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A WAGE EARNERS' INVESTMENT FUND UNDER
STEADY-STATE INFLATION AND GROWTH

Hans Brems

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A WAGE EARNERS' INVESTMENT FUND UNDER STEADY-STATE INFLATION AND GROWTH

By HANS BREMS

Summary

To a wage earners' investment fund all employers contribute compulsorily a fraction of their wage bill. To the employees the fund issues nonnegotiable fund certificates, redeemable after a specified number of years. Within the framework of a simple neoclassical model of steady-state inflation and growth and using Danish data, the article determines the size of such a fund as well as its effects upon the marginal productivity of capital, disposable-income distribution between capital and labor, the propensity to save national output, and the real wage rate.

A WAGE EARNERS' INVESTMENT FUND UNDER STEADY-STATE INFLATION AND GROWTH

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October 16, 1973

A WAGE EARNERS' INVESTMENT
FUND UNDER STEADY-STATE
INFLATION AND GROWTH

By HANS BREMS*

With the purpose of paring down consumer demand to wartime output of consumers' goods, Keynes proposed, in How to Pay for the War [9], a "deferred-pay" scheme calling for £550 million in annual compulsory saving. The complete scheme, including "the accumulation of working-class wealth under working-class control," would embody, Keynes said in his preface, "an advance towards economic equality greater than any which we have made in recent times."

U. S. labor unions often find themselves at odds with the administration on questions of national economic policy. By contrast, Western European labor unions are used to influencing national policy by working with the government, Barbash [2]. An example of such collaboration is the re-emergence in Western Europe of the Keynesian idea a third of a century later under a new name and serving new purposes.

I. A WAGE EARNERS' INVESTMENT FUND

Serving the dual purpose of giving labor a share of, first, the

capital gains accruing to stockholders in an inflationary economy and, second, the co-determination rights inherent in stock ownership, a wage earners' investment fund would work as follows. Primarily in the form of corporate stock all employers would contribute compulsorily a fraction of their wage bill¹ to the fund. The fund would belong to the employees. To the individual employees, in turn, the fund would issue nonnegotiable fund certificates. A specified number of years after its issue a fund certificate would become redeemable in cash at a price which would include the share of that certificate in all capital gains and dividends made by the fund during the lifetime of the certificate. The fund would be allowed to sell contributed corporate stock at any time and buy other stock.

Neither in Europe itself nor in the U. S. has the idea attracted theoretical interest. In view of its sheer order of magnitude and of its simultaneous emergence in several advanced countries, this might seem surprising. The purpose of the present article is to make a beginning by analyzing the effects of a wage earners' investment fund upon the physical marginal productivity of capital, disposable-income distribution between labor and capital, the propensity to save national output, and the real wage rate.

What would be a suitable theoretical framework for our analysis?

From the purposes of the fund it follows that our model should be capable of accommodating inflation and that its capitalists should be stockholders rather than bondholders. We choose the simplest possible one-sector neoclassical model of steady-state inflation and growth. Its capitalists are capitalist-entrepreneurs producing a single good from labor and an immortal capital stock of that good, hence investment is the act of setting aside part of output for installation as capital stock. Capital stock is the result of accumulated savings—voluntary as well as forced. Technology, available labor force, and the money wage rate are growing autonomously.

What would be a suitable institutional framework? For numerical solution of our key transcendental equation we shall need empirical values of parameters and shall use stylized Danish ones.

II. NOTATION

Variables

c \equiv propensity to consume national output

C \equiv consumption

ϕ \equiv size of wage earners' investment fund relative to capital stock

Φ \equiv absolute size of wage earners' investment fund

- g_v \equiv proportionate rate of growth of variable $v \equiv c, C, \Phi, I, i,$
 $\kappa, L, P, S, \theta, W, X, Y_1, Y_2,$ and Y
- I \equiv investment
- i \equiv internal rate of return
- k \equiv present gross worth of a physical unit of capital stock
- κ \equiv physical marginal productivity of capital stock
- L \equiv labor employed
- v \equiv propensity to save national output
- P \equiv price of good
- r \equiv discount rate applied by capitalist-entrepreneurs
- S \equiv physical capital stock
- θ \equiv disposable-income to output ratio, called "the payout ratio"
- W \equiv wage bill including employers' contribution to fund per year
- X \equiv physical output
- Y \equiv disposable money income
- Z \equiv profits bill

Parameters

- α, β \equiv exponents of production function
- b \equiv employers' contribution to fund as a fraction of wage bill

- c_d \equiv propensity to consume national disposable real income
 e \equiv Euler's number, the base of natural logarithms
 F \equiv available labor force
 g_p \equiv proportionate rate of growth of parameter $p \equiv F, M,$ and w
 M \equiv multiplicative factor of production function
 ρ \equiv redemption period
 w \equiv money wage rate including employers' contribution to fund per man year

Parameters listed are stationary except $F, M,$ and $w,$ whose growth rates $g_F, g_M,$ and g_w are stationary. Time coordinates are t and $\tau.$ The unit of time is the year.

III. THE EQUATIONS OF THE MODEL

1. Definitions

Fifteen variable growth rates are listed in Sec. II. To all apply the definition

$$(1) \text{ through } (15) \quad g_v \equiv \frac{dv}{dt} \frac{1}{v}$$

Define investment as the derivative of capital stock with

respect to time:

$$(16) \quad I \equiv ds/dt$$

2. Production

Let the capitalist-entrepreneurs apply the Cobb-Douglas production function

$$(17) \quad X = ML^{\alpha}S^{\beta}$$

where $0 < \alpha < 1$; $0 < \beta < 1$; $\alpha + \beta = 1$; and $M > 0$. Define the money wage rate w as including employers' contribution per man year to the investment fund. Then let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

$$(18) \quad \frac{w}{P} = \frac{\partial X}{\partial L} = \alpha \frac{X}{L}$$

Physical marginal productivity of capital is defined:

$$(19) \quad \kappa \equiv \frac{\partial X}{\partial S} = \beta \frac{X}{S}$$

Multiply (19) by price of output P to find value marginal productivity of capital. Define money profits earned on each physical unit of capital stock S as its value marginal productivity. Then multiply by S to find money profits earned on capital stock S

$$(20) \quad Z \equiv \kappa PS = \beta PX$$

Under full employment, available labor force must equal labor employed:

$$(21) \quad F = L$$

Define the wage bill as the money wage rate times employment:

$$(22) \quad W \equiv wL$$

3. Absolute Size of Fund

At time t , let the employers contribute the amount $bW(t)$ to the wage earners' investment fund. Let $bW(\tau, t)$ be the value at time τ of the amount $bW(t)$ contributed at time t . How does $bW(t)$ grow to become $bW(\tau, t)$? Assume the wage earners to have the same motivation and skill as the capitalist-entrepreneurs hence, like the capitalist-entrepreneurs, to be making the internal rate of

return i on the money value of the capital stock they own, i. e. the wage earners' investment fund. Let the earnings of the fund be compounded continuously, then

$$(23) \quad bW(\tau, t) = e^{i(\tau - t)} bW(t)$$

Let all wage earners present their fund certificates for redemption as soon as the latter become redeemable. Redemption at time τ is the accumulated value at time τ of the contribution made at time $\tau - \rho$, where ρ is the redemption period. The size of the fund at time τ is the value at time τ of all contributions made from $t = \tau - \rho$ to $t = \tau$:

$$(24) \quad \Phi(\tau) \equiv \int_{\tau - \rho}^{\tau} bW(\tau, t) dt$$

The wage bill out of which the contributions to the fund are made, is growing at the proportionate rate g_W , hence

$$(25) \quad W(\tau) = e^{g_W(\tau - t)} W(t)$$

Insert (23) and (25) into (24) and find the size of the fund

$$(26) \quad \Phi(\tau) = \int_{\tau - \rho}^{\tau} e^{(1 - g_W)(\tau - t)} bW(t) dt$$

The integration would be facilitated by assuming the internal rate of return i and the proportionate rate of growth of the wage bill g_W to be stationary:

$$(27) \quad di/dt = 0$$

$$(28) \quad dg_W/dt = 0$$

The integration will have to be carried out separately for $i \neq g_W$ and $i = g_W$. Find all variables in the outcome referring to the same time τ , purge it of τ , and write the size of the fund

$$(29) \quad \Phi = bW[e^{(1 - g_W)\rho} - 1]/(1 - g_W) \text{ for } i \neq g_W$$

$$\Phi = bW\rho \text{ for } i = g_W$$

4. Disposable-Income Distribution between Labor and Capital

Redemption at time τ is the accumulated value at time τ of the contribution made at time $\tau - \rho$. That value we write $bW(\tau, \tau - \rho)$

and define labor's disposable income at time τ as the wage bill minus contribution plus redemption at that time:

$$Y_1(\tau) \equiv W(\tau) - bW(\tau) + bW(\tau, \tau - \rho)$$

Insert (25) into (23), replace t by $\tau - \rho$, and find redemption

$$bW(\tau, \tau - \rho) = e^{(1 - g_W)\rho} bW(\tau)$$

Use this, (18), and (22) to write labor's disposable income

$$(30) \quad Y_1 = \theta_1 PX, \text{ where}$$

$$\theta_1 \equiv \alpha + \alpha b [e^{(1 - g_W)\rho} - 1]$$

The capitalist-entrepreneurs are making the internal rate of return i on the money value of the capital stock they own, i. e., all capital stock minus the wage earners' investment fund. The internal rate of return i includes profits and capital gains, as we shall see in Eq. (57) in Sec. V. Follow convention² and exclude capital gains from disposable income. According to (20) profits are earned at the rate κ , so define disposable income of the capitalist-entrepreneurs as

$$Y_2 \equiv \kappa(PS - \Phi)$$

Remembering the two separate forms of (29), insert (18), (20), (22), and (29) and write the disposable income of the capitalist-entrepreneurs

$$(31) \quad Y_2 = \theta_2 PX, \text{ where}$$

$$\theta_2 \equiv \beta - \alpha b \kappa [e^{(1 - g_W)\rho} - 1] / (1 - g_W) \text{ for } i \neq g_W$$

$$\theta_2 \equiv \beta - \alpha b \kappa \rho \text{ for } i = g_W$$

Add (30) and (31) and find national disposable money income

$$(32) \quad Y \equiv Y_1 + Y_2 = \theta PX, \text{ where}$$

$$\theta \equiv \theta_1 + \theta_2 = 1 + \alpha b (1 - g_W - \kappa) [e^{(1 - g_W)\rho} - 1] / (1 - g_W) \text{ for } i \neq g_W$$

$$\theta \equiv \theta_1 + \theta_2 = 1 - \alpha b \kappa \rho \text{ for } i = g_W$$

θ is a disposable-income to output ratio, a "payout" ratio.

5. Consumption

Let the parameter c_d be the propensity to consume national disposable real income:

$$(33) \quad C = c_d Y/P$$

Define the variable c as the propensity to consume national output:

$$(34) \quad C \equiv cX$$

Take (32), (33), and (34) together and find

$$(35) \quad c = c_d \theta$$

Define the variable v as the propensity to save national output:

$$(36) \quad v \equiv 1 - c$$

6. Equilibrium

Finally, output equilibrium requires output to equal the sum

of consumption and investment demand for it:

$$(37) \quad X = C + I$$

IV. SOLUTIONS FOR PROPORTIONATE RATES OF GROWTH

Define, as Hahn and Matthews [8] did, steady-state growth as stationary proportionate rates of growth. Our system (1) through (37) possesses the following set of steady-state solutions:

$$(38) \quad g_c = 0$$

$$(39) \quad g_C = g_X$$

$$(40) \quad g_\phi = g_W$$

$$(41) \quad g_I = g_X$$

$$(42) \quad g_l = 0$$

$$(43) \quad g_\kappa = 0$$

$$(44) \quad g_L = g_F$$

$$(45) \quad g_P = g_W - g_M/\alpha$$

$$(46) \quad g_S = g_F + g_M/\alpha$$

$$(47) \quad g_\theta = 0$$

$$(48) \quad g_W = g_F + g_w$$

$$(49) \quad g_X = g_S$$

$$(50) \quad g_{Y1} = g_W$$

$$(51) \quad g_{Y2} = g_W$$

$$(52) \quad g_Y = g_W$$

To convince himself that those are indeed solutions, the reader should take derivatives with respect to time of (16) through (37). He should then use the definitions (1) through

(15), insert the solutions (38) through (52), and convince himself that each equation is satisfied. Thus our auxiliary assumptions (27) and (28)—consistent with (42) and (48), respectively—have paid off handsomely. But there is more to growth theory than finding proportionate rates of growth. Our purpose was to find the effects of a wage earners' investment fund upon the physical marginal productivity of capital, disposable-income distribution between labor and capital, the propensity to save national output, and the real wage rate. Those effects are effects upon levels in a growing economy. In determining such levels our solutions for proportionate rates of growth (38) through (52) will be useful.

V. PHYSICAL MARGINAL PRODUCTIVITY κ AND INTERNAL RATE OF RETURN i

According to our solution (43) a physical unit of capital stock added at time τ would have the physical marginal productivity κ at any time from $t = \tau$ to $t = \infty$. What sort of value marginal productivity will it have? Let it be perfectly foreseen by the entrepreneurs that price is growing at the proportionate rate g_p :

$$(53) \quad P(t) = e^{g_P(t - \tau)} P(\tau)$$

But let the capitalist-entrepreneurs be purely competitive ones, hence price is beyond their control. At time t , value marginal productivity is, then

$$\frac{\partial [P(t)X(t)]}{\partial S(t)} = P(t)\kappa$$

As seen from the present time τ , value marginal productivity at time t is $e^{-r(t - \tau)} P(t)\kappa$, where r is the discount rate applied by the capitalist-entrepreneurs. Define present gross worth k at time τ of the physical unit of capital stock as the present worth of all its future value marginal productivities:

$$(54) \quad k(\tau) \equiv \int_{\tau}^{\infty} e^{-r(t - \tau)} P(t)\kappa dt$$

Let the rate of inflation be less than the discount rate:

$$(55) \quad g_P < r$$

Insert (53) into (54) and use (55) to carry out the integration.

Since in the outcome all variables refer to the same time τ , we may purge it of τ :

$$(56) \quad k = PK/(r - g_p)$$

Define the present net worth of the physical unit of capital stock as gross worth minus price:

$$n \equiv [\kappa/(r - g_p) - 1]P$$

Define the internal rate of return i as that value of r which makes net worth equal to zero, hence

$$(57) \quad i \equiv \kappa + g_p$$

where g_p stands for (45). In an inflationary economy, then, the internal rate of return of a physical unit of capital stock equals the physical marginal productivity of that unit plus the proportionate rate of capital gain (45).

VI. A TRANSCENDENTAL EQUATION IN PHYSICAL MARGINAL PRODUCTIVITY κ

Use (34), (36), and (37) to find $I = vX$ and (1) through (16) to find that $I = g_S S$, hence $S = vX/g_S$. Insert that into (19) and find

$$(58) \quad \kappa = \beta g_S / v$$

Insert (45), (46), (48), and (57) into $1 - g_W$ and find

$$(59) \quad 1 - g_W = \kappa - g_S$$

Remembering the two separate forms of (32), insert (32), (35), (36), and (59) into (58) and find the transcendental equation in κ :

$$(60) \quad \kappa \left\{ (1 - c_d) / g_S + abc_d [e^{(\kappa - g_S)\rho} - 1] / (\kappa - g_S) \right\} = \beta \text{ for } 1 \neq g_W$$

$$\kappa \left\{ (1 - c_d) / g_S + abc_d \rho \right\} = \beta \text{ for } 1 = g_W$$

An explicit solution of (60) is beyond reach. But our appendix proves the existence of a unique and positive solution for κ . And once we had empirical values of the parameters entering (60) we could find that solution numerically. Let us choose such values, then.

VII. EMPIRICAL VALUES OF PARAMETERS

Denmark is one country currently considering a wage earners' investment fund, so let us choose stylized Danish values of the parameters entering (60). The relevant wage bill is the wage bill of employees alone—not including the estimated wage bill of proprietors. In this narrow sense labor's share of Danish national income is one-half, Det økonomiske Råd [6], 40, and we shall use that value for α —thereby in effect classifying all proprietors' income as a return to capital and thus exaggerating the physical marginal productivity of capital now to be determined.

1953-69 Denmark's net domestic fixed asset formation in 1963 prices was 13.1% of her net national product in 1963 prices, Brems [4], 33-39, or roughly $1/8$. If, like a conventional neoclassical growth model, Denmark had had neither a government nor a wage earners' investment fund, her propensity to consume national disposable real income would have been roughly $7/8$.

1960-69 the Danish labor force in terms of number of men was growing at 1% per annum, Det økonomiske Råd [5], 55, but hours per man are now declining by the same percentage, Det økonomiske Råd [7], 80. Technological progress is perhaps 2% per annum, so

let us adopt

$$\begin{aligned}\alpha &= 1/2 \\ c_d &= 7/8 \\ g_F &= 0 \\ g_M &= 1/50\end{aligned}$$

From our assumption that $\alpha + \beta = 1$ and from (46) it follows that

$$\begin{aligned}\beta &= 1/2 \\ g_S &= 1/25\end{aligned}$$

VIII. NUMERICAL SOLUTIONS FOR LEVELS

1. The Physical Marginal Productivity of Capital

Insert our empirical Danish parameter values into our transcendental equation (60) and find

$$(61) \quad \kappa\{50 + 7b[e^{(\kappa - 0.04)\rho} - 1]/(\kappa - 0.04)\} - 8 = 0$$

Eq. (61) contains the two structural characteristics of a wage

earners' investment fund, i. e., $b \equiv$ employers' contribution as a fraction of the wage bill and $\rho \equiv$ the redemption period. Both the Danish bill, Arbejdsministeriet [1], 1, and the union proposal, Landsorganisationen [10], 37, suggested a contribution fraction of 5%. The bill, Arbejdsministeriet [1], 2, suggested a redemption period of seven years, whereas the union proposal, Landsorganisationen [10], 38, suggested a five-year redemption period. We should like to examine a rather wide range of alternative structural characteristics of such a fund, say

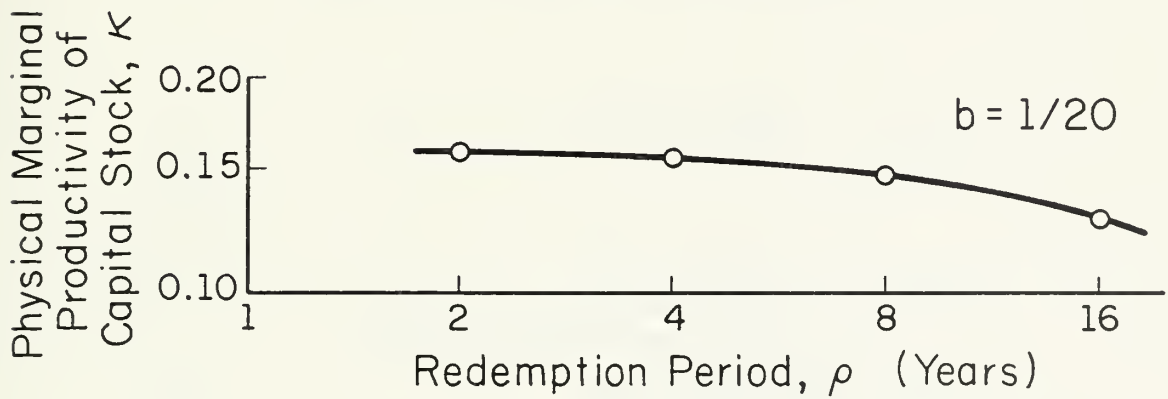
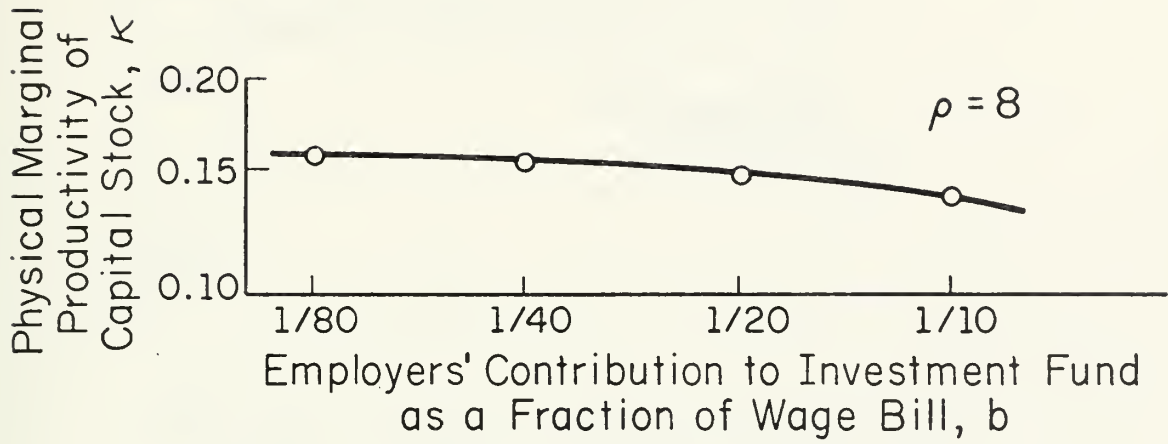
$$b = 1/80, \quad 1/40, \quad 1/20, \quad \text{and} \quad 1/10$$

$$\rho = 2, \quad 4, \quad 8, \quad \text{and} \quad 16$$

Inserting these alternative values into (61), can the latter be solved for κ ? Indeed it can—and was, using an IBM 360/75 at the Computer Services Office of the University of Illinois. The results are shown in Column 3 of Table 1 and in Figure 1. As one would expect, the higher the employers' contribution fraction b and the longer the redemption period ρ are, the lower is the physical marginal productivity of capital stock κ . But the elasticities of the latter with respect to b and ρ —apparent as

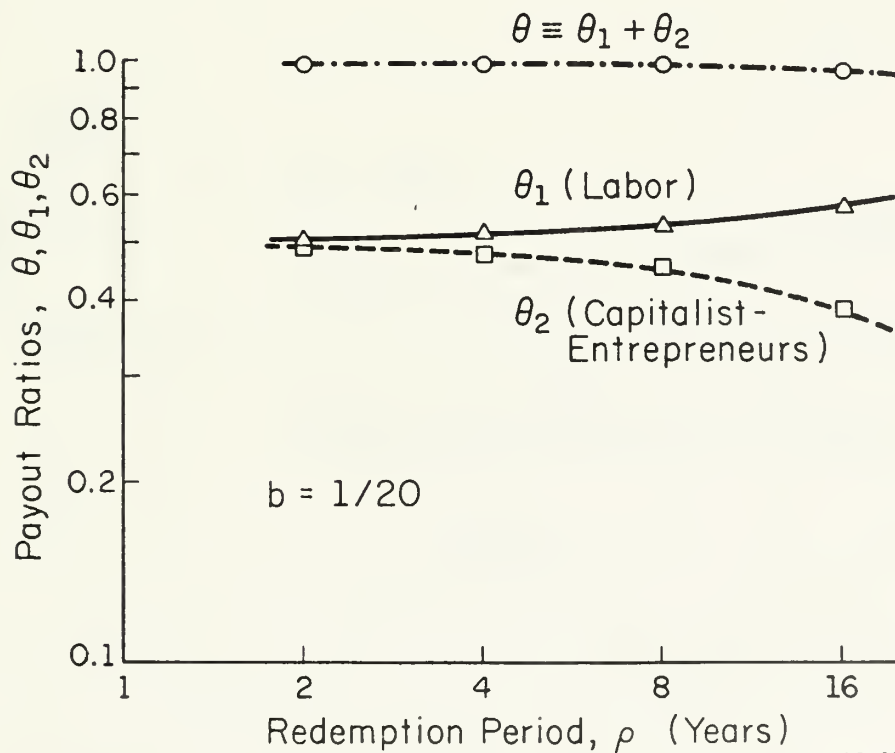
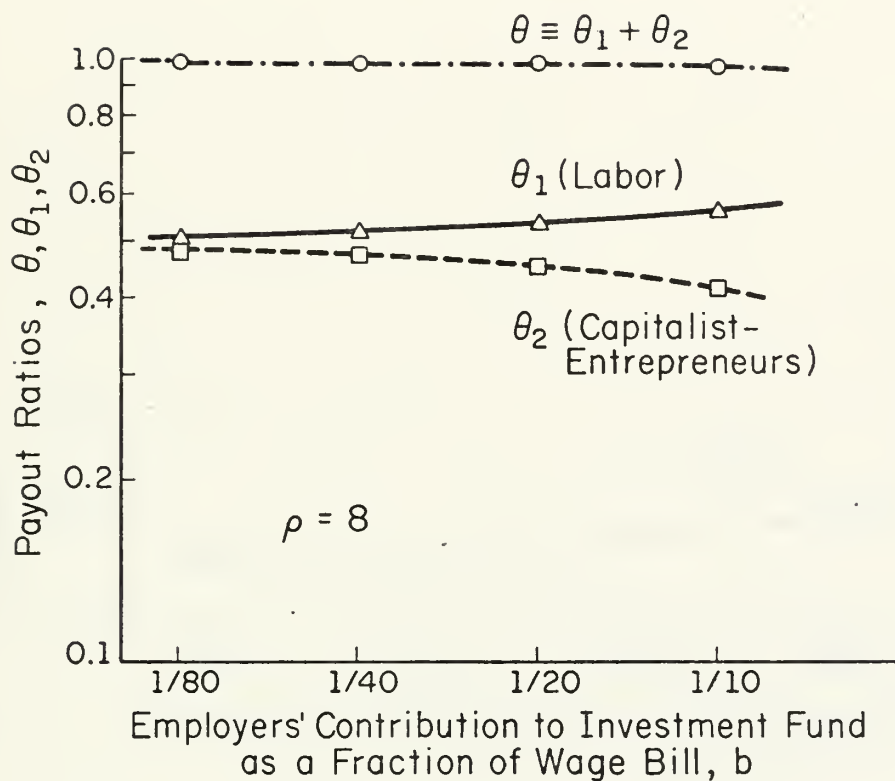
TABLE 1—ECONOMIC EFFECTS OF ALTERNATIVE CONTRIBUTION FRACTIONS AND REDEMPTION PERIODS

Contribution Fraction	Redemption Period	Physical Marginal Productivity of Capital	Labor Payout Ratio	Capital-Payout Ratio	Overall Payout Ratio	Propensity to Save National Output	Fund as a Fraction of Capital Stock
b	ρ	κ	θ_1	θ_2	θ	v	ϕ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1/80	8	0.156	0.510	0.487	0.997	0.128	0.026
1/40	8	0.153	0.518	0.475	0.994	0.131	0.050
1/20	8	0.147	0.534	0.454	0.987	0.136	0.093
1/10	8	0.137	0.559	0.417	0.976	0.146	0.165
1/20	2	0.158	0.507	0.491	0.998	0.127	0.018
1/20	4	0.155	0.515	0.480	0.995	0.129	0.039
1/20	16	0.128	0.578	0.387	0.965	0.156	0.226



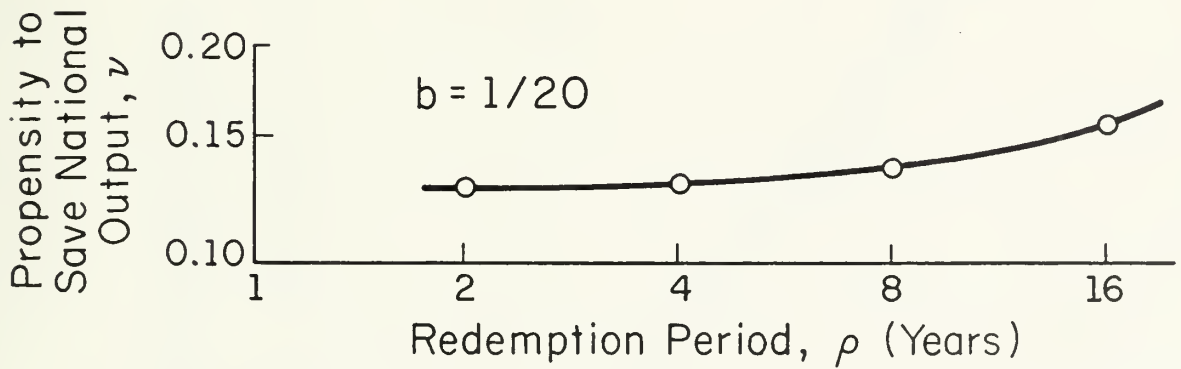
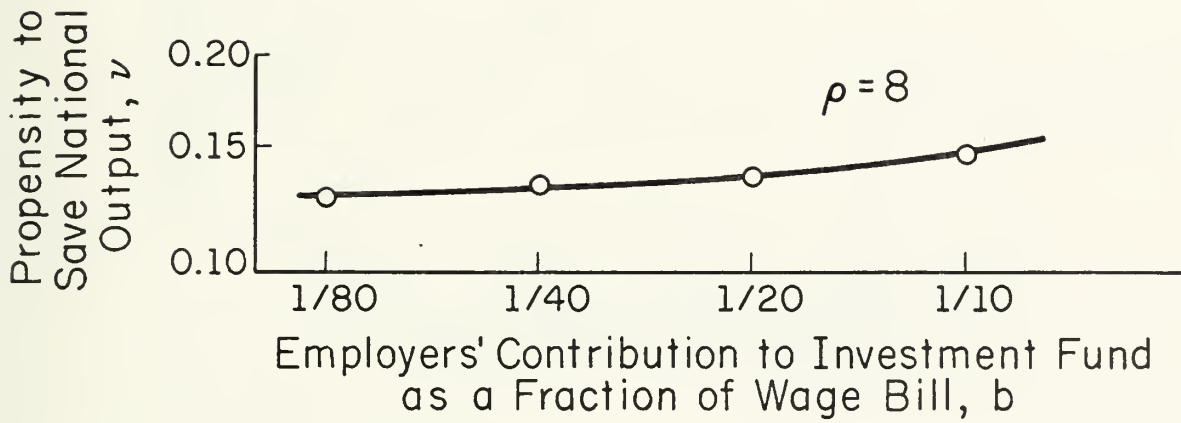
QP-51

Figure 1



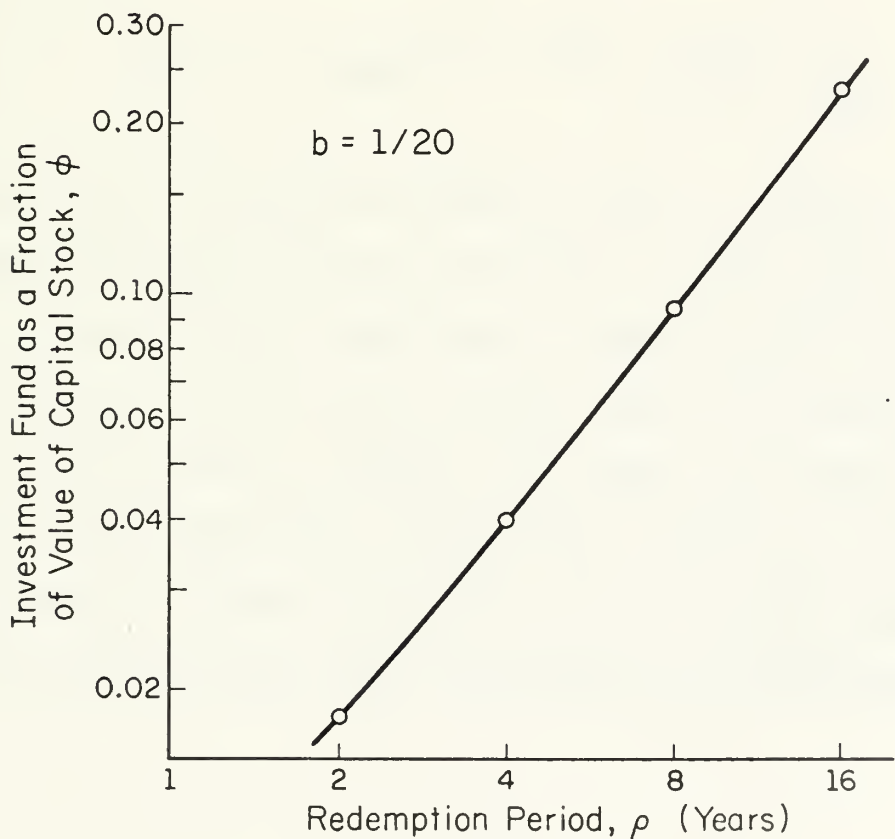
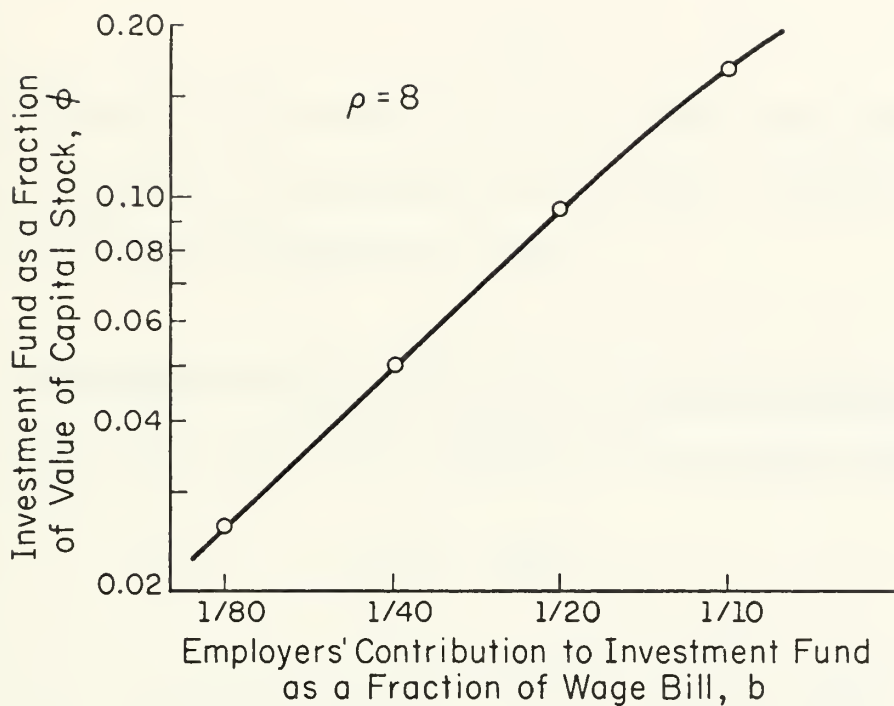
QP-52

Figure 2



QP-53

Figure 3



QP-54

Figure 4

the steepness of the curves on their double-logarithmic scale—are modest in the range considered politically. Beyond $b = 1/20$ and $\rho = 8$ they become noticeably higher.

2. Disposable-Income Distribution between Labor and Capital

Insert (59) into (30) through (32) and write the payout ratios

$$(62) \quad \theta_1 = \alpha + \alpha b [e^{(\kappa - g_S)\rho} - 1]$$

$$(63) \quad \theta_2 = \beta - \alpha b \kappa [e^{(\kappa - g_S)\rho} - 1] / (\kappa - g_S)$$

$$(64) \quad \theta = 1 - \alpha b g_S [e^{(\kappa - g_S)\rho} - 1] / (\kappa - g_S)$$

Here are two opposing forces at work: Rising b or ρ will at the same time make (62) rise and (63) and (64) fall (because b and ρ are rising) and make (62) fall and (63) and (64) rise, because κ is falling! But the former force wins in the practical range, as seen from Columns 4 through 6 of Table 1 and from Figure 2: The higher the employers' contribution fraction b and the longer the redemption period ρ are, the higher is labor's payout ratio and the lower is that of the capitalist-entrepreneurs. Labor wins,

and the capitalist-entrepreneurs lose. But labor wins slightly less than the capitalist-entrepreneurs are losing, so the overall ratio is lower. The elasticities of the payout ratios of labor and the capitalist-entrepreneurs with respect to b and ρ are considerable, especially beyond $b = 1/20$ and $\rho = 8$. We conclude that the redistributive effects of a wage earners' investment fund may well be considerable.

3. The Propensity to Save National Output

Insert (35) and (64) into (36) and write the propensity to save national output

$$(65) \quad v = 1 - c_d + abc_d g_s [e^{(\kappa - g_s)\rho} - 1] / (\kappa - g_s)$$

Again, two opposing forces are at work: Rising b or ρ will at the same time make (65) rise, because b and ρ are rising and fall, because κ is falling! But the former force wins in the practical range, as seen from Column 7 of Table 1 and from Figure 3: The higher the employers' contribution fraction b and the longer the redemption period ρ are, the higher is the propensity to save national output. The elasticities of the latter with respect to b and ρ are

modest in the range considered politically. Beyond $b = 1/20$ and $\rho = 8$ they become noticeably higher.

4. The Real Wage Rate

Use (34), (36), and (37) to find $I = vX$ and (1) through (16) to find that $I = g_S S$, hence $S = vX/g_S$. Insert that into (17), insert the outcome into (18) and find the real wage rate

$$(66) \quad w/P = \alpha [M(v/g_S)^\beta]^{1/\alpha}$$

including, we recall, employers' contribution per man hour to the wage earners' investment fund. Using our empirical parameter values $\alpha = \beta = 1/2$ we find the elasticity β/α of the real wage rate with respect to the propensity to save national output to be unity—a particularly simple Wicksell Effect. Consequently, the elasticities of the real wage rate with respect to b and ρ would be the same as those of the propensity to save national output, apparent from Figure 3, and it is unnecessary to draw a separate diagram for the real wage rate.

5. Relative Size of Fund

Insert (18), (20), (22), (58), and (59) into (29) and write size of the fund

(67) $\Phi = \phi PS$, where

$$\phi \equiv \alpha b g_S [e^{(\kappa - g_S)\rho} - 1] / [(\kappa - g_S)v]$$

There are several opposing forces at work here: Rising b or ρ will at the same time make (67) rise, because b and ρ are rising, and fall, because κ is falling and v rising. But the former force wins very easily in the practical range, as seen from Column 8 of Table 1 and from Figure 4: The higher the employers' contribution fraction b and the longer the redemption period ρ are, the larger is the investment fund as a fraction of the value of capital stock. The elasticities of the fraction ϕ with respect to b and ρ are considerable.

IX. THE SOUNDNESS OF SOME UNDERLYING ASSUMPTIONS

Such conclusive findings are, no doubt, the results of

using both a very simple growth model and very simple underlying assumptions about the behavior of wage earners owning an investment fund. Let us discuss briefly the soundness of two such assumptions.

1. Prompt Redemption

We have assumed wage earners to present their fund certificates for redemption as soon as the latter become redeemable. Much of the redemption is a capital gain, and our knowledge of the disposal of capital gains is sporadic, Bhatia [3]. Keynes would have questioned our assumption: "The argument is, I suppose, that savings deferred in this way are more likely than normal savings to be spent by their owners as soon as they are free to do so. How far this will prove to be true in fact, I am not sure. It may be that the blocked deposits will be instrumental in spreading the habit of small savings more widely..." Keynes [9], 47. Because Keynes' proposal was adopted to less than a quarter of his suggested sum, Maital [12], 166, and because it was adopted under wartime conditions, the issue remained unsettled.

2. Identical Motivations of Wage Earners and Capitalist-Entrepreneurs

We have assumed wage earners to have the same motivation and

skill as the capitalist-entrepreneurs, hence to be earning the same internal rate of return on the capital they own. Leaving aside skill, is the assumption of identical motivation a sound one? Two observations are in order.

First, the Danish union proposal, Landsorganisationen [10], Sec. 14, as well as the Danish bill, Arbejdsministeriet [1], Sec. 22, specifically demand an "active" placement of the fund and define "active" as a placement guaranteeing, first, a share of the capital gains of the economy and, second, a maximum dividend. So far, then, our assumption of identical motivation seems sound.

Second, however, such a motivation would require the fund to sell quite a bit of the contributed corporate stock and buy other stock. The reason is an inherent anomaly in the fund: Contributions are in proportion to the wage bill, hence less capital-intensive firms—with less investment need—will contribute proportionately more than more capital-intensive ones—with more investment need. Maximization of capital gains would require the fund to buy stock in more capital-intensive firms and sell stock in less capital-intensive ones, in less rapidly growing ones, and in less well-managed ones. But the employees of less capital-intensive, less rapidly growing, or less well-managed firms are

the very ones whose employment is most vulnerable. To such employees the exercise of their co-determination rights may well look like a last defense—to be taken away from them if the fund sells "their" stock! They would no doubt demand a hearing, and such a hearing is indeed suggested in an official comment to the Danish bill, Arbejdsministeriet [1], 22. Hearing or no hearing, a conflict remains between the interests of the wage earner qua owner of the investment fund and qua holder of a particular job. Or, which is the same thing, a conflict remains between the two purposes of a wage earners' investment fund, i. e., giving labor a share of, first, the capital gains and, second, the co-determination rights inherent in stock ownership. If widespread, such a conflict could play havoc with our assumption of identical motivation in wage earners and capitalist-entrepreneurs.

A P P E N D I X

THE EXISTENCE OF A UNIQUE AND POSITIVE SOLUTION OF EQUATION (60)

1. The Function $(e^{f\rho} - 1)/f$

Define $f \equiv \kappa - g_s$. Then Eq. (60) has in it a function $G(f)$ defined

$$G(f) \equiv (e^{f\rho} - 1)/f \text{ for } f \neq 0$$

$$G(0) \equiv \rho \text{ for } f = 0$$

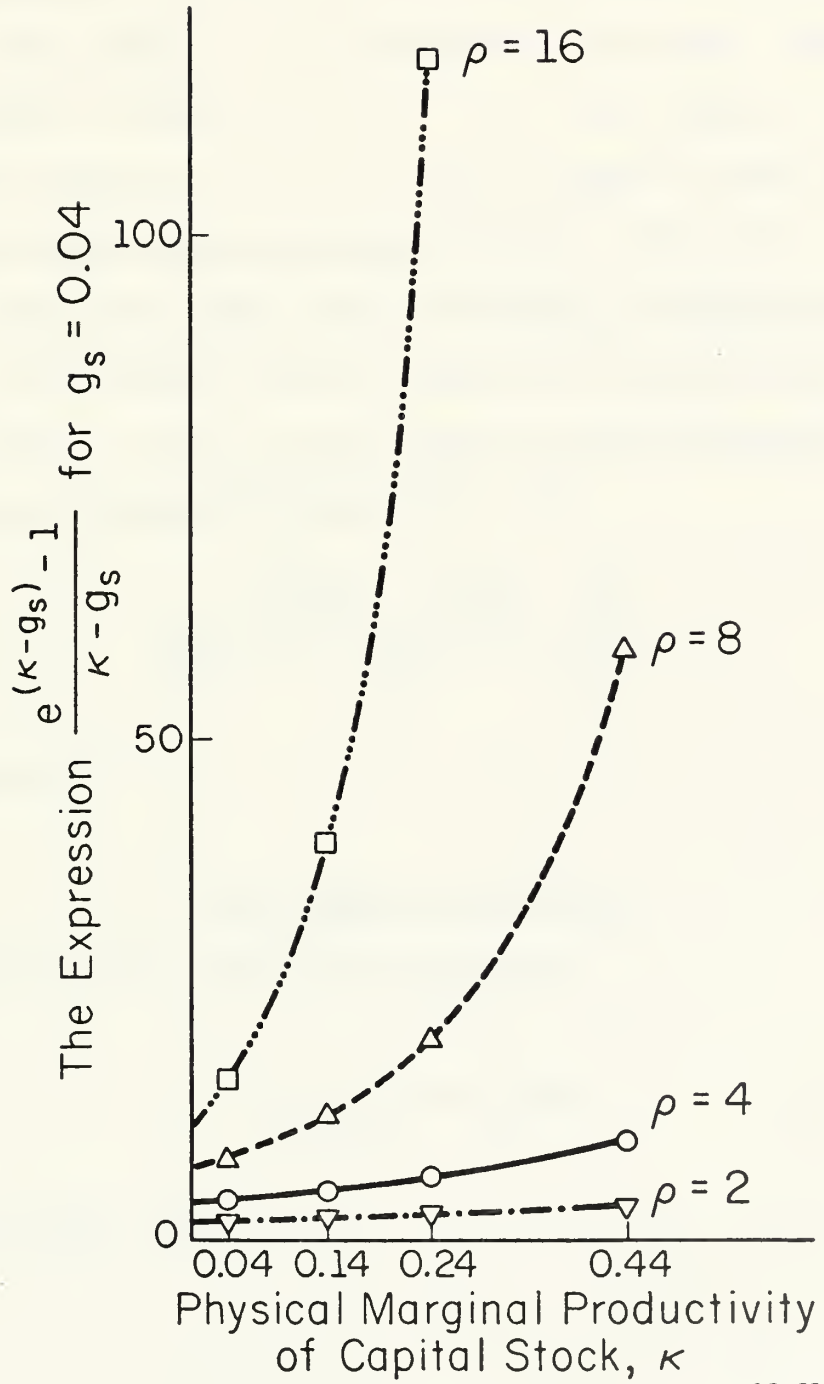
Assume $\rho > 0$. Then $G(0)$ is positive. And $G(f)$ is positive for $f < 0$, because then $e^{f\rho} < 1$, and also positive for $f > 0$, because then $e^{f\rho} > 1$. The limit of $G(f)$ for $f \rightarrow 0$ is found by L'Hôpital's Rule:

$$\lim_{f \rightarrow 0} G(f) = \rho$$

But if $G(f)$ has both the value and the limit ρ at $f = 0$, it is continuous at $f = 0$. The function is shown in Figure 5.

To see how $G(f)$ varies with f , differentiate with respect to f :

$$(68) \quad \frac{d[(e^{f\rho} - 1)/f]}{df} = \frac{f\rho e^{f\rho} - (e^{f\rho} - 1)}{f^2} = \frac{e^{-f\rho} - (1 - f\rho)}{e^{-f\rho} f^2}$$



QP-55

Figure 5

The denominator of (68) is nonnegative: It is positive for all values of f other than $f = 0$, for which it is zero. The numerator is also nonnegative. Write it $u \equiv e^{-x} - (1 - x)$, where $x \equiv f\rho$. Take the derivative $du/dx = -e^{-x} + 1$, set it equal to zero, and find $x = 0$. Take the second derivative $d^2u/dx^2 = e^{-x} > 0$. Consequently u satisfies the first-order and second-order conditions for a minimum at $x = 0$: u is positive for all values of f other than $f = 0$, for which it is zero. For $f = 0$ the limit of the derivative (68) can be found by using L'Hôpital's Rule twice:

$$\lim_{f \rightarrow 0} \frac{d[(e^{f\rho} - 1)/f]}{df} = \frac{\rho^2}{2}$$

which is positive.

2. The Brace of Eq. (60)

The brace of Eq. (60) may be written

$$(69) \quad (1 - c_d)/g_s + abc_d G(f)$$

Realistically assume that

$$0 < \alpha < 1$$

$$0 < b$$

$$0 < c_d < 1$$

$$0 < g_S$$

$$0 < \rho$$

Then the brace (69) is positive for all values of f . At $f = 0$ it has both the value and the limit $(1 - c_d)/g_S + abc_d\rho$, hence is continuous.

3. The Entire Eq. (60)

But if the brace is always positive, then κ times the brace is negative, zero, and positive for $\kappa < 0$, $\kappa = 0$, and $\kappa > 0$, respectively. Moreover, since the derivative (68) has a positive limit at $\kappa = g_S$ and is positive at all other values of κ , the brace is rising with rising κ , and κ times the brace is rising in more than proportion to κ . Consequently, if we draw the left-hand side of (60) as a function of κ , the function will be continuous, will be located in the third quadrant for $\kappa < 0$, will pass through the origin for $\kappa = 0$, and will be located in the first quadrant for $\kappa > 0$. It is rising without bounds as κ rises without bounds.

The right-hand side of (60) can be drawn as a horizontal line at the positive distance β from the κ -axis. Curve and line must intersect, will do so only once, and will do so in the first quadrant. This proves the existence of a unique and positive solution for κ .

F O O T N O T E S

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¹If the employer contribution is a fixed fraction of the wage bill, let us call it an investment wage; if it is a fixed fraction of the profits bill, let us call it profit sharing; and if it is a fixed fraction of equity, let us call it equity sharing.

An investment wage is on the statute books of the German Federal Republic (since 1961) and Italy, constituted a bill, Arbejdsministeriet [1], before the Danish Parliament in 1973, and was proposed by labor in the Netherlands in 1964 and by Professor Lundberg [11] in Sweden in 1973. Profit sharing is on the statute books of France (since 1967) and proposed by labor in the Netherlands also in 1964. Equity sharing was proposed by labor in Britain in 1973. The present article confines

itself to the investment wage.

²Convention is possibly ill-advised, Bhatia [3].

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