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The Value Allocation of an Economy with Asymmetric Information

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We wish to thank David Ballard, Leonidas Koutsougeras, Wayne Shafer, and Anne Villamil for useful comments, discussions and suggestions. As usual we are responsible for any remaining shortcomings.

ABSTRACT

We analyze the Shapley value allocation of an economy with differential information. Since the intent of the Shapley value is to measure the sum of the expected marginal contributions made by an agent to any coalition that he/she belongs to, the value allocation of an economy with differential information provides an interesting way to measure the information advantage of an agent. This feature of the Shapley value allocation is not necessarily shared by the Walrasian equilibrium and the core. Thus, we analyze the informational structure of an economy with asymmetric information from a different and new viewpoint.

In particular we address the following questions: How do coalitions of agents share their private information? How can one measure the information advantage or superiority of an agent? Is each agent's private information verifiable by other members of a coalition? Do coalitions of agents pool their private information? Do agents have an incentive to report their true private information? What is the correct concept of a value allocation in an economy with asymmetric information? Do value allocations exist in an economy with private information? We provide answers to each of these questions.

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1 Introduction

The concept of a (cardinal) value allocation was first introduced by Shapley (1969). Roughly speaking it is defined as a feasible allocation which yields to each agent in the economy a "utility level" which is equal to the sum of the agent's expected marginal contributions to all coalitions that he/she is a member of. Because the Shapley value measures the sum of the expected marginal contributions of an agent made to any coalition that he/she belongs to, we argue in this paper that it provides an interesting way to measure the "worth" of an agent's information advantage in an economy with differential information. This feature of the Shapley value allocation is not necessarily shared by the Walrasian equilibrium and the core. Thus, we analyze the informational structure of an economy with differential information from a different and new viewpoint. In particular, we address the following questions: How can one measure the information advantage or superiority of an agent? How do coalitions of agents share their private information? Is each agent's private information verifiable by all other members of a coalition? Do agents have an incentive to report their private information truthfully? What is the correct concept of a value allocation with private information? Do value allocations exist in an economy with private information? We provide answers to each of these questions.

We consider a two-period economy where each agent i is characterized by a utility function, a random second period endowment, a prior belief about the distribution of all agent's second period endowments, and private information about the actual endowment realizations after uncertainty is revealed in the second period. Let $\mathcal{F}_i$ denote the private information set of agent i (which is a partition of a probability space) and S denote a coalition of agents. We first show that the notions of a value allocation for an economy with differential information which correspond to Wilson's (1978) concepts of the coarse and the fine core, respectively, are problematic. Indeed, we show that the *coarse value allocation,* where the net-trade of each agent is $\bigwedge_{i \in S} \mathcal{F}_i$-measurable[1] (and hence information is verifiable by each member

[1]The symbol $\bigwedge_{i \in S} \mathcal{F}_i$ denotes the "meet" which is the maximal partition which is contained in all partitions $\mathcal{F}_i$. Intuitively, this is the maximum amount of information which all agents share. By a slight abuse of notation, we will denote the σ-algebra generated by a partition $\mathcal{F}_i$ also by $\mathcal{F}_i$. If Ω is a countable set then clearly every sub-σ-algebra of $\mathcal{F}$ is generated by a partition ($\mathcal{F}$ denotes the σ-algebra of measurable subsets of Ω).

of the coalition), and the *fine value allocation,* where the net-trade of each agent is $\bigvee_{i \in S} \mathcal{F}_i$-measurable[2] (and hence coalitions of agents pool their information), do not reflect in any interesting way the information advantage or superiority of an agent. This is rather surprising: Since the value allocation is a cooperative solution concept, intuition suggests that the rational behavior by members of a coalition is either to pool their private information (fine value allocation) or to base their decision on public information (common knowledge) of all members of a coalition (coarse value allocation). However, we not only show that a coarse value allocation does not exist generically (unless the information asymmetry is trivial)—we show that if a coarse value allocation exists for some private information economy, then the emergence of a player with trivial (i.e., bad or no) information destroys existence immediately. In the case of the fine value allocation, we show that the private information of a particular agent is completely irrelevant, i.e., the value allocation corresponds to the complete information case. Furthermore, whenever agents in a coalition are required to pool their private information (as the fine value allocation demands) they may not have an incentive to report their information truthfully.

Fortunately, the problems noted above do not arise if we assume that the net-trade of each agent is $\mathcal{F}_i$-measurable because the information advantage of each agent is taken into account in this setting. This $\mathcal{F}_i$-measurable value allocation concept corresponds to the core notion introduced by Yannelis (1991) and subsequently examined by Allen (1991).[3] We show that this concept has nice incentive properties: it fulfills a coalitional incentive compatibility property that we introduce in this paper. Roughly speaking, the coalitional incentive compatibility property captures the idea that within a coalition it is impossible for a subcoalition to cheat the remaining agents in the coalition by misreporting their private information, and thus making themselves better off (i.e., there is truthful revelation of information in each coalition). We call such a value allocation "coalitionally incentive compatible," and give several examples which show its nice properties. In particular, we show that the coalitional incentive compatible value allocation provides more plausible outcomes than the core or the Walrasian equilibrium of a

[2] $\bigvee_{i \in S} \mathcal{F}_i$ denotes the "join" of the $\mathcal{F}_i$. That is the minimal partition containing all $\mathcal{F}_i$.

[3] Note that since the net-trade of each agent is $\mathcal{F}_i$-measurable it is always the case that $|S-1|$ members (where $|A|$ denotes the cardinality of the set A) of coalition S can pool their private information to verify the private information of the remaining agent.

differential information economy. This suggests to us that the coalitional incentive compatible value may (at least in certain cases) serve as a good substitute for the more traditional equilibrium concepts in economies with differential information. Finally, under standard assumptions (i.e., continuity and concavity of the utility functions) we prove the existence of coalitional incentive compatible value allocations. In the presence of finitely many states of nature the existence of such value allocations follows readily from Shapley's (1969) result. However, with a continuum of states, functional analytic and measure theoretic arguments seem to be required. For the technical reader we collect the basic mathematical results required, and provide a rigorous proof in Section 8. Non-technical readers can skip this section. Indeed they only need to know that coalitional incentive compatible value allocations exist in an economy with differential information under standard (continuity and concavity) assumptions on the utility functions even in the case of a continuum of states.

The rest of the paper proceeds as follows: Section 2 contains notation. In section 3 we outline the model (i.e., the exchange economy with differential information). In section 4 we define rigorously the concepts of the coarse and the fine value allocations. Their interpretation is discussed in section 5. In section 6 we introduce the notion of a coalitional incentive compatible value allocation and show that it results in truthful revelation of information within each coalition. The interpretation of this value allocation is discussed in section 7—we argue that the coalitional incentive compatible value allocation sheds some light on the debate on value allocations started by the examples of Roth (1980) and Shafer (1980). A rigorous proof that coalitional incentive compatible value allocations exist is given in section 8. Section 9 contains some concluding thoughts and finally in the Appendix we extend the examples discussed in sections 5 and 7 to a more general class of economies.

2 Notation

$\mathbb{R}^n$ denotes the n-dimensional Euclidean space.
$\mathbb{R}^n_+$ denotes the positive cone of $\mathbb{R}^n$.
$\mathbb{R}^n_{++}$ denotes the strictly positive cone of $\mathbb{R}^n$.
2^A denotes the set of all subsets of the set A.

$\emptyset$ denotes the empty set.
$\setminus$ denotes set-theoretic subtraction.
$|A|$ denotes the cardinality of the set A.
Let $(\Omega, \mathcal{F}, \mu)$ denote a measure space. Then $\mathcal{F}_i$ will always denote a measurable partition[4] of Ω and $E_i(\omega)$ will denote the element of the partition $\mathcal{F}_i$ which contains ω. If X is a linear topological space, its dual is the space X^* of all continuous linear functionals on X.

3 The Exchange Economy with Differential Information

Let Y denote the commodity space. For simplicity we will identify Y with $\mathbb{R}^l_+$, however, for our main existence result in section 8, Y may be infinite dimensional. Hence infinitely many commodities are permissible.[5] We will consider an exchange economy which extends over two time periods $t = 0, 1$ where consumption takes place in $t = 1$. At $t = 0$ there is uncertainty over the state of nature described by a probability space $(\Omega, \mathcal{F}, \mu)$. Let $I = \{1, \ldots, n\}$ denote the set of all agents. In $t = 0$ agents will agree on net-trades which may be contingent on the state of nature in $t = 1$. However, they have differential information with respect to the true state of nature. This is modeled as follows: At $t = 1$ agents do not necessarily know which state $\omega \in \Omega$ has actually occurred, i.e., they know their own endowment realization and every agent i might have some additional information about the state described by a measurable partition $\mathcal{F}_i$ of Ω.[6] Since agents can always observe their own endowment realization we can assume without loss of generality that agent i's endowment is measurable with respect to $\mathcal{F}_i$.

In summary, an *exchange economy with differential information* is given by $\mathcal{E} = \{(X_i, u_i, e_i, \mathcal{F}_i, \mu): i = 1, \ldots, n\}$ where
(1) $X_i: \Omega \to 2^Y$ is the *consumption set* of agent i;

[4] A measurable partition of Ω is a collection of sets $A_i \in \mathcal{F}$, $i \in \mathbb{N}$ such that $\bigcup_{i=1}^{\infty} A_i = \Omega$ and $A_i \cap A_j = \emptyset$ for all $i \neq j$.

[5] Y can be any separable Banach lattice with an order continuous norm, whose dual Y^* has the Radon-Nikodym Property (see section 8 for the appropriate definitions).

[6] Hence if $\bar{\omega}$ is the true state of the economy in $t = 1$ then agent i observes the event $E_i(\bar{\omega})$ in the partition $\mathcal{F}_i$ which contains $\bar{\omega}$.

(2) $u_i: Y \to \mathbb{R}$ is the *utility function* of agent i;[7]
(3) $\mathcal{F}_i$ is a partition of Ω denoting the *private information* of agent i;
(4) $e_i: \Omega \to Y$ is the *initial endowment* of agent i, where each e_i is $\mathcal{F}_i$-measurable and $e_i(\omega) \in X_i(\omega)$ μ-a.e.;
(5) μ is a probability measure on Ω denoting the *common prior* of each agent.

The expected utility of agent i of a is given by

$$\int_{\Omega} u_i(x_i(\omega))\, d\mu(\omega).^{8}$$

4 The Coarse and the Fine Value Allocation

In this section we introduce two different notions of the value allocation for our asymmetric information economy. The difference stems from the (measurability) restriction on the type of allocations that are allowed. Both notions are analogs of the coarse and the fine cores of Wilson (1978). We begin by defining these concepts. In section 5 we show that each is problematic, and in section 6 we define a third concept which has better features. The strategy in this section is to derive a game with transferable utility from the economy with differential information, $\mathcal{E}$, in which each agent's utility is weighted by a factor λ_i which allows interpersonal utility comparisons. In the value allocation itself no side-payments are necessary. At this point we appeal to the *principle of irrelevant alternatives:* "If restriction of the feasible

[7] One may also assume that the utility function is random, i.e., u_i is a real valued function defined on $\Omega \times Y$. All the results of the paper will remain valid.

[8] Bayesian updating of priors can be introduced as follows: Let $q_i: \Omega \to \mathbb{R}_{++}$ be a Radon-Nikodym derivative (density function) denoting the prior of agent i. For each $i = 1, \ldots, n$, denote by $E_i(\omega)$ the event in $\mathcal{F}_i$ containing the realized state of nature $\omega \in \Omega$ and suppose that $\int_{t \in E_i(\omega)} q_i(t)\, d\mu(t) > 0$. Given $E_i(\omega) \in \mathcal{F}_i$ define the *conditional expected utility* of agent i as follows:

$$\int_{t \in E_i(\omega)} u_i(t, x_i(t)) q_i(t|E_i(\omega))\, d\mu(t),$$

where

$$q_i(t|E_i(\omega)) = \begin{cases} 0 & \text{if } t \notin E_i(\omega) \\ \dfrac{q_i(t)}{\int_{t \in E_i(\omega)} q_i(t)\, d\mu(t)} & \text{if } t \in E_i(\omega). \end{cases}$$

All the results of the paper remain valid if we use the above conditional expected utility formulation. However, for the simplicity of the exposition we do not do so.

set, by eliminating side-payments, does not eliminate some solution point, then that point remains a solution" [Shapley (1969)].[9] We thus get a game with side-payments as follows:

Definition 1. *A game with side-payments* $\Gamma = (I, V)$ *consists of a finite set of agents* $I = \{1, \dots, n\}$ *and a superadditive, real valued function* V *defined on* 2^I *such that* $V(\emptyset) = 0$. *Each* $S \subset I$ *is called a coalition and* $V(S)$ *is the "worth" of the coalition* S.

The Shapley value of the game Γ, [Shapley (1953)] is a rule which assigns to each agent i a "payoff" Sh_i given by the formula[10]

$$\mathrm{Sh}_i(V) = \sum_{\substack{S \subset I \\ S \supset \{i\}}} \frac{(|S|-1)!(|I|-|S|)!}{|I|!}[V(S) - V(S \setminus \{i\})].$$

The Shapley value has the property that $\sum_{i \in I} \mathrm{Sh}_i(V) = V(I)$, i.e., the Shapley value is efficient. For each economy with differential information $\mathcal{E}$ and each set of weights $\{\lambda_i : i = 1, \dots, n\}$, we associate a game with side-payments (I, V_λ^c), [we also refer to this as a "transferable utility" (TU) game] according to the rule:

For every coalition $S \subset I$ let

$$V_\lambda^c(S) = \max_{x_i} \sum_{i \in S} \lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) \tag{4.1}$$

subject to

(i) $\sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega)$, μ-a.e.

(ii) $x_i - e_i$ is $\bigwedge_{i \in S} \mathcal{F}_i$-measurable for every $i \in S$.

We are now ready to give a formal definition of the coarse value allocation.

Definition 2. *An allocation* $x : \Omega \to \prod_{i=1}^n X_i$ *is said to be a coarse value allocation of the economy with differential information* $\mathcal{E}$ *if the following holds:*
(i) Each net-trade $x_i - e_i$ *is* $\bigwedge_{i=1}^n \mathcal{F}_i$*-measurable.*

[9] See Emmons and Scafuri (1985, p. 60) or Shafer (1980, p. 468) for further discussion.

[10] The Shapley value measures is the sum of the expected marginal contributions an agent can make to all the coalitions that he/she is a member of [see Shapley (1953)].

(ii) $\sum_{i=1}^{n} x_i(\omega) = \sum_{i=1}^{n} e_i(\omega)$, μ-a.e.

(iii) *There exist* $\lambda_i \geq 0$, $(i = 1, \ldots, n)$ *with* $\lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) = \mathrm{Sh}_i(V_\lambda^c)$ *for all* i, *where* $\mathrm{Sh}_i(V_\lambda^c)$ *is the Shapley value of agent* i *derived from the game* (I, V_λ^c), *defined in (4.1) and* $\mathrm{Sh}_i(V_\lambda^c) \geq \lambda_i \int u_i(e_i)\, d\mu$ *for every* i.

Condition (i) says that net-trades can only be based on common knowledge of the grand coalition. (ii) is the market clearing condition. (iii) says that the expected utility of each agent multiplied with his/her weight λ_i must be equal to his/her Shapley value derived from the TU game (I, V_λ^c) which requires net-trades of all members of a coalition to be based on common knowledge of the coalition. Moreover the value allocation is required to be individually rational. Finally, note that the efficiency of the Shapley value for games with side payments immediately implies that the value allocation is constrained Pareto efficient.

The second concept that we introduce in this section is the fine value. For each economy with differential information $\mathcal{E}$ and each set of weights $\{\lambda_i : i = 1, \ldots, n\}$, we associate a game with side-payments (I, V_λ^f) according to the rule:

$$V_\lambda^f(S) = \max_{x_i} \sum_{i \in S} \lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) \tag{4.2}$$

subject to

(i) $\sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega)$, μ-a.e.

(ii) $x_i - e_i$ is $\bigvee_{i \in S} \mathcal{F}_i$-measurable for every $i \in S$.

We now give a formal definition of the fine value allocation:

Definition 3. *An allocation* $x : \Omega \to \prod_{i=1}^{n} X_i$ *is said to be a fine value allocation of the economy with differential information* $\mathcal{E}$ *if the following holds:*

(i) *Each net-trade* $x_i - e_i$ *is* $\bigvee_{i=1}^{n} \mathcal{F}_i$*-measurable.*

(ii) $\sum_{i=1}^{n} x_i(\omega) = \sum_{i=1}^{n} e_i(\omega)$, μ-a.e.

(iii) *There exist* $\lambda_i \geq 0$, $(i = 1, \ldots, n)$ *with* $\lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) = \mathrm{Sh}_i(V_\lambda^f)$ *for all* i, *where* $\mathrm{Sh}_i(V_\lambda^f)$ *is the Shapley value of agent* i *derived from the game* (I, V_λ^f), *defined in (4.2) and* $\mathrm{Sh}_i(V_\lambda^f) \geq \lambda_i \int u_i(e_i)\, d\mu$ *for every* i.

The only difference between Definition 2 and Definition 3 is the measurability assumption, i.e., net-trades are now based on pooled information of a coalition.

We now consider existence of the coarse value allocation and the fine value allocation in an economy with differential information. The following Theorem, which we present here in the notation describing a game derived from an exchange economy, is proved in Ichiishi (1983, Theorem 6.1):

Theorem 1. *For every S let $U(S)$ be the set of all utility allocations which a coalition S can attain. Assume that $U(S)$ fulfills the following conditions.*
(i) $U(S) \neq \emptyset$ for every coalition S.
(ii) $U(S)$ is compact, convex, and comprehensive.[11]
(iii) U is superadditive [i.e., let S, T be coalitions such that $S \cap T = \emptyset$ then $U(S) \times U(T) \subset U(S \cup T)$].

Then there exists a value allocation. This allocation is Pareto optimal and individually rational.

Clearly, for the definition of the fine and the coarse value allocation the attainable utility allocations are given as follows:

$U^c(S) = \{(w_1, \ldots, w_m) \in I\!R^n$: there exist net-trades z_i, where $\sum_{i \in S} z_i = 0$, where z_i is $\bigwedge_{i \in S} \mathcal{F}_i$-measurable, and $w_i \leq \int u(e_i + z_i)\, d\mu\}$.

$U^f(S) = \{(w_1, \ldots, w_m) \in I\!R^n$: there exist net-trades z_i, where $\sum_{i \in S} z_i = 0$, where z_i is $\bigvee_{i \in S} \mathcal{F}_i$-measurable, and $w_i \leq \int u(e_i + z_i)\, d\mu\}$.

Theorem 1 immediately implies that there always exists a fine value allocation in our economy provided that the state space Ω is finite.[12] For $U^c(S)$ the theorem does not apply since $U^c(S)$ violates condition (iii) of Theorem 1: Consider for example an economy with three agents denoted by I, J and K. Assume that I and J have full information, and that agent K has only trivial information (i.e., $\mathcal{F}_K = \{\Omega, \emptyset\}$). Clearly $U^c(\{K\}) \times U^c(\{I, J\}) \not\subset U^c(\{I, J, K\})$, and hence condition (iii) of Theorem 1 is violated.

5 Interpretation of Coarse and Fine Values

In this section we discuss the properties of the coarse and the fine value for our economy with differential information. The straightforward explanation of the coarse value is that it must be possible for each member of a coalition

[11] A subset $A \subset I\!R^n$ is comprehensive if $a \in A$ implies $a' \in A$ for every $a' \leq a$.

[12] Existence also holds for arbitrary state spaces, however, as this concept turns out to be uninteresting we do not elaborate further on existence.

to verify the net-trades of all other members (i.e., to be able to check whether the net-trades which are actually executed correspond to the net-trades that agents agreed on before the agents observed their endowment realizations and obtained additional information about the true state of nature). This obviously becomes a problem once an agent enters a coalition who has "bad" information (i.e., a very coarse $\mathcal{F}_i$). The presence of this agent and the assumption that he/she must be able to verify the net-trades of all other agents makes the whole coalition worse off. Nevertheless, one of the central ideas underlying the concept of the value is that we wish to allocate to every agent a consumption bundle which corresponds to his/her marginal contribution to every coalition that he/she is a member of. The appropriateness of the concept should therefore be judged according to this criterion. Thus, agents with superior information should be assigned a higher Shapley value. In fact, for a given differential information economy and for given utility transfer weights this is the case. Unfortunately, the coarse value has a serious problem: It does not exist in general unless the information asymmetry is trivial. We show this by means of the following example (Proposition 1 in the Appendix provides a more general result):

Example 1. Consider an economy with three agents denoted by I, J, and K, and four states of nature a, b, c and d. Assume that there is only one consumption commodity in each state. The random endowment of the agents are given by $(4,4,1,1)$ for I; $(4,1,4,1)$ for J; and $(1,1,1,1)$ for K. Agent K has an information set $\mathcal{F}_K$. We consider the cases where $\mathcal{F}_K$ is trivial—i.e., $\mathcal{F}_K = \{\{a,b,c,d\},\emptyset\}$—and the case where $\mathcal{F}_K$ corresponds to full information—i.e., $\mathcal{F}_K = \{\{a\},\{b\},\{c\},\{d\}\}$. Further, assume that agent I cannot distinguish state a from b, and state c from d; and finally, agent J cannot distinguish a from c, and b from d, i.e., $\mathcal{F}_I = \{\{a,b\},\{c,d\}\}$ and $\mathcal{F}_J = \{\{a,c\},\{b,d\}\}$. Assume that all agents are described by the Neuman-Morgenstern utility function $\sqrt{x}$. Each state occurs with the same probability. We now analyze the coarse value allocation for this economy.

First consider the case where K has full information. Let $U^c(S)$ denote the utility allocations coalition S can attain. Then $U^c(\{i\}) = \{w_i \leq 1.5\}$, for $i = I, J$ and $U^c(\{K\}) = \{w_K \leq 1\}$. Further, $U^c(\{I,J\}) = \{(w_I, w_J)\colon w_I \leq 1/2\sqrt{4+t} + 1/2\sqrt{1+t}$, and $w_J \leq 1/2\sqrt{4-t} + 1/2\sqrt{1-t}$, $|t| \leq 1\}$, since all net-trades must be state-independent.[13] On the other

[13]Clearly, t denotes the state independent net-transfer.

hand, net-trades between I and K, or J and K can be state dependent. However, they must be measurable with respect to the information of agents I and J, respectively. Hence, $U^c(\{I,K\}) = U^c(\{J,K\}) = \{(w_i, w_K): w_i \leq 1/2\sqrt{4+t_1} + 1/2\sqrt{1+t_2}$, and $w_K \leq 1/2\sqrt{1-t_1} + 1/2\sqrt{1-t_2}$, $|t_i| \leq 1\}$, where $i = I, J$. Finally, in the grand coalition the only possible net-trades are state-independent since the information sets $\mathcal{F}_I$ and $\mathcal{F}_J$ are independent. Hence, $U^c(\{I,J,K\}) = \{(w_I, w_J, w_K): w_i \leq 1/2\sqrt{4+t_i} + 1/2\sqrt{1+t_i}$, for $i = I, J$, $w_K \leq \sqrt{1+t_K}$, $\sum_{i=I,J,K} t_i = 0$, and $|t_i| \leq 1\}$. It is easy to see that all sets $U^c(S)$ are comprehensive, convex, and compact. However, note that superadditivity (i.e., condition 3 in the Theorem) is violated since $U^c(\{I,K\}) \times U^c(\{J\}) \not\subset U^c(\{I,J,K\})$. In fact, we show that there does not exist a coarse value allocation. Recall that $V_\lambda^c(S)$ maximizes the weighted sum of the utility allocations in $U^c(S)$. From the definition of the Shapley value, it follows immediately that $\mathrm{Sh}_K > \lambda_K \int u_K(e^K)\, d\mu$.[14] Hence, in a value allocation agent K must get a positive net-transfer. Since only state-independent net-transfers are allowed in the grand coalition, either agent I or agent J must get a lower utility than he/she derives from the initial endowments. In other words, individual rationality would be violated, and a coarse value allocation therefore does not exist.

[14]This can be established as follows: Given the information structure of the example only state independent net-trades are possible between members of the grand coalition. In order to have individual rationality, and since $\sum_{i=I,J,K} \mathrm{Sh}_i = V_\lambda(\{I,J,K\})$ must hold for every coarse value allocation, the weights λ_i must be chosen such that the maximum in (4.1) is attained when the agents get their initial endowment. This immediately implies that it is also optimal for the coalition $\{I,J\}$ to choose their initial endowment. Thus, $V_\lambda^c(S) = \sum_{i\in S} \lambda_i \int u_i(e_i)\, d\mu$ for the grand coalition, for all coalitions in which K is not a member, and for the one-agent coalitions. However, it is also easy to see that $V_\lambda(S) > \sum_{i\in S} \lambda_i \int u_i(e_i)\, d\mu$, for $S = \{I,K\}$ and for $S = \{J,K\}$. [Note that we solve

$$\max_{t_1,t_2} \lambda_i/2\sqrt{4+t_1} + \lambda_i/2\sqrt{1+t_2} + \lambda_K/2\sqrt{1-t_1} + \lambda_K/2\sqrt{1-t_2};$$

for $i = I, J$. It follows from the first order conditions that

$$\frac{\lambda_i^2}{\lambda_K^2} = \frac{4+t_1}{1-t_1}, \text{ and } \frac{\lambda_i^2}{\lambda_K^2} = \frac{1+t_2}{1-t_2}.$$

Thus, $t_1 \neq t_2$ for all λ_i, λ_K. Hence the assertion follows.] Hence we can conclude that $V_\lambda(S) - V_\lambda(S\setminus\{K\}) \geq \lambda_K \int u_K(e_K)\, d\mu$ with the strict inequality holding for the coalitions $\{I,K\}$ and $\{J,K\}$. Hence, $\mathrm{Sh}_K > \lambda_K \int u_K(e_K)\, d\mu$.

If agent K has trivial information one can check that a coarse value allocation exists. In this case the coarse value allocation assigns every agent simply the initial endowment (and the λ's are chosen such that no trades takes place). The same is true for other information sets $\mathcal{F}_K$ which allow only trivial net-trades. On the other hand, if $\mathcal{F}_K$ is sufficiently fine to allow non-trivial net-trades within some coalitions, then a coarse value allocation does not exist. This follows immediately from the above argument.

Example 1 shows that the coarse value allocation does not exist unless the information structure is essentially uninteresting. In particular, a value allocation exists only if K has trivial information, and further since the information of I and J are independent, no non-trivial trades are possible between agents. This non-existence example casts serious doubts on the usefulness of the coarse value. In fact, the case against the coarse value is even more persuasive as our example is not pathological. Proposition 1 in the Appendix shows that this type of example is generic. Further, existence of a coarse value allocation is in general immediately destroyed if a player with no (or bad) information is introduced: Note that in an economy as in Example 1, but without agent I, a value allocation always exists. Consider the case where K has full information. As we have shown above, a value allocation does not exist any more if we introduce agent I. It is easy to see that the same is true if we introduce any player with trivial information. Again, this argument can be generalized along the lines of Proposition 1.

We next consider the fine value. Existence is not a problem for this concept. Unfortunately, it does not measure the information advantage of an agent in an appropriate way because the fine value allocation always coincides with the value allocation of the corresponding complete information economy. We show this for an example, however, this phenomenon also applies in general as Proposition 2 in the Appendix shows.

Example 2. Consider the same differential information economy as in Example 1. First consider the case where agent K has full information. Let $U^f(S)$ denote the utility allocations which coalition S can attain. For one-agent coalitions, $U^f(S)$ is the same as for the coarse value. For all other coalitions we have immediately full information. Hence all utility vectors derived from feasible allocation can be attained. Because of Theorem 1 we know that there exists a value allocation. However, as we noted, the game corresponds to the game with complete information, hence the value allocation will correspond to the value allocation of the economy with full information, and consequently does not take into account the fact that I and J do not have full information.

Next consider the case where agent K has trivial information. Here $U^f(\{I,K\})$ and $U^f(\{J,K\})$ differ from the previous case. However, for any weight λ it follows that $U^f(\{I,K\})$ and $U^f(\{J,K\})$ are the same as in the case where K has full information because it is never optimal for the coalition consisting of $i = I, J$ and K to make net-trades which are not measurable with respect to $\mathcal{F}_i$. Hence, we get the same value allocation as in the case where K has full information. The fine value allocation therefore ignores the information advantages or disadvantages of agent K.

As the Examples show, neither the coarse nor the fine value give interesting results. Information advantages and disadvantages are not reflected in the fine value allocation (this is shown in general in Proposition 2 in the Appendix) and the coarse value allocation does not exist unless the information asymmetry is uninteresting. The problem for the coarse value is of course that we measure the negative externality an agent imposes on the coalition by having information which is independent of other agent's information. However, the marginal contribution an agent makes to any coalition should be the guideline for a definition of the value. This critique does not apply to the fine value because agents contribute to the coalition by sharing their information with other agents. However, Example 2 shows that we can get odd results in this case: Agents I and J can pool their information to attain complete information and agent K is not needed—if we assume that agents report their private information truthfully. The crucial question, of course, is will agents report their (non-verifiable) private information truthfully? Consider the following argument. Assume that the net-trades of agent I is given by $(0, -1.5, 1.5, 0)$, and that the net-trade of agent J is given by $(0, 1.5, -1.5, 0)$. The net-trade of agent K is 0 in every state. Assume that state a is realized and that I announces $\{a, b\}$. Agent J could now announce $\{b, c\}$ thus increasing his net-transfer, without I being able to tell that J misreported. Thus, although information pooling takes place, agents might not have an incentive to reveal their information truthfully. This makes the information sharing assumption in the definition of the fine value somewhat problematic, and this is also responsible for the fact that the fine value allocation does not take information asymmetries into account.

6 Coalitional Incentive Compatible Values

When agents have differential information, an arbitrary allocation may not be viable since agents might have an incentive to misreport the state. In other words, arbitrary allocations might not be incentive compatible in the sense that groups of agents might be able to misreport their information without other agents noticing it, and hence achieve different payoffs ex post. Before we introduce our key incentive compatibility criterion, we introduce the notion of a value allocation which corresponds to the core notion of Yannelis (1991). We will show below (Lemma 2) that this value allocation fulfills the strong incentive compatibility criterion (introduced below) and in section 8 we show that it exists under very mild assumptions. This value allocation concept will turn out to be quite appealing in an economy with differential information.

For each economy with differential information $\mathcal{E}$ and each set of weights $\{\lambda_i : i = 1, \ldots, n\}$, we associate a game with side-payments (I, V_λ) according to the rule: For $S \subset I$ let

$$V_\lambda(S) = \max_{x_i} \sum_{i \in S} \lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) \tag{6.1}$$

subject to

(i) $\sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega)$, μ-a.e.

(ii) $x_i - e_i$ is $\mathcal{F}_i$-measurable for every i.

Definition 4. *An allocation $x : \Omega \to \prod_{i=1}^n X_i$ is said to be a coalitional incentive compatible value allocation of the economy with differential information $\mathcal{E}$ if the following holds:*

(i) Each net-trade $x_i - e_i$ is $\mathcal{F}_i$-measurable.

(ii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$, μ-a.e.

(iii) There exist $\lambda_i \geq 0$, $(i = 1, \ldots, n)$ with $\lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) = \mathrm{Sh}_i(V_\lambda)$ for every i, where $\mathrm{Sh}_i(V_\lambda)$ is the Shapley value of agent i derived from the game (I, V_λ), defined in (6.1) and $\mathrm{Sh}_i(V_\lambda) \geq \lambda_i \int u_i(e_i)\, d\mu$ for every i.

The interpretation of conditions (i), (ii) and (iii) is the same as in Definition 2. Again, note that the value allocation is constrained Pareto efficient. We now introduce two notions of incentive compatibility.

Definition 5. *A feasible allocation fulfills strong coalitional incentive compatibility if and only if the following does not hold:*
There exists a coalition $S \subset I$ and two states a and b which members of $I \setminus S$ cannot distinguish (that is, a and b are in the same event of the partition for every agent not in S) and such that after appropriate side-payments members of S are strictly better off by announcing b whenever a has actually occurred. Formally, strong coalitional incentive compatibility implies that there do not exist a coalition S, states a, b with $a \in E_i(b)$ for every $i \notin S$, and a net-trade vector z^i, $i \in S$ such that $\sum_{i \in S} z^i = 0$ and

$$u^i(e^i(a) + (x^i(b) - e^i(b)) + z^i) > u^i(x^i(a)), \text{ for every } i \in S. \qquad (C1)$$

Strong coalitional incentive compatibility models the idea that it is impossible for any subcoalition to cheat the remaining players by misreporting the state and making side-payments to each other which agents who are not members of this subcoalition cannot observe. If side-payments can be observed we get the following weaker notion of incentive compatibility. (Set $z_i = 0$ for every $i \in S$ in (C1) to get weak coalitional incentive compatibility.)

Definition 6. *A feasible allocation fulfills weak coalitional incentive compatibility if and only if the following does not hold:*
There exists a coalition $S \subset I$ and two states a and b which members of $I \setminus S$ cannot distinguish and such that members of S are strictly better off by announcing b whenever a has actually occurred. Formally, weak coalitional incentive compatibility implies that there do not exist a coalition S and states a, b with $a \in E_i(b)$ for every $i \notin S$, such that

$$u^i(e^i(a) + (x^i(b) - e^i(b)) > u^i(x^i(a)), \text{ for every } i \in S. \qquad (C2)$$

Weak incentive compatibility is weaker in the sense that allocations which fulfill weak coalitional incentive compatibility must also fulfill strong coalitional incentive compatibility. It is easy to find examples where the reverse is not true. Of course, for the case of two agents, both concepts coincide. In what follows we will discuss both notions. It is going to turn out that in certain cases that strong coalitional incentive compatibility is analytically more tractable since it corresponds to individual measurability of the net-trade of each agent. We begin our analysis with some notation.

Let $U^w(S)$ and $U^s(S)$ denote the utility allocations which coalition S can attain, and which fulfill weak and strong coalitional incentive compatibility, respectively. Then

$U^w(S) = \{(w_1, \ldots, w_n) \in I\!R^n$: there exist net-trades z_i, where $\sum_{i \in S} z_i = 0$, where z_i fulfill weakly coalitional incentive compatibility, and where $w_i \leq \int u_i(e_i + z_i)\, d\mu\}$.

$U^s(S) = \{(w_1, \ldots, w_n)$: there exist net-trades z_i, where $\sum_{i \in S} z_i = 0$, where z_i fulfill strong coalitional incentive compatibility, and $w_i \leq \int u_i(e_i + z_i)\, d\mu\}$.

Next we define the weak and strong incentive compatible value allocation: For each economy with differential information $\mathcal{E}$ and for each set of weights $\{\lambda_i : i = 1, \ldots, n\}$ we associate a TU game (I, V_λ^w) as follows: For $S \subset I$ let

$$V_\lambda^w(S) = \max_{x_i} \sum_{i \in S} \lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) \tag{6.2}$$

subject to

(i) $\displaystyle\sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega)$ μ-a.e.

(ii) $x_i - e_i$ fulfills weak coalitional incentive compatibility

Definition 7. *An allocation $x: \Omega \to \prod_{i=1}^n X_i$ is said to be a weak coalitional incentive compatible value allocation of the economy with differential information $\mathcal{E}$ if the following holds:*

(i) The allocation x_i fulfills weak coalitional incentive compatibility.

(ii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$, μ-a.e.

(iii) There exist $\lambda_i \geq 0$, $(i = 1, \ldots, n)$ with $\lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) = \mathrm{Sh}_i(V_\lambda^w)$ for all i, where $\mathrm{Sh}_i(V_\lambda^w)$ is the Shapley value of agent i derived from the game (I, V_λ^w), defined in (6.2) and $\mathrm{Sh}_i(V_\lambda^w) \geq \lambda_i \int u_i(e_i)\, d\mu$ for every i.

To define the *strong incentive compatible value allocation* replace condition (ii) in (6.2) condition (i) in Definition 7 by: The allocation x_i fulfills strong coalitional incentive compatibility.

We now continue by characterizing the sets $U^s(S)$ and $U^w(S)$. It will turn out that $U^s(S)$ corresponds to the attainable utility allocations which fulfill individual measurability of the net-trade of each agent for an economy with one commodity per state.

Let $U(S) = \{(w_1, \ldots, w_n)$: there exist net-trades z_i such that $\sum_{i \in S} z_i = 0$, where z_i is $\mathcal{F}_i$-measurable , and $w_i \leq \int u_i(e_i + z_i)\, d\mu\}$.

Lemma 1. *The sets $U(S)$, $U^w(S)$ and $U^s(S)$ are comprehensive. If the state space Ω is finite then $U(S)$, $U^w(S)$ and $U^s(S)$ are also compact.*

Proof. In both cases compactness follows immediately from the continuity of u_i. It remains to prove that the sets are comprehensive. This follows, however, immediately form the definition of U, U^w and U^s.

Lemma 2. *The following relation holds: $U(S) \subset U^s(S) \subset U^w(S)$ for every coalition $S \subset I$. Moreover, if there is only one commodity per state then the first inclusion holds with equality, i.e., $U(S) = U^s(S)$.*

Proof. It is obvious that $U^s(S) \subset U^w(S)$. We now show that $U(S) \subset U^s(S)$. Let $w \in U(S)$. Then there exist net-trades z_i such that z_i is $\mathcal{F}_i$-measurable for every agent i, and $\sum_{i \in S} z_i = 0$, and $w_i \leq \int u_i(e_i + z_i)\, d\mu$. For every subcoalition $T \subset S$ let $z^T = \sum_{i \in T} z_i$. Since the feasibility constraint holds with equality it follows that $z^{S \setminus T} = -z^T$. Since $z^{S \setminus T}$ is $\bigvee_{i \notin S} \mathcal{F}_i$-measurable it follows therefore that z^S is also $\bigvee_{i \in S} \mathcal{F}_i$-measurable. Hence, members of T cannot gain from misreporting their private information. This proves strong incentive compatibility.

Finally we show the equality for the case of an economy with one commodity per state. Since we have already proved that $U(S) \subset U^s(S)$ for every S it remains to prove that $U^s(S) \subset U(S)$ for every S. Suppose by way of contradiction that the net-trade of one agent, say agent j is not $\mathcal{F}_j$-measurable. Hence, there exist two states a and b which agent j cannot distinguish and for which $z^j(a) \neq z^j(b)$. Without loss of generality assume that $z^j(a) > z^j(b)$. Then in state b the coalition $T = S \setminus \{j\}$ can announce state a without agent j being able to notice it. Thus, they can redistribute their excess income and make all members of T better off. This provides the contradiction to strong incentive compatibility. Hence $U^s(S) \subset U(S)$. This concludes the proof.

Lemma 2 implies that when there is one commodity per state of nature the sets $U^s(S)$ and $U(S)$ coincide for every S. Therefore existence of an incentive compatible value allocation for $\mathcal{E}$ implies the existence of a strong incentive compatible value allocation in such a case. However, even when there is one commodity per state the presence of an arbitrary state space makes the consumption space infinite dimensional and therefore the proof of a strong incentive compatible value allocation is non-trivial (this is done in section 8). For a finite state space Ω, the existence of an incentive compatible value allocation and hence of a strong incentive compatible value allocation follows from Theorem 1. Unfortunately, for weak coalitional incentive compatibility, convexity can be violated, and hence the proof does not go through. In some cases we can therefore get non-existence of weak coalitional incentive compatible value allocations.

7 Interpretation of Weak, Strong, and Coalitional Incentive Compatible Values

In this section we continue with the economy of Example 1 and derive the games for the strong, the weak and the coalitional incentive compatible value allocation.

Example 3. Consider the same economy as in Example 1. We now analyze the strong incentive compatible value allocation, i.e., the value allocation of the game $(I, V_\lambda^s(S))$ and the coalitional incentive compatible value allocation, i.e., the value allocation of the game $(I, V_\lambda(S))$. By Lemma 2 both are the same. As in the previous examples we work with the set of attainable utility allocations $U(S) = U^s(S)$.

The payoffs to the one-agent coalitions are the same as in the previous cases. Further, $U(\{I,J\}) = U^c(\{I,J\})$, because $\mathcal{F}_I$ and $\mathcal{F}_J$ are independent. This, however, does not apply to the other two agent coalitions. $U(\{I,K\}) = U(\{J,K\}) = \{(w_1, w_2)\colon w_1 \leq 1/2\sqrt{4+t_1} + 1/2\sqrt{1+t_2},\ w_2 \leq 1/2\sqrt{1-t_1} + 1/2\sqrt{1-t_2}$, such that $|t_i| \leq 1$, for $i = 1,2\}$. Similarly, we derive that $U(\{I,J,K\}) = \{(w_1, w_2, w_3)\colon$ there exist state independent net-trades z_i, $i = I, J, K$ where z_i is $\mathcal{F}_i$-measurable for $i = J, K$, where $\sum_{i=I,J,K} z_i = 0$, and $w_i \leq \sum_{s=a,b,c,d} 1/4\sqrt{e_i + z_i}$, for $i = I, J, K\}$. Because of Theorem 1 (or Theorem 2 of Section 8) there exists a coalitional incentive compatible value allocation. Further, it is obvious that $V_\lambda(\{I,J,K\}) > V_\lambda(\{I,J\}) + V_\lambda(\{K\})$ for all $\lambda > 0$.[15] Hence, $\mathrm{Sh}_K(V_\lambda) > \int u_K(e_K)\, d\mu$, i.e., agent K must get a higher utility than he/she derives from the initial endowment.

[15]Note that $V_\lambda(\{I,J\}) + V_\lambda(\{K\})$ is less or equal to the payoff of the grand coalition if agents are restricted to state-independent net-trades. Optimal risk sharing when agent i's

Now consider the case where $\mathcal{F}_K$ is trivial. In such a case only constant net-trades are possible and the value allocation assigns to every agent his/her initial endowment.

In contrast to the previous examples, the information of agent K now matters. If agent K has full information then he/she can use this information advantage to act as an intermediary to allow trade between agent I and agent J. This becomes clear when looking at $V_\lambda(\{I, J, K, \})$ and $V_\lambda(\{I, J\})$. Without agent K, agents I and J cannot do any risk sharing. This changes when agent K enters the coalition. Now all trades basically go via agent K since $V_\lambda(\{I, J, K, \})$ is essentially the union of $V_\lambda(\{I, K\})$ and $V_\lambda(\{J, K\})$. In addition, the agent is compensated for his/her intermediation service by getting a strictly higher utility than in the case where he/she is less well informed. However, it is essential for agent K to have a strictly positive endowment in every state. If agent K's endowment is for example 0, then he/she is not able to insure agent I and J, i.e., to increase their consumption in the low-income state and increase it in the high-income state since this would require agent K to hold a positive initial endowment in state d. Thus, the coalitional incentive compatible value allocation in such a case would assign to every agent the initial endowment. This changes immediately if we consider endowments which are not independent.

Example 3a. Consider an economy as in Example 3 but assume that the endowments of agent I and J are given by $(4,4,1,4)$ and $(4,1,4,4)$, respectively. Assume that $\mathcal{F}_I = \{\{a, b, d\}, \{c\}\}$, $\mathcal{F}_J = \{\{a, c, d\}, \{b\}\}$, and that agent K has full information. We also assume that agent K has zero endowment in all states. Since the derivation of $V_\lambda(S)$ is the same as in Example 3 we just sketch the argument that $\mathrm{Sh}_K(V_\lambda) > 0$ for every $\lambda > 0$, and for every λ possible in the value allocation (even if $\lambda_K = 0$).

Consider the grand coalition. Let t_i be the net-trade of agent $i = I, J$ in the low-income, and let t_i' denote the net-trade of agent $i = I, J$ in the high-income state. If $\lambda_K > 0$ then the first order conditions immediately

net-trade must be $\mathcal{F}_i$-measurable, implies that agent K's consumption in state d must be strictly lower than his/her consumption in state a. Hence, such an allocation cannot be achieved via state-independent net-transfers, and the inequality follows.

imply that it is never optimal to choose $t_i = t'_i$ for $i = I, J$.[16] If $\lambda_K = 0$ then the first order conditions imply that it is never optimal to choose $t_i = t'_i$ for $i = I, J$ unless $t_i = t'_i \neq 0$.[17] However, this implies that one of the agents must get a negative net-transfer in all states, and hence individual rationality must be violated,[18] and the associated weights λ cannot occur as transfer weights in a value allocation. Thus, $V_\lambda(\{I, J, K\}) - V_\lambda(\{I, J\}) > 0$ for all possible weights λ, since the coalition $\{I, J\}$ is restricted to state-independent net-transfers. Therefore Agent K has a positive Shapley value, and he/she must get a positive consumption in the value allocation. Note, that any notion of a Walrasian equilibrium in this economy will give zero consumption to agent K since his/her budget set is always zero.

The reason why the value allocation assigns positive consumption to agent K if the endowments of agents I and J are not independent, whereas his/her consumption must be zero if the endowments are independent, has the following interpretation: In both cases agents I and J attempt to insure against low-income realizations. Because of differential information, however, they need agent K as an intermediary to execute the correct trades.

[16]The agents solve

$$\max_{t_i, t'_i} \sum_{i=I,J} \frac{3\lambda_i}{4}\sqrt{4+t'_i} + \frac{\lambda_i}{4}\sqrt{1+t_i} + \frac{\lambda_K}{2}\sqrt{-t'_I - t'_J} + \frac{\lambda_K}{4}\sqrt{-t'_I - t_J} + \frac{\lambda_K}{4}\sqrt{-t_I - t'_J}.$$

Without loss of generality assume that $\lambda_I > 0$. Then the first order conditions are

$$\begin{aligned} \frac{3\lambda_I}{\sqrt{4+t'_I}} &= \frac{2\lambda_K}{\sqrt{-t'_I - t'_J}} + \frac{\lambda_K}{\sqrt{-t'_I - t_J}} \\ \frac{\lambda_I}{\sqrt{1+t_I}} &= \frac{\lambda_K}{\sqrt{-t_I - t'_J}}. \end{aligned}$$

Clearly, there does not exist a solution if $t'_I = t_I$ and if $t'_J = t_J$.

[17]The argument is similar as above. Just consider the first order conditions of

$$\max_{t_I, t'_I} \frac{3\lambda_I}{4}\sqrt{4+t'_I} + \frac{\lambda_I}{4}\sqrt{1+t_I} + \frac{3\lambda_J}{4}\sqrt{4-t_I} + \frac{\lambda_J}{4}\sqrt{1-t'_I},$$

and choose $t_I = t'_I = 0$.

[18]Since $\sum_{i=I,J,K} \mathrm{Sh}_i(V_\lambda) = V_\lambda(\{I, J, K\})$, we conclude that $\lambda_i \int u_i(x_i)\, d\mu = \mathrm{Sh}_i(V_\lambda)$, where x_i is the consumption of agent i after the transfers t_i and t'_i. Hence the assertion follows.

This arrangement works even if agent K has zero-endowment as long as only one of the agents has a low endowment realization, because the claim of this particular agent can then be covered by the agent who has the high endowment-realization. This is basically the setting of Example 3a. If both agents have low endowment realizations at the same time (which can occur if endowments are independent) then they both want a positive net-transfer. Agent K cannot fulfill his/her payment obligations because his/her endowment is zero, and K would have to default. The problem is therefore to find an incentive compatible way of letting agent K announce default in such a case. Clearly this is possible if agent I and agent J are able to observe state d.[19] Another possibility is to weaken the incentive compatibility requirements. We do this in the following example.

Example 4. Consider the economy of Example 1, except assume that agent K's endowment is given by $(0,0,0,0)$ but the agent has full information. We now analyze the weak coalitional incentive compatible value.

It is clear that the $U^s(S) = U^w(S)$ for the coalitions $S = \{I\}$, $S = \{J\}$ and $S = \{I, J\}$. Further, for the coalitions $S = \{I, K\}$ and $S = \{J, K\}$ we can derive $U^w(S)$ by a similar procedure as $U^s(S)$, taking into account that agent K has zero endowment. The attainable utility allocations differ in an interesting way when we consider the grand coalition. We show that $U^w(\{I, J, K\})$ corresponds to the attainable utility allocations under full information:

Consider an allocation (x_I, x_J, x_K) which is Pareto optimal under full information. Let $x_i(s)$ denote the consumption of agent i in state s. We now show that this allocations fulfills weak coalitional incentive compatibility. Clearly, $x_i(b) = x_i(c)$ for $i = I, J, K$, since the aggregate endowment in states a and b coincides. Further, $x_K(a) \geq x_K(b) \geq x_K(d)$. Note that agent K cannot misreport if state d occurs because one of the other agents will disagree.[20] The same is true if state b or state c occurs. Finally, agent K

[19]This can be done simply by assuming in Example 3 that the information partitions of agents I and J are given by $\{\{a,b\},\{c\},\{d\}\}$ and $\{\{a,c\},\{b\},\{d\}\}$, respectively. The proof that agent K's Shapley value (and hence consumption) is strictly positive is along the same lines as Example 3a.

[20]Agent K can only announce either states b or c. Hence either agent I or J must agree. This, however, is impossible since they would have to pay a net-transfer corresponding to a high income state which is always strictly higher than the net-transfer in a low income state.

has no incentive to misreport in state a since this is the state where he/she gets the highest net-transfer.

Next note that the sets $U^w(S)$ fulfill the conditions of Theorem 1. Hence there exists a value allocation. We now show that agent K must get a strictly positive consumption of the good in each state of nature in the value allocation. This, however, follows immediately since the above computation imply that $V_\lambda^w(\{I,J,K\}) - V_\lambda^w(\{I,J\}) > 0$ for every λ and hence $\mathrm{Sh}_K(V_\lambda^w) > 0$.[21] Since $\mathrm{Sh}_K(V_\lambda^w) = \lambda_k \int u_K(x_K)\, d\mu$, where x_K is the consumption assigned to agent K in the value allocation, x_K must be strictly positive.

In contrast to the fine and the coarse value allocation, the weak, the strong, and the coalitional incentive compatible value allocation give very plausible and interesting results for our differential information economy and information superiority of an agent is now taken into account explicitly. Example 3 shows that the information of agent K matters (and the same is true for all other agents). In particular, if agent K has complete information, he/she can act as an intermediary between agent I and agent J. In the case of the strong and the coalitional incentive compatible value agent K uses his/her own endowment to insure the other agents and all trades go via agent K. Furthermore, as Examples 3a and 4 show, agent K can still be an "intermediary" between agent I and agent J even without having a positive endowment: Agent K simply announces the true state of nature and as compensation for this service gets a positive net-transfer.

This role of agent K in Examples 3a and 4 is interesting in connection with the literature on financial intermediation. For example, Boyd and Prescott (1986) argue that coalitional structures, i.e., cooperative games with differential information, are important for understanding financial intermediation. They use an economy with differential information, however, they assume that there exists an "evaluation technology" to verify the true state of the economy—using this technology is costly.[22] In their analysis they propose the core as a solution concept. As our examples indicate, the Shapley value is an interesting tool for the analysis of problems of financial intermediation

[21] The inequality follows since the grand coalition can attain all unconstrained Pareto efficient allocations. None of them can be attained via state-independent net-transfers.

[22] Boyd and Prescott (1986, p. 212) note that their evaluation technology is endogenous in the sense that it depends on the state of the economy. Nevertheless, the specific structure of the cost function as assumed at the outset.

and might work better than the core for the following reasons.

First, incentive compatible value allocations obviate the need to assume the existence of a state verification technology at the outset. Rather, verification evolves endogenously in our model. Whenever there is "doubt" about the state of the economy, agent I and agent J can turn to agent K who then announces the true state. Further, agent K is compensated for this service by a positive net-transfer. This net-transfer to agent K can be interpreted as a "cost of state verification" paid by agent I and agent J, and the magnitude of this cost is determined endogenously. Thus, our results in Examples 3, 3a, and 4 provide insight into models with costly state verification introduced by Townsend (1979). Instead of exogenously assuming the existence of a costly state verification technology (as is standard in this literature), one can assume as in our example that there is one (or more) additional agent(s) with zero endowment and "better" (or complete) information. The weak and the coalitional incentive compatible value allocation assigns positive consumption to this agent for announcing the true state of nature, and this transfer is interpreted as the cost of state verification paid by the other agents.

The second reason that we believe that the incentive compatible value may be preferable to the core for intermediation problems is demonstrated by the following example.

Example 5. Consider an economy with four agents denoted by I, J, K_1, K_2. Assume that the endowment, distribution, and information structure for I and J is the same as in example 3a. Furthermore, assume that K_1 and K_2 both have full information and zero endowments (so there are now two intermediaries instead of one). We wish to show that the value allocation in this example is more plausible than either the Wilson-type cores [Wilson (1978)], or the Yannelis-type core [Yannelis (1991)], or even the Walrasian equilibrium (i.e., any rational expectation equilibrium concept). In any core allocation, agents K_1 and K_2 both get zero. This follows immediately from the following argument: Assume by way of contradiction that K_1 has positive consumption in a core allocation. Then the coalition consisting of I, J and K_2 can block this allocation denoted by $(x_I, x_J, x_{K_1}, x_{K_2})$. For example, choose $(x_I + \varepsilon x_{K_1}, x_J + \varepsilon x_{K_1}, 0, x_{K_2} + (1 - 2\varepsilon)x_{K_1})$ for $\epsilon > 0$. Moreover, since the budget sets of agents K_1 and K_2 contain only the zero consumption vector, any rational expectations equilibrium concept will give zero to both agents.

In contrast, the value allocation still assigns a positive consumption to K_i, $i = 1,2$. This follows immediately from the argument we used in Example 3a. For all possible transfer-weights λ we still get $V_\lambda(\{I,J,K_i\}) - V_\lambda(\{I,J\}) > 0$, $i = 1,2$. Hence, $\mathrm{Sh}_{K_i}(V_\lambda) > 0$ for $i = 1,2$. Since $\mathrm{Sh}_K(V_\lambda) = \lambda_k \int u_K(x_K)\,d\mu$, K_1 and K_2 must get a positive level of consumption.

In Example 5 the Shapley value still assigns positive net-payments to the intermediaries—even when there is more than one intermediary. This is the case since each of the intermediaries makes a "contribution to society" due to his/her superior information. In this example the core and the Walrasian equilibrium seem to be less plausible. If the intermediaries get zero payoff for their service, the question is of course why they should serve as intermediaries. The reader should also note that a similar result can be derived for the weak coalitional incentive compatible value allocation in a setting as in Example 3.

Note that Examples 3a, 4, and 5 resemble the phenomena first observed by Roth (1980) and Shafer (1980) for the ordinal and cardinal value allocation for an economy without differential information, i.e., an agent with zero initial endowment may end up with a positive consumption of each good in the economy. This was shown to be true for *some* choice of weights. Of course, in their examples there also exists a value allocation which assigns zero consumption to the agents with zero endowment [see also the subsequent discussion in Aumann (1985, 1987), Roth (1983), Scafuri and Yannelis (1984), and Yannelis (1983)]. Despite the fact that our Example 5 has the same flavor as that of the Roth-Shafer examples, the differential economy framework seems to provide now a nice interpretation of the agent with no endowment, i.e., this agent can be viewed as a financial intermediary.[23] It is also important to note that now the intermediary (despite the fact that he/she has zero initial endowment) ends up with positive consumption of each good in the economy *for any choice* of the weights in contrast to the Roth-Shafer examples.

[23]The financial intermediary interpretation in the Roth-Shafer examples is more subtle, but it works in a similar way: Due to the fact that the agent has a "better" utility function (is less risk averse) he/she can make a positive contribution to all two agent coalitions. This contribution may be viewed as facilitating risk sharing.

8 Existence of Coalitional Incentive Compatible Value Allocations

The goal of this section is to provide a general existence result. In particular we show that coalitional incentive compatible value allocations exist in a setting where there is an infinite number of commodities and an infinite number of states of nature. Before stating our existence result, we outline some mathematical preliminaries. For the reader who is only interested in an existence result for finite dimensional commodity spaces, all he/she has to know is that order intervals are weakly compact in $L^1_{\mathbb{R}^n}(\mu)$ (which follows from Cartwright's Theorem stated below).

8.1 Mathematical Preliminaries

Let $(\Omega, \mathcal{F}, \mu)$ be a probability space, and X be a Banach space. For $1 \leq p < \infty$, we denote by $L^p_X(\mu)$ the space of all equivalence classes of X-valued Bochner integrable functions $f: T \to X$ normed by

$$\|x\|_p = \left(\int \|x(t)\|^p \, d\mu(t) \right)^{\frac{1}{p}}.$$

It is a standard result that when normed by the functional $\|\cdot\|_p$ above, $L^p_X(\mu)$ becomes a Banach space [see Diestel and Uhl (1977), p. 50]. Recall that a correspondence $\phi: T \to 2^X$ is said to be *integrably bounded* if there exists a map $h \in L^1_{\mathbb{R}}$ such that $\sup\{\|x\|: x \in \phi(t)\} \leq h(t)$, μ-a.e.

A Banach space has the *Radon-Nikodym Property* with respect to the measure space (T, β, μ) if for each μ-continuous X-valued measure on T with bounded variation G there exists a $g \in L^1_X$ such that $G(A) = \int_A g(t)\, d\mu(t)$ for all $A \in \beta$. A Banach space X has the Radon-Nikodym Property (RNP) if X has the RNP with respect to every finite measure space. Recall now [see Diestel-Uhl (1977, Theorem 1, p. 98)] that if (T, β, μ) is a finite measure space $1 \leq p < \infty$, and X is a Banach space, then X^* has the RNP if and only if $(L^p_X)^* = L^q_{X^*}$ where $\frac{1}{p} + \frac{1}{q} = 1$.

We close this section by collecting some basic results on Banach lattices [for an excellent treatment see Aliprantis and Burkinshaw (1985)]. Recall that a Banach space X is a *Banach lattice* if there exists an ordering $\geq$ on X with the following properties:

(i) $x \geq y$ implies $x + z \geq y + z$ for every $z \in X$;
(ii) $x \geq y$ implies $\lambda x \geq \lambda y$ for every scalar $\lambda \geq 0$;
(iii) for all $x, y \in X$ there exists a supremum (denoted by $x \vee y$) and an infimum (denoted by $x \wedge y$).
(iv) $|x| \geq |y|$ implies $\|x\| \geq \|y\|$ for all $x, y \in X$.

As usual, $x^+ = x \vee 0$, $x^- = (-x) \vee 0$, and $|x| = x^+ + x^-$, we call x^+ and x^- the positive and negative parts of x, respectively and $|x|$ the absolute value of x. For x, $y \in X$ we define the *order interval* $[x, y]$ as follows:

$$[x, y] = \{z \in X : x \leq z \leq y\}.$$

Note that $[x, y]$ is convex and norm closed, hence weakly closed (recall Mazur's Theorem). A Banach lattice L is said to have an order continuous norm if $x_n \downarrow 0$ in L implies $\|x_n\| \downarrow 0$. A very useful result which will play an important role is that if X is a Banach lattice then the fact that X has an order continuous norm is equivalent to the weak compactness of order intervals [see for example Aliprantis and Burkinshaw (1985)].

We finally note that Cartwright (1974) has shown that if X is a Banach lattice with order continuous norm (or equivalently X has weakly compact order intervals) then $L^1_X(\mu)$ has weakly compact order intervals, as well. Cartwright's Theorem will play a crucial role in our existence proof.

8.2 The Existence Proof

Let the commodity space Y be the positive cone of a separable Banach lattice Z. Assume that Z has an order continuous norm, and that its dual Z^* has the Radon-Nikodym Property. We now state our main existence result.

Theorem 2. *Let $\mathcal{E} = \{(X_i, u_i, e_i, \mathcal{F}_i, \mu): i = 1, \ldots, n\}$ be a finite exchange economy with differential information (as described in Section 3), satisfying the following assumptions for each agent:*
(A1) $X_i: \Omega \to 2^Y$ is a convex, closed, non-empty valued correspondence.
(A2) $u_i: Y \to I\!R$ is weakly continuous, integrably bounded and concave.
(A3) $e_i: \Omega \to Y$ is integrably bounded.
Then a coalitional incentive compatible value allocation exists in $\mathcal{E}$.

Proof. Let $\{(X_i, u_i, e_i): i = 1, 2, \ldots, n\}$ be an exchange economy where
(a) $X_i \subset I\!R^l$ is the *consumption set* of agent i;

(b) $u_i: X_i \to I\!R$ is the *utility function* of agent i;

(c) $e_i \in X_i$ is the *initial endowment* of agent i.

Given an economy $\{(X_i, u_i, e_i): i = 1, 2, \ldots, n\}$ and a set of weights $\{\lambda_i: i = 1, \ldots, n\}$, where $\lambda_i \geq 0$ for every i and $\sum_{i=1}^n \lambda_i = 1$, define the game

$$V_\lambda(S) = \max_{x_i \in X_i} \sum_{i \in S} \lambda_i u_i(x_i), \text{ subject to } \sum_{i \in S} x_i = \sum_{i \in S} e_i.$$

Denote by $\mathrm{Sh}_i(V_\lambda)$ the Shapley value of agent i. The allocation

$$x = (x_1, \ldots, x_n) \in \prod_{i=1}^n X_i$$

is said to be a *λ-transfer value allocation* or a *cardinal value allocation* for the economy $\{(X_i, u_i, e_i): i = 1, \ldots, n\}$ if

(i) $\sum_{i=1}^n x_i = \sum_{i=1}^n e_i$, and

(ii) there exist $\{\lambda_i \geq 0: i = 1, \ldots, n\}$ with $\sum_{i=1}^n \lambda_i = 1$ such that $\lambda_i u_i(x_i) = \mathrm{Sh}_i(V_\lambda)$, and $\mathrm{Sh}_i(V_\lambda) \geq \lambda_i u_i(e_i)$ for each i.

Emmons and Scafuri (1985) or Shapley (1969) show that if u_i is concave, and continuous; and if X_i is bounded from below, closed and convex; then a cardinal value allocation exists for the economy $\{(X_i, u_i, e_i):, i = 1, \ldots, n\}$. Let L_{X_i} denote the set of all functions $x_i: \Omega \to Y$ which are $\mathcal{F}_i$-measurable, and for which $x_i(\omega) \in X_i(\omega)$, μ-a.e. Define $W_i: L_{X_i} \to I\!R$ by $W_i(x) = \int u_i(x_i(\omega))\, d\mu(\omega)$.

We will prove our Theorem by considering its trace in finite dimensions and appealing to the Emmons and Scafuri (1985) existence result. We first need to prove some simple facts:

(1) L_{X_i} is non-empty.

(2) L_{X_i} is convex, closed and bounded from below.

(3) W_i is weakly continuous on L_{X_i}.

(4) W_i is concave on L_{X_i}.

Fact (1) follows immediately. Since by assumption e_i is $\mathcal{F}_i$-measurable and integrably bounded, we can conclude that $e_i \in L_{X_i}$. Fact (2) follows directly from assumption (A.1). Fact (3) is proved in Yannelis (1991, Claim 4.1) and (4) follows directly from the concavity of u_i.

Now consider the economy $\bar{\mathcal{E}} = \{(L_{X_i}, W_i, e_i): i = 1, \ldots, n\}$, where L_{X_i} denotes the consumption set of agent i, where W_i is the utility function of agent i, and where $e_i \in L_{X_i}$ denotes the initial endowment of agent i. Note that the existence of a value allocation in $\bar{\mathcal{E}}$ implies the existence of a value allocation for the original economy $\mathcal{E}$.

Let $\mathcal{A}$ be the set of all finite dimensional subspaces of $L^1_Y(\mu)$ containing the initial endowments. For each $\alpha \in \mathcal{A}$, let $L^\alpha_{X_i} = L_{X_i} \cap \alpha$ be the consumption set of agent i and $W^\alpha_i : L^\alpha_{X_i} \to I\!R$ be the utility function of agent i. For each $\alpha \in A$, we have an economy $\bar{\mathcal{E}}^\alpha$ with a finite dimensional consumption space. Further, for each $\alpha \in \mathcal{A}$, the economy $\bar{\mathcal{E}}^\alpha$ fulfills the assumptions of Emmons and Scafuri (1985). Hence there exists a value allocation, i.e., there exist $x^\alpha \in \prod_{i=1}^n L^\alpha_{X_i}$ such that

(i) $\sum_{i=1}^n x_i^\alpha = \sum_{i=1}^n e_i$;

(ii) there exist $\lambda_i^\alpha \geq 0$ with $\sum_{i=1}^n \lambda_i^\alpha = 1$, such that $\lambda_i^\alpha W_i^\alpha(x_i^\alpha) = \mathrm{Sh}_i(V_{\lambda W^\alpha})$ for every i, where $\mathrm{Sh}_i(V_{\lambda W^\alpha})$ is the Shapley value of agent i derived from the game $(I, V_{\lambda W^\alpha})$,[24] and $\mathrm{Sh}_i(V_{\lambda W^\alpha}) \geq \lambda_i W_i^\alpha(e_i)$.

By (i) we have that

$$0 \leq \sum_{i=1}^n x_i^\alpha = \sum_{i=1}^n e_i = e.$$

Hence each x_i^α lies in the order interval $[0, e]$ in $\sum_{i=1}^n L_{X_i} \subset L^1_Y(\mu)$, which is weakly compact by Cartwright's Theorem [see Cartwright (1974) or Section 8.1].

Order the set $\mathcal{A}$ by inclusion. Then $\{(x_1^\alpha, \ldots, x_n^\alpha, \lambda_1^\alpha, \ldots, \lambda_n^\alpha) : \alpha \in \mathcal{A}\}$ is a net in $K = \prod_{i=1}^n [0, e] \times \Delta$, where Δ denotes the $(n-1)$-dimensional simplex. Since K is compact there exists a subnet which converges to a point $(x_1, \ldots, x_n, \lambda_1, \ldots, \lambda_n)$. To complete the proof we must show that this limit is a value allocation of our original economy $\mathcal{E}$, i.e., that conditions (i) and (ii) hold. (i) follows immediately. (ii) follows from the weak continuity of W_i and from the continuity of the Shapley value in λ. This concludes the proof.

Corollary 1. *Let $\mathcal{E} = \{(X_i, u_i, e_i, \mathcal{F}_i, \mu) : i = 1, \ldots, n\}$ be a finite exchange economy with differential information, satisfying the assumptions of the main*

[24] Clearly, the game $(I, V_{\lambda W^\alpha})$ is defined as follows: For every coalition $S \subset I$ let

$$V_{\lambda W^\alpha}(S) = \max_{x_i \in L^\alpha_{X_i}} \sum_{i \in S} \lambda_i^\alpha W_i^\alpha(x_i) \text{ subject to } \sum_{i \in S} x_i = \sum_{i \in S} e_i.$$

theorem. Suppose there is one commodity per state. Then there exists a strong coalitional incentive compatible value allocation in $\mathcal{E}$.

Proof. Combine Theorem 2 with Lemma 2.

9 Concluding Remarks

In this paper we studied the cardinal value allocation in an economy with differential information. We showed that whenever coalitions of agents either pool their private information (fine value allocation) or whenever coalitions of agents base their decision on common knowledge (coarse value allocation) the information advantage or superiority of an agent is not reflected in the above concepts in any interesting way. Moreover, for the fine value allocation there is not necessarily truthful revelation of information within a coalition, and the coarse value allocation does not exist in general. However, the coalitional incentive compatible value allocation seems to provide not only an interesting way to measure the "worth" of the information advantage of an agent (a property not necessarily shared by the Walrasian equilibrium or the core in a differential information economy framework) but also ensures the truthful revelation of information within a coalition, because incentive compatibility is now inherent in this concept. We also indicated (Examples 3a, 4 and 5) that the coalitional incentive compatible value allocation provides more plausible outcomes than the Walrasian equilibrium and the core. Furthermore, our examples suggests that the value allocation may be suitable for analyzing problems of financial intermediation. Finally, we note that the coalitional incentive compatible value allocation exists under very mild assumptions and it provides plausible outcomes in situations where the more traditional concepts fail to do so. We believe that this concept has great potential in applications to a broad class of problems in economies with differential information.

After obtaining the main results of our paper, we became aware of independent work by Beth Allen (1991a) which examines some of the concepts that we analyze in this paper in a game theoretic setting. It is important to note that our results are not coincident with hers. Rather, they complement each other. In particular, we concentrate on the interpretation of the different value allocation concepts, and on the problem of truthful revelation of information within each coalition, whereas Allen's main focus is to provide existence theorems for TU and NTU games, with additional information sharing rules other than those examined in our paper. Moreover, it is important to note that the techniques used in either paper are different.

10 Appendix

The purpose of this section is to establish that the examples which we use are not "pathological." Instead, the central ideas which we show via these examples hold in much more general environments. We start by showing that the non-existence result of Example 1 is generic for a broader class of differential information economies.[25] For simplicity we consider economies with three agents I, J and K where $\bigwedge_{i=I,J,K} \mathcal{F}_i$ is trivial and with only one commodity per state. However, it is straightforward to extend this argument to more general classes of economies.

Proposition 1. *Assume that there are three agents such that $\bigwedge_{i=I,J,K} \mathcal{F}_i$ is trivial and such that $\mathcal{F}_i \wedge \mathcal{F}_j$ is non-trivial for some $i \neq j$. Further, assume that there is one commodity per state and at least three states of nature. Then a coarse value allocation does not exist generically.*

Proof. Denote by W_i the expected utility of agent i. Since by definition the value allocation must be individually rational, the weights λ_i must be chosen such that (e_I, e_J, e_K) maximizes

$$\sum_{i=I,J,K} \lambda_i W_i(e_i + t_i), \text{ subject to } \sum_{i=I,J,K} t_i = 0,$$

where the t_i are state independent net-transfers. Normalize the λ_i by choosing $\lambda_K = 1$. Then it follows that

$$\lambda_i = \frac{\sum_{\omega \in \Omega} \partial W_K / \partial x_\omega(e_K)}{\sum_{\omega \in \Omega} \partial W_i / \partial x_\omega(e_i)} \qquad i = I, J. \tag{A1}$$

[25] "Generic" means that if we fix the information structure of an economy and perturb the endowments of the agents such that the endowment of every agent remains measurable with respect to his/her information then non-existence of a coarse value allocation holds for an open set of economies whose complement has measure zero.

Further, in a value allocation $\mathrm{Sh}_i = \lambda_i W_i(e_i)$ for every agent i. Thus, we have more than two equations which must be fulfilled for the λ_i. Hence, if we can show that the derivative of (A1) and of the functions $\mathrm{Sh}_i - \lambda_i W_i(e_i)$ has a rank of at least three, a solution cannot exist for a generic choice of initial endowments.

Clearly, by taking the derivative of λ_i with respect to e_i we get a matrix of full rank. It is therefore sufficient to show that there exists a vector $h \in I\!R^{3|\Omega|}$ such that the derivative of λ_i and u_i in the direction h is zero and such that the derivative of Sh_i in the direction of h is non-zero. This can be achieved as follows. Assume without loss of generality that $\mathcal{F}_I \wedge \mathcal{F}_J$ is non-trivial. Hence there exists a partition of Ω into two sets T_1 and T_2 such that agent I and J can attain arbitrary income transfers which are constant on T_1 and T_2. It is easy to see that maximizing the weighted sum of utilities of agent I and J requires

$$\sum_{\omega \in T_i} \partial W_I / \partial x_\omega(x_I) = \sum_{\omega \in T_i} \partial W_J / \partial x_\omega(x_J), \text{ for } i = 1, 2. \qquad (A2)$$

The fact that a derivative in the direction h should not change the utility and the weights of agents I and J can be expressed by four equations which h must fulfill. Since h is an element of at least a six-dimensional Euclidean space we have sufficient freedom to choose h such that the derivatives of the left- and right hand-side of (A2) in the direction h differ. This however means that the derivative of Sh_I or Sh_J in the direction h is non-zero. This concludes the proof.

Next we establish in general that the fine value allocation does not take information asymmetries into account.

Proposition 2. *Let $\mathcal{E}$ be an economy with differential information, and let $\hat{\mathcal{E}}$ be the corresponding full information economy (i.e, the economy where $\mathcal{F}_i = \mathcal{F}$ for all agents i). Then all fine value allocations of the two economies coincide.*

Proof. The essential idea is already contained in Example 2. Let $\hat{\mathcal{F}}_i$ denote the σ-algebra generated by e_i. Concavity of the utility functions of all agents implies that in the full information case, agents in a coalition S will never choose net-trades which are not measurable with respect to $\bigvee_{i \in S} \hat{\mathcal{F}}_i$.

Since $\hat{\mathcal{F}}_i \subset \mathcal{F}_i$ it follows that all such allocations can be also achieved in the economy with differential information. Hence $V_\lambda^f(S)$ will be the same in the full information and in the differential information case. All value allocations must therefore coincide.

The other examples also immediately generalize to more general cases. For example, we can choose in Example 4, n agents with independent endowments and whose information is the σ-algebra generated by the endowment, and an additional agent who has "better information" (though not necessarily complete information). Then a weakly coalitional incentive compatible value allocation will exist, and the agent with the better information will serve as an intermediary again receiving a positive net-payment for his/her service.

References

C.D. Aliprantis and O. Burkinshaw (1985), "Positive Operators," Academic Press: New York, London.

B. Allen (1991), "Market Games With Asymmetric Information and Nontransferable Utility: Representation Results and the Core," CARESS Working paper 91-09, University of Pennsylvania.

B. Allen (1991a), "Market Games with Asymmetric Information: The Value," mimeo, University of Pennsylvania.

R.J. Aumann (1985), "On the Non-transferable Utility Value: A Comment on the Roth-Shafer Examples," *Econometrica* **53**, 667–677.

R.J. Aumann (1987), "Value, Symmetry and Equal Treatment: A Comment on Scafuri-Yannelis," *Econometrica* **55**, 1461–1464.

J. Boyd and E.C. Prescott (1986), "Financial Intermediary Coalitions," *Journal of Economic Theory* **38**, 211–232.

D. Cartwright (1974), "The Order Completeness of Some Spaces of Vector Valued Functions," *Bulletin of the Australian Mathematical Society* **11**, 57–61.

J. Diestel and J.J. Uhl (1977), "Vector Measures," Mathematical Surveys, **15**, *American Mathematical Society*, Providence, Rhode Island.

D. Emmons and A.J. Scafuri (1985), "Value Allocations—An Exposition," In: Advances in Equilibrium Theory, ed.: C.D. Aliprantis et al., Springer: Berlin, Heidelberg, New York.

T. Ichiishi (1983), "Game Theory for Economic Analysis," Academic Press: New York.

A. Roth (1980), "Values of Games Without Side Payments: Some Difficulties With Current Concepts," *Econometrica* **48**, 457–465.

A. Roth (1983), "Some Further Thoughts on the NTU Value and Related Matters: A Rejoinder to Aumann," mimeo University of Pittsburgh, a shortened version appeared in *Econometrica (1986)*, **55**, 981–984.

A.J. Scafuri and N.C. Yannelis (1984), "Non-Symmetric Cardinal Value Allocations," *Econometrica* **52**, 1365–1368.

W.J. Shafer (1980), "On the Existence and Interpretation of Value Allocation," *Econometrica* **48**, 467–474.

L.A. Shapley (1953), "A Value of n-Person Games," in H.W. Kuhn and A.W. Tucker (Eds.), *Contributions to the Theory of Games*, Vol. II, 307–317, Princeton University Press.

L.A. Shapley (1969), "Utility Comparisons and the Theory of Games," in *La Decision*, Guilbaud ed., Edition du CNRS, Paris, France.

R.M. Townsend (1979), "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory* **21**, 1–29.

R. Wilson (1978), "Information, Efficiency, and the Core of an Economy," *Econometrica* **46**, 807–816.

N.C. Yannelis (1983), "Existence and Fairness of Value Allocation Without Convex Preferences," *Journal of Economic Theory* **31**, 282–292.

N.C. Yannelis (1991), "The Core of an Economy with Differential Information," *Economic Theory* **1**, 183–198.