. If ordinary care is not exercised and a plant by reason of neglect gets into a condition which requires special attention to restore it to an average condition, all circumstances of use and age being considered, the prospective outlay to accomplish this rehabilitation is called deferred maintenance. But even when there is no deferred maintenance there may have been a lessening of worth due to the fact that on account of wear and tear, or by reason of obsolescence, or inadequacy, or from any other cause, the time of probable serviceability of any article in question is being continually lessened. Loss of value from such cause is depreciation. Or again, if cost is made a starting point, it may be found that there is included unused property perhaps held to meet a future demand, but which, when valuations as a basis for rates are under consideration, may have to be omitted or deducted from the aggregate of the listed properties. And, finally, the property under consideration may include items designed of a capacity to meet future requirements and so far in excess of a reasonable allowance for growth in the near future that it must be regarded as overbuilt and not, therefore, entitled to be taken into account at its full cost.

Without attempting a full discussion of all of these factors which must be brought under review by an appraiser, particularly when the establishment of a rate-base is involved, attention will be given briefly to property not required for immediate use, and this will be followed by a discussion of depreciation including some matter relating to amortization and the replacement requirement, here conveniently considered though not all strictly pertaining to reduction of value.

The Overbuilt Plant and Property not Required for Immediate Use

Plant Capacity in Relation to Requirement. — Ordinarily, and with good reason, public utilities are constructed of a capacity for service somewhat in excess of immediate requirements. It is customary in anticipation of the increased demand for service which comes with the growth of the community to give the utility ample capacity. A part of the utility plant is built, in other words, to meet the demands of the future rate-payer. The question may well be asked to what extent this is justified and to what extent the rate-payer of today should be made to bear a burden for the benefit of the rate-payer of tomorrow.

Prudent foresight in such matters is always commendable. It should be exercised by the private owner just as it would be exercised by wisely managed municipal public utility departments. Future requirements should be foreseen and provision should be made for expansion of works. This may sometimes require the acquisition of lands, of rights of way, of water-rights, of reservoir sites, and the construction of works beyond immediate requirements. Outlays in this direction, within limits, are generally more economical than deferred acquisition or deferred construction, at the advanced value of land and the higher cost of construction of the future.

While not endeavoring to set up a rule for the determination of the extent to which it may be proper to take future requirements into account in planning any installation, attention is called to the fact that any such provision for anticipated future service adds to the investment of capital and thereby makes higher earnings necessary than would be required, if, at all times, the plant capacity could be kept exactly at the momentary requirement. Here then is an additional call upon the rate-payers, which, naturally, will fall heaviest upon those of the early years, at which time the disparity between actual demand and the provided capacity is generally greatest. There lies herein a further reason why the rate-payer in the early years of the life of any public utility should be relieved, as elsewhere suggested, of the requirement to meet amortization of capital. These rate-payers will ordinarily have done their full share if they contribute operating expenses, interest on the investment, and whatever may be necessary to anticipate and to meet replacements of discarded parts as such replacements become necessary.

Property not in Use. — Guided by the decisions of the courts and by the rulings of public service boards and commissions, it has become customary to discriminate with care between the property of a public utility which is in use and the property which may not strictly be regarded as in use. Property which is in no way related to the service rendered and which is of no prospective necessity will be left out of consideration in this discussion. But there is another type of property owned, but not in use, as for example a reservoir site, acquired by a water works owner at a favorable moment but not likely to be made a part of the system for an indefinite period, or a terminal right of way for a railroad, or an undeveloped water-power which according to sound judgment must be regarded as necessary for future expansion though not in active use as a part of the present system. The wisdom of early acquisition of such properties at opportune moments can hardly be questioned. How to deal with them in making an appraisal for rate-fixing purposes is the question. If such properties are included in the valuation of the rate-base as of date when purchased, the rate-payer is taxed for the benefit of the future. If, on the other hand, they are not included in valuations until they are actually put into use, then they must ultimately be included at original cost plus interest, or as seems to be a more general practice, at their value at the time they go into use. Sometimes by such procedure a large unearned increment represented by increase of value goes to the public utility owner and generally he is entitled thereto as a reward for his foresight.

As a general principle the holding of a reasonable amount of property which will at an early day be in use, should be considered legitimate and in conformity with the practice that would be followed by any prudent management, and it will be proper to place such property in the same category as property in actual use. This, in its essence, is no different from allowing full value of the pumping plant which has twice the capacity immediately required, but which was planned with due regard to future demand upon it.

When any property which is thus held to meet prospective requirements is not included in the appraisal, the burden of holding the same available for future use falls on the owner of the property. When he thus acquires rights, lands or other elements which are of value to his system of works, but are perhaps of a character from which no return can be expected, he puts the community under obligation to suitably recompense him for his foresight and for the material service which he thereby renders. By judicious investment from time to time in property which a prudent agent under like circumstances would acquire, the public utility owner not only provides for the expansion of his system of works, but he hopes to profit by the increased value that comes to all property located in prosperous sections that are well served by public utilities.

The ideal arrangement would be to have all acquisitions of property of whatever nature that appertain to future service made subject to the approval of competent representatives of those who are ultimately to pay the bills, and, in that case, there would be no question as to the propriety of including property held for future use. The only question would be whether the present rates should yield interest on the full investment in such property or whether the owner should be required to carry the same, in whole or in part, for the benefit of the community until such time as the greater demand for the service and consequent greater earnings will justify the inclusion of the property at cost, plus interest, plus such additional amount as may be thought a fair profit allowance to the owner in each particular case. Approval of this character in the past has been out of the question. To a large extent it will be impractical, too, in the future. It frequently happens that the purpose of the acquisition of property must be kept under cover, because, if disclosed, the prices would become prohibitive. For this reason the publicity incidental to a preliminary approval by representatives of the public would often prove embarrassing. It will no doubt be suggested that condemnation proceedings should be resorted to whenever property is to be acquired for the use of a public utility. No one who is familiar with the conduct and results of such proceedings in the courts of this country would be willing to admit the advisability of this procedure in all cases. The jury which is required to base its findings on the evidence submitted in such proceedings frequently reaches conclusions that are not fair. Not until value of the property taken, and the amount of damage to remaining property, is made determinable by impartial experts, not selected by the litigants but appointed by the courts, will there be any hope of securing through court proceedings, at a reasonable cost, the rights and properties that may be required by public utilities.

Discussion of Overbuilt Plants by the Wisconsin R. R. Commission. — In the discussion of the value of the LaCrosse Gas and Electric Co. properties the Wisconsin R. R. Commission says (Wis. R. C. R., Vol. 2, p. 5):

"Duplication of such plants is a waste of capital whenever service can be adequately furnished by one plant. . . . Competition in this service therefore usually means a bitter struggle and low rates, until one of the contestants is forced out of the field, when the rates are raised to the old level, if not above it, or to a combination of some kind between them which also ultimately results in higher rates."

The Commission in the case of the City of Racine *vs.* Racine Gas Light Co. (Wis. R. C. R., Vol. 6, p. 286) calls attention to the fact that the investment in physical properties is somewhat greater than the amount ordinarily needed. The investment was made to meet anticipated needs. Latitude must be given in such cases to owners but it is questioned whether the return allowed on the investment should be at such high rates as prevail in other cities of the same size.

Referring to non-operating property the Commission in the LaCrosse Gas and Electric Co. case (Wis. R. C. R., Vol. 8, p. 164) expresses the view that the value of such property can be made a part of the value used in fixing rates only when the income therefrom is added to actual income or deducted from operating expenses, even though future use of the same is anticipated.

Discarded Property. — In Wisconsin it is required by law that discarded property must be left out of consideration in making appraisals of value as a basis for fixing rates. However there are nevertheless cases in which equipment, not actively a part of the plant, but held in reserve for emergency use, may properly be included in the base value (Wis. R. C. R., Vol. 5, p. 24).

Depreciation, Amortization and the Replacement Requirement

Depreciation. — Depreciation is a lessening of worth with age which deserves special consideration in any discussion of rate regulation. The matter of amortization and the making of provision for renewals or replacements is so intimately interwoven with the depreciation question, as ordinarily handled, that no attempt will be made to restrict the discussion to depreciation. Information bearing upon amortization and the replacement requirement will here be found intermingled with the general discussion of the depreciation question.

Assumptions Made in the Discussion. — The general presentation of the problems involved in determining what should be the fair earnings of a public utility will be simplified by assuming that the actual useful life or term of service of each of its parts will conform with the probable life term predicted for these parts. This is not in reality the case. In the preliminary presentation of the subject, however, this assumption has been strictly adhered to. How the departure of the actual term of usefulness from the probable term will affect the computation of the annual replacement requirement will be considered later.

It has been found convenient to use an interest rate of 6 per cent throughout this volume for purposes of illustration and this rate is to be understood when no other rate is mentioned.

Hypothetical Case; 20 Year Life, 10 Years Old. — Take the case of a plant, all parts of which have a life of 20 years, all constructed at one time and owned by a prudent owner who sets apart at 6 per cent interest, as an amortization fund, each year $0.027185 for every dollar invested therein. If the plant is one which will actually net 6 per cent on the invested capital, then the apparent excess of the annual earnings, over expenses, should be $6 + $2.72 = $8.72 continuously during the life of the plant, and the owner, in estimating the price at which he can sell it without loss at the end of any period, as, for example, at the end of 10 years, would figure as follows (for each $100 of original investment):

Investment (original)	$100.00
In the amortization fund: being the amount of a 10-year annuity of $2.7185 at 6 per cent interest	35.83
Remaining value	$ 64.17

A prospective purchaser would figure that the plant should be worth at least as much as the present value of $8.72 per annum treated as an annuity for the remaining 10 years, which, at 6 per cent per annum, is $64.17.

At the end of 10 years, the original owner, keeping for his own use the money in the replacement fund, will be satisfied to sell at $64.17. The purchaser, content in this case with the assumed rate of interest of 6 per cent, will be willing to pay $64.17, because at the end of the plant's useful life, he will have recovered his investment with 6 per cent interest compounded annually. He will then be under the same necessity of replacing the plant, making a new investment of $100, as the original owner would have been if he had remained in possession.

During the entire 20 years of usefulness the plant has been rendering adequate service. The efficiency of the service is independent of, and bears no relation to, the useful life of the plant, nor to the fact that some or all of its parts were gradually deteriorating.

Interchange of Terms — Depreciation, Amortization and Replacement. — It cannot be known just how, nor at what rate, the actual deterioration of a plant takes place. This may be rapid at some period of its life, and slow at another, but, as the plant is supposed, at all times during its life, to be adequately performing the service expected of it, variations in this rate of deterioration are immaterial. In other words, the amortization of capital is a question which may be considered without regard to the physical condition of a plant at any period of its life. Nevertheless many engineers and economists have found it convenient to consider the actual, or the theoretical accumulation in an amortization fund as the measure of plant depreciation with a consequent interchange of terms. The term "depreciation" is frequently used when the term "amortization" would be more appropriate.

There is a clear distinction between amortization and replacement. The amortization deals with the retirement of the invested capital. This may be in installments in uniform or in unequal annual amounts, or in a lump sum at the end of useful life. The replacement may mean the substitution of a new identical plant, but at a cost dependent on new conditions, new prices of labor and material, or it may mean the substitution of new devices rendering equivalent service. In either event the replacement may be at a greater or less cost than the original cost, with, therefore, a corresponding increase or decrease of capital invested. Expenditures for new parts of a plant, which take the place of old parts which are retired for any cause, should be charged to replacement only to the extent of capital represented by the part of the plant thus retired. Any excess of the expenditure for replacement over the cost of the discarded part of a plant should be treated as an addition to, and any less cost as a deduction from, the invested capital. The term "replacement" should not be used in the sense of retirement of invested capital, which deals with the cost of the replaced part and not with the cost of the new equivalent installation. Theoretically, the amount which should go into an amortization fund should be estimated on the basis of invested capital, or cost, and not on the cost of replacement.

In the case of the supposed valuation by a seller and by a purchaser of a plant with a 20-year useful life, at the end of a 10-year period, there is no need of assuming that an amortization fund has actually been created. The amortization annuity, instead of actually appearing in a fund, may be otherwise invested.

Example of Insufficient Amortization. — When the owner of a steamboat which has a limited life and which is yielding 6 per cent per annum of its cost and nothing for amortization, sets apart, out of the 6 per cent, an annual amount, also bearing interest at 6 per cent, to meet its replacement at the end of the steamboat's life, he will have invested not only the original cost of the steamboat, no part of which comes back to him in the annual 6 per cent return, but also a gradually increasing sum which in the life of the steamboat will become adequate to replace it. At the end of the steamboat's usefulness, after replacing it with a new one, the total original investment will be doubled without any increase of earning capacity, and the owner will have, in effect, lost his original investment.

It follows from this that a return of 6 per cent per annum, without anything for amortization, or for replacement, on an investment in a perishable article, when money is worth 6 per cent, is inadequate. The excess of earnings over expenditures must be at least equal to the current interest rate on safe money investments plus an increment depending on the useful life of the plant. This increment must be such that, within the life of the plant, it will either return to the owner his original investment or will be adequate to replace the article in service with a new one.

Had the owner borrowed money for the acquisition of the article, and were he paying interest on the borrowed money at 6 per cent, this fact would be self-evident. The 6 per cent earnings would then be required to meet interest payments, and, at the time when the article has reached the end of its life and must be replaced with a new one, he would find himself, not only in debt for the original article but would have to duplicate the indebtedness to make the replacement.

Amortization and the Value of Stock. — The amortization increment is ordinarily expected to appear in the earnings as that sum which, at compound interest during the life of the article, will be adequate to retire the original investment.

To illustrate these points further, let it be supposed that ownership is represented by capital stock of a corporation. If a plant owned by the corporation and built with funds contributed by the stockholders earns just enough to net 6 per cent without any allowance for amortization, the stock which at the outset may have been worth 100 per cent will gradually decrease in value until, at the end of the plant's usefulness, it will be worth nothing.

The situation is quite different when the earnings net 6 per cent plus an annual amortization increment here supposed to be paid into a special fund. In this case, the stockholder receives 6 per cent each year, and the amortization grows while the plant depreciates in value. The stock, if fully paid up, will be at par from the beginning to the end of the plant's usefulness, and the money in the fund at the end of the period is available either for distribution to the stockholders, being a return of the money advanced by them or it is available for reinvestment in a new plant to replace the original one. Should a sale be made at any time while the plant is in service, with due allowance for its depreciation (offset by the amortization fund) and, this value being recognized by a purchaser and the price paid, there would again be 100 per cent available for distribution to the stockholders, the deficiency of the selling price being made up by the accumulation in the amortization fund.

In the case of net earnings amounting to less than interest on the invested capital plus current depreciation, the valuation of the plant by a purchaser would be at all times less than the value determined by deducting accrued depreciation from cost. In the case of earnings amounting to a proper interest return on the investment plus an adequate allowance for amortization or for replacement the valuation would be, as already explained, capital invested (or the replacement cost) less depreciation.

The Use of an Amortization Fund. — Theoretically, then, a part of the earnings each year may be placed in an amortization fund as a repayment of capital invested, and this fund may be used for the replacement of the parts of the plant as they go out of service or of the entire plant when it has reached the end of its life.

The accumulation of an amortization fund for such use, however, while theoretically sound policy, is a measure not always adopted in actual practice, particularly when the properties owned are of a complex character — when they are made up of numerous parts of various periods of probable usefulness. Municipalities, State and National Governments, do not set apart funds for the replacement of worn-out or antiquated buildings, parts of water-works, street pavements, sewers, and the like, until the replacement is necessary. They do not maintain funds at interest representing accrued depreciation out of which to reconstruct their public works. The sinking fund required to retire bonds which may have been issued to construct these works originally must not be confounded with a replacement fund. The one may be necessary to pay for the works in the first instance, the other to maintain them for all time. The annual contribution to the sinking fund is a partial payment for the original work. The contribution to a replacement fund, in the case of a plant which is to serve without time limit, is for the purpose of perpetuating the work, because in that case the replacement fund, as far as it will go or as far as it is required, will be used for making replacements.

The Wisconsin R. R. Commission in the case of the Superior Commercial Club *vs.* Duluth Street Railway Co. (Wis. R. C. R., Vol. 11, pp. 1 to 21) elsewhere quoted makes clear the distinction between depreciation and amortization. Referring to depreciation the Commission in the matter of the Fennimore Mutual Water & Light Plant, in 1913 (Wis. R. C. R., Vol. 12, p. 209), warns against the confusion of the depreciation fund with the depreciation reserve. The "fund" is actually created by setting a part of the income aside. The "reserve" is merely a book account which designates the amount and character of various transactions bearing upon depreciation and replacement expenditures.

Though it may be difficult to make satisfactory forecasts with reference to necessary reinvestments to replace discarded parts of a plant, the requirements for amortization, being based on cost, are usually readily determinable with some degree of precision.

Application to Complex Plants. — Thus far, the plant is assumed to have been constructed and put into use all at once, and is of such a character that all its parts have the same life. The same principles will apply when a plant is made up of many elements or parts having various periods of usefulness. The amortization or the replacement annuity is, in such case, determined for each part and from the sum of the annuities thus ascertained the minimum earnings which will prevent loss are determined.

Mathematical Determination of the Replacement Fund. — The following problem presents itself: In the case of a plant of gradual development but of full growth and mature age, composed of numerous units, the useful life of all the units or parts of which is n years, it is desired to know what amount is in the replacement fund at any time, that fund being assumed to receive such an increment at the end of each year that, during the life of each unit, this annuity, with interest, will amount to the original cost of this unit.

Being composed of a large number of elements — each year having added new ones — the addition to it per year will be taken for the purpose of this illustration at one-nth of the total plant as it stands at the end of the nth year.

For each dollar invested on this assumption in the first year, there will be \$1 invested in each succeeding year, and for each dollar thus invested there will be n dollars of total investment.

Let a represent the annual contribution to the replacement fund for each dollar invested.

Assume this contribution to be available at the end of each year.

Then after n years, na will be the annual contribution to the amortization fund for each dollar of the annual investment.

Let m represent any number of years greater than n.

Let i represent the interest rate expressed in hundredths, *i.e.*, for 6 per cent, $i = 0.06$.

During the first n years, after beginning the construction of the plant, there will be no replacements, and the replacement fund continues to grow. At the end of the nth year the replacement requirement, assuming permanency in character and cost, will be \$1 for each dollar of annual investment, and this replacement requirement will continue at this rate thereafter.

At the end of the nth year the replacement fund will contain:

For each dollar invested the first year:

$$a\,(1 + i)^{n-1} + a\,(1 + i)^{n-2} + \cdots + a \text{ dollars}$$

or

$$\frac{a}{i}[(1 + i)^n - 1] \text{ dollars.}$$

For each dollar invested the second year:

$$a(1+i)^{n-2} + a(1+i)^{n-3} + \cdots + a \text{ dollars}$$

or

$$\frac{a}{i}[(1+i)^{n-1} - 1] \text{ dollars.}$$

For each dollar invested the nth year: a dollars.

Therefore the total amount S_n in the replacement fund at the end of the nth year, after deducting the $1 replacement requirement of that year:

$$S_n = \frac{a}{i}[(1+i)^n - 1 + (1+i)^{n-1} - 1 + \cdots + (1+i) - 1] - 1. \quad (1)$$

$$S_n = \frac{a}{i}\left\{\frac{1}{i}[(1+i)^{n+1} - (1+i)] - n\right\} - 1. \quad (2)$$

$$S_n = \frac{a}{i^2}[(1+i)^{n+1} - (1+i) - ni] - 1. \quad (3)$$

There will be in the replacement fund for each dollar annually invested:

At the end of the $(n+1)$st year:

$$S_{n+1} = S_n(1+i) + na - 1. \quad (4)$$

At the end of the $(n+2)$d year:

$$S_{n+2} = S_n(1+i)^2 + (na-1)(1+i) + na - 1. \quad (5)$$

At the end of the $(n+3)$d year:

$$S_{n+3} = S_n(1+i)^3 + (na-1)(1+i)^2 + (na-1)(1+i) + na - 1, \quad (6)$$

and so on; and at the end of the mth year:

$$S_m = S_n(1+i)^{m-n} + (na-1)(1+i)^{m-n-1} + (na-1)(1+i)^{m-n-2} + \cdots + na - 1. \quad (7)$$

Substituting the value of S_n and summarizing the series:

$$S_m = \frac{a}{i^2}[(1+i)^{m+1} - (1+i)^{m-n+1} - ni(1+i)^{m-n}] - (1+i)^{m-n} + \frac{1}{i}[(1+i)^{m-n} - 1](na-1), \quad (8)$$

which may be reduced to

$$S_m = \frac{a}{i^2}[(1 + i)^{m+1} - (1 + i)^{m-n+1}] - \frac{1}{i}[(1 + i)^{m-n+1} + na - 1]. \quad (9)$$

For the interest rate of 6 per cent: $i = 0.06$ and

$$S_m = \frac{a}{0.0036}[1.06^{m+1} - 1.06^{m-n+1}] - \frac{1}{0.06}(1.06^{m-n+1} + na - 1). \quad (10)$$

For $m = n$ and $i = 0.06$

$$S_n = \frac{a}{0.0036}(1.06^{n+1} - 1.06) - \frac{1}{0.06}(0.06 + na) \quad (11)$$

or

$$S_n = \frac{a}{0.0036}(1.06^{n+1} - 1.06) - \frac{na}{0.06} - 1. \quad (12)$$

If now the total amount in the replacement fund be compared with the total investment which, on the assumption made, will be n dollars for each dollar of annual investment, the relation between the replacement and the investment expressed in percentage and called R will be found to be:

$$R_m = \frac{100\,a}{ni^2}[(1 + i)^{m+1} - (1 + i)^{m-n+1}] - \frac{100}{ni}[(1 + i)^{m-n+1} + na - 1] \quad (13)$$

and for the interest rate of 6 per cent or $i = 0.06$,

$$R_m = \frac{100\,a}{0.0036\,n}(1.06^{m+1} - 1.06^{m-n+1}) - \frac{100}{0.06\,n}(1.06^{m-n+1} + na - 1) \quad (14)$$

and for $m = n$ and $i = 0.06$,

$$R_n = \frac{100\,a}{0.0036\,n}(1.06^{n+1} - 1.06) - \frac{100\,a}{0.06} - \frac{100}{n} \quad (15)$$

or

$$R_n = 27{,}778\frac{a}{n}(1.06^{n+1} - 1.06) - 1667\,a - \frac{100}{n}. \quad (16)$$

Table 2 is based on the foregoing formulæ. For comparison two rates of interest 6 per cent and 4 per cent per annum have been used in estimating the amount which should be in the replacement fund.

TABLE 2. THE REPLACEMENT FUND

For plants made up of numerous parts, all having the same probable life new, all serving their full probable term of usefulness and no longer. The plants are assumed to have an age equal to or greater than the number of years in the probable life term of their parts. The total invested capital in each case will be *n* dollars for each dollar of annual investment, when *n* represents the number of years in the probable life term.

At the end of the year.	Amount in the replacement fund expressed in percentage of the total investment.							
	Prob. life 5 years.		Prob. life 10 years.		Prob. life 20 years.		Prob. life 40 years.	
	4 per cent.	6 per cent.	4 per cent.	6 per cent.	4 per cent.	6 per cent.	4 per cent.	6 per cent.
5	38.4	37.6	.....	.....	.....	.....	.....	.....
10	38.6	37.7	41.7	40.2	.....	.....	.....	.....
15	38.4	37.6	41.8	40.2	.....	.....	.....	.....
20	38.4	37.8	41.8	40.0	41.1	38.0	.....	.....
30	....	......	.....	.....	41.0	37.9	.....	.....
40	....	......	41.8	40.3	41.0	37.9	36.1	31.3
60	....	......	.....	.....	41.1	38.0	36.0	30.9
80	....	......	.....	.....	41.1	.....	36.1	30.9

In the foregoing mathematical analysis, a plant has been assumed which has reached its full growth and which has an age equal to or greater than the number of years in the useful life term of its parts. Moreover it has been assumed that all of its parts have the same probable life when new.

The same formulæ will apply to any number of articles of the same probable life installed at a uniform rate per year, even when the plant of which they form a part is still being extended, because in this case the articles may be separated into two groups, one being composed of all articles *n* years of age and less, which have not yet been replaced and the other group of those articles which have replaced discarded articles. To each of these groups taken separately the above formulæ apply.

It is noteworthy in the assumed case of a plant which has attained full growth and is made up of numerous parts that, when the replacement requirement is computed from the beginning by the compound interest method, the amount in the replacement fund should theoretically vary between comparatively narrow limits; at 6 per cent interest from 31 to 40 per cent for life terms ranging from 5 to 40 years. But in reality there can never be absolute agreement between the actual useful life and the probable life of all parts of the plant. The formulæ noted in this chapter are not therefore strictly applicable. They are nevertheless valuable in illustrating a principle.

Application of Earnings to Replacement and Amortization. — The demands upon the replacement fund usually begin long before the end of the probable life term is reached and may be quite irregular in amount. The non-existence of a replacement fund in the full amount indicated by mathematical and theoretical consideration does not, therefore, prove that the deficiency has been distributed as profit, nor yet that there has been any waiver of the right to have the earnings cover a fair replacement increment.

Furthermore, if the earned annual replacement increment be treated as amortization of capital and be immediately applied for this purpose, it will thereby be removed from all further consideration. The interest on any increment thus applied is not available to retire more capital. Treated as an annuity and remaining in the business, interest may be compounded so long as the fund is held for its intended purpose, that is, for retirement of capital at the end of the useful life of the item which is being retired. Interest ceases to accumulate the moment the fund is applied to retire the investment in whole or in part. Consequently, if a uniform annual amortization increment bearing interest compounded annually be determined from amortization tables based on the probable life of a new article and if it be covered by the earnings from year to year, even though the amortization increment as earned be reinvested in the property, it cannot rightfully be classed as a repayment of invested capital until the end of the probable life term. If the fund resulting from the accumulation of such increments be applied at any earlier date, a new amortization annuity, based on the remaining value and the remaining life, must be computed.

Incomplete Amortization. — It will be seen that if an amortization annuity thus determined from probable life when an article goes into service be deducted from the investment from year to year, the result will be incomplete amortization. In the case of an article with a probable life of forty years, the amortization rate thus computed at 6 per cent interest would be 0.6462 per cent per annum. The amount of capital returned in forty years would be $25.85 on each $100 of capital invested and there would remain $74.15 still to be made good at that time.

These facts make clear the point that, whenever amortization in lieu of replacement is accomplished by annuities bearing compound interest, the appraisal for rate-fixing purposes must be of the entire investment without deduction of accrued depreciation.

Second Mathematical Determination of the Replacement Fund. — The foregoing mathematical demonstration that the accumulation in a replacement fund for a plant of mature age, when computed by the compound interest sinking fund method, and actually earned should amount to a considerable sum, confirms a conclusion which can be reached in a more direct way.

In the assumed case of a plant which has a life of n years, and of which one-nth has been constructed each year, after n years there will have to be replaced one-nth thereof each year. Because the annual investment in the installation has been uniform there will be, for each dollar invested per year, a total investment of n dollars.

The annual replacement after n years, for each dollar annually invested, will be $1. If now the annuity to replace the several parts of the plant in n years is a dollars for each dollar of the annual investment, then after n years the annual amount received as annuity will be na, and this will fall short of meeting the actual expenditures by an amount expressed by $(1 - na)$ which, at 6 per cent per annum, is the interest on $\frac{100\,(1 - na)}{0.06}$ dollars; or, expressed in percentage of the cost, is $\frac{100^2\,(1 - na)}{6\,n}$ per cent of the total investment in the plant.

For a plant not subject to further growth, with a uniform useful life of all its parts, and constructed progressively, there should be, at 6 per cent interest, an unexpended interest-bearing balance in the replacement fund as follows:

When the useful life is five years:

$$\frac{100^2\,(1 - 0.8870)}{30} = 37.7 \text{ per cent of the total investment.}$$

When the useful life is ten years:

$$\frac{100^2\,(1 - 0.7587)}{60} = 40.2 \text{ per cent of the total investment.}$$

When the useful life is twenty years:

$$\frac{100^2\,(1 - 0.5437)}{120} = 38.0 \text{ per cent of the total investment.}$$

When the useful life is forty years:

$$\frac{100^2\,(1 - 0.2585)}{240} = 30.9 \text{ per cent of the total investment.}$$

If earnings have been adequate to provide an interest-bearing replacement fund, then these percentages represent the probable accumulation in such a fund.

Some amount such as shown by these figures, depending on the expectancy, represents the accumulation of replacement annuities during that period of the plant's life during which the actual replacement expenditures were less than the annuity. If the annual allowance for maintenance in the past has been based on the requirements of operation and repair without surplus to meet future replacements and if there has been no special allowance for amortization, the current allowance for amortization and replacements should not be determined by the interest bearing sinking fund method based on original probable life but should be otherwise determined, as hereafter shown.

When, in other words, opportunity is not given to accumulate the 40 per cent of the invested capital (approximately), which, for ordinary periods of useful life of perishable properties, should in the course of time be in a replacement, depreciation or amortization fund, any amount estimated from amortization tables on the original full period of useful life will fall short of the real replacement requirement.

Illustration of the Replacement Requirements. — Let it be assumed that a conduit, such as a cast-iron pipe, used for any purpose, has a length of 40 miles. Let it be also assumed that the pipe is not being further extended, that the life of this pipe is 40 years, no more and no less, and that it was constructed progressively, one mile each year. It took 40 years to install the pipe, and at the end of this time the first mile of pipe laid was ready for replacement — it had served its time. Each year thereafter, one mile of pipe has to be replaced, and the replacement at this rate will continue indefinitely. The annual replacement expenditure during the first 40 years is nothing, but, thereafter, it is the cost of installing one mile of pipe. If prices of labor and material have remained constant, and if conditions have otherwise remained as they were when the first mile of pipe was laid, then the annual replacement expenditure will be one-fortieth of the total amount invested in the pipe line.

Provision for this replacement must be made if the pipe is to continue in service. If, now, the extension of the pipe progresses beyond the 40-year period at the same rate, before assumed, of one mile per year, there will be no changes in the annual replacement requirement during a second period of 40 years, but at the end of this second period — at the end of 80 years — there will be 80 miles of pipe in service, and thereafter during the third 40-year period there will have to be replaced annually 2 miles of pipe, or one-fortieth of 80 miles, or twice the amount of pipe extension per annum.

Determination of the Replacement Requirement. — It is possible, by such analysis, when a plant is growing at a fixed rate and has attained an age exceeding the life of its perishable parts, to prescribe a rule for determining the replacement requirement; but it must be remembered that a rule thus determined can be strictly correct only for the impossible hypothetical case of service in exact conformity with the assumed probable life, and that for practical application a rule thus determined may require some modification as explained in Chapter VI.

For each group of parts having the same length of life, there is to be determined: first, the average annual capital invested, using, however, replacement cost instead of the actual investment; and second, the full number of times that the age of the plant is greater than the useful life of the particular group of parts under consideration. The replacement requirement (for the hypothetical case, in which actual service conforms throughout with the assumed probable life) is then ascertained by multiplication.

A pipe line may again serve as an illustration: Suppose it is desired to know the replacement requirement for a pipe line 300 miles long, which has been extended 2 miles each year, the age of the oldest portion of which, therefore, is 150 years.

The life of the pipe being taken at 40 years, the full number of times this is contained in 150 years is three. The annual replacement requirement will be three times two, or 6 miles of pipe.

The 6 miles of pipe requiring replacement were constructed 40 years ago, and the conditions under which this was done may have been materially at variance with those prevailing at the time of their replacement. Consequently, in the determination of the replacement requirement, expressed in dollars instead of in miles of pipe, the replacement cost of the system and not the original cost of capital invested should be taken into account. Expressed as a percentage of the total length of pipe in service, or of the total cost of replacing the entire pipe line, this would be 2 per cent.

By the compound interest annuity method of computation, in the selected illustration at 6 per cent interest, the allowance for replacement would be 0.646 per cent of the cost of the system, which is only one-third of the actual requirement, and this allowance, as already explained, would only then be justified if amortization had covered the entire period in the life of each part of the pipe during which there was no expenditure for replacements, so that the inadequate annual allowance could be supplemented by the earnings of an accumulated replacement fund.

In a plant which is made up of a multiplicity of parts of various periods of usefulness, those which have the same expectancy should, as before stated, be grouped together. For each group, the replacement requirement can then be estimated separately, and from the several amounts thus ascertained the total requirement is determined.

The rule previously laid down for a hypothetical case is not strictly applicable under the conditions as they actually present themselves. There can be no absolute conformity between the assumed period of usefulness of the various parts of a plant and the time during which they actually prove useful.

The probable useful life or expectancy is merely the average life, which is often not reached and is just as often exceeded. Thus, again referring to the pipe line, it is to be assumed that while some of it may serve beyond the average period of usefulness of such pipe, other parts thereof, from one cause or another, will require replacement early in its life. Consequently, any rule such as that previously laid down, which indicates a uniform replacement requirement in successive periods, with a sudden rise in the requirement at the beginning of each new period, if the plant be one that is steadily growing, will require some modification.

The simplest modification of the foregoing rule is to assume gradual changes in the annual replacement requirement as the age of the plant increases, instead of the sudden changes, and then to call this requirement at all times inversely proportional to the useful life of any group of parts. This is sometimes referred to as the "Straight Line Method." It might with equal propriety be called a direct percentage method, as the inverse ratio is usually expressed in percentage.

Under this direct percentage method, there would be allowed 2.5 per cent per annum of the replacement cost of all parts of a plant having a 40-year life; 3.33 per cent per annum of the replacement cost of all parts having a 30-year life; 5 per cent per annum of the replacement cost of all parts having a 20-year life, and so on.

This method, applied to the hypothetical case of a pipe line, constructed and extended one mile per year, and each mile thereof having a useful life of exactly forty years, would, at the end of the fortieth year, make the replacement requirement 2.5 per cent per annum, or one mile of pipe. At the end of the sixtieth year, the requirement thus determined would be 2.5 per cent of the 60 miles of pipe then in service, or 1.5 miles of pipe. This would be 50 per cent in excess of the amount actually replaced, which at that time would be only one mile. This would also apply for any time before the pipe first laid has reached the limit of its usefulness, as at 20 years. In the assumed case there is no replacement requirement at 20 years; yet the straight percentage method indicates 2.5 per cent of 20 miles of pipe, or 0.5 mile of pipe. It follows from this illustration that the Straight Line Method would give results somewhat too high.

By further analysis of this problem, the following formulæ have resulted, which are free from this objection and fulfill every ordinary requirement. In devising these formulæ, the fact was taken into account that there may be some replacement requirement in the early years of a plant's life, and that this requirement gradually increases. These formulæ apply only to plants which have been developed gradually and are being extended at a uniform annual rate.

Using the notation already introduced, and designating with C the total cost of replacing the group of items, the probable useful life of which, when new, was n years, with c the annual renewal requirement, and with g the average annual investment in extensions, the formulæ are:

For m less than n,

$$c = \frac{mg}{2n} = \frac{C}{2n}. \qquad (17)$$

For m greater than n,

$$c = \frac{C}{n} - \frac{g}{2}. \qquad (18)$$

For very large values of m in relation to n (n being the years of probable usefulness), the value of this expression approaches $\frac{C}{n}$ which is the mathematical equivalent of the Straight Line Method.

However desirable it might otherwise appear to introduce a method of computing the replacement requirement by recourse to amortization tables, to do this satisfactorily, in the case of a complex plant, is usually out of the question, when past earnings have been inadequate to accomplish the desired amortization. In such cases the use of some formula, as above noted, for estimating the probable replacement requirement is to be recommended and its application would be equitable from the standpoints of both the owner and the rate-payer.

The Interest Bearing Annuity and the Replacement Requirement. — When an annuity, bearing interest compounded annually, is allowed to accumulate in a fund to retire invested capital, the demand upon the rate-payer is in annual installments. It would be equally proper to make the demand upon the rate-payer for the replacement of each individualized article, at the full cost of replacement, at the time when the article is discarded. In this case the lump sum cost of replacement, equivalent to the amount of the annuity, will take the place of the installments. If the annuity installments are forthcoming as they are due, then the annuity method is adequate. If the owner of the property does not get them, recourse should be had to the lump sum allowance due at the time of failure if capital is to be kept unimpaired.

It is perfectly reasonable, moreover, to assume, unless there is evidence to the contrary, that the method of estimating and providing for replacement requirements, which prevails in any case, has been introduced deliberately. The owner of the public service property may be perfectly willing to waive collection of the annuity installments if he knows that what they will amount to, that is, the actual annual replacement, will be covered by the gross earnings when the time comes for discarding parts of his plant. In other words, he may be willing to accept the amount of an annuity in lieu of the annuity itself; and the rate-payer may desire such an arrangement, because, in the early days of the plant's life, he may be unable to pay a sufficient amount for the service to cover the replacement annuity. It must be remembered, however, that such an arrangement burdens the future rate-payer to some extent for the benefit of the rate-payer in the early days of a plant's life. Not more so, however, than when, as is done by some appraisers, early losses are used as a measure of "going value."

It follows directly from the foregoing that even when earnings cover current average annual replacement requirements, the appraisal for rate-fixing purposes may still be the entire investment without deduction for depreciation. This will be the case whenever it can be shown that past earnings were inadequate to permit the accumulation of a fund, out of earnings in excess of reasonable interest on the investment, which, if it existed, would offset in whole or in part the so-called accrued depreciation.

CHAPTER VI

THE EFFECT OF NON-AGREEMENT OF ACTUAL WITH PROBABLE LIFE UPON THE DETERMINATION OF THE DEPRECIATION OR REPLACEMENT REQUIREMENT

Depreciation Estimates are Approximations. — Consideration is now to be given to depreciation as a factor affecting the required earnings. It is not enough to know what the theoretical depreciation will be if estimated from the probable life of any article. Any article in use may be considered as being gradually consumed in the service. When no longer useful, it must be replaced. The replacement requirement therefore must be estimated. To do this properly something more must be known besides the cost, probable life, and age. It will be necessary to take the condition of the article into account — to give consideration in other words to the question of whether it will outlast its probable term of usefulness or not. To disregard this fact results in crude approximation and loose methods of accounting which are undesirable.

Probable Life is Based on Experience. — The sum of all experience, so far as the same has been made a matter of record, fixes the probable life of various classes of articles. Some articles in every class will fail early, others will survive their probable life term. When any system of accounting is adopted under which the capital invested in individualized articles is to be retired during their probable life, some articles will fail before their cost is completely amortized and there will be others continuing to render efficient service after their cost has been completely amortized. The anomaly results of having to carry in the accounts a part of the cost of articles no longer in use but still in the process of amortization and also of having wiped out the entire cost of others still in service, which judged by the accounts should have no value.

Effect of Expectancy upon Present Value. — In order to weigh the advantages and disadvantages of these methods in their practical application some attention must now be given to the probable life and to the expectancy of the various elements that go to make up a public service property.

When it is desired to know the present value of any article, the question is not "how old is it and what did it cost?" but "how much longer will it serve and what will it cost to replace it?". The first question might be asked when the accrued amortization of capital is to be estimated. The second question is to be answered when the current depreciation or the current replacement requirement are to be estimated.

How then shall the expectancy be determined?

All estimates of annual depreciation and of accrued depreciation are based on premises which cannot be determined with accuracy. The probable life of any article when new and the life expectancy of any article which has been in use for some time cannot be determined with any great degree of precision. Consequently estimates of depreciation are only approximations.

Academic Discussion Justified. — There is much uncertainty in such estimates under the ordinary conditions under which public utilities are operated. A wide range in the method of making the estimates has been the result. When, therefore, the correctness of methods is under discussion, this difficulty of making close estimates should not be lost sight of. Nevertheless, the academic discussion which is being indulged in by the engineer and the economist relating to best and most convenient methods of procedure is justified, because the same will lead to an ultimate standardization of methods and finally to the general adoption of the most convenient and generally best method, fair to both the rate-payer and the owner.

Although this limitation be recognized as well as the uncertainties that result from imperfect knowledge relating to the actual and to the probable life of the elements of any public service plant, and to the difficulty of determining the expectancy of those articles which have already been in use for some time, it is nevertheless important that the whole question be fully considered in order that a framework may be constructed into which the best data furnished by experience can be fitted.

The non-agreement of actual life of individual items with their probable life and the extent to which this lack of agreement should be taken into account in estimating present worth and in estimating replacement requirements, in line with this thought, has been studied on various assumptions with interesting results. These will be briefly referred to, and the resulting tables are presented for use until, in the light of larger experience, they can be replaced with better ones.

Assumptions Relating to Departure of Actual from Probable Life. — Unfortunately there are no records available from which absolutely dependable tables of expectancy could be prepared for each class of perishable articles in use in connection with public service properties, such as have been prepared by actuaries for human beings. Any assumption in this regard is more or less conjecture. Nevertheless, it is interesting to note what the expectancy would be at various ages, if certain definite reasonable assumptions are made.

When any large number of articles which have the same probable life, as, for example, ten years, is under consideration, there will be as many service years in the aggregate, represented by the failures to reach the probable life term of ten years, as there will be service years represented by those articles which outlast the ten-year term. It may also be accepted as a certainty that there will be a greater number of articles per year to go out of use in the years just preceding and just following the term limit than at any other time. This suggests conformity with the law of probabilities.

All articles in a group having a ten-year probable life might fail (*a*) exactly at the end of ten years, but this is highly improbable. The individual articles might fail (*b*) at a uniform rate per year, one-twentieth each year, the last going out of service in the twentieth year. Or (*c*) there might be no failures at all for a number of years, as, for example, during the first half of the probable life term, and thereafter a uniform or an irregular rate of failures until some time after the end of the probable life term. Or (*d*) there might be a gradually increasing number of failures per year from the beginning to the end of the term of the probable life, and thereafter a gradually decreasing number of failures.

Of these various possible distributions of failures to a series of years the most probable one is unquestionably (*d*). For the sake of a definite basis for calculation it has been assumed that the increase in the number of annual failures up to the maximum and thereafter the decrease in the number of annual failures is uniform; that annual increase before the maximum and annual decrease thereafter are the same and that the maximum rate is one-tenth of the whole number and occurs in the tenth year. On these assumptions studies have been made to determine the expectancy of any article which has reached any age with rather interesting results. These are presented not only as an improvement in the method of estimating probable remaining life or expectancy of any article which is no longer new, though still in good condition, but also to encourage further study along these lines, in order that, where necessary, closer approximation of the actual accrued and annual depreciation can be made than has heretofore been attempted.

It should be stated that other hypotheses in the matter of the rate of failures were tried, but that no other gave results that appeared as reasonable as those based on the hypothesis as just stated. Thus, for example, the assumption was tried that $\frac{1}{2}$ per cent of all articles of ten-year life fail in the first year, $1\frac{1}{2}$ per cent in the second year, and so on, $9\frac{1}{2}$ per cent in the tenth, $9\frac{1}{2}$ per cent in the eleventh, $8\frac{1}{2}$ per cent in the twelfth, etc., and $\frac{1}{2}$ per cent in the twentieth, but with less satisfactory results for application in estimating remaining value and replacement requirements.

And, again, the failures were estimated for each year on the assumption that they will occur according to the law of probabilities, coupled with the assumption that all failures will occur within a period twice as long as the probable life term. The results on this assumption will be again referred to. In so far as the results relating to replacement requirements obtained by the law of probabilities is concerned, it may be stated that these, while not at great variance with the adopted hypothesis, were yet too irregular in amount to warrant the use of the law of probabilities in place of the other more readily applied hypothesis. Either hypothesis of rates of failure may be used as a fair basis for approximation to actual conditions. There has appeared no good reason, therefore, for confining the study to the law of probabilities which after all would have to be applied with some such assumptions as made, of practically no survival beyond twice the probable life term and 50 per cent of all failures within a period equal to four-tenths of the probable life term, or with other assumptions which would also be more or less of a conjectural nature.

Tabular Illustration of Expectancy for 10,000 Articles. — In order to further consider the question of the expectancy of any article which is no longer new, let it be assumed that 10,000 articles are installed at the same time and that all of these have the same probable life. Let the probable life term be divided into ten periods. Then according to the hypothesis relating to the annual failures, as already stated, there will be 100 of these articles going out of use during the first period; 200 in the second period; and so on to 1000 in the tenth period; thereafter 900 in the eleventh period; 800 in the twelfth period, etc., and 100 in the nineteenth period.

On this hypothesis results are readily found as shown in Table 3, in which years may be regarded as representing periods.

TABLE 3. EXPECTANCY

The probable life of each article is 10 years or periods. For terms other than 10 years, each year in the table may be regarded as a period equal to one-tenth of the probable life term.

(Based on the special hypothesis of failures as explained in the text)

Year or period.	For 10,000 articles.			Single article.
	Number of failures.	Remaining number of articles at beginning of year.	Remaining service years at beginning of year.	Expectancy at beginning of year or period.
1	100	10,000	100,000	10.0
2	200	9,900	90,000	9.09
3	300	9,700	80,100	8.27
4	400	9,400	70,400	7.46
5	500	9,000	61,000	6.77
6	600	8,500	52,000	6.12
7	700	7,900	43,500	5.51
8	800	7,200	35,600	4.95
9	900	6,400	28,400	4.44
10	1000	5,500	22,000	4.00
11	900	4,500	16,500	3.67
12	800	3,600	12,000	3.33
13	700	2,800	8,400	3.00
14	600	2,100	5,600	2.67
15	500	1,500	3,500	2.33
16	400	1,000	2,000	2.00
17	300	600	1,000	1.67
18	200	300	400	1.33
19	100	100	100	1.00
20	0	0	0	0

Merit of the Assumed Law of Failures. — Although, under the hypothesis of failures on which Table 3 is based, there may still be considerable departure from the actual number of failures in any year, there can be no question that this hypothesis is, as already stated, a much nearer approach to the truth than the other hypothesis heretofore generally accepted as a basis for calculation, that each article will fail theoretically at exactly the end of its probable life term.

The last column of Table 3 shows that on the hypothesis as explained, an article which has a probable life when new of ten years will, if it is still in service and in good condition at the beginning of the tenth year, have an expectancy of four years and at the beginning of the fifteenth year, its expectancy will still be 2.33 years.

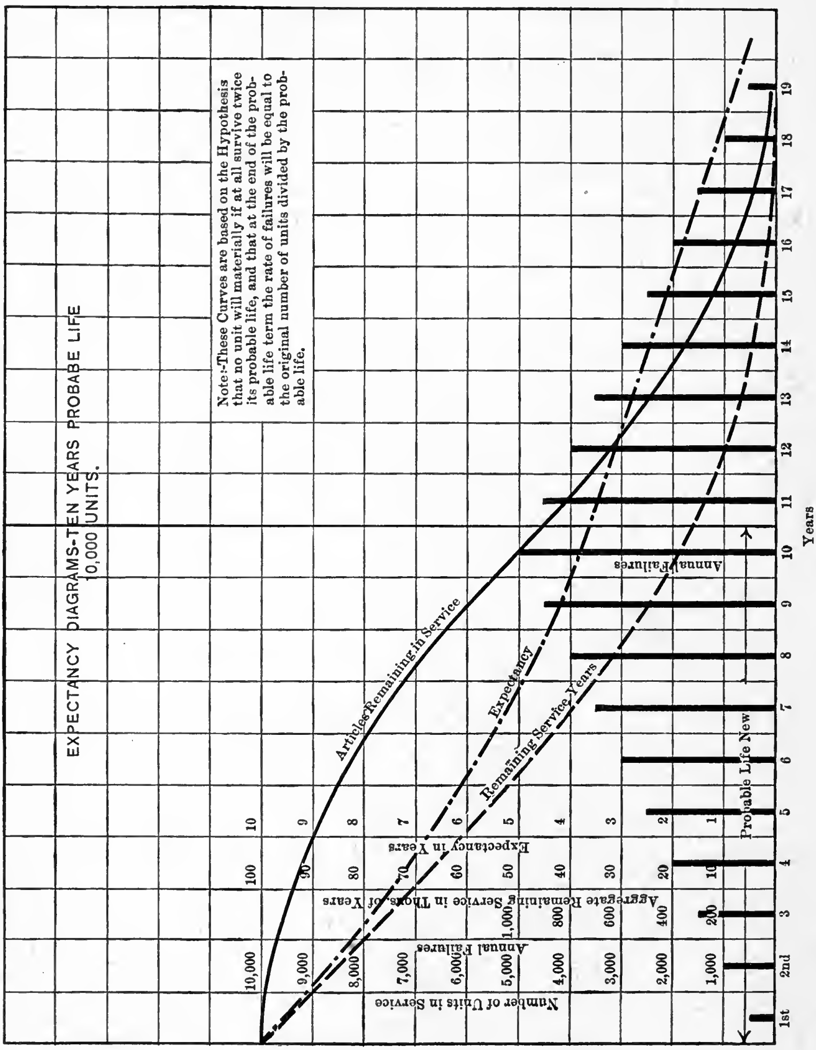

EXPECTANCY DIAGRAMS-TEN YEARS PROBABE LIFE
10,000 UNITS.
Note:-These Curves are based on the Hypothesis that no unit will materially if at all survive twice its probable life, and that at the end of the probable life term the rate of failures will be equal to the original number of units divided by the probable life.
Articles Remaining in Service
Expectancy
Remaining Service Years
Annual Failures
Probable Life New
Number of Units in Service
Annual Failures
Aggregate Remaining Service in Thous. of Years
Expectancy in Years
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
1,000
800
600
400
200
100
90
80
70
60
50
40
30
20
10
10
9
8
7
6
5
4
3
2
1
1st
2nd
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
Years

FIG. 1.

If the reasonableness of the assumption on which Table 3 is based be admitted, or if it should be possible to prove by actual records of failures that these assumptions are near enough to the truth to be accepted as giving results substantially correct, then a further analysis will show that the actual replacement requirements under various conditions of investment will be as shown in Tables 4 to 6. In the preparation of these tables account has been taken of the failures that will occur among the replacements as well as among the units of the original installations.

Diagrammatic Illustration of the Assumed Rate of Failures.— The basis for the results in Table 3 for articles with a probable life of ten years is shown diagrammatically in Fig. 1. The expectancy is found by dividing the remaining service years at any time by the corresponding number of surviving units. The reversed curve marked "Articles remaining in service" clearly indicates the hypothesis of failures on which the table is based. It is to be noted that under this hypothesis there is no serious departure from the results that were obtained by assuming that the law of probabilities would apply.

Tabular Presentation of Replacement Requirements for Groups of Articles. — The replacement requirements, as shown in Table 5, for numerous articles which when new have a probable life of ten years, if failures occur substantially as assumed, and if each failing article be at once replaced, would increase from $1 in the first year to about $10 in the ninth year for $100 of original investment, fluctuating thereafter between $9 and nearly $12 per year and gradually settling down to $10 per year. For an annual investment of $100 per year (*i.e.*, for a growing plant), the replacement requirements would gradually increase from $1 per year in the first year to $463 in the fiftieth year, or from $1 per $100 of investment in the first year to $6.01 in the tenth year to $8.16 in the twentieth and to $9.27 in the fiftieth year.

In practical application, in other words, the annual replacement requirement in the case of a plant of full growth all parts of which have a probable life of n years, after the plant is n years older than any of these parts, will be about one nth of their replacement cost.

In a plant which continues to grow, the theoretical annual replacement requirement will gradually approach but can never quite reach one nth of the total replacement cost. (See Tables 4 to 6.)

TABLE 4. REPLACEMENT REQUIREMENTS

NUMEROUS ARTICLES. PROBABLE LIFE 5 YEARS

Each article is replaced as it goes out of use. For an original investment of \$100 with no betterments or additions. Also for an investment growing at the uniform rate of \$100 per year.

(For the special hypothesis as stated in the text.)

Year.	Plant of full growth, original investment \$100.	Growing plant, annual investment \$100.	
	Replacements per year.	Replacements per year.	Replacements per \$100 of investment.
1	\$ 4.00	\$ 4.00	\$ 4.00
2	8.16	12.16	6.08
3	12.65	24.81	8.27
4	17.64	42.45	10.61
5	23.34	65.78	13.16
6	21.97	87.75	14.62
7	21.16	108.91	15.56
8	20.54	129.45	16.18
9	19.73	149.18	16.58
10	18.31	167.49	16.75
11	19.73	187.22	17.02
12	20.37	207.60	17.30
13	20.45	228.05	17.54
14	20.21	248.25	17.73
15	19.90	268.16	17.88
16	19.87	288.03	18.00
17	19.91	307.94	18.11
18	20.03	327.92	18.22
19	20.06	347.95	18.31
20	20.02	368.00	18.40
21	20.00	388.02	18.48
22	20.00	408.00	18.55
23	20.00	428.00	18.61
24	20.00	448.00	18.67
25	20.00	468.00	18.72

TABLE 5. REPLACEMENT REQUIREMENTS

Numerous Articles. Probable Life 10 Years

Each article is replaced as it goes out of use. For an original investment of $100 with no betterments or additions. Also for an investment growing at the uniform rate of $100 per year.

(For the special hypothesis as stated in the text.)

Year.	Plant of full growth, original investment $100.	Growing plant, annual investment $100.	
	Replacements per year.	Replacements per year.	Replacements per $100 of investment.
1	$ 1.00	$ 1.00	$1.00
2	2.01	3.01	1.51
3	3.04	6.05	2.02
4	4.10	10.15	2.54
5	5.20	15.35	3.07
6	6.36	21.71	3.62
7	7.57	29.28	4.18
8	8.87	38.15	4.77
9	10.25	48.39	5.38
10	11.73	60.12	6.01
11	11.33	71.46	6.50
12	11.03	82.48	6.87
13	10.79	93.27	7.18
14	10.60	103.88	7.42
15	10.46	114.34	7.63
16	10.28	124.61	7.80
17	10.09	134.70	7.93
18	9.85	144.55	8.08
19	9.53	154.08	8.12
20	9.10	163.18	8.16
21	9.53	172.72	8.23
22	9.84	182.56	8.30
23	10.05	192.60	8.37
24	10.17	202.77	8.45
25	10.22	212.99	8.52
26	10.22	223.21	8.59
27	10.17	233.38	8.64
28	10.11	243.49	8.69
29	10.03	253.52	8.74
30	9.97	263.49	8.78
35	9.99	313.31	8.95
40	10.01	363.40	9.09
45	10.00	413.40	9.19
50	10.00	463.40	9.27

TABLE 6. REPLACEMENT REQUIREMENTS

Numerous Articles. Probable Life 20 Years

Each article is replaced as it goes out of use. For an original investment of $100 with no betterments or additions. Also for an investment growing at the uniform rate of $100 per year.

(For the special hypothesis as stated in the text.)

Year.	Plant of full growth, original investment $100.	Growing plant, annual investment $100.	
	Replacements per year.	Replacements per year.	Replacements per $100 of investment.
1	$0.25	$ 0.25	$0.25
2	0.50	0.75	0.38
3	0.75	1.50	0.50
4	1.01	2.51	0.63
5	1.26	3.77	0.76
6	1.52	5.29	0.89
7	1.79	7.08	1.01
8	2.06	9.14	1.15
9	2.33	11.47	1.28
10	2.61	14.08	1.41
11	2.90	16.98	1.55
12	3.19	20.17	1.69
13	3.49	23.66	1.82
14	3.80	27.46	1.97
15	4.12	31.58	2.11
16	4.45	36.03	2.26
17	4.69	40.72	2.41
18	5.15	45.87	2.56
19	5.51	51.38	2.77
20	5.89	57.27	2.86
21	5.74	63.01	3.00
22	5.67	68.68	3.12
23	5.60	74.28	3.23
24	5.53	79.81	3.33
25	5.46	85.27	3.41
26	5.40	90.17	3.49
27	5.36	96.03	3.55
28	5.31	101.34	3.62
29	5.27	106.61	3.68
30	5.22	111.83	3.73
35	4.98	137.28	3.92
40	4.56	161.04	4.03
45	4.97	185.20	4.12
50	5.10	210.54	4.21
55	5.07	236.00	4.30
60	4.99	261.20	4.36
65	4.98	286.09	4.40
70	5.00	311.04	4.45

TABLE 7. THE REPLACEMENT REQUIREMENT IN THE CASE OF NUMEROUS ARTICLES AND A COMPARISON OF METHODS OF PROCEDURE

PROBABLE LIFE 10 YEARS

With due regard to expectancy at various ages. Numerous articles. All installed at same time. No additions to plant. Articles replaced as worn out. Original investment $100. Interest 6 per cent. Net earnings 6 per cent.

(Based on the hypothesis of failures as explained in the text.)

Year.	Probable repl't requ't.	Straight Line Method.			Equal Annual Payment Method.			Unlimited Life Method.		
		Rem'g value.	Amortization allowance.	Req'd earnings.	Rem'g value.	Amortization allowance.	Req'd earnings.	Rem'g investment.	Repl't allowance.	Requ'd earnings.
1	$1.00	$100	$10	$16.00	$100	$7.59	$13.59	$100	$1.00	$7.00
2	2.01	91	10	15.46	93	8.01	13.59	100	2.01	8.01
3	3.04	83	10	14.98	87	8.37	13.59	100	3.04	9.04
4	4.10	76	10	14.56	81	8.73	13.59	100	4.10	10.10
5	5.20	70	10	14.20	76	9.03	13.59	100	5.20	11.20
6	6.36	65	10	13.90	71	9.33	13.59	100	6.36	12.36
7	7.57	62	10	13.72	68	9.51	13.59	100	7.57	13.57
8	8.87	59	10	13.54	66	9.63	13.59	100	8.87	14.87
9	10.25	58	10	13.48	64	9.75	13.59	100	10.25	16.25
10	11.73	58	10	13.48	64	9.75	13.59	100	11.73	17.73
11	11.33	60	10	13.60	65	9.69	13.59	100	11.33	17.33
12	11.03	61	10	13.66	66	9.63	13.59	100	11.03	17.03
13	10.79	62	10	13.72	67	9.57	13.59	100	10.79	16.79
14	10.60	63	10	13.78	68	9.51	13.59	100	10.60	16.60
15	10.46	64	10	13.84	69	9.45	13.59	100	10.46	16.46
16	10.28	64	10	13.84	....	....	13.59	100	10.28	16.28
17	10.09	65	10	13.90	....	....	13.59	100	10.09	16.09
18	9.85	65	10	13.90	....	....	13.59	100	9.85	15.85
19	9.53	65	10	13.90	....	....	13.59	100	9.53	15.53
20	9.10	64	10	13.84	69	9.45	13.59	100	9.10	15.10
21	9.53	63	10	13.78	....	....	13.59	100	9.53	15.53
22	9.84	63	10	13.78	....	....	13.59	100	9.84	15.84
23	10.05	63	10	13.78	....	....	13.59	100	10.05	16.05
24	10.17	63	10	13.78	....	....	13.59	100	10.17	16.17
25	10.22	63	10	13.78	68	9.51	13.59	100	10.22	16.22
26	10.22	63	10	13.78	....	....	13.59	100	10.22	16.22
27	10.17	63	10	13.78	....	....	13.59	100	10.17	16.17
28	10.11	63	10	13.78	....	....	13.59	100	10.11	16.11
29	10.03	64	10	13.84	....	....	13.59	100	10.03	16.03
30	9.97	64	10	13.84	....	....	13.59	100	9.97	15.97

Departure of Actual Failures from Assumed Laws.—While it may be granted that in the long run the failures of individual articles in any class will follow some definite law (perhaps a law similar to that which has above been cited as more probable than failure always at the end of the probable life term), the fact remains that in no particular case, no matter how large a plant may be, will there be absolute conformity with any assumed law. Consequently figures determined on the basis of any reasonable hypothesis of failures can be used to prepare smoothed-out curves and from such curves, tables for general use can be prepared. It is enough to know for the present that ordinarily it may be assumed that the replacement requirement of a large number of articles with a probable life of n years should increase progressively year by year to about one nth of the cost of effecting complete replacement of all articles and that this rate of one nth would be reached at about the nth year.

Table 7 has been prepared to show for numerous articles, all of which when new have a probable life of 10 years, the replacement requirements on the assumption that failures actually occur according to the hypothesis already explained, and to show their relation to the remaining value, the current amortization, and the required annual earnings estimated by the Straight Line and by the Equal Annual Payment methods of procedure, also the remaining investment, the replacement allowance, and the required earnings if estimated by the Unlimited Life Method. These methods of procedure, when rates for public utilities are to be fixed, are explained in Chapter IX.

Expectancy of an Equivalent Single Article.—When numerous articles are under consideration, it may be desirable to know the expectancy of an equivalent single article, representing the aggregate of all the separate articles, which may be used to simplify the estimates of accrued and current depreciation. If the Straight Line Method of procedure is adopted and if numerous articles of the same kind and same individual cost are involved, and if it be assumed that actual life in each case will agree with the estimated probable life, then the expectancy of the equivalent single article will be the average of the remaining ages of the individual articles. But under all other methods of procedure, even with the assumption of agreement between actual and probable life, this will not be the case. A full discussion of this fact would be superfluous, but it is here stated as a caution against a possible erroneous assumption.*

However, in view of the fact that the termination of service or failure of any article does not occur at the exact end of its probable life term, there would be no sense in attempting to establish the remaining probable life for group values unless hard and fast appraisal rules are laid down by those charged with the regulation of public utility rates coupled with adequate assurance that such rules will be adhered to.

The Assumed Hypothesis of Failures. — It may be repeated that the foregoing Tables 4 to 7 are based on the hypothesis that failures of any group of articles of the same probable life will be most numerous at the end of the probable life term; that there will be a gradual uniform increase in the number of annual failures from the beginning to the end of this term; and that but few, if any, of the articles will have a life in excess of double that of the probable life of the article new.

It may be repeated, too, that this hypothesis is not based on adequate experience, that it probably departs further from average results than would be found under strict adherence to the law of probabilities, which is the basis of Table 13, and that it remains subject to modification as experience may determine. Until suitably modified, it offers, however, a better means of approximating remaining value of any article than is afforded when calculations are made from probable life tables without regard to the condition of the article at the time of the valuation.

Failures and Expectancy according to the Law of Probability. — A study has been made, as already stated, to see where the law of probability would lead, and the result of the comparison will be of interest.

* The remaining life of an equivalent single article, when computations are made by the Sinking Fund Compound Interest Methods, is noted in a paper in Transactions American Society of Civil Engineers, Vol. XXV, p. 836, with examples on a 4 per cent interest basis.

Let it again be assumed that practically no article of a large group, all of which have a 10-year probable life, will survive 20 years or twice the probable life term, and that one-half or very nearly one-half of the failures occur within the two years just preceding and the two years just following the end of the probable life term. Then, according to the law of probability, and on the assumption that the failures may be bunched at the end of the successive years, there will be failures in each successive year as shown in Table 8. These are noted only to the nearest 5 in 10,000, and in other respects are offered only as approximations to demonstrate a law rather than the result of accurate computation.

TABLE 8. FAILURES AND EXPECTANCY ACCORDING TO THE LAW OF PROBABILITY

ON THE ASSUMPTION THAT NO ARTICLE SURVIVES TWICE THE PROBABLE LIFE TERM. 10,000 ARTICLES. PROBABLE LIFE = 10 YEARS

Year.	For 10,000 articles.			Single article.
	Number of failures.	Remaining number of articles, beginning of year.	Remaining service years, beginning of year.	Expectancy at beginning of year.
1	15	10,000	100,000	$10.00
2	35	9,985	90,000	9.00
3	85	9,950	80,015	8.05
4	180	9,865	70,065	7.11
5	330	9,685	60,200	6.22
6	550	9,355	50,515	5.40
7	805	8,805	41,160	4.78
8	1065	8,000	32,355	4.04
9	1265	6,935	24,355	3.52
10	1340	5,670	17,420	3.08
11	1265	4,330	11,750	2.71
12	1065	3,065	7,420	2.42
13	805	2,000	4,355	2.18
14	550	1,195	2,355	1.97
15	330	645	1,160	1.80
16	180	315	515	1.64
17	85	135	200	1.50
18	35	50	65	1.25
19	15	15	15	1.00
20	....	0	0	0

In addition to the probable failures from year to year, Table 8 also shows the number of original articles in a group of 10,000 that will be still in service in any year; the probable remaining service years in the surviving articles of the original group; and the expectancy of any single article at any age.

TABLE 9. REMAINING VALUE AND REPLACEMENT REQUIREMENT

REMAINING VALUE OF NUMEROUS ARTICLES WHICH HAVE ALL COME INTO USE AT THE SAME TIME AND ALSO THE REPLACEMENT REQUIREMENT. REMAINING VALUE OF A SINGLE ARTICLE. ORIGINAL INVESTMENT $100. PROBABLE LIFE 10 YEARS

Failures assumed to occur according to the law of probability. 6 per cent interest.

Year.	Single article.			Numerous articles.	
	Expectancy years. Beginning of year.	Remaining value.		Remaining value Equal Annual Payment Method.	Replacement requirement Unlimited Life Method.
		Straight Line Method. Beginning of year.	Equal Annual Payment Method. Beginning of year.		
1	$10.00	$100.00	$100.00	$100.00	$ 0.15
2	9.00	90.00	92.41	92.55	0.35
3	8.05	80.50	84.57	84.94	0.85
4	7.11	71.10	76.79	78.00	1.80
5	6.22	62.20	68.80	69.61	3.31
6	5.40	54.00	61.06	63.34	5.53
7	4.78	47.80	55.00	59.63	8.12
8	4.04	40.40	47.49	56.23	10.80
9	3.52	35.20	41.92	56.56	12.95
10	3.08	30.80	37.18	58.99	13.96
11	2.71	27.10	33.01	......	13.62
12	2.42	24.20	29.70	......	12.23
13	2.18	21.80	26.96	......	10.47
14	1.97	19.70	24.53	......	9.01
15	1.80	18.00	22.50	81.67	8.11
16	1.64	16.40	20.55	......	8.04
17	1.50	15.00	18.87	......	8.53
18	1.25	12.50	15.84	......	9.33
19	1.00	10.00	12.82	......	10.16
20	......	......	......	62.86	10.67
21	......	......	......	......	10.93
22	......	......	......	......	10.87
23	......	......	......	......	10.56
24	......	......	......	......	10.18
25	......	......	......	65.85	9.82
26	......	......	......	......	9.58
27	......	......	......	......	9.56
28	......	......	......	......	9.64
29	......	......	......	......	9.82
30	......	......	......	64.47	9.92

Remaining Value and the Replacement Requirement according to the Law of Probability. — Table 9 has been prepared to show for numerous articles, all of which have a probable life term of 10 years, the probable annual replacement requirement on the assumption that failures occur according to the law of probabilities, all articles going out of use within 20 years. There is also shown in the table the remaining value of these articles if estimated by the Equal Annual Payment Method, and also the expectancy of a single article with a 10-year probable life and its remaining value if computed by the Straight Line Method and by the Equal Annual Payment Method.

Computation of Annuities to Meet Actual Replacement Requirement. — The computation of the annuities which would replace each lot of annually failing articles, if the same be assumed to fail on any hypothesis similar to those already suggested, can readily be made and will prove instructive. It will be found that in every case the sum of all such annuities will exceed the annuity computed in the ordinary way from the average or probable life. If the computation be then extended to cover all articles remaining in service from year to year and to include also the new articles which have been added to replace the failures, it will be found that in the early years the sum of the annuities is larger than the annuity computed by the use of probable life, in the ordinary way, that after a period in excess of the probable life term, the sum of the annuities will be a minimum and somewhat less than that computed in the ordinary way, and that thereafter it will increase again to about the amount computed by the Straight Line Method.

It will be unnecessary to introduce a complete calculation to illustrate this point and only the results for articles with a probable life of 10 years will be briefly referred to.

Let it be supposed that of 10,000 articles, which all have a probable life of 10 years, 100 fail at the end of the first year; 200 at the end of the second, 300 at the end of the third and so on to 1000 at the end of the tenth, 900 at the end of the eleventh, 800 at the end of the twelfth, and so on to the last 100 at the end of the nineteenth.

By substituting dollars for articles and computing the annuity at 6 per cent interest required for the replacements of each year it will be found that an annuity of $94 will be required to replace the failures of the first year, an annuity of $97 to replace those of the second year, an annuity of $94 to replace those of the third year, an annuity of $91 to replace those of the fourth year, and so on to the nineteenth year. The sum of these annuities is $1101 or 11.01 per cent.

If the articles which fail each year are replaced and the required annuities for these replacements are also taken into account then the sum of the annuities at 6 per cent interest will be: 11.01 per cent the first year, 10.17 per cent the second, 9.41 per cent the third, 8.78 per cent the fourth, 8.26 per cent the fifth, 7.86 per cent the sixth and so on to a minimum in about the fourteenth year of a little over 7 per cent, and thereafter gradually increasing to about 11 per cent in the thirtieth year.

Had the determination been made in the usual way, based on the assumption that no distinction need be made between the actual and the probable life, the allowance for replacements would have been 7.59 per year, continuously from the beginning. Conceding a moderate or even a wide range of error due to an almost arbitrary though reasonable assumption of the probable annual failures when average life is known, the result of the computation of what should be set aside annually on the annuity basis, to meet the replacements, when compared with the annuity determined from probable life new, shows that the latter is insufficient.

The computation also shows that the required earnings based on this determination would be undesirably high at the beginning, in the early years, when they should be low.

No further demonstration than the above will be needed to show the futility of depending on the Sinking Fund Method and the Equal Annual Payment Method as usually applied, when estimating the replacement requirements.

The legitimate and always applicable method of procedure when public utility rates are to be fixed, is that which the author calls the Unlimited Life Method. This does not involve any estimate of accrued depreciation, and only reasonable care in determining the annual replacement requirement. The replacement fund is kept apart and if error has been made in the assumption of the life of the items and the fund becomes either too large, or inadequate, the necessary correction can be applied in subsequent years.

CHAPTER VII

THE PURPOSE OF THE APPRAISAL

General Statement

Various Purposes of Appraisal. — The owner of an operating property is interested more in knowing what it has cost him than he is in the value of the property, until the time comes when a transfer of ownership is to be made or the business is to be capitalized. Generally, however, when any enterprise of considerable magnitude is involved the owner should, at all times, have a clear conception of its value. This can only be obtained by making the analysis of the investment and of operating expenses in relation to revenue, and involves much more than a study of financial records. The condition and adaptability of a plant to the uses to which it is being put are involved and must be considered by the appraiser. Valuation, therefore, may be required:

a. As a basis for a purchase and sale transaction.

b. When the property is to be pledged as a security for a loan or as the basis for a bond issue.

c. When the rates of a public utility are to be fixed or regulated, because the earnings resulting from the rates should be adequate to bring a suitable return on the investment and because the charge for the service should be reasonable.

d. As a basis for taxation.

The taxation value bears or is intended to bear some definite relation to market value. This is not only true in the case of ordinary taxation, for the purpose of carrying on government, but also in the innumerable cases in which it becomes necessary to apportion to the property which is benefitted, the cost of some improvement in proportion to the benefits which it confers.

Elements of Value. — No definite and final rules can be laid down as a guide for the appraiser in reaching his conclusions relating to value. He is directly concerned with the ascertainment, as definitely as circumstances will warrant, of the net revenue both present and prospective. He must give consideration to:

The cost of construction.

The cost to reproduce the property new.

The relation of the property to actual or possible competing properties.

The condition of the property, including its adaptability to the intended purpose.

The weight to be given to these items and the method of applying them in making valuations will be discussed in the proper chapters of this publication.

When property with a salable output is valued for purchase or sale, or for capitalization, or bonding, the investigation must be extended to cost of operation and the market value, present and prospective of the service rendered or of the commodity furnished.

In the regulation of rates, also, consideration should be given to the value that will result from the earnings and this likewise involves a comparison of operating costs with earnings, present and prospective, from whatever source.

Valuation for Purchase or Sale

When property is to be valued for purchase or sale, both the seller and the purchaser desire to know the value as determined from the excess of the earnings over the cost of operation.

First Step — Determination of the Cost. — The first question to be answered in determining the amount that may reasonably be assumed to be invested in the property will be what has the property cost or what may it reasonably be assumed to have cost. Perhaps this can be ascertained from the cost records with due consideration of losses from unprofitable operation and with proper allowance for excessive promotion costs, for excessive salary payments, for unprofitable or useless expenditures and losses by accident. When cost cannot be thus ascertained or whenever there is a doubt and the importance of a close approximation warrants such procedure, the cost of reproducing the property is to be estimated. Proper allowance must be made, too, for all expenses of whatever nature connected with construction and with the establishment of the business.

Second Step — Deferred Maintenance and Depreciation. — The second step will be to ascertain the deferred maintenance, if any, and the accrued depreciation. The accrued depreciation will be the difference between the cost of reproduction new and the present or remaining value of the items which make up the property and which are subject to depreciation.

Third Step — Cost of Operation. — The third step will be the determination of the cost of operation. According to the nature of the business this may be directed to the aggregate output of commodities or service, or it may cover a segregated analysis of cost of operation for a variety of services or commodities. But the essential fact to be ascertained is the total outlay including interest on the investment, salaries and wages, supplies, maintenance and repairs, current depreciation or replacement requirements, and in the case of a franchise, or patent right with a limited life term, or in the case of an oil well or a mine with a limited oil or ore body, including also an amortization increment.

Fourth Step — Earnings, Present and Prospective. — The fourth step will be the determination of the earnings, current and prospective, that will result from such charges for service or for the commodity as may reasonably be assumed to be proper and dependable. It is here, in the case of the public utility, that there may be much uncertainty due to the difficulty of forecasting the attitude of the rate-regulating authorities toward the public service corporation. For while it is true that private property may not be taken for public use without due process of law and that there must be no confiscation through inadequate earnings, there is yet some uncertainty relating to the compensation which the owner may expect for establishing and managing the utility and also as to whether, in every case, the value of the service rendered has really justified the enterprise. There is much uncertainty, in other words, in relation to the earnings in excess of interest on the investment that will be permitted by the public service commissions, and, when value depends largely upon prospective business, there may be uncertainty, too, relating to the forecast of earnings on which the estimates of profits are based.

Fifth Step — Determination of Profits. — The fifth step involves a comparison of the cost of operation with the anticipated gross income. The excess of the income over cost of operation represents the profits of the business. The profits may be actual or they may be prospective. The profits both current and prospective in excess of interest on the investment, when capitalized, create an increment of value in excess of the investment. This increment of value may, according to circumstances, be apportioned to "franchise," "going value," "good-will," "patent right," or any other classification of value that may seem appropriate.

Real Estate Value. — The valuation of real estate may acquire special consideration. But here, as in the case of other revenue-producing property, the real test of value is the amount of the revenue which the property will produce. What is the rental value of the property? This is the question to be answered. The appraiser will direct his inquiry to the gross income, immediate or prospective, which is dependable. He will ascertain the taxes and probable assessments for improvements to which the property is liable, and he will determine from these various items the net revenue which the property will produce. This net revenue, immediate and prospective including appreciation, will then determine the value.

When the right of eminent domain is exercised to acquire real estate for some special purpose due consideration must be given to its serviceability for the intended use. This subject is further considered in Chapter X.

Appraisals as a Basis for Fixing Rates

Protection of the Investment. — In determining the investment on which the investor in public service properties should be allowed a reasonable return, all attendant circumstances must be duly considered. It may be stated, however, that, apart from the determination of the rate of interest which should result from the investment, it will be equitable and fair to consider the public service corporation as the agent of the State or municipality, as the case may be, and to determine in what situation the State or municipality would have found itself had there been no intermediate owner or public service corporation.

Let it be assumed that the owner of a public service plant has made his investment under good expert advice, and that the plant is in every respect the same as, or equal to, what the people would have constructed for themselves. Let it be further assumed that the plant is free from debt, that it will have no residual value, and that it and all its parts will have an actual useful life of n years. The owner will then be entitled:

First. — To a reasonable interest on his investment;

Second. — To operating expenses; including maintenance, repair and replacement expenditures;

Third. — To an annuity which, in n years, at compound interest, will amount to his investment;

Fourth. — To reasonable compensation for managing the business which may be based on the volume of business transacted, and on the general prosperity of the community.

Purpose of the Accumulating Replacement Fund. — If it be now supposed that the owner actually received these amounts, estimated on a proper basis, and that he allows the annuity at compound interest to accumulate so that amortization will be an accomplished fact at the end of n years, then, as he has command of the amortization fund, he will have a decreasing amount of capital actually tied up in the plant. This decreasing capital or remaining value of the plant is the complement of the growing amortization fund. This fund, in the case of a plant which is to continue in service beyond the term of n years, is supposed to be held inviolable for the replacement of the plant at the end of its life. The owner reaps no benefit from it whatever, beyond holding it as the means for replacing a worn-out plant.

The value of the plant in its varied stages of depreciation, plus such amortization fund, should at all times be at least equal to the original investment. The owner, even if he gets an annuity, as here assumed, is entitled at all times to the interest, not on a plant valued at first cost or investment less depreciation, but on the entire first cost. Had he determined, instead of building the plant, to keep his funds invested in safe securities at ordinary interest rates, he would, at the end of n years, have been in possession of his entire capital plus interest on the full amount thereof for the entire time. If, under the assumed facts, he were not allowed interest on the full amount invested in the public service plant, an injustice would be done.

This is true even when replacement takes the place of amortization. The owner in this case is entitled to interest on the entire capital invested in the plant, and, at the end of the plant's usefulness, he is also entitled to a return of the capital itself. Suppose that a city constructs a plant, paying cash for it, and collects rates which will just yield a fair rate of interest on the investment. At the end of n years the plant is replaced with a new one of the same capacity. As the city has not included in its rates, theretofore charged, an increment for amortization, it now finds itself in possession of a new plant and a total investment twice as great as the cost of the first plant. Applying the same principles to the second plant, as to the first, rates should be doubled. This, of course, would be an absurdity.

A charge by a city for service rendered which is less than sufficient to amortize the cost of the plant within its life may yet be equitable and proper, when the cost of the plant is intentionally put upon the whole community and not upon the ratepayer.

Rate-Base with and without Deduction of Depreciation. — The application of the fundamental principles, elsewhere noted, to any public utility plant will show that the determination of a rate-base without deduction from the capital actually but reasonably invested is a proper proceeding, provided, of course, that the replacement requirements are not overlooked and are computed by some proper method.

It may be repeated that, when depreciated value or investment less accrued depreciation is taken into account as a basis for computing necessary earnings, the current depreciation or amortization must be computed on the basis of depreciated value or investment and the *remaining* life of the plant or of its parts.

This can best be made clear by an illustration: Let it be supposed that the passenger rates and the freight tariff on a steamboat line are subject to regulation, and that some one going into the steamboat business builds a steamer for the service. Let it be assumed, too, that in connection with this business he requires no capital investment other than the cost of the steamer; that terminal facilities, office space, and whatever else he needs, are obtainable by rental. For the purpose of this illustration, let it be further assumed that the volume of business is such that there is no doubt about the income, so that the element of hazard is eliminated.

If the steamboat has a life of 20 years, it will gradually depreciate in value and will go out of service at the end of a 20-year period. Ignoring its possible scrap value, which is immaterial for the purpose of this illustration, the following questions are to be considered.

At the end of 10 years, with interest at 6 per cent per annum, and earnings just sufficient to yield interest plus an amortization, figured for a 20-year life at $0.027185 on each dollar of the investment:

1. What will be the value of the steamboat to the owner at the end of 10 years?

2. What will be the amount that a purchaser can afford to pay for the steamboat at the end of 10 years?

3. What should be the earnings during the time the steamboat is in possession of the original owner?

4. What should be the earnings during the time the steamboat is in the possession of a purchaser after 10 years of service?

The first and second questions are answered elsewhere. The owner, by one line of reasoning, finds the remaining value in the steamboat to be 64.17 per cent; the purchaser, by a different line of reasoning, finds the same value.

The third question, too, is answered elsewhere. The original owner is entitled to a net return during the entire period of his ownership of 6 per cent on his investment, which is at all times 100 per cent. No deduction is to be made for depreciation because the fund which results from the accumulation of the amortization annuity, together with its interest, is available for no other purpose than the replacement of the steamboat at the end of its period of usefulness. It is dead capital, and remains dead until the property is disposed of or until required to replace the worn-out steamboat. The original owner, therefore, is entitled to a return of $6 + 2.72 = 8.72$ per cent per annum on his investment.

In considering the fourth question, it may at first appear as though the purchaser, having invested only $64.17 on each $100 of original cost, could claim a return on this investment alone — that he should be allowed, in addition to the amortization as above determined, net earnings of $3.85 (6 per cent on $64.17) per annum on what he paid for each $100 of the original cost of the steamboat; that the valuation for rate-fixing purposes, in other words, should be the original investment less depreciation. Under the adoption of this view, it will be seen that, if the steamboat were sold repeatedly, there would be a constantly decreasing appraisal for rate-fixing purposes.

In the last year of its service the valuation entitled to consideration in fixing earnings would be only 8.23 per cent. This view is unfair to the owner of the property, who should be assumed to be planning a continuation of the steamboat business. When he takes possession of the steamer, its value to him, as already set forth, is $64.17 on each $100 of original cost, but, as owner, he at once finds that, of his capital ordinarily available for other purposes, an amount equal to 35.83 per cent of the cost of a new steamboat is, to all intents and purposes, tied up in his steamboat business. It has become dead capital, for all purposes except replacement, so long as he remains in the steamboat business. This 35.83 per cent at interest at 6 per cent is necessary to supplement the annuity regularly going into the amortization fund, together with which at the end of the 20-year period it will just replace the steamer. Whether or not the 35.83 per cent is actually set apart is immaterial; the fact remains that ownership of the depreciating steamer renders this amount of capital as already stated unavailable or dead for any purpose other than replacement, and the new owner is entitled to interest on this 35.83 per cent just as well as on the 64.17 per cent which he paid for the steamer.

The demonstration of this fact may be made as follows: The purchaser of the steamboat, who buys the boat when it has a remaining period of usefulness of 10 years, invests, as has been explained, $64.17 for each $100.00 of the original cost of the steamboat. He is unquestionably entitled to interest on this sum, together with amortization, which at the assumed interest rate of 6 per cent will be:

Interest at 6 per cent on $64.17 per annum..................	$3.85
Amortization annuity for the remaining 10 years, during which the investment of $64.17 is paid back to the purchaser ($7.59 on each $100)..	4.87
Total..	$8.72

This is exactly the same as though, instead of the value of the steamboat, the capital originally invested had been taken into account, in which case the original owner or purchaser would be allowed:

Interest at 6 per cent on the investment of $100 per annum......	$6.00
Amortization annuity to retire $100 of the investment within the life of the steamboat, that is, within 20 years...............	2.72
Total..	$8.72

Although it may be superfluous, one more illustration of this principle will be given: Let it be supposed that the owner borrows money from a bank at 6 per cent per annum to build a steamboat, and that he earns 6 per cent plus the amortization increment of 2.72 per cent.

Of the $8.72 to his credit at the end of each year's business for every $100 of capital invested, he pays the bank $2.72 on account of principal and so much of the remaining $6 as may be necessary to meet the interest then due. This will be all of the $6 the first year, and a decreasing amount thereafter until the end of the 20-year period, when his steamboat is retired. He then finds that he has paid back to the bank on account of the borrowed capital twenty annuity increments of $2.72, amounting to $54.40, and that there is still due to the bank $45.50. He also finds that the various amounts remaining in his hands from year to year, $0.16 at the end of the second year, $0.34 at the end of the third year, $0.52 at the end of the fourth year, and so on, together with interest thereon at 6 per cent, when computed for the 20-year period, will amount to the $45.50, the balance due at the bank. The owner finds he has earned nothing. He has invested no money of his own and has received no return, which is as it should be in this hypothetical case. The rates, however, throughout the entire 20 years were fixed on the principle that 6 per cent per annum should always be allowed on 100 per cent of the capital invested, together with the amortization annuity, but without any deduction for depreciation. They could not have been fixed lower without entailing loss to the owner.

Unlimited Life. — The value of a revenue-producing property when the earnings thereof include an amortization annuity has already been discussed. It remains to consider the case of a property which, in addition to the accepted reasonable rate of interest (net), is without limit as to its time of serviceability, earning the estimated annual replacement requirement determined by some formula, as above explained, instead of the annuity computed from amortization tables.

In this event the plant may be regarded as having unlimited life. Each part thereof as it wears out is replaced out of current earnings. The owner does not maintain an amortization fund, neither is any of his capital rendered dead or unavailable. To him the value of the property is at all times 100 per cent, so, too, in the case of a purchaser. Knowing that all replacement requirements are fully covered by the earnings, the purchaser is willing to pay 100 per cent for the plant, regardless of the amount of accrued depreciation.

The case may be considered of a public service property whose earnings have been inadequate to supply any amortization increment, but which will in the future be able to earn the actual annual replacement requirement. The original investment in this case having been 100 per cent and there having been no amortization annuity in the past, there can be no transfer of the property at less than 100 per cent without loss; but if, by reason of inadequate returns, the market value could not be maintained at 100 per cent, and a sale has been made at less than this sum, the new owner will be compensated and protected if, on his investment which is not original cost, he earns reasonable interest and an adequate amount for replacements. This must be so, because, in the future, actual replacement requirements being covered by the earnings, the worn-out parts will be replaced without cost to the owner. This replacement neither increases nor decreases his investment; but, if the property is extended and new parts are added, such additions represent newly invested capital to the full amount of their cost, and in such a case his investment, expressed as a percentage of the total cost, will gradually increase.

At all times, however, without causing loss to the new owner, that part of the plant which he bought at a depreciated value could be valued at his purchase price, while all extensions subsequent to the purchase, on the assumption that replacements are met out of earnings and that there is no amortization of capital, should, for rate-fixing purposes, be appraised at 100 per cent. Such a course, however, would deprive the new owner of the opportunity for profit, of which he probably thought to avail himself when he bought a plant of depreciated value, and would place the rate-payer in the position of having made a profit at the expense of the original owner. This fact explains why the market value of stocks and bonds is cited so frequently as an indication of value.

Improper Use of the Term "Value." — It may be held that a determination of value for rate-fixing purposes, on the principles herein set forth, is not a determination of value at all. This may be true, but it then becomes a matter of defining "value," as used by the courts in order that a distinction may be made between value and the appraisal of the investment on which rates may be properly based.

The term "value" has been very generally used in matters involving the fixing of rates in the past. When fundamental principles are better understood, more attention will be paid to the capital reasonably and properly invested.

Under a system of permitting the owner of public service properties to earn from year to year the actual average replacement requirements, the necessity for a close distinction between repair and replacement disappears. This is of some advantage, as it is at best difficult to discriminate between small items of replacement and large repair items.

Basis of Rates in the Knoxville Case. — The United States Supreme Court in the Knoxville case Knoxville *vs.* Knoxville Water Co. (212 U. S. 1; 29 Sup. Ct. Rep. 148) says:

"A water plant, with all its additions, begins to depreciate in value from the moment of its use. Before coming to the question of profit at all the company is entitled to earn a sufficient sum annually to provide not only for current repairs, but for making good the depreciation and replacing the parts of the property when they come to the end of their life. The company is not bound to see its property gradually waste, without making provision out of earnings for its replacement. It is entitled to see that from earnings the value of the property invested is kept unimpaired, so that, at the end of any given term of years the original investment remains as it was at the beginning. It is not only the right of the company to make such a provision, but it is its duty to its bond and stock holders, and, in the case of a public service corporation, at least, its plain duty to the public. If a different course were pursued, the only method of providing for replacement of property which has ceased to be useful would be the investment of new capital and the issue of new bonds or stocks. This course would lead to a constantly increasing variance between present value and bond and stock capitalization — a tendency which would inevitably lead to disaster either to the stock-holders or to the public, or both. If, however, a company fails to perform this plain duty and to exact sufficient returns to keep the investment unimpaired, whether this is the result of unwarranted dividends upon over issues of securities, or of omission to exact proper prices for the output, the fault is its own. When, therefore, a public regulation of its prices comes under question, the true value of the property then employed for the purpose of earning a return cannot be enhanced by a consideration of the errors of management which have been committed in the past."

According to this statement by the Court, the owner of a public utility should be required to exact prices for his output, which, at the beginning of operation and at all times thereafter, will cover the current depreciation of the physical elements of his plant. Yet, as elsewhere explained, in the early years, this is frequently impossible, and in nearly every case is inadvisable and would work unnecessary hardship upon the consumer. Losses in the early years when rate-payers are few are oftentimes unavoidable and these losses deserve consideration. Past history cannot be ignored if rates are to be so fixed as to be fair alike to the owner and to the rate-payer. In other words, not all shortage of earnings in the past is to be ascribed to errors of management. It is difficult to reconcile the language of the Court with this principle even as it is difficult to understand why so many of the Courts have held that value which results from earnings must be made the starting point when rates are to be fixed.

"Value" Defined in the Minnesota Rate Cases. — Justice Hughes in delivering the opinion of the U. S. Supreme Court in the Minnesota Rate Cases (June 9, 1913) (230 U. S. 352) said:

"The depreciation in question is not that which has been overcome by repairs and replacements, but is the actual existing depreciation in the plant as compared with a new one. It would seem to be inevitable that in many parts of the plant there should be such depreciation, as, for example, in old structures and equivalent remaining on hand, and when an estimate of value is made on the basis of reproduction new the extent of existing depreciation should be shown and deducted. . . . And when particular physical items are estimated as worth so much new, if in fact they be depreciated, this amount should be found and allowed for. If this is not done, the physical valuation is manifestly incomplete. And it must be regarded incomplete in this case."

In the Minnesota Rate Cases the Master, in ascertaining a basis for rate fixing, had allowed the cost of reproduction new without making any deduction for accrued depreciation. The Master did not deny that there was depreciation but found that the same was more than offset by the appreciation in value of certain items. It was this finding which the Supreme Court refused to approve.

The Supreme Court in its decision as usual makes value the starting point. Consequently the court says that depreciation must be deducted from the cost of reproduction new. But cost of reproduction is not "value." Not even the value of the property new is exactly determined by its original cost. The opinion of the court becomes illogical if it be conceded to be unnecessary to make value the starting point when rates are to be fixed. But the Master, too, seems to have overlooked this point and sought to find something, appreciation in this case, to offset depreciation. He was quite as much at fault as the Supreme Court. And yet the allowance of appreciation in determining present value as a basis for fixing rates or for the issue of bonds is not uncommon. Railroads claim appreciation of their road-beds. Though it may be granted that the compacted road-bed, long in service, is worth more than a recently constructed bed, this should have nothing to do with the fixing of rates, no more than accrued depreciation should have. The new road-bed requires more careful watching and greater expense for upkeep. Consequently the net earnings are lower than they would be with the same volume of business, the same rates and a well-compacted old road-bed. If the resulting earnings are inadequate in the early years of business, the deficiency may be treated as a temporary investment to be amortized out of the larger net earnings of later years.

Depreciation in the Pocatello Water Company Case. — The Supreme Court of Idaho in a recent decision (1915) in the Pocatello Water Company case, Murray *vs.* Public Utilities Commission (150 Poc. Rep. 47, p. 50), reversing the Public Service Commission of that State says in reference to depreciation:

"So far as the question of depreciation is concerned, we think deduction should be made only for actual tangible depreciation and not for theoretical depreciation, sometimes called 'accrued depreciation.' In other words, if it be demonstrated that the plant is in good operating condition and giving as good service as a new plant, then the question of depreciation may be entirely disregarded."

This decision is in substantial accord with the minority report of the Commission which had been made by A. P. Ramstedt and in which he said:

"A person having invested his money in a continuous business enterprise for the benefit of others must always be ready to replace the constructive portions of his plant as they wear out. A person having embarked on such an enterprise is justly entitled to compensation to cover this depreciation in addition to a fair return, over and above expenses, upon the reasonable value of the property. Allowance for depreciation cannot, in my judgment, be considered as profit or an earning factor in the business. . . . The fact that in an investigation of the petitioner's property . . . it is found that the market value of his physical property employed for public use has depreciated $77,188.39 does not, in my judgment, justify the commission, in its determination of a fair value for rate purposes, to deduct the amount of depreciation from the present estimated cost of reproducing the property new, and thereby reduce the earning power of his property."

The fact is evidently being recognized in Idaho as elsewhere, that the service rendered should always be 100 per cent good even though rendered by a plant whose physical parts are perishable and that if a 100 per cent valuation for rate purposes is fair when the plant is new it is equally fair when the plant is old.

Wisconsin Railroad Commission on Investment as the Rate-base. — The view which the author takes relating to making the necessary original cost and not "value" the starting point when rates are to be fixed finds some support in the following statement by the Wisconsin R. R. Commission in the City of Appleton Case (Wis. R. C. R., Vol. 5, p. 220).

"For rate-making purposes the actual total investment, subject to certain qualifications, seems to be the basis for determining the reasonableness of the charges that may be exacted of the public for the services rendered or product furnished in certain jurisdictions. Of course where such information is not available, the reasonable value of the investment would have to be ascertained by some method of appraisement, and in such event the 'actual total investment' doctrine would be inapplicable."

Appraisals for Taxation Purposes

The Taxation Value. — The tax value is usually intended to bear some definite relation to the market value. In some of the older Eastern cities of this country the tax value is intended to represent the full market value. Generally, however, this is not the case. Real estate will usually be found to be valued for taxation purposes at from 50 to 75 per cent of its market value and personal property sometimes still lower.

The appraiser of tax values is therefore interested in market value and must give consideration to all the elements that go to establish market value, such as supply and demand, earning capacity present and prospective, rate of appreciation and the like. But this is a subject which is foreign to the purposes of this volume and may be dismissed with the above broad statement.

Appraisals for the Purpose of Capitalization

Capitalization. — The primary matter to receive consideration when a property is to be valued for the purpose of capitalization is its earning capacity. What are the net earnings, present and prospective? When this question can be satisfactorily answered the problem has been solved. Inquiry must therefore be made relating to the total annual outlay involved in conducting the business enterprise. Operating expense of every kind including an adequate annual replacement requirement and an amortization increment if the life of the enterprise is limited, must be weighed against the assured present and prospective income. The resulting net earnings capitalized at the interest rate, which an enterprise of the nature under consideration should yield, will indicate the upper limit of value for capitalization.

When a public utility is under consideration the problem may be complicated by the fact that rates and therefore the earnings are subject to regulation and in consequence of the control and regulation exercised by the public through its public service commissions, any great precision in estimating the gross present and prospective earnings may be out of the question. This may prove embarrassing particularly in those cases where the actual investment of capital is small but the volume of current business is large. In all cases the actual properly invested capital will be a first approximation of value for capitalization purposes and usually the lower limit of such value if the property is one in successful operation.

CHAPTER VIII

THE FIXING OF RATES

Public Utilities and the Regulation of Public Service

Ownership of Public Utilities. — In the older countries the public utilities are generally owned and managed by the State or municipality. In the countries, on the other hand, which are but sparsely populated and in which the potentiality of natural resources is large, private enterprise is usually depended upon to make the development.

Certain utilities are almost universally publicly owned. This is true of city streets, very largely of country roads and bridges and, in most countries, of the sewers. There is probably not a city in the United States in which a charge is made for the service rendered by the sewer system. The benefit, in this case, to the community as a whole, of properly disposing of human excrement and such domestic waste as can be floated away with water, is generally recognized. There is no apportionment of cost to individuals, in other words, according to the value of the service. The cost of establishing and maintaining the system is raised by taxation.

The streets and public roads, too, are generally built at public expense. But there are other utilities such as water-works, electric light and power works, gas-works, telephone systems, railroads and other transportation systems which may be either publicly or privately owned. The private ownership is usually exercised through a corporation. That this should be so is natural for the reason that stability of management is thereby secured, the element of uncertainty in the matter of the life of the owner being eliminated.

Quasi-public Character of Public Utilities. — While the public service corporation is subject to the same general laws which control other corporations operating for profit, they are accorded certain rights and privileges which give them a quasi-public character. They have the right of eminent domain, *i.e.*, the privilege of taking, under due process of law, private property for their uses (subject, of course, to the making of adequate compensation therefor) and they may be accorded the privilege of occupying streets, roads and public places subject to reasonable regulations.

The tendency of the day is to give the public service corporations more and more monopolistic character and to trust to regulation and control for the protection of the public interest. This seems wise and should make for good service at reasonable rates, always provided that regulation be not too stringent and onerous and that the public service corporation be allowed to make a fair profit.

The Disadvantage of Frequent Modification of Rates. — In some of the States, as until within recent years in California, the right to regulate has found its main expression in laws requiring the rates of certain utilities to be fixed annually. The water and gas rates have thus been under annual investigation in San Francisco, and the occasional establishment of rates which were thought inadequate has led to much long drawn-out litigation which has rarely resulted satisfactorily to either of the litigants.

As the result of such litigation relating to water rates established by San Francisco for 1902–03 and for the years subsequent thereto, the court has allowed, temporarily, the collection of rates by the Spring Valley Water Company in excess of the limit fixed by the city authorities and the excess collected has been impounded and now (1915) amounts to over $1,500,000. It is only fair to say, however, that by recent amendment of the charter of the city, the rates will hereafter be subject to regulation by the California State Railroad Commission (Public Service Commission).

When the established rate is made subject to frequent change and the owner of the utility is not allowed to earn more than some predetermined interest return on his investment, the incentive to improve the service falls away, because, if the owner succeeds in economizing, in producing his output at a lower cost, the immediate result will be lower rates and less earnings. The gross income may be reduced though, perhaps, the value of the service to the rate-payer may remain unaffected or may even be increased. Rates should not therefore be subject to change too frequently. If there must be a term during which the established rates will apply, let it be at least 5 years.

Control and Regulation. — The acceptance of the privilege to operate a public utility carries with it a submission to such regulation as may be demanded by the public which grants the privilege. This regulation must be reasonable. The public utility owner is in business for profit. He invests his capital and applies his ability and experience to the development of enterprises that would otherwise remain dormant for a longer or shorter period. He brings within reach of the community, means of transportation, an adequate water supply, light, power and heat, telegraph and telephone facilities which all contribute to the growth and prosperity of the community. He does this because he considers his enterprise a good business venture.

In the case of the commercial and industrial enterprise, too, a tendency may be noted toward monopoly which is likely to lead in time to regulation and control similar to that of the public utility. Whenever the operation of any concern acquires such magnitude that it can and does destroy the business of its competitors, it becomes monopolistic in character and the state is bound, sooner or later, to exercise its right to regulate and control not only the method of conducting business, but in the end, perhaps, the prices to be charged for its output.

Excessive and Fictitious Profits. — Where, in the rapid transition from a frontier region to a state with stable industrial and commercial conditions, the advance in the value of real estate has been rapid and the opportunity for profit in many lines of business has been large, the owner of the public utility has frequently taken advantage of the opportunity and has capitalized inordinate profits, sometimes real and sometimes fictitious. Where there were no public service commissions — and these are a relatively new institution, — the accounts were kept in a fashion to suit the manipulator. Apparent profits were used as the basis for a distribution of dividends and for the issuance of stocks and bonds, and the rate-payer was charged to the limit.

The Right to Regulate Rates. — The right to regulate rates is now quite generally recognized. Without such right every corporation which has a monopoly would have the public at its mercy and could charge for its service all that the traffic will bear.

The right to regulate, if properly exercised, should in the long run prove of advantage to the owner of the utility. Fair regulation will insure to the owner what he is entitled to and will protect him against unfair competition such as has frequently been the result of opportunity to capture with a relatively small outlay the cream of the business.

Under proper regulation the owner of every public utility whose existence is justified by all attendant circumstances, should get the protection of his legitimate reasonable investment, also a fair interest return on this investment and should recover all the necessary operating expenses including replacement requirements. He should receive such treatment that at some time, by amortization of the capital which he has invested, or by a sale to some purchaser, he can withdraw from the business without loss, provided of course, that he has exercised due foresight and has not met with unavoidable losses such as may be caused by floods or earthquakes.

Regulation Essential. — The necessity for regulation has now become apparent. The practice of granting perpetual franchises with unwise privileges has fallen into disfavor. This first became manifest in laws permitting the periodical regulation of public service rates by local, municipal and county authorities. Thus, for example, in California the Constitution of the State (in effect Jan. 1, 1880 but now modified) provided:

"The use of all water now appropriated, or that may hereafter be appropriated, for sale, rental, or distribution, is hereby declared to be a public use and subject to the regulation and control of the State, in the manner to be prescribed by law; provided, that the rates or compensation to be collected by any person, company or corporation in this State for the use of water supplied to any city and county, or city or town, or the inhabitants thereof, shall be fixed annually by the Board of Supervisors, or City and County, or City or Town Council, or other governing body of such city and county, or city or town, by ordinance or otherwise, in the manner that other ordinances or legislative acts or resolutions are passed by such body, and shall continue in force for one year and no longer. Such ordinances or resolutions shall be passed in the month of February of each year, and take effect on the first day of July thereafter. (Art. XIV, Sec. 1) (1880)

"The right to collect rates or compensation for the use of water supplied to any county, city and county, or town, or the inhabitants thereof, is a franchise, and cannot be exercised except by authority of and in the manner prescribed by law." (Art. XIV, Sec. 2) (1880)

"In any city where there are no public works owned and controlled by the municipality, for supplying the same with water or artificial light, any individual, or any company duly incorporated for such purpose, under and by authority of the laws of this State, shall under the direction of the Superintendent of Streets, or other officer in control thereof, and under such general regulations as the municipality may prescribe for damages and indemnity for damages, have the privilege of using the public streets and thoroughfares thereof, and of laying down pipes and conduits therein, and connection therewith as far as may be necessary for introducing into and supplying such city and its inhabitants either with gas light or other illuminating light, or with fresh water for domestic and all other purposes, upon the condition that the municipal government shall have the right to regulate the charge thereof." (Art. XI, Sec. 19) (1884)

San Francisco Charter Provisions relating to Rates. — The Charter of the City and County of San Francisco (in effect January 8, 1900) mentions, among the powers of the Supervisors:

"To fix and determine by ordinance in the month of February of each year, to take effect on the first day of July thereafter, the rates of compensation to be collected by any persons, company or corporation in the City and County, for the use of water, heat, light or power, supplied to the City and County or to the inhabitants thereof, and to prescribe the quality of the service."

The enforcement of such laws as these has, at times, proven embarrassing to the public service corporations, and has given rise to charges of corruption, to wrangling and to litigation, unfortunate alike for the corporations and for the public. The remedy has been sought in national and state control, this control being exercised through National and State Commissions.

The Interstate Commerce Act. — The Interstate Commerce Act in line with this idea was approved on February 4, 1887. The purpose of this Act may be stated as follows:

"The principal objects of the Interstate Commerce Act were to secure just and reasonable charges for transportation, to prohibit unjust discrimination in the rendition of like services under similar circumstances and conditions; to prevent undue or unreasonable preferences to persons, corporations or localities; to inhibit greater compensation for a shorter than for a longer distance over the same line; and to abolish combinations for the pooling of freight." (Interstate Commerce Commission *vs.* Baltimore & Ohio Railroad Co., etc., 145 U. S. 263.)

Public Service Control in Massachusetts. — The creation of a Board of Railway Commissioners in Massachusetts in the early sixties may be noted. A Gas and Electric Light Commission followed in 1885. The Board of Highway Commissioners in that State controls the Telegraph and Telephone Companies.

Public Service Control in California. — The Constitution of California, which went into effect on Jan. 1, 1880, contained a provision for a Railroad Commission. But its powers were limited and its work was not effective. By an amendment to the Constitution in 1911 the powers of the Commission were more clearly defined and its work speedily met with deserved approval. By further amendment the powers of the Railroad Commission were extended (Mar. 28, 1911) to all other public utilities so that the Railroad Commission of California is now in effect a public service commission.

Sec. 23 of Art XII of the California Constitution now provides:

"Every private corporation, and every individual or association of individuals, owning, operating, managing or controlling any commercial railroad, inter-urban railroad, street railroad, canal, pipe line, plant or equipment, or any part of such railroad, canal, pipe line, plant or equipment, within this State, for the transportation or conveyance of passengers or express matter, or freight of any kind, including crude oil, or for the transmission of telephone or telegraph messages, or for the production, generation, transmission, delivery or furnishing of heat, light, water or power or for the furnishing of storage or wharfage facilities, either directly or indirectly, to or for the public, and every common carrier is hereby declared to be a public utility subject to such control and regulation by the Railroad Commission as may be provided by the Legislature, and every class of private corporations, individuals or association of individuals hereafter declared by the Legislature to be public utilities shall likewise be subject to such control and regulation. The Railroad Commission shall have and exercise such power and jurisdiction to supervise and regulate public utilities, in the State of California, and to fix the rates to be charged for commodities furnished, or services rendered by public utilities as shall be conferred upon it by the Legislature, and the right of the Legislature to confer powers upon the Railroad Commission respecting public utilities is hereby declared to be plenary and to be unlimited by any provision of this Constitution."

The California Public Utilities Act by an amendment in effect August 10, 1913, provides:

"The Term 'public utility,' when used in this act, includes every common carrier, pipe line corporation, gas corporation, electrical corporation, telephone corporation, telegraph corporation, water corporation, wharfinger and warehouseman, where the service is performed for, or the commodity delivered to, the public or any portion thereof. The term 'public or any portion thereof,' as herein used, means the public generally, or any limited portion of the public including a person, private corporation, municipality or other political subdivision of the state, for which the service is performed or to which the commodity is delivered, and whenever any common carrier, pipe line corporation, gas corporation, electrical corporation, telephone corporation, telegraph corporation, water corporation, wharfinger or warehouseman performs a service or delivers a commodity to the public or any portion thereof for which any compensation or payment whatsoever is received, such common carrier, pipe-line corporation, gas corporation, electrical corporation, telephone corporation, telegraph corporation, water corporation, wharfinger or warehouseman is hereby declared to be a public utility subject to the jurisdiction, control and regulation of the commission and the provisions of this act. Furthermore, when any person or corporation performs any service or delivers any commodity to any person or persons, private corporation or corporations, municipality or other political subdivision of the state, which in turn either directly or indirectly, mediately or immediately, perform such service or deliver such commodity to or for the public or some portion thereof, such person or persons, private corporation or corporations and each thereof is hereby declared to be a public utility and to be subject to the jurisdiction, control and regulation of the commission and to the provisions of this act."

Public Service Control in New York. — In New York there has been for many years a Railroad Commission. In 1905 a Commission was established for the control of gas and electric corporations. In 1907 these commissions were superseded by two public service commissions, which were not however given control over water companies.

Public Service Control in Wisconsin. — In Wisconsin, too, the need for regulation of transportation rates first found expression in the creation of a railroad commission and, as in the case of California, the powers of this Commission were subsequently extended so that it is now a public service commission. This Commission exercises the same control over municipally owned utilities as over those which are privately owned. It has accomplished much of the pioneer work in standardizing methods of procedure and its work is of great value in establishing the fundamental principles of valuation as a basis for the fixing of rates.

Public Service Control in Other States. — Many other states have public service commissions or railroad commissions with duties and powers of a public service commission. Georgia, Oregon, Nebraska, Oklahoma, Maryland, Colorado, Florida, Illinois, Indiana, Iowa, Maine, Missouri, Pennsylvania, Rhode Island, South Carolina, Vermont and other states started off with a railroad commission. Arizona, Kansas, Nevada, New Jersey, New Hampshire and Oklahoma have their public service commissions more or less liberally endowed with powers of regulation and control.

Supreme Court of Wisconsin on the Powers of the Railroad Commission. — All the powers which a public service commission has are conferred upon it directly by the people or by legislative enactment. Its function is to make administrative regulations. Its powers are not legislative. On this point the Supreme Court of Wisconsin says (Minn. St. Paul and Sault Ste Marie Ry. Co. *vs*. Railroad Com. of Wis., 136 Wis. 146):

"The division of the governmental powers into executive, legislative and judicial, while of great importance in the creation or organization of a State, and form the viewpoint of institutional law and otherwise, is not an exact classification. No such exact delimitation of governmental powers is possible. In the process of enacting a law there is frequently necessary the preliminary determination of a fact or a group of facts by the Legislature, and it is well settled that the Legislature may declare the general rule of law to be in force and take effect upon the subsequent establishment of the facts necessary to make it operative or to call for its application. . . . The Legislature may delegate any power, not legislative, which it may itself rightfully exercise. This power to ascertain facts is such a power as may be delegated. . . . This law established, and thenceforth assumes the existence of rates, charges, classifications and services, discoverable by investigation but undisclosed, which are exactly reasonable and just. It commits to the Railroad Commission the duty to ascertain and disclose that particular rate, charge, classification or service. The law intends that there is only one rate charge or service that is reasonable and just. When the order of the Commission is set aside by the Court, it is because this reasonable and just rate, charge, classification or service has not yet been correctly ascertained. When the order of the Commission has been rescinded or changed by the Commission because of the changed conditions it is because there is a new reasonable rate to be ascertained and disclosed, applicable to such new conditions and fixed by force of law immediately when the new conditions came into existence. But the theory and the mandate of the law is that this point is always discoverable although not always discovered. Until it is discovered and made known the former rates and service prevail. The order of the Commission is prima facie evidence that the rate, charge or service found and fixed by it is the particular rate, charge or service declared by the Legislature in general terms to be lawful and to be in force. If it were conceded that the Commission had power or discretion to fix one of several rates, either of which would be just and reasonable, it would be hard to say that this was not a delegation of pure legislative power to the Commission. But the theory of this law is to delegate to the Commission the power to ascertain facts and to make mere administrative regulations.

. . . .

"The notion that commissions of this kind should be closely restricted by the courts, and that justice in our day can be had only in courts, is not conducive to the best results. Justice dwells with us as with the fathers; it is not exclusively the attribute of any office or class, it responds more rapidly to confidence than to criticism, and there is no reason why the members of the great Railroad Commission of this State should not develop and establish a system of rules and precedents as wise and beneficent within their sphere of action as those established by the early common-law judges. We find the statute well framed to bring this about."

Competition in the Public Service. — Before the advantage of coöperation and combination leading to monopolistic control was fully realized by the owners of business ventures, it was common belief that free and unrestricted competition should be encouraged and would result in permanent advantage to the public. The fallacy of this view when large public service concerns are involved is now generally recognized. The competition in the case of such concerns may result in a momentary lessening of rates but in the long run the rate-payer or the security holder will be the sufferer. The temporary advantages may become a permanent burden.

Coöperation Between he Public and the Owner. — The treatment of the public service corporation should be such as to encourage the undertaking of other public service enterprises. It should not be a hard and fast rule that a fixed rate of return will be allowed in all cases. The owner should feel that good management will be rewarded. There is perhaps no better way of securing the desired hearty and cordial coöperation between the owner of the utility and the public than by the introduction of a system of profit sharing. This has been done with apparently great success in Chicago and other cities where revenue in excess of an agreed interest return on an appraisal of street car lines is divided on a reasonable basis between the city and the owner. So, too, in Boston, for every decrease of 5 cents per 1000 cubic feet in the charge for gas the Gas Company is permitted to increase its dividend rate one per cent. The consumer benefits by the lower rate, the Gas Company by the larger dividend.

Attitude of the Public Service Commissions. — Public service commissions were not created to assume an attitude hostile to the public service corporations. While they must protect the public against excessive charges for the service rendered and must see to it that there will be no discrimination against individuals, they are likewise charged with the duty of allowing the corporations to do business on a fair basis. It must not be expected, however, that mistakes will not be made. As the records to date are read, such mistakes will be found in both directions. On the whole the attempt to be fair is generally recognized and the confidence of the people in their public service commissions is growing from day to day.

The Rate-Base

Net Earnings as a Basis of Value. — When there is no competition and no restriction by law or otherwise upon the charges which may be made by public utilities for commodities furnished or for service rendered, it is safe to assume that these charges will be fixed on the broad principle of "all the traffic will bear." This does not mean all that the rate-payer would be willing to pay rather than to be deprived of the commodity or of the service, but that amount which the owner finds to be best adapted to maintain and increase his aggregate profits. He will in determining this amount give consideration to all elements of cost, to his investment, to the hazards of the business, to the possible profitable extension of his business, to the value of the service and to profit. When rates are thus established by an owner, the net earnings bear no definite or fixed relation to the investment and the value of the business is to be determined from a consideration of the present and prospective net earnings and of those elements which determine the permanency of the business. In such a case, in other words, it is proper to capitalize assured net earnings to determine value. When value thus ascertained is in excess of cost, the excess represents profit and such excess has been all too frequently used as a basis for "watering" stock and for over-bonding. Not only has this been the case, but, by ignoring depreciation, or by making inadequate provision for the replacement of worn-out parts, it has sometimes been possible to make the net earnings appear greater than they actually were, and thus to further swell the apparent though, in part, fictitious value.

It is fundamentally sound policy to encourage the development of the natural resources of any country, state or region by permitting those who establish public utilities to earn a reasonable profit on the investment made for the benefit of the public, and to earn a fair interest rate, provided that the investment has been wisely made.

The Public Utility Affects the Value of Other Property. — No necessary public utility can be established without at once adding to the value of all real property in the district which it serves. The profit resulting from the establishment of the utility is general. No water-supply system can be put in to serve a municipality, no work for the improvement of a water-supply, such as high pressure fire protection can be added to the system without adding value to vacant lots as well as to those already built upon. Value is added when gas and electricity are brought within reach, when streets are improved, when sewers are built, when tunnels or bridges for better intercommunication are constructed. The addition of this value cannot always be immediately capitalized but it is nevertheless an asset and should not be entirely lost sight of when rates are to be fixed.

This point is here brought out in order that the reasonableness of allowing the owner of the public utility a fair profit in addition to ordinary interest on his investment may be realized.

Public Contribution to the Revenue of Public Utilities. — The owner of real estate or other property even when not served by the utility, may reasonably be required to make a direct contribution toward its earnings. Not all of the revenue should, in other words, come from the rate-payers. It is frequently practicable to secure a contribution from the property owner who is not a rate-payer by establishing a high rate for the service rendered to the public and thereby reducing the rate for the service to the rate-payer, or by a remission of local taxes in whole or in part, or by granting a bonus when a new installation is involved.

When an effort was made some years ago by a rival telephone company to secure permission to operate in San Francisco, the inducement of free service for all city departments was held out. The advisability of accepting the offer of the outside company was passed upon by the author, who was at that time City Engineer of San Francisco, and the disadvantage of competition in the telephone service was pointed out. The new company did not acquire the requested privilege at that time, though it did so under a later administration. The acceptance of the offer to render service free to the City, while attractive, was unfair to the rate-payer who is thereby made to carry, not only the full burden entailed by the telephone service of which a part should be carried by the vacant lot, but also the additional amount which is saved to the City by free service.

While it would be correct in principle and economically sound to let a suitable part of the revenue of every public utility be provided by a general tax, the application of this principle is not as simple nor as easy as appears at first view. The main difficulty lies in the fact that tax rates are already high and that the layman sees the force of the argument that the payment for the individual service rendered should be measured by the cost of the service. It is convenient to ignore the unearned increment that goes to the parties who are not served by the utility and it has become customary to ignore the same, consequently, the practice of letting the rate-payer carry the full burden may be expected to continue with only rare exceptions.

Cost of Rendering the Service as a Modifier of Rates. — In the case of the service rendered by certain utilities special effort is made to determine the cost of serving various classes of customers in order that the charge for the service may bear some direct relation to the cost of rendering the service in each class. This may be unwise. The prime consideration is what should the rate-payer in each class be asked to pay when his ability to pay is weighed against that of the rate-payers in other classes. Perhaps the establishment of class service is not always the best procedure, but it is legitimate and at present apparently the only practicable way of conducting certain business enterprises. There is no intent to find fault with the system except only to say that, if cost to serve the individual rate-payer were made the criterion of the charge for the service, there would be practically as many rates as there are rate-payers. In the application of the principle, therefore, whether always sound or not, it has become customary to classify the service and within the established classes to make the charge uniform.

Consideration is to be given, then, when rates are to be fixed, not alone to the total amount of the earnings, but also to their distribution to various classes of consumers. This is a subject which will not be taken up in this volume, in which the problem under discussion is concerned with aggregate earnings and not with the distribution thereof to the individual rate-payer.

Deficient Earnings in Early Years. — Nearly every public utility begins operating at a loss. There may be a number of years during which the earnings will not be sufficient to cover ordinary operating expenses (including depreciation) and these years are generally followed by others in which the return is not commensurate with what the investment should earn. The deficiency in the earnings of the early years in the case of any legitimate public utility should be made good to its owner in some way at some time. This may be done by treating any ascertainable deficiency in the earnings as a permanent investment representing cost of establishing the business or by making suitable provision to amortize it in a reasonable time. When, therefore, the appraisal of a rate-base or the fixing of rates for a long-established public utility is in question, it should be assumed to be quite as likely that past history will show a deficiency as that it will show an excess of earnings over what should be considered a reasonable return. It follows that past history relating to insufficient earnings should be brought under review and that there may be cases in which the contrary opinion of the U. S. Supreme Court, as expressed in the case of Knoxville *vs.* Knoxville Water Company (212 U. S., p. 14) (29 Sup. Ct. Rep., p. 148), does not strictly apply. In this case, quoted at greater length on p. 134, the Court says:

> " If, however, a company fails to perform this plain duty and to exact sufficient returns to keep the investment unimpaired, whether this is the result of unwarranted dividends upon overissues of securities, or of omission to exact proper prices for the output, the fault is its own."

The omission to exact prices for the output that would keep the investment unimpaired may be due to the fact that the service rates necessary to accomplish this would have been greater than the service would bear. The failure to secure adequate earnings is not always the fault of the owner of the property. It may even be the deliberate program to let the ample returns of future years make good the early deficiency as, for example, when the Unlimited Life Method of procedure is adopted.

In the Kennebec Water District Case (97 Maine 185; 54 Atlantic 6) the Court in its instructions to the appraisers of the properties of the Maine Water Co. said:

"The actual rates which may have been charged heretofore, and the actual earnings, are both admissible and material in determining the value of the plant. The value of the evidence, however, will depend upon whether the appraisers shall find that the rates charged have been reasonable."

The Basis of the Calculation when Rates are to be Fixed. — Enough has been said to show that, when an appraisal is to be made as a basis for establishing the rates to be charged for the commodity or the service output of a public utility, something else is needed besides value. The starting point in the last analysis is not the value of the property, but the amount of the legitimate investment necessary to establish the business supplemented by the volume of the business transacted. The legitimate investment is not necessarily the actual cost of constructing and developing the property and bringing it to a paying basis, but it is the amount which may reasonably be assumed to have been properly and legitimately invested in the public service. This amount may for convenience be called the "rate-base," and as here defined is to be distinguished from "value."

It is to be understood that this definition of the rate-base is more or less academic and has not yet obtained general recognition. The ordinary practice has been to attempt to make "present value" the basis of the calculation when fixing rates. The endeavor to do this, with inclusion in the amount on which an interest return is to be computed, of certain elements of value which can have no existence unless created by adequate earnings, has met with only indifferent success and has led the rate-fixing bodies into much confusion from which there now appears to be a strong desire to escape.

The "rate-base," as thus defined, may include some or all of the cost of establishing and developing the business. It would be unwise to lay down an inflexible rule that the entire cost of establishing and developing the business should be included in the rate-base because any such rule would result in placing a premium upon inefficient management. The aim should be to include a reasonable allowance for this item which in one case may be determined from cost records while in other cases it may have to be estimated from less dependable data and assumptions.

The rate-base should be made the basis of the calculation. The earnings present and prospective should be adequate to yield a proper return on the rate-base and they should moreover also be adequate to create some value in addition thereto which will be compensation to the owner for having established and for managing the utility and may be in lieu of appreciation which under this method of procedure would not always appear in the rate-base.

The Effect of a Bonus upon the Rate-base. — Attention has already been called to the fact that when the owner has received a bonus, in order to encourage construction, the original cost to him is reduced by the amount of the bonus. This circumstance should not be overlooked in determining the rate-base.

How the bonus paid to the owner may affect the rate-base can be made clear by an illustration. The case can readily be conceived of an irrigation system constructed at an average cost of $30 per acre. The right to take water from this canal system is made the matter of contract, each land owner who takes water for a given tract paying to the canal owner a bonus of $20 per acre. When all the land within the area to be served by the canal has acquired the right to take water from the canal, the owner will be out of pocket only $10 per acre and not $30. Full justice will be done to both the owner and to the irrigator if the rate-base in this case be estimated at $10 and not at $30 per acre, due consideration being given to all other elements that should be included in the rate-base.

Two Procedures may be Followed in Determining a Rate-base. — There are two standpoints from which the matter of determining the rate-base may be approached and the procedure in the matter of making an appraisal which is to serve as a basis for fixing rates will vary according to the standpoint taken.

In the one case, the investment is regarded as unimpaired, and no deduction is made for amortization or depreciation, and the amount of the investment properly determined is made the rate-base.

In the other case the amounts earned as depreciation and remaining unexpended are considered as being applied to the retirement of capital, in which event the rate-base is the properly invested capital less the accrued depreciation (it being assumed that current depreciation has actually at all times been earned and collected).

These two views have led to various procedures when the fair earnings of a public utility are to be determined — these will be given further attention in following pages.

The late Commissioner J. W. Eshleman, in writing the decision of the Railroad Commission of California in the matter of the application of James A. Murray and Edward Fletcher for an increase of water rates in the county of San Diego, Cal., says: " My own view is that the nearest and fairest approximation which may be made to a correct ' value ' upon which a public utility shall be allowed to earn is the amount of the investment wisely made, and this view is not at all in conflict with the position of the courts in this regard." The Commissioner as appears from this statement recognizes the desirability of making " investment " and not " value " the basis of the calculation but apparently feels the necessity of reconciling the use of the amount invested with the decisions of the courts, which, for the most part, still hold " value " to be the starting point.

Factors to be Considered in Establishing the Rate-base. — In determining what should be regarded as the legitimate reasonable investment in any public utility to serve as a basis for computing rates, some consideration may be given to certain factors which aid in determining what the value of the property resulting from net earnings should be. Such factors are the capitalization of the business enterprise, the original cost, the cost of reproduction new, the depreciated or present value, the taxation value and the reasonable charge for the service rendered. There should be taken into account, too, in the broad problem of determining net earnings, the past history, sacrifices made by the owners to establish the business, financial aid by the public, the quality of the service, the cost of like service rendered elsewhere and other similar matters. The aim will be to establish, first, a rate-base on which the owner is to get a fair interest return and, second, to ascertain what the net earnings should be in addition to this return to create such a value as will be fair to both the owner and to the public.

Comments on Base Physical Cost by the Wisconsin Railroad Commission. — Referring to the use of the original cost of the properties of the LaCrosse Gas and Electric Co. as the basis of the calculation the Wisconsin Railroad Commission states (Wis. R. C. R., Vol. 2, p. 16) in substance that it is quite likely that, because of local conditions or the manner in which the plants have been acquired, the cost of these plants to the present owner is considerably higher than the figures given (cost of reproduction). Owing to progress in the electrical field most plants have had to be rebuilt or re-equipped. The cost to present owners of such plants is likely therefore to be much greater than either the cost of the original plant or the cost of reproduction new.

The Commission in the Antigo Water Case (Wis. R. C. R., Vol. 3, p. 631) says:

"The original cost, the cost of reproduction new and the present value bear a very close relation to the physical property of the plants and are therefore of the greatest importance in determining the value of the same. As to which one of these three elements is of the greatest importance in fixing this value, is a matter that largely depends upon the circumstances in each case and may also be more or less affected by the purposes for which the valuation is intended. . . . No matter what the differences between these three cost values may amount to, each one of these items constitutes evidence of the true value and should therefore be carefully considered."

The Commission also says that the burden of unreasonably high construction cost should not be charged to the ratepayers.

Comments on Value and Depreciation by the Wisconsin Railroad Commission. — Substantial recognition of the principle that present value is not a proper rate-base is found in the findings of the Wisconsin Railroad Commission in the case of the Superior Commercial Club *vs.* Duluth Street Railway Co. (Wis. R. C. R., 1912, Vol. 11, p. 1 to 21). The Commission makes the following statement:

" A valuation of the physical property of the Superior division of the company as of June 30, 1911, showed a cost new of $717,538 and a present value of $487,236. When the present value of the physical property of 1911 is increased by the present value of that part of the property located in Duluth, but chargeable to Superior and which cannot greatly exceed $70,000, when additions of about $10,000 are made for working capital and when proper allowances for depreciation and going value are added, it will be found that the total amount does not quite reach the cost value new. In fact, it does not greatly exceed $700,000. This sum finds support in the cost of reproduction of the plant and the business as well as in their original cost. . . . As under normal conditions investors are entitled to have their property or investment kept intact, it follows that the amounts, which have been properly set aside for such purposes or for depreciation, in accordance with the provisions of the law and the rules of the Commission, should in the instant case be included in the amount on which returns are allowed. On the other hand amounts earned for depreciation but withdrawn or used for other purposes than provided by law should not be so included."

Again in the case of the Stevens Point Lighting Co. relating to service and rates the Commission says (Wis. R. C. R., Vol. 14, p. 364):

" The failure of a utility to make allowance for depreciation if the earnings have been sufficient is tantamount to a withdrawal of capital from the business and the cost of reproduction new must be diminished in determining the fair value upon which the reasonable return allowed is to be based when an adequate reserve for depreciation has not been provided. The utility is, however, entitled to earn an amount sufficient to offset future depreciation."

Other Factors Affecting Rates. — While it is important to establish a rate-base whenever rates are to be fixed, there may be cases in which other circumstances are of equal moment with the rate-base as a guide to the allowable earnings. It may happen that the public service requires only a small investment of capital compared with the volume of the business that is transacted, and it may then be more desirable and equitable to bring the compensation of the owner into some relation to the volume of business transacted rather than to the capital which is invested in the business.

The case may readily be conceived of a concern such as an express company which rents its office facilities and operates under contract with railroad and steamship companies and which, outside of its trucks and other vehicles for the local distribution of the parcels entrusted to its care, has made no investment of any moment. It would be vain in such a case to attempt a regulation of rates based solely upon a fair return upon the invested capital. The whole field must be brought into view. The volume of business transacted, and the value that would be created if earnings are allowed which exceed, in some definite fashion, the cost of conducting the business, should receive due consideration. If earnings are thus allowed which exceed the cost of operation by 10 to 15 per cent, this would not seem unreasonable unless the resulting rates are, in fact, more than the traffic can or should bear.

Compensation for Hazard. — The risk of loss assumed by the owner when he undertakes the development of a public service enterprise is an element for consideration when a limit is to be set upon the allowable earnings. This fact is generally recognized. The compensation for this risk should be determined on the basis of the risk that may be justly contemplated by those who enter upon similar ventures under like conditions. This principle is clearly set forth by the Supreme Judicial Court of Maine in its instructions to the appraisers of the Maine Water Company's properties in the Kennebec Water District Case (97 Maine 185; 54 Atlantic 6):

"The reasonableness of the rate may also be affected, for a time, by the degree of hazard to which the original enterprise was naturally subjected; that is, such hazard only as may have been justly contemplated by those who made the original investment, but not unforeseen or emergent risks. And such allowance may be made as is demanded by an ample and fair public policy. If allowance be sought on account of this element, it would be permissible at the same time to inquire to what extent the company has already received income at rates in excess of what would otherwise be reasonable, and thus has already received compensation for this hazard."

Earnings in Relation to the Rate-base and to the Volume of Business. — If it can be accomplished without making rates unreasonable, there should be not only an interest allowance upon the rate-base equal to the current rate on money invested in industrial and related enterprises, preferably about 6 per cent per annum, but also some additional allowance, perhaps generally less than 5 per cent, unless conditions are unusual, as a reasonable participation in the general prosperity of the community (this in lieu of appreciation), plus some percentage allowance on the reasonable cost of operation (not including in this cost the interest on the invested capital). This latter allowance should, perhaps, be so graded that it will be small when the investment is large in relation to cost of operation and that it will approach 10 per cent or, in some cases, be even more, if the investment in relation to the cost of operation is small. The objection which may be urged to such an arrangement, that it will make it to the interest of the owner to inflate the cost of operation, is to be weighed against the danger of discouraging investment in public utilities if any less liberal policy is pursued. Moreover, under the control which is now exercised over the public utility by the public service commissions, this objection will lose much of the force which it might otherwise have.

The capitalization of the percentage allowance on the cost of operation, plus any other allowance (except amortization) in excess of a fair interest on the investment, will represent the sum of intangible values, in excess, of course, of such portions thereof as have found a place in the rate-base.

The Franchise Value and the Rate-base. — While recognizing that earnings may be so high that a franchise has value the Wisconsin Railroad Commission places itself squarely on record against the inclusion of such value in the value which is made the basis for fixing rates. The Commission in the Antigo Water Co. case (Wis. R. C. R., Vol. 3, p. 727) says in substance that though earnings may be high enough to yield a surplus that may be made the basis for determining a franchise value, properly subject to taxation, this fact by no means implies that these earnings or any value based thereon should also be the basis for rate-fixing. Taxation, according to the Commission, is based on the ability to pay. Rates should be based on the cost of service (measured by expenses) including a fair return on the investment. The Commission apparently recognizes, what the author endeavors to make clear, that "taxation value" and therefore "market value" really depends on the earnings and therefore, too, upon the rates.

In the case of the City of Appleton *vs.* Appleton Waterworks Co. the Commission again says (Wis. R. C. R., Vol. 5, p. 282) that in estimating capital upon which return is to be made the franchise has no value, because franchise value is produced by the rates charged. It is proper, however, to tax the company on its franchise value which would be paid for if the business were sold.

The Commission in the case of the State Journal Printing Co. *vs.* Madison Gas and Electric Co. (Wis. R. C. R., Vol. 4, p. 586) declares franchises to be monopolistic in their nature. They belong to the community. Their value has been created by the growth of population and by economic and social developments rather than by individual effort. Belonging to the public, the right of control and of the disposal thereof also rests in the community. Under these conditions according to the Commission, there appear to be no good grounds upon which the value of exclusive privileges of this kind should become private property.

CHAPTER IX

POSSIBLE PROCEDURES WHEN THE RATES FOR A PUBLIC SERVICE ARE TO BE FIXED

The Prime Consideration. — When a method of procedure is to be selected under which the necessary earnings of a public utility are to be determined, the first consideration will be the fairness of this procedure both to the rate-payer and to the owner of the utility. Although there are various methods of procedure which have been adopted throughout the country, and which may be shown to be correct under various restrictions as to application and under the assumption that their application is continuous from the beginning, it will not do to accept any particular one as equitable or advisable in every case.

It is elsewhere shown that when capital is assumed to be retired at a rate which keeps pace with theoretical depreciation, serious consideration must be given to the departure of actual life from the probable life of the various parts of the utility which have a limited period of serviceability. It is made clear that a proper accounting system must be adhered to and that a change of procedure may result to the disadvantage either of the owner or of the rate-payer.

The Preferable Method of Procedure. — Apart from the fairness which is essential, the adopted method of procedure should not impose unnecessary burdens upon either party. That method which requires the least earnings in the early years of their life will be preferable in the case of newly established enterprises. In the case of properties that have been long in use, past history must be taken into account, as it will not do to arbitrarily assume that the business has been a profitable one from the beginning. The presumption should rather be the other way; the probability that there were losses in the early years should not be overlooked.

The various methods of procedure have had their origin in the various plans that have been adopted for making provision for amortization of capital and the replacement or renewal of discarded articles. When the distinction between amortization and the replacement requirements is disregarded, such methods of procedure as the Straight Line Method and Equal Annual Payment Method are the result. When amortization is treated as a matter apart, such methods as the Sinking Fund Method and the Unlimited Life Method are the natural outcome.

The Sinking Fund Method. — The "Sinking Fund Method" of making appraisals and of determining necessary earnings is a method under which the annual allowance for replacements is uniform in amount. This replacement allowance (frequently referred to as depreciation) is the annuity which, together with compound interest at the rate of the net earnings of the property, will during the probable life of an article amount to its replacement cost. The replacement, it is assumed, is to be accomplished *at the end of the probable life of that article.* There is no repayment of capital. The investment remains undiminished. This is, therefore, a 100 per cent valuation method.*

The amount which goes into the replacement fund does not retire capital. It generally remains in the business and may be assumed to earn the same amount as any other capital used in the business. Nevertheless, it is held by some authorities that this fund should be treated as though invested in absolutely safe securities and that, therefore, the interest rate introduced into the calculation should be about 4 per cent. But whether the fund be invested in outside securities or in the business, it is a fund which should be separately accounted for. Its earnings are earnings of the business and make the amount which must be collected from the rate-payer correspondingly less. When the fund remains in the business, it should be charged with the regular interest rate of return and this amount of the earnings

* Trans. Am. Soc. C. E., Vol. LXXV, p. 828.

should be added to the fund and will not be available for distribution as profit.

To illustrate — if the investment is $1,000,000 and out of surplus earnings $100,000 are invested in betterments, the total investment becomes $1,100,000 of which $100,000 is due to the replacement fund. If the net earnings are 6 per cent or in the aggregate $66,000 per annum, only $60,000 of this will be available for distribution to the owners and the other $6000 must be added to the replacement fund.

Equal Annual Payment Method. — The " Equal Annual Payment Method," as described in the report of the American Society of Civil Engineers' Special Committee on Valuation, presented to the Annual Meeting of January 21, 1914, refers to a method which makes the annual depreciation, or amortization increment an amount increasing, from year to year according to a definite law. The annual depreciation is estimated by sinking fund methods. It is equal to the annuity which will retire the remaining value in the remnant of the original probable life term and, when added to the interest on the remaining value (uniformity of interest rates being assumed), the sum will be uniform from year to year throughout the probable life term. When depreciation allowances computed by this method are actually earned, they are considered as refunds of invested capital. The remaining investment in that case decreases as the earned depreciation decreased by expenditures for replacements accrues. Depreciation earnings may be regarded as being thus applied to retire capital, but this method of estimating what should be earned from year to year is undesirable, because it involves frequent, elaborate and cumbersome re-estimates of remaining value, requiring the service of experts, while as usually applied, it is identical in results with the Sinking Fund Method, which has the advantage of simplicity.

The Straight Line Method. — The " Straight Line Method " of estimating the annual depreciation or the annual amortization installment is that method which makes the annual amortization of capital uniform throughout the probable life of an article. The annual amortization installment is estimated from the original cost of an article less residual value and its probable life, by dividing the cost less residual value by the number of years of probable life. This method, like any other amortization method, is justified on the theory of the immediate application of the annual depreciation installment to the retirement of capital. In this respect both this method and the Equal Annual Payment Method differ essentially from the Sinking Fund Method and from the Unlimited Life Method, under neither of which any retirement of invested capital is taken into consideration.

Unlimited Life Method. — The "Unlimited Life Method" of procedure, when rates are to be fixed, is justified by the fact that public utilities may generally be regarded as having perpetual life and that, ordinarily, capital need not be retired unless the property is to be purchased. A public service property taken in its entirety may be treated, therefore, as though it had unlimited life. The property is kept in good condition by the repair of its parts and by replacements, as these parts become useless and have to be discarded. No part of the investment, if there be unlimited life, need be returned to the owner, but as the plant grows old and one part after another has to be replaced, he must be allowed to recover in the earnings the cost of each article as replaced. It is on this principle that many complex and, particularly, publicly owned properties are operated. In the early years of the life of a property made up of many parts, such as rails and ties, the replacement requirements will be small. As the property acquires age, the replacements — provided that extensions are relatively unimportant and negligible in comparison with the extent of the property under consideration — will gradually increase to nearly the amount indicated by the Straight Line Method. The departure from this amount will be dependent upon the annual extensions of the system in relation to the entire investment. There will be ultimate agreement between the replacement requirement and the amount estimated by the Straight Line Method if the plant is one that has ceased to grow. The valuation for rate-fixing purposes under the Unlimited Life Method of procedure will be the amount of the investment without deduction of depreciation, and the replacement requirements, until definitely ascertained by experience, will be approximated from the estimated cost of effecting the replacements, with due consideration of the age and expectancy and the probable life new of the individual parts of which the property is made up.

Sinking Fund Method Illustrated. — Some of the principles that govern the establishment of rates may be made clear by the use of an illustration. An electric generator with a 20-year life will serve the purpose. Assume, in the absence of any accepted method of procedure, that remaining value is determined by deducting from cost the accrued depreciation estimated by compound interest sinking fund methods. Suppose the generator to be a part of a light and power system and suppose, further, that it has reached the last year of its useful life and will have no scrap value. A purchaser will value the generator, if he estimates interest on a 6 per cent per annum basis, at about 8.23 per cent of its original value and this is all that he will pay for the same. He takes upon himself, when he buys the light and power plant, an obligation to replace the generator with a new one at the end of another year. He must then renew the investment represented by this article. The obligation which he voluntarily takes upon himself at the time of his purchase to replace the generator in a year is 91.77 per cent of the cost of a new generator. At the end of the life of the generator, he will have received in his earnings the last increment of the original investment in the generator, or 8.23 per cent of its cost, and he will meet his obligation to continue in business by acquiring a new generator and thereby renewing the full original investment in this particular appliance.

It makes no difference whether there is only one generator, or whether there are 20 in use. The principle is always the same. Moreover in such a simple case as that of 20 generators of all possible ages, each with a useful life of exactly 20 years, the accrued obligation to replace these generators when worn out (6 per cent per annum interest being again used for purposes of illustration) will be 38 per cent as shown by the formula on page 93.

In other words, the remaining value of the 20 generators (cost less depreciation, as ordinarily noted) will be about 62 per cent. No purchaser would include the 20 generators in his valuation of the property at more than 62 per cent of their aggregate cost; but he would, nevertheless, and with reason, expect to be allowed to earn interest on 100 per cent of their cost new, claiming rightfully that he is entitled to the same rate of income return on unexpended annual replacement increments as he is entitled to earn on the rest of his invested capital. He will justify this claim by pointing out that the earnings on any accumulated replacement fund are not available for any other use than the replacement of worn-out property; that such earnings do not, therefore, represent income; and that it is for this reason he is entitled to have the aggregate annual replacement allowance, together with accumulated interest, treated as interest-bearing capital.

California Law Restrictions upon the Depreciation Fund. — In this connection attention may be called to the laws of California (Statutes of 1911, First Extra Session, Ch. 14, Sec. 49), which prescribes that the Railroad Commission (having the duties of a public service commission):

> "may from time to time ascertain and determine and by order fix the proper and adequate rates of depreciation of the several classes of property of each public service utility. Each public service utility shall conform its depreciation accounts to the rates, so ascertained, determined and fixed, and shall set aside the moneys so provided for out of earnings, and carry the same in a depreciation fund, and expend such fund only for such purposes and under such rules and regulations, both as to the original expenditures and subsequent replacement as the Commission may prescribe. The income upon investments of moneys in such fund shall likewise be carried in such fund."

It is here apparently recognized that the depreciation fund should be used for the sole purpose of replacing worn-out parts and that in regulating rates, consideration need be given only to the amount of capital reasonably and properly invested without any deduction for depreciation.

The earnings, in other words, to be adequate, must include at least operating expenses of every character, a proper allowance for present and prospective replacements (so-called current depreciation) and a reasonable income computed at the proper interest rate on the invested capital.

Comparison of Sinking Fund and Equal Annual Payment Methods. — Enough has already been said to show that the so-called Sinking Fund Method of procedure, when no distinction is made between actual and probable life, is exactly equivalent in its results to the Equal Annual Payment Method. The former, in which the rate-base is 100 per cent of the investment, involves a single computation of the annuity which at compound interest will amount in the probable life term to the capital invested in any article. The latter, under which accrued depreciation is deducted from the investment, requires a re-estimate of the current amortization (depreciation) increment from year to year and a new annual estimate of accrued depreciation.

Amount of Amortization or Amount in the Replacement Fund. — If under any method of procedure the owner of a plant of numerous parts and all possible ages is required to place all sums earned as " depreciation " or earned to meet "replacement requirements" into a special fund, and the depreciation or replacement requirement has been actually earned from the beginning, the amount of amortization accomplished out of such a fund or the theoretical balance on hand, after meeting all current replacement requirements, will be as follows:

Under the Straight Line Method of estimating amortization for old plants of full growth, the theoretical accumulation should be 50 per cent of the investment in perishable property — this accumulation is independent of the interest rate and this fund's earnings are of no concern to the rate-payer.

Under the method of estimating the " accrued obligation to replace" by the Sinking Fund Method, for old plants, with interest at 6 per cent and the hypothesis that actual life will conform to probable life, the theoretical accrued obligation to replace or what is equivalent, the theoretical accumulation in the fund as shown in Table 2, page 94, will be:

					Per cent
For articles	having	5	year	life, about	38
"	"	10	"	"	40
"	"	20	"	"	38
"	"	30	"	"	34
"	"	40	"	"	31
"	"	50	"	"	28
"	"	60	"	"	25

These percentages apply strictly, of course, only to cases in which the age of the plant is greater than the life of its parts. They indicate the extent, however, to which an accumulating replacement or depreciation fund may exceed the demands upon it. In some measure, approximating this unexpended surplus, such a fund is available for outside investments. Interest on the same, when it represents the obligation to replace, must be earned by the owner for the benefit of the plant, consequently, it makes no difference to what purpose he applies this surplus, provided only that ample assurance remains that replacements will actually be made when necessary. Interest is to be added to this fund even when the same is invested in the plant and by the amount of such interest, the earnings will not be available for distribution as profit.

When the capital is being amortized, the accumulations in the amortization fund are the absolute property of the owner. They are not subject to control by the rate-payer, yet, here, too, proper assurance may be expected that replacements will be duly made and it would not be unreasonable to expect an adequate special fund to be maintained to meet emergencies.

Soundness of 100 Per Cent Appraisal as Rate-base. — In order to show beyond all question that the method of valuing at the investment without deduction of the depreciation in fixing rates, under continuous application from the beginning, is correct and proper, an electric generator with a life of 20 years, which has served 15 years, may again be taken as a basis for an illustration.

The usual assumption is made, for the purpose of this illustration, that there is no change in the cost of this article during its life, that it has no scrap value and that it will go out of service when exactly 20 years old. Interest in this illustration is taken at 6 per cent per annum.

By the "Equal Annual Payment Method" (correct if applied from the day the article went into use and if there were, in fact, agreement between actual and probable life):

Original investment		$100.00	
Life (new)	20 years		
Time in service	15 years		
Remaining life	5 years		
Accrued depreciation (amortization to date)		63.27	
Remaining value		36.73	
Interest on remaining value			$2.20
Annual depreciation or annual amortization increment for sixteenth year			6.52
Required net earnings			$8.72

In this case the depreciation in the 16th year is that amount which, invested annually at 6 per cent, will retire the remaining value $36.73 in the remaining 5 years of life.

By the "Sinking Fund Method" (correct under continuous application from the beginning and agreement between actual and probable life) the computation is as follows:

Permanent investment		$100	
Life (new)	20 years		
Time in service	15 years		
Remaining life	5 years		
Interest on the investment			$6.00
Annual depreciation or annual replacement increment for any year			2.72
Required net earnings			$8.72

While it can be shown that for each year the earnings should be $8.72 on $100 of original capital investment, a new calculation is necessary, as already stated, for each perishable article and for each year under the Equal Annual Payment Method, while for the more rational Sinking Fund Method, a single simple calculation suffices for each group of articles of the same probable life term.

For comparison the application of the other methods of procedure to the same generator may be of interest.

By the "Straight Line Method" (correct under continuous application from the beginning and agreement between actual and probable life) there will be:

Original investment		$100	
Life (new)	20 years		
Time in service	15 years		
Remaining life	5 years		
Accrued depreciation (amortization to date)		75	
Remaining value		$25	
Interest on remaining value			$1.50
Annual depreciation or amortization increment for sixteenth year			$5.00
Required net earnings			$6.50

By the "Unlimited Life Method" on the assumption that there is only a single generator and that the annual replacement increment is estimated by the compound interest annuity method:

Original investment		$100
Life (new)	20 years	
Time in service	15 years	
Remaining life	5 years	
Interest on the investment		$6.00
Annual replacement increment		2.72
Required net earnings		$8.72

The foregoing comparisons, as above stated, are based on the assumption, which can never be fully realized, that there is absolute agreement between the actual and the probable life of each article.

Effect of Departure of Actual Life from Probable Life. — To illustrate the application of the various methods of procedure with some regard to the fact that of many articles having the same probable life when new, some will actually serve a shorter and others a longer time, let it be assumed that a group of articles is under consideration whose average life, and therefore whose probable life, is 5 years. Let it be further assumed in order to give definiteness to the problem that there will go out of use and be replaced 4 per cent of these articles at the end of the first year; 8 per cent at the end of the second year; 12 per cent at the end of the third year; 16 per cent at the end of the fourth year; 20 per cent at the end of the fifth year; 16 per cent at the end of the sixth year; 12 per cent at the end of the seventh year; 8 per cent at the end of the eighth year; and 4 per cent at the end of the ninth year.

This hypothesis of failures has already been referred to and its basis explained in Chapter VI. Two courses are open. Either the allowance for amortization or so-called depreciation is extended for each article throughout the period of its probable life, regardless of whether the article fails early or survives, or, as an alternative, the annual allowance of amortization or depreciation is continued throughout actual life and stops with the failure of each article. Each article which takes the place of another which is discarded is supposed to receive the same consideration and the same treatment in the accounts as an original article.

The result is shown in Table 10.

Explanation of Table 10. — In Table 10 there is noted for the Equal Annual Payment Method in column A the amortization requirement computed from amortization tables, entering the tables not with age, but with probable life new less the expectancy; and in column B, the amortization requirement as determined from Table 20 by taking the difference between the remaining value at the beginning of successive years, giving, as explained, consideration to the new articles introduced each year to replace discarded articles. When amortization of capital continues during the actual life of each article, terminating when the article goes out of use, the Equal Annual Payment Method is no longer, strictly speaking, an equal annual payment method because the interest on the remaining value plus the annual depreciation is no longer constant in amount.

TABLE 10. COMPARISON OF RESULTS

THE SINKING FUND, EQUAL ANNUAL PAYMENT AND UNLIMITED LIFE METHODS

When consideration is given to the requirements for periods determined either by probable life or by the actual life of articles.

FIVE-YEAR PROBABLE LIFE. NUMEROUS ARTICLES

Failures as noted in the text — 6 per cent interest

Year.	New invest. for each $100 of orig. invest. beginning of year	Sinking Fund Method.		Equal Annual Payment Method.			Unlimited Life Method
		Repl. allowance during prob. life	Repl. allowance during actual life	Amort. during prob. life.	Amortization during actual life.		Replacement requirement
					A	*B*	
1	$100.00	$17.74	$17.74	$17.74	$17.74	$15.10	$ 4.00
2	4.00	18.45	17.74	19.51	18.66	17.46	8.16
3	8.16	19.89	17.74	22.13	19.31	19.65	12.65
4	12.65	22.13	17.74	25.72	20.09	17.30	17.64
5	17.64	25.26	17.74	30.40	19.89	19.40	23.34
6	23.34	11.66	17.74	12.63	19.64	20.30	21.97
7	21.97	14.85	17.74	16.32	19.69	20.60	21.16
8	21.16	17.17	17.74	19.11	19.34	21.10	20.54
9	20.54	18.58	17.74	20.98	20.41	19.70	19.73
10	19.73	19.95	17.74	21.46	20.44	18.30	18.31

For comparison with the Sinking Fund Method and the Equal Annual Payment Method, there are noted in the last column of Table 10 the annual replacement requirements, which are the amounts to be provided if the Unlimited Life Method of procedure is adopted. By the plan of amortizing the cost of each article or for providing a fund for its replacement in the exact term of its probable life, there will be a rapid accumulation in the replacement or depreciation fund, or a rapid amortization of capital in the early years, which is an undesirable feature of operation. To find the necessary earnings by each of the several methods, interest on the capital remaining as an investment is to be added to the amounts noted in the table.

Amortization During the Probable Life Term. — When only a single article is involved, as in the case of a steamboat, the disadvantage of adopting the plan of amortizing its cost or of providing a replacement fund in the exact term of its probable life is apparent. The steamboat may meet with an accident in the early years of its life. If it does and is replaced with a new one, amortization of the remaining value of the first steamboat plus the amortization of the cost of the new steamboat will be necessary. The burden will fall upon the rate-payer. Under the alternative plan of continuing a uniform annual amortization allowance during the actual term of the steamboat's service there will be no effect apparent upon the required earnings by an early failure of the steamboat. The loss of the first steamboat will fall upon the owner, but, if rates are equitably established, the loss will be made good to him in the course of time by reason of the survival of other steamboats and the continuance of the depreciation or amortization allowance after original cost has been amortized.

The wise plan is the one in which there is the least disturbance of the rates and in which, so far as may be, the required earnings will be least in the early years.

The Book Accounts Under Various Methods of Procedure. — The book accounts relating to the foregoing tabular illustration when numerous articles all with a probable life new of 5 years are involved would show the following for each $100 of original investment:

a. Sinking Fund Method with a replacement allowance for the term of the probable life of each article, regardless of the time of its actual failure. (Each article which replaces a failing article is here treated as a new article.)

	Dr. side of Ledger			*Cr. side of Ledger*	
1st yr.	To depr. or amort. allowance		$17.74	By renewals	$4.00
2nd yr.	To int. on balance	$13.74	0.82		
	To depr. or amort. allowance		18.45	By renewals	8.16
3rd yr.	To int. on balance	24.85	1.49		
	To depr. or amort. allowance		19.89	By renewals	12.65
4th yr.	To int. on balance	33.58	2.01		
	To depr. or amort. allowance		22.13	By renewals	17.64

5th yr.	To int. on balance 40.08	2.40		
	To depr. or amort. allowance	25.26	By renewals	23.34
6th yr.	To int. on balance 44.40	2.67		
	To depr. or amort. allowance	11.66	By renewals	21.97
7th yr.	To int. on balance 36.76	2.21		
	To depr. or amort. allowance	14.85	By renewals	21.16
8th yr.	To int. on balance 32.66	1.96		
	To depr. or amort. allowance	17.17	By renewals	20.54
9th yr.	To int. on balance 31.25	1.88		
	To depr. or amort. allowance	18.58	By renewals	19.73
10th yr.	To int. on balance 31.98	1.92		
	To depr. or amort. allowance	19.95	By renewals	18.31
	Totals	$203.04		$167.50
	Balance			35.54

At the end of the tenth year the amount in the depreciation fund would be $35.54 for each $100 of original investment.

b. The Sinking Fund Method accounts, if the replacement requirement or depreciation were estimated during the actual life of each article, would be charged, at the end of each year with $17.74 and interest on the annual balance. It would be given credit for $4.00 renewals the first year, $8.16 the second year; $12.65 the third year and so on.

	Dr. side of Ledger		*Cr. side of Ledger*	
1st yr.	To repl. allowance	$17.74	By renewals	$4.00
2nd yr.	To int. on $13.74	0.82		
	To repl. allowance	17.74	By renewals	8.16
3rd yr.	To int. on $24.14	1.45		
	To repl. allowance	17.74	By renewals	12.65
4th yr.	To int. on $30.68	1.84		
	To repl. allowance	17.74	By renewals	17.64
5th yr.	To int. on $32.62	1.96		
	To repl. allowance	17.74	By renewals	23.34
6th yr.	To int. on $28.98	1.74		
	To repl. allowance	17.74	By renewals	21.97
7th yr.	To int. on $26.49	1.59		
	To repl. allowance	17.74	By renewals	21.16
8th yr.	To int. on $24.66	1.48		
	To repl. allowance	17.74	By renewals	20.54
9th yr.	To int. on $23.34	1.40		
	To repl. allowance	17.74	By renewals	19.73
10th yr.	To int. on $22.75	1.37		
	To repl. allowance	17.74	By renewals	18.31
	Totals	$191.05		$167.50
	Balance			23.55

At the end of the tenth year there would be in the replacement fund $23.55 for each $100 of original investment.

c. The amortization and replacement account in the case of the Equal Annual Payment Method, if amortization be allowed during the probable life term of each article, regardless of whether the article fails early or survives, would be about as follows: (Every article which replaces another is here treated as a new article.)

	Dr. side of Ledger		*Cr. side of Ledger*	
1st yr.	To allowance for amort. and repl.	$17.74	By renewals	$4.00
2nd yr.	To allowance for amort. and repl.	19.51	By renewals	8.16
3rd yr.	To allowance for amort. and repl.	22.13	By renewals	12.65
4th yr.	To allowance for amort. and repl.	25.72	By renewals	17.64
5th yr.	To allowance for amort. and repl.	30.40	By renewals	23.34
	Totals	$115.50		$55.79
	Balance			$59.71
	Balance	$59.71		
6th yr.	To allowance for amort. and repl.	12.63	By renewals	$21.97
7th yr.	To allowance for amort. and repl.	16.32	By renewals	21.16
8th yr.	To allowance for amort. and repl.	19.11	By renewals	20.54
9th yr.	To allowance for amort. and repl.	20.98	By renewals	19.73
10th yr.	To allowance for amort. and repl.	21.46	By renewals	18.31
	Totals	$150.21		$101.71
	Balance			48.50

The amount available for amortization at the end of the fifth year should be according to this account about $59.71 for each $100 of original investment and $48.50 at the end of the tenth year. In actual bookkeeping the account would have been balanced at the end of each year. The remaining investment, or the present value, is shown by this account to have been $40.29 on each $100 at the end of the fifth year and $51.60 at the end of the tenth year. The large reduction of capital which results from this method of applying the Equal Annual Payment Method shows its undesirability.

d. The amortization and replacement account in the case of the Equal Annual Payment Method, if amortization be estimated during the actual life of each article (plan A, Table 10) and the balance is applied at the end of each year to retire capital, would show:

	Dr. side of Ledger		*Cr. side of Ledger*	
1st yr.	To allowance for amort. and repl.	$17.74	By renewals	$4.00
2nd yr.	To allowance for amort. and repl.	18.66	By renewals	8.16
3rd yr.	To allowance for amort. and repl.	19.31	By renewals	12.65
4th yr.	To allowance for amort. and repl.	20.09	By renewals	17.64
5th yr.	To allowance for amort. and repl.	19.89	By renewals	23.34
	Totals	$95.69		$65.79
	Balance			19.90
	Balance	$19.90		
6th yr.	To allowance for amort. and repl.	19.64	By renewals	$21.97
7th yr.	To allowance for amort. and repl.	19.69	By renewals	21.16
8th yr.	To allowance for amort. and repl.	19.34	By renewals	20.54
9th yr.	To allowance for amort. and repl.	20.41	By renewals	19.73
10th yr.	To allowance for amort. and repl.	20.44	By renewals	18.31
	Totals	$119.42		$101.71
	Balance			17.71

In actual bookkeeping the account would have been balanced every year. The amount available for amortization at the end of the fifth year should be about $19.90 and at the end of the tenth year about $17.71.

e. The amortization and replacement account, in the case of the Equal Annual Payment Method, if amortization be estimated during the actual life of each article (plan *B*, Table 10) and the balance is applied at the end of each year to retire capital, would show:

	Dr. side of Ledger		*Cr. side of Ledger*	
1st yr.	To allowance for amort. and repl.	$15.10	By renewals	$4.00
2nd yr.	To allowance for amort. and repl.	17.46	By renewals	8.16
3rd yr.	To allowance for amort. and repl.	19.65	By renewals	12.65
4th yr.	To allowance for amort. and repl.	17.30	By renewals	17.64
5th yr.	To allowance for amort. and repl.	19.40	By renewals	23.34
	Totals	$88.91		$65.79
	Balance			23.12
	By Balance	$23.12		
6th yr.	To allowance for amort. and repl.	$20.30	By renewals	$21.97
7th yr.	To allowance for amort. and repl.	20.60	By renewals	21.16
8th yr.	To allowance for amort. and repl.	21.10	By renewals	20.54
9th yr.	To allowance for amort. and repl.	19.70	By renewals	19.73
10th yr.	To allowance for amort. and repl.	18.30	By renewals	18.31
	Totals	$122.82		$101.71
	Balance			21.11

The amount available for amortization at the end of the fifth year is shown by this account to be $23.12 for each $100 of original investment and $21.11 at the end of the tenth year.

f. The amortization and replacement account in the case of the Straight Line Method (5-year probable life) would show the following:

	Dr. side of Ledger		*Cr. side of Ledger*	
1st yr.	To allowance for amort. and repl.	$20.00	By renewals	$4.00
2nd yr.	To allowance for amort. and repl.	20.00	By renewals	8.16
3rd yr.	To allowance for amort. and repl.	20.00	By renewals	12.65
4th yr.	To allowance for amort. and repl.	20.00	By renewals	17.64
5th yr.	To allowance for amort. and repl.	20.00	By renewals	23.34
	Sub-Totals	$100.00		$65.79
	Balance			34.21
	By Balance	$34.21		
6th yr.	To allowance for amort. and repl.	20.00	By renewals	$21.97
7th yr.	To allowance for amort. and repl.	20.00	By renewals	21.16
8th yr.	To allowance for amort. and repl.	20.00	By renewals	20.54
9th yr.	To allowance for amort. and repl.	20.00	By renewals	19.73
10th yr.	To allowance for amort. and repl.	20.00	By renewals	18.31
	Totals	$134.21		$101.71
	Balance			32.50

These figures show that at the end of the fifth year about 34 per cent of the total cost of the depreciating articles would have been returned to the owner and at the end of the tenth year about 32.5 per cent. The remaining investment at 5 years would have been about 66 per cent and at the end of the tenth year about 67.5 per cent.

g. The replacement account in the case of the Unlimited Life Method would show the following:

	Dr. side of Ledger		*Cr. side of Ledger*	
1st yr.	To allowance for repl.	$4.00	By renewals	$4.00
2nd yr.	To allowance for repl.	8.16	By renewals	8.16
3rd yr.	To allowance for repl.	12.65	By renewals	12.65
4th yr.	To allowance for repl.	17.64	By renewals	17.64
5th yr.	To allowance for repl.	23.34	By renewals	23.34
6th yr.	To allowance for repl.	21.97	By renewals	21.97
7th yr.	To allowance for repl.	21.16	By renewals	21.16
8th yr.	To allowance for repl.	20.54	By renewals	20.54
9th yr.	To allowance for repl.	19.73	By renewals	19.73
10th yr.	To allowance for repl.	18.31	By renewals	18.31

Theoretically there would be no accumulation in the replacement fund because in the case of numerous articles the annual demand on the fund would be offset by the replacement allowance.

The simplicity of the Unlimited Life Method, as well as its advantage in requiring least earnings in the early years, appears from a comparison of such accounts as above presented.

Before leaving this subject attention may be called to the fact that the balance which these figures show to be probable in a replacement fund under the Sinking Fund Method of procedure and the amount of the amortization under the Equal Annual Payment Method do not agree with what would be found on the impossible hypothesis that all articles serve throughout their probable terms of usefulness and no longer. On this purely hypothetical assumption (see page 94) the fund should contain about 38 per cent of the cost of the depreciating articles after five years of operation (5 years probable life being here under consideration).

If the various methods of procedure and fixed rules of accounting are closely adhered to and a comparison of results be made at the end of the tenth year, numerous articles all having a probable life of 5 years being under consideration, the results in so far as the amount of amortization or the amount in the replacement funds are concerned should be about as follows:

On the hypothetical assumption of agreement between actual and probable life, and an amortization allowance estimated by compound interest methods, the original investment should have been reduced by about. .	38 per cent
By the Sinking Fund Method, if the replacement allowance continues during probable life, the amount in the replacement fund should be about.	35 per cent
By the Sinking Fund Method, if the replacement allowance continues during the actual life, the amount in the replacement fund should be about.	24 per cent

By the Equal Annual Payment Method, if the amortization allowance be continued during probable life, the original investment should have been reduced (so-called accrued depreciation) by about....................	48 per cent
By the Equal Annual Payment Method, if the amortization allowance be continued during the actual useful life, the original investment should have been reduced (so-called depreciation) as follows:	
a. According to the plan *A* of estimating the annual allowance by about.....	18 per cent
b. According to the plan *B* of estimating the annual allowance by about.....	21 per cent
By the Straight Line Method, the original investment should have been reduced by about................................	33 per cent
By the Unlimited Life Method there would be no reduction of the invested capital and there would theoretically be no accumulation in the replacement fund.	

Rental Value as an Aid in Determining Present Value. — Rental value is a convenient aid in forming a clear conception of present or remaining value of any item of property. Take, for example, a high-duty pump and assume that the same be rented by Smith, the owner of a water-works property, from Jones on such terms that Jones will recover the cost of the pump during its life and interest on his investment. The care and the maintenance of the pump falls upon Smith. The rental value may be so determined that as the business grows the return to Jones of capital will be increased, but ordinarily, the pump being assumed to be in full service all the time, the rental value remains uniform throughout the period that the pump renders efficient service. If the interest rate agreed upon is 6 per cent and if it be further agreed that the probable life of the pump is 25 years, Jones will expect $7.82 per annum throughout the probable life term of the pump for each $100 of its cost. The agreement will provide that he gets this sum annually so long as the pump remains an efficient appliance and that the payment shall cease as soon as the pump becomes useless. The present value of this pump to Jones for each $100 of its cost, at any time of its life, will be the present value of an annuity of $7.82 for the expectancy or remaining life of the pump. This will be true if he is under no obligation to replace it and if scrap value be disregarded.

When the pump is new, the probable life or expectancy is 25 years and the present value will be 100 per cent.

When the pump is 10 years old, its expectancy may be about 17 years and the remaining value at that time $82 for each $100 of first cost or 82 per cent.

When the pump is 25 years old and still in good condition, its expectancy may be about 9 years and the remaining value of each $100 of original cost at that time would be the present value of an annuity of $7.82 for 9 years or $53 or when compared with the total original cost, 53 per cent thereof.

If the pump is defective or is put out of service by some accident before it has served 25 years, Jones will be a loser. If it serves beyond the term assumed as the limit of its usefulness or beyond 25 years, Jones will be the gainer. If Jones is in the business of supplying pumps in large numbers on these terms he will find, if probable life has been correctly determined, that his capital is yielding 6 per cent per annum.

Further Comparison of Methods with Tables and Diagrams. — A further comparison of the various methods of procedure which have found favor when rates for the output of public service properties are to be fixed, is given in the following pages with tables and diagrams where practicable. These comparisons are strictly theoretical and are based in each case as noted on the impossible assumption that the actual life of every item which goes to make up the property will coincide with the probable life of that item. The comparison nevertheless shows where these methods, if consistently applied from the beginning, would lead and are therefore of value to the appraiser and to the property owner, as well as to the rate-regulating authority.

Tables 11 and 12 illustrate forcefully the fact that under the Equal Annual Payment Method the amount to be earned annually from the beginning of operations is uniform and that under the Straight Line Method the amount of earnings should be greatest at the beginning.

TABLE 11. STRAIGHT LINE METHOD
(Hypothetical)

Cost of plant $100. Probable life 10 years. Interest 6 per cent. No distinction is made between probable and actual life. The annual amortization or depreciation increment is $10.

Year.	Accrued depreciation or amortization at end of year.	Present value, remaining investment or rate-base at beginning of year.	Interest on rate-base.	Annual amortization and interest.
1	$10	$100	$6.00	$16.00
2	20	90	5.40	15.40
3	30	80	4.80	14.80
4	40	70	4.20	14.20
5	50	60	3.60	13.60
6	60	50	3.00	13.00
7	70	40	2.40	12.40
8	80	30	1.80	11.80
9	90	20	1.20	11.20
10	100	10	0.60	10.60

TABLE 12. EQUAL ANNUAL PAYMENT METHOD
(Hypothetical)

Cost of plant $100. Probable life 10 years. Interest 6 per cent. No distinction is made between probable and actual life.

Year.	Annual depreciation or amortization increment.	Accrued depreciation or amortization at end of year.	Present value, remaining investment or rate-base at beginning of year.	Interest on rate-base.	Annual amortization and interest.
1	$7.59	$7.59	$100.00	$6.00	$13.59
2	8.04	15.63	92.41	5.55	13.59
3	8.53	24.15	84.37	5.06	13.59
4	9.04	33.19	75.85	4.55	13.59
5	9.58	32.77	66.81	4.01	13.59
6	10.15	52.92	57.24	3.43	13.59
7	10.76	63.68	47.08	2.83	13.59
8	11.41	75.09	36.32	2.18	13.59
9	12.09	87.18	24.91	1.50	13.59
10	12.82	100.00	12.82	0.77	13.59

The Sinking Fund Method, if correctly applied, involves, as already stated, only a one time calculation of the annual depreciation or replacement increment and its result agrees with that of the Equal Annual Payment Method.

The Unlimited Life Method is the most flexible. It may be so applied as to give identical results with the Equal Annual Payment and the Sinking Fund Method, or it may be applied to vary somewhat therefrom so as to make the earnings requirements least in the early years.

Under the Sinking Fund Method, on the above assumption with reference to absolute agreement of actual with probable life, the annual requirement for $100 of cost would be interest of $6 and the amortization or depreciation increment of $7.59 for each of the 10 years of life making a total of $13.59 for each year.

Under the Unlimited Life Method, the replacement increment may be estimated as in the case of the Sinking Fund Method, in which event the required annual earnings would be $13.59 or they may be graded from a smaller amount in the earlier years to a larger amount in the later years, in which event the required annual earnings would appear on an increasing scale.

The extreme case has been assumed for the illustration of the Unlimited Life Method in Table 13, that no provision whatever is made for replacement until the article to be replaced fails. This will account for the sudden increase noted for the eleventh and twenty-first years, in each of which the replacement requirement is increased by $100. The first of the original articles fails in the tenth year (actual life being assumed the same as the probable life), and thereafter $100 of the original investment goes out of use and has to be replaced annually until and after the twenty-first year when the annual replacement requirement is $200.

TABLE 13. COMPARISON OF METHODS OF PROCEDURE
(Hypothetical)

Annual investment $100. Probable life of all parts of the property 10 years. Interest 6 per cent. No distinction is made between probable and actual life.

Year.	Straight Line Method.		Equal Annual Payment Method.		Sinking Fund Method.		Unlimited Life Method.	
	Rate-base beginning of year.	Required earnings.	Rate-base beginning of year.	Required earnings.	Rate-base beginning of year.	Required earnings.	Rate-base beginning of year.	Required earnings.
1	$100	$16.00	$100.00	$13.59	$100.00	$13.59	$100	$6.00
2	190	31.40	192.41	27.17	200.00	27.17	200	12.00
3	270	46.20	276.78	40.76	300.00	40.76	300	18.00
4	340	60.40	352.63	54.35	400.00	54.35	400	24.00
5	400	74.00	419.44	67.94	500.00	67.94	500	30.00
6	450	87.00	476.68	81.53	600.00	81.53	600	36.00
7	490	99.40	523.76	95.12	700.00	95.12	700	42.00
8	520	111.20	560.08	108.69	800.00	108.69	800	48.00
9	540	122.40	584.99	122.28	900.00	122.28	900	54.00
10	550	133.00	597.81	135.87	1000.00	135.87	1000	60.00
11	650	149.00	697.81	149.46	1100.00	149.46	1100	166.00
12	740	164.40	790.22	163.05	1200.00	163.05	1200	172.00
13	820	179.20	874.59	176.63	1300.00	176.63	1300	178.00
14	890	193.40	950.44	190.22	1400.00	190.22	1400	184.00
15	950	207.00	1017.25	203.81	1500.00	203.81	1500	190.00
16	1000	220.00	1074.49	217.40	1600.00	217.40	1600	196.00
17	1040	232.40	1111.57	230.99	1700.00	230.99	1700	202.00
18	1070	244.20	1147.89	244.56	1800.00	244.56	1800	208.00
19	1090	255.40	1172.80	258.15	1900.00	258.15	1900	214.00
20	1100	266.00	1185.62	271.74	2000.00	271.74	2000	220.00
21	1190	282.00	1278.03	285.33	2100.00	285.33	2100	326.00

One more comparison may be of interest as shown in Table 14.

TABLE 14. COMPARISON OF METHODS OF PROCEDURE
(Hypothetical)

The property is made up of numerous items of all possible ages, the combined cost of which is $100. The probable life of each item is 10 years. All considerations are theoretical and no distinction is made between actual and probable life. Interest 6 per cent. The property is supposed to have reached its full growth and to be more than 10 years old.

Method of procedure.	Rate-base beginning of year.	Interest on rate-base.	Annual depreciation or replacement increment.	Required earnings.
Straight line amortization method.	$55.00	$3.30	$10.00	$13.30
Equal annual payment amortization method..................	59.78	3.59	10.00	13.59
Sinking fund replacement method.	100.00	6.00	7.59	13.59
Unlimited life replacement method.	100.00	6.00	10.00	16.00

The total required earnings appear largest for the Unlimited Life Method for the reason that the required earnings in the early years of the plant's life are supposed to have been low.

When rates are to be fixed for a public service property which is long established, concerning which past records are unreliable, but which is legitimate, rendering a necessary service and entitled to fair income, the question as to which method of procedure should be adopted presents itself.

Let it be supposed that the records indicate a fairly normal development and growth with some years of inadequate earnings and no conclusive evidence that any part of the invested capital has been repaid, but that worn-out parts have been renewed as necessary and that the service rendered has been satisfactory.

In this assumed case the application of the various methods of procedure will theoretically produce results as shown in Table 14. If any of the methods except the Unlimited Life Method be adopted, recourse will probably be had to an addition of large values for intangible elements in order that no injustice may be done to the owner of the property. In most cases the Unlimited Life Method will be indicated as most nearly equitable. This will be yet more apparent when the effect of the non-agreement of actual with probable life upon the application of the various methods of procedure is taken into account.

The comparison of results by the methods of procedure which have been under discussion has been visualized, for the special case of a single item which cost $100, in a diagram, Fig. 2. It has been assumed in the preparation of this diagram that in the application of the Unlimited Life Method to a single article the replacement requirement will be anticipated and will be estimated by the compound interest Sinking Fund Method. In the practical application of this method a close approximation of the replacement requirements is not essential because no part of the replacement increment goes to the repayment of capital. This entire increment remains in the replacement account and will be subject to being decreased or increased from time to time as this account shows to be necessary.

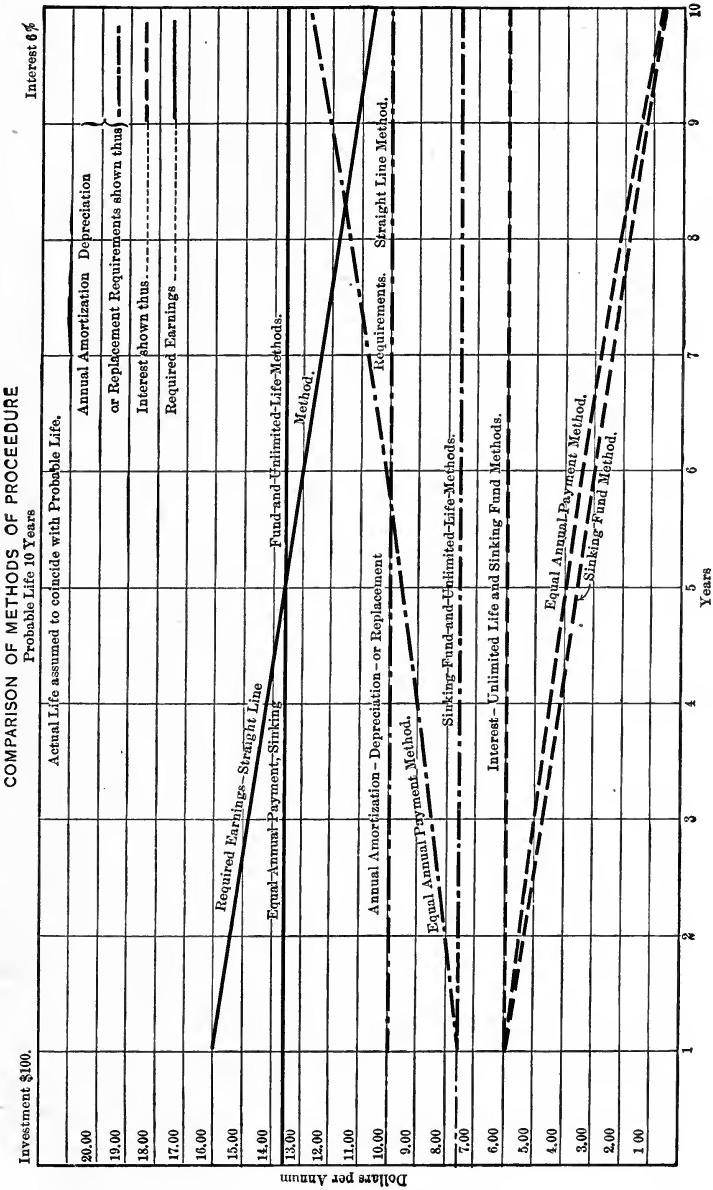

COMPARISON OF METHODS OF PROCEEDURE
Probable Life 10 Years
Investment $100.
Interest 6%
Actual Life assumed to coincide with Probable Life.
Annual Amortization Depreciation or Replacement Requirements shown thus
Interest shown thus
Required Earnings
Required Earnings–Straight Line
Equal-Annual-Payment, Sinking-Fund-and-Unlimited-Life-Methods.
Method.
Annual Amortization–Depreciation–or Replacement Requirements. Straight Line Method.
Equal Annual Payment Method.
Sinking-Fund-and-Unlimited-Life-Methods.
Interest–Unlimited Life and Sinking Fund Methods.
Equal Annual Payment Method.
Sinking-Fund Method.
Dollars per Annum
20.00
19.00
18.00
17.00
16.00
15.00
14.00
13.00
12.00
11.00
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
1
2
3
4
5
6
7
8
9
10
Years

FIG. 2.

The Obligation to Replace. — In further substantiation of the advantage which results from use of a rate-base computed from the investment without any deduction of depreciation, attention may be called to the obligation to replace worn-out or discarded essential parts of every public utility plant which goes with the ownership thereof. It does not matter, in the case of any individualized article, such as a steamboat, whether a replacement fund is being set apart to be kept inviolate and is accumulating interest at a rate which will bring it to the value of the steamboat in its life or whether there is no such fund. The owner of the steamboat is burdened with the obligation to replace and this obligation is as real and as binding as though it were represented by an accumulated fund. It is optional with him whether to set apart a fund if none exists and let its earnings go toward a new steamboat or to simply let the obligation stand and to provide funds for the new steamboat when the old one goes out of use. To the extent of this obligation, that is, to the amount which should be in a replacement fund, any capital which he commands is available for no other use than the replacement when the time comes. The interest on this fund, real or imaginary, is available for this use only and as the fund, together with the remaining physical value of the steamboat, is equal to the amount originally invested in the steamboat, it is plain that there is no need of annually drawing the dividing line between the remaining physical value and the amount which should be in the replacement fund (the so-called accrued depreciation). The remaining physical value plus the obligation to replace is the invested capital on which the interest is to be computed whenever earned depreciation or replacement allowances are not regarded as amortizing capital.

Period and Rate of Amortization. — When a municipality constructs improvements under a bond issue or otherwise, suitable provision is made for the replacement of any of the worn-out parts of these improvements at the time that these parts go out of use. This is in strict conformity with the procedure under the Unlimited Life Method. But in the case of the municipality the bond issue, if any, is also to be taken care of. The cost of the improvement is, in other words, to be distributed fairly to those that will benefit thereby. This is usually done by so fixing the term of the bonds that the cost will be distributed over a sufficiently long period of time. The determination of this time period need not be in any definite relation to the life of the parts of the improvement. The improvement itself will usually be one that may be regarded as having unlimited life, such as parks, playgrounds, streets, and the like. When the term of the bonds of longest life has been fixed on the basis of the probable life of the main elements of the improvement, or in some other way, then a determination must be reached as to the best and most equitable rate of amortization.

This amortization may take place at a uniform rate per year, bearing heavily on present day property owners — Straight Line Method.

It may take place at an increasing rate per year:

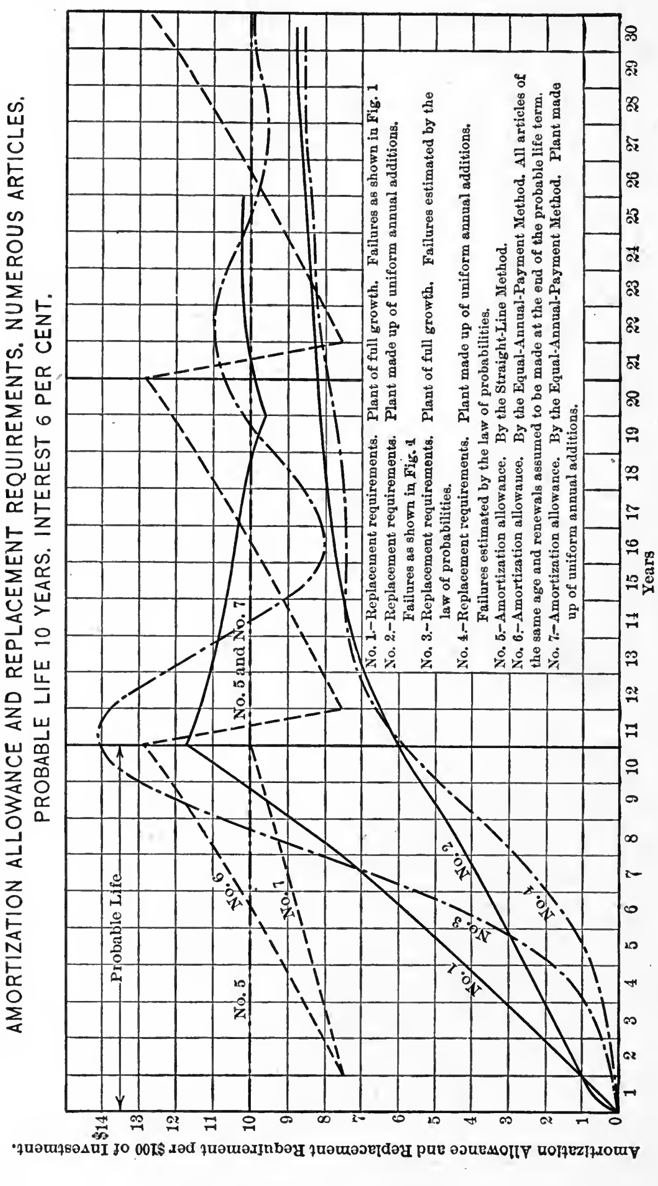
(*a*) According to the scheme outlined under the Equal Annual Payment Method.

(*b*) According to any arbitrary scheme that will approximate the compound interest Sinking Fund Method of estimating the annual amortization increment.

Or, it may be deferred for a time and then take place according to either of these methods.

In the case of a public service property constructed by a municipality the amortization of capital usually begins at or soon after the acquisition of the property and in the case of a utility constructed by a private owner, the amortization of capital should begin under the Straight Line Method and the Equal Annual Payment Method at the beginning of operation, and, theoretically under the Sinking Fund Method or the Unlimited Life Method as herein fully explained, not at all in so far as rate-base determination is concerned, during the continuance of full private ownership.

In Fig. 3 a comparison is made between the annual amortization allowance computed by the Straight Line Method and the



AMORTIZATION ALLOWANCE AND REPLACEMENT REQUIREMENTS. NUMEROUS ARTICLES.
PROBABLE LIFE 10 YEARS. INTEREST 6 PER CENT.
Amortization Allowance and Replacement Requirement per $100 of Investment.
$14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Years
Probable Life
No. 1
No. 2
No. 3
No. 4
No. 5
No. 6
No. 7
No. 5 and No. 7
No. 1.-Replacement requirements. Plant of full growth. Failures as shown in Fig. 1
No. 2.-Replacement requirements. Plant made up of uniform annual additions. Failures as shown in Fig. 1
No. 3.-Replacement requirements. Plant of full growth. Failures estimated by the law of probabilities.
No. 4.-Replacement requirements. Plant made up of uniform annual additions. Failures estimated by the law of probabilities.
No. 5.-Amortization allowance. By the Straight-Line Method.
No. 6.-Amortization allowance. By the Equal-Annual-Payment Method. All articles of the same age and renewals assumed to be made at the end of the probable life term.
No. 7.-Amortization allowance. By the Equal-Annual-Payment Method. Plant made up of uniform annual additions.

Fig. 3.

Equal Annual Payment Method and the probable annual replacement requirements as these would be estimated by the hypotheses of failures which are referred to in Chapter VI.

These curves show the wide departure of the results by the Straight Line Method and by the Equal Annual Payment Method from the actual maintenance requirement and demonstrate the desirability of proceeding under the Unlimited Life Method. The replacement requirements line represents approximately the expenditures which are necessary from time to time for renewals to keep the plant in an efficient condition. The wide departure of these from the amortization lines determined by the common methods of estimating depreciation are made apparent by the diagram. Further comment is hardly necessary, except to say that under the Unlimited Life Method the actual replacement requirements may be assumed to approximate lines 1 and 3 for a number of articles of the same probable life in a plant of full growth and to approximate lines 2 and 4 for articles distributed in equal amounts to all possible ages (plant made up of uniform annual additions).

Advantage of the Unlimited Life Method. — The comparison of methods of procedure when valuations are to be used as the basis of fixing rates may be summed up in the broad statement that a close approximation of the rate-base and of the necessary annual earnings is not possible by any method of appraisal which makes the ascertainment of accrued depreciation necessary; that the use of any such method requires trained experts and involves cumbersome calculations and that the uncertainties of the determination of depreciation affect not alone the valuation but also the required annual earnings, while, on the other hand, the methods which make use of the amount of capital reasonably and properly invested as a rate-base are simpler and free from uncertainties except in the matter of the provision which should be made either for current depreciation under the Sinking Fund Method or for replacements under the Unlimited Life Method. The last-named method has the unique advantage of easy adaptation to any situation that may develop, particularly in the matter of adjusting the amount which annually goes into the replacement fund to the amount found, by actual experience, to be necessary for any special plant. This method, when selected for a new plant, also has the advantage over other methods that the required earnings in the early years are less than those estimated by any of the others. It will therefore show for the same amount of earnings a smaller annual loss or a larger annual profit in the early years than the other methods.

Careful investigation might show that the Unlimited Life Method is but an old method under a new name. Industrial establishments and public utilities could without doubt be found which had been operating in substantial conformity with this procedure before they came under control of public service commissions. As the method is theoretically sound and has weighty advantages in its favor both from the standpoint of the rate-payer who wants the burden light in the early years of the utility's life and of the owner who wants his investment protected without the uncertainty and confusion of present value considerations, it seems probable that it may come into general favor despite the present-day leaning of the public service commissions and the courts in another direction.

A good illustration of the difficulty of dealing with depreciating property and of the undesirability of using depreciated or remaining value of physical property in the rate-base will be found in the general principles enunciated by the Engineering Board, Division of Valuation, Interstate Commerce Commission, as submitted to the Commission in November, 1915. The Board lays down the following general rules:

"When depreciating property under the several accounts, the following general rules shall apply:

"1. Ordinarily, service condition per cent shall be the ratio between the remaining service life and the total service life. When the depreciation of an item of property is based upon the weighted average of the parts, the service condition of each part shall be determined by this ratio.

"2. When a normal life for a particular item of property has been prescribed for use in determining depreciation, that life must not be departed from unless an investigation of the records of the carrier or actual inspection in the field, or the two combined, warrant such departure. In no case shall a remaining service life of an item of property be taken at more than the prescribed normal life.

"3. When no normal life is prescribed, the total service life and remaining service life shall be determined from observation of actual conditions and the examination of records and data from reliable sources.

"4. Salvage and scrap will be allowed in cases where such values actually exist. Whether allowance shall be made in a given case for salvage or scrap shall be left to the discretion of the member of the Engineering Board. If an allowance is made this fact shall in all cases be stated upon the pricing sheet even though at the time the amount of the allowance cannot be given."

The complex operations involved in applying these rules when rates are to be fixed, is in strong contrast with the simple operations involved in proceeding under the Unlimited Life Method which requires no estimate of *accrued* depreciation.

The Use of Cost Records and Cost of Reproduction New. — The purpose of the valuation of the public utility may be as already stated:

a. To fix a selling price.

b. To establish a basis for an issue of securities.

c. To establish a rate-base.

d. To serve as a basis for taxation.

The ascertainment of the rate-base is important. By adding to or subtracting from the same, various facts relating to values can be ascertained. It may be possible to determine cost of construction from the cost records. When this can be done, there should be a check by means of estimates of the cost of reconstruction to make certain that the actual reported cost is legitimate and that it does not include too much overhead expense nor too large expenditures for abandoned or discarded items, *i.e.*, for items that were intended for temporary use only or that failed for any reason to fulfill their intended purpose.

The cost records are not always dependable and they may have to be either entirely disregarded or largely supplemented by cost estimates. Recourse may be had in such cases to the cost of reproduction as a means of approximating the amount that may reasonably be assumed to be properly invested in the property. In making the estimate of the cost of reproduction it is not advisable to use the prices of materials and labor momentarily prevailing but rather the average for a considerable time period, preferably about 5 years.

Tabulation of Field Results. — The results of the field examination, including a classified enumeration of the physical items that go to make up the property, should be tabulated in convenient form on sheets which will show in appropriate columns information about as follows:

1. The year of installation.
2. The age.
3. The probable life new.
4. The expectancy or estimated remaining years of service.
5. The cost to reconstruct, itemized.
6. The sub-totals of (5).
7. The contractor's profit.
8. The totals of (6) and (7).
9. The allowance for overhead expenses.
10. The total investment, estimated as the sum of (8) and (9).
11. The residual or scrap value.
12. The present value, in per cent, computed from (3) and (4) (sometimes called condition per cent).
13. The remaining or present value in dollars (computed from (10), (11), and (12)).
14. The accrued depreciation (10) – (13).
15. The current rate of depreciation (computed from (3), (4), (10) and (11)).

It will be noted that according to this tabulation the cost to reconstruct and the residual or scrap value are considered when estimating the remaining value from the remaining years of service. Theoretically this cost of effecting the replacement should be taken into account, but it is sometimes convenient to let the original cost less residual value take its place.

According to the selected method of procedure, the rate-base is computed from the tabulated information by adding the cost of establishing the business and of any other definitely ascertainable items of intangible character, such as the cost of the franchise, or the cost of the water-right, to the sum of items in column (10) for the Sinking Fund Method or for the Unlimited Life Method of procedure; or by adding these items of intangible character to the sum of the items in column (13) when the Equal Annual Payment Method or the Straight Line Method of procedure is to be adopted.

The Effect of Method of Procedure on Market Value. — When a market value of a successfully operating plant is to be fixed on the assumption that there is accrued depreciation but that there is no deferred maintenance, special consideration must be given to the method of procedure followed by the rate-fixing authority:

If the Unlimited Life Method has been the procedure, the owner may not have recovered any part of the accrued depreciation in the earnings but only enough to meet replacement requirements. The market value in that event should not be less than the rate-base in column (10) determined from the investment without deduction of depreciation, plus some addition for value due to net earnings in excess of interest on the rate-base.

But if, under the Unlimited Life Method or under the Sinking Fund Method of procedure, the owner has received some annual amount to forestall the replacement requirement and retains possession of whatever amount there may be in the replacement fund, the market value will be less than the amount determined on the other assumption by the amount which should theoretically be in the replacement fund.

If the Equal Annual Payment or the Straight Line Method of procedure has prevailed from the beginning and a continuance thereof is a certainty, the market value will be ascertained from the rate-base, column (13), determined from the investment less depreciation to which may be added something for value due to net earnings in excess of interest on the rate-base.

Rate-base Determination. — When a rate-base is to be established, the totals of columns (10) or (13) will be used in combination with all or a part of any allowance for establishing the business and the cost of the ascertainable items of intangible character such as franchises and water-rights. The sum of these items with the total of column (10) will be the rate-base for the Unlimited Life Method and the Sinking Fund Method of procedure, and the sum of these items with the total of column (13) will be the rate-base for the Equal Annual Payment Method and the Straight Line Method of procedure. This is subject to the proviso that past history will show that accruing depreciation has actually been offset by earnings. Unless there has been a surplus in the earnings over a fair net return which is to be allowed on the investment, there will have been no amortization, despite the fact that depreciation is obvious. The accrued depreciation as ascertained for any particular time does not always measure the amount of accomplished amortization. This is only the case when the allowance for depreciation, under these two methods of procedure, has actually been earned and has been collected.

CHAPTER X

NOTES ON THE DETERMINATION OF THE VALUE OF REAL ESTATE IN EMINENT DOMAIN PROCEEDINGS AND FOR RATE-FIXING PURPOSES

Market Value of Real Estate and the Rate-base. — The market value of real estate is not to be confounded with the amount at which it is carried in the rate-base. While consideration must sometimes be given to market value in fixing the rate-base this is not always necessary. In fact the determination of the rate-base should ordinarily be independent of market value because it is not "value" but "investment" which should be made the guide and control when rates are to be fixed. When there is uncertainty about the reasonable cost or when the first dependable valuation for rate-fixing purposes is made long after the acquisition of the property, circumstances may, however, point to market value at some agreed time as the best starting point.

Market Value Defined in Court Decisions. — The U. S. Supreme Court in Boom Co. *vs.* Patterson (98 U. S. 403, 408; 25 L. Ed., 206) says in discussing market value:

> "The inquiry in such cases must be what is the property worth in the market, viewed not merely with reference to the uses to which it is at the time applied but with reference to the uses to which it is plainly adapted; that is to say, what is it worth from its availability for valuable uses. . . . Its capability of being made thus available gives it a market value which can be readily estimated."

In this case the Boom Company had brought suit in Minnesota against Patterson, a citizen of Illinois, to condemn three islands which were desired for use in connection with a series of log booms. A verdict was returned by the jury for $9358.33, the value of the islands having been found to be $300 aside from any consideration of their value for boom purposes and $9058.33 due to their adaptability for this special use.

The Court granted a motion for a new trial, unless the owner would consent to accept $5500. This amount was acceptable to the owner and judgment for this amount was entered in his favor. The Boom Company then appealed.

The Court is perhaps a little over-confident in saying that the market value can be readily estimated. In all other respects the statement is clear and logical.

The proposition is being generally recognized by the courts that, when the right of eminent domain is exercised, the question to be considered is "What is the value of the property for the most advantageous uses to which it may be applied?"

See Goodin *vs.* Cincinnati and Whitewater Canal Co. (18 Ohio St. 169).

Young *vs.* Harrison (17 Ga. 30).

U. S. *vs.* Chandler-Dunbar Water Power Co. (229 U. S. 53, 76).

In this last-named case referring to the award of special value for canal and lock purposes it was said:

> "The exception taken to the inclusion as an element of value of the availability of these parcels of land for lock and canal purposes must be overruled. That this land had a prospective value for the purpose of constructing a canal and lock parallel with those in use had passed beyond the region of the purely conjectural or speculative. That one or more additional parallel canals and locks would be needed to meet the increasing demands of lake traffic was an immediate probability. This land was the only land available for the purpose."

See also Shoemaker *vs.* U. S. (147 U. S. 282).

In the case of "San Diego Land and Town Co. *vs.* George Neale *et al*" (78 Cal. 63, 68; 30 Pac. Rep. 372; 3 L. R. A. 372) the majority of the Court says:

> "The consensus of the best-considered cases is that for the purposes in hand the value to be taken is the market value, by

which is undoubtedly meant, not what the owner would realize at a forced sale, but 'the price that he could obtain after reasonable and ample time, such as would ordinarily be taken by an owner to make sale of like property.' . . . But in many instances, as in the case before us, there is no actual demand or current rate of price, either because there have been no sales of similar property, or because the particular piece is the only thing of its kind in the neighborhood, and no one has been able to use it for the purpose for which it is suitable and for which it may be highly profitable to use it. . . . From the necessity of the case the value must be arrived at from the opinions of well-informed persons, based upon the purposes for which the property is suitable. . . . What is done is merely to take into consideration the purposes for which the property is suitable, as a means of ascertaining what reasonable purchasers would in all probability be willing to give for it, which, in a general sense, may be said to be the market value. And in such an inquiry it is manifest that the fact that the property has not previously been used for the purposes in question is irrelevant. The current of authority sustains these views."

What a Purchaser can Afford to Pay is not Market Value. — The value to the person who desires to acquire the property, the amount, in other words, which such person can afford to pay for it, is not its market value. In the Chandler-Dunbar Water Power Co. case above referred to the U. S. Supreme Court says (229 U. S. 80):

"In a condemnation proceeding the value of the property to the Government for its particular use is not a criterion. The owner must be compensated for what is taken from him, but that is done when he is paid its fair market value for all available uses and purposes."

Reference may also be had to the "Minnesota Rate Cases" (230 U. S. 352, 451).

And also to "Five Tracts of Land in Cumberland Tp., Adams Co. Pa. *vs.* U. S." (101 Fed. 661, 664).

U. S. *vs.* Honolulu Plantation Co. (122 Fed. 581, 584; 58 Circuit Ct. Appeals 279).

The proposition that the necessity of the party desiring to acquire property by eminent domain cannot be made the measure or market value is further made clear in:

Tidewater Canal Co. *vs.* Archer (9 Gill and J. Md. 481; 22 Md. 307).

Gardner *vs.* Inhabitants of Brookline (127 Mass. 358).

Burt *vs.* Wigglesworth (117 Mass. 302).

Reading and Pottsville R. R. Co. *vs.* Balthasser (126 Pa. St. 1).

Dorlan *vs.* East Brandywine and W. R. 46 Pa. 520. The Stockton and Copperopolis Railroad Co. *vs.* Vincent Galgiani (49 Cal. 139).

Admissibility of Evidence Relating to Value for a Special Purpose. — The question relating to the admissibility of evidence bearing directly upon the value of a tract of land for reservoir purposes is quite fully discussed in Spring Valley Water Works *vs.* Drinkhouse (92 Cal. 528, 532; 28 Pac. Rep. 681). The view expressed in that case that value of the land for reservoir purposes might be shown appears however to be controverted in the more recent case of "Sacramento Southern Railroad Co. *vs.* Heilbron" (156 Cal. 408) which involves the condemnation of a strip of land for railroad purposes. It was contended in this case that the rule as laid down in California permits evidence of value for the use of the land for a particular purpose in terms of money. In reference to this contention the Court says:

"It is seen, therefore, that this Court by its latest utterances has definitely aligned itself with the great majority of the courts in holding that damages must be measured by the market value of the land at the time it is taken, that the test is not the value for a special purpose, but the fair market value of the land in view of all the purposes to which it is naturally adapted; that therefore while evidence that it is 'valuable' for this or that or another purpose may always be given and should be freely received, the value in terms of money, the price, which one or another witness may think the land would bring for this or that or the other specific purpose is not admissible as an element in determining that market value."

Apparently, if this rule is strictly adhered to, it will debar from consideration the economic features which determine whether and to what extent the enterprise which involves the acquisition of the land can be made profitable. It may make it impracticable to get before the court that information which a prudent purchaser would seek when making up his mind relating to the price which he would be justified in paying.

The presentation, then, in court of the evidence on which the valuation of real estate is to be based is not always a simple matter. The information wanted is the market value. The expert who testifies to value must inform himself what the value is, all purposes for which the property is suitable being taken into account. Recent court rulings appear, as above shown, to be against allowing evidence which will show the value in money for any special purpose.

The local dealer in real estate who knows what sales have been made in recent years and who knows or is supposed to know the effect that the adaptability of any particular tract of land to a particular purpose has upon the market value of that land is, according to such rules of procedure, the proper value expert. His opinion relating to the money value of an island for boom purposes, or of a tract suitable for a dam site or for a reservoir site or for some other public use, is allowed to go before the judge or jury that fixes the value while the trained engineering expert who may have made a careful analysis of all economic features involved but who may have no knowledge of the land value for other than one special purpose, is not allowed to testify to the value which the same property would have for the special use to which it is about to be put for the benefit of the public. He is restricted to a statement of the nature of such use and may present facts relating thereto but he must not express an opinion relating to value in terms of money unless he is in a position to say that he has given consideration to all possible uses.

This rule of the courts is intended to stop the introduction of evidence too speculative and remote in character. The possibility of growing a special crop at an estimated annual profit per acre and a determination of the land value from the net return resulting from growing such a crop is held to be too speculative and no doubt with reason. But the court should not be denied such information as would be collected by a prudent purchaser who neglects no source of information when he makes up his mind what would be a reasonable price to pay for the property sought to be acquired, and there should be no testimony excluded which will throw light on the circumstances which affect or fix the market value. The favorably disposed purchaser will not restrict himself to the questioning of those only who are supposed to have general knowledge of the market value resulting from every possible use. He will rather make inquiry along every possible line and will not neglect special adaptability to a particular purpose. He will want to know the strategic value of the property when made use of for the special purpose to which it is supposed to be best adapted. If the value for such a special purpose can be ascertained with due consideration of all elements involved — adaptability to special uses with due allowance for risk, the cost of development, the cost of operation, the immediate or deferred market for the output or service and the prospective net return that will result from the use of the property, — the knowledge so obtained will aid the purchaser in reaching his conclusion concerning what a well-informed public would consider such a property worth.

The amount which a particular municipality or a person who is seeking to condemn a property can afford to pay cannot, however, be made the measure of the market value. This may define an upper limit. More cannot be paid by that municipality or person. The purchase at the full amount determined by such necessity would deprive the person, who wishes to utilize the property for the special purpose to which it is adapted, of a margin of profit to which such person is entitled.

In the light of the foregoing it would seem reasonable to exclude from the consideration of the courts in condemnation proceedings evidence relating to the value expressed in money for such special uses as can be exercised only by the person who seeks to condemn the property, as in the case of land required by the Government for fortification purposes, or for a light house, but in all cases where the adaptability to the special use is recognized and the property can be applied to that use by any one suitably circumstanced, the evidence relating to the effect of such adaptability upon market value should be admissible. It should be admitted even though somewhat speculative. The court must determine what weight to give to evidence of value for special uses which only remotely affect the market value.

In the case of a mine on land whose surface has value for no other purpose than grazing, it will, unquestionably, be proper to ascertain the value of the land for the special purpose of mining. If the land has value for mining and at the same time for reservoir purposes or as a dam site, there should be no objection to a consideration of these values separately for each specific purpose, just as the same would be considered by the prudent purchaser when he makes a study of market value. The consideration of the effect of availability for these purposes upon market value is not to be classed as too speculative. If use for any such purpose lies in the future, due allowance must be made for the lapse of time before the ultimate value resulting from that use can be realized. If the time is uncertain and remote, the effect of the special adaptability upon market value will be small when compared with what this effect might be if immediate use were a certainty.

The Value Multiple. — Other considerations, too, may limit this market value. It has, for example, become a practice, almost standard, to value rights of way for railroads and highways at from 1.5 to 3.0 times the market value of adjacent lands of similar character. Custom has given land required for such uses such values. There are no frequent transfers of such property as in the case of town lots or farms. Consequently the determination of the value multiple which may be regarded as generally customary in the region where a right of way is to be valued may be the best guide in fixing this value.

In the case of a storage reservoir from which water may be required for an immediate high use, such as a municipal supply, and where the most profitable other use to which the reservoir land can be put is grazing, there may be a wide difference between values if determined for these two purposes. And yet in such a case the fact that this particular property will sooner or later be used for water development purposes may have given it a market value that cannot be ignored when the right of eminent domain is exercised. If the public has correctly determined this market value, it will lie somewhere between the value of the land for grazing and the amount which a person can afford to pay who wishes to use it for the storage of water. This does not mean that the original owner is to share the increment of special value equally with the party who is going to put the land to a special use but merely that he should share, to some extent, in the same.

When the value of land for a special purpose is but little in excess of the value of the land for ordinary uses, the division of the excess might, perhaps, equitably be on the basis of an equal division between the original owner and the person who desires to acquire the land. When, however, this excess is relatively large, then it may be proper and fair to assume that the larger portion of the excess value created by the enterprise should go to the person who is ready to take the risk of a successful carrying out of the enterprise and this fact should not be lost sight of in estimating market value.

When lands are to be taken for fortification and other similar purposes where consideration of the value expressed in money for the special purpose is not admissible, consideration should be given to the fact that the owner is forced to part with property for the good of the public and is entitled to receive for it somewhat more than the price which would obtain between a willing seller and a buyer, and also to the fact that whenever the adaptability of a piece of property for a special use is generally recognized, such property must have acquired greater value than other similar property not available for such use.

The owner who is forced to part with his property in such cases should obtain a reasonable reward for his foresight in acquiring property of special adaptability to some important use. This reward should, whenever practicable, bear some reasonable relation to the value for other uses to which the property can be put. There may be cases in which a 25 or 50 per cent allowance will be adequate, and there may be others in which this allowance may exceed the value for other uses two-, five- or even ten-fold, and there may also be occasional cases where a reasonable allowance not capable of demonstration, but yet fair when all circumstances are considered may be out of all relation to value for other uses.

The value multiple, as applied in any locality to railroad rights of way, and the unit price per pole of a power line are the outcome of numerous adjustments resulting from a regard to considerations as above set forth, and where such multiples are recognized, they, in turn, as already stated, have an effect upon market value of other similar easements.

The Value Multiple in the Minnesota Rate-Cases. — In connection with the valuation of rights of way and other lands as a part of the appraisal to be taken into account when the rates of public utilities are to be fixed, the decision of the United States Supreme Court in the Minnesota Rate-Cases is of interest (230 U. S. 352; 33 Sup. Ct. 729, June 9, 1913). Justice Hughes, in writing the decision of the court, says:

"The increase sought for 'railway value' in these cases is an increment over all outlays of the carrier and over the values of similar land in the vicinity. It is an increment which can not be referred to any known criterion, but must rest on a mere expression of judgment which finds no proper test or standard in the transactions of the business world. It is an increment which in the last analysis must rest on an estimate of the value of the railroad use as compared with other business uses, it involves an appreciation of the return from rates (when rates themselves are in dispute) and a sweeping generalization embracing substantially all the activities of the community. For an allowance of this character there is no warrant.

"Assuming that the company is entitled to a reasonable share in the general prosperity of the communities which it serves, and thus to attribute to its property an increase in value, still the increase so allowed, apart from any improvements it may make, cannot properly extend beyond the fair average of the normal market value of land in the vicinity having a similar character. Otherwise we enter into the realm of mere conjecture. We, therefore, hold that it was error to base the estimates of the value of the right-of-way, yards and terminals upon the so-called 'railway value' of the property. The company would certainly have no ground of complaint if it were allowed a value for these lands equal to the fair average market value of similar land in the vicinity, without additions by the use of multipliers, or otherwise, to cover hypothetical outlays."

The Court in this decision may be correct in stating that market value should be determined without the use of multipliers. Nevertheless the fact that rights of way are actually costing from 25 to 200 per cent more than lands of similar character in the same vicinity will have an unquestioned effect upon the market value of lands required for other rights of way and this effect cannot well be ignored when such lands are to be valued.

Right-of-Way Value in the Georgia Railway Case. — Special Master Thorington, in the Georgia Railway Case (Central of Georgia Railway Company *vs.* Railroad Commission of Alabama, U. S. Dist. Court, Middle Dist. of Ala., Northern Division, Report of Wm. S. Thorington, Special Master, Jan. 8, 1912), after stating that the fact that the railroad company is compelled to pay in addition to its market value a further sum due to damages or because it is a railroad company making the purchase adds nothing whatever to the actual acreage value, says:

"It is, however, proper to add that right-of-way values, including estimates for damages to property not taken, or excess cost that railroads are compelled to pay in order to acquire right-of-way property needed by them for railroad use, have been recognized by some courts, and some railroad commissions, and such excess cost was held to properly constitute part of the right-of-way valuation for rate purposes. In Shephard *vs.* Northern Pacific Railway Co. et al 184 Fed. 765, it is said the evidence was conclusive 'that every railroad company is compelled to pay more than the normal market value of property in sales between private parties for the irregular tracts it needs and acquires for rights-of-way, yards and station grounds. . . . The measure of the value of real estate is its market value for its most available use.' "

California Railroad Commission on the Right-of-Way Value. — In discussing the valuation of a railroad right of way, the California Railroad Commission says (Stockton Terminal and Eastern Railroad. Decision No. 618):

" After ascertaining the market value of the property at the time of its acquisition, the department (engineering department) also ascertained the market value as of June 30, 1912, and then multiplied that value by 1.5. This multiple was applied for the reason that the investigations of the department throughout the State show that on an average it costs one and one-half times the normal market value of abutting property to acquire rights of way in country districts by purchase or condemnation for railroad purposes. In the absence of more definite information as affecting this particular railroad, this average multiple was used."

The same commission again refers to and approves the use of this multiple in the matter of ascertaining the value of the property of the Nevada County Narrow Gauge R. R. Co. (Decision No. 1384).

CHAPTER XI

THE VALUE OF A WATER-RIGHT AND OF RESERVOIR AND WATERSHED LANDS

Value of Irrigation Water. — When water is used for irrigation, it makes the intense cultivation of the soil possible. It aids in producing crops which can be marketed at prices not subject to regulation except by the law of supply and demand. The availability and use of the irrigation water modify the character and increase the amount and consequently the value of the crop. These elements may thus add an increment of value to the irrigated land. Under such use the value of the water at the field and, by comparison with the cost of development, its value at its source can be determined. Water and water-rights in districts where water is used for irrigation acquire, in consequence, a recognized market value depending upon the appreciation of the land that results from irrigation, upon the value of the crops harvested and upon the cost of developing and making available the irrigation water.

Payment for Water-Rights. — Ordinarily when water is to be taken from a stream for uses which decrease or otherwise modify the natural flow of the stream below the point of diversion, the riparian rights of lower land owners are thereby affected. The diversion cannot be made in such cases without making compensation to the riparian owners for the damage to their property which results from the taking, except, of course, when such owners sleep upon their rights, virtually admitting too small a damage to make it worth while to attempt to recover compensation.

To the extent of the cost of securing the riparian rights and possibly of securing other water-rights whose use is secondary or which for any reason should be merged in one holding, there is then — a public utility being under consideration — an investment to be assumed in that intangible element, the water-right.

Sometimes by reason of local development and high values of riparian lands and an already established use of the stream flow for power, the cost of settling with the riparian owners and of quieting title to adverse users of the water may be large. At other times the situation is such that equally good rights to use water may be secured without any cost except the cost incident to the construction of the project and the acquisition of the necessary lands and rights of way.

Water-Rights have Value. — The fact that in the first case it will have to be conceded that the owner of the public utility is entitled to have the cost of the water-right which he holds made a part of the rate-base and that at least to the extent of cost (reasonable and actual proper cost being assumed), this water-right has or should be made to have value, justifies the public in concluding that the other water-right which has cost nothing should have a similar value, whether the same be made a part of the rate-base or not. Water-rights, then, are to be regarded as having market value. When the water is developed and is actually being put to use or when the need of putting the water to beneficial use is proximate, the existence of such value is easily recognized. When an investment has been necessary to quiet title to adverse rights and to meet other expense of securing the water-right, the propriety of including its cost in the rate-base is unquestioned.

Water-right Value in the San Joaquin and Kings River Canal Case. — The Supreme Court of the United States in "San Joaquin and Kings River Canal and Irrigation Co. *vs.* The County of Stanislaus" (233 U. S. 458) in reference to the fundamental principle of taking the value of water-rights into account when rates are to be fixed, says, in its decision reversing the decree of the lower court:

"By a statute of March 12, 1885, the boards (of County Supervisors) are authorized to fix these rates for their several counties, but so that the returns to the parties furnishing the water shall not be less than 6 per cent upon the value of the canals, ditches, flumes, chutes, and all other property actually used and useful to the appropriation and furnishing of such water.' The rates, when fixed are binding for one year and until established anew or abrogated. . . . The question before the court has been narrowed to a single issue. If the plaintiff is entitled to 6 per cent upon its tangible property alone, it is agreed that the order must stand. But if the plaintiff has water-rights that are to be taken into account, the rates fixed will fall short of giving it what it is entitled to and must be set aside. . . .

"It is not disputed that the plaintiff has a right as against riparian proprietors to withdraw the water that it distributes through its canals. Whether the right was paid for, as the plaintiff says, or not, it has been confirmed by prescription and is now beyond attack. It is not disputed either that if the plaintiff were the owner of riparian lands to which its water was distributed it would have a property in the water that could not be taken without compensation. But it is said that as the plaintiff appropriates this water to distribution and sale it thereby dedicates it to public use under California law and so loses its private right in the same. . . .

"It seems unreasonable to suppose that the Constitution meant that if a party instead of using the water on his own land, as he may, sees fit to distribute it to others, he loses the rights that he has bought or lawfully acquired. Recurring to the fact that in every instance only a few specified individuals get the right to a supply, and that it clearly appears from the latest statement of the Supreme Court of California (Palmer *vs.* Railroad Commission, Jan. 20, 1914 (47 Cal. 201)), that the water when appropriated is private property, it is unreasonable to suppose that the constitutional declaration meant to compel a gift from the former owner to the users and that in dealing with water 'appropriated for sale' it means that there should be nothing to sell. (See San Diego Water Co. *vs.* San Diego, 118 Cal. 556, 567; 50 Pac. Rep. 633; 38 L. R. A. 460; 62 Am. St. Rep. 261; Fresno Canal and Irrigation Co. *vs.* Park, 129 Cal. 437, 443; 62 Pac. Rep. 87; Stanislaus Water Co. *vs.* Bachman, 152 Cal. 716; Leavitt *vs.* Lassen Irrigation Co., 157 Cal. 82.)"

According to this decision the water-right must receive the same consideration as other property when rates are to be fixed.

But the court does not attempt to settle the question relating to how a water-right is to be valued.

California Railroad Commission on Water-right Values. — In the matter of valuing water-rights Commissioner Thelen, in writing the decision of the Railroad Commission of California in the San Diego case (Decision No. 1465), sounds a note of warning, when he says:

"This case illustrates clearly the tremendous importance to the people of this State of the claim made by certain water companies and other utilities that the value which adheres to the water which they convey to their customers belongs to the utility, and that the utility is entitled to capitalize the full value of that water, entirely irrespective of its cost to the utility, and to collect a charge for water high enough to yield a return on such amount as the experts for the utility estimate to be the value of the water or of the water-right. I do not deem it necessary at this point to discuss the authorities both in the State and federal courts bearing on this question, for the reason that, according to press despatches this question has now been decided by the Supreme Court of the United States in the case of San Joaquin and Kings River Canal and Irrigation Co. *vs.* County of Stanislaus. The question of the amount of value to be allowed is, of course, a question of fact, the determination of which still rests in this Commission. I desire, at this time, to draw attention to the grave consequences which may follow if the theories of value of water-rights urged by various public utilities before this Commission are adopted. If it is true that the entire value of the water which a public utility secured by appropriation or otherwise belongs to the utility and that the public must pay rates on such value, it follows that, where there is only one source of water supply for a municipality, the water utility has the right to capitalize the entire life of the municipality. And, in any case, the utility will have the right to take for itself the entire increased value of land due to the placing thereon of this water, entirely irrespective of the fact that the people of this State have given to the utility the right to appropriate the water and that the actual price paid for the water may have been absolutely insignificant as against the amount claimed by the utility."

The Free Grant of Water-Rights in Western States. — The right to appropriate flowing water and to put the same to beneficial use is given by law, in most of the Western states, to any one who will construct proper works for the development of the water and for its transmission to places of use. The water of the stream belongs to the public. The grant of the right to put it to some beneficial use is on a par with a franchise to construct a highway or to build a railroad. This right has value as a franchise has value when the earnings are sufficient to create a value. As in the case of a franchise, so in the case of the water-right, the cost thereof becomes a part of the rate-base in the event that investment and not value be made the starting point.

Determination of the Water-right Value. — The courts and the rate-fixing authorities accepting the view of the public are showing a tendency to allow earnings which will give the water-right value. But neither the courts, nor public service commissions, nor experts have yet agreed upon any method of determining the water-right value. The method of ascertaining the water-right value, in the case of water used to supply the needs of an urban population, by comparison with the ordinary cost of developing water in the same region in like amount, of like quality and under similar conditions of delivery has occasionally been applied but not with entirely satisfactory results.

To illustrate, let it be assumed that the average ordinary cost of making water for domestic use available for distribution, in the region in which a water-right is to be valued, is 10 cents per 1000 gallons. This cost is here supposed to include interest on the investment and the outlay of whatever nature connected with operation. Let it be further assumed that the cost of making available the water which is to be valued has been found to be 9 cents per 1000 gallons. It will readily be seen that under such circumstances a rise of 1 cent per 1000 gallons in the average regional cost of water production would have the absurd effect of doubling the value of the water-right. Furthermore, the water-right of any supply whose cost of development exceeds the average or ordinary cost would, under strict application of this test of value, prove to be a liability and not an asset. This, too, is an absurdity and condemns the method.

Neither this method nor a comparison with the cost of developing the next most available supply can be used as a dependable method for determining water-right values.

In some sections of the country, as, for example, in portions of California, the demand for irrigation water has nearly, if not quite, reached the limit of supply. In such regions the depressing effect upon the value of water which results from large undeveloped available sources is no longer felt. The earnings that result from the use of the water have become the measure of its value and this value is consequently relatively high.

The recognized value of water-rights in such sections has an effect upon the value of water-rights elsewhere and for uses other than irrigation. The value of water used for domestic purposes, similar general conditions being assumed, should not be less than that of water used for irrigation, and, if for this purpose it has a high value in one part of the state, — the question is asked, why not in another? Such considerations as these are not without effect upon the market value of water-rights.

The fact should not be overlooked that the inclusion of a water-right value in the rate-base of a public service property, to the extent that this value exceeds cost, would be in the nature of an allowance to be regarded as part compensation for having undertaken the water development, and this compensation increment might reasonably be brought into some definite relation to the general cost of developing water in any region.

Water-right Values in Relation to Cost of Works. — It has above been stated that the allowance of a fair reward for the successful development and beneficial use of water is legitimate. The making of a reasonable allowance, preferably based upon some definite percentage allowance on the ordinary regional cost of developing water or of developing hydro-electric power as the case may be, should be encouraged. If this principle were generally recognized, it would result in fixing with some definiteness the value of water at its source and would remove much of the uncertainty that now obtains in relation to the value of water-rights. If thus determined, the value of the water-right will not be subject to unreasonable fluctuation nor to too wide a range. Where the average regional cost of development, including everything necessary to make water available for distribution, is 10 cents per 1000 gallons and the allowance for water-rights is to be about 10 per cent of this amount, or 1 cent per 1000 gallons, a change of 1 per cent in the cost of water development would only modify the value of the water at the source by 0.01 cent per 1000 gallons. A 10 per cent increase or decrease in the regional cost of development would be necessary to affect this value by 0.1 cent. In other words, when, in a certain region, an amount has been agreed upon and generally accepted as a proper allowance to be made for the value of developed water, or rather, when such value is to be created by a suitable allowance of earnings, this value will be fairly stable and will thereafter pass as the market value whether or not the cost of development is below or above the average.

Strategic Value of Water-Rights. — In addition to the basic value at its source, a water supply may have additional value, due to an inherent advantage of quality and location and other circumstances that determine its development cost in comparison with the development cost of competing supplies. Such value is properly termed "strategic value."

To illustrate, a riparian ownership which controls a water-power may be cited. The case may readily be conceived of a water-power, limited in amount, but completely controlled by the riparian owner. When such a source of power is to be valued in a region where the market for power is good, where, for example, the water-power will be delivered to a market in which it displaces a like amount of power generated by steam, the cost of the latter in comparison with the cost of the former affords a legitimate means of determining value, or, better stated, an upper limit of value. The valuation becomes a simple matter when, under such circumstances, the power is already fully developed and is in use or is being supplied to a market which takes it all. But when the power is undeveloped, some consideration must be given to the uncertainty of achieving the expected results and due allowance must be made for the time that will have to elapse before a return from the sale of power can be realized.

There will, of course, be cases in which an analysis of the cost of generating and delivering power will show the advantage to be with the power developed by steam. In such cases the hydro-electric enterprise may nevertheless be a legitimate one. It may have been initiated when the price of fuel for generating steam was such that the advantage of cost was temporarily with the water-power; or the margin in favor of the steam-power may be so small that the recognized advantage and economic value to society in conserving the energy which annually reappears in the water of the stream outweighs any financial disadvantage that may appear from a comparison with steam as a source of power, and justifies earnings that might not under other circumstances be considered reasonable.

Furthermore, if the market for the output of a hydro-electric installation has been established, there is no certainty that the market of a competing plant, at a different cost and sale price of power, would be the same. This is a circumstance which should be duly weighed in making the comparison.

In any event, the owner of such utility should be recognized as engaged in a meritorious enterprise, deserving not only adequate protection, but also such reward for having developed the water-power and having made an investment for the benefit of the public as the circumstances may justify. Under this view, even when the water-right which makes the development of power possible would appear to be without market value at current fuel prices, it will be reasonable to allow to the owner, not alone a rate-base increment equal to the cost of securing the water-right, if there has been any such cost, but also, if this cost has been legitimate and reasonable, some excess allowance in the earnings, perhaps proportional to the amount of power developed rather than to the actual investment in works for developing and marketing this power. But this can only be done within limits or so long as the rates for the service remain reasonable. When it would require excessive rates, the owner must suffer the penalty of having made an untimely if not an unwise investment.

Illustration of Strategic Value. — As an illustration of special water-right value the case of a water supply for general and domestic use which affords water of prime quality in limited amount may be taken, but which, when compared with other sources in use in the same community, has the advantage of proximity, elevation and reliability of service.

Let it be assumed, for example, that such a supply was the first to come into use, that its water was distributed as required throughout the built-up section of a growing town, but that at length a time came when additional water had to be brought in by a second system from some remote source, and that at the time of the valuation the distributing pipes of each of the two systems cover practically the entire built-up territory. The original water-works may now be supplying only a small fraction of the aggregate amount of water being used. Undoubtedly under such circumstances, the charge for water by the two concerns would be the same or very nearly the same. The water from the newer works could not be supplied at a low enough rate to drive the earlier concern out of business. Without any reduction of rates, this original utility should hold its customers. There need be no falling off in the amount of water which it supplies, assumed to be the limit of its capacity. But, if, as assumed, the rates charged by the two concerns are the same, the relative amount of net earnings will be greater for the original than for the new water-works. If it costs the original concern 17.5 cents per thousand gallons to develop and market its water crop (interest on the investment included) and it is costing the new concern 20 cents to do the same, and if this larger cost has been taken into account in fixing the water rates, then the water-right and other intangible elements of value of the original concern may reasonably be valued at (\$200 − \$175) \$25 per day per million gallons of daily delivery more than the water-right and other intangible elements of value of the new concern. This is interest on about \$150,000, if 6 per cent per annum be made the basis of the calculation.

If, in other words, rates are allowed which in the case of the new or main water-works system will create a water-right value of \$50,000 per million gallons of daily delivery for the new water-works, then the value of the water-rights controlled by the original system may be about \$200,000 per million gallons per day.

In the case of a water-power, too, there may be a pronounced and easily recognized strategic value. The usual distinction is to be made, however, between the power development with an established market and that in which the power output is not yet in full demand.

When there is no question about the market for the power, the problem will have to be solved on the basis of a comparison of the cost of utilizing the water-power as compared with power from other sources and this comparison may show more or less strategic value. Where there is no such strategic value, the water-right should be considered as a privilege similar to a franchise and should be treated accordingly.

It must be remembered in this connection, however, and in any analysis of this character, that the advantage that one concern may have over another in the amount that net earnings exceed interest on the investment, is to be applied to all elements of value in excess of the capital actually invested and can not always be assigned to water-rights alone.

The Time Element in Valuing Water-Rights. — The water-right as thus far discussed is the right to put water to a continuing beneficial use without limit as to the time during which such right may be exercised. There will be cases of reversion of the right within a fixed time to the public which has made the grant thereof and there will be other cases in which a superior supply of water, later to be developed, may at some time throw the original source out of use altogether or leave it available for only inferior uses. Where a water-right thus limited in life is to be valued, the question not only arises as to its strategic value, but cognizance must be taken of the fact that the life of the right is limited and that it will not be a source of revenue for all time.

The value of any water-right, in excess of cost, like the value of a franchise, results from earnings in excess of a fair interest on the investment. This value is, therefore, directly dependent upon the rates established by the rate-fixing bodies or, in the case of the restricted franchise, upon the rates allowed to be charged under such franchise. Large power is in the hands of the rate-fixing authorities to make or to destroy the value of water at its source and until a definite policy has been adopted by such authorities, there will continue to be more or less uncertainty relating to such value. The real value of property of this character will, for the present, remain somewhat speculative, particularly in cases where the development of the water or of a water-power lies in an uncertain future. This can hardly be otherwise because it is not yet certain that the tendency of today to allow something for the water-right, practically as compensation for making the development, will be adhered to. When it is fully understood that such an allowance will be made and when a definite limit is set to the amount of such an allowance, the valuation experts will be relieved of much embarrassment.

In the case of the established utility much of the difficulty ordinarily encountered when water-rights or franchises are to be valued as a basis for fixing rates will fall away if the method of procedure which the author recommends be followed and the invested capital and not present value be made the rate-base.

Views of the Wisconsin Railroad Commission on Water-power Value. — The attitude of the Wisconsin Railroad Commission toward the determination of the value of a water-power by a comparison with the cost of steam power appears in the following quotation from the Commission's decision in the case of the City of Beloit *vs.* Beloit Water Gas and Electric Co. (Wis. R. C. R., Vol. 7, p. 247).

"It seems clear from the expression of opinions thus made and from the general practice of engineers and other men in valuing water powers that the saving effected by the use of the water power over steam power, especially, measures the values of the water power. Other methods of appraisal are used and have been mentioned by the witnesses in these proceedings, namely, rental value and market value. These latter methods, however, are quite often open to objections which destroy their reliability and it appears that it is almost always necessary to fall back upon the method of calculating the saving over steam power and then by capitalizing this saving, arrive at the total value of the water power. The Commission has commented upon this and other methods of determining the value of water power in earlier decisions:

"'From a purely commercial point of view this method of estimating the value of water-power rights may, in the main, be sound. But it is not so clear that this can be said for it when the question is regarded from the point of view of public policy. . . . it appears to deprive a locality of the natural advantages it might otherwise derive from being located near such water powers. If water-rights are private property under the law, then all the benefits which accrue from these rights would probably go to their private owners. If, on the other hand, water-power rights are public rights rather than private rights, then it would also seem that the public ought to share in any benefits that may be derived therefrom.' Ross *et al vs.* Burkhardt Milling and Electric Power Co. (Wis. R. C. R., Vol. 5, p. 139, 147)."

On the subject of water-power value the Commission says further in the case of City of Rhinelander *vs.* Rhinelander Lighting Co. (Wis. R. C. R., Vol. 9, p. 424):

"While calculations of the saving produced by the use of water-power instead of steam-power are of much importance in private and public undertakings in showing the financial feasibility of hydraulic construction, the title of the owners in utility business to the entire savings so produced has not been clearly demonstrated. Indeed, the respondent's claims seem to go so far as to preclude the public from any share in economical methods of service and seem to place upon users of utility service the burden of maximum costs of operation."

Valuation of Reservoir Sites in Relation to Water-right Value. — Passing now to the consideration of the value of reservoir sites it may be broadly stated that, in some measure, any value thus or otherwise ascertained as appertaining to water-rights may serve as an aid in determining the value of water-shed and reservoir lands or of other lands whose use is necessary to make the development of the water possible.

The value of such lands should not, however, be measured by the necessity of the community which needs the water. Their value is not what the community can afford to pay for them; but they have at least the value which would be determined by the market for similar lands devoted to other uses. They also have an additional value due to special adaptability for use in developing a water supply.

It is owing to the desirability of bringing this excess value of reservoir lands into some relation to the value of the water whose development their ownership makes possible that it may sometimes be found desirable to make the value of the water-right a measure of the excess of value (sometimes perhaps only of the upper limit of the excess) of an assembled reservoir property over the value of the land for other purposes. While this is not an established practice, it is one which appears to have some merit. It is to be understood that the excess of the value of reservoir lands over similar lands not available for reservoir use as here considered is apart from and in addition to the value of the water-right.

Where the water development requires only a few acres of ground, as in the case of artesian supplies, particularly if the land remains available for other uses, or when the topographic situation is unusually favorable, a small fraction of the value of the water-right might prove to be an adequate allowance for special adaptability.

Value Multiple Applied to Reservoir Lands. — In such special cases some use might be made of a value multiple such as has become customary when rights of way are to be secured for railroads and canals which are acquired usually at some increase over ordinary values, *i.e.*, over the value of grazing and farm lands crossed by the railroads and canals. In the case of railroads the excess is generally 50 per cent to 200 per cent. This would probably also be a fair assumption in the case of canal rights of way. In the case of reservoirs the value of the reservoir site as compared with that of other land of similar location and quality may go to much higher limits, but not enough data are at hand to justify an attempt to give reliable averages. Where the site is large and the use one that may not prove highly remunerative, due to remoteness of location, high cost of construction, scant rainfall on the watershed or other modifying causes, such as the availability of alternative reservoir sites, a bare allowance to the owner for forced abandonment of his holdings may represent the limit of what any prudent purchaser would pay. In other cases a multiple of 5 or even more may not be unreasonable.

When there is also a strategic value due to relatively large earnings that will result, perhaps, from rates that have been or that must be so fixed that they will yield a fair return on some other less favorably situated property, then there will be an excess of value determinable from the large present or prospective earnings, and a part of this excess would naturally be allotted to the person who owns the reservoir site, the rest thereof going to the party who actually makes the water development.

Certain Increments of Reservoir Land Value. — It is generally recognized that the problem of valuing reservoir lands is one of the most difficult that can be presented to the engineer. None of the facts relating to the availability of a reservoir site taken by itself determines its market value. But every such fact must have some effect upon the minds of an intelligent public and therefore influences the market value.

Tracts separately owned which must all be combined under one ownership to be available for use as a reservoir will have less value in separate ownership than after being assembled in one holding. The value of reservoir lands will be less if there are other similar reservoir properties available for alternative use. The time when the utilization of the property is a necessity, if this time can be definitely or even approximately fixed, will affect its market value. The value will be higher after actual construction has demonstrated that a reservoir will hold water than before. If there have been any judicial determinations of the market value or of the value for rate-fixing purposes of the property to be valued or of similarly situated or otherwise comparable property or if valuations thereof have been made by authorities charged with the regulation of rates or by other public authorities, all such determinations will have an effect upon the market and would be given consideration by a prudent purchaser. They are also, therefore, proper elements for consideration by an appraiser. The cost of the lands in a site already acquired is a factor that should be given due weight with proper allowance for the circumstances attending the purchases. Appreciation or depreciation that may have taken place subsequent to such purchase may also have to be given consideration.

The value of a parcel of land which is required in connection with others to make a reservoir site available is, as above stated, worth less by itself than it will be when united with other tracts into the one holding which makes the development possible. While the individual tract is reservoir land and due to this fact may have acquired a market value in excess of the value which it would have if not located in a reservoir site, this excess value would be estimated by a prudent purchaser at less than the excess value which it acquires when brought into the same ownership as all the rest of the lands which make the storage of water possible. At how much less it should be valued cannot be stated with any degree of confidence for general guidance. And yet it may tentatively be suggested that the party who assembles the property should get the benefit of at least one half and, in many cases, much more than this proportion of the excess of its value, in the constructed reservoir, over its value for ordinary uses.

Effect of Various Factors on Reservoir Value. — Too much weight is sometimes attached to the influence of the cost of developing a water supply upon the market value of the land which is to be acquired to make the water development possible. Whether this cost be great or small, the custom prevails of allowing the owner of the water-works to recover, in the earnings, the cost of operation including replacement requirements and interest on the cost of structures. Whenever, therefore, the necessity of using the land is unquestioned, the cost of developing the water should have but little effect on the market value of the reservoir land. The amount of the water made marketable is, on the other hand, a circumstance which will have greater or less effect on this value, depending upon the value which will be allowed in the rate-base for the developed water-right. There are cases, too, in which the nearness of a storage site to the place of use may give it special value, due to the fact that it adds to the reliability of the service and, finally, there will be cases in which the property to be used for water storage or already in use as a reservoir has acquired greater value for residential or other purposes than, by any fair line of reasoning, could be determined for its use in developing or storing water. When this is the case, the market value of the reservoir land (always to be distinguished from the amount at which it is included in the rate-base) is determined by the other uses to which it might be put. If such a reservoir is a necessity and no other equivalent structure can be substituted for it, the appraisal of market value would be fixed by these other uses. If on the other hand it is possible to substitute for the reservoir some other structure located elsewhere, and costing less though of equivalent service value, then in the case of the constructed reservoir the time will have come when its use should be discontinued or in the case of the reservoir site, not yet in use, the project plans should be so modified as to eliminate the reservoir.

It may be well to repeat that when dealing with water not yet developed, proper allowance must be made in all appraisals of water-right and of reservoir and watershed values for the probable lapse of time before the water will actually be in use.

Watershed Land Value. — When storage reservoirs for water for domestic use are involved and the value of the watershed lands is to be determined, it would be legitimate to give consideration to the modification of the cost of operation which might result from the ownership of these lands. When such ownership would result in safeguarding the quality of water so that filtration and other treatment to make the water attractive and wholesome would become unnecessary, the limiting value of such ownership can be measured by the added cost that would be incurred for a purification of the supply if the water were in danger of pollution due to the use of lands in the watershed for human habitation or for other purposes that would detract from the wholesomeness of the runoff waters. Consideration would certainly be given by a prudent purchaser who is weighing the desirability of acquiring watershed lands to this matter and, therefore, this is properly an element to be considered in determining the value of watershed lands whether already in use or required for early use.

In this connection consideration would be given: first, to the degree of protection which will result from such ownership, because, after all, the ownership may not be an absolute insurance against the necessity of some treatment, such as filtration, sooner or later; second, to the sentimental value attaching to any drinking water not subject to the danger of pollution in comparison with a water known to have been polluted but made wholesome by suitable treatment; third, to the time in the future when the expenditures for filtration and other treatment will begin; fourth, also to any protection which the ownership of the watershed would give against an adverse use of the waters originating therein, and against their diversion from the watershed and, finally, to any other benefits that might result from such ownership, such as uses not incompatible with the development of the water supply.

Frequently, of course, the value of watershed lands, owing to their use for farming purposes and owing to the inclusion of densely populated areas, is so high that their ownership for the protection of the quality of the runoff waters is entirely out of the question and they do not then come under consideration in the determination of the value of the opportunity to make a water supply available.

Notes relating to Some Water-right Values in California.— The following relating to the cost and value of certain water-rights in California is from the testimony of engineers who appeared before the Master in Chancery in the proceeding entitled "Spring Valley Water Company *vs.* The City and County of San Francisco" which was on trial from July, 1915, to May, 1916.

Mr. G. G. Anderson in his testimony referred to the value of water-rights in southern portions of California as determined from the value of the shares of stock in the various mutual irrigation water companies, and he says "care was exercised in the analysis of individual cases to limit the values of these water-rights to terms of the right to divert and use only, excluding all interest in attached lands or ditch systems or any assets other than actual rights of diversion and use." He cites:

	Per million gals. per day
Duarte Mutual Co. near San Gabriel	$154,800
Covina Co. near San Gabriel	117,957
Del Monte Co. near Pomona	98,220
Canyon Water Co. near Pomona	104,490
San Antonio Co. near Ontario	144,738
Bear Valley Co. near Redlands	90,248
Mill Creek Co. near Redlands	92,880
Gage Canal Co. near Riverside	77,400
Temescal Co. near Corona	96,750

The foregoing values Mr. Anderson states "attach to the service for the particular purpose (irrigation) which entails delivery of water during the irrigation season or ordinarily 240 days per annum, rarely does the season extend to 270 days, and the service during the limited period does not, in all cases, yield full efficiency on the water-right."

For the irrigation plants in the Santa Clara Valley Mr. Anderson develops values ranging from $31,442 to $67,724 per million gallons of daily supply from gravity systems, over periods up to a maximum of 115 days per annum; and $167,709 to $242,786 for pumping systems.

He also finds that the original cost to the Spring Valley Water Company of acquiring water-rights on both sides of San Francisco bay (about 1865 to 1913) averaged nearly $38,000 per million gallons per day, of water delivered to the inhabitants of San Francisco on the date of valuation, Dec. 31, 1913. By a different analysis and with some allowance for uncertainties Mr. Chas. H. Lee testifying in the same case finds this cost to have been about $48,000 per million gallons per day.

Mr. F. C. Herrmann calls attention to the purchase by the Pacific Gas and Electric Company from the Livermore Water and Power Company of rights to about one million gallons of water per day at about $100,000 in 1913. The water-rights involved in this transaction are for the Mocho and Positas Creeks near Livermore, Cal.

He also refers to a number of sales of springs and wells with small yield and to the following sales in southern portions of California:

In condemnation proceedings by the City of Sierra Madre the value of water for domestic use was placed by the court at $270,760 per million gallons per day.

Near Alhambra, in 1892, Richard Garvey bought from De Barth Shorb about 390,000 gallons of water per day at $64,595 per million gallons per day.

At Montecito, Mr. Knapp bought 12,900 gallons of water per day a one-third right in the Warm Springs tunnel, at $386,000 per million gallons per day.

The value of water fixed by the California Railroad Commission in the Glendale case, in addition to the allowance for structures was $154,720 per million gallons per day.

While each specific instance of a sale of water-rights, as cited by Mr. Herrmann, should be considered in the light of all circumstances attending each such sale and of the specific needs of the purchaser in each case, these sales, nevertheless, have some effect upon the market value of water-rights showing as they do what a purchaser may, under certain circumstances, be willing to pay for water.

Both Mr. Herrmann and Mr. Anderson were witnesses for the plaintiff in the Spring Valley case. Mr. Chas. H. Lee appearing for the defendant, that is for San Francisco, added the following estimates of net water-right values:

1. Culture exclusively citrus (Southern Cal.):

	Per million gals. per day
Lugonia Water Co. near Redlands	$154,000
San Antonio Water Co. near Ontario	142,800
Del Monte Irrig. Co. near Pomona	90,900
Temescal Water Co. near Corona	69,600
Gage Canal Co. near Riverside	73,500
Redlands Water Co. near Redlands	52,400
San Dimas Irrig. Co. near San Dimas	51,500

2. Citrus and diversified crop (Southern Cal.):

	Per million gals. per day
Santa Ana Irrig. Co. near Redlands	$43,500
Alta Mutual Water Co. near Riverside	42,500
Thermal Belt Water Co. in Santa Clara River Valley	42,500
Riverside Water Co. at Riverside	14,700
Los Nietos Ditch Co. near Whittier	13,200
South Side Improvement Co. in Santa Clara River Valley	2,860

3. Diversified crops, no citrus (Southern Cal.):

	Per million gals. per day
Banning Water Co. near Banning	$37,400
Moneta Water Co. near Redondo	23,200
McKenzie Ditch Co. near San Bernardino	13,200
Stout Ditch Co. near Redlands	12,100
Puente Water Co. near Puente	2,400
Arroyo Ditch Co. near Downey	2,300
Little Lake Irrig. Co. near Norwalk	1,000

For more northerly portions of the State of California where water is more abundant and where the demand does not yet approach the limit of possible development, Mr. Lee lists the following:

Culture exclusively citrus:

	Per million gals. per day
Lemon Cove Ditch Co. on Kaweah River	$25,400
Rosedale Water Co. on Tule River	23,200

Citrus and diversified crops:

	Per million gals. per day
South Tule Independent Ditch on Tule River	5,070

Diversified crops, no citrus:

	Per million gals. per day
Bishop Creek Ditch in Owens Valley	3,100
Clark Colony Water Co. in Owens Valley	3,100
McNally Ditch Co. in Owens Valley	3,100
Owens River Canal Co. in Owens Valley	3,100
Roberts Ditch Co. near Colusa	1,160
Watson Ditch Co. near Visalia	2,560
Evans Ditch Co. near Visalia	2,012
Murphy Slough Assoc. on lower Kings River	1,940
Rawson Ditch in Owens Valley	1,940
Consolidated Peoples Ditch on lower Kings River	870
Oakes Ditch Co. near Visalia	580
Poplar Ditch Co. on Tule River	217

Mr. Lee cites also a few canal companies for whose water-rights no value is demonstrable from the market value of stock in the canal company. These were omitted from the above enumeration.

The above figures are not presented to show the value of water in regions where society already demands full utilization, but rather to show that with the demand for a higher use of water which must come as the pioneer region gradually changes to a densely populated territory, the value of the water-right in any such country as that under consideration, where nature has set a limit to the available supply, must go up.

CHAPTER XII

THE ACCOUNTING SYSTEM

Purpose of the Accounts. — It is not proposed to take up the matter of accounting in connection with public utility enterprises any further than to indicate its purpose and to refer briefly to certain instructions in relation thereto which have been issued by public service commissions.

The accounts should show clearly and in sufficient detail the facts relating to the investment, to operating expenses and to income. The natural division is therefore into two groups, the one relating to the investment, the other to income and operating expenses.

The accounts relating to investment are designed to show the investment in the property which is devoted to public use. These accounts should be kept in such form that the additions and betterments from year to year will be clearly apparent. They will include not alone the actual cost of all physical properties but also any amount paid for franchises or for water-rights and rights of way.

The income accounts are those which are designed to show, for each year, the amount of money earned for services rendered or for commodity delivered and the cost of rendering the service or furnishing the commodity. They will include the returns from outside investments and other sources on the one hand and on the other every expenditure necessary to render the service or to supply the commodity including taxes, insurance, rents and the like. The net balance of the operating and income accounts is profit or loss.

Construction Account, Interstate Commerce Commission. — In the general instructions of the Interstate Commerce Commission relating to the accounting system of railroads, construction is defined as including all processes connected with the acquisition of the original road and equipment, road extensions, additions and betterments. The following is from these instructions:

"*Costs* shall be actual money costs to the carrier. When a portion of the funds expended by the carrier has been obtained through donations by the states, municipalities, individuals or others, no deduction on account of such donations shall be made in stating the costs. Contributions for joint expenditures should not be considered as donations. The carrier's proportion only of the cost of joint projects, such as construction of jointly owned tracks and elimination of highway crossings at joint expense, shall be included in these accounts.

"The charges to the accounts of this classification shall be based upon the cost of the property acquired. When the consideration given for the purchase or the improvement of property the cost of which is chargeable to the accounts of this classification is other than money, the money value of the consideration at the time of the transaction shall be charged to these accounts, and the actual consideration shall be described in the record in sufficient detail to identify it. The carrier shall be prepared to furnish the Commission, upon demand, the particulars of its determination of the actual cash value of the consideration, if other than money.

"It is intended that the accounts for fixed improvements and equipment shall include the cost of construction of such property. The cost of construction shall include the cost of labor, materials and supplies, work-train service, special machine service, transportation contract work, protection from casualties, inquiries and damages, privileges, and other analogous elements in connection with such work."

Treatment of Depreciation and Replacement, Interstate Commerce Commission. — The attitude of the Interstate Commerce Commission with reference to the depreciation and replacement of property appears from the following instructions:

"When a unit of property other than land or equipment — such as a section of road, side or yard track, shop or power plant machine, building, or other structure — is retired from service and replaced with property of like purpose, the ledger value of the retired property shall be credited to the appropriate accounts of this classification at the time that the property is retired from service. The amount of this credit shall be charged concurrently as follows:

"An amount equal to the credit balance in the accrued depreciation balance-sheet account with respect to the property thus retired shall be charged to that account and the remainder (less salvage and insurance recovered, if any), together with the cost of demolishing the property, if demolished by or for the carrier, shall be charged to the accounts in Operating Expenses appropriate for the cost of repairs of the property before retirement. The accounting for the salvage shall be in accordance with the disposition made of the material recovered.

"If, however, the property retired and replaced with property of like purpose is of minor importance, such as a small roadway building or other small structure, and is replaced in kind without betterment, the cost of the replacement shall be charged to operating expense accounts, and no adjustment made in the road and equipment accounts.

"If so authorized by the Interstate Commerce Commission, the carrier may charge to Profit and Loss any extraordinarily large item representing the cost of property retired and replaced, instead of charging such item to Operating Expenses. The carrier shall file with the Commission a statement of the cost and a description of the property retired and the reasons which, in its judgment, indicate the propriety of charging the cost of such property to Profit and Loss.

"The provisions of this section are applicable in accounting (at the time of retirement) for the cost of property abandoned, even though the new property has been actually installed previously to the date of the demolishment of the abandoned property.

"When the renewals to be made to an important building or other structure will constitute the major portion of its value when renewed, the property, when taken out of service, shall be considered as retired and accounted for as provided above, and for the purposes of this classification the renewed property shall be considered as an addition, and the appraised cost thereof shall be included in the accounts of this classification, consideration being given to the second-hand portions remaining therein. In no case shall the charge for the renewed property exceed the cost (at current market prices of labor and material) of new property of equal capacity and equal expectation of life in service, less a suitable allowance on account of the second-hand parts remaining therein.

"When a unit of property other than land or equipment — such as a section of road, side or yard track, shop or power plant machine, building, or other structure — is retired from service and not replaced, the ledger value shall be credited to the appropriate property accounts at the time that the property is retired from service. The amount of this credit shall be concurrently charged as follows:

"An amount equal to the credit balance in the accrued depreciation balance-sheet account with respect to the property thus retired shall be charged to that account, and the remainder (less salvage and insurance recovered, if any), together with the cost of demolishing the property if demolished by or for the account of the carrier, shall be charged to the appropriate profit and loss account. The accounting for the salvage shall be in accordance with the disposition made of the material recovered."

Retirement of Land, Interstate Commerce Commission. — In the matter of land retired from use the Commission says:

"When any land, the cost of which is included in the accounts of this classification, is retired, the ledger value shall be credited to account 'Land for transportation purposes.' If the land is retained by the carrier, its estimated value shall be charged to balance-sheet account 'Miscellaneous physical property,' the necessary adjustment of the difference between the ledger value and the estimated value on account of the loss in the property due to its retirement from transportation service shall be made through Profit and Loss. If sold, the difference between the ledger value . . . and the amount received for the land shall be adjusted in Profit and Loss."

Engineering Account, Interstate Commerce Commission. — Relating to engineering on the construction of a railroad, the Commission says:

"This account shall include the pay and expenses of engineers, assistants, and clerks engaged in the survey and construction of new lines and extensions, or in making additions to and betterments of the carrier's road, including wharves and docks.

"When employees . . . are engaged in the maintenance of the road, their pay and expenses while thus employed shall be charged to Operating Expenses.

"Expenditures for tentative or preliminary surveys shall be carried in a suspense account until it is determined whether or not to continue the work. If the project is continued, expenditures for all surveys in connection therewith shall then be transferred to this account, and, if abandoned, to Operating Expenses, Income, or Profit and Loss, as may be appropriate."

Interest during Construction, Interstate Commerce Commission. — The Interstate Commerce Commission in the instructions relating to "interest during construction" says:

"When any bonds, notes, or other evidences of indebtedness are sold, or any interest-bearing debt is incurred for acquisition and construction of original road and equipment, extensions, additions, and betterments, the interest, accruing on the part of the debt representing the cost of property chargeable to road and equipment accounts (less interest, if any, allowed by depositaries on unexpended balances) after such funds become available for use and before the receipt or the completion or coming into service of the property so acquired shall be charged to this account.

"When such securities are sold at a premium the proportion of such premium assignable to the time between the date of the actual issuance of the securities and the time when the property acquired or the improvement made becomes available for service shall be credited to this account.

"This account shall also include such proportion of the discount and expense on funded debt issued for the acquisition of original road, original equipment, road extensions, additions, and betterments, as is equitably assignable to the period between the date of the actual issuance of securities and the time when the property acquired or the improvement made becomes available for the service for which it is intended. The proportion of discount and expense thus chargeable shall be determined by the ratio between the period prior to the completion or coming into service of the facilities or improvements acquired and the period of the entire life of the securities issued.

"This account shall also include reasonable charges for interest, during the construction period before the property becomes available for service, on the carrier's own funds expended for construction purposes."

Application of Accounting Principles to the Unlimited Life Method. — Under the Unlimited Life Method of procedure, which is a new suggestion and which is not contemplated in the foregoing instructions, but which is the one that should find preference for all complex public utility properties made up of a large number of individual items, there will be no depreciation to write off. It will be unnecessary to estimate accrued depreciation except when the transfer of the property is involved or a valuation is to be applied to individual articles. The current replacement requirement, however, will have to be estimated; but the estimate need be only approximate. The replacement fund will not be available for the retirement of capital. Neither will it be available for betterments or additions to the property. If it grows too rapidly, if the accumulation in it becomes unnecessarily large, the current replacement requirement may have been over-estimated; if it is depleted there may have been an underestimate of the replacement requirement. Excessive accumulation in the replacement fund should be checked by reducing the amount annually set apart for the fund. In the accounting system, the cost of each article as it goes out of use is written off the books and each article which replaces a discarded article is entered as a renewal and is treated practically as though it were new construction. Its cost is offset by the amount written off for the discarded article and the replacement fund is depleted by this cost. The capital investment account remains unaffected unless the cost of the new article is greater or less than that of the discarded article. The excess cost, if any, should not come out of the replacement fund but from new capital. The deficiency, if any, represents a reduction of the invested capital.

The accounting system under the Unlimited Life Method of procedure, as will be seen from the above statements, is much simpler than that for procedures which take cognizance of the accrued depreciation and of the constantly changing present value of the physical elements.

The replacement account, if the Unlimited Life Method be adopted, will be credited with all expenditures that are made for the renewal of items which can be conveniently individualized and which serve or are expected to serve for a number of years, and it will be charged with the amount annually set apart out of earnings for replacements. This account should be kept separate from repair accounts although there is no fundamental difference between the replacement of a broken spoke in a wheel and the replacement of a discarded generator. It is fundamentally immaterial whether the replacement relates to single lengths of pipe, to broken window panes and the like or whether they are large as in the case of the dismantled steamboat. The only difference lies in the classification of the accounts. In either event funds for the replacement must be made available at the proper time. It is in the making of suitable provision for the replacement of the individualized items that expert advice should be sought in order that a proper distinction may be made between the apparent and the real profits of the business. Without a proper analysis, an apparent profit has, too frequently, been found to be in fact a loss.

Application of Accounting Principles to Other Methods. — Under any method of procedure, other than the Unlimited Life Method, the difficulty is presented of dealing with articles which do not serve throughout their full probable life terms and with articles which remain in service beyond these terms. In the one case the cost of an article may have to be carried for complete amortization long after the article has gone out of use. In the other case an article with many more years of usefulness should be rated as of no value. Accountants have found methods to overcome these difficulties but the bookkeeping which is involved becomes complicated.

When a public utility is established and its existence is justified by all attendant circumstances, the owner has reason to assume that the prospective business will prove profitable. There will usually, however, be a certain time during which his operating expenditures will exceed the revenue produced by the property. This time may extend over a period of years. Due consideration must be given to such facts as this as well as to the estimates of the prospective business when the cost of a commodity or of a service is to be ascertained and its sale price is to be fixed. The cost records should be so kept that they will give all needed information relating to past financial history as well as to the current cost of operation.

Any reduction in the value of an operative property such as may result from an advance in the art of the manufacture of its output or due to decrease in value of real estate and like causes, may result in an operating loss. In the case of a public service corporation where the investment is of real concern, such possible reduction of value is one of the elements covered by the hazards of the business and should be forestalled or should in the course of time be amortized by the excess of earnings over and above earnings on ordinary safe investments.

Any increase in value, no matter what it may result from, belongs among the earnings. Despite the rulings which have been had on this point by competent authority, it would seem preferable not to include appreciation in the rate-base but as elsewhere suggested to permit earnings that will give the owner a limited share in the general prosperity of the community.

CHAPTER XIII

THE VALUATION OF MINES AND OIL PROPERTIES

General Statement

Purposes of the Valuation. — Valuations of mines and oil properties as in case of other properties are needed for many purposes, the most general being, (1) the purchase or sale of such properties or portions thereof, (2) the information of owners or others interested in the property as a guide to the proper operation thereof or to the financial operations connected with such operation and (3) the determining of a basis on which taxation can be figured. The methods used to accomplish purposes (1) and (2) are closely associated and in many cases identical, but the proper method or methods for determining what the basis for taxation shall be, as shown later, must differ from valuations for other purposes.

Limitations upon Accuracy. — Because of the impossibility of fixing definitely the economic value of the mineral deposit, the extent of which may be indeterminable, valuations of mining and oil properties must be less definite than the valuations of industrial and public utility properties, the value of whose physical elements, at least, can usually be determined with considerable accuracy.

In underground metal mining, the development is in many instances kept just a short distance in advance of productive operations. This is sometimes due to the irregular nature of the ore bodies, but generally can be explained as a result of the method of financing mining properties. This is particularly true in small operations where the money earned is used to carry the expense of the development and the necessity to raise additional funds for extensive development is avoided.

This is not always true of extensive mining properties representing large investments. The Utah Copper Company in the United States and the Braden and Chicaquamata copper properties in Chile have ore bodies blocked out sufficient for 45 to 70 years of operation. This extensive development facilitates the valuation of these ore bodies. It should be noted, however, that two mines having equal ore reserves might be valued at very different figures because one may promise a future beyond its reserves while for the other there may be no hope of further production when the reserves in sight are exhausted.

The difficulty that is met in estimating the value of the mineral deposit is apparent also at oil properties when only a number of wells sufficient to develop enough oil to supply the existing market and to provide for a limited period in advance are drilled. Because of the usual great depth of oil wells, the sinking of new bores becomes an item of large expense and the development to a considerable extent of the holdings of an oil producing company might financially embarrass an otherwise profitable concern.

Placer gold deposits (hydraulic and dredging) in most cases are thoroughly prospected by pits or drill-holes and the value ascertained before mining operations are commenced. The drift placer mines of the Western United States form an exception and it is in most cases extremely difficult if not impossible to determine the value of such deposits.

Methods Commonly Employed in Valuation

Enumeration of Methods of Valuation. — Numerous methods of valuation have been employed and these have been modified by such variations as have suited the views of the valuating engineer or valuating body. That this has resulted in confusion is shown by recent court decisions. The engineer or valuer who does not definitely state the methods used by him in obtaining his valuation of the property seems to be the most likely to have his estimate supported by the findings of the court when appeal has been taken.

Among methods that have been used the following are the most important:

(1) Valuation by empirical methods.
(2) Valuation based on market value.
(3) Valuation based on royalty value.
(4) Valuation by capitalizing profits.
(5) Valuation by estimation method.

(1) *Valuation by Empirical Methods.* — An important example of an empirical method is the so-called "foot-acre" valuation of coal measures. A certain value per foot-acre of coal has been established in a certain district and the value of any property is based on this unit disregarding the fact that thin beds of coal are not worth as much per acre-foot as thick beds.

Another empirical method is that under which a value is placed on a mineral property of so many times the annual proceeds or the annual profits regardless of the actual expectation of life of the mine. It is needless to say that this method is used only for taxation purposes.

(2) *Valuation based on Market Value.* — A method of valuation of coal lands based on the use of the results of sales of neighboring properties has been found acceptable to the courts. The attempt is made to fix the actual sale value of a coal property by examining the price at which other property similar to that under consideration has been sold. This method is derived from existing methods of real-estate valuation and from an engineer's standpoint is acceptable only when checked or supplemented by a valuation on some other basis. Such checks may demonstrate that recent sales were not made at the actual value of the properties sold. Further than this no recent sales of similar properties may be available for purposes of comparison.

(3) *Valuation Based on Royalty Value.* — The royalty value method of appraising the value of mining properties makes use of the established royalty value per ton. This method of

valuation has been applied to some extent in coal regions. Because coal becomes more valuable from year to year the value of a leasehold will increase. The annual profits exceed the royalties by continuously increasing amounts and a valuation based on the average royalty payments in a district cannot be a gauge of the actual values, particularly districts where the leases have been in effect for some length of time. In addition to the value of the mine or colliery determined from royalty values, the leasehold itself has a value which increases as the value of the mineral product goes up and the actual value of the property must be the summation of these two values. As the leasehold value is determined by subtracting the valuation based on royalty values from a valuation of the property, itself, by some other method, it is evident that the royalty value method cannot be independently used to determine the value of the property.

(4) *Valuation by Capitalizing Profits.* — By this method the valuation is based on a capitalization of the net annual profits. It requires an estimate of the remaining life of the property but does not take into account the fact that incompetent management may so deplete the profits that a valuation with these as a basis may not approximate the actual value of the property in competent hands.

This method should only be adopted when estimated profits instead of actual profits are used as a basis. It is then applicable in the case of oil properties and other deposits the extent of which is indeterminable. If the life of adjoining or neighboring properties has been proven, an approximate basis may be had for estimating the life of the property which is to be valued. The depths to which mineralization extends in neighboring mines often is used to determine the expectation of a prospect when such information supplements the geological features of the deposits.

(5) *Valuation by Estimation Method.* — This method is adopted by mining engineers in the valuation of mineral properties when these are to be bought, sold or financed. Deter-

mination of all the elements that have a bearing on the values is necessary. The estimated profits capitalized furnish the foundation for a valuation of the ore bodies or deposits. These profits are dependent on the sales value of the product, the geölogical structure of the deposit, the cost of mining (present and future), the cost of surface treatment, and other factors.

In fixing the value of coal lands use can be made of the "foot-acre" method of valuing coal lands provided account is taken of the fact that a foot-acre of coal in a thin vein is not so valuable as it is in a thick vein. The sales of adjoining properties will be of assistance to the engineer in fixing a value, particularly in case of coal lands which have not been explored but which contain the same beds and show geological conditions similar to properties that have recently changed hands. The "royalty value" should not be ignored and when properly determined furnishes a valuable check on the valuation.

When account has been taken of the risks of operation, the geological features of the deposit and the economics of the necessary surface plant, the total prospective estimated profit can be suitably discounted and a certain definite valuation placed on the property. In determining the present value of a property for which the profits have been estimated, it is necessary to remember that certain fixed charges such as replacements, sinking fund payments (paid or not paid), etc., are to be deducted before the profits are capitalized.

A mining property to be a good investment must promise to an investor the return of his capital in addition to interest on the capital at a rate commensurate with the risk he takes in investing in such a property. What the risk will be should be determined by the investor before he makes the investment. Comparison of the proposed investment with certain standard investments such as government bonds, railroad bonds and real-estate mortgages should be made.

Proper Rate of Interest. — The proper rate of interest on mining investments has been and still remains the subject of much discussion among engineers. A reasonable solution of the problem has not been advanced and it is the belief of the author that it is not possible to determine a rate of interest that would apply equally well to all mining investments. The risk incurred by investing in a property that has been slightly developed is much greater than the risk in the case of a well-prospected ore body even when the mines contain similar ores and will be operated under similar conditions. Then again the average risks in copper mining differ from the average risks in gold mining and so on.

Mr. H. C. Hoover has tabulated the risks of mining as follows:*

"1. The risk of continuity in metal contents beyond the sample faces.
"2. The risk of continuity in volume through the blocks estimated.
"3. The risk of successful metallurgical treatment.
"4. The risk of metal prices, in all but gold.
"5. The risk of properly estimating costs.
"6. The risk of extension of the ore beyond exposures.
"7. The risk of management."

Several of these risks are found in industrial enterprises (Risks 4, 5 and 7). The risks of continuity of ore body and of ore values are peculiar to the mining industry. The limited market for the mineral products and the effect of the volume of the output on the prices that can be obtained increases the risk that capital must take. The problem of obtaining proper metallurgical treatment is an important one particularly when starting operations at new properties. The fact that the mineral constituents of the ore may change as greater depths are reached and that the previously satisfactory flow sheet may no longer realize the percentage of extraction on which the profits were based cannot be ignored by the investor. The interest return that might be attractive to an investor in a proven district might not be sufficient to attract capital in a district where the mines are still prospects and where the depth of the mineralization has not been tested.

* H. C. Hoover, "Principles of Mining," 1909.

It is a fact that industrial enterprises because of additional risks demand greater interest returns than Government bonds and it seems reasonable that mining investments taken as a class should call for a greater rate of interest than industrial enterprises.

This claim for a higher rate of interest is opposed by such an authority on mine valuation as Mr. J. R. Finlay * who can be quoted as follows:

"I have generally assumed that 5 per cent was a normal interest — or discount rate. If that is so, it is a fair figure to use in a mine valuation, which should be nothing but a candid inquiry into the present value of expected profits."

If Mr. Finlay's statement is correctly understood, he is willing to ignore the risk that these prospective profits may be diminished or may entirely be cut off before the estimated life of the property has been accomplished. If it is true that at any mining property risks exist over and above the risks existing in so-called "safe" investments, then a 5 per cent discount or interest rate is not sufficient to induce sane investment. Mr. Finlay's 5 per cent interest rate, as applied to the iron mines of Michigan in his appraisal made for the State Tax Commission in 1911, was changed by that Commission to a 6 per cent basis in 1913.

Other authorities have gone on record as advocating higher interest rates and in this connection the following will be found of interest:

Mr. J. H. Curle † states that a suitable mining investment must fulfill the following requirements:

"1st. The development in the bottom must be good;
"2nd. The mine must pay 10 per cent per annum;
"3rd. There must be 60 per cent of the price of the shares in sight."

Mr. Hoover in his admirable work on mine valuation says: ‡

* J. R. Finlay, "Valuation of Iron Mines," Trans. A.I.M.E., Vol. 45, p. 295.
† J. H. Curle, "The Economist," London, Sept. 15, 1903.
‡ H. C. Hoover, "Principles of Mining," 1909.

"What rate of excess return the mine must yield is a matter of the risks in the venture and the demand of the investor. Mining business is one where 7 per cent above provision for capital return is an absolute minimum demanded by the risks inherent in mines, even where the profit in sight gives warranty to the return of capital."

Mr. G. A. Denny, an English engineer, says:

"A normal mining risk stated in terms of interest may be taken at 10 per cent per annum on the capital expended plus a rate for the redemption of capital."

John Hays Hammond † expressed his views on this question as follows:

"In many mines persistency of the ore deposits and, therefore, the reliability of the mines as dividend payers, justified the investment upon a basis in some instances as low as 8 per cent, dividends to which, of course, must be added a certain percentage to provide for the amortization of the capital. Generally speaking, however, investments in mining securities are not to be regarded as attractive unless they return from 10 per cent to 15 per cent in dividends, in addition to the profits to be set aside for amortization."

Price of the Mineral Product. — Because of wide fluctuations in the prices that minerals bring in the market, the use of the current market price of such a product in measuring the value of a mineral deposit is not proper for valuation purposes in general. No hardship would be forced on the owner or operator if, for taxation purposes, valuations were made annually based on the current price of the mineral produced. This method, however, results in a valuation fluctuating in amount from year to year and because of the constant changes required, does not seem to be practical.

The "normal" price of a mineral may be defined as the average market price over a certain definite period of years. Periods of time five or ten years in length are in general use. The

* G. A. Denny, Mexican Mining Journal, July, 1910.

† John Hays Hammond, Engineering and Mining Journal, Jan. 1, 1910.

"basic" price represents the point at which the mineral production falls off to such an extent that a rise in price results.

Mr. Hoover in his "Principles of Mining" states that safety lies somewhere between the "basic" and "normal" prices. No such limitation should be placed on the exercise of the judgment of the valuating engineer, as this assumption cannot hold good during the years following protracted periods of financial depression such as the hard times of 1893 to 1898. The "normal" price of many minerals estimated on a 10-year or 20-year average basis was for a considerable period of time following this depression less than the actual value of the mineral.

For illustrative purposes certain tables of statistics giving the production and average prices of four important metals (copper, lead, silver and zinc) from 1880 to 1914 are given at the close of this Chapter. Curves have been plotted which illustrate the relative increase in the production of these metals from year to year and also the changes in the average annual price. (Fig. 4 to Fig. 7.) For comparison normal curves of production and price (based on ten-year averages) have been plotted assuming that the average figure for production and price are applicable midway of the ten-year period. By continuing these normal curves to date on the assumption of a five-year future history for the metal, a present "normal" price can be obtained that will be as close an approximation as can be made.

Valuations for Purchase and Sale. — When a valuation of a mining property is needed because of a contemplated change in ownership, the most satisfactory method of valuation is the straightforward one involving the determination of the value of the ore body on the basis of its production past, present and expected.

Such a valuation includes an estimate of the probable life or expectancy of the property under the actual or an assumed rate of production, an estimate of the price that the mineral product will bring in the markets of the world and an estimate of the cost of producing the mineral. This method requires the use of all available information as to geology, actual operating costs, adjacent developed properties, recent sales of same, determination of the "normal" price of the mineral, recent sales of metal or of ore at the property, determination of the proper rate of interest to be used as a basis for capitalization, and the like.

The rate of production that is properly brought under consideration must be determined from consideration of the fact that the operating costs per ton decrease as the capacity increases, while the fixed charges increase when the cost of the installation is increased. It may be found that the rate of production actually in use is not that which brings out the full value of the property and an assumption of a proper rate should be made in determining the profits that can be expected. In the case of certain minerals having a limited market, the output is controlled by market conditions which must be considered in determining the rate of production.

Valuations for Owner's Information. — Valuations made for the purpose of informing the owner as to the condition of the property, although made in a similar way to those for purchase and sale purposes, may be limited by the special purpose for which they are required.

During the operation of a property it may be desirable to determine from time to time the present value of the property, taking into account the new underground developments, improved methods of ore treatment and changed market conditions. A desire on the part of the owner to ascertain on what basis he might, with advantage to himself, join in coöperative operations with adjacent properties, may lead to a valuation. This often occurs in the case of oil lands in small holdings that for reasons of economy can only market their product by cooperative means. The rental of coal lands and other deposits that contain large mineral bodies to other operators requires a determination of the "royalty" that should be charged and this should be based on an actual valuation and not on the average royalty in current use in the district although this value may be used for a check.

Valuation for Taxation Purposes. — Whether it is desirable from an economic standpoint to tax the ore bodies in the possession of mining companies is an open question that has been hotly debated by engineers and economists. Different methods of taxation have been suggested:

(1) Taxation based on surface improvements only.

(2) Taxation based on estimated value of the ore body and surface improvements.

(3) Taxation based on "foot-acre" or "royalty" value and on empirical methods of valuation.

(4) Taxation based on annual production or gross proceeds.

(5) Taxation based on annual profits or net proceeds.

Probably the earliest popular method of taxing mining properties was to tax only their surface improvements. It was not considered advisable to tax an industry that was of such economic value to a community. Also because of the risks of the business it was deemed necessary to make mining investments as attractive as possible to the capitalist and not to hamper operations, particularly in the early stages, when an enterprise was just getting on its feet. The history of the mining industry in this respect is similar to that of public utilities, which have anticipated and facilitated the growth of population in the territory served. Just so long as the operations were conducted at a risk the public showed no desire to interfere or to assume responsibility, but once the investments have become firmly established and are earning a reasonable interest on the capital investment regulation of the utility with its accompanying embarrassments has been commenced and the practice of allowing the company a return on its depreciated "fair" value and not on its investment has sprung up.

In the case of the mining properties it became apparent that the large coal, iron and copper operations after a successful struggle for existence had become profitable and represented a very large portion of the business of certain communities. Taxation above a mere tax on surface improvements was proposed and has become an established practice in many states.

Valuation of Mines for Taxation in Various Localities

In a general way some methods of valuing mining property for taxation purposes will now be outlined.

Alabama, California, Iowa, New Jersey, New York, Oregon, Washington, and West Virginia appraise mining property in a similar manner to real estate and other property. They have no laws pertaining to the valuation of mining property. This method requires a perfunctory valuation of the surface improvements and of the ore body, the latter in some cases being entirely ignored.

Michigan, Minnesota and Wisconsin assess their mining properties on a basis of the estimated value of the ore body and the surface improvements giving weight to all the elements that enter into value.

In Michigan the present system was introduced by Mr. J. R. Finlay in 1911. Data are compiled from the reports of mine operators in sworn statements. Then an inspection is made by the State Geologist and his assistants to estimate the reserves and check up other data. The State Tax Commission passes on the figures and after a public hearing the final results are reported to the local assessors. The valuation is based on four elements — ore reserves (developed and prospective), average annual profits (five-year average), life or expectancy, and the rate of interest. The developed reserves consist of ore blocked out above the bottom levels. Prospective reserves consist of the expected extension of the ore bodies downward or laterally beyond existing workings. The average annual profits are estimated from a five-year record when such exists. Undeveloped properties are judged by the operations of other mines. The expectancy of a property is taken at the ratio of the estimated reserves to an average five-year shipment. The life of the undeveloped mines is estimated by comparison with adjacent operations. The interest rate is now 6 per cent for both principal and sinking fund. It was 5 per cent and 4 per cent respectively in 1911 when Mr. Finlay had charge. The criticism of this system is that the values of the ore bodies cannot be accurately determined, that by using annual profits as a basis a premium is placed on inefficient operations, that the life cannot be determined with any degree of certainty and that the rate of interest is too low, making the present estimate of value too high. It has several times been proposed to introduce a tonnage tax in Michigan. Recently the State Grange proposed to invoke the Initiative in order to obtain a tax bill proposing a levy of ½ cent per pound on copper and 10 cents per ton on iron ore. This method of taxing would penalize the concern that was mining with only a small margin of profit and would tend to unnecessarily hamper the mining industry in the State.

Minnesota has the engineers of the State Tax Commission measure up the tonnage of ore underground and then an ad valorem tax on 50 per cent of the present indicated value is levied. Mineral lands that are not productive are taxed. A tax of 4 to 4½ cents per ton has been proposed on stock piles.

In Wisconsin the 1913 Legislature passed a law assessing mining properties or mineral lands upon the estimated value of the ore underground. A report based on production profits, etc., was made by W. L. Uglow, engineer for the State Tax Commission, but his recommendations were greatly altered by the Commission. It was found necessary to reduce the figures for developed properties and to entirely cut out the estimates on undeveloped properties. The system he adopted might be called a semi-empirical method which is based on figures from a theoretical zinc mine. His theoretical or "hypothetical" mine was the result of combining the figures from several actual operating properties. The assumed life was taken as four years which is the average as determined from the previous experience of the district. Mr. Uglow's method of ascertaining the value of a mine as applied consisted of multiplying the annual profit by the factor 2.43 which is the figure obtained by using the "hypothetical" mine and assuming that profits do not fluctuate from year to year. He claims as advantages that his method is based on actual profit and average life and that it bases the assessment on a multiple of the profits rather than on the market value of the mine.

Pennsylvania taxes its coal lands under the acts of 1841 and 1842 which require the taxation of "every subject of taxation at the actual value thereof." The foot-acre method of assessment was in general use until 1907 when a big jump in value was made. Appeal to the state courts resulted in decisions which declared the "foot-acre" and "royalty" methods unsatisfactory and dependence is now placed mainly on recent sales in the vicinity to determine the market value of a property. Although fairly satisfactory for real estate appraisals this method cannot be applied with equal success to lands whose coal deposits, of indeterminate extent, in many instances give them value. Further than this recent sales in the vicinity may be scarce and the resulting standard of value because of this fact very unsatisfactory.

Arizona and Colorado tax mining properties on the basis of production.

Arizona divides mining properties into two general classes, productive and non-productive. Productive properties include those which yield net proceeds during the year, after deducting cost of active operation, cost of transportation, cost of marketing, smelting and refining, and expenses for betterments and repairs. The return of the capital cannot be considered as part of the cost nor can the salaries of those not continuously engaged in the enterprise within the state be included as an element of expense. The assessed value is arbitrarily taken as one-eighth of the gross value plus 4 times the net value of the annual output plus the value of the surface improvements. Non-producing mines are taxed in the same way as other real property in the State.

Colorado determines the net proceeds by deducting the actual cost of extraction, after eliminating unnecessary expenditures and the actual cost of transportation to place of sale, from the gross value of the annual production (the sum actually obtained by the owner). In 1915, the assessed value was fixed at 25 per cent of the gross annual proceeds or the net annual proceeds whenever this exceeds 25 per cent of the gross proceeds. Improvements are assessed at the same rate as other property in the vicinity is assessed. It is claimed in Colorado that this law permits the owner of a large idle group of claims to pay no greater tax than a prospector actually working a single claim.

Idaho, Montana, Nevada, New Mexico and Utah tax the net proceeds of mining properties.

In Idaho the net proceeds are used as a basis for the assessment. Improvements are assessed at full cash value. The ground of undeveloped claims is taxed on the prices paid to the U. S. Government when they were purchased.

Montana bases its mine tax on the net proceeds. The surface is assessed at the price paid to the U. S. Government ($5 per acre for metal mines and $2 per acre for coal). If the surface is more valuable for other purposes, it is assessed at the market value of such property. Surface improvements are taxed at market or local value.

Nevada assesses all patented claims at $500 unless it can be proved that over $100 worth of work has been done during the year. Improvements are assessed at local value. A tax is levied on the net proceeds of all operating mines. By deducting such charges for operation, transportation and reduction or sale of output as the State Tax Commission permits, from gross value of the production, the net proceeds are estimated. No deduction for office expenses other than those of operating offices are permitted. The salaries of mine officials are limited. The investment in surface plant is not depreciated nor is the redemption of any investment in mine ground or prior development permitted. By an agreement between the Tax Commission and operators, future levies were to be made only on 60 per cent of the net profits or the same percentage which is applied in assessment of other property. During 1915, this percentage has been raised to 70 per cent by the Tax Commission.

New Mexico has placed its mine taxation in the hands of a

State Tax Commission. The owners or lessees of mines are ordered to furnish annual statements supplying the Commission with the necessary data to determine the net proceeds. Surface improvements are taxed as other like property. Unproductive patented mining claims are assessed and taxed on a reasonable valuation as undeveloped mineral lands in addition to their surface value whatever it may be. The law for the taxation of mining property was drawn up by the attorney of one of the large mining companies in New Mexico and is believed to be satisfactory to the mining interests.

Utah has a system for assessing on the basis of the net proceeds but the State Board of Equalization does not allow deduction for interest, taxes, insurance, legal expenses, etc., so the mines are really being taxed on more than their net proceeds.

Mexico prior to the Carranza regime placed an annual surface tax of 6 pesos per pertenencia (100 meters square) on all claims of less than 25 pertenencias in size. On each pertenencia in excess of 25 the annual charge is only 3 pesos. A state or federal tax on output could be levied not to exceed $1\frac{1}{2}$ per cent of the assay value. The tax on reduction plants was 0.6 per cent of the valuation but other improvements were not taxed. When ore was exported unrefined it was taxed $3\frac{1}{2}$ per cent of the assay value but when it was refined in Mexico the export tax was only $2\frac{1}{2}$ per cent of the assay value.

Since the de facto government has been controlled by Carranza, an annual tax has been established at the rate of \$6 per hectare on a property of not over 10 hectares, \$7.50 per hectare on the acreage in excess of 10 hectares and below 20 hectares; \$9 per hectare on that from 20 to 50 hectares; and \$12 per hectare on all above 50 hectares.

Recent developments in Mexico are such that it cannot be stated with definiteness what is the method of mine taxation.

German South-West Africa. — British consular reports state that the mines of German South-West Africa were taxed by an empirical method. The tax amounted to the difference between 66 per cent of the gross value of the output and 70 per cent of the cost of operating. This amounts to no tax when the profits are 6 per cent or less, to 10 per cent tax on gross production when the profits are 20 per cent of the gross, to 31 per cent when the profits are 50 per cent and so on up to the limit of 66 per cent of the gross production when the operating expenses approach zero.

Propriety of Taxing Mining Property. — The question of the propriety of taxing mining properties is an important one. In states such as Michigan and Minnesota where mines make up a large proportion of the industry of the state it is necessary that mining properties bear a large share in the maintenance of the government which exists largely for their protection. In the states just mentioned a closer approximation of the value of the ore underground is possible than in states where the mining interests are largely concerned with precious metals. This should not be construed to mean that the methods of assessment now in vogue in the former should be considered the proper ones.

There is no doubt that taxation of mines in any form must retard to some extent the development of the mining industry which is of such economic value to the country. It also must be granted that in certain communities the mining interests are so large that it is only proper that they pay their share of the governmental burden. Injustice can be avoided only by the exercise of caution when mineral lands are to be taxed. That taxation may run riot has been shown in Minnesota in the case of mines located within the boundaries of municipalities.

In Buhl, Minnesota, the 1913 municipal tax levy was $125,000 or $125 per inhabitant. The town had 172 voters and had over 100 persons on the pay roll during the year. Only $1\frac{1}{3}$ per cent of the assessment was paid by the inhabitants and $98\frac{2}{3}$ per cent by the Mining Companies. In Keewatin the levy amounted to $374.10 per inhabitant. The township of Stutz in 1914 spent more than $900 for every voter. By court order some of these levies have been modified but they show what extremes are possible. It is understood that the per capita expenditures of municipalities have now been limited to \$25 per inhabitant by the State Legislature.

The general principles of mine taxation are not comparable with those controlling the taxation of unimproved real estate which is held idle waiting for the unearned increment. Mining properties may be allowed to remain idle because there is no present market for their product at a price that will permit a profit but they cannot be held for any length of time as an investment waiting for a chance to make a large turn because under ordinary circumstances no important permanent increase in value is to be expected. When large increases in the price of minerals take place, new substances at more reasonable prices are apt to be supplied to do the same work. Water power takes the place of mineral fuels. Oil takes the place of coal in certain localities where coal is high and so on.

Improvements in the methods of production as well as increased production have resulted in decreased prices of the output rather than in increased returns. Furthermore as compared with farming and other manufacturing industries mining is not as attractive a field as it formerly was and in another decade will probably not be as attractive as it is at the present time.

Mr. T. A. Rickard, in a recent paper before the International Engineering Congress, points out that a mine is not an investment but is a speculation. He writes:

> "The act of mining cannot be applied on scientific principles until two basic ideas are fully comprehended: (1) A mine is a wasting asset and (2) mining is a speculative business. To treat a mine as an investment, and to appraise it on that basis is to ignore the cumulative facts of to-day and of other days. Mining is a speculation that can be wise or foolish according as a man recognizes the inherent risk and takes his chances accordingly."

Taxation of certain deposits such as coal-beds, oil fields, etc., on the basis of the present value of future production tends to increase the current output and might easily lead to the destructive working of such deposits if vigorously enforced, *e.g.*, the abandonment of thin beds of coal or the failure to take care of the water in the oil well. The same is true but probably not to the same extent of other minerals.

The unfair taxation of precious metals will result in the hampering of the organization and the financing of promising prospects or the shut down of properties that are running very close to the margin of no profits.

The following quotation from Mr. T. C. Bonney * emphasizes the difference between real estate and mineral lands:

"The mineral store of each district and of the whole earth is practically limited in quantity, be it gold, or any other metal, be it coal or any fuel. The formation of a fresh supply is a process so slow that, for all practical purposes, it may be excluded from consideration. . . . Hence the store, sooner or later, must be exhausted, now in this country, now in that. In agriculture, provided manures can be obtained, the land seems never to lose its productive power. The mine or quarry, once worked out, has played its part for good in the economy of the earth."

Conclusion Relating to Taxation of Mines. — In conclusion, it may be stated that the fairest way to tax mining properties of any class would be on their annual profits, modified by the regulating body in the case of wasteful or careless operations, capitalized at a rate of interest commensurate with the average mining risk for that class of property. By repeating these valuations at frequent intervals no hardship will be thrown on the property whose profits are on the downward grade. When it is found impossible to estimate the life of the property involved with definiteness as is usually the case in the mining of precious metals, empirical or semi-empirical methods of valuation must be adopted. Such method as establishing a taxable value equal to a percentage of or a multiple of one year's net or gross proceeds would probably be preferred to the use of an estimated market value by the mining companies. The percentage or multiple used should be fixed so as to distribute the taxes in the proper proportion between mine owners and other taxpayers. The effort to tax non-productive properties cannot be endorsed

* T. G. Bonney, D.Sc., F.R.S., Dictionary of Political Economy, 1910, p. 768.

except in cases where it can be clearly demonstrated that deposits economically available do exist.

Methods of Valuing Oil Properties

In the *Appalachian oil fields*, oil properties have been valued on the basis of the daily production of the wells measured after the output has reached a settled stage. The valuation is so fixed as to permit the purchaser to recover his capital in 5 to 10 years and to obtain a reasonable return on his investment. It is stated that the average sale price of producing lands varied from $800 to $1000 per barrel-day a few years ago.

In *Illinois*, many properties changed hands during the early stages of the operations at a rate that would insure the return or amortization of capital within 18 months. This was before the life of the wells had been given a practical test.

Two large valuations of oil properties have been made in *California* since 1900 and the methods that were adopted are of particular interest to the valuation engineer.

In 1910 the Kern River oil-field was valued because of the intended consolidation of several separate holdings within the field. Because of the object of the investigation only values bearing comparison with one another were required. The values of all the properties involved were expressed in terms of the value of a certain selected tract of oil land located in the approximate center of the field. Concentric circles drawn about the center of the field were used to express the relative value of individual tracts as affected by their distance from the center of greatest productivity. A measurement of the productivity of all the producing wells was made covering a month's time, thus determining the production at that time and experiments were conducted on samples of the oil-bearing strata to determine the amount of oil that could be recovered from a known volume. The results were used as a measure of the extractable oil which could be pumped from the oil strata, the volume and area of which could be estimated. A safety factor was introduced by assuming that only one-half of the oil measures would be productive. Subsequent experience indicates that this safety factor was not sufficient.

A method of oil land valuation, which has been applied more recently, is that adopted by the engineers in valuing the properties which market their product through the Independent Oil Producers Agency, which valuation had in view the merging of a number of companies in one large organization or corporation. The method used was based on the determination of a production curve for each property. A study of the history of producing wells has given basis for the assumption that the wells in a field constantly decrease in production at a rate the changes of which can be plotted in curves and, using these curves as a basis, old wells, new wells and wells not yet drilled can all be taken into account in estimating the future production of the oil lands. The total operating expenses have been estimated from existing records and the prospective future net receipts have been discounted in obtaining the present value of the properties. Amortization of all capital except the salvage of surface improvements has been assumed to take place in ten years. "History" is also prerequisite to this method and attention should be called to the fact that, because of the inaccessibility of the reservoirs of oil, the more extensive the record of previous operations, the more satisfactory will be the valuation no matter what method may be adopted.

PRODUCTION AND PRICE OF COPPER, SILVER, LEAD AND ZINC IN THE UNITED STATES

Plates and Tables

Accompanying Chapter on Valuation of Mines and Oil Properties

Figures 4 to 7, and Tables 15 to 18 which directly follow this page deal only with four of the principal metals, copper, silver, lead and zinc. The statistics given are based on the best available data and the figures are believed to be sufficiently accurate to justify their use by valuating engineers who have in hand problems in which these metals have a part.

The figures submitted for the United States prices and production are those of the United States Geological Survey.

Other available sources were found to be less complete and no discrepancies sufficiently large to question the reliability of these figures could be located. The world production of these metals has been taken from the annual reports of the U. S. Geological Survey supplemented by the annual statements of the Director of the U. S. Mint, Henry R. Merton & Co. (Limited) of London, and the Metallgesellschaft and Metallurgische Gesellschaft, A.-G., Frankfort-am-Main, Germany.

TABLE 15. COPPER IN THE UNITED STATE

ANNUAL PRODUCTION

(Based on smelter returns and prices at New York City)

(1880–1914)

Year.	Production.		Per cent of world production.	Average annual price at N. Y.	Price received for Lake copper.
	Pounds.	Total value.			
1880	60,480,000	$ 11,491,000	17.4	$0.190	$0.195
1881	71,680,000	12,176,000	19.5	0.170	0.180
1882	91,646,000	16,038,000	22.7	0.175	0.180
1883	117,152,000	18,065,000	26.1	0.154	0.154
1884	145,222,000	17,790,000	30.0	0.122	0.131
1885	170,963,000	18,293,000	34.1	0.107	0.110
1886	161,235,000	16,528,000	33.4	0.102	0.108
1887	185,227,000	21,116,000	36.9	0.114	0.120
1888	231,271,000	33,834,000	39.7	0.146	0.152
1889	231,246,000	26,908,000	40.1	0.116	0.120
1890	265,115,000	30,849,000	44.0	0.116	0.150
1891	295,812,000	38,455,000	47.2	0.130	0.125
1892	352,972,000	37,977,000	50.2	0.107	0.115
1893	339,786,000	32,055,000	50.1	0.094	0.105
1894	364,867,000	33,141,000	51.6	0.091	....
1895	385,913,000	38,012,000	52.3	0.098	....
1896	460,061,000	49,457,000	54.4	0.107	....
1897	494,078,000	54,080,000	57.2	0.109	....
1898	526,513,000	61,865,000	54.7	0.117	....
1899	568,667,000	101,223,000	54.8	0.178	....
1900	606,117,000	98,494,000	55.6	0.162	0.166
1901	602,073,000	87,301,000	51.8	0.145	0.150
1902	659,509,000	76,569,000	53.5	0.116	0.119
1903	698,045,000	91,506,000	53.2	0.131	0.133
1904	812,537,000	105,630,000	55.8	0.130	0.132
1905	888,784,000	137,762,000	57.5	0.155	0.156
1906	917,806,000	177,596,000	57.5	0.193	0.191
1907	868,996,000	173,799,000	54.7	0.200	0.184
1908	942,571,000	124,419,000	56.0	0.132	0.134
1909	1,092,952,000	142,084,000	58.4	0.130	0.133
1910	1,080,160,000	137,180,000	56.8	0.127	0.130
1911	1,097,233,000	137,154,000	56.0	0.125	0.128
1912	1,243,269,000	205,139,000	55.0	0.165	0.164
1913	1,224,484,000	189,795,000	55.7	0.155	0.154
1914	1,150,137,000	152,968,000	....	0.133	0.136

TABLE 15. COPPER IN THE UNITED STATES (*Continued*)

NORMAL PRODUCTION AND PRICE

(Normals for ten-year periods)

(1880–1914)

Ten-year periods (inclusive).	Production.		Normal production, pounds.	Normal price (ten-year average).
	Pounds.	Value.		
1880–1889	1,466,122,000	$ 192,239,000	146,612,200	$0.131
1881–1890	1,670,757,000	211,597,000	167,075,700	0.127
1882–1891	1,894,889,000	237,876,000	189,488,900	0.126
1883–1892	2,156,215,000	259,815,000	215,621,500	0.121
1884–1893	2,378,849,000	273,805,000	237,884,900	0.115
1885–1894	2,598,494,000	289,156,000	259,849,400	0.111
1886–1895	2,813,444,000	308,875,000	281,344,400	0.110
1887–1896	3,112,270,000	341,804,000	311,227,000	0.110
1888–1897	3,421,121,000	374,768,000	342,112,100	0.110
1889–1898	3,716,363,000	402,799,000	371,636,300	0.108
1890–1899	4,053,784,000	477,114,000	405,378,400	0.118
1891–1900	4,394,786,000	544,759,000	439,478,600	0.124
1892–1901	4,701,047,000	593,605,000	470,104,700	0.126
1893–1902	5,007,584,000	632,197,000	500,758,400	0.126
1894–1903	5,365,843,000	691,648,000	536,584,300	0.129
1895–1904	5,813,513,000	764,137,000	581,351,300	0.131
1896–1905	6,316,384,000	863,887,000	631,638,400	0.137
1897–1906	6,774,129,000	992,026,000	677,412,900	0.146
1898–1907	7,149,047,000	1,111,745,000	714,904,700	0.155
1899–1908	7,565,105,000	1,174,299,000	756,510,500	0.155
1900–1909	8,089,390,000	1,215,160,000	808,939,000	0.150
1901–1910	8,563,433,000	1,253,846,000	856,343,300	0.146
1902–1911	9,058,593,000	1,303,699,000	905,859,300	0.144
1903–1912	9,642,353,000	1,432,269,000	964,235,300	0.149
1904–1913	10,168,792,000	1,530,558,000	1,016,879,200	0.151
1905–1914	10,506,392,000	1,577,896,000	1,050,639,200	0.149

TABLE 16. SILVER IN THE UNITED STATES

ANNUAL PRODUCTION

(1880–1914)

Year.	Annual production.		Per cent world production.	Average commercial value per ounce.
	Troy ounces.	Total value.		
1880	30,318,700	$34,717,000	40.6	$1.15
1881	32,257,800	37,657,500	40.8	1.13
1882	36,196,900	41,105,900	41.9	1.14
1883	35,732,800	39,618,400	40.1	1.11
1884	37,743,800	41,921,300	46.3	1.11
1885	39,909,400	42,503,500	43.6	1.07
1886	39,694,000	39,482,400	42.5	0.99
1887	41,721,600	40,887,200	42.9	0.98
1888	45,792,700	43,045,100	42.2	0.94
1889	50,094,500	46,838,400	41.7	0.94
1890	54,516,300	57,242,100	43.2	1.05
1891	58,330,000	57,630,000	42.5	0.99
1892	63,500,000	55,662,500	41.5	0.87
1893	60,000,000	46,800,000	36.4	0.78
1894	49,500,000	31,422,100	30.1	0.63
1895	55,727,000	36,445,500	33.2	0.65
1896	58,834,800	39,654,600	37.5	0.68
1897	53,860,000	32,316,000	33.6	0.60
1898	54,438,000	32,118,400	32.2	0.59
1899	54,764,500	32,858,700	32.5	0.60
1900	57,647,000	35,741,100	33.2	0.62
1901	55,214,000	33,128,400	31.9	0.60
1902	55,500,000	29,415,000	34.1	0.53
1903	54,300,000	29,322,000	32.4	0.54
1904	57,682,800	33,456,000	35.1	0.58
1905	56,101,600	34,222,000	32.6	0.61
1906	56,517,900	38,256,400	34.2	0.68
1907	56,514,700	37,299,700	30.7	0.66
1908	52,440,800	28,050,600	25.8	0.53
1909	54,721,500	28,455,200	25.8	0.52
1910	57,137,900	30,854,500	25.8	0.54
1911	60,399,400	32,615,700	26.7	0.53
1912	63,766,800	39,197,500	28.4	0.615
1913	66,801,500	40,348,100	....	0.604
1914	72,455,100	40,067,700	....	0.553

TABLE 16. SILVER IN THE UNITED STATES (*Continued*)

NORMAL PRODUCTION AND PRICE

(Normals Based on ten-year periods)

(1880–1914)

Ten-year periods (inclusive).	Production.		Normal production, pounds.	Normal price (ten-year average).
	Pounds.	Value.		
1880–1889	389,462,200	$407,776,700	38,946,220	$1.045
1881–1890	413,659,800	430,301,800	41,365,980	1.04
1882–1891	439,732,000	450,274,300	43,973,200	1.025
1883–1892	467,035,100	464,830,900	46,703,510	0.995
1884–1893	491,302,300	472,012,500	49,130,230	0.96
1885–1894	503,058,500	461,513,300	50,305,850	0.915
1886–1895	518,876,100	455,455,300	51,887,610	0.88
1887–1896	538,016,900	455,627,500	53,801,690	0.845
1888–1897	550,155,300	447,056,300	55,015,530	0.815
1889–1898	558,800,600	436,129,600	55,880,060	0.78
1890–1899	563,470,600	422,149,900	56,347,060	0.75
1891–1900	566,601,300	400,648,900	56,660,130	0.705
1892–1901	563,485,300	376,147,300	56,348,530	0.67
1893–1902	555,485,300	349,899,800	55,548,530	0.63
1894–1903	549,785,300	332,421,800	54,978,530	0.605
1895–1904	557,968,100	334,455,700	55,796,810	0.60
1896–1905	558,342,700	332,232,200	55,834,270	0.595
1897–1906	556,025,800	330,834,000	55,602,580	0.595
1898–1907	558,680,500	335,817,700	55,868,050	0.60
1899–1908	556,683,300	331,749,900	55,668,330	0.595
1900–1909	556,640,300	327,346,400	55,664,030	0.59
1901–1910	556,131,200	322,459,800	55,613,120	0.58
1902–1911	561,316,600	321,947,100	56,131,660	0.575
1903–1912	569,583,400	331,729,600	56,958,340	0.58
1904–1913	582,084,900	342,755,700	58,208,490	0.59
1905–1914	596,857,200	349,367,400	59,685,700	0.585

TABLE 17. LEAD IN THE UNITED STATES

ANNUAL PRODUCTION

(Based on smelter returns and prices at New York City)

(1880–1914)

Year.	Annual production.		Per cent world production.	Average annual price at N. Y.
	Pounds.	Total value.		
1880	195,650,000	$ 9,782,500	21.7	$0.050
1881	234,170,000	11,240,160	24.4	0.048
1882	265,780,000	12,624,550	26.2	0.049
1883	287,914,000	12,322,719	29.3	0.043
1884	279,794,000	10,537,042	29.1	0.037
1885	258,824,000	10,469,431	26.4	0.039
1886	261,258,000	12,200,749	24.3	0.046
1887	291,400,000	13,113,000	24.9	0.045
1888	302,838,000	13,399,256	24.8	0.044
1889	312,794,000	13,794,235	26.0	0.039
1890	287,260,000	12,668,166	21.6	0.045
1891	357,108,000	15,534,198	26.4	0.043
1892	347,308,000	13,892,320	22.7	0.040
1893	327,964,000	11,839,590	23.3	0.037
1894	318,662,000	9,942,254	23.4	0.033
1895	340,000,000	11,220,000	23.8	0.032
1896	376,000,000	10,528,000	25.3	0.030
1897	424,000,000	14,885,728	26.8	0.036
1898	444,000,000	16,650,000	25.9	0.038
1899	421,000,000	18,945,000	23.3	0.045
1900	541,648,000	23,561,688	29.0	0.044
1901	541,400,000	23,280,200	27.6	0.043
1902	540,000,000	22,140,000	27.3	0.041
1903	564,000,000	23,520,000	28.0	0.042
1904	614,000,000	26,402,000	29.4	0.043
1905	604,000,000	28,690,000	29.0	0.047
1906	700,306,000	39,917,442	33.7	0.057
1907	750,198,000	39,760,494	35.0	0.053
1908	621,524,000	26,104,008	27.0	0.042
1909	726,638,000	31,245,434	32.6	0.043
1910	778,422,000	34,250,568	32.2	0.044
1911	811,726,000	36,527,670	33.1	0.045
1912	830,790,000	37,385,550	32.4	0.045
1913	872,860,000	38,405,840	34.4	0.044
1914	1,025,588,000	39,997,932	....	0.039

TABLE 17. LEAD IN THE UNITED STATES (*Continued*)

NORMAL PRODUCTION AND PRICE

(Normals for ten-year periods)

(1880–1914)

Ten-year periods (inclusive).	Production.		Normal production, pounds.	Normal price (ten-year average).
	Pounds.	Value.		
1880–1889	2,690,602,000	$119,483,500	269,060,200	$0.044
1881–1890	2,782,212,000	122,369,200	278,221,200	0.044
1882–1891	2,905,150,000	126,663,200	290,515,000	0.044
1883–1892	2,986,678,000	127,931,000	298,667,800	0.043
1884–1893	3,026,728,000	127,447,900	302,672,800	0.042
1885–1894	3,065,596,000	126,853,200	306,559,600	0.041
1886–1895	3,146,772,000	127,603,800	314,677,200	0.041
1887–1896	3,261,514,000	126,003,100	326,151,400	0.039
1888–1897	3,394,114,000	127,703,800	339,411,400	0.038
1889–1898	3,535,276,000	130,954,500	353,527,600	0.037
1890–1899	3,643,302,000	136,105,300	364,330,200	0.037
1891–1900	3,897,690,000	146,998,800	389,769,000	0.038
1892–1901	4,081,982,000	154,744,800	408,098,200	0.038
1893–1902	4,274,674,000	162,992,500	427,467,400	0.038
1894–1903	4,510,710,000	174,572,900	451,071,000	0.039
1895–1904	4,806,048,000	191,132,600	480,604,800	0.040
1896–1905	5,060,048,000	208,602,600	506,004,800	0.041
1897–1906	5,394,354,000	237,992,000	539,435,400	0.044
1898–1907	5,720,552,000	262,866,800	572,055,200	0.046
1899–1908	5,898,076,000	272,320,800	589,807,600	0.046
1900–1909	6,203,714,000	284,621,200	620,371,400	0.046
1901–1910	6,440,488,000	295,310,100	644,048,800	0.046
1902–1911	6,710,814,000	308,557,600	671,081,400	0.046
1903–1912	7,001,604,000	323,803,200	700,160,400	0.046
1904–1913	7,310,464,000	338,689,000	731,046,400	0.046
1905–1914	7,722,052,000	352,284,938	772,205,200	0.046

TABLE 18. ZINC IN THE UNITED STATES

Annual Production

(Based on smelter returns and prices at St. Louis)

(1880–1914)

Year.	Annual production.		Per cent world production.	Average annual price at N. Y.
	Pounds.	Total value.		
1880	46,478,000	$ 2,277,432	9.1	$0.055
1881	53,600,000	2,680,000	9.3	0.052
1882	67,530,000	3,646,620	11.1	0.053
1883	73,744,000	3,311,106	11.2	0.045
1884	77,088,000	3,422,707	11.8	0.044
1885	81,376,000	3,539,856	12.4	0.043
1886	85,282,000	3,752,408	12.7	0.044
1887	100,680,000	4,782,300	14.1	0.046
1888	111,806,000	5,500,855	15.3	0.049
1889	117,720,000	5,791,824	15.7	0.05
1890	127,366,000	6,266,407	16.4	0.055
1891	161,746,000	8,033,700	19.9	0.05
1892	174,520,000	8,027,920	20.8	0.046
1893	157,664,000	6,306,560	18.7	0.04
1894	150,656,000	5,288,026	17.8	0.035
1895	179,372,000	6,278,020	19.4	0.036
1896	162,998,000	6,519,920	17.4	0.039
1897	199,960,000	8,498,300	20.4	0.041
1898	230,798,000	10,385,910	22.3	0.046
1899	258,102,000	14,840,865	23.9	0.058
1900	247,772,000	10,654,196	23.5	0.044
1901	281,644,000	11,265,760	25.0	0.041
1902	313,854,000	14,625,596	26.0	0.048
1903	318,438,000	16,717,995	25.2	0.054
1904	373,404,000	18,670,200	26.9	0.051
1905	407,698,000	24,054,182	28.0	0.059
1906	399,388,000	24,362,668	25.7	0.061
1907	447,490,000	26,401,910	27.5	0.059
1908	381,498,000	17,930,406	23.9	0.047
1909	460,450,000	24,864,300	27.0	0.054
1910	504,958,000	27,267,732	27.6	0.054
1911	543,242,000	30,964,794	27.6	0.057
1912	647,814,000	44,699,166	30.3	0.069
1913	674,504,000	37,772,224	30.8	0.056
1914	686,836,000	35,028,636	....	0.051

TABLE 18. ZINC IN THE UNITED STATES (*Continued*)

NORMAL PRODUCTION AND PRICE

(Normals for ten-year period)

(1880–1914)

Ten-year periods (inclusive).	Production.		Normal production.	Normal price (ten-year average).
	Pounds.	Value.		
1880–1889	815,304,000	$ 38,795,100	81,530,400	$0.047
1881–1890	896,192,000	42,694,100	89,619,200	0.048
1882–1891	1,004,338,000	48,047,800	100,433,800	0.048
1883–1892	1,111,328,000	52,428,100	111,132,800	0.047
1884–1893	1,195,248,000	55,424,600	119,524,800	0.046
1885–1894	1,268,816,000	57,289,900	126,881,600	0.045
1886–1895	1,366,812,000	60,028,000	136,681,200	0.044
1887–1896	1,444,528,000	62,795,500	144,452,800	0.043
1888–1897	1,543,808,000	66,511,500	154,380,800	0.043
1889–1898	1,662,800,000	71,396,500	166,280,000	0.043
1890–1899	1,803,182,000	80,445,600	180,318,200	0.045
1891–1900	1,923,588,000	84,833,400	192,358,800	0.044
1892–1901	2,043,486,000	88,065,500	204,348,600	0.043
1893–1902	2,182,820,000	94,663,200	218,282,000	0.043
1894–1903	2,343,594,000	105,084,600	234,359,400	0.045
1895–1904	2,566,342,000	118,456,800	256,634,200	0.046
1896–1905	2,794,668,000	136,233,000	279,466,800	0.049
1897–1906	3,031,058,000	154,075,800	303,105,800	0.051
1898–1907	3,278,588,000	171,979,400	327,858,800	0.0525
1899–1908	3,429,288,000	179,523,900	342,928,800	0.052
1900–1909	3,631,636,000	189,547,300	363,163,600	0.052
1901–1910	3,888,822,000	206,160,800	388,882,200	0.053
1902–1911	4,150,420,000	225,859,800	415,042,000	0.054
1903–1912	4,484,380,000	255,933,400	448,438,000	0.057
1904–1913	4,840,446,000	276,987,600	484,044,600	0.057
1905–1914	5,153,878,000	293,346,000	513,387,800	0.057

COPPER 1880-1914

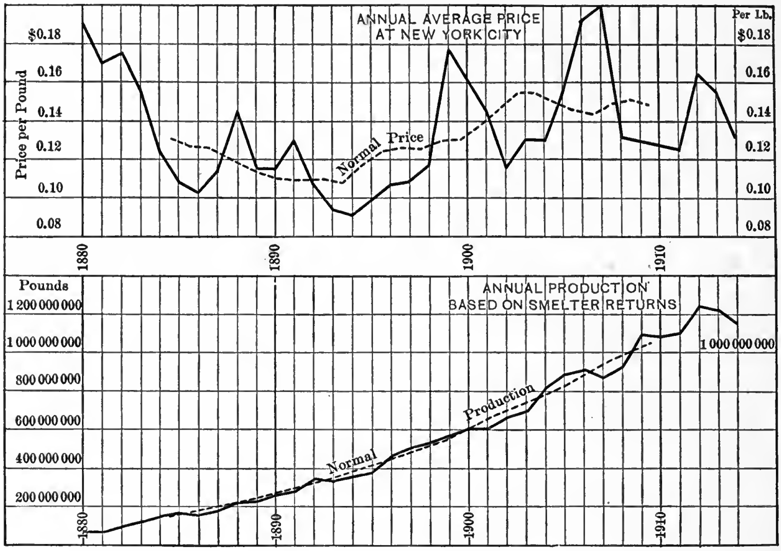


FIG. 4.

SILVER 1880-1914

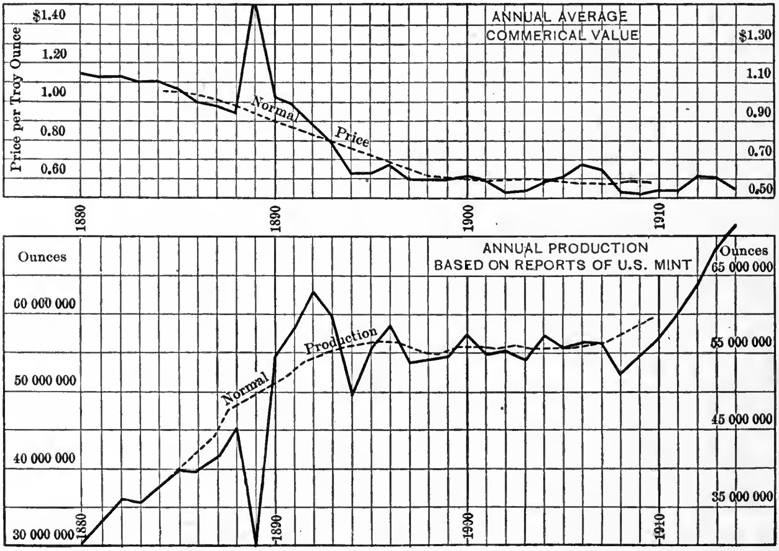


FIG. 5.

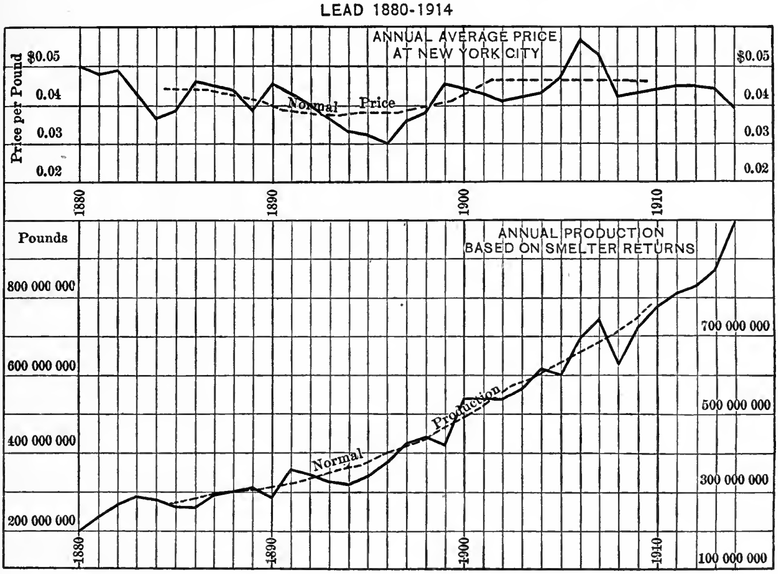

LEAD 1880-1914
ANNUAL AVERAGE PRICE AT NEW YORK CITY
Price per Pound
$0.05
0.04
0.03
0.02
Normal Price
1880
1890
1900
1910
Pounds
ANNUAL PRODUCTION BASED ON SMELTER RETURNS
800 000 000
600 000 000
400 000 000
200 000 000
700 000 000
500 000 000
300 000 000
100 000 000
Normal Production

FIG. 6.

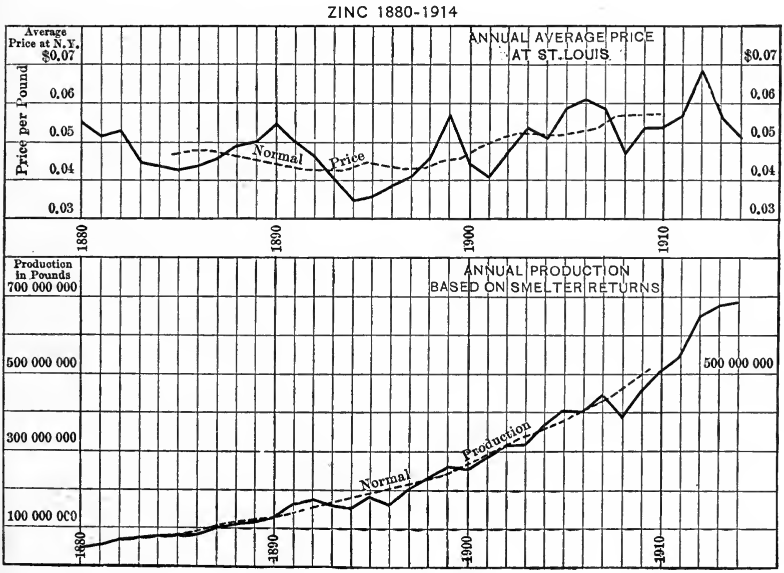

ZINC 1880-1914
Average Price at N.Y.
ANNUAL AVERAGE PRICE AT ST. LOUIS
Price per Pound
$0.07
0.06
0.05
0.04
0.03
Normal Price
1880
1890
1900
1910
Production in Pounds
700 000 000
500 000 000
300 000 000
100 000 000
ANNUAL PRODUCTION BASED ON SMELTER RETURNS
Normal Production

FIG. 7.

BIBLIOGRAPHY

Valuation of Mines and Oil Properties

"The Valuation of Metal Mines," by T. A. Rickard, International Engineering Congress, Oct., 1915.

"Principles of Mine Valuation," by J. R. Finlay, Columbia School of Mines Quarterly, Jan., 1913.

"The Valuation of Mines," by J. B. Kendall, Mining Magazine, Nov. 21, 1914.

"Valuation of Mining Shares," by Newton B. Knox, Mining and Scientific Press, Vol. 96, p. 773.

"Appraisal of Mining Properties of Michigan," State Board of Tax Commissioners, 1911.

"Study of Method of Mine Valuation and Assessment," Wisconsin Geological and Natural History Survey, 1914.

"Principles of Mine Taxation," by E. B. Kirby, Engineering and Mining Journal, Oct. 28, 1911.

"Mine Taxation," by Heath Steele, Engineering and Mining Journal, Aug. 29, 1914.

"Initial Report of Nevada Tax Commission, 1913–1914," Nevada Tax Commission.

"Assessment of Mines, 1915," Nevada Tax Commission.

"Suggestions Regarding Mining Investments," John Hays Hammond (address before Financial Forum, N. Y.), Engineering and Mining Journal, Vol. 89, p. 8.

"Safety of Judicious Mining Investments," by J. Parke Channing, Engineering and Mining Journal, Vol. 89, p. 211.

"The Classification of Public Lands," by G. O. Smith and others, Bulletin No. 537, United States Geological Survey.

"The Cost of Mining," by J. R. Finlay, McGraw-Hill Publishing Co., 1909.

"Principles of Mining," by H. C. Hoover, McGraw-Hill Publishing Co., 1909.

"Valuation of Mines and Mineral Lands," by H. M. Chance, Trans. A.I.M.E., 1904, Abstract from same, Engineering and Mining Journal, Vol. 78, p. 515.

"Valuation of Coal Lands," by H. M. Chance, Trans. A.I.M.E., Vol. 47, p. 111.

"Valuation of Public Coal Lands," by G. H. Ashley, Bulletin No. 424, U. S. Geological Survey.

"The Valuation of Anthracite Mines," by R. V. Norris, International Engineering Congress, Oct., 1915.

"Evaluating Coal Properties in Western Canada," by R. W. Coulthard, International Engineering Congress, Oct., 1915.

"Mine Taxation," by H. M. Chance, Proceedings American Mining Congress, Oct., 1913.

"Taxation of Coal Lands," by R. V. Norris, Proceedings American Mining Congress, Oct., 1913, also Colliery Engineer, Feb., 1914.

"Ore Valuation of Gold Mines," by C. B. Horwood, Trans. A.I.M.E., Vol. 39, p. 685.

"Appraisal of Michigan Mines," by J. R. Finlay, Engineering and Mining Journal, Sept. 9 to Oct. 14, 1911. (Serial.)

"Valuation of Iron Mines," by J. R. Finlay, Trans. A.I.M.E., Vol. 45, p. 282.

"Observations on the Appraisal of the Iron Mines of Michigan," Engineering and Mining Journal, March 6, 1915.

"Michigan Iron Ore Reserves and Method of Appraisal for Taxation," by R. C. Allen, Mining and Engineering World, Sept. 12, 1914.

"Iron Mine Assessments in Minnesota," by D. E. Woodbridge, Engineering and Mining Journal, Vol. 84, p. 967.

"Valuation of Oil Lands and Properties," by M. E. Lombardi, Trans. International Engineering Congress, Oct., 1915.

"The Valuation of Oil Lands," by Wm. Forstner, Mining and Scientific Press, Nov. 4, 1911.

"Examination of Petroleum Properties," by Chas. Janin, Mining and Scientific Press, Vol. 101, p. 269.

"Valuation of Oil Wells," Mining and Scientific Press, Vol. 104, p. 284. (Editorial.)

"The Valuation of Mining Areas on the Rand," by W. F. Wilkinson, T.I.M. and M. Vol. 18, p. 348 — 6 pages.

"Valuation of Oil Wells in the Appalachian Fields," Mining and Scientific Press, Feb. 17, 1912.

"Petroleum Mining and Oil Field Development," by A. Beeby Thompson, London, Lockwood, 1910. ("Production and Life of Oil Fields," pp. 83–88.)

"The Oil Fields of Russia and the Russian Petroleum Industry," by A. Beeby Thompson, C. Lockwood & Son, 1904. ("Valuation of Oil Properties," pp. 382–4.)

"Present Conditions in the California Oil Fields," by M. L. Requa, Trans. A.I.M.E., Vol. 42, p. 837.

"Estimating Productive Capacity of Oil Lands," by R. P. McLaughlin, Western Engineering, July, 1914.

"Estimation of Oil Reserves," by C. W. Washburne, Bull. A.I.M.E., Feb., 1915.

"Factors in Cost of Oil Production," by Wm. R. Jewell, Mining and Scientific Press, July 8, 1911.

"Depreciation as applied to Oil Properties," by P. W. Henry, Bull. A.I.M.E., Jan., 1915.

"Comparative Costs of Rotary and Standing Drilling," by M. L. Requa, Bull. A.I.M.E., Feb., 1915.

CHAPTER XIV

TABLES

There have been prepared by the author and are here presented a set of tables which are intended to meet the requirements of the valuation engineer and the economist.

Where values are not noted for every year from 1 to 100, as is the case in some of the tables, methods are indicated whereby other tables can be called to aid.

It is believed that all values have been noted with sufficient accuracy to meet every ordinary requirement.

The following tables with explanations preceding each will be found in this chapter:

EXPLANATION OF TABLE 19

The Probable Useful Life of Various Articles of the Character Most Frequently Encountered in Public Utility Valuations

The probable life of any article does not depend merely upon its wearing qualities when in service, but upon a number of other factors as elsewhere explained. Its useful life under average conditions is determinable with a greater or less degree of approximation from a study of all available experience.

Without entering upon a full discussion of the basic available data, there is here presented a table in which the probable useful term of life for many articles, appliances, machinery and other things likely to be connected with industrial establishments or with public utilities is noted, due credit being given, so far as this is practicable, for the source of information in each case.

Under ordinary conditions and particularly when used in connection with the Unlimited Life Method of procedure, the life terms here presented may be accepted as covering failure from any cause such as physical deterioration, destruction by accident, inadequacy and obsolescence.

It is to be noted that in some of the sources from which information presented in this table was drawn, the depreciation was expressed in percentage by the Straight Line Method. This percentage has been translated into years of probable life.

Summary of Authorities Quoted in Table 19 and Reference to Publication

Alvord.	John W. Alvord in Proceedings American Water Works Association, 1903.
Arbitrators.	Board of Arbitration in street lighting controversy at Atlanta, Georgia, 1899.
Arnold.	Bion J. Arnold. Coney Island and Brooklyn figures adopted by Public Service Commission of New York. Four Chicago appraisals quoted by H. A. Foster.
Bryan.	Wm. H. Bryan. "The Appraisal and Depreciation of Water Works." Journal of Association of Engineering Societies, Dec., 1907.
Burdick.	Chas. B. Burdick. Appraisal of Mt. Vernon Water Works, Illinois, 1906.
Cal. R. R. Com.	California Railroad Commission — Reports 1913-4. Calistoga Electric Co. *vs.* Napa Valley Electric Co., May, 1914. Cuyamaca Water Co. Case, March, 1913. San Jose Water Co. Case, May, 1914. Stockton Terminal & Eastern R. R., April, 1913.
Chi. Tele. Com.	Chicago Telephone Commission. Published 1908.
Chi. Trac. Com.	Chicago Traction Commission. Quoted by G. W. Cravens, Electrical Review, April 23, 1910.

Chi. U. T. Co.	Chicago Union Traction Co. Quoted in Report Wisconsin R. R. Commission, Vol. 10, p. 220.
Connette.	E. G. Connette. 3rd Ave. Case figures adopted by Public Service Commission of New York.
Cooley.	Mortimer E. Cooley. Milwaukee 3¢ Fare Case. Report Wisconsin R. R. Com., Vol. 10, p. 226.
Dodge.	B. L. Dodge. Mt. Vernon Water Works Appraisal, Illinois, 1906.
Floy.	Trans. American Institute of Electrical Engineers, June, 1911. Third Avenue Case figures adopted by Public Service Commission of New York.
Foster.	H. A. Foster. "Engineering Valuation of Public Utilities and Factories," Van Nostrand, 1912.
Gillette.	H. P. Gillette. Appraisal of Great Northern & Northern Pacific R. R.'s in Washington. Direction of Washington Railroad Commission, 1906.
Hammond.	Robt. Hammond. Journal of Institute of Electrical Engineers, England, April 25, 1907.
Kiersted.	W. Kiersted. Galena Appraisal, Kansas, 1905.
Metcalf.	Leonard Metcalf. Transactions American Society of Civil Engineers, 1909.
Mil. E. R. & L. Co.	Milwaukee Electric Railway and Lighting Co., in the Milwaukee 3¢ Fare Case, Aug., 1912.
P. G. & E. Co.	Pacific Gas & Electric Company. Figures used in rate hearings before Master in Chancery, San Francisco, 1913-4.
Parsons.	T. C. Parsons. Discussion of Mr. Hammond's paper, Journal Institute Electric Engineers, England, April 25, 1907.
Preece.	Sir Wm. Preece. Report of Bristol Corporation, England, 1906.
Rosecrans.	W. H. Rosecrans. Galena Appraisals, Kansas, 1905.
S. J. L. & P. Corp.	San Joaquin Light & Power Corporation. Rate hearing before California Railroad Commission, 1915.
Starrett.	Milton G. Starrett. Milwaukee 3¢ Fare Case. Report Wisconsin R. R. Commission, Vol. 10, p. 226.
St. Louis P. S. C.	St. Louis Public Service Commission. Case of Union Electric Light & Power Co.
Stone & Webster.	Figures for Chicago Union Traction Co. Case. Report Wisconsin Railroad Commission, Vol. 10, p. 220.
Telephone "Data."	Compiled from practice A. T. & T. Co., Wisconsin R. R. Commisison and others. Quoted by H. A. Foster.
Trac. Val. Com.	Traction Valuation Commission. Case of the Chicago Consolidated Traction Co.
Wilgus.	W. G. Wilgus. Appraisal of Lehigh Valley Railroad, January, 1914.
Williams.	Benezette Williams. Galena Appraisal, Kansas, 1905.

Wis. R. R. Com.	Wisconsin Railroad Commission. Quoted by Floy and by Foster and in published reports. Madison Electric Railway & Light Co. Case, March 8, 1910. Milwaukee Electric Railway & Lighting Co. Case, Nov. 25, 1913. Milwaukee 3¢ Fare Case. Aug. 23, 1912. Wisconsin Telephone Co. Case, Aug. 3, 1909.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Accumulators	15		Hammond	Jour. Inst. Elec. Engrs.
	15		Parsons	Jour. Inst. Elec. Engrs.
	15	10%	Preece	Report Bristol Corp., 1906.
Air brakes	20		Wis. R. R. Com.	Quoted by Henry Floy.
Air compressors	20	5%	P. G. & E. Co.	S. F. rate hearing, 1913-14.
	20-25		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
Arc lamps	6⅔		Arbitrators	Ata. St. Lgt. Contrvsy, 1899.
	12	5%	Preece	Report Bristol Corp., 1906.
	12½		St. Louis P. S. C.	Union E. L. & P. Co. Case.
	15		Wis. R. R. Com.	Quoted by Henry Floy.
	15		P. G. & E. Co.	S. F. rate hearings, 1913-14.
Arc lamp posts	40	5%	Preece	Report Bristol Corp., 1906.
Ammonia concentrators and tanks	15		Wis. R. R. Com.	Quoted by H. A. Foster.
Belting	8		P. G. & E. Co.	S. F. rate hearings, 1913-14.
	20		Wis. R. R. Com.	Quoted by Henry Floy.
	10-20		Wis. R. R. Com.	Mil. 3¢ Fare Case.
with shafting and ropes	15		Chi. U. T. Co.	Chi. Con. Trac. Co. Case.
with shafting and ropes	20		Stone & Webs.	Chi. Con. Trac. Co. Case.
(leather)	20-25		Foster	" Eng. Valuation," etc.
Benches (gas plant)	25		Wis. R. R. Com.	Quoted by H. A. Foster.
Blowers (gas plant)	15		Wis. R. R. Com.	Quoted by H. A. Foster.
	20		P. G. & E. Co.	S. F. rate hearings, 1913-14.
Boilers	10		Arnold	Coney Isl. & Brooklyn Case.
	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
	15		Alvord	Aver. 32-Proc. A. W. W. A.
	15		St. Louis P. S. C.	Union Elec. L. & P. Co. Case.
	12-16		Metcalf	Trans. Am. S. C. E., 1909.
	16.7		Dodge	Mt. Vernon W. W. Ill., 1906.
	15-20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	20		Stone & Webs.	Chi. Union Trac. Co. Case.
	20	0%	P. G. & E. Co.	S. F. rate hearings, 1913-14.
	20		Hammond	Jour. Inst. Elec. Engrs., 1907.
	10-28		Chi. Trac. Com.	Chicago Trac. Com., 1908.
	25		Williams	Galena Appraisal, Kan., 1905.
	25		Burdick	Mt. Vernon W. W. Ill., 1906.
	28.57		Arnold	4 Chicago Appraisals.
	25-30		Trac. Val. Com.	Chicago Con. Trac. Co. Case.
	30		Kiersted	Galena Appraisal, Kan., 1905.
	25-40		Rosecrans	Galena Appraisal, Kan., 1905.
water-tube	20		Floy	3rd Ave. Case, N. Y.
water-tube	20		Wis. R. R. Com.	Quoted by Henry Floy.
water-tube	25	5%	Preece	Report Bristol Corp., 1906.
water-tube	20-30		Foster	" Eng. Valuation," etc.
fire-tube	10		Arbitrators	Ata. St. Lgt. Contrvsy, 1899.
fire-tube	10-15		Foster	" Eng. Valuation," etc.
fire-tube electric light	16-30		Wis. R. R. Com.	Quoted by Henry Floy.
fire-tube water works	20-25		Wis. R. R. Com.	Quoted by Henry Floy.
Lancashire	22	3%	Preece	Report Bristol Corp., 1906.
boilers and accessories	20		S. J. L. & P. Cor.	Cal. R. R. C. rate hearings.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Boilers (continued)				
steel breechings	10		Arnold	4 Chicago Appraisals.
breechings and connections	10–30		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
Bonds	20		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
	20		Wis. R. R. Com.	Quoted by Henry Floy.
Bridges				
Howe truss R. R.	10		Gillette	Gt. Nor. & Nor. Pac. apprais.
trestles and wooden bridges	10		Cal. R. R. Com.	Stockton Term. & East. Case.
Buildings	25–50		Wis. R. R. Com.	Quoted by Henry Floy.
	50		St. Louis P. S. C.	Union Elec. L. & P. Case.
	50		Connette	3rd Ave. Case, N. Y. City.
railroad station	10		Cal. R. R. Com.	Stockton Term. & East R. R. Case.
railroad	33⅓		Gillette	Gt. Nor. & N. Pac. Appraisal.
street railway	50		Cooley	Milwaukee 3¢ Fare Case.
street railway	50		Starrett	Milwaukee 3¢ Fare Case.
street railway	50		Stone & Webs.	Chi. Union Trac. Co. Case.
street railway	50		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
street railway	35–75		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
street railway power plants	20		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
street railway power plants	25		Cooley	Milwaukee 3¢ Fare Case.
street railway power plants	50		Stone & Webs.	Chi. Union Trac. Co. Case.
street railway power plants	60		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
sub-station	50	0%	Preece	Report Bristol Corp., 1906.
sub-station	50	1%	Cal. R. R. Com.	Calistoga E. Co. *vs.* Napa Val. E. Co.
wooden, wood frame	20		P. G. & E. Co.	S. F. rate hearings, 1913–14.
wooden	25		Rosecrans	Galena Appraisal, Kan., 1905.
wooden frame	50		Arbitrators	Ata. St. Lgt. Contrvsy, 1899.
frame dwellings	35		Wis. R. R. Com.	Quoted by H. A Foster.
frame stables and sheds	20–25		Wis. R. R. Com.	Quoted by H. A. Foster.
corrugated iron	20		P. G. & E. Co.	Wooden frame and floor.
corrugated iron	25		P. G. & E. Co.	Wooden frame, conc. floor.
corrugated iron	30		P. G. & E. Co.	Steel frame, wood floor.
corrugated iron	36		P. G. & E. Co.	Steel frame, conc. floor.
brick	14		Rosecrans	Galena Appraisal, Kan., 1905.
brick	30		Williams	Galena Appraisal, Kan., 1905.
brick	30		Kiersted	Galena Appraisal, Kan., 1905.
brick	30		Dodge	Mt. Vernon W. W., Ill., 1906.
brick	30		P. G. & E Co.	Corr. roof, steel roof, frame, conc. floor.
brick	30		P. G. & E. Co.	Wood r., wood r-f., wood f.
brick	36		P. G. & E. Co.	Slate r., wood r-f., and floor.
brick	40		P. G. & E. Co.	Corr. r., steel r-f., conc. f.
brick	50		P. G. & E. Co.	Slate r., steel r-f., conc. f.
brick	50		P. G. & E. Co.	Rein. conc. r., steel r-f., conc. f.
brick	35		Burdick	Mt. Vernon W. W., Ill., 1906.
brick	60		Hammond	Jour. Inst. E. E., 1907.
brick	66⅔		Trac. Val. Com.	Chi. Union Trac. Co. Case.
brick	80	0%	Preece	Report Bristol Corp., 1906.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Buildings (continued)				
brick gas retort	30		Wis. R. R. Com.	Quoted by H. A. Foster.
1st class stone and brick	75		Wis. R. R. Com.	Quoted by H. A. Foster.
2nd class shops, etc.	50		Wis. R. R. Com.	Quoted by H. A. Foster.
fireproof	40	0%	Chi. Tel. Com.	Chicago Tel. Com., 1908.
reinforced concrete	50		P. G. & E. Co.	Steel r-f., conc. r., and floor.
foundations	100	0%	Preece	Report Bristol Corp., 1906.
Cables				
mains	30	15%	Parsons	Jour. Inst. E. E., 1907.
mains	20		Chi. Tel. Com.	Chicago Telephone Com.
armored mains	25		Hammond	Jour. Inst. E. E., 1907.
solid mains	30		Hammond	Jour. Inst. E. E., 1907.
armored	35	15%	Preece	Report Bristol Corp., 1906.
cables and feeders	20		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
cables and feeders	15–25		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
main lead-covered	20	26%	P. G. & E. Co.	S. F. rate hearings, 1913–14.
subsidiary cables	15	40%	Chi. Tel. Com.	Chicago Telephone Com.
aerial	15		Chi. Tel. Com.	Chicago Telephone Com.
aerial lines	20		St. L. P. S. C.	Union Elec. L. & P. Case.
aerial exchange	12–15	30%	Tel. " Data "	Quoted by H. A. Foster.
aerial lead-covered	10–15		Foster	" Eng. Valuation," etc.
aerial lead-covered	15		Wis. R. R. Com.	Wisconsin Tel. Co. Case.
aerial terminals	12	0%	Chi. Tel. Com.	Chicago Telephone Com.
aerial terminals	10–12		Tel. " Data "	Quoted by H. A. Foster.
underground lead covered	20		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
underground lead covered	25		Wis. R. R. Com.	Quoted by Floy.
underground lead covered	20		Wis. R. R. Com.	Quoted by H. A. Foster.
underg'd main exchange	20–25	40%	Tel. " Data "	Quoted by H. A. Foster.
underground sub-exchange	13–20	40%	Tel. " Data "	Quoted by H. A. Foster.
toll	30	40%	Tel. " Data "	Quoted by H. A. Foster.
submarine	10		Tel. " Data "	Quoted by H. A. Foster.
Coal and ash machinery	10		Wis. R. R. Com.	Quoted by Henry Floy.
	10		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
	14		Trac. Val. Com.	Chicago Con. Trac. Co. Case.
	14.2		Arnold	4 Chicago Appraisals.
	15		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
	20		Stone & Webs.	Chi. Union Trac. Co. Case.
	20		Henry Floy	3rd Ave. Case, N. Y. City.
Compressors (air)	20	5%	P. G. & E. Co.	S. F. rate hearings, 1913–14.
	20–25		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
Compressor station				
natural gas	10	50%	Cal. R. R. Com.	Midway Gas Co., *et al.*
Condensers	10		Arbitrators	Ata. St. Lgt. Controversy.
	15		St. L. P. S. C.	Union E. L. & P. Co. Case.
	20		Wis. R. R. Com.	Quoted by Henry Floy.
	20		Floy	3rd Ave. Case, N. Y. City.
	20		Arnold	Coney Isl. & Brooklyn Case.
	20	0%	P. G. & E. Co.	S. F. rate hearings, 1913–14.
	25		S. J. L. & P. Co.	Cal. R. R. Com. rate hear'gs.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Condensers (continued)				
old equipment	20		S. J. L. &. P. Co.	Cal. R. R. Com. rate hear'gs.
	25		Trac. Val. Com.	Chi. Union Trac. Co. Case.
Conduits	50		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	50		St. L. P. S. C.	Union Elec. L. & P. Co., Case.
	100		Floy	3rd Ave. Case N. Y. City.
mains	30	15%	Parsons	Jour. Inst. E. E., 1907.
ducts	30		Hammond	Jour. Inst. E. E., 1907.
"solid" system	30		Hammond	Jour. Inst. E. E., 1907.
"ducts" system	30		Hammond	Jour. Inst. E. E., 1907.
"solid" sys. in wood	40	12%	Preece	Report Bristol Corp., 1906.
clay in concrete	50	0%	Chi. Tel. Com.	Chicago Telephone Com.
fibre in concrete	20	0%	Chi. Tel. Com.	Chicago Telephone Com.
subsidiary	20	0%	Chi. Tel. Com.	Chicago Telephone Com.
main-vitrified clay	50	0%	Tel. "Data"	Quoted by H. A. Foster.
main-concrete	55	0%	Tel. "Data"	Quoted by H. A. Foster.
main-fibre	20	0%	Tel. "Data"	Quoted by H. A. Foster.
main-iron	20	0%	Tel. "Data"	Quoted by H. A. Foster.
main-creosoted wood	20	0%	Tel. "Data"	Quoted by H. A. Foster.
subsidiary	15	0%	Tel. "Data"	Quoted by H. A. Foster.
vitrified tile	50		P. G. & E. Co.	S. F. rate hearings, 1913-14.
vitrified tile and fibre	40		P. G. & E. Co.	S. F. rate hearings, 1913-14.
fibre duct	30		P. G. & E. Co.	S. F. rate hearings, 1913-14.
wrought iron pipe	20		P. G. & E. Co.	S. F. rate hearings, 1913-14.
Edison tube and fittings	20	15%	P. G. & E. Co.	S. F. rate hearings, 1913-14.
paper conduit	25		P. G. & E. Co.	S. F. rate hearings, 1913-14.
wood conduit	25		P. G. & E. Co.	S. F. rate hearings, 1913-14.
w. i. lateral pipes	15		P. G. & E. Co.	S. F. rate hearings, 1913-14.
conduit under tracks	40		P. G. & E. Co.	S. F. rate hearings, 1913-14.
manholes and paving	40		P. G. & E. Co.	S. F. rate hearings, 1913-14.
service holes and paving	40		P. G. & E. Co.	S. F. rate hearings, 1913-14.
Converters				
static transformers	15		Hammond	Jour. Inst. E. E., 1907.
rotary	20		Hammond	Jour. Inst. E. E., 1907.
Conveyors	10		Hammond	Jour. Inst. E. E., 1907.
Cranes	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
	20		Stone & Webs.	Chi. Union Trac. Co. Case.
	50		Wis. R. R Com.	Milwaukee 3¢ Fare Case.
	50		Arnold	4 Chicago Appraisals.
wooden	15		P. G. & E. Co.	S. F. rate hearings, 1913-14.
steel	same as bldg		P. G. & E. Co.	S. F. rate hearings, 1913-14.
Culverts				
cast iron	16		Cal. R. R. Com.	Stockton Term. & East Case.
log and timber	16⅔		Gillette	Gt. Nor. & N. P. R. R. Appraisal.
Dams				
earth	100		Cal. R. R. Com.	Cuyumaca Water Co. Case.
earth and loose rock	50		S. J. L. & P. Co.	Cal. R. R. Com. rate hear'gs.
concrete	50		S. J. L. & P. Co.	Cal. R. R. Com. rate hear'gs.
concrete diverting	100		Cal. R. R. Com.	Cuyumaca Water Co. Case.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Distribution system				
street railway	$12\frac{1}{2}$		Cooley	Milwaukee 3¢ Fare Case.
street railway	14		Starrett	Milwaukee 3¢ Fare Case.
distrib. and trans.	30.03		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
Ditches	50		S. J. L. & P. Co.	Cal. R. R. Com. rate hearing.
concrete lining	20		S. J. L. & P. Co.	Cal. R. R. Com. rate hearing.
Economizers	10–20		Floy	4 Chicago Appraisals.
	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
	20		Stone & Webs.	Chi. Union Trac. Co. Case.
Electrical machinery				
dynamos and alternators	25		Hammond	Jour. Inst. Elec. E., 1907.
dynamos and alternators	30	8%	Preece	Report Bristol Corp., 1906.
generators	10		Arbitrators	Ata. St. Lgt. Controversy.
generators	15		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
generators	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
generators	20		Stone & Webs.	Chi. Union Trac. Co. Case.
generators	15–20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
generators	20	2¢ lb.	P. G. & E. Co.	S. F. rate hearings, 1913–14.
generators	20		Arnold	Coney Isl. & Brooklyn Case.
generators	20		Floy	3rd Ave. Case, N. Y. City.
generators	20		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
generators	13–33		Trac. Val. Com.	Chicago Con. Trac. Co. Case.
generators, modern types	20		Wis. R. R. Com.	Quoted by Henry Floy.
generators, obsolete types	15		is. R. R. Com.	Quoted by Henry Floy.
generators belted	10–20		Chi. Trac. Com.	Quoted by G. W. Cravens.
generators direct conn.	20		Chi. Trac. Com.	Quoted by G. W. Cravens.
generators, telephone plant	20	20%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.
motors	10		Arbitrators	Ata. St. Lgt. Controversy.
motors	20		Hammond	Jour. Inst. Elec. Eng., 1907.
motors	25	9%	Preece	Report Bristol Corp., 1906.
motors	25	9%	Parsons	Jour. Inst. Elec. Eng., 1907.
motors	20	2¢ lb.	P. G. & E. Co.	S. F. rate hearings, 1913–14.
motors, modern types	20		Foster	"Eng. Valuation," etc.
motors, obsolete	15		Foster	"Eng. Valuation," etc.
motors, street railway	20		Floy	3rd Ave. Case, N. Y. City.
motors, street railway	20		Wis. R. R. Com.	Quoted by Henry Floy.
motors, street railway	30		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
motors, small, misc.	20		P. G. & E. Co.	S. F. rate hearings, 1913–14.
Engines	15		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
	20		Arbitrators	Ata. St. Lgt. Controversy.
	25		Hammond	Jour. Inst. E. E., 1907.
engines and machinery	25	6%	Preece	Report Bristol Corp., 1906.
	$10–33\frac{1}{3}$		Chi. Trac. Com.	Quoted by G. W. Cravens.
	13–20		Arnold	Coney Isl. & Brooklyn Case.
	$20–33\frac{1}{3}$		Arnold	4 Chicago Appraisals.
	15–20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	20–33		Trac. Val. Com.	Chicago Con. Trac. Co. Case.
machinery	27	6%	Parsons	Jour. Inst. Elec. E., 1907.
steam	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
steam	20		Stone & Webs.	Chi. Union Trac. Co. Case.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Engines (continued)				
steam	20	.	Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
steam	20	5%	P. G. & E. Co.	S. F. rate hearings, 1913-14.
steam	20		Floy	3rd Ave. Case, N. Y. City.
steam	20		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
steam	15-25		Metcalf	Trans. Am. Soc. C. E., 1909.
steam, slow speed	20		Wis. R. R. Com.	Quoted by Henry Floy.
Corliss slow speed	25-30		Foster	"Eng. Valuation," etc.
steam high speed	15-20		Foster	"Eng. Valuation," etc.
steam, high speed	15		Wis. R. R. Com.	Quoted by Henry Floy.
gas	10-15		Foster	"Eng. Valuation," etc.
gas	15		Wis. R. R. Com.	Quoted by Henry Floy.
steam turbines	20		Stone & Webs.	Chi. Union Trac. Co. Case.
steam turbines	20		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
steam turbines	15		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
steam turbines	20		Wis. R. R. Com.	Quoted by Henry Floy.
steam turbines	20		Hammond	Jour. Inst. E. E., 1907.
steam turbines	30	1¢ lb.	P. G. & E. Co.	S. F. rate hearings, 1913-14.
Exhausters				
gas plant	25	1¢ lb.	Wis. R. R. Com.	Quoted by H. A. Foster.
Fences				
wooden	12		P. G. & E. Co.	S. F. rate hearings, 1913-14.
wire mesh	12		P. G. & E. Co.	S. F. rate hearings, 1913-14.
railroad	14.3		Gillette	Gt. Nor. & N. P. Appraisal.
railroad	15		Cal. R. R. Com.	Stockton Term. & East. Case.
Filter beds				
water filters	15-20		Bryan	"Appraisal of water works."
water works	30-50		Foster	"Eng. Valuation," etc.
Flumes				
wooden	30		Cal. R. R. Com.	Cuyumaca Water Co. Case.
steel	25		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
concrete	50		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
Foundations	Same as life of article supported		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
			Floy	3rd Ave. Case, N. Y. City.
Fire protection apparatus	12		P. G. & E. Co.	S. F. rate hearings, 1913-14.
Fuel oil handling apparatus	25		Trac. Val. Com.	Chicago Con. Trac. Co.
Fire hydrants	40		Williams	Galena Appraisal, Kan., 1905.
	40-50		Metcalf	Trans. Am. Soc. C. E., 1909.
Furniture and fixtures	12½		Cooley	Milwaukee 3¢ Fare Case.
	20		Starrett	Milwaukee 3¢ Fare Case.
	20		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
	20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
Gates and valves				
water works	40		Williams	Galena Appraisal, Kan., 1905.
water works	40-50		Metcalf	Trans. Am. Soc. C. E., 1909.
Gas holders	50		Wis. R. R. Com.	Quoted by H. A. Foster.
Governors				
gas plant	50		Wis. R. R. Com.	Quoted by H. A. Foster.
consumers	25		Wis. R. R. Com.	Quoted by H. A. Foster.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Head gates				
on ditches	30		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
Heaters (feed water)	15		C. U. T. Co.	Chi. Union Trac. Co. Case.
	20		Stone & Webs.	Chi. Union Trac. Co. Case.
	20		P. G. & E. Co.	S. F. rate hearings, 1913–14.
	16–25		Trac. Val. Com.	Chicago Con. Trac. Co. Case.
	15–30		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	30		Wis. R. R. Com.	Quoted by Henry Floy.
	33⅓		Arnold	4 Chicago Appraisals.
Lamps				
arc	6⅔		Arbitrators	Ata. St. Lgt. Controversy.
arc	12	5%	Preece	Report Bristol Corp., 1906.
arc	12½		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
arc	15		P. G. & E. Co.	S. F. rate hearings, 1913–14.
arc	15		Wis. R. R. Com.	Quoted by Henry Floy.
Nernst	8–10		Foster	"Eng. Valuation," etc.
arc lamp posts	40	5%	Preece	Report Bristol Corp., 1906.
Lighting systems				
incandescent street	15		P. G. & E. Co.	S. F. rate hearings, 1913–14.
arc (commercial & mun.)	15		P. G. & E. Co.	S. F. rate hearings, 1913–14.
municipal street	20	5%	Cal. R. R. Com.	Calis. E. C. *vs.* Napa Val. E. C.
Lighting protection	10		P. G. & E. Co.	S. F. rate hearings, 1913–14.
	15		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
	15–20		Foster	"Eng. Valuation," etc.
	10–12		Foster	"Eng. Valuation," etc.
Machinery	27	6%	Parsons	Jour. Inst. Elec. E., 1907.
engines and machinery	25	6%	Preece	Report Bristol Corp., 1906.
Meters				
electric	10		Hammond	Jour. Inst. Elec. E., 1907.
electric	12	2%	Preece	Report Bristol Corp., 1906.
electric	12	5%	Parsons	Jour. Inst. Elec. E., 1907.
electric	12.5		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
electric	15		P. G. & E. Co.	S. F. rate hearings, 1913–14.
electric	23.6	10%	Cal. R. R. Com.	Calis. E. C. *vs.* Napa Val. E. C.
electric-switchboard	20		Wis. R. R. Com.	Quoted by Henry Floy.
electric service	15		Wis. R. R. Com.	Quoted by Henry Floy.
gas-station (drums)	20		Wis. R. R. Com.	Quoted by H. A. Foster.
gas-station (cases)	50		Wis. R. R. Com.	Quoted by H. A. Foster.
gas-consumers	25		Wis. R. R. Com.	Quoted by H. A. Foster.
water	20		Williams	Galena Appraisal, Kan., 1905.
water	20–30		Metcalf	Trans. Am. Soc. C. E., 1909.
water	50		Cal. R. R. Com.	Cuyumaca W. Co. Case.
Paving	10		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
	10		Cooley	Milwaukee 3¢ Fare Case.
	12.5		Starrett	Milwaukee 3¢ Fare Case.
	10–12.5		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	10–26		Stone & Webs.	Chi. Union Trac. Co. Case.
	10–25		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
asphalt	10		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
asphalt	10		Stone & Webs.	Chi. Union Trac. Co. Case.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Paving (continued)				
asphalt	12		Wis. R. R. Com.	Milwaukee E. R. & L. Case.
brick	12		Wis. R. R. Com.	Milwaukee E. R. & L. Case.
cobble	25		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
cobble	26		Stone & Webs.	Chi. Union Trac. Co. Case.
creosoted blocks	12		Wis. R. R. Com.	Milwaukee E. R. & L. Case.
granite	16		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
granite	16		Stone & Webs.	Chi. Union Trac. Co. Case.
granite	21		Wis. R. R. Com.	Mil. El. Ry. & Lt. Co. Case.
Pipe				
water pipe:				
cast iron	75		Kiersted	Galena Appraisal, Kan., 1905.
cast iron	80		Williams	Galena Appraisal, Kan., 1905.
cast iron	100		Burdick	Mt. Vernon W. W., Ill., 19c6.
cast iron	100		Dodge	Mt. Vernon W. W., Ill., 1906.
cast iron	100		Alvord	Proc. Am. W. W. Assoc., 1903.
cast iron mains	50–75		Metcalf	Trans. Am. Soc. C. E., 1909.
cast iron mains	100		Cal. R. R. Com.	San Jose Water Co. Case, 1914.
cast iron, small size	20–40		Metcalf	Trans. Am. Soc. C. E., 1909.
wrought iron	20		Kiersted	Galena Appraisal, Kan., 1905.
wrought iron	20		Dodge	Mt. Vernon W. W., Ill., 1906.
wrought iron underground	30		Williams	Galena Appraisal, Kan., 1905.
wrought iron services	15–30		Metcalf	Trans. Am. Soc. C. E., 1909.
galv. wrought iron incl. fit's.	30–50		Foster	"Eng. Valuation," etc.
black w. iron and services.	25–35		Foster	"Eng. Valuation," etc.
steel pipe	25–50		Metcalf	Trans. Am. Soc. C. E., 1909.
services	20		Dodge	Mt. Vernon W. W., Ill., 1906.
wood-stave	20–30		Metcalf	Trans. Am. Soc. C. E., 1909.
wood-stave	30		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
distributing system	16.5		Cal. R. R. Com.	Cuyumaca Water Co. Case.
power-plant pen stocks	30		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
gas pipe:				
w. i. & steel under 3″ diam.	20		Wis. R. R. Com.	Quoted by H. A. Foster.
w. iron & steel 3″ and over	30		Wis. R. R. Com.	Quoted by H. A. Foster.
c. iron mains 3″ & 4″ diam.	50		Wis. R. R. Com.	Quoted by H. A. Foster.
cast iron mains over 6″	75		Wis. R. R. Com.	Quoted by H. A. Foster.
services	20		Wis. R. R. Com.	Quoted by H. A. Foster.
miscellaneous:				
pipe and covering	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
pipe and covering	17		Arnold	Coney Isl. & Brooklyn Case.
pipe and covering	20		Stone & Webs.	Chi. Union Trac. Co. Case.
pipe and covering	20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
pipe and covering	20		Floy	3rd Ave. Case, N. Y. City.
pipe and covering	28.5		Arnold	4 Chicago Appraisals.
pipe and covering	22–25		Trac. Val. Com.	Chicago Con. Trac. Co. Case.
power stations	20		P. G. & E. Co.	S. F. rate hearings, 1913–14.
Poles	10	0%	Chi. Tel. Com.	Chicago Telephone Com.
wooden	10		Arbitrators	Ata. St. Lgt. Controversy.
wooden	13⅓		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
wooden in earth	12–18		Wis. R. R. Com.	Quoted by Henry Floy.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Poles				
wooden in concrete........	20		Wis. R. R. Com.	Quoted by Henry Floy.
cedar.....................	10	1¢–7¢ ft.	P. G. & E. Co.	S. F. rate hearing 1913–14.
cedar with cross arms.....	12	0%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.
cedar under 35′ long.......	10–15	0%	Tel. "Data"	Quoted by H. A. Foster.
cedar 35′ long and over....	15–20	0%	Tel. "Data"	Quoted by H. A. Foster.
cedar in earth............	10–18		Foster	"Eng. Valuation," etc.
cedar in concrete..........	12–20		Foster	"Eng. Valuation," etc.
chestnut under 35′ long....	8–12		Tel. "Data"	Quoted by H. A. Foster.
chestnut 35′ and over......	12–15		Tel. "Data"	Quoted by H. A. Foster.
pine-creosoted............	20		Tel. "Data"	Quoted by H. A. Foster.
redwood..................	16	6¢–25¢ ft.	P. G. & E. Co.	S. F. rate hearing, 1913–14.
telephone.................	12–15		Wis. R. R. Com.	Quoted by H. A. Foster.
telephone cross arms......	8–12		Wis. R. R. Com.	Quoted by H. A. Foster.
average exchange..........	10	0%	Tel. "Data"	Quoted by H. A. Foster.
average toll..............	15	0%	Tel. "Data"	Quoted by H. A. Foster.
iron......................	20		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
iron......................	40		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
iron......................	40		Arnold	4 Chicago Appraisals.
steel.....................	50		Floy	3rd Ave. Case, N. Y. City
Power plant equipment......	12½		Cooley	Milwaukee 3¢ Fare Case.
	20		Starrett	Milwaukee 3¢ Fare Case.
	21.01		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
Pumps.....................	15		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
	20		Floy	3rd Ave. Case, N. Y. City.
	20		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
	20		Arnold	Coney Isl. & Brooklyn Case.
	20		Arnold	4 Chicago Appraisals.
	20		Arbitrators	Ata. St. Ltg. Controversy.
	21.3		Alvord	Proc. Am. W. W. Assoc., 1903.
	22		Rosecrans	Galena Appraisal, Kan., 1905.
	25		Hammond	Jour. Inst. Elec. E., 1907.
	25	6%	Preece	Report Bristol Corp., 1906.
	25–30		Williams	Galena Appraisal, Kan., 1905.
	30		Kiersted	Galena Appraisal, Kan., 1905.
	30		Burdick	Mt. Vernon W. W., Ill., 1906.
	40		Dodge	Mt. Vernon W. W., Ill., 1906.
and auxiliary machinery..	20–30		Metcalf	Trans. Am. Soc. C. E., 1909.
and condensers...........	15–25		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
small steam..............	15		Wis. R. R. Com.	Quoted by Henry Floy.
general service...........	16	5%	P. G. & E. Co.	S. F. rate hearings, 1913–14.
centrifugal...............	16	5%	D. G. & E. Co.	S. F. rate hearings, 1913–14.
centrifugal...............	20–30		Foster	"Eng. Valuation," etc.
geared power.............	20–30		Foster	"Eng. Valuation," etc.
boiler feed...............	15–20		Foster	"Eng. Valuation," etc.
oil.......................	16	5%	P. G. & E. Co.	S. F. rate hearings, 1913–14.
Purifiers				
gas—modern.............	50		Wis. R. R. Com.	Quoted by H. A. Foster.
Reservoirs...................	50–100		Metcalf	Trans. Am. Soc. C. E., 1909.
	100		Burdick	Mt. Vernon W. W., Ill., 1906.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Reservoirs (continued)				
earthen	100		Cal. R. R. Com.	Cuyumaca Water Co. Case.
Retort house				
floors	15-30		Wis. R. R. Com.	Quoted by H. A. Foster.
Rolling stock				
street railway	12.5		Cooley	Milwaukee 3¢ Fare Case.
street railway	16.7		Starrett	Milwaukee 3¢ Fare Case.
street railway	15-20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
railroad	28		Gillette	Gt. Nor. & N. P. Appraisal.
railroad locomotives	20-35		Wilgus	Lehigh Val. R. R. App.
railroad passenger cars	35-40	30-40%	Wilgus	Lehigh Val. R. R. App.
railroad freight cars	30-50	25%	Wilgus	Lehigh Val. R. R. App.
street railway cars	30		Arnold	Coney Isl. & Brooklyn Case.
street railway bodies	15		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
street railway open bodies	25		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
street ry. closed bodies	20		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
street ry. bodies and trucks	13.3		M. E. R. & L. Co.	Milwaukee 3¢ Fare Case.
street ry. bodies and trucks	20		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
street ry. bodies and trucks	20		Stone & Webs.	Chi. Union Trac. Co. Case.
street railway trucks	15-20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
street railway trucks	30		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
street railway trucks	30		Arnold	Coney Isl. & Brooklyn Case.
street railway elec. equip.	10		M. E. R. & L. Co.	Milwaukee 3¢ Fare Case.
street railway elec. equip.	12-15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
street railway elec. equip.	12-15		Stone & Webs.	Chi. Union Trac. Co. Case.
street railway elec. equip.	10-20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
st. ry. fenders and registers	15		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
Scrubbers and condensers	30		Wis. R. R. Com.	Quoted by H. A. Foster.
Services (electric)	15		P. G. & E. Co.	S. F. rate hearings, 1913-14.
	20		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
	21.6	30%	Cal. R. R. Com.	Calis. E. Co. *vs.* Napa Val. E. C.
Snowsheds (*R. R.*)	25		Gillette	Gt. Nor. & N. P. Appraisal.
Stacks				
brick	33		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
brick	33.3		Floy	4 Chicago Appraisals.
steel	12		P. G. & E. Co.	S. F. rate hearings, 1913-14.
steel	14.2		Floy	4 Chicago Appraisals.
Stand pipes	25		Rosecrans	Galena Appraisal, Kan., 1905.
	30		Kiersted	Galena Appraisal, Kan., 1905.
	40		Williams	Galena Appraisal, Kan., 1905.
	25-40		Metcalf	Trans. Am. Soc. C. E., 1909.
Station buildings and R. R. structures	10		Cal. R. R. Com.	Stockton Term & East Case.
Steam vessels				
on Great Lakes	40	15%	Wilgus	Lehigh Val. R. R. Appraisal.
on tide water	32	15%	Wilgus	Lehigh Val. R. R. Appraisal.
Stokers				
fixed parts	20		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
moving parts	5		Trac. Val. Com.	Chi. Con. Trac. Co. Case.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Storage batteries	10		M. E. R. & L. Co.	Milwaukee 3¢ Fare Case.
	10		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	12.5	6%	P. G. & E. Co.	S. F. rate hearings, 1913-14.
	15	5%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.
	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
	20		Stone & Webs.	Chi. Union Trac. Co. Case.
	20		St. Louis P. S. C.	Union Elec. L. & P. Co. Case.
	20		Floy	3rd Ave. Case, N. Y. City.
Sub-station equipment	20		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
	25		Preece	Report Bristol Corp., 1906.
	50		Cal. R. R. Com.	Calis. E. C. *vs.* Napa Val. E. C.
Sumps and wells				
at gas plant	30		P. G. & E. Co.	S. F. rate hearings, 1913-14.
tar and ammonia	50		Wis. R. R. Com.	Quoted by H. A. Foster.
Surge tank				
concrete	30		S. J. L. & P. Co.	Cal. R. R. Com. rate hearing.
Superheaters	20		P. G. & E. Co.	S. F. rate hearings, 1913-14.
Switchboards	12½		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
	15		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
	16.7		Arnold	Coney Isl. & Brooklyn Case.
	20		Stone & Webs.	Chi. Union Trac. Co. Case.
	20		M. E. R. & L. Co.	Milwaukee 3¢ Fare Case.
	20		Floy	3rd Ave. Case, N. Y. City.
	20		P. G. & E. Co.	S. F. rate hearings, 1913-14.
	20		Hammond	Jour. Inst. Elec. E., 1907.
	15-20		S. J. L. & P. Co.	Cal. R. R. Com. rate hearing.
	15-20		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	33.3		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
	50		Arnold	4 Chicago Appraisals.
	50		Chi. Trac. Com.	Quoted by G. W. Cravens.
	50		Wis. R. R. Com.	Quoted by G. W. Cravens.
telephone, central	8	20%	Chi. Tel. Com.	Chicago Telephone Com.
telephone, central	8-10	15%	Tel. "Data"	Quoted by H. A. Foster.
telephone, central	12	20%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.
telephone P. B. X.	8	20%	Chi. Tel. Com.	Chicago Telephone Com.
telephone P. B. X.	10	10%	Tel. "Data"	Quoted by H. A. Foster.
telephone P. B. X.	10	15%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.
Systems and plants				
electric light and power	17.46		Wis. R. R. Com.	Fon du Lac W. Co. Case aver. 6 plants.
electric light and power	18		Wis. R. R. Com.	Madison Gas & E. Co. Case.
electric light and power	20		Mass. G. & E. Com.	Mass. Gas & Elec. Com.
electric light and power	20-25		Wis. R. R. Com.	Jefferson Mun. E. L. & W. plant.
electric light and power	22.0		P. G. & E. Co.	S. F. rate hearing, 1913-14.
electric railway	18.02		Wis. R. R. Com.	Fon du Lac W. Co. Case aver. 7 plants.
electric railway	18.6		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Systems and plants (con.)				
electric railway	19.8		Wis. R. R. Com.	Duluth St. Ry. Co. Case.
telephone	10–25		Wis. R. R. Com.	Oregon Telephone Co. Case.
telephone	14–15		Wis. R. R. Com.	Various cases in 1912–13.
water supply	50–65		Wis. R. R. Com.	Various cases in 1910–11.
overhead system	33⅓	35%	Cal. R. R. Com.	Calis. E. C. *vs.* Napa Val. E.C.
st. ry. distrib. system	12½		Cooley	Milwaukee 3¢ Fare Case.
	14		Starrett	Milwaukee 3¢ Fare Case.
st. ry. distrib. & trans. sys.	30.03		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
Tanks				
wooden	12		P. G. & E. Co.	S. F. rate hearings, 1913–14.
steel	30		P. G. & E. Co.	S. F. rate hearings, 1913–14.
Tar extractors				
P. & A.	40		Wis. R. R. Com.	Quoted by H. A. Foster.
Telephone equipment				
street railway	10–12		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
street railway	13.3		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
electric power plants	12		P. G. & E. Co.	S. F. rate hearings, 1913–14.
elec. power plant lines	20		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
elec. power plant instrum'ts	20		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
elec. power plant equipm't.	8–14		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
farmers line	15		Wis. R. R. Com.	Wautoma & Mt. Morris Tel. Line.
subscribers instruments	8–10	10%	Tel. "Data"	Quoted by H. A. Foster.
	10	5%	Chi. Tel. Com.	Chicago Tel. Commission.
	10	5%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.
telephone drop wires	8	15%	Chi. Tel. Com.	Chicago Tel. Commission.
Testing instruments				
electric	10	20%	Cal. R. R. Com.	Calis. E. C. *vs.* Napa Val. E. Co.
Tools and shop machinery	5–25		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
	12½		Cooley	Milwaukee 5¢ Fare Case.
	14.2		Starrett	Milwaukee 3¢ Fare Case.
	13.3		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
	20		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
	20		Stone & Webs.	Chi. Union Trac. Co. Case.
tools, teams and furniture	4	10%	Chi. Tel. Com.	Chicago Telephone Com.
tools and furniture	7		Wis. R. R. Com.	Quoted by H. A. Foster.
tools and sundries	10	5%	Preece	Report Bristol Corp., 1906.
Track				
rails, ties and bonding	12½		Wis. R. R. Com.	Milwaukee 3¢ Fare Case.
rails, ties and bonding	12½		Cooley	Milwaukee 3¢ Fare Case.
rails, ties and bonding	12½		Starrett	Milwaukee 3¢ Fare Case.
rails, ties and bonding	12.85		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
rails, ties and bonding	13.86		Stone & Webs.	Chi. Union Trac. Co. Case.
rails, ties and bonding	13.33		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
straight	18		Wis. R. R. Com.	Quoted by Henry Floy.
special work	8.3		Mil. E. R. & L.	Milwaukee 3¢ Fare Case.
special work	10		Arnold	4 Chicago Appraisals.
special work	12		Wis. R. R. Com.	Quoted by Henry Floy.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Continued*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
Track (continued)				
special work	12.85		Chi. U. T. Co.	Chi. Union Trac. Co. Case.
special work	13.86		Stone & Webs.	Chi. Union Trac. Co. Case.
ties	20		Trac. Val. Com.	Chi. Con. Trac. Co. Case.
railroad rails, new	25	26%	Cal. R. R. Com.	Stockton Term. & East. Case.
railroad rails, relay	15	48%	Cal. R. R. Com.	Stockton Term. & East. Case.
R. R. rails and track fast.	40		Gillette	Gt. Nor. & N. P. Appraisal.
railroad track fastenings	12½		Cal. R. R. Com.	Stockton Term. & East. Case.
railroad ties	8		Gillette	Gt. Nor. and N. P. Appraisal.
railroad ties, redwood	16		Cal. R. R. Com.	Stockton Term. & East. Case.
railroad ties, creosoted	19.35		O. Chanute	Houston & Texas Cent. R. R.
trestles and bridges	10		Cal. R. R. Com.	Stockton Term. & East. Case.
ballast	10		Cal. R. R. Com.	Stockton Term. & East. Case.
frogs and switches	25	36%	Cal. R. R. Com.	Stockton Term. & East. Case.
cast-iron culverts	16		Cal. R. R. Com.	Stockton Term. & East. Case.
Transformers	15		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
	15		Wis. R. R. Com.	Quoted by Henry Floy.
station service	20		Wis. R. R. Com.	Quoted by Henry Floy.
station	25		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
line	20	10%	Cal. R. R. Com.	Calis. E. C. *vs.* Napa Val. E. Co.
distribution	20	$1.00 kw.	P. G. & E. Co.	S. F. rate hearings, 1913–14.
power	20	$ .50 kw.	P. G. & E. Co.	S. F. rate hearings, 1913–14.
static transformers	15		Hammond	Jour. Inst. E. E., 1907.
Tunnels				
power plants	50		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
Turbines				
steam	15		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
steam	20		Wis. R. R. Com.	Quoted by Henry Floy.
steam	20		Hammond	Jour. Inst. Elec. E., 1907.
steam	30	1¢ lb.	P. G. & E. Co.	S. F. rate hearings, 1913-14.
water	30		Wis. R. R. Com.	Quoted by Henry Floy.
water-type prior 1900	25–40		Foster	"Eng. Valuation," etc.
water-type after 1900	30–50		Foster	"Eng. Valuation," etc.
Washers (gas plant)				
cast iron	40		Wis. R. R. Com.	Quoted by H. A. Foster.
Water gas machines	30		Wis. R. R. Com.	Quoted by H. A. Foster.
Watt meters				
service	10–15		Foster	"Eng. Valuation," etc.
Wells				
water	30		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
water, driven or drilled	50–75		Foster	"Eng. Valuation," etc.
water, large open masonry	75-100		Foster	"Eng. Valuation," etc.
water well pumps	10		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearings.
gas wells	10	0%	Cal. R. R. Com.	Midway Gas Co. *et al.* Case.
gas well drilling equip.	10	25%	Cal. R. R. Com.	Midway Gas Co. *et al.* Case.
Wharves and docks	33⅓		Gillette	Gt. Nor. & N. P. Appraisal.
Wire				
aerial	20		St. L. P. S. C.	Union Elec. L. & P. Co. Case.
aerial copper	15		Chi. Tel. Com.	Chicago Telephone Com.

TABLE 19. THE PROBABLE USEFUL LIFE OF VARIOUS ARTICLES (*Concluded*)

Description.	Expectancy.	Residual value.	Authority.	Remarks.
	Years			
Wire (continued)				
bare	25	35%	P. G. & E. Co.	S. F. rate hearings, 1913-14.
insulated	20	15%	P. G. & E. Co.	S. F. rate hearings, 1913-14.
copper, weatherproof	13		Arbitrators	Ata. St. Lgt. Controversy.
copper, weatherproof	16		Wis. R. R. Com.	Quoted by Henry Floy.
copper, weatherproof	10-15		Foster	"Eng. Valuation," etc.
2-kv. distribution	15		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
4-kv. distribution	15		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
10-kv. distribution	20		S. J. L. & P. Cor.	Cal. R. R. Com. rate hearing.
telephone, iron	10	0%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.
telephone, iron	8-15		Wis. R. R. Com.	Quoted by H. A. Foster.
telephone, weatherproof, iron	15		Wis. R. R. Com.	Quoted by H. A. Foster.
telephone, copper	20	75%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.
telephone exch., bare copper	10-15	40%	Tel. "Data"	Quoted by H. A. Foster.
telephone exch., insul. cop.	10	10%	Tel. "Data"	Quoted by H. A. Foster.
telephone exch., bare iron	8-10	0%	Tel. "Data"	Quoted by H. A. Foster.
telephone toll, bare copper	40	40%	Tel. "Data"	Quoted by H. A. Foster.
telephone toll, bare iron	15	0%	Tel. "Data"	Quoted by H. A. Foster.
telephone, distribution	10	15%	Wis. R. R. Com.	Wisconsin Tel. Co. Case.

EXPLANATION OF TABLE 20

Expectancy and Remaining Value

(Approximate Values Only)

The expectancy of any article in use, which is still in serviceable condition, can be approximated from the following tables, when the probable life new of other articles of the same class is known. There is noted in these tables also the approximate remaining value, although this may also be obtained from the Amortization and Depreciation Table 27 by entering the same with the years of expectancy as elsewhere explained. The expectancy noted in Table 20 is based on a reasonable assumption of the rate at which a large number of articles of the same probable life would go out of use (see Chapter VI) and is in fair accord with the expectancy that would be found if all the articles were assumed to fail according to the law of probabilities within a period equal to twice the probable life term. The strict application of the law of probability under assumptions, relating not only to the probable life of any article in question, but also to the probability of survivals beyond some period, as, for example, twice the probable life, would be an unnecessary refinement not justified until a vastly greater mass of data than now available, relating to actual life of individual articles in each class, has been assembled.

It may be claimed for this table in its present form that it will afford means of making much closer approximation to the actual remaining worth of any article than is possible by the use of the ordinary amortization tables in which age alone is taken into account and in which no distinction is made between actual and probable life.

In this table the expectancy and remaining value are noted for any single article, whose probable life when new is known. The table has been computed for 4 per cent, 5 per cent, 6 per cent and 7 per cent interest per annum. For other interest rates, values may be interpolated.

In Tables 4 to 7, Chapter VI, the annual replacement requirement is noted for groups of numerous articles, all of the same probable life new. These tables cover the two cases of a plant which has attained its full growth and of the plant to which a uniform annual addition is being made. The annual replacement requirement is based on the probable annual number of failures. Similar tables can readily be prepared for other terms of usefulness than 5, 10 and 20 years covered by these tables and for any other hypothesis of the rate at which actual failures will take place.

All figures noted in Tables 4 to 6 and in Table 20 are approximation figures derived from smoothed-out curves. They are, in fact, modifications of the values which were obtained as a result of the assumption already fully discussed that failures among any large group of articles will be greatest in number at or near the end of the probable life term, that practically no articles will survive twice the probable life term, and that there will be a uniform increase in the annual rate of failures from the beginning to the year of maximum number of failures and that the decrease in the number of annual failures will follow a similar law.

To find the expectancy of any perishable article which has a probable life term new of n years, not covered by any of the subdivisions of the table here published, the ten-year life subdivision of Table 20 may be called to aid.

Let e = the expectancy which is to be determined,
m = the age of the article,
n = the probable life new of the article,
m' = the relative age of an article whose probable life new was 10 years such that

$$m' = \frac{10\,m}{n}. \tag{19}$$

Let e' = the relative expectancy at the age m' of an article whose probable life new was 10 years.

Then the expectancy to be determined will be found from

$$e = \frac{ne'}{10}. \tag{20}$$

Example. — What is the expectancy of an article 8 years old, in fair condition, whose probable life new was 12 years? Here $n = 12$ and $m = 8$.

$$m' = \frac{10 \times 8}{12} = 6.67.$$

For the age 6.67 years in the ten-year subdivision of Table 20 (this being in the seventh year) the expectancy 5.17 years is found by interpolation. The required expectancy is, therefore,

$$e = 5.17 \times \frac{12}{10} = 6.2 \text{ years,}$$

which would be found in a twelve-year probable life table for the end of the eighth year or for the beginning of the ninth year.

For general use the following formulæ are recommended which will be found to agree fairly well with the results presented in Table 20. They are applicable to any probable life term and any age to the limit beyond which it has been assumed no article will continue in service.

When the age of an article is less than its probable life term new, that is, when $m < n$

$$e = n - 0.93\,m + 0.30\frac{m^2}{n}. \tag{21}$$

When the age of an article is greater than its probable life term new, that is, when $m > n$

$$e = 0.72\,n - 0.35\,m. \tag{22}$$

Example. — What is the expectancy of an article whose probable term of usefulness when new was 12 years, which is 8 years old and apparently in good condition?

Here $n = 12$ and $m = 8$.

By equation (21)

$$e = 12 - 7.44 + 1.60 = 6.16 \text{ years.}$$

Example. — What is the expectancy of a similar article 15 years old, which has survived its probable term of usefulness but is still in service and in good condition?

Here $n = 12$ and $m = 15$.

By equation (22)

$$e = 8.64 - 5.25 = 3.39 \text{ years.}$$

Example. — What is the expectancy of an article 40 years old, whose probable life new was 60 years and which is in good condition?

Here $n = 60$ and $m = 40$.

By equation (21)

$$e = 60 - 37.20 + 8.00 = 30.80 \text{ years.}$$

To find the remaining value of an original investment of $100 in any article or the accrued depreciation when the expectancy of the article is known and the probable life new is different from any covered by the tables here published, proceed as follows:

Find the annuity which in n years will amount to $100. From tables, such as Tables 20 and 27 in this volume, or by calculation, find the amount of this annuity for $(n - e)$ years. This amount will represent the accrued depreciation and $100 less this amount will be the remaining value of an original investment.

Example. — What is the remaining value (6 per cent interest) of an article which cost $100, whose probable life new was 12 years, which is 8 years old and which is still in fair condition?

The expectancy of this article as shown in a preceding example is about 6.2 years. The annuity which in 12 years at 6 per cent will amount to $100 is $5.93; this annuity in (12 – 6.2) years amounts to $40.44, the accrued depreciation. The present worth will be $100 – $40.44 = $59.56.

By use of the ten- and fifteen-year subdivisions of the table and interpolation the procedure would be as follows:

Eight years in a twelve-year probable life table is fairly comparable with 6.67 years in a ten-year table and with 10 years in a fifteen-year table.

Remaining value (at 6 per cent):

For age 6.67 years from the ten-year table	$58.80
For age 10 years from the fifteen-year table	62.30
Difference	$ 3.50

or approximately $58.80 + \frac{2}{5} \times 3.50 = \$60.20$.

It remains to be noted that when remaining value is to be determined age may be entirely disregarded when expectancy can be determined by an inspection of the article in question. The condition of the article and all other circumstances affecting its serviceability being taken into consideration its probable remaining term of service or its expectancy is estimated. In this case it will be more convenient to use Table 27 than Table 20.

Select the subdivision of Table 27, corresponding with the probable life new of the article, and use only the headings of the columns at the bottom of the page. For the expectancy noted in the right-hand column, the remaining value and the accrued depreciation will be found in their respective columns. Interpolation may be resorted to for fractional years if this refinement be thought desirable.

Because expectancy in the case of individualized articles which have been long in use can best be determined by inspection and by a consideration of local circumstances affecting continued serviceability, it has been thought sufficient to note values in the long-term subdivisions of Table 20 and for articles long in service only for every fifth or every tenth year.

To find the current depreciation of any single article in any year by the use of Table 20 subtract from its remaining value at the beginning of that year the remaining value at the beginning of the following year.

Example. — What is the current depreciation in the sixth year, at 6 per cent interest, of an article whose probable life is 15 years?

Remaining value at the beginning of the sixth year	$76.90
Remaining value at the beginning of the seventh year	72.60
Depreciation in the sixth year	$ 4.30

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100

All values in this table are based on the assumption that in a group of many articles, no article will survive twice its probable life and that the number of annual failures will increase at a uniform rate to a maximum at the end of the probable life term and will thereupon decrease at a uniform rate.

Probable Life 5 Years

5 Years | 5 Years

Beginning of year.	Expectancy years.	Remaining value. 4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
1	5.00	$100.00	$100.00	$100.00	$100.00
2	4.15	84.30	84.60	84.90	85.20
3	3.40	70.00	70.50	71.00	71.50
4	2.75	57.30	57.90	58.40	59.00
5	2.25	47.40	47.90	48.50	49.10
6	1.85	39.30	39.80	40.30	40.90
7	1.55	33.00	33.50	34.00	34.50
8	1.25	26.80	27.20	27.60	28.00
9	1.00	21.60	22.00	22.40	22.80

Probable Life 10 Years

10 Years | 10 Years

1	10.00	100.00	100.00	100.00	100.00
2	9.10	92.50	92.90	93.20	93.50
3	8.25	85.20	85.80	86.40	87.00
4	7.50	78.50	79.30	80.10	81.00
5	6.80	72.10	73.10	74.00	75.00
6	6.10	65.60	66.70	67.70	68.80
7	5.50	59.90	60.90	62.00	63.10
8	5.00	54.90	56.10	57.20	58.40
9	4.50	49.80	51.00	52.20	53.30
10	4.10	45.90	46.90	48.10	49.20
11	3.70	41.60	42.70	43.90	45.00
12	3.35	37.90	39.00	40.10	41.20
13	3.00	34.20	35.30	36.30	37.40
14	2.65	30.40	31.40	32.30	33.30
15	2.30	26.50	27.40	28.30	29.30
16	2.00	23.30	24.10	24.90	25.70
17	1.65	19.50	20.20	20.90	21.60
18	1.35	15.70	16.30	16.90	17.50
19	1.00	11.90	12.30	12.80	13.30

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100 (*Continued*)

PROBABLE LIFE 15 YEARS

15 YEARS 15 YEARS

Beginning of year.	Expectancy years.	Remaining value.			
		4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	4 per cent per annum.
1	15.00	$100.00	$100.00	$100.00	$100.00
2	14.10	95.50	95.80	96.10	96.40
3	13.20	90.90	91.50	92.10	92.60
4	12.40	86.60	87.40	88.30	89.00
5	11.65	82.40	83.50	84.50	85.50
6	10.90	78.20	79.50	80.70	81.80
7	10.15	73.90	75.30	76.70	78.00
8	9.45	69.60	71.10	72.60	74.00
9	8.85	65.90	67.50	69.10	70.60
10	8.25	62.10	63.80	65.50	67.10
11	7.75	58.90	60.60	62.30	64.00
12	7.25	55.60	57.40	59.10	60.80
13	6.75	52.30	54.00	55.80	57.50
14	6.30	49.20	51.00	52.70	54.40
15	5.90	46.40	48.20	49.90	51.60
16	5.55	44.00	45.70	47.40	49.10
17	5.20	41.50	43.20	44.80	46.50
18	4.85	38.90	40.60	42.20	43.80
19	4.50	36.40	37.90	39.50	41.10
20	4.15	33.70	35.30	36.80	38.40
21	3.80	31.10	32.60	34.00	35.50
22	3.45	28.40	29.80	31.20	32.60
23	3.10	25.70	27.00	28.30	29.60
24	2.75	22.90	24.20	25.40	26.60
25	2.40	20.20	21.20	22.30	23.40
26	2.05	17.40	18.30	19.30	20.30
27	1.70	14.50	15.30	16.10	17.00
28	1.35	11.60	12.20	12.90	13.60
29	1.00	8.60	9.20	9.70	10.30

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100 (*Continued*)

PROBABLE LIFE 20 YEARS

20 YEARS 20 YEARS

Beginning of year.	Expectancy years.	Remaining value.			
		4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
1	20.00	$100.00	$100.00	$100.00	$100.00
2	19.10	97.00	97.30	97.60	97.80
3	18.20	93.90	94.40	95.00	95.50
4	17.35	90.80	91.60	92.40	93.10
5	16.50	87.60	88.70	89.70	90.70
6	15.75	84.80	86.10	87.30	88.40
7	15.00	81.80	83.30	84.70	86.00
8	14.30	79.00	80.60	82.10	83.60
9	13.60	76.00	77.80	79.50	81.10
10	12.90	73.00	75.00	76.80	78.50
11	12.20	69.90	72.00	73.90	75.80
12	11.60	67.20	69.30	71.40	73.30
13	11.00	64.50	66.70	68.80	70.80
14	10.50	62.10	64.30	66.50	68.50
15	10.00	59.70	62.00	64.20	66.30
16	9.50	57.20	59.50	61.70	63.90
17	9.00	54.70	57.00	59.30	61.50
18	8.60	52.60	55.00	57.20	59.40
19	8.20	50.60	52.90	55.20	57.40
20	7.80	48.50	50.80	53.00	55.30
21	7.40	46.30	48.60	50.90	53.10
22	7.00	44.20	46.40	48.70	50.90
23	6.60	41.90	44.20	46.40	48.50
24	6.25	40.00	42.20	44.30	46.50
25	5.90	38.00	40.10	42.30	44.40
26	5.55	36.00	38.00	40.10	42.20
27	5.20	33.90	35.90	38.00	40.00
28	4.85	31.90	33.90	35.80	37.70
29	4.50	29.70	31.60	33.50	35.30
30	4.15	27.60	29.40	31.20	33.00
31	3.80	25.40	27.10	28.80	30.50
32	3.45	23.30	24.80	26.40	28.00
33	3.10	21.10	22.50	24.00	25.50
34	2.75	18.80	20.10	21.50	22.90
35	2.40	16.50	17.70	18.90	20.20
36	2.05	14.20	15.30	16.40	17.50
37	1.70	11.80	12.70	13.70	14.60
38	1.35	9.50	10.20	11.00	11.70
39	1.00	7.10	7.60	8.20	8.80

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100 (*Continued*)

PROBABLE LIFE 25 Years

25 YEARS | 25 YEARS

Beginning of year.	Expectancy years.	Remaining value.			
		4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
1	25.00	$100.00	$100.00	$100.00	$100.00
2	24.10	97.80	98.10	98.40	98.60
3	23.20	95.60	96.10	96.60	97.10
4	22.30	93.30	94.10	94.80	95.40
5	21.45	91.00	92.00	92.90	93.80
6	20.60	88.70	90.00	91.10	92.10
7	19.80	86.40	87.90	89.20	90.50
8	19.05	84.20	85.90	87.40	88.80
9	18.30	81.90	83.80	85.50	87.20
10	17.60	79.80	81.80	83.60	85.30
11	16.90	77.60	79.70	81.70	83.50
12	16.20	75.20	77.50	79.60	81.60
13	15.55	73.00	75.45	77.70	79.80
14	14.90	70.80	73.30	75.60	77.80
15	14.30	68.70	71.20	73.70	76.00
16	13.75	66.70	69.30	71.90	74.20
17	13.20	64.70	67.30	69.90	72.40
18	12.70	62.80	65.50	68.10	70.60
19	12.20	61.80	63.60	66.30	68.90
20	11.70	58.90	61.70	64.40	67.00
21	11.25	57.10	59.90	62.70	65.30
22	10.80	55.20	58.10	60.90	63.50
23	10.40	53.60	56.40	59.20	61.90
24	10.00	51.90	54.80	57.60	60.30
25	9.60	50.20	53.00	55.80	58.50
26	9.25	48.70	51.50	54.30	57.00
27	8.90	47.10	49.90	52.70	55.40
28	8.50	45.30	48.10	50.90	53.60
29	8.15	43.80	46.50	49.30	51.90
30	7.80	42.20	44.90	47.60	50.10
31	7.40	40.30	43.00	45.60	48.20
36	5.60	31.50	33.90	36.30	38.60
41	3.80	22.10	24.00	25.90	27.80
46	2.05	12.40	13.50	14.70	15.90
49	1.00	6.20	6.80	7.40	8.00

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100 (*Continued*)

PROBABLE LIFE 30 YEARS

30 YEARS | 30 YEARS

Beginning of year.	Expectancy years.	Remaining value.			
		4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
1	30.00	$100.00	$100.00	$100.00	$100.00
2	29.10	98.40	98.60	98.90	99.00
3	28.20	96.70	97.20	97.70	98.00
4	27.30	95.00	95.80	96.40	97.00
5	26.45	93.30	94.30	95.20	95.90
6	25.60	91.60	92.80	93.80	94.70
7	24.80	89.90	91.30	92.50	93.60
8	24.00	88.20	89.80	91.20	92.40
9	23.30	86.60	88.40	89.90	91.30
10	22.55	84.90	86.80	88.50	90.10
11	21.80	83.10	85.20	87.10	88.80
12	21.05	81.30	83.50	85.60	87.40
13	20.30	79.40	81.80	84.00	86.00
14	19.60	77.50	80.10	82.40	84.50
15	18.90	75.70	78.40	80.80	83.10
16	18.25	73.90	76.70	79.30	81.60
17	17.65	72.20	75.10	77.80	80.20
18	17.05	70.50	73.50	76.40	78.80
19	16.50	68.90	71.90	74.80	77.40
20	15.95	67.20	70.40	73.30	76.00
21	15.40	65.50	68.70	71.70	74.50
22	14.90	64.00	67.20	70.30	73.10
23	14.40	62.40	65.70	68.70	71.70
24	13.95	60.90	64.20	67.40	70.30
25	13.50	59.40	62.70	65.90	68.90
26	13.05	57.60	60.90	64.50	67.50
27	12.60	56.50	59.70	62.90	66.00
28	12.20	55.00	58.40	61.60	64.70
29	11.80	53.60	56.90	60.20	63.30
30	11.45	52.30	55.70	58.90	62.00
31	11.10	51.00	54.40	57.70	60.80
32	10.75	49.70	53.10	56.30	59.50
33	10.40	48.40	51.70	55.00	58.10
34	10.00	46.90	50.20	53.50	56.60

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100 (*Continued*)

PROBABLE LIFE 30 YEARS

30 YEARS | 30 YEARS

Beginning year.	Expectancy years.	Remaining value.			
		4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
35	9.65	$45.50	$48.80	$52.10	$55.20
36	9.30	44.20	47.40	50.60	53.70
37	8.95	42.80	46.00	49.20	52.30
38	8.55	41.20	44.30	47.50	50.50
39	8.20	39.70	42.90	46.00	49.00
40	7.85	38.30	41.40	44.40	47.40
41	7.50	36.80	39.80	42.80	45.80
46	5.70	28.90	31.60	34.10	36.80
51	3.90	20.50	22.60	24.60	26.70
56	2.10	11.40	12.70	13.90	15.20
59	1.00	5.60	6.20	6.90	7.50

PROBABLE LIFE 40 YEARS

40 YEARS | 40 YEARS

1	40.00	100.00	100.00	100.00	100.00
2	39.10	99.10	99.30	99.40	99.60
3	38.20	98.10	98.50	98.80	99.10
4	37.30	97.10	97.70	98.20	98.60
5	36.40	96.00	96.80	97.50	98.00
6	35.50	94.90	95.90	96.80	97.50
7	34.70	93.90	95.10	96.10	96.90
8	33.90	92.90	94.30	95.40	96.40
9	33.10	91.80	93.40	94.70	95.80
10	32.30	90.70	92.50	93.90	95.10
11	31.50	89.60	91.50	93.10	94.40
12	30.75	88.50	90.60	92.30	93.80
13	30.00	87.40	89.60	91.50	93.10
14	29.30	86.30	88.60	90.70	92.40
15	28.55	85.10	87.60	89.80	91.60
16	27.80	83.80	86.50	88.80	90.80
17	27.10	82.70	85.50	87.90	90.00
18	26.40	81.50	84.40	87.00	89.20
19	25.70	80.20	83.30	86.00	88.30
20	25.00	78.90	82.10	85.00	87.40
21	24.40	77.80	81.10	84.00	86.60
22	23.80	76.60	80.10	83.10	85.70
23	23.20	75.50	79.00	82.10	84.90
24	22.60	74.20	77.90	81.10	83.90

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100 (*Continued*)

PROBABLE LIFE 40 YEARS

40 YEARS | 40 YEARS

Beginning of year.	Expectancy years.	Remaining value.			
		4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
25	22.00	$73.00	$76.70	$80.00	$83.00
26	21.45	71.80	75.60	79.00	82.00
27	20.95	70.80	74.60	78.10	81.20
28	20.45	69.70	73.60	77.10	80.30
29	19.95	68.60	72.50	76.10	79.40
30	19.45	67.40	71.40	75.10	78.40
31	19.00	66.40	70.40	74.20	77.50
36	16.80	60.90	65.20	69.10	72.80
41	14.80	55.60	59.90	64.00	67.80
46	12.85	50.00	54.30	58.40	62.20
51	10.95	44.10	48.20	52.30	56.10
56	9.10	37.90	41.80	45.60	49.30
61	7.30	31.40	34.90	38.40	41.70
66	5.55	24.70	27.60	30.60	33.50
71	3.80	17.50	19.70	22.00	24.30
76	2.05	9.70	11.10	12.50	13.90
79	1.00	4.90	5.60	6.30	7.00

PROBABLE LIFE 50 YEARS

50 YEARS | 50 YEARS

1	50.00	100.00	100.00	100.00	100.00
2	49.05	99.40	99.50	99.70	99.80
3	48.15	98.70	99.10	99.40	99.50
4	47.25	98.10	98.60	99.00	99.30
5	46.35	97.50	98.10	98.60	99.00
6	45.45	96.80	97.70	98.30	98.70
7	44.60	96.00	97.00	97.80	98.40
8	43.75	95.40	96.60	97.50	98.10
9	42.90	94.70	96.00	97.10	97.80
10	42.10	94.10	95.50	96.60	97.50
11	41.30	93.30	94.90	96.20	97.20
12	40.50	92.60	94.40	95.80	96.80
13	39.75	91.90	93.80	95.30	96.50
14	39.00	91.20	93.20	94.80	96.10
15	38.25	90.40	92.60	94.40	95.70

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100 (*Continued*)

PROBABLE LIFE 50 YEARS

50 YEARS — 50 YEARS

Beginning of year.	Expectancy years.	Remaining value.			
		4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
16	37.50	$89.60	$92.00	$93.80	$95.30
17	36.75	88.80	91.30	93.30	94.90
18	36.00	88.00	90.60	92.80	94.50
19	35.30	87.20	90.00	92.20	94.00
20	34.60	86.40	89.30	91.70	93.50
21	33.90	85.60	88.60	91.10	93.10
22	33.20	84.70	87.90	90.50	92.60
23	32.50	83.80	87.10	89.80	92.00
24	31.80	82.90	86.30	89.20	91.50
25	31.15	82.10	85.60	88.50	90.90
26	30.50	81.20	84.80	87.90	90.40
27	29.90	80.30	84.10	87.20	89.80
28	29.30	79.50	83.30	86.60	89.20
29	28.70	78.60	82.50	85.90	88.70
30	28.10	77.70	81.70	85.20	88.00
31	27.50	76.80	80.90	84.40	87.40
36	24.85	72.50	77.00	80.90	84.20
41	22.45	68.10	72.90	77.10	80.80
46	20.35	64.00	69.00	73.40	77.40
51	18.50	60.00	65.10	69.70	73.90
61	14.80	51.20	56.30	61.10	65.50
71	11.10	41.10	45.80	50.30	54.60
81	7.50	29.60	33.50	37.40	41.20
91	3.90	16.50	19.00	21.50	24.00
99	1.00	4.50	5.20	6.00	6.80

PROBABLE LIFE 60 YEARS

60 YEARS — 60 YEARS

1	60.00	100.00	100.00	100.00	100.00
2	59.10	99.60	99.70	99.80	99.90
3	58.15	99.20	99.50	99.60	99.80
4	57.25	98.80	99.20	99.50	99.60
5	56.35	98.40	98.90	99.30	99.50
6	55.45	97.90	98.60	99.10	99.40
7	54.60	97.50	98.30	98.80	99.20
8	53.75	97.10	98.00	98.60	99.10
9	52.90	96.60	97.70	98.40	98.90
10	52.05	96.20	97.30	98.20	98.80

TABLE 20. EXPECTANCY AND REMAINING VALUE OF ANY ARTICLE WHOSE REPLACEMENT COST IS $100 (*Concluded*)

PROBABLE LIFE 60 YEARS

60 YEARS | 60 YEARS

Beginning of year.	Expectancy years.	Remaining value.			
		4 per cent per annum.	5 per cent per annum.	6 per cent per annum.	7 per cent per annum.
11	$51.20	$95.70	$97.00	$97.90	$98.60
12	50.40	95.20	96.60	97.70	98.40
13	49.50	94.60	96.20	97.40	98.20
14	48.75	94.10	95.90	97.10	98.00
15	48.00	93.70	95.50	96.80	97.80
16	47.25	93.20	95.10	96.60	97.60
17	46.50	92.70	94.70	96.30	97.40
18	45.75	92.10	94.30	95.90	97.10
19	45.00	91.60	93.90	95.60	96.90
20	44.25	91.00	93.50	95.30	96.60
21	43.50	90.40	93.00	94.90	96.40
22	42.75	89.80	92.50	94.60	96.10
23	42.00	89.20	92.00	94.20	95.80
24	41.30	88.60	91.60	93.80	95.50
25	40.60	88.00	91.10	93.40	95.20
26	39.90	87.40	90.60	93.00	94.90
27	39.20	86.80	90.10	92.60	94.60
28	38.55	86.10	89.50	92.20	94.30
29	37.90	85.50	89.00	91.80	93.90
30	37.25	84.90	88.50	91.30	93.60
31	36.60	84.20	87.90	90.90	93.20
36	33.60	80.90	85.10	88.60	91.30
41	30.80	77.50	82.20	86.10	89.20
46	28.30	74.10	79.10	83.30	86.80
51	26.10	70.80	76.10	80.60	84.30
56	24.10	67.60	73.10	77.80	81.80
61	22.20	64.20	69.90	74.80	79.10
71	18.50	57.00	62.80	68.00	72.60
81	14.80	48.70	54.30	59.60	64.40
91	11.10	39.00	44.20	49.10	53.10
101	7.50	28.10	32.40	36.50	40.50
111	3.90	15.70	18.30	21.00	23.60
119	1.00	4.30	5.00	5.80	6.70

EXPLANATION OF TABLE 21

Compound Interest

Amount of one dollar at interest compounded annually

In Table 21 the amount is given of one dollar at interest rates from 2 to 10 per cent, interest being compounded annually for any number of years up to 100.

The formula on which this table is based is the following:

Let A' represent the amount at the end of nth year of one dollar plus interest compounded annually.

Let n represent the number of years.

Let i represent the interest rate expressed decimally as 0.05 for 5 per cent.

Then $$A' = (1 + i)^n \qquad (23)$$

Example. — What is the amount in 7 years of $400 at 6 per cent interest compounded annually?

From Table 21 the amount of $1 at 6 per cent interest in 7 years is found to be $1.503630 and the amount of $400, therefore

$$\$400 \times \$1.503630 = \$601.45.$$

The section of Table 21 covering interest rates in excess of 10 per cent will be found serviceable for certain special purposes such as the determination of the present value of mining properties. This section of the table has been abbreviated by the omission of certain individual years but it can nevertheless be made to serve in finding the amount of $1 at compound interest for any year by going into the table with any two years whose sum is equal to the number of years for which the amount is to be determined and obtaining the product of values found in the table for these two years. This product will be the required amount.

Example. — What is the amount of $1 at 15 per cent compound interest in 46 years?

The amount of $1 for 16 years at 15 per cent is found to be $9.357621.

The amount of \$1 for 30 years at 15 per cent is found to be \$66.211772.

Then $$9.357621 \times 66.211772 = \$619.58,$$

which is the amount of \$1 at 15 per cent in 46 years.

While occasion may rarely arise when it is necessary to know the amount of \$1 at these high rates of interest for a long term of years this section of the table is valuable for other purposes. It can be used to determine the present value of \$1 due at a future date; the amount of an annuity of \$1, and also the present value of an annuity of \$1.

The method of use for these purposes is explained in connection with the respective tables which follow.

To Illustrate. — What is the present value of a mine yielding \$10,000 net per annum whose estimated life is 10 years and which, owing to the hazards of the enterprise, should yield a net return of 20 per cent per annum?

From Table 21 it is found that \$1 at 20 per cent compound interest will amount to \$6.191736 in 10 years.

According to formula (27) in the explanation of Table 23 the amount of an annuity of \$1 for 10 years will be

$$\frac{6.191736 - 1}{0.20} = \$25.95868.$$

And by formula (29) in the explanation of Table 24 the present value of an annuity of \$1 for 10 years

$$\frac{25.95868}{6.191736} = \$4.192149.$$

Consequently the value of the mine equipped to produce the net annual amount of \$10,000:

$$10,000 \times 4.192149 = \$41,921.49.$$

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY

(Amounts are noted for the end of each year)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	$1.020 000	$1.025 000	$1.030 000	$1.035 000	$1.040 000	$1.045 000	$1.050 000
2	1.040 400	1.050 625	1.060 900	1.071 225	1.081 600	1.092 025	1.102 500
3	1.061 208	1.076 891	1.092 727	1.108 718	1.124 864	1.141 166	1.157 625
4	1.082 432	1.103 813	1.125 509	1.147 523	1.169 859	1.192 519	1.215 506
5	1.104 080	1.131 408	1.159 274	1.187 686	1.216 653	1.246 182	1.276 282
6	1.126 162	1.159 693	1.194 052	1.229 255	1.265 319	1.302 260	1.340 096
7	1.148 686	1.188 686	1.229 874	1.272 279	1.315 932	1.360 862	1.407 100
8	1.171 659	1.218 403	1.266 770	1.316 809	1.368 569	1.422 101	1.477 455
9	1.195 093	1.248 863	1.304 773	1.362 897	1.423 312	1.486 095	1.551 328
10	1.218 994	1.280 085	1.343 916	1.410 599	1.480 244	1.552 969	1.628 895
11	1.243 374	1.312 087	1.384 234	1.459 970	1.539 454	1.622 853	1.710 339
12	1.268 242	1.344 889	1.425 761	1.511 069	1.601 032	1.695 881	1.795 856
13	1.293 607	1.378 511	1.468 534	1.563 956	1.665 074	1.772 196	1.885 649
14	1.319 479	1.412 974	1.512 590	1.618 695	1.731 676	1.851 945	1.979 932
15	1.345 868	1.448 298	1.557 967	1.675 349	1.800 944	1.935 282	2.078 928
16	1.372 786	1.484 506	1.604 706	1.733 986	1.872 981	2.022 370	2.182 875
17	1.400 241	1.521 618	1.652 848	1.794 676	1.947 900	2.113 377	2.292 018
18	1.428 246	1.559 659	1.702 433	1.857 489	2.025 817	2.208 479	2.406 619
19	1.456 811	1.598 650	1.753 506	1.922 501	2.106 849	2.307 860	2.526 950
20	1.485 947	1.638 616	1.806 111	1.989 789	2.191 123	2.411 714	2.653 298
21	1.515 666	1.679 582	1.860 295	2.059 431	2.278 768	2.520 241	2.785 963
22	1.545 980	1.721 571	1.916 103	2.131 512	2.369 919	2.633 652	2.925 261
23	1.576 899	1.764 611	1.973 587	2.206 114	2.464 716	2.752 166	3.071 524
24	1.608 437	1.808 726	2.032 794	2.283 328	2.563 304	2.876 014	3.225 100
25	1.640 606	1.853 944	2.093 778	2.363 245	2.665 836	3.005 434	3.386 355

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (*Continued*)

(Amounts are noted for the end of each year)

Year.	2 per cent.	$2\frac{1}{2}$ per cent.	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
26	$1.673 418	$1.900 293	$2.156 591	$2.445 959	$2.772 470	$3.140 679	$ 3.555 673
27	1.706 886	1.947 800	2.221 289	2.531 567	2.883 369	3.282 010	3.733 456
28	1.741 024	1.996 495	2.287 928	2.620 172	2.998 703	3.429 700	3.920 129
29	1.775 845	2.046 407	2.356 566	2.711 878	3.118 651	3.584 036	4.116 136
30	1.811 362	2.097 568	2.427 262	2.806 794	3.243 398	3.745 318	4.321 942
31	1.847 589	2.150 007	2.500 080	2.905 031	3.373 133	3.913 857	4.538 039
32	1.884 541	2.203 757	2.575 083	3.006 708	3.508 059	4.089 981	4.764 941
33	1.922 231	2.258 851	2.652 335	3.111 942	3.648 381	4.274 030	5.003 189
34	1.960 676	2.315 322	2.731 905	3.220 860	3.794 316	4.466 362	5.253 348
35	1.999 890	2.373 205	2.813 862	3.333 590	3.946 089	4.667 348	5.516 015
36	2.039 887	2.432 535	2.898 278	3.450 266	4.103 933	4.877 378	5.791 816
37	2.080 685	2.493 349	2.985 227	3.571 025	4.268 090	5.096 860	6.081 407
38	2.122 299	2.555 682	3.074 783	3.696 011	4.438 813	5.326 219	6.385 477
39	2.164 745	2.619 574	3.167 027	3.825 372	4.616 366	5.565 899	6.704 751
40	2.208 040	2.685 064	3.262 038	3.959 260	4.801 021	5.816 365	7.039 989
41	2.252 200	2.752 190	3.359 899	4.097 834	4.993 061	6.078 101	7.391 988
42	2.297 244	2.820 995	3.460 696	4.241 258	5.192 784	6.351 615	7.761 588
43	2.343 189	2.891 520	3.564 517	4.389 702	5.400 495	6.637 438	8.149 667
44	2.390 053	2.963 808	3.671 452	4.543 342	5.616 515	6.936 123	8.557 150
45	2.437 854	3.037 903	3.781 596	4.702 359	5.841 176	7.248 248	8.985 008
46	2.486 611	3.113 851	3.895 044	4.866 941	6.074 823	7.574 420	9.434 258
47	2.536 344	3.191 697	4.011 895	5.037 284	6.317 816	7.915 268	9.905 971
48	2.587 070	3.271 490	4.132 252	5.213 589	6.570 528	8.271 456	10.401 270
49	2.638 812	3.353 277	4.256 219	5.396 065	6.833 349	8.643 671	10.921 333
50	2.691 588	3.437 109	4.383 906	5.584 927	7.106 683	9.032 636	11.467 400

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (*Continued*)

(Amounts are noted for the end of each year)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
51	$2.745 420	$3.523 036	$4.515 423	$ 5.780 399	$ 7.390 951	$ 9.439 105	$12.040 770
52	2.800 328	3.611 112	4.650 886	5.982 713	7.686 589	9.863 865	12.642 808
53	2.856 335	3.701 390	4.790 412	6.192 108	7.994 052	10.307 739	13.274 949
54	2.913 461	3.793 925	4.934 125	6.408 832	8.313 814	10.771 587	13.938 696
55	2.971 731	3.888 773	5.082 149	6.663 141	8.646 367	11.256 308	14.635 631
56	3.031 165	3.985 992	5.234 613	6.865 301	8.992 222	11.762 842	15.367 412
57	3.091 789	4.085 642	5.391 651	7.105 587	9.351 910	12.292 170	16.135 783
58	3.153 624	4.187 783	5.553 401	7.354 282	9.725 987	12.845 318	16.942 572
59	3.216 697	4.292 478	5.720 003	7.611 682	10.115 026	13.423 357	17.789 701
60	3.281 031	4.399 790	5.891 603	7.878 091	10.519 627	14.027 408	18.679 186
61	3.346 651	4.509 784	6.068 351	8.153 824	10.940 413	14.658 641	19.613 145
62	3.413 584	4.622 529	6.250 402	8.439 208	11.378 029	15.318 280	20.593 802
63	3.481 856	4.738 092	6.437 914	8.734 580	11.833 150	16.007 603	21.623 493
64	3.551 493	4.856 545	6.631 051	9.040 291	12.306 476	16.727 945	22.704 667
65	3.622 523	4.977 958	6.829 983	9.356 701	12.798 735	17.480 702	23.839 901
66	3.694 974	5.102 407	7.034 882	9.684 185	13.310 685	18.267 334	25.031 896
67	3.768 873	5.229 967	7.245 929	10.023 132	13.843 112	19.089 364	26.283 490
68	3.844 251	5.360 717	7.463 307	10.373 941	14.396 836	19.948 385	27.597 665
69	3.921 136	5.494 734	7.687 206	10.737 029	14.972 710	20.846 063	28.977 548
70	3.999 558	5.632 103	7.917 822	11.112 825	15.571 618	21.784 136	30.426 426
71	4.079 549	5.772 905	8.155 357	11.501 774	16.194 483	22.764 422	31.947 747
72	4.161 140	5.917 228	8.400 017	11.904 336	16.842 262	23.788 821	33.545 134
73	4.244 363	6.065 159	8.652 018	12.320 988	17.515 953	24.859 318	35.222 391
74	4.329 250	6.216 788	8.911 578	12.752 223	18.216 591	25.977 987	36.983 511
75	4.415 835	6.372 207	9.178 926	13.198 550	18.945 255	27.146 996	38.832 686

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (*Continued*)

(Amounts are noted for the end of each year)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
76	$4.504 152	$ 6.531 513	$ 9.454 293	$13.660 500	$19.703 065	$28.368 611	$ 40.774 320
77	4.594 235	6.694 800	9.737 922	14.138 617	20.491 187	29.645 199	42.813 036
78	4.686 120	6.862 170	10.030 060	14.633 469	21.310 835	30.979 233	44.953 688
79	4.779 842	7.033 725	10.330 962	15.145 640	22.163 268	32.373 298	47.201 372
80	4.875 439	7.209 568	10.640 891	15.675 738	23.049 799	33.830 096	49.561 441
81	4.972 948	7.389 807	10.960 117	16.224 388	23.971 791	35.352 451	52.039 513
82	5.072 407	7.574 552	11.288 921	16.792 242	24.930 663	36.943 311	54.641 489
83	5.173 855	7.763 916	11.627 588	17.379 970	25.927 889	38.605 760	57.373 563
84	5.277 332	7.958 014	11.976 416	17.988 269	26.965 005	40.343 019	60.242 241
85	5.382 879	8.156 964	12.335 709	18.617 859	28.043 605	42.158 455	63.254 353
86	5.490 536	8.360 888	12.705 780	19.269 484	29.165 349	44.055 586	66.417 071
87	5.600 347	8.569 911	13.086 953	19.943 916	30.331 963	46.038 087	69.737 925
88	5.712 354	8.784 158	13.479 562	20.641 953	31.545 242	48.109 801	73.224 821
89	5.826 601	9.003 762	13.883 949	21.364 421	32.807 051	50.274 742	76.886 062
90	5.943 133	9.228 856	14.300 467	22.112 176	34.119 333	52.537 105	80.730 365
91	6.061 996	9.459 578	14.729 481	22.886 102	35.484 107	54.901 275	84.766 883
92	6.183 236	9.696 067	15.171 366	23.687 116	36.903 471	57.371 832	89.005 228
93	6.306 900	9.938 469	15.626 507	24.516 165	38.379 610	59.953 565	93.455 489
94	6.433 038	10.186 931	16.095 302	25.374 231	39.914 794	62.651 475	98.128 263
95	6.561 699	10.441 604	16.578 161	26.262 329	41.511 386	65.470 792	103.034 676
96	6.692 933	10.702 644	17.075 506	27.181 510	43.171 841	68.416 977	108.186 410
97	6.826 792	10.970 210	17.587 771	28.132 863	44.898 715	71.495 741	113.595 731
98	6.963 328	11.244 465	18.115 404	29.117 513	46.694 664	74.713 049	119.275 517
99	7.102 594	11.525 577	18.658 866	30.136 626	48.562 450	78.075 137	125.239 293
100	7.244 646	11.813 716	19.218 632	31.191 408	50.504 948	81.588 518	131.501 258

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (*Continued*)

(Amounts are noted for the end of each year)

Years.	$5\frac{1}{2}$ per cent.	6 per cent.	$6\frac{1}{2}$ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	$1.055 000	$1.060 000	$1.065 000	$1.070 000	$1.080 000	$1.090 000	$ 1.100 000
2	1.113 025	1.123 600	1.134 225	1.144 900	1.166 400	1.188 100	1.210 000
3	1.174 241	1.191 016	1.207 950	1.225 043	1.259 712	1.295 029	1.331 000
4	1.238 825	1.262 477	1.286 466	1.310 796	1.360 489	1.411 582	1.464 100
5	1.306 960	1.338 226	1.370 087	1.402 552	1.469 328	1.538 624	1.610 510
6	1.378 843	1.418 519	1.459 142	1.500 730	1.586 874	1.677 100	1.771 561
7	1.454 679	1.503 630	1.553 987	1.605 781	1.713 824	1.828 039	1.948 717
8	1.534 687	1.593 848	1.654 996	1.718 186	1.850 930	1.992 563	2.143 589
9	1.619 094	1.689 479	1.762 570	1.838 459	1.999 005	2.171 893	2.357 948
10	1.708 144	1.790 848	1.877 137	1.967 151	2.158 925	2.367 364	2.593 742
11	1.802 092	1.898 299	1.999 151	2.104 852	2.331 639	2.580 426	2.853 117
12	1.901 207	2.012 196	2.129 096	2.252 192	2.518 170	2.812 665	3.138 428
13	2.005 774	2.132 928	2.267 488	2.409 845	2.719 624	3.065 805	3.452 271
14	2.116 091	2.260 904	2.414 874	2.578 534	2.937 194	3.341 727	3.797 498
15	2.232 476	2.396 558	2.571 841	2.759 032	3.172 169	3.642 482	4.177 248
16	2.355 263	2.540 352	2.739 011	2.952 164	3.425 943	3.970 306	4.594 973
17	2.484 802	2.692 773	2.917 046	3.158 815	3.700 018	4.327 633	5.054 470
18	2.621 466	2.854 339	3.106 654	3.379 932	3.996 020	4.717 120	5.559 917
19	2.765 647	3.025 600	3.308 587	3.616 528	4.315 701	5.141 661	6.115 909
20	2.917 757	3.207 135	3.523 645	3.869 684	4.660 957	5.604 411	6.727 500
21	3.078 234	3.399 564	3.752 682	4.140 562	5.033 834	6.108 808	7.400 250
22	3.247 537	3.603 537	3.996 606	4.430 402	5.436 540	6.658 600	8.140 275
23	3.426 152	3.819 750	4.256 386	4.740 530	5.871 464	7.257 874	8.954 302
24	3.614 590	4.048 935	4.533 051	5.072 367	6.341 181	7.911 083	9.849 733
25	3.813 392	4.291 871	4.827 699	5.427 433	6.848 475	8.623 081	10.834 706

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (*Continued*)

(Amounts are noted for the end of each year)

Years.	$5\frac{1}{2}$ per cent.	6 per cent.	$6\frac{1}{2}$ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
26	$ 4.023 129	$ 4.549 383	$ 5.141 500	$ 5.807 353	$ 7.396 353	$ 9.399 158	$ 11.918 177
27	4.244 401	4.822 346	5.475 697	6.213 868	7.988 061	10.245 082	13.109 994
28	4.477 843	5.111 687	5.831 617	6.648 838	8.627 106	11.167 140	14.420 994
29	4.724 124	5.418 388	6.210 672	7.114 257	9.317 275	12.172 182	15.863 093
30	4.983 951	5.743 491	6.614 366	7.612 255	10.062 657	13.267 678	17.449 402
31	5.258 069	6.088 101	7.044 300	8.145 113	10.867 669	14.461 770	19.194 342
32	5.547 262	6.453 387	7.502 179	8.715 271	11.737 083	15.763 329	21.113 777
33	5.852 362	6.840 590	7.989 821	9.325 340	12.676 050	17.182 028	23.225 154
34	6.174 242	7.251 025	8.509 160	9.978 114	13.690 134	18.728 411	25.547 670
35	6.513 825	7.686 087	9.062 255	10.676 581	14.785 344	20.413 968	28.102 437
36	6.872 085	8.147 252	9.651 301	11.423 942	15.968 172	22.251 225	30.912 681
37	7.250 050	8.636 087	10.278 636	12.223 618	17.245 626	24.253 835	34.003 949
38	7.648 803	9.154 252	10.946 747	13.079 271	18.625 276	26.436 680	37.404 343
39	8.069 487	9.703 507	11.658 286	13.994 820	20.115 298	28.815 982	41.144 778
40	8.513 309	10.285 718	12.416 075	14.974 458	21.724 522	31.409 420	45.259 256
41	8.981 541	10.902 861	13.223 119	16.022 670	23.462 483	34.236 268	49.785 181
42	9.475 526	11.557 033	14.082 622	17.144 257	25.339 482	37.317 532	54.763 699
43	9.996 679	12.250 455	14.997 993	18.344 355	27.366 640	40.676 110	60.240 069
44	10.546 497	12.985 482	15.972 862	19.628 460	29.555 972	44.336 960	66.264 076
45	11.126 554	13.764 611	17.011 098	21.002 452	31.920 449	48.327 286	72.890 484
46	11.738 515	14.590 487	18.116 820	22.472 623	34.474 085	52.676 742	80.179 532
47	12.384 133	15.465 917	19.294 413	24.045 707	37.232 012	57.417 649	88.197 485
48	13.065 260	16.393 872	20.548 550	25.728 907	40.210 573	62.585 237	97.017 234
49	13.783 849	17.377 504	21.884 205	27.529 930	43.427 419	68.217 908	106.718 957
50	14.541 961	18.420 154	23.306 679	29.457 025	46.901 613	74.357 520	117.390 853

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (*Continued*)

(Amounts are noted for the end of each year)

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
51	$15.341 769	$19.525 364	$ 24.821 613	$ 31.519 017	$ 50.653 742	$ 81.049 697	$ 129.129 938
52	16.185 566	20.696 885	26.435 018	33.725 348	54.706 041	88.344 170	142.042 932
53	17.075 773	21.938 698	28.153 294	36.086 122	59.082 524	96.295 145	156.247 225
54	18.014 940	23.255 020	29.983 258	38.612 151	63.809 126	104.961 708	171.871 948
55	19.005 762	24.650 322	31.932 170	41.315 001	68.913 856	114.408 262	189.059 143
56	20.051 079	26.129 341	34.007 761	44.207 052	74.426 965	124.705 005	207.965 057
57	21.153 888	27.697 101	36.218 265	47.301 545	80.381 122	135.928 456	228.761 562
58	22.317 352	29.358 927	38.572 452	50.612 653	86.811 612	148.162 017	251.637 719
59	23.544 806	31.120 463	41.079 662	54.155 539	93.756 540	161.496 598	276.801 491
60	24.839 770	32.987 691	43.749 840	57.946 427	101.257 064	176.031 292	304.481 640
61	26.205 958	34.966 952	46.593 579	62.002 677	109.357 629	191.874 108	334.929 804
62	27.647 286	37.064 969	49.622 162	66.342 864	118.106 239	209.142 778	368.422 784
63	29.167 886	39.288 868	52.847 603	70.986 865	127.554 738	227.965 628	405.265 062
64	30.772 120	41.646 200	56.282 697	75.955 945	137.759 117	248.482 535	445.791 569
65	32.464 587	44.144 972	59.941 072	81.272 861	148.779 847	270.845 963	490.370 725
66	34.250 139	46.793 670	63.837 242	86.961 962	160.682 234	295.222 099	539.407 798
67	36.133 896	49.601 290	67.986 662	93.049 299	173.536 813	321.792 088	593.348 578
68	38.121 261	52.577 368	72.405 795	99.562 750	187.419 758	350.753 376	652.683 436
69	40.217 930	55.732 010	77.112 172	106.532 142	202.413 339	382.321 180	717.951 779
70	42.429 916	59.075 931	82.124 463	113.989 392	218.606 406	416.730 086	789.746 957
71	44.763 562	62.620 486	87.462 553	121.968 650	236.094 918	454.235 794	868.721 652
72	47.225 558	66.377 715	93.147 619	130.506 455	254.982 512	495.117 015	955.593 818
73	49.822 963	70.360 378	99.202 215	139.641 907	275.381 113	539.677 547	1051.153 199
74	52.563 226	74.582 001	105.650 359	149.416 840	297.411 602	588.248 526	1156.268 519
75	55.454 204L	79.056 921	112.517 632	159.876 019	321.204 530	641.190 893	1271.895 371

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (*Continued*)

(Amounts are noted for the end of each year)

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
76	$ 58.504 185	$ 83.800 336	$119.831 128	$171.067 341	$ 346.900 892	$ 698.898 074	$ 1,399.084 909
77	61.721 915	88.828 356	127.620 311	183.042 054	374.652 964	761.798 900	1,538.993 399
78	65.116 620	94.158 057	135.915 631	195.854 998	404.625 201	830.360 801	1,692.892 739
79	68.698 034	99.807 541	144.750 147	209.564 848	436.995 217	905.093 273	1,862.182 013
80	72.476 426	105.795 993	154.158 907	224.234 388	.71.954 834	986.551 668	2,048.400 215
81	76.462 630	112.143 753	164.179 236	239.930 795	509.711 221	1075.341 318	2,253.240 236
82	80.668 074	118.872 378	174.850 886	256.725 950	550.488 119	1172.122 037	2,478.564 260
83	85.104 819	126.004 721	136.216 194	274.696 767	594.527 168	1277.613 021	2,726.420 686
84	89.785 584	133.565 004	198.320 246	293.925 540	642.089 342	1392.598 192	2,999.062 754
85	94.723 791	141.578 904	211.211 062	314.500 328	693.456 489	1517.932 029	3,298.969 030
86	99.933 599	150.073 639	224.939 781	336.515 351	748.933 008	1654.545 912	3,628.865 933
87	105.429 947	159.078 057	239.560 867	360.071 426	808.847 649	1803.455 044	3,991.752 526
88	111.228 594	168.622 740	255.132 323	385.276 426	873.555 461	1965.765 998	4,390.927 778
89	117.346 167	178.740 105	271.715 924	412.245 775	943.439 897	2142.684 938	4,830.020 556
90	123.800 206	189.464 511	289.377 459	441.102 980	1018.915 089	2335.526 582	5,313.022 612
91	130.609 217	200.832 382	308.186 994	471.980 188	1100.428 296	2545.723 975	5,844.324 873
92	137.792 724	212.882 325	328.219 149	505.018 801	1188.462 560	2774.839 132	6,428.757 360
93	145.371 324	225.655 264	349.553 394	540.370 118	1283.539 565	3024.574 654	7,071.633 096
94	153.366 747	239.194 580	372.274 364	578.196 026	1386.222 730	3296.786 373	7,778.796 406
95	161.801 918	253.546 255	396.472 198	618.669 748	1497.120 549	3593.497 147	8,556.676 047
96	170.701 024	268.759 030	422.242 891	661.976 630	1616.890 192	3916.911 890	9,412.343 651
97	180.089 580	284.884 572	449.688 679	708.314 994	1746.241 408	4269.433 960	10,353.578 016
98	189.994 507	301.977 646	478.918 443	757.897 044	1885.940 720	4653.683 016	11,388.935 818
99	200.444 205	320.096 305	510.048 142	810.949 837	2036.815 978	5072.514 488	12,527.829 400
100	211.468 636	339.302 083	543.201 271	867.716 326	2199.761 256	5529.040 792	13,780.612 340

TABLE 21. AMOUNT OF ONE DOLLAR AT INTEREST COMPOUNDED ANNUALLY (*Concluded*)

(Amounts are noted for the end of each year)

Years.	12 per cent.	15 per cent.	20 per cent.	25 per cent.
1	$1.12	$1.15	$1.20	$1.25
2	1.2544	1.3225	1.440	1.5625
3	1.404928	1.520875	1.7280	1.953125
4	1.573519	1.749006	2.07360	2.441406
5	1.762342	2.011357	2.488320	3.051758
6	1.973823	2.313061	2.985984	3.814697
7	2.210681	2.660020	3.583181	4.768372
8	2.475963	3.059023	4.299817	5.960464
9	2.773079	3.517876	5.159780	7.450581
10	3.105848	4.045558	6.191736	9.313226
11	3.478550	4.652391	7.430084	11.641532
12	3.895976	5.350250	8.916100	14.551915
13	4.363493	6.152788	10.699321	18.189894
14	4.887112	7.075706	12.839185	22.737368
15	5.473566	8.137062	15.407022	28.421709
16	6.130394	9.357621	18.488426	35.527137
17	6.866041	10.761264	22.186111	44.408921
18	7.689966	12.375454	26.623333	55.511151
19	8.612762	14.231772	31.948000	69.388939
20	9.646293	16.366537	38.337600	86.736174
21	10.803848	18.821518	46.005120	108.420217
22	12.100310	21.644746	55.206144	135.525271
23	13.552347	24.891458	66.247373	169.406589
24	15.178629	28.625176	79.496847	211.758236
25	17.000064	32.918953	95.396217	264.697796
30	29.959922	66.211772	237.376314	807.793567
35	52.799620	133.175523	590.668229	2,465.190326
40	93.050970	267.863546	1,469.771568	7,523.163845
45	163.987604	538.769269	3,657.261988	22,958.874023
50	289.002190	1,083.657442	9,100.438150	70,064.923216
60	897.596933	4,383.998746	56,347.514353	652,530.446800
70	2,787.799828	17,735.720039	348,888.956932	6,077,163.357286
80	8,658.483093	71,750.879401	2,160,228.462010	56,597,994.242667
90	26,891.934202	290,272.325206	13,375,565.248934	527,109,897.161526
100	83,522.265659	1,174,313.450700	82,817,974.522015	4,909,093,465.297726

EXPLANATION OF TABLE 22

THE PRESENT VALUE OF ONE DOLLAR DUE AT A FUTURE DATE

The present value of \$1 due at some future time is the sum which placed at compound interest will amount to \$1 at that time.

The formula on which Table 22 is based is as follows:

Let P represent the present value of \$1 due at the end of n years.

Let n represent any number of years.

Let i represent the interest rate expressed decimally as 0.05 for 5 per cent.

Then
$$P = \frac{1}{(1 + i)^n} \tag{24}$$

Table 22 has been prepared for only a few selected years because the present value of \$1 due at a future time is also readily obtainable from Table 21. According to equation (23), A' may be substituted for $(1 + i)^n$, equation (24) may then be written:

$$P = \frac{1}{A'} \tag{25}$$

That is to say, the present value of \$1 due at any future time is the reciprocal of \$1 at compound interest for the same time.

Example. — What is the present value of \$600 due in 8 years at 4 per cent interest?

From Table 21 the amount of \$1 at 4 per cent compound interest in 8 years is \$1.368569, consequently the present value of \$1 due in 8 years will be

$$1 \div 1.368569 = 0.730690$$

and the present value of \$600 due in 8 years at 4 per cent will be

$$0.730690 \times 600 = \$438.41.$$

Note. — To find the present value of \$1 due at the end of any number of years n not noted in this table, select two or more lines from the table the sum of whose years is equal to n

and multiply the values found in the interest column on these lines.

Example.—What is the present value of $1 at 5% due in 36 years?

Present value of $1 due in 30 years at 5% = 0.231377
Present value of $1 due in 6 years at 5% = 0.746215
0.231377 × 0.746215 = $0.172657 the required present value.

TABLE 22. THE PRESENT VALUE OF ONE DOLLAR DUE AT A FUTURE DATE

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	$.980 392	$.975 610	$.970 874	$.966 184	$.961 538	$.956 938	$.952 381
2	.961 169	.951 814	.942 596	.933 511	.924 556	.915 730	.907 029
3	.942 322	.928 599	.915 142	.901 943	.888 996	.876 297	.863 838
4	.923 845	.905 951	.888 487	.871 442	.854 804	.838 561	.822 702
5	.905 731	.883 854	.862 609	.841 973	.821 927	.802 451	.783 526
6	.887 971	.862 297	.837 484	.813 501	.790 315	.767 896	.746 215
7	.870 560	.841 265	.813 092	.785 991	.759 918	.734 828	.710 681
8	.853 490	.820 747	.789 409	.759 412	.730 690	.703 185	.676 839
9	.836 755	.800 728	.766 417	.733 731	.702 587	.672 904	.644 609
10	.820 348	.781 198	.744 094	.708 919	.675 564	.643 928	.613 913
15	.743 015	.690 466	.641 862	.596 891	.555 265	.516 720	.481 017
20	.672 971	.610 271	.553 676	.502 566	.456 387	.414 643	.376 889
25	.609 531	.539 391	.477 606	.423 147	.375 117	.332 731	.295 303
30	.552 071	.476 743	.411 987	.356 278	.308 319	.267 000	.231 377
35	.500 028	.421 371	.355 383	.299 977	.253 415	.214 254	.181 290
40	.452 890	.372 431	.306 557	.252 572	.208 289	.171 929	.142 046
45	.410 197	.329 174	.264 439	.212 659	.171 198	.137 964	.111 297
50	.371 528	.290 942	.228 107	.179 053	.140 713	.110 710	.087 204
60	.304 782	.227 284	.169 733	.126 934	.095 060	.071 289	.053 536
70	.250 028	.177 554	.126 297	.089 986	.064 219	.045 905	.032 866
80	.205 110	.138 705	.093 977	.063 793	.043 384	.029 559	.020 177
90	.168 261	.108 356	.069 928	.045 224	.029 309	.019 034	.012 387
100	.138 033	.084 647	.052 033	.032 060	.019 800	.012 257	.007 604

TABLE 22. THE PRESENT VALUE OF ONE DOLLAR DUE AT A FUTURE DATE (*Continued*)

Years.	$5\frac{1}{2}$ per cent.	6 per cent.	$6\frac{1}{2}$ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	$.947 867	$.943 396	$.938 967	$.934 579	$.925 926	$.917 431	$.909 091
2	.898 452	.889 996	.881 659	.873 439	.857 339	.841 680	.826 446
3	.851 614	.839 619	.827 849	.816 298	.793 832	.772 183	.751 315
4	.807 217	.792 094	.777 323	.762 895	.735 030	.708 425	.683 013
5	.765 134	.747 258	.729 881	.712 986	.680 583	.649 931	.620 921
6	.725 246	.704 961	.685 334	.666 342	.630 170	.596 267	.564 474
7	.687 437	.665 057	.643 506	.622 750	.583 490	.547 034	.513 158
8	.651 599	.627 412	.604 231	.582 009	.540 269	.501 866	.466 507
9	.617 629	.591 898	.567 353	.543 934	.500 249	.460 428	.424 098
10	.585 431	.558 395	.532 726	.508 349	.463 193	.422 411	.385 543
15	.447 933	.417 265	.388 827	.362 446	.315 242	.274 538	.239 392
20	.342 729	.311 805	.283 797	.258 419	.214 548	.178 431	.148 644
25	.262 234	.232 999	.207 138	.184 249	.146 018	.115 968	.092 296
30	.200 644	.174 110	.151 186	.131 367	.099 377	.075 371	.057 309
35	.153 520	.130 105	.110 348	.093 663	.067 635	.048 986	.035 584
40	.117 463	.097 222	.080 541	.066 780	.046 031	.031 838	.022 095
45	.089 875	.072 650	.058 785	.047 613	.031 328	.020 692	.013 719
50	.068 767	.054 288	.042 906	.033 948	.021 321	.013 449	.008 519
60	.040 258	.030 314	.022 857	.017 257	.009 876	.005 681	.003 284
70	.023 569	.016 927	.012 177	.008 773	.004 574	.002 400	.001 266
80	.013 798	.009 452	.006 487	.004 460	.002 119	.001 014	.000 488 2
90	.008 078	.005 278	.003 456	.002 267	.000 981 4	.000 428 2	.000 188 2
100	.004 729	.002 947	.001 804	.001 152	.000 454 6	.000 180 9	.000 072 6

EXPLANATION OF TABLE 23

The Amount of an Annuity of One Dollar

An annuity is a sum uniform in amount due annually.

The amount of an annuity in any term or number of years is the sum of the several annual installments with interest thereon during the term compounded annually.

Table 23 shows the amount of an annuity of one dollar paid at the *end* of each year with the earned interest increments of each year added at the end of the year.

To find the amount of an annuity of one dollar paid at the *beginning* of each year subtract \$1 from the figures noted in the table and the result will then apply at the beginning of each year, or, which is the same, at the end of the preceding year. Thus for 6 per cent interest at the beginning of the year 1, the amount is zero; at the end of year 11, or beginning of the year 12, it is \$16.87 − 1.00 = \$15.87.

The values given in Table 23 are calculated by the following formula:

Let A'' represent the amount of an annuity of one dollar paid at the end of each year.

Let i represent the interest rate expressed in percentage, as 0.05 for 5 per cent.

Let n represent the number of years.

Then

$$A'' = \frac{(1+i)^n - 1}{i} \tag{26}$$

and it follows from equation (23) that:

$$A'' = \frac{A' - 1}{i} \tag{27}$$

The amount of an annuity of \$1 paid at the end of each year can be found, in other words, from any table giving the amount of \$1 at compound interest by subtracting \$1 from the amount found in the compound interest table and dividing the remainder by the rate of interest expressed decimally.

Example. — What is the amount of an annuity of \$35 at the end of 17 years at 5 per cent interest?

From Table 23 the amount of an annuity of \$1 for 17 years at 5 per cent is found to be \$25.84037, consequently the amount of an annuity of \$35 will be

$$35 \times 25.84037 = \$904.41.$$

Or from Table 21 the amount of \$1 at 5 per cent compound interest for 17 years is found to be 2.292018, therefore, according to equation (27),

$$A'' = \frac{2.292018 - 1}{0.05} = 25.84036$$

and the amount of the annuity of \$35 will be

$$35 \times 25.84036 = \$904.41.$$

TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR
(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. Amounts are noted for the end of each year)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	$ 1.000 00	$ 1.000 00	$ 1.000 00	$ 1.000 00	$ 1.000 00	$ 1.000 00	$ 1.000 00
2	2.020 00	2.025 00	2.030 00	2.035 00	2.040 00	2.045 00	2.050 00
3	3.060 40	3.075 63	3.090 90	3.106 22	3.121 60	3.137 03	3.152 50
4	4.121 61	4.152 52	4.183 63	4.214 94	4.246 46	4.278 19	4.310 13
5	5.204 04	5.256 33	5.309 14	5.362 47	5.416 32	5.470 71	5.525 63
6	6.308 12	6.387 74	6.468 41	6.550 15	6.632 98	6.716 89	6.801 91
7	7.434 28	7.547 43	7.662 46	7.779 41	7.898 29	8.019 15	8.142 01
8	8.582 97	8.736 12	8.892 34	9.051 69	9.214 23	9.380 01	9.549 11
9	9.754 63	9.954 52	10.159 11	10.368 50	10.582 80	10.802 11	11.026 56
10	10.949 72	11.203 38	11.463 88	11.731 39	12.006 11	12.288 21	12.577 89
11	12.168 72	12.483 47	12.807 80	13.141 99	13.486 35	13.841 18	14.206 79
12	13.412 09	13.795 55	14.192 03	14.601 96	15.025 81	15.464 03	15.917 13
13	14.680 33	15.140 44	15.617 79	16.113 03	16.626 84	17.159 91	17.712 98
14	15.973 94	16.518 95	17.086 32	17.676 99	18.291 91	18.932 11	19.598 63
15	17.293 42	17.931 93	18.598 91	19.295 68	20.023 59	20.784 05	21.578 56
16	18.639 28	19.380 22	20.156 88	20.971 03	21.824 53	22.719 34	23.657 49
17	20.012 07	20.864 73	21.761 59	22.705 02	23.697 51	24.741 71	25.840 37
18	21.412 31	22.386 35	23.414 44	24.499 69	25.645 41	26.855 08	28.132 38
19	22.840 56	23.946 01	25.116 87	26.357 18	27.671 23	29.063 56	30.539 00
20	24.297 37	25.544 66	26.870 37	28.279 68	29.778 08	31.371 42	33.065 95
21	25.783 31	27.183 27	28.676 49	30.269 47	31.969 20	33.783 14	35.719 25
22	27.298 98	28.862 86	30.536 78	32.328 90	34.247 97	36.303 38	38.505 21
23	28.844 96	30.584 43	32.452 88	34.460 41	36.617 89	38.937 03	41.430 48
24	30.421 86	32.349 04	34.426 47	36.666 53	39.082 60	41.689 20	44.502 00
25	32.030 30	34.157 76	36.459 26	38.949 86	41.645 91	44.565 21	47.727 10

TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (*Continued*)
(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. Amounts are noted for the end of each year)

Years.	2 per cent.	$2\frac{1}{2}$ per cent.	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
26	$33.670 91	$36.011 71	$ 38.553 04	$ 41.313 10	$ 44.311 74	$ 47.570 64	$ 51.113 45
27	35.344 32	37.912 00	40.709 63	43.759 06	47.084 21	50.711 32	54.669 13
28	37.051 21	39.859 80	42.930 92	46.290 63	49.967 58	53.993 33	58.402 58
29	38.792 23	41.856 30	45.218 85	48.910 80	52.966 29	57.423 03	62.322 71
30	40.568 08	43.902 70	47.575 42	51.622 68	56.084 94	61.007 07	66.438 85
31	42.379 44	46.000 27	50.002 68	54.429 47	59.328 34	64.752 39	70.760 79
32	44.227 03	48.150 28	52.502 76	57.334 50	62.701 47	68.666 25	75.298 83
33	46.111 57	50.354 04	55.077 84	60.341 21	66.209 53	72.756 23	80.063 77
34	48.033 80	52.612 89	57.730 18	63.453 15	69.857 91	77.030 26	85.066 96
35	49.994 48	54.928 21	60.462 08	66.674 01	73.652 22	81.496 62	90.320 31
36	51.994 37	57.301 41	63.275 94	70.007 60	77.598 31	86.163 97	95.836 32
37	54.034 25	59.733 95	66.174 22	73.457 87	81.702 25	91.041 34	101.628 14
38	56.114 94	62.227 30	69.159 45	77.028 89	85.970 34	96.138 20	107.709 55
39	58.237 24	64.782 98	72.234 23	80.724 91	90.409 15	101.464 42	114.095 02
40	60.401 98	67.402 55	75.401 26	84.550 28	95.025 52	107.030 32	120.799 77
41	62.610 02	70.087 62	78.663 30	88.509 54	99.826 54	112.846 69	127.839 76
42	64.862 22	72.839 81	82.023 20	92.607 37	104.819 60	118.924 79	135.231 75
43	67.159 47	75.660 80	85.483 89	96.848 63	110.012 38	125.276 40	142.993 34
44	69.502 66	78.552 32	89.048 41	101.238 33	115.412 88	131.913 84	151.143 01
45	71.892 71	81.516 13	92.719 86	105.781 67	121.029 39	138.849 97	159.700 16
46	74.330 56	84.554 03	96.501 46	110.484 03	126.870 57	146.098 21	168.685 16
47	76.817 18	87.667 89	100.396 50	115.350 97	132.945 39	153.672 63	178.119 42
48	79.353 52	90.859 58	104.408 40	120.388 26	139.263 21	161.587 90	188.025 39
49	81.940 59	94.131 07	108.540 65	125.601 85	145.833 73	169.859 36	198.426 66
50	84.579 40	97.484 35	112.796 87	130.997 91	152.667 08	178.503 03	209.348 00

TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (*Continued*)
(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. Amounts are noted for the end of each year)

Year.	2 per cent.	$2\frac{1}{2}$ per cent.	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
51	$ 87.270 99	$100.921 46	$117.180 77	$136.582 84	$159.773 77	$187.535 66	$220.815 40
52	90.016 41	104.444 49	121.696 20	142.363 24	167.164 72	196.974 78	232.856 17
53	92.816 74	108.055 61	126.347 08	148.345 95	174.851 31	206.838 63	245.498 97
54	95.673 07	111.757 00	131.137 49	154.538 06	182.845 36	217.146 37	258.773 92
55	98.586 53	115.550 92	136.071 62	160.946 89	191.159 17	227.917 96	272.712 62
56	101.558 26	119.439 69	141.153 77	167.580 03	199.805 54	239.174 27	287.348 25
57	104.589 43	123.425 69	146.388 38	174.445 33	208.797 76	250.937 11	302.715 66
58	107.681 22	127.511 33	151.780 03	181.550 92	218.149 67	263.229 28	318.851 44
59	110.834 84	131.699 11	157.333 43	188.905 20	227.875 66	276.074 60	335.794 02
60	114.051 54	135.991 59	163.053 44	196.516 88	237.990 69	289.497 95	353.583 72
61	117.332 57	140.391 38	168.945 04	204.349 97	248.510 31	303.525 36	372.262 90
62	120.679 22	144.901 16	175.013 39	212.548 80	259.450 73	318.184 00	391.876 05
63	124.092 81	149.523 69	181.263 79	220.988 01	270.828 75	333.502 28	412.469 85
64	127.574 66	154.261 78	187.701 71	229.722 59	282.661 90	349.509 89	434.093 34
65	131.126 16	159.118 33	194.332 76	238.762 88	294.968 38	366.237 83	456.798 01
66	134.748 68	164.096 29	201.162 74	248.119 58	307.767 12	383.718 53	480.637 91
67	138.443 65	169.198 70	208.197 62	257.803 76	321.077 80	401.985 87	505.669 81
68	142.212 53	174.428 66	215.443 55	267.826 89	334.920 91	421.075 23	531.953 30
69	146.056 78	179.789 38	222.906 86	278.200 84	349.317 75	441.023 61	559.550 96
70	149.977 91	185.284 11	230.594 06	288.937 86	364.290 46	461.869 68	588.528 51
71	153.977 47	190.916 22	238.511 89	300.050 69	379.862 08	483.653 82	618.954 94
72	158.057 02	196.689 12	246.667 24	311.552 46	396.056 56	506.418 24	650.902 68
73	162.218 16	202.606 35	255.067 26	323.456 80	412.898 82	530.207 06	684.447 82
74	166.462 52	208.671 51	263.719 28	335.777 79	430.414 77	555.066 38	719.670 21
75	170.791 77	214.888 30	272.630 86	348.530 01	448.631 37	581.044 36	756.653 72

TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (*Continued*)
(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. Amounts are noted for the end of each year)

Year.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
76	$175.207 6	$221.260 5	$281.809 8	$361.728 6	$ 467.576 6	$ 608.191 4	$ 795.486 4
77	179.711 8	227.792 0	291.264 1	375.389 1	487.279 7	636.560 0	836.260 7
78	184.306 0	234.486 8	301.002 0	389.527 7	507.770 9	666.205 2	879.073 8
79	188.992 1	241.349 0	311.032 1	404.161 1	529.081 7	697.184 4	924.027 5
80	193.772 0	248.382 7	321.363 0	419.306 8	551.245 0	729.557 7	971.228 8
81	198.647 4	255.592 3	332.003 9	434.982 5	574.294 8	763.387 8	1020.790 3
82	203.620 3	262.982 1	342.964 0	451.206 9	598.266 6	798.740 2	1072.829 8
83	208.692 8	270.556 6	354.252 9	467.999 2	623.197 2	835.683 6	1127.471 3
84	213.866 6	278.320 6	365.880 5	485.379 1	649.125 1	874.289 3	1184.844 8
85	219.143 9	286.278 6	377.857 0	503.367 4	676.090 1	914.632 3	1245.087 1
86	224.526 8	294.435 5	390.192 7	521.985 3	704.133 7	956.790 8	1308.341 4
87	230.017 4	302.796 4	402.898 4	541.254 7	733.299 1	1000.846 4	1374.758 5
88	235.617 7	311.366 3	415.985 4	561.198 7	763.631 0	1046.884 5	1444.496 4
89	241.330 1	320.150 5	429.465 0	581.840 6	795.176 3	1094.994 3	1517.721 2
90	247.156 7	329.154 3	443.348 9	603.205 0	827.983 3	1145.269 0	1594.607 3
91	253.099 8	338.383 1	457.649 4	625.317 2	862.102 7	1197.806 1	1675.337 7
92	259.161 8	347.842 7	472.378 9	648.203 3	897.586 8	1252.707 4	1760.104 6
93	265.345 0	357.538 8	487.550 2	671.890 4	934.490 2	1310.079 2	1849.109 8
94	271.651 9	367.477 2	503.176 7	696.406 6	972.869 9	1370.032 8	1942.565 3
95	278.085 0	377.664 2	519.272 0	721.780 8	1012.784 6	1432.684 3	2040.693 5
96	284.646 7	388.105 8	535.850 2	748.043 1	1054.296 0	1498.155 1	2143.728 2
97	291.339 6	398.808 4	552.925 7	775.224 7	1097.467 9	1566.572 0	2251.914 6
98	298.166 4	409.778 6	570.513 5	803.357 5	1142.366 6	1638.067 8	2365.510 3
99	305.129 7	421.023 1	588.628 9	832.475 0	1189.061 3	1712.780 8	2484.785 9
100	312.232 3	432.548 7	607.287 7	862.611 7	1237.623 7	1790.856 0	2610.025 2

TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (*Continued*)
(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. Amounts are noted for the end of each year)

Year.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	$ 1.000 00	$ 1.000 00	$ 1.000 00	$ 1.000 00	$ 1.000 00	$ 1.000 00	$ 1.000 00
2	2.055 00	2.060 00	2.065 00	2.070 00	2.080 00	2.090 00	2.100 00
3	3.168 03	3.183 60	3.199 23	3.214 90	3.246 40	3.278 10	3.310 00
4	4.342 27	4.374 62	4.407 17	4.439 94	4.506 11	4.573 13	4.641 00
5	5.581 09	5.637 09	5.693 64	5.750 74	5.866 60	5.984 71	6.105 10
6	6.888 05	6.975 32	7.063 73	7.153 29	7.335 93	7.523 33	7.715 61
7	8.266 89	8.393 84	8.522 87	8.654 02	8.922 80	9.200 43	9.487 17
8	9.721 57	9.897 47	10.076 86	10.259 80	10.636 63	11.028 47	11.435 89
9	11.256 26	11.491 32	11.731 85	11.977 99	12.487 56	13.021 04	13.579 48
10	12.875 35	13.180 79	13.494 42	13.816 45	14.486 56	15.192 93	15.937 42
11	14.583 50	14.971 64	15.371 56	15.783 60	16.645 49	17.560 29	18.531 17
12	16.385 59	16.869 94	17.370 71	17.888 45	18.977 13	20.140 72	21.384 28
13	18.286 80	18.882 14	19.499 81	20.140 64	21.495 30	22.953 38	24.522 71
14	20.292 57	21.015 07	21.767 29	22.550 49	24.214 92	26.019 19	27.974 98
15	22.408 66	23.275 97	24.182 17	25.129 02	27.152 11	29.360 92	31.772 48
16	24.641 14	25.672 53	26.754 01	27.888 05	30.324 28	33.003 40	35.949 73
17	26.996 40	28.212 88	29.493 02	30.840 22	33.750 23	36.973 70	40.544 70
18	29.481 21	30.905 65	32.410 07	33.999 03	37.450 24	41.301 34	45.599 17
19	32.102 67	33.759 99	35.516 72	37.378 96	41.446 26	46.018 46	51.159 09
20	34.868 32	36.785 59	38.825 31	40.995 49	45.761 96	51.160 12	57.275 00
21	37.786 08	39.992 73	42.348 95	44.865 18	50.422 92	56.764 53	64.002 50
22	40.864 31	43.392 29	46.101 64	49.005 74	55.456 76	62.873 34	71.402 75
23	44.111 85	46.995 83	50.098 24	53.436 14	60.893 30	69.531 94	79.543 02
24	47.538 00	50.815 58	54.354 63	58.176 67	66.764 76	76.789 81	88.497 33
25	51.152 59	54.864 51	58.887 68	63.249 04	73.105 94	84.700 90	98.347 06

TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (*Continued*)
(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. Amounts are noted for the end of each year)

Year.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
26	$ 54.965 98	$ 59.156 38	$ 63.715 38	$ 68.676 47	$ 79.954 42	$ 93.323 98	$ 109.181 77
27	58.989 11	63.705 77	68.856 88	74.483 82	87.350 77	102.723 13	121.099 94
28	63.233 51	68.528 11	74.332 57	80.697 69	95.338 83	112.968 22	134.209 94
29	67.711 35	73.639 80	80.164 19	87.346 53	103.965 94	124.135 36	148.630 93
30	72.435 48	79.058 19	86.374 86	94.460 79	113.283 21	136.307 54	164.494 02
31	77.419 43	84.801 68	92.989 23	102.073 04	123.345 87	149.575 22	181.943 42
32	82.677 50	90.889 78	100.033 53	110.218 15	134.213 54	164.036 99	201.137 77
33	88.224 76	97.343 16	107.535 71	118.933 43	145.950 62	179.800 32	222.251 54
34	94.077 12	104.183 75	115.525 53	128.258 76	158.626 67	196.982 34	245.476 70
35	100.251 36	111.434 78	124.034 69	138.236 88	172.316 80	215.710 75	271.024 37
36	106.765 19	119.120 87	133.096 95	148.913 46	187.102 15	236.124 72	299.126 81
37	113.637 27	127.268 12	142.748 25	160.337 40	203.070 32	258.375 95	330.039 49
38	120.887 32	135.904 21	153.026 88	172.561 02	220.315 95	282.629 78	364.043 43
39	128.536 13	145.058 46	163.973 63	185.640 29	238.941 22	309.066 46	401.447 78
40	136.605 61	154.761 97	175.631 92	199.635 11	259.056 52	337.882 44	442.592 56
41	145.118 92	165.047 68	188.047 99	214.609 57	280.781 04	369.291 86	487.851 81
42	154.100 46	175.950 54	201.271 11	230.632 24	304.243 52	403.528 13	537.636 99
43	163.575 99	187.507 58	215.353 73	247.776 50	329.583 01	440.845 66	592.400 69
44	173.572 67	199.758 03	230.351 72	266.120 85	356.949 65	481.521 77	652.640 76
45	184.119 17	212.743 51	246.324 59	285.749 31	386.505 62	525.858 73	718.904 84
46	195.245 72	226.508 12	263.335 68	306.751 76	418.426 07	574.186 02	791.795 32
47	206.984 23	241.098 61	281.452 50	329.224 39	452.900 15	626.862 76	871.974 85
48	219.368 37	256.564 53	300.746 92	353.270 09	490.132 16	684.280 41	960.172 34
49	232.433 63	272.958 40	321.295 47	378.999 00	530.342 74	746.865 65	1057.189 57
50	246.217 48	290.335 90	343.179 67	406.528 93	573.770 16	815.083 56	1163.908 53

TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (*Continued*)
(Annual Installments of One Dollar and Accrued Interest Installments at end of each year.
Amounts are noted for the end of each year)

Year.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
51	$260.759 44	$ 308.756 06	$ 366.486 35	$ 435.985 95	$ 620.671 8	$ 889.441 1	$ 1281.299 4
52	276.101 21	328.281 42	391.307 96	467.504 97	671.325 5	970.490 8	1410.429 3
53	292.286 77	348.978 31	417.742 98	501.230 32	726.031 6	1058.834 9	1552.472 3
54	309.362 55	370.917 01	445.896 28	537.316 44	785.114 1	1155.130 1	1708.719 5
55	327.377 49	394.172 03	475.879 53	575.928 59	848.923 2	1260.091 8	1880.591 4
56	346.383 25	418.822 35	507.811 70	617.243 59	917.837 1	1374.500 6	2069.650 6
57	366.434 33	444.951 69	541.819 46	661.450 65	992.264 0	1499.205 1	2277.615 6
58	387.588 21	472.648 79	578.037 73	708.752 19	1072.64[illegible] 1	1635.133 5	2506.377 2
59	409.905 57	502.007 72	616.610 18	759.364 84	1159.456 8	1783.295 5	2758.014 9
60	433.450 37	533.128 18	657.689 84	813.520 38	1253.213 3	1944.792 1	3034.816 4
61	458.290 14	566.115 87	701.439 68	871.466 81	1354.470 4	2120.823 4	3339.298 0
62	484.496 10	601.082 82	748.033 26	933.469 49	1463.828 0	2312.697 5	3674.227 8
63	512.143 39	638.147 79	797.655 42	999.812 35	1581.934 2	2521.840 3	4042.650 6
64	541.311 27	677.436 66	850.503 03	1070.799 22	1709.489 0	2749.805 9	4447.915 7
65	572.083 39	719.082 86	906.785 72	1146.755 16	1847.248 1	2998.288 5	4893.707 3
66	604.547 98	763.227 83	966.726 80	1228.028 02	1996.027 9	3269.134 4	5384.078 0
67	638.798 12	810.021 50	1030.564 04	1314.989 98	2156.710 2	3564.356 5	5923.485 8
68	674.932 01	859.622 79	1098.550 70	1408.039 28	2330.247 0	3886.148 6	6516.834 4
69	713.053 27	912.200 16	1170.956 49	1507.602 03	2517.666 7	4236.902 0	7169.517 8
70	753.271 20	967.932 17	1248.068 67	1614.134 17	2720.080 1	4619.223 2	7887.469 6
71	795.701 12	1027.008 10	1330.193 13	1728.123 57	2938.686 5	5035.953 3	8677.216 5
72	840.464 68	1089.628 59	1417.655 68	1850.092 22	3174.781 4	5490.189 1	9545.938 2
73	887.690 24	1156.006 30	1510.803 30	1980.598 67	3429.763 9	5985.306 1	10501.532 0
74	937.513 20	1226.366 68	1610.005 52	2120.240 58	3705.145 0	6524.983 6	11552.685 2
75	990.076 43	1300.948 68	1715.655 88	2269.657 42	4002.556 6	7113.232 1	12708.953 7

TABLE 23. AMOUNT OF AN ANNUITY OF ONE DOLLAR (*Concluded*)
(Annual Installments of One Dollar and Accrued Interest Installments at end of each year. Amounts are noted for the end of each year)

Year.	$5\frac{1}{2}$ per cent.	6 per cent.	$6\frac{1}{2}$ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
76	$1045.530 6	$1380.005 6	$1828.173 5	$ 2429.533 4	$ 4323.761 2	$ 7754.423 0	$ 13980.849 1
77	1104.034 8	1463.805 9	1948.004 6	2600.600 8	4670.662 0	8453.321 1	15379.934 0
78	1165.756 7	1552.634 3	2075.624 9	2783.642 8	5045.315 0	9215.120 0	16918.927 4
79	1230.873 4	1646.792 4	2211.540 6	2979.497 8	5449.940 2	10045.480 8	18611.820 1
80	1299.571 4	1746.599 9	2356.290 7	3189.062 7	5886.935 4	10950.574 1	20474.002 1
81	1372.047 8	1852.395 9	2510.449 6	3413.297 1	6358.890 2	11937.125 8	22522.402 3
82	1448.510 4	1964.539 6	2674.628 9	3653.227 9	6868.601 5	13012.467 1	24775.642 6
83	1529.178 5	2083.412 0	2849.479 8	3909.953 8	7419.089 6	14184.589 1	27254.206 8
84	1614.283 3	2209.416 7	3035.695 9	4184.650 6	8013.616 8	15462.202 1	29980.627 5
85	1704.068 9	2342.981 7	3234.016 2	4478.576 1	8655.706 1	16854.800 3	32979.690 3
86	1798.792 7	2484.560 6	3445.227 3	4793.076 5	9349.162 6	18372.732 4	36278.659 3
87	1898.726 3	2634.634 3	3670.167 0	5129.591 8	10098.095 6	20027.278 3	39907.525 2
88	2004.156 3	2793.712 3	3909.727 9	5489.663 2	10906.943 3	21830.733 3	43899.277 7
89	2115.384 9	2962.335 1	4164.860 2	5874.939 7	11780.498 7	23796.499 3	48290.205 5
90	2232.731 0	3141.075 2	4436.576 2	6287.185 4	12723.938 6	25939.184 2	53120.226 1
91	2356.531 2	3330.539 7	4725.953 6	6728.288 4	13742.853 7	28274.710 8	58433.248 7
92	2487.140 4	3531.372 1	5034.140 6	7200.268 6	14843.282 0	30820.434 8	64277.573 5
93	2624.933 2	3744.254 4	5362.359 8	7705.287 4	16031.744 6	33595.273 9	70706.330 9
94	2770.304 5	3969.909 7	5711.913 1	8245.657 5	17315.284 1	36619.848 6	77777.964 0
95	2923.671 2	4209.104 2	6084.187 5	8823.853 5	18701.506 9	39916.635 0	85556.760 4
96	3085.473 2	4462.650 5	6480.659 7	9442.523 3	20198.627 4	43510.132 1	94113.436 4
97	3256.174 2	4731.409 5	6902.902 6	10104.499 9	21815.517 6	47427.044 0	103525.780 1
98	3436.263 8	5016.294 1	7352.591 3	10812.814 9	23561.759 0	51696.478 0	113879.358 1
99	3626.258 3	5318.271 8	7831.509 7	11570.712 0	25447.699 7	56350.161 0	125268.293 9
100	3826.702 5	5638.368 1	8341.557 9	12381.661 8	27484.515 7	61422.675 5	137796.123 3

EXPLANATION OF TABLE 24

Annuity Which Will Amount to One Dollar in a Given Time

Table 24 shows the annual investment necessary to accumulate one dollar in a given number of years at interest rates ranging from 2 to 10 per cent per annum. The annuity is assumed to be applied at the end of each year. The table shows in each case the sum of the annual installments plus the interest earnings.

This table is based on the following formula:

Let a_n' represent the annual installment which at compound interest in n years will amount to \$1.

Let n represent the number of years required by the annuity to amount to \$1.

Let i represent the interest rate expressed in per cent, as 0.05 for 5 per cent.

Then

$$a_n' = \frac{i}{(1+i)^n - 1} \tag{28}$$

and it follows from equation (26) that

$$a_n' = \frac{1}{A}. \tag{29}$$

In other words the annuity which will amount to \$1 in a given time can be found with the aid of a table showing the amount of an annuity of \$1. It is the reciprocal of the latter.

Example. — What annuity will amount at 5 per cent compound interest to \$7500 in 31 years?

From Table 24 the annuity which will amount to \$1 in 31 years at 5 per cent is found to be 0.014132. The annuity which will amount to \$7500 in 31 years is, therefore,

$$7500 \times 0.014132 = \$105.99.$$

Or from Table 23 the amount of an annuity of \$1 for 31 years at 5 per cent is found to be \$70.76079, therefore, the annuity which will amount to \$7500 in 31 years will be

$$\frac{7500}{70.76079} = \$105.99.$$

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME

(Annuity Applied at End of each Year)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	$1.000 000	$1.000 000	$1.000 000	$1.000 000	$1.000 000	$1.000 000	$1.000 000
2	.495 050	.493 827	.492 611	.491 400	.490 196	.488 998	.487 805
3	.326 755	.325 137	.323 530	.321 934	.320 349	.318 773	.317 209
4	.242 624	.240 818	.239 027	.237 251	.235 490	.233 744	.232 012
5	.192 158	.190 247	.188 355	.186 481	.184 627	.182 792	.180 975
6	.158 526	.156 550	.154 597	.152 668	.150 762	.148 878	.147 017
7	.134 512	.132 495	.130 506	.128 545	.126 610	.124 701	.122 820
8	.116 510	.114 467	.112 456	.110 477	.108 528	.106 610	.104 722
9	.102 515	.100 457	.098 434	.096 446	.094 493	.092 574	.090 690
10	.091 327	.089 259	.087 231	.085 241	.083 291	.081 379	.079 505
11	.082 178	.080 106	.078 077	.076 092	.074 149	.072 248	.070 389
12	.074 560	.072 487	.070 462	.068 484	.066 552	.064 666	.062 825
13	.068 118	.066 048	.064 030	.062 062	.060 144	.058 275	.056 456
14	.062 602	.060 537	.058 526	.056 571	.054 669	.052 820	.051 024
15	.057 825	.055 766	.053 767	.051 825	.049 941	.048 114	.046 342
16	.053 650	.051 599	.049 611	.047 685	.045 820	.044 015	.042 270
17	.049 970	.047 928	.045 953	.044 043	.042 199	.040 418	.038 699
18	.046 702	.044 670	.042 709	.040 817	.038 993	.037 237	.035 546
19	.043 782	.041 761	.039 814	.037 940	.036 139	.034 407	.032 745
20	.041 157	.039 147	.037 216	.035 361	.033 582	.031 876	.030 243
21	.038 785	.036 787	.034 872	.033 037	.031 280	.029 601	.027 996
22	.036 631	.034 647	.032 747	.030 932	.029 199	.027 546	.025 971
23	.034 668	.032 696	.030 814	.029 019	.027 309	.025 682	.024 137
24	.032 871	.030 913	.029 047	.027 273	.025 587	.023 987	.022 471
25	.031 220	.029 276	.027 428	.025 674	.024 012	.022 439	.020 952

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (*Continued*)

(Annuity Applied at End of each Year)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
26	$.029 699	$.027 769	$.025 938	$.024 205	$.022 567	$.021 021	$.019 564
27	.028 293	.026 377	.024 564	.022 852	.021 239	.019 719	.018 292
28	.026 990	.025 088	.023 293	.021 603	.020 013	.018 521	.017 123
29	.025 778	.023 891	.022 115	.020 445	.018 880	.017 414	.016 046
30	.024 650	.022 778	.021 019	.019 371	.017 830	.016 392	.015 051
31	.023 596	.021 739	.019 999	.018 372	.016 855	.015 443	.014 132
32	.022 611	.020 768	.019 047	.017 442	.015 949	.014 563	.013 280
33	.021 687	.019 859	.018 156	.016 572	.015 104	.013 745	.012 490
34	.020 819	.019 007	.017 322	.015 760	.014 315	.012 982	.011 755
35	.020 002	.018 206	.016 539	.014 998	.013 577	.012 270	.011 072
36	.019 233	.017 452	.015 804	.014 284	.012 887	.011 606	.010 434
37	.018 507	.016 741	.015 112	.013 613	.012 240	.010 984	009 840
38	.017 821	.016 070	.014 459	.012 982	.011 632	.010 402	.009 284
39	.017 171	.015 436	.013 844	.012 388	.011 061	.009 856	.008 765
40	.016 556	.014 836	.013 262	.011 827	.010 523	.009 343	.008 278
41	.015 972	.014 268	.012 712	.011 298	.010 017	.008 862	.007 822
42	.015 417	.013 729	.012 192	.010 798	.009 540	.008 409	.007 395
43	.014 890	.013 217	.011 698	.010 325	.009 090	.007 982	.006 993
44	.014 388	.012 730	.011 230	.009 878	.008 665	.007 581	.006 616
45	.013 910	.012 268	.010 785	.009 453	.008 262	.007 202	.006 262
46	.013 453	.011 827	.010 363	.009 051	.007 882	.006 845	.005 928
47	.013 018	.011 407	.009 961	.008 669	.007 522	006 507	.005 614
48	.012 602	.011 006	.009 578	.008 306	.007 181	.006 189	.005 318
49	.012 204	.010 623	.009 213	.007 962	.006 857	.005 887	.005 040
50	.011 823	.010 258	.008 865	.007 634	.006 550	.005 602	.004 777

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (*Continued*)

(Annuity Applied at End of each Year)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
51	$.011 459	$.009 909	$.008 534	$.007 322	$.006 259	$.005 332	$.004 529
52	.011 109	.009 574	.008 217	.007 024	.005 982	.005 077	.004 294
53	.010 774	.009 254	.007 915	.006 741	.005 719	.004 835	.004 073
54	.010 452	.008 948	.007 626	.006 471	.005 469	.004 605	.003 864
55	.010 143	.008 654	.007 349	.006 213	.005 231	.004 388	.003 667
56	.009 847	.008 372	.007 084	.005 967	.005 005	.004 181	.003 480
57	.009 561	.008 102	.006 831	.005 732	.004 789	.003 985	.003 303
58	.009 287	.007 842	.006 588	.005 508	.004 584	.003 799	.003 136
59	.009 022	.007 593	.006 356	.005 294	.004 388	.003 622	.002 978
60	.008 768	.007 353	.006 133	.005 089	.004 202	.003 454	.002 828
61	.008 523	.007 123	.005 919	.004 894	.004 024	.003 295	.002 686
62	.008 286	.006 901	.005 714	.004 705	.003 854	.003 143	.002 552
63	.008 058	.006 688	.005 517	.004 525	.003 692	.002 998	.002 424
64	.007 839	.006 482	.005 328	.004 353	.003 538	.002 861	.002 304
65	.007 626	.006 285	.005 146	.004 188	.003 390	.002 730	.002 189
66	.007 421	.006 094	.004 971	.004 030	.003 249	.002 606	.002 081
67	.007 223	.005 910	.004 803	.003 879	.003 115	.002 488	.001 978
68	.007 032	.005 733	.004 642	.003 734	.002 986	.002 375	.001 880
69	.006 847	.005 562	.004 486	.003 595	.002 863	.002 267	.001 787
70	.006 668	.005 397	.004 337	.003 461	.002 745	.002 165	.001 699
71	.006 494	.005 238	.004 193	.003 333	.002 633	.002 068	.001 616
72	.006 327	.005 084	.004 054	.003 210	.002 525	.001 975	.001 536
73	.006 165	.004 936	.003 921	.003 092	.002 422	.001 886	.001 461
74	.006 007	.004 792	.003 792	.002 978	.002 323	.001 802	.001 390
75	.005 855	.004 654	.003 668	.002 869	.002 229	.001 721	.001 322

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (*Continued*)

(Annuity Applied at End of each Year)

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
76	$.005 707 5	$.004 519 6	$.003 548 5	$.002 764 5	$.002 138 7	$.001 644 2	$.001 257 1
77	.005 564 5	.004 390 0	.003 433 3	.002 663 9	.002 052 2	.001 570 9	.001 195 8
78	.005 425 8	.004 264 6	.003 322 2	.002 567 2	.001 969 4	.001 501 0	.001 137 6
79	.005 291 2	.004 143 4	.003 215 1	.002 474 3	.001 890 1	.001 434 3	.001 082 2
80	.005 160 7	.004 026 0	.003 111 7	.002 384 9	.001 814 1	.001 370 7	.001 029 6
81	.005 034 0	.003 912 5	.003 012 0	.002 298 9	.001 741 3	.001 310 0	.000 979 6
82	.004 911 1	.003 802 5	.002 915 8	.002 216 3	.001 671 5	.001 252 0	.000 932 1
83	.004 791 7	.003 696 1	.002 822 8	.002 136 8	.001 604 6	.001 196 6	.000 886 9
84	.004 675 8	.003 593 0	.002 733 1	.002 060 2	.001 540 5	.001 143 8	.000 844 0
85	.004 563 2	.003 493 1	.002 646 5	.001 986 6	.001 479 1	.001 093 3	.000 803 2
86	.004 453 8	.003 396 3	.002 562 8	.001 915 8	.001 420 2	.001 045 2	.000 764 3
87	.004 347 5	.003 302 5	.002 482 0	.001 847 6	.001 363 7	.000 999 2	.000 727 4
88	.004 244 2	.003 211 7	.002 403 9	.001 781 9	.001 309 5	.000 955 2	.000 692 3
89	.004 143 7	.003 123 5	.002 328 5	.001 718 7	.001 257 6	.000 913 2	.000 658 9
90	.004 046 0	.003 038 1	.002 255 6	.001 657 8	.001 207 8	.000 873 2	.000 627 1
91	.003 951 0	.002 955 2	.002 185 1	.001 599 2	.001 160 0	.000 834 9	.000 596 9
92	.003 858 6	.002 874 9	.002 116 9	.001 542 7	.001 114 1	.000 798 3	.000 568 1
93	.003 768 7	.002 796 9	.002 051 1	.001 488 3	.001 070 1	.000 763 3	.000 540 3
94	.003 681 2	.002 721 3	.001 987 4	.001 435 9	.001 027 9	.000 729 9	.000 514 8
95	.003 596 0	.002 647 9	.001 925 8	.001 385 5	.000 987 4	.000 698 0	.000 490 0
96	.003 513 1	.002 576 6	.001 866 2	.001 336 8	.000 948 5	.000 667 5	.000 466 5
97	.003 432 4	.002 507 5	.001 808 6	.001 289 9	.000 911 2	.000 638 3	.000 444 1
98	.003 353 8	.002 440 3	.001 752 8	.001 244 8	.000 875 4	.000 610 5	.000 422 7
99	.003 277 3	.002 375 2	.001 698 9	.001 201 2	.000 841 0	.000 583 8	.000 402 4
100	.003 202 7	.002 311 9	.001 646 7	.001 159 3	.000 808 0	.000 558 4	.000 383 1

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (*Continued*)

(Annuity Applied at End of each Year)

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	$1.000 000	$1.000 000	$1.000 000	$1.000 000	$1.000 000	$1.000 000	$1.000 000
2	.486 618	.485 437	.484 262	.483 092	.480 769	.478 469	.476 190
3	.315 654	.314 110	.312 576	.311 052	.308 034	.305 055	.302 115
4	.230 295	.228 591	.226 903	.225 228	.221 921	.218 669	.215 471
5	.179 176	.177 396	.175 635	.173 891	.170 456	.167 092	.163 797
6	.145 179	.143 363	.141 568	.139 796	.136 315	.132 920	.129 607
7	.120 964	.119 135	.117 331	.115 553	.112 072	.108 691	.105 405
8	.102 864	.101 036	.099 237	.097 468	.094 015	.090 674	.087 444
9	.088 839	.087 022	.085 238	.083 486	.080 080	.076 799	.073 641
10	.077 668	.075 868	.074 105	.072 378	.069 029	.065 820	.062 745
11	.068 571	.066 793	.065 055	.063 357	.060 076	.056 947	.053 963
12	.061 029	.059 277	.057 568	.055 902	.052 695	.049 651	.046 763
13	.054 684	.052 960	.051 283	.049 651	.046 522	.043 567	.040 779
14	.049 279	.047 585	.045 940	.044 345	.041 297	.038 433	.035 746
15	.044 626	.042 963	.041 353	.039 795	.036 830	.034 059	.031 474
16	.040 583	.038 952	.037 378	.035 858	.032 977	.030 300	.027 817
17	.037 042	.035 445	.033 906	.032 425	.029 629	.027 046	.024 664
18	.033 920	.032 357	.030 855	.029 413	.026 702	.024 212	.021 930
19	.031 150	.029 621	.028 156	.026 753	.024 128	.021 730	.019 547
20	.028 679	.027 185	.025 756	.024 393	.021 852	.019 546	.017 460
21	.026 465	.025 005	.023 613	.022 289	.019 832	.017 617	.015 624
22	.024 471	.023 046	.021 691	.020 406	.018 032	.015 905	.014 005
23	.022 670	.021 278	.019 961	.018 714	.016 422	.014 382	.012 572
24	.021 036	.019 679	.018 398	.017 189	.014 978	.013 023	.011 300
25	.019 549	.018 227	.016 981	.015 811	.013 679	.011 806	.010 168

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (*Continued*)

(Annuity Applied at End of each Year)

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
26	$.018 193	$.016 904	$.015 695	$.014 561	$.012 507 1	$.010 715 4	$.009 159 0
27	.016 952	.015 697	.014 523	.013 426	.011 448 1	.009 734 9	.008 257 6
28	.015 814	.014 593	.013 453	.012 392	.010 488 9	.008 852 0	.007 451 0
29	.014 679	.013 580	.012 474	.011 449	.009 618 5	.008 055 7	.006 728 1
30	.013 805	.012 649	.011 577	.010 586	.008 827 4	.007 336 4	.006 079 2
31	.012 917	.011 792	.010 754	.009 797	.008 107 3	.006 685 6	.005 496 2
32	.012 095	.011 002	.009 997	.009 073	.007 450 8	.006 096 2	.004 971 7
33	.011 335	.010 273	.009 299	.008 408	.006 851 6	.005 561 7	.004 499 4
34	.010 630	.009 598	.008 656	.007 797	.006 304 1	.005 076 6	.004 073 7
35	.009 975	.008 974	.008 062	.007 234	.005 803 3	.004 635 8	.003 689 7
36	.009 366	.008 395	.007 513	.006 715	.005 344 7	.004 235 0	.003 343 1
37	.008 800	.007 857	.007 005	.006 237	.004 924 4	.003 870 3	.003 029 9
38	.008 272	.007 358	.006 535	.005 795	.004 538 9	.003 538 2	.002 746 9
39	.007 780	.006 894	.006 099	.005 387	.004 185 1	.003 235 6	.002 491 0
40	.007 320	.006 462	.005 694	.005 009	.003 860 2	.002 959 6	.002 259 4
41	.006 891	.006 059	.005 318	.004 660	.003 561 5	.002 707 9	.002 049 8
42	.006 489	.005 683	.004 968	.004 336	.003 286 8	.002 478 1	.001 860 0
43	.006 113	.005 333	.004 644	.004 036	.003 034 1	.002 268 4	.001 688 0
44	.005 761	.005 006	.004 341	.003 758	.002 801 5	.002 076 7	.001 532 2
45	.005 431	.004 700	.004 060	.003 500	.002 587 3	.001 901 7	.001 391 0
46	.005 122	.004 415	.003 797	.003 260	.002 389 9	.001 741 6	.001 263 0
47	.004 831	.004 148	.003 553	.003 037	.002 208 0	.001 595 2	.001 146 8
48	.004 559	.003 898	.003 325	.002 831	.002 040 3	.001 461 4	.001 041 5
49	.004 302	.003 664	.003 112	.002 639	.001 885 6	.001 338 9	.000 945 9
50	.004 061	.003 444	.002 914	.002 460	.001 742 9	.001 226 9	.000 859 2

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (*Continued*)

(Annuity Applied at End of each Year)

Years.	$5\frac{1}{2}$ per cent.	6 per cent.	$6\frac{1}{2}$ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
51	$.003 835 0	$.003 238 8	$.002 728 6	$.002 293 7	$.001 611 2	$.001 124 3	$.000 780 46
52	.003 621 9	.003 046 2	.002 555 5	.002 139 0	.001 489 6	.001 030 4	.000 709 00
53	.003 421 3	.002 865 5	.002 393 8	.001 995 1	.001 377 4	.000 944 4	.000 644 13
54	.003 232 5	.002 696 0	.002 242 7	.001 861 1	.001 273 7	.000 865 7	.000 585 23
55	.003 054 6	.002 537 0	.002 101 4	.001 736 3	.001 178 0	.000 793 6	.000 531 75
56	.002 887 0	.002 387 6	.001 969 2	.001 620 1	.001 089 5	.000 727 5	.000 483 17
57	.002 729 0	.002 247 4	.001 845 6	.001 511 8	.001 007 8	.000 667 0	.000 439 06
58	.002 580 1	.002 115 7	.001 730 0	.001 410 9	.000 932 3	.000 611 6	.000 398 98
59	.002 439 6	.001 992 0	.001 621 8	.001 316 9	.000 862 5	.000 560 8	.000 362 58
60	.002 307 1	.001 875 7	.001 520 5	.001, 229 2	.000 797 9	.000 514 2	.000 329 51
61	.002 182 0	.001 766 4	.001 425 6	.001 147 5	.000 738 3	.000 471 5	.000 299 46
62	.002 064 0	.001 663 7	.001 336 8	.001 071 3	.000 683 1	.000 432 4	.000 272 17
63	.001 952 6	.001 567 0	.001 253 7	.001 000 2	.000 632 1	.000 396 5	.000 247 36
64	.001 847 4	.001 476 2	.001 175 8	.000 933 9	.000 585 0	.000 363 7	.000 224 82
65	.001 748 0	.001 390 7	.001 102 8	.000 872 0	.000 541 3	.000 333 5	.000 204 34
66	.001 654 1	.001 310 2	.001 034 4	.000 814 3	.000 501 0	.000 305 9	.000 185 73
67	.001 565 4	.001 234 5	.000 970 3	.000 760 5	.000 463 7	.000 280 6	.000 168 82
68	.001 481 6	.001 163 3	.000 910 3	.000 710 2	.000 429 1	.000 257 3	.000 153 45
69	.001 402 4	.001 096 3	.000 854 0	.000 663 3	.000 397 2	.000 236 0	.000 139 48
70	.001 327 5	.001 033 1	.000 801 2	.000 619 5	.000 367 6	.000 216 5	.000 126 78
71	.001 256 8	.000 973 7	.000 751 8	.000 578 7	.000 340 3	.000 198 6	.000 115 24
72	.001 189 8	.000 917 7	.000 705 4	.000 540 5	.000 315 0	.000 182 1	.000 104 76
73	.001 126 5	.000 865 0	.000 661 9	.000 504 9	.000 291 6	.000 167 1	.000 095 22
74	.001 066 7	.000 815 4	.000 621 1	.000 471 6	.000 269 9	.000 153 3	.000 086 56
75	.001 010 0	.000 768 7	.000 582 9	.000 440 6	.000 249 8	.000 140 6	.000 078 68

TABLE 24. ANNUITY WHICH WILL AMOUNT TO ONE DOLLAR IN A GIVEN TIME (*Concluded*)

(Annuity Applied at End of each Year)

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
76	$.000 956 5	$.000 724 6	$.000 547 0	$.000 411 60	$.000 231 28	$.000 128 96	$.000 071 53
77	.000 905 8	.000 683 2	.000 513 4	.000 384 53	.000 214 10	.000 118 30	.000 065 02
78	.000 857 8	.000 644 1	.000 481 8	.000 359 24	.000 198 20	.000 108 52	.000 059 11
79	.000 812 4	.000 607 2	.000 452 2	.000 335 63	.000 183 49	.000 099 55	.000 053 73
80	.000 769 5	.000 572 5	.000 424 4	.000 313 57	.000 169 87	.000 091 32	.000 048 84
81	.000 728 8	.000 539 8	.000 398 3	.000 292 97	.000 157 26	.000 083 77	.000 044 40
82	.000 690 4	.000 509 0	.000 373 9	.000 273 73	.000 145 59	.000 076 85	.000 040 36
83	.000 653 9	.000 480 0	.000 350 9	.000 255 76	.000 134 79	.000 070 50	.000 036 69
84	.000 619 5	.000 452 6	.000 329 4	.000 238 97	.000 124 79	.000 064 67	.000 033 35
85	.000 586 8	.000 426 8	.000 309 2	.000 223 29	.000 115 53	.000 059 33	.000 030 32
86	.000 555 9	.000 402 5	.000 290 3	.000 208 63	.000 106 96	.000 054 43	.000 027 56
87	.000 526 7	.000 379 6	.000 272 5	.000 194 95	.000 099 03	.000 049 93	.000 025 06
88	.000 499 0	.000 357 9	.000 255 8	.000 182 16	.000 091 68	.000 045 81	.000 022 78
89	.000 472 7	.000 337 6	.000 240 1	.000 170 21	.000 084 89	.000 042 02	.000 020 71
90	.000 447 9	.000 318 4	.000 225 4	.000 159 05	.000 078 59	.000 038 55	.000 018 83
91	.000 424 4	.000 300 3	.000 211 6	.000 148 63	.000 072 77	.000 035 37	.000 017 11
92	.000 402 1	.000 283 2	.000 198 6	.000 138 88	.000 067 37	.000 032 45	.000 015 56
93	.000 381 0	.000 267 1	.000 186 5	.000 129 78	.000 062 38	.000 029 77	.000 014 14
94	.000 361 0	.000 251 9	.000 175 1	.000 121 28	.000 057 75	.000 027 31	.000 012 86
95	.000 342 0	.000 237 6	.000 164 4	.000 113 33	.000 053 47	.000 025 05	.000 011 69
96	.000 324 1	.000 224 1	.000 154 3	.000 105 90	.000 049 51	.000 022 98	.000 010 63
97	.000 307 1	.000 211 4	.000 144 9	.000 098 97	.000 045 84	.000 021 09	.000 009 66
98	.000 291 0	.000 199 4	.000 136 0	.000 092 48	.000 042 44	.000 019 34	.000 008 78
99	.000 275 8	.000 188 0	.000 127 7	.000 086 42	.000 039 30	.000 017 75	.000 007 98
100	.000 261 3	.000 177 4	.000 119 9	.000 080 76	.000 036 38	.000 016 28	.000 007 26

EXPLANATION OF TABLE 25

The Present Value of an Annuity of One Dollar

The present value of an annuity is the sum of the present values of the several annuity installments.

The present value of an annuity of \$1 receivable at the end of each year is presented for a few selected years in Table 25.

This table is based on the following formula:

Let P' represent the present value of an annuity of \$1 receivable at the end of each year during n years.

Let n represent any number of years, the term of the annuity.

Let i represent the rate of interest expressed decimally; thus for 5 per cent, $i = 0.05$.

Then:

$$P' = \frac{1}{i} - \frac{1}{i(1+i)^n} \quad \text{or} \quad \frac{1}{i}\left(1 - \frac{1}{(1+i)^n}\right). \qquad (30)$$

This equation may be written:

$$P' = \frac{(1+i)^n - 1}{i} \times \frac{1}{(1+i)^n}. \qquad (31)$$

Or by reference to equations (23) and (24)

$$P' = \frac{A''}{A'}. \qquad (32)$$

In other words the present value of the annuity of \$1 is the amount of an annuity of \$1 in n years divided by the amount of \$1 at compound interest in n years.

The present value of an annuity of \$1 receivable at the end of each year is therefore ascertainable for any number of years to 100 from Tables 21 and 23 by dividing the amount of an annuity of \$1 found in Table 23 by the amount of \$1 at compound interest found in Table 21.

Example. — What is the present value of an annuity of \$53 receivable at the end of each year for 20 years at 5 per cent interest?

The amount of an annuity of \$1 for 20 years at 5 per cent interest is found in Table 23 to be \$33.06595. The amount of \$1 at 5 per cent compound interest for 20 years is found in Table 21 to be \$2.653298, consequently, the present value of an annuity of \$1 at 5 per cent for 20 years:

$$33.06595 \div 2.653298 = \$12.46221.$$

And the present value of the annuity of \$53 for 20 years is

$$12.46221 \times 53 = \$660.50.$$

TABLE 25. THE PRESENT VALUE OF AN ANNUITY OF ONE DOLLAR

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	$ 0.980 392	$ 0.975 610	$ 0.970 874	$ 0.966 184	$ 0.961 538	$ 0.956 938	$ 0.952 381
2	1.941 561	1.927 424	1.913 470	1.899 694	1.886 095	1.872 668	1.859 410
3	2.883 883	2.856 024	2.828 611	2.801 637	2.775 091	2.748 964	2.723 248
4	3.807 729	3.761 974	3.717 098	3.673 079	3.629 895	3.587 526	3.545 950
5	4.713 460	4.645 829	4.579 707	4.515 052	4.451 822	4.389 977	4.329 477
6	5.601 431	5.508 125	5.417 191	5.328 553	5.242 137	5.157 872	5.075 692
7	6.471 991	6.349 391	6.230 283	6.114 544	6.002 055	5.892 701	5.786 373
8	7.325 481	7.170 137	7.019 692	6.873 956	6.732 745	6.595 886	6.463 213
9	8.162 237	7.970 866	7.786 109	7.607 686	7.435 332	7.268 790	7.107 822
10	8.982 585	8.752 064	8.530 203	8.316 605	8.110 896	7.912 718	7.721 735
15	12.849 263	12.381 378	11.937 935	11.517 411	11.118 387	10.739 546	10.379 658
20	16.351 433	15.589 162	14.877 475	14.212 403	13.590 326	13.007 936	12.462 210
25	19.523 457	18.424 376	17.413 148	16.481 515	15.622 080	14.828 209	14.093 945
30	22.396 456	20.930 293	19.600 441	18.392 045	17.292 033	16.288 889	15.372 451
35	24.998 619	23.145 157	21.487 220	20.000 661	18.664 613	17.461 012	16.374 194
40	27.355 479	25.102 775	23.114 772	21.355 072	19.792 774	18.401 584	17.159 086
45	29.490 160	26.833 024	24.518 713	22.495 450	20.720 040	19.156 347	17.774 070
50	31.423 606	28.362 312	25.729 764	23.455 618	21.482 185	19.762 008	18.255 925
60	34.760 887	30.908 657	27.675 564	24.944 734	22.623 490	20.638 022	18.929 290
70	37.498 620	32.897 857	29.123 421	26.000 397	23.394 515	21.202 112	19.342 677
80	39.744 514	34.451 817	30.200 763	26.748 776	23.915 392	21.565 345	19.596 460
90	41.586 929	35.665 768	31.002 407	27.279 311	24.267 278	21.799 241	19.752 262
100	43.098 352	36.614 105	31.598 905	27.655 425	24.504 999	21.949 853	19.847 910

TABLE 25. THE PRESENT VALUE OF AN ANNUITY OF ONE DOLLAR (*Continued*)

Years.	5½ per cent.	6 per cent.	6½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	$ 0.947 867	$ 0.943 396	$ 0.938 967	$ 0.934 579	$ 0.925 926	$ 0.917 431	$0.909 091
2	1.846 320	1.833 393	1.820 626	1.808 018	1.783 265	1.759 111	1.735 537
3	2.697 933	2.673 012	2.648 476	2.624 316	2.577 097	2.531 295	2.486 852
4	3.505 150	3.465 106	3.425 799	3.387 211	3.312 127	3.239 720	3.169 865
5	4.270 284	4.212 364	4.155 679	4.100 197	3.992 710	3.889 651	3.790 787
6	4.995 530	4.917 324	4.841 014	4.766 540	4.622 880	4.485 919	4.355 261
7	5.682 967	5.582 381	5.484 520	5.389 289	5.206 370	5.032 953	4.868 419
8	6.334 566	6.209 794	6.088 751	5.971 299	5.746 639	5.534 819	5.334 926
9	6.952 195	6.801 692	6.656 104	6.515 232	6.246 888	5.995 247	5.759 024
10	7.537 626	7.360 087	7.188 830	7.023 582	6.710 081	6.417 658	6.144 567
15	10.037 581	9.712 249	9.402 669	9.107 914	8.559 479	8.060 689	7.606 080
20	11.950 382	11.469 921	11.018 507	10.594 014	9.818 148	9.128 546	8.513 564
25	13.413 933	12.783 356	12.197 877	11.653 583	10.674 776	9.822 579	9.076 840
30	14.533 745	13.764 836	13.058 676	12.409 041	11.257 783	10.273 654	9.426 914
35	15.390 552	14.498 246	13.686 957	12.947 672	11.654 568	10.566 821	9.644 159
40	16.046 125	15.046 297	14.145 527	13.331 709	11.924 613	10.757 360	9.779 051
45	16.547 726	15.455 832	14.480 228	13.605 522	12.108 402	10.881 197	9.862 788
50	16.931 518	15.761 861	14.724 521	13.800 746	12.233 485	10.961 683	9.914 814
60	17.449 854	16.161 428	15.032 966	14.039 181	12.376 552	11.047 991	9.967 157
70	17.753 304	16.384 544	15.197 282	14.160 389	12.442 820	11.084 449	9.987 338
80	17.930 953	16.509 131	15.284 900	14.222 005	12.473 514	11.099 849	9.995 118
90	18.034 954	16.578 699	15.331 451	14.253 328	12.487 732	11.106 354	9.998 118
100	18.095 840	16.617 546	15.356 293	14.269 251	12.494 318	11.109 102	9.999 274

EXPLANATION OF TABLE 26

The Annuity Which $1 Will Purchase

The annuity receivable at the end of each year which $1 will purchase is noted in Table 26.

This table contains the reciprocals of the numbers appearing in Table 25. It is based on the following formula:

Let a_n'' represent the annuity receivable at the end of each year which $1 will buy for n years.

Let n represent any number of years, the term of the annuity.

Let i represent the rate of interest expressed decimally; thus for 5 per cent, $i = 0.05$.

Then:

$$a_n'' = \frac{i(1+i)^n}{(1+i)^n - 1}. \tag{33}$$

And by reference to equations (23) and (24)

$$a_n'' = \frac{A'}{A''}. \tag{34}$$

In other words the annuity receivable at the end of each year which $1 will buy for n years is the amount of $1 at compound interest divided by the amount of an annuity of $1 in n years.

The annuity which $1 will purchase for any number of years is, therefore, ascertainable from Tables 21 and 23 by dividing the amount of $1 at compound interest found in Table 21, by the amount of an annuity of $1 found in Table 23.

Example. — What annuity for 20 years can be purchased for $500 at 5 per cent interest?

The amount of $1 at 5 per cent compound interest for 20 years in Table 21 is found to be $2.653298. The amount of an annuity of $1 for 20 years at 5 per cent interest, as found in Table 23, is $33.06595; consequently the annuity which $1 will purchase for 20 years at 5 per cent is

$$2.653298 \div 33.06595 = 0.0802426.$$

And the annuity which can be purchased for $500 will be

$$0.0802426 \times 500 = \$40.12.$$

TABLE 26. THE ANNUITY WHICH ONE DOLLAR WILL BUY

Years.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	$1.020 000	$1.025 000	$1.030 000	$1.035 000	$1.040 000	$1.045 000	$1.050 000
2	.515 050	.518 827	.522 611	.526 400	.530 196	.533 998	.537 805
3	.346 755	.350 137	.353 530	.356 934	.360 349	.363 773	.367 209
4	.262 624	.265 818	.269 027	.272 251	.275 490	.278 744	.282 012
5	.212 158	.215 247	.218 355	.221 481	.224 627	.227 792	.230 975
6	.178 526	.181 550	.184 598	.187 668	.190 762	.193 878	.197 017
7	.154 512	.157 495	.160 506	.163 544	.166 610	.169 701	.172 820
8	.136 510	.139 467	.142 456	.145 477	.148 528	.151 610	.154 722
9	.122 515	.125 457	.128 434	.131 446	.134 493	.137 574	.140 690
10	.111 327	.114 259	.117 231	.120 241	.123 291	.126 379	.129 505
15	.077 825	.080 766	.083 767	.086 825	.089 941	.093 114	.096 342
20	.061 157	.064 147	.067 216	.070 361	.073 582	.076 876	.080 243
25	.051 220	.054 276	.057 428	.060 674	.064 012	.067 439	.070 952
30	.044 650	.047 778	.051 019	.054 371	.057 830	.061 392	.065 051
35	.040 002	.043 206	.046 539	.049 998	.053 577	.057 270	.061 072
40	.036 556	.039 836	.043 262	.046 827	.050 523	.054 343	.058 278
45	.033 910	.037 268	.040 785	.044 453	.048 262	.052 202	.056 262
50	.031 823	.035 258	.038 865	.042 634	.046 550	.050 602	.054 777
60	.028 768	.032 353	.036 133	.040 089	.044 202	.048 454	.052 828
70	.026 668	.030 397	.034 337	.038 461	.042 745	.047 165	.051 699
80	.025 161	.029 026	.033 112	.037 385	.041 814	.046 371	.051 030
90	.024 046	.028 038	.032 256	.036 658	.041 208	.045 873	.050 627
100	.023 203	.027 312	.031 647	.036 159	.040 808	.045 558	.050 383

TABLE 26. THE ANNUITY WHICH ONE DOLLAR WILL BUY (*Continued*)

Years.	$5\frac{1}{2}$ per cent.	6 per cent.	$6\frac{1}{2}$ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	$1.055 000	$1.060 000	$1.065 000	$1.070 000	$1.080 000	$1.090 000	$1.100 000
2	.541 618	.545 437	.549 262	.553 092	.560 769	.568 469	.576 190
3	.370 654	.374 110	.377 576	.381 052	.388 034	.395 055	.402 115
4	.285 294	.288 591	.291 903	.295 228	.301 921	.308 669	.315 471
5	.234 176	.237 396	.240 635	.243 891	.250 456	.257 092	.263 797
6	.200 179	.203 363	.206 568	.209 796	.216 315	.222 920	.229 607
7	.175 964	.179 135	.182 331	.185 553	.192 072	.198 691	.205 406
8	.157 864	.161 036	.164 237	.167 468	.174 015	.180 674	.187 444
9	.143 839	.147 022	.150 238	.153 486	.160 080	.166 799	.173 641
10	.132 668	.135 868	.139 105	.142 378	.149 029	.155 820	.162 745
15	.099 626	.102 963	.106 353	.109 795	.116 830	.124 059	.131 474
20	.083 679	.087 185	.090 756	.094 393	.101 852	.109 546	.117 460
25	.074 549	.078 227	.081 981	.085 811	.093 679	.101 806	.110 168
30	.068 805	.072 649	.076 577	.080 586	.088 827	.097 336	.105 879
35	.064 975	.068 974	.073 062	.077 234	.085 803	.094 636	.103 690
40	.062 320	.066 462	.070 694	.075 009	.083 860	.092 960	.102 259
45	.060 431	.064 700	.069 060	.073 500	.082 587	.091 902	.101 391
50	.059 061	.063 444	.067 914	.072 460	.081 743	.091 227	.100 859
60	.057 307	.061 876	.066 520	.071 229	.080 798	.090 514	.100 330
70	.056 328	.061 033	.065 801	.070 620	.080 368	.090 216	.100 127
80	.055 769	.060 573	.065 424	.070 314	.080 170	.090 091	.100 049
90	.055 448	.060 318	.065 225	.070 159	.080 079	.090 039	.100 019
100	.055 261	.060 177	.065 120	.070 081	.080 036	.090 016	.100 007

EXPLANATION OF TABLE 27

Amortization and Depreciation

Definitions of amortization and depreciation have already been given

Table 27 is an amortization table or a set of tables computed for the interest rates 4 per cent to 7 per cent per annum and for all terms of indebtedness, terms of bonds or probable life terms of perishable articles, from 2 to 75 years, which are likely to come under consideration.

By adding to this table headings at the bottom of the columns it has also been made convenient for use as a depreciation table. Care must be taken in its use to preserve the distinction indicated by the top and bottom headings and not to assume that the years of expectancy in the last column apply to the age or number of years noted in the first column.

All values noted in this table are referred to $100 as the basic amount in order that the figures in the table may be used as percentages when so desired.

The table has been computed by the use of the following formulæ which may be used when values are to be ascertained for interest rates or terms not covered by the table.

Let a_n be the amortization installment which must be invested annually in order to amount at compound interest to $100 in n years.

Let i be the interest rate expressed decimally (thus 0.05 for 5 per cent).

Let A_m be the accrued amortization in m years when the annual amortization installment is a_n and the interest rate is i.

Let m be any number of years.

Let n represent the amortization term expressed in years.

Let a_m be the current amortization in the mth year, *i.e.*, the amortization increment a_n plus interest on the amortization fund already accumulated.

Let e be the remaining years of usefulness of any article whose probable life new is n years.

The basic formula for \$100 derived from (28) from which all other formulæ are derived will be

$$a_n = \frac{100\, i}{(1+i)^n - 1}. \tag{35}$$

Note. — This formula is for the special case in which the amount of the annuity is \$100 which fact should not be overlooked in applying it.

The accrued amortization in m years will be:

$$A_m = \frac{100\,(1+i)^m - 1}{(1+i)^n - 1}. \tag{36}$$

The remaining investment at the end of the mth year will be:

$$100 - A_m = 100\left[1 - \frac{(1+i)^m - 1}{(1+i)^n - 1}\right]. \tag{37}$$

The current amortization a_m in the mth year will be:

$$a_m = 100\left[\frac{(1-i)^m - (1+i)^{m-1}}{(1+i)^n - 1}\right]. \tag{38}$$

For depreciation these formulæ may be written as follows:

The basic depreciation increment for a value of \$100 will be:

$$a_n = \frac{100\, i}{(1+i)^n - 1}. \tag{35}$$

The accrued depreciation will be:

$$A_m = \frac{100\,(1+i)^{n-e} - 1}{(1+i)^n - 1}. \tag{39}$$

The remaining or present value at the time when the expectancy is e years will be

$$100 - A_m = 100\left[1 - \frac{(1+i)^{n-e} - 1}{(1+i)^n - 1}\right]. \tag{40}$$

The current annual depreciation when the expectancy is e years will be

$$a_m = 100\left[\frac{(1+i)^{n-e} - (1+i)^{n-e-1}}{(1+i)^n - 1}\right]. \tag{41}$$

The remaining value of any perishable article depends upon the cost, all circumstances considered, of replacing it when it ceases to be useful, and the time when the article will go out of use. But this value is dependent too upon the probable life of a new article of the same kind. The remaining value of several articles with the same expectancy of which one is in the 10-year life class, another in the 20-year class and another in the 40-year class will not be the same, because the proportional service yet to be expected when compared with that of new articles will, in these cases, vary inversely as 10 to 20 to 40, and the remaining values will depart widely from each other. If the expectancy, for example, of each of three such articles is 5 years, and 6 per cent interest be made the basis of the calculation, the remaining values will be 57.23 per cent, 36.73 per cent and 28.00 per cent of the cost of replacement. The article with the probable life new of 10 years has the highest value for the reason that its remaining service years are a larger proportion of its probable life new than in the case of the other two articles with longer probable life new.

Illustration of the Use of Table 27

What amount at the end of the 26th year will be in a sinking fund for the retirement of a bond issue of $100,000 running 40 years, if the money accumulating in the sinking fund earns 6 per cent per annum?

On page 361 in the 40-year life section of Table 27 at year 27 (beginning of year 27 is the end of year 26) in the left-hand column, the accrued amortization in the 6 per cent column for each $100 will be found to be $41.1637. Consequently, the amount in the sinking fund to retire $100,000 will be $41,163.70. The amount which will be added to the sinking fund in the 27th year will be $2939.60.

Let it be assumed that by any means, such as inspection by experts, the probable remaining term of usefulness of an electric generator 13 years old has been found to be 11 years and that the type of generator to which it belongs has a probable life new of 20 years. What at 6 per cent interest is the accrued depreciation if the cost of the generator was $3000 and what is its remaining value if the cost of replacement at the end of its period of usefulness may be estimated at $2500?

On page 353 in the 20-year life section of Table 27 at 11 years in the right-hand column there are found in the column with the bottom heading "Present Value" the figures 68.7614 which represent percentage. The remaining value estimated from cost would be $0.687614 \times 3000 = \$2062.84$; consequently the accrued depreciation, estimated from cost, would be $3000 - 2062.84 = \$937.16$. The remaining value estimated from the cost to replace will be $0.687614 \times 2500 = \$1719.04$.

The actual accrued depreciation, therefore, will be

$$3000 - 1719.04 = \$1280.96.$$

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES

The Current Annual and Total Amortization and the Remaining Investment also the Current Annual and Accrued Depreciation and the Present Value for Each $100 of Bonds or of Investment.

Computed by the compound interest sinking fund method

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
2 Year Life							**2 Year Life**
	4 per cent.			5 per cent.			
1	$100.0000	$49.0196	$49.0196	$100.0000	$48.7805	$48.7805	2
2	50.9804	50.9804	100.0000	51.2195	51.2195	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$48.5437	$48.5437	$100.0000	$48.3092	$48.3092	2
2	51.4563	51.4563	100.0000	51.6908	51.6908	100.0000	1
3 Year Life							**3 Year Life**
	4 per cent.			5 per cent.			
1	$100.0000	$32.0349	$32.0349	$100.0000	$31.7209	$31.7209	3
2	67.9651	33.3162	65.3511	68.2791	33.3069	65.0278	2
3	34.6489	34.6489	100.0000	34.9722	34.9722	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$31.4110	$31.4110	$100.0000	$31.1052	$31.1052	3
2	68.5890	33.2956	64.7066	68.8948	33.2825	64.3877	2
3	35.2934	35.2934	100.0000	35.6123	35.6123	100.0000	1
4 Year Life							**4 Year Life**
	4 per cent.			5 per cent.			
1	$100.0000	$23.5490	$23.5490	$100.0000	$23.2012	$23.2012	4
2	76.4510	24.4910	8.0400	76.7988	24.3612	47.5624	3
3	51.9600	25.4706	73.5106	52. 376	25.5793	73.1417	2
4	26.4894	26.4894	100.0000	26.8583	26.8583	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$22.8591	$22.8591	$100.0000	$22.5228	$22.5228	4
2	77.1409	24.2307	47.0898	77.4772	24.0994	46.6222	3
3	52.9102	25.6845	72.7743	53.3778	25.7864	72.4086	2
4	27.2257	27.2257	100.0000	27.5914	27.5914	100.0000	1
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
5 YEAR LIFE							5 YEAR LIFE
	4 per cent.			5 per cent.			
1	$100.0000	$18.4627	$18.4627	$100.0000	$18.0975	$18.0975	5
2	81.5373	19.2012	37.6639	81.9025	19.0023	37.0998	4
3	62.3361	19.9693	57.6332	62.9002	19.9525	57.0523	3
4	42.3668	20.7680	78.4012	42.9477	20.9501	78.0024	2
5	21.5988	21.5988	100.0000	21.9976	21.9976	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$17.7396	$17.7396	$100.0000	$17.3891	$17.3891	5
2	82.2604	18.8041	36.5437	82.6109	18.6063	35.9954	4
3	63.4563	19.9322	56.4759	64.0046	19.9087	55.9041	3
4	43.5241	21.1282	77.6041	44.0959	21.3024	77.2065	2
5	22.3959	22.3959	100.0000	22.7935	22.7935	100.0000	1
6 YEAR LIFE							6 YEAR LIFE
	4 per cent.			5 per cent.			
1	$100.0000	$15.0762	$15.0762	$100.0000	$14.7017	$14.7017	6
2	84.9238	15.6792	30.7554	85.2983	15.4368	30.1385	5
3	69.2446	16.3064	47.0618	69.8615	16.2087	46.3472	4
4	52.9382	16.9587	64.0205	53.6528	17.0191	63.3663	3
5	35.9795	17.6370	81.6575	36.6337	17.8701	81.2364	2
6	18.3425	18.3425	100.0000	18.7636	18.7636	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$14.3363	$14.3363	$100.0000	$13.9796	$13.9796	6
2	85.6637	15.1964	29.5327	86.0204	14.9581	28.9377	5
3	70.4673	16.1082	45.6409	71.0623	16.0052	44.9429	4
4	54.3591	17.0747	62.7156	55.0571	17.1256	62.0685	3
5	37.2844	18.0992	80.8148	37.9315	18.3244	80.3929	2
6	19.1852	19.1852	100.0000	19.6071	19.6071	100.0000	1
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
7 Year Life							**7 Year Life**
	4 per cent.			5 per cent.			
1	$100.0000	$12.6610	$12.6610	$100.0000	$12.2820	$12.2820	7
2	87.3390	13.1674	25.8284	87.7180	12.8961	25.1781	6
3	74.1716	13.6941	39.5225	74.8219	13 5409	38.7190	5
4	60.4775	14.2419	53.7644	61.2810	14.2179	52.9369	4
5	46.2356	14.8115	68.5759	47.0631	14.9288	67.8657	3
6	31.4241	15.4040	83.9799	32.1343	15.6753	83.5410	2
7	16.0201	16.0201	100.0000	16.4590	16.4590	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$11.9135	$11.9135	$100.0000	$11.5553	$11.5553	7
2	88.0865	12.6283	24.5418	88.4447	12.3642	23.9195	6
3	75.4582	13.3860	37.9278	76.0805	13.2297	37.1492	5
4	62.0722	14.1892	52.1170	62.8508	14.1558	51.3050	4
5	47.8830	15.0405	67.1575	48.6950	15.1467	66.4517	3
6	32.8425	15.9430	83.1005	33.5483	16.2069	82.6586	2
7	16.8995	16.8995	100.0000	17.3414	17.3414	100.0000	1
8 Year Life							**8 Year Life**
	4 per cent.			5 per cent.			
1	$100.0000	$10.8528	$10.8528	$100.0000	$10.4722	$10.4722	8
2	89.1472	11.2869	22.1397	89.5278	10.9958	21.4680	7
3	77.8603	11.7384	33.8781	78.5320	11.5456	33.0136	6
4	66.1219	12.2079	46.0860	66.9864	12.1229	45.1365	5
5	53.9140	12.6962	58.7822	54.8635	12.7290	57.8655	4
6	41.2178	13.2041	71.9863	42.1345	13.36[illegible]4	71.2309	3
7	28.0137	13.7322	85.7185	28.7691	14.0337	85.2646	2
8	14.2815	14.2815	100.0000	14.7354	14.7354	100.0000	1
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy, Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(Continued)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
8 Year Life							**8 Year Life**
	6 per cent.			7 per cent.			
1	$100.0000	$10.1036	$10.1036	$100.0000	$ 9.7468	$ 9.7468	8
2	89.8964	10.7098	20.8134	90.2532	10.4290	20.1758	7
3	79.1866	11.3524	32.1658	79.8242	11.1591	31.3349	6
4	67.8342	12.0335	44.1993	68.6651	11.9402	43.2751	**5**
5	55.8007	12.7556	56.9549	56.7249	12.7760	56.0511	4
6	43.0451	13.5209	70.4758	43.9489	13.6704	69.7215	3
7	29.5242	14.3321	84.8079	30.2785	14.6273	84.3488	2
8	15.1921	15.1921	100.0000	15.6512	15.6512	100.0000	1
9 Year Life							**9 Year Life**
	4 per cent.			5 per cent.			
1	$100.0000	$ 9.4493	$ 9.4493	$100.0000	$ 9.0690	$ 9.0690	9
2	90.5507	9.8273	19.2766	90.9310	9.5225	18.5915	8
3	80.7234	10.2204	29.4970	81.4085	9.9986	28.5901	7
4	70.5030	10.6292	40.1262	71.4099	10.4985	39.0886	6
5	59.8738	11.0543	51.1805	60.9114	11.0234	50.1120	**5**
6	48.8195	11.4965	62.6770	49.8880	11.5746	61.6866	4
7	37.3230	11.9564	74.6334	38.3134	12.1533	73.8399	3
8	25.3666	12.4346	87.0680	26.1601	12.7610	86.6009	2
9	12.9320	12.9320	100.0000	13.3991	13.3991	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$ 8.7022	$ 8.7022	$100.0000	$ 8.3486	$ 8.3486	9
2	91.2978	9.2244	17.9266	91.6514	8.9330	17.2816	8
3	82.0734	9.7778	27.7044	82.7184	9.5584	26.8400	7
4	72.2956	10.3645	38.0689	73.1600	10.2275	37.0675	6
5	61.9311	10.9864	49.0553	62.9325	10.9434	48.0109	**5**
6	50.9447	11.6455	60.7008	51.9891	11.7094	59.7203	4
7	39.2992	12.3443	73.0451	40.2797	12.5291	72.2494	3
8	26.9549	13.0849	86.1300	27.7506	13.4061	85.6555	2
9	13.8700	13.8700	100.0000	14.3445	14.3445	100.0000	1
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
10 YEAR LIFE						10 YEAR LIFE	
	4 per cent.			5 per cent.			
1	$100.0000	$ 8.3291	$ 8.3291	$100.0000	$ 7.9505	$ 7.9505	10
2	91.6709	8.6623	16.9914	92.0495	8.3480	16.2985	9
3	83.0086	9.0088	26.0002	83.7015	8.7654	25.0639	8
4	73.9998	9.3690	35.3692	74.9361	9.2037	34.2676	7
5	64.6308	9.7439	45.1131	65 7324	9.6638	43.9314	6
6	54.8869	10.1336	55.2467	56.0686	10.1470	54.0784	5
7	44.7533	10.5389	65.7856	45.9216	10.6544	64.7328	4
8	34.2144	10.9606	76.7462	35.2672	11.1871	75.9199	3
9	23.2538	11.3989	88.1451	24.0801	11.7464	87.6663	2
10	11.8549	11.8549	100.0000	12.3337	12.3337	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$ 7.5868	$ 7.5868	$100.0000	$ 7.2377	$ 7.2377	10
2	92.4132	8.0420	15.6288	92.7623	7.7444	14.9821	9
3	84.3712	8.5245	24.1533	85.0179	8.2865	23.2686	8
4	75.8467	9.0360	33.1893	76.7314	8.8666	32.1352	7
5	66.8107	9.5782	42.7675	67.8648	9.4872	41.6224	6
6	57.2325	10.1528	52.9203	58.3776	10.1513	51.7737	5
7	47.0797	10.7620	63.6823	48.2263	10.8619	62.6356	4
8	36.3177	11.4078	75.0901	37.3644	11.6223	74.2579	3
9	24.9099	12.0922	87.1823	25.7421	12.4358	86.6937	2
10	12.8177	12.8177	100.0000	13.3063	13.3063	100.0000	1
15 YEAR LIFE						15 YEAR LIFE	
	4 per cent.			5 per cent.			
1	$100.0000	$ 4.9941	$ 4.9941	$100.0000	$ 4.6342	$ 4.6342	15
2	95.0059	5.1939	10.1880	95.3658	4.8660	9.5002	14
3	89.8120	5.4016	15.5896	90.4998	5.1092	14.6094	13
4	84.4104	5.6177	21.2073	85.3906	5.3646	19.9740	12
5	78.7927	5.8424	27.0497	80.0260	5.6330	25.6070	11
6	72.9503	6.0760	33.1257	74.3930	5.9146	31.5216	10
7	66.8743	6.3192	39.4449	68.4784	6.2103	37.7319	9
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
15 YEAR LIFE							15 YEAR LIFE
	4 per cent.			5 per cent.			
8	$60.5551	$6.5719	$46.0168	$62.2681	$6.5209	$44.2528	8
9	53.9832	6.8348	52.8516	55.7472	6.8468	51.0996	7
10	47.1484	7.1081	59.9597	48.9004	7.1892	58.2888	6
11	40.0403	7.3925	67.3522	41.7112	7.5487	65.8375	5
12	32.6478	7.6882	75.0404	34.1625	7.9261	73.7636	4
13	24.9596	7.9957	83.0361	26.2364	8.3224	82.0860	3
14	16.9639	8.3156	91.3517	17.9140	8.7385	90.8245	2
15	8.6483	8.6483	100.0000	9.1755	9.1755	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$4.2963	$ 4.2963	$100.0000	$ 3.9795	$ 3.9795	15
2	95.7037	4.5540	8.8503	96.0205	4.2580	8.2375	14
3	91.1497	4.8273	13.6776	91.7625	4.5561	12.7936	13
4	86.3224	5.1170	18.7946	87.2064	4.8750	17.6686	12
5	81.2054	5.4239	24.2185	82.3314	5.2162	22.8848	11
6	75.7815	5.7493	29.9678	77.1152	5.5814	28.4662	10
7	70.0322	6.0944	36.0622	71.5338	5.9722	34.4384	9
8	63.9378	6.4601	42.5223	65.5616	6.3901	40.8285	8
9	57.4777	6.8476	49.3699	59.1715	6.8375	47.6660	7
10	50.6301	7.2584	56.6283	52.3340	7.3160	54.9820	6
11	43.3717	7.6941	64.3224	45.0180	7.8282	62.8102	5
12	35.6776	8.1555	72.4779	37.1898	8.3762	71.1864	4
13	27.5221	8.6450	81.1229	28.8136	8.9625	80.1489	3
14	18.8771	9.1636	90.2865	19.8511	9.5899	89.7388	2
15	9.7135	9.7135	100.0000	10.2612	10.2612	100.0000	1
20 YEAR LIFE							20 YEAR LIFE
	4 per cent.			5 per cent.			
1	$100.0000	$3.3582	$ 3.3582	$100.0000	$3.0243	$ 3.0243	20
2	96.6418	3.4925	6.8507	96.9757	3.1755	6.1998	19
3	93.1493	3.6322	10.4829	93.8002	3.3342	9.5340	18
4	89.5171	3.7775	14.2604	90.4660	3.5010	13.0350	17
5	85.7396	3.9286	18.1890	86.9650	3.6760	16.7110	16
6	81.8110	4.0857	22.2747	83.2890	3.8598	20.5708	15
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
20 YEAR LIFE							20 YEAR LIFE
	4 per cent.			5 per cent.			
7	$77.7253	$4.2492	$26.5239	$79.4292	$4.0528	$24.6236	14
8	73.4761	4.4191	30.9430	75.3764	4.2554	28.8790	13
9	69.0570	4.5959	35.5389	71.1210	4.4682	33.3472	12
10	64.4611	4.7797	40.3186	66.6528	4.6916	38.0388	11
11	59.6814	4.9709	45.2895	61.9612	4.9262	42.9650	10
12	54.7105	5.1698	50.4593	57.0350	5.1725	48.1375	9
13	49.5407	5.3766	55.8359	51.8625	5.4311	53.5686	8
14	44.1641	5.5916	61.4275	46.4314	5.7027	59.2713	7
15	38.5725	5.8152	67.2427	40.7287	5.9878	65.2591	6
16	32.7573	6.0479	73.2906	34.7409	6.2872	71.5463	5
17	26.7094	6.2898	79.5804	28.4537	6.6016	78.1479	4
18	20.4196	6.5414	86.1218	21.8521	6.9317	85.0796	3
19	13.8782	6.8031	92.9249	14.9204	7.2782	92.3578	2
20	7.0751	7.0751	100.0000	7.6422	7.6422	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$2.7185	$ 2.7185	$100.0000	$2.4393	$ 2.4393	20
2	97.2815	2.8815	5.6000	97.5607	2.6100	5.0493	19
3	94.4000	3.0545	8.6545	94.9507	2.7928	7.8421	18
4	91.3455	3.2377	11.8922	92.1579	2.9882	10.8303	17
5	88.1078	3.4320	15.3242	89.1697	3.1974	14.0277	16
6	84.6758	3.6379	18.9621	85.9723	3.4213	17.4490	15
7	81.0379	3.8562	22.8183	82.5510	3.6607	21.1097	14
8	77.1817	4.0875	26.9058	78.8903	3.9170	25.0267	13
9	73.0942	4.3328	31.2386	74.9734	4.1911	29.2178	12
10	68.7614	4.5928	35.8314	70.7822	4.4846	33.7024	11
11	64.1686	4.8683	40.6998	66.2976	4.7984	38.5008	10
12	59.3002	5.1604	45.8602	61.4992	5.1344	43.6352	9
13	54.1398	5.4701	51.3303	56.3648	5.4937	49.1289	8
14	48.6697	5.7982	57.1285	50.8711	5.8783	55.0072	7
15	42.8715	6.1462	63.2747	44.9928	6.2898	61.2970	6
16	36.7253	6.5149	69.7896	38.7030	6.7301	68.0271	5
17	30.2104	6.9058	76.6954	31.9729	7.2012	75.2283	4
18	23.3046	7.3202	84.0156	24.7717	7.7053	82.9336	3
19	15.9844	7.7594	91.7750	17.0664	8.2446	91.1782	2
20	8.2250	8.2250	100.0000	8.8218	8.8218	100.0000	1
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
25 YEAR LIFE							25 YEAR LIFE
	4 per cent.			5 per cent.			
1	$100.0000	$2.4012	$ 2.4012	$100.0000	$2.0952	$ 2.0952	**25**
2	97.5988	2.4972	4.8984	97.9048	2.2001	4.2952	24
3	95.1016	2.5972	7.4956	95.7047	2.3100	6.6053	23
4	92.5044	2.7010	10 1966	93.3947	2.4254	9.0307	22
5	89.8034	2.8091	13.0057	90.9693	2.5468	11.5775	21
6	86.9943	2.9214	15.9271	88.4225	2.6742	14.2517	**20**
7	84.0729	3.0383	18.9654	85.7483	2.8078	17.0595	19
8	81.0346	3.1598	22.1252	82.9405	2.9482	20.0077	18
9	77.8748	3.2862	25.4114	79.9923	3.0957	23.1034	17
10	74.5886	3.4176	28.8290	76.8966	3.2504	26.3538	16
11	71.1710	3.5544	32.3834	73.6462	3.4129	29.7667	**15**
12	67.6166	3.6965	36.0799	70.2333	3.5836	33.3503	14
13	63.9201	3.8444	39.9243	66.6497	3.7627	37.1130	13
14	60.0757	3.9982	43.9225	62.8870	3.9509	41.0639	12
15	56.0775	4.1581	48.0806	58.9361	4.1485	45.2124	11
16	51.9194	4.3244	52.4050	54.7876	4.3559	49.5683	**10**
17	47.5950	4.4974	56.9024	50.4317	4.5736	54.1419	9
18	43.0976	4.6773	61.5797	45.8581	4.8024	58.9443	8
19	38.4203	4.8643	66.4440	41.0557	5.0424	63.9867	7
20	33.5560	5.0590	71.5030	36.0133	5.2946	69.2813	6
21	28.4970	5.2613	76.7643	30.7187	5.5593	74.8406	**5**
22	23.2357	5.4718	82.2361	25.1594	5.8373	80.6779	4
23	17.7639	5.6906	87.9267	19.3221	6.1291	86.8070	3
24	12.0733	5.9183	93.8450	13.1930	6.4356	93.2426	2
25	6.1550	6.1550	100.0000	6.7574	6.7574	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$ 1.8227	$ 1.8227	$100.0000	$1.5811	$ 1.5811	**25**
2	98.1773	1.9320	3.7547	98.4189	1.6917	3.2728	24
3	96.2453	2.0480	5.8027	96.7272	1.8101	5.0829	23
4	94.1973	2.1708	7.9735	94.9171	1.9369	7.0198	22
5	92.0265	2.3011	10.2746	92.9802	2.0724	9.0922	21
6	89.7254	2.4391	12.7137	90.9078	2.2175	11.3097	**20**
7	87.2863	2.5855	15.2992	88.6903	2.3728	13.6825	19
8	84.7008	2.7406	18.0398	86.3175	2.5388	16.2213	18
9	81.9602	2.9051	20.9449	83.7787	2.7165	18.9378	17
10	79.0551	3.0794	24.0243	81.0622	2.9067	21.8445	16
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(Continued)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
25 Year Life							25 Year Life
	6 per cent.			7 per cent.			
11	$75.9757	$3.2641	$27.2884	$78.1555	$3.1102	$24.9547	**15**
12	72.7116	3.4600	30.7484	75.0453	3.3279	28.2826	14
13	69.2516	3.6675	34.4159	71.7174	3.5608	31.8434	13
14	65.5841	3.8877	38.3036	68.1566	3.8101	35.6535	12
15	61.6964	4.1208	42.4244	64.3465	4 0768	39.7303	11
16	57.5756	4.3682	46.7926	60 2697	4.3622	44.0925	**10**
17	53.2074	4.6302	51.4228	55.9075	4.6675	48.7600	9
18	48.5772	4.9081	56.3309	51.2400	4.9942	53.7542	8
19	43.6691	5.2025	61.5334	46.2458	5.3439	59.0981	7
20	38.4666	5.5147	67.0481	40.9019	5.7179	64.8160	6
21	32.9519	5.8455	72.8936	35.1840	6.1181	70.9341	5
22	27.1064	6.1963	79.0899	29.0659	6.5465	77.4806	4
23	20.9101	6.5681	85.6580	22.5194	7.0047	84.4853	3
24	14.3420	6.9621	92.6201	15.5147	7.4950	91.9803	2
25	7.3799	7.3799	100.0000	8.0197	8.0197	100.0000	1
30 Year Life							30 Year Life
	4 per cent.			5 per cent.			
1	$100.0000	$1.7830	$ 1.7830	$100.0000	$1.5051	$ 1.5051	**30**
2	98.2170	1.8543	3.6373	98.4949	1.5804	3.0855	29
3	96.3627	1.9285	5.5658	96.9145	1.6595	4.7450	28
4	94.4342	2.0057	7.5715	95.2550	1.7424	6.4874	27
5	92.4285	2.0859	9.6574	93.5126	1.8295	8.3169	26
6	90.3426	2.1693	11.8267	91.6831	1.9210	10.2379	**25**
7	88.1733	2.2560	14.0827	89.7621	2.0170	12.2549	24
8	85.9173	2.3463	16.4290	87.7451	2.1179	14.3728	23
9	83.5710	2.4402	18.8692	85.6272	2.2238	16.5966	22
10	81.1308	2.5378	21.4070	83.4034	2.3349	18.9315	21
11	78.5930	2.6393	24.0463	81.0685	2.4517	21.3832	**20**
12	75.9537	2.7449	26.7912	78.6168	2.5743	23.9575	19
13	73.2088	2.8545	29.6457	76.0425	2.7031	26.6606	18
14	70.3543	2.9689	32.6146	73.3394	2.8381	29.4987	17
15	67.3854	3.0877	35.7023	70.5013	2.9801	32.4788	16
16	64.2977	3.2111	38.9134	67.5212	3.1291	35.6079	**15**
17	61.0866	3.3395	42.2529	64.3921	3.2855	38.8934	14
	Present value.	Current depreciation during year.	Accrued depreciation	Present value.	Current depreciation during year.	Accrued depreciation	Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
30 Year Life.						30 Year Life	
	4 per cent.			5 per cent.			
18	$57.7471	$3.473	$45.7260	$61.1066	$3.4499	$42.3433	13
19	54.2740	3.6121	49.3381	57.6567	3.6223	45.9656	12
20	50.6619	3.7565	53.0946	54.0344	3.8034	49.7690	11
21	46.9054	3.9068	57.0014	50.2310	3.9936	53.7626	**10**
22	42.9986	4.0631	61.0645	46.2374	4.1932	57.9558	9
23	38.9355	4.2256	65.2901	42.0442	4.4030	62.3588	8
24	34.7099	4.3946	69.6847	37.6412	4.6231	66.9819	7
25	30.3153	4.5704	74.2551	33.0181	4.8542	71.8361	6
26	25.7449	4.7532	79.0083	28.1639	5.0969	76.9330	**5**
27	20.9917	4.9433	83.9516	23.0670	5.3518	82.2848	4
28	16.0484	5.1411	89.0927	17.7152	5.6194	87.9042	3
29	10.9073	5.3467	94.4394	12.0958	5.9004	93.8046	2
30	5.5606	5.5606	100.0000	6.1954	6.1954	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$1.2649	$ 1.2649	$100.0000	$1.0586	$ 1.0586	**30**
2	98.7351	1.3408	2.6057	98.9414	1.1328	2.1914	29
3	97.3943	1.4212	4.0269	97.8086	1.2120	3.4034	28
4	95.9731	1.5065	5.5334	96.5966	1.2969	4.7003	27
5	94.4666	1.5969	7.1303	95.2997	1.3877	6.0880	26
6	92.8697	1.6927	8.8230	93.9120	1.4848	7.5728	**25**
7	91.1770	1.7943	10.6173	92.4272	1.5887	9.1615	24
8	89.3827	1.9019	12.5192	90.8385	1.6999	10.8614	23
9	87.4808	2.0161	14.5353	89.1386	1.8190	12.6804	22
10	85.4647	2.1370	16.6723	87.3196	1.9462	14.6266	21
11	83.3277	2.2652	18.9375	85.3734	2.0825	16.7091	**20**
12	81.0625	2.4011	21.3386	83.2909	2.2283	18.9374	19
13	78.6614	2.5452	23.8838	81.0626	2.3843	21.3217	18
14	76.1162	2.6980	26.5818	78.6783	2.5512	23.8729	17
15	73.4182	2.8597	29.4415	76.1271	2.7297	26.6026	16
16	70.5585	3.0314	32.4729	73.3974	2.9208	29.5234	**15**
17	67.5271	3.2133	35.6862	70.4766	3.1253	32.6487	14
18	64.3138	3.4061	39.0923	67.3513	3.3440	35.9927	13
19	60.9077	3.6104	42.7027	64.0073	3.5782	39.5709	12
20	57.2973	3.8271	46.5298	60.4291	3.8286	43.3995	11
	Present value.	Current depreciation during year.	Accrued depreciation	Present value.	Current depreciation during year.	Accrued depreciation	Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
30 Year Life							30 Year Life
	6 per cent.			7 per cent.			
21	$53.4702	$4.0566	$50.5864	$56.6005	$4.0966	$47.4961	**10**
22	49.4136	4.3001	54.8865	52.5039	4.3883	51.8794	9
23	45.1135	4.5581	59.4446	48.1206	4.6902	56.5696	8
24	40.5554	4.8316	64.2762	43.4304	5.0186	61.5882	7
25	35.7238	5.1214	69.3976	38.4118	5.3698	66.9580	6
26	30.6024	5.4288	74.8264	33.0420	5.7457	72.7037	**5**
27	25.1736	5.7544	80.5808	27.2963	6.1479	78.8516	4
28	19.4192	6.0998	86.6806	21.1484	6.5782	85.4298	3
29	13.3194	6.4657	93.146	14.5702	7.0387	92.4685	2
30	6.8537	6.8537	100.0000	7.5315	7.5315	100.0000	1
35 Year Life							35 Year Life
	4 per cent.			5 per cent.			
1	$100.0000	$1.3577	$1.3577	$100.0000	$1.1072	$1.1072	**35**
2	98.6423	1.4121	2.7698	98.8928	1.1625	2.2697	34
3	97.2302	1.4685	4.2383	97.7303	1.2207	3.4904	33
4	95.7617	1.5273	5.7656	96.5096	1.2816	4.7720	32
5	94.2344	1.5884	7.3540	95.2880	1.3458	6.1178	31
6	92.6460	1.6519	9.0059	93.8822	1.4131	7.5309	**30**
7	90.9941	1.7179	10.7238	92.4691	1.4837	9.0146	29
8	89.2762	1.7867	12.5105	90.9854	1.5579	10.5725	28
9	87.4895	1.8581	14.3686	89.4275	1.6358	12.2083	27
10	85.6314	1.9325	16.3011	87.7917	1.7176	13.9259	26
11	83.6989	2.0098	18.3109	86.0741	1.8035	15.7294	**25**
12	81.6891	2.0902	20.4011	84.2706	1.8936	17.6230	24
13	79.5989	2.1737	22.5748	82.3770	1.9883	19.6113	23
14	77.4252	2.2607	24.8355	80.3887	2.0877	21.6990	22
15	75.1645	2.3512	27.1867	78.3010	2.1922	23.8912	21
16	72.8133	2.4452	29.6319	76.1088	2.3017	26.1929	**20**
17	70.3681	2.5430	32.1749	73.8071	2.4168	28.6097	19
18	67.8251	2.6447	34.8196	71.3903	2.5377	31.1474	18
19	65.1804	2.7505	37.5701	68.8526	2.6645	33.8119	17
20	62.4299	2.8605	40.4306	66.1881	2.7978	36.6097	16
21	59.5694	2.9750	43.4056	63.3903	2.9376	39.5473	**15**
22	56.5944	3.0939	46.4995	60.4527	3.0845	42.6318	14
23	53.5005	3.2178	49.7173	57.3682	3.2388	45.8706	13
	Present value.	Current depreciation during year.	Accrued depreciation	Present value.	Current depreciation during year.	Accrued depreciation	Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(Continued)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
35 YEAR LIFE							35 YEAR LIFE
	4 per cent.			5 per cent.			
24	$50.2827	$3.3464	$53.0637	$54.1294	$3.4007	$49.2713	12
25	46.9363	3.4803	56.5440	50.7287	3.5708	52.8421	11
26	43.4560	3.6195	60.1635	47.1579	3.7492	56.5913	**10**
27	39.8365	3.7642	63.9277	43.4087	3.9367	60.5280	9
28	36.0723	3.9149	67.8426	39.4720	4.1337	64.6617	8
29	32.1574	4.0714	71.9140	35.3383	4.3403	69.0020	7
30	28.0860	4.2343	76.1483	30.9980	4.5572	73.5592	6
31	23.8517	4.4037	80.5520	26.4418	4.7851	78.3443	**5**
32	19.4480	4.5798	85.1318	21.6557	5.0244	83.3687	4
33	14.8682	4.7630	89.8948	16.6313	5.2756	88.6443	3
34	10.1052	4.9535	94.8483	11.3557	5.5394	94.1837	2
35	5.1517	5.1517	100.0000	5.8163	5.8163	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$0.8974	$ 0.8974	$100.0000	$0.7234	$0.7234	**35**
2	99.1026	0.9512	1.8486	99.2766	0.7740	1.4974	34
3	98.1514	1.0083	2.8569	98.5026	0.8282	2.3256	33
4	97.1431	1.0688	3.9257	97.6744	0.8862	3.2118	32
5	96.0743	1.1329	5.0586	96.7882	0.9483	4.1601	31
6	94.9414	1.2010	6.2596	95.8399	1.0146	5.1747	**30**
7	93.7404	1.2729	7.5325	94.8253	1.0856	6.2603	29
8	92.4675	1.3493	8.8818	93.7397	1.1616	7.4219	28
9	91.1182	1.4303	10.3121	92.5781	1.2429	8.6648	27
10	89.6879	1.5162	11.8283	91.3352	1.3300	9.9948	26
11	88.1717	1.6071	13.4354	90.0052	1.4230	11.4178	**25**
12	86.5646	1.7034	15.1388	88.5822	1.5226	12.9404	24
13	84.8612	1.8058	16.9446	87.0596	1.6293	14.5697	23
14	83.0554	1.9140	18.8586	85.4303	1.7432	16.3129	22
15	81.1414	2.0289	20.8875	83.6871	1.8653	18.1782	21
16	79.1125	2.1506	23.0381	81.8218	1.9959	20.1741	**20**
17	76.9619	2.2797	25.3178	79.8259	2.1356	22.3097	19
18	74.6822	2.4165	27.7343	77.6903	2.2851	24.5948	18
19	72.2657	2.5614	30.2957	75.4052	2.4450	27.0398	17
20	69.7043	2.7152	33.0109	72.9602	2.6162	29.6560	16
21	66.9891	2.8780	35.8889	70.3440	2.7993	32.4553	**15**
22	64.1111	3.0507	38.9396	67.5447	2.9953	35.4506	14
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
35 YEAR LIFE						35 YEAR LIFE	
	6 per cent.			7 per cent.			
23	$61.0604	$3.2338	$42.1734	$64.5494	$3.2049	$38.6555	13
24	57.8266	3.4278	45.6012	61.3445	3.4293	42.0848	12
25	54.3988	3.6334	49.2346	57.9152	3.6693	45.7541	11
26	50.7654	3.8515	53.0861	54.2459	3.9262	49.6803	**10**
27	46.9139	4.0826	57.1687	50.3197	4.2010	53.8813	9
28	42.8313	4.3275	61.4962	46.1197	4.4951	58.3764	8
29	38.5038	4.5871	66.0833	41.6236	4.8097	63.1861	7
30	33.9167	4.8624	70.9457	36.8139	5.1464	68.3325	6
31	29.0543	5.1541	76.0998	31.6675	5.5067	73.8392	**5**
32	23.9002	5.4634	81.5632	26.1608	5.8922	79.7314	4
33	18.4368	5.7911	87.3543	20.2686	6.3046	86.0360	3
34	12.6457	6.1387	93.4930	13.9640	6.7459	92.7819	2
35	6.5070	6.5070	100.0000	7.2181	7.2181	100.0000	1
40 YEAR LIFE						40 YEAR LIFE	
	4 per cent.			5 per cent.			
1	$100.0000	$1.0524	$ 1.0524	$100.0000	$0.8278	$ 0.8278	**40**
2	98.9476	1.0944	2.1468	99.1722	0.8692	1.6970	39
3	97.8532	1.1382	3.2850	98.3030	0.9127	2.6097	38
4	96.7150	1.1838	4.4688	97.3903	0.9583	3.5680	37
5	95.5312	1.2311	5.6999	96.4320	1.0062	4.5742	36
6	94.3001	1.2803	6.9802	95.4258	1.0565	5.6307	**35**
7	93.0198	1.3316	8.3118	94.3693	1.1093	6.7400	34
8	91.6882	1.3848	9.6966	93.2600	1.1648	7.9048	33
9	90.3034	1.4402	11.1368	92.0952	1.2230	9.1278	32
10	88.8632	1.4978	12.6346	90.8722	1.2842	10.4120	31
11	87.3654	1.5577	14.1923	89.5880	1.3484	11.7604	**30**
12	85.8077	1.6201	15.8124	88.2396	1.4158	13.1762	29
13	84.1876	1.6848	17.4972	86.8238	1.4866	14.6628	28
14	82.5028	1.7523	19.2495	85.3372	1.5610	16.2238	27
15	80.7505	1.8223	21.0718	83.7762	1.6390	17.8628	26
16	78.9282	1.8952	22.9670	82.1372	1.7210	19.5838	**25**
17	77.0330	1.9710	24.9380	80.4162	1.8070	21.3908	24
18	75.0620	2.0499	26.9879	78.6092	1.8974	23.2882	23
19	73.0121	2.1319	29.1198	76.7118	1.9923	25.2805	22
20	70.8802	2.2171	31.3369	74.7195	2.0919	27.3724	21
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
40 YEAR LIFE							40 YEAR LIFE
	4 per cent.			5 per cent.			
21	$68.6631	$2.3059	$33.6428	$72.6276	$2.1965	$29.5689	**20**
22	66.3572	2.3980	36.0408	70.4311	2.3063	31.8752	19
23	63.9592	2.4940	38.5348	68.1248	2.4216	34.2968	18
24	61.4652	2.5937	41.1285	65.7032	2.5427	36.8395	17
25	58.8715	2.6975	43.8260	63.1605	2.6698	39.5093	16
26	56.1740	2.8054	46.6314	60.4907	2.8033	42.3126	**15**
27	53.3686	2.9176	49.5490	57.6874	2.9434	45.2560	14
28	50.4510	3.0343	52.5833	54.7440	3.0906	48.3466	13
29	47.4167	3.1557	55.7390	51.6534	3.2451	51.5917	12
30	44.2610	3.2819	59.0209	48.4083	3.4074	54.9991	11
31	40.9791	3.4132	62.4341	45.0009	3.5778	58.5769	**10**
32	37.5659	3.5498	65.9839	41.4231	3.7567	62.3336	9
33	34.0161	3.6916	69.6755	37.6664	3.9445	66.2781	8
34	30.3245	3.8393	73.5148	33.7219	4.1417	70.4198	7
35	26.4852	3.9930	77.5078	29.5802	4.3488	74.7686	6
36	22.4922	4.1526	81.6604	25.2314	4.5662	79.3348	**5**
37	18.3396	4.3189	85.9793	20.6652	4.7946	84.1294	4
38	14.0207	4.4915	90.4708	15.8706	5.0343	89.1637	3
39	9.5292	4.6712	95.1420	10.8363	5.2860	94.4497	2
40	4.8580	4.8580	100.0000	5.5503	5.5503	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$0.6462	$ 0.6462	$100.0000	$0.5009	$ 0.5009	**40**
2	99.3538	0.6849	1.3311	99.4991	0.5360	1.0369	39
3	98.6689	0.7260	2.0571	98.9631	0.5735	1.6104	38
4	97.9429	0.7696	2.8267	98.3896	0.6136	2.2240	37
5	97.1733	0.8157	3.6424	97.7760	0.6566	2.8806	36
6	96.3576	0.8647	4.5071	97.1194	0.7026	3.5832	**35**
7	95.4929	0.9166	5.4237	96.4168	0.7517	4.3349	34
8	94.5763	0.9716	6.3953	95.6651	0.8044	5.1393	33
9	93.6047	1.0299	7.4252	94.8607	0.8606	5.9999	32
10	92.5748	1.0916	8.5168	94.0001	0.9210	6.9209	31
11	91.4832	1.1572	9.6740	93.0791	0.9853	7.9062	**30**
12	90.3260	1.2266	10.9006	92.0938	1.0544	8.9606	29
13	89.0994	1.3002	12.2008	91.0394	1.1281	10.0887	28
14	87.7992	1.3782	13.5790	89.9113	1.2071	11.2958	27
15	86.4210	1.4609	15.0399	88.7042	1.2917	12.5875	26
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
40 YEAR LIFE							40 YEAR LIFE
	6 per cent.			7 per cent.			
16	$84.9601	$1.5485	$16.5884	$87.4125	$1.3820	$13.9695	**25**
17	83.4116	1.6415	18.2299	86.0304	1.4788	15.4483	24
18	81.7701	1.7399	19.9698	84.5517	1.5823	17.0306	23
19	80.0302	1.8443	21.8141	82.9694	1.6930	18.7236	22
20	78.1859	1.9550	23.7691	81.2764	1.8116	20.5352	21
21	76.2309	2.0724	25.8415	79.4648	1.9384	22.4736	**20**
22	74.1585	2.1966	28.0381	77.5264	2.0741	24.5477	19
23	71.9619	2.3284	30.3665	75.4523	2.2192	26.7669	18
24	69.6335	2.4682	32.8347	73.2331	2.3746	29.1415	17
25	67.1653	2.6162	35.4509	70.8585	2.5408	31.6823	16
26	64.5491	2.7732	38.2241	68.3177	2.7187	34.4010	**15**
27	61.7759	2.9396	41.1637	65.5990	2.9090	37.3100	14
28	58.8363	3.1160	44.2797	62.6900	3.1126	40.4226	13
29	55.7203	3.3029	47.5826	59.5774	3.3305	43.7531	12
30	52.4174	3.5011	51.0837	56.2469	3.5636	47.3167	11
31	48.9163	3.7112	54.7949	52.6833	3.8131	51.1298	**10**
32	45.2051	3.9339	58.7288	48.8702	4.0800	55.2098	9
33	41.2712	4.1699	62.8987	44.7902	4.3656	59.5754	8
34	37.1013	4.4200	67.3187	40.4246	4.6712	64.2466	7
35	32.6813	4.6853	72.0040	35.7534	4.9982	69.2448	6
36	27.9960	4.9664	76.9704	30.7552	5.3480	74.5928	**5**
37	23.0296	5.2644	82.2348	25.4072	5.7225	80.3153	4
38	17.7652	5.5802	87.8150	19.6847	6.1229	86.4382	3
39	12.1850	5.9151	93.7301	13.5618	6.5516	92.9898	2
40	6.2699	6.2699	100.0000	7.0102	7.0102	100.0000	1
45 YEAR LIFE							45 YEAR LIFE
	4 per cent.			5 per cent.			
1	$100.0000	$0.8262	$0.8262	$100.0000	$0.6262	$0.6262	**45**
2	99.1738	0.8593	1.6855	99.3738	0.6575	1.2837	44
3	98.3145	0.8937	2.5792	98.7163	0.6903	1.9740	43
4	97.4208	0.9294	3.5086	98.0260	0.7249	2.6989	42
5	96.4914	0.9666	4.4752	97.3011	0.7611	3.4600	41
6	95.5248	1.0053	5.4805	96.5400	0.7992	4.2592	**40**
7	94.5195	1.0455	6.5260	95.7408	0.8392	5.0984	39
8	93.4740	1.0872	7.6132	94.9016	0.8810	5.9794	38
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES

(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
45 YEAR LIFE							45 YEAR LIFE
	4 per cent.			5 per cent.			
9	$92.3868	$1.1308	$ 8.7440	$94.0206	$0.9251	$ 6.9045	37
10	91.2560	1.1760	9.9200	93.0955	0.9714	7.8759	36
11	90.0800	1.2230	11.1430	92.1241	1.0200	8.8959	**35**
12	88.8570	1.2720	12.4150	91.1041	1.0710	9.9669	34
13	87.5850	1.3229	13.7379	90.0311	1.1245	11.0914	33
14	86.2621	1.3757	15.1136	88.9086	1.1807	12.2721	32
15	84.8864	1.4308	16.5444	87.7279	1.2398	13.5119	31
16	83.4556	1.4880	18.0324	86.4881	1.3018	14.8137	**30**
17	81.9676	1.5476	19.5800	85.1863	1.3668	16.1805	29
18	80.4200	1.6094	21.1894	83.8195	1.4352	17.6157	28
19	78.8106	1.6738	22.8632	82.3843	1.5070	19.1227	27
20	77.1368	1.7408	24.6040	80.8773	1.5823	20.7050	26
21	75.3960	1.8104	26.4144	79.2950	1.6614	22.3664	**25**
22	73.5856	1.8828	28.2972	77.6336	1.7445	24.1109	24
23	71.7028	1.9581	30.2553	75.8891	1.8317	25.9426	23
24	69.7447	2.0365	32.2918	74.0574	1.9223	27.8659	22
25	67.7082	2.1180	34.4098	72.1341	2.0195	29.8854	21
26	65.5902	2.2026	36.6124	70.1146	2.1205	32.0059	**20**
27	63.3876	2.2907	38.9031	67.9941	2.2264	34.2323	19
28	61.0969	2.3824	41.2855	65.7677	2.3378	36.5701	18
29	58.7145	2.4777	43.7632	63.4299	2.4548	39.0249	17
30	56.2368	2.5768	46.3400	60.9751	2.5773	41.6022	16
31	53.6600	2.6798	49.0198	58.3978	2.7062	44.3084	**15**
32	50.9802	2.7870	51.8068	55.6916	2.8417	47.1501	14
33	48.1932	2.8986	54.7054	52.8499	2.9837	50.1338	13
34	45.2946	3.0144	57.7198	49.8662	3.1328	53.2666	12
35	42.2802	3.1350	60.8548	46.7334	3.2896	56.5562	11
36	39.1452	3.2605	64.1153	43.4438	3.4539	60.0101	**10**
37	35.8847	3.3909	67.5062	39.9899	3.6267	63.6368	9
38	32.4938	3.5264	71.0326	36.3632	3.8081	67.4449	8
39	28.9674	3.6676	74.7002	32.5551	3.9983	71.4432	7
40	25.2998	3.8142	78.5144	28.5568	4.1984	75.6416	6
41	21.4856	3.9669	82.4813	24.3584	4.4083	80.0499	**5**
42	17.5187	4.1255	86.6068	19.9501	4.6286	84.6785	4
43	13.3932	4.2905	90.8973	15.3215	4.8600	89.5385	3
44	9.1027	4.4621	95.3594	10.4615	5.1032	94.6417	2
45	4.6406	4.6406	100.0000	5.3583	5.3583	100.0000	1
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES

(*Continued*)

45 YEAR LIFE

Year.	Remaining investment beginning of year. (6 per cent.)	Amortization during year. (6 per cent.)	Total amortization end of year. (6 per cent.)	Remaining investment beginning of year. (7 per cent.)	Amortization during year. (7 per cent.)	Total amortization end of year. (7 per cent.)	
1	$100.0000	$0.4700	$ 0.4700	$100.0000	$0.3500	$ 0.3500	**45**
2	99.5300	0.4983	0.9683	99.6500	0.3744	0.7244	44
3	99.0317	0.5281	1.4964	99.2756	0.4007	1.1251	43
4	98.5036	0.5599	2.0563	98.8749	0.4287	1.5538	42
5	97.9437	0.5934	2.6497	98.4462	0.4587	2.0125	41
6	97.3503	0.6290	3.2787	97.9875	0.4908	2.5033	**40**
7	96.7213	0.6668	3.9455	97.4967	0.5252	3.0285	39
8	96.0545	0.7068	4.6523	96.9715	0.5620	3.5905	38
9	95.3477	0.7492	5.4015	96.4095	0.6013	4.1918	37
10	94.5985	0.7941	6.1956	95.8082	0.6434	4.8352	36
11	93.8044	0.8418	7.0374	95.1648	0.6884	5.5236	**35**
12	92.9626	0.8923	7.9297	94.4764	0.7366	6.2602	34
13	92.0703	0.9458	8.8755	93.7398	0.7882	7.0484	33
14	91.1245	1.0026	9.8781	92.9516	0.8433	7.8917	32
15	90.1219	1.0628	10.9409	92.1083	0.9024	8.7941	31
16	89.0591	1.1265	12.0674	91.2059	0.9655	9.7596	**30**
17	87.9326	1.1941	13.2615	90.2404	1.0331	10.7927	29
18	86.7385	1.2657	14.5272	89.2073	1.1055	11.8982	28
19	85.4728	1.3417	15.8689	88.1018	1.1828	13.0810	27
20	84.1311	1.4222	17.2911	86.9190	1.2657	14.3467	26
21	82.7089	1.5075	18.7986	85.6533	1.3542	15.7009	**25**
22	81.2014	1.5979	20.3965	84.2991	1.4490	17.1499	24
23	79.6035	1.6939	22.0904	82.8501	1.5504	18.7003	23
24	77.9096	1.7954	23.8858	81.2997	1.6590	20.3593	22
25	76.1142	1.9032	25.7890	79.6407	1.7751	22.1344	21
26	74.2110	2.0174	27.8064	77.8656	1.8994	24.0338	**20**
27	72.1936	2.1385	29.9449	75.9662	2.0323	26.0661	19
28	70.0551	2.2667	32.2116	73.9339	2.1746	28.2407	18
29	67.7884	2.4028	34.6144	71.7593	2.3268	30.5675	17
30	65.3856	2.5469	37.1613	69.4325	2.4897	33.0572	16
31	62.8387	2.6997	39.8610	66.9428	2.6640	35.7212	**15**
32	60.1390	2.8617	42.7227	64.2788	2.8504	38.5716	14
33	57.2773	3.0334	45.7561	61.4284	3.0499	41.6215	13
34	54.2439	3.2154	48.9715	58.3785	3.2635	44.8850	12
35	51.0285	3.4083	52.3798	55.1150	3.4920	48.3770	11
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
45 YEAR LIFE							45 YEAR LIFE
	6 per cent.			7 per cent.			
36	$47.6202	$3.6129	$55.9927	$51.6230	$3.7363	$52.1133	**10**
37	44.0073	3.8296	59.8223	47.8867	3.9979	56.1112	9
38	40.1777	4.0594	63.8817	43.8888	4.2777	60.3889	8
39	36.1183	4.3030	68.1847	39.6111	4.5772	64.9661	7
40	31.8153	4.5611	72.7458	35.0339	4.8976	69.8637	6
41	27.2542	4.8348	77.5806	30.1363	5.2404	75.1041	**5**
42	22.4194	5.1249	82.7055	24.8959	5.6073	80.7114	4
43	17.2945	5.4324	88.1379	19.2886	5.9997	86.7111	3
44	11.8621	5.7583	93.8962	13.2889	6.4197	93.1308	2
45	6.1038	6.1038	100.0000	6.8692	6.8692	100.0000	1
50 YEAR LIFE							50 YEAR LIFE
	4 per cent.			5 per cent.			
1	$100.0000	$0.6550	$ 0.6550	$100.0000	$0.4777	$ 0.4777	**50**
2	99.3450	0.6812	1.3362	99.5223	0.5015	0.9792	49
3	98.6638	0.7085	2.0447	99.0208	0.5267	1.5059	48
4	97.9553	0.7368	2.7815	98.4941	0.5529	2.0588	47
5	97.2185	0.7663	3.5478	97.9412	0.5806	2.6394	46
6	96.4522	0.7969	4.3447	97.3606	0.6097	3.2491	**45**
7	95.6553	0.8288	5.1735	96.7509	0.6401	3.8892	44
8	94.8265	0.8620	6.0355	96.1108	0.6722	4.5614	43
9	93.9645	0.8964	6.9319	95.4386	0.7057	5.2671	42
10	93.0681	0.9323	7.8642	94.7329	0.7410	6.0081	41
11	92.1358	0.9696	8.8338	93.9919	0.7781	6.7862	**40**
12	91.1662	1.0084	9.8422	93.2138	0.8170	7.6032	39
13	90.1578	1.0487	10.8909	92.3968	0.8578	8.4610	38
14	89.1091	1.0906	11.9815	91.5390	0.9008	9.3618	37
15	88.0185	1.1343	13.1158	90.6382	0.9457	10.3075	36
16	86.8842	1.1797	14.2955	89.6925	0.9931	11.3006	**35**
17	85.7045	1.2268	15.5223	88.6994	1.0427	12.3433	34
18	84.4777	1.2759	16.7982	87.6567	1.0948	13.4381	33
19	83.2018	1.3270	18.1252	86.5619	1.1496	14.5877	32
20	81.8748	1.3800	19.5052	85.4123	1.2071	15.7948	31
21	80.4948	1.4353	20.9405	84.2052	1.2674	17.0622	**30**
22	79.0595	1.4926	22.4331	82.9378	1.3307	18.3929	29
23	77.5669	1.5523	23.9854	81.6017	1.3974	19.7903	28
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(Continued)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
50 Year Life							50 Year Life
	4 per cent.			5 per cent.			
24	$76.0146	$1.6144	$25.5998	$80.2097	$1.4672	$21.2575	27
25	74.4002	1.6791	27.2789	78.7425	1.5405	22.7980	26
26	72.7211	1.7461	29.0250	77.2020	1.6176	24.4156	**25**
27	70.9750	1.8161	30.8411	75.5844	1.6984	26.1140	24
28	69.1589	1.8887	32.7298	73.8860	1.7834	27.8974	23
29	67.2702	1.9642	34.6940	72.1026	1.8726	29.7700	22
30	65.3060	2.0427	36.7367	70.2300	1.9661	31.7361	21
31	63.2633	2.1245	38.8612	68.2639	2.0645	33.8006	**20**
32	61.1388	2.2095	41.0707	66.1994	2.1677	35.9683	19
33	58.9293	2.2979	43.3686	64.0317	2.2761	38.2444	18
34	56.6314	2.3897	45.7583	61.7556	2.3899	40.6343	17
35	54.2417	2.4854	48.2437	59.3657	2.5095	43.1438	16
36	51.7563	2.5847	50.8284	56.8562	2.6347	45.7785	**15**
37	49.1716	2.6882	53.5166	54.2215	2.7666	48.5451	14
38	46.4834	2.7957	56.3123	51.4549	2.9050	51.4501	13
39	43.6877	2.9075	59.2198	48.5499	3.0501	54.5002	12
40	40.7802	3.0238	62.2436	45.4998	3.2027	57.7029	11
41	37.7564	3.1448	65.3884	42.2971	3.3629	61.0658	**10**
42	34.6116	3.2706	68.6590	38.9342	3.5309	64.5947	9
43	31.3410	3.4013	72.0603	35.4033	3.7075	68.3042	8
44	27.9397	3.5375	75.5978	31.6958	3.8929	72.1971	7
45	24.4022	3.6789	79.2767	27.8029	4.0875	76.2846	6
46	20.7233	3.8261	83.1028	23.7154	4.2919	80.5765	**5**
47	16.8972	3.9791	87.0819	19.4235	4.5065	85.0830	4
48	12.9181	4.1383	91.2202	14.9170	4.7319	89.8149	3
49	8.7798	4.3038	95.5240	10.1851	4.9683	94.7832	2
50	4.4760	4.4760	100.0000	5.2168	5.2168	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$0.3444	$0.3444	$100.0000	$0.2460	$0.2460	**50**
2	99.6556	0.3651	0.7095	99.7540	0.2632	0.5092	49
3	99.2905	0.3870	1.0965	99.4908	0.2816	0.7908	48
4	98.9035	0.4102	1.5067	99.2092	0.3014	1.0922	47
5	98.4933	0.4349	1.9416	98.9078	0.3224	1.4146	46
6	98.0584	0.4609	2.4025	98.5854	0.3450	1.7596	**45**
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
50 Year Life							50 Year Life
	6 per cent.			7 per cent.			
7	$97.5975	$0.4886	$ 2.8911	$98.2404	$0.3692	$ 2.1288	44
8	97.1089	0.5179	3.4090	97.8712	0.3950	2.5238	43
9	96.5910	0.5489	3.9579	97.4762	0.4226	2.9464	42
10	96.0421	0.5819	4.5398	97.0536	0.4522	3.3986	41
11	95.4602	0.6169	5.1567	96.6014	0.4839	3.8825	**40**
12	94.8433	0.6538	5.8105	96.1175	0.5178	4.4003	39
13	94.1895	0.6931	6.5036	95.5997	0.5540	4.9543	38
14	93.4964	0.7346	7.2382	95.0457	0.5928	5.5491	37
15	92.7618	0.7787	8.0169	94.4529	0.6343	6.1814	36
16	91.9831	0.8255	8.8424	93.8186	0.6786	6.8600	**35**
17	91.1576	0.8749	9.7173	93.1400	0.7262	7.5862	34
18	90.2827	0.9275	10.6448	92.4138	0.7771	8.3633	33
19	89.3552	0.9831	11.6279	91.6367	0.8314	9.1947	32
20	88.3721	1.0421	12.6700	90.8053	0.8896	10.0843	31
21	87.3300	1.1046	13.7746	89.9157	0.9519	11.0362	**30**
22	86.2254	1.1710	14.9456	88.9638	1.0185	12.0547	29
23	85.0544	1.2411	16.1867	87.9453	1.0898	13.1445	28
24	83.8133	1.3156	17.5023	86.8555	1.1661	14.3106	27
25	82.4977	1.3946	18.8969	85.6894	1.2477	15.5583	26
26	81.1031	1.4782	20.3751	84.4417	1.3351	16.8934	**25**
27	79.6249	1.5670	21.9421	83.1066	1.4285	18.3219	24
28	78.0579	1.6610	23.6031	81.6781	1.5285	19.8504	23
29	76.3969	1.7606	25.3637	80.1496	1.6355	21.4859	22
30	74.6363	1.8662	27.2299	78.5141	1.7500	23.2359	21
31	72.7701	1.9783	29.2082	76.7641	1.8725	25.1084	**20**
32	70.7918	2.0969	31.3051	74.8916	2.0036	27.1120	19
33	68.6949	2.2227	33.5278	72.8880	2.1438	29.2558	18
34	66.4722	2.3561	35.8839	70.7442	2.2939	31.5497	17
35	64.1161	2.4975	38.3814	68.4503	2.4545	34.0042	16
36	61.6186	2.6473	41.0287	65.9958	2.6263	36.6305	**15**
37	58.9713	2.8061	43.8348	63.3695	2.8101	38.4406	14
38	56.1652	2.9745	46.8093	60.5594	3.0068	42.4474	13
39	53.1907	3.1530	49.9623	57.5526	3.2173	45.6647	12
40	50.0377	3.3422	53.3045	54.3353	3.4425	49.1072	11
41	46.6955	3.5427	56.8472	50.8928	3.6835	52.7907	**10**
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
50 Year Life				**50 Year Life**			
	6 per cent.			7 per cent.			
42	$43.1528	$3.7553	$60.6025	$47.2093	$3.9414	$56.7321	9
43	39.3975	3.9805	64.5830	43.2679	4.2172	60.9493	8
44	35.4170	4.2194	68.8024	39.0507	4.5124	65.4617	7
45	31.1976	4.4726	73.2750	34.5383	4.8284	70.2901	6
46	26.7250	4.7410	78.0160	29.7099	5.1662	75.4563	**5**
47	21.9840	5.0253	83.0413	24.5437	5.5280	80.9843	4
48	16.9587	5.3270	88.3683	19.0157	5.9149	86.8992	3
49	11.6317	5.6465	94.0148	13.1008	6.3289	93.2281	2
50	5.9852	5.9852	100.0000	6.7719	6.7719	100.0000	1
60 Year Life				**60 Year Life**			
	4 per cent.			5 per cent.			
1	$100.0000	$0.4202	$ 0.4202	$100.0000	$0.2828	$ 0.2828	**60**
2	99.5798	0.4370	0.8572	99.7172	0.2970	0.5798	59
3	99.1428	0.4544	1.3116	99.4202	0.3118	0.8916	58
4	98.6884	0.4727	1.7843	99.1084	0.3274	1.2190	57
5	98.2157	0.4916	2.2759	98.7810	0.3437	1.5627	56
6	97.7241	0.5112	2.7871	98.4373	0.3610	1.9237	**55**
7	97.2129	0.5317	3.3188	98.0763	0.3790	2.3027	54
8	96.6812	0.5529	3.8717	97.6973	0.3980	2.7007	53
9	96.1283	0.5750	4.4467	97.2993	0.4178	3.1185	52
10	95.5533	0.5981	5.0448	96.8815	0.4388	3.5573	51
11	94.9552	0.6220	5.6668	96.4427	0.4606	4.0179	**50**
12	94.3332	0.6468	6.3136	95.9821	0.4837	4.5016	49
13	93.6864	0.6728	6.9864	95.4984	0.5079	5.0095	48
14	93.0136	0.6996	7.6860	94.9905	0.5333	5.5428	47
15	92.3140	0.7276	8.4136	94.4572	0.5600	6.1028	46
16	91.5864	0.7567	9.1703	93.8972	0.5880	6.6908	**45**
17	90.8297	0.7870	9.9573	93.3092	0.6174	7.3082	44
18	90.0427	0.8185	10.7758	92.6918	0.6481	7.9563	43
19	89.2242	0.8512	11.6270	92.0437	0.6807	8.6370	42
20	88.3730	0.8853	12.5123	91.3630	0.7146	9.3516	41
21	87.4877	0.9207	13.4330	90.6484	0.7504	10.1020	**40**
22	86.5670	0.9575	14.3905	89.8980	0.7880	10.8900	39
23	85.6095	0.9958	15.3863	89.1100	0.8273	11.7173	38
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(Continued)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
60 Year Life							60 Year Life
	4 per cent.			5 per cent.			
24	$84.6137	$1.0356	$16.4219	$88.2827	$0.8686	$12.5859	37
25	83.5781	1.0771	17.4990	87.4141	0.9122	13.4981	36
26	82.5010	1.1201	18.6191	86.5019	0.9577	14.4558	**35**
27	81.3809	1.1650	19.7841	85.5442	1.0056	15.4614	34
28	80.2159	1.2115	20.9956	84.5386	1.0559	16.5173	33
29	79.0044	1.2600	22.2556	83.4827	1.1087	17.6260	32
30	77.7444	1.3104	23.5660	82.3740	1.1641	18.7901	31
31	76.4340	1.3629	24.9289	81.2099	1.2223	20.0124	**30**
32	75.0711	1.4173	26.3462	79.9876	1.2835	21.2959	29
33	73.6538	1.4741	27.8203	78.7041	1.3476	22.6435	28
34	72.1797	1.5329	29.3532	77.3565	1.4149	24.0584	27
35	70.6468	1.5944	30.9476	75.9416	1.4858	25.5442	26
36	69.0524	1.6580	32.6056	74.4558	1.5600	27.1042	**25**
37	67.3944	1.7245	34.3301	72.8958	1.6380	28.7422	24
38	65.6699	1.7934	36.1235	71.2578	1.7200	30.4622	23
39	63.8765	1.8651	37.9886	69.5378	1.8059	32.2681	22
40	62.0114	1.9397	39.9283	67.7319	1.8962	34.1643	21
41	60.0717	2.0173	41.9456	65.8357	1.9911	36.1554	**20**
42	58.0544	2.0980	44.0436	63.8446	2.0906	38.2460	19
43	55.9564	2.1820	46.2256	61.7540	2.1951	40.4411	18
44	53.7744	2.2692	48.4948	59.5589	2.3049	42.7460	17
45	51.5052	2.3599	50.8547	57.2540	2.4201	45.1661	16
46	49.1453	2.4544	53.3091	54.8339	2.5411	47.7072	**15**
47	46.6909	2.5526	55.8617	52.2928	2.6682	50.3754	14
48	44.1383	2.6546	58.5163	49.6246	2.8016	53.1770	13
49	41.4837	2.7609	61.2772	46.8230	2.9416	56.1186	12
50	38.7228	2.8712	64.1484	43.8814	3.0888	59.2074	11
51	35.8516	2.9861	67.1345	40.7926	3.2432	62.4506	**10**
52	32.8655	3.1056	70.2401	37.5494	3.4053	65.8559	9
53	29.7599	3.2298	73.4699	34.1441	3.5756	69.4315	8
54	26.5301	3.3590	76.8289	30.5685	3.7544	73.1859	7
55	23.1711	3.4933	80.3222	26.8141	3.9421	77.1280	6
56	19.6778	3.6331	83.9552	22.8720	4.1392	81.2672	**5**
57	16.0447	3.7784	87.7337	18.7328	4.3462	85.6134	4
58	12.2663	3.9295	91.6632	14.3866	4.5635	90.1769	3
59	8.3368	4.0867	95.7499	9.8231	4.7917	94.9686	2
60	4.2501	4.2501	100.0000	5.0314	5.0314	100.0000	1
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.	Accrued depreciation	Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
60 YEAR LIFE							60 YEAR LIFE
	6 per cent.			7 per cent.			
1	$100.0000	$0.1876	$ 0.1876	$100.0000	$0.1229	$ 0.1229	**60**
2	99.8124	0.1988	0.3864	99.8771	0.1316	0.2545	59
3	99.6136	0.2108	0.5972	99.7455	0.1407	0.3952	58
4	99.4028	0.2234	0.8206	99.6048	0.1506	0.5458	57
5	99.1794	0.2368	1.0574	99.4542	0.1611	0.7069	56
6	98.9426	0.2510	1.3084	99.2931	0.1724	0.8793	**55**
7	98.6916	0.2661	1.5745	99.1207	0.1845	1.0638	54
8	98.4255	0.2820	1.8565	98.9362	0.1974	1.2612	53
9	98.1435	0.2989	2.1554	98.7388	0.2112	1.4724	52
10	97.8446	0.3169	2.4723	98.5276	0.2260	1.6984	51
11	97.5277	0.3360	2.8083	98.3016	0.2418	1.9402	**50**
12	97.1917	0.3560	3.1643	98.0598	0.2587	2.1989	49
13	96.8357	0.3775	3.5418	97.8011	0.2768	2.4757	48
14	96.4582	0.4000	3.9418	97.5243	0.2963	2.7720	47
15	96.0582	0.4241	4.3659	97.2280	0.3169	3.0889	46
16	95.6341	0.4495	4.8154	96.9111	0.3392	3.4281	**45**
17	95.1846	0.4765	5.2919	96.5719	0.3629	3.7910	44
18	94.7081	0.5051	5.7970	96.2090	0.3882	4.1792	43
19	94.2030	0.5354	6.3324	95.8208	0.4154	4.5947	42
20	93.6671	0.5675	6.8999	95.4053	0.4446	5.0393	41
21	93.1001	0.6016	7.5015	94.9607	0.4756	5.5149	**40**
22	92.4985	0.6377	8.1392	94.4851	0.5089	6.0239	39
23	91.8608	0.6759	8.8151	93.9761	0.5446	6.5685	38
24	91.1849	0.7165	9.5316	93.4315	0.5827	7.1512	37
25	90.4684	0.7594	10.2910	92.8488	0.6235	7.7747	36
26	89.7090	0.8051	11.0961	92.2253	0.6672	8.4419	**35**
27	88.9039	0.8533	11.9494	91.5581	0.7138	9.1557	34
28	88.0506	0.9045	12.8539	90.8443	0.7638	9.9196	33
29	87.1461	0.9589	13.8128	90.0804	0.8173	10.7369	32
30	86.1872	1.0163	14.8291	89.2631	0.8745	11.6114	31
31	85.1709	1.0773	15.9064	88.3886	0.9357	12.5471	**30**
32	84.0936	1.1420	17.0484	87.4529	1.0012	13.5483	29
33	82.9516	1.2104	18.2588	86.4517	1.0713	14.6196	28
34	81.7412	1.2831	19.5419	85.3804	1.1463	15.7659	27
35	80.4581	1.3601	20.9020	84.2341	1.2265	16.9924	26
36	79.0980	1.4417	22.3437	83.0076	1.3124	18.3048	**25**
37	77.6563	1.5282	23.8719	81.6952	1.4043	19.7091	24
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
60 Year Life				60 Year Life			
	6 per cent.			7 per cent.			
38	$76.1281	$1.6199	$25.4918	$80.2909	$1.5025	$21.2116	23
39	74.5082	1.7171	27.2089	78.7884	1.6078	22.8194	22
40	72.7911	1.8201	29.0290	77.1806	1.7203	24.5397	21
41	70.9710	1.9293	30.9583	75.4603	1.8407	26.3804	**20**
42	69.0417	2.0452	33.0035	73.6196	1.9695	28.3499	19
43	66.9965	2.1677	35.1712	71.6501	2.1074	30.4573	18
44	64.8288	2.2978	37.4690	69.5427	2.2550	32.7123	17
45	62.5310	2.4357	39.9047	67.2877	2.4127	35.1250	16
46	60.0953	2.5819	42.4866	64.8750	2.5817	37.7067	**15**
47	57.5134	2.7367	45.2233	62.2933	2.7624	40.4691	14
48	54.7767	2.9010	48.1243	59.5309	2.9557	43.4248	13
49	51.8757	3.0750	51.1993	56.5752	3.1627	46.5875	12
50	48.8007	3.2596	54.4589	53.4125	3.3841	49.9716	11
51	45.5411	3.4551	57.9140	50.0284	3.6209	53.5925	**10**
52	42.0860	3.6624	61.5764	46.4075	3.8744	57.4669	9
53	38.4236	3.8821	65.4585	42.5331	4.1456	61.6125	8
54	34.5415	4.1151	69.5736	38.3875	4.4358	66.0483	7
55	30.4264	4.3620	73.9356	33.9517	4.7463	70.7946	6
56	26.0644	4.6237	78.5593	29.2054	5.0786	75.8732	**5**
57	21.4407	4.9012	83.4605	24.1268	5.4340	81.3072	4
58	16.5395	5.1952	88.6557	18.6928	5.8144	87.1216	3
59	11.3443	5.5069	94.1626	12.8784	6.2215	93.3431	2
60	5.8374	5.8374	100.0000	6.6569	6.6569	100.0000	1
75 Year Life				75 Year Life			
	4 per cent.			5 per cent.			
1	$100.0000	$0.2229	$0.2229	$100.0000	$0.1322	$0.1322	**75**
2	99.7771	0.2318	0.4547	99.8678	0.1387	0.2709	74
3	99.5453	0.2411	0.6958	99.7291	0.1457	0.4166	73
4	99.3042	0.2507	0.9465	99.5834	0.1530	0.5696	72
5	99.0535	0.2608	1.2073	99.4304	0.1607	0.7303	71
6	98.7927	0.2712	1.4785	99.2697	0.1686	0.8989	**70**
7	98.5215	0.2820	1.7605	99.1011	0.1772	1.0761	69
8	98.2395	0.2934	2.0539	98.9239	0.1859	1.2620	68
9	97.9461	0.3050	2.3589	98.7380	0.1953	1.4573	67
10	97.6411	0.3172	2.6761	98.5427	0.2050	1.6623	66
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
75 YEAR LIFE							75 YEAR LIFE
	4 per cent.			5 per cent.			
11	$97.3239	$0.3299	$3.0060	$98.3377	$0.2153	$1.8776	**65**
12	96.9940	0.3432	3.3492	98.1224	0.2260	2.1036	64
13	96.6508	0.3568	3.7060	97.8964	0.2374	2.3410	63
14	96.2940	0.3712	4.0772	97.6590	0.2492	2.5902	62
15	95.9228	0.3860	4.4632	97.4098	0.2616	2.8518	61
16	95.5368	0.4014	4.8646	97.1482	0.2749	3.1267	**60**
17	95.1354	0.4175	5.2821	96.8733	0.2884	3.4151	59
18	94.7179	0.4342	5.7163	96.5849	0.3029	3.7180	58
19	94.2837	0.4515	6.1678	96.2820	0.3181	4.0361	57
20	93.8322	0.4696	6.6374	95.9639	0.3339	4.3700	56
21	93.3626	0.4884	7.1258	95.6300	0.3507	4.7207	**55**
22	92.8742	0.5080	7.6338	95.2793	0.3682	5.0889	54
23	92.3662	0.5282	8.1620	94.9111	0.3866	5.4755	53
24	91.8380	0.5494	8.7114	94.5245	0.4059	5.8814	52
25	91.2886	0.5714	9.2828	94.1186	0.4262	6.3076	51
26	90.7172	0.5942	9.8770	93.6924	0.4476	6.7552	**50**
27	90.1230	0.6180	10.4950	93.2448	0.4699	7.2251	49
28	89.5050	0.6427	11.1377	92.7749	0.4934	7.7185	48
29	88.8623	0.6684	11.8061	92.2815	0.5181	8.2366	47
30	88.1939	0.6952	12.5013	91.7634	0.5440	8.7806	46
31	87.4987	0.7229	13.2242	91.2194	0.5712	9.3518	**45**
32	86.7758	0.7519	13.9761	90.6482	0.5997	9.9515	44
33	86.0239	0.7819	14.7580	90.0485	0.6298	10.5813	43
34	85.2420	0.8132	15.5712	89.4187	0.6612	11.2425	42
35	84.4288	0.8458	16.4170	88.7575	0.6943	11.9368	41
36	83.5830	0.8796	17.2966	88.0632	0.7290	12.6658	**40**
37	82.7034	0.9148	18.2114	87.3342	0.7654	13.4312	39
38	81.7886	0.9513	19.1627	86.5688	0.8037	14.2349	38
39	80.8373	0.9894	20.1521	85.7651	0.8440	15.0789	37
40	79.8479	1.0290	21.1811	84.9211	0.8861	15.9650	36
41	78.8189	1.0702	22.2513	84.0350	0.9304	16.8954	**35**
42	77.7487	1.1129	23.3642	83.1046	0.9769	17.8723	34
43	76.6358	1.1575	24.5217	82.1277	1.0258	18.8981	33
44	75.4783	1.2038	25.7255	81.1019	1.0771	19.9752	32
45	74.2745	1.2519	26.9774	80.0248	1.1309	21.1061	31
46	73.0226	1.3020	28.2794	78.8939	1.1875	22.2936	**30**
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
75 YEAR LIFE							75 YEAR LIFE
	4 per cent.			5 per cent.			
47	$71.7206	$1.3541	$29.6335	$77.7064	$1.2468	$23.5404	29
48	70.3665	1.4082	31.0417	76.4596	1.3092	24.8496	28
49	68.9583	1.4646	32.5063	75.1504	1.3746	26.2242	27
50	67.4937	1.5232	34.0295	73.7758	1.4434	27.6676	26
51	65.9705	1.5840	35.6135	72.3324	1.5155	29.1831	**25**
52	64.3865	1.6475	37.2610	70.8169	1.5913	30.7744	24
53	62.7390	1.7133	38.9743	69.2256	1.6709	32.4453	23
54	61.0257	1.7819	40.7562	67.5547	1.7545	34.1998	22
55	59.2438	1.8532	42.6094	65.8002	1.8421	36.0419	21
56	57.3906	1.9272	44.5366	63.9581	1.9343	37.9762	**20**
57	55.4634	2.0044	46.5410	62.0238	2.0309	40.0071	19
58	53.4590	2.0845	48.6255	59.9929	2.1326	42.1397	18
59	51.3745	2.1680	50.7935	57.8603	2.2391	44.3788	17
60	49.2065	2.2546	53.0481	55.0212	2.3511	46.7299	16
61	46.9519	2.3448	55.3929	53.2701	2.4687	49.1986	**15**
62	44.6071	2.4387	57.8316	50.8014	2.5921	51.7907	14
63	42.1684	2.5361	60.3687	48.2093	2.7217	54.5124	13
64	39.6323	2.6376	63.0053	45.4876	2.8577	57.3701	12
65	36.9947	2.7432	65.7485	42.6299	3.0007	60.3708	11
66	34.2515	2.8528	68.6013	39.6292	3.1507	63.5215	**10**
67	31.3987	2.9670	71.5683	36.4785	3.3082	66.8297	9
68	28.4317	3.0856	74.6539	33.1703	3.4737	70.3034	8
69	25.3461	3.2091	77.8630	29.6966	3.6473	73.9507	7
70	22.1370	3.3374	81.2004	26.0493	3.8297	77.7804	6
71	18.7996	3.4709	84.6713	22.2196	4.0212	81.8016	**5**
72	15.3287	3.6097	88.2810	18.1984	4.2222	86.0238	4
73	11.7190	3.7542	92.0352	13.9762	4.4334	90.4572	3
74	7.9648	3.9043	95.9395	9.5428	4.6550	95.1122	2
75	4.0605	4.0605	100.0000	4.8878	4.8878	100.0000	1
	6 per cent.			7 per cent.			
1	$100.0000	$0.0769	$0.0769	$100.0000	$0.0441	$0.0441	**75**
2	99.9231	0.0814	0.1583	99.9559	0.0471	0.0912	74
3	99.8417	0.0864	0.2447	99.9088	0.0504	0.1416	73
4	99.7553	0.0916	0.3363	99.8584	0.0540	0.1956	72
5	99.6637	0.0970	0.4333	99.8044	0.0578	0.2534	71
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Continued*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
75 YEAR LIFE							75 YEAR LIFE
	6 per cent.			7 per cent.			
6	$99.5667	$0.1029	$0.5362	$99.7466	$0.0618	$0.3152	**70**
7	99.4638	0.1090	0.6452	99.6848	0.0661	0.3813	69
8	99.3548	0.1156	0.7608	99.6187	0.0707	0.4520	68
9	99.2392	0.1225	0.8833	99.5480	0.0757	0.5277	67
10	99.1167	0.1299	1.0132	99.4723	0.0810	0.6087	66
11	98.9868	0.1376	1.1508	99.3913	0.0867	0.6954	**65**
12	98.8492	0.1459	1.2967	99.3046	0.0927	0.7881	64
13	98.7033	0.1547	1.4514	99.2119	0.0992	0.8873	63
14	98.5486	0.1640	1.6154	99.1127	0.1062	0.9935	62
15	98.3846	0.1738	1.7892	99.0065	0.1136	1.1071	61
16	98.2108	0.1842	1.9734	98.8929	0.1216	1.2287	**60**
17	98.0266	0.1952	2.1686	98.7713	0.1301	1.3588	59
18	97.8314	0.2070	2.3756	98.6412	0.1392	1.4980	58
19	97.6244	0.2194	2.5950	98.5020	0.1489	1.6469	57
20	97.4050	0.2326	2.8276	98.3531	0.1593	1.8062	56
21	97.1724	0.2465	3.0741	98.1938	0.1705	1.9767	**55**
22	96.9259	0.2613	3.3354	98.0233	0.1824	2.1591	54
23	96.6646	0.2770	3.6124	97.8409	0.1952	2.3543	53
24	96.3876	0.2936	3.9060	97.6457	0.2089	2.5632	52
25	96.0940	0.3113	4.2173	97.4368	0.2235	2.7867	51
26	95.7827	0.3299	4.5472	97.2133	0.2391	3.0258	**50**
27	95.4528	0.3497	4.8969	96.9742	0.2559	3.2817	49
28	95.1031	0.3707	5.2676	96.7183	0.2738	3.5555	48
29	94.7324	0.3929	5.6605	96.4445	0.2929	3.8484	47
30	94.3395	0.4165	6.0770	96.1516	0.3134	4.1618	46
31	93.9230	0.4414	6.5184	95.8382	0.3354	4.4972	**45**
32	93.4816	0.4680	6.9864	95.5028	0.3589	4.8561	44
33	93.0136	0.4961	7.4825	95.1439	0.3840	5.2401	43
34	92.5175	0.5258	8.0083	94.7599	0.4109	5.6510	42
35	91.9917	0.5574	8.5657	94.3490	0.4396	6.0906	41
36	91.4343	0.5908	9.1565	93.9094	0.4704	6.5610	**40**
37	90.8435	0.6262	9.7827	93.4390	0.5033	7.0643	39
38	90.2173	0.6638	10.4465	92.9357	0.5386	7.6029	38
39	89.5535	0.7037	11.1502	92.3971	0.5763	8.1792	37
40	88.8498	0.7459	11.8961	91.8208	0.6166	8.7958	36
41	88.1039	0.7906	12.6867	91.2042	0.6598	9.4556	**35**
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

TABLE 27. AMORTIZATION AND DEPRECIATION TABLES
(*Concluded*)

Year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	Remaining investment beginning of year.	Amortization during year.	Total amortization end of year.	
75 YEAR LIFE							75 YEAR LIFE
	6 per cent.			7 per cent.			
42	$87.3133	$0.8381	$13.5248	$90.5444	$0.7059	$10.1615	34
43	86.4752	0.8883	14.4131	89.8385	0.7554	10.9169	33
44	85.5869	0.9417	15.3548	89.0831	0.8082	11.7251	32
45	84.6452	0.9982	16.3530	88.2749	0.8648	12.5899	31
46	83.6470	1.0580	17.4110	87.4101	0.9253	13.5152	**30**
47	82.5890	1.1215	18.5325	86.4848	0.9901	14.5053	29
48	81.4675	1.1888	19.7213	85.4947	1.0594	15.5647	28
49	80.2787	1.2602	20.9815	84.4353	1.1336	16.6983	27
50	79.0185	1.3357	22.3172	83.3017	1.2129	17.9112	26
51	77.6828	1.4159	23.7331	82.0888	1.2979	19.2091	**25**
52	76.2669	1.5009	25.2340	80.7909	1.3887	20.5978	24
53	74.7660	1.5909	26.8249	79.4022	1.4859	22.0837	23
54	73.1751	1.6864	28.5113	77.9163	1.5899	23.6736	22
55	71.4887	1.7875	30.2988	76.3264	1.7012	25.3748	21
56	69.7012	1.8948	32.1936	74.6252	1.8203	27.1951	**20**
57	67.8064	2.0085	34.2021	72.8049	1.9477	29.1428	19
58	65.7979	2.1290	36.3311	70.8572	2.0841	31.2269	18
59	63.6689	2.2567	38.5878	68.7731	2.2299	33.4568	17
60	61.4122	2.3921	40.9799	66.5432	2.3860	35.8428	16
61	59.0201	2.5357	43.5156	64.1572	2.5531	38.3959	**15**
62	56.4844	2.6878	46.2034	61.6041	2.7318	41.1277	14
63	53.7966	2.8491	49.0525	58.8723	2.9230	44.0507	13
64	50.9475	3.0200	52.0725	55.9493	3.1276	47.1783	12
65	47.9275	3.2012	55.2737	52.8217	3.3466	50.5249	11
66	44.7263	3.3933	58.6670	49.4751	3.5808	54.1057	**10**
67	41.3330	3.5969	62.2639	45.8943	3.8315	57.9372	9
68	37.7361	3.8127	66.0766	42.0628	4.0997	62.0369	8
69	33.9234	4.0415	70.1181	37.9631	4.3867	66.4236	7
70	29.8819	4.2839	74.4020	33.5764	4.6937	71.1173	6
71	25.5980	4.5410	78.9430	28.8827	5.0233	76.1406	**5**
72	21.0570	4.8135	83.7565	23.8594	5.3738	81.5144	4
73	16.2435	5.1022	88.8587	18.4856	5.7500	87.2644	3
74	11.1413	5.4084	94.2671	12.7356	6.1525	93.4169	2
75	5.7329	5.7329	100.0000	6.5831	6.5831	100.0000	1
	Present value.	Current depreciation during year.		Present value.	Current depreciation during year.		Expectancy. Years.

INDEX

(Reference is to Page.)