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Wage Theory and Growth Theory

*Hans Brems*

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Wage Theory and Growth Theory

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## WAGE THEORY AND GROWTH THEORY

HANS BREMS

### Abstract

According to the "natural" rate hypothesis in the short run, by accepting a "natural" rate of less than full employment, labor can have a real wage rate higher than under full employment. To see that hypothesis in a long-run perspective the paper solves a neoclassical growth model for capital stock, output, and factor prices and finds that in the long run, by accepting a "natural" rate of less than full employment, labor can have a real wage rate no higher than under full employment: levels of capital stock and output are correspondingly lower. Nobody benefits.

\*

Under profit maximization, pure competition, and a given capital stock, demand for labor is simply labor's marginal-productivity curve. As a result, in the short run, by accepting a "natural" rate of less than full employment, labor can have a real wage rate higher than under full employment.



But only in the short run may capital stock be considered given. The purpose of the paper is to examine how much of such a short-run wage-employment tradeoff will survive once capital stock has become a variable. We shall solve a neoclassical growth model for its capital stock, output, and factor prices and examine the sensitivities of such solutions to a "natural" rate of less than full employment. The model is this.

## I. THE MODEL

### 1. Variables

$C \equiv$  physical consumption

$g_v \equiv$  proportionate rate of growth of variable  $v$

$I \equiv$  physical investment

$\kappa \equiv$  physical marginal productivity of capital stock

$L \equiv$  labor employed

$P \equiv$  price of goods and services

$r \equiv$  nominal rate of interest

$\rho \equiv$  real rate of interest

$S \equiv$  physical capital stock

$w \equiv$  money wage rate

$X \equiv$  physical output

$Y \equiv$  money national income

## 2. Parameters

$a \equiv$  multiplicative factor of production function

$\alpha, \beta \equiv$  exponents of production function

$c \equiv$  propensity to consume

$F \equiv$  available labor force

$\lambda \equiv$  "natural" fraction of available labor force employed

$M \equiv$  supply of money

$V \equiv$  velocity of money

## 3. National Income

Money national income defined as the aggregate earnings arising from current production is identically equal to national product defined as the market value of physical output:

$$Y \equiv PX$$

(1)

#### 4. Production Function

We must be careful with our aggregation and begin at the firm level. Let the inputs of an individual firm be labor  $L$  and physical capital stock  $S$  and its physical output be  $X$ . Then let a Cobb-Douglas production function be common to all firms:

$$X = aL^{\alpha}S^{\beta} \quad (2)$$

where  $0 < \alpha < 1$ ,  $0 < \beta < 1$ ,  $\alpha + \beta = 1$ , and  $a$  is what growth measurement [Maddison (1987: 658)] calls "joint factor productivity."

#### 5. Demand for Labor

Demand for labor is a short-run commitment to be determined by maximization of profits. Here the firm may consider its physical capital stock  $S$  a constant and ignore the effect of investment  $I$  upon it. Maximizing its gross profits  $PX - wL$  with respect to employment  $L$ , the firm will then hire labor until the last man costs as much as he contributes, and under pure competition the real wage rate will then equal the physical marginal productivity of labor:

$$\frac{w}{P} = \frac{\partial X}{\partial L} = a\alpha L^{\alpha-1} S^{\beta} \quad (3)$$

Since  $\alpha + \beta = 1$ ,  $\alpha - 1 = -\beta$ , so raise to power  $-1/\beta$ , rearrange, and write firm demand for labor

$$L = (a\alpha)^{1/\beta} \left(\frac{w}{P}\right)^{-1/\beta} S \quad (4)$$

On the right-hand side of (4) everything except  $S$  is common to all firms. Factor out all such common factors and sum (4) over firms. Then  $S$  becomes aggregate physical capital stock and  $L$  aggregate demand for labor.

## 6. Supply of Labor

Current labor-market literature, e.g., Lindbeck and Snower (1986) and Blanchard and Summers (1988) distinguish between "insiders," who are employed hence decision-making, and "outsiders," who are unemployed hence disenfranchised. Facing our short-run demand (4) the decision-making insiders can, in the short run, have a higher real wage rate by

accepting less employment. Let them accept the fraction  $\lambda$  employed of available labor force, where  $0 < \lambda \leq 1$ . In other words, if  $L > \lambda F$  insiders will insist on a higher real wage rate. If

$$L = \lambda F \tag{5}$$

they will be happy with the existing real wage rate. If  $L < \lambda F$  they will settle for a lower real wage rate.

Consider the fraction  $\lambda$  a parameter, then (5) will be a solution for employment corresponding to Friedman's (1968: 8) "natural" rate  $1 - \lambda$  of unemployment. The fraction  $\lambda$  would reflect institutional dimensions of the labor market such as union density. Cross-country measurement of movements in employment and union densities is reproduced in Appendix I and found to be in good accordance with our interpretation of labor supply (5).

## 7. The Wage-Employment Tradeoff

The real wage rate insiders will be happy with, given their natural rate  $\lambda$  of employment, is found by inserting (5) into (3):

$$\frac{w}{P} = a\alpha(\lambda F)^{-\beta} S^{\beta} \quad (6)$$

What is the implied slope of the Phillips curve? As long as the ratio  $w/P$  satisfies (6) the levels of the money wage rate  $w$  and price  $P$  can be anything: the Phillips curve is vertical. Where labor cannot negotiate real but only money wage rates, short contract periods will have to do, and a temporarily finite slope of the Phillips curve is possible until successive rounds of collective bargaining have restored levels of the money wage rate  $w$  and price  $P$  satisfying our wage-employment tradeoff (6). Cross-country measurement of movements in employment and real wage rates is reproduced in Appendix II and found to be in good accordance with our wage-employment tradeoff (6).

## 8. Physical Output

Write the firm production function (2) as

$$X = a \left( \frac{L}{S} \right)^{\alpha} S \quad (7)$$



and the firm demand for labor (4) as the factor proportion

$$\frac{L}{S} = (a\alpha)^{1/\beta} \frac{w}{P}^{-1/\beta} \quad (4)$$

Insert (4) into (7), then on the right-hand side of (7) everything except  $S$  is common to all firms. Factor out all such common factors and sum (7) over firms. Then  $S$  becomes aggregate physical capital stock and  $X$  aggregate physical output. We already know that the factor proportion (4) holds for the firm as well as for the economy at large. Read it for the economy at large, multiply out in (7), and arrive at a production function of the form (2) now holding for the economy at large. Into such an aggregated (2) insert (5) and write physical output:

$$X = a(\lambda F)^{\alpha} S^{\beta} \quad (8)$$

## 9. Desired Capital Stock and Investment

Desired capital stock and investment are long-run commitments to be determined by maximization of present net worth. Here the firm can

no longer consider its physical capital stock  $S$  a constant or ignore the effect of investment  $I$  upon it.

We begin by defining the rate of growth of a variable  $v$  as the derivative of its logarithm with respect to time:

$$g_v \equiv \frac{d \log_e v}{dt} \tag{9}$$

To find the capital stock desired by the firm define physical marginal productivity of capital stock as

$$\kappa \equiv \frac{\partial X}{\partial S} = a \beta L^\alpha S^{\beta - 1} = \beta \frac{X}{S} \tag{10}$$

Firms were purely competitive; then price  $P$  of output is beyond their control. At time  $t$ , then, marginal value productivity of capital stock is  $\kappa(t)P(t)$ .

Let there be a market in which money may be placed or borrowed at the stationary nominal rate of interest  $r$ . Let that rate be applied when discounting future cash flows. As seen from the present time  $\tau$ ,

then, marginal value productivity of capital stock is  $\kappa(t)P(t)e^{-r(t - \tau)}$ . Define present gross worth of another physical unit of capital stock as the present worth of all future marginal value productivities over its entire useful life:

$$k(\tau) \equiv \int_{\tau}^{\infty} \kappa(t)P(t)e^{-r(t - \tau)} dt$$

Let firms expect physical marginal productivity of capital stock to be growing at the stationary rate  $g_{\kappa}$ :

$$\kappa(t) = \kappa(\tau)e^{g_{\kappa}(t - \tau)}$$

and price of output to be growing at the stationary rate  $g_p$ :

$$P(t) = P(\tau)e^{g_p(t - \tau)}$$

Insert these, define

$$\rho \equiv r - (g_{\kappa} + g_p) \tag{11}$$

and write the integral as

$$\kappa(\tau) = \int_{\tau}^{\infty} \kappa(\tau)P(\tau)e^{-\rho(t - \tau)} dt$$

Neither  $\kappa(\tau)$  nor  $P(\tau)$  is a function of  $t$  hence may be taken outside the integral sign. Our  $g_{\kappa}$ ,  $g_p$ , and  $r$  were all said to be stationary; hence the coefficient  $\rho$  of  $t$  is stationary, too. Assume  $\rho > 0$ . As a result find the integral to be

$$k = \kappa P / \rho$$

Find present net worth of another physical unit of capital stock as its gross worth minus its price:

$$n \equiv k - P = (\kappa / \rho - 1)P$$

Capital stock desired by the firm is the size of stock at which the present net worth of another physical unit of capital stock would be zero:

$$\kappa = \rho$$

.

Insert (10) and find capital stock desired by the firm

$$S = \beta X / \rho \tag{12}$$

Define investment desired by the firm

$$I \equiv g_S S = \beta g_S X / \rho \tag{13}$$

What is  $g_S$ ? Let it be correctly foreseen by firms that because  $\alpha + \beta = 1$  the economy at large will have the solution (20), to be found presently, and let that solution be common to all firms.

What is  $\rho$ ? In its definition (11) let it be correctly foreseen by firms that because  $\alpha + \beta = 1$  the economy at large will have the solution (23), to be found presently, and let that solution be common to all firms. Historically the marginal productivity  $\kappa$  of capital has indeed remained stationary. In that case (11) simply collapses into the real rate of interest, common to all firms.

On the right-hand sides of (12) and (13), then, everything except  $X$  is common to all firms. Factor out all such common factors and sum (12) and (13) over firms. Then  $X$  becomes aggregate physical output and (12) and (13) aggregate desired capital stock and investment, respectively.

10. Consumption; Equilibrium; Money

Let the aggregate consumption function be

$$C = cX \tag{14}$$

where  $0 < c < 1$ .

Aggregate equilibrium requires aggregate supply to equal aggregate demand:

$$X = C + I \tag{15}$$

To determine the rate of inflation we must, first, define the velocity of money as the number of times per year a stock of money transacts money national income:

$$Y \equiv MV \tag{16}$$

and, second, consider the money supply  $M$  and its velocity  $V$  to be parameters growing at the rates  $g_M$  and  $g_V$ , respectively.

## II. SOLUTIONS

### 1. Convergence

The key to our solutions for growth rates and levels is Solow's (1956) convergence proof. We apply it as follows. Differentiate aggregate physical output (8) with respect to time, consider our natural rate  $\lambda$  of employment a stationary parameter, and find

$$g_X = g_a + \alpha g_F + \beta g_S \quad (17)$$

Insert (14) and the definitional part of (13) into (15), rearrange, and write the rate of growth of physical capital stock as

$$g_S = (1 - c)X/S \quad (18)$$

Differentiate with respect to time, use (17) recalling that  $\alpha + \beta = 1$ , and express the proportionate rate of acceleration of physical capital stock as

$$g_{gS} = g_X - g_S = \alpha(g_a/\alpha + g_F - g_S) \quad (19)$$

In (19) there are three possibilities: if  $g_S > g_a/\alpha + g_F$ , then  $g_{gS} < 0$ . If

$$g_S = g_a/\alpha + g_F \quad (20)$$

then  $g_{gS} = 0$ . Finally, if  $g_S < g_a/\alpha + g_F$ , then  $g_{gS} > 0$ . Consequently, if greater than (20)  $g_S$  is falling; if equal to (20)  $g_S$  is stationary; and if less than (20)  $g_S$  is rising. Furthermore,  $g_S$  cannot alternate around (20), for differential equations trace continuous time paths, and as soon as a  $g_S$ -path touched (20) it would have to stay there. Finally,  $g_S$  cannot converge to anything else than (20), for if it did, by letting enough time elapse we could make the left-hand side of (19) smaller than any arbitrarily assignable positive constant  $\epsilon$ , however small, without the same being possible for the right-hand side. We conclude that  $g_S$  must either equal  $g_a/\alpha + g_F$  from the outset or, if it does not, converge to that value.

Once such convergence has been established we may easily find the corresponding values of other growth rates: insert (20) into (17), recall that  $\alpha + \beta = 1$ , and find the long-run growth rate of physical output



$$g_X = g_S \quad (21)$$

Differentiate (6) with respect to time, use (20), and find the long-run growth rate of the real wage rate

$$g_{w/P} = g_a/\alpha \quad (22)$$

Differentiate (10) with respect to time, use (21), and find the long-run growth rate of the physical marginal productivity of capital stock

$$g_K = 0 \quad (23)$$

As we recall from the definition (11),  $g_K$  was one part of the definition of the real rate of interest. To solve for the other part, insert (1) into (16), differentiate with respect to time, use (21), and find the long-run rate of inflation

$$g_P = g_M + g_V - g_S \quad (24)$$

where  $g_S$  stands for the solution (20).

We have found our natural rate  $\lambda$  to be absent from all our long-run growth rates (20) through (24). But might it be present in the long-run levels at which our variables are growing? We shall see.

## 2. Real Rate of Interest

To solve for the long-run level of the real rate of interest insert (13) and (14) into (15), divide any nonzero  $X$  away, and find

$$\rho = \frac{\beta g_S}{1 - c} \tag{25}$$

where  $g_S$  stands for our solution (20). Our solution (25) has no  $\lambda$  in it: the long-run real rate of interest is invariant with the natural rate  $\lambda$  of employment. Differentiating our solution (25) with respect to time, we find it to be stationary--as we assumed in Sec. I, 9 above.

### 3. Physical Capital Stock

To solve for the long-run level of physical capital stock insert (8) and (25) into (12) and find

$$S = \left( a \frac{1 - c^{1/\alpha}}{g_S} \right) \lambda F \quad (26)$$

where  $g_S$  stands for our solution (20). Our solution (26) does have  $\lambda$  in it: the long-run physical capital stock is in direct proportion to the natural rate  $\lambda$  of employment. Differentiating our solution (26) with respect to time, we find it growing at the rate (20), invariant with  $\lambda$ --as it should.

### 4. The Real Wage Rate

To solve for the long-run level of the real wage rate insert (26) into the short-run level (6) and find

$$\frac{w}{P} = \alpha a^{1/\alpha} \left( \frac{1 - c}{g_S} \right)^{\beta/\alpha} \quad (27)$$

where  $g_S$  stands for our solution (20). Our solution (27) has no  $\lambda$  in it: the long-run real wage rate is invariant with the natural rate  $\lambda$  of employment. Differentiating our solution (27) with respect to time, we find it growing at the rate (22), invariant with  $\lambda$ --as it should.

### 5. Physical Output

To solve for the long-run level of physical output insert (26) into the short-run level (8) and find

$$X = a^{1/\alpha} \left( \frac{1 - c}{g_S} \right)^{\beta/\alpha} \lambda F \quad (28)$$

where  $g_S$  stands for our solution (20). Our solution (28) does have  $\lambda$  in it: the long-run physical output is in direct proportion to the natural rate  $\lambda$  of employment. Differentiating our solution (28) with respect to time, we find it growing at the rate (21), invariant with  $\lambda$ --as it should.

### III. CONCLUSION

We have found a stark contrast between the short-run and the long-run scope for wage policy. The simple mathematics of the contrast is this.

In our real wage rate (6) neither  $a$ ,  $\alpha$ ,  $\beta$ , nor  $F$  is a function of the natural rate  $\lambda$ . Physical capital stock  $S$  may or may not be. In general differentiate the natural logarithm of (6) with respect to  $\lambda$  and find the elasticity of the real wage rate with respect to the natural rate  $\lambda$  to be

$$\frac{\partial \log_e (w/P)}{\partial \log_e \lambda} = -\beta + \beta \frac{\partial \log_e S}{\partial \log_e \lambda} \quad (29)$$

In the short run physical capital stock  $S$  can be considered a constant depending on nothing:

$$\frac{\partial \log_e S}{\partial \log_e \lambda} = 0 \quad (30)$$

Insert (30) into (29) and find the latter collapsing into  $-\beta$ : labor can have a  $\beta$  percent higher real wage rate by accepting a one percent lower natural rate  $\lambda$  of employment.

By contrast, in the long run physical capital stock  $S$  cannot be considered a constant but is a variable to be solved for. When we solved for it we found (26) whose elasticity with respect to  $\lambda$  was

$$\frac{\partial \log_e S}{\partial \log_e \lambda} = 1 \quad (31)$$

Insert (31) into (29) and now find the latter collapsing into  $-\beta + \beta = 0$ : labor can have a no higher real wage rate by accepting a lower natural rate  $\lambda$  of employment.

In plain English the reason for the stark contrast is that in the long run the levels (26) and (28) of capital stock and output simply adjust to  $\lambda$  and are correspondingly lower: the economy is impoverished, accumulates less capital stock and produces less output. Labor does not benefit. Nobody benefits.

APPENDIX I. U.S.-EUROPEAN DIFFERENCES IN EMPLOYMENT AND UNION DENSITY

Friedman (1968: 9) never meant his natural rate to be "immutable and unchangeable." Indeed, over the thirteen years 1973-1986 Freeman (1988b: 294-295) found actual employment as a fraction of working-age population declining steadily in OECD-Europe as a whole but largely rising in the United States.

Among the institutional dimensions reflected by the natural rate Friedman (1968: 9) mentioned union density. One would expect the employment fraction and union density to be moving in opposite directions. Roughly speaking, so they did: over the fifteen years 1970-1985 Freeman (1988a: 69) found union density rising sharply in Denmark, Finland, and Sweden; rising moderately in Australia, Canada, France, Germany, Ireland, Italy, New Zealand, and Switzerland; rising slightly in Norway and the United Kingdom; declining slightly in Austria and the Netherlands; declining moderately in Japan and sharply in the United States. In Sweden, however, the employment fraction and union density moved in the same direction. Allowing for Sweden, Barro (1988: 36) found the persistence of low employment to go with high union density and large size of government but only in countries lacking centralized bargaining.

APPENDIX II. U.S.-EUROPEAN DIFFERENCES IN WAGE-EMPLOYMENT TRADEOFF (6)

Differentiating our wage-employment tradeoff (6) with respect to time would suggest increases in employment and the real wage rate to be of opposite orders of magnitude, and so they were: over the twenty-five years 1960-1985 Freeman (1988b: 296-297) indeed found countries in most of OECD-Europe to have larger increases in their real wage rates and smaller increases in their employment than had the United States and Sweden. The pairing of the United States and Sweden was also noticed by Ergas and Shafer (1987-1988).

Economists from Keynes (1936: 14) to Summers (1988) have insisted that relative real wages do matter. Under decentralized bargaining, wage restraint by an individual union may lower its relative real wages. Centralized bargaining removes such fears. Sweden with her centralized bargaining and very high union density did show more wage restraint than countries with decentralized bargaining--as Freeman and Ergas-Shafer found.



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